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KARLHEINZ STOCKHAUSEN'S "KLAVIERSTUCK XI": AN ANALYSIS OF ITS COMPOSITION VIA A MATRIX SYSTEM OF SERIAL POLYPHONY AND THE TRANSLATION OF RHYTHM INTO PITCH

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THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

KARLHEINZ STOCKHAUSEN'S KLAVIERSTÜCK XI: AN ANALYSIS OF ITS COMPOSITION VIA A MATRIX SYSTEM OF SERIAL POLYPHONY AND THE TRANSLATION OF RHYTHM INTO PITCH

A DOCUMENT

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By

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KARLHEINZ STOCKHAUSEN'S KLAVIERSTÜCK XI: AN ANALYSIS OF ITS COMPOSITION VIA A MATRIX SYSTEM OF SERIAL POLYPHONY AND THE TRANSLATION OF RHYTHM INTO PITCH

A DOCUMENT

APPROVED FOR THE DEPARTMENT OF MUSIC

By

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The author wishes to anknowledge Karlheinz Stockhausen's generosity in providing the original manuscript and sketches for Klavierstück XI, access to his personal archive, and use of the photo-copy machine in his home. Study and travel with Stockhausen, at his personal invitation, during the 1980 European tour with Ensemble InterContemporain was extremely beneficial; further study at his home after the tour and a continuing correspondence have provided additional insights and inspiration.

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CHAPTER I

BACKGROUND AND ORIENTATION

The musical stature of Karlheinz Stockhausen is exemplary both in the quantity and quality of his activities as a composer, conductor, performer, theoretician, teacher and publisher of music. At the age of fifty-five Stockhausen has completed eighty major works for various ensembles, soloists, electronic media, and combinations of live performers and electronic media. His symphonic works, which place him in the mainstream of major symphonists since Mahler, include: Jubilaum (1977), Inori (1973/74), Ylem (1972), Trans (1971), Stop (1965), Mixtur (1964), Gruppen (1955/57), Spiel (1952), Punkte (1952/62), and Formel (1951). His music for solo instruments, aside from those scores which do not designate the instrument (Plus Minus, Solo, and Spiral), constitutes an enrichment of both piano and clarinet repertoire. Harlekin (1975) and The Little Harlekin (1975), both for solo clarinet, incorporate dance and a form of mime into the musical structure.

Stockhausen was born in Modrath, Germany, on August 22,

1928. He attended primary school at Altenberg, and at the age of ten was enrolled in an Oberschule in Burscheid, where he stayed until the age of twelve. From 1941 to October, 1944 Stockhausen attended a Teacher Training College at Xanten. After entering the Grammar School for the Humanities at Bergisch Gladbach in February, 1946, Stockhausen graduated in March, 1947. That same year he began his studies at the State Academy of Music in Cologne, and also studied philosophy, musicology and German at Cologne University. After graduation from the State Academy of Music in 1951, Stockhausen attended the Darmstadt New Music Courses. His subsequent association with Darmstadt International Vacation Courses for New Music, as they are now called, has become legendary. During the years 1952/1953 Stockhausen studied with the French composer Olivier Messiaen at the Paris Conservatoire of Music. Stockhausen's last formal university studies were communications and phonetics at Bonn University from 1954 to 1956 with Dr. Werner Meyer-Eppler.

Stockhausen's pianistic studies began at the age of six while living in Altenberg, his first teacher being a church organist. Stockhausen continued to study piano throughout his formal education, finally participating in Hans Otto Schmidt-Neuhaus's piano class in the Cologne Musikhochschule. At Xanten Stockhausen expanded his musical activities to include violin lessons and playing the oboe in the college symphony orchestra, in addition to being pianist for the college dance band. During his studies at Bergisch Gladbach Stockhausen gained positions as both the pianist and accompanist for a dancing studio and became the musical director of an amateur operetta society. While studying at Cologne he also found employment as a jazz pianist and accompanied a magician, Adrion, by improvising at the piano.

The two most significant influences on Stockhausen's use of rhythm stem from his studies (circa 1951-1956) with Messiaen and Meyer-Eppler. The Analysis and Aesthetics course with Messiaen focused on rhythmic studies of various composers's music.[1] Messiaen also analyzed his own music from an aesthetic standpoint which taught Stockhausen to view a work of music as an organic whole, which can be perceived through an analytic orientation that seeks to discover how the various parameters involved combine in order to create both interior structures and sometimes the larger form of an entire work.[2] As a result of his studies with Meyer-Eppler, which began in 1953,[3] Stockhausen incorporated mathematical concepts into his musical thinking.

Twenty-six pages of unpublished stketches reveal the processes Stockhausen used to to organize the rhythmic structure of Klavierstück XI before composing the pitch content of the

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[1]Concerning the influences of jazz music, his 1951 Darmstadt studies, his principal instrumental orientation being the piano keyboard, and the Paris study with Messiaen, see Maconie, R., The Works of Karlheinz Stockhausen. (London: Oxford University Press, 1976), pp. 5-9, 30-40; and concerning the Darmstadt and Messiaen studies see Wörner, K.H., Stockhausen: Life and Work. (Berkeley: University of California Press, (1973) pp. 252-253.

[2]Stockhausen in conversation with the author, Denver, August, 1980. Also see Wörner, p. 253.

[3]Wörner, p. 237.

piece. In November of 1980 I researched Klavierstück XI in Stockhausen's personal archive at Kurten, West-Germany. Later at his home Stockhausen removed the folder for Klavierstück XI from his vault which contains original manuscripts and the sketches for his compositions. As source material for this study, Stockhausen then allowed me to photocopy the original manuscript of Klavierstück XI and 26 pages of notes and sketches. These unpublished sketches, as well as sketches for other works by Stockhausen, including Gruppen, are now directly available from the Stockhausen Verlag.

Stockhausen's Klavierstück I-XI, composed and revised from 1952 to 1961, represent a significant expansion of serial processes to parameters other than pitch organization. The drama of the evolution of musical thought which threads its way throughout the Klavierstück reflects Stockhausen's concern for a unified compositional vision which evolves from work to work. His piano music has also been enriched by the transference of concepts and processes from para-musical realms such as acoustics, electronic music, mathematics, and phonetics and communication research.

Klavierstück XI was composed in 1956. Other works by Stockhausen dating from that year are Zeitmasze (1955/56), Gruppen (1955/57) and the electronic piece Gesang der Junglinge (1955/56). Maconie has discussed the fact that Klavierstück XI is more akin to Gruppen in initial theoretical conception than the preceding piano works.[4] The aleatoric character of

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[4] Maconie, p. 100.

Klavierstück XI further distinguishes it from all the other works Stockhausen was composing during the same era (1950-1957). It is the eleventh in a projected cycle of twenty-one pieces, thirteen having now been published. Further works in the cycles constituting Klavierstück XIV-XXI may also incorporate aleatoric functions.[5]

Klavierstück XI is published on a single page of very thick paper which is supported by a wooden frame which comes with the score.[6] The frame stabilizes the score in an upright position on the piano rack. This is necessary because the piece, according to performance instructions, should not be played from memory. The variable form of the work from one performance to the next is determined by random visual associations which make the score necessary for each performance. After choosing one of six predetermined tempi, which range from very fast to very slow, the performer plays the first of nineteen separate music fragments upon which the visual attention falls. The fragments are of different lengths and are randomly distributed throughout the page. After performing the first fragment the pianist encounters a set of tempo, attack, and dynamic instructions for performing the next fragment which the visual attention randomly alights upon. Fragments may be repeated, and when this occurs bracketed one or two octave transpositions are then observed. Some fragments may be played only once or not at all. When the performer arrives at a fragment

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[5]Henck, Herbert, Stockhausen's Klavierstück X, p. 76.

[6]Published by Universal Edition (London).

for the third time the piece is then concluded without the fragment being played a third time. Stockhausen eventually gave pianists permission to plan a predetermined course through the great many possibilities of different sequences in which the nineteen fragments can be encountered during performance. This avoids giving the piece the character of a "suite" of small pieces, which often happened when pianists took inordinate amounts of time between fragments while preparing themselves to play fragments with different tempi, registers, etc., than they had previously employed in the same fragment.

The polyvalent nature of the performance instructions for Klavierstück XI results in a changing sequence in which the nineteen separate fragments of the piece are ordered from one performance to the next. This relates to the mathematical concepts of random structures and statistics.[7] This performance concept can also be perceived as a logical extension of several separate serial processes--discussed in Chapter 2--which were combined polyphonically by the superimposition of their respective matrices during composition.

After a study of the twenty-six pages of Stockhausen's sketches--which reveal the step by step conception and composition of Klavierstück XI--the nineteen fragments can also be seen as a particular localization of space within a larger, unified, multidimensional matrix of overlapping serial processes. In addition, this spatial localization is reflected in both the placement of each fragment in the score, and in the

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[7] Wörner, p. 237.

manner in which the performer determines the sequence in which to play the fragments. The concept that such insights into the compositional craft of the composer may subsequently inform and aid performance practice seems especially relevant concerning the study of Stockhausen's sketches.

In the performance instructions printed on the back of the published score of Klavierstück XI, Stockhausen suggests that the piece be played more than once during the same concert. This would highlight the polyvalent form of the piece. Some fragments could appear in one version and be absent in another. Identical fraqments occurring in both performances might happen at different points in time, be framed by different fragments on either side, and have different tempo, attack, and dynamic instructions. The length of the two performances could also vary noticeably. The essential character of the form of the piece, a directionless time-field[8], could then begin to be aurally perceived in a single concert.

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[8]Stockhausen, K., "...How Time Passes..." Die Reihe 3 (1959), p. 36.

CHAPTER TWO

A MATRIX SYSTEM OF SERIAL POLYPHONY

Introduction

Interrelationships of Matrices

The nineteen music fragments of Klavierstück XI were composed by a matrix system of serial polyphony in which rhythm matrices, often serially reordered by number matrices, were combined together into a larger Final Rhythm Matrix[9] of proportions similar to the earlier ones. A series of element matrices, containing trills, pauses, and other types of various elements, were added together to form a Final Element Matrix.

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[9]The first page of Stockhausen's sketches contains the rhythm matrix, shown in Example 3, used in constructing the sixth column of the Final Rhythm Matrix. A succession of increasingly smaller rhythm matrices was employed in constructing columns 5 to 1 of the Final Rhythm Matrix. The analysis of the construction of this matrix will therefore also proceed from column 6 to column 1 in Chapter II.

The rhythmic cells in each column of the Final Rhythm Matrix correspond in number to the total number of columns in the rhythm matrix from which the rhythmic cells are derived. This is because an entire rhythm matrix is transferred to a single column within the Final Rhythm Matrix. Each successive rhythm matrix Stockhausen used in constructing the Final Rhythm Matrix has one less column. Therefore, column 6 of the Final Rhythm Matrix has 7 rhythmic cells inside each block; column 5 has 6 rhythmic cells, and so on to column 1, which has only 2 rhythmic cells per block.

In all matrices, both of numbers and of rhythms, the term "row" will designate a horizontal succession and the term "column" will designate a vertical succession. The top row of a matrix will be designated "row 1" and the column at the left side of the matrix will be designated "column 1".

The Final Rhythm Matrix and Final Element Matrix were then added together. This multiple matrix system is reminiscent of similar systems which can be observed in the application of matrix algebra,[10] yet the particular manner in which the elements of these various matrices are first organized and later combined with one another appears to be original with Stockhausen.

Stockhausen employed three different types of matrices in constructing the six columns of the Final Rhythm Matrix: rhythm matrices, number matrices, and Roman numeral with exponent matrices. As an aid to more detailed analyses of matrix processes, occurring later in Chapter II, the manner in which these matrices operate upon one another will now be discussed. Example 1, shown on page 10, contains column 5 of the Final Rhythm Matrix and the three matrices Stockhausen employed in constructing it. Since many examples in this document directly relate to one another, this necessitates their being viewed simultaneously. Appendix II, which is bound separately from the text of this document, provides additional copies of selected examples: examples referred to in the text, but not present on the same page, can be simultaneously viewed by consulting the appropriate example in Appendix II.

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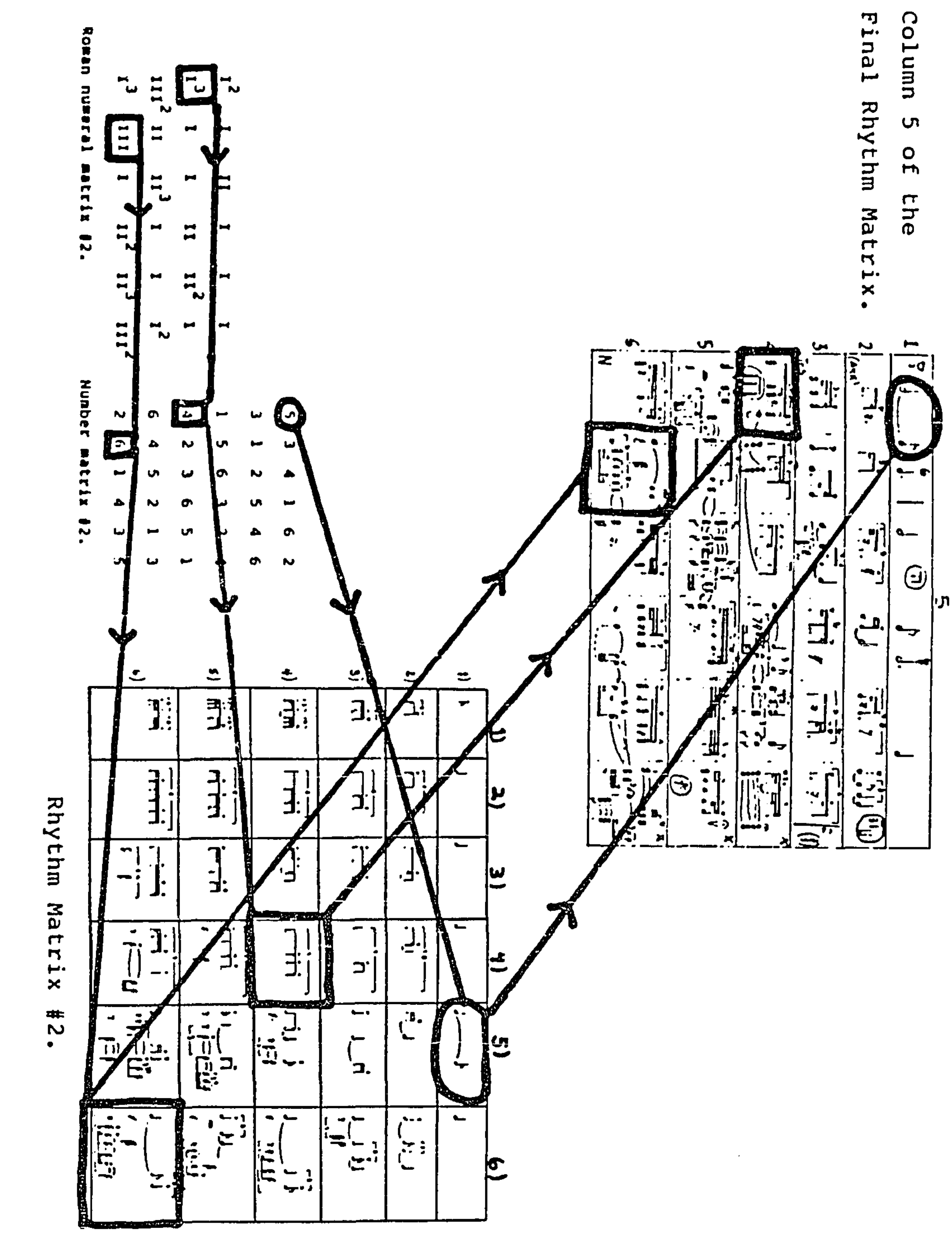
[10] Andree, R.V., Selections From Modern Abstract Algebra, Chapters 5 and 9.

Example 1: Interrelationships of Roman numeral matrix #2, number matrix #2, and rhythm matrix #2 in the construction of column 5 of the Final Rhythm Matrix.



The manner in which the number matrices operate upon the rhythm matrices is illustrated by circled elements in Example 1. The number 5, circled in the top row of number matrix #2, is employed in shifting the fifth block of the top row of rhythm matrix #2 to the initial position of the top row of the fifth column of the Final Rhythm Matrix. The number matrix number is used to locate the block, found in the identical row of the rhythm matrix, which has a column number identical with the number matrix number--in this instance the number 5. Once the specific block is located in the rhythm matrix, it is then shifted to a new position--relative to the other blocks in the same row--identical with the position of the number matrix number within its own row. Since the number 5 is in the first position, the rhythm matrix block is shifted to the first position in the identical row of column 5 in the Final Rhythm Matrix. In the same manner the number 3, being the second number of the top row of number matrix #2, shifts the dotted quarter note in the interior of the third block of the identical row in rhythm matrix #2 to the second position in column 5 of the Final Rhythm Matrix. Note the dotted quarter note immediately following the circled element in the top row of column 5 of the Final Rhythm Matrix.

The blocks in Example 1 indicated by darkened squares illustrate how the Roman numeral matrices operate upon the rhythm matrices via the number matrices. The I^3 in the square in column 1 of Roman numeral matrix #2 occupies the first position in its row. By association with the number 4, pointed out by a darkened square in the same position of the identical row in number matrix #2, the I^3 is further associated with the block at row 4 column 4 of rhythm matrix #2. The Roman numeral I indicates the interior of this block in the rhythm matrix has only one voice. The exponent (3) associated with the Roman numeral operates upon the rhythm matrix block by indicating that each interior attack will have 3 note heads instead of only one when this block is transferred to a new row position in the Final Rhythm Matrix. This is evident when the corresponding block at the beginning of row 4 in column 5 of the Final Rhythm Matrix is examined. The exponents in the Roman numeral matrices only affect the upper voice in those rhythm matrix blocks having more than one voice. This can be verified by using number matrix #2 to identify the upper voice of the block in column 5 of the Final Rhythm Matrix which originated at row 6 column 5 in rhythm matrix #2.[11]

The Construction of the Rhythm Matrices

Example 3, having a 6 x 7 dimension, is the first rhythm matrix Stockhausen employed in order to arrive at portions of the rhythmic structure of Klavierstück XI. The number matrix shown in Example 2 was used to reorder rhythm matrix #1 when column 6 of the Final Rhythm Matrix was constructed.

(For Examples 2 and 3, see page 13)

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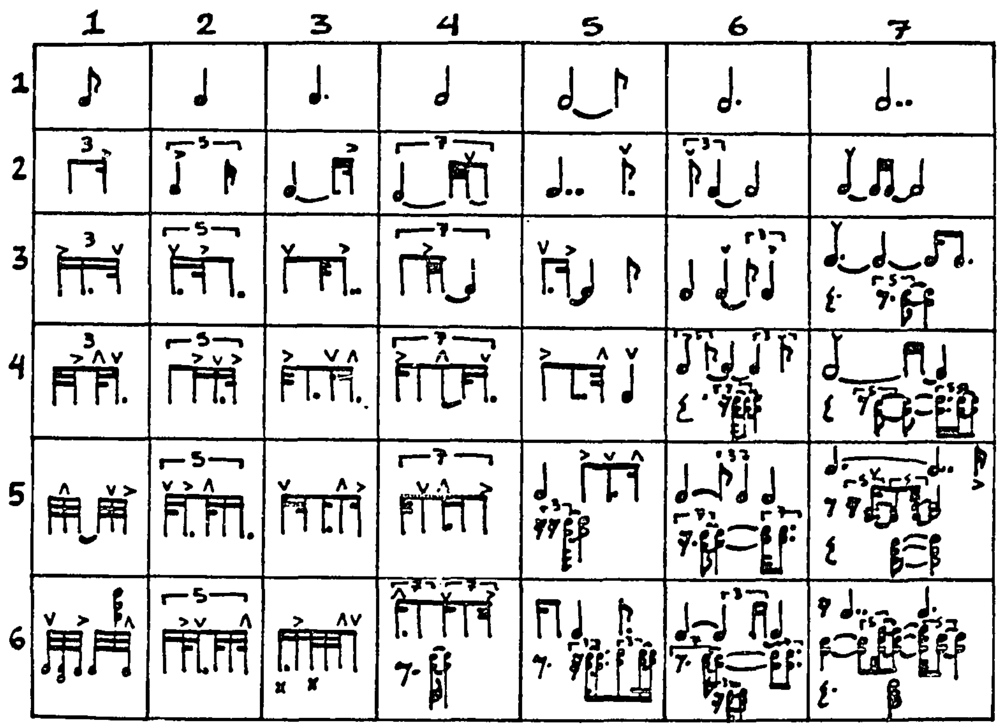
[11]Interior rhythmic alterations within many blocks transferred from rhythm matrix #2 to columns of the Final Rhythm Matrix may be noticed in Example 1, including those blocks pointed out by darkened squares. A discussion of these rhythmic alterations in transference will be presented in detail later in Chapter II.

Example 2: Number matrix #1.

6	1	4	3	7	5	2
1	3	6	5	2	7	4
4	6	2	1	5	3	7
3	5	1	7	4	2	6
7	2	5	4	1	6	3
5	7	3	2	6	4	1

Example 3: Rhythm matrix #1.



The interior of each block across the top row of the rhythm matrix of Example 3 contains a single note value. Each block's interior note value is one eighth note longer than the interior of the preceding block, starting with an eighth note in block one of row one. The interior of each block in every column contains composed rhythmic subdivisions which equal in duration the note value in the block at the top of the column. The number of durations in each rhythmic subdivision are increased by one duration in each descending block. Thus in row 2 there are 2 durations in each block; in row 3 there are 3 durations in each block, etc.

Pitch enrichment processes

Roman numeral matrix #1, associated with the number and rhythm matrices shown in Examples 2 and 3, was employed in determining the number of pitches to be added to the upper voice of certain rhythmic blocks in rhythm matrix #1.

Example 4: Roman numeral matrix #1.

(Row)							
(1)							
(2)	I^2	I	I	I	I	I	I
(3)	I^3	I	I	I	I	I	I^2
(4)	I^2	I	I	II^3	I	I^2	II
(5)	III^3	I^2	II^2	I	I	II	I^3
(6)	II^2	III	I^3	I	III^2	II^3	I^2

This matrix has the appearance of a 5 x 7 format, but subsequent analysis shows it to be part of a 6 x 7 matrix with the top row not written out. The first written row in the sketches aligns evenly with the second row of the number matrix. The omitted row, if written out, would consist solely of the Roman numeral I without an exponent, placed in the seven corresponding positions. Roman numerals in Example 4 denote the number of voices present in the block in the rhythm matrix to which the corresponding number in the number matrix refers. For example, the Roman numeral with exponent II^2 is found in column 3 of the fourth row of Example 4, which aligns with row 5 of Example 2. The number at row 5 column 3 in Example 2 is 5. Using the number 5 to locate the block in the 5th column of the corresponding row in Example 3, which is the function of this number before rearranging the located block's chronological position when transferred to the Final Rhythm Matrix, one obtains a block with 2 voices. Thus the Roman numeral II indicates the presence of two voices in the block at row 5 column 5 of Example 3. The exponent indicates the number of pitches to be contained in each duration in the upper voice in the same block when it is transferred to the Final Rhythm Matrix. An exponent of 2 doubles the number of pitches by the addition of one pitch per duration; exponents of 3 add two pitches per duration. Stockhausen's omission of the top row of the Roman numeral matrix can now be seen to be logical, since without exponents the top row would have no function, altering nothing in the corresponding blocks of the rhythm matrix. For the same reason Stockhausen also omitted nonfunctional rows in subsequent Roman numeral matrices.

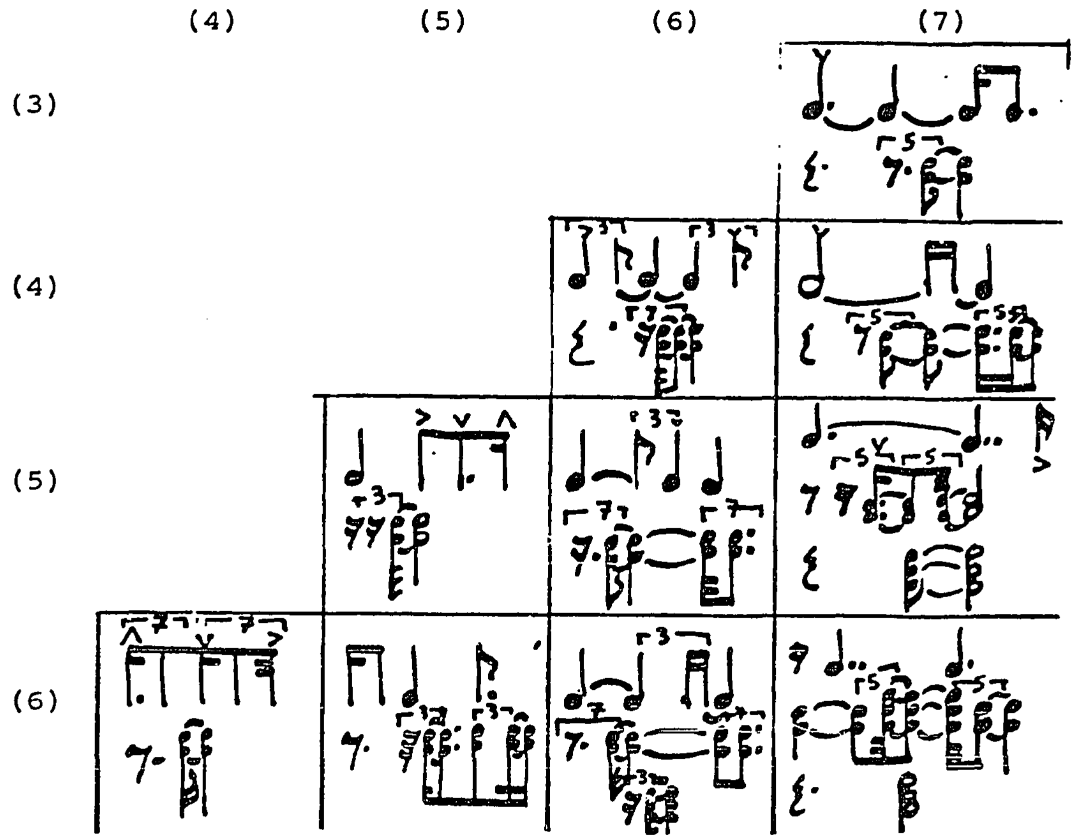
Example 5b illustrates how the durations in the block at row 2 column 1 of Example 3 appear in the Final Rhythm Matrix after being altered by the exponent (2) of the top row in Example 4. Example 5a is from the rhythm matrix of Example 3. Example 5b is from the sixth column of the Final Rhythm Matrix (Example 10). When a two or three voice texture is involved it is always the upper voice which is altered by the addition of pitches.

Example 5: Pitch enrichment.

a)  b) 

The pitch content of the middle and lower voices, in those blocks in the rhythm matrices which contain two or three voices, was also enriched by the addition of tones when Stockhausen composed the rhythm matrices.

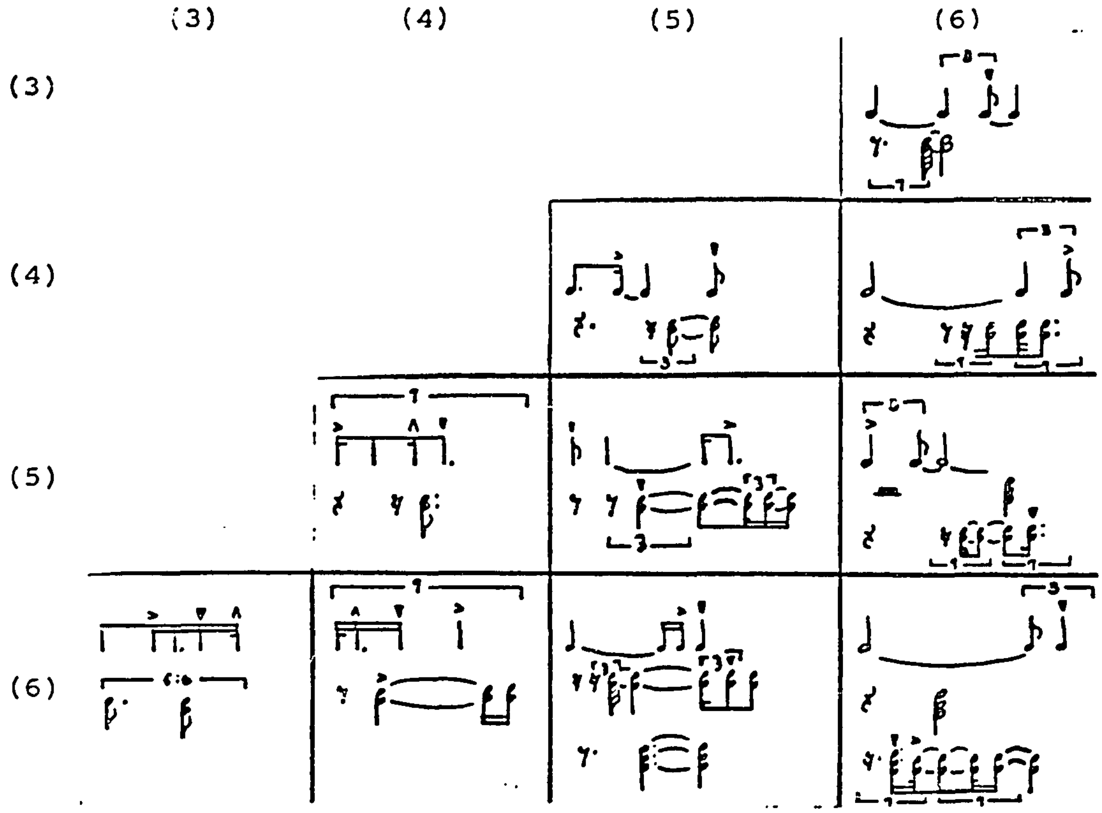
Example 6: Polyphonic blocks from rhythm matrix #1 (Example 3).



Polyphonic rhythm with up to three simultaneous rhythmic voices begins at row 6 column 4 of rhythm matrix #1, as shown in Example 6 above. Polyphony starts at row 6 column 3 of the 6 x 6 rhythm matrix #2, of which the lower right corner is shown in Example 7.

(For Example 7, see page 17)

Example 7: Polyphonic blocks from rhythm matrix #2, shown in Example 13.



After introducing polyphony in the single respective blocks of row 6 in Examples 6 and 7, Stockhausen added one additional polyphonic block to each successive column extending upwards from row 6. In Example 7 column 4 has two polyphonic blocks, column 5 has three, etc.

It was Stockhausen's choice to create polyphony at the point where the increasing number of single linear subdivisions of a fundamental duration would otherwise become cumbersome, both in the individual blocks and in later matrices to which these blocks were to be transferred. For rhythmic variety to be maintained in such a single voice texture an increasing number of durations with smaller values would become necessary. Within a series of several given durations the effect of increasing numbers of notes with smaller values is toward a textural/rhythmic homogeneity. But with polyphony Stockhausen was able to enrich the rhythmic texture of the blocks while simultaneously maintaining larger note values. As seen in the lower voices of the polyphonic blocks at rows 5 and 6, columns 5 and 6 in Example 3, and with many other voices throughout the same matrix, beginning with the first occurrence of this process at row 2 column 3, smaller note values are often tied to much larger ones in order to create longer durations of greater rhythmic variety.

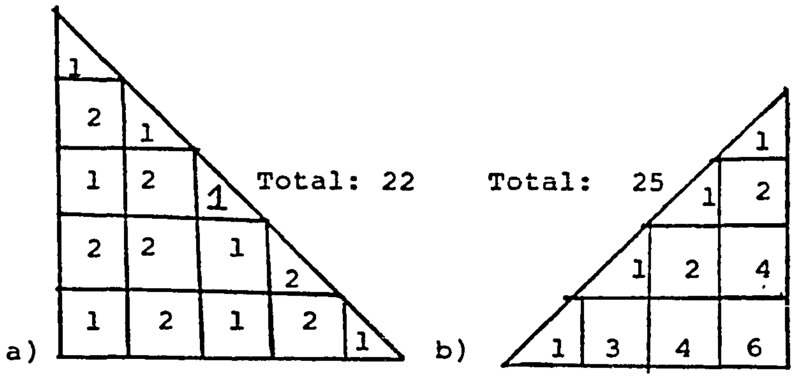
An examination of the rhythmic block located at row 6 column 5 of Example 3 reveals a characteristic not found within any other block in the matrices associated with Examples 6 and 7. If the total number of single durations in all voices in this block are added together, the resulting number is 7, not 6 as it should be if corresponding with the chronological number for the row (6) the block is located in. Yet this apparent discrepancy is reconciled by noting the fact that the fourth duration in the upper voice occurs simultaneously with the second duration in the lower voice. The combined effect of the two voices, when realized as pitch structures, will be that of six different attacks within the vertical dimension. This is consistent with the method associated with rhythmic formation in this block.

It was Stockhausen's decision for the lower voices to contain two or more pitches in those blocks within the rhythm matrices which exhibit polyphony. If the polyphony is two-voiced, as in Example 6, the lower voice always has two pitches per duration. Within this matrix blocks exhibiting three-voice polyphony always have middle voices with two pitches per duration and lower voices with three pitches per duration. In the second rhythm matrix shown in Example 7, within the blocks at rows 5 and 6, column 6, Stockhausen typically introduces greater rhythmic variety by reversing the vertical densities of the lower voices. Here the middle voices have three pitches per duration and the lower voices have two pitches.

This method of pitch enrichment within a polyphonic conception of initial rhythmic cells, in addition to the free control of register from tone to tone during the pitch composition of Klavierstück XI, allowed Stockhausen to have great control of texture and harmonic sonority in this work. Example 8 illustrates how the two separate pitch enrichment techniques associated with Examples 6 and 7 in one instance, and Example 4 in another, mirror one another in initial conceptions as to spatial dispositions of resultant pitches and relative numbers of pitches involved.

(For Example 8, see page 20)

Example 8: Pitch enrichment processes. (Examples 8a and 8b were constructed by the author of this paper, and are not to be found in Stockhausen's sketches.)



c) I^2

I^3 II^2

I^2 II^3 I^2

III^3 I^2 II^2 I^3

II^2 I^3 III^2 II^3 I^2

The bottom row of Example 8b represents the total number of pitches which were added to each block of row 6, columns 4 through 7, of the rhythm matrix on page 1 of Stockhausen's sketches (Example 3). Within each block durations with 3 pitches are considered having 2 pitches added by the pitch enrichment process; durations with 2 pitches are counted having 1 pitch added. Durations in this analysis are identical with attacks, whether they are single notes or two or more tied vertical sonorities. For example, the number 3 in the bottom row of Example 8b indicates 3 pitches added to the block at row 6 column 5 in Example 3. An examination of the lower voice in this block reveals 3 attacks (durations), each having 2 pitches. Three pitches, one for each separate attack, are therefore considered added by the pitch enrichment process.

Example 8a is an expression of the total number of pitches added to certain blocks of the same rhythm matrix via the process discussed in association with Example 4. Example 8c is Stockhausen's method of placing together only those Roman numerals from Example 4 which have exponents. He wrote this particular type of grouping just to the right of the Roman numeral matrix shown in Example 4. Although their dimensions are unequal, the triangular shape of Example 8a is a mirror of the triangle of Example 8b. If the pitch enrichment process of Example 8a also mirrored in spatial disposition that of Example 8b, this being implied by the contrasting shapes of the two triangles, the two processes could be understood as ways to separately enrich the pitch content of the right (Example 8b) and left (Example 8a) sides of the same rhythm matrix. However, the spatial disposition of the pitches within the rhythm matrix associated with Examples 8a and 8c is identical with the asymmetrical spatial disposition of the exponents in Example 4. Example 9 illustrates how the two pitch enrichment processes overlap one another when combined in the same 6 x 7 rhythm matrix.

(For Example 9, see page 22)

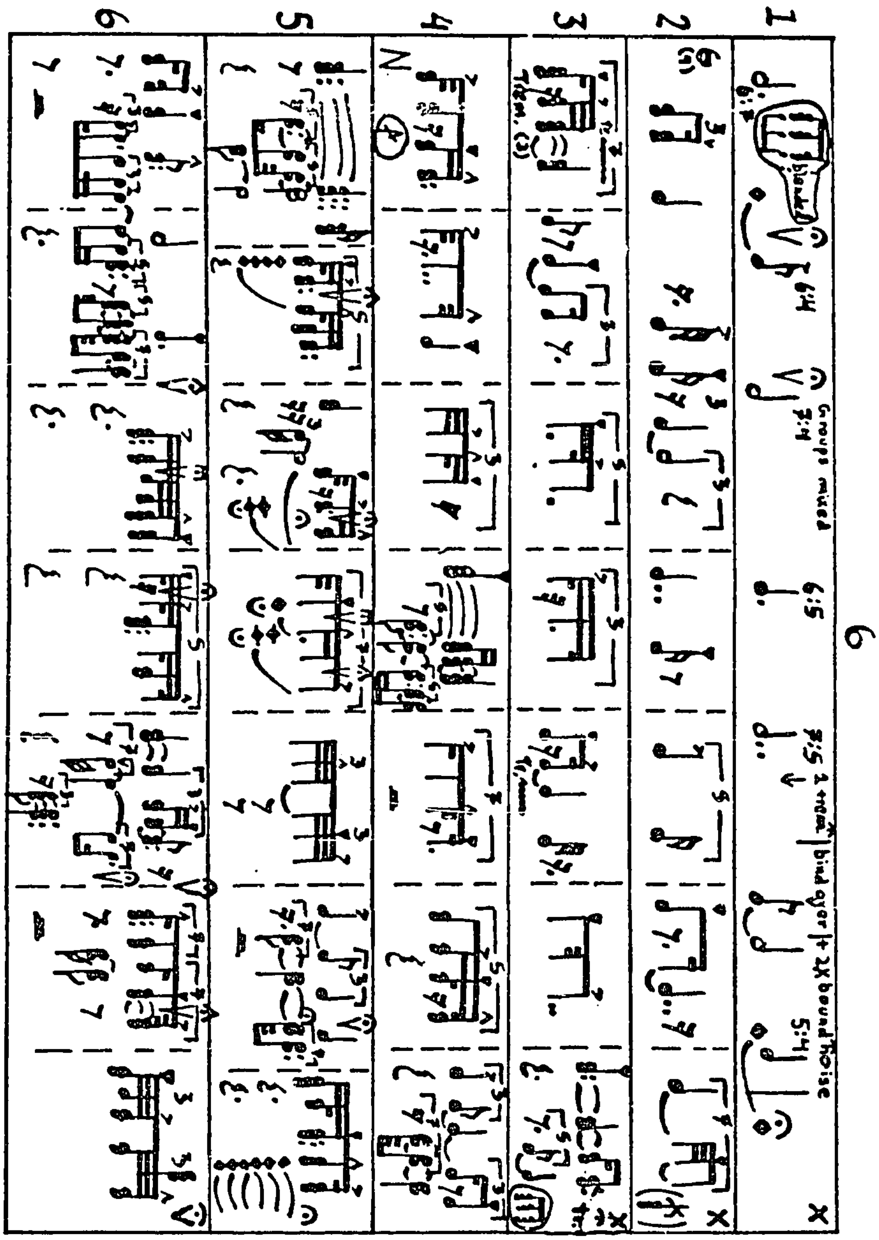
Example 9: Overlapping pitch enrichment processes of Examples 8a and 8c. (This example is not found in Stockhausen's sketches.)

6	1	4	3	7	5	2
1 (I^{2})	3 (I)	6 (I)	5 (I)	2 (I)	7 (I)	4 (I)
4 (I^{3})	6 (I)	2 (I)	1 (I)	5 (I)	3 (I)	7(II^{2})
3 (I^{2})	5 (I)	1 (I)	7(II^{3})	4 (I)	2 (I^{2})	6 (II)
7(III^{3})	2 (I^{2})	5 (II^{2})	4 (I)	1 (I)	6 (II)	3 (I^{3})
5(II^{2})	7(III)	3 (I^{3})	2 (I)	6(III^{2})	4(II^{3})	1 (I^{2})

The Arabic numerals in Example 9 represent the number matrix of Example 2. Roman numerals with and without exponents are those from Example 4, placed in the matrix blocks which correspond to identical blocks of the number matrix. Blocks with notes having stems and flags represent that portion of the same matrix affected by the pitch enrichment process of Example 6. In those blocks where the two processes overlap, such as row 6, columns 5-7, there is no interference between the two enrichment processes because one (Example 8a) only affects the upper voice, and the other (Example 8b) only affects the middle and lower voices.

The number matrix in Example 9 was employed in reordering the block within rhythm matrix #1 (Example 3). The result of this reordering process is column 6 of the Final Rhythm Matrix.

Example 10: Column 6 of the Final Rhythm Matrix.



The chronological position of rhythmic blocks in the successive rows of rhythm matrix #1 (Example 3) were matched with numbers in the corresponding rows of number matrix #1 (Example 2). Number matrix numbers were matched with blocks in the rhythm matrix having columns with numerical values identical with the number matrix numbers. When Stockhausen transferred the entire rhythm matrix of Example 3 to a single column of the Final Rhythm Matrix, shown in Example 10, the chronological arrangement of the rhythmic blocks was reordered to match the chronological arrangement of the number matrix numbers. This process is illustrated in Example 1.

Random pitch deletions

A comparison of the rhythmic cell at row 5 column 7 in Example 3, and the corresponding beginning rhythmic cell in row 5 of Example 10, reveals the tied duration of the lower voice with 2 fewer pitches in the Final Rhythm Matrix. The lower voice of the block at row 5 column 5 of Example 3 also has 1 missing pitch when transferred to the Final Rhythm Matrix. The 3 durations of the lower voice of the block at row 6 column 5 in Example 3 are each minus 1 pitch in the Final Rhythm Matrix. The block at row 6 column 6 is minus 1 pitch in each duration of the middle voice when transferred.[12]

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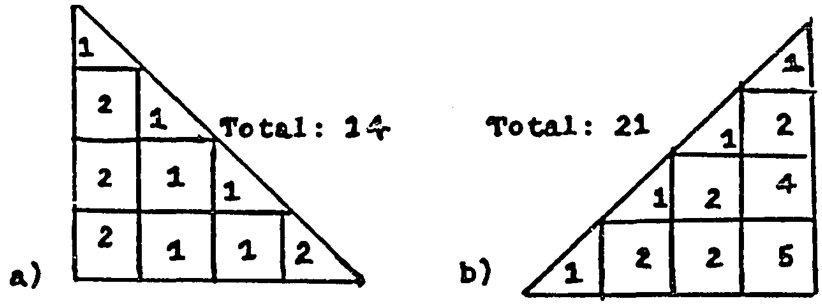
[12]Since tied notes sustain an initial pitch, every untied note head in any given voice of these rhythmic cells generates an additional pitch. Therefore, the absence of any duration, whether its form is that of a single note or a tied series of notes, will eliminate one pitch when the rhythms are translated into pitches. The method by which Stockhausen translated rhythmic proportions into pitch relationships will be explained in Chapter 3.

As there is no additional process apparent in the sketches by which Stockhausen deleted tones when transferring rhythmic cells from one matrix to another, this was obviously a decision he made randomly during the act of writing out the Final Rhythm Matrix. Other contexts, to be discussed later in this chapter, also arose in which Stockhausen reconsidered the relative pitch densities of the various rhythmic cells, as well as their interior rhythmic structures, while transferring them from one matrix to another.

In addition to thinning out the pitch density at those points in the piece where Stockhausen translated these rhythmic cells into pitches, these pitch deletions create a shift in the total numerical balance of pitches employed in the two separate processes. Since a total of eight pitches are deleted from the process of Example 8b, the total number of pitches added to the Final Rhythm Matrix falls from 25 to 17. There are no deletions of pitches associated with the process of Example 8a, so that number remains 22. Thus the pitch enrichment process of Example 8a contains the majority of pitches employed in the Final Rhythm Matrix.

(For Example 11, see page 26)

Example 11: Pitch enrichment processes associated with rhythm matrix #2. (This example is not found in Stockhausen's sketches.)



c)

I^2

$\mathrm{I}^3 \quad \mathrm{II}^2$

$\mathrm{II}^3 \quad \mathrm{I}^2 \quad \mathrm{III}^2$

$\mathrm{I}^3 \quad \mathrm{II}^2 \quad \mathrm{III}^2 \quad \mathrm{II}^3$

(Example 11c is from Roman numeral matrix #2, shown in Example #1.)

The most distinguishing feature of Example 11 is the greater difference between the total number of pitches the two processes add to the rhythm matrix, since they are both identical in conception to the pitch enrichment processes of Example 8a and 8b. The Roman numeral with exponent process of Example 11a and 11c contains only two-thirds the number of pitches found in the process of Example 11b. An examination of column 5 of the Final Rhythm Matrix, shown in Example 14, reveals one less pitch deleted from the Example 11b process than was deleted from the corresponding process of Example 8b. The result is an equal number of pitches (14) from both processes added to rhythmic cells when written out in the Final Rhythm Matrix.

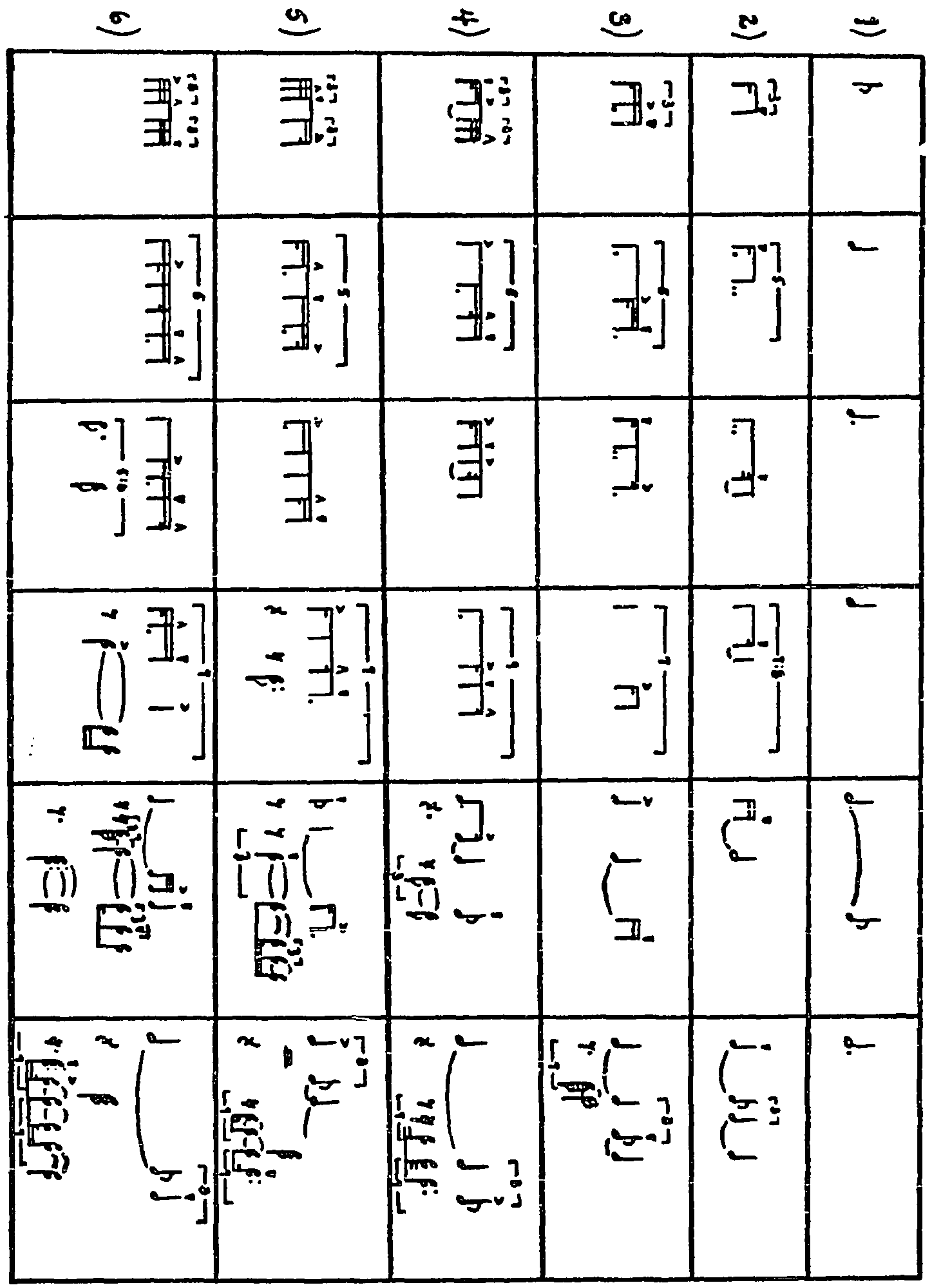
Pitch deletions will now be discussed in conjunction with the 5th column of the Final Rhythm Matrix (Example 14), and its derivative number and rhythm matrices, shown in Examples 12 and 13 respectively. (For Example 12, see page 27)

Example 12: Number matrix #2.

5	3	4	1	6	2
3	1	2	5	4	6
1	5	6	3	2	4
4	2	3	6	5	1
6	4	5	2	1	3
2	6	1	4	3	5

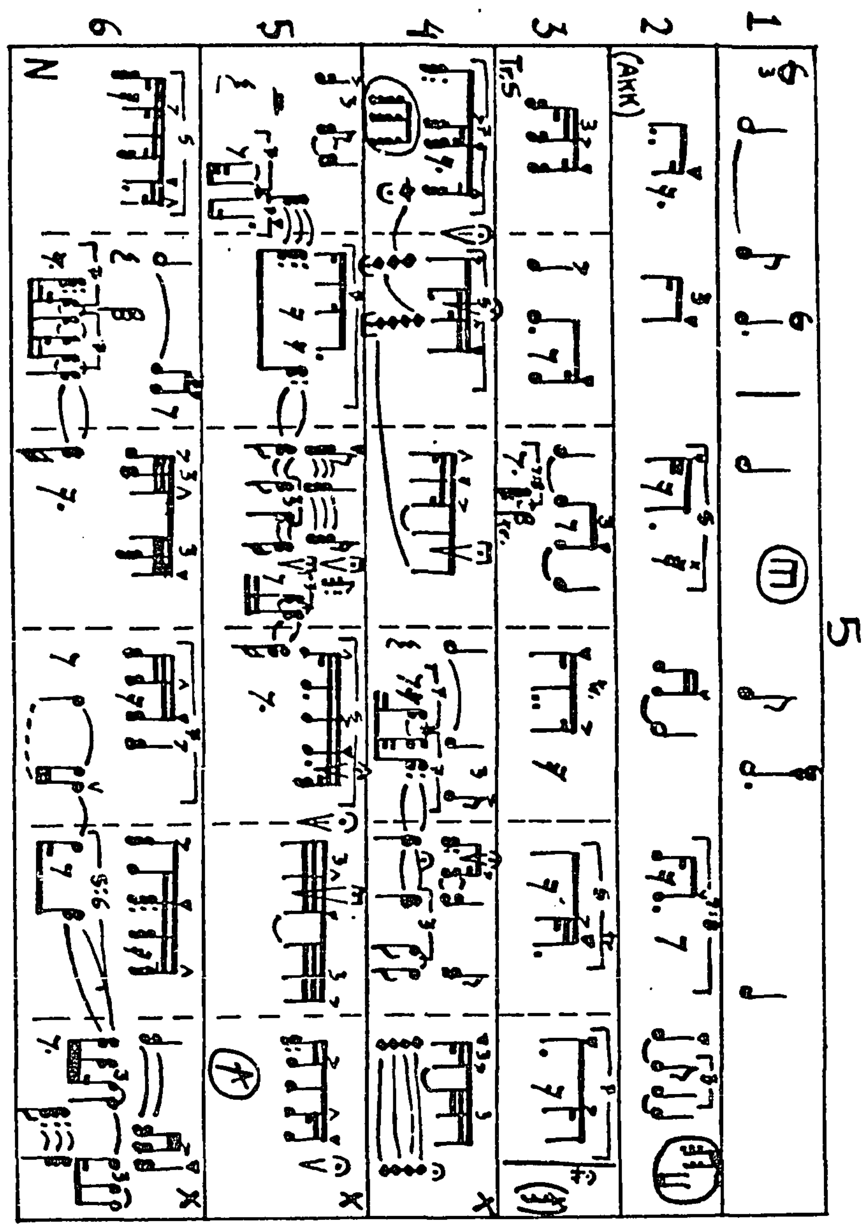
(For Example 13, see page 28)

Example 13: Rhythm matrix #2.



(For Example 14, see page 29)

Example 14: Column 5 of the Final Rhythm Matrix.



The number matrix of Example 12 can be used to locate the position in the Final Rhythm Matrix of the block originating at row 4 column 5 of Example 13. The number 5 is placed in row 4 column 5 in the number matrix of Example 12.

This means the rhythmic cell at row 4 column 5 of Example 13 will be the fifth rhythmic cell of row 4 of Example 14. When this rhythmic cell is examined in Example 14, it becomes apparent that the lower voice has one less pitch than its counterpart in Example 13. This pitch deletion obviously occurred when Stockhausen transferred the rhythmic cell to the Final Rhythm Matrix, and was the result of a decision made independently of any process visible in the sketches. Other rhythmic cells with fewer pitches can be located by the above method, utilizing the number matrix of Example 12. The origins of these additional random pitch deletions can be found at row 5 column 6, and at row 6 columns 4 and 5 of Example 13. Matching these rhythmic cells with their counterparts in Example 14 will reveal the specific pitch deletions.

Example 15: Number matrix #3 and Roman numeral matrix #3, both associated with Rhythm matrix #3.

a)

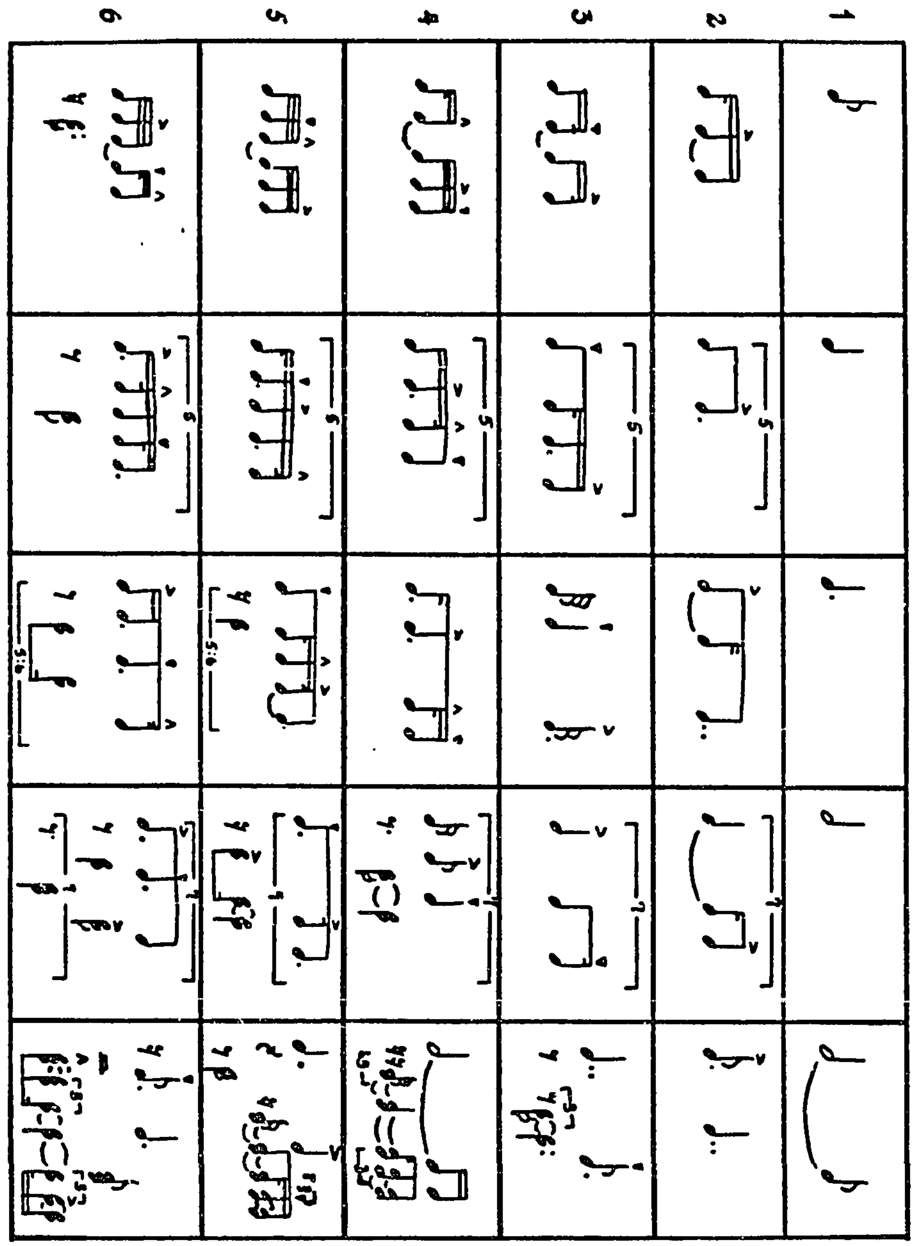
4	5	2	1	3
1	4	5	3	2
3	1	2	5	4
2	5	1	4	3
4	2	3	1	5
5	3	4	2	1

b)

I	I	I	II	I
I(2)	II	I	II	I
II	I	II	I	III
III	I	III	II	II

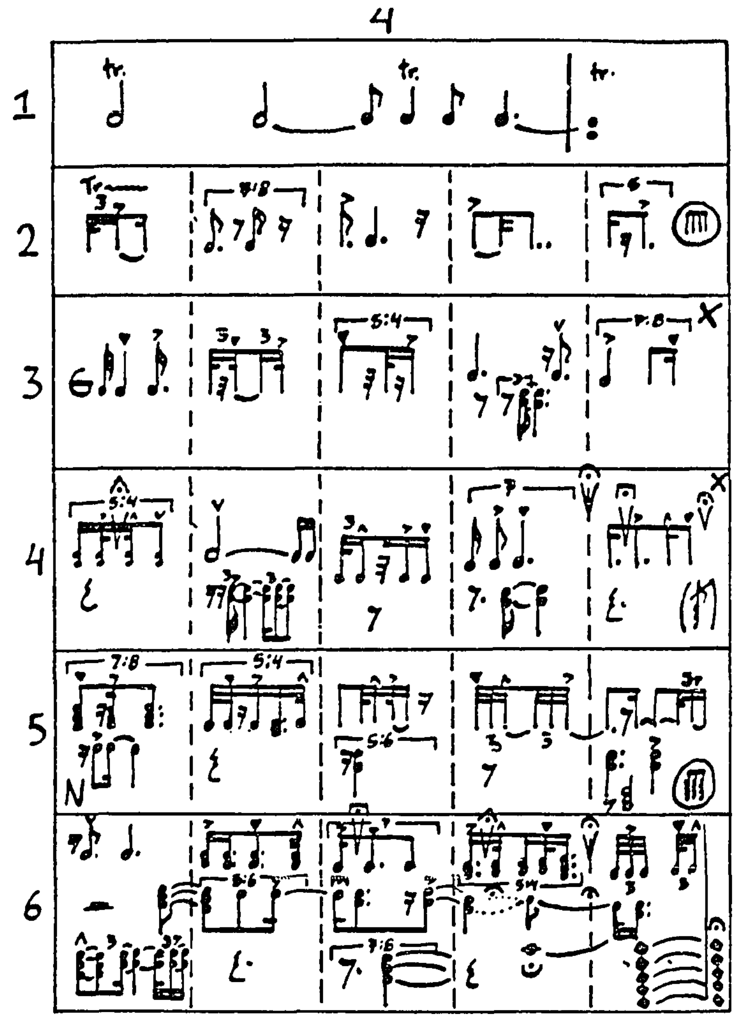
(For Example 16, see page 31)

Example 16: Rhythm matrix #3.



(For Example 17, see page 32)

Example 17: Column 4 of the Final Rhythm Matrix.



<u>Pitch enrichment in conjunction with switched pitch characteristics between voices</u>

Stockhausen changed the pitch enrichment process for the upper voice in column 4 of the Final Rhythm Matrix, shown in Example 17. The Roman numeral with exponent process is employed only once within this rhythm matrix. The exponent (2) in Example 15b is in parenthesis because Stockhausen put no exponents in this matrix in the sketches, but its inclusion is proper when considering the exponent process involved at this one point.[13] The upper voice of the first rhythmic cell of row 4 contains 2 pitches, reflecting the Roman numeral with exponent process. This can be verified by examining the rhythm matrix of Example 16 in conjunction with the associated matrices of Example 15.

Stockhausen then adopted a new method of enriching the upper voices of polyphonic blocks by having them switch pitch characteristics with the middle voice. The beginning rhythmic cell of row 5 in Example 17 has 3 pitches per duration in the upper voice. In the rhythm matrix from which Example 17 was formed (Example 16), the corresponding rhythmic cell at row 5 column 4 has only one pitch per duration in the upper voice and 2 pitches per duration in the lower voice. Stockhausen not only switched pitch characteristics between voices at this point, but also added an additional third pitch per duration to the upper voice after the switch.

The second rhythmic cell of row 5 in Example 17 occupies the same chronological position in relation to the other rhythmic cells that it had in Example 16. This block in Example 16, at row 5 column 2, is minus rhythmic polyphony for a reason which will be discussed later. There is therefore no lower voice from which to transfer pitches to the upper voice. Yet Stockhausen added one pitch to the fourth duration of this

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[13]It is typical of Stockhausen to leave a remnant of one process which thus becomes conjunct with another process in the same matrix. The significance of other similar instances will be discussed later in this paper.

block when he transferred it to the rhythm matrix of Example 17! When considering the chronological placement of this rhythmic cell, the one added pitch within it seems to serve as a reminder that this is the very point at which a new process was exchanged for a previous one. Stockhausen considers such additions to a musical fabric, which is already determined by previous processes and is also complete within itself, to be a form of ornamentation. His personal name for this type of ornamentation is "little flowers".[14]

Other rhythmic cells with similar added pitches occur only in rows 5 and 6 of the Final Rhythm Matrix. They are easily identified in the fifth and sixth rows of Examples 10, 14, 17, 26, and 28, because the added pitches are only found among groups of durations which share beams in common with one another. It is interesting to observe an added pitch at the fourth duration of the second rhythmic cell of row 5 in Example 10. This point is identical chronologically, regarding both rhythmic cell placement and its interior duration, to an added duration in row 5 of Example 17.

When transferring the rhythmic cell at row 5 column 6 of Example 16, to row 5 of Example 17, Stockhausen switched the relative positions of the two upper voices. This block is the last rhythmic cell of row 5 in Example 17. The pitches per duration of these two voices were also switched, so the upper voice in Example 17 also has one pitch per duration, and the middle voice has two pitches per duration.

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[14]From a lecture by Stockhausen at Denver University, August, 1980.

Suspensions

An additional characteristic exhibited by the last rhythmic cell of row 5 in Example 17 is the tied note in the upper voice connecting it to the last duration of the rhythmic cell to the left, which occupies the fourth chronological position for rhythmic cells in this row. This tied note in the fifth rhythmic cell represents a suspension of the last duration in the fourth rhythmic cell, since there is no corresponding duration at that same point in the originating middle voice of the rhythmic cell at row 5 column 6 in Example 16.

This method of linking adjacent rhythmic cells by means of similar suspensions can also be found between every rhythmic cell in row 6 of Example 17, thus linking them all together. This is the only such rhythmic block in the entire Final Rhythm Matrix. Suspensions can also be found between various rhythmic cells in Example 10 (row 6, rhythmic cells 1-3) and Example 14 (rows 4-6), as well as in columns 1-3 of the Final Rhythm Matrix.

The second and fourth rhythmic cells of row 4 in Example 17 have two pitches per duration in their lower voices as a result of the pitch enrichment process identical with Examples 8b and 11b. The rhythmic polyphony in Example 16 extends from row 6 column 1, to include rows 3 through 6 in column 5. In order to make this pitch enrichment process avoid affecting exactly half of the blocks in Example 16, Stockhausen decided to restrain the previous rate of expansion of rhythmic polyphony found in Examples 3 and 13. Starting with the block at row 6 column 1 in Example 16, rhythmic polyphony is restrained to a single block in row 6 column 2. Column 3 has rhythmic polyphony in blocks at rows 5 and 6; column 4 likewise at rows 4-6. Column 5 has rhythmic polyphony from rows 3-6. The total number of blocks in Example 16 that exhibit rhythmic polyphony is 11, four less than half the blocks in this matrix.

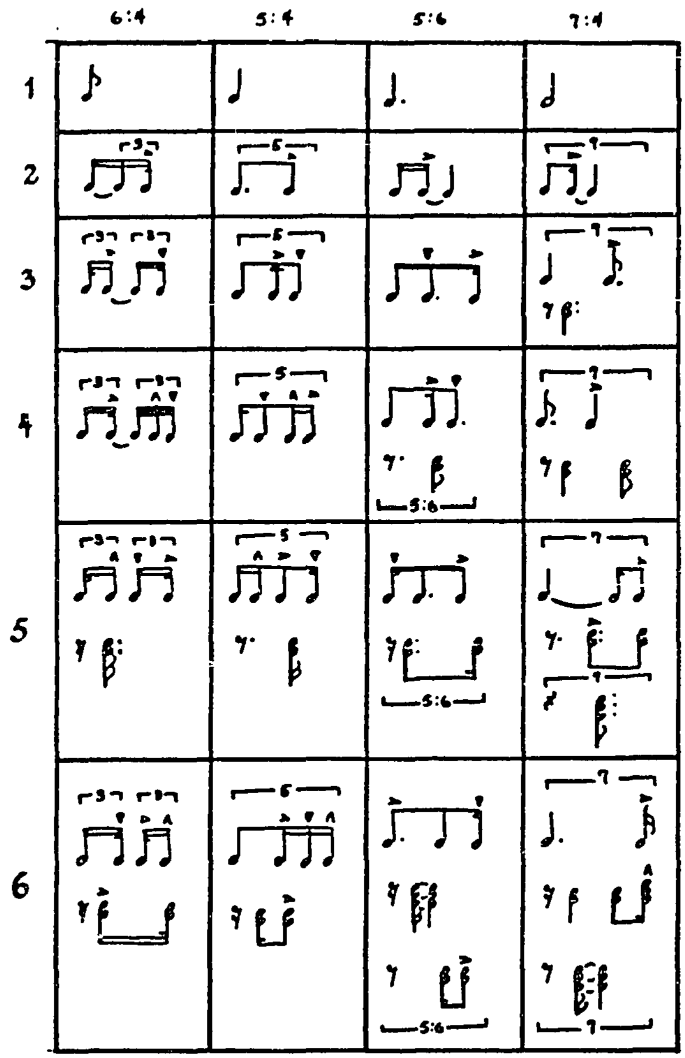
Example 18: Number matrices #'s 4-6.

a) #4.	b) #5.	c) #6.
3 4 2 1	3 4 2 1	4 1 3 2
4 1 3 2	4 1 3 2	1 2 4 3
1 2 4 3	2 3 1 4	3 4 2 1
2 3 1 4	1 4 2 3	2 3 1 4
3 2 4 1	3 2 4 1	3 2 4 1
4 1 3 2	4 3 1 2	4 3 1 2

Example 18a is the number matrix Stockhausen used to reorder the rhythmic cells of Example 19a. Examples 18b and 18c were alternate number matrices Stockhausen considered and then rejected. Example 18b coincides with the reordering of Example 19a which constitutes Example 19b.

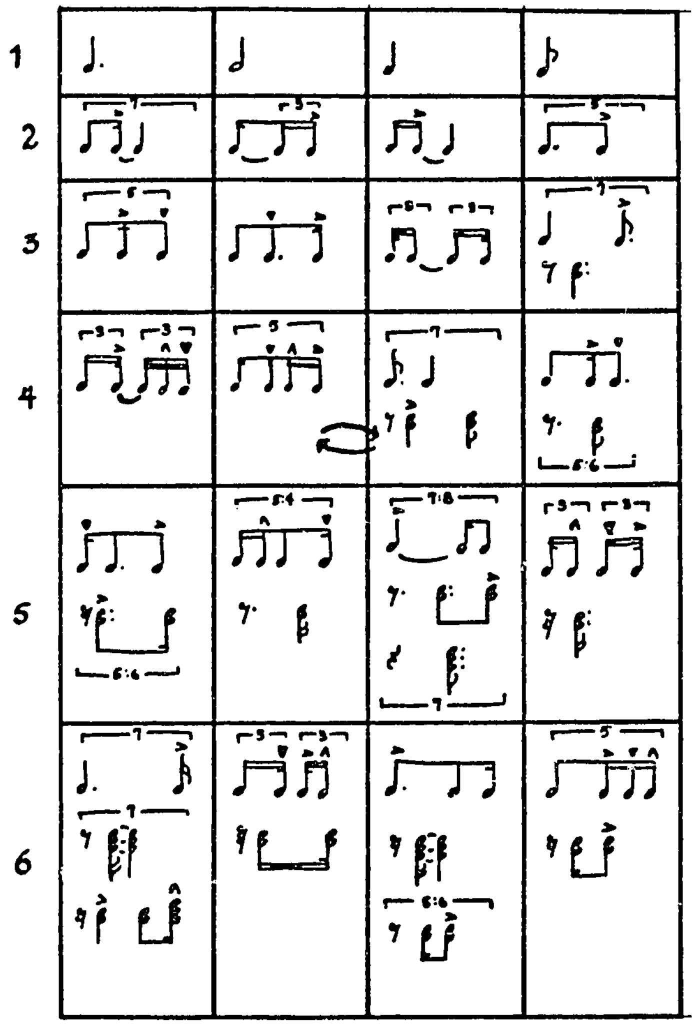
(For Examples 19a-c, see pages 37-39)

Example 19a: Rhythm matrix #4.



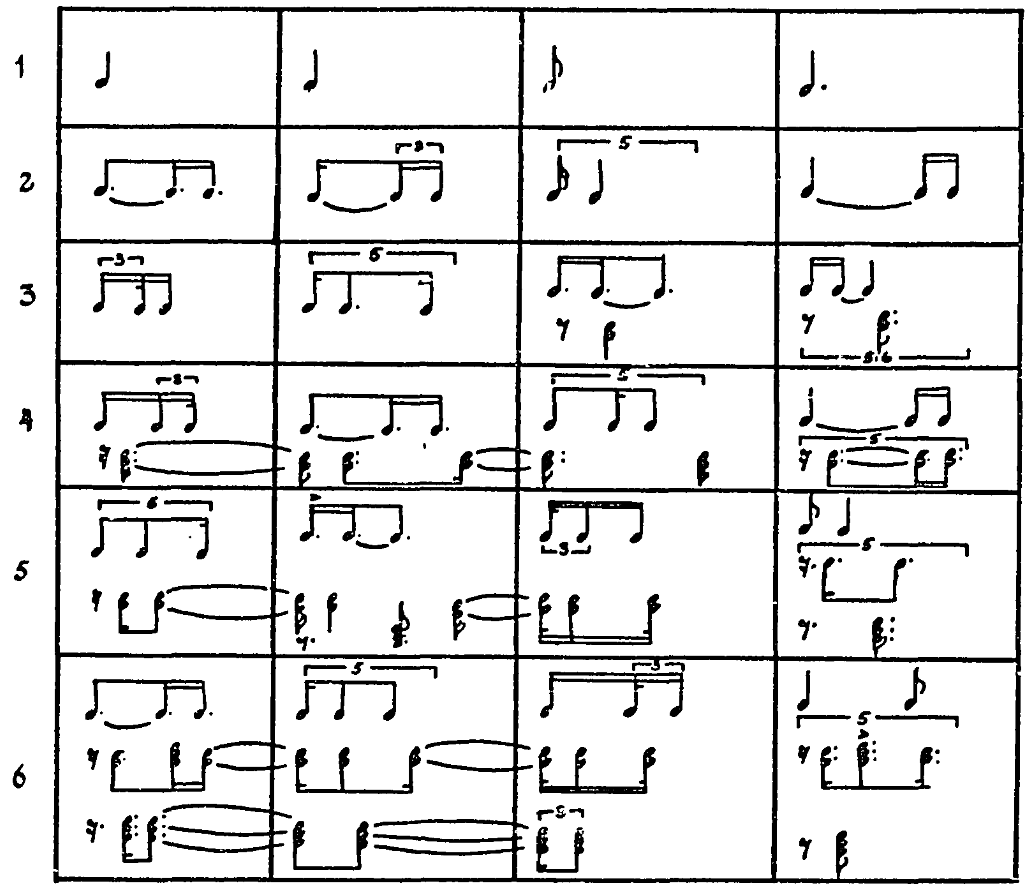
(For Example 19b, see page 38)

Example 19b: Rhythm matrix #5.



(For Example 19c, see page 39)

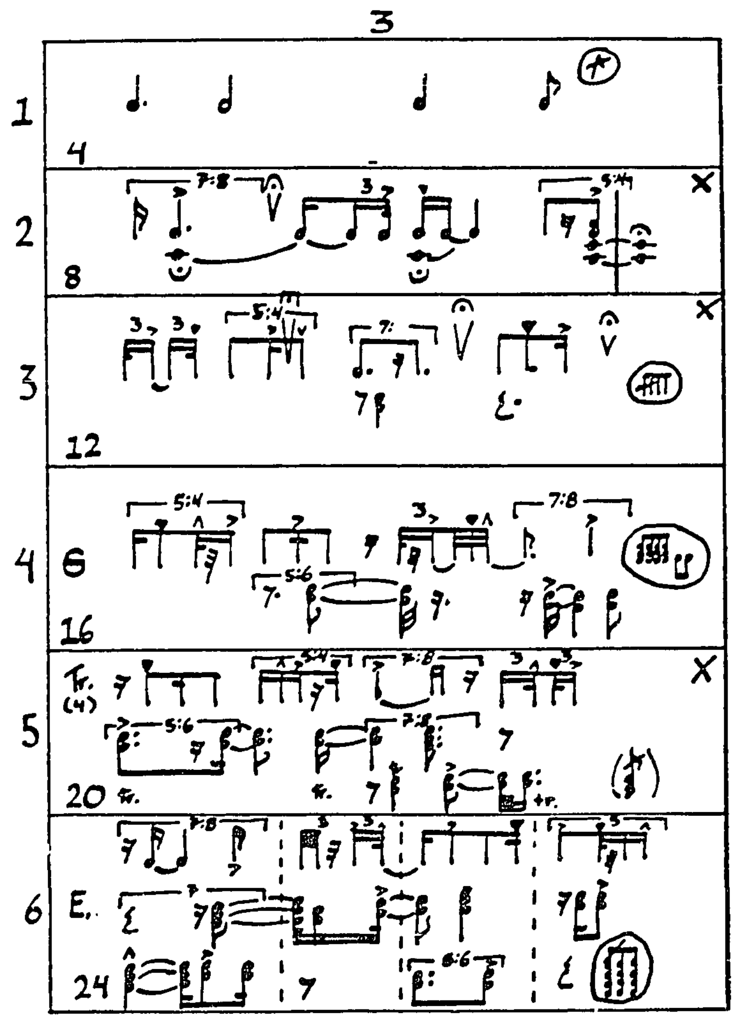
Example 19c: Rhythm matrix #6.



A different scheme for the expansion of rhythmic polyphony was employed in Example 19a, which contains the rhythm matrix Stockhausen employed in constructing column 3 of the Final Rhythm Matrix: two blocks with rhythmic polyphony in the bottom 2 rows of column 1, two similar blocks in the bottom 2 rows of column 2, three similar blocks in column 3, and four similar blocks in column 4. Like Example 16, the total number of blocks having rhythmic polyphony is 11, but the total number of blocks in this matrix of 6 x 4 dimensions is only 24. This is six less than the 30 blocks contained in Example 16. Thus starting with column 6 of the Final Rhythm Matrix and going back through column 1, the proportion of the number of rhythmic cells having rhythmic polyphony increases relative to the total number of rhythmic cells contained in each succeeding column.

The rhythm matrix of Example 19c reveals that Stockhausen was considering using a rhythmic scheme for column 3 of the Final Rhythm Matirx which had the _same_ duration values in each descending block. The suspensions linking various rhythmic cells from one column to the next also indicate that he was composing the third column of the Final Rhythm Matrix at this point, rather than creating a rhythm matrix he intended to reorder later. A distinguishing feature of this example is the fact that column 4 contains the 5:6 rhythm, columns 3 and 2 have 5:4 rhythms, and column 1 has a 5:4 rhythm. The rhythm matrix used for column 3 of the Final Rhythm Matrix, shown in Example 19a, has 7:4 rhythms in the fourth column, 5:6 rhythms in the third column, 5:4 rhythms in the second column, and 6:4 rhythms in the first column. Stockhausen obviously decided that column 3 of the Final Rhythm Matrix should be more rhythmically complex than he originally intended in Example 19c. This decision is fundamental to the general level of rhythmic complexity of parts of _Klavierstück XI_. The chains of rhythmic cells in the Final Rhythm Matrix were used to create the pitch content, excepting grace notes, of the separate fragments which constitute the published form of the piece. Three of the blocks of column 3 of the Final Rhythm Matrix were used in composing three of the nineteen music fragments of the piece. These blocks, as well as the other 16 blocks from the Final Rhythm Matrix used in composing the piece, will be identified after the discussion of the formation of the Final Rhythm Matrix is concluded.

Example 20: Column 3 of the Final Rhythm Matrix.



<u>Durations shortened by the inclusion of rests</u>

In addition to the suspension of certain durations from one rhythmic cell to the beginning of the succeeding rhythmic cell, several rhythmic cells in Example 20 also exhibit interior alterations when compared with their counterparts in the rhythm matrix of Example 19a. Throughout the Final Rhythm Matrix durations within certain rhythmic cells are shortened by the inclusion of rests. The durations involved in this process always shrink in exact proportion to the size of the rest being interjected. This makes the total duration of the rhythmic cells always stay the same, no matter whether the rests are interjected before, in between, or after the affected durations. This process of further sophisticating rhythmic cells when transferring them from one matrix to another can be viewed as a fundamental characteristic of Stockhausen's compositional method for Klavierstück XI.

A comparison of the rhythmic cell at row 2 column 2 of Example 19a, with the fourth rhythmic cell of row 2 in Example 20, reveals a rhythmic alteration. By the inclusion of an interior sixteenth rest, the dotted eighth note at the beginning of the rhythmic cell in Example 19a is shortened to an eighth note in Example 20. The first rhythmic cell of row 2 in Example 20 is also altered, but in a different manner: The eighth note value of the first duration is halved and the resulting sixteenth value is transferred to the second duration, with all other characteristics (the accent over the second duration and the 7:8 rhythm) remaining the same. A sixteenth rest shortens the length of the second duration of the top voice of the third rhythmic cell of row 3. Silence is thus interjected between durations which were formerly connected. The rhythmic cell at the beginning of row 4 has its third duration shortened by the inclusion of a thirty-second rest. This further separates this duration from the fourth duration in the same rhythmic cell. A similar thirty-second rest can be observed in the interior of the third rhythmic cell in this block.

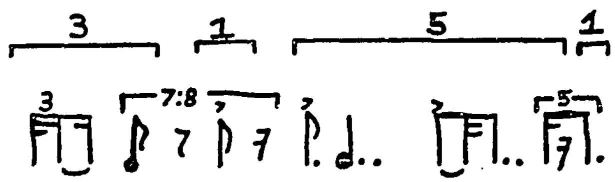
The rhythmic alterations of row 5 in Example 20 are more extensive than those of the upper blocks. Both voices of the first rhythmic cell have been transformed by switching certain rhythmic characteristics with one another. The sixteenth rest before the first duration in the upper voice preceded the first duration of the lower voice in row 5 column 3 of Example 19a. Since the first duration of this same cell in Example 19a is also a sixteenth note, it would be eliminated, not shortened, by the inclusion of a sixteenth rest at the same point. So Stockhausen changed the entire rhythmic scheme of the upper voice from a sixteenth followed by a dotted eighth and an eighth (Example 19a), to an eighth followed by a sixteenth and another eighth (Example 20). The alteration of the lower voice is less extensive. The same time values remain, with the first duration merely shifted to the left by one sixteenth and a rest of the same dimension placed immediately after it. This separates the two durations of this voice by the inclusion of a sixteenth rest between them. The final duration of the lower voice is also expanded by suspension to the beginning of the following rhythmic cell. The resulting tied dotted eighth duration occupies the space held by the dotted eighth rest in row 5 column 2 of Example 19a. A sixteenth rest is inserted within the top voice of the second rhythmic cell of this same block in Example 20, and a dotted rhythm transforms the first two durations of the top voice of the fourth rhythmic cell. The top voice of the third rhythmic cell of row 5 of Example 20 has been shortened in its last duration by one sixteenth rest. The first duration of the bottom voice, which was the middle voice in Example 19a, has been changed from a dotted eighth to a quarter, with a corresponding change in the time value of the preceding rest.

A comparison of the first rhythmic cell of row 6 in Example 20 with its counterpart at row 6 column 4 of Example 19a, reveals alterations in all three voices. These alterations have been achieved by switching various characteristics of the three voices in both examples with one another. The middle voice in Example 19a is switched to the lower voice in Example 20. The sixteenth rest of the middle voice in Example 19a is placed at the beginning of the upper voice of Example 20, thus altering the length of the first duration in this same voice. To the same extent as the top voice's duration is shortened in Example 20, the bottom voice is lengthened (a sixteenth) when compared with its counterpart in Example 19a (the middle voice). The final duration of the middle voice in Example 19a has 3 note heads. This characteristic is switched to the beginning of the rhythmic cell when transferred to the bottom voice in Example 20. The bottom voice of Example 19a is altered internally when transferred to the middle voice in Example 20. Since the time values of the respective rest and duration of the lower voice are simply reversed with one another, no characteristics are switched with any other voices. The other rhythmic cells in this block exhibit less extensive alterations.

The rhythmic cell at row 2 column 4 of rhythm matrix #3, shown in Example 16, exhibits an extreme transformation when its counterpart is examined in the second rhythmic cell of row 2 column 4 of the Final Rhythm Matrix (Example 17). The initial duration of this cell is shortened, from a quarter tied to a sixteenth, to an untied dotted eighth. An eighth rest, equal to the degree to which the initial duration has been shortened, is interjected between the two durations of this cell. The final duration of this rhythmic cell is also shortened, from an eighth to a sixteenth, followed by a rest of the same value.

The rhythmic transformation of the cell at row 2 column 4 of Example 16 is modest in comparison with the second cell of this block. Basically, it consists of the inclusion of a sixteenth rest at the end of the cell. This reduces the double-dotted quarter note of Example 16 to merely a dotted quarter in Example 17. The significance of the sixteenth rest in the third rhythmic cell of this block, however, greatly increases when the surrounding rhythmic cells are considered simultaneously. If there were no sixteenth rest at the end of the third rhythmic cell, there would also be no break in the continuous succession of durations in this block until the interjected sixteenth rest within the final rhythmic cell. If Stockhausen had chosen this block for the rhythmic structure of one of the nineteen fragments constituting Klavierstück XI, the textural effect without the above mentioned sixteenth rest would be one of four groups[15] of notes separated by rests. This hypothetical version of row 2 of the fourth column of the Final Rhythm Matrix is shown in Example 21.

Example 21: Row 2 of Column 4 of the Final Rhythm Matrix with the double-dotted quarter note, originating in row 2 column 5 of Example 16, unaltered.



The initial group would have 3 durations, as illustrated by the number 3 above the brackets indicating the first 3 durations in Example 21. The second group would contain only 1 duration, followed by a group of 5 durations, and finally another single duration. Due to its greater length the central group of five durations would be prominent and contrast with the much smaller groups at either end of the fragment. The sixteenth rest introduced at the end of the third rhythmic cell thus insures a more homogenous rhythmic texture throughout the entire block while also introducing rhythmic variety at the same time.

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[15]The significance of this type of "duration-group" is discussed later in this paper, beginning on page 49. See Stockhausen, K., "...How Time Passes..." Die Reihe 3 (1959), p. 28 for similar concepts concerning the introduction of ties and rests into a previously conceived rhythmic fabric. In Stockhausen's Example 19 rests replace durations rather than shorten them, and tied durations are expressed as longer single note values rather than being several smaller tied notes.

Example 22: Rhythmic cells in the Final Rhythm Matrix having rhythmic transformations in transference from their original matrices. (This example is not found in Stockhausen's sketches.)

	1	2	3	4	5	6
1						
2	1	12	14	235	135	2346
3	1	23	3	234	123456	1245
4	1	12	134	123	13	123567
5	1	12	1234	125	35	37
6	1	123	1234	3	1245	23456

The numbers in the various blocks of Example 22 identify the rhythmic cells in the Final Rhythm Matrix which have been transformed when transferred from their originating rhythm matrices. For example, the numbers 1, 3, and 5 in the block at row 2 column 5 indicate that rhythmic transformation has occurred in the first, third, and fifth rhythmic cells of the block at row 2 column 5 of the Final Rhythm Matrix. As has been previously explained, each column of the Final Rhythm Matrix has a separate originating rhythm matrix, reordered by a corresponding number matrix. Rhythm matrix #2 is the originating matrix for the fifth column of the Final Rhythm Matrix. Therefore, using the number matrices, in conjunction with the numbers in Example 22, to locate the above designated rhythmic cells in their originating rhythm matrices, will enable the reader to compare corresponding rhythmic cells in the originating matrices and the Final Rhythm Matrix. This will facilitate the identification of specific alterations in each affected rhythmic cell.

The absence of numbers in the top row of Example 22 reflects the fact that Stockhausen left these durations rhythmically unaltered. The alterations of rhythmic cells within columns 4-6 of the Final Rhythm Matrix, found in Examples 17, 14, and 10 respectively, are less complex than those of columns 1-2. As they are easily identified with the aid of Example 22, there is no further need in this paper for an analysis of each rhythmic cell's transformation in these columns.

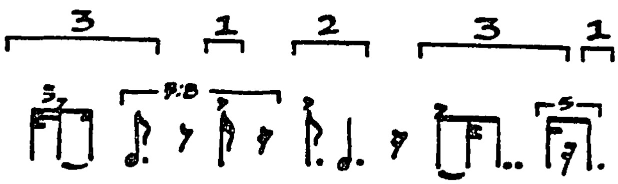
Fermati as organizers of rhythmic texture

Row 4 of Example 14 has rhythmic alterations involving the first and third rhythmic cells, yet the homogeneity of the rhythmic texture of the upper voice seems to be opposite in nature to that of row 2 in Example 17. Two duration-groups of radically different sizes, three durations and eighteen durations respectively, are separated by the dotted sixteenth rest in the first rhythmic cell. However, this block also contains a characteristic, absent from the above mentioned block in Example 17, which alters the rhythmic flow with a regularity similar to interjected rests: fermati are inserted into the block between rhythmic cells one and two, and within the second, third, and fifth rhythmic cells. These fermati originate in an element matrix of identical dimensions (6 x 6). When this matrix, which will be called the Final Element Matrix, is superimposed over the Final Rhythm Matrix, the contents of those blocks which coincide are combined with one another. (The construction of the Final Element Matrix will be discussed later in this chapter, beginning on page 88.)

Duration-Groups

When the durations of each voice in the respective blocks of Example 17 are grouped toether by ties, either linking initial rhythmic cells or creating longer duations within rhythmic cells, and by rests which have been interjected within rhythmic cells, the number of durations in each resultant duration-group seems to fluctuate in a significant manner, as shown in Example 23.

Example 23: Row 2 of Example 17 with brackets over duration-groups indicating the number of durations contained in each duration-group.



The brackets over duration-groups in Example 23 indicate the number of durations in each duration-group of row 2 in Example 17. A succession of duration-groups having three durations, one duration, two durations, three durations, and one duration is revealed.

(For Example 24, see page 50)

Example 24: Numbers series indicating the number of durations in each duration-group in the top voices of all blocks within the Final Rhythm Matrix. (This Example is not found in Stockhausen's sketches.)

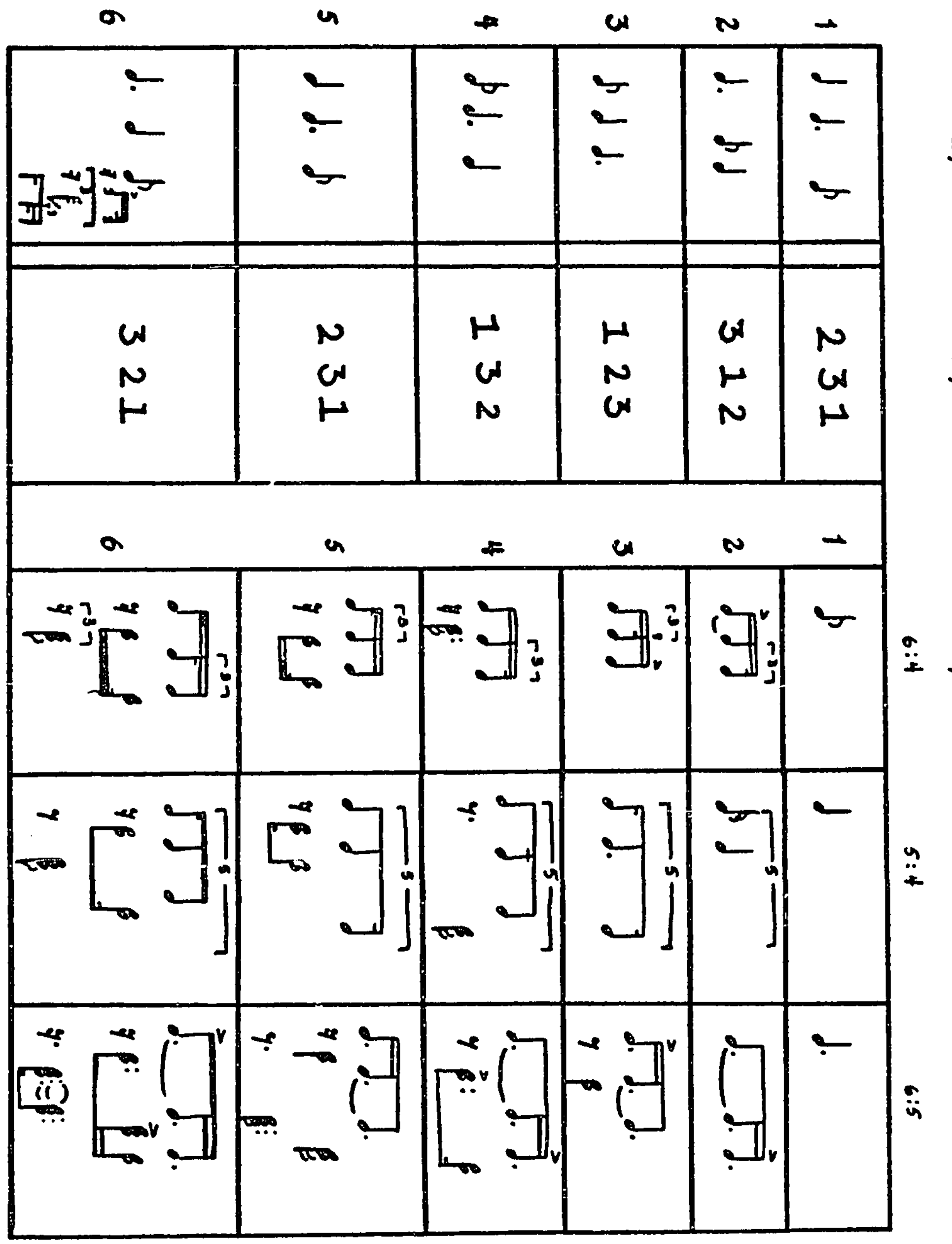
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	1 3	3 3	7 1	31231	23512	3212312
3	4	43	7 4	43124	524132	1324325
4	4	26	3414	8 9	3 18	2375431
5	22	53	634	14752	23	9 20
6	4	35	472	19	17864	13 8 11

Example 24 is a 6 x 6 matrix which expresses the number of durations in each duration-group in the top voices of all blocks in the Final Rhythm Matrix. Although no implementation of a number matrix was involved in creating the number of durations in the successive duration-groups, the visual appearance of these groups is reminiscent of such a process. This results from the general lack of repetition in the number of durations contained in the various duration-groups within each block. The general absence of large numerical gaps in a chronological consideration of the varying numbers in Example 24 also contributes to the serialized appearance of the duration-groups. For example, the numbers in the block at row 2 column 5 create a chronological series, when rearranged, from 1 to 5 with the number 4 missing. The block directly below also contains a number series from 1 to 5 with a repetition of the number 2. The blocks having fewer duration-groups, such as those of columns 1-3 are less significant in this regard, due to the much smaller total number of durations involved.

In columns 4-6 of Example 24 the greatest difference between the size of duration-groups in the same blocks occurs at row 4 column 5, and row 5 column 6. These large differences are logical due to the similarity in length of the larger duration-groups involved. Stockhausen evidently wanted several duration-groups which were exceptional in length. These occur in blocks at row 4 column 5, row 5 columns 5 and 6, and row 6 columns 4 and 6. The large duration-group at row 6 column 4 is similar to the one at row 5 column 5, in that each block has only one duration-group. The duration-groups of row 4 column 5, and row 5 column 6 also have similar characteristics of two duration-groups with similar sizes (3-18 and 9-20 respectively). The three duration-groups of the block at row 6 column 6 are close to one another in size, being separated by numerical distances of 3 and 2 when their chronological arrangement is considered with the starting point being the duration-group with 8 durations (8-11-13). It is also significant to consider the various sizes of the largest duration-groups, for there is no repetition in their respective sizes. Rearranged chronologically in respect to their various sizes they also make a more or less consistent appearing spectrum of numbers between 8 and 23: 8-11-13-18-19-23.

The serialized appearance of the respective duration-groups in the entire Final Rhythm Matrix is typical of the way Stockhausen often composes: A characteristic of a strictly controlled process is transferred to a more randomly controlled context somewhere else, often to a different musical parameter. The varying sizes of the duration-groups at row 2 column 6 of Example 24 are an illustration of this concept. The first set of three numbers in this block (3-2-1), and the second set of three numbers in the same block (2-3-1), are both different orderings of the same chronological numerical series (1-2-3). The seventh number in this block (2) is also one of the numbers in the same chronological series. Assuming the block contained nine numbers, the last two would either be 3-1 or 1-3 if the third set of numbers were to have numerical components identical to the first two sets. Since the numbers 3-1 have already occurred in the second three-number set, the logical choice for the last two numbers is 1-3. This block is the only block in the entire Final Rhythm Matrix which expresses this serial logic without any contradiction in the manner in which the respective numbers proceed one after the other. The block at row 2 column 4 is also composed only of the numbers 1 through 3, yet the first two numbers of the first set of three (3-1) are immediately repeated by the last two numbers in the block (3-1). This implies, according to the same logic three different arrangements of the numbers 1 through 3 per set, an exact repetition of the same series (3-1-2) immediately following the first three numbers. Stockhausen utilized this logic in the numerical ordering he employed in creating fragment #17 of Klavierstück XI. In this fragment--which is derived from the block directly above the previously discussed, serially significant block at row 2 column 6 of the Final Rhythm Matrix (see Examples 10 and 24)--a different ordering of an identical set of numbers normally occurs if the respective sets immediately follow one another. (The analysis of the construction of fragment #17 begins on page 64 of this paper.)

Example 25: Duration scheme (Example 25a) and number matrix #7 (Example 25b) used in reordering rhythm matrix #7 (Example 25c).

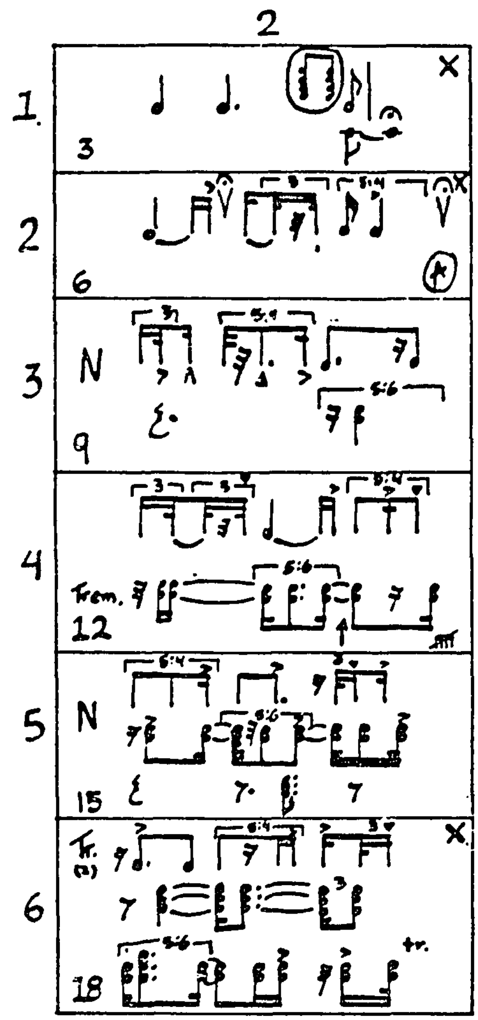


Stockhausen employed the duration matrix of Example 25a in reordering the rhythmic cells of Example 25c, when transferring them to the Final Rhythm Matrix. To show the creation of column 2 of the Final Rhythm Matrix it was helpful to reconstruct the number matrix implied by Example 25a: Number matrix #7 is not found in Stockhausen's sketches, but is placed here to express the numerical reordering accomplished by Example 25a.

The rhythmic cell Stockhausen wrote out beneath the last duration of row 6 of Example 25a is a rearrangment of the corresponding rhythmic cell found at row 6 column 1 of Example 25c. It was used in creating the third rhythmic cell of row 6 column 2 of the Final Rhythm Matrix. As Example 26 illustrates, the only difference is the top and bottom voices of the rhythmic cell in Example 25a being switched with one another when transferred to the Final Rhythm Matrix.

(For Example 26, see page 55)

Example 26: Column 2 of the Final Rhythm Matrix.



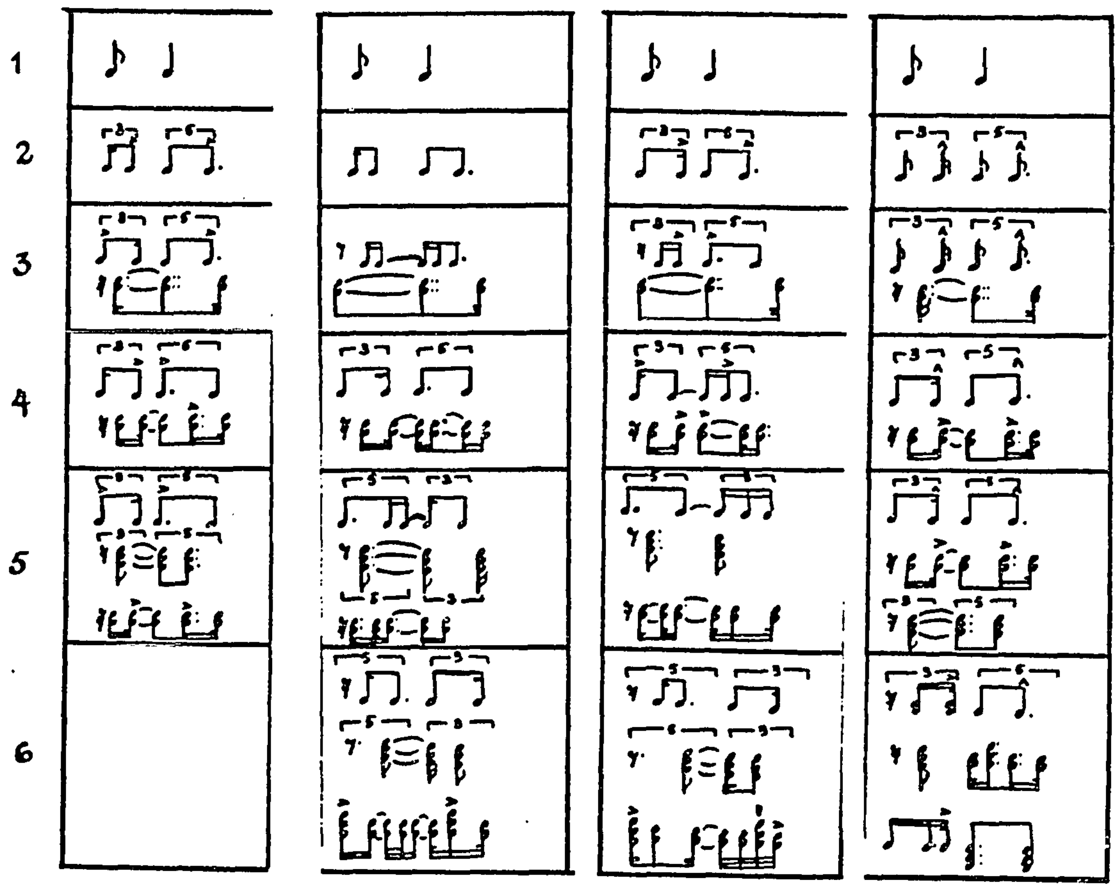
A comparison of the block at row 6 of Example 26 with row 6 of Example 25c reveals the two bottom voices of all three rhythmic cells to be switched with one another. The preponderance of dotted notes in column 3 of Example 25c is absent in this block, as well as throughout the entire column constituting Example 26.

The duration matrix of Example 25a or the number matrix of Example 25b can be used to determine the origin, from Example 25c, of each of the three rhythmic cells in each block of Example 26. The rhythmic alterations which occur in the majority of these rhythmic cells when transferred to the Final Rhythm Matrix can be studied with the aid of Example 22, which identifies those rhythmic cells in the Final Rhythm Matrix that have been altered. An important characteristic of the rhythmic alterations is the fact that no matter how dramatic the alteration may be (as in the top voice of the second rhythmic cells of row 6, Examples 25c and 26), rhythmic proportions such as 5:4 are always maintained. The number of durations also remains the same. It is the value of durations and/or their relationship with attendant rests that constitutes the alterations.

(For Example 27, see page 57)

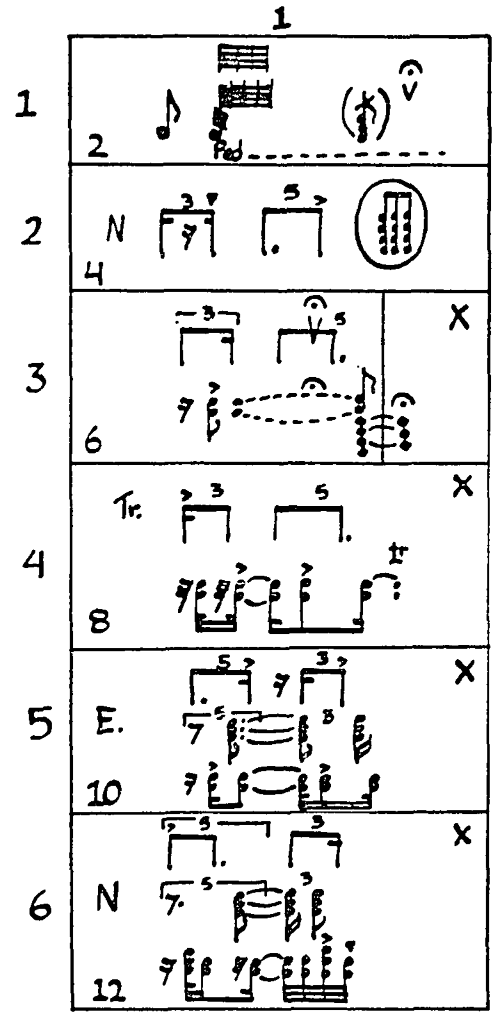
Example 27: Rhythm matrices #'s 8-11.

a) #8 b) #9 c) #10 d) #11



Examples 27a-c were used in constructing column 1 of the Final Rhythm Matrix. Row 6 of rhythm matrix #8 is empty in Stockhausen's sketches. Although its dimensions are the same as Examples 27a-c, Example 27d is found on a different page of the sketches and was not employed in constructing the Final Rhythm Matrix; it has been included in Example 27 since it provides the reader with further evidence of the extensive effort Stockhausen made in bringing rhythmic sophistication to a column of the Final Rhythm Matrix which had no associated number matrix that could further sophisticate the originating rhythm matrix. Since column 1 of the Final Rhythm Matrix contains only pairs of rhythmic cells, a number matrix was not used for the purpose of rearranging them in transference from the originating rhythm matrices.

Example 28: Column 1 of the Final Rhythm Matrix.



The blocks at rows 1-4 in column 1 of the Final Rhythm Matrix maintain the duration scheme found in the rhythm matrices of Example 27a-c: an eighth's duration followed by a quarter. The remaining two bottom blocks of column 1 rows 5 and 6 have the duration scheme, and hence the rhythmic cells, reversed when compared with the corresponding blocks in Examples 27a-c.

When column 1 of the Final Rhythm Matrix is compared with the rhythm matrices of Examples 27a-c, some rhythmic cells are identical to corresponding cells in the originating matrix, yet others share characteristics with two or more matrices. It is difficult to determine the exact origin of such a rhythmic cell. For example, the bottom voice of the first rhythmic cell in the block at row 4 column 1 of Example 28 has been transformed in transference from the originating rhythm matrix, but it shares characteristics with corresponding cells in Examples 27a-c. The bottom voice of the second rhythmic cell in the same block shares a common characteristic with Examples 27a and 27b: the last duration of the first rhythmic cell is tied to the first duration of the second cell in the bottom voice of both examples. The bottom voice of the corresponding block in Example 27c is minus this tied characteristic between rhythmic cells.

Example 29: Originating rhythmic cells for column 1 of the Final Rhythm Matrix, located in Examples 27a-c. (This example is not found in Stockhauen's sketches.)

1	abc	
2	abc	abc
3	a a	a a
4	a abc	ab ab
5	b b a c	b b a c
6	bc bc c	bc bc c

Example 29 identifies the origin(s) of the respective voices in the rhythmic cells of Example 28. The letters a, b, or c indicate an origin in corresponding rhythmic cells in Examples 27a-c. Letters placed above and below one another indicate corresponding voices in the respective rhythmic matrices. Those voices in Example 28 sharing characteristics with more than one rhythm matrix will have more than one letter indicating their origin. For example, the bottom voice of row 4 in Example 28 is shown to have originating rhythmic matrices from Examples 27a-c in the first rhythmic cell, as indicated by the letters (abc) in the lower left portion of the block at row 4 in Example 29. The dotted line in Example 29 separates the letters for indicated rhythmic cells in the same manner in which Stockhausen employed a dotted line in order to distinguish separate rhythmic cells within the same block in columns 4-6 of the Final Rhythm Matrix, as shown in Examples 17, 14, and 10 respectively.

Aspects relating to Klavierstück X

The notation of the second duration of the block at row 1 column 1 in the Final Rhythm Matrix (Example 28) indicates Stockhausen was planning to transform this duration from a quarter note to a series of repeated tones. It is notated with a series of slashes through the stem of the note. Two groups of repeated sixty-fourth notes that imply the same pitch are written directly above it. The repeated notes alternate successively between the upper and lower groups of sixty-fourth notes, just as a pianist could alternate hands if performing a very fast series of repeated tones on the same note.

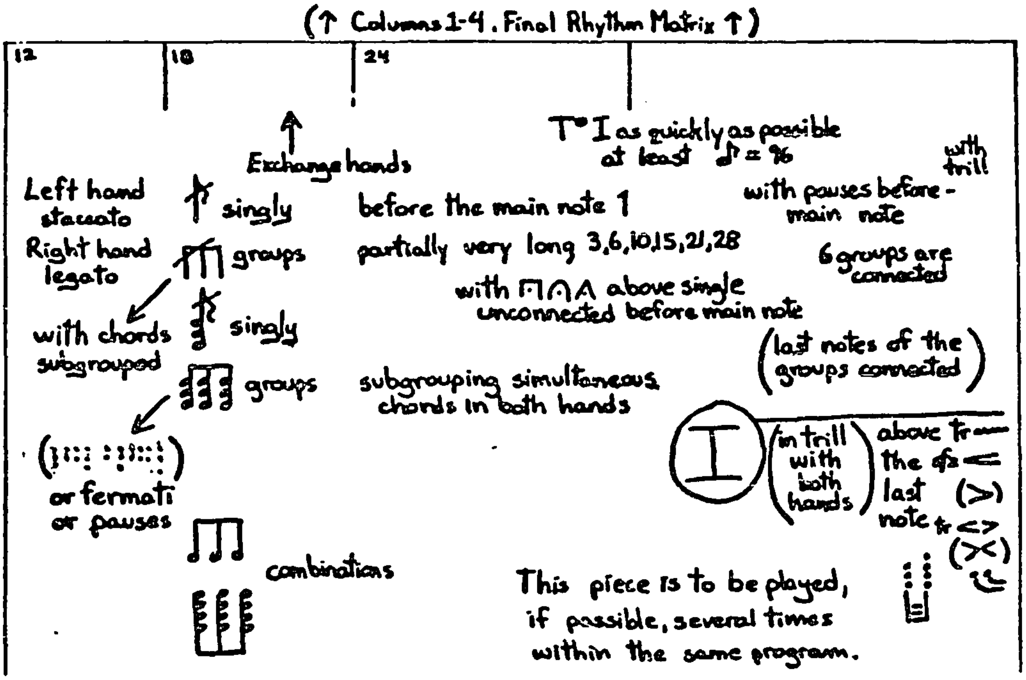
This is one of several characteristics this block shares with the repeated tones on pages 28 and 29 of the published score of Klavierstück X. Two other shared characteristics are the held pedal and the fact that the repeated notes immediately follow another duration. In Klavierstück X these repeated notes always begin inside the preceding vertical sonority. Whether Stockhausen intended this to occur with the repeated tones of Example 28, or whether the first duration in the same block would have been a different pitch, will have to remain speculation. Although this block with repeated tones was not used in creating one of the fragments of Klavierstück XI, an examination of its written-out rhythmic form in Stockhausen's sketches reveals the repeated note idea was abandoned and the original quarter note duration was retained in an unaltered form.

This comparison with an aspect of Klavierstück X is logical because, as will be shown later during the discussion concerning the composition of fragment #17, the use of durations in another block, at row 1 column 6 of the Final Rhythm Matrix (Example 10), directly relates to a much more extensive aspect of Klavierstück X: long durations used to measure the length of large groups of notes having much smaller time values. The two blocks thus exhibit a symmetry when considered together, being at opposite ends of the row they share in common.

Written instructions below the Final Rhythm Matrix

Except for the unused repeated note alteration, no other additional written instructions, which would dramatically alter the form of the fragment, or change interior characteristics of certain rhythms, are written out inside other blocks in the Final Rhythm Matrix. However, Stockhausen wrote out many diverse suggestions about different aspects of the piece directly below the first four columns of the Final Rhythm Matrix. Many of these suggestions, the majority of which are reproduced in translation in Example 30, were to be incorporated in conjunction with elements originating in the Final Element Matrix: single grace notes, grace note groups, duration-groups, and fermati of different shapes are the principal items included in this catagory. Other items, such as the tempo statement and a request for the piece to be performed more than once on the same program, also originate here.

Example 30: Suggestions written by Stockhausen directly below columns 1-4 of the Final Rhythm Matrix.



The Roman numeral I, circled in Example 30 with a connecting horizontal line which extends to the right edge of the page, is an indication this is the first of 2 pages of the sketches which contain the 6 columns of the Final Rhythm Matrix. Although some instructions shown in Example 30 must remain ambiguous, the entire example is indicative of the thorough manner in which Stockhausen contemplated the translation of the contents of the Final Rhythm Matrix into musical fragments.

Serial ordering of all durations in the Final Rhythm Matrix

An additional serial system expresses the ordering of the number of durations in each block of the Final Rhythm Matrix, shown in Examples 10, 14, 17, 20, 26, and 28. This system can be explained by shifting the last 6 column numbers (1 2 3 4 5 6 7) for the initial 6 x 7 matrices (rhythm matrix #1 and number matrix #1), beginning with the number 2, into the corresponding topmost blocks of the 6 x 6 format of the Final Rhythm Matrix. Then instead of using the chronological number of the corresponding rows for determining the number of durations in the interior of each descending block, as in rhythm matrix #1 (Example 2), the numerical value for the number of durations in each descending block is successively increased by the value of the number in the block at the top of each respective column. Example 31 is the number matrix which expresses this duration series.

(For Example 31, see page 64)

Example 31: Number matrix #8.

2	3	4	5	6	7
4	6	8	10	12	14
6	9	12	15	18	21
8	12	16	20	25	28
10	15	20	25	30	35
12	18	24	30	36	42

Stockhausen wrote out the corresponding numbers from Example 31 inside the blocks of columns 1-3 of the Final Rhythm Matrix. However, when the durations of columns 4-6 of the Final Rhythm Matrix are counted, they also equal the numbers found in the corresponding blocks of Example 31.

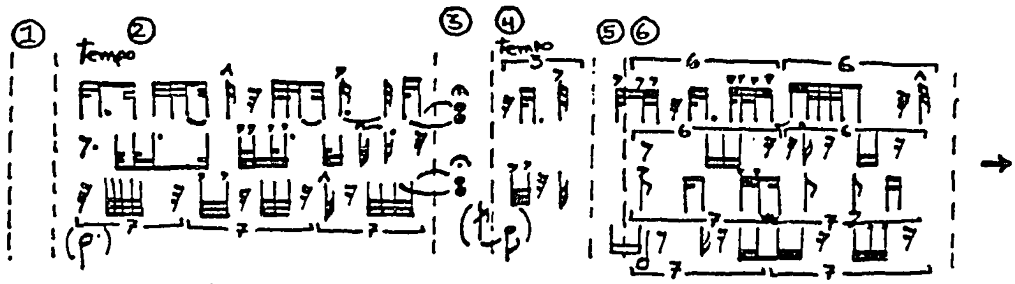
The Construction of Fragment #17

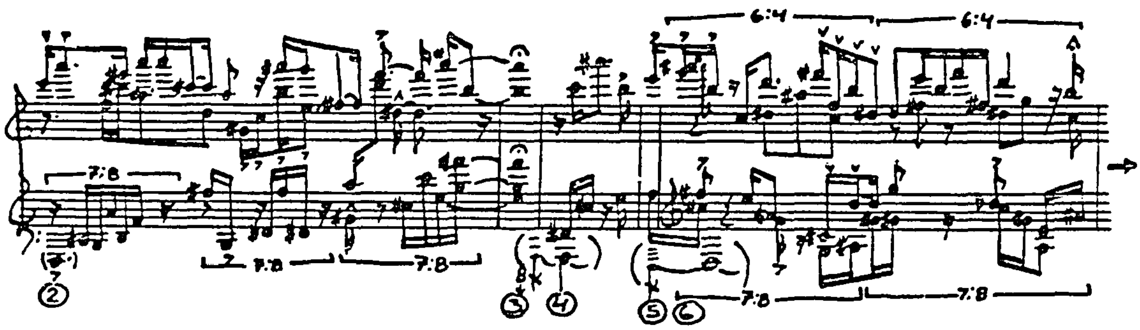
The block at row 1 column 6 of the Final Rhythm Matrix was employed in creating Fragment #17 of Klavierstück XI. The varying durations in this block were used in a manner similar to the way Stockhausen composed Klavierstück X:[16] In addition to being translated into 7 different pitches, the size of these durations determined regions into which groups of notes with much smaller time values were also composed. This contrasts with only being translated into successions of pitches like the durations of all other blocks in the Final Rhythm Matrix. Just as the number series within the block at row 2 column 6 of Example 24 can be seen as a conceptual link to the block directly above it, Stockhausen's use of this block at row 1

[16] Cf. Henck, H., Stockhausen's Klavierstück X for a marvelous account of this work concerning its history, theory, analysis, practice, and documentation.

column 6 in Example 10 can be understood as a conceptual link with the piano piece which chronologically precedes Klavierstück XI.[17]

Example 32: Fragment #17 without grace note groups. Original rhythmic scheme placed above the pitch realizations of the published score.

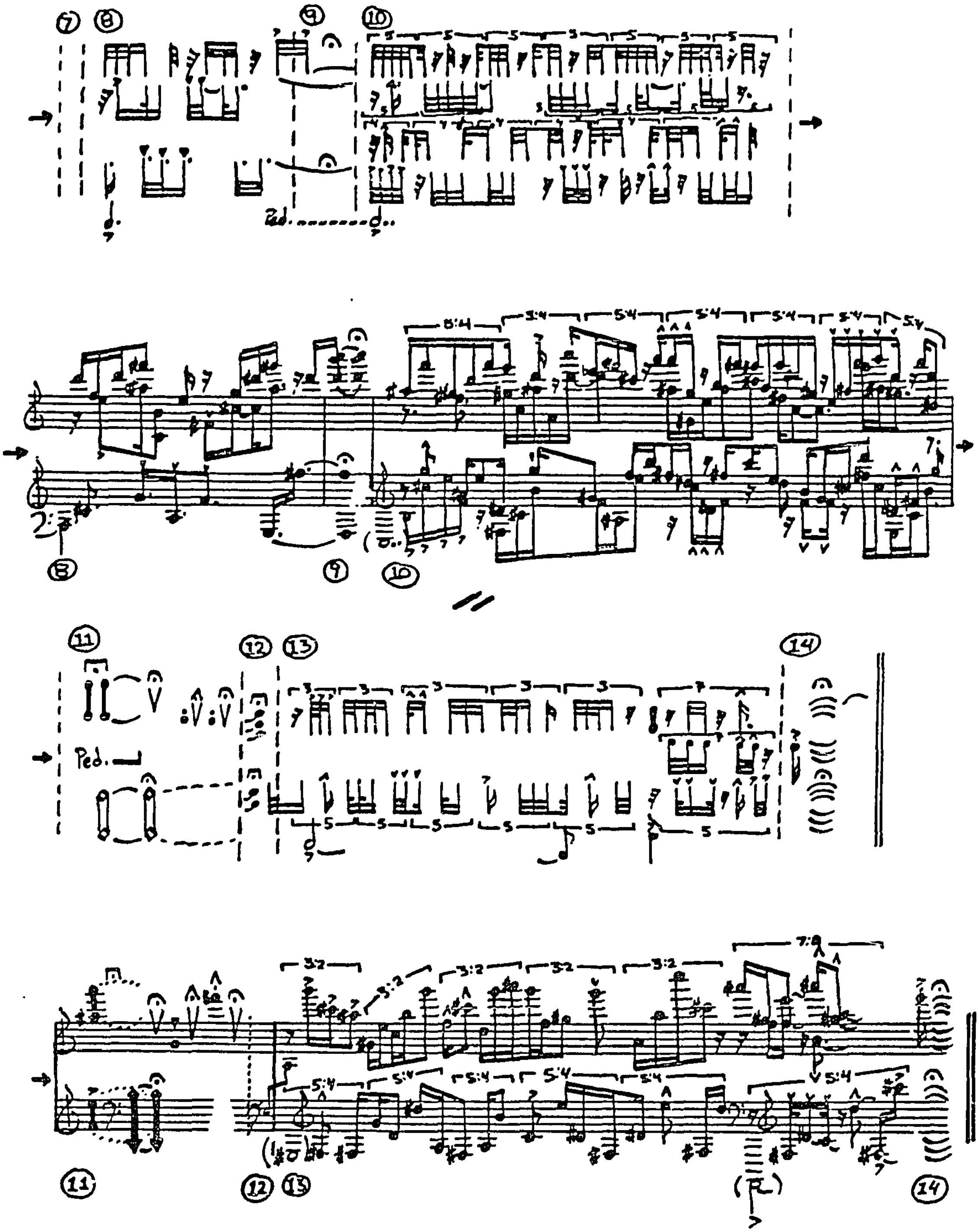




(For the continuation of Example 32, see page 66)

[17]Cf. Volans, K., "The Klavierstück -- Stockhausen's Microcosm" for a general study of common theoretical concepts which develop throughout the Klavierstücke.

Example 32: continued.



Example 32 contains the rhythmic schemes of fragment #17 placed above the pitch realizations of the published score. Grace note groups have been omitted from this example. Fragment #17 was composed in 14 sections which can be identified by the circled numbers in Example 32. Dotted lines also demarcate the sections of the fragment in the rhythmic schemes. Sections 1, 3, 5, 7, and 12 of the published score are filled with six different note groups in small type, also called grace notes by Stockhausen.[18] The grace note groups, which originate in the Final Element Matrix, will be discussed later in conjunction with that matrix. Sections 2, 4, 6, 8, 10, and 13 contain groups of polyphonic durations in addition to the seven durations from row 1 column 6 of the Final Rhythm Matrix. Sections 9, 12, and 14 contain pauses which sustain certain tones from the previous respective sections with the aid of the damper pedal.

The six grace note groups in fragment #17 are composed of separate attacks totalling, respectively, 28, 3, 15, 10, 6, and 21 in number. Example 33a is the first version of the initial grace note group of 28 attacks which Stockhausen rejected after considering it in the sketches.

(For Example 33, see page 68)

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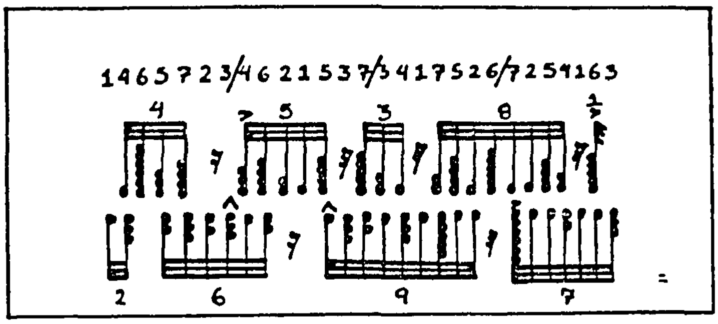
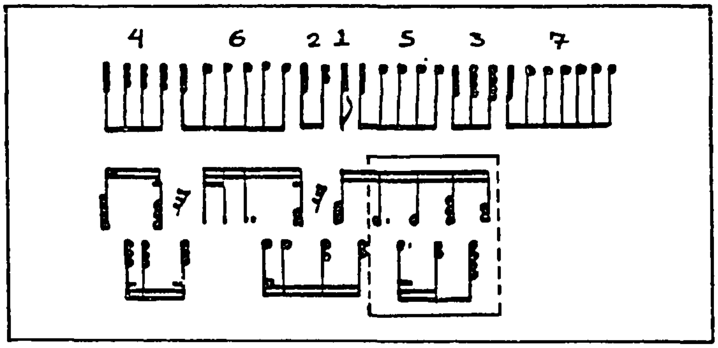
[18]Stockhausen in conversation with the author, Denver, 1980.

Example 33: Three versions for the initial 28 grace note attacks of fragment #17.

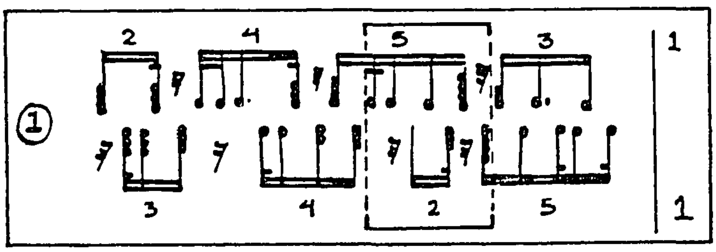
a)



b)



c)



The four separate number series above the rhythms in Example 33a have the same lengths as the number series of Example 2. Although the first three number series are different from any in Example 2, the fourth (7254163) is identical to row 5 of Example 2. Each of the numbers in the four series was used to determine the number of pitches contained in each of the 28 attacks in this grace note group. For this group Stockhausen used another numbering system which caused the entire grace note group to consist of 9 smaller sub-groups of notes, each of which contains a different number of attacks ranging from 1 to 9 in total number. These numbers were written out by Stockhausen above and below the grace note sub-groups. This serial method ensures that each of the 28 separate attacks would have a different number of pitches than either of its surrounding neighbors. This would perhaps distinguish this particular grace note goup as the most technically difficult passage to perform in the entire piece. Yet Stockhausen's decision to recompose this group in a different manner may also have been influenced by the fact that a group of 9 attacks, the third group in the bottom voice of Example 33a, is employed in order for this serial method to remain consistent. The number 9 as a quantifier of attacks in specific duration groups occurs nowhere else in the various serial systems Stockhausen employed in this work. The largest matrix employed has 6 x 7 dimensions. The number 9 never occurs in any of the number matrices used in reordering rhythm matrices, nor does it appear in the additive series, shown in Example 39, employed in generating the number of attacks within grace note groups. The number 9 occurs twice in Example 24 as a result of analysis; it occurs in Example 31, but only as a quantifier for three specific rhythmic cells which have already been composed (located at row 3 of Example 26). Stockhausen may have felt the presence of a 9-note grace note sub-group to be both incongruous with the size of the other grace note groups in Klavierstück XI, and inconsistent with the various serial systems employed throughout the composition of the piece.

Stockhausen chose another numerical series, the third row of Example 1, in order to compose a second 28-note grace note group which he subsequently used for the beginning of fragment #17. He first wrote out the note groups, each with the number of attacks determined by the number series, found at the top of Example 33b. This method of arriving at 28 separate attacks results from the fact that the numerical total of the 7 separate numbers in this seven-number series, when added to one another, is 28 (1+2+3+4+5+6+7=28). The method employed in determining the number of pitches in each attack appears to be random. The first attack in the first group of 4 attacks at the top of Example 33b is a cluster. The following 3 attacks in the same group have 3 note heads which will be later translated into 3 separate pitches. Each of the 7 groups of attacks begins with a cluster.

Stockhausen created rhythmic polyphony for the second group of 28 grace notes (Example 33c) with a number system similar to the one he employed for the first group in Example 33a. Instead of using a single number series once in determining the number of attacks the various groups of an upper and lower voice would contain, as in Example 33a, Stockhausen used the same set of numbers twice in Example 33c. One ordering of the numbers 1 through 5 was used for the upper voice (2-4-5-3-1): another ordering was employed for the lower voice (3-4-2-5-1). As Example 33c indicates, the number 1 was written out in both voices just beyond the vertical line, although it was not used in determining any note groups. The rhythms of the two voices were composed by Stockhausen as he fitted the various note groups together throughout the space of 28 thirty-second note durations. (Although the grace notes are also written as thirty-second notes in small type in the score of the piece, they are performed "in as quick succession as possible"[19] as grace note groups could also be interpreted.) The two voice version at the bottom of Example 33b was abandoned at the point it ends in the example. The final version is that of Example 33c. The first two note groups are identical in both voices of the two versions found in Examples 33b and 33c. A comparison of the bracketed areas with dotted lines in the two examples will reveal the rhythmic changes Stockhausen made in both voices, beginning with the second attack of the 5-note group in the upper voice in Example 33c.

(For Example 34, see page 72)

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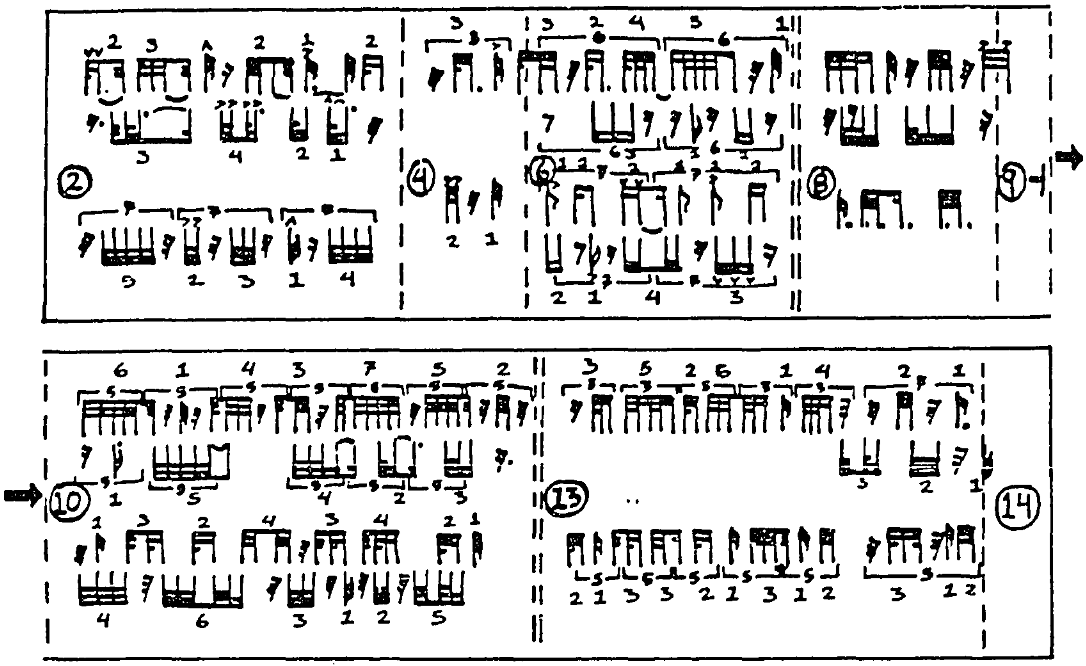
[19] From the performance instructions for Klavierstück XI.

Example 34: Original rhythmic schemes for sections 2, 4, 6, 8, 10, and 13 of fragment #17.



The second section of fragment #17 immediately follows the beginning grace note group of 28 attacks. A comparison of section 2 in Example 32 and the identical section in Example 34 reveals the absence of a tie between the second and third durations of the upper voice in Example 32. Example 34 occurs first in the sketches on the same page as the note groups of Example 33, and is the original form of these durations. The rhythmic schemes in Example 32 were written out later. A tie is also absent between the third and fourth durations of the middle voice in section 2 of Example 32, between the tenth and eleventh durations of the middle voice in section 10, and between the fifteenth and sixteenth durations of the bottom voice in section 13. If, after writing out the respective voices in such a single polyphonic passage, Stockhausen reconsidered the interrelationships of the various voices before recopying the passage, as from Examples 34 to 32, he could have decided at this point to omit certain ties. The second omitted tie from section 2 of Example 32 overlapped with a tied duration in the upper voice. Yet this is the only instance in this example in which an omitted tie overlapped another tie. Whether or not the omitted ties are copying mistakes, the result is the same: by omitting a tie between two notes, one duration is changed into two durations. This calls for the creation of an extra pitch at the point of the second duration thus formed. When the score of Klavierstück XI is consulted it becomes apparent that Stockhausen retained the absence of ties at the above mentioned points and composed additional tones.

Although the dotted half note in parenthesis in section 2 of Example 32 is absent in section 2 of Example 34, Stockhausen was aware of its value being that of the first duration in the block at row 1 column 6 of the Final Rhythm Matrix, shown in Example 10. The total duration of section 2 in Example 34 is therefore equal to the dotted half note; it was composed with this time value in mind.

A serial system similar to the system employed in the creation of the two voices of Example 33c was also used in composing durations in sections of fragment #17 having fundamental durations in parenthesis. As seen by the various number sets above and below the durations in sections 2, 4, 6, 8, 10, and 13 of Example 34, various sets of numbers were employed in determining the number of durations in each of the respective voices. The number sets also determine the sequence in which the durations occur. A number set of 2-3-1 implies a sequence of 2 durations with connected beams, followed by 3 durations with connected beams, followed by 1 duration. As with the duration-groups of Example 24, a duration here may be a single note, or two or more notes tied to one another. Having this choice allowed Stockhausen greater possibilities for rhythmic sophistication than would otherwise have been possible. Instead of using the same number set for each of the voices in a given section, here Stockhausen employed a different number system for each voice. The upper voice of section 2 of Example 34 is composed of two orderings of the same set (2-3-1 and 3-1-2), the middle voice contains one set (3-4-2-1), and the bottom voice has one set (5-2-3-1-4). Stockhausen wrote out the set numbers above the durations in section 2 of Example 34 only, although a few of the other set numbers are placed together higher on the same page. The only time an immediate repetition of a set occurs is with the last 3 durations of the bottom voice of section 13.

The accumulative total of the durations of sections 2, 4, 6, 8, 10, and 13 equal in turn the 7 durations of the block at row 1 column 6 of the Final Rhythm Matrix, shown in Example 10. The 7 durations are contained within only six sections in Example 34 because section 13 contains two of the durations from the Final Rhythm Matrix. These larger durations are written with parentheses around them in the score of Klavierstück XI. This indicates they should be omitted the first time the fragment is performed, and they are included if the same fragment is repeated again during the same performance. (The note values of these sections in Examples 10 and 34 are doubled in the score.)

In the block from the Final Rhythm Matrix he employed in composing fragment #17, Stockhausen indicated many characteristics to be associated either with or placed between the various durations in the same block. The majority of these characteristics are in addition to those originating in the Final Element Matrix. The rhythmic proportions found in those sections of fragment #17 containing durations in parentheses relate in part to similar or identical proportions Stockhausen wrote out just above or under the same durations in the Final Rhythm Matrix. In the block mentioned above, 7 proportions are indicated for the respective durations: 6:7 (dotted half note), 6:4 (eighth note tied to a grace note), 7:4 (half note), 6:5 (dotted quarter), 7:5 (double dotted half), no proportion indicated for the sixth duration (eighth tied to a half), and 5:4 (quarter). Thus the lower voice of section 2 of Example 32 can be observed to consist of 5 duration-groups composed within 3 bracketed rhythmic proportions, each consisting of 7 thirty-second notes in the time of 6 thirty-seconds. In relation to the fundamental duration of a dotted half note, the 6 unbracketed thirty-second notes in section 2 equal 1/3 of the dotted half note--a quarter note--in duration.

The projected rhythmic proportions are consistent in relation to subdivisions of a fundamental counting unit in all but two of the other sections of Example 32 having bracketed fundamental tones. Section 8 consists of a fundamental duration of a dotted quarter note. The associated rhythmic proportion is 6:5. The three voices in this section are basically conceived within the space of 12 thirty-second notes that equal a dotted quarter note. The final duration of the top voice extends into the space of section 9. Although the 6:5 rhythmic proportion is absent in this section, counting the number of durations in each voice is revealing in terms of sixes and fives. The connected beams of the upper voice reveal a 4-1-3-2 duration-group ordering. But the two thirty-second rests also separate the first two duration-groups from the third duration-group. The total number of durations in the first two duration-groups (4+1) is 5. The total number for durations in the third and fourth duration-groups (3+2) is also 5. The total number for durations in the middle voice is 6. Six durations are also present in the lower voice, all of which are either dotted sixteenth or dotted thirty-second notes. The dotted rhythms plus the three accents further distinguish the 6 durations of the lower voice from the 6 durations of the middle voice.

The 7:5 proportion is not present in section 10 of Example 32, although 7 duration-groups composed within the 5:4 proportion are present in the upper voice. The voice just below the top voice is composed of 5 duration-groups written within the same rhythmic proportion. Stockhausen felt it necessary to put brackets above the duration-groups of the third voice from the top in order to emphasize the fact that this voice was composed within a 4:4 proportion, as was the bottom voice also. Section 10 is the most contrapuntally complex section of fragment #17. This results from having four voices with much larger duration-groups than section 6, the other four-voiced section.

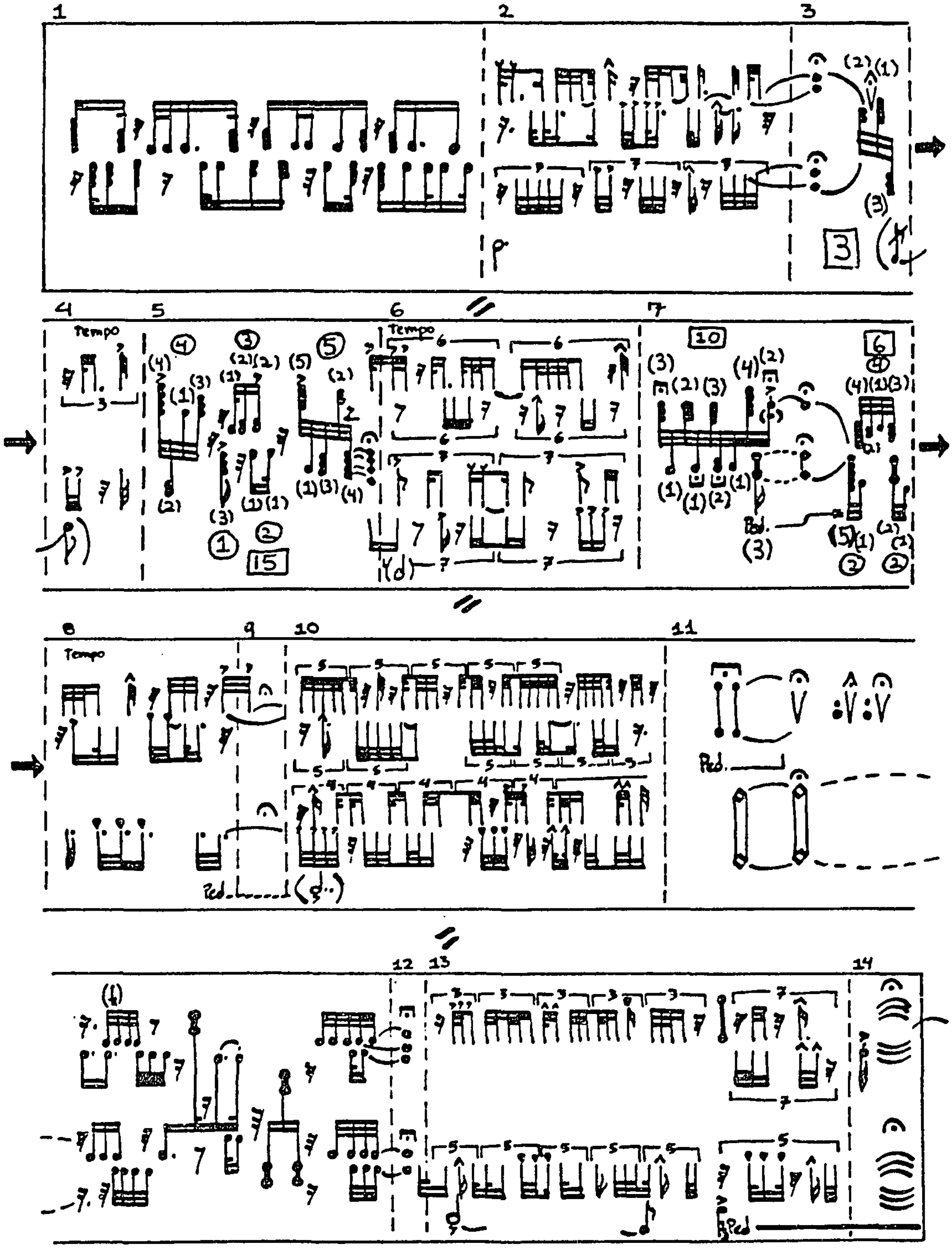
In Example 32 section 4 of fragment #17 is composed of three duration-groups organized into 2 voices. One group of 3 durations in the upper voice, and two groups of 2 durations and 1 duration in the lower voice. Both voices occur within the time of an eighth note. The durations in the upper voice are composed as six thirty-seconds to be played in the time of four. This is the rhythmic proportion written above the second duration in the related block in the Final Rhythm Matrix.

Each voice exhibits different characteristics in those sections of Example 32 having fundamental durations in parentheses. The two bottom voices of section 6 both exhibit the 7:4 rhythmic proportion, but the lower voice contains rests while the voice just above it has none. The two upper voices in this section both exhibit a 6:4 rhythmic proportion, but the upper voice is composed of thirty-seconds, sixteenths, a dotted sixteenth and an eighth, while the voice just below it is composed solely of sixteenth notes.

Stockhausen purposely changed the number of separate voices composed in each of the sections of Example 32 having fundamental durations in parentheses. As with the organization of many other aspects of the composition of this fragment, the numerical expression of this constant alternation of the number of voices can be grouped into sets: Sections 2, 4, and 6 comprise a set of 3, 2, and 4 voices respectively; sections 8, 10, and 13 comprise a set of 3, 4, 2, and 3 voices respectively. A change from a 3-voice beginning to a 2-voice texture occurs in the first set (3-2-4). This is balanced by the final group of voices (3-4-2-3) in the second set, as the group which precedes it also has 2 voices (both occur in section 13). This balances the change from 3 voices to 2 voices at the beginning of the first set. The final 3-voice group in the second set also implies the beginning of a third set, again having 3-2-4 as its numerical expression of the respective voices in order to contrast with the 3-4-2 set which occurred immediately before it. This method of procedure, continuing a certain consistency in organizational logic even though a process is sometimes terminated before its final completion, is a compositional characteristic Stockhausen often exhibits. The previous analysis of the similar grouping of numbers for duration-groups into sets and an implied set, in the block at row 2 column 6 in Example 24, also illustrates this aspect of Stockhausen's methodology.

(For Example 35, see page 79)

Example 35: Rhythmic scheme for fragment #17 with grace notes.



Example 35 is the rhythmic scheme, with grace notes included, Stockhausen used when composing the pitch realizations for fragment #17. The grace notes in section 3 of Example 35 herald a new type of pitch organization which begins at this point in the fragment: each succeeding attack has a different number of note heads. This is a characteristic identical with the abandoned 28 attacks of Example 33a. The first grace note attack in section 3 has 2 note heads and the third attack has 3 note heads. The second attack is a cluster. The number of note heads in each of the 3 attacks can be seen to exhibit a vertical number set organization of 3-1-2. The fact that the second attack is a cluster, instead of having only 1 note head as indicated by the set number for this attack (1), results from Stockhausen's practise of randomly inserting clusters into sets having the number of note heads determined by serial logic. As a result, clusters play a very important textural role in Klavierstück XI, particularly in fragments #12, #14, #17, #18, and #19.

The grace note groups of section 5 have attacks organized in vertical sets of 4-2-1-3, 3-1-2/1-2-1, and 5-1-3-2-4. The last grace note attack in section 5 is tied to a duration from the Final Element Matrix. Numbers in parentheses in Example 35 have been added above and below individual attacks in grace note groups to facilitate the analysis of the vertical dimension in group sets. Circled numbers, when applicable, are placed immediately above or below sub-groups to indicate the number of attacks in each sub-group. Numbers in squares indicate the number of attacks contained in each grace note group.

The grace note groups of section 7 have 10 and 6 attacks respectively and relate to each other structurally when an apparent inconsistency in the serial logic employed in organizing the number of note heads per attack is observed: The first attack of the second grace note group of 6 attacks has 5 note heads; it is also the first of a sub-group of 2 attacks in the lower voice. Since the second attack of this sub-group contains one note head, this implies either 2 note heads for the first attack, or a cluster. If the attack with 5 note heads is considered as belonging to the sub-group of the upper voice, which contains a vertical set ordering of 4-2-1-3, it would fit numerically as a logical fifth set number. Yet it shares beams with the second attack in the lower voice, and another sub-group of 2 attacks also follows in the lower voice at the end of the group's 6 attacks.[20]

The first grace note group in section 7 has 10 attacks with varying numbers of note heads resulting from the numerical sets 3-1-2/1-3-2/1-4-3-2. The set 1-4-3-2 contrasts with the 4-2-1-3 set in the upper voice of the second grace note group in section 7. As the numbers in parentheses indicate, the attacks of the first grace note group basically alternate above and below the thirty-second note beams they share in common. Thus the first attack, having 3 note heads, is above the beams; the second attack with 1 note head immediately follows below

---

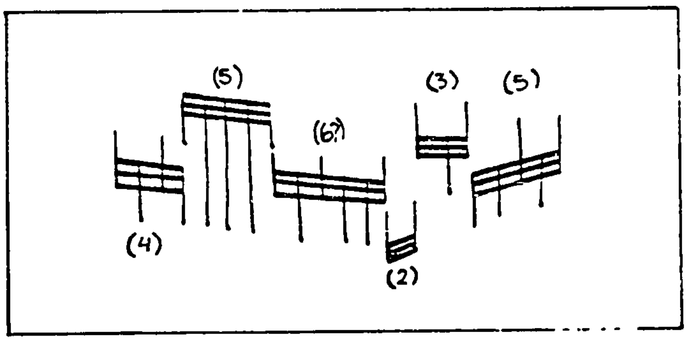
[20]Simultaneous attacks between upper and lower voices of this group are counted as 1 attack because in this way a total of 6 attacks in the vertical dimension is determined. Six is the number of attacks employed by Stockhausen at this point in fragment #17 because the number he used for determining the attacks, originating in the Final Element Matrix, is also 6.

the beams. The third attack of this set, located above the beams, is a cluster although the set number above the attack is (2). This is a continuation of Stockhasuen's practise of randomly inserting clusters into sets having note heads organized by serial logic.

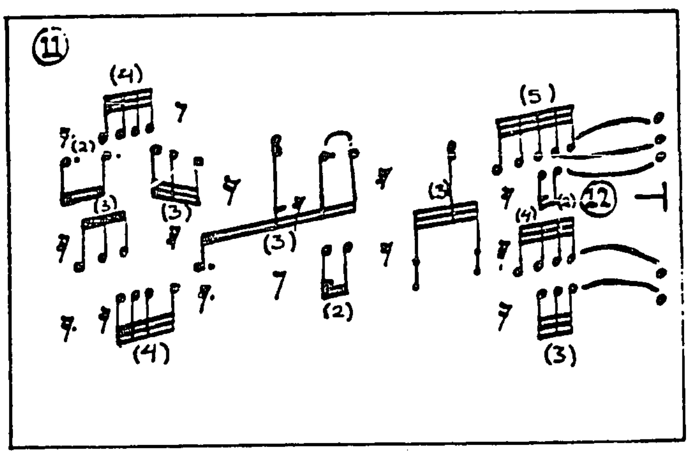
The group of grace notes that fills out the second portion of section 11 is organized into 4 voices. Although a number from the Final Element Matrix calls for 21 attacks, there are only 20 separate attacks present in Example 35. This is also the form in which this grace note group was originally written in the sketches.

Example 36: Initial conception (36a) and final version (36b) of grace note group in sectiion 11 of fragment #17.

a)



b)



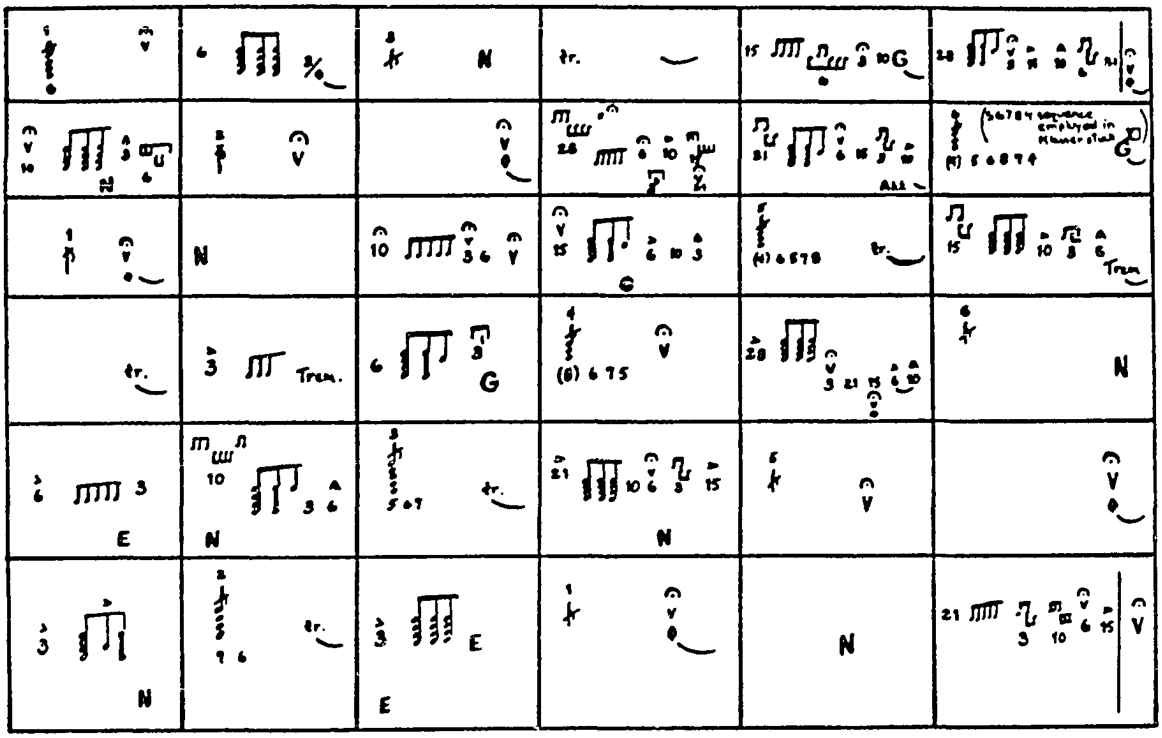
Example 36a is an initial, incomplete sketch for the beginning of this grace note group. It reveals Stockhausen to be considering a two voice texture at this point. The slanted beams and corresponding note heads with different heights in Example 36b indicate that Stockhausen was thinking in terms of the direction the resulting succession of pitches would move. When he transferred this grace note group to a later page in the sketches, the beams were no longer slanted and all note heads in the individual sub-groups had the same height. However, a comparison of the implied pitch directionality of Example 36b with the score of Klavierstück XI reveals a direct correspondance. When individual pitches within the separate sub-groups of attacks are compared, a movement in the direction implied by Example 36b can be observed. When Stockhausen composed the pitch realizations for this fragment, he added an additional smaller grace note between the first and second attacks of the upper voice of the group of 20 grace notes, thus providing for the twenty-first attack the Final Element Matrix calls for! This grace note is illustrated in parenthesis in section 11 of Example 35.

The fermata separating the first attack from the second and third attacks of the grace note group in section 3 of Example 35 originated in the Final Element Matrix. The fact the 3 attacks in this grace note group have a common beam, and thus function as one voice, originates in the additional instructions Stockhausen wrote out in the top block of the sixth column of the Final Rhythm Matrix. The instructions for the grace note groups to be "blended" together was written inside the circle which also contains notes arranged in two voices, shown in Example 10. The encircled grace notes are written above and between the first two durations which adjoin the grace notes both in this block and in the score for Klavierstück XI.

Instructions for the enigmatic beginning of section 11 of Example 35 were also written out in the same block at row 1 column 6 of the Final Rhythm Matrix. An arrow indicates the precise point at which this section should begin, written directly after the double-dotted half note in this block. See Example 10. The portion of section 11 which precedes the group of 21 grace notes originates in this set of instructions. The translation of these instructions is "1 tremolo/bind over/ + 2X bound noise". Although other tremolos originating in the Final Element Matrix were employed in other fragments, the tremolo indication at this location in the sketches was abandoned in the pitch realization of section 11. The two clusters at the beginning of this section are bound over in sound by means of the two silently depressed clusters, indicated by diamond-shaped note heads, which immediately follow. The staccato note heads without stems (pitches: g and f# in the score, also accented) complete the pitch content of this section. The absence of stems is a characteristic associated with an "echo" effect Stockhausen also employed in fragment #3, which is analyzed in Chapter 3 (Example 51). Although the symbol for echo (E) is absent from the block in the Final Element Matrix which corresponds with fragment #17, this symbol is present in two other blocks of the Final Element Matrix.[21]

Example 37: The Final Element Matrix (37a); definitions of elements (37b).

a)



(For Example 37b, see page 86)

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[21]The analysis of the construction of the Final Element Matrix begins on page 88.

(Example 37 continued):

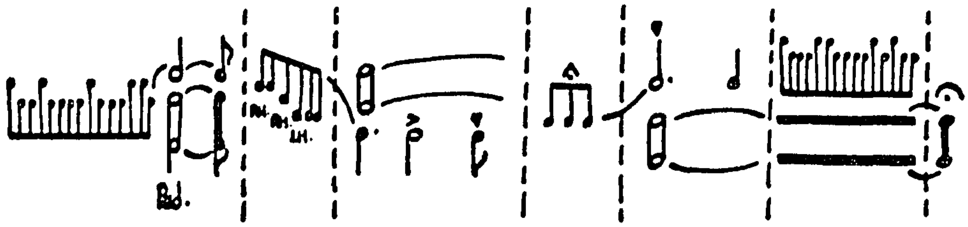
b)

3	= 3 large clusters (etc.) tied through a certain region. Not tied to one another.
4 (8)675	= 4 grace note attacks placed at different points, having 8,6,7, and 5 pitches respectively.
	= tied elements (single pitches, chords, trills or tremolos) which are placed at the end of a fragment and linked ("binden") to the following fragment that is subsequently played.
tr.	= trill
N	= Neutral
	= 3 grace notes, each having 1 pitch, placed at different points in the fragment.
	= tied chords or single pitches which are silently depressed.
10 6	= grace note groups of single pitches per attack: 10 attacks begin the fragment, and groups of 3 and 6 attacks occur within the fragment.
V	= fermata(s) to be inserted within note groups
E	= an echo effect
28	= a group of 28 single gracenote attacks to be written with connected stems above and below the note heads in order to divide notes between hands.
15	= a group of 15 grace note attacks which fluctuate between single pitches, clusters, and chords.
trem.	= tremolo
Akk.	= a chord tied over at the end of the fragment.

The E in the block at row 5 column 1 of the Final Element Matrix was employed in fragment #3. The E's in the block at row 6 column 3 correspond to an unpublished fragment which was written out in rhythmic form in the sketches. The echo effect is always achieved by the suspension of certain notes by means of ties. Then groups of staccato notes without stems, or grace note groups in three unpublished rhythmic fragments, provide the "echoes" throughout the tied region.

(For Example 38, see page 87)

Example 38: Rhythmic scheme for an unpublished fragment originating in the block at row 1 column 5 of the Final Rhythm Matrix (Example 14).



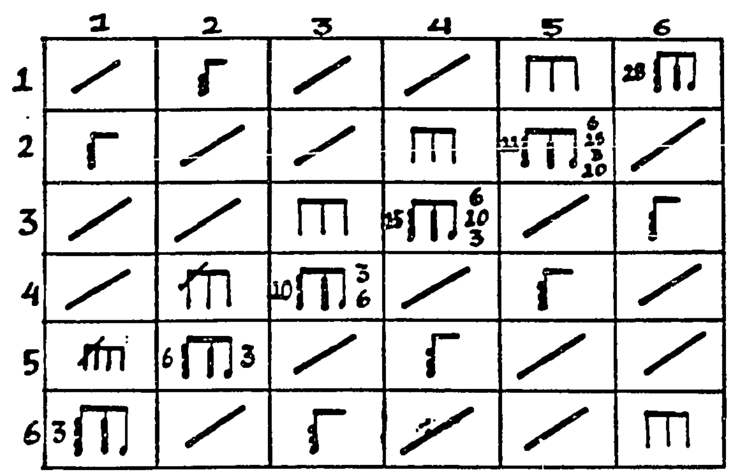
Example 38, although much more modest in size than fragment #17, shows another rhythmic fragment from the sketches which provides a contrasting "echo" section. This fragment is one of a group of 17 rhythmic fragments, which, although they originated within the Final Rhythm Matrix, were not used for the pitch realizations of the published fragments constituting Klavierstück XI. The "echo" attribute of this fragment originates with the E at row 1 column 5 of the Final Element Matrix (Example 37a). The "echo" section occurs in section 6 of Example 38, immediately after the quarter note duration in the fifth section. The "echoes" are provided by means of grace notes above a tied cluster.

Since the beginning of section 11 of fragment #17 (Example 35) is neither implied by the durations in the Final Rhythm Matrix, nor indicated by any element in the corresponding block in the Final Element Matrix (row 1 column 6 in Example 37), it can be understood in formal terms as an "insert" within the formal structure of fragment #17. Its effect, both aurally, and in the context of visual pianistic gestures, is that of a more sudden and dramatic interruption of the flow of musical activity than is achieved by the various other pauses in this fragment.[22] The length of the very active preceding section (10), which has the longest fundamental duration in the entire fragment (a double-dotted half note), and the pauses which separate every pianistic gesture in the "insert", both contribute to the effectiveness of this section.

The Construction of the Final Element Matrix

Example 39: Element matrix #1 (39a), addendum to element matrix #1 (39b), number matrix #9 (39c), number matrix #10 (39d).

a)



(For Examples 39b-d, see page 89)

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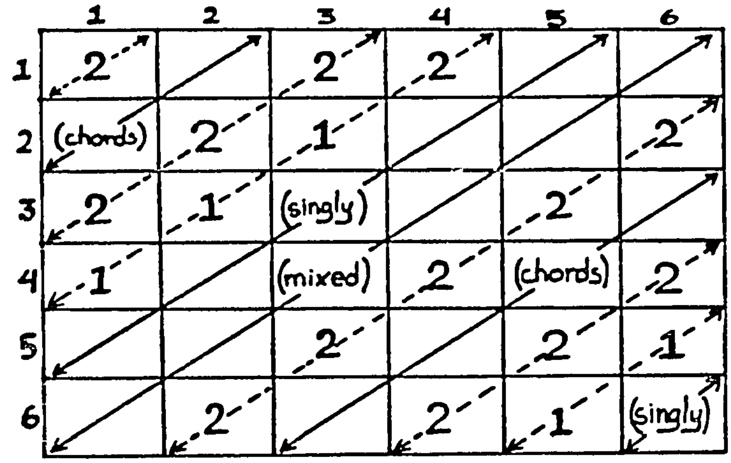
[22] Cf. Henck, H., pp. 79-80 concerning sudden stops that interrupt the flow of musical activities in many of Stockhausen's pieces.

(Example 39 continued):

b)



c)

3	6	3 6 10	3 6 10 15	3 6 10 15 21	3 6 10 15 21 28
6	3 10	3 6 15	3 6 10 21	3 6 10 15 28	
10	3 15	3 10 15	3 6 15 21	3 6 10 21 28	
3	3 6	3 6 10	3 6 10 15	3 6 10 15 21	3 6 10 15 21 28
6	6 10	6 10 15	6 10 15 21	6 10 15 21 28	6 10 15 21 28 3
10	10 15	10 15 21	10 15 21 28	10 15 21 28 3	10 15 21 28 3 6
3	6 3	10 3 6	15 6 3 10	21 6 3 10 15	28 10 3 6 15 21
6	10 6	15 6 10	21 10 6 15	28 10 6 15 21	3 15 6 10 21 28
10	15 10	21 10 15	28 15 10 21	3 15 10 21 28	6 21 10 15 28 3

d)

				15 6 10 3	21 6 15 3 10	28 3 15 10 6 21
				21 3 10 6 15	28 6 10 15 3 21	15 6 3 10
				28 3 21 15 6 10	15 10 3 6	21 10 6 3 15
mixed	3	6 3	10 3 6	15 10 6 3	21 15 10 6 3	28 21 15 10 6 3
singly	6 3	10 3 6	3	21 15 10 6 3	28 21 15 10 6 3	15 10 6 3
chords	10 3 6	3	6 3	28 21 15 10 6 3	15 10 6 3	21 15 10 6 3

Example 39a represents the initial conception of Stockhausen's plan to organize the contents of the diagonal blocks from lower left to upper right in the Final Element Matrix, shown in Example 37a. Example 39b has been included to illustrate elements present in the Final Element Matrix not found in Example 39a. The Final Element Matrix and the Final Rhythm Matrix (Examples 10, 14, 17, 20, 26, and 28 combined as one 6-column matrix) both have 6 x 6 dimensions. This allows them to be superimposed over one another. In this fashion both matrices were "added" to one another. The contents of each corresponding block within the two matrices were then combined with one another.

The diagonal lines in certain blocks of Example 39a indicate blocks into which Stockhausen planned to put various elements other than the three types of grace note groups inside the other blocks in the same example. The organization of the various elements in the Final Element Matrix, like the direction implied by the diagonal lines in Example 39a, was conceived as a series of diagonal arrays of similar elements. The direction of these arrays is from lower left to upper right. The first such array visible in Example 39a is the eighth note chords in the blocks at row 1 column 2, and row 2 column 1. The next diagonal, consisting of 3 blocks starting at row 3 column 1 and ending at row 1 column 3, contains elements other than grace note groups in the Final Element Matrix, although varying numbers of single grace notes comprise one of the 2 elements indicated in each block in this diagonal in Example 39b. Diagonals in Example 39b having dotted lines as their expressions indicate the other blocks with diverse elements not represented in Example 39a. The numbers in the blocks with dotted lines indicate how many different types of elements are present in the Final Element Matrix (Example 37a).

The three rows in columns 1-3, and the top three rows in columns 4-6 of Example 39d, are the final arrangements of numbers Stockhausen used in determining how many attacks would be present in various grace note groups which were to be added to certain blocks from the Final Rhythm Matrix. (This was done when the rhythmic schemes were written, with grace notes included, after the completion of the Final Rhythm Matrix.) All columns in Example 39c, moving from left to right, have an additional number in each block. In Example 39d Stockhausen changed the total amount of various numbers in each column in a manner similar to previously discussed serial orderings, such as associated with Example 34. For example, the top row of Example 39d consists of 1 number in column 1, 2 numbers in column 2, and so on until column 6 which has 6 numbers in the top block. This same arrangement of numbers is evident in the central diagonal from lower left to upper right in Example 39a. However, when this diagonal is examined in the Final Element Matrix (Example 37a) the positions of the numbers 6-3 and 10-3-6 in their respective blocks have been switched with one another. This changes the total number of different numbers which would have then been present in the second and third blocks in row 4 columns 2 and 3 of Example 39d, had the switched number sets been written in this position in Stockhausen's sketches. Instead of an arrangement which can be expressed by the numbers 1-2-3, the total number of separate numbers extracted from the initial additive series and then rearranged would be 1-3-2, representing the numbers 3, 10-3-6, and 6-3. This contrasts with 2-3-1, which expresses the totality of different numbers in the first three blocks of the secoond row (6-3, 10-3-6, and 3), and 3-1-2, which would be corresponding numbers for the first three blocks of the bottom row (10-3-6, 3, and 6-3).

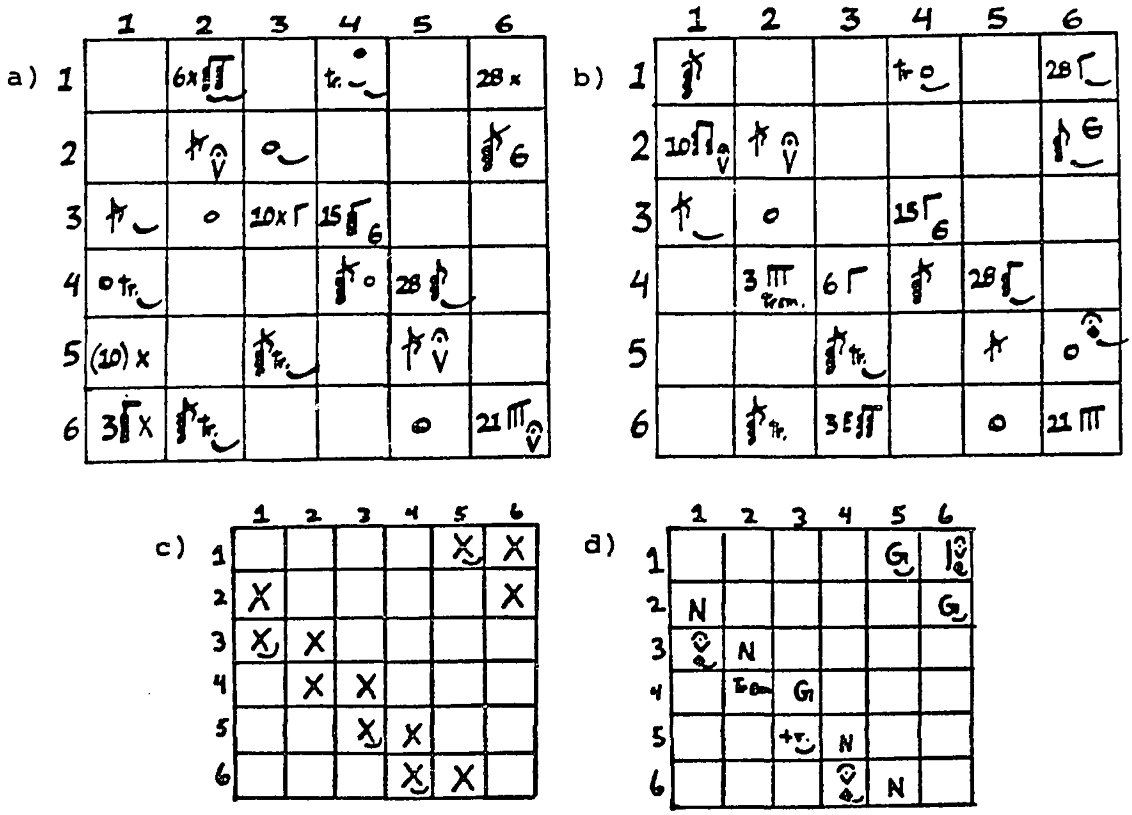
Example 39e: Blocks in Final Rhythm Matrix indicated to which were transferred additive series blocks from Example 39d. (This example is not found in Stockhausen's sketches.)

	1	2	3	4	5	6
1				(4) R3C4	(5) R2C5	(6) R1C6
2				(5) R6C6	(6) R2C4	(4) R1C5
3				(6) R4C5	(4) R3C6	(5) R5C4
4 mixed	(1) R6C1	(2) R4C3	(3) R5C2			
5 singly	(2) R5C1	(3) R3C3	(1) R4C2			
6 chords	(3) R2C1	(1) R6C3	(2) R1C2			

Numbers in parentheses in Example 39e indicate how many numbers from the additive series are included in those blocks of Example 39d which were transferred to blocks in the Final Element Matrix. The additional letters and numbers in Example

39e indicate the specific block in the Final Element Matrix, shown in Example 37a, to which the transference took place. For example, R4C3 located at row 4 column 2 in Example 39e indicates the 2 numbers in the corresponding block in Example 39d were transferred to the block at row 4 column 3 of the Fianl Element Matrix.

Example 40: Element Matrix #2 (40a), Element Matrix #3 (40b), step function patterns in Final Element Matrix (40c), step function elements in Final Element Matrix (40d).



Examples 40a and 40b are element matrices written out by Stockhausen before constructing the Final Element Matrix. A comparison of these matrices with Example 39a will reveal some identical elements along the long diagonal. Other elements appear in these matrices for the first time, and were transferred to corresponding blocks in the Final Element Matrix. The X's in Example 40c indicate the path taken by two series of elements Stockhausen decided to organize into the Final Element Matrix via a step function pattern.[23] The largest area so mapped out in Example 40c begins in the block at row 2 column 1. The function is to proceed from the first block down one block, across one block, down one block, and so forth until reaching the block at row 6 column 5. This step function is also visible beginning with the block at row 1 column 5 and extending to column 6 row 2. In this instance, the step function begins by moving across first, and then proceeds downwards. Example 40d contains those elements found in the Final Element Matrix (Example 39a) which follow the pattern indicated by the step functions of Example 40c.

Example 41: Composite of elements from Examples 39 and 40 occurring also in the Final Element Matrix. (This example is not found in Stockhausen's sketches.)

	1	2	3	4	5	6
1	40b	39c/d 40a	(39b)	40a/b	39a/c/d 40c	39a/c/d 40a/b/c
2	39c/d 40b/c	40a/b	40a	39c/d	39a/c/d	40a/b/c
3	40a/b/c	40b/c	39c/d 40a	39a/c/d	(39b)	39a/c/d
4	40a	39c/d 40b/c	39a/c/d 40b/d	40a/b	39a/c/d 40a/b	(39b)
5	39a/c/d 40a	39a/c/d	40b/c/d	39c/d 40c	40a/b	40b (o)?
6	39a/c/d 40a	40a/b/c	39a/c/d 40b	40c	40a/b (o)?	39c/d 40a/b

23Andree, p. 81 concerning functions and mappings.

The (39b) in blocks at row 1 column 3, row 3 column 5, and row 4 column 6 of Example 41 indicates the fact that although no elements are present in the corresponding blocks of any matrix in the sketches, elements were implied in these blocks by Stockhausen's serial ordering of the elements as shown in Example 39b. The composition of the elements in these blocks must have occurred during the time Stockhausen was constructing the Final Element Matrix.

The circle in the block at row 6 column 5 of Examples 40a-b and 41 is not present in the Final Element Matrix, nor are any of the other similar circles in Examples 40a-b. These circles may indicate places in the matrix where Stockhausen intended to make decisions later as to which element(s) he would choose. For example, the circle in the block at row 6 column 5 of Example 40a is replaced by the letter N in the corresponding block in the Final Element Matrix. The circle in the block at row 4 column 4 of Example 40a is replaced by a fermata in the Final Element Matrix. The circle in the block at row 3 column 2 of Example 40b is replaced by another letter N in the Final Element Matrix, shown in Example 37a.

The letter N is used by Stockhausen in the published score of Klavierstück XI to indicate a "normal" touch, as opposed to a legato or staccato touch. When the rhythmic fragments from the Final Rhythm Matrix are examined in their written out form with grace note groups, no indication of the N is ever used. Those fragments of Klavierstück XI having the performance indication N immediately following them, do not match with the corresponding derivative blocks in the Final Rhythm Matrix (Examples 10, 14, 17, 26, and 28) containing the letter N. However, all other elements in the Final Element Matrix which were added to the Final Rhythm Matrix are evident in the corresponding fragments of Klavierstück XI.

When elements from the Final Element Matrix (Example 37a) are compared with the contents of corresponding rhythmic fragments, or the fragments of Klavierstück XI, the character of certain elements often changes. For example, a trill indication in the Final Element Matrix may become a tremolo in a rhythmic fragment, as in fragment #11 (Example 59). The number of times a certain element is employed may also vary unless a specific number is indicated in the Final Element Matrix. The symbol ϴ occurs only once in the block at row 1 column 5 of the Final Element Matrix. Yet in the corresponding block of the Final Rhythm Matrix, shown in Example 14, it first occurs with the number 3 written directly under it. The number 3 indicates the same symbol should occur 3 times in the same block. It thus occurs at that point and twice again in the same block. Elements from the Final Element Matrix often appear in rhythmic fragments which have no such elements in corresponding blocks of the Final Element Matrix. The "echoes" in section 11 of fragment #17, shown in Example 65, are not indicated either in the Final Element Matrix or in the Final Rhythm Matrix.

The 19 Fragments of Klavierstück XI

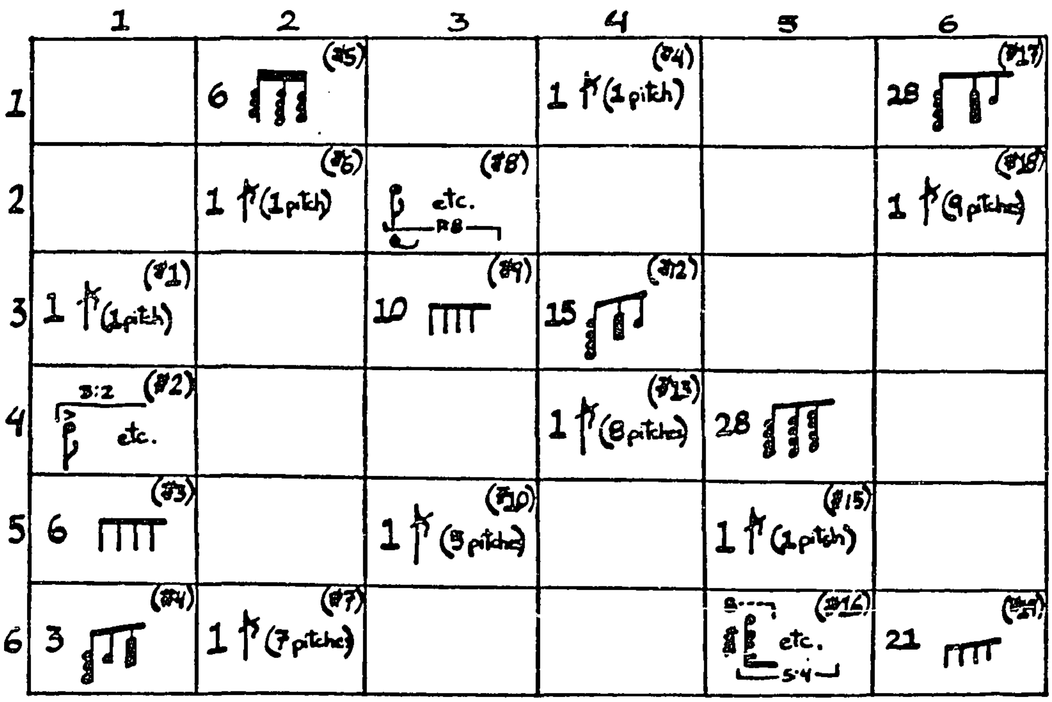
Example 42 indicates those blocks from the Final Rhythm Matrix which were chosen to provide the rhythmic material for the 19 fragments of Klavierstück XI.

Example 42: Matrix indicating blocks in Final Rhythm Matrix used as rhythmic sources for the 19 published fragments of Klavierstück XI. (This example is not found in Stockhausen's sketches.)

	1	2	3	4	5	6
1		#5		#11		#17
2		#6	#8			#18
3	#1		#9	#12		X
4	#2			#13	#14	
5	#3		#10		#15	
6	#4	#7			#16	#19

The X at the right edge of the block at row 3 column 6, taken from the corresponding block in the Final Rhythm Matrix, indicates the fact that Stockhausen originally intended to use this block for composing fragment #19. His rejection of this block may be due to certain numerical relationships which are created by the choice of the block at row 6 column 6 instead. When one considers the number of blocks in each column of Example 42 which were employed in composing fragments, all columns have 3 such blocks except column 1, which has 4 such blocks. All rows in the same example have 3 such blocks except for row 6, which, like column 1, also has 4 such blocks. Thus both the column and row having 4 chosen blocks occupy outer edges of the matrix. This would not be the case if the block at row 3 column 6 were chosen instead. Another factor in the choice of the block for fragment #19 is the large size of Fragments #14 and #17, each of which contains six grace note groups from the Final Element Matrix. The block at row 6 column 6 of the Final Element Matrix contains 5 groups of grace notes, as opposed to only 4 grace note groups in the block at row 3 column 6. The number of grace note groups in the chosen block for fragment #19 more nearly matches the total size of the grace note groups of fragments #14 and #17, especially since it has the largest group of grace notes, 21. The largest grace note group in the block at row 3 column 6 contains only 15 attacks. The character of the initial 21 grace notes in the block at row 6 column 6, being single pitches, also provides a balancing contrast with the initial grace note attacks of fragment #14 (chords) and fragment #17 (mixed).

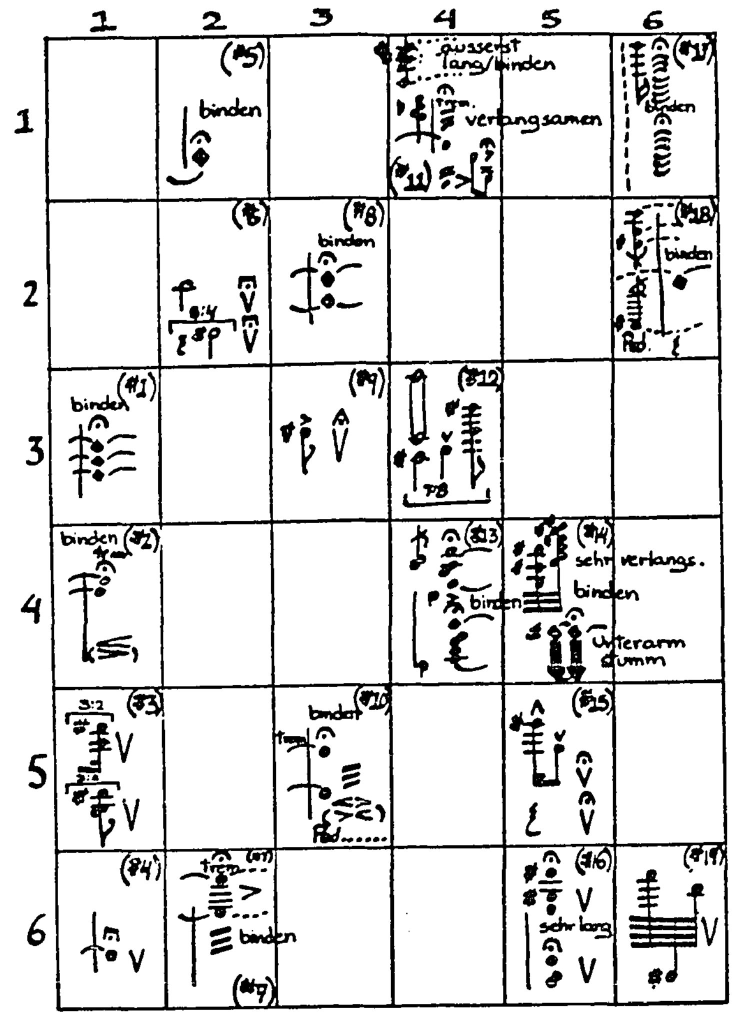
Example 43: Matrix illustrating beginning attacks of the 19 fragments of Klavierstück XI. (This example is not found in Stockhausen's sketches.)



Example 43 illustrates the carefully considered, serially organized appearance of the beginning attacks in the 19 fragments of <u>Klavierstück XI</u>. The serialized appearance results from the variety of different numbers associated with both the grace note groups and single grace notes. Fragments #2, #8, and #16, having no grace note groups or single grace notes at their beginnings, also have a serialized character when considered together. Fragment #2 starts with an accented eighth note in a rhythmic proportion of 3:2. Fragment #8 varies by beginning with an unaccented eighth note under which another pitch is silently depressed; the rhythmic proportion is 7:8. Fragment #16 begins with a sixteenth-note chord of 3 pitches; the rhythmic proportion is 5:4. The lack of repetition in the beginnings of these fragments, as well as the varying similarities, contribute to the serialized appearance.

(For Example 44, see page 100)

Example 44: Matrix illustrating ending attacks of the 19 fragments of <u>Klavierstück XI</u>. (This example is not found in <u>Stockhausen's</u> sketches.)



An examination of the various endings of the fragments of <u>Klavierstück XI</u> also reveals a serialized character similar to that of the fragments's respective beginnings. Fragments #4, #13, and #16 have principal ending elements not found either in the Final Element Matrix or in the rhythm fragments with grace notes used in composing the pitch realizations. These endings can therefore be seen as additional "little flowers" that Stockhausen has added at his own discretion. The note heads without stems in sections 9 and 11 of fragment #17, shown in Example 32, are other "little flowers".

Summary

Chapter 2 of this study has analyzed several serial systems, including number matrices, rhythm matrices, and element matrices, which, although being intricately constructed independently of one another, were then combined in various ways. Additional serial systems, which often interrelate in various ways, were analyzed concerning the construction of fragment #17. The particular manner in which Stockhausen produced more sophisticated matrices by combining two or more together into a single matrix, often transforming various internal elements of some matrices during the combinative process, can be understood as a form of serial polyphony he employed in a unique way when composing Klavierstück XI.

One primary aspect seems evident, relative to the study of Stockhausen's sketches for Klavierstück XI and concerning his compositional methodology: the tremendous extent to which Stockhausen further sophisticates systems, in their respective realizations, after having thought through the particulars of their initial conceptions with great depth of logic. The discussions associated with Examples 22 and 24 seem particularly relevant to this regard.

A thoughtful consideration of Stockhausen's decisions concerning alterations or adjustments made during the processes of both sophisticating internal particulars and initiating changes which often affect the formal structures of various fragments, may also provide a very illuminating, instructive insight into the depth of Stockhausen's musical considerations. The decision to reject the contents of Example 33a seems particularly relevant, since a completely different serial system was then substituted for determining the contents alternate sub-groups--only 28 attacks, as required by the element matrix number, were retained from the initial conception concerning the basic structure of the beginning of fragment #17. The final version for the initial 28 attacks, shown in Example 33c, exhibits a pitch density that contrasts sharply with Example 33a both sonically and pianistically.[24] The profound extent to which Stockhausen evaluates and often reshapes materials that other composers might allow to remain unchanged, reveals the presence of a composer who, although greatly occupied with multitudinous architectural considerations relative to parameters that are often conceived both polyphonically and with mathematical precision, is foremost a great musician who molds and shapes the resultant materials of his systems into music of great profundity and astounding variety.

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[24] Whether or not Stockhausen considered the pitch realization necessary for Example 33a to be excessively difficult must remain speculation, although this is doubtful due to the performance instruction that grace notes be played "as fast as possible". The word "possible" apparently allows for mini-pauses, when necessary, within grace note groups having wide leaps which make it impossible for the performer to maintain a constant, uninterrupted tempo throughout the entire grace note group. This is demonstrated in the performance of <u>Klavierstück XI</u> by Aloys Kontarsky on CBS Records #32 11 0034-1.

CHAPTER THREE

THE TRANSLATION OF RHYTHM INTO PITCH

The pitch structure of Klavierstück XI was composed by a method which reflects, but does not always exactly equal, the harmonic relationships of overtones to one another within the natural harmonic series. This was achieved by translating the duration proportions of the rhythmic structures into resultant pitch fluctuations which correspond intervalically with intervals having identical numerical relationships to one another within the series.[25] If the fundamental tone, or partial, is given the numerical value of 1, and successive ascending partials are numbered from 2 to 12, it can be shown that the chronological numbers also approximate the vibration proportions of the partials to both the fundamental and to one another. The second partial in the harmonic series vibrates twice as fast as the first partial, and the third partial vibrates three times as fast the first partial. The third partial also vibrates three times during the time the second

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[25]Stockhausen stated, in a post card to the author dated March 11, 1983, that "Electronic Study I is the first example for identical proportions of the overtone series for pitch, durations, dynamics, form. Gruppen is another." The fact that both mentioned works were written early in Stockhausen's career may be given significance by his statement to the author in Denver, 1980, that "It was not the music of Webern--which to this day remains the most beautiful and mysterious music of that era--but what I realized I could do with the atomic structure of sound, that caused me to decide to become a composer."

partial vibrates twice. Thus the proportion between the second and third partials is that of 2:3. The proportion becomes 3:2 if the third partial is considered first. The interval between the second and third partials is a perfect 5th. When Stockhausen encountered a duration proportion[26] of 2:3 or 3:2 between any two notes in the rhythmic structures, he used this proportion to imply an interval of a perfect 5th, with a possible "adjustment"[27] of the interval to that of a diminished or augmented 5th. Similarly, the 2:1 duration proportion either yielded an octave or an "adjustment" resulting in a major 7th or minor 9th.

Early in the sketches Stockhausen presented the concept of more than one interval being the possible outcome of a duration proportion pitch realization: a comment concerning the

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[26]On pp. 50-51 of New Musical Resources, completed in 1919, Henry Cowell prophetically discussed the interrelationship of duration and pitch: "Assume that we have two melodies moving parallel to each other, the first written in whole notes, and the second in half-notes. If the time for each note were to be indicated by the tapping of a stick, the taps for the second melody would recur with double the rapidity of those for the first. In now the taps were to be increased greatly in rapidity without changing the relative speed, it will be seen that when the taps for the first melody reach sixteen to the second, those for the second melody will be thirty-two to the second. In other words, the vibrations from the taps of one melody will give the musical tone C, while those of the other will give the tone C one octave higher. Time has been translated, as it were, into musical tone. . . . a parallel can be drawn between the ratio of rhythmical beats and the ratio of musical tones by virtue of the common mathematical basis of both musical time and musical tone. The two times, in this view, might be said to be 'in harmony', the simplest possible."

[27]Stockhausen, in conversation with the author, Paris, 1980, repeatedly referred to these pitch variables as being "adjustments", and considered this to be "the most fascinating aspect of the study".

possibility of 2 intervals accompanies a musical example consisting of the pitch d with c sharps written a major 7th above and a minor 9th below it. This also indicates the possibility of calculating the interval either above or below the initial pitch. The "adjustments"[28] were made in order to maintain atonality by avoiding octaves or triadic harmony whenever Stockhausen considered it appropriate. A possible genesis for Stockhausen's practise of allowing a duration proportion to imply more than one possible pitch is to be found in the following quote from one of his most famous articles:

> Such a switch, from "pointillist" to "statistical" perception of time has become a further occasion for the statistical composition of fields. But this means that the elements themselves are no longer presented as discrete degrees of some scale or other (whether as discrete pitches or durations . . .). rather, a field-size in the sense described above, is substituted for each discrete value . . . Such field-sizes are now the "elements", and composition thus includes the statistical character of mass-structure among the elements.[29]

The field-size of possible pitch fluctuations resulting from duration proportions is three chromatic pitches. One of these pitches expresses the basic interval which results from harmonic relationships within the overtone series, and the other two are chromatic adjustments of this interval a half-step in either direction.

In calculating duration proportions between adjacent

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[28] In a letter to the author, dated March 23, 1984, Stockhausen made a further comment concerning adjustments, relative to "the generation of pitch through rhythm in Klavierstück XI. They are not all 'off' but approximate in the case of octaves (and sometimes fifths), and that is exactly how our hearing sense has historically evolved."

[29] "...How Time Passes..." Die Reihe 3 (1959): 14-15.

pitches in the 19 fragments of Klavierstück XI, the largest possible common denominators will generally be used. For example, the duration proportion 7:3 exists between a double-dotted quarter note ( ) and a dotted eighth note ( ). In this instance sixteenth note durations are the common denominator. The total number of sixteenth note durations, contained within the respective note values of the two above mentioned durations, determines the numbers, 7 and 3, for each side of the duration proportion 7:3. Sixteenth notes are the common denominator because they are the largest common note values to which both a double-dotted quarter note (=) and a dotted eighth note (=) can be reduced.

The 7:3 duration proportion can also exist between a double-dotted half note ( ) followed by a dotted quarter ( ). The duration proportion in this instance is determined by eighth note durations: (=):(=). If sixteenth note durations are considered as common denominators, the same two notes can be understood as having a 14:6 duration proportion: (=):(=). A comparison of this alternate 14:6 proportion with the proportion 7:3 reveals an identical interval in the harmonic series.[30] Thus the

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[30]The numbered harmonic series located on page 33 of Appendix II, has been provided for matching the numbers which express the duration proportions with the respective partials having identical chronological numbers in the harmonic series. The interval between the two indicated partials is then observed. The number 7 from the duration proportion 7:3 yields the pitch b♭ in this particular series; the number 3 yields the pitch g, located a minor 10th below the b♭. Matching partials with the 14:6 duration proportion numbers also yields a minor 10th between pitches located an octave above the pitches for the 7:3 proportion.

largest common denominator for calculating duration proportions are determined by the largest identical note value to which both durations can be mutually reduced. The largest possible common denominator produces the smallest possible numbers for expressing duration proportions. Smaller numbers are used to facilitate analysis when long successions of durations in the fragments of Klavierstück XI are analyzed at the end of this chapter. The exception to this practise will occur when it is revealing to compare certain durations by expressing the total numbers for a smaller common note value to which two durations can be reduced. For example, this occurs in the analysis in Example 60 between the first two durations in the bass clef. The 12:12 duration proportion, if reduced to its smallest possible numerical expression, would be 1:1. The duration proportions 12:12 and 1:1 both express the note value of a dotted half note. Using the 12:12 expression for this duration proportion is revealing, since an examination of the derivative rhythmic scheme, shown in the same example, proves both durations in the music fragment to be augmentations of corresponding durations having smaller note values in the rhythmic scheme. In order to compare the original sizes of the two durations, as well as their respective augmentations, sixteenth notes are necessary as common denominators.

As shown in the above discussion, Stockhausen used the proportions of harmonic relationships between non-adjacent partials for determining pitch fluctuations. For example, the intervallic relationship of the 5th partial to the 1st partial is that of a major 17th. Since Stockhausen often changed the register of pitches determined by duration proportions, a major 17th, for example, could be reduced to a major 10th. The 2nd and 3rd pitches (d# and g#) of fragment #6, shown in Example 54, have a 5:1 duration proportion, which generates the interval of a major 17th or major 10th. If e^b as the enharmonic spelling of the d# is considered, the interval between the two pitches can be understood as being originally a major 10th, which, during composition, was adjusted upwards one half-step from the pitch g to g#.[31] The registral disposition of these two pitches also illustrates another aspect of Stockhausen's use of rhythmic duration proportions in relation to the natural harmonic series: notes with longer values have lower registers than adjacent notes with shorter values. This reflects the fact that lower partials, especially the fundamental, or first partial, generally sound both louder and longer than the higher partials in most ordinary sounds.

Since the pitches of Klavierstück XI were derived from duration proportions, and not 12-tone sets, a discussion of the relationship of the duration proportions of the original rhythmic schemes to the pitches of the separate voices within the framgents will be helpful in understanding Stockhausen's procedure. Fragment #1 and portions of other fragments will be

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[31]In this particular instance the g# was g natural in the original pitch realization of this fragment, and the following pitch was g#. When Stockhausen wrote out the fragment with grace notes included, he changed the third pitch to g# and altered the fourth pitch down a half step to g natural, as shown in Example 54.

analyzed in order to make the reader familiar enough with the proportional system so that an informed study of the analysis of all the rhythmic schemes and the corresponding pitches of the 19 fragments can be achieved. Examples 49-67 include an analysis of the relationship of the duration proportions to their resultant pitches, with all intervallic adjustments identified and measured.

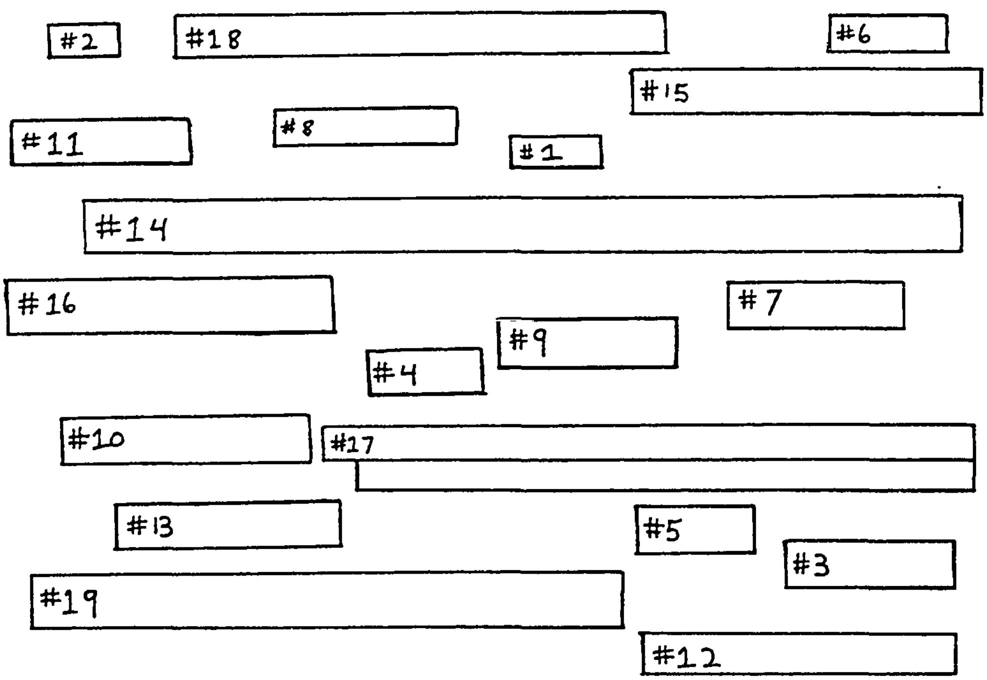
The rhythmic schemes are written directly above their pitch realizations in the examples. This facilitates the analysis because the separate voices of the music fragments are easily identified by their counterparts in the rhythmic schemes. The voices in the rhythmic schemes are always written out on separate horizontal planes, while the voices of the musical fragments often intermix or cross lines during their respective motions. This is especially apparent in the larger, more polyphonically complex fragments. The fact that the durations of the rhythmic schemes are exactly half those of the music fragments in all examples is irrelevant since the proportions of these durations are the same in both instances.

After its completion, Stockhausen retained identical note values from the Final Rhythm Matrix when he composed the pitch realizations for fragments 1-16 and 18-19 without grace note groups.[32] He then doubled the note values for the 19 fragments when writing their pitch realizations with grace note groups included. He then cut the completed versions out

[32]Fragment #17 was originally composed with grace note groups included, but the note values were still one half those of the original manuscript.

separately in order to disperse them throughout the original manuscript of *Klavierstück XI*. Although the arrangement of fragments according to relative sizes is basically identical to the published score of *Klavierstück XI*, some individual fragments occupied different positions relative to one another in the original manuscript.

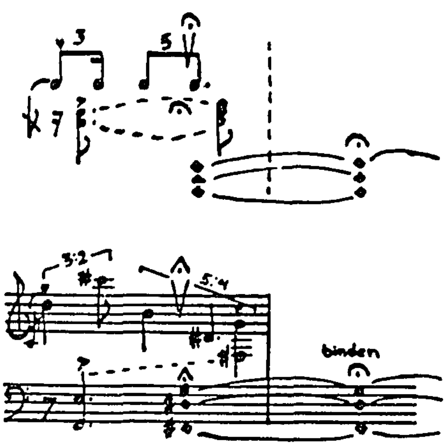
Example 45: Spatial disposition of the 19 fragments in the original manuscript of *Klavierstück XI*. (Relative sizes of individual fragments in this example only approximate those of the original manuscript.)



Example 45 illustrates the spatial disposition of the 19 fragments in the original manuscript of *Klavierstück XI*. The interested reader may compare this example with the published score of *Klavierstück XI* to determine the alternate spatial disposition Stockhausen chose for the published score. Examples 49-67 present the fragments in their chronological numbering shown in Example 42. This makes it possible to easily locate the individual fragments in the published score. After numbering the fragments in the published score, their spatial disposition can then be compared with Example 45.

Since their pitches were not determined by a varying proportional process, grace note groups and notes in small type will generally be omitted from the following examples. The pitches for grace notes, grace note groups, and notes in small type were "improvised".[33] Their omission will facilitate the reader's matching of the rhythms to the pitches of the musical fragments written directly below the rhythms.

Example 46a: Fragment #1 with rhythmic scheme directly above it.



The two segments of the top voice, in either the rhythmic

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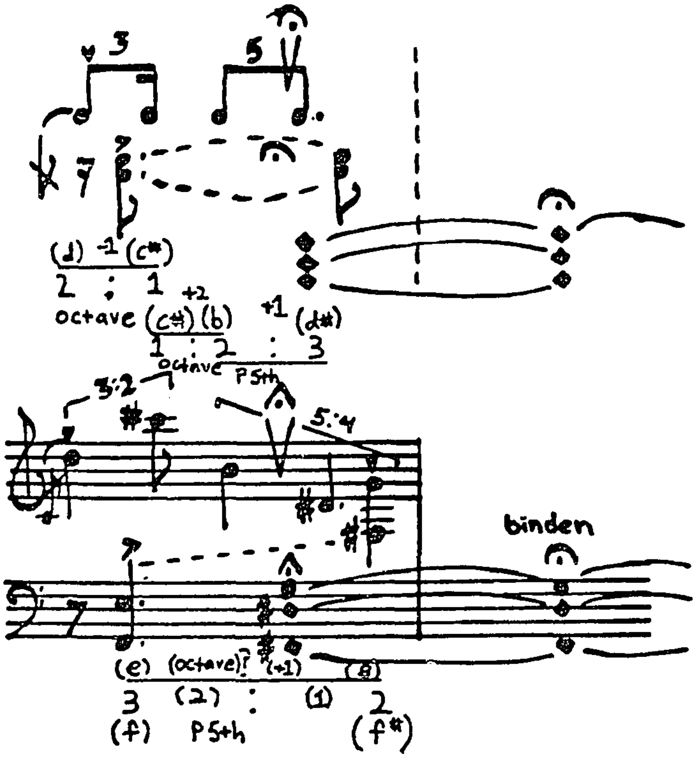
[33] Stockhausen in conversation with the author, Denver, 1980. Pitch improvisation is also logical in this context because the proportion 1:1, being identical with that of several notes in small type which have identical values and follow one another successively, also yields a "free" change of pitch in the duration proportion system Stockhausen used in composing the pitch realizations of notes in larger type. Notes in larger type, in the published score, originate in the Final Rhythm Matrix; notes in smaller type generally originate in the Final Element Matrix, as discussed in Chapter 2.

or pitch version in Example 46, both yield the proportions 2:1 between the first two durations, 1:2 between the second duration and the third, and 2:3 between the third and fourth durations. In this and subsequent examples, the analysis of the duration proportions and Stockhausen's adjustments is given above and below the staves of the pitch portion of the examples.

Example 46b: Fragment #1 with analysis of duration proportions written above and below the pitch realizations.



Example 46b includes the duration proportion analysis in the basic form it will take in Examples 49-67 . The horizontal line above the duration proportions can be used to identify duration proportions, and links related pitches and adjustments to them. The pitches, although not always indicated, will often be written in parentheses directly above both the horizontal line and the related duration proportion number. The number which indicates the size of the intervallic adjustment will be indicated with a mathematical symbol(+, -, or =) and centered directly above the colon for the duration proportion. A +1, for example, will indicate an adjustment one half step upwards; conversely, a -2 will indicate two half steps downwards. Additional information may or may not be placed above or below the horizontal line, such as the name for the interval the duration proportion implies, or identification of specific pitches in the vertical dimension when necessary. As the reader becomes increasingly familiar with the intervals associated with the duration proportions, writing the name of this interval below each proportion becomes less necessary. In the larger fragments with many notes having small values, space also becomes problematic when writing information beyond the duration proportion and adjustment number.

As indicated in the analysis shown in Example 46b, the 2:1 duration proportion between the pitches d and c# implies the interval of an octave between them. The number -1, centered above the colon and between the letter names for the two resulting pitches, indicates an adjustment of the octave interval to that of a major 7th, one half step smaller. The +2 between the pitches c# and b in parenthesis indicates the 1:2 duration proportion yielded an octave which was then augmented 2 half steps, avoiding repetition of the pitch c already written in the same fragment.

The 2:3 duration proportion in the top voice of Example 46b implies a resultant interval of a perfect 5th. The +1 in the analysis indicates this interval was adjusted a half step further downwards to the pitch d#. When Stockhausen first composed this fragment he wrote the pitch e^b instead of d#. Since the pitch e is a perfect 5th below the pitch b directly above it, the presence of an e^b would illustrate that Stockhausen was thinking in terms of a 5th, and not a 6th, which is what the d# would otherwise imply.[34]

An examination of the note values of the first duration in the lower voice (pitches: f and e in the vertical dimension) and the second duration in the same voice (pitches: f# and g in the vertical dimension, found in the treble staff and connected to the first duration by a dotted line), reveals a duration proprotion of 3:2 if the respective note values are solely considered. However, the pitches of the second duration are composed as if it was an interior part of the bracketed 5:4

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[34]Stockhausen changed the vast majority of flats to sharps when he wrote out the pitch realizations of the fragments with grace notes included. This eliminates the occasional appearance of tonal relations between individual pitches the presence of flats can occasion. For example, the pitch g followed by the pitch b^b can form a minor 3rd. This can imply a triadic appearing relationship with the pitch g as a root. But changing the b^b to a# changes this appearance by turning the interval into an augmented 2nd. In the analysis of the pitch realizations at the end of this chapter, the enharmonic spellings of intervals should thus be considered equivalent and interchangeable. For example, a duration proportion between two pitches of 7:3 calls for an interval of a minor 10th. The written expression for this interval, if placed under the horizontal line in the analysis, would be m10th. An = sign above the horizontal line will still be centered there, signifying an unadjusted interval, if the pitches are g and a# instead of g and b^b.

rhythmic portion of the upper voice in the same staff. Whether or not it is to be played as belonging within the 5:4 rhythm, an examination of the rhythmic scheme in the same example shows this duration was originally an eighth note having a 3:2 duration proportion relationship with the dotted eighth note in the same voice. If the pitches of this second duration had been composed with the 3:2 duration proportion in mind, one of the pitches of this duration would have to be a perfect 5th, or an adjustment thereof, above the first duration. The fact that it would need to be above, and not below the first duration results from the practice of having the smaller note values higher than larger note values in the same originating duration proportion. A perfect 5th above the pitch f in the first duration would be the pitch c; adjusted it could become either b or c#--pitches already used in this very short fragment. However, it could have been adjusted to the pitch b^b, and the lower tone of the same duration could have been the pitch a, located a minor 9th below the b^b. This would have avoided the repetition of pitches and also would be playable above and below the pitch d#, which occurs in the lower stave immediately before the pitch d# in the upper stave. The diamond shape of the note heads, coupled with the absence of a stem, indicates the tones of this duration are to be depressed silently. As these tones are tied to another identical duration, extending beyond the length of the original rhythmic fragment, the overtones of the corresponding piano strings will be sounding after the pitches of the other durations of this fragment have died away. This effect is insured by the fermata above the second of the two tied durations. Perhaps Stockhausen originally considered the pitches a and b^b for inclusion in this duration, but preferred the sonority that is produced by the pitches f# and g. The pitch g, in relation to the d# below it, is a major third higher. The pitch b^b would be a perfect 5th above if the d# were considered as being sonically identical with its enharmonic double, e^b. The sonority thus produced would have tonal implications when considered in conjunction with the lowest pitch in the fragment (f), which would be the root of V in the key of B^b. The a-b^b sonority would reinforce the dominant quality of the pitch f, since its root, the pitch a, would be located a major 3rd above the pitch f in the bass clef. The pitch f# which Stockhausen chose instead pulls against the low pitch f in the bass. The pitch g in the same duration has a triadic relationship with the pitches d and b in the upper voice, but this is offset by the intervening pitches and their resulting sonorities. This includes the pitches c# and d# in the upper stave, and f and e in the lower stave.

The numbers (2) and (1), placed in parentheses on either side of the colon for the 3:2 duration proportion below the lower voice in Example 46b, indicate the fact that Stockhausen composed the pitch realization for the second duration of this voice as though it resulted from a 2:1 duration proportion. The (+1) in parenthesis above the horizontal line indicates the degree of adjustment made upon the substituted duration proportion. The question mark to the left of the adjustment number indicates the absence of normal intervallic relationships between duration proportions and resultant pitches. The word (octave) in parenthesis above the horizontal line indicates the interval the substituted duration proportion implies.

Since Stockhausen had free control of register when composing the pitch realizations, a duration proportion such as 3:1 could result in either a perfect 12th or a perfect 5th, if the pitch realization of the second duration is dropped an octave. This is assuming the second pitch is composed above the preceding pitch, which is in accordance with Stockhausen's general practise of giving longer durations lower pitch realizations than shorter durations. But if Stockhausen were to drop the second pitch <u>two</u> octaves in this instance, the resulting interval would be a perfect 4th lower than the preceding pitch. As this type of situation sometimes occurs, I have used the symbol of an arrow written directly to the right of the letter for the pitch involved. This clarifies the fact that the interval for the resulting pitch was calculated in the opposite direction, relative to the preceding pitch, in which it is finally situated. In Example 55 the arrow found accompanying the duration proportion above the first duration in the treble clef is pointing downwards. This indicates the pitch c was calculated a perfect 5th <u>above</u> the pitch f in the first duration, and then placed a perfect 4th below the f.

The fact that Stockhausen chose to cross voices in Example 46b and used a duration proportion other than the one existing between the two note values involved, illustrates that, from one step to another during the formative process of composition, he will occasionally alter the attributes of certain particulars in order to achieve his musical goals. The discussion of the further sophistication of rhythmic blocks within the rhythm matrices of Chapter 2 is a good example. The repeated note of row 1, shown in Example 28, which Stockhausen considered and then rejected, is another.

In polyphonic passages the overlapping sequence in which the durations of the various voices occur may result in cross relations between the duration proportions of the various voices, and thus affect the choice of resultant pitches. A duration from one voice in the rhythmic schemes may then relate to a duration in another voice, rather than result from the duration proportion formed with the succeeding duration in the same voice. This often happens when a duration in a second voice enters before the succeeding duration in the first voice occurs. So many such overlapping relationships can occur in polyphonic passages as to render it difficult to determine if only one or perhaps several duration proportions relate to the same duration. This can occur with single pitches or vertical sonorities of two pitches or more. In such instances the analysis for all possible duration proportion pitch realization relationships has been written out.

Although this situation happens often in various fragments, the first 6 durations in Example 55 are a typical instance. The initial vertical sonority in fragment #7, consisting of the pitches f-e, exhibits duration proportion pitch realization relationships with the two succeeding durations. These have pitch realizations consisting of the pitches c and g-g#-a respectively. Like the f-e sonority, the g-g#-a sonority originates in the *lower* voice of the rhythmic scheme written above the pitch realizations. The pitch c immediately following the f-e sonority is the first duration from the *upper* voice of the rhythmic scheme. As shown in the duration proportion pitch realization analysis, the pitch f in the initial duration exhibits a 1:3 duration proportion relationship with the pitch c. The arrow next to the (c)↓ indicates this pitch could have been originally conceived as the pitch c''', located a perfect 12th *above* the pitch f', and then could have been dropped 3 octaves downwards when realized as the pitch c'.[35]

The pitch e in the initial duration of fragment #7 is shown in the analysis having an unadjusted duration proportion pitch realization relationship (=) with the pitch a in the g-g#-a sonority. In a similar manner the fourth duration (pitches: d-a#-d#) exhibits an unadjusted duration proportion pitch realization relationship (3:4) with the g-g#a sonority, although it originates in the *middle* voice of the rhythmic scheme. As the analysis indicates, both the pitch c and the g-g#-a sonorities exhibit normal duration proportion pitch

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[35] The only other explanation for the pitch c' at this point in the fragment is that Stockhausen randomly chose it from the available chromatic space. Since the initial entries of new voices, or reentries of voices which have dropped out temporarily, often exhibit duration proportion pitch realization relationships with immediately preceding sonorities, such durations are considered in this way in the analysis.

realization relationships with the durations which immediately follow in their respective voices.

Two duration proportions may be logically associated with the 23rd duration of the upper voice in Example 64. The pitch d, being the lower member of a vertical sonority consisting of the pitches d-d#, is pointed out by an arrow in the example, as is the corresponding duration in the rhythmic scheme. Although this pitch belongs to a vertical sonority found in the lowest register employed at this point in the fragment, an examination of the rhythmic scheme reveals it to be a member of the upper voice which suddenly changes register for this one duration. An immediate return to the upper register, with the vertical sonority f#-f, then follows in the same voice. Although the correct duration proportion, 5:1, is written in the analysis <u>above</u> the music staves, an alternate duration proportion linking the pitch d with the pitch g is also written <u>below</u> the d. The vertical sonority which follows the d-d# sonority in the same staff contains 3 pitches, g-c#-a. This sonority suddenly appears in the bottom voice at this point in the fragment, and is therefore presented in the analysis having possible duration proportion pitch realization relationships to the d-d# sonority (duration proportion: 5:7), to the preceding b-a vertical sonority from the same voice in the rhythmic scheme (duration proportion: 6:7), and to the pitch e which immediately precedes it in the upper staff (duration proportion: 11:14).

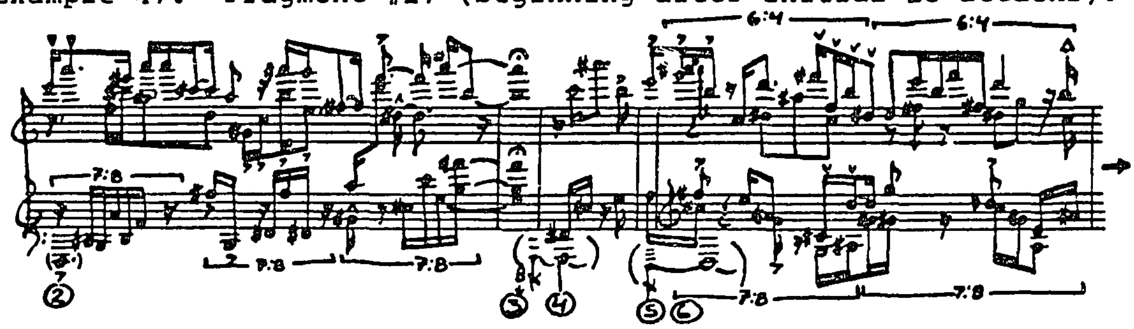
A discussion of pitch priority in vertical sonorities will conclude the present discussion of duration proportion analysis. The bottom pitch of a vertical sonority is often, but not always, used to calculate the interval for a succeeding pitch realization. An interior pitch in a vertical sonority may be used, but the top pitch is more commonly chosen as an alternative to the bottom pitch. The top pitch in the first duration of Example 63 relates in this fashion to the bottom pitch of the second duration. The beginning durations in Example 62 illustrate the fact that the topmost pitches in a succession of vertical sonorities can result from the duration proportion process, while the additional tones in the vertical sonorities continue to fill out chromatic space with sonorities Stockhausen prefers.

Often when one pitch of a two-note vertical sonority was determined by a previous duration proportion, the second pitch of the same sonority is used to calculate a pitch in the following sonority. This process can be observed between the first three durations of the upper voice in the last (9th) barred region of Example 62. As indicated in the analysis, the d# of the a-d# sonority has an unadjusted duration proportion pitch realization relationship with the pitch g in the following a#-g sonority. The pitch a# in the a#-g sonority is then used in determining the pitch realization of the 4:1 duration proportion which exists between the a#-g sonority and the next duration from the rhythmic scheme. The following pitch in the upper voice, as it originally appeared in the sketches, was b'''. This results from a +1 adjustment to the 4:1 duration proportion relative to the pitch a♯ in the previous sonority. Whether the original pitch b''' was changed to d'''' by Stockhausen, or was perhaps a copying error must remain speculation.

A fundamental aspect of Stockhausen's pitch adjustments was the continuous consideration of the constant filling out of chromatic space in order to maintain atonality and avoid the repetition of pitches. This consideration is also a characteristic of orthodox 12-tone composition. Since Stockhausen had no tone row to consider, nor combinations of several rows occurring simultaneously, recently sounded pitches attained the same status. The additional improvised pitches which accompany the pitch realizations of the derivative rhythmic scheme, shown in Example 59 and analyzed later in this paper, are a very instructive example of how Stockhausen considered the filling out of chromatic space while composing Klavierstück XI. The pitch adjustments throughout the entire piece would not otherwise have been necessary.

An exception to the avoidance of repetition of pitches occurs between the fourth and fifth durations of the upper voice of the beginning of fragment #17, immediately after the initial group of 28 notes in small type, omitted here.

Example 47: Fragment #17 (beginning after initial 28 attacks).

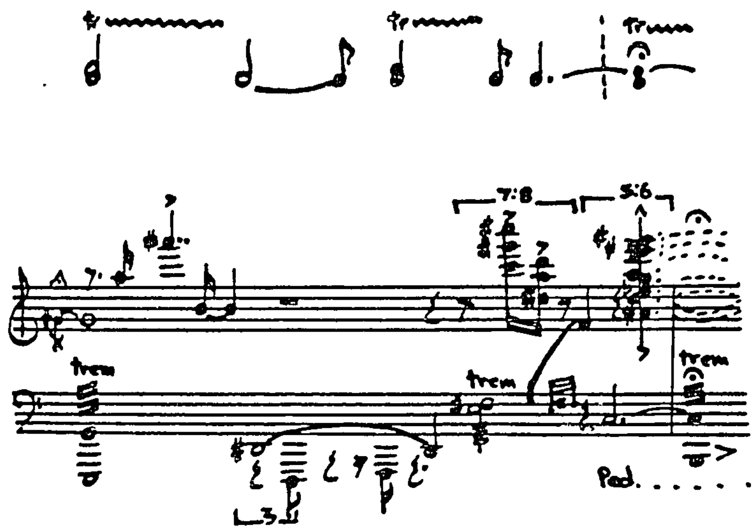


The pitch c'''' occurs twice at this point, and the same pitch also occurred as the second duration in the same voice. Stockhausen here, and again later in the same fragment, created, for a brief moment, a certain "melodic" contour that catches the listener's ear in a manner otherwise impossible without the repetition of pitches.

Fragment #17, as has been shown in Chapter 2, is also exceptional in the manner in which its rhythmic structure was composed. The pitch realization analysis of this fragment, shown in Example 65, will also reveal more exceptions to the associated duration proportions than any other fragment. The extremely high incidence of these exceptions indicates the fact that Stockhasuen regularly abandoned the duration proportion process, and then came back to it, all through the fragment. The very dense texture of this fragment, often having three or four voices that cross with one another, may be the principal reason for the many duration proportion exceptions, in addition to the fact that alternate formative processes of several kinds are also involved with this fragment. Indeed, the mixture of free intervals and regularly composed pitch realizations resulting from duration proportions seems to set off in harmonic relief the fact that this fragment is unique and special. The intense drama which pervades the mood of this fragment is also highlighted by the inclusion of oft-repeated f#'s, which form a kind of pedal point in the bottom voice of section 13 of this fragment, shown in Example 65. The seven times the f#'s are sounded is equal to the number of durations from the Final Rhythm Matrix (row 1 of Example 10) which were used in determining the lengths of certain regions of this fragment, as well as being included within it.

The following example contains a fragment which is also exceptional in the manner in which it was composed. This fragment forms a companion-piece to fragment #17 in that it contrasts in size, texture, and mood with fragment #17, while being also unique among the remaining 17 fragments.

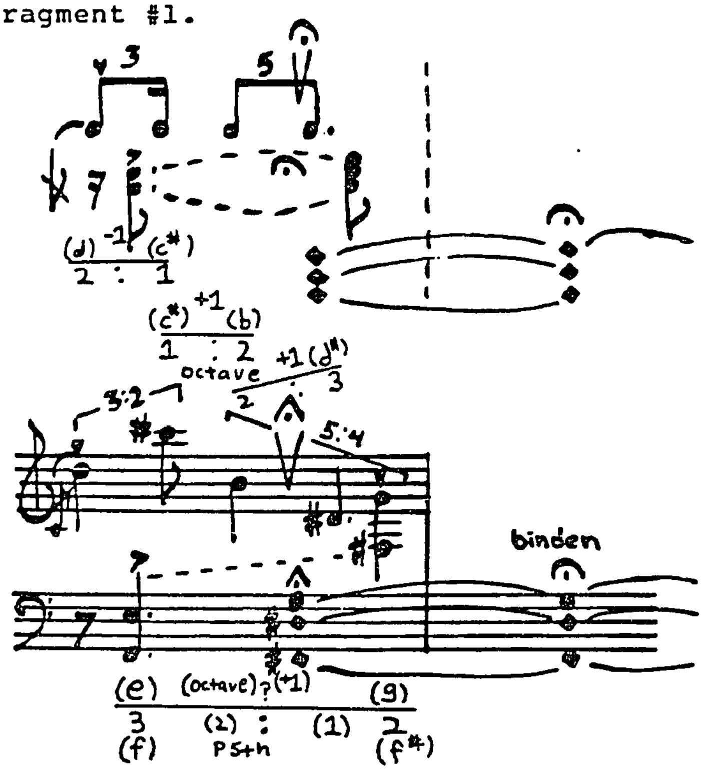
Example 48: Fragment #11 with original rhythmic scheme.



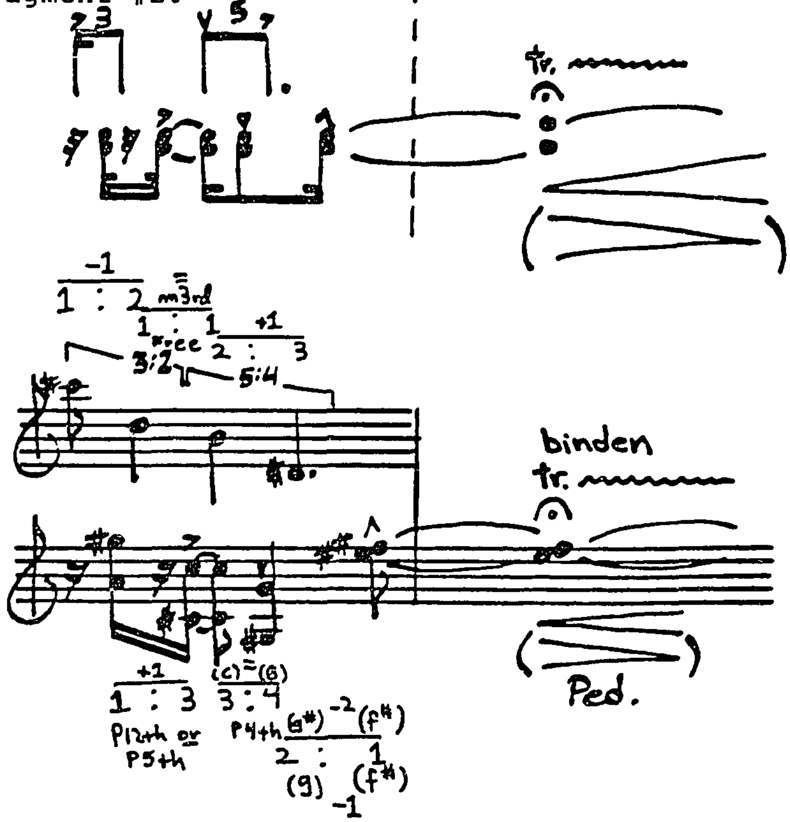
Fragment #11 is perhaps the most mysterious in mood of all the fragments of <u>Klavierstück XI</u>. Not only does it <u>sound</u> mysterious, there is also a mystery associated with the pitches of the upper staff, and two pitches in the lower. With the exception of the pitch f′, the seventh duration, the pitches of the upper staff do not originate in the Final Rhythm Matrix discussed in Chapter 2. This is also true of the second and third durations in the lower staff. One may thus conclude that these additional pitches were improvised around the other durations which come from the Final Rhythm Matrix. The intervals of these pitches also do not conform to the duration proportions implied by their respective durations. Although the interval between the improvised pitches in the bottom staff is a minor 3rd (pitches: d and f), a general chromatic tendency can be observed in the pitches of the improvised durations in the upper staff. The pitches g#-a-a#-b constitute the first four improvised durations. The following chromatic pitches c and c# are present in the next duration. The seventh duration in the upper staff, the pitch f, originated in the Final Rhythm Matrix. The eighth duration contains further pitches which continue to fill out the chromatic space from the point left off at the sixth duration: d-e^{b}-e-f#-g-g#-a-a#. The absence of the pitch f within the sixth duration is accounted for by observing this pitch as being the preceding duration, hence the avoidance of the repetition of pitches. If one considers the mysterious mood of this entire fragment, as well as the final tremolo which ends it, what a mysterious ending for a performance of <u>Klavierstück XI</u> were this fragment to be the final fragment played in a particular performance!

The following examples contain the 19 musical fragments of <u>Klavierstück XI</u>, the rhythmic schemes from the matrices placed directly above the pitch realizations, and an analysis of the duration proportions and their resultant pitches and pitch adjustments. The numbered harmonic series on page 33 of Appendix II will facilitate the reader's acquaintance with the intervals associated with specific duration proportions.

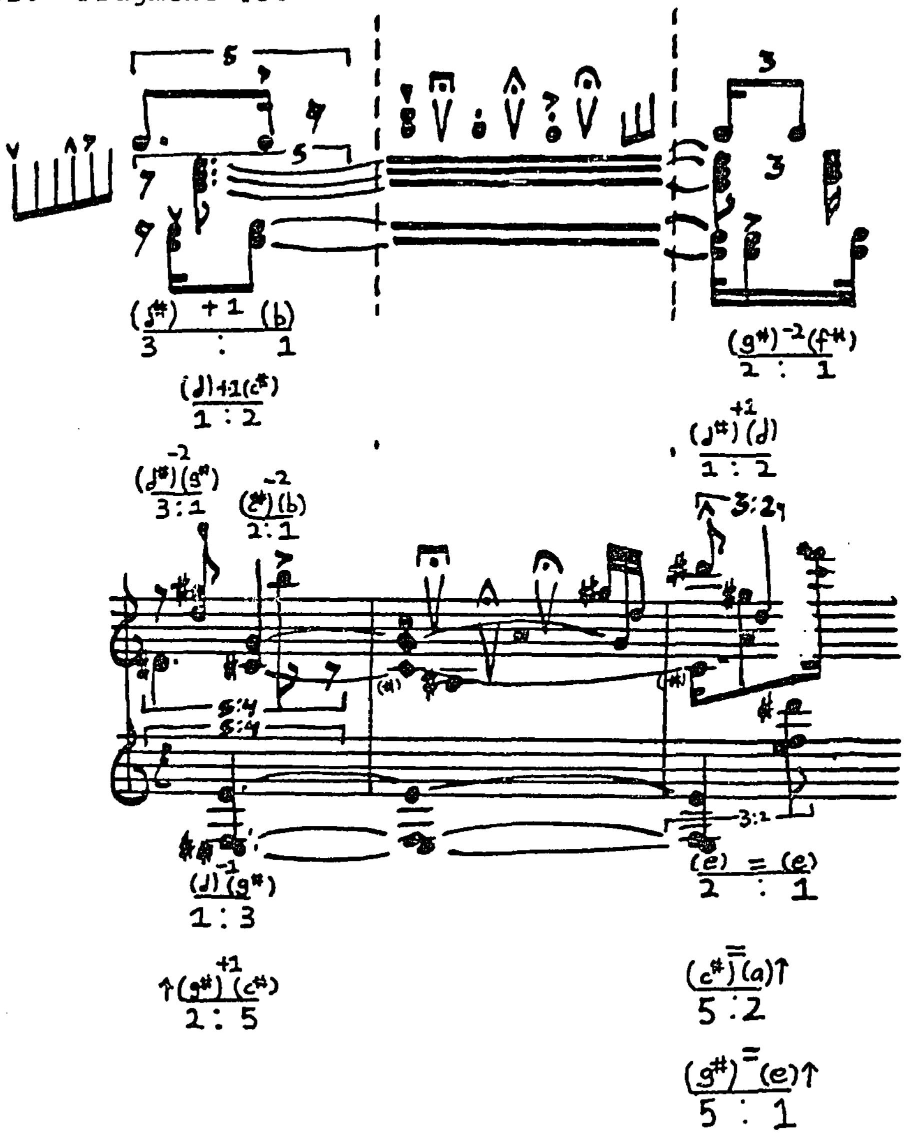
Example 49: Fragment #1.



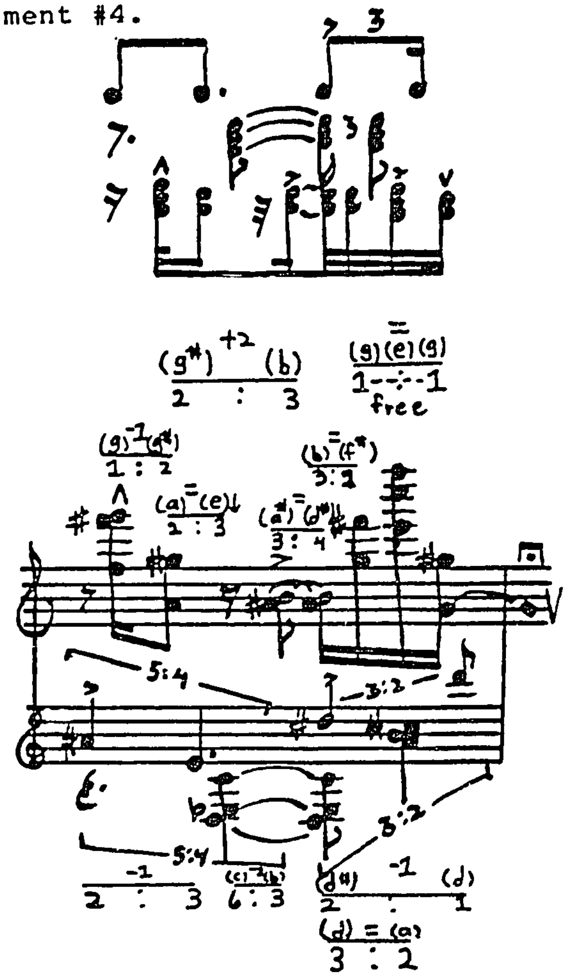
Example 50: Fragment #2.



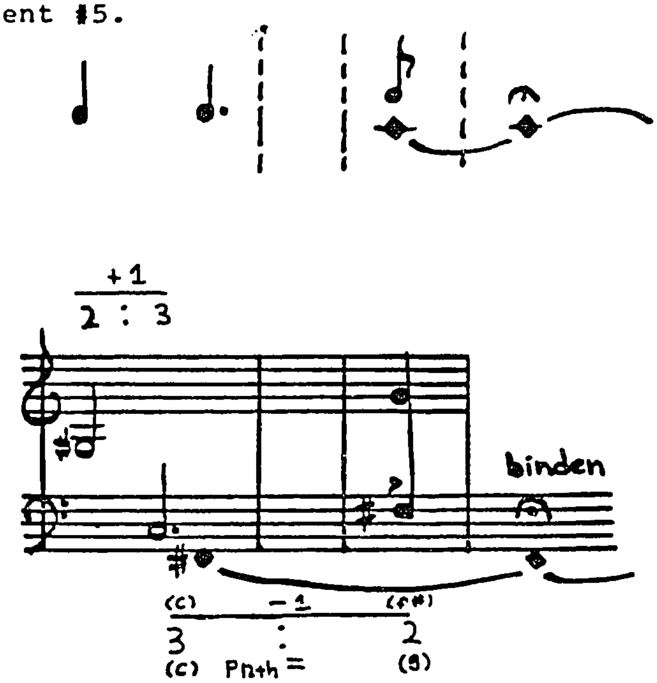
Example 51: Fragment #3.



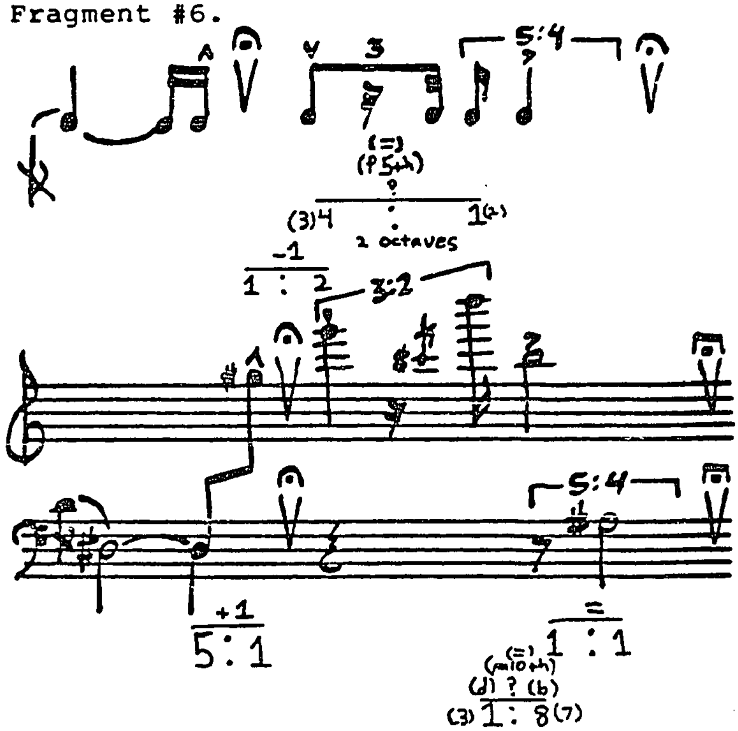
Example 52: Fragment #4.



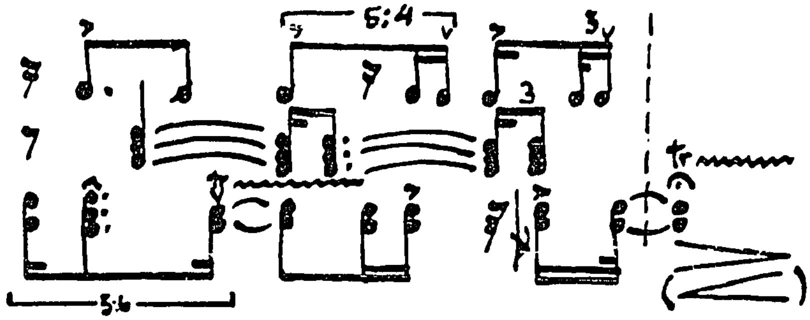
Example 53: Fragment #5.

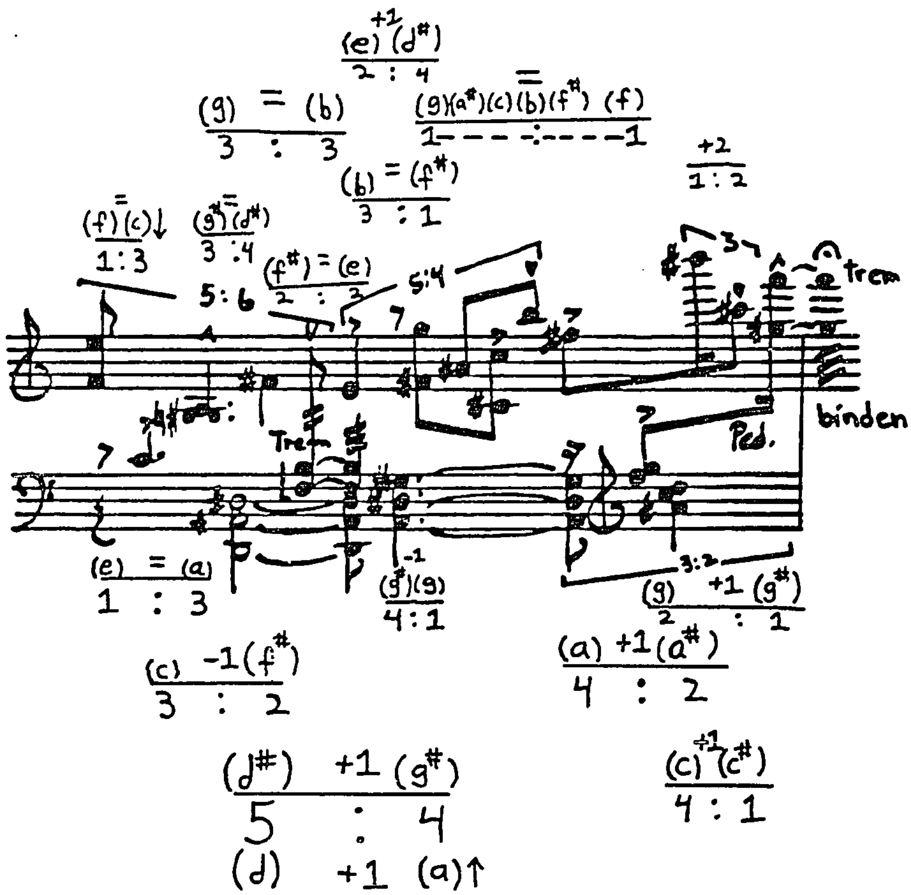


Example 54: Fragment #6.

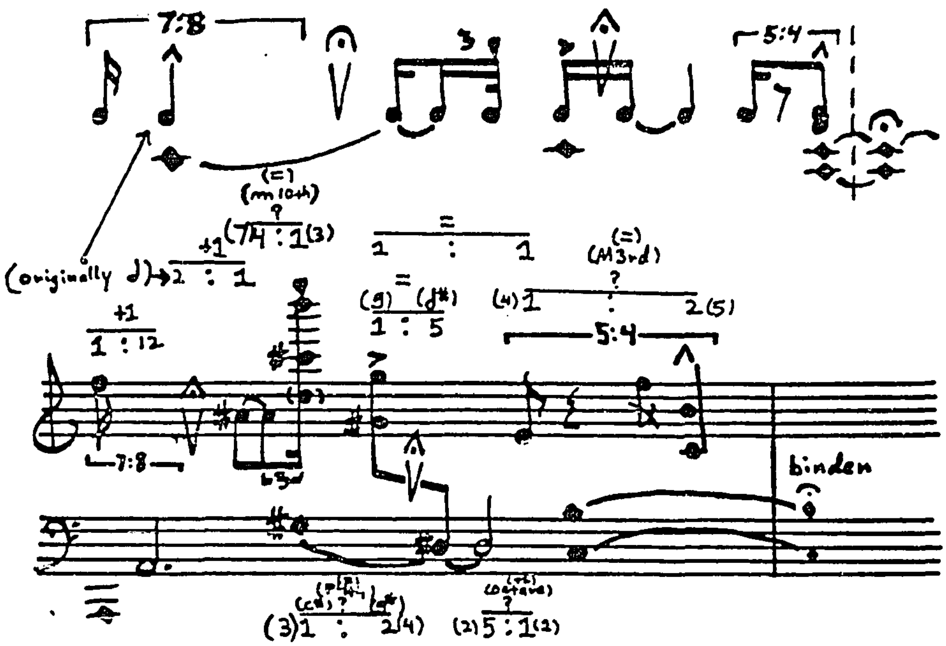


Example 55: Fragment #7.

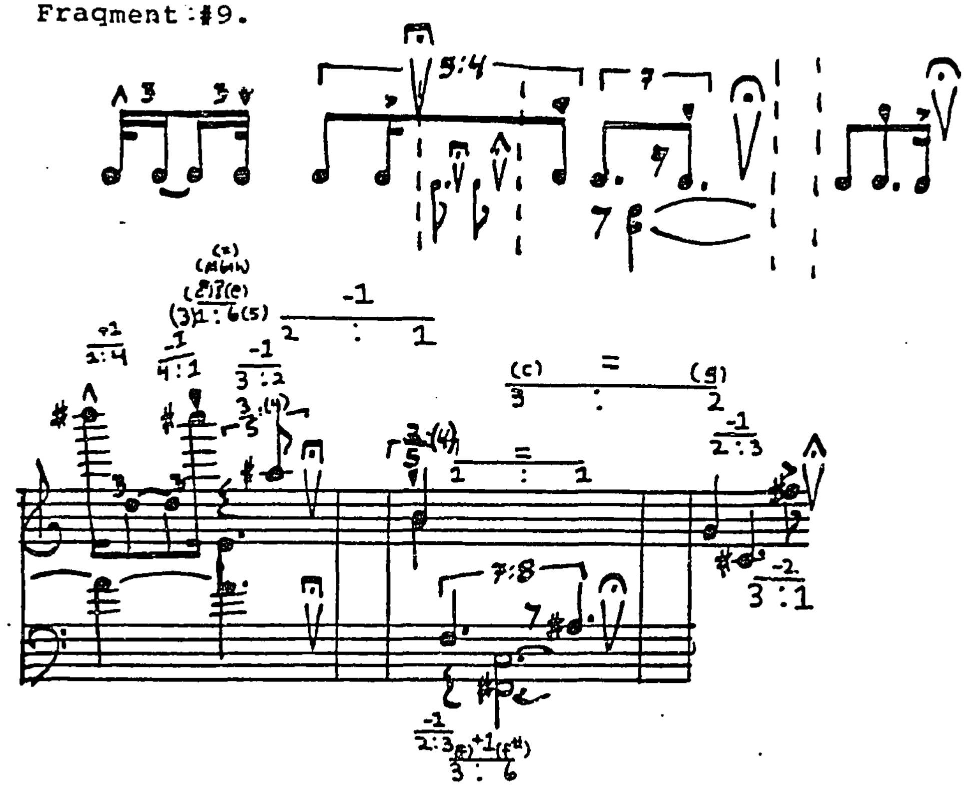




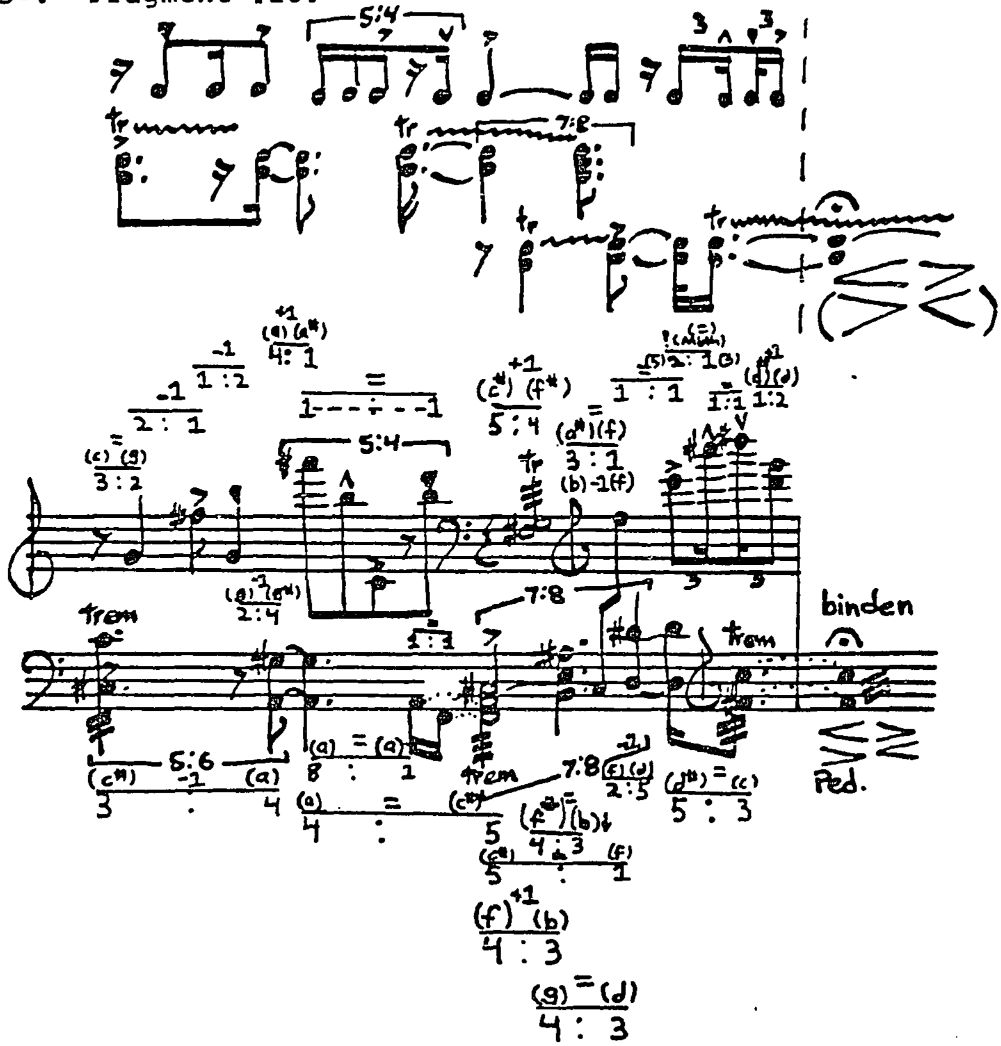
Example 56: Fragment #8.



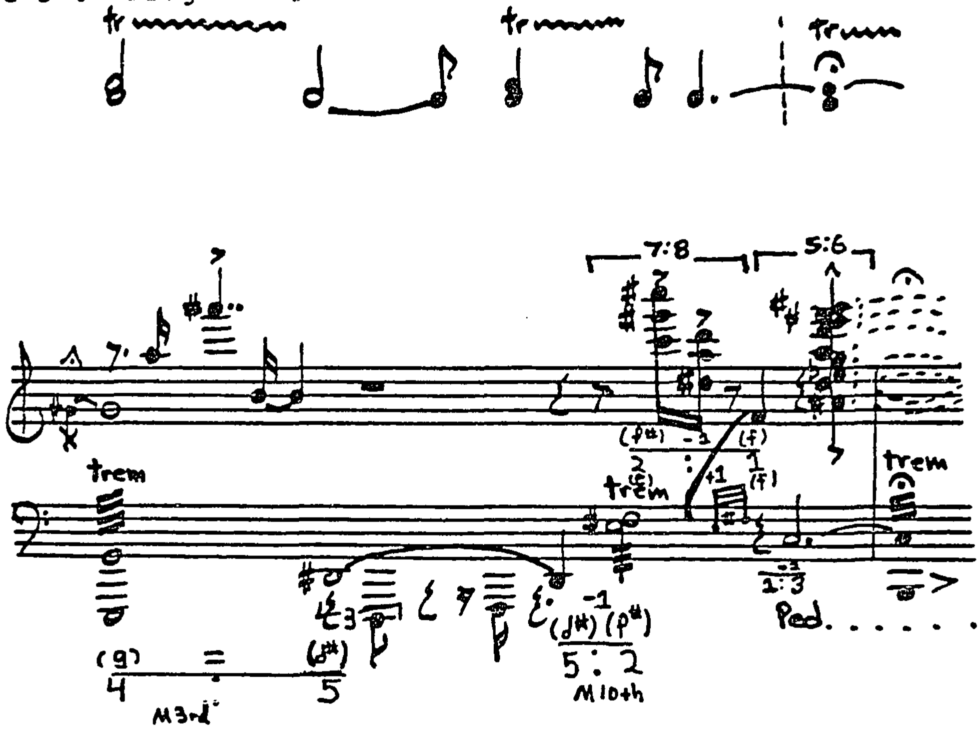
Example 57: Fragment #9.



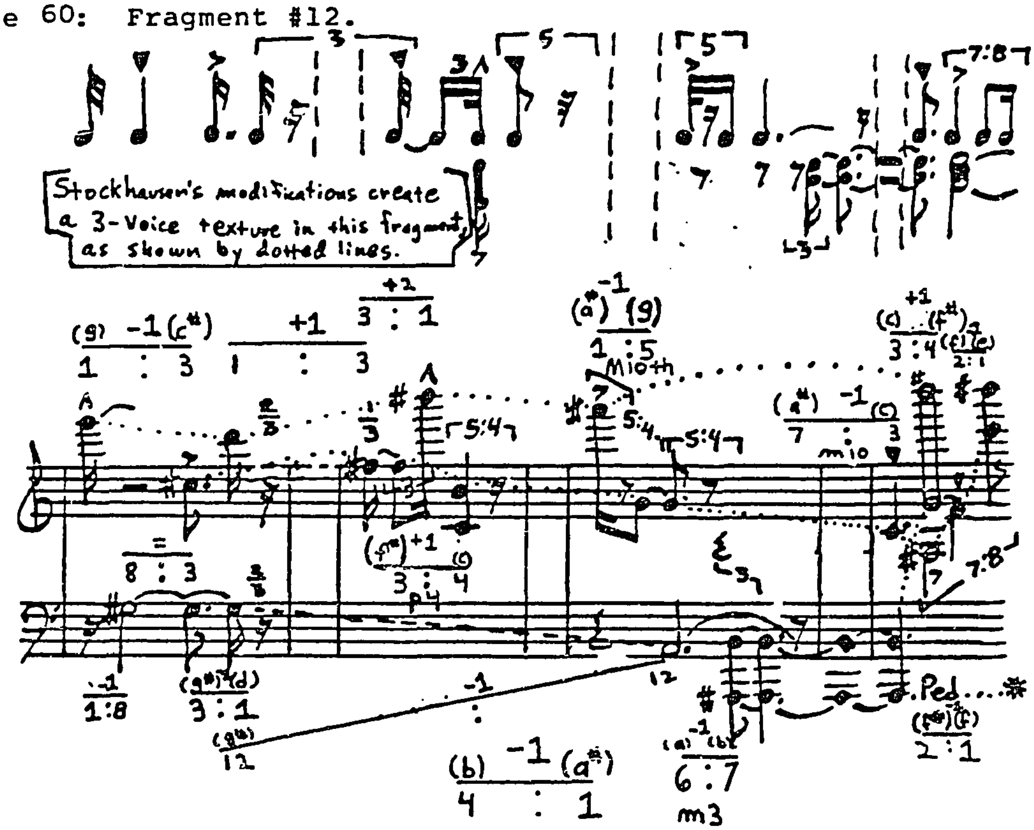
Example 58: Fragment #10.



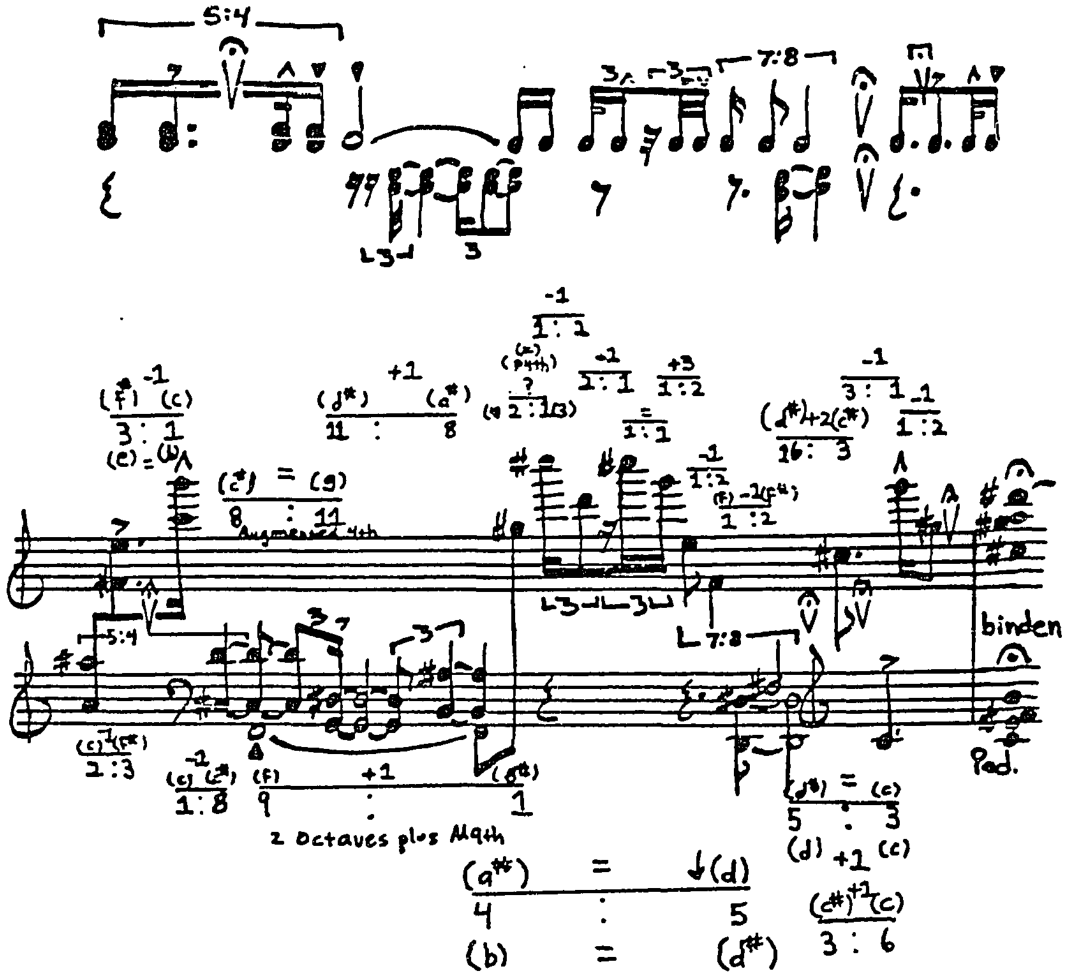
Example 59: Fragment #11.



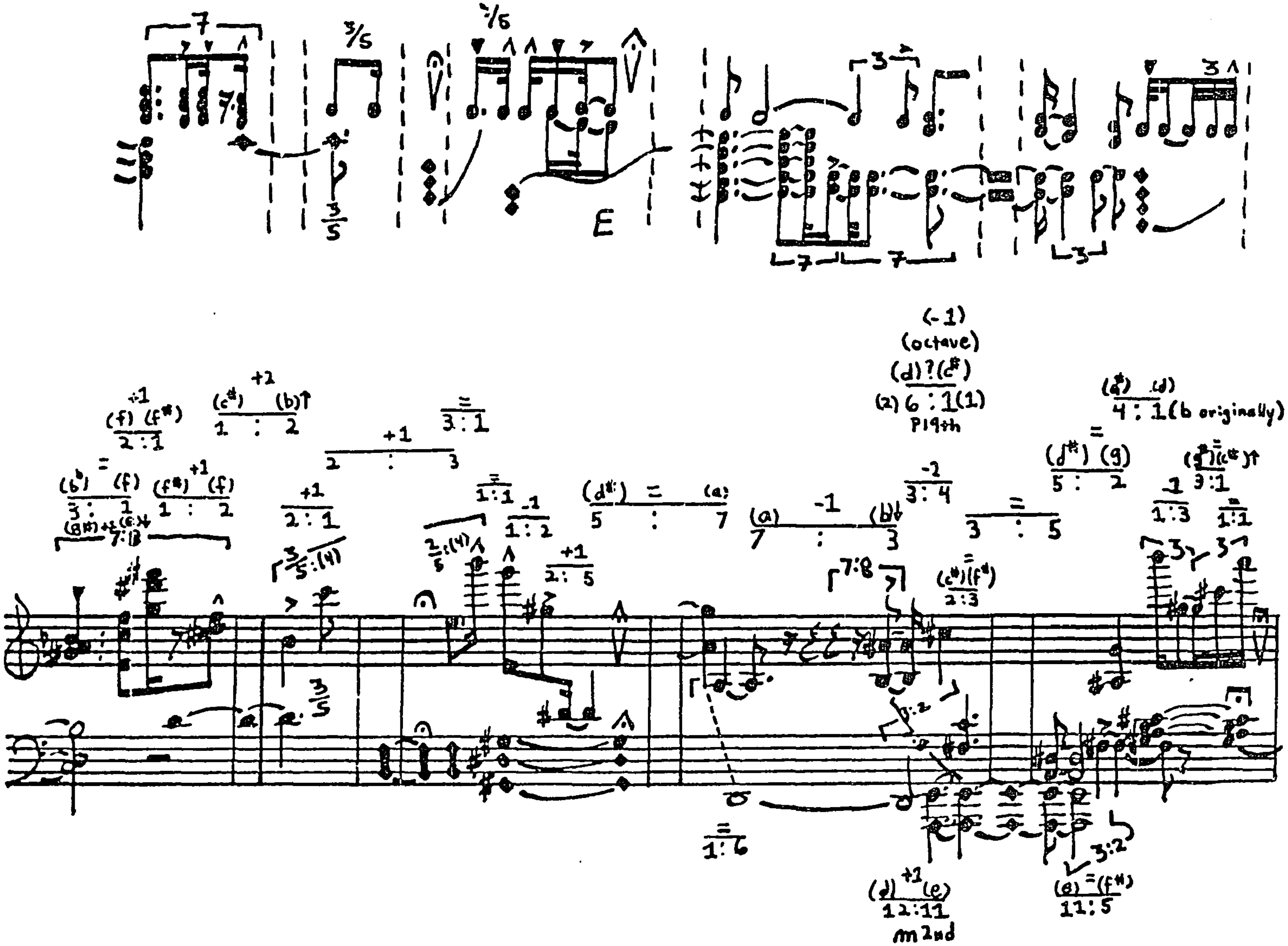
Example 60: Fragment #12.



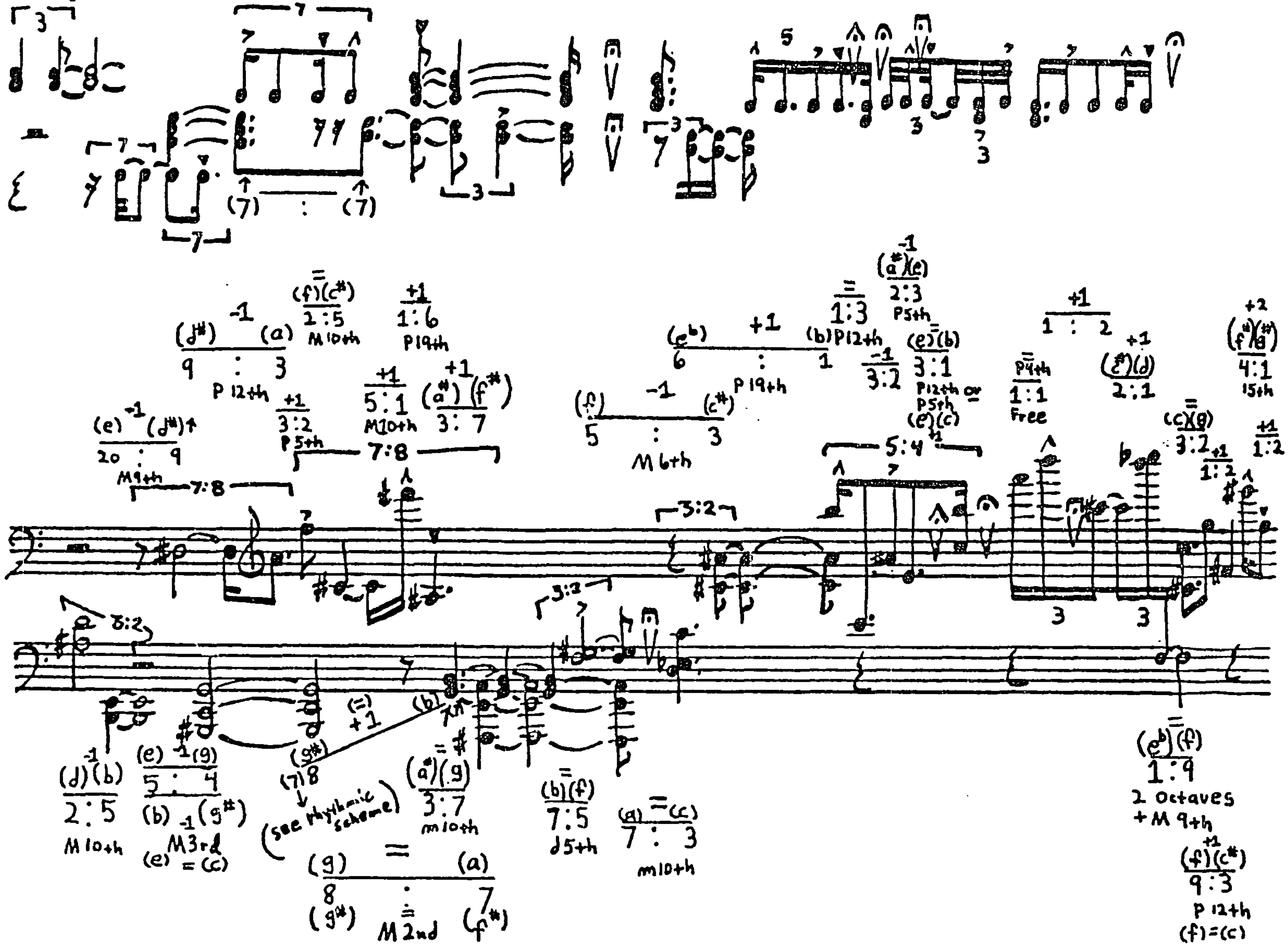
Example 61: Fragment #13.



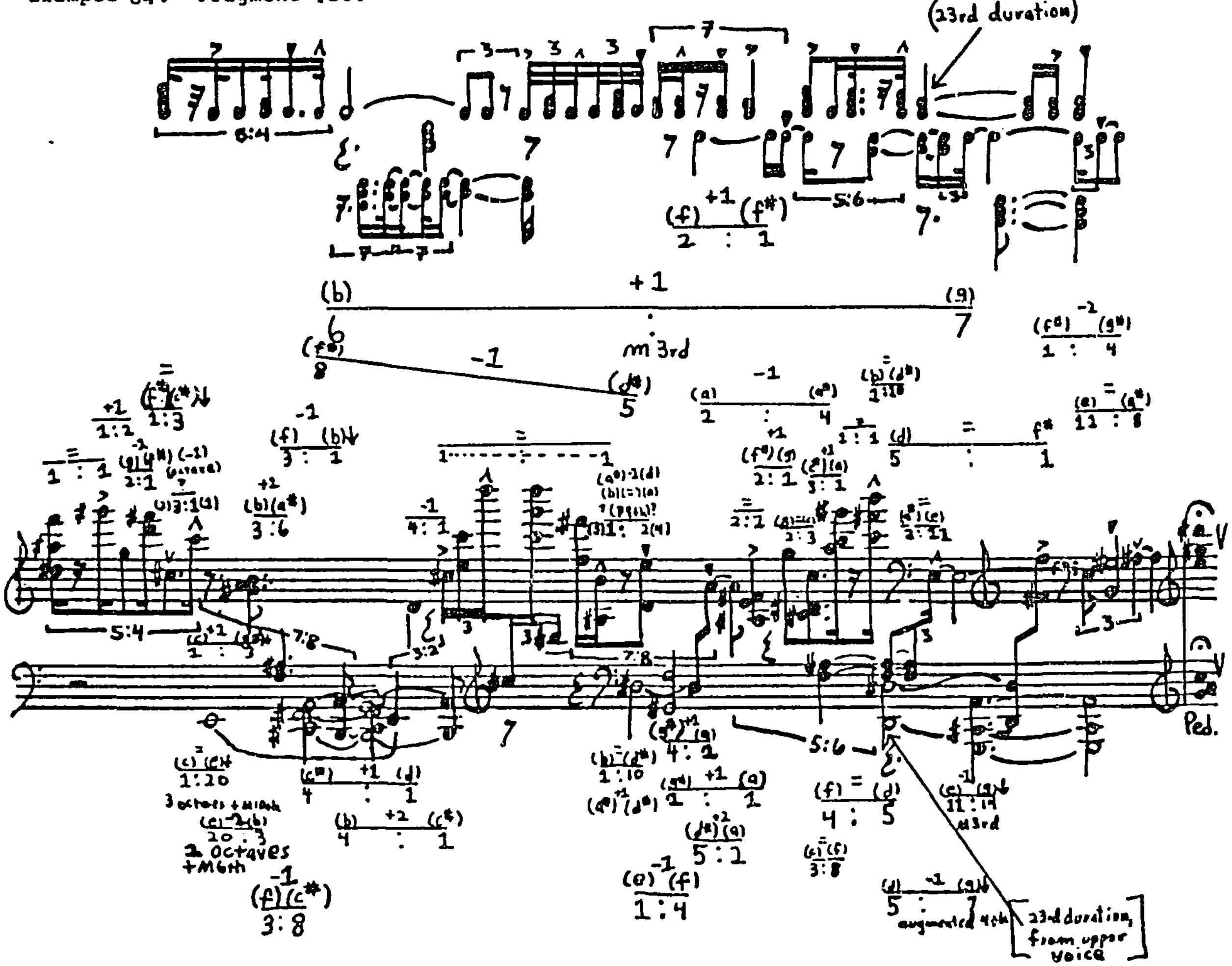
Example 62: Fragment #14.



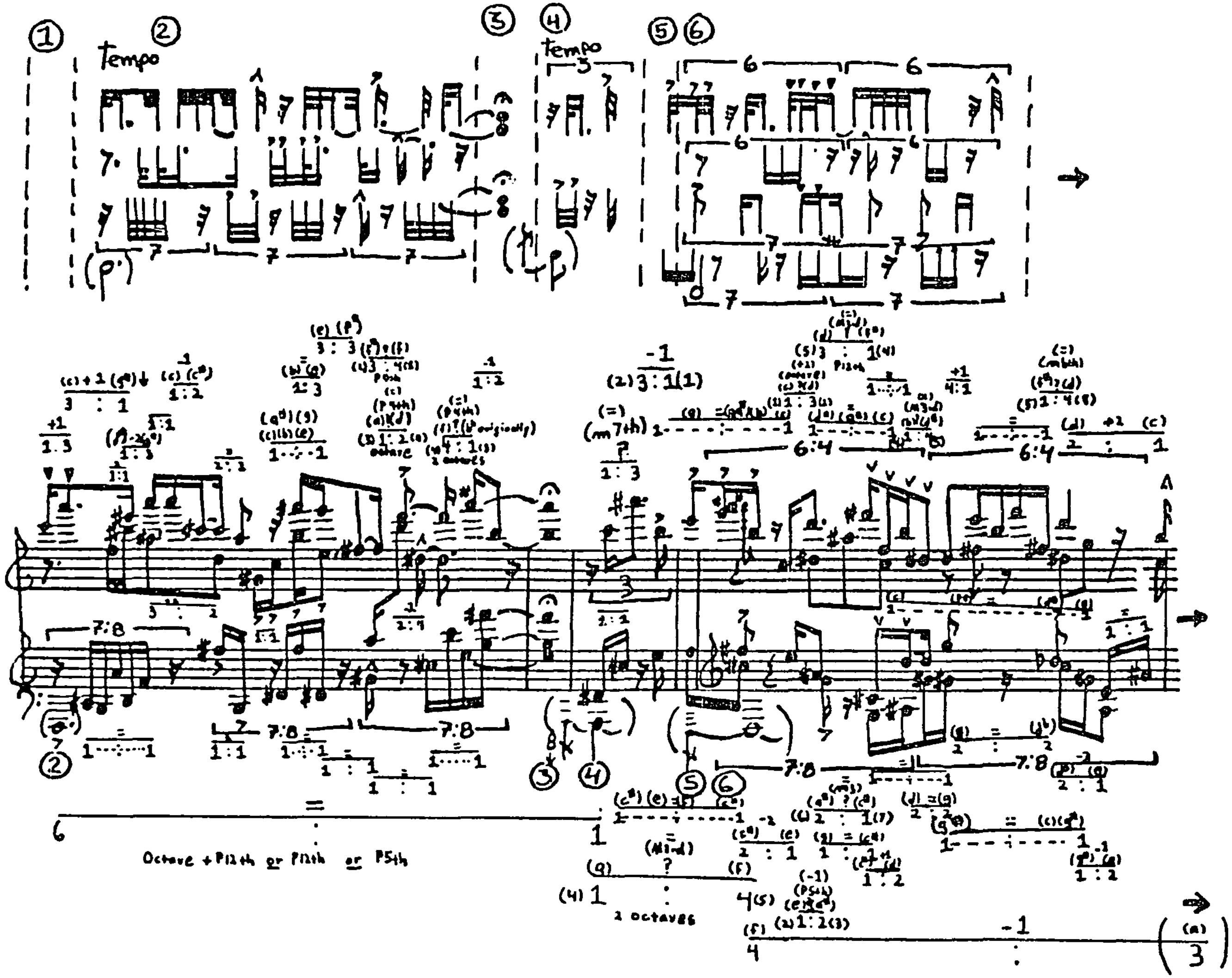
Example 63: Fragment #15.



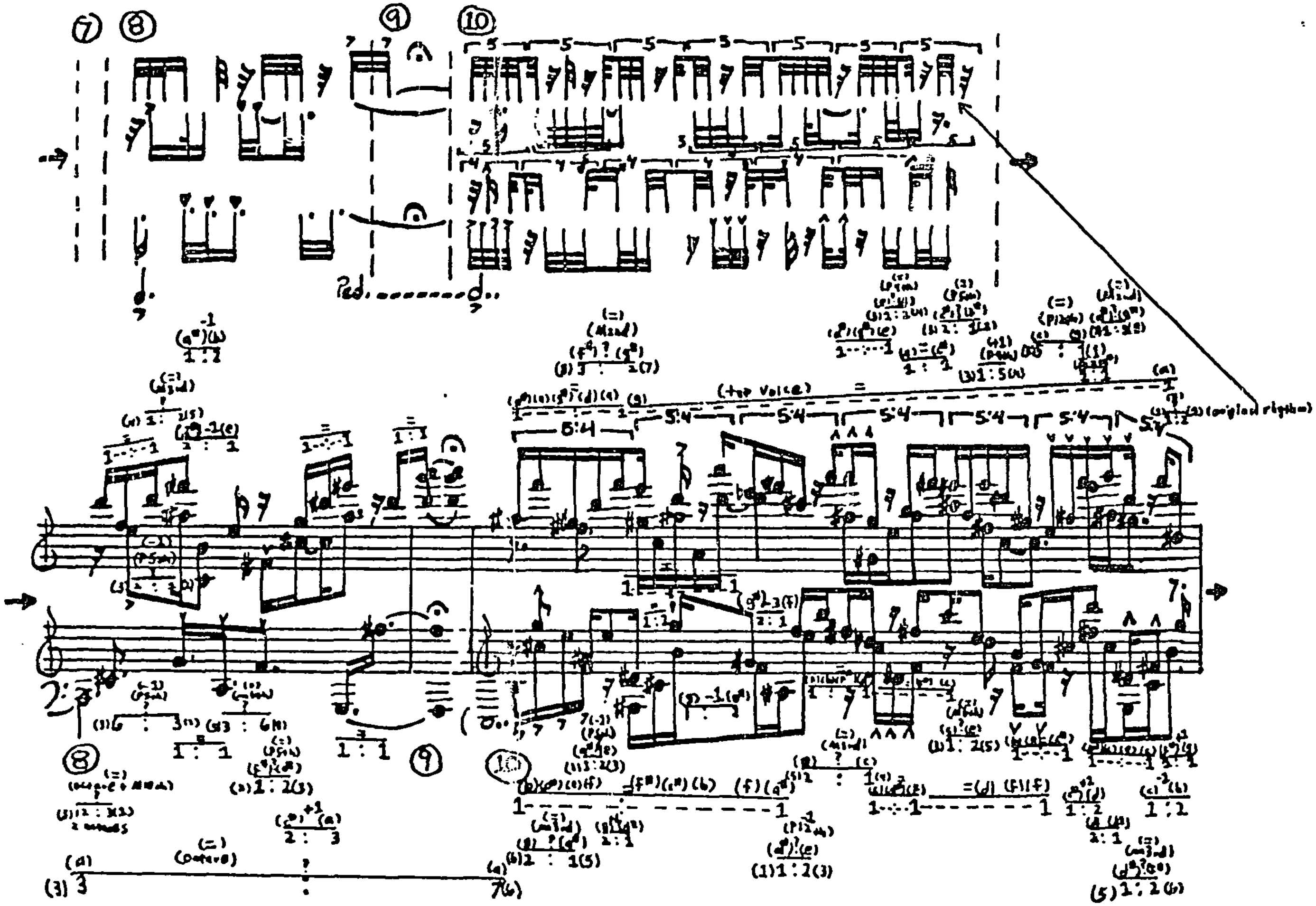
Example 64: Fragment #16.



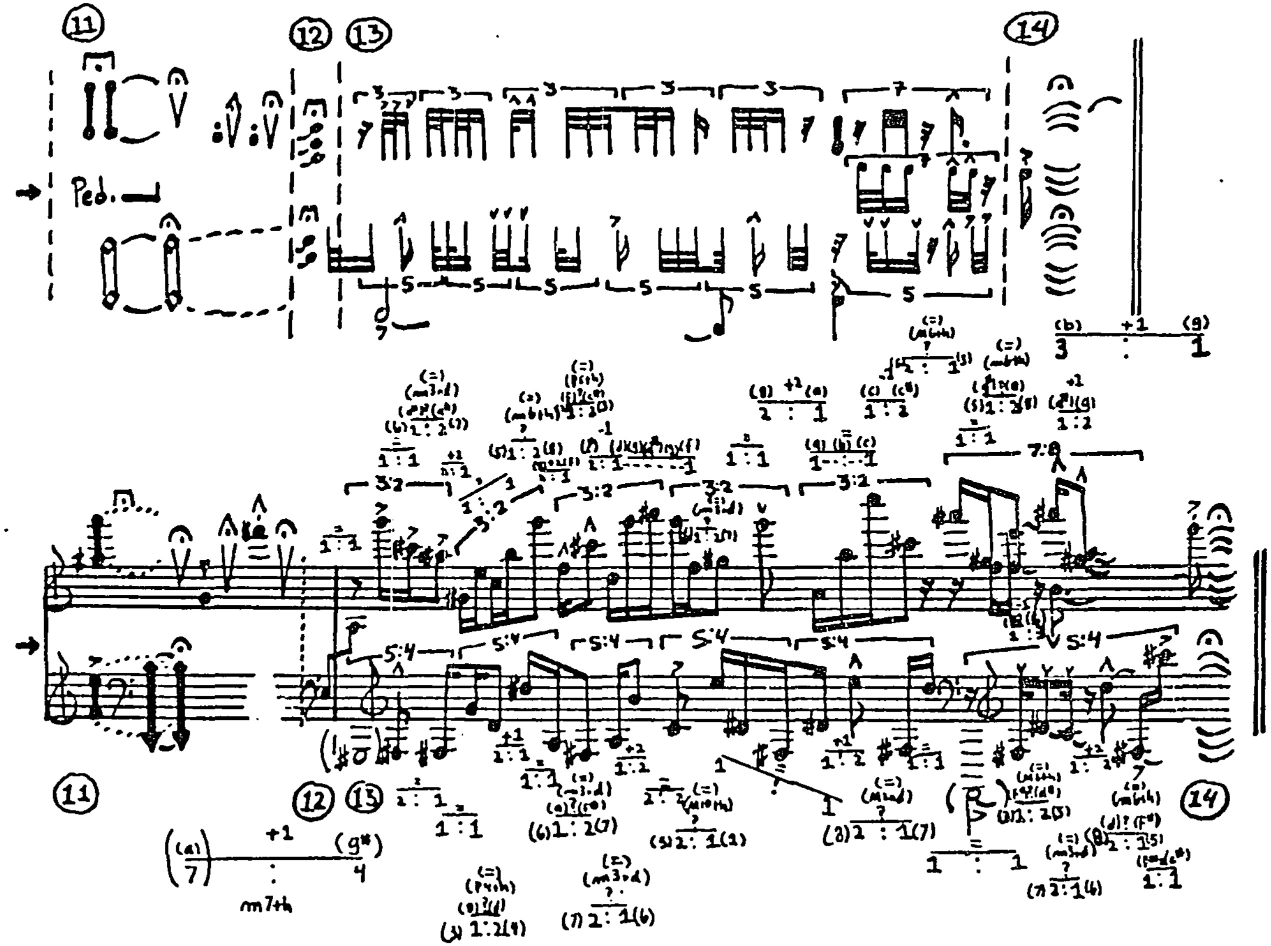
Example 65: Fragment #17 (beginning).



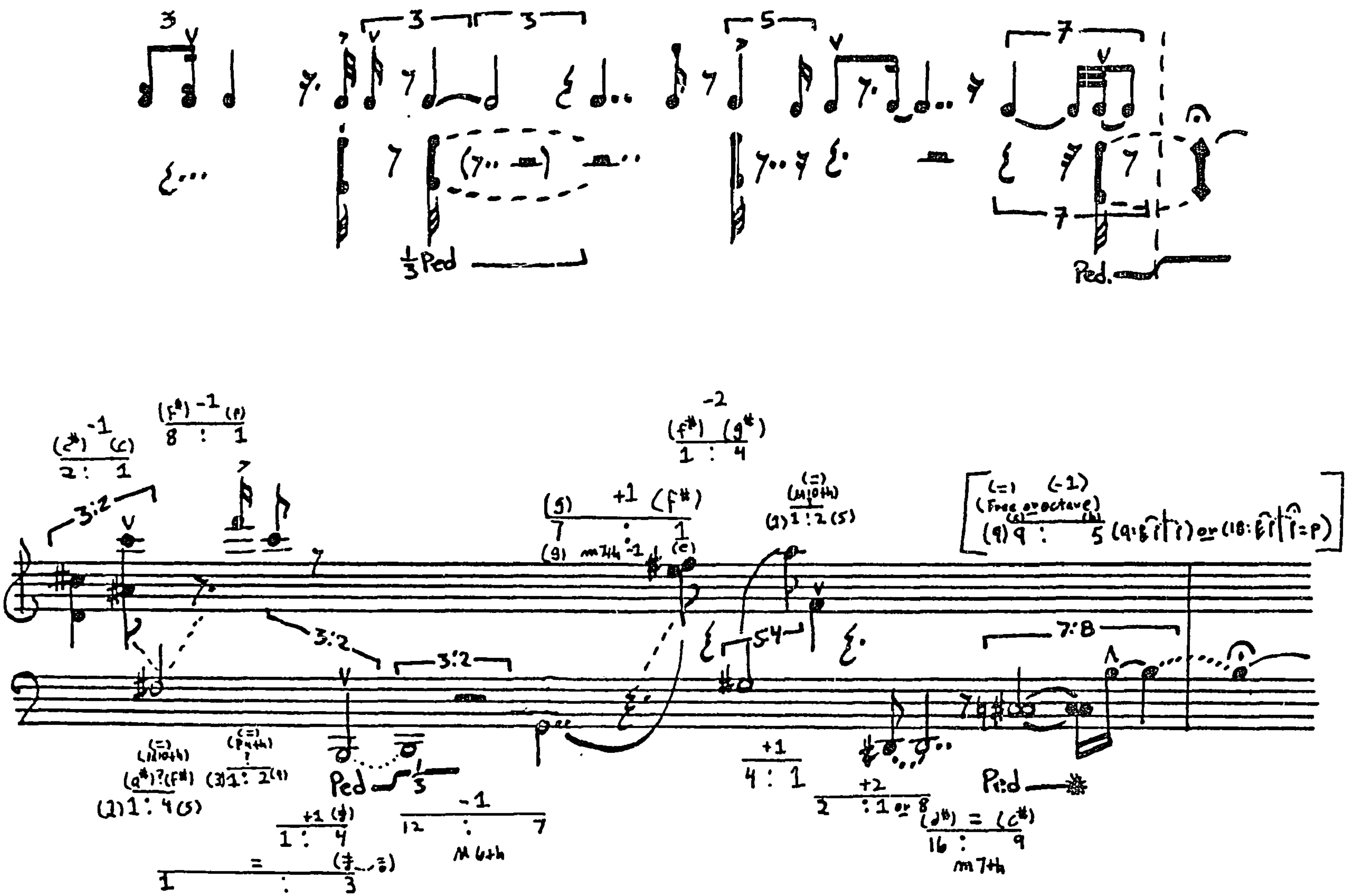
Example 65: Fragment #17 (continuation).



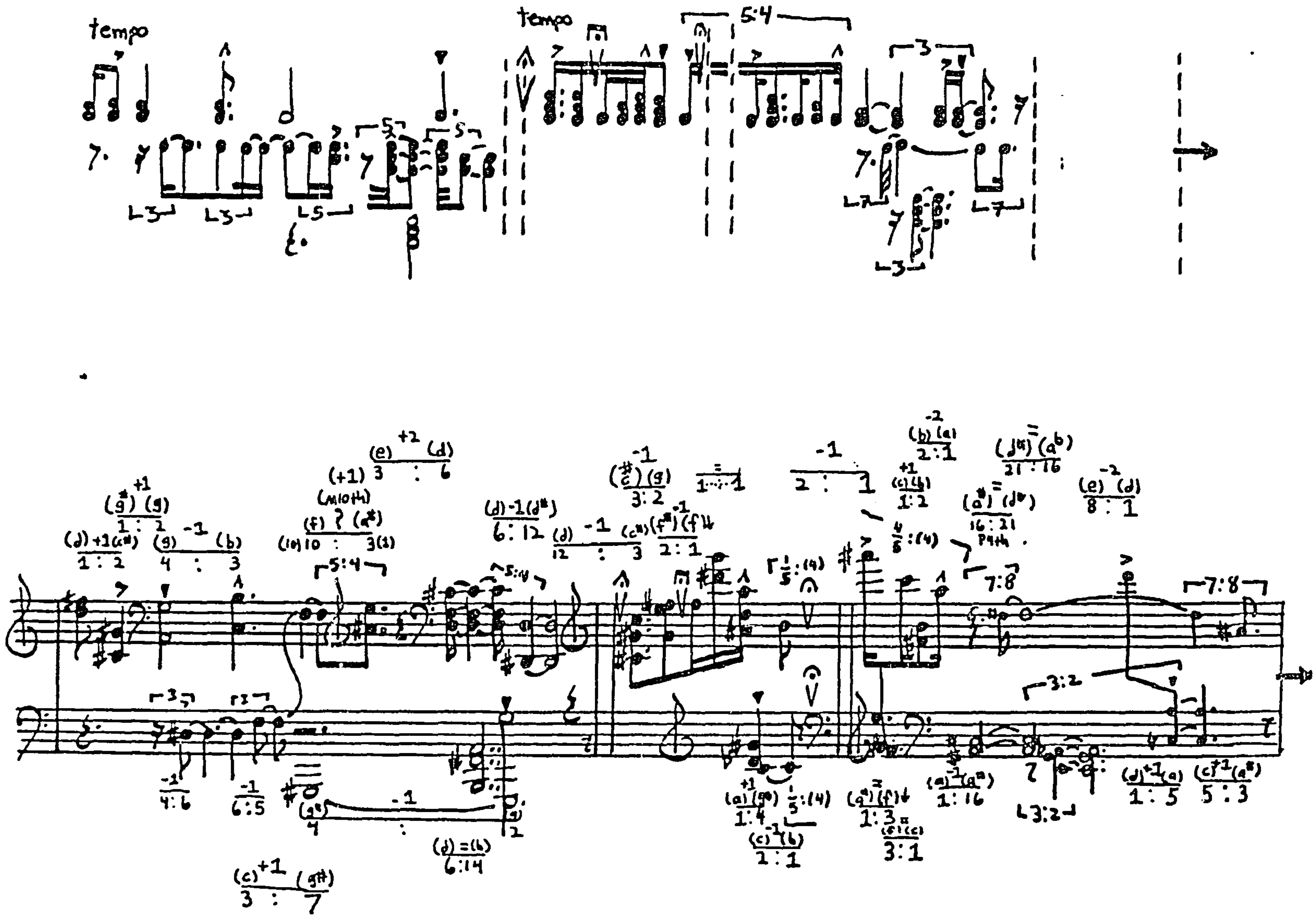
Example 65: Fragment #17 (conclusion).



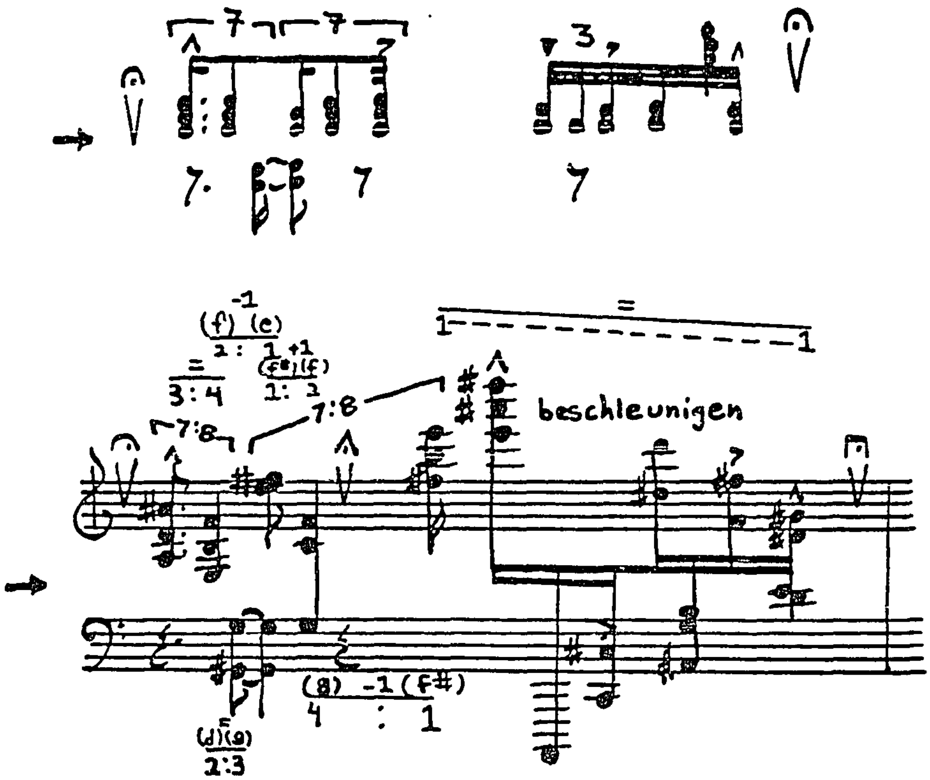
Example 66: Fragment #18.



Example 67: Fragment #19 (beginning).



Example 67: Fragment #19 (conclusion).



APPENDIX I

A BRIEF DISCUSSION OF MATHEMATICAL ASPECTS OF NUMBER MATRICES EMPLOYED IN THE COMPOSITION OF KLAVIERSTÜCK XI

As shown in Chapter 2, Stockhausen employed number matrices during the construction of the Final Rhythm Matrix. In addition to their operating functions relative to the rhythm matrices, some mathematical aspects of several number matrices are interesting enough to warrant discussion in this Appendix. Perhaps the most significant aspect concerns the descending level of mathematical complexity exhibited as the number matrices successively shrink in size, beginning with number matrix #1. This can be partially explained by the fact that as number matrices shrink in size, the smaller amount of numbers contained therein tends to create more simplified numerical relationships. Yet the manner in which matrices are organized also affects their mathematical complexity, far beyond the scope of this specific discussion. As will be shown in the following discussion, Stockhausen used a symmetrical matrix first, then employed various contrasting types of successively smaller matrices.

After beginning with a number and rhythm matrix, each having 7 columns, succeeding matrices of smaller dimensions were used in constructing the Final Rhythm Matrix, which has only six columns. The initial number matrix, considering its size and symmetry, is not only the most mathematically intricate number matrix, but also shares, with rhythm matrix #1 and Roman numeral matrix #1, the only dimension numerically identical with the work number Stockhausen assigned to Klavierstück XI: Number matrix #1 is the only number matrix associated with this work having 7 columns; Klavierstück XI is designated by Stockhausen as Work No. 7.

Example 2: Number matrix #1.[36]

6	1	4	3	7	5	2
1	3	6	5	2	7	4
4	6	2	1	5	3	7
3	5	1	7	4	2	6
7	2	5	4	1	6	3
5	7	3	2	6	4	1

Unlike the 6 x 6 number matrix which follows in page 3 of Stockhausen's sketches, this initial number matrix exhibits certain symmetrical aspects. There is, however, no apparent correlation between these symmetries and the manner in which Stockhausen used this particular matrix in organizing portions of the rhythmic structure of the piece. Counting rows and columns one often obtains an identical number if a mirror image of an identical path is traced from the same starting point. For example, starting from the number 6 located at row 1 column 1, if one counts 5 units across (counting the number 6 as the first unit) and 2 units down (counting the number 7 as the

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[36]This Appendix will retain identical example numbers used in Chapter 2.

first unit), the number 2 is obtained. Starting from the same beginning point and counting down 5 units and across 2 units obtains the same number. This symmetrical property is partially due to the fact that column 1 down (6 1 4 3 7 5) and row 1 across (6 1 4 3 7 5 2) are identical, except for the fact that rows will have one additional number due to the 6 x 7 structure of the matrix. As each row across follows the same numerical relationships as the top row, further symmetries are found between certain rows and columns. For example, column 6 down is identical with row 6 across; column 5 down is identical with row 5 across, etc.

As shown in Example 2a the interior numerical relationships, or differences modulo 7, were written by Stockhausen just above and below the matrix,[37] conveniently placed between the numbers of rows 1 and 6. The modulo differences written above the matrix, however, differ in appearance from those found below it.

Example 2a: Top and bottom rows of number matrix #1 with differences modulo 7.

$$\begin{array}{c}
6^{+2}1^{+3}4^{-1}3^{+4}7^{-2}5^{-3}2 \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
5_{+2}7_{-4}3_{-1}2_{+4}6_{-2}4_{-3}1
\end{array}$$

The +3 associated with the top row expresses the same relationship of the numbers 1 to 4 in this system as the -4 associated

[37] See Andree, R.V. Selections From Modern Abstract Algebra, Chapter 1, concerning modulo systems.

with the bottom row, since $-4 \equiv +3$ modulo 7. This is true because in a modulo system containing the numbers 1 through 7 (with no zero used) any additive process which extends beyond the number 7 immediately continues with the number 1. Subtraction works the same way in the opposite direction. Therefore, if the +3 and -4 were switched with one another, the resulting numbers would still be the same: in this system $(1-4 \equiv 4)$ and $(7+3 \equiv 3)$ modulo 7.

This manner of expressing the interior numerical relationships within the initial number series occurs only once in Stockhausen's sketches, and is never used as in independent series. The function of this modulo expression was to aid Stockhausen in constructing the interior of the number matrix accurately, since the modulo differences are the same for each interior row.

Example 12: Number matrix #2.

5	3	4	1	6	2
3	1	2	5	4	6
1	5	6	3	2	4
4	2	3	6	5	1
6	4	5	2	1	3
2	6	1	4	3	5

Unlike the identical sequences of numbers across and down the first row and column of number matrix #1, a comparison of row 1 with column 1 in Example 12 reveals the third and fourth numbers of each sequence to be switched with one another. But row 1 from left to right and row 6 from right to left are identical. The same relationship exists between rows

2 and 5, and rows 3 and 4. The diagonal formed by the numbers between the upper left and lower right corners of this matrix also exhibits a reverse ordering when the first set of three numbers is compared with the second set of three numbers: 5-1-6/6-1-5. The same is true of the diagonal extending from the lower left to the upper right: 2-4-3/3-4-2.

The number matrices of Example 18b and 18c have the same series of numbers descending the first column as those which constitute row 1. The numbers in these matrices in column 1 at rows 5 and 6 both coincide with the first two numbers of the same series. These two matrices also have no number series in their respective rows which duplicate one another. Rows 2 and 6 of the number matrix Stockhausen finally used, Example 18a, are identical.

Example 18: Number matrices #'s 4-6.

a) #4	b) #5	c) #6
3 4 2 1	3 4 2 1	4 1 3 2
4 1 3 2	4 1 3 2	1 2 4 3
1 2 4 3	2 3 1 4	3 4 2 1
2 3 1 4	1 4 2 3	2 3 1 4
3 2 4 1	3 2 4 1	3 2 4 1
4 1 3 2	4 3 1 2	4 3 1 2

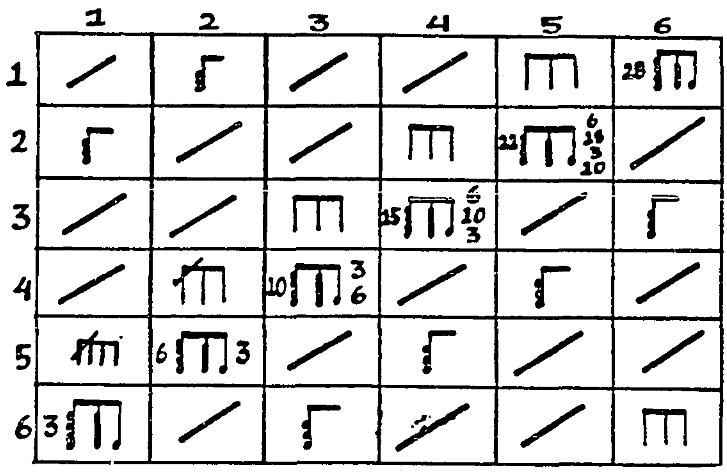
The number matrix of Example 18a has a different property relative to its outside columns (1 and 4): Beginning with the number 3 at row 1 column 1, every successive number descending along the column increases by 1 until the number 4 is reached. The number 4 corresponds with the total number of rhythmic cells to be reordered in each block of column 3 of the Final Rhythm Matrix. After the number 4 the number series starts over again with the number 1 at row 3 column 1. A second number 4 is reached at row 6 of this column, and the series starts over again with the number 1 at row 1 column 4, and continues on again to the bottom of column 4.

It was necessary to match the rhythmic cells of Example 19a with those of Example 20 in order to determine the number matrix of Example 18a, because number matrix #4 was not written out in Stockhausen's sketches. Number matrix #5, although not chosen for operation upon rhythm matrix #4, is the only matrix with symmetrical properties in Example 18. This can be determined by checking successive diagonals, extending from lower left to upper right beginning with the number at row 1 column 1 in each matrix. Number matrix #4, when submitted to this process, yields the numbers 3, 4-4, 1-1-2, 2-2-3-1, 3-3-4-2, etc. Although a symmetry is observed in the 4-4 pair, there are no other symmetries evident. Number matrix #5, however, yields several symmetrical sets of numbers by the same process: 3, 4-4, 2-1-2, 1-3-3-1, 4-2-2-4, 3-4-3, and 1-1. The numbers in the four outer corners of number matrices #4 and #5 are identical, ranging from 1 to 4. The numbers 4 and 2 are found twice in the outer corners of number matrix #6.

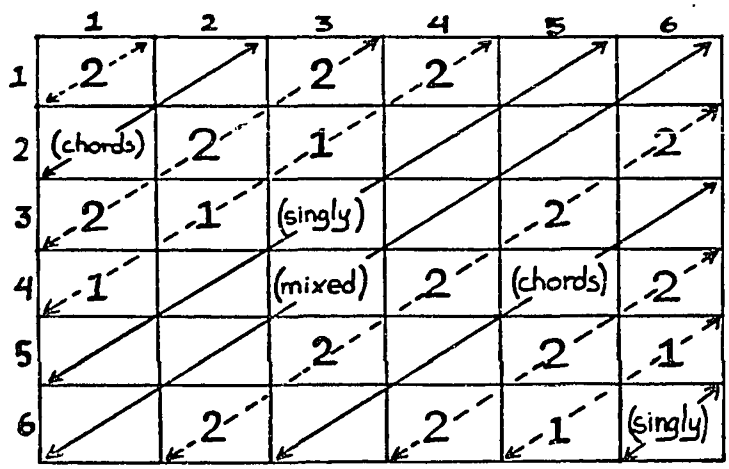
An additional aspect of the organization of the succeeding diagonals in Examples 39a and 39b may be observed: if the blocks at the upper left corner and the lower right corner are also counted, the total number of blocks in each successive diagonal from left to right creates a number series which expands from 1 block to 6 blocks and then contracts back to 1 block again (1-2-3-4-5-6-5-4-3-2-1). Any matrix of identical dimensions would exhibit this characteristic, yet it is significant here because Stockhausen organized the contents of these diagonal blocks accordingly, as shown in Examples 39a and 39b.

Example 39: Element matrix #1 (39a) and addendum to element matrix #1 (39b).

a)



b)



The 3 groups of number series shown in Example 39c are various ways Stockhausen initially considered rearranging all or part of the number series 3-5-10-21-28. Its original form is 0-1-3-6-10-21-28. This additive series is formed by starting with the initial number 0. The number 1 is then added to 0. (The number 1, although absent from the number series of Examples 39c and 39d, is represented in the form of single grace note vertical sonorities in Example 39a.) Each successive number in such a number series is then arrived at by first adding 1 to the previous number that was employed in expanding the series. The number so obtained is then added to the last number in the series to obtain the following number for the series. As the number 1 was the first such number employed in this series, adding 1 to the number 1 equals 2. The number 2 is then added to the number 1 and the next number in the series becomes 3. In like manner the number 3 is added to the number 3 to obtain the number 6 for the series, 4 is added to 6 to obtain the number 10, and so on to the number 28, which is the largest number used by Stockhausen in this series.

Example 39: Number matrix #9 (39c) and number matrix #10 (39d).

c)

3	6	3 6 10	3 6 10 15	3 6 10 15 21	3 6 10 15 21 28
6	3 10	3 6 15	3 6 10 21	3 6 10 15 28	
10	3 15	3 10 15	3 6 15 21	3 6 10 21 28	
3	3 6	3 6 10	3 6 10 15	3 6 10 15 21	3 6 10 15 21 28
6	6 10	6 10 15	6 10 15 21	6 10 15 21 28	6 10 15 21 28 3
10	10 15	10 15 21	10 15 21 28	10 15 21 28 3	10 15 21 28 3 6
3	6 3	10 3 6	15 6 3 10	21 6 3 10 15	28 10 3 6 15 21
6	10 6	15 6 10	21 10 6 15	28 10 6 15 21	3 15 6 10 21 28
10	15 10	21 10 15	28 15 10 21	3 15 10 21 28	6 21 10 15 28 3

(For Example 39d, see page 155)

Example 39: (continued).

d)

				15 6 10 3	21 6 15 3 10	28 3 15 10 6 21
				21 3 10 6 15	28 6 10 15 3 21	15 6 3 10
				28 3 21 15 6 10	15 10 3 6	21 10 6 3 15
mixed	3	6 3	10 3 6	15 10 6 3	21 15 10 6 3	28 21 15 10 6 3
singly	6 3	10 3 6	3	21 15 10 6 3	28 21 15 10 6 3	15 10 6 3
chords	10 3 6	3	6 3	28 21 15 10 6 3	15 10 6 3	21 15 10 6 3

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APPENDIX II

SELECTED EXAMPLES

AN AID IN THE STUDY OF EXAMPLES WHICH

RELATE TO ONE ANOTHER IN

KARLHEINZ STOCKHAUSEN'S KLAVIERSTÜCK XI: AN ANALYSIS

OF ITS COMPOSITION VIA A MATRIX SYSTEM OF SERIAL

POLYPHONY AND THE TRANSLATION OF RHYTHM

INTO PITCH

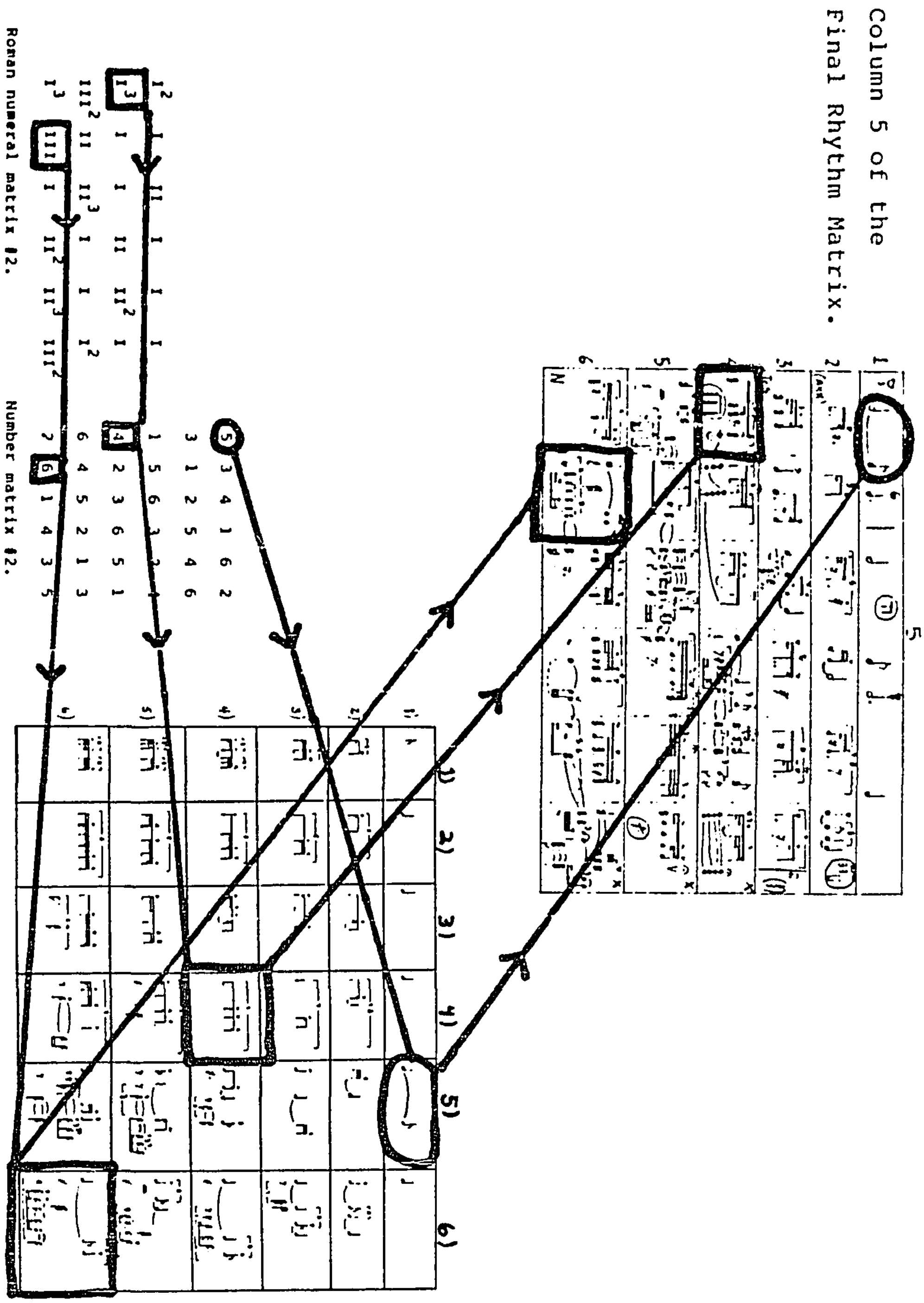
by

Nathan M. Stephen Truelove

Oklahoma City, Oklahoma

1984

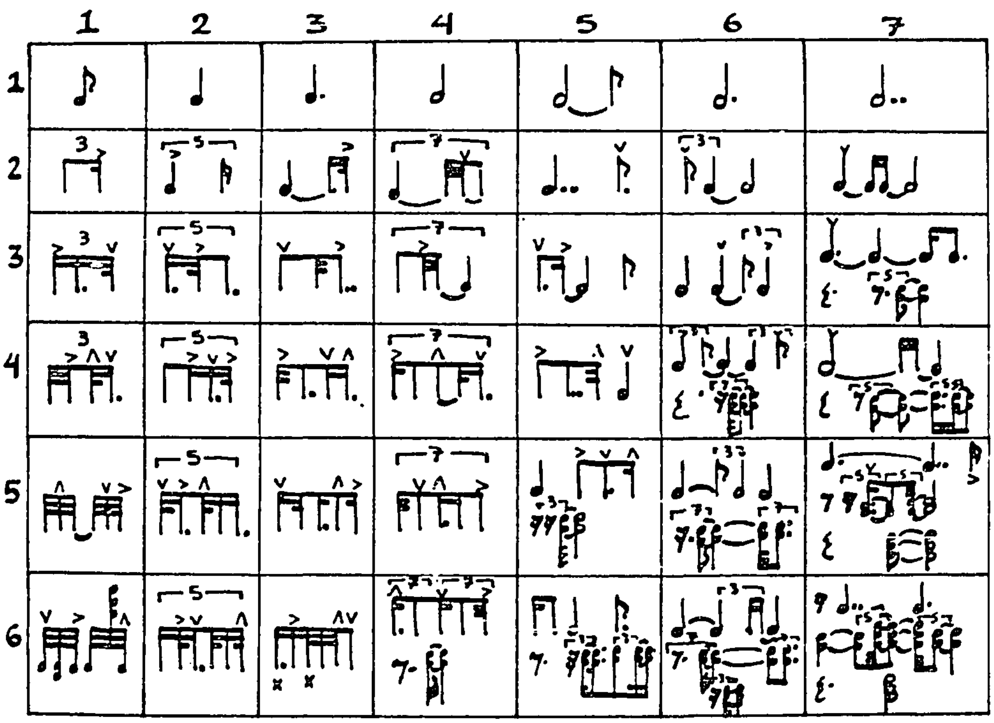
Example 1: Interrelationships of Roman numeral matrix #2, number matrix #2, and rhythm matrix #2 in the construction of column 5 of the Final Rhythm Matrix.



Example 2: Number matrix #1.

6	1	4	3	7	5	2
1	3	6	5	2	7	4
4	6	2	1	5	3	7
3	5	1	7	4	2	6
7	2	5	4	1	6	3
5	7	3	2	6	4	1

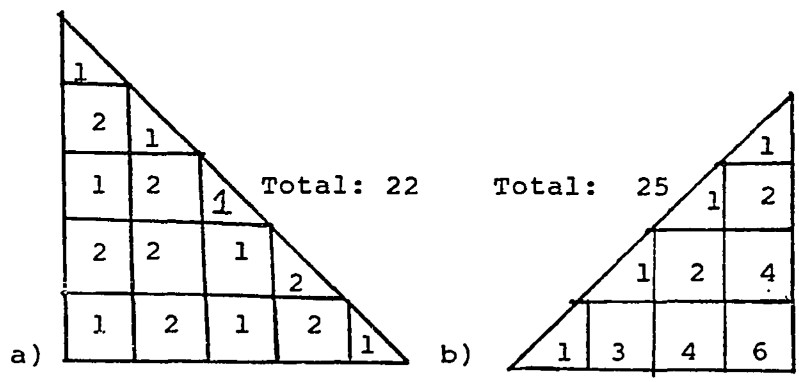
Example 3: Rhythm matrix #1.



Example 4: Roman numeral matrix #1.

(Row)							
(1)							
(2)	I^2	I	I	I	I	I	I
(3)	I^3	I	I	I	I	I	I^2
(4)	I^2	I	I	II^3	I	I^2	II
(5)	III^3	I^2	II^2	I	I	II	I^3
(6)	II^2	III	I^3	I	III^2	II^3	I^2

Example 8: Pitch enrichment processes. (Examples 8a and 8b were constructed by the author of this paper, and are not to be found in Stockhausen's sketches.)



c)

I^{2}

I^{3} II^{2}

I^{2} II^{3} I^{2}

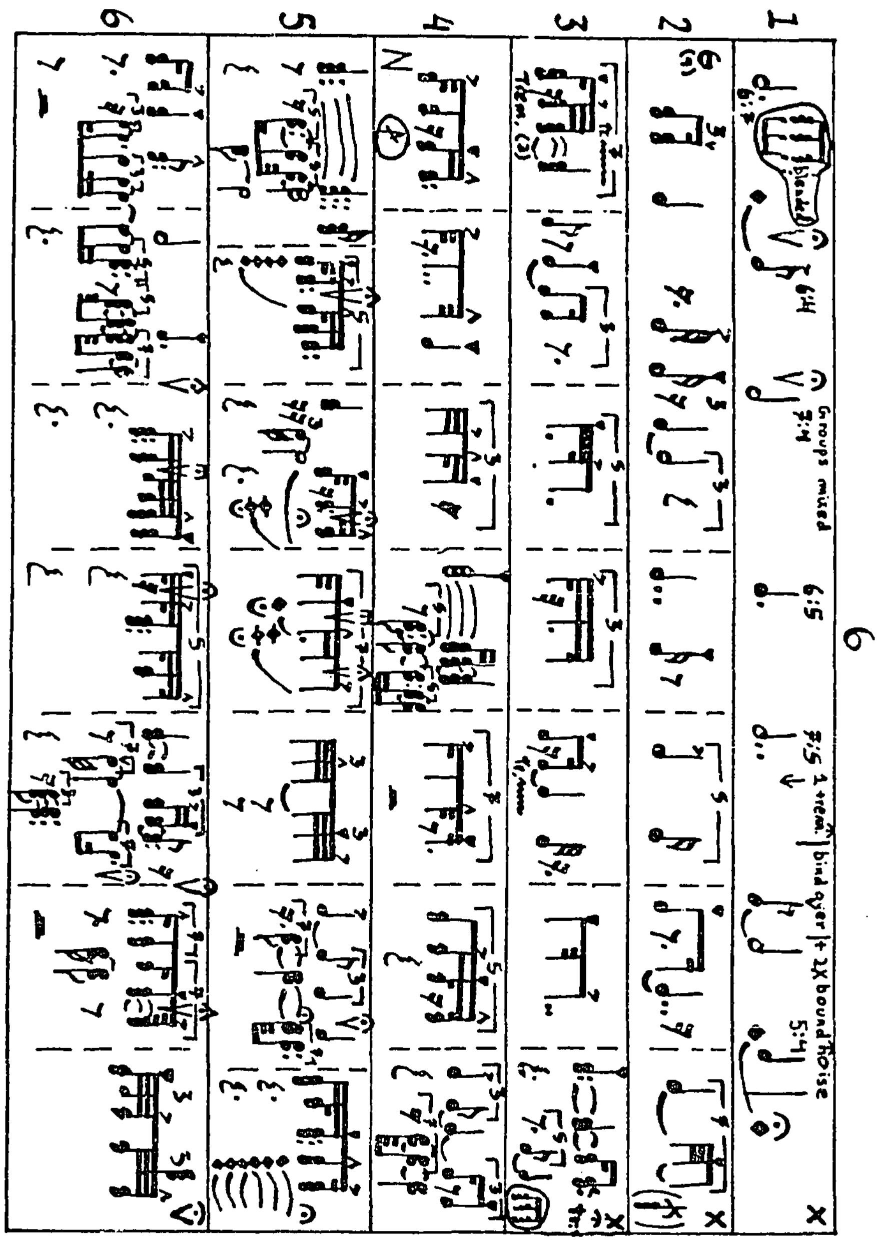
III^{3} I^{2} II^{2} I^{3}

II^{2} I^{3} III^{2} II^{3} I^{2}

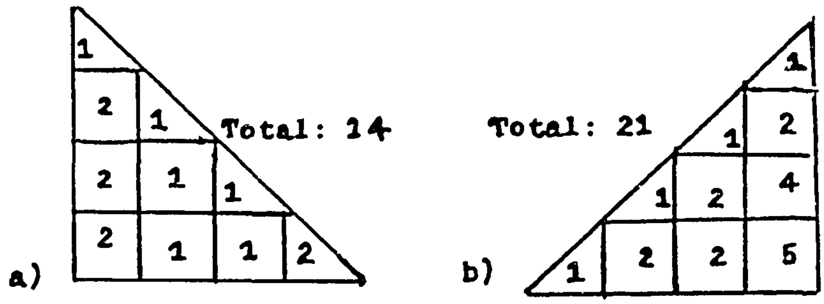
Example 9: Overlapping pitch enrichment processes of Examples 8a and 8c. (This example is not found in Stockhausen's sketches.)

6	1	4	3	7	5	2
1 (I^2)	3 (I)	6 (I)	5 (I)	2 (I)	7 (I)	4 (I)
4 (I^3)	6 (I)	2 (I)	1 (I)	5 (I)	3 (I)	7(II^2)
3 (I^2)	5 (I)	1 (I)	7(II^3)	4 (I)	2 (I^2)	6 (II)
7(III^3)	2 (I^2)	5 (II^2)	4 (I)	1 (I)	6 (II)	3 (I^3)
5(II^2)	7(III)	3 (I^3)	2 (I)	6(III^2)	4(II^3)	1 (I^2)

Example 10: Column 6 of the Final Rhythm Matrix.



Example 11: Pitch enrichment processes associated with rhythm matrix #2. (This example is not found in Stockhausen's sketches.)



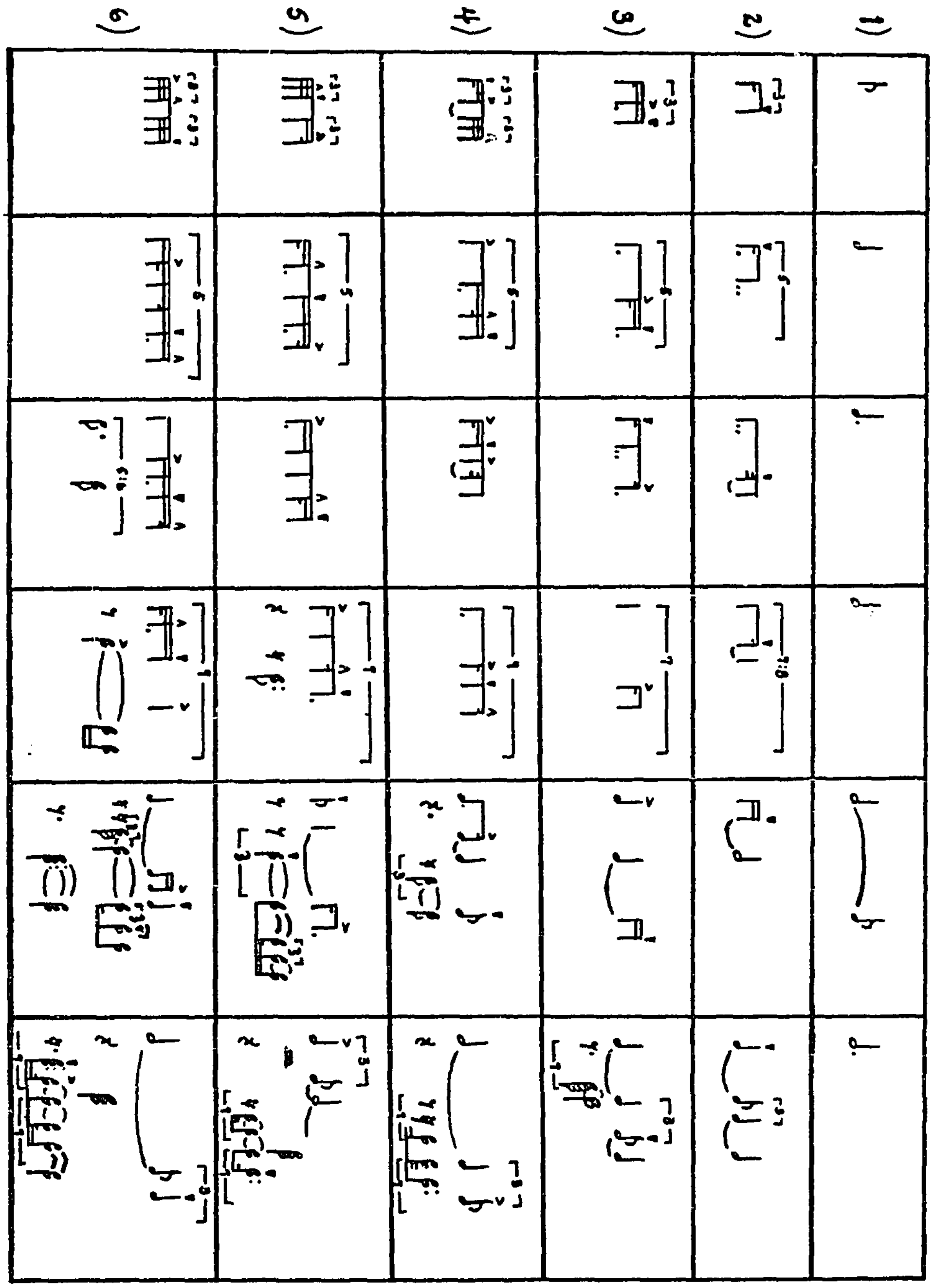
c) I^2
I^3 II^2
II^3 I^2 III^2
I^3 II^2 III^2 II^3

(Example 11c is from Roman numeral matrix #2, shown in Example #1.)

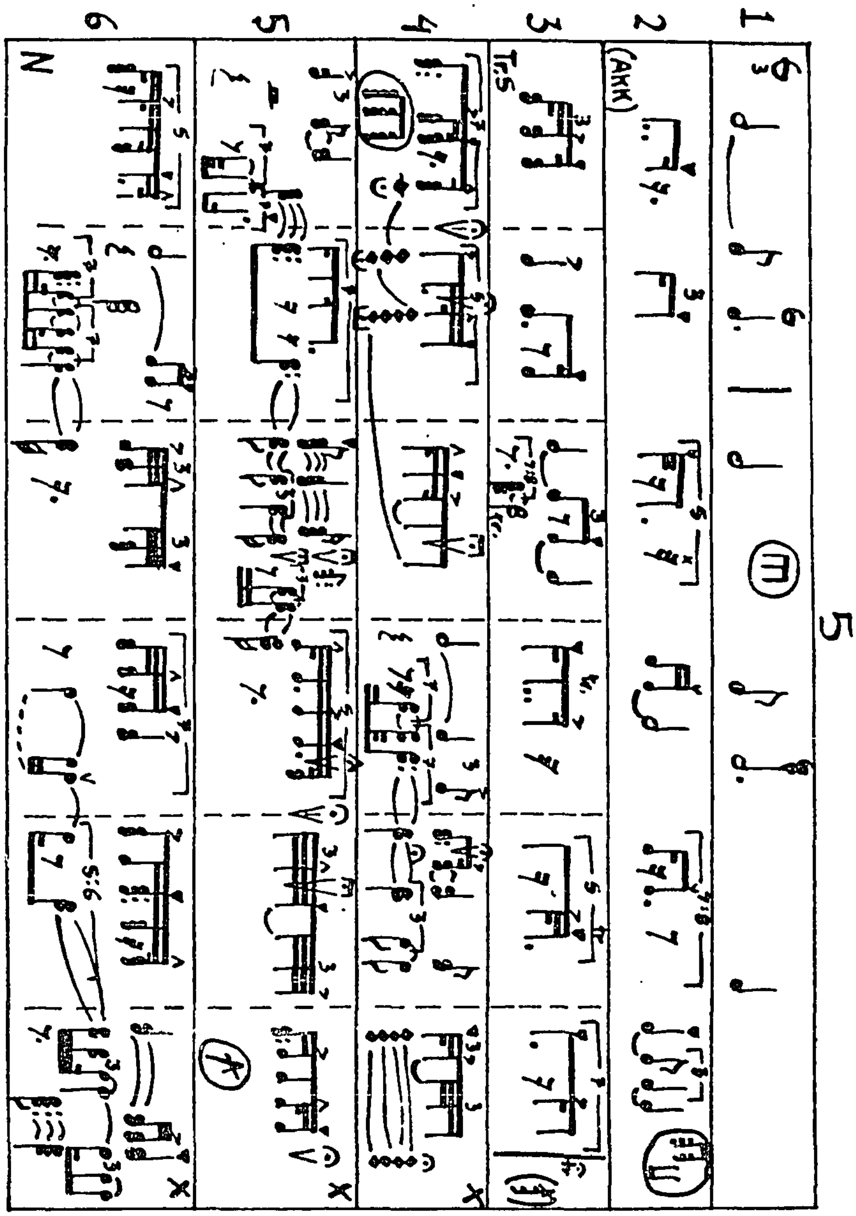
Example 12: Number matrix #2.

5	3	4	1	6	2
3	1	2	5	4	6
1	5	6	3	2	4
4	2	3	6	5	1
6	4	5	2	1	3
2	6	1	4	3	5

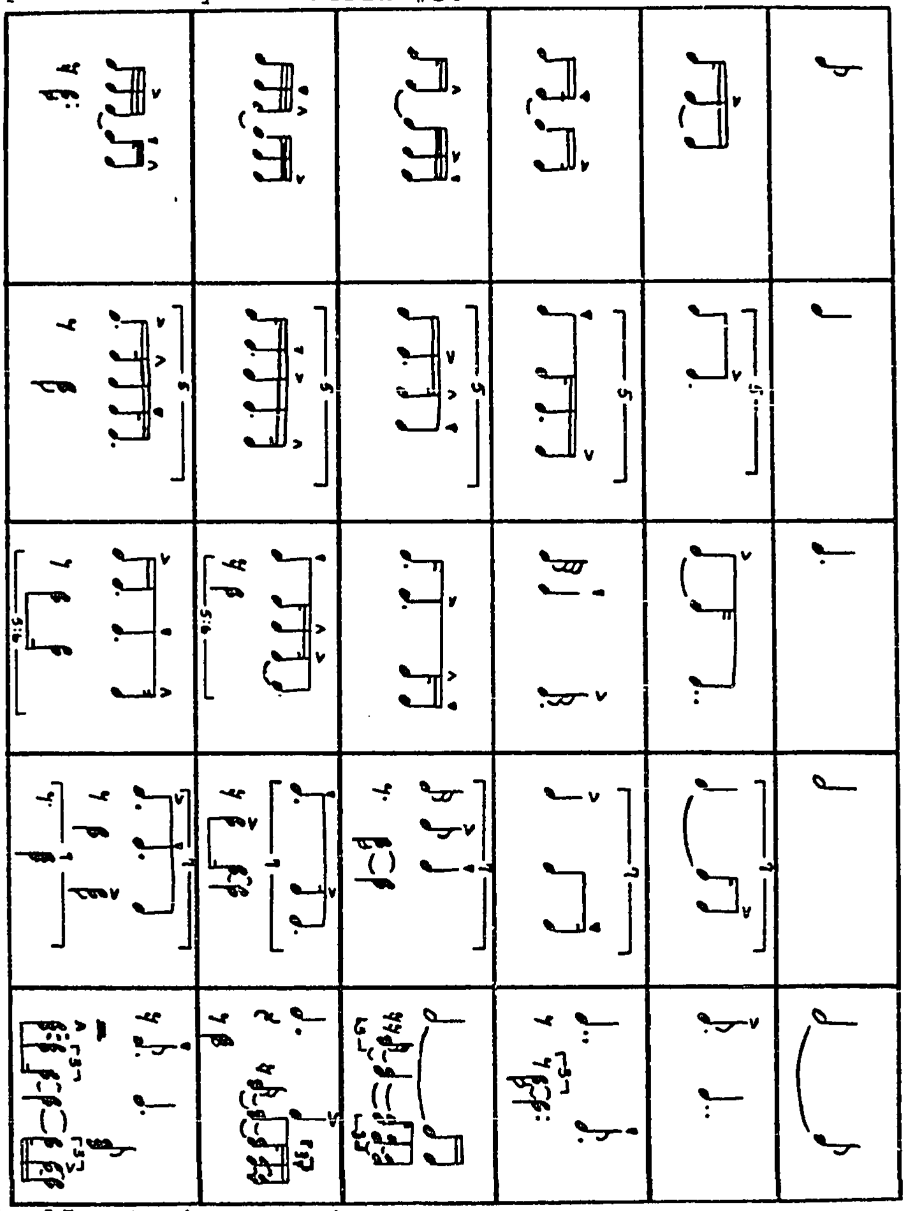
Example 13: Rhythm matrix #2.



Example 14: Column 5 of the Final Rhythm Matrix.



Example 16: Rhythm matrix #3.



Example 15: Number matrix #3 and Roman numeral matrix #3, both associated with Rhythm matrix #3.

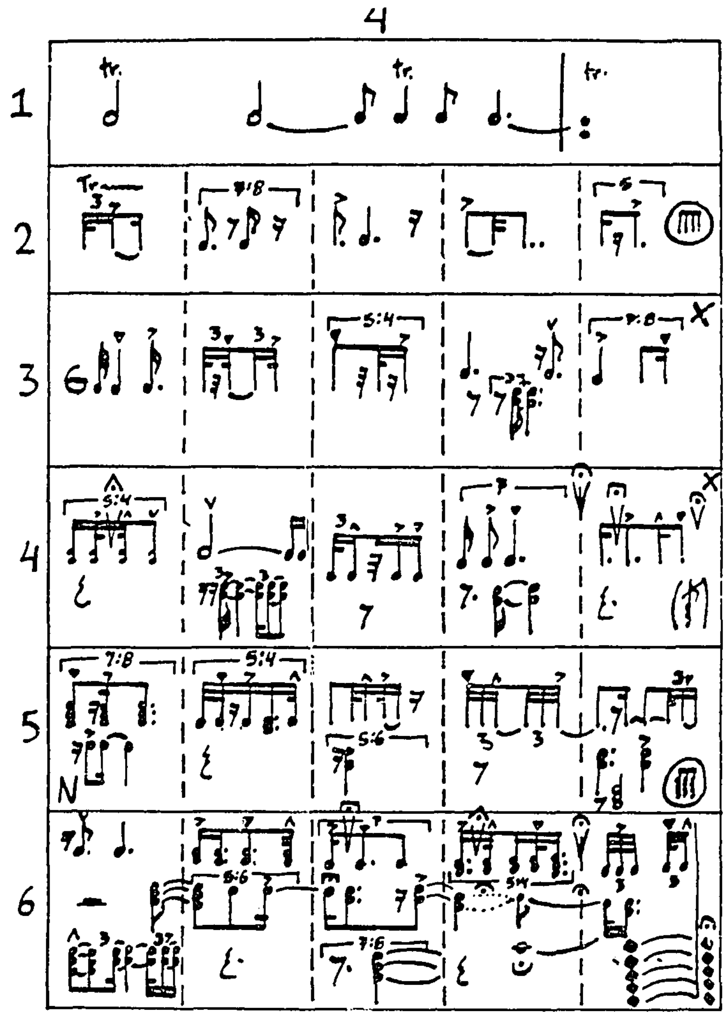
a)

4	5	2	1	3
1	4	5	3	2
3	1	2	5	4
2	5	1	4	3
4	2	3	1	5
5	3	4	2	1

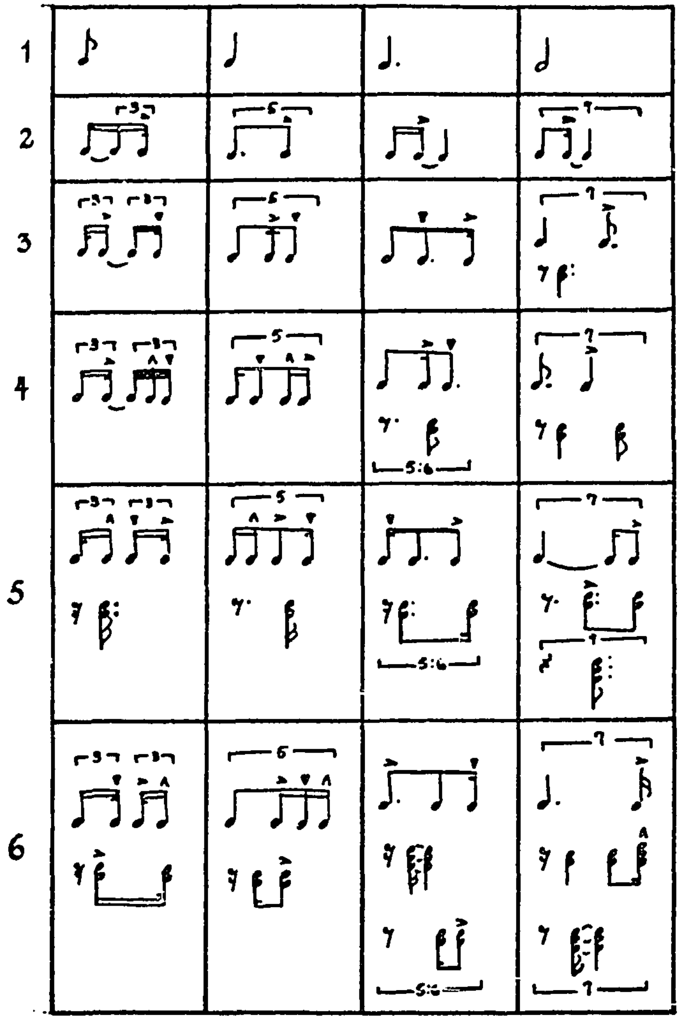
b)

I	I	I	II	I
I(2)	II	I	II	I
II	I	II	I	III
III	I	III	II	II

Example 17: Column 4 of the Final Rhythm Matrix.



Example 19a: Rhythm matrix #4.



Example 18: Number matrices #'s 4-6.

a) #4.

3	4	2	1
4	1	3	2
1	2	4	3
2	3	1	4
3	2	4	1
4	1	3	2

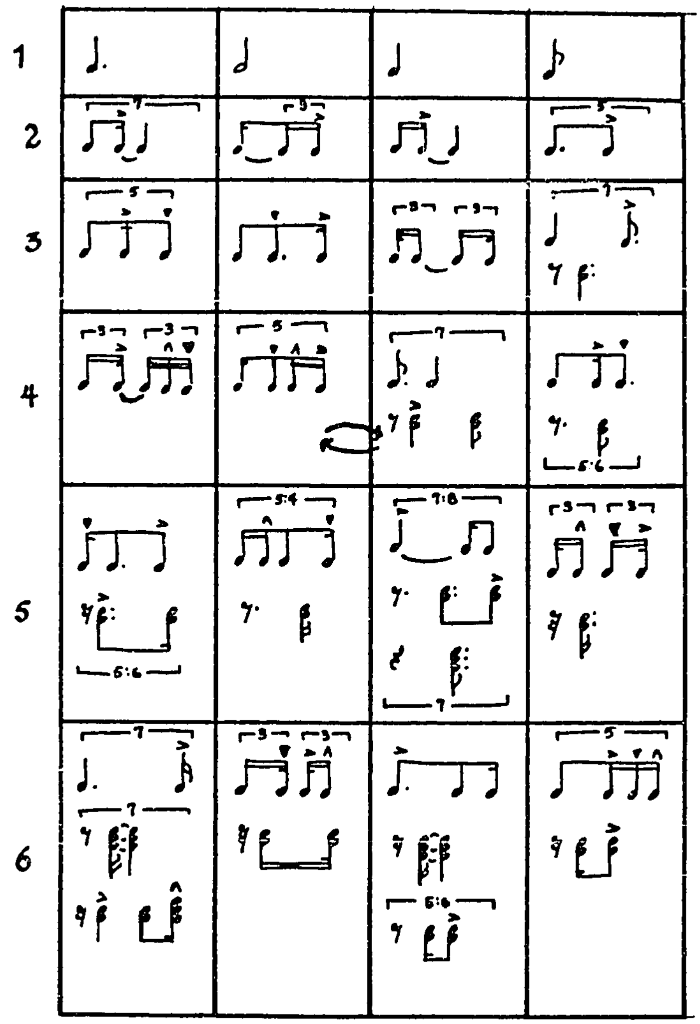
b) #5.

3	4	2	1
4	1	3	2
2	3	1	4
1	4	2	3
3	2	4	1
4	3	1	2

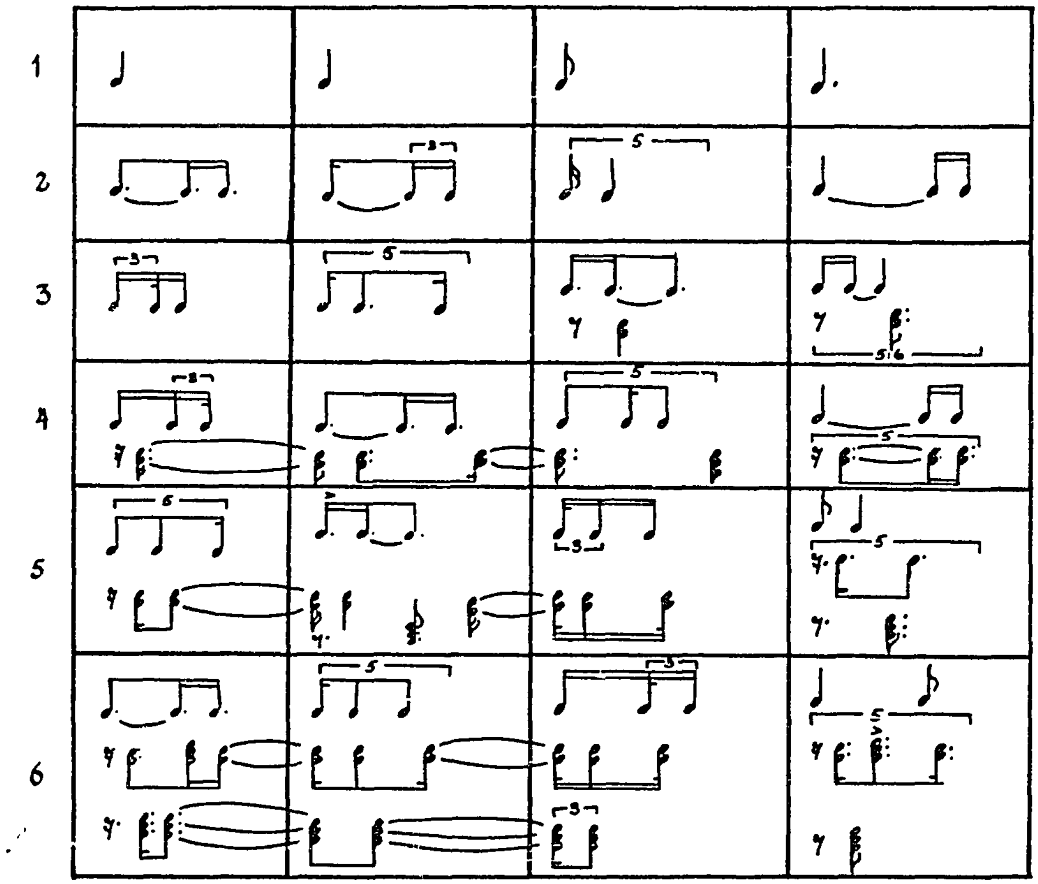
c) #6.

4	1	3	2
1	2	4	3
3	4	2	1
2	3	1	4
3	2	4	1
4	3	1	2

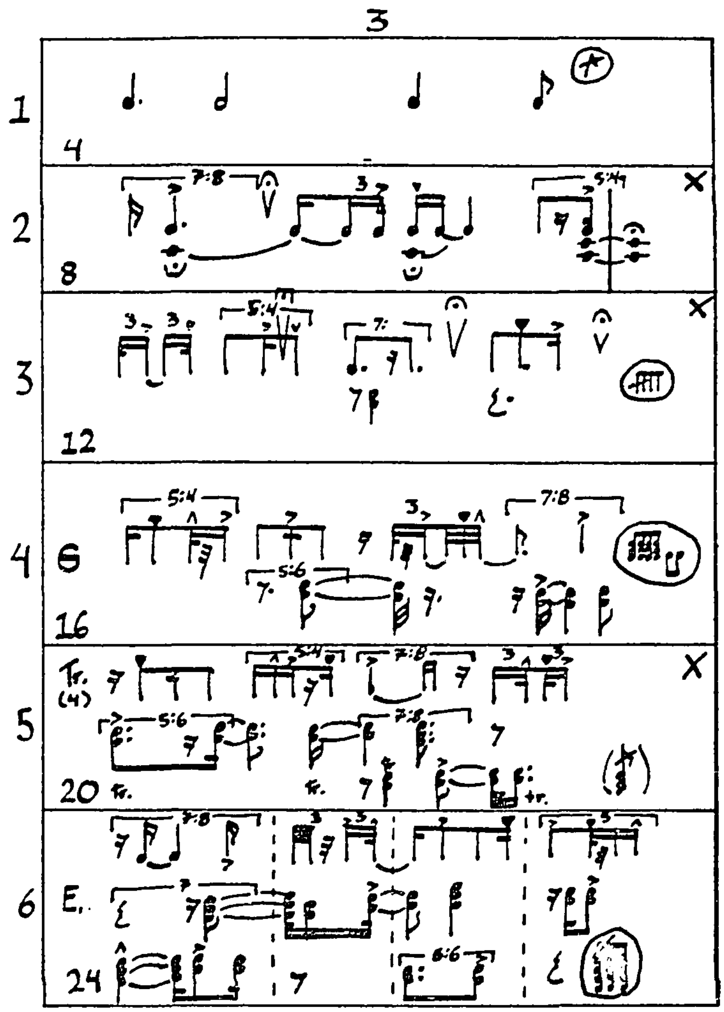
Example 19b: Rhythm matrix #5.



Example 19c: Rhythm matrix #6.



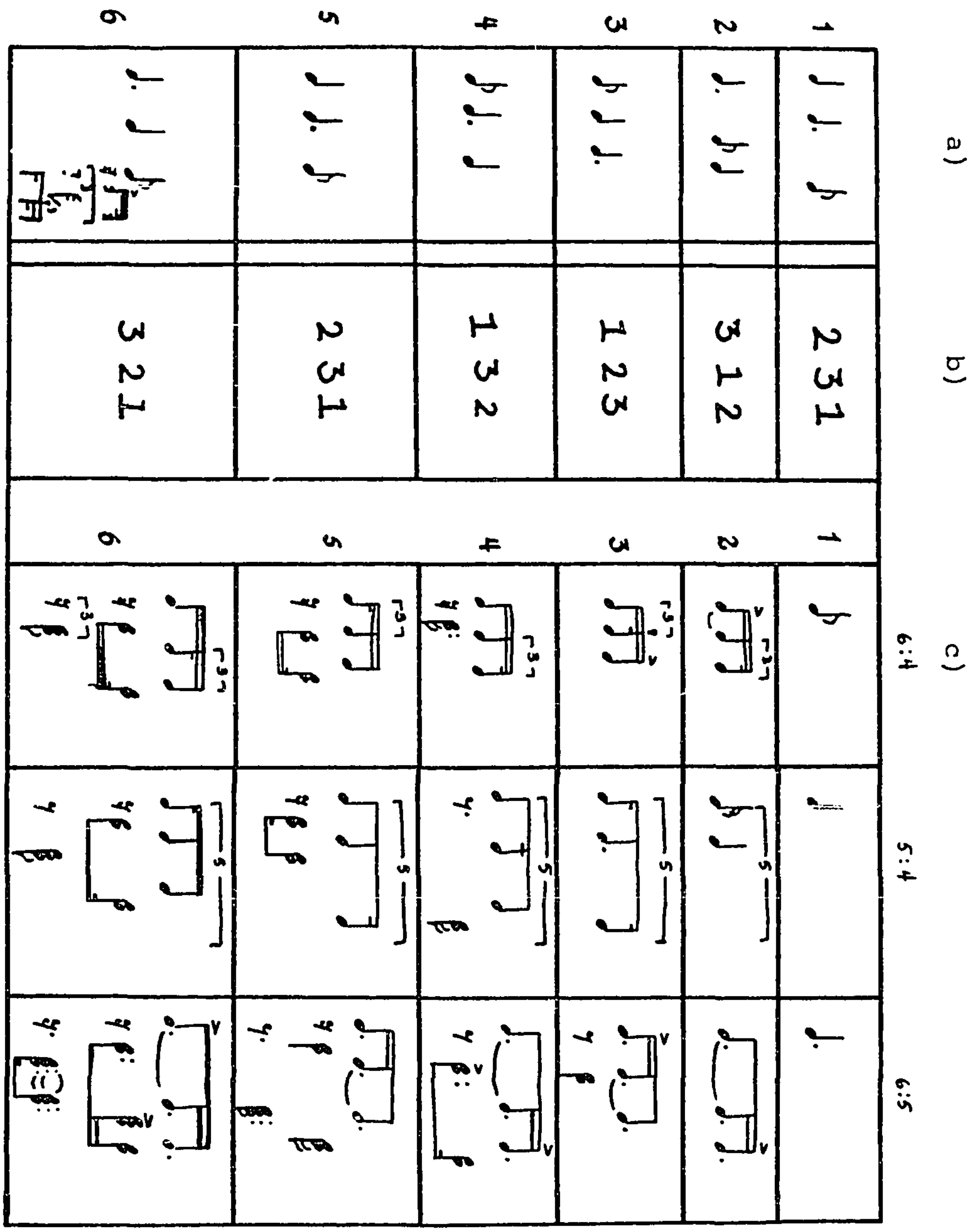
Example 20: Column 3 of the Final Rhythm Matrix.



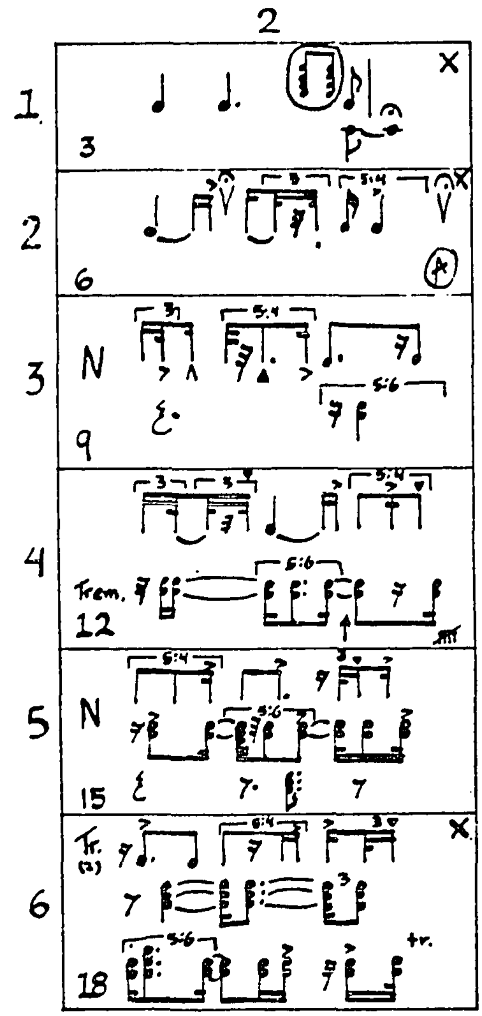
Example 24: Numbers series indicating the number of durations in each duration-group in the top voices of all blocks within the Final Rhythm Matrix. (This Example is not found in Stockhausen's sketches.)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	1 3	3 3	7 1	31231	23512	3212512
3	4	43	7 4	43124	524132	1324325
4	4	26	3414	8 9	3 $\underline{18}$	2375431
5	22	53	634	14752	$\underline{23}$	9 $\underline{20}$
6	4	35	472	$\underline{19}$	17864	$\underline{13}$ 8 $\underline{11}$

Example 25: Duration scheme (Example 25a) and number matrix #7 (Example 25b) used in reordering rhythm matrix #7 (Example 25c).

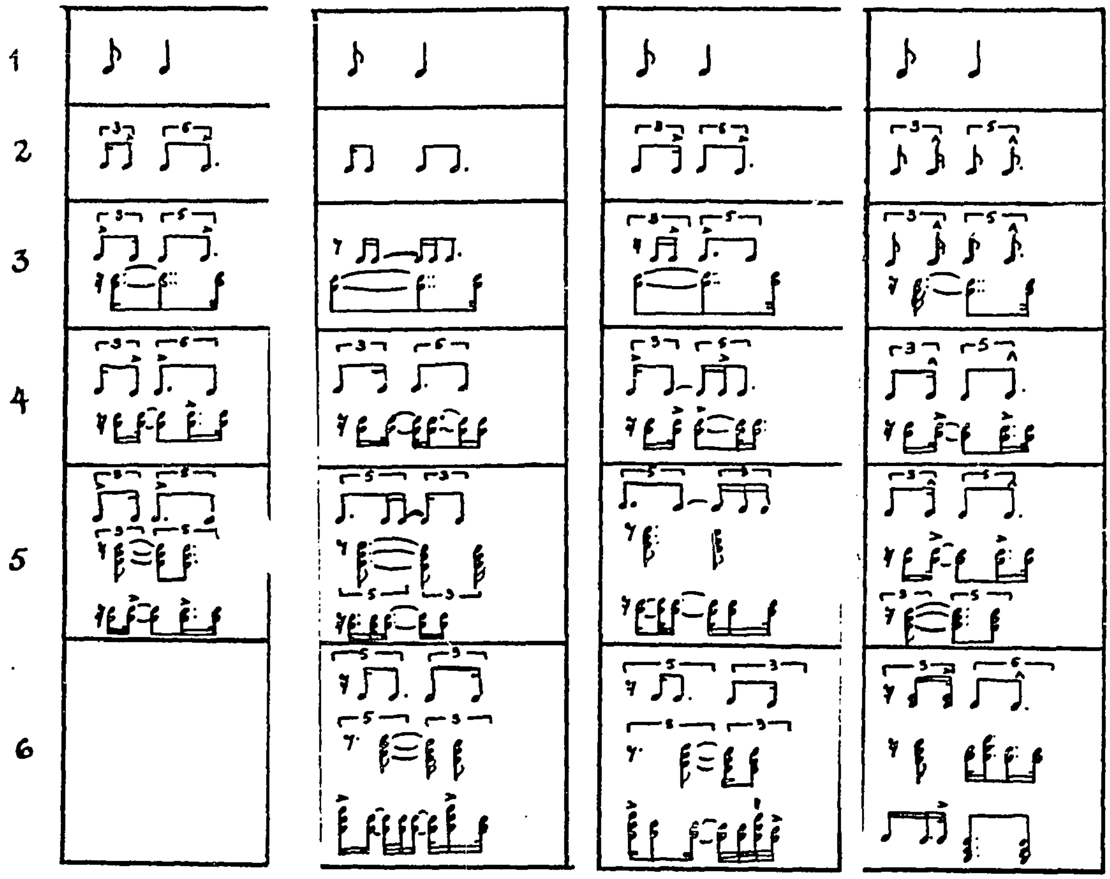


Example 26: Column 2 of the Final Rhythm Matrix.

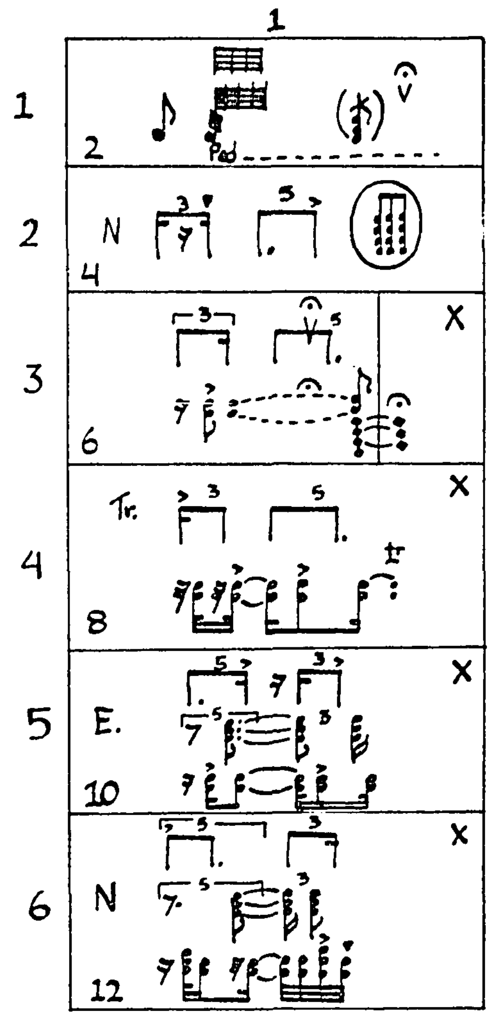


Example 27: Rhythm matrices #'s 8-11.

a) #8 b) #9 c) #10 d) #11



Example 28: Column 1 of the Final Rhythm Matrix.



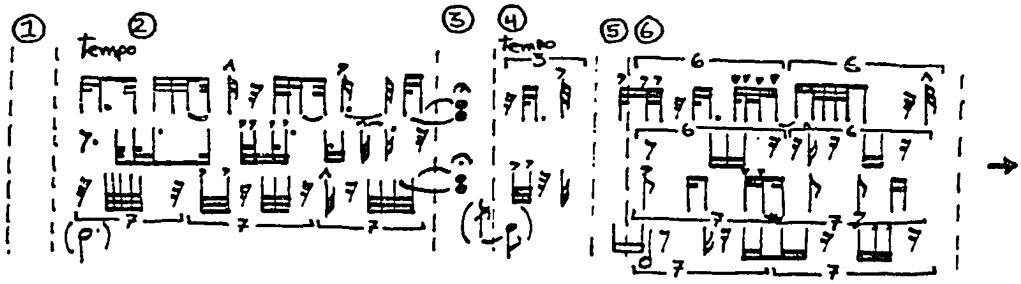
Example 29: Originating rhythmic cells for column 1 of the Final Rhythm Matrix, located in Examples 27a-c. (This example is not found in Stockhauen's sketches.)

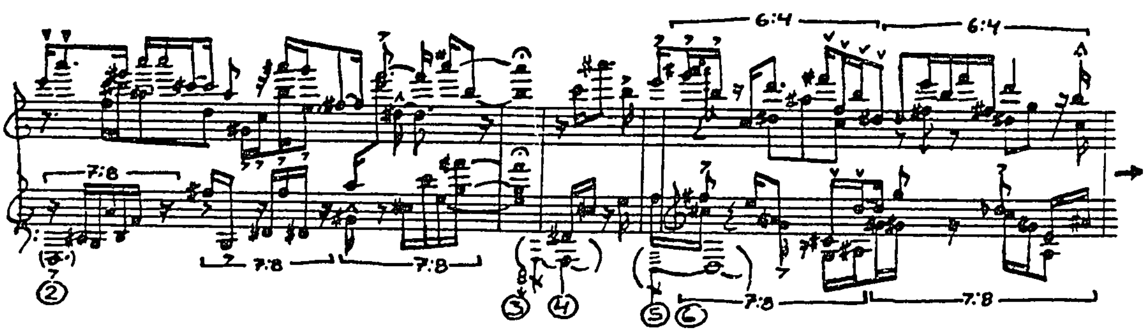
1	abc	
2	abc	abc
3	a a	a a
4	a abc	ab ab
5	b b a c	b b a c
6	bc bc c	bc bc c

Example 31: Number matrix #8.

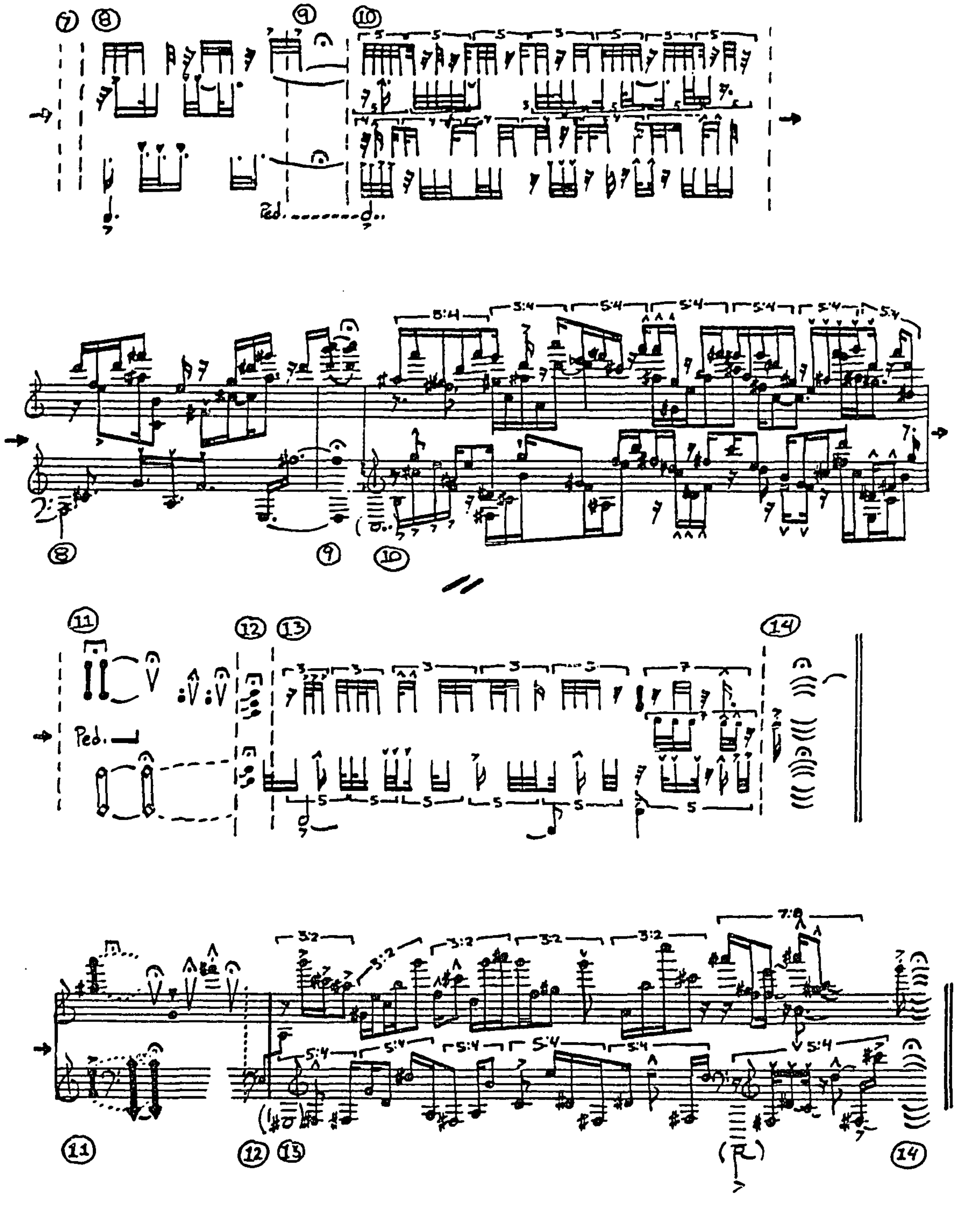
2	3	4	5	6	7
4	6	8	10	12	14
6	9	12	15	18	21
8	12	16	20	25	28
10	15	20	25	30	35
12	18	24	30	36	42

Example 32: Fragment #17 without grace note groups. Original rhythmic scheme placed above the pitch realizations of the published score.



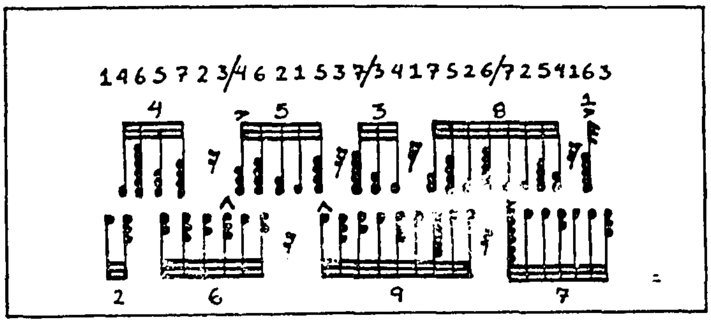


Example 32: continued.

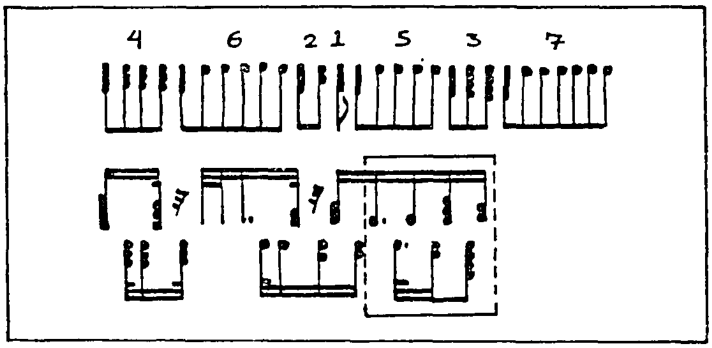


Example 33: Three versions for the initial 28 grace note attacks of fragment #17.

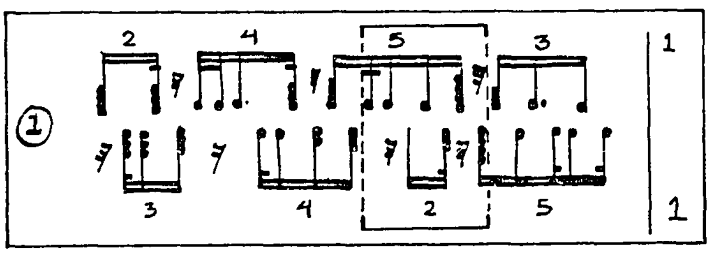
a)



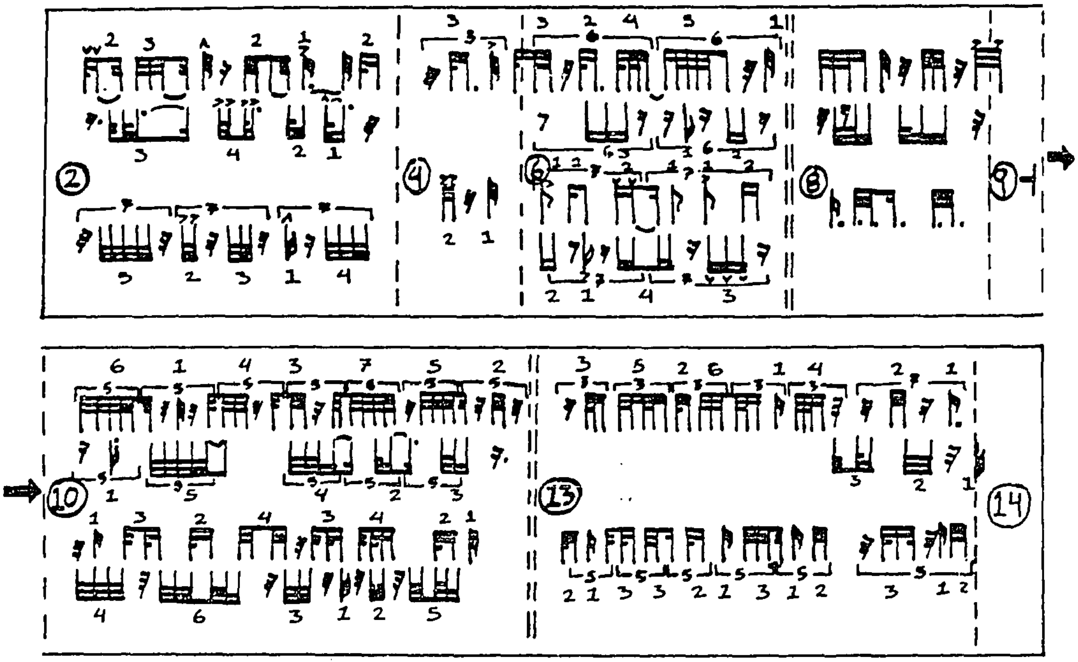
b)



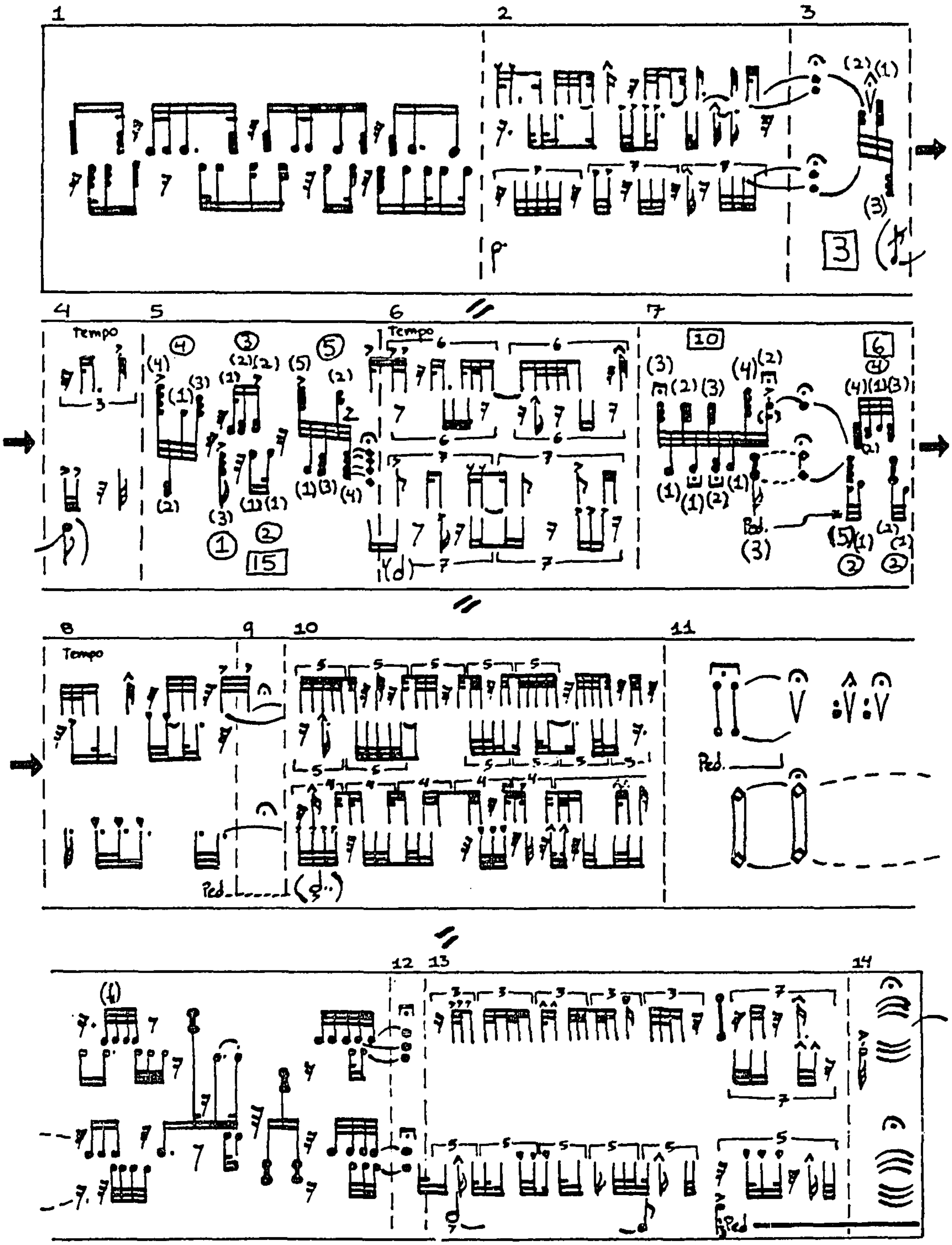
c)



Example 34: Original rhythmic shemes for sections 2, 4, 6, 8, 10, and 13 of fragment #17.

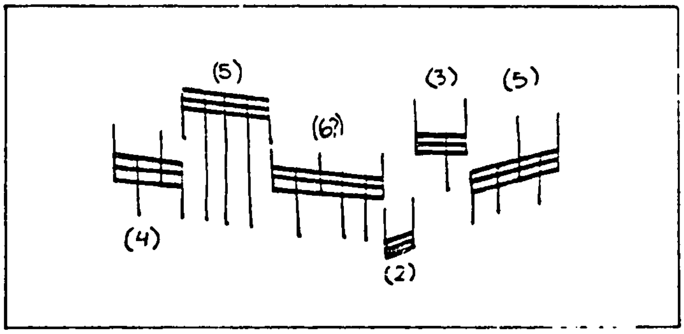


Example 35: Rhythmic scheme for fragment #17 with grace notes.

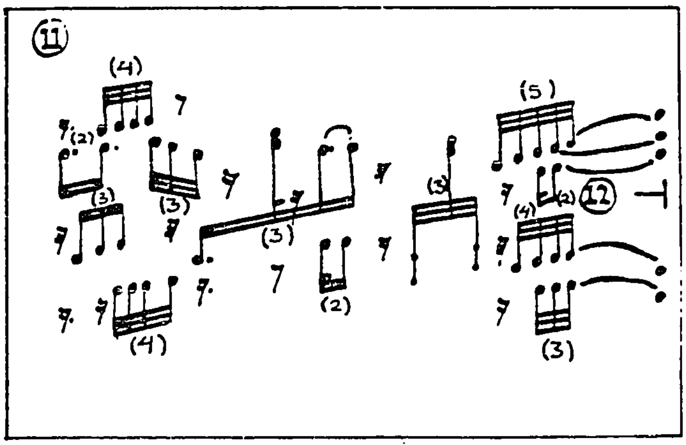


Example 36: Initial conception (36a) and final version (36b) of grace note group in sectiion 11 of fragment #17.

a)



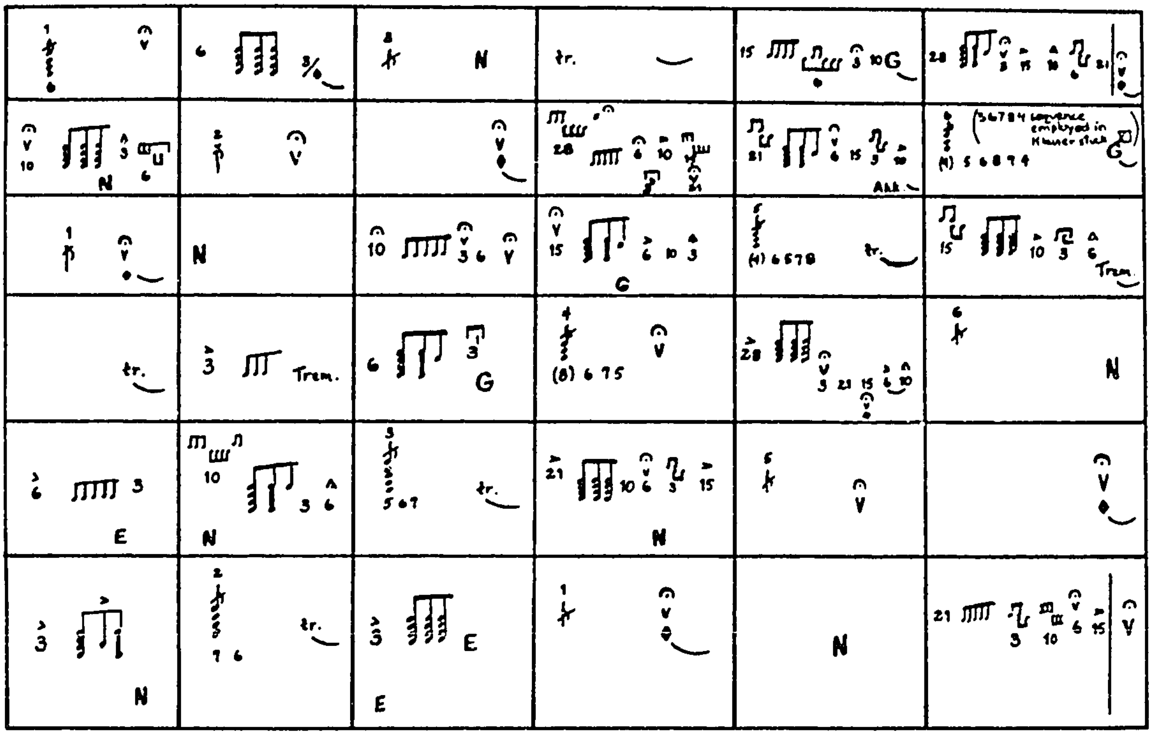
b)



two other blocks of the Final Element Matrix.[21]

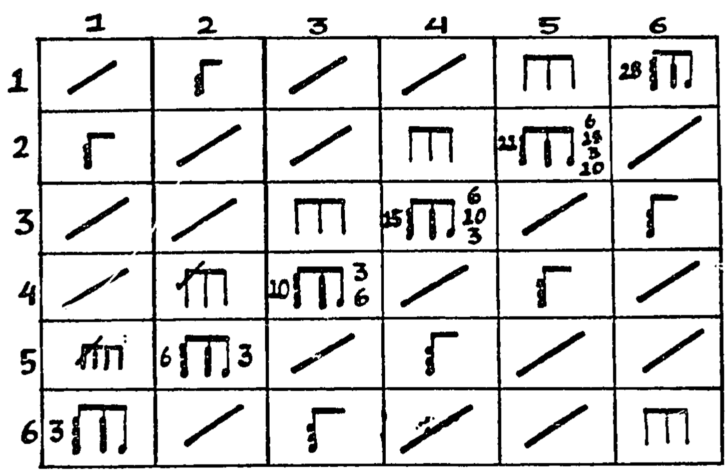
Example 37: The Final Element Matrix (37a); definitions of elements (37b).

a)



Example 39: Element matrix #1 (39a), addendum to element matrix #1 (39b), number matrix #9 (39c), number matrix #9 (39d)

a)



(Example 39 continued):

b)

	1	2	3	4	5	6
1	2		2	2		
2	(chords)	2	1			2
3	2	1	(singly)		2	
4	1		(mixed)	2	(chords)	2
5			2		2	1
6		2		2	1	(singly)

c)

3	6	3 6 10	3 6 10 15	3 6 10 15 21	3 6 10 15 21 28
6	3 10	3 6 15	3 6 10 21	3 6 10 15 28	
10	3 15	3 10 15	3 6 15 21	3 6 10 21 28	
3	3 6	3 6 10	3 6 10 15	3 6 10 15 21	3 6 10 15 21 28
6	6 10	6 10 15	6 10 15 21	6 10 15 21 28	6 10 15 21 28 3
10	10 15	10 15 21	10 15 21 28	10 15 21 28 3	10 15 21 28 3 6
3	6 3	10 3 6	15 6 3 10	21 6 3 10 15	28 10 3 6 15 21
6	10 6	15 6 10	21 10 6 15	28 10 6 15 21	3 15 6 10 21 28
10	15 10	21 10 15	28 15 10 21	3 15 10 21 28	6 21 10 15 28 3

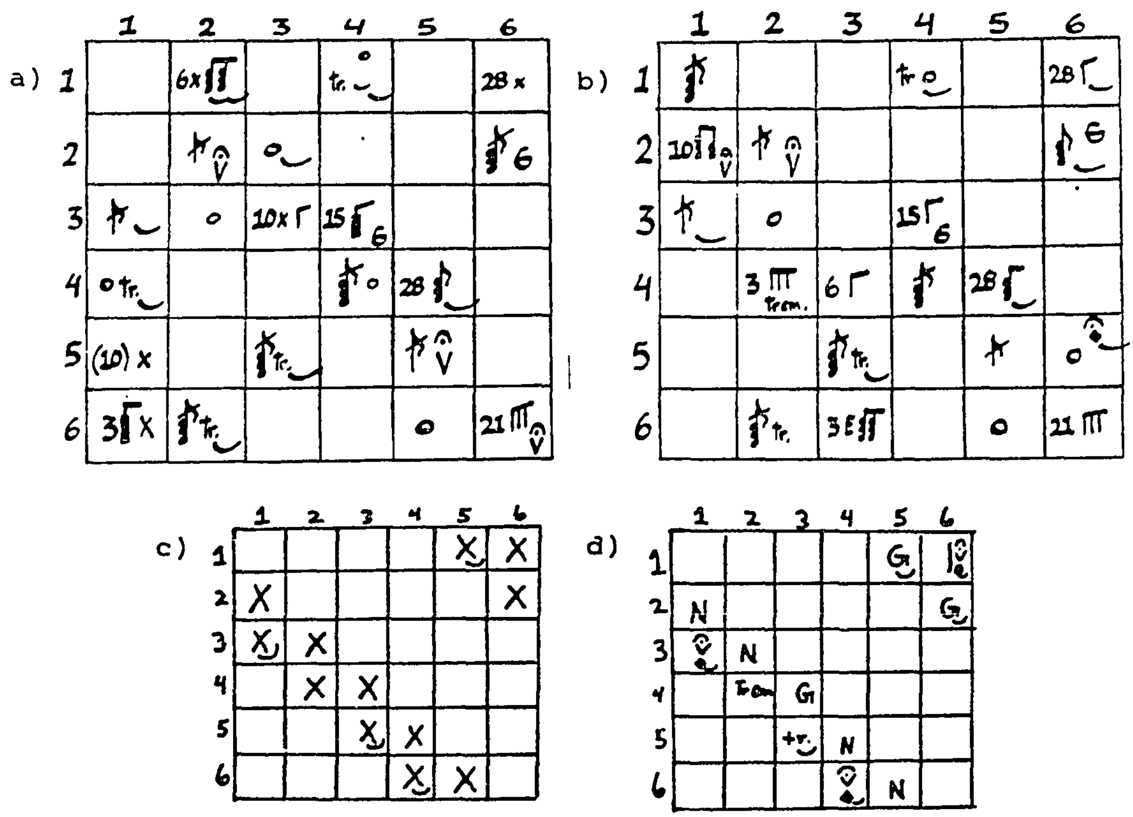
d)

				15 6 10 3	21 6 15 3 10	28 3 15 10 6 21
				21 3 10 6 15	28 6 10 15 3 21	15 6 3 10
				28 3 21 15 6 10	15 10 3 6	21 10 6 3 15
mixed	3	6 3	10 3 6	15 10 6 3	21 15 10 6 3	28 21 15 10 6 3
singly	6 3	10 3 6	3	21 15 10 6 3	28 21 15 10 6 3	15 10 6 3
chords	10 3 6	3	6 3	28 21 15 10 6 3	15 10 6 3	21 15 10 6 3

Example 39e: Blocks in Final Rhythm Matrix indicated to which were transferred additive series blocks from Example 39d. (This example is not found in Stockhausen's sketches.)

	1	2	3	4	5	6
1				(4) R3C4	(5) R2C5	(6) R1C6
2				(5) R6C6	(6) R2C4	(4) R1C5
3				(6) R4C5	(4) R3C6	(5) R5C4
4 mixed	(1) R6C1	(2) R4C3	(3) R5C2			
5 singly	(2) R5C1	(3) R3C3	(1) R4C2			
6 chords	(3) R2C1	(1) R6C3	(2) R1C2			

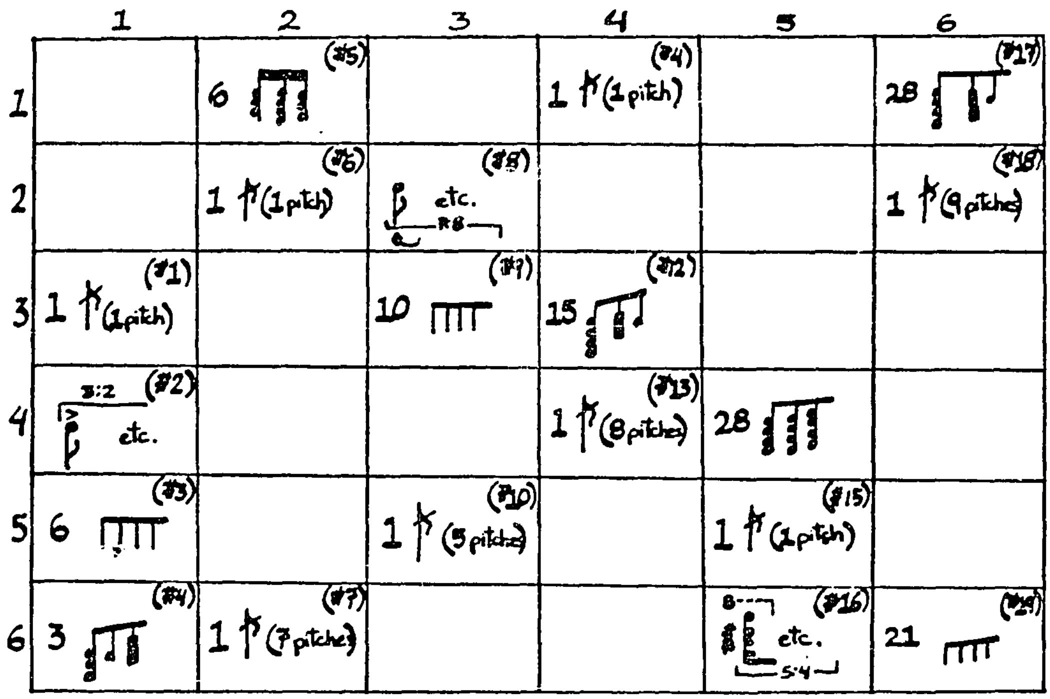
Example 40: Element Matrix #2 (40a), element matrix #3 (40b), step function patterns in Final Element Matrix (40c), step function elements in Final Element Matrix (40d).



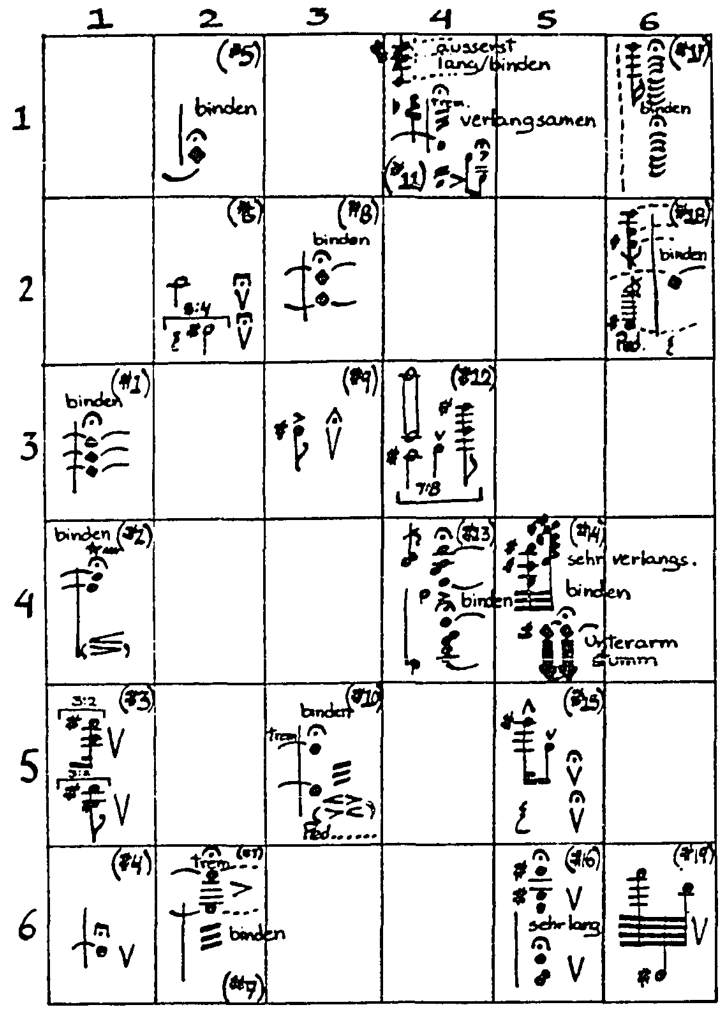
Example 41: Composite of elements from Examples 39 and 40 occurring also in the Final Element Matrix. (This example is not found in Stockhausen's sketches.)

	1	2	3	4	5	6
1	40b	39c/d 40a	(39b)	40a/b	39a/c/d 40c	39a/c/d 40a/b/c
2	39c/d 40b/c	40a/b	40a	39c/d	39a/c/d	40a/b/c
3	40a/b/c	40b/c	39c/d 40a	39a/c/d	(39b)	39a/c/d
4	40a	39c/d 40b/c	39a/c/d 40b/d	40a/b	39a/c/d 40a/b	(39b)
5	39a/c/d 40a	39a/c/d	40b/c/d	39c/d 40c	40a/b	40b (o)?
6	39a/c/d 40a	40a/b/c	39a/c/d 40b	40c	40a/b (o)?	39c/d 40a/b

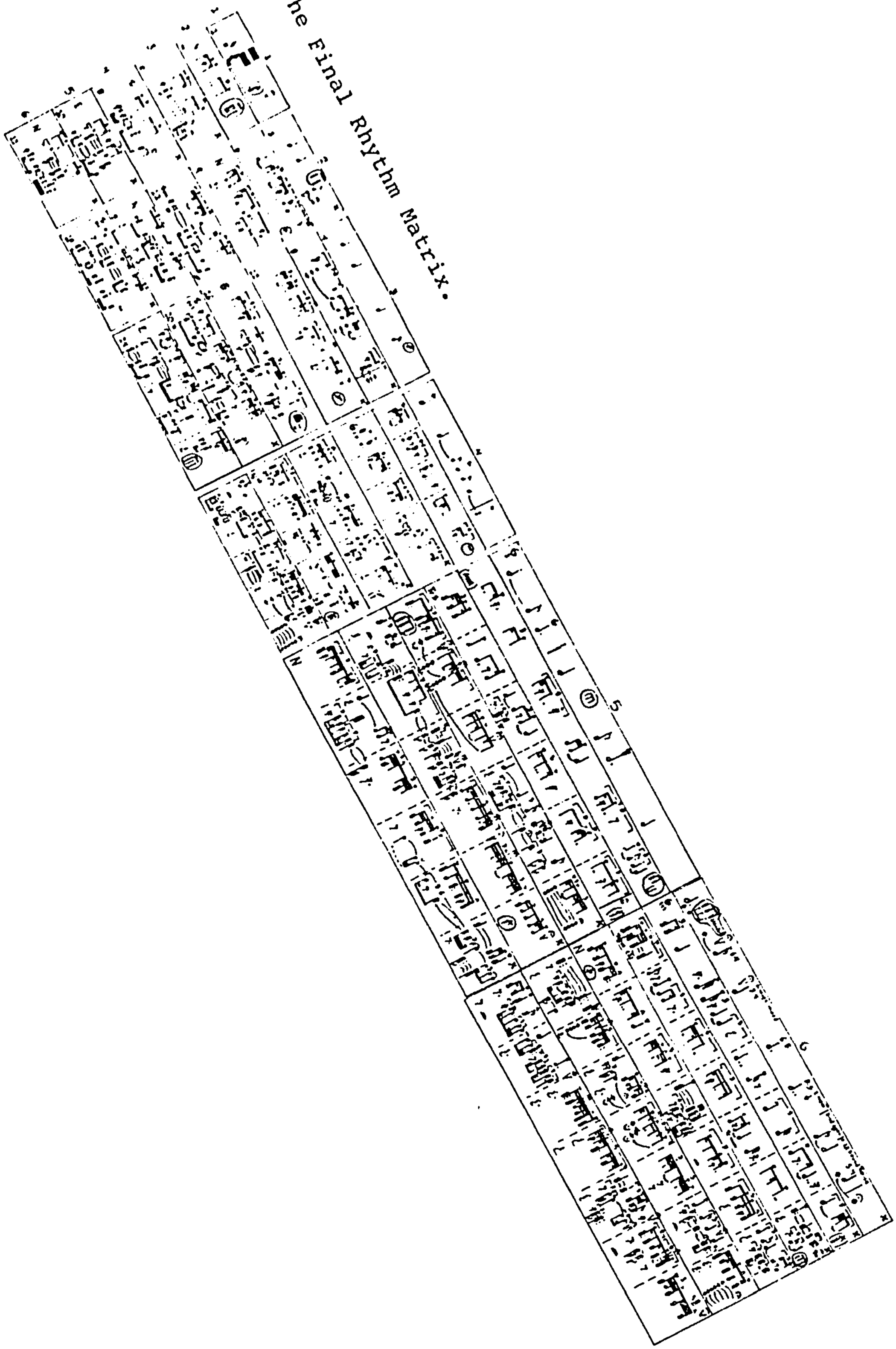
Example 43: Matrix illustrating beginning attacks of the 19 fragments of Klavierstuck XI. (This example is not found in Stockhausen's sketches.)



Example 44: Matrix illustrating ending attacks of the 19 fragments of Klavierstuck XI. (This example is not found in Stockhausen's sketches.)



The Final Rhythm Matrix.



Numbered Harmonic Series.