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RISKY ASSET PRICE EXPECTATION FORMATION
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RISKY ASSET PRICE EXPECTATIONS FORMATION AND EMERGENT MARKET BEHAVIORS

ABSTRACT

We model how individuals with diverse beliefs form their price expectations in the light of events in a market that is perpetually novel and constantly evolving. Our model is unique in that it take into consideration the fact that people, when making decisions in an ill-defined and complex environment, will exploit their innate ability to think in fuzzy notions as well as reason inductively in these fuzzy notions. The apparatus we employ to model learning and expectations formation is the genetic-fuzzy classifier system. The formulation of this apparatus, and the study of the complications that can arise when investors hold heterogeneous expectations that may change over time as well as their implications for security prices constitute the primary focus of this dissertation.

We find several interesting and intriguing results. First, results from our computer simulations reveal that market behaviors which are otherwise treated as anomalies in standard asset pricing models emerge naturally in our model. Second, our results provide support for the two diametric views held by academician and traders—that is, while academic theorists in general view the market as rational and efficient, market traders typically see the market as psychological, organic, and imperfectly efficient. Lastly, our simulations produce time series behaviors of prices and trading volume that bear strong resemblance to corresponding time series behaviors observed in real financial markets. In summary, we find 1) positive autocorrelation in trading volume, 2) positive contemporaneous correlation between trading volume and volatility, and 3) slightly excess kurtosis, ARCH-liked features, and low autocorrelation in the returns time series.

1. INTRODUCTION

What is finance? Campbell, Lo and MacKinlay (1997) eloquently summarized the essence of finance as a discipline in the following few sentences.

> *The starting point for every financial model is the uncertainty facing investors, and the substance of every financial model involves the impact of uncertainty on the behavior of investors and, ultimately, on market prices. Indeed, in the absence of uncertainty, the problems of financial economics reduce to exercises in basic microeconomics. The very existence of financial economics as a discipline is predicated on uncertainty.* (Campbell, Lo and MacKinlay 1997, p.3)

Indeed, this is the very spirit behind this dissertation. In this dissertation, we focus on the implications upon asset prices and market behaviors of an element of uncertainty that has emerged because investors are unable to form objective and precise price expectations in real financial markets[1] (see Section 1.2 in this chapter). We capture this element of uncertainty with an alternative model of the process that determines how price expectations are formed. What makes our model unique is that it takes into consideration the recognized fact that investors will take advantage of their innate ability to reason inductively and analyze in fuzzy terms when they have to make decisions in a complex and ill-defined environment.

Our approach is inspired by research findings about human learning behavior from psychology and is based on techniques that have emerged from recent advances in machine learning and Artificial Intelligence research. Like most recent

[1] It is important to point out that this breakdown in our ability to form objective price expectations is not due to limitations in our computational ability; it happens because the problem is inherently ill-defined making it impossible for us to pin down a price expectations from a sea of many plausible price expectations that everyone can agree upon objectively. We will say more about this in Section 1.2.

contributions in this area, we are encouraged by the promise of fruitful results offered by these new methods of analysis[2]. The apparatus we employ to model expectations formation is a hybrid system called the genetic-fuzzy classifier[3]. We will argue later in this chapter that our approach will not only address the criticisms of existing learning models, but will also provide a more accurate picture of how investors actually form their expectations in real life.

It is important to accurately capture the way in which investors form their price expectations in real life because, like Shiller (1984, 1989) and Keynes (1936), we suspect that the so called anomalies and empirical puzzles[4] in real financial markets are somehow related to the manner in which price expectations are formed in real markets (see our discussion in Section 1.2). In addition, since asset prices are ultimately driven by price expectations, a model that accurately captures how expectations are shaped will help to shed light on how prices are formed in real markets.

Despite the gravity of these issues, theoretical finance and economic literature have largely ignored the need to model in a realistic manner the way that investors form their price expectations in real life[5]. Instead, theorists have mainly focused on

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[2] Varian (1993, p.1) argued that the recent upsurge in interest in this and other traditionally difficult research questions is most likely due to the availability of better methods of analysis that promise researchers more fruitful ways of addressing these questions.

[3] See Cox (1994).

[4] Some examples are market crashes, mean reversion, relatively high level of trading, presence of technical trading, excess volatility, volatility clustering etc. These behaviors are curious from the stand point of standard neoclassical models with strong underlying rationality assumptions.

[5] Varian (1993) has argued that this is because theorists did not have the right tools to address the problem and it is not because they are not interested in the problem.

equilibrium situations in which they can design clever ways to circumvent the difficult question of how price expectations are formed. To this end, a common approach in many neoclassical models is to invoke the rational expectations argument[6]. But these models typically impose strong rationality assumptions about the abilities and behaviors of economic agents which some of us have found to be rather unappealing. For instance, Herbert Simon[7], one of the most well known and vocal dissidents, has argued that:

> *A comparative examination of the models of adaptive behaviour employed in psychology (e.g. learning theories), and of the models of rational behaviour employed in economics, shows that in almost all respects the latter postulate a much greater complexity in the choice mechanisms, and a much larger capacity in the organism for obtaining information and performing computations, than do the former. Moreover, in the limited range of situations where the predictions of the two theories have been compared (See Thrall et al. 1954, Chapters 9, 10, 18), the learning theories appear to account for the observed behaviour rather better than do the theories of rational behaviour.*
>
> *Both from these scanty data and from an examination of the postulates of the economic models it appears probable that, however adaptive the behaviour of organisms in learning and choice situations, this adaptiveness falls short of the ideal of 'maximizing' postulated in economic theory. Evidently, organisms adapt well enough to 'satisfice'; they do not, in general, 'optimize'.*
> (Simon 1992, p.39)

Despite these criticisms, many theorists have maintained that the underlying

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[6] This entails making strong rationality assumptions so that every agents in the model will arrive at the same price expectations. Another common approach in neoclassical models is to assume a tatonnement process, see Arrow and Hahn (1971).

[7] Furthermore, according to Simon:

Neoclassical economic theory assumes that the problem agenda, the way in which problems are represented, the values to be achieved (utility function), and the alternatives available for choice have all been given in advance. It has no systematic way of explaining how problems get on the agenda ..., what is it that people value and how values change, or how action alternatives are created .... Hence it is incapable of creating a genuine economic dynamics. (Simon et al. 1992, p.5).

For more information, see the bounded rationality works of Herbert Simon—Simon 1947, 1955, 1957, 1959, 1976, 1982, 1986, Simon et al. 1992.

rationality assumptions are sound because evolutionary forces in the markets will eventually select for rational behaviors[8]. Most of us are familiar with this argument and have in general accepted its validity. But Blume and Easley (1992) recently challenged the validity of this conventional wisdom by demonstrating clearly in several different cases that this conventional intuition in general is not valid. Yet the above hand waving by theorists is not a serious issue if the concern is equilibrium analysis. However, if the intent is to explain real market phenomena, neoclassical rationality models will undoubtedly be inappropriate because real markets can hardly be considered as equilibrium systems[9].

To model expectations formation in the presence of disequilibrium, we make use of a genetic-fuzzy classifier system.[10] Even though our classifier system is similar in many aspects to that used in the Santa Fe Institute Artificial Stock Market model (the SFI model), there is a crucial difference. We have allowed the economic agents in our model to use fuzzy rules of thumb instead of the more restrictive conventional rules used in the SFI model. To illustrate the difference between conventional rules and fuzzy rules, consider the following conventional rule as compared to the fuzzy rule.

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[8] See for instance the arguments of Alchian (1950), Friedman (1953), or a recent discussion of Alchian's contribution to economics by Demsetz (1996).

[9] Kenneth Arrow, one of the pioneer architects of the neoclassical framework, recently raiseds the concern that "the attainment of equilibrium (inevitably) requires a disequilibrium process", and in the presence of disequilibrium it will not be meaningful to talk about rational behavior in the sense defined in neoclassical theory (Arrow 1986, p.S387).

[10] A Classifier system is a machine learning system that is capable of learning syntactically simple rules so that it may operate in an arbitrary environment. What we have done differently is to replace the conventional rules typically found in these systems with fuzzy rules.

Conventional Rule: *If market index is greater than (0.9), then a is (0.2) and b is (0.8)*

Fuzzy Rule: *If market index is (high), then a is (low) and b is (high)*

In these rules, a and b are the parameters to be used in a linear forecast equation for forecasting next period price. The market index, both a and b, and the fuzzy sets (low and high) are assumed to have been scaled to operate over the interval [0,1] (also known as the universe of discourse). In our application, a fuzzy set is a mathematical mapping that transforms the magnitude of a variable into a qualitative expression that describes how large or small the variable is. This can be easily accomplished by, for instance, drawing a curve over the universe of discourse, [0,1], and letting the vertical height spanned by this curve be constrained to the membership values of [0%,100%]. The membership value indicates to what degree a variable is a member of a fuzzy set; a membership value of 100% will indicate a 100% member whereas a membership value of 0% will represent a non-member.

It is obvious that the difference between these two rules lies in the way the second rule uses fuzzy sets instead of crisp numbers to define the states. An apparent problem with a conventional rule is it implies a sudden and definite change in decision at the cut-off point. For instance, had the market index been a tiny fraction less than 0.9, say market index is 0.899, this condition in the rule will not be fulfilled and the rule will not be activated. Our introspection should convince us that we do not make such extreme decision changes over a tiny change in the relevant decision variable. In contrast, a fuzzy rule allows for a more gradual change in decision. A decline in the market index value to 0.899 will still satisfy the condition in the fuzzy rule although it now satisfies the condition to a lesser degree. This will in turn result in a proportionate change in the values for the forecast parameters, *a* and *b*. There are other advantages to using fuzzy rules and these will be discussed in Section 1.3.

The way our genetic-fuzzy classifier system operates is straight forward. Expectations formation is effected by a set of fuzzy rules of thumb contained in the system and learning is simulated by systematically evolving the rules with the aid of a genetic algorithm (GA). Each rule contains a set of conditions and a set of forecast parameters presented in the format "If *conditions*, then *forecast parameters*". Whenever the conditions in a fuzzy rule are matched to the prevailing state in the market, the forecast parameters in that same rule are used in a linear forecast equation to forecast next period price[11]. This forecast then forms the new price expectations at that point in time.

One important contribution of this work is the demonstration that behaviors which would have been considered anomalies (for instance, market crashes, mean reversion, relatively high level of trading, presence of technical trading, excessive volatility, etc.) within the standard asset pricing framework will emerge naturally in a framework where individuals are allowed to form their expectations using the approach we have proposed. Another equally intriguing result is that this model lends support to the two diametric views held by academic theorists and market traders[12]. Academic theorists in general see the market as rational and efficient, but market

[11] The forecasting equation we will use is: $E_t(p_{t+1}+d_{t+1})=a(p_t+d_t)+b$.

[12] This is the same result obtained by Arthur et al. 1996, 1997, and Palmer et al. 1994 in their Santa Fe Institute Artificial Stock Market Model.

traders typically view the market as psychological, organic, and imperfectly efficient. Both views are certainly consistent with the behaviors we have seen in real financial markets. If we were to examine historical financial market data, we will see that financial markets, while most of the time, seem to be quite efficient, on occasions, can also exhibit moods and personality[13]. In addition, the statistical behaviors of the time series of prices and trading volume, arising from our simulations, are also comparable to those from real financial markets. Returns are found to have low autocorrelations and slightly excess kurtosis (although the magnitude for kurtosis is still smaller than in actual returns). The returns series also appear to exhibit the signature "ARCH" behaviors commonly seen in time series of actual stock returns. Volatility in returns are found to be contemporaneously correlated with trading volume and trading volume is autocorrelated.

To sum up, the formulation of our model, and the study of the complications that can arise when investors hold heterogeneous expectations that may change over time as well as their implications for security prices, will constitute the primary focus of this research.

The rest of this chapter is devoted to a critique of some existing learning models and a discussion of the roots of market anomalies and the motivations for our

[13] This is especially evident during market crashes. One good example is the recent market crash on October 27, 1997. On that day, the Dow Jones Industrial Average suffered its biggest point drop in history. This occurred despite the fact that fundamentals of the United States economy are strong (and have been for the past several years) and the prospects for continued growth, with low inflation and low unemployment, are equally great. On the following day, October 28, 1997, the U.S. stock markets soared back with the Dow Jones Industrial Average posting its biggest point gain in history amid record volume.

genetic-fuzzy approach to modeling expectations formation. In Chapter 2, we present an informal tutorial on fuzzy logic and genetic algorithm. The intent of Chapter 2 is not to provide a rigorous theoretical underpinnings for these methods but to present a practical and yet intuitive approach to these methods. Chapter 3 describes the modeling of expectations formation in more details and sets up the framework of our artificial stock market model. The basic framework of the market is similar to a typical neoclassical two-asset market. The various controlled experiments that are conducted are also discussed in this chapter. Chapter 4 presents and discusses the results. Chapter 5 concludes with a summary of the main results and offers suggestions for future research.

Before we proceed any further, we would like to state that many of the ideas and views presented here are not original. They represent the collective wisdom of our predecessors in various disciplines. Nonetheless, this dissertation is unique in that it brings to bear the knowledge scattered amongst various disciplines to provide a unified approach to model economic decision making under uncertainty.

1.1 On the Modeling of Learning Behavior

The literature on learning has distinguished existing approaches as either based on *rational learning* or *ad hoc learning.*[14] In the *rational learning* literature, we have models that derive learning behavior from Savage's axioms about preferences (Savage 1954). Learning in these models occurs as individuals repeatedly update their priors

[14] Sargent (1993) and Kirman and Salmon (1995) provide good reviews of recent learning models in economics.

using Bayes's rule in the light of new information as they seek to maximize their expected utility under uncertainty. Since Bayesian learning is a consequence of assumptions about preferences, this approach has been referred to as *rational learning*[15]. In contrast, in the *ad hoc learning* literature, we have models that typically employ non-bayesian learning mechanisms borrowed from the literature on statistics, econometrics, and machine learning and Artificial Intelligence. Because there appears to be no unifying principle that underlies the construction of these models, unlike in the former case, this approach has consequently been labeled as *ad hoc learning*[16].

Although both streams of literature, in general, acknowledge that agents are boundedly rational and face informational constraints, the learning mechanisms considered in the literature thus far are not entirely satisfactory. For instance, there are substantial evidence that Bayes theorem lacks empirical relevance and yet Bayesian learning remains a very popular approach[17]. Salmon (1995, p.236) suggested that, perhaps, this is because theorists still seem to be more concerned with how people

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[15] Examples of rational learning models include the earlier works of Prescott (1972), Cyert and DeGroot (1974) (section titled "Consistent Model"), Rothschild (1974) and Townsend (1978), and more recent works of Frydman (1981), Townsend (1983a, 1983b), McLennan (1987), Mirman et al., (1984), Bray and Kreps (1986), Easley and Kiefer (1989), Kiefer and Nyarko (1988, 1989), and Blume and Easley (1993) among many others.

[16] Examples of ad hoc learning models include Radner (1972), Cyert and DeGroot (1974) (section titled "Inconsistent Model"), DeCanio (1979), Evans (1985), Bray and Savin (1986), Lucas (1986), Marcet and Sargent (1988, 1989a, 1989b, 1989c), Nyarko (1990, 1991a, 1991b), Woodford (1990), Linn and Stanhouse (1997), etc.

[17] Kahneman et al. (1982) show that likelihoods and preferences expressed by individual experimental subjects do not satisfy the coherence properties that are necessary for the existence of subjective probabilities. Moreover they are found to update probabilities but they do not appear to use information efficiently in that posterior probabilities differ quantitatively and systematically from those predicted by the use of Bayes's rule (Nelson and Winter 1982, Edwards 1968, Tversky and Kahneman 1974, Cyert and DeGroot 1987). Grether (1992) also describes experiments in which it seems clear that individuals systematically fail to take "proper" account of prior probabilities.

should behave rather than how they *actually* do behave. Salmon also pointed out that:

> *It seems strange from the behavioral point of view, for instance, to assume, as is the case with the standard statistical models of "rational" learning, that agents have complete knowledge of the relevant economic structure and yet are assumed to be completely ignorant of perhaps just a subset of the parameter values within that structure.*[18] *The economic interactions that have taken place in the past to have left an individual in such an odd state are unspecified.* (Salmon 1995, p.237)

In a similar vein, Bullard lamented that:

> *Many theorists choose to suppose that agents ignore the interaction of beliefs and outcomes—they ignore behavioral uncertainty—leading to learning schemes that are inherently misspecified (Bray 1982). The misspecification causes these decision rules to be biased, although they are often shown to converge to MMIE eventually. In fact, this kind of interpretation implies that agents in many models, since they all use the same method of forming expectations, collectively adhere to biased forecast functions—a requirement that is especially dubious considering that no agent is allowed to respecify the forecast function if the bias is detected.* (Bullard 1990,p.333)[19]

To resolve these inconsistencies, Salmon suggested that,

> *A more reasonable position might be that agents' knowledge of the structure and their learning activity evolve symbiotically and the manner by which learning takes place adapts to their increased understanding of their economic environment which in turn may grow, according to economic incentives, through deliberately increased interaction with that environment. Some flexibility within the method of learning is then needed as the agent's approximation to reality improves.* (Salmon 1995, p.237-238)

There are at present two methodologies in the learning literature that can potentially address these problems and they are the artificial neural networks (see Cho 1992 and

[18] Often for instance the "regression parameters" are assumed to be unknown but the residual variance is assumed to be known.

[19] MMIE refers to Mills-Muth implicit expectations. Bullard gives it a special name to distinguish it from the concept of rational expectations. The intention is to emphasize that there is a process that drives the formation of MMIE. Mathematically, it is described by: $X^*_{t+s}=E_t(X_{t+s})$, where X^* is the forecast of X, and E_t is the mathematical expectations operator based on information available at time t.

Salmon 1995) and the classifier system (see for instance Arifovic 1991, 1994, 1995, 1996, Arifovic and Eaton 1995, Arthur 1995, and Arthur et al. 1996, 1997). In comparison to the other methods (Bayesian learning, least square learning etc.), these two approaches generally assume much less mathematical sophistication (in the conventional sense) on the part of the agents. Rather than relying on conventional mathematics or statistics, these mathematically less sophisticated agents solve complex problems using intuitive methods. But this should not be taken to imply that these agents are inferior to the mathematically more sophisticated agents in solving the complex decision problems they are faced with. In fact, we will argue in the next two sections that these intuitive methods are actually more suited to dealing with the sort of complex and ill-defined problems that the agents encountered.

Both these approaches in general allow agents to learn about the structure of their forecast functions as well as the relevant parameters of these functions. However, in an artificial neural networks, all that agents can see are the inputs and outputs to the neural networks. Learning takes place in a black box so that agents may still be unaware of what their forecast functions look like or what the relevant parameters are even if they have already learned the correct structure for their forecast functions.

In contrast, learning is transparent to the agents in a classifier system. A classifier system allows agents to hold a multitude of different forecast functions which they repeatedly test and revise as they learn from their interactions with the environment. Hence, agents know precisely what their forecast functions look like, what the parameters are, and which of the forecast functions works best at any point in time.

Another interesting feature intrinsic in a genetic classifier system is its forward looking characteristic. A genetic classifier system constantly creates and holds on to new forecast functions which may not be useful at the present time but might become useful at some point in time in the future. This feature is not presence in an artificial neural networks which functions more like a curve fitting machine. In addition, Arthur et al. (1996) have argued that the characteristics of a genetic classifier system closely resemble the induction process that people use to make decisions when they are confronted with an ill-defined environment (see the next section).

Therefore, a classifier system is a more appealing approach than the artificial neural networks, and it is for these reasons that we have decided to use a variation of a classifier system to model expectations formation in our model.

1.2 Expectations Formation and Market Created Uncertainty

As yet, no intuition has been provided to explain why the approach we have proposed for modeling learning and expectations formation might explain the anomalies and transition dynamics we see in real markets. In order to understand why our model will work, we need to get to the roots of these market anomalies[20]. It is instructive at this point to take a step back and ask what it will take for a model to account for the market anomalies we have seen. Since models are necessarily

[20] We should emphasize that the so called market anomalies are not really anomalies per se. They are called "anomalies" only because they cannot be explained by standard asset pricing models.

abstractions of the real world, their successes will ultimately hinge on incorporating those elements that are essential to explaining what they are intended to explain. Although no one knows for certain what the essential ingredients for explaining market anomalies are , empirical evidence (see Shiller 1989) seem to suggest that a certain type of *market created uncertainty* (see Peck and Shell 1991), intentionally sidestepped in standard asset pricing models (probably because of its analytical intractability), may potentially explain the seemingly anomalous behaviors in real financial markets.

Specifically, we are referring to the uncertainty created as a result of the interactions (either directly and/or indirectly) among heterogeneous market participants who have to learn to form their expectations in a market environment that is inherently ill-defined. The difficulty here is that under such circumstances, people will not be able to deduce their expectations logically[21] (see Arthur 1994, 1995). Consequently, each market participant will have to form his price expectations based on his subjective forecast of the expectations of the rest of the market participants. When this is the case, and when no one is absolutely certain of what true fundamental values are, the market can develop a life of its own and respond in ways that are not correlated with movements in fundamental values[22]. According to Arthur,

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[21] Arthur finds that in markets where individuals form their expectations heterogeneously, deductive reasoning will not provide any closure and will result in indeterminacy (see also discussions by Arrow 1986, Blume and Easley 1995).

[22] Gennotte and Leland (1990), and Jacklin, Kleidon and Pfleiderer (1992) have demonstrated that uncertainty among market participants about the proportions of investors who follow various investment strategies is sufficient to produce market crashes, even if investors rationally update their beliefs over time.

... the sense he makes of the **Rorschach** [bold added to replace italics] *pattern of market information* I_t *is influenced by the sense he believes others may make of the same pattern. If he believes that others believe the price will increase, he will revise his expectations to anticipate upward-moving prices (in practice helping validate such beliefs). If he believes others believe a reversion to lower values is likely, he will revise his expectations downward. All we need to have self reinforcing suspicions, hopes, and apprehensions rippling through the subjective formation of expectations (as they do in real markets) is to allow that* I_t *contains hints—and imagined hints—of others' intentions.* (Arthur 1995, p.23)

Hence, the process of expectations formation under such circumstances can be precarious. This view of the market is akin to that of Keynes's (1936, p.150). Keynes has regarded asset prices as "the outcome of the mass psychology of a large number of ignorant individuals," with professional speculators mostly trying to outguess the future moods of irrational traders, and thereby reinforcing asset price bubbles. In a similar vein, Dreman (1977, p.99) maintained that individual investors, including professionals do not form opinions on independently obtained information[23]. Their forecasts of future events are heavily influenced by "the thinking of the group." Similar views have also been advanced by Black (1986), De Long et al. (1989, 1990), Shiller (1984, 1989), and most recently, Soros (1994).

But what makes the problem worse is that such behavioral uncertainties diminish the incentives for arbitrage which in turn impair the market natural tendency to return itself to its fundamentals. In particular, these uncertainties create two types of risk for potential arbitrageurs. Shleifer and Summers (1990) identified these risks as identification risk and noise trader risk (future resale price risk).

Identification risk arises because uncertainty in the market makes it difficult for

[23] See also Dreman (1982).

potential arbitrageurs to distinguish between price movements driven by noise trader actions and price movements driven by pieces of private information which they have not yet received. Hence it is difficult for potential arbitrageurs to exploit noise traders because they can never completely assure themselves that the price movement was driven by noise, which create profit opportunities, and not by news that the market knows but they have not yet heard.

In addition, there is also the risk that price may move further from fundamental value by the end of the speculator's investment horizon. This latter risk is known as the noise trader risk or future resale price risk. On that account, an investor who knows, even with certainty, that an asset is overvalued will still take only a limited short position because noise traders may push prices even further from their fundamental values when it comes time to close the arbitrage position.

Another type of risk, fundamental risk, although not due to the market uncertainty we have discussed, can also limit arbitrage. Fundamental risk is inherent in the market. It is the possibility that the fundamental value of the stock may change against the arbitrage position before the position is closed. Even if noise traders do not move prices away from fundamental values, changes in the fundamentals themselves might move the price against the investor.

Altogether, these problems make arbitrage risky and limit arbitrage. Because arbitrage plays an "error-correction" role in the market to bring asset prices in line with their fundamental values, this role will be hampered when arbitrage is limited. As a result, asset prices may deviate from fundamental values and such deviations may persist hence weakening whatever correlation there may have been between movements in asset prices and movements in their fundamental values.

The implication of this discussion is that when we impose the assumption of "mutual consistency in perceptions" in rational expectations models, we leave miss out the *market created uncertainty* outlined above which is precisely what is needed to explain the seemingly anomalous behaviors in real markets. Therefore, to account for market anomalies, we must allow the agents in our model the opportunity to form their expectations independently based on their subjective evaluations. But how do we model such expectations formation? Before we can answer this question, we need to first investigate how humans reason in situations that are ill-defined and uncertain.

We have earlier alluded to the fact that deductive reasoning will break down in an environment that is ill-defined. But if deductive reasoning will not work, how then can individuals form their expectations? Arthur et al. (1996) argues that individuals will form their expectations by induction (see also Arthur 1991, 1992, Blume and Easley 1995, Rescher 1980). So what is induction or inductive reasoning? Nicholas Rescher defined it as follows:

> *Induction is **an ampliative method of reasoning**—it affords means for going beyond the evidence in hand in endeavor to answer our questions about how things stand in the world. Induction affords the methodology we use in the search for optimal answers.*
>
> *Induction as a cognitive method proceeds by way of **the systematization of question-resolving conjecture with experience**, by fitting conjectural extensions sufficiently tightly into the overall setting of our other (generally tentative) commitments. Though induction always involves a leap beyond the information in hand, it only endorses these leaps when the fit is sufficiently close.* (Rescher 1980, p.87)

Simply put, induction is a means for finding the *best available* answers to our questions that transcend the information at hand. In other words, the conclusions we draw in an induction are suggested by the data at hand rather than logically deduced from them. Logical deduction fails because the information we have at hand leave some gaps in our reasoning. In order to complete our reasoning, we fill those gaps in the least risky, minimally problematic way, as determined by plausibilistic best-fit considerations. Although this may sound like guesswork, it is really more than guess work; it is responsible estimation in the sense that we are willing to commit ourselves to the tenability of the answer which we put forth. In other words, we must find the answers to be both sensible and defensible.

Inductive reasoning follows a two-step process: possibility-elaboration and possibility-reduction. The first step involves creating a spectrum of plausible alternatives based on our experience and the information available. In the second step, these alternatives are tested to see how well they answer "the question" or how well they connect the existing incomplete premises to explain the data observed. The *best fit connection* is then accepted as a viable explanation for the data observed. Subsequently when new information become available or when the underlying premises change, the *fit* of the current *connection* may not be good anymore. When this happens a new alternative will take over.

So how can induction be implemented in economic models? Arthur et al. (1996) visualize induction taking place as follows. Under this scheme of rationalizing, each individual in the market continually creates a multitude of "market hypotheses" (this corresponds to the possibility-elaboration step discussed above). These hypotheses which represent the individuals' subjective expectational models of what moves the market price and dividend are then simultaneously tested for their predictive ability in the market. In the end, those that perform well in predicting market movements will be retained and acted upon in buying and selling decisions, and the others that perform badly will be dropped (this corresponds to the possibility-reduction step). In addition, as new information enter or emerge from the market, other new hypotheses will be generated and be tested as above. This process is carried out repeatedly as individuals learn and adapt in a constantly evolving market[24].

The expectations formation process we have just described can be adequately modeled by letting each individual forms his expectations using his personal genetic-fuzzy classifier system. Each genetic-fuzzy classifier system contains a set of *conditional forecast* rules that guide the decision making. We can think of these rules as the subjective "market hypotheses" held by each individual. Inside the classifier system is a genetic algorithm that is responsible for generating new rules, testing all existing rules in the market place, and weeding out bad rules. The possibility-elaboration step is then captured by the constant formulation of new *conditional forecast* rules in the system and the possibility-reduction step is represented by the subsequent testing of these *conditional forecast* rules and the eventual removal of the bad ones. The next section will discuss in more depth the rationale behind our approach.

[24] See also a related discussion by Baumol and Quandt (1964).

1.3 Rationale for our Genetic-Fuzzy Approach

We have briefly touched on the motivations for using the genetic-fuzzy system back in the last two sections, we now develop a more complete argument in the following two sub-sections. The first sub-section discusses the rationale for incorporating fuzzy reasoning in the system and the second sub-section explains the reasons for using a genetic classifier system.

1.3.1 Why Use Fuzzy Logic?

Some cognitive psychologists have indicated that fuzzy logic offers a reasonably accurate model of the way humans think and reason, and they further suggested that perhaps our ability to efficiently process an immense amount of complex information, some of which are intrinsically vague, is the outcome of applying fuzzy logic to our reasoning and thought processes. However, for us to have confidence in using fuzzy set theory to model the way humans think and reason, there are two questions that need to be addressed. Smithson (1987) drew our attention to the following two questions. First, "are there evidence that support the hypothesis that at least some categories of human thought are fuzzy?" and second, "are the mathematical operations of fuzzy sets as prescribed by fuzzy set theory a realistic description of how humans manipulate fuzzy concepts?" For answers to these questions, we refer to the evidence cited in Smithson.

> *There is a considerable body of psychological research which demonstrates that prototypicality in natural semantic categories is a graded concept (e.g., Rosch (1973a), Rosch and Mervis (1975), Hersh and Caramazza (1976)), and that people widely agree and show reliability in ranking or rating the exemplarity of stimuli in semantic categories. Likewise, some anthropological research has*

shown that gradedness applies across cultures. Kay and McDaniel (1975) found the fuzzy set representation of gradedness in color categories more suitable than the earlier (Berlin and Kay 1969) notions of "focus" and "boundary". Kempton (1978) was able to extend traditional cognitive anthropological folk taxonomic methods via fuzzy set methods in elicting taxonomic judgments about footwear and pottery, in two cultures. Burgess et al. (1983) discovered that the Tarhumara color-terms carry obligatory modifiers which specify the grade of membership of a stimulus in a color-category. (Smithson 1987, p. 55)

Fuzzy negation has not been very systematically critiqued either philosophically or psychologically. Hersh and Caramazza (1976) are among the few researchers to have empirically investigated the fit between subjects' own ratings of membership in A and A', and fuzzy-set predictions for membership in A'. They found a high degree of correspondence between standard fuzzy negation (that is, $m_{A'} = 1 - m_A$*) and the proportion of people who indicated a stimulus was not a member of set A. However, their investigation did not use direct membership ratings and did not investigate any other kinds of fuzzy negation.* (Smithson 1987, pp. 59-60)

Apparently, the standard fuzzy set account of intersection and union does not always apply to concepts that most people would agree involve conjunction and disjunction. However, at least in some cases modified versions of the theory may work well. And it is worth bearing in mind that several empirical investigations have found quite a good fit between fuzzy set operators and data. Oden (1977) found that the product operator fit better than the "min" for "and," but the difference was not large. Thole et al. (1979), on the other hand, found that the "min" fit the best (and this was confirmed in a more rigorous reanalysis in Smithson 1984). Zimmerman and Zysno's (1980) generalized connective was based on the product operator, which caused them to conclude that this operator corresponds most closely with human judgment. However, my reanalysis of their data using least-squares estimates of an alternative generalized connective indicated that the difference in fit between a connective based on "min" and one based on the product was negligible. Furthermore, in at least one application I have found that the bounded sum works best. The question of which intersection and union operators best reflect psychological reality still is open, and the answer may well turn out to be context-dependent. (Smithson 1987, pp. 64-65)

Perhaps the most interesting conceptual discussion about various multivalent and fuzzy logics for behavioral scientists is Dubois and Prades's (1980, pp. 155-169) assessment of the compatibility of several such logics with Piagetian criteria for human reasoning. Briefly, Piaget claimed that adult reasoning requires the capacity to distinguish among and relate together four kinds of transformations: (1) identity, (2) negation, (3) reciprocity, and (4) correlativity. ...

They find that the Arithmetic Rule for implication using either the min-max, product, or bounded sum operators is compatible with Piagetian criteria, but the Maxmin Rule using the min-max operator is not. ...
From this kind of evidence and ...by writers such as Gaines (1975), it appears that the Arithmetic Rule probably holds up the most consistently under conceptual scrutiny. ... Aside from apparent consistency with Piagetian theory, however, little is known about which of these logics, if any, really models human reasoning.
(Smithson 1987, pp. 73-74)

All in all, the evidence presented do seem to provide support for the use of fuzzy set theory to model the way humans think and reason.

Additional evidence that are more relevant to economics are the survey findings of Katona (1975). Katona (1975) pointed out that "while most people can be induced to make a guess as to the direction of change in the near future of major macroeconomic variables, they are reluctant to give quantitative estimates of the extent of the change." Based on decades of survey research on the general public in the United States, Katona concluded that "the majority knew whether unemployment had increased or decreased in the preceding months, whether profits or retail sales had gone up or down, and also whether interest rates had risen or fallen, but did not know how much larger or smaller any of these magnitudes were." Therefore, Katona's findings also seem to suggest that people think in terms of fuzzy notions.

We turn our attention now to the other considerations that have motivated us to use fuzzy logic. The task of modeling expectations formation in an environment that is continually evolving and novel poses two unique difficulties that conventional mathematics are ill-equipped to handle. First, the problem is intrinsically ill-defined. It is ill-defined because investors who may hold diverse beliefs, clearly do not have an objective method to form their expectations as this would require each of them to perform the impossible task of forecasting the expectations of all the other investors. Conventional mathematics is not tuned to handling ill-defined problems because we are not able to precisely define such problems in the language of conventional mathematics. And when we cannot precisely define the problems, we obviously are not going to be able to solve them. Second, inherent in this problem is a source of uncertainty that cannot be appropriately modeled by conventional probability theory. This is the vagueness that permeates human thoughts and human discourse. Since human thoughts and human discourse mold expectations, it is crucial to have a proper model of this element of uncertainty. These difficulties are not unique to our problem. They are in fact common in most complex systems, especially humanistic systems.

In order to cope with the immense burden of modeling complex systems using conventional approaches, scientists have often opted to sacrifice the realism of the models in favor of unrealistic simplifying assumptions and attribute the unmodeled portion of the systems to random noise. Although such approaches have worked well with simple mechanistic systems which can be precisely defined and where the inherent uncertainties arise primarily from random noise, such approaches are totally unsuited for modeling complex systems for the reasons discussed above as well as the common wisdom that in complex systems, we cannot hope to get meaningful results unless our assumptions are also realistic or relevant in the context being modeled. A case in point is the rational choice models in economics which we already know have been unable to account for various phenomena in the real economy.

It is in fact such difficulties that have motivated Lotfi Zadeh to develop his fuzzy set theory. In his struggle to develop a better approach to model complex systems, Zadeh came to notice how easily humans make decision based on imprecise, non-numerical information in complex situations, and it hit upon him that at the heart of the problem is the precision that is demanded by conventional mathematics. The stumbling block in conventional mathematics is the need to precisely define a problem before we can solve it. As we have argued, in some cases, we simply are not able to precisely define the problems. The key to overcoming this stumbling block is to create a new branch of mathematics that is capable of handling imprecise or vague non-numerical data. Zadeh has clearly articulated the need for such a mathematics in his paper titled "From Circuit Theory to System Theory." In that paper, Zadeh argued that:

> *There is a fairly wide gap between what might be regarded as "animate" systems theorists and "inanimate" systems theorists at the present time, and it is not at all certain that this gap will be narrowed, much less closed, in the near future. There are some who feel this gap reflects the fundamental inadequacy of the conventional mathematics—the mathematics of precisely-defined points, functions, sets, probability measures, etc.—for coping with the analysis of biological systems, and that to deal effectively with such systems, which are generally orders of magnitude more complex than man-made systems, we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions. Indeed, the need for such mathematics is becoming increasingly apparent even in the realm of inanimate systems, for in most practical cases the a priori data as well as the criteria by which the performance of a man-made system is judged are far from being precisely specified or having accurately known probability distributions.* (Zadeh 1962, p. 857)

The need for a mathematics of "cloudy" quantities led Zadeh to develop the concept of fuzzy sets and the related mathematics for manipulating such sets.

According to Ebrahim Mamdani, the genius of the fuzzy approach is "the possibilities of implementing 'rules of thumb' experience, intuition, heuristics, and the fact that it does not need a model of the process" (Kosko 1993, p.169). But what is even more important is a recent result from Kosko. Kosko (1992) has demonstrated clearly that fuzzy systems are universal approximators. In simple terms, it means that fuzzy systems are able to approximate general nonlinear functions to any desired degree of accuracy. This makes fuzzy systems particularly valuable for modeling complex nonlinear relationships which we either do not know how to specify or do not know how to solve analytically in the language of conventional mathematics.

The strength of fuzzy logic has attracted many compliments. For instance, Goguen remarked that:

> *The inexactness of description is not a liability; on the contrary, it is a blessing in that sufficient information can be conveyed with less effort. The vague description is also easier to remember. That is, inexactness makes for greater efficiency.* (Goguen 1981)

A down to earth analogy here will help to bring his point across. Suppose your friend sees a truck speeding towards you. A precise way of conveying the information to you could be, "A 6-wheeled 3-ton black truck traveling at 50 mph is accelerating towards you at a rate of 10 mph". The fuzzy way of conveying the same information would be "Watch out! A truck is speeding towards you." It is clear how you would have preferred to be alerted under such circumstance[25]. The ingenuity here is that by

[25] To give a second analogy, suppose you are given the task of explaining in 300 words the contents of a 10 page essay. This can be accomplished by lifting sentences out of the essay to make up the 300 words, or by writing a 300 words summary for the 10 pages essay. The first approach (analogous to the conventional approach) would be very precise because the sentences are exact duplicate of those in the essay, however, its content would be not as relevant or as meaningful as it

being less precise in our description, we can be more relevant in our communication (given our constraints). In a similar vein, economist Riccardo Viale has argued that:

> *... if classical logic were accepted as the canon of deductive inferential rationality one would reach the absurdity of having to accept the countless conclusions, trivial but correct, which are implied by a set of valid premises. This would have fatal consequences for man's ability to adapt to his environment. Other inferential rules are therefore needed to select and skim significant deductions from trivial ones .... Secondly, it is not clear why one should favor classical logic over any types of logic, which offer advantage of formalizing the concepts of possibility and necessity, or non-monotonic types of logic and "fuzzy logic," which can emulate the ambiguity, poor resolution and contradictoriness of human reasoning.* (Viale 1992, pp.174-175)

The core of these observations is what Zadeh had, in his earlier work, referred to as the *principle of incompatibility.*

> *..., the essence of this principle is that as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics. It is in this sense that precise quantitative analyses of the behavior of humanistic systems are not likely to have much relevance to the real world societal, political, economic, and other types of problems which involve humans either as individuals or in groups."* (Zadeh 1973, p. 28)

Other researchers have also voiced similar sentiments. For instance, Tong pointed out that:

> *The [complex] process is often highly nonlinear and has large number of variables. It is often hard to discover the underlying mathematical structures, and there is often a large amount of process knowledge expressible only in linguistic terms. There are many other reasons but they all basically derive from the sheer complexity of the process ... as systems become more complex, it becomes increasingly difficult to make mathematical statements about them which are both meaningful and precise.* (Tong 1978, p. 143)

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will merely contain a collection of scattered thoughts. On the contrary, the second approach (analogous to the fuzzy logic approach) would not be as precise, but its content will be more meaningful as it will attempt to connect the ideas in the entire essay and make sense out of it. The latter approach will not be as precise because in the summary you will need to use words or phrases that encompass a fuzzier and larger set of thoughts (see Zadeh 1973, pp. 28-29).

To sum up, the reasons leading to our decision to use a fuzzy system are: 1) it is a better model of certain aspects of human thoughts and reasoning, 2) it is a better approach for dealing with problems that are ill-defined and inherently vague, and 3) it is a lot simpler and more efficient that a conventional system. A fuzzy system is simpler and more efficient than a conventional system because fuzzy sets allow us to compress a lot of information into very few simple fuzzy notions. This is one of the reasons why conventional expert systems have had limited success in most real world applications thus far. A conventional expert system typically requires hundreds of rules to simulate real-world situations, while in contrast, a fuzzy expert system would generally requires only tens of rules to perform similar tasks (see Moffat 1990). Given the above reasons, it is not difficult to understand why fuzzy systems have grown in popularity over the years. Zadeh foresaw more than 30 years ago that an electromechanical controller would respond better to imprecise input if its behavior was modeled on spontaneous human reasoning. Zadeh's vision has finally materialized. Fuzzy logic based controllers have proven their worth in many areas where conventional logic based controllers have failed.

1.3.2 Why Use a Genetic Classifier System?

The two reasons for using a genetic classifier system are: 1) it offers a general and yet robust framework for modeling decision making and learning in a perpetually novel environment, and 2) it provides a fairly accurate representation of the reasoning systems that humans use.

The design of a classifier system is modeled after the simplistic but unusually robust survival machinery employed by living organisms. This survival machinery is evidently robust because organisms have relied on it not only to survive but to prosper for millions of years in a harsh and constantly evolving natural environment. At the heart of this machinery are two components: a stimulus-response system which tells the organisms how to behave under various conditions, and an algorithm for modifying the stimulus-response system as the organisms adapt to changes in the environment.

Our classifier system models this survival machinery as a system of fuzzy *conditional action* rules that are systematically evolved by a genetic algorithm (GA). A system of conditional action rules capture the essence of a stimulus-response system while a GA mimics the optimization process that takes place at the genetic level of an organism. Like the stimulus-response system, a set of fuzzy *conditional action* rules determines how the system should respond under various environmental conditions. This gives us a very general but simple framework for modeling complex decision making. A GA makes use of artificial *production*, *crossover* and *mutation* operators to systematically improve the *conditional action* rules as the environment changes. Chapter 2 will discuss GAs in more depth. For now, we will simply state that Holland has shown that a general and yet robust parallel search and optimization algorithm can be created from combining these three artificial genetic operators together. Holland's schema theorem basically asserts that a GA is capable of finding the optimal solutions to any optimization problem and that it will arrive at these solutions at an exponential rate[26]. This result was subsequently confirmed by De Jong (1975) when he successfully applied GAs to the optimization of various complicated functions with the following characteristics: 1) continuous/discontinuous, 2) convex/nonconvex, 3) unimodal/multimodal, 4) quadratic/nonquadratic, 5) low-dimensionality/high-dimensionality, and 6) deterministic/stochastic. Such strengths make a GA an invaluable tool for modeling learning.

Now turning our attention to the second reason, we are interested in whether there are reasons to believe that a genetic classifier system captures certain key features of the reasoning systems humans actually use. Psychologists have argued that our mind holds two different reasoning systems[27]. To illustrate, consider the problem of figuring out the change at the cash register[28]. It is not uncommon to find that sometimes the answer will spring to our mind intuitively, but at other times we will have to do some mental arithmetic to arrive at the answer.

Sloman (1996) recently distinguished these two reasoning systems as associative and rule-based. An associative system operates reflexively and can handle both concrete images as well as abstract notions. It also allows us to produce "quick and dirty" answers based on heuristics just as in the example above. As the name

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[26] See for instance, Holland (1992), Goldberg (1989) and the discussion in Section 2.6 of this dissertation.

[27] However, psychologists have yet to agree on a precise definition of these two systems, or on where to draw the line that distinguishes one system from the other. For instance, Smolensky (1988) distinguishes these systems as intuitive processor versus conscious rule interpreter, Hinton (1990) calls it intuitive and rational processors, Schneider and Shiffrin (1977) use the terms automatic and controlled processing, Evans (1989) describes them as perceptually based matching process and linguistic-logical process, and Shastri and Ajjanagadde (1993) distinguish them as reflexive and reflective reasoning.

[28] This example is from Sloman (1996, p.1).

suggests, an associative system helps us make sense of new information by associating the new information with the knowledge already existing in our mind. The way an associative system processes information is analogous to the process used by police officers to compose a picture of a suspect by putting together pieces of different parts of a face based on accounts given by eye witnesses. This process allows police officers to create a picture that closely resemble what a suspect might looked like even though none of the officers may have seen the suspect before. Likewise, a similar process help us interpret new information that we may have never encountered before. People in general are unconscious of associative processing in the mind. In contrast, people are usually conscious of rule-based processing which is known to be responsible for the "logical, hierarchical and casual-mechanical" aspects of humans reasoning. Our ability to perform mathematical calculation via systematic application of rules is a good example of rule-based processing.

But how do we use these two systems and how do these two systems interact to influence our decision? Sloman maintained that the two systems do not have their own exclusive problem domain, but rather they have overlapping domains; domains that differ depending on the individual's knowledge, skill and experience. Sloman explained that:

> *Together, they lend their different computational resources to the task at hand; they function as two experts who are working cooperatively to compute sensible answers. One system may be able to mimic the computation performed by the other, but only with effort and inefficiency and even then not necessarily reliably. The systems have different goals and are specialists at different kinds of problems. When a person is given a problem, however, both systems may try to solve it: Each may compute a response, and those responses may not agree.* (Sloman 1996, p.6)

A classifier system with *conditional action* rules that are evolved systematically in a parallel fashion (via a GA) resembles the associative system that Sloman talked about. There are two levels of association at play when agents use classifier systems to guide their decision making. At a macro level, agents in the model attempt to *associate* the rules to the states of the market they have observed by matching the conditions in the rules with the current market state.

However, what is more important is the next level of association which takes place in the background (not noticed by the agents). This is found in the GA within the classifier system where association is done at the level of the schemas. In a classifier system, rules are coded as strings of numbers. A schema is a similarity template which we can generalize from a collection of strings representing the rules. For instance, a possible schema for the strings {00110, 00111, 01110} is (0*11*), where the symbol "*" represents a "wild card". Schemas are useful in that they enable a GA to quickly find general patterns that lead to better decision making. Furthermore, in a GA, the search take places in parallel among all existing schemas. This phenomenon is known as implicit parallelism[29]. Both the use of schemas and the implicit parallelism make a GA very efficient in processing information.

Our mind also appears to order and reorder concepts in successively more abstract form like the schema representations in a GA. The advantages of such sub-

[29] Holland has shown in his Schema theorem that the implicit parallelism inherent in a GA allows it to process on the order of N^3 schemata per generation where N is the number of strings in the population. To put this in perspective, if we have 100 strings, a GA will process on the order of a million schemata.

conceptual representations are twofold. Like the schemata in a GA, these sub-conceptual representations in our mind allow more efficient storage and processing of new information. Instead of having to store every piece of new information as it is, our mind can instead represent the new and perhaps complex information as sub-conceptual models. These sub-conceptual models serve as building blocks for bigger and more complex notions. In addition, such sub-conceptual models not only symbolize a concept but also represent some of its internal structure. They constitute an analysis of a concept. The advantage of including such analyses in a representation is to permit simpler and faster processing of reasoning[30]. This process is termed representational redescription by Clark and Karmiloff-Smith. Our capacity to generalize and to use analogy are all a part of this redescription process.

Once the association is done and a rule is selected, agents in our financial market model then proceed to calculate their holdings of assets in a systematic mechanical fashion. This part of the thought process resembles the rule-based system Sloman has discussed. To sum up, our artificial agents, in making their decisions, make use of the same two systems of reasoning in a cooperative fashion just as humans would in their reasoning and decision making processes.

A more general framework for studying human reasoning is the mental model framework as Garnham and Oakhill have argued recently(1994, p.341). They have urged psychologists to use this framework for unifying research on thinking and

[30] This is the same reason why a GA so efficient. This will become obvious after our discussion of the Schema Theorem in Chapter 2.

reasoning in psychology. As a matter of fact, both the associative system and rule-based system outlined above can be interpreted in terms of a mental model framework. Mental models are cognitive views that humans construct to try to make sense of how things work. When faced with a problem, our mind constructs mental models to resolve the problem by integrating in novel ways the representations stored in our memory (the same sort of representations we have discussed previously). For instance, suppose we are given a task of supporting a coffee cup several inches above a surface using only a sheet of paper[31]. Solving this problem would require much more than activating the schemas for "paper" and "cup". In this case, a useful mental model might contain information about operations that can be performed with paper, the weight of the cup, and so on.

In our model, individuals' mental models of what moves prices and dividends in the market are represented by rules in a classifier system. The manner in which the rules are evolved with a GA resembles the manner in which mental models are formulated in our minds. For instance, the construction of a mental model involves an incremental updating of the representation on the basis of the present and past input. So the resultant representation in any given moment guides the interpretation of subsequent input. This gradual updating of mental models is captured by the use of the reproduction and crossover operators in a GA. In constructing a new rule (new mental model), the reproduction operator determines which existing rules (existing mental models) will be allowed to contribute to the new rule. The selection is based

[31] This example is from Holland et al. (1989).

on the rules' predictive ability. The crossover operator then genetically crossovers the selected rules to create a new rule. Therefore the new rule is a hybrid of those existing rules which have proven to be successful in the past, and this new rule in turn influences how the system interacts with subsequent inputs.

Another important point to keep in mind is that the rules in our classifier system do not cover all possible contingencies, and this is typical of decision-making in the real world. For instance, Hogarth (1980) has argued that:

> *... under most circumstances it is not reasonable to talk about finding 'all the alternatives'. The generation of alternatives is a lengthy and costly process, and one where, in real-world situations, even minimal completeness can seldom be guaranteed. Theories of optimal search can cast some light on such processes, but, because of limits on complexity, human alternative-generating behavior observed in the laboratory is usually best described as heuristic search aimed at finding satisfactory alternatives, or alternatives that represent an improvement over those previously available.* (Hogarth 1980, p.5)

The manner in which we have allowed the rules to be evolved by a GA in our model is also in agreement with Hogarth's interpretation; that is, "it is a heuristic search aimed at finding satisfactorily alternatives that represent an improvement over those previously available." Recent studies in cognitive psychology suggest that this heuristic search is conducted in parallel rather than sequentially. This is captured by the parallel search characteristics intrinsic in a GA (See Rumelhart, McClelland and the PDP research group 1987, and Holland, Holyoak, Nisbett and Thagard 1986).

2. FUZZY LOGIC AND GENETIC ALGORITHMS

The objective of this chapter is to provide a tutorial on fuzzy logic and genetic algorithms. The intention here is not to provide a rigorous discussion of the theory of fuzzy sets[32] or genetic algorithms but to present a practical and intuitive approach to these methods. The presentation will begin with fuzzy logic by introducing the concepts of fuzzy sets in Section 2.1, presenting the concepts of membership function and fuzzy logical operators in Section 2.2, illustrating how to construct a fuzzy inference system in Section 2.3, and finally, discussing recent developments in Section 2.4. This is followed by the presentation of genetic algorithms. Section 2.5 provides an overview of genetic algorithms and Section 2.6 discusses the fundamental theorem of genetic algorithms. In Section 2.7 we compare the GA to conventional optimization methods. Section 2.8 describes the basic elements in a genetic algorithm. In Section 2.9, we look at a simple application of a GA, and finally, in Section 2.10 we discuss the crux of a genetic-fuzzy classifier system.

2.1 What is a Fuzzy Set?

A fuzzy set differs from a classical set in that it has no well-defined boundary. To appreciate the gist of Lotfi A. Zadeh's innovation, close your eyes for a moment, and picture in your mind, a group of "old" persons around you. You will realize, in

[32] Klir and Folger (1988) have shown that an isomorphism exists between logic and set theory. As such the term fuzzy logic and fuzzy set theory will be used interchangeably. To be more precise, some may argue that it is really a fuzzy subset theory and not fuzzy set theory. However, like the majority, we prefer to use the term fuzzy set theory. Both Klir and Folger (1988) and Klir and Yuan (1995) are good introductory text on fuzzy sets theory.

your attempt to classify this set of "old" persons, that the boundary of this set is not "crisp". There is no clear cut-off for classifying whether a person is old or not old. For instance, we do not think of a 40 year-old person as old and a 39 year-old as not old. Instead, what we have in our mind, is a fuzzy boundary that allows for varying degree of membership to the set of "old" persons. Consequently, even though both a 39 year-old person and a 40 year-old person may be members of the set of "old" persons, they will not have the same degree of membership. The 39 year-old person would be a member of this set to a lesser degree than the 40 year-old.

Moreover, the 39 year-old and 40 year-old persons may also, at the same time, be members of the reciprocal set—the set of "young" [33] persons. In this case, the 39 year-old person would be a member of the set of "young" persons to a greater degree than the 40 year-old. Therefore, within this new paradigm, an entity may belong to both a set and its complement at the same time[34]. It follows then that, within this paradigm, a statement like—"A 40 year-old person is old"—can no longer be an absolutely true or an absolutely false statement any more. It has to fall somewhere between false and truth. The notion of fuzzy sets has therefore extended the arena of classical two-valued Boolean logic to one where there is a gradual and perhaps even continuous transition between the false state and the truth state. Instead of absolute truth or false, "everything is [now] a matter of degree" (Kosko 1993, p. 18). Needless

[33] "Young" is taken to be synonymous with "not old" (i.e. the complementary set) here.

[34] Such inconsistency would not have survived in the Boolean logic paradigm for Boolean logic requires an entity to be either a member (truth) or not a member (false) of a set, and cannot permit any overlap between a set and its complement – "truth" and "false". It is therefore interesting to note how such seemingly inconsistent ideas can fit in nicely within the fuzzy set paradigm.

to say, such gradual transition between false and truth will allow us to better represent the vague or imprecise concepts inherent in our natural language. However, as we have seen in Chapter 1, this innovation is more than just an interesting exercise in modeling imprecise linguistic concepts. It is actually an integral component of a very powerful theory for modeling complex and ill-defined systems. We will explore how it could be applied later in Section 2.3 and Section 2.10. In the next section we look at how to set up membership functions and how logical operators work on fuzzy sets.

2.2 Membership Function and Fuzzy Set Operators

2.2.1 Membership Function

We have mentioned in the last section that a fuzzy set has no well-defined boundary. To formalize the concept of fuzzy set, Zadeh (1965) introduced the notion of graded membership or a membership function. A membership function is a mathematical function that maps elements from a crisp set into real numbers in the interval [0,1]. Figure 1 and 2 illustrate the difference between a classical set and a fuzzy set of "tall" ("not tall") persons. In a classical set, an element either belongs to or does not belong to a set. Hence if we consider anyone who is at least 6' tall as "tall", and anyone who is less than 6' tall as "short" or "not tall", we can then represent this concept by the crisp cut-off at 6' in figure 1.

This is unlike in the fuzzy world where an element can be a member of a set and its complement at the same time. "Fuzziness" is represented by drawing a curve to allow for a varying degree in tallness as shown in figure 2. This curve is called a membership function. The membership function tells us to what degree an element belongs to a fuzzy set. It assigns to each element in the fuzzy set a number in the interval [0,1]. The degree of membership in a fuzzy set can be read off the vertical axis. A membership value of 0 means that an element does not belong to the fuzzy set, and a membership value of 1 means an element is a complete member of the fuzzy set.

Take for instance, a person of height 5' 6". Most would agree that this person is *not very* tall. By that we are merely expressing our opinion of the *degree of tallness*, and not whether this person is tall or not tall. In the fuzzy set representation of tall persons, this height might have a membership value of 0.6 (see figure 2). This means that this person is a 60% member of the set of "tall" persons (notice that this person is also a 40% member of the set of "not tall" persons[35]). In this example, a fuzzy set has allowed us to properly describe the vague notion—"tall". In contrast, we are not able to faithfully represent this notion by probability theory. It is not the same to say that *there is a 60% chance that this person is tall.* By saying this, we are implying that this person could be tall or short, and it is more likely that this person is tall. Obviously, this does not capture the true meaning of what we have in mind. Hence, a vague concept is best represented by a fuzzy set.

2.2.2 Fuzzy Set Operators

As with conventional set theory, the concepts of complement, intersection, and union can also be defined for fuzzy sets. In the following definitions, μ denotes the

[35] See Section 2.2.2 for the relationship between a set and its complement.

membership function and X the universe of discourse[36].

Complementation (*NOT*)

Complementation corresponds to the logical operator "not." The complement ($\neg$ A) of

a fuzzy set A is defined by

(2.1) $$\mu_{\neg A}(x) = 1 - \mu_A(x), \qquad x \in X$$

***Intersection* (*AND*)**

The intersection corresponds to the logical operator "*AND*." The intersection of two

fuzzy sets A and B on X, denoted A $\cap$ B, is defined by

(2.2) $$\mu_{A \cap B}(x) = \min\{\mu_A(x); \mu_B(x)\}, \qquad x \in X$$

***Union* (*OR*)**

The union corresponds to the logical operator "*OR*." The union of two fuzzy sets A

and B on X, denoted A$\cup$B, is defined by

(2.3) $$\mu_{A \cup B}(x) = \max\{\mu_A(x); \mu_B(x)\}, \qquad x \in X$$

We illustrate these concepts by applying the above operators to two fuzzy sets, A and B. Figures 3 and 4 show the fuzzy sets A and B prior to applying these operators, and figures 5, 6 and 7 show graphically the outcomes after applying the complementation, intersection, and union operators respectively to the fuzzy sets A and B.

[36] The universe of discourse of any fuzzy set A in X is defined to be the set of points in X for which the membership function for A is positive).

Having defined the fuzzy logical operators, a few comments are in order before we conclude this section. Although the above definitions are commonly used for logical operators, they are by no means the only operational definitions. Some other alternative definitions for the pair of *AND* and *OR* operators are the **Product** operators and the **Bounded Sum/Difference** Operators[37]. However, unlike the **Min-Max** operators defined above, these other alternatively defined operators do not satisfy idempotency and distributivity. In addition, if there are existing errors associated with the membership grades, these alternative operators will further compound the existing errors. Fortunately, this error compounding does not happen with the use of the **Min-Max** operators. Nevertheless, the alternatively defined *AND* and *OR* operators may be preferred for other reasons. For example, the *AND* operator of the **Product** pair is useful because it resolves the Eubulidean paradox (Smithson 1987, p. 27). There are also alternative definitions for the Complementation operator and they are the **Sugeno Class** complements and the **Yager Class** complements[38].

So far we have only discussed how to represent simple vague adjectives with fuzzy sets and how logical operators work on these fuzzy sets. However, our natural

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[37] **Product Operators**

AND: $\mu_{A\cap B}(x) = \mu_A(x) \bullet \mu_B(x)$; $\quad x \in X$

OR: $\mu_{A\cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \bullet \mu_B(x)$; $\quad x \in X$

Bounded Sum/Difference Operators

AND: $\mu_{A\cap B}(x) = \max\{0, \mu_A(x) + \mu_B(x) - 1\}$; $\quad x \in X$

OR: $\mu_{A\cup B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\}$; $\quad x \in X$

[38] **Sugeno Class Complements:** $\mu_{\neg A}(x, \lambda) = \dfrac{1 - \mu_A(x)}{1 + \lambda \mu_A(x)}, \quad x \in X$

Yager Class Complements: $\mu_{\neg A}(x, \lambda) = (1 - \mu_A(x)^\lambda)^{1/\lambda}, \quad x \in X$

As a matter of fact, the standard complementation operator we have defined is a special case of the *Yager class* complements; it is obtained by setting $\lambda = 1$.

languages are certainly more interesting than just the plain adjectives we have considered. In our natural languages, we often use linguistic hedges such as "not very", "very", "somewhat", "more or less" etc. Zadeh refers to this class of hedges as Type I hedges and has suggested representing these linguistic hedges as operators on fuzzy sets (Zadeh 1972). The operators for representing these linguistic hedges are : Normalization, Concentration, Dilation, Contrast Intensification and Fuzzification. Zadeh has also discussed another class called Type II hedges which is used to describe hedges like "technically", "essentially", "practically" etc.. Readers interested in the modeling of these hedges may want to consult Zadeh's original paper—"A Fuzzy-Set-Theoretic Interpretation of Linguistic Hedges" (Zadeh 1972). In the next section we show how the operators defined above can be used in a fuzzy inference system or a fuzzy controller to guide decision making.

2.3 Constructing a Fuzzy Inference System

A fuzzy inference system is a type of expert system, and like all expert systems, it is in essence a computer-based system that is designed to emulate the reasoning process of a human expert within a specific domain of knowledge. The design of a fuzzy inference system is quite simple conceptually. It consists of four modules: a fuzzification module, a fuzzy rule base, a fuzzy inference engine, and a defuzzification module. The relationships among these four modules and the working environment are illustrated in a schematic diagram in figure 8.

The fuzzification module is one of the two modules that interfaces directly with the external environment. As the name suggests, the function of the fuzzification module is to convert the values of the relevant decision variables it has measured in the environment into appropriate fuzzy sets (actually truncated fuzzy sets). The fuzzy rule base contains a set of fuzzy decision making or control rules. These rules are typically expressed as *if-then* rules. For instance, one such rule might look like "If it is warm and the room is full then set the speed of the air conditioner's compressor high." The inference engine then takes the fuzzified inputs and combines them with the relevant fuzzy rules to make inferences regarding the output variables. The product of the inference engine are fuzzy outputs which will need to be transformed back into non-fuzzy values to guide decision making or for control purposes. This last step is accomplished by the defuzzification module which is the second module that interfaces directly with the external environment.

To fully appreciate the inner workings of a fuzzy inference system, let us now discuss the basic steps involved in the design of a fuzzy inference system and explore how it works in practice. For the purpose of illustration, we will consider the design of a fuzzy system for trading a stock.

The first step in the design involves identifying the relevant input and output variables and their ranges of values (which include setting up the universe of discourse), and selecting the fuzzy sets to represent the possible states of these variables. To keep things simple, we will restrict our fuzzy trading system to only two inputs—a price indicator and a volume indicator. The price indicator measures the percentage change in price, and the volume indicator measures the level of trading volume. The price indicator and the volume indicator are allowed to have three possible states ("low", "normal" and "high") and two possible states ("normal" and "low") respectively. We assume that these two variables have been scaled to operate over the universe of discourse of [-0.5,0.5] and [0,1] respectively.

Next we construct membership functions for both inputs. We use three membership functions for the price indicator to represent "low" (closer to -0.5), "normal" (around 0), and "high" (closer to 0.5). For the volume indicator, we use two membership functions to represent "high" (closer to 1), and "normal" (closer to 0). Various shapes , such as triangular, trapezoidal, bell curve, sigmoid curve, etc., have been used in practice for membership functions. But the shape is generally less important than the number of membership functions and their placement. We will use the triangular membership functions for the price indicator and the trapezoidal membership functions for the volume indicator. Figures 9 and 10 show the membership functions for both inputs.

The output we want obviously is a trading decision—that is, "buy", "sell", or "hold." We will use three triangular membership functions to represent these decisions and we restrict the corresponding universe of discourse to [-1,1]. Figure 11 shows the membership functions for the trading decision. We will also assume that a defuzzified output value of less than -0.3 represents a sell decision, and of greater than 0.3 represents a buy decision. Any value between these two values will result in a decision to hold.

Having set up the fuzzy sets, we can now explore how the fuzzification module works. Suppose that the price and volume indicators have the values of 0.2 and 0.65 respectively. To fuzzify these input values, we must first determine for each variable if the fuzzy set representing the antecedent in a given rule include the given value in its range. If it does, fuzzification entails truncating the height of this fuzzy set to the point where the input value intersects this fuzzy set. The effect of fuzzification is illustrated by the top two pictures in figures 12 to15. Referring to figure 12, we can see that the input value of 0.2 for the price indicator has caused the corresponding fuzzy set to be truncated at a membership value 0f 0.4. In the case of the volume indicator, it is truncated at a membership value of 0.8. Therefore, the effect of fuzzification is to transform the crisp input values into membership values that indicate the degree to which these values have satisfied the fuzzy antecedents in the rule. If the given input value falls outside the range of the corresponding fuzzy set in a rule, as is the case in the top left picture in figure 15, the outcome will be a zero membership value.

The next step in the design is to construct a set of fuzzy rules to guide the trading decision. In practice, these rules are set up by consulting experts in the area we are modeling. Alternatively, these rules can also be learned from past data (see Kosko 1993, pp. 157-171, 214-222). But when neither approach is possible, the rules can be set up by trial and error. To illustrate, suppose our intention is to mimic a positive feedback trading strategy. A positive feedback strategy involves buying when there is an upward price momentum and selling when there is a downward price momentum. An upward (downward) price momentum is identified by a volume-driven price increases (decreases). Accordingly, we can construct the rules for this strategy as follows.

Rule 1: If the price indicator is high and the volume indicator is high, then the trading decision is buy.

Rule 2: If the price indicator is normal then the trading decision is hold.

Rule 3: If the volume indicator is normal, then the trading decision is hold.

Rule 4: If the price indicator is low and the volume indicator is high, then the trading decision is sell.

The third step involves the design of the inference engine. Here we decide on how to interpret the logical connectives in the rules, the implication method and how to aggregate the outcomes from each rules. As we have mentioned earlier, there are several possible definitions for the fuzzy logical operators '*AND*', '*OR*' and '*NOT*' (complement). For reasons discuss in the last section, we will use the **Min-Max** logical operators in our example. Recall that the effect of the '*AND*' operator is to pick out the minimum membership value from a group of fuzzy elements connected by the '*AND*' operator. Since we have used the '*AND*' operator in the first and last rule above, the joint effect of the antecedents in each rule will be determined by the minimum membership values of the two fuzzy antecedents in each of these two rules. Going back to figure 12 again, the membership values for the antecedents have been determined to be 0.4 and 0.8 for the price and volume indicator respectively. Therefore, the joint effect should be a membership value of 0.4 (the minimum of the two values).

Implication involves computing the effect this joint membership value of the antecedents has on the consequent in the same rule. There are numerous ways in which this can be achieved and one such approach is motivated by the parallels in classical logic. It is well known in classical logic (i.e. binary logic) that the truth value of $p \Rightarrow q$ is the same as either $\neg p \vee q$ or $(p \wedge q) \vee \neg p$. Alternative fuzzy implications can be derived by substituting the various definitions for the fuzzy logical operators $\wedge$ (*AND*) and $\vee$ (*OR*) into these two expressions. Some examples are the Kleene-Dienes implication, the Lukasiewicz implication, and Zadeh implication (for more details see Driankov, Hellendoorn and Reinfrank 1993, pp.85-87).

Of all the implication operators, the most important one known in the fuzzy control literature is the Mamdani implication[39]. It is based on the intersection operator, i.e., $p \Rightarrow q \equiv p \wedge q$. We have used the Mamdani implication in our example. In practice, the Mamdani implication is implemented with the *min* operator which has the effect of truncating the height of the fuzzy set for the consequent to the joint membership value of the antecedents determined above. This explains why the bottom picture in figure 12 shows that the consequent has been truncated at the membership value of 0.4.

Figures 12 to 15 show how each rule has responded to the given input values, and the effect of implication is illustrated by the bottom picture in each of these figures. Notice that there is no logical connective in Rule 2 and 3. In these cases, the joint membership value of the antecedents is just the membership value of the only

[39] The Mamdani implication is important because it will not have "interaction." Basically, this means that it will produce the same result regardless of whether it uses composition based inference or individual rule based inference. See Driankov, Hellendoorn and Reinfrank (1993, pp.101-102).

antecedent that is present in each rule. In the case of Rule 4, although there are two antecedents, the first one is not satisfied by the given value. The first antecedent therefore yields a membership value of zero. Consequently, the joint membership value is zero and the fuzzy set for the consequent is cut off at zero. This implies therefore that the output from Rule 4 is an empty fuzzy set.

The next task that has to be performed by the inference engine is to consolidate the truncated fuzzy set from each rule. This can be accomplished by either taking the union or the intersection of all the resulting truncated fuzzy sets that represent the consequent. The choice will depend on whether we treat the rules as disjunctive or conjunctive. If we treat the rules as disjunctive, we will obtain a conclusion for a set of given input values whenever the joint membership value of the antecedents is non-zero for at least one rule. On the other hand, if we consider the rules as conjunctive, then we will obtain a conclusion only if the joint membership value of the antecedents is non-zero for all the rules. The interpretation of the rules as either disjunctive or conjunctive depends on their intended use and the way the joint membership value for the antecedents in each rule is determined. For the example we are considering, clearly we want to treat the rules as disjunctive. We therefore take the union of all the truncated fuzzy consequent. The result is the largest fuzzy set that can be formed from superimposing the truncated fuzzy sets of the consequent on each other. This final output fuzzy set is illustrated in figure 16.

The final step is to design the defuzzification module. The purpose as we have mentioned before is to convert the final fuzzy set obtained by the inference engine to a real value so that it may be used for control purposes or guide decision making. Of the numerous defuzzification methods that have been proposed, the six most popular ones are: Centroid or Center-of-Area method, Center-of-Sums method, Center-of-Largest-Area method, First-of-Maxima method, Middle-of-Maxima method, and Height defuzzification method. These defuzzification methods are illustrated in figures 17 to 22[40].

In most practical applications, a major consideration in the choice of a defuzzification method is its computational complexity. The Height method, Middle-of-Maxima method and the First-of-Maxima method are the faster ones among the six methods listed above. Other considerations to keep in mind are Continuity, Disambiguity and Plausibility. Continuity will ensure that a small change in the input will not result in a large change in the output. The problem of ambiguity arises when the result of the defuzzification method is not unique. This happens when for instance the Center-of-Largest-Area method is applied to two fuzzy sets that have identical areas. A defuzzified output is considered plausible if it lies approximately in the middle of the support of the corresponding fuzzy set and has a high degree of membership in the same fuzzy set. We apply the Centroid method to our example. It is apparent from figure 16 (since the bulk of the set is concentrated on the right side), that even without carrying out the defuzzification, the decision is to buy. The calculation of the centroid gives a value of 0.333 which confirms a buy decision as

[40] See Driankow, Hellendoorn and Reinfrank (1993, pp.132-141) for details on how to carry out these calculations.

expected[41]. In the next section, we explore recent developments in fuzzy logic.

2.4 Recent Developments

It is interesting to note the following comments by Kosko on the earlier development of fuzzy logic.

> *Zadeh saw scientists throwing ever more math at problems and trying to think and run the business of science with the black-white reasoning that computers and adding machines used. He chose the word "fuzzy" to spit in the eyes of modern science.*
>
> *The term fuzzy invited the wrath of science and received it. It forced the new field to grow up with all the problems of a "boy named Sue." The fuzzy movement in those days was a small cult and it went underground. It grew and matured without the usual support of subsidized science. That made it stronger.*
>
> *Fuzzy logic did not come of age at universities. It came of age in the commercial market and leapfrogged the philosophical objections of Western Scientists.* (Kosko 1993, p. 20)

As Kosko has clearly stated, the application of fuzzy logic in the commercial sector is where it gained most of its fame and subsequently caught the attention of most western academics. Over the last decade, fuzzy logic has increasingly been used in commercial applications; particularly in control systems which have to deal with vague inputs. The current interest in the commercial application of fuzzy logic was possibly triggered by two events in the 1980s; one in 1985 and another in 1987. In 1985, Shoji Miyamoto and Seiji Yasunobu of Hitachi, in their simulations,

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[41] The equation for calculating the Centroid is given by:

$$u^{*} = \frac{\int u.\mu(u)du}{\int \mu(u)du}$$

where u^{*} is the centroid, u are values within the domain of the final fuzzy set, and $\mu(u)$ is the membership value for a given u.

demonstrated that a fuzzy control system is far superior than the conventional control system for the Sendai subway in Japan (see Seiji Yasunobu and Shoiji Miyamoto 1985). Although they ran into problems initially, they had a breakthrough when they realized that they should design the controller to anticipate rather than react to events as they happen. Their ideas were adopted and fuzzy systems were used to control accelerating, braking, and stopping when the line opened in 1987. The implementation was a phenomenal success.

Later in the same year (1987), at the second annual International Fuzzy Systems Association (IFSA) conference in Tokyo, Takeshi Yamakawa demonstrated the use of fuzzy control for a classic control problem—the balancing of an inverted pendulum (see Takeshi Yamakawa 1989). This is a very difficult task to accomplish with conventional control method but Yamakawa was able to maintain the pendulum upright with ease using a fuzzy controller. But that was not all. The most astonishing discovery at the conference was certainly the serendipitous demonstration that a fuzzy system can continue to function satisfactorily even when it has been degraded (McNeill and Freiberger 1993, p. 157). At the conference, a curious observer had requested Yamakawa to remove a computer board from his fuzzy control system as an attempt to investigate how the system would respond to a degradation of the information provided to the controller. The general expectations at that time were that the pendulum would drop immediately when the board was removed. However, to everyone's amazement, the pendulum continued to maintain its balance, proving forcefully that a fuzzy control system can continue to make decisions even with partial information.

Such sensational demonstrations quickly set the stage for a broad-based research effort in fuzzy logic. Some examples of how Fuzzy Logic has been applied in practice include[42]:

- Automatic control of dam gates for hydroelectric-powerplants (Tokio Electric Power.)
- Simplified control of robots (Hirota, Fuji Electric, Toshiba, Omron)
- Camera aiming for the telecast of sporting events (Omron)
- Substitution of an expert for the assessment of stock exchange activities (Yamaichi, Hitachi)
- Preventing unwanted temperature fluctuations in air-conditioning systems (Mitsubishi, Sharp)
- Efficient and stable control of car-engines (Nissan)
- Cruise-control for automobiles (Nissan, Subaru)
- Improved efficiency and optimized function of industrial control applications (Aptronix, Omron, Meiden, Sha, Micom, Mitsubishi, Nisshin-Denki, Oku-Electronics)
- Positioning of wafer-steppers in the production of semiconductors (Canon)
- Optimized planning of bus time-tables (Toshiba, Nippon-System, Keihan-Express)
- Archiving system for documents (Mitsubishi Elec.)
- Prediction system for early recognition of earthquakes (Inst. of Seismology Bureau of Metrology, Japan)
- Medicine technology: cancer diagnosis (Kawasaki Medical School)
- Combination of Fuzzy Logic and Neural Nets (Matsushita)
- Recognition of handwritten symbols with pocket computers (Sony)
- Recognition of motives in pictures with video cameras (Canon, Minolta)
- Automatic motor-control for vacuum cleaners with recognition of surface condition and degree of soiling (Matsushita)
- Back light control for camcorders (Sanyo)
- Compensation against vibrations in camcorders (Matsushita)
- Single button control for washing-machines (Matsushita, Hitatchi)
- Recognition of handwriting, objects, voice (CSK, Hitachi, Hosai Univ., Ricoh)
- Flight aid for helicopters (Sugeno)
- Simulation for legal proceedings (Meihi Gakuin Univ, Nagoya Univ.)
- Software-design for industrial processes (Aptronix, Harima, Ishikawajima-OC Engineering)
- Controlling of machinery speed and temperature for steel-works (Kawasaki Steel, New-Nippon Steel, NKK)

[42] These examples are from Bauer, Nouak, and Winkler 1996, and Kosko 1993, pp. 184-187.

- Controlling of subway systems in order to improve driving comfort, precision of halting and power economy (Hitachi)
- Improved fuel-consumption for automobiles (NOK, Nippon Denki Tools)
- Improved sensitiveness and efficiency for elevator control (Fujitec, Hitachi, Toshiba)
- Improved safety for nuclear reactors (Hitachi, Bernard, Nuclear Fuel div.)

This concludes the presentation on fuzzy logic. The remaining of this chapter is devoted to discussing genetic algorithms. This next section provides an overview of genetic algorithms.

2.5 Genetic Algorithms—An Overview

Genetic Algorithms[43] (GAs) are robust parallel search and optimization algorithms that are inspired by Darwin's notion of the "survival of the fittest" at the genetic level. At this (genetic) level, the "survival of the fittest" refers to how the "fittest" genes are sought out in nature through a combined processes of selection, crossover, and mutation. Since in principle, the search for the "fittest" genes in nature is no different from the search for optimal solutions in an optimization problem, we could potentially exploit this robust search algorithm of nature for solving very general optimization problems. But of course, this is possible only if we could represent our tentative solutions (search objects) in an optimization problem in the form of chromosome-liked objects suitable for manipulation by artificial selection, crossover and mutation operators.

[43] GA is the brainchild of John H. Holland at the University of Michigan (see Holland 1970a, 1970b, 1970c, and 1975). The initial ideas were first conceived by Holland in the early 1960s. Robust is used here to mean efficient and efficacious. We will follow Goldberg's (1989) footsteps in justifying that a GA is indeed efficient and efficacious later. Davis (1991) is a good reference for GAs.

In GAs, this is typically accomplished by transforming the search objects into binary strings that mimic the chromosomes. Chromosomes are entities composed of genes. If we draw an analogy between a chromosome and a binary string, then the genes in the chromosomes are like the binary bits within the binary string. The position of a gene in the chromosome is called its locus and the value taken by each gene is called its allele. Among other functions, these genes hold crucial hereditary information which can affect the development and survival of an organism. The genetic makeup of an organism is known as its genotype. Similarly, in a GA, the bits in a binary string hold important information which can determine whether the string will be selected for reproduction or be discarded. In the case of a GA, the genotype refers to the binary structure or representation of the search objects.

There is however another class of properties of an organism which also can affect the way an organism adapts and survives as the genotype does. But unlike the genotype, this set of properties is not heritable. This is the phenotype, a set of properties or behaviors which an organism acquires from its interaction with the environment (as it learns to adapt). The parallel of the phenotype in GAs is the actual representation of the search objects (i.e., that which the binary representation decode into—it could be a set of parameters, solution alternatives, or points in the solution space).

In genetics, genotype properties are inherited by future generations through iteration after iteration of the combined processes of selection, crossover, and mutation. GAs mimic this natural evolutionary process by subjecting a population of binary strings to repeated operations which resemble those of selection, crossover and mutation. In the selection process, search objects or binary strings (chromosomes) are selected for reproduction based on their "fitness". "Fitness" in a GA is measured by how well the search objects have solved the problem at hand, or simply, by how close they approximate the true solution of the problem. Generally, the more 'fit" ones are more likely to be selected for reproduction than the less "fit" ones. Those selected for reproduction are subsequently paired off randomly and subjected to crossover. In a crossover, each pair of strings are chopped off at a randomly chosen point, and the segments from each parent are then combined to create new offspring. Although no new genes are produced, a crossover can potentially create many different combinations of truly unique strings. For instance, in human reproduction where 23 out of the 46 chromosomes from each parent are recombined and passed down to offspring, trillions of totally unique combinations can possibly be produced. In other words, trillions of truly unique individuals can be created due to crossover alone.

A numerical example here will help to provide a more definitive illustration of the effectiveness of a crossover operator. Consider a binary string with 3 bits. The entire space that can be spanned by a 3-bit binary string is: 000, 001, 010, 011, 100, 101, 110, and 111 (the number of possible unique strings is equal to 2^n, where n is the number of bits). This same space could also be spanned by a crossover operator with only two linearly independent strings—i.e., two strings with different allele at each locus. For instance, consider the pair of strings 000, and 111 in figure 23. Using this pair of strings as the parents, figure 23 shows the possible offspring obtainable from crossover at different points (the boxes in dark outline in figure 23 enclose the binary bits that have been crossed-over).

A, B and C are three unique pairs of offspring obtainable from crossover at different sites. Note that the creation of B requires a two point crossover, unlike that of A and C which only requires a single point crossover. It is clear from this example that a crossover operator alone (if we allow for two-point crossover) can indeed create a complete set of truly unique strings starting from only two linearly independent strings. In this case, it takes only three unique crossovers to produce a complete set of strings which span the entire space. Of course the number of times we need to crossover will vary with the length of the string. In general the number of unique crossovers needed to generate a complete set of totally unique binary strings starting from just two linearly independent strings, is equivalent to $(2^{n-1}-1)$ where n is the number of bits in the string.

We have just seen that the key condition necessary for the crossover operator to span the space is to have a set of linearly independent strings at the start. When this condition is not satisfied, the crossover operator can only access a subset of the total space, and when this is the case, there is no guarantee that a crossover operator will be able to find the optimal string(s) for it(they) may lie outside the spaces which are accessible by the crossover operator. Fortunately GAs have more to offer than just crossover. The aforementioned constraint can be removed with a mutation operator. In a mutation, information held in genes are altered at random. In a binary string, this amounts to switching one or more bits in a string from 0 to 1 or vice versa.

Continuing with the previous example, if the starting strings had been 000 and 110, a mutation operator could step in, and, with some probability, alter either the first string to 001 or the second string to 111, and in doing so create a set of unique strings to satisfy the above condition. Although this discussion has not been mathematically rigorous, it should be intuitively clear that the combined operation of crossover and mutation is indeed capable of searching the entire solution space. Therefore there should be no doubt as to the effectiveness of a GA as a search algorithm.

In most applications, the population of strings evolved in a GA is rather large and therefore will be likely to contain at least two linearly independent strings. Hence, mutation operators are not applied frequently. The probability for mutation is usually set to less than 0.01. Another good reason for keeping the mutation probability low is to improve the final convergence behavior. If the mutation probability is set too high, the frequent perturbation will not allow the strings to settle down at the correct solution(s). On the contrary, crossover is applied more frequently, usually with a probability of about 0.7 but may be higher if necessary. Frequent crossovers are desirable as they ensure that the entire solution space is searched thoroughly for good solutions.

However, what we have established so far with the searching ability of the combination of crossover and mutation (i.e., the ability to span the entire solution space) is merely a basic requirement of any useful search strategy. It is well known that any random search strategy is also capable of searching the entire solution space with some probability. So the more pragmatic question here is whether a GA is any better than a random search strategy. This question relates to the efficiency of a GA and will be discussed in details in the next section when we discuss the Schema Theorem.

To sum up this section, GAs are search and optimization algorithms based on natural genetics. The four key steps in a GA are: 1) encoding of initial trial solutions (search objects), 2) fitness evaluation, 3) selection for reproduction, and 4) crossover and mutation. In its search for better solutions, a GA repeats the last three of these four steps until an adequate solution has been found.

2.6 The Schema Theorem

The Schema theorem was constructed by Holland(1975) to make a statement about the efficiency of a GA as an optimization algorithm. This theorem asserts that in a GA the genetic makeup of a population of individuals will evolve towards that of the fitter individuals at an exponential rate, or in other words, tentative solutions will improve at an exponential rate. Goldberg (1989) has presented an excellent discussion of the Schema Theorem, and the discussion here will primarily follow his[44]. In relation to the previous discussion, we will stay with the binary representation, that is, we will consider only strings which are constructed over the binary alphabet V = {0, 1}. We further define a schema H taken from the three-letter alphabet W = {0, 1, *}. where '0' and '1' are the usual binary bits and '*' represents a wildcard, or "a don't care" symbol. Schemata are simply patterns or similarity templates which we can

[44] See also the original work of Holland (1975). For a quick overview see Mitchell (1995). Vose (1991) has a generalization of schema and genetic algorithms.

generalize from a collection of strings in a population. For example, consider a collection of strings 00110, 00111, 01110, and 01111. A possible schema for this population is H = 0*11*. Some other possible schema for the same set of strings are 0****, and ***1*.

A useful way to characterize a schema is to define its order and defining length. The order of a schema H, denoted by O(H), refers to the number of fixed positions present in the template. For the above example of 0*11*, the order is 3 (symbolically, we represent it as O(0*11*)=3). The defining length of a schema H, denoted by δ(H), is the number of bits between the first and last specific string position. For example, the schema 0*11* has defining length δ = 3 because the last specific position is 4 and the first specific position is 1. When there is only one specific bit like in schemata 0**** and ***1*, the defining length δ is equal to zero.

Having established convenient notations for describing the schemata, we can now focus on analyzing the net effect of the reproduction, crossover and mutation operators on the dynamics of schemata. Let A(t) represents a population of binary strings at time t, and m(H,t) represent m examples of a particular schema H contained within A(t) at time t. Since reproduction involves copying strings according to their fitness values, if we denote the fitness of the i^{th} string by f_i, the probability that the i^{th} string will be selected for reproduction is given by $p_i = f_i \Big/ \sum_{i=1}^{N} f_i$. Consequently, after we have selected a new population of size N with replacement from the initial population, we should on average have m(H, t+1) = m(H,t) $N \; f(H)/\sum f_i$

representatives of the schema H in the new population at time t+1 where $f(H)$ is the average fitness of the strings representing schema H at time t. It is instructive to rewrite this equation in term of the average fitness of the entire population, $\overline{f}$, where $\overline{f} = \Sigma f_j / N$. With this substitution we obtain:

(2.4) $$m(H, t+1) = m(H, t)\ \frac{f(H)}{\overline{f}}$$

The effect of the reproduction operator is now obvious. Under reproduction, a particular schema grows in direct proportion to the ratio of the average fitness of the schema to the average fitness of the population. As a result, schemata with fitness values above the population average will be more represented in the next generation, while schemata with fitness values below the population average will be less represented. Keep in mind that the same phenomenon applies simultaneously to every schema H contained in a particular population A. As such, all the schemata in a population will grow or decay according to their schema averages under the operation of reproduction alone. In summary, above-average schemata grow and below-average schemata die off.

Consider now the growth of a particular schema H which remains above the average fitness by an amount $c\overline{f}$ with c is a constant. The difference equation describing the growth of this schema would then be:

(2.5) $$m(H, t+1) = m(H, t)\ \frac{\overline{f} + c\overline{f}}{\overline{f}} = (1+c)\ m(H, t)$$

However, m(H,t) can also be expressed as (1+c)m(H, t-1) so that:

(2.6a) $$m(H, t+1) = (1+c)^2\, m(H, t-1)$$

Repeating this backward substitution until t=0 results in:

(2.6b) $$m(H,t) = m(H, 0)\, (1+c)^t$$

We can therefore conclude that reproduction will allocate exponentially increasing (decreasing) numbers of trials to above- (below-) average schemata.

However, reproduction alone does not promote exploration of new regions of the search space as we have discussed in the previous section. In order to encourage exploration of new regions, we need the crossover and mutation operators. To analyze the effect of crossover on the dynamics of schemata, we consider a particular string, A, of length $l = 5$ and two representative schemata of this string:

$$A = 0\,1\,1\,1\,0$$

$$H_1 = *\,1\,*\,*\,0$$

$$H_2 = *\,*\,*\,1\,0$$

Both schemata H_1 and H_2 are represented in the string A. Assume that string A has been chosen for mating and that crossover will take place between position 3 and 4. The effect of this crossover on our two schemata H_1 and H_2 is illustrated below. The separator symbol $|$ is used to mark the crossing site.

$$A = 0\,1\,1\,|\,1\,0$$

$$H_1 = *\,1\,*\,|\,*\,0$$

$$H_2 = *\,*\,*\,|\,1\,0$$

It is obvious that unless string A is crossed-over with another string that is identical to itself, the schema H_1 will be destroyed because the 1 at position 2 and the 0 at position 5 will be placed in different offspring. In contrast, schema H_2 will survive because the 1 at position 4 and the 0 at position 5 will be carried intact to a single offspring. Despite the fact that this example is rather specific, we can see that in general schema H_1 is less likely to survive a crossover than schema H_2 because on average the cut point is more likely to fall between the extreme fixed position. We can calculate the probability that schema H_1 will be destroyed as follows. Note that H_1 has a defining length of 3. If the crossover site is selected uniformly at random among the 4 available sites, then clearly schema H_1 is destroyed with probability $P_d = \delta(H_1)/(l-1) = 3/4$ (or it survives with probability $P_s = 1 - P_d = 1/4$). Likewise, the probability that schema H_2 will be destroyed is $P_d = \delta(H_2)/(l-1) = 1/4$ (and the survival probability is 3/4). The survival probability for a schema H of length l, is in general given by $P_s = 1 - \delta(H)/(l-1)$. However, if crossover is applied at a probability P_c, then the total effect on the survival probability will be:

(2.7) $$P_s \geq \left[1 - P_c \frac{\delta(H)}{\ell - 1}\right]$$

Up till now we have only consider the effect of crossover. There is one more operator we have yet to consider. This is the mutation operator. Recalling that a mutation operator randomly alters a single bit with probability P_m, the survival probability of a single allele should be $(1-P_m)$. For a particular schema to survive a mutation, each of the O(H) fixed positions within the schema must survive. If we multiply the survival probability of each allele by itself O(H) times, we have the survival probability of a schema of order O(H) given by: $P_s = (1 - P_m)^{O(H)}$. For sufficiently small values of P_m the schema survival probability can be approximated by $P_s = 1 - O(H) \cdot P_m$.

If we combined the effects of reproduction, crossover and mutation, the equation describing the growth dynamics of schemata becomes:

$$\text{(2.8)} \qquad m(H, t+1) \geq m(H, t)\ \frac{f(H)}{\bar{f}}\left[1 - P_c \frac{\delta(H)}{\ell - 1} - O(H) \cdot P_m\right]$$

This equation clearly shows that the growth of a particular schema depends primarily on 1) whether the schema is above or below the population average, 2) whether the schema has relatively short or long defining length, and 3) how specific the schema is (that is, how many fixed bits a schema has) as measured by its order, O(H). In general, less specific schemata with above-average fitness values and shorter defining lengths will enjoy exponential growth. This important conclusion is called the Schema Theorem, or the Fundamental Theorem of Genetic Algorithms[45].

At this point, it is worthwhile to remind ourselves that H is only one of the many schemata that are processed by a GA simultaneously. In general Holland's implicit parallelism result shows that for a population of N strings, a GA implicitly processes on the order of N^3 schemata per generation (see Goldberg 1985, and Goldberg 1989, p.20 and 40). When we put this in perspective, a population of 100 strings will enable a GA to process on the order of a million schemata. As the GA

[45] This result has been rigorously proven in theory (see Holland 1970b, 1970c, 1975, and 1987, and Bethke 1981).

processes this vast collection of schemata, short, low order, above-average schemata, that represent better solutions, are given exponentially increasing representations in subsequent generations. A GA is therefore a very efficient search algorithm in comparison to purely random search strategies.

Putting this result together with our earlier illustration of GAs' effectiveness, we can conclude that GAs are indeed very robust search and optimization algorithms. De Jong (1975) has established this point conclusively when he successfully applied GAs to the optimization of complicated functions with the following characteristics: 1) continuous/discontinuous, 2) convex/nonconvex, 3) unimodal/multimodal, 4) Quadratic/nonquadratic, 5) Low-dimensionality/high-dimensionality, and 6) Deterministic/stochastic[46].

Finally, these results have also emphasized two important principles to keep in mind when designing a GA code. These are:

> *The user should select a coding so that short, low-order schemata are relevant to the underlying problem and relatively unrelated to schemata over other fixed positions.*
>
> *The user should select the smallest alphabet that permits a natural expression of the problem.* (Goldberg 1989, p.80)

2.7 GA Versus Conventional Optimization and Search Methods

Given that we have asserted that a GA is a robust search and optimization strategy, it is natural to question whether a GA is indeed more robust than conventional search and optimization methods. Goldberg (1989) argued that

[46] See also Grefenstette (1985, 1987) for demonstrations of the robustness of GAs.

conventional optimization and search methods are inferior to GAs because they are based on sequential search and they lack a practical and robust guiding mechanism to direct the search to regions of the space which are more likely to result in fruitful outcomes.

Conventional search and optimization methods in the literature can in general be classified as calculus-based, enumerative and random[47]. A common theme among these conventional methods is the use of a serial approach in their search for the optimal solution. In comparison with the parallel search approach of a GA, which can process on the order of N^3 schemata simultaneously (recall the implicit parallelism result), the serial approach is highly inefficient. Since many practical problems have spaces which are simply too large to be searched sequentially, it is not practical in real applications to rely on the serial approach. For example, even the highly acclaimed enumerative scheme of *dynamic programming* is known to break down on problems of moderate size and complexity[48]. In addition, the localized nature of the search in a serial approach also implies that it has the tendency to be stuck to local optimum rather than the global optimum. Such a problem does not arise in a GA which uses a parallel approach in its search for the optimal solution.

The calculus-based method is the only one among the three alternative conventional approaches that makes use of a guiding mechanism to direct its search. It

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[47] We need to distinguish between a random search strategy and other methods that make use of randomness. For instance, a GA uses randomness but is not what we would called a random search strategy. That is, although it has some degree of randomness, a GA's search is not directionless.

[48] Bellman (1961) called this the "curse of dimensionality".

uses the derivatives of the objective function to guide the search for an optimal solution. For instance, in the "hill climbing" method, search is conducted in the direction in which the slope is the steepest (could be steepest ascent or descent depending on the nature of the problem). But, because it relies on derivatives to direct the search, this approach is not suited for working with objective functions whose derivatives are ill-defined or too difficult to evaluate. This is not a problem in a GA. In a GA, no calculation of derivatives is ever needed to direct the search. All that is needed in a GA is simply an evaluation of the objective function and the manipulation of the tentative solutions using genetic operators, and yet, tentative solutions will evolve towards better solutions at an exponential rate as we have illustrated in the last section.

Neither the enumerative nor the random algorithms employs a guiding mechanism to direct its search towards the best solution. The enumerative algorithm systematically evaluates and compare the objective function at every point in the search space to determine the optimal solution. The random search algorithm searches the entire space at random and continually keep track of the best solution found as they proceed. However, without a guiding scheme, time will be wasted in exploring unfruitful regions of the search space. Thus these two approaches will not be as efficient as a GA.

2.8 The Elements of a Genetic Algorithm

2.8.1 Encoding of Initial Trial Solutions

We have mentioned previously that the most common encoding method is to transform the tentative or trial solutions into binary strings. This transformation is straightforward for numerical value types of trial solutions and needs no further elaboration here. However, some concern has been raised regarding the representational bias in conventional binary representation because the Hamming distance between adjacent values is not constant[49] (Hollstien 1971). Caruana and Schaffer (1988) have found that large Hamming distances in the standard binary representation can result in the search process being deceived hence keeping it from efficiently locating the global minimum. This problem can be resolved by the use of Gray coding which may also help to speed up convergence. As an illustration, the list in Table 1 shows the corresponding relations between Binary-coded integers and Gray-coded integers, for integers ranging from 0 to 15. Notice that for the Gray-coded integers, adjacent integers differ by a single bit (i.e. a hamming distance of 1).

In practice, Gray encoding is initially applied to the entire population of strings (tentative solutions). Decoding (re-encoding) is then carried out systematically at the step right before (after) fitness evaluation. The algorithm for Gray encoding and decoding is straightforward. Let $A_{i,j}$ represents the i^{th} bit in the j^{th} string, and $G_{i,j}$ represents a similarly positioned bit for a Gray coded string. The algorithm is as

[49] Hamming distance is used here to refer to the number of bits that separates two binary integers. For instance if we compare the strings *0101* and *0111*, there is a hamming distance of 1 between them because only one bit needs to be flipped to make the two strings identical.

follows.

(2.9) *For Gray encoding*: if $A_{i,j} \neq A_{i,j-1}$, then $G_{i,j} = 1$, else $G_{i,j} = 0$

For Gray decoding: if $G_{i,j} = 0$, then $A_{i,j} = A_{i,j-1}$, else $A_{i,j} \neq A_{i,j-1}$

In addition to binary coding, there is an increasing interest in alternative coding strategies such as integer and real-valued representations. One argument in favor of these alternative coding strategies is they may be more convenient, more efficient or more natural than binary coding in representing the problem. For instance, Wright (1991) has argued that real-value coding is more efficient as there is no need to convert the chromosomes to phenotypes before each function evaluation. There is also no loss in precision by representing continuous values as discrete binary or other values, and there is greater freedom to use different genetic operators. The use of real-valued encodings is described in details by Michalewicz (1992) and others in the literature on evolution strategies (see for example, Back, Hoffmeister, and Schwefel 1991).

Once a decision is made on the representation, the next step is to generate an initial population of individuals (chromosomes or genotypes). Unless there is prior knowledge on what the approximate solution should be, the initial population is usually generated using a random number generator.

2.8.2 Fitness Evaluation

The fitness of each individual string is related to how well it satisfies the objective function. Hence a straightforward measure of relative fitness would be to compare the raw value of the objective function contributed by each individual. However, on those occasions when the objective function is not a convenient or suitable measure of fitness, a fitness function constructed from the objective function may be used to evaluate the relative fitness of the individuals. A commonly used transformation is

$$F(x_i) = \frac{f(x_i)}{\sum_{i=1}^{N} f(x_i)} \tag{2.10}$$

where N is the population size, x_i is the phenotype value of individual i, and $f(x_i)$ is the fitness function or objective function. This transformation allows offspring to be selected in direct proportion to an individual's relative fitness. However, this transformation is not suited for objective functions with negative values. In such instances, instead of using the actual objective function for $f(x_i)$ in the above equation, a linear transformation of the actual objective function may be used (such as, $af(x_i) + b$; where a and b are appropriately chosen constants). Another concern that may surface is that the range of $f(x_i)$ may be too wide. When this is the case, highly fit individuals in early generations can dominate the reproduction process and may cause the algorithm to result in premature convergence to some sub-optimal solution. Baker (1985) has suggested overcoming this problem by assigning fitness value according to the individuals' ranking within the population rather than basing the fitness on their raw performance. This is accomplished by using an equation similar to the following for calculating fitness.

$$F(x_i) = 2 - MAX + 2(MAX - 1)\left(\frac{x_i - 1}{N - 1}\right) \tag{2.11}$$

MAX is typically chosen to be in the interval [1.1, 2.0] and is used for controlling the selective pressure toward the most fit individuals. x_i is the ranking, or the position in the ordered population of individuals. As an illustration of how this transformation works, consider two individuals – the highest ranked and the lowest ranked – in a population of N individuals. The variable x_i will be equal to N for the highest ranked and be equal to 1 for the lowest ranked. Substituting these values into the equation will give us fitness values of MAX and 2- MAX respectively. The difference in fitness value between the highest ranked and the lowest ranked individual is therefore 2MAX - 2. Using a larger value for MAX will expand this difference and hence put more selective pressure in the directions of the most fit individuals, while using a smaller value for MAX will do the opposite.

2.8.3 Selection for Reproduction

This step controls the number of offspring that each individual will contribute to the new generation. The idea here is to allow more fit individuals to contribute more offspring than the less "fit" individuals. This is in essence an artificial version of Darwin's game of the "survival of the fittest". Methods for selecting individuals usually use some form of a "roulette wheel" mechanism to probabilistically select individuals based on their fitness. The Basic Roulette Wheel Selection Method and the Stochastic Universal Sampling method are two commonly used selection techniques.

Basic Roulette Wheel Selection

The goal of these selection techniques is to design a mechanism which will select individuals for reproduction based on their fitness values. A roulette wheel selection method accomplishes this by first dividing a roulette wheel into N sectors (where N corresponds to the total number of individuals) and then assigning to each individual a sector with an area which is proportionate to its fitness (see illustration in figure 24). In Figure 24, we see that individual 5 has the highest level of fitness, as it occupies the biggest sector. The circumference of the roulette wheel is set equal to the sum of all the individual's fitness (denoted as *Sum*). The process of finding an individual for mating involves, first, generating a random number in the interval [0,*Sum*], and then, selecting the individual whose sector spans that random number. This process is repeated until the desired number of individuals have been selected. Note that sampling is done with replacement.

Several variations of this basic roulette wheel selection method have emerged with the sole purpose of minimizing the spread and the bias in the sampling while maintaining or improving the efficiency of the algorithm. Bias is the absolute difference between an individual's actual and expected selection probability. Spread is the range in the possible number of trials that an individual may achieve. A "minimum spread" is the smallest spread that theoretically permits zero bias. Bias is therefore an indicator of accuracy, while the spread is a measure of its consistency. An efficient single-phase sampling algorithm which has a zero bias and minimum spread is the Stochastic Universal Sampling method. This is described next.

Stochastic Universal Sampling (SUS)

In the basic roulette wheel selection method, the process has to be repeated until the desired number of individuals have been selected. In this method, all of the individuals are selected in one step. SUS uses N equally spaced pointers, where N is the desired number of individuals. At the beginning, the population is shuffled randomly and a random number in the interval [0, *Sum*/N] is generated, φ. The N individuals are then chosen by generating N pointers spaced by 1 (i.e., φ, $\varphi+1$, ..., $\varphi+N-1$), and selecting the individuals whose fitness sectors on the roulette wheel span the positions of the pointers.

2.8.4 Crossover and Mutation

The individuals selected in the previous step are then subjected to crossover and mutation. The purpose of crossover and mutation is twofold – to improve the genetic structure of the individuals (or to recover those good genetic materials lost in the process) and to allow for sufficient diversity in the population genetic structure. Crossover involves slicing each chromosome into two or more segments then recombining pieces of different chromosome segments into new chromosomes. Mutation is simply the alteration of one or more bits (genes) in an individual string (chromosome).

Crossover

Several crossover schemes are available. The main difference among the various schemes is in the number of crossover points each allows (for e.g., single point crossover, multi-point crossover (see Spears and De Jong 1991), and uniform crossover (see Syswerda 1989)). The simplest form of crossover is the single-point crossover which is shown in figure 25. Multi-point crossover, illustrated in figure 26, is a straight forward extension of the single-point crossover. In a uniform crossover, a crossover template or mask is used to determine which parent will supply the offspring with which bits. The crossover template has the same length as the chromosome structure and contains binary bits which are created at random. Bits of ones (zeros) mean that genetic material in those positions will be supplied by the first (second) parent (see figure 27). Uniform crossover has been said to reduce the bias associated with the length of the binary representation used and the particular coding for a given parameter set.

These crossover schemes are not appropriate, however, for real-value encoded chromosomes because such an operation will not search the relevant real-valued space efficiently or, worse yet, invalid values may be produced as a result of the operation. Instead of crossover operators, other recombination operators are used for real-value encoded chromosomes (such as an intermediate recombination operator, or a linear recombination operator). An intermediate recombination operator produces offspring according to the following:

(2.12) $$O_i = P_i \bullet \alpha(P_j - P_i)$$

where the Ps are the parents, Os are the offsprings, and α is a scaling factor chosen randomly from a uniform distribution over some interval, usually [-0.25, 1.25] (see Muhlenbein and Schlierkamp-Voosen 1993). A linear recombination operator is a special case of the intermediate recombination operator. The scaling factor, α, is a constant for the linear recombination operator, instead of a random value.

Mutation

Mutation is a random process where one allele of a gene is replaced by another to produce a new genetic structure. In a binary representation, this involves the switching of chosen bit(s) from 1 to 0 or vice versa. In GAs, mutation is applied at random with low probability, typically in the range 0.001 and 0.01, and it modifies elements in the chromosomes. The role of mutation is often seen as providing a guarantee that the probability of searching any given string is always positive and will never be zero. Mutation acts as a safety net to recover good genetic material that may be lost through the action of selection and crossover. With non-binary representations, mutation is achieved by either perturbing the gene values or random selection of new values within the allowed range. In general it has been found that for codings more complex than binary, high mutation rates can be both desirable and necessary (see Tate and Smith 1993, Wright 1991, Janikow and Michalewicz 1991).

As for actually building the new generation, the newly made individuals are usually simply inserted into the new population and the process is repeated until the new population is of the desired size. More elaborate strategies have been devised in which, for instance, an offspring is inserted into the new population only if it is fitter than its parents or if it is sufficiently different from the rest of the population (anticrowding). Several replacement strategies have been proposed to maintain genetic diversity in order to prevent the GA from converging prematurely (Eshelman and Schaffer 1991). Most of these methods depend on a similarity measure between individuals. In the next section, we discuss the application of a GA to find the minimum of a function.

2.9 A Simple Application of a GA

In this Section, we look at how a GA can be used to solve a simple optimization problem. The problem we will consider is the minimization of the function $f(x) = x^2$, where $0 \le x \le 10$. This function is displayed in figure 28. It is clear that the minimum of this function occurs at $x = 0$, and $f(0) = 0$. With such a trivial problem as an example, our intention is of course, not to demonstrate the power of a GA[50]. My purpose here is twofold: 1) to show the structure of a GA computer program and to relate it to the discussion in the previous section, and 2) to make sense of the GA simulation results from the perspective of the Schema Theorem.

This section is divided into two subsections. The first part discusses the key steps in the GA computer program that we have used to solve the above problem—the minimization of $f(x) = x^2$. The second part discusses the results from the simulation.

[50] Besides, as we have mentioned earlier, more definitive and careful experiments confirming the robustness of GAs have already been conducted by De Jong (1975). There is very little incremental value in carrying out such a demonstration here.

2.9.1 A GA computer program

We discuss below a simple MATLAB program for solving this minimization problem. The subroutines in the program are taken from the Genetic Algorithm Toolbox for use with MATLAB developed by Chipperfield et al.(1995) at the University of Sheffield. Rather than going through every step of our GA code[51], we will focus only on the crucial steps and relate it to the discussion in the previous section.

Figure 29 shows the result of the GA simulations. It is clear from the plot that the crossover and mutation operators are very effective as the GA converges very quickly (after about 70 generations, less than 10 seconds running time on a Pentium 120) to the correct solution.

<u>*Explaining The Key Steps In Our GA Program*</u>

1. Initialization of population.

   Chrom = [ones (NIND, NVAR*PRECI)]
   NIND = 20, is the number of individuals in the population
   PRECI = 8, sets the length of each individual string
   NVAR = 1, is the number of variables encoded in each string.

   This statement generates a matrix, ***Chrom***, of size 20x8, containing 20 strings of ones. Each string has a length of 8 bits. Note that we have intentionally set the starting population to a collection of strings of ones, which is the furthest in hamming distance to the correct solution of strings of zeros, to demonstrate the effectiveness and efficiency of the crossover

[51] The GA code is a simple modification of the SGA code in the toolbox.

and mutation operators in searching the solution space. We will see in the next subsection that the population converges rather quickly to the correct solution.

2. Evaluate initial population.

   temp = bs2rv(Chrom, FieldD)
   ObjV = temp * temp

   The first statement uses the routine ***bs2rv*** to convert the binary strings into real-valued decimal numbers and store them as temp (temp is the variable denoted as x in the objective function). ***FieldD*** is the field description. It contains information about the variable and provides an option for Gray coding. For reasons discussed earlier, Gray coding was used in the simulation.

   The second statement calculates the value of the objective function for each decoded string (recall that the objective function is x^2)

3. Assign fitness-value to entire population

   FitnV = ranking(ObjV)

   Here fitness is assessed using the routine ***ranking*** which ranks the individuals and subsequently transforms the relative ranking to fitness values using linear ranking with a selective pressure of 2. The parameter MAX is set to 2. So the equation for fitness calculation reduces to:

   $F(x_i) = 2\left[\frac{x_i - 1}{N - 1}\right]$ where N is equal to 20, and x_i is the ranking that runs from 1 to 20. This implies that the fitness values assigned will follow the following sequence (in order of increasing ranking):

[0.1053 0.2105 0.3158 0.4211 0.5263 0.6316 0.7368 0.8421 0.9474 1.0526 1.1579 1.2632 1.3684 1.4737 1.5789 1.6842 1.7895 1.8947 2.0000]

Note also that in a situation where subsets of strings are identical, the fitness value for each string in each subset of identical strings is determined by the average fitness value of the subset. For instance if the strings ranked 1, 2, and 3 are identical, then the fitness value is : (0.1053+0.2105+0.3158)/3. Ranking in this routine is designed for minimization of objective functions. Hence if we have a maximization problem, we will need to recast the problem as a minimization prior to using the ***ranking*** routine.

4. Select individuals for breeding

   SelCh = select ('sus', Chrom, FitnV, GGAP)

   This step uses the routine ***select*** to pick out individuals for breeding based on the individual's fitness. The method employed here is the Stochastic Uniform Sampling method which was discussed in the previous section.

5. Recombine selected individuals (crossover)

   SelCh = recombin ('xovsp', SelCh, 0.7)

   recombin allows for different crossover operations. In our case, we chose the '***xovsp***', which is the single point crossover. The crossover probability is set to 70%. This routine actually uses a lower level routine which performs a uniform crossover. With appropriately specified options, it will behave like a single point crossover (i.e. the mask will contain a substring of zeros next to a substring of ones).

6. Perform mutation on offspring

   SelCh = mut (SelCh, 0.003)

   Mutation changes the bits in the population of strings and the mutation probability is set to 0.3% in this case.

7. Evaluate offspring

   templ = bs2rv (SelCh, FieldD)
   ObjVSel = templ * templ)

   This is similar to step 2 above.

8. Reinsert offspring into current population

   [Chrom ObjV] = reins (Chrom,SelCh,1,1,ObjV,ObjVSel)

   reins allows two options for offspring to be reinserted back into the population. One option is to allow offspring to replace parents uniformly at random. Another option is to allow offspring to replace least fit parents. Here, the latter option is used. ***ObjV*** and ***ObjVSel*** contain the fitness values of the parents and the offspring respectively.

2.9.2 Simulation Results

Figure 29 shows that the objective function converges very quickly to the correct solution of zero (after about 70 generations which take less than 10 seconds to run on a Pentium 120). Table 2-4 show how the population of strings evolve over time. The population consists of 20 individual strings, and the crossover and mutation probabilities were set to 0.7 and 0.003 respectively. The length of each string, which determines the precision of the binary representation, was set to 8 bits. we also used a generation gap of 0.9, which mean that new individuals totaling 90% of the population are created and reinserted into the population at each generation.

We observed in Section 2.6 that in general, schemata which have fitness values above (below) that of the average fitness of the population should grow (decay) exponentially. In Table 2, we see that at Generation 10, the more fit strings are those which belong to the schema [* * * * 0 * * *] (keep in mind that the purpose is to minimize the objective function, so the more fit strings will have lower f(x) values). Five generations later, at Generation 15, the entire population has converged to the schema [* * * * 0 * * *] confirming the prediction of the Schema Theorem. In Table 3, at Generation 35, note that the more fit schema is the one that has a zero in the last position, i.e. [0 0 * * 0 * * 0]. Just as the Schema Theorem predicted the population again converges to this schema by Generation 55 . At Generation 55, a new schema with better fitness value has been discovered. This schema puts an additional zero in the third position. Following the evolution to Generation 60, we see again that the population has quickly converged to this new schema. This is the same pattern that we see over and over again as we trace the evolution of the population over time. The conclusion we can draw from these observations is exactly what the Schema Theorem has predicted, that is, in a genetic algorithm tentative solutions improve at an exponential rate.

2.10 A Genetic Fuzzy Classifier System

We have argued earlier in this chapter that fuzzy logic allows us to construct much better control systems than we can achieve using conventional mathematical control theory. However, the quality of a fuzzy control system, like most rule-based systems, ultimately relies on a good set of control rules (among other factors). Unfortunately a fuzzy system is not capable of learning these rules on its own. This self-learning ability is not as crucial in applications in which the operating environment is fairly stable and does not evolve over time. But for applications to situations in which the environment is constantly evolving, for instance, our economy, a fuzzy control system with a fixed set of control rules is likely to be less effective.

To overcome this particular limitation of a fuzzy system, various approaches have been developed. One fruitful approach involves the use of a neural network to learn the rules over time. Although this approach has been successful, it has an important disadvantage which we have already discussed back in Chapter 1. In a neural network, the learning that is taking place happens in a black box and it is not transparent to the modeler. For that reason, the modeler will have no intuitive feel as to why the control system may behave in the way it has. In light of this criticism, the GA approach has been proposed. In a Genetic-Fuzzy classifier system, a GA is used to evolve the fuzzy inference rules through a combined process of reproduction, crossover and mutation. The advantage of GA over neural network is it affords complete transparency to the process which is going on. This transparency allows the modeler to check on whether the evolution that is taking place or the rules themselves, is sensible or not, and it in turn provides the modeler an opportunity to take appropriate actions to correct the model whenever it is necessary.

There are two obvious ways in which a GA can be implemented to evolve the fuzzy inference rules. One way is to manipulate the antecedents or consequent in each rule directly. Suppose we have the following two rules (chromosomes):

R1: *If the price indicator is high and the volume indicator is high, then the trade is buy.*

R2: *If the price indicator is low and the volume indicator is low, then the trade is hold.*

Applying crossover can, for instance, result in the formation of the following two rules:

R1′: *If the price indicator is high and the volume indicator is* ***low****, then the trade is buy.*

R2′: *If the price indicator is low and the volume indicator is* ***high****, then the trade is hold.*

Notice that the antecedents in bold have been swapped between the two rules. On the other hand applying mutation to a rule, let's say R1, can give us:

R1″: *If the price indicator is* ***normal*** *and the volume indicator is high, then the trade is buy.*

Another possibility is to use a GA to evolve the parameters that define the fuzzy sets representing the fuzzy antecedents or consequent. Consider the fuzzy set in figure 30. This fuzzy set can be formalized by the following equation:

(2.13) $$\mu(x) = \frac{x}{(1-a)} - \frac{a}{(1-a)}, \qquad a \le x \le 1.0$$

Since the parameter 'a' is just a numerical value, the implementation of a GA to evolve this parameter is straight forward. We have a choice of either casting its value in binary representation or keeping it in its original real-valued representation. The steps involved are similar to what we have described in Section 2.9. The objective function will be determined by what this fuzzy set is to be used for.

In practice, to facilitate easy coding, the linguistic fuzzy rules in a genetic-fuzzy classifier are usually transformed into bit strings. Consider the fuzzy rule base we have constructed in Section 2.3.

Rule 1: If the price indicator is high and the volume indicator is high, then the trade is buy.

Rule 2: If the price indicator is normal then the trade is hold.

Rule 3: If the volume indicator is normal, then the trade is hold.

Rule 4: If the price indicator is low and the volume indicator is high, then the trade is sell.

This set of rules may for instance be represented by the matrix in figure 31. The numbers in the first two columns represent the states of the antecedents, which are the price and volume indicators respectively. The third column captures the state of the consequent which is the trading decision. The fourth column gives the weight assigned to each rule in the rule base, and the last column specifies the type of logical operator in use. Consider the first column. Since the price indicators can have three possible outcomes, that is, "low", "normal", and "high", these can be denoted by "1", "2", and "3" respectively. So a "3" in the first column would mean that the price indicator is "high" while a "2" would imply that the price indicator has been normal. A "0" means that this antecedent is absent from the rule. The same explanation also applies to the second column and the third column. In the second column, since the volume indicator can only be "normal" or "high", we denote these membership functions by "1" and "2" respectively. In the third column, "1", "2" and "3" are used to denote "sell", "hold" and "buy" respectively. The '1s' in the fourth column means that all the rules are given equal weighting. Finally in the last column, a '1' stands for the '*AND*' operator and a '2' stands for an '*OR*' operator. In the next chapter, we will discuss in more depth how we have implemented a genetic-fuzzy classifier system to model the learning behavior of artificial economic agents in our model.

3. MODEL AND EXPERIMENTS

We have argued in Chapter 1 that in order to account for the anomalies and empirical puzzles in real financial markets, we must have an accurate model of the process that determines how price expectations are formed. In particular, we emphasized that such a model must take into consideration the fact that investors will rely on their innate abilities to reason inductively and analyze in fuzzy terms[52]. We subsequently justified that a genetic-fuzzy classifier system can faithfully capture these traits. In this chapter we will discuss in details how we have adapted the genetic-fuzzy classifier system to accomplish this goal. To illustrate that our intuition is a plausible explanation for market anomalies, we populate an artificial stock market with agents who form their expectations using the genetic-fuzzy classifier system and we investigate the implications of this expectations formation mechanism on market dynamics[53]. The basic framework of our artificial stock market is borrowed from a typical neoclassical two-asset market.

Section 3.1 will describe the structure of this artificial stock market in more details. In Section 3.2, we discuss in depth how we have adapted a genetic-fuzzy classifier to model the process that generates price expectations in the market. Section 3.3 will outline the various controlled experiments we have conducted to illustrate that our intuition is indeed plausible.

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[52] To recap, investors have to rely on inductive reasoning because the ill-defined environment they operate in prohibits the use of deductive reasoning. The use of fuzzy notions is necessary because it allows investors to efficiently process the immense amount of information that enters the market.

[53] Our model is based on the Santa Fe Artificial Stock Market model by Arthur et. al (1996, 1997). Another work that investigates similar issues is Beltratti and Margarita (1992).

3.1 The Market Environment

What we have in mind is a neoclassical two-asset market very similar to that of Bray (1982) or Grossman and Stiglitz (1980) except for a little twist; we deviate from these traditional models by allowing agents in our model to form their expectations inductively using a genetic-fuzzy classifier system in the manner we have outlined in Chapter 1.

The only two tradeable assets in the market are a risky stock and a risk-free bond. We assume that the risk-free bond is in infinite supply and it pays a constant interest rate r. But only N units of the risky stock are available, and each pays a dividend of d_t, which is driven by an exogenous stochastic process $\{d_t\}$ not known to the agents. The dividend process is arbitrary; and in the Santa Fe Institute experiments, Arthur et al. (1996, 1997) have considered the following AR(1) process[54]

(3.1) $$d_t = \bar{d} + \rho(d_{t-1} - \bar{d}) + \varepsilon_t,$$

where ε_t is Gaussian, i.i.d., has zero mean, and variance σ_ε^2. The subscript, t, indexed time. We assume that time is discrete and we have an infinite horizon.

There are N heterogeneous agents in the market. These agents form their expectations individually and independent of each other. In other words, they do not communicate their buying or selling intentions to each other. Thus, they will quite

[54] Besides studying the outcomes under this dividend process, part of our experiments also include investigating the consequences of alternative dividend processes on the model's behaviors. Specifically, we have investigated the consequences of including a cyclical drift term as well as a linear growth drift term in the dividend process. More will be said about these alternative processes in Section 3.3.

likely hold different expectations from each other. Other than the heterogeneity in their expectations, these agents are otherwise identical to each other. They all share a similar constant absolute risk aversion (CARA) utility function $U(W) = -exp(-\lambda W)$. At each period, upon observing the information available to them, they will make decisions on their desired holdings of each of the two assets to maximize their utilities.

Assuming that agent i's predictions at time t of the next period's price and dividend are normally distributed with (conditional) mean, $E_{i,t}[p_{t+1} + d_{t+1}]$ and variance, $\sigma^2_{t,i,p+d}$, then agent i's demand, $x_{i,t}$, for holding shares of the risky asset is given by[55]:

(3.2)
$$x_{i,t} = \frac{E_{i,t}[p_{t+1} + d_{t+1}] - p_t(1+r)}{\lambda\sigma^2_{i,t,p+d}},$$

where p_t is the price of the risky asset at time t, and λ is the degree of relative risk aversion. Since total demand must equal the total number of shares issued for the market to clear,

(3.3)
$$\sum_{i=1}^{N} x_{i,t} = N .$$

This last equation closes the model and determines the clearing price, p_t, in equation (3.2).

[55] This optimal demand function is derived from the first order condition of expected utility maximization of agents with CARA utility under the condition that the forecasts follow a Gaussian distribution (see Grossman 1976 for details). But when the distribution of stock prices is non-Gaussian (as we will see in our simulations) the above connection to the maximization of a CARA utility function no longer exists, so in these cases we simply take this demand function as given.

Now let us turn our attention to the timing of the various events in the model. The current dividend, d_t, is announced at the start of time period t, and this is public information. Agents then form their expectations of the next period's price and dividend $E_{i,t}[p_{t+1} + d_{t+1}]$ based on this information and other general information on the state of the market (which includes the historical dividend sequence $\{\ldots d_{t-2}, d_{t-1}, d_t\}$ and price sequence $\{\ldots p_{t-2}, p_{t-1}\}$). Once their price expectations are established, agents will use equation (3.2) to calculate their desired holdings of the two assets. This information is in turn conveyed to a Walrasian auctioneer who then declares a price p_t that will clear the market. The sequence is then repeated. One final thing to keep in mind is that in the process, agents keep track of the forecasting abilities of their genetic-fuzzy classifiers which they have relied upon to generate their price expectations. As agents learn about the forecasting abilities of these classifiers, those unreliable classifiers will be weeded out to make room for classifiers with new and perhaps better rules.

3.2 Modeling the Formation of Expectations

The structure of our genetic-fuzzy classifier is based on the design of the classifier system originally developed by Holland (see Goldberg 1989, Holland and Reitman 1978, or Holland et al. 1989). At the heart of a Holland's classifier system are three essential components: a set of conditional action rules, a credit allocation system (Holland called this the Bucket Brigade algorithm) and a genetic algorithm (GA). The behavior of the system ultimately is determined by its rules. Each rule contains a set of conditions and an action or a combination of actions. Its operation is straightforward. Whenever the prevailing state in the environment matches all the conditions in a rule, the system adopts the actions prescribed in the rule. The function of the credit allocation system is to systematically keep track of the relative effectiveness of each rule in the classifier. This information is in turn used to guide a GA in the invention of new rules and the elimination of ineffective rules. Together they make it possible for the system to learn about the environment and adapt to innovations in the environment.

The genetic-fuzzy classifier we have employed to model expectations formation is a simple modification of the Holland's classifier system. We replace the conventional rules in Holland's classifier with fuzzy rules to create our genetic-fuzzy classifier. These fuzzy rules still use a similar condition-action format as the conventional rules, but they differ from the conventional rules in that the conditions and actions are now described by fuzzy terms rather than precise terms.

Recall that what we want to get out of the system are price expectations. We accomplish this by replacing the "action" part of the rules with a set of forecast parameters. These forecast parameters are then substituted in a linear forecasting equation to generate the price expectations we are after. The forecast equation we have used is:

(3.4) $$E_t(p_{t+1} + d_{t+1}) = a(p_t + d_t) + b\,,$$

where a and b are the forecast parameters to be obtained from the activated rule, and the variables p_t and d_t are the price and dividend at time, t. Therefore, the format of the rules now looks like,

*If **conditions** then **forecast parameters**.*

Here is an example of such a rule,

*If **{price/fundamental value}** is **low**, then a is **low** and b is **high**.*

But before a GA can operate on these rules, we must transform them into bits strings. We use five bits to specify the conditions in a rule. These five bits represent five market descriptors and they include one fundamental factor and four other technical factors. We use another two bits to represent the forecast parameters a and b. Altogether, we use a string of seven bits to represent each conditional forecast rule[56].

3.2.1 Conditions and Forecast Parameters

Specifically, the five market descriptors we have used for the conditional part of a rule are: $p*r/d$, $p/MA(5)$, $p/MA(10)$, $p/MA(100)$, and $p/MA(500)$. The variables r, p and d are the interest rate, price, and dividend respectively. The variable MA(n) in the denominator denotes a n-period moving average of prices. We organize the positions of the five bits so that they refer to the market descriptors in the same order as above[57]. Thus, the first bit reflects the current price in relation to the current dividend and it indicates whether the stock is above or below the fundamental value at

[56] In practice, we have two additional bits to denote the weight of each rule in a Rule Base and the logical connective used in the *conditions* of the rules. The interpretation of these two bits has already been discussed in Section 2.10. In our experiment, we have kept the weights the same and we have used the logical '*OR*' operator in all the rules.

[57] In other words, the five bits for the conditional part of a rule would be arranged according to:
[$p*r/d$ $p/MA(5)$ $p/MA(10)$ $p/MA(100)$ $p/MA(500)$]

the current price. Clearly this is a "fundamental" bit. The remaining four bits, bits 2-5, are "technical" bits which indicate whether a trend in the price is under way. These "technical" bits will be ignored if useless and acted upon if the technical-analysis trend actually emerges.

To transform these market descriptors into fuzzy sets, we need to set up appropriate universe of discourse and decide on the number and type of fuzzy sets to use for each of these market variables. We set the universe of discourse for each of these variables to [0,1][58]. We let the possible states of each market descriptor be represented by a set of four membership functions—two trapezoidal and two triangular fuzzy sets, and we label them as "low", "moderately low", "moderately high" and "high". The shapes and locations of these fuzzy sets along the universe of discourse are as illustrated in figure 32. When we represent these fuzzy sets as bits, they are coded as "1", "2", "3" and "4" for "low", "moderately low", "moderately high" and "high" respectively. A "0" is reserved to record the absence of a fuzzy set. A "0" has the same interpretation as the "#" (don't care) symbol used by Arthur et al. (1996, 1997).

To give an example, if the conditional part of the rule is coded as [0 1 3 0 2], this would mean that $p*r/d$ and $p/MA(100)$ are not present in the conditional part of the rule, and that $p/MA(5)$ are "low", $p/MA(10)$ is "moderately high" and $p/MA(500)$ is "moderately low". In other words, this corresponds to a state in which

[58] When we set the universe of discourse to the interval [0,1], we have implicitly multiplied each of the market descriptors by 0.5. So if a market descriptor is equal to 0.5, it means that market price is exactly equal to the benchmark that is referred to in the market descriptor.

the market price is less than $MA(5)$ but somewhat greater than $MA(10)$ and is slightly less than $MA(500)$. As long as the prevailing state in the market matches the conditions for $p/MA(5)$,. $p/MA(10)$, and $p/MA(500)$, the conditional part of the rule will be fulfilled and this rule will be activated regardless of what the values for $p*r/d$ and $p/MA(100)$ might be (which is why we said that a "0" is like a "don't care" symbol).

However, we need to point out that because these market descriptors are intrinsically fuzzy, the conditions described by them will be likely to match many states in the market. Hence, what really matters is the degree to which each of these conditions is fulfilled and not so much whether each condition is indeed matched or not matched by the prevailing state.

Now we turn to the modeling of the forecast part of the rule. We allow the possible states of each forecast parameter to be represented by five fuzzy sets. The fuzzy membership functions used for this purpose are a Z-shaped function, three Gaussian functions and a S-curve function. These fuzzy sets are labeled as "low", "moderately low", "average", "moderately high", and "high". The universe of discourse for parameter, a and b, are set to [0.65,1.25] and [-12,22] respectively[59]. The shapes and locations of these fuzzy membership functions are as illustrated in figures 33 and 34. When we represent these fuzzy sets as bits, we code them as "1", "2", "3", "4" and "5" for "low", "moderately low", "average", "moderately high", and

[59] These intervals are chosen so that the HREE (homogeneous rational expectation equilibrium) values are centered in these intervals.

"high" respectively. An example will make this clear. If the forecast part of the rule is coded as [2 5], it means that the forecast parameters a is "moderately low" and b is "high". Following from the example above, when we put together the conditions and the forecast parameters, we will get a complete rule which we would code as: [0 1 3 0 2 | 2 5]. In general, we can write it as: $[x_1, x_2, x_3, x_4, x_5 \; |y_1, y_2]$, where $x_1, x_2, x_3, x_4, x_5 \in \{0,1,2,3,4\}$ and $y_1, y_2 \in \{1,2,3,4,5\}$. Although it is not explicit in our notation, we have used the "*OR*" operators as the logical connectives among the conditions. We should therefore interpret the rule $[x_1, x_2, x_3, x_4, x_5 \; |y_1, y_2]$ as:

"If $p*r/d$ is x_1 or $p/MA(5)$ is x_2 or $p/MA(10)$ is x_3 or $p/MA(100)$ is x_4 or $p/MA(500)$ is x_5, then a is y_1 and b is y_2"

3.2.2 Fuzzy Rule Bases As Market Hypotheses

A genetic-fuzzy classifier contains a set of fuzzy rules that jointly determines what the price expectations should be for a given state of the market. We call a set of rules a rule base. Each rule base represents an investor's tentative hypothesis of the market and it is supposed to stand for a complete and consistent belief. This point needs further clarification.

Take for instance a fuzzy rule like "*If* $p*r/d$ *is high than a is low and b is high*". This rule by itself does not make much sense as a hypothesis because it does not specify what the forecast parameters should be for other contingencies, for example the case where $p*r/d$ is "low", "moderately low" or "moderately high". We will need three additional rules to cover these other possible states in order to form a complete belief. For this reason, we have designed the rule base so that each rule base contains four fuzzy rules. In evolving these rules with a GA, care is also taken to ensure that the rules in the same rule base will never be inconsistent with each other[60]. That is, we cannot have rules that share identical conditions and yet suggest different forecast parameters within the same rule base. Figure 35 shows an example of a rule base, coded as a set of four bit strings, that is both complete and consistent.

However we do allow each agent in our model to work in parallel with several distinct rule bases . To be specific, we have allowed each agent in our model to work with three rule bases. The implication of this is that, at any given moment, agents may entertain several different market hypotheses in their minds. Hence, it is quite possible that each agent may derive several different price expectations at any given time. To sort out which of these price expectations to believe, an agent looks at the relative forecast accuracies of these rule bases and act on the one that has recently proven to be the most accurate. Sub-section 3.2.4 discusses how we measure forecast accuracies and calculate the fitness values of rule bases. In the next sub-section, we take a look at a simple example to illustrate how the system works.

3.2.3 An Example

To demonstrate how our genetic-fuzzy expectational system works, consider a simple fuzzy rule base with the following four rules.

If $0.5^* p/MA(5)$ is *low* then a is *average* and b is *moderately high*.

[60] Nonetheless, fuzzy logic does allow for internal inconsistency if we were to compare it to Boolean logic. Recall that we have argued in Chapter 2 that a variable may be a member of both a set and its complement.

If $0.5^* p/MA(5)$ is *moderately low* then a is *moderately high* and b is *high*.

If $0.5^* p/MA(5)$ is *high* then a is *low* and b is *low*.

If $0.5^* p/MA(5)$ is *moderately high* then a is *high* and b is *high*.

Now suppose that the current state in the market is given by p=80, d=10, and $MA(5)$=100. This gives us, $0.5 \times p/MA(5)$=0.4. The response of each rule and the resultant fuzzy sets for the two forecast parameters, given this state of the market, are illustrated in Figures 36-41. In particular, pay attention to the responses for the 1st, 3rd and 4th rules. In these cases, the membership value for those fuzzy sets representing $p/MA(5)$ is zero since 0.4 is outside their domains, consequently the forecast parameters associated with these rules will also have zero membership values. Thus, only the 2nd rule contributes to the resultant fuzzy sets for the forecast parameters a and b. When we defuzzify these resultant fuzzy sets using the Centroid method, we obtain 1.1 and 19.6 for the parameters 'a' and 'b' respectively. This is illustrated in figures 40 and 41. Substituting these forecast parameters into equation (3.4) gives us the forecast for the next period price and dividend of: $E(p+d) = 1.1(80+10) + 19.6 = 118.6$.

3.2.4 Forecast Accuracies and Fitness Values

Forecast accuracy is measured by the inverse of $e^2_{t,i,j}$. The variable $e^2_{t,i,j}$ .is the moving average of squared forecast error and is defined as:

(3.5) $$e^2_{t,i,j} = (1-\theta)e^2_{t-1,i,j} + \theta[(p_{t+1} + d_{t+1}) - E_{t,i,j}(p_{t+1} + d_{t+1})]^2,$$

where θ is a weight (a constant), subscript i and j denote the i^{th} individual and the j^{th}

rule base, and t indexes the time. In each period, agents refer to $e^2_{t,i,j}$ to decide which price expectations to believe and act upon.

The variable $e^2_{t,i,j}$ is also used for two other purposes. First, it is used as a proxy for the forecast variance $\sigma^2_{t,i,j}$ which is needed to solve equation (3.2). This equation tells the agents how many risky shares to hold in each period. Second, it contributes to the fitness measure which is defined as:

(3.6) $$f_{t,i,j} = -e^2_{t,i,j} - \beta s.$$

The parameter β is a constant and s is the specificity. Specificity is the number of bits which are set (i.e., not 0's) in the conditional part of a rule base. The parameter β is introduced to penalize specificity. The purpose is to discourage agents from carrying bits that are superfluous or redundant. Thus, the more specific the conditions are in a rule base, the lower its fitness will be, keeping other things constant. The net effect of this is to ensure that a bit is used only if agents genuinely find it useful in predictions and in doing so introduces a weak drift towards the all 0's configuration. The fitness measure is used to guide the selection of rule bases for 'crossover' and 'mutation' in the GA. A GA creates new rule bases by "mutating" the values in the rule base array, or by "crossover"—combining part of one rule base array with the complementary part of another[61]. In general, the more fit ones will be more likely to reproduce whereas

[61] Note that in our experiment, we do not allow crossover to happen between different agents. Crossover only takes place among the rule bases held by the same agent. We do this so that we can investigate how agents learn by observing only the common information in the market and not by exchanging ideas with each other.

the less fit ones will have higher probability of being eliminated.

3.2.5 *Recapitulate*

Our model begins with a dividend, d_t, announced publicly at time period t. Based on this information and the various moving averages of historical market price, agents generate several different price expectations using their genetic-fuzzy classifiers. They forecast next period's price and dividend ($E_{i,t}[p_{t+1} + d_{t+1}]$) by using the forecast parameters from the rule base that has proven to be the most accurate recently. With this expectation and its variance, they use equation (3.2) to calculate their desired stock holdings. This information is then passed on to a Walrasian auctioneer who calculates a price to clear the market. Once the market clears, the next period's price and dividend are revealed and the accuracies of the rule bases are updated.

Learning in the model happens at two different levels. On the surface, learning happens rapidly as agents experiment with different rule bases and over time discover which rule bases are accurate and worth acting upon and which should be ignored. At a deeper level, learning takes place on a slower time scale as a GA from time to time discards unreliable rule bases to make room for new ones through crossover and mutation. The new, untested rule bases that are created from time to time will not cause disruptions because they will be acted upon only if they prove to be accurate. This avoids brittleness and provides what machine-learning theorists call "gracefulness" in the learning process.

3.3 Experiments

This section describes the controlled experiments we have conducted to demonstrate that our intuiton about the roots of market anomalies and empirical puzzles in real financial markets is plausible. In these experiments, we kept almost all of the model's parameters the same so that comparisons can be made of the market outcomes using the model under identical conditions with only controlled changes. The primary control parameter is the learning frequency.

Learning frequency refers to the frequency at which a GA is invoked in the model. When the learning frequency is high, a GA is invoked more frequently and agents will revise their rule bases more often. On the contrary, when it is low, a GA is invoked less often, so agents will revise their rule bases at a slower pace. Recall that agents are not able to use deductive reasoning to shape their price expectations. Instead, they use inductive reasoning which basically amounts to formulating tentative hypotheses and testing these hypotheses again and again in the market. Under such a scheme, it is intuitively clear that the learning frequency will play a key role in determining the structure of the rule bases and how well the agents are able to coordinate their price expectations. When the learning frequency is high, agents will be revising their beliefs quite frequently so they will be unlikely to have adequate time to fully explore whether their market hypotheses are consistent with those belonging to the other agents. At the same time, if agents revise their hypotheses at shorter horizon, their hypotheses will also be likely to be based on the transient shorter horizon features of the time series of market variables. These factors together make it difficult for agents to converge on an equilibrium price expectation even if it is present. In contrast, when the learning frequency is low, agents will have more time between revising their rule bases to explore their hypotheses. Furthermore, their hypotheses will also tend to be based on the longer horizon features in the time series of market variables. Consequently, agents are more likely to locate an equilibrium price expectation if it is present in the market.

In our core experiments (see description below) we used the dividend process given by the AR(1) process we have presented as equation (3.1) in Section 3.1. In addition to these core experiments, we have also investigated the impacts of an alternative dividend process on market outcomes. We are interested in investigating whether agents in our model are able to learn and adapt in an environment that exhibits regular patterns. In particular, we looked at an alternative variation of the AR(1) dividend process that exhibits cyclical behaviors. This was accomplished by adding a cyclical drift term to the AR(1) process. We intentionally set the period of the cycle so that only the "slow learning"[62] agents will have the opportunity to observe a complete cycle of the dividend process between revising their rule bases. The "fast-learning" agents on average will not have the opportunity to observe the complete cycle of the dividend process between revising their hypotheses. We then studied the impacts of the frequency of learning on market outcomes under this alternative dividend process.

3.3.1 Core Experiments

What we called the core experiments are the controlled experiments that have

[62] The slow (fast) learning agents are those who revise their hypotheses less (more) frequently.

been conducted by Arthur et al. (1997). We do this so that we will have some benchmarks to compare our results. In these experiments, the only parameter that changes is the learning frequency. The model's parameters that are common to all these experiments are tabulated in Table 5. We conducted two sets of experiments; one for a learning frequency, $k = 250$, where the agents learn on average once every 250 time periods and the other for a learning frequency, $k = 1000$, where they learn on average once every 1000 periods. We will follow Arthur et al. (1997) in referring these two cases as "fast learning" and "slow learning". In these experiments, learning takes place asynchronously for the agents. In other words, not all the agents in the model will update their rule bases simultaneously.

We began with a random initial configuration of rules and we ran each experiment for 200,000 periods to allow asymptotic behavior to emerge if it is present. Subsequently, starting with the configuration attained at t = 200,000 we ran an additional 10,000 periods to collect the data for statistical analysis. We repeat the simulations 20 times under different random seeds to collect cross-sectional statistics.

3.3.2 Experiment With An Alternative Dividend Process

We also investigated the impacts on market outcomes of an dividend process with the following specification:

$$d_t = 4 * Sin(0.0065 \cdot t) + \bar{d} + \rho(d_{t-1} - 2 * Sin(0.03 \cdot t) - \bar{d}) + \varepsilon_t \quad (3.7)$$

The added cyclical drift term will cause the dividend to oscillate with a magnitude of +/- 4 from its mean. The period of a complete cycle is about 970 time periods. Thus, those agents who are learning at a frequency, k =1000, will on average observe about one complete cycle before they revise their beliefs. In contrast, those agents who are learning at the higher frequency of k =250, will on average observe only about 1/4 cycle before they revise their beliefs.

4. RESULTS

Simulation results from our experiments show that our model is able to generate behaviors that bear strong resemblance to many of the anomalies that have been observed in real financial markets. We discuss these results in the following sections. For ease of exposition, we will refer to the high learning frequency experiment and low learning frequency experiment as the "fast learning" and "slow learning" cases respectively. In addition, we will let REE stands for Rational Expectation Equilibrium.

4.1 Statistical Analysis and Times Series Behaviors

This section looks at the behaviors of market variables averaged over the 20 runs.

4.1.1 Asset Price and Return

Figure 42 and 43 present snapshots of observed price behavior over typical windows for both experiments. These graphs present the price series over a shortened window, so that the visual relation between the market price and the REE price is not obscured by the compression necessary when presenting the entire history. These graphs seem to suggest that the market price is more volatile than the REE price.

To make a more precise statement, we compute the mean and standard deviation for the market price across the 20 runs for both sets of experiments and we present this result in the first two rows in Table 6. Judging from the standard deviation alone, it is clear that the market price in both sets of our experiments are more volatile than the REE price, hence confirming our observation above[63,64]. We also noted that the market price in the fast learning case is more volatile than the market price in the slow learning case. The higher volatility in asset price in the former case can be attributed to the more frequent revision of rules by agents in this set of experiments. Another reason is that the rules in the fast learning case are more likely to be based on the transient shorter horizon features in the time series of market variables. This then makes it necessary for the agents to employ different rule bases to form their expectations at different times. This regular switching in the agents' beliefs can give rise to higher volatility because they need time to adapt to the changes. This effect is more pronounced in the experiment with the cyclical dividend process. We will say more about this below.

The remaining rows in Table 6, except for the last row, looks at the behavior of the residual series (ε_t) obtained from regressing the market price and dividend as follows.

(4.1) $$p_{t+1} + d_{t+1} = a + b(p_t + d_t) + \varepsilon_{t+1}$$

We know that in the homogeneous REE, the residual series should be independent and identically distributed as N(0,4), for the dividend process we have used[65]. This means that the theoretical standard deviation for ε_t should be 2. Furthermore, under a

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[63] We estimated the standard deviation of the REE price to be 5.4409.

[64] For evidence on volatility of market price and related tests, see Leroy and Porter (1981a, 1981b) and Shiller (1981). Shiller (1988) is a discussion of the volatility debate..

[65] Note that $\sigma^2_{p+d} = (1+f)^2\sigma^2_\varepsilon = [\rho/(1+r-\rho)]^2\sigma^2_\varepsilon$. This result is derived in Appendix A. This equation will give us a variance of 4 when we substitute into this equation the parameter values listed on Table 5.

gaussian distribution its kurtosis should be zero. We compare these theoretical results to those in the third and fifth rows in Table 6. It is apparent from the values in the third row that the residual series from both sets of experiments are more volatile than their theoretical counterpart. The fifth row shows that that the residuals from the fast learning experiment exhibit slightly excess kurtosis. Although this is consistent with the fact that real asset returns are leptokurtotic, the magnitude is still smaller than those for daily asset returns.

To facilitate comparison with real data, we present summary statistics for Disney, Exxon, IBM and Intel in Table 7.[66] These results were computed from daily data over the last five and a half years, from January 1993 to June 1998. It is obvious from the third row in Table 7 that the magnitude of excess kurtosis is much larger in these data.

The sixth row in Table 6 looks at the autocorrelation in the residuals. This value will tell us if there are any linear structure remaining in the residuals. Our result shows that there is little autocorrelation remaining. This corresponds to the low autocorrelations for actual stock returns presented in the fourth row of Table 7.

Several authors have shown that security returns exhibit conditional time-varying variability (for instance, Engle 1982, Bollerslev 1986, Bollerslev, Chou and Kroner 1992, Glosten, Jaganathan, and Runkle 1994, Nelson 1991). We therefore test for ARCH dependence in the residuals in row 7 and 8 on Table 6. We test for this in two different ways. In row 7, we investigate the first order autocorrelation in squared

[66] There stocks were selected to present the results for a variety of the major industries in the economy.

residuals. In row 8, we perform the ARCH LM test proposed by Engle (1982). Both of these tests reveal that there are ARCH dependence in the residuals. However, the effect is more pronounced for the fast learning case. In this case, all of the 20 runs rejected the null hypothesis of "no ARCH" at the 95% confidence level in our ARCH LM tests. In the slow learning case, only 15% of the runs rejected the null. The first order autocorrelation of the squared residuals is a little larger than the slow learning case, and it is barely significant at the 95% confidence level.

The last row in Table 6 compares the mean excess return for the fast learning and slow learning experiments.[67] The mean excess return is higher for the fast learning case with a value of 3.15% as compared with 2.71% in the slow learning case. There is therefore an increase in the equity premium in the fast learning case. Both these values are higher than the estimated value for the REE case which is 2.52%.

4.1.2 Trading Volume

Figure 44 and 45 show snapshots of observed trading volume over a typical window for both the fast learning and slow learning cases. Clearly, trading volume is not zero. As a matter of fact, we find that the volume of trades, on occasions, can be as high as 40% of the total number of shares available in the market[68]. But on average, for the fast learning case, the volume traded is about 2.53% of the total number of shares available in the market. In the slower learning case, the average

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[67] Excess Return is calculated as $\frac{(p_{t+1}+d_{t+1}-p_t)}{p_t}-r_f$.

[68] In the experiments, we have placed an upper bound on the number of shares that can be traded at 10 shares per period, which represents 40% of the number of shares available in the market.

volume traded is 0.1637 shares which is about one fourth the value of the fast learning case. The summary statistics for volume are presented in Table 8.

Figure 46 and 47 plot the volume autocorrelations for both the fast learning and slow learning experiments. In these two plots, the broken lines are one standard deviation away from the continuous line which is the mean. These plots shows that the trading volume is autocorrelated. This result lines up well with the positive autocorrelations usually found in time series of the volume traded for common stocks[69]. To compare, we plot the volume autocorrelations for Disney, Exxon, IBM and Intel in figure 48. The features in these two plots are strikingly similar.

Figure 49, 50 and 51 look at the cross correlation between volume traded and volatility. In figure 49 and 50, the cross correlation is between volume traded and squared residuals. In figure 51, which shows the results for actual stocks, we take the squared returns to be the volatility and we compute the cross correlation between it and the volume traded. We find that volume traded is contemporaneously correlated with volatility for both the fast and slow learning cases but the results is stronger for the former. Again, these results are strikingly similar to those for actual securities presented in figure 51.

4.1.3 Market Efficiency

Figure 52 plots a snapshot of the difference between the REE price and the market price over a typical window. This plot displays periods in which the market appears to be rather efficient and the market price tracks the REE price quite well. But

[69] See Karpov (1987).

this is intersperse with sporadic wild fluctuations where the market price would break away from the REE price and do something different for a short period of time. This is most apparent in the fast learning case. This result implies that the market moves in and out of various states of efficiency. The figure also shows that the market price has a tendency to return near to the REE price. Such behaviors are common in real financial markets.

4.1.4 Dividend Process with Cyclical Drift Term

The dividend process we look at oscillates with a magnitude that is equal to 40% its long run mean value. We have intentionally set the period of the oscillatory term such that only the slow learning agents have the opportunity to observe the complete cycle between revising their rules. Figure 53 displays the behavior of the dividend process. The shaded regions in figure 53 and the next few figures mark the periods for the oscillations. The duration of each period is about 970 time periods.

Our results in general reveal that the learning mechanism used by the agents is quite robust. Agents are able to track the REE price quite well despite the large oscillatory disturbances[70]. Figure 58 plots the difference between the REE price and the market price. It is clear that the slow learning agents perform better than the fast learning agents in tracking the REE price. This is also evident in figures 56 and 57

[70] Under the alternative dividend process, the conventional Rational Expectations Equilibrium (REE) will not make sense because conventional REE is a static concept. Nonetheless, it is intuitively clear that a dynamic equilibrium can exist. This will be an equilibrium in which every agents correctly forecast the ups and downs in the market at each point in time. Our results seem to suggest that this is what the agents are trying to achieve. We should also point out that our calculation of the REE price is still based on the parameters derive under the previous conditions. For ease of exposition, we will continue to refer to it as the REE price although we recognize that it does not make sense to talk about a REE.

which show close up the time series behaviors of the market price relative to the REE price.

When there are persistent oscillatory features in the time series for market price and dividend, agents must employ rule bases that are sensitive to such periodic fluctuations. Agents will most likely have to rely on several different rule bases in order to keep up with the periodic fluctuations. For instance, agents may hold rule bases for the up hill phase, the down hill phase and the turning point phase of the cycle. Figure 54 and 55 display snapshots of the time series behaviors of the forecast parameters '*a*' and '*b*'. The oscillatory behaviors in these time series suggest that the agents systematically rotate the rule bases they have used to form their forecast in order to keep up with the periodic changes in the market.

Figure 59 plots the time series of volume traded. It is interesting to note that trading are clustered near the turning point phases of the cycle. This makes sense because the turning points are where the market price makes the most drastic change during its course. The trading volume in the fast learning case is higher than that for the slow learning case. This is expected because those agents who revise their rule bases frequently tend to focus on the short horizon features in the time series and therefore will be likely to construct expectations that tend to over-estimate the movements of market price, especially at the turning point.

4.2 Long Run Behaviors

This section focuses on the behaviors of some variables over the 200,000 time periods. Figures 60 and 61 show the percentage of bits set for the two sets of experiments. The percentage of bits set refer to the non-zero bits averaged over all the rule bases held by the agents in each period. In the case of the fundamental bits set, we consider only the bits in the first position, and we calculate the fraction of the non-zero fundamental bits over all the available fundamental bits. In a similar fashion, we compute the fraction of technical bits set by taking into consideration only the second, third, fourth and fifth bits in each rule base. Figures 60 and 61 portray the behaviors for the fundamental bits set and the technical bits set respectively. The results show that these bits did not converge to zeros even after a relatively long time. This indicates that there is strong persistence in the use of both fundamental and technical information despite the fact that the market price seems to track the REE price quite well (this is evident in figures 62 and 63)[71]. Keep in mind that these information should have been irrelevant if the agents were in a Homogeneous REE.

Figures 62 and 63 portray the convergence behaviors of the average of each of the forecast parameters—'*a*' and '*b*', in each period. It is clear that the mean values of the parameters approach quite close to the theoretical HREE values of 9.5 and 4.5 for '*a*' and '*b*' respectively.

[71] See Brock, Lakonishok and LeBaron (1992), Sweeney (1986, 1988) and Taylor and Allen (1992) for evidence on technical trading. For evidence on the predictive value of Price-Dividend ratio see Campbell and Shiller (1988a, 1988b). For predictability in the market in general, see Campbell, Lo and MacKinlay (1997). See also the related works of Cutler et. al (1989, 1991), Fama and French (1988), Fama (1991) and Lo and Mackinlay (1988).

5. SUMMARY AND CONCLUSIONS

This dissertation argues that the so called market anomalies can be explained by allowing agents in the model to form their expectations in a manner akin to how investors would form their expectations in real life. In particular, because the environment that investors operate in is ill-defined, they will have to rely on their innate abilities to analyze in fuzzy terms and reason inductively. We showed that these traits can be faithfully captured by a genetic-fuzzy classifier system. We subsequently asserted that we should be able to account for some of the documented anomalies and empirical puzzles by allowing agents in our model to form their price expectations using a genetic-fuzzy classifier. The model we have constructed was indeed capable of replicating some of the anomalies and stylized time series behaviors we have seen in real financial markets.

Although we did not intentionally try to calibrate our market to fit real data, we are pleased that some of the results quantitatively came out to be relatively closed to those in real financial data. We are referring to the results for volume autocorrelations and volume and volatility cross-correlations. However, there are other aspects of our results that are not entirely satisfactory. In particular, we find the kurtosis in the returns series to be too low. We have some idea on how to improve on this aspect of the model. We suspect that it is partly related to how we have set up the fuzzy sets for the conditional part of a rule. During our simulations we have observed that the market indicators tend to vary between the value of 0.4 and 0.6. The location of our fuzzy membership functions along the universe of discourse are not particular sensitive to changes within the range of 0.4 and 0.6. We can improve the sensitivity by adding more membership functions within this range, while at the same time making those existing membership functions operating within this range to be narrower.

Nevertheless, this alone might not be sufficient to generate the rather large kurtosis we see in real data. We think it will be helpful to introduce some means for agents to coordinate their price expectations directly. In a separate paper, Scott and I (see Linn and Tay 1998) have proposed such a model. We develop a model of investor behavior based on endogenous influence through interaction. In that model, the individual trader's choice of which way to trade depends upon the level of uncertainty present in the market, the extent of agreement on the direction of trade reflected in the choices of other traders, and on the extent of price persistence at the time of the decision. We feel that combining the present model with a model like the one we have just described will probably give us the best chance at explaining the huge kurtosis we see in real financial markets. But we are not motivated merely by the end results. The approach we have suggested is also grounded on the behavior we typically see in real markets. That is, investors do not act alone, they do communicate with each other, and to some extent their communication will influence their trading behaviors.

So far, we have only focused on the time series behaviors of a few market variables. As a whole, our model can also account for several anomalies in real market qualitatively. First, our model can give rise to rather active trading. Second, our model seems to support the views of both academician and market traders. Academic theorists in general view the market as rational and efficient. But market traders typically see the market as psychological and imperfectly efficient. In our model, we find that the market moves in and out of various states of efficiency. This is obvious in figure 52. Furthermore, we find that by slowing down the speed of learning, the market can approach the efficiency of a REE. Third, we find evidence of ARCH effects in the returns, low autocorrelation in returns, and persistent technical trading behaviors. All in all, we were able to account for several features in the real markets and our results are consistent with what Arthur et al. have found.

BIBLIOGRAPHY

Alchian, A. A. "Uncertainty, Evolution, and Economic Theory." *Journal of Political Economy* 58 (1950): 211-221.

Arifovic, J. "Learning By Genetic Algorithms In Economic Environments." Ph.D. diss., University of Chicago, 1991.

Arifovic, J. "Genetic Algorithm Learning and the Cobweb Model." *Journal of Economic Dynamics and Control* 18 (1994): 3-28.

Arifovic, J. "Genetic Algorithms and Inflationary Economies." *Journal of Monetary Economics* 36 (1995): 219-243.

Arifovic, J. "The Behavior of the Exchange Rate in the Genetic Algorithm and Experimental Economies." *Journal of Political Economy* 104 (1996): 510-541.

Arifovic, J. and C. Eaton. "Coordination Via Genetic Learning." *Computational Economics* 8 (1995): 181-203.

Arrow, K. J. "Rationality of Self and Others in an Economic System." *Journal of Business* 59 (1986): S385-S398.

Arrow, K. J. and F. Hahn. *General Competitive Analysis*. San Francisco.: Holden-Day, 1971.

Arthur, W. B. "Designing Economic Agents that Act Like Human Agents: A Behavioral Approach to Bounded Rationality." *American Economic Review* 81 (1991): 353-359.

Arthur, W. B. "On Learning and Adaptation in the Economy." Working Paper 92-07-038, Santa Fe Institute, 1992.

Arthur, W. B. "Inductive Reasoning and Bounded Rationality." *American Economic Review* 84 (1994): 406-411.

Arthur, W. B. "Complexity in Economic and Financial Markets." *Complexity* 1 (1995): 20-25.

Arthur, W. B., B. LeBaron, and R. Palmer. "Time Series Properties of an Artificial Stock Market." SSRI Working Paper 9725, University of Wisconsin, Madison, 1997.

Arthur, W. B., J. H. Holland, B. LeBaron, R. Palmer, and P. Tayler. "Asset Pricing Under Endogeneous Expectations in an Artificial Stock Market." Working Paper 96-12-03, Santa Fe Institute, 1996.

Back, T., F. Hoffmeister, and H. P. Schwefel. "A Survey of Evolution Strategies." *Proceedings of the International Conference on Genetic Algorithm* 4 (1991): 2-10.

Baker, J. E. "Adaptive Selection Methods for Genetic Algorithms." *Proceedings of an International Conference on Genetic Algorithms and Their Applications* 1 (1985): 101-111.

Bauer, P., Nouak, S. and R. Winkler. <HTTP://www.flll.uni-linz.ac.at:80/fuzzy /fuzzy.html>, 1996

Baumol, W.J., and R. E. Quandt. "Rules of Thumb and Optimally Imperfect Decision Rules." *American Economic Review* 54 (1964): 23-46.

Bellman, R. E. *Adaptive Control Processes: A Guided Tour.* Princeton, NJ: Princeton University Press,1961.

Beltratti, A. and S. Margarita. "Evolution of Trading Strategies Among Heterogeneous Artificial Economic Agents." Technical Report, Instituto di Ecoomia G. Prato, Universta di Torino, 1992.

Berlin, B. and P. Kay. *Basic Color Terms: Their Universality and Evolution.* Berkeley: University of California Press, 1969.

Bethke, A. D. "Genetic Algorithms as Function Optimizers." Ph.D. diss., University of Michigan, 1981.

Black, F. "Noise." *Journal of Finance* 41 (1986): 529-544.

Blume, L. E. and D. Easley. "Evolution of Market Behavior." *Journal of Economic Theory* 58 (1992): 9-40.

Blume, L. E. and D. Easley. "Rational expectations and rational learning." Working Paper, Department of Economics, Cornell University,1993

Blume, L. and D. Easley. "What Has the Rational Learning Literature Taught Us?." In *Learning and Rationality in Economics*, eds. A. Kirman, and, M. Salmon. Cambridge: Basil Blackwell,1995.

Bollerslev, T. "Generalized Autoregressive Conditional Heteroskedasticity." *Journal* *of Econometrics* 31 (1986): 307-327.

Bollerslev, T, R. Y. Chou, and K. F. Kroner. "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence." *Journal of Econometrics* 52 (1992): 5-59.

Bray, M. M. "Learning, Estimation and the Stability of Rational Expectations." *Journal of Economic Theory* 26 (1982): 318-339.

Bray, M. M. and D. M. Kreps. "Rational Learning and Rational Expectations." In *Essays in Honour of K. J. Arrow*, eds. W. Heller, R. Starr and D. Starett. Cambridge: Cambridge University Press,1986.

Bray, M. M. and N. E. Savin. "Rational Expectations and Equilibria, Learning and Model Specification." *Econometrica* 54 (1986): 1129-1160.

Brock, W. A., Lakonishok, J. and B. LeBaron. "Simple Technical Trading Rules and the Stochastic Properties of Stock Returns." *Journal of Finance* 47 (1992): 1731-1764.

Bullard, J. "Rethinking Rational Expectations." In *Acting Under Uncertainty: Multidisciplinary Conceptions*, ed. George M. Von Furstenberg, 325-354. Norwell, MA: Kluwer Academic Publishers,1990.

Burgess, D., W. Kemption, and R. E. MacLaury. "Tarahumara Color Modifiers: Category Structure Resaging Evolutionary Change." *American Ethnologist* 10 (1983): 133-149.

Campbell, J. Y. and R. Shiller. "Stock Prices, Earnings and Expected Dividends." *Journal of Finance* 63 (1988a): 661-676.

Campbell, J. Y. and R. Shiller. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *Review of Financial Studies* 1 (1988b): 195-277.

Campbell, J. Y., A. W. Lo, and A. Craig MacKinlay. *The Econometrics of Financial Markets*. Princeton, NJ: Princeton University Press, 1997.

Caruana, R. A. and J. D. Schaffer. "Representation and Hidden Bias: Gray Vs. Binary Coding." *Proceedings of the 6th International Conference on Machine Learning* (1988): 153-161.

Cho, I. K. "Perceptrons Play the Repeated Prisoner's Dilemma." Working Paper, University of Chicago, September, 1992.

Chipperfield, A., P. Fleming, H. Pohlheim, and C. Fonseca. *Genetic Algorithm Toolbox For Use With MATLAB*. Department of Automatic Control and Systems Engineering, University of Sheffield, 1995.

Cox, E. *The Handbook of Fuzzy Systems*. New York: Academic Press, 1994.

Cutler, D. M., J. M. Poterba, and L. H. Summers. "What Moves Stock Prices?." *Journal of Portfolio Management* 15 (1989): 4-12.

Cutler, D. M., J. M. Poterba, and L. H. Summers. "Speculative Dynamics." *Review of Economic Studies* 58 (1991): 529-546.

Cyert, R. M. and DeGroot, M. M. "Rational Expectations and Bayesian Analysis." *Journal of Political Economy* 82 (1974): 521-536.

Cyert, R. M. and M. M. DeGroot. *Bayesian Analysis and Uncertainty in Economic Theory*. Totowa, NJ: Rowman and Littlefield, 1987.

Davis, L. *Handbook of Genetic Algorithms*. New York: Van Nostrand Reinhold, 1991.

DeCanio, S. J. "Rational Expectations and Learning from Experience." *Quarterly Journal of Economics* 93 (1979): 47-57.

De Jong, K. A. "An Analysis of the Behavior of a Class of Genetic Adaptive Sytems." Ph.D. diss., University of Michigan, 1975.

De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann. "The Size And Incidence of the Losses from Noise Trading." *Journal of Finance* 44 (1989): 681-696.

De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann. "Positive Feedback Investment Strategies and Destabilizing Rational Speculation." *Journal of Finance* 45 (1990): 379-396.

Demsetz, H. "Rationality, Evolution and Acquisitivenss." *Economic Inquiry* 34 (1996): 484-496.

Dreman, D. *Psychology and the Stock Market: Investment Strategy Beyond Random Walk*. New York: Amaco, 1977.

Dreman, D. *The New Contrarian Investment Strategy*. New York: Random House, 1982.

Driankov, D., H. Hellendoorn and M. Reinfrank. *An Introduction to Fuzzy Control.* New York: Springer Verlag, 1993.

Dubois, D., and H. Prade. *Fuzzy Sets and Systems: Theory and Applications.* New York: Academic Press, 1980.

Easley, D. and N. Kiefer. "Controlling a Stochastic Process With Unknown Parameters." *Econometrica* 56(1989): 963-978.

Edelman, G. M. *Neural Darwinism.* New York: Basic Books, 1987.

Edwards, W. "Conservation in Human Information Processing." In *Formal Respresentation of Human Judgement*, ed. B. Kleinmuntz. New York: Wiley, 1968.

Engle, R. F. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica* 50 (1982): 987-1007.

Eshelman, L. J. and J. D. Schaffer. "Preventing Premature Convergence by Preventing Incest." In *Proceeding of the Fourth International Conference on Genetic Algorithms.* San Mateo: Morgan Kaufmann, 1991

Evans, G. "Expectational Stability and the Multiple Equilibria Problem in Linear Rational Expectations Models." *Quarterly Journal of Economics* 100 (1985): 1217-1234.

Evans, G. "The Fragility of Sunspots and Bubbles." *Journal of Monetary Economics* 23 (1989): 297-318.

Fama, E. F. "Efficient Capital Markets: II." *Journal of Finance* 46 (1991): 1575-1618.

Fama, E. F. and K. R. French. "Permanent and Temporary Components of Stock Prices." *Journal of Political Economy* 96 (1988): 246-273.

Friedman, M. *Essays in Positive Economics.* Chicago, IL: University of Chicago Press, 1953.

Frydman, R. "Sluggish Price Adjustments and the Effectiveness of Monetary Policy Under Rational Expectations." *Journal of Money, Credit, and Banking* 13 (1981): 94-104.

Gaines, B. R. "Stochastic and Fuzzy Logics." *Electronic Letters* 11 (1975): 188-189.

Gaines, B. R., L. A. Zadeh, and H. J. Zimmerman. "Fuzzy Sets and Decision

Analysis." *TIMS/Studies in the Management Science* 20 (1984): 3-4.

Garnham, A. and J. Oakhill. *Thinking and Reasoning*. Cambridge: Blackwell Publisher, 1994

Gennotte, G. and H. Leland. "Market Liquidity, Hedging and Crashes." *American Economic Review* 80 (1990): 999-1021.

Glosten, L. R., R. Jagannathan, and D. Runkle. "Relationship Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks." Northwestern University, Mimeo, 1989.

Goguen, J. A. "Concept Representation in Natural and Artificial Languages: Axioms, Extensions and Applications for Fuzzy Sets." In *Fuzzy Reasoning and its Applications*, eds. E. H. Mamdani and B. Gaines, 67-115. New York: Academic Press, 1981.

Goldberg, D. E. "Optimal Initial Population Size for Binary-Coded Genetic Algorithms." TCGA Report No. 85001, University of Alabama, Tuscaloosa, The Clearinghouse for Genetic Algorithms, 1985.

Goldberg, D. E. *Genetic Algorithms in Search, Optimization and Machine Learning*. Reading, MA: Addison-Wesley, 1989.

Grefenstette, J. J. ed. *Proceedings of the International Conference on Genetic Algorithms and Their Applications*. Hillsdale: Lawrence Erlbaum Associates, 1985.

Grefenstette, J. J. ed. *Genetic algorithms and their applications: Proceedings of the Second International Conference on Genetic Algorithms*. Hillsdale: Lawrence Erlbaum Associates, 1987.

Grether, D. M. "Testing Bayes Rule and the Representativeness Heuristic: Some Experimental Evidence." *Journal of Economic Behavior and Organization* 17 (1992): 31-57.

Grossman, S. J. "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information." *Journal of Finance* 31 (1976): 573-585.

Grossman, S. and J. E. Stiglitz. "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70 (1980): 393-408.

Hersch, H. M. and A. A. Caramazza. "A Fuzzy Set Approach to Modifiers and Vagueness in Natural Language." *Journal of Experimental Psychology: General*

105 (1976): 254-276.

Hinton, G. E. "Mapping Part--Whole Hierachies Into Connectionist Networks." *Artificial Intelligence* 46 (1990): 47-76.

Hogarth, R. M. *Judgement and Choice: The Psychology of Decision*. New York: Wiley, 1980.

Holland, J. H. "Hierarchical Descriptions of Universal Spaces and Adaptive Systems." In *Essays on Cellular Automata*, ed. A. W. Burks. Urbana, IL: University of Illinois Press, 1970a.

Holland, J. H. "Robust Algorithms for Adaptation Set in General Format Framework." *Proceedings of the IEEE Symposium on Adaptive Processes Decision and Control.* XVII (1970b): 5.1-5.5.

Holland, J. H. "Processing and Processors for Schemata." In *Associative Information Processing*, ed. E. L. Jacks, New York: American Elsevier, 1970c.

Holland, J. H. *Adaptation in Natural and Artificial Systems*. Ann Arbor, MI: University of Michigan Press, 1975.

Holland, J. H. "Genetic Algorithms and Classifier Systems: Foundations and Future Directions." *Genetic Algorithms and their Applications: Proceedings of the Second International Conference on Genetic Algorithms* (1987) 82-89.

Holland, J. H. "Genetic Algorithm." *Scientific American* (July 1992): 66-72.

Holland, J. H., K. J. Holyoak, R. E. Nisbett, and P. R. Thagard eds. *Induction: Processes of Inference, Learning, and Discovery*. Cambridge: MIT Press, 1989.

Holland, J. H. and J. Reitman. "Cognitive Systems Based on Adaptive Algorithms." In *Pattern-Directed Inference Systems*. New York: Academic Press, 1978.

Hollstien, R. B. "Artificial Genetic Adaptation in Computer Control Systems." Ph.D. diss., University of Michigan, 1971.

Jacklin, C. J., A. W. Kleidon, and P. Pfleiderer. "Under Estimation Of Portfolio Insurance and the Crash of October 1987." *Review of Financial Studies* 5 (1992): 35-63.

Janikow, C. Z. and Z. Michalewicz. "An Experimental Comparison of Binary and Floating Point Representations in Genetic Algorithms." *Proceedings of the International Conference on Genetic Algorithm* 4 (1991): 31-36.

Kahneman, D., P. Slovic, and A. Tversky. "Judgement Under Uncertainty: Heuristics and Biases." Cambridge: Cambridge University Press, 1982.

Karpov, J. M. "The Relation Between Price Changes and Trading Volume: A Survey." *Journal of Financial and Quantitative Analysis* 22 (1987): 109-126.

Katona, G. "*Psychological Economics*." New York: Elsevier, 1975.

Kay, P. and C. McDaniel. "Color Categories As Fuzzy Sets." Working Paper #44, Berkeley: Language Behavior Research Laboratory, University of California, 1975.

Kempton, W. "Category Grading and Taxonomic Relations: A Mug is a Sort of a Cup." *American Ethnologist* 5 (1978): 44-65.

Keynes, J. M. *The General Theory of Employment, Interest and Money*. London: Macmillan, 1936.

Kiefer, N. M. and Y. Nyarko. "Control of a Linear Regression Process With Unknown Parameters." In *Dynamic Econometric Modeling, Proceedings of the Third International Symposium in Economic Theory and Econometrics*, eds. W. A. Barnett, E. R. Berndt and H. White. Cambridge: Cambridge University Press, 1988.

Kiefer, N. M. and Y. Nyarko. "Optimal Control of an Unknown Linear Process with Learning." *International Economic Review* 30 (1989): 571-586.

Kirman, A. and M. Salmon, eds. *Learning and Rationality in Economics*. Cambridge: Basil Blackwell, 1995.

Klir, G. J. and T. A. Folger. *Fuzzy Sets, Uncertainty, and Information*. Englewood Cliffs, NJ: Prentice Hall, 1988.

Klir, G. J. and B. Yuan. *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. Upper Saddle River, NJ: Prentice Hall, 1995.

Kosko, B. *Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence*. Englewood Cliffs: Prentice Hall, 1992.

Kosko, B. *Fuzzy Thinking: The New Science of Fuzzy Logic*. New York: Hyperion, 1993.

Leroy, S. F. and R. D. Porter. "Stock Price Volatility: Tests Based on Implied

Variance Bounds." *Econometrica* 49 (1981a): 97-113.

Leroy, S. F. and R. D. Porter. "The Present-Value Relation: Tests Based on Implied Variance Bounds." *Econometrica* 49 (1981b): 555-574.

Linn, S. C. and B. E. Stanhouse. "The Economic Advantage of Least Squares Learning in a Risky Asset Market." *Journal of Economics and Business* 49 (1997): 303-319.

Linn, S. C. and N. S. Tay. Investor Interaction and the Dynamics of Security Prices." CFS Working Paper, Finance Division, University of Oklahoma, 1998.

Lo, A. W. and A. C. Mackinlay. "Stock Prices Do Not Follow Random Walk: Evidence From a Simple Specification Test." *Review of Financial Studies* 1 (1988): 41-66.

Lucas, R. E. Jr. "Adaptive Behavior and Economic Theory." *Journal of Business* 59 (1986): 401-426.

Marcet, A. and T. J. Sargent. "The Fate of Systems With Adaptive Expectations." *American Economic Review* 78 (1988): 168-172.

Marcet, A. and T. J. Sargent. "Convergence of Least Squares Learning Mechanism in Self Referential Linear Stochastic Models." *Journal of Economic Theory* 48 (1989a): 337-368.

Marcet, A. and T. J Sargent. "Convergence of Least Squares Learning in Environments With Hidden State Variables and Private Information." *Journal of Political Economy* 97 (1989b): 1306-1322.

Marcet A. and T. J. Sargent. "Least Square Learning and the Dynamics of Hyperinflation." In *Economic Complexity: Chaos, Sunspots, Bubbles and Nonlinearity*, eds. W. A. Barnett, J. Geweke and K. Shell. Cambridge: Cambridge University Press, 1989c.

McLennan, A. "Incomplete Learning in a Repeated Statistical Decision Problem." Working Paper, University of Minnesota, 1987.

McNeill, D. and P. Freiberger. *Fuzzy Logic*. New York: Simon and Schuster, 1993.

Michalewicz, Z. *Genetic Algorithm + Data Structure = Evolution Programs*. North Holland: Springer Verlag, 1992.

Mirman, L., A. Postlewaite and R. Kihlstrom. "Experimental Consumption and the

'Rothschild Effect'." In *Bayesian Models in Economic Theory*, eds. M. Boyer and R. E. Kihlstrom. 1984.

Mitchell, M. *An Introduction to Genetic Algorithms*. Cambridge, MA: MIT Press, 1995.

Moffat, S. "Fuzzy Thinking." *Fortune* (17 December 1990): 173-174.

Muhlenbein, H. and D. Schlierkamp-Voosen. "Predictive Models for the Breeder Genetic Algorithm: I. Continuous Parameter Optimization." *Evolutionary Computation* 1 (1993): 25-49.

Nelson, D. "Conditional Heteroskedasticity in Asset Returns: A New Approach." *Econometrica* 59 (1991): 347-70.

Nelson, R. and S. Winter. *An Evolutionary Theory of Economic Change*. Cambridge: Harvard University Press, 1982.

Nyarko, Y. "Bayesian Rationality and Learning Without Common Priors." C. V. Starr Center for Applied Economics Working Paper 90-45, New York University, 1990.

Nyarko, Y. "Bayesian Learning Without Common Priors and Convergence to Nash Equilibrium." C. V. Starr Center for Applied Economics Working Paper 91-6, New York University, 1991a.

Nyarko, Y. "Learning in Mis-Specified Models and the Possibility Of Cycles." *Journal of Economic Theory*, 51 (1991b) 416-427.

Oden, G. C. "Integration of Fuzzy Logical Information." *Journal of Experimental Psychology (General)* 106 (1977): 565-575.

Palmer, R. G., W. B. Arthur, J. H. Holland, B. LeBaron, and P. Tayler. "Artificial Economic Life: A Simple Model of A Stock Market." Physica D, 75 (1994): 264-274.

Peck, J. and K. Shell. "Market Uncertainty: Correlated And Sunspot Equilibria In Imperfectly Competitive Economies." *Review of Economic Studies* 58 (1991): 1011-1029.

Prescott, E. "The Multiperiod Control Problem Under Uncertainty." *Econometrica* 40 (1972): 1043-1058.

Radner, R. "Existence of Equilibrium of Plans, Prices, and Price Expectations."

Econometrica 40 (1972): 289-304.

Rescher, N. *Induction: An Essay on the Justification of Inductive Reasoning* Pittsburgh: University of Pittsburgh Press, 1980.

Rosch, E. "Natural Categories." *Cognitive Psychology* 4 (1973a): 328-350.

Rosch, E. "On The Internal Structure of Perceptual and Semantic Categories." In *Cognitive Development and the Acquistion of Language*, ed. T. E. Moore. New York: Academic Press, 1973b.

Rosch, E. and C. B. Mervis. "Family Resemblance: Studies in the Internal Structure of Categories." *Cognitive Psychology* 7 (1975): 573-605.

Rothschild, M.. "A Two-Armed Bandit Theory of Market Pricing." Journal of Economic Theory 9 (1974): 185-202.

Rumelhart, D. E., J. L., McClelland, and, the PDP Research Group, eds. "*Parallel Distributed Processing*. Cambridge: MIT Press, 1987

Salmon, M. "Bounded Rationality and Learning: Procedural Learning." In *Learning and Rationality in Economics*, eds. A. Kirman and M. Salmon, 236-274. Cambridge: Basil Blackwell, 1995.

Savage, L. J.. "The Foundations of Statistics." New York: Wiley, 1954

Sargent, T. J. "*Bounded Rationality in Macroeconomics*." Oxford: Clarendon Press, 1993.

Schneider, W., and R. M. Shiffrin.. "Controlled and Automatic Human Information Processing: I. Detection, Search, and Attention." *Psychological Review* 84 (1977): 1-66.

Shastri, L. and V. Ajjanagadde. "From Simple Associations to Systematic Reasoning: A Connectionist Representation of Rules, Variables and Dynamic Bindings Using Temporal Synchrony." *Behavioral and Brain Science* 16 (1993): 417-494.

Shiller, R. J. "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review* 71 (1981): 421-436.

Shiller, R. J. "Stock Prices and Social Dynamics." *Brookings Papers on Economic Activity* 2 (1984): 457-498.

Shiller, R. J.. "The Volatility Debate. *American Journal of Agricultural Economics.*"

70 (1988): 1057-1063.

Shiller, R. J. "*Market Volatility*." Cambridge, Mass: MIT Press, 1989.

Shleifer, A., and, L. H. Summers. "The Noise Trader Approach to Finance. *Journal of Economic Perspectives,* 4 (1990): 19-34.

Simon, H. A.. "*Administrative Behavior*." New York: Macmillan, 1947.

Simon, H. A.. "A Behavioral Model of Rational Choice." *Quarterly Journal of Economics* 69 (1955): 99-118.

Simon, H. A.. "*Models of Man: Social and Rational*." New York: Wiley. 1957.

Simon, H. A.. "Theories of Decision Making in Economics. *American Economic Review* 49 (1959): 253-283.

Simon, H. "From Substantive to Procedural Rationality." In S. J. Latsis ed., Method and Appraisal in Economics, Cambridge: Cambridge University Press, 1976; reprinted in Models of Bounded Rationality, Boston: MIT Press, 1982.

Simon, H. A. "*Models of Bounded Rationality*." Cambridge: MIT Press, 1982

Simon, H. A. "Rationality in Psychology and Economics." In *Rational Choice: The Contrast Between Economics and Psychology*, eds. R. Hogarth and M. Reder, 25-40. Chicago: University of Chicago Press, 1986.

Simon, H. A., M., Egidi, R., Marris, and R. Viale.. *Economics, Bounded Rationality and the Cognitive Revolution*. Vermont: Edward Elgar, 1992.

Sloman, S.A.. "The Empirical Case for Two Systems of Reasoning." *Psychological Bulletin* 119 (1996): 3-22.

Smithson, M. "Multvariate Analysis Using 'And' and 'Or'." *Journal of Mathematical Social Science* 7 (1984): 231-251.

Smithson, M.. "*Fuzzy Set Analysis for Behavioral and Social Sciences*. New York: Springer Verlag, 1987.

Smolensky, P. "On the Proper Treatment of Connectionism." *Behavioral and Brain Science* 11 (1988):1-23.

Soros, G. *The Alchemy of Finance: Reading the Mind of the Market*. New York: J. Wiley, 1994.

Spears, W. M., and, K. A. De Jong. "An Analysis of Multi-Point Crossover." In *Foundations of Genetic Algorithms*, ed. J. E. Rawlins, (1991) 301-315.

Sweeney, R. J. "Beating the Foreign Exchange Market." *Journal of Finance* 41 (1986): 163-182.

Sweeney, R. J. "Some New Filter Rule Tests: Methods and Results." *Journal of Financial and Quantitative Analysis* 23 (1988): 285-300.

Syswerda, S. "Uniform Crossover in Genetic Algorithms." *Proceedings of the International Conference on Genetic Algorithm* 3 (1989): 2-9.

Taylor, M. and H. Allen. "The Use of Technical Analysis in the Foreign Exchange Market." *Journal of International Money and Finance* 11 (1992): 304-314.

Tate, D. M., and A. E. Smith. "Expected Allele Convergence and The Role of Mutation in Genetic Algorithms." *Proceedings of the International Conference on Genetic Algorithm* 5 (1993): 31-37.

Thole, U., H.J., Zimmermann, and P. Zysno. "On the Suitability of Minimum and Product Operators for the Intersection of Fuzzy Sets." *Fuzzy Sets and Systems* 2 (1979): 167-180.

Thrall, R. M., C. H. Coombs, and R. L. Davis (eds.). *Decision Processes*. New York: Wiley, 1954.

Tong, R. M. "Synthesis of Fuzzy Models for Industrial Processess--Some Recent Results." *International Journal of General Systems* 4 (1978): 2-10.

Townsend, R. M. "Market Anticipation, Rational Expectations and Bayesian Analysis." *International Economic Review* 19 (1978): 481-494.

Townsend, R. M. "Forecasting the Forecasts of Others." *Journal of Political Economy* 91 (1983a): 545-588.

Townsend, R. M. "Equilibrium Theory With Learning and Disparate Expectations: Issues and Methods." In *Individual Forecasting and Aggregate Outcomes*, eds. R. Frydman and E. S. Phelps. Cambridge: Cambridge University Press, 1983b.

Tversky, A. and D. Kahneman. "Judgement Uncertainty: Heuristics And Biases." *Science* 185 (1974): 1124-1130.

Varian, H. R. "What Use is Economic Theory?" Working Paper, Department of

Economics, University of Michigan, 1993.

Viale, R. "Cognitive Constraints of Economic Rationality." In *Economics, Bounded Rationality and the Cognitive Revolution*, eds. H. A. Simon., M. Egidi, R. Marris, and R. Viale, 174-193. Brookfield, Vermont: Edward Elgar, 1992.

Vose, M. D. "Generalizing the Notion of Schema in Genetic Algorithms." *Artificial Intelligence* 50 (1991): 385-396.

Woodford, M. "Learning to Believe in Sunspots." *Econometrica* 58 (1990): 277-307.

Wright, A. H. "Genetic Algorithms for Real Parameter Optimization." In *Foundations of Genetic Algorithms*, ed. J. E. Rawlins. San Mateo: Morgan Kaufmann, 1991.

Yamakawa, T. "Stabilization of an Inverted Pendulum by a High-Speed Fuzzy Logic Controller." *Hardware System, Fuzzy Sets and Systems* 32 (1989): 161-180.

Yasunobu, S., and S. Miyamoto. "Automatic Train Operation System by Predictive Fuzzy Control." In *Industrial Applications of Fuzzy Control*, ed. M. Sugeno. New York: North Holland, 1985.

Zadeh, L. A. "From Circuit Theory to System Theory." *Proceedings of the Institute of Radio Engineers* 50 (1962): 856-865.

Zadeh, L. A. "Fuzzy Sets." *Information and Control* 8 (1965): 338-353.

Zadeh, L. A. "A Fuzzy-Set-Theoretic Interpretation of Linguistic Hedges." *Journal of Cybernetics* 2 (1972): 4-34.

Zadeh, L. A. "Outline of a New Approach to the Analysis Of Complex Systems and Decisions Processes." *IEEE Transactions on Systems, Man, and Cybernetics* 1 (1973): 28-44.

Zimmermann, H. J. and P. Zysno. "Latent Connectives in Human Decision Making." *Fuzzy Sets and Systems* 4 (1980): 37-51.

FIGURES

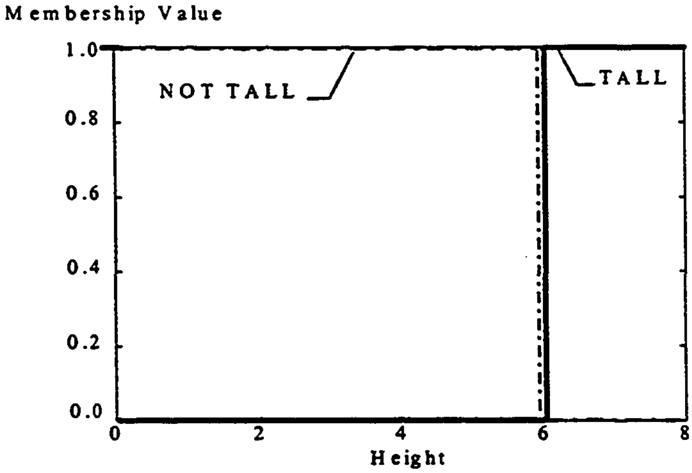

Fig. 1. Classical Sets of Tall and Not-Tall person

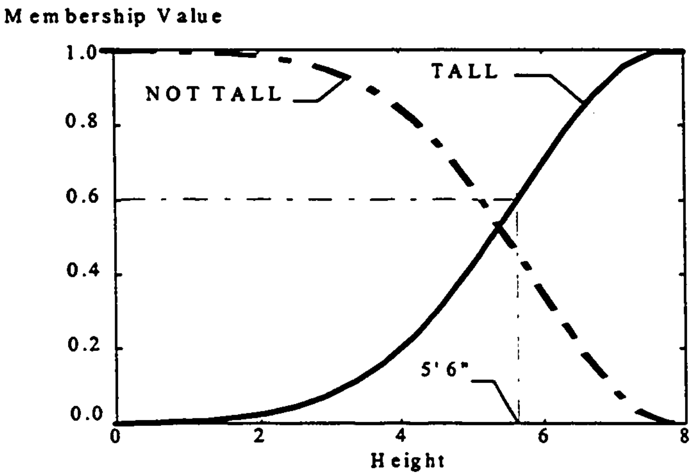

Fig. 2. Fuzzy Sets of Tall and Not-Tall person

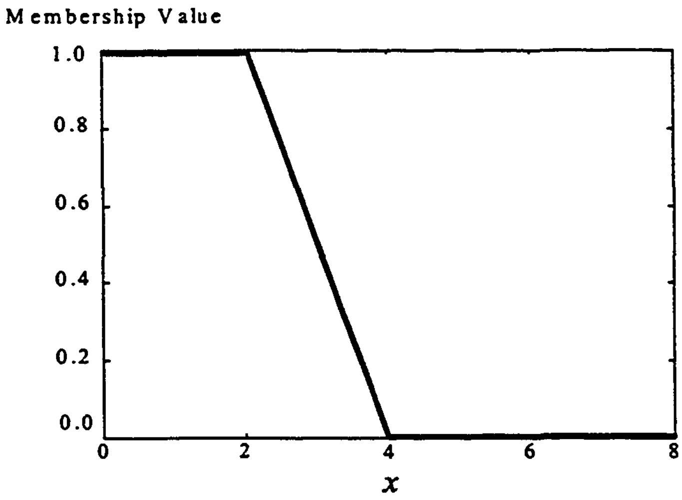

Fig. 3. Fuzzy Set A

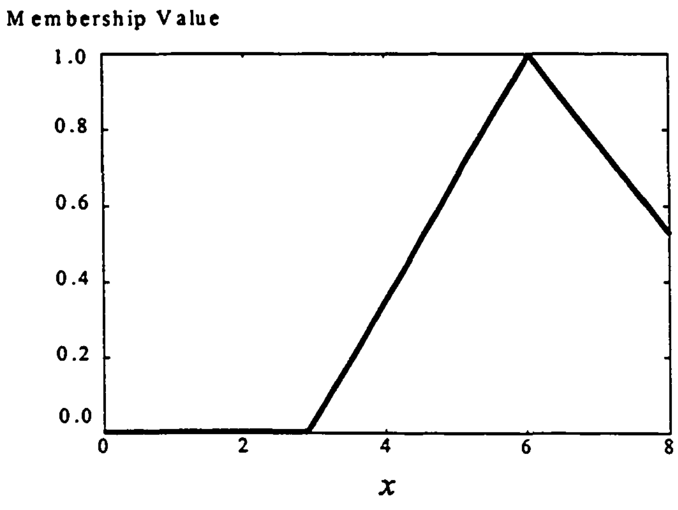

Fig. 4. Fuzzy Set B

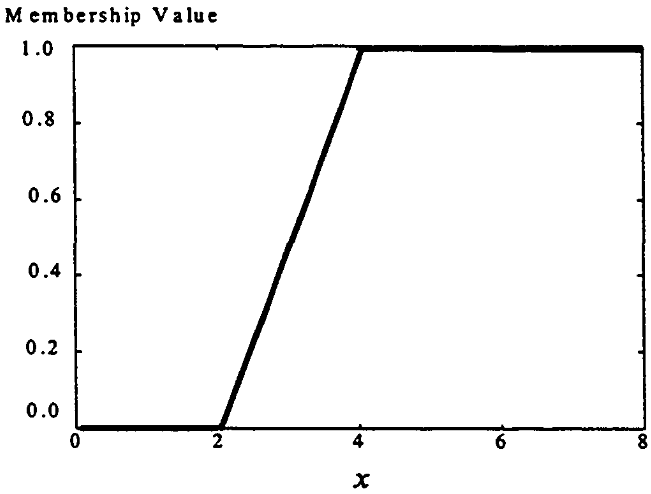

Fig. 5. Fuzzy Set for **Not-A** ($\neg$A)

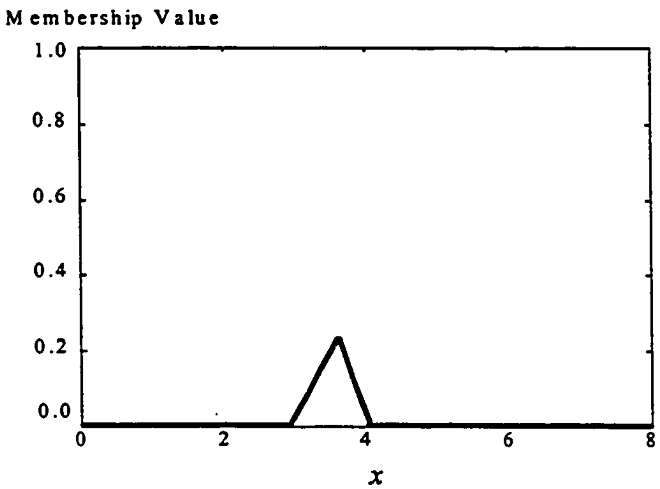

Fig. 6. Fuzzy Set for **Intersection**: A $\cap$ B (A **AND** B)

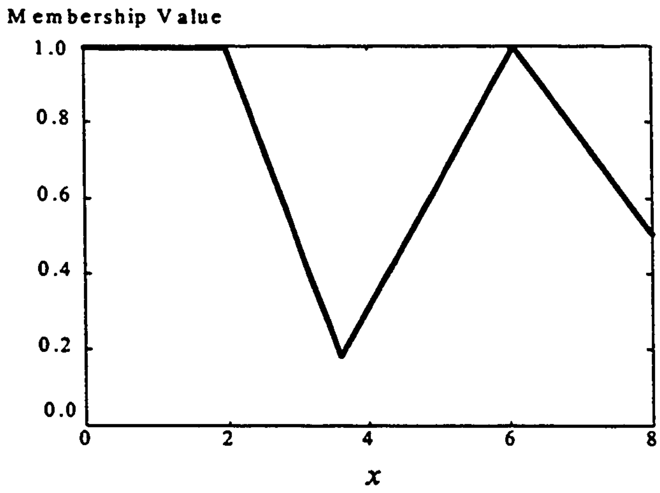

Fig. 7. Fuzzy Set for **Union** : A∪B (A **OR** B)

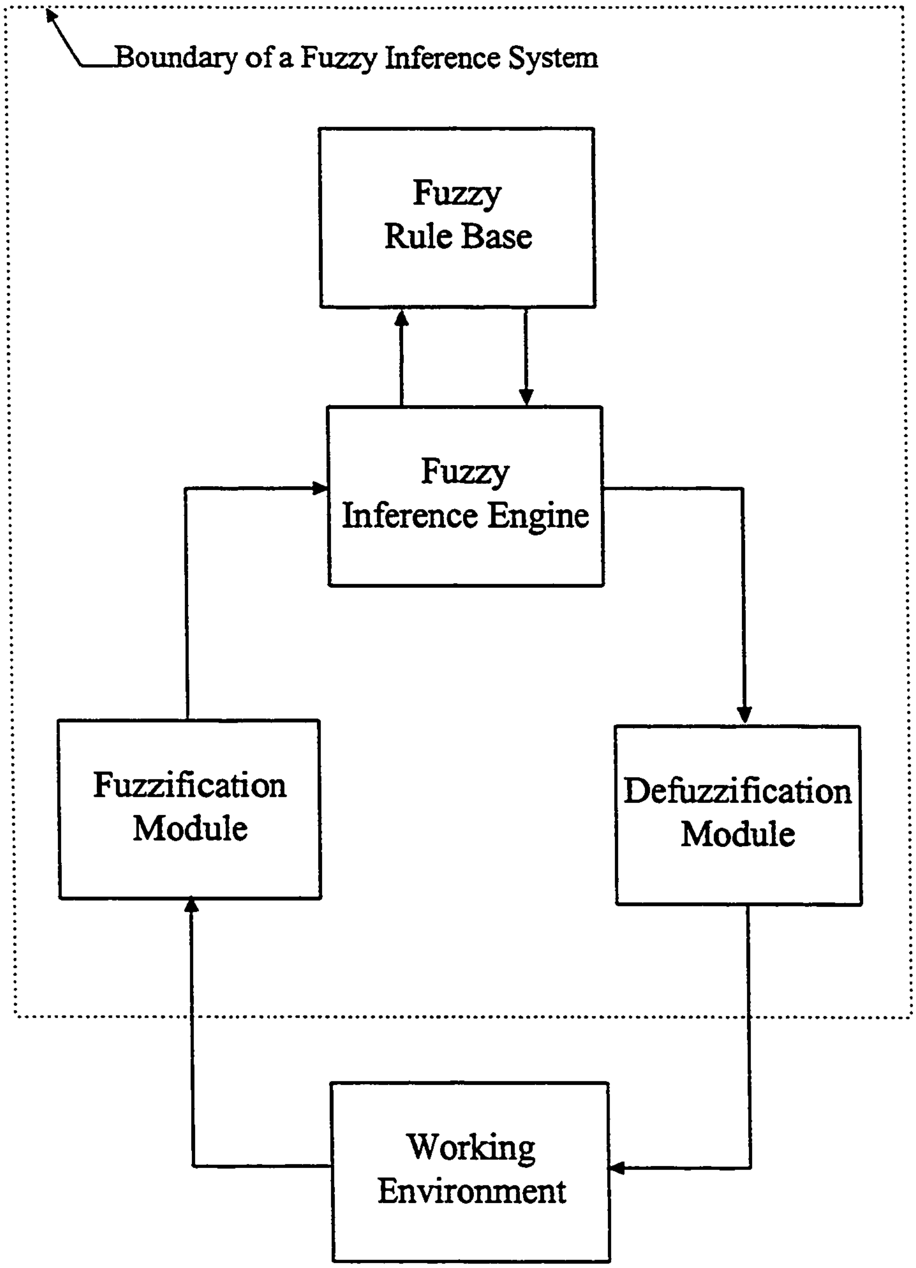

Fig. 8 A Schematic Diagram of A Fuzzy Inference System

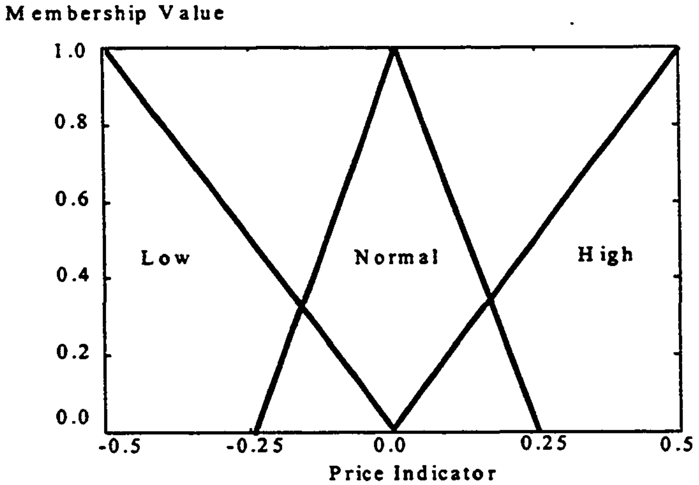

Fig. 9. Fuzzy Sets for Input 1: Price Indicator

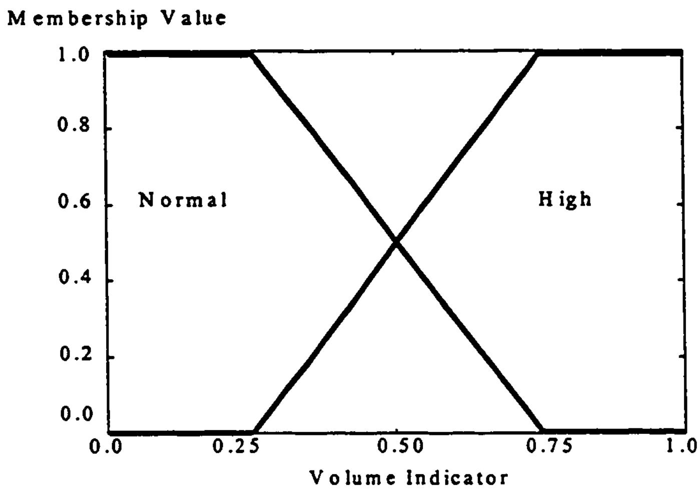

Fig. 10. Fuzzy Sets for Input 2: Volume Indicator

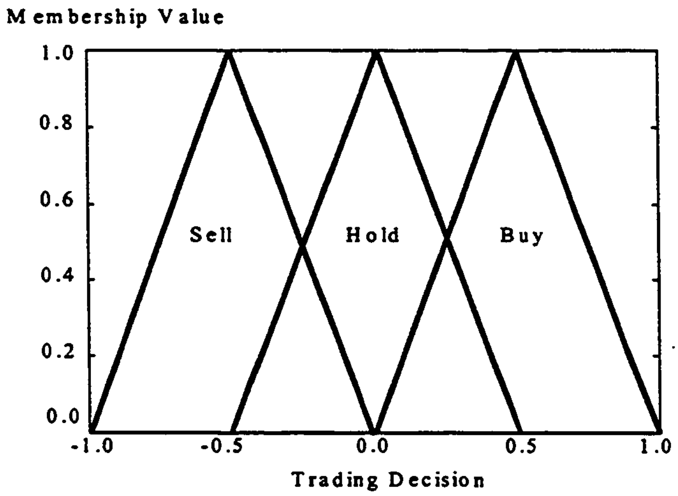

Fig. 11. Fuzzy Sets for Output: Trading Decision

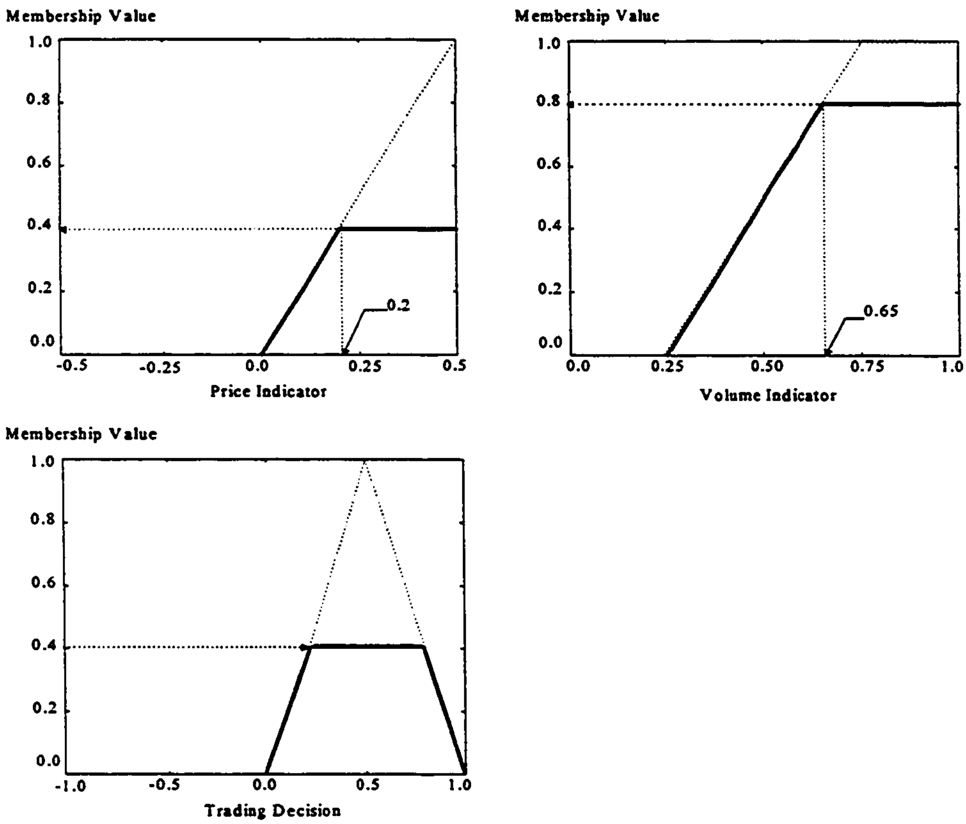

Fig. 12. Response of Rule 1

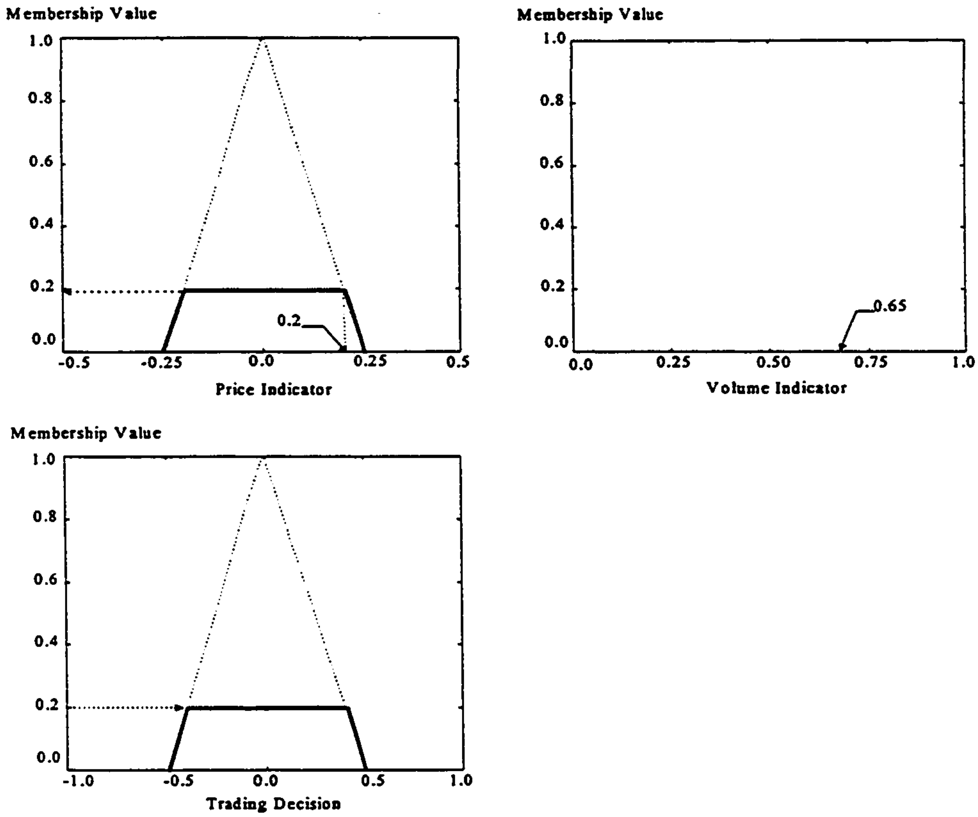

Fig. 13. Response of Rule 2

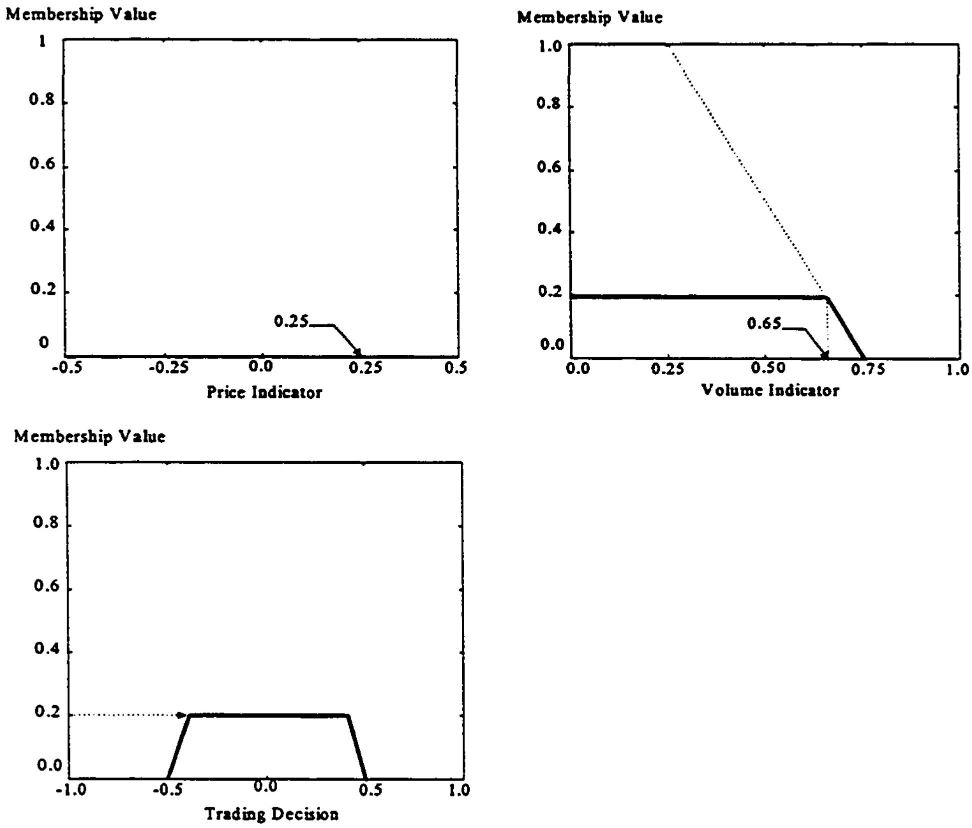

Fig. 14. Response of Rule 3

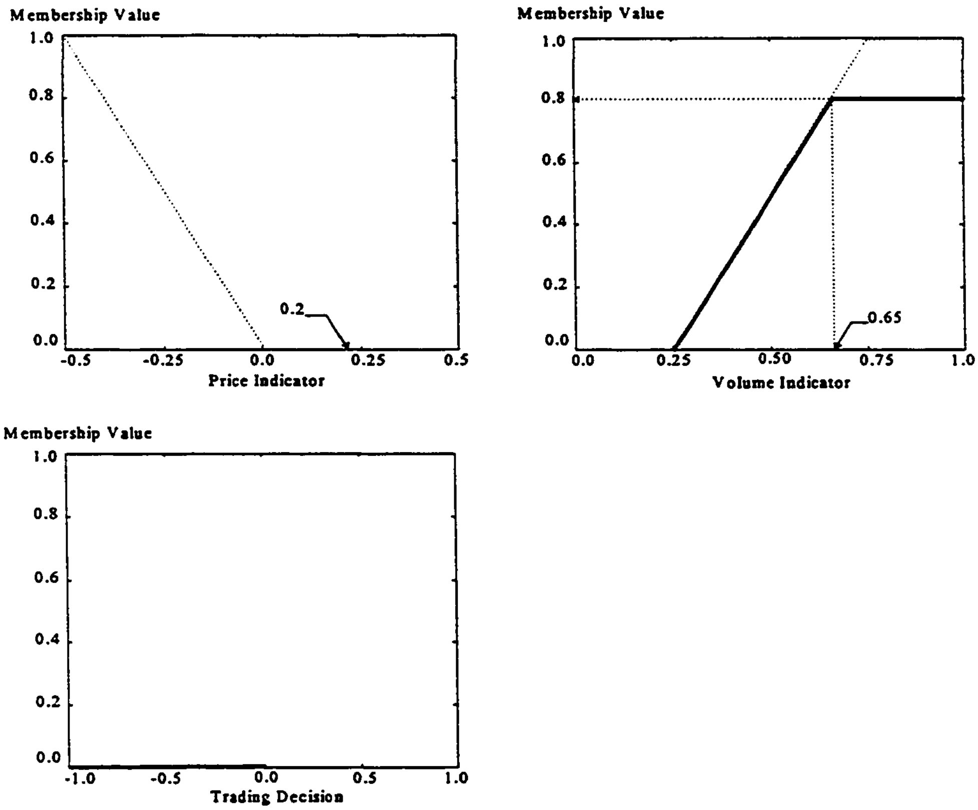

Fig. 15. Response of Rule 4

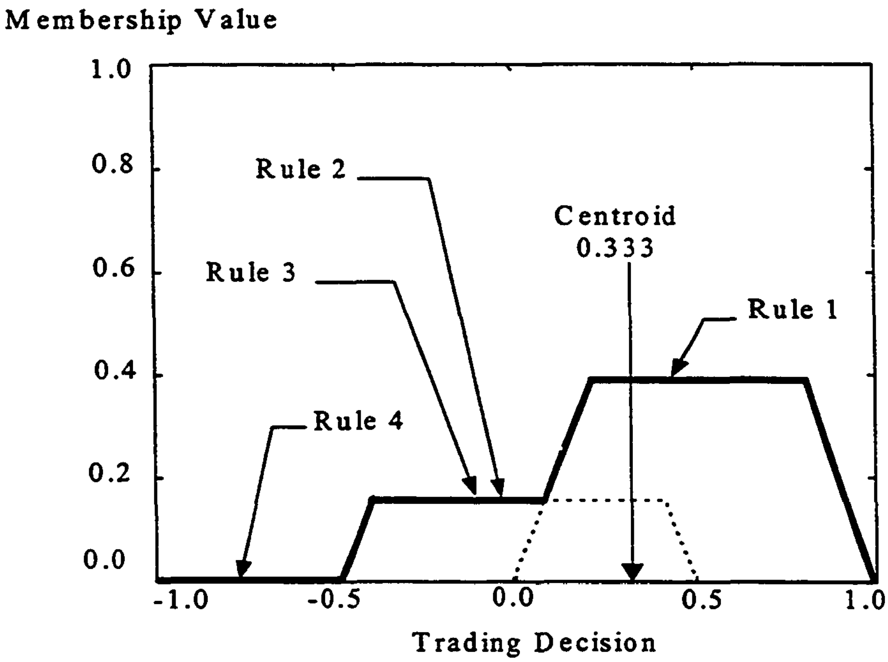

Fig. 16. Aggregate Output/Defuzzification

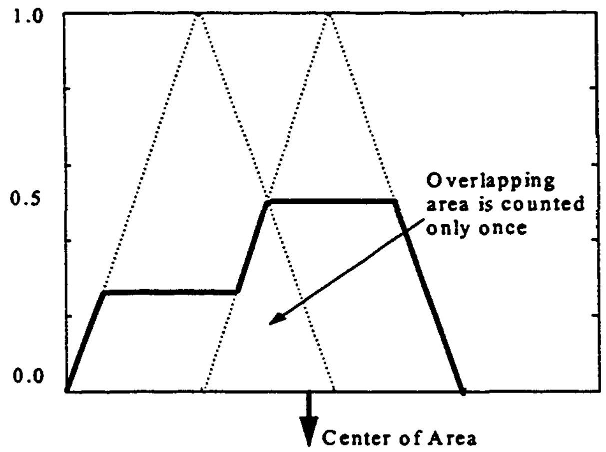

Fig. 17. Centroid Defuzzification Method

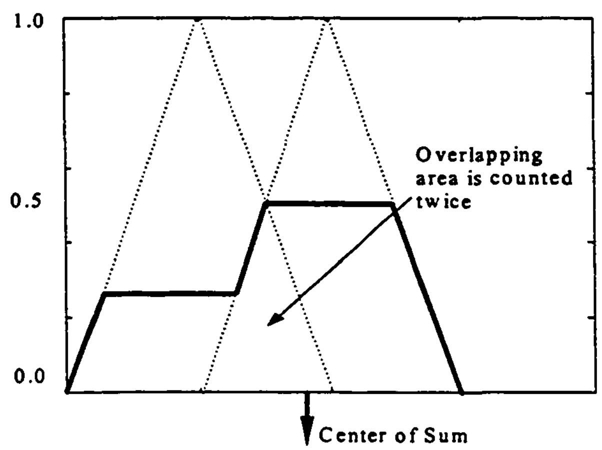

Fig. 18. Center of Sum Defuzzification Method

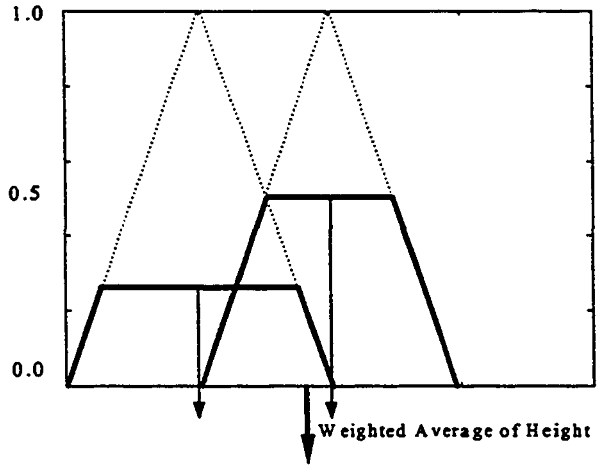

Fig. 19. Height Defuzzification Method

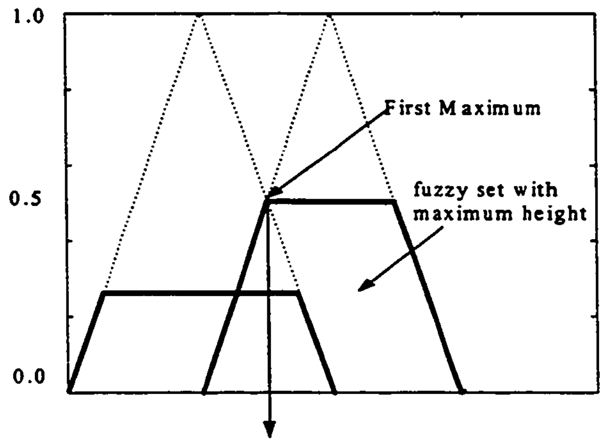

Fig. 20. First-of-Maxima Defuzzification Method

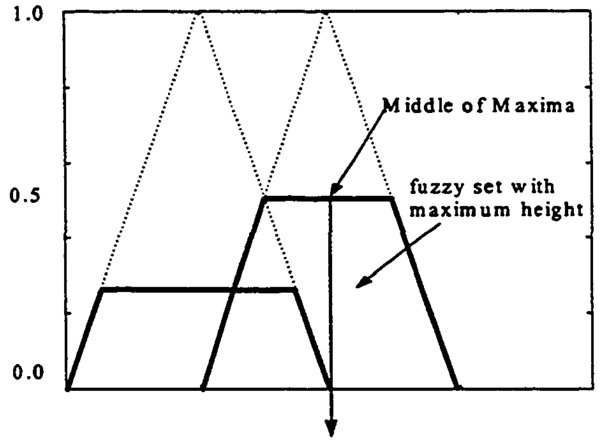

Fig. 21. Middle-of-Maxima Defuzzification Method

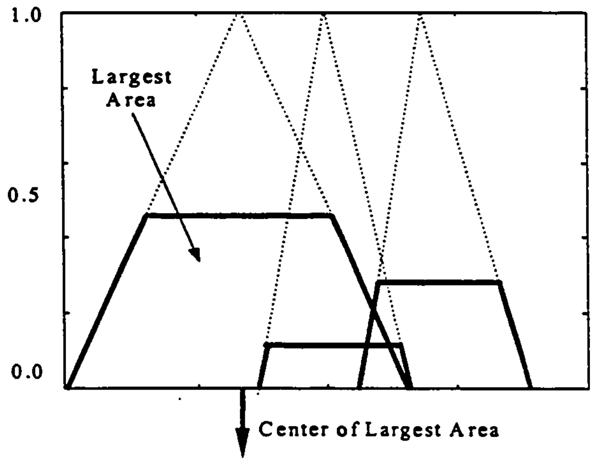

Fig. 22. Center-of-Largest Area Defuzzification Method

Parent 1	1	1	1
Parent 2	0	0	0
Offspring A1	1	1	0
Offspring A2	0	0	1
Offspring B1	1	0	1
Offspring B2	0	1	0
Offspring C1	0	1	1
Offspring C2	1	0	0

Fig. 23. Spanning of parameter space by Crossover operator

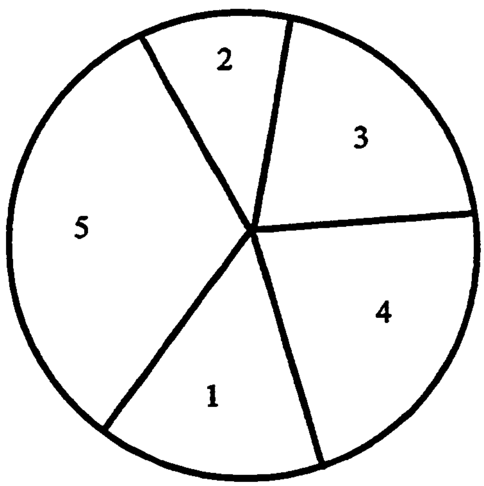

Fig. 24. A Roulette Wheel

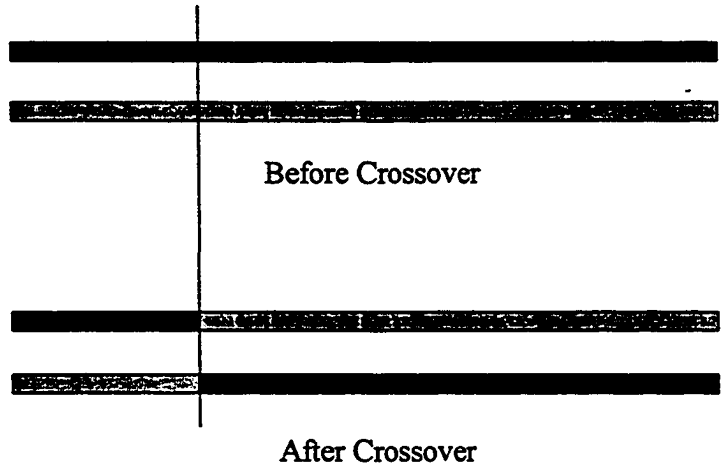

Fig. 25. Single-Point Crossover

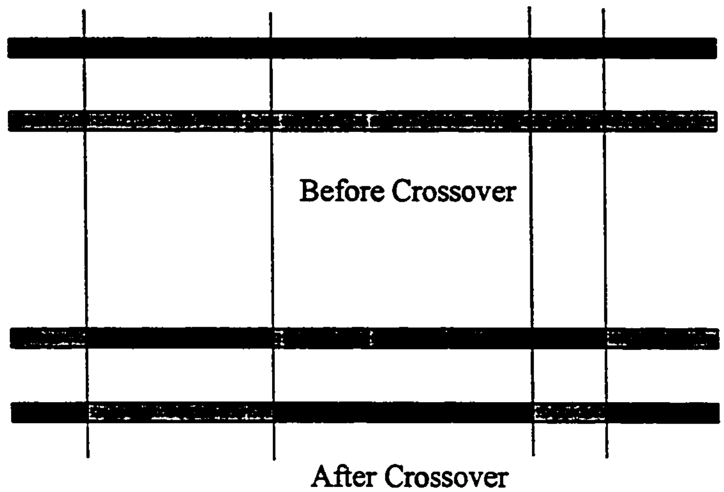

Fig. 26. Multi-Point Crossover

Parent 1	=	1	0	1	1	0	0	0	1	1	1
Parent 2	=	0	0	0	1	1	1	1	0	0	0
MASK	=	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**0**	**0**
Offspring 1	=	0	0	1	1	1	1	0	1	0	0
Offspring 2	=	1	0	0	1	0	0	1	0	1	1

Fig. 27. Uniform Crossover

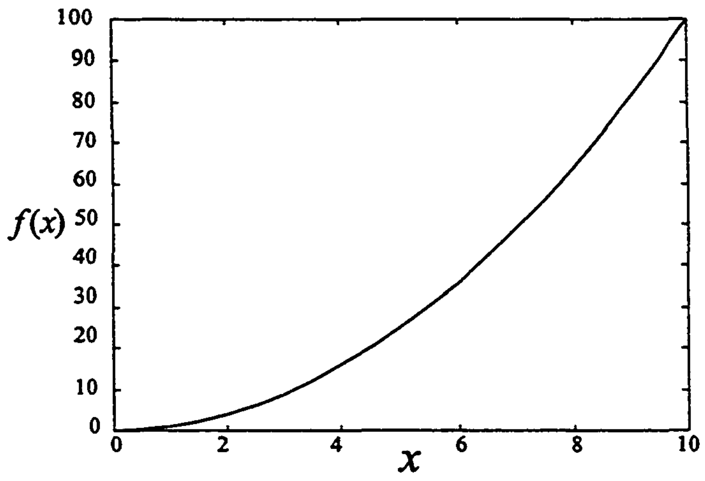

Fig. 28. Plot of $f(x) = x^2$

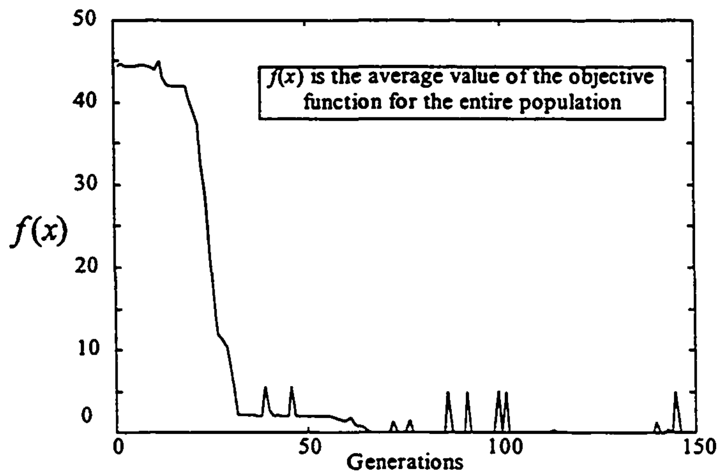

Fig. 29. Result of GA simulation

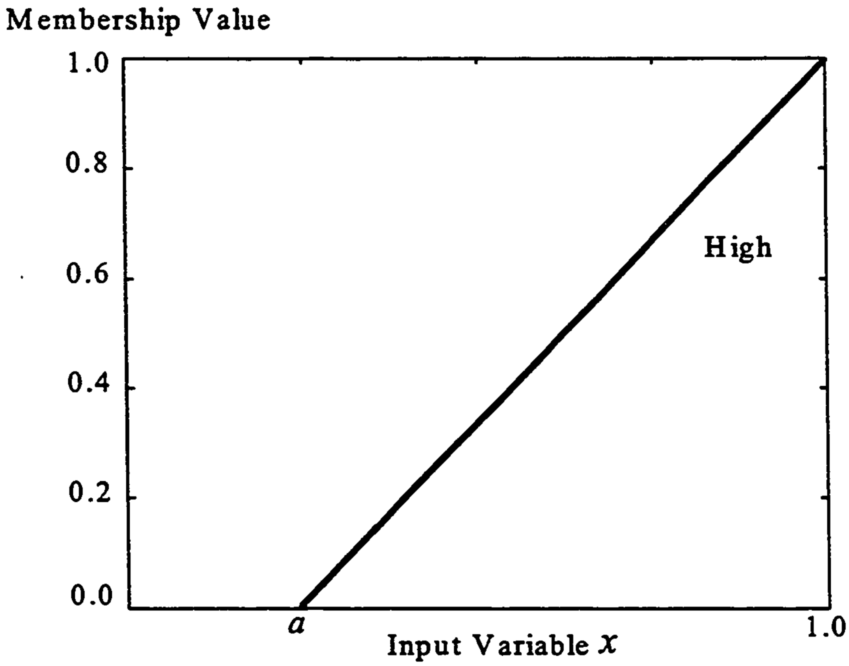

Fig. 30. Fuzzy Set for Input x

$$\begin{vmatrix} 3 & 2 & 3 & 1 & 1 \\ 2 & 0 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \end{vmatrix}$$

Fig. 31. A Fuzzy Rule Base Coded in Bit Strings

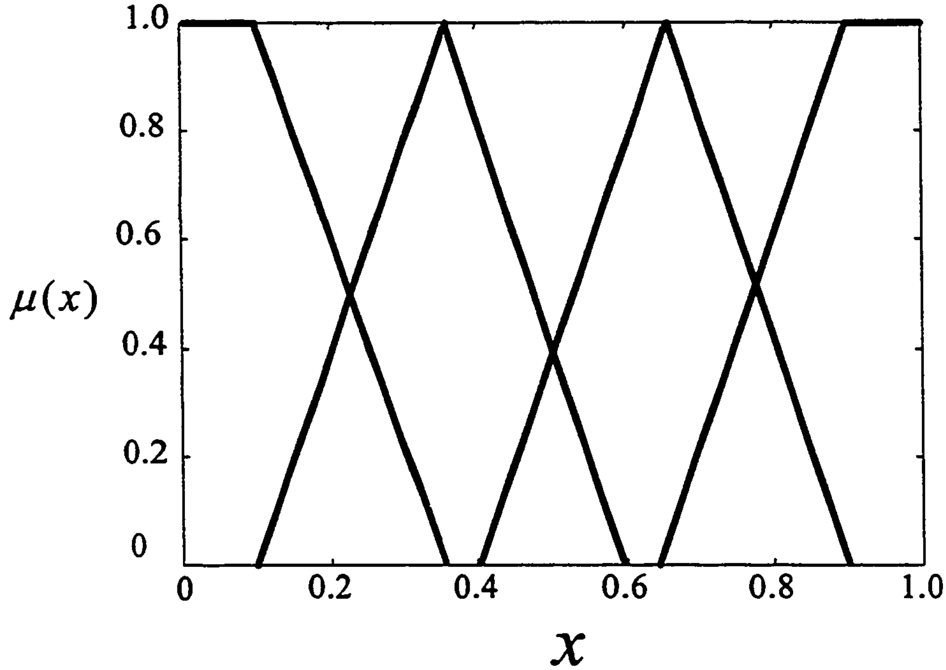

Fig. 32. Fuzzy Sets for the States of the Market Descriptors

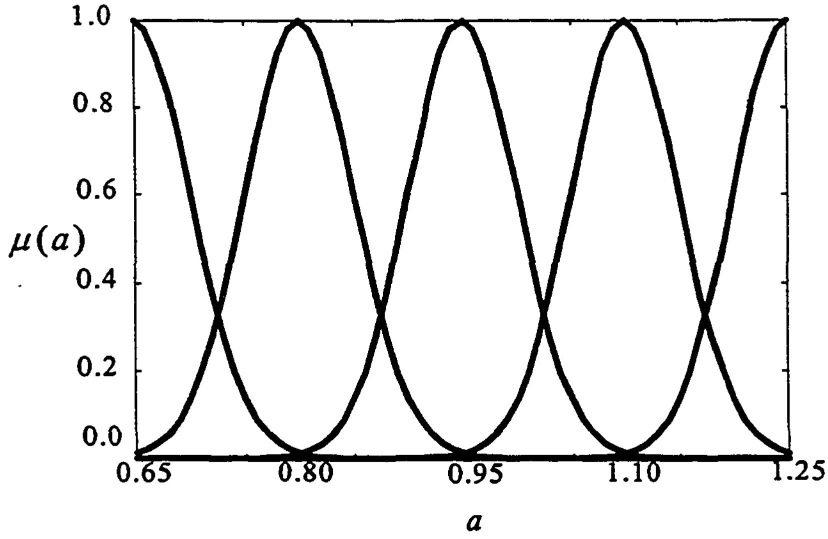

Fig. 33. Fuzzy Sets for Forecast Parameter '*a*'

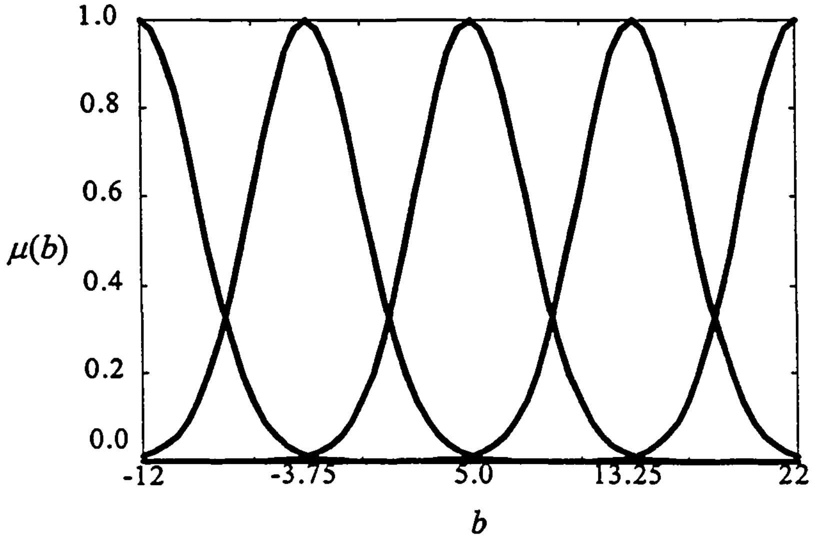

Fig. 34. Fuzzy Sets for Forecast Parameters '*b*'

		Conditions			Forecast Parameters	
1	0	0	2	3	3	5
4	0	0	1	4	1	3
2	0	0	4	2	2	4
3	0	0	3	1	5	1

Fig. 35. A Complete and Consistent Rule Base

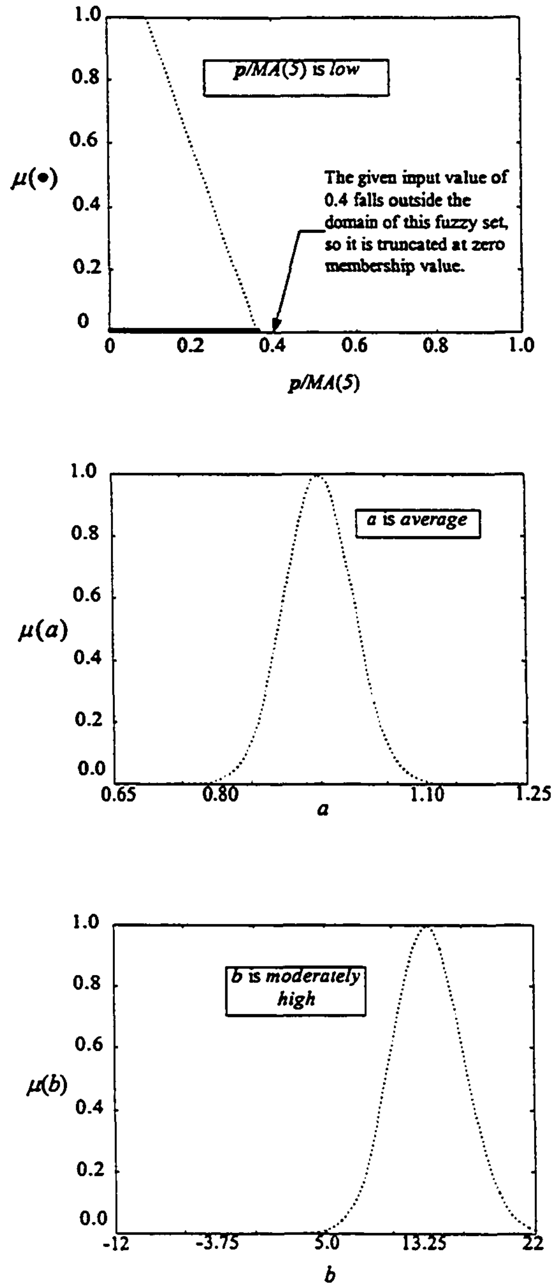

Fig. 36. Response of 1st Rule

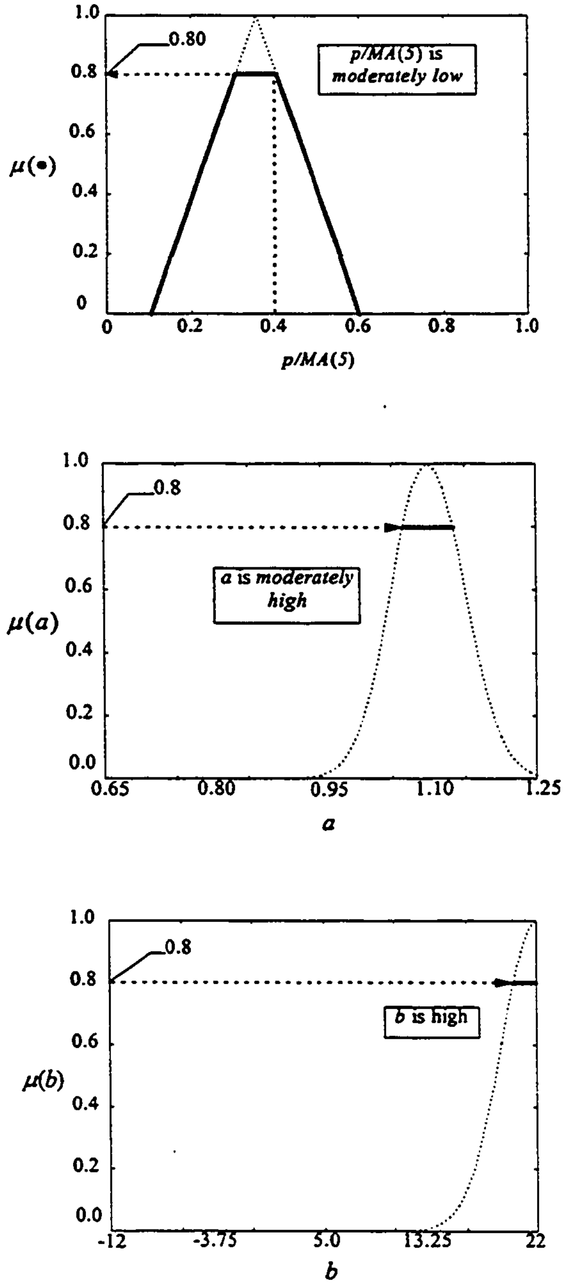

Fig. 37. Response of 2nd Rule

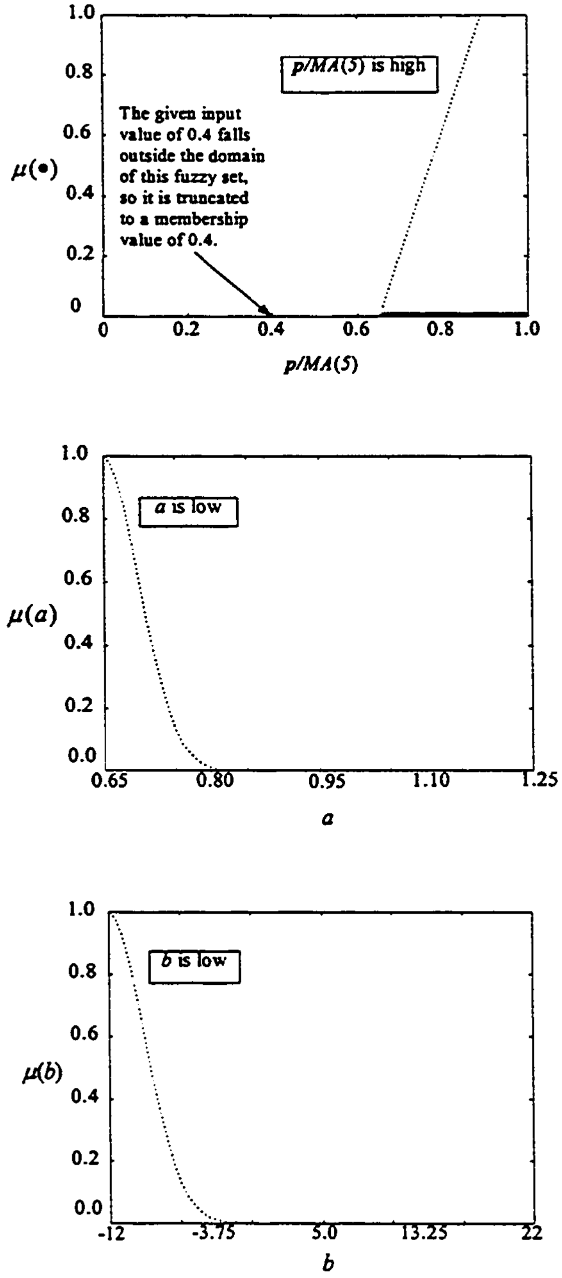

Fig. 38. Response of 3rd Rule

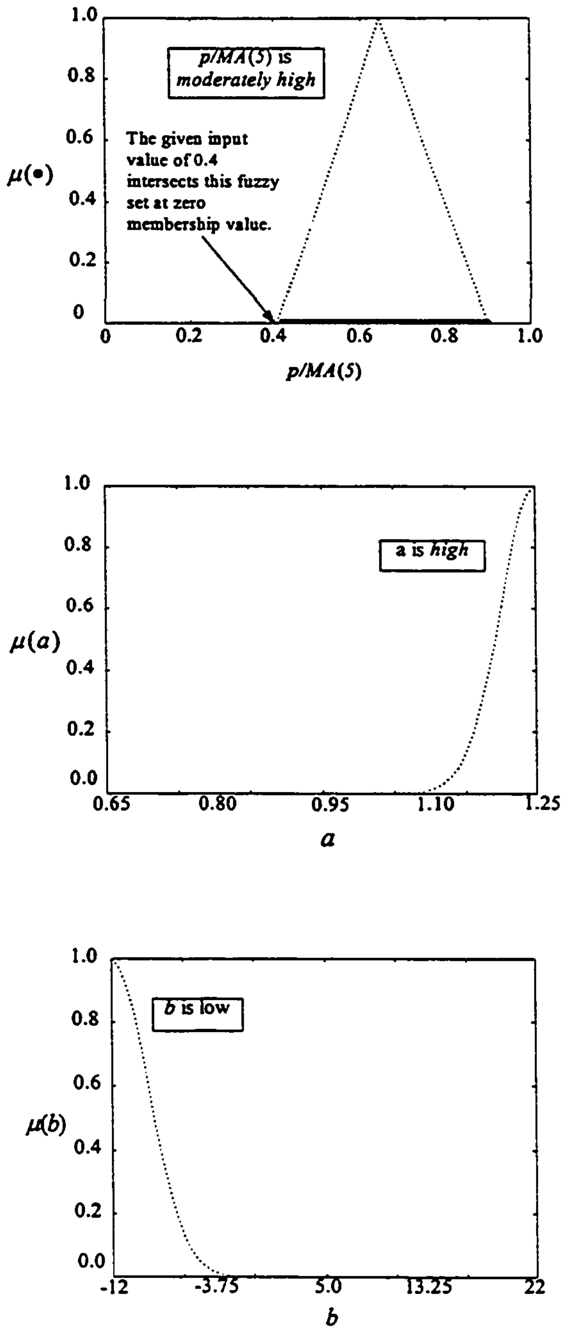

Fig. 39. Response of 4^{th} Rule

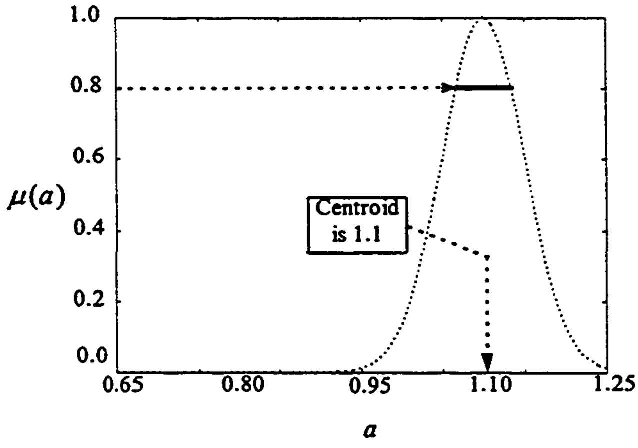

Fig. 40. Resultant Fuzzy Set for Forecast Parameter '*a*'

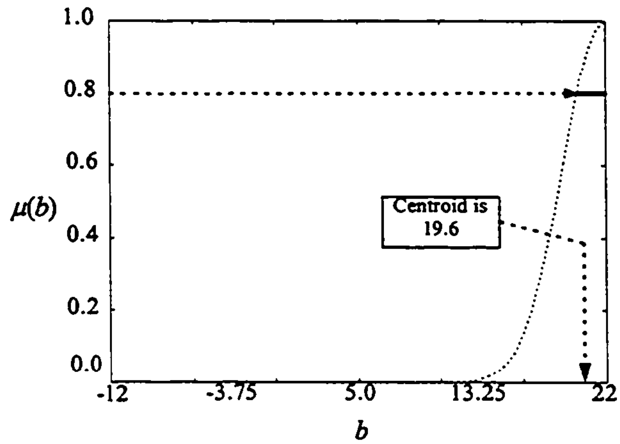

Fig. 41. Resultant Fuzzy Set for Forecast Parameter '*b*'

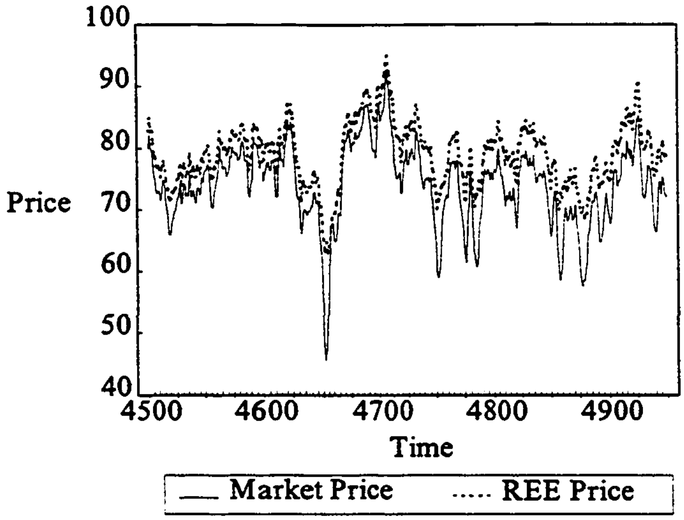

Fig. 42. Time Series Record of the Market Price Vs. the REE Price Over a Typical Window in the High Learning Frequency Case

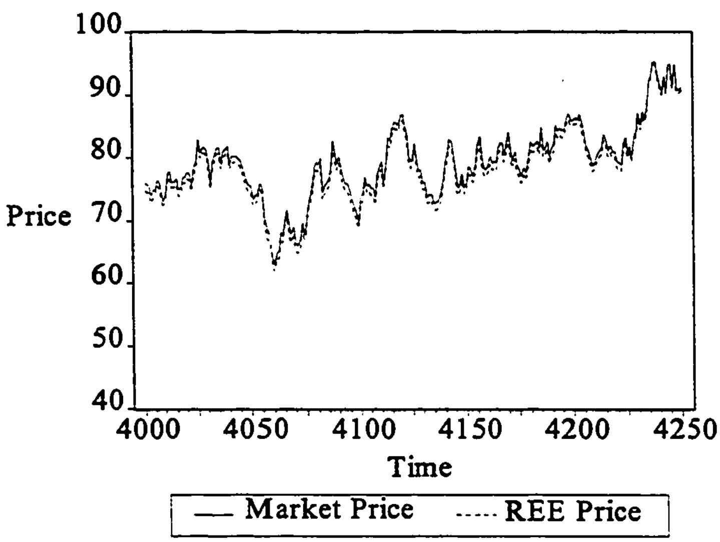

Fig. 43. Time Series Record of the Market Price Vs. the REE Price Over a Typical Window in the Low Learning Frequency Case

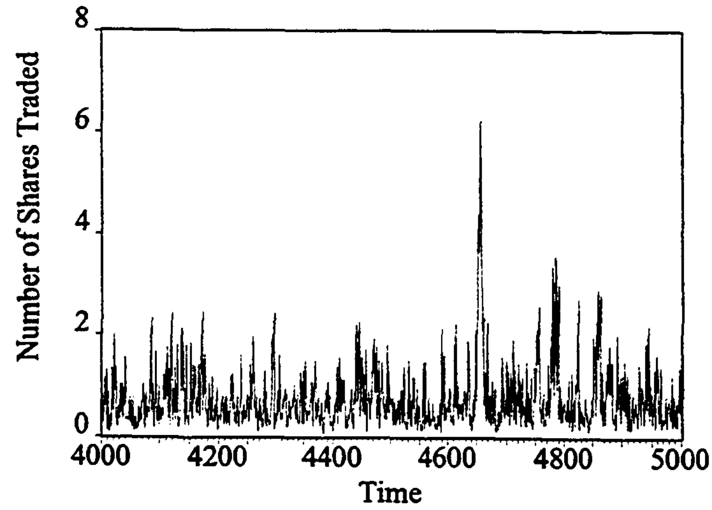

Fig. 44. Time Series Record of the Volume Traded Over a Typical Window in the High Learning Frequency Case

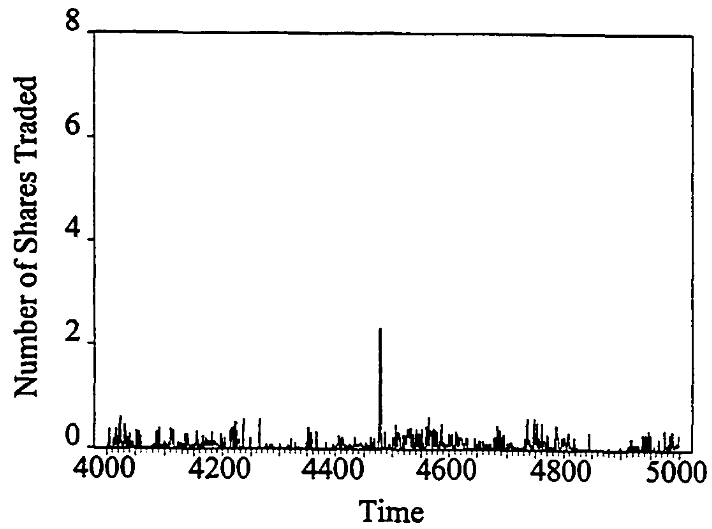

Fig. 45. Time Series Record of the Volume Traded Over a Typical Window in the Low Learning Frequency Case

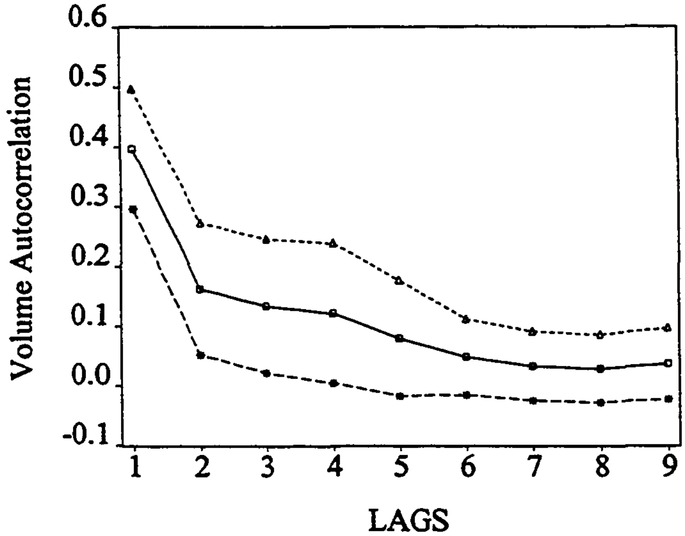

Fig. 46. Volume Autocorrelations in the High Learning Frequency Case

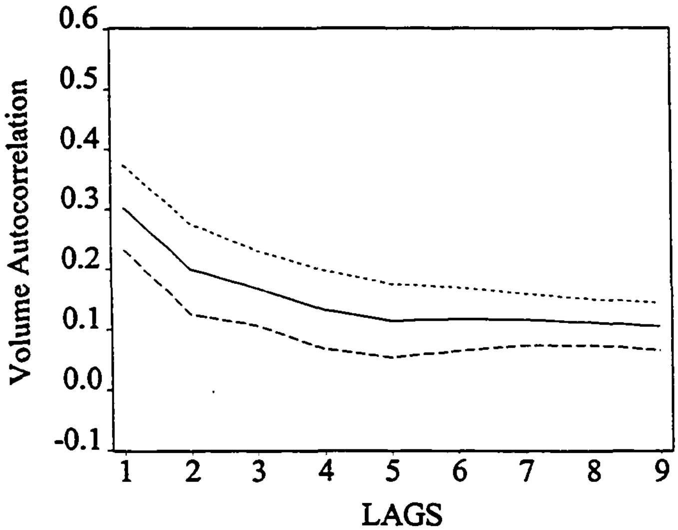

Fig. 47. Volume Autocorrelations in the Low Learning Frequency Case

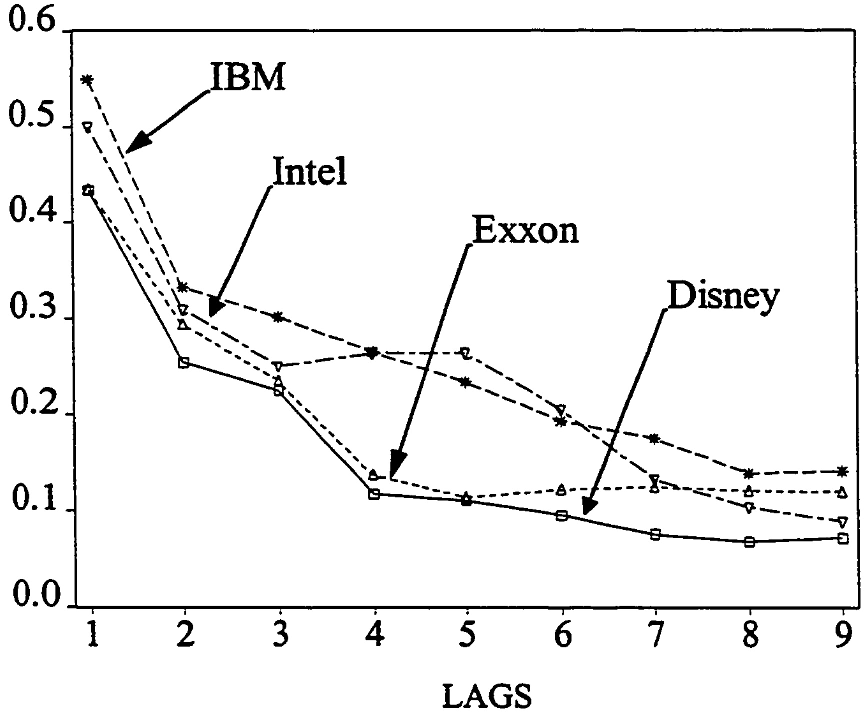

Fig. 48. Volume Autocorrelations for Disney, Exxon, IBM and Intel

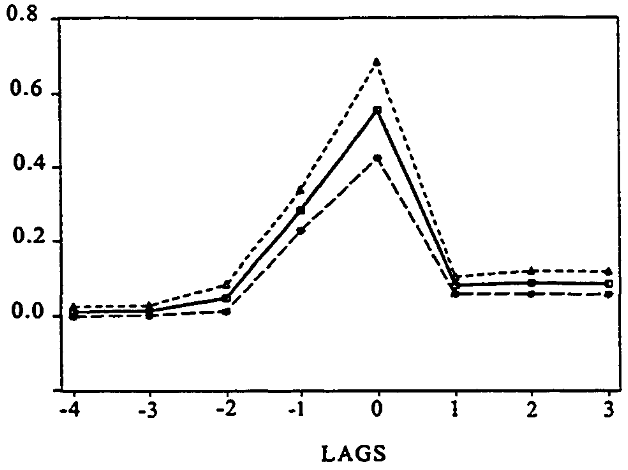

Fig. 49. Cross Correlation Between Volume and Volatility

in the High Learning Frequency Case

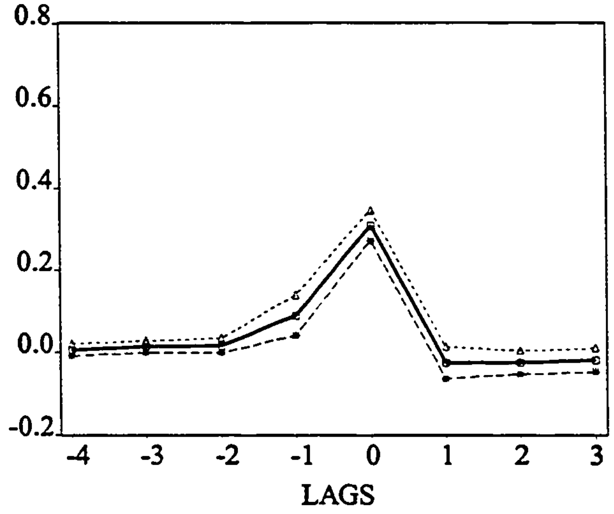

Fig. 50. Cross Correlation Between Volume and Volatility

in the Low Learning Frequency Case

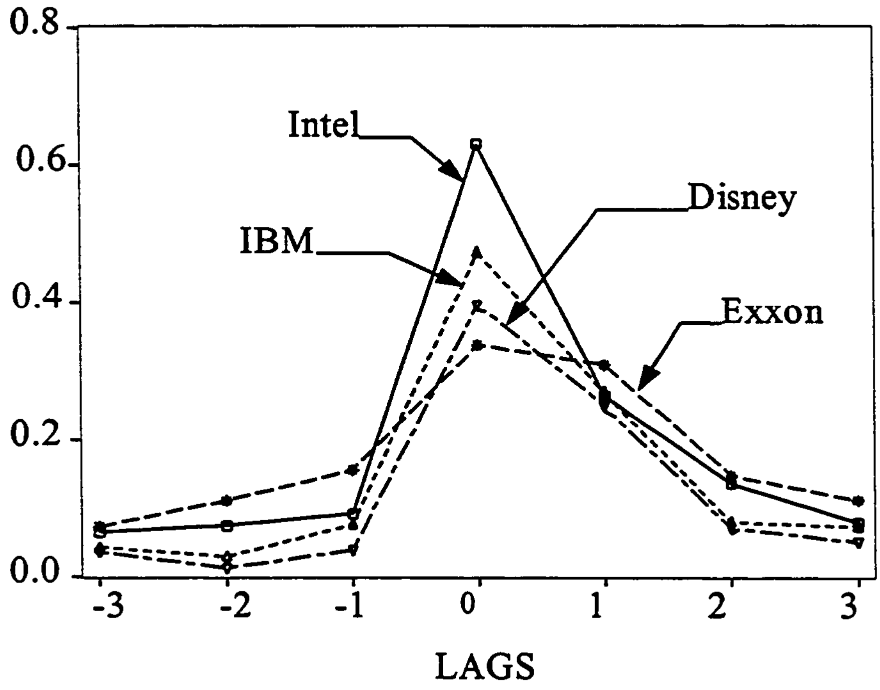

Fig. 51. Cross Correlation Between Volume and Volatility for

Disney, Exxon, IBM and Intel

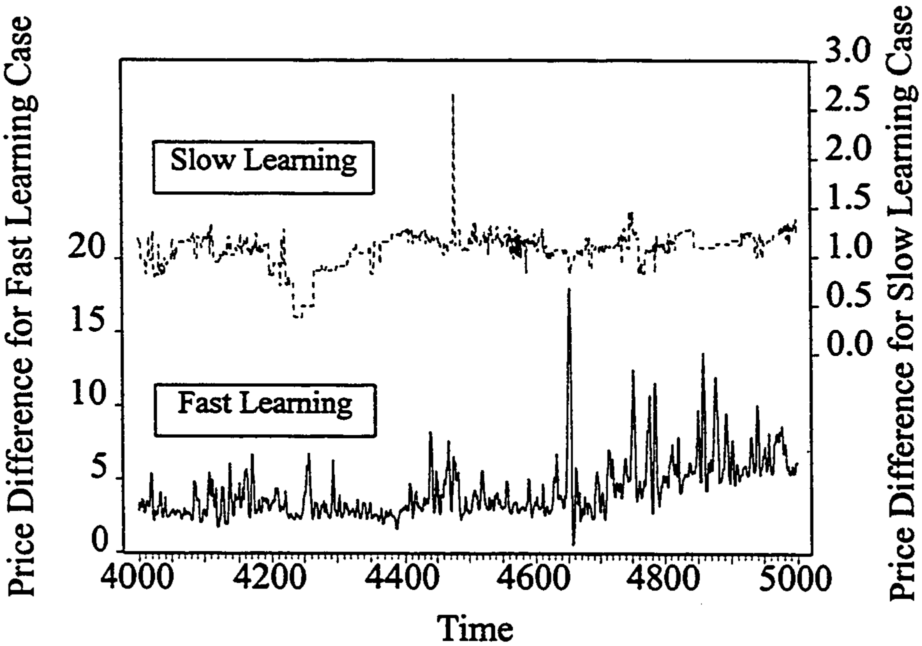

Fig. 52. Time Series Record of the Difference between the Market Price and the REE Price Over a Representative Window

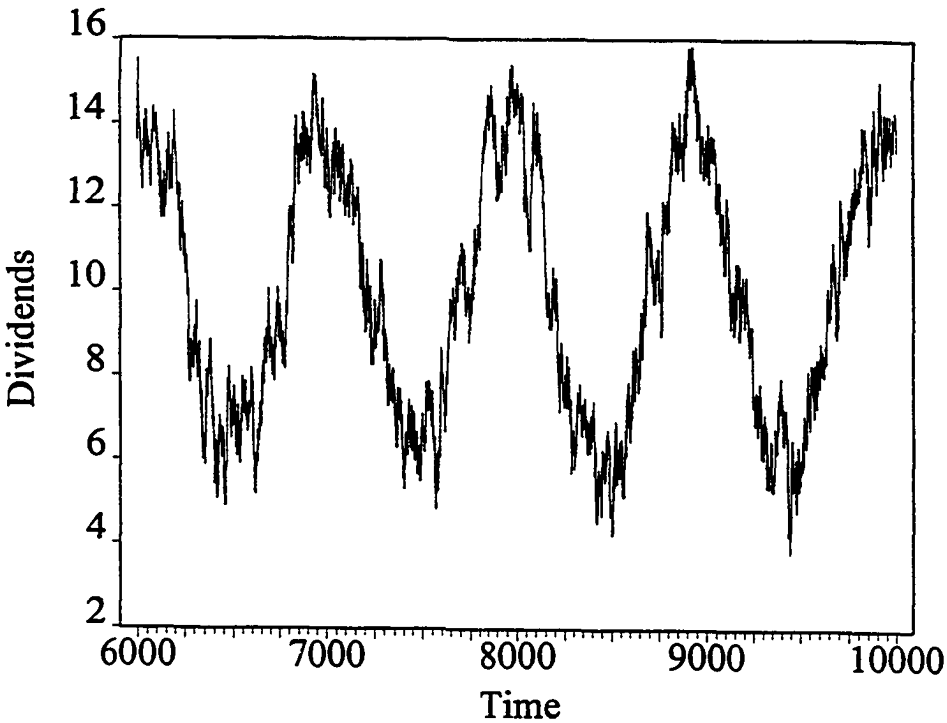

Fig. 53 Time Series Record of the Cyclical Dividend Process Over a Typical Window

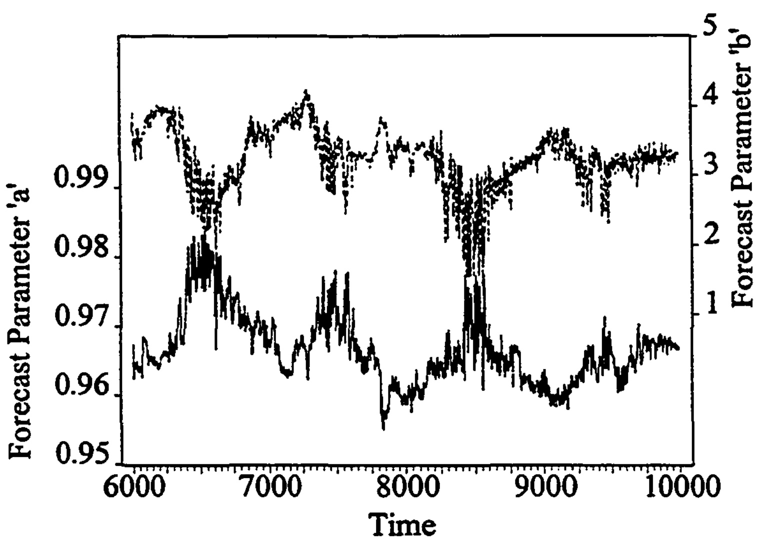

Fig. 54 Time Series Record of the Forecast Parameters Over a Typical Window for the High Learning Frequency Case with a Cyclical Dividend Process

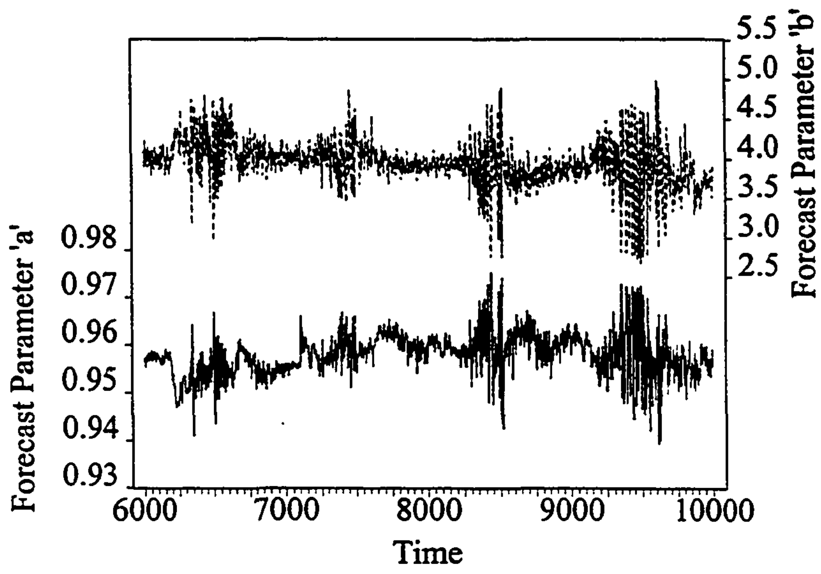

Fig. 55 Time Series Record of the Forecast Parameters Over a Typical Window for the Low Learning Frequency Case with Cyclical Dividend Process

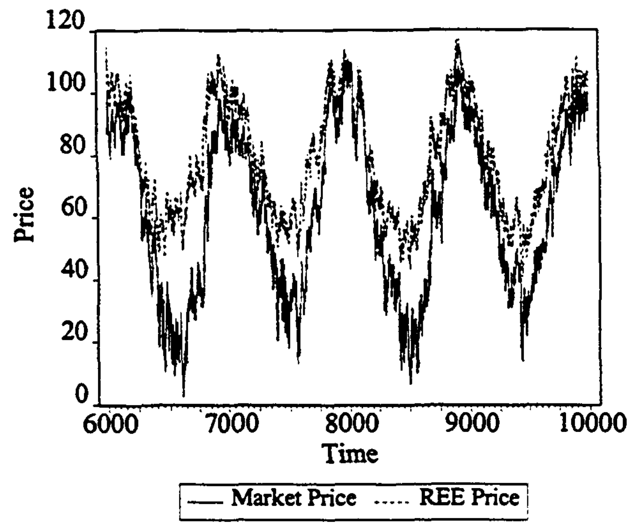

Fig. 56 Time Series Record of the REE Price and the Market Price Over a Typical Window for the High Learning Frequency Case with Cyclical Dividend Process

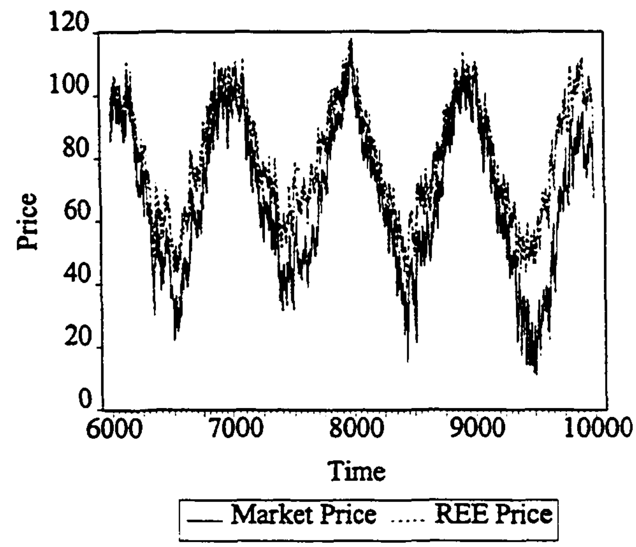

Fig. 57 Time Series Record of the REE Price and the Market Price Over a Typical Window for the Low Learning Frequency Case with Cyclical Dividend Process

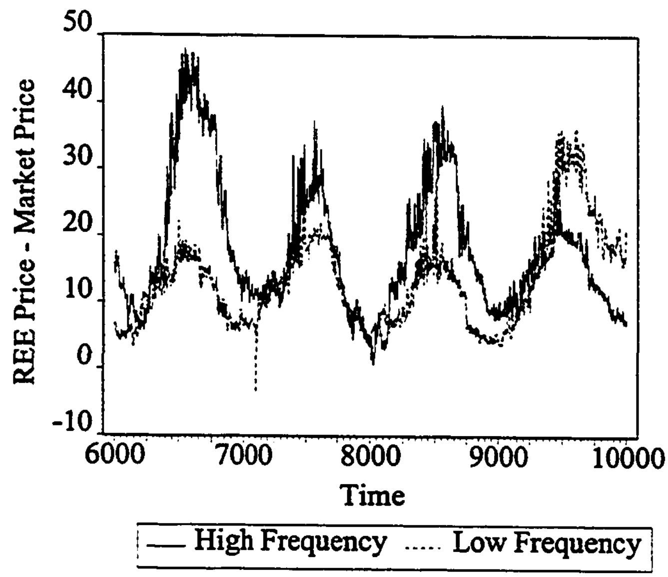

Fig. 58 Time Series Record of the Difference Between the REE Price and Market Price Over a Typical Window Under a Cyclical Dividend Process

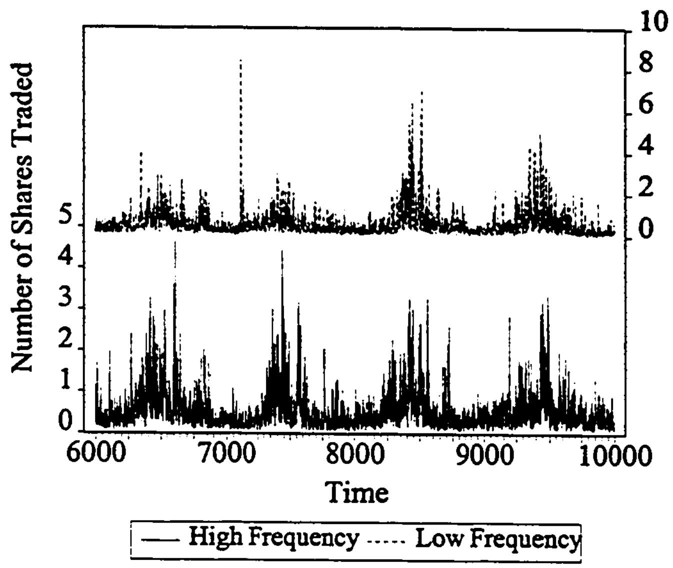

Fig. 59 Time Series Record of the Volume Traded Over a Typical Window Under a Cyclical Dividend Process

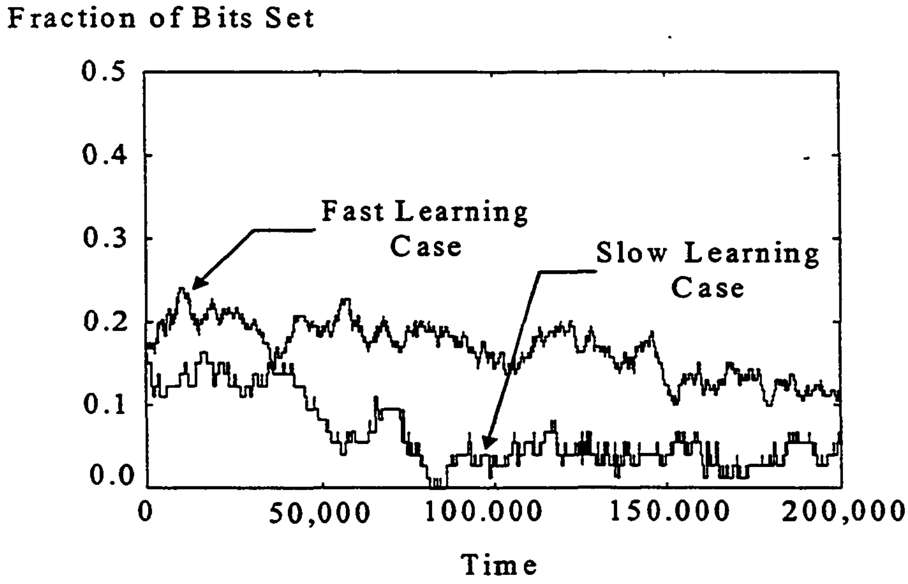

Fig. 60 Time Series Record of Fraction of Fundamental Bits Set

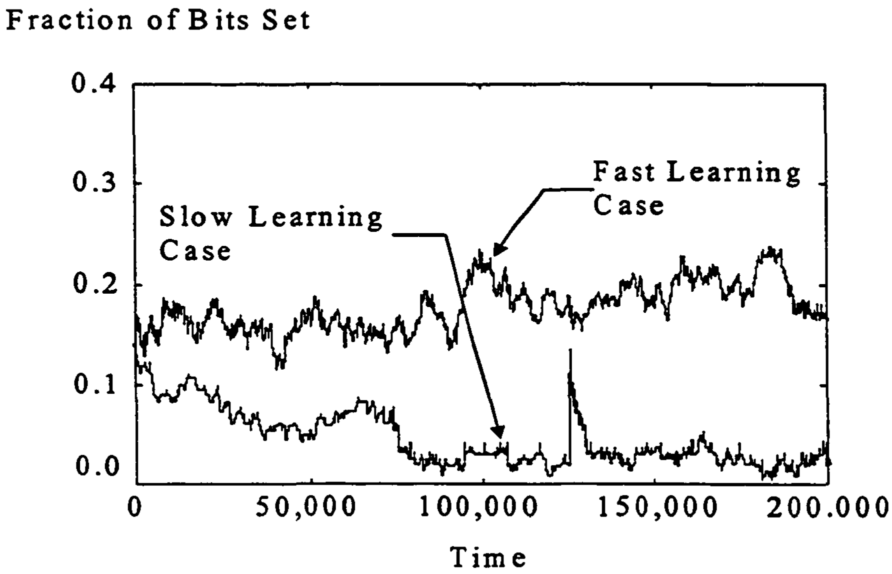

Fig. 61 Time Series Record of Fraction of Technical Bits Set

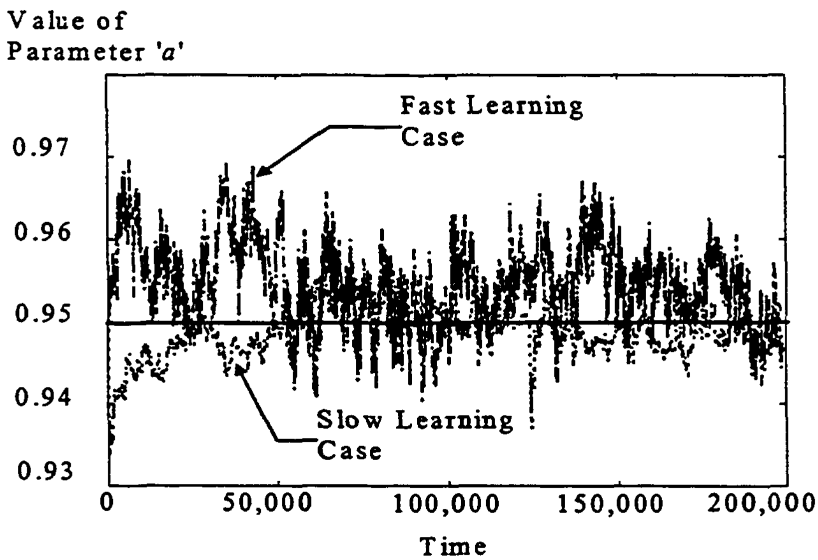

Fig. 62 Time Series Record of the Average of Forecast Parameter '*a*'

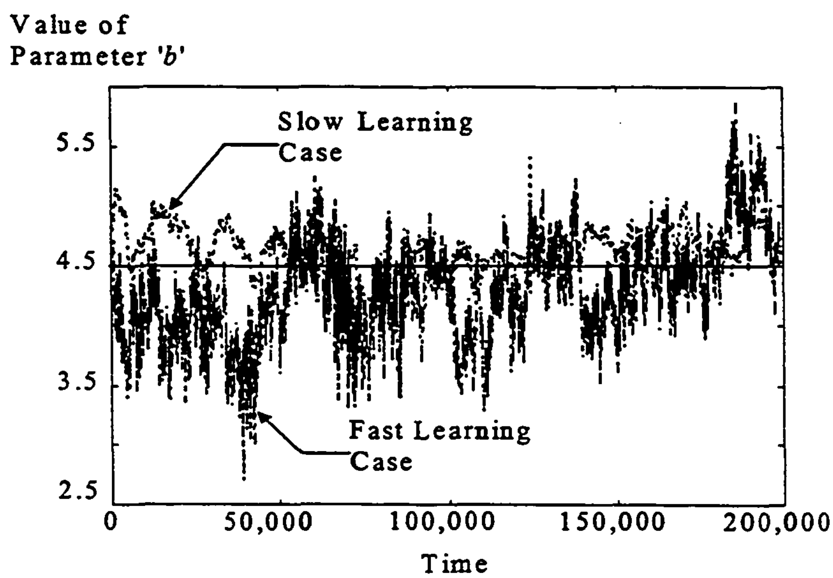

Fig. 63 Time Series Record of the Average of Forecast Parameter '*b*'

TABLES

TABLE 1

Relations between Gray and Binary representation

Gray	Binary
0000	0000
0001	0001
0010	0011
0011	0010
0100	0110
0101	0111
0110	0101
0111	0100
1000	1100
1001	1101
1010	1111
1011	1110
1100	1010
1101	1011
1110	1001
1111	1000

TABLE 2

Evolution of the population of strings over time

Generation 5									Generation 10									Generation 15								
								f(x)									f(x)									f(x)
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5	1	1	1	1	0	1	1	1	6.5
1	1	1	1	1	1	1	1	6.7	1	1	1	1	1	1	1	1	6.7	1	1	1	1	0	1	1	1	6.5

TABLE 3

Evolution of the population of strings over time

Generation 25								f(x)	Generation 35								f(x)	Generation 55								f(x)
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	0	1.4	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	0	3.6	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	1	1.5	0	0	0	1	0	1	1	0	1.1
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	0	1.4	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	0	1.4	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	0	1.4	0	0	1	1	0	1	1	0	1.4
1	1	1	1	0	1	1	1	6.5	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	0	3.6	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
1	1	1	1	0	1	1	1	6.5	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	0	1.4	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	0	1.4	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	0	1	1	3.5	0	0	1	1	0	1	1	0	1.4	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	0	3.6	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	0	3.6	0	0	1	1	0	1	1	1	1.5	0	0	1	1	0	1	1	0	1.4
0	1	1	1	0	1	1	1	3.5	0	0	1	1	0	1	1	0	1.4	0	0	1	1	0	1	1	0	1.4

TABLE 4

Evolution of the population of strings over time

Generation 60									Generation 70									Generation 80								
								f(x)									f(x)									f(x)
0	1	0	1	0	1	1	0	3.9	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	1	1	0.08	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	1	0	0.1
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	0	0	0	1	0	0.1	0	0	0	0	0	0	1	1	0.08	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	1	0	0.1	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	1	1	0.08	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	0	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	1	1	0.08	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	1	0	0.1	0	0	0	0	0	0	0	0	0.0
0	0	0	1	0	1	1	0	1.1	0	0	0	0	0	0	0	0	0.0	0	0	0	0	0	0	0	0	0.0

TABLE 5

Common Parameter Values in the Experiments

Description of Parameters	Values
Mean Dividend ($\bar{d}$ in eq. (3.1))	10
Autoregressive Parameter (ρ in eq. (3.1))	0.95
Variance (σ_ε^2 in eq. (3.1))	0.0743
Interest Rate (r)	0.1
Risk Aversion Parameter (λ in eq. (3.2))	0.5
Weight (θ in eq. (3.5))	1/75
Cost of specificity (β in eq. (3.6))	0.004
Number of Bits for Market Descriptors	5
Number of Bits for Forecast Parameters	2
Crossover Probability	0.1
Mutation Probability[1]	0.9
Number of Agents (also Number of Shares)	25
Number of Rule Bases per Agent	3
Number of Fuzzy Rules per Rule Base	4

[1] This is the probability that an agent will have one of his rule bases subjected to mutation. When a particular rule base is selected for mutation, the probability that each bit is mutated is 0.03, and the probability that a bit will be transformed from 0's to non-0's or vice versa is 0.5.

TABLE 6

Summary Statistics of Market Price and Residuals

Variables	Fast Learning Experiment	Slow Learning Experiment	REE (Theoretical)
Price (P_t)			
Mean ($\overline{P}$)	75.6795	78.8940	80.000
Std. Dev.(σ_P)	5.9420	5.7674	5.528
Residual (ε_t)			
Standard Deviation	2.0950 (0.0107)	2.0160 (0.02080)	2.0
Excess Kurtosis	0.0839 (0.0849)	0.0563 (0.04232)	0.0
$\rho_1(\varepsilon_t)$	0.1097 (0.0867)	0.0555 (0.01612)	0.0
$\rho_1(\varepsilon_t^2)$	0.0387** (0.01382)	-0.0120 (0.00730)	0.0
ARCH LM(1)[1]	18.1886*** [1.00]	1.9471 [0.15]	
Mean Excess Return[2]	3.154% (0.2527)	2.710% (0.0336)	2.5%

The numbers in parenthesis are the standard errors.
The numbers in square bracket are the percentage of the number of tests that reject the null hypothesis of "no ARCH".

[1]The number for the ARCH LM(1) tests are the mean of the χ^2-statistics for the 20 runs.

[2]Excess Return is calculated as $\dfrac{(p_{t+1}+d_{t+1}-p_t)}{p_t}-r_f$.

** denotes significance at the 2% confidence level.
*** denotes significance at the 1% confidence level.

TABLE 7

Summary Statistics of Returns

for Disney, Exxon, IBM, and Intel

Variables	Disney	Exxon	IBM	Intel
		Return (R_t)		
Mean	0.00085	0.00067	0.00130	0.00100
Standard Deviation	0.01548	0.01221	0.01930	0.02296
Excess Kurtosis	2.8444	1.9933	5.2202	3.4910
$\rho_1(R_t)$	-0.031 (0.244)	-0.101*** (0.000)	-0.019 (0.492)	0.033 (0.227)
$\rho_1(R_t^2)$	0.045* (0.097)	0.110*** (0.000)	0.073*** (0.007)	0.069*** (0.011)
ARCH LM(1)	2.7949* (0.095)	64.9086*** (0.000)	7.3189*** (0.007)	7.8167*** (0.005)

The numbers given in parenthesis are the p-values.
* denotes significance at the 10% confidence level.
*** denotes significance at the 1% confidence level.

TABLE 8

Summary Statistics of Trading Volume

Variables	Fast Learning Experiment	Slow Learning Experiment
Mean	0.6326 (0.0775)	0.1637 (0.0409)
Maximum	10	6.206
Minimum	0.01497	0.0

APPENDIX

SOLUTION FOR A LINEAR HOMOGENEOUS RATIONAL EXPECTATIONS EQUILIBRIUM

Recall that the dividend process and the demand are given by:

$$d_t = \bar{d} + \rho(d_{t-1} - \bar{d}) + \varepsilon_t$$

$$x_{i,t} = \frac{E_{i,t}[p_{t+1} + d_{t+1}] - p_t(1+r)}{\lambda\sigma^2_{i,t,p+d}}$$

Now to solve for a homogeneous linear rational expectations equilibrium, we conjecture that price is a linear function of the dividend, that is,

$$p_t = fd_t + e.$$

This allows us to write the conditional expectation and conditional variance of $(p_{t+1} + d_{t+1})$ as

$$\begin{aligned} E_{i,t}[p_{t+1} + d_{t+1}] = E_{i,t}[(1+f)d_{t+1} + e] &= E_{i,t}[(1+f)(\bar{d} + \rho(d_t - \bar{d}) + \varepsilon_{t+1}) + e] \\ &= (1+f)(\bar{d} + \rho(d_t - \bar{d})) + e \end{aligned}$$

$$\begin{aligned} Var_{i,t}[p_{t+1} + d_{t+1}] = Var_{i,t}[(1+f)d_{t+1} + e] &= Var_{i,t}[(1+f)(\bar{d} + \rho(d_t - \bar{d}) + \varepsilon_{t+1}) + e] \\ \sigma^2_{p+d} \quad &= Var_{i,t}[(1+f)\varepsilon] = (1+f)^2\sigma^2_\varepsilon \end{aligned}$$

In equilibrium, each agent must hold the same number of shares (since all the agents are equally risk averse). Given that the total number of shares is equal to the total number of agents, each agent must hold only one share at all times when they are in equilibrium. This allows us to set the demand equation to one. We can then substitute into the demand equation the above expression for the one-period ahead forecast to get,

$$1=\frac{(1+f)(\bar{d}+\rho(d_t-\bar{d}))+e-(fd_t+e)(1+r)}{\lambda\sigma_{p+d}^2}$$

Since the LHS is a constant, there must not be any dependence on time on the RHS, so terms containing d_t must vanish. This leads us to

$$(1+f)\rho-(1+r)f=0$$

$$f=\frac{\rho}{(1+r-\rho)}$$

To solve for 'e', we substitute f back into the demand equation and set it to 1 to get

$$e=\frac{\bar{d}(f+1)(1-\rho)-\gamma\sigma_{p+d}{}^2}{r}.$$

Now to obtain the relationship between the forecast parameters 'a' and 'b' in our model and these HREE parameters, we write the one-period ahead optimal forecast for price and dividend as:

$$E(p_{t+1}+d_{t+1})=\rho(p_t+d_t)+(1-\rho)[(1+f)\bar{d}+e]$$

Comparing this equation to $E(p_{t+1}+d_{t+1})=a(p_t+d_t)+b$, it is obvious that

$$a=\rho$$
$$b=(1-\rho)[(1+f)\bar{d}+e]$$

IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6"

APPLIED IMAGE . Inc
1653 East Main Street
Rochester, NY 14609 USA
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female -- reduces gender to essentialized identities on the nation's linear path from feminine victim or Goddess to masculine nation-state. Chatterjee traces this development through anti-colonial to postcolonial Indian nationalism. In order to protect a sense of their own identity in the face of colonial power, he argues, indigenous colonized communities separate the material and spiritual: "The material is the domain of the 'outside,' of the economy and of statecraft, of science and technology, a domain where the West had proved its superiority and the East had succumbed...The spiritual, on the other hand, is an 'inner' domain bearing the 'essential' marks of cultural identity. The greater one's success in imitating Western skills in the material domain, therefore, the greater the need to preserve the distinctness of one's spiritual culture" (6).

The model appears to create a double-bind for postcolonial nationalism. Internal identity, no matter how complicated or fraught with difference, becomes essential and mythic while external economic, political, and social identities are evaluated according to standards of modern liberal and capitalist ideology. As Radhakrishnan writes, "Woman becomes the allegorical name for a specific historical failure: the failure to coordinate the political or the ontological with the epistemological within an undivided agency" (85). This failure is both essentially gendered and unavoidable as the model itself seemingly forecloses the possibility of alternative national identities.

Reading national identity in these gendered terms means that if the nation is an imagined community, then as Chatterjee notes, "(e)ven our imaginations remain forever colonized" (5). Colonization occurs through the metaphor of the family and its gendered signifiers of the nation. The paternal family effectively fuses two narratives of national identity: legal-political and ethnic. Whereas the legal-political nation legitimates itself on the basis of history, territory, and, in modern liberal terms, the promise to protect private difference in return for public allegiance, the ethnic nation "is seen as a fictive 'super-family', and it boasts pedigrees and genealogies to back up its claims...the nation can trace its roots to an imputed common ancestry and...therefore its members are brothers and sisters, or at least cousins, differentiated by family ties from outsiders."[7] In the case of India, that rhetoric anoints a state created by decree with a notion of national solidarity united more in opposition to the British than in ethnic, linguistic, or religious purity or cohesion.[8]

Rushdie's response to this model is to render it both literal and fantastic so that we are forced to see how it is constructed and to account for our own reliance on it. Saleem underscores this duality when he looks back on the eve of independent India:

> there was an extra festival on the calendar, a new myth to celebrate, because a nation which had never previously existed was about to win its freedom, catapulting us into a world which, although it had five thousand years of history, although it had invented the game of chess and traded with Middle Kingdom Egypt, was nevertheless quite imaginary; into a mythical land, a country which would never exist except by the efforts of a phenomenal collective will -- except in a dream we all agreed to dream...(129)

Here Rushdie collapses the distinctions between postcolonial India's mythic and political formation. He intersperses passages such as the one above with official state rhetoric, making it easy to imagine Nehru or Mohandas K. Gandhi as national fathers or Indira Gandhi's Emergency as a parental response to the nation's difficult adolescence (when, as she explained her suspension of civil and political rights during the Emergency, "there comes a time in the life of the nation when hard decisions have to be taken").[9]

Mohandas Gandhi was himself adept at fusing conflicting symbols of the nation into those of a supposedly essential Indian identity to legitimate and bolster the cause of nationalism. Although he launched his political movement from the city of Ahmedabadi and was himself educated abroad, as was Nehru, he crafted his political identity as oppositional to Western modernization. Khilnani lauds the skill with which Gandhi "reversed priorities, and embraced the very values the colonial imagination rejected" by promoting an image of rural life: "He composed his own pastorale, and used it both to disrupt the order and regularities of the colonial city and to ridicule the hollow mimicry of the Indian elites and middle classes. He brought the nationalist idea from the city to the villages, and through the long foot marches he took across the countryside...he constructed a new topography of India, defined not by the railway tracks that linked cities but by the routes that connected villages" (125). This strategy does not detract from the sincerity of Gandhi's commitment to political self-determination through rural life; rather, it exemplifies the way in which the gendered model of the nation (here figured as the opposition between rural life and pacification on the one hand and urban colonial violence on the other) may create a foundation for political resistance.

While Rushdie's own ideal image of India corresponds more closely to Nehru's vision of a specifically Indian modernity than it does to Gandhi's rural vision, he does not condemn Gandhi's nationalist program. Instead, Rushdie warns of the dangers of conflating the political and the mythic through the figure of Indira Gandhi. When she appears in *Midnight's Children* as the Widow, she is both Prime Minister and the "Mother goddess," Kali (known for her destructive powers).[10] In his study of Hindu goddesses, David Kinsley describes Kali as a terrifying figure, her blackness bejeweled by children's corpses, serpents, and skulls and her long fingernails and teeth smeared with the blood of her victims (116). She is often pictured alone or with the God Shiva, the only one who can temper her excesses when she threatens to destroy the world she helped create. According to Joanna Liddle and Rama Joshi, she functions within rather than against a patriarchal religious structure: "The story of Kali -- India's matriarchal myth -- is that she was created to save the gods from their more powerful enemies, but having done so, she continued on a rampage of uncontrollable killing, which could only be stopped by her husband Shiva lying down in front of her" (55). Kinsley reads more flexibility in that image, noting that she often appears to be dancing on top of the prostrate or even corpse-like Shiva. In both cases, she remains the mythological embodiment of unrestrained feminine fury.

Kali's counterpart is Parvati, the goddess of domesticity who softens Shiva's destructiveness and who, in Kinsley's words, "persuaded or provoked him into creating a child, who was necessary for the preservation of the world" (41). Shiva and Parvati as the other two most important of midnight's children provide a parallel context for the figure of Kali in the novel. Shiva is Saleem's enemy, rival, and midnight twin, and Parvati is Saleem's one true ally, whose liaison with Shiva produces Aadam (Saleem's surrogate son and the representative of the next generation of Indian independence).

Indira Gandhi as Kali first appears to Saleem in a dream. Dressed in the colors of the national flag and foreshadowing the future she has in store for midnight's children, the Widow

> sits on a high chair the chair is green the seat is black the Widow's hair has a centre-parting it is green on the left and on the right black. High as the sky the chair is green the seat is black the Widow's arm is long as death its skin is green the fingernails are long and sharp and black. Between the walls the children green the walls are green the Widow's arm comes snaking down the snake is green the children scream the fingernails are black they scratch the Widow's arm is hunting see the children run and scream the Widow's hand curls round them green and black. (249)

With this malevolent image, Rushdie denounces Indira Gandhi's attempts to legitimate her power -- as captured in the slogan "Indira is India" -- through mythic references which place her own harsh measures within the context of Hinduism and its "reconciliation of extremes: erotic passion and ascetic renunciation, frenzied motion and unmoving calm, violence and passivity."[11] By refering to her only as the Widow, Rushdie insists such reconciliation is impossible: her usurpation of traditionally masculinized state power and sexual power goes unchecked in the absence of a husband. Shiva in the novel serves not as her husband but as her son -- perhaps the true child of midnight as well as the Widow's henchman.

When Rushdie replicates the Hindu dualities of the Widow or Kali (destruction) and Parvati (benevolence), he does so to warn of these rhetorical and metaphorical ploys; at the same time, by scripting these religious figures according marital status, he reinforces a patriarchal critique of Indira Gandhi's authority.[12] While Parvati does give birth to the next generation of midnight's children, she is nonetheless destroyed in one of the Widow's campaigns. Wifely obedience and loyalty are no match for unrestrained, insatiable feminine appetite for power. Rather than serve to define and protect some form of inclusive Indian community, these references ultimately mythologize authoritarian rule and the suppression of difference while reinforcing a patriarchal political ideology.

What we see in this example -- both in the role Gandhi defines for herself and Rushdie's characterization of it -- is an attempt to bridge pre-colonial and post-colonial identities through the metaphor of the family tree. By representing herself as simultaneously timeless and modern, Indira Gandhi legitimates her power through a seamless construction of Indian identity, one that connects, in Chatterjee's terms, spiritual and material domains. She is "Mother India" in its mythic and modern form.

As both the daughter of Nehru and, through her marriage, the namesake (though not a relative) of Gandhi, figures who have themselves gained near mythic status, Indira represents the contemporary scion of a lineage of nationalist power. In encouraging her image as "Mother India" (Indira-Mata), she hopes to capitalize on the beneficent associations with the image.

Whereas the language of the family should describe a national community that is continuous and united, in *Midnight's Children* it reveals the desire for those attributes and their costs. The Widow uses her power to suppress figurative and literal challenges to her authority: she dismisses parliament, suspends civil rights, and, true to Rushdie's metaphor, sterilizes the children of midnight's independence to drain them of their magic and fertility. Saleem, like Rushdie, wavers between this hopeless view of modern India, where the only "purpose of Midnight's Children might be annihilation" (274), and the endless possibilities for alternative stories permeating the narrative form itself.

We see another example of the centrality and failure of the family metaphor at the highpoint of Saleem's powers: when he nightly convenes the Midnight Children's Conference (MCC), acting as translator, parliamentarian, president, and d.j. for all the voices in his head of the children born at the moment of India's independence. Upon recognizing his power to tune into and connect all the thought communications of midnight's children, Saleem conceives of himself as the foundation for an ideal bourgeois public sphere: "I had in mind...a sort of loose confederation of equals, all points of view given equal expression" (263).

For Habermas, defining the liberal bourgeois model which informs the MCC, the ideal public sphere is a space, between the private domain of the home and the official public domain of the state and courts, in which individuals converse as generic citizens on the common good. Communication itself is disinterested and undistorted so that identity remains noncorporeal and discourse alone constitutes the group. Although Habermas recognizes this as ideal, he nonetheless sees it as a crucial component of completing the heretofore incomplete project of modernity.

Rushdie illustrates the inherent *failures* of the model: the impossibility of erasing identity (Saleem transmits visual and oral communications; the children are motivated by self-interest), of delineating this sphere from others, and of creating a neutral, equal space (although "ungrounded" in many respects, the conference takes place in Saleem's mind).[13] Almost as soon as this microcosm of a secular and participatory public sphere commences, it is fractured by power struggles, conflicts of belief, and an inability to act.

The first to speak out is Shiva, Saleem's rival in magic powers (as opposed to Saleem's telepathy, Shiva has the gift of physical prowess, used to destroy enemies and to father an untold number of illegitimate children), whose denied birthright has condemned him to life on the streets. In response to Saleem's idea for the MCC, Shiva demands, "What we ever goin' to do with a gang like that? Gangs gotta have gang bosses." While Saleem tries to maintain order through his role as "big brother" over the family of magically-gifted children who together symbolize the birth of the new nation, Shiva disrupts it with ridicule and contempt. In response to Saleem's search for the common good, his rival thunders, "Rich kid, you don't know one damn thing! What *purpose*, man? What thing in the whole sister-sleeping world got *reason*, yara?...(Y)ou got to get what you can, do what you can with it, and then you got to die. That's reason, rich boy. Everything else is only mother-sleeping *wind*!" (263-4). Saleem eventually fails at maintaining the allegiance of his "siblings" in this national family and the conference, as reflected in Saleem's own body, gradually disintegrates. In an ironic twist on the family metaphor and its capacity for "mother-sleeping" and "sister-sleeping" corruption, Shiva becomes the Widow's enforcer, leading the sterilization program (as did Indira Gandhi's son, Rajiv) and thereby escaping its ultimate effects. His lasting legacy is the horde of children he fathers (before voluntarily undergoing sterilization), including Aadam, his son with Parvati (his figurative sister in the family of midnight's children).

Once again the mythic allusions to Shiva and Parvati and to the "reconciliation of extremes" are domesticated through the nation as family metaphor. It is no surprise then that Shiva and Aadam survive while Parvati dies in the Widow's destruction of the magicians' ghetto. By juxtaposing the Widow and the MCC, Rushdie shows how the nation as family serves multiple political agendas. It is at once a critique of the metaphor used to legitimate political power and an acknowledgment of the necessity of metaphor in creating a sense of belonging. The novel stops short of providing alternatives to the model it criticizes, although it offers the imaginative space for such an alternative to exist.

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Radhakrishnan and Chatterjee call for a recognition of communal identities outside the spectrum defined by the Enlightenment subject and his nation, identities which may form the basis for alternative constructions of national identity. Chatterjee's historiographic project illuminates examples of specifically Indian modernity to unravel "an inelegant braiding of an idea of community with the concept of capital" (237). Instead of beginning to read modern postcolonial nationalism according to Western standards of political organization, he looks at formations of nationalism within the "spiritual domain" during the colonial era (6). Thus, in a reformulation of the traditional gender roles used to substantiate national identity, the inner or spiritual domain becomes the foundation for alternative modernities.

This approach, combined with Radhakrishnan's insistence on reading nation and gender simultaneously, suggests a way out of the dilemma posed by Jameson's approach to national allegory and Chatterjee's own dualistic model. Here the connections between the libidinal or private and the national are neither refuted nor evaluated solely in terms of their relationship to the imperial center; rather, alternative historiographies expand and critique the category of the modern nation "from within itself," to include non-Western examples drawn from community identifications (237). Despite his attempts to theorize community as foundational to Indian modernity, however, Chatterjee ends on a note of disappointment:

> The irony is, of course, that this other narrative is again violently interrupted once the postcolonial nation state attempts to resume its journey along the trajectory of world-historical development. The modern state, embedded as it is within the universal narrative of capital, cannot recognize within its jurisdiction any form of community except the single, determinate, demographically enumerable form of the nation. It must therefore subjugate, if necessary by the use of state violence, all such aspirations of community identity. These other aspirations, in turn, can give to themselves a historically valid justification only by claiming an alternative nationhood with rights to an alternative state. (238)

He concludes that efforts to de-colonize the imagination of the modern postcolonial national subject are ultimately thwarted by an inability to theorize nation and community simultaneously.[14] Modernity itself in this reading becomes synonymous with subject-nation identifications defined by essentialized genders and capital expansion.

Rushdie does not offer successful mediating communities in the subject-nation relationship; he makes the relationship itself suspect through its literalization. As Chatterjee predicts, communities like the magicians' ghetto -- home of "conjurers and contortionists and jugglers and fakirs" (461) who "disbelieved, with the absolute certainty of illusionists-by-trade, in the possibility of magic" (462) -- which challenge the Widow's mythic stature are destroyed; those which flourish, such as the pickle factory, support the prevailing ideology of economic growth and historical progress.

The transformation of Saleem's childhood home, Methwold's Estate, into the pickle factory, site of his current literary and culinary endeavors, illustrates how that ideology is domesticated and maintained. When William Methwold sold his estate of four villas to Indian families he stipulated that "the houses be bought complete with every last thing in them, that the entire contents be retained by the new owners; and that the actual transfer should not take place until midnight on August 15th" (109). As Methwold intended, his estate (in addition to his unacknowledged fathering of Saleem) exerts his influence long after independence, determining much of the flavor of Saleem's childhood "kingdom":

> the Estate, Methwold's Estate, is changing them. Every evening at six they are out in their gardens, celebrating the cocktail hour, and when William Methwold comes to call they slip effortlessly into their imitation Oxford drawls...and Methwold, supervising their transformation, is mumbling under his breath. Listen carefully: what is he saying? Yes, that's it. 'Sabkuch ticktock hai,' mumbles William Methwold. All is well. (113)

When family fortunes decline in political, religious and economic turmoil, a group of enterprising, industrializing women inherit the estate and transform it first into a land reclamation project (a miniaturized version of the British construction of Bombay) and later the pickle factory. Thus, Saleem ends his story back "home" where he began it, as manager of the factory under his old nurse Mary Pereira by day and writer by night.

Although geographically circular, Saleem's path traces the linear development of the modernizing nation-state, the legacy of Methwold and his estate. What has changed in the process, however, is the gendering of public and private worlds, rather than the phallic power which prevails, as the women control the process of capitalization Methwold began.

The transformation of the domestic into the commercial has a revisionary potential in terms of the gendered signifiers of the nation as family metaphor. That potential is limited, however, by the depiction of the "formidable" and "strong armed" women who took over Methwold's Estate, marking their endeavor with a pink obelisk, and by Saleem's own insistence at remaining at the center of the story as the preserver of history.[15]

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As bridges between readers, subjects, and nations, the narrators are successful to the extent that they can define themselves as spokesmen for the nation in order to naturalize the nation as family metaphor. Challenges to that metaphor appear simultaneously as challenges to the narrators' authority and, thus, to our own reading pleasure such that we work with them to preserve the illusion of narrative authority. When we identify with the narrators in this role, they function as cinematic sutures, roles that seems particularly appropriate for an author who honors the Bombay film industry with cinematic images, language, and processes of identification.[16] In *Midnight's Children*, time passes through images such as "a calendar ruffled by a breeze, its pages flying off in rapid succession to denote the passing of the years" (414), and chapters "fade out." Rushdie creates a relationship between text and reader that is both constructed and real when he invites readers to approach the screen until the images dissolve and "the illusion itself is reality" (197). We are urged to surrender ourselves temporarily to the text as we might to a film in a darkened theater.

Rushdie makes this relationship seductive through magic realism with its destabilizing conflation of the fantastical, historical, and quotidian; we enter a world where meaning depends on our willingness to accede to the interdependence of the fantastic and the "real." Since the novel refuses any return to secure epistemological or ontological foundations, we must rely on the terms the narrative provides for our identifications. At the same time, *Midnight's Children* and *The Moor's Last Sigh* are historically grounded and motivated. Magic realism forces us to recognize, however, that we conceptualize our histories through ideologically-laden metaphors, that identification takes place in the (Lacanian) Imaginary. While we can never step outside of the beliefs that structure our realities, we can learn to recognize our complicity with those signifying systems.

Rushdie's novels can seem at once profoundly misogynist and self-critical. They present the authoritative masculine mind as constituting and constructing the narrative of the modern nation against as well as for a sensual, material, feminine ground. Saleem and the Moor are literally writing for their lives and their texts substitute for children they cannot father. While Saleem writes for Padma, his female audience and caretaker, the "lotus-goddess of the present," the Moor writes to keep alive himself and Aoi Uë, his fellow prisoner, comforter, and conscience who "dragged him down to earth."

Particularly in *Midnight's Children*, the relationship between author and audience replicates the gendered divisions of national identity described by Radhakrishnan and Chatterjee: Saleem has agency to write the nation while Padma maintains his literally essential foundation of rural, feminine national spirit. Without Padma, her "ignorance and superstition" and her "paradoxical earthiness of spirit," Saleem loses his inspiration. It is only when she returns to care for him that he says, "I am balanced once more -- the base of my isosceles triangle is secure. I hover at the apex, above past and present, and feel fluency returning to my pen" (232). At the same time, his faulty memories, labyrinthine discourse, impotence, and impending disintegration mock the ideology of the modern nation as nationalism-patriarchy-progress.

Saleem's authority becomes suspect when he separates it from historical "truth." In its chronicle of Indian independence, for example, the story jumps from the Amritsar massacre of 1919 (when General Dyer ordered Indian troops to open fire on unarmed civilians gathered for a Hindu festival) to 1942 (the year of Gandhi's "Quit India" campaign). Brennan notes, in *Salman Rushdie and the Third World*, that in the historical gap, with its virtual excision of Gandhi's role in the independence movement, "the story of Indian nationalism is erased from the book that documents its sad outcome, and the most dramatic illustration of Rushdie's argument is an absence" (84).

This erasure is not the same critique of nationalism Rushdie makes through the character of the Widow; there authoritarian nationalism, rationalized in mythic terms, works in the name of purity to suppress difference. In the example above, eliding Gandhian political forces of nationalism (while noting the religious factionalism among Indians) has two distinct effects: it privileges Jawaharlal Nehru's vision of Indian nationhood over Gandhi's and it leaves a void Saleem fills with his own claims to power. Even as Saleem's telepathy brings him into the minds of the Prime Minister and his cabinet, however, he reminds us of his own fallibility:

> Re-reading my work, I discovered an error in chronology. The assassination of Mahatma Gandhi occurs, in these pages, on the wrong date. But I cannot say, now, what the actual sequence of events might have been; in my India, Gandhi will continue to die at the wrong time.
>
> Does one error invalidate the entire fabric? Am I so far gone, in my desperate need for meaning, that I am prepared to distort everything -- to rewrite the whole history of my times purely in order to place myself in a central role? (198)

Although he is quite willing to keep himself at the center of the story at all costs, Saleem's questions serve as a reminder that other sources of knowledge (as well as other kinds of political nationalism, such as Nehru's) exist, though within the text only he can provide access to them.

These lies, faults, or failures, however, do not invalidate his narrative in favor of another; he emphasizes in his parodies of "official" histories and by interweaving historical and his own fictional renderings of events that "(n)obody, no country, has a monopoly of untruth" (389). As David Lipscomb makes clear, Rushdie's amalgamation of perspectives, "denies the reader the comfort of settled 'truths,' 'truths' that some claim to find in univocal political or disciplinary discourses."[17] Rushdie himself suggests in his "Errata'" essay in *Imaginary Homelands*, "The reading of Saleem's unreliable narration might be...a useful analogy for the way in which we all, every day, attempt to 'read' the world" (25).

By continually asserting and subverting the narrator's power, Rushdie employs two literary suturing techniques defined by Brian Finney: point-of-view narration and meta-fiction. In point-of-view narration, the protagonist gives us an unfettered view of events, thereby masking the presence of the author: "(W)e oscillate between anxiety at the threatened intrusion of the narrator's voice and pleasure (*jouissance*) once we have sutured over this intrusion by occupying a similar locus to that of the protagonist" (138). While the idea that point-of-view narration could succeed in fully masking the author's presence only makes more sense theoretically than practically, Rushdie presents an egregious example of this perspective to call its effects into question. Saleem, for example, asks the reader to "imagine yourself inside me somehow, looking out through my eyes, hearing the noise, the voices..." (202). The reader's pleasure and satisfaction necessitate a willingness to trust the process of signification, even as the suture itself (by addressing the reader directly) hints at the fissures underlying of our desire for narrative cohesion.

Point-of-view narration in the novels promotes the identification between narrator and reader in the paternalistic terms Silverman outlines. By contrasting the feminine audiences' desire for simple linearity with the narrators' convoluted and metaphorical texts, the reader marks his or her own sophistication at the expense of that fictional, feminine audience.[18] When Saleem, frustrated by Padma's demands for a more straightforward tale, wishes for "a more discerning audience, someone who would understand the need for rhythm, pacing, the subtle introduction of minor chords which will later rise" (116), he is, of course, invoking and flattering the contemporary reader.

The second narrative technique, meta-fiction, works in opposition to point-of-view narration to align the reader with the author at the expense of the fictional subject. Rushdie does this by regularly addressing the reader in a voice which does not quite match that of the ostensible narrator. In *The Moor's Last Sigh*, Rushdie writes: "And so for the yarn of the Moor: if I were forced to choose between logic and childhood memory, between head and heart, then sure; in spite of all the foregoing, I'd go along with the tale" (85-6). The passage reads contextually as the Moor's musing on what to believe about his past, though it may also read as an authorial reminder to the reader that he or she is also *choosing* to "go along with the tale." At other times, the novel addresses the reader directly in anticipation of questions he or she is probably asking: "Control, please, your horses," the meta-narrator insists to ward off impatience and retain the reader's allegiance.

In an example from *Midnight's Children*, Rushdie and/as Saleem seem to speak in tandem when the narrator describes the metaphorical possibilities of midnight's children:

> Reality can have a metaphorical content; that does not make it less real. A thousand and one children were born; there were a thousand and one possibilities which had never been present in one place at one time before; and there were a thousand and one dead ends. Midnight's children can be made to represent many things, according to your point of view; they can be seen as the last throw of everything antiquated and retrogressive in our myth-ridden nation, whose defeat was entirely desirable in the context of a modernizing, twentieth-century economy; or as the true hope of freedom, which is now forever extinguished; but what they must not become is the bizarre creation of a rambling, diseased mind. (240)

Rather than enabling the reader to experience the pleasure of identification with the protagonist, meta-fiction, according to Finney, "constitutes its readers as intellectual problem solvers," as "participants in" rather than "consumers of" the text (140); the resulting pleasure masks its origins in the metatextual strategy itself. In the quote above, we can indulge with Saleem in a philosophical moment which defends his own narrative or we can separate ourselves from Saleem and join with

Rushdie in deciphering the puzzles of Indian independence. What we must not do, however, is dismiss them both.

Rushdie also disrupts the authority of point-of-view narration with his diverse literary allusions which emphasize narrative construction. Such references, drawn often from canonical Western texts, presumably resonate loudest with Western readers. At the same time, since the premise of the novels is the relationship between the protagonists and modern India, readers may find themselves trying to solve the "riddle of midnight" -- of independent India -- with "external" clues. In the name of hybridity, this cross-cultural puzzle underscores the interpenetrability of cultures which Rushdie self-consciously represents: "I did not have to make a synthesis. I was merely the echo of a culture which is really plural."[19] Read critically, the puzzle reproduces the terms of Jameson's model of postcolonial literature in that what counts as a modern nation is measured by Western standards. Aijaz Ahmad, for example, cites Rushdie's cosmopolitan literary tastes, as Brennan does, as an evasion of political responsibility for counteracting imperialism, a self-indulgent reflection of Rushdie's own privileged migrancy. While Rushdie's allusions stem no doubt at least in part from British schooling and expatriate life, they do not reconstruct a homogeneous Western tradition; rather, much like Saleem's own mixed legacy, they emphasize the lack of epistemological purity in narrating the nation, a lack which corresponds to the nation's own history.

In a final example of the variety of meta-fictional effects at work, in *The Moor's Last Sigh*, Rushdie's own current political predicament sometimes leaps out of the narrative: "(H)ad I slipped accidentally from one page, one book of life on to another, -- in my wretched, disoriented state, had my reading finger perhaps slipped from the sentence of my own story on to this other, outlandish, incomprehensible text that had been lying, by chance, just underneath?" (285). These diverse strategies challenge the reader's complacent assumption of a stable perspective vis à vis the text, yet all work simultaneously to involve the reader in the stakes defined by the narrative.

Point-of-view narration and meta-fiction compete for the reader's loyalty. As we shift from the narrator to the author's perspective(s) and back again, we are reminded again and again of how our "national longing for form" supersedes our, and the text's, ability to meet it. The tension or play of perspectives merely reproduces our desire for allegorical purity and narrative cohesion on more than one level. We look for Saleem and the Moor to show us what India "looks like" in the familial terms both we and they understand, and we turn to Rushdie for an invitation to help "solve" the riddle of midnight as posed by the characters. In this way, we rely on the division of material and spiritual, masculine and feminine national identities because our own perspectives align with those of the empowered and modern "authors." Despite the allegorical frameworks of the novels, Rushdie insists that "they don't work in any kind of exact formal sense; you cannot translate the structure of [*Midnight's Children*] into the secret meaning, the book is not a code."[20] The conflicting paths of identification disrupt any pat formulas; the two kinds of suture continually draw attention to the very wound they seek to bind.

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Revisiting the wound reminds the reader of the historical crises in the nation's history, moments when the nation as family failed to hold its members together: its colonial past, interreligious strife, the Emergency, the rise of the neo-colonial elite, and the secessionist wars. It reminds us of the impossible task of definitively solving the riddle of midnight. In moving from point-of-view narration to meta-fiction, we recognize the spatial difference between our own corporeal coordinates and those of the subject of the text and of the temporal lapses implicit in these shifting identifications. In other words, although identification is necessarily unconconscious, we become aware of its processes and their dependence upon the text's mnemonic structure.

Memory, Saleem and Rushdie's, forms the connective tissue of *Midnight's Childen*: "Morality, judgment, character...it all starts with memory," Saleem writes, "...and I am keeping the carbons" (253). The story is "(m)emory's truth, because memory has its own special kind. It selects, eliminates, alters, exaggerates, minimizes, glorifies, and vilifies also; but in the end it creates its own reality, its heterogeneous but usually coherent version of events; and no sane human being ever trusts someone else's version more than his own" (253). Caught between Saleem and Rushdie's memory, the coherence we seek continually unravels from the instigatory mnemic traces driving the narrative itself.

In *Midnight's Children,* memory is neither trustworthy nor nostalgic, but it is productive, even as it calls forth ruptures in coherent subjectivity and national identity. Memory's implicit heterogeneity holds the promise for pluralism which the text itself attempts to deliver. As Joseph Swann argues, memory in the text spurs "the reproductive cycle of art," the only reproductive capacity Saleem retains and Rushdie can offer (260). Just as Silverman outlines in her aesthetic theory how the aesthetic works through the stimulation of "new" and barred memories to displace the subject from normative identifications, in the novel we see how memory's productive capacity -- creating its own truths, stimulating the narrative itself, presenting other ways of seeing as well as the terms of its own critique -- serves as an antidote to national purity.

Rushdie recognizes the danger implicit in such a reading of turning his aesthetic (with its selective memories) into an escape from history rather than an entry into it. He notes in *Imaginary Homelands,* for instance, that "imaginative truth is simultaneously honourable and suspect" (10), though he defends his "broken mirror" of memory as a useful reflection of the "provisional nature of all truths" (12). Gorra raises the question more pointedly in his discussion of Rushdie's aesthetic principle: "to bend Indian life this way or that, to make us believe in the illusions of telepathy or in metaphors that seem to come literally true -- and always to remember what reality is. The illusion becomes not an aspect of the country's corruption but a comment on it" (146). While this strategy aims to make us "think critically not only about Indian politics and identity but also about the terrible seductive force of Saleem's -- of

[Rushdie's] own -- desire to encapsulate the whole of reality" (146), Gorra remains "troubled that a book about the nightmare of history cannot make me care about the individual characters to whom that history happens" (147).

Gorra's definition of Rushdie's aesthetic principle is a useful one, particularly in its characterization of the ways in which illusion and metaphor refocus our attention on the nature of reality. His definition elucidates how meta-fictional perspectives eventually surpass Saleem's point-of-view in garnering our attention. I am not convinced, however, that the privileging of intellect over emotion necessarily limits the aesthetic value or the historical commitment of the text. Rushdie's aesthetic insists that ideological beliefs work in collaboration with rather than despite libidinal stimulation to create pleasure, that both aesthetics and history share a set of signifiers and images through which we see the world. Indeed, his project seems less about separating aesthetics and history than in showing how we can understand one through the other. The memories which fuel Saleem's overflowing story may replicate the images provided by the dominant fiction; but it is Rushdie's aesthetic focus on those images, and his manipulation of them through illusion and metaphor, that allows us to achieve the critical distance necessary to see how they are deployed historically and ideologically.

Unsurprisingly, then, Saleem's amnesia, suffered between the two wars with Pakistan (he was "only wiped clean whilst others, less fortunate, were wiped out," 413), is both purifying and nearly lethal. While the blow that makes him "empty and free, because all the Saleems go pouring out...restored to innocence and purity" (419-20) initally provides relief from his weighty inheritance, it turns into a "seceding from history" and, with it, political responsibility. "But how convenient this amnesia is, how much it excuses!" Saleem later exclaims looking back on his shifting political allegiances and his wartime activities (426). Amnesia facilitates his ultimate political submission (he becomes a citizen and "soldier" of Pakistan) and reduces him to his most basic desires. His extraordinary sense of smell the only vestige of his former self, he is used as a tracking dog by Pakistani forces during the second war (resulting in the independence of Bangladesh) with India.

When he finally leads his three companions astray from their wartime mission in the jungle, without hope of being found and divorced from all ties to their former lives and from "the type of memories which give men a firm hold on reality" (417), all four fall prey to the immediacy of their own fantasies. Instead of trying to rescue themselves by remembering who they are and what their responsibilities might be, they worship at Kali's jungle temple for the illusory rewards of nightly visits from four "soft women." It is only when they suddenly recognize their growing transparence that "they understood that this was the last and worst of the jungle's tricks, that by giving them their heart's desire it was fooling them into using up their dreams, so that as their dream-life seeped out of them they became as hollow and translucent as glass" (439). Kali's dissipation of their dreams parallels the Widow's sterilization campaign, which Saleem terms, "Sperectomy: the draining out of hope" (521). Both seek to eliminate heterogeneous beliefs, memories, and histories in the name of ideological purity and to channel the subject's desires into forms easily satisfied or contained by the state.

These ectomies initiate the spreading of invisible cracks that threaten to turn Saleem into "specks of voiceless dust" (552): "Because in drainage lie the origins of the cracks: my hapless, pulverized body, drained above and below, began to crack because it was dried out" (550). To forestall the surfacing of lack that threatens both his and the nation's unity, he spins memories into a national and personal narrative of unity. During Padma's temporary absence, for example, he complains, "A balance has been upset; I feel cracks widening down the length of my body; because suddenly I am alone, without my necessary ear, and it isn't enough" (177). While the narrative process can only delay Saleem's inevitable break-up, Rushdie intends its form, modelled on the oral narrative, to capture "the Indian talent for non-stop self-regeneration...The form -- multitudinous, hinting at the infinite possibilities of the country -- is the optimistic counterweight to Saleem's personal tragedy" (*IH*, 16).

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Aesthetic form has a similar sustaining and procreative function in *The Moor's Last Sigh*. Although the novel begins as the Moor's chronicle of his family history, interwoven with that of the nation, it ends with the revelation that he is imprisoned by his mother's former protégé, Vasco Miranda, who will kill him as soon as the narrative ends: "He had made a

Scheherazade of me. As long as my tale held his interest he would let me live" (421). Other parallels with *Midnight's Children* abound, situating *The Moor's Last Sigh* as a kind of sequel to the chronicle of capital and narrative expansion that are at once the nation's only hopes and greatest threats. Saleem and Aadam (now Adam) reappear, along with the flourishing pickle factory in contemporary Bombay. Competition for the role of Mother India resurfaces in expanded form to include not just Indira Gandhi and various Hindu goddesses, but also a film star and her movie role, the Moor's mother, Aurora, and his lover, Uma. Meanwhile, the historical parameters of India's story have grown to include early Portuguese spice traders and Moorish conquerors on one end and the age of information and fluid capital on the other.

If *Midnight's Children* chronicles both the yearning for a pure and stable identity and its inevitable failure, *The Moor's Last Sigh* investigates impurity in all its forms: it details how love of country eroticizes the nation as family metaphor, resulting in a seemingly endless array of sexual, economic, political, and religious corruptions. At the same time, the novel, like *Midnight's Children,* is a paean to the revelatory potential of the aesthetic. Through the image of the palimpsest, which runs throughout the novel to characterize the city, markets, paintings, politics, and characters, Rushdie insists that aesthetic texts can reveal what usually remains hidden, that within their impurities lie other truths.

The Moor is our guide through a series of false Edens in which the "romantic myth of the plural, hybrid nation" (227) gives way to "debauchery and crime" (303). Descended from Cochin Jews, Moorish

Sultans, Christians, and, significantly, perhaps Prime Minister Nehru himself, the Moor defines himself as "a jewholic-anonymous, a cathjew nut, a stewpot, a mongrel cur. I was -- what's the word these days? -- *atomised*. Yessir: a real Bombay mix" (104). Born a decade after independence, he represents the city itself, his own fantastical growth rate (he ages at twice the average speed) a mirror of urban sprawl: "I grew in all directions, willy-nilly. My father was a big man but by the age of ten my shoulders had grown wider than his coats. I was a skyscraper freed of all legal restraints, a one-man population explosion, a megalopolis, a shirt-ripping, button-popping Hulk" (188).

The Moor's embodiment of Bombay provides insight into the kind of Indian modernity Rushdie prizes. The city, whose long history of commercial and industrial growth sets it apart from the modernity of the nation-state's bureaucracy in New Delhi or the colonial history of Calcutta, was "the great powerhouse of Indian economic modernization": it "became permanently lodged in the popular imagination," according to Khilnani, "as a totem of modern India itself" (136). Its mixture of economic growth, cosmopolitanism, and congested class diversity finds its greatest expression of the national imagination in the film industry located there. Particularly in a nation with widespread illiteracy and 22,000 distinct dialects, film provides the kind of common cultural ground necessary for an imagined national community that Benedict Anderson ascribes to print capitalism. The Moor, as we shall see, is himself located at the intersection of these facets of national identity represented by and in Bombay, embodying both their promises and failures.[21]

In addition to his accelerated development, the Moor is distinguished by his deformed right hand. Like Saleem's nose, this deformity symbolizes and substitutes for the phallic power the narrator wants yet can never wholly achieve, particularly after he, too, becomes impotent. As Silverman notes in *Male Subjectivity at the Margins,* ideological consistency depends upon the aligament of phallic power with the male sexual organ. In making that metaphorical connection literal in the texts, Rushdie once again makes it available for conscious scrutiny. The lack which "drainage above and drainage below" occasioned in Saleem is replaced here by the Moor's inability to assume his legacy as the only son of Aurora and Abraham, to pass on the tradition of economic growth (their wealth comes from a long history of spice trading) and imaginative renewal they possess.

Once again the image of Mother India determines the contemporary terms of that legacy. The Moor, who insults his mother and trades an eroticized relationship with her for a doomed affair with multi-personality Uma, is disinherited for his disloyalty and cast into the underworld from the Eden of his mother's artistic salon. The cost of losing his mother's love, or of forsaking it, is his identity. Finding himself in the hidden bowels of the central jail, beneath the city he thought he knew in its entirety, he dreamt "that my skin was indeed coming away from my body, as I had dreamed so long ago that it would. But in this version of the dream, my peeling skin took with it all elements of my personality. I was becoming nobody, nothing; or, rather, I was becoming what had been made of me. I was what the Warder saw, what my nose smelled on my body, what the rats were beginning, with growing enthusiasm, to approach. I was scum" (288). Without the protective and unifying image of Aurora, the Moor finds himself at the disposal of competing ideological factions.

This tension, between the eroticization of the nation as family and the need for the metaphor in maintaining a sense of self, is replayed throughout the novel. What varies, however, is the image of Mother India herself. The following scene illustrates the centrality of the nation as family metaphor on the level of plot and structure, as well as the way in which Rushdie transfigures its terms. Aurora, whose paintings present her son's life against an expressionist national backdrop, presides over one of her infamous soirées. Within the novel as a whole, she represents an irreverent and urban alternative to Indira Gandhi's "Indira is India" and the hit film *Mother India*'s feminine symbolics (which rely on images of Hindu mythology and rural spirit). At the party, Aurora addresses the leading lady of *Mother India*, who plays Radha, and her husband Sunil who plays the wayward son Birju:

> The first time I saw that picture, I took one look at your Bad Son, Birju, and I thought, O boy, what a handsome guy -- too much sizzle, too much chilli, bring water. He may be a thief and a bounder, but that is some A-class loverboy goods. And now look -- you have gone and marry-o'ed him! What sexy lives you movie people leadofy: to marry your own son, I swear, wowie.

Despite the guests' shocked protestations regarding the difference between "fictions" and "flesh and blood," Aurora insists on conflating them:

"'Even in the picture, but, I knew right off that bad Birju had the hots for his gorgeous ma.'"

Another painter -- Aurora's devotee and, later, the Moor's captor -- Vasco Miranda, encourages the analogy at the guests' expense:

> Sublimation, of mutual parent-child longings, is deep-rooted in the national psyche. The use of names in the picture makes the meaning clear. This "Birju" moniker is also used by God Krishna, isn't it, and we know that milky "Radha" is the blue chap's one true love...*Mother India* is the dark side of the Radha-Krishna story, with the subsidiary theme of forbidden love added on. But what the hell; Oedipus-schmoedipus! (138)

In this scene, Rushdie intertwines the actual film (arguably the most widely recognized national icon produced by the Bombay film industry) and its actors with a fictitious storyline (made, in the case of the relationship of Aurora and the Moor, to suggest the film version) in order to make literal the metaphors of national identity and to make us ask if any of these incarnations of Mother India has a greater claim on "reality" than the others.

This strategy forces readers to reconsider readings of the film itself, its importance as an image of national identity, and the wider validity of the nation as family narrative as embodied by the maternal image. The film, actually made in 1957, the year of the fictitious Moor's birth, achieves its success through the displacement of minority identities in favor of Hindu nationalism. According to Nalini Natarajan in her analysis of the film's bid to define a coherent national identity, the lead actress's Muslim identity is coopted and forgotten in her marriage to her Hindu co-star and in the film's Hindu "cultural message...with its echoes of Radha, Parvati, Sita with all of the traditional self-sacrificing virtues ascribed to these women. We have, then, a nationalist articulation of Hindu religion and culture focusing on the figure of a Muslim actress" (85). In Rushdie's hands, the film's image of Mother India is complicit with rather an oppositional to the dominant Western model of the nation as family. While Mother India might seem to provide the basis for alternative national identifications, based on the split between femininized tradition and masculinized modernity, in fact that image works to promote both majoritarian politics and the normative identifications of the Oedipal complex.

In the novel, Rushdie plays off and subverts the familiarity of the image of Mother India by revealing the palimpsest of conflicting meanings it contains. Instead of one stable image of Mother India which pre-dates and survives colonial experience and includes (consumes) all of the nation's religions and tongues, we are forced to choose between competing images. With the exception of Indira Gandhi, moreoever, as in the film none of those images is represented by Hindu women. Just how elastic is the image, Rushdie seems to be asking. How far will it bend to accommodate the plurality of the nation while still maintaining a sense of national belonging amenable to the values of the Hindu majority?

On one level of the plot, Rushdie contrasts Indira Gandhi's authoritarianism with the plurality of Aurora's painting, noting that, as discussed previously, they are mutually exclusive ideologically and public favor wavers from one to the other. Through the incompatibility of Aurora and Indira's visions of the nation (Aurora's paintings are panned by the public when Indira's authority heightens), Rushdie shows how aesthetic value remains inevitably tied to political context.

Further confusing the imagery of the age, while the public rejects Aurora's exhibit, it embraces Uma's abstract sculptures on the themes of religion and motherhood. Thus, the dichotomy of Parvati and Kali resurfaces in the fortunes of Uma and the Prime Minister. Despite her artistic success and ability to insinuate herself into positions of power, however, Uma fails to maintain a stable alternative identity even as Parvati. Her aesthetic depiction of religion and motherhood is just one of many veneers that she dons in order to get what she wants. Those veneers mask a dangerous and hollow core dedicated, much like Kali herself, to consuming and destroying everything in her path in an insatiable bid for control. It is only after she has contrived to destroy the Moor's relationship with his family and finally kills herself that the Moor sees her as a warning against facile multiplicity: "what had happened was, in a way, a defeat for the pluralist philosophy on which we had all been raised. For in the matter of Uma Sarasvati it had been the pluralist Uma, with her multiple selves, her highly inventive commitment to the infinite malleability of the real, her modernistically provisional sense of truth, who had turned out to be the bad egg" (272).

When difference is reduced to equivalence, or multiculturalism becomes only a matter of style and not belief, Rushdie warns, it loses its historical and political foundations. Uma easily exchanges one identity for another, depending on political expediency, thereby emptying them all of real significance. Nothing binds them together except her own self-interest. Once again, Rushdie points to the dangers of abandoning a metaphor of belonging to bind disparate identities together. Although traditional Mother India works in conjunction with a patriarchal ideology, without that image -- or a substitute for it -- identity becomes a pliant tool in service of self-promotion. Uma's assumption of the Mother India image, in this regard, functions analagously to the transformation discussed earlier of traditional communal identifications, based on religion or caste, for instance, into vehicles for national political ascendancy.

Despite her apparent political irrelevance during Indira Gandhi's rule, Aurora's remains the most important alternative to the rural matriarch of the film and to Gandhi's own authoritarianism. Whereas *Mother India* idealizes "the Indian peasant woman...as bride, mother, and producer of sons; as long-suffering, stoical, loving, redemptive, and conservatively wedded to the maintenance of the social status quo," Aurora, the Moor says, "was a city girl, perhaps *the* city girl, as much the incarnation of the smartyboots metropolis as Mother India was village earth made flesh" (139). The implicit and ironic comparison is between the cosmopolitan Bombay film industry and its most popular product.

Wealthy, headstrong, and visionary, Aurora refuses to bend her artistic and personal attitudes to prevailing tastes. Although she flouts tradition as a matter of course -- from her marriage to the Jewish duty manager in her father's vast export company to her artistic flamboyance to her many lovers (ranging from a Hindu nationalist leader to the first Prime Minister), she remains committed to the ideal image, promulgated by Nehru, of a secular India committed to protecting its diverse community interests. That same commitment to political communion distinguishes Aurora's form of aesthetic pluralism from Uma's.

Their aesthetic difference also finds a parallel in the distinction between Nehru's secularism and that of, say, the United States. Rather than the legal and bureaucratic protection of individual rights guaranteed by the U.S. Constitution (captured by the notion of one's "inalienable rights"), Nehru developed a strong central state dedicated, constitutionally and through the power it shared with regional governments, to protecting specific forms of communal cultural identities. Khilnani reads Nehru's strategy as a specifically Indian response to the problem posed by modernity of forging diverse cultural traditions into a secular unity. The effectiveness of the strategy, according to Khilnani, was its flexibility to adapt to changing social conditions in the formation of nationhood: "Nehru's idea of Indianness emerged through improvised responses to constrained circumstances: its strength was not its ideological intensity, but its ability to steer towards an Indianness seen as layered, adjustable, imagined, not as fixed property" (167). The image of India promoted by Aurora and Nehru, then, is unified, but not static.

As a member of an (elite) economic, religious, and ethnic minority, Aurora tries to incorporate her family history into a national aesthetic vision. Uma, and Adam as we shall see, representatives of the next generation of Indians, define themselves through a seemingly ahistorical internationalization of languages and images rather than their plural or hybrid forms. The familial and cultural contexts of both Uma and Adam are either wholly fabricated (on Uma's part) or elided (Adam's quasi-mythic parentage from Shiva and Parvati and his rearing by Saleem and the pickle factory women remain the concerns of *Midnight's Children* rather than this novel).

Aurora's paintings reflect the changing fortunes of her family and the nation, within and against the image of Mother India. Her "career" begins with the mural she paints across her room after her mother Belle's death dispels the idyllic trance of childhood. The mural incorporates the stories of her childhood without their sanitizing gloss: Vasco de Gama, her ancestor, arriving in India, smelling spices and money; the Last Supper with her family members attending their feasting servants; the masons of the Taj Mahal losing their hands to prevent any finer construction; the approaching war for independence; erotic temple imagery through a child's eye and her own fanciful gods.

Like the crowd that swallows Saleem Sinai in *Midnight's Children* and Rushdie himself in *The Riddle of Midnight*, the mural draws Aurora's astounded father "onward" into "the crowd without boundaries": "Aurora had composed her giant work in such a way that the images of her own family had to fight their way through this hyperabundance of imagery, she was suggesting that the privacy of Cabral Island was an illusion and this mountain, this hive, this endlessly metaphoric line of humanity was the truth" (60). It was Mother India in all her manifestations -- "Mother India with her garishness and her inexhaustible motion, Mother India who loved and betrayed and ate and destroyed and again loved her children, and with whom the children's passionate conjoining and eternal quarrel stretched long beyond the grave" (61) -- presided over from the center and height of the ceiling by a Mother India with Belle's face. Personal and national longing unite in the mural in recognition of the lack that can never be made good, the melancholia that can never be resolved; her over-abundant, imaginative depictions of histories and identities offer the only possible compensation. Aurora pursues this joint political and aesthetic vision throughout the rest of her life, rising to prominence as both an artist and agitator for a secular independent India and refusing to be claimed by any one school, group, or party along the way.

While Aurora continually paints portraits of her family against the backdrop of the nation and the fortunes of the two are linked throughout the story, her aesthetic aims are communal rather than comprehensive. In other words, her paintings explore the problem of imagining the nation rather than present a definitive, singular perspective of it. After independence, for example, Aurora finds herself in a creative conundrum, caught between "Vasco Miranda's playful influence, his fondness for imaginary worlds whose only natural law was his own sovereign whimsicality, and Abraham's dogmatic insistence on the importance, at that historical juncture, of a clear-sighted naturalism that would help India describe herself to herself" (173). It is, of course, the same problem of satisfying "the national longing for form" that drives *The Riddle of Midnight* and *Midnight's Children,* presented in another medium.

Aurora's artistic style emphasizes, through its use of layers, mirrors, diptychs and triptychs, the multi-faceted dimensions of the real and the need to look beyond the surface to find it.

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Aurora's subsequent artistic periods loosely mark the Moor's development and the political fortunes of India's "first family," that of Indira Gandhi.[22] These paintings, divided into her early period (1957-1977, from the Moor's birth to the ousting of Indira Gandhi after the Emergency), high period (1977-1981, until Gandhi regained power), and dark period (1981-1987, from the Moor's disinheritance through the assassinations of Indira and Rajiv marking the end of the Gandhi "dynasty"), all focus on the Moor. He functions not only as a representative of the nation and as a window into the family's affairs, but also, in his depiction as Sultan Boabdil (the last Sultan of Granada) from whom he is descended on his father's side, as a symbol of the nation's long history.

In the early period, as if to record the hopefulness of India and her only son as well as of the dawn she personifies, Aurora paints eroticized portraits of herself and the Moor in which his deformed hand, otherwise a sign of colonialism's disfigurement of the national body, "was transformed into a series of miracles" (224). As torches of light and symbols of power and fertility, these depictions of the Moor's hand (and his rapid growth) make the vestiges of the colonial past into a new source of strength. That strength lasts as long as he remains loyal to his mother as his one true love, and her paintings reflect the seemingly limitless potential of his future and the diversity of the country.

In these paintings, his deformity, which marks him, as does his heritage, as marginal and possibly even cursed (the family name translates as "unlucky"), transforms the world around him into his kingdom. Aurora's masterpieces of the period portray their affluent Malabar Hill home as the Moor's palace, a vestige of the Moorish invaders' sister forts in Delhi and Alhambra, overlooking the Chowpatty Beach slums below:

> "Call it Mooristan," Aurora told me. "This seaside, this hill, with the fort on top...Place where worlds collide, flow in and out of one another, and washofy away...One universe, one dimension, one country, one dream, bumpo'ing into one another, or being under, or on top of. Call it Palimpstine. And above it all, in the palace, you" (226).

Palimpstine and Mooristan encapsulate the romantic myth of Indian pluralism, and throughout her life Aurora urges the Moor to search for them. Even after the Emergency forever ends that period of hopefulness, and her paintings turn from palimpsests to apocalyptic images of division, she gives him a passport with a Spanish visa and one-way ticket: "Always keep it valid...Only don't go to the English. We have had enough of them. Go find Palimpstine; go see Mooristan" (235).

Aurora invokes the symbol of the Moorish invaders' power, Delhi's Red Fort, to represent what Nehru called in his independence address "the noble mansion of free India where all her children may dwell." Rushdie himself creates an historical palimpsest, layering, as it were, architectural references to India's political history stretching from the Moorish invasion (which brought Islam to India) to the Mughal empire (with the Taj Mahal) to colonial power to the current seat of the central state government. He circulates the image of the Red Fort in an attempt to accomplish aesthetically what Nehru tried politically: to forge symbols of a united India which simultaneously reflect its complicated past. Aware of the need for symbolic identification to compensate for the lack of historical unity, Nehru himself inaugurated the state ritual of raising the national flag from the fort on each anniversary of independence. The Red Fort thus forces its readers into the conflicted arena of memory and forgetting, as outlined by Benedict Anderson in *Imagined Communities*, making identification with the unified nation a matter of suspended disbelief.

Khilnani describes the ritual flag-raising (15) as just one aspect of Nehru's program to create a foundation for Indian national identity. Nehru also pursued this program through the exercise of state power, itself a legacy of the political arena left over from the British Raj, and through his history, *The Discovery of India* (1946). In both cases, Khilnani writes, Nehru "relied on a compelling, if imaginary, story of the Indian past, told as a tale of cultural mixing and fusion, a civilizational tendency towards unification that would realize itself within the frame of a modern nation state. He located this story of an internal impulse towards Indian unity within a larger story of the movement of world history, a narrative of diverse peoples coming to determine their own futures and to participate in the benefits of economic progress" (166). This strategy, with its "internal impulse" toward a Western model of modernity based on economic expansion and state power, reformulates the model of postcolonial national identity presented by Chatterjee and Radhakrishnan. Nehru, and Rushdie through the figure of Aurora, undoes the opposition between passive femininized tradition and masculinized modernity, making way for a specifically Indian version of political modernity.

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Despite the transformative potential of these reworked images of national identity, Rushdie never completely abandons the paternalistic nation as family model of Indian history. In *Midnight's Children,* Rushdie regularly associates history with Saleem's disfigurement. From his sterilization to the spreading of the cracks to an array of other scars, disfiguration plagues Saleem as it plagues the nation: "life has been transmuted into grotesquery by the irruption into it of history" (61). Rushdie encourages us to read the Moor's disfigurement as a similar conflation of personal and national disaster: "Banished from the natural, what choice did I have but to embrace its opposite? Which is to say, *unnaturalism,* the only real ism of these back-to-front and jabberwocky days. Placed beyond the Pale, would you not seek to make light of the Dark? Just so. Moraes Zogoiby, expelled from his story, tumbled towards history" (5).

Rushdie's criticism of the effects of history on the subject-nation derive not from a desire to separate aesthetics and history, but from the historical and political events themselves. When the Moor transfers his devotion from his mother to Uma (coinciding with the Emergency), Aurora's paintings turn dark and threatening. In place of the multiple worlds fading in and out of one another, she paints jagged fissures swallowing up her fantastical creatures and shows the fort crumbling into rubble. Gradually her style becomes more naturalistic as she shows herself watching the Moor watching Uma. It reveals Aurora's self-conscious awareness of her ebbing power to hold his love and allegiance.

After Uma engineers the Moor's expulsion from his family, Aurora's aesthetic eye trails him into the underworld where she watches his decline with increasing horror. He later sees himself in these last paintings as "(m)otherless...his previous metaphorical rôle as a unifier of opposites, a standard-bearer of pluralism, ceasing to stand as a symbol -- however approximate -- of the new nation, and being transformed, instead, into a semi-allegorical figure of decay" (302).

To survive in the underworld beneath the cosmopolitanism of Bombay, he learns to use his deformed hand as a club, enforcing the will of a Hindu nationalist leader. Here we find, what Norman Rush calls in his review of the novel, "a mordant reflection on the final outlook for religious nationalism in India, whose most cheering conclusion is that any hope for the downfall of that institution lies in the infinite mercenary corruptibility of the human species" (7). The only escape from the underworld is, paradoxically, up through its ranks of corruption. At the top the Moor finds his father, Abraham, presiding over a corporate empire which stretches from land development projects to drug smuggling to weapons production.

Rushdie presents religious nationalism and economic corruption as the alternatives to the modern plurality of Aurora's vision and Bombay's cosmopolitanism. The Moor's downfall mirrors the changing fortunes of the city itself, as Khilnani describes them; his underworld experiences find political expression in the rise of Bombay's Shiv Sena party. In the vacuum left by the organizational collapse of the Congress Party, begun by Indira Gandhi's restructuring of the central party's regional alliances after Nehru's death in 1964, and by the increasing disparity in Bombay between the political power of rich and poor, the Shiv Sena movement provides a source of communal identification and political will.

Khilnani charts the Shiv Sena ("army of Shivaji") party's rise from the 1960s to the present as the triumph of religious sectarianism over pluralism. What began as an "anti-immigrant party, dedicated to protecting employment and educational opportunities for Bombay's Marathi-speakers" has, with a keen sense of political opportunity, increased its power by targeting Tamils, other English-speaking migrants, and, most recently, Muslims (142). It has increased its political might by providing basic services to its constituents and, in the 1980s, by fostering sectarianism through violent riots against Muslims and their property. The alliance the Shiv Sena forged with the BJP in 1984 provides a further example of the way in which, at least for Rushdie, the transformation of Bombay into Mumbai reflects the wider political climate of the nation.

Khilnani supports this comparison, as he cites Rushdie in his analysis of the Shiv Sena:

> 'In Bombay all Indias met and merged. In Bombay, too, all-India met what-was-not-India...what was beautiful in Bombay was that it belonged to nobody and to all' -- that old nationalist dream of Bombay, and the sense of its end, suffuses Salman Rushdie's lament for the city...The Shiv Sena visualizes India not as a land of cosmopolitan miscegenation, but as a hierarchical grid that contains internally homogenous communities, each insulated from the others. This idea seeks to efface Bombay's cosmopolitanism, to annex its modernity and distribute the benefits of it to one, closed community (143-4).

Just as Rushdie represents the differences between Nehru and Indira Gandhi's national visions through Aurora and Uma, he shows the more recent conflict between national and international identifications through Aurora and Vasco Miranda's paintings. Aurora's final painting, which the Moor sees only after her death and whose title the Moor appropriates for his own story, shows mother and son reunited in the same panel but not reconciled. While she holds out her hand in forgiveness, he is in the foreground, "lost in limbo like a wandering shade" (315).

The Moor's Last Sigh is also the title of Vasco Miranda's homage to Aurora's palimpsests. Initially commissioned by Abraham in 1947 to do a portrait of Aurora with her first child, Ina, Miranda painted a bare-breasted Aurora cradling air. Abraham, incensed by this apparent insult to his wife and child, rejects the portrait and sends Miranda back into the studio where he paints a self-portrait as the last Sultan -- *The Artist as Boabdil, the Unlucky (el-Zogoybi), Last Sultan of Granada, Seen Departing from the Alhambra.* Or, *The Moor's Last Sigh* -- over the rejected Madonna tableau. Miranda's self-portrait quickly confirms his commercial potential, launching his career as an internationally renowned muralist of airports and corporate headquarters. In the aesthetic comparisons Rushdie makes between Aurora, Uma and Miranda, he represents the dual challenge to pluralism by rising communitarianism and the international spread (and corruption, in Rushdie's view) of capitalism. Both drain an aesthetic of difference of its historical and cultural substance.

The multiple meanings of the title converge at the end of the novel when, seeking to escape from the collapse of his family empire and the city he loves, the Moor travels to Spain, hoping to find in Miranda's "Little Alhambra" Aurora's four stolen paintings. "Go find your precious Palimpstina. Go see Mooristan," his father urges just before the city explodes (371). The Moor locates Miranda ensconced in what appears to be a fantastical tribute to Aurora's imagination and his own obsession with her. Gradually the Moor sees the vulgarity of the images and decay emerges from the illusion: "No, it was not a miracle, after all, my first impressions had been illusory, and the illusion had already faded. The 'Little Alhambra', for all its size and flamboyance, was no New Moorusalem, but an ugly, pretentious house" (409). This realization comes too late to save him from imprisonment. He finds himself locked in a garish tower with a painting restorer, Aoi Uë, both prisoners of

Miranda's vengeful obsession. While Aoi Uë works each day to uncover the madonna portrait of Aurora from beneath Miranda's self-portrait, the Moor is forced to record his family history as his own "last sigh."

The Moor's pilgrimage ends in failure because he finds "an anti-Jerusalem: not a home, but an away. A place that did not bind, but dissolved" (388). The fantasy world Miranda created to enliven the Moor's childhood (and alleviate the self-doubt caused by his deformity) cannot sustain him now, nor substitute for the healing power of the maternal image. Ultimately only Aurora's aesthetic visions, with their melding of history, myth, and imagination, can mend his physical and psychological wounds. Here the narrative imitates the potential of the palimpsest as an aesthetic form. After Aurora's death, Miranda's destruction of his portrait of her, the theft, and loss of her own paintings, only the text itself retains her image(s).

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Rushdie presents alternatives to Aurora's secular pluralism bound by the maternal metaphor as nothing more than false Edens. In addition to the religious singularism Rushdie countermands for obvious reasons, he presents unscrupulous and unrestrained capitalization as yet another national affliction. Adam and Abraham epitomize this most recent corruption. Last seen as an infant in the pickle factory of *Midnight's Children*, Adam reappears here as the symbol of the global market. At only 17, he has amassed a private fortune with his business savvy and, like Uma, he can use any linguistic trope to ingratiate himself into wealth and opportunity. His commercial flair seduces Abraham who adopts him into the family and business:

> "'We're going to put our *footprint* on the world,' beamed Abraham, proud of knowing the word's new connotation. 'What villagers these locals are with their talk of the rule of Ram! Not Ram Rayja but RAM Rayja -- that is our ace in the hole.'
>
> "*Not Ram but RAM*: I recognised at once the young fellow's sloganising touch."

"There was a new Adam in Eden," the Moor writes after he is displaced by the adoption. "The future had arrived. There was a generation waiting to inherit the earth, caring nothing for old-timers' concerns: dedicated to the pursuit of the new, speaking the future's strange, binary, affectless speech -- quite a change from our melodramatic garam-masala exclamations" (343). The distinction the Moor makes between his own cross-cultural borrowings and lineage and Adam's internationalism parallels the argument Rushdie makes for his own discursive hybridity.

Adam's international tongue masks a new form of corporate colonialism dedicated to the growing empire of capital. His memos

> quickly became legendary. 'To optimise manpower utilisation, engendering of a we-feeling is key,' they typically said...Further 'encouragement' was given to the idea that each employee should offer monthly 'evaluations' of his fellows' strengths and weaknesses -- thus turning the building into a tower of hypercritical

> (overtly huggy-wuggy, secretly stabby-wabby) sneaks. 'We will be a listening corporation,' Adam informed us all (343).

That language works hand in hand with Abraham's ability to lose his "humble origins" in becoming a corporate legend, his shady economic interests and a compliant political climate. When the government declares, for instance, that city dwellers not listed on the recent census (that is, the street dwellers) do not exist, Abraham and his colleagues see opportunity, "hiring as many phantoms as they could to work on the huge construction sites springing up on every inch of the new land, and even going so far – O philanthropists! – as to pay them small amounts of cash for their work. 'Nobody ever heard of paying spooks until we began the practice,' said ancient Abraham, cackling wheezily. 'But naturally we accepted no responsibility in case of ill-health or injury. It would have been, if you follow my line, illogical" (187). Only after the foundations of his wealth begin to emerge does Abraham feel regret: "The magic stops working when people start seeing the strings," he tells the Moor. "To hell! I had a damn fine run. Have a bloody apple" (187).

Adam and Abraham represent the threat corporate colonialism poses to national identity. As opposed to the religious nationalism discussed previously (with respect to the parallel storylines of the Moor in the underworld and the changing political climate in Bombay), we find economic post-nationalism. Similarly, we might find in Khilnani's analysis of India's cities an urban counterpart to Adam and Abraham in Bangalore. Populated by a relatively new entrepreneurial, industrial, and technical professional class, Bangalore is the home of many of the largest multinational corporations operating in India. While India provides highly trained, relatively inexpensive labor for corporations such as IBM and Hewlett-Packard, they, in turn, offer workers salaries unmatchable in other parts of the country or other sectors of the economy. Since these corporations wield an economic power which both carries with it and produces international identifications, the city itself "has become the capital of Non-Resident India...[and] this new class too has a secessionist understanding of the idea of India" (148).

The image of false Edens, economic, political and aesthetic, whose idyllic veils are eventually stripped away apportions blame for corruption on Indians themselves rather than on outside forces. Those "sequestered, serpented, Edenic-infernal private universes" (15) that enclose the Moor's family history are sites of privilege and opportunity squandered by greed and corruption. As alternately (elite) minority and national representative, the Moor insists that there are no pure lineages and that all bear responsibility for the nation's fate. His Moorish lineage does not absolve the majority from political responsibility; instead it draws attention to the spread of neo-colonialism in business and politics.

The Moor with whom he is continually compared, Boabdil, Last Sultan of Granada, effectively ended his empire by betraying his father and then capitulating to the Spanish. Rushdie is clearly aware of the ironies of colonialism intersecting in the Sultan's story. Boabdil is the last trace of Arab power in Europe which once competed with Vasco de Gama for trade routes to Asia and which stretched from the Iberian Peninsula to the Sultanate of Delhi, and his decline signals the aggrandizement of Spanish and Portuguese global power. These two colonial narratives intersect in Christopher Columbus, who, in Spain to seek Isabella's patronage for his intended voyage to India, attends the ceremony marking Boabdil's abdication to Catholic rule over Granada in 1491, thus marking the beginning of a whole other colonial narrative.[23]

The Moor bears the weight of colonial history, yet insists upon taking responsibility for national affairs rather than attributing them to the legacies of that past; and, all the while, he retains his faith in his mother's passionate embrace of plurality. As he departs with his stuffed dog Jawaharlal (a sad commentary on the Prime Minister's legacy) from Bombay for Spain, for example, he ruminates on Macaulay's 1835 "Minute on Education" with its encapsulation of the British colonial mentality, concluding "a class of 'Macaulay's Minutemen' [Indians educated by the British to facilitate colonization] would hate the best of India...We [his family] were not, had never been, that class. The best, and worst, were in us, and fought in us, as they fought in the land at large. In some of us, the worst triumphed; but still we could say -- and say truthfully -- that we had loved the best" (376).

The love that perseveres at the end of the novel is for Aurora's mongrelized Mother India. Just as Aoi Uë painstakingly unveils her from beneath the Sultan's image, the Moor tries to reconstruct her in his narrative. Yet Aurora has already died, the painting will be destroyed before it is restored, and, as Rushdie has shown throughout the text, the image of Mother India is at once redemptive, ferocious, and submissive. This conclusion, much like Aurora's childhood mural, focuses attention on the lack that can never be filled and, therefore, on the aesthetic processes enabling us to suture it. In the same way that *Midnight's Children*'s optimism lies in the narrative's productive form, *The Moor's Last Sigh* retains hope in the aesthetic's capacity for imaginative renewal and penetrating vision, powers which depend upon an acknowledgement of the lack they try to overcome.

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The continual reminder of lack has a productive function in the creation of the distance and deferral (between reader and text) necessary for re-vision of the nation. In *The Moor's Last Sigh,* the recognition of distance and deferral comes through the shifting narrative perspectives and the trope of the palimpsest which always hints at another vision lying just below the surface. According to Silverman, as previously discussed, the re-visionary potential of distance and deferral stems from the way memory functions in aesthetic texts. Whether by uncovering otherwise repressed memories or instilling "new" memories in the unconscious, Silverman argues, aesthetic texts open up paths of identification which are normally foreclosed by the dominant fiction.

In *The Moor's Last Sigh,* Rushdie constructs the living presence of India's mongrelized history out of word plays, parodies, and images; that history refuses to conform to a nostalgic vision of a unified past. Since aesthetic value depends upon pleasure and desire, when a text stimulates these other memories it potentially expands our libidinal range. Rushdie makes this possible through the nation as family metaphor whose familiarity invites our identification with it. Although that metaphor attempts to domesticate difference in order to create a unified national identity, by showing the process of domestication at work, both its success and failures, he calls attention to its newly expanded range of identifications. In becoming conscious of that expanded range, we may learn to idealize what is outside of ourselves. Distance and deferral are the conditions necessary for this ex-corporative idealization and, therefore, identification. The aesthetic can intervene in these normative processes by simultaneously allowing conscious "scrutiny" of their terms and unconscious libidinal stimulation necessary for identification.

Silverman describes the process of ex-corporative identification in terms of sublimation. Following Lacan, she defines sublimation as the "shift away from the impossible non-object of desire which is produced with entry into language and the 'fading' of the real to a nameable and specific object." That new object becomes laden with the responsibility of "making good the subject's lack." Narrative or aesthetic compensation, then, offers the subject images with which to satisfy his or her foundational desires: "When one treats an object in this way, one of course idealizes it. To sublimate is thus to confer ideality on that someone or something through which the subject articulates his or her ineffable desire" (75).

Rather than view the Moor's "atomised" identity and his deformity as failures in his bid to represent the modern nation, or as failures of the nation itself, Rushdie asks us to make a positive identification between the Moor and India through precisely the same terms. In order to do so, we must reconsider the terms of national identity itself. Since the Moor ends up disinherited, impotent, and exiled he fails to complete the equation of paternity, progress, and power with which we started. Similarly, the novel refutes attempts at locating national identity within a feminized, mythic sphere since the images of Mother India are neither consistent nor necessarily alternative to the dominant nation as family model. What remains is the aesthetic unveiling of memories and histories, of the horrific alongside the beautiful. Our identifications, Rushdie insists, can only take place on the common meeting ground of those memories. By bringing the terms of those identifications to the forefront, he enables us to take a conscious second look at the ideological tenets of the nation as family. Even as his narrative depends upon the reproduction of those terms, it makes possible our intervention in their circulation.

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Despite the concluding hope (and, indeed, echo) of *The Moor's Last Sigh*, that we might "hope to awaken, renewed and joyful, into a better time," when Rushdie invites us into the aesthetic realm to escape from the political present, he simultaneously insists that we take responsibility for the aesthetic images of ourselves we sanction and condemn. The enormous challenge of this charge manifests itself in the political (in the sense of the actualization of state power) and aesthetic limitations of Rushdie's pluralist ideal.

Nehru's tremendous gift as a statesman, particularly as India's first Prime Minister, was his recognition of the need to create both an idea of India available for collective identification and a state apparatus and agenda capable of rewarding subjects for that identification in their daily lives (through political and economic opportunity, education and health care, and so forth). In the realm of the imaginary, Nehru, like Rushdie, adopted the image of the palimpsest as a metaphor for Indian heterogeneity and unity: "India," he wrote in *The Discovery of India*, "...was like an ancient palimpsest on which layer upon layer of thought and reverie had been inscribed, and yet no succeeding layer had completely hidden or erased what had been written previously." Nehru used that image as part of his effort to construct (some would say fabricate) a history of diverse peoples who *always* held a sense of "oneness," arguing that "(s)ome kind of dream of unity has occupied the mind of India since the dawn of civilization."[24]

It would be unfair to read the decline of the Congress Party and the rise of religious nationalism and global capitalism, or even of the lapsing fortunes of Nehru's pluralism more generally, as the product solely of a failure of imagination. Whether attributed to the immense task of governing close to a billion people or to specific political failures, the fact that India retains gross inequalities of wealth, education, and opportunity provides an impetus for political unrest. In other words, disparities between the ideal of the nation and the actual compensation the state offers facilitate subjects' withdrawal from the nation's founding ideology. Moreoever, although the examples from *The Moor's Last Sigh* discussed above focus on the relationship between Rushdie's aesthetic and Nehru's urban modernity, it is important to remember that the idea of India was founded on the collaborative visions of Nehru and Gandhi and, thus, on urban and rural identifications.

In drawing attention to these two constitutive tenets of modern India -- the idea of unity awaiting political (national) expression and the joint set of identifications proffered by the "founding fathers," Nehru and Gandhi -- I want to emphasize the access they give us to the nature of the nation's lack. Both Judith Butler and William Connolly, in their psychoanalytic and political analyses of how the state exercises power to maintain the allegiance of its citizens, stress the problem inherent in the legitimation of that power. While the nation, as an ideal, inaugurates the subject's desire for and, thus identification with it, the state (the actuality of that ideal) will always fall short and substitute coercive power for it. The relationship between the nation's ideal and coercive powers creates a parallel set of powers within its citizen-subjects. As Butler explains (and as discussed in Chapter One), an ideal such as nation sets the terms for subjection (*assujetissement*) to the symbolic order that makes possible the subject's (self-) expression; at the same time, it constitutes a subject capable of wielding power (*pouvoir*). The state, Butler argues, at once promotes its status as an ideal and "cultivates melancholia among its citizenry" in order to exercise its power. Citizens, meanwhile, constituted in terms of the ideal forever try to satisfy the desires spurred by its inaugural and inevitable loss.

The exact nature of lack remains inarticulable and unthinkable, precisely because it is foundational to subjectivity. Yet we may approach it through analysis of the cultural narratives that attempt to compensate for the melancholia and desires such loss generates. In Rushdie's celebration of plurality (through Saleem's narrative superfluity, Aurora's imaginative abundance, and the Moor's embodiment of cosmopolitan Bombay) we see the yearning for a unity constructed out of the combined visions of Nehru and Gandhi. This yearning, moreover, points to two "unassimilable" (Caruth) traumas which both found and undermine the nation. The first, and perhaps most wrenching, is Partition, which Khilnani terms, "the unspeakable sadness at the heart of the idea of India: a *memento mori* that what made India possible also profoundly diminished the integral value of the idea" (201-2). The second and related event is Gandhi's assassination by a Hindu militant, who thought Gandhi too pro-Muslim, five months after independence. Both traumas point to the impossibility of the ideal of unity and tolerance.

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Both Partition and Gandhi's assassination suggest that future struggles over identification will be fought in terms of religious difference and Nehru versus Gandhi's visions of modern India. We see these struggles in Rushdie's novels in numerous ways: the virtual elision of

Gandhi from the story of Indian independence (as Brennan remarks); the privileging of an urban modernity that simultaneously neglects the conditions of rural Indians (although it shows the negative effects of modernization and capitalism, particularly on the urban poor); and the critique of sectarianism's rise in pluralism's wake. The sad irony of the novels, however, is that their social critiques -- of capitalism, militant religious fundamentalism, and narratives of purity -- sometimes appear in the all-too-familiar terms of ethnic and religious stereotyping, in the divisive terms the novels otherwise contest.

Rushdie's hope, like Nehru's, is for an Indian pluralism that will provide a viable alternative vision of the modern nation. He outlined this hope in an NPR interview on *The Moor's Last Sigh*: "What India at its best represents is a wonderful demonstration of multiplicity; it's a wonderful demonstration of how people of completely different backgrounds and faiths and ideas can live together and form a composite culture which is greater than the sum of its parts." His own writing attempts to capture that multiplicity through excess and flamboyance which strains against yet ultimately produces a sense of coherence. The narrative pleasures of his aesthetic, thus, provide the compensation for and identification with the terms of the ideal nation. More importantly, it is that aesthetic excess which fosters the kind of critical distance necessary for conscious, and therefore ethical, identifications.

While these teeming narratives work stylistically, their political effects are more ambiguous. The difference underscores the limitations of the aesthetic in challenging the prevailing social order. In the case of *The*

Satanic Verses, of course, imaginative license led to retrenched political and religious strictures and to the state's ultimate expression of its terrorizing potential. What began as, in Rushdie's words, a "lovesong to our mongrel selves" (*IH*, 394), turned into a vehicle for defining cultural animosity between Islam and the West; and that animosity echoes the sectarianism of Partition and Gandhi's death. The "Rushdie affair," moreover, helped define Islam as the West's new bugaboo, replacing the "evil empire" of the Soviet Union with a new supposed threat to freedom and prosperity. Current political rhetoric in the United States barely distinguishes between the Ayatollah Khomeini, the World Trade Center bombing, Saddam Hussein and biological weapons: all are evidence of an insidious attack on "our values" and, in George Bush's famous wartime words, on "the family of nations."

Rushdie himself is only too aware of how political reaction detracts from his literary intentions, how it provides a lens through which to read the book that obscures its aesthetic aims. It is worth quoting his interview with James Fenton in 1991 (two years after the *fatwa* began) at length:

> What I feel most of all, and really this is not bullshit, is a sense of overwhelming failure. [W]hat I tried to do was to bring...India, Islam, the West -- these three worlds, all of which are present inside me in a very vivid way -- I tried to bring them together. I tried to describe each in terms of the other. Well, it didn't work. Whatever one may feel about *The Satanic Verses* as a novel, if you look at the event of *The Satanic Verses* it pushed those worlds further apart...

> There were many times in the months after this began that I said to myself that I no longer wished to be a writer. I felt that everything I had put into the act of being a writer had failed, had simply been invalidated by what had happened. You write out of what you think of as your best self, the best there is in you. If the upshot of that is that the whole planet thinks of you as a complete bastard, you wonder what it's about, what it was for, and why do it.

His attempt to describe different worlds "in terms of each other" is undoubtedly a difficult strategy but perhaps an unavoidable one in a "mongrel" world, plural nation, or textual palimpsest. Much of the time, his overt metaphors bring our prejudices and otherwise naturalized ideologies to the fore, enabling us to look anew at them. Thus, the gradual cracking of Saleem in *Midnight's Children* or the Oedipal jokes in *The Moor's Last Sigh* promote humor and understanding, rather than outrage. In the case of religion, however, as *The Satanic Verses* "event" makes clear, readers are less willing to forgive, perhaps, a similar literary play. There we see that the pleasure and compensation promised by the aesthetic is thwarted by the very losses it seeks to assuage.

Given all this, it is difficult to understand why Rushdie would turn in *The Moor's Last Sigh* to the figure of Abraham to father and to represent capitalist corruption. Even though Rushdie's metaphors are always suggestive rather than prescriptive, using the Jewish patriarch to epitomize the inherent dangers of rampant, corrupt capitalism seems too conveniently to exacerbate anti-semitic stereotypes. Rushdie's critique of capitalism is based in part on its homogenizing effects on cultural difference, its devaluing of the historical and material components of identity. It is unfortunate, then, that his gambit to celebrate impurity of subject, nation and language reinforces ethnic and religious animosities and expands the realms in which they operate. Perhaps the problem of imagining alternative images of subject and nation marks the limits, both our own and Rushdie's, of coming to terms either politically or aesthetically with our foundational loss; for as Saleem says, "New myths are needed; but that's none of my business" (546).

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[1]Perhaps the most widely noted critique issues from Aijaz Ahmad's chapter on Jameson in *In Theory*. Ahmad focuses on the ways in which the "Three Worlds" model necessarily privileges questions of nationalism at the expense of those of material and economic collectivities. As Ahmad notes, "If this Third World is *constituted* by the singular 'experience of colonialism and imperialism', and if the only possible response is a nationalist one, then what else is there that is more urgent to narrate than this 'experience'? In fact, there is *nothing else* to narrate" (102). Neil Lazarus, reviewing Ahmad's book in "Postcolonialism and the Dilemma of Nationalism," points out that Ahmad is critiquing not the 'Three Worlds' model as such, but Jameson's definition of it "in terms not of production but of imperialism" (376). Thus, Ahmad disputes Jameson's paradigm of 'First World'/capitalism, 'Second World'/socialism, and 'Third World'/nationalism on the grounds that it ignores differences between nationalist movements and defines the 'Third World' only according to the effects of colonialism and imperialism (Jameson's foundation for 'Third World' nationalism). Madhava Prasad argues that Ahmad, in subscribing to the same set of terms despite their different definitions, fails to offer a viable alternative to Jameson.

[2]Madhava Prasad, for example, suggests abandoning the "Three Worlds" model in favor of redefining "the libidinal/private in its allegorical status (its relation to particular nations but especially to particular classes -- a class allegory) and collapsing the distinction which originates in capitalist ideology" (78). By replacing the nationalistic effects of public/private distinctions with class ones, Prasad exposes one of the foundational dualities of bourgeois thinking. This approach has the advantage of rethinking public and private, although it does so at the expense of analyzing the national or aesthetic.

[3]Much of the scholarship on *Midnight's Children* focuses on traces in the text of *The Tin Drum*, *Tristram Shandy*, and *Kim*, among others. See, for example, Michael Gorra's *After Empire: Scott, Naipaul, Rushdie*, Richard Cronin's "The Indian English Novel: *Kim* and *Midnight's Children*," Clement Hawes's "Leading History by the Nose: The Turn to the Eighteenth Century in *Midnight's Children*," Michael Harris's *Outsiders and Insiders: Perspective of Third World Culture and British Post-Colonial Fiction*, Colin Smith's "The Unbearable Lightness of Salman Rushdie," K. J. Phillips's "Salman Rushdie's *Midnight's Children*: Models for Storytelling, East and West," and Rudolph Bader's "Indian *Tin Drum*." In "Caught in a Strange Middle Ground: Contesting History in Salman Rushdie's *Midnight's Children*," David Lipscomb also provides a detailed analysis of Rushdie's historiographic borrowings from Stanley Wolpert's *A New History of India*.

[4]Timothy Brennan, Cosmopolitans and Celebrities," *Race and Class* 31.1 (1989): 2, 4. It is important to note that while Brennan here groups Mukherjee and Rushdie into a single category of international cosmopolitan writers, they see themselves quite differently. Mukherjee at the time of Brennan's article had already moved from defining herself as an expatriate Indian to an American. Rushdie, during the same period, considered himself Indian first.

[5]In *Questions of Cinema*, Stephen Heath notes that the shot/reverse shot formation, as described in Oudart and Dayan's systems of suture, does not account for the multiplicity of techniques used to create the identificatory relationship between viewer and cinematic subject (98). At the same time, Heath warns against reading all editing techniques that construct a narrative out of a series of shots (a definition that empties suture of any significance); he maintains a strict reading of suture as the name of the relationship of lack to its ideological system of signification (100-101).

[6]Salman Rushdie, "Talk of the Nation," *National Public Radio* (15 January 1996).

[7]Anthony D. Smith, *National Identity* (Reno: University of Las Vegas Press, 1991), 11-12. Smith emphasizes that both legal-political and ethnic nationalisms depend for success

upon shared collective historical memories: "Transcending oblivion through posterity; the restoration of collective dignity through an appeal to a golden age; the realization of fraternity through symbols, rites and ceremonies, which bind the living to the dead and fallen of the community; these are the underlying functions of national identity and nationalism in the modern world, and the basic reasons why the latter have proved so durable, protean and resilient through all vicissitudes" (163). I want to push this model a step further to look at the way the rhetoric of the family binds the nation, legal-political and ethnic, as a site of collective meaning. Rushdie's work is especially good at showing the struts and rivets that hold these two models together, and it is this exposure of construction itself that makes it possible to imagine other models.

[8]See, for example, Partha Chatterjee's *Nationalist Thought and the Colonial World: A Derivative Discourse?* (London: Zed Books, 1986) for a discussion of Gandhi's use of kinship rhetoric in support of Indian nationalism.

[9]Neil Ten Kortenaar, "'Midnight's Children' and the Allegory of History," *ARIEL* 26.2 (April 1995),45.

[10]See David W. Price, "Salman Rushdie's 'Use and Abuse of History' in 'Midnight's Children'," *ARIEL* 25.2 (April 1994): 91-107. Price uses Nietzsche's division of antiquarian, monumental, and critical history to discuss competing historical narratives within the novel. In Price's schema, William Methwold (Saleem's British progenitor) represents the antiquarian, Indira Gandhi the monumental, and Saleem the critical historian. Whereas the first two betray the ideals of the nation, Saleem represents "the potential to contribute to life" (92). Price discusses Saleem's imaginative construction of the past as a way of making "true narratives" (102) to counter the official "truths" proffered by the government. While this celebration of the imaginary and the metaphoric is in keeping with Rushdie's own faith, captured in narrative form, in "the Indian talent for non-stop regeneration" (*Imaginary Homelands*, 16), it elides the way in which Saleem's errors, lies, and impending dissolution undercut his own authority.

[11]Stanley Wolpert, *A New History of India* (New York: Oxford University Press, 1993), 82.

[12]Rushdie invokes Hindu deities throughout the novels to establish a link between the mythic, narrative, and national "realities." We see this not only in the relationship between Kali and Parvati, but also in Parvati and Shiva's child Adam (described in terms of his mythological counterpart, Ganesh) and in the relationship between Saleem and Shiva. These rivals for leadership over the Midnight Children's Conference, both of whom share the distinction of being born at the exact moment of Indian independence, represent the dichotomy between Brahma and Shiva. As Timothy Brennan says in *Salman Rushdie and the Third World*, "Brahma, as we know, is the god who *dreams* the world. Shiva, we learn, is the god who allows it to exist by declining to use his immeasurable power for destroying it" (113).

[13]Bruce Robbins's collection, *The Phantom Public Sphere* provides an excellent overview of the limits and continued usefulness of the term "public sphere." Contributors show how Habermas's model constitutes the participatory public as white, male, literate, and propertied even while it claims to be noncoporeal and open. Robbins and Nancy Fraser, in particular, respond to the restrictions of the Habermasian public phere with their own models of multiple, shifting spheres. Negt and Kluge's *Public Sphere and Experience* provides one of the fullest elaborations of Habermas, although it falls prey to the same gender exclusions. They rework the ideal public sphere into three overlapping spheres -- the liberal bourgeois sphere, public sphere of production, and the proletarian public sphere -- in order to show how public opinion is constituted in multiple commercial, social, and political domains rather than in the idealized (no)place between public and private. They

also keep these publics firmly attached to contemporary social conditions and media technologies. Arguing that bourgeois society replicates the patriarchal family structure, Negt and Kluge say that the decline of the family and/or entry into the labor force transfers control of patriarchal fantasy-production to the "consciousness industry" (a conglomeration of macro-capitalist interests and media which address the consumer). The "consciousness industry" provides consumers with the "libidinal compensation" for desires constructed in patriarchal terms.

[14]As Chatterjee notes in his introduction, "If the nation is an imagined community and if nations must also take the form of states, then our theoretical language must allow us to talk about community and state at the same time. I do not think our present theoretical language allows us to do this" (11).

[15]In a further revision of gender roles, when Adam and the pickle factory reappear in *The Moor's Last Sigh* he, at only 17, has assumed control of entering the pickles in the global market. With his flair for slogans and marketing, he is the epitome of the self-made man. Mary Periera and her sister are reduced to the "old ladies" (MLS, 349).

[16]In "*The Satanic Verses* as a Cinematic Narrative," Nicholas D. Rombes, Jr. quotes a 1983 interview with Rushdie in *Contemporary Authors*. Telling Jean W. Ross about his use of cinematic techniques and terminology, Rushdie notes, "The whole experience of montage technique, split screens, dissolves, and so on, has become a film language which translates quite easily into fiction and gives you an extra vocabulary of literature." Rombes uses this quote to launch his analysis of cinematic techniques in *The Satanic Verses*. According to Rombes, Rushdie creates a "dichotomy between the narrator's omniscient point of view and the constricted point of view of the 'camera eye'" (50). Understanding and truth come from the narrator's ability to look within characters. The camera's point of view, on the other hand, is limited by the economic and physical conditions of production. Rombes attributes this dichotomy to Rushdie in order to argue that it distinguishes "between seeing as a method of discovering the truth and seeing as a means of recording to speculate on the dangerous potential for manipulating what we see and how we see it via technology" (52). I have a completely different reading of Rushdie's use of the cinematic. As I argue in this chapter, Rushdie appropriates these techniques in celebration, not in criticism. While Rombes sees the coexistence of a narrator and a camera's "eye" as a mark of their difference, I see them as parallel functions. Rushdie uses these "eyes" to underscore the partiality of all vision, regardless of any claims of omniscience. We find this partiality not only in the fallibility of all of Rushdie's narrators, but also in his exuberant magic realism where truth and perception are always in play.

[17]David Lipscomb, "Caught in a Strange Middle Ground: Contesting History in Salman Rushdie's *Midnight's Children*," *Diaspora* 1.2 (1991), 164. Lipscomb is particularly helpful in filtering out Rushdie's references and borrowings from Wolpert's *A New History of India* in *Midnight's Children*.

[18]See, for example, Keith Wilson's analysis (33) of the reader's relationship to Padma in *Midnight's Children*.

[19]Interview, *La Croix* (30 April 1984), 24. Quoted in and translated by Jean-Pierre Durix, "Salman Rushdie's Declaration of Kaleidoscopic Identity," *Declarations of Cultural Independence in the English Speaking World*, ed. Luigi Sampietro (Milan: D'Imperio Editore Novaram 1989), 173. Rushdie has been critiqued for his celebration of plurality on the grounds that it represents his own class privilege. See, for example, Timothy Brennan, "Cosmopolitans and Celebrities," Race and Class 31.1 (1989), 1-19. There Brennan describes Rushdie's exoticism as a marketable mark of difference in the Western literary marketplace. Whereas as Brennan reads Rushdie as "deny[ing] the old pattern of the need

to create a national *mythos* in the country of origin" (4), I see him as pointing out just how difficult that process is. As Saleem notes at the end of book, "New myths are needed; but that's none of my business" (546).

[20]Salman Rushdie, "*Midnight's Children* and *Shame*," *Kunapipi* 7 (1985),3.

[21]Rushdie himself retains a nostalgic longing for the city and its former cosmopolitanism. It's no surprise, for example, that young Saleem of *Midnight's Children*, Rushdie's spokesman, finds his greatest joy when the family returns to the city of his childhood dreams: "Back to Bom! Back to Bom!" he shouts in an uncharacteristic moment of abandon. Rushdie's own love of the Indian film industry is well known and finds its clearest expression in his incorporation of cinematic techniques and allusions in his novels.

[22]Stanley Wolpert, *A New History of India* (Oxford: Oxford University Press, 1993): 410.

[23]Richard Fletcher, in his description of the transfer of power, notes that, "curiously enough, [the Catholic Monarchs] had chosen to dress themselves in Moorish costume for the ceremony" (165). This idiosyncratic appropriation of Moorish dress finds its way into *The Moor's Last Sigh* in Miranda's self-portrait and Aurora's thematic paintings.

In "Christopher Columbus and Queen Isabella Consummate Their Relationship (*Santa Fe, AD* 1492)," Rushdie plays on the interstices of colonial narratives with an eroticized story of colonizing appetites. He presents a frustrated Columbus imagining the Queen at the take-over: "*See: there at the gates of the Alhambra is Boabdil the Unlucky, the last Sultan of the last redoubt of all the centuries of Arab Spain. Behold: now, at this very instant, he surrenders the keys to the citadel into her grasp...there! And as the weight of the keys falls from his hand into hers, she...she...*yawns" (114). The story sexualizes imperial desire so that Columbus and the Queen need each other for their conquests. "'The loss of money and patronage,' Columbus says, 'is as bitter as unrequited love'" (115). Columbus, resenting his dependence on the Queen, fantasizes about refusing her if she calls for him. When the summons finally comes, however, he answers, "*Yes. I'll come*" (119).

[24]Quoted in Shashi Tharoor's *India: From Midnight to the Millenium* (New York: Arcade Publishing, 1997), 127. Khilnani also cites this passage in his analysis of "the idea of India." Both historians read Nehru's image as an attempt to satisfy the condition of historical amnesia (amnesia of a fractured populace) Ernst Renan says is necessary for modern nationhood.

Bibliography

Abel, Elizabeth. "Black Writing, White Reading: Race and the Politics of Feminist Interpretation." *Critical Inquiry* 19. 3 (Spring 1993): 470-498.

Afzal-Khan, Fawzia. *Cultural Imperialism and the Indo-English Novel.* University Park, PA: Pennsylvania State University Press, 1993.

——"Myth De-Bunked: Genre and Ideology in Rushdie's *Midnight's Children* and *Shame*." *The Journal of Indian Writing in English* 14.1 (January 1986): 49-60.

Ahmad, Aijaz. *In Theory: Classes, Nations, Literatures.* New York: Verso, 1992.

Alam, Faruk. *Bharati Mukherjee.* New York: Twayne Publishers, Simon & Schuster Macmillan, 1996.

Ali, Tariq. "*Midnight's Children.*" *New Left Review* 136 (November-December 1982): 87-95.

Anderson, Benedict. *Imagined Communities: Reflections on the Origin and Spread of Nationalism.* New York: Verso, 1991.

Appadurai, Arjun. "The Heart of Whiteness." *Callaloo* 16. 4 (1993): 796-807.

Appiah, K. Anthony. "Is the Post- in Postmodernism the Post- in Postcolonial?" *Critical Inquiry* 17.2 (1991): 336-357.

——"Giving Up the Perfect Diamond." Review of *The Holder of the World.* *New York Times Book Review.* 10 October 1993: 7.

Appignanesi, Lisa and Sara Maitland. *The Rushdie File.* Syracuse: Syracuse University Press, 1990.

Awkward, Michael. "Negotiations of Power: White Critics, Black Texts, and the Self-Referential Impulse." *American Literary History* 2. 4 (Winter 1990): 581-606.

—"Race, Gender, and the Politics of Reading." *Black American Literature Forum* 22. 1 (Spring 1988): 5-27.

Bader, Rudolph. "Indian Tin Drum." *International Fiction Review* 11.2 (Summer 1984): 75-83.

Banville, John. "An Interview with Salman Rushdie." *The New York Review of Books* 40.5 (4 March 1993): 34-36.

Becker, Carol (ed.). *The Subversive Imagination: Artists, Society, and Social Responsibility*. New York: Routledge, 1994.

Benston, Kimberly W. "I yam what I yam: the yopos of (un)naming in Afro-American literature." *Black Literature and Literary Theory*. Ed. Henry Louis Gates, Jr. New York: Methuen, 1984. 151-172.

Berlant, Lauren. *The Anatomy of National Fantasy: Hawthorne, Utopia, and Everyday Life*. Chicago: University of Chicago Press, 1991.

——"National Brands/National Body." In *The Phantom Public Sphere*. Ed. Bruce Robbins. Minneapolis: University of Minnesota, 1993. 173-208.

Bhabha, Homi K. *Nation and Narration*. New York: Routledge, 1990.

——"Interrogating Identity: The Postcolonial Prerogative." In *Anatomy of Racism*. Ed. David Theo Goldberg. Minneapolis: University of Minnesota Press, 1990.

——*The Location of Culture*. New York: Routledge, 1994.

——"Of Mimicry and Man: The Ambivalence of Colonial Discourse." *October* 28 (Spring 1984).

——"'Race', Time and the Revision of Modernity." *Oxford Literary Review* 13.1-2 (1991): 193-219.

——"Signs Taken for Wonders: Questions of Ambivalence and Authority Under a Tree Outside Delhi, May 1817." In *Race, Writing, and Difference*. Ed. Henry Louis Gates, Jr. Chicago: University of Chicago Press, 1986. 163-184.

Birch, David. "Postmodernist Chutneys." *Textual Practice* 15.1 (Spring 1991): 1-7.

Blaise, Clark. *Resident Alien*. Ontario, Canada: Penguin Books, 1986.

Bodnar, John E. *Remaking America: Public Memory, Commemoration and Patriotism in the Twentieth Century*. Princeton: Princeton University Press, 1992.

Booker, M. Keith. "Beauty and the Beast: Dualism as Despotism in the Fiction of Salman Rushdie." *English Literary History* 57.4 (Winter 1990): 977-997.

Booth, Wayne. *The Company We Keep: An Ethics of Fiction*. Berkeley: University of California Press, 1988.

Brennan, Timothy. "Cosmopolitans and Celebrities." *Race and Class* 31.1 (1989): 1-19.

----"India, Nationalism, and Other Failures." *The South Atlantic Quarterly* 87.1 (Winter 1988): 131-146.

----*Salman Rushdie and the Third World: Myths of Nation*. New York: St. Martin's Press, 1989.

Brodhead, Richard H. *The School of Hawthorne*. New York: Oxford University Press, 1986.

Brooks, David. "An Interview with Salman Rushdie." *Helix* 19 (1984): 55-69.

Bruner, Jerome. *Actual Minds, Possible Worlds*. Cambridge: Harvard University Press, 1986.

Buell, Frederick. *National Culture and the New Global System*. Baltimore: Johns Hopkins University Press, 1994.

Butler, Judith. *Bodies That Matter: On the Discursive Limits of "Sex."* New York: Routledge, 1993.

----*Excitable Speech: A Politics of the Performative*. New York: Routledge, 1997.

----*Gender Trouble: Feminism and the Subversion of Identity*. New York: Routledge, 1990.

----"Imitation and Gender Insubordination." *The Lesbian and Gay Studies Reader*. Ed. Henry Abelove, Michèle Aina Barale, and David Halperin. New York: Routledge, 1993.

----*The Psychic Life of Power*. Stanford: Stanford University Press, 1997.

Butler, Thomas (ed.). *Memory: History, Culture and the Mind.* Oxford: Basil Blackwell, 1989.

Butts, Dennis. "From Newbury to Salman Rushdie: Teaching the Literature of Imperialism in Higher Education." *Literature and Imperialism.* Ed. Robert Giddings. New York: St. Martin's Press, 1991: 64-74.

Campbell, Elaine. "Beyond Controversy: Vidia Naipaul and Salman Rushdie." *Literary Half-Yearly* 27 (July 1986): 42-49.

Carter-Sanborn, Kristin. "We Murder Who We Were: *Jasmine* and the Violence of Identity." *American Literature* 66. 3 (September 1994): 573-593.

Caruth, Cathy. *Unclaimed Experience: Trauma, Narrative, and History.* Baltimore: The Johns Hopkins University Press, 1996.

Caton, Louis F. "Romantic Struggles: The Bildungsroman and Mother-Daughter Bonding in Jamaica Kincaid's *Annie John.*" *MELUS* 21.3 (Fall 1996): 125-142.

Caygill, Howard. *The Art of Judgment.* Oxford: Basil Blackwell, 1989.

Chakrabarty, Dipesh. "Postcoloniality and the Artifice of History: Who Speaks for 'Indian' Pasts?" *Representations* 37 (1992): 1-26.

Chatterjee, Partha. *The Nation and Its Fragments: Colonial and Postcolonial Histories.* Princeton: Princeton University Press, 1993.

----*Nationalist Thought and the Colonial World: A Derivative Discourse?* London: Zed Books, 1986.

Chrisman, Laura. "The Imperial Unconscious?" *Critical Quarterly* 32. 3 (1990): 38-58.

Christian, Barbara. "Community and Nature: The Novels of Toni Morrison." *Journal of Ethnic Studies* 7. 4 (1980): 65-78.

Chua, C.L. "Passages from India: Migrating to America in the Fiction of V.S. Naipaul and Bharati Mukherjee." In *Reworlding: The Literature of the Indian Diaspora*. Ed. Emmanuel S. Nelson. New York: Greenwood Press, 1992. 51-61.

Cincotti, Joseph A. "Same Trip, Opposite Direction." *New York Times Book Review*, 10 October 1993: 7.

Connerton, Paul. *How Societies Remember*. Cambridge: Cambridge University Press, 1989.

Connolly, William E. *Identity/Difference: Democratic Negotiations of Political Paradox*. Ithaca: Cornell University Press, 1991.

—*Political Theory and Modernity*. New York: Basil Blackwell, 1988.

Cornwell-Giles, JoAnne. "Afro-American Criticism and Western Consciousness: The Politics of Knowing." *Black American Literature Forum* 24. 1 (Spring 1990): 85-98.

Crighton, S. and L. Shapiro. "An Exclusive Talk with Salman Rushdie." *Newsweek* (12 February 1990): 46-49.

Cronin, Richard. "The Indian English Novel: *Kim* and *Midnight's Children*." *Modern Fiction Studies* 33.2 (Summer 1987): 201-213.

Crosby, Christina. "Dealing with Differences." *Feminists Theorize the Political*. Ed. Judith Butler and Joan W. Scott. Routledge: New York, 1992. 130-143.

Cunningham, Valentine. "Nosing Out the Indian Reality." *Times Literary Supplement*. 15 May 1981: 38.

Danto, Arthur C. "Mughal Art." *The Nation* (February 2/9, 1998): 64-67.

Dayal, Samir. "Talking Dirty: Salman Rushdie's *Midnight's Children*." *College English* 54.4 (April 1992): 431-445.

Degabriele, Maria. "Trafficking Culture in Postcolonial Literature: Postcolonial Fiction and Salman Rushdie's *Imaginary Homelands*." *SPAN* (October 1992): 60-70.

Deleuze, Gilles and Félix Guattari (ed.). *Anti-Oedipus: Capitalism and Schizophrenia*. Minneapolis: University of Minnesota Press, 1983.

Derrida, Jacques. "Economimesis." *Diacritics* vol. 11 (June 1985): 3-25.

---*The Truth in Painting*. Trans. Geoff Bennington and Ian McLeod. Chicago: University of Chicago Press, 1987.

Dharker, Rani. "An Interview with Salman Rushdie." *New Quest* 42 (November-December 1983): 351-360.

Dingwaney, Anuradha. "Author(iz)ing *Midnight's Children* and *Shame*: Salman Rushdie's Constructions of Authority." In *Reworlding: The Literature of the Indian Diaspora*. Ed. Emmanuel S. Nelson. New York: Greenwood Press, 1992. 157-167.

DuBois, W.E.B. *The Souls of Black Folk*. New York: Signet, 1969.

Durix, Jean Pierre. "Magic Realism in Midnight's Children." *Commonwealth Essays and Studies* 8.1 (Autumn 1985): 57-63.

---"Salman Rushdie's Declaration of Kaleidoscopic Identity." *Declarations of Cultural Independence in the English-Speaking World: A Symposium*. Ed. Luigi Sampietro. Milan: D'Imperio Editore Novare, 1989: 173-184.

Eagleton, Terry. *The Ideology of the Aesthetic*. Oxford: Basil Blackwell, 1990.

Esonwanne, Uzo. "'Race' and Hermeneutics: Paradigm Shift--From Scientific to Hermeneutic Understanding of Race." *African American Review* 26. 4 (1992): 565-581.

Ewing, Katherine Pratt. *Arguing Sainthood: Modernity, Psychoanalysis, and Islam*. Durham: Duke University Press, 1997.

Fanon, Frantz. *Black Skin, White Masks*. Trans. Charles Lam Markmann. New York: Grove Press, 1967.

---*The Wretched of the Earth*. Transl. Constance Farrington. New York: Grove Press, 1963.

Fenton, James. "Keeping up with Salman Rushdie." *The New York Review of Books* 38.6 (28 March 1991): 26-34.

Feroza, Jussawalla. "Chiffon Saris: The Plight of South Asian Immigrants in the New World." *The Massachusetts Review* 29.4 (1988): 583-595.

Finney, Brian. "Suture in Literary Analysis." *LIT: Literature Interpretation Theory* v. 2 (1990): 131-144.

Fischer, Michael M.J. "Ethnicity and the Post-Modern Arts of Memory." In *Writing Culture: The Poetics and Politics of Ethnography*. Ed. James Clifford and George Marcus. Berkeley: University of California Press, 1986. 194-233.

Flanagan, Kathleen. "The Fragmented Self in Salman Rushdie's *Midnight's Children*." *The Commonwealth Novel in English* 5.1 (Spring 1992): 38-45.

Fletcher, D. M. (ed). *Reading Rushdie: Perspectives on the Fiction of Salman Rushdie*. Atlanta: Rodopi, 1994.

Fletcher, Richard. *Moorish Spain*. Berkeley: University of California Press, 1992.

Forster, E. M. *A Passage to India*. New York: Harcourt Brace, 1924.

Freud, Sigmund. *The Freud Reader*. Ed. Peter Gay. New York: W. W. Norton & Co., 1989.

—*The Interpretation of Dreams*. Transl. and ed. James Strachey. New York: Avon Books, 1965.

—*Moses and Monotheism*. Transl. Katherine Jones. New York: Alfred A. Knopf, 1939.

Friedlander, Saul, Geulie Arad, and Dan Diner (eds.). *History and Memory: Studies in Representation of the Past*. Bloomington: Indiana University Press, 1994.

Fuss, Diana. *Essentially Speaking: Feminism, Nature & Difference*. New York: Routledge, 1989.

—*Identification Papers*. New York: Routledge, 1995.

Gates, Henry Louis, Jr. *Figures in Black: Words, Signs, and the "Racial" Self.* Oxford: Oxford University Press, 1987.

----(ed.) *"Race," Writing, and Difference.* University of Chicago Press: Chicago, 1985.

----*The Signifying Monkey: A Theory of African-American Literary Criticism.* Oxford: Oxford University Press, 1988.

Glendenning, Victoria. "A Novelist in the Country of the Mind." *Sunday Times*, October 25, 1981: 28.

Gorra, Michael. *After Empire: Scott, Naipaul, Rushdie.* Chicago: University of Chicago Press, 1997.

----"Call it Exile, Call it Immigration." Review of *Jasmine. New York Times Book Review.* 10 September 1989: 9.

Gramsci, Antonio. *Selections from the Prison Notebooks.* Ed. and transl. Quintin Hoare and Geoffrey Nowell Smith. New York: International Publishers, 1971.

Greene, Gayle. "Feminist Fiction and the Uses of Memory." *Signs* 16: 2 (1991): 290-321.

Grossberg, Lawrence. "Cultural Studies and/in New Worlds." In *Race, Identity, and Representation in Education.* Ed. Cameron McCarthy and Warren Crichlow. New York: Routledge, 1993. 89-108.

Habermas, Jurgen. *The Philosophical Discourse of Modernity.* Cambridge: MIT Press, 1978.

----*The Structural Transformation of the Public Sphere.* Transl. Thomas Burger with Frederick Lawrence. Cambridge: MIT Press, 1991.

Haffenden, John. *Novelists in Interview.* New York: Methuen, 1985.

Hall, Stuart. "Ethnicity: Identity and Difference." *Radical America* 23. 4 (1991): 9-20.

——"The Television Discourse--Encoding and Decoding." In *Studying Culture*. Ed. Ann Gray and Jim McGuigan. New York: Edward Arnold, 1993. 28-34.

Haraway, Donna. "Ecce Homo, Ain't (Arn't) I a Woman, and Inappropriate/d Others." In *Feminists Theorize the Political*. Ed. Judith Butler and Joan W. Scott. New York: Routledge, 1992. 86-100.

——*Simians, Cyborgs, and Women*. New York: Routledge, 1992.

Harris, Michael. *Outsiders and Insiders: Perspectives of Third World Culture and British Post-Colonial Fiction*. New York: Peter Lang, 1992.

Harvey, David. *The Condition of Postmodernity*. Oxford: Blackwell, 1990.

Hawes, Clement. "Leading History by the Nose: The Turn to the Eighteenth Century in *Midnight's Children*." *Modern Fiction Studies* (Winter 1993) 39.1: 147-168.

Hawthorne, Nathaniel. *The Great Short Works of Hawthorne*. New York: Harper Row, 1967.

Healey, Beth. "Mosaic vs. Melting Pot." *New York Times Book Review*. June 1988: 22.

Heath, Stephen. *Questions of Cinema*. Bloomington: Indiana University Press, 1981.

Henderson, Mae Gwendolyn. "Speaking in Tongues: Dialogics, Dialectics, and the Black Woman Writer's Tradition." *Reading Black, Reading Feminist*. Ed. Henry Louis Gates, Jr. New York: Meridian, 1990. 116-142.

Hewson, Kelly. "Opening Up the Universe a Little More: Salman Rushdie and the Migrant as Story-Teller." *SPAN: Journal of the South Pacific Association for Commonwealth Literature and Language Studies*, 29 (October 1989): 82-93.

Holston, James and Arjun Appadurai. "Cities and Citizenship." *Public Culture* 8.2 (Winter 1966): 187-204.

Homans, Margaret. "'Women of Color' Writers and Feminist Theory." *New Literary History* 25.1 (Winter 1994): 73-94.

hooks, bell. "Choosing the Margin as a Space of Radical Openness." *Yearning: Race, Gender and Cultural Politics.* Boston: South End Press, 1990.

Hutcheon, Linda. *A Poetics of Postmodernism.* New York: Routledge, 1988.

——"Subject in/of/to History and His Story." *Diacritics* 16. 1 (Spring 1986): 78-91.

Irving, T.B. "The Rushdie Confrontation: A Clash in Values." *Iowa Review* 20. 1 (Winter 1990): 175-184.

Iyer, Nalini. "American/Indian: Metaphors of the Self in Bharati Mukherjee's 'The Holder of the World.'" *ARIEL* 27.4 (1996): 29-44.

Jameson, Frederic. *The Political Unconscious.* Ithaca: Cornell University Press, 1981.

——"Third World Literature in the Era of Multinational Capitalism." *Social Text* 15 (Fall 1986): 65-88.

Joyce, James. *A Portrait of the Artist as a Young Man.* New York: Penguin, 1964.

Juan-Novarro, Santiago. "The Dialogic Imagination of Salman Rushdie and Carlos Fuentes: National Allegories and the Scene of Writing in *Midnight's Children* and *Cristobal Nonato.*" *Neohelicon* 20.2 (1994): 257-311

Jussawalla, Feroza. "Beyond Indianness: The Stylistic Concerns of 'Midnight's Children.'" *The Journal of Indian Writing in English* 12.2 (July 1984): 26-47.

Kammen, Michael. *Mystic Chords of Memory: The Transformation of Tradition in American Culture.* New York: Knopf, 1991.

Kasson, John F. *Amusing the Million.* New York: Hill & Wang, 1978.

Kaufman, Michael T. "Author from Three Countries." *New York Times Book Review* (November 13, 1983): 3, 22-23.

Kehde, Suzanne. "Colonial Discourse and Female Identity: Bharati Mukherjee's *Jasmine*." *International Women's Writing: New Landscapes of Identity*. Ed. Anne E. Brown and Marjanne E. Goozé. Westport, CT: Greenwood Press, 1995: 70-77.

Khilnani, Sunil. *The Idea of India*. New York: Farrar, Straus Giroux, 1997.

Kinsley, David. *Hindu Goddesses: Visions of the Divine Feminine in the Hindu Religious Tradition*. Berkeley: University of California Press, 1986.

Kolodny, Annette. "The Integrity of Memory: Creating a New Literary History of the United States." *American Literature* 57 (1985): 291-307.

Kortenaar, Neil Ten. "'Midnight's Children' and the Allegory of History." *ARIEL: A Review of International English Literature* 26.2 (April 1995): 41-62.

Krishnaswamy, Revathi. "Mythologies of Migrancy: Postcolonialism, Postmodernism and the Politics of (Dis)location." *ARIEL: A Review of International English Literature* 26.1 (January 1995): 125-146.

Lacan, Jacques. *Ecrits: A Selection*. New York: W.W. Norton, 1977.

——*Four Fundamental Concepts of Psycho-Analysis*. Ed. Jacques-Alain Miller. Transl. Alan Sheridan. New York: W.W. Norton and Co., 1981.

Lakshmi, Vijay. "Rushdie's Fiction: The World Beyond the Looking Glass." In *Reworlding: The Literature of the Indian Diaspora*. Ed. Emmanuel S. Nelson. New York: Greenwood Press, 1992. 149-155.

Laplanche, J. and J.-B. Pontalis. *The Language of Psychoanalysis*. New York: W.W. Norton, 1973.

Lazarus, Neil. "Postcolonialism and the Dilemma of Nationalism: Aijaz Ahmad's Critique of Third-Worldism." *Diaspora* 2.3 (1993): 373-400.

Leong, Liew-Geok. "Bharati Mukherjee." *International Literature in English: Essays on the Modern Writers*. Ed. Robert L. Ross. New York: St. James Press, 1991. 487-500.

Levine, George (ed.). *Ideology and Aesthetics*. New Brunswick, NJ: Rutgers University Press, 1994.

Liddle, Joanna and Rama Joshi. *Daughters of Independence: Gender, Caste and Class in India*. London: Zed Books, 1986.

Lim, Shirley Geok-Lin. "Assaying the Gold; or, Contesting the Grounds of Asian-American Literature." *New Literary History* 24 (1993): 147-169.

Lipscomb, David. "Caught in a Strange Middle Ground: Contesting History in Salman Rushdie's *Midnight's Children*." Diaspora 1.2 (1991): 163-189.

Low, Gail Ching-Liang. "In a Free State: Post-Colonialism and Postmodernism in Bharati Mukherjee's Fiction." *Women: A Cultural Review* 4.1 (Spring 1993): 8-18.

Marzorati, Gerald. "Salman Rushdie: Fiction's Embattled Infidel." *The New York Times Magazine*. 29 January 1989: 24.

Mason, Roger Burford. "Salman Rushdie." *PN Review* 15.4 (1989): 15-19.

McClintock, Anne. *Imperial Leather: Race, Gender and Sexuality in the Colonial Context*. New York: Routledge, 1995.

-----"'No Longer in a Future Heaven': Gender, Race and Nationalism." *Dangerous Liaisons: Gender, Nation, and Postcolonial Perspectives*. Ed. Anne McClintock, Aamire Mufti, and Ella Shohat. Minneapolis: University of Minnesota Press, 1997. 89-112.

Meer, Ameena. "Bharati Mukherjee: An Interview." *BOMB* 29 (1989):46-47.

-----"Salman Rushdie: An Interview." *BOMB: Interviews*. New York: New Art Publications, 1992. 61-74.

Messud, Claire. "The Emperor's Tear." Review of *The Holder of the World*. *Times Literary Supplement*, 12 November 1993: 23.

Michaels, Walter Benn. "Race into Culture: A Critical Genealogy of Cultural Identity." *Critical Inquiry* 18 (1992): 655-685.

Miller, Donald. "Omnipotence and Its Enemies." *Third Text* 11 (1990): 135-43.

Miller, Jacques-Alain. "Suture (elements of the logic of the signifier)." *Screen* 18.4 (Winter 1977-78): 24-34.

Mishra, Vijay. "Postcolonial Differend: Diasporic Narratives of Salman Rushdie." *ARIEL: A Review of International English Literature* 26.3 (July 1995): 7-45.

Morley, David and Kevin Robins (eds.). *Spaces of Identity: Global Media, Electronic Landscapes and Cultural Boundaries.* New York: Routledge, 1995.

Morrison, Toni. *Beloved.* New York: Plume Books, 1987.

—*Sula.* New York: Plume Books, 1973.

Mouffe, Chantal. "Citizenship and Political Identity." *October* 61 (Summer 1992): 28-32.

Mukherjee, Arun. *Towards an Aesthetics of Opposition: Essays on Literature, Criticism, and Cultural Imperialism.* Stratford, Ontario: Williams-Wallace, 1988.

Mukherjee, Arun P. "Characterization in Salman Rushdie's *Midnight's Children*: Breaking out of the Hold of Realism and Seeking the 'Alienation Effect.'" *The New Indian Novel in English: A Study of the 1980s.* Ed. Virney Kirpal. New Delhi: Allied Publishers, 1990: 109-119.

Mukherjee, Bharati. "After the Fatwa." With Clark Blaise. *Mother Jones* 15.3 (April-May 1990): 28-31, 61-65.

—*Conquering America with Bharati Mukherjee.* Videocassette. Produced by Bill Moyers. Public Affairs Television, 1990.

—"A Conversation with V.S. Naipaul." With Robert Boyers. *Salmagundi* 50-51 (Fall 1980-Winter 1981): 153-171.

—*Darkness.* New York: Fawcett Crest, 1985.

—*Days and Nights in Calcutta.* With Clark Blaise. New York: Doubleday & Co., 1977.

—"A Four-Hundred-Year-Old Woman."

----*The Holder of the World.* New York: Fawcett Columbine, 1993.

----"Immigrant Writing: Give Us Your Maximalists!" *New York Times Book Review,* August 28, 1988: 1, 28-29.

----"An Interview with Bharati Mukherjee." With Alison B. Carb. *Massachusetts Review* 29.4 (1988): 645-654.

----"An Interview with Bharati Mukherjee." With Michael Connell, Jessie Grearson and Tom Grimes. *Iowa Review* 20.3 (Spring 1990): 7-32.

----"An Interview with Bharati Mukherjee." With Geoff Hancock. *Canadian Fiction Magazine* 59 (May 1987): 30-44.

----"An Interview with Bharati Mukherjee." *View* 20. 3 (1990).

----"An Invisible Woman." *Saturday Night* 96 (March 1981): 36-40.

----*Jasmine.* New York: Fawcett Crest, 1989.

----*Leave It to Me.* New York: Alfred A. Knopf, 1997.

----*The Middleman and Other Stories.* New York: Fawcett Crest, 1988.

----"Prophet and Loss: Salman Rushdie's Migration of Souls." *Village Voice Literary Supplement,* 72 (March 1989): 9-12.

----*The Sorrow and the Terror: The Haunting Legacy of the Air India Tragedy.* With Clark Blaise. Markham, Ontario: Viking Penguin, 1987.

----*The Tiger's Daughter.* New York: Fawcett Crest, 1971.

----*Wife.* New York: Fawcett Crest, 1975.

Mulvey, Laura. "Visual Pleasure and Narrative Cinema." *Screen,* 16. 3 (1975): 8-18.

Naik, M. K. "A Life in Fragments: The Fate of Identity in *Midnight's Children.*" *Indian Literary Review* 3. 3 (October 1985): 63-68.

Natarajan, Nalini. "Women, Nation, and Narration in Midnight's Children." *Scattered Hegemonies.* Ed. Inderpal Grewal and Caren Kaplan. Minneapolis: University of Minnesota Press, 1994.

Nazareth, Peter. "Total Vision." *Canadian Literature* 110 (Fall 1986): 184-191.

Needham, Anurdha Dingwaney. "The Politics of Post-Colonial Identity in Salman Rushdie." *Massachusetts Review* 29.4 (Winter 1988-1989): 609-624.

Negt, Oskar and Alexander Kluge. *The Public Sphere and Experience.* Minneapolis: University of Minnesota Press, 1993.

Nelson, Cecil. "New Englishes, New Discourses, New Speech Acts." *World Englishes: Journal of English as an International Language* 10.3 (Winter 1991): 317-323.

Nelson, Emmanuel S. *Bharati Mukherjee: Critical Perspectives.* New York: Garland Publishing, Inc., 1993.

—-"Kamala Markandaya, Bharati Mukherjee, and the Indian Immigrant Experience." *Toronto South Asian Review* 9 (Winter 1991): 1-9.

—-"Troubled Journeys: Indian Immigrant Experience in Kamala Markandaya's *Nowhere Man* and Bharati Mukherjee's *Darkness.*" *From Commonwealth to Post-Colonial.* Ed. Anna Rutherford. Sydney, Australia: Dangaroo Press, 1992: 53-59.

Nietzsche, Friedrich. "On the Truth and Lies in a Nonmoral Sense." *Philosophy and Truth: Selections from Nietzsche's notebooks of the Early 1870's.* Ed. and transl. Daniel Breazeale. New Jersey: Humanities Press, 1979. 79-100.

Nussbaum, Martha C. *Poetic Justice: The Literary Imagination and Public Life.* Boston: Beacon Press, 1995.

Ondaatje, Michael. "Michael Ondaatje: Interview by Linda Hutcheon." *Other Solitudes: Canadian Multicultural Fictions.* Ed. Linda Hutcheon and Marion Richmond. Toronto: Oxford University Press, 1990. 196-202.

Parameswaran, Uma. "Handcuffed to History: Salman Rushdie's Art." *ARIEL: A Review of International English Literature* 14.4 (October 1983): 34-45.

—-"'Lest He Returning Chide': Saleem Sinai's Inaction in Salman Rushdie's *Midnight's Children.*" *The Literary Criterion* 18.3 (1983): 57-66.

Pathak, R. S. "Identity Crisis in the Novels of Salman Rushdie." *Language Forum* 18.1-2 (1992): 112-134.

Pattanayak, Chandrabhanu. "Interview with Salman Rushdie." *The Literary Criterion* 18.3 (1983):19-22.

Phillips, K.J. "Salman Rushdie's Midnight's Children: Models for Storytelling, East and West." *Comparative Literature East and West: Traditions and Trends. Selected Conference Papers.* Ed. Moore, Cornelia N. and Raymond A. Moody. Honolulu: University of Hawaii Press, 1989: 202-207.

Pinckney, Darryl. *High Cotton.* New York: Penguin, 1992.

----"Slouching Toward Washington." *The New York Review of Books,* (December 21, 1995): 73-82.

Piwiniski, David J. "Losing Eden in Modern Bombay: Rushdie's *Midnight's Children.*" *Notes on Contemporary Literature* 23.3 (May 1993): 10-12.

Prakash, Gyan. "Writing Post-Orientalist Histories of the Third World: Perspectives from Indian Historiography." *Comparative Studies in Society and History* 32.2 (April 1990): 383-408.

Prasad, Madhava. "On the Question of a Theory of (Third World) Literatures." *Social Text* 31/32 (1992): 57-83.

Price, David W. "Salman Rushdie's 'Use and Abuse of History' in 'Midnight's Children'." *ARIEL: A Review of International English Literature* 25.2 (April 1994): 91-107.

Raban, Jonathan. "Savage Boulevards, Easy Streets." Review of *The Middleman and Other Stories. New York Times Book Review.* June 19, 1988: 1, 22-3.

Radhakrishnan, R. "Nationalism, Gender, and the Narrative of Identity." *Nationalisms and Sexualities.* Ed. Andrew Parker, Mary Russo, Doris Somer, and Patricia Yaeger. New York: Routledge, 1992. 77-95.

Rahman, Tariq. "Politics in the Novels of Salman Rushdie." *Commonwealth Novel in English* 4.1 (Spring 1991): 24-37.

Rajan, Gita. "Bharati Mukherjee." *Writers of the Indian Diaspora: A Bibliographical Critical Sourcebook.* Ed. Emmanuel S. Nelson. Westport, CT: Greenwood Press, 1993. 235-242.

Rao, K.B. "Asia and the Pacific: Midnight's Children." *World Literature Today* 56 (Winter 1982): 181.

Rao, Madhusudana. "Quest for Identity: A Study of the Narrative in Rushdie's *Midnight's Children.*" *Literary Criterion* 25. 4 (1990): 31-42.

Reimenschneider, Dieter. "History and the Individual in Anita Desai's *Clear Light of Day* and Salman Rushdie's *Midnight's Children.*" *World Literature Written in English* 23.1 (Winter 1984): 196-207.

Renan, Ernest. "Qu'est-ce qu'une nation?" *Oeuvres Complètes* (Paris, 1947-61), vol. 1, pp. 887-907. Reprinted as "What is a nation?" Transl. Martin Thom. *Nation and Narration.* Ed. Homi K. Bhabha. New York: Routledge, 1990: 8-22.

Robbins, Bruce (ed.). *Intellectuals: Aesthetics, Politics, Academics.* Minneapolis: University of Minnesota, 1990.

—"Introduction." *The Phantom Public Sphere.* Minneapolis: University of Minnesota, 1993.

Rombes, Nicholas D., Jr. "The Satanic Verses as a Cinematic Narrative." *Literature/Film Quarterly* 21.1 (1993): 47-53.

Rosen, Jonathan. "Our Jerusalem."*The New York Times Book Review.* 23 June 1996: 91.

Ross, Bruce M. *Remembering the Personal Past: Descriptions of Autobiographical Memory.* New York: Oxford University Press, 1991.

Ross, Jean W. "Interview with Salman Rushdie." *Contemporary Authors* 111 (1983): 414-417.

Roy, Anindo."The Aesthetics of an (Un)willing Immigrant: Bharati Mukherjee's *Days and Nights in Calcutta* and *Jasmine.*" *Bharati Mukherjee: Critical Perspectives.* Nelson, Emmanuel S. New York: Garland Publishing, Inc., 1993.

Rubenstein, Roberta. *Boundaries of the Self: Gender, Culture, Fiction.* Urbana: University of Illinois Press, 1987.

Rush, Norman. "Doomed in Bombay." Review of *The Moor's Last Sigh. The New York Times Book Review,* January 14, 1996: 7.

Rushdie, Salman. "Damme, This Is the Oriental Scene for You!" *The New Yorker* (June 23 & 30, 1997): 50-61.

----"A Dangerous Art Form." *Third World Book Review* 1 (1984): 3-5.

----*East, West.* New York: Pantheon Books, 1994.

----"In Conversation: 'Fictions are Lies that Tell the Truth." With Günter Grass. *The Listener* (27 June 1985): 14-15.

----"Goodness: The American Neurosis." *The Nation* 242 (22 March 1986): 344.

----*Imaginary Homelands.* New York: Penguin Books, 1991.

----"The Indian Writer in England."*The Eye of the Beholder: Indian Writing in English.* Ed. Maggie Butcher. London: Commonwealth Institute, 1983. 75-83.

----"Interview." National Public Radio, *All Things Considered,* January 17, 1996.

----"Interview." National Public Radio, *Talk of the Nation,* January 15, 1996.

----"Introduction." Ali, Tariq. *An Indian Dynasty.* New York: G. P. Putnam's Sons, 1985.

----*Midnight's Children.* New York: Penguin Books, 1980.

----"*Midnight's Children* and *Shame.*" *Kunapipi* 7 (1985): 1-19.

----"Introduction." *Mirrorwork: 50 Years of Indian Writing, 1947-1997.* Ed. with Elizabeth West. New York: Henry Holt and Co., 1997. vii-xx.

----*The Moor's Last Sigh.* New York: Pantheon Books, 1995.

---"The Riddle of Midnight." Public Media Video, 1988.

---"Salman Rushdie." With Charlotte Cornwall. *Writers Talk -- Ideas of Our Time.* Writers in Conversation Series. ICA Video, 1989.

---"Salman Rushdie: *The Satanic Verses.*" With W. L. Webb. *Writers Talk -- Ideas of Our Time.* Writers in Conversation Series. ICA Guardian Video, 1989.

Rustomji-Kerns, Roshni. "Expatriates, Immigrants, and Literature: Three South Asian Women Writers." *Massachusetts Review* 29.4 (Summer 1988): 655-665.

Sage, Vic. "The God-Shaped Hole': Salman Rushdie and the Myth of Origins." *Hungarian Studies in English* 22 (1991): 9-21.

Said, Edward. "Third World Intellectuals and Metropolitan Culture." *Raritan* 9.3 (Winter 1990): 27-50.

---*The World, the Text and the Critic.* Cambridge, MA: Harvard University Press, 1983.

Saldivar, José. *The Dialectics of Our America: Genealogy, Cultural Critique, and Literatary History.* Durham: Duke University Press, 1991.

Sangari, Kumkum. "The Politics of the Possible." *Cultural Critique* 7 (Fall 1987): 157-186.

Sant-Wade, Arvindra and Karen Marguerite Radell. "Refashioning the Self: Immigrant Women in Bharati Mukherjee's New World." *Studies in Short Fiction* v. 29 (1992): 11-17.

Scarry, Elaine. *The Body in Pain: The Making and Unmaking of the World.* New York: Oxford University Press, 1985.

Sen, Suchismita. "Memory, Language, and Society in Salman Rushdie's *Haroun and the Sea of Stories.*" *Contemporary Literature* 36.4 (Winter 1995): 654-675.

Scott, Joan W. "Multiculturalism and the Politics of Identity." *October* 61 (1992): 12-19.

Sen, Suchismita. "Memory, Language, and Society in Salman Rushdie's Haroun and the Sea of Stories." *Contemporary Literature* 36.4 (Winter 1995): 654-673.

Sennett, Richard. "The Identity Myth." *The New York Times.* January 30, 1994. p 17.

Sethi, Sunil. "After Midnight." *India Today* (15 April 1983): 136-7.

Shepherd, Ron. "*Midnight's Children* as Fantasy."*The Commonwealth Review* 1.2 (1990): 33-43.

Shulman, Polly. "Home Truth's: Bharati Mukherjee, World Citizen." *Voice Literary Supplement.* June 1988: 19.

Silverman, Kaja. *Male Subjectivity at the Margins.* New York: Routledge, 1992.

----*The Subject of Semiotics.* New York: Oxford University Press, 1983.

----*Threshold of the Visible World.* New York: Routledge, 1995.

Singh, Amritjit, Joseph T. Skerrett, Jr., and Robert E. Hogan (ed.). *Memory & Cultural Politics: New Approaches to American Ethnic Literatures.* Boston: Northeastern University Press, 1996.

----*Memory, Narrative, & Identity: New Approaches in Ethnic American Literatures.* Boston: Northeastern University Press, 1994.

Singh, Sushila. "Salman Rushdie's Novels from Fantasy to Reality." *Commonwealth Review* 1.1 (1989): 111-23.

Slemon, Stephen. "Post-Colonial Allegory and the Transformation of History." *Journal of Commonwealth Literature* 23.1 (1988): 157-168.

Smith, Anthony D. *National Identity.* Reno: University of Las Vegas Press, 1991.

Smith, Colin. "The Unbearable Lightness of Salman Rushdie." Selected Papers of the 10th Annual Conference on Commonwealth Literature and Language Studies, Konigstein, 11-14 June 1987. *Critical Approaches to the New Literatures in English.* Ed. Dieter Riemenschneider. Essen: Die Blau Eule: 1989: 104-115.

Spiegelman, Art. *Maus: A Survivor's Tale, II: And Here My Troubles Began.*
New York: Pantheon, 1991.

Spillers, Hortense J. "'All the Things You Could Be by Now, If Sigmund Freud's Wife Was Your Mother': Psychoanalysis and Race." *Boundary 2* 23.3 (Fall 1996): 75-141.

---*Comparative American Identities: Race, Sex, and Nationality in the Modern Text*. New York: Routledge, 1991.

---"Mama's Baby, Papa's Maybe." *Diacritics* 17.2 (Summer 1987): 65-81.

Spivak, Gayatri Chakravorty. *Outside in the Teaching Machine*. New York: Routledge, 1993.

---*The Post-Colonial Critic*. Ed. Sarah Harasym. New York: Routledge, 1990.

---"Reading *The Satanic Verses*." *Public Culture* 2. 1 (Fall 1989): 79-99.

Srivastava, Aruna. "'The Empire Writes Back': Language and History in *Shame* and *Midnight's Children*." I*Past the Last Post: Theorizing Post-Colonialism and Post-Modernism*. Ed. Ian Adam and Helen Tiffin. Alberta: University of Calgary Press: 1990. 65-77.

St. Andrews, B. A. "Co-Wanderers Kogawa and Mukherjee: New Immigrant Writers." *World Literature Today* 66.1 (1992): 56-58.

Steinberg, Sybil. "Bharati Mukherjee." *Publisher's Weekly*, 25 August 1989, 46-47.

Suleri, Sara. "Contraband Histories: Salman Rushdie and the Embodiment of Blasphemy." *Yale Review* 78. 4 (Summer 1989): 604-624.

---*The Rhetoric of English India*. Chicago: University of Chicago Press, 1992.

Sunder Rajan, Rajeswari. *Real and Imagined Women: Gender, Culture and Postcolonialism*. New York: Routledge, 1993.

Swann, Joseph. "'East Is East and West Is West'? Salman Rushdie's *Midnight's Children* as an Indian Novel." *World Literature Written in English* 26. 2 (Autumn 1986): 353-362.

Tapping, Craig. "South Asia/North America: New Dwellings and the Past." In *Reworlding: The Literature of the Indian Diaspora*. Ed. Emmanuel S. Nelson. Westport, CT: Greenwood Press, 1992. 35-42.

Taylor, Charles. *Sources of the Self: The Making of the Modern Identity.* Cambridge: Harvard University Press, 1989.

Tharoor, Shashi. *India: From Midnight to Millenium.* New York: Arcade Publishing, 1997.

Tompkins, Jane. *Sensational Designs: The Cultural Work of American Fiction, 1790-1860.* New York: Oxford University Press, 1985.

Viswanathan, Gauri. *Masks of Conquest: Literary Study and British Rule in India.* New York: Columbia University Press, 1989.

Walcott, Derek. *Collected Poems, 1948-1984.* New York: Noonday Press, 1992.

Wall, Cheryl. A. (ed.). *Changing Our Own Words: Essays on Criticism, Theory, and Writing by Black Women.* New Brunswick: Rutgers University Press, 1990.

Walton, Jean. "Re-Placing Race in (White) Psychoanalytic Discourse: Founding Narratives of Feminism." *Critical Inquiry* 21 (Summer 1995): 775-804.

Warner, Michael. "The Mass Public and the Mass Subject." In *The Phantom Public Sphere.* Ed. Bruce Robbins. Minneapolis: University of Minnesota, 1993. 234-256.

Wieseltier, Leon. "Midnight's Other Children." *New Republic* (December 1983): 32-34.

West, Cornel. "Black Culture and Postmodernism." *Remaking History.* Ed. Barbara Kruger and Phil Mariani. Seattle: Bay Press, 1989. 87-96.

----"The Dilemma of the Black Intellectual." *Breaking Bread: Insurgent Black Intellectual Life*. Ed. bell hooks and Cornel West. Boston: South End Press, 1991. 131-146.

----*Race Matters*. New York: Vintage Books, 1994.

White, Jonathan. "Politics and the Individual in the Modernist Historical Novel." *Recasting the World: Writing After Colonialism*. Baltimore: The Johns Hopkins University Press, 1993: 208-240.

Wickramagamage, Carmen. "Relocation as Positive Act: The Immigrant Experience in Bharati Mukherjee's Novels." *Diaspora* 2.2 (1992): 171-200.

Wilson, Keith. "*Midnight's Children* and Reader Responsibility." *Critical Quarterly* 26.3 (Autumn 1984): 23-37.

Wolpert, Stanley. *A New History of India*. New York: Oxford University Press, 1993.

Yates, Frances Amelia. *The Art of Memory*. Chicago: Chicago University Press, 1966.

Zizek, Slavoj. *Looking Awry: An Introduction to Jacques Lacan Through Popular Culture*. Cambridge: MIT Press, 1991.

----*The Sublime Object of Ideology*. New York: Verso, 1989.

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