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EFFECTING CHANGE ON THE MATHEMATICS TEACHING EFFICACY OF PRESERVICE ELEMENTARY TEACHERS

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Dedicated to my parents, Stanley and Billye Mae Sayers

Daddy, you taught me how to write, encouraged me every day to be the woman God meant for me to be, and told me that when I accomplished my goals to give all the glory back to God. Mother, you taught me to love teaching mathematics and continue to be a living example for me of what a Christian wife, mother, and grandmother should be. Your daily prayers for me support me through even the most difficult paths in life. Daddy, you called me Doctor Bowman the day you died. With God's help and your confidence in me, I accomplished my goals and now I give all the glory back to God.

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Abstract

This sequential mixed methods case study was conducted in a mathematics class in a Midwestern university to determine whether exposure to constructivist mathematics teaching would influence change in the mathematics teaching efficacy beliefs of preservice elementary teachers. The study examined the instructor's beliefs and pedagogical practices and how they affected her students in two sections of geometry for a sixteen-week semester. Qualitative data collected and analyzed included a course syllabus, instructor-selected textbook, observation notes, instructor and student reflections, photos of student work, and interviews. Quantitative data was collected using a version of the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) (Enochs, Smith, & Huinker, 2000) adapted and validated for preservice elementary teachers. Qualitative data from the instructor indicated recurring themes of the instructor's use of *humor*, *wait-time*, *questioning*, *persistence*, *encouragement*, *negotiation*, and *repetition*. These themes fall generally into two broad pedagogical categories: *care* and *technique*. The quantitative data on the students from the MTEBI indicated insignificant ($p > .05$) positive change in both the personal mathematics teaching efficacy and the mathematics teaching outcome expectancy. However, the qualitative data on the students indicated significant positive effect on their mathematics teaching beliefs as indicated throughout the semester by the words they used in their reflections, their engagement in classroom community, and their conversation and questions during class. Recurring themes observed throughout the study indicated a progression in student response from *struggle* and *frustration* to *confidence* and *community*, a progression which can be interpreted as an indication of positive change in the preservice teachers.

Keywords: preservice elementary teachers, mathematics teaching efficacy, mathematics beliefs, effecting teacher change, learner-centered, geometry

Chapter 1: Introduction

Any fundamental change in the intellectual outlook of human society must necessarily be followed by an educational revolution. (Whitehead, 1949, p. 77)

Where does one start to explain the motivation behind such a study? My background in mathematics education as both a student and a teacher is generations deep and varied, including experience in public and private schools. Forty plus years since becoming certified to teach mathematics, however, I find myself yearning to understand how students learn and how teachers teach even more so than at the beginning. While assisting another professor in her research I was led to a book published by the Conference Board of the Mathematical Sciences (CBMS), *The Mathematical Education of Teachers – MET* (2001), later revised and updated (Conference Board of the Mathematical Sciences, 2012) to address the introduction and national adoption of the *Common Core State Standards for Mathematics* (CCSSM) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These two books rekindled my excitement for research in a way nothing else has been able to do, perhaps because I was able to read in print from reputable and research-based sources what I had experienced and deeply believed with respect to the teaching and learning of mathematics, and more specifically, the mathematics teaching and learning occurring in elementary schools.

Problem

Prospective elementary teachers often enter college with only a “superficial knowledge of K-12 mathematics, including the mathematics that they intend to teach” (Conference Board of the Mathematical Sciences, 2012, p. 4). Teachers whose knowledge in mathematics lacks depth have also been shown to lack confidence in their mathematics teaching ability. When knowledge and/or confidence wane in teachers, it can impact their students' learning, because:

> Students learn mathematics through the experiences that teachers provide. Thus students' understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school. The improvement of mathematics education for all students requires effective mathematics teaching in all classrooms. (NCTM, 2000, pp. 16–17)

Children are perceptive of nuances adults have learned to ignore. If a teacher does not know or like the mathematics she teaches, she can unknowingly plant seeds for dislike and deficit in the minds of her students.

According to significant research, confidence in mathematics teaching ability – teaching efficacy – comes through self-efficacy and mathematics knowledge beyond the level of courses taught (e.g. Swars, Daane, & Giesen, 2006; Thompson, 1992; Tschannen-Moran, Hoy, & Hoy, 1998). Furthermore, studies show a positive relationship between both teacher achievement and attitude toward mathematics and student mathematical achievement (Ball, 1988; Schofield, 1981). The implication of this is profound. Over the course of a career, one inadequately-prepared elementary teacher who lacks confidence in mathematics can influence the mathematical ability of hundreds of students. As a secondary mathematics teacher, it is extremely difficult, sometimes impossible, to impact change on students' negative attitudes toward mathematics and to fill in the mathematical deficits they form prior to middle school and high school. Stopping this cycle requires a change in the mathematical teaching efficacy of preservice elementary teachers before they ever enter the classroom.

> Extensive research has been conducted and documents of recommendations have been published regarding ways to bring about change in how future

> teachers of mathematics are prepared (Conference Board of the Mathematical Sciences, 2001, 2012). However, recommendations do not always indicate compliance and “the mathematical content preparation of preservice elementary teachers still varies widely across the nation” (Matthews & Seaman, 2007, p. 3). In 2008, the National Council on Teacher Quality (NCTQ) surveyed schools of education with elementary education programs:[NCTQ] found that in the United States, there is extreme variability in what is required in mathematics courses for pre-service elementary teachers. Specifically, NCTQ found that 15 out of 77 of the education schools sampled required no specialized mathematics courses, 11 schools required only one course, 42 schools require two courses, and only 9 schools required at least 3 courses as the CBMS guidelines suggested. (Matthews, Rech, & Grandgenett, 2010, p. 2)

For example, although the state of Oklahoma requires twelve hours of college mathematics for certification in elementary education, the choice of those courses, not to mention their development and implementation, is entirely left up to the colleges of education across the state. A recent syllabus study conducted with universities and colleges across the state found consistency and compliance with the recommendations weak to nonexistent (Conrady, 2016; Conrady & Bowman, 2015), which indicates a significant problem exists even eight years after the above cited study by Matthews, et al. (2010).

Effecting Change

A past president of the National Council of Teachers of Mathematics described the problem of effecting change in mathematics education in "Beyond Pockets of Wonderfulness" (Seeley, 2015). In this message to teachers, Seeley expressed admiration for observing an innovative lesson teaching multiplication, a new approach to teaching algebraic functions, and a mathematician who contacts a high school to gather data; however, Seeley also counters with the problem of isolation and calls the events "pockets of wonderfulness." After setting the stage, Seeley continues:

> The problem is that when great events happen in isolation from the larger system within which they operate, we fall short of what might be possible otherwise. Educators generate tremendous power by talking to one another and working together. Articulation and collaboration are important tools for making *lasting systemic change*. When educators fail to take advantage of these tools, students are destined to have to start over, lose ground, and miss opportunities to connect mathematical ideas. [Emphasis added.] (p. 171)

Developing insight to what might bring *lasting systemic change* in mathematics education is a worthy goal of research. Mathematics education research reveals over one hundred years of cyclic changes in mathematical content and pedagogy continually bringing us back to where we began, rather like how a river pushed out of its natural path eventually returns to where it began. Why does history continue to repeat itself in mathematics education? Why do we just have "pockets of wonderfulness" and not widespread wonderfulness for all students to experience? Perhaps like Nature itself, humans and the systems in which we operate simply resist change.

Evolution, as a form of change, is a slow, arduous process, because Nature is innately reticent to change. Sir Isaac Newton expressed this fact in his First Law, "Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon" (Newton, Chittenden, Adee, Motte, & Hill, c1846, p. 83). Nature's earth-moving forces have consisted of melting glaciers, volcanoes, meteors, and other extreme events. Although Newton's reference was directed specifically toward inanimate physical objects, it applies to human nature in much the same way. The quandary over what kind of mathematics students need to learn and how mathematics should be taught formally surfaced in the United States in 1892, when the National Education Association (NEA) appointed a *Committee of Ten* (Briggs, 1931) “tasked with developing a plan for the nationwide standardization," (Fiss, 2011, p. 1185) and this debate continues today, over one-hundred twenty years later.

The cyclic changes over mathematical content continually oscillate between teaching only the arithmetic needed for daily living to requiring algebra and geometry courses that lead to science, technology, engineering, and mathematics (STEM) careers. The dilemmas over how the predetermined mathematics should be taught range from memorize, drill, and kill to presenting mathematics in context only. The quandary we face today is not new, as Brownell evinced a similar thought in 1947:

> To classify arithmetic as a tool subject, or as a skill subject, or as a drill subject is to court disaster. Such characterizations virtually set mechanical skills and isolated facts as the major learning outcomes, prescribe drill as the method of teaching, and encourage memorization through repetitive practice as the chief or sole learning process. In such programs, arithmetical meanings of the kinds mentioned above have little or no place. Without these meanings to hold skills and ideas together in an intelligible, unified system, pupils in our schools for too long a time have 'mastered' skills which they do not understand, which they can use only in situations closely paralleling those of learning, and which they must soon forget. (p.11)

Today, an educator might refer to Brownell's perspective for how mathematics should be taught as constructivist, learner-active, or student-centered.

Key names in the development of constructivist learning in mathematics are Ernst von Glasersfeld (Noddings, 1990), Lev Vygotsky, and Jean Piaget (Dimitriadis & Kamberelis, 2006). These three researchers, among others, developed theories as to how people learn, aligning around the tenet that learners construct knowledge through experience and acting on objects. Defining constructivism, Schwant (2015) says "most of us would agree that knowing is not passive … that human beings do not find or discover knowledge so much as construct or make it" (p. 36). However, despite decades of work on how children learn and efforts to adapt teaching to align with the findings, classroom teaching remains virtually unchanged. Even with access to the latest technology and a vast amount of research on problem-based learning, teachers continue to lecture, question, and assign while students sit in rows writing with tools a century old. The learner-active, hands-on, student-centered learning methods proven to promote deeper and long-lasting learning are difficult to implement, except, perhaps, in those "pockets of wonderfulness" Seeley (2015) mentioned. The general public's resistance to the implementation of the Common Core State Standards for Mathematics is the most recent indication of this trend in mathematics education. Given this dilemma, what does it take to effect systemic change in mathematics education? According to Newtonian principles (1846), to effect change in the vicious cycle in mathematics education requires a force greater than the forces holding it in stasis.

During the last three decades, there have been numerous attempts to effect radical change in the curriculum and pedagogy of mathematics education. Those changes, primarily motivated by prioritizing student college- and career-readiness, have made the mathematical teaching and learning for elementary students a constant focus of concern. One result of this concern, for example, has been pushing algebraic concepts down into even early childhood mathematics. College mathematics relies on the fundamental mathematics students learn in secondary schools; likewise, secondary mathematics relies on the fundamental mathematics students learn in primary schools. For some students, instruction begins in home settings prior to formal education, while other students begin their instruction as they enter preschool or kindergarten. The remaining students begin their mathematical learning in elementary school, where teachers who love children have dedicated themselves to teaching all subjects to them, including mathematics. Therefore, it logically follows that in order to have mathematically prepared college students requires mathematically prepared elementary teachers. Certainly, then, goals of an elementary teacher education program would include enhancing the "preservice teachers' beliefs about science and mathematics and their ability to teach these subjects" (Huinker & Madison, 1997, p. 107) as well as increasing their self-efficacy in mathematics. Therefore, it is essential that "teacher educators must be aware of their students' beliefs and plan for experiences which will have positive impact on teacher self-efficacy and outcome expectancy," (Enochs & Riggs, 1990, p. 701).

One problem that resonates through studies addressing mathematics teaching efficacy is that elementary teachers who lack confidence in their mathematics teaching ability have the potential to engender mathematical weakness in the students in their classrooms (e.g. Bates, Latham, & Kim, 2011; Briley, 2012; Eddy & Easton-Brooks, 2010; Enochs, Smith, & Huinker, 2000; Gibson & Dembo, 1984; Harkness, D'ambrosio, & Morrone, 2007; Huinker & Madison, 1997; Moseley & Utley, 2006; Swars et al., 2006; Tschannen-Moran et al., 1998; Utley, Moseley, & Bryant, 2005). While each of the cited studies related teacher efficacy to teacher performance, Swars, et al. (2006) focused on the relevance of mathematics anxiety on teacher efficacy and Briley (2012) focused on teacher beliefs. Together these studies provoke the researcher to ask--given this collective knowledge of relationships affecting the mathematics teaching efficacy of preservice elementary teachers--why is change so difficult to achieve?

Existing research indicates preservice elementary teachers confident in their mathematics teaching ability would improve the mathematics education of all students. This research project took place in what has been identified as an isolated potential "pocket of wonderfulness" and aims to add to the body of research in mathematics education for preservice elementary teachers. By concentrating on the mathematics content courses instead of the methods courses, the research observes how university-level mathematics taught to preservice elementary teachers in a learner-centered environment might create a path of improvement for their teaching-efficacy and outcome expectancy in mathematics.

Mathematics Knowledge

University level mathematics constitute the primary audience of *The Mathematical Education of Teachers II* (Conference Board of the Mathematical Sciences, 2012). This report from the Conference Board of Mathematical Sciences (CBMS) stresses:

> A major advance in teacher education is the realization that teachers should study the mathematics they teach in depth, and from the perspective of a teacher. There is widespread agreement among mathematics education researchers and mathematicians that it is not enough for teachers to rely on their past experiences as learners of mathematics. It is also not enough for teachers just to study mathematics that is more advanced than the mathematics they will teach. Importantly, mathematics courses and professional development for elementary teachers should not only aim to remedy weaknesses in mathematical knowledge, but also help teachers develop a deeper and more comprehensive view and understanding of the mathematics they will or already do teach. (p. 23)

According to this passage, preservice elementary teachers do not just need more mathematics; they need different mathematics. Furthermore, CBMS emphasizes the recommended mathematics needs to be learned from the perspective of a teacher. Thus, preservice teachers need to learn the mathematics the way they will be teaching it – in a constructivist, learner-active setting, as opposed to a traditional university lecture setting.

The earlier 2001 report from CBMS, *The Mathematical Education of Teachers*, provided recommendations for the types and amounts of mathematics all preservice teachers should take through the mathematics departments in their colleges and universities. One of the revisions in the 2012 document was increasing the number of hours of mathematics that each category of preservice teachers should take. The courses are broken down into three categories by the level of mathematics the teachers will be teaching as:

> 1. Prospective elementary grade teachers should be required to take at least 12 semester-hours on fundamental ideas of elementary school mathematics, their early childhood precursors, and middle school successors.
> 2. Prospective middle grades (5-8) teachers of mathematics should be required to take at least 24 semester-hours of mathematics, that includes at least 15 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.
> 3. Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics that includes three courses with a primary focus on high school mathematics from an advanced viewpoint. (Conference Board of the Mathematical Sciences, 2012, p. 18)

This passage, again, stresses the courses should be specifically tied to the mathematics the teachers will be teaching in their own classrooms: however, should not be confused with methods courses. Both reports from the CBMS emphasized these recommended courses were mathematical content courses and should be taught by university mathematics instructors.

The Study

Guided by three research questions, this study employed a sequential strategy of mixed-methods case study (Creswell, 2014), which examined preservice elementary teachers (PSETs) and their instructor in a Midwestern state university mathematics course. The mathematics course is one of four mathematics courses, three of twelve hours, required for all PSETs at the university and is a partial fulfillment of the state's mathematics requirement for elementary teacher certification. This particular instructor was chosen because of her reported beliefs and practices in constructivist learning in the mathematics classroom. The research includes how those beliefs impact her teaching of university level mathematics classes, as well as how the PSETs responded to her teaching methods.

Research Questions

The primary questions guiding the research are:

1. When a mathematics instructor's beliefs about student learning are constructivist in nature, what are the features of and pedagogic practices utilized in her university mathematics course for preservice elementary teachers?
2. What is the perspective of preservice elementary teachers in a university mathematics course taught in this manner?
3. What impact does a university mathematics course taught from a constructivist-learner perspective have on preservice elementary teachers' self-efficacy in mathematics?

The Chapters

Chapters two, three, and four consist of three publication-ready articles outlining the theory, methodology, and findings of the study. The first article, *Chapter Two*, explores theory surrounding the reticence to change in education. The article in *Chapter Three* addresses the first of the three research questions by examining the beliefs, characteristics, and pedagogy of a dedicated university mathematics instructor who teaches mathematics to preservice elementary teachers. The article found in *Chapter Four* addresses the second and third research questions by describing the thoughts, actions, and mathematics teaching efficacy of preservice elementary teachers in the instructor's university mathematics class. Because the three chapters are intended as stand-alone articles, readers may observe overlap in necessary methodology and background literature for the study.

Finally, *Chapter Five* provides a global view of the research and offers insight as to how the study potentially enriches the body of research in mathematics education for preservice elementary teachers. Additionally, the chapter offers suggestions for further research along with questions left unanswered by this study.

Chapter 2: Reticence to Change in Education

Abstract for Article 1

In "Beyond Pockets of Wonderfulness"(2015), Seeley expresses the problem of effecting change in mathematics education. In her message to teachers, Seeley expresses admiration for innovative lessons taught in various classrooms, and also conveys concerns regarding the problem of isolation calling these classrooms "pockets of wonderfulness." After setting the stage, Seeley continues:

> The problem is that when great events happen in isolation from the larger system within which they operate, we fall short of what might be possible otherwise. Educators generate tremendous power by talking to one another and working together. Articulation and collaboration are important tools for making *lasting systemic change*. When educators fail to take advantage of these tools, students are destined to have to start over, lose ground, and miss opportunities to connect mathematical ideas. [Emphasis added.] (p. 171)

Working toward *lasting systemic change* in mathematics education requires stamina, persistence, and an understanding of the complex interaction between culture, schools, and the curriculum. Research into the history of mathematics education reveals over one hundred years of cyclic changes in mathematical content and pedagogy continually bringing us back to where essentially we began, rather like how a river pushed out of its natural path eventually returns to where it began. Why does history continue to repeat itself in mathematics education? Why does change only come in small pockets of change and as a mathematics education community we cannot seem to support and sustain systemic change? Is it perhaps, like Nature itself, humans and the systems in which we operate simply resist change?

Reticence to Change in Education

A Metaphor of Change

The complexity of an educational ecosystem is difficult to understand from outside the system. Consider the tranquility of watching fish swimming in an aquarium. At first glance, the aquarium may be thought of as a single simple system where one need only add water, a filtering system, and fish, and then hours of enjoyment for the owner ensue. Aquarium owners, however, know that this not the case at all. The complexity of a healthy aquarium requires careful thought about the quality of the water, types of fish, appropriate food, filtering systems, and lighting. External concerns include the location of the aquarium away from direct sunlight, heavy pedestrian traffic, and overly curious pets.

The first decision in purchasing fish is whether to choose saltwater or freshwater fish. Once one makes a decision regarding the water type, water quality is a key factor to a healthy aquarium. Water must first be tested, treated, and then retested. Appropriate plants, coral or rocks, and other underwater furnishings are added. After a few days, while still monitoring the water, one may slowly add fish to the tank. The keeper of the aquarium must give careful thought to the collection of fish, as not all fish get along together. Certain fish will hover near the bottom to keep things clean, while others school together in spiral patterns all over the tank. Some fish must be purchased in pairs, yet others are solitary and prefer to have no one else like them in the aquarium. All fish must be slowly introduced to their new home; otherwise, they will suffer shock and die.

Each aquarium has a fish population it can manage. Overcrowding the water causes pollution beyond what a filter is able to clear, and continually adding new additives to the water will not correct the quality. A limit exists for how many fish, coral, snails, and flora can be added before changes must occur. Regular and consistent maintenance of the tank requires removing a quantity of water and replacing it with pure water. However, if toxins build up for too long, no amount of filtering or water replacement can purify the tank so the fish can thrive. Fish must be removed, water siphoned, tank scrubbed, and the process restarted. Although viewers from the outside see only the aquarium's beauty, the keeper understands the vast complexity of this ecosystem.

Just as different aquariums require different amounts of work, so too do different schools require varying levels of engagement to bring about change. The filtering system in a school can be thought of as its communication network, constantly assuring toxins are not building up. The larger the school, the more complex the need for creative communication options becomes. For example, a complex population – young teachers, older teachers, administrators – all at different levels of education and experience, will not perceive digital communication equally. Thus, a simple e-mail request of "Come to my office to discuss this" can create unwarranted panic for some, while others totally ignore the message. The population has come from dissimilar locations, with diverse mindsets and goals. Some of those administrators will not work well with some of the teachers. There is truly a place for everyone, but within a context of community and complexity, the chosen fish must be able to thrive in this aquarium together. Before adding new hires to the population, one must consider the environment. Will the Clownfish fit in with the Anemone? Will this Damselfish interact well with the Yellow Tang? Sometimes when one determines a new fish might be a bad fit for the aquarium, it is not added. Often, however, when a new fish is highly desirable, then fish perceived to react badly with the new one are removed from the tank. Regardless of the size of the tank, maintaining and sustaining the health of the population is a complex endeavor.

A similar complexity exists with the curriculum and pedagogy in schools. Changes are difficult because the system is complex. As in an aquarium, where one cannot change the food without considering the fish or change the fish without considering the environment, likewise, when one considers the complexity in education one cannot change what happens in elementary schools without considering what happens at other educational levels. Changing curriculum at one level causes ripples throughout the system. Although a specific change may make it easier for the teachers and/or administrators at one level, it may not be best for the students, or vice versa. The change has to be systemic and universal; the process should consider the needs of all levels, all students, and all teachers. Everyone in the system must be engaged in the change process. Research suggests that to truly change the way elementary students learn mathematics, one must also change the way elementary teachers are taught mathematics at the university level (e.g. Association of Mathematics Teacher Educators, 2017; Conference Board of the Mathematical Sciences, 2001, 2012; Ma, 2010).

Reticence to Change

Evolution is a slow, arduous process, because Nature is innately reticent to change. Mountains, rivers, streams, and even great canyons have remained where they were born for centuries. Their birth and alteration have only occurred through Nature's extreme events – earth-moving forces in the form of melting glaciers, volcanoes, and meteors. Sir Isaac Newton expressed this in his First Law: "*Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon*" (Newton et al., c1846, p. 83). Other scientists, such as Max Planck and Albert Einstein, imagined less fixed notions in the realm of physics (Hayles, 1991), lending substance for a more chaotic state of the coming and continuance of the cosmos. Human nature, however, tends to be more consistent with Newton's Law of stasis.

People--and the systems they create--resist change, and most systems are not fond of chaos. People carve out comfortable niches for themselves and find contentment in familiarity and the status quo. Change, whether in location, occupation, social status, or political leadership, creates conflict inside comfort zones, and conflict often creates chaos. Systems, as well as organizations, experience this chaos with regularity as people move in and out and up and down, thus disrupting the flow of status quo. Nonetheless, disrupting the flow is necessary for growth in any system. Waldrop (1992) suggested "species evolve for better survival in a changing environment … so do corporations and industries." He added, "complex systems are more spontaneous, more disorderly, more alive," than static objects and somehow have managed to find a niche at "the edge of chaos" where the components of the systems "never quite lock into place, and yet never quite dissolve into turbulence" (Waldrop, 1992, pp. 11–12). Organisms in systems at the *edge of chaos* do not indicate or even create the hopefulness they will be willing to change. Achieving *lasting systemic change* in people may require similar interactive forces basic to Newton's Laws of Motion.

Change Theories

Searching for mechanisms to achieve *lasting systemic change* in education leads researchers away from physical science toward biological science – psychology. Changing people is a markedly different process than changing the location of a hill, perhaps because people tend to have more to say about what happens to them. Should a highway department decide to move a hill to make a new road more conducive to travel, no concern is given to the will of the hill. Rather, with a few well-placed explosives, the hill has been moved and road construction begins. However, should a more willful organism, such as a school district, decide to restructure its curriculum or staff, a few well-placed explosives will not have the desired effect. Chaos will ensue and undoubtedly, change will happen, but not in any controlled manner. Change within systems involving people requires careful study, intentional consistency, engagement in the process at all levels, and knowledgeable leadership.

The works of Whitehead, Piaget, and Bandura have contributed to theories of change and are all familiar to educators. Other change theorists, such as Lewin, Lippitt, Prochaska, and DiClemente, are perhaps more familiar to medical professionals. Each of these theorists added worthy knowledge to the field of change, often building on the work of one another. However, Bandura's work remains the most relevant if one seeks to create *lasting systemic change*. His cardinal defining properties of a genuine stage theory include “qualitative transformations across stages, invariant sequence of change, and nonreversibility” (Bandura, 1997, p. 412), and describe the traits required for *lasting systemic change*.

Developing a Stance for Change

Research on change theories began to emerge shortly after the end of World War II. In the late 1940s both Alfred North Whitehead and Kurt Lewin published their views on how change should occur; however, their foci were quite different (Lewin, 1947; Whitehead, 1949). While Whitehead focused his views of change on the development of children, Lewin examined societal changes. Both views are crucial to how change occurs in education. Lewin’s three step process for permanent change – unfreeze, move, and refreeze – sounds simplistic in nature, but because the process is forced change, it can cause considerable disruption to the system it is forced upon. For example, even something as seemingly innocuous as rearranging the teachers in a building by grade instead of subject can be unwarranted change for the teachers. Regarding such cases, Lewin cautioned, “since any level is determined by a force field, permanency implies that the new force field is made relatively secure against change” (Lewin, 1947, p. 35). In the unfreezing step, Lewin advised that problems can arise in different cases--including the removal of prejudices, complacency, and self-righteousness--in order to “bring about deliberately an emotional stir-up” (p. 35). The reverse process of stabilization promises the same potentiality for conflict, and one should prepare appropriately in advance.

While Lewin’s view of change describes moving people like cars, Whitehead’s view of change in individuals is organic and can only happen when individuals are ready. He wrote in his essay, *The Rhythm of Education*, “life is essentially periodic,” (Whitehead, 1949, p. 17) and to create intellectual progress teachers must be aware of the periodic stages of romance, precision, and generalization. In the romance stage, one lures or *hooks* the individuals he wishes to change. One creates interest in the romance phase for it is the *sine qua non* or essential part “for attention and apprehension,” (p. 31). One must constantly bear in mind “the pupil’s mind is a growing organism,” (p. 30) and the “natural mode by which living organisms are excited towards suitable self-development is enjoyment,” (p. 31). Whether the pupil is a young child, a teacher, or a concerned citizen, this principle applies. One must romance an individual to bring about an interest in change and then, once interest is properly aroused, the next stage must come soon, before the interest dies.

Whitehead reiterated each of these stages in his essay on *The Rhythmic Claims of Freedom and Discipline* (Whitehead, 1949) writing “when this stage of romance has been properly guided another craving grows,” in reference to the stage of precision. The precision stage follows romance as the interest has grown to a point of a craving desire to know more, to have more, to explore deeper into the knowledge base that has been introduced. Just as in a relationship, however, the romancing must continue to keep the precision stage alive. The romancing, at this point, works to “discover in practice the exact balance between freedom and discipline which will give the greatest rate of progress,” (p. 35). When proper balance is achieved the individual will be ready to move on to the final stage of the rhythmic cycle.

Generalization is the stage in which the individual is effective at what he has been working on and is ready to begin to show what he can do. In teacher education, this might be considered the first-year teacher or in medical fields the first year as a physician. With the internship completed, the desire for the occupation in his heart, "he relapses into the discursive adventures of the romantic stage with the advantage that this mind is now a disciplined regiment instead of a rabble" (p. 37).

Strongly believing change and learning are directly linked, the researcher considers the work of Jean Piaget. Piaget approached his ideas about change from a biological perspective and believed change occurs through self-regulation as an individual's knowledge or schemata is forced to a state of disequilibrium by encountering contradicting information to their schemata (Bandura, 1997; Dimitriadis & Kamberelis, 2006). He claimed changes to one's schemata are actively constructed and adjusted in response to "external *perturbances*," (Dimitriadis & Kamberelis, 2006, p. 171) and then the schemata become reorganized with the concepts of assimilation, accommodation, disequilibrium, and equilibrium.

Assimilation differs from accommodation to the extent of whether it is the schema or the experience that requires adjustment. Fitting a new experience into an old schema is assimilation whereas making an old schema fit a new experience is accommodation. Considering the changes through which systems of organisms are subjected to in society, a worthy example to clarify would be helpful. A new teacher would experience assimilation upon entry into an established school district, where on the other hand an established school district would experience accommodation at a sudden change in leadership.

> Adaptation is typically motivated by the experience of disequilibrium, the uncomfortable sense that one's experience is at odds with one's capacity to understand and explain it. When individuals experience disequilibrium (for

> whatever reason), they engage in the dual processes of assimilation and accommodation until they reach a new state of equilibrium where they feel they have developed good (or good enough) naïve theories of experience and the world (Dimitriadis & Kamberelis, 2006, p. 171).

These are not truly *stages*, because they occur over and over again as an individual experiences new situations and must make adaptations or else remain in a state of disequilibrium.

Nearly ten years after their writings, an expanded version of change emerged as a fusion of Lewin's and Whitehead's theoretical claims. Although perhaps better known for their seven phase schemata, Lippitt, Watson, and Wesley first suggested a "five general phases of change process" (Lippitt, Watson, & Westley, 1958, p. 130). Their five phase process included (a) development of a need for change or *unfreezing*; (b) establishment of a change relationship; (c) working toward a change or *moving*; (d) generalization and stabilization of change or *freezing*; and, finally, (e) achieving a terminal relationship.

The key defining element of the change theory of Lippitt, et al. is that the person, organization, or system being changed must be first be convinced change is necessary. Development of a need for change includes not only problem awareness, but also a desire to both change and to seek help from other sources outside the defined system. The authors stressed, "problem awareness is not automatically translated into a desire for change" (p. 131), but feasibility for change and confidence that obstacles can be overcome are also key elements to reach the *desire* phase. Too often those in leadership believe they are solely responsible for the condition of their systems and try to keep everything in-house. One can see this as a reoccurring situation in systems of all sizes – families, classrooms, businesses, and government. The *desire* for change is the primary requisite before moving on to any other phases.

Prochaska and DiClemente spent over a decade trying to define a set of stages one must go through to escape addictive behaviors (DiClemente & Prochaska, 1982; Prochaska & DiClemente, 1982; Prochaska, DiClemente, & Norcross, 1992). Although addictive behaviors are not always the impetus for necessary change in systems, there are significant similarities making their model worthy of consideration. Their set of five stages include: (a) pre-contemplation, (b) contemplation, (c) preparation, (d) action, and (e) maintenance (Prochaska et al., 1992).

In the pre-contemplative stage, a person is satisfied with stasis and has no intention to change his or her behavior. It is not a case of not being able to see a solution, but rather a case of not being able to recognize the existence of a problem. Similar to the first phase that Lippit, et al. describe, to move from the pre-contemplation stage to the contemplation stage, when a person acknowledges a problem exists, often requires pressure and/or coercion from an outside source, such as a family member or close friend. Even at this point, an individual must decide whether or not a change is merited or worth the effort. A person can remain in the contemplation stage for months or even years before moving on to the next stage.

The third stage – and it is with hesitancy they are numbered – preparation, is where one begins to form a plan for change. It is in this stage where one intends a definite action within a short time, and perhaps one even takes small steps to reduce the frequency of participation in the behavior. This stage must quickly lead to the action stage, or else the person reverts to an earlier stage. The action stage is one in which “individuals must modify their behavior, experiences, or environment to overcome their problems,” (p. 1104). This stage is not to be equated with change, but rather the acting stage of the process of change. Mistaking it for change often occurs; as a result, one never reaches the final stage of maintenance, but rather a relapse into an earlier stage. The maintenance stage is not evidence of a change’s finality, but rather that one has reached a phase where the work to avoid relapse must begin. Since relapse to addiction is “the rule rather than the exception,” (p. 1104) continued support is needed.

When Prochaska and DiClemente began their early studies, they believed their five stages were linear; however, after more than a decade of repeated studies, they came to the conclusion the stages were spiral in nature. Prochaska and DiClemnte also posited that within this spiral context, individuals could enter, leave, or reenter the stages at any point along the spiral and could even repeat the stages multiple times. The success of lasting change continued to depend, nonetheless, on appropriate interventions occurring at appropriate times during the change process. Because of the conclusion Prochaska and DiClemente reached, Bandura believed their change stages were not a set of stages at all. He based his criticism on the biological definition of stages, such as the one a larva experiences as it goes through as it transforms from a caterpillar to a butterfly (Bandura, 1997), stating true stages must be performed sequentially, and that no repeating was possible. Considering his example of the transformation of larva to butterfly, one would have difficulty arguing his point of view.

According to Bandura (1997), efficacy beliefs affect each phase of personal change: the adoption of new behavior patterns, their generalized use under different circumstances, and their maintenance over time. Bandura asserts, “people’s beliefs that they can motivate themselves and regulate their own behavior plays a crucial role in whether they even consider changing" (p. 279). Outcome expectation is the second component of Bandura's self-efficacy model for change. He defines efficacy beliefs as "a judgment of one's ability to organize and execute given types of performances" and "outcome expectation is a judgment of the likely consequence such performances will produce," (p. 21). When efficacy belief is paired with outcome expectation the results of change can more accurately be predicted for an individual or group of individuals in a systemic organization. Both efficacy beliefs and outcome expectations can show either negative or positive aspects, but "productive engagement" (p. 20) ensues only when both are positive.

In order to bring about lasting systemic change in mathematics education, one might need to consider aspects of several theories of change. The ways children learn, adults think, and schools are organized are all essential components when considering the complexity of change in education. Attempting to alter the manner in which teachers should deliver lessons without considering the children to which the lessons will be delivered is a fruitless effort. Failing to consider the teachers while attempting reorganization of schools produces chaotic levels of stress and dissatisfaction. Attempting to adopt new mathematics curricula for students, whether at the national, state, or local level, without considering the schools' abilities to support a new adoption can result in epic failure for all stakeholders. Change just for the sake of change is questionable.

Systemic Change in Mathematics Education

> The history of mathematical teaching in this country is yet to be written. It is necessary to pay some attention to this history in writing upon the theory; as the traditions of the elders have a great influence, partly good and partly injurious. If we find that a tradition in mathematical teaching arose from definite reasons still in force, we must be cautious about rejecting it as useless; but such are not all the methods which have been handed down. T. H. Safford, Williamstown, December 2, 1886. (Safford, 1888, p. 5)

It seems as though the history of mathematics education has been written on a Mobius strip – there is really only one side, but people keep seeing two. Nearly one hundred thirty years have passed since Safford penned the above passage, when the country was in the midst of what has been referred to as an educational revolution. Still a young country, secondary schools were growing exponentially across the United States of America, and the concern over content taught in those schools was a great concern of universities. In 1892, in order to address inconsistencies in secondary school offerings and college entry requirements, the National Education Association appointed a Committee of Ten "tasked with developing a plan for the nationwide standardization" (Fiss, 2011, p. 1185), and the educational world has been seemingly unhappy ever since. As with all committees, some groups are overrepresented, while others are underrepresented, and the Committee of Ten was no exception. The committee was headed by the president of Harvard University, Charles W. Eliot, and consisted of four other prominent college presidents, a college professor, the US Commissioner of Education, and three secondary principals – two public and one private. To fulfill its goals, the committee "formulated eleven questions" (Briggs, 1931, p. 135) ranging from what ages students should begin formal studies of certain subjects to how much time should be allotted to study each subject. These eleven questions led to the formation of nine more committees of ten members – eighty-nine men and one woman – mostly gathered from the East Coast. Only forty-two of the committee members represented secondary schools and of those, only seventeen were from public schools.

One of the nine committees specifically focused on mathematics, and their report contains significant irony. Although there were no representatives from the lower schools, the key focus of their endeavors was on coursework for elementary education, such as concrete geometry and some elements of algebra (Briggs, 1931). Again, the committee members were primarily from universities; however, only one of the committee members was trained in pure mathematics. The remainder of the members, (save one who was actually teaching mathematics and recruited by Eliot), were focused on the fields of physical science and contended mathematics might be learned incidentally through those studies (Fiss, 2011).

Citing a monograph by T.H. Safford (1888) as a cornerstone, the committee decided both arithmetic and mathematics should be taught through application and investigation. Elementary students would learn by measuring their classroom or the playground and estimating weights of various objects, while secondary students would benefit from constructions, physics laboratory experimentation, and practical astronomy. Furthermore, the committee members thought it would be a wonderful idea if students were tested over geometry theorems which they had never proved so their true knowledge could be ascertained. They went on to suggest college entrance exams in mathematics might be done orally, as all students would benefit from the oratory skills. Therefore, to the committee members, the primary benefit of the study of mathematics was not as a mental exercise alone, but rather as a useful tool to understand everyday objects (Fiss, 2011). They conclude:

> The method of teaching should be throughout objective, and such as to call into exercise the pupil’s mental activity. The text-books should be subordinate to the living teacher. The illustrations and problems should, so far as possible, be drawn from familiar objects; and the scholar himself should be encouraged to devise as many as he can. So far as possible, rules should be derived inductively, instead of being stated dogmatically. On this system the rules will come at the end, rather than at the beginning, of a subject. (NEA, 1893, p. 105)

Whereas in 2017, the above conversation sounds quite normal and wonderful, at the end of the 1800s the mere suggestion of knowledge not being epistemic to teachers and textbooks neared heresy. In the letter of transmittal accompanying the report, W.T. Harris, Commissioner of Education, remarked, “I consider this the most important educational document ever published in this country” (National Education Association of the United States, 1893, p. ii). Colonel Francis W. Parker, of Cook County Normal School in Illinois, recommended the report for all educators and administrators to be read in small reading groups, like today’s *professional learning communities*. This opinion, however, was not the favorable reaction to the report. Some educators had a considerable number of concerns, including the time-saving omission of key mathematical sequences necessary for understanding concepts. In this regard, the single most outspoken critic of the report from the Mathematics Conference was Superintendent J.M. Greenwood of the Kansas City, Missouri, High School. Disagreeing with Colonel Parker, Greenwood said, “to the Committee of Ten, and to the Committee of Ninety [i.e. the subject conferences], I will say, that the only way a boy can learn arithmetic is to study arithmetic and not to mix it up with other things” (Fiss, 2011, p. 1193). Greenwood went on to become the president of the National Education Association (NEA) in 1898.

Global Changes

Any fundamental change in the intellectual outlook of human society must necessarily be followed by an educational revolution. (Whitehead, 1949, p. 77)

The following decades saw tumultuous times. The United States' involvement in World War I, the Wall Street Crash, the Great Depression, immigration, industrialization, and urbanization all had an immense effect on mathematics education and schools in general. Sustenance and survival occupied the minds of the populace, rather than assuring mathematics with its rigor was taught in the schools. These events and others brought on vast changes to both the quantity and quality of the schools' populations. Each major historical event seemed to be followed by concerns that the education students were receiving was either not enough or inappropriate for the times. The mid-century period saw the beginnings of the race for space, which triggered even more changes in mathematics education. However, not all of these changes pushed mathematics education forward.

During this same time period, mathematics curriculum reform was occurring in other nations. Alfred North Whitehead, renowned mathematician-turned-philosopher, was writing and speaking on the subject extensively. In an essay, *The Mathematical Curriculum* included in *The Aims of Education*, he warned, "any fundamental change in the intellectual outlook of human society must necessarily be followed by an educational revolution" (Whitehead, 1949, p. 77). In agreement with the results from the Mathematics Committee, whether coincidental or not, Whitehead stressed that due to the changes "mathematics, if it is to be used in general education, must be subjected to a rigorous process of selection and adaptation" (p.79). He recognized current reform efforts in mathematical instruction and acknowledged "changing a well-established curriculum entrenched behind public examinations" (p. 79) was difficult to do in a short time. Additionally, he warned "knowledge does not keep any better than fish" (p. 98), and the continuation of knowledge required change. Whitehead was also convinced mathematical concepts did not exist in a vacuum, but that number, quantity, and space were all interconnected relational concepts. He strongly believed little knowledge could be gained by teaching children "disconnected ideas" and teaching in such manner would lead to "mental dryrot" (p. 2).

This history of mathematics education is essential in understanding why change is challenging. As Klein argues, "the education wars of the past century are best understood as a protracted struggle between content and pedagogy" (Klein, 2003, p. 176), and those wars or struggles are continuing today. The above passages from Whitehead (1949) could have well been applied in 2010 to convince the public of the necessity of a Common Core Curriculum for mathematics. Concurrent with Whitehead, another mathematician, textbook author, and university professor, wrote an article that would become a classic in mathematics education, *The Place of Meaning in the Teaching of Arithmetic* (Brownell, 1947). Doubters in Brownell's era asked questions similar to those asked by parents, politicians, and some teachers today in response to the curricular changes they are being asked to make, including:

- Are meanings really necessary in the learning of arithmetic?
- Are not meanings of the kind now called for really too difficult for children to learn?
- Does it not take an undue amount of time to teach meanings – so much that other more important aspects suffer?
- Suppose that meanings are learned: do they actually function; are they really used; may they not interfere with effective thinking? (p. 11)

A Nation at Risk: The Imperative for Educational Reform (U.S. National Commission on Excellence in Education, 1983) captured the nation's attention with the blatant statement in its first paragraph: "the educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people" (Denning, 1983, p. 469). It is no wonder this report was considered to be the most humbling on the nation since the launch of Sputnik in 1957. After delving into all the findings and deficits in education, *A Nation at Risk* (1983) recommended five areas needing urgent improvement: (a) Content, (b) Standards and Expectations, (c) Time, (d) Teaching, and (e) Leadership and Fiscal Support (Denning). As one reads through the list of specific recommendations under each category, it is difficult to find argument with the tenets of such a great wish list – increased teacher pay, smaller class sizes, more secondary mathematics, rigorous textbook choices, commitment of the public. The question might be, however, what happened to all of these good intentions given the nation responded so strongly to the report?

In 1989, when the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards for School Mathematics*, its members were sharing the culmination of "three years of planning, writing, and consensus-building among the membership of NCTM and the broader mathematics, science, engineering, and education communities, the business community, parents, and school administrators" (NCTM & The Commission, 1991, p. 1). Key goals of the NCTM's released standards were to grant students "*mathematical power*…the ability to explore, conjecture, and reason logically; to solve nonroutine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity" (p. 1). This *mathematical power* would lead to students' "development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions" (p. 1). It would bring about "perseverance, interest, curiosity, and inventiveness" (p. 1), or perhaps it would just result in teacher frustration. In 1991 NCTM released a publication to guide teachers on how to teach the 1989 standards, *Professional Standards for Teaching Mathematics* (NCTM & The Commission, 1991). This publication included worthwhile tasks to engage students, instructions on conducting student discourse, descriptions of a proper learning environment, and an overview of what mathematics teachers should do to help students develop the mathematical power the 1989 standards would provide.

In 1998, NCTM produced a 342-page document entitled *Principles and Standards for School Mathematics: Discussion Draft*, and although it has a copyright date of 1998 and NCTM's logo, it also includes a disclaimer on the title page: "this Discussion draft is a working document and does not represent official policy of the National Council of Teachers of Mathematics. Comments and reactions are welcome" (NCTM, 1998b). The document also provided a form to send in comments and/or reactions, gave a URL to the upcoming "Standards 2000 Web" for public comments, and listed an email address. Articles came out in NCTM publications encouraging teachers to take part in the national standards setting for mathematics. One such article, "Give your feedback on basic skills!" urged elementary teachers to let their voices be heard on how the draft handled the basics (NCTM, 1998a). The writers made every effort to represent all of the stakeholders, and the 2000 publication of *Principles and*

Standards for School Mathematics (NCTM, 2000) remains a remarkable feat. But the ink was hardly dry before The No Child Left Behind Act of 2001 was released as Public Law 107-110 (Boehner, 2002).

The No Child Left Behind Act of 2001 soon became known as simply NCLB. The NCLB, is a 670-page document full of promises, goals, and ultimatums. Its primary goal, listed at the top of the first page under the title, “to close the achievement gap with accountability, flexibility, and choice, so that no child is left behind” (p.1) is as desirable today as it was at the time of the law’s incipience. However, whatever it takes to close that gap requires something educators, politicians, parents, corporations, taxpayers, students, and every stakeholder in education simply cannot uncover. When President George W. Bush signed the NCLB Act into law in January 2002, from a political viewpoint, it “represented a sweeping reauthorization of the Elementary and Secondary Education Act, which was originally enacted in 1965 as part of Lyndon Johnson’s War on Poverty” and was the “cumulative result of a standards-and-testing movement that began with the release of the report *A Nation at Risk* by the Reagan administration in 1983” (Rudalevige, 2006). However, from a classroom educator’s viewpoint, it quickly became a four-letter-word that made educators’ lives absolutely miserable.

The bill’s verbiage and its mandate that all states implement accountability systems so that schools and teachers are held accountable for the education of *all* students seemed like an insult to teachers and schools. Mathematics educators had worked hard to establish standards and also to make provisions for implementation in teachers’ classrooms. Many states had used the NCTM standards to create their own state standards, and by following NCTM's leadership and guidelines, they had the resources available to use in the classrooms to meet those standards. Teachers work hard to educate all of their students. The fact that some children get left behind is multifaceted and not always an indication teachers need a new accountability system; and certainly not a system that takes students away from educational opportunities by establishing more high-stakes testing.

One would find it extremely difficult to debate NCLB's initial goal "to close the achievement gap…" (Boehner, 2002, p. 1); however, as with many legislative mandates, there remains a disconnect between design, interpretation, implementation, and funding. Although NCLB "brought test-based accountability" (Dee, Jacob, Hoxby, & Ladd, 2010, p. 149) to schools across the nation, it also created additional per pupil spending for schools to provide more direct instruction and student support without providing adequate funding. Student motivation and attendance were impacted as a result of deemphasizing non-tested subjects of arts, social studies, and science to focus on the targeted subjects of reading and mathematics (Dee, et al., 2010). As had his predecessors, President Barack Obama expressed his "strong commitment to academic standards as a fundamental element of his educational reform agenda" (Mathis, 2010, p. 1) for the nation's children, saying:

> Because economic progress and educational achievement go hand in hand, educating every American student to graduate prepared for college and success in a new work force is a national imperative. Meeting this challenge requires that state standards reflect a level of teaching and learning needed for students to graduate ready for success in college and careers. (Barack Obama, White House Statement, February 22, 2010)

Some celebrated the release of the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices &

Council of Chief State School Officers, 2010) while others boycotted it. Those who embraced the document shared their beliefs through writing and presentations (Bowman, 2015; Bowman & Conrady, 2014) because they saw it as a continuation of the goals for which mathematics educators and NCTM have worked so hard. Others, however, perhaps just weary from attending to yet another set of standards, set out to block its acceptance in their states. Regardless of which side of the CCSSM one supported, NCTM continued to support the nation's mathematics teachers by releasing a guide to aid the implementation of the CCSSM, *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). This book is a necessary tool for those who teach mathematics, K-12 and beyond, and those just preparing to teach, even if not following the CCSSM.

Principles to Actions names six guiding principles for school mathematics addressed in the guide: teaching and learning, access to equity, curriculum, tools and technology, assessment, and professionalism (p. 5). Additionally, it emphasizes and supports the CCSSM mathematical practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. (NCTM, 2014, p. 8)

After one-hundred plus years in mathematics education, very little appears to have changed. How different is this from what the Committee of Ten called for in 1893? As a reminder, they asserted:

> The method of teaching should be throughout objective, and such as to call into exercise the pupil's mental activity. The text-books should be subordinate to the living teacher. The illustrations and problems should, so far as possible, be drawn from familiar objects; and the scholar himself should be encouraged to devise as many as he can. So far as possible, rules should be derived inductively, instead of being stated dogmatically. On this system the rules will come at the end, rather than at the beginning, of a subject. (NEA, 1893, p. 105)

The call for sense-making in student learning continues to be a primary focus, as well as modeling and structure. Students continue to be encouraged to construct methods rather than follow supplied rules. While the tools have changed, the focus remains on the child and a call for his mental activity – a return to beginnings, over a century ago.

Perhaps *lasting systemic change* in mathematics education is difficult to attain because, like the ecosystem in the aquarium, everything is in a continual state of flux. The addition of each new piece of legislation, standardization, curriculum, or administration, causes a ripple effect, which in turn results in systemic disequilibrium. Finding ourselves at the edge of chaos we grasp for the closest anchor, something familiar we can believe in – thus the cycle begins again – stuck forever on the Mobius strip of the history of mathematics education.

Chapter 3: A Nontraditional University Mathematics Instructor

Abstract for Article 2

Traditional university mathematics instructors are rarely dissimilar in their teaching techniques or pedagogical practices. The typical technique follows this sequence: a lecture over new material, problems or proofs worked out on some type of media board, a few well-chosen examples, time for questions, and then assigned homework. Granted, some students have learned to be adept at learning through this style of instruction. Nonetheless, “lecturing can overwhelm students with too much information, whereas hands-on learning strategies and instruction focused on meaningful conceptualization have been shown to increase student achievement” (Zimmermann, Carter, Kanold, & Toncheff, 2012, p. 46). University students preparing to teach mathematics may need different experiences with learning mathematics. Extensive research has made it clear teachers must possess a different kind of mathematical knowledge (Ball, 1990; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Ma, 2010; Shulman, 1986). *The Mathematical Education of Teachers* (2001) and *The Mathematical Education of Teachers II* (2012) set forth specific guidelines for the mathematics preservice elementary teachers should be taught at the university level and how their mathematics should be learned. This article reports on a study of a nontraditional university mathematics instructor in a Midwestern university in her geometry classes for preservice elementary teachers. Further, it explores the features of and pedagogic practices of her learner-centered beliefs in mathematics teaching at the university level. This research found recurring themes of the instructor’s use of *humor, wait-time,*

questioning, *persistence*, *encouragement*, *negotiation*, and *repetition*. These themes fall generally into two broad pedagogical categories: *care* and *technique*.

A Nontraditional University Mathematics Instructor

Introduction

> *One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it.* (Hersh, 1979, p. 33)

If as researchers and teacher educators we believe the mathematical beliefs of our preservice teachers affect the students they will teach, then it is imperative we consider the beliefs of the mathematics instructors who are teaching university mathematics to the preservice teachers. In his work on the philosophy of mathematics, Ernest (1988) listed three mathematical beliefs systems to which mathematics teachers might ascribe. The first of these he referred to as the "instrumentalist view," in which one views mathematics as "an accumulation of facts, rules, and skills to be used in the pursuance of some external end." Ernest called the second belief system the "Platonist view," in which mathematics is a "static, but unified body of certain knowledge" which is "discovered, not created". Finally, he identified the third belief system as the "problem solving view," in which mathematics is said to be a "dynamic, continually expanding field of human creation and invention, a cultural product". This third belief system implies that "mathematics is a process of enquiry and coming to know, not a finished product, for its results remain open to revision" (Ernest, 1988, 1989).

This investigation examined the pedagogical beliefs and actions of a university mathematics instructor who taught a course in geometry to preservice elementary teachers. The case reported in this article is part of a larger study which also examined student reactions and effects in mathematics teaching efficacy of the students to the learner-centered teaching style of the instructor. Findings of the full study may be found in *Student-Centered Learning in University Mathematics* (Bowman, under review).

Literature

During the last three decades, mathematics education has had numerous attempts at radical changes in curriculum and pedagogy (Ball et al., 2005; Ma, 2010; Reeder & Bateiha, 2016; Shulman, 1986; J. Utley & Reeder, 2012). These changes, primarily motivated by the aim to have all students college-ready, have made the mathematical teaching and learning for elementary students a constant focus of concern (Greenberg & Walsh, 2008). These concerns have manifested in the grand attempts in recent years for our nation to adopt common standards across the states (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) to realign and resequence when and where mathematics topics are taught in PK-12 curriculum. For example, pushing the introduction of algebraic concepts into early childhood mathematics is one of the responses to this concern. College mathematics relies on the fundamental mathematics students learn in secondary schools; secondary mathematics relies on the fundamental mathematics students learn in primary schools. For some students, instruction begins in home settings prior to formal education, while other students begin theirs as they enter preschool or kindergarten. The remaining students begin their mathematical learning in elementary schools where teachers have dedicated themselves to teaching children a variety of subjects, including mathematics. Therefore, it seems only logical: to create mathematically-prepared college students requires mathematically-prepared elementary teachers. Based on the multitier aspects of Bandura's definition of teacher efficacy (Bandura, 1977, 1981, 1997):

> One would predict that teachers who believe student learning can be influenced by effective teaching, and who also have confidence in their own teaching abilities, should persist longer, provide a greater academic focus in the classroom, and exhibit different types of feedback than teachers who have lower expectations to their ability to influence student learning. (Gibson & Dembo, 1984, p. 570)

Certainly, then, goals of an elementary teacher education program would include enhancing the "preservice teachers' beliefs about science and mathematics and their ability to teach these subjects" (Huinker & Madison, 1997, p. 107) as well as increasing their self-efficacy in mathematics. Consequently, it is essential that "teacher educators must be aware of their students' beliefs and plan for experiences which will have positive impact on teacher self-efficacy and outcome expectancy," (Enochs & Riggs, 1990, p. 701).

One problem that resonates through studies addressing mathematics teaching efficacy is that elementary teachers who lack confidence in their mathematics teaching ability have the potential to engender mathematical weakness in the students in their classrooms (e.g. Bates et al., 2011; Briley, 2012; Bursal & Paznokas, 2006; Eddy & Easton-Brooks, 2010; Enochs et al., 2000; Harkness et al., 2007; Moseley & Utley, 2006; Swars et al., 2006; Tschannen-Moran et al., 1998; Utley et al., 2005). One of the reasons for this may be that teacher efficacy beliefs frequently direct classroom pedagogy. Teachers with stronger efficacy beliefs often conduct learner-centered, student-owned lessons; whereas those with weaker efficacy beliefs often prefer to lecture and assign readings (Czerniak & Schriver, 1994). Additionally, those with weaker efficacy beliefs tend to avoid both taking and teaching the mathematics they perceive difficult (Bursal & Paznokas, 2006; Harkness et al., 2007; Philipp, 2007). While each of these studies relates teacher efficacy to teacher performance, some focus on the relevance of mathematics anxiety to teacher efficacy, (Swars, et al. 2006; Bursal & Paznokas, 2006), while others examine teacher beliefs (Briley, 2012), or motivation (Harkness et al., 2007). Why, with this collective knowledge of relationships affecting the mathematics teaching efficacy of preservice elementary teachers, is change so difficult to achieve?

The *Mathematical Education of Teachers II* (*MET II*) states those who choose to teach elementary children often possess only a superficial knowledge of K-12 mathematics, including the mathematics they will be teaching (Conference Board of the Mathematical Sciences, 2012). The authors stress, "a strong understanding in the mathematics a teacher will teach is necessary for good teaching" and every "student deserves a teacher who knows, very well, the mathematics that the student is to learn" (p. 24). Unfortunately, the ideal does not always match the actual. Addressing this issue head-on, the authors set forth recommendations for not only what mathematics preservice elementary teachers should learn, but how mathematics learning should occur. In part, they state:

> A major advance in teacher education is the realization that teachers should study the mathematics they teach in depth, and from the perspective of a teacher. There is widespread agreement among mathematics education researchers and mathematicians that it is not enough for teachers to rely on their past experiences as learners of mathematics. It is also not enough for teachers just to study mathematics that is more advanced than the mathematics they will teach. Importantly, mathematics courses and professional development for elementary teachers should not only aim to remedy weaknesses in mathematical knowledge, but also help teachers develop a deeper and more comprehensive view and understanding of the mathematics they will or already do teach. (Conference Board of the Mathematical Sciences, 2012, p. 23)

Crucial in the above passage is the realization that preservice elementary teachers do not just need *more* mathematics; they require *different* mathematics. Responding to this section of *MET II*, one of the authors for the Common Core State Standards for Mathematics (CCSSM) remarked, “to be clear, what the report is describing here is not college algebra, abstract algebra, calculus, liberal arts mathematics, or mathematical modeling” (Zimba, 2016, p. 157). Zimba (2016) listed specific considerations for what created courses should and should not be, and specified that the courses should be *mathematically rigorous*. He noted, “‘rigorous’ here refers to quality of mathematical thought, not sophistication of topics, techniques, or notation” (p. 157).

This plea for a different kind of mathematical rigor for preservice teachers is consistent with the research in mathematical knowledge begun by Shulman (1986), who differentiated between content and pedagogical knowledge for all content areas. Ball and others further developed Shulman’s ideas about teacher knowledge, but specifically for mathematics (Ball, 1990; Ball et al., 2005, 2008). Ball and her colleagues divided the knowledge needed for teaching into six distinct sections: common content knowledge, horizon content knowledge, specialized content knowledge, knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum (Ball et al., 2008). The mathematical community has widely accepted this way of thinking about mathematical knowledge. Relying on Ball’s work, Ma (2010) reiterated that the kind of knowing elementary mathematics teachers need goes far deeper than algorithms, saying they must have a *profound understanding of fundamental mathematics*. She defined *profound understanding of fundamental* *mathematics* by claiming it "goes beyond being able to compute correctly and to give a rationale for computational algorithms." Ma asserts that it also includes being "aware of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics," as well as being able to teach it to students (Ma, 2010, p. xxxiii).

In an effort to continue to provide direction for the instruction leading preservice teachers to this level of mathematical knowledge, the 2012 report from CBMS, *MET II*, made recommendations for the types and amounts of mathematics all preservice teachers should take through the mathematics departments in their colleges and universities. Although the authors stress that the quality of the courses is more important than the quantity, the suggested coursework is broken down for each level:

1. Prospective elementary grade teachers should be required to take at least 12 semester-hours on fundamental ideas of elementary school mathematics, their early childhood precursors, and middle school successors.
2. Prospective middle grades (5-8) teachers of mathematics should be required to take at least 24 semester-hours of mathematics, that includes at least 15 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.
3. Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics that includes three courses with a primary focus on high school mathematics from an advanced viewpoint. (Conference Board of the Mathematical Sciences, 2012, p. 18)

This passage, again, emphasizes that these courses should be specifically tied to the mathematics the teachers will be teaching in their own classrooms. These recommendations should not be confused with methods courses. Both reports from the CBMS, the 2001 report and the revised 2012 report, emphasized that these recommended courses were mathematical content courses and should be taught by university mathematics instructors. Because CBMS stresses the mathematics the students need must be learned from the perspective of a teacher, it follows that preservice teachers need to learn the mathematics the way they will be teaching it – in a constructivist, learner-active setting, as opposed to a traditional university lecture setting.

A recent research study by Jaworski, Mali, and Petropoulou (2016) poignantly describes traditional university level mathematics teaching. The authors write:

> Traditionally, at university level, students are taught in large groups, often with hundreds of students. Most of the teaching is delivered through uni-vocal lecture format often described as transmission teaching: the teacher is a lecturer who exposes the mathematics for the students who listen, copy from the board and go away to make their own meanings from the experience. (para. 3)

In this article, the authors accentuate they are not referring to graduate students who often teach undergraduate mathematics, but rather to university professors in the mathematics department.

In a letter to the members in 2004, the president of the National Council of Teachers of Mathematics, Seeley, addressed the problem of effecting change in mathematics education in "Beyond Pockets of Wonderfulness," which she later included in a book of messages for mathematics instructors (2015). In this brief message, Seeley expressed admiration for an innovative lesson teaching multiplication, a new approach to teaching algebraic functions, and a mathematician who contacts a high school to gather data; however, Seeley also countered with the problem of isolation and called these events *pockets of wonderfulness*. After setting the stage, Seeley (2015) continued:

> The problem is that when great events happen in isolation from the larger system within which they operate, we fall short of what might be possible otherwise. Educators generate tremendous power by talking to one another and working together. Articulation and collaboration are important tools for making *lasting systemic change* [emphasis added]. When educators fail to take advantage of these tools, students are destined to have to start over, lose ground, and miss opportunities to connect mathematical ideas. (p. 171)

However, assuming they exist, how does one find such *pockets of wonderfulness* and then, perhaps, more importantly, how does one help such *pockets of wonderfulness* become widespread?

Methodology

This research project took place in a potential isolated *pocket of wonderfulness* aimed to add to the body of research in mathematics education for preservice elementary teachers. Such research has the potential to highlight a path of improvement for their self-efficacy in mathematics, impact how university-level mathematics is taught to preservice elementary teachers, and improve the mathematics education of all students. The primary question guiding the research was, “When a mathematics instructor’s beliefs about student learning are constructivist in nature, what are the features of and pedagogic practices utilized in her university mathematics course for preservice elementary teachers?”

Guided by this question, a sequential strategy of mixed-methods case study (Creswell, 2014) was designed, which examined preservice elementary teachers (PSETs) and their instructor in a Midwestern state university mathematics course. The mathematics course studied is one of four mathematics courses, three of twelve hours, required for all PSETs at the university and is a partial fulfillment of the state’s mathematics requirement for elementary teacher certification. This particular instructor was chosen because of her reputation as a teacher who believes in constructivist learning and implements pedagogic practices that reflect those beliefs in the mathematics classroom.

Each semester, the instructor, Dr. Mu (pseudonym) teaches two sections of a university mathematics class designed to focus on the concepts of geometry. Although enrollment is not limited to preservice elementary teachers, the course is designed for them and to meet the twelve hours of mathematics required by state legislation. Each of the two sections of the class meets twice a week, with one section meeting in the morning and the other in the afternoon. The observed classes met for sixteen weeks during the fall 2016 semester. The morning class consisted of twenty-six students and the afternoon class, sixteen.

Understanding how an instructor views mathematics and comparing her stated view to how she teaches can inform the researcher's perspective to what occurs in the classroom. Thompson (1992) also stressed that the researcher's belief system in mathematics must be examined and revealed to position the researcher's perspective in her research. The researcher for this project entered the research project with a secondary mathematics teaching background, as well as one that includes teaching preservice elementary teachers. She is a proponent of constructivist learning in the mathematics classroom. As both teacher and researcher, she had to step out of the teaching role and take the role of observer in an attempt to prevent researcher bias in the study. Agreeing with Thompson's premise, "the relationship between beliefs and practice suggest that belief systems are dynamic, permeable mental structures, susceptible to change in light of experience" (1992, p. 140), this researcher finds herself hard-pressed to describe her beliefs position in mathematics, however. Clinging to a belief that mathematics is alternately discovered and created, this researcher is perhaps still searching for ideological solid ground.

Data Collection

Data were collected for an entire 16-week semester via weekly class observations and direct interactions with the instructor and her students. Specifically, the following types of data were collected: the course syllabus and course description provided by Dr. Mu, handwritten observer notes, audio recordings, face-to-face interviews, prompt-driven electronically submitted reflections, digital photos of students' work, and excerpts from the instructor-chosen course textbook (Aichele & Wolfe, 2008). Because a *constant comparative method* was used for data analysis, the reflection questions for Dr. Mu developed organically from the researcher observations. The instructor reflection prompts were given once monthly, beginning with the week before the semester began.

Data Analysis

Data analysis began at the beginning of the study using a *constant comparative method* (Glaser, 1965; Schwandt, 2015) which allowed for the systematic development of themes by jointly coding and analyzing collected data throughout the research. Although Glaser, Corbin, and Strauss are traditionally connected with grounded theory, this particular methodology lends itself appropriately to this study (Corbin & Strauss, 1990a, 1990b), as the ongoing analysis of the collected data drove questions and further data collection.

Reflections, observation and interview notes, course syllabus, instructor-chosen textbook, and audio transcripts of Dr. Mu's classes were examined to look for evidence to answer the guiding research question. As recurrent themes arose, they were coded and compared to other similar words, phrases, and actions that could be associated with

Dr. Mu's pedagogical beliefs. This cycle of data collection and analysis via searching for themes that would then feed back into the data collection continued throughout the entirety of the semester. For example, throughout the study, the researcher not only observed Dr. Mu's teaching and interactions with students, but also presented the instructor with questions and clarifications about classroom observations.

Findings

The first meeting with Dr. Mu revealed her passion for teaching mathematics and concern for preservice elementary teachers. Her reputation as a nontraditional university mathematics instructor led to the choice to select her for the study, but only through conversation and observation could the researcher come to better understand what that meant for Dr. Mu's teaching and for her students' learning. The findings of this study revealed features of the course as well as Dr. Mu's beliefs about teaching and learning. These features and beliefs include her development and use of her course syllabus, intentional textbook selection, and her care and techniques with teaching which includes the implementation of *humor, wait-time, questioning, persistence, encouragement, negotiation,* and *repetition* while conducting the classes. Together, these combine to create the observed nontraditional college mathematics course.

Course Syllabus

Dr. Mu set the tone for her nontraditional college mathematics course on the first day of the semester when she distributed the syllabus (See Appendix A) to her students. The syllabus directed the students to read a section of their course textbook, "Making Sense of Geometry in an Inquiry Class" (Aichele & Wolfe, 2008, pp. 641–650). This section, along with the syllabus, made it clear that students would be responsible for their own learning, a task many college students have not been given in their prior coursework. The syllabus stated that "the value of this course will depend mostly on you – your involvement, effort, and creativity" and included a specific sequence of expectations prior to coming to class. According to the syllabus, students would be given a daily set of activity pages to complete on their own time using the following series of options:

> First try these by yourself in order to present your initial thoughts, understanding, and ideas. Treat each page as a quiz of your own initial understanding and reactions. Feel free to think outside the box and tinker. After working by yourself, feel free to use other resources – friends, classmates, or the internet – to further develop your answers before class. Some questions and needed clarifications can be left until class, but the majority of your answers and ideas should be well developed after your completing the assignment. Be sure to bring any needed manipulatives with you to class. (Instructor Syllabus)

The syllabus further explained these completed pages would drive class discussions and would be graded twice – once for completion during the group discussion prior to whole class discussion and then once for accuracy when turned in at the end of each class.

Dr. Mu realized her expectations were likely different than those of other instructors and spent a considerable amount of time the first day of class going over each element of the syllabus, including late work (not accepted), contact information, needed materials, and her grading scale. As the semester progressed, she would refer back to the syllabus to address class expectations and exceptions. The syllabus also included a link to a website specifically prepared for preservice teachers taking this foundational course in geometry and measurement. The website included textbook tips, geometry definitions, external website links, and other helpful information for students needing additional assistance in their learning.

The Textbook

One of the first conversations the researcher had with Dr. Mu revolved around the textbook she had chosen for the course. Dr. Mu chose *Geometric Structures: An Inquiry-Based Approach for Prospective Elementary and Middle School Teachers* by Douglas B. Aichele and John Wolfe (2008, Pearson). In the first class, Dr. Mu explained to the students it was the “best” book for what they were learning. When later asked why she felt it was the best book, she replied:

> I said I thought the book was best because its instructions and activities are organized so that I can teach with this cycle. Traditional textbooks organize homework as practice problems. This textbook organizes questions and experiences for students to investigate at home, and which we then use as the springboard for our class discussions. Rather than telling students geometric definitions and having them practice them, this book organizes its questions to have the students solve problems and make sense of situations. Each topic is organized to develop and fortify students’ conceptual understanding. The problems grow in complexity across the topic while also revisiting problems discussed in class. (Instructor Reflection)

Dr. Mu’s choice to use this textbook in her class, as described by her reflection, was a key curricular decision for her course.

The textbook by Aichele and Wolfe (2008) presents each topic in a manner uniquely different from traditional mathematics textbooks. For example, a traditional mathematics textbook for college students typically introduces new topics with several paragraphs of information tying in past topics, a few worked out examples, and then problem sets for students to work out on their own. This textbook, however, introduces new topics with activities for the students to work through on their own. The activity pages begin with a list of materials students will need to do the activity and continue with progressively challenging steps and thought questions to engage the students in sense-making. The absence of descriptive information that traditional textbooks include encourage the students to research other sources, discuss their work with other students, and pose questions to the instructor when returning to class.

Beliefs and Pedagogical Practices of the Instructor

Dr. Mu's pedagogical practices were driven by her beliefs about mathematics and students' learning, and included her intentional use of *humor, wait-time*, *questions*, *persistence*, *encouragement*, *negotiation*, and *repetition*. When asked what she believed about mathematics, her reply was quite lengthy and included a metaphor with three levels of understanding. She replied, in part:

> I see mathematics as a language. I teach students to be literate in that language, and being literate comes at three levels. First level – to learn a language, a student must learn symbols used to communicate in the language….Second level – but even as we wouldn't consider someone literate in German just because they could read and write some German words, neither do I expect someone to be literate in mathematics because they know the vocabulary and symbols….Third level – every language has invisible nuances, idioms, and figures of speech. The more literate a person is in a language, the more accurately they read and provide the correct meanings. This type of literacy doesn't come from a dictionary or a textbook. Generally, *it comes from being saturated in numerous situations with native* speakers to build an understanding of its meaning. [Emphasis added.] (Instructor Reflection)

Dr. Mu's last phrase above, *it comes from being saturated in numerous situations with native speakers*, sheds light on how her beliefs about mathematics influenced her efforts to help her students become more mathematically literate during each class period. She intentionally created a community of learners where all students' answers were valued and no questions were deemed foolish or unnecessary.

Dr. Mu's beliefs in student learning directly contributed to the creation of this community of learners. When prompted, "Before I met you, others described you as a 'constructivist' teacher. How do you see yourself as a teacher and why?" She replied:

> I'm not sure my first reaction would be to say that I'm a constructivist teacher. I'm not sure that's the title I would give myself. However, if someone said that

> about my teaching, I wouldn't correct them. Constructivism as a theory describes learning as an active, "constructive" process. As such, learners are described as *making* sense of the content for themselves and *linking* instruction with their prior knowledge. I see student learning in this way—active and personal. I believe everyone comes to a class with unique prior knowledge and learns at their own rate. As a result, I organize my instruction to draw out their prior knowledge and to provide students time to think, opportunities for processing, and feedback on the plausibility and correctness of their ideas. (Instructor Reflection)

The observations of Dr. Mu's pedagogic practices reflected this belief about student learning on a regular basis. Her way of *drawing out prior knowledge* was made clear on the first day of class, both on the syllabus and in her explanations of daily expectations. A key point she stressed to students regarding class preparation was for students to *persevere to the point of questions* and those questions would drive the next class period.

Dr. Mu characterized her teaching as nontraditional by first describing a traditional university mathematics class, including some she had taught in the past. Just as the researcher, Dr. Mu had taught secondary mathematics before her graduate work. Her secondary teaching of mathematics was, in her words, "traditional", but her experiences in her graduate work shaped her beliefs about student learning, leading to her nontraditional methods observed. In her new role, she referred to herself as a *coach*. She explained:

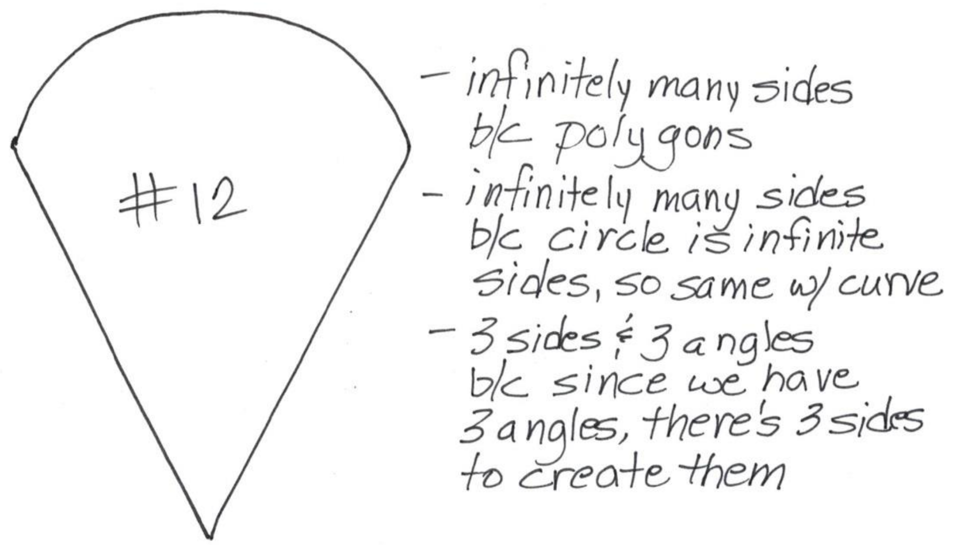
> Generally, in the cycle of traditional math instruction, a teacher verbally describes the mathematical content, shows students example problems, provides students a limited amount of in-class practice, and assigns more practice problems as homework. Someone who observed my teaching would probably notice I organize instruction in a different cycle than "traditional teaching." While traditional instruction usually begins verbally describing mathematical content and showing students example problems, I generally begin instruction by asking students to venture possible solutions to a seeming riddle, wonder through possible contradictions, or speculate the meanings of words prior to any of my own instruction. When I taught traditionally, students' time in class was spent listening to me and practicing problems, but with my current cycle of

> instruction, students' time in class is spent actively talking, processing, and arguing about the mathematical topics, and I spend class helping *coach* students in their mathematical justifications and conjectures. As their *coach*, I stay neutral while negotiating class arguments, I don't answer questions, [but] bear a poker face or a face of ignorance while craftily teasing out mathematical ideas or misconceptions that are surfacing in their talking, processing, and arguing. [Emphasis added.] (Instructor Reflection)

An example of Dr. Mu's neutrality Dr. Mu *used to tease out mathematical ideas* occurred on the first day of class. Dr. Mu passed bags of precut shapes from black-line masters provided in *Teaching Student-Centered Mathematics, Volume 1* (Van de Walle, Lovin, Karp, & Bay-Williams, 2014) to each group of four students. She assigned several shape-sorting activities, through which a sort of *number talk* (Humphreys & Parker, 2015) immerged. Although the key aim of number talks is to develop computational fluency, the students' rich conversations with Dr. Mu about geometric qualities were comparable. Through this process, Dr. Mu provided the learning environment in which students were able to "acquire basic concepts, algorithmic skills, heuristic processes, and habits of cooperation and reflection" (Davis, Maher, & Noddings, 2006, p. 187).

As she walked around the room, Dr. Mu continually listened to her students' conversations. Several minutes into this particular task, she interrupted their table discussions: "I hear a lot of you saying 3 sides or 4 sides. Look at shape 12 [a shape formed by two line segments that met at a vertex with its two diverging ends connected by an arc]. Does it have 3 sides?" Student answers included: infinitely many sides, two sides, and three sides. After each student responded, an opportunity was provided to defend their point of view, but at no time did Dr. Mu validate or invalidate any of their answers; she simply wrote them on the board beside the shape she had sketched (see Figure 1).

Figure 1. Determining a Reliable Definition for a Triangle



After collecting and recording several of their comments and reasons, Dr. Mu continued, “Guess what? Class is over. You have two days to ponder this and we will start with your thoughts on Thursday.” (Class Lecture 08/23/2016). The class period ended with questions, not answers, which encouraged the continuation of curiosity and critical thinking for the next class. The next class period began precisely where the previous one had left off, with students discussing in their small groups whether or not figure 12 (see Figure 1) might be a triangle.

Similarly, in the second day of discussions over the definition of a triangle, the students made comments and asked questions, including: “If a curve is a side, we should be able to measure the angles.” “Does a line have to be straight?” “Is the curve at the top a side?” “Is a line a side?” These questions, and others, led Dr. Mu to discuss the need for standards, such as those provided by NCTM (NCTM, 2000). She explained, “Mathematics is not black and white. It’s very cultural. Who decides what a side is? In this class, we will agree: sides are straight” (Class Lecture 08/25/2016). She continued to share what she called *invisible words,* e.g. *straight* sides, but the *straight* is invisible, emphasizing that “sides” would always be straight and there was no need to write or say the word “straight”.

Consistent with her beliefs, Dr. Mu explained that this process of negotiating definitions is her way of linking their prior knowledge to what is to be learned. She clarified:

> For example, because I believe I should link instruction with prior knowledge, I will ask students to define vocabulary words before I provide the definitions. I begin by asking everyone to define it individually and then offer their definitions to the class for us to compare and contrast against one another. Through this process, I am asking them to refine the class-created definitions. Finally, I provide the formal definition. In this way, I hope students come to recognize how accurate or inaccurate their prior knowledge was and to understand how each detail of the definition is essential for naming/describing the vocabulary word. When I submit the formal definition to them, I posit it as a socially constructed consensus of many mathematicians—recognizing that some mathematicians still argue definitions. While creating a class definition, students often will have opposing views with strong mathematical thinking as evidence for both sides. Students need to believe their ideas were sound—they just didn’t live at the time when mathematicians had discussions and came to the consensus.

Her consistent practice allowed students to gain confidence and construct meanings. Each new definition was tested for mathematical soundness by intentional, instructor-chosen examples. The example of the shape created by two line segments and an arc tested the soundness of the students’ definition of a triangle. Given a student definition of a triangle as “a shape made from three sides,” with no attention to the specificity of the word *sides*, this non-polygon becomes a triangle, underscoring the need to strengthen the definition.

Care and Technique

Several themes related to Dr. Mu's pedagogical practice revealed in the data--*humor*, *wait-time*, *questioning*, *persistence*, *encouragement*, *negotiation*, and *repetition*--can be placed into two general pedagogical categories – *care* and *technique*.

Dr. Mu's intentional use of *humor* to ease the students' anxiety was seen in both statements and gestures. Statements such as "playing the devil's advocate here..." or "if you thought I was listening, you're mistaken," broke the tension in the classroom and opened the floor for more candid discussion. Dr. Mu's ability to patiently provide the students with *wait-time* to think and construct meaning in the problems would often carry over into the next class period. Dr. Mu continually emphasized that students should own the mathematics through her *questioning* and *persistence* in making them explain why they believed or did not believe an answer was correct. She stressed the difference between simply *feeling* an answer was correct and *knowing* an answer was correct by forcing them to cite class-negotiated definitions that defended their answers.

Students, trusting their prior knowledge was intact, often resisted the process of defending answers. However, Dr. Mu was able to take student comments such as "I hate definitions!" and turn them into positive teaching moments using *encouragement* to coax the students into owning their mathematical knowledge. Through the answer *negotiation* process she had students use daily to make sense of the mathematics, the students came to connect their often imperfect prior knowledge of mathematics to definitions and processes that made sense to them.

Repetition was a key component of Dr. Mu's daily classes. As students were asked to repeat definitions to verify their claims, she would ask questions such as, "and why do you say it is a prism? Use your definitions." This repetition, however, should not be confused with the rote "drill and kill" over which educators disagree. The repetition Dr. Mu employed in her classes was aimed at the possibility of sense-making leading to student ownership of mathematical knowledge.

Dr. Mu's care and technique in each of these intentional pedagogical practices were further observed by her attention to body language and verbal responses as she interacted with the students. For example, instead of standing over the students when probing their knowledge or demonstrating possible techniques for individuals to try, she pulled up a chair and sat down beside them in their workspaces. Never raising her voice or attempting to talk over students' discussions, she would pull the class back together with a raise of her hands accompanied with phrases such as "give me your eyes" or "thumbs up." When they made and recognized mistakes, she responded "part of the learning process is learning where you went awry" or "know why it was wrong before you change your answer."

Discussion

A nontraditional syllabus and/or textbook do not ensure a course might be taught any differently than any other university mathematics class. The power for difference resides in the instructor's beliefs about mathematics teaching and learning, and in her ability to engage students in learning experiences that reflect those beliefs. This study sought to examine the following question: "When a mathematics instructor's beliefs about student learning are constructivist in nature, what are the features of and pedagogic practices utilized in her university mathematics course for preservice elementary teachers?" Even simple statements such as, "I believe I should link instruction with prior knowledge" or "someone who observed my teaching would probably notice I organize instruction in a different cycle than *traditional teaching*" are void without evidence of practice. However, Dr. Mu's beliefs and practices revealed congruence in her beliefs about mathematics teaching and learning and the experiences she provided for her students.

As instructors are teaching students to teach mathematics, the human element cannot be ignored. Just as elementary students have specific individual needs, so too do those who will teach them. During the course, Dr. Mu exhibited care alongside technique through her intentional spoken words, visual demonstrations, and body language. These themes confirm her beliefs in students as constructivist learners. Even though Dr. Mu does not describe herself as a constructivist teacher, her methodology for student learning very much aligns with constructivist beliefs for mathematical learning. Davis, Maher, and Noddings (2006) wrote:

> Constructivists agree that mathematical learning involves the active manipulations of meanings, not just numbers and formulas. … Every stage of learning involves a search for meaning, and the acquisition of rote skills in no way ensures that learners will be able to use these skills intelligently in mathematical settings. Misconceptions may develop anywhere in the process, and constructivist teachers are continually watching for them and planning activities that will lead students to challenge their own faulty conceptions. (Davis et al., 2006, p. 187)

Dr. Mu demonstrated these traits in her daily interactions with and reactions to students' answers, as well as their questions. There was not an observed instance in which she gave them a formula to rotely manipulate numbers to obtain answers. Rather, as students constructed their own meanings and understandings of the geometry they were exploring, she addressed misconceptions in such a way that meanings were clear and concise for the students. Students spent time in class comparing, contrasting, questioning, and constructing meanings, rather than reading them from a book or being given them by an instructor.

Students need to see nonexamples [as demonstrated in Figure 1], along with examples to construct and strengthen meaningful definitions. The geometry standards for grades Pre-K – 2 from *Principles and Standards for School Mathematics* state: “through class discussions of such examples and nonexamples, geometric concepts are developed and refined” (NCTM, 2000, p. 98). When exploring new knowledge, such as the relationship between area and perimeter of a closed figure, Liping Ma (2010) assigned four sequential levels to the mathematical understanding of elementary teachers. She described the first level of understanding as “disproving the claim” through the use of counterexamples, the second level as “identifying the possibilities” by exploring various relationships, the third level as “clarifying the conditions” under which the identified possibilities would hold true, and the fourth and final level as “explaining the conditions” and why they were chosen over others (Ma, 2010, pp. 93–98).

It is difficult to inspire students to persevere to this level of understanding in a traditional classroom setting; however, Dr. Mu created a community where students were motivated to stay with a definition until the entire group agreed and understood. One of the ways Dr. Mu created community was through her choice of textbooks for the course. Throughout the reform efforts in mathematics curriculum, educators have continued to argue the importance of adherence to textbooks as a curriculum guide. Textbooks do have the ability to frame the curriculum and determine what, when, and how students are taught various concepts (Nicol & Crespo, 2006); when appropriately designed, textbooks can even place teachers in the center of the curriculum (Ball & Cohen, 1996).

Especially for preservice or novice teachers, the textbook choice may be crucial in determining both what and how mathematics is taught and learned. Dr. Mu's course textbook, by Aichele and Wolfe (2008), was written, "to provide a creative, inquiry-based experience with geometry that is appropriate for prospective elementary and middle school teachers" (p. xi). The textbook does lend to community making as Dr. Mu created, but it is only in combination with her beliefs and pedagogical practice of persistence in encouraging students to engage in sense-making that the text is successful in the course. If one were to simply pick up the book and attempt the course as an independent study or online course, the result would not be the same.

This opens the question as to how widespread sustained change in the mathematical education of elementary teachers might occur. As another study notes,

> It would be difficult to reproduce or replicate this course by simply adopting the course text and implementing the activities described within this study. It requires creating an environment that questions and defies the very definitions of teaching and learning. The creation of an active-learning environment fueled by the continuous collaboration of all members of the class was an essential element of this course. (Bates, 2014, p. 108)

Even when an instructor attempts to repeat her own lesson in a different setting or with a different set of students, the results are not the same as when she first taught it. Historically, researchers have described this phenomenon as a man not being able to step into the same river twice, for on the second entry both the river and the man will have changed. This analogy, paraphrased from Plato's *Cratylus*, poignantly describes the complexity of classroom dynamics: "Heracleitus is supposed to say that all things are in motions and nothing at rest; he compares them to the stream of a river, and says that you cannot go into the same water twice" (Plato, 1952, p. 94).

So should we just abandon all hope for change? If this level of change in teacher education requires a unique blend of mathematical beliefs, research-grounded pedagogical learner-centered practices, and curricular decisions, can we hope to replicate such *pockets of wonderfulness*? This level of change to mathematics teacher education requires intentionality, cooperation, and hard work. Some universities are already seeing the benefits of coordinating efforts between mathematics and education departments, such as the University of Nebraska at Omaha and Oklahoma State University (e.g. Matthews & Seaman, 2007; Matthews, Rech, & Grandgenett, 2010; Utley, 2004), and together are following the recommendations of the Conference Board of Mathematical Sciences (2001, 2012). Other schools are examining alternate models, such as blended content and methods courses (Burton, Daane, & Giesen, 2008). Some individual mathematicians and mathematics educators, such as Bass and Ball (2014), have worked in partnership to research mathematical learning in the field. Expressing their surprising collaboration after over two decades of work, Ball and Bass stated:

> Improving mathematics learning depends on intertwining deep expertise in the practice of both mathematics and instruction. These connections can be built strongly when collaborations engage in practice—through direct engagement in instruction, through artifacts that can be discussed, studied, and re-examined over and over. This involves cross-disciplinary and new interdisciplinary work—about what are the key questions, what counts as a claim, and what counts as evidence and warrants. A crucial foundation for such collaboration is mutual respect; another is solid grounding in the domains of mathematics as a discipline and in the actual practice of instruction as well as its close and disciplined study. With almost twenty years of experience with this work, we can see our progress as well as the hard knocks of the arguments it has taken to get here. We are encouraged by the results and interested in articulating more fully the methods involved so that others can also engaged [sic] in such partnerships. (p. 311)

Research continues in varied venues for those who choose to do something different to make a change, but it is essential the research is presented and published because “there exists a need in mathematics education for the telling and sharing of the stories of teachers who have transformed their pedagogic practices and have successfully enacted significantly different ways of teaching mathematics” (Reeder, Cassel, Reynolds, & Fleener, 2006, p. 66). The recent release of *Standards for Preparing Teachers of Mathematics* (Association of Mathematics Teacher Educators, 2017) offers hope and encouragement that through dissemination of current research and trends, even more colleges and universities will follow in these first steps of collaboration for change for mathematics teacher education. Collaboration will not be without struggles, such as claiming ownership to common ground and agreeing on the essence of mathematics (Fried, 2014; Marzocchi, Miller, & Silber, 2016), between mathematicians and mathematics educators. Nonetheless, if the focus is on the students, *and it should always be on the students*, hope for change remains.

Chapter 4: Student-Centered Learning in University Mathematics

Abstract for Article 3

The fact cannot be ignored that there are and there will be preservice teachers who are extremely anxious about mathematics, and they will soon be teaching in schools (Bursal & Paznokas, 2006, p. 177). Decades of research show students learn mathematics best in student-centered learning settings (e.g. Harper & Daane, 1998; Ma, 2010; NCTM, 2000; Reeder & Bateiha, 2016; Reynolds, 2010), so researched-based educators teach their teacher candidates that their mathematics classes should be student-centered. However, in their university mathematics courses, preservice teachers traditionally do not experience student-centered learning. Although education teachers may teach preservice teachers how to teach so their students can learn, what preservice teachers learn by example is how they have been taught themselves. This mixed-methods research examined whether or not the mathematical teaching efficacy of preservice elementary teachers might be influenced in a learner-centered university mathematics class designed specifically for them. Quantitative results from the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) indicated an insignificant positive effect ($p > .05$) on both the mathematics teaching outcomes expectancy subscale and the personal mathematics teaching efficacy subscale. Additionally, the qualitative results indicated a positive effect on mathematics teaching self-efficacy by the change in tone and language in preservice teacher conversations and reflections. Recurring themes throughout the study showed a progression in student response from struggle and frustration to confidence and community.

Student-Centered Learning in University Mathematics

Introduction

> In mathematics classrooms, students co-construct their knowledge through collaboration on meaningful tasks. When they do so, they make connections to previous mathematical understanding and refine their thinking; they are not empty vessels waiting for information deposits and accumulation. If teachers focus their instruction on meaningful mathematics, use real-world problems, and let students reconsider their own understanding in light of new experiences, the students will be motivated. (Harkness, D'ambrosio, & Morrone, 2007, p. 237)

Extensive research has made it clear teachers must possess a different kind of mathematical knowledge (Ball, 1990; Ball et al., 2005, 2008; Ma, 2010; Shulman, 1986). *The Mathematical Education of Teachers* (*MET*) (2001) and *The Mathematical Education of Teachers II* (*MET II*) (2012) set forth specific guidelines for what mathematics preservice elementary teachers should be taught at the university level and how that mathematics should be learned. Yet, research into how these two vital reports are directing the mathematical education of future teachers in the field produces sparse findings (e.g. Ma, 2010; Matthews & Seaman, 2007; Matthews et al., 2010). Several decades ago, Hersh (1979) recognized that mathematicians continue to struggle over the nature of mathematics and have developed a "just *do* it" (p. 35) philosophy, thereby avoiding committing to one or another prevailing philosophies of mathematics. However, teachers of PK-12 mathematics--elementary especially--do not have that luxury. They are tasked with the responsibility of nurturing a mathematical medium in which all future mathematics can grow. Without this initial nurturing of minds, children may have difficulty making necessary mathematical connections as they mature. To help students make those connections, those who teach them mathematics must understand how the mathematics they are teaching intertwines with the mathematics students have already learned, as well as the mathematics students will later learn (Conference Board of the Mathematical Sciences, 2001, 2012). This research study conducted in a potential isolated *pocket of wonderfulness* (Seeley, 2015) aims to add to the body of research in mathematics education for preservice elementary teachers (PSETs) by creating a pathway of improvement for their self-efficacy in mathematics, impacting how university-level mathematics is taught to preservice elementary teachers, and providing insight as to how the mathematics education of all students might be improved.

Literature

Numerous studies over the past three decades have sought to find connections between teacher attitudes, beliefs, and practices in mathematics education (e.g. Briley, 2012; Goldin et al., 2016; Harper & Daane, 1998; Philipp, 2007; Philipp et al., 2007; Raymond, 1997; Thompson, 1992). Thompson (1992) wrote in her synthesis of the research up to that point, "belief systems are dynamic, permeable mental structures, susceptible to change in light of experience" and the "relationship between beliefs and practice is a dialectic, not a simple cause-and-effect relationship" (p. 140). In her studies with beginning elementary teachers, Raymond (1997) described their beliefs about mathematics as the outcome of prior encounters with mathematics. She defined *mathematical beliefs* to be "personal judgments about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics," and, based on her definition, concluded that "*mathematics beliefs* are central to the beliefs-practice relationship" (p. 552).

Philipp et al. (2007) found in their study that (PSETs) “who studied children’s mathematical thinking while learning mathematics developed more sophisticated beliefs about mathematics, teaching, and learning and improved their mathematical content knowledge than those who did not” (p. 438). Briley (2012) discovered in his study with PSETs, those “who reported stronger beliefs in their capabilities to teach mathematics effectively were more likely to possess more sophisticated beliefs as well as were more likely to have more confidence in solving mathematical problems.” Briley also reported in his study that the PSETs’ mathematical beliefs “had a statistically significant effect on mathematics teaching efficacy and on mathematics self-efficacy” (p. 438).

One problem that resonates throughout studies addressing mathematics teaching efficacy is elementary teachers who lack confidence in their mathematics teaching ability are prone to share weak mathematical thinking with their students (e.g. Bates et al., 2011; Briley, 2012; Bursal & Paznokas, 2006; Eddy & Easton-Brooks, 2010; Enochs et al., 2000; Harkness et al., 2007; Moseley & Utley, 2006; Swars et al., 2006; Tschannen-Moran et al., 1998; Utley et al., 2005). A reason for this may be that teacher efficacy beliefs frequently direct classroom pedagogy. Teachers with stronger efficacy beliefs often conduct learner-centered, student-owned lessons; while those with weaker efficacy beliefs often prefer to lecture and assign readings (Czerniak & Schriver, 1994), resulting in teaching facts rather than concepts (Harper & Daane, 1998). Additionally, those teachers with weaker efficacy beliefs tend to avoid both taking and teaching the mathematics they perceive difficult (Bursal & Paznokas, 2006; Harkness et al., 2007; Philipp, 2007). While each of these studies relates teacher efficacy to teacher performance, some focus on the relevance of mathematics anxiety to teacher efficacy, (Swars, et al. 2006; Bursal & Paznokas, 2006), while others examine teacher beliefs (Briley, 2012), or motivation (Harkness et al., 2007). Surveying these studies raises the question, why with this collective knowledge of relationships affecting the mathematics teaching efficacy of preservice elementary teachers is change so difficult to achieve?

These studies – and many more – regarding the nature of mathematical beliefs, efficacy, and affect help educational researchers to recognize that the beliefs of mathematics instructors affect the mathematical beliefs of PSETs and that the mathematical beliefs of PSETs will affect the students they will one day teach. With the exception of a few (e.g. Moseley & Utley, 2006; Briley, 2012), the studies above were conducted in PSETs methods courses. If we hope, however, to increase the mathematical learning of elementary students, our work must begin in studying how PSETs learn mathematics in their university mathematics courses.

Methodology

The fact cannot be ignored that there are and there will be preservice teachers who are extremely anxious about mathematics, and they will soon be teaching in schools.
(Bursal & Paznokas, 2006, p. 177)

The study took place in two sections of a geometry course for preservice elementary education majors, taught by one mathematics professor at a Midwestern university. This research study employed a sequential strategy of mixed-methods case study (Creswell, 2014), which examined PSETs and their instructor in a learner-centered university mathematics course. The mathematics course is one of four mathematics courses, three of twelve hours, required for all PSETs at the university and constitutes a partial fulfillment of the state's mathematics requirement for elementary teacher certification. This particular instructor, Dr. Mu (pseudonym), was chosen because of her reputed beliefs and pedagogical practices in constructivist learning in the mathematics classroom. The students were chosen by their enrollment in her two courses.

The instructor taught two sections of the course, which was designed to teach the concepts of geometry to preservice elementary teachers. Although enrollment was not limited to preservice elementary teachers, the course was designed specifically for them and to meet the state's requirements for elementary teacher certification. Each of the two sections of the course met twice a week, with one section meeting in the morning and the other in the afternoon. The observed classes met for sixteen weeks during the fall 2016 semester. The morning class consisted of 26 students and the afternoon class, 16. Although 35 of the 42 students consented to participate in the study, pre/post data was only obtained from 25 of those. Additionally, four of the 25 were found to be secondary mathematics education majors, so their data was excluded from the quantitative data analysis. The remaining 21 students, primarily Caucasian women, contributed sufficient data to observe common tendencies among the group. Data were both collected and analyzed using a *constant comparative* method to explore answers to the following questions:

1. What are the perspectives of preservice elementary teachers in a university mathematics course taught in this manner?
2. What impact does a university mathematics course taught from a constructivist-learner perspective have on preservice elementary teachers' self-efficacy in mathematics?

Understanding how an instructor views mathematics and comparing her stated view to how she teaches can inform the researcher's perspective to what occurs in the classroom. Thompson (1992) also stressed that the researcher's belief system in mathematics must be examined and revealed to position the researcher's perspective in her research. The researcher for this project entered the research project with a secondary mathematics teaching background, as well as one that includes teaching preservice elementary teachers. She is a proponent of constructivist learning in the mathematics classroom. As both teacher and researcher, she had to step out of the teaching role and take the role of observer in an attempt to prevent researcher bias in the study. The full study included how Dr. Mu's beliefs impacted her design and teaching of university level mathematics classes, as well as how PSETs responded to her teaching methods. Although this article focuses on the students, the instructor and her beliefs remain vital to the study. The complete data describing the instructor and the learner-centered facets of her course may be found in *A Nontraditional University Mathematics Instructor* (Bowman, under review).

Data Collection

Data were collected throughout a 16-week semester via weekly class observations and direct interactions with the instructor and her students. Specifically, the following types of data were collected: the instructor-designed course syllabus and course description provided by Dr. Mu, handwritten observer notes, audio recordings, face-to-face interviews, prompt-driven electronically submitted reflections, digital photos of students' work, pre/post survey data from the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) (Enochs et al., 2000), and excerpts from the instructor-chosen course textbook (Aichele & Wolfe, 2008). Because a *constant comparative method* was used for data analysis, the reflection questions for Dr. Mu and her students developed organically from the researcher observations and interviews. The instructor reflection prompts were given once monthly, beginning with the week before the semester began and the student reflection prompts were given every three weeks, beginning the second week of the semester.

The researcher's theoretical lens as a pragmatic mathematics teacher urged her to use the most efficient and readily available tool to accomplish the goals of the study. Quantitative tools, such as the MTEBI, allowed an efficient way to obtain a beginning and ending snapshot of the self-efficacy in mathematics teaching of the participants in the class. Additionally, the data from the initial administration of the MTEBI aided in selecting the participants for reflection questions; and by being sequenced in this way guided the qualitative portion of the mixed-methods study.

The Mathematics Teaching Efficacy Beliefs Instrument

During the first week of classes, the researcher administered a quantitative pre-assessment to the students to measure their self-confidence in mathematics using the MTEBI (Enochs et al., 2000; Huinker & Enochs, 1995; Huinker & Madison, 1997). The initial MTEBI, which uses a 5-point Likert-scaled survey, helped select the participants to include in the selective prompt-driven reflections for the qualitative portion of the data. At the end of the semester, the MTEBI was re-administered by the researcher as a posttest for quantitative comparison with the pretest.

The MTEBI examines the two components of self-efficacy Bandura (1977) identified as meaningful to teaching: efficacy expectations and outcome expectations (Bandura, 1977). He differentiated between outcome and efficacy expectations in that:

> Outcome expectancy is defined as a person's estimate that a given behavior will lead to certain outcomes. An efficacy expectation is the conviction that one can successfully execute the behavior required to produce the outcomes. Outcome and efficacy expectations are differentiated, because individuals can believe that a particular course of action will produce certain outcomes, but if they entertain serious doubts about whether they can perform the necessary activities such information does not influence their behavior. (p. 193)

The MTEBI quantitative data collection instrument and scoring instructions were modified (Huinker & Madison, 1997) for use with preservice elementary teachers in their mathematics courses. Thirteen of the 21 items on the MTEBI are on the personal mathematics teaching efficacy (PMTE) subscale, and eight of the items are on the mathematics teaching outcome expectancy (MTOE) subscale (Huinker & Madison, 1997, p. 110). Each of the 21 items has five response options, ranging in degree from *strongly agree* to *strongly disagree* (Enochs et al., 2000, p. 195). The MTEBI was validated for assessing the mathematics teaching efficacy of preservice elementary teachers in an extensive study by its developers (Enochs et al., 2000), and since its development has been used by numerous researchers, for multiple comparisons, and with various results.

Huinker and Madison (1997) structured their study to assess the self-efficacy of PSETs in both mathematics and science while enrolled in methods courses for both subjects. The two instruments they used were the MTEBI for the mathematics teaching self-efficacy and the Science Teaching Efficacy Beliefs Instrument (STEBI-B) (Riggs & Enochs, 1990) for preservice elementary teachers to assess the science teaching self-efficacy. The MTEBI was modified from the STEBI-B for use with PSETs specifically with the speculative outcome, “improving science and mathematics teaching efficacy will ultimately improve instruction and student achievement in elementary classrooms” (Huinker & Madison, 1997, p. 109).

Utley (2004) used the MTEBI in her study along with an instrument she developed and validated explicitly for geometry attitudes (Utley, 2007) to assess beliefs and attitudes toward geometry in PSETs. Her study explored the effects of a nontraditional summer geometry course on PSETs. The results of her study revealed a positive effect on student’s geometry self-efficacy.

Data Analysis

Two types of data analysis began at the beginning of the study. The qualitative data were analyzed using a *constant comparative method* (Glaser, 1965; Schwandt, 2015). A constant comparative method allowed for the systematic development of themes by jointly coding and analyzing collected data throughout the study. Although Glaser, Corbin, and Strauss are traditionally connected with grounded theory, this particular methodology lends itself appropriately to this study (Corbin & Strauss, 1990a, 1990b), as the ongoing analysis of the collected data drove questions and further data collection. Reflections, observation notes, interview notes, course syllabus, instructor-chosen textbook, and audio transcripts of the classes were examined to look for evidence to answer the guiding research questions. Words students used in their reflections were color-coded and listed in tables to count for frequencies. Comparisons of those words to observer notes of students’ interactions and work in class began to cluster into theme-sets. As recurrent themes arose, they were coded and compared to other similar words, phrases, and actions that could be associated with the students' mathematics efficacy beliefs.

Quantitative analysis was enacted on the initial MTEBI survey data running a series of parametric t-tests with SPSS, version 24 (IBM Corp., 2016) to check for possible initial differences between the two classes. The initial data from the MTEBI's administration was also used to select students for further qualitative study. The final data from the administration of the MTEBI was compared to the initial data with paired t-tests to provide insight as to whether quantitative change could be validated in the students' efficacy. Additionally, paired t-tests were run between the pre/post means of each of the 21 questions to glean information about individual gains or losses on specific question items. It is noteworthy to mention that the two subscale means, PMTE and MTOE, are found separately and are not summed at any point in the analysis.

Findings

The data collected from the study included quantitative data in the form of a pre/post survey over teaching efficacy beliefs and qualitative data in the form of observer notes, photographs of students' work, student reflections, course syllabus, and conversation with students. Together these findings revealed recurring themes of *struggle*, *frustration*, *confidence*, and *community* among the students in the class.

Findings from Quantitative Data

The MTEBI (Huinker & Madison, 1997) was administered at the beginning and end of the semester to 25 students. However, four of the students were later found to be secondary mathematics education majors, so their data was removed from the analysis. The comparisons from the beginning and end of the semester administration of the

MTEBI to the remaining students (n=21) are detailed in the discussion and tables. The two separate subscale scores of the MTEBI are not combined – the PMTE subscale has a possible score that ranges from 13 to 65 and the MTOE subscale has a possible score that ranges from 10 to 50 (Huinker & Enochs, 1995). The initial range for the PMTE subscale was from 41 to 61 at the beginning of the semester, and from 40 to 63 at the end of the semester. The initial range for the MTOE subscale was from 19 to 34 at the beginning of the semester, and from 19 to 39 at the end of the semester. Individual students' scores for both beginning and ending subscales are reported in Table 1.

Consistent with other studies, the data show small gains as well as losses throughout. For personal efficacy, 10 of the 21 students showed a gain, one showed no change, and the remaining 10 showed a loss. For outcome expectancy, 13 showed a gain, two showed no change, and the remaining six showed a loss. The discussion section (which follows the findings) provides commentary on the significance of exhibiting either a gain or loss on either of the subscales. Although most losses in subscale scores were negligible, Phi (pseudonym) exhibited a significant loss in both subscales. Her results are further discussed later under the heading *A Case in Point*.

A paired t-test, run with SPSS, version 24 (IBM Corp., 2016), exhibited an overall loss for the PMTE, but an overall gain for the MTOE. Nevertheless, as shown in Table 2, both the loss and the gain were insignificant at the 95% confidence level ($p > 0.05$) for both subscale scores. A careful examination of the data reveals that if Phi were excluded from the data set then the results would be considerably more positive (see parenthetical data in Table 2) toward showing a change in the students' efficacy toward teaching mathematics. However, Phi's data is important to this study and one of the reasons why the qualitative data is essential to the study.

Table 1. Pre/Post Subscale Scores for PMTE and MTOE from the MTEBI

Participants	Pre – PMTE	Post – PMTE	Pre – MTOE	Post – MTOE
Epsilon	57	48	19	28
Psi	43	42	24	27
Tau	52	53	26	19
Zeta	49	48	29	30
Lambda	53	52	25	30
Upsilon	61	59	26	25
Delta	48	49	29	32
Beta	41	40	29	28
Omicron	41	46	34	34
Rho	58	58	31	31
Kappa	52	62	24	21
Chi	59	60	27	32
Gamma	46	50	28	30
Eta	42	45	32	31
Nu	50	46	25	29
Sigma	47	51	24	30
Xi	56	63	33	39
Theta	52	46	28	29
Alpha	54	57	30	27
Iota	54	51	33	34
Phi	52	40	27	22

.

Table 2. MTEBI Subscale Comparisons and Results

Table 2. Subscale scores, means, standard deviations, and results for PMTE and MTOE for pre/post administration of MTEBI (Data in parenthesis exclude Phi's scores.)

Subscale	M	SD	MDiff	t(20)	p
PMTE-pre	50.8095	5.91286			
PMTE-post	50.7619	6.91307	-.04762	-.043	.966
(PMTE)			(.5500)	(.671)	(.510)
MTOE-pre	27.7619	3.70006			
MTOE-post	28.9524	4.57686	1.19048	1.387	.181
(MTOE)			(1.5000)	(1.783)	(.091)

Table 3. MTEBI Questions & Pre/Post Mean Differences

Item	Questions	M	SD	p
1.	When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.	.286	1.056	.229
2.	I will continually find better ways to teach mathematics.	-.048	.384	.576
3.	Even if I try very hard, I will not teach mathematics as well as I will most subjects.	.381	1.284	.189
4.	When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.	.048	1.203	.858
5.	I know how to teach mathematics concepts effectively.	.333	.856	.090
6.	I will not be very effective in monitoring mathematics activities.	-.048	1.117	.847
7.	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.	.000	1.095	1.000
8.	I will generally tech mathematics ineffectively.	-.286	1.419	.367
9.	The inadequacy of a student's mathematics background can be overcome by good teaching.	-.095	.700	.540
10.	When a low-achieving child progresses in mathematics, it is usually due to extra attention by the teacher.	-.190	.814	.296
11.	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	.238	.831	.204
12.	The teacher is generally responsible for the achievement of students in mathematics.	.619	1.024	.012
13.	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.	.095	1.221	.724
14.	If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.	.476	1.209	.086
15.	I will find it difficult to use manipulative to explain to students why mathematics works.	-.476	1.289	.106
16.	I will typically be able to answer students' questions.	-.286	.902	.162
17.	I wonder if I will have the necessary skills to teach mathematics.	.476	1.030	.047
18.	Given a choice, I will not invite the principal to evaluate my mathematics teaching.	-.476	1.209	.086
19.	When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.	.095	.995	.666
20.	When teaching mathematics, I will usually welcome student questions.	-.476	1.078	.056
21.	I do not know what to do to turn students on to mathematics.	.333	.577	.016

On the item-by-item pre/post analysis, 12 of the 21 items (See Table 3) indicated a positive effect, with three of those significant at the 95% confidence interval, ($p < .05$). One of the eight items (12) designed to measure MTOE showed a significant positive effect ($p = .012$) and two of the 13 questions (17 & 21) designed to measure PMTE also showed significant positive effect ($p = .047$, $p = .016$).

Findings from Qualitative Data

Even though mathematics is the researcher's primary field of study, she does not believe a number defines a person. This belief led to a primarily qualitative study, with only the MTEBI adding a quantitative data component. The qualitative data shared in this report are from classroom observations of the students and Dr. Mu, written reflections from the students, audio recordings of the classes, and conversations with the students about their work during class. Because the researcher chose a *constant comparative* method for analyzing the qualitative data, the analysis of data from the beginning drove the collection of other data, which in turn revealed interesting facets of the study. For example, examining Phi's scoring on the MTEBI contrasted with her daily growth in the classroom requires a brief diversion from the data analysis for the whole class to look at the data analysis for *a case in point* – Phi.

A Case in Point

A close examination of the MTEBI results in Tables 1 and 2 reveals an interesting anomaly in Phi's scores. If only provided with her quantitative data, then it would appear the semester-long class was detrimental to both her personal teaching efficacy and outcome expectancy. Phi's PMTE pretest subscale score was twelve points higher than her posttest subscale score, and her MTOE pretest subscale score five points higher than her posttest subscale score. Nonetheless, her daily interaction and growth in mathematical sense-making was evidenced by the conversations and attitude monitored throughout the semester.

Phi sat at the same back table in the afternoon classroom throughout the semester. At the beginning of the semester, Phi would often have a single earbud from her phone inserted in one ear while the other earbud dangled from the split in the wire. Up through mid-semester, she often complained about the required work and supporting explanations for given answers with phrases like:

> I hate definitions! I hate this! I will not be teaching this! In the program [education department] we learn how to teach math. In here we have to do math. I don't need to do math. I know it. But when you ask me to explain it, I can't. But I'm not going to be teaching this the first day of third grade. I just know. I always get them right, but I can't tell you how I know. I know what I know and that's it.

While teaching, Dr. Mu continually stressed that understanding of definitions would help the students to make sense of the mathematics, and proof of understanding would be their answer explanations. In response to researcher concerns after class about Phi's remarks, Dr. Mu said, "In reality, if you can't explain it, you don't know it."

Evidence of a change of attitude for Phi appeared quite gradually during the last month of classes. She once again began to mention the course she was taking concurrently in the college of education, but rather than contrasting them, she was comparing them and recognizing the common goals of the two professors. She still did not enjoy following directions and preferred her own methods, but she was listening to her peers during group work and questioning their methods and solutions. Her reasoning showed a depth of knowledge and determination to learn not obvious during the earlier observations. For example, to justify an area problem on dot paper, she told the group she had found an old geoboard and rubber bands at home and tried it until she was convinced it was impossible to construct a triangle having an area of one-third. During the last observed class session, Phi was working on finding the surface area and volume of pyramids and prisms. As she worked with the other students at her table, she remarked:

> Oh! Length times base times height is for a rectangle, but this is a triangle. So we have to divide by two. I see that now. But how did you see that? Now, I'm going to self-correct here. Tell me if I'm doing it right. I think this stuff is so common sense that it makes you feel dumb when you get it wrong.

The conversation no longer consisted of obstinate statements, such as, "*I just know what I know and that's it,"* but rather community-driven curiosity, "*How did you get that answer?"*

When Phi took the initial administration of the MTEBI at the beginning of the semester, she was confident she knew all of the mathematics necessary to teach mathematics to elementary students. Her perception of her strengths caused her initial scores to be overly high. By the end of the semester, however, she realized she possessed mathematical weaknesses. Her final score, while lower than her beginning score, was a hopeful indication she will continue growing where she left off. A number alone does not appropriately describe the growth Phi experienced during this course.

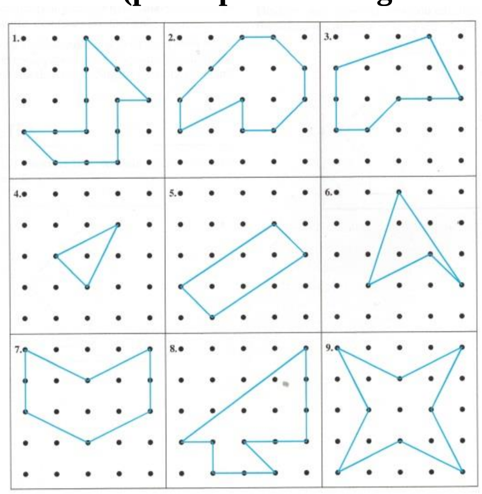
Classroom Observations

The structure of constructivist classrooms encourages mastery of skills because students are not only allowed, but encouraged to share ideas, make mistakes, ask questions, and change answers. Through group work opportunities, students learn to work hard at sense-making, with an emphasis on effort rather than product (Murhukrishna & Borkowski, 1996). As in previous studies with preservice teachers by

Harkness, et al., the students in Dr. Mu's classes also "learned that the teacher expected them to work hard to understand the mathematics embedded within tasks. They were free to share their ideas and strategies, to make mistakes, and to ask questions of not only the teacher but also of their classmates" (Harkness et al., 2007, p. 250).

During the course of the semester, the researcher became highly interested in observing the students as they struggled to make sense of structure in geometry. While Dr. Mu was discussing problems with the students, the researcher was a silent observer; but once the students began working on problems, the researcher became an active participant, roaming from table to table, taking both pictures and notes of their work and engaging in conversations. One of the most interesting observations came from a task Dr. Mu gave the students to find the areas of various polygons on a dot-grid matrix, and then explain or defend their solutions. A sample page from the student text, *Geometric Structures: An Inquiry-Based Approach for Prospective Elementary and Middle School Teachers* (Aichele & Wolfe, 2008, p. 211), is shown in Figure 2, with permission of both authors and the publisher.

Figure 2. Areas on a Dot Matrix (photo permission granted by Aichele & Pearson)

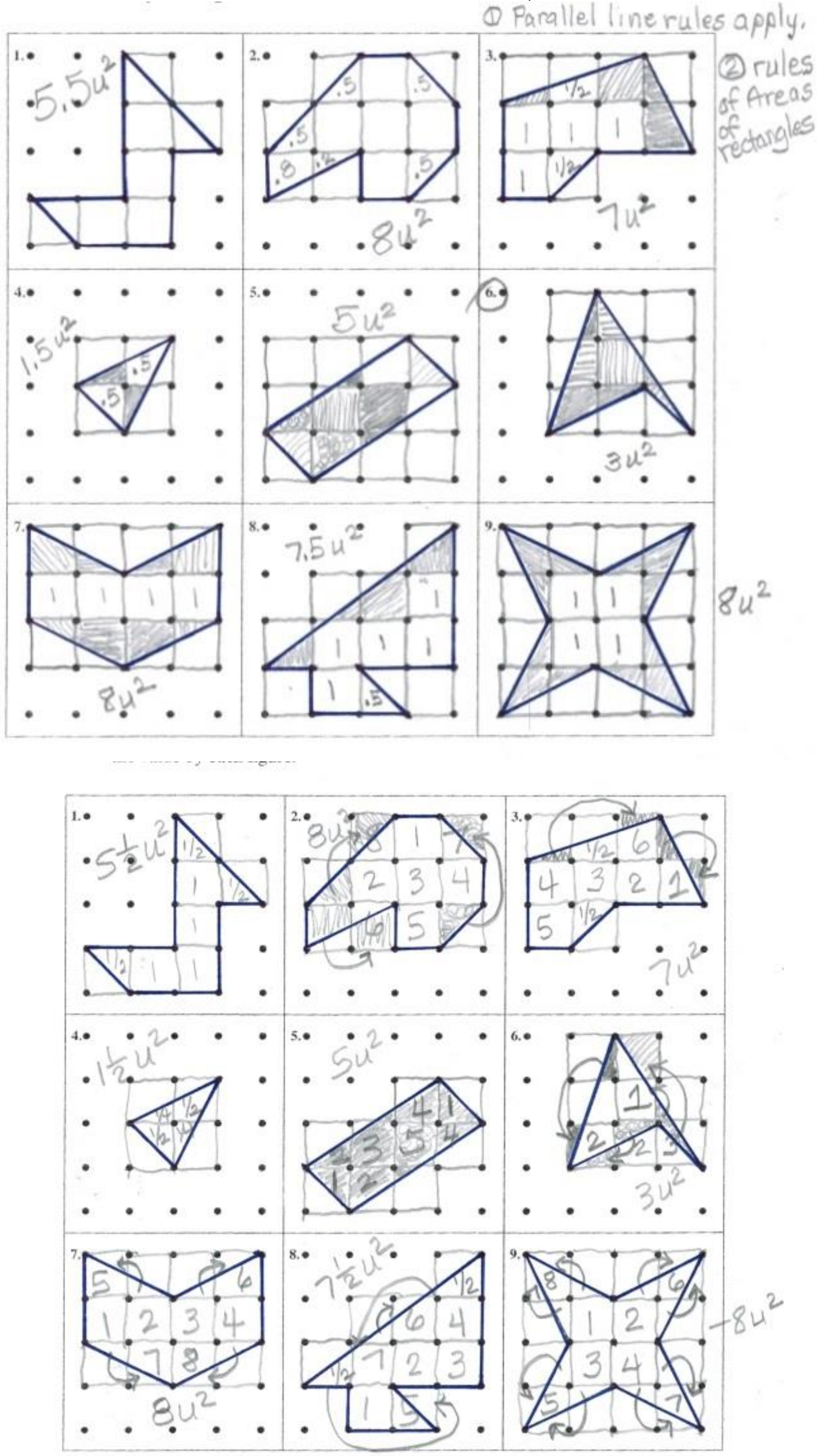


The students found the areas of the figures by using the *take-away* or *cut-up* methods; no formulas were allowed. The *take-away* method process begins as one finds the area of the largest rectangle that encloses the shape, and then *takes away* the extra parts. With the *cut-up* method, one draws lines or uses scissors to cut the shape into pieces that can make square units. As the researcher observed the students' work, she questioned their techniques for finding the areas. Table discussions with students about their different answers and preferred methodologies were intriguing and lengthy.

As shown in Figure 3, students had unique ways of marking and figuring out the areas in square units. Interestingly, a majority of the students preferred the *cut-up* method over the *take-away* method. One student remarked she never would have thought of the *take-away* method on her own. Other students simply said they thought the *cut-up* method was easier to visualize. As the students discussed their different approaches, first in their small groups and then as a whole class, individual students took time to defend their own answers or else argue for or against another student's answers.

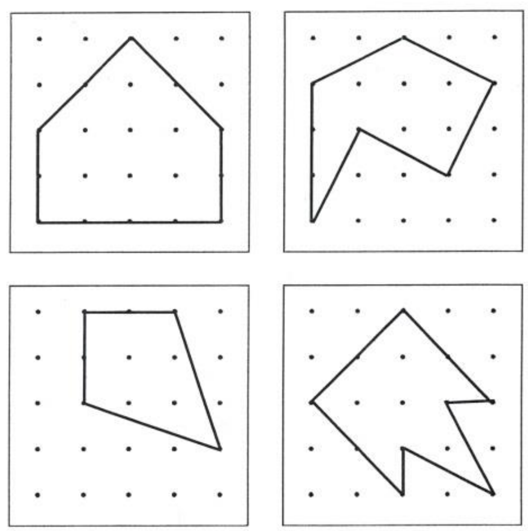
Dr. Mu used similar figures on dot-grid matrices to emphasize how one could use slope to find the areas of various polygons when all edges are drawn on the slant. Through the first set of activities for irregular polygons, the students developed the realization the area of a triangle was half the area of a rectangle. Once they had that realization, they went on to develop the formula for the area of a triangle as base times height divided by two. Although many of the students knew the formula before the activities began, they were not allowed to use it until they could explain how and why it worked.

Figure 3. Finding Areas of Irregular Polygons (photo permission granted by Aichele & Pearson)



After the formula had been derived from the activities and the students had practiced using it, Dr. Mu gave the students the instructor-created worksheet [Figure 4] and asked the students whether they could use the formula they had derived to find the areas of the shapes. After a few frustrated minutes without any solutions, she then asked them to try to find the perimeter of each shape. Unlike the shapes they had previously been given, neither the *take-away* nor the *cut-up* models were completely successful for this activity.

Figure 4. Areas & Perimeters with Slant Heights (photo used with permission)



As students struggled to find the areas and perimeters, the results of these two activities together were used to develop and prove the Pythagorean Theorem in a later class. By giving the students time to develop the sense-making required to understand the geometric relationships in the drawings, the resulting theorem came as a natural outcome of their *struggle*. The students were able to recognize the slant edges as hypotenuses of right triangles, then by constructing the legs of the triangle, compute the length of the slant edges. The Pythagorean Theorem was not given as a formula to be committed to memory and practiced, but rather a sum of squares made from the edges of triangles that could be used to find the lengths of the edges. Watching the students develop this understanding was entertaining and memorable.

Student Reflections

In addition to class observations, students responded to questions sent to them through electronic communications. The researcher sent questions out in sets of two, four times during the semester, for a total of eight questions. Several of the questions pertained to their thoughts and feelings about teaching, while a few about their personal learning. A few select questions and student answers are included here to construct a picture of students' perception of mathematics.

Question: "What was the hardest mathematics concept for you to learn in your K-12 schooling? Do you understand it now? If so, what changed?"

Eta: "Algebra and geometry. I'm understanding more of geometry because of Dr. Mu."

Nu: "Algebra was very difficult for me. Now I love it. I think I got older and got better teachers. I enjoy the assuredness I find when completing an equation."

Xi: "The hardest problem I encountered was multiplying decimals. I would literally convert my decimals to fractions, multiply the fractions and convert the fractions back to decimals. I do understand it now. My issue was that I could see the parts more easily in a fraction than I could visualize with a decimal. Now I am able to see the parts represented with a decimal."

Iota: "The hardest concept was in high school with the introduction to algebra and geometry. It was a breakaway from the standard adding, subtracting, multiplication, and division and a move towards the more difficult. It was just such a new territory and I don't remember much. I still struggle with it. Math is just not easy to me."

Rho: "I think Geometry was hardest for me to learn. I had names for shapes but did not know why or the definition of the shape which made it harder. I am now learning more and growing as a student."

Pi: "Slope in 7^{th} grade was pretty rough. I understand it now because my dad tutored me intensely."

Omega: "When I was in elementary school, long division was very difficult for me. Yes, I understand it now. My mom helped me learn it and through practice, I was able to understand it."

When researchers cite studies that indicate missing attributes of PSET mathematical knowledge, it is essential to understand what that missing knowledge might entail. While fractional measures on geometric shapes continually caused moans and groans among the students during the semester's observations, none of the students' answers mentioned fractions as being problematic. Additionally, algebraic struggles are often related to fractional misunderstanding. Students know what mathematics they struggle with, but they do not always know why. It is worthy to note that in the students' answers to this question they attributed a change in their understanding to an individual – a parent or teacher – and not to a program. It is essential for all preservice teachers to recognize that it is a teacher who makes a definitive difference in student learning.

A question regarding students' perception of their own learning during the semester's course also produced interesting answers.

Question: "What mathematics have you learned during this course that you did not understand at the beginning of the semester and what helped you to learn it?"

Eta: "There wasn't just one thing that was difficult for me. Dr. Mu is very encouraging and she has this was of making us figure it out on our own instead of her telling us how to do it when we come to her with questions. I hope to be a creative and fun teacher like her."

Nu: "I didn't know much when I joined the class. The most helpful tool I've found is class discussion. I do better when I'm fully engaged."

Pi: "I learned a lot of neat tricks about finding triangles in polygons. I got a stronger understanding of different definitions and more of the reasons behind the simple math that we are learning."

Specific student remarks during the semester, either in isolated reflection comments or face-to-face conversations, that indicated a positive mathematics teaching efficacy included:

Sigma: "I feel confident that if I'm not familiar with the concepts or terms that I have enough knowledge that I can figure it out."

Xi: "I feel excited to be able to get to the point where I will be able to answer questions in a new exciting, and unique approach to mathematics. Something that was hard for some of my teachers to accomplish and a challenge that I look forward to."

Iota: “Having learned math myself with negative feelings but having a positive outcome gives me hope that I can be sympathetic and successful at helping my students.”

Rho: “If I was to not know an answer the student and I could look it up together and get the answer.”

Pi: “I can correct any errors in thinking and inspire a new generation of kids who enjoy math.”

Eta: “I'm more of a hands on person so I would try that approach with children. Like those cubes Dr. Mu has that connect together. Stuff like that to help kids be more hands on and interactive and make math fun. Math wasn't really fun for me growing up but in this class it has been more so than ever before, even when I get stuck.”

Discussion

It is perhaps difficult to evaluate the perceptions and effect on mathematics teaching efficacy of students in such a short time, especially when only accessing brief glimpses of their work and thoughts. However, the researcher observed recurrent themes throughout the study that indicated a progression in student response from *struggle* and *frustration* to *confidence* and *community*, and she interprets that progression as an indication of positive change in the preservice teachers. Student reflections and conversations linked to feelings about doing or teaching mathematics at the beginning of the semester frequently featured words such “nervous,” “scared,” “embarrassed,” “upset,” and “frustrated.” By the end of the semester, students’ feelings toward doing and teaching mathematics shifted; their later conversations and reflections included words such as “excited,” “challenge,” “joy,” “enjoy,” “fun,” “comfortable,” and “understand.” The qualitative element of this study was able to capture this shift in student mathematics teaching self-efficacy in a way that the quantitative element was not.

The study was designed to answer the following questions:

1. How do preservice elementary teachers perceive a university mathematics course taught from a constructivist perspective?
2. What affect does a university mathematics course taught from a constructivist perspective have on preservice elementary teachers’ self-efficacy in mathematics?

Preservice Teacher Perceptions

The perceptions of the preservice elementary teachers in this study of their nontraditional mathematics course varied throughout the semester. At one extreme, some of the students dropped the course, while some students found joy in learning mathematics for the first time. In answering the question, “How do preservice elementary teachers perceive a university mathematics course taught from a constructivist perspective?” several contextual observations must be considered. First, the students in the two sections of the geometry class were not typical college sophomores, the student level expected for this course. Rather, many of the students were married with families and full-time jobs, in addition to attending college. The afternoon section, specifically, had adult learners who were entering or returning to college to pursue second careers. It is worthy to note that nontraditional students often have different expectations for college classes and may have significant barriers to success traditional students do not have (Jesnek, 2012). While nontraditional students are typically delineated by age, those 25 and older (Wyatt, 2011), adult learners are "those whose prior knowledge includes a significant element derived from work or life experience in addition to, or instead of, any prior formalized study" and "choosing to enter or re-enter higher education adds to this list their personal expectations of the learning process" (Toynton, 2005, p. 107).

Several of these barriers were observed during the semester. One of the afternoon students, Chi, arrived thirty minutes late each day. A mother of teens, she was teaching math in an urban middle school and had to travel across the city to get to class after school dismissed. Chi was alternatively certified and seeking standard certification to teach. An older student, Iota, expressed that finding time to do homework was challenging. A single mother, working three jobs, and going to school fulltime, Iota remarked she found the method of teaching in the class *frustrating*, and, describing herself as "old-fashioned," said she preferred a more traditional text featuring more examples and a glossary so she could figure out things on her own, rather than rely on classroom discussions. Another student, Tau, got married during the early part of the semester. Her focus and conversation were often on the trials of being married and living with parents, rather than on the mathematics at hand. Tau was absent from class several times during the semester, thus missing out on essential classroom discussion. Her *frustration* was both visible and vocal. These barriers and others are often not considered in traditional university courses.

Students *struggled* for the first few weeks to adjust to the course's nontraditional structure. Unlike other university mathematics courses, students were expected to be active learners during the class period. By the second week of classes, students spontaneously engaged with peer checking and discussing homework as they entered the classroom and by the end of the third week, students were willing to participate in sharing their answers with the class *community* as a whole. Although two of the students verbally expressed their *frustration* with the text and format, the majority of the class seemed to enjoy the interaction and hands-on engagement in their learning. The remarks of one student, “I hope to be this kind of teacher in my classroom,” indicate the difference the course made for her.

By the end of the semester, students were *confident* one person does not have all the *right* answers. The students were continually comparing, defending, explaining, questioning, and making sense of the answers, finding a common solution where each student was prepared to defend her answer to the class. The amount of sense-making and understanding that stemmed from this type of discussion would have never occurred if homework was simply done, handed in, graded, and passed back. In their answers to their reflection questions, students expressed learner-centered class techniques that both engage learners and encourage struggle as the keys to their increases in understanding and geometrical sense-making. It is hoped they will carry these valuable tools into their own classrooms to develop strong mathematical learners of their students.

Effects on Teacher Efficacy Beliefs

The quantitative data from the MTEBI, though determined statistically insignificant within a 95% confidence interval, was encouraging toward a positive change in mathematics teaching efficacy beliefs. Without Phi’s data, both the personal mathematics teaching efficacy and the mathematics teaching outcome efficacy showed positive gains after the one-semester class. The collected quantitative data supported a positive change in both beliefs and attitudes toward geometry and mathematics in general, although the change was not statically significant ($p > .05$).

When teaching produces negative effects, such as in a pre/post assessment, how one interprets the findings may rest on the researcher's or teachers' beliefs. For example, if after teaching a unit on factoring quadratics, several students score lower on the posttest than on the pretest, one might ask whether the teaching effort actually decreased the knowledge of those students. It is more likely, however, that those students made no learning gains, and one must look further into what might have occurred to make their learning gains appear negative. One might draw similar considerations in attitudinal or belief surveys where a negative effect occurs. In such a survey where one is choosing between levels on a Likert-type scale, especially when the survey is administered at the end of a class period, a slight negative effect is more likely a non-effect and should be considered as such. Unfortunately, however, those small negative effects sum to decrease the sum of the gains significantly. In the case of this research study, the qualitative data presents a much more accurate picture of what effect the class had on the students' self-efficacy in teaching mathematics.

In addition to the comparison of pre/post subscale scores, paired t-tests were also run on an item-by-item analysis on the pre/post means for each question (see table 4). Several of the individual questions did show significant gains ($p < .05$) at the 95% confidence interval. It seems worthy to look at those individual questions with respect to the overall analysis: Question 12, "The teacher is generally responsible for the achievement of students in mathematics" (p = .012), question 17, "I wonder if I will have the skills necessary to teach mathematics" (p = .047), and question 21, "I do not know what to do to turn students on to mathematics" (p = .016). Each of these questions addresses an element of efficacy that would directly affect their mathematical teaching presence in the classroom, and viewing significant improvement in each one is encouraging.

Often, however, it is interesting to examine questions that indicate a negative effect on belief or attitudinal surveys. In this case, two questions designed to measure MTOE (9 & 10) and seven of the questions designed to measure PMTE (2, 6, 8, 15, 16, 18, & 20) indicated a negative effect on efficacy. Even though the negative effect was insignificant ($p > 0.05$) at the 95% confidence interval, the specific questions are valuable so the researcher might gain insight to what might have occurred. Questions nine and ten are particularly interesting because they describe the outcome efficacy a teacher might expect from her own teaching. The remaining seven questions show negative effects on PMTE, and may indicate the presence of an underlying mathematics anxiety issue not examined in the study. Several of the questions are reverse coded [see Appendix C] before scoring, so that students' answers are more reliable.

Conclusion

In qualitative studies, researchers often avoid drawing conclusions. Instead, they might say "So what?" or "What does all of this mean?" or "What do you think happened here?" Students in this study showed evidence of an increase in mathematics teaching efficacy through the qualitative data collected. Their feelings toward mathematics teaching changed throughout the semester, as indicated by the words they used in their reflections, their engagement in classroom community, and their conversation and questions during class. Although the quantitative data did not show a significant change, when Phi's data is excluded from the data set, positive changes in both the personal mathematics teaching efficacy subscale score and the mathematics teaching outcome expectancy subscale score are observed. Phi's quantitative data is not consistent with her qualitative data, and based on the amount of qualitative data collected on Phi, that data is a better representative of her change in efficacy during the course.

This research indicates that in this setting with a nontraditional university mathematics instructor, the mathematics teaching efficacy of preservice elementary teachers can be positively changed, even if only slightly, in one semester. If the students were to have access to such teaching methods as Dr. Mu provided for this course, one would venture to guess greater gains could be made in the full twelve hours of mathematics required for teacher certification. Granted, this was a small sample of students, but each of them will have the opportunity to impact hundreds of elementary students. Often when good things happen in small spaces, some think it does not matter in the larger scheme of things; however, this opportunity mattered for these students.

Research Design Missteps and Questions for Further Research

In efforts to get the IRB research approval for this project, the researcher vowed not to change anything in the instructor's regular teaching nor add anything to the students' load. This meant the reflections the researcher received from the students were completely voluntary. Although the reflections were sent out to all 35 students who consented to the study, only ten replied to the first set, eight to the second set, five to the third, and four to the fourth. Even though the researcher encouraged the students to respond to the reflections multiple times during the semester, both in class and by email, there was no recourse if they did not. The responses received were quite informative, but the study would have been richer with a total response. Other studies (e.g. Harkness et al., 2007) were able to negotiate daily reflections as part of the students' coursework with the instructor. Merit certainly exists in that element of design.

Additional complications included students who dropped the course after consenting to the study. Because the instructor did not know which students in the two sections were participating in the study, the researcher did not find out until the end of the semester eight of the 35 had dropped the course. As a result, of the quantitative data collected from the MTEBI, eight pretests had no matching posttest. Of the remaining students, one student did not take the pretest and another did not complete the back side of the posttest, which left 25 of the original set from which to draw quantitative data.

At the completion of this study, many questions remain. Past studies have shown the association between mathematics anxiety and efficacy beliefs in teaching mathematics (Bursal & Paznokas, 2006; Stoehr, 2017; Swars et al., 2006). Can efficacy be considered without considering anxiety as a factor? Why does constructivist learning have a greater positive effect on outcome efficacy than on personal efficacy? Can the personal efficacy be changed for preservice elementary education teachers in one semester? If it cannot be changed in one semester, how many semesters might it take? How do these questions affect our approach to designing curriculum and coursework for preservice elementary education majors? How can longitudinal studies be designed to follow groups of students with the current number of students who drop out of programs?

There is much to be learned and more to accomplish to improve the mathematics teaching occurring in elementary classrooms. Regardless of how many curriculums are mandated at the national, state, or local levels, the mathematics teaching efficacy of the teachers teaching those classes will determine the learning that occurs. We must continue to encourage young teacher candidates and praise and support the university mathematics and mathematics education teachers who are striving to practice change in their classrooms.

Chapter 5: Making Sense of the Journey

This study set out to answer three questions about change in the mathematics teacher preparation of preservice elementary teachers through their mathematics content courses. Encouraged – yet puzzled – by discovering the published works of *MET* (2001) and *MET II* (2012), along with my experience in teaching mathematics to preservice elementary teachers, I designed a sequential mixed-methods case study guided by the following three research questions:

1. When a mathematics instructor's beliefs about student learning are constructivist in nature, what are the features of and pedagogic practices utilized in her university mathematics course for preservice elementary teachers?
2. What is the perspective of preservice elementary teachers in a university mathematics course taught in this manner?
3. What impact does a university mathematics course taught from a constructivist-learner perspective have on preservice elementary teachers' self-efficacy in mathematics?

The study was conducted in a Midwestern state university in two sections of a geometry course designed to help preservice elementary teachers prepare to teach mathematics and also to meet the state's twelve hours of mathematics coursework requirement for teacher certification. The course instructor was chosen for her reputation as a teacher who believes in constructivist learning and implements pedagogic practices that reflect those beliefs in the mathematics classroom. The student participants in the study were chosen by their enrollment in the instructor's two classes.

When I say I was both encouraged and puzzled by the two published works, I wondered why, if researchers and educators recognize and identify good mathematics teaching for preservice elementary teachers, is change so difficult to achieve? This puzzlement led me to study theories of change, especially in the field of mathematics education. Using an aquarium as a metaphor of the complex system of education, in Chapter Two, I discussed how difficult it is to achieve lasting system-wide change. Perhaps *lasting systemic change* in mathematics education is difficult to attain because, like the ecosystem in the aquarium, everything is in continual flux. Each new addition of legislation, standards, curriculum, administration, and so on, causes a ripple of effects that result in systemic disequilibrium. Finding ourselves at the edge of chaos we grasp for the closest anchor, something familiar in which we can believe. Thus the cycle begins again, and we are stuck forever on the Mobius strip of the history of mathematics education.

In Chapter Three, I examined the pedagogical practices of a nontraditional university mathematics instructor to identify the nontraditional elements of her class. Through observations and written reflections, I was able to view her beliefs about mathematics and student learning through her actions. The data revealed several themes related to the instructor's pedagogical practice: *humor*, *wait-time*, *questioning*, *persistence*, *encouragement*, *negotiation*, and *repetition*, and these traits can be placed into two general pedagogical categories – *care* and *technique*. The instructor's nontraditional syllabus and choice of textbook were crucial to her course design, but it was her elements of care and technique with the students that made the course a unique and nontraditional university mathematics class. The community she facilitated allowed for the students to ask questions and pose possible solutions in a nonthreatening environment unlike any they had experienced prior to this class.

Finally, in Chapter Four, I was able to explain what I witnessed as the effect of a nontraditional university mathematics course on the preservice elementary teachers. During the semester of observations, I collected both quantitative and qualitative data. The quantitative data was collected through a pre/post administration of the MTEBI, the Mathematics Teaching Efficacy Beliefs Instrument. The data from the MTEBI showed statistically insignificant ($p > .05$) gains in both the Personal Mathematics Teaching Efficacy Beliefs, as well as the Outcome Expectancies subscale scores of the instrument. The qualitative data collected included researcher observation notes, photographs of students' work, excerpts from the textbook, written student reflections, and student conversations. The recurring themes observed throughout the study indicated a progression in student response from *struggle* and *frustration* to *confidence* and *community*, and I interpreted that progression as an indication of positive change in the preservice teachers.

Returning to the Metaphor

When I say I see the education system, and mathematics education in particular, as a semi-closed ecosystem – an aquarium – I would like to talk about what that means for individual teachers. We view a whole school district as a large aquarium being carefully tended by someone outside the aquarium, an administrator or school board, perhaps. The keepers are in charge of choosing the fish, monitoring the water, and maintaining all of the intricacies described in Chapter Two. However, let us think for a moment about how the metaphor changes as we look at one individual fishbowl, with one teacher tending her fish. She did not get to choose the fish; in fact it is possible no one considered which fish or how many were placed into her small fishbowl. She must do her best to create safe spaces in the fishbowl, for aggressive fish more often target shyer ones. She must feed them all and do her best to create an environment in which they can all thrive and grow. She will become attached to many of them and yet, without warning, they will be snatched from her fishbowl without her say. More will be added and she will nurture them the best way she knows, feeding them tidbits of mathematics, reading, social studies, and science, all while protecting them and teaching them to get along with one another.

The job given to elementary teachers is overwhelming, and they are seldom given the tools and resources they need to make their job easier. It is their love of teaching, learning, and children that called them to this career and it is their devotion to children that will keep them there. It is our obligation as teacher educators to make certain they are prepared for their content; nothing, however, can prepare them for the keepers of the fishbowl.

Implications of this Study

If as mathematics educators we truly care about our children's mathematics education, then we will be concerned with how their teachers are learning mathematics. When mathematicians and mathematics educators care about the mathematics learning of their preservice elementary teachers, they can create learner-centered spaces where preservice teachers can learn mathematics in a nonthreatening environment. These learner-centered spaces do not have to be isolated *pockets of wonderfulness*, however, for us to have the ability "to imagine and create curriculum alternatives and develop ideas about curriculum that are substantially and qualitatively different, we may be required to challenge and question our traditional ideas about what mathematics is and what it means to know mathematics" (Reeder et al., 2006, p. 66). The mathematics that preservice teachers bring from their K-12 education is not sufficient to prepare them for teaching mathematics to children and the way mathematics has been and continues to be taught to preservice teachers in many colleges and universities is not sufficient to bring about change in either their content knowledge or their confidence and feelings about mathematics. Dr. Mu's choice of textbook, development of syllabus, development of community in her classroom to support mathematical questioning and understanding, reveals that teaching mathematics for preservice teachers can be accomplished in ways that align with the vision set forth by the Conference Board of Mathematical Sciences (2001, 2012).

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Appendix A: Instructor Syllabus

Foundations of Geometry and Measurement
Fall 2016 Syllabus

Instructor:	**Email:**
Office:	**Telephone:**

**Preferred Correspondence is email. Please do not leave voicemails.*

Office Hours: T/Th 3:20 – 4:20 & various other hours on T/Th by appointment
Don't hesitate to email me and ask for an appointment. I'm often free before/after class.

Required Text and Materials

- *Geometric Structures – An Inquiry-Based Approach for Prospective Elementary and Middle School Teachers* by Douglas B. Aichele and John Wolfe.
- Required manipulatives:
  scissors **protractor** **calculator** (no cell phone calculators)
  compass **ruler** **mira** (or image reflector)
  The mira can be found at the bookstore. You can also find miras on the internet at ____________ (search for "geo-reflector mira").
  Other helpful supplies are **tracing paper** (or transparencies) and **colored pencils**.

Prerequisite(s)

No formal prerequisite is listed, but prior enrollment Structures of Mathematics, is highly recommended. This course is an extension of the problem solving strategies and skills of Structures of Mathematics.

Course Objectives

Upon completion of the course, the student should:

1. have been introduced to basic concepts of geometry and measurement.
2. be enabled to think conceptually about mathematics and apply the concepts learned in real-world problem-solving situations.
3. have developed multiple perspectives and solution methods when dealing with mathematics.

General Comments

The content and instructional delivery of this course models the current professional thinking and standards endorsed by the National Council of Teachers of Mathematics (NCTM). *The value of this course will depend mostly on you - your involvement, effort, and creativity.*

As a point of beginning, you should become familiar with some necessary material:
Making Sense of Geometry in an Inquiry-based Class – pgs. 641-650

Internet Resources for Students – pg. 651

Desire to Learn Information

The full homework schedule, the project schedule, course handouts, exam reviews and other various documents are posted on D2L for you to print off your own copies. To access our class's page, login to _______ and click on the 'My Courses' tab. Click the link labeled 'Click here to:'. Lastly, click on the link for our class. A second way to enter ________ is by going to (website masked).

Daily Expectations

Before class, you will be expected to complete a set of activity pages (about 5-7 pages). First try these *by yourself* in order to present your initial thoughts, understanding, and ideas. Treat each page as a quiz of your own initial understanding and reactions. Feel free to think outside the box and tinker. After working by yourself, feel free to use other resources – friends, classmates, or the internet—to further develop your answers before class. Some questions and needed clarifications can be left until class, but the majority of your answers and ideas should be well developed after your completing the assignment. Be sure to bring any needed manipulatives with you to class.

During class, these activity pages are central to our class discussion in developing your understanding. Because your completion of these activity pages prior to class is of utmost importance to the class conversation, these activity pages will be checked for completion at the beginning of class. As class progresses and you find that your understanding develops, feel free to change your answers and make any adjustments to what you have written previously.

At the end of class, these activity pages will be collected as a homework grade. All homework should be stapled in numerical order according to page numbers. If any CD problems were assigned, these are to be stabled at the back and according to their own numerical order.

Homework Grade

After discussing the activity pages during class, they will be collected to be graded based on correctness. Selected problems will be chosen to be graded and returned to you. Each set will be worth 20 points, and at least the lowest one will be dropped at the end of the semester. Late homework is not accepted. Hard copies of the activity pages can be turned in early. You may turn them into your instructor's office prior to class or send them with a classmate to class (due on arrival).

Class Attendance/Participation/Quizzes

Class attendance is *essential* for developing your and your classmates' understanding. Your active participation during class is also *integral* to your and your classmates' understanding. As a result, your attendance and participation will be graded daily. Your instructor will keep track of participation. The following rubric describes points to be awarded.

Class participation is graded on a scale from 0 (lowest) through 4 (highest), using the criteria below. The criteria focus on what you *demonstrate*—not your level of understanding. I expect the usual level of participation to satisfy the criteria for a "3".

Grade	In-Class Participation & Homework Criteria
0	Absent.
1	Present, not disruptive. Little or no homework preparation. Demonstrates very infrequent involvement in small-group or large-group discussions.
2	Demonstrates incomplete homework preparation. Does not offer to contribute to small-group discussion, but contributes to a moderate degree when addressed and generally offers comments that are straightforward and without elaboration. Demonstrates sporadic involvement and does not show evidence of trying to interpret or analyze class topics.
3	Demonstrates good homework preparation: making sense of material with some implications. Contributes well to small-group discussion in an ongoing way: responds to other students' points, thinks through own points, questions others in a constructive way, offers and supports suggestions that may run counter to majority opinion, and offers interpretations of material (more than just facts). Demonstrates consistent ongoing involvement in large-group discussion.
4	Demonstrates excellent preparation: making sense of material and comes ready with questions. Contributes in a significant way to small-group discussion: analyzes and synthesizes other students' points, thinks through own points, questions others in a constructive way, offers and supports suggestions that may run counter to majority opinion, and keeps group analysis focused. Contributes with active involvement in large-group discussion: responds very thoughtfully to other students' comments, puts together pieces of the discussion to develop new approaches that take the class further, contributes to the cooperative argument-building, suggests alternative ways of approaching material and helps class analyze which approaches are appropriate, etc.

If you are less than 5 minutes late to class, you will be penalized half a point; if you are more than 5 minutes late, you will be penalized a full point. Leaving early follows the similar point deductions.

Quizzes will be given irregularly and are generally unannounced. They serve to provide some feedback in your participation of understanding the material. Quizzes are worth 4pts and will contribute to the overall total of participation points for the semester. Quizzes will be graded on a 4 point rubric with 4 expressing excellent understanding, 3 expressing good understanding with some clarifications/adjustments needed, 2 expressing significant understanding is missing, 1 expressing a general lack of understanding.

Project Grade

During the semester, there will be some out of class projects assigned. Take them seriously but have fun and be creative! Projects will be accepted late for reduced credit up to one week after due date. Due dates for projects are to be announced when assigned.

Exam Grade

There will be three (3) examinations (50 min) and a comprehensive final examination. Students are required to provide *PRIOR* notice to the instructor if conflicts with test or exam days will occur. The more prior notice, the more likely permission will be granted. Without prior notice, make-up exams will not be offered. Dates are subject to change. Currently, they are fixed to be on the following Thursdays:

	Exam Dates
Exam 1	Thursday, 15 September
Exam 2	Tuesday, 18 October
Exam 3	Tuesday, 22 November

If your final exam is higher than one of your earlier test grades, it may replace your lowest test grade. *Everyone is required to take the final exam*. There will be no curving of examination scores. The final examination is scheduled as follows:

Class Time	**Final Exam Date and Time**
TR, 11:00 am	Thursday, December 15th from 11:00am to 12:50pm
TR, 4:30 pm	Thursday, December 15th from 5:30 – 7:20pm

Course Evaluation

Course grades will be determined according to the following distribution.

Attend/Participation/Quiz	9%
Homework	9%
Projects	12%
Exam 1	17%
Exam 2	17%
Exam 3	17%
Final Exam	19%
TOTAL	100%

Letter grades will be assigned according to the normal grading scale: 90% – 100% = A, 80% – 89% = B, etc.

TOTAL 100%

Academic Dishonesty/Misconduct

Working with another person or in study groups on problems can be helpful in learning the material. I encourage you to work together if you find it helpful. However, all written work submitted must be your own. Copying someone else's problem solution or allowing your written solution to be copied is prohibited. In order to be successful in learning the material and doing well on examinations you must think about the problems themselves before discussing them with anyone else. *The minimum penalty for an act of academic dishonesty will be the assignment of a grade of 0 on the examination or homework assignment*.

Special Accommodations for Students

The University ____________ complies with Section 504 of the Rehabilitation Act of 1973 and the American with Disabilities Act of 1990. Students with disabilities who need special accommodations must make their requests by contacting Disability Support Services, at ____________. The DSS Office is located in the __________, Room ____. Students should also notify the instructor of special accommodation needs by the end of the first week of class.

Student Information Sheet and Syllabus Attachment

Important information on university policies, including withdrawals, incomplete grades, university emergencies, weather information, final exam daily limits, course evaluations, phone numbers, and other miscellaneous information can be found by downloading the Student Information Sheet and Syllabus Attachment at (website masked)

School Closing Information

Students, faculty, and staff may call the _____ Closing Line at ______ or check the ___ Home Page at (website masked). You may also check with local media.

If a university emergency occurs that prevents the administration of a final examination, the student's final course grade will be calculated based on the work in the course completed to that point in time and the faculty member's considered judgment. Final exams will not be rescheduled, and a grade of "I" will not be given as a result of a missed exam.

Final Note: Any changes in this syllabus will be communicated to you in class by the instructor.

Appendix B: Mathematics Teaching Efficacy Beliefs Instrument

MATHEMATICS TEACHING EFFICACY BELIEFS INSTRUMENT
(MTEBI - Preservice)

Name___________________________________ Date:___________________

Please indicate the degree to which you agree or disagree with each statement below by circling the appropriate number to the right of the statement.

1	2	3	4	5
Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree

1. When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.	1	2	3	4	5
2. I will continually find better ways to teach mathematics.	1	2	3	4	5
3. Even if I try very hard, I will not teach mathematics as well as I will most subjects.	1	2	3	4	5
4. When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.	1	2	3	4	5
5. I know how to teach mathematics concepts effectively.	1	2	3	4	5
6. I will not be very effective in monitoring mathematics activities.	1	2	3	4	5
7. If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.	1	2	3	4	5
8. I will generally teach mathematics ineffectively.	1	2	3	4	5
9. The inadequacy of a student's mathematics background can be overcome by good teaching.	1	2	3	4	5
10. When a low-achieving child progresses in mathematics, it is usually due to extra attention by the teacher.	1	2	3	4	5
11. I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	1	2	3	4	5
12. The teacher is generally responsible for the achievement of students in mathematics.	1	2	3	4	5
13. Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.	1	2	3	4	5

14. If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.	1	2	3	4	5
15. I will find it difficult to use manipulatives to explain to students why mathematics works.	1	2	3	4	5
16. I will typically be able to answer students' questions.	1	2	3	4	5
17. I wonder if I will have the necessary skills to teach mathematics.	1	2	3	4	5
18. Given a choice, I will not invite the principal to evaluate my mathematics teaching.	1	2	3	4	5
19. When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.	1	2	3	4	5
20. When teaching mathematics, I will usually welcome student questions.	1	2	3	4	5
21. I do not know what to do to turn students on to mathematics.	1	2	3	4	5

Appendix C: MTEBI Scoring Instructions

- Step 1. Item Scoring: Items must be scored as follows: Strongly Agree = 5; Agree = 4; Uncertain = 3; Disagree = 2; and Strongly Disagree = 1.
- Step 2. The following items must be reversed scored in order to produce consistent values between positively and negatively worded items. Reversing these items will produce high scores for those high and low scores for those low in efficacy and outcome expectancy beliefs.

  Item 3 Item 17

  Item 6 Item 18

  Item 8 Item 19

  Item 15 Item 21

  In SPSSx, this reverse scoring can be accomplished by using the recode command. For example, recode ITEM3 with the following command: RECODE

  ITEM3 (5=1) (4=2) (2=4) (1=5)
- Step 3. Items for the two scales are scattered randomly throughout the MTEBI. The items designed to measure Personal Mathematics Teaching Efficacy Belief (SE) are as follows: Items 2, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, and 21. Items designed to measure Outcome Expectancy (OE) are as follows: Items 1, 4, 7, 9, 10, 12, 13, and 14.

  Note: In the computer program, DO NOT sum scale scores before the RECODE procedures have been completed. In SPSSx, this summation may be accomplished by the following COMPUTE command:

COMPUTE SESCALE = ITEM2 + ITEM3 + ITEM5 + ITEM6 + ITEM8 + ITEM11 + ITEM15 + ITEM16 + ITEM17 + ITEM18 + ITEM19+ITEM20+ITEM21

COMPUTE OESCALE = ITEM1 + ITEM4 + ITEM7 + ITEM9 + ITEM10 + ITEM12 + ITEM13 + ITEM14