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THE RELIABILITY OF INTERNAL ACCOUNTING CONTROL SYSTEMS: DESIGN AND ANALYSIS

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THE RELIABILITY OF INTERNAL ACCOUNTING
CONTROL SYSTEMS: DESIGN AND ANALYSIS

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THE RELIABILITY OF INTERNAL ACCOUNTING

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CHAPTER I

INTRODUCTION

In general there are two kinds of internal controls - administrative controls and accounting controls. There is no distinct demarcation between the two types of controls. In fact their functions are not mutually exclusive as can be seen from the following definitions by the AICPA (SAS No. 1 § 320.27 and § 320.28):

> Administrative control includes, but is not limited to, the plan of organization and the procedures and records that are concerned with the decision processes leading to management's authorization of transactions. Such authorization is a management function directly associated with the responsibility for achieving the objectives of the organization and is the starting point for establishing accounting control of transactions.
>
> Accounting control comprises the plan of organization and the procedures and records that are concerned with the safeguarding of assets and the reliability of financial records and consequently are designed to provide reasonable assurance that:
>
> a. Transactions are executed in accordance with management's general or specific authorization.
>
> b. Transactions are recorded as necessary (1) to permit preparation of financial statements in conformity with generally accepted accounting principles or any other criteria applicable to such statements and (2)

to maintain accountability for assets.

c. Access to assets is permitted only in accordance with management's authorization.

d. The recorded accountability for assets is compared with the existing assets at reasonable intervals and appropriate action is taken with respect to any differences.

The present study concerns accounting controls. Such controls play important roles in the financial information system. Yu and Neter (1973) describe them as follows:

> The primary purpose of incorporating a set of internal controls in the financial information system is to enhance the system's reliability- i.e., to maintain a high probability of preventing, detecting, and eliminating errors, irregularities and fraud in the financial information system.

It is well accepted that a more reliable control system provides a better quality of output. Recognizing this, the American Institute of CPAs has incorporated (AICPA 1973) an evaluation of the reliability of the internal controls as a requirement for attest audits. The reliability of the internal control system helps the external auditor determine the extent and nature of substantive audit tests. More reliable systems require less extensive substantive testing by the auditor.

The Institute of Internal Auditors has also recognized the importance of internal controls and has incorporated this concept in its definition (given below) of the nature of the responsibilities of the internal auditors (Brink, Cashin and Witt 1973):

> Internal auditing is an independent appraisal activity within an organization for the review of operations as a service to management. It is a managerial control which functions by measuring and evaluating the effectiveness of other controls.

Furthermore, it is evident from the objectives of internal auditing (Brink, Cashin and Witt 1973) that the main tasks of an internal auditor are to assist management to design effective and efficient internal control systems and evaluate independently their effectiveness and efficiencies, e.g., reliabilities:

> The objective of internal auditing is to assist all members of management in the effective discharge of their responsibilities, by furnishing them with analyses, appraisals, recommendations and pertinent comments concerning the activities reviewed. ... The attainment of this overall objective involves such activities as:
>
> - Reviewing and appraising the soundness, adequacy, and application of accounting, financial, and other operating controls, and promoting effective control at reasonable cost.
> - Ascertaining the extent of compliance with established policies, plans, and procedures.
> - Ascertaining the extent to which company assets are accounted for and safeguarded from losses of all kinds.
> - Ascertaining the reliability of management data developed within the organization.
> - Appraising the quality of performance in carrying out assigned responsibilities.
> - Recommending operating improvements.

A problem confronting an auditor, whether he is an external or internal auditor, is the evaluation of the reliability of an internal control system. A traditional approach is to use questionnaires, flow charts, and tests of transactions for evaluation purposes (Loebbecke and Zuber 1980, see also Felix 1981 for recent works). With the exception of statistical attribute sampling tests, these methods do not provide objective and quantitative evaluation; they depend on subjective judgments (Yu and Neter 1973). The main purpose of this study is to develop reliability models for internal control systems that would help analyse, compare and evaluate such systems in a more objective manner. Further elaboration on the purpose of this study is presented later in this chapter.

The remainder of this chapter is divided into four sections. The first section deals with the research issues and the statement of the problems associated with existing reliability theories as applied to internal accounting control systems. The second section descusses the purpose and significance of the study. The third section presents a review of the previous works that are pertinent to the present study. The fourth section summarizes the advantages of developing reliability models using the current approach.

A. Statement of the Problem

In recent years, interest in the study of internal accounting control systems using reliability theory has grown. The common trend seems to be to apply reliability results from engineering to internal control systems without being critical about the basic differences between the two systems (i.e. engineering systems[1] and internal accounting control systems). Such an approach may lead to problems. As stated by Mautz and Sharaf (1964, pp. 15-16):

> ... successful adoption of this kind requires an understanding of the subject's own problems as much as an understanding of the nature of the borrowed tools. Rarely are ideas and methods in other fields such that they can be accepted without some modification.

In an auditing context they (Mautz and Sharaf 1964, p. 16) also state that:

> ... in order to successfully adopt sampling techniques in audit verification, serious attention must be given to the nature of business data and the characteristics which differentiate them from the data of other fields of inquiry. Unless this principle is kept under continuous study in theory and observed in practice, more harm than benefit may result.

In the present context, Bodnar (1975) seems to have borrowed reliability results from engineering for application to internal accounting control systems with a paradoxical result. He states (Bodnar 1975, p. 756):

> ... fewer rather than more people (controls) increase reliability. This is in direct contrast to traditional views on internal control (dual control (see Sec. III.C for definition) in particular), and was difficult to defend as it goes against "common sense".

Bodnar believes that the product rule in probability theory is responsible for this paradox. However, I believe that a proper reliability theory which takes basic differences in accounting and engineering systems into account may, when developed exclusively for internal accounting control systems, provide results that are free from paradoxes. In the light of the statements by Mautz and Sharaf presented earlier, the reason for the paradox may not be the product rule as pointed out by Bodnar (1975), but an <u>improper</u> theoretical mapping of the engineering properties to the accounting control components.

The following list presents the important differences[2] between the properties of models borrowed to date from engineering and properties related to accounting control systems:

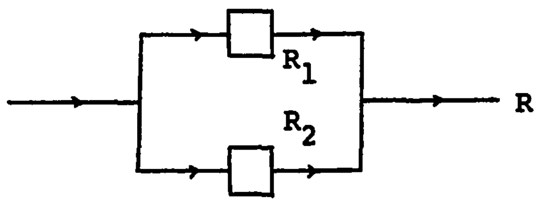
(i) In engineering systems[3], no direct human interaction is considered, whereas in an internal control system (ICS) human interaction may be present at any stage.

(ii) Elements of an engineering system[4] either work or do not work; they have no decision making capabilities (Barlow, Proschan and Hunter 1965, and Amstadter 1971). Whereas the elements of an ICS, not only work or not, but also have, in general, decision making capability (Cushing 1974).

(iii) The reliability characteristic of an engineering component is given by one parameter (for a given failure mode) which represents the probability that the component works properly (see any book on the mathematical theory of reliability for engineering systems, e.g. Barlow, Proschan and Hunter (1965) and Amstadter (1975)). It is also considered to be independent[5] of the input. Whereas, the reliability characteristic of an ICS element has to be given, in general, by several parameters which represent: (a) probability that the control element is in <u>operation</u>, (b) the probability that the element works properly given that it is <u>in operation</u> and the input information is correct, and (c) the probability that the element works properly given that it is in operation and the input information is incorrect (some of these concepts are discussed by (Cushing (1974) and Soliman (1979)). Furthermore these parameters, in general, depend on the state of the input information (Cushing 1974), unlike the reliability parameters of engineering components.

(iv) An engineering system[4], usually contains one kind of component, whereas an ICS contains two kinds of components: one that just completes a procedure without any decision feature and one that not only completes a procedure but also makes a decision based on the state of the input information (see chapter II for details).

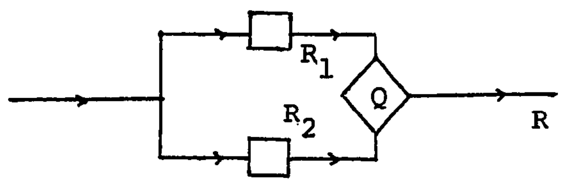
These differences have important implications for modeling and specifying reliability systems in the two domains (Engineering and Accounting). For example, a parallel switch circuit in an electronic system has the following reliability, R, that a current will flow through the unit:



$$R = 1 - (1 - R_1)(1 - R_2) = R_1 + R_2 - R_1 R_2$$

That is, output flow reliability, R is equal to unity less the chance, $(1 - R_1)(1 - R_2)$ that <u>both</u> parallel switches will fail to allow current to flow.

In the case of an ICS, an equivalent diagram might be:



where one channel is a redundant[6] channel and the reliabilities of the output information from the two channels are R_1 and R_2. For a control purpose the information from the two channels are compared and corrected (in case of a discrepancy) at the junction component (i.e. a component with two or more input channels). The control component at the junction is not considered in the current literature. Instead the reliability of this unit is taken from the engineering result as $R = 1 - (1 - R_1)(1 - R_2)$ (Barrett, Baker and Ricketts 1977, pp. 37-38). This approach implicitly assumes that: (a) the element at the junction works correctly with 100% accuracy when one or both pieces of the input information are correct, and (b) it does <u>not</u> work properly <u>at all</u> when both inputs are incorrect (see Section II.C.2.b for detailed discussion).

In general this will not be true in ICS's. The reliability of the output information, in this case, will not only depend on R_1 and R_2 but also on the reliability vector $\vec{Q}$ (see Section II.C.2.b) of the component at the junction. The consequences of this incongruity in reliability mapping properties have not been investigated fully. Their impact on the management, control and audit of accounting systems is unknown.

B. Purpose and Significance of the Study

The purpose of this study is to develop and discuss the basic concepts for internal accounting control processes and to use these concepts to develop a reliability theory for internal accounting control systems. First the basic components of an internal accounting control system will be identified and the corresponding reliability models will be developed. Second, the reliability models of more complex systems will be developed using the models of the basic components as building blocks. The potential utility of studying the sensitivity of internal accounting control systems to changes in design and/or reliability of components will also be addressed, from the perspectives of both internal and external audit objectives. Finally a field study will be reported to support empirically three representative theoretical models.

The study provides the formal explanation necessary to compare the relative reliabilities of alternative control system designs. It also offers the opportunity to compare the cost effectiveness of alternative control systems.

In addition, the study performs sensitivity analyses on the reliabilty models of two control systems in order to identify the most and the least important elements of those control systems, and to compare the reliabilities of alternative control system designs. A component is said to be important if a small increase in the value of its reliability parameter causes a relatively large increase in the output reliability in comparision to similar increases caused by the same percentage increase in the value of other component parameters.

The above information seems to be very useful for both internal and external auditors. The internal auditor can use these results to effectively manage time and resources to improve the output reliability. This can be achieved by improving reliability of the relatively more important components, by providing personnel training, and/or replacing machines. The external auditor can use the sensitivity results in making a decision about the required precision of reliability estimates for individual components when evaluating the overall reliability of a system. A more important component may need a more accurate estimation of its reliability to obtain a more precise evaluation of overall reliability. Sensitivity analysis can be used to target the most cost effective investigation and/or redesign strategies when resource constraint prevent involvement with more than one or a few individual system's component. The auditor can also investigate the potential for error in decisions related to the use of overall reliabilities by considering the sensitivity of such decisions to the precision of available component reliability estimates.

C. Background Research

There are two types of research works pertinent to the present study. First, empirical works that show that the traditional approach leads to incoherent decision outcomes. Second, studies that use quantitative models for analyzing and evaluating internal accounting controls.

In the first category of work, Weber (1978), while studying auditor's decision making on overall system reliability, concluded that the auditors did not carry out the assessment in a uniform manner. Joyce (1976), in an earlier study, also found that significant difference exists between individuals' judgments. There are several recent empirical studies (e.g. Ashton and Brown 1980, Weber 1980, and Mock and Turner 1981) that support the same view. Recently, Mock and Turner (1981), in a comprehensive empirical study of internal control evaluation and auditor judgment, came to a similar conclusion, i.e. the present approach (using questionnaires and interviews) for evaluating internal controls leads to incoherent audit judgments, as seen in the following excerpt (Mock and Turner 1981, p. 122):

> ... the actual sample size decision and rationale documentation exhibited a great deal of variability among auditors. Variability was observed in terms of factors that are inputs into auditor's sample size recommendations, including their interpretation of the nature (substantive, compliance, dual purpose) of the audit procedure, their judgments about appropriate alpha risk, beta risk, and materiality, the relevance of various internal control strengths and the amount of the reliance that they were willing to place on the compliance tested strengths, and their information search strategies, as evidenced in a protocol study.

Thus, it appears that the subjective nature of the present approach to internal control evaluation is an inherent weakness in the usual audit process. To eliminate some of the problems associated with subjective evaluation, researchers have explored more objective procedures. For example, one could consider mathematical models of internal control systems having different control 'components' with determinable reliabilities. These reliabilities of different control components could then be combined to yield the overall reliability of the system. However, there are two very basic questions that arise at this stage. The first one regards the process of objectively and cost effectively evaluating the reliability of an individual control 'component', and the second one concerns the methodology to be used in combining these individual reliabilities for determining the overall reliability of the system. While very little effort has been expended to answer the first question, a considerable amount of effort has been devoted to the second question. The research works pertaining to the latter question are summarized below.

1. Yu and Neter's Work

Yu and Neter (1973) developed a stochastic reliability model of an internal control system by treating the quality of accounting data as a stochastic variable and the flow of error states as a stochastic process. The model basically deals with the flow of error in an accounting information system. Each operating element of the system has some propensity to introduce, change and eliminate errors in the input information. This propensity depends on the error state of input information and on the characteristic of the operating element. Yu and Neter have expressed these propensities of an element to introduce, change and eliminate a given error in the input information in terms of probabilities.

They have considered two types of operating elements. One, where the element performs only a transformation operation i.e. it converts input data into output in accordance with certain rules. The other, where the element performs a decision operation. According to Yu and Neter, the decision operation performs basically a sorting function. It allocates information to various output classifications each of which requires a different action.

For elements performing transformation operations, the propensities to introduce, change and eliminate error are given by the different elements of a 'Transformation Probability Matrix,' T. An example using a 4 x 4 matrix is considered by Yu and Neter. In that example a payroll clerk performs a transformation operation on payroll data with four error state classifications: no errors, monetary error only, non-monetary error only, both monetary and non-monetary errors. The effect of the transformation operation is obtained by multiplying T with the 'input vector' (an 'input vector' represents the different probabilities with which the error states are present in the input information). The above multiplication yields the 'output vector' which represents the probabilities with which different error states are present in the output data.

For an element performing a <u>decision operation</u>, the propensities to introduce, change and eliminate errors are given by a 'Decision Probability Matrix,' D. This matrix, in fact, consists of two diagonal matrices: one for the correct decision and the other for the incorrect decision. The effect of decision operations on the input data can be obtained by multiplying D with the 'input vector'. The resulting vector is an 'output vector' which, in this case, consists of two vectors, one for the correct decision and the other for the incorrect decision.

Yu and Neter have also developed models for branching operations and merging operations. They have considered two types of merging operations. One, where similar documents representing separate transactions of different departments are merged together. The other, where different documents pertaining to the same transaction are combined to form one complete document. They have also modeled feedback operation for correction.

Thus Yu and Neter modeling provides an objective way to evaluate the reliability of output information from an internal accounting control system knowing the different probability matrices (T & D) of the operating elements. However, there are several weaknesses with their models that limit their applicability. These weaknesses are discussed below.

The assumption that different operating elements are independent is not valid in most real situations. Also in the case in which different input data are merged, the probabilities with which errors are present in the different input data are not necessarily independent of each other. For example, in a disbursement voucher preparation and review process, there are three input vectors. One for each of the following documents: 1) the Purchase Order (approved), 2) the Vendor's Invoice and 3) the Receiving Report. The three input vectors are not mutually independent; an error in quantity ordered in a PO (Purchase Order) may create an error in the quantity on the corresponding vendor's invoice (from the vendor's point of view there is no error, but the customer is not getting what he really wanted). Thus, the reliability model developed by Yu and Neter for an input vector which is comprised of several documents pertaining to the same transaction, is incomplete. The models developed in the present work incorporate these dependencies (see Section B of Chapter V for details).

Yu and Neter have also implicitly assumed that the propensities of an element to introduce, change and eliminate errors from input data are independent of the number of input channels. This assumption seems unrealistic because the operating element may not function the same way as it would in the single input case. Also it may use one channel's input to make its decision on the accuracy of the input data from the other channels. The present work has developed models for components with multi-channel inputs, and demonstrated the validity of some of the models by conducting a field study. (see Chapters II, III and V).

It is interesting to note that the element performing a decision operation (as defined by Yu and Neter) can never improve the reliability of the input data even if it were operating with 100% reliability. This means that the element will further contaminate the input data if it is not working correctly in every input case. It appears from this result that the system would be more reliable if such decision operations were omitted. This is contrary to the control objective.

2. Cushing's Work

A simple stochastic model adapted from the field of reliability engineering (Lloyd and Lipow 1962) was used by Cushing (1974) to develop reliability models of internal accounting control systems. Basically the model, like the Yu and Neter models, deals with the reliability of information as the data flow through a system of internal controls. The reliability of the output data is given in terms of the reliability of the input data and the reliability parameters of the control elements that process the input data. The parameters represent the probability that the element completes the control process correctly for given initial conditions.

Unlike Yu and Neter (1973), Cushing has considered two steps in the control task: one, where the control element signals the existence of an error of a given type and the other, where the element initiates a proper action for a given error signal so that the output data is error free. Cushing has further considered that the probabilities of completing the above tasks depend on the states of the input data. More explicity, the probability that the control component signals the existence of an error in the input data given that the error exists (correct functioning of the signaling process) is different from the probability that the component signals the existence of an error in the input information given that no error exists (improper operation of the signaling process). Also, when the existence of an error is signaled, the probability of taking proper actions so that the output information is error free is different in the two cases: 1) when the input contains no error and 2) when the input contains an error.

It is interesting to note that the Cushing model is equivalent to the Yu and Neter model for the simple one input case. This can be demonstrated by combining non-monetary and monetary errors in the Yu and Neter model into one type of error and by replacing the two step control process (signaling and correcting) in the Cushing model by a one step process (detection-correction). When the Cushing model is rewritten in a matrix form, it takes the same form as that of the Yu and Neter model.

Cushing also developed reliability models for control systems, representing (a) single control-single error, (b) single control-multiple error, (c) multiple control-single error and (d) multiple control-multiple error. He also discussed several extensions of the model and incorpcrated cost-benefit analysis. The cost-benefit analysis provides a decision tool to management and/or the internal auditor. This helps them in making decisions about installing and/or improving their control system.

While Cushing's work provides reliability models for simple systems, his approach of using decision trees to develop models becomes very cumbersome in complex cases,e.g. control components with multi-channel inputs, control components in parallel and etc. Cushing did not develop models for these complex cases. A set theoretical approach is used in the present study to develop models. This approach is more versatile. It provides models for any complex system even with interdependent control components and input information channels (see Chapters II, III and V).

3. Bodnar's Work

Bodnar (1975) made an attempt to extend Cushing's approach to develop reliability models for internal control systems with more complex structures, by considering human behavior in the performance of control tasks. He considered two types of systems. One where the control tasks are completed by two separate channels (each channel consisting of several components in series), with one being redundant. The other, where the control tasks are performed in a 'dual' form. Each 'dual' element consists of two components in parallel; each performing a similar task (i.e. one is redundant). The 'dual' elements are connected in series to form a 'dual' system.

The reliability model for a 'parallel' system, as given by Bodnar, is $R_p = 1 - (1 - r^n)^2$ where R_p is the system's reliability, r is the component's reliability and n represents the number of components in each channel. All the components are assumed to be identical in terms of their reliabilities. For a 'dual' system, the reliability is $R_d = \left[1 - (1 - r)^2\right]^n$ where n represents the number of 'dual' elements. Again, all the components are assumed to have the same value, r, for their reliabilities.

It is seen that R_p and R_d both tend to zero as $n \rightarrow \infty$.

This leads to the paradoxical conclusion that fewer rather than more people (controls) increase reliability. Bodnar recognized this paradox. He explained it partly by the product rule in probability theory and partly as a result of human behavior (i.e., it is easier for a lesser number of people to collude). He pointed out as a result of his study that effort is warranted to reduce the number of work stages in order to have higher reliability. Also, since his results led to a paradoxical conclusion, he asserted that (Bodnar 1975, p. 756): "... in many cases, the results of reliability modeling may not correspond to common beliefs held by management."

Bodnar did not discuss explicitly his method of developing models. As mentioned earlier, it appears that he has borrowed the reliability results from engineering for similarily structured systems. This may pose some problems, especially when the two systems (internal control system and engineering system) have the same structure (component wise) but differ in their logic flow. There are situations where the system has a series structure but the logic is parallel and *vice versa*. For example, a set of capacitors in parallel will have a series logic for the set to function i.e. if any capacitor fails (shorts), all of them fail. Thus, it is important that before a result is borrowed from other disciplines based on system's structures, the logic flows of the two systems be studied. The difference in the logic should then be implemented in the model. In fact, a more direct approach would be to look for logic similarity between the two systems when borrowing results.

In the present work, logic flow technique is used to develop the models. Unlike Bodnar's result, this approach has yielded results that are in accordance with intuition and common sense.

4. Other Relevant Works

Stratton (1977, 1981) and Soliman (1979) have also extended the application of reliability theory to more complex systems. Their treatments again lack discussion and inclusion of all the control features discussed in Chapter I, Section A. Soliman (1979) starts with the basic concepts of a control process in a control system, but when he derives the reliability of a complex system he treats the control components as if they were engineering components in the same manner as Bodnar (1975) and Stratton (1980), i.e. he fails to incorporate the different reliability parameters associated with a control component into the reliability models of complex control systems. For example he does not recognize the probability that a control component operates correctly given that the input information is <u>correct</u> and the control is in operation, P_c, and the probability that a control component operates correctly given that the input information is in <u>error</u> and the control is in operation, P_e. Rather, he uses a single parameter (which is independent of the state of the input information) for the reliability of a control component in the treatment of complex systems (see Soliman 1979, PP. 37, 43 and 46).

Hamlen (1980) has proposed a linear programming model for the design of an internal control system, which minimizes a system's cost subject to management established error reduction probability goals for designated error types. However, she does not consider the basic problem of combining the reliabilities of the individual components.

Vasarhelyi (1980) while examining the nature and multiplicity of internal control procedures and errors, briefly mentioned the complexities and difficulties involved in combining the reliabilities of internal control components. He classified the relationships of different components of an internal control system into five groups and suggested that an empirical study be conducted in order to answer the question of how to combine the reliabilities of the components to find the overall system's reliability. In fact, he has given some possible analytical relationships for combining the reliabilities when the controls are connected in series and in parallel, but has provided no theoretical justification for the relationships.

Baber (1980 and 1981) developed decision theory models to determine policies regarding implementation of parallel processing accounting controls and investigation of the true value of a transaction when a control is (or is not) in operation. He investigated the nature of optimal policies under various assumptions and special cases. While Baber's study is useful in formulating control policies by management, it does not provide a method to combine reliabilities of different control elements to obtain an overall reliability of a control system; nor does it yield any information about the overall accuracy of an account balance. These omissions are of significance to the external auditor when making an audit plan, and to the manager and/or the internal auditor when planning and designing a more reliable control system. Although the cost-benefit of a control system is an important point to consider when deciding about implementing the controls in an accounting system, it is even more important to know how a given control system, with a fixed operating cost, can be made more efficient by rearranging the different elements of the control system.

Knechel (1981) has studied the relationship between processing errors and aggregate errors in the financial statement accounts. He determined the distribution of aggregate errors under a variety of conditions, e.g. the presence or absence of certain internal controls and a normal or uniform distribution of processing errors. In his study, he also illustrates the use of simulation techniques for developing these distributions. He simulated the errors in a revenue system to demonstrate the effect of controls on the aggregate error distribution, for the analysis and evaluation of internal controls. Knechel used Cushing's (1974) reliability models of internal controls, for studying the effects of internal controls on the distribution of aggregate errors. Since Cushing's reliability models are limited to simple control systems, reliability modeling of complex systems seems warranted in order to use Knechel's approach in complex internal control situations.

Recently Weber (1982) extended Cushing's approach using reliability theory to develop probability models of internal control systems for asset safeguarding and maintenance of data integrity. However, his treatment is again limited to very simple cases.

From the above discussion, it is evident that considerable effort has been expended to develop quantitative models for evaluating the reliability of a control system. However, very little coherence exists between different works. The main reason seems to be a lack of formal concepts associated with the control process.

D. Advantages of Reliability Theory

There are several advantages from applying reliability theory to internal accounting control systems. Some of these advantages are common to all the theoretical approaches that provide an objective estimation of the reliability of a system; and some are more specific to the particular approach. In the following list, I present the specific advantages of the present approach along with some of the general advantages.

(i) The present approach is more complete and robust compared to previous approaches. Here, the basic concepts of a control process are developed in an accounting control environment and are used to develop reliability models for the basic systems' components that can exist in any accounting control system.

(ii) The present work uses information flow diagrams with proper logic for completion of the control process in developing reliability models. The information flow can be easily drawn by knowing the flow of information in the system. In many instances such diagrams are already in existance (see e.g. Brink, Cashin, and Witt 1973).

(iii) The present approach is easier to understand. It requires only an understanding of the control process and some elementary knowledge of probability theory.

(iv) In the present approach, reliability models of complex systems can be easily obtained by combining the reliability models of the basic components of a control system.

(v) In general, the reliability parameters of a system's components are not assumed to be independent of each other in the present work. For example, the reliability parameters P_c and P_e (see Table I for definitions) of a control component depend on the state of the input information. Whereas, in the previous works (e.g. Bodnar 1975, Stratton 1977, and Soliman 1979) the reliabilities of different components are assumed to be independent.

(vi) An external auditor should be able to use the reliability models for an objective evaluation of the reliability of an internal accounting control system when making an audit decision.

(vii) Reliability models can be useful in designing a reliable and cost effective internal accounting control system. A configuration change (a change in the arrangement of a system's components) of a system changes the reliability of the system. The effects of such changes on the reliabilty of output can be studied in advance using the reliability models developed here. Such an analysis is presented in Section B of Chapter IV.

(viii) Sensitivity analysis of a reliability model of an internal accounting control system can provide information useful to both the external and internal auditors (see Chapter IV for details).

(ix) Variance analysis can be performed on different reliability parameters. This helps management determine those parameters that are not up to standard.

(x) The field study experience demonstrated that the reliability parameters can be easily evaluated from past data.

(xi) The present work uses limiting cases which yield intuitively appealing results to demonstrate the validity of the models.

(xii) The models developed in previous works are special case of more general models developed here (e.g. see Chapter III).

CHAPTER II

DEFINITIONS, CONCEPTS AND RELIABILITY MODELS OF BASIC COMPONENTS

This chapter is divided into two sections. The first section presents the definitions and concepts associated with the terms that are used in this work in developing reliability models. The second section is devoted to the development of reliability models of the basic components of an internal accounting control system. These models are then used later in the work (see Chapter III) as building blocks to develop reliability models of complex systems.

A. Definitions and Concepts

Definitions and concepts occupy a key position in any theoretical development. Mautz and Sharaf (1964) have emphasized that concepts provide a basis for advancement in a field of knowledge by facilitating communication about a subject matter and its problems. Thus, with this view in mind, I begin by defining the terms that are associated with the internal control process and then discuss the related concepts.

System: A system normally would consist of several components (see below); each performing some type of activity (or types of activities) in the completion of a control process. An internal accounting system will be referred to, here, as a system.

System Component: A system component is an element (e.g. module, procedure, task) of a structural set of activities related to accomplishment of end results. Basically there are two kinds of system components: Performance components and control components.

Performance Component: A system component whose inherent purpose contributes directly to completion of a substantive accounting objective.

Control Component: A system component whose inherent purpose is prevention, detection, and/or correction of errors.

A 'performance' component is similar to an electronic component in an electronic device. Failure of an electronic component will cause failure in the electronic device. Similarly, failure of a performance component will introduce substantive contamination in information as it leaves the component. Once the contamination is introduced, only a control component can detect it and/or correct it.

A 'control' component is very different; it has no common analog in electronic systems. As mentioned earlier, control components are used to prevent errors, if present, in input information. A failure of a control component may arise from two sources: one when the component is not working properly i.e., improper performance or operative failure, and the other when the component is not functioning at all - inoperative failure. Operative failure of a control component will create and/or introduce substantive contamination. Whereas, inoperative failure produces a condition favorable to substantive contamination but will not always create and/or introduce error in the output information. In other words, when a control component is not working at all, then the prevention, detection, and/or correction of error is not possible by that component but the output information is not further contaminated as a result of exposure to the component.

However, when a control component is working, but not working properly (operative failure), it may introduce and/or create substantive contamination by making wrong judgments. For instance, a control component with a single input information channel can contaminate the output information by detecting and subsequently correcting an 'error' that is not present in the input information. Operative failure may also arise from improper correction of properly detected errors in input information or by accepting an incorrect input information as a correct one. The situations under which an operative failure occurs become more complex when the control component has two or more input information channels. A further discussion on this can be found in Sections C.2.b and C.2.c of Chapter II.

Reliability of Information: The probability that the information is error free for a given substantive objective. This reliability value may change when the substantive objective is changed.

Reliability of a System Component: The probability that a particular component completes its process properly for a given control objective. In the case of a 'performance' component, the reliability is expressed by one parameter. However, in the case of a 'control' component, the reliability is expressed by several parameters (Cushing 1974). These parameters are as follows:

(i) The first parameter of the reliability 'vector' of a 'control' component for a given control objective represents the probability that the control is in operation. When the control is not in operation, then the prevention, detection and/or correction of error is not possible by that control component. Furthermore, the control component will not create and/or introduce substantive contamination into the input information if it is not in operation.

(ii) The second parameter represents the probability that the 'control' component operates properly given that the control is in operation and the input information is correct.

(iii) The third parameter represents the probability that the 'control' component works properly given that the control is in operation and the input information is incorrect (see Table I for details).

A list of definitions of the reliability parameters is presented in Table I along with the symbols used for those definitions for different situations.

TABLE I

List of Symbols* Representing Characteristic Probabilities

I. Characteristic Probabilities of 'Performance' Components

A. For Single Channel Input Case:

$P = P(S_p)$ - Probability that a 'Performance' component completes its process correctly.

$P_j = P(S_{pj})$ - Probability that jth performance components completes its process correctly.

$P_{off} = P(S_{off}/\bar{S}_p \cap \bar{S}_i)$ - Probability that the error in the performance process offsets the error in the input information given that the input is incorrect and the performance component is operating improperly.

B. For Multi-Channel Input Case:

$Q = P(S_p)$ - Probability that a performance component completes its process correctly when multiple input information channels are present.

$Q_j = P(S_{pj})$ - Similar to Q except it is for the jth performance component.

II. Characteristic Probabilities of 'Control' Components

A. For Single Channel Input Case:

$P_w = P(S_w)$ - Probability that a control component works, i.e. remains in operation during a control period when only one input channel exists.

$P_c = P(S_c/S_i \cap S_w)$ - Probability that a control component operates correctly given that the input information is correct and the control is in operation.

$P_e = P(S_c/\bar{S}_i \cap S_w)$ - Probability that a control component operates correctly given that the input information is in error and the control is in operation.

* The symbols as defined in this list stand for one type of error.

TABLE I (CONTINUED)

$\vec{P}$ - Probability vector for a control component with one input channel. It has three elements: P_w, P_c, and P_e defined above.

P_{wj} - Similar to P_w except it is for the jth control component.

P_{cj} - Similar to P_c except it is for the jth control component.

P_{ej} - Similar to P_e except it is for the jth control component.

$\vec{P}_j$ - Similar to $\vec{P}$ except it is for the jth control component.

B. For Multi-Channel Input Case:

$Q_w = P(S_w)$ - Similar to P_w except number of input information channels is more than one.

$Q_c = P(S_c/S_{i1} \cap S_{i2} \cap . . . \cap S_{in} \cap S_w)$ - Similar to P_c but with multiple input channels.

$Q_{1e} = P(S_c/\bar{S}_{i1} \cap S_{i2} \cdots \cap S_w) = P(S_c/S_{i1} \cap \bar{S}_{i2} \cap \ldots S_{in} \cap S_w) = P(S_c/S_{i1} \cap S_{i2} \cdots \cap \bar{S}_{in} \cap S_w)$ - Probability that a control component operates correctly given that one of the input channels contains an error in its information.

. . . .

Q_{ke} - Probability that a control component operates correctly given that k of the input channels contain errors in their information. k may take integer values ranging from 1 to n where n represents the total number of input channels.

$\vec{Q}$ - Probability vector for a control component with more than one input information channel. Its elements are : Q_w, Q_c, Q_{1e}, ..Q_{ne} where n represents the total number of input channels.

Q_{wj} - Similar to Q_w except it is for the jth control component.

Q_{cj} - Similar to Q_c except it is for the jth control component.

TABLE I (CONTINUED)

Q_{1ej} - Similar to Q_{1e} except it is for the jth control component.

. . . .

Q_{kej} - Similar to Q_{ke} except it is for the jth control component.

$\vec{Q}_j$ - Probability vector for a control component at the jth junction in a 'dual' system. The elements of this vector are: Q_{wj}, Q_{cj}, Q_{1ej} and Q_{2ej}.

III. Characteristic Probabilities of Input and Output Information:

$R_i = P(S_i)$ - Reliability of the input information to a component. In other words it represents the probability that the input information is free from a particular error being considered.

$R_o = P(S_o)$ - Reliability of the output information from a component, a subsystem or a system.

$R_{ik} = P(S_{ik})$ - Reliability of the input information of the kth channel where k may take integer values ranging from 1 to n, n being the total number of input information channels in parallel.

R_{oj} - Reliability of the output information from the jth component.

$\mathcal{R}_{oj}$ - Reliability of the output information from the jth dual element.

' - Prime is used as a superscript on most of the symbols listed above when the component is connected in parallel with another component.

B. General Approach to Quantitative Development of Reliability Models for the Basic Components

The main purpose of this section is to present a discussion on the method that has been used in the present study to derive reliability models of the basic internal accounting control components. The present approach is a little different from previous approaches. Cushing (1974) used a decision tree approach whereas Stratton (1977 and 1981) and Soliman (1979) used logic diagrams. The present work uses a set theoretical approach to develop the logic and uses probability theory for obtaining reliability models. The following steps illustrate the procedures to obtain a reliability model for an assumed control objective:

(i) Identify the error type of concern.

(ii) Draw a logic flow diagram of the system, subsystem or a system component showing the component(s) and the inputs and outputs of the unit.

(iii) Identify the function (functions) of the component (components of a system or a subsystem) under study.

(iv) Write down the basic states of the input information (i.e. correct or incorrect).

(v) Write down the basic operational and functional states of the system components.

(vi) Write down the state of the output information that produces correct output.

(vii) Determine all feasible combinations of input information (given in (iii)) with the operational and functional states in (iv) that produce correct output information rather than erroneous output. This step is important, because a clear understanding of the process is required in order to determine all the possible mutually exclusive combinations.

(viii) Use probability algebra to transform the above relation between different states to a relation between different probabilities of the states. This provides the desired reliability model. It should be pointed out that in order to understand the above procedure, one does not need a detailed understanding of probability theory. An understanding at an introductory level is sufficient. The following theorems are most frequently used in the model development process here, and hence it would be necessary to understand these theorems (Meyer, 1970).

(a) <u>Sum Rule</u> - If an 'event' D is expressed in terms of the mutually exclusive 'events' A, B, and C as $D = A \cup B \cup C$ then $P(D) = P(A \cup B \cup C) = P(A) + P(B) + P(C)$ where P stands for the probability of the 'event' in the argument, and $\cup$ represents <u>or</u> as used in set theory.

(b) <u>Product Rule</u> - If an event D is expressed in terms of the 'events' A, B, and C as

$D = A \cap B \cap C$ then

$P(D) = P(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$

where $\cap$ stands for 'and'. The 'events' A, B, and C are not necessarily independent.

The above approach is used in deriving the reliability models for specific components as presented in the next section.

C. Reliability Models of Basic Components

Here, the reliability models of all the basic elements of an internal control system are developed. In the previous section I discussed two types of components: 'performance' components and 'control' components. Both types of components are essential parts of any internal accounting control system.

In order to develop reliability models, it is very important that we understand the control process and the function of each of the system components for a given objective. The following discussion presents reliability models for the basic system components.

1. Performance Component

As defined in the previous section, a performance component should treat incoming information in a manner consistent with a given substantive objective. Performance components have no decision making capability and therefore their reliabilities are represented by single parameters. In this section, I derive and discuss the reliability models of performance components for the following cases.

a. Performance Component with Single Input Information Channel

An example of such a component is the process of preparing a purchase order after receiving an approved purchase requisition. The input, in this example is the approved purchase requisition, the process is the preparation of the purchase order and the output information is the completed purchase order.

The relation between the reliabilities of the input and output information and the reliability of the performance component (A schematic representation of a performance component will be given by the symbol □ ; see Fig. 1) is given by

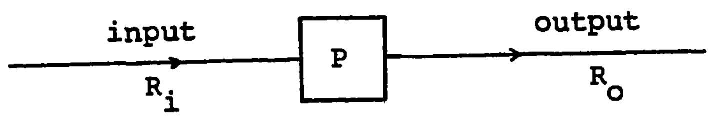


Figure 1: Reliabilities in Relation to a Performance Component.

the following proposition.

<u>Proposition</u> 1: The reliability, R_o, of output information from a performance component is given in terms of the reliability, R_i, of the input information, the reliability, P, of the performance component, and the probability, P_{off}, that the performance process offsets the input error, as:

$$R_o = R_i P + (1 - R_i)(1 - P)P_{off} \qquad \text{(II-1)}$$

<u>Proof</u>: Consider the following 'events':

S_i - The input information is correct.

S_p - The performance component completes its process properly.

S_{off} - The error in the performance process offsets the error in the input information.

S_o - The output information is correct.

The correct state of the output information for a given control objective can be given in terms of the following relation:

$$S_o = (S_i \cap S_p) \cup (\bar{S}_i \cap \bar{S}_p \cap S_{off}) \qquad \text{(II-2)}$$

where $\bar{S}_i$ and $\bar{S}_p$ are opposite 'events' to that of S_i and S_p, respectively. The set theoretical notation is used for conjunction. The above equation means that output information is correct if the input is correct <u>and</u> the performance component completes its process correctly, <u>or</u> if the input is incorrect <u>and</u> the performance component completes its process incorrectly <u>and</u> the error in the performance process offsets the error in the input.

Writing Eq.(II-2) in probability form by using the product rule one obtains:

$$P(S_o) = P(S_i \cap S_p) + P(\bar{S}_i \cap \bar{S}_p \cap S_{off}) =$$

$$P(S_i)P(S_p/S_i) + P(\bar{S}_i)P(\bar{S}_p/\bar{S}_i)P(S_{off}/\bar{S}_p \cap \bar{S}_i) \qquad \text{(II-3)}$$

where P stands for the probability with which the 'event' in the argument occurs, $P(S_p/S_i)$ represents the conditional probability of 'events' S_p given that 'event' S_i occurs and $P(S_{off}/\bar{S}_p \cap \bar{S}_i)$ represents the conditional probability of events S_{off} given $\bar{S}_i$ and $\bar{S}_p$. It is assumed[7] here that S_p is independent of S_i; i.e., the accurate application of the process does not depend on the state of the input information. In other words, the likelihood of proper performance is not dependent on whether input is correct. This independence between S_p and S_i leads Eq. (II-3) to:

$$P(S_o) = P(S_i)P(S_p) + P(\bar{S}_i)P(\bar{S}_p)P(S_{off}/\bar{S}_i \cap \bar{S}_p) \qquad \text{(II-4)}$$

or in terms of the symbols defined in Table I, one gets:

$$R_o = R_i P + (1 - R_i)(1 - P)P_{off} \qquad \text{(II-5)}$$

Q.E.D.

The first element of this result suggests that the reliability of the output information is further contaminated due to the errors committed in completing the task or procedure at the performance component. However, the second term in Eq.(II-5) represents an increase in reliability of the output information due to the offsetting effect of the error in the performance process. The probability, P_{off}, that the error in the input is offset by the error in the performance process is usually so small that the net effect of the second term in Eq.(II-5) on R_o is neglegible. This offsetting process <u>will not be considered in remaining cases</u> because the reliability models would become too complex without much added precision. If the offsetting process is considered to be important in some individual case then the process can be easily incorporated in the model in the same manner as done in the present case.

For P=1, i.e., when the performance component completes its procedure correctly in every input case, the reliability of the output is then simply equal to R_i. This is what one expects on an intuitive basis; the reliability of the output is equal to the reliability of the input if no further contamination is added to the input information by a performance component.

In the case of a perfectly reliable input (R_i=1), the reliability, R_o, of the output is equal to the reliability parameter P of the performance component. Furthermore, for R_i=1, and P=1, one obtains R_o=1 which implies that when a performance component is 'perfect' and the input information is error free then the output contains no error. Again, these results are intuitively appealing.

b. Performance Component with Two Input Information Channels and no Redundancy

Performance components with two input information channels with no redundancy are quite common in accounting systems. An example of such a component is a person who computes the total cost of inventory received, based on quantity counts provided by one source and prices obtained from another source. In this case the two inputs are not redundant. A schematic representation of such a component is given in Figure 2.

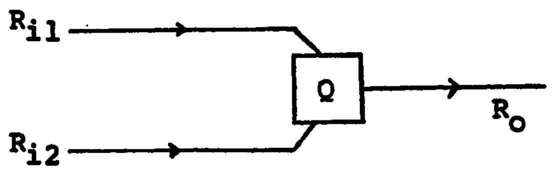


Figure 2: A performance component with two inputs and no redundancy.

In Fig. 2, R_{i1}, R_{i2} and R_o, respectively, represent the reliabilities of the two inputs and the output. The reliability parameter of the performance component is given by Q (see Table I for definition). The reliability model of this component can be given in terms of the following proposition.

<u>Proposition</u> 2: The reliability R_o of output information from a performance component of reliability parameter Q is given by

$$R_o = R_{i1}R_{i2}Q \qquad \text{(II-6)}$$

Here, Q is assumed to be independent of R_{i1}, and R_{i2} i.e. the probability, Q of performing a procedure by the performance component is independent of the state of the input information[8].

<u>Proof</u>: Consider the following 'events':

S_{i1} - The input information in channel one is correct.

S_{i2} - The input information in channel two is correct.

S_p - The performance component completes its process properly.

S_o - The output information is correct.

The state, that the output information is correct for a given performance objective can be written in the following form (using set theoretical notations):

$$S_o = S_{i1} \cap S_{i2} \cap S_p \qquad \text{(II-7)}$$

The above relation implies that the output information is correct when the two inputs are correct and the performance component is completing its process correctly.

Using the product rule and writing Eq. (II-7) in probability form one gets:

$$P(S_o) = P(S_{i1})P(S_{i2}/S_{i1})P(S_p/S_{i1} \cap S_{i2}) \qquad \text{(II-8)}$$

$P(S_{i2}/S_{i1})$ and $P(S_p/S_{i1} \cap S_{i2})$, respectively, represent the conditional probabilities of occurance of S_{i2} given S_{i1} and of S_p given S_{i1} and S_{i2}. As discussed earlier, it can be assumed that S_{i1}, S_{i2} and S_p are mutually independent. Thus Eq. (II-8) becomes:

$$P(S_o) = P(S_{i1})P(S_{i2})P(S_p) \qquad \text{(II-9)}$$

Rewriting this in terms of the symbols defined in Table I, one obtains:

$$R_o = R_{i1}\ R_{i2}Q \qquad \text{(II-10)}$$

Q.E.D.

Equation (II-10) suggests that if two pieces of information are combined by a performance component to produce an output, then the reliability of the output is the product of the three reliabilities. When $R_{i1} = R_{i2} = 1$, then $R_o = Q$. This means that, in the case of error free inputs, the reliability of the output information is equal to the reliability parameter of the performance component.

When one of the channels provides error free information (i.e. either $R_{i1} = 1$ or $R_{i2} = 1$) and the performance component is operating correctly all the time ($Q = 1$), then the reliability of the output information is equal to the reliability of the input information of the channel containing the contaminated information. For example, if the information in channel one is error free (i.e. $R_{i1} = 1$) and $Q = 1$ then from Eq.(II - 10) R_o equals R_{i2}. This is what one expects in such cases.

c. Performance Component with n Parallel Input Channels and no Redundancy

Here, a generalized reliability model of a performance component with n parallel input channels is developed. It is assumed that all the input inforamtion channels are non-redundant. The reliability of the output information from such a component (see Fig.3.) is given by the following proposition:

<u>Propostion</u> 3: The reliability, R_o, of the output information from a performance component with n input channels is equal to the product of all the reliabilities of the input information from the different channels and the reliability parameter of the component:

$$R_o = \prod_{j=1}^{n} R_{ij} Q \qquad \text{(II-11)}$$

where R_{ij}'s and Q are defined in Table I.

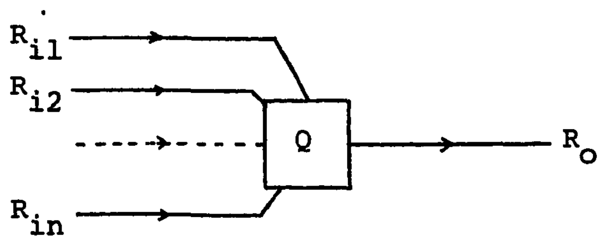


Figure 3: A performance component with n input non-redundant channels.

<u>Proof</u>: Consider the following 'events':

S_{i1} - The input information in channel one is correct.

S_{i2} - The input information in channel two is correct.

. . .

. . .

S_{in} - The input information in channel n is correct.

S_p - The performance component completes it process correctly.

S_o - The output information is correct.

One can write the state, S_o that the output information is correct for a given control objective, as follows:

$$S_o = S_{i1} \cap S_{i2} \cap S_{i3} \cap \ldots \cap S_{in} \cap S_p \qquad \text{(II-12)}$$

This relation implies that the output information is correct only when all the inputs are correct and the performance component is functioning properly, i.e. it is completing its process correctly.

It should be clear that the 'events' S_{i1}, $S_{i2} \ldots S_{in}$ are mutually independent because they each represent the correct state of the input in the corresponding channel where each channel is independent of the other channel. Also, the 'event' S_p that the performance component completes its process correctly, can be assumed to be independent of the states of the inputs with the same argument as the one presented in Sec.II.C.la. Thus, with the above assumptions, one can rewrite Eq.(II-12) in terms of the probabilities of occurrance of the corresponding 'events' as follows:

$$P(S_o) = P(S_{i1})P(S_{i2})...P(S_{in})P(S_p) \qquad \text{(II-13)}$$

In terms of the symbols defined in Table I, one can express the above equation as:

$$R_o = R_{i1}R_{i2}....R_{in}Q \qquad \text{(II-14)}$$

or

$$R_o = \prod_{j=1}^{n} R_{ij}Q \qquad \text{(II-15)}$$

Q.E.D.

The above result is a general result for n parallel nonredundant channels where n can take any integer value. Special cases can be obtained by substituting different values for n. For instance, n = 1 and n = 2 give us the reliability results for single channel and two channels cases, respectively (see Eqs. II -5 and II -10).

From Eq.(II-15), one finds that the reliability, R_o of the output information decreases as the number of channels increases for all $R_i < 1.0$. This means that if an additional non-redundant information channel is used in completing a process, the output is further contaminated due to the error in the additional information, even though the process was completed correctly.

2. Control Component

A control component constitutes the most important element of a control system; when working properly it reduces the error of a particular kind in the output information. As discussed earlier control components, unlike the performance components, have decision making capabilities. This property makes the reliability modeling of control components more difficult. This difficulty increases with the increase in the number of parallel redundant information channels. The following discussion presents the different reliability models for different cases.

a. A Control Component with One Input Information Channel

An example of a control component with one input information channel is a person approving a purchase requisition. The input information is an unapproved requistion. In general, such an element will be characterized by three reliability parameters: P_w, P_c and P_e (see Table I for definitions). A schematic diagram of a control element will be represented by the symbol ◇ (see Fig. 4).

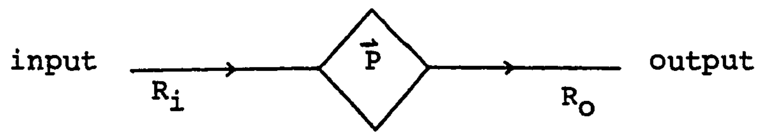


Figure 4: A control component with one input channel.

In Fig. 4, R_i and R_o represent the reliability of the input and the output information respectively, and vector $\vec{P}$ represents the reliability vector for the control component having P_w, P_c and P_e as the three components (see Table I for definitions). The following proposition presents the relationship between the different reliabilities mentioned in this case.

Propostion 4: The reliability R_o of output information from a control component with reliability parameters: P_w, P_c and P_e is given by

$$R_o = R_i[1 - P_w + P_w(P_c - P_e)] + P_eP_w \qquad \text{(II-16)}$$

for a given control objective. The reliability R_o applies to a particular control objective.

Proof : Consider the following 'events':

S_i - The input information is correct.

S_w - The control component is in operation.

S_c - The control component operates properly.

S_o - The output information is correct.

The state of the output information depends on the state of the input information and the subsequent operation of the control element. Thus using set notation, one can write S_o in terms of the following mutually exclusive and exhaustive sets:

$$S_o = (S_i \cap \bar{S}_w) \cup (S_i \cap S_w \cap S_c) \cup (\bar{S}_i \cap S_w \cap S_c) \qquad \text{(II-17)}$$

The above equation says that the output information is correct if the input is correct and the control is not in operation (inoperative failure) or the input is correct and the control is in operation and the control operates properly or the input is incorrect and the control is in operation and the control operates properly.

Writing Eq. (II-17) in probability form, one gets:

$$P(S_o) = P(S_i)P(\bar{S}_w) + P(S_i)P(S_w)P(S_c/S_i \cap S_w) + P(\bar{S}_i)P(S_w)P(S_c/\bar{S}_i \cap S_w) \qquad \text{(II-18)}$$

Here, it is assumed that S_w is independent of S_i, i.e. whether the control operates does not depend on the state of the input information[9]. In other words, the control is as likely to function in an input error circumstance as in a no input error circumstance. It is also assumed that S_c, the proper functioning of a control depends on S_i. This assumption implies that the two conditional probabilities: $P(S_c/S_i \cap S_w)$ and $P(S_c/\bar{S}_i \cap S_w)$, in general, will not be equal, i.e. $P_c \neq P_e$ (in terms of the symbols defined in Table I). Although the above assumption ($P_c \neq P_e$) is still subject to empirical test, it seems intuitively appealing. A control component, espcially a person, may not work with the same 'alertness' in the two circumstances (when the input is correct and when it is not correct).

In terms of the symbols defined in Table I, Eq. (II-18) becomes:

$$R_o = R_i(1 - P_w) + R_iP_wP_c + (1 - R_i)P_wP_e \qquad \text{(II-19)}$$

Rearranging this equation, one gets:

$$R_o = R_i\left[1 - P_w + P_w(P_c - P_e)\right] + P_wP_e \qquad \text{(II-20)}$$

It is interesting to note that the last two terms: $R_iP_wP_c + (1 - R_i)P_wP_e$ in Eq.(II-19) represent the conditional probability of satisfactory control performance. This would be useful information to the management. A further discussion is presented in Chapter IV.

Special Cases:

(i) Assume: $P_w = 0$ i.e. the control is never in operation. This condition yields:

$$R_o = R_i \qquad \text{(II-21)}$$

which means that the reliability of the output information is the same as that of the input information. This is what one would expect if the control never works or is not installed.

(ii) Assume a perfect control component i.e. $P_w = P_c = P_e = 1$. This gives:

$$R_o = 1 \qquad \text{(II-22)}$$

This again is an intuitively expected result because when the control is perfect the reliability of the output information should always be unity irrespective of the state of the input information.

(iii) Assume: $P_w = 1$, $P_e = 0$ i.e. the component is always in operation and has no corrective capability. The reliability of the output information, in this case becomes:

$$R_o = R_i P_c \qquad \text{(II-23)}$$

This result implies that the input information is further contaminated by the control component making incorrect decisions when the input is correct.

It will be shown in Chapter IV, through sensitivity analysis, that the relative importance of P_c and P_e changes as the reliability of the input information changes. When $R_i > 0.5$, P_c is more important than P_e but when $R_i < 0.5$, P_e becomes more important than P_c. This result has important implications in personnel training programs (see Chapter IV for further details)

b. Control Component with Two Input Information Channels (One Being Redundant)

A reliability model of a control component with two input information channels with one being redundant is derived here. The control element receives two inputs and performs an 'operation' (attempted prevention, detection and/or correction of an error in the input) and produces an output. In fact it may produce several identical outputs to be sent to different sources. An example of such a component is a voucher clerk in an inventory control system where the clerk determines the correct quantity of items for which payment should be made, by comparing two independent pieces of input information: (1) the quantity received as reported on the receiving report, and (2) the quantity shipped by the vendor as shown on the vendor's invoice.

There are several operations performed on the input information by the control element depending on the state of the input. The schematic representation of such an element is shown in Fig. 5.

The reliability vector $\vec{Q}$ in Fig. 5 represents the reliability parameters of the control component. It has, in general, four components: Q_w, Q_c, Q_{1e} and Q_{2e} where Q_{1e} is assumed to be independent of the channel containing error, i.e. Q_{1e} is constant regardless of which input channel contains erroneous data. The reliability model for this component is presented through the following proposition.

Proposition 5: The reliability, R_o, of the output information for a given control objective is given by the following relation:

$$R_o = R_{i1}R_{i2}\left[1 - Q_w + Q_w(Q_c - 2Q_{1e} + Q_{2e})\right] + (R_{i1} + R_{i2})Q_w(Q_{1e} - Q_{2e}) + Q_{2e}Q_w \qquad \text{(II-24)}$$

where Q_w, Q_c, Q_{1e} and Q_{2e} represent the reliability parameters of the control component as discussed above, and R_{i1} and R_{i2} are the reliabilities of the two inputs.

Proof: Consider the following 'events':

S_{i1} - The input information in channel one is correct.

S_{i2} - The input information in channel two is correct.

S_w - The control component is in operation.

S_c - The control component operates properly.

S_o - The output information is correct.

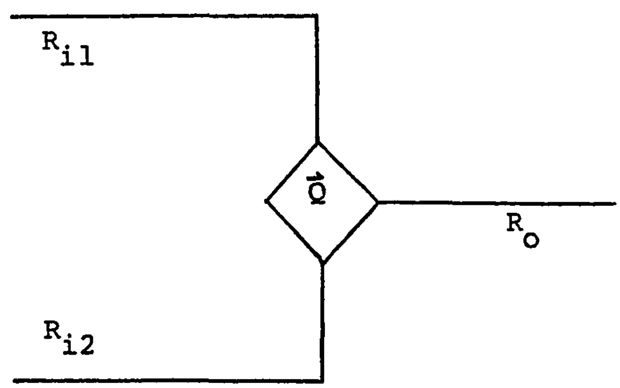


Figure 5: A control component with two inputs with one being redundant

The state of the output information can be written in terms of the states of the input information and the control component as follows:

$$S_o = (S_{i1} \cap S_{i2} \cap \bar{S}_w) \cup (S_{i1} \cap S_{i2} \cap S_w \cap S_c) \cup (\bar{S}_{i1} \cap S_{i2} \cap S_w \cap S_c)$$
$$\cup (S_{i1} \cap \bar{S}_{i2} \cap S_w \cap S_c) \cup (\bar{S}_{i1} \cap \bar{S}_{i2} \cap S_w \cap S_c) \qquad \text{(II-25)}$$

The combination of states in each paranthesis in Eq.(II-25) are mutually exclusive and thus the equation can be written in terms of the corresponding probabilities as:

$$\begin{aligned} P(S_o) &= P(S_{i1})P(S_{i2}/S_{i1})P(\bar{S}_w) \\ &+ P(S_{i1})P(S_{i2}/S_{i1})P(S_w)P(S_c/S_{i1} \cap S_{i2} \cap S_w) \\ &+ P(\bar{S}_{i1})P(S_{i2}/\bar{S}_{i1})P(S_w)P(S_c / \bar{S}_{i1} \cap S_{i2} \cap S_w) \\ &+ P(S_{i1})P(\bar{S}_{i2}/S_{i1})P(S_w)P(S_c/S_{i1} \cap \bar{S}_{i2} \cap S_w) \\ &+ P(\bar{S}_{i1})P(\bar{S}_{i2}/\bar{S}_{i1})P(S_w)P(S_c/\bar{S}_{i1} \cap \bar{S}_{i2} \cap S_w) \qquad \text{(II-26)} \end{aligned}$$

where S_w (but not S_c) is assumed to be independent of the states of the input.

This result is general and can be applied to any control component with two input channels. In the above equation the 'events' S_{i1} and S_{i2} are not assumed to be independent. In fact, one event may depend on the other but not vice versa. For example the correct state of a purchase order would not depend on the correctness of the corresponding vendor's invoice. However, the correctness of a vendor's invoice may depend on the correctness of the corresponding purchase order.

In the present derivation of the model, it is assumed, for simplicity that two 'events' S_{i1} and S_{i2} are independent of each other. As an example, consider a payroll system where one channel contains the number of hours worked by each employee based on the time cards and the other channel contains number of hours worked based on job cards. The two pieces of information are independent and are compared for control purposes. The present model is suited to this situation.

By assuming independence of inputs, one can write Eq. (II-26) in the following form using the parameters defined in Table I:

$$R_o = R_{i1}R_{i2}(1 - Q_w) + R_{i1}R_{i2}Q_wQ_c + (1 - R_{i1})R_{i2}Q_wQ_{1e} + R_{i1}(1 - R_{i2})Q_wQ_{1e} + (1 - R_{i1})(1 - R_{i2})Q_wQ_{2e} \quad \text{(II-27)}$$

Rearranging, one obtains:

$$R_o = R_{i1}R_{i2}\left[1 - Q_w + Q_w(Q_c - 2Q_{1e} + Q_{2e})\right] + (R_{i1} + R_{i2})Q_w(Q_{1e} - Q_{2e}) + Q_wQ_{2e} \quad \text{(II-28)}$$

Q.E.D.

Special Cases:

(i) Assume a perfect control component, i.e. $Q_w = Q_c = Q_{1e} = Q_{2e} = 1$. Then the reliability of the output information becomes:

$$R_o = 1 \quad \text{(II-29)}$$

This result implies that the output is always correct when the control is perfect.

(ii) Assume: $Q_w = Q_c = Q_{1e} = 1$ and $Q_{2e} = 0$.

This means that the control is always in operation and the control element always works properly when either or both inputs are correct, but that the control never works properly if both the inputs are incorrect. This condition yields (by substitution in Eq.II-24):

$$R_o = R_{i1} + R_{i2} - R_{i1}R_{i2} \qquad \text{(II-30)}$$

This is the traditional result (Bodnar 1975 and Barett, Baker, and Richett 1977) advanced without explication of the limiting assumptions, i.e. $Q_w = Q_c = Q_{1e} = 1$, and $Q_{2e} = 0$.

c. Control Component with n Parallel Independent Input Information Channels (n - 1 Channels Being Redundant)

A reliability model of a control component with n parallel independent input information channels with n-1 channels being redundant, as a general case, is developed here. This model will yield reliability results for special cases, e.g. two channels, three channels and so on when n=2, 3 and etc..

There are several situations in accounting systems where we have components with n>2. For example, in a voucher system, a clerk reviewing voucher data for the correct 'nature' of the material requires three inputs: (1) a bill of lading, (2) a receiving report and (3) an inspection report. He prepares a voucher only after finding all the information is in agreement. In the case of a discrepancy, the voucher clerk should try to find it and correct it before completing the voucher.

The schematic representation of a control element with n input information sources is presented in Fig. 6. Here, all the input channels are assumed to be independent and have different reliabilities. The assumption regarding the channels

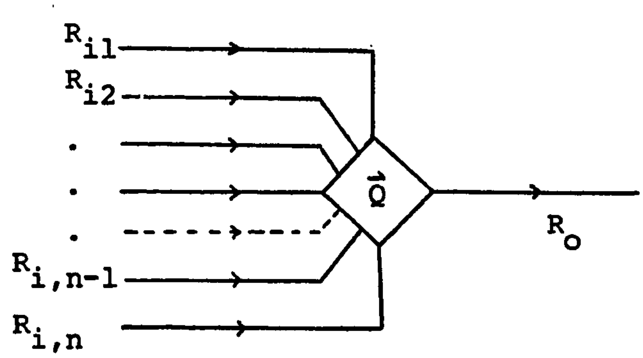


Figure 6: A control component with n input channels; n-1 channels being redundant.

being independent can be easily relaxed by redefining the reliability parameters in terms of the conditional probabilities as discussed in Sec.C.1.b of this chapter.

The reliability model of the control component under consideration is given in terms of the following proposition.

<u>Proposition</u> 6: The reliability, R_o, of the output information can be written in terms of the reliabilities of the input channels and the reliability parameters of the control element as:

$$R_o = \prod_{j=1}^{n} R_{ij}(1 - Q_w) + \prod_{j=1}^{n} R_{ij} Q_w Q_c$$

$$+ \sum_{j=1}^{n} (1 - R_{ij}) \prod_{\substack{k=1 \\ k \neq j}}^{n} R_{ik} Q_w Q_{1e}$$

$$+ \sum_{j=1}^{n} \sum_{k=j+1}^{n} (1 - R_{ij})(1 - R_{ik}) \prod_{\substack{\ell=1 \\ \ell \neq j \\ \ell \neq k}}^{n} R_{i\ell} Q_w Q_{2e}$$

$$+ \sum_{j=1}^{n} \sum_{k=j+1}^{n} \sum_{\ell=k+1}^{n} (1-R_{ij})(1-R_{ik})(1-R_{i\ell}) \prod_{\substack{m=1 \\ m \neq j \\ m \neq k \\ m \neq \ell}}^{n} R_{im} Q_w Q_{3e}$$

$$+ \dots\dots\dots\dots\dots\dots\dots$$

$$+ \prod_{j=1}^{n} (1 - R_{ij}) Q_w Q_{ne} \qquad \text{(II-31)}$$

where Σ stands for the summation and Π for the product. The different reliability symbols are defined in Table I.

<u>Proof</u>: Consider the following 'events':

S_{i1} - The input information in channel one is correct.

S_{i2} - The input information in channel two is correct.

. . . .

. . . .

. . . .

S_{in} - The input information in channel n is correct.

S_w - The control component is in operation.

S_c - The control component operates properly.

S_o - The output information is correct.

The output information will be correct in the following (n+2) state combinations: 1) all the input information from different channels are correct and the control element is not in operation, 2) all the inputs are correct, the control is in operation and it operates properly, 3) all the inputs except one are correct, the control is in operation and it operates properly, 4) all the inputs except two are correct, the control is in operation and it operates properly.... n+2) all n inputs are incorrect but the control is in operation and it operates properly. This statement can be written in terms of the 'events' defined above as:

$$S_o = (\bigcap_{j=1}^{n} S_{ij} \cap \bar{S}_w) \cup (\bigcap_{j=1}^{n} S_{ij} \cap S_w \cap S_c)$$

$$\cup (\bigcup_{k=1}^{n} (\bar{S}_{ik} \cap (\bigcap_{\substack{j=1 \\ j \neq k}}^{n} S_{ij}) \cap S_w \cap S_c))$$

$$\cup \left[\bigcup_{j=1}^{n} \left(\bigcup_{k=j+1}^{n} (\bar{S}_{ij} \cap \bar{S}_{ik} \cap (\bigcap_{\substack{\ell=1 \\ \ell \neq j \\ \ell \neq k}}^{n} S_{i\ell}) \cap S_w \cap S_c)\right)\right]$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cup ((\bigcap_{j=1}^{n} \bar{S}_{ij}) \cap S_w \cap S_c) \qquad \text{(II-32)}$$

where $\bigcup_{j=1}^{n} S_{ij}$ stands for $S_{i1} \cup S_{i2} \cup S_{i3} \cup \ldots \cup S_{in}$ and $\bigcap_{j=1}^{n} S_{ij}$ stands for $S_{i1} \cap S_{i2} \cap S_{i3} \cap \ldots \cap S_{in}$. Using the property of mutually exclusive events or combination of events, and the product rule, one can write Eq. (II-32) in terms of the respective probabilities of occurance of the corresponding events, in the following form:

$$
\begin{aligned}
P(S_o) = {} & \prod_{j=1}^{n} P(S_{ij}) P(\bar{S}_w) + \prod_{j=1}^{n} P(S_{ij}) P(S_w) P(S_c / \bigcap_{j=1}^{n} S_{ij} \cap S_w) \\
& + \sum_{k=1}^{n} \Big[P(\bar{S}_{ik}) \prod_{\substack{j=1 \\ j \neq k}}^{n} P(S_{ij}) P(S_w) P(S_c / \bar{S}_{ik} \cap (\bigcap_{\substack{\ell=1 \\ \ell \neq k}}^{n} S_{i\ell}) \cap S_w) \Big] \\
& + \sum_{j=1}^{n} \sum_{k=j+1}^{n} P(\bar{S}_{ij}) P(\bar{S}_{ik}) \prod_{\substack{\ell=1 \\ \ell \neq j \\ \ell \neq k}}^{n} P(S_{i\ell}) P(S_w) \\
& \quad x P\Big(S_c / \bar{S}_{ij} \cap \bar{S}_{ik} \cap (\bigcap_{\substack{m=1 \\ m \neq j,k}}^{n} S_{im} \cap S_w)\Big) \\
& + \ldots\ldots\ldots\ldots\ldots\ldots \\
& + \prod_{j=1}^{n} P(\bar{S}_{ij}) P(S_w) P(S_c / S_w \bigcap_{\ell=1}^{n} \bar{S}_{i\ell})
\end{aligned}
\qquad \text{(II-33)}
$$

In the above equation it is assumed that the state of the input information from each channel is independent of the state of the input information from the other channels. Equation(II-33) yields Eq.(II-31) when the reliability symbols defined in Table I are used instead of the probabilities symbols given in Eq.(II-33). Q.E.D.

The reliability model in Eq.(II-31) is for a general case. Specific models can be obtained by selecting a value for n. For example, if n = 1, the reliability result in Eq.-(II-31) reduces to:

$$R_o = R_{i1}(1 - Q_w) + R_{i1}Q_wQ_c + (1 - R_{i1})Q_wQ_{1e} \qquad \text{(II-34)}$$

which is equivalent to Eq.(II-19), the reliability model for a control component with one input channel.

Similarly, for n = 2, Eq.(II-31) reduces to

$$\begin{aligned} R_o &= R_{i1}R_{i2}(1 - Q_w) + R_{i1}R_{i2}Q_wQ_c \\ &+ [(1 - R_{i1})R_{i2} + (1 - R_{i2})R_{i1}]Q_wQ_{1e} \\ &+ (1 - R_{i1})(1 - R_{i2})Q_wQ_{2e} \end{aligned} \qquad \text{(II-35)}$$

This result is identical to the result derived in Sec.II.C.2.b.

<u>Special Case</u>:

For $R_{i1} = R_{i2} = \ldots = R_{in} = R$ i.e. all the input channels have the same reliability and for $Q_{1e} = Q_{2e} = \ldots Q_{ne} = Q_e$, the reliability R_o of the output information becomes (Eq.II-31):

$$R_o = R^n(1 - Q_w) + R^n Q_w Q_c + \Big[n(1 - R)R^{n-1} + \frac{n(n-1)}{2!}(1-R)^2 R^{n-2} + \frac{n(n-1)(n-2)}{3!}(1-R)^3 R^{n-3} + \ldots\ldots\ldots\ldots\ldots + (1-R)^n \Big] Q_w Q_e$$

or

$$R_o = R^n \left[1 - Q_w + Q_w(Q_c - Q_e) \right] + \left[R + (1-R) \right]^n Q_w Q_e$$

This result reduces to:

$$R_o = R^n \left[1 - Q_w + Q_w(Q_c - Q_e) \right] + Q_w Q_e \qquad \text{(II-36)}$$

Here again for $Q_w = Q_c = Q_e = 1$, i.e. for a perfect control component with n input channels, the reliability, R_o becomes unity, as expected.

3. Parallel Inputs Merged with no Redundancy and no Component at the Junction

The reliability model developed here deals with a system of parallel inputs merged with no redundancy and no component at the junction. A payroll system in a company with several departments is a good example of such a system. The payroll office receives similar payroll information (Number of employees, their wage rates, number of hours worked, etc.) from each department. All these inputs from different departments work like a single input for the payroll office. The schematic diagram for such a merger is given in Fig. 7.

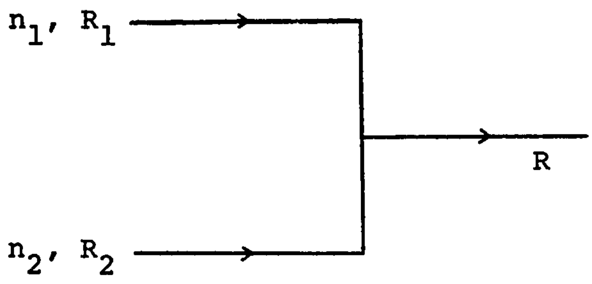


Figure 7: Merger of similar information

Where R_1 and R_2 are the reliabilities of items within two input data sets, R is the reliability of the combined information and n_1 and n_2 stand for the amount of data from the two channels, respectively. The reliability model for the combined information of this kind has been discussed by Yu and Neter (1973), and the reliability is given by the weighted average of the reliabilities:

$$R = (n_1R_1 + n_2R_2)/(n_1 + n_2) \qquad \text{(II-36)}$$

The result can be easily generalized for more than two channels. But it is not considered here because it does not add new insight to the model.

CHAPTER III

RELIABILITY MODELS OF COMPLEX SYSTEMS OR SUBSYSTEMS

This chapter discusses methods of combining the reliability models of the basic components (discussed in Chapter II) to develop models of complex systems or subsystems. To make the problem simpler, I will consider only one type of control objective. Generalization to multiple control objectives is possible provided one keeps in mind that a change in the control objective may change the structure of the system and hence the reliability model. One simply needs to multiply the individual reliabilities for different control objectives to obtain an overall reliability (see Cushing 1974).

This chapter is divided into three sections. The first section discusses models of systems having components in series. The second part deals with models of systems with components in parallel. The third section is devoted to systems which have components in 'dual'.

A. Components in Series

1. Performance Components in Series

Consider a system of n performance components connected in series. The schematic diagram of such a system is given in Fig. 8.

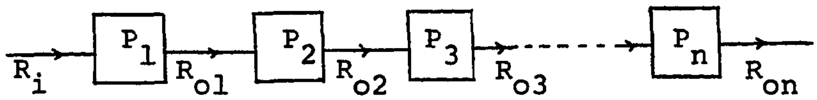


Figure 8: Performance components in series.

The symbols in Fig. 8 representing different reliabilities are defined in Table I. The following proposition presents the reliability for such a system.

Proposition 7: The reliability of output informtion after it has passed though n performance components is given by

$$R_{on} = R_i \prod_{j=1}^{n} P_j \qquad \text{(III-1)}$$

where R_i is the reliability of the input information to the first component.

Proof: Using Eq. (II-1), one can write the reliability of the output information from the first component as

$$R_{o1} = R_i P_1 \qquad \text{(III-2)}$$

considering this information as input for the second element we can write R_{o2} as

$$R_{o2} = R_{o1} P_2 = R_i P_1 P_2 \qquad \text{(III-3)}$$

Continuing this process of considering the output from the previous component as the input for the next component up to n components, one obtains:

$$R_{on} = R_i P_1 P_2 P_3 \ldots P_n \qquad \text{(III-4)}$$

or

$$R_{on} = R_i \prod_{j=1}^{n} P_j \qquad \text{(III-5)}$$

Q.E.D.

Since all P_j's and R_i are less than unity, the reliability of the final output information, R_{on} decreases as n (the number of performance components) increases. This result is similar to the reliability of an electronic device with n components in series. As the number of components increases, the reliability of the device decreases.

An example of such a system is a payroll system with no 'control' components. This is illustrated in Fig. 9 by dividing the job processes into different performance elements. Here, the first element receives the information on different clock cards and sorts them by employee number. The next element calculates the hours worked by each employee. This information is then used by the 3rd component to calculate the grand total of hours worked, and so on until in the last component payroll checks are prepared for the employees. In this system, error can be introduced at each step of the process, i.e., at each component. If one component fails to operate, the whole system fails. For example, if the element calculating the total hours worked per employee,

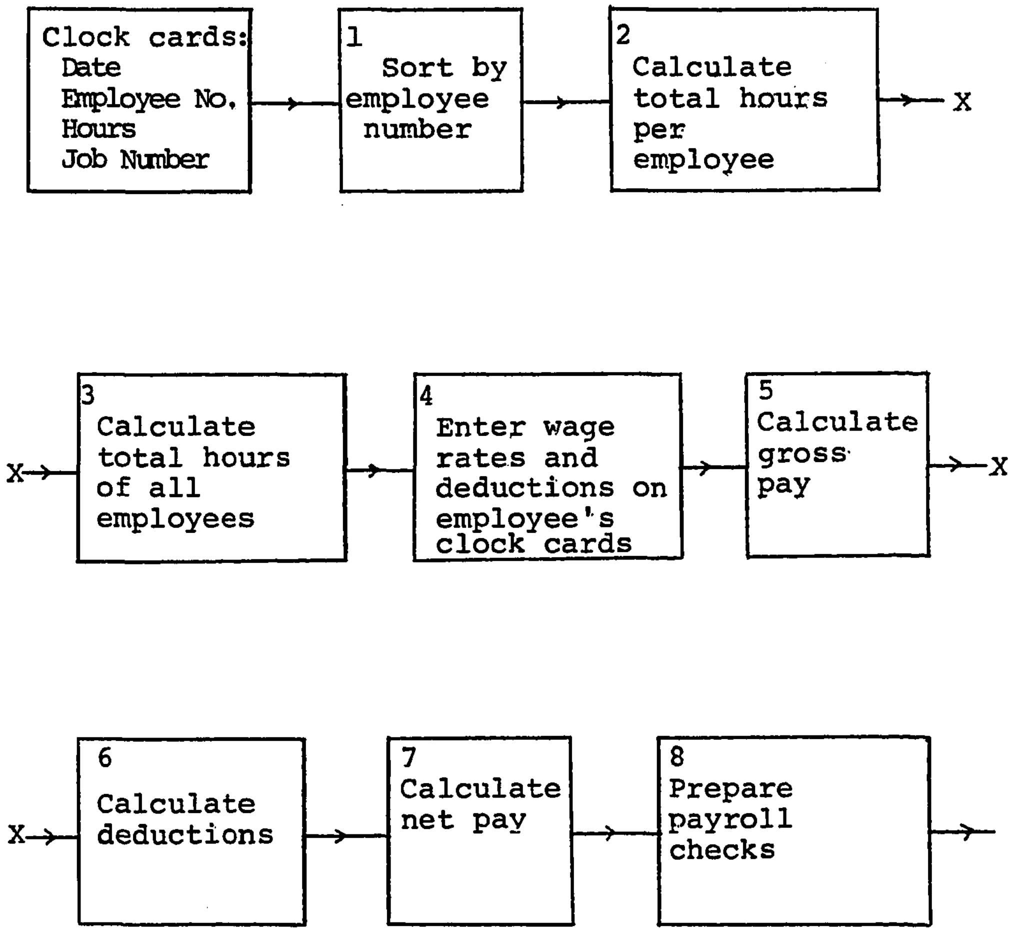


Figure 9: A payroll system with no control component.

computes the hours worked incorrectly, then the information flowing through the system beyond that element is always in error.

2. Control Components in Series

Here, I want to discuss the reliability model of a control system or subsystem which consists of control components in series (see Figure 10).

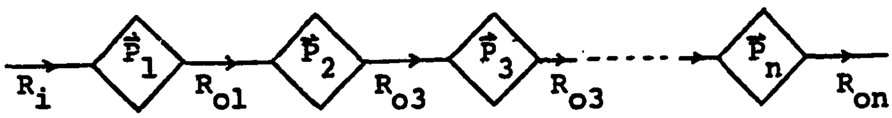


Figure 10: Control components in series.

The reliability model for this system can be given by the following propostion.

<u>Proposition</u> 8: The reliability R_{on} of the final output information after it has passed through n control components is given by

$$R_{on} = R_i \prod_{j=1}^{n} x_j + \sum_{j=1}^{n-1} y_j \left(\prod_{k=j+1}^{n} x_k \right) + y_n \qquad \text{(III-6)}$$

where

$$x_j = 1 - P_{wj} + P_{wj}(P_{cj} - P_{ej}); \; j = 1,2...n \qquad \text{(III-7)}$$

$$y_j = P_{ej}P_{wj} \; ; \; j = 1,2...n \qquad \text{(III-8)}$$

<u>Proof</u>: Using the reliability model of a single control component one can write R_{o1} as

$$R_{o1} = R_i[1 - P_{w1} + P_{w1}(P_{c1} - P_{e1})] + P_{e1}P_{w1} \qquad \text{(III-9)}$$

or

$$R_{o1} = R_i x_1 + y_1 \qquad \text{(III-10)}$$

where x_1 and y_1 are defined in Eqs. (III-7) and (III-8).

Considering the output of the first component as the input for the second one, one can write the reliability of output information from the second component as

$$R_{o2} = R_{o1}x_2 + y_2 \tag{III-11}$$

Using the result of Eq. (III-10), one can write

$$R_{o2} = R_i x_1 x_2 + y_1 x_2 + y_2 \tag{III-12}$$

Iterating this procedure n times, one obtains:

$$R_{on} = R_i \prod_{j=1}^{n} x_j + \sum_{j=1}^{n-1} \left[y_j \left(\prod_{k=j+1}^{n} x_k \right) \right] + y_n \tag{III-13}$$

Q.E.D.

<u>Special Case</u>:

$P_{wj} = P_w$, $P_{cj} = P_c$ and $P_{ej} = P_e$ $\forall$ j, then

$$x_j = x \ \forall j$$
$$y_j = y \ \forall j$$

This implies that with regard to reliability, all the components are identical. For this case R_{on} reduces to:

$$R_{on} = R_i x^n + y \frac{1 - x^n}{1 - x} \tag{III-14}$$

where

$$x = 1 - P_w + P_w(P_c - P_e) \tag{III-15}$$

and

$$y = P_e P_w \tag{III-16}$$

Equation (III-14) provides interesting results that are discussed below.

It can be shown[10] that x must fall in the range: $-1 \leqslant x \leqslant 1$. For $-1 < x < 1$, as $n \rightarrow \infty$, $x^n \rightarrow 0$ therefore R_{on} reduces to:

$$R_{o\infty} = P_e/(1 - P_c + P_e) \qquad \text{(III-17)}$$

This result suggests that even after infinitely many controls in a series, the output is not error free if $P_c \neq 1$, i.e., if the controls have the potential to falsely signal error when the input is correct. It is interesting to note that as $n \rightarrow \infty$, R_{on} becomes insensitive to P_w and for $P_c = 1$, $R_{on} \rightarrow 1$ for any value of P_e. This suggests that as the number of controls in a series increases, it becomes far less important for a control component to operate than for a control to be designed so that when it does operate it avoids the potential of falsely signaling an error. The result is in accordance with intuition and "common sense" as opposed to Bodnar's result (Bodnar 1975). The graphs[11] presented in Figs. 11 - 13 elaborate the above results. For example, one can see from Fig. 11 that when $P_w = 1$ and $P_c = 1$, i.e., when the control is in operation and it can not falsely signal error, the reliability, R_{on}, of the output information increases rapidly as n, the number of control components in the series increases, and it approaches unity for large n. The graph also indicates that R_{on} approaches unity more quickly for larger P_e.

When $P_c \neq 1$, two situations are of interest: one where $P_c > P_e$ and the other where $P_e > P_c$. In the former case if we assume $P_w = 1$, then x becomes positive and R_{on} varies monotonically with n. For certain values of P_e, it decreases with increasing

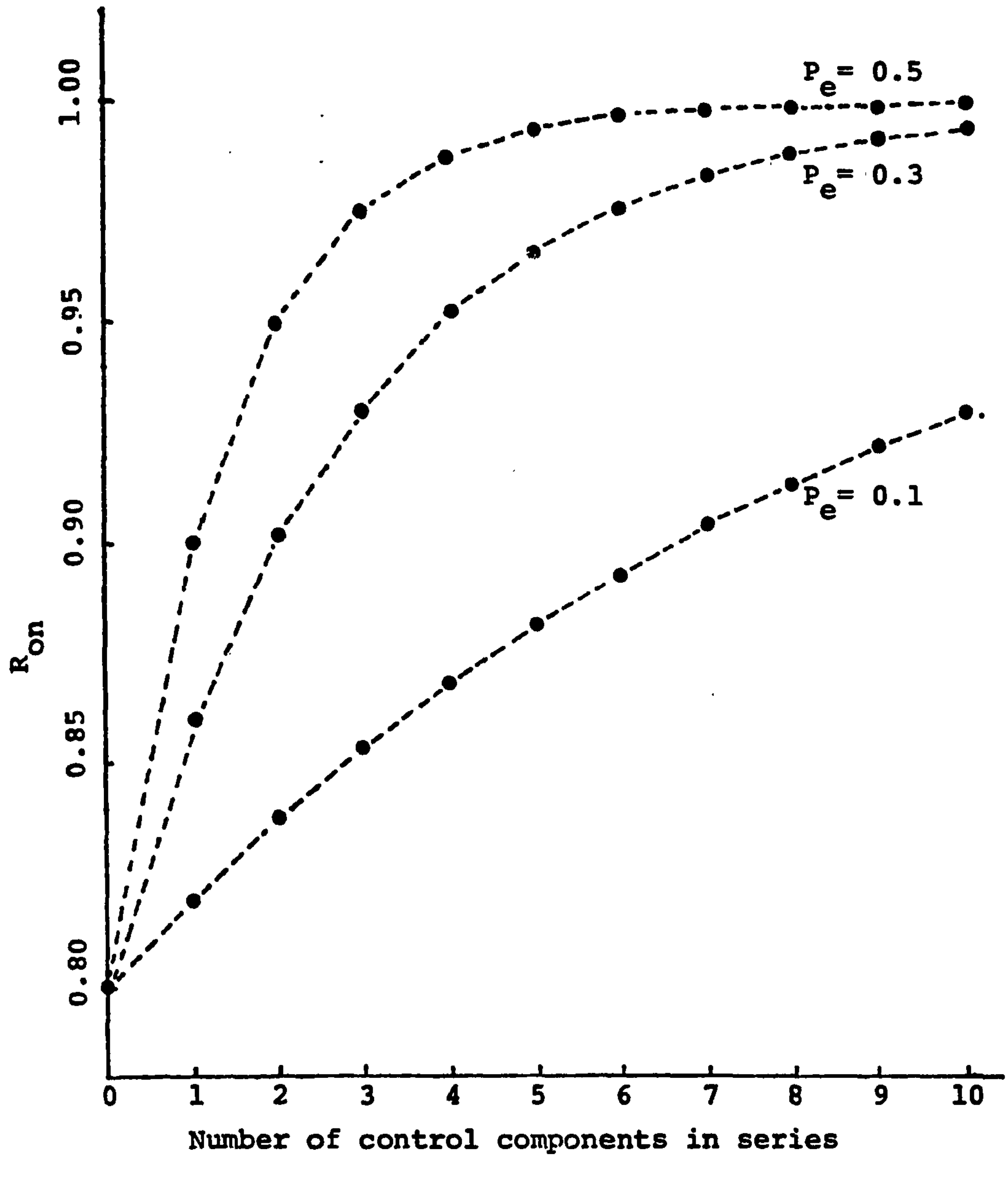


Figure 11: Variation of R_{on}, the reliability of the output information with the number of control components in series for R_i=0.8, P_W=1.0, P_C=1.0 and for different values of P_e. All the components are assumed to be identical in terms of their reliability parameters.

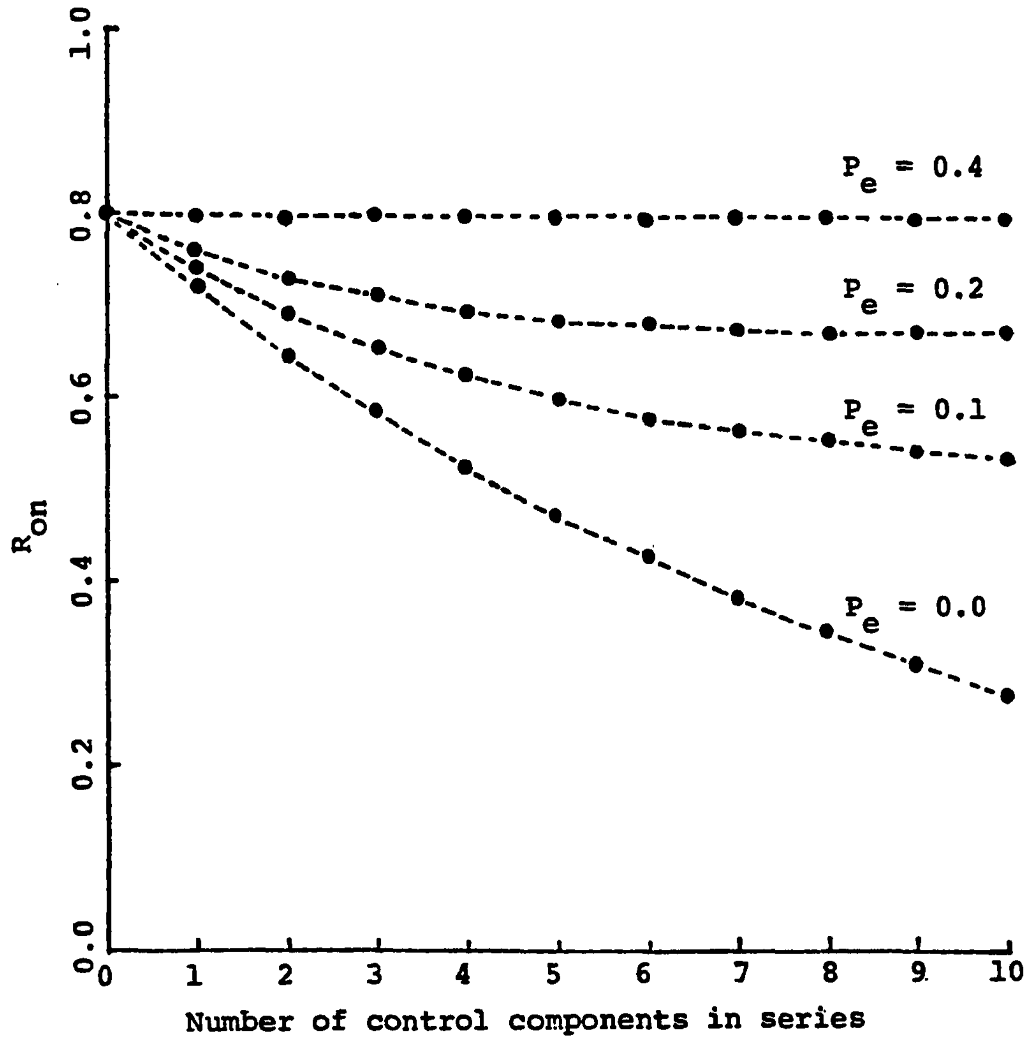


Figure 12: Variation of R_{on}, the reliability of the output information with the number of control components in series for R_i=0.8, P_w=1.0, and P_c=0.9 and for different values of P_e (0.0 to 0.4). All the components are identical in terms of their reliability parameters.

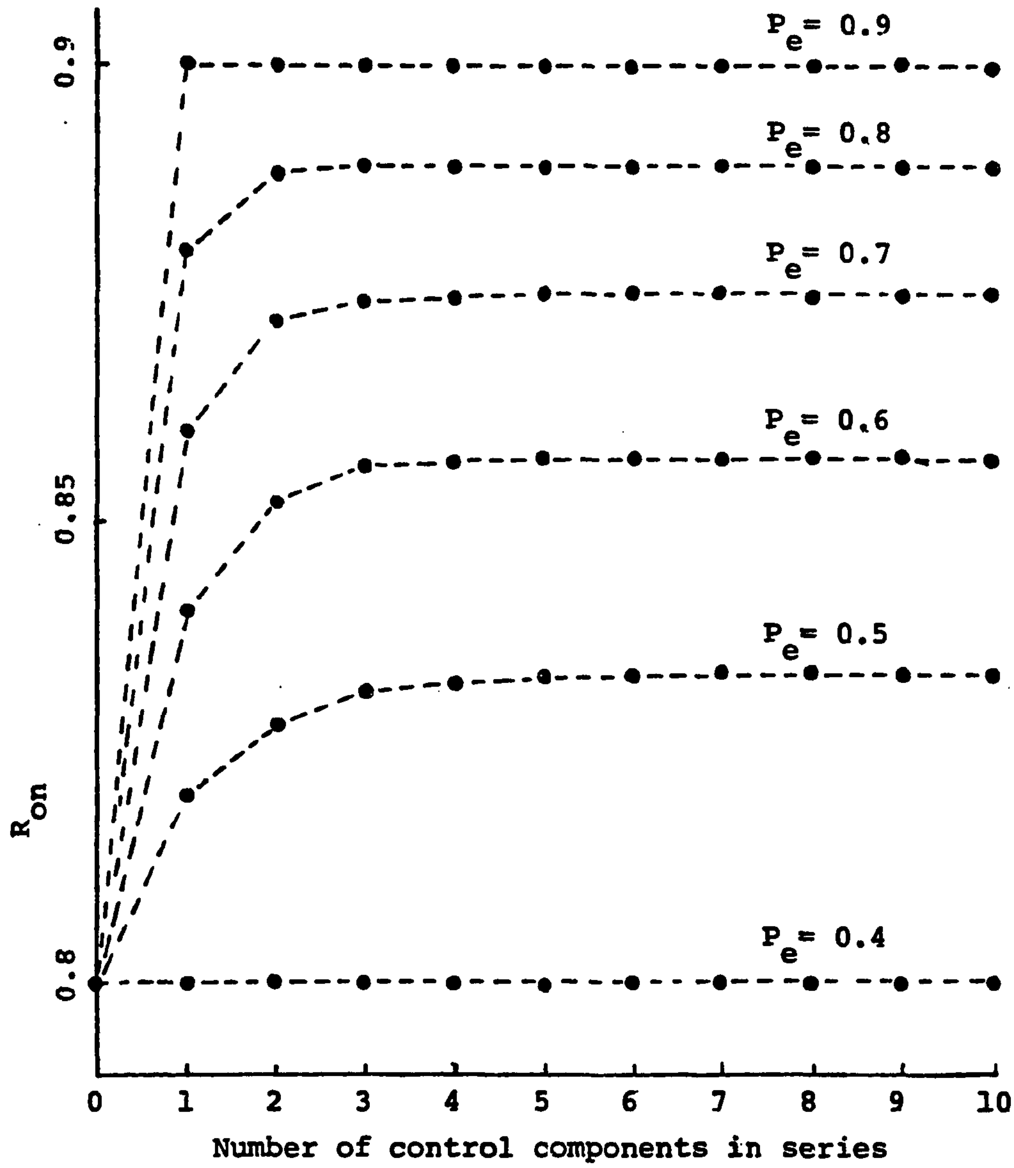


Figure 13: Variation of R_{on}, the reliability of the output information with the number of control components in series for R_i=0.8, P_w=1.0, P_c=0.9 and for different values of P_e. All the components are identical in terms of their reliability parameters.

n and for others it increases with n (see Figures 12 and 13). These results are important for control design purposes. A control system will be dysfunctional if the final reliability of the output information decreases with the increase in the number of control components in series. If such is the case, management should provide proper training and/or replace personnel to improve the reliability parameters P_c and P_e so that R_{on} increases with increasing n as in Fig. 13. However, it should be clear from Fig. 13 that for $P_c<1$, R_{on} never reaches unity for any n regardless of P_e. But when $P_c = 1$, R_{on} approaches unity (see Fig. 11) for any value of P_e as n is increased. This clearly shows the importance of P_c over P_e in improving the reliability of a control system. The value of n, required to drive R_{on} to unity, increases as P_e decreases.

When $x<0$, i.e. for $P_w=1$ and $P_c<P_e$, R_{on} as plotted in Fig. 14 'oscillates' with increasing n for some values of P_c. The 'oscillation' never attenuates for $P_c= 0$ and $P_e = 1$, and R_{on} takes values $1 - R_i$ and R_i alternately as n changes. The physical reasoning for this result is that if a control operates in such a manner that it contaminates 100% of the correct input and corrects 100% of the incorrect inputs then the reliability of the output information from such an element would be $1 - R_i$ where R_i is the reliability of the input information. The control elements are functional[12] ($R_{on} > R_i$) for $P_c \geq 1 - (1 - R_i)P_e/R_i$, although R_{on} reaches a saturation limit very rapidly as n is increased (see Fig. 14). This limit is not

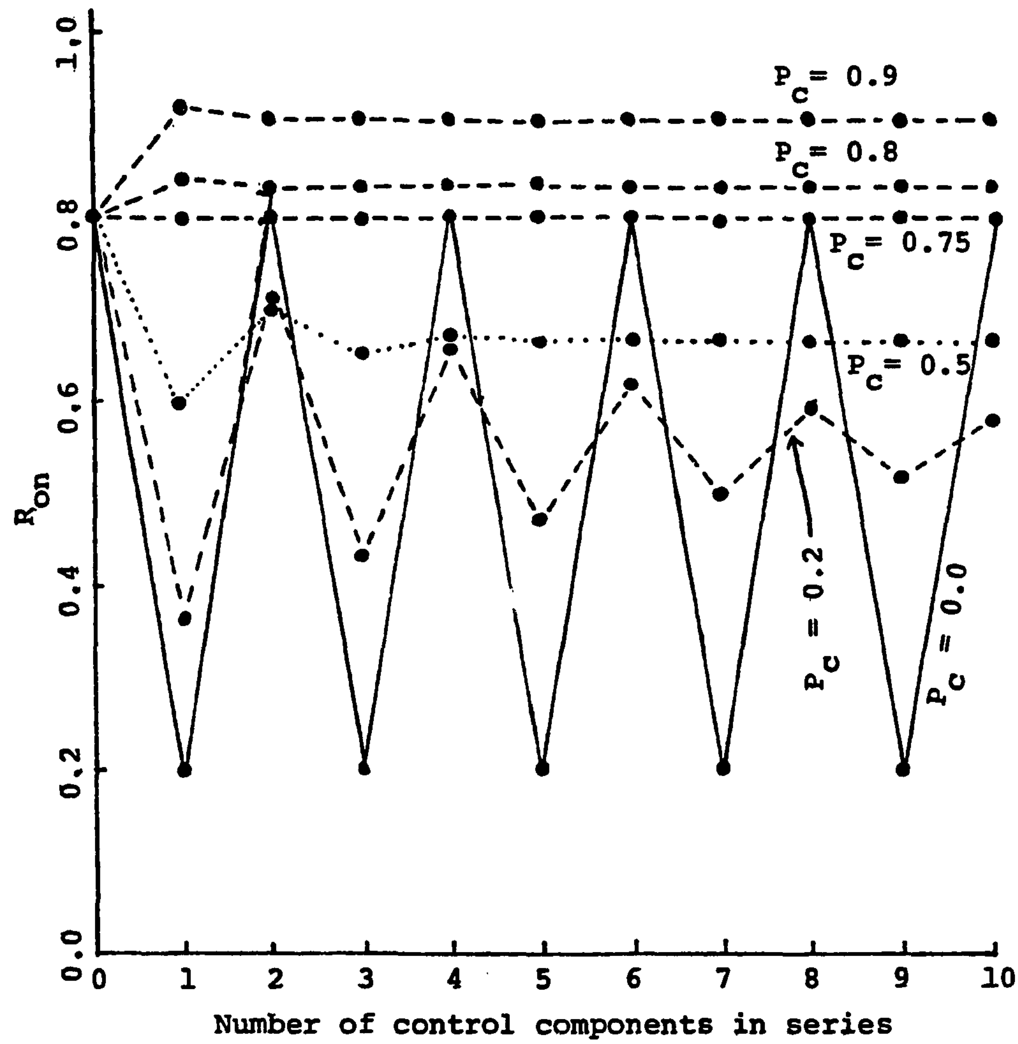


Figure 14: Variation of R_{on}, the reliability of the output information with the number of control components in series for R_i=0.8, P_w=1.0, P_e=1.0 and for different values of P_c. All the components are identical in terms of their reliability parameters.

usually unity unless $P_c = 1$.

When $P_c = 1$ and $P_e = 0$, i.e., the control elements do not contaminate correct inputs and at the same time do not correct any input errors, then $R_{on} = R_i$ for all n. This result makes sense, because the control is neither introducing error nor correcting error that may exist in the input. Thus, the reliability of the output remains unchanged as the information flows through such a system.

3. Performance and Control Components in Series

This section deals with reliability models for systems containing a mixture of performance and control components connected in series. There are several examples of such systems in real cases (Arens and Loebbecke, 1980). An example, part of a payroll system, is presented in Figure 15.

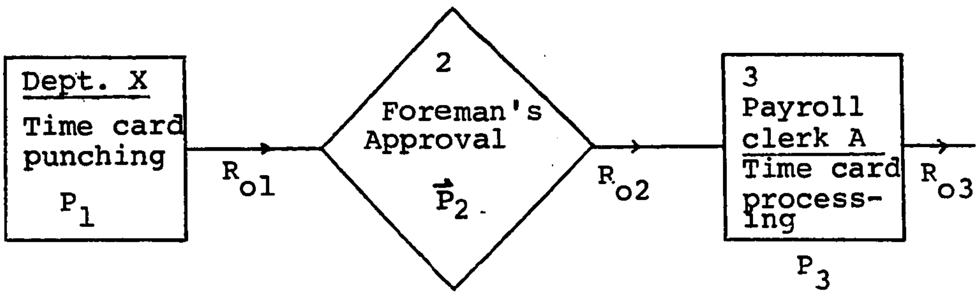


Figure 15: Payroll subsystem.

The reliability model for any particular control objective in such a system can be easily obtained from the general result derived in Sec. III.A.2. (Eq. III-6) by setting P_{ej}'s equal to zeros and P_{wj}'s = 1 for those components (jth element) which behave as performance components.

As an example of the above approach, consider the subsystem in Fig. 15. The first and third components are 'performance' components and the second one is a control component. Thus setting $n = 3$, $P_{c1} = P_1$, $P_{e1} = P_{e3} = 0$, $P_{c3} = P_3$ and $P_{w1} = P_{w3} = 1$, one obtains the following reliability model for the subsystem:

$$R_{o3} = [P_1(1 - P_{w2}) + P_1P_{w2}P_{c2} + (1 - P_1)P_{w2}P_{e2}]P_3 \quad \text{(III-20)}$$

This result can also be derived using the basic approach of combining reliabilities of different components by considering the output of one component as an input to the other. Using this approach one gets

$$R_{o1} = P_1 \quad \text{(III-21)}$$

$$R_{o2} = R_{o1}(1 - P_{w2}) + R_{o1}P_{w2}P_{c2} + (1 - R_{o1})P_{w2}P_{e2} \quad \text{(III-22a)}$$

and

$$R_{o3} = R_{o2}P_3 \quad \text{(III-22b)}$$

Combining the above equations one obtains the result of Eq. (III-20). This model is further discussed in Chapter IV.

B. Components in Parallel

This section is devoted to the discussion of reliability models of systems with redundant parallel components. The general case and one special case of control components in parallel with n elements in one channel and m elements in the other with a control component at the junction (see Fig. 16) are discussed first. This is followed by a discussion of the general case and one special case of performance components in parallel preceding a control element.

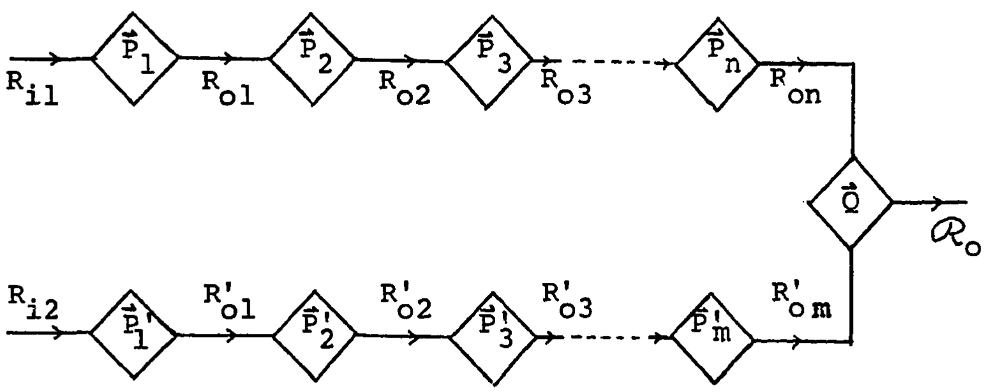


Figure 16: Control components in parallel with a control component at the junction and one channel being redundant.

1. Control Components in Parallel

The reliability model for a system of control components connected in parallel as shown in Fig. 16 can be easily obtained by using the results derived earlier in Eqs. (II-24) and (III-6). The result is:

$$\mathcal{R}_o = R_{on}R'_{om}\,[1 - Q_w + Q_w(Q_c - 2Q_{1e} + Q_{2e})]$$

$$+ (R_{on} + R'_{om})Q_w(Q_{1e} - Q_{2e}) + Q_{2e}Q_w \qquad \text{(III-23)}$$

R_{on} and R'_{om} are given by the following equations (using Eq. III-6):

$$R_{on} = R_{i1}\prod_{j=1}^{n} x_j + \sum_{j=1}^{n-1} y_j\left(\prod_{k=j+1}^{n} x_k\right) + y_n \qquad \text{(III-24)}$$

$$R'_{om} = R_{i2}\prod_{\ell=1}^{m} x'_\ell + \sum_{\ell=1}^{m-1} y'_\ell\left(\prod_{k=\ell+1}^{m} x'_k\right) + y'_m \qquad \text{(III-25)}$$

where

$$x_j = 1 - P_{wj} + P_{wj}(P_{cj} - P_{ej});\ j = 1, 2, \ldots n \qquad \text{(III-26)}$$

$$y_j = P_{ej}P_{wj}$$

$$x'_\ell = 1 - P'_{w\ell} + P'_{w\ell}(P'_{c\ell} - P'_{e\ell});\ \ell = 1, 2, .\ m \qquad \text{(III-27)}$$

and

$$y'_\ell = P'_{e\ell}P'_{w\ell} \qquad \text{(III-28)}$$

Special Case:

In order to interpret some of the results obtained here, let us assume that all of the control components are identical in terms of their reliability parameters, except the one at the junction; i.e. $x_j = x'_\ell = x$; $y_j = y'_\ell = y$ for all ℓ and j. This reduces Eqs.(III-24) and (III-25) to (see Eq. III-14):

$$R_{on} = R_{i1}x^n + y\frac{1 - x^n}{1 - x} \qquad \text{(III-29)}$$

and

$$R'_{om} = R_{i2}x^m + y\frac{1 - x^m}{1 - x} \qquad \text{(III-30)}$$

It has been demonstrated in Sec. III.A.2. that reliability of the output information can be improved significantly by selecting proper control components (see Fig. 11) and increasing the number of control components in series. However when this highly reliable information is combined by the control component at the junction in Fig. 16, the output may not be as reliable as the inputs because of the possibility of substantive contamination being created and/or introduced at the junction.

Consider the following example to illustrate the above point. Let $R_{on} = 0.99$, $R'_{om} = 0.98$, $Q_w = 1$, $Q_c = 0.95$, $Q_{1e} = 0.9$, and $Q_{2e} = 0.85$. This yields:

$$\mathcal{R}_o = 0.9485 \qquad \text{(III-31)}$$

This result is definitely below the reliability of either of the inputs. Therefore the parallel combination of two channels of control connected in series (Fig. 16) may seem to be an undesirable combination if the control element at the junction is not highly reliable. However, one should keep in mind at this stage that the presence of a control component at the junction may control the behavior of the two elements in the two channels, and therefore eliminating the control at the junction may create an environment where the two elements may not perform with the same high reliability.

For example, in a payroll system of a manufacturing firm, a control component, determining the 'correct' total hours worked by employees, compares two inputs: 1) the total hours worked by employees according to the employee cards and 2) the total hours worked according to the job cards. Since it is known that the two data sources are compared, the personnel responsible for each input component would be careful not to introduce any error. This may be due to the fear that an error may be caught by the control element and attributed to the related data source. On the other hand, if there is no control, i.e. only one of the data channels is available and no control is built to verify that number, then the source generating the data need not fear such attribution. Thus the presence of a control component in this example motivates the two data generating sources to produce highly reliable data.

2. Performance Components in Parallel Preceding a Control Component

The general schematic diagram for a system with redundant performance components in parallel preceding a control is given in Fig. 17.

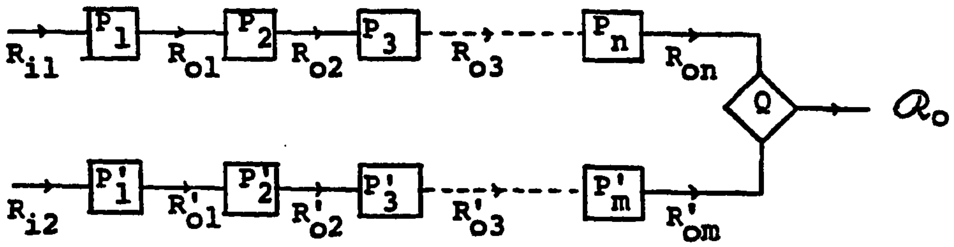


Figure 17: Performance components in parallel with a control component at the junction.

The reliability logic for this system can be obtained directly from Eq.(III-23) by simply substituting $P_{ej} = P'_{ek} = 0$, $P_{wj} = P'_{wk} = 1$, $P_{cj} = P_j$ and $P_{ck} = P'_k$ for $j = 1, 2, \ldots n$ and $k = 1, 2, \ldots m$. The substituiton of P_e's $= 0$ implies that performance components do not have error correcting capability and P_w's$=$ 1 implies that performance components are always in operation while their respective processes are being completed. If a performance component of a control system fails to operate, the system will collapse i.e. information will not flow through the system unless the performance components are in operation.

Using the above substitutions in Eq.(III-23), one obtains the following result for the reliability of the output information from the system shown in Fig. 17:

$$\mathcal{R}_o = R_{on}R'_{om}\,[1 - Q_w + Q_w(Q_c - 2Q_{1e} + Q_{2e})] + (R_{on} + R'_{om})Q_w(Q_{1e} - Q_{2e}) + Q_{2e}Q_w \qquad \text{(III-32)}$$

where

$$R_{on} = R_{i1}\prod_{j=1}^{n} P_j \qquad \text{(III-33)}$$

and

$$R'_{om} = R_{i2}\prod_{k=1}^{m} P'_k \qquad \text{(III-34)}$$

Special Case:

Consider $P_j = P'_k = P \;\forall\; j$ and k, i.e., all the performance components are identical in terms of reliability parameters. This reduces Eq. (II-32) to:

$$\mathcal{R}_o = R_{i1}R_{i2}P^{n+m}[1 - Q_w + Q_w(Q_c - 2Q_{1e} + Q_{2e})] + (R_{i1}P^n + R_{i2}P^m)Q_w(Q_{1e} - Q_{2e}) + Q_{2e}Q_w \qquad \text{(III-35)}$$

This result is important. It demonstrates that the reliability of output information from a series of performance components can be improved significantly if a redundant parallel channel is created and the results of the two channels are compared. In case of a discrepancy, the discrepancy is searched out and the error is corrected. This point is illustrated in the following example.

Example: Assume $P = 0.9$, $n = 8$, $m = 4$, $R_{i1} = R_{i2} = 0.8$, $Q_w = 1$, $Q_c = .92$, $Q_{1e} = 0.90$, $Q_{2e} = 0.85$. From Eqs. (III-33)-(III-35) one gets:

$$R_{on} = 0.3444 \qquad \text{(III-36)}$$

$$R'_{om} = 0.52488 \qquad \text{(III-37)}$$

and

$$\mathcal{R}_o = 0.863 \qquad \text{(III-38)}$$

In this case the output reliability was increased. The reliability, $\mathcal{R}_o$ of the information after it has gone through the control component at the junction (see Fig. 17) is higher than R_{on} and R'_{om}, which are the reliabilities of the information coming out of the two channels. But the improvement in reliability depends on the values of the reliability parameters of the control component. In certain situations, their $\vec{Q}$ values may be so low that it may further reduce the reliability of the output information by introducing and/or creating errors.

C. 'Dual' Elements in Series

Here, the reliability model of a control system with control components connected in 'dual' is considered. A schematic representation of the system is given in Fig. 18.

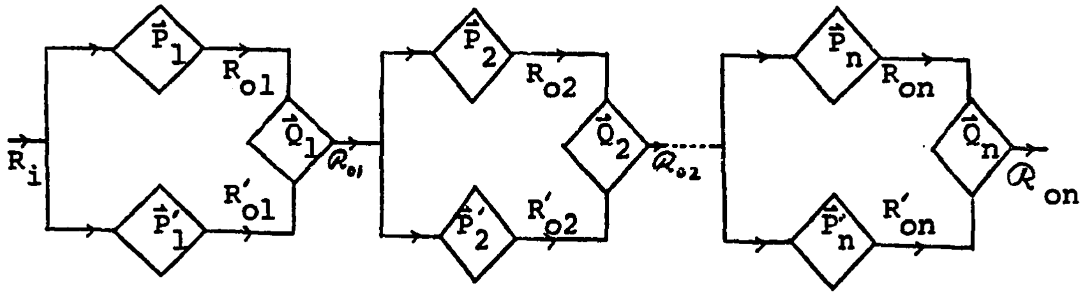


Figure. 18: A series of 'dual' control elements.

The reliability of the output information from this system can be obtained by considering the reliability model of an individual 'dual' element. Consider a jth 'dual' element (Fig. 19) which consists of two control elements in parallel and one control element at the junction.

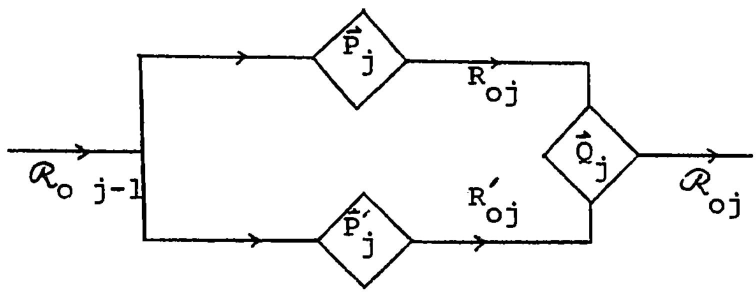


Figure 19: jth 'dual' element consisting of control components.

An example of such an element is a price control system where a control component (a person) in one channel checks the price of an item against a catalog price and the other redundant control component, a computer, compares the input price with its stored information on prices. These two parallel (one being redundant) pieces of price information go to a third control component (e.g., a supervisor or a programmed computer edit check). This component compares the two pieces of information and makes a decision. The output, in this case is the final price of the item.

The reliability of the input information to the 'dual' element in Fig. 19 is $\mathcal{R}_{o\ j-1}$ which is the reliability of the output from the (j - 1)th 'dual' element. From Eq.(II-28) one can write the reliability of the output from the jth 'dual' element as:

$$\mathcal{R}_{oj} = R_{oj}R'_{oj}\,X_j + (R_{oj} + R'_{oj})Y_j + Z_j \qquad \text{(III-39)}$$

where

$$X_j = 1 - Q_{wj} + Q_{wj}(Q_{cj} - 2Q_{1ej} + Q_{2ej}) \qquad \text{(III-40)}$$

$$Y_j = Q_{wj}(Q_{1ej} - Q_{2ej}) \qquad \text{(III-41)}$$

$$Z_j = Q_{wj}Q_{2ej} \qquad \text{(III-42)}$$

and, R_{oj} and R'_{oj} are given by (Eq. II-16):

$$R_{oj} = \mathcal{R}_{o\ j-1}x_j + y_j \qquad \text{(III-43)}$$

$$R'_{oj} = \mathcal{R}_{o\ j-1}x'_j + y'_j \qquad \text{(III-44)}$$

where

$$x_j = 1 - P_{wj} + P_{wj}(P_{cj} - P_{ej}) \qquad \text{(III-45)}$$

$$y_j = P_{ej}P_{wj} \qquad \text{(III-46)}$$

One can iterate this procedure, starting from j = 1 to j = n to obtain $\mathcal{R}_{on}$ as shown in Fig. 18. For the first dual element in Fig. 18, one obtains:

$$R_{o1} = R_i x_1 + y_1$$

$$R'_{o1} = R_i x'_1 + y'_1$$

$$\mathcal{R}_{o1} = R_{o1}R'_{o1}X_1 + (R_{o1} + R'_{o1})Y_1 + Z_1 \qquad \text{(III-47)}$$

For the second element:

$$R_{o2} = R_{o1}x_2 + y_2$$

$$R'_{o2} = R_{o1}x'_2 + y'_2$$

$$\mathcal{R}_{o2} = R_{o2}R'_{o2}X_2 + (R_{o2} + R'_{o2})Y_2 + Z_2 \qquad \text{(III-48)}$$

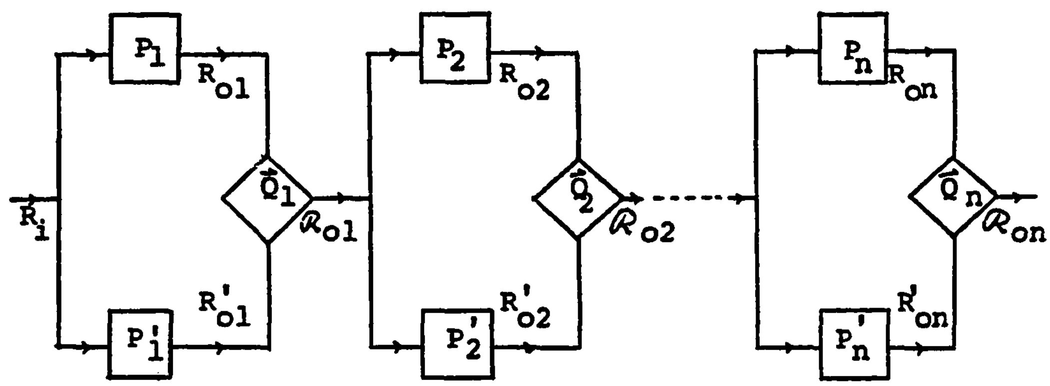


Figure 20: n 'dual' elements in series. Each 'dual' element contains two performance components in parallel and one control component at the junction.

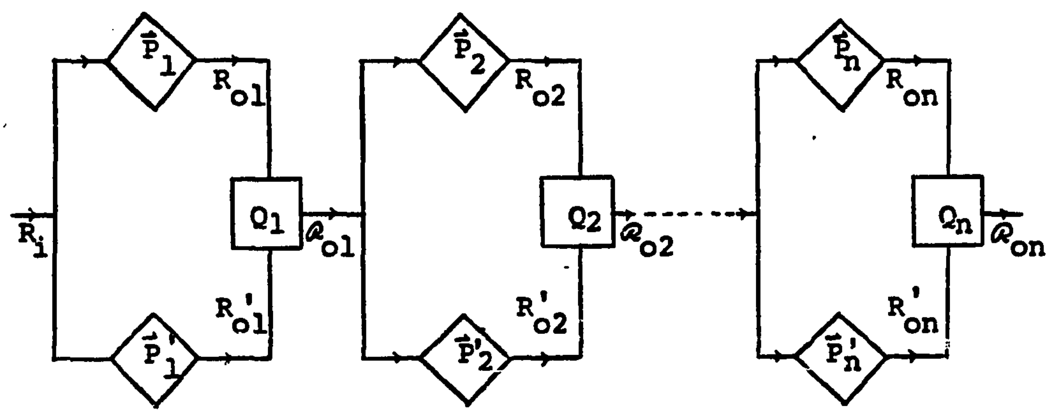


Figure 21: n 'dual' elements in series. Each 'dual' element contains two control components in parallel and a performance component at the junction.

Similarly, one can write the reliability of the output information from the nth 'dual' element as:

$$\mathcal{R}_{on} = R_{on}R'_{on}X_n + (R_{on} + R'_{on})Y_n + Z_n \qquad \text{(III-49)}$$

where

$$\begin{aligned} R_{on} &= \mathcal{R}_{o\,n-1}x_n + y_n \\ R'_{on} &= \mathcal{R}_{o\,n-1}x'_n + y'_n \end{aligned} \qquad \text{(III-50)}$$

Equation (III-49) represents a general result. A special case where all the control components in 'dual' redundant channels are identical and all the control components at the junction are identical in terms of their reliability parameters, can be studied by appropriately selecting values for the reliability parameters in Eq. (III-49).

Here all the elements are control elements and the reliability of the output information may increase or even decrease with the increase in the number of 'dual' elements in series. For example, when the control elements are in operation all the time (i.e. $P_w = Q_w = 1$) and they correctly perform their operation when the input(s) are correct (i.e. $P_c = Q_c = 1$); but $P_e = 0.4$ and $Q_{1e} = Q_{2e} = 0.0$, then the reliability $\mathcal{R}_{on}$ of the output information decreases rapidly to zero as the number of 'dual' elements increases. This effect can be seen in the graph in Fig. 22. But when the parameter Q_{1e} ($= Q_{2e}$) is improved slightly, i.e., $Q_{1e} = Q_{2e} = 0.2$, $\mathcal{R}_{on}$ will increase with increasing n (see curve 2 in Fig. 22).

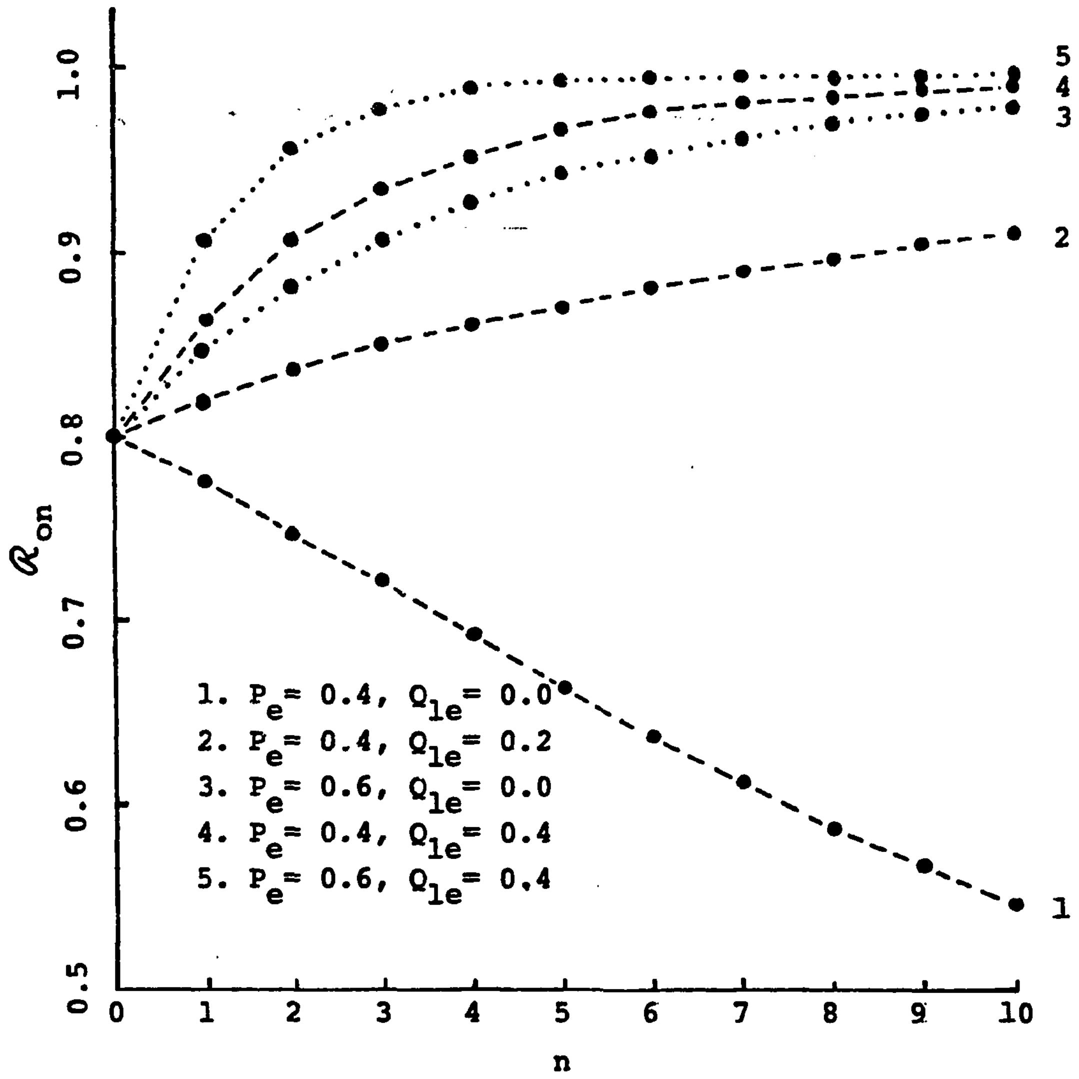


Figure 22: Variation of $\mathcal{R}_{on}$ with n, where n represents the number of identical dual control elements in series for $P_w = Q_w = P_c = Q_c = 1$, and $Q_{1e} = Q_{2e}$.

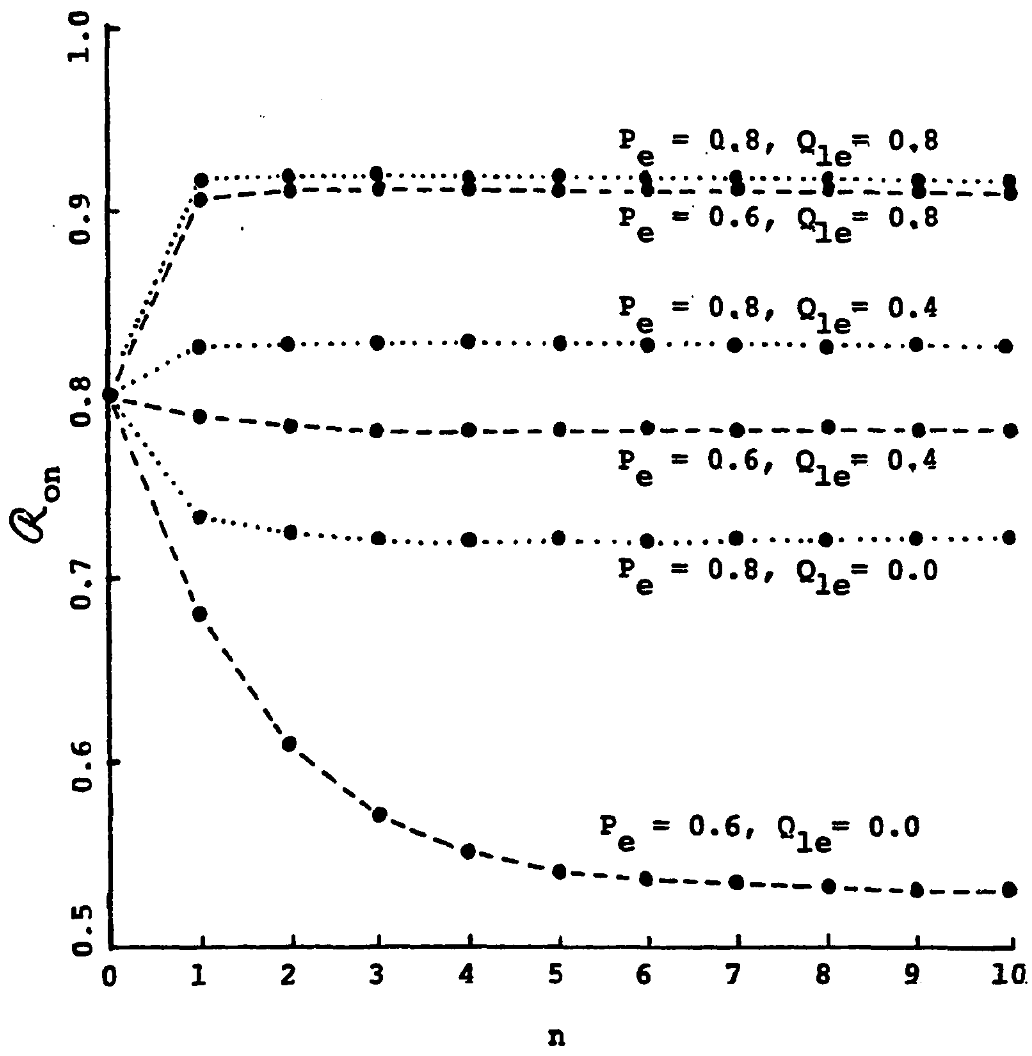


Figure 23: Variation of $\mathcal{R}_{on}$ with n where n is the number of identical dual control elements in series for $P_w = Q_w = 1$, $P_c = 0.9$, $Q_c = 0.95$, and $Q_{1e} = Q_{2e}$.

When the parameters P_e, Q_{1e} and Q_{2e} are increased together then $\mathcal{R}_{on}$ increases very rapidly and approaches unity as n increases (see curves 4 and 5 in Fig. 22). It is interesting to note that when $P_c < 1$ and $Q_c < 1$, $\mathcal{R}_{on}$ never approaches unity for any n, P_e, Q_{1e} and Q_{2e} (see the graphs in Fig. 23). This result suggests that in order for $\mathcal{R}_{on}$ to approach unity, one must improve the parameters P_c and Q_c, i.e. improve the correct judgment process when the inputs are correct rather than adding more 'dual' elements to the system without improving P_c and Q_c. These results are important for control system design purposes. Because the general notion that adding more controls would increase the output reliability is not necessarily true. Also adding new controls to increase the output reliability may be costlier than improving the reliability parameters P_c and Q_c of the existing control components through such steps as providing additional job training.

1. 'Dual' Performance Elements in Series with Control Components at the Junctions

The reliability model of a system consisting of n 'dual' performance elements in series with control components at the junctions (see Fig. 20) is discussed here. The reliability results for this system can be obtained directly from the results developed in the previous case (Eqs. III-39 - III-50) by substituting $P_{ej} = P'_{ej} = 0$, $P_{wj} = P'_{wj} = 1$, $P_{cj} = P_j$ and $P'_{cj} = P'_j$ for all j where j = 1, 2,... n (see the discussion in Sec. III.B.2 for the logic in support of this approach). This procedure yields the reliability, $\mathcal{R}_{on}$ of the output information from the nth 'dual' element of the system (using Eqs. III-49 and III-50) as:

$$\mathcal{R}_{on} = R_{on}R'_{on}X_n + (R_{on} + R'_{on})Y_n + Z_n \tag{III-51}$$

where

$$X_n = 1 - Q_{wn} + Q_{wn}(Q_{cn} - 2Q_{1en} + Q_{2en}) \tag{III-52}$$

$$Y_n = Q_{wn}(Q_{1en} - Q_{2en}) \tag{III-53}$$

$$Z_n = Q_{wn}Q_{2en} \tag{III-54}$$

$$R_{on} = \mathcal{R}_{on-1}P_n \tag{III-55}$$

$$R'_{on} = \mathcal{R}_{on-1}P'_n \tag{III-56}$$

The above symbols are defined in Table I (see also Fig. 20).

A computer program (see Appendix B) was written to study the variation of the output reliability, $\mathcal{R}_{on}$ with the number (n) of 'dual' elements in the system (see Fig. 20). Figure

24 shows this variation for different values of the reliability parameters of the components under the assumption that all the control components are identical and that the performance components of the system are also identical to each other (i.e. $P_j = P_j' = P$ and $Q_{cj} = Q_c$, $Q_{1ej} = Q_{1e}$, and $Q_{2ej} = Q_{2e}$ for all j).

It is observed from Fig. 24 that as the probabilities Q_{1e} and Q_{2e} increase, the output reliability increases for a given number of 'dual' elements in the system. It is also observed that for smaller values of Q_{1e} and Q_{2e}, $\mathcal{R}_{on}$ decreases with increasing n (see curves 1-3 in Fig. 24) reaching a lower limit. This is an undesirable feature of a control system, as one would like the output reliability $\mathcal{R}_{on}$ to increase as the number of controls are increased. However, when Q_{1e} and Q_{2e} are large, the output reliability, $\mathcal{R}_{on}$ increases rapidly with increasing n, reaching an upper limit (see curves 4-6 in Fig. 24). It is interesting to note that the upper limit of $\mathcal{R}_{on}$ never approaches unity for $P<1$ and $Q_c<1$ (see curves 1-4 in Fig. 24). However, when $P = 1$ and $Q_c = 1$, the upper limit becomes unity. This result implies that a high reliability in the output information can be acheived by increasing the number of dual elements in series, <u>only</u> when the correct input information is not further contaminated. In this case (i.e. when $P = 1$, and $Q_c = 1$), the output reliability, $\mathcal{R}_{on}$ approaches

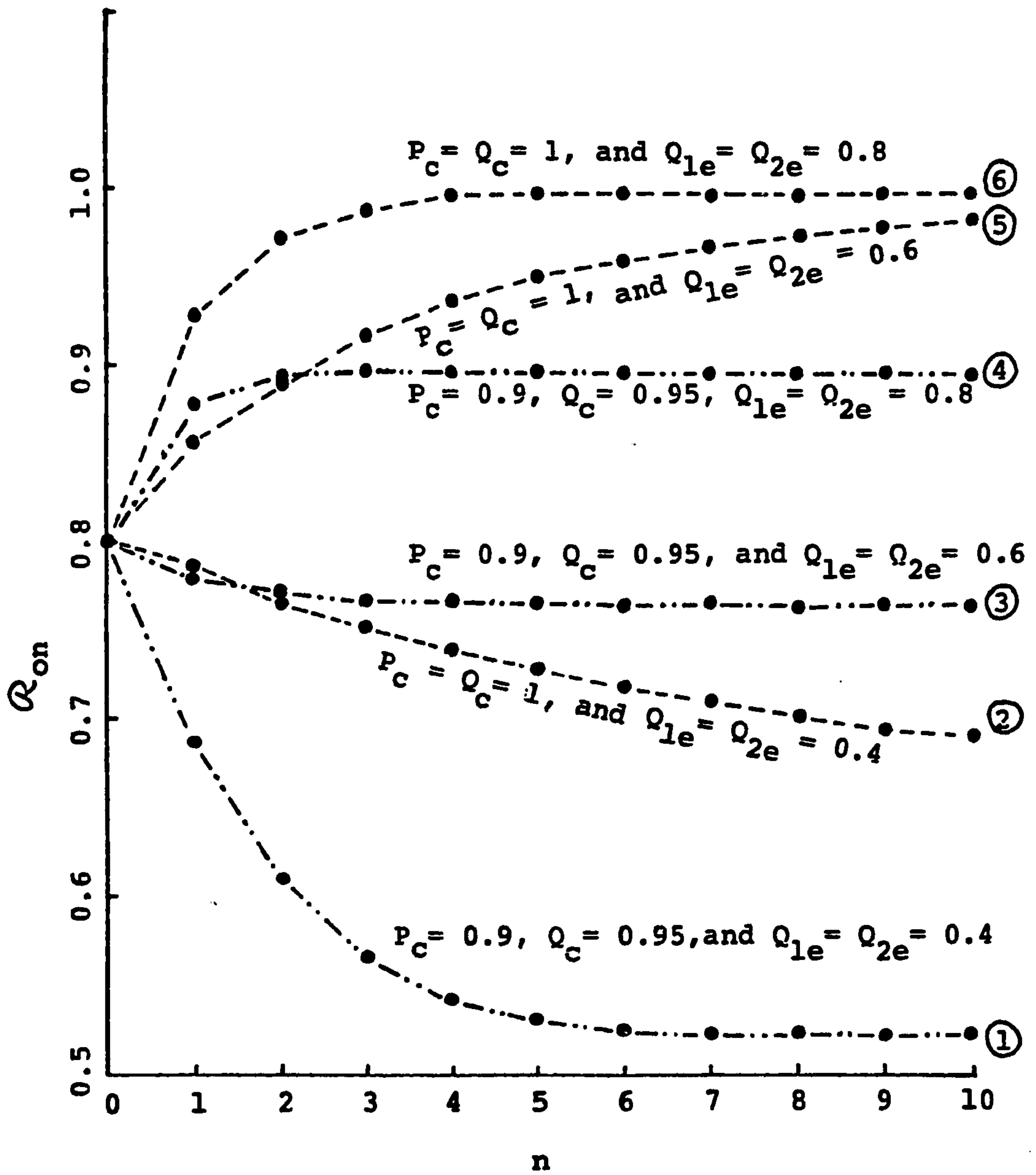


Figure 24: Variation of $\mathcal{R}_{on}$ with n, where n represents the number of identical dual performance elements in series with control components at the junctions (Fig. 20) for $R_i = 0.8$.

unity very quickly as the number of dual elements increases (see curves 5 and 6 in Fig. 24).

2. 'Dual' Control Elements in Series with Performance Components at the Junction

This subsection is devoted to the development and discussion of a reliability model of a system consisting of 'dual' control elements in series with performance components at the junctions. A schematic representation of such a system is given in Fig. 21. The reliability results for this system can be obtained directly from Eqs.(III-39)-(III-50) by substituting $Q_{1ej} = Q_{2ej} = 0$, $Q_{wj} = 1$ and $Q_{cj} = Q$ for all j where j = 1, 2,... n (see the discussion in Sec.III.B.2 for the logic in support of this approach). Thus, one obtains from Eq. (III-49), the output reliability $\mathcal{R}_{on}$ from the nth element as:

$$\mathcal{R}_{on} = R_{on}R_{on}'Q_n \qquad \text{(III-57)}$$

where

$$R_{on} = \mathcal{R}_{on-1}\,[1 - P_{wn} + P_{wn}(P_{cn} - P_{en})] + P_{en}P_{wn} \qquad \text{(III-58)}$$

and

$$R_{on}' = \mathcal{R}_{on-1}\,[1 - P_{wn}' + P_{wn}'(P_{cn}' - P_{en}')] + P_{en}'P_{wn}' \qquad \text{(III-59)}$$

The different symbols used above are defined in Table I (see also Fig. 21).

A computer program (see Appendix B) was used to compute values for the output reliability $\mathcal{R}_{on}$ for different values of n, P_c, and P_e. The results were plotted in Fig. 25 against n. It was assumed in the above computation that all the control components of the system are identical to each other, similarly all the performance components are identical to each other (i.e. $P_{wj} = P'_{wj} = P_w$; $P_{cj} = P'_{cj} = P_c$; $P_{ej} = P'_{ej} = P_e$ and $Q_j = Q$ for all j).

It is observed that for $P_c = 0.9$ and for smaller values of P'_e, the output reliability, $\mathcal{R}_{on}$, initially decreases rapidly with an increase in the number of 'dual' elements in the system. However, it fairly quickly approaches a lower limit as n increases further (see curves 1 and 2 in Fig. 25). But for larger values of P_e and for $P_c = 1.0$, $\mathcal{R}_{on}$ increases with increasing n reaching an upper limit. The upper limit depends on Q (see Eq. III-57). As the likelihood of correctly completing performance tasks by the performance components at the junctions increases, the output reliability increases.

An important implication of the above findings is that when a control system is designed as depicted in Fig. 21, the output reliability cannot be increased to unity by just adding more 'dual' elements. In fact the additional gain in the output reliability by increasing n may not be significant, if it ($\mathcal{R}_{on}$) has already approached the upper limit (see curves 3 and 4 in Fig. 25). Also in certain cases where the reliability parameters of the system are not high enough, the output reliability decreases with increasing n. Thus the increase in n, in this case, is dysfunctional (see curves 1 and 2 in Fig. 25).

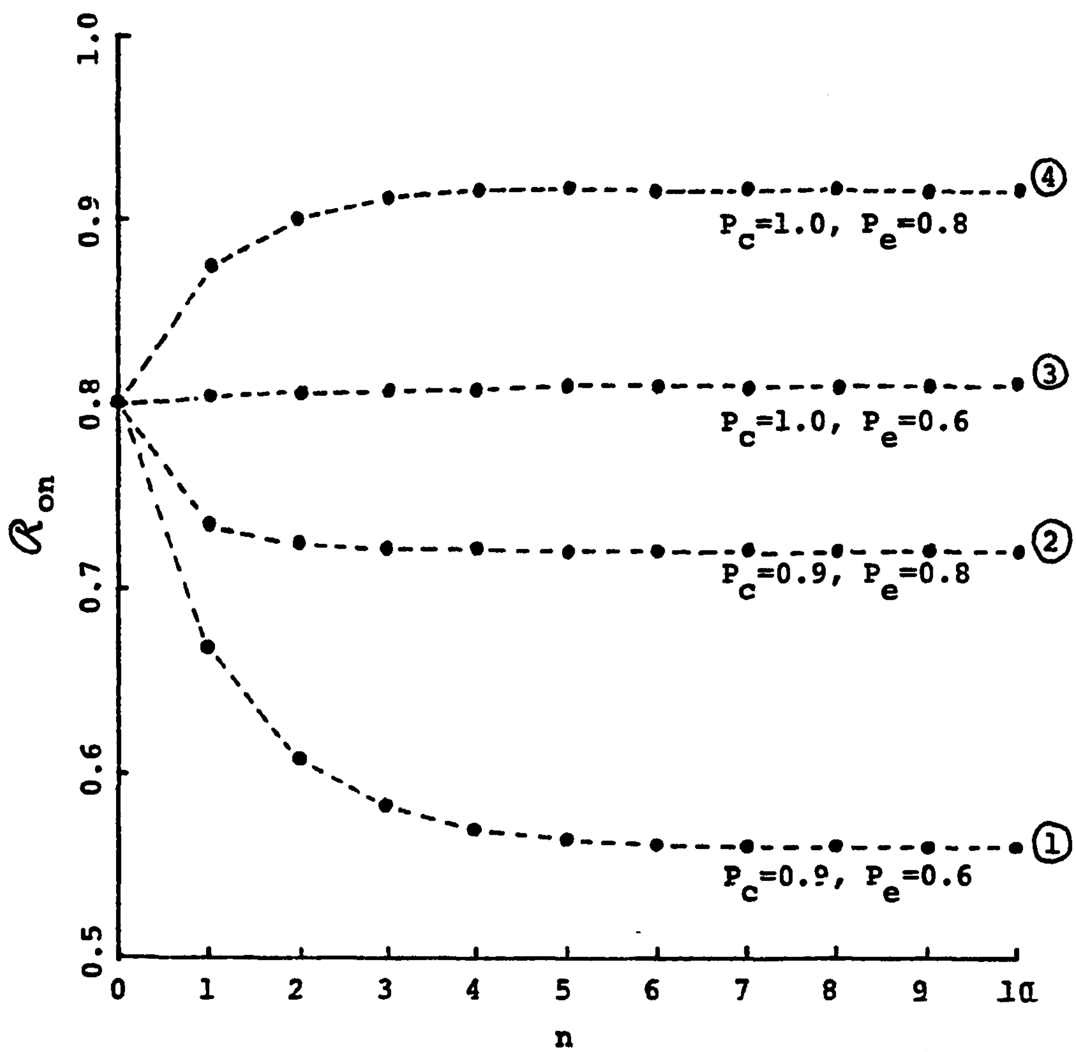


Figure 25: Variation of $\mathcal{R}_{on}$ with n, where n represents the number of identical dual control elements in series with performance components at the junctions (Fig. 21) for P_w = 1.0 and Q = 0.95.

Chapter III has provided reliability models of complex forms of internal control systems. It has also analyzed the effects on output reliability of input reliability parameters and of the different control parameters. The effects of increasing the number of control components in a system with various configurations were also discussed. Several limiting cases were considered. These results provide intuitive support for the models. A field study conducted to validate some of the models is presented in Chapter V.

CHAPTER IV

SENSITIVITY AND COMPARATIVE STRUCTURAL ANALYSES

This chapter presents the results of sensitivity analysis and structural analysis of certain control systems (or subsystems). The detailed procedures for the analyses are also discussed here. The results of sensitivity analysis provide information regarding the relative sensitivity of the output reliability in terms of the other reliability parameters. These results and the related procedures are presented below in Section A. The results of structural analysis allow comparison of the output reliability of alternative subsystem configurations. This analysis is illustrated in Section B.

A. Sensitivity Analysis

This Section demonstrates the usefulness of analyzing the sensitivity of output reliabilities with respect to changes in input and component reliability parameters. Such information should be useful in the design of an internal control system. Also the result should help management in making decisions about personnel training for improving existing control systems.

To illustrate the above assertion, I propose to analyze the sensitivity of the reliability of output information from the following systems or subsystems:

1. A control component system (Fig. 4).
2. A three component system (two performance and one control type) (Fig. 16).
3. A two control component system for disbursement voucher review of quantity of items purchased (Fig. 17).

The reliability, R_o, of output information from a control system depends on the reliability parameters of all components and the reliability of input information. This dependency can be expressed as a function of $\vec{p}$:

$$R_o = f(\vec{p}) \tag{IV-1}$$

where $\vec{p}$ represents a reliability 'vector' consisting of the reliability parameters of the components of the system or subsystem under study and of the input reliability (the reliability of the input information).

A change in R_o can be written in terms of the changes in the reliability parameters as:

$$\Delta R_o = \sum_{j=1}^{n} I_j r_j p_j \tag{IV-2}$$

where

$$I_j = \partial R_o / \partial p_j, \text{ and } r_j = \Delta p_j / p_j \tag{IV-3}$$

$\Delta R_o(j) = I_j r_j p_j$ represents the increase in R_o due to increase in p_j by r_j percent.

In order to develop a reliability importance measure (Birnbaum, 1969) one can set $\Delta R_o(i) = \Delta R_o(j)$ and obtain a ratio of the percentage increases in p_i and p_j:

$$r_i/r_j = I_j p_j / I_i p_i \tag{IV-4}$$

which means that a r_i percent increase in p_i will increase R_o as much as a r_j percent increase in p_j. If $r_i/r_j = 5$, this implies that p_j is more important than p_i; a 1% increase in p_j is equivalent to a 5% increase in p_i. One can define a matrix 'reliability importance ratio matix', representing these ratios as:

$$[c_{ij}] = [r_i/r_j] = [I_j p_j / I_i p_i]$$

or

$$[c_{ij}] = \left[\frac{p_j \partial R_o / \partial p_j}{p_i \partial R_o / \partial p_i}\right] \tag{IV-5}$$

The elements of this matrix can give information at a glance about the relatively important parameters. This concept should become more clear when specific examples are considered later.

1. A Control Component

Consider a control component with input information having reliability R_i, and an output having reliability R_o. The reliability parameters of the control component are represented by P_w, P_c and P_e. The reliability, R_o, of the output information is given by Eq. (II-16):

$$R_o = R_i \left[1 - P_w + P_w(P_c - P_e)\right] + P_e P_w \qquad \text{(IV-6)}$$

The partial derivatives of R_o with respect to different parameters can be written as:

$$\partial R_o/\partial R_i = 1 - P_w + P_w(P_c - P_e) \qquad \text{(IV-7)}$$

$$\partial R_o/\partial P_w = P_e - R_i + (P_c - P_e)R_i \qquad \text{(IV-8)}$$

$$\partial R_o/\partial P_c = R_i P_w \qquad \text{(IV-9)}$$

$$\partial R_o/\partial P_e = (1 - R_i)P_w \qquad \text{(IV-10)}$$

From Eqs.(IV-9) and (IV-10), one knows that $\partial R_o/\partial P_c$ and $\partial R_o/\partial P_e$, both are positive. However, one is larger than the other depending on the value of R_i: for $R_i > 0.5$, $(\partial R_o/\partial P_c)/(\partial R_o/\partial P_e) > 1$ and for $R_i < 0.5$, $(\partial R_o/\partial P_c)/(\partial R_o/\partial P_e) < 1$. As an example, consider $R_i = 0.8$ ($R_i > 0.5$) then, $\partial R_o/\partial P_c = 0.8\ P_w$ and $\partial R_o/\partial P_e = 0.2\ P_w$ i.e. $\partial R_o/\partial P_c$ is four times larger than $\partial R_o/\partial P_e$. This result implies that a change in P_c will make a change in R_o four times as big as the change produced by a similar change in P_e. In other words, R_o is more sensitive to changes in P_c than it is to P_e when $R_i > 0.5$.

However, when $R_o < 0.5$, the relative importance of the two parameters: P_c and P_e is reversed. This is illustrated through the following example. Assume, $R_i = 0.4$, then from Eqs. (IV-9) and (IV-10), one obtains: $\partial R_o/\partial P_c = 0.4\ P_w$ and $\partial R_o/\partial P_e = 0.6\ P_w$ i.e. $(\partial R_o/\partial P_c)/(\partial R_o/\partial P_e) < 1$. This implies that R_o is more sensitive to P_e than it is to P_c. This result has an important implication. When it is known that the input information has a reliability more than 0.5, then P_c (the probability that the control works properly given that the input is correct) is more important than P_e. But when $R_i < 0.5$, P_e (the probability that the control works properly given that the input is <u>incorrect</u>) becomes more important. This result is intuitively appealing. It says that <u>when the input is more likely to be incorrect than correct, the detection and correction or errors (P_e) becomes more important than the process of recognizing correct input (P_c).</u>

The partial derivative, $\partial R_o/\partial P_w$ in Eq. (IV-8) can be either positive or negative depending on the relative magnitude of R_i, P_c and P_e. This provides an interesting result. The negative value of $\partial R_o/\partial P_w$ means that if the probability that the control is in operation increases, the reliability of the output decreases, implying that the control is dysfunctional. That is, it introduces more error than it corrects when $\partial R_o/\partial P_w < 0$.

To illustrate the above point consider an example where

$R_i = 0.8$ and $P_c = 0.9$. These values yield $\partial R_o/\partial P_w = -0.2$ for $P_e = 0.3$ (see Eq. IV-8). In fact the slope, $\partial R_o/\partial P_w$ is always negative for $R_i = 0.8$ and $P_c = 0.9$ if $P_e < 0.4$. This implies that for the parameter values given above the output reliability, R_o is always less than the input reliability R_i ($R_o = 0.78$ for $R_i = 0.8$, $P_c = 0.9$, $P_e = 0.3$ and $P_w = 1.0$; use Eq. IV) for any value of P_w in the range: $1 > P_w > 0$. Whereas, when $P_w = 0$, i.e. when the control is not in operation, $R_o = R_i$. This suggests that the existance of a control with the given parameters further contaminates the input information and reduces the output reliability below the input value. Such a control is dysfunctional.

In order for a control to be effective, we must demand $\partial R_o/\partial P_w > 0$. In general, this condition is achieved when (see Eq. IV-8)

$$P_e - R_i + (P_c - P_e)R_i > 0 \qquad \text{(IV-11)}$$

For $P_c = P_e$, the control will be functional only when $P_c = P_e > R_i$. This result should be useful in deciding about a personnel training program for completing a control process. Again consider the example presented earlier where the control component was dysfunctional when $R_i = 0.8$, $P_c = 0.9$ and $P_e < 0.4$. When P_e is increased such that $P_e > 0.4$, in the example, the control becomes functional (i.e. $\partial R_o/\partial P_w > 0$ for $P_e > 0.4$, $R_o = 0.8$ and $P_c = 0.9$]. This means that the presence of the control component will improve the output reliability. As P_w increases the output reliability increases and reaches a maximum at $P_w=1$.

Thus a higher likelihood (P_w) of a control component being in operation produces a higher output reliability, if the control is functional (i.e. $\partial R_o/\partial P_w > 0$).

The above example demonstrates that by simply increasing P_e from 0.4 to a higher value, the control component can be made functional. Such increases might be achieved by training personnel more thoroughly in techniques for error detection and correction or by improving automated procedures for error detection and correction.

A convenient approach to display and utilize these partial derivatives can be illustrated using the results from our example. Using the partial derivatives of R_o (Eq. IV-6) with respect to the different component parameters (Eq. IV-8, 9, and 10) and the partial derivative with respect to R_i (Eq. IV-7), one can obtain the following expression for the 'reliability importance ratio matrix', C, using Eq. (IV-5):

$$C = \begin{array}{c} \\ R_i \\ P_w \\ P_c \\ P_e \end{array} \begin{array}{c} \begin{array}{cccc} \quad R_i \quad & \quad P_w \quad & \quad P_c \quad & \quad P_e \quad \end{array} \\ \begin{bmatrix} 1 & \dfrac{R_o - R_i}{R_o - P_eP_w} & \dfrac{P_cR_iP_w}{R_o - P_eP_w} & \dfrac{P_eP_w(1-R_i)}{R_o - P_eP_w} \\ \dfrac{R_o - P_eP_w}{R_o - R_i} & 1 & \dfrac{P_cR_iP_w}{R_o - R_i} & \dfrac{P_eP_w(1-R_i)}{R_o - R_i} \\ \dfrac{R_o - P_eP_w}{P_cR_iP_w} & \dfrac{R_o - R_i}{P_cR_iP_w} & 1 & \dfrac{P_eP_w(1-R_i)}{P_cR_iP_w} \\ \dfrac{R_o - P_eP_w}{P_eP_w(1-R_i)} & \dfrac{R_o - R_i}{P_eP_w(1-R_i)} & \dfrac{P_cR_iP_w}{P_eP_w(1-R_i)} & 1 \end{bmatrix} \end{array} \qquad \text{(IV-12)}$$

For $R_i = 0.8$, $P_w = 0.98$, $P_c = 0.95$, and $P_e = 0.9$, the matrix in Eq. (IV-12) becomes:

$$C = \begin{array}{c} \\ R_i \\ P_w \\ P_c \\ P_e \end{array} \begin{array}{c} \begin{array}{cccc} R_i & P_w & P_c & P_e \end{array} \\ \begin{bmatrix} 1.000 & 2.486 & 13.493 & 3.196 \\ 0.402 & 1.000 & 5.429 & 1.286 \\ 0.074 & 0.184 & 1.000 & 0.237 \\ 0.313 & 0.778 & 4.223 & 1.000 \end{bmatrix} \end{array} \qquad \text{(IV-13)}$$

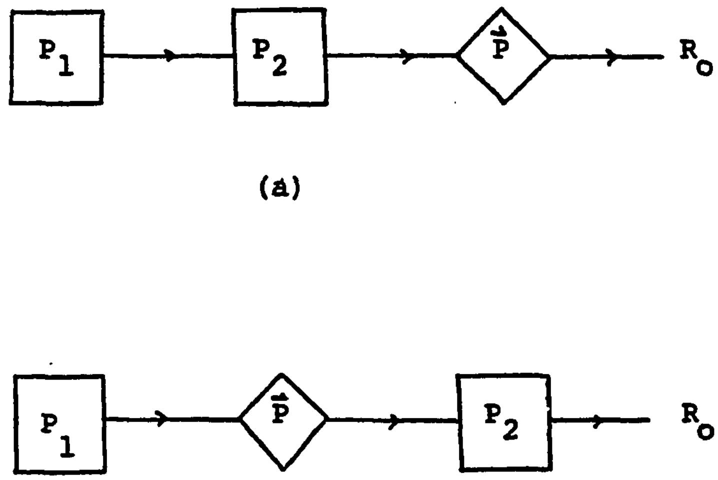
The interpretation of the numbers in different columns can be illustrated through the following example. The numbers in the third column mean that a 1.0% increase in P_c would increase R_o as much as a 13.49% increase in R_i, a 5.43% in P_w, or a 4.22% in P_e. It should be noted that the numbers in one row are reciprocals of the numbers in the corresponding column. For instance, the numbers in the third row are reciprocals of the numbers in the third column. After examining all elements of the matrix, one can tell that parameter P_c is the most important parameter, because the èlements in the corresponding column are the largest or the numbers in the corresponding row are the smallest. This result would be important when management wants to efficiently improve the reliability of the output information, relative to changes in input or component reliabilities.

2. Three Components (Two Performance and One Control Type) in Series

In this section, I want to present a sensitivity analysis of a subsystem consisting of three components: two performance types and one control type. These components are connected in series (Fig. 26). The main reason for presenting the sensitivity analysis for this subsystem is to demonstrate that the relative importance of the reliability parameters of a system or subsystem changes as the configuration of the system or subsystem changes, i.e. as the components of the system or subsystem are rearranged.

Here, I have chosen two of the possible configurations of the three components connected in series, for this study. In the first configuration (Fig. 26a), the first two components are of the performance type. They are connected in series. This combination is then connected to a third element, a control component, also in series.

In the second configuration (Fig. 26.b), the control component is placed between the two performance components. This implies that the information generated by the first element is being checked for accuracy by the second element, a control component. The resulting information is then passed on to a third element (Performance type) to complete the task. However, the situation is different in the first configuration (Fig. 26.a) case. There, the first element (performance type) generates a piece of information, and the second element (per-



(a)

(b)

Figure 26: Three components (two performance type and one control type) in series.

formance type) completes a task using this information. The resulting information goes to a third element, the control component, which makes a decision about whether the input information is correct and then takes appropriate actions based on its findings.

It is shown through the present study that a configurational change in a system or subsystem will not only change the relative importance of the different reliability parameters, but would also affect the overall reliability of the output information. The sensitivity analysis for the two cases are presented below.

<u>Configuration (a)</u>: Consider first the subsystem in Fig. 26.a. The reliability of the output information from this subsystem can be written in terms of the reliability parameters of the system components as:

$$R_o = P_1P_2\left[1 - P_w + P_w(P_c - P_e)\right] + P_eP_w \qquad \text{(IV-14)}$$

where P_1 and P_2 are the reliability parameters of the two performance components and P_w, P_c, and P_e represent the characteristic reliability parameters of the control component (see Table I for detail definitions). The above relation (Eq. IV-14) is easily derived by using the approach adopted in Chapter III.

The 'reliability importance ratio matrix,' C (Eq. IV-5) can be determined by first obtaining the following partial derivatives of R_o (see Eq. IV-14).

$$P_1(\partial R_o/\partial P_1) = R_o - P_eP_w \qquad \text{(IV-15)}$$

$$P_2(\partial R_o/\partial P_2) = R_o - P_eP_w \qquad \text{(IV-16)}$$

$$P_c(\partial R_o/\partial P_c) = P_1P_2P_cP_w \qquad \text{(IV-17)}$$

$$P_e(\partial R_o/\partial P_e) = (1 - P_1P_2)P_eP_w \qquad \text{(IV-18)}$$

$$P_w(\partial R_o/\partial P_w) = R_o - P_1P_2 \qquad \text{(IV-19)}$$

and then substituting them into Eq. (IV-5):

$$C = \begin{array}{c} \\ P_1 \\ P_2 \\ P_c \\ P_e \\ P_w \end{array} \begin{array}{c} \begin{array}{ccccc} P_1 & P_2 & P_c & P_e & P_w \end{array} \\ \begin{bmatrix} 1 & 1 & \frac{P_1P_2P_cP_w}{R_o-P_eP_w} & \frac{(1-P_1P_2)P_eP_w}{R_o-P_eP_w} & \frac{R_o-P_1P_2}{R_o-P_eP_w} \\ 1 & 1 & \frac{P_1P_2P_cP_w}{R_o-P_eP_w} & \frac{(1-P_1P_2)P_eP_w}{R_o-P_eP_w} & \frac{R_o-P_1P_2}{R_o-P_eP_w} \\ \frac{R_o-P_eP_w}{P_1P_2P_cP_w} & \frac{R_o-P_eP_w}{P_1P_2P_cP_w} & 1 & \frac{(1-P_1P_2)P_eP_w}{P_1P_2P_cP_w} & \frac{R_o-P_1P_2}{P_1P_2P_cP_w} \\ \frac{R_o-P_eP_w}{(1-P_1P_2)P_eP_w} & \frac{R_o-P_eP_w}{(1-P_1P_2)P_eP_w} & \frac{P_1P_2P_cP_w}{(1-P_1P_2)P_eP_w} & 1 & \frac{R_o-P_1P_2}{(1-P_1P_2)P_eP_w} \\ \frac{R_o-P_eP_w}{R_o-P_1P_2} & \frac{R_o-P_eP_w}{R_o-P_1P_2} & \frac{P_1P_2P_cP_w}{R_o-P_1P_2} & \frac{(1-P_1P_2)P_eP_w}{R_o-P_1P_2} & 1 \end{bmatrix} \end{array}$$

(IV-20)

The above matrix can now be used to discuss special cases to show the relative importance of the different reliability parameters. For example consider the following values of the parameters: $P_1 = 0.85$, $P_2 = 0.9$, $P_c = 0.95$, $P_e = 0.9$, and $P_w = 0.95$. These values yield:

$$R_o = 0.9296 \tag{IV-21}$$

and

$$C = \begin{array}{c} \\ P_1 \\ P_2 \\ P_c \\ P_e \\ P_w \end{array} \begin{array}{c} \begin{array}{ccccc} P_1 & P_2 & P_c & P_e & P_w \end{array} \\ \begin{bmatrix} 1.000 & 1.000 & 9.256 & 2.694 & 2.207 \\ 1.000 & 1.000 & 9.256 & 2.694 & 2.207 \\ 0.108 & 0.108 & 1.000 & 0.291 & 0.238 \\ 0.371 & 0.371 & 3.436 & 1.000 & 0.819 \\ 0.453 & 0.453 & 4.195 & 1.221 & 1.000 \end{bmatrix} \end{array} \tag{IV-22}$$

As discussed in the previous section, the parameter associated with the column containing the biggest numbers is the most important parameter. Here, parameter P_c seems to be the most important one, as the column corresponding to P_c(3rd column in Eq. IV-22) contains the largest numbers. The interpretation of the numbers in column three of matrix C is that a one percent increase in P_c would increase R_o as much as a 9.256% increase in P_1, 9.256% in P_2, 3.436% in P_e or a 4.195% increase in P_w. Thus for the given configuration of the subsystem and given parameter values, P_c is the most important parameter. The next most important parameter in this case is P_e. However, the relative importance of the parameters may change when the values of the parameters change without any configurational change. This is demonstrated through the following example.

Consider the following values of the parameters: $P_1 = 0.45$, $P_2 = 0.9$, $P_c = 0.95$, $P_e = 0.9$, and $P_w = 0.95$. Substituting these values in Eqs.(IV-14) and (IV-20), one gets:

$$R_o = 0.8945 \qquad \text{(IV-23)}$$

and

$$C = \begin{array}{c} \\ P_1 \\ P_2 \\ P_c \\ P_e \\ P_w \end{array} \begin{array}{c} \begin{array}{ccccc} P_1 & P_2 & P_c & P_e & P_w \end{array} \\ \begin{bmatrix} 1.000 & 1.000 & 9.256 & 14.315 & 12.396 \\ 1.00 & 1.000 & 9.256 & 14.315 & 12.396 \\ 0.108 & 0.108 & 1.000 & 1.546 & 1.339 \\ 0.0699 & 0.0699 & 0.647 & 1.000 & 0.866 \\ 0.0807 & 0.0807 & 0.747 & 1.155 & 1.000 \end{bmatrix} \end{array} \qquad \text{(IV-24)}$$

By an inspection of the elements in different columns of matrix C, one finds that P_e, P_w, and P_c are the parameters in the decreasing order of their importance. This ordering is quite different from the one obtained in the previous case (P_c, P_e and P_w in order of decreasing importance; see Eq. IV-22). This change in the relative importance of the parameters P_c, P_e and P_w can be interpreted in the following way. In the former case where $P_1 = 0.85$ and $P_2 = 0.9$, the reliability of the input information to the control component was 0.765 i.e. the input to the control element was more than 50% correct. Therefore, it is important that when the control works it does not further contaminate the information by making a wrong judgment when the input is correct. The parameter associated with this process is P_c and thus P_c is the most important parameter in this case. In the latter case where $P_1 = 0.45$ and $P_2 = 0.9$ the input information to the control element has reliability 0.405; i.e. the input information is less than 50% correct. Therefore, the correction process by the control element when the input is in error is more important than the correct judgment when the input is error free. This implies that P_e is more important than P_c. Also one finds that P_w is more important than P_c, in the latter case. This result means that the probability that the control is in operation is more important than the probability that the control operates properly when the input is correct. This is again due to the fact that the input to the control element contains more incorrect information than correct information, and therefore the correction process and the process that the control is in operation become important. A similar result was obtained in the case of a single control component as discussed in Sec. IV. A.1. A change in the relative importance of the parameter due to a change in the configuration of the system is demonstrated in the following subsection.

Configuration (b): Here the control component is placed between the two performance components (Fig. 16.b). The reliability of the final output information from this subsystem can be again obtained by using the approach of Chapter III. The result is

$$R_o = P_1P_2\left[1 - P_w + P_w(P_c - P_e)\right] + P_2P_eP_w \qquad \text{(IV-25)}$$

where P_1 and P_2 are the reliability parameters of the two performance components and P_w, P_c and P_e are the characteristic reliabiltiy parameters of the control component.

The following partial derivatives of R_o (Eq. IV-25) are used in determining the 'reliability importance ratio matrix' C for this case:

$$P_1\left(\frac{\partial R_o}{\partial P_1}\right) = R_o - P_2P_eP_w \qquad \text{(IV-26)}$$

$$P_2\left(\frac{\partial R_o}{\partial P_2}\right) = R_o \qquad \text{(IV-27)}$$

$$P_c\left(\frac{\partial R_o}{\partial P_c}\right) = P_1P_2P_cP_w \qquad \text{(IV-28)}$$

$$P_e\left(\frac{\partial R_o}{\partial P_e}\right) = (1-P_1)P_eP_2P_w \qquad \text{(IV-29)}$$

$$P_w\left(\frac{\partial R_o}{\partial P_w}\right) = R_o - P_1P_2 \qquad \text{(IV-30)}$$

The resulting matrix is:

$$C = \begin{array}{cc} & \begin{array}{ccccc} P_1 & P_2 & P_c & P_e & P_w \end{array} \\ \begin{array}{c} P_1 \\ \\ P_2 \\ \\ P_c \\ \\ P_e \\ \\ P_w \end{array} & \begin{bmatrix} 1 & \dfrac{R_o}{R_o-P_2P_eP_w} & \dfrac{P_1P_2P_cP_w}{R_o-P_2P_eP_w} & \dfrac{(1-P_1)P_2P_eP_w}{R_o-P_2P_eP_w} & \dfrac{R_o-P_1P_2}{R_o-P_2P_eP_w} \\ \dfrac{R_o-P_2P_eP_w}{R_o} & 1 & \dfrac{P_1P_2P_cP_w}{R_o} & \dfrac{(1-P_1)P_2P_eP_w}{R_o} & \dfrac{R_o-P_1P_2}{R_o} \\ \dfrac{R_o-P_2P_eP_w}{P_1P_2P_cP_w} & \dfrac{R_o}{P_1P_2P_cP_w} & 1 & \dfrac{(1-P_1)P_2P_eP_w}{P_1P_2P_cP_w} & \dfrac{R_o-P_1P_2}{P_1P_2P_cP_w} \\ \dfrac{R_o-P_2P_eP_w}{(1-P_1)P_2P_eP_w} & \dfrac{R_o}{(1-P_1)P_2P_eP_w} & \dfrac{P_1P_2P_cP_w}{(1-P_1)P_2P_eP_w} & 1 & \dfrac{R_o-P_1P_2}{(1-P_1)P_2P_eP_w} \\ \dfrac{R_o-P_2P_eP_w}{R_o-P_1P_2} & \dfrac{R_o}{R_o-P_1P_2} & \dfrac{P_1P_2P_cP_w}{R_o-P_1P_2} & \dfrac{(1-P_1)P_2P_eP_w}{R_o-P_1P_2} & 1 \end{bmatrix} \end{array} \tag{IV-31}$$

For the following values of the parameters: $P_1 = 0.85$, $P_2 = 0.9$, $P_c = 0.95$, $P_e = 0.9$ and $P_w = 0.95$, R_o and C become:

$$R_o = 0.8441 \tag{IV-32}$$

$$C = \begin{array}{cc} & \begin{array}{ccccc} P_1 & P_2 & P_c & P_e & P_w \end{array} \\ \begin{array}{c} P_1 \\ P_2 \\ P_c \\ P_e \\ P_w \end{array} & \begin{bmatrix} 1.000 & 11.316 & 9.256 & 1.548 & 1.06 \\ 0.088 & 1.000 & 0.818 & 0.137 & 0.094 \\ 0.108 & 1.223 & 1.000 & 0.167 & 0.115 \\ 0.646 & 7.313 & 5.981 & 1.000 & 0.685 \\ 0.943 & 10.673 & 8.730 & 1.46 & 1.000 \end{bmatrix} \end{array} \tag{IV-33}$$

Inspecting the columns of the above matrix, one finds that P_2, $P_c^{'}$, P_e, P_w and P_1 are the parameters in decreasing order of their importance. Whereas in the previous case (configuration (a), Fig. 16.a) for the same values of the parameters, the order was: P_c, P_e, P_w and P_1 or P_2 (see matrix columns in Eq. IV-22). This comparison shows that the relative importance of the parameters changes as the configuration of systems changes. It is also observed from Eqs. (IV-21) and (IV-32) that, for the same values of the reliability parameters of the components, the output reliability, R_o is higher for configuration (a) (Fig. 26.a) than what it is for configuration (b) (Fig. 26.b).

The above information seems useful when management is redesigning a control system and has a limited budget. The management can choose the control configuration that yields higher output reliability and then allocate resources to improve those parameters which are relatively more important in terms of absolute reliability. Use of the importance ratio as a guide to the allocation of resources should be tempered by cost considerations (which are beyond the scope of this study) unless management is willing to assume that the effectiveness of resources so consumed is proportional to the importance ratio. That is, that a dollar spent on P_2 is 11.316 times as effective with respect to R_o as would be the same dollar spent on P_1, etc.

3. A Disbursement Voucher

Review Illustration

The sensitivity analysis of a review process for the quantity of items purchased, in the preparation of a disbursement voucher is presented here. Generally, a voucher clerk receives three inputs: an approved purchase order, a vendor's invoice and a receiving report. In order to determine the correct quantity of items for payment, he compares the three pieces of input information to assume agreement in terms of the quantity of items ordered, shipped (by the vendor), and received. A schematic diagram for such a system is given in Fig. 27.

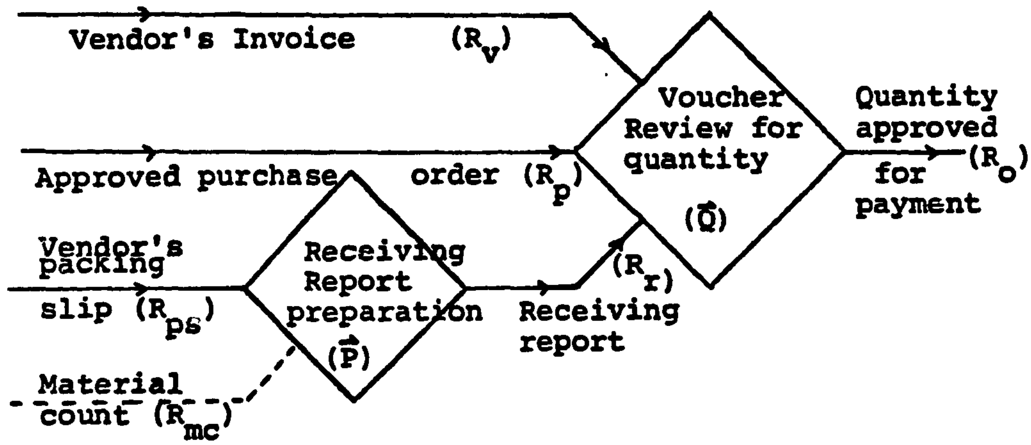


Figure 27: A subsystem for approving quantity of items purchased for payment.

The receiving report preparation process functions like a control component for the error type (error in the quantity of items shipped and/or received) considered here. In general this component has two inputs: 1) the number of items shipped per the vendor's packing slip and 2) the number of items counted physically by the receiver. There are two possible structures and hence two different models of this component based on the order in which the material counting is done. In one case the material counting is done before seeing the quantity on the vendor's packing slip. In the other, the counting is done after seeing the information on the packing slip. In the former case, the result of material counting is independent of the information on the packing slip. But in the later case, the result may be influenced by the information on the packing slip. A detailed comparison of the two reliability models of this component is presented in Section B of this chapter. This comparison illustrates the effect on output reliability associated with structural changes in a system (or subsystem).

For the present study (sensitivity analysis), it is assumed that the material counting is done independently of the information on the packing slip. It is also assumed that the component uses the number obtained through the material count, as a standard for making the receiving report. This assumption makes the reliability, R_{mc} of the information obtained through the material count, unity. This effectively reduces the number of input channels, in the receiving report subsystem from two to one. This can be demonstrated by setting one of the input reliability parameters to unity in the model (Eq. II-24) of a two input control component developed in Section II.C.2.b.

The entire two control element system (Fig. 27) consists of the receiving report preparation element and its relation to the voucher review control component. The voucher review control component has two inputs in addition to the output from the receiving report preparation element. For simplicity, it is assumed that the probability that the two controls are in operation is unity. Therefore, the parameters that are of interest are : R_v, R_p, R_{ps}, P_c, P_e, R_r, Q_c, Q_{1e}, Q_{2e} and Q_{3e}, where R_v, R_p, R_{ps}, and R_r are respectively the reliability parameters of the vendor's invoice, approved purchase order, vendor's packing slip and of the receiving report. The other parameters that are characteristic of the control elements are defined in Table I.

The reliability of the receiving report for the quantity of items received can be written in the following form (using Eq. II-16 and $P_w = 1$):

$$R_r = R_v P_c + (1 - R_v) P_e \qquad \text{(IV-34)}$$

where R_{ps}, the reliability of the packing slip is replaced by R_v, the reliability of the vendor's invoice. This can be done when the two documents are identical (packing slip is a carbon copy of the vendor's invoice) in terms of their information regarding the quantity of material shipped.

The reliability, R_o of the output information (the quantity approved for payment) from the voucher review element can be determined in terms of the reliability parameters of the inputs and of the control components by using Eq.(II-31) with $n = 3$, and $Q_w = 1$. The result is

$$R_o = R_v R_p R_r Q_c + Q_{1e}\left[(1-R_v)R_p R_r + (1-R_p)R_v R_r + (1-R_r)R_p R_v\right]$$
$$+ Q_{2e}\left[(1-R_v)(1-R_p)R_r + (1-R_v)(1-R_r)R_p + (1-R_p)(1-R_r)R_v\right]$$
$$+ Q_{3e}\left[(1-R_v)(1-R_p)(1-R_r)\right] \qquad \text{(IV-35)}$$

One can, now, perform the sensitivity analysis by the usual method of taking the partial derivatives and studying the relative slopes. The second method (Birnbaum 1969), using the 'reliability importance ratio matrix' C, however, presents a direct result regarding the relative importance of the different parameters. The following partial derivatives are used in obtaining matrix C (Eq. IV-5).

$$R_v(\partial R_o/\partial R_v) = R_o - Q_{1e}R_p R_r - Q_{2e}\left[(1-R_r R_p + (1-R_p)R_r\right]$$
$$- Q_{3e}(1-R_r)(1-R_p) \qquad \text{(IV-36)}$$

$$R_p(\partial R_o/\partial R_p) = R_o - Q_{1e}R_v R_r - Q_{2e}\left[(1-R_r)R_v + (1-R_v)R_r\right]$$
$$- Q_{3e}(1-R_r)(1-R_v) \qquad \text{(IV-37)}$$

$$R_r(\partial R_o/\partial R_r) = R_o - Q_{1e}R_v R_p - Q_{2e}\left[(1-R_v)R_p + (1-R_p)R_v\right]$$
$$- Q_{3e}(1-R_v)(1-R_p) \qquad \text{(IV-38)}$$

$$P_c(\partial R_o/\partial P_c) = P_c(\partial R_r/\partial P_c)(\partial R_o/\partial R_r)$$
$$= (P_c R_v/R_r)R_r(\partial R_o/\partial R_r) \qquad \text{(IV-39)}$$

$$P_e(\partial R_o/\partial P_e) = P_e(\partial R_r/\partial P_e)(\partial R_o/\partial R_r)$$
$$= \left(P_e(1-R_v)/R_r\right)R_r(\partial R_o/\partial R_r) \qquad \text{(IV-40)}$$

$$Q_c(\partial R_o/\partial Q_c) = Q_c R_v R_p R_r \tag{IV-41}$$

$$Q_{1e}(\partial R_o/\partial Q_{1e}) = Q_{1e}\Big[(1-R_v)R_pR_r + (1-R_r)R_vR_p + (1-R_p)R_vR_r\Big] \tag{IV-42}$$

$$Q_{2e}(\partial R_o/\partial Q_{2e}) = Q_{2e}\Big[(1-R_v)(1-R_p)R_r + (1-R_v)(1-R_r)R_p + (1-R_p)(1-R_r)R_v\Big] \tag{IV-43}$$

$$Q_{2e}(\partial R_o/\partial Q_{3e}) = Q_{3e}(1-R_v)(1-R_p)(1-R_r) \tag{IV-44}$$

The numerical values of the reliability importance matrix C for different values of the parameters are given in Tables II - VI.

Table II suggests that for the given values of the parameters, the relative importance of these parameters are given as follows: $Q_c > Q_{1e} > R_p > R_v > P_c > Q_{2e} > P_e > Q_{3e}$. The most and the least important parameters are Q_c and Q_{3e}, respectively. This result can be interpreted as follows. Since, all three inputs to the voucher review element are highly reliable ($R_v = 0.92$, $R_p = 0.95$, $R_r = 0.96$) it is much more important that the reviewing element makes a correct decision given the inputs are correct (process associated with Q_c) than the process of correcting errors when all the inputs are incorrect (process associated with Q_{3e}). It is also observed that the

TABLE II

The Reliability Importance Ratio Matrix for $R_v = 0.92$, $R_p = 0.95$, $P_c = 0.96$, $P_e = 0.93$, $Q_c = 0.97$, $Q_{1e} = 0.94$, $Q_{2e} = 0.9$, and $Q_{3e} = 0.85$

	R_v	R_p	P_c	P_e	Q_c	Q_{1e}	Q_{2e}	Q_{3e}
R_v	1.000	1.043	0.972	0.082	28.54	5.08	0.285	0.0051
R_p	0.959	1.000	0.932	0.079	27.369	4.877	0.273	0.0049
P_c	1.029	1.073	1.000	0.084	29.367	5.233	0.293	0.0052
P_e	12.217	12.738	11.871	1.000	348.620	62.117	3.480	0.0619
Q_c	0.035	0.037	0.034	0.003	1.000	0.178	0.010	0.0002
Q_{1e}	0.197	0.205	0.191	0.016	5.612	1.000	0.056	0.0010
Q_{2e}	3.511	3.661	3.412	0.287	100.191	17.852	1.000	0.0178
Q_{3e}	197.350	205.762	191.759	16.154	5631.469	1003.409	56.207	1.0000

TABLE III

The Reliability Importance Ratio Matrix for $R_v = 0.55$, $R_p = 0.6$, $P_c = 0.96$, $P_e = 0.93$, $Q_c = 0.97$, $Q_{1e} = 0.94$, $Q_{2e} = 0.9$, $Q_{3e} = 0.85$

	R_v	R_p	P_c	P_e	Q_c	Q_{1e}	Q_{2e}	Q_{3e}
R_v	1.000	1.107	1.070	0.848	15.951	23.826	9.315	0.4310
R_p	0.904	1.000	0.967	0.766	14.413	21.529	8.417	0.3894
P_c	0.934	1.034	1.000	0.793	14.904	22.263	8.704	0.4027
P_e	1.179	1.305	1.262	1.000	18.804	28.088	10.981	0.5080
Q_c	0.063	0.069	0.067	0.053	1.000	1.494	0.584	0.0270
Q_{1e}	0.042	0.046	0.045	0.036	0.669	1.000	0.391	0.0181
Q_{2e}	0.107	0.119	0.115	0.091	1.712	2.558	1.000	0.0463
Q_{3e}	2.320	2.568	2.483	1.968	37.014	55.287	21.615	1.0000

parameter P_c is more important than P_e. Here again, since the input information to the receiving element is highly reliable ($R_v = 0.92$), the correct decision process when the input is correct becomes more important than the process of correcting errors when the input is in error.

When R_v and R_p are changed to 0.55 and 0.6, respectively, leaving the other parameter values unchanged, the order of importance of the parameters becomes (see Table III): $Q_{1e} \succ Q_c \succ Q_{2e} \succ R_p \succ P_c \succ R_v \succ P_e \succ Q_{3e}$. This time the most and the least important parameters are Q_{1e} and Q_{3e}, respectively. The parameter Q_c is now in second place, because the values of the reliabilities of the inputs have decreased considerably (R_v has changed from 0.92 to 0.55 and R_p from 0.95 to 0.6) and thus the error correction process when one of the inputs is in error has become more important than the process of making correct decision when the inputs are correct (the process associated with Q_c). Also Q_{2e} is in the third position in relative importance order, a considerably higher rank than it had in the previous case. This is again because the two inputs are not very reliable ($R_v = 0.55$ and $R_p = 0.6$).

The value of R_v and R_p were further decreased to 0.4 and 0.45, respectively, keeping all other parameter values unchanged. The 'reliability importance ratio matrix', C in Table IV for this case suggests the following order of the parameters for their relative importance: $Q_{1e} \succ Q_{2e} \succ Q_c \succ P_e \succ R_p \succ Q_{3e} \succ P_c \succ R_v$. As expected, the parameters Q_{1e} and Q_{2e} are the first

TABLE IV

The Reliability Importance Ratio Matrix for $R_v = 0.4$, $R_p = 0.45$, $P_c = 0.96$, $P_e = 0.93$, $Q_c = 0.97$, $Q_{1e} = 0.94$, $Q_{2e} = 0.9$, and $Q_{3e} = 0.85$.

	R_v	R_p	P_c	P_e	Q_c	Q_{1e}	Q_{2e}	Q_{3e}
R_v	1.000	1.141	1.104	1.605	11.397	30.745	21.158	1.1273
R_p	0.877	1.000	0.968	1.407	9.992	26.955	18.550	0.9844
P_c	0.906	1.033	1.000	1.453	10.321	27.843	19.161	1.0209
P_e	0.623	0.711	0.688	1.000	7.103	19.161	13.186	0.7026
Q_c	0.088	0.100	0.097	0.141	1.000	2.698	1.857	0.0990
Q_{1e}	0.033	0.037	0.036	0.052	0.371	1.000	0.688	0.0367
Q_{2e}	0.047	0.054	0.052	0.076	0.539	1.453	1.000	0.0533
Q_{3e}	0.887	1.012	0.980	1.423	10.110	27.273	18.769	1.0000

and second most important parameters. The correction process is more important when the inputs have very low reliabilities ($R_v = 0.4$, $R_p = 0.45$). Also Q_{3e} is no longer the least important parameter. It is also interesting to note the change in the order of P_c and P_e. Here P_e is more important than P_c. The reason is again obvious. It is because the input information to the receiving element is highly unreliable ($R_v = 0.4$) and hence the correction process when the input is in error (the process associated with P_e), becomes more important than the process of making a correct decision when the input is correct (the process associated with P_c).

Table V presents the elements of the matrix C for $R_v = 0.45$ and $R_p = 0.9$ with the other parameter values unchanged. The following order represents the relative importance of the different parameters: $Q_{1e} \succ Q_c \succ Q_{2e} \succ R_p \succ P_e \succ P_c \succ R_v \succ Q_{3e}$. Since one of the inputs to the voucher review element is highly unreliable ($R_v = 0.45$), Q_{1e} becomes the most important parameter. Also P_e is more important than P_c. Again it is due to R_v being less than 0.5. It is observed that R_p and R_v occupy the 4th and 7th positions, respectively, in their relative importance order. If one follows the relative orders of R_p and R_v in all the previous cases, one finds that when $R_p > R_v$, R_p is more important than R_v. However, when $R_p < R_v$ their relative importance order is reversed as seen below.

In the last case considered here, where R_v and R_p are changed to 0.9 and 0.45, respectively and the other parameter

TABLE V

The Reliability Importance Ratio Matrix for $R_v = 0.45$, $R_p = 0.9$, $P_c = 0.96$, $P_e = 0.93$, $Q_c = 0.97$, $Q_{1e} = 0.94$, $Q_{2e} = 0.9$, $Q_{3e} = 0.85$.

	R_v	R_p	P_c	P_e	Q_c	Q_{1e}	Q_{2e}	Q_{3e}
R_v	1.000	2.285	1.110	1.314	26.095	35.231	5.221	0.1860
R_p	0.438	1.000	0.486	0.575	11.419	15.418	2.285	0.0814
P_c	0.901	2.058	1.000	1.184	23.507	31.737	4.703	0.1675
P_e	0.761	1.739	0.845	1.000	19.853	26.804	3.972	0.1415
Q_c	0.038	0.088	0.043	0.050	1.000	1.350	0.200	0.0071
Q_{1e}	0.028	0.065	0.032	0.037	0.741	1.000	0.148	0.0053
Q_{2e}	0.192	0.438	0.213	0.252	4.998	6.748	1.000	0.0356
Q_{3e}	5.378	12.288	5.970	7.068	140.326	189.458	28.077	1.0000

TABLE VI

The Reliability Importance Ratio Matrix for $R_v = 0.9$, $R_p = 0.45$, $P_c = 0.96$, $P_e = 0.93$, $Q_c = 0.97$, $Q_{1e} = 0.94$, $Q_{2e} = 0.9$, $Q_{3e} = 0.85$.

	R_v	R_p	P_c	P_e	Q_c	Q_{1e}	Q_{2e}	Q_{3e}
R_v	1.000	0.437	0.975	0.105	11.626	15.529	2.111	0.0622
R_p	2.286	1.000	2.230	0.240	26.582	35.504	4.827	0.1422
P_c	1.025	0.448	1.000	0.108	11.922	15.923	2.165	0.0638
P_e	9.526	4.167	9.290	1.000	110.755	147.929	20.112	0.5922
Q_c	0.086	0.038	0.084	0.009	1.000	1.336	0.182	0.0054
Q_{1e}	0.064	0.028	0.063	0.007	0.749	1.000	0.136	0.0040
Q_{2e}	0.474	0.207	0.462	0.050	5.507	7.355	1.000	0.0295
Q_{3e}	16.086	7.036	15.687	1.689	187.020	249.791	33.961	1.0000

values are left unchanged, the order of relative importance of the different parameters becomes (see Table VI): $Q_{1e} > Q_c > Q_{2e} > R_v > P_c > R_p > P_e > Q_{3e}$. The first three parameters and the last one, Q_{3e} are in the same order as they were in the previous case. However, the order of R_p and R_v, and the order of P_c and P_e are reversed in relation to the previous case, i.e. $R_p > R_v$ and $P_c > P_e$ here, whereas in the previous case $R_p < R_v$ and $P_c < P_e$. The reason for this reversal is the change in the values of R_v and R_p.

This study shows how sensitivity analysis can be used to obtain the relative importance of parameters. This result can then be used by management in providing job training and improving the performance of the system. As explained, next, the relative importance of these reliability parameters can also be used in comparative sensitivity analysis when studying alternative system configurations.

B. Comparative Structural Analysis

This section shows how to study the relative sensitivity of structure selection decisions to changes in reliability values. The example considered here compares two alternative configurations for a control component preparing a receiving report in an inventory acquisition system. The alternative configurations will be set out prior to examining the sensitivity of the selection decision to changes in reliability levels.

As assumed earlier in Section IV.A.3., the receiving report preparation component has two inputs: the number of items on the vendor's packing slip and the number obtained by direct material count. However, there are two possible structures of the component depending on the order in which the material counting is done in relation to seeing the number on the packing slip. In one case the material count will be independent of the information on the packing slip and in the other case it may be influenced by the information on the packing slip. The reliability models in the two cases are different. Also the output reliabilities in the two cases are significantly different. The details are presented below.

Structure (a): In this case, the control component, preparing the receiving report, receives two inputs. The two inputs are considered to be independent of each other. That is the material counting is done before seeing the number on the packing slip. A schematic diagram of the information flow for this case is presented in Fig. 28. Since the two input channels are independent of each other and one of them

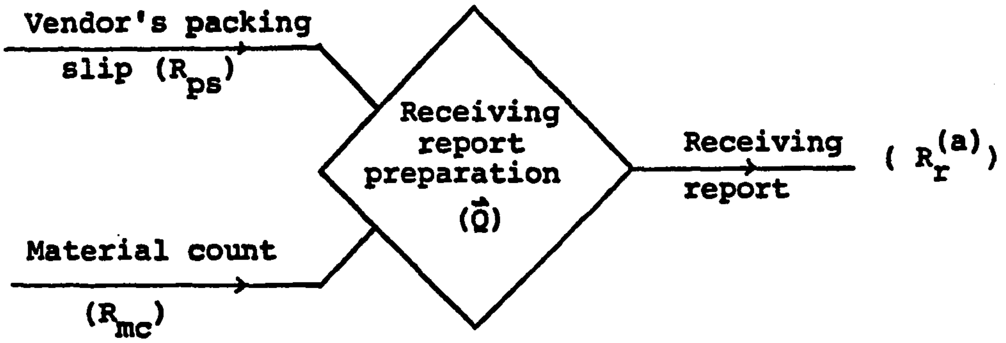


Figure 28: A control component preparing a receiving report. Physical count is taken before seeing the vendor's packing slip.

is redundant, the reliability model for this case can be obtained directly from Eq.(II-27) by substituting the corresponding reliability parameters (Q_w is assumed to be unity, i.e. the control is always in operation):

$$R_r^{(a)} = R_{ps}R_{mc}Q_c + (1-R_{ps})R_{mc}Q_{1e}^{m} + R_{ps}(1-R_{mc})Q_{1e}^{p} + (1-R_{ps})(1-R_{mc})Q_{2e} \qquad \text{(IV-45)}$$

where R_{ps} and R_{mc} are the reliabilities of the two inputs: the packing slip and the initial material count, respectively. Q_{1e}^{m} and Q_{2e}^{p} respectively, represent the reliability parameters of the component associated with the process of preparing a 'correct' receiving report when only the material count is correct and when only the packing slip is correct. The parameters Q_c and Q_{2e} are defined in Table I.

By defining the following parameters:

$$P_c' = R_{mc}Q_c + (1-R_{mc})Q_{1e}{}^p \qquad \text{(IV-46)}$$

and

$$P_e' = R_{mc}Q_{1e}{}^m + (1-R_{mc})Q_{2e} \qquad \text{(IV-47)}$$

one can rewrite Eq. (IV-45) as:

$$R_r^{(a)} = R_{ps}P_c' + (1-R_{ps})P_e' \qquad \text{(IV-48)}$$

This equation, which represents the reliability model of the component in Fig. 28, is used later in this section for comparison with structure (b) (Fig. 29).

It is important to point out, here, that the different parameters (Q_c, $Q_{1e}{}^p$, $Q_{1e}{}^m$, and Q_{2e}) associated with the preparation of a correct receiving report and the various input conditions, usually will take high values because the report maker may decide to recount the material in case the first count disagrees with the packing slip. This process, it seems, would increase the liklihood of preparing a correct receiving report which in turn would lead to high values of the control parameters (Q_c, $Q_{1e}{}^p$, $Q_{1e}{}^m$, and Q_{2e}). The effect of high values of these parameters on the output reliability, $R_r{}^{(a)}$ is very significant. As one can see through Eq. (IV-45) or Eq. (IV-48). Even if the input reliabilities (R_{ps} and R_{mc}) are small, the output reliability, $R_r{}^{(a)}$ will be high if the control parameters are high. This can be demonstrated through the following example.

Let $Q_c = 0.95$, ${Q_{1e}}^p = 0.9$, ${Q_{1e}}^m = 0.92$, and $Q_{2e} = 0.8$. For $R_{mc} = 0.4$ and $R_{ps} = 0.5$, from Eqs. (IV-46)-(IV-48), one obtains: $P_c' = 0.92$, $P_e' = 0.848$ and $R_r^{(a)} = 0.884$. Similarly one can show in general that P_c', P_e' and $R_r^{(a)}$ take high values if Q_c, ${Q_{1e}}^p$, ${Q_{1e}}^m$ and Q_{2e} are high, irrespective of the values of the input reliabilities, R_{ps} and R_{mc}. These results will be useful when a comparison is made between the two output reliabilities for the two structures, later in this section.

Structure (b): In this case, the two inputs are not independent of each other. To be more explicit, the material count will be influenced by the information on the packing slip. Thus the correctness of the number determined through the material count may depend on the correctness of the number on the packing slip. The information flow diagram in this case can be schematically presented as Figure 29.

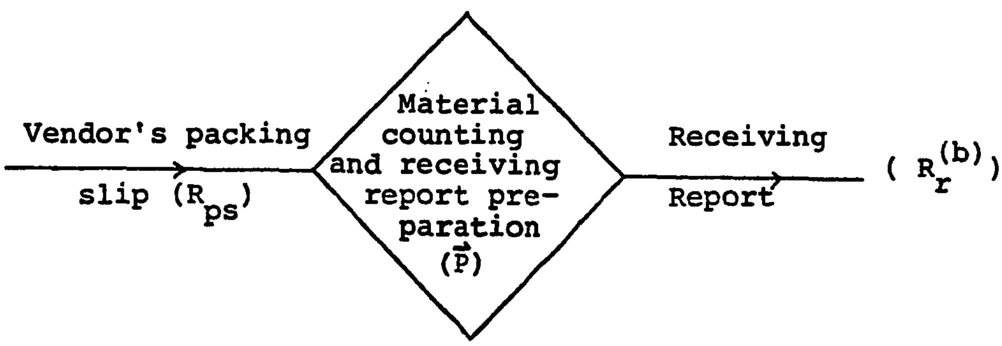


Figure 29: A control component preparing a receiving report. Physical count is taken after seeing the vendor's packing slip.

Here, in effect, the control component receives one input which is the number on the vendor's packing slip. Then, the component determines the number through a material count and prepares the receiving report. The reliability model for this structure can be obtained directly from Eq. (II-16) by substituting appropriate parameters (it is assumed that $P_w = 1$, i.e., the control is always in operation):

$$R_r^{(b)} = R_{ps}P_c + (1-R_{ps})P_e \qquad \text{(IV-49)}$$

where P_c represents the reliability of the component given that the vendor's packing slip is correct, and P_e is the likelihood of preparing a correct receiving report when the input (packing slip) is wrong. Since, in this case, the outcome of the material count is dependent on the information on the packing slip, it is reasonable to assume that the likelihood, P_c, of preparing a correct receiving report when the number on the packing slip is correct is much higher than P_e.

The above result (i.e. $P_c \gg P_e$) has important implications for the output reliability of the component. As one can see from Eq. (IV-49) (see also lines FD and ED in Fig. 30) for small values of R_{ps}, the output reliability $R_r^{(b)}$ will be low if P_c is assumed to be high and P_e low. For example, assume that $P_c = 0.98$, $P_e = 0.1$ and $R_{ps} = 0.5$, then from Eq.(IV-49) $R_r^{(b)} = 0.54$. One can also see that if there is a complete dependency (i.e. $P_c = 1$, and $P_e = 0$) then $R_r^{(b)} = R_{ps}$, which implies that the output reliabiltiy is the same as the input reliability and thus the control has no effect or useful purpose.

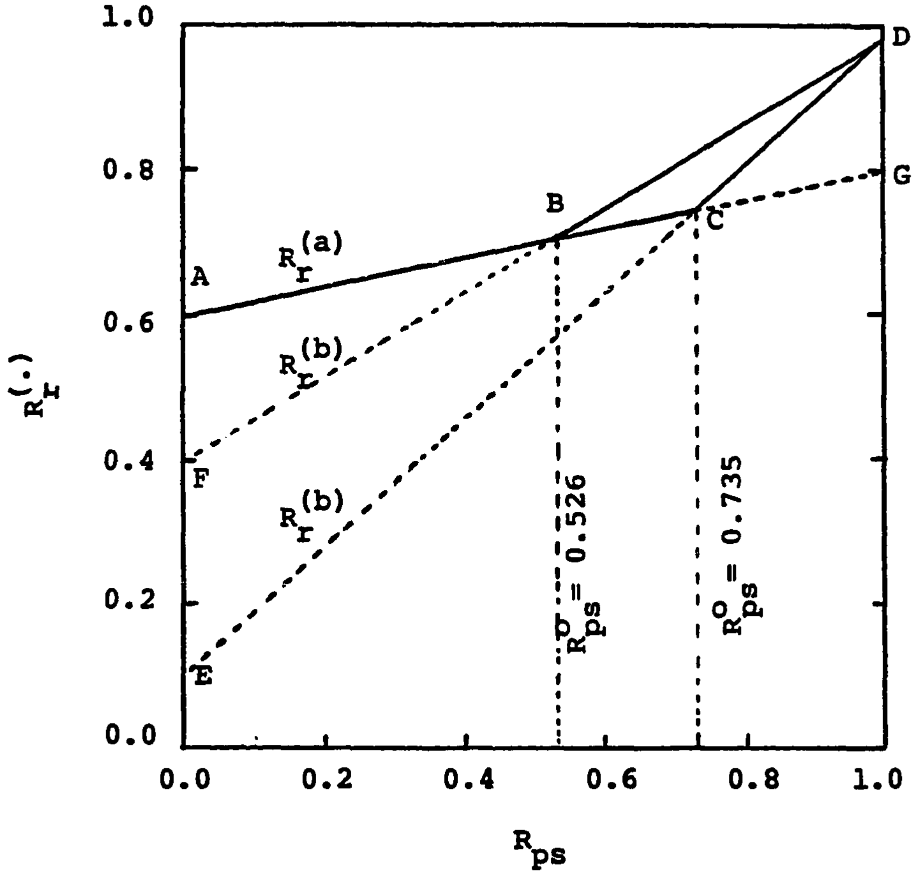


Figure 30: Variation of receiving report reliability, $R_r^{(\cdot)}$, with the input reliability, R_{ps}, for the two configurations, a and b. Line AG represents variation of R_r for configuration (a) with $P'_c = 0.8$ and $P'_e = 0.6$. Lines ED and FD represent variation of R_r for configuration (b) with parameter values: $P_c = 0.98$ and $P_e = 0.1$, and; $P_c = 0.98$ and $P_e = 0.4$, respectively.

A comparison of the output reliabilities for the two structures is presented below. It shows that one structure of the control component is preferred to the other for a certain range of values of the input reliability parameter, R_{ps}.

Comparison of the two Structures: As assumed earlier, the parameters P'_c and P'_e defined in the case of structure (a), have high values. As a result, $R_r^{(a)}$, the output reliability for structure (a) is also high irrespective of the value of the reliability of the packing slip, R_{ps}. On the other hand, the output reliability, $R_r^{(b)}$ for structure (b) depends heavily on R_{ps}. As discussed earlier, it is low for low values of R_{ps} and high for high values of R_{ps}. A direct comparison between the two output reliabilities, $R_r^{(a)}$ and $R_r^{(b)}$, shown in Fig. 30 suggests that for given values of parameters P_c, P_e, P'_c and P'_e, $R_r^{(a)}$ is higher than $R_r^{(b)}$ at low values of R_{ps} (the packing slip reliability) and vice versa at large values of R_{ps}. Thus structure (a) is preferred to (b) if input reliability, R_{ps} is low, but structure (b) is preferred over (a) when input reliability is high.

The above result is of great importance in the design and implementation of a control system, because it provides information regarding the preferability of a certain structure over another structure for a control system with given control parameters. It should also be pointed out that such structural changes may not involve additional cost to management. Rather, it may provide conditions for behavioral changes that may improve output reliability.

In Fig. 30 line AG represents the variation of the output reliability, $R_r^{(a)}$ with respect to R_{ps} for structure (a) with parameter values: $P_c' = 0.8$ and $P_e' = 0.6$ (e.g. $Q_c = 0.8$, $Q_{1e}{}^{p} = 0.8$, $Q_{1e}^{m} = 0.6$, $Q_{2e} = 0.6$ and $0 \leqslant R_{mc} \leqslant 1$). Lines ED and FD represent, respectively, the variation of $R_r^{(b)}$ with R_{ps} for $P_c = 0.98$ and $P_e = 0.1$, and $P_c = 0.98$ and $P_e = 0.4$. It is observed (see Fig. 30) that structure (a) with given parameter values is uniformly dominant over structure (b) in the region AB. Also it is seen that structure (a) is uniformly inferior to (b) in the region CD. But in the region BC the choice of structure (a or b) depends on the particular choice of parameter values for structure (b). For instance when $P_e = 0.1$, structure (a) is preferable to (b) in the region BC, but when $P_e = 0.4$, structure (b) is preferable to (a). This preference for different structures for different regions of R_{ps} is shown by solid lines in Fig. 30. The point where the two structures are indifferent (R_{ps}^{o}) varies when the parameter values are changed. For example, when $P_e = 0.1$ is changed to $P_e = 0.4$, the indifference point moved from C to B.

One can determine an analytical expression for the indifference point, R_{ps}^{o} for this case by setting the two output reliabilities equal to each other, i.e., $R_r^{(a)} = R_r^{(b)}$. This yields

$$R_{ps}^{o} = \frac{P_e' - P_e}{(P_c - P_c') + (P_e' - P_e)} \qquad \text{(IV-49)}$$

When $P_e' = P_e$, that is when the likelihoods of correctly preparing the receiving report given that the information on the packing slip is wrong, are equal in the two cases, then $R_{ps}^o=0$. This means that one structure is always preferred over the other depending on the relative values of P_c' and P_c. For example, if $P_c > P_c'$ and $P_e = P_e'$ then structure (b) is preferred for all values of R_{ps}. However, when $P_c < P_c'$ and $P_e = P_e'$ (i.e. $R_{ps}^o = 0$), then structure (a) is preferred over structure (b) of the control component for all values of R_{ps}. Similarly, for $P_c' = P_c$, i.e. $R_{ps}^o = 1$, one can show that one structure is preferred over the other for all values of input reliability, R_{ps}. In this case, the relative values of P_e' and P_e determine the preferable structure. For example, in this case, when $P_e' > P_e$ structure (a) is always preferred.

The results obtained here are congruent with intuitive results. For example, it was observed in the above analysis that when the input reliability of the packing slip is high, structure (b), in which the material count is influenced by the information on the slip, is preferred. This is what one would expect when it is known that the vendor's packing slip is always correct. In other words, it is far better to depend on the vendor's slip when it is more or less always correct than to make an independent count and risk contaminating the informational output as a result.

In addition to the above results, Fig. 30 provides information on how accurately one needs to evaluate the input reliability, R_{ps}, when making decision about which structure to implement. To illustrate this point, consider the case where control parameters' values are: $P_c' = 0.8$, $P_e' = 0.6$, $P_c = 0.98$ and $P_e = 0.4$. Thus, lines AG and FD are of interest for the present discussion. It is clear from Fig. 30 that when the estimated value of the input reliability, R_{ps} falls far away from the indifference point B then even a relatively large estimation error in R_{ps} would be tolerable, i.e., even a relatively large estimation error in R_{ps} would not affect the decision about whether to implement structure (a) or (b). But when the estimated value of R_{ps} falls near point B, a large estimation error is intolerable because it could affect the decision outcome.

In general, the largest estimation error that will not affect the decision is $|R_{ps} - R_{ps}^{o}|$. This can be expressed in terms of percentage of R_{ps} as:

$$\text{marginal estimation error} \doteq \frac{\left|R_{ps} - R_{ps}^{o}\right|}{R_{ps}} \qquad \text{(IV-50)}$$

For the control parameter values considered above, the indifference point, $R_{ps}^{o} = 0.526$. Thus the 'marginal estimation error' is 31.5% (Eq. IV-50) when $R_{ps} = 0.4$. This means that an estimation error in R_{ps} of up to 31.5% when $R_{ps} = 0.4$, will not affect the decision. But when R_{ps} is estimated to be 0.5 then the 'marginal estimation error' becomes 5.2%, meaning that an error of more than 5.2% in the estimation of R_{ps} could affect the decision outcome.

CHAPTER V

FIELD STUDY

This chapter presents the method and the results of a field study conducted in support of the reliability models developed in the previous chapters. The chapter is divided into two sections. The first section deals with the approach used to collect the data for the study. The second section presents a discussion of the results of the field study.

The following control and performance components of an inventory acquisition system were chosen for the field study:

Control Components:

1. Disbursement Voucher Review for Quantity: A single input control component.
2. Disbursement Voucher Review for Price: A two input control component.

Performance Components:

1. Inventory Pricing: A performance component with two input channels.

Detailed discusssions of these components are presented later in the chapter.

The main objective of the field study was to compare the reliability values developed using the proposed reliability model with direct measurement of the reliability parameters for the system being studied. To accomplish this objective, reliability parameters associated with input information were evaluated. A major portion of this chapter is devoted to explaining the modeling and evaluation process.

A. Method of Field Study

There were several principal steps involved in completing the field study. These were: 1) selection of a firm; 2) selection and modeling of internal accounting control components; 3) collection of data; and 4) evaluation of reliability parameters. Each of these steps warrants further discussion. The following paragraphs present these discussions.

The main criterion in selecting a firm for the study was the degree of deviation from prescribed norms in the company's internal control system. A company with a moderate rate of deviation in one of its internal accounting control systems was preferred, in order to avoid studying and sampling rare events. Also a heavy volume of routine inventory purchases, controlled with a voucher system, was required in order to avoid sampling of rare events (errors). It was also required that the purchase orders (POs) or purchase requisitions (PRs) be numbered serially. This was required to facilitate the sampling procedure. A preliminary interview was arranged with the Vice President of Finance of the candidate company to discuss the feasibility of conducting the study using their data and to find out whether the company's internal accounting controls met our objectives.

Subsequently, a meeting was arranged with the company's internal audit manager to discuss and obtain a preliminary understanding of the firm's internal accounting controls. This step helped in making a decision about what components to study. The ease of obtaining the required data was a deciding factor in selecting the components. These components were listed earlier. The modeling procedures for these components are presented in the next section.

The next step in the field stduy was to collect data from the system. The first step in this process was to determine, in detail, the needed information. The following list presents the data required for the study:

1. The quantity (q_{po}) of items on the purchase order (PO) to be studied.
2. The quantity (q_{vi}) of items on the vendor's invoice (VI).
3. The quantity (q_{ra}) of items received and accepted by the receiver on the receiving report (RR).
4. The quantity (q_{vo}) of items on the approved voucher (VO) for disbursement.
5. The per unit price, p_{po}, of items on the PO.
6. The per unit price, p_{vi}, of items on the VI.
7. The per unit price, p_{vo}, of items on the VO for disbursement.

There were 1202 POs and the corresponding documents (VIs, RRs and VOs) of interest for the period (October 1981-

January 1982) under study. Only 400 of these were randomly selected for the study. Of these 400 POs only 379 POs were used in the study, the remaining 21 POs were not yet completely processed and thus they were excluded from the sample. A four digit random number table (Arkin 1974) was used to select the sample of the PO numbers. Once a PO number was selected, the corresponding vendor's name was obtained from a log book maintained in the purchasing department. Knowing the vendor's name, the voucher for disbursement and the related PO, RR and VI were pulled from the voucher file maintained under the vendor's name. A careful study of the above documents provided the information needed for the study. To facilitate the data collection process, the data for each PO selected was recorded on a work sheet as shown in Fig. 31.

To expedite processing and to provide uniformity among sampling units, certain field work processing rules were developed. Many times, for one PO there were several VIs, RRs and VOs. Thus to collect all the information related to a PO one had to trace all the different VIs, RRs and VOs that corresponded to that particular PO. Similarly, one PO often contained several different items. In such cases only one line item for which the total purchasimg cost was the highest was selected for study. The reason for selecting one line item was to treat each PO equivalently, i.e. each PO studied, supposedly, contained one kind of item. The reason for selecting the line item with highest purchasing cost was materiality, i.e. an error in such

Purchase Order No. & Date	Vendor's Name	Part Number	Correct Quantity (q_c)	Correct Price (p_c)	PO(approved)		Vendor's Invoice		Disbursement Voucher		Quantity Received and accepted (q_{ra})	Remarks
					q_{po}	p_{po}	q_{vi}	p_{vi}	q_{vo}	p_{vo}		

Figure 31: Worksheet for data collection.

cases could have material impact. If the total costs of different items were the same then the first item was selected.

On many occasions, as a matter of general policy one PO was used to order one kind of item in different amount at different times. Thus one PO might have several VIs, RRs and VOs depending on the number of scheduled deliveries. In such situations, the delivery schedule for which the total purchasing cost was the highest was selected. If all the scheduled deliveries were equally costly then the first delivery time was considered for the study.

The last phase of the field study was the evaluation of the reliability parameters of the input information and of the system components. Since the processes for which the reliability parameters are defined (see definitions in Chapter II) are Bernouli processes (absence or presence of a given error type in the input information or in the completion of a task), the parameters can be evaluated by their unbiased estimators (Larsen and Marx 1981). As an example, the unbiased estimator for the reliability of POs for correct pricing can be given as:

$$\hat{R}_{po} = \frac{\text{(Number of POs with no price deviation from the correct price)}}{\text{(Total number of POs investigated)}} \qquad \text{(V-1)}$$

The other parameters (estimated by similar difinitions) are given in the next section.

B. Reliability Models and Field Study Results

This section describes the models of the components studied and presents the results of the field study. The field data are used here to evaluate the reliability parameters. These parameters are then used in the reliability models to predict the reliability of the output information. These predicted values are then compared with the values obtained directly from the output data. These steps are discussed below in detail for the three components tested.

1. Disbursement Voucher Review for Quantity: A Single Input Control Component

This component was established by management to review VIs, POs and RRs as a means for determining the correct quantity of material for which the firm would pay. Two criteria were used in deciding the 'correct' quantity (q_c). In one criterion, the quantity for which payment is made shall not exceed the lessor of the quantity ordered (q_{po}) or the quantity received and accepted (q_{ra}). The second criterion, which was the company's policy, differed slightly when the material was overshipped; an overshipment of 10% or less was allowed to be received, and if accepted (acceptance depends on the quality and nature of material), it was considered to be the 'correct' quantity for payment. The first rule is a stricter criterion than the company's policy. It was selected because it would provide a higher error rate than the company's policy and thus a smaller sample size would suffice.

It appears that the disbursement voucher review component is a control component with three input channels. The three inputs being q_{po}, q_{vi} and q_{ra} for each PO. However, since the two channels (PO and RR) are used in deciding the correct amount of material for payment, the component effectively becomes a single input channel control component with q_{vi} as the only input information. The output information is q_{vo}, the quantity on the disbursement voucher (see Fig. 32).

Figure 32: Disbursement Voucher review for correct quantity for payment.

The reliability model for such a component was developed in Chapter II. The result is (Eq. II-16):

$$R_{vo} = R_{vi}P_c + (1 - R_{vi})P_e \qquad \text{(V-2)}$$

where R_{vi} and R_{vo} are the reliabilities of the input and the output information, respectively. The above parameters are evaluated by the following estimators:

$$\hat{R}_{vi} = N_{vi}(q_{vi}=q_c)/N_{vi} \qquad \text{(V-3)}$$

$$\hat{P}_c = \frac{N_{vo}(q_{vo}=q_c/q_{vi}=q_c)}{N_{vo}(q_{vi}= q_c)} \qquad \text{(V-4)}$$

$$\hat{P}_e = \frac{N_{vo}(q_{vo} = q_c/q_{vi} \neq q_c)}{N_{vo}(q_{vi} \neq q_c)} \qquad \text{(V-5)}$$

$$\hat{R}_{vo} = \frac{N_{vo}(q_{vo} = q_c)}{N_{vo}} \qquad \text{(V-6)}$$

where the different Ns are defined in Table VII.

The total sample of 379 POs, VIs, and associated RRs and VOs were picked from a pool of such documents processed during the four month period. This sample was divided into two groups corresponding to two, two-month periods. The sample size for the first two-month period was 186 and 193 for the second period. For each VI and VO, q_{vi} and q_{vo} were matched with the corresponding q_c (the correct quantity for disbursement) to search for deviations. The number of VIs and VOs that had no deviations were noted. The number of VOs that had no deviation were divided into two groups: one in which the corresponding q_{vi}s = q_cs and the other where q_{vi}s $\neq$ q_cs. There are two managerial policy concepts that were considered in deciding the correct quantity, q_c for payment (Both of these policies were discussed earlier in this section). Table VIII presents the results for both the cases.

The reliability parameters R_{vi}, P_c, P_e and R_{vo} are computed using Eqs. (V-3)-(V-6) and Table VIII and the results are presented in Table IX. The values of R_{vo}, evaluated directly, for the two periods are within 3.5% of each other.

TABLE VII

List of Definitions of Different Numbers (Ns)

N_{po}	- Total number of Purchase Orders (POs) in the sample.
N_{vi}	- Total number of Vendor's Invoices (VIs) in the sample.
N_{vo}	- Total number of Disbursement Vouchers (VOs) in the sample.
$N_{po}(p_{vi}= p_c)$	- Number of Purchase Orders (POs) for which the corresponding VIs have correct price i.e. $p_{vi}= p_c$.
$N_{po}(p_{vi}\neq p_c) = N_{po} - N_{po}(p_{vi}= p_c)$	
$N_{vi}(p_{vi}= p_c)$	- Number of VIs with correct Vendor's Invoice price.
$N_{vo}(p_{vo}= p_c)$	- Number of VOs with correct voucher price.
$N_{vi}(q_{vi}= q_c)$	- Number of VIs with correct quantity of material.
$N_{vo}(q_{vi}= q_c)$	- Number of VOs for which the corresponding VIs have correct quantities i.e. $q_{vi}= q_c$.
$N_{vo}(q_{vi}\neq q_c) = N_{vo} - N_{vo}(q_{vi}= q_c)$	
$N_{vo}(q_{vo}=q_c)$	- Number of Vos with correct quantity for disbursement.
$N_{vo}(p_{vi}=p_c,\ p_{po}= p_c)$	- Number of VOs for which the corresponding VIs and POs have correct prices.
$N_{vo}(p_{vi}=p_c,\ p_{po}\neq p_c)$	- Number of VOs for which the correponding VIs prices are correct but the POs prices are not.
$N_{vo}(p_{vi}\neq p_c,\ p_{po}= p_c)$	- Number of VOs for which the corresponding POs have correct prices but not the VIs.

TABLE VII (CONTINUED)

$N_{vo}(p_{vi} \neq p_c, p_{po} \neq p_c)$ - Number of VOs for which both the prices, p_{vi} and p_{po} are incorrect.

$N_{vo}(p_{vo} = p_c, q_{vo} = q_c)$ - Number of VOs with correct prices and quantities.

$N_{po}(p_{po} = p_c/p_{vi} = p_c)$ - Number of POs with correct prices given that the corresponding VI's prices are correct.

$N_{po}(p_{po} = p_c/p_{vi} \neq p_c)$ - Number of POs with $p_{po} = p_c$ given that the corresponding VI's prices are incorrect.

$N_{vo}(q_{vo} = q_c/p_{vo} = p_c)$ - Number of VOs with correct quantities given that the voucher's prices are correct.

$N_{vo}(q_{vo} = q_c/q_{vi} = q_c)$ - Number of VOs with correct quantities given that the corresponding VI's quantities are also correct.

$N_{vo}(q_{vo} = q_c/q_{vi} \neq q_c)$ - Number of VOs with $q_{vo} = q_c$ given that the corresponding VIs quantities are in error.

$N_{vo}(p_{vo} = p_c/p_{vi} = p_{po} = p_c)$ - Number of VOs with $p_{vo} = p_c$ given that the corresponding p_{vi}s and p_{po}s are equal to p_cs.

$N_{vo}(p_{vo} = p_c/p_{vi} \neq p_c, p_{po} = p_c)$ - Number of VOs with $p_{vo} = p_c$ given that the corresponding $p_{vi} \neq p_c$ and $p_{po} = p_c$.

$N_{vo}(p_{vo} = p_c/p_{vi} = p_c, p_{po} \neq p_c)$ - Number of VOs with $p_{vo} = p_c$ given that the corresponding $p_{vi} = p_c$ and $p_{po} \neq p_c$.

$N_{vo}(p_{vo} = p_c/p_{vi} \neq p_c, p_{po} \neq p_c)$ - Number of VOs with $p_{vo} = p_c$ given that the corresponding $p_{vi} \neq p_c$ and $p_{po} \neq p_c$.

$N_{vo}(qxp = q_c x p_c/q_{vo} = q_c, p_{vo} = p_c)$ - Number of VOs with correct inventory values given that the vouchers have correct prices and quantities.

TABLE VIII

Number of Vendor's Invoices and Disbursement Vouchers with and without Deviations in Their Respective Quantities

Mgt. Policy for payment	Period	N_{vi}	$N_{vi}(q_{vi} = q_c)$	N_{vo}	$N_{vo}(q_{vi} = q_c)$	$N_{vo}(q_{vo} = q_c / q_{vi} = q_c)$	$N_{vo}(q_{vi} \neq q_c)$	$N_{vo}(q_{vo} = q_c / q_{vi} \neq q_c)$	N_{vo} for $q_{vo} = q_c$
No payment for overshipped materials	First Period	186	153	186	153	153	33	12	165
	Second Period	193	164	193	164	163	29	12	175
No payment for over 10% over-shipped materials	First Period	186	173	186	173	173	13	10	183
	Second Period	193	173	193	173	173	20	12	185

TABLE IX

Reliability Parameters for Disbursement Voucher Review for Quantity

Mgt. Policy for payment	Period	R_{vi}	P_c	P_e	R_{vo} (Direct)	R_{vo} (Predicted)	% diff.
No payment for overshipped materials	First Period	0.8226	1.0	0.3636	0.8871	0.891	0.4
	Second Period	0.8497	0.9939	0.4138	0.9067	0.9043	-0.3
No payment for over 10% over-shipped materials	First Period	0.9301	1.0	0.7692	0.9839	0.972	-1.2
	Second Period	0.8964	1.0	0.6	0.9585	0.9761	1.8

This suggests that the system can be assumed to be stable during the two periods. It is also noted (see Table IX) that R_{vi} and P_c did not change significantly over the periods. However, there is a 13.8% increase in P_e from the first period to the second period (see Table IX) as computed for the policy where no payment is allowed for overshipped materials. Therefore, it is likely that the process of correcting errors when the input is in error has improved in the second period by about 13.8%. When the second criterion (no payment for over 10% overshipped material) is considered for deciding a correct quantity, q_c, the parameter, P_e decreases by 22% from the first period to the second period. This shows that with the second criterion, the process of correcting errors in the input information deteriorated during the second period compared to what it was during the first period.

In general, the following steps were involved in validating a reliability model of a component. First, the output reliability for a period was computed, using: 1) the model, 2) the estimated input reliability for the period, and 3) the estimated values of the component's parameters (for another period). Second this predicted value of the output reliability (for a period) was compared with the estimated value from the data of that period. In this case, the predicted values of R_{vo} (the output reliability) for the two periods and for the two q_c decision criteria, were within 1.8% of their direct estimates (see Table IX). This result shows that the model used here (Eq. V -2) appears to be a valid model for this component. A more detailed study is needed to validate the model statistically.

It should be pointed out that the values of P_c and P_e for each period are significantly different (see Table IX). This suggests that the likelihood of a correct review given that the input is correct does differ from the likelihood of a correct review when the input is incorrect, as assumed in Chapter II.

2. Disbursement Voucher Review for Price: A Two Input Control Component

The task of this component is to establish a price per unit for disbursement after having reviewed the related PO and VI. This element has two inputs (see Fig. 33): the price p_{po} from the PO and the price, p_{vi} from the VI. The output is the voucher price, p_{vo} used for disbursement purposes. The following criteria are used in deciding the 'correct' price for payment.

1. When the Vendor's Invoice price is equal to the Purchase Order price (i.e. $p_{vi} = p_{po}$) then PO price is considered to be the correct price unless it is detected that both the prices are incorrect and a third price is correct.
2. When the Vendor's Invoice price is less than Purchase Order price (i.e. $p_{vi} < p_{po}$) then the VI price is taken to be correct i.e. $p_c = p_{vi}$.

3. When the Vendor's Invoice price is greater than the PO price (i.e. $p_{vi} > p_{po}$) then the PO price is taken to be correct i.e. $p_c = p_{po}$.

The above policy is only an assumed policy for the present study. The actual policy in practice by the company was a little different. They differed on the third point where $p_{vi} > p_{po}$; they allowed payment for any unfavorable price variance of up to $20 or 10%. The former criterion is used in evaluating the reliability parameters.

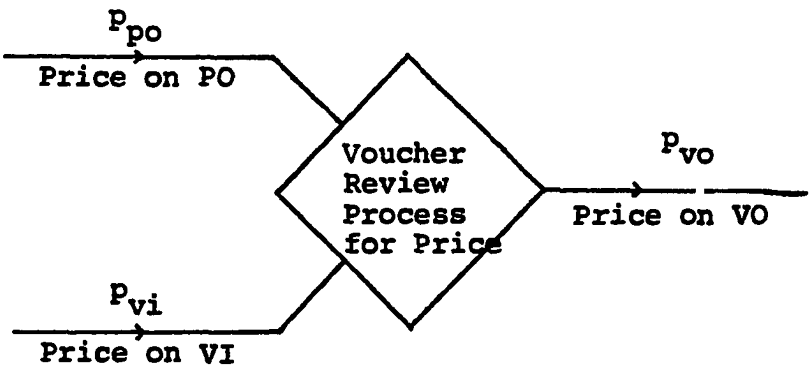


Figure 33: Disbursement Voucher review for price.

The reliability model developed in Chapter II for a control component with two input channels similar to the component in Fig. 33 is not applicable here. The reason being that the two input channels are assumed to be independent in the earlier derivation. In the present case, they are not in fact, independent. Thus, an appropriate reliability model was developed before the field data was analysed. The following paragraphs develop this model.

Theoretical Model: Consider the following 'events' related to this component:

S_{vi} - The correct state of the vendor's invoice i.e. $p_{vi} = p_c$ (the correct price).

S_{po} - The correct state of the purchase order i.e. $p_{po} = p_c$.

S_c - The voucher review process functions properly.

S_{vo} - The correct state of the voucher i.e. $p_{vo} = p_c$.

The state of the output information can be written in terms of S_{vi}, S_{po} and S_c as:

$$S_{vo} = (S_{vi} \cap S_{po} \cap S_c) \cup (\bar{S}_{vi} \cap S_{po} \cap S_c) \cup (S_{vi} \cap \bar{S}_{po} \cap S_c) \cup (\bar{S}_{vi} \cap \bar{S}_{po} \cap S_c) \quad \text{(V-7)}$$

which states that the output information is correct when the two inputs are correct <u>and</u> the voucher review process functions properly <u>or</u> the price on the vendor's invoice is incorrect <u>and</u> the price on the purchase order is correct <u>and</u> the voucher review process is functioning properly <u>or</u> the vendor's price is correct <u>and</u> the purchase order price is wrong <u>and</u> the review process is correct <u>or</u> both the prices are incorrect but the review process functions properly and finds the correct price.

Rewriting Eq. (V-7) in probability form, one gets:

$$P(S_{vo}) = P(S_{vi} \cap S_{po} \cap S_c) + P(\bar{S}_{vi} \cap S_{po} \cap S_c) + P(S_{vi} \cap \bar{S}_{po} \cap S_c) + P(\bar{S}_{vi} \cap \bar{S}_{po} \cap S_c) \quad \text{(V-8)}$$

where P represents the probability of the event given in the argument of P.

Using the product rule in probability Eq. (V-8) can be written as:

$$
\begin{aligned}
P(S_{vo}) &= P(S_{vi})P(S_{po}/S_{vi})P(S_c/S_{vi} \cap S_{po}) \\
&+ P(\bar{S}_{vi})P(S_{po}/\bar{S}_{vi})P(S_c/\bar{S}_{vi} \cap S_{po}) \\
&+ P(S_{vi})P(\bar{S}_{po}/S_{vi})P(S_c/S_{vi} \cap \bar{S}_{po}) \\
&+ P(\bar{S}_{vi})P(\bar{S}_{po}/\bar{S}_{vi})P(S_c/\bar{S}_{vi} \cap \bar{S}_{po})
\end{aligned}
\qquad \text{(V-9)}
$$

This equation represents the reliability model for the component considered here. In order that the above equation be interpreted properly, the detailed definitions of the different probabilities are given in Table X. Based on these definitions, the reliability parameters in Eq. (V-9) can be evaluated using the following estimators:

$$\hat{P}(S_{vo}) = N_{vo}(p_{vo} = p_c)/N_{vo} \qquad \text{(V-10)}$$

$$\hat{P}(S_{vi}) = N_{vi}(p_{vi} = p_c)/N_{vi} \qquad \text{(V-11)}$$

$$\hat{P}(S_{po}/S_{vi}) = \frac{N_{po}(p_{po} = p_c/p_{vi} = p_c)}{N_{po}(p_{vi} = p_c)} \qquad \text{(V-12)}$$

$$\hat{P}(S_{po}/\bar{S}_{vi}) = \frac{N_{po}(p_{po}= p_c/p_{vi} \neq p_c)}{N_{po}(p_{vi} \neq p_c)} \qquad (V\text{-}13)$$

$$\hat{P}(S_c/S_{vi} \cap S_{po}) = \frac{N_{vo}(p_{vo}= p_c/p_{vi}= p_{po}= p_c)}{N_{vo}(p_{vi}= p_{po}= p_c)} \qquad (V\text{-}14)$$

$$\hat{P}(S_c/\bar{S}_{vi} \cap S_{po}) = \frac{N_{vo}(p_{vo}= p_c/p_{vi} \neq p_c, p_{po}= p_c)}{N_{vo}(p_{vi} \neq p_c, p_{po}= p_c)} \qquad (V\text{-}15)$$

$$\hat{P}(S_c/S_{vi} \cap \bar{S}_{po}) = \frac{N_{vo}(p_{vo}= p_c/p_{vi}= p_c, p_{po} \neq p_c)}{N_{vo}(p_{vi}= p_c, p_{po} \neq p_c)} \qquad (V\text{-}16)$$

$$\hat{P}(S_c/\bar{S}_{vi} \cap \bar{S}_{po}) = \frac{N_{vo}(p_{vo}= p_c/p_{vi} \neq p_c, p_{po} \neq p_c)}{N_{vo}(p_{vi} \neq p_c, p_{po} \neq p_c)} \qquad (V\text{-}17)$$

The detailed definitions of the different numbers used in the above equations are given in Table VII.

TABLE X

Definitions of Reliability Parameters used in the Reliability Model of Disbursement Voucher Review for Price

$P(S_{vo})$	Probability that the voucher price is correct.
$P(S_{vi})$	Probability that the vendor's price is correct.
$P(S_{po}/S_{vi})$	Probability that the PO price is correct given that the vendor's price is correct.
$P(S_{po}/\bar{S}_{vi})$	Probability that the PO price is correct given that the vendor's price is incorrect.
$P(S_c/S_{vi} \cap S_{po})$	Probability that the voucher review process is correct given that both the input prices, p_{vi} and p_{po} are correct.
$P(S_c/\bar{S}_{vi} \cap S_{po})$	Probability that the voucher review process is correct given that $p_{vi} \neq p_c$ and $p_{po} = pc$.
$P(S_c/S_{vi} \cap \bar{S}_{po})$	Probability that the voucher review process is correct given that $p_{vi} = p_c$ and $p_{po} \neq p_c$.
$P(S_c/\bar{S}_{vi} \cap \bar{S}_{po})$	Probability that the voucher review process is correct given that the two input prices are incorrect i.e. $p_{vi} \neq p_c$ and $p_{po} \neq p_c$.

Field Study Results: The sample that was used in the case of the previous component is used again for this study. The sample for the two periods is the same (186 documents for the first period and 193 for the second). For each PO and related VI and VO, a correct price (p_c) was established based on the first criterion discussed earlier. The number of POs, VIs and VOs that had no deviations from the correct price were noted for different given conditions. The results are listed in Table XI. The reliability parameters for this were then evaluated using Eqs. (V-10)-(V-17) and Table XI. The values are presented in Table XII.

It is observed that this subsystem is also stable during the two periods since the values of the reliability parameters for the two periods are close to each other (within 2.8% except for one parameter, see Table XII). The predicted values of $P(S_{vo})$ (the reliability of the output information) for the two periods using the model (Eq. V-9) were within 0.3% of the directly measured values (see Table XII). Thus, the model developed here appears to be an appropriate model for the component.

It may be of interest to note that the two probabilities $P(S_c/\bar{S}_{vi} \cap S_{po})$ and $P(S_c/S_{vi} \cap \bar{S}_{po})$ are significantly different (see Table XII). This result is unlike the assumption made in Chapter II. There, the two probabilities were assumed to be equal. No general conclusion can be drawn from this finding because the decision rule used in this case differed from that of the company. However, if it were to be generalizable it would mean that the likelihood of a correct review depends on

TABLE XI

Number of Vendor's Invoices, Purchase Orders and Disbursement Vouchers with and without Deviation in their Respective Prices for Different given Conditions

Mgt. Policy for payment assumed	No payment for any over priced material	
Ns \ Periods	First Period	Second Period
N_{vo}	186	193
$N_{vo}(p_{vo}= p_c)$	182	192
N_{vi}	186	193
$N_{vi}(p_{vi}= p_c)$	179	191
$N_{po}(p_{vi}= p_c)$	179	191
$N_{po}(p_{vi}\neq p_c)$	7	2
$N_{po}(p_{po}= p_c/p_{vi}= p_c)$	170	184
$N_{po}(p_{po}= p_c/p_{vi}\neq p_c)$	7	2
$N_{vo}(p_{vi}= p_{po}= p_c)$	170	184
$N_{vo}(p_{vi}= p_c, p_{po}\neq p_c)$	9	7
$N_{vo}(p_{vi}\neq p_c, p_{po}= p_c)$	7	2
$N_{vo}(p_{vi}\neq p_c, p_{po}\neq p_c)$	0	0
$N_{vo}(p_{vo}=p_c/p_{po}=p_{vi}=p_c)$	170	184
$N_{vo}(p_{vo}=p_c/p_{vi}\neq p_c, p_{po}=p_c)$	3	1
$N_{vo}(p_{vo}=p_c/p_{vi}=p_c, p_{po}\neq p_c)$	9	7
$N_{vo}(p_{vo}=p_c/p_{vi}\neq p_c, p_{po}\neq p_c)$	0	0

TABLE XII

Reliability Parameters for Disbursement Voucher Review for Price

	First Period	Second Period	Percentage Difference
$P(S_{vi})$	0.9624	0.9896	2.8
$P(S_{po}/S_{vi})$	0.9497	0.9634	1.4
$P(S_{po}/\bar{S}_{vi})$	1.0	1.0	0.0
$P(S_c/S_{vi} \cap S_{po})$	1.0	1.0	0.0
$P(S_c/\bar{S}_{vi} \cap S_{po})$	0.4286	0.5	16.7
$P(S_c/S_{vi} \cap \bar{S}_{po})$	1.0	1.0	0.0
$P(S_c/\bar{S}_{vi} \cap \bar{S}_{po})$	undefined	undefined	-
$P(S_{vo})$ Predicted	0.9812	0.9955	1.5
$P(S_{vo})$ Direct	0.9785	0.9948	1.7

which channel's information is correct.

3. Inventory Pricing: A Performance Component with two Input Channels

The function of this component is to determine the value of inventory by multiplying p_{vo} (the price on the disbursement voucher) with q_{vo} (the quantity on the disbursement voucher). Thus it has two input channels: one being the information on p_{vo}s and the other being information on q_{vo}s (see Fig.34). A reliability model for a performance component with two input channels was developed in Chapter II (Proposition 2). But the assumption made then that the two channels are independent is not valid here. In the present case the two channels have 100% overlapping data. Therefore, the reliability model developed earlier (Eq. II-10) in Chapter II can not be used for this component. However, the result of Eq.(II-8), which is a general result, can still be used in the present case in developing a reliability model. The details of this procedure are presented below.

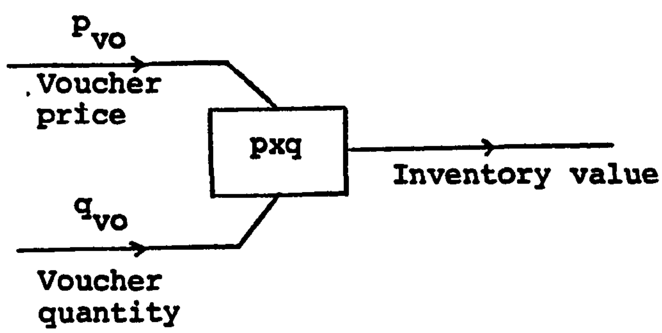


Figure 34: Logic diagram for inventory pricing.

<u>Theoretical Model</u>: Rewriting Eq. (II-8) in the following form, one gets the reliability model for the present component:

$$P(S_{inv}) = P[S_{vo}(p)]\,P[S_{vo}(q)/S_{vo}(p)]\,P[S/S_{vo}(p)\cap S_{vo}(q)] \quad \text{(V-18)}$$

where the different symbols are defined as follows:

S_{inv} - The inventory is valued correctly i.e. $p x q = p_c x q_c$.

$S_{vo}(p)$- The price on the disbursement voucher is correct, i.e. $p_{vo} = p_c$.

$S_{vo}(q)$- The quantity on the disbursement voucher is correct, i.e. $q_{vo} = q_c$.

S - The performance component performs its task correctly, i.e. it multiplies p_{vo} and q_{vo} correctly.

and

$P(S_{inv})$ - Probability that the inventory is valued properly.

$P[S_{vo}(p)]$ - Probability that $p_{vo} = p_c$.

$P[S_{vo}(q)/S_{vo}(p)]$ - Probability that $q_{vo} = q_c$ given that $p_{vo} = p_c$.

$P[S/S_{vo}(p)\cap S_{vo}(q)]$ - Probability that the multiplication process is completed properly given that $p_{vo} = p_c$ & $q_{vo} = q_c$.

The above probabilities can be evaluated by the following estimators.

$$P(S_{inv}) = N_{vo}(p_{vo}= p_c, q_{vo}= q_c)/N_{vo} \qquad \text{(V-19)}$$

$$P[S_{vo}(p)] = N_{vo}(p_{vo}= p_c)/N_{vo} \qquad \text{(V-20)}$$

$$P[S_{vo}(q)/S_{vo}(p)] = \frac{N_{vo}(q_{vo}= q_c/p_{vo}= p_c)}{N_{vo}(p_{vo}= p_c)} \qquad \text{(V-21)}$$

$$P[S/S_{vo}(q) \cap S_{vo}(p)] = \frac{N_{vo}(qxp=q_c x p_c/q_{vo}=q_c, p_{vo}=p_c)}{N_{vo}(q_{vo}=q_c, p_{vo}=p_c)} \qquad \text{(V-22)}$$

where Ns are defined in Table VII. The results of the field study are discussed below.

Field Study Results : The data were taken from the same sample that was used in the previous cases. The first period data are used to evaluate the reliability parameters of the system. These parameter values were then used in the reliability model to predict the reliability of the output information for the second period. This result is compared with the directly measured value from the data of the second period. The number of disbursement vouchers were determined for the two periods for the given conditions and are listed in Table XIII. Using these numbers and Eqs. (V-19) - (V-22) the reliability parameters of the system for the two periods were calculated and presented in Table XIV.

It is observed again that the system is stable during the two periods as the values of the reliability parameters for the two periods are within 3.5% of each other (see Table

TABLE XIII

Number of Disbursement Vouchers with Different Given Conditions

Ns	First Period	Second Period
N_{vo}	186	193
$N_{vo}(p_{vo}=p_c)$	182	192
$N_{vo}(q_{vo}=q_c, p_{vo}=p_c)$	162	174
$N_{vo}(q_{vo}=q_c/p_{vo}=p_c)$	162	174

TABLE XIV

Reliability Parameters for Inventory Pricing Component

Reliability Parameters	First Period	Second Period	% diff.
$P[S_{vo}(p)]$	0.9785	0.9948	1.7
$P[S_{vo}(q)/S_{vo}(p)]$	0.8901	0.9063	1.8
$P[S_c/S_{vo}(p)\ S_{vo}(q)]$	1.0	1.0	0.0
$P[S_{inv.}]$ Predicted	0.871	0.9016	3.5
$P[S_{inv.}]$ Direct	0.871	0.9016	3.5

XIV). The reliability parameters $P(S_{vo}(p))$ and $P(S_{vo}(q)/S_{vo}(p))$, increased by 1,7% and 1,8%, respectively from the first period to the second period, whereas $P(S_{inv})$ increased by 3,5%. If we assume that the process of multiplying q_{vo} with p_{vo} is always correct, then the predicted values of the reliability of the value of inventory for the two period are in substantial agreement with direct values. In fact they are equal because of the assumption that the multiplication process is always correct.

CHAPTER VI

CONCLUDING REMARKS

The main purpose of the study as stated in Chapter I was to develop reliability models of internal control systems with different configurations and to show their usefulness in the design and analysis of a control system. In addition, the study was also intended to provide additional support (besides the intuitive reasoning and the limiting results) in validating some of the reliability models developed here, by conducting a field study. The above objectives have been fulfilled. A summary of the results and a discussion of the limitations of the study are presented in the following sections. Also some interesting problems for future research are presented.

A. Summary

The recognition of the importance of internal accounting controls by the accounting profession (e.g. The American Institute of Certified Public Accountants, and The Institute on Internal Auditors) has aroused interest in accounting academicians to study control systems. In particular several attempts have been made to develop reliability models that would provide an objective way to evaluate internal control systems.

In Chapter II, a list of definitions of different terms used in the study, was presented. Also a step-by-step general procedure was described for developing reliability models of systems, subsystems or components.

Two types of components were recognized: performance type and control type. Reliability models for these components were developed for one-, two- and n- input information channels. In the multi-channel input case, it was assumed that the different input channels are independent. This assumption is not valid in general. Therefore, separate models were developed for such cases in Chapter V for the field study.

The reliability models of the two components ('performance' and 'control') with one and more than one input channel, were used in Chapter III as building blocks to develop reliability models for more complex systems. The following configurations were considered in the study for developing reliability models:

1. Performance components in series (Fig. 8).
2. Control components in series (Fig. 10).
3. Performance and control components in series (Fig. 15).
4. Control components in parallel (Fig. 16).
5. Performance components in parallel (Fig. 17).
6. 'Dual' control elements in series (Fig. 18).
7. 'Dual' performance elements in series (Fig. 20).

Several special cases were also considered. In all the cases, the models yielded intuitively appealing results for the reliability of the output information. Limiting cases, where the parameters were either set to zero or one, were considered. This provides intuitive support for the model.

Sensitivity and structural analyses were discussed in Chapter IV. Through several examples, it was demonstrated that sensitivity analysis can provide information about the relative importance of different components and their reliability parameters. Two approaches to achieve this goal were discussed. In the first, partial derivatives of the reliability of the final output data with respect to different parameters were evaluated and compared with each other for a given set of values of parameters. The parameter for which the corresponding derivative was the largest, was determined to be the most important parameter for the system. This result meant that, for a given increase in the value of this parameter, the final reliability increased the most in comparison with a similar increase in any of the other parameters. In the other approach (Birnbaum 1969), a 'reliability importance ratio matrix' was defined (see Sec. IV.A). The elements of one column were compared with the elements of the other columns. The parameter corresponding to the column that had the biggest elements, was the most important parameter. The second approach, though a bit more cumbersome than the first one, was found to be more direct.

It is worth emphasizing that the above result has important applications in the design and analysis of a control system. For example, it was shown in Chapter IV that the relative importance of different parameters of a control system change when the configuration of the system changes or when the reliability of the input data is changed. An important application of this result is that a knowledge of the relatively important parameters can help an internal auditing manager in concentrating his efforts and resources on improving those parameters that are relatively more important in order to achieve a higher output reliability. Also, an external auditor can use this information in allocating his resources to evaluate those parameters that will most influence the output reliability of the system.

A structural analysis of a control subsystem was also presented in Chapter IV to demonstrate the effects of changes in the structure of a subsystem on its output reliability. A control component that prepares receiving reports in an inventory acquisition system, was considered for the analysis. Two possible structures of the component were analysed. In one structure, the material count was taken independent of the number on the vendor's packing slip (i.e. counting was done before seeing the packing slip). In the other cases, the material count was assumed to be dependent on the information on the packing slip (i.e. counting was done after seeing the packing slip). The two models for the two structures were developed and discussed. It was argued on intuitive grounds that the parameter values of the control component would be significantly different in the two cases. This, in turn, would lead to significantly different values of the output reliability. It was shown that for a given set of values of the control parameters, one structure was preferred over the other depending on the value of the input reliability. Analysis of this kind appears to be important to management and internal auditors who can use the results in the design and implementation of a control system. It should be noted that a structural change may not involve much cost, but may improve the output reliability significantly. For example, the structural change considered in the present study does not involve any cost, rather it influences the behavioral action of the 'task performer', the component.

Analysis of a reliability model of a given structure can also provide useful information to management. For example, management can set a goal for the level of reliability to be maintained in the output information. Any unfavorable deviation from this goal can become a serious threat to management activity such as control of assets. Therefore, it is important that reasons for the deviations should be traced and the problems be eliminated by taking proper actions. Since, the reliability of output information is constituted by the reliability of the input information and the reliability parameters of the system, it is not difficult to trace the source of the unfavorable deviation in output reliability. Once the source is known, its performance can be improved by providing proper training or by replacing the element.

A field study was conducted to provide empirical support of the reliability models developed here. The following control and performance components of an inventory acquisition system were chosen for the study:

1. Disbursement Voucher Review for Quantity: A single input control component (Fig. 32).
2. Disbursement Voucher Review for Price: A two input control component (Fig. 33).
3. Inventory Pricing: A performance component with two input channels (Fig. 34).

It should be mentioned that the last two components in the above list are not like the components with two input channels discussed in Chapter II, because the assumption that the two input channels are independent was not valid here. The reliability models, incorporating this dependency, were developed in Chapter V for these components (Nos. 2 and 3 in the above list).

The field data were divided into two periods of equal duration. The data of the two periods were used to determine the reliability parameters of the components. The values of the parameters for the two periods were compared for the stability of the system's performance. It was observed from this comparison that the system showed good stability over the two periods. The first period values of the parameters of a component were used to predict the reliability of the output data for the second period using the second period's input reliability (reliability of the input data). This result was then compared with the value measured directly from the second period's data. In all the three cases, the predicted values were within 1.8% of the directly estimated values. This suggests that the field study results support the reliability model derived here. However, a more detailed study is needed to validate the models statistically.

B. Limitations and Scope for Future Study

Beside the limitations imposed by the explicit assumptions made during the development of the different reliability models here, there are other limitations of which one ought to be aware in order to effectively use these models. The following list presents such limitations:

(i) In the present study, reliability parameters of system components are considered to be time independent. This assumption is not true in general. For example, a person completing a task may not work with the same effectiveness when the work-stress is low (not much work) and when it is high. The dependence of the work-stress on the amount of business (i.e. volume of work) which itself may depend, for example, on the time of the year, makes work-stress and consequently the reliability parameters, time dependent. Learning curve effects i.e., improvement in reliability of performance as one gains experience over time, may also cause time dependence. The time dependent characteristic of the reliability parameters can be incorporated in the models developed here, by developing cycle or learning curve dynamics or by restricting application of the model to conclusions about contemporaneous conditions.

(ii) Structures of internal accounting control systems may change over time. These structural changes should be incorporated in the reliability models when evaluating the reliabilities. In addition assurance that the system tested is stable in design over time during operation should be obtained.

(iii) Values of the reliability parameters of a control system change with a change in the control objective of the control system (the same control system may be used for several control objectives). Moreover, the configuration of a control system for a control objective may also change when the control objective is changed. These changes may cause difficulty in evaluating the reliability parameters for different control objectives, and may make reliability modeling cost ineffective.

(iv) A highly reliable control system may not necessarily be what management wants. The ultimate interest of management is in the dollar value impact of the non-reliability of a control system on the final dollar value of output information. Even though a control system may be highly reliable for a given control objective, a single error that causes a significant impact on the dollar value of output information, can not be ignored from a decision maker's point of view. The reliability models developed here do not provide for the impact of control failure on the dollar value of output information from a control system. Further research in this area seems warranted.

There are several research problems that are not dealt with in the present work, but which should make interesting future projects. First of all, a comprehensive field study is needed to validate the models statistically. Weber (1982) has developed reliability models for asset safeguarding and maintenance of data integrity in EDP systems using Cushing's approach. He has considered only simple systems. With the use of the present work, Weber's work can be extended to more complex systems. The present approach can also be applied to develop reliability models for collusion processes when several persons are involved. Theoretical works on probabilistic models of collusion processes are almost non-existent in the accounting literature. Bodnar (1975) has very briefly discussed such a model, but again was confronted with non-intuitive results. Another area that can be further extended is the variance analysis of reliability parameters.

Furthermore, since large scale internal accounting control systems that are now in common use were not analyzed in the present work, a complete analysis based on their reliability models would be useful for practicing accountants. Cost-benefit analysis is also important in the control design and implementation process. Therefore a study incorporating this aspect in the reliability model is needed.

C. Conclusion

This study has presented a general approach for developing a reliability model for any control system with any configuration. The reliability results obtained in the limiting cases were shown to be intuitively appealing. The results of the field study also supported the models. Usefulness of the models were demonstrated through sensitivity and structural analyses.

FOOTNOTES

[1]The general meaning of engineering systems includes both the electrical and physical systems.

[2]The list representing the important differences is not necessarily complete. However, it illuminates most of the important differences.

[3]The engineering systems that I am concerned with, here, are those which are used to borrow the reliability results. In fact, there are more complex engineering systems where human interactions are considered (Smith and Green, 1981) but they are not of our interest for the present study.

[4]The engineering systems considered here are those systems from which the reliability results have been borrowed. In a complex engineering system, decision making components are present, especially at the junction point of several parallel connected electronic components in a logic diagram. However, the reliability of such a component is usually assumed to be unity (Lerner 1981) and so one does not see the problem being investigated.

[5]In evaluating the reliability of an engineering component, the failure due to the wrong kind of input signal is excluded from consideration (See Amstadter 1971, p. 104).

[6]An information channel is said to be redundant when only one channel's information is required to complete a process or task but several information channels are available.

[7]There are several examples where this assumption is valid. For instance, in a payroll system (see Figure 9), the task of calculating gross pay by a performance component (a clerk) is independent of the state of the input information. It should be noted that in situations where the assumption is not valid, one can use a general result with the conditional probabilities.

[8]This can be justified by considering the example cited earlier in this section. In this example, a person computes the total cost of inventory by multiplying the quantity received with the corresponding price. The correctness of the multiplication process does not depend on the state of the two inputs: the price and the quantity.

[9]This assumption can again be validated through an example. Consider a control component, a voucher clerk reviewing the voucher by comparing the information on the purchase order, vendor's invoice and the receiving report. The process of reviewing the voucher represents the state S_w (the control is in operation). It is clear that the state that the review is in process does not depend on the correctness or incorrectness of the input information on the three documents.

[10]Since $x = 1 - P_w + P_w(P_c - P_e)$, x takes the following values for all possible extreme values of P_w, P_c and P_e:

x	P_w	P_c	P_e
0	1	1	1
1	1	1	0
-1	1	0	1
0	1	0	0
1	0	1	1
1	0	1	0
1	0	0	1
1	0	0	0

[11]A computer program (See Appendix A) was written to compute the output reliability for different n and different values of the reliability parameters.

[12]$R_{on} = R_i x^n + P_e P_w(1-x^n)/(1-x)$ and $x = 1-P_w + P_w(P_c - P_e)$. we want $R_{on} > R_i$ for the control system to be functional, i.e.

$$R_i x^n + P_e P_w(1-x^n)/(1-x) > R_i$$

or

$$P_e P_w(1-x^n)/(1-x) > R_i(1-x^n)$$

or

$$P_e P_w > R_i(1-x) = R_i(P_w - P_w P_c + P_w P_e)$$

or

$$P_e > R_i(1 - P_c + P_e)$$

This leads to the desired result.

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APPENDIX A

COMPUTER PROGRAM FOR CALCULATING OUTPUT RELIABILITY FOR A SYSTEM WITH CONTROL COMPONENTS IN SERIES

The computer program given below was used to compute values for the output reliability R_{on} plotted in Figs. 11-14 for the system shown in Fig. 10. The different values of the parameters used in the computation are given along with the corresponding curves in Figs. 11-14.

```
      RI=0.8
      PE=0.9
      PW=1.0
      PC=0.0
      DPC=0.1
      DO 10 I=1,10
      PC=PC+DPC
      X=1.0-PW+PW*(PC-PE)
      Y=PW*PE
      X1=1.0-X
      WRITE(6,22)RI,PC,PW,PE,X,X1,Y
      DO 20 N=1,10
      XN=X**N
         RON=RI*XN+Y*(1.0-XN)/X1
      WRITE(6,21)N,RON
20    CONTINUE
10    CONTINUE
21    FORMAT(5X,I2,5X,F7.4)
22    FORMAT(1X,7(F7.4,1X))
      STOP
      END
```

APPENDIX B

COMPUTER PROGRAM FOR CALCULATING OUTPUT RELIABILITY FOR A SYSTEM WITH 'DUAL' ELEMENTS

The computer program given below was used to compute the values for the output reliability $\mathcal{R}_{on}$ plotted in Figs. 22 - 25 for the systems shown in Figs. 18, 20, and 21. The different values of the parameters used in the computation are given with the corresponding curves.

```
$JOB
      DIMENSION R(15),SR(15)
      RI=0.8
      PW=1.0
      PC=0.8
      PE=0.0
      DPE=0.2
      QW=1.0
      QC=0.85
      WRITE(6,51)RI,PW,PC,QW,QC
      DO 10 I=1,5
      PE=PE+DPE
      X=1.0-PW+PW*(PC-PE)
      Q1E=0.0
      DQ1E=0.2
      WRITE(6,52)PE,X
      DO 20 J=1,5
      Q1E=Q1E+DQ1E
      Q2E=0.0
      CX=1.0-QW+QW*(QC-2.0*Q1E+Q2E)
      Z=Q2E*QW
      Y=QW*(Q1E-Q2E)
      WRITE(6,51)Q1E,Q2E,CX,Y,Z
      SR(1)=RI
      R(1)=0.0
      DO 40 M=2,15
      R(M)=0.0
   40 SR(M)=0.0
      DO 30 N=1,11
      NN=N+1
      IF(R(N).LE.1.0E-6)R(N)=0.0
      IF(SR(N).LE.1.0E-6)SR(N)=0.0
      R(NN)=SR(N)*X+PE
      SR(NN)=R(NN)**2*CX+2.0*R(NN)*Y+Z
      WRITE(6,53)NN,R(NN),SR(NN)
   30 CONTINUE
   20 CONTINUE
   10 CONTINUE
   51 FORMAT(1X,5(F7.4,1X))
   52 FORMAT(5X,2(F7.4,1X))
   53 FORMAT(7X,I3,2(F8.4,1X))
      STOP
      END
$EXEC
```