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HEDGING IN THE INTEREST RATE MARKETS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

By

SUSAN JANE CRAIN
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HEDGING IN THE INTEREST RATE MARKETS

A DISSERTATION
APPROVED FOR THE DEPARTMENT OF FINANCE

BY

Louis Ederington

Bobbie Foote

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HEDGING IN THE INTEREST RATE MARKETS

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HEDGING IN THE INTEREST RATE MARKETS

Abstract

This study explores how market makers in the Eurodollar options on futures hedge their option positions using Eurodollar futures. The delta neutrality hedging model based on Black's option pricing model (OPM) is examined as well as several hedging methodologies extensively studied for spot/futures portfolios. These include naive, risk-minimization, and bivariate GARCH models. A bivariate EGARCH hedge is developed as an alternative hedging model. Results suggest that the OPM delta hedge is the most effective on both a within-sample and out-of-sample basis. Consistent with previous studies. the time-varying hedge ratio models outperform the risk-minimization and naive models with constant hedge ratios. In the option/futures framework, the superior performance of the bivariate EGARCH hedge looks promising for extensions to spot/futures portfolios and other derivatives.

The suitability of GARCH and EGARCH models in a cross-hedging framework is also examined whereby a T-bill spot position is hedged with Eurodollar futures. The naive and risk-minimization models are also included in this extension. The T-bill spot asset is chosen for this study because managers of T-bill portfolios commonly cross-hedge with Eurodollar futures. Evidence of cointegration between the two markets is factored in with an error correction representation. Consistent with previous results, the time-varying hedge ratio models outperform the constant hedge ratio models. Unlike the Eurodollar option/futures results where EGARCH is superior to GARCH, the within-sample and out-of-sample tests in the T-bill/Eurodollar cross-hedge show that GARCH and EGARCH hedging performance is virtually identical.

CHAPTER 1: INTRODUCTION

1.1 Hedging with Futures and Options

The explosive growth in the use of financial futures has been explained as the fulfillment of a need to manage portfolio risk generated by economic instability, mounting federal debt, spiraling inflation, and volatile interest rates.[1] Those involved in portfolio risk management include securities dealers, investment banks, insurance companies, pension funds, trust funds, mutual funds, corporations, thrift institutions, and individual investors.

The objectives of hedging through the use of futures include protecting the value of portfolio assets and limiting opportunity losses. With these objectives in mind, several hedging theories have been developed and tested in recent years. The earliest hedging strategy employing futures was based strictly on risk minimization while later approaches incorporated portfolio theory with both risk and return as part of the optimization problem. The most recent studies of hedging effectiveness have been built on the assumption that spot and futures prices are driven by a Bivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and considerable evidence in support of this latest assumption now exists.

In a typical market an investor can use not only a futures contract to hedge a spot position, but he/she also has two other derivative instruments available. The first is the option on the spot and the second is the option on futures contracts. Since option contracts do not carry the obligation to perform, as futures contracts do, they more closely resemble an insurance policy where the hedger pays a premium for the insurance and has the choice of either exercising the option or allowing it to expire as worthless.

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[1]Chicago Board of Trade (1990): *Treasury Futures for Institutional Investors.*

Natenberg (1994) describes hedgers utilizing the options markets as either natural longs or natural shorts as dictated by normal business activities. Commodity producers and lenders are natural longs in the cash position whereas commodity users and borrowers are natural shorts. Other potential hedgers include those who have voluntarily taken long or short positions in the market such as speculators or portfolio managers. He describes hedging decisions as a tradeoff between what the hedger is willing to give up under one set of market conditions in order to protect himself under another set, with the amount of protection purchased depending upon the amount of risk that the hedger is willing to bear. The options trader has a dual purpose--to hedge downside risk while allowing upside profit potential.

Not only have futures markets grown substantially in recent years, but there has also been a proliferation in the introduction of various options on spot instruments as well as options on futures. Thompkins (1994) attributes the growth in options usage to three factors: 1) the development of the Black-Scholes (1973) option pricing model, 2) the introduction of option contracts on regulated exchanges, and 3) advances in computer technology.

When options are the focus of a hedging study, the common approach involves the use of an option pricing model (OPM) for the calculation of hedge ratios such as in Hsin, Kuo, and Lee (1994) and Hancock and Weise (1994). In the case of options on futures, five inputs are required for the Black (1976) option pricing model, 1) the current futures price, 2) the option strike price, 3) the days to expiration, 4) the risk-free interest rate, and 5) the volatility of the underlying futures contract. The hedge ratio, or delta, is determined by measuring the relative change in the option price for a given change in the futures price.

Numerous studies have examined hedging strategies for investors who hold the spot position as fixed and use the option on spot or a futures contract as the hedging instrument. However, very little attention has been devoted to options on futures or to the professional market maker's hedging behavior. The market maker is willing and ready at any time to buy at a bid price or sell at an asked price, thus meeting otherwise unfilled orders, providing a ready and liquid market, and reducing price volatility. According to Baird (1993), the *options* market makers are distinguished from the others due to the nature of the instrument in which they deal. The many strike prices and calendar months available result in options that may trade infrequently and as a consequence, the options market makers are often faced with carryover positions on a regular basis that expose them to a level of risk that other financial dealers may not experience. Hedging provides a means to manage this risk exposure. At a mechanical level, the hedging instrument of choice for the options market maker is normally a futures contract.

1.2 The Market Maker's Hedging Strategy

For market makers in options on futures, the hedging strategy most widely adopted is the delta neutral stance based on Black's (1976) option pricing model (OPM).The method is convenient and relatively simple to implement. Since this method utilizes the option strike price and the days to expiration as inputs, the hedge ratio is contract specific. Is it reasonable to assume, therefore, that the hedging results from this method will be superior to any other hedging model that does not incorporate the same level of information? Perhaps not since there are several reasons for questioning the effectiveness of OPM hedge ratios.

Even though a delta hedge has the advantage of being contract specific, anyone who maintains a portfolio of options rather than a single contract will typically offset the "combined" delta position of the portfolio with the appropriate number of futures contracts to establish a delta neutral stance. This "position" delta could possibly lessen the contract-specific advantage of the OPM delta hedge.

Next consider that the delta hedging strategy is model specific and relies on the assumption that the theoretical option pricing model is correct. Further, at the foundation of most option pricing models are a set of assumptions that are consistently violated, including frictionless markets, constant interest rates, constant volatilities, and continuous trading.

With respect to the last item, the question of discrete rebalancing has been addressed by several authors. Robins and Schachter (1994) argue that the distributional properties of discretely rebalanced hedges are such that delta-based hedging is not an optimal strategy. Figlewski (1989) found evidence that discrete rebalancing is one factor that makes it possible to only establish bounds rather than uniquely determined option prices, resulting in options hedge ratios that are not well defined. Boyle and Emanuel (1980) analyze the distribution of the returns on hedge portfolios when rebalancing takes place at discrete points and find that the distribution is strongly skewed and leptokurtic.

Recalling the five inputs listed above for the Black OPM, one of these inputs in particular can be incorrectly estimated. When discussing the input of a volatility estimate, Baird (1993) states that market makers almost always initially set bid/asked prices around recent implied volatility levels, using historical levels for reference. In addition, Natenberg (1994) also recognizes the mean-reverting tendencies of long-term volatilities as well as the apparent serial correlation of volatility. Natenberg, a floor trader, states that the options trader intuitively incorporates all of this information when making volatility forecasts. This subjective approach could be regarded as the "gut level" method of volatility estimation. The obvious question becomes, could an options trader utilize a mathematical model to improve volatility estimates and hedging performance?

Another point to consider is that for the trader to effectively hedge an options portfolio, he must consider the complication arising from the fact that an option's delta is not constant. That being the case, the market maker must decide how often to compute the delta risk of his portfolio, and further, how often he should update his hedged position.

Given these many questions and issues related to the use of option pricing models for hedging, further study is warranted. Mean reversion and serial correlation of volatilities imply some form of the Autoregressive Conditional Heteroskedasticity (ARCH) family of models. An investigation along these lines could also address the relationship between the volatilities of an option and its underlying asset, with that relationship modeled as a bivariate GARCH model, such as Baillie and Myers (1991) and others have done to determine hedge ratios in spot/futures portfolios. Further, it is possible that other hedging strategies could also be applied to the options portfolio. With recent advances in computer technology, these alternative methodologies are no longer a cumbersome task to practitioners compared to the OPM delta hedge.

1.3 Thesis Contribution

This paper utilizes several well-known hedging methodologies to examine a relatively unexplored subset of hedging activity. The primary focus is on the trader who owns an *option* as a primary asset and who subsequently hedges the option with futures. The options market maker most readily fits within this category. For the options market maker, OPM delta neutrality hedging is the norm, but as suggested by Hsin, Kuo, and Lee (1994), the perfect option delta neutrality hedge can only be achieved if the assumptions of the option pricing model are correct and if the hedge ratios can be continuously adjusted.

So why do options market makers use this method? One possibility is that option pricing models are relatively easy to program and use whereas other models, such as GARCH, appear rather formidable. Another possibility is that delta neutrality hedging is "the best" method, which means that those traders who utilize alternative less effective strategies would either change tactics or be forced out of the market completely.

With these thoughts in mind, this study focuses on strategies that have been applied to futures market hedging to see if any conclusions can be drawn for options hedging. Included are Ederington's (1979) risk minimization model for hedging options portfolios in the interest rate market. The more recent bivariate GARCH hedging model is also studied to see if it could be of benefit in an options portfolio framework. Evidence from GARCH hedging studies in spot/futures portfolios seem to suggest some benefit to using this model.

An issue that has not been addressed in the hedging literature is the suitability of the Exponential-GARCH model. EGARCH should be superior if the time series reflect an asymmetric relation between volatility and past returns. Empirical evidence in support of univariate EGARCH effects has been found in numerous financial time series where negative return shocks introduce more volatility than positive return shocks. The markets examined include monthly U. S. stock returns by Pagan and Schwert (1990), the daily French CAC 40 index by Rabemananjara and Zakoian (1993), a daily CRSP value-weighted market index by Nelson (1991), and the daily S&P 500 index for both spot and futures by Koutmos and

Tucker (1996). This evidence in other financial markets warrants univariate EGARCH analysis for the option and futures time series chosen here, and the development of a bivariate EGARCH model. This extension will be the first application of an EGARCH model to the hedging problem.

The use of options in a portfolio leads to some complications that do not surface in the spot/futures framework. In the application of the hedging models to the option/futures portfolio, great care must be taken to synchronize the option and futures prices. As mentioned previously, the many strike prices and calendar months can lead to infrequent trading, and in fact, it isn't unusual for an option contract to trade only once or twice on any given day. If the last option trade occurs at say, 9:00 am, then the use of the *closing* futures price for that day will bias the results. To avoid this potential problem, when the options are sampled, the time is noted for the last option trade of the day and the futures price is then time-matched.

Another issue of importance for the options/futures portfolio relates to the specification of the time series used for the application of a hedging model. Should price levels, price changes, or returns be used? The spot/futures hedging literature contains examples of primarily price changes and returns.[2] When returns are used, the futures return and the spot return are generally of the same magnitude. Options returns, on the other hand,

---

[2]

Returns are used by Figlewski (1984), Cecchetti, Cumby and Figlewski (1988) and Hancock and Weise (1994). The change in the logarithm of price (percentage change) is used by Baillie and Myers (1991), Park and Switzer (1995), and Kroner and Sultan (1993). Price changes are used by Ederington (1979), Hill and Schneeweis (1984), Toevs and Jacob (1986), and Saunders and Sienkiewicz (1988).

are considerably different from the returns of the underlying instrument. For this reason, the hedging models for the options/futures portfolio must utilize price changes.

Previous authors have considered both in-sample and out-of-sample hedging results, and this study also addresses those issues. For the in-sample study, the computation of the optimal hedging ratio utilizes all of the prices from the sample as if they were known in advance. In other words, this method assumes perfect foresight. This is obviously far from reality and in fact, the hedger must utilize information available at one point in time to compute a hedge ratio that will hopefully remain appropriate for future price moves. Whereas most previous studies within the GARCH framework have used weekly prices to update weekly hedges, a more optimal strategy would include more frequent updating. In fact, Myers (1991) recognizes that an efficient use of available information is achieved if a model's parameter estimates are updated as each new observation becomes available, although he feels that the cost of this strategy is prohibitive. However, a market maker whose income depends on trading profits would surely opt for a utilization of more data points as opposed to less.

With this thought in mind, out-of-sample tests are therefore based on a moving window of *daily* information to be used for the computation of the parameter estimates. This method is in contrast to previous studies. For example, Cecchetti, Cumby, and Figlewski (1988) determine parameter estimates for monthly data using the first six years of their sample period and use those same estimates in conjunction with realized values of returns over the final two years of the sample. Myers (1991) also computes post-sample hedge ratios using parameter estimates from the first six years of his sample period. These are then used along with realized weekly values of cash and futures prices available up to the time that the portfolio is being adjusted over the final two years of the sample period. Baillie and Myers (1991) use daily cash and futures prices over a 1986 contract period to obtain model estimates that are used to simulate implied hedging rules over a 1982 contract data period. Kroner and Sultan (1993) and Park and Switzer (1995) improve on the previous method by re-computing parameter estimates on a weekly basis to forecast the hedge for the following week. However, instead of using daily observations, both of these studies use weekly spot and futures data.

Given the assumption that options and futures prices have some form of structural relationship between them, it is conceivable that this relationship could change over time, and daily prices will more readily detect those changes. Engle and Mezrich (1995) relate their view that the arrival of news in the marketplace results in patterns of volatility clustering and that the frequency of a data sample dictates the type of cluster that can be seen and measured. Higher frequency data are more revealing about the volatility properties. For this reason, the use of daily prices is a more realistic approach for the computation of hedge ratios on a weekly basis. This method also provides a compromise for the cost issue raised by Myers (1991). Figure 1 illustrates the concept.

In this simple illustration, the entire sample period is assumed to be 15 trading days. The window of information to compute the first hedge ratio spans the first 10 trading days with the hedging ratio computed at the close of trading on day 10, denoted as trading day t. The hedge is simultaneously placed at the close of day t and kept in place through the end of day $t+1$. At the close of day $t+1$, the portfolio is liquidated and a new hedge is placed and kept until the close of day $t+2$. The portfolio liquidation recurs on a daily basis. The hedge ratio calculated on day t is used for each day through day $t+5$. Then at the close of trading on day $t+5$, the closing price information from the previous five days is used to establish a new 10-day window, dropping the first week's prices from the information set. This new information set includes prices from day $t-5$ through day $t+5$. A new hedge ratio is then computed based on this updated information through the end of day $t+5$, and the hedge is simultaneously placed and kept through the end of day $t+6$. In this manner, the hedging horizon is one day with an updated hedge ratio computed on a weekly basis.

Two markets are investigated in this paper. The Eurodollar is chosen in particular since most of the hedging literature to date has ignored this particular instrument, concentrating only on agriculturals, currencies, treasury securities, and stock indices.[3] This study will therefore provide new insights into this particular market. In terms of market size and trading volume, the Eurodollar futures contract is the most actively traded contract with an annual Chicago Mercantile Exchange trading volume in 1996 of 99.6 million contracts, averaging over $390 billion daily. For the same time period, a total of 21.8 million put and call options on futures were traded for an average of $85 billion per day. The Eurodollar option on futures is the most heavily traded option market on the Chicago Mercantile

[3] Authors investigating agricultural commodities include Baillie and Myers (1991), Myers (1991), and Witt, Schroeder, and Hayenga (1987). Currencies have been investigated by Lien and Luo (1994), Hsin, Kuo and Lee (1994), Kroner and Sultan (1993), and Herbst, Swanson, and Caples (1992). Treasury security studies have been done by Cecchetti, Cumby, and Figlewski (1988), Toevs and Jacob (1986), Hill and Schneeweis (1984), and Ederington (1979). Analyses of stock indices have been conducted by Park and Switzer (1995), Ghosh (1993), Junkus and Lee (1985), and Figlewski (1984, 1985).

Exchange and it represents a very important market segment for practitioners.

Although this paper starts with a study of the hedging of an option on Eurodollar futures with a Eurodollar futures contract, another very different hedging strategy is also explored, namely, the hedging of a spot position in Treasury bills with Eurodollar futures. Even though T-bill portfolio managers can choose T-bill futures for hedging purposes, they commonly use Eurodollar futures contracts because of their superior liquidity. Hedge ratio considerations in this cross-hedging framework have not been previously addressed.

This second area of study utilizes the same hedging models as before, but since the two instruments are not as closely related, the results could differ substantially from that of the option/futures portfolio. In particular, the GARCH models have not been used in the study of cross hedges, and the question of whether GARCH models with time-varying hedge ratios are superior to the risk minimization alternative with a constant hedge ratio remains to be answered.

The rest of the paper is constructed as follows. Chapter 2 provides a literature review including a discussion of the evolution of hedging methodology over the past 40 years, as well as descriptions and empirical tests of various hedging models. Chapter 3 includes applications of the hedging models to the Eurodollar options on futures. Chapter 4 extends the same analysis to the T-bill spot market. Chapter 5 provides conclusions.

CHAPTER 2: LITERATURE REVIEW

This chapter describes the progression of hedging methodologies from the simplest early models to the more recent computationally challenging models. The discussion includes a brief explanation for the calculation of hedge ratios and hedging effectiveness under three general headings termed risk minimization, GARCH methods, and option delta neutrality. In addition, several empirical studies are cited for each. Two other topics are also reviewed, the issue of time-varying hedge ratios and the issue of cointegration. However, before getting into the details of these models and the other related issues, a brief summary of the hedging literature is in order.

2.1 The Hedging Literature

Perhaps the first question that needs to be answered is, why do investors hedge? The obvious answer is to minimize risk, but Working (1953) analyzes hedging behavior from a different perspective than risk minimization. In his view, hedging activity is related to anticipated changes in the basis, suggesting a profit maximization motive. The *basis* is defined simply as the difference between the cash and futures prices. Through personal interviews with brokerage firm representatives, Johnson (1960) finds the existence of both risk minimization as well as profit maximization motives. His model, as well as a later one by Stein (1961), incorporate portfolio theory and the risk minimization/return maximization relationship into the hedging decision.

Whatever the motivation for hedging, the next question to be answered is, what hedge ratio should be used? And after answering this question, if it *can* be accurately answered, the next question is, how well does the strategy perform? The simplest solution to the hedge ratio question is the “naive” strategy where the spot position is offset with an equal and opposite position in the futures market. If both prices rise or fall by the exact same amount, then the hedge is perfect and losses in one market are exactly offset by profits in the other, resulting in the complete elimination of risk. We know, however, that this price pattern is not realized, so more sophisticated methods are required.

Ederington (1979) implicitly assumes risk minimization motives in his portfolio model when he solves for hedge ratios and further develops a measure of hedging effectiveness. Howard and D'Antonio (1984) recognize the short comings of a model that ignores return requirements and thus develop an optimization technique that incorporates both risk and return to solve for hedge ratios. In addition, they construct a measure of hedging effectiveness that they later revise (Howard and D'Antonio (1987)) in response to a correction by Chang and Shanker (1987).

Both of these models assume constant hedge ratios, but the question of whether hedge ratios are time-varying has been raised by several authors, including Grammatikos and Saunders (1983) and Malliaris and Urrutia (1991a, 1991b). The method to solve for these changing hedge ratios was developed by Engle (1982), but it wasn’t utilized in the hedging arena until Cecchetti, Cumby, and Figlewski (1988) used an ARCH model of time varying estimates of the covariance matrix of returns on cash and futures as a means of determining an optimal hedge. Baillie and Myers (1991) extend the analysis by relaxing the restrictive assumption of constant correlation.

Another issue of prime importance deals with the long-run relationship between the two time-series under consideration. If the two variables are cointegrated, Engle and

Granger (1987) suggest that the bivariate model should include an error correction term. Ghosh (1993) applies this reformulation to the minimum risk model, Kroner and Sultan (1993) introduce an error correction term for cointegration into the GARCH model, and Lien and Luo (1993) extend this same model by moving into a multi-period framework.

From the early 1950's through the mid 1990's, several strides were made in the area surrounding the strategies for hedging with futures. In another arena, delta neutrality hedging found its start in the development of the Black-Scholes (1973) option pricing model. With the formula in hand, it's a rather straight-forward process to differentiate with respect to the variables and determine the set of risk factors (called delta, gamma, vega, theta, rho, and lambda) that are used by options traders.

Since the original model evaluated European options on non-dividend paying stock, subsequent studies have addressed such issues as the underlying instrument and early exercise. Black (1976) provided a modification for pricing options on futures contracts while both Cox, Ross, and Rubenstein (1979) and Barone-Adesi and Whaley (1987) developed models to value American options with an early exercise feature.

Given this brief background, the issues of time-varying hedge ratios and cointegration are now discussed, followed by more complete details of the risk minimization, GARCH, and option delta neutrality models.

2.2 The Issue of Time-varying Hedge Ratios

After a trader computes a hedge ratio and places the hedge, can he just forget it? Is a hedge ratio computed on February 28 good enough for a position taken on April 15? The answer to both of these questions is no. In fact, several studies have shown that hedge ratios are unstable over time.

Grammatikos and Saunders (1983) use three econometric approaches to examine the question of hedge ratio stability for five foreign currency futures contracts. The first method is a moving window regression procedure whereby hedge ratios are initially estimated for a two-year period and then reestimated every quarter by adding the new quarter's data and deleting the initial quarter's data. The second method examines whether significant shifts have taken place in hedge ratios over various subperiods (chosen based on economic events) where the regression model includes dummy variables for the chosen subperiods. The third method uses a random coefficients model where the hedging equation allows the hedge ratio to vary over time. They conclude that hedge ratios are stable in the Canadian dollar, vary randomly in the Swiss franc, and increase significantly over time in the British pound and Japanese yen.

Marmer (1986) tests the stability of hedge ratios in the Canadian dollar market in the same fashion as Grammatikos and Saunders but extends the comparison to different hedge lengths and varying delivery dates. The regression includes two dummy variables for different time periods as follows

$$\Delta S_t = A + B_1\Delta F_t + B_2(D_1\Delta F_t) + B_3(D_2\Delta F_t)$$

where

ΔS_t = the change in spot prices

ΔF_t = the change in futures prices

D_1 = 1 for t equal to July 10, 1981-February 4, 1983 and 0 for all other t

D_2 = 1 for t equal to July 10, 1981-August 26, 1983 and 0 for all other t

The equation divides the full sample period of 7/10/81 through 9/7/84 into three periods. The estimated hedge ratio over the entire period is $\hat{B}_1$. $\hat{B}_1 + \hat{B}_2$ is the estimated optimal hedge ratio over the first half of the period, and $\hat{B}_1 + \hat{B}_3$ is the estimated optimal hedge ratio over the first two thirds of the period. If the hedge ratio is stable over time then $\hat{B}_2 = \hat{B}_3 = 0$. Marmer finds no evidence of instability in the hedge ratios over the subperiods and he offers the possible explanation that the subperiods chosen may not be matched with times having significant economic events.

Malliaris and Urrutia (1991a) suggest that both the hedge ratio and hedging effectiveness measure (from a risk minimization model) follow a random walk process and test this hypothesis in two stock indices and four foreign currency markets. Weekly spot and futures data are used and a two-week hedging horizon is assumed. A moving window regression procedure is employed whereby the parameters are reestimated every quarter by adding a new quarter of spot and futures data and deleting the initial quarter's data, keeping a one-year estimation period. Results for both a Dickey and Fuller test as well as a variance ratio test of random walk are reported. The Dickey and Fuller methodology consists of the following regression:

$$Y_t = b_0 + b_1 Y_{t-1} + b_2 T + \epsilon_t$$

where Y_t, Y_{t-1} = hedge ratio or measure of hedging effectiveness

T = time trend

ϵ_t = residual term at time t

The null hypothesis that $(b_0, b_1) = (0,1)$ is not rejected at the 1 percent level suggesting that the hedge ratio and hedging effectiveness measure in each of the markets follow a random walk process. The variance ratio test reinforces the Dickey/Fuller result.

In a subsequent study, Malliaris and Urrutia (1991b) postulate that if hedge ratios are constant over time, then the longest possible estimation period should result in the best estimate of the hedge ratio and the most effective hedge. Once again, a moving window regression is used where the hedging horizons are one week and four weeks. On an *ex post* and *ex ante* basis, the hedging effectiveness was only weakly impacted by the length of the estimation suggesting some support for the notion that hedge ratios are unstable over time.

The previous studies have employed various methods to address the issue of time-varying hedge ratios, but other authors have used the ARCH/GARCH models to provide the same information. Cecchetti, Cumby, and Figlewski (1988) determine hedge ratios through an ARCH specification for T-Bonds and find that the optimal ratio varies from 0.52 to over 0.91 for their sample period. Myers (1991) illustrates the hedge ratio path for wheat through the use of figures with the constant hedge ratio as a reference point. Baillie and Myers (1991) also use this same illustration technique in the commodity markets of beef, coffee, corn, cotton, gold, and soybeans. Each of the figures seem to illustrate considerable hedge ratio variability.

2.3 The Issue of Cointegration

As defined in Hamilton (1994), an $(nx1)$ vector time series $\mathbf{y}_t$ is said to be cointegrated if each of the series taken individually is integrated of order one, $I(1)$, or is nonstationary with a unit root, while some linear combination of the series $\mathbf{a}'\mathbf{y}_t$ is stationary, $I(0)$, for some nonzero $(nx1)$ vector $\mathbf{a}$. To understand what this definition means requires some background in the analysis of time series data. The following discussion is taken from

Hamilton (1994), Greene (1993), and Campbell and Perron (1991).

The best place to start is to first define the terms used in the above definition. First, a stochastic process, y_t, is defined to be *covariance stationary* if it satisfies three requirements: 1) $E[y_t]$ is independent of t, 2) $Var[y_t]$ is a constant, independent of t, and 3) $Cov[y_t, y_s]$ is a function of $t-s$, but not of t or s. The covariance between y_t and y_{t-s} depends only on the length of time separating the observations and not on the date of the observation. Second, if a series is found to be non-stationary but it becomes stationary after differencing once, then it is said to be *integrated of order one, I*(1). Third, to define a *unit root* process requires a little more discussion.

Many financial time series, when graphed, seem to exhibit either increasing or decreasing patterns. Two popular approaches to modeling these patterns are deterministic time trends and unit root processes. Consider a time series that can be represented as follows

$$y_t = TD_t + Z_t$$

In this situation, TD_t is a deterministic trend in y_t which could be represented as

$$TD_t = \kappa + \delta t$$

which is linear in time. Z_t is the noise function and the unit root hypothesis concerns the behavior of the noise function. Further, we can assume that Z_t is described by an autoregressive-moving average process:

$$A(L)Z_t = B(L)e_t$$

where A(L) and B(L) are polynomials in the lag operator L of order p and q, respectively, and e_t is a sequence of i.i.d. innovations.

$$Z_t - \alpha_1 Z_{t-1} - \ldots - \alpha_p Z_{t-p} = e_t + \beta_1 e_{t-1} + \ldots + \beta_q e_{t-q}$$

Since the deterministic trend TD_t includes the mean of y_t, the noise function Z_t is assumed to have a zero mean. In addition, the roots of the moving average polynomial B(L) are assumed to lie outside the unit circle.

The distinction between a trend-stationary and a difference-stationary model lies in the roots of the autoregressive polynomial, A(L). In the first case, the roots are strictly outside the unit circle (are greater than one) so that Z_t is a stationary process and y_t is stationary around a trend. In the second case, Z_t has one unit autoregressive root and all other roots lie strictly outside the unit circle. In this case, the first difference of Z_t is a stationary process and the first difference of y_t is stationary around a fixed mean. Therefore, the unit root hypothesis is that y_t is difference stationary.

From a practical standpoint, if a series is non-stationary then a transformation is required, but which transformation should be made? If the series is trend stationary, then the correct treatment is subtracting δt from y_t to produce stationarity. On the other hand if the series exhibits a unit root process, the correct treatment is to difference the series to achieve stationarity.

The Phillips and Perron test statistic for a unit root is

$$T(\hat{\rho}-1)-(1/2)(T^2\hat{\sigma}^2_{\hat{\rho}}/s^2)(\hat{\lambda}^2-\hat{\gamma}_0).$$

The variables for this test statistic are computed in a multi-step process that starts with a regression of y_t on y_{t-1}

$$y_t = \alpha + \rho y_{t-1} + u_t$$

The OLS estimation provides $\hat{\rho}$, its standard error $\hat{\sigma}_{\hat{\rho}}$, and the standard error, s, of the regression. $\hat{\gamma}_0$ is a consistent estimate of $E(u_t^2)$ given by

$$\hat{\gamma}_0 = T^{-1}\sum_{t=1}^{T} \hat{u}_t^2$$

The computation of $\hat{\lambda}^2$ uses the value of $\hat{\gamma}_0$ and also requires the input of the jth autocovariance of the residual ($\hat{u} = y_t - \hat{\alpha} - \hat{\rho}y_{t-1}$) which is calculated from the following equation for $\hat{\gamma}_j$

$$\hat{\gamma}_j = T^{-1}\sum_{t=j+1}^{T} \hat{u}_t\hat{u}_{t-j}$$

$\hat{\lambda}^2$ is then computed as

$$\hat{\lambda}^2 = \hat{\gamma}_0 + 2\sum_{j=1}^{q} [1 - j/(q+1)]\hat{\gamma}_j$$

where q denotes the number of autocovariances used. If the computed test statistic exceeds the critical value, then the null hypothesis of a unit root process is accepted and the proper correction for the non-stationarity is first differencing.

Tests for unit roots have been conducted in spot and futures of several financial and commodity markets. Evidence of unit roots were found by Kroner and Sultan (1993) in the British pound, Canadian dollar, Japanese yen and Swiss franc; Park and Switzer (1995) in the S&P 500 Index and Toronto 35 Index; Baillie and Myers (1991) in beef, coffee, corn, cotton, gold and soybeans; and Ghosh (1993) in the S&P 500 Index, Dow Jones Industrial Average, and New York Stock Exchange composite index.

Getting back to cointegration, in the case of two time series, cointegration means that there is some long-term equilibrium relationship between the two even though permanent changes may occur to each series individually. If the series are cointegrated, then the manner in which the two variables drift together can be distinguished from the deviations of the individual series from their long-run trends. For the regression model

$$y_t = \beta x_t + \epsilon_t$$

if both series y and x were $I(1)$, there may be a β such that

$$\epsilon_t = y_t - \beta x_t$$

is stationary, $I(0)$. If the two series are both $I(1)$, this difference between them may be stable around a fixed mean.

On the other hand, if the two series are integrated of different orders, say $I(2)$ and $I(1)$, then they *must* be drifting apart and the distance between them would be increasing with time. Typically, when two series are integrated of different orders, then linear combinations of them will be integrated to the higher of the two orders and thus the series would not be considered as cointegrated.

The procedure for modeling cointegration of two time series calls for first, a test for unit roots in the individual series, second, a regression of one series on the other, third, a test of the residuals, and fourth, if cointegration is found, the residuals are used as an error correction term in a first-difference regression. In the application to the hedging problem, if both the futures and spot time series possess unit roots, then the procedure to test for cointegration first requires a regression of the spot on futures, with a collection of the residuals $\hat{u}_t$. The residual $\hat{u}_t$ is then regressed on its own lagged value $\hat{u}_{t-1}$.

$$\hat{u}_t = \rho \hat{u}_{t-1} + e_t$$

If the two time series are not cointegrated, then the original regression is spurious and $\hat{\rho}_T$ should be near 1. On the other hand, if $\hat{\rho}_T$ is well below 1, then the null hypothesis of a spurious regression is rejected and the variables are considered to be cointegrated. The test statistic is

$$(T-1)(\hat{\rho}_T-1)$$

Critical values for the test statistic are calculated by generating a sample of the appropriate size for y_{1t}, y_{2t} independent Gaussian random walks, estimating

$$y_{1t} = \alpha + \beta y_{2t} + u_t \quad \text{and} \quad \hat{u}_t = \rho \hat{u}_{t-1} + e_t$$

by OLS, and tabulating the distribution of $(T-1)(\hat{\rho}_T-1)$. In the event of cointegration, the exact specification of the error correction term is determined from the analysis, with the objective that the term will maintain the long-run relationship between the two series. According to Kroner and Sultan (1993), if cointegration is ignored in the regression of price changes to establish a hedge ratio, then the model is misspecified because the data is overdifferenced thus obscuring the long-run relationship between the two time series and a downward bias is implied in the hedge ratio.

Tests for cointegration between the spot and futures have been conducted in several financial and commodity markets. Evidence of cointegration was found by Park and Switzer (1995) in the S&P 500 Index and the Toronto 35 Index, and by Kroner and Sultan (1993) in the British pound, Canadian dollar, German mark, Japanese yen, and Swiss franc. On the other hand, Baillie and Myers (1991) find no cointegration in beef, coffee, corn, cotton, gold, or soybeans. Ghosh (1993) tests for cointegration between the S&P 500 futures and three spot indices-the Dow Jones Industrial Average, S&P 500, and New York Stock Exchange composite, and finds evidence that each spot index is cointegrated with the S&P 500 futures contract.

2.4 Option Delta Neutrality Model

As the price of an underlying security changes, the theoretical value of a call also changes. The amount of the change depends upon the "moneyness" of the contract. When the call is deep in-the-money, its value changes at a rate almost identical to that of the underlying, and when it is deep out-of-the-money, its value may change only slightly with large changes in the price of the underlying. The measure of how an option's value changes for a given change in the price of the underlying contract is called delta,

$$\Delta = \frac{\textit{Dollar change of option price}}{\textit{Positive dollar change of asset price}}$$

which is also referred to as the hedge ratio. In theory, an option can never gain or lose value more quickly than the underlying, so the upper bound of delta is 1.00. A call also cannot theoretically move in the opposite direction of the underlying market, so the lower bound of delta is zero. Most calls will have deltas somewhere between zero and 1.0. A call with a delta of .25 is expected to change its value at 25 percent of the rate of the underlying asset. When hedging in the options market, if an increase (decrease) in the option position exactly offsets the decrease (increase) in the opposite underlying asset position, then the hedge is neutral as to the direction of the underlying asset.

If an option position is hedged against the underlying asset, the delta conveys the proper ratio of underlying contracts to options required to establish a neutral stance. The underlying asset always has a delta of 1.0, so the proper hedge ratio is determined by dividing 1.0 by the option's delta. For example, a call with a delta of 0.5 requires a hedge ratio of 2.0 (1.0/0.5), or for every two options purchased, the hedger should sell one underlying contract to achieve neutrality. In a *portfolio* of multiple options and/or underlying assets, the delta is computed as a weighted average of the individual deltas, and if this "position delta" equals zero, then the portfolio is considered to be delta neutral.

The neutral position delta provides no particular preference as to the direction in which the underlying instrument will move. This is in keeping with the market makers' objective of trading options to derive the bid-ask spread as a primary source of income. They generally do not speculate on the capital gains due to option price changes. As such, the delta neutral hedge position assumes risk minimization as a goal. When the delta values are extracted from an option pricing model, the hedging methodology for purposes of this study is called "option delta neutrality".

Analysis of hedging techniques through the use of options and options on futures has focused on the delta-hedging ratios derived from various option pricing models such as Black and Scholes (1973), Black (1976), and Barone-Adesi and Whaley (1987). The original Black-Scholes (1973) option pricing model evaluated European options on non-dividend paying stock. A later revision incorporated the dividend component, Black (1976) modified the model to price options on futures contracts, and Garman and Kohlhagen (1983) evaluated options on foreign currencies. Cox, Ross, and Rubenstein (1979) developed a binomial model to value American options while Barone-Adesi and Whaley (1987) utilize a quadratic model to value American options.

The Black (1976) model for evaluating European options on futures contracts is as follows:

$$C=Ue^{-rt}N(d)-Ee^{-rt}N[d-v(t)^{1/2}]$$

$$P=-Ue^{-rt}N(-d)+Ee^{-rt}N[v(t)^{1/2}-d]$$

$$\text{where } d = \frac{\ln(U/E)+(v^2/2)t}{v(t)^{1/2}}$$

$$\text{call delta} = e^{-rt}N(d)$$

$$\text{put delta} = -e^{-rt}N(-d)$$

C = theoretical value of a call
U = price of the underlying contract
t = time to expiration in years
e = base of the natural logarithm
ln = natural logarithm
N(x)=the normal cumulative distribution function

P = theoretical value of a put
E = exercise price
v = annual volatility expressed as a decimal fraction
r = risk-free interest rate expressed as a decimal fraction

Delta neutral hedge ratios are equal to the call delta and put delta.

According to Natenberg (1994), a floor trader, the American option pricing models which allow for the possibility of early exercise are not worth the additional effort, particularly in the futures options markets. The additional early exercise value is so small, that there is virtually no difference between the Black-Scholes model and an American pricing model.

Empirical testing of American versus European option pricing models has been conducted in various markets. Blomeyer and Johnson (1988) compare the Black and Scholes

(1973) European model to the Geske and Johnson (1984)American model on put options for four stocks. They concluded that both models tended to undervalue put options, although the Geske/Johnson values were significantly closer to market prices. In a study of the FTSE-100 stock index, Dawson (1994) concludes that American call options are frequently overpriced. Shastri and Tandon (1986) consider options on futures for the S&P 500 and German Mark. They compare the Black (1976) model with the Geske and Johnson (1984) model and find that the European model performs as well as the American model in replicating prices observed in the market.

As mentioned previously, a change in any of the inputs to the model results in a change to delta. That being the case, the option market maker must continually readjust his position to remain delta neutral. Obviously, that would be impossible, but some decision must be made as to just how often the rebalancing should occur. If the price changes are small and stay within a tight range, then delta-neutrality hedging will probably work relatively well with very little rebalancing. However, when this isn't the case, Thompkins (1994) suggests that three methods are used. The first consists of a rebalancing decision that occurs at approximately the same time once a day. The second approach pre-defines an acceptable delta level and rebalances the position back to zero when the level is exceeded. The third technique is the daily standard deviation principle where the market maker first converts his predicted annual volatility to a daily volatility with a division by the square root of 260 (number of trading days). This number is then multiplied by the current level of the underlying instrument to determine the number of ticks represented. When the underlying instrument's price varies by more than this range, the trader rebalances back to delta neutrality.

Natenberg (1994) also suggests three approaches for portfolio adjustments. The first is to adjust at regular intervals based on the interval of the trader's volatility estimate. For example, if the volatility estimate is based on daily price changes, then the adjustment is done daily. If the estimate is based on weekly price changes, then the adjustment is done weekly. The second approach adjusts when the position becomes a predetermined number of deltas long or short. In this case, the trader doesn't know how often the adjustment will be made. In some instances, the adjustments are frequent while in other instances, no adjustment is made for long periods of time. The predetermined number of deltas depends on the size of the positions as well as the trader's capitalization. The third approach is to adjust by *feel*, for those traders who can sense when the market is about to move.

Hsin, Kuo, and Lee (1994) conduct an empirical test of delta hedging methodology by finding the solutions for the Biger and Hull (1983) European option pricing model and then using the Barone-Adesi and Whaley (1987) American option pricing model to find option prices and delta hedging ratios for four foreign currencies. They develop a measure of hedging effectiveness similar to that used by Cecchetti, Cumby, and Figlewski (1988) that assumes that the hedger determines the hedge ratios by maximizing expected utility which is completely ordered by mean and variance. A negative exponential utility function with constant absolute risk aversion is assumed and the function, $V(E(r),\sigma;A)$ is a monotone increasing function of the expected utility where A denotes the absolute risk aversion coefficient. Hedging effectiveness is measured by the difference of the certainty equivalent returns between the hedged position and the spot position.

$$HE^* = V(E(r_H), \sigma_H; A_o) - V(E(r_S), \sigma_S; A_o)$$

$$HE^* = V(r_H^{ce}, 0; A_o) - V(r_S^{ce}, 0; A_o)$$

$$HE^* = r_H^{ce} - r_S^{ce}$$

If HE* is positive, certainty equivalent returns are higher and the hedge is effective. The option delta hedging strategy is compared to a currency futures market hedge in four foreign currencies and the futures hedge is found to be superior.

Hancock and Weise (1994) use the Black (1976) model to determine a hedge ratio for the S&P index options on futures and use the Black-Scholes (1973) model to determine a hedge ratio for the S&P index options. These two options strategies are then compared to a futures hedge. Their results show that all of the hedging instruments are equally effective for hedging a spot position in the S&P 500.

2.5 Naive Model

The naive model is the simplest to use in terms of hedge ratio computation in the spot/futures portfolio framework. In fact, the naive model dictates that the value of the spot position should be exactly offset by a futures position, resulting in a hedge ratio of one. The underlying assumption is that risk reduction is the primary goal of the hedger. And in fact, if price movements in the spot market are exactly equal to price movements in the futures market, then price risk can be completely eliminated. The naive hedge is often used as a point of reference for testing the effectiveness of other hedging models.

2.6 Risk Minimization Model

Ederington (1979) uses a portfolio model wherein the spot position is considered fixed and the optimal hedge ratio, or number of futures contracts is determined per unit of spot. The hedge ratio is obtained from the Ordinary Least Squares regression of spot price changes on futures price changes. The optimal hedge ratio results in the minimum risk level for the portfolio and consists of the covariance between the spot and futures divided by the variance of the futures. The measure of hedging performance in this framework is the coefficient of determination (R^2) from the regression.

The objective of the hedger is to minimize the variance of the price changes for the portfolio. The expected price change and variance of the hedged position are

$$E(\Delta_p) = C_s E(P_{s,t+1} - P_{s,t}) + C_f E(P_{f,t+1} - P_{f,t})$$

$$var(\Delta_p) = C_s^2\sigma_s^2 + C_f^2\sigma_f^2 + 2C_sC_f\sigma_{sf}$$

where C_s,C_f are the number of units of the spot and futures holdings, P_s, P_f are the prices of the spot and futures, σ_s^2, σ_f^2 are the variances of the spot and futures price changes, and σ_{sf} is the covariance of spot and futures price changes.

To minimize the variance, the equation is differentiated with respect to C_f, and the partial derivative is set equal to zero.

$$\frac{d\ var(\Delta_p)}{dC_f} = 2C_f\sigma_f^2 + 2C_s\sigma_{sf} = 0$$

$$C_f^* = \frac{-2C_s\sigma_{sf}}{2\sigma_f^2}$$

Assuming that the number of units of the spot held, C_s, equals 1, then the optimal number of futures contracts per spot is equal to

$$C_f^* = -\frac{\sigma_{sf}}{\sigma_f^2}.$$

This result can be obtained by regressing spot price changes on futures price changes as follows:

$$(P_{s,t+1} - P_{s,t}) = \alpha + \beta(P_{f,t+1} - P_{f,t}) + \epsilon_t$$

where

$P_{s,t}$ = the spot price at time t

$P_{f,t}$ = the futures price at time t

and β equals C_f^* as shown above.

To measure the effectiveness of the hedge ratio, the hedger can compute the proportional reduction in the variance of price changes in the hedged portfolio to the variance of the unhedged position

$$e = 1 - \frac{var(\Delta_p)^*}{\sigma_s^2}$$

$$e = 1 - \frac{\sigma_s^2(1 - \rho^2)}{\sigma_s^2}$$

$$e = \rho^2$$

where ρ equals the correlation coefficient of spot price changes and futures price changes. The effectiveness of the hedge, e, is the correlation coefficient squared, R^2, from the regression.

Subsequent to Ederington's article, other authors have provided refinements to his technique. For instance, Herbst, Kare, and Caples (1989) point out that time series data on spot and futures can exhibit serial correlation, resulting in violation of the basic assumptions of the OLS model. In particular, the R^2 statistic is overestimated, the variance of the error term is underestimated, and the slope coefficient (optimal hedge ratio) is inefficient. A simple refinement to correct for serial correlation utilizes an autoregressive procedure where the error term is modeled as

$$\epsilon_t = \nu_t - \alpha_1\epsilon_{t-1} - \dots - \alpha_p\epsilon_{t-p}$$

with ν_t normally and independently distributed with a mean of zero and a variance of σ^2. They test this simple autoregressive model but find that the error terms are still correlated across time so they ultimately use an autoregressive integrated moving average (ARIMA) specification

$$(1\text{-}B)SP_t = \Gamma(B) \cdot FU_t + \theta(B)/\Phi(B) \cdot e_t$$

where SP_t and FU_t are spot and futures rates at time t, B is the backshift operator, such that $B(X_t) = X_{t-1}$, $\Gamma(B)$ is the transfer function, $\theta(B)$ is the autoregressive operator, and $\Phi(B)$ is the moving average operator. Yau (1993) also recognizes serial correlation as an important source of hedge ratio instability and uses a maximum-likelihood method of correction.

Ghosh (1993) extends the analysis in another direction with tests for cointegration between the spot and futures. After finding evidence of unit roots and cointegration, he proposes the following error correction model:

$$\Delta P_{s,t}=\alpha u_{t-1}+\beta\Delta P_{f,t}+\sum_{i=1}^{m}\delta_i\Delta P_{f,t-i}+\sum_{j=1}^{n}\theta_j\Delta P_{s,t-j}+e_t$$

where $P_{s,t}$ is the spot price series, $P_{f,t}$ is the futures price series, and u_t is the cointegrating residual.

Empirical studies of the risk minimization model include Ederington (1979) who computes hedge ratios and the hedging effectiveness of two-week and four-week hedges in the GNMA and T-Bill markets. His conclusions include that in most cases, the estimated hedge ratio is less than one, and that both the GNMA and T-Bill hedging effectiveness is greater over long (four-week) periods than short.

Hill and Schneeweis (1984) extend the Ederington study into additional markets by assuming a spot and futures position in five foreign currencies and three equity indices. They find significant reductions in variability by using a risk minimizing hedge over a naive strategy. They also address the issue of cross hedging by assuming a spot position in corporate bonds and a position in either GNMA or T-Bond futures. These results also indicate significant reductions in variability.

Ghosh (1993) tests for the presence of cointegration between spot and futures in the S&P 500 index, Dow Jones Industrial Average, and New York Stock Exchange composite index. He then incorporates an error correction term into the risk minimizing model and finds that optimal hedge ratios as well as adjusted R^2s are higher. Also, out-of-sample forecasts are better with an error correction term.

Numerous other empirical studies have also utilized the risk minimizing model in various markets, including foreign currencies by Hill and Schneeweis (1982), Dale (1981), and Marmer (1986); treasury bills by Lasser (1987); European Currency Unit by Saunders and Sienkiewicz (1988); stock index futures by Figlewski (1984, 1985) and Junkus and Lee (1985); and the Hong Kong Hang Seng Index by Yau (1993).

2.7 Autoregressive Conditional Heteroskedasticity (ARCH) Models

2.71 General Description

Before discussing hedge ratio computations and hedging effectiveness measures, a brief background of ARCH models is helpful. A quick look at a chart of prices for any financial instrument reveals periods of considerable turbulence as well as periods of relative calm. The assumption of time-varying volatility seems evident, and furthermore, the patterns suggest that volatility occurs in clusters. For instance, high volatility periods seem to persist for some amount of time before falling to lower levels. Engle and Mezrich (1995) state that historical data show that some clusters are short-lived, say for a few hours, while others last for years. So what causes these clusters? They explain that the primary source of changes in market prices is the arrival of news, and if the news arrives in rapid succession, the returns will exhibit a volatility cluster. At the highest frequency, such as intraday, the sources of changes to market price are the pressures and turbulence of trading, called noise. At a lower frequency, macroeconomic and institutional changes are the most likely source.

The next step beyond recognizing these patterns, of course, is to answer the question of whether volatility in the future can be predicted. A model with this objective in mind should determine whether recent information is more important than old information and assess how fast information decays. This is the idea behind the Autoregressive Conditional Heteroskedasticity (ARCH) model proposed by Engle (1982) which assumes that today's volatility is measurably related to yesterday's volatility and price movement. According to Engle, the ARCH process is a mean zero, serially uncorrelated process with non-constant variance conditional on the past. Unconditional variances, however, are constant. How these conditional and unconditional variances fit into the analysis was explained by Engle (1993). If y_t is the return on an asset received in period t, then the unconditional mean is

$$E(y_t) = \mu.$$

The conditional mean, c_t, however, uses information, ψ_{t-1}, from the previous period,

$$c_t = E[y_t \mid \psi_{t-1}] = E_{t-1}[y_t].$$

The unconditional variance can be defined as

$$\sigma^2 = E[y_t - \mu]^2 = E[y_t - c_t]^2 + E[c_t - \mu]^2$$

and the conditional variance can be defined as

$$h_t = E_{t-1}[y_t - c_t]^2.$$

The conditional variance depends upon the information set. The idea here is that investors will be able to utilize the information known and forecast more accurately with h_t, implying that volatility is predictable.

2.72 Univariate Models

A formal representation of Engle's model is as follows:

$$y_t = \beta x_t + \epsilon_t$$

$$\epsilon_t = z_t \sqrt{h_t}$$

$$z_t \text{ i.i.d.}, \quad E(z_t)=0, \; Var(z_t)=1$$

$$\epsilon_t \mid \epsilon_{t-1} \sim N(0,h_t)$$

If h_t evolves according to

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2$$

then ϵ_t follows an ARCH process of order q, ARCH(q). This specification suggests that the conditional variance is simply a weighted average of past squared forecast errors. Different values of q reflect how fast volatility is changing.

If the last equation is rewritten as

$$\epsilon_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + [\epsilon_t^2 - h_t]$$

the bracketed term is considered the innovation in the autoregression for ϵ^2. According to Engle (1993), this is the source of the name ARCH.

Evidence of ARCH effects are confirmed with Engle's (1982) ARCH test in a two step process. First, the time series of price changes is modeled as

$$R_t = \alpha + u_t$$

The residuals are saved and squared to be used in a second regression

$$u_t^2 = \delta + a_1 u_{t-1}^2 + \ldots + a_m u_{t-m}^2 + e_t$$

The sample size T times the uncentered R_u^2 from the regression then converges in distribution to a χ^2 variable with m degrees of freedom. The null hypothesis is that u_t is i.i.d.~N(0,σ^2).

In those models where a long lag length q is required, the Generalized ARCH, or GARCH model developed by Bollerslev (1986) provides an alternative. If h_t evolves according to

$$h_t=\alpha_0+\sum_{j=1}^{q}\alpha_j\epsilon_{t-j}^2+\sum_{k=1}^{p}\delta_k h_{t-k}$$

then ϵ_t follows a GARCH(p,q) process. In this representation, past conditional variances are also allowed to impact the present conditional variance.

A potential loss of information occurs in the previous ARCH and GARCH models, however, because the residuals are squared prior to estimation. Thus, an extension to the GARCH model was formulated by Nelson (1991) to account for the fact that information may result in asymmetric volatility behavior. For instance, a negative surprise may increase volatility while a positive surprise of the same magnitude may decrease volatility.[4] The result of this intuition was the Exponential GARCH model. If h_t evolves according to

$$\log\ h_t=\alpha_0+\sum_{k=1}^{p}\delta_k\log\ h_{t-k}+\sum_{j=1}^{q}\alpha_j(\phi z_{t-j}+\gamma[|z_{t-j}|-E|z_{t-j}|])$$

where $z_t=\epsilon_t/\sqrt{h_t}$,

then ϵ_t follows an EGARCH(p,q) process. The use of logarithms allows the parameters to be negative without the variance becoming negative. The asymmetric relation is captured in the third term on the right hand side of the above equation which must be a function of both the magnitude and the sign of z_t. Over the range $0<z_t<\infty$, the term is linear in z_t with a slope of $\phi+\gamma$, and over the range $-\infty<z_t\leq 0$, the term is linear in z_t with a slope is $\phi-\gamma$. The

4 As recapped by Bollerslev, Chou & Kroner (1992), asymmetric volatility reactions in equity markets are attributed to a so-called "leverage effect" whereby a reduction in the equity value raises the debt-to-equity ratio resulting in an increased riskiness of the firm and an increase in future volatility.

term $[|z_t| - E|z_t|]$ represents the magnitude effect and the sign effect is captured in the ϕ coefficient.

The impact of this relationship upon variance is demonstrated in Table 1. For this particular example, γ equals 1 and the α_1, ϕ, and z_{t-1} coefficients values are assumed to be 0.20, 0.80, and 0.02, any of which can be either positive or negative. The first two lines illustrate the situation where both α_1 and ϕ are positive. In this case, a positive standardized residual, z_{t-1}, results in a higher variance than a negative residual of the same magnitude. The third and fourth lines continue the assumption of a positive magnitude effect, α_1, but now the sign effect, ϕ, is negative. In this instance, a negative standardized residual results in a higher variance than a positive residual of the same magnitude. The last four lines of the example assume a negative magnitude effect, α_1. If the sign effect, ϕ, is positive, then a negative residual results in higher variance than a positive residual. If the sign effect is negative, then a positive residual results in higher variance than a negative residual.

Engle and Mezrich (1995) describe the estimation procedure for these GARCH models. Using historical data, a set of parameters are chosen to compute volatility for every day over the sample. The resulting volatilities are then compared to the observed volatility clusters, and if they fail to match, new parameters are chosen. In this way, the parameters are both estimated and checked using the same data. The method of maximum likelihood provides a systematic approach to the estimation by postulating a well-defined objective function and then maximizing it with respect to the unknown parameters. The solution is obtained through an iterative algorithm.

Within the maximum likelihood method, an assumption must be made concerning the distribution of z_t. Hamilton (1994) explains the issue as follows. The *unconditional* distribution of many financial time series seems to exhibit kurtosis, some of which can be explained by the presence of ARCH. In that event, the *conditional* distribution is normally distributed and the density is

$$\frac{1}{\sqrt{2\pi h_t}}\exp\left(\frac{-(y_t-x_t'\beta)^2}{2h_t}\right)$$

The parameters of the univariate GARCH specification under the assumption of normality are obtained through maximization of the log likelihood

$$\sum_{t=1}^{T}\log f(y_t|x_t,Y_{t-1};\theta)=-(T/2)\log(2\pi)-(1/2)\sum_{t=1}^{T}\log(h_t)-(1/2)\sum_{t=1}^{T}(y_t-x_t'\beta)^2/h_t$$

Empirical studies of financial time series utilizing the Autoregressive Conditional Heteroskedasticity (ARCH) model by Engle (1982) and the Generalized (GARCH) model by Bollerslev (1986) have been extensive.[5] Engle (1982) uses the ARCH model to estimate the means and variances of inflation in the United Kingdom and finds the ARCH effect to be significant. Bollerslev (1986) applies the GARCH model to explain the uncertainty of the inflation rate and finds it to provide a better fit than the ARCH model. Nelson (1991) uses the EGARCH method to estimate a model of the risk premium on the CRSP Value-Weighted Market Index. Pagan and Schwert (1990) compare various measures of stock volatility including GARCH and EGARCH and find EGARCH to be superior to GARCH because it

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See Bollerslev, Chou and Kroner (1992) for an extensive discussion of the models and empirical testing.

reflects the asymmetric relation between volatility and past returns.

2.73 Multivariate Models

For the multivariate extension of GARCH, the form becomes:

$$\mathbf{y}_t = \boldsymbol{\beta}'\mathbf{x}_t + \epsilon_t$$

$$\epsilon_t | \psi_{t-1} \sim N(\mathbf{0}, \mathbf{H}_t)$$

where $\mathbf{y}_t$ is an $(n \times 1)$ vector, $\boldsymbol{\beta}$ is an $(n \times k)$ matrix of coefficients, $\mathbf{x}$ is a $(k \times 1)$ vector of explanatory variables, ϵ_t is an $(n \times 1)$ vector of white noise residuals, and $\mathbf{H}_t$ is an $(n \times n)$ conditional variance-covariance matrix of residuals.

Several parameterizations of the multivariate GARCH model have been suggested. Engle and Kroner (1993) proposed[6]

$$\mathbf{H}_t = \mathbf{C} + \mathbf{B}_t\mathbf{H}_{t-1}\mathbf{B}_t' + \mathbf{A}_t\epsilon_{t-1}\epsilon_{t-1}'\mathbf{A}_t'$$

where $\mathbf{C}$, $\mathbf{B}_t$, and $\mathbf{A}_t$ denote $(n \times n)$ matrices of parameters. For hedging applications, n=2.

Another parameterization suggested by Bollerslev (1990) is the constant correlation with the following matrix representation

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} h_{1,t} & 0 \\ 0 & h_{2,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{1,t} & 0 \\ 0 & h_{2,t} \end{bmatrix}$$

This results in the following two equations

$$h^2_{1,t} = c_1 + a_1\epsilon^2_{1,t-1} + b_1h^2_{1,t-1}$$

$$h^2_{2,t} = c_2 + a_2\epsilon^2_{2,t-1} + b_2h^2_{2,t-1}$$

Bollerslev, Engle, and Wooldridge (1988) suggest the vech($\mathbf{H}$)

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[6]All parameterizations shown are for the GARCH (1,1).

$$\text{vech}(\mathbf{H}_t) = \mathbf{C} + \mathbf{A}\ \text{vech}(\epsilon_{t-1}\epsilon_{t-1}') + \mathbf{B}\ \text{vech}(\mathbf{H}_{t-1})$$

where vech is the column stacking operator that stacks the lower triangular portion of a symmetric matrix. For the GARCH (1,1), the matrix representation is as follows

$$\begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} \epsilon^2_{1,t-1} \\ \epsilon_{1,t-1}\epsilon_{2,t-1} \\ \epsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}$$

where $h_{11,t}$ equals the variance of $(\epsilon_{1,t})$ at time t, $h_{12,t}$ equals the covariance of $(\epsilon_{1,t},\epsilon_{2,t})$ at time t, and $h_{22,t}$ equals the variance of $(\epsilon_{2,t})$ at time t. Due to the large number of parameters, a simplification suggested by the authors assumes that the A and B matrices are all taken to be diagonal, implying that each variance and covariance depends only on its own past values and prediction errors. This reduces the number of parameters from 21 to 9.

Recalling that the univariate E-GARCH model assumes that h_t evolves as

$$\log\ h_t = \alpha_0 + \sum_{k=1}^{p} \delta_k \log\ h_{t-k} + \sum_{j=1}^{q} \alpha_j(\phi z_{t-j} + \gamma[|z_{t-j}| - E|z_{t-j}|])$$

where $z_t = \epsilon_t/\sqrt{h_t}$,

under the assumption of constant correlation, the second moments for the bivariate representation can be modeled as

$$h_{11,t} = \exp\{a_{1,0} + a_{1,1}\log h_{11,t-1} + a_{1,2}(|z_{1,t-1}| - E|z_{1,t-1}| + a_{1,3}z_{1,t-1})\}$$

$$h_{22,t} = \exp\{a_{2,0} + a_{2,1}\log h_{22,t-1} + a_{2,2}(|z_{2,t-1}| - E|z_{2,t-1}| + a_{2,3}z_{2,t-1})\}$$

$$h_{12,t} = \rho(h_{11,t}h_{22,t})^{1/2}$$

This representation was first suggested by Koutmos and Tucker (1996) in a study of the interactions between the spot and futures equity markets. In the case of varying correlation, the bivariate matrix representation becomes

$$\begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \exp\left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \log \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \right.$$

$$\left. \left(abs \begin{bmatrix} \epsilon_{11,t-1}/\sqrt{h_{11,t-1}} \\ \epsilon_{1,t-1}\epsilon_{2,t-1}/h_{12,t-1} \\ \epsilon_{22,t-1}/\sqrt{h_{22,t-1}} \end{bmatrix} - \begin{bmatrix} E|z_1| \\ E|z_1|E|z_2| \\ E|z_2| \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{11,t-1}/\sqrt{h_{11,t-1}} \\ \epsilon_{1,t-1}\epsilon_{2,t-1}/h_{12,t-1} \\ \epsilon_{22,t-1}/\sqrt{h_{22,t-1}} \end{bmatrix} \right) \right\}$$

As explained in Johnson and Kotz (1970), if z has a normal distribution, then $|z|$ is said to have a *folded* normal distribution whereby the distribution is formed by folding the part corresponding to negative values of z about the vertical axis and then adding it to the positive part. Leone, Nelson and Nottingham (1961) showed that if z is normally distributed with mean μ and variance σ^2, then $E(|z|) = \sqrt{2/\pi}\,\sigma e^{-\mu^2/2\sigma^2} + \mu[1-2N(-\mu/\sigma)]$ where $N(d)$ is the cumulative unit normal distribution with upper integral limit d. If the mean and variance of z are 0 and 1, respectively, then $E(|z|)$ simplifies to $\sqrt{2/\pi}$. Once again, the *B*, *C*, & *D* matrices are considered to be diagonal to reduce the number of parameters and give a more parsimonious result.

Chowdhury, Kroner, and Sultan (1995) suggest an evaluation of the validity of the GARCH model by using post-estimation diagnostics on the standardized residuals $(\epsilon_{ij,t}/\sqrt{h_{ij,t}})$ and $(\epsilon^2_{ij,t}/\sqrt{h_{ij,t}})$. The Ljung-Box (Q) statistic provides evidence of any lingering serial correlation in the residuals.

2.74 Hedging Applications

The application of these ARCH/GARCH models to the hedging problem was first done by Cecchetti, Cumby, and Figlewski (1988) who recognized that the joint distribution of cash and futures prices is not constant, and therefore that the hedge ratio must also vary over time. In a hedged portfolio, variations in the hedge ratio will change the expected return and variance of the portfolio, thus resulting in a risk/return frontier. The optimal futures hedge is the one that maximizes expected utility by setting the marginal rate of substitution between risk and return equal to the slope of the risk/return frontier. This intersection point also corresponds to the maximum certainty equivalent return. Sub-optimal hedge ratios (and certainty equivalent returns) occur at other points of intersection between lesser indifference curves and the frontier. Within this framework, the optimal futures hedge is found in a two step process. 1) Estimation of the joint distribution of return which is needed to construct a risk-return frontier, and 2) Optimization by finding a hedge ratio that maximizes expected utility.

For the estimation, they first calculate the difference of realized returns from their ex-ante means, which are assumed to be equal to the riskless rate plus a time invariant premium. Then a three equation, third-order linear Autoregressive Conditional Heteroskedasticity (ARCH) model is estimated which yields time-varying estimates of the covariance matrix of returns on spot and futures. The correlation is assumed to be constant so changes in the covariance are due only to changes in the standard deviations. The optimization procedure consists of using the estimates of the covariance matrix to obtain a time series of expected utility maximizing hedge ratios for a logarithmic investor:

$$\max_{h(t)} \iint_{R_f R_s} \log[1 + R_s(t) - h(t)R_f(t)] \cdot f_t(R_s, R_f) dR_s dR_f$$

where $h(t)$ is the hedge ratio, R_s and R_f are the returns on spot and futures, and $f_t(R_s, R_f)$ is the bivariate normal density with covariance matrix determined by the ARCH model. The integral is solved by numerical methods and yields the optimal hedge ratio which is used to determine a value for maximum expected utility and ultimately the certainty-equivalent return. The ex-ante hedging effectiveness for the Cecchetti, Cumby, and Figlewski (1988) model is measured by the certainty equivalent return.

Whereas Cecchetti, Cumby, and Figlewski assume constant correlation, Baillie and Myers (1991) extend the methodology through the use of a multivariate (GARCH) specification with non-constant correlation. Both a diagonal vech parameterization as well as a positive definite parameterization are tested. They contend that if the expected return to holding futures is zero, then the minimum variance hedging rule is also generally the expected utility maximizing rule. This provides a hedge ratio which depends solely on the elements of the conditional covariance matrix of spot and futures returns, H_t.

$$b_t = h_{21,t}/h_{22,t}$$

where b is the hedge ratio, h_{21} is the conditional covariance between spot and futures, and h_{22} is the conditional variance of futures.[7] Estimated parameters from the bivariate GARCH model are used as a basis for measuring hedging effectiveness. The variance of the return on the hedged portfolio, conditional on information available at time $t-1$, is given by

[7]The hedge ratio for the bivariate EGARCH model can be computed in the same manner.

$$Var(R_t|\Omega_{t-1}) = Var(R_t^s|\Omega_{t-1}) + b_{t-1}^2 Var(R_t^f|\Omega_{t-1}) - 2b_{t-1} Cov(R_t^s, R_t^f|\Omega_{t-1})$$

where R_t^s, R_t^f are the return to the spot and futures, and Ω_{t-1} is the information set at time *t-1*. The percentage reduction in the conditional variance of the portfolio return is first compared to the no-hedge outcome. Next, the variance reductions for each period are averaged to get a summary measure. Then the measure for the GARCH portfolio is compared to the same measure for a portfolio with a constant hedge ratio. This *ex-ante* evaluation procedure is the same as that chosen by Myers (1991) who looks at wealth levels instead of returns. In addition, Myers also supplements it with an ex-post method to measure actual wealth levels that would have been achieved under each hedging rule.

Another method for evaluating hedging performance was used by Kroner and Sultan (1993) and Park and Switzer (1995). Both studies construct the portfolios implied by the computed hedge ratios and then calculate the variance of returns to the portfolios over the sample period. They further assume an investor with a mean-variance expected utility function of $EU(x) = E(x) - \gamma Var(x)$ where x is the return from the spot-futures hedged portfolio and γ is the degree of risk aversion. Given that the expected return of the hedged portfolio is zero and the risk aversion is $\gamma=4$, the portfolio variance under each hedging strategy is inserted into the equation to arrive at an average utility. In addition, the equation allows for analysis of reduced returns due to the introduction of transactions costs.

Another issue addressed in the literature concerns the long-run relationship between the two time series under consideration. To explore this question, Kroner and Sultan (1993) conduct cointegration analysis and propose a bivariate GARCH error-correction model.

They assume the constant correlation parameterization, and model the first moments as

$$s_t = \alpha_{0s} + \alpha_{1s}(S_{t-1} - \delta F_{t-1}) + \epsilon_{st}$$

$$f_t = \alpha_{0f} + \alpha_{1f}(S_{t-1} - \delta F_{t-1}) + \epsilon_{ft}$$

where s and f signify spot and futures first differences, S and F are spot and futures prices, and $(S_{t-1} - \delta F_{t-1})$ is the error correction term.

Empirical testing of GARCH hedging models includes Cecchetti, Cumby, and Figlewski (1988) who were among the first to utilize an ARCH model to estimate time-varying estimates of the covariance matrix of returns on cash and futures. A time series of expected utility maximizing one-month hedge ratios are constructed for 20-year Treasury bonds with results showing that over both in-sample and post-sample periods, investors with log utility would prefer the utility maximizing hedge to a variance minimizing hedge by a certainty equivalent return of, on average, 20 basis points.

Baillie and Myers (1991) estimate bivariate GARCH models for cash and futures prices in six commodity markets. In-sample results show that the GARCH hedging model performs better than a constant hedge ratio but there are marked differences from commodity to commodity. Out-of-sample results, however, show the GARCH model to be significantly better than constant optimal hedge ratios for almost every commodity.

Myers (1991) compares a bivariate GARCH model to both a constant conditional covariance and a moving sample variances and covariances model in constructing optimal hedge ratios for wheat storage hedging and finds the GARCH model to be only marginally better.

Park and Switzer (1995) study the economic viability of the bivariate GARCH hedging method in the presence of transactions costs using stock index futures. Through the use of a dynamic hedging strategy, investors rebalance their hedges only if potential utility gains offset losses due to transaction costs. Results show that investors prefer the Bivariate GARCH method over three other strategies, 1) Naive, 2) Ordinary Least Squares, and 3) Ordinary Least Squares with Cointegration between spot and futures.

Kroner and Sultan (1993) use a bivariate error correction model with a GARCH error structure on five foreign currencies and compare it to a constant naive hedge, a constant conventional hedge, and a constant hedge from an error correction model. Within-sample results show variance improvement in all currencies with the conditional hedge, ranging from 2.5 percent over the conventional hedge to 6 percent over the naive hedge. For out-of-sample testing, the conditional hedge again outperforms all other hedges except in the case of the British Pound conventional hedge, where it shows no improvement. Investor utility analysis which incorporates transactions costs shows the conditional hedge to be superior in all cases.

2.8 Comparison of the Risk Minimization Model and the GARCH Model

Comparing the optimal hedge ratio computed with the risk minimization model to the optimal hedge ratio computed with the GARCH model reveals a great deal of similarity.

$$b^* = \sigma_{sf}/\sigma_f^2 \qquad \text{Risk Minimization}$$

$$b^*_t = h_{21,t}/h_{22,t} \qquad \text{GARCH}$$

Both can be described as the covariance between spot and futures divided by the variance of futures. The difference lies in what *kind* of variance is used. In the first case, the variances are unconditional and constant through time. In the second case, the subscript indicates that the variances are conditional on the information available at time t. Recalling the explanation by Engle (1993), the conditional variance can be thought of as one of the components that goes into making up the total unconditional variance.

Kroner and Sultan (1993) provide an excellent explanation of the difference between these two models. First assume that the investor owns one unit of the spot asset and -b units of the futures contract. Then the total change in value of the portfolio can be represented as

$$x = s - bf$$

where s equals the change in spot price from time period 0 to time period 1, $s = (S_1 - S_0)$, and f equals the change in the futures price from time period 0 to time period 1, $f = (F_1 - F_0)$. Next assume that the investor has an expected utility function,

$$EU(x) = E(x) - \gamma Var(x)$$

where γ equals the degree of risk aversion. The investor then maximizes his expected utility with respect to b

$$\text{Max } \{E(s) - bE(f) - \gamma\ [\sigma_s^2 + b^2\, \sigma_f^2 - 2b\sigma_{sf}]\}$$

which gives the optimal number of futures contracts in the portfolio,

$$b^* = \frac{E(f) + 2\gamma\sigma_{sf}}{2\gamma\sigma_f^2}$$

If we assume that $E(F_1) = F_0$, then the equation reduces to

$$b^* = \frac{\sigma_{sf}}{\sigma_f^2}$$

If the joint distribution of spot and futures is constant over time, then $b_1 = b_2 = \ldots = b_T$ and the hedge ratio can be computed as the least squares estimator from the regression of ΔS_t on ΔF_t.

Now if we assume that the distribution of spot and futures prices is time varying, then the total change in value of the portfolio can be represented as

$$x_t = s_t - b_{t-1} f_t.$$

In this example, the investor chooses the optimal one-period holding of futures at each time t by maximizing a slightly different utility function,

$$E_t U(x_{t+1}) = E_t (x_{t+1}) - \gamma \sigma_t^2 (x_{t+1}).$$

Now the risk is measured by conditional variances and the terms are subscripted with t to indicate that they are calculated based on the information available at time t. The hedge ratio at time t is

$$b_t^* = \frac{E_t(f_{t+1}) + 2\gamma \sigma_t(s_{t+1}, f_{t+1})}{2\gamma \sigma_t^2(f_{t+1})}.$$

Again assuming that $E_0(F_1) = F_0$, then the equation reduces to

$$b_t^* = \frac{\sigma_t(s_{t+1}, f_{t+1})}{\sigma_t^2(f_{t+1})}$$

which differs from the previous representation in that the time-varying conditional moments have replaced the time-invariant unconditional moments so that the risk-minimizing hedge ratio will change through time as new information arrives in the market. If the joint distribution of spot and futures is constant through time, then the conditional model reduces to the unconditional model. In both cases, the utility maximizing hedge ratio equals the risk minimizing hedge ratio if the futures are assumed to follow a martingale process (i.e., $E_0(F_1) = F_0$). This simplifying assumption eliminates the requirement of knowing the investor's utility function.

CHAPTER 3: HEDGING OPTIONS ON FUTURES WITH FUTURES

3.1 Data Description, Sample Statistics, and Univariate GARCH Testing

Daily prices of futures and options on futures for the Eurodollar contract are sampled for the period of January 2, 1990 through November 30, 1994. There are four delivery months for both the futures and options, March, June, September, and December, and the last trading day for both is the second London bank business day before the third Wednesday of the contract month. The futures price is quoted as 100 minus the annualized futures interest rate (three-month LIBOR). The trading unit for the Eurodollar futures and options contracts is a Eurodollar time deposit with a principal value of $1,000,000 and a three-month maturity. The minimum fluctuation is 1 point (.01) which is equal to $25 per contract.

For the option on futures, a two-step process is used to choose one strike price for each contract month. First, the closing futures price is checked on each contract switching date, for example, the June futures price on the day following expiration of the March contract. Second, several strikes around that price are sampled. Since some strike prices are traded more frequently than others depending on the movement in the underlying futures, the objective here is to find the most liquid contract.

The sample of prices starts with the March 90 contract on January 2, 1990 and continues until its expiration in March. Then the June 90 contract is used through its expiration, then the September 90 contract, and so on through each year until the end of the sample period. Every trade for the chosen strike price is sampled from the Time and Sales Database of the Chicago Mercantile Exchange. For the daily sample, only the last trade of each day is included, but since the time of the last option trade varies from day to day, the corresponding time is also sampled. This step is necessary so that the futures price can be time-matched to avoid bias due to non-synchronization. In addition, any final trade that does not occur exactly on the hour is matched to the next hour. For instance, if the final trade occurs at 12:47, it is considered as a 1:00 price to be matched with a 1:00 futures price.

For the futures, tick-by-tick prices from the Chicago Mercantile Exchange are sampled on an hourly basis for the nearby contract. As with the options, the sample runs through the expiration date for each contract before switching to the next contract month. The futures prices are then merged with the options prices according to hour so that the appropriate time-matched price is included in the daily sample. Although this procedure does not result in an exact synchronization unless the option trade occurs exactly on the hour, it is much more accurate than a procedure that utilizes only closing futures prices.

The standard practice in much of the hedging literature is to convert the prices to logarithms. As explained by Hamilton (1994), then for small changes, the first difference of the log is approximately the same as the percentage change:

$$\begin{aligned}(1\text{-}L)\log(y_t) &= \log(y_t/y_{t-1}) \\ &= \log\{1 + [(y_t - y_{t-1})/y_{t-1}]\} \\ &\cong (y_t - y_{t-1})/y_{t-1}.\end{aligned}$$

This follows from the fact that for x close to zero, $\log(1 + x) \cong x$. In addition, several authors then multiply the log by some scale number ranging from 100 to 1000. If $\log(y_t)$ is multiplied by 100, then the changes are measured directly in units of percentage change. So if $(1\text{-}L)[100 \times \log(y_t)] = 1.0$, then y_t is 1 percent higher than y_{t-1}.

When futures are used to hedge a spot position, the two returns are both generally of the same magnitude. Options, on the other hand, create a problem in the use of returns. As an example, the prices of the options on January 2, 1990 and January 3, 1990 were 0.36 and 0.26, while the prices of the futures on the same two dates were 91.99 and 91.85. The return on options is -27.8 percent and the return on futures is -0.15 percent. In this instance, the options return is 185 times larger than the futures return, suggesting that 185 futures contracts are required to hedge one option contract. Hedge ratios of this magnitude are obviously impractical. To alleviate this undesirable result, daily price *changes* are used instead of returns. For the same example above, the option price change of -0.10 and the futures price change of -0.14 suggests a hedge ratio of 0.71 futures contracts (-0.10/-0.14) to one option contract.

Another advantage to using price changes versus returns relates to the empirical evidence of an inverse relationship between the level of prices and the rate of return variance.[8] It is recognized that as the expiration date of the option approaches, option prices tend to be very small, implying a possible bias due to high volatility. For example, if the price change of an option is +1/8 and the beginning price is 1/8, then the return is +100 percent whereas a beginning price of 20 equates to a return of only +0.625 percent. The use of price changes alleviates the problem because a one tick increase on an option with a price close to zero is the same to the option market maker as a one tick increase for an option with a price far greater than zero. When using returns, the standard solution to the bias created by

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Cox and Ross (1976) develop the Constant Elasticity of Variance (CEV) model for pricing options where the standard deviation of the return distribution moves inversely with the level of the stock price, thus incorporating the effects of operating and/or financial leverage.

increased volatility is to roll to the next contract at one to two weeks prior to expiration. However, this method fails to address the fact that the market maker must continue to trade in the expiring option up to and including the day of expiration. The use of price changes rather than returns is therefore supported in this framework.

Another issue of importance is that on the day of the contract rollover, the computed price changes are meaningless. So to avoid the elimination of these 20 days from the sample, two prices are included on each expiration date, one for the expiring contract and one for the new contract. For example, if the March contract expires on March 19, the June contract is also sampled on March 19. Then on March 20, the price change is calculated for the June contract using the June contract prices from March 20 and March 19.

Since the OPM delta hedging model requires the input of a risk-free asset, the 90-day spot T-bill is chosen. The hourly prices are collected from the Daily Information Bulletin of the Chicago Mercantile Exchange International Monetary Market. In the same procedure as mentioned before, these prices are also time-matched to the futures and options price series.

Table 2 reports descriptive statistics for both the futures and option on futures of the nearby three-month Eurodollar contract traded on the Chicago Mercantile Exchange. There are 1,158 observations for both. The mean price change over the sample period is 0.0008 for the options and 0.0047 for the futures. The futures mean is significantly different from zero at the 1 percent level. The variance of the option price changes is smaller than the futures, 0.0009 compared to 0.0028. Both the option and futures price changes show a high degree of both skewness and kurtosis at the 0.1 percent level.

To get a better idea of what this data looks like, the raw futures prices, raw options prices, futures price changes, and options price changes are shown in Figures 2 through 5. The futures price series seems to exhibit a definite upward trend from the beginning date of January 2, 1990 through approximately December of 1992. The options price series, on the other hand, appears to be stationary. Figure 4 shows the futures price changes, and the positive drift is no longer evident. The options price changes in Figure 5 also seem to exhibit a stationary pattern.

Several other diagnostic tests are run on the Eurodollar futures and options series, starting with the unit root test which is run on the raw price series. The results are shown in Table 2. The Phillips and Perron test statistics are -78.3810 and -1.8383 for the options and futures respectively. These numbers indicate that a unit root exists in the futures but not the options, which is not too surprising given the evidence from Figures 2 and 3. Since the raw price series are integrated of different orders, the series are drifting apart over time, and this deems the cointegration test unnecessary. Although the prices of futures and options are certainly related, two arguments against cointegration are suggested.

The first is due to the deterioration of time value over the life of the option. In the case where the futures price trends upward over time, the upward movement in the options price is partially offset by the decrease in time value. In the case where the futures price trends downward over time, the downward movement in the options price is magnified by the decrease in time value. The second concerns that portion of the option price derived from the volatility parameter. For example, suppose a call option is purchased when the volatility level is at 15 percent. If the price of the underlying asset falls, the value of the call option should decrease. However, if the implied volatility increases to 30 percent at the same time as the fall of the underlying asset's price, then the value of the option could actually increase.

To test for serial correlation, the Ljung-Box Q statistic for 24 lags is

$$T(T+2)\sum_{j\leq 24}\frac{1}{T-j}\rho_j^2$$

where T is the number of observations, j is the number of lags, and ρ is the correlation. Under a null hypothesis of no serial correlation, Q is asymptotically distributed as a χ^2. With a test statistic of 48.7106 and 26.6925 for the options and futures respectively, the Ljung-Box test shows evidence of serial correlation in the options market but not in the futures market.

Table 2 also displays the Engle tests for ARCH effects at 1, 3, 5, and 10 orders. The options show significance in each case, however, the futures exhibit ARCH effects only at the 5th and 10th orders. These Engle test results suggest that prior to computing hedge ratios within the bivariate model, further univariate testing of the futures and options time series is necessary to determine whether ARCH, GARCH, and EGARCH effects are prominent. These results are shown in Table 3.[9] Since the Engle ARCH test above suggested significant ARCH effects of the 5th order in futures, the ARCH(5) model is estimated. For the options, the ARCH model coefficients are 0.0451, 0.1326, 0.1802, 0.1045, and 0.4975 for the 5 lags. The 2nd, 3rd, 4th, and 5th lags are highly significant, but the first lag is not.

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Although the results from Table 2 are computed on the entire sample period of 1,158 observations, all subsequent results are based only on the first half of the sample. The change is made to be consistent with the in-sample hedging results which are also computed on only the first half of the sample.

The coefficients suggest that yesterday's errors are virtually lacking in information for the determination of today's variance, but the errors from all previous days are relevant. In fact, the lag from day 5 is larger in magnitude than the first 4 days' lags, with no decay evident.

For the futures, the ARCH model shows coefficients of -0.0097, -0.0376, 0.1095, -0.0138, and 0.1277 for the five lags. Although three of these coefficients are negative, a check of the variance series confirms that none of the daily variance values are negative. The 2nd lag is significant at the 0.1 percent level, the 3rd is significant at the 5 percent level, and the 5th is significant at the 1 percent level. The 3rd and 5th lags are of a much greater magnitude than the other three lags and once again, there is no decay evident.[10]

The parameters of the univariate GARCH (1,1) specification under the assumption of normality are also shown in Table 3. The GARCH model for options shows all of the parameters to be highly significant. The coefficient on the lagged variance term is 0.8737 while the coefficient of the lagged error term is 0.0800. These results seem to confirm the previous significant ARCH effects. For the futures, however, the GARCH model coefficients are all insignificant.

Turning now to the results of the EGARCH model for the Eurodollar options, these results are shown at the bottom of Table 3. The coefficient of the magnitude, a_l, for the options is 0.1895 which is significant at the 0.1 percent level and the coefficient that

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A ten lag ARCH model is also tested. For the futures, a high degree of significance is found for lags 2, 5, 7, and 9. Marginal significance is found for lags 3 and 10. Lags 1, 4, 6, and 8 are insignificant. The log likelihood value is 1345. For the options, a high degree of significance is found for lags 3, 4, 5, 7, 8, and 9. Marginal significance is found for lag 2, and lags 1, 6, and 10 are insignificant. The log likelihood value is 1665.

measures the sign effect, c, has a value of 0.7777, which is also significant at the 0.1 percent level. Since both of these coefficients are positive, this suggests that positive price changes result in higher volatility than negative price changes of an equal magnitude. Although the "leverage effect" explanation for asymmetric volatility reactions in the equity markets does not apply here, the EGARCH process is indeed supported by Eurodollar options prices.

One possible explanation for EGARCH effects could be the consequence of market making costs which are reflected in the bid-ask spread. Jameson and Wilhelm (1992) note that the variation in spreads is related to three costs faced by the market maker: inventory carrying costs, asymmetric information costs, and order processing costs. George and Longstaff (1993) further suggest that inventory holding costs may be dominant and that these costs could be related to option value. If frictions in money markets imply that market makers' working capital is not perfectly liquid at zero cost, then changing the value of net inventory is costly and these costs would be positively related to the price of the option traded because higher priced options imply a greater change in the value of inventory. When option prices get higher, inventory carrying costs also become higher, which leads to a wider bid-ask spread. The wider the bid-ask spread, the higher the price volatility will be. George and Longstaff (1993) test this hypothesis in the S&P 100 index options market and find that bid-ask spreads are positively related to the option price.

The EGARCH model coefficients for futures are all insignificant with the exception of the constant term, a_0. The magnitude effect, a_1, is negative at -0.0936. The coefficient that measures the sign effect, c, has a value of 1.3357. The combination suggests that the volatility tends to be higher when price changes are negative compared to positive price changes of an equal magnitude, although this asymmetry effect appears statistically insignificant.

The likelihood ratio test statistic (LR) for EGARCH vs. GARCH is 58 (=2(1699-1670)) for the options, which is significant at the 0.1 percent level, and it is 6 (=2(1352-1349)) for the futures, which is insignificant. These results suggest than an EGARCH model improves, relative to a GARCH model, a measure of fit substantially for the Eurodollar options on futures. Overall these univariate results are indicative of ARCH effects in the options but not in the futures. However, the bivariate relationship between options and futures must be addressed for the computation and evaluation of hedge ratios.

3.2 Option Delta Neutrality

Since the option contract under consideration has a futures contract as the underlying instrument, the appropriate option pricing model is Black's (1976). Although Black's model is developed for European options, given the conclusions of Shastri and Tandon (1986) and Natenberg (1994) concerning its performance for American options, it is used here for the computation of hedging ratios under delta-neutrality. The hedge ratio is determined as $e^{-rt}N(d)$ as shown in Section 2 above. It should be noted here that the hedge ratio computed with the Black OPM is only valid for small futures price changes. This fact provides additional support for the use of daily prices versus the weekly prices used in numerous prior hedging studies. It is naturally assumed that daily price changes would be small compared to weekly price changes.

Black's OPM requires an estimate of volatility, and the question arises as to which measure to use. Myers and Hanson (1993) suggest a moving sample variance of past price changes over the most recent 30 days, while Hancock and Weise (1994) use the variance of daily returns over the previous contract period. For this study, the volatility is estimated from the previous 30 days of futures prices for the relevant contract month. The use of 30 trading days equates to approximately six weeks of price observations which should be representative of both low and high volatility days. The idea here is to capture the high volatility days generated by the monthly release of government reports such as employment, the consumer price index, etc.[11] In order to compute a call delta on the first day of the option sample of January 2, 1990, it is necessary to have the previous 30 daily prices for the underlying futures contract. To accommodate this requirement and avoid the loss of the first 30 call deltas in January, the daily prices for futures contracts are sampled for the last 30 days of 1989. In this manner, the sample size is maintained at 1,158 observations.

While the other models lend themselves to the use of all future data points (perfect foresight) to construct a hedge ratio on a within-sample basis, the delta neutrality model does not. For instance, the risk minimization model assumes perfect foresight and utilizes every data point over the sample period within a regression estimation to compute a constant hedge ratio, and the GARCH models employ every observation in the estimation of coefficients for the lagged error and variance terms. The options delta neutrality model, on the other hand,

[11] Since the hedge ratio is being extracted from the OPM for hedging effectiveness comparisons, it is important that the input variables used result in an accurate option price. When the option prices are computed through the Black (1973) OPM using 30 days of historical prices for the volatility of the Eurodollar, the mean option price is 0.166247 and the variance is 0.017516 for the 1/2/90-11/30/94 period. The mean of the actual option prices is 0.156588 with a variance of 0.018587. These values are quite close and provide confidence for the chosen volatility measure.

uses only single known data points at each moment in time along with a volatility estimate computed from strictly historical information. This difference in information utilization on a within-sample basis has the potential to diminish the perceived effectiveness of the OPM when compared to other models.

Since a change in any of the inputs to the Black model results in a change to delta, the hedged position must be readjusted to maintain a neutral stance. For this analysis, the option/futures portfolio position is assumed to be rebalanced on a daily basis. Specifically, at the end of the first day, one Eurodollar call option is purchased and the appropriate number of futures contracts as determined by Black's OPM are shorted. The position is maintained until the end of the next day, when the procedure is repeated. The change in the value of the delta neutral portfolio from day t to day t+1 is represented as:

$$\Delta Portfolio_{t,t+1} = [P_{o,t+1} - P_{o,t}] - e^{-r_t T_t} N(d_t)[P_{f,t+1} - P_{f,t}].$$

The performance of the OPM delta hedge can be compared to the value of an unhedged option position that varies over time as $\Delta P_{o,t,t+1} = P_{o,t+1} - P_{o,t}$.

The sample period (1/2/90 - 11/30/94) is divided into a within-sample period (1/2/90 - 5/15/92) and an out-of-sample period (5/16/92 - 11/30/94). During the 1/2/90-5/15/92 period, there are a total of 577 observations of daily price changes. The variance of the change in the unhedged position value, $\Delta P_{o,t,t+1}$, is 0.001161. If the market maker reduces the variance of his portfolio via the Black OPM delta hedging strategy by updating the hedge ratio on a daily basis with the volatility forecast computed from the past 30 daily returns, the variance of the $\Delta Portfolio_{t,t+1}$ is 0.000127. The OPM strategy results in a 89.06 percent reduction in the variance of the option price changes where the percentage is computed as (1 - (0.000127/0.001161)).[12] This result achieves the market makers' goal of minimizing the risk of the option portfolio due to the futures price movement while preserving their bid-ask spread income.

3.3 Naive Hedge

The traditional approach for the naive model suggests that a spot position should be exactly offset by a futures position which results in a hedge ratio of one. The assumption is that price movements in the spot and futures position will be approximately equal. When the switch is made to an options framework, however, these assumptions are no longer valid. The relationship between the price of the option and the price of the underlying futures contract depends upon the extent to which the option is in- or out-of-the money. For an option that is deep in-the-money, the correspondence between the price of the option and the price of the futures contract will be close to one. When the option is at-the-money, the option price move will be approximately half of the futures price move, and when the option is deep out-of-the money, the option price move may be close to zero.

A naive hedge ratio of one would make sense only if the options in the portfolio are deep-in-the-money. Since the sample of options used for this study include in-the-money, out-of-the-money, and a preponderance of at-the-money cases, the hedge ratio chosen for the naive strategy is 0.50.

To measure the performance of the naive hedging strategy, a portfolio is formed by purchasing one call option and selling 0.50 futures contracts. The variance of the daily

[12]The out-of-sample results are shown in Section 3.7.

portfolio value change over the 1/2/90-5/15/92 within-sample period is 0.000376. This compares to the variance of the unhedged option position change of 0.001161 for a variance reduction of 67.61 percent (1 - 0.00376/0.001161).

3.4 Risk Minimization

The regression format for the option/futures portfolio is

$$(P_{o,t} - P_{o,t-1}) = \alpha + \beta(P_{f,t} - P_{f,t-1}) + \epsilon_t$$

where the error term is modeled as $\epsilon_t = \alpha_1\epsilon_{t-1} + \alpha_2\epsilon_{t-2} + ... + \alpha_p\epsilon_{t-p} + \eta_t$. Results of the minimum risk strategy on a within-sample basis over daily observations from 1/2/90-5/15/92 are shown in Table 4. The Durbin Watson statistic at 1.8643 is insignificant, implying that there is no sign of autoregressive disturbances. The hedge ratio, β, is -0.4824 and is significant at the 0.1 percent level. This implies that 0.4824 contracts of the Eurodollar futures needs to be shorted for a long position of 1 call option to minimize the variance of the hedged position value change. Since the hedge ratio is considerably less than one, this indicates that the Eurodollar futures contract is more volatile than the Eurodollar option contract. The measure of hedging effectiveness, R^2 at 0.6726, shows that there is a substantial 67.26 percent reduction in the variance of option price changes achieved with the futures hedge. That is, a portfolio of 1 call option (long) and 0.4824 contracts of the futures (short) should lead to a reduction of 67.26 percent in the variance of the portfolio value change. When keeping track of the actual portfolio value changes for the 1/2/90-5/15/92 period, the variance is 0.000376 which implies a 67.61 percent variance reduction from the unhedged position (1-0.000376/0.001161). This is consistent with the regression result, where the discrepancy between 67.26 percent and 67.61 percent is due to rounding.

3.5 *Bivariate GARCH*

The GARCH model used for this study is the non-constant correlation parameterization used by Baillie and Myers (1991) and Lien and Luo (1994), where the price changes, $R_{1,t}$ and $R_{2,t}$ on options and futures are modeled as

$$R_{1,t} = \mu_1 + \epsilon_{1,t}$$

$$R_{2,t} = \mu_2 + \epsilon_{2,t}$$

$$\epsilon_t | \psi_{t-1} \sim N(\mathbf{0}, \mathbf{H}_t)$$

where ϵ_t is a (2x1) vector of residuals, ψ_{t-1} is the information set at time t-1, and $\mathbf{H}_t$ is a (2x2) conditional variance-covariance matrix of residuals. The within-sample results of this model for 1/2/90-5/15/92 are shown in Table 5, where the coefficients are estimated using the Berndt, Hall, Hall, and Hausman (BHHH) algorithm. The intercept terms for the options and futures price changes are 0.0019 and 0.0066 with only the futures showing significance at the 5.0 percent level. The constant term for the options variance, C_1, is 0.0005, the constant term for futures variance, C_3, is 0.0008, and the constant term for the covariance, C_2, is 0.0007. All of these terms are significant at the 0.1 percent level. One point of interest here is that the unconditional covariance and variance of the futures can be constructed as $C_2/(1-A_{22}-B_{22}) = 0.00163944$ and $C_3/(1-A_{33}-B_{33}) = 0.00345153$, respectively.[13] The unconditional hedge ratio is therefore 0.00163944/0.00345153 = 0.47498997 which is very close to the hedge ratio of 0.4824 computed with the risk minimization model.

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This holds if $A_{ii} + B_{ii} < 1$. See *The Econometric Modeling of Financial Time Series* (1993) by Terence C. Mills, page 103.

The coefficients for the lagged error terms, A_{11}, A_{22}, and A_{33} are 0.1000, 0.0742, and 0.0430. All are significant at the 0.1 percent level. The coefficients for the lagged variance terms, B_{11}, B_{22}, and B_{33} are 0.4857, 0.4708, and 0.7348. These are also significant at the 0.1 percent level. Since the covariance coefficients, A_{22} and B_{22}, are both significant, this model suggests that there is a high degree of interaction between futures and options prices. Post-estimation diagnostics of the standardized residuals consist of the Ljung-Box Q(24) statistics for up to 24th order serial correlation in the first moments and the Ljung-Box $Q^2(24)$ statistics for serial correlation in the second moments. All are insignificant suggesting that there is no evidence of lingering serial correlation in either the first or second moments. Overall, the bivariate GARCH model seems to provide a good fit of the options and futures data.

To test the hedging effectiveness of the GARCH model, the option/futures portfolio is formed using the daily time-varying hedge ratio, $h_{12,t}/h_{22,t}$, from the variance/covariance matrix of residuals. Similar to the OPM delta hedge, the option/futures position is updated and held for one day. The variance of the change in the portfolio value for the period of 1/2/90-5/15/92 is 0.000353. The percent reduction of the variance from the unhedged option position is 69.63 percent (1 - 0.000353/0.001161). This reduction is less than the 89.06 percent of the OPM but more than the 67.61 percent of the risk minimization hedge. It should be noted, however, that the GARCH and OPM models assume daily rebalancing of the option/futures portfolio with the optimal hedge ratio that changes every day, whereas the risk minimization hedge does not rebalance during the 1/2/90-5/15/92 period, but remains constant. This incomparability issue will be addressed in section 3.7.

3.6 *Bivariate E-GARCH*

The bivariate E-GARCH model uses the same mean equations as the GARCH model, but with a different specification of the error structure. The estimated model under the assumption of constant correlation on a within-sample basis over the period of 1/2/90-5/15/92 is shown in Table 6. Neither of the means, μ_1 and μ_2, are significant but all of the coefficients of the variance equations are highly significant. The lagged variance terms for the options and futures, $a_{1,1}$ and $a_{2,1}$ are 0.9191 and 0.8362, respectively. The terms that measure the magnitude of past innovations, $a_{1,2}$ and $a_{2,2}$, are 0.3838 and 0.3129. Since these coefficients are positive, the innovations in log h_{11} and log h_{22} are positive when the magnitude of z_t is larger than its expected value. The sign terms, $a_{1,3}$ and $a_{2,3}$, are 0.2207 and -0.5556 for the options and futures, respectively. For the options, both the magnitude and sign effect terms are positive which suggests that negative price changes do not increase volatility. For the futures, the magnitude term is positive while the sign term is negative. In the bivariate framework, this measure of asymmetry suggests that the futures exhibit a larger degree of volatility when price changes are negative. The correlation coefficient is 0.8827 and is significant at the 0.1 percent level. Tests of the residuals, Ljung-Box Q(24) and $Q^2(24)$ are insignificant for the first and second moments in both the futures and options. Based on these residual tests, the EGARCH model also appears to provide a good fit of the data.

The results of the bivariate EGARCH model under the assumption of non-constant correlation on a within sample basis are shown in Table 7. This model does not appear to provide as good a fit as the constant correlation model shown above, as evidenced by the

Ljung-Box Q statistic for the residuals on the options. Both of the means are significant at 0.0081 and 0.0160. The lagged variance terms for the options and futures, b_{11} and b_{33}, are 0.8534 and 0.9973 and both are significant at the 0.1 percent level. The magnitude coefficients for the options and futures, c_{11} and c_{33}, are 0.1405 and 0.0588. The first is significant at the 5.0 percent level and the second at the 0.1 percent level. The sign effect for the options and futures are 0.0985 and 0.1309, neither of which is significant. Since both the futures and options have positive magnitude and sign effects, negative price changes do not tend to cause increased volatility in this framework.

Turning now to the covariance, the lagged covariance term, b_{22}, is 0.9598, the magnitude coefficient, c_{22}, is 0.2963, and the sign effect, d_{22}, is -0.8707. All three of these are significant at the 0.1 percent level which indicates a high degree of interaction between the futures and options. The magnitude and sign effect require a different interpretation than before, however, since the covariance term reflects the interaction between the markets. The relevant portion of the equation is shown below.

$$c_{22}[\,|\epsilon_{1,t-1}\epsilon_{2,t-1}/h_{12,t-1}| \;-\; E|z_1|E|z_2| + d_{22}(\epsilon_{1,t-1}\epsilon_{2,t-1}/h_{12,t-1})]$$

In this case, a negative numerator value occurs when the error terms for the options and futures are of different signs indicating that prices have changed in opposite directions. Now the positive magnitude coefficient and negative sign coefficient suggest that volatility is higher when this opposite price change pattern exists.[14] As mentioned previously, this model

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Referring to Table 1, this situation occurs when α_1 is positive, ϕ is negative, and z_{t-1} is negative.

does not seem to be as effective as the constant correlation EGARCH model shown above.

The hedging effectiveness of the bivariate EGARCH models can be assessed in the same manner as the bivariate GARCH model. The optimal hedge ratio is available on a daily basis and the option/futures position is updated at the end of everyday with a holding period of 1 day. Over the period of 1/2/90-5/15/92, the variance of the unhedged option position can be reduced by 79.67 percent (1-0.000236/0.001161) with the constant correlation EGARCH model, and a less impressive 67.36 percent reduction (1-0.000379/0.001161) with the nonconstant correlation EGARCH model. These compare to the variance reduction with the GARCH model of 69.63 percent.

3.7 Out-of-Sample Hedging Effectiveness

A variety of methods have been utilized in previous research to compare alternative hedging strategies. Hancock and Weise (1994) construct difference *t* tests to determine if significant differences exist between the risk free rate and the returns on their hedged portfolios. They also test for differences in returns from one hedged portfolio to the next. Their analysis considers only returns and not risk. On the other hand, most of the GARCH hedging literature considers only the reduction in variance as a measure of hedging performance. Baillie and Myers (1991) argue that the minimum variance objective is consistent with expected utility maximization provided that the expected return to holding futures is zero.

The previous sections have examined six hedging strategies, the Black OPM delta hedge, the naive hedge, the risk minimization hedge, the bivariate GARCH hedge, the bivariate EGARCH constant correlation hedge, and the bivariate EGARCH non-constant correlation hedge. The results are on a within-sample basis over the period of 1/2/90-5/15/92 and the computations of hedge ratios utilize all of the prices as if they were known in advance. The optimal hedge ratio is updated at the end of every day for the OPM delta and bivariate GARCH and EGARCH hedges,[15] while the risk minimization model uses a constant hedge ratio during the within-sample period. The hedging effectiveness is measured by the percent reduction in the variance of the unhedged option position. This method is assumed to be the most appropriate since the option market maker's objective is to generate profits from the bid/ask spread and not the returns generated from market moves. The goal is to keep the change in the portfolio value to a minimum. Daily hedging is assumed and the result of using the hedge is computed as $(P_{option,t+1} - P_{option,t}) - b_t^*(P_{futures,t+1} - P_{futures,t})$ where b_t^* is the computed hedge ratio from each hedging method. For the Eurodollar, this results in a net position on a tick basis. The results are summarized in Panel A of Table 8.

The mean of the daily net tick position for the unhedged call option for the sample period of 1/2/90-5/15/92 is 0.002662. The other means on a decreasing basis are, Black OPM at -0.000425, EGARCH non-constant correlation at -0.000665, EGARCH constant correlation at -0.000749, GARCH at -0.000888, risk minimization at -0.001158, and the naive hedge at -0.001297.

As explained above, the hedging effectiveness is measured by the percent reduction in the variance of the unhedged option position. If one call option is purchased with no

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Since daily hedge ratios can be extracted from the GARCH and EGARCH models on a within-sample basis, hedges are updated daily to be consistent with the OPM model. In out-of-sample testing, weekly hedge ratio updates are employed for the GARCH, EGARCH, and OPM models.

hedge, the variance of the option price changes for the sample period is 0.001161. The risk of the hedged position is substantially lower with the various strategies; in descending order, 89.06 percent by the OPM delta hedge, 79.67 percent by the constant correlation bivariate EGARCH hedge, 69.63 percent by the bivariate GARCH hedge, 67.61 percent by the risk-minimization hedge, 67.53 percent by the naive hedge, and 67.36 percent by the nonconstant correlation bivariate EGARCH hedge.

These results are comparable to the spot/futures hedging results reported by Baillie and Myers (1991) and Kroner and Sultan (1993) who found that GARCH model hedges perform better than constant hedge ratio models with respect to reducing the variance of portfolio returns. The superior performance of the EGARCH-constant correlation model looks promising for extensions to spot/futures portfolios and cross hedging portfolios.

Although these results may be interesting in their own right, the true test is in the comparison of these models within a more realistic framework, namely on an out-of-sample basis. The hedger utilizes all information available at one point in time to compute a hedge ratio that will hopefully remain appropriate for future price movement. Ideally, the market markers should update the hedge ratio whenever new information arrives in the market, but the continuous updating requires prohibitively high transaction costs. The optimal frequency of the option/futures portfolio rebalancing should depend on the bid-ask spread and the cost of hedging, including the transaction costs involved in buying and selling the futures contracts. In this paper, the market maker is assumed to update the hedge ratio on a weekly basis.

As explained in section 1.3, the out-of-sample tests of hedging effectiveness use daily price changes for the computation of the parameter estimates. The hedge ratio is initially computed for the first half (1/2/90-5/15/92) of the entire sample. The hedge ratio is then used to form a portfolio of 1 call option and the optimal number of futures (short). The hedged position is kept in place for one day, at which time the position is liquidated and a new hedge is placed using the same hedge ratio as the previous day. This process continues on a daily basis, using the same hedge ratio for a period of one week (five days). After one week's time, a new hedge ratio is computed utilizing a new moving window of information (1/9/90-5/22/92) that picks up the latest week of daily prices and removes the initial week of prices.[16] In this manner, each computation is based on the same number of days of daily price information. The hedge ratio is computed on the same day of each week except in those cases where the market is closed on that particular day. In that case, the hedge ratio is computed on the prior day, resulting in only 4 days of new price information. The next week once again reverts back, resulting in 5 days of new price information. The same number of trading days is maintained in both of these instances, however, whereby the first 4 days are eliminated from the sample in the first case and the first 5 days are eliminated from the sample in the second case. This process results in a total of 129 hedge ratio computations over the sample period of 1/2/90-11/30/94.

For the within-sample testing, the hedge ratios for the OPM, GARCH, and EGARCH models change on a daily basis, while the hedge ratio for the risk minimization and naive models remain constant. This implies that transactions costs are much greater for the time-

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The previous window of information covers the 1/2/90-5/15/92 period, while the next window is for the period of 1/9/90-5/22/92.

varying models, and a market maker would need to factor in the additional cost when making comparisons of effectiveness among the alternatives. In the out-of-sample tests, each hedging model considered[17] is reestimated on a weekly basis for the rebalancing of the hedged position, i.e., the number of rebalancings is identical across the different hedging models. This method eliminates the problem of comparing the risk minimization and naive models with their constant hedge ratios to the time-varying OPM and GARCH models. Furthermore, the same number of updates means that any transaction costs involved in implementing the hedging strategies must be roughly of the same magnitude in the out-of-sample tests.

The weekly out-of-sample hedge ratios for each of the models are shown in Figure 6. Several points are worth noting. First, the naive hedge ratio of 0.50 provides a very good point of reference for all of the models. The risk minimization model shows the least variability as it hovers quite close to the constant naive hedge ratio. The Black OPM appears to have the most variability with hedge ratios ranging from a value of virtually zero to a value of one. For the most part, the GARCH and EGARCH models do a good job of mimicking the Black OPM, however, they provide hedge ratios with less extreme variation. One exception is noted in the vicinity of September and October of 1992.

Comparisons of the hedging models on an out-of-sample basis are shown in Panel B of Table 8. For all of the models, the daily change in the value of the hedged

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Since the EGARCH model with non-constant correlation is considerably inferior to the constant correlation model on a within-sample basis, it is eliminated from the analysis on an out-of-sample basis.

option/futures position has been traced with the optimal hedge ratio being updated weekly for the out-of-sample period which covers from 5/18/92 through 11/30/94.

The mean of the net tick position for the unhedged call option is -0.001080. The means of the hedged portfolios in descending order are the Black OPM at -0.000610, GARCH at -0.000820, constant correlation EGARCH at -0.001060, risk minimization at -0.001700, and the naive hedge at -0.001550.

The variance of the unhedged option position during the period is 0.000707. The overall results show that the OPM delta hedge is the most effective strategy for market makers in the Eurodollar options on futures market. The OPM delta hedge reduces the variance of the unhedged position by 71.29 percent (1 - 0.000203/0.000707) even on an out-of-sample basis. Both the GARCH and EGARCH models, which incorporate time-varying volatility, perform better than the risk minimization and naive hedging models which assume constant volatility. Nevertheless, the risk minimization model still reduces the risk substantially at 60.11 percent. The bivariate EGARCH model outperforms the bivariate GARCH model, 65.77 percent versus 63.65 percent, probably due to its ability to control for an asymmetric response of volatility to new information. Although the option market makers in this framework can stick to their most popular OPM delta strategies, the outstanding performance of the most up-to-date bivariate EGARCH (65.77 percent versus 71.29 percent) warrants further investigation and extension of its application to spot/futures portfolios and cross hedging portfolios.

3.8 Conclusions

This investigation differs from prior studies in many respects. First, hedging strategies that have been used extensively in the spot/futures markets (i.e. risk minimization, risk/return optimization, and GARCH) are analyzed for the first time in the options/futures markets. Second, this is the first extensive hedging study applied to the Eurodollar market. Third, daily data is used rather than weekly data. Fourth, the analysis utilizes price changes whereas many previous studies have used returns. Fifth, the bivariate EGARCH model is applied for the first time to the hedging problem.

In the initial stage of this analysis, the price behavior of the futures and options is examined to detect any possible long-run relationship between the two. The options price structure is stationary in nature. This pattern can be explained by the fact that the option price is a positive function of the time to maturity for the contract; therefore, as maturity approaches, the time value erodes. The pattern is most evident for an option that is out-of-the-money where the price gradually approaches zero. On the other hand, the futures prices exhibit a non-stationary pattern, precluding a cointegrating relationship between futures and options. Further study of the pattern of daily price changes in these two contracts reveals another difference in that the options exhibit significant ARCH effects while these same effects are only marginally detected in the futures contract. In a bivariate framework, significant GARCH and EGARCH effects are detected in well-fitting models. The strong ARCH, GARCH, and EGARCH behavior in the options is probably the driving force behind the bivariate results.

Application of the hedging models on a within-sample and out-of-sample basis reinforce previous studies' findings that time-varying hedge ratios are superior to constant hedge ratios. A graph of the weekly hedge ratios reveals that the GARCH models follow a pattern that is quite similar to the Black OPM. The similarity may be explained by the fact that both of these methodologies rely on time-varying variances. The differences may be explained by the fact that one utilizes conditional variances while the other utilizes unconditional variances. Another possible explanation is that the Black OPM is contract specific and the hedge ratio reflects the additional information in terms of various option parameters.

While the Black OPM delta hedging strategy is somewhat challenged by the EGARCH model, it is still the most effective based on both within- and out-of-sample results. However, the superiority of the newly attempted bivariate EGARCH model over the bivariate GARCH model as well as the risk minimization model looks promising for extension to spot/futures portfolios and other derivatives markets.

CHAPTER 4: HEDGING A SPOT POSITION WITH FUTURES

4.1 Cross Hedging - Considerations and Empirical Evidence

In this extension, it is assumed that an investor holds a spot position in Treasury bills and wishes to hedge his/her exposure through the use of a Eurodollar futures contract. This scenario therefore moves into the realm of cross-hedging. In general, a cross-hedge occurs when the cash market instrument differs from the underlying deliverable item for the futures contract. According to Chance (1991), a hedger must choose a contract with a high level of liquidity so that the contract can be closed easily. Obviously, the contract chosen should also have a high correlation with the commodity being hedged. Kawaller and Koch (1992) relate that a hedger wishing to hedge the risk of a short-term interest rate exposure could use either a three-month T-bill or a three-month Eurodollar futures contract since both satisfy the correlation requirements imposed by accounting regulators.[18] That being the case, the Eurodollar may be a better choice for hedging because of its higher level of liquidity. As an example, on November 15, 1996, the total open interest in the December T-bill was 2,870 contracts while the open interest in the December Eurodollar was 439,867 contracts. The total one-day volume of trading in all T-bill contracts was 317 compared to 371,418 contracts for the Eurodollar. A representative of the marketing department of the Chicago

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Statements of Financial Accounting Standards (SFAS) No. 80 specifies that cross-hedging is permitted if there is a clear economic relationship between the item underlying the future and the item being hedged, and if high correlation is probable. According to Her and Tanker (1996), regression analysis is the most commonly used method to measure expected future correlation. An R^2 of 0.80 is generally considered to meet the test of high correlation. The R^2 from a regression of T-bill spot yields on Eurodollar futures yields is 0.9984 over the sample period of 1/2/90 through 12/31/94.

Mercantile Exchange relates that hedging T-bill portfolios with Eurodollar futures is a common practice, although volume statistics are unavailable that would isolate hedging activity from TED spread activity.[19] Further confirmation of T-bill/Eurodollar cross-hedging is provided by an officer of Harris Trust and Savings of Chicago[20] who says that Eurodollar futures have two advantages; first, contract expirations extend out much further, and second, the contracts are highly liquid. He relates that traders recognize that some basis risk is assumed due to the use of Eurodollar futures rather than T-bill futures against the T-bill spot position, but if the basis risk becomes uncomfortable, it can be hedged with a TED spread.

Schwarz, Hill, and Schneeweis (1986) suggest that the London Interbank Offer Rate (LIBOR) is very similar to the prime rate in the United States and that Eurodollar borrowing can be done at rates competitive with U.S. short-term loan rates. However, Eurodollar deposit accounts tend to earn higher interest rates than those available in the U.S. bank Certificate of Deposit (CD) market due to such factors as liquidity, international crises, and other risk premium considerations. In addition, the spread between T-bill and Eurodollar deposit rates are related to the general level of short-term T-bill rates and to the value of the dollar. When interest rates are high, the LIBOR rate increases relative to the Treasury bill rate.

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The TED spread is defined as (T-bill futures price - Eurodollar futures price) where both contracts have the same expiration month. The TED spread is believed to represent a risk premium which depends on the stability of the world's financial system. Trading the TED spread allows an investor to speculate on general economic conditions and the soundness of banks.

20 Tom Dickinson, Vice President, Treasurer's Department, Fixed Income Portfolios.

Empirical studies within the cross-hedging realm have explored many markets, however, the risk minimization model is used most prevalently with no extensions to GARCH frameworks. Hill and Schneeweis (1984) test the effectiveness of hedging a spot position in corporate bonds with a position in GNMA or U.S. Treasury bond futures. In most cases, the use of interest rate futures reduces the variability of spot price changes by over 50 percent. Figlewski (1984) uses an S&P 500 futures contract to hedge the underlying portfolios of the S&P 500 index, the New York Stock Exchange composite, the American Stock Exchange composite, the National Association of Securities Dealers Automated Quotation System (NASDAQ) index of over-the-counter stocks, and the Dow Jones Industrials index. Figlewski reports mean returns and standard deviations for the unhedged and hedged portfolios. A further computation of hedging effectiveness based on the measure, 1- (variance of the hedged position / variance of the unhedged position), shows a variance reduction of 94 percent for the S&P 500 index, 93 percent for the NYSE composite, 65 percent for the AMEX composite, 62 percent for the OTC index, and 91 percent for the Dow index. Park, Lee, and Lee (1987) test the effectiveness of German mark futures contracts to hedge the seven currencies of the European Monetary System, namely the German mark, French franc, Belgian franc, Danish crown, Italian lira, Irish pound, and Dutch guilder. They explore one-week, two-week, and four-week hedges in nearby, middistant, and distant contracts and find percentage reductions in variability in the spot position ranging from 5 percent to 94 percent. Saunders and Sienkiewicz (1988) cross-hedge nine individual currencies with the European Currency Unit (ECU) futures contract for three holding period lengths. The daily holding period results show hedging effectiveness ranging from 30.99

percent to 74.96 percent. The effectiveness measure for each individual currency increases for the one-week holding period and increases further still for the two-week holding period.

4.2 Data Description, Sample Statistics, and Univariate GARCH Testing

New York money market interest rates are collected from the Daily Information Bulletin of the International Monetary Market of the Chicago Mercantile Exchange. The bulletin quotes T-bill yields based on bank discount rates. The bank discount rate is the interest rate used by market participants when they buy and sell bills and the IMM obtains the data from Telerate Corp. of New York on a real time basis and samples them hourly. The rates represent the average bid quotations of a sample of primary government securities dealers for the bills auctioned during that specific week. To ensure that the time until maturity represented by the series is approximately constant, a different bill is quoted each week from the Tuesday open to the following Monday close.

For the Eurodollar futures, four delivery months exist, March, June, September, and December, and the last trading day for each contract is the second London bank business day before the third Wednesday of the contract month. Tick-by-tick prices from the Chicago Mercantile Exchange are sampled on an hourly basis for the nearby contract with the sample running through the expiration date for each contract before switching to the next contract month. The futures price is then subtracted from 100 to obtain the annualized futures interest rate (three-month Libor).

The daily 2:00 p.m. rates of the U.S. 90-day Treasury bill and the Eurodollar three-month Libor are sampled for the period of 1/2/90 to 12/30/94. Figure 7 displays the Libor versus T-bill rate over the sample period. The two interest rates appear to be closely related with both exhibiting a decreasing trend from approximately 5/90 through 10/92 with an increasing trend starting around 2/94. This pattern suggests that neither time series is stationary and that cointegration is also likely. In addition, Figure 8 shows that the basis (computed as Eurodollar yield minus T-bill yield) is consistent with Schwarz, Hill, and Schneeweis' (1986) statement that the Libor rate increases relative to the T-bill rate when interest rates are high. When the rates are lowest (from 1/92 - 2/94), the basis at contract expiration is approximately 0.3 but when the rates are highest (from 1/90 - 12/91 and 3/94 - 12/94), the basis at expiration ranges from 0.4 to as high as 0.6.

Table 9 reports descriptive statistics for both the T-bill spot and Eurodollar futures yield. There are 1,172 observations over the sample period. The unit root tests on yields confirm non-stationarity for both with Phillips and Perron test statistics of -1.6810 and -1.4454 for the T-bill spot and Eurodollar futures respectively. First differences of the T-bill yield and Libor are shown in Figures 9 and 10 with both suggesting that the trends have been eliminated.

The Phillips Z_p statistic of -47.4932 rejects the null hypothesis of no cointegration between the T-bill spot and Eurodollar futures.[21] This result confirms previous studies of the relationship between Eurodollar rates and domestic interest rates. Fung and Isberg (1992) find significant cointegration in daily observations of three-month maturity yields on Eurodollar deposits and negotiable U.S. certificate of deposit rates over a sample period of

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[21] The coefficient (β) of the cointegrating regression ($\Delta TB_t = \alpha + \beta \, \Delta ED_t + \epsilon_t$) over the sample period of 1/2/90 through 12/31/94 is 0.9217. In the futures markets, Tse and Booth (1995) obtained a β of 0.92 over a sample period of 1/2/87 - 12/31/92 and a β of 0.97 over a period of 1/2/88 - 7/30/93.

1981-1988. Tse and Booth (1995) use daily prices of T-bill and Eurodollar futures from 1/87 through 7/93 and find cointegration with the TED spread as the cointegrating vector. Fung and Lo (1995) find cointegration in daily prices of Eurodollar and T-bill futures contracts over the sample period of 1982 through 1991. Booth and Tse (1995) test daily prices of Eurodollar and T-Bill futures over the sample period of 3/82 through 2/94 and find evidence of a cointegrating vector that possesses long memory suggesting a fractionally integrated type process.

As shown in Table 9, the mean yield change for the T-bill is -0.0024 and -0.0047 for the Eurodollar, with the latter being significant at the 1.0 percent level. The variance of the T-bill yield change is 0.0020 compared to 0.0028 for the Eurodollar. A high degree of skewness and kurtosis is evident in both markets. The Ljung-Box Q statistic for serial correlation of 45.1911 for the T-bill is significant at the 1.0 percent level while the test statistic of 32.1632 for the Eurodollar is insignificant.

Engle's test for ARCH effects shows insignificance at orders of 1 and 3, however, the 5th order is significant at the 5 percent level in the T-bill and at the 1 percent level in the Eurodollar. The 10th order is insignificant in the T-bill and shows 5 percent significance in the Eurodollar. The results of further univariate testing for ARCH, GARCH, and EGARCH effects are shown in Table 10. The ARCH(5) model coefficients for the T-bill are 0.1635, 0.1333, 0.0723, 0.0420, and -0.0069 for the five lags. The first three lags are significant at the 0.1, 1.0, and 5.0 percent levels respectively. The coefficients suggest that yesterday's errors contain the most information for the determination of today's variance with an obvious pattern of decay from the first lag to the 5th lag. The ARCH(5) model coefficients for the Eurodollar are 0.0048, -0.0297, 0.0474, 0.0609, and 0.0952 for the five lags. The 2nd, 4th, and 5th lags are significant at the 5 percent level, but the first lag is insignificant. The coefficients increase in absolute value from the first to the 5th lag suggesting no decay in information.

The coefficients of the univariate GARCH (1,1) model for the T-bill are all significant at the 0.1 percent level. The coefficient of the lagged variance term is 0.5130 and the coefficient of the lagged error term is 0.1931. For the futures, the coefficient of the lagged variance term is 0.9156 which is significant at the 0.1 percent level while the coefficient of the lagged error term is 0.0238 which is significant at the 5.0 percent level.

The results of the EGARCH model for the T-bill indicate a high level of significance on all of the parameters. The magnitude coefficient, a_1, is 0.2592 and the coefficient that measures the sign effect, c, has a value of -1.1219. This combination suggests that a positive price change surprise reduces volatility while a negative price change surprise increases volatility. This result is consistent with Brenner, Harjes, and Kroner (1996) who find asymmetric volatility reactions in weekly observations of the T-bill over a sample period of 2/9/73 to 7/6/90. Brunner and Simon (1996) also report volatility asymmetries for excess returns on 10-year Treasury notes and 30-year Treasury bonds and offer the possible explanation that inflation volatility tends to increase as the level of inflation increases.

The Eurodollar futures magnitude coefficient is 0.1252 and the sign effect coefficient is 0.3574. Neither is significant, but the positive signs of the coefficients suggest that positive price changes result in higher volatility than negative price changes of an equal magnitude. The coefficient for the lagged variance term is -0.7336 which is significant at the 1.0 percent level. The likelihood ratio test statistic (LR) for the EGARCH vs. GARCH is 24 (=2(1531-1519)) for the T-bill, which is significant at the 0.1 percent level, and is -2 (=2(1381-1382)) for the Eurodollar futures, which is insignificant. Therefore, the EGARCH model improves the measure of fit for the T-bill spot but does not improve the fit for the Eurodollar futures.

Overall, these results in Table 10 indicate that ARCH, GARCH, and EGARCH effects are highly prominent in the T-bill spot, but are only marginally suggested for the Eurodollar futures.

4.3 Naive Hedge

Under the assumption of a naive hedge ratio of shorting one Eurodollar futures contract (principal value of $1,000,000) for each T-bill portfolio of $1,000,000 held, the variance of the daily portfolio value change over the 1/2/90-6/19/92 within-sample period is 0.002417. This compares to the variance of the unhedged T-bill position change of 0.002117, resulting in a variance *increase* of 14.17 percent (1 - 0.002417/0.002117) for the hedged portfolio.

4.4 Risk Minimization

The regression format for the T-bill spot/Eurodollar futures portfolio is

$$Y_{s,t} - Y_{s,t-1} = \alpha_0 + \beta(Y_{f,t} - Y_{f,t-1}) + \alpha_1(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_t$$

where the error term is modeled as $\epsilon_t = \alpha_1\epsilon_{t-1} + \alpha_2\epsilon_{t-2} + \ldots + \alpha_p\epsilon_{t-p} + \eta_t$. The error correction term is $Y_{s,t-1} - \delta Y_{f,t-1}$ where δ has a value of 0.9217. Results of the minimum risk strategy over the sample period of 1/2/90 - 6/19/92 are shown in Table 11. The coefficient of the error correction term is -0.0187 with a significance of 5 percent suggesting that the term belongs in the model. The hedge ratio is -0.4607 and is significant at the 0.1 percent level.[22] Eurodollar futures should be shorted at 0.4607 contracts per $1,000,000 in T-bill spot value. The Durbin-Watson statistic of 1.9420 is insignificant which indicates no evidence of autoregressive disturbances. The measure of hedging effectiveness, R^2, is 0.3249 implying that a 32.49 percent reduction in the variance of T-bill spot yield changes is achieved with the hedge.[23]

4.5 Bivariate GARCH

The GARCH model used for the T-bill spot/Eurodollar futures portfolio is

$$R_{1,t} = \mu_1 + \alpha_1(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_{1,t}$$

$$R_{2,t} = \mu_2 + \alpha_2(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_{2,t}$$

$$\epsilon_t | \Omega_{t-1} \sim N(\mathbf{0},\mathbf{H}_t)$$

where $R_{1,t}$ and $R_{2,t}$ are the spot and futures yield changes, ϵ_t is a (2x1) vector of residuals, Ω_{t-1} is the information set at time t-1, and $\mathbf{H}_t$ is a (2x2) conditional variance-covariance matrix of residuals. $Y_{s,t-1} - \delta Y_{f,t-1}$ is the error correction term. The 1/2/90-6/19/92 within-sample results for this model are shown in Table 12. The intercept terms for the spot and futures yield changes are -0.0055 and -0.0022 with the futures having significance at the 5 percent level. The coefficients for the error correction term, α_1 and α_2, are -0.0017 and

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The higher volatility of the Eurodollar futures which results in a hedge ratio of less than 0.50 explains why the naive hedge with a hedge ratio of 1.0 results in a variance increase over the unhedged position.

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This is a relatively small improvement compared to the reductions of 70 percent in the Eurodollar option/futures hedges. It is also disappointing compared to the results of previously mentioned cross-hedging studies.

0.0566 for the spot and futures. The Eurodollar coefficient is significant at the 0.1 percent level while the T-bill coefficient is insignificant. Tse and Booth (1995) suggest that this result is consistent with T-bill yields Granger-causing Eurodollar yields.

The constant term for the spot variance, C_1, is 0.0007, the constant term for the futures variance, C_3, is 0.0030, and the constant term for the covariance, C_2, is 0.0013. Both of the variance constants are significant at the 0.1 percent levels while the covariance constant is not significant. The unconditional covariance can be constructed as $C_2/(1-A_{22}-B_{22}) = 0.00153745$ while the unconditional variance of the futures is $C_3/(1-A_{33}-B_{33}) = 0.00331286$. The unconditional hedge ratio is therefore 0.00153745/0.00331286 = 0.46408565 which is consistent with the hedge ratio of 0.4607 computed with the risk minimization model.

The coefficients for the lagged error terms, A_{11}, A_{22}, and A_{33}, are 0.2039, 0.0051, and 0.0885. The first is significant at the 0.1 percent level, the second is insignificant and the third is significant at the 1.0 percent level. The coefficients for the lagged variance terms, B_{11}, B_{22}, and B_{33}, are 0.4697, 0.1601, and 0.0199 with only the T-bill showing significance at the 0.1 percent level. Since the covariance coefficients, A_{22} and B_{22}, are both insignificant, this model suggests that there is very little interaction between T-bill spot and Eurodollar futures yield changes. The Ljung-Box Q(24) and $Q^2(24)$ statistics for post estimation diagnostics of the standardized residuals are all insignificant so no lingering serial correlation is detected in either the first or second moments.

The hedging effectiveness of the GARCH model is tested by forming a portfolio of the T-bill spot and Eurodollar futures using the daily time-varying hedge ratio, $h_{12,t}/h_{22,t}$ from the conditional variance-covariance matrix of residuals. The variance of the change in the portfolio value for the period of 1/2/90-6/19/92 is 0.001392. The percent reduction of the variance from the unhedged spot position is 32.12 percent (1-0.001392/0.002117). This is only marginally better than the 32.11 percent shown with the risk minimization hedge. Given that the GARCH model results show insignificance on three of the four coefficients that make up the conditional hedge ratio, these results are not too surprising. Additionally, the variance reduction of 32.12 percent might be considered to be quite good given the insignificance of the GARCH model covariance coefficients implying very little interaction between the T-bill spot and Eurodollar futures yield changes.

4.6 Bivariate EGARCH

The bivariate EGARCH model with error correction term and constant correlation is shown in Table 13. The results are on a within-sample basis over the sample period of 1/2/90-6/19/92. The mean of the T-bill spot is -0.0056 which is significant at the 1.0 percent level, and the mean of the Eurodollar futures is -0.0025 which is insignificant. The coefficients for the error correction terms, α_1 and α_2 are 0.0042 and 0.0487. Like the GARCH model, the T-bill term is insignificant while the Eurodollar term is significant. All of the variance equation coefficients are significant at either the 1.0 or 0.1 percent levels. The lagged variance terms for the spot and futures, $a_{1,1}$ and $a_{2,1}$, are 0.8785 and 0.9439, respectively. The term measuring the magnitude of past innovations in the T-bill, $a_{1,2}$, is 0.2321 and the sign effect, $a_{1,3}$, is -0.6398. This suggests that the T-bill exhibits a larger degree of volatility when yield changes are negative compared to positive yield changes of the same magnitude. For the futures, the magnitude term, $a_{2,2}$, is 0.1186 and the sign term,

a_{23}, is 0.6073. Since both are positive, negative yield changes do not increase volatility. The correlation coefficient is 0.6203. Tests of the residuals, Ljung-Box Q(24), in the T-bill suggest some lingering serial correlation in the first moment. The other tests, Q(24) and $Q^2(24)$ are insignificant.

The hedging effectiveness of the EGARCH constant correlation model is determined in the same manner as the GARCH model. The optimal hedge ratio is computed on a daily basis and the spot/futures position is updated at the end of everyday. The variance of the unhedged T-bill position can be reduced by 32.59 percent (1-0.001427/0.002117). This compares to a variance reduction of 32.12 percent with the GARCH model, meaning the EGARCH and GARCH models are virtually identical in hedging effectiveness.

The results of the bivariate EGARCH model under the assumption of non-constant correlation on a within sample basis are shown in Table 14. The means of the spot and futures are -0.0067 and -0.0053. The first is significant at the 1.0 percent level. Once again, only the Eurodollar error correction term, α_2, is significant at the 1.0 percent level. The lagged variance terms, b_{11} and b_{33}, are 0.7193 and -0.9537 and both are significant at the 0.1 percent level. The magnitude coefficients, c_{11} and c_{33}, are 0.2453 and 0.0449. The T-bill term is significant at the 0.1 percent level while the Eurodollar term is significant at the 5.0 percent level. The sign effects for the spot and futures are -0.5663 and 0.2235 with the first showing significance at the 0.1 percent level. For the T-bill, the magnitude is positive and the sign effect is negative which suggests that negative yield changes tend to cause increased volatility compared to positive yield changes of the same magnitude. However, for the Eurodollar, both terms are positive, so negative yield changes do not cause increased volatility.

The lagged covariance term, b_{22}, is -0.7964 and is significant at the 0.1 percent level. The magnitude coefficient, c_{22}, is -0.0296 and the sign effect, d_{22} , is -0.2159. Since both are negative, this suggests that volatility is higher when the error terms for the spot and futures are of the same sign, however neither the magnitude nor the sign coefficient is significant..

The hedging effectiveness of the non-constant correlation EGARCH model reduces the variance of the unhedged position by 30.00 percent (1-0.001482/0.002117) which is less than the 32.59 percent reduction achieved with the constant correlation EGARCH model.

4.78 Out-of-Sample Hedging Effectiveness

The within-sample hedging results discussed in the previous sections are recapped in Panel A of Table 15. Daily hedging is assumed and hedging effectiveness is measured as the percent reduction in the variance of the unhedged spot position. The mean of the daily net tick position for the unhedged T-bill spot for the sample period of 1/2/90-6/19/92 is -0.007130. The others in decreasing order are the naive hedge at 0.001323, risk minimization at -0.003270, EGARCH at -0.003350, GARCH at -0.003440, and EGARCH constant correlation at -0.003500. The percentage reductions in variance for the hedged portfolios in decreasing order are: EGARCH constant correlation at 32.59 percent, GARCH at 32.12 percent, risk minimization at 32.11 percent, and EGARCH non-constant correlation at 30.00 percent. The naive hedge results in a variance increase of 14.17 percent. The risk of an unhedged T-bill spot position can be lowered by hedging with Eurodollar futures, although the improvements are rather small. In view of the cointegration results and the obvious linkage between these two markets, these results are rather disappointing.

Moving now to out-of-sample hedging results, the hedge ratio is initially computed for the first half (1/2/90-6/19/92) of the entire sample, with a recomputation of hedge ratios on a weekly basis. Each recomputation updates the information set through a moving window technique whereby the latest week of yields is included and the initial week of yields is deleted. Daily hedging of the portfolio is assumed with the results shown in Panel B of Table 15.

The mean of the net tick position for the unhedged T-bill spot is 0.001767. The means of the hedged portfolios in descending order are the naive at 0.003354, the risk minimization at 0.002622, GARCH at 0.002568, and EGARCH constant correlation at 0.002285. The variance of the unhedged T-bill position is 0.001690. The overall results show that the GARCH model is the most effective hedging strategy with a variance reduction of 34.88 percent (1-0.001101/0.001690). The EGARCH model is the next best with a variance reduction of 34.03 percent (1-0.001115/0.001690) followed by the risk minimization model with a variance reduction of 32.96 percent (1-0.001133/0.001690). Unlike the within-sample results where the naive hedge results in a variance increase, now there is a variance reduction of 20.30 percent (1-0.001347/0.001690). These results are consistent with other studies that have found time-varying hedge ratio models (GARCH and EGARCH) to provide superior performance over the constant hedge ratio models (risk minimization and naive). Unlike the Eurodollar option/futures hedge, however, where the EGARCH model was significantly better than the GARCH model, in this T-bill/Eurodollar cross-hedge, the two models are virtually equivalent.

4.8 Conclusions

The primary motivation for this study of T-bill spot/Eurodollar futures hedged portfolios is to test alternative hedging models within a cross-hedging framework. In light of the excellent hedging results provided by the EGARCH model in the Eurodollar options/futures portfolio, this extension is warranted. This is the first application of the GARCH and EGARCH time-varying hedging models to the cross-hedging problem.

The price behavior of the T-bill spot and Eurodollar futures reveals that both have a unit root and that cointegration is confirmed. The T-bill exhibits a high degree of ARCH, GARCH, and EGARCH effects while the Eurodollar has only marginal evidence of the same effects. An error correction representation is used to account for cointegration with evidence that T-bill yields Granger-cause Eurodollar yields. In the bivariate GARCH framework, there doesn't appear to be much interaction between yield changes in the two markets as evidenced by the insignificant covariance coefficients.

On both a within-sample and out-of-sample basis, the variance reduction for the hedged portfolio is in the low to mid-30 percent range for all of the models. The results of prior studies are confirmed whereby time-varying hedge ratios are more effective than constant hedge ratios. The hedging performance of the EGARCH model is virtually equivalent to the GARCH model in within-sample and out-of-sample testing. This result is inconsistent with the Eurodollar option/futures result where the EGARCH was superior in both cases.

CHAPTER 5: DISSERTATION SUMMARY

This study explores various hedging strategies within two different environments. The hedging models are first analyzed from the perspective of market makers in the Eurodollar options who hedge with Eurodollar futures, and the second perspective moves into a cross-hedging environment where T-bill spot portfolios are hedged with Eurodollar futures. Hedging strategies used extensively in the spot/futures markets are included--the naive, risk minimization, and bivariate GARCH models. In addition, a bivariate EGARCH model is developed and applied to the hedging problem, which is the first attempt in the literature. In addition to the previously mentioned models, the study also includes Black's option pricing model (OPM) delta hedge which is the most frequently adopted strategy by market makers. All models include both within-sample and out-of-sample results.

In the Eurodollar options/futures framework, there is no evidence of a long-run cointegrating relationship. GARCH and EGARCH effects are detected in well-fitting bivariate models with evidence of a great deal of interaction between the options and futures price changes. Tests of the hedging models confirm that time-varying hedge ratios are superior to constant hedge ratios, both on a within-sample and out-of-sample basis. Variance reductions range from 67.53 percent to 89.06 percent on a within-sample basis and from 57.57 percent to 71.29 percent on an out-of-sample basis. Although the Black OPM is the most effective hedging strategy, the EGARCH model looks promising for extension into other markets.

For the cross-hedge of a T-bill spot portfolio with Eurodollar futures, an error correction model is used to incorporate the long-run cointegrating relationship between the yields in the two markets. The bivariate GARCH model suggests that there is little interaction between the T-bill spot and Eurodollar futures yield changes. Consistent with previous studies, time-varying hedge ratios are superior to constant hedge ratios, both on a within-sample and out-of-sample basis. Excluding the naive hedge, variance reductions range from 30.00 percent to 32.59 percent on a within-sample basis and from 32.96 percent to 34.88 percent on an out-of-sample basis. The EGARCH model provides only a 0.57 percent improvement over the GARCH model on a within-sample basis, making the two virtually equivalent. The out-of-sample results are similar with the GARCH model providing a 0.85 percent improvement over the EGARCH. Cross-hedging effectiveness (in the 30 percent range) for the T-bill spot/Eurodollar futures portfolios is less than the direct hedging effectiveness (in the 70 percent range) for the Eurodollar option/futures portfolios.

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Figure 1
Hedging Window

The information window is assumed to be 10 days. The first hedge ratio is computed at the close of trading on day 10, denoted t. The hedge is simultaneously placed at the close of day t and kept in place through the end of day t+1. At the close of day t+1, the portfolio is liquidated and a new hedge is placed and kept until the close of day t+2. The portfolio liquidation recurs on a daily basis. The hedge ratio calculated on day t is used for each day through day t+5. A new hedge ratio is computed at the close of trading on day t+5, utilizing a new 10-day window which includes prices from day t-5 through day t+5. Again, the hedge is simultaneously placed at the close of day t+5 and kept in place through the end of day t+6.

```
|----------|----------|
t-10       t         t+1
       Compute      Lift
         Hedge     Hedge
         Ratio
          and
         Place
         Hedge     |-----------|
                  t+1         t+2
                 Place        Lift
                 Hedge       Hedge
                              |-----------|
                             t+2         t+3
                            Place        Lift
                            Hedge       Hedge
                                         |-----------|
                                        t+3         t+4
                                       Place        Lift
                                       Hedge       Hedge
                                                    |-----------|
                                                   t+4         t+5
                                                  Place        Lift
                                                  Hedge       Hedge
                                                               |-----------|
                                                              t+5         t+6
                                                            Compute       Lift
                                                             Hedge       Hedge
                                                             Ratio
                                                              And
                                                             Place
                                                             Hedge
```

Figure 2
Eurodollar Futures Prices

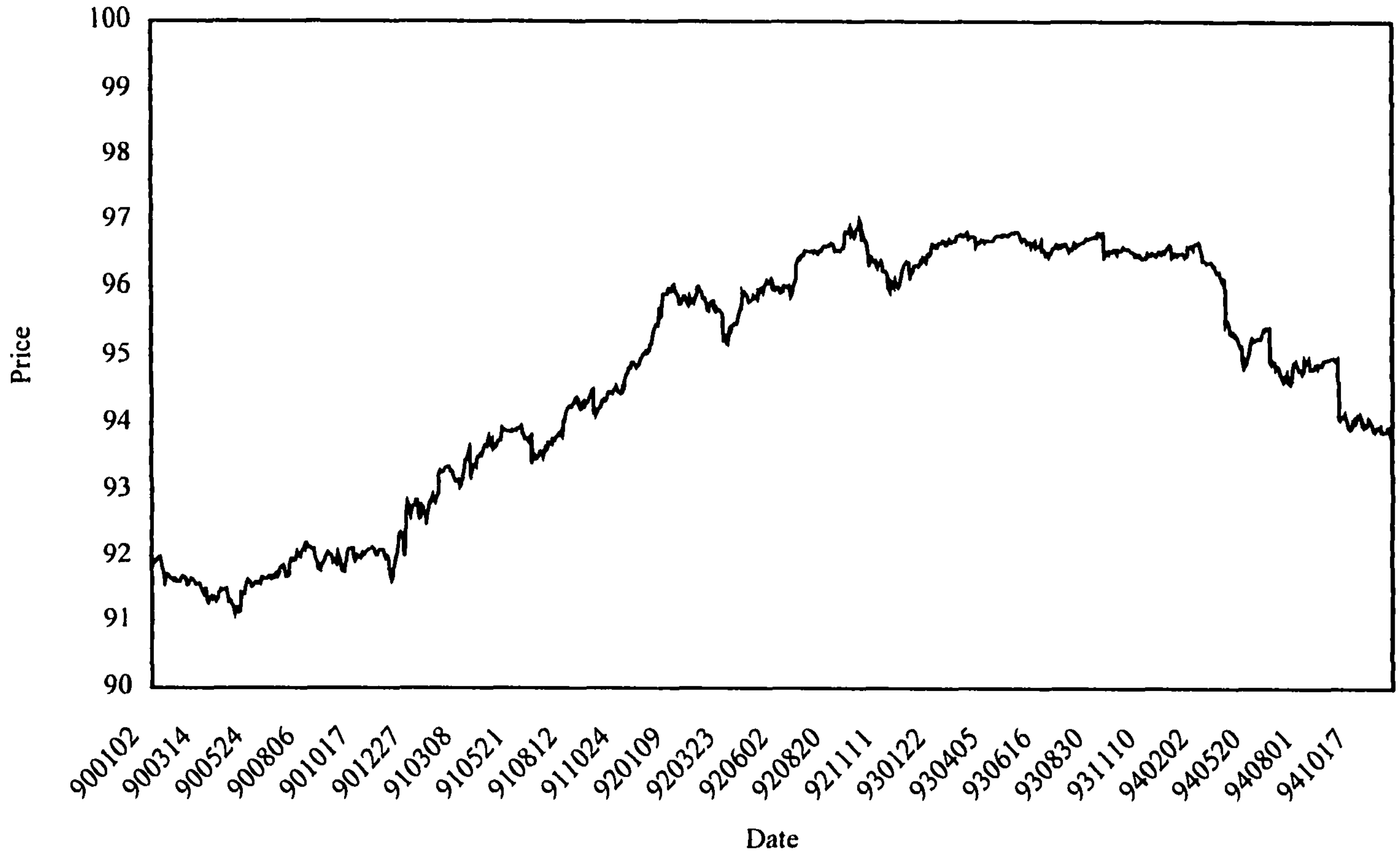

Figure 3
Eurodollar Options Prices

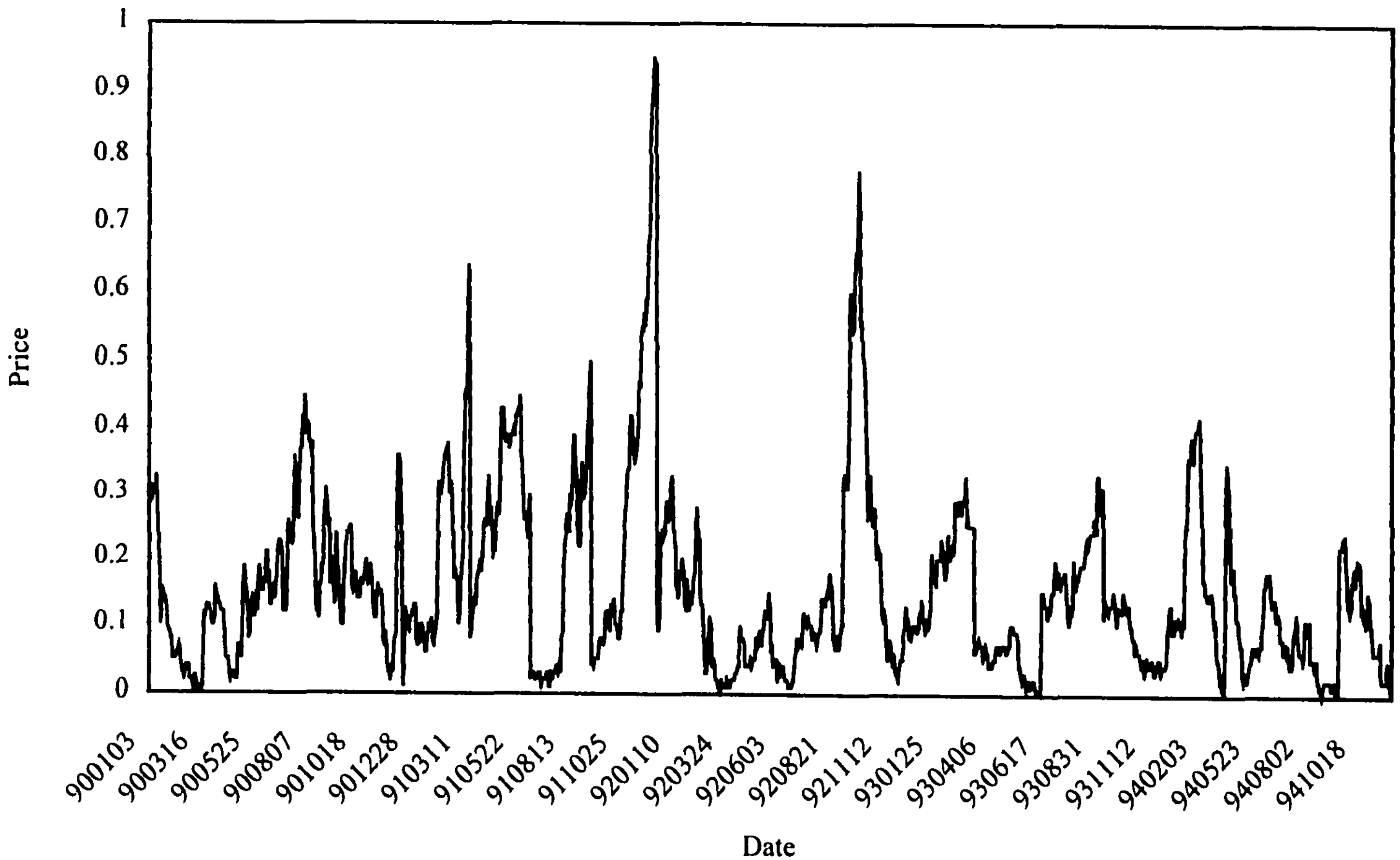

Figure 4
Eurodollar Futures Price Changes

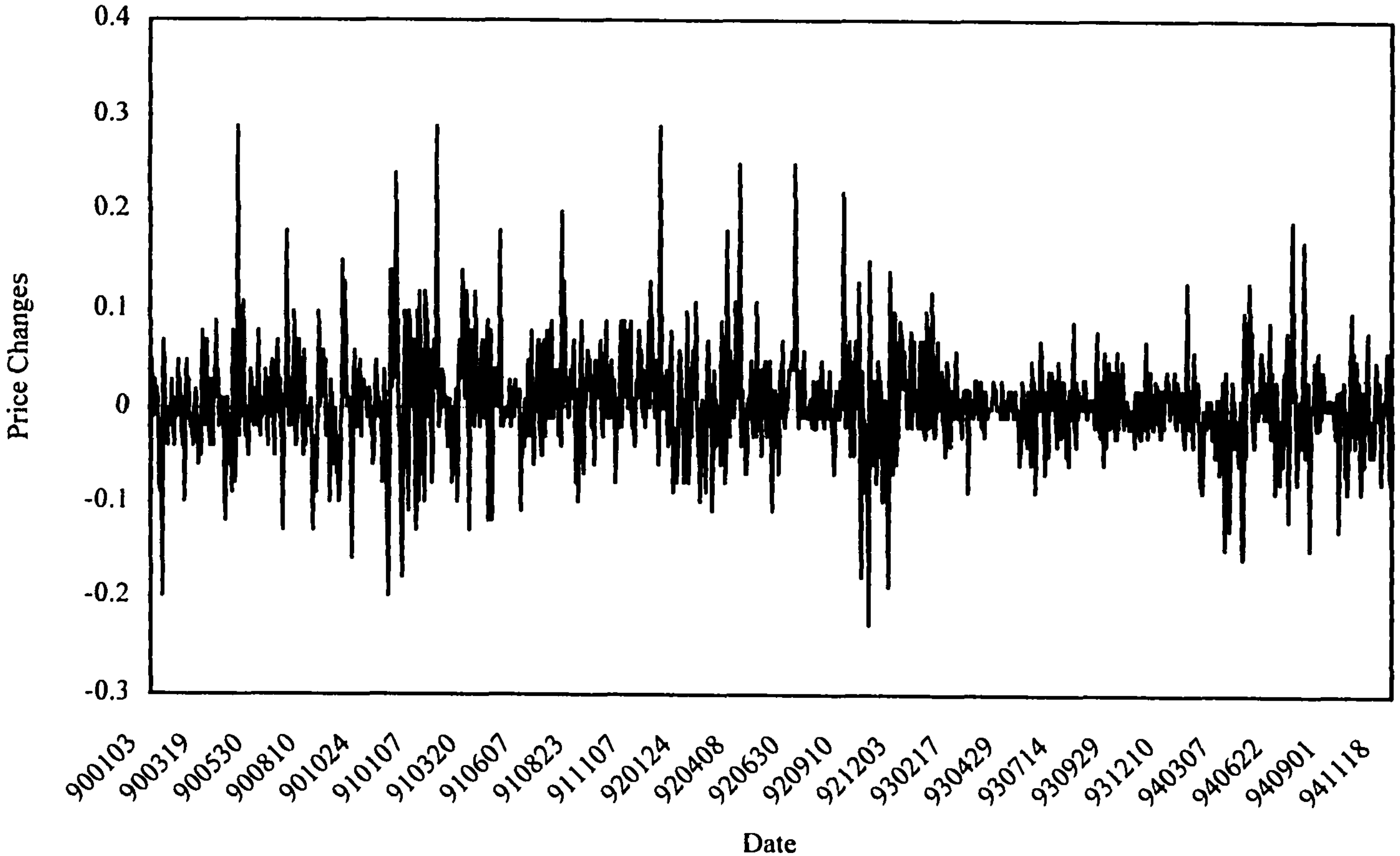

Figure 5
Eurodollar Options Price Changes

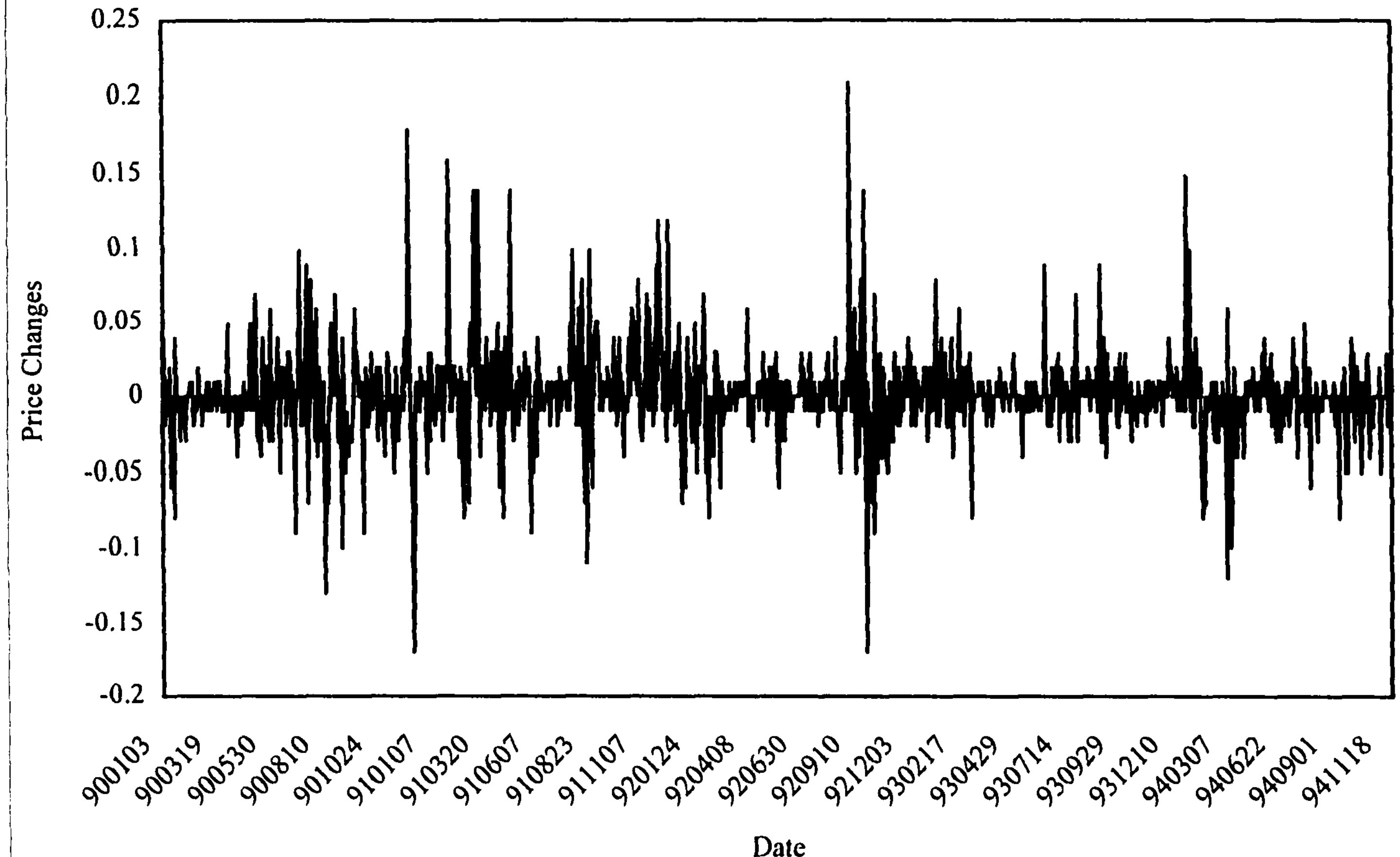

Figure 6
Eurodollar Futures/Options Out-Of-Sample Hedge Ratios

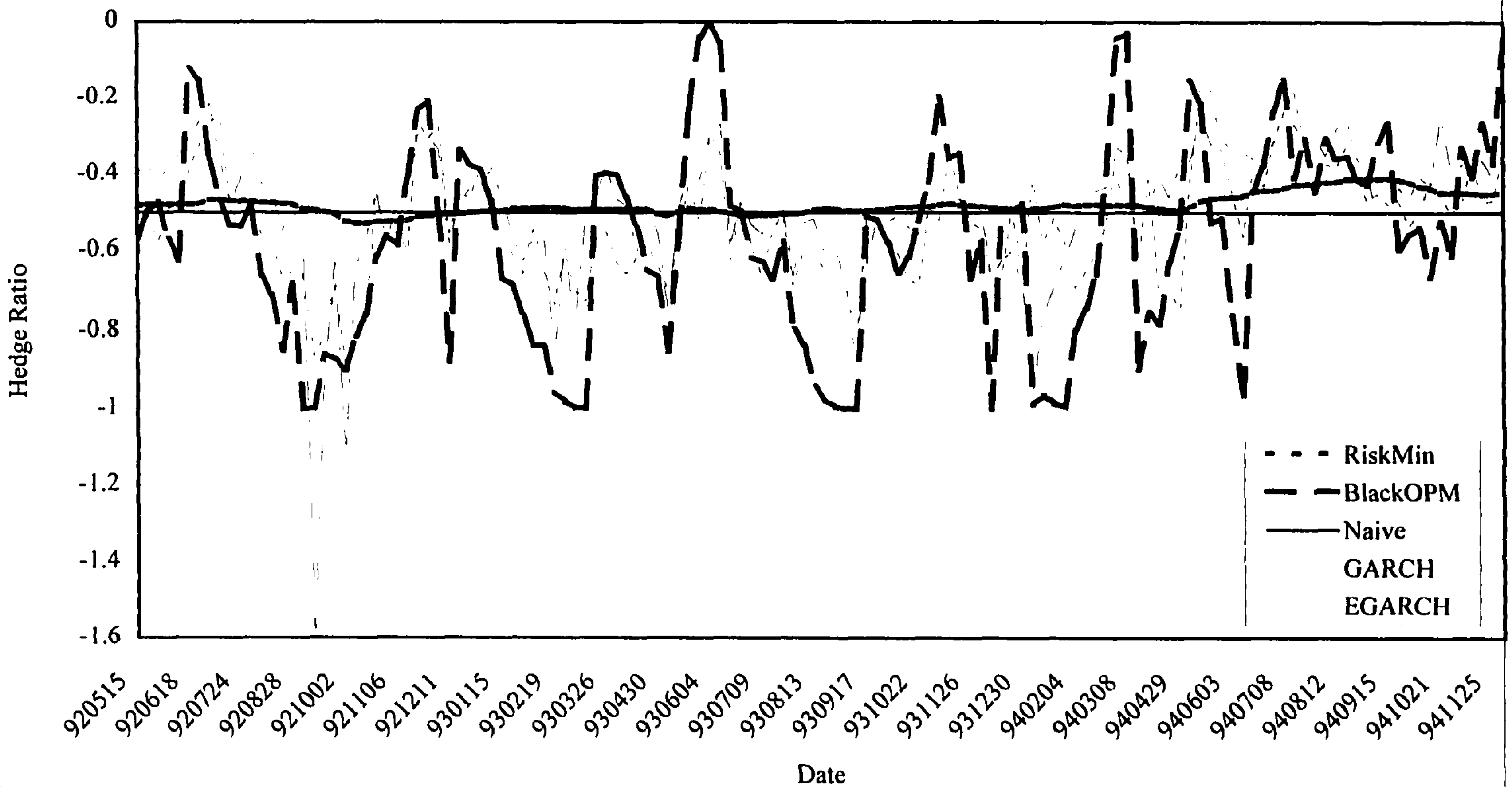

Figure 7
Eurodollar Futures Yield Versus Treasury Bill Spot Yield

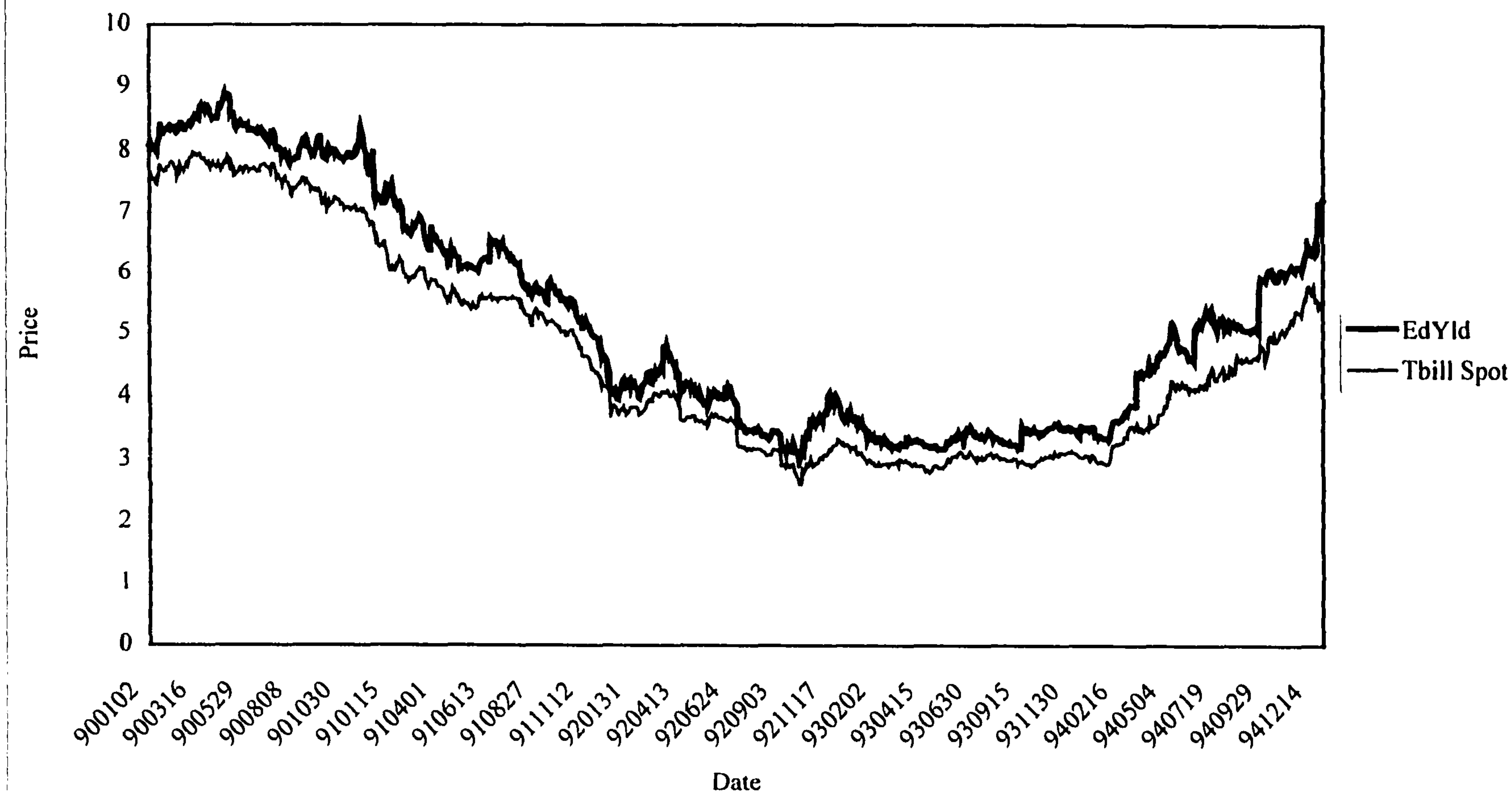

Figure 8
Treasury Bill Spot/Eurodollar Futures Basis

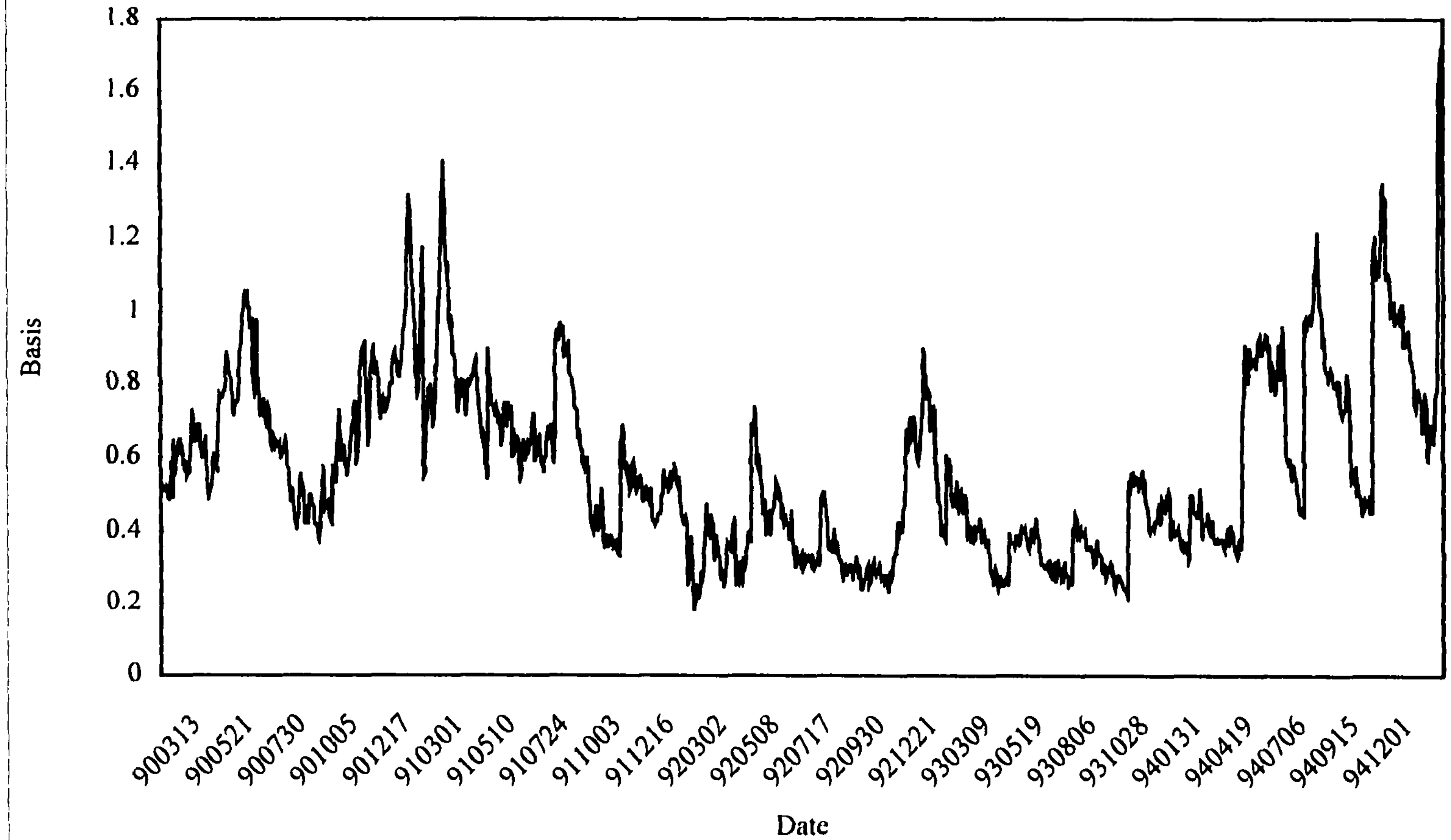

Figure 9
Spot Treasury Bill Yield Changes

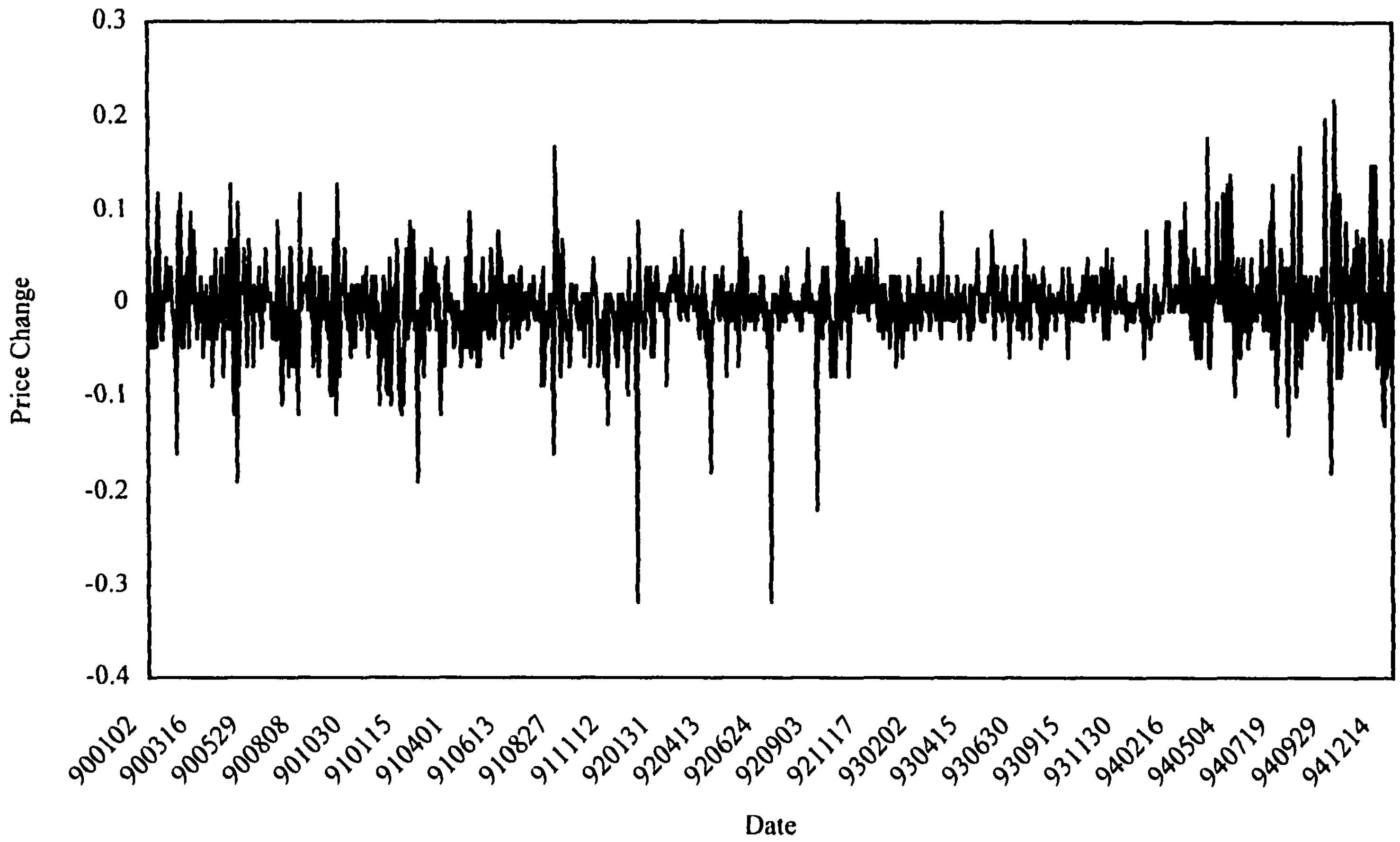

Figure 10
Eurodollar Futures Yield Changes

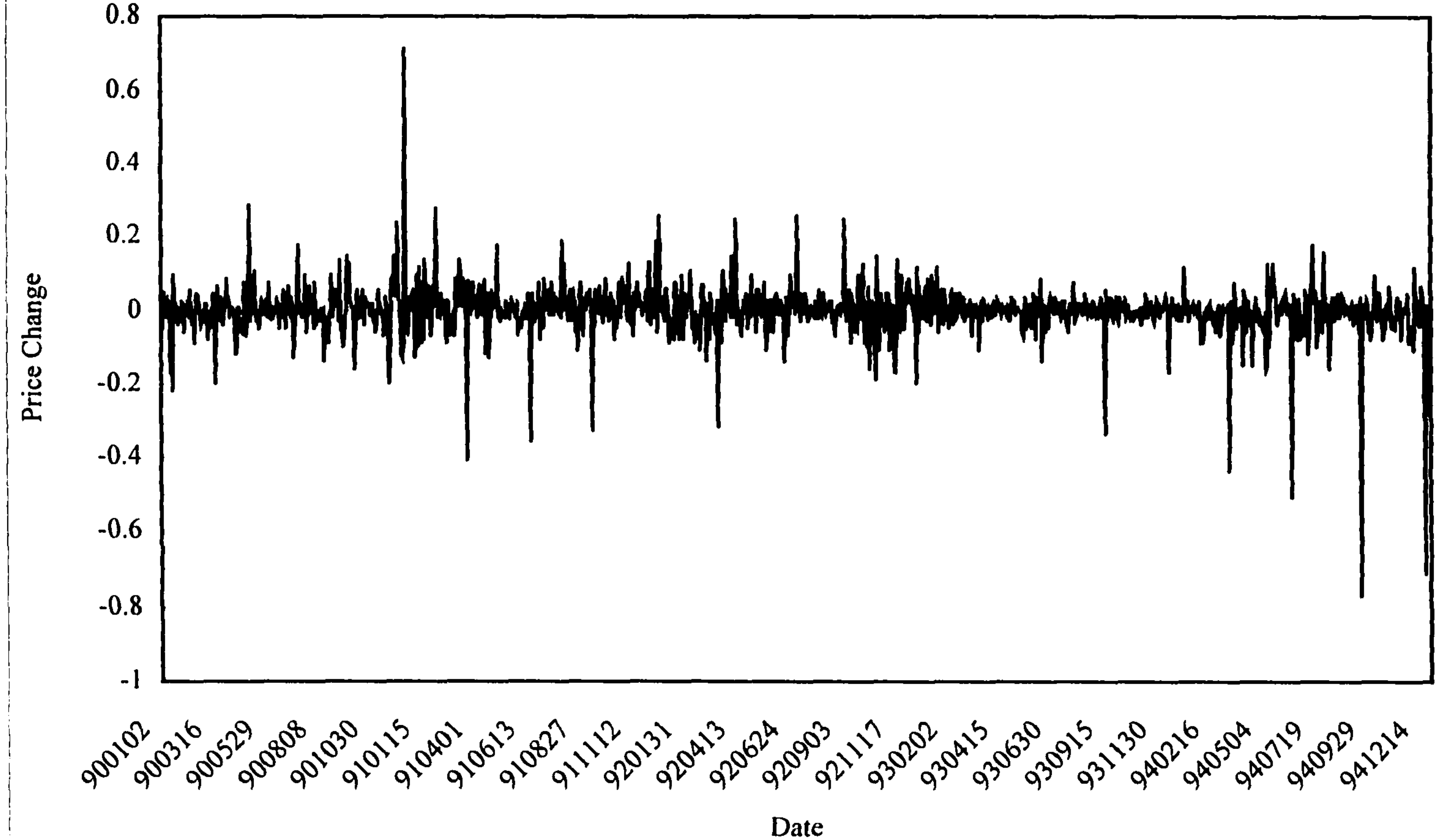

Table 1
EGARCH Parameters

Conditional variance computations based on the asymmetric relation term of the EGARCH model,

$$h = \exp\{\alpha_1(\phi z_{t-1} + [|z_{t-1}| - E|z_{t-1}|])\}$$

are shown below for positive and negative variations of the parameters. Under the condition of normality, $E|z_{t-1}|$ is equal to $(2/\pi)^{1/2}$.

α_1	ϕ	z_{t-1}	h
+0.20	+0.80	+0.02	0.85866
+0.20	+0.80	-0.02	0.85318
+0.20	-0.80	+0.02	0.85318
+0.20	-0.80	-0.02	0.85866
-0.20	+0.80	+0.02	1.16460
-0.20	+0.80	-0.02	1.17208
-0.20	-0.80	+0.02	1.17208
-0.20	-0.80	-0.02	1.16460

Table 2
Descriptive Statistics
Eurodollar Options on Futures and Eurodollar Futures

The options data is the daily price change for the most liquid strike price over the sample period of 1/2/90 to 11/30/94. The option corresponds to the nearby futures contract. The futures data is the daily price change for the nearby contract. Price changes on both the options and futures are calculated as (P_t-P_{t-1}) where P equals price. Unit root tests are based on both raw prices and price changes while the other tests are based on price changes only. The Phillips and Perron (1988) test statistic for a unit root is used with a truncation lag of 4, where the null hypothesis is that a unit root exists. Q(24) is the Ljung-Box test for up to 24th order serial correlation. ARCH(x) is the Engle (1982) LM test for ARCH effects. *, **, *** indicate significance at the 5.0, 1.0, and 0.1 per cent levels respectively.

	Options	Futures
Unit Root Test on Price	-78.3810***	-1.8383
Mean	0.0008	0.0047**
Variance	0.0009	0.0028
Skewness	0.5941***	0.4837***
Kurtosis	8.0069***	4.8076***
Q(24)	48.7106**	26.6925
ARCH(1)	9.3201**	0.8538
ARCH(3)	16.4999**	1.7323
ARCH(5)	39.1389***	12.1764*
ARCH(10)	40.9826***	18.7605*
No. of observations	1158	1158

Table 3
Univariate Conditional Heteroskedasticity Models
Eurodollar Options on Futures and Eurodollar Futures

Univariate testing of ARCH, GARCH, and EGARCH effects are shown. The tests are run on price changes computed as (P_t-P_{t-1}) where P_t is the price on day t. The models are:

ARCH(5): $h_t = a_0 + a_1\epsilon^2_{t-1} + a_2\epsilon^2_{t-2} + a_3\epsilon^2_{t-3} + a_4\epsilon^2_{t-4} + a_5\epsilon^2_{t-5}$

GARCH(1,1): $h_t = a_0 + a_1\epsilon^2_{t-1} + b_1h_{t-1}$

EGARCH(1,1): $\log h_t = a_0 + a_1(cz_{t-1} + [\,|z_{t-1}| - E|z_{t-1}|\,]) + b_1\log h_{t-1}$

*, **, *** indicate significance at the 5.0, 1.0, and 0.1 percent levels, respectively.

	Options	Futures
ARCH(5):		
a_0	0.0004***	0.0029***
a_1	0.0451	-0.0097
a_2	0.1326***	-0.0376***
a_3	0.1802***	0.1095*
a_4	0.1045**	-0.0138
a_5	0.4975***	0.1277**
Log Likelihood	1670	1350
GARCH(1,1)		
a_0	0.0001***	0.0027
a_1	0.0800***	-0.0174
b_1	0.8737***	0.2088
Log Likelihood	1676	1349
EGARCH(1,1)		
a_0	-0.4199***	-4.2761**
a_1	0.1895***	-0.0936
b_1	0.9367***	0.2520
c	0.7777***	1.3357
Log Likelihood	1699	1352

Table 4
Minimum Risk
Eurodollar Options on Futures and Eurodollar Futures
(Within Sample)

Minimum risk hedge ratio and hedging effectiveness for the nearby Eurodollar futures is shown for the period from 1/2/90 to 5/15/92. The hedge ratio is derived from the OLS regression of daily options price changes on futures price changes.

$$P_{o,t} - P_{o,t-1} = \alpha + \beta(P_{f,t} - P_{f,t-1}) + \epsilon_t$$

Hedging effectiveness is measured by the R^2 of the regression. N is the number of observations and DW is the Durbin Watson statistic. *, **, and *** denote significance at the 5.0, 1.0, and 0.1 percent levels respectively.

Hedge Ratio	-0.4824***
R^2	0.6726
N	577
DW	1.8643

Table 5
Estimation of the Bivariate GARCH Model
Eurodollar Options on Futures and Eurodollar Futures
(Within Sample)

The following bivariate GARCH model is estimated:

$$R_{1,t} = \mu_1 + \epsilon_{1,t}$$
$$R_{2,t} = \mu_2 + \epsilon_{2,t}$$
$$\epsilon_t | \Omega_{t-1} \sim N(0,H_t)$$

$$\begin{bmatrix} h_{11t} \\ h_{12t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} + \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \begin{bmatrix} \epsilon^2_{1t-1} \\ \epsilon_{1t-1}\epsilon_{2t-1} \\ \epsilon^2_{2t-1} \end{bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \begin{bmatrix} h_{11t-1} \\ h_{12t-1} \\ h_{22t-1} \end{bmatrix}$$

$R_{1,t}$ and $R_{2,t}$ are the options and futures price changes.*, **, and *** denote significance at the 5, 1, and 0.1 percent levels, respectively. LB is the Ljung-Box statistic for 24 lags.

Conditional Mean and Variance Equations	
μ_1	0.0019
μ_2	0.0066*
C_1	0.0005***
C_2	0.0007***
C_3	0.0008***
A_{11}	0.1000***
A_{22}	0.0742***
A_{33}	0.0430***
B_{11}	0.4857***
B_{22}	0.4708***
B_{33}	0.7348***
Log Likelihood	3358
LB(24) for (ϵ_t/σ_t) options	35.5694
LB(24) for $(\epsilon_t/\sigma_t)^2$ options	26.4625
LB(24) for (ϵ_t/σ_t) futures	15.5195
LB(24) for $(\epsilon_t/\sigma_t)^2$ futures	13.7010

Table 6
Estimation of the Bivariate EGARCH Model
Eurodollar Options on Futures and Eurodollar Futures
Constant Correlation
(Within Sample)

The following bivariate EGARCH model was estimated:

$$R_{1,t} = \mu_1 + \epsilon_{1,t}$$
$$R_{2,t} = \mu_2 + \epsilon_{2,t}$$
$$\epsilon_t | \Omega_{t-1} \sim N(0,H_t)$$

$$h_{11t}=\exp\{a_{1,0} + a_{1,1}\log h_{11,t-1} + a_{1,2}(|z_{1,t-1}| - E|z_{1,t-1}| + a_{1,3}z_{1,t-1})\}$$
$$h_{22t}=\exp\{a_{2,0} + a_{2,1}\log h_{22,t-1} + a_{2,2}(|z_{2,t-1}| - E|z_{2,t-1}| + a_{2,3}z_{2,t-1})\}$$
$$h_{12}=\rho(h_{11t}h_{22t})^{1/2}$$

$R_{1,t}$ and $R_{2,t}$ are options and futures price changes. For a normal distribution, $E|z| = (2/\pi)^{1/2}$. LB is the Ljung-Box statistic for 24 lags. *, **, and *** indicate significance at the 5.0, 1.0, and 0.1 percent levels, respectively.

Conditional Mean and Variance Equations	
μ_1	-0.0001
μ_2	0.0028
$a_{1,0}$	-0.5048***
$a_{1,1}$	0.9191***
$a_{1,2}$	0.3838***
$a_{1,3}$	0.2207**
$a_{2,0}$	-0.8745***
$a_{2,1}$	0.8362***
$a_{2,2}$	0.3129***
$a_{2,3}$	-0.5556***
ρ	0.8827***
Log Likelihood	3464
LB(24) for (ϵ_t/σ_t) options	22.6269
LB(24) for $(\epsilon_t/\sigma_t)^2$ options	14.6761
LB(24) for (ϵ_t/σ_t) futures	12.1389
LB(24) for $(\epsilon_t/\sigma_t)^2$ futures	12.0873

Table 7
Estimation of the Bivariate EGARCH Model
Eurodollar Options on Futures and Eurodollar Futures
Non-constant Correlation
(Within Sample)

$$R_{1,t} = \mu_1 + \epsilon_{1,t}$$
$$R_{2,t} = \mu_2 + \epsilon_{2,t}$$
$$\epsilon_t | \Omega_{t-1} \sim N(0,H_t)$$

$$\begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \exp\left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \log \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} + \begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{22} & 0 \\ 0 & 0 & c_{33} \end{bmatrix} \right.$$

$$\left. \left(abs \begin{bmatrix} \epsilon_{1,t-1}/\sqrt{h_{11,t-1}} \\ \epsilon_{1,t-1}\epsilon_{2,t-1}/h_{12,t-1} \\ \epsilon_{2,t-1}/\sqrt{h_{22,t-1}} \end{bmatrix} - \begin{bmatrix} E|z_1| \\ E|z_1|E|z_2| \\ E|z_2| \end{bmatrix} + \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t-1}/\sqrt{h_{11,t-1}} \\ \epsilon_{1,t-1}\epsilon_{2,t-1}/h_{12,t-1} \\ \epsilon_{2,t-1}/\sqrt{h_{22,t-1}} \end{bmatrix} \right) \right\}$$

*, **, and *** indicate significance at the 5.0, 1.0, and 0.1 percent levels, respectively.

Conditional Mean and Variance Equations

μ_1	0.0081**	c_{11}	0.1405*
μ_2	0.0160***	c_{22}	0.2963***
a_1	-0.8689***	c_{33}	0.0588***
a_2	-0.1099	d_{11}	0.0985
a_3	-0.0079	d_{22}	-0.8707***
b_{11}	0.8534***	d_{33}	0.1309
b_{22}	0.9598***		
b_{33}	0.9973***	Log Likelihood	3167

LB(24) for (ϵ_t/σ_t) options	38.9488*
LB(24) for $(\epsilon_t/\sigma_t)^2$ options	28.4892
LB(24) for (ϵ_t/σ_t) futures	17.2847
LB(24) for $(\epsilon_t/\sigma_t)^2$ futures	16.8818

For a normal distribution, $E|Z| = (2/\pi)^{1/2}$

Table 8
Comparisons of Hedging Effectiveness
Eurodollar Options on Futures and Eurodollar Futures

Variances of the net tick change in the hedged option/futures portfolio value as well as the unhedged option position value are reported. The net tick position is $(P_{o,t+1} - P_{o,t}) - b_t^*(P_{f,t+1} - P_{f,t})$ where b_t^* is the computed hedge ratio from each hedging method, $P_{o,t}$ is the options price, and $P_{f,t}$ is the futures price. The within-sample results are computed based on daily hedge ratio updates for the 1/2/90-5/15/92 period and the out-of-sample results are based on weekly hedge ratio updates for the 5/18/92-11/30/94 period. The percent reduction in variance is computed as:

1 - (variance of the hedged position / variance of the unhedged position)

The values shown are the estimates times 10^2.

Method	Mean	Variance	Percent Reduction In Variance
Panel A: Within-Sample			
Unhedged	0.2662	0.1161	
Black OPM	-0.0425	0.0127	89.06%
Naive Hedge (b=0.5)	-0.1297	0.0377	67.53%
Risk Minimization	-0.1158	0.0376	67.61%
GARCH	-0.0888	0.0353	69.63%
EGARCH (Constant Correlation)	-0.0749	0.0236	79.67%
EGARCH	-0.0665	0.0379	67.36%
Panel B: Out-of-Sample			
Unhedged	-0.1080	0.0707	
Black OPM	-0.0610	0.0203	71.29%
Naive Hedge (b=0.5)	-0.1550	0.0300	57.57%
Risk Minimization	-0.1700	0.0282	60.11%
GARCH	-0.0820	0.0257	63.65%
EGARCH (Constant Correlation)	-0.1060	0.0242	65.77%

Table 9
Descriptive Statistics
Treasury Bill Spot and Eurodollar Futures

Statistics are computed for the 90 day Treasury Bill spot and 3-month Eurodollar futures contract yields over the sample period of 1/2/90 to 12/31/94. Eurodollar yields are computed as (100 minus futures price). Yield changes on both the T-bill and Eurodollars are calculated as $(Y_t - Y_{t-1})$ where Y equals yield. Unit root tests are based on both raw yields and yield changes and the cointegration test is based on raw yields, while the other tests are based on yield changes only. The Phillips and Perron (1988) test statistic for a unit root is used with a truncation lag of 4, where the null hypothesis is that a unit root exists. The Phillips Z_ρ statistic tests the null hypothesis of no cointegration. Q(24) is the Ljung-Box test for up to 24th order serial correlation. ARCH(*x*) is the Engle (1982) LM test for ARCH effects. *, **, *** indicate significance at the 5.0, 1.0, and 0.1 per cent levels respectively.

	T-Bill Spot	Eurodollar Futures
Unit Root Test on Yield	-1.6810	-1.4454
Mean	-0.0024	-0.0047**
Variance	0.0020	0.0028
Skewness	-0.6026***	-0.5187***
Kurtosis	7.1490***	4.3021***
Q(24)	45.1911**	32.1632
ARCH(1)	1.6079	2.8664
ARCH(3)	3.8594	4.3305
ARCH(5)	12.5910*	17.2425**
ARCH(10)	18.0332	22.6319*
Cointegration Test	-47.4932***	
No. of observations	1172	1172

Table 10
Univariate Conditional Heteroskedasticity Models
Treasury Bill Spot and Eurodollar Futures

Univariate testing of ARCH, GARCH, and EGARCH effects are shown. The tests are run on yield changes computed as $(Y_t - Y_{t-1})$ where Y_t is the yield on day *t*. The models are:

ARCH(5):	$h_t = a_0 + a_1\epsilon^2_{t-1} + a_2\epsilon^2_{t-2} + a_3\epsilon^2_{t-3} + a_4\epsilon^2_{t-4} + a_5\epsilon^2_{t-5}$
GARCH(1,1):	$h_t = a_0 + a_1\epsilon^2_{t-1} + b_1 h_{t-1}$
EGARCH(1,1):	$\log h_t = a_0 + a_1(cz_{t-1} + [\lvert z_{t-1}\rvert - E\lvert z_{t-1}\rvert]) + b_1 \log h_{t-1}$

*, **, *** indicate significance at the 5.0, 1.0, and 0.1 percent levels, respectively.

	T-Bill Spot	Eurodollar Futures
ARCH(5):		
a_0	0.0014***	0.0027***
a_1	0.1635***	0.0048
a_2	0.1333**	-0.0297*
a_3	0.0723*	0.0474
a_4	0.0420	0.0609*
a_5	-0.0069	0.0952*
Log Likelihood	1514	1379
GARCH(1,1)		
a_0	0.0007***	0.0002*
a_1	0.1931***	0.0238*
b_1	0.5130***	0.9156***
Log Likelihood	1519	1382
EGARCH(1,1)		
a_0	-2.3388***	-9.9179***
a_1	0.2592***	0.1252
b_1	0.6233***	-0.7336**
c	-1.1219**	0.3574
Log Likelihood	1531	1381

Table 11
Minimum Risk
Treasury Bill Spot and Eurodollar Futures
(Within Sample)

Minimum risk hedge ratio and hedging effectiveness for the nearby Eurodollar futures is shown for the period from 1/2/90 to 6/19/92. The hedge ratio is derived from the OLS regression of daily T-bill yield changes on Eurodollar futures yield changes.

$$Y_{s,t} - Y_{s,t-1} = \alpha_0 + \beta(Y_{f,t} - Y_{f,t-1}) + \alpha_1(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_t$$

Hedging effectiveness is measured by the R^2 of the regression. N is the number of observations and DW is the Durbin Watson statistic. *, **, and *** denote significance at the 5.0, 1.0, and 0.1 percent levels respectively.

Hedge Ratio	-0.4607***
α_1	-0.0187*
R^2	0.3249
N	586
DW	1.9420

Table 12
Estimation of the Bivariate GARCH Model
Treasury Bill Spot and Eurodollar Futures
(Within Sample)

The following bivariate GARCH model is estimated:

$$R_{1,t} = \mu_1 + \alpha_1(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_{1,t}$$
$$R_{2,t} = \mu_2 + \alpha_2(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_{2,t}$$
$$\epsilon_t|\Omega_{t-1} \sim N(0,H_t)$$

$$\begin{bmatrix} h_{11t} \\ h_{12t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} + \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \begin{bmatrix} \epsilon^2_{1t-1} \\ \epsilon_{1t-1}\epsilon_{2t-1} \\ \epsilon^2_{2t-1} \end{bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \begin{bmatrix} h_{11t-1} \\ h_{12t-1} \\ h_{22t-1} \end{bmatrix}$$

$R_{1,t}$ and $R_{2,t}$ are the spot and futures yield changes.*, **, and *** denote significance at the 5, 1, and 0.1 percent levels, respectively. LB is the Ljung-Box statistic for 24 lags.

Conditional Mean and Variance Equations	
μ_1	-0.0055*
μ_2	-0.0022
α_1	-0.0017
α_2	0.0566***
C_1	0.0007***
C_2	0.0013
C_3	0.0030***
A_{11}	0.2039***
A_{22}	0.0051
A_{33}	0.0885**
B_{11}	0.4697***
B_{22}	0.1601
B_{33}	0.0199
Log Likelihood	3034
LB(24) for (ϵ_t/σ_t) spot	35.4629
LB(24) for $(\epsilon_t/\sigma_t)^2$ spot	10.0274
LB(24) for (ϵ_t/σ_t) futures	27.1731
LB(24) for $(\epsilon_t/\sigma_t)^2$ futures	17.9479

Table 13
Estimation of the Bivariate EGARCH Model
Treasury Bill Spot and Eurodollar Futures
Constant Correlation
(Within Sample)

The following bivariate EGARCH model was estimated:

$$R_{1,t} = \mu_1 + \alpha_1(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_{1,t}$$
$$R_{2,t} = \mu_2 + \alpha_2(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_{2,t}$$
$$\epsilon_t | \Omega_{t-1} \sim N(0,H_t)$$

$$h_{11t}=\exp\{a_{1,0} + a_{1,1}\log h_{11,t-1} + a_{1,2}(|z_{1,t-1}| - E|z_{1,t-1}| + a_{1,3}z_{1,t-1})\}$$
$$h_{22t}=\exp\{a_{2,0} + a_{2,1}\log h_{22,t-1} + a_{2,2}(|z_{2,t-1}| - E|z_{2,t-1}| + a_{2,3}z_{2,t-1})\}$$
$$h_{12t}=\rho(h_{11t}h_{22t})^{1/2}$$

$R_{1,t}$ and $R_{2,t}$ are spot and futures yield changes. For a normal distribution, $E|z| = (2/\pi)^{1/2}$. LB is the Ljung-Box statistic for 24 lags. *, **, and *** indicate significance at the 5.0, 1.0, and 0.1 percent levels, respectively.

Conditional Mean and Variance Equations	
μ_1	-0.0056**
μ_2	-0.0025
α_1	0.0042
α_2	0.0487***
$a_{1,0}$	-0.7425***
$a_{1,1}$	0.8785***
$a_{1,2}$	0.2321***
$a_{1,3}$	-0.6398***
$a_{2,0}$	-0.3131**
$a_{2,1}$	0.9439***
$a_{2,2}$	0.1186***
$a_{2,3}$	0.6073**
ρ	0.6203***
Log Likelihood	3055
LB(24) for (ϵ_t/σ_t) spot	37.6670*
LB(24) for $(\epsilon_t/\sigma_t)^2$ spot	10.4636
LB(24) for (ϵ_t/σ_t) futures	20.9427
LB(24) for $(\epsilon_t/\sigma_t)^2$ futures	13.6105

Table 14
Estimation of the Bivariate EGARCH Model
Treasury Bill Spot and Eurodollar Futures
Non-constant Correlation
(Within Sample)

$$R_{1,t} = \mu_1 + \alpha_1(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_{1,t}$$
$$R_{2,t} = \mu_2 + \alpha_2(Y_{s,t-1} - \delta Y_{f,t-1}) + \epsilon_{2,t}$$
$$\epsilon_t | \Omega_{t-1} \sim N(0,H_t)$$

$$\begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \exp\left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \log \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} + \begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{22} & 0 \\ 0 & 0 & c_{33} \end{bmatrix} \right.$$
$$\left. \left(abs \begin{bmatrix} \epsilon_{1,t-1}/\sqrt{h_{11,t-1}} \\ \epsilon_{1,t-1}\epsilon_{2,t-1}/h_{12,t-1} \\ \epsilon_{2,t-1}/\sqrt{h_{22,t-1}} \end{bmatrix} - \begin{bmatrix} E|z_1| \\ E|z_1|E|z_2| \\ E|z_2| \end{bmatrix} + \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t-1}/\sqrt{h_{11,t-1}} \\ \epsilon_{1,t-1}\epsilon_{2,t-1}/h_{12,t-1} \\ \epsilon_{2,t-1}/\sqrt{h_{22,t-1}} \end{bmatrix} \right) \right\}$$

*, **, and *** indicate significance at the 5.0, 1.0, and 0.1 percent levels, respectively.

Conditional Mean and Variance Equations

μ_1	-0.0067**	c_{11}	0.2453***
μ_2	-0.0053	c_{22}	-0.0296
α_1	-0.0014	c_{33}	0.0449*
α_2	0.0357**	d_{11}	-0.5663***
a_1	-1.7263***	d_{22}	-0.2159
a_2	-11.6263***	d_{33}	0.2235
a_3	-11.1189***		
b_{11}	0.7193***		
b_{22}	-0.7964***	Log Likelihood	3045
b_{33}	-0.9537***		

LB(24) for (ϵ_t/σ_t) options	36.8348*
LB(24) for $(\epsilon_t/\sigma_t)^2$ options	11.1699
LB(24) for (ϵ_t/σ_t) futures	26.1274
LB(24) for $(\epsilon_t/\sigma_t)^2$ futures	18.6169

For a normal distribution, $E|Z| = (2/\pi)^{1/2}$

Table 15
Comparisons of Hedging Effectiveness
Treasury Bill Spot and Eurodollar Futures

Variances of the net tick change in the hedged spot/futures portfolio value as well as the unhedged spot position value are reported. The net tick position is $(Y_{s,t+1} - Y_{s,t}) - b_t^*(Y_{f,t+1} - Y_{f,t})$ where b_t^* is the computed hedge ratio from each hedging method, $Y_{s,t}$ is the T-bill spot yield, and $Y_{f,t}$ is the Eurodollar futures yield. The within-sample results are computed based on daily hedge ratio updates for the 1/2/90-6/19/92 period and the out-of-sample results are based on weekly hedge ratio updates for the 6/22/92-12/31/94 period. The percent reduction in variance is computed as:

1 - (variance of the hedged position / variance of the unhedged position)

The values shown are the estimates times 10^2.

Method	Mean	Variance	Percent Reduction in Variance
Panel A: Within-Sample			
Unhedged	-0.7130	0.2117	
Naive Hedge (b=1.0)	0.1323	0.2417	-14.17%
Risk Minimization	-0.3270	0.1437	32.11%
GARCH	-0.3440	0.1392	32.12%
EGARCH (Constant Correlation)	-0.3500	0.1427	32.59%
EGARCH	-0.3350	0.1482	30.00%
Panel B: Out-of-Sample			
Unhedged	0.1767	0.1690	
Naive Hedge (b=1.0)	0.3354	0.1347	20.30%
Risk Minimization	0.2622	0.1133	32.96%
GARCH	0.2568	0.1101	34.88%
EGARCH (Constant Correlation)	0.2285	0.1115	34.03%