

A Bayesian Analysis of Carbon Dioxide Emissions in States Under the Regional

Greenhouse Gas Initiative

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## Table of Contents

I – Abstract.....	3
II – Theory and Development of Cap and Trade... ..	4
III – The Regional Greenhouse Gas Initiative.....	9
IV – Bayesian Statistical Methods .....	13
V – Methods.....	17
VI – Results.....	24
VII – Conclusions.....	26
VIII – References.....	29
IX – Appendices.....	31

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Sincerely,

Derek Christian Wietelman

# I

## Abstract

In the field of environmental economics, cap and trade systems are an attempt at using a marketplace with as little government restriction as possible in order to reduce emissions of an undesirable pollutant. A hard cap on the amount of pollution allowed is set, and pollution permits allowing the holder to legally pollute are distributed in a variety of ways. Each year, the cap is lowered and fewer permits are allocated, forcing firms to streamline their processes to be more efficient so that they will need to purchase fewer permits each year. Several cap and trade programs like the EPA's Acid Rain Program and the European Union's Emissions Trading System have been implemented over the course of the last few decades. One such system that has been adopted in the Northeastern United States is the Regional Greenhouse Gas Initiative (RGGI), a group of nine U.S. states that have agreed to form a cap and trade program regulating carbon dioxide emissions from power plants. While cap and trade makes sense theoretically, experiments in public policy are carried out in the real world, not a controlled laboratory setting. This paper applies Bayesian modeling techniques using the statistical package R and its ability to interface with JAGS (Just Another Gibbs Sampler) to the yearly CO<sub>2</sub> emissions data from power plants located in the RGGI member states to assess whether or not RGGI has been successful at reducing carbon emissions. JAGS software uses Markov Chain Monte Carlo algorithms, specifically Gibbs Sampling methods, to generate posterior distributions from the data given prior distributions. If it is ultimately determined that emissions have been falling under the RGGI, it may provide a template for other states and nations to design their own cap and trade programs.

## II

### Theory and Development of Cap and Trade

While the push for the implementation of cap and trade systems in the United States began during the 1970s and 1980s, the preliminary ideas that would later evolve into what we know as cap and trade today originated with the writings of the economist Arthur Pigou. In his work *The Economics of Welfare* (1932), Pigou articulated for the first time a need for “internalizing the externalities.” In economics, an externality is an undesirable byproduct of production that is not accounted for in standard measures of marketplace activity like demand, supply, and price. One common example of an externality is pollution. When a new coal generating plant is built, the factory owners must pay typical fixed and variable costs such as property taxes, equipment fees, and employee salaries. However, there are many more costs that the factory causes but is not accountable for, such as the increased medical costs for the surrounding community due to the decrease in air quality from pollutants emitted by the factory. Pigou argued that producers had absolutely no incentive to stop performing the undesirable activity causing the externality (in this case, the carbon dioxide emissions from the coal plant) and that intervention in the marketplace is necessary. Pigou suggested that this intervention should come in the form of a tax on the polluting activity. By setting a per-unit tax that is equivalent to the marginal external social damage caused by producing pollutants past the economically efficient equilibrium, firms would begin to scale back and make their processes more efficient rather than pay the tax.

The Pigouvian approach of pollution taxation, while not a cap and trade system, paved the way for future economists and scholars to work with the idea of quantifying the price of pollution and designing methods to help with the abatement of pollution levels. Another

economist who followed in Pigou's footsteps yet criticized many of his ideas was Ronald Coase. The approach that Coase took to abate pollution levels contrasted with that of Pigou, and argued that strict pollution taxation or governmental intervention in general was not necessary to achieve efficiency in the pollution marketplace. Known in policy and economics circles as the Coase Theorem, Coase argued that the way for a society or community to reach its efficient level of pollution control is to first determine which firms hold the relevant property rights to certain undesirable externalities, and then the firms or marketplace participants can engage in strict contract negotiations to reach the most efficient outcome (Coase 1960). Economist Steven Hackett (2011) illustrates the Coase Theorem using the example of a car repair shop and a local housing community. In Hackett's example, the auto body shop holds the legal property rights and is allowed to operate, although they produce 120 decibels of noise pollution which is under the city's legal limit for noise pollution. The housing community stands to gain \$1 million if they can eliminate the noise from the car repair shop. Since the car repair shop owns the property right, they are legally not bound to do anything. Therefore, the housing community has two options: they can pay the shop \$300,000 dollars to relocate somewhere else, or they can pay \$100,000 dollars to equip the shop with sound-proofing materials. The Coase Theorem shows here that the efficient solution, without any government assistance necessary, would be for the two to enter into a contract where the housing community pays \$100,000 dollars for the sound-proofing materials, and gains \$1 million from the reduced noise, resulting in a net benefit of \$900,000. Conversely, if the car repair shop is producing 130 decibels of noise, which is over the legal limit, the two would reach an agreement where the repair shop would have to pay for the sound-proofing materials.

The example above illustrates how the Coasian approach can be applied to situations involving air pollution. The car repair shop can be replaced with a coal plant that emits CO<sub>2</sub> into the air, and the housing community still remains a housing community. The only regulation necessary is a simple cap on the amount of pollution. If the amount of pollution that the plant permits is less than the legal limit but still at a level undesirable by the community, then the community will pay the plant to streamline its processes and make them cleaner. If the amount of pollution that the plant emits is greater than the legal limit, then the plant will pay out of its own pocket to lower their emissions levels, or face a lawsuit from the community for violating the pollution standard. In general, the Coase Theorem purports itself to hold as long as it is feasible to assign property rights to certain participants, there is a net benefit to be had by environmental improvement, transaction costs are low, agreements are enforceable under law, and that free-riders who are not paying for a benefit but reap its rewards are eliminated as much as possible (Hackett 2011).

Yet another alternative to both the Pigouvian and Coasian approaches to dealing with pollution through the marketplace is the “command and control” approach of governmental regulation. Command and control differs from the Pigou and Coase models in that it takes a heavier-handed approach in regulating exactly how firms reduce pollution levels. Rather than the Pigouvian approach of simply taxing firms if they over-pollute, or letting firms reach an optimal point of pollution through Coasian negotiations, command and control calls for specific guidelines as to how pollution will be reduced that are applied to all firms and strictly enforced by government regulators. One common example of command and control regulation includes requiring that car manufacturers install catalytic converters on vehicles. Another less strict form

of command and control comes in the form of uniform efficiency requirements on products like air conditioners and refrigerators.

While the Pigouvian, Coasian, and command and control frameworks help paved the way for cap and trade to be developed, the theoretical ideas of the three approaches still needed to be fully fleshed out and developed into the cap and trade programs like those in place today. One work that is often referred to as the origin of cap and trade is J.H. Dales' *Pollution, Property, and Prices* (1968). Dales, a University of Toronto economist, was in the process of researching a solution for water pollution issues in the Great Lakes areas around Toronto. Dales, in his own words, called pollution “a reflection of the amount of waste generated by a population and of the geographical concentration of population and production.” (Dales 1968). Deeply committed to using the study of economics to help with pollution abatement, Dales acknowledged the Pigouvian approach of taxing polluters but was not satisfied with that approach. Dales instead offered his own design of a policy that “refers only to water-pollution; but the argument should apply equally well to air-pollution problems” (Dales 1968). Dales' solution that he proposed was one of the first academic works to describe what is known today as cap and trade – creating a carbon market where firms can buy and sell pollution permits. Take the following quote from Dales as an example:

Let us try to set up a “market” in “pollution rights.” The Board starts the process by creating a certain number of Pollution Rights, each Right giving whoever buys it the right to discharge one equivalent ton of wastes into natural waters during the current year...if half a million tons of wastes are currently being dumped into the water system, the Board would issue half a million Rights. All waste dischargers would then be required to buy whatever number of Rights they need...To put the market into operation, let us say that the Board decides to withhold 5 percent of the Rights in order to allow for the growth of production and population during the first year, and therefore offers 475,000 Rights for sale. Since the demand is for 500,000, the Rights will immediately command some positive price – say, 10 cents each.



In just a few sentences, Dales took elements from each of the three pollution abatement frameworks discussed earlier and developed for the first time a cap and trade program. Dales argued that by establishing a regional price per ton for water pollution in the Great Lakes and charging firms for each ton of pollution that they produced through a permit system, over time growth in the industry would result in an increased demand for pollution permits, “and since the number of Rights issued by the Board cannot be increased the price of Rights will move upward” (Dales 1968). Given that the price of the permits would rise, firms would accordingly streamline their processes and make them cleaner so as to not have to purchase as many permits as before. Dales’ version of cap and trade was well thought out given that it was such a novel concept at the time, as he even foresaw the need to clarify that the market would set the prices for the permits, not the central regulatory board that would enforce compliance.

Ultimately, the approach that Dales outlined takes from each the Pigouvian and Coasian frameworks, but not the command and control approach. Cap and trade has traces of Pigouvian elements in the fact that it quantifies a price for pollution, something that had never been done before Pigou. Cap and trade’s Coasian roots stem from the fact that government regulation is kept to a minimum, with firms not being told how to reduce pollution and a marketplace being established to allow for firms to buy, sell, and trade permits as necessary. Dales thought that the costs of government regulation were far too high and accordingly shied away from incorporating command and control aspects into his cap and trade design, stating that “control by regulation...[is] likely to be unfair and inefficient (i.e., more costly to the polluters than some other scheme that would achieve the same reduction in pollution)” (Dales 1968).

After the work done by Dales and other similar work with initial cap and trade ideas by Thomas Crocker (1966) and W.D. Montgomery (1972), the next step was to get a cap and trade

system off of the pages of economics journals and into the real world. One of the first wide-spread cap and trade programs was adopted by the United States in 1990, in the form of the Environmental Protection Agency's Acid Rain Program. Throughout the 1970s and 1980s, it became apparent that American power plants were producing an abundance of sulfur dioxide, which when emitted would make its way back to Earth in the form of ecologically devastating acid rain. Seeking to solve the problem, the Bush White House under the counsel of various environmentalist groups such as the Environmental Defense Fund began looking at the implementation of cap and trade as an alternative to heavy-handed command and control regulation, which had been in place before and had not been working. Against the advice of some of his closest advisors, President Bush went with the cap and trade program, and it has worked well by most standards. Acid rain emissions fell by three million tons in the first year of the cap alone (1995) and continued falling steadily after that. By 2009, the Acid Rain program has been estimated to save the United States \$122 billion annually in benefits from increased health of not only humans but natural ecosystems free from the harmful effects of acid rain (Conniff 2009). While far from the only example of cap and trade in the real world, the EPA's Acid Rain Program was significant in that it showed the world that cap and trade could work in the real world, not just in the pages of an academic paper.

### III

#### The Regional Greenhouse Gas Initiative

The Regional Greenhouse Gas Initiative (RGGI) is one such example of a cap and trade program like the one outlined by J.H. Dales that has been implemented in the United States, and is the cap and trade program of interest for this paper. The Regional Greenhouse Gas Initiative is an association of northeastern U.S. states that was originally formed in 2003, and began operating as a full cap and trade program at the start of 2009. So far, the program has operated

for two full control periods (2009-2011 and 2012-2014). The third control period began on January 1<sup>st</sup>, 2015 and will run through the end of 2017. Member states include Connecticut, Delaware, Maine, Maryland, Massachusetts, New Hampshire, New York, Rhode Island, and Vermont. New Jersey was at one time a part of the program, but Governor Chris Christie pulled the state from the program at the end of 2011.

The design of the Regional Greenhouse Gas Initiative cap and trade system involves three central components. The first is the cap that is set on the total permissible amount of carbon dioxide tons that can be emitted into the atmosphere. The original cap had been set at approximately 188 million tons in 2009, but by the end of 2012 the cap was leaving far too many permits unsold, undesirable in an auction system since unsold permits drive down the price of permits that are eventually sold. In the year 2013, the cap was scheduled to be approximately 165 million tons, but only 91 million tons had been sold in all of 2012. Starting in 2014, the cap was lowered to the 2012 emissions level, 91 million tons, and lowers by 2.5% each year from 2015 to 2020, at which point the cap will be re-evaluated. In order to meet the requirements of the cap, the program is divided into three-year control periods. The first control period lasted from 2009 through 2011 and the second period lasted from 2012 through 2014. Regulated plants must prove that they have purchased a number of permits that is equal to the amount of tons that they emitted over the course of the three-year period.

The second major component to RGGI is its regulation of carbon-emitting sources in the member states. For the cap to work properly, accurate tracking of carbon emissions is absolutely necessary. If firms are able to emit carbon dioxide without it being tracked or reported properly, they will purchase far fewer permits than necessary and the program would be rendered meaningless. RGGI regulates carbon dioxide emissions from fossil fuel power plants located

within the member states that have a capacity of 25 MW or greater. Referred to as “CO<sub>2</sub> Budget Sources” or more simply “sources,” the RGGI tracks emissions from each source within the states over the course of the control period. New York is the largest of the states in the RGGI, with 81 plants large enough to be defined as a source under the RGGI that purchased approximately 39% of the total permits auctioned off in 2014. The smallest RGGI member state is Vermont, which only has two regulated sources that purchased only approximately 1% of the total permits that were auctioned off in 2014. As stated previously, sources must prove at the end of each three year control period that they have purchased the requisite number of permits that is equal to the tons of CO<sub>2</sub> that were emitted by that source over the course of the control period.

The third major component of the RGGI is the physical allocation or distribution of the allowances to the sources themselves. The RGGI ultimately leaves the authority of how to divide the permits up to the states, but each state has chosen to allocate a large majority of their permits through an auction method. In the first control period, about 90% of the total permits were distributed through competitive auctions. In 2014, 91% of permits were distributed through auctions, with the rest being distributed through accounts that individual states chose to set up and set aside some permits for future discretionary use. The average auction clearing price for permits in 2013 averaged out over the four quarters to approximately \$2.92 per permit.

Blas Luis Perez Henriquez, the director of the Center for Environmental Public Policy at UC-Berkeley, identifies two more aspects of RGGI that are crucial to its operation in his 2013 work *Environmental Commodities Markets and Emissions Trading*. The first is the RGGI established “reserve price” of \$1.86 per ton. The reserve price serves an important function within the overall cap and trade framework because, as with any market based system, there exists the question of what to do once the problem is solved. The reserve price of \$1.86 was set

in place in case emissions are reduced at such a high level that firms no longer wish to buy many, if any at all, of the pollution permits. In microeconomic theory, the reserve price acts as a price floor, since the price of permits may not fall below this price even if natural market demand dictates that the demand should fall below the price. The reserve price exists so that even if demand falls below the established reserve price, there will still be a mechanism in place for RGGI to earn the revenue necessary to continue operation. The reserve price is key to keeping administrative costs manageable, and Henriquez notes that the RGGI currently has “relatively low administrative costs of about 0.05 percent of the overall emissions allowance revenue.” (Henriquez 2013).

The other important factor of RGGI that Henriquez discusses is the program’s use of offset allowances for other projects outside the capped electric power generating sector. Sources are allowed to use up to 3.3% of their carbon permits on offset projects that seek to reduce pollution levels of either CO<sub>2</sub>, methane (CH<sub>4</sub>), or sulfur hexafluoride (SF<sub>6</sub>). These projects must take place within the nine-state region of the RGGI and similar reporting requirements to the standard capped power generating stations are applicable. While this aspect of RGGI has not been taken advantage of quite as much as policymakers would have anticipated, as firms attempt to lower CO<sub>2</sub> levels, they may start taking some of the permits that they have already purchased but do not need and applying them to these sorts of projects.

The Regional Greenhouse Gas Initiative, while not the first cap and trade system implemented, or the largest, is an excellent case study example to measure the feasibility of such programs. The challenges that RGGI faces, such as cooperation across nine autonomous U.S. states, political pressures (including New Jersey’s withdrawal from the program under a Republican administration), and a complex system of regulation, reporting, auction-proctoring,

and cap-setting that all must work in unison together, are not challenges that are unique to RGGI alone. However, over the six years of its operation the Regional Greenhouse Gas Initiative has amassed a wealth of data from the budget sources over the nine states that can be neatly observed and analyzed.

## IV

### Bayesian Statistical Methods

This paper applies the use of Bayesian methods of statistical analysis to construct a method by which it can be determined whether or not the CO<sub>2</sub> emissions among electric power plants in the nine member states have been reducing in RGGI member states. Before an outline of the methods used in this project, a background on Bayesian methods is necessary.

The origins of the idea that spawned the entire field of Bayesian statistics can be traced back to the works of two men, Thomas Bayes and Pierre-Simon Laplace. English statistician and Presbyterian minister Thomas Bayes, for whom the theory is named after, was one of the early probability theorists seeking to solve previously intractable problems in the realm of probability. Published posthumously in 1763 by a good friend and mathematician Richard Price, Bayes' "An Essay towards solving a problem in the Doctrine of Chances" spawned an entire subfield of mathematical statistical analysis and a statistical framework that challenged classical or frequentist notions of statistical methods. Bayes sets up the terms of the problem that he is attempting to solve in the first lines of the essay – "Given the number of times in which an unknown event has happened and failed: Required the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named." In other words, Bayes was attempting to learn the probability of a certain event occurring in the future based off of the number of times the event had either occurred or not

occurred in the past. The solution that he posed was both simple and yet radical in nature: incorporate prior information known about the problem, and if no prior information is known, simply make an educated guess, then go from there.

Bayes used the following example to illustrate his point: if he had his back turned to a table and had a colleague drop the ball anywhere on the table, he has no way of knowing where the ball is. Given that he has no prior information on the ball's whereabouts, he assigns an equal probability (or likelihood that the ball lies on a certain point) to each point on the table. Next, his colleague drops a second ball onto the table and reports the location of the second ball relative to the first ball to Bayes. If he reports that the second ball lies above the first ball, then he has reason to believe that the first ball may lie on the bottom half of the table. If the colleague drops a third ball and reports that that ball also lies above the first ball, then that belief is only strengthened. This process can be repeated many times, and with each trial more information is gathered and factored into the decision that Bayes makes (Bayes 1763). Interestingly enough, Bayes' *Essay* was not revolutionary at the time of its release. Bayes himself never published it, and after its posthumous publication it sat largely unread for about a decade. It was the mathematician Pierre-Simon Laplace that took the concept Bayes invented and brought it into mainstream mathematics. Pierre-Simon Laplace was a mathematician that independently rediscovered what Bayes had outlined in 1774. Laplace, who was known for contributing to both Bayesian and frequentist statistical methods, mathematically defined the work that he and Thomas Bayes had done in the field of conditional probability: the probability that event A occurs conditional on event B occurring is the probability of event B occurring conditional on event A times the probability of event A occurring divided by the probability of event B

occurring. The following is an expression of Bayes' Theorem in its most elementary form given the probabilities of two events A and B occurring:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

A more complex version of Bayes' Theorem dealing with probability distributions rather than single events can be rewritten in the following way, given observed data  $y$  and random parameter  $\theta$ :

$$f(\theta|y) = \frac{f(y|\theta)f(\theta)}{f(y)}$$

The difference between Bayesian methods and "classical" or frequentist statistics lies in how the two methods utilize prior information. D.J. Bartholomew in his work "A Comparison of some Bayesian and Frequentist Inferences" illustrated the difference between the two.

Bartholomew considers a general problem in which we have a random sample of observations  $x = \{x_1, x_2, x_3, \dots, x_n\}$  on a continuous random variable with density function  $f(x, \theta)$ . We wish to make an inference about the parameter  $\theta$  based off the random sample of observations that we have. If we follow the Bayesian approach, we will first specify a prior distribution for  $\theta$  based off what we know about the parameter. Then we multiply by the likelihood function of the parameter and arrive at new distribution, referred to as the "posterior" distribution. This new distribution reflects the prior information that we incorporated into our problem. If we gain new information about the event we are studying, we can update the probability again, with our previous posterior distribution becoming the new prior. This can be expressed formally as:

$$\text{posterior} \propto \text{prior} \times \text{likelihood}$$



This fits with Bayes example of the ball dropping scenario. As Bayes gains new information about other balls relative to the first ball, he updates his probability of the ball being located at a certain point based off of this information. In contrast, the frequentist approach to this problem does not treat the parameter  $\theta$  as a random variable, and does not incorporate prior information into any probability distributions derived in the course of the problem. All inference is based off various test statistics calculated using sample data, not any prior information known about the samples.

Bayesian methods are applicable to this project as well as countless others through the use of Markov Chain Monte Carlo (MCMC) methods, which are used to generate samples from posterior distributions of models both simple and complex. One specific software that can conduct Bayesian analysis through interfacing with the statistical package R is JAGS (Just Another Gibbs Sampler). Before we discuss the finer details of using JAGS for the analysis of RGGI data, MCMC methods in addition to Gibbs sampling must be discussed.

Markov Chain Monte Carlo (MCMC) algorithms are not new, as they were first developed by a team of physicists in 1953 (Metropolis et. al, 1953). However, over the last two decades the use of MCMC algorithms has increased greatly. The goal of an MCMC algorithm is to construct a Markov chain that will converge to a target distribution. In the case of a Bayesian analysis, this is the previously discussed posterior distribution  $f(y|\theta)$ . A Markov chain is a certain type of stochastic process where the distribution of a parameter  $\theta$  at a certain sequence of time  $t$  depends only the immediately previous sequence of the random variable, and none other. (Ntzoufras 2009). For this project, Gibbs sampling is the specific form of MCMC algorithms that will be utilized. Gibbs sampling was developed by Stuart and Donald Geman in their 1984 paper “Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images.” The

original paper developed the Gibbs sampling subset of Metropolis' MCMC algorithms as an application to be applied to the restoration of degraded images in the digital picture processing field. Statistician Jim Albert explains a more general use for Gibbs sampling. The Gibbs sampling method starts with a parameter vector of interest that takes the form  $\theta = (\theta_1, \dots, \theta_p)$ . The joint posterior distribution of  $\theta$ , denoted as  $[\theta|data]$ , can be found by constructing a Markov chain simulation algorithm from the joint posterior distribution by simulating individual parameters from the set of all  $p$  conditional distributions of the form  $[\theta_1|\theta_2, \dots, \theta_p, data]$  (Albert 2009). By simulating one value of each parameter from the conditional distributions, we complete one cycle of Gibbs sampling. Under these conditions, the output from the Gibbs algorithm will eventually converge to the posterior distribution that we are interested in. Gibbs sampling methods have become extremely popular, and the package JAGS has made Gibbs sampling even more accessible to researchers.

## V

### Methods

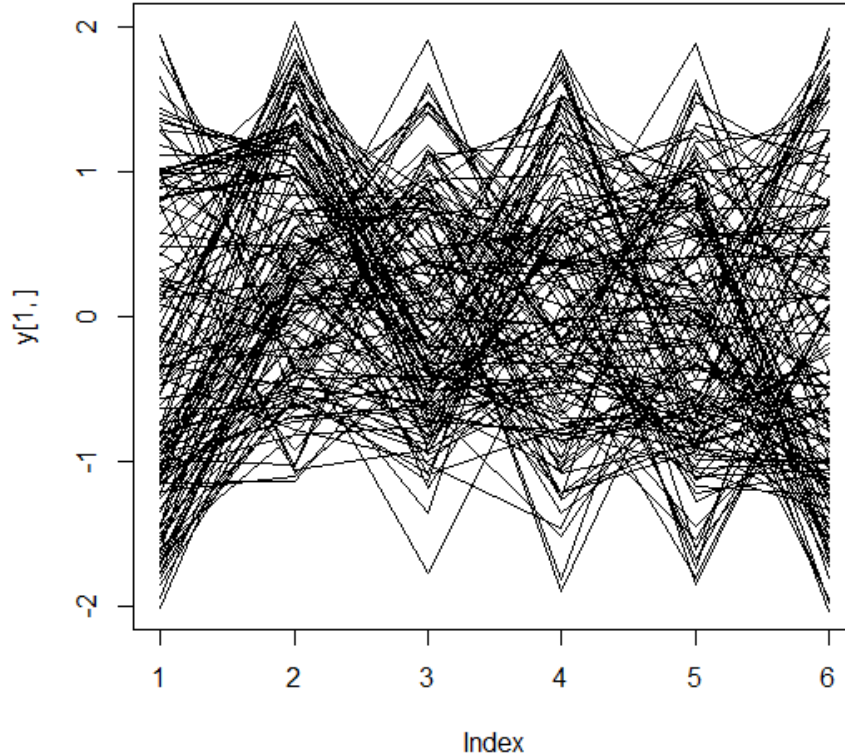
The Regional Greenhouse Gas Initiative, as a part of the strict monitoring aspect of the program, has collected carbon emissions data in the form of mass tons of CO<sub>2</sub> emitted per year on 172 budget sources over the course of the past six years (2009-2014). The budget sources are power plants in the states of Connecticut, Delaware, Maine, Maryland, Massachusetts, New Hampshire, New York, Rhode Island, and Vermont that have an operating capacity of 25 megawatts (MW) or higher. The only missing data is for certain years when particular power plants chose not to operate. Over the course of six years, the 172 budget sources provided a total of 999 emissions data points and 33 missing points, for a total of 1032 total data points. To begin the analysis, profile plots of the emissions for each of the states were constructed using the

statistical package R. The year (2009-2014) was plotted on the x-axis and the amount of carbon dioxide emitted by each individual site was plotted on the y-axis, with a contiguous line connecting the six emission points for each specific site. All sites within an individual state were included on the profile plot for that state. The profile plots for each state are included in Appendix A at the conclusion of the paper. Once plotted, the profile plots can give a rough outline of the trends that I seek to analyze using MCMC methods and Gibbs sampling. From the profile plots, a general downward sloping trend can be observed in several of the states (Massachusetts, Maryland, Maine, and Rhode Island are a few where a clear decreasing trend is observed).

The brunt of the statistical analysis comes in the form of using JAGS (Just Another Gibbs Sampler) and its ability to interface with R. By defining a simple Bayesian model, reading in the data, and setting the parameters of interest, R interfaces with JAGS through the “rjags” package to simulate a specified number of samples from the posterior distribution we are seeking to define. In order to utilize the Gibbs sampling, several steps were necessary. The first is to clean the raw data and format it into an accessible form. From the Regional Greenhouse Gas Initiative’s website, the emissions values in Mass Tons of CO<sub>2</sub> per year at each budget source were obtained. Data for all 172 budget sources over the course of the six years was compiled into a single comma-separated value (.csv) file as it is convenient to import .csv files into R.

The next step is to define the Bayesian model to use in the analysis. If we let  $Y_{it}$  equal the single emission value for the  $i$ -th site ( $i = 1, 2, \dots, 172$ ), and year  $t$ , then we can set up a number of possible models to attempt to predict posterior values of  $Y_{it}$ . The model chosen in this case is based off of the plot containing all of the raw data for all 172 sites onto a single profile plot. The

raw data plot for all 172 sites was constructed in R and appears below. From this data, a model must be proposed.



While the raw data plot is cluttered with 172 separate profile lines, if one follows the thickest cluster of lines, it appears that one trend that the data follows is first increasing, then decreasing, then a flattening out at the end. A possible model that could capture this trend of increase, then decrease, then flattening out is a polynomial of degree 3, or a cubic polynomial.

The model that I propose is as follows:

$$Y_{it} = \beta_{0i} + \beta_{1i}t + \beta_{2i}t^2 + \beta_{3i}t^3 + \varepsilon_{it}$$

where  $Y_{it}$  is the individual emissions variable defined above and the parameters

$\beta_{0i}$ ,  $\beta_{1i}$ ,  $\beta_{2i}$ , and  $\beta_{3i}$  are site-specific random parameters that affect the shape of the distribution.

Once prior distributions are assigned, this model becomes a Bayesian model that we can apply Gibbs Sampling techniques to. We can assign the following prior distributions to the parameters of the model:

$$\beta_{0i} \sim N(\theta_0, \tau_0^2)$$

$$\beta_{1i} \sim N(\theta_1, \tau_1^2)$$

$$\beta_{2i} \sim N(\theta_2, \tau_2^2)$$

$$\beta_{3i} \sim N(\theta_3, \tau_3^2)$$

$$\varepsilon_{it} \sim N(0, \tau^2)$$

For these prior distributions,  $\theta_0, \theta_1, \theta_2,$  and  $\theta_3$  represent the mean values of the posterior mean for  $\beta_{0i}, \beta_{1i}, \beta_{2i},$  and  $\beta_{3i}$ , and  $\tau_0^2, \tau_1^2, \tau_2^2,$  and  $\tau_3^2$  represent the mean value of the variance of the same posterior distributions. It becomes necessary to set prior distributions for these parameters as well, as these are the parameters we ultimately wish to estimate. The following priors are assigned:

$$\theta_0 \sim N(0, 10^6)$$

$$\theta_1 \sim N(0, 10^6)$$

$$\theta_2 \sim N(0, 10^6)$$

$$\theta_3 \sim N(0, 10^6)$$

$$\tau_0^2 \sim \text{Gamma}(.1, .1)$$

$$\tau_1^2 \sim \text{Gamma}(.1, .1)$$

$$\tau_2^2 \sim \text{Gamma}(.1, .1)$$

$$\tau_3^2 \sim \text{Gamma}(.1, .1)$$

$$\tau^2 \sim \text{Gamma}(.1, .1)$$

where  $\tau^2$  represents the variance of the error term in the regression term in the cubic polynomial model. Now that all necessary priors have been defined, the next step in the process is to translate the model defined above into appropriate code that can be read by R and JAGS. The following is the code that describes our model. A few comments on the code follow.

```

model{

    # N observations

    for (i in 1:N) {

        y[ind[i, 1], ind[i, 2]] ~ dnorm ( mu[i], tau )

        mu[i] <- beta0[ind[i, 1]] + beta1[ind[i, 1]] * ind[i, 2] + beta2[ind[i, 1]] * pow(ind[i, 2],2) +
        beta3[ind[i, 1]] * pow(ind[i, 2],3)

    }

    # Priors

    for (i in 1:m){

        beta0[i] ~ dnorm(theta0, tau0)
        beta1[i] ~ dnorm(theta1, tau1)
        beta2[i] ~ dnorm(theta2, tau2)
        beta3[i] ~ dnorm(theta3, tau3)

    }

    theta0 ~ dnorm (0, 1.0E-6)
    theta1 ~ dnorm (0, 1.0E-6)
    theta2 ~ dnorm (0, 1.0E-6)
    theta3 ~ dnorm (0, 1.0E-6)
    tau0 ~ dgamma(.1, .1)
    tau1 ~ dgamma(.1, .1)
    tau2 ~ dgamma(.1, .1)
    tau3 ~ dgamma(.1, .1)
    tau ~ dgamma(.1, .1)

}

```

The indicator matrix notation at the top of the code is necessary because the data is not exactly symmetric. This will come into play when we run the R script in the next step. An additional note should be made regarding the monitoring of our various  $\tau$  parameters. When our model is run, we will want the values of these parameters to be somewhat large, because JAGS

measures the precision parameter,  $1/\tau$ . In other words, whenever we monitor the tau parameter in our output, we note that  $\tau$  is equivalent to  $\sigma^{-2}$ , or  $1/\sigma^2$ . If the value is extremely small, then 1 over that value will result in an extremely large error, meaning that our model is near useless as the error component accounts for almost all of the variation.

The final step in the process is to define the code to use to interface between JAGS and R. The following is the script that was run in R to produce the results described in Section VI. A few comments follow after the code.

```
rawdata <- read.csv("allstatesEmissionsData.csv")

emission=rawdata[,3]
emission[emission==0]=NA

##### Set-Up of Emission Matrix
y=matrix(NA,172,6)
for (i in 1:172){
  y[i,]=emission[((i-1)*6+1):(i*6)]
}

##### Standardize values due to large variance in emissions across sites
for (i in 1: 172){
  y[i,]=(y[i,]-mean(y[i,]))/sqrt(var(y[i,]))
}
plot(y[1,],type="l",ylim=c(-2,2))
for (i in 2:172){
  lines(y[i,])
}

##### An indicator matrix is necessary given that we have some missing values
ind=c(NA,NA)
for (i in 1:172){
  for (j in 1:6){
    if (!is.na(y[i,j])) {
      ind=rbind(ind,c(i,j))
    }
  }
}
ind=ind[-1,]

N=dim(ind)[1] # Total Missing Values
m=172 # Total Number of Sites

##### JAGS: Gibbs Sampling Using JAGS
library("rjags")
```

```

library("coda")

data <- list(y=y,ind=ind,N=N,m=m)
inits <- list(list(tau0=1,tau1=1,tau2=1,tau3=1,tau=1,theta0=0,theta1=0,theta2=0,theta3=0))
parameters<-c("tau","tau0","tau1","tau2","tau3","theta0","theta1","theta2","theta3")

model <- jags.model("rggimodelcode.txt",data=data,inits=inits,n.chains=1,n.adapt=0)
update(model,n.iter=50000) # burn-in samples
output <- coda.samples(model,parameters,n.iter=50000,thin=1)

```

The values are standardized by subtracting each value by the quantity of its expected value divided by its standard deviation. This becomes necessary due to the large amount of variation between sites (some sites only emit a few thousand tons of CO<sub>2</sub>, while others emit hundreds of millions of tons). The indicator matrix in the model code is set up in the R script as well. Because the data is not symmetric, we need a loop that will read through the indicator matrix and detect missing values, denoted by “NA” in the comma-separated value master data file.

The last few lines of code dealing with JAGS specifically are the key to producing the output necessary to conduct the analysis. First, the R libraries “rjags” and “coda” are loaded. “rjags” is the library that allows the user to interface between R and JAGS. “coda” is a package that contains many useful output analysis functions that can be used for MCMC output. Functions in “coda” include the ability to calculate means, standard deviations, and to produce various plots from the posterior distributions of the specific parameters that we are monitoring in the MCMC analysis. The next steps are to define the data, the initial values to start the algorithm with, and the parameters to monitor. As defined earlier, we are monitoring the mean value of the posterior means for the site-specific parameters and mean values of the posterior variances of the site-specific random parameters as well as  $\tau$ . We set the initial values for the chain to begin at 0 for each of the means, and 1 for each of the variances. Next, the text file where the Bayesian model is defined is imported in, and the Gibbs sampler is ready to be run. We specify that 50,000



random sample iterations will be taken, and then thrown out, upon which a second round of 50,000 sample iterations will be taken. This process is called “burning-in” and is common when running MCMC algorithms. To be absolutely sure that the samples have properly converged to the posterior distribution that we are trying to measure, we take an extremely large number of samples and throw out the first round of samples, as they could have been biased by the initial values chosen for the parameters. With the last line of the code, we use “coda” to take the second round of 50,000 iterations, as well as to create the object “output” that will contain the results of the MCMC algorithm to be summarized. With these steps, the algorithm is ready to be run. The results will be summarized in the following sections.

## VI

### Results

The following output was yielded after the R script defined above was run through the R and JAGS interface. The dimension of the output was a 50,000 sample iteration chain with 9 monitored parameters. The mean and standard deviations of the monitored parameters are presented below:

Monitored Parameter	Mean	Standard Deviation
$\tau^2$	1.40725	0.080695
$\tau_0^2$	18.13165	9.375206
$\tau_1^2$	61.51307	17.858926
$\tau_2^2$	149.48072	28.858246
$\tau_3^2$	637.00078	75.272043

$\theta_0$	-1.36612	0.176942
$\theta_1$	1.47257	0.186335
$\theta_2$	-0.41247	0.059559
$\theta_3$	0.03357	0.006556

Histograms for the distribution of each of the parameters are provided in Appendix Ce.

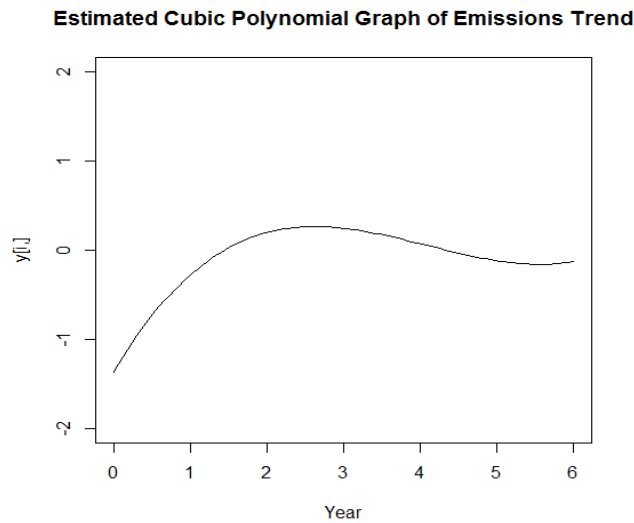
The parameters that we are most interested in for the purposes of the analysis are  $\theta_0$  through  $\theta_3$  as they are the estimated posterior means of the random site-specific  $\beta$  parameters. To recall, the proposed model for the raw data is

$$Y_{it} = \beta_{0i} + \beta_{1i}t + \beta_{2i}t^2 + \beta_{3i}t^3 + \varepsilon_{it}$$

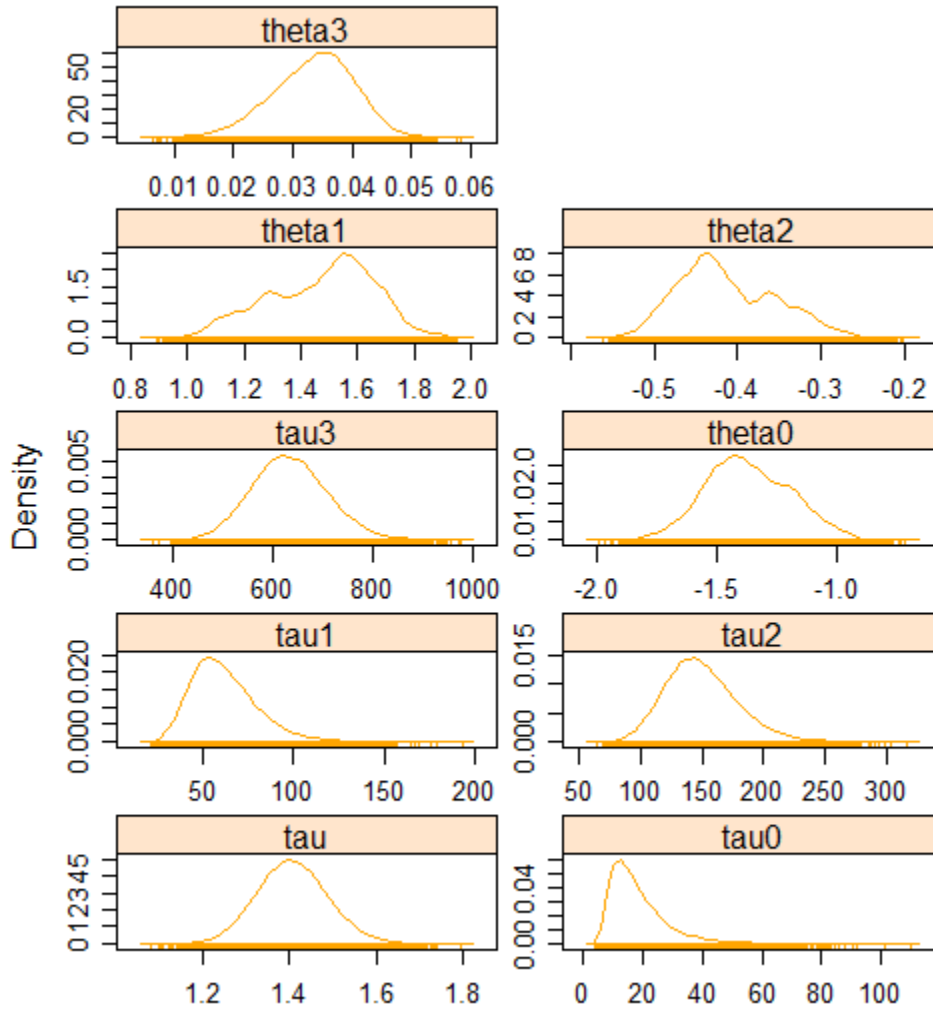
Taking from the table of estimated posterior means, the algorithm has suggested that an equation to represent the trend of carbon emissions over time across the budget sources of the Regional Greenhouse Gas Initiative is

$$Y_{it} = -1.36612 + 1.47257t - 0.41247t^2 + 0.03357t^3$$

The graph of the equation is as follows:



Finally, density plots of the nine monitored parameters are produced:



## VII

### Conclusions

From the results presented in Section VI, several conclusions can be drawn. First, we can see that the posterior distributions for the tau parameters converged correctly, as their density curves all take the general shape of a gamma distribution with a right-skewed distribution. If we recall, the prior distribution for each of these parameters was set as a gamma distribution. The posterior distributions for the means of the site-specific theta parameters all take approximately

normal distribution shapes as expected, but are not as smooth as the tau parameter density curves, and have several observed spikes and valleys. For example, while  $\theta_0$  and  $\theta_3$  are relatively smooth normal curves,  $\theta_1$  has a valley near its mean around 1.4 and then a spike around the 1.6 mark.  $\theta_2$  experiences a similar trend, but in the reverse direction, with a spike coming first and then a valley at approximately the mean value of -0.4.

The mean value of the parameter tau is of special interest. As mentioned earlier, in JAGS the parameter  $\tau$  represents the precision parameter and is denoted by  $1/\sigma^2$ . Therefore, since  $\tau = 1.40725$ , we can derive that the posterior mean of the variance of the error term is  $1/1.40725 = .7106$ . This is desirable, since if we had a large variance then that would mean that almost all of the model would have been explained by the error term, and little or none explained the rest of the site-specific random parameters. Based off these results, there is evidence to show that the model that we arrived at from the posterior distribution

$$Y_{it} = -1.36612 + 1.47257t - 0.41247t^2 + 0.03357t^3$$

is a model that describes the trend that emissions at site i and year t follow. Based off of the graph, we see the increasing trend up to year two, then a slight decreasing trend until year 5, and then a flattening off at the end of year 6.

This trend supports the claim that, since the adoption of RGGI by the nine member states, the trend of steadily increasing yearly carbon emissions has begun to reverse. One explanation is that after imposition of the RGGI at year 0, it took a period of two years to start to see an effect in emissions. After the two year period, firms began to feel the crunch of having to purchase permits at an auction system, and began to emit less. Additionally, it likely took the firms a year or two to update the technology within the power plants to emit fewer mass tons of carbon dioxide. Despite this, the results of the MCMC output do not by any means purport to

conclusively prove that RGGI is the reason for this trend. There are a multitude of other factors that could come into play when it comes to explaining the decreasing trend in carbon emissions, such as energy prices, world economic conditions (2009 and 2010 were marked by the aftermath of the economic downturn of 2007 and 2008, for example), political conditions (New Jersey was pulled from the RGGI by its Governor in 2011 and was accordingly left out of this dataset), and technological advancements that could affect energy and carbon demand. A full economic benefit-cost analysis review of the RGGI could help to determine the role that these other factors could be playing, but MCMC and Gibbs sampling have been used to show that there is an identifiable trend in carbon emissions over the six operating years of the Regional Greenhouse Gas Initiative.

## VIII

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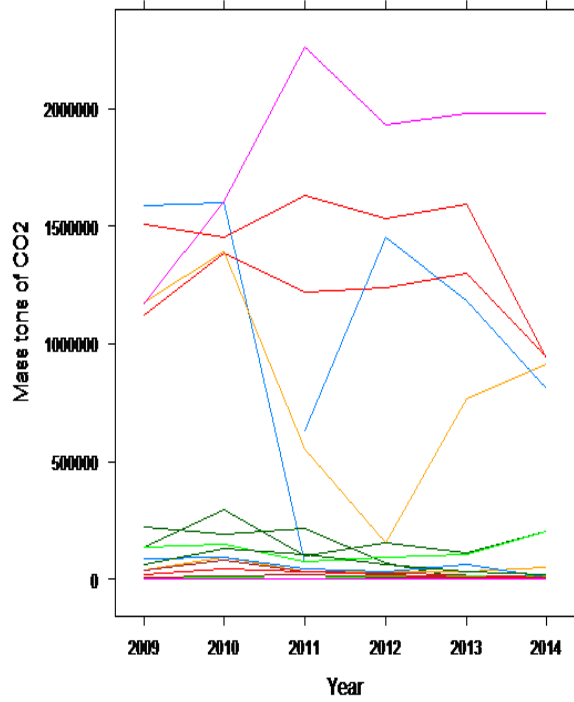
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# IX

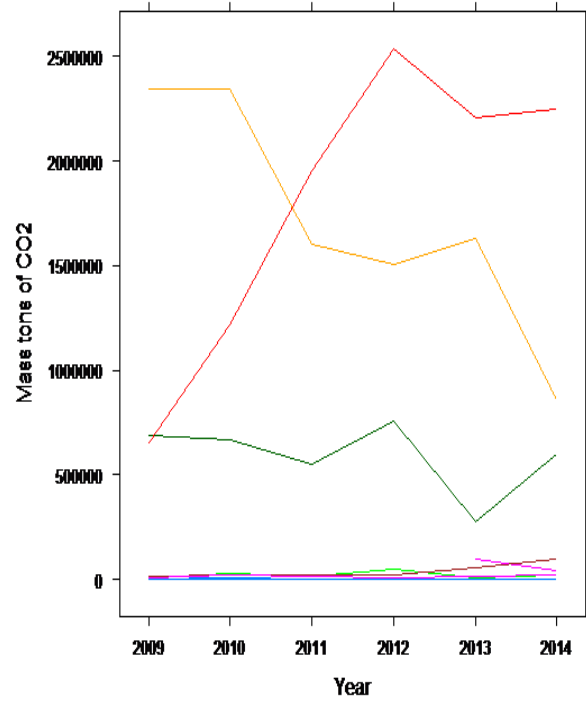
## Appendices

### Appendix A – Profile Plots of Original Emissions Data by State

**Annual Carbon Emissions By Facility (Connecticut)**

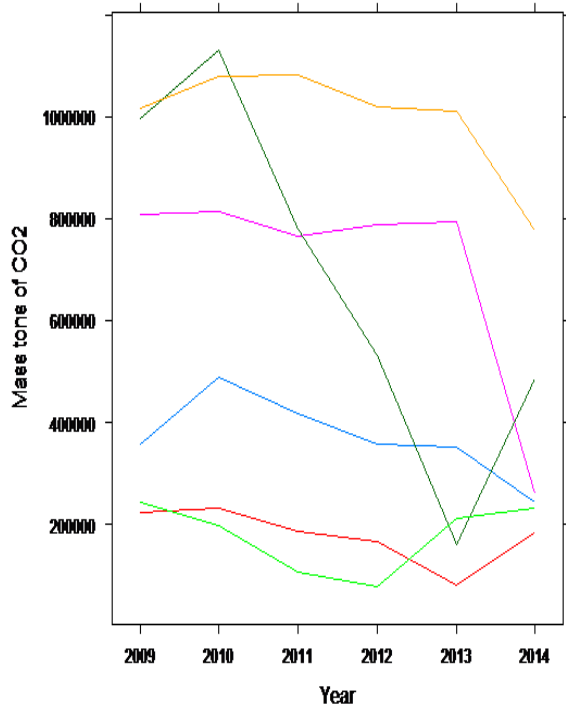


**Annual Carbon Emissions By Facility (Delaware)**

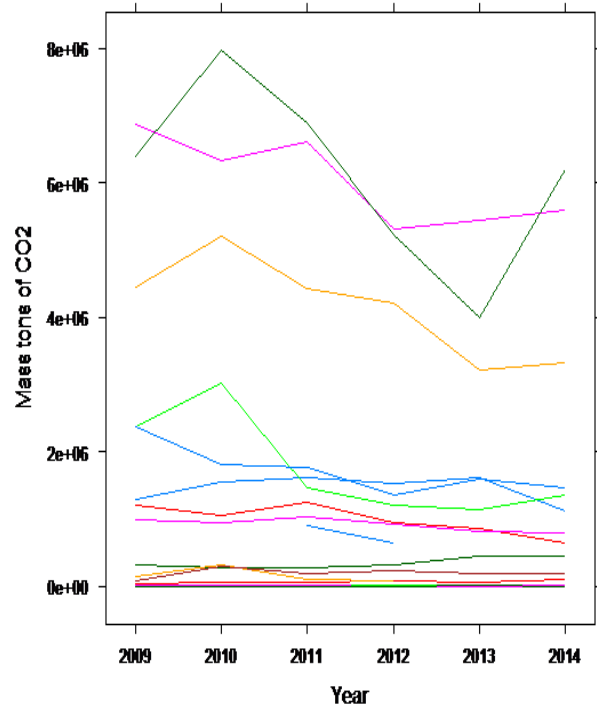




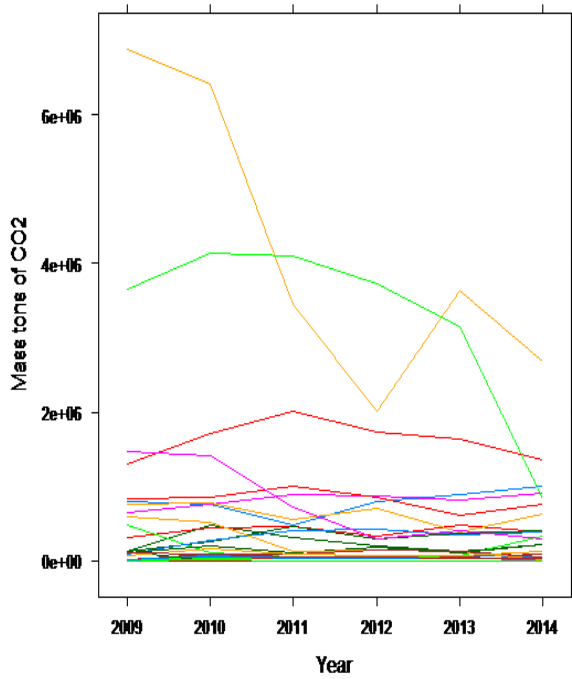
**Annual Carbon Emissions By Facility (Maine)**



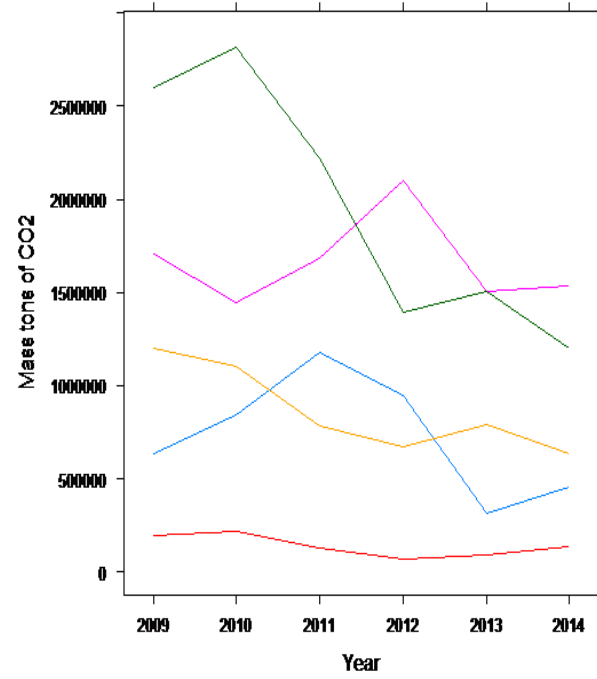
**Annual Carbon Emissions By Facility (Maryland)**



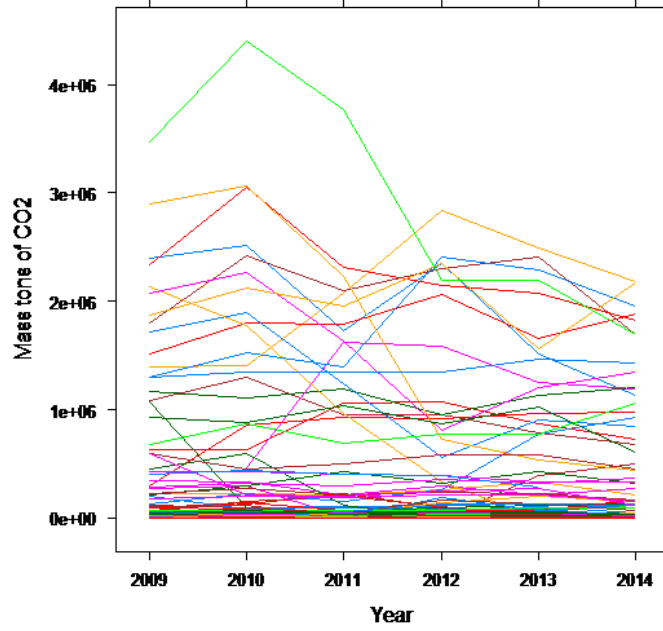
**Annual Carbon Emissions By Facility (Massachusetts)**



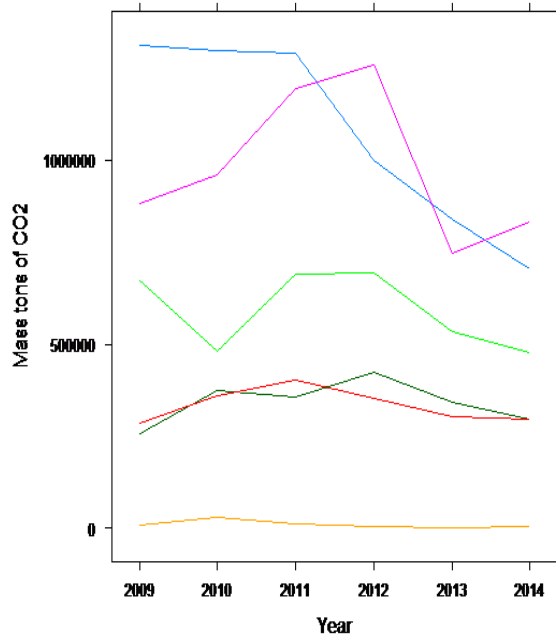
**Annual Carbon Emissions By Facility (New Hampshire)**



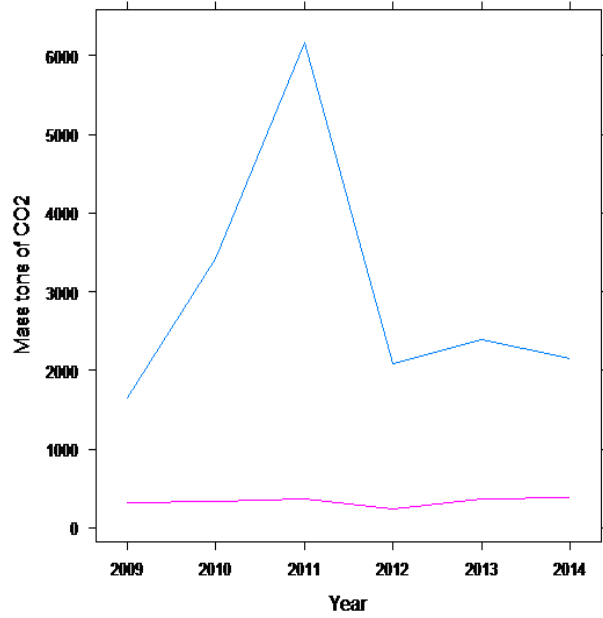
**Annual Carbon Emissions By Facility (New York)**



**Annual Carbon Emissions By Facility (Rhode Island)**



**Annual Carbon Emissions By Facility (Vermont)**



## Appendix B – Raw Data

The following is the raw data used in the analysis. The format of the data is as follows: each line represents one specific budget source. The six numbers listed in each line are the six emissions values (in mass tons of CO<sub>2</sub>) for that specific site over the course of 2009-2014. An 'x' denotes a missing value. I have separated the data by state for comparative purposes.

### Connecticut:

8816.913, 13298.557, 17420.909, 5289.988, 8958.307, 19050.855,  
37598.673, 94974.059, 29822.296, 32971.112, 30946.44, 50624.194,  
40037.228, 79355.764, 29794.375, 25028.604, 20398.163, x,  
138706.313, 293961.399, 98389.585, 151335.782, 111514.632, 204283.777,  
2201.2, 2733.4, 2837.2, 2005.4, 3612.8, 2942.1,  
1174538.335, 1389597.279, 554068.133, 156454.144, 765427.358, 910431.528,  
132750.417, 149117.046, 75347.012, 90539.455, 105511.332, 202434.845,  
3923.85, 5443.175, 3470.135, 4428.614, 6258.867, 9088.839,  
221521.825, 189124.425, 214275.15, 66007.875, 941.6, 5384.025,  
1588593.801, 1596730.456, 68066.659, x, x, x,  
4254.325, 6245.325, 3356.425, 14771.225, 16098.079, 11132.812,  
87315.009, 93122.474, 45773.253, 31600.867, 60494.425, 4058.951,  
1122589.016, 1387544.509, 1217775.3, 1240495.679, 1301248.899, 946551.509,  
1507231.775, 1450649.45, 1628812.418, 1529513.814, 1593751.11, 936228.109,  
936228.109, 1168751.142, 1603614.869, 2260904.429, 1931244.831, 1977547.848, 1978767.008,  
63547.308, 130267.86, 105167.064, 63059.12, 31188.341, 21794.082,  
19986.4, 41322.187, 30160.512, 19905.502, 9990.584, 9298.561,  
x, x, 633057.535, 1454502.535, 1180471.842, 806932.416,

### Delaware:

720.2, 1009.9, 449.6, 574.6, 193.6, 3406.4,  
685345.65, 665965.45, 548042.825, 755064.975, 271833.825, 597705.664,  
2345073.7, 2342049.025, 1602430.4, 1502778.75, 1630927.875, 858849.2,  
5284.776, 29286.784, 15545.951, 51264.581, 8726.882, 23647.66,  
651583.304, 1218247.035, 1952486.783, 2536173.116, 2205734.677, 2250268.531,  
511.307, 3683.57, 2200.305, 692.721, 1503.564, 1778.84,  
7995.031, 18547.282, 11846.876, 7918.66, 14022.307, 23712.328,  
11817.163, 20480.441, 17393.261, 21054.522, 53520.697, 99648.512,  
x, x, x, x, 98597, 38818,

### Maine:

242370.257, 198691.014, 107641.962, 77824.537, 211640.686, 231609.629,  
809077.204, 813063.986, 766547.607, 787071.331, 793406.434, 259499.339,  
357730.12, 489272.653, 416387.104, 357370.991, 352862.397, 241987.115,  
995235.229, 1130402.253, 778158.393, 532676.38, 161783.128, 485856.856,  
223947.846, 232582.932, 187548.532, 166212.43, 81648.687, 182987.859,  
1015132.021, 1079444.68, 1081176.299, 1018916.747, 1011081.964, 775593.17,

### Maryland:

6860722.39, 6329727.781, 6610419.074, 5322511.886, 5452082.252, 5597267.314,  
1216247.191, 1047894.313, 1242441.56, 944062.864, 862110.143, 652192.742,  
5710.072, 18141.758, 18499.38, 29033.373, 12767.037, 13101.96,  
2379234.629, 1818364.332, 1760349.375, 1360548.515, 1607263.174, 1472680.048,  
36687.651, 68506.891, 59305.997, 75942.847, 73306.144, 100321.32,  
2731.501, 11881.736, 14040.28, 19710.179, 14457.489, 17070.882,  
0, 7566.076, 410.185, 9270.704, 19921.211, 5775.482,  
25300.075, 11920.45, 18142.2, 5203.35, 7484.725, 21064.953  
142375.156, 313114.047, 108129.996, 92982.151, x, x,  
4452075.443, 5197729.604, 4421483.07, 4201655.099, 3228435.958, 3326883.065,  
2378038.742, 3019789.381, 1469866.972, 1205436.613, 1149403.715, 1357912.728,  
6382061.111, 7969125.452, 6895629.344, 5216654.12, 3986507.823, 6192869.979,  
82654.096, 294107.771, 183915.577, 244563.735, 193914.035, 188681.793,  
x, x, 896818.081, 655870.484, x, x,  
1291525.165, 1562314.757, 1626206.833, 1539522.399, 1624845.293, 1119915.97,  
995739.92, 950699.213, 1034649.823, 937960.491, 811734.715, 805683.735,  
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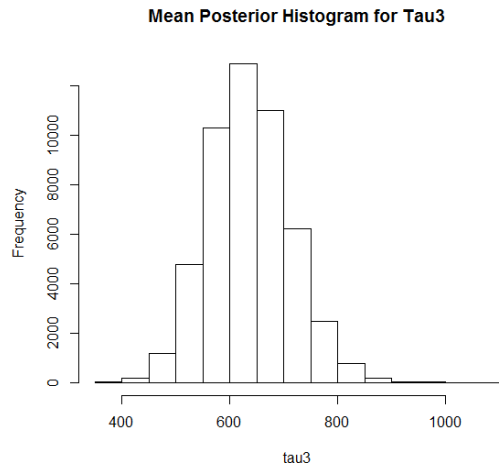
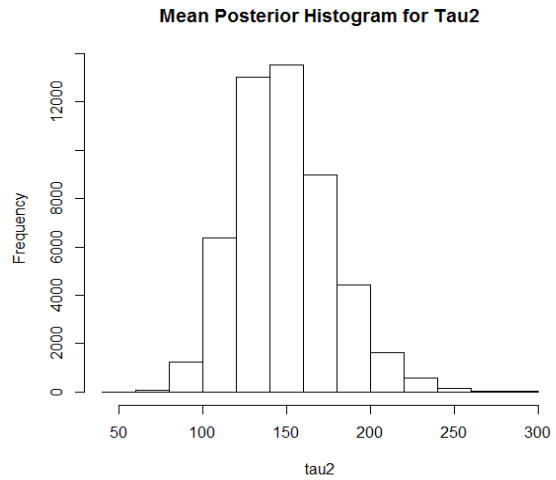
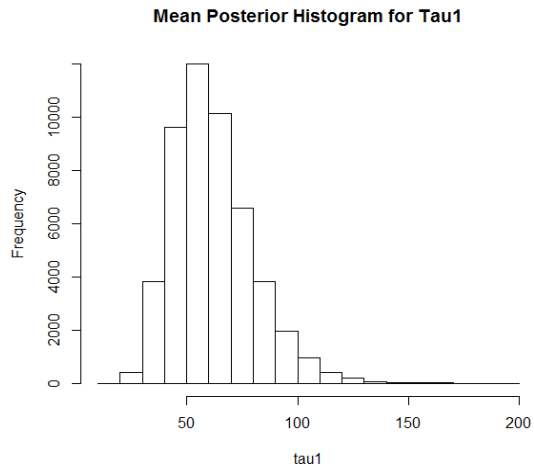
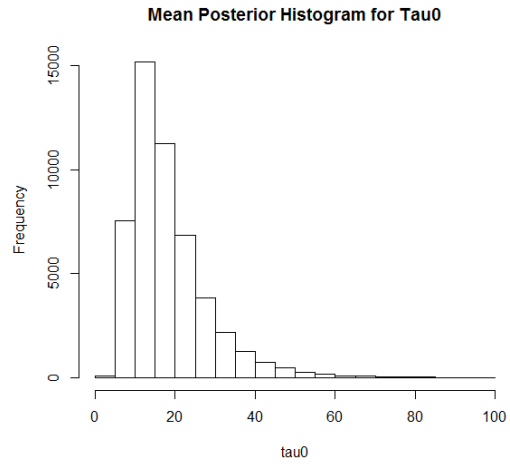
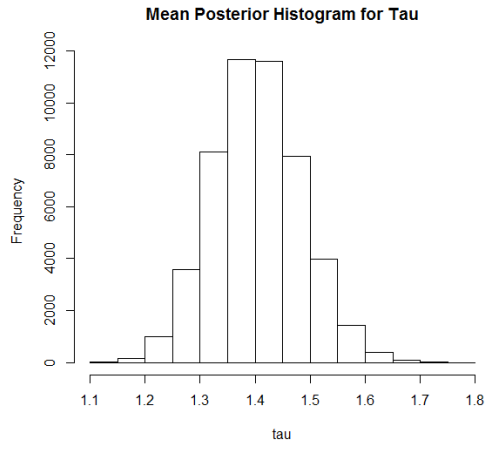
### Massachusetts:

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763347.781, 788764.267, 560386.129, 717225.806, 394812.098, 634430.142,  
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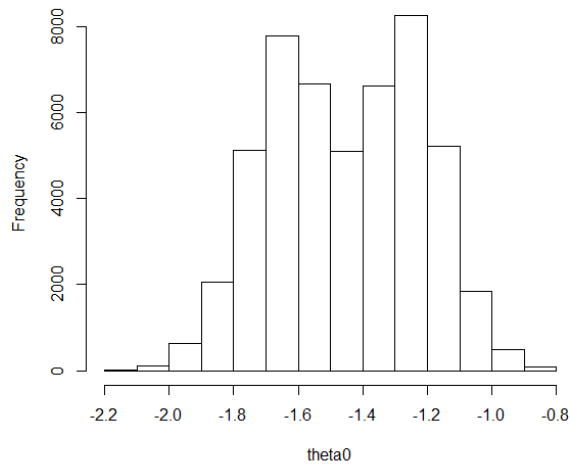
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1481923.046, 1421294.046, 725807.559, 302615.367, 412068.294, 304011.297,  
106258.925, 95540.975, 56573.628, 56824.25, 32007.343, 31596.626,  
14880.2, 72868.705, 34518.373, 41692.35, 43845.878, 44679.275,  
2065.9, 10341.3, 6655.2, 6447.6, 3455.4, 4921,  
21525.173, 64275.137, 40260.909, 55918.281, 59763.575, 99581.252,  
70946.575, 172839.575, 89477.6, 71720.175, 76079.95, 132606.3,  
4432.7, 6149.8, 1121.1, 896.8, 362.8, 3705.1,  
136875.44, 477526.908, 309498.719, 196869.568, 134069.813, 215699.484,  
91249.175, 277924.675, 413130.375, 428435.3, 353423.6, 385508.45,  
1600.449, 955.311, 3517.709, 2469.818, 0, x,  
1395.68, 15444.466, 2577.306, 2547.255, 2324.597, 1695.058,  
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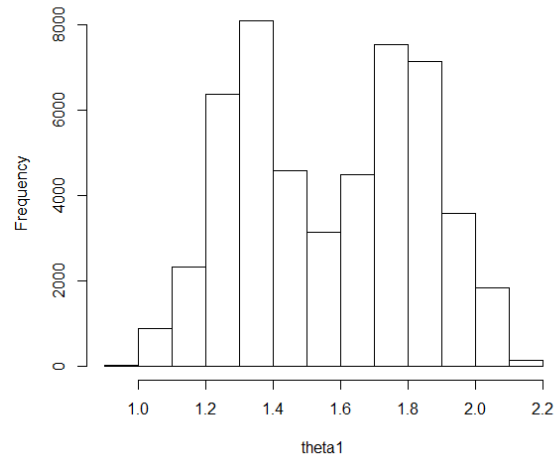
## Appendix C – Histograms of Posterior Outputs



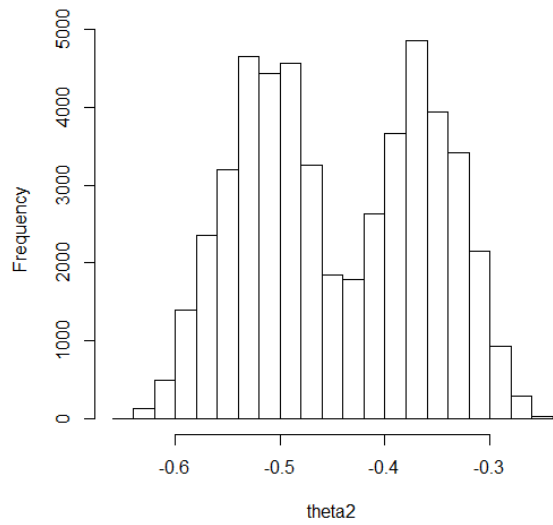
Mean Posterior Histogram for Theta0



Mean Posterior Histogram for Theta1



Mean Posterior Histogram for Theta2



Mean Posterior Histogram for Theta3

