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A GENERAL STREAMLINE MODELLING TECHNIQUE FOR HOMOGENEOUS AND HETEROGENEOUS POROUS MEDIA, WITH APPLICATION TO STEAMFLOOD PREDICTION

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A GENERAL STREAMLINE MODELLING TECHNIQUE FOR HOMOGENEOUS AND HETEROGENEOUS POROUS MEDIA, WITH APPLICATION TO STEAMFLOOD PREDICTION

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DOCTOR OF PHILOSOPHY

By

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A GENERAL STREAMLINE MODELLING TECHNIQUE FOR HOMOGENEOUS AND HETEROGENEOUS POROUS MEDIA, WITH APPLICATION TO STEAMFLOOD PREDICTION

A DISSERTATION

APPROVED FOR THE DEPARTMENT OF PETROLEUM AND GEOLOGICAL ENGINEERING

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ABSTRACT

A general streamline/streamtube simulation method applicable to homogeneous, anisotropic, as well as piecewise homogeneous porous media is described with its application to steamflood recovery prediction. The boundaries of the media can be of any arbitrary but smooth shape with boundary conditions that can be any of or combination of either Neuman, Dirichlet, or mixed boundary conditions.

The (Laplace's) equation describing the distribution of potential was solved by the Boundary Element Method (BEM) also called the Boundary Integral Element Method (BIEM). This method utilizes the superposition of singular solutions whose distributions are determined by the boundary conditions of the problem. The streamlines are generated assuming single phase flow (unit mobility assumption). The streamtubes are assumed to be linear tubes of rectangular cross section. Streamlines and streamtubes were generated for a homogeneous reservoir assuming it was bounded on all sides by sealing faults. Then, it was assumed to have an oil-water contact (constant pressure) boundary on one side, while the remainder of the boundary remained sealed. Oil recovery calculations were made for a steam drive process

in each streamtube generated and added together to give the recovery in the entire field or pilot area. Finally, the reservoir was treated as a piecewise homogeneous reservoir having two regions of unequal permeabilities. The streamlines and streamtubes were again generated and oil recovery calculated for each of the two types of boundary arrangements mentioned above.

Steamflood prediction was made assuming each streamtube existed in isolation. Thus, there was no heat exchange between streamtubes. For homogeneous reservoirs, recovery was calculated using the Marx and Langenheim equations. For the piecewise homogeneous reservoir, new equations were derived for the location and rate of advance of the steam front while it is in the second permeability region of a streamtube containing two permeability regions. Comparison of the potentials obtained by the BEM and those obtained analytically for simple domains confirm the validity of the Boundary Element Method. This method provides a general technique to properly model different reservoirs with arbitrary boundaries and boundary conditions, particularly, piecewise homogeneous reservoirs which had not been possible using the method of images.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES

LIST OF FIGURES

A GENERAL STREAMLINE MODELLING TECHNIQUE FOR HOMOGENEOUS AND HETEROGENEOUS POROUS MEDIA, WITH APPLICATION TO STEAMFLOOD PREDICTION

CHAPTER I

INTRODUCTION

Steamflood recovery performance can be predicted either by the use of analytical equations derived for linear and radial homogeneous porous media, or by the use of numerical simulators. The numerical simulators have the benefits of giving more detailed and more reliable results. They also have the advantage of being applicable to a wide variety of types of porous media. Unfortunately, these simulators require a great deal of detailed reservoir data, making them expensive to run. Quite often, the available geological and production data are not sufficient, and economic considerations prohibit the collection of such extensive data. In such situations, simulators are often run with some of the data being assumed. This reduces the advantages of the numerical simulator. Improving the accuracy by obtaining more data may make the cost of running one unjustifiable.

The analytical methods have the advantage of simplicity and therefore are relatively cheap. But, in order to predict recovery using analytic equations for an entire field with multiple sources and sinks, the field must be reduced to an equivalent linear or radial system. One way to achieve this is by the so-called streamline/streamtube technique. Previous methods of determining the streamlines and streamtubes for reservoirs with regular as well as irregular boundaries were based on the technique of image wells. According to the well-image theory, hydrogeologic boundaries may be replaced, for analytical purposes, by imaginary wells which produce the same distrubance effects as the boundaries. Boundary problems are thereby simplified to the consideration of an infinite reservoir in which real and image wells operate simultaneously and their effects are superposed to give the potentials and streamlines at desired locations. This image-well method is inadequate in the following respects:

1. Theoretically, the number of image wells and their locations extend to infinity. The actual number of image wells used in the solution of a particular problem becomes a matter of experience or judgement.
2. It has only been applied to homogeneous reservoirs. Extension to heterogeneous systems is theoretically possible but the calculations very quickly become tedious.

3. Irregular boundaries are generally modelled by arbitrary locations of image wells whose rates are determined as those necessary to confine as much of the streamlines as desired. Hence, the derived potentials from which the streamlines and streamtubes are calculated do not represent the true potential solutions to the problem with its boundary conditions.

It is therefore easy to see that a new modelling technique is needed for cases where current streamline methods would either be over-simplifications or incorrect, and numerical simulators uneconomical. These would include:

(i) Homogeneous reservoirs with irregularly shaped boundaries having a variety of boundary conditions.

(ii) Heterogeneous reservoirs whose properties are such that they could be naturally divided into a number of subregions with homogeneous properties.

The research work described here develops such a model. A heterogeneous reservoir is assumed to be made up of a few subregions with constant material properties such as transmissibility, porosity, etc. The model exploits the simplicity of the analytical models by utilizing as its basis the point source solution in infinite medium, from which it determines other solutions by superposition. At the same time, it uses a numerical technique where the boundaries of the region are discretized, called the Boundary Element

Method (BEM) or the Boundary Integral Element Method (BIEM). This technique is used to generate the streamtubes (or stream channels) for heterogeneous reservoirs of any arbitrary shape or boundary conditions. These streamtubes form the conduits through which the steam displacement processes take place from injectors to producers.

CHAPTER II

LITERATURE SURVEY

In the year 1899, Slichter[1] used the concept of steady state streamlines in his theoretical investigation of the motion of ground waters. In 1962, Ferris et al[2] published a comprehensive study of ground water flow in bounded systems with various kinds of boundary conditions. The effects of the boundaries were simulated by the use of image wells. This image-well technique is now widely used by ground water hydrologists as well as petroleum engineers.[2,3,4,5]

Petroleum engineers have extended the streamline concept to include so-called streamtubes or streamchannels whose dimensions are determined from the knowledge of the streamlines.[6,7] The streamtubes act as the channels, inside which oil displacement processes are assumed to be taking place. The use of image-wells to simulate the boundary conditions has generally been limited to homogeneous domains with boundaries of simple geometrical shapes such as squares, rectangles, circles, etc. Even though, in theory, the image well method can be applied to heterogeneous media[4] as well as to arbitrary boundary shapes, in practice, the calculations quickly become tedious. This is partly due to the fact that the image wells and their locations extend to infinity even for the simplest closed systems. This problem was partially addressed by Chan,[8] who in 1975 presented an improved image-well method for straight-line boundaries that involves a rearrangement of the image pattern depending on the boundary configurations. However, even with this improvement, the number of image wells required still prohibits its use in all but the simplest geometrical shapes. Secondly, the improved method does not account for heterogeneous media. The limitation to simple geometrical shapes was removed by Leblanc[9] and Lin.[10] Leblanc used the image-well technique to bound the streamlines in a homogeneous reservoir of arbitrarily shaped boundary by using a trial and error procedure to determine the image well rates required to adequately confine the streamlines. Lin extended the method to include reservoirs which are not sealed but have some restricted flow as in the case of partial water encroachment. Each method is unsatisfactory because locations of the image wells (and therefore the potentials) are determined by how well the streamlines are confined rather than by the true boundary conditions of the physical system. Thus, the image-well technique is inadequate for reservoirs that are: (a) heterogeneous and (b) have arbitrarily shaped boundaries.

The Boundary Element Method (BEM), also called the Boundary Integral Element Method (BIEM), is a technique that can handle several kinds of shapes and heterogeneities. It is an integral method that utilizes the superposition of singular solutions of the partial differential equations of the system. The method has recently emerged as an important numerical technique for the solution of linear elliptic equations such as occur in potential flow and elastostatics.[11,14] During the last decade, the BEM has been extended to solve linear parabolic equations as well.[15,16] The extension of the method to piece-wise non-homogeneous bodies of arbitrary shape has been attempted by Banerjee,[17] and Jaworski[18] while Butterfield and Tomlin[19] have used the method for solving zoned anisotropic continuum problems. The literature on the BEM is large and growing rapidly as more researchers develop new methods of application. Reference 17 lists more than 70 references on this subject and contains a good review of the literature on BEM up to the year 1976. The Boundary Element Method has been used in this research work to generate the steady state streamlines and associated streamtubes for heterogeneous reservoirs. The streamtubes serve as the channels inside which any displacement process (steam drive in this case) takes place. In what follows, a brief review of the literature on steam drive processes is given.

The benefits of applying heat to an oil reservoir to aid oil recovery were foreseen as early as 1917,[59] and in the 1920's and 1930's, steam was used to remove paraffin from the wellbore.[22] Since Shell Oil Company's successful stimulation of a California oil well in the early 1960's, steam injection has steadily gained prominence as an oil recovery method. In fact, steam injection is currently the most popular of all enhanced oil recovery methods--producing 405 million barrels per day (MMBPD) worldwide as compared with 221 MMBPD from all other enhanced recovery methods combined.[22]

Even before Shell Oil Company's successful stimulation of the early 1960's, technical papers began to be published describing the transport of injected heat in porous media. One of these papers was by Lauwerier,[23] whose model for the injection of hot water into an oil bearing formation was published in 1955. He assumed a linear homogeneous reservoir of constant thickness, sandwiched between two oil sands. The thermal conductivity of the oil sand in the vertical direction was assumed equal to that of the cap and base rock which were identical. The horizontal thermal conductivity in the oil sand was assumed to be zero. The vertical thermal conductivity in the water sand was assumed infinite so that the temperature in the vertical direction for the water sand was always uniform. Instantaneous equilibrium was assumed between sand grains and reservoir fluid so that the sand grains had the same temperature as the reservoir fluid throughout. Finally, he formulated his problem by assuming that at time t, the temperature at the boundaries between the two zones were elevated and kept at a constant temperature due to the injection of hot water. By making separate heat balances within the oil and water sands, two equations were obtained. They were solved using the Laplace transform technique to obtain the temperature distributions in the two sands. In 1959, Rubenstein[24] improved Lauwerier's model by assuming constant, isotropic thermal conductivities in the reservoir as well as in the confining cap and base rock.

In 1959, Marx and Langenheim[25] presented a model for calculating the heated area and its rate of advance based on idealized step temperature at the steam front. They also presented an equation for the temperature gradient for a linear temperature distribution in the steam zone. Finally, they proposed a method of predicting the oil recovery. In 1961, Willman, et al[26] presented data on oil recovery mechanisms for hot water and steam injection for a variety of oils and sand types. They presented a modified Buckley-Leverett method to predict the oil recovery for a radial system. Both the Marx and Langenheim, and the Willman, et al models have been further explained and expanded by several authors.[27-30] Baker[31] reported on an experimental study of heat flow in steam flooding in a radial system. Among some of Baker's results were: (a) the fraction of injected heat lost when expressed as a function of time did not depend on injection rate. (b) A significant portion of injected heat was contained in the hot water zone ahead of and underlying the steam zone. (c) The division of heat between the steam and hot water zones did depend on injection rate. In the same year, Mandle and Volek[32] made a rigorous calculation of the transport of heat during steamflooding. They found that earlier theory, based on neglecting the heat transport from the steam zone into the oil/water region was consistent with the physical model up to a critical time t_c. t_c marks the time at which the mode of heat transfer from the steam zone into the oil/water region changes from being predominantly conductive to being predominantly convective. The equation describing the growth of the steam zone changes accordingly at $t = t_c$. They presented approximate equations to describe the steam zone growth after $t = t_c$.

In 1970, Shutler and Boberg[33] presented a graphical method for calculating oil recovery during steamflooding. The method was a modification of the Buckley-Leverett method for isothermal, two phase flow in porous media. The 1970's saw continued and more diverse interest in steam drive mechanisms. Previous efforts had been directed more on obtaining analytical solutions to simplified models. But in the 1970's the analytical, experimental, as well as numerical models, became more sophisticated. Some of the experimental and analytical research reported were:

Neuman[34] presented a theoretical analysis for a steam flood model that included the effects of gravity override. Van Lookeren[35] derived equations to estimate the approximate shape of the steam/liquid interface. Huygen[36] who fitted curves to recovery and sweep data for a three dimensional model of a five-spot pattern. Gomaa[37] used a numerical method to determine the effects of various parameters on steamflood recovery. He reported the following: (a) above 150 ft., the reservoir thickness had little effect on vertical heat loss. (b) Pattern shape and spacing had insignificant effect on steamflood oil recovery if injection rate per unit reservoir volume was fixed and well productivity was not a limiting factor. Chu[38] studied the effects of well patterns on steamflood performance. He reported that the alteration of one pattern resulted in substantial loss of production in the surrounding patterns. He also reported that a 5-spot pattern produced more oil, the production getting progressively less in the inverted 7-spot, the inverted 13-spot and the inverted 9-spot, respectively. Atkinson and Ramey[39] presented a general relationship between the various mathematical models of heat transfer in porous media. They also presented new models for fractured and non-fractured porous media. Rhee and Doscher[40] presented a semi-analytical method for calculating oil recovery by steamflooding that combined the effects of steam distillation and gravity override. Myhill and

Stegemeier[41] published correlations for the effects, of thermal, reservoir petrophysical, and steam properties on the equivalent oil/steam ratio.

CHAPTER III

RESEARCH OBJECTIVE AND PROPOSED MODEL

The objective of this research work is to develop a streamline model to predict oil recovery during steamflooding in heterogeneous porous media of arbitrary shapes and boundary conditions. The prediction model is desired to be able to handle the following types of heterogeneities:

1. Piece-wise homogeneous reservoirs. That is, heterogeneous reservoirs that can be split into subregions such that rock properties such as permeability, porosity, etc. can be considered as uniform within each subregion, but only be different from region to region.
2. Anisotropic reservoirs.
3. Homogeneous reservoirs with impermeable shale inclusions.

The external boundaries of these reservoirs can be

1. completely closed
2. partially closed, with the remainder at infinity.

The boundary conditions can be any of Neuman, Dirichlet, mixed, or any combination of the aforementioned.

In this research work, only the piecewise homogeneous reservoirs will be treated in detail, the others are handled by minor modifications. The proposed model uses the Boundary Element Method to generate steady state streamlines for the above mentioned heterogeneous reservoirs. As each streamline is traced, the dimensions of a hypothetical streamtube surrounding it are calculated. Thus, for each streamline, an associated streamtube is calculated. When this is done for all the streamlines, the heterogeneous system is now assumed to be replaced by a series of straight conduits connecting injectors to producers. Each of these conduits is considered to be an isolated system inside which the steam displacement processes take place. Lateral heat losses from the sides of the streamtubes are ignored. For piecewise homogeneous reservoirs, some, but not all, of the streamtubes will consist of regions of differing permeabilities. For these cases, a new generalized derivation of the equations of Marx and Langenheim or Mandle and Volek was made. The rate of advance of the steam front and recovery in each of the tubes is accumulated for any time to complete the desired model.

CHAPTER IV

STATEMENT OF THE PROBLEM

Consider a two-dimensional reservoir domain (D), of arbitrary shape enclosed by the boundary surface (S). The domain D is made up of a number of homogeneous sub-domains, D_i for i = 1,..., number of sub-domains (Figure 1). Thus, $D = D_1 + D_2 + \cdots + D_n$. Each subdomain has its own value of material properties such as permeability, porosity etc., that may or may not be different from those of the adjacent region or regions.

The surface S is assumed to be sufficiently smooth and can be such that part or all of it is at infinity. S can be made up of any combination of three kinds of surfaces S_Φ, S_n, and $S_{\Phi,n}$. Each specifies a different kind of boundary condition. $S = S_\Phi + S_n + S_{\Phi,n}$. Over S_Φ, the potentials are specified. Over S_n, the gradients $\left(\frac{\partial \Phi}{\partial n}\right)$ are specified; and over $S_{\Phi,n}$, a mixed boundary condition is specified.

Inside any or all of the domains are arbitrarily located sources and sinks. The first problem is to find the steady state potential, streamline and streamtube distributions for the whole system subject to any combination of the three types of boundary conditions:

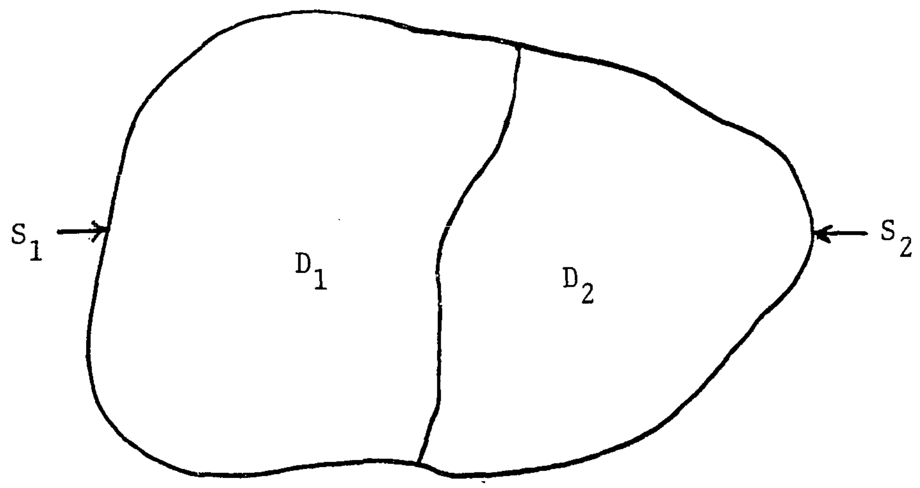


FIGURE 1: Piecewise-Homogeneous Domain

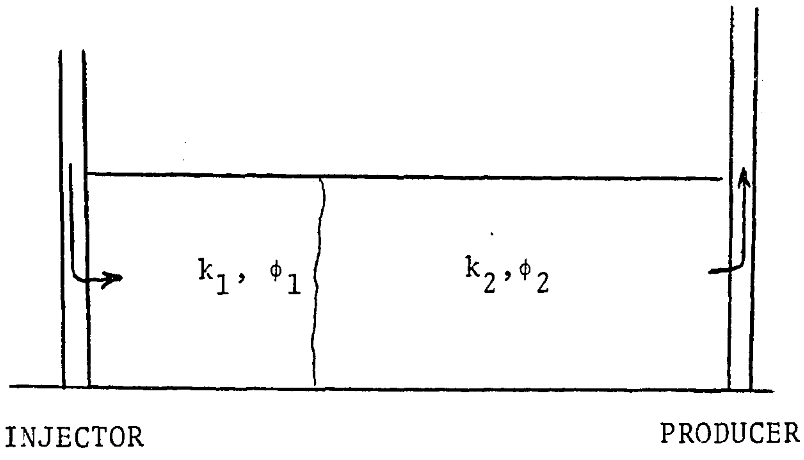


FIGURE 2

Linear one-dimensional model of piecewise-homogeneous reservoir

a. The Dirichlet condition prescribes the potential on the S_Φ boundary. That is,

$$\Phi = \bar{\Phi}(x,y) \text{ on } S = S_\Phi \tag{1}$$

b. The Neumann condition prescribes the normal derivative of the potential on the S_n boundary. That is,

$$\frac{\partial \Phi}{\partial n} = \frac{\partial \bar{\Phi}}{\partial n} \text{ on } S = S_n \tag{2}$$

c. The mixed boundary condition prescribes a relation between Φ and $\frac{\partial \Phi}{\partial n}$ on the $S_{\Phi,n}$ boundary. That is,

$$\Phi = f\left(\frac{\partial \Phi}{\partial n}\right) \text{ on } S = S_{\Phi,n} \tag{3}$$

The third condition is found on the free-water surface in an aquifer and is not generally of concern to Petroleum Engineers.

The second half of the problem to be solved to complete the model is to derive equations that describe the rate of advance of the steam zone in a linear system made up of regions of different permeability (Fig. 2). These equations will be used to predict the oil recovery inside the generated streamtube.

CHAPTER V

MATHEMATICAL DEVELOPMENT

A common method of formulating the equations that describe the characteristics of a physical system is by considering some elemental portion of its domain and applying some physical law to it, such as the the conservation principle. This method of formulation generally results in differential equations which can be solved analytically or numerically. The most widely-used numerical technique is the method of finite differences. In this method, the domain is divided into elements or cells and the differential operators are approximated by difference operators resulting in a

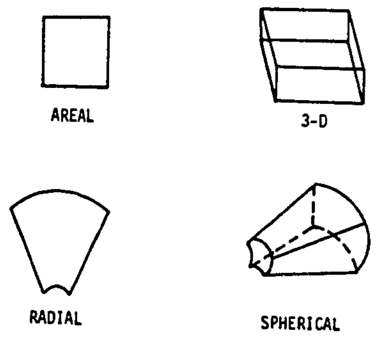


FIGURE 3. Elemental volumes[45]

system of algebraic equations that are valid at the "nodes" of the cells within the domain. Because the domain of the problem is discretized, the method will be termed a domain method. Another way of solving the differential equations is by the Boundary Element Method. In this method, the differential equations in the domain are first transformed into integral equations which relate the unknown function and possibly certain of its derivatives to the given values on the boundary. The method of solution utilizes the superposition of known basic solutions to give the complete solution. After the differential equations that describe the system have been derived, they are first transformed into integral equations on the boundary before being solved by the boundary element technique.

The equations describing the potential distribution and flow of a single phase, incompressible fluid in a homogeneous porous medium is developed by taking an elemental volume of the reservoir and applying the law of conservation of mass to it. This is a procedure that can be found in standard texts.[44,45,46] The result is called the continuity equation and can be expressed in general form as:

$$\nabla \cdot (\rho u) = - \frac{\partial}{\partial t} (\rho \phi) \qquad \text{(V.1.1)}$$

Equation (V.1.1) is called the continuity equation where

$\nabla \cdot (\rho u)$ = divergence of (ρu)

ρ = density

u = volume flux per unit area

t = time

ϕ = porosity

Darcy's law relates the volume flux (u) to the potential gradient.

$$u = -\nabla\left(\frac{k}{\mu}\,\Phi\right) \qquad (V.1.2)$$

where

k = permeability

Φ = potential

μ = fluid viscosity

Substituting Equation (V.1.2) into Equation (V.1.1) gives:

$$\nabla\cdot\left[\frac{\rho}{\mu}\,\nabla(k\Phi)\right] = \frac{\partial}{\partial t}\,(\rho\phi) \qquad (V.1.3)$$

Equation (V.1.3) is completely general. No assumptions have been made about the nature of the medium. It can be applied to any coordinate system, and is valid at any point of a three dimensional domain except at locations where there are sources or sinks.

The following simplifying assumptions are made in order to formulate the equations describing the physical system.

1. Incompressible, homogeneous, single phase fluid is assumed to be flowing in the system.
2. The medium is homogeneous and isotropic.
3. Two-dimensional flow exists. This means that flow

in the vertical direction is negligible. Therefore, flow is only in the horizontal x and y directions.

4. Gravitational effects are neglected.

Introducing these simplifying assumptions gives, in two dimensions:

$$\frac{\rho}{\mu}\frac{\partial}{\partial x}\left(k_x \frac{\partial \Phi}{\partial x}\right) + \frac{\rho}{\mu}\frac{\partial}{\partial y}\left(k_y \frac{\partial \Phi}{\partial y}\right) = -\frac{\partial}{\partial t}(\rho \phi) \qquad \text{(V.1.4)}$$

Making the further assumption of steady state flow, Equation (V.1.4) reduces to:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial \Phi}{\partial y}\right) = 0 \qquad \text{(V.1.5)}$$

For a homogeneous reservoir, $k_x = k_y = k$. Therefore, Equation (V.1.5) becomes:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \qquad \text{(V.1.6)}$$

Equation (V.1.6) is known as Laplace's equation. It applies everywhere in the domain except where sources and sinks exist. When sources and sinks are present in the domain, Equation (V.1.6) changes to:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\mu}{k}\sum_{j=1}^{N} q_j \, \delta(x - x_j, y - y_j) = 0 \qquad \text{(V.1.7)}$$

which is known as Poisson's equation, where

q_j = the volumetric flow rate per unit reservoir thickness of the jth source or sink

N = the number of sources and sinks

δ = the Dirac delta function

x_j, y_j = the coordinates of the j'th source or sink location

For a heterogeneous reservoir made up of two or more homogeneous subregions, Darcy's law applies in each subregion. This means that Equation (V.1.6) as well as Equation (V.1.7) are applicable depending on whether there are sources or not. To obtain the potential distribution for the heterogeneous system, either Equation (V.1.6) or Equation (V.1.7) (depending upon the presence or absence of sources and sinks) is applied to each region. They are combined by taking care of the compatibility and continuity conditions on the various interfaces between adjacent regions.

V.2 TRANSFORMATION TO AN INTEGRAL FORMULATION

As stated earlier, when the domain is a heterogeneous one, made up of homogeneous subdomains, the method is to apply Laplace's or Poisson's equation to each subdomain. They are then combined (making sure to satisfy compatibility and continuity conditions on the common boundaries between the regions). Thus, the homogeneous subregions form the basic building blocks for the heterogeneous system. For this reason, the theory will be developed for a homogeneous domain.

Extensions to heterogeneous systems will be discussed in a later section. The objective, therefore, is to find potential distributions that satisfy the differential equation

$$L\Phi \equiv \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = -\frac{\mu}{k}\sum_{j=1}^{N} q_j\ \delta(x - x_j,\ y - y_j) \qquad \text{(V.2.1)}$$

subject to:

$$\Phi|_{S_\Phi} = \bar{\Phi}_{S_\Phi} \qquad \text{(V.2.2)}$$

$$\left.\frac{\partial\Phi}{\partial n}\right|_{S_n} = (\bar{\Phi}_n)_{S_n} \qquad \text{(V.2.3)}$$

where L is the differential operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$.

N is the number of sources and sinks

$\bar{\Phi}_{S_\Phi}$ is the specified potential on the S_Φ boundary

$(\bar{\Phi}_n)_{S_n}$ is the specified normal gradinet $\left(\frac{\partial\Phi}{\partial n}\right)$ on the S_n boundary

The first step in the integral formulation of the problem is to form the product of a function W with both sides of Equation (V.2.1) and integrate over the domain of interest. This is called the inner, or scalar, or dot product of W with $L\Phi$ and is denoted $\langle W, L\Phi\rangle$.[47] By definition,

$$\langle W, L\Phi\rangle = \int_D W\{L\Phi\}\ dD \qquad \text{(V.2.4)}$$

L is the linear differential operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ on Φ.

W is an arbitrary function which is sufficiently differentiable for $L^*\{W\}$ to exist, where L^* is the formal adjoint differential operator associated with L. Integration by parts is performed on (V.2.4) until the operator on Φ disappears (i.e., integrating twice by parts for two-dimensional problems). The result is of the form

$$\int_D W\{L\Phi\}\, dD = \{\text{boundary terms}\} + \int_D \Phi\{L^*W\}\, dD \quad \text{(V.2.5)}$$

In two dimensions, the boundary terms are line integrals; and in three dimensions, they are surface integrals. Forming the inner product $\langle L\Phi, W\rangle$ for the left hand side of Equation (V.1.2) gives

$$\langle L\Phi, W\rangle = \int_D \left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2}\right) W\, dD \quad \text{(V.2.6)}$$

where D represents the domain of integration. Integration of (V.2.6) by parts is done by first rewriting it as a double integral:

$$\langle L\Phi, W\rangle = \int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} \frac{\partial^2\Phi}{\partial x^2} W\, dx\, dy + \int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} \frac{\partial^2\Phi}{\partial y^2} W\, dx\, dy \quad \text{(V.2.7)}$$

Integrate each term by parts to give, for the first term,

$$\int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} \frac{\partial^2\Phi}{\partial x^2} W\, dx\, dy = \int_{y_1}^{y_2} \left[W \frac{\partial\Phi}{\partial x}\bigg|_{x_1}^{x_2} - \int_{x_1(y)}^{x_2(y)} \frac{\partial\Phi}{\partial x}\frac{\partial W}{\partial x}\, dx\right] dy \quad \text{(V.2.8)}$$

Integrate the last term on the R.H.S. of (V.2.8) by parts and substitute back into (V.2.8) to give

$$\int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} \frac{\partial^2\Phi}{\partial x^2} W \, dx \, dy = \int_{y_1}^{y_2} \left[W\left(\frac{\partial\Phi}{\partial x}\right)\Big|_{x_1(y)}^{x_2(y)} - \frac{\partial W}{\partial x}(\Phi)\Big|_{x_1(y)}^{x_2(y)} + \int_{x_1(y)}^{x_2(y)} \Phi \frac{\partial^2 W}{\partial x^2} dx \right] dy$$

$$= \int_{y_1}^{y_2} W\left(\frac{\partial\Phi}{\partial x}\Big|_{x_1(y)}^{x_2(y)}\right) dy - \int_{y_1}^{y_2} \frac{\partial W}{\partial x}\left(\Phi\Big|_{x_1(y)}^{x_2(y)}\right) dy$$

$$+ \int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} \Phi \frac{\partial^2 W}{\partial x^2} dx \, dy \qquad \text{(V.2.9)}$$

Now

$$dy = ds \cos\theta$$

and

$$\cos\theta = \hat{i} \cdot \hat{n}$$

$$\therefore \; dy = \hat{i} \cdot \hat{n} \, ds \qquad \text{(V.2.10)}$$

where $\hat{n}$ is the outward unit normal and ds is the differential element of arc length along the boundary S. $\hat{i}$ is the unit vector in the x direction.

Substituting for dy into (V.2.9) gives

$$\int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} \frac{\partial^2\Phi}{\partial x^2} W \, dx \, dy = \int_S \left(W \frac{d\Phi}{dx}\right)\hat{i}\cdot\hat{n} \, ds - \int_S \left(\frac{\partial W}{\partial x} \Phi\right)\hat{i}\cdot\hat{n} \, ds$$

$$\int_D \Phi \frac{\partial^2 W}{\partial x^2} dD \qquad \text{(V.2.11)}$$

By a similar procedure

$$\int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} \frac{\partial^2\Phi}{\partial x^2} W \, dx \, dy = \int_S \left(W \frac{\partial\Phi}{\partial y}\right)\hat{j}\cdot\hat{n} \, ds - \int_S \left(\frac{\partial W}{\partial y} \Phi\right)\hat{j}\cdot\hat{n} \, ds$$

$$+ \int_D \frac{\partial^2 W}{\partial y^2} \, dD \qquad (V.2.12)$$

Combining (V.2.11) and (V.2.12) gives

$$\int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} \left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2}\right) W \, dx \, dy = \int_S W\left(\frac{\partial\Phi}{\partial x} \hat{i} + \frac{\partial\Phi}{\partial y} \hat{j}\right)\cdot\hat{n} \, ds$$

$$- \int_S \Phi\left(\frac{\partial W}{\partial x} \hat{i} + \frac{\partial W}{\partial y} \hat{j}\right)\cdot\hat{n} \, ds + \int_D \Phi\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) dD$$

$$(V.2.13)$$

where $\hat{j}$ is the unit vector in the y-direction.

Therefore, the inner product of the left hand side of Equation (V.2.1) gives:

$$\langle L\Phi, W\rangle = \int_S W \frac{\partial\Phi}{\partial n} \, ds - \int_S \Phi \frac{\partial W}{\partial n} \, ds + \int_D \left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) dD$$

$$(V.2.14)$$

The inner product of the right hand side of Equation (V.2.1) is:

$$\langle \textstyle\sum_{j=1}^{N} q_j \, \delta(x - x_j, y - y_j), W\rangle \qquad (V.2.15)$$

$$= \frac{\mu}{k} \int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} \textstyle\sum_{j=1}^{N} q_j \, \delta(x - x_j, y - y_j) \, W \, dx \, dy$$

which is simply,

$$\frac{\mu}{k} \sum_{j=1}^{N} q_j(x_j,y_j)\ W\ (x_j,y_j) \qquad \text{(V.2.16)}$$

Therefore, from Equations (V.2.14) and (V.2.16), the inner product of both sides of Equation (V.2.1) is:

$$\int_S W \frac{\partial \Phi}{\partial n}\, ds - \int_S \Phi \frac{\partial W}{\partial n}\, ds + \int_D \Phi\ \nabla^2 W\ dD$$

$$= - \frac{\mu}{k} \sum_{j=1}^{N} q_j(x_j,y_j)\ W\ (x_j,y_j) \qquad \text{(V.2.17)}$$

where ∇^2 is the two-dimensional Laplacian differential operator denoting the vector dot product $\nabla \cdot \nabla$. ∇ is the gradient operator defined in cartesian coordinates as

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} .$$

Thus,

$$\nabla^2 = \nabla \cdot \nabla = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y}\right) \cdot \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y}\right) = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$\hat{i}$ and $\hat{j}$ are unit vectors in the x and y directions, respectively. Equation (V.2.17) can be rewritten as:

$$\int_D \Phi\ \nabla^2 W\ dD = \int_S \Phi \frac{\partial W}{\partial n}\, ds - \int_S W \frac{\partial \Phi}{\partial n}\, ds$$

$$- \frac{\mu}{k} \sum_{j=1}^{N} q_j(x_j,y_j)\ W\ (x_j,y_j) \qquad \text{(V.2.18)}$$

Now, by a clever choice of W, the left part of Equation (V.2.18), that is, the domain integral, can be reduced to the potential Φ only. When this is done, the original differential equation would have been transformed into an integral equation involving just boundary integrals only. The value of W to achieve this is that W which satisfies the equation

$$\nabla^2 W = - \delta(x - x_i, y - y_i) \qquad \text{(V.2.19)}$$

without satisfying any boundary conditions. Equation (V.2.19) happens to be the equation describing the potential distribution that would occur due to the application of a unit charge at (x_i, y_i), and the solution to it is variously called the fundamental solution, the free-space Green's function, or the principal solution. Using the right side of Equation (V.2.19) to substitute for $\nabla^2 W$ in Equation (V.2.18) gives:

$$-\int_D \Phi\, \delta(x - x_i, y - y_i)\, dD = \int_S W \frac{\partial \Phi}{\partial n}\, ds - \int_S \Phi \frac{\partial W}{\partial n}\, ds$$

$$+ \frac{\mu}{k} \sum_{j=1}^{N} q_j(x_i, y_j)\; W\, (x_j, y_j) \qquad \text{(V.2.20)}$$

The left side of Equation (V.2.20) is simply $-\Phi(x_i, y_i)$; thus

$$\Phi(x_i, y_i) = \int_S W \frac{\partial \Phi}{\partial n}\, ds - \int_S \Phi \frac{\partial W}{\partial n}\, ds$$

$$+ \frac{\mu}{k} \sum_{j=1}^{N} q_j(x_j, y_j) W(x_j, y_j) \qquad \text{(V.2.21)}$$

Equation (V.2.21) relates the potential at an interior point (x_i, y_i) to both its value on the boundary and that of its normal derivative to the boundary. It contains the function W which is the solution to Equation (V.2.19) and for this work will be called the fundamental solution given as,[48]

$$W = \frac{1}{2\pi} \ln \frac{1}{r} \qquad (V.2.22)$$

where r is the distance between an observation point (x,y) and the point of application of the unit charge (x_i, y_i). If the observation point is anywhere within the domain (except at the source/sink locations),

$$r = r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \qquad (V.2.23)$$

and

$$W = \frac{1}{2\pi} \ln \frac{1}{\sqrt{(x - x_i)^2 + (y - y_i)^2}} \qquad (V.2.24)$$

Let

$$r_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \qquad (V.2.25)$$

be the distance between the unit charge point (x_i, y_i) and the source point (x_j, y_j); then

$$W(x_j, y_j) = \frac{1}{2\pi} \ln \frac{1}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}} \qquad (V.2.26)$$

Equation (V.2.21) can be rewritten as:

$$\Phi(x_i, y_i) = \frac{1}{2\pi} \int_S \frac{\partial \Phi}{\partial n} \left(\ln \frac{1}{r_i}\right) ds - \frac{1}{2\pi} \int_S \Phi \frac{\partial}{\partial n}\left(\ln \frac{1}{r_i}\right) ds$$

$$+ \frac{\mu}{2\pi k} \sum_{j=1}^{N} q_j(x_j, y_j) \ln \frac{1}{r_{i,j}} \qquad (V.2.27)$$

It is of interest to note that Equation (V.2.27) could have been obtained by different methods, such as application of Green's second identity directly to Equation (V.1.7), or by the method of weighted residuals.[12]

V.3 REPRESENTATION AS A BOUNDARY INTEGRAL EQUATION

Equations (V.2.21) or (V.2.27) are integral formulations of the differential equation (V.2.1). In order to apply the boundary element solution method, Equation (V.2.22) or (V.2.27) must be transformed to a boundary integral equation. This means that the unknown $\Phi(x_i, y_i)$ on the left of Equation (V.2.22) or (V.2.27) should be in terms of boundary coordinates. Therefore, the point of application of the unit charge (Equation V.2.20) has to be on the boundary. Let the boundary point of application of the unit charge be denoted (x_b, y_b), then the fundamental solution is, then:

$$W(x_b, y_b) = \frac{1}{2\pi} \ln \frac{1}{r_b} \qquad \text{(V.3.1)}$$

where r_b is the distance from any point (x,y) to the boundary unit charge point (x_b, y_b) and is given as

$$r_b = \sqrt{(x - x_b)^2 + (y - y_b)^2} \qquad \text{(V.3.2)}$$

The distance from the source/sink locations (x_j, y_j) to the boundary unit charge point is:

$$r_{b,j} = \sqrt{(x_b - x_j)^2 + (y_b - y_j)^2} \qquad \text{(V.3.3)}$$

By taking the point of application of the unit charge to the boundary, the boundary integral formulation of the problem becomes:

$$\Phi(x_b,y_b) = \int_S W \frac{\partial\Phi}{\partial n}\, ds - \int_S \frac{\partial W}{\partial n}\, ds + \frac{\mu}{k} \sum_{j=1}^{N_j} q_j(x_j,y_j)W(x_j,y_j) \qquad (V.3.4)$$

Substituting for $W(x_b,y_b)$ and $W(x_j,y_j)$ using Equations (V.3.2) and (V.3.3) gives:

$$\Phi(x_b,y_b) = \frac{1}{2\pi} \int_S \frac{\partial\Phi}{\partial n} \ln\frac{1}{r_b}\, ds - \frac{1}{2\pi} \int_S \Phi \frac{\partial}{\partial n}\left(\ln\frac{1}{r_b}\right) ds + \frac{\mu}{2\pi k}\sum_{j=1}^{N} q_j(x_j,y_j) \ln\frac{1}{r_{b,j}} \qquad (V.3.5)$$

where r_b and $r_{b,j}$ are given by Equations (V.3.2) and (V.3.3). The evaluation of the right hand side of Equation (V.3.5) gives the potential at any boundary point with coordinates (x_b,y_b).

V.3.1 EVALUATION OF THE IMPROPER INTEGRALS

All the integrals involve the term $\ln\frac{1}{r_b}$ which has a singularity at the point $r_b = 0$. These kinds of integrals whose integrands become infinite at some point in the domain of integration are known as improper integrals. The usual method of evaluating such integrals is as follows:[12,48]

a. For a two-dimensional problem, take a small circle or semi-circle of radius ε around the point of singularity. This divides the domain of integration (the boundary) into two parts; one part contains the singularity and the other part does not (Figure 4).

b. Evaluate both integrals and take the limit as ε tends to zero. The result is the required value of the integral.

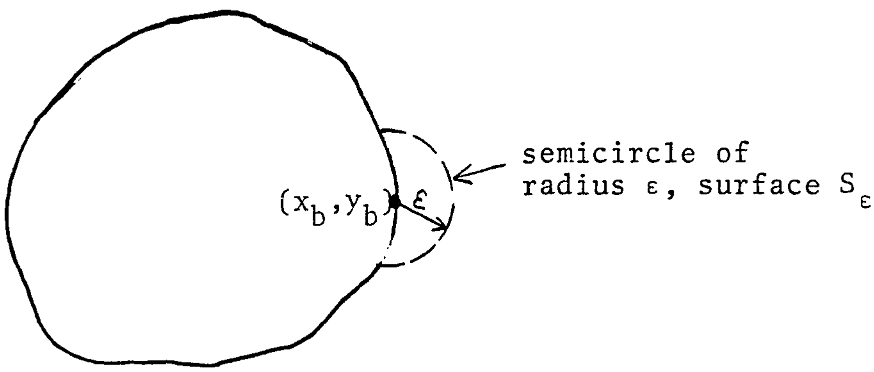


FIGURE 4. Method of evaluating improper integral.

On the part of the boundary $S - 2\varepsilon$, $\ln \frac{1}{r_b}$ is continuous. Following the procedure outlined above, the integrals in Equation (V.3.5) can be evaluated as follows: For the second integral on the right of Equation (V.3.5),

$$\frac{1}{2\pi}\int_S \Phi \frac{\partial}{\partial n}\left(\ln \frac{1}{r_b}\right) ds = \frac{1}{2\pi}\int_{S-S_\varepsilon} \Phi \frac{\partial}{\partial n}\left(\ln \frac{1}{r_b}\right) ds$$

$$+ \frac{1}{2\pi}\int_{S_\varepsilon} \Phi \frac{\partial}{\partial n}\left(\ln \frac{1}{r_b}\right) ds \qquad (V.3.6)$$

For the last integral of Equation (V.3.6), the normal direction is the same as the radial direction. That is,

$$\frac{\partial}{\partial n}\left(\ln \frac{1}{r_b}\right) = \frac{\partial}{\partial r_b}\left(\ln \frac{1}{r_b}\right)\Bigg|_{r_b=\varepsilon} = -\frac{1}{\varepsilon} \qquad (V.3.7)$$

The last term of Equation (V.3.6) now becomes:

$$\frac{1}{2\pi}\int_{S_\varepsilon} \Phi \frac{\partial}{\partial n}\left(\ln \frac{1}{r_b}\right) ds = -\frac{1}{2\pi\varepsilon}\int_{S_\varepsilon} \Phi\, ds \qquad (V.3.8)$$

For the case where the surface S_ε is a semi-circle, Equation (V.3.8) becomes:

$$\frac{1}{2\pi\varepsilon}\int_{S_\varepsilon} \Phi \frac{\partial}{\partial n}\left(\ln \frac{1}{r_b}\right) ds = -\frac{1}{2\pi\varepsilon}\int_{\frac{2\pi\varepsilon}{2}} \Phi\, ds = -\frac{1}{2\pi\varepsilon}\left(\frac{2\pi\varepsilon}{2}\,\Phi^*\right)$$

$$= -\frac{1}{2}\,\Phi^* \qquad (V.3.9)$$

where Φ^* is an average value of $\Phi(x,y)$ on S_ε, and in the limit as ε tends to zero

$$\lim_{\varepsilon\to 0}\left(-\frac{1}{2}\Phi^*\right) = -\frac{1}{2}\,\Phi(x_b,y_b) \qquad (V.3.10)$$

The first integral on the right of Equation (V.3.5) is split into two integrals as:

$$\frac{1}{2\pi}\int_S \ln\frac{1}{r_b}\ \frac{\partial\Phi}{\partial n}\,ds = \frac{1}{2\pi}\int_{S-S_\varepsilon} \ln\frac{1}{r_b}\left(\frac{\partial\Phi}{\partial n}\right)ds$$

$$+\ \frac{1}{2\pi}\int_{S_\varepsilon} \ln\frac{1}{r_b}\left(\frac{\partial\Phi}{\partial n}\right)ds \qquad \text{(V.3.11)}$$

Following the procedure leading to Equation (V.3.9) gives

$$\frac{1}{2\pi}\int_{S_\varepsilon} \ln\frac{1}{r_b}\left(\frac{\partial\Phi}{\partial n}\right)ds = -\ \frac{1}{2\pi\varepsilon}\left(\frac{2\pi\Phi}{2}\right)\left(\frac{\partial\Phi}{\partial n}\right)^*$$

$$= -\ \frac{1}{2}\ \frac{\partial\Phi^*}{\partial n} \qquad \text{(V.3.12)}$$

where $\frac{\partial\Phi^*}{\partial n}$ is an average value of $\frac{\partial\Phi}{\partial n}$ on S_ε and

$$\lim_{\varepsilon\to 0}\left(-\ \frac{1}{2}\ \frac{\partial\Phi^*}{\partial n}\right) = 0 \qquad \text{(V.3.13)}$$

From (V.3.6)

$$\lim_{\varepsilon\to 0}\frac{1}{2\pi}\int_{S-S_\varepsilon} \Phi\ \frac{\partial}{\partial n}\left(\ln\frac{1}{r_b}\right)ds = \frac{1}{2\pi}\int_S \Phi\ \frac{\partial}{\partial n}\left(\ln\frac{1}{r_b}\right)ds \quad \text{(V.3.14)}$$

From (V.3.11)

$$\lim_{\varepsilon\to 0}\frac{1}{2\pi}\int_{S-S_\varepsilon} \ln\frac{1}{r_b}\left(\frac{\partial\Phi}{\partial n}\right)ds = \frac{1}{2\pi}\int_S \ln\frac{1}{r_b}\left(\frac{\partial\Phi}{\partial n}\right)ds \qquad \text{(V.3.14a)}$$

Substituting (V.3.10), (V.3.13), (V.3.14), and (V.3.14a) into (V.3.5) gives the potential at any boundary point (x_b, y_b) as:

$$\Phi(x_b,y_b) = \frac{1}{2\pi}\int_S \Phi \frac{\partial}{\partial n}\left(\ln \frac{1}{r_b}\right) ds - \frac{1}{2\pi}\int_S \left(\ln \frac{1}{r_b}\right)\frac{\partial \Phi}{\partial n} ds$$

$$+ \frac{1}{2}\Phi(x_b,y_b) + \sum q_j(x_j,y_j)\frac{1}{2\pi}\ln\frac{1}{r_{b_j}} \qquad \text{(V.3.15)}$$

Therefore,

$$\frac{1}{2}\Phi(x_b,y_b) = \frac{1}{2\pi}\int_S \ln\frac{1}{r_b}\left(\frac{\partial \Phi}{\partial n}\right) ds - \frac{1}{2\pi}\int_S \Phi\frac{\partial}{\partial n}\left(\ln\frac{1}{r_b}\right) ds$$

$$+ \frac{\mu}{k}\sum q_j(x_j,y_j)\frac{1}{2\pi}\ln\frac{1}{r_{b,j}} \qquad \text{(V.3.16)}$$

Equation (V.3.16) is the boundary integral formulation of the problem. The boundary S is made up of two parts, S_1 and S_2 such that $S = S_1 + S_2$. On S_1, Φ is prescribed and $\frac{\partial \Phi}{\partial n}$ is unknown. On S_2, $\frac{\partial \Phi}{\partial n}$ is prescribed and Φ is unknown.

CHAPTER VI

SOLUTION BY DISCRETIZATION OF THE BOUNDARY

The integral equations (V.2.26) and V.3.16) are solved numerically be discretizing the boundaries of the domain into elements (hence the name: Boundary Element Method). A homogeneous medium will be assumed. Later, the method will be modified to handle heterogeneous media made up of homogeneous regions. The boundary of the domain is divided into M straight line segments as shown in Figure 5.

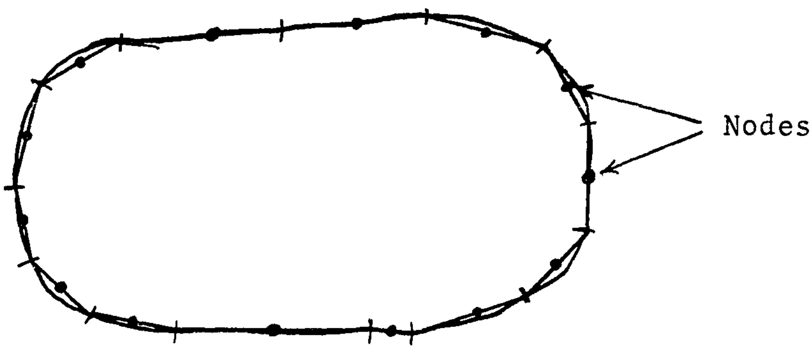


FIGURE 5

Straight-line boundary elements or segments

The unknown values of Φ and $\frac{\partial\Phi}{\partial n}$ are evaluated at the mid-points of each segment. These points are called the node points. In the present work, the node points will always be at the mid-points of straight line boundary segments. The cases where the boundary segments are curved and cases where the node points are at the intersections of the boundary elements are treated in Brebbia.[12] Of the M boundary segments, M_1 belong to the S_Φ type boundary where Φ is specified as constant ($\bar{\Phi}$) on each segment while $\frac{\partial\Phi}{\partial n}$ is unknown. The remaining M_2 segments belong to the S_n type boundary where $\frac{\partial\Phi}{\partial n}$ is specified as constant $\left(\frac{\partial\bar{\Phi}}{\partial n}\right)$ on each segment and Φ is unknown. Equation (V.3.16), when put in discrete form over the two types of boundaries, is:

$$\frac{1}{2}\,\Phi(x_b,y_b) = \frac{1}{2\pi}\sum_{L=1}^{M_1}\left(\frac{\partial\Phi}{\partial n}\right)_L\int_{S_L} \ln\frac{1}{r_{b,L}}\,dS_L$$

$$+\ \frac{1}{2\pi}\sum_{L=1}^{M_2}\left(\frac{\partial\bar{\Phi}}{\partial n}\right)_L\int_{S_L} \ln\frac{1}{r_{b,L}}\,dS_L$$

$$-\ \frac{1}{2\pi}\sum_{L=1}^{M_1}\bar{\Phi}_L\int_{S_L}\frac{\partial}{\partial n}\left(\ln\frac{1}{r_{b,L}}\right)dS_L$$

$$-\ \frac{1}{2\pi}\sum_{L=1}^{M_2}\Phi_L\int_{S_L}\frac{\partial}{\partial n}\left(\ln\frac{1}{r_{b,L}}\right)dS_L$$

$$+\ \frac{\mu}{k}\sum_{j=1}^{N} q_j(x_j,y_j)\,\frac{1}{2\pi}\ln\frac{1}{r_{b,j}} \qquad \text{(VI.0.1)}$$

which can be combined as:

$$\frac{1}{2}\,\Phi(x_b,y_b) = \frac{1}{2\pi}\sum_{L=1}^{M}\left(\frac{\partial\bar{\Phi}}{\partial n}\right)_L\int_{S_L}\ln\frac{1}{r_{b,L}}\,dS_L$$

$$-\frac{1}{2\pi}\sum_{L=1}^{M}\bar{\Phi}_L\int_{S_L}\frac{\partial}{\partial n}\left(\ln\frac{1}{r_{b,L}}\right)dS_L$$

$$+\frac{\mu}{k}\sum_{j=1}^{N}q_j(x_j,y_j)\,\frac{1}{2\pi}\ln\frac{1}{r_{b,j}} \qquad \text{(VI.0.2)}$$

where

$M = M_1 + M_2$

$S = S_\Phi + S_n$

$\bar{\Phi}_L$ = given boundary condition of Φ on S_Φ of the Lth segment

$\left(\frac{\partial\bar{\Phi}}{\partial n}\right)_L$ = given boundary condition of $\frac{\partial\Phi}{\partial n}$ on S_n of the Lth segment

Let

$$G_{b,L} = \frac{1}{2\pi}\int_{S_L}\ln\frac{1}{r_{b,L}}\,dS_L \qquad \text{(VI.0.3)}$$

$$H_{b,L} = \frac{1}{2\pi}\int_{S_L}\frac{\partial}{\partial n}\left(\ln\frac{1}{r_{b,L}}\right)dS_L \qquad \text{(VI.0.4)}$$

Equation (VI.0.2) then becomes:

$$\frac{1}{2}\,\Phi(x_b,y_b) = \sum_{L=1}^{M}\left(\frac{\partial\bar{\Phi}}{\partial n}\right)_L[G_{b,L}] - \sum_{L=1}^{M}\bar{\Phi}_L[H_{b,L}]$$

$$+\frac{\mu}{2\pi k}\sum_{j=1}^{N}q_j(x_j,y_j)\ln\frac{1}{r_{b,j}} \qquad \text{(VI.0.5)}$$

$G_{b,L}$ and $H_{b,L}$ are evaluated for all the segments all around the boundary, including the setment containing the charge point (x_b, y_b). Therefore, there are two kinds of integrals to be evaluated, namely:

1. Integration over the segment b that contains the unit charge point. This segment contains a singularity point at (x_b, y_b). Integration over this segment happens when L = b.
2. Integration over the rest of the segments. These are the condition when L ≠ b.

Because of this need to evaluate two kinds of integrals, the boundary integral equation (VI.0.5) can be rewritten to reflect these two kinds of integrals by defining the following:

$$G_{b,L} = \begin{cases} \breve{G}_{b,L} & \text{when } L \neq b \\ G_{b,b} & \text{when } L = b \end{cases}$$

$$H_{b,L} = \begin{cases} \breve{H}_{b,L} & \text{when } L \neq b \\ H_{b,b} & \text{when } L = b \end{cases}$$

Equation (VI.0.5) now becomes:

$$\begin{aligned} \frac{1}{2}\,\phi(x_b, y_b) = & \left[\left(\frac{\partial \Phi}{\partial n}\right)_{L=b} G_{b,b} + \sum_{L=1}^{M} \left(\frac{\partial \Phi}{\partial n}\right)_{L\neq b} \breve{G}_{b,L}\right] \\ & - [\phi(x_b, y_b) H_{b,b} + \sum_{L=1}^{M} (\phi)_{L\neq b} \breve{H}_{b,L}] \\ & + \frac{\mu}{2\pi k} \sum_{j=1}^{N} q_j(x_j, y_j) \ln \frac{1}{r_{b,j}} \end{aligned} \qquad \text{(VI.0.6)}$$

which can be rearranged to give (where $\Phi(x_b,y_b) = \Phi_{L=b}$)

$$\left[\Phi_{L=b}\left(H_{b,b} + \frac{1}{2}\right) + \sum_{L=1}^{M} (\Phi)_{L\neq b}\, \breve{H}_{b,L}\right]$$

$$- \left[\left(\frac{\partial\Phi}{\partial n}\right)_{L=b} G_{b,b} + \sum_{L=1}^{M} \left(\frac{\partial\Phi}{\partial n}\right)_{L\neq b} \breve{G}_{b,L}\right]$$

$$= \frac{\mu}{2\pi k}\sum_{j=1}^{N} q_j(x_j,y_j)\, \ln \frac{1}{r_{b_j}} \qquad \text{(VI.0.7)}$$

V.1 EVALUATION OF THE INTEGRALS

Equation (VI.0.7) requires the evaluation of the following integrals:

$$G_{b,L} = \frac{1}{2\pi}\int_{S_L} \ln \frac{1}{r_{b,L}}\, dS_L \begin{cases} \text{for all } L \neq b, \text{ denoted as } \breve{G}_{b,L} \\ \text{for all } L = b, \text{ denoted as } G_{b,b} \end{cases}$$

$$H_{b,L} = \frac{1}{2\pi}\int_{S_L} \frac{\partial}{\partial n} \ln \frac{1}{r_{b,L}}\, dS_L \begin{cases} \text{for all } L \neq b, \text{ denoted as } \breve{H}_{b,L} \\ \text{for } L = b, \text{ denoted as } H_{b,b} \end{cases}$$

These integrals can easily be evaluated analytically for the present case where the segments S_L are straight lines. However, for generality, they are expressed in dimensionless form and evaluated numerically using a four-point Gaussian quadrature method with the formula:

$$\int_{-1}^{1} f(\alpha)\, d\alpha = \sum_{k=1}^{n} W_k f(\alpha_k) + E_n \qquad \text{(VI.1.1)}$$

where

W_k = weighting factor

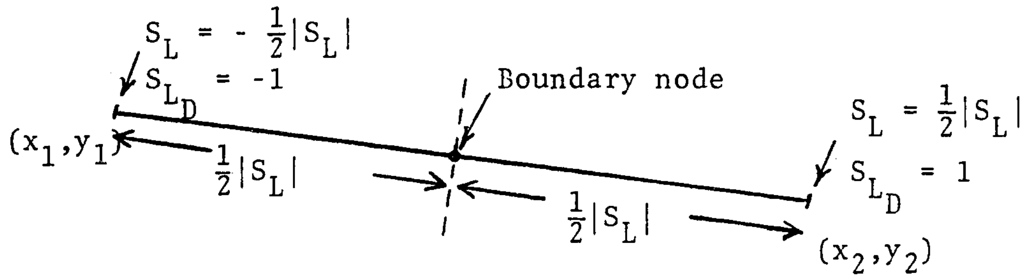
α_k = coordinate of the k'th integration point

n = total number of integration points (n = 4)

E_n = error term

In order to apply the quadrature formula to $G_{i,L}$ and $H_{i,L}$, they must be transformed to integrals with limits from -1 to +1. This is achieved by expressing the variable of integration dS in dimensionless form as:

<u>For any element where L ≠ b:</u>



define a dimensionless variable S_{L_D} as:

$$S_{L_D} = \frac{S_L}{\frac{1}{2}|S_L|}, \qquad \text{then} \quad dS_L = \frac{1}{2}|S_L|\, dS_{L_D}$$

where

$S_{L_D} = 0$ when $S_L = 0$

$S_{L_D} = -1$ when $S_L = -\frac{1}{2}|S_L|$

$S_{L_D} = 1$ when $S_L = \frac{1}{2}|S_L|$

and

$$S_L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore, from Equation (A.1.10) of Appendix A,

$$H_{b,L} = \frac{1}{2\pi} \int_{S_L} \frac{\partial}{\partial n_L}\left(\ln \frac{1}{r_{b,L}}\right) ds \tag{VI.1.2}$$

$$= - \frac{1}{4\pi} \int_{-1}^{1} \frac{(x_L - x_b)(y_2 - y_1) + (y_L - y_b)(x_2 - x_1)}{(x_L - x_b)^2 + (y_L - y_b)^2} dS_D$$

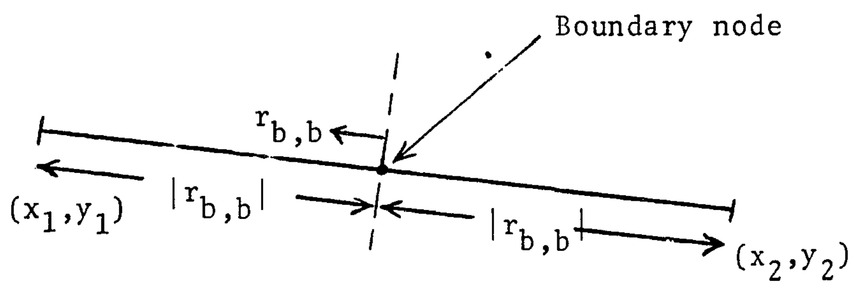
From Equation (A.1.11) of Appendix A

$$G_{b,L} = \frac{1}{2\pi} \int_{S_L} \ln \frac{1}{r_{b,L}} ds \tag{VI.1.3}$$

$$= \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{8\pi} \int_{-1}^{1} \ln \frac{1}{\sqrt{(x_L - x_b)^2 + (y_L - y_b)^2}} dS_D$$

<u>For the element where L = b:</u>

For this case, the vector r_b is along the length of the segment and is equal to one-half the length of the segment. Therefore, ds = dr.



Since $r_{b,b}$ is along the segment,

$$\frac{\partial}{\partial n_b} = \frac{\partial}{\partial r_{b,b}} \cdot \frac{\partial r_{b,b}}{\partial n_b}$$

but $\frac{\partial r_{b,b}}{\partial n_b} = 0$ because $r_{b,b}$ is perpendicular to n_b. Define the dimensionless variable

$$r_D = \frac{r_{b,b}}{|r_{b,b}|} \qquad dr = |r_{b,b}| dr_D$$

From Equations (A.1.12) and (A.1.15) of Appendix A,

$$H_{b,b} = \frac{1}{2\pi} \int_{S_b} \frac{\partial}{\partial n_b} \left(\ln \frac{1}{r_{b,b}} \right) ds_b = 0 \quad \text{(VI.1.4)}$$

$$G_{b,b} = \frac{1}{2\pi} \int_{S_b} \ln \frac{1}{r_{b,b}} ds_b = \frac{|r_{b,b}|}{\pi} \left(\ln \frac{1}{|r_{b,b}|} - 1 \right)$$

(VI.1.5)

$$G_{b,b} = \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{2\pi} \left(\ln \frac{1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} - 1.0 \right)$$

(VI.1.6)

No integration is therefore required for the case when $L = b$. Applying the quadrature formula to Equations (VI.1.2) and (VI.1.3) result in:

$$H_{b,L} = -\frac{1}{4\pi} \sum_{k=1}^{4} W_k \left(\frac{(x_{L_k} - x_b)(y_2 - y_1) + (y_{L_k} - y_b)(x_2 - x_1)}{(x_{L_k} - x_b)^2 + (y_{L_k} - y_b)^2} \right)$$

(VI.1.7)

$$G_{b,L} = \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{4\pi} \sum_{k=1}^{4} W_k \left[\ln \left(\frac{1}{\sqrt{(x_{L_k} - x_b)^2 + (y_{L_k} - y_b)^2}} \right) \right] \qquad \text{(VI.1.8)}$$

where (x_{L_k}, y_{L_k}) are the quadrature points. The values of $H_{b,L}$ and $G_{b,L}$ for all the boundary segments can now be calculated using Equations (VI.1.4) through (VI.1.8). Since $H_{b,b} = 0$, Equation (VI.0.10) becomes:

$$\frac{1}{2} \Phi_b = \sum_{L=1}^{M} \left(\frac{\partial \Phi}{\partial n}\right)_L G_{b,L} - \sum_{L=1}^{M} \Phi_L H_{b,L} + \frac{1}{2\pi} \sum_{j=1}^{n} q_j (x_j, y_j) \times \ln \frac{1}{r_{b,j}} \qquad \text{(VI.1.9)}$$

which is put in the form

$$\sum_{L=1}^{M} \Phi_L H_{b,L} - \sum_{L=1}^{M} \left(\frac{\partial \Phi}{\partial n}\right)_L G_{b,L} = \frac{\mu}{2\pi k} \sum_{j=1}^{n} q_j (x_j, y_j) \ln \frac{1}{r_{b,j}} \qquad \text{(VI.1.10)}$$

Note that when L = b, the first term on the left of Equation (VI.1.10) is $\Phi_b(\frac{1}{2} + H_{b,b})$. But, since $H_{b,b} = 0$ when L = b, the first term on the left of Equation (VI.1.10) becomes $\frac{1}{2} \Phi_b$.

CHAPTER VII

DETERMINATION OF THE POTENTIAL AND VELOCITY AT INTERIOR POINTS

Equation (VI.1.10) is written for every node in the system; that is, for b = 1,2,...,m (number of nodes). This will result in m equations in m unknowns which can be put in matrix notation (Brebbia[12]) as:

$$[H]\Phi = [G]\Phi_n + B \tag{VII.0.1}$$

where $\Phi_n = \frac{\partial \Phi}{\partial n}$ = the derivative normal to the boundary

$$B = \frac{\mu}{2\pi k}\sum_{j=1}^{n} q_j(x_j,y_j) \ln \frac{1}{r_{b,j}}$$

B is a column vector, H and G are m by m matrices.

Let $m = m_1 + m_2$ where m = number of elements. In the L.H.S. of (VII.0.1), m_1 values of Φ are specified as boundary conditions, leaving m_2 unknown values of Φ. Within the R.H.S. of (VII.0.1), m_2 values of $\left(\frac{\partial \Phi}{\partial n}\right)$ are specified in the boundary conditions, leaving m_1 unknown values of $\left(\frac{\partial \Phi}{\partial n}\right)$. All the values of the B vector are known. Therefore, Equation (VII.0.1) has m equations and a total of m unknowns which can be rearranged as:

$$Ax = F \tag{VII.0.2}$$

where x now is a vector containing the m unknowns, m_1 of which are the unknown $\frac{\partial\Phi}{\partial n}$'s and m_2 of which are the unknown Φ's on the boundaries. $m_1 + m_2 = m$. The matrix system of Equation (VII.0.2) can be solved by Gauss elimination method to give the m_1 values of $\frac{\partial\Phi}{\partial n}$ and m_2 values of Φ on the boundaries.

VII.1 POTENTIAL

After the solution, all the boundary values of Φ and $\frac{\partial\Phi}{\partial n}$ are now known and are used to determine the potentials and velocities at any desired interior point.

Interior points where the potentials are desired are denoted with the coordinates (x_i, y_i). Therefore, in this case,

$$r_{i,L} = \sqrt{(x_L - x_i)^2 + (y_L - y_i)^2} \tag{VII.1.1}$$

$$r_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - x_i)^2} \tag{VII.1.2}$$

The potential at any interior point can be determined by Equation (V.2.27) which in discrete form is:

$$\Phi(x_i, y_i) = \sum_{L=1}^{M} \left(\frac{\partial\Phi}{\partial n}\right)_L G_{i,L} - \sum_{L=1}^{M} \Phi_L H_{i,L} + \frac{\mu}{2\pi k}\sum_{j=1}^{n} q_j(x_j, y_j) \ln \frac{1}{r_{i,j}} \tag{VII.1.3}$$

where:

$$G_{i,L} = \frac{1}{2\pi} \int_{S_L} \frac{\partial}{\partial n}\left(\ln \frac{1}{r_{i,L}}\right) dS \qquad \text{(VII.1.4)}$$

$$H_{i,L} = \frac{1}{2\pi} \int_{S_L} \ln \frac{1}{r_{i,L}} dS \qquad \text{(VII.1.5)}$$

$r_{i,L}$ is the distance from the interior point (x_i, y_i) to the mid-point of the L'th boundary point (x_L, y_L). $r_{i,L}$ and r_{ij} are given by Equations (VII.1.1) and (VII.1.2) respectively.

After the application of the quadrature formula, $H_{i,L}$ and $G_{i,L}$ can be evaluated as:

$$H_{i,L} = - \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{4\pi} d_{i,L} \times \sum_{k=1}^{4} W_k \frac{1}{\sqrt{(x_i - x_{L_k})^2 + (y_i - y_{L_k})^2}} \qquad \text{(VII.1.6)}$$

$$G_{i,L} = \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{4\pi} \sum_{k=1}^{4} W_k \ln \frac{1}{\sqrt{(x_{L_k} - x_i)^2 + (y_{L_k} - y_i)^2}} \qquad \text{(VII.1.7)}$$

VII.2 VELOCITY

The velocities in the x and y directions at any point in the interior of the domain are obtained by Darcy's law:

$$v_x = - \frac{k}{\mu\phi} \frac{\partial}{\partial x} \Phi(x,y) \qquad \text{(VII.2.1)}$$

$$v_y = -\frac{k}{\mu\phi}\frac{\partial\Phi}{\partial y} \tag{VII.2.2}$$

Utilizing (VII.1.4) and (VII.1.5), the potential at an interior point (x_i, y_i) can be written as:

$$\Phi(x_i,y_i) = \frac{1}{2\pi}\sum_{L=1}^{m}\left(\frac{\partial\Phi}{\partial n}\right)_L \int_{S_L} \ln\left(\frac{1}{r_{i,L}}\right) dS_L$$

$$- \frac{1}{2\pi}\sum_{L=1}^{m} \Phi_L \int_{S_L} \frac{\partial}{\partial n}\left(\ln\frac{1}{r_{i,L}}\right) dS_L + \frac{\mu}{k}\frac{1}{2\pi}\sum_{j=1}^{n} q_j \ln\left(\frac{1}{r_{i,j}}\right)$$

(VII.2.3)

From Appendix A,

$$\frac{\partial}{\partial n}\left(\ln\frac{1}{r_{i,L}}\right) = -\frac{d_{i,L}}{r_{i,L}^2} \tag{VII.2.4}$$

Substituting (VII.2.4) into (VII.2.3) and differentiating with respect to x and y gives:

$$\frac{\partial\Phi(x_i,y_i)}{\partial x_i} = \frac{1}{2\pi}\sum_{L=1}^{m}\left(\frac{\partial\Phi}{\partial n}\right)_L \int_{S_L} \frac{\partial}{\partial x_i}\left(\ln\frac{1}{r_{i,L}}\right) dS_L$$

$$- \frac{1}{2\pi}\sum_{L=1}^{m} \Phi_L \int_{S_L} \frac{\partial}{\partial x_i}\left(\frac{-d_{i,L}}{r_{i,L}^2}\right) dS_L + \frac{\mu}{k}\frac{1}{2\pi}\sum_{j=1}^{n} q_j \frac{\partial}{\partial x_i}\ln\left(\frac{1}{r_{i,j}}\right)$$

(VII.2.5)

Similarly,

$$\frac{\partial\Phi(x_i,y_i)}{\partial y_i} = \frac{1}{2\pi}\sum_{L=1}^{m}\left(\frac{\partial\Phi}{\partial n}\right)_L \int_{S_L} \frac{\partial}{\partial y_i}\left(\ln\frac{1}{r_{i,L}}\right) dS_L$$

$$- \frac{1}{2\pi}\sum_{L=1}^{m} \Phi_L \int_{S_L} \frac{\partial}{\partial y_i}\left(\frac{-d_{i,L}}{r_{i,L}^2}\right) dS_L + \frac{\mu}{k}\frac{1}{2\pi}\sum_{j=1}^{n} q_j \frac{\partial}{\partial y_i}\ln\left(\frac{1}{r_{i,j}}\right)$$

(VII.2.6)

where:

$$r_{i,L} = \sqrt{(x_i - x_L)^2 + (y_i - y_L)^2}$$

$$r_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

$$d_{i,L} = \frac{\left(\frac{y_2 - y_1}{x_2 - x_1}\right)x_i - y_i + y_1 - \left(\frac{y_2 - y_1}{x_2 - x_1}\right)x_1}{\pm\sqrt{\left(\frac{y_2 - y_1}{x_2 - x_1}\right)^2 + 1}} \qquad \text{(VII.2.7)}$$

The sign is chosen so that $d_{i,L}$ is always positive. The terms

$$\frac{\partial}{\partial x_i}\left(-\frac{d_{i,L}}{r_{i,L}^2}\right), \text{ and } \frac{\partial}{\partial y_i}\left(-\frac{d_{i,L}}{r_{i,L}^2}\right)$$

are given in Appendix A as:

$$\frac{\partial}{\partial x_i}\left(-\frac{d_{i,L}}{r_{i,L}^2}\right) = \frac{2(x_i - x_L)d_{i,L}}{r_{i,L}^4} - \left(\frac{1}{r_{i,L}^2}\right)\left(\frac{A}{\pm\sqrt{A^2 + 1}}\right) \qquad \text{(VII.2.8)}$$

$$\frac{\partial}{\partial y_i}\left(-\frac{d_{i,L}}{r_{i,L}^2}\right) = \frac{2(y_i - y_L)d_{i,L}}{r_{i,L}^4} - \left(\frac{1}{r_{i,L}^2}\right)\left(\frac{A}{\pm\sqrt{A^2 + 1}}\right) \qquad \text{(VII.2.9)}$$

where

$$A = \frac{y_2 - y_1}{x_2 - x_1}$$

Since

$$\frac{\partial}{\partial x_i}\left(\ln \frac{1}{r_{i,L}}\right) = -\frac{\partial}{\partial x_i}\left(\ln r_{i,L}\right) = -\frac{1}{r_{i,L}}\left(\frac{dr_{i,L}}{dx_i}\right) = -\frac{x_i - x_L}{r_{i,L}^2}$$

(VII.2.10)

$$\frac{\partial}{\partial y}\left(\ln \frac{1}{r_{i,L}}\right) = -\frac{y_i - y_L}{r_{i,L}^2} \tag{VII.2.11}$$

Equations (VII.2.8) to (VII.2.11) are substituted into (VII.2.5) and (VII.2.6) to give:

$$\begin{aligned}\frac{\partial\Phi(x_i,y_i)}{\partial x_i} &= \frac{1}{2\pi}\sum_{L=1}^{m}\left(\frac{\partial\Phi}{\partial n}\right)_L \int_{S_L} -\frac{x_i - x_L}{r_{i,L}^2}\, dS_L \\ &- \frac{1}{2\pi}\sum_{L=1}^{m} \Phi_L \int_{S_L}\left[\frac{2(x_i - x_L)d_{i,L}}{r_{i,L}^4} - \left(\frac{1}{r_{i,L}^2}\right)\left(\frac{A}{\pm\sqrt{A^2+1}}\right)\right] dS_L \\ &+ \frac{\mu}{2\pi k}\sum_{j=1}^{n} q_j \left(\frac{x_i - x_j}{r_{i,j}^2}\right)\end{aligned} \tag{VII.2.12}$$

$$\begin{aligned}\frac{\partial\Phi(x_i,y_i)}{\partial y_i} &= \frac{1}{2\pi}\sum_{L=1}^{m}\left(\frac{\partial\Phi}{\partial n}\right)_L \int_{S_L} -\frac{y_i - y_L}{r_{i,L}^2}\, dS_L \\ &- \frac{1}{2\pi}\sum_{L=1}^{m} \Phi_L \int_{S_L}\left[\frac{2(y_i - y_L)d_{i,L}}{r_{i,L}^4} - \left(\frac{1}{r_{i,L}^2}\right)\left(\frac{1}{\pm\sqrt{A^2+1}}\right)\right] dS_L \\ &+ \frac{\mu}{2\pi k}\sum_{j=1}^{n} q_j \left(\frac{y_i - y_j}{r_{i,j}^2}\right)\end{aligned} \tag{VII.2.13}$$

When Equations (VII.2.12) and (VII.2.13) are evaluated and substituted into Equations (VII.2.1) and (VII.2.2), the velocities in the x and y directions are obtained at any given locations in the interior of the domain. The integrals in Equations (VII.2.12) and (VII.2.13) are evaluated numerically using a four point Gaussian quadrature formula as given earlier in Section 5.1.

In matrix representation, Equations (VII.2.12) and (VII.2.13) can be expressed as:

$$\frac{\partial \Phi(x_i, y_i)}{\partial x_i} = \sum_{L=1}^{m} [G'_x]_{i,L} \left(\frac{\partial \Phi}{\partial n}\right)_L - \sum_{L=1}^{m} [H'_x]_{i,L} \Phi_L + B' \quad \text{(VII.2.14)}$$

$$\frac{\partial \Phi(x_i, y_i)}{\partial y_i} = \sum_{L=1}^{m} [G'_y]_{i,L} \left(\frac{\partial \Phi}{\partial n}\right)_L - \sum_{L=1}^{m} [H'_y]_{i,L} \Phi_L + B' \quad \text{(VII.2.15)}$$

After applying the quadrature formula,

$$H'_x\ i,L = -2 \sum_{k=1}^{4} W_k \left[\frac{d_{i,L}(x_i - x_{L_k})}{r^4_{i,L_k}} + \frac{A}{r^2_{i,L_k} \pm \sqrt{A^2+1}}\right] \left(\frac{1}{2}|S_L|\right)$$

(VII.2.16)

$$H'_y\ i,L = -2 \sum_{k=1}^{4} W_k \left[\frac{d_{i,L}(y_i - y_{L_k})}{r^4_{i,L_k}} + \frac{A}{r^2_{i,L_k} \pm \sqrt{A^2+1}}\right] \left(\frac{1}{2}|S_L|\right)$$

(VII.2.17)

$$G'_x\ i,L = -\sum_{k=1}^{4} \frac{x_i - x_{L_k}}{r_{i,L_k}} \left(\frac{1}{2}|S_L|\right) \quad \text{(VII.2.18)}$$

$$G'_y\ i,L = -\sum_{k=1}^{4} \frac{y_i - y_{L_k}}{r_{i,L_k}} \left(\frac{1}{2}|S_L|\right) \quad \text{(VII.2.19)}$$

where (x_{L_k}, y_{L_k}) are the coordinates of the kth quadrature point on the Lth boundary segment.

r_{i,L_k} = the distance from an interior point (x_i, y_i) to the kth quadrature point (x_{L_k}, y_{L_k}) on the Lth boundary segment

$|S_L|$ = the length of the Lth segment

$$= \sqrt{(x_{2_L} - x_{1_L})^2 + (y_{2_L} - y_{1_L})^2}$$

$d_{i,L}$ = the perpendicular distance from an interior point (x_i, y_i) to the Lth boundary segment

CHAPTER VIII

EXTENSION TO HETEROGENEOUS POROUS MEDIA

Two kinds of heterogeneities can be considered. They are:

1. Piece-wise homogeneous systems
2. Homogeneous systems with impermeable inclusions

VIII.1 PIECEWISE HOMOGENEOUS POROUS MEDIA

The domain of interest is made up of a number of homogeneous sub-domains. For the purposes of illustration, only two sub-domains will be treated in this work, but the theory applies to any finite number of sub-domains. Each sub-domain has a different permeability and each sub-domain has three kinds of boundaries enclosing it; namely (Figure 6) for the pth subdomain:

1. S_{Φ}^{p} boundary where values of potentials are specified.
2. S_{n}^{p} boundary where values of the normal derivatives of potential $\left(\frac{\partial \Phi}{\partial n}\right)$ are specified.
3. S_{I}^{p} boundary is the interface with the adjacent sub-domain. On S_{I}^{p} both Φ and $\frac{\partial \Phi}{\partial n}$ are unknown.

where p = 1, 2,..., number of sub-domains.

All the equations derived for a single domain (or sub-domain) in the previous sections apply to each one of the p sub-domains in turn.

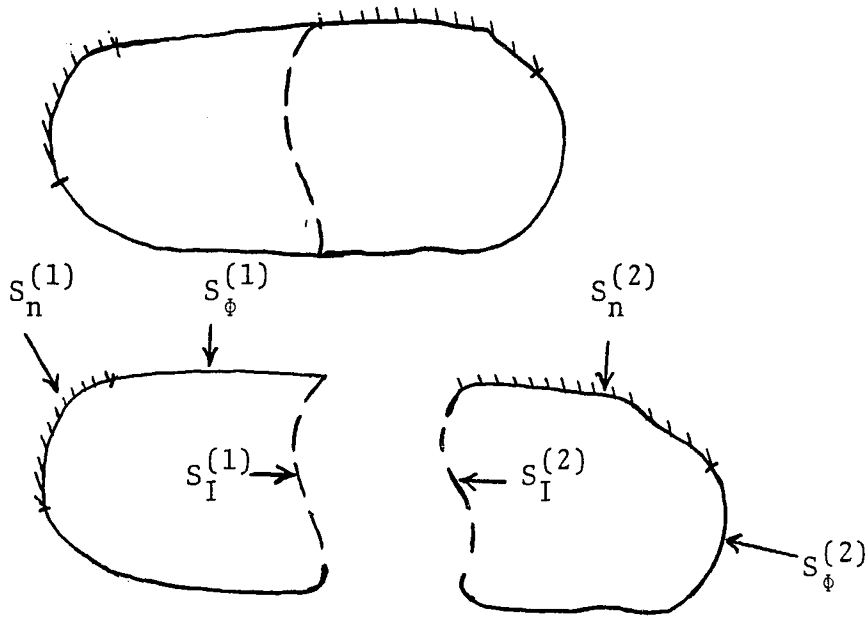


FIGURE 6. Piecewise homogeneous domain showing types of boundary conditions

Considering any arbitrary p'th sub-domain, Equation (VII.0.1) can be written as:

$$[H]^{(p)}\{\Phi\}^{(p)} = [G]^{(p)}\{\Phi_n\}^{(p)} + B \qquad \text{(VIII.1.1)}$$

Equation (VIII.1.1) is applied to each sub-domain as if it exists in isolation, and then all the sub-domain matrices are assembled into one by utilizing continuity of flux conditions and compatibility conditions on the common boundaries. The continuity condition on the common boundary shown in Figure 6 as:

$$-T_1\left(\frac{\partial\Phi_I}{\partial n_I}\right)^{(1)} = -T_2\left(\frac{\partial\Phi_I}{\partial n_I}\right)^{(2)} \qquad \text{(VIII.1.2)}$$

The compatibility condition on the common boundary is:

$$\Phi_I^{(1)} = \Phi_I^{(2)} \qquad \text{(VIII.1.3)}$$

For each sub-domain, the boundary elements are numbered such that the sub-domain is always on the left as we go round the boundary. This ensures that the direction of the normal on the boundary surface of each element is always pointing away from the interior of the sub-domain.

For each sub-domain, the vector $\{\Phi\}^{(p)}$ is made up of the unknown Φ's from the S_n boundary (where $\frac{\partial\Phi}{\partial n}$ is specified) and from the common boundary S_I (where both Φ and $\frac{\partial\Phi}{\partial n}$ are unknown). Therefore:

$$\{\Phi\}^{(p)} = \begin{Bmatrix} \Phi_{S_n} \\ \Phi_{S_I} \end{Bmatrix}^{(p)}$$

Hence the L.H.S. of (VIII.1.1) is expanded as:

$$[H_{S_n} \quad H_{S_\Phi} \quad H_{S_I}]^{(p)} \begin{Bmatrix} \Phi_{S_n} \\ \bar{\Phi}_{S_\Phi} \\ \Phi_{S_I} \end{Bmatrix}^{(p)}$$

where $\bar{\Phi}_{S_\Phi}$ indicates known (specified) values of Φ. Similar treatment of the $\left\{\frac{\partial\Phi}{\partial n}\right\}^{(p)}$ gives the following

$$[G_{S_n} \quad G_{S_\Phi} \quad G_{S_I}] \begin{Bmatrix} (\bar{\Phi}_n)_{S_n} \\ (\Phi_n)_{S_\Phi} \\ (\Phi_n)_{S_I} \end{Bmatrix}^{(p)}$$

where $\bar{\Phi}_n$ denotes known (specified) values of $\frac{\partial \Phi}{\partial n}$.

Equation (VIII.1.1) then becomes, for a single subregion (the p'th subregion),

$$[H_{S_n} \quad H_{S_\Phi} \quad H_{S_I}]^{(p)} \begin{Bmatrix} \Phi_{S_n} \\ \bar{\Phi}_{S_\Phi} \\ \Phi_{S_I} \end{Bmatrix}^{(p)} = [G_{S_n} \quad G_{S_\Phi} \quad G_{S_I}]^{(p)} \begin{Bmatrix} (\bar{\Phi}_n)_{S_n} \\ (\Phi_n)_{S_\Phi} \\ (\Phi_n)_{S_I} \end{Bmatrix}^{(p)} + \{B\} \tag{VIII.1.4}$$

The system of matrices represented by Equation (VIII.1.4) is set up for every one of the sub-domains and assembled using Equations (VIII.1.2) and (VIII.1.3), (consider the case where the number of sub-domains = 2, i.e., p = 2). For the first sub-domain, p = 1, putting all unknowns in the G-matrix gives:

$$[G^{(1)}_{S_\Phi} \quad G^{(1)}_{S_I} \quad -H^{(1)}_{S_I} \quad -H^{(1)}_{S_n}] \begin{Bmatrix} (\Phi_n)^{(1)}_{S_\Phi} \\ (\Phi_n)^{(1)}_{S_I} \\ \Phi^{(1)}_{S_I} \\ \Phi^{(1)}_{S_n} \end{Bmatrix} = [H^{(1)}_{S} \quad - G^{(1)}_{S_n}] \begin{Bmatrix} \bar{\Phi}^{(1)}_{S_\Phi} \\ \bar{\Phi}^{(1)}_{n} \end{Bmatrix} - \{B^{(1)}\} \tag{VIII.1.5}$$

For the second sub-domain, p = 2.

$$[G_{S_\Phi}^{(2)} \;\; G_{S_I}^{(2)} \; - \; H_{S_I}^{(2)} \; - \; H_{S_n}^{(2)}]\begin{Bmatrix} \Phi_{n_{S_\Phi}}^{(2)} \\ \Phi_{n_{S_I}}^{(2)} \\ \Phi_{S_I}^{(2)} \\ \Phi_{S_n}^{(2)} \end{Bmatrix} = [H_{S_\Phi}^{(2)} \; - \; G_{S_n}^{(2)}]\begin{Bmatrix} \bar{\Phi}_{S_\Phi}^{(2)} \\ \bar{\Phi}_{n}^{(2)} \end{Bmatrix} - \{B^{(1)}\} \qquad \text{(VIII.1.6)}$$

The continuity condition on the common boundary is:

$$\Phi_{S_I}^{(1)} = \Phi_{S_I}^{(2)} \qquad \text{(VIII.1.7)}$$

The compatibility condition is:

$$-T_1\Phi_{n_{S_I}}^{(1)} = T_2\Phi_{n_{S_I}}^{(2)}$$

$$\therefore \; \Phi_{n_{S_I}}^{(2)} = -\frac{T_1}{T_2}\,\Phi_{n_{S_I}}^{(1)} \qquad \text{(VIII.1.8)}$$

The assembled system matrix is:

$$\begin{bmatrix} -H_{S_n}^{(1)} & G_{S_\Phi}^{(1)} & -H_{S_I}^{(1)} & G_{S_I}^{(1)} & 0 & 0 \\ 0 & 0 & -H_{S_I}^{(2)} & -\frac{T_1}{T_2}G_{S_I}^{(2)} & -H_{S_n}^{(2)} & G_{S_\Phi}^{(2)} \end{bmatrix}\begin{Bmatrix} \Phi_{S_n}^{(1)} \\ \Phi_{n_{S_\Phi}}^{(1)} \\ \Phi_{S_I}^{(1)} \\ \Phi_{n_{S_I}}^{(1)} \\ \Phi_{S_n}^{(2)} \\ \Phi_{n_{S_\Phi}}^{(2)} \end{Bmatrix} \qquad \text{(VIII.1.9)}$$

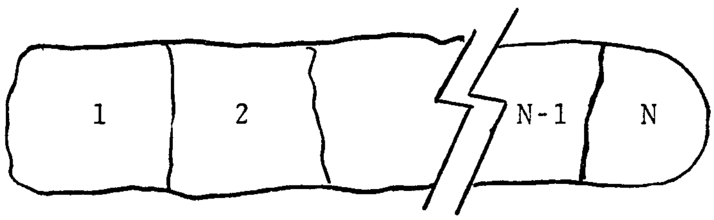
$$= \begin{bmatrix} H_{S_\Phi}^{(1)} & -G_{S_n}^{(1)} & 0 & 0 \\ 0 & 0 & H_S^{(2)} & -G_{S_n}^{(2)} \end{bmatrix}\begin{Bmatrix} \bar{\Phi}_{S_\Phi}^{(1)} \\ \bar{\Phi}_{n_{S_n}}^{(1)} \\ \bar{\Phi}_{S_\Phi}^{(2)} \\ \bar{\Phi}_{n_{S_n}}^{(2)} \end{Bmatrix} - \{B\}$$

The matrix system given by (VIII.1.9) is solved for the boundary values of Φ and $\frac{\partial\Phi}{\partial n}$. These values are used to calculate the interior values of $\Phi(x,y)$, $\frac{\partial\Phi}{\partial x}$, and $\frac{\partial\Phi}{\partial y}$ within any of the sub-domains of interest by applying Equations (VII.1.3), (VII.2.14) and (VII.2.15).

The matrix system given by (VIII.1.9) can be expressed schematically as:

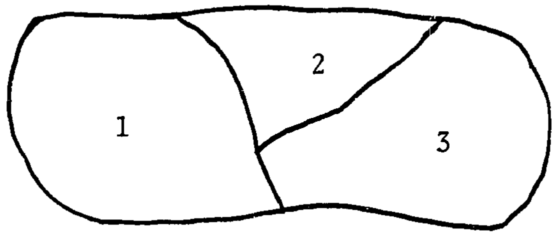
$$\begin{bmatrix} X & X & X & X & 0 & 0 \\ 0 & 0 & X & X & X & X \end{bmatrix} \left\{ \begin{matrix} \\ \\ \end{matrix} \right\} = \left\{ \begin{matrix} Y \\ Y \end{matrix} \right\}$$

This can be generalized for other kinds of systems. For example, for a system having n regions, we have



$$\begin{bmatrix} X & X & X & X & & & & & & & & \\ 0 & 0 & X & X & X & X & & & & & & \\ 0 & 0 & 0 & 0 & X & X & X & X & & & & \\ & & & & & \cdot & \cdot & \cdot & & & & \\ & & & & & & \cdot & \cdot & \cdot & & & \\ & & & & & & & \cdot & \cdot & \cdot & & \\ & & & & & & & & X & X & X & X \end{bmatrix} \left\{ \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \end{matrix} \right\} = \left\{ \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \end{matrix} \right\}$$

For a system of the type



the matrix system will be of the form

$$\begin{bmatrix} X & X & X & X & X & X & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X & X & 0 & 0 & X & X & X & X & 0 & 0 \\ 0 & 0 & 0 & 0 & X & X & X & X & 0 & 0 & X & X \end{bmatrix} \left\{ \quad \right\} = \left\{ \quad \right\}$$

(a) (b) (c)

(a) = interface between regions 1 and 2

(b) = interface between regions 1 and 3

(c) = interface between regions 2 and 3

VIII.2 HOMOGENEOUS SYSTEM WITH IMPERMEABLE INCLUSIONS

This kind of system can easily be handled by the Boundary Element Method. Instead of one, there are now three surfaces that bound the domain of interest (Figure 8). All that is required is to maintain the numbering convention on each of the surfaces so that the domain is on the left hand side.

It is observed in Figure 8 that, to satisfy the numbering convention,

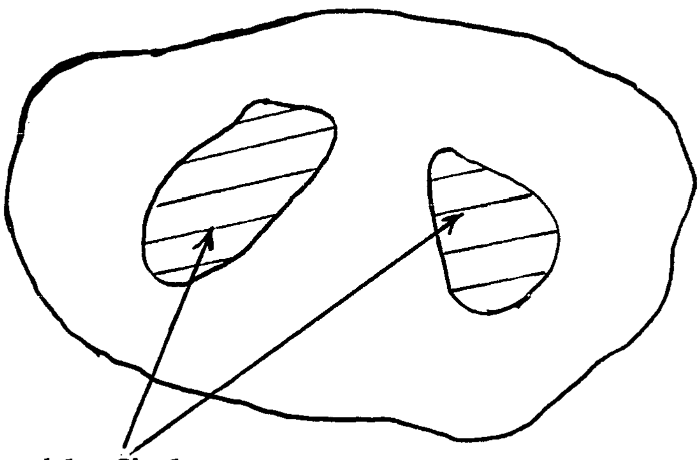


FIGURE 7. Homogeneous porous medium with impermeable shale inclusions.

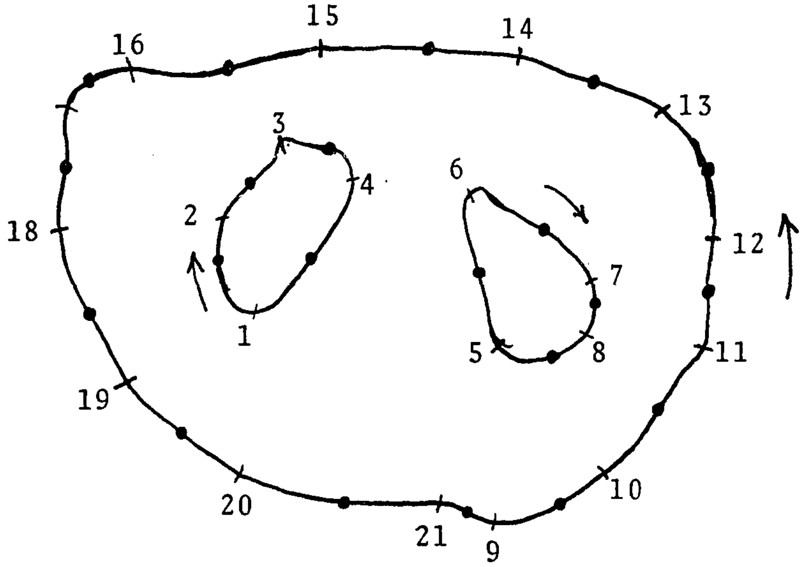


FIGURE 8. Homogeneous porous medium with impermeable inclusions showing discretization and numbering scheme.

- Internal boundaries are numbered in the clockwise direction.
- External boundaries are numbered in the counter-clockwise direction.

VIII.3 STREAMLINE AND STREAMTUBE CALCULATION PROCEDURE

The streamlines are determined by following the path of a hypothetical fluid particle from the injector to the producer. For any injection well, the number of streamlines emanating from that well is independent of its injection rate. For a system with multiple injection wells, a fixed number of streamlines is assigned to each well. Thus, assigning the fixed number of streamlines (N_L) to an injector with rate q_L, the injection rate into each streamtube emanating from it is:

$$q_s = \frac{q_L}{N_L} \qquad \text{(VIII.3.1)}$$

The injection rate into the streamtubes emanating from any other injector with injection rate q_1 becomes:

$$q_{s_1} = \frac{q}{q_L} \times q_s \qquad \text{(VIII.3.2)}$$

The points of origin of the fluid particles (and therefore the streamlines) are distributed equidistant along a small arbitrary circle centered at the source. At any current position, the potential is determined using Equation (VII.1.3) and velocity vectors v_x and v_y in the x and y directions are determined using Equations (VII.2.1) to (VII.2.15). In order to determine the next location of the fluid particle, the assumption is made that the velocity remains constant over a finite distance increment Δs. At any point, the resultant particle velocity (v_t) is the vectorial sum of v_x and v_y. Since v_x and v_y and thus v_t are assumed constant over Δs, the time of travel can be calculated as:

$$\Delta t = \frac{\Delta s}{v_t} \qquad \text{(VIII.3.3)}$$

Hence, from any starting point denoted by (x_i, y_i), the particle would move to a new point given by:

$$x_{i+1} = x_i + v_{x_i} \Delta t \qquad \text{(VIII.3.4)}$$

$$y_{i+1} = y_i + v_{y_i} \Delta t \qquad \text{(VIII.3.5)}$$

The distance step increment Δs is chosen to be constant for convenience. As successive steps are taken, a curve is generated and is commonly called a streamline.

The width of the streamtube at any position of the fluid particle can be determined from the injection rate and the velocity. For each injector, the volume of injected fluid associated with a streamline is:

$$q_s = \frac{q}{N} \qquad \text{(VIII.3.6)}$$

where N is the number of streamlines emanating from the injector and is given by Equation (VIII.3.2). Since this volume of fluid must be conserved at all positions along the streamline, then for any arbitrary position i of coordinates (x_i, y_i)

$$q_s = v_{t_i} A_i \qquad \text{(VIII.3.7)}$$

where v_{t_i} is the resultant velocity at position i

A_i is the vertical cross sectional area at position i, and

ϕ is the porosity, assumed constant

Making the assumption that the vertical thickness of the reservoir (and therefore the streamtube) is a known constant h, the width of the streamtube at any position i (x_i, y_i) can be determined as:

$$W_i = \frac{q_s}{v_{t_i} h} \qquad \text{(VIII.3.7)}$$

Due to the fact that analytical solutions to the equations that describe the rate of advance of a steam front in porous medium have only been obtained for linear or radial models, the streamtube is modified to be linear as follows:

Using Equation (VIII.3.7), the width of the streamtube is determined at any given streamline location and averaged along the length of the entire streamline to give a single width for every streamtube. The streamtubes are then assumed to have rectangular cross sections. The streamlines and associated streamtubes are terminated by defining an arbitrary capture radius around the producer. When the calculated position of the streamline gets within the capture radius, the streamline is terminated.

VIII.4 CONTINUATION OF STREAMLINES IN ADJACENT REGIONS

For piece-wise homogeneous reservoirs, some of the streamlines may cross into adjacent regions where the permeabilities are different. In such cases, continuation of the streamlines into adjacent regions can be achieved by noting the interface boundary segment where the streamline crossed as well as the coordinates of this crossing point. Since the potential derivative normal to every segment is known as part of the solution, this normal gradient can be used with Darcy's law to determine the velocities and distances needed to continue tracing the streamlines.

In practice, however, the streamlines could not be traced all the way until they met any boundary. The potentials exhibited fluctuations at distances closer than half an element length away from any boundary. The reason for this is the singular nature of the fundamental solution $\ln \frac{1}{r_{i,j}}$ as i approaches j. Therefore, the method adopted here was to test every new position to see if it is less than half an element length away from the mid point of any boundary. If it is, a perpendicular line was dropped from the previous position to the interface segment. This is consistent with the fact that the streamline must be normal to a flow boundary. The point of intersection of this normal with the interface segment is taken as the starting point for continuation of the streamline into the adjacent region and is given as:

$$x_c = \frac{(c_1 - y_{1_L})x_i - c_2 x_{1_L} + y_{1_L} - y_i}{c_1 - c_2}$$

$$y_c = c_1(x_{1_L} - x_i) + y_i$$

where:

(x_c, y_c) = coordinates of the point where the streamline crosses the interface boundary

$$c_1 = \frac{x_{1_L} - x_{2_L}}{y_{2_L} - y_{1_L}}$$

$$c_2 = \frac{y_{2_L} - y_{1_L}}{x_{2_L} - x_{1_L}}$$

(x_i, y_i) = coordinates of last position of streamline

(x_{1_L}, y_{1_L}), (x_{2_L}, y_{2_L}) = coordinates of the extreme points of the Lth interface element

The normal derivative of the potential at the Lth interface is already known as part of the solution. This normal derivative is introduced into Darcy's law to determine the next position away from the boundary into the adjacent region. A check is made to ensure that this position is further from the boundary than half the interface segment length.

A package of computer programs has been written in FORTRAN to generate and plot the streamlines, and to calculate the dimensions of the associated streamtubes. Details of the algorithms and flow charts are listed in Appendix E. The computer program itself is listed in Appendix O.

The final part of this modelling effort is its application to a continuous steam drive process. Since some of the streamtubes in a piecewise homogeneous reservoir will contain two or more permeability regions, the next section develops the appropriate equations for the rate of advance of the steam front in a linear streamtube of constant cross section having two or more permeability regions.

CHAPTER IX

RATE OF ADVANCE OF STEAM FRONT IN A PIECE-WISE HOMOGENEOUS LINEAR POROUS MEDIUM

In order to design a steamflood project or to predict recovery from such projects, a knowledge of the distribution and movement of temperature fronts in the reservoir is required. This information can be obtained by taking a control volume within the medium (reservoir) and applying the general and particular laws of physics to it. A general law is one whose application is independent of the nature of the medium under consideration. A particular law is one whose application is dependent on the nature of the medium. In the formulation of a steam drive model, the general laws employed are:

a. the law of conservation of energy (heat balance)

b. the law of conservation of mass

and the particular laws employed are:

c. Fourier's law of conduction

In modelling steam drive processes, some basic as-asumptions are made such as:

1. Changes in viscosity and density due to changes in temperature have negligible effect on energy transfer. This

assumption enables energy balance equations to be uncoupled from mass balance equations.[39]

2. The thermal conductivity in the direction perpendicular to the direction of fluid flow within the reservoir is infinite. This assumption means that the temperature in the reservoir at any cross section perpendicular to the direction of fluid flow is uniform.

3. The sand grains and the reservoir fluids maintain instantaneous thermal equilibrium. This means that the sand grains and surrounding fluids were always at the same temperature.

A general heat balance for thermal recovery projects may be expressed as:

$$\begin{Bmatrix}\text{The amount of heat} \\ \text{(enthalpy) contained} \\ \text{within a given vol-} \\ \text{ume at any time}\end{Bmatrix} = \begin{Bmatrix}\text{heat} \\ \text{injected}\end{Bmatrix} - \begin{Bmatrix}\text{heat} \\ \text{produced}\end{Bmatrix}$$

$$+ \begin{Bmatrix}\text{heat} \\ \text{generated}\end{Bmatrix} - \begin{Bmatrix}\text{heat} \\ \text{lost}\end{Bmatrix}$$

Mathematically, it can be expressed in integral form as[42]

$$\int_V \rho c T \, dV = \int_0^t \dot{Q}(\tau) \, d\tau - \int_0^t \left[\int_A \left(-k \frac{\partial T}{\partial n} + V_f \rho_f c_f T \right) dA \right] dt$$

In this equation, V is the volume over which the balance is made (the entire steam zone). Since V varies with time for the present problem, it will be more convenient to make an instantaneous heat balance as:

$$\int_V \rho c T \, dV = \dot{Q}(\tau) - \int_A \left(-k \frac{\partial T}{\partial n} + V_f \rho_f c_f T\right) dt$$

This equation and the one preceding it are completely general and include the effects of variations in formation properties with location and time.

Assuming the confining layers are semi-infinite planes with constant, homogeneous thermal properties, assuming further that the temperature difference imposed on the boundaries of the confining layers is directly proportional to the amount of heat stored in the reservoir, the solution of the instantaneous heat balance equation is:[42]

$$H(t) = \int_0^t \dot{Q}(\tau) \, e^{\theta(t-\tau)} \, \mathrm{erfc} \sqrt{\theta(t-\tau)} \, d\tau$$

$\dot{Q}(\tau)$ = rate of heat injection - rate of heat produced

$H(t)$ = heat contained in the reservoir at time t

= (heat content of rock + fluids) vol/sec of steam zone

Since vol/sec = velocity x cross-sectional area, this equation can be used to calculate the velocity of the steam front.

The next section makes such an instantaneous heat balance across the condensation front for a piecewise homogeneous linear porous medium.

IX.1 INSTANTANEOUS HEAT AND MASS BALANCE ACROSS THE MOVING CONDENSATION FRONT

Consider a condensation front located at position z(t) in the N'th region of a multiregion composite porous medium. Take two arbitrary fixed cross sections at positions z_{N_a} and z_{N_b} on the upstream and downstream sides of the condensation front respectively. (See Figure 9.)

The region behind the condensation front is completely covered by steam. In this work, the terms condensation front and steam front will be used interchangeably.

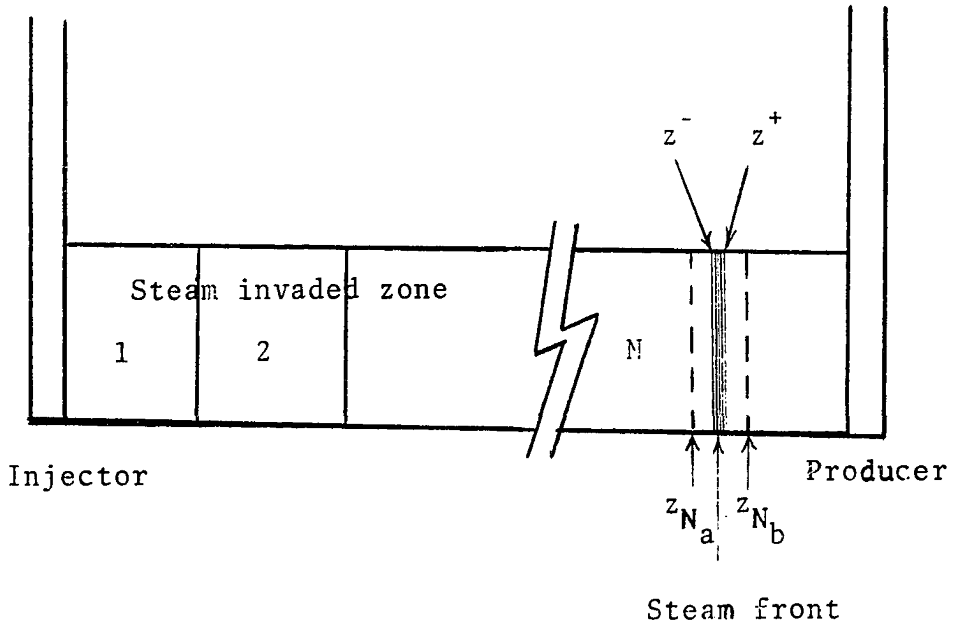


FIGURE 9. Reservoir cross section showing location of steam front.

The objective is to determine the velocity of the steam or condensation front as it moves within any arbitrary region of a multi-region heterogeneous linear porous medium. To achieve this, an elemental volume is taken on the steam front such that one half of it is behind the steam front and the other half is ahead of the steam front. The law of conservation of energy is applied in the form of a heat balance to the elemental volume enclosed by the two fixed cross sections at z_{N_a} and z_{N_b}. The upstream plane of the condensation front is denoted z^-, while the downstream plane is denoted z^+.

A heat balance in the fixed elemental area between the two fixed cross sections z_{N_a} and z_{N_b} is:

$$\begin{bmatrix}\text{Heat flux}\\ \text{in at } z_{N_a}\end{bmatrix} - \begin{bmatrix}\text{Heat flux}\\ \text{out at } z_{N_b}\end{bmatrix} = \begin{bmatrix}\text{Rate of}\\ \text{accumulation}\\ \text{of heat}\end{bmatrix} + \begin{bmatrix}\text{rate of loss}\\ \text{of heat from}\\ \text{the volume}\end{bmatrix}$$

Following Mandle and Volek,[32] the heat balance can be expressed mathematically as:

$$Q_N(z_{N_a},t) - Q_N(z_{N_b},t) = \frac{d}{dt}\int_{z_{N_a}}^{z_N^-(t)} H_N(z_{N_a},t)\, dz_N$$

$$+ \frac{d}{dt}\int_{z_N^+(t)}^{z_{N_b}} H_N(z_{N_b},t)\, dz_N + Q_{L_N}(t) \qquad \text{(IX.1.1)}$$

where, for the steam front in the N'th region,

$H_N(z_{N_a},t)$ denotes the heat content per unit volume of fluid/solid system between z_{N_a} and z_N^-.

$H_N(z_{N_b},t)$ denotes the heat content per unit volume of fluid/solid system between z^+ and z_{N_b}

$Q_N(z_{N_a},t)$ = the heat flux in at z_{N_a} per unit cross sectional area

$Q_N(z_{N_b},t)$ = the heat flux out at z_{N_b} per unit cross sectional area

The application of the Leibnitz rule to Equation (IX.1.1) and considering that z_{N_a} and z_{N_b} are fixed values, gives

$$Q_N(z_{N_a},t) - Q_N(z_{N_b},t) = \int_{z_{N_a}}^{z_N^-(t)} \frac{d}{dt} H_N(z_{N_a},t)\, dz$$

$$+ H_N(z_N^-(t),t) \frac{d}{dt} z_N^-(t) + \int_{z_N^+(t)}^{z_{N_b}} \frac{d}{dt} H_N(z_{N_b},t)\, dz$$

$$- H_N(z_N^+(t),t) \frac{d}{dt} z_N^+(t) + Q_{L_N}(t) \qquad \text{(IX.1.2)}$$

Equation (IX.1.2) is the heat balance across two fixed cross sections containing the steam front in the N'th region. In the limit as z_{N_a} tends to $z_N^-(t)$ and z_{N_b} tends to $z_N^+(t)$, the heat balance equation becomes:

$$Q_N(z_N^-(t),t) - Q_N(z_N^+(t),t) = H_N(z_N^-(t),t) \frac{d}{dt} z_N^-(t)$$

$$- H_N(z_N^+(t),t) \frac{d}{dt} z_N^+(t) + Q_{L_N}(t)$$

Making the assumption that $\frac{d}{dt} z_N^-(t) = \frac{d}{dt} z_N^+(t) = \frac{d}{dt} z_N(t)$ and no heat is lost within the thin condensation front, and dropping the time function of z^- and z^+ for convenience, the heat balance equation becomes the heat balance across a moving condensation front which is instantaneously frozen in time.

$$Q_N(z_N^-,t) - Q_N(z_N^+,t) = [H_N(z_N^-,t) - H_N(z_N^+,t)]v_N(t) \qquad \text{(IX.1.3)}$$

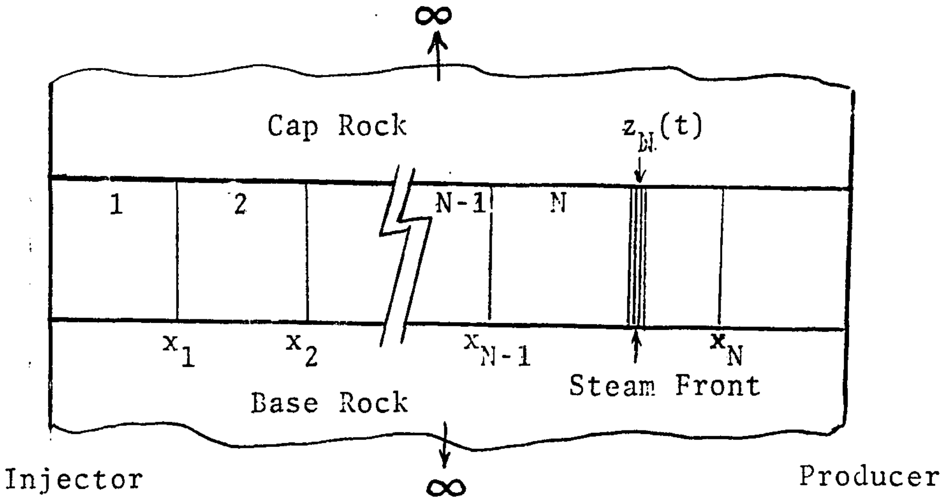


FIGURE 10. Reservoir cross section for heat balance.

That is, the instantaneous heat balance across a moving condensation front is described as:

$$\begin{bmatrix}\text{Heat flux}\\ \text{in at } z_N^-\end{bmatrix} - \begin{bmatrix}\text{Heat flux}\\ \text{out at } z_N^+\end{bmatrix} = \begin{bmatrix}\text{Rate of change of heat}\\ \text{content between } z_N^- \text{ and } z_N^+\end{bmatrix} \qquad \text{(IX.1.4)}$$

where, since there are no heat sources or sinks in the steam zone,

$$Q_N(z_N^-,t) = \begin{bmatrix}\text{Heat flux}\\ \text{in at } z_N^-\end{bmatrix} = \begin{bmatrix}\text{Rate of heat}\\ \text{injection}\end{bmatrix} - \begin{bmatrix}\text{Rate of heat loss}\\ \text{in N - 1 regions}\end{bmatrix}$$

$$- \begin{bmatrix}\text{Rate of heat loss}\\ \text{in N'th region}\end{bmatrix} - \begin{bmatrix}\text{Rate of change of heat}\\ \text{content in N - 1 regions}\end{bmatrix}$$

$$- \begin{bmatrix}\text{Rate of change of heat}\\ \text{content in N'th region}\end{bmatrix} \qquad \text{(IX.1.5)}$$

$$Q_N(z_N^+,t) = \begin{bmatrix}\text{Heat flux}\\ \text{out at } z_N^+\end{bmatrix} = \begin{bmatrix}\text{Heat flux out}\\ \text{by convection}\\ \text{of the fluids}\end{bmatrix} + \begin{bmatrix}\text{Heat flux out}\\ \text{by conduction}\end{bmatrix}$$

$$\text{(IX.1.6)}$$

and

$$\begin{bmatrix}\text{Rate of change}\\ \text{of heat content}\\ \text{between } z^- \text{ and } z^+\end{bmatrix} = \begin{bmatrix}\text{Heat}\\ \text{content}\\ \text{at } z^-\end{bmatrix} - \begin{bmatrix}\text{Heat}\\ \text{content}\\ \text{at } z^+\end{bmatrix} * \begin{bmatrix}\text{Velocity of}\\ \text{the steam}\\ \text{front}\end{bmatrix}$$

$$\text{(IX.1.7)}$$

Equations (IX.1.4) through (IX.1.7) apply to mass balances as well as to heat balances. When making mass balances "heat" is replaced by "mass". Furthermore, for mass balances, the second and third terms on the right hand side of Equation (IX.1.5) disappear because there are no mass losses behind the steam front as is the case with heat. Also, Equation (IX.1.5) shows that the regions behind the steam front are handled in two parts. The first part is made up of all the N - 1 regions behind the steam front. The second part is the part of the N'th region behind the steam front.

IX.2 HEAT BALANCE

For a condensation front at any arbitrary location $z_N(t)$ in the N'th region of a multi-region composite medium, the heat balance for an elemental area taken across the condensation front (Figure 10) has been shown by Equation (IX.1.3) to be

$$Q_N(z_N^-,t) - Q_N(z_N^+,t) = v_N(t)[H_N(z_N^-,t) - H_N(z_N^+,t)] \qquad \text{(IX.2.1)}$$

for a unit reservoir cross section area. Following Equation (IX.1.5) the rate of heat input $Q_N(z_N^-,t)$ per unit cross section area at z_N^- is:

$$\begin{aligned} Q_N(z_N^-,t) &= Q(0,t) - \textstyle\sum_{j=1}^{N-1} (Q_L(t))_j - (Q_L(t))_N \\ &- \rho_{st}(L_{st} + c_w\Delta T(z^-))\left[\sum_{j=1}^{N-1} \phi_j \int_{x_{j-1}}^{x_j} \frac{d}{dt}[S_{st_j}(z_j,t)]dz_j \right. \\ &\left. - \phi_N \int_{x_{n-1}}^{z_N(t)} \frac{d}{dt}[S_{st_j}(z_N,t)]dz_N\right] \\ &- \rho_w(T_{st})c_w\Delta T(z^-) \sum_{j=1}^{N-1} \phi_j \int_{x_{j-1}}^{x_j} \frac{d}{dt}[S_{w_j}(z_j,t)]dz_j \\ &- \rho_w(T_{st})c_w\phi_N\Delta T(z^-) \int_{x_{N-1}}^{z_N(t)} \frac{d}{dt}[S_{w_j}(z_N,t)]dz_N \\ &- \rho_o(T_{st})c_o\Delta T(z^-) \sum_{j=1}^{N-1} \phi_j \int_{x_{j-1}}^{x_j} \frac{d}{dt}[S_{o_j}(z_j,t)]dz_j \\ &- \rho_o(T_{st})c_o\Delta T(z^-)\phi_N \int_{x_{N-1}}^{z_N(t)} \frac{d}{dt}[S_{o_j}(z_N,t)]dz_N \qquad \text{(IX.2.2)} \end{aligned}$$

where:

$Q_N(z_N^-,t)$ = the rate of flow of heat per unit cross sectional area through the vertical cross sectional area at z_N^- into the elemental volume in the N'th region

$Q_N(z_N^+,t)$ = the rate of flow of heat per unit cross sectional area out of the elemental volume through the vertical cross section at z_N^+ in the N'th region

$v_N(t)$ = the velocity of the steam front in the N'th region

$H_N(z^-,t)$ = heat content per unit volume of reservoir rock/fluid system at the upstream face of the steam front in the N'th region

$H_N(z^+,t)$ = heat content per unit volume of reservoir rock/fluid system at the downstream face of the steam front in the N'th region

$Q(0,t)$ = rate of injection of heat per unit cross sectional area

$(Q_L(t))_j$ = rate of loss of heat per unit cross sectional area to overburden and underburden from the j'th region

$\rho_m(T)$ = density of m at temperature T, where m can be oil, water, or steam

L_{st} = latent heat of steam

c_m = specific heat of m, where m = oil, water, steam

$\Delta T(z^-)$ = change in temperature at the upstream face of the steam front

$\Delta T(z^+)$ = change in temperature at the downstream face of the steam front

ϕ_j = porosity of the rock in the j'th region

$S_m(z_j,t)$ = saturation distribution of m in the j'th region, where m = oil, water, steam

The rate of heat injection per unit cross sectional area Q(0,t) is obtained as the sum of the rate of heat injection in the form of water and the rate of heat injection in the form of steam.

$$Q(0,t) = M_{st}(0,t)[L_{st} + c_w T_{st}] + M_w(0,t)c_w T_{st} \qquad \text{(IX.2.3)}$$

The heat flux out of the condensation front

$Q_N(z_N^+,t)$ = the heat flux out by horizontal conduction + convective heat flux associated with mass flux of fluids out of the front

Mathematically, the heat flux out of the condensation front is written as:

$$Q_N(z_N^+,t) = M_w(z_N^+,t)c_w \Delta T(z_N^+) \qquad \text{(IX.2.4)}$$

$$+ M_o(z_N^+,t)c_o \Delta T(z_N^+) + \left(-\hat{K} \left.\frac{\partial T}{\partial x}\right|_{z_N^+}\right)$$

The mass rates $M_w(z_N^+,t)$ and $M_o(z_N^+,t)$ are obtained from mass balances across the condensation front.

IX.3 MASS BALANCES

A mass balance for any fluid on an elemental volume taken across the condensation front from z_N^- to z_N^+ in the N'th region is given as:

mass flux in at z_N^- - mass flux out at z_N^+

= rate of change of mass content in the elemental area

The rate of change of mass content is expressed in terms of the velocity of the condensation front. The mass flux in at z_N^- is the difference between the mass rate of injection, and the rate of accumulation of mass in the swept portion.

IX.3.1 Oil

For oil, the mass rate of injection = 0, and the expression for the mass balance is:

$$-\left\{\rho_o(T_{st}) \sum_{j=1}^{N-1} \phi_j \int_{x_{j-1}}^{x_j} \frac{d}{dt}\,[S_o(z_j,t)]\, dz_j \right.$$

$$\left. + \rho_o(T_{st})\phi_N \int_{x_{N-1}}^{z_N(t)} \frac{d}{dt}\,[S_o(z_N,t)]\, dz_N \right\} - M_o(z_N^+,t)$$

$$= \phi_N v_N(t)\,[\rho_o(T_{z^-}) S_{o_N}(z_N^-,t) - \rho_o(T_{z_N^+}) S_{o_N}(z_N^+,t)] \qquad \text{(IX.3.1)}$$

from which,

$$M_o(z_N^+,t) = -\rho_o(T_{st}) \sum_{j=1}^{N-1} \phi_j \int_{x_{j-1}}^{x_j} \frac{d}{dt} [S_o(z_j,t)]\, dz_j$$

$$- \rho_o(T_{st})\phi_N \int_{x_{N-1}}^{z_N(t)} \frac{d}{dt} [S_o(x,t)]\, dz_N$$

$$- \phi_N v_N(t)[\rho_o(T_{z^-})S_{o_N}(z_N^-,t) - \rho_o(T_{z^+})S_{o_N}(z_N^+,t)]$$

(IX.3.2)

IX.3.2 Water

The mass flux of water into the elemental area at z_N^- is from a combination of sources:

(a) The mass fraction of steam that was injected as water.

(b) The mass of water that condensed out of the injected steam.

(c) Displaced connate water.

The mass flux of water that condensed from the injected steam is the difference between the mass rate of injection of steam and the rate of change of steam saturation in the steam zone. Mathematically, the mass balance for water is:

$$q(0,t)[x\rho_{st}+1-x\rho_w(T_{st})]-\rho_w(T_{st})\sum_{j=1}^{N-1}\phi_j\int_{x_{j-1}}^{x_j} \frac{d}{dt}[S_w(z_j,t)]\, dz_j$$

$$-\rho_w(T_{st})\phi_N\int_{x_{N-1}}^{z_N(t)}\frac{d}{dt}[S_w(z_N,t)dz_N-\rho_{st}\sum_{j=1}^{N-1}\phi_j\int_{x_{j-1}}^{x_j}\frac{d}{dt}[S_{st}(z_j,t)dz_j$$

$$-\rho_{st}\phi_N\int_{x_{N-1}}^{z_N(t)}\frac{d}{dt}[S_{st}(z_N,t)dz_N - M_w(z_N^+,t) = \phi_N v_N(t)[\rho_w(T_{z_N^-})S_w(z_N^-,t)$$

$$+ \rho_{st}S_{st}(z_N^-,t) - \rho_w(T_{z_N^+})S_w(z_N^+,t)] \qquad \text{(IX.3.3)}$$

from which, the mass flux of water out of the elemental area at z_N^+ can be derived as:

$$M_w(z_N^+,t)=Q(0,t)[x\rho_{st}+1-x\rho_w]-\rho_w(T_{st})\sum_{j=1}^{N-1}\phi_j\int_{x_{j-1}}^{x_j}\frac{d}{dt}[S_w(z_j,t)]dz_j$$

$$-\rho_w(T_{st})\phi_N\int_{x_{N-1}}^{z_N(t)}\frac{d}{dt}[S_w(z_N,t)]\ dx$$

$$-\rho_{st}\sum_{j=1}^{N-1}\phi_j\int_{x_{j-1}}^{x_j}\frac{d}{dt}[S_{st}(z_j,t)]\ dx$$

$$-\rho_{st}\phi_N\int_{x_{N-1}}^{z_N(t)}\frac{d}{dt}[S_{st}(z_N,t)\ dz_N$$

$$-\phi_N v_N(t)[\rho_w(T_{z_N^-})S_w(z_N^-,t)+\rho_{st}S_{st}(z_N^-,t)$$

$$-\rho_w(T_{z_N^+})S_w(z_N^+,t)] \qquad (IX.3.4)$$

All the component terms that make up the heat balance have now been defined. Equations (IX.3.4) and (IX.3.2) can be substituted into Equation (IX.2.4). The resulting expression, as well as Equation (IX.2.2) can be substituted into Equation (IX.2.1) to give a general heat balance equation. But first, the integrals in the heat and mass balance equations are evaluated so as to simplify those equations.

Applying the result of Appendix B, we observe that, for the N - 1 regions behind the steam front, their limits of integration are constants. Therefore,

$$\int_{x_{j-1}}^{x_j}\frac{d}{dt}[S_m(z_j,t)]dz_j = (x_{j-1}-x_j)\frac{d}{dt}\bar{S}_{m_j}(t) \qquad (IX.3.6)$$

For the N'th region (which contains the steam front), the lower limit of integration, x_{N-1}, is a constant, but the upper limit, $z_N(t)$, varies with time. Therefore,

$$\int_{x_{N-1}}^{z_N(t)} \frac{d}{dt} [S_m(z_N,t)]dz_N = v_N(t)[\bar{S}_{m_N}(t) - S_m(z_N^-,t)]$$

$$+ \frac{d}{dt} \bar{S}_{m_N}(t)[z_N(t) - x_{N-1}] \qquad \text{(IX.3.7)}$$

Combining (IX.3.6) and (IX.3.7) gives:

$$\sum_{j=1}^{N-1} {}_j\int_{x_{j-1}}^{x_j} \frac{d}{dt}[S_m(z_j,t)]dz_j + \phi_N \int_{x_{N-1}}^{z_N(t)} \frac{d}{dt}[S_m(z_N,t)dz_N$$

$$+ \sum_{j=1}^{N-1} \phi_j[(x_j - x_{j-1}) \frac{d}{dt} \bar{S}_{m_j}(t)] + \phi_N v_N(t)[\bar{S}_{m_N}(t)$$

$$- S_{m_N}(z_N^-,t)] + \phi_N \frac{d}{dt} \bar{S}_{m_N}(t)[z_N(t) - x_{N-1}] \qquad \text{(IX.3.8)}$$

where m = oil, water, or steam. $S_{m_j} = (S_o + S_w + S_t)_j$

Making the assumption that $\frac{d}{dt} \bar{S}_m(z,t) = 0$ for each region

$$\sum_{j=1}^{N} \phi_j \int_{x_{j-1}}^{x_j} \frac{d}{dt}[S_{m_j}(z_j,t)]dz_j + \phi_N\int_{x_{N-1}}^{z_N(t)} \frac{d}{dt}[S_{m_N}(z_N,t)]dz_N$$

$$= \phi_N v_N(t)[\bar{S}_{m_N} - S_m(z_N^-,t)] \qquad \text{(IX.3.9)}$$

Introducing Equation (IX.3.9) into Equation (IX.2.2) gives the heat flux per unit cross section at the inlet of the steam front as:

$$Q_N(z_N^-,t) = Q(0,t) - \textstyle\sum_{j=1}^{N} Q_{L_j}(t) - \rho_{st}[L_{st} + c_{st}(T_{st} - T_i)]$$

$$\times\ \phi_N v_N(t)[\bar{S}_{st_N} - S_{st}(z_N^-,t)] - \rho_w(T_{st})c_w(T_{st} - T_i)\phi_N v_N(t)$$

$$\times\ [\bar{S}_{w_N} - S_{w_N}(z_N^-,t)] - \rho_o(T_{st})c_o(T_{st} - T_i)\phi_N v_N(t)$$

$$\times\ [\bar{S}_{o_N} - S_{o_N}(z_N^-,t)] \qquad \text{(IX.3.10)}$$

Note that this equation still contains the unknown heat lost rates per unit cross sectional area $Q_{L_j}(t)$ for j = 1, 2, ..., number of regions.

Noting that $T_{z^-} = T_{st}$, Equation (IX.3.9) is substituted into the equations describing the mass fluxes out of the condensation front as:

$$M_{o_N}(z_N^+,t) = \phi_N v_N(t)[\rho_o(T_{st})\bar{S}_{o_N} - \rho_o(T_{z^+})S_{o_N}(z_N^+,t)] \qquad \text{(IX.3.10a)}$$

$$M_{w_N}(z_N^+,t) = Q(0,t)[x\rho_{st}+1-x\rho_w(T_{st})] - {}_N v_N(t)[\ {}_w(T_{st})\bar{S}_{w_N}$$

$$+\ \rho_{st}\bar{S}_{st_N} - \rho_w(T_{z^+})S_{w_N}(z_N^+,t)] \qquad \text{(IX.3.11)}$$

Equations (IX.3.10) and (IX.3.11) are substituted into Equation (IX.2.4) to give the heat flux out of the condensation front as:

$$Q_N(z_N^+,t) = c_o\Delta T(z_N^+)\phi_N v_N(t)[\rho_o(T_{st})\bar{S}_{o_N} - \rho_o(T_{z^+})S_{o_N}(z_N^+,t)]$$

$$+\ \{c_w\Delta T(z_N^+)(1-x)\rho_w + x\rho_{st}L_{st}\}Q(0,t) - c_w\Delta T(z_N^+)\phi_N v_N[\rho_w(T_{st})\bar{S}_{w_N}$$

$$+\ \rho_{st}\bar{S}_{st_N} - \rho_w(T_{st})S_{w_N}(z_N^+,t)] \qquad \text{(IX.3.12)}$$

IX.3.3 Enthalpy

The enthalpy of the rock and associated fluids at the inlet and outlet faces of the elemental area are:

$$H_N(z_N^-,t) = \rho_{r_N} c_{r_N}(1 - \phi_N)(T_{st} - T_i) + \rho_{st}\phi_N L_{st} S_{st_N}(z_N^-,t)$$
$$+ \rho_{st} c_w (T_{st} - T_i)\phi_N S_{w_N}(z_N^-,t) + \rho_w c_w (T_{st} - T_i)\phi_N S_{w_N}(z_N^-,t)$$
$$+ \rho_o c_o(T_{st})(T_{st} - T_i)\phi_N S_{o_N}(z_N^-,t) \qquad \text{(IX.3.13)}$$

$$H(z^+,t) = \rho_r c_{r_N}(1 - \phi_N)\Delta T(z_N^+) + \rho_w(T_{z_N^+}) c_w \Delta T(z_N^+)\phi_N S_{w_N}(z_N^+,t)$$
$$+ \rho_o(T_{z_N^+}) c_o \Delta T(z_N^+)\phi_N S_{o_N}(z_N^+,t) \qquad \text{(IX.3.14)}$$

Recall that the heat balance equation is:

$$Q_N(z_N^-,t) - Q_N(z_N^+,t) = v_N(t)[H_N(z_N^-,t) - H_N(z_N^+,t)] \qquad \text{(IX.3.15)}$$

where:

$Q_N(z_N^-,t)$ is given by Equation (IX.3.10) but contains the still unknown rates of heat loss $Q_{L_j}(t)$ for $j = 1,\ldots,N$

$Q_N(z_N^+,t)$ is given by Equation (IX.3.12)

$H_N(z_N^-,t)$ is given by Equation (IX.3.11)

$H_N(z_N^+,t)$ is given by Equation (IX.3.12)

The assumption is made that there is no convective heat flux across the steam front. This means that all the heat arriving at the steam front is used up in heating the rock and fluids. The condensate leaves at z_N^+ having the original reservoir temperature T_i. Thus $\Delta T(z_N^+) = 0$.

This assumption makes $Q_N(z_N^+,t) = 0$, and $H_N(z_N^+,t) = 0$. Therefore, the heat balance equation becomes:

$$Q_N(z_N^-,t) = v_N(t)H_N(z_N^-,t) \qquad \text{(IX.3.16)}$$

The unknown heat loss rates contained in the equation for $Q_N(z_N^-,t)$ is calculated next.

IX.4 RATE OF HEAT LOSS TO CAP AND BASE ROCK IN A PIECE-WISE HOMOGENEOUS LINEAR POROUS MEDIUM

The properties of the cap and base rock are assumed to be identical and constant even though those of the porous medium vary in regions (Figure 11). The cap and base rock are assumed to extend to infinity in either direction. The objective here becomes the calculation of the rate of heat loss to semi-infinite slabs on each side that represent the cap and base rocks.

Since the cap and base rock are assumed identical, the heat loss derivations will be made using only the cap rock, but the result is simply doubled to account for the base rock as well.

The steam front is assumed to be located in the N'th region of a piece-wise homogeneous linear porous medium. Each of the N steam-invaded regions behind the front are divided into equal elemental areas (Figure 11). Let there be P_j such elemental areas in each j'th region where $j = 1, 2, \ldots, N$.

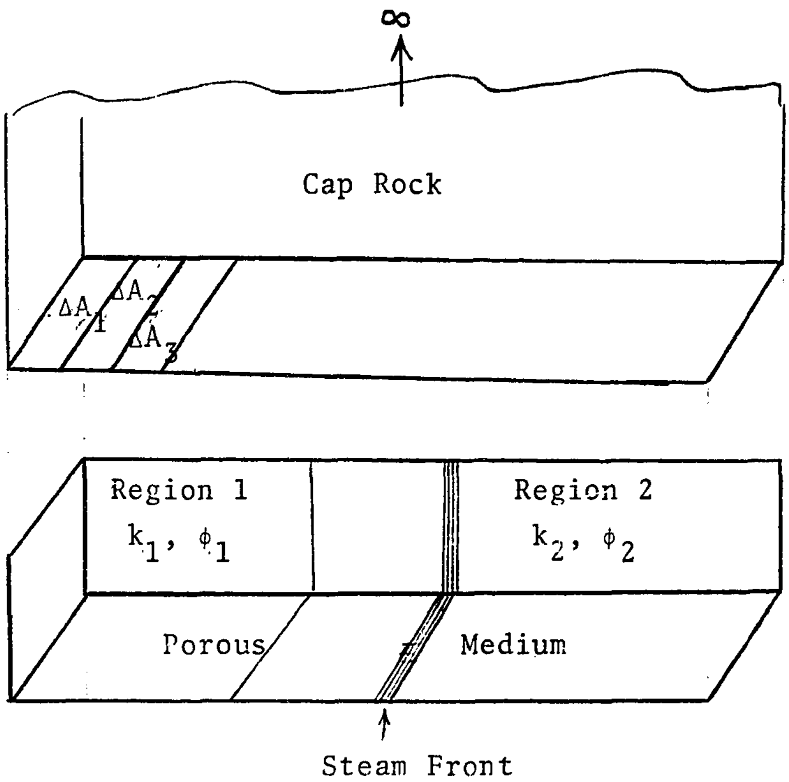


FIGURE 11. Schematic of linear porous medium for heat loss calculations.

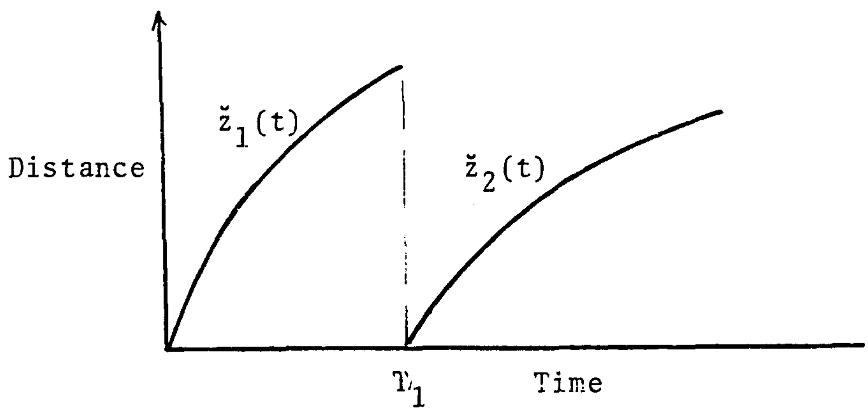


FIGURE 12. Distance as a function of time in each region of the linear piece-wise homogeneous porous medium.

The rate of heat loss per unit cross sectional area to the cap and base rock from an elemental area in an arbitrary j'th region is:

$$Q_{L_j}(t)\Big|_{\Delta A_j} = -2k\Delta A_j \left.\frac{dT}{dz}\right|_{z=0} \tag{IX.4.1}$$

which, after substituting for $\left.\frac{dT}{dz}\right|_{z=0}$ from Appendix D, becomes:

$$Q_{L_j}(t)\Big|_{\Delta A_j} = -\frac{2k\Delta A_j}{Wh}\left(\frac{T_s - T_i}{\sqrt{\pi\alpha(t-\tau_j)}}\right) \tag{IX.4.1a}$$

Summing for all P_j such elemental areas in the j'th region gives:

$$Q_{L_j}(t) = -\sum_{m=1}^{P_j} \frac{2k\Delta A_{j,m}}{Wh}\left(\frac{T_s - T_i}{\sqrt{\pi\alpha(t-\tau_{jm})}}\right) \tag{IX.4.2}$$

Since $A_j = W z_j$, in the limit as A_j tends to zero,

$$\lim_{\Delta A \to 0} Q_{L_j}(t) = -\frac{2k(T_s - T_i)}{L}\int_0^{L_j} \frac{1}{\sqrt{\pi\alpha(t-\tau_j)}}\, dz_j(\tau_j) \tag{IX.4.3}$$

where L_j is the length of the j'th region.

Summing for all N regions behind the steam front gives the rate of heat loss per unit cross sectional area to cap and base rock from all N regions behind the steam front as:

$$Q_L(t) = \sum_{j=1}^{N} Q_{L_j}(t) = -\frac{2k(T_s-T_i)}{L}\sum_{j=1}^{N}\int_{t_{j-1}}^{t_j} \frac{\frac{dz_j(\tau_j)}{d\tau_j}\, d\tau_j}{\sqrt{\pi\alpha(t-\tau_j)}} \tag{IX.4.4}$$

where $A_j(\tau) = Wz_j(\tau)$

W = width of the porous medium

$$\alpha = \frac{k}{\rho_{cb} c_{cb}}$$

ρ_{cb} = density of cap and base rock

c_{cb} = heat capacity of cap and base rock

k = thermal conductivity of cap and base rock

h = thickness of porous medium

In Equation (IX.4.4), each $z_j(\tau_j)$ is only defined and continuous in the region $t_{j-1} < \tau_j < t_j$. Since a function can have a differential quotient only at points where the function is continuous, the differential quotient $\frac{dz_j(\tau_j)}{d\tau_j}$ can only be taken at points within the region $t_{j-1} < \tau < t_j$ and not outside the region because $z_j(\tau_j)$ is not defined there. However, if $z_i(\tau_j)$ is expressed in terms of the Heaviside unit step function $H(t)$, then each $z_j(\tau_j)$ can be defined for all $\tau > 0$ with discontinuities at $\tau = t_{j-1}$ and $\tau = t_j$. A "generalized derivative" can be taken by product differentiation where generalization means a definition of the derivative at the points of discontinuity as well. Thus, introducing the Heaviside unit step function $H(t)$ into Equation (IX.4.4) gives:

$$Q_L(t) = \\ = \frac{-2k(T_s - T_i)}{L} \sum_{j=1}^{N} \int_0^t \frac{\frac{d}{dt}\{[H(\tau-\tau_{j-1}) - H(\tau-\tau_j)]\xi_j(\tau)\}}{\sqrt{\pi\alpha(t-\tau)}} d\tau \tag{IX.4.5}$$

The summation can be taken inside the integral and $Q_L(t)$ can be expressed as:

$$Q_L(t) = \frac{-2k(T_s - T_i)}{h} \int_0^t \frac{\frac{d}{d\tau}\{\sum_{j=1}^{N}[H(\tau-\tau_{j-1}) - H(\tau-\tau_j)]\xi_j(\tau)\}}{\sqrt{\pi\alpha(t-\tau)}} d\tau \tag{IX.4.6}$$

Let

$$z(\tau) = \sum_{j=1}^{N}[H(\tau-\tau_{j-1}) - H(\tau-\tau_j)]\xi_j(\tau) \tag{IX.4.7}$$

where each $\xi_j(t)$ is continuous and defined for all $\tau > 0$.

$$Q_L(t) = \frac{-2k(T_s - T_i)}{h} \int_0^t \frac{\frac{d}{d\tau}\, z(\tau)}{\sqrt{\pi\alpha(t-\tau)}} d\tau \tag{IX.4.8}$$

$\frac{dz(\tau)}{d\tau}$ is obtained by the product differentiation of Equation (IX.4.7) to give

$$\frac{dz(\tau)}{d\tau} = \sum_{j=1}^{N}\{[H(\tau-\tau_{j-1}) - H(\tau-\tau_j)]\frac{d\xi_j}{d\tau}$$

$$+ v(\tau)[\delta(\tau-\tau_{j-1}) - \delta(\tau-\tau_j)]\xi_j(\tau)\} \tag{IX.4.9}$$

$$= \{[H(\tau-\tau_{N-1}) - H(\tau-\tau_N)]\frac{d\xi_N}{d\tau}$$

$$+ [\delta(\tau-\tau_{N-1}) - \delta(\tau-\tau_N)]\xi_N(\tau)\} \tag{IX.4.10}$$

where $H(\tau)$ is the Heaviside unit step function

$\delta(\tau)$ is the Dirac delta function

Equation (IX.4.8) is substituted into Equation (IX.3.10). The result is $Q_N(z_N^-,t)$ which is substituted into the left side of Equation (IX.3.16). When the expression for $H_N(z_N^-,t)$ is substituted into the right side of Equation (IX.3.16), the result is:

$$
\begin{aligned}
Q(0,t) &+ \frac{2k(T_s - T_i)}{h} \int_0^t \frac{\frac{d}{d\tau} z(\tau)}{\sqrt{\pi\alpha(t - \tau)}} d\tau \\
&- \rho_{st}[L_{st} + c_w(T_{st} - T_i)]\phi_N v_N(t)[\bar{S}_{st_N} - S_{st}(z_N^-,t)] \\
&- \rho_w(T_{st})c_w(T_{st} - T_i)\phi_N v_N(t)[\bar{S}_{w_N} - S_{w_N}(z_N^-,t)] \\
&- \rho_o(T_{st})c_o(T_{st})(T_{st} - T_i)\phi_N v_N(t)[\bar{S}_{o_N} - S_{o_N}(z_N^-,t)] \\
&= V_N(t)[\rho_{\gamma_N} c_{\gamma_N}(1 - \phi_N)(T_{st} - T_i) + \rho_{st}L_{st}\phi_N S_{st_N}(z_N^-,t) \\
&\quad + \rho_{st}c_s(T_{st} - T_i)\phi_N S_{w_N}(z_N^-,t) \\
&\quad + \rho_w c_w(T_{st} - T_i)\phi_N S_{w_N}(z_N^-,t) \\
&\quad + \rho_o(T_{z_N^+})c_o(T_{st})(T_{st} - T_i)\phi_N S_{o_N}(z_N^-,t)]
\end{aligned}
\qquad \text{(IX.4.10)}
$$

Considering that

$$Q(0,t) = M_{st}(0,t)[L_{st} + c_w(T_{st}-T_i)] + M_w(0,t)c_w(T_{st}-T_i) \qquad \text{(IX.4.11)}$$

where

$$M_{st}(0,t) = \rho_{st}\ xq(0,t) \qquad \text{(IX.4.12)}$$

$$M_w(0,t) = \rho_w(T_{st})(1-x)q(0,t) \qquad \text{(IX.4.13)}$$

then Equation (IX.4.10) simplifies to:

$$q(0,t)[xL_{st}\rho_{st} + \rho_w c_w(T_{st} - T_i)(1 - x)] + \frac{2k(T_s - T_i)}{h\sqrt{\pi\alpha}} \int_0^t \frac{\frac{d}{d\tau}\, z(\tau)}{\sqrt{t - \tau}}\, d\tau = V_N(t)[\rho_{r_N} c_{r_N}(1 - \phi_N)$$

$$\times(T_{st} - T_i) + \rho_{st}L_{st}\phi_N\bar{S}_{st_N} + \rho_w(T_{st})c_w(T_{st} - T_i)\phi_N\bar{S}_{w_N}$$

$$+ \rho_o(T_{st})c_o(T_{st})(T_{st} - T_i)\phi_N\bar{S}_{o_N}] \qquad \text{(IX.4.13)}$$

Let

$$\sum \overline{\rho c} = \rho_{r_N} c_{r_N}(1 - \phi_N) + \rho_w(T_{st})c_w\bar{S}_{w_N}\phi_N$$

$$+ \rho_o(T_{st})c_o(T_{st})\bar{S}_{o_N}\phi_N + \rho_{st}c_w\bar{S}_{st_N}\phi_N \qquad \text{(IX.4.14)}$$

Also let

$$\frac{dz(\tau)}{d\tau} = v(\tau) \qquad \text{(IX.4.15)}$$

Then, introducing (IX.4.14) and (IX.4.15) into (IX.4.13) gives

$$q(0,t)[x\rho_{st}L_{st} + \rho_w c_s(1-x)(T_{st}-T_i)] + \frac{2k(T_{st}-T_i)}{h\sqrt{\pi\alpha}} \int_0^t \frac{v(\tau)}{\sqrt{t-\tau}}\, d\tau$$

$$= V_N(t)\left\{\sum \overline{\rho c} + \frac{\rho_{st}L_{st}\bar{S}_{st_N}\phi_N}{T_{st}-T_i}\right\}(T_{st}-T_i) \qquad \text{(IX.4.16)}$$

where

$$v(\tau) = \sum_{j=1}^{N}\{[H(t-t_{j-1}) - H(t-t_j)]\frac{d\xi_j(\tau)}{d\tau}$$

$$+ [\delta(t-t_{j-1}) - \delta(t-t_j)]\xi_j(\tau)\} \qquad \text{(IX.4.17)}$$

Equation (IX.4.16) is a generalization of the equation of Marx and Langenheim,[25] and Mandle and Volek[32] derived for a piece-wise homogeneous linear porous medium. If the steam front is within the first region of a piece-wise homogeneous linear porous medium, the N = 1, and

$$\left.\begin{aligned} V_N(\tau) &= V_1(\tau) \\ V(\tau) &= V_1(\tau) \end{aligned}\right\} \quad \text{A continuous function of time}$$

Then Equation (IX.4.16) becomes

$$q(0,t)[\rho_{st}xL_{st} + c_w(T_{st} - T_i)(1 - x)\rho_w(T_{st})]$$

$$+ \frac{2k(T_{st} - T_i)}{h\sqrt{\pi\alpha}} \int_0^t \frac{V_1(\tau)}{\sqrt{t - \tau}}\, d\tau = V_1(t)\left[\sum \overline{\rho c}\right.$$

$$\left. + \frac{\rho_{st}L_{st}\bar{S}_{st}\phi}{T_{st} - T_i}\right](T_{st} - T_i) \qquad \text{(IX.4.18)}$$

which is the same as the equation of Mandle and Volek. However, for a piece-wise homogeneous linear porous medium, $V(\tau)$ is a sectionally continuous function as given by (IX.4.17). After transformation to dimensionless form and simplification, Equation (IX.4.16) can be solved to give the velocity and distance of the steam front in any region of a piece-wise homogeneous porous medium.

Since $\alpha = \frac{k}{\rho_{cb}c_{cb}}$, Equation (IX.4.16) can be expressed as

$$q(0,t)\ [\rho_{st}xL_{st} + c_w(T_{st} - T_i)(1 - x)\rho_w(T_{st})]$$

$$- \sqrt{\frac{4k\rho_{cb}c_{cb}(T_{st} - T_i)^2}{\pi h^2}} \int_0^t \frac{v(\tau)}{\sqrt{t - \tau}}\, d\tau$$

$$= v_N(t)(T_{st} - T_i)\left[\sum\overline{\rho c} + \frac{\rho_{st}L_{st}\bar{S}_{st}\phi_N}{T_{st} - T_i}\right] \qquad \text{(IX.4.19)}$$

Define dimensionless variables.

Dimensionless time:

$$t_D = \left(\frac{4k}{\rho_{cb}c_{cb}h^2}\right)t \quad \text{or} \quad \tau_D = \left(\frac{4k}{\rho_{cb}c_{cb}h^2}\right)\tau \qquad \text{(IX.4.20)}$$

Dimensionless distance:

$$z_D(t) = \frac{z(t)}{L} \qquad \text{(IX.4.21)}$$

where L is the length of the entire porous medium.

Dimensionless velocity:

$$v_D(t) = \frac{dz_D(t)}{dt_D} = \frac{dz_D(t)}{dt}\frac{dt}{dt_D}$$

$$= \frac{1}{L}\frac{dz(t)}{dt}\frac{\rho_{cb}c_{cb}h^2}{4k}$$

$$= \left(\frac{\rho_{cb}c_{cb}h^2}{4kL}\right)v(t) \qquad \text{(IX.4.22)}$$

Introducing the dimensionless variables into Equation (IX.4.19) gives:

$$q(0,t)\ [\rho_{st} x L_{st} + c_w (T_{st} - T_i)(1 - x)\rho_w(T_{st})]$$

$$- \left(\frac{4k\rho_{cb}c_{cb}}{\pi h^2}\right)^{1/2} \int_0^{t_D} \left(\frac{4kL}{\rho_{cb}c_{cb}h^2}\right) \frac{\frac{dz(\tau_D)}{d\tau_D}\left(\frac{\rho_{cb}c_{cb}h^2}{4k}\right)}{\left(\frac{\rho_{cb}c_{cb}h^2}{4k}\right)^{1/2}\sqrt{t_D - \tau_D}}\, d\tau_D$$

$$= \left(\frac{4kL}{\rho_{cb}c_{cb}h^2}\right)(T_{st} - T_i)\, \frac{dz_N(t_D)}{dt_D}\left[\sum\overline{\rho c} + \frac{\rho_{st}L_{st}\bar{S}_{st_N}\phi_N}{T_{st} - T_i}\right]$$

(IX.4.23)

which is expressed as

$$\frac{q(0,t)\ [\rho_{st}xL_{st}+\rho_w c_w(T_{st} - T_i)(1 - x)]\rho_{cb}c_{cb}h^2}{4kL(T_{st} - T_i)\left[\sum\overline{\rho c} + \frac{\rho_{st}L_{st}S_{st_N}\phi_N}{T_{st} - T_i}\right]}$$

$$- \frac{\rho_{cb}c_{cb}}{\sqrt{\pi}\left[\sum\overline{\rho c} + \frac{\rho_{st}L_{st}S_{st_N}\phi_N}{T_{st} - T_i}\right]} \int_0^{t_D} \frac{\frac{dz_D(\tau_D)}{d\tau_D}}{\sqrt{t_D - \tau_D}}\, d\tau_D$$

$$= \frac{d}{dt_D}\, z_{N_D}(t_D) \qquad \text{(IX.4.24)}$$

Let

$$F_N = \frac{[\rho_{st}xL_{st}+\rho_w c_w(T_{st} - T_i)(1 - x)]\rho_{cb}c_{cb}h^2}{4kL(T_{st} - T_i)\left[\sum\overline{\rho c} + \frac{\rho_{st}L_{st}S_{st_N}\phi_N}{T_{st} - T_i}\right]} \qquad \text{(IX.4.25)}$$

Let

$$\lambda_N = \frac{\rho_{cb} c_{cb}}{\left[\sum \overline{\rho c} + \frac{\rho_{st} L_{st} \bar{S}_{st} \phi_N}{T_{st} - T_i}\right]} \qquad \text{(IX.4.26)}$$

Introducing (IX.4.25) and (IX.4.26) into (IX.4.24) gives:

$$F_N q(0,t) - \frac{\lambda_N}{\sqrt{\pi}} \int_0^{t_D} \frac{v_D(\tau_D)}{\sqrt{t_D - \tau_D}} \, d\tau_D = v_{ND}(t_D) \qquad \text{(IX.4.27)}$$

Equation (IX.4.27) is the dimensionless form of Equation (IX.4.16) describing the velocity of the steam front in the Nth region of a piece-wise homogeneous linear porous medium. Note that $v_D(t_D) = \sum_{j=1}^{m} v_{jD}(t_D)$. Therefore, when the steam front is within the first region, N = 1, Equation (IX.4.27) becomes:

$$F_1 q_{st}(0,t) - \frac{\lambda_1}{\sqrt{\pi}} \int_0^{t_D} \frac{v_{1D}(\tau_D)}{\sqrt{t_D - \tau_D}} \, d\tau_D = v_{1D}(t_D) \qquad \text{(IX.4.28)}$$

where v_{1D} is a sectionally continuous function. If the reservoir is homogeneous, then v_{1D} becomes the continuous ous function $\check{v}_{1D}$. Dropping the subscripts, we have, for a homogeneous reservoir,

$$Fq(0,t) = \frac{\lambda}{\sqrt{\pi}} \int_0^{t_D} \frac{\check{v}_D(\tau_D)}{\sqrt{t_D - \tau_D}} \, d\tau_D = \check{v}_D(t_D) \qquad \text{(IX.4.29)}$$

The solution to Equation (IX.4.29) is obtained by Laplace transform method and is given in several references.[25,32] For the case of constant injection rate [q(0,t) = q],

$$\xi_D(t_D) = \frac{Fq}{\lambda^2}\left(e^{\lambda^2 t_D} \operatorname{erfc}\sqrt{\lambda^2 t_D} + \frac{2\sqrt{\lambda^2 t_D}}{\sqrt{\pi}} - 1\right) \quad \text{(IX.4.30}$$

$$v_D(t_D) = Fq\left(e^{\lambda^2 t_D} \operatorname{erfc}\sqrt{\lambda^2 t_D}\right) \quad \text{(IX.4.31)}$$

The solution to Equation (IX.4.28) for the case where the steam front is within the first region of a piece-wise homogeneous linear system can also be obtained by the Laplace transform method as well.

Consider the case of a linear piece-wise homogeneous porous medium having only two regions (N = 2). When the steam front is in the second region, Equation (IX.4.27) becomes:

$$F_2 q(0,t) - \frac{\lambda_2}{\sqrt{\pi}}\int_0^{t_D} \frac{v_D(\tau_D)}{\sqrt{t_D - \tau_D}}\, d\tau_D = v_{2D}(t_D) \quad \text{(IX.4.32)}$$

where

$$v_{2_D}(t_D) = \{[H(t_D - t_{D_1})]\check{v}_{2D}(t_D) + [\delta(t_D - t_{D_1})$$

$$- \delta(t_d - t_{D_2})]\xi_{2D}(t_D)\} \quad \text{(IX.4.33)}$$

and

$$v_D(t_D) = \frac{d\xi_D(t_D)}{dt_D} = \sum_{j=1}^{2} \{[H(t_D - t_{D_{j-1}}) - H(t_D - t_{D_j})]$$

$$\times \check{v}_{jD}(t_D) + [\delta(t_D - t_{D_{j-1}}) - \delta(t_D - t_{D_j})]\xi_{jD}\}$$

$$= v_{1D}(t_D) + v_{2D}(t_D) \qquad \text{(IX.4.34)}$$

The expressions for $\xi_{1D}(t_D)$ and $v_{1D}(t_D)$ are known and given by Equations (IX.4.30) and (IX.4.31), respectively. Substituting for $v_D(t_D)$ from Equation (IX.4.34) into (IX.4.32) gives:

$$F_2q(0,t) - \frac{\lambda_2}{\sqrt{\pi}} \int_0^{t_D} \frac{v_{1D}(\tau_D)}{\sqrt{t_D - \tau_D}} d\tau_D - \frac{\lambda_2}{\sqrt{\pi}} \int_0^{t_D} \frac{v_{2D}(\tau_D)}{\sqrt{t_D - \tau_D}} d\tau_D$$

$$= v_{2D}(t_D) \qquad \text{(IX.4.35)}$$

The second term on the left of Equation (IX.4.34) gives the rate of heat loss from the first region while the front is in the second region. In order to obtain an analytical solution to Equation (IX.4.35), the assumption is made that no heat is lost from the first region after the steam front clears it. Equation (IX.4.35) simplifies to:

$$F_2q_{st}(0,t) = \frac{\lambda_2}{\sqrt{\pi}} \int_0^{t_D} \frac{v_{2D}(\tau_D)}{\sqrt{t_D - \tau_D}} d\tau_D = v_{2D}(t_D) \quad \text{(IX.4.36)}$$

In general, the assumption is made that if the steam front is moving within the Nth region, the rate of heat loss from the N-1 regions behind the steam front is negligible. In other words, only the region containing the steam front is losing heat at any given time. With this assumption, Equation (X.0.27) becomes:

$$F_n q(0,t) - \frac{\lambda_N}{\sqrt{\pi}} \int_0^{t_D} \frac{v_{ND}(\tau_D)}{\sqrt{t_D - \tau_D}} \, d\tau_D = v_{ND}(t_D) \quad \text{(IX.4.37)}$$

IX.5 SOLUTION BY LAPLACE TRANSFORM METHOD

The Laplace transform of each of the terms of Equation (IX.4.37) gives

$$\mathcal{L}\{F_N q_{st}(0,t)\} = F_N q(s) \qquad \text{(IX.5.1)}$$

$$\int_0^{t_D} \frac{v_{ND}(\tau_D)}{\sqrt{t_D - \tau_D}} \, d\tau_D$$

is a convolution integral. Therefore,

$$\mathcal{L}\left\{\frac{-\lambda_N}{\sqrt{\pi}} \int_0^{t_D} \frac{v_{ND}(\tau_D)}{\sqrt{t_D - \tau_D}} \, d\tau_D\right\} = \frac{-\lambda_N}{\sqrt{\pi}}\left[\mathcal{L}\{v_{ND}(t_D)\} \cdot \mathcal{L}\left\{\frac{1}{\sqrt{t_D}}\right\}\right] \quad \text{(IX.5.2)}$$

Since

$$v_{ND}(t_D) = \{[H(t_D - t_{D_{N-1}})]v_{ND}(t_D)$$
$$+ [\delta(t_D - t_{D_{N-1}}) - \delta(t_D - t_{D_N})]\xi_{ND}(t_D)\} \quad \text{(IX.5.3)}$$

$$\mathcal{L}\{v_D(t_D)\} = \mathcal{L}\{[H(t_D - t_{D_{N-1}})]\frac{d\xi_{ND}(t_D)}{dt_D}$$

$$+ [\delta(t_D - t_{D_{N-1}}) - \delta(t_D - t_{D_N})]\xi_{ND}(t_D)\} \qquad \text{(IX.5.3a)}$$

From Appendix D,

$$\mathcal{L}\{v_D(t_D)\} = S\xi_D(s) + \xi_D(0^-) \qquad \text{(IX.5.4)}$$

where

$$\xi_{ND}(0^-) = 0$$

$$\xi_{ND}(s) = \int_{t_{D_{N-1}}}^{t_{D_N}} \xi_{ND}(\tau_D) e^{-s\tau_D} \, d\tau_D$$

From standard tables of Laplace transforms,[49]

$$\mathcal{L}\left\{\frac{1}{\sqrt{t_D}}\right\} = \frac{\sqrt{\pi}}{\sqrt{s}} \qquad \text{(IX.5.5)}$$

Introducing (IX.5.4) and (IX.5.5) into the left of Equation (IX.4.28) gives:

$$\mathcal{L}\left\{\frac{-\lambda_N}{\sqrt{\pi}} \int_0^{t_D} \frac{v_D(\tau_D)}{\sqrt{t_D - \tau_D}} \, d\tau_D\right\} = \frac{-\lambda_N}{\sqrt{\pi}} \, s\xi_D(s) \, \frac{\sqrt{\pi}}{\sqrt{s}}$$

$$= -\lambda_N \sqrt{s}\, \xi_D(s) \qquad \text{(IX.5.6)}$$

Considering Equations (IX.5.4), (IX.5.5) and (IX.5.6) the Laplace transform of Equation (IX.4.28) becomes:

$$F_N q(s) - \lambda_N \sqrt{s}\, \xi_{ND}(s) = s\xi_D(s) \qquad \text{(IX.5.7)}$$

$$\xi_D(s) = \frac{F_N q(s)}{\sqrt{s}(\sqrt{s} + \lambda_N)} \qquad \text{(IX.5.8)}$$

In order to take the inverse transform of Equation (IX.5.8) it is rewritten as:

$$\xi_{ND}(s) = \frac{F_N}{\lambda_N^2} \frac{\lambda_N^2 q(s)}{\sqrt{s}(\sqrt{s} + \lambda_N)} \qquad \text{(IX.5.9)}$$

The inverse transform of Equation (IX.5.9) is:

$$z_{N_D}(t_D) = \frac{F_N}{\lambda_N^2} \int_0^{t_D} q(0,t)\ e^{\lambda_N(t_D - \tau_D)}\ \mathrm{erfc}\ \sqrt{\lambda_N(t_D - \tau_D)}\ d\tau_D$$

(IX.5.10)

where

$$z_D(t_D) = \textstyle\sum_{j=1}^{N} [H(t_D - t_{D_{j-1}}) - H(t_D - t_{D_j})]\xi_{Dj}(t_D)$$

(IX.5.11)

IX.6 CONSTANT INJECTION RATE

In the case that the injection rate is constant,

$$q_{st}(0,t) = \text{constant} = q$$

Therefore, the Laplace transform of Equation (IX.4.28) is:

$$\frac{F_N q}{s} - \lambda_N \sqrt{s}\ \xi_D(s) = s\xi_D(s) \qquad \text{(IX.6.1)}$$

from which,

$$\xi_D(s) = \frac{F_N q}{\sqrt{s}(s)(\sqrt{s} + \lambda_N)} \qquad \text{(IX.6.3)}$$

The inverse transform of which is:

$$z_{N_D}(t_D) = \frac{F_N q}{\lambda_N^2}\left[e^{\lambda_N^2 t_D} \operatorname{erfc} \sqrt{\lambda_N^2 t_D} + \frac{2\sqrt{\lambda_N^2 t_D}}{\sqrt{\pi}} - 1\right] \qquad \text{(IX.6.4)}$$

For the case of constant injection rates, the velocity is:

$$v_{N_D}(t_D) = \frac{dz_{N_D}(t_D)}{dt_D} = F_N q [e^{\lambda_N^2 t_D} \operatorname{erfc} \sqrt{\lambda_N^2 t_D}\ ] \qquad \text{(IX.6.5)}$$

where

$$v_D(t_D) = \sum_{j=1}^{N} \{[H(t_d - t_{D_{j-1}}) - H(t_D - t_{D_j})] \frac{d\xi_{Dj}(t_D)}{dt_D}$$
$$+ [\delta(t_D - t_{D_{j-1}}) - \delta(t_D - t_{D_j})]\xi_{Dj}(t_D)\}$$

IX.7 OIL RECOVERY CALCULATION

Oil recovery calculations are made based on the knowledge of the steam invaded volume and the residual oil saturation. This is a very simplified approach which ignores the oil recovery by the other mechanisms such as recovery by the hot and cold water zones ahead of the steam front; recovery by steam distillation; recovery by viscosity reduction; recovery by thermal expansion; and recovery by the gas drive mechanism of the steam. If it is assumed that the oil saturation is reduced from the initial saturation (S_{o_i}) to the residual saturation (S_{o_r}) for each of N permeability zones in a streamtube swept by steam, and the steam front is at position $z_N(t)$ in the Nth zone at time t, then the oil recovered up to time t in a streamtube is:

$$\text{Rec} = \frac{1}{5.61}\left[\sum_{j=1}^{N-1}\{xWh\phi(S_{o_i} - S_{o_r})\}_j + z_N W_N h \phi_N (S_{o_i} - S_{o_r})_N\right] \quad \text{(barrels)} \qquad \text{(IX.7.1)}$$

If steam has broken through in a streamtube at time t_b, Equation IX.7.1 becomes:

$$\text{Rec} = \frac{1}{5.61} \sum_{j=1}^{N}\{xWh\phi(S_{o_i} - S_{o_r})\}_j \quad \text{(barrels)} \qquad \text{(IX.7.2)}$$

After the breakthrough time (t_b) for any streamtube, no more oil is recovered from that streamtube. Thus, the breakthrough times for each of the zones in a streamtube are required for the recovery calculation. Since the length of each permeability zone in a streamtube is known, by starting from time t = 0, the time for the steam front to clear each zone in sequence is calculated through the knowledge of the steamfront velocity with time. Equation IX.7.1 is applied to each streamtube in a producer and summed for all streamtubes in the producer. The results are added for all wells to give the recovery from the field at the specified time. For any streamtube where steam has broken through, Equation IX.7.2 is applied.

A computer program was written in FORTRAN to perform the oil recovery calculations with time. Appendix F gives a short description of the program, a listing of the flow chart. A listing of the program itself is given in Appendix O.

CHAPTER X

TEST FOR THE VALIDITY OF BOUNDARY ELEMENT METHOD OF SOLUTION

The validity of the boundary element technique and the correctness of the solution algorithm were verified by applying the method to a simple problem where the analytical solution is known. The example is taken from Street[58] and involves the steady two-dimensional irrotational flow of an ideal fluid in a corner (Figure 13).

The mathematical model is:

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \quad 0 < x < \pi,\ 0 < y < \pi$$

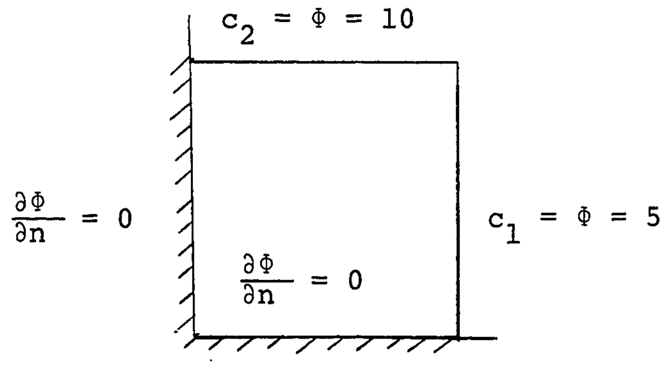


FIGURE 13. Steady flow of an ideal fluid in a corner.

with boundary conditions:

$$\frac{\partial\Phi(0,y)}{\partial x} = 0 \qquad 0 < y < \pi$$

$$\Phi(\pi,y) = 5 \qquad 0 < y \leq \pi$$

$$\frac{\partial\Phi(x,0)}{\partial y} = 0 \qquad 0 < x < \pi$$

$$\Phi(x,\pi) = 10 \qquad 0 < x < \pi$$

The solution to this problem is given[58] as:

$$\Phi(x,y) = C_1 + \frac{4(C_2-C_1)}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}\cosh[(n-\frac{1}{2})y]\cos[(n-\frac{1}{2})x]}{(2n-1)\cosh[(n-\frac{1}{2})\pi]}$$

where $C_1 = 5$ and $C_2 = 10$.

The boundary element method of solution is obtained by dividing each side into three equal segments. The nodes are taken at the mid-points of the segments as shown in Figure 14.

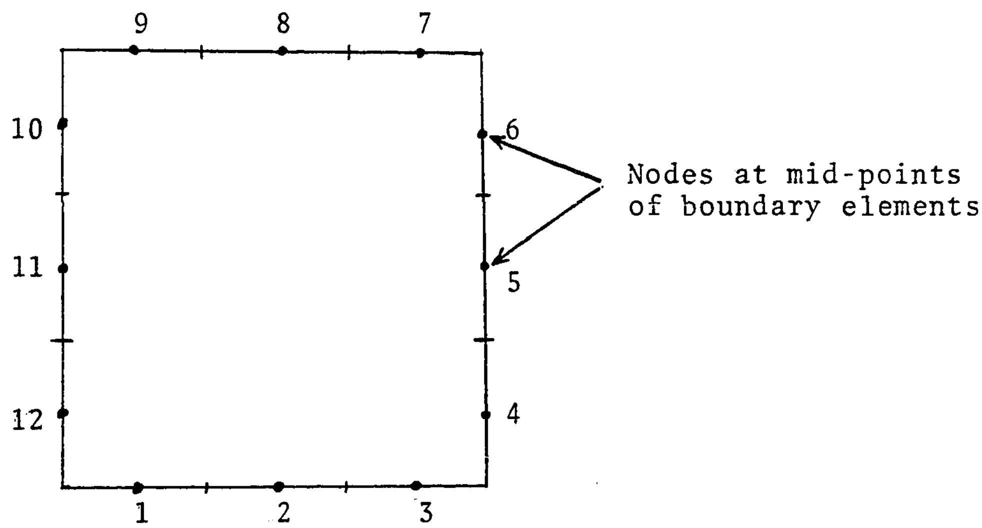


FIGURE 14. Division of square domain into segments and numbering scheme.

The results from the boundary element solution method are shown below. Table 1 lists the input data. Table 2 shows the calculated potentials at the boundary nodes and the potentials calculated at selected interior points. Table 3 compares the solutions obtained by the boundary element method at the interior points to that obtained analytically. The distribution of the absolute values of the errors $|\varepsilon|$ at the interior points are calculated and plotted in Figure 15 where $|\varepsilon|$ is defined as:

$$|\varepsilon| = \left| \frac{\Phi_{(BEM)} - \Phi_{(Analytical)}}{\Phi_{(Analytical)}} \right|$$

It can be seen from Table 3 that at points farther than half an element length away from any boundary, the maximum error was less than 0.4 percent. Figure 15 shows that the error gets very large on and near the boundaries up to half an element length away from the boundaries.

TABLE 1. Input Data for Test Program

TEST FOR THE VALIDITY OF THE BOUNDARY ELEMENT METHOD(BEM)
CALCULATION OF POTENTIALS IN A SQUARE USING THE (BEM)
RESULTS COMPARED WITH ANALYTICAL SOLUTION GIVEN BY STREET

INPUT DATA

NUMBER OF BOUNDARY ELEMENTS=12
NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS CALCULATED=15
NUMBER OF SOURCES AND SINKS= 0

THE CORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELEMENTS

POINT	X	Y
1	0.0000E 00	0.0000E 00
2	0.1047E 01	0.0000E 00
3	0.2094E 01	0.0000E 00
4	0.3142E 01	0.0000E 00
5	0.3142E 01	0.1047E 01
6	0.3142E 01	0.2094E 01
7	0.3142E 01	0.3142E 01
8	0.2094E 01	0.3142E 01
9	0.1047E 01	0.3142E 01
10	0.0000E 00	0.3142E 01
11	0.0000E 00	0.2094E 01
12	0.0000E 00	0.1047E 01

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	1	0.0000E 00
2	1	0.0000E 00
3	1	0.0000E 00
4	0	0.5000E 01
5	0	0.5000E 01
6	0	0.5000E 01
7	0	0.1000E 02
8	0	0.1000E 02
9	0	0.1000E 02
10	1	0.0000E 00
11	1	0.0000E 00
12	1	0.0000E 00

TABLE 2 RESULTS AT BOUNDARY NODES

BOUNDARY NODES

X	Y	POTENTIAL	DERIVATIVE
0.5235E 00	0.0000E 00	0.7431629E 01	0.0000000E 00
0.1570E 01	0.0000E 00	0.6809022E 01	0.0000000E 00
0.2618E 01	0.0000E 00	0.5646283E 01	0.0000000E 00
0.3142E 01	0.5235E 00	0.5000000E 01	-0.1448967E 01
0.3142E 01	0.1570E 01	0.5000000E 01	-0.1798650E 01
0.3142E 01	0.2618E 01	0.5000000E 01	-0.7014348E 01
0.2618E 01	0.3142E 01	0.1000000E 02	0.7014794E 01
0.1570E 01	0.3142E 01	0.1000000E 02	0.1794951E 01
0.5235E 00	0.3142E 01	0.1000000E 02	0.1448620E 01
0.0000E 00	0.2618E 01	0.9354136E 01	0.0000000E 00
0.0000E 00	0.1570E 01	0.8193880E 01	0.0000000E 00
0.0000E 00	0.5235E 00	0.7569907E 01	0.0000000E 00

POTENTIALS AT SELECTED INTERNAL POINTS

X	Y	POTENTIAL
0.1000E 01	0.1000E 01	0.7501E 01
0.1500E 01	0.1500E 01	0.7501E 01
0.2000E 01	0.2000E 01	0.7500E 01
0.2500E 01	0.2500E 01	0.7500E 01
0.3000E 01	0.3000E 01	0.7033E 01
0.1000E 01	0.1571E 01	0.7927E 01
0.1500E 01	0.1571E 01	0.7566E 01
0.2000E 01	0.1571E 01	0.7013E 01
0.2500E 01	0.1571E 01	0.6232E 01
0.3000E 01	0.1571E 01	0.4877E 01
0.1000E 01	0.5000E 00	0.7282E 01
0.1500E 01	0.5000E 00	0.6928E 01
0.2000E 01	0.5000E 00	0.6446E 01
0.2500E 01	0.5000E 00	0.5841E 01
0.3000E 01	0.5000E 00	0.4825E 01

TABLE 3 COMPARISON WITH ANALYTICAL SOLUTION

INTERNAL POINTS X	Y	POTENTIAL(BEM)	POTENTIAL(ANALYTICAL)
0.1000E 01	0.1000E 01	0.7501070E 01	0.7500757E 01
0.1500E 01	0.1500E 01	0.7500750E 01	0.7500802E 01
0.2000E 01	0.2000E 01	0.7500497E 01	0.7500597E 01
0.2500E 01	0.2500E 01	0.7500035E 01	0.7500886E 01
0.3000E 01	0.3000E 01	0.7033089E 01	0.7504594E 01
0.1000E 01	0.1571E 01	0.7926839E 01	0.7919108E 01
0.1500E 01	0.1571E 01	0.7566373E 01	0.7565387E 01
0.2000E 01	0.1571E 01	0.7012587E 01	0.7021172E 01
0.2500E 01	0.1571E 01	0.6232046E 01	0.6254806E 01
0.3000E 01	0.1571E 01	0.4877094E 01	0.5291536E 01
0.1000E 01	0.5000E 00	0.7282336E 01	0.7292308E 01
0.1500E 01	0.5000E 00	0.6928425E 01	0.6944761E 01
0.2000E 01	0.5000E 00	0.6445957E 01	0.6463688E 01
0.2500E 01	0.5000E 00	0.5841250E 01	0.5867875E 01
0.3000E 01	0.5000E 00	0.4825499E 01	0.5196271E 01

DISTRIBUTION OF ABSOLUTE ERROR

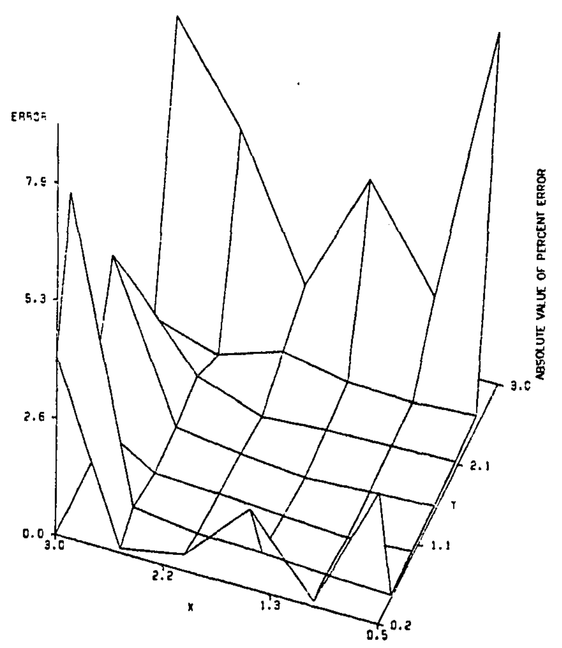


FIGURE 15. Distribution of error.

CHAPTER XI

APPLICATIONS AND RESULTS

XI.1 SAMPLE FIELD PROBLEM: SHIELLS CANYON FIELD[50]

A steam-distillation drive pilot project in zone 203 of the Shiells Canyon field, Ventura County, California, has been reported by Konopnicki, et al.[50] from which this example is taken. Zone 203 is described as part of the Sespe formation of Oligocene age located in the Shiells Canyon field. The average sand thickness is 48.8 m (160 ft). The reservoir is bordered on all sides by faults that are assumed to be fluid sealing at the flood pressure (Fig. 16) The reservoir has a 35° dip, but for purposes of this application, it is assumed to be horizontal.

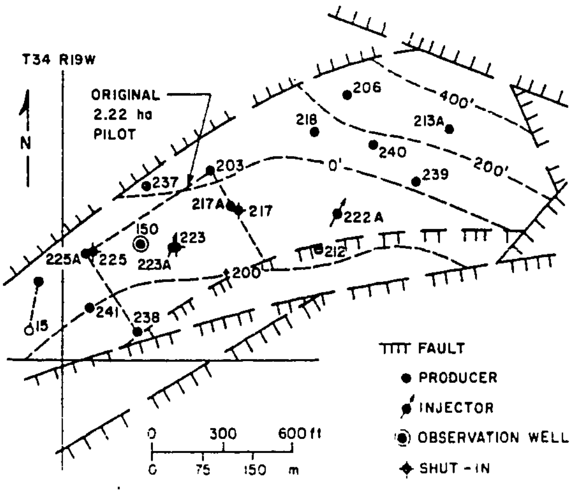


FIGURE 16. Shiells Canyon Field--Zone 203

(After Konopnicki, et al[50])

The primary producing mechanism has been solution-gas drive. The cumulative production before steam injection was 42,500 m^3 (267,000 bbl) which is 9.5% of the estimated 449,175 m^3 (2,825,000 bbl) of oil originally in place. The pilot is an inverted type pattern originally consisting of four producing wells (217A, 203, 237 and 225A) down-dip from the injection well, one producing well (238) up-dip from the injection well, and one thermal observation well (150). Two additional producers (241 and 15) were drilled during the flood (Fig. 16).

XI.1.1 Boundary Conditions

Konopnicki et al. report that the reservoir is bordered on all sides by faults that are assumed to be fluid sealing. However, their figure (Fig. 16) shows the west boundary to be undefined. In the present analysis, two kinds of conditions are assigned at the west boundary for comparative analysis. The two types of boundary conditions are:

(a) The west boundary is assumed to be an oil-water contact; thus, it is a constant-pressure boundary.

(b) The west boundary is assumed to be fluid sealing, therefore making all the boundaries fluid sealing.

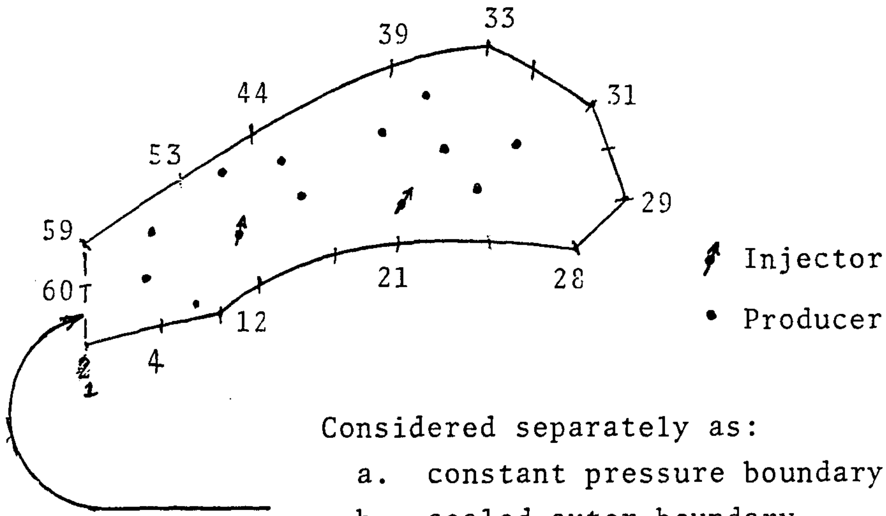
In what follows, the boundary element method of generating streamtubes and the steamflood calculation method are applied to the Shiells Canyon reservoir by considering it as:

1. A homogeneous reservoir with
    a. sealed outer boundary
    b. the west boundary assumed to be an oil/water contact and therefore at original reservoir pressure, the rest of the boundary sealed.
2. A piece-wise homogeneous reservoir consisting of two regions that have unequal permeabilities with
    a. sealed outer boundary
    b. the west boundary at constant pressure, the remainder of the boundary sealed.

XI.2 CASE 1:

XI.2.1 Shiells Canyon Field Analysed as a Single Region Homogeneous Reservoir

The discretization of the boundary and the scheme of numbering are shown in Figure 17. The numbering scheme was such that the domain under consideration was always on the left hand side. This scheme ensures that the direction of the normal gradient at any point of the boundary (or boundaries) is always away from the domain. There are a total of sixty elements taken on the boundary, but only a select few of them are indicated in Figure 17. The coordinates of the extreme points of all the boundary elements are shown in Table 4, while Table 5 gives the coordinates and strengths of the injectors and producers.



Scale: 1 inch = 738.46 ft

FIGURE 17. Discretization and numbering scheme for Shiells Canyon Field as a homogeneous reservoir.[50]

TABLE 4. The Coordinates of the End Points of the Boundary Elements for Shiells Canyon Reservoir Analysed as a Single Homogeneous Region

POINT	X(INCH)	Y(INCH)
1	0.1000	0.1500
2	0.2500	0.2000
3	0.4000	0.2500
4	0.5500	0.2800
5	0.6500	0.3000
6	0.7000	0.3250
7	0.7500	0.3300
8	0.8000	0.3400
9	0.8500	0.3500
10	0.9000	0.4000
11	1.0000	0.4500
12	1.1000	0.5250
13	1.2500	0.6000
14	1.4000	0.6500
15	1.5500	0.6800
16	1.6500	0.6900
17	1.7000	0.7000
18	1.7500	0.7100
19	1.8000	0.7200
20	1.8500	0.7250
21	1.9000	0.7300
22	1.9500	0.7350
23	2.1000	0.7450
24	2.2500	0.7500
25	2.4000	0.7500
26	2.6000	0.7400
27	2.7500	0.7300
28	2.9000	0.7250
29	3.1600	1.0000
30	3.0600	1.2500
31	2.9500	1.5500
32	2.7500	1.6500
33	2.4000	1.8200
34	2.2500	1.8000
35	2.1000	1.7500
36	2.0500	1.7450
37	2.0000	1.7250
38	1.9500	1.7100
39	1.8500	1.6900
40	1.7500	1.6500
41	1.6500	1.6100
42	1.5000	1.5500
43	1.3500	1.4800
44	1.2500	1.4400
45	1.2000	1.4000
46	1.1500	1.3800
47	1.0500	1.3250
48	0.9500	1.2600
49	0.9000	1.2500
50	0.8500	1.2000
51	0.8000	1.1700
52	0.7500	1.1500
53	0.6500	1.0800
54	0.5500	1.0200
55	0.5000	0.9900
56	0.4500	0.9500
57	0.4000	0.9200
58	0.3000	0.8500
59	0.1000	0.7300
60	0.1000	0.5000

TABLE 5. Coordinates of Sources and Sinks for Shiells Canyon Field Analysed as a Single Homogeneous Region

X(INCH)	Y(INCH)	RATE(BBL/D)
0.4500	0.5500	-69.1000
0.5000	0.8000	-69.1000
0.7500	0.4000	-69.1000
0.9000	1.1500	-69.1000
1.0000	0.8000	560.0000
1.2500	1.2000	-69.1000
1.3500	1.0000	-69.1000
1.9000	0.9500	200.0000
1.8000	1.3500	-69.1000
2.0500	1.5500	-69.1000
2.1500	1.2500	-69.1000
2.3500	1.0500	-69.1000
2.5750	1.3000	-69.1000

THE AVERAGE RADIUS OF THE WELLS(INCH) IS= 0.500
THE CHARACTERISTIC PRESSURE(PSI) IS= 85.0000

XI.3 RESULTS FOR CASE 1: SHIELLS CANYON FIELD MODELLED AS A HOMOGENEOUS RESERVOIR

XI.3.1 Sealed Outer Boundary

Table 6 lists the prescribed boundary conditions according to a preassigned code. If the potential is prescribed on the boundary segment, the code = 0. If the potential gradient is prescribed, the code = 1.

A plot of the streamlines generated is shown in Figure 18, while Table 7 gives the dimensions of the streamtubes generated which are read as input to the steamflood recovery calculation program. The input data to the steamflood program are given in Table 8. Figure 19 gives a plot of the oil recovery as a function of time. A complete listing of the outputs from the streamline modelling program is presented in Appendix G while the detailed output from the steamflood program is listed in Appendix H.

XI.3.2 Part of the Boundary at Constant Pressure, the Remainder Sealed

The only difference between the data for this case and that of the sealed boundary is the constant potential specified to boundary segments numbered 59 and 60. These two segments are assigned the initial pressure of 85 psi. Figure 20 shows a plot of the streamlines generated. Table 9 gives the dimensions of the streamtubes generated and Figure 21 gives a plot of the oil recovery as a function

TABLE 6. The Prescribed Boundary Conditions for Shiells Canyon Reservoir Analysed as a Homogeneous Reservoir with Sealed Boundary

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	1	0.0
2	1	0.0
3	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0
7	1	0.0
8	1	0.0
9	1	0.0
10	1	0.0
11	1	0.0
12	1	0.0
13	1	0.0
14	1	0.0
15	1	0.0
16	1	0.0
17	1	0.0
18	1	0.0
19	1	0.0
20	1	0.0
21	1	0.0
22	1	0.0
23	1	0.0
24	1	0.0
25	1	0.0
26	1	0.0
27	1	0.0
28	1	0.0
29	1	0.0
30	1	0.0
31	1	0.0
32	1	0.0
33	1	0.0
34	1	0.0
35	1	0.0
36	1	0.0
37	1	0.0
38	1	0.0
39	1	0.0
40	1	0.0
41	1	0.0
42	1	0.0
43	1	0.0
44	1	0.0
45	1	0.0
46	1	0.0
47	1	0.0
48	1	0.0
49	1	0.0
50	1	0.0
51	1	0.0
52	1	0.0
53	1	0.0
54	1	0.0
55	1	0.0
56	1	0.0
57	1	0.0
58	1	0.0
59	1	0.0
60	1	0.0

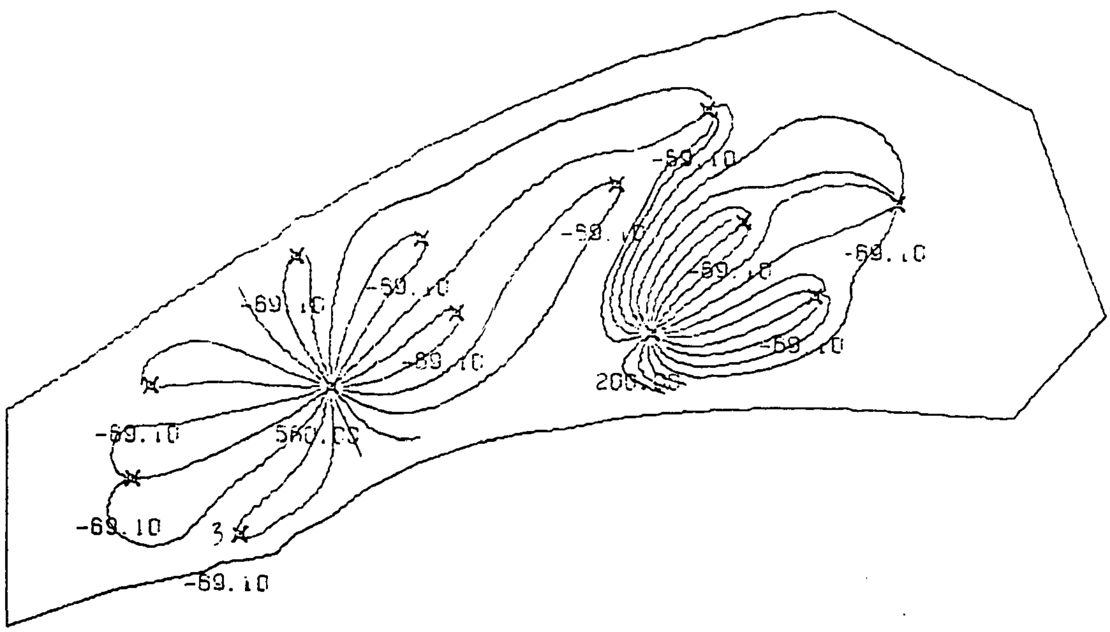


FIGURE 18. Boundary element modelling of Shiells Canyon Field as a single homogeneous reservoir with sealed boundary.

TABLE 7. Calculated Streamtube Dimensions for Shiells Canyon Field Analysed as a Homogeneous Reservoir with Sealed Outer Boundary

WELL NO.	S/L NO.	CODE	LENGTH (ft)	WIDTH (ft)	RATE (bbl/d)
1	1	1	538.10	45.18	28.00
1	2	1	425.00	23.75	28.00
1	3	1	686.10	79.51	28.00
2	1	1	441.40	27.76	28.00
2	2	1	344.70	19.18	28.00
3	1	1	334.80	18.06	28.00
3	2	1	384.80	21.76	28.00
4	1	1	269.70	13.51	28.00
4	2	1	269.70	14.88	28.00
5	1	1	359.80	15.89	28.00
5	2	1	338.10	14.26	28.00
6	1	1	294.70	12.69	28.00
6	2	1	288.10	11.96	28.00
7	1	1	809.80	29.41	28.00
7	2	1	759.80	28.63	28.00
8	1	1	112.00	38.76	28.00
8	2	1	950.00	34.34	28.00
8	3	1	588.10	15.61	10.00
8	4	1	569.70	12.31	10.00
8	5	1	598.00	13.78	10.00
9	1	1	266.40	6.27	10.00
9	2	1	291.40	6.74	10.00
9	3	1	378.30	8.90	10.00
9	4	1	288.10	6.46	10.00
10	1	1	463.10	10.18	10.00
10	2	1	369.70	8.22	10.00
10	3	1	331.60	7.53	10.00
10	4	1	334.80	7.39	10.00
10	5	1	403.30	10.92	10.00
11	1	1	716.40	22.42	10.00
11	2	1	903.30	34.43	10.00
11	3	1	713.10	19.76	10.00
11	4	1	538.10	15.60	10.00
11	5	1	606.60	22.23	10.00

TABLE 8. Input Data to Steamflood Recovery Prediction Program

TOTAL NUMBER OF PRODUCERS=	11
STEAM TEMPERATURE (DEG. F)=	430.0000
STEAM QUALITY=	0.7000
CONVERGENCE LIMIT=	0.0010
RESERVOIR THICKNESS (FT)=	160.0000
STEAM PRESSURE (PSI)=	344.0000
INITIAL RESERVOIR TEMPERATURE(DEG F)=	105.0000

FLUID PROPERTIES

	WATER	OIL	STEAM
DENSITY(LB/CU FT) AT STD. TEMP.	62.4000	55.0000	
DENSITY(LB/CU FT) AT STEAM TEMP.	52.3800	48.7000	0.7431
SPECIFIC HEAT(BTU/LB *F)	1.0000	0.4880	1.0000
LATENT HEAT(BTU/LB)			789.5100

PROPERTIES OF THE CAP AND BASE ROCK

DENSITY(LB/CU. FT)	149.0000
SPECIFIC HEAT(BTU/LB-*F)	0.2130
THERM. COND.(BTU/HR-FT-*F)	1.1000

RESERVOIR ROCK PROPERTIES

	REGION 1	REGION 2
POROSITY	0.2050	0.1800
DEN.(LB/CU FT)	165.0000	165.0000
SPEC. HEAT(BTU/LB*F)	0.2000	0.2000
PERMEABILITY(MD)	140.0000	70.0000

INITIAL FLUID SATURATIONS

	REGION 1	REGION 2
WATER	0.5500	0.5500
OIL	0.4500	0.4500
STEAM	0.0	0.0

AVERAGE RESIDUAL SATURATIONS BEHIND THE FRONT

	REGION 1	REGION 2
WATER	0.5800	0.5800
OIL	0.1800	0.1800
STEAM	0.2400	0.2400

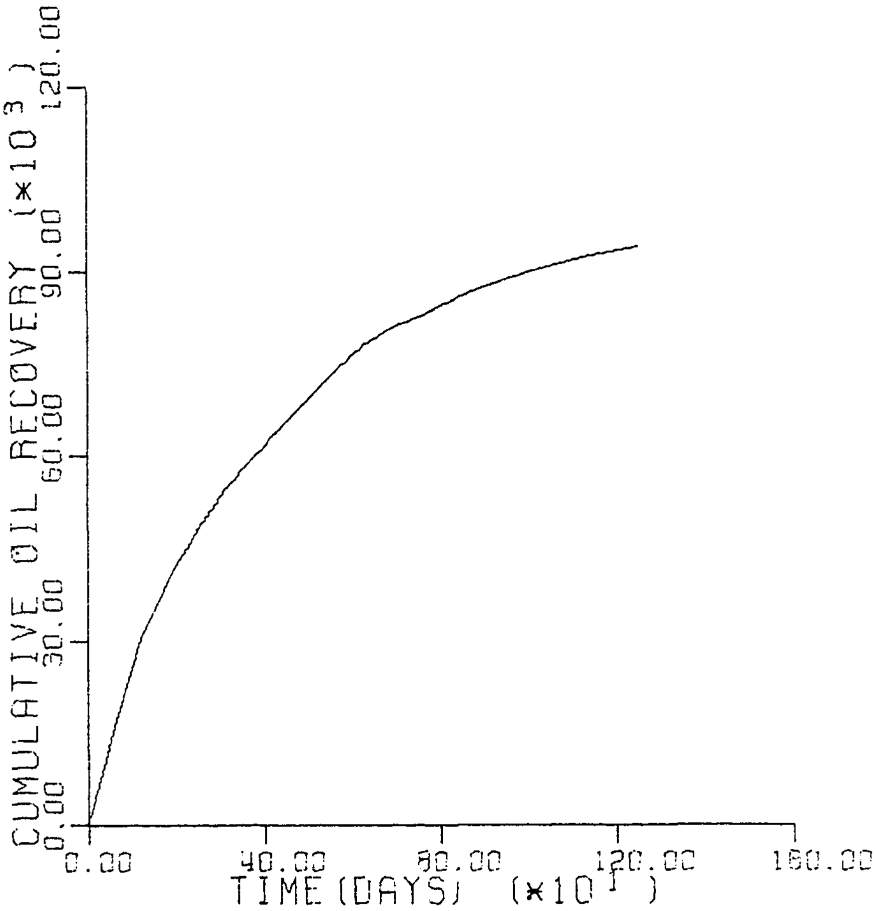


FIGURE 19. Steamflood recovery prediction for Shiells Canyon Field as a homogeneous reservoir with sealed outer boundary.

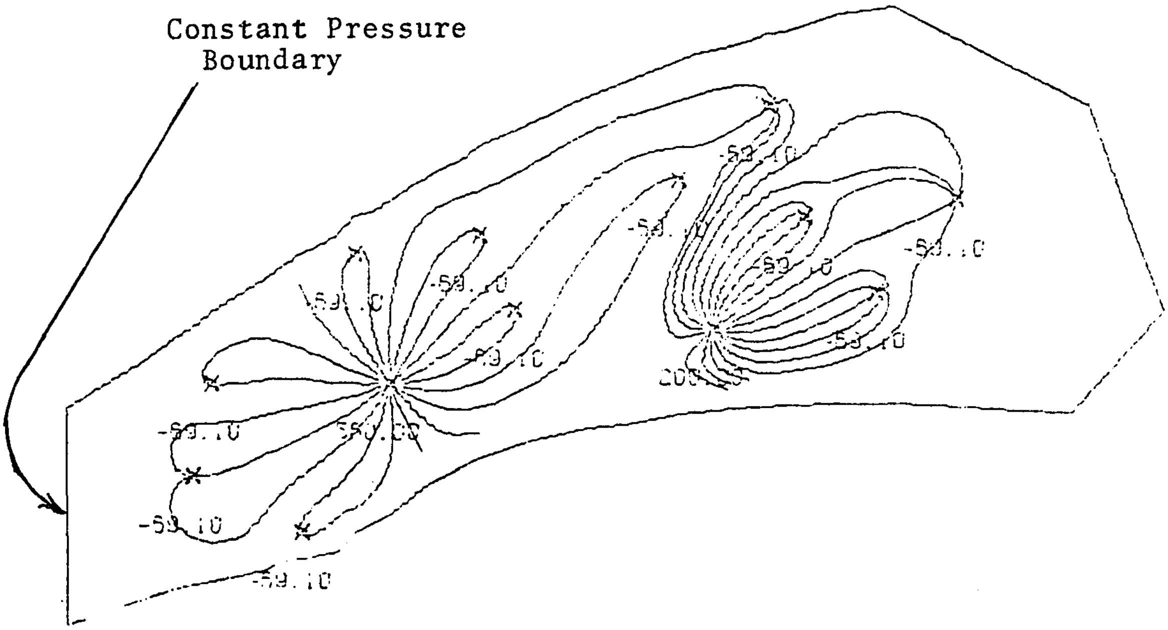


FIGURE 20. Boundary element modelling of Shiells Canyon Field as a homogeneous reservoir. Part of the boundary at constant pressure, part sealed.

TABLE 9. Calculated Streamtube Dimensions for Shiells Canyon Field Analysed as a Homogeneous Reservoir. Part of the Boundary at Constant Pressure, the Remainder Sealed.

WELL NO.	S/L NO.	CODE	LENGTH (ft)	WIDTH (ft)	RATE (bbl/d)
1	1	1	541.40	45.84	28.00
1	2	1	425.00	23.67	28.00
1	3	1	678.30	73.69	28.00
2	1	1	441.40	27.62	28.00
2	2	1	344.70	19.14	28.00
3	1	1	338.10	18.06	28.00
3	2	1	384.80	21.62	28.00
4	1	1	269.70	13.51	28.00
4	2	1	269.70	14.91	28.00
5	1	1	359.80	15.91	28.00
5	2	1	338.10	14.26	28.00
6	1	1	294.70	12.70	28.00
6	2	1	288.10	11.96	28.00
7	1	1	809.80	29.43	28.00
7	2	1	759.80	28.67	28.00
8	1	1	1120.00	38.69	28.00
8	2	1	950.00	34.38	28.00
8	3	1	588.10	15.55	10.00
8	4	1	569.70	12.31	10.00
8	5	1	598.00	13.79	10.00
9	1	1	266.40	6.27	10.00
9	2	1	291.40	6.75	10.00
9	3	1	378.30	8.92	10.00
9	4	1	288.10	6.46	10.00
10	1	1	463.10	10.17	10.00
10	2	1	369.70	8.22	10.00
10	3	1	331.60	7.53	10.00
10	4	1	334.80	7.39	10.00
10	5	1	403.30	10.97	10.00
11	1	1	716.40	22.47	10.00
11	2	1	903.30	34.84	10.00
11	3	1	713.10	19.81	10.00
11	4	1	538.10	15.61	10.00
11	5	1	606.60	22.34	10.00

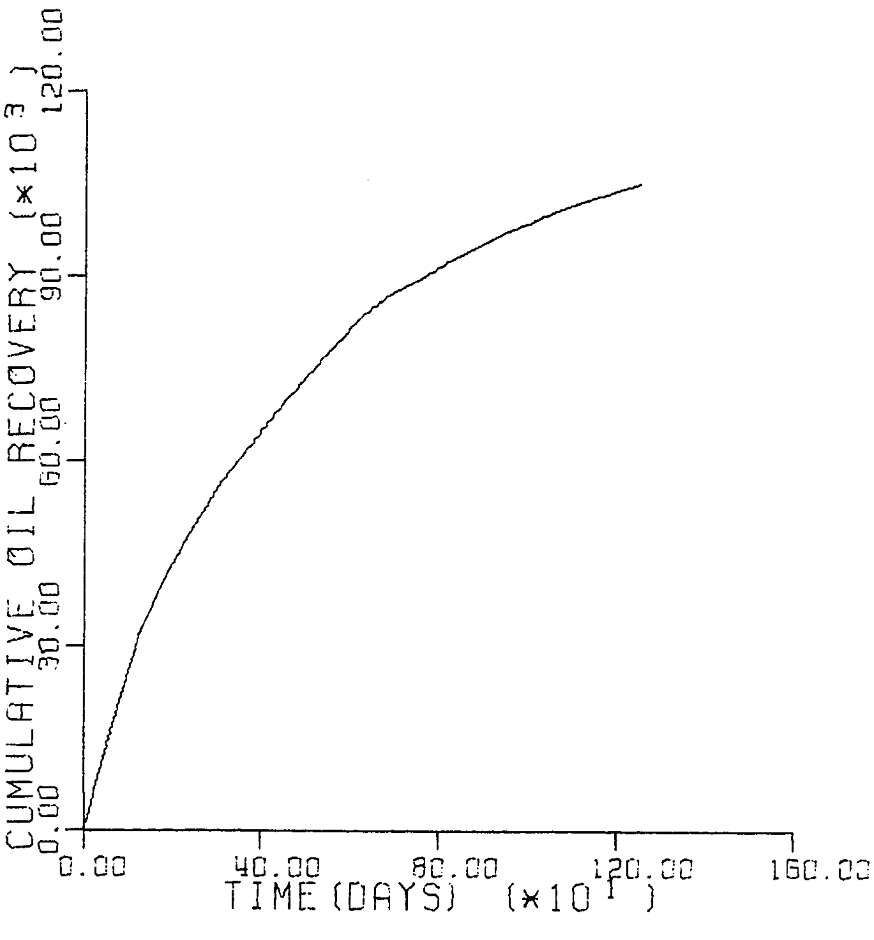


FIGURE 21. Steamflood recovery prediction for Shiells Canyon Field as a homogeneous reservoir with part of the boundary at constant pressure, part sealed.

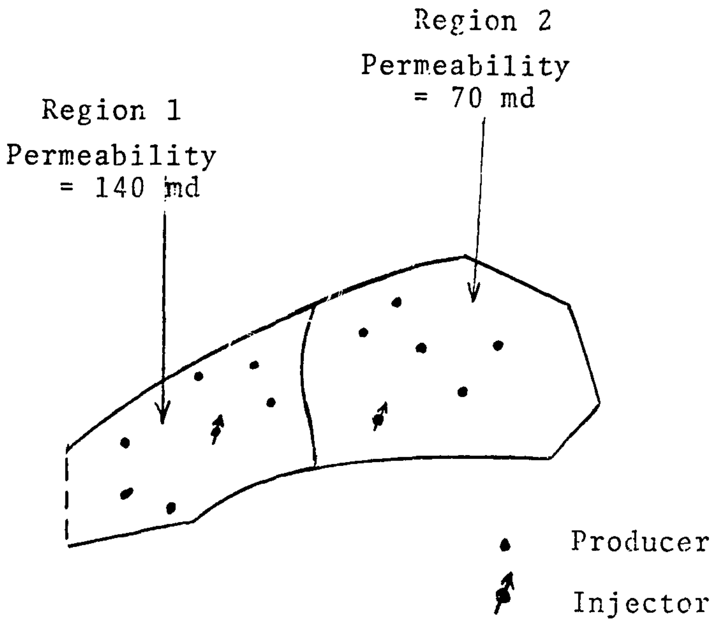
of time. The detailed output from the streamline modelling program is listed in Appendix I, while Appendix J presents the detailed output of the steamflood recovery program.

XI.4 CASE 2:

XI.4.1 Shiells Canyon Field Analysed as a Piecewise Homogeneous Reservoir

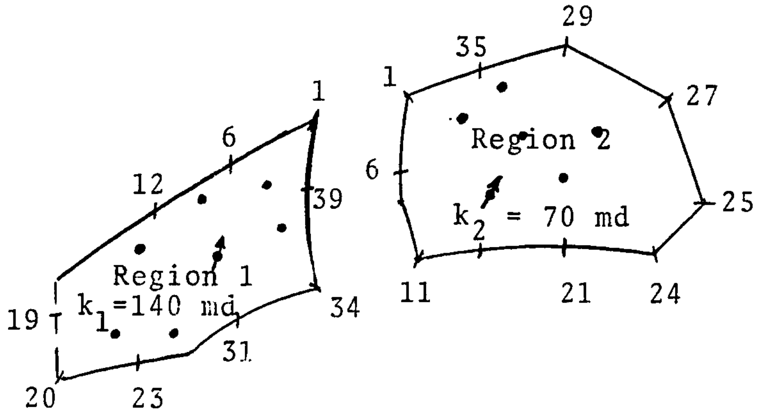
The Shiells Canyon field has been modified to consist of two regions with different permeabilities (Figure 22). Region 1 has an average permeability of 0.138 μm^2 (140 md) while the second region is assigned a permeability half as much, 0.69 μm^2 (70 md). Figure 23 shows the numbering scheme employed. The scheme was such that the domain of interest was always on the left as the numbering progressed. It is to be observed that the numbering of the second region starts where that for the first ends on the interface between the two regions. This was done purely for convenience in assembling the two matrices into one. The reservoir is analysed assuming first that the entire reservoir is enclosed by sealing faults, and then by assuming that the west boundary is at constant pressure.

The coordinates of the end points of the boundary elements for both regions are listed in Table 10. The coordinates and strengths of the injectors and producers are listed in Table 11.



Scale: 1" = 738.46 ft

FIGURE 22. Shiells Canyon Field (Zone 203) arbitrarily divided into two regions with different material properties.



Scale: 1" = 738.46 ft

FIGURE 23. Shiells Canyon Field (Zone 203) as a piecewise-homogeneous field showing discretization and numbering scheme.

TABLE 10. Case 2:

THE COORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELEMENTS

Region 1			Region 2		
POINT	X(INCH)	Y(INCH)	POINT	X(INCH)	Y(INCH)
1	1.5000	1.5500	1	1.5000	1.5500
2	1.3500	1.4800	2	1.5000	1.5000
3	1.2500	1.4400	3	1.4800	1.4500
4	1.2000	1.4000	4	1.4600	1.3500
5	1.1500	1.3800	5	1.4600	1.2500
6	1.0500	1.3250	6	1.4800	1.1500
7	0.9500	1.2600	7	1.5000	1.0500
8	0.9000	1.2500	8	1.5000	0.9500
9	0.8500	1.2000	9	1.5400	0.8500
10	0.8000	1.1700	10	1.5500	0.7500
11	0.7500	1.1500	11	1.5500	0.6800
12	0.6500	1.0800	12	1.6500	0.6900
13	0.5500	1.0200	13	1.7000	0.7000
14	0.5000	0.9900	14	1.7500	0.7100
15	0.4500	0.9500	15	1.8000	0.7200
16	0.4000	0.9200	16	1.8500	0.7250
17	0.3000	0.8500	17	1.9000	0.7300
18	0.1000	0.7300	18	1.9500	0.7350
19	0.1000	0.5000	19	2.1000	0.7450
20	0.1000	0.1500	20	2.2500	0.7500
21	0.2500	0.2000	21	2.4000	0.7500
22	0.4000	0.2500	22	2.6000	0.7400
23	0.5500	0.2800	23	2.7500	0.7300
24	0.6500	0.3000	24	2.9000	0.7250
25	0.7000	0.3250	25	3.1600	1.0000
26	0.7500	0.3300	26	3.0600	1.2500
27	0.8000	0.3400	27	2.9500	1.5500
28	0.8500	0.3500	28	2.7500	1.6500
29	0.9000	0.4000	29	2.4000	1.8200
30	1.0000	0.4500	30	2.2500	1.8000
31	1.1000	0.5250	31	2.1000	1.7500
32	1.2500	0.6000	32	2.0500	1.7450
33	1.4000	0.6500	33	2.0000	1.7250
34	1.5500	0.6800	34	1.9500	1.7100
35	1.5500	0.7500	35	1.8500	1.6900
36	1.5400	0.8500	36	1.7500	1.6500
37	1.5000	0.9500	37	1.6500	1.6100
38	1.5000	1.0500			
39	1.4800	1.1500			
40	1.4600	1.2500			
41	1.4600	1.3500			
42	1.4800	1.4500			
43	1.5000	1.5000			

TABLE 11. Case 2:

COORDINATES OF SOURCES AND SINKS AND THEIR STRENGTHS

Region 1

X(INCH)	Y(INCH)	RATE(BBL/D)
0.4500	0.5500	-69.1000
0.5000	0.8000	-69.1000
0.7500	0.4000	-69.1000
0.9000	1.1500	-69.1000
1.0000	0.8000	560.0000
1.2500	1.2000	-69.1000
1.3500	1.0000	-69.1000

THE AVERAGE RADIUS OF THE WELLS(INCH) IS= 0.500
THE CHARACTERISTIC PRESSURE(PSI) IS= 85.0000

Region 2

COORDINATES OF SOURCES AND SINKS AND THEIR STRENGTHS

X(INCH)	Y(INCH)	RATE(BBL/D)
1.9000	0.9500	200.0000
1.8000	1.3500	-69.1000
2.0500	1.5500	-69.1000
2.1500	1.2500	-69.1000
2.3500	1.0500	-69.1000
2.5750	1.3000	-69.1000

THE AVERAGE RADIUS OF THE WELLS(INCH) IS= 0.500
THE CHARACTERISTIC PRESSURE(PSI) IS= 85.0000

XI.5 RESULTS FOR CASE 2: SHIELLS CANYON RESERVOIR ANALYSED AS A PIECEWISE HOMOGENEOUS RESERVOIR

XI.5.1 Sealed Boundary

The prescribed boundary conditions are assigned on each boundary segment according to the code described earlier under Case 1. For the interface boundary, a code of 2 is assigned implying that the potential and the normal derivative of the potential are unknown at the interface. A listing of the boundary conditions for both regions is given in Table 12. A plot of the streamlines obtained are shown in Figure 24. Table 13 gives the dimensions of the streamtubes associated with the streamlines. These dimensions are read into the steamflood program to calculate recovery as a function of time which is plotted in Figure 25.

XI.5.2 Part of the Boundary at Constant Pressure, the Remainder Sealed

The coordinates of the end points of the boundary segments are the same as in Table 10. Also, the coordinates and strengths of the injectors and producers are the same as given in Table 11. The left boundary of region 1 is assumed to be at the initial reservoir pressure of 85 psi. Thus, segment numbers 18 and 19 are assigned this constant pressure. The result from the streamline simulation program is plotted in Figure 26. The dimensions of the streamtubes obtained are listed under Table 14. The petrophysical

TABLE 12. Boundary Conditions for Shiells Canyon Reservoir Analysed as a 2-Region Piecewise Homogeneous Reservoir

Region 1

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	1	0.0
2	1	0.0
3	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0
7	1	0.0
8	1	0.0
9	1	0.0
10	1	0.0
11	1	0.0
12	1	0.0
13	1	0.0
14	1	0.0
15	1	0.0
16	1	0.0
17	1	0.0
18	1	0.0
19	1	0.0
20	1	0.0
21	1	0.0
22	1	0.0
23	1	0.0
24	1	0.0
25	1	0.0
26	1	0.0
27	1	0.0
28	1	0.0
29	1	0.0
30	1	0.0
31	1	0.0
32	1	0.0
33	1	0.0
34	2	0.0
35	2	0.0
36	2	0.0
37	2	0.0
38	2	0.0
39	2	0.0
40	2	0.0
41	2	0.0
42	2	0.0
43	2	0.0

Region 2

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	2	0.0
2	2	0.0
3	2	0.0
4	2	0.0
5	2	0.0
6	2	0.0
7	2	0.0
8	2	0.0
9	2	0.0
10	2	0.0
11	1	0.0
12	1	0.0
13	1	0.0
14	1	0.0
15	1	0.0
16	1	0.0
17	1	0.0
18	1	0.0
19	1	0.0
20	1	0.0
21	1	0.0
22	1	0.0
23	1	0.0
24	1	0.0
25	1	0.0
26	1	0.0
27	1	0.0
28	1	0.0
29	1	0.0
30	1	0.0
31	1	0.0
32	1	0.0
33	1	0.0
34	1	0.0
35	1	0.0
36	1	0.0
37	1	0.0

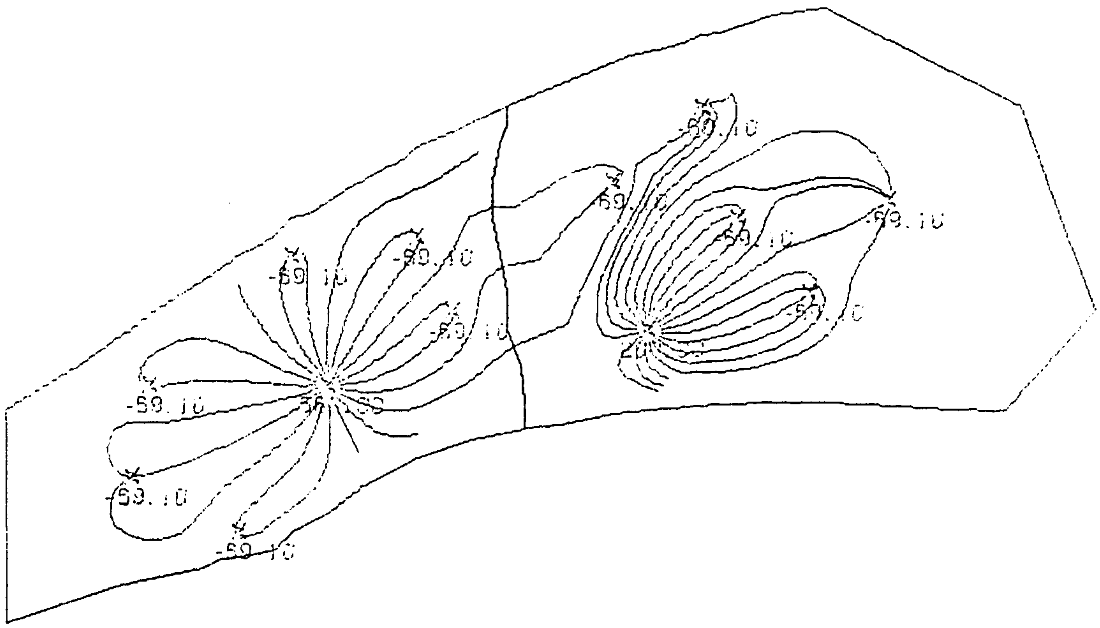


FIGURE 24. Boundary element modelling of Shiells Canyon Field as two-region piecewise homogeneous reservoir. Sealed outer boundary.

TABLE 13. Calculated Streamtube Dimensions for Shiells Canyon Reservoir Analysed as a Piecewise Homogeneous Reservoir with Sealed Boundary

			REGION 1			REGION 2		
WELL NO.	S/L NO.	CODE	LENGTH	WIDTH	RATE	LENGTH	WIDTH	RATE
1	1	1	534.80	45.44	28.00	0.0	0.0	0.0
1	2	1	425.00	23.89	28.00	0.0	0.0	0.0
1	3	1	700.00	85.96	28.00	0.0	0.0	0.0
2	1	1	441.40	27.95	28.00	0.0	0.0	0.0
2	2	1	344.70	19.24	28.00	0.0	0.0	0.0
3	1	1	334.80	18.07	28.00	0.0	0.0	0.0
3	2	1	391.40	21.94	28.00	0.0	0.0	0.0
4	1	1	269.70	13.51	28.00	0.0	0.0	0.0
4	2	1	269.70	14.84	28.00	0.0	0.0	0.0
5	1	1	359.80	15.86	28.00	0.0	0.0	0.0
5	2	1	338.10	14.26	28.00	0.0	0.0	0.0
6	1	1	294.70	12.69	28.00	0.0	0.0	0.0
6	2	1	288.10	11.95	28.00	0.0	0.0	0.0
7	1	2	376.90	51.38	28.00	403.30	42.31	28.00
7	2	2	411.10	79.31	28.00	313.10	32.17	28.00
8	1	2	602.00	58.92	28.00	319.70	38.38	28.00
8	2	2	427.10	58.81	28.00	484.80	46.20	28.00
8	3	1	581.60	14.81	10.00	0.0	0.0	0.0
8	4	1	566.40	12.23	10.00	0.0	0.0	0.0
8	5	1	594.70	13.71	10.00	0.0	0.0	0.0
9	1	1	266.40	6.20	10.00	0.0	0.0	0.0
9	2	1	291.40	6.74	10.00	0.0	0.0	0.0
9	3	1	378.30	9.01	10.00	0.0	0.0	0.0
9	4	1	288.10	6.44	10.00	0.0	0.0	0.0
10	1	1	463.10	10.12	10.00	0.0	0.0	0.0
10	2	1	369.70	8.20	10.00	0.0	0.0	0.0
10	3	1	331.60	7.52	10.00	0.0	0.0	0.0
10	4	1	334.80	7.39	10.00	0.0	0.0	0.0
10	5	1	403.30	11.19	10.00	0.0	0.0	0.0
11	1	1	719.70	22.68	10.00	0.0	0.0	0.0
11	2	1	913.10	36.20	10.00	0.0	0.0	0.0
11	3	1	713.10	19.91	10.00	0.0	0.0	0.0
11	4	1	538.10	15.62	10.00	0.0	0.0	0.0
11	5	1	609.80	22.74	10.00	0.0	0.0	0.0

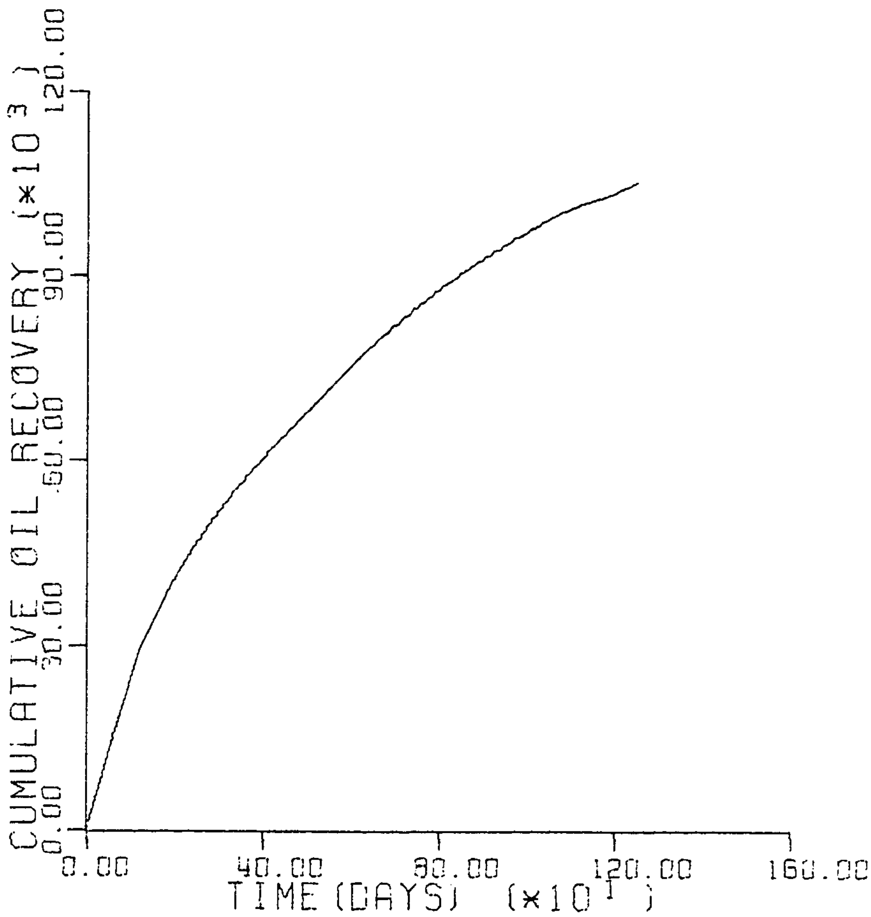


FIGURE 25. Steamflood recovery prediction of Shiells Canyon Field as two-region piecewise homogeneous reservoir. Sealed outer boundary.

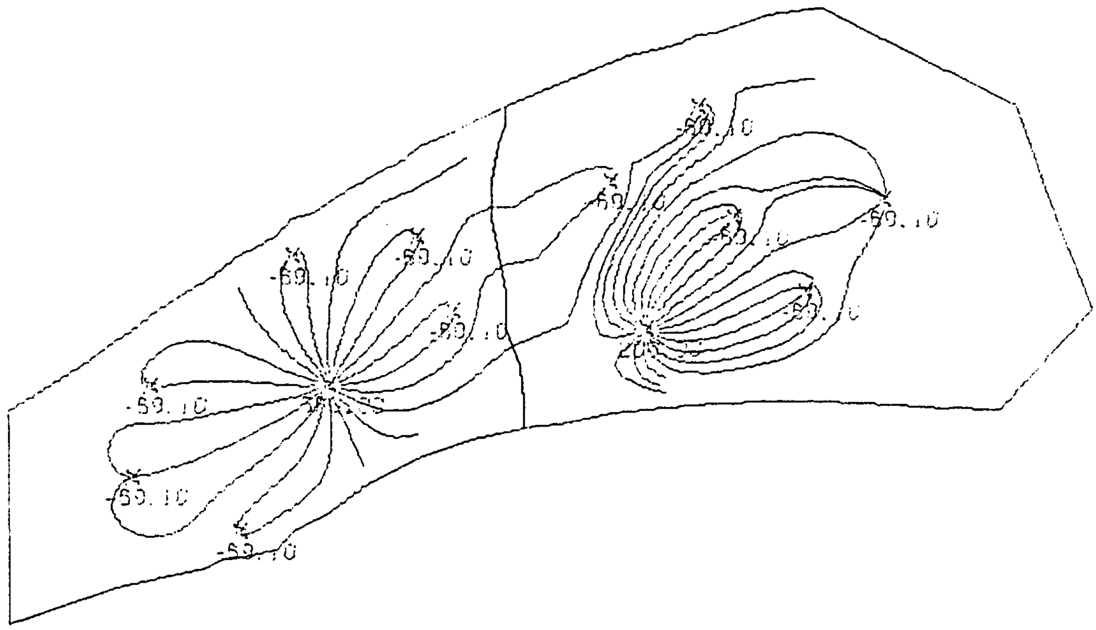


FIGURE 26. Boundary element modelling of Shiells Canyon Field as two-region piecewise homogeneous reservoir. Part of the boundary at constant pressure, part sealed.

TABLE 14. Calculated Streamtube Dimensions for Shiells Canyon Field Analysed as a Piecewise Homogeneous Reservoir. Part of the Boundary at Constant Pressure, the Remainder Sealed

			REGION 1			REGION 2		
WELL NO.	S/L NO.	CODE	LENGTH	WIDTH	RATE	LENGTH	WIDTH	RATE
1	1	1	553.30	49.91	28.00	0.0	0.0	0.0
1	2	1	425.00	23.40	28.00	0.0	0.0	0.0
1	3	1	656.60	66.11	28.00	0.0	0.0	0.0
2	1	1	444.70	27.95	28.00	0.0	0.0	0.0
2	2	1	344.70	19.06	28.00	0.0	0.0	0.0
3	1	1	338.10	17.95	28.00	0.0	0.0	0.0
3	2	1	381.60	21.11	28.00	0.0	0.0	0.0
4	1	1	269.70	13.31	28.00	0.0	0.0	0.0
4	2	1	269.70	14.75	28.00	0.0	0.0	0.0
5	1	1	366.40	15.86	28.00	0.0	0.0	0.0
5	2	1	331.60	13.79	28.00	0.0	0.0	0.0
6	1	1	291.40	12.62	28.00	0.0	0.0	0.0
6	2	1	294.70	12.12	28.00	0.0	0.0	0.0
7	1	2	451.30	60.06	28.00	216.40	30.08	28.00
7	2	2	478.40	58.33	28.00	248.00	31.38	28.00
8	1	2	376.40	61.82	28.00	541.40	43.88	28.00
8	2	1	591.40	13.37	10.00	0.0	0.0	0.0
8	3	1	619.70	14.81	10.00	0.0	0.0	0.0
9	1	1	266.40	6.40	10.00	0.0	0.0	0.0
9	2	1	288.10	6.87	10.00	0.0	0.0	0.0
9	3	1	363.10	8.27	10.00	0.0	0.0	0.0
9	4	1	288.10	6.64	10.00	0.0	0.0	0.0
10	1	1	478.30	11.11	10.00	0.0	0.0	0.0
10	2	1	384.80	8.51	10.00	0.0	0.0	0.0
10	3	1	334.80	7.71	10.00	0.0	0.0	0.0
10	4	1	331.60	7.47	10.00	0.0	0.0	0.0
10	5	1	384.80	9.71	10.00	0.0	0.0	0.0
11	1	1	703.30	21.65	10.00	0.0	0.0	0.0
11	2	1	834.80	27.73	10.00	0.0	0.0	0.0
11	3	1	719.70	19.85	10.00	0.0	0.0	0.0
11	4	1	538.10	16.01	10.00	0.0	0.0	0.0
11	5	1	591.40	20.86	10.00	0.0	0.0	0.0

data input to the steamflood program are the same as those given under Table 8. The calculated recovery of oil as a function of time is presented in Figure 27.

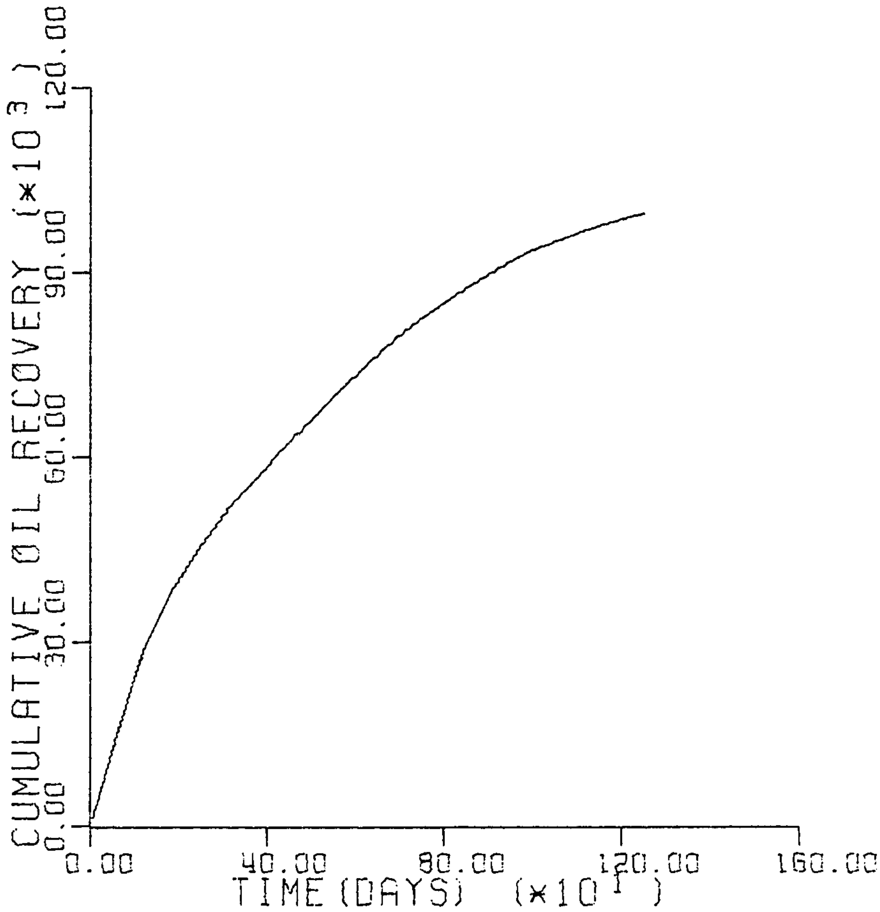


FIGURE 27. Steamflood recovery prediction for Shiells Canyon Field as a two-region piecewise homogeneous reservoir. Part of the boundary at constant pressure, part sealed.

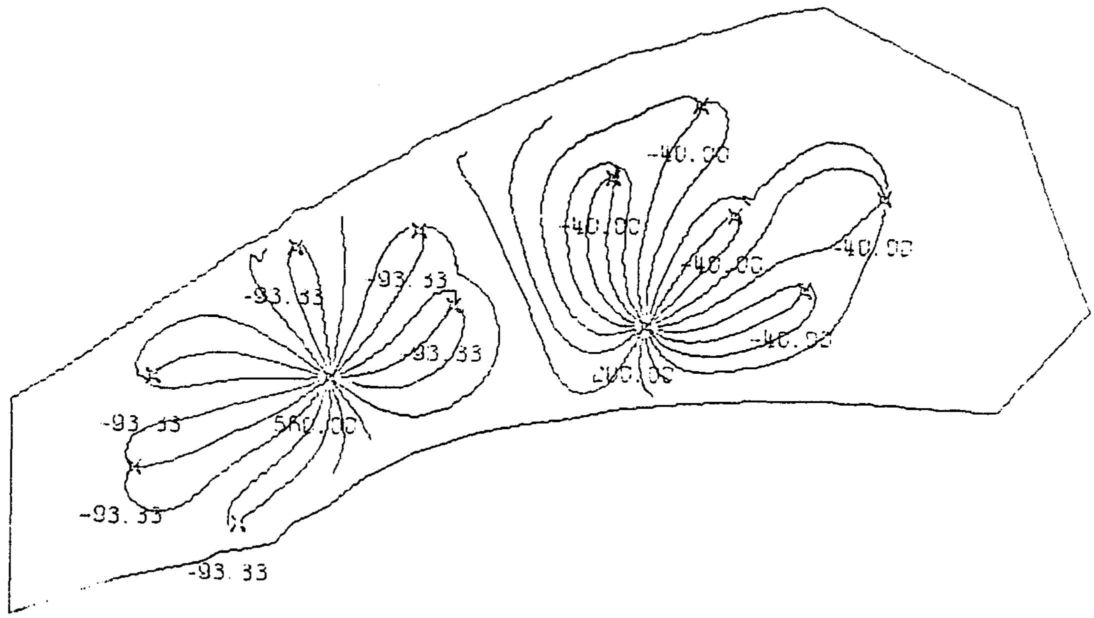


FIGURE 28. Boundary element streamline modelling of Shiells Canyon Field as a homogeneous reservoir with sealed outer boundary. Unequal production rates.

XI.6 DISCUSSION OF RESULTS

The pattern of streamlines obtained by Konopnicki et al, is shown in Figure 28. This pattern differs considerably from those obtained in this study using the Boundary Element Method for a homogeneous reservoir (Figures 18 and 20). However, a direct comparison between the two studies is inappropriate for the following reasons:

a. The production rates assigned to each producer by Konopnicki et al. in their streamline model is unknown. In this study, equal rates, determined as the sum of all the injection rates divided by the number of producers were assigned to each producer. This is important because the rates assigned to the producers significantly affect the pattern of streamlines developed. This effect can be observed in Figure 28 where the producers surrounding each injector have been assigned rates equal to the injection rate divided by the number of producers around the injector.

b. There is one fewer producer in this study than that published by Konopnicki et al.

c. Even though Konopnicki et al. state that the boundary of the reservoir is assumed to be fluid sealing all around, it is evident from Figure 29 that the boundary on the west was certainly not a no-flow boundary. In this study, the west boundary is first assumed to be fluid sealing and then assumed to be at constant pressure (initial pressure of 85 psi). Considering the level of accuracy obtained for the test case (Section X.1), this author feels that the streamlines obtained in this study are representative of the true streamlines under the assumed boundary conditions.

Comparing the results of the sealed outer boundary and that where the west boundary is kept at constant pressure, it was observed that the values of potentials (Φ) and the normal gradients of potentials $\left(\frac{\partial \Phi}{\partial n}\right)$ obtained on the boundary "nodes" are markedly different for the two cases. However, given such differences, the streamline patterns look identical to each other. Small differences are observed in the widths of the streamtubes generated. To investigate further, more of the boundary segments were changed to the constant pressure conditions. It was observed that the streamlines deviated more from the sealed boundary case as more of the boundary segments were changed to constant boundary segments. Further testing led this author to conclude that changes in the boundary conditions affected the pattern of the streamlines generated. The magnitude of the effect depended on (a) the proportion of the entire boundary that had been changed, (b) the strengths of the sources and sinks present, (c) the proximity to any boundary.

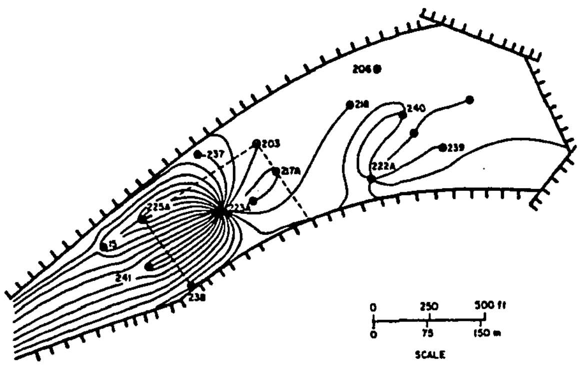


FIGURE 29. Streamlines for Shiells Canyon Steamflood (After Konopnicki, et al.[50])

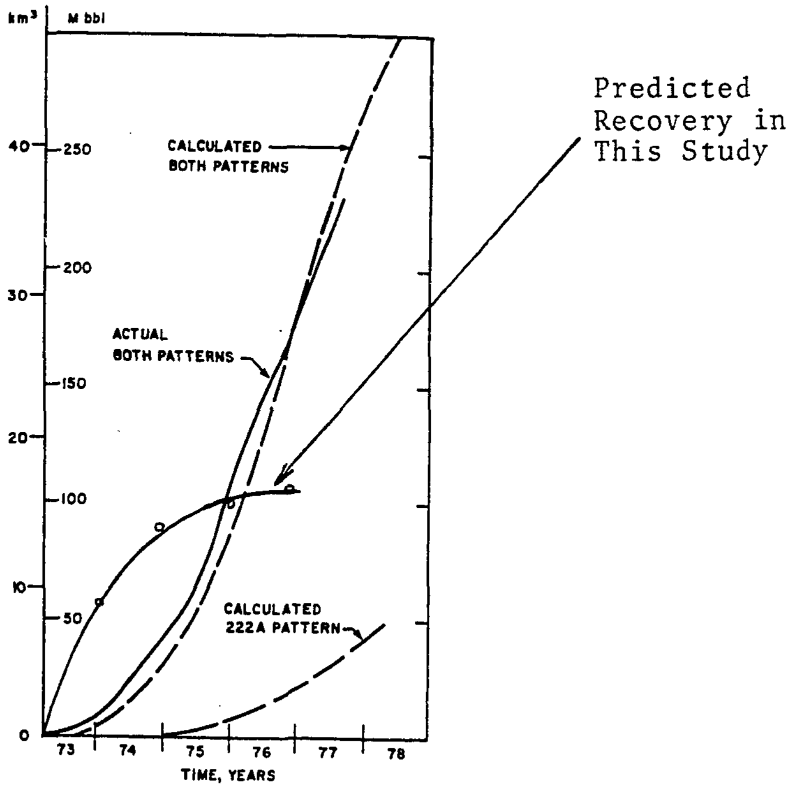


FIGURE 30. Comparison with Field Performance(50)

It was observed that the reason for the streamlines looking nearly identical to each other for the different boundary conditions was because the effects of the sources and sinks was so large in comparison to the boundary effects that the boundary effects were masked. Only near boundaries that are far from sources and sinks were the boundary effects felt.

The results from the analysis of the reservoir as a 2-region piecewise homogneous reservoir showed the same similarity in streamline patterns for the two kinds of boundary conditions (Figures 24 and 26) as observed earlier for the same reasons. Figure 31 gives a comparison of the oil recovery versus time for the two systems under the two kinds of boundary conditions. It can be observed that the recovery from the single homogeneous reservoir was higher than the recovery from the 2-region piecewise homogeneous reservoir for the two kinds of boundary conditions applied. This agreed with expectations since one of the regions of the 2-region reservoir has a permeability that is half the homogeneous reservoir. For the homogeneous reservoir, the summulative oil recovered was higher when part of the boundary was at constant pressure, the remainder sealed, than that when all the boundary was sealed. The reverse was the case in the 2-region piecewise homogeneous reservoir where the recovery from the sealed boundary condition was higher.

The cause of this is the loss of one streamline (Figure 26) in the case when part of the boundary was at constant pressure. Figure 30 shows a comparison between the predicted performance from this study and the actual field performance. The figure shows a high rate of recovery at early times in this study. This is consistent with the fact that the steam front velocity is relatively high at early times and declines exponentially with time. The recovery prediction by this study does not match the field performance. However, this was expected for several reasons: (a) The actual injection rates were neither steady nor continuous (Figure 32) whereas a constant injection rate was assumed for this study. (b) The prediction model used in this study assumes that the oil saturation reduces to irreducible in the steam swept area. (c) The production rates used to generate the streamlines for this study was assumed. Since the streamlines are very sensitive to the injection and production rates, this could introduce substantial errors.

CUMMULATIVE RECOVERY(BBLS*10E5) VS TIME(DAYS)

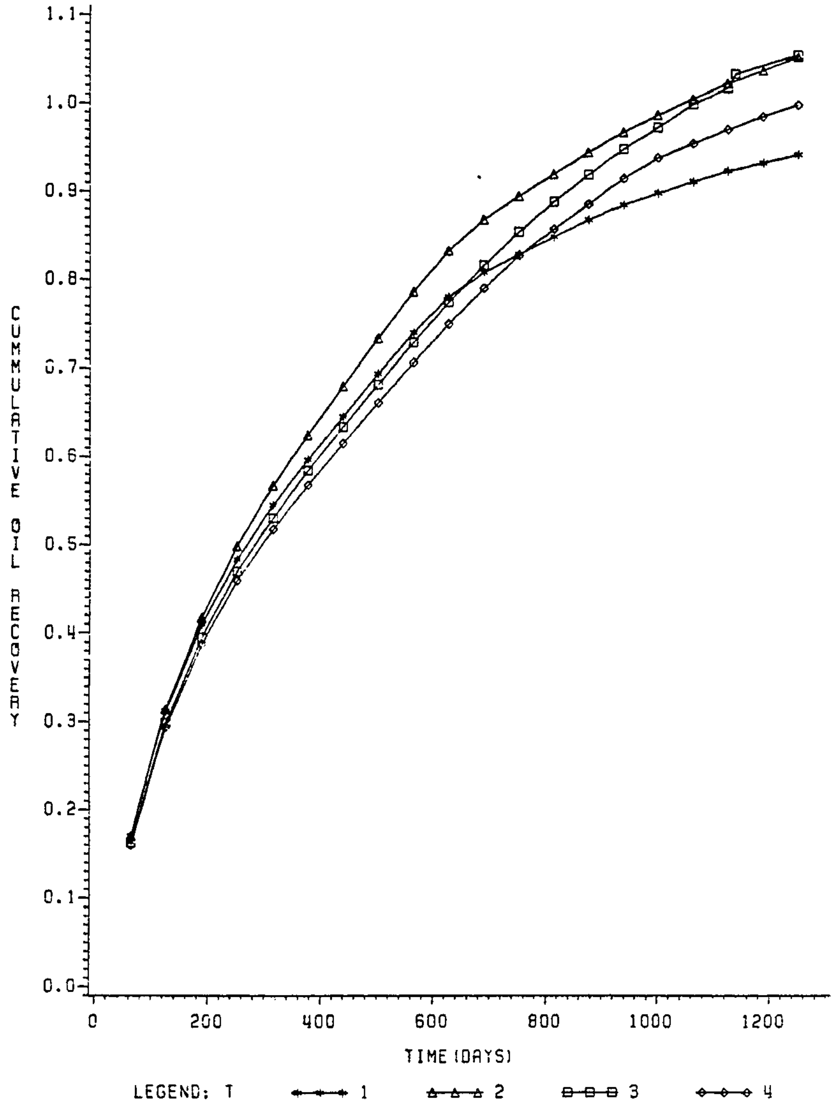


FIGURE 31. Comparison of results

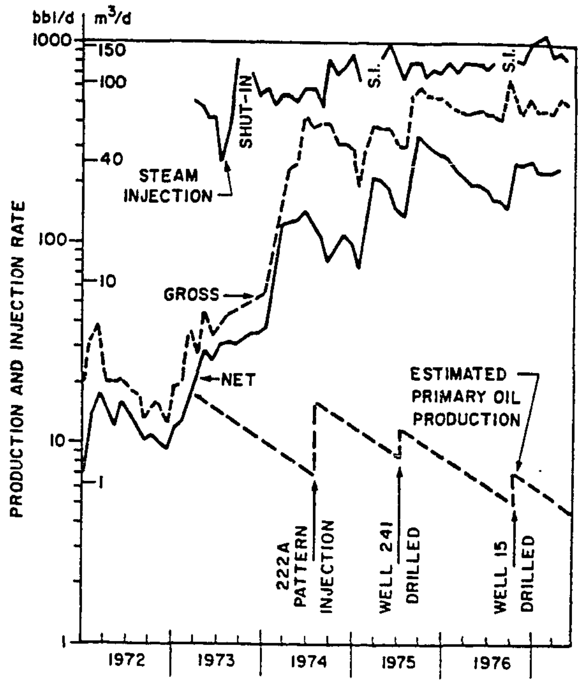


FIGURE 32. Field injection history (50)

XI.7 COMPUTATIONAL EFFICIENCY OF THE BOUNDARY ELEMENT METHOD

In practical terms, the most important question that any numerical method must answer is: "How does it compare with the commonly used methods with regard to accuracy and computational effort?" A comparison with the "method of images" is not appropriate here because the image method is only applicable to homogeneous reservoirs. Furthermore, it is not a numerical method. However, it can be said that the Boundary Element Method is superior to the image method in terms of accuracy. In fact, it is claimed to be superior to both the finite difference and finite element methods in terms of accuracy and finer resolution in the interior.[12,54,56] Part of the reason for this is because the error of discretization is confined to points on and near boundaries since only the boundary is discretized as opposed to discretizing the entire domain. Brebbia[12] reports that finite element results are usually accurate for the original variables under consideration (potentials) in this case, but when these variables are differentiated (to obtain fluxes), the results are much less accurate and are usually discontinuous between elements.

Several authors[21,54,55] have reported on the comparative computational efforts associated with the solution of the coefficient matrix arising in the Boundary Element Method. For a homogeneous medium, the coefficient matrix arising from the Boundary Element Method (Equation VII.0.1) is generally non-symetric, fully populated, non-singular, well conditioned but not diagonally dominant even though the largest terms are the diagonal terms. Hess[21] compared the computing effort involved in a direct Gauss elimination procedure to that of the Gauss-Siedel iterative method for both the interior and exterior problem. As reported by Hess, the computing effort involved in a direct elimination is proportional to N^3, where N is the number of unknowns. The computing effort for an iterative solution is proportional to IFN^2, where I is the number of iterations required for convergence, and F is the number of solutions being obtained. Thus, if I was independent of N, the iterative method would require less computing effort than the direct method for sufficiently large N. He further reported that for the exterior flow problem, the Gauss-Siedel procedure converged in a number of iterations that was indepedent of N but dependent on the body shape. Thus, he found that the iterative procedure required less effort if N was as large as 100-300 depending on F, but had the considerable disadvantage that the required computing time was less predictable than was the case when a direct elimination was used. For the interior flow problem, I increases linearly with N, so that the iterative procedure is usually not competitive with direct elimination.

Bettes[54] compared the computational work involved in the Boundary Integral Element Method with that of the finite element method. He found that for a square n x n two-dimensional problem, the finite element method required fewer storage locations for all meshes smaller than n = 11, and fewer arithmetic operations for all meshes smaller than n = 35. For three dimensional problems (n by n by n cube), the finite element method required fewer storage locations if n is less than 30, and fewer arithmetic operations if n is less than 135. Bettes made the conclusion that problems have to be very large before the Boundary Integral Element method is computationally cheaper than the finite element method.

Mukherjee and Morjaria[55] have also compared the accuracy and computational efficiency of the boundary element and finite element methods for problems of time-dependent inelastic torsion of prismatic shafts. After making the comparisons for solid shafts with circular, square, elliptical, and triangular cross sections, they concluded that (a) the CPU times on an IBM 370/168 are of the same order for both BEM and FEM methods but with the BEM program generally running somewhat faster than the FEM program with the same internal mesh; (b) the discretization and input data preparation is much easier for the BEM than for the FEM.

CHAPTER XII

CONCLUSIONS AND RECOMMENDATIONS

XII.1 CONCLUSIONS

1. A streamline simulation model has been developed that is applicable to homogeneous as well as piecewise homogeneous porous media having arbitrarily shaped boundaries and under different kinds of boundary conditions. Such a general capability was not previously possible.

2. The model has successfully been used to predict steamflood recovery in both homogeneous and piecewise homogeneous porous media assuming streamtubes that are thermally isolated from each other.

3. The model can be used to predict oil recovery from any other form of secondary or tertiary recovery.

4. Interpretation of the results from this model should be done with the limitations in mind. It is recommended that the model be used primarily as a diagnostic tool prior to full scale simulation.

XII.2 LIMITATIONS OF THE MODEL

1. Streamlines are invariant with time.

2. Assumption of single phase flow is incorrect.

3. Assumption of zero heat transfer between streamtubes does not represent the physical condition.

4. The boundary element model can only be applied to systems described by linear differential equations.

5. Any region less than half an element size away from the boundary gives wrong results.

6. Cannot be used on domains with rapidly changing heterogeneities.

7. Cannot exploit existing sparse-matrix techniques.

XII.3 SUGGESTIONS FOR FURTHER STUDY

There are several areas in this research work that can be extended, improved upon. Some of the areas where further work is needed are:

1. The application of the Boundary Element Method to other equations such as parabolic equations.

2. Application to anisotropic porous media. This can very easily be done by making the appropriate transformation.

3. The search for a suitable analytical solution to the equation describing the velocity of the steam front in the second region without ignoring the heat losses in the first region.

4. The extension of the Boundary Element Method to handle reservoirs with part of their boundaries at infinity.

NOMENCLATURE

(x,y)	= coordinate axis
(x_i,y_i)	= coordinates of interior points
(x_b,y_b)	= coordinates of boundary points
(x_j,y_j)	= coordinates of sources and sinks
ϕ	= porosity
k	= permeability
K	= thermal conductivity
D	= domain
S	= boundary
Φ	= potential
S_Φ	= boundary where potential is specified
S_n	= boundary where potential gradient $\frac{\partial\Phi}{\partial n}$ is specified
$S_{\Phi,n}$	= boundary where mixed boundary conditions are specified
ρ	= density
u	= volume flux
$\nabla\cdot$	= divergence operator
t	= time
∇	= gradient operator
μ	= viscosity

N = number of sources and sinks

δ = Dirac delta function

q_j = volumetric rate of flow of the jth source or sink

Φ_n = potential gradient $\frac{\partial \Phi}{\partial n}$

L = two-dimensional difference operator, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

W = potential distribution due to a unit charge in an infinite system

$\hat{i}$ = unit vector in the x direction

$\hat{j}$ = unit vector in the y direction

$\hat{r}$ = unit vector normal to a surface

r = radius

r_i = distance between an observation point (x,y) and an internal charge point (x_i,y_i)

= $\sqrt{(x - x_i)^2 + (y - y_i)^2}$

$r_{i,j}$ = distance between a source or sink point (x_j,y_j) and an internal charge point (x_i,y_i)

= $\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$

$r_{b,j}$ = distance between a source or sink point (x_j,y_j) and a boundary charge point (x_b,y_b)

= $\sqrt{(x_j - x_b)^2 + (y_j - y_b)^2}$

$r_{b,L}$ = distance between a boundary node point (x_L,y_L) and a boundary charge point (x_b,y_b)

= $\sqrt{(x_L - x_b)^2 + (y_L - y_b)^2}$

ε = radius of a small circle on the surface (Figure 4)

S_ε = the surface of the semicircle of radius ε (Figure 4)

Φ^* = average value of Φ on S_ε

(x_{L_k}, y_{L_k}) = coordinates of Gaussian quadrature points

$\bar{\Phi}$ = known value of potential specified as a boundary condition

$\frac{\partial \bar{\Phi}}{\partial n}$ = known value of potential gradient specified as a boundary condition

M_1 = number of boundary segments in which potentials are specified and potential gradients are unknown

M_2 = number of boundary segments in which potential derivatives are specified and potentials are unknowns

$G_{b,L}$ = defined by Equation (VI.0.3)

$H_{b,L}$ = defined by Equation (VI.0.4)

$\check{G}_{b,L}$ = $G_{b,L}$ when $L \neq b$

$G_{b,b}$ = $G_{b,L}$ when $L = b$

$G_{i,L}$ = defined by Equation (VII.1.4)

$H_{i,L}$ = defined by Equation (VII.1.5)

v_x = velocity in the x-direction

v_y = velocity in the y-direction

S_L = the surface of the Lth line segment

S_{L_D} = dimensionless form of S_L

$Q_N(\xi^-, t)$ = the rate of flow of heat per unit cross sectional area through the vertical cross sectional area at ξ^- into the elemental volume in the Nth region

$Q_N(\xi^+,t)$ = the rate of flow of heat per unit cross sectional area out of the elemental volume through the vertical cross section at ξ^+ in the Nth region

$V_N(t)$ = the velocity of the steam front in the Nth region

$H_N(\xi^-,t)$ = heat content per unit volume of reservoir rock/fluid system at the upstream face of the steam front in the Nth region

$H_N(\xi^+,t)$ = heat content per unit volume of reservoir rock/fluid system at the downstream face of the steam front in the Nth region

$Q(0,t)$ = rate of injection of heat per unit cross sectional area

$[Q_L(t)]_j$ = rate of loss of heat per unit cross sectional area to overburden and underburden from the jth region

$\rho_m(T)$ = density of m at temperature T, where m can be oil, water, or steam

L_{st} = latent heat of steam

c_m = specific heat of m, where m = oil, water, steam

$\Delta T(\xi^-)$ = change in temperature at the upstream face of the steam front

$\Delta T(\xi^+)$ = change in temperature at the downstream face of the steam front

ϕ_j = porosity of the rock in the jth region

$S_m(x,t)$ = saturation distribution of m, where m = oil, water, steam

$M_{st}(0,t)$ = mass rate of injection of steam per unit cross sectional area

$M_w(0,t)$ = mass rate of injection of water per unit cross sectional area

T_{st} = $(T_{st} - T_i)$ where $T_i = 0$ and T_{st} is the temperature of steam
= $\Delta T(\xi^-)$

$\mathcal{k}$ = thermal conductivity of overburden and underburden

$\xi_j(t)$ = the location of the steam front in the jth region at time t

T_{ξ^+} = the temperature at the downstream face of the steam front

T_{ξ^-} = the temperature at the upstream face of the steam front

$q_{st}(0,t)$ = rate of injection of steam per unit cross sectional area

k_j = permeability of the jth region

h_j = thickness of the jth region

$\overline{S}_m(t)$ = average saturation of phase m, where m = oil, water, steam

c_{r_N} = specific heat of reservoir rock of the Nth region

$[Q_L(t)]_{\Delta A}$ = rate of heat loss per unit cross section area from an elemental area of size ΔA

d = thermal diffusivity of the overburden or underburden $(k/\rho_{cb}c_{cb})$

P_j = number of elemental areas (or number of time steps in the jth region)

t = time

τ = variable of time

Δt = time step

W_j = width of the jth region of the porous medium

t_D = dimensionless time

ξ_D = dimensional distance

v_D = $\frac{d\xi_D}{dt_D}$ = dimensionless velocity

L_j = length of the jth region of the porous medium

x = fraction of injected steam that is water

λ_j = ratio of heat capacity of cap and base rock to that of the jth region of the reservoir

$\check{z}_j(t)$ = discontinuous distance function defined only for the period $t_{j-1} < t < t_j$ and not defined outside this interval

$z_j(t)$ = step-wise continuous distance function defined for all time $t > 0$, but equal to zero outside the time interval $t_{j-1} < t < t_j$.

$$z_j(t) = \{H(t - t_{j-1}) - H(t - t_j)\}\xi_j(t)$$

$\xi_j(t)$	=	continuous distance function with time
$z(t)$	=	$\sum_{j=1}^{N}\{H(t - t_{j-1}) - H(t - t_j)\}\xi_j(t)$

Subscripts

+	=	downstream face of the condensation front
-	=	upstream face of the condensation front
w	=	water
st	=	steam
o	=	oil
j	=	denotes the region
ΔA	=	elemental area
cb	=	cap and base rock (overburden and underburden)
D	=	dimensionless

APPENDIX A

DERIVATIONS OF $H_{b,L}$, $G_{b,L}$, $H'_{b,L}$, $G'_{b,L}$

APPENDIX A

DERIVATIONS OF $H_{b,L}$, $G_{b,L}$, $H'_{b,L}$ AND $G'_{b,L}$

$H_{b,L}$ is defined as

$$\frac{1}{2\pi}\int_{S_L}\frac{\partial}{\partial n_L}\left(\ln\frac{1}{r_{b,L}}\right)dS_L$$

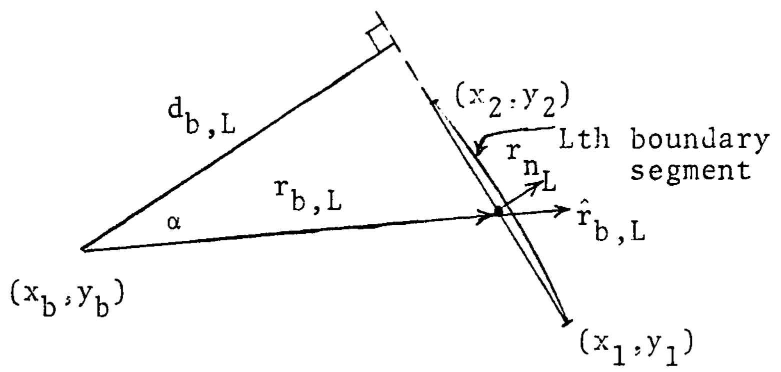
$$\frac{\partial}{\partial n}\left(\ln\frac{1}{r_{b,L}}\right) = \frac{\partial}{\partial\hat{r}_{b,L}}\left(\ln\frac{1}{r_{b,L}}\right)\cos\alpha$$

Since

$$\frac{\partial}{\partial n}(\ln u) = \frac{1}{u}\frac{\partial u}{\partial n}$$

then

$$\frac{\partial}{\partial n}\left(\ln\frac{1}{r_{b,L}}\right) = -\frac{1}{r_{b,L}}\cos\alpha = \frac{-d_{b,L}}{r_{b,L}^2} \qquad \text{(A-1)}$$



Therefore,

$$H_{b,L} = -\frac{1}{2\pi}\int_{S_L} \frac{d_{b,L}}{r_{b,L}^2}\, dS_L \tag{A-2}$$

where $d_{b,L}$ is the perpendicular distance from a point (x_b, y_b) to the line $Ax + By + C = 0$ and is given as:

$$d_{b,L} = \frac{Ax_b + By_b + C}{\pm\sqrt{A^2 + B^2}} \tag{A-3}$$

where the appropriate sign is chosen so as to make $d_{b,L}$ always positive.

The equation of the line joining the points (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{y_2 - y_1}{x_2 - x_1}\right)x - y + \left[y_1 - x_1\left(\frac{y_2 - y_1}{x_2 - x_1}\right)\right] = 0$$

Therefore,

$$A = \frac{y_2 - y_1}{x_2 - x_1}$$

$$B = -1$$

$$C = y_1 - x_1\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$d_{b,L} = \frac{\left(\frac{y_2 - y_1}{x_2 - x_1}\right)x_b - y_b + y_1 - x_1\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}{\pm\sqrt{\left(\frac{y_2 - y_1}{x_2 - x_1}\right)^2 + 1}} \tag{A-4}$$

In order that $d_{b,L}$ is always positive, the following convention is used:

If the numerator, $\left(Ax_b + By_b + C\right) < 0$, then use $-\sqrt{A^2 + B^2}$ as the denominator.

If the numerator, $\left(Ax_b + By_b + C\right) > 0$, then use $+\sqrt{A^2 + B^2}$ as the denominator.

$G_{b,L}$ is defined as

$$\frac{1}{2\pi} \int_{S_L} \ln \frac{1}{r_{b,L}} \, dS_L \qquad \text{(A-5)}$$

$\{H'_x\}_{b,L}$ is defined as

$$\int_{S_L} \frac{\partial}{\partial x}\left[\frac{\partial}{\partial n}\left(\ln \frac{1}{r_{b,L}}\right)\right] dS_L$$

which is evaluated as follows:

$$\frac{\partial}{\partial x}\left[\frac{\partial}{\partial n}\left(\ln \frac{1}{r_{b,L}}\right)\right] = \frac{\partial}{\partial x}\left(\frac{d_{b,L}}{r_{b,L}^2}\right) = -\left[d_{b,L} \frac{\partial}{\partial x}\left(\frac{1}{r_{b,L}^2}\right) + \frac{1}{r_{b,L}^2} \frac{\partial}{\partial x}(d_{b,L})\right]$$

$$= \left[-d_{b,L} \frac{2(x_b - x_L)}{r_{b,L}^4} + \frac{A}{r_{b,L}^2 \pm \sqrt{A^2 + 1}}\right]$$

Therefore,

$$\{H'_x\}_{b,L} = \int_{S_L} \left[\frac{-d_{b,L} \cdot 2(x_b - x_L)}{r_{b,L}^4} + \frac{A}{r_{b,L}^2 \pm \sqrt{A^2 + 1}}\right] dS_L \qquad \text{(A-6)}$$

Similarly,

$$\{H'_y\}_{b,L} = \int_{S_L} \left[\frac{-d_{b,L}\cdot 2(y_b - y_L)}{r^4_{b,L}} + \frac{A}{r^2_{b,L} \pm \sqrt{A^2 + 1}}\right] dS_L \quad \text{(A-7)}$$

$\{G'_x\}_{b,L}$ is defined as

$$\int_{S_L} \frac{\partial}{\partial x}\left(\ln \frac{1}{r_{b,L}}\right) dS_L$$

and is given as follows:

$\frac{\partial}{\partial x}\left(\ln \frac{1}{r_{b,L}}\right)$ is given

$\frac{-(x_b - x_L)}{r^2_{b,L}}$.

$$\therefore \quad \{G'_x\}_{b,L} = -\int_{S_L} \frac{(x_b - x_L)}{r^2_{b,L}} dS_L \qquad \text{(A-8)}$$

Similarly,

$$\{G'_y\}_{b,L} = -\int_{S_L} \frac{y_b - y_L}{r^2_{b,L}} dS_L \qquad \text{(A-9)}$$

The integrals in Equations (A-2) through (A-9) are evaluated by applying the quadrature formula given in Section IX.1 Equation (IX.1.1).

APPENDIX B

EVALUATION OF TERMS UNDER THE INTEGRAL SIGNS OF THE HEAT BALANCE

APPENDIX B

EVALUATION OF THE TERMS UNDER THE INTEGRAL SIGNS

The average saturation within any arbitrary region from x_a to x_b is:

$$\bar{S}(t) = \frac{1}{x_b - x_a} \int_{x_a}^{x_b} S(x,t)\, dx \qquad \text{(b-1)}$$

Since S(x,t) is continuous within each region behind the steam front but discontinuous at the interfaces between the regions, the average saturation in all regions behind

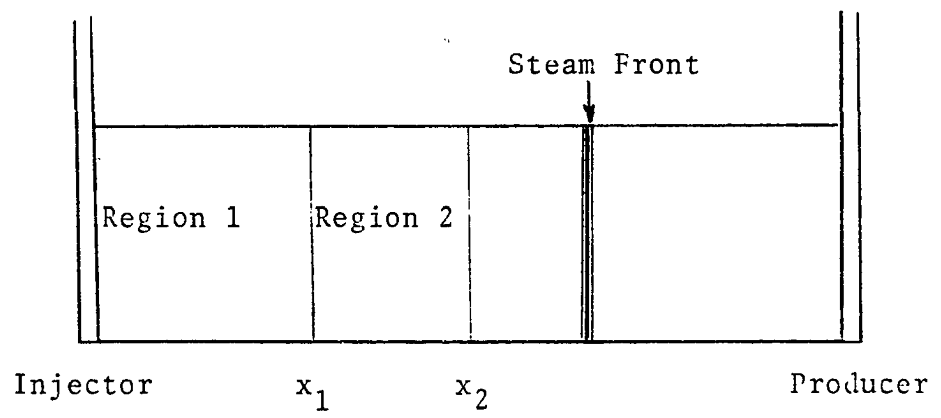


FIGURE B-1

the steam front can be obtained by applying Equation (B-1) to every region behind the front and taking the average of these averages.

Let

$$\bar{\bar{S}}(t) = \frac{\sum_{k=1}^{j} S_k(t)}{j} \tag{B-2}$$

Rewrite B-1 as

$$\bar{S}(t)(x_b - x_a) = \int_{x_a}^{x_b} S(x,t)\ dx$$

and differentiate both sides to obtain

$$\bar{S}(t)\ \frac{\partial}{\partial t}\ (x_b - x_a) + (x_b - x_a)\ \frac{\partial}{\partial t}\ \bar{S}(t) = \frac{\partial}{\partial t} \int_{x_a}^{x_b} S(x,t)\ dx \tag{B-3}$$

Apply Leibnitz's theorem to the RHS of Equation (B-3) to obtain

$$\frac{\partial}{\partial t} \int_{x_a}^{x_b} S(x,t)\ dx = \int_{x_a}^{x_b} \frac{\partial}{\partial t}\ S(x,t)\ dx + S(x_b,t)\ \frac{\partial x_b}{\partial t} - S(x_a,t)\ \frac{\partial x_a}{\partial t} \tag{B-4}$$

Equation B-4 is substituted into B-4 to give

$$\bar{S}(t)\ \frac{\partial}{\partial t}\ (x_b - x_a) + (x_b - x_a)\ \frac{\partial}{\partial t}\ \bar{S}(t) = \int_{x_a}^{x_b} \frac{\partial}{\partial t}\ S(x,t)\ dx + S(x_b,t)\ \frac{\partial x_b}{\partial t} - S(x_a,t)\ \frac{\partial x_a}{\partial t} \tag{B-5}$$

Now if both x_b and x_a are constants, Equation B-5 reduces to:

$$(x_b - x_a) \frac{\partial}{\partial t} \overline{S}(t) = \int_{x_a}^{x_b} \frac{\partial}{\partial t} S(x,t)\, dx \qquad \text{(B-6)}$$

For all regions behind the steam front except the region containing the steam front, the limits x_a and x_b are constants. Therefore, Equation B-6 is applied to as many such regions. For the region containing the condensation front, the limit x_a is a constant, but x_b is the location of the condensation front $\xi(t)$ which is a function of time. Therefore, Equation B-5 becomes:

$$\overline{S}(t)V(t) + [\xi(t) - x_a] \frac{\partial}{\partial t} \overline{S}(t) - S(\xi,t)V(t)$$

$$= \int_{x_a}^{\xi(t)} \frac{\partial}{\partial t} S(x,t)\, dx \qquad \text{(B-7)}$$

APPENDIX C

TEMPERATURE DISTRIBUTION IN THE CAP AND BASE ROCK

APPENDIX C

TEMPERATURE DISTRIBUTION IN THE CAP AND BASE ROCK

A general heat balance on any medium is (Van Poolen, Spillete):

$$\frac{\partial}{\partial t}(MT) + \nabla\cdot(\rho_f c_f T)V = \nabla\cdot\hat{k}\nabla T \qquad \text{(C-1)}$$

For the cap or base rock, there are no fluids,

$$\therefore\ M = \rho_{cb}c_{cb}$$

$$\rho_f c_f = \rho_o c_o + \rho_w c_w$$

for an oil water system

$$V = 0 = \text{velocity of the fluids}$$

Thus, Equation (C-1) becomes:

$$\rho_{cb}c_{cb}\frac{\partial T}{\partial t} = \nabla\cdot\hat{k}\nabla T \qquad \text{(C-2)}$$

For a one-dimensional medium in the z-direction

$$\frac{\partial T}{\partial t} = \frac{\hat{k}}{\rho_{cb}c_{cb}}\frac{\partial^2 T}{\partial z^2} \qquad \text{(C-3)}$$

Defining $\alpha = \frac{k}{\rho_{cb}c_{cb}}$, then

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{C-4}$$

The equation describing the temperature distribution in the overburden or underburden is:

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{C-5}$$

Subject to:

$$T(z,0) = T_i \qquad \text{when } t = 0 \tag{C-6}$$

$$T(0,t) = T_{st} \qquad \text{at } z = 0,\ t > 0 \tag{C-7}$$

$$T(\infty,t) = T_i \qquad \text{at } z = \infty,\ t > 0 \tag{C-8}$$

Define $\theta(z,t)$ as

$$\frac{T(z,t) - T_i}{T_{st} - T_i}$$

Thus, the differential equation becomes:

$$\frac{\partial^2 \theta}{\partial z^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \tag{C-9}$$

subject to:

$$\theta(z,0) = 0 \tag{C-10}$$

$$\theta(0,t) = 1 \tag{C-11}$$

$$\theta(\infty,t) = 0 \tag{C-12}$$

The solution to Equation (C-4) with its boundary conditions (Equations C-5 to C-6) are obtainable from standard texts such as Carslaw and Jaeger.[58] It is given as:

$$\theta(z,t) = \text{erfc}\left(\frac{z}{\sqrt{4\alpha t}}\right) \tag{C-13}$$

Differentiating (C-13) with respect to z gives

$$\frac{d}{dz}\,\theta(z,t) = \frac{d}{dz}\left[\text{erfc}\left(\frac{z}{\sqrt{4\alpha t}}\right)\right]$$

$$= \frac{d}{dz}\left[-\frac{2}{\sqrt{\pi}}\int_0^{\frac{z}{\sqrt{4\alpha t}}} e^{-t^2}\,dt\right] = -\frac{2}{\sqrt{\pi}}\left[-e^{-\left(\frac{z}{\sqrt{4\alpha t}}\right)^2}\frac{1}{\sqrt{4\alpha t}}\right]$$

$$\frac{d}{dz}\,\theta(z,t) = -\frac{1}{\sqrt{\pi\alpha t}}\,e^{-\left(\frac{z}{\sqrt{4\alpha t}}\right)^2} \tag{C-14}$$

Since

$$\frac{dT(z,t)}{dz} = (T_{st} - T_i)\,\frac{d\theta(z,t)}{dz}$$

$$\left.\frac{dT(z,t)}{dz}\right|_{z=0} = -\frac{T_{st} - T_i}{\sqrt{\pi\alpha t}} \tag{C-15}$$

For an area that received heat initially at time τ,

$$\left.\frac{dT(z,t)}{dz}\right|_{z=0} = -\frac{T_{st} - T_i}{\sqrt{\pi\alpha(t-\tau)}} \tag{C-16}$$

APPENDIX D

LAPLACE TRANSFORM OF A SECTIONALLY CONTINUOUS FUNCTION

APPENDIX D

LAPLACE TRANSFORM OF A SECTIONALLY CONTINUOUS FUNCTION

For a function $\xi_D(t_D)$ which is sectionally continuous and continuously differentiable in the domain $t > 0$:

$$\xi_D(\tau_D) = \sum_{j=1}^{N} \xi_{D_j}(\tau_D), \quad t_{D_{j-1}} < \tau_D < t_{D_j}$$

where the functions $\xi_{D_j}(\tau_D)$ are continuous in the domain $t_{D_{j-1}} < \tau_D < t_{D_j}$. At the points t_j for $j = 1,\ldots,N$, $\xi_D(\tau_D)$ has finite steps. Incorporating the Heaviside unit step function,

$$\xi_D(\tau_D) = \sum_{j=1}^{N} \{H(t_D - t_{D_{j-1}}) - H(t_D - t_{D_j})\}\xi_{D_j}(\tau_D)$$

The generalized derivative of this function is:

$$V_D(\tau_D) = \frac{d\xi_D(\tau_D)}{d\tau_D} = \sum_{j=1}^{N} \{[H(t_D - t_{D_{j-1}}) - H(t_D - t_{D_j})]$$

$$\frac{d\xi_{D_j}(\tau_D)}{d\tau_D} + [\delta(t_D - t_{D_{j-1}}) - \delta(t_D - t_{D_j})]\ \xi_{D_j}(\tau_D)\}$$

$$= \sum_{j=1}^{N} \{[H(t_D - t_{D_{j-1}}) - H(t_D - t_{D_j})]V_{D_j}(\tau_D)$$

$$+ \xi_{D_j}(\tau_{D_{j-1}}) - \xi_{D_j}(\tau_{D_j})\}$$

where H is the Heaviside unit step function

δ is the Dirac delta function.

The Laplace transform of any function $v_D(t_D)$ is defined as

$$\mathcal{L}\{v_D(t_D)\} = \int_0^\infty v_D(t_D)\; e^{-st_D}\, dt_D$$

that is,

$$\mathcal{L}\{V_D(\tau_D)\} = \sum_{j=1}^{N}\left\{\left[\int_{t_{D_{j-1}}}^{t_{D_j}} V_{D_j}(\tau_D)\; e^{-s\tau_D}\, d\tau_D\right] + \xi_{D_j}(\tau_{D_{j-1}})\; e^{-s\tau_{D_{j-1}}} - \xi_{D_j}(\tau_{D_j})\; e^{-s\tau_{D_j}}\right\}$$

$$- \xi_o(0)$$

Integrate by parts. Let

$$u = e^{-s\tau_D}$$

$$dv = V_{D_j}(\tau_D)$$

$$\mathcal{L}\{V_D(\tau_D)\} = \sum_{j=1}^{N}\left[e^{-s\tau_D}\xi_{D_j}(\tau_D)\Big|_{t_{D_{j-1}}}^{t_{D_j}} + s\int_{t_{D_{j-1}}}^{t_{D_j}} \xi_{D_j}(\tau_D)\right.$$

$$\left.\times e^{-s\tau_D}\, d\tau_D + \xi_{D_j}(\tau_{D_{j-1}})e^{-s\tau_{D_{j-1}}} - \xi_{D_j}(\tau_{D_j})e^{-s\tau_{D_j}}\right] - \xi_o(0)$$

$$= \sum_{j=1}^{N}\left[e^{-s\tau_{D_j}}\xi_{D_j}(\tau_{D_j}) - e^{-s\tau_{D_{j-1}}}\xi_{D_j}(\tau_{D_{j-1}}) + \xi_{D_j}(\tau_{D_{j-1}})\right.$$

$$\left.\times e^{-s\tau_{D_{j-1}}} - \xi_{D_j}(\tau_{D_j})e^{-s\tau_{D_j}} + sZ(s)\right] - \xi_o(0)$$

Since

$$\sum_{j=1}^{N} \int_{t_{D_{j-1}}}^{t_{D_j}} \xi_{D_j}(\tau_D)\ e^{-s\tau_D}\ d\tau_D = Z_D(s) = \mathcal{L}\{\xi_{D_j}(\tau_D)\}$$

Thus,

$$\mathcal{L}\{V_D(\tau_D)\} = sZ_D(s) - \xi_o(0)$$

where

$$Z_D(s) = \sum_{j=1}^{N} \int_{t_{D_{j-1}}}^{t_{D_j}} \xi_{D_j}(\tau_D)\ e^{-s\tau_D}\ d\tau_D$$

APPENDIX E

COMPUTER IMPLEMENTATION OF BOUNDARY ELEMENT METHOD OF STREAMLINE SIMULATION

APPENDIX E

COMPUTER IMPLEMENTATION OF BOUNDARY ELEMENT METHOD OF STREAMLINE SIMULATION

A package of computer programs has been written in FORTRAN to perform all the calculations necessary to generate the streamlines and calculate the dimensions of the streamtubes using the Boundary Element Method. The program can handle:

1. Single homogeneous porous medium with or without sources and sinks.

2. A two-region piecewise homogeneous porous medium with or without sources and sinks.

The boundary conditions can be either:

1. sealed
2. constant pressure
3. combination of (1) and (2)

and consists of a main program that calls the following 15 subroutines.

1. INPUT
2. CONVRS
3. MATRIX
4. INTE

5. INLO
6. SOURCE
7. LOWVEL
8. ASEMBL
9. SLNPD
10. SPLIT
11. INTER
12. OUTPUT
13. BDRY
14. STRM
15. COMPAT

Subroutines INPUT, INTE, SLNPD, INTER and INLO are modifications of the subroutines INPUT, INTE, SLNPD, INTER, and INLO published in reference 12.

Subroutine INPUT reads all the input data required, such as the coordinates of the ends of the boundary segments, the coordinates and strengths of the sources and sinks, the boundary conditions, etc.

Subroutine MATRIX performs the integrations to determine the values of $H_{b,L}$ and $G_{b,}L$ which make up the elements of the [H] and [G] matrices. It does this by calling two other subroutines, INTE and INLO. Subroutine INTE calculates the values $\check{H}_{b,L}$ and $\check{G}_{n,L}$ (that is, $H_{b,L}$ and $G_{b,L}$ when $L \neq b$), while subroutine INLO calculates $H_{b,b}$ and $G_{b,b}$. $\check{H}_{b,L}$ and $\check{G}_{b,L}$ form the off-diagonal elements of the [H] and [G] matrices while $H_{b,b}$ and $G_{b,b}$ form the diagonal elements.

Both subroutines are modified from reference 12. Subroutine MATRIX also calls the subroutine SOURCE to calculate the contributions of the sources and sinks, and rearranges the [H] and [G] matrices according to the prescribed boundary conditions such that all the elements of [H] and [G] corresponding to unknown boundary conditions are put in the matrix [G]. Finally, it calculates the $[H_x']$, $[G_x']$, $[H_y']$ and $[G_y']$ matrices required to calculate the velocities in the x and y directions at interior points.

Subroutine ASEMBL is called for a piecewise homogeneous reservoir having two regions. It arranges the new [G] matrices for both regions (obtained from routine MATRIX) into a single matrix [A]. The arrangement is such that continuity and compatibility conditions at the common interface between the two regions are accounted for.

Subroutine SLNPD solves the resulting [A] matrix in double precision using a Gaussian elimination procedure that allows for row interchange in case of zero pivot elements. After solution, subroutine SPLIT divides the solution vector into the two vectors representing the boundary potentials and gradients for each region.

Subroutine INTER uses these vectors to determine the potential at internal points. It calls both subroutine INTE and subroutine INLO to supply the [H] and [G] matrices needed.

Subroutine OUTPUT is called to give a printed output of the calculated values.

Subroutine BDRY plots the boundaries of the reservoir system.

Subroutine STRM generates the streamlines and plots them. It also calculates the lengths and widths of the streamtubes. It calls the following subroutines: (a) subroutine LOWVEL to calculate the veloctiy near the lowest producer; (b) subroutine INTE and subroutine INLO. The steps of operations used by subroutine STRM can be summarized as follows:

a. Define a capture radius (say 20 times the wellbore radius) and calculate the velocity at this radius for the lowest producer.

b. For each injector, calculate the number of streamlines emanating from it. The starting points for each streamline are spaced evenly on the wellbore radius.

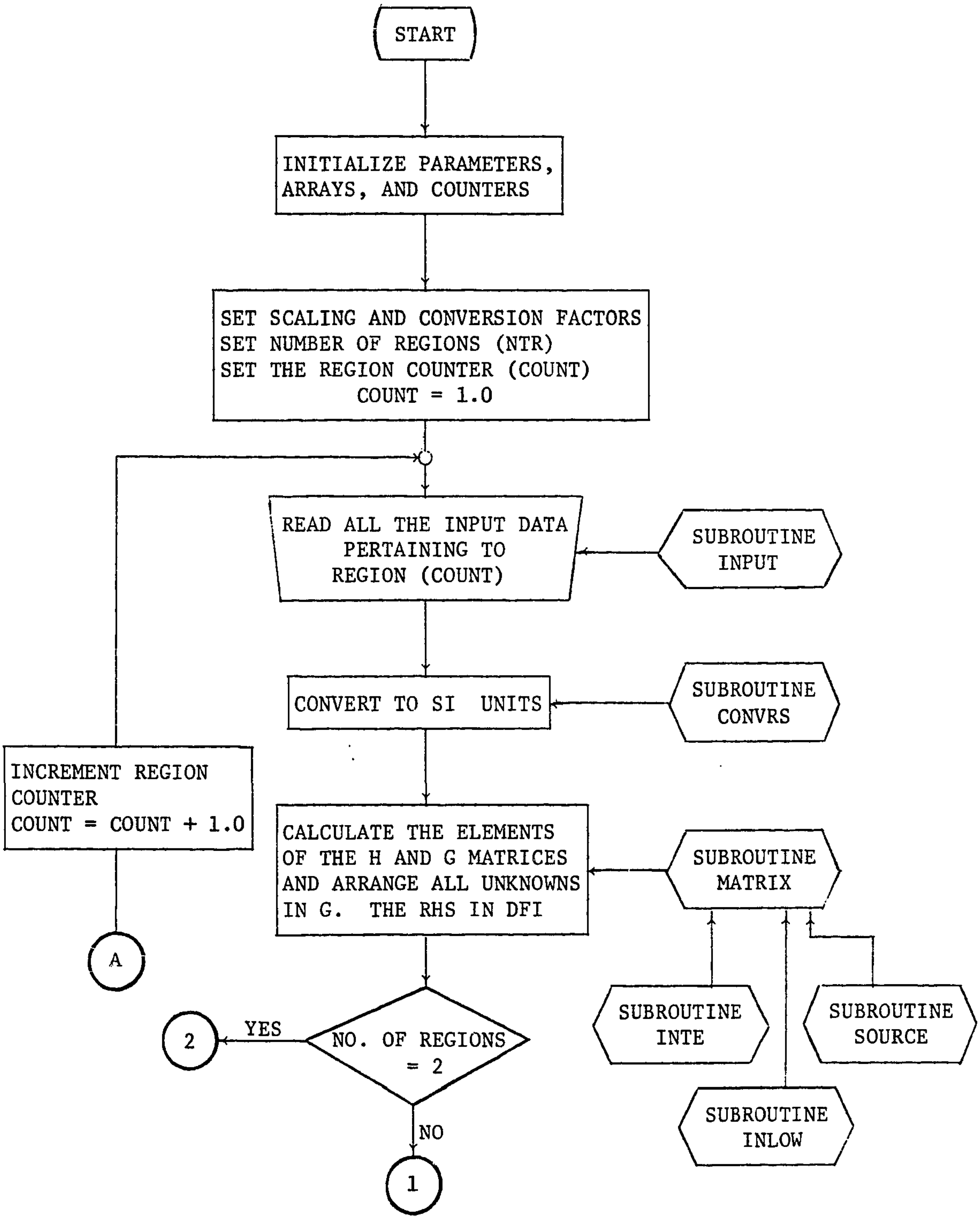
c. Calculate the velocity vectors and the next position. Check to see if the new position is less than a half element length away from any boundary. If the boundary is not an interface, stop the streamline. If it is an interface, calculate the point where the streamline is assumed to cross the interface. This point is obtained as the point of intersection of the normal from the last streamline point to the nearest boundary segment. If the point is not near any boundary, the length and width of the streamline at this point are calculated. The point is plotted and the next position calculated and step (c) repeated for all streamlines and all wells.

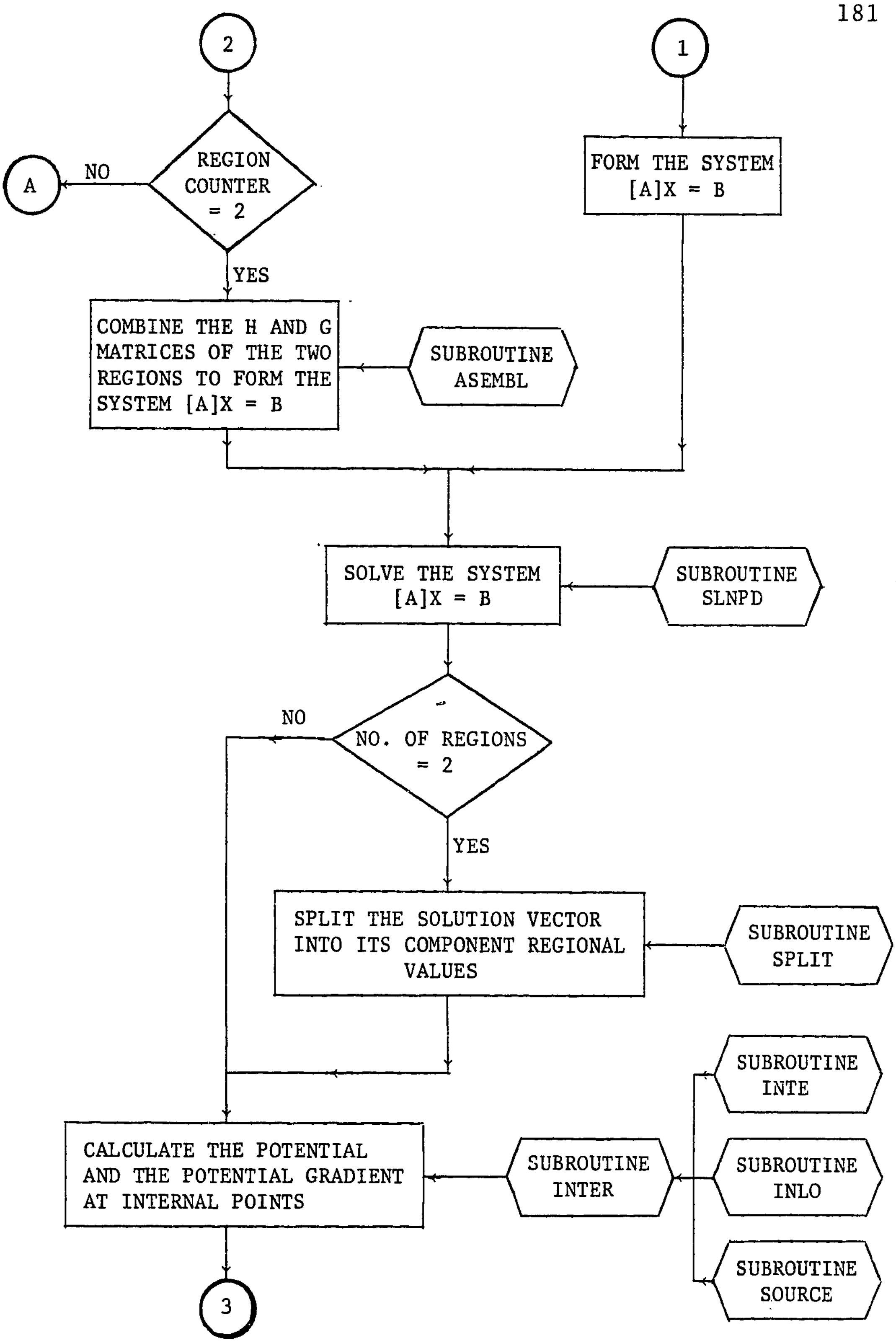
Subroutine COMPAT continues the generation of the streamlines after they cross into adjacent regions. The procedure followed by subroutine COMPAT is similar to that of subroutine STRM except that the starting point is on the interface boundary. The next streamline position away from the interface boundary was calculated using the procedure outlined in Section VIII.4.

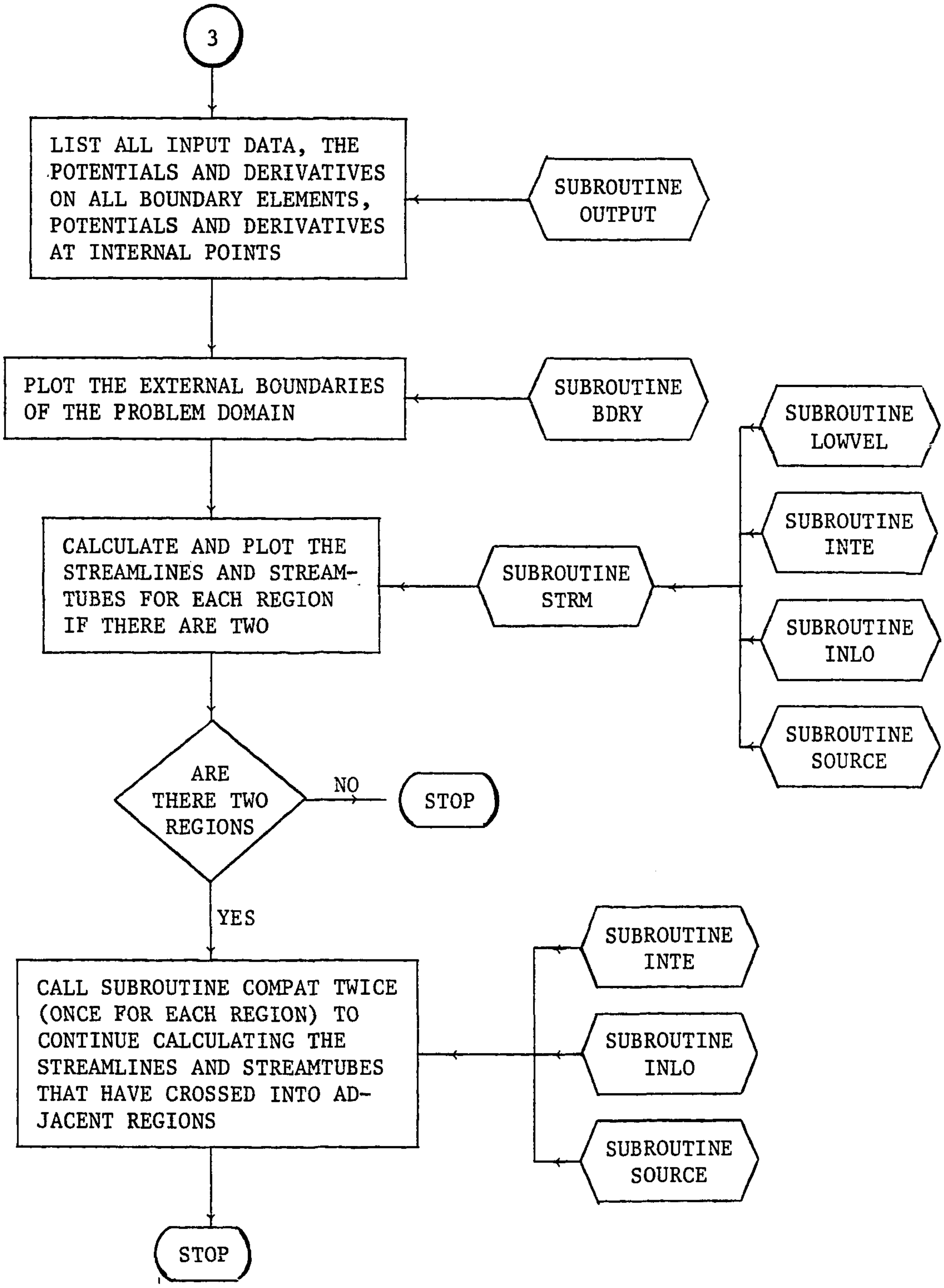
In what follows, a listing of the flow charts for all the important subroutines is given.

FLOW CHART FOR MAIN PROGRAM OF ELEMENT

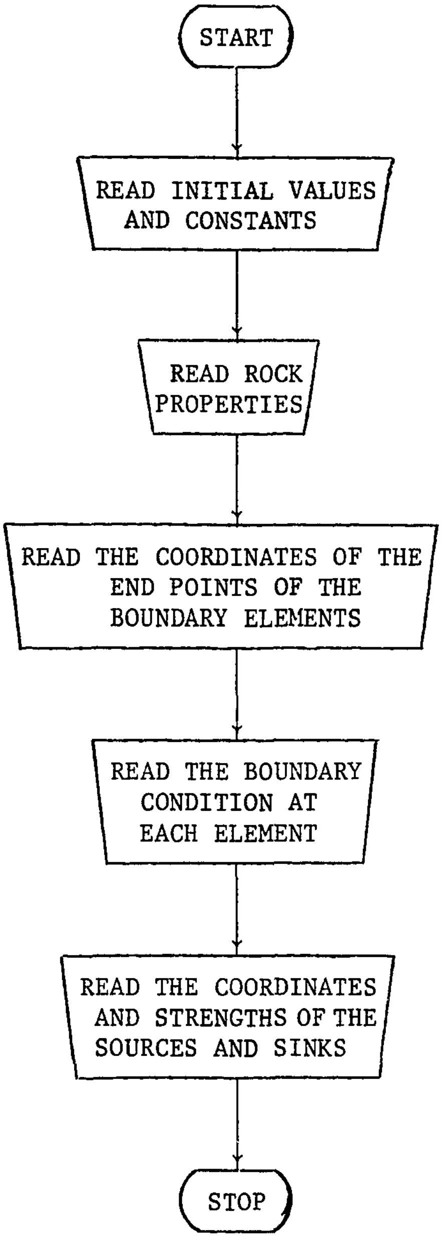
METHOD OF STREAMLINE SIMULATION



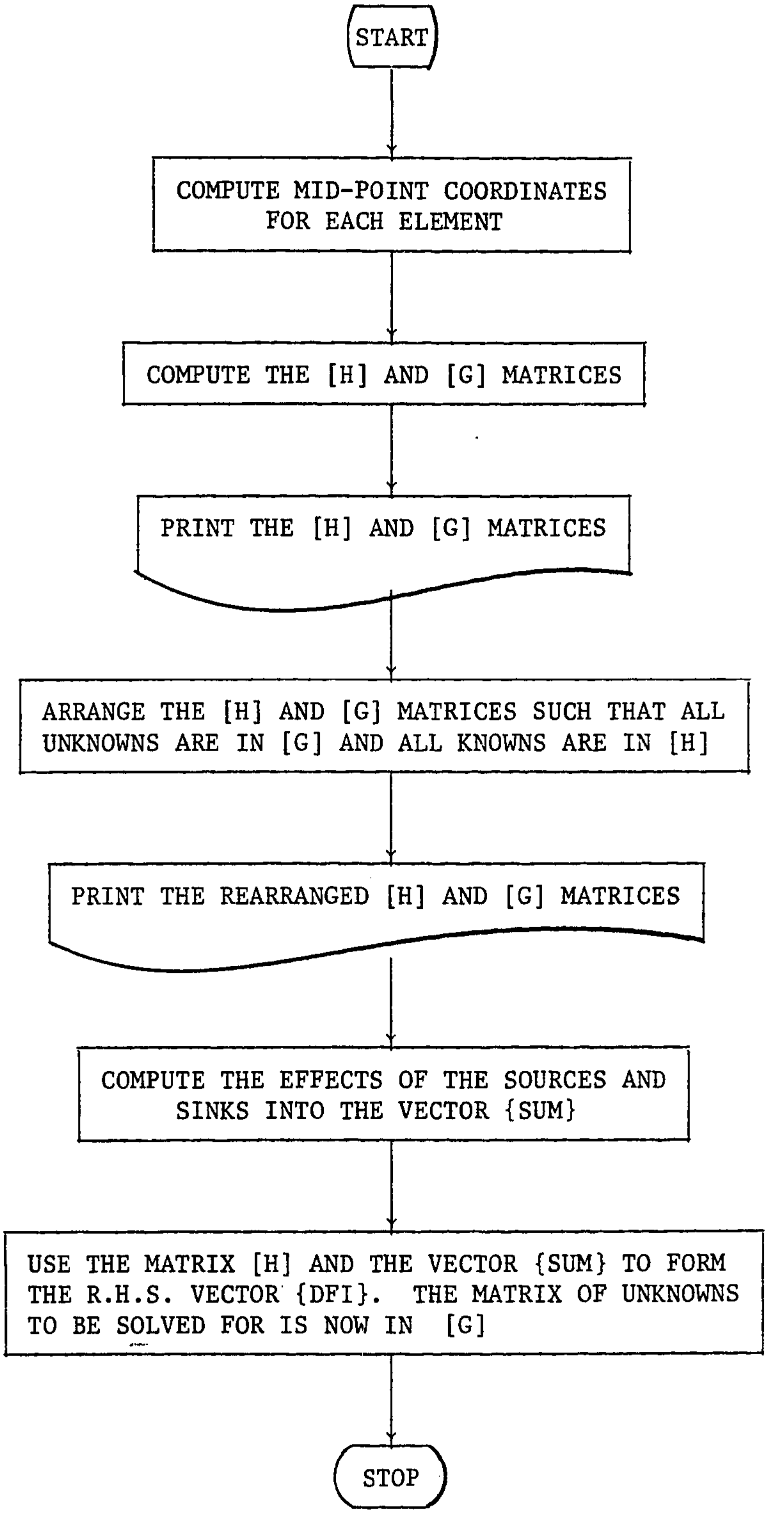

2
1
REGION COUNTER = 2
NO
A
YES
FORM THE SYSTEM [A]X = B
COMBINE THE H AND G MATRICES OF THE TWO REGIONS TO FORM THE SYSTEM [A]X = B
SUBROUTINE ASEMBL
SOLVE THE SYSTEM [A]X = B
SUBROUTINE SLNPD
NO. OF REGIONS = 2
NO
YES
SPLIT THE SOLUTION VECTOR INTO ITS COMPONENT REGIONAL VALUES
SUBROUTINE SPLIT
SUBROUTINE INTE
CALCULATE THE POTENTIAL AND THE POTENTIAL GRADIENT AT INTERNAL POINTS
SUBROUTINE INTER
SUBROUTINE INLO
SUBROUTINE SOURCE
3


3
LIST ALL INPUT DATA, THE POTENTIALS AND DERIVATIVES ON ALL BOUNDARY ELEMENTS, POTENTIALS AND DERIVATIVES AT INTERNAL POINTS
SUBROUTINE OUTPUT
PLOT THE EXTERNAL BOUNDARIES OF THE PROBLEM DOMAIN
SUBROUTINE BDRY
SUBROUTINE LOWVEL
SUBROUTINE INTE
CALCULATE AND PLOT THE STREAMLINES AND STREAM-TUBES FOR EACH REGION IF THERE ARE TWO
SUBROUTINE STRM
SUBROUTINE INLO
SUBROUTINE SOURCE
ARE THERE TWO REGIONS
NO
STOP
YES
SUBROUTINE INTE
CALL SUBROUTINE COMPAT TWICE (ONCE FOR EACH REGION) TO CONTINUE CALCULATING THE STREAMLINES AND STREAMTUBES THAT HAVE CROSSED INTO AD-JACENT REGIONS
SUBROUTINE INLO
SUBROUTINE SOURCE
STOP

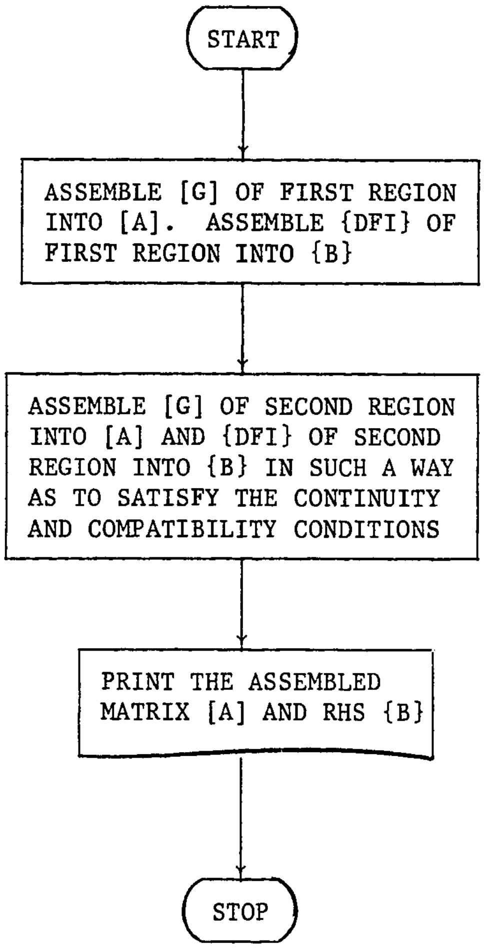
FLOW CHART FOR SUBROUTINE INPUT



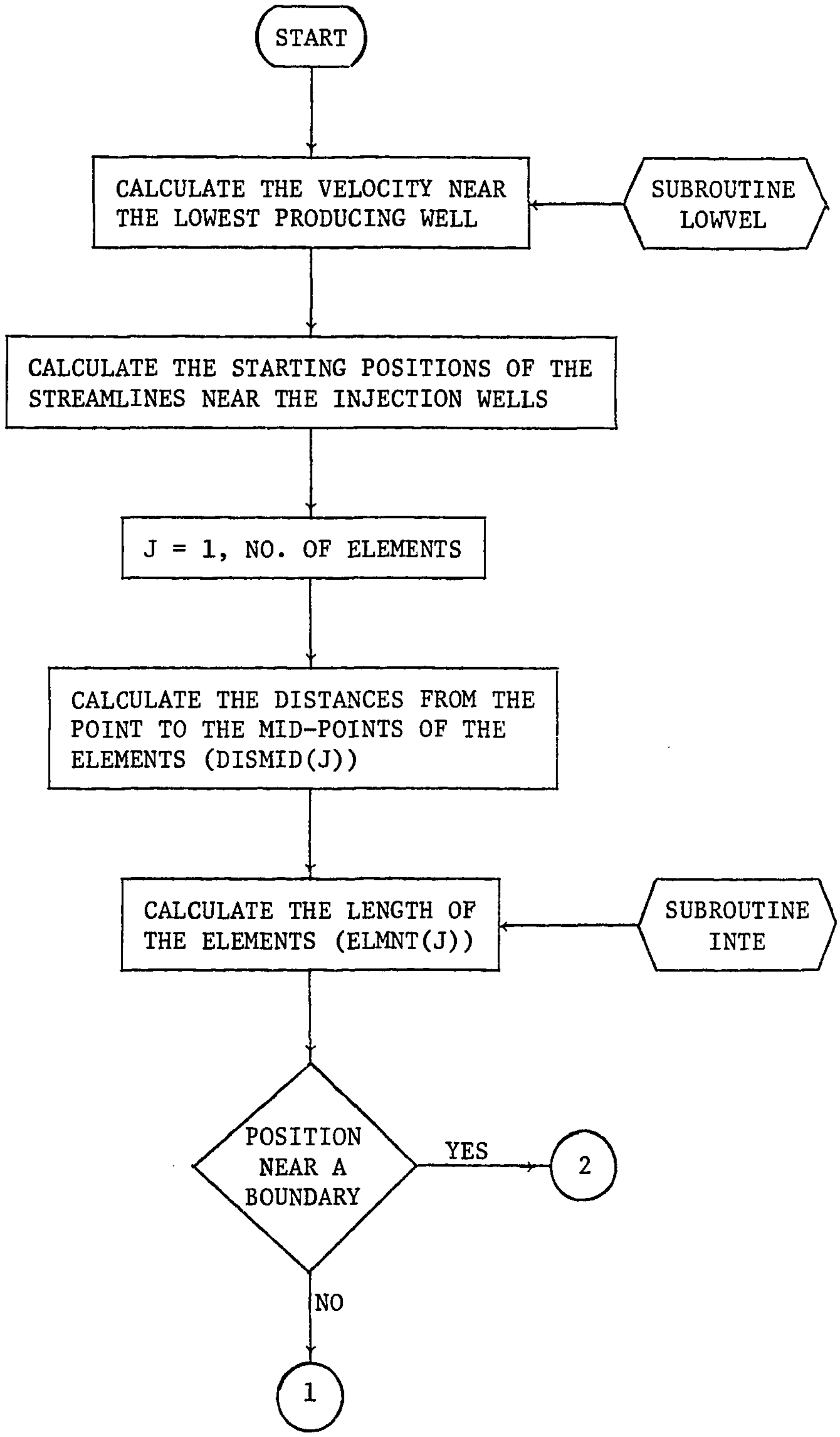
FLOW CHART FOR SUBROUTINE MATRIX

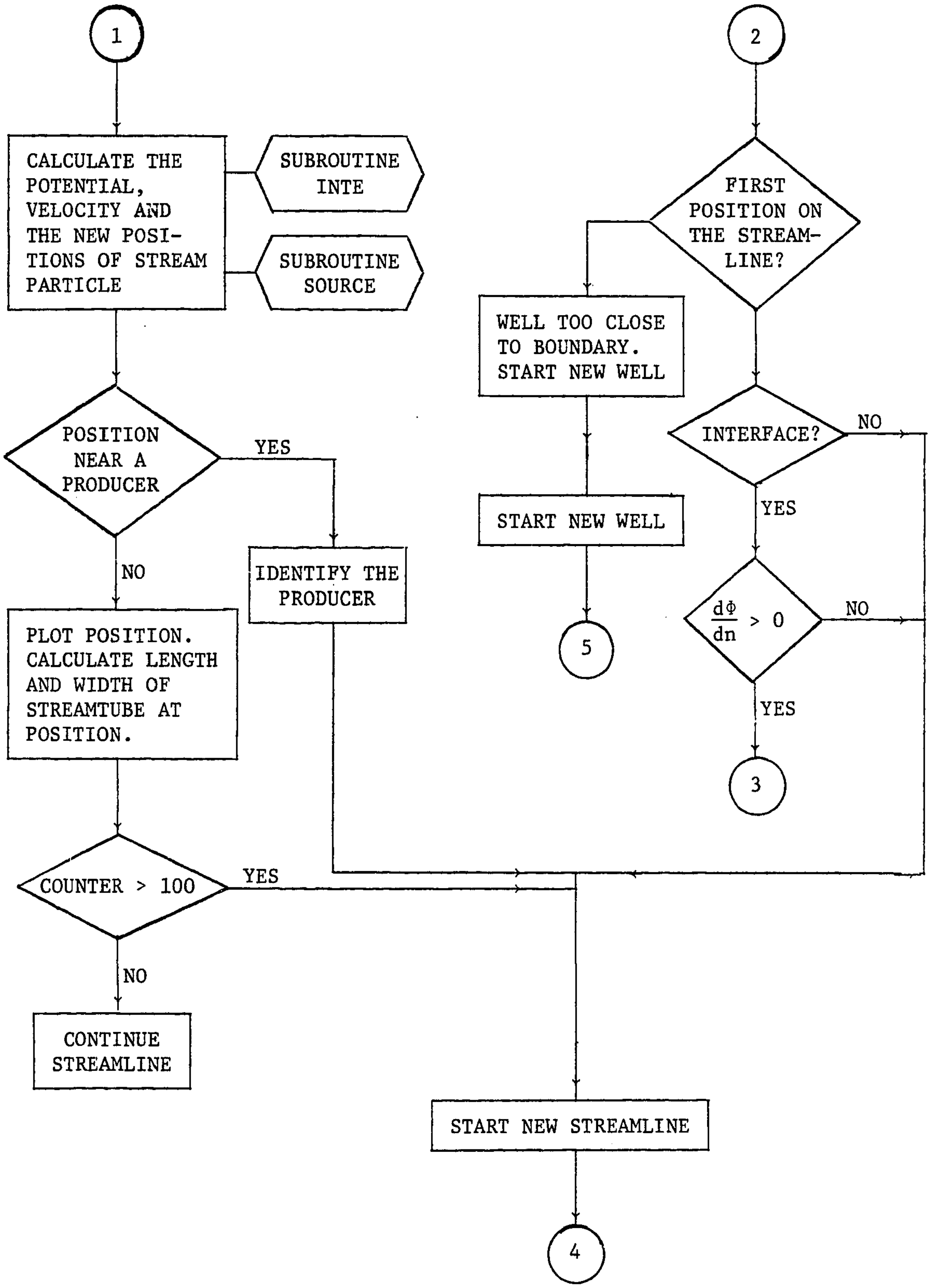


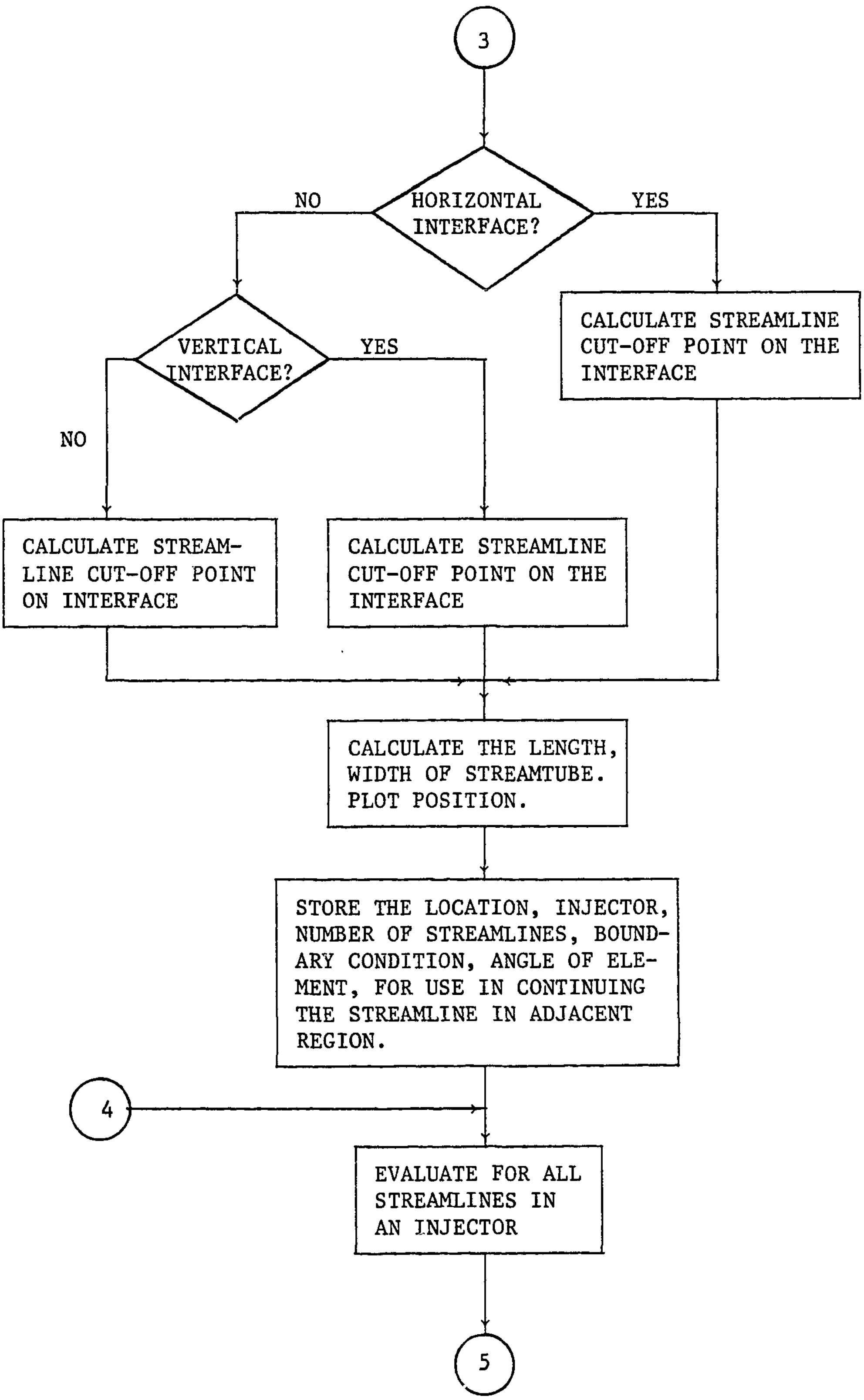
FLOW CHART FOR SUBROUTINE ASEMBL

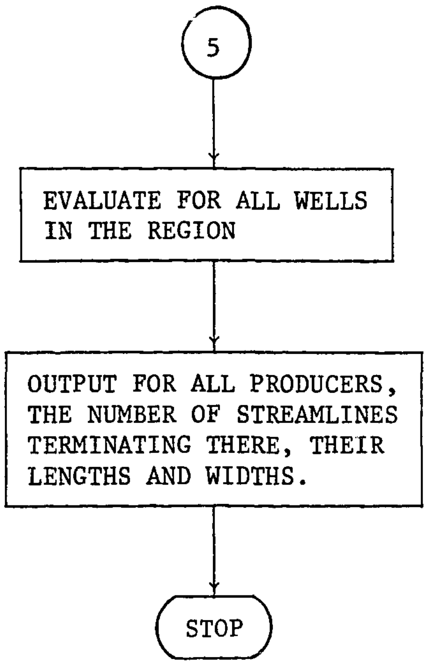


FLOW CHART FOR SUBROUTINE STRM

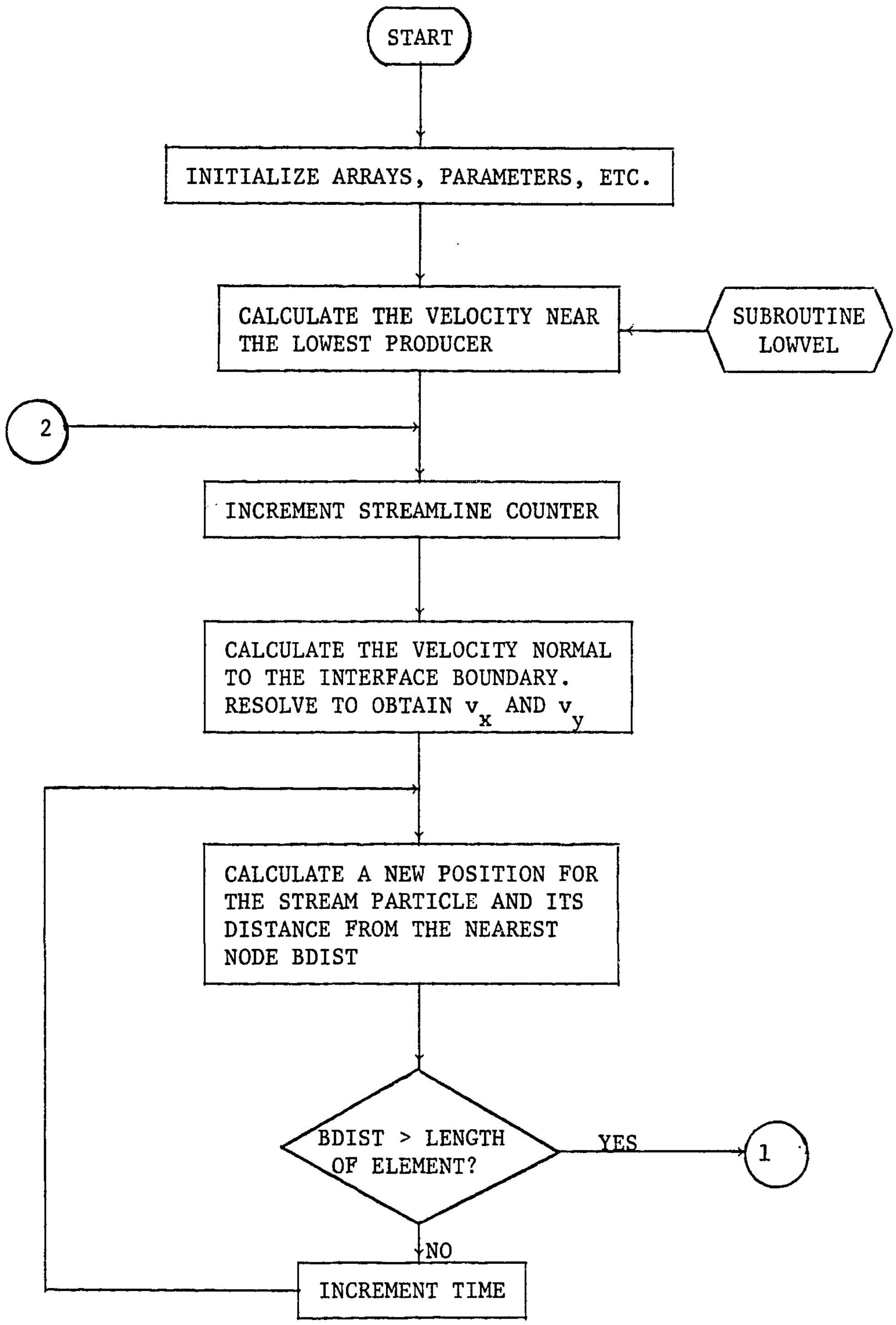


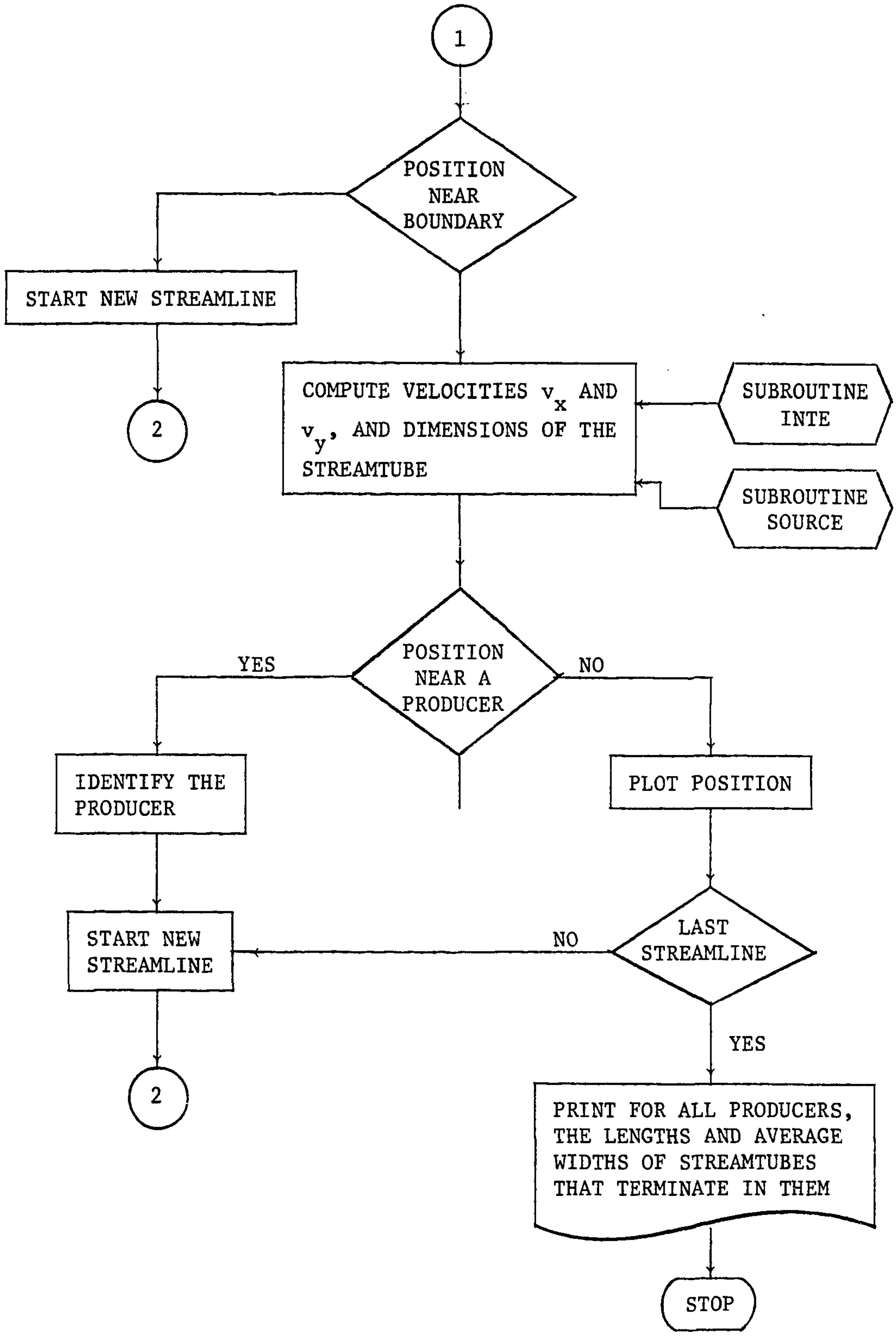

1
CALCULATE THE POTENTIAL, VELOCITY AND THE NEW POSITIONS OF STREAM PARTICLE
SUBROUTINE INTE
SUBROUTINE SOURCE
POSITION NEAR A PRODUCER
YES
IDENTIFY THE PRODUCER
NO
PLOT POSITION. CALCULATE LENGTH AND WIDTH OF STREAMTUBE AT POSITION.
COUNTER > 100
YES
NO
CONTINUE STREAMLINE
2
FIRST POSITION ON THE STREAMLINE?
WELL TOO CLOSE TO BOUNDARY. START NEW WELL
START NEW WELL
5
INTERFACE?
NO
YES
$\frac{d\Phi}{dn} > 0$
NO
YES
3
START NEW STREAMLINE
4


3
HORIZONTAL INTERFACE?
NO
YES
VERTICAL INTERFACE?
YES
NO
CALCULATE STREAMLINE CUT-OFF POINT ON THE INTERFACE
CALCULATE STREAM-LINE CUT-OFF POINT ON INTERFACE
CALCULATE STREAMLINE CUT-OFF POINT ON THE INTERFACE
CALCULATE THE LENGTH, WIDTH OF STREAMTUBE. PLOT POSITION.
STORE THE LOCATION, INJECTOR, NUMBER OF STREAMLINES, BOUNDARY CONDITION, ANGLE OF ELEMENT, FOR USE IN CONTINUING THE STREAMLINE IN ADJACENT REGION.
4
EVALUATE FOR ALL STREAMLINES IN AN INJECTOR
5


5
EVALUATE FOR ALL WELLS IN THE REGION
OUTPUT FOR ALL PRODUCERS, THE NUMBER OF STREAMLINES TERMINATING THERE, THEIR LENGTHS AND WIDTHS.
STOP

FLOW CHART FOR SUBROUTINE COMPAT




1
POSITION NEAR BOUNDARY
START NEW STREAMLINE
2
COMPUTE VELOCITIES v_x AND v_y, AND DIMENSIONS OF THE STREAMTUBE
SUBROUTINE INTE
SUBROUTINE SOURCE
POSITION NEAR A PRODUCER
YES
NO
IDENTIFY THE PRODUCER
PLOT POSITION
START NEW STREAMLINE
NO
LAST STREAMLINE
YES
2
PRINT FOR ALL PRODUCERS, THE LENGTHS AND AVERAGE WIDTHS OF STREAMTUBES THAT TERMINATE IN THEM
STOP

APPENDIX F

COMPUTER IMPLEMENTATION OF STEAMFLOOD RECOVERY CALCULATION

APPENDIX F

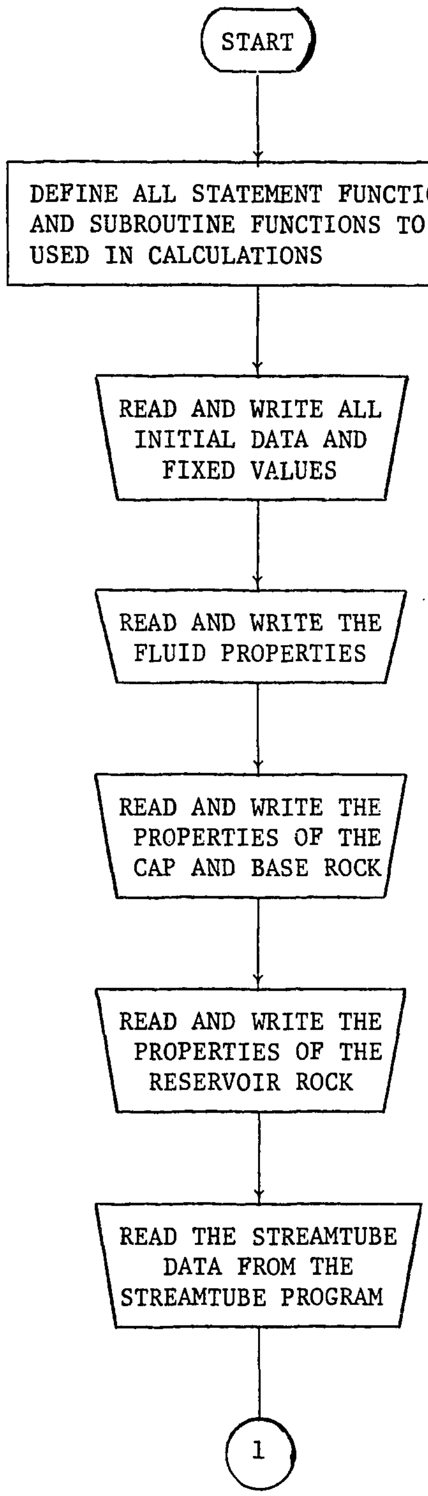
COMPUTER PROGRAM FOR STEAMFLOOD RECOVERY CALCULATION

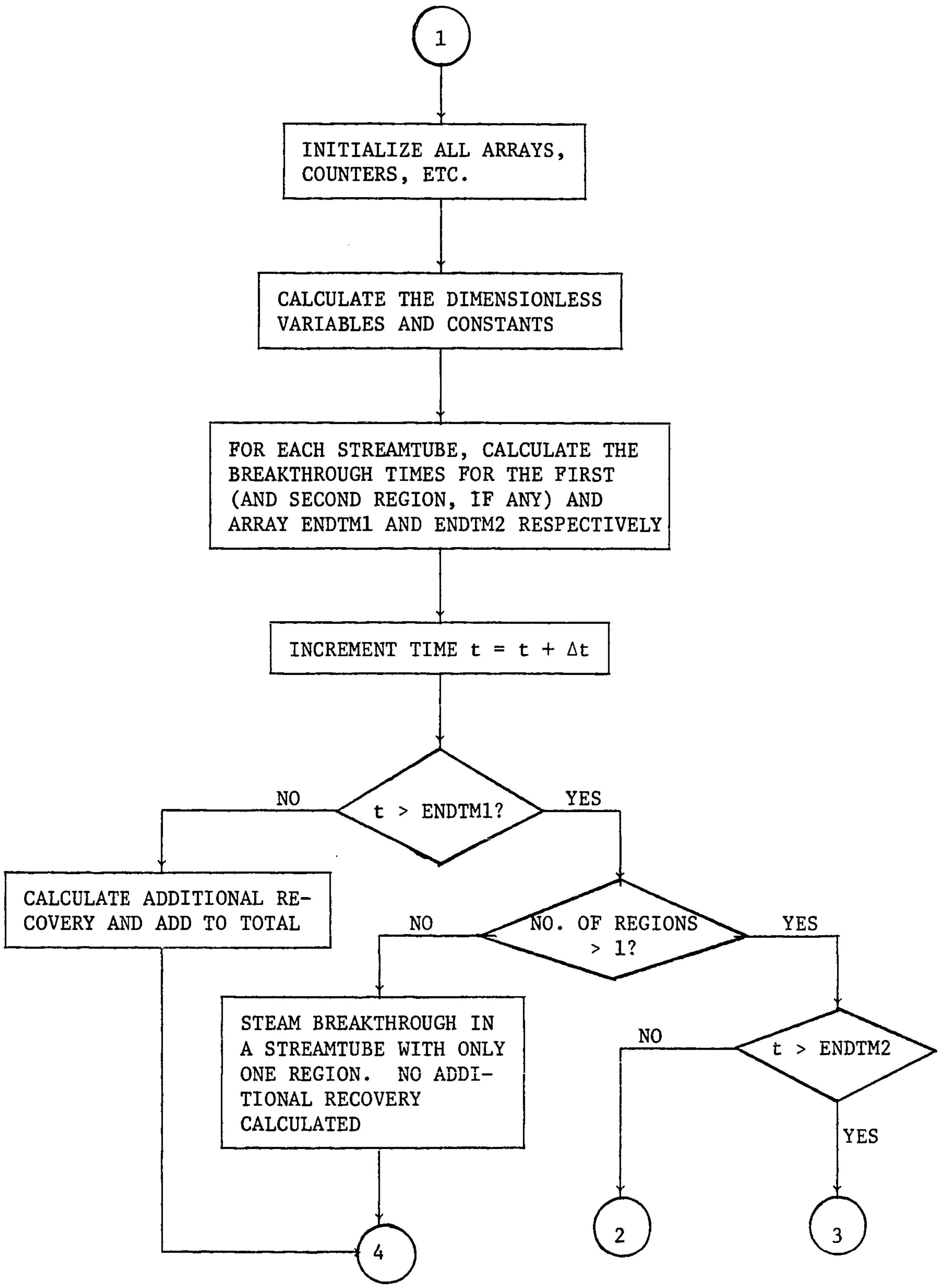
The steamflood recovery program requires as part of its input data, the dimensions of the streamtubes which are obtained from the output of the streamline/streamtube program. Briefly, it performs the following operations:

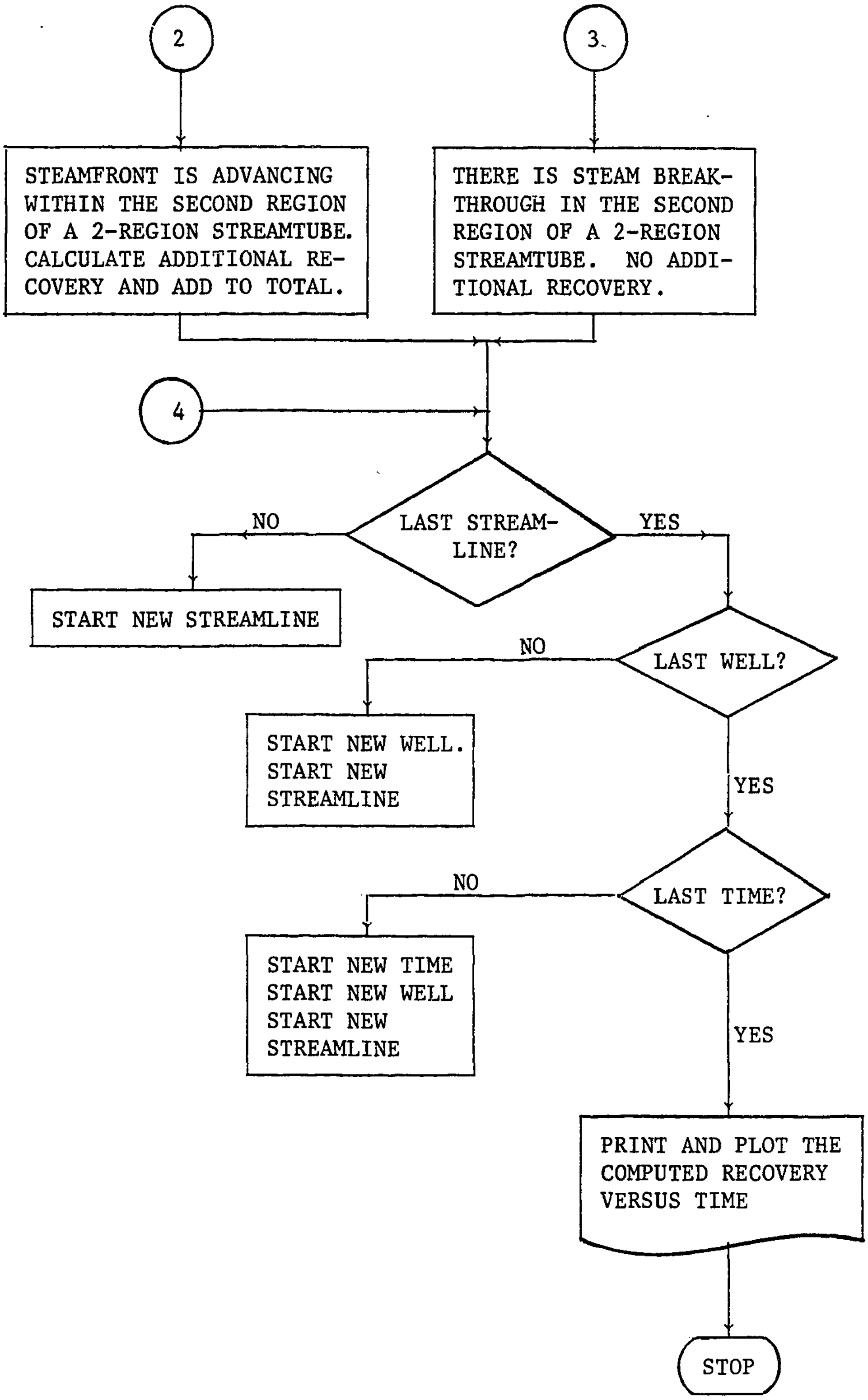
1. Reads the input data under the following format:
    a. initial and fixed values
    b. fluid properties
    c. properties of the cap and base rock
    d. reservoir rock properties
    e. initial and residual fluid saturations
    f. streamtube dimensions from streamline program
2. Calculates the sweep out time for each of the permeability zones in the streamtube.
3. Calculates the recovery from each streamtube, well, and field for any time period. It prints out the results and plots them.

A listing of the flow chart follows.

FLOW CHART FOR STEAMFLOOD RECOVERY CALCULATION




1
INITIALIZE ALL ARRAYS, COUNTERS, ETC.
CALCULATE THE DIMENSIONLESS VARIABLES AND CONSTANTS
FOR EACH STREAMTUBE, CALCULATE THE BREAKTHROUGH TIMES FOR THE FIRST (AND SECOND REGION, IF ANY) AND ARRAY ENDTM1 AND ENDTM2 RESPECTIVELY
INCREMENT TIME t = t + Δt
t > ENDTM1?
NO
YES
CALCULATE ADDITIONAL RECOVERY AND ADD TO TOTAL
NO. OF REGIONS > 1?
NO
YES
STEAM BREAKTHROUGH IN A STREAMTUBE WITH ONLY ONE REGION. NO ADDITIONAL RECOVERY CALCULATED
t > ENDTM2
NO
YES
4
2
3


2
3
STEAMFRONT IS ADVANCING WITHIN THE SECOND REGION OF A 2-REGION STREAMTUBE. CALCULATE ADDITIONAL RECOVERY AND ADD TO TOTAL.
THERE IS STEAM BREAKTHROUGH IN THE SECOND REGION OF A 2-REGION STREAMTUBE. NO ADDITIONAL RECOVERY.
4
LAST STREAMLINE?
NO
YES
START NEW STREAMLINE
LAST WELL?
NO
START NEW WELL. START NEW STREAMLINE
YES
LAST TIME?
NO
START NEW TIME START NEW WELL START NEW STREAMLINE
YES
PRINT AND PLOT THE COMPUTED RECOVERY VERSUS TIME
STOP

APPENDIX G

RESULTS OF STREAMLINE MODELLING OF SHIELLS CANYON FIELD AS A SINGLE HOMOGENEOUS RESERVOIR

BOUNDARY ELEMENT MODELLING OF SHIELLS CANYON(203) FIELD

ANALYSIS AS A SINGLE HOMOGENEOUS RESERVOIR

SEALED OUTER BOUNDARY

BY: D. T. NUMBERE, UNIVERSITY OF OKLAHOMA, JAN., 1982

REGION 1.0

DATA

NUMBER OF BOUNDARY ELEMENTS= 60
NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS CALCULATED= 0
NUMBER OF SOURCES AND SINKS= 13
NUMBER OF INTERFACE NODES= 0

PERMEABILITY(MD)= 140.0000

THICKNESS(FT)= 160.0000

POROSITY= 0.2050

SCALE: 1 INCH = 738.46FEET

THE COORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELEMENTS

POINT	X(INCH)	Y(INCH)
1	0.1000	0.1500
2	0.2500	0.2000
3	0.4000	0.2500
4	0.5500	0.2800
5	0.6500	0.3000
6	0.7000	0.3250
7	0.7500	0.3300
8	0.8000	0.3400
9	0.8500	0.3500
10	0.9000	0.4000
11	1.0000	0.4500
12	1.1000	0.5250
13	1.2500	0.6000
14	1.4000	0.6500
15	1.5500	0.6800
16	1.6500	0.6900
17	1.7000	0.7000
18	1.7500	0.7100
19	1.8000	0.7200
20	1.8500	0.7250
21	1.9000	0.7300
22	1.9500	0.7350
23	2.1000	0.7450
24	2.2500	0.7500
25	2.4000	0.7500
26	2.6000	0.7400
27	2.7500	0.7300
28	2.9000	0.7250
29	3.1600	1.0000
30	3.0600	1.2500
31	2.9500	1.5500
32	2.7500	1.6500
33	2.4000	1.8200
34	2.2500	1.8000
35	2.1000	1.7500
36	2.0500	1.7450
37	2.0000	1.7250
38	1.9500	1.7100
39	1.8500	1.6900
40	1.7500	1.6500
41	1.6500	1.6100
42	1.5000	1.5500
43	1.3500	1.4800

44	1.2500	1.4400
45	1.2000	1.4000
46	1.1500	1.3800
47	1.0500	1.3250
48	0.9500	1.2600
49	0.9000	1.2500
50	0.8500	1.2000
51	0.8000	1.1700
52	0.7500	1.1500
53	0.6500	1.0800
54	0.5500	1.0200
55	0.5000	0.9900
56	0.4500	0.9500
57	0.4000	0.9200
58	0.3000	0.8500
59	0.1000	0.7300
60	0.1000	0.5000

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	1	0.0
2	1	0.0
3	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0
7	1	0.0
8	1	0.0
9	1	0.0
10	1	0.0
11	1	0.0
12	1	0.0
13	1	0.0
14	1	0.0
15	1	0.0
16	1	0.0
17	1	0.0
18	1	0.0
19	1	0.0
20	1	0.0
21	1	0.0
22	1	0.0
23	1	0.0
24	1	0.0
25	1	0.0
26	1	0.0
27	1	0.0
28	1	0.0
29	1	0.0
30	1	0.0
31	1	0.0
32	1	0.0
33	1	0.0
34	1	0.0
35	1	0.0
36	1	0.0
37	1	0.0
38	1	0.0
39	1	0.0
40	1	0.0
41	1	0.0
42	1	0.0
43	1	0.0

44	1	0.0
45	1	0.0
46	1	0.0
47	1	0.0
48	1	0.0
49	1	0.0
50	1	0.0
51	1	0.0
52	1	0.0
53	1	0.0
54	1	0.0
55	1	0.0
56	1	0.0
57	1	0.0
58	1	0.0
59	1	0.0
60	1	0.0

COORDINATES OF SOURCES AND SINKS AND THEIR STRENGTHS

X(INCH)	Y(INCH)	RATE(BBL/D)
0.4500	0.5500	-69.1000
0.5000	0.8000	-69.1000
0.7500	0.4000	-69.1000
0.9000	1.1500	-69.1000
1.0000	0.8000	560.0000
1.2500	1.2000	-69.1000
1.3500	1.0000	-69.1000
1.9000	0.9500	200.0000
1.8000	1.3500	-69.1000
2.0500	1.5500	-69.1000
2.1500	1.2500	-69.1000
2.3500	1.0500	-69.1000
2.5750	1.3000	-69.1000

THE AVERAGE RADIUS OF THE WELLS(INCH) IS= 0.500
THE CHARACTERISTIC PRESSURE(PSI) IS= 85.0000

RESULTS

BOUNDARY NODES

X(INCH)	Y(INCH)	PRESSURE(PSI)	NORMAL GRADIENT
0.1750E+00	0.1750E+00	0.8469E+02	0.0
0.3250E+00	0.2250E+00	0.8510E+02	0.0
0.4750E+00	0.2650E+00	0.8524E+02	0.0
0.6000E+00	0.2900E+00	0.8547E+02	0.0
0.6750E+00	0.3125E+00	0.8556E+02	0.0
0.7250E+00	0.3275E+00	0.8565E+02	0.0
0.7750E+00	0.3350E+00	0.8588E+02	0.0
0.8250E+00	0.3450E+00	0.8643E+02	0.0
0.8750E+00	0.3750E+00	0.8697E+02	0.0
0.9500E+00	0.4250E+00	0.8830E+02	0.0
0.1050E+01	0.4875E+00	0.8940E+02	0.0
0.1175E+01	0.5625E+00	0.8961E+02	0.0
0.1325E+01	0.6250E+00	0.8772E+02	0.0
0.1475E+01	0.6650E+00	0.8565E+02	0.0
0.1600E+01	0.6850E+00	0.8446E+02	0.0
0.1675E+01	0.6950E+00	0.8398E+02	0.0
0.1725E+01	0.7050E+00	0.8370E+02	0.0
0.1775E+01	0.7150E+00	0.8344E+02	0.0
0.1825E+01	0.7225E+00	0.8315E+02	0.0
0.1875E+01	0.7275E+00	0.8283E+02	0.0
0.1925E+01	0.7325E+00	0.8243E+02	0.0
0.2025E+01	0.7400E+00	0.8129E+02	0.0
0.2175E+01	0.7475E+00	0.7940E+02	0.0
0.2325E+01	0.7500E+00	0.7791E+02	0.0
0.2500E+01	0.7450E+00	0.7689E+02	0.0
0.2675E+01	0.7350E+00	0.7642E+02	0.0
0.2825E+01	0.7275E+00	0.7623E+02	0.0
0.3030E+01	0.8625E+00	0.7610E+02	0.0
0.3110E+01	0.1125E+01	0.7601E+02	0.0
0.3005E+01	0.1400E+01	0.7582E+02	0.0
0.2850E+01	0.1600E+01	0.7579E+02	0.0
0.2575E+01	0.1735E+01	0.7598E+02	0.0
0.2325E+01	0.1810E+01	0.7614E+02	0.0
0.2175E+01	0.1775E+01	0.7637E+02	0.0
0.2075E+01	0.1747E+01	0.7668E+02	0.0
0.2025E+01	0.1735E+01	0.7673E+02	0.0
0.1975E+01	0.1717E+01	0.7699E+02	0.0
0.1900E+01	0.1700E+01	0.7748E+02	0.0
0.1800E+01	0.1670E+01	0.7816E+02	0.0
0.1700E+01	0.1630E+01	0.7900E+02	0.0
0.1575E+01	0.1580E+01	0.8013E+02	0.0
0.1425E+01	0.1515E+01	0.8143E+02	0.0
0.1300E+01	0.1460E+01	0.8246E+02	0.0
0.1225E+01	0.1420E+01	0.8296E+02	0.0
0.1175E+01	0.1390E+01	0.8350E+02	0.0
0.1100E+01	0.1352E+01	0.8418E+02	0.0
0.1000E+01	0.1292E+01	0.8510E+02	0.0
0.9250E+00	0.1255E+01	0.8565E+02	0.0
0.8750E+00	0.1225E+01	0.8572E+02	0.0
0.8250E+00	0.1185E+01	0.8626E+02	0.0
0.7750E+00	0.1160E+01	0.8677E+02	0.0
0.7000E+00	0.1115E+01	0.8714E+02	0.0
0.6000E+00	0.1050E+01	0.8695E+02	0.0
0.5250E+00	0.1005E+01	0.8648E+02	0.0
0.4750E+00	0.9700E+00	0.8610E+02	0.0
0.4250E+00	0.9350E+00	0.8569E+02	0.0

0.3500E+00	0.8850E+00	0.4501E+02	0.0
0.2000E+00	0.7900E+00	0.4476E+02	0.0
0.1000E+00	0.6150E+0C	0.4469E+02	0.0
0.1000E+00	0.3250E+00	0.4464E+02	0.0

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PRODUCER NUMBER 1

NUMBER OF STREAMLINES= 3

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.5381E+03	0.4518E+02	0.2800E+02
2	0.4250E+03	0.2375E+02	0.2800E+02
3	0.6881E+03	0.7951E+02	0.2800E+02

PRODUCER NUMBER 2

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4414E+03	0.2776E+02	0.2800E+02
2	0.3447E+03	0.1918E+02	0.2800E+02

PRODUCER NUMBER 3

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.3348E+03	0.1806E+02	0.2800E+02
2	0.3848E+03	0.2176E+02	0.2800E+02

PRODUCER NUMBER 4

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2697E+03	0.1351E+02	0.2800E+02
2	0.2697E+03	0.1488E+02	0.2800E+02

PRODUCER NUMBER 6

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.3598E+03	0.1589E+02	0.2800E+02
2	0.3381E+03	0.1426E+02	0.2800E+02

PRODUCER NUMBER 7

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2947E+03	0.1269E+02	0.2800E+02
2	0.2881E+03	0.1196E+02	0.2800E+02

PRODUCER NUMBER 9

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.8098E+03	0.2941E+02	0.2800E+02
2	0.7598E+03	0.2863E+02	0.2800E+02

PRODUCER NUMBER 10

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.1120E+04	0.3876E+02	0.2800E+02
2	0.9500E+03	0.3434E+02	0.2800E+02
3	0.5881E+03	0.1561E+02	0.1000E+02
4	0.5697E+03	0.1231E+02	0.1000E+02
5	0.5980E+03	0.1378E+02	0.1000E+02

PRODUCER NUMBER 11

NUMBER OF STREAMLINES= 4

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2664E+03	0.6268E+01	0.1000E+02
2	0.2914E+03	0.6742E+01	0.1000E+02
3	0.3783E+03	0.8902E+01	0.1000E+02
4	0.2881E+03	0.6459E+01	0.1000E+02

PRODUCER NUMBER 12

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4631E+03	0.1018E+02	0.1000E+02
2	0.3697E+03	0.8221E+01	0.1000E+02
3	0.3316E+03	0.7526E+01	0.1000E+02
4	0.3348E+03	0.7388E+01	0.1000E+02
5	0.4033E+03	0.1092E+02	0.1000E+02

PRODUCER NUMBER 13

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.7164E+03	0.2242E+02	0.1000E+02
2	0.9033E+03	0.3443E+02	0.1000E+02
3	0.7131E+03	0.1976E+02	0.1000E+02
4	0.5381E+03	0.1560E+02	0.1000E+02
5	0.6066E+03	0.2223E+02	0.1000E+02

APPENDIX H

RESULTS OF STEAMFLOOD PREDICTION OF SHIELLS CANYON FIELD AS A HOMOGENEOUS RESERVOIR WITH SEALED OUTER BOUNDARY

APPENDIX H

STEAMFLOOD PREDICTION OF SHIELLS CANYON(203) FIELD

METHOD: STREAMLINE/STREAMTUBE METHOD USING (BEM)TECHNIQUE

ANALYSIS AS A SINGLE HOMOGENOUS RESERVOIR

BOUNDARY CONDITION: SEALED OUTER BOUNDARY

PRODUCTION RATES: EQUAL RATES ASSIGNED TO ALL PRODUCERS IN ENTIRE REGION

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DATA

TOTAL NUMBER OF PRODUCERS=	11
STEAM TEMPERATURE (DEG. F)=	430.0000
STEAM QUALITY=	0.7000
CONVERGENCE LIMIT=	0.0010
RESERVOIR THICKNESS (FT)=	160.0000
STEAM PRESSURE (PSI)=	344.0000
INITIAL RESERVOIR TEMPERATURE(DEG F)=	105.0000

FLUID PROPERTIES

	WATER	OIL	STEAM
DENSITY(LB/CU FT) AT STD. TEMP.	62.4000	55.0000	
DENSITY(LB/CU FT) AT STEAM TEMP.	52.3800	48.7000	0.7431
SPECIFIC HEAT(BTU/LB *F)	1.0000	0.4880	1.0000
LATENT HEAT(BTU/LB)			789.5100

PROPERTIES OF THE CAP AND BASE ROCK

DENSITY(LB/CU. FT)	149.0000
SPECIFIC HEAT(BTU/LB-*F)	0.2130
THERM. COND.(BTU/HR-FT-*F)	1.1000

RESERVOIR ROCK PROPERTIES

	REGION 1	REGION 2
POROSITY	0.2050	0.1800
DEN.(LB/CU FT)	165.0000	165.0000
SPEC. HEAT(BTU/LB*F)	0.2000	0.2000
PERMEABILITY(MD)	140.0000	70.0000

INITIAL FLUID SATURATIONS

	REGION 1	REGION 2
WATER	0.5500	0.5500
OIL	0.4500	0.4500
STEAM	0.0	0.0

AVERAGE RESIDUAL SATURATIONS BEHIND THE FRONT

	REGION 1	REGION 2
WATER	0.5800	0.5800
OIL	0.1800	0.1800
STEAM	0.2400	0.2400

INPUT DATA FROM STREAMLINE PROGRAM

			REGION 1			REGION 2		
WELL NO.	S/L NO.	CODE	LENGTH	WIDTH	RATE	LENGTH	WIDTH	RATE
1	1	1	538.10	45.18	28.00	0.0	0.0	0.0
1	2	1	425.00	23.75	28.00	0.0	0.0	0.0
1	3	1	686.10	79.51	28.00	0.0	0.0	0.0
2	1	1	441.40	27.76	28.00	0.0	0.0	0.0
2	2	1	344.70	19.18	28.00	0.0	0.0	0.0
3	1	1	334.80	18.06	28.00	0.0	0.0	0.0
3	2	1	384.80	21.76	28.00	0.0	0.0	0.0
4	1	1	269.70	13.51	28.00	0.0	0.0	0.0
4	2	1	269.70	14.88	28.00	0.0	0.0	0.0
5	1	1	359.80	15.89	28.00	0.0	0.0	0.0
5	2	1	338.10	14.26	28.00	0.0	0.0	0.0
6	1	1	294.70	12.69	28.00	0.0	0.0	0.0
6	2	1	288.10	11.96	28.00	0.0	0.0	0.0
7	1	1	809.80	29.41	28.00	0.0	0.0	0.0
7	2	1	759.80	28.63	28.00	0.0	0.0	0.0
8	1	1	112.00	38.76	28.00	0.0	0.0	0.0
8	2	1	950.00	34.34	28.00	0.0	0.0	0.0
8	3	1	588.10	15.61	10.00	0.0	0.0	0.0
8	4	1	569.70	12.31	10.00	0.0	0.0	0.0
8	5	1	598.00	13.78	10.00	0.0	0.0	0.0
9	1	1	266.40	6.27	10.00	0.0	0.0	0.0
9	2	1	291.40	6.74	10.00	0.0	0.0	0.0
9	3	1	378.30	8.90	10.00	0.0	0.0	0.0
9	4	1	288.10	6.46	10.00	0.0	0.0	0.0
10	1	1	463.10	10.18	10.00	0.0	0.0	0.0
10	2	1	369.70	8.22	10.00	0.0	0.0	0.0
10	3	1	331.60	7.53	10.00	0.0	0.0	0.0
10	4	1	334.80	7.39	10.00	0.0	0.0	0.0
10	5	1	403.30	10.92	10.00	0.0	0.0	0.0
11	1	1	716.40	22.42	10.00	0.0	0.0	0.0
11	2	1	903.30	34.43	10.00	0.0	0.0	0.0
11	3	1	713.10	19.76	10.00	0.0	0.0	0.0
11	4	1	538.10	15.60	10.00	0.0	0.0	0.0
11	5	1	606.60	22.23	10.00	0.0	0.0	0.0

CALCULATED BREAKTHROUGH TIMES(HOURS) FOR EACH STREAMTUBE

PROD. NO.	STREAMLINE NO.	CODE	ENDTIME(1)	ENDTIME(2)
1	1	1	15868.5625	0.0
1	2	1	6144.6992	0.0
1	3	1	39442.2148	0.0
2	1	1	7555.3711	0.0
2	2	1	3929.7900	0.0
3	1	1	3578.2664	0.0
3	2	1	5040.6680	0.0
4	1	1	2110.3652	0.0
4	2	1	2332.9001	0.0
5	1	1	3374.3901	0.0
5	2	1	2823.9495	0.0
6	1	1	2163.1919	0.0
6	2	1	1991.6033	0.0
7	1	1	15514.1211	0.0
7	2	1	14048.2070	0.0
8	1	1	2531.5093	0.0
8	2	1	21971.1758	0.0
8	3	1	16871.8437	0.0
8	4	1	12564.0469	0.0
8	5	1	14984.0547	0.0
9	1	1	2734.8716	0.0
9	2	1	3240.8657	0.0
9	3	1	5716.1836	0.0
9	4	1	3062.2073	0.0
10	1	1	8182.2500	0.0
10	2	1	5127.8672	0.0
10	3	1	4165.2852	0.0
10	4	1	4126.5156	0.0
10	5	1	7606.9062	0.0
11	1	1	31527.6133	0.0
11	2	1	67928.6875	0.0
11	3	1	27188.5703	0.0
11	4	1	15291.1250	0.0
11	5	1	25876.1133	0.0

REAL TIME(HCURS)= 6000.0000

DIMENSIONLESS TIME= 0.0325

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TCTAL
1	1	1	0.1557E+05		
1	2	1	0.1557E+05		
1	3	1	0.1557E+05		
				0.4670E+05	
2	1	1	0.1557E+05		
2	2	1	0.1043E+05		
				0.2599E+05	
3	1	1	0.9537E+04		
3	2	1	0.1321E+05		
				0.2274E+05	
4	1	1	0.5747E+04		
4	2	1	0.6330E+04		
				0.1208E+05	
5	1	1	0.9017E+04		
5	2	1	0.7604E+04		
				0.1662E+05	
6	1	1	0.5898E+04		
6	2	1	0.5435E+04		
				0.1133E+05	
7	1	1	0.1557E+05		
7	2	1	0.1557E+05		
				0.3113E+05	
8	1	1	0.6847E+04		
8	2	1	0.1557E+05		
8	3	1	0.5560E+04		
8	4	1	0.5560E+04		
8	5	1	0.5560E+04		
				0.3909E+05	
9	1	1	0.2634E+04		
9	2	1	0.3099E+04		
9	3	1	0.5311E+04		
9	4	1	0.2935E+04		
				0.1398E+05	
10	1	1	0.5560E+04		
10	2	1	0.4794E+04		
10	3	1	0.3936E+04		
10	4	1	0.3901E+04		
10	5	1	0.5560E+04		
				0.2375E+05	
11	1	1	0.5560E+04		
11	2	1	0.5560E+04		
11	3	1	0.5560E+04		
11	4	1	0.5560E+04		
11	5	1	0.5560E+04		
				0.2780E+05	
					0.2712E+06

REAL TIME(HCURS)= 12000.0000

DIMENSIONLESS TIME= 0.0650

WELL	STR/L	NO. OF REGIONS	RECUVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.2969E+05		
1	2	1	0.1592E+05		
1	3	1	0.2969E+05		
				0.7530E+05	
2	1	1	0.1933E+05		
2	2	1	0.1043E+05		
				0.2975E+05	
3	1	1	0.9537E+04		
3	2	1	0.1321E+05		
				0.2274E+05	
4	1	1	0.5747E+04		
4	2	1	0.6330E+04		
				0.1208E+05	
5	1	1	0.9017E+04		
5	2	1	0.7604E+04		
				0.1662E+05	
6	1	1	0.5898E+04		
6	2	1	0.5435E+04		
				0.1133E+05	
7	1	1	0.2969E+05		
7	2	1	0.2969E+05		
				0.5938E+05	
8	1	1	0.6847E+04		
8	2	1	0.2969E+05		
8	3	1	0.1060E+05		
8	4	1	0.1060E+05		
8	5	1	0.1060E+05		
				0.6835E+05	
9	1	1	0.2634E+04		
9	2	1	0.3099E+04		
9	3	1	0.5311E+04		
9	4	1	0.2935E+04		
				0.1398E+05	
10	1	1	0.7436E+04		
10	2	1	0.4794E+04		
10	3	1	0.3936E+04		
10	4	1	0.3901E+04		
10	5	1	0.6946E+04		
				0.2701E+05	
11	1	1	0.1060E+05		
11	2	1	0.1060E+05		
11	3	1	0.1060E+05		
11	4	1	0.1060E+05		
11	5	1	0.1060E+05		
				0.5302E+05	
					0.3896E+06

REAL TIME(HCURS)= 18000.0000

DIMENSIONLESS TIME= 0.0975

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.3834E+05		
1	2	1	0.1592E+05		
1	3	1	0.4299E+05		
				0.9726E+05	
2	1	1	0.1933E+05		
2	2	1	0.1043E+05		
				0.2975E+05	
3	1	1	0.9537E+04		
3	2	1	0.1321E+05		
				0.2274E+05	
4	1	1	0.5747E+04		
4	2	1	0.6330E+04		
				0.1208E+05	
5	1	1	0.9017E+04		
5	2	1	0.7604E+04		
				0.1662E+05	
6	1	1	0.5898E+04		
6	2	1	0.5435E+04		
				0.1133E+05	
7	1	1	0.3756E+05		
7	2	1	0.3431E+05		
				0.7187E+05	
8	1	1	0.6847E+04		
8	2	1	0.4299E+05		
8	3	1	0.1448E+05		
8	4	1	0.1106E+05		
8	5	1	0.1300E+05		
				0.8838E+05	
9	1	1	0.2634E+04		
9	2	1	0.3099E+04		
9	3	1	0.5311E+04		
9	4	1	0.2935E+04		
				0.1398E+05	
10	1	1	0.7436E+04		
10	2	1	0.4794E+04		
10	3	1	0.3936E+04		
10	4	1	0.3901E+04		
10	5	1	0.6946E+04		
				0.2701E+05	
11	1	1	0.1535E+05		
11	2	1	0.1535E+05		
11	3	1	0.1535E+05		
11	4	1	0.1324E+05		
11	5	1	0.1535E+05		
				0.7466E+05	
					0.4657E+06

REAL TIME(HOURS)= 24000.0000

DIMENSIONLESS TIME= 0.1300

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.3834E+05		
1	2	1	0.1592E+05		
1	3	1	0.5569E+05		
				0.1099E+06	
2	1	1	0.1933E+05		
2	2	1	0.1043E+05		
				0.2975E+05	
3	1	1	0.9537E+04		
3	2	1	0.1321E+05		
				0.2274E+05	
4	1	1	0.5747E+04		
4	2	1	0.6330E+04		
				0.1208E+05	
5	1	1	0.9017E+04		
5	2	1	0.7604E+04		
				0.1662E+05	
6	1	1	0.5898E+04		
6	2	1	0.5435E+04		
				0.1133E+05	
7	1	1	0.3756E+05		
7	2	1	0.3431E+05		
				0.7187E+05	
8	1	1	0.6847E+04		
8	2	1	0.5145E+05		
8	3	1	0.1448E+05		
8	4	1	0.1106E+05		
8	5	1	0.1300E+05		
				0.9684E+05	
9	1	1	0.2634E+04		
9	2	1	0.3099E+04		
9	3	1	0.5311E+04		
9	4	1	0.2935E+04		
				0.1398E+05	
10	1	1	0.7436E+04		
10	2	1	0.4794E+04		
10	3	1	0.3936E+04		
10	4	1	0.3901E+04		
10	5	1	0.6946E+04		
				0.2701E+05	
11	1	1	0.1989E+05		
11	2	1	0.1989E+05		
11	3	1	0.1989E+05		
11	4	1	0.1324E+05		
11	5	1	0.1989E+05		
				0.9279E+05	
					0.5050E+06

REAL TIME(HOURS)= 30000.0000

DIMENSIONLESS TIME= 0.1625

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.3834E+05		
1	2	1	0.1592E+05		
1	3	1	0.6789E+05		
				0.1222E+06	
2	1	1	0.1933E+05		
2	2	1	0.1043E+05		
				0.2975E+05	
3	1	1	0.9537E+04		
3	2	1	0.1321E+05		
				0.2274E+05	
4	1	1	0.5747E+04		
4	2	1	0.6330E+04		
				0.1208E+05	
5	1	1	0.9017E+04		
5	2	1	0.7604E+04		
				0.1662E+05	
6	1	1	0.5898E+04		
6	2	1	0.5435E+04		
				0.1133E+05	
7	1	1	0.3756E+05		
7	2	1	0.3431E+05		
				0.7187E+05	
8	1	1	0.6847E+04		
8	2	1	0.5145E+05		
8	3	1	0.1448E+05		
8	4	1	0.1106E+05		
8	5	1	0.1300E+05		
				0.9684E+05	
9	1	1	0.2634E+04		
9	2	1	0.3099E+04		
9	3	1	0.5311E+04		
9	4	1	0.2935E+04		
				0.1398E+05	
10	1	1	0.7436E+04		
10	2	1	0.4794E+04		
10	3	1	0.3936E+04		
10	4	1	0.3901E+04		
10	5	1	0.6946E+04		
				0.2701E+05	
11	1	1	0.2425E+05		
11	2	1	0.2425E+05		
11	3	1	0.2222E+05		
11	4	1	0.1324E+05		
11	5	1	0.2127E+05		
				0.1052E+06	
					0.5296E+06

PREDICTED RECOVERY

TIME(DAYS)	RECOVERY(BBLS)
62.5000	0.1697E+05
125.0000	0.3120E+05
187.5000	0.4079E+05
250.0000	0.4830E+05
312.5000	0.5455E+05
375.0000	0.5967E+05
437.5000	0.6456E+05
500.0000	0.6938E+05
562.5000	0.7399E+05
625.0000	0.7808E+05
687.5000	0.8089E+05
750.0000	0.8293E+05
812.5000	0.8490E+05
875.0000	0.8685E+05
937.5000	0.8858E+05
1000.0000	0.8993E+05
1062.5000	0.9127E+05
1125.0000	0.9245E+05
1187.5000	0.9340E+05
1250.0000	0.9432E+05

APPENDIX I

RESULTS OF STREAMLINE MODELLING OF SHIELLS CANYON FIELD AS A HOMOGENEOUS RESERVOIR HAVING A PART OF THE BOUNDARY AT CONSTANT PRESSURE WHILE THE REMAINDER IS SEALED

BOUNDARY ELEMENT MODELLING OF SHIELLS CANYON(203) FIELD

ANALYSIS AS A SINGLE HOMOGENEOUS RESERVOIR

PART OF THE BOUNDARY AT CONSTANT PRESSURE, THE REMAINDER SEALED

BY: D. T. NUMBERE, UNIVERSITY OF OKLAHOMA, JAN., 1982

REGION 1.0

DATA

NUMBER OF BOUNDARY ELEMENTS= 60
NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS CALCULATED= 0
NUMBER OF SOURCES AND SINKS= 13
NUMBER OF INTERFACE NODES= 0

PERMEABILITY(MD)= 140.0000

THICKNESS(FT)= 160.0000

POROSITY= 0.2050

SCALE: 1 INCH = 738.46FEET

THE COORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELEMENTS

POINT	X(INCH)	Y(INCH)
1	0.1000	0.1500
2	0.2500	0.2000
3	0.4000	0.2500
4	0.5500	0.2800
5	0.6500	0.3000
6	0.7000	0.3250
7	0.7500	0.3300
8	0.8000	0.3400
9	0.8500	0.3500
10	0.9000	0.4000
11	1.0000	0.4500
12	1.1000	0.5250
13	1.2500	0.6000
14	1.4000	0.6500
15	1.5500	0.6800
16	1.6500	0.6900
17	1.7000	0.7000
18	1.7500	0.7100
19	1.8000	0.7200
20	1.8500	0.7250
21	1.9000	0.7300
22	1.9500	0.7350
23	2.1000	0.7450
24	2.2500	0.7500
25	2.4000	0.7500
26	2.6000	0.7400
27	2.7500	0.7300
28	2.9000	0.7250
29	3.1600	1.0000
30	3.0600	1.2500
31	2.9500	1.5500
32	2.7500	1.6500
33	2.4000	1.8200
34	2.2500	1.8000
35	2.1000	1.7500
36	2.0500	1.7450
37	2.0000	1.7250
38	1.9500	1.7100
39	1.8500	1.6900
40	1.7500	1.6500
41	1.6500	1.6100
42	1.5000	1.5500
43	1.3500	1.4800

44	1.2500	1.4400
45	1.2000	1.4000
46	1.1500	1.3800
47	1.0500	1.3250
48	0.9500	1.2600
49	0.9000	1.2500
50	0.8500	1.2000
51	0.8000	1.1700
52	0.7500	1.1500
53	0.6500	1.0800
54	0.5500	1.0200
55	0.5000	0.9900
56	0.4500	0.9500
57	0.4000	0.9200
58	0.3000	0.8500
59	0.1000	0.7300
60	0.1000	0.5000

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	1	0.0
2	1	0.0
3	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0
7	1	0.0
8	1	0.0
9	1	0.0
10	1	0.0
11	1	0.0
12	1	0.0
13	1	0.0
14	1	0.0
15	1	0.0
16	1	0.0
17	1	0.0
18	1	0.0
19	1	0.0
20	1	0.0
21	1	0.0
22	1	0.0
23	1	0.0
24	1	0.0
25	1	0.0
26	1	0.0
27	1	0.0
28	1	0.0
29	1	0.0
30	1	0.0
31	1	0.0
32	1	0.0
33	1	0.0
34	1	0.0
35	1	0.0
36	1	0.0
37	1	0.0
38	1	0.0
39	1	0.0
40	1	0.0
41	1	0.0
42	1	0.0
43	1	0.0

44	1	0.0
45	1	0.0
46	1	0.0
47	1	0.0
48	1	0.0
49	1	0.0
50	1	0.0
51	1	0.0
52	1	0.0
53	1	0.0
54	1	0.0
55	1	0.0
56	1	0.0
57	1	0.0
58	1	0.0
59	0	0.8500000E+02
60	0	0.8500000E+02

COORDINATES OF SOURCES AND SINKS AND THEIR STRENGTHS

X(INCH)	Y(INCH)	RATE(BBL/D)
0.4500	0.5500	-69.1000
0.5000	0.8000	-69.1000
0.7500	0.4000	-69.1000
0.9000	1.1500	-69.1000
1.0000	0.8000	560.0000
1.2500	1.2000	-69.1000
1.3500	1.0000	-69.1000
1.9000	0.9500	200.0000
1.8000	1.3500	-69.1000
2.0500	1.5500	-69.1000
2.1500	1.2500	-69.1000
2.3500	1.0500	-69.1000
2.5750	1.3000	-69.1000

THE AVERAGE RADIUS OF THE WELLS(INCH) IS= 0.500
THE CHARACTERISTIC PRESSURE(PSI) IS= 85.0000

RESULTS

BOUNDARY NODES

X(INCH)	Y(INCH)	PRESSURE(PSI)	NORMAL GRADIENT
0.1750E+00	0.1750E+00	0.4448E+02	0.0
0.3250E+00	0.2250E+00	0.4472E+02	0.0
0.4750E+00	0.2650E+00	0.4487E+02	0.0
0.6000E+00	0.2900E+00	0.4510E+02	0.0
0.6750E+00	0.3125E+00	0.4520E+02	0.0
0.7250E+00	0.3275E+00	0.4527E+02	0.0
0.7750E+00	0.3350E+00	0.4551E+02	0.0
0.8250E+00	0.3450E+00	0.4605E+02	0.0
0.8750E+00	0.3750E+00	0.4660E+02	0.0
0.9500E+00	0.4250E+00	0.4793E+02	0.0
0.1050E+01	0.4875E+00	0.4903E+02	0.0
0.1175E+01	0.5625E+00	0.4923E+02	0.0
0.1325E+01	0.6250E+00	0.4734E+02	0.0
0.1475E+01	0.6650E+00	0.4527E+02	0.0
0.1600E+01	0.6850E+00	0.4408E+02	0.0
0.1675E+01	0.6950E+00	0.4360E+02	0.0
0.1725E+01	0.7050E+00	0.4332E+02	0.0
0.1775E+01	0.7150E+00	0.4305E+02	0.0
0.1825E+01	0.7225E+00	0.4277E+02	0.0
0.1875E+01	0.7275E+00	0.4244E+02	0.0
0.1925E+01	0.7325E+00	0.4204E+02	0.0
0.2025E+01	0.7400E+00	0.4090E+02	0.0
0.2175E+01	0.7475E+00	0.3901E+02	0.0
0.2325E+01	0.7500E+00	0.3752E+02	0.0
0.2500E+01	0.7450E+00	0.3650E+02	0.0
0.2675E+01	0.7350E+00	0.3603E+02	0.0
0.2825E+01	0.7275E+00	0.3584E+02	0.0
0.3030E+01	0.8625E+00	0.3571E+02	0.0
0.3110E+01	0.1125E+01	0.3561E+02	0.0
0.3005E+01	0.1400E+01	0.3543E+02	0.0
0.2850E+01	0.1600E+01	0.3540E+02	0.0
0.2575E+01	0.1735E+01	0.3560E+02	0.0
0.2325E+01	0.1810E+01	0.3579E+02	0.0
0.2175E+01	0.1775E+01	0.3599E+02	0.0
0.2075E+01	0.1747E+01	0.3625E+02	0.0
0.2025E+01	0.1735E+01	0.3634E+02	0.0
0.1975E+01	0.1717E+01	0.3659E+02	0.0
0.1900E+01	0.1700E+01	0.3709E+02	0.0
0.1800E+01	0.1670E+01	0.3778E+02	0.0
0.1700E+01	0.1630E+01	0.3862E+02	0.0
0.1575E+01	0.1580E+01	0.3974E+02	0.0
0.1425E+01	0.1515E+01	0.4104E+02	0.0
0.1300E+01	0.1460E+01	0.4208E+02	0.0
0.1225E+01	0.1420E+01	0.4259E+02	0.0
0.1175E+01	0.1390E+01	0.4313E+02	0.0
0.1100E+01	0.1352E+01	0.4381E+02	0.0
0.1000E+01	0.1292E+01	0.4473E+02	0.0
0.9250E+00	0.1255E+01	0.4525E+02	0.0
0.8750E+00	0.1225E+01	0.4535E+02	0.0
0.8250E+00	0.1185E+01	0.4588E+02	0.0
0.7750E+00	0.1160E+01	0.4641E+02	0.0
0.7000E+00	0.1115E+01	0.4677E+02	0.0
0.6000E+00	0.1050E+01	0.4659E+02	0.0
0.5250E+00	0.1005E+01	0.4611E+02	0.0
0.4750E+00	0.9700E+00	0.4574E+02	0.0
0.4250E+00	0.9350E+00	0.4534E+02	0.0

0.3500E+00	0.8850E+00	0.8537E+02	0.0
0.2000E+00	0.7900E+00	0.8509E+02	0.0
0.1000E+00	0.6150E+00	0.8500E+02	-0.4122E-03
0.1000E+00	0.3250E+00	0.8500E+02	0.2567E-03

****

PRODUCER NUMBER 1

NUMBER OF STREAMLINES= 3

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.5414E+03	0.4584E+02	0.2800E+02
2	0.4250E+03	0.2367E+02	0.2800E+02
3	0.6783E+03	0.7369E+02	0.2800E+02

PRODUCER NUMBER 2

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4414E+03	0.2762E+02	0.2800E+02
2	0.3447E+03	0.1914E+02	0.2800E+02

PRODUCER NUMBER 3

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.3381E+03	0.1806E+02	0.2800E+02
2	0.3848E+03	0.2162E+02	0.2800E+02

PRODUCER NUMBER 4

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2697E+03	0.1351E+02	0.2800E+02
2	0.2697E+03	0.1491E+02	0.2800E+02

PRODUCER NUMBER 6

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.3598E+03	0.1591E+02	0.2800E+02
2	0.3381E+03	0.1426E+02	0.2800E+02

PRODUCER NUMBER 7

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2947E+03	0.1270E+02	0.2800E+02
2	0.2881E+03	0.1196E+02	0.2800E+02

PRODUCER NUMBER 9

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.8098E+03	0.2943E+02	0.2800E+02
2	0.7598E+03	0.2867E+02	0.2800E+02

PRODUCER NUMBER 10

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WICTH	INJ. RATE
1	0.1120E+04	0.3869E+02	0.2800E+02
2	0.9500E+03	0.3438E+02	0.2800E+02
3	0.5881E+03	0.1555E+C2	0.1000E+02
4	0.5697E+03	0.1231E+02	0.1000E+02
5	0.5980E+03	0.1379E+02	0.1000E+02

PRODUCER NUMBER 11

NUMBER OF STREAMLINES= 4

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2664E+03	0.6270E+01	0.1000E+02
2	0.2914E+03	0.6746E+01	0.1000E+02
3	0.3783E+03	0.8922E+01	0.1000E+02
4	0.2881E+03	0.6459E+C1	0.1000E+02

PRODUCER NUMBER 12

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4631E+03	0.1017E+02	0.1000E+02
2	0.3697E+03	0.8223E+01	0.1000E+02
3	0.3316E+03	0.7528E+01	0.1000E+02
4	0.3348E+03	0.7392E+01	0.1000E+02
5	0.4033E+03	0.1097E+02	0.1000E+02

PRODUCER NUMBER 13

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.7164E+03	0.2247E+02	0.1000E+02
2	0.9033E+03	0.3484E+02	0.1000E+02
3	0.7131E+03	0.1981E+02	0.1000E+02
4	0.5381E+03	0.1561E+02	0.1000E+02
5	0.6066E+03	0.2234E+02	0.1000E+02

APPENDIX J

RESULTS OF STEAMFLOOD PREDICTION OF SHIELLS CANYON FIELD AS A HOMOGENEOUS RESERVOIR. PART OF ITS BOUNDARY AT CONSTANT PRESSURE, THE REMAINDER SEALED

APPENDIX J

STEAMFLOOD PREDICTION OF SHIELLS CANYON(203) FIELD

METHOD: STREAMLINE/STREAMTUBE METHOD USING (BEM)TECHNIQUE

ANALYSIS AS A SINGLE HOMOGENEOUS RESERVOIR

BOUNDARY CONDITION: PART OF THE BOUNDARY AT CONSTANT

PRESSURE, THE REMAINDER IS SEALED.

PRODUCTION RATES: ASSIGNED EQUAL IN ENTIRE REGION

BY: D. T. NUMBERE, UNIVERSITY OF OKLAHOMA,1982

DATA

TOTAL NUMBER OF PRODUCERS=	11
STEAM TEMPERATURE (DEG. F)=	430.0000
STEAM QUALITY=	0.7000
CONVERGENCE LIMIT=	0.0010
RESERVOIR THICKNESS (FT)=	160.0000
STEAM PRESSURE (PSI)=	344.0000
INITIAL RESERVOIR TEMPERATURE(DEG F)=	105.0000

FLUID PROPERTIES

	WATER	OIL	STEAM
DENSITY(LB/CU FT) AT STD. TEMP.	62.4000	55.0000	
DENSITY(LB/CU FT) AT STEAM TEMP.	52.3800	48.7000	0.7431
SPECIFIC HEAT(BTU/LB *F)	1.0000	0.4880	1.0000
LATENT HEAT(BTU/LB)			789.5100

PROPERTIES OF THE CAP AND BASE ROCK

DENSITY(LB/CU. FT)	149.0000
SPECIFIC HEAT(BTU/LB-*F)	0.2130
THERM. COND.(BTU/HR-FT-*F)	1.1000

RESERVCIR ROCK PROPERTIES

	REGION 1	REGION 2
POROSITY	0.2050	0.1800
DEN.(LB/CU FT)	165.0000	165.0000
SPEC. HEAT(BTU/LB*F)	0.2000	0.2000
PERMEABILITY(MD)	140.0000	70.0000

INITIAL FLUID SATURATICNS

	REGION 1	REGICN 2
WATER	0.5500	0.5500
OIL	0.4500	0.4500
STEAM	0.0	0.0

AVERAGE RESIDUAL SATURATIONS BEHIND THE FRONT

	REGION 1	REGICN 2
WATER	0.5800	0.5800
OIL	0.1800	0.1800
STEAM	0.2400	0.2400

INPUT DATA FROM STREAMLINE PROGRAM

			REGION 1			REGION 2		
WELL NO.	S/L NO.	CODE	LENGTH	WIDTH	RATE	LENGTH	WIDTH	RATE
1	1	1	541.40	45.84	28.00	0.0	0.0	0.0
1	2	1	425.00	23.67	28.00	0.0	0.0	0.0
1	3	1	678.30	73.69	28.00	0.0	0.0	0.0
2	1	1	441.40	27.62	28.00	0.0	0.0	0.0
2	2	1	344.70	19.14	28.00	0.0	0.0	0.0
3	1	1	338.10	18.06	28.00	0.0	0.0	0.0
3	2	1	384.80	21.62	28.00	0.0	0.0	0.0
4	1	1	269.70	13.51	28.00	0.0	0.0	0.0
4	2	1	269.70	14.91	28.00	0.0	0.0	0.0
5	1	1	359.80	15.91	28.00	0.0	0.0	0.0
5	2	1	338.10	14.26	28.00	0.0	0.0	0.0
6	1	1	294.70	12.70	28.00	0.0	0.0	0.0
6	2	1	288.10	11.96	28.00	0.0	0.0	0.0
7	1	1	809.80	29.43	28.00	0.0	0.0	0.0
7	2	1	759.80	28.67	28.00	0.0	0.0	0.0
8	1	1	1120.00	38.69	28.00	0.0	0.0	0.0
8	2	1	950.00	34.38	28.00	0.0	0.0	0.0
8	3	1	588.10	15.55	10.00	0.0	0.0	0.0
8	4	1	569.70	12.31	10.00	0.0	0.0	0.0
8	5	1	598.00	13.79	10.00	0.0	0.0	0.0
9	1	1	266.40	6.27	10.00	0.0	0.0	0.0
9	2	1	291.40	6.75	10.00	0.0	0.0	0.0
9	3	1	378.30	8.92	10.00	0.0	0.0	0.0
9	4	1	288.10	6.46	10.00	0.0	0.0	0.0
10	1	1	463.10	10.17	10.00	0.0	0.0	0.0
10	2	1	369.70	8.22	10.00	0.0	0.0	0.0
10	3	1	331.60	7.53	10.00	0.0	0.0	0.0
10	4	1	334.80	7.39	10.00	0.0	0.0	0.0
10	5	1	403.30	10.97	10.00	0.0	0.0	0.0
11	1	1	716.40	22.47	10.00	0.0	0.0	0.0
11	2	1	903.30	34.84	10.00	0.0	0.0	0.0
11	3	1	713.10	19.81	10.00	0.0	0.0	0.0
11	4	1	538.10	15.61	10.00	0.0	0.0	0.0
11	5	1	606.60	22.34	10.00	0.0	0.0	0.0

CALCULATED BREAKTHROUGH TIMES(HOURS) FOR EACH STREAMTUBE

PROD. NO.	STREAMLINE NO.	CODE	ENDTIME(1)	ENDTIME(2)
1	1	1	16232.3555	0.0
1	2	1	6122.6797	0.0
1	3	1	35560.2891	0.0
2	1	1	7514.6562	0.0
2	2	1	3921.2864	0.0
3	1	1	3615.2166	0.0
3	2	1	5006.4922	0.0
4	1	1	2110.3652	0.0
4	2	1	2337.7622	0.0
5	1	1	3378.9150	0.0
5	2	1	2823.9495	0.0
6	1	1	2169.8333	0.0
6	2	1	1991.6033	0.0
7	1	1	15525.5352	0.0
7	2	1	14069.7266	0.0
8	1	1	30226.8555	0.0
8	2	1	21999.7695	0.0
8	3	1	16800.4453	0.0
8	4	1	12564.0469	0.0
8	5	1	14995.8437	0.0
9	1	1	2735.7625	0.0
9	2	1	3243.0010	0.0
9	3	1	5729.8828	0.0
9	4	1	3062.2073	0.0
10	1	1	8173.4727	0.0
10	2	1	5129.1250	0.0
10	3	1	4166.4844	0.0
10	4	1	4128.8320	0.0
10	5	1	7644.0664	0.0
11	1	1	31607.5586	0.0
11	2	1	68893.5000	0.0
11	3	1	27266.1172	0.0
11	4	1	15301.9453	0.0
11	5	1	26020.0859	0.0

REAL TIME(HOURS)= 6000.0000

DIMENSIONLESS TIME= 0.0325

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.1557E+05		
1	2	1	0.1557E+05		
1	3	1	0.1557E+05		
				0.4670E+05	
2	1	1	0.1557E+05		
2	2	1	0.1041E+05		
				0.2597E+05	
3	1	1	0.9631E+04		
3	2	1	0.1312E+05		
				0.2275E+05	
4	1	1	0.5747E+04		
4	2	1	0.6342E+04		
				0.1209E+05	
5	1	1	0.9029E+04		
5	2	1	0.7604E+04		
				0.1663E+05	
6	1	1	0.5903E+04		
6	2	1	0.5435E+04		
				0.1134E+05	
7	1	1	0.1557E+05		
7	2	1	0.1557E+05		
				0.3113E+05	
8	1	1	0.1557E+05		
8	2	1	0.1557E+05		
8	3	1	0.5560E+04		
8	4	1	0.5560E+04		
8	5	1	0.5560E+04		
				0.4781E+05	
9	1	1	0.2634E+04		
9	2	1	0.3100E+04		
9	3	1	0.5323E+04		
9	4	1	0.2935E+04		
				0.1399E+05	
10	1	1	0.5560E+04		
10	2	1	0.4795E+04		
10	3	1	0.3937E+04		
10	4	1	0.3903E+04		
10	5	1	0.5560E+04		
				0.2375E+05	
11	1	1	0.5560E+04		
11	2	1	0.5560E+04		
11	3	1	0.5560E+04		
11	4	1	0.5560E+04		
11	5	1	0.5560E+04		
				0.2780E+05	
					0.2800E+06

REAL TIME(HCURS)= 12000.0000

DIMENSIONLESS TIME= 0.0650

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.2969E+05		
1	2	1	0.1587E+05		
1	3	1	0.2969E+05		
				0.7525E+05	
2	1	1	0.1923E+05		
2	2	1	0.1041E+05		
				0.2963E+05	
3	1	1	0.9631E+04		
3	2	1	0.1312E+05		
				0.2275E+05	
4	1	1	0.5747E+04		
4	2	1	0.6342E+04		
				0.1209E+05	
5	1	1	0.9029E+04		
5	2	1	0.7604E+04		
				0.1663E+05	
6	1	1	0.5903E+04		
6	2	1	0.5435E+04		
				0.1134E+05	
7	1	1	0.2969E+05		
7	2	1	0.2969E+05		
				0.5938E+05	
8	1	1	0.2969E+05		
8	2	1	0.2969E+05		
8	3	1	0.1060E+05		
8	4	1	0.1060E+05		
8	5	1	0.1060E+05		
				0.9119E+05	
9	1	1	0.2634E+04		
9	2	1	0.3100E+04		
9	3	1	0.5323E+04		
9	4	1	0.2935E+04		
				0.1399E+05	
10	1	1	0.7428E+04		
10	2	1	0.4795E+04		
10	3	1	0.3937E+04		
10	4	1	0.3903E+04		
10	5	1	0.6978E+04		
				0.2704E+05	
11	1	1	0.1060E+05		
11	2	1	0.1060E+05		
11	3	1	0.1060E+05		
11	4	1	0.1060E+05		
11	5	1	0.1060E+05		
				0.5302E+05	
					0.4123E+06

REAL TIME(HOURS)= 18000.0000

DIMENSIONLESS TIME= 0.0975

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.3914E+05		
1	2	1	0.1587E+05		
1	3	1	0.4299E+05		
				0.9800E+05	
2	1	1	0.1923E+05		
2	2	1	0.1041E+05		
				0.2963E+05	
3	1	1	0.9631E+04		
3	2	1	0.1312E+05		
				0.2275E+05	
4	1	1	0.5747E+04		
4	2	1	0.6342E+04		
				0.1209E+05	
5	1	1	0.9029E+04		
5	2	1	0.7604E+04		
				0.1663E+05	
6	1	1	0.5903E+04		
6	2	1	0.5435E+04		
				0.1134E+05	
7	1	1	0.3759E+05		
7	2	1	0.3436E+05		
				0.7195E+05	
8	1	1	0.4299E+05		
8	2	1	0.4299E+05		
8	3	1	0.1442E+05		
8	4	1	0.1106E+05		
8	5	1	0.1301E+05		
				0.1245E+06	
9	1	1	0.2634E+04		
9	2	1	0.3100E+04		
9	3	1	0.5323E+04		
9	4	1	0.2935E+04		
				0.1399E+05	
10	1	1	0.7428E+04		
10	2	1	0.4795E+04		
10	3	1	0.3937E+04		
10	4	1	0.3903E+04		
10	5	1	0.6978E+04		
				0.2704E+05	
11	1	1	0.1535E+05		
11	2	1	0.1535E+05		
11	3	1	0.1535E+05		
11	4	1	0.1325E+05		
11	5	1	0.1535E+05		
				0.7466E+05	
					0.5026E+06

REAL TIME(HOURS)= 24000.0000

DIMENSIONLESS TIME= 0.1300

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.3914E+05		
1	2	1	0.1587E+05		
1	3	1	0.5569E+05		
				0.1107E+06	
2	1	1	0.1923E+05		
2	2	1	0.1041E+05		
				0.2963E+05	
3	1	1	0.9631E+04		
3	2	1	0.1312E+05		
				0.2275E+05	
4	1	1	0.5747E+04		
4	2	1	0.6342E+04		
				0.1209E+05	
5	1	1	0.9029E+04		
5	2	1	0.7604E+04		
				0.1663E+05	
6	1	1	0.5903E+04		
6	2	1	0.5435E+04		
				0.1134E+05	
7	1	1	0.3759E+05		
7	2	1	0.3436E+05		
				0.7195E+05	
8	1	1	0.5569E+05		
8	2	1	0.5151E+05		
8	3	1	0.1442E+05		
8	4	1	0.1106E+05		
8	5	1	0.1301E+05		
				0.1457E+06	
9	1	1	0.2634E+04		
9	2	1	0.3100E+04		
9	3	1	0.5323E+04		
9	4	1	0.2935E+04		
				0.1399E+05	
10	1	1	0.7428E+04		
10	2	1	0.4795E+04		
10	3	1	0.3937E+04		
10	4	1	0.3903E+04		
10	5	1	0.6978E+04		
				0.2704E+05	
11	1	1	0.1989E+05		
11	2	1	0.1989E+05		
11	3	1	0.1989E+05		
11	4	1	0.1325E+05		
11	5	1	0.1989E+05		
				0.9280E+05	
					0.5546E+06

REAL TIME(HOURS)= 30000.0000

DIMENSIONLESS TIME= 0.1625

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.3914E+05		
1	2	1	0.1587E+05		
1	3	1	0.6789E+05		
				0.1229E+06	
2	1	1	0.1923E+05		
2	2	1	0.1041E+05		
				0.2963E+05	
3	1	1	0.9631E+04		
3	2	1	0.1312E+05		
				0.2275E+05	
4	1	1	0.5747E+04		
4	2	1	0.6342E+04		
				0.1209E+05	
5	1	1	0.9029E+04		
5	2	1	0.7604E+04		
				0.1663E+05	
6	1	1	0.5903E+04		
6	2	1	0.5435E+04		
				0.1134E+05	
7	1	1	0.3759E+05		
7	2	1	0.3436E+05		
				0.7195E+05	
8	1	1	0.6789E+05		
8	2	1	0.5151E+05		
8	3	1	0.1442E+05		
8	4	1	0.1106E+05		
8	5	1	0.1301E+05		
				0.1579E+06	
9	1	1	0.2634E+04		
9	2	1	0.3100E+04		
9	3	1	0.5323E+04		
9	4	1	0.2935E+04		
				0.1399E+05	
10	1	1	0.7428E+04		
10	2	1	0.4795E+04		
10	3	1	0.3937E+04		
10	4	1	0.3903E+04		
10	5	1	0.6978E+04		
				0.2704E+05	
11	1	1	0.2425E+05		
11	2	1	0.2425E+05		
11	3	1	0.2228E+05		
11	4	1	0.1325E+05		
11	5	1	0.2137E+05		
				0.1054E+06	
					0.5916E+06

PREDICTED RECOVERY

TIME(DAYS)	RECOVERY(BBLS)
62.5000	0.1697E+05
125.0000	0.3142E+05
187.5000	0.4171E+05
250.0000	0.4986E+05
312.5000	0.5674E+05
375.0000	0.6249E+05
437.5000	0.6800E+05
500.0000	0.7343E+05
562.5000	0.7865E+05
625.0000	0.8334E+05
687.5000	0.8689E+05
750.0000	0.8950E+05
812.5000	0.9205E+05
875.0000	0.9456E+05
937.5000	0.9687E+05
1000.0000	0.9877E+05
1062.5000	0.1007E+06
1125.0000	0.1024E+06
1187.5000	0.1039E+06
1250.0000	0.1054E+06

APPENDIX K

RESULTS OF STREAMLINE MODELLING OF SHIELLS CANYON FIELD AS A PIECEWISE HOMOGENEOUS RESERVOIR WITH SEALED OUTER BOUNDARY

BOUNDARY ELEMENT MODELLING OF SHIELLS CANYON(203) FIELD

ANALYSIS AS A PIECEWISE-HOMOGENEOUS RESERVIOR

SEALED OUTER BOUNDARY

BY: D. T. NUMBERE, UNIVERSITY OF OKLAHOMA, 1981

REGION 1.0

DATA

NUMBER OF BOUNDARY ELEMENTS= 43
NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS CALCULATED= 0
NUMBER OF SOURCES AND SINKS= 7
NUMBER OF INTERFACE NODES= 10

PERMEABILITY(MD)= 140.0000

THICKNESS(FT)= 160.0000

POROSITY= 0.2050

SCALE: 1 INCH = 738.46FEET

THE COORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELEMENTS

POINT	X(INCH)	Y(INCH)
1	1.5000	1.5500
2	1.3500	1.4800
3	1.2500	1.4400
4	1.2000	1.4000
5	1.1500	1.3800
6	1.0500	1.3250
7	0.9500	1.2600
8	0.9000	1.2500
9	0.8500	1.2000
10	0.8000	1.1700
11	0.7500	1.1500
12	0.6500	1.0800
13	0.5500	1.0200
14	0.5000	0.9900
15	0.4500	0.9500
16	0.4000	0.9200
17	0.3000	0.8500
18	0.1000	0.7300
19	0.1000	0.5000
20	0.1000	0.1500
21	0.2500	0.2000
22	0.4000	0.2500
23	0.5500	0.2800
24	0.6500	0.3000
25	0.7000	0.3250
26	0.7500	0.3300
27	0.8000	0.3400
28	0.8500	0.3500
29	0.9000	0.4000
30	1.0000	0.4500
31	1.1000	0.5250
32	1.2500	0.6000
33	1.4000	0.6500
34	1.5500	0.6800
35	1.5500	0.7500
36	1.5400	0.8500
37	1.5000	0.9500
38	1.5000	1.0500
39	1.4800	1.1500
40	1.4600	1.2500
41	1.4600	1.3500
42	1.4800	1.4500
43	1.5000	1.5000

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	1	0.0
2	1	0.0
3	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0
7	1	0.0
8	1	0.0
9	1	0.0
10	1	0.0
11	1	0.0
12	1	0.0
13	1	0.0
14	1	0.0
15	1	0.0
16	1	0.0
17	1	0.0
18	1	0.0
19	1	0.0
20	1	0.0
21	1	0.0
22	1	0.0
23	1	0.0
24	1	0.0
25	1	0.0
26	1	0.0
27	1	0.0
28	1	0.0
29	1	0.0
30	1	0.0
31	1	0.0
32	1	0.0
33	1	0.0
34	2	0.0
35	2	0.0
36	2	0.0
37	2	0.0
38	2	0.0
39	2	0.0
40	2	0.0
41	2	0.0
42	2	0.0
43	2	0.0

COORDINATES OF SOURCES AND SINKS AND THEIR STRENGTHS

X(INCH)	Y(INCH)	RATE(BBL/D)
0.4500	0.5500	-69.1000
0.5000	0.8000	-69.1000
0.7500	0.4000	-69.1000
0.9000	1.1500	-69.1000
1.0000	0.8000	560.0000
1.2500	1.2000	-69.1000
1.3500	1.0000	-69.1000

THE AVERAGE RADIUS OF THE WELLS(INCH) IS= 0.500
THE CHARACTERISTIC PRESSURE(PSI) IS= 85.0000

REGION 2.0

DATA

NUMBER OF BOUNDARY ELEMENTS= 37
NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS CALCULATED= 0
NUMBER OF SOURCES AND SINKS= 6
NUMBER OF INTERFACE NODES= 10

PERMEABILITY(MD)= 70.0000

THICKNESS(FT)= 160.0000

POROSITY= 0.2050

SCALE: 1 INCH = 738.46FEET

THE COORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELEMENTS

POINT	X(INCH)	Y(INCH)
1	1.5000	1.5500
2	1.5000	1.5000
3	1.4800	1.4500
4	1.4600	1.3500
5	1.4600	1.2500
6	1.4800	1.1500
7	1.5000	1.0500
8	1.5000	0.9500
9	1.5400	0.8500
10	1.5500	0.7500
11	1.5500	0.6800
12	1.6500	0.6900
13	1.7000	0.7000
14	1.7500	0.7100
15	1.8000	0.7200
16	1.8500	0.7250
17	1.9000	0.7300
18	1.9500	0.7350
19	2.1000	0.7450
20	2.2500	0.7500
21	2.4000	0.7500
22	2.6000	0.7400
23	2.7500	0.7300
24	2.9000	0.7250
25	3.1600	1.0000
26	3.0600	1.2500
27	2.9500	1.5500
28	2.7500	1.6500
29	2.4000	1.8200
30	2.2500	1.8000
31	2.1000	1.7500
32	2.0500	1.7450
33	2.0000	1.7250
34	1.9500	1.7100
35	1.8500	1.6900
36	1.7500	1.6500
37	1.6500	1.6100

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	2	0.0
2	2	0.0
3	2	0.0
4	2	0.0
5	2	0.0
6	2	0.0
7	2	0.0
8	2	0.0
9	2	0.0
10	2	0.0
11	1	0.0
12	1	0.0
13	1	0.0
14	1	0.0
15	1	0.0
16	1	0.0
17	1	0.0
18	1	0.0
19	1	0.0
20	1	0.0
21	1	0.0
22	1	0.0
23	1	0.0
24	1	0.0
25	1	0.0
26	1	0.0
27	1	0.0
28	1	0.0
29	1	0.0
30	1	0.0
31	1	0.0
32	1	0.0
33	1	0.0
34	1	0.0
35	1	0.0
36	1	0.0
37	1	0.0

COORDINATES OF SOURCES AND SINKS AND THEIR STRENGTHS

X(INCH)	Y(INCH)	RATE(BBL/D)
1.9000	0.9500	200.0000
1.8000	1.3500	-69.1000
2.0500	1.5500	-69.1000
2.1500	1.2500	-69.1000
2.3500	1.0500	-69.1000
2.5750	1.3000	-69.1000

THE AVERAGE RADIUS OF THE WELLS(INCH) IS= 0.500
THE CHARACTERISTIC PRESSURE(PSI) IS= 85.0000

RESULTS

BOUNDARY NODES

X(INCH)	Y(INCH)	PRESSURE(PSI)	NORMAL GRADIENT
0.1425E+01	0.1515E+01	-0.5609E+02	0.0
0.1300E+01	0.1460E+01	-0.5464E+02	0.0
0.1225E+01	0.1420E+01	-0.5398E+02	0.0
0.1175E+01	0.1390E+01	-0.5335E+02	0.0
0.1100E+01	0.1352E+01	-0.5260E+02	0.0
0.1000E+01	0.1292E+01	-0.5159E+02	0.0
0.9250E+00	0.1255E+01	-0.5107E+02	0.0
0.8750E+00	0.1225E+01	-0.5091E+02	0.0
0.8250E+00	0.1185E+01	-0.5035E+02	0.0
0.7750E+00	0.1160E+01	-0.4976E+02	0.0
0.7000E+00	0.1115E+01	-0.4942E+02	0.0
0.6000E+00	0.1050E+01	-0.4958E+02	0.0
0.5250E+00	0.1005E+01	-0.5004E+02	0.0
0.4750E+00	0.9700E+00	-0.5041E+02	0.0
0.4250E+00	0.9350E+00	-0.5080E+02	0.0
0.3500E+00	0.8850E+00	-0.5113E+02	0.0
0.2000E+00	0.7900E+00	-0.5137E+02	0.0
0.1000E+00	0.6150E+00	-0.5143E+02	0.0
0.1000E+00	0.3250E+00	-0.5139E+02	0.0
0.1750E+00	0.1750E+00	-0.5119E+02	0.0
0.3250E+00	0.2250E+00	-0.5141E+02	0.0
0.4750E+00	0.2650E+00	-0.5127E+02	0.0
0.6000E+00	0.2900E+00	-0.5104E+02	0.0
0.6750E+00	0.3125E+00	-0.5092E+02	0.0
0.7250E+00	0.3275E+00	-0.5087E+02	0.0
0.7750E+00	0.3350E+00	-0.5064E+02	0.0
0.8250E+00	0.3450E+00	-0.5009E+02	0.0
0.8750E+00	0.3750E+00	-0.4956E+02	0.0
0.9500E+00	0.4250E+00	-0.4820E+02	0.0
0.1050E+01	0.4875E+00	-0.4710E+02	0.0
0.1175E+01	0.5625E+00	-0.4686E+02	0.0
0.1325E+01	0.6250E+00	-0.4866E+02	0.0
0.1475E+01	0.6650E+00	-0.5054E+02	0.0
0.1550E+01	0.7150E+00	-0.5120E+02	-0.8409E-02
0.1545E+01	0.8000E+00	-0.5147E+02	-0.7983E-02
0.1520E+01	0.9000E+00	-0.5197E+02	-0.8892E-02
0.1500E+01	0.1000E+01	-0.5283E+02	-0.4986E-02
0.1490E+01	0.1100E+01	-0.5365E+02	-0.8297E-02
0.1470E+01	0.1200E+01	-0.5434E+02	-0.9221E-02
0.1460E+01	0.1300E+01	-0.5503E+02	-0.9038E-02
0.1470E+01	0.1400E+01	-0.5580E+02	-0.9774E-02
0.1490E+01	0.1475E+01	-0.5652E+02	-0.1061E-01
0.1500E+01	0.1525E+01	-0.5741E+02	-0.1380E-01

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RESULTS

BOUNDARY NODES

X(INCH)	Y(INCH)	PRESSURE(PSI)	NORMAL GRADIENT
0.1500E+01	0.1525E+01	-0.5741E+02	0.2760E-01
0.1490E+01	0.1475E+01	-0.5652E+02	0.2121E-01
0.1470E+01	0.1400E+01	-0.5580E+02	0.1955E-01
0.1460E+01	0.1300E+01	-0.5503E+02	0.1808E-01
0.1470E+01	0.1200E+01	-0.5434E+02	0.1844E-01
0.1490E+01	0.1100E+01	-0.5365E+02	0.1659E-01
0.1500E+01	0.1000E+01	-0.5283E+02	0.9972E-02
0.1520E+01	0.9000E+00	-0.5197E+02	0.1778E-01
0.1545E+01	0.8000E+00	-0.5147E+02	0.1597E-01
0.1550E+01	0.7150E+00	-0.5120E+02	0.1682E-01
0.1600E+01	0.6850E+00	-0.5169E+02	0.0
0.1675E+01	0.6950E+00	-0.5239E+02	0.0
0.1725E+01	0.7050E+00	-0.5283E+02	0.0
0.1775E+01	0.7150E+00	-0.5326E+02	0.0
0.1825E+01	0.7225E+00	-0.5374E+02	0.0
0.1875E+01	0.7275E+00	-0.5431E+02	0.0
0.1925E+01	0.7325E+00	-0.5507E+02	0.0
0.2025E+01	0.7400E+00	-0.5723E+02	0.0
0.2175E+01	0.7475E+00	-0.6092E+02	0.0
0.2325E+01	0.7500E+00	-0.6386E+02	0.0
0.2500E+01	0.7450E+00	-0.6587E+02	0.0
0.2675E+01	0.7350E+00	-0.6679E+02	0.0
0.2825E+01	0.7275E+00	-0.6714E+02	0.0
0.3030E+01	0.8625E+00	-0.6741E+02	0.0
0.3110E+01	0.1125E+01	-0.6763E+02	0.0
0.3005E+01	0.1400E+01	-0.6795E+02	0.0
0.2850E+01	0.1600E+01	-0.6800E+02	0.0
0.2575E+01	0.1735E+01	-0.6759E+02	0.0
0.2325E+01	0.1810E+01	-0.6705E+02	0.0
0.2175E+01	0.1775E+01	-0.6675E+02	0.0
0.2075E+01	0.1747E+01	-0.6637E+02	0.0
0.2025E+01	0.1735E+01	-0.6602E+02	0.0
0.1975E+01	0.1717E+01	-0.6551E+02	0.0
0.1900E+01	0.1700E+01	-0.6445E+02	0.0
0.1800E+01	0.1670E+01	-0.6301E+02	0.0
0.1700E+01	0.1630E+01	-0.6123E+02	0.0
0.1575E+01	0.1580E+01	-0.5877E+02	0.0

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PRODUCER NUMBER 1

NUMBER OF STREAMLINES= 3

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.5533E+03	0.5142E+02	0.2800E+02
2	0.4250E+03	0.2374E+02	0.2800E+02
3	0.6697E+03	0.7554E+02	0.2800E+02

PRODUCER NUMBER 2

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4447E+03	0.2874E+02	0.2800E+02
2	0.3447E+03	0.1926E+02	0.2800E+02

PRODUCER NUMBER 3

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.3381E+03	0.1804E+02	0.2800E+02
2	0.3881E+03	0.2164E+02	0.2800E+02

PRODUCER NUMBER 4

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2730E+03	0.1335E+02	0.2800E+02
2	0.2664E+03	0.1458E+02	0.2800E+02

PRODUCER NUMBER 6

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.3664E+03	0.1561E+02	0.2800E+02
2	0.3348E+03	0.1371E+02	0.2800E+02

PRODUCER NUMBER 7

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2881E+03	0.1253E+02	0.2800E+02
2	0.2947E+03	0.1203E+02	0.2800E+02

PRODUCER NUMBER 2

NUMBER OF STREAMLINES 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4523E+03	0.5838E+02	0.2800E+02
1	0.2164E+03	0.2901E+02	0.2800E+02
2	0.4779E+03	0.5484E+C2	0.2800E+02
2	0.2816E+03	0.4460E+02	0.2800E+02

PRODUCER NUMBER 3

NUMBER OF STREAMLINES= 3

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.6447E+03	0.2909E+02	0.1000E+02
2	0.5881E+03	0.1317E+02	0.1000E+02
3	0.6164E+03	0.1461E+02	0.1000E+02
1	0.3765E+03	0.5957E+02	0.2800E+02
1	0.5348E+03	0.4241E+02	0.2800E+02

PRODUCER NUMBER 4

NUMBER OF STREAMLINES= 4

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2664E+03	0.6362E+01	0.1000E+02
2	0.2881E+03	0.6823E+01	0.1000E+02
3	0.3664E+03	0.8241E+01	0.1000E+02
4	0.2881E+03	0.6602E+01	0.1000E+02

PRODUCER NUMBER 5

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4750E+03	0.1096E+02	0.1000E+02
2	0.3848E+03	0.8447E+01	0.1000E+02
3	0.3348E+03	0.7668E+01	0.1000E+02
4	0.3316E+03	0.7443E+01	0.1000E+02
5	0.3881E+03	0.9796E+01	0.1000E+02

PRODUCER NUMBER 6

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.7033E+03	0.2170E+02	0.1000E+02
2	0.8447E+03	0.2877E+02	0.1000E+02
3	0.7197E+03	0.1975E+02	0.1000E+02
4	0.5381E+03	0.1590E+02	0.1000E+02
5	0.5914E+03	0.2104E+02	0.1000E+02

NO STREAMLINE FROM REGION 2 CROSSES INTO REGION 1

APPENDIX L

STEAMFLOOD PREDICTION OF SHIELLS CANYON FIELD ANALYSED AS A PIECEWISE HOMOGENEOUS RESERVOIR HAVING SEALED BOUNDARY

APPENDIX L

STEAMFLOOD PREDICTION OF SHIELLS CANYON(203) FIELD

METHOD: STREAMLINE/STREAMTUBE METHOD USING (BEM)TECHNIQUE

ANALYSIS AS A PIECE-WISE HOMOGENEOUS RESERVOIR

HAVING TWO REGIONS OF UNEQUAL PERMEABILITY

BOUNDARY CONDITION: SEALED OUTER BOUNDARY

PRODUCTION RATES: ASSIGNED EQUAL IN ENTIRE REGION

BY: D. T. NUMBERE, MARCH 1982

DATA

TOTAL NUMBER OF PRODUCERS=	11
STEAM TEMPERATURE (DEG. F)=	430.0000
STEAM QUALITY=	0.7000
CONVERGENCE LIMIT=	0.0010
RESERVOIR THICKNESS (FT)=	160.0000
STEAM PRESSURE (PSI)=	344.0000
INITIAL RESERVOIR TEMPERATURE(DEG F)=	105.0000

FLUID PROPERTIES

	WATER	OIL	STEAM
DENSITY(LB/CU FT) AT STD. TEMP.	62.4000	55.0000	
DENSITY(LB/CU FT) AT STEAM TEMP.	52.3800	48.7000	0.7431
SPECIFIC HEAT(BTU/LB *F)	1.0000	0.4860	1.0000
LATENT HEAT(BTU/LB)			789.5100

PROPERTIES OF THE CAP AND BASE ROCK

DENSITY(LB/CU. FT)	149.0000
SPECIFIC HEAT(BTU/LB-*F)	0.2130
THERM. COND.(BTU/HR-FT-*F)	1.1000

RESERVOIR ROCK PROPERTIES

	REGION 1	REGION 2
POROSITY	0.2050	0.1800
DEN.(LB/CU FT)	165.0000	165.0000
SPEC. HEAT(BTU/LB*F)	0.2000	0.2000
PERMEABILITY(MD)	140.0000	70.0000

INITIAL FLUID SATURATIONS

	REGION 1	REGION 2
WATER	0.5500	0.5500
OIL	0.4500	0.4500
STEAM	0.0	0.0

AVERAGE RESIDUAL SATURATIONS BEHIND THE FRONT

	REGION 1	REGION 2
WATER	0.5800	0.5800
OIL	0.1800	0.1800
STEAM	0.2400	0.2400

INPUT DATA FROM STREAMLINE PROGRAM

			REGION 1			REGION 2		
WELL NO.	S/L NO.	CODE	LENGTH	WIDTH	RATE	LENGTH	WIDTH	RATE
1	1	1	553.30	51.42	28.00	0.0	0.0	0.0
1	2	1	425.00	23.74	28.00	0.0	0.0	0.0
1	3	1	669.70	75.54	28.00	0.0	0.0	0.0
2	1	1	444.70	28.74	28.00	0.0	0.0	0.0
2	2	1	344.70	19.26	28.00	0.0	0.0	0.0
3	1	1	338.10	18.04	28.00	0.0	0.0	0.0
3	2	1	388.10	21.64	28.00	0.0	0.0	0.0
4	1	1	273.00	13.35	28.00	0.0	0.0	0.0
4	2	1	266.40	14.58	28.00	0.0	0.0	0.0
5	1	1	366.40	15.51	28.00	0.0	0.0	0.0
5	2	1	334.80	13.71	28.00	0.0	0.0	0.0
6	1	1	288.10	12.53	28.00	0.0	0.0	0.0
6	2	1	294.70	12.03	28.00	0.0	0.0	0.0
7	1	2	452.30	58.38	28.00	216.40	29.01	28.00
7	2	2	477.90	54.84	28.00	281.60	44.60	28.00
8	1	2	376.50	59.57	28.00	534.80	42.41	28.00
8	2	1	644.70	29.09	10.00	0.0	0.0	0.0
8	3	1	588.10	13.17	10.00	0.0	0.0	0.0
8	4	1	616.40	14.61	10.00	0.0	0.0	0.0
9	1	1	266.40	6.36	10.00	0.0	0.0	0.0
9	2	1	288.10	6.82	10.00	0.0	0.0	0.0
9	3	1	366.40	8.24	10.00	0.0	0.0	0.0
9	4	1	288.10	6.60	10.00	0.0	0.0	0.0
10	1	1	475.00	10.96	10.00	0.0	0.0	0.0
10	2	1	384.80	8.45	10.00	0.0	0.0	0.0
10	3	1	334.80	7.67	10.00	0.0	0.0	0.0
10	4	1	331.60	7.44	10.00	0.0	0.0	0.0
10	5	1	388.10	9.80	10.00	0.0	0.0	0.0
11	1	1	703.30	21.70	10.00	0.0	0.0	0.0
11	2	1	844.70	28.77	10.00	0.0	0.0	0.0
11	3	1	719.70	19.75	10.00	0.0	0.0	0.0
11	4	1	538.10	15.90	10.00	0.0	0.0	0.0
11	5	1	591.40	21.04	10.00	0.0	0.0	0.0

CALCULATED BREAKTHROUGH TIMES(HOURS) FOR EACH STREAMTUBE

PROD. NO.	STREAMLINE NO.	CODE	ENDTIME(1)	ENDTIME(2)
1	1	1	18872.6953	0.0
1	2	1	6142.0391	0.0
1	3	1	36053.1523	0.0
2	1	1	7903.7891	0.0
2	2	1	3947.0635	0.0
3	1	1	3611.0315	0.0
3	2	1	5056.8008	0.0
4	1	1	2110.9402	0.0
4	2	1	2255.1360	0.0
5	1	1	3375.7742	0.0
5	2	1	2682.9524	0.0
6	1	1	2090.1030	0.0
6	2	1	2051.3933	0.0
7	1	2	17379.4336	21095.7500
7	2	2	17236.3633	24982.4414
8	1	2	14525.8516	29210.0820
8	2	1	37627.2305	0.0
8	3	1	14001.4336	0.0
8	4	1	16518.7578	0.0
9	1	1	2777.7727	0.0
9	2	1	3242.8152	0.0
9	3	1	5092.4102	0.0
9	4	1	3133.0645	0.0
10	1	1	9101.8555	0.0
10	2	1	5505.5273	0.0
10	3	1	4291.3516	0.0
10	4	1	4117.0391	0.0
10	5	1	6502.2969	0.0
11	1	1	29753.0977	0.0
11	2	1	50784.7969	0.0
11	3	1	27456.6445	0.0
11	4	1	15614.1445	0.0
11	5	1	23645.6953	0.0

REAL TIME(HOURS)= 6000.0000

DIMENSIONLESS TIME= 0.0325

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.1557E+05		
1	2	1	0.1557E+05		
1	3	1	0.1557E+05		
				0.4670E+05	
2	1	1	0.1557E+05		
2	2	1	0.1047E+05		
				0.2604E+05	
3	1	1	0.9620E+04		
3	2	1	0.1325E+05		
				0.2287E+05	
4	1	1	0.5748E+04		
4	2	1	0.6126E+04		
				0.1187E+05	
5	1	1	0.9021E+04		
5	2	1	0.7240E+04		
				0.1626E+05	
6	1	1	0.5694E+04		
6	2	1	0.5592E+04		
				0.1129E+05	
7	1	2	0.1557E+05		
7	2	2	0.1557E+05		
				0.3113E+05	
8	1	2	0.1557E+05		
8	2	1	0.5560E+04		
8	3	1	0.5560E+04		
8	4	1	0.5560E+04		
				0.3225E+05	
9	1	1	0.2673E+04		
9	2	1	0.3100E+04		
9	3	1	0.4762E+04		
9	4	1	0.3000E+04		
				0.1354E+05	
10	1	1	0.5560E+04		
10	2	1	0.5127E+04		
10	3	1	0.4049E+04		
10	4	1	0.3893E+04		
10	5	1	0.5560E+04		
				0.2419E+05	
11	1	1	0.5560E+04		
11	2	1	0.5560E+04		
11	3	1	0.5560E+04		
11	4	1	0.5560E+04		
11	5	1	0.5560E+04		
				0.2780E+05	
					0.2639E+06

REAL TIME(HOURS)= 12000.0000

DIMENSIONLESS TIME= 0.0650

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.2969E+05		
1	2	1	0.1591E+05		
1	3	1	0.2969E+05		
				0.7529E+05	
2	1	1	0.2016E+05		
2	2	1	0.1047E+05		
				0.3063E+05	
3	1	1	0.9620E+04		
3	2	1	0.1325E+05		
				0.2287E+05	
4	1	1	0.5743E+04		
4	2	1	0.6126E+04		
				0.1187E+05	
5	1	1	0.9021E+04		
5	2	1	0.7240E+04		
				0.1626E+05	
6	1	1	0.5694E+04		
6	2	1	0.5592E+04		
				0.1129E+05	
7	1	2	0.2969E+05		
7	2	2	0.2969E+05		
				0.5938E+05	
8	1	2	0.2969E+05		
8	2	1	0.1060E+05		
8	3	1	0.1060E+05		
8	4	1	0.1060E+05		
				0.6150E+05	
9	1	1	0.2673E+04		
9	2	1	0.3100E+04		
9	3	1	0.4762E+04		
9	4	1	0.3000E+04		
				0.1354E+05	
10	1	1	0.8211E+04		
10	2	1	0.5127E+04		
10	3	1	0.4049E+04		
10	4	1	0.3893E+04		
10	5	1	0.5996E+04		
				0.2728E+05	
11	1	1	0.1060E+05		
11	2	1	0.1060E+05		
11	3	1	0.1060E+05		
11	4	1	0.1060E+05		
11	5	1	0.1060E+05		
				0.5302E+05	
					0.3829E+06

PEAL TIME(HCURS)= 18000.0000

DIMENSIONLESS TIME= 0.0975

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.4299E+05		
1	2	1	0.1591E+05		
1	3	1	0.4299E+05		
				0.1019E+06	
2	1	1	0.2016E+05		
2	2	1	0.1047E+05		
				0.3063E+05	
3	1	1	0.9620E+04		
3	2	1	0.1325E+05		
				0.2287E+05	
4	1	1	0.5748E+04		
4	2	1	0.6126E+04		
				0.1187E+05	
5	1	1	0.9021E+04		
5	2	1	0.7240E+04		
				0.1626E+05	
6	1	1	0.5694E+04		
6	2	1	0.5592E+04		
				0.1129E+05	
7	1	2	0.4283E+05		
7	2	2	0.4279E+05		
				0.8562E+05	
8	1	2	0.4207E+05		
8	2	1	0.1535E+05		
8	3	1	0.1222E+05		
8	4	1	0.1420E+05		
				0.8384E+05	
9	1	1	0.2673E+04		
9	2	1	0.3100E+04		
9	3	1	0.4762E+04		
9	4	1	0.3000E+04		
				0.1354E+05	
10	1	1	0.8211E+04		
10	2	1	0.5127E+04		
10	3	1	0.4049E+04		
10	4	1	0.3893E+04		
10	5	1	0.5996E+04		
				0.2728E+05	
11	1	1	0.1535E+05		
11	2	1	0.1535E+05		
11	3	1	0.1535E+05		
11	4	1	0.1349E+05		
11	5	1	0.1535E+05		
				0.7491E+05	
					0.4800E+06

REAL TIME(HOURS)= 24000.0000

DIMENSIONLESS TIME= 0.1300

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.4487E+05		
1	2	1	0.1591E+05		
1	3	1	0.5569E+05		
				0.1165E+06	
2	1	1	0.2016E+05		
2	2	1	0.1047E+05		
				0.3063E+05	
3	1	1	0.9620E+04		
3	2	1	0.1325E+05		
				0.2287E+05	
4	1	1	0.5748E+04		
4	2	1	0.6126E+04		
				0.1187E+05	
5	1	1	0.9021E+04		
5	2	1	0.7240E+04		
				0.1626E+05	
6	1	1	0.5694E+04		
6	2	1	0.5592E+04		
				0.1129E+05	
7	1	2	0.5034E+05		
7	2	2	0.5395E+05		
				0.1043E+06	
8	1	2	0.5323E+05		
8	2	1	0.1989E+05		
8	3	1	0.1222E+05		
8	4	1	0.1420E+05		
				0.9954E+05	
9	1	1	0.2673E+04		
9	2	1	0.3100E+04		
9	3	1	0.4762E+04		
9	4	1	0.3000E+04		
				0.1354E+05	
10	1	1	0.8211E+04		
10	2	1	0.5127E+04		
10	3	1	0.4049E+04		
10	4	1	0.3893E+04		
10	5	1	0.5996E+04		
				0.2728E+05	
11	1	1	0.1989E+05		
11	2	1	0.1989E+05		
11	3	1	0.1989E+05		
11	4	1	0.1349E+05		
11	5	1	0.1963E+05		
				0.9278E+05	
					0.5468E+06

REAL TIME(HOURS)= 30000.0000

DIMENSIONLESS TIME= 0.1625

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.4487E+05		
1	2	1	0.1591E+05		
1	3	1	0.6789E+05		
				0.1287E+06	
2	1	1	0.2016E+05		
2	2	1	0.1047E+05		
				0.3063E+05	
3	1	1	0.9620E+04		
3	2	1	0.1325E+05		
				0.2287E+05	
4	1	1	0.5748E+04		
4	2	1	0.6126E+04		
				0.1187E+05	
5	1	1	0.9021E+04		
5	2	1	0.7240E+04		
				0.1626E+05	
6	1	1	0.5694E+04		
6	2	1	0.5592E+04		
				0.1129E+05	
7	1	2	0.5034E+05		
7	2	2	0.5873E+05		
				0.1091E+06	
8	1	2	0.6678E+05		
8	2	1	0.2425E+05		
8	3	1	0.1222E+05		
8	4	1	0.1420E+05		
				0.1174E+06	
9	1	1	0.2673E+04		
9	2	1	0.3100E+04		
9	3	1	0.4762E+04		
9	4	1	0.3000E+04		
				0.1354E+05	
10	1	1	0.8211E+04		
10	2	1	0.5127E+04		
10	3	1	0.4049E+04		
10	4	1	0.3893E+04		
10	5	1	0.5996E+04		
				0.2728E+05	
11	1	1	0.2407E+05		
11	2	1	0.2425E+05		
11	3	1	0.2242E+05		
11	4	1	0.1349E+05		
11	5	1	0.1963E+05		
				0.1039E+06	
					0.5928E+06

PREDICTED RECOVERY

TIME(DAYS)	RECOVERY(BBLS)
62.5000	0.1624E+05
125.0000	0.2988E+05
187.5000	0.3953E+05
250.0000	0.4700E+05
312.5000	0.5309E+05
375.0000	0.5847E+05
437.5000	0.6338E+05
500.0000	0.6819E+05
562.5000	0.7294E+05
625.0000	0.7746E+05
687.5000	0.8167E+05
750.0000	0.8548E+05
812.5000	0.8893E+05
875.0000	0.9201E+05
937.5000	0.9490E+05
1000.0000	0.9738E+05
1062.5000	0.1001E+06
1125.0000	0.1019E+06
1187.5000	0.1035E+06
1250.0000	0.1056E+06

APPENDIX M

RESULTS OF STREAMLINE MODELLING OF SHIELLS CANYON FIELD AS A PIECEWISE HOMOGENEOUS RESERVOIR. PART OF ITS BOUNDARY AT CONSTANT PRESSURE, THE REMAINDER SEALED

APPENDIX M

BOUNDARY ELEMENT MODELLING OF SHIELLS CANYON(203) FIELD

ANALYSIS AS A PIECEWISE-HOMOGENEOUS RESERVIOR

PART OF THE BOUNDARY AT CONSTANT PRESSURE, THE REMAINDER SEALED

BY: D. T. NUMBERE, UNIVERSITY OF OKLAHOMA, 1982

REGION 1.0

DATA

NUMBER OF BOUNDARY ELEMENTS= 43
NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS CALCULATED= 0
NUMBER OF SOURCES AND SINKS= 7
NUMBER OF INTERFACE NODES= 10

PERMEABILITY(MD)= 140.0000

THICKNESS(FT)= 160.0000

POROSITY= 0.2050

SCALE: 1 INCH = 738.46FEET

THE COORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELEMENTS

POINT	X(INCH)	Y(INCH)
1	1.5000	1.5500
2	1.3500	1.4800
3	1.2500	1.4400
4	1.2000	1.4000
5	1.1500	1.3800
6	1.0500	1.3250
7	0.9500	1.2600
8	0.9000	1.2500
9	0.8500	1.2000
10	0.8000	1.1700
11	0.7500	1.1500
12	0.6500	1.0800
13	0.5500	1.0200
14	0.5000	0.9900
15	0.4500	0.9500
16	0.4000	0.9200
17	0.3000	0.8500
18	0.1000	0.7300
19	0.1000	0.5000
20	0.1000	0.1500
21	0.2500	0.2000
22	0.4000	0.2500
23	0.5500	0.2800
24	0.6500	0.3000
25	0.7000	0.3250
26	0.7500	0.3300
27	0.8000	0.3400
28	0.8500	0.3500
29	0.9000	0.4000
30	1.0000	0.4500
31	1.1000	0.5250
32	1.2500	0.6000
33	1.4000	0.6500
34	1.5500	0.6800
35	1.5500	0.7500
36	1.5400	0.8500
37	1.5000	0.9500
38	1.5000	1.0500
39	1.4800	1.1500
40	1.4600	1.2500
41	1.4600	1.3500
42	1.4800	1.4500
43	1.5000	1.5000

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	1	0.0
2	1	0.0
3	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0
7	1	0.0
8	1	0.0
9	1	0.0
10	1	0.0
11	1	0.0
12	1	0.0
13	1	0.0
14	1	0.0
15	1	0.0
16	1	0.0
17	1	0.0
18	0	0.8500000E+02
19	0	0.8500000E+02
20	1	0.0
21	1	0.0
22	1	0.0
23	1	0.0
24	1	0.0
25	1	0.0
26	1	0.0
27	1	0.0
28	1	0.0
29	1	0.0
30	1	0.0
31	1	0.0
32	1	0.0
33	1	0.0
34	2	0.0
35	2	0.0
36	2	0.0
37	2	0.0
38	2	0.0
39	2	0.0
40	2	0.0
41	2	0.0
42	2	0.0
43	2	0.0

COORDINATES OF SOURCES AND SINKS AND THEIR STRENGTHS

X(INCH)	Y(INCH)	RATE(BBL/D)
0.4500	0.5500	-69.1000
0.5000	0.8000	-69.1000
0.7500	0.4000	-69.1000
0.9000	1.1500	-69.1000
1.0000	0.8000	560.0000
1.2500	1.2000	-69.1000
1.3500	1.0000	-69.1000

THE AVERAGE RADIUS OF THE WELLS(INCH) IS= 0.500
THE CHARACTERISTIC PRESSURE(PSI) IS= 85.0000

REGION 2.0

DATA

NUMBER OF BOUNDARY ELEMENTS= 37
NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS CALCULATED= 0
NUMBER OF SOURCES AND SINKS= 6
NUMBER OF INTERFACE NODES= 10

PERMEABILITY(MD)= 70.0000

THICKNESS(FT)= 160.0000

POROSITY= 0.2050

SCALE: 1 INCH = 738.46FEET

THE COORDINATES OF THE EXTREME POINTS OF THE BOUNDARY ELEMENTS

POINT	X(INCH)	Y(INCH)
1	1.5000	1.5500
2	1.5000	1.5000
3	1.4800	1.4500
4	1.4600	1.3500
5	1.4600	1.2500
6	1.4800	1.1500
7	1.5000	1.0500
8	1.5000	0.9500
9	1.5400	0.8500
10	1.5500	0.7500
11	1.5500	0.6800
12	1.6500	0.6900
13	1.7000	0.7000
14	1.7500	0.7100
15	1.8000	0.7200
16	1.8500	0.7250
17	1.9000	0.7300
18	1.9500	0.7350
19	2.1000	0.7450
20	2.2500	0.7500
21	2.4000	0.7500
22	2.6000	0.7400
23	2.7500	0.7300
24	2.9000	0.7250
25	3.1600	1.0000
26	3.0600	1.2500
27	2.9500	1.5500
28	2.7500	1.6500
29	2.4000	1.8200
30	2.2500	1.8000
31	2.1000	1.7500
32	2.0500	1.7450
33	2.0000	1.7250
34	1.9500	1.7100
35	1.8500	1.6900
36	1.7500	1.6500
37	1.6500	1.6100

BOUNDARY CONDITIONS

NODE	CODE	PRESCRIBED VALUE
1	2	0.0
2	2	0.0
3	2	0.0
4	2	0.0
5	2	0.0
6	2	0.0
7	2	0.0
8	2	0.0
9	2	0.0
10	2	0.0
11	1	0.0
12	1	0.0
13	1	0.0
14	1	0.0
15	1	0.0
16	1	0.0
17	1	0.0
18	1	0.0
19	1	0.0
20	1	0.0
21	1	0.0
22	1	0.0
23	1	0.0
24	1	0.0
25	1	0.0
26	1	0.0
27	1	0.0
28	1	0.0
29	1	0.0
30	1	0.0
31	1	0.0
32	1	0.0
33	1	0.0
34	1	0.0
35	1	0.0
36	1	0.0
37	1	0.0

COORDINATES OF SOURCES AND SINKS AND THEIR STRENGTHS

X(INCH)	Y(INCH)	RATE(BBL/D)
1.9000	0.9500	200.0000
1.8000	1.3500	-69.1000
2.0500	1.5500	-69.1000
2.1500	1.2500	-69.1000
2.3500	1.0500	-69.1000
2.5750	1.3000	-69.1000

THE AVERAGE RADIUS OF THE WELLS(INCH) IS= 0.500
THE CHARACTERISTIC PRESSURE(PSI) IS= 85.0000

RESULTS

BOUNDARY NODES

X(INCH)	Y(INCH)	PRESSURE(PSI)	NORMAL GRADIENT
0.1425E+01	0.1515E+01	0.8076E+02	0.0
0.1300E+01	0.1460E+01	0.8205E+02	0.0
0.1225E+01	0.1420E+01	0.8266E+02	0.0
0.1175E+01	0.1390E+01	0.8327E+02	0.0
0.1100E+01	0.1352E+01	0.8402E+02	0.0
0.1000E+01	0.1292E+01	0.8501E+02	0.0
0.9250E+00	0.1255E+01	0.8560E+02	0.0
0.8750E+00	0.1225E+01	0.8568E+02	0.0
0.8250E+00	0.1185E+01	0.8623E+02	0.0
0.7750E+00	0.1160E+01	0.8675E+02	0.0
0.7000E+00	0.1115E+01	0.8713E+02	0.0
0.6000E+00	0.1050E+01	0.8695E+02	0.0
0.5250E+00	0.1005E+01	0.8648E+02	0.0
0.4750E+00	0.9700E+00	0.8610E+02	0.0
0.4250E+00	0.9350E+00	0.8569E+02	0.0
0.3500E+00	0.8850E+00	0.8537E+02	0.0
0.2000E+00	0.7900E+00	0.8509E+02	0.0
0.1000E+00	0.6150E+00	0.8500E+02	-0.3533E-03
0.1000E+00	0.3250E+00	0.8500E+02	0.3379E-03
0.1750E+00	0.1750E+00	0.8468E+02	0.0
0.3250E+00	0.2250E+00	0.8510E+02	0.0
0.4750E+00	0.2650E+00	0.8524E+02	0.0
0.6000E+00	0.2900E+00	0.8548E+02	0.0
0.6750E+00	0.3125E+00	0.8557E+02	0.0
0.7250E+00	0.3275E+00	0.8566E+02	0.0
0.7750E+00	0.3350E+00	0.8590E+02	0.0
0.8250E+00	0.3450E+00	0.8645E+02	0.0
0.8750E+00	0.3750E+00	0.8700E+02	0.0
0.9500E+00	0.4250E+00	0.8834E+02	0.0
0.1050E+01	0.4875E+00	0.8946E+02	0.0
0.1175E+01	0.5625E+00	0.8972E+02	0.0
0.1325E+01	0.6250E+00	0.8794E+02	0.0
0.1475E+01	0.6650E+00	0.8608E+02	0.0
0.1550E+01	0.7150E+00	0.8560E+02	-0.1048E-01
0.1545E+01	0.8000E+00	0.8519E+02	-0.6458E-02
0.1520E+01	0.9000E+00	0.8469E+02	-0.8446E-02
0.1500E+01	0.1000E+01	0.8383E+02	-0.4569E-02
0.1490E+01	0.1100E+01	0.8302E+02	-0.7717E-02
0.1470E+01	0.1200E+01	0.8234E+02	-0.8584E-02
0.1460E+01	0.1300E+01	0.8167E+02	-0.8253E-02
0.1470E+01	0.1400E+01	0.8094E+02	-0.7529E-02
0.1490E+01	0.1475E+01	0.8047E+02	-0.6497E-02
0.1500E+01	0.1525E+01	0.8102E+02	-0.2877E-01

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RESULTS

BOUNDARY NODES

X(INCH)	Y(INCH)	PRESSURE(PSI)	NORMAL GRADIENT
0.1500E+01	0.1525E+01	0.8102E+02	0.5754E-01
0.1490E+01	0.1475E+01	0.8047E+02	0.1299E-01
0.1470E+01	0.1400E+01	0.8094E+02	0.1506E-01
0.1460E+01	0.1300E+01	0.8167E+02	0.1651E-01
0.1470E+01	0.1200E+01	0.8234E+02	0.1717E-01
0.1490E+01	0.1100E+01	0.8302E+02	0.1543E-01
0.1500E+01	0.1000E+01	0.8383E+02	0.9137E-02
0.1520E+01	0.9000E+00	0.8469E+02	0.1689E-01
0.1545E+01	0.8000E+00	0.8519E+02	0.1292E-01
0.1550E+01	0.7150E+00	0.8560E+02	0.2096E-01
0.1600E+01	0.6850E+00	0.8512E+02	0.0
0.1675E+01	0.6950E+00	0.8438E+02	0.0
0.1725E+01	0.7050E+00	0.8395E+02	0.0
0.1775E+01	0.7150E+00	0.8353E+02	0.0
0.1825E+01	0.7225E+00	0.8306E+02	0.0
0.1875E+01	0.7275E+00	0.8251E+02	0.0
0.1925E+01	0.7325E+00	0.8177E+02	0.0
0.2025E+01	0.7400E+00	0.7962E+02	0.0
0.2175E+01	0.7475E+00	0.7595E+02	0.0
0.2325E+01	0.7500E+00	0.7303E+02	0.0
0.2500E+01	0.7450E+00	0.7103E+02	0.0
0.2675E+01	0.7350E+00	0.7012E+02	0.0
0.2825E+01	0.7275E+00	0.6975E+02	0.0
0.3030E+01	0.8625E+00	0.6950E+02	0.0
0.3110E+01	0.1125E+01	0.6931E+02	0.0
0.3005E+01	0.1400E+01	0.6897E+02	0.0
0.2850E+01	0.1600E+01	0.6892E+02	0.0
0.2575E+01	0.1735E+01	0.6933E+02	0.0
0.2325E+01	0.1810E+01	0.6975E+02	0.0
0.2175E+01	0.1775E+01	0.7019E+02	0.0
0.2075E+01	0.1747E+01	0.7075E+02	0.0
0.2025E+01	0.1735E+01	0.7096E+02	0.0
0.1975E+01	0.1717E+01	0.7149E+02	0.0
0.1900E+01	0.1700E+01	0.7255E+02	0.0
0.1800E+01	0.1670E+01	0.7403E+02	0.0
0.1700E+01	0.1630E+01	0.7589E+02	0.0
0.1575E+01	0.1580E+01	0.7864E+02	0.0

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PRODUCER NUMBER 1

NUMBER OF STREAMLINES= 3

STREAMLINE NUMBER	TOTAL [illegible]	[illegible] WIDTH	INJ. RATE
1	[illegible]	[illegible]	0.2800E+02
2	[illegible]	[illegible]	0.2800E+02
3	[illegible]	[illegible]	0.2800E+02

PR[illegible]

NU[illegible]

STREAMLIN[illegible]	[illegible]	[illegible]	[illegible] RATE
1	[illegible]	[illegible]	[illegible]800E+02
2	[illegible]	[illegible]	[illegible]2800E+02

PRODUCER NUMBER [illegible]

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.3381E+03	0.1795E+02	0.2800E+02
2	0.3816E+03	0.2111E+02	0.2800E+02

PRODUCER NUMBER 1

NUMBER OF STREAMLINES= 3

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.5533E+03	0.4991E+02	0.2800E+02
2	0.4250E+03	0.2343E+02	0.2800E+02
3	0.6566E+03	0.6611E+02	0.2800E+02

PRODUCER NUMBER 2

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4447E+03	0.2795E+02	0.2800E+02
2	0.3447E+03	0.1906E+02	0.2800E+02

PRODUCER NUMBER 3

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.3381E+03	0.1795E+02	0.2800E+02
2	0.3816E+03	0.2111E+02	0.2800E+02

PRODUCER NUMBER 4

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2697E+03	0.1331E+02	0.2800E+02
2	0.2697E+03	0.1475E+02	0.2800E+02

PRODUCER NUMBER 6

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.3664E+03	0.1586E+02	0.2800E+02
2	0.3316E+03	0.1379E+02	0.2800E+02

PRODUCER NUMBER 7

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2914E+03	0.1262E+02	0.2800E+02
2	0.2947E+03	0.1212E+02	0.2800E+02

PRODUCER NUMBER 2

NUMBER OF STREAMLINES= 2

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4513E+03	0.6006E+02	0.2800E+02
1	0.2164E+03	0.3008E+02	0.2800E+02
2	0.4784E+03	0.5833E+02	0.2800E+02
2	0.2480E+03	0.3138E+02	0.2800E+02

PRODUCER NUMBER 3

NUMBER OF STREAMLINES= 3

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.5914E+03	0.1337E+02	0.1000E+02
2	0.6197E+03	0.1481E+02	0.1000E+02
1	0.3764E+03	0.6182E+02	0.2800E+02
1	0.5414E+03	0.4388E+02	0.2800E+02

PRODUCER NUMBER 4

NUMBER OF STREAMLINES= 4

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.2664E+03	0.6399E+01	0.1000E+02
2	0.2881E+03	0.6867E+01	0.1000E+02
3	0.3631E+03	0.8268E+01	0.1000E+02
4	0.2881E+03	0.6640E+01	0.1000E+02

PRODUCER NUMBER 5

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.4783E+03	0.1111E+02	0.1000E+02
2	0.3848E+03	0.8507E+01	0.1000E+02
3	0.3348E+03	0.7707E+01	0.1000E+02
4	0.3316E+03	0.7468E+01	0.1000E+02
5	0.3848E+03	0.9714E+01	0.1000E+02

PRODUCER NUMBER 6

NUMBER OF STREAMLINES= 5

STREAMLINE NUMBER	TOTAL LENGTH	AVERAGE WIDTH	INJ. RATE
1	0.7033E+03	0.2165E+02	0.1000E+02
2	0.8348E+03	0.2773E+02	0.1000E+02
3	0.7197E+03	0.1985E+02	0.1000E+02
4	0.5381E+03	0.1601E+02	0.1000E+02
5	0.5914E+03	0.2086E+02	0.1000E+02

NO STREAMLINE FROM REGION 2 CROSSES INTO REGION 1

APPENDIX N

RESULTS OF STEAMFLOOD PREDICTION FOR SHIELLS CANYON FIELD AS A PIECEWISE HOMOGENEOUS RESERVOIR. PART OF ITS BOUNDARY AT CONSTANT PRESSURE, THE REMAINDER SEALED

APPENDIX N

STEAMFLOOD PREDICTION OF SHIELLS CANYON(203) FIELD

METHOD: STREAMLINE/STREAMTUBE METHOD USING (BEM)TECHNIQUE

ANALYSIS AS A PIECE-WISE HOMOGENEOUS RESERVOIR

HAVING TWO REGIONS OF UNEQUAL PERMEABILITY

BOUNDARY CONDITION: PART OF THE BOUNDARY AT CONSTANT

PRESSURE, THE REMAINDER IS SEALED.

PRODUCTION RATES: ASSIGNED EQUAL IN ENTIRE REGION

DATA

TOTAL NUMBER OF PRODUCERS=	11
STEAM TEMPERATURE (DEG. F)=	430.0000
STEAM QUALITY=	0.7000
CONVERGENCE LIMIT=	0.0010
RESERVOIR THICKNESS (FT)=	160.0000
STEAM PRESSURE (PSI)=	344.0000
INITIAL RESERVOIR TEMPERATURE(DEG F)=	105.0000

FLUID PROPERTIES

	WATER	OIL	STEAM
DENSITY(LB/CU FT) AT STD. TEMP.	62.4000	55.0000	
DENSITY(LB/CU FT) AT STEAM TEMP.	52.3800	48.7000	0.7431
SPECIFIC HEAT(BTU/LB *F)	1.0000	0.4880	1.0000
LATENT HEAT(BTU/LB)			789.5100

PROPERTIES OF THE CAP AND BASE ROCK

DENSITY(LB/CU. FT)	149.0000
SPECIFIC HEAT(BTU/LB-*F)	0.2130
THERM. COND.(BTU/HR-FT-*F)	1.1000

RESERVCIR ROCK PROPERTIES

	REGION 1	REGION 2
POROSITY	0.2050	0.1800
DEN.(LB/CU FT)	165.0000	165.0000
SPEC. HEAT(BTU/LB*F)	0.2000	0.2000
PERMEABILITY(MD)	140.0000	70.0000

INITIAL FLUID SATURATIONS

	REGION 1	REGION 2
WATER	0.5500	0.5500
OIL	0.4500	0.4500
STEAM	0.0	0.0

AVERAGE RESIDUAL SATURATIONS BEHIND THE FRONT

	REGION 1	REGION 2
WATER	0.5800	0.5800
OIL	0.1800	0.1800
STEAM	0.2400	0.2400

INPUT DATA FROM STREAMLINE PROGRAM

			REGION 1			REGION 2		
WELL NO.	S/L NO.	CODE	LENGTH	WIDTH	RATE	LENGTH	WIDTH	RATE
1	1	1	553.30	49.91	28.00	0.0	0.0	0.0
1	2	1	425.00	23.40	28.00	0.0	0.0	0.0
1	3	1	656.60	66.11	28.00	0.0	0.0	0.0
2	1	1	444.70	27.95	28.00	0.0	0.0	0.0
2	2	1	344.70	19.06	28.00	0.0	0.0	0.0
3	1	1	338.10	17.95	28.00	0.0	0.0	0.0
3	2	1	381.60	21.11	28.00	0.0	0.0	0.0
4	1	1	269.70	13.31	28.00	0.0	0.0	0.0
4	2	1	269.70	14.75	28.00	0.0	0.0	0.0
5	1	1	366.40	15.86	28.00	0.0	0.0	0.0
5	2	1	331.60	13.79	28.00	0.0	0.0	0.0
6	1	1	291.40	12.62	28.00	0.0	0.0	0.0
6	2	1	294.70	12.12	28.00	0.0	0.0	0.0
7	1	2	451.30	60.06	28.00	216.40	30.08	28.00
7	2	2	478.40	58.33	28.00	248.00	31.38	28.00
8	1	2	376.40	61.82	28.00	541.40	43.88	28.00
8	2	1	591.40	13.37	10.00	0.0	0.0	0.0
8	3	1	619.70	14.81	10.00	0.0	0.0	0.0
9	1	1	266.40	6.40	10.00	0.0	0.0	0.0
9	2	1	288.10	6.87	10.00	0.0	0.0	0.0
9	3	1	363.10	8.27	10.00	0.0	0.0	0.0
9	4	1	288.10	6.64	10.00	0.0	0.0	0.0
10	1	1	478.30	11.11	10.00	0.0	0.0	0.0
10	2	1	384.80	8.51	10.00	0.0	0.0	0.0
10	3	1	334.80	7.71	10.00	0.0	0.0	0.0
10	4	1	331.60	7.47	10.00	0.0	0.0	0.0
10	5	1	384.80	9.71	10.00	0.0	0.0	0.0
11	1	1	703.30	21.65	10.00	0.0	0.0	0.0
11	2	1	834.80	27.73	10.00	0.0	0.0	0.0
11	3	1	719.70	19.85	10.00	0.0	0.0	0.0
11	4	1	538.10	16.01	10.00	0.0	0.0	0.0
11	5	1	591.40	20.86	10.00	0.0	0.0	0.0

CALCULATED BREAKTHROUGH TIMES(HOURS) FOR EACH STREAMTUBE

PROD. NO.	STREAMLINE NO.	CODE	ENDTIME(1)	ENDTIME(2)
1	1	1	18260.6484	0.0
1	2	1	6048.5781	0.0
1	3	1	30286.2461	0.0
2	1	1	7671.5547	0.0
2	2	1	3904.0466	0.0
3	1	1	3592.2112	0.0
3	2	1	4838.9414	0.0
4	1	1	2078.0183	0.0
4	2	1	2311.8730	0.0
5	1	1	3432.4089	0.0
5	2	1	2672.3630	0.0
6	1	1	2130.7292	0.0
6	2	1	2067.1299	0.0
7	1	2	17888.3867	21748.8750
7	2	2	18472.7227	23131.5273
8	1	2	15123.5234	30573.1289
8	2	1	14321.5000	0.0
8	3	1	16866.7812	0.0
9	1	1	2794.5066	0.0
9	2	1	3264.7046	0.0
9	3	1	5061.4687	0.0
9	4	1	3151.8491	0.0
10	1	1	9305.1289	0.0
10	2	1	5547.0469	0.0
10	3	1	4314.2812	0.0
10	4	1	4131.6172	0.0
10	5	1	6386.0742	0.0
11	1	1	29675.5391	0.0
11	2	1	47986.7930	0.0
11	3	1	27613.4141	0.0
11	4	1	15732.7773	0.0
11	5	1	23419.4766	0.0

REAL TIME(HOURS)= 6000.0000

DIMENSIONLESS TIME= 0.0325

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.1557E+05		
1	2	1	0.1557E+05		
1	3	1	0.1557E+05		
				0.4670E+05	
2	1	1	0.1557E+05		
2	2	1	0.1036E+05		
				0.2593E+05	
3	1	1	0.9572E+04		
3	2	1	0.1271E+05		
				0.2228E+05	
4	1	1	0.5662E+04		
4	2	1	0.6274E+04		
				0.1194E+05	
5	1	1	0.9165E+04		
5	2	1	0.7212E+04		
				0.1638E+05	
6	1	1	0.5800E+04		
6	2	1	0.5633E+04		
				0.1143E+05	
7	1	2	0.1557E+05		
7	2	2	0.1557E+05		
				0.3113E+05	
8	1	2	0.1557E+05		
8	2	1	0.5560E+04		
8	3	1	0.5560E+04		
				0.2669E+05	
9	1	1	0.2689E+04		
9	2	1	0.3120E+04		
9	3	1	0.4735E+04		
9	4	1	0.3017E+04		
				0.1356E+05	
10	1	1	0.5560E+04		
10	2	1	0.5163E+04		
10	3	1	0.4070E+04		
10	4	1	0.3906E+04		
10	5	1	0.5560E+04		
				0.2426E+05	
11	1	1	0.5560E+04		
11	2	1	0.5560E+04		
11	3	1	0.5560E+04		
11	4	1	0.5560E+04		
11	5	1	0.5560E+04		
				0.2780E+05	
					0.2581E+06

REAL TIME(HOURS)= 12000.0000

DIMENSIONLESS TIME= 0.0650

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.2969E+05		
1	2	1	0.1569E+05		
1	3	1	0.2969E+05		
				0.7506E+05	
2	1	1	0.1960E+05		
2	2	1	0.1036E+05		
				0.2997E+05	
3	1	1	0.9572E+04		
3	2	1	0.1271E+05		
				0.2228E+05	
4	1	1	0.5662E+04		
4	2	1	0.6274E+04		
				0.1194E+05	
5	1	1	0.9165E+04		
5	2	1	0.7212E+04		
				0.1638E+05	
6	1	1	0.5800E+04		
6	2	1	0.5633E+04		
				0.1143E+05	
7	1	2	0.2969E+05		
7	2	2	0.2969E+05		
				0.5938E+05	
8	1	2	0.2969E+05		
8	2	1	0.1060E+05		
8	3	1	0.1060E+05		
				0.5090E+05	
9	1	1	0.2689E+04		
9	2	1	0.3120E+04		
9	3	1	0.4735E+04		
9	4	1	0.3017E+04		
				0.1356E+05	
10	1	1	0.8381E+04		
10	2	1	0.5163E+04		
10	3	1	0.4070E+04		
10	4	1	0.3906E+04		
10	5	1	0.5896E+04		
				0.2742E+05	
11	1	1	0.1060E+05		
11	2	1	0.1060E+05		
11	3	1	0.1060E+05		
11	4	1	0.1060E+05		
11	5	1	0.1060E+05		
				0.5302E+05	
					0.3713E+06

REAL TIME(HOURS)= 18000.0000

DIMENSIONLESS TIME= 0.0975

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.4299E+05		
1	2	1	0.1569E+05		
1	3	1	0.4299E+05		
				0.1017E+06	
2	1	1	0.1960E+05		
2	2	1	0.1036E+05		
				0.2997E+05	
3	1	1	0.9572E+04		
3	2	1	0.1271E+05		
				0.2228E+05	
4	1	1	0.5662E+04		
4	2	1	0.6274E+04		
				0.1194E+05	
5	1	1	0.9165E+04		
5	2	1	0.7212E+04		
				0.1638E+05	
6	1	1	0.5800E+04		
6	2	1	0.5633E+04		
				0.1143E+05	
7	1	2	0.4296E+05		
7	2	2	0.4299E+05		
				0.8595E+05	
8	1	2	0.4223E+05		
8	2	1	0.1247E+05		
8	3	1	0.1448E+05		
				0.6918E+05	
9	1	1	0.2689E+04		
9	2	1	0.3120E+04		
9	3	1	0.4735E+04		
9	4	1	0.3017E+04		
				0.1356E+05	
10	1	1	0.8381E+04		
10	2	1	0.5163E+04		
10	3	1	0.4070E+04		
10	4	1	0.3906E+04		
10	5	1	0.5896E+04		
				0.2742E+05	
11	1	1	0.1535E+05		
11	2	1	0.1535E+05		
11	3	1	0.1535E+05		
11	4	1	0.1359E+05		
11	5	1	0.1535E+05		
				0.7500E+05	
					0.4648E+06

REAL TIME(HCURS)= 24000.0000

DIMENSIONLESS TIME= 0.1300

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.4355E+05		
1	2	1	0.1569E+05		
1	3	1	0.5569E+05		
				0.1149E+06	
2	1	1	0.1960E+05		
2	2	1	0.1036E+05		
				0.2997E+05	
3	1	1	0.9572E+04		
3	2	1	0.1271E+05		
				0.2228E+05	
4	1	1	0.5662E+04		
4	2	1	0.6274E+04		
				0.1194E+05	
5	1	1	0.9165E+04		
5	2	1	0.7212E+04		
				0.1638E+05	
6	1	1	0.5800E+04		
6	2	1	0.5633E+04		
				0.1143E+05	
7	1	2	0.5176E+05		
7	2	2	0.5479E+05		
				0.1066E+06	
8	1	2	0.5339E+05		
8	2	1	0.1247E+05		
8	3	1	0.1448E+05		
				0.8034E+05	
9	1	1	0.2689E+04		
9	2	1	0.3120E+04		
9	3	1	0.4735E+04		
9	4	1	0.3017E+04		
				0.1356E+05	
10	1	1	0.8381E+04		
10	2	1	0.5163E+04		
10	3	1	0.4070E+04		
10	4	1	0.3906E+04		
10	5	1	0.5896E+04		
				0.2742E+05	
11	1	1	0.1989E+05		
11	2	1	0.1989E+05		
11	3	1	0.1989E+05		
11	4	1	0.1359E+05		
11	5	1	0.1946E+05		
				0.9271E+05	
					0.5275E+06

REAL TIME(HOURS)= 30000.0000

DIMENSIONLESS TIME= 0.1625

WELL	STR/L	NO. OF REGIONS	RECOVERY	WELL TOTAL	RESERVOIR TOTAL
1	1	1	0.4355E+05		
1	2	1	0.1569E+05		
1	3	1	0.6789E+05		
				0.1271E+06	
2	1	1	0.1960E+05		
2	2	1	0.1036E+05		
				0.2997E+05	
3	1	1	0.9572E+04		
3	2	1	0.1271E+05		
				0.2228E+05	
4	1	1	0.5662E+04		
4	2	1	0.6274E+04		
				0.1194E+05	
5	1	1	0.9165E+04		
5	2	1	0.7212E+04		
				0.1638E+05	
6	1	1	0.5800E+04		
6	2	1	0.5633E+04		
				0.1143E+05	
7	1	2	0.5176E+05		
7	2	2	0.5479E+05		
				0.1066E+06	
8	1	2	0.6412E+05		
8	2	1	0.1247E+05		
8	3	1	0.1448E+05		
				0.9106E+05	
9	1	1	0.2689E+04		
9	2	1	0.3120E+04		
9	3	1	0.4735E+04		
9	4	1	0.3017E+04		
				0.1356E+05	
10	1	1	0.8381E+04		
10	2	1	0.5163E+04		
10	3	1	0.4070E+04		
10	4	1	0.3906E+04		
10	5	1	0.5896E+04		
				0.2742E+05	
11	1	1	0.2402E+05		
11	2	1	0.2425E+05		
11	3	1	0.2253E+05		
11	4	1	0.1359E+05		
11	5	1	0.1946E+05		
				0.1038E+06	
					0.5616E+06

PREDICTED RECOVERY

TIME(DAYS)	RECOVERY(BBLS)
62.5000	0.1598E+05
125.0000	0.2940E+05
187.5000	0.3882E+05
250.0000	0.4596E+05
312.5000	0.5177E+05
375.0000	0.5682E+05
437.5000	0.6153E+05
500.0000	0.6613E+05
562.5000	0.7066E+05
625.0000	0.7503E+05
687.5000	0.7906E+05
750.0000	0.8277E+05
812.5000	0.8580E+05
875.0000	0.8868E+05
937.5000	0.9160E+05
1000.0000	0.9394E+05
1062.5000	0.9557E+05
1125.0000	0.9718E+05
1187.5000	0.9866E+05
1250.0000	0.1000E+06

APPENDIX O

LISTING OF COMPUTER PROGRAMS

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C***************************************************************************
C  PROGRAM FOR THE SOLUTION OF TWO DIMENSIONAL POTENTIAL
C  PROBLEMS BY THE BOUNDARY ELEMENT METHOD. APPLICABLE TO
C HOMOGENEOUS AS WELL AS 2-REGION PIECEWISE HOMOGENEOUS RESERVOIR
C NEUMANN AND DIRICHLET CONDITIONS SPECIFIED ON THE BOUNDARIES
C***************************************************************************
C
      DIMENSION X1(120),Y1(120),XM1(120),YM1(120)
      DIMENSION FI1(120),DFI1(120),KODE1(120),CX1(120)
      DIMENSION SOL1(120),H1(120,120),A(120,120),B(120)
      DIMENSION G1(120,120),CY1(120),SUM1(120),CX2(120)
      DIMENSION RO(120),X2(120),Y2(120),FI2(120),DFI2(120)
      DIMENSION CY2(120),SOL2(120),XSS1(120),YSS1(120)
      DIMENSION XSS2(120),YSS2(120),QSS2(120),SUM2(120)
      DIMENSION XM2(120),YM2(120),XI1(120),YI1(120)
      DIMENSION QSS1(120),KODE2(120),DYBLE1(120)
      DIMENSION DGDX1(120),DGDY1(120),DHDX1(120),DHDY1(120)
      DIMENSION AQSS2(120),DXBLE1(120),DXBLE2(120),DISMID(120)
      DIMENSION DGDX2(120),DGDY2(120),DHDX2(120),DHDY2(120)
      DIMENSION VT1(120),YI2(120),DYBLE2(120),H2(120,120)
      DIMENSION VX2(120),VY2(120),VT2(120),VX1(120),VY1(120)
      DIMENSION G2(120,120),XI2(120)
      DIMENSION DPHIDX(120),DPHIDY(120),VLCP1(120),VLCP2(120)
      DIMENSION DPDX(120),DPDY(120),XO1(120),YO1(120),XO2(120)
      DIMENSION ANSL1(120),ANSL2(120),YO2(120),A1(120,120)
      DIMENSION AQSS1(120),F(120),INFO(18),DIST(120),ELMNT1(120
      DIMENSION SINK(120),STRML(120),STRMW(120),AVWIDT(120)
      DIMENSION MM(120),A2(120),LENTH(120,120),WIDTH(120,120)
      DIMENSION ATETA1(120),ATETA2(120),ADFI1(120),ADFI2(120)
      DIMENSION XIF(120),YIF(120),ELMNT2(120),VEL1(120)
      DIMENSION VEL2(120),BETA(120),FIPLOT(120),DFIPLT(120)
      DIMENSION QSSS1(120),QSSS2(120)
      DIMENSION XAVE1(120),XAVE2(120),YAVE1(120),YAVE2(120)
      DIMENSION OLDL1(120),OLDL2(120),OLDW1(120),OLDW2(120)
      COMMON /BLK1/LEC,IMP,VICSTM,PI
      COMMON /BLK2/DISFAC,VICFAC,DENFAC,TRFAC,RATEF,SCALE
      COMMON /BLK3/PRESSF,DPRESF,PORFAC,SCLFAC
      INTEGER TITLE(18),TITLE1(18),TITLE2(18),TITLE3(18)
C
C...::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C       CONVERSION FACTORS
C   DISFAC=CONVERSION FROM (FT) TO (M)
C   TRFAC=CONVERSION FROM (MD) TO (SQ M)
C   RATEF=CONVERSION FROM (BBL/DAY) TO (CU. M/SEC)
C   SCALE=SCALE FACTOR OF THE FIELD
C   PRESSF=PRESSURE CONVERSION FROM (LBF/SQ IN) TO (NEWTON/SQ M)
C   DPRESF=PRESSURE GRADIENT CONVERSION FROM ((LB/SQ IN)/FT)
C        TO ((NEWTON/SQ M)/M)
C   PORFAC=POROSITY CONVERSION FACTOR=1
C...:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C  NTR=TOTAL NUMBER OF REGIONS
      NTR=2
      LEC=5
      IMP=6
      PI=3.1415926
      VICSTM=1.0
C
      DISFAC=.3048
```

```
0038              VICFAC=0.001
0039              TRFAC=9.869E-16
0040              RATEF=1.8401E-6
0041              SCALE=738.46
0042              PRESSF=6894.76
0043              DPRESF=PRESSF/DISFAC
0044              PORFAC=1.0
            C
0045              COUNT=0.
0046              SCLFAC=DISFAC*SCALE
            C
            C WRITE HEADINGS OR TITLES
            C
0047              WRITE(IMP,100)
0048          100 FORMAT(1H1,//,80('*'))
            C  READ THE NAME OF THE JOB
0049              READ(LEC,150)TITLE
0050              READ(LEC,150)TITLE1
0051              READ(LEC,150)TITLE2
0052              READ(LEC,150)TITLE3
0053          150 FORMAT(18A4)
0054              WRITE(IMP,250)TITLE
0055              WRITE(IMP,250)TITLE1
0056              WRITE(IMP,250)TITLE2
0057              WRITE(IMP,250)TITLE3
0058          250 FORMAT(/15X,18A4)
            C
            C  READ INPUT DATA FOR FIRST REGION
            C
0059            7 COUNT=COUNT+1.
0060              WRITE(IMP,153)COUNT
0061          153 FORMAT(//,30X,'REGION',F4.1,/)
            C
0062              CALL INPUT(CX1,CY1,X1,Y1,KODE1,FI1,XSS1,YSS1,QSS1,NSS1,
                &  N1,NIF,L1,T1,THICK1,POR1,RI1,TOPP1)
            C
            C...CONVERT TO SI UNITS
            C
0063              CALL CONVRS(CX1,CY1,X1,Y1,KODE1,FI1,XSS1,YSS1,QSS1,NSS1,
                &  N1,L1,T1,THICK1,POR1,RI1,TOPP1)
            C
            C CALCULATE THE ELEMENTS OF THE H AND G MATRICES AND ARRANGE
            C THEM SO AS TO PUT ALL UNKNOWNS IN G AND KNOWNS IN H.
            C THE ELEMENTS OF H AND THE SOURCE VECTOR FORM THE RHS VECTOR DFI
            C
0064              CALL MATRX(X1,Y1,XM1,YM1,G1,H1,FI1,DFI1,KODE1,XSS1,
                &  YSS1,QSS1,NSS1,IJ1,NIF,SUM1,T1,N1,L1,CX1,CY1)
            C
            C..IF THERE ARE TWO REGIONS REPEAT THE ABOVE STEPS FOR SECOND
            C..REGION
            C
0065              IF(NTR .LT. 2)GO TO 155
0066              COUNT=COUNT+1.
0067              WRITE(IMP,154)COUNT
0068          154 FORMAT(1H1,//,30X,'REGION',F4.1,/)
            C
0069              CALL INPUT(CX2,CY2,X2,Y2,KODE2,FI2,XSS2,YSS2,QSS2,NSS2,
                &  N2,NIF,L2,T2,THICK2,POR2,RI2,TOPP2)
```

```
C
            CALL CONVRS(CX2,CY2,X2,Y2,KODE2,FI2,XSS2,YSS2,QSS2,NSS2,
     &  N2,L2,T2,THICK2,POR2,RI2,TOPP2)
C
            CALL MATRX(X2,Y2,XM2,YM2,G2,H2,FI2,DFI2,KODE2,XSS2,
     &  YSS2,QSS2,NSS2,IJ2,NIF,SUM2,T2,N2,L2,CX2,CY2)
C
C ASSEMBLE THE G MATRICES(CONSIDERING CONTINUITY AND
C COMPATIBILITY ON THE INTERFACE) INTO THE GLOBAL MATRIX (A)
C THE RHS (B) NOW CONSISTS OF THE SUM OF THE DFI VECTOR AND THE
C VECTOR (SUM) DUE TO THE SOURCE/SINK TERMS.
C...NOTE ..IF THE BOUNDARY IS COMPLETELY CLOSED, DFI WOULD EQUAL
C ZERO
C
            CALL ASEMBL(G1,G2,DFI1,DFI2,N1,N2,A,B,NAA,NIF,NNN,
     &  NOM,NNI,NNM,T1,T2)
            GO TO 156
C
C STORE THE A MATRIX AND B VECTOR FOR FUTURE CHECK OF THE SOLUTIO
C SOLVE AND CHECK THE SOLUTION FOR A HOMOGENEOUS SINGLE REGION
  155       DO 158 I=1,N1
            A2(I)=DFI1(I)
            DO 157 J=1,N1
            A1(I,J)=G1(I,J)
  157       CONTINUE
  158       CONTINUE
            N2=0
            CALL SLNPD(G1,DFI1,D,N1,N2,NJ)
            WRITE(IMP,159)
  159       FORMAT(1H1,///20X,'CHECKING THE SOLUTION',//,
     &  9X,'RHS VECTOR',10X,'VALUE FROM SOLUTION VECTOR',//)
            DO 162 I=1,NJ
            F(I)=0.
            DO 161 J=1,NJ
            F(I)=F(I)+(A1(I,J)*DFI1(J))
  161       CONTINUE
            WRITE(IMP,163)A2(I),F(I)
  163       FORMAT(5X,E14.3,14X,E14.3)
  162       CONTINUE
            GO TO 164
C
C  SOLVE AND CHECK THE SOLUTION IF A 2-REGION COMPOSITE
C
  156       DO 76 I=1,NAA
            A2(I)=B(I)
            DO 75 J=1,NAA
            A1(I,J)=A(I,J)
  75        CONTINUE
  76        CONTINUE
C
C SOLVE THE SYSTEM
C
            CALL SLNPD(A,B,D,N1,N2,NJ)
C
C CHECK THE SOLUTION
C
            WRITE(IMP,108)
  108       FORMAT(1H1,///20X,'CHECKING THE SOLUTION VECTOR'//,
```

```
                 &  9X,'RHS VECTOR',10X,'VALUE FROM SOLUTION VECTOR',//)
0102                DO 72 I=1,NAA
0103                F(I)=0.
0104                DO 73 J=1,NAA
0105                F(I)=F(I)+(A1(I,J)*B(J))
0106          73    CONTINUE
0107                WRITE(IMP,74)A2(I),F(I)
0108          74    FORMAT(5X,E14.3,13X,E14.3)
0109          72    CONTINUE
            C
            C SPLIT THE SOLUTION VECTOR INTO ITS COMPONENT REGIONAL VALUES
            C
0110                CALL SPLIT(N1,N2,NIF,NNN,NOM,NNI,NNM,NAA,KODE1,KODE2,
                 &  FI1,FI2,DFI1,DFI2,B,T1,T2)
0111          164   CONTINUE
0112                IF(NTR .GT. 1)GO TO 168
            C
            C  REORDER THE FI AND DFI ARRAYS BY PUTTING ALL POTENTIALS IN FI
            C  AND ALL DERIVATIVES IN DFI
            C
0113                DO 169 I=1,N1
0114                IF(KODE1(I))20,20,10
0115          10    CH=FI1(I)
0116                FI1(I)=DFI1(I)
0117                DFI1(I)=CH
0118          20    CONTINUE
0119          169   CONTINUE
0120          168   CONTINUE
            C
            C  PRINT OUT THE BOUNDARY VALUES AND COMPUTE THE POTENTIAL
            C  VALUES AT DESIGNATED INTERNAL POINTS IF ANY FOR THE FIRST REGI
            C
0121                CALL OUTPUT(X1,Y1,XM1,YM1,FI1,DFI1,CX1,CY1,SOL1,N1,
                 &  XSS1,YSS1,L1,NSS1,QSS1,TOPP1,THICK1,T1,QSSS1)
            C
0122                IF(L1 .LE. 0)GO TO 173
            C
            C  COMPUTE THE POTENTIAL VALUES AT INTERNAL POINTS FOR
            C  THE FIRST REGION
            C
0123                CALL INTER(FI1,DFI1,L1,N1,CX1,CY1,X1,Y1,
                 &  XSS1,YSS1,QSS1,NSS1,SOL1)
            C
0124         173    IF(NTR .LT. 2)GO TO 165
            C
            C  PRINT OUT THE BOUNDARY VALUES FOR THE SECOND REGION AND
            C  COMPUTE THE POTENTIAL VALUES AT DESIGNATED INTERNAL POINTS
            C  IF ANY.
            C
0125                CALL OUTPUT(X2,Y2,XM2,YM2,FI2,DFI2,CX2,CY2,SOL2,N2,
                 &  XSS2,YSS2,L2,NSS2,QSS2,TOPP2,THICK2,T2,QSSS2)
            C
0126                IF(L2 .LE. 0)GO TO 165
            C
            C COMPUTE THE POTENTIAL VALUES AT INTERNAL POINTS FOR THE
            C SECOND REGION
            C
0127                CALL INTER(FI2,DFI2,L2,N2,CX2,CY2,X2,Y2,
```

```
                  &   XSS2,YSS2,QSS2,NSS2,SOL2)
            C PLOT BOUNDARIES AND WELL LOCATIONS
            C
0128          165     CONTINUE
0129                  CALL BDRY(X1,Y1,XSS1,YSS1,NSS1,QSSS1,N1)
0130                  IF(NTR .LT. 2)GO TO 166
0131                  CALL BDRY(X2,Y2,XSS2,YSS2,NSS2,QSSS2,N2)
            C
            C GENERATE THE STREAMLINES
            C
0132          166     CONTINUE
0133                  CALL STRM(NSS1,QSS1,FI1,DFI1,X1,Y1,XSS1,YSS1,
                  &   N1,T1,KODE1,IP2,XO2,YO2,XM1,YM1,AQSS2,ANSL2,MM,
                  &   ATETA1,ADFI1,ELMNT1,THICK1,POR1,RI1,QSSS1,
                  &   OLDL1,OLDW1)
0134                  IF(NTR .LT. 2)GO TO 873
            C
0135                  IF(IP2 .LE. 0)GO TO 878
0136                  CALL COMPAT(NSS2,QSS2,FI2,DFI2,X2,Y2,XSS2,YSS2,
                  &   N2,T2,KODE2,IP2,XO2,YO2,XM2,YM2,AQSS2,ANSL2,MM,
                  &   ATETA1,ADFI1,ELMNT1,THICK2,POR2,RI2,T1,
                  &   OLDL1,OLDW1)
0137                  GO TO 872
            C
0138          878     WRITE(IMP,871)
0139          871     FORMAT(/5X,'NO STREAMLINE FROM REGION 1 CROSSES',
                  &   ' INTO REGION 2',/)
0140          872     CALL PLOT(0.0,0.0,3)
            C
0141                  CALL STRM(NSS2,QSS2,FI2,DFI2,X2,Y2,XSS2,YSS2,
                  &   N2,T2,KODE2,IP1,XO1,YO1,XM2,YM2,AQSS1,ANSL1,MM,
                  &   ATETA2,ADFI2,ELMNT2,THICK2,POR2,RI2,QSSS2,
                  &   OLDL2,OLDW2)
            C
0142                  IF(IP1 .LE. 0)GO TO 879
0143                  CALL COMPAT(NSS1,QSS1,FI1,DFI1,X1,Y1,XSS1,YSS1,
                  &   N1,T1,KODE1,IP1,XO1,YO1,XM1,YM1,AQSS1,ANSL1,MM,
                  &   ATETA2,ADFI2,ELMNT2,THICK1,POR1,RI1,T2,
                  &   OLDL2,OLDW2)
0144                  GO TO 873
            C
0145          879     WRITE(IMP,874)
0146          874     FORMAT(/5X,'NO STREAMLINE FROM REGION 2 CROSSES',
                  &   ' INTO REGION 1'/)
0147          873     CALL PLOT(0.0,0.0,999)
0148          79      STOP
0149                  END
```

```
      SUBROUTINE INPUT(CX,CY,X,Y,KODE,FI,XSS,YSS,QSS,NSS,N,
     & NIF,L,T,THICK,POR,RI,TOPP)
C*********************************************************************
C
      COMMON /BLK1/LEC,IMP,VICSTM,PI
      COMMON /BLK2/DISFAC,VICFAC,DENFAC,TRFAC,RATEF,SCALE
      DIMENSION CX(120),CY(120),X(120),Y(120),KODE(120)
      DIMENSION FI(120),XSS(120),YSS(120),QSS(120)
C
C  THIS SUBROUTINE READS ALL THE INPUT DATA FOR A REGION
C
C  N=THE NUMBER OF BOUNDARY ELEMENTS=NUMBER OF NODES
C  L=THE NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS
C  CALCULATED
C  NSS= TOTAL NUMBER OF SOURCES AND SINKS
C  QSS= THE STRENGHTS OF THE SOURCES AND SINKS
C  NIF=NUMBER OF INTRFACE ELEMENTS OR NODES
C  T=PERMEABILITY
C
C  READ BASIC PARAMETERS
C
      READ(LEC,200)N,L,NSS,NIF
  200 FORMAT(4I5)
      WRITE(IMP,300)N,L,NSS,NIF
  300 FORMAT(/,30X,'DATA'//8X,'NUMBER OF BOUNDARY ELEMENTS=',
     & I3/8X,'NUMBER OF INTERNAL POINTS WHERE THE FUNCTION IS '
     & ,'CALCULATED=',I3/8X,'NUMBER OF SOURCES AND SINKS=',I5
     & /8X,'NUMBER OF INTERFACE NODES=',I3,/)
C
C....READ RESERVOIR PROPERTIES
C
      READ(LEC,109)T,THICK,POR
  109 FORMAT(3F10.2)
      WRITE(IMP,107)T,THICK,POR
  107 FORMAT(20X,'PERMEABILITY(MD)=',F10.4,//20X,
     & 'THICKNESS(FT)=',F10.4,//20X,'POROSITY=',F10.4,//)
      WRITE(IMP,498)SCALE
  498 FORMAT(/20X,'SCALE: 1 INCH = ',F7.2,'FEET',//)
C
      IF(L .LE. 0)GO TO 499
C
C READ INTERNAL POINTS CORDINATES
      DO 1 I=1,L
    1 READ(LEC,400)CX(I),CY(I)
  400 FORMAT(2F10.4)
C
C  READ AND WRITE THE CORDINATES OF THE EXTREME POINTS
C  OF THE BOUNDARY ELEMENTS INTO ARRAYS X AND Y
C
  499 WRITE(IMP,500)
  500 FORMAT(1H1,///8X,'THE COORDINATES OF THE EXTREME POINTS',
     & ' OF THE BOUNDARY ELEMENTS',//9X,'POINT',11X,'X(INCH)',
     & 12X,'Y(INCH)',/)
      DO 10 I=1,N
      READ(LEC,600)X(I),Y(I)
  600 FORMAT(2F10.4)
   10 WRITE(IMP,700)I,X(I),Y(I)
  700 FORMAT(9X,I3,2(9X,F10.4))
```

```
                C
                C  READ THE BOUNDARY CONDITIONS INTO FI(I) ACCORDING TO A
                C  PRE-ASSIGNED CODE. IF THE POTENTIAL(PRESSURE) IS SPECIFIED,
                C  THE BOUNDARY IS ASSIGNED A CODE=0. IF THE POTENTIAL GRADIENT
                C  IS SPECIFIED, IT IS ASSIGNED A CODE=1. IF THE BOUNDARY IS ON
                C  AN INTERFACE BETWEEN TWO REGIONS(BOTH POTENTIAL AND POTENTIAL
                C  GRADIENT ARE UNKNOWN),IT IS ASSIGNED A CODE=2
                C
0027                    WRITE(IMP,800)
0028              800   FORMAT(1H1,///14X,'BOUNDARY CONDITIONS'//8X,'NODE',6X,
                    &   'CODE',5X,'PRESCRIBED VALUE')
0029                    DO 20 I=1,N
0030                    READ(LEC,900)KODE(I),FI(I)
0031              900   FORMAT(I5,F10.4)
0032              20    WRITE(IMP,950)I,KODE(I),FI(I)
0033              950   FORMAT(8X,I3,8X,I1,8X,E14.7)
                C
                C READ THE COORDINATES AND STRENGTHS OF THE SOURCES AND SINKS
                C
0034                    WRITE(IMP,556)
0035              556   FORMAT(1H1,//8X,'COORDINATES OF SOURCES AND SINKS',1X,
                    &   'AND THEIR STRENGTHS',//)
0036                    WRITE(IMP,557)
0037              557   FORMAT(20X,'X(INCH)',5X,'Y(INCH)',7X,'RATE(BBL/D)',/)
0038                    DO 444 JJ=1,NSS
0039                    READ(LEC,333)XSS(JJ),YSS(JJ),QSS(JJ)
0040              333   FORMAT(  3F10.4)
0041                    WRITE(IMP,666)XSS(JJ),YSS(JJ),QSS(JJ)
0042              666   FORMAT(15X,2F12.4,6X,F12.4)
0043              444   CONTINUE
                C  READ THE AVERAGE RADIUS OF THE WELLS AND THE CHARACTERISTIC
                C  PRESSURE
                C
0044                    READ(LEC,448)RI,TOPP
0045              448   FORMAT(2F10.4)
0046                    WRITE(IMP,449)RI,TOPP
0047              449   FORMAT(/20X,'THE AVERAGE RADIUS OF THE WELLS(INCH) IS=',
                    &   F7.3,/20X,'THE CHARACTERISTIC PRESSURE(PSI) IS=',F10.4//)
0048                    RETURN
0049                    END
```

```
      SUBROUTINE CONVRS(CX,CY,X,Y,KODE,FI,XSS,YSS,QSS,NSS,N,
     & L,T,THICK,POR,RI,TOPP)
C*****************************************************************
C
      COMMON /BLK1/LEC,IMP,VICSTM,PI
      COMMON /BLK2/DISFAC,VICFAC,DENFAC,TRFAC,RATEF,SCALE
      COMMON /BLK3/PRESSF,DPRESF,PORFAC,SCLFAC
      DIMENSION CX(120),CY(120),X(120),Y(120),KODE(120)
      DIMENSION FI(120),XSS(120),YSS(120),QSS(120)
C
      T=T*TRFAC
      RI=RI*DISFAC
      THICK=THICK*DISFAC
      POR=POR*PORFAC
      TOPP=TOPP*PRESSF
      IF(L .LE. 0)GO TO 15
      DO 10 I=1,L
      CX(I)=CX(I)*DISFAC*SCALE
      CY(I)=CY(I)*DISFAC*SCALE
   10 CONTINUE
   15 DO 20 I=1,N
      X(I)=X(I)*DISFAC*SCALE
      Y(I)=Y(I)*DISFAC*SCALE
   20 CONTINUE
C
C  AFTER CONVERTING THE PRESSURE AND PRESSURE GRADIENTS TO
C  SI UNITS,DIVIDE THEM BY THE CHARACTERISTIC PRESSURE
C
      DO 40 I=1,N
      IF(KODE(I) .EQ. 1)GO TO 30
      FI(I)=FI(I)*PRESSF/TOPP
      GO TO 40
   30 FI(I)=FI(I)*DPRESF/TOPP
   40 CONTINUE
      DO 50 I=1,NSS
      XSS(I)=XSS(I)*DISFAC*SCALE
      YSS(I)=YSS(I)*DISFAC*SCALE
      QSS(I)=QSS(I)*RATEF
C
C USE THE RATE TO DETERMINE THE STRENGTH
      QSS(I)=QSS(I)*((VICSTM*VICFAC)/(THICK*T))
C
C  DIVIDE THE STRENGTH BY THE CHARACTERISTIC PRESSURE
      QSS(I)=QSS(I)/TOPP
   50 CONTINUE
      RETURN
      END
```

```
0001                 SUBROUTINE MATRX(X,Y,XM,YM,G,H,FI,DFI,KODE,XSS,YSS,
                 &   QSS,NSS,IJ,NIF,SUM,T,N,L,CX,CY)
           C*****************************************************************
           C
0002                 COMMON /BLK1/LEC,IMP,VICSTM,PI
0003                 DIMENSION X(120),Y(120),XM(120),YM(120),G(120,120)
0004                 DIMENSION H(120,120),DYBLEN(120)
0005                 DIMENSION FI(120),KODE(120),DFI(120),A(120,120),B(120)
0006                 DIMENSION XSS(120),YSS(120),QSS(120),SUM(120),SIG(120)
0007                 DIMENSION SUMC(120),CX(120),CY(120),DXBLEN(120)
0008                 DIMENSION DGDX(120),DGDY(120),DHDX(120),DHDY(120)
0009                 DIMENSION THETA(120),DIST(120),ELMNT(120)
           C
           C  THIS SUBROUTINE COMPUTES THE H AND G MATRIX ELEMENTS BY
           C  THE METHOD OF GAUSSIAN QUADRATURE ALONG THE BOUNDARY
           C  ELEMENTS. IT THEN FORMS THE SYSTEM AX=F
           C  'INTE' CALCULATES ALL THE OFF-DIAGONAL ELEMENTS WHILE
           C  'INLO' CALCULATES ONLY THE DIAGONAL ELEMENTS
           C
           C  COMPUTE THE MID-POINT COORDINATES AND STORE IN ARRAY
           C  XM AND YM
           C
0010                 X(N+1)=X(1)
0011                 Y(N+1)=Y(1)
0012                 DO 10 I=1,N
0013                 XM(I)=(X(I)+X(I+1))/2
0014         10      YM(I)=(Y(I)+Y(I+1))/2
           C
           C  COMPUTE THE G AND H MATRICES AND THE B VECTOR
           C
0015                 DO 31 I=1,N
0016                 DO 30 J=1,N
0017                 IF(I-J)20,25,20
0018         20      CALL INTE(XM(I),YM(I),X(J),Y(J),X(J+1),Y(J+1),H(I,J)
                &    ,G(I,J),DGDX(I),DGDY(I),DHDX(I),DHDY(I),DIST(J),
                &    ELMNT(J),THETA(J))
0019                 GO TO 30
0020         25      CALL INLO(X(J),Y(J),X(J+1),Y(J+1),G(I,J))
0021                 H(I,J)=PI
0022         30      CONTINUE
0023         31      CONTINUE
           CC  WRITE PORTIONS OF THE MATRICES OBTAINED
           C         WRITE(IMP,34)
           C  34      FORMAT(1H1,1X,'THE H MATRIX IS'//)
           C         NNA=1
           C         NA=0
           C  102     NA=NA+12
           C         DO 32 I=1,N
           C         WRITE(IMP,33)(H(I,J),J=NNA,NA)
           C  33      FORMAT(1X,12F10.4)
           C  32      CONTINUE
           C         IF(NA .GE. N)GO TO 101
           C         NNA=NNA+12
           C          GO TO 102
           C  101      CONTINUE
           C         WRITE(IMP,36)
           C  36      FORMAT(//,1X,'THE G MATRIX IS'//)
           C         NNA=1
```

```
                C       NA=0
                C  104   NA=NA+12
                C       DO 38 I=1,N
                C       WRITE(IMP,39)(G(I,J),J=NNA,NA)
                C  39    FORMAT(1X,12E10.2)
                C  38    CONTINUE
                C       IF(NA .GE. N)GO TO 103
                C       NNA=NNA+12
                C       GO TO 104
                C  103   CONTINUE
                C
                C ARRANGE THE SYSTEM TO PUT ALL THE UNKNOWNS IN G(I,J) ON THE LHS
                C
 0024                   IJ=0
 0025                   NEIF=N+NIF
 0026                   DO 51 J=1,N
 0027                   IF(KODE(J) .EQ. 1)GO TO 41
 0028                   IF(KODE(J) .EQ. 0)GO TO 51
 0029                   IJ=IJ+1
 0030                   DO 50 I=1,N
 0031                   CH=H(I,J)
 0032                   G(I,N+IJ  )=-CH
 0033                   H(I,J)=0.
 0034             50    CONTINUE
 0035             40    GO TO 51
 0036             41    DO 52 I=1,N
 0037                   CH=G(I,J)
 0038                   G(I,J)=-H(I,J)
 0039                   H(I,J)=-CH
 0040             52    CONTINUE
 0041             51    CONTINUE
                C       WRITE(IMP,46)
                C  46    FORMAT(1H1,1X,'THE REARRANGED G MATRIX IS'//)
                C       NNA=1
                C       NAA=0
                C  106   NAA=NAA+12
                C       DO 48 I=1,N
                C       WRITE(IMP,47)(G(I,J),J=NNA,NAA)
                C  47    FORMAT(1X,12E10.2)
                C  48    CONTINUE
                C       IF(NAA .GE. N)GO TO 105
                C       NNA=NNA+12
                C       GO TO 106
                C  105   CONTINUE
                C       WRITE(IMP,42)
                C  42    FORMAT(//,1X,'THE REARRANGED H MATRIX IS'//)
                C       NNA=1
                C       NAA=0
                C  110   NAA=NAA+12
                C       DO 43 I=1,N
                C       WRITE(IMP,44)(H(I,J),J=NNA,NAA)
                C  44    FORMAT(1X,12E10.2)
                C  43    CONTINUE
                C       IF(NAA .GE. N)GO TO 109
                C       NNA=NNA+12
                C       GO TO 110
                C  109   CONTINUE
                C
```

```
                C CALCULATE THE CONTRIBUTIONS OF THE SOURCES AND SINKS
                C
 0042                 DO 55 I=1,N
 0043                 SUM(I)=0.
 0044                 DO 54 JJ=1,NSS
 0045                 CALL SOURCE(XM(I),YM(I),XSS(JJ),YSS(JJ),QSS(JJ),NSS,
                     & BLENT,DXSURS,DYSURS)
 0046                 SUM(I)=SUM(I)+BLENT
 0047             54  CONTINUE
 0048             55  CONTINUE
                C     WRITE(IMP,56)
                C  56  FORMAT(//,1X,'THE VALUES DUE TO SOURCES AND SINKS ARE'//
                C     DO 58 I=1,N
                C     WRITE(IMP,57)SUM(I)
                C  57  FORMAT(30X,F12.4)
                C  58  CONTINUE
                C
                C  COMPUTE THE R.H.S. VECTOR
                C
                C     WRITE(IMP,71)
                C  71  FORMAT(1H1,//10X,'THE RHS WITHOUT EFFECTS OF ',
                C    & 'SOURCES AND SINKS'//)
 0049                 DO 61 I=1,N
 0050                 DFI(I)=0.
 0051                 DO 60 J=1,N
 0052                 DFI(I)=DFI(I)+H(I,J)*FI(J)
 0053             60  CONTINUE
                C     WRITE(IMP,72)DFI(I)
                C  72  FORMAT(10X,E14.7)
 0054                 DFI(I)=DFI(I)-SUM(I)
 0055             61  CONTINUE
 0056                 RETURN
 0057                 END
```

```
      SUBROUTINE INTE(XP,YP,X1,Y1,X2,Y2,H,G,DGDX,DGDY,
     &  DHDX,DHDY,DIST,ELMNT,THETA)
C*******************************************************************
C
      DIMENSION XCO(4),YCO(4),GI(4),OME(4)
      COMMON /BLK1/LEC,IMP,VICSTM,PI
C
C  ALL THE OFF-DIAGONAL ELEMENTS OF MATRICES H AND G ARE
C  CALCULATED BY THIS SUBROUTINE
C  DIST=PERPENDICULAR DISTANCE FROM THE NODE POINT UNDER
C  CONSIDERATION TO THE BOUNDARY ELEMENTS
C  RA=DISTANCE FROM THE POINT UNDER CONSIDERATION TO THE
C  INTEGRATION POINTS IN THE BOUNDARY ELEMENTS
C
      GI(1)=0.86113631
      GI(2)=-GI(1)
      GI(3)=0.33998104
      GI(4)=-GI(3)
      OME(1)=0.34785485
      OME(2)=OME(1)
      OME(3)=0.65214515
      OME(4)=OME(3)
      AX=(X2-X1)/2
      BX=(X2+X1)/2
      AY=(Y2-Y1)/2
      BY=(Y2+Y1)/2
      ELMNT=SQRT((X1-X2)**2+(Y1-Y2)**2)
      IF(AX)10,20,10
   10 TA=AY/AX
      THETA=ATAN(TA)
      XLEN=(TA*XP-YP+Y1-TA*X1)/SQRT(TA**2+1.0)
      DIST=ABS(XLEN)
      DDISTX=(TA/SQRT(TA**2+1.0))
      DDISTY=(-1.0/SQRT(TA**2+1.0))
      IF(XLEN .GT. 0.0)GO TO 30
   36 DDISTX=-DDISTX
      DDISTY=-DDISTY
      THETA=-THETA
      GO TO 30
   20 ALEN=XP-X1
      DIST=ABS(ALEN)
      THETA=PI/2.0
      DDISTX=1.0
      DDISTY=0.0
      IF(ALEN .GT. 0.0)GO TO 30
      DDISTX=-DDISTX
      DDISTY=-DDISTY
      THETA=-THETA
   30 SIG=(X1-XP)*(Y2-YP)-(X2-XP)*(Y1-YP)
      IF(SIG)31,32,32
   31 DIST=-DIST
      DDISTX=-DDISTX
      DDISTY=-DDISTY
      THETA=-THETA
   32 G=0.
      H=0.
      DGDX=0.
      DGDY=0.
```

```
0048              DHDX=0.
0049              DHDY=0.
0050              DO 40 I=1,4
0051              XCO(I)=AX*GI(I)+BX
0052              YCO(I)=AY*GI(I)+BY
0053              RA=SQRT((XP-XCO(I))**2+(YP-YCO(I))**2)
0054              G=G+ALOG(1/RA)*OME(I)*SQRT(AX**2+AY**2)
0055              DGDX=DGDX+(-1.0*(XP-XCO(I))/RA**2)*OME(I)*SQRT(AX**2+
               &  AY**2)
0056              DGDY=DGDY+(-1.0*(YP-YCO(I))/RA**2)*OME(I)*SQRT(AX**2+
               &  AY**2)
0057              H=H+(-1.0*(DIST*OME(I)*SQRT(AX**2+AY**2))/RA**2)
0058              DHDX=DHDX+((-1.0)*(((1.0/RA**2)*DDISTX)+(DIST*(-2.0)
               &  *(XP-XCO(I))/RA**4)))*OME(I)*SQRT(AX**2+AY**2)
0059              DHDY=DHDY+((-1.0)*(((1.0/RA**2)*DDISTY)+(DIST*(-2.0)
               &  *(YP-YCO(I))/RA**4)))*OME(I)*SQRT(AX**2+AY**2)
0060        40    CONTINUE
0061              RETURN
0062              END
```

```
0001          SUBROUTINE INLO(X1,Y1,X2,Y2,G)
        C*************************************************************
        C
        C  THIS SUBROUTINE COMPUTES THE DIAGONAL ELEMENTS OF THE
        C  'G' MATRIX. THE DIAGONAL ELEMENTS OF THE 'H' MATRIX
        C  ARE ALL ZERO.
        C
0002          AX=(X2-X1)/2
0003          AY=(Y2-Y1)/2
0004          SR=SQRT(AX**2+AY**2)
0005          G=2*SR*(ALOG(1/SR)+1)
0006          ELMNT=SQRT((X1-X2)**2+(Y1-Y2)**2)
0007          RETURN
0008          END
```

```
0001          SUBROUTINE SOURCE(XM,YM,XSS,YSS,QSS,NSS,BLENT,DXSURS,
             & DYSURS)
        C*************************************************************
0002          COMMON /BLK1/LEC,IMP,VICSTM,PI
0003          DENT=SQRT((XM-XSS)**2+(YM-YSS)**2)
0004          BLENT=QSS*ALOG(1.0/DENT)
0005          DXSURS=QSS*(-1.0)*(XM-XSS)/DENT**2
0006          DYSURS=QSS*(-1.0)*(YM-YSS)/DENT**2
0007          RETURN
0008          END
```

```
      SUBROUTINE ASEMBL(G1,G2,DFI1,DFI2,N1,N2,A,B,NAA,NIF,
     & NNN,NOM,NNI,NNM,T1,T2)
C**********************************************************************
C
      COMMON /BLK1/LEC,IMP,VICSTM,PI
      DIMENSION G1(120,120),G2(120,120),DFI1(120),DFI2(120)
      DIMENSION A(120,120),B(120)
C
C STORE THE MATRIX G(I,J) INTO A(I,J) AND DFI(I,J) INTO B(I,J)
C
C INITIALIZE THE ARRAYS
C
      NAA=N2+N1
      NNN=N1-NIF
      NDIF=N2-NIF
      NOM=N1+NIF
      NNI=NIF+1
      NNM=N2+NIF+1
      DO 6 I=1,NAA
      DO 7 J=1,NAA
      A(I,J)=0.0
    7 CONTINUE
      B(I)=0.0
    6 CONTINUE
C
C ASSEMBLE G1 INTO A
C
  806 DO 801 J=1,NOM
      DO 800 I=1,N1
      A(I,J)=G1(I,J)
  800 CONTINUE
  801 CONTINUE
      DO 805 J=1,NDIF
      DO 804 I=1,N1
      A(I,NOM+J)=0.
  804 CONTINUE
  805 CONTINUE
      DO 808 J=1,N1
  808 B(J)=DFI1(J)
C
C ASSEMBLE G2 INTO A
C
  810 DO 812 J=1,NNN
      DO 812 I=1,N2
      A(N1+I,J)=0.
  812 CONTINUE
      DO 814 J=1,NIF
      DO 813 I=1,N2
      A(N1+I,NNN+J)=-G2(I,NNI-J)*(T1/T2)
  813 CONTINUE
  814 CONTINUE
      DO 816 J=1,NIF
      DO 815 I=1,N2
      A(N1+I,N1+J)=G2(I,NNM-J)
  815 CONTINUE
  816 CONTINUE
      DO 818 J=1,NDIF
      DO 817 I=1,N2
```

```
                A(N1+I,NOM+J)=G2(I,NIF+J)
          817   CONTINUE
          818   CONTINUE
                DO 830 I=1,N2
                B(N1+I)=DFI2(I)
          830   CONTINUE
      C
      C  WRITE OUT THE FINAL RESULTING MATRIX
      C
      C  821     WRITE(IMP,62)
      C   62     FORMAT(1H1,1X,'THE FINAL ASSEMBLED MATRIX IS :'//)
      C         NB=1
      C         NBB=0
      C  740     NBB=NBB+12
      C         DO 1101 I=1,NAA
      C         WRITE(IMP,1100)(A(I,J),J=NB,NBB),B(I)
      C 1100     FORMAT(1X,12F9.3,4X,F9.4)
      C 1101     CONTINUE
      C         IF(NB .GE. 48)GO TO 741
      C         NB=NB+12
      C         GO TO 740
      C  741     CONTINUE
                RETURN
                END
```

```
              SUBROUTINE SLNPD(A,B,D,N,NC,NJ)
C*****************************************************************
C
              COMMON /BLK1/LEC,IMP,VICSTM,PI
              DIMENSION A(120,120),B(120)
              DOUBLE PRECISION AA(120,120),BB(120),C
C
C  THIS IS A SUBROUTINE THAT CAN SOLVE THE EQUATIONS BY
C  GAUSSIAN ELIMINATION
C
C  A=THE SYSTEM MATRIX
C  B=ORIGINALLY IT CONTAINS THE RHS COEFFICIENTS. AFTER
C    SOLUTION IT CONTAINS THE VALUES OF THE SOLUTION VECTOR
C  N=ACTUAL NUMBER OF UNKNOWNS
C  NX=ROW AND COLUMN DIMENSION OF MATRIX 'A'
C
              NJ=N+NC
              N11=NJ-1
C
C CHANGE TO DOUBLE PRECISION VARIABLES
C
              DO 11 I=1,NJ
              DO 12 J=1,NJ
              AA(I,J)=A(I,J)
   12         CONTINUE
              BB(I)=B(I)
   11         CONTINUE
C
C  INTERCHANGE ROWS TO GET NON ZERO DIAGONAL COEFFICIENT
C
              DO 100 K=1,N11
              K1=K+1
              C=A(K,K)
              IF(DABS(C)-0.000001)1,1,3
    1         DO 7 J=K1,NJ
              IF(DABS(AA(J,K))-0.000001)7,7,5
    5         DO 6 L=K,NJ
              C=AA(K,L)
              AA(K,L)=AA(J,L)
    6         AA(J,L)=C
              C=BB(K)
              BB(K)=BB(J)
              BB(J)=C
              C=AA(K,K)
              GO TO 3
    7         CONTINUE
    8         WRITE(IMP,2)K
    2         FORMAT('***** SINGULARITY IN ROW',I5)
              D=0.
              GO TO 300
C
C  DIVIDE ROW BY DIAGONAL COEFFICIENT
C
    3         C=AA(K,K)
              DO 4 J=K1,NJ
    4         AA(K,J)=AA(K,J)/C
              BB(K)=BB(K)/C
C
```

```
C  ELIMINATE UNKNOWN X(K) FROM ROW 1
C
         DO 10 I=K1,NJ
         C=AA(I,K)
         DO 9 J=K1,NJ
   9     AA(I,J)=AA(I,J)-C*AA(K,J)
  10     BB(I)=BB(I)-C*BB(K)
 100     CONTINUE
C
C
C  COMPUTE LAST UNKNOWN
C
         IF(DABS(AA(NJ,NJ))-0.000001)14,14,101
 101     BB(NJ)=BE(NJ)/AA(NJ,NJ)
C
C  APPLY BACK SUBSTITUTION PROCESS TO COMPUTE REMAINING UNKNOWNS
C
         DO 200 L=1,N11
         K=NJ-L
         K1=K+1
         DO 200 J=K1,NJ
 200     BB(K)=BB(K)-AA(K,J)*BB(J)
         WRITE(IMP,1307)
1307     FORMAT(1H1,//,2X,'THE SOLUTION VECTOR IS'//)
         DO 1300 I=1,NJ
         WRITE(IMP,1301)BB(I)
1301  FORMAT(10X,E14.7)
1300  CONTINUE
C
C RETURN TO SINGLE PRECISSION VARIABLES
C
 255     DO 13 J=1,NJ
         B(J)=BB(J)
  13     CONTINUE
C
C  COMPUTE VALUE OF DETERMINANT
C
C        D=1.
C        DO 250 I=1,NJ
C  250    D=D*AA(I,I)
         GO TO 300
  14     WRITE(IMP,15)K
  15     FORMAT('*****SINGULARITY IN ROW',I5)
         D=0.
 300     RETURN
         END
```

```
            SUBROUTINE SPLIT(N1,N2,NIF,NNN,NOM,NNI,NNM,NAA,KODE1,
     &      KODE2,FI1,FI2,DFI1,DFI2,B,T1,T2)
C*****************************************************************
C
            COMMON /BLK1/LEC,IMP,VICSTM,PI
            DIMENSION KODE1(120),KODE2(120),FI1(120),FI2(120)
            DIMENSION DFI1(120),DFI2(120),B(120)
C
            DO 50 I=1,N1
            IF(KODE1(I) .EQ. 2)GO TO 50
            IF(KODE1(I) .EQ. 0)GO TO 52
            CH=FI1(I)
            DFI1(I)=CH
            FI1(I)=B(I)
            GO TO 50
   52       DFI1(I)=B(I)
   50       CONTINUE
   54       DO 60 I=1,NIF
            DFI1(NNN+I)=B(NNN+I)
            DFI2(I)=-B(N1+1-I)*(T1/T2)
            FI2(I)=B(NOM+1-I)
            FI1(NNN+I)=B(N1+I)
   60       CONTINUE
            NDIF=N2-NIF
            DO 70 I=1,N2
            IF(KODE2(I) .EQ. 2)GO TO 70
            IF(KODE2(I) .EQ. 0)GO TO 61
            CH=FI2(I)
            FI2(I)=B(N1+I)
            DFI2(I)=CH
            GO TO 70
   61       DFI2(I)=B(N1+I)
   70       CONTINUE
   51       CONTINUE
            RETURN
            END
```

```
0001                SUBROUTINE INTER(FI,DFI,L,N,CX,CY,X,Y,
                 &  XSS,YSS,QSS,NSS,SOL)
           C*****************************************************************
           C
0002                COMMON /BLK1/LEC,IMP,VICSTM,PI
0003                COMMON /BLK2/DISFAC,VICFAC,DENFAC,TRFAC,RATEF,SCALE
0004                COMMON /BLK3/PRESSF,DPRESF,PORFAC,SCLFAC
0005                DIMENSION FI(120),DFI(120),CX(120),CY(120),X(120)
0006                DIMENSION Y(120),DYBLEN(120),ELMNT(120)
0007                DIMENSION SOL(120),SUM(120),DGDX(120),DGDY(120)
0008                DIMENSION DHDX(120),DHDY(120),SIG(120),DXBLEN(120)
0009                DIMENSION THETA(120),XSS(120),YSS(120),QSS(120)
0010                DIMENSION XIF(120),YIF(120),FIPLOT(120),DIST(120)
           C
           C  THIS SUBROUTINE COMPUTES POTENTIAL VALUES FOR INTERNAL POINTS
           C
0011                DO 40 K=1,L
0012                SOL(K)=0.
0013                DO 30 J=1,N
0014                CALL INTE(CX(K),CY(K),X(J),Y(J),X(J+1),Y(J+1),HI,
                 &  GI,DGDX(K),DGDY(K),DHDX(K),DHDY(K),DIST(J),ELMNT(J),
                 &  THETA(J))
0015                SOL(K)=SOL(K)+DFI(J)*GI-FI(J)*HI
0016         30     CONTINUE
           C CALCULATE THE VALUES DUE TO SOURCES AND SINKS AT INTERNAL POINT
0017                SUM(K)=0.
0018                DO 90 JJ=1,NSS
0019                CALL SOURCE(CX(K),CY(K),XSS(JJ),YSS(JJ),QSS(JJ),NSS,
                 &  BLENT,DXSURS,DYSURS)
0020                SUM(K)=SUM(K)+BLENT
0021         90     CONTINUE
0022                SOL(K)=(SOL(K)+SUM(K))/(2.0*PI)
0023         40     CONTINUE
0024                WRITE(IMP,300)
0025        300     FORMAT(//,2X,'INTERNAL POINTS',//11X,'X',18X,'Y',
                 &  14X,'POTENTIAL',/)
0026                DO 20 K=1,L
0027                XIF(K)=CX(K)/SCLFAC
0028                YIF(K)=CY(K)/SCLFAC
0029                FIPLOT(K)=SOL(K)/PRESSF
0030         20     WRITE(IMP,400)XIF(K),YIF(K),FIPLOT(K)
0031        400     FORMAT(3(5X,E14.7))
0032                RETURN
0033                END
```

```
0001            SUBROUTINE OUTPUT(X,Y,XM,YM,FI,DFI,CX,CY,SOL,N,XSS,
              &  YSS,L,NSS,QSS,TOPP,THICK,T,QSSS)
          C*****************************************************************
          C
0002            COMMON /BLK1/LEC,IMP,VICSTM,PI
0003            COMMON /BLK2/DISFAC,VICFAC,DENFAC,TRFAC,RATEF,SCALE
0004            COMMON /BLK3/PRESSF,DPRESF,PORFAC,SCLFAC
0005            DIMENSION XM(120),YM(120),FI(120),DFI(120),CX(120)
0006            DIMENSION SOL(120),XSS(120),YSS(120),XIF(120)
0007            DIMENSION CY(120),QSSS(120),X(120),Y(120),QSS(120)
0008            DIMENSION YIF(120),FIPLOT(120),DFIPLT(120)
          C
0009            WRITE(IMP,100)
0010        100 FORMAT(1H1,80('*')//30X,'RESULTS'//25X,'BOUNDARY NODES'
              &  //11X,'X(INCH)',13X,'Y(INCH)',10X,'PRESSURE(PSI)',7X,
              &  'NORMAL GRADIENT'/)
          C
0011            DO 10 I=1,N
0012            FI(I)=FI(I)*TOPP
0013            DFI(I)=DFI(I)*TOPP
          C
          C FOR SAKE OF OUTPUT ONLY RETURN THE FOLLOWING TO FPS UNITS
0014            XIF(I)=XM(I)/SCLFAC
0015            YIF(I)=YM(I)/SCLFAC
0016            FIPLOT(I)=FI(I)/PRESSF
0017            DFIPLT(I)=DFI(I)/DPRESF
0018            WRITE(IMP,200)XIF(I),YIF(I),FIPLOT(I),DFIPLT(I)
0019        200 FORMAT(4(8X,E12.4))
0020         10 CONTINUE
0021            DO 30 J=1,NSS
0022            QSS(J)=QSS(J)*TOPP
          C
          C DETERMINE THE RATE FROM THE STRENGTH
0023            QSSS(J)=QSS(J)/(VICSTM*VICFAC/(THICK*T))
0024         30 CONTINUE
0025            WRITE(IMP,500)
0026        500 FORMAT(' ',80('*'))
0027            RETURN
0028            END
```

```
      SUBROUTINE BDRY(X,Y,XSS,YSS,NSS,QSSS,N)
C*****************************************************************
C
      COMMON /BLK1/LEC,IMP,VICSTM,PI
      COMMON /BLK2/DISFAC,VICFAC,DENFAC,TRFAC,RATEF,SCALE
      COMMON /BLK3/PRESSF,DPRESF,PORFAC,SCLFAC
      DIMENSION X(120),Y(120),XSS(120),YSS(120),QSS(120)
      DIMENSION QSSS(120)
C
C SUBROUTINE BDRY PLOTS THE BOUNDARY AND WELL LOCATIONS
C
      FACPLO=2.0
      N1=N+1
      CALL FACTOR(FACPLO)
      CALL PLOT(X(N1)/SCLFAC,Y(N1)/SCLFAC,3)
      DO 741 I=1,N1
      CALL PLOT(X(I)/SCLFAC,Y(I)/SCLFAC,2)
  741 CONTINUE
      DO 742 I=1,NSS
      FACPLO=2.0
      XSSI=XSS(I)/SCLFAC
      YSSI=YSS(I)/SCLFAC
      QSSI=QSSS(I)/RATEF
      PN1=XSSI*FACPLO
      PN2=YSSI*FACPLO
      FACPLO=1.0
      CALL FACTOR(FACPLO)
      CALL NUMBER(PN1-0.15,PN2-0.15,0.08,QSSI,0.0,2)
      IF(QSS(I) .GE. 0.0) GO TO 744
      CALL SYMBOL(PN1,PN2,0.08,11,0,-1)
      GO TO 742
  744 CALL SYMBOL(PN1,PN2,0.08,10,0,-1)
  742 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STRM(NSS,QSS,FI,DFI,X,Y,XSS,YSS,
     & N,T,KODE,IPE,XOE,YOE,XM,YM,AQSS,ANSL,MM,ATETA,
     & ADFI,ELMNT,THICK,POR,RI,QSSS,OLDL,OLDW)
C*******************************************************************
C
      COMMON /ELK1/LEC,IMP,VICSTM,PI
      COMMON /BLK2/DISFAC,VICFAC,DENFAC,TRFAC,RATEF,SCALE
      COMMON /BLK3/PRESSF,DPRESF,PORFAC,SCLFAC
      DIMENSION X(120),Y(120),FI(120),DFI(120),XSS(120)
      DIMENSION YSS(120),G(120,120),DHDY(120),RO(120)
      DIMENSION QSS(120),XI(120),YI(120),KODE(120)
      DIMENSION MM(120),VX(120),VY(120),VT(120),H(120,120)
      DIMENSION DIST(120),XM(120),YM(120),SUM(120),DISMID(120)
      DIMENSION ANSL(120),DGDX(120),DGDY(120),DHDX(120)
      DIMENSION DXBLEN(120),DPDX(120),DPDY(120),DYBLEN(120)
      DIMENSION DPHIDX(120),DPHIDY(120),VLCP(120),XOE(120)
      DIMENSION SINK(120),STRML(120),STRMW(120),AVWIDT(120)
      DIMENSION AQSS(120),ELMNT(120),LENTH(120,120)
      DIMENSION QSSS(120),CQSS(120,120),WIDTH(120,120)
      DIMENSION ADFI(120),ATETA(120),THETA(120),VEL(120)
      DIMENSION YOE(120),ZETA(120),XIF(120),YIF(120)
      DIMENSION OLDL(120),OLDW(120)
      INTEGER SINK
      REAL LENTH
C
C FACT=THE CONVERSION FACTOR CHANGING (ATM/CM) TO
C (NEWTON/SQ M)/M
      FACT=1.0333E+7
C CALCULATE THE VELOCITY NEAR THE LOWEST PRODUCING WELL
C
      CALL LOWVEL(NSS,QSS,FI,DFI,X,Y,XSS,YSS,N,T,VLOW,RI,
     & QMNI,RAD,POR,THICK)
C
C  INITIALIZE ARRAYS
C
      NST=20
      DO 206 I=1,NSS
      SINK(I)=0.0
  206 CONTINUE
      IPE=0
      DO 220 I=1,NSS
      IF(QSSS(I) .LE. 0.0) GO TO 220
      NSL=ABS((-FLOAT(NST)*QSSS(I)/QMNI)+0.5)
      IF(NSL .LT. 10) NSL=10
      IF(NSL .GT. 20) NSL=20
C
C CALCULATE THE STARTING POSITIONS ON A STREAMLINE
C
      DO 210 K=1,NSL
      ICANT=0
      XI(K)=XSS(I)+RAD*COS(1.0+(K*2.0*PI)/NSL)
      YI(K)=YSS(I)+RAD*SIN(1.0+(K*2.0*PI)/NSL)
      STRML(K)=0.0
      STRMW(K)=(2.0*PI*RI)/NSL
      AVWIDT(K)=STRML(K)
      FACPLO=2.0
      CALL FACTOR(FACPLO)
      CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,3)
```

```
C
C CALCULATE THE POTENTIAL AND VELOCITY AT THIS POSITION
C
C IPE IS THE COUNTER FOR STREAMLINES THAT CROSS INTO NEW REGIONS
C ICANT IS THE COUNTER FOR THE NUMBER OF POINTS ON A STREAMLINE
C        WRITE(IMP,10)
C  10     FORMAT(6X,'XI',8X,'YI',7X,'DPDX',5X,'DPDY',5X,
C      $  'DXBLEN',5X,'DYBLEN',7X,'VT',7X,'DT',7X,'DSX',7X,'DSY')
  171    ICANT=ICANT+1
         IF(ICANT .GT. 100) GO TO 210
         VX(K)=0.
         VY(K)=0.
         VT(K)=0.
         DPDX(K)=0.
         DPDY(K)=0.
         DO 172 J=1,N
         DISMID(J)=SQRT((XI(K)-XM(J))**2+(YI(K)-YM(J))**2)
         CALL INTE(XI(K),YI(K),X(J),Y(J),X(J+1),Y(J+1),H(K,J),
     &   G(K,J),DGDX(J),DGDY(J),DHDX(J),DHDY(J),DIST(J),
     &   ELMNT(J),THETA(J))
         ELMNT2=0.5*ELMNT(J)
C....CHECK IF POINT IS NEAR A BOUNDARY. IF SO GO  TO 347
C...OTHERWISE FIND THE VELOCITY AT THE CURRENT POSITION
         IF(DISMID(J) .LT. ELMNT2)GO TO 347
         GO TO 748
  347    IF(ICANT .GT. 1)GO TO 258
         WRITE(IMP,259)I
  259    FORMAT(/8X,'INJECTOR NUMBER',I2,' IS TOO CLOSE TO',
     &   ' A BOUNDARY',/)
         GO TO 220
  258    IF(KODE(J) .GT. 1) GO TO 749
         GO TO 210
  749    IF(DFI(J) .LT. 0.0)GO TO 348
         GO TO 210
  748    DPDX(K)=DPDX(K)+DFI(J)*DGDX(J)-FI(J)*DHDX(J)
         DPDY(K)=DPDY(K)+DFI(J)*DGDY(J)-FI(J)*DHDY(J)
  172    CONTINUE
C
C CALCULATE THE CONTRIBUTIONS OF THE SOURCES AND SINKS
C
         DXBLEN(K)=0.
         DYBLEN(K)=0.
         DO 54 JJ=1,NSS
         CALL SOURCE(XI(K),YI(K),XSS(JJ),YSS(JJ),QSS(JJ),NSS,
     &   BLENT,DXSURS,DYSURS)
         DXBLEN(K)=DXBLEN(K)+DXSURS
         DYBLEN(K)=DYBLEN(K)+DYSURS
  54     CONTINUE
         DPHIDX(K)=(DPDX(K)+DXBLEN(K))/(2.0*PI)*FACT
         DPHIDY(K)=(DPDY(K)+DYBLEN(K))/(2.0*PI)*FACT
         VX(K)=-1.0*(T/VICSTM*VICFAC)*DPHIDX(K)
         VY(K)=-1.0*(T/VICSTM*VICFAC)*DPHIDY(K)
         VT(K)=SQRT(VX(K)**2+VY(K)**2)
         DT=50.0*RI/ABS(VT(K))
         DSX=DT*VX(K)
         DSY=DT*VY(K)
         DS=DT*VT(K)
         XIF(K)=XI(K)/SCLFAC
```

```
              YIF(K)=YI(K)/SCLFAC
              IF(ICANT .GT. 5)GO TO 3005
C             WRITE(IMP,11)XIF(K),YIF(K),DPDX(K),DPDY(K),DXBLEN(K),
C          &  DYBLEN(K),VT(K),DT,DSX,DSY
C  11         FORMAT(2X,10E10.3)
C...CHECK IF POINT IS NEAR A PRODUCER, IF SO BRANCH TO 333
C...OTHERWISE CALCULATE THE LENGTH AND WIDTH AT THE CURRENT
C...POSITION. PLOT THE CURRENT POSITION
  3005        CONTINUE
              IF(VT(K) .GT. VLOW) GO TO 333
  111         STRML(K)=STRML(K)+(VT(K)*DT)
              STRMW(K)=STRMW(K)+QSSS(I)/(NSL*THICK*POR*VT(K))
              AVWIDT(K)=STRMW(K)/ICANT
              CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,2)
C...CALCULATE NEW POSITION
              XI(K)=XI(K)+VX(K)*DT
              YI(K)=YI(K)+VY(K)*DT
              XIF(K)=XI(K)/SCLFAC
              YIF(K)=YI(K)/SCLFAC
              GO TO 171
  333         DO 200 LL=1,NSS
              IF(QSS(LL) .GE. 0.0)GO TO 200
              RO(LL)=SQRT((XI(K)-XSS(LL))**2+(YI(K)-YSS(LL))**2)
              IF(RAD .GE. RO(LL)) GO TO 208
   200        CONTINUE
C...POINT IS NOT NEAR PRODUCER, BUT RATHER, IT IS NEAR INJECTOR
              GO TO 111
C IDENTIFY THE PRODUCER AT WHICH THE STREAMLINE TERMINATED
   208        SINK(LL)=SINK(LL)+1
              LAST=SQRT((XI(K)-XSS(LL))**2+(YI(K)-YSS(LL))**2)
              WLAST=(2.0*PI*RI)/FLOAT(NSL)
              STRMW(K)=STRMW(K)+WLAST
              AVWIDT(K)=STRMW(K)/(FLOAT(ICANT)+1.0)
              LENTH(LL,SINK(LL))=STRML(K)+LAST
              WIDTH(LL,SINK(LL))=AVWIDT(K)
              CQSS(LL,SINK(LL))=QSSS(I)/FLOAT(NSL)
              CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,2)
              GO TO 210
C
C...POINT IS NEAR AN INTERFACE BOUNDARY
C
   348        IPE=IPE+1
              MM(IPE)=J
              EX=XI(K)
              EY=YI(K)
C
C...CHECK FOR HORIZONTAL BOUNDARY
C
              IF(ABS(Y(J+1)-Y(J)) .GT. 0.001)GO TO 244
   243        XI(K)=XI(K)
              YI(K)=Y(J)
              XOE(IPE)=XI(K)
              YOE(IPE)=YI(K)
              AQSS(IPE)=QSSS(I)
              ANSL(IPE)=NSL
              ADFI(IPE)=DFI(J)
C
C  CALCULATE THE ANGLE THE OUTWARD NORMAL MAKES WITH THE
```

```
                C  X-DIRECTION
                C
0125                   ATETA(IPE)=0.5*PI
0126                   IF(Y(J) .LT. EY)ATETA(IPE)=-ATETA(IPE)
0127                   VT(K)=-1.0*(T/VICFAC*VICSTM)*DFI(J)
0128                   VEL(IPE)=VT(K)
0129                   CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,2)
0130                   STRML(K)=STRML(K)+DIST(J)
0131                   STRMW(K)=STRMW(K)+QSSS(I)/(NSL*THICK*POR*VT(K))
0132                   AVWIDT(K)=STRMW(K)/ICANT
0133                   OLDL(IPE)=STRML(K)
0134                   OLDW(IPE)=AVWIDT(K)
0135                   EX=EX/SCLFAC
0136                   EY=EY/SCLFAC
0137                   XIF(K)=XI(K)/SCLFAC
0138                   YIF(K)=YI(K)/SCLFAC
0139              714  GO TO 2121
                C
                C...CHECK FOR VERTICAL BOUNDARY
                C
0140              244  IF(ABS(X(J+1)-X(J)) .GT. 0.001)GO TO 245
0141              246  XI(K)=X(J)
0142                   YI(K)=YI(K)
0143                   XOE(IPE)=XI(K)
0144                   YOE(IPE)=YI(K)
0145                   AQSS(IPE)=QSSS(I)
0146                   ANSL(IPE)=NSL
0147                   ADFI(IPE)=DFI(J)
                C
                C  THETA IS THE ANGLE THAT THE BOUNDARY ELEMENT MAKES WITH
                C THE X-AXIS
                C  ATETA IS THE ANGLE THE OUTWARD NORMAL MAKES WITH THE
                C  X-AXIS
                C
0148                   ATETA(IPE)=0.0
0149                   IF(EX .GT. X(J))GO TO 3001
0150                   ATETA(IPE)=-ATETA(IPE)
0151                   ADFI(IPE)=-ADFI(IPE)
0152             3001  VT(K)=-1.0*(T/VICFAC*VICSTM)*DFI(J)
0153                   VEL(IPE)=VT(K)
0154                   CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,2)
0155                   STRML(K)=STRML(K)+DIST(J)
0156                   STRMW(K)=STRMW(K)+QSSS(I)/(NSL*THICK*POR*VT(K))
0157                   AVWIDT(K)=STRMW(K)/ICANT
0158                   OLDL(IPE)=STRML(K)
0159                   OLDW(IPE)=AVWIDT(K)
0160                   EX=EX/SCLFAC
0161                   EY=EY/SCLFAC
0162                   XIF(K)=XI(K)/SCLFAC
0163                   YIF(K)=YI(K)/SCLFAC
0164                   GO TO 2121
0165              245  XI(K)=(((X(J)-X(J+1))/(Y(J+1)-Y(J)))*EX-((Y(J+1)-Y(J))/
                     &  (X(J+1)-X(J)))*X(J)+Y(J)-EY)/((X(J)-X(J+1))/(Y(J+1)-Y(J)
                     &  )-((Y(J+1)-Y(J))/(X(J+1)-X(J))))
0166                   YI(K)=((X(J)-X(J+1))/(Y(J+1)-Y(J)))*XI(K)-((X(J)-X(J+1))/
                     &  (Y(J+1)-Y(J)))*EX+EY
0167                   XOE(IPE)=XI(K)
0168                   YOE(IPE)=YI(K)
```

```
               C  X-DIRECTION
               C
0125                 ATETA(IPE)=0.5*PI
0126                 IF(Y(J) .LT. EY)ATETA(IPE)=-ATETA(IPE)
0127                 VT(K)=-1.0*(T/VICFAC*VICSTM)*DFI(J)
0128                 VEL(IPE)=VT(K)
0129                 CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,2)
0130                 STRML(K)=STRML(K)+DIST(J)
0131                 STRMW(K)=STRMW(K)+QSSS(I)/(NSL*THICK*POR*VT(K))
0132                 AVWIDT(K)=STRMW(K)/ICANT
0133                 OLDL(IPE)=STRML(K)
0134                 OLDW(IPE)=AVWIDT(K)
0135                 EX=EX/SCLFAC
0136                 EY=EY/SCLFAC
0137                 XIF(K)=XI(K)/SCLFAC
0138                 YIF(K)=YI(K)/SCLFAC
0139            714  GO TO 2121
               C
               C...CHECK FOR VERTICAL BOUNDARY
               C
0140            244  IF(ABS(X(J+1)-X(J)) [illegible] TO 245
0141            246  XI(K)=X(J)
0142                 YI(K)=YI(K)
0143                 XOE(IPE)=X[illegible]
0144                 YOE(IPE)=[illegible]
0145                 AQSS(IP[illegible]
0146                 ANSL(I[illegible]
0147                 ADFI([illegible]
               C
               C  THETA IS [illegible]
               C THE X-AX[illegible]
               C  ATETA I[illegible]
               C  X-AXIS [illegible]
               C
0148                 A[illegible]
0149                 IF[illegible]
0150                 A[illegible]
0151                 AD[illegible]
0152           3001  VT[illegible]
0153                 VEL[illegible]
0154                 CALL[illegible]
0155                 STRM[illegible]
0156                 STRMW[illegible]
0157                 AVWID[illegible]
0158                 OLDL(IP[illegible]
0159                 OLDW(IPE[illegible]
0160                 EX=EX/SCLF[illegible]
0161                 EY=EY/SCLFAC
0162                 XIF(K)=XI(K)/SC[illegible]
0163                 YIF(K)=YI(K)/SCLFAC
0164                 GO TO 2121
0165            245  XI(K)=(((X(J)-X(J+1))/(Y(J+1)-Y(J)))*EX-((Y(J+1)-Y(J))/
                   & (X(J+1)-X(J)))*X(J)+Y(J)-EY)/(((X(J)-X(J+1))/(Y(J+1)-Y(J)
                   & )-((Y(J+1)-Y(J))/(X(J+1)-X(J))))
0166                 YI(K)=((X(J)-X(J+1))/(Y(J+1)-Y(J)))*XI(K)-((X(J)-X(J+1))/
                   & (Y(J+1)-Y(J)))*EX+EY
0167                 XOE(IPE)=XI(K)
0168                 YOE(IPE)=YI(K)
```

```
      AQSS(IPE)=QSSS(I)
      ANSL(IPE)=NSL
      ADFI(IPE)=DFI(J)
      VT(K)=-1.0*(T/VICFAC*VICSTM)*DFI(J)
C
C..ADJUST FOR STREAMLINES APPROACHING FROM THE LEFT
      IF(XI(K) .GT. EX)ADFI(IPE)=-ADFI(IPE)
      VEL(IPE)=VT(K)
      STRML(K)=STRML(K)+(VT(K)*DT)
      STRMW(K)=STRMW(K)+QSSS(I)/(NSL*THICK*POR*VT(K))
      AVWIDT(K)=STRMW(K)/ICANT
      OLDL(IPE)=STRML(K)
      OLDW(IPE)=AVWIDT(K)
C
C  CALCULATE THE ANGLE THE OUTWARD NORMAL MAKES WITH THE
C  X-AXIS
C
      ZETA(J)=-1.0/TAN(THETA(J))
      ATETA(IPE)=ATAN(ZETA(J))
      CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,2)
      XIF(K)=XI(K)/SCLFAC
      YIF(K)=YI(K)/SCLFAC
      EX=EX/SCLFAC
      EY=EY/SCLFAC
      XIF(K)=XI(K)/SCLFAC
      YIF(K)=YI(K)/SCLFAC
 2121 WRITE(IMP,2111)IPE,XIF(K),YIF(K),EX,EY
 2111 FORMAT(//8X,'STREAMLINE INTERFACE BEGINING POINTS ARE',
     & I5,2F9.3,/8X,'PRECEEDING INTERIOR POINT IS',2F10.3)
      WRITE(IMP,605)VEL(IPE),ADFI(IPE),ATETA(IPE),AQSS(IPE)
     & ,ANSL(IPE)
  605 FORMAT(/2X,5F10.4)
  210 CONTINUE
  220 CONTINUE
C...WRITE OUT THE ANSWERS
      DO 440 I=1,NSS
      IF(QSSS(I) .GE. 0.0) GO TO 440
      WRITE(IMP,441)I,SINK(I)
  441 FORMAT(1H1,////15X,'PRODUCER NUMBER',I3,//15X,
     & 'NUMBER OF STREAMLINES=',I3,//)
      WRITE(IMP,443)
  443 FORMAT(8X,'STREAMLINE NUMBER',3X,'TOTAL LENGTH',3X,
     & 'AVERAGE WIDTH',6X,'INJ. RATE',//)
      ITEMP=SINK(I)
      IF(ITEMP .LE. 0) GO TO 440
      DO 444 J=1,ITEMP
      LENTH(I,J)=LENTH(I,J)/DISFAC
      WIDTH(I,J)=WIDTH(I,J)/DISFAC
      CQSS(I,J)=CQSS(I,J)/RATEF
      WRITE(IMP,445)J,LENTH(I,J),WIDTH(I,J),CQSS(I,J)
  445 FORMAT(14X,I3,10X,E12.4,4X,E12.4,5X,E12.4)
  444 CONTINUE
  440 CONTINUE
      RETURN
      END
```

```
0001                SUBROUTINE LOWVEL(NSS,QSS,FI,DFI,X,Y,XSS,YSS,
                 &  N,T,VLOW,RI,QMNI,RAD,POR,THICK)
             C
             C*****************************************************************
0002                COMMON /BLK1/LEC,IMP,VICSTM,PI
0003                COMMON /BLK2/DISFAC,VICFAC,DENFAC,TRFAC,RATEF,SCALE
0004                COMMON /BLK3/PRESSF,DPRESF,PORFAC,SCLFAC
0005                DIMENSION X(120),Y(120),FI(120),DFI(120),XSS(120)
0006                DIMENSION YSS(120),DXBLEN(120),ELMNT(120)
0007                DIMENSION QSS(120),XI(120),YI(120),DPDX(120),DPDY(120)
0008                DIMENSION VX(120),VY(120),VT(120),A(120,120),G(120,120)
0009                DIMENSION DGDX(120),DGDY(120),DHDX(120),DHDY(120)
0010                DIMENSION DPHIDX(120),DPHIDY(120),VLCP(120),DYBLEN(120)
0011                DIMENSION RO(120),THETA(120),SUM(120),DIST(120)
             C
             C
0012                NST=20
0013                FACT=1.0333E+7
             C  DEFINE A CAPTURE RADIUS (RAD)
0014                RAD=RI*50.0
             C
             C CALCULATE THE VELOCITY NEAR THE LOWEST PRODUCTION WELL
             C
0015                QMNI=1.0E+10
0016                DO 110 KK=1,NSS
0017                IF(QSS(KK) .GT. 0.0) GO TO 110
0018                IF(ABS(QSS(KK)) .GT. ABS(QMNI))GO TO 110
0019                QMNI=QSS(KK)
0020                K=KK
0021           110  CONTINUE
0022                VX(K)=0.
0023                VY(K)=0.
0024                VT(K)=0.
0025                DPDX(K)=0.
0026                DPDY(K)=0.
0027                XI(K)=XSS(K)+RAD
0028                YI(K)=YSS(K)+RAD
0029                DO 172 J=1,N
0030                CALL INTE(XI(K),YI(K),X(J),Y(J),X(J+1),Y(J+1),A(K,J),
                 &  G(K,J),DGDX(J),DGDY(J),DHDX(J),DHDY(J),DIST(J),
                 &  ELMNT(J),THETA(J))
0031                DPDX(K)=DPDX(K)+DFI(J)*DGDX(J)-FI(J)*DHDX(J)
0032                DPDY(K)=DPDY(K)+DFI(J)*DGDY(J)-FI(J)*DHDY(J)
0033           172  CONTINUE
             C
             C CALCULATE THE CONTRIBUTIONS OF THE SOURCES AND SINKS
             C
0034                DXBLEN(K)=0.
0035                DYBLEN(K)=0.
0036                SUM(K)=0.
0037                DO 54 JJ=1,NSS
0038                CALL SOURCE(XI(K),YI(K),XSS(JJ),YSS(JJ),QSS(JJ),NSS
                 &  ,BLENT,DXSURS,DYSURS)
0039                DXBLEN(K)=DXBLEN(K)+DXSURS
0040                DYBLEN(K)=DYBLEN(K)+DYSURS
0041           54   CONTINUE
0042                DPHIDX(K)=(DPDX(K)+DXBLEN(K))/(2.0*PI)*FACT
0043                DPHIDY(K)=(DPDY(K)+DYBLEN(K))/(2.0*PI)*FACT
```

```
0044            VX(K)=-1.0*(T/VICSTM*VICFAC)*DPHIDX(K)
0045            VY(K)=-1.0*(T/VICSTM*VICFAC)*DPHIDY(K)
0046            VLCP(K)=SQRT(VX(K)**2+VY(K)**2)
0047            VLOW=VLCP(K)
0048            QMNI=QMNI/(VICSTM*VICFAC/(THICK*T))
0049            QMNIW=QMNI/RATEF
0050            VELO=VLOW/DISFAC
0051            WRITE(IMP,113)QMNIW,VELO
0052      113   FORMAT(1H1,///3X,'THE RATE OF THE LOWEST PRODUCER(BBL/D)=
              &   F12.3/3X,'THE VELOCITY NEAR THE LOWEST PRODUCER(FT/SEC)='
              &   E12.3,/)
0053            RETURN
0054            END
```

```
0001              SUBROUTINE COMPAT(NSS,QSS,FI,DFI,X,Y,XSS,YSS,N,T,
                &  KODE,IPC,XOC,YOC,XM,YM,BQSS,BNSL,MMX,BETA,BDFI,ELT,
                &  THICK,POR,RI,TADJ,OLDL,OLDW)
           C
           C****************************************************************
0002              COMMON /BLK1/LEC,IMP,VICSTM,PI
0003              COMMON /BLK2/DISFAC,VICFAC,DENFAC,TRFAC,RATEF,SCALE
0004              COMMON /BLK3/PRESSF,DPRESF,PORFAC,SCLFAC
0005              DIMENSION X(120),Y(120),FI(120),DFI(120),XSS(120)
0006              DIMENSION XI(120),YI(120),QSS(120),KODE(120)
0007              DIMENSION YSS(120),G(120,120),DISMID(120)
0008              DIMENSION VX(120),VY(120),VT(120),H(120,120)
0009              DIMENSION DIST(120),XM(120),YM(120),SUM(120)
0010              DIMENSION BNSL(120),DGDX(120),DGDY(120),DHDX(120)
0011              DIMENSION DXBLEN(120),DPDX(120),DPDY(120),DYBLEN(120)
0012              DIMENSION DHDY(120),RO(120),YOC(120),AVWIDT(120)
0013              DIMENSION DPHIDX(120),DPHIDY(120),VLCP(120),XOC(120)
0014              DIMENSION MMX(120),SINK(120),STRML(120),STRMW(120)
0015              DIMENSION BQSS(120),ELMNT(120),LENTH(120,120)
0016              DIMENSION XIF(120),YIF(120),THETA(120),BDFI(120)
0017              DIMENSION WIDTH(120,120),VEL(120),ELT(120),BETA(120)
0018              DIMENSION OLDL(120),OLDW(120),CQSS(120,120)
0019              DIMENSION OLENTH(120,120),OWIDTH(120,120)
0020              INTEGER SINK
0021              REAL LENTH
           C
           C...THIS SUBROUTINE CALCULATES THE STREAMLINES THAT ORIGINATED
           C....IN THE ADJACENT REGION INTO THE REGION UNDER CONSIDERATION
           C....THEIR STARTING POINTS ARE ON THE INTERFACE BOUNDARY
           C
0022              FACT=1.0333E+7
0023              WRITE(IMP,107)THICK,POR
0024         107  FORMAT(1H1,//20X,'CONTINUATION IN ADJACENT REGION',//20X,
                &  'THICKNESS=',F10.4,//20X,'POROSITY=',F10.4,///)
           C
0025              WRITE(IMP,64)
0026          64  FORMAT(30X,'INTERFACE POINTS',//2X,'COORDINATES',
                &  3X,'VELOCITY',2X,'BOUNDARY CONDS.',5X,'ANGLE',
                &  2X,'FLOW RATE AND NO OF STRMLNS'//)
0027              DO 66 I=1,IPC
0028              BDFI(I)=-1.0*(TADJ/T)*BDFI(I)
0029              VEL(I)=-1.0*(T/(VICSTM*VICFAC))*BDFI(I)*FACT
0030              WRITE(IMP,63)XOC(I),YOC(I),VEL(I),BDFI(I),BETA(I),
                &  BQSS(I),BNSL(I)
0031          63  FORMAT(7F10.4)
0032          66  CONTINUE
           C
           C CALCULATE THE VELOCITIES NEAR THE PRODUCING WELLS
           C
0033              CALL LOWVEL(NSS,QSS,FI,DFI,X,Y,XSS,YSS,N,T,VLOW,RI,
                &  QMNI,RAD,POR,THICK)
           C
           C  INITIALIZE ARRAYS
           C
0034              DO 206 I=1,NSS
0035              SINK(I)=0.0
0036         206  CONTINUE
           C
```

```
C...XOC,YOC ARE THE INTERFACE BOUNDARY STARTING COORDINATES
C...XI,YI ARE UNSCALED INTERNAL CALCULATION COORDINATES
C...XIF,YIF ARE SCALED COORDINATES USED ONLY FOR OUTPUT
C
          DO 210 K=1,IPC
          ICANT=0
          FACPLO=2.0
          CALL FACTOR(FACPLO)
          CALL PLOT(XOC(K)/SCLFAC,YOC(K)/SCLFAC,3)
          STRML(K)=0.0
          STRMW(K)=0.0
          AVWIDT(K)=0.0
  171     ICANT=ICANT+1
          IF(ICANT .GT. 100) GO TO 210
          VX(K)=0.
          VY(K)=0.
          VT(K)=0.
          DPDX(K)=0.
          DPDY(K)=0.
          IF(ICANT .GT. 1)GO TO 501
          ANGLE=BETA(K)
          VX(K)=VEL(K)*COS(ANGLE)
          VY(K)=VEL(K)*SIN(ANGLE)
          VT(K)=SQRT(VX(K)**2+VY(K)**2)
          STRML(K)=0.0
          STRMW(K)=STRMW(K)+BQSS(K)/(BNSL(K)*THICK*POR*VT(K))
          AVWIDT(K)=STRMW(K)/ICANT
          DT=50.0*RI/ABS(VT(K))
          DO 502 L=1,1000
          DTO=DT+(L*0.005*DT)
          XI(K)=XOC(K)+(VX(K)*DTO)
          YI(K)=YOC(K)+(VY(K)*DTO)
          XIF(K)=XI(K)/SCLFAC
          YIF(K)=YI(K)/SCLFAC
          IF(L .GT. 10)GO TO 503
          WRITE(IMP,646)VX(K),VY(K),XIF(K),YIF(K)
  646     FORMAT(/2X,4F10.4)
  503     CONTINUE
          BDIST=SQRT((XI(K)-XOC(K))**2+(YI(K)-YOC(K))**2)
          IF(BDIST .GT. ELT(K))GO TO 608
  502     CONTINUE
          WRITE(IMP,609)XIF(K),YIF(K)
  609     FORMAT(/2X,'POSITION AFTER 100 TRIALS IS',2F10.4,/)
          GO TO 210
  608     CONTINUE
          CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,2)
          XIF(K)=XI(K)/SCLFAC
          YIF(K)=YI(K)/SCLFAC
C         WRITE(IMP,606)XIF(K),YIF(K)
C 606      FORMAT(/2X,'XNEW=',F10.4,5X,'YNEW=',F10.4,/)
 501      DO 172 J=1,N
          DISMID(J)=SQRT((XI(K)-XM(J))**2+(YI(K)-YM(J))**2)
          CALL INTE(XI(K),YI(K),X(J),Y(J),X(J+1),Y(J+1),H(K,J),
     &    G(K,J),DGDX(J),DGDY(J),DHDX(J),DHDY(J),DIST(J),
     &    ELMNT(J),THETA(J))
          ELMNT2=0.5*ELMNT(J)
C..IF POINT IS NEAR ANY  BOUNDARY, STOP AND START NEW
C..STREAMLINE
```

```
0085               IF(DISMID(J) .LT. ELMNT2)GO TO 210
0086       360     DPDX(K)=DPDX(K)+DFI(J)*DGDX(J)-FI(J)*DHDX(J)
0087               DPDY(K)=DPDY(K)+DFI(J)*DGDY(J)-FI(J)*DHDY(J)
0088       172     CONTINUE
          C
          C CALCULATE THE CONTRIBUTIONS OF THE SOURCES AND SINKS
          C
0089               DXBLEN(K)=0.
0090               DYBLEN(K)=0.
0091               DO 54 JJ=1,NSS
0092               CALL SOURCE(XI(K),YI(K),XSS(JJ),YSS(JJ),QSS(JJ),NSS,
               &   BLENT,DXSURS,DYSURS)
0093               DXBLEN(K)=DXBLEN(K)+DXSURS
0094               DYBLEN(K)=DYBLEN(K)+DYSURS
0095       54      CONTINUE
0096               DPHIDX(K)=(DPDX(K)+DXBLEN(K))/(2.0*PI)*FACT
0097               DPHIDY(K)=(DPDY(K)+DYBLEN(K))/(2.0*PI)*FACT
0098               VX(K)=-1.0*(T/VICSTM*VICFAC)*DPHIDX(K)
0099               VY(K)=-1.0*(T/VICSTM*VICFAC)*DPHIDY(K)
0100               VT(K)=SQRT(VX(K)**2+VY(K)**2)
          C...CHECK IF POINT IS NEAR A PRODUCER, IF SO BRANCH TO 333
          C...OTHERWISE CALCULATE THE LENGTH AND WIDTH AT THE CURRENT
          C...POSITION. PLOT THE CURRENT POSITION
0101               DT=50.0*RI/ABS(VT(K))
0102               IF(VT(K) .GT. VLOW) GO TO 333
0103       111     STRML(K)=STRML(K)+(VT(K)*DT)
0104               STRMW(K)=STRMW(K)+BQSS(IPC)/(BNSL(IPC)*THICK*POR*VT(K))
0105               AVWIDT(K)=STRMW(K)/ICANT
          C...CALCULATE NEW POSITION
0106               XI(K)=XI(K)+VX(K)*DT
0107               YI(K)=YI(K)+VY(K)*DT
0108               CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,2)
0109               XIF(K)=XI(K)/SCLFAC
0110               YIF(K)=YI(K)/SCLFAC
0111               WRITE(IMP,332)K,XIF(K),YIF(K),VX(K),VY(K),VT(K)
0112      332      FORMAT(6X,I3,2X,5E10.3)
0113               GO TO 171
0114      333      DO 200 LL=1,NSS
0115               IF(QSS(LL) .GE. 0.0)GO TO 200
0116               RO(LL)=SQRT((XI(K)-XSS(LL))**2+(YI(K)-YSS(LL))**2)
0117               IF(RAD .GE. RO(LL)) GO TO 208
0118       200     CONTINUE
0119               GO TO 111
          C..IDENTIFY THE PRODUCER AT WHICH THE STREAMLINE TERMINATED
0120       208     SINK(LL)=SINK(LL)+1
0121               LAST=SQRT((XI(K)-XSS(LL))**2+(YI(K)-YSS(LL))**2)
0122               LENTH(LL,SINK(LL))=STRML(K)+LAST
0123               WLAST=(2.0*PI*RI)/BNSL(IPC)
0124               STRMW(K)=STRMW(K)+WLAST
0125               AVWIDT(K)=STRMW(K)/(FLOAT(ICANT)+1.0)
0126               WIDTH(LL,SINK(LL))=AVWIDT(K)
0127               OLENTH(LL,SINK(LL))=OLDL(K)
0128               OWIDTH(LL,SINK(LL))=OLDW(K)
0129               CQSS(LL,SINK(LL))=BQSS(K)/BNSL(K)
0130               CALL PLOT(XI(K)/SCLFAC,YI(K)/SCLFAC,2)
0131       210     CONTINUE
          C...WRITE OUT THE ANSWERS
0132               DO 440 I=1,NSS
```

```
0133          IF(QSS(I) .GE. 0.0) GO TO 440
0134          WRITE(IMP,441)I,SINK(I)
0135     441  FORMAT(1H1,15X,'PRODUCER NUMBER',I3,///15X,
            &  'NUMBER OF STREAMLINES',I3,///)
0136          WRITE(IMP,443)
0137     443  FORMAT(8X,'STREAMLINE NUMBER',3X,'TOTAL LENGTH',3X,
            &  'AVERAGE WIDTH',6X,'INJ. RATE',///)
0138          ITEMP=SINK(I)
0139          IF(ITEMP .LE. 0) GO TO 440
0140          DO 444 J=1,ITEMP
0141          LENTH(I,J)=LENTH(I,J)/DISFAC
0142          WIDTH(I,J)=WIDTH(I,J)/DISFAC
0143          OLENTH(I,J)=OLENTH(I,J)/DISFAC
0144          OWIDTH(I,J)=OWIDTH(I,J)/DISFAC
0145          CQSS(I,J)=CQSS(I,J)/RATEF
0146          WRITE(IMP,445)J,OLENTH(I,J),OWIDTH(I,J),CQSS(I,J)
0147          WRITE(IMP,445)J,LENTH(I,J),WIDTH(I,J),CQSS(I,J)
0148     445  FORMAT(14X,I3,8X,E12.4,6X,E12.4,5X,E12.4)
0149     444  CONTINUE
0150     440  CONTINUE
0151     272  CONTINUE
0152          RETURN
0153          END
```

```
      DIMENSION RL1(330,30),TTD(30),TD(10),WR1(30,30)
      DIMENSION RHOCR(30),PERM(4),WIDTH(30),FB(30,30)
      DIMENSION AVSATW(4),AVSATC(4),AVSATG(4),FA(30,30)
      DIMENSION DENR(4),CR(4),POR(4),THICKR(4),LAMDA(4)
      DIMENSION VA(30,30),VB(30,30),INJECA(30,30)
      DIMENSION DELTD(30),RTIME(30),RL2(30,30),WR2(30,30)
      DIMENSION DIST1(30,30),CUMREC(30),QST(30)
      DIMENSION INJECB(30,30),NSTLNE(30),OILREC(30,30)
      DIMENSION TOTREC(30),ENDTM1(30,30),ENDTM2(30,30)
      DIMENSION TZTD1(30,30),DIST2(30,30),CODE(30,30)
      DIMENSION TZTD2(30,30),SWI(30),SOI(30),SGI(30)
      DIMENSION QSTRM1(30,30),QSTRM2(30,30),END1(30,30)
      DIMENSION END2(30,30)
      INTEGER TITLE1(18),TITLE2(18),TITLE3(18),TITLE4(18)
      INTEGER TITLE5(18),TITLE6(18),TITLE7(18)
      REAL LAMDA,LAMDA1,LAMDA2,INJEC,INJEC1,INJEC2,INJEC3
      REAL INJEC4,INJECA,INJECB
      INTEGER CODE
C*****************************************************************
C...OBJECTIVE: TO CALCULATE OIL RECOVERY BY STEAMFLOODING
C              IN STREAMTUBES THAT CONSIST OF ONE OR TWO
C              PERMEABILITY REGIONS.
C*****************************************************************
C
C....DEFINE STATEMENT FUNCTIONS AND SUBROUTINE FUNCTIONS USED
C
      RTD(A,B)=(2.0*SQRT((A**2)*B)/1.7725)-1.0
      VELTD(A,B)=EXP((A**2)*B)*ERFC(SQRT((A**2)*B))
      ZTD(A,B,C)=C*(VELTD(A,B)+RTD(A,B))
      VTD(A,B,D)=D*VELTD(A,B)
C
C...READ THE TITLES FOR THE JOB
C
      READ(5,1005)TITLE1
      READ(5,1005)TITLE2
      READ(5,1005)TITLE3
      READ(5,1005)TITLE4
      READ(5,1005)TITLE5
      READ(5,1005)TITLE6
      READ(5,1005)TITLE7
 1005 FORMAT(18A4)
      WRITE(6,1006)TITLE1
      WRITE(6,1007)TITLE2
      WRITE(6,1007)TITLE3
      WRITE(6,1007)TITLE4
      WRITE(6,1007)TITLE5
      WRITE(6,1007)TITLE6
      WRITE(6,1007)TITLE7
 1006 FORMAT(////10X,18A4)
 1007 FORMAT(/10X,18A4)
C
C..... READ AND WRITE INITIAL AND FIXED VALUES
C
      READ(5,5)NPROD,PST,TST,TI,X,EPS,THICK,NREG
    5 FORMAT(I5,6F10.4,I1)
      WRITE(6,6)NPROD,TST,X,EPS,THICK,PST,TI
    6 FORMAT(///40X,'DATA',/39X,'------',//6X,
     & 'TOTAL NUMBER OF PRODUCERS=',15X,I3,/6X,
```

```
                    &   'STEAM TEMPERATURE (DEG. F)=',14X,F10.4,/6X,
                    &   'STEAM QUALITY=',25X,F10.4,/6X,'CONVERGENCE LIMIT=',
                    &   21X,F10.4,/6X,'RESERVOIR THICKNESS (FT)=',15X,F10.4/
                    &   6X,'STEAM PRESSURE (PSI)=',20X,F10.4,/6X,
                    &   'INITIAL RESERVOIR TEMPEFATURE(DEG F)=',4X,F10.4,//)
              C
              C..... READ AND WRITE FLUID PROPERTIES
              C
0044                    READ(5,10)DENW1,DENO1,DENW2,DENO2
0045            10      FORMAT(4F10.4)
0046                    READ(5,12)CW,CO,CG
0047            12      FORMAT(3F10.4)
              C
              C CALCULATE THE DENSITY AND LATENT HEAT OF STEAM USING
              C CORRELATIONS
              C
0048                    AAA=-0.9588
0049                    BBB=-0.08774
0050                    DENST=1.0/(363.9*(PST**AAA))
0051                    STLAT=1318.0*(PST**BBB)
              C
0052                    WRITE(6,20)DENW1,DENO1,DENW2,DENO2,DENST,CW,CO,CG,STLAT
0053            20      FORMAT(//40X,'FLUID PROPERTIES',/39X,'-----------------
                    &   //,44X,'WATER',8X,'OIL',8X,'STEAM',/43X,'-------',6X,
                    &   '-----',
                    &   6X,'-------',//,6X,'DENSITY(LB/CU FT) AT STD. TEMP.',2X,
                    &   F10.4,2X,F10.4,/6X,'DENSITY(LB/CU FT) AT STEAM TEMP.',1X
                    &   F10.4,2X,F10.4,2X,F10.4,/6X,
                    &   'SPECIFIC HEAT(BTU/LB *F)',8X,F10.4,2X,F10.4,3X,F10.4,/6
                    &   'LATENT HEAT(BTU/LB)',40X,F10.4,//)
              C
              C......READ AND WRITE THE PROPERTIES OF THE CAP AND BASE ROCK
              C
0054                    READ(5,70)DENCB,CCB,CBK
0055            70      FORMAT(3F10.4)
0056                    WRITE(6,72)DENCB,CCB,CBK
0057            72      FORMAT(30X,'PROPERTIES OF THE CAP AND BASE ROCK',/29X,
                    &   '-------------------------------------',//6X,
                    &   'DENSITY(LB/CU. FT)',
                    &   8X,F10.4,/6X,'SPECIFIC HEAT(BTU/LB-*F)',F10.4,/6X,
                    &   'THERM. COND.(BTU/HR-FT-*F)',F10.4//)
              C
              C .....READ AND WRITE  RESERVOIR ROCK PROPERTIES
              C
0058                    DO 43  I=1,NREG
0059                    READ(5,44)POR(I),DENR(I),CR(I),PERM(I)
0060            44      FORMAT(4F10.4)
0061            43      CONTINUE
0062                    WRITE(6,50)
0063            50      FORMAT(1H1,////,30X,'RESERVOIR ROCK PROPERTIES',/29X,
                    &   '--------------------------',//31X,'REGION 1',7X,
                    &   'REGION 2',/30X,'----------',5X,'----------',//)
0064                    WRITE(6,60)(POR(I),I=1,NREG),(DENR(I),I=1,NREG),
                    &   (CR(I),I=1,NREG),(PERM(I),I=1,NREG)
0065            60      FORMAT(/6X,'POROSITY',13X,2(F10.4,7X),/6X,
                    &   'DEN.(LB/CU FT)',8X,2(F10.4,7X),/6X,'SPEC. HEAT(BTU/LB*F
                    &   ,2X,2(F10.4,7X),/6X,'PERMEABILITY(MD)',6X,2(F10.4,7X),//
              C
```

```
                C READ AND WRITE INITIAL FLUID SATURATIONS
                C
 0066                 READ(5,38)(SWI(I),I=1,2)
 0067                 READ(5,38)(SOI(I),I=1,2)
 0068                 READ(5,38)(SGI(I),I=1,2)
 0069             38  FORMAT(2F10.4)
 0070                 WRITE(6,39)(SWI(I),I=1,NREG),(SOI(I),I=1,NREG),
                    & (SGI(I),I=1,NREG)
 0071             39  FORMAT(//20X,'INITIAL FLUID SATURATIONS',/19X,
                    & '---------------------------',
                    & //,23X,'REGION 1',3X,'REGION 2',/22X,'----------',1X,
                    & '----------',
                    & //16X,'WATER',2(F10.4),/16X,'OIL',2X,2(F10.4),/16X,
                    & 'STEAM',2(F10.4),//)
                C
                C......READ AND WRITE THE AVERAGE RESIDUAL SATURATIONS AFTER
                C      STEAM FLOODING FOR EACH REGION
                C
 0072                 READ(5,82)(AVSATW(I),I=1,2)
 0073                 READ(5,82)(AVSATO(I),I=1,2)
 0074                 READ(5,82)(AVSATG(I),I=1,2)
 0075             82  FORMAT(2F10.4)
 0076                 WRITE(6,84)(AVSATW(I),I=1,NREG),(AVSATO(I),I=1,NREG),
                    & (AVSATG(I),I=1,NREG)
 0077             84  FORMAT(//,20X,'AVERAGE RESIDUAL SATURATIONS BEHIND THE '
                    & ,'FRONT',/19X,'--------------------------------------'
                    & ,'----------',//,23X,'REGION 1',3X,'REGION 2',/22X,
                    & '----------',1X'----------',
                    & //16X,'WATER',2(F10.4),/16X,'OIL',2X,2(F10.4),/16X,
                    & 'STEAM',2(F10.4),//)
                C
                C...READ AND WRITE INPUT VALUES FROM THE STREAMLINE PROGRAM
                C
 0078                 WRITE(6,18)
 0079             18  FORMAT(1H1,///20X,'INPUT DATA FROM STREAMLINE PROGRAM',
                    & /19X,'------------------------------',//32X,'REGION 1',
                    & 20X,'REGION 2'/31X,'----------',19X,'----------',/5X,
                    & 'WELL NO.',2X,'S/L NO.',2X,'CODE',2X,'LENGTH',
                    & 2X,'WIDTH',2X,'RATE',6X,'LENGTH',2X,'WIDTH',2X,'RATE'//)
 0080                 DO 45 M=1,NPROD
 0081                 READ(5,13)NSTLNE(M),QST(M)
 0082             13  FORMAT(I3,F10.4)
 0083                 NTEMP=NSTLNE(M)
 0084                 DO 15 K=1,NTEMP
 0085                 READ(5,16)CODE(M,K),RL1(M,K),WR1(M,K),QSTRM1(M,K)
 0086             16  FORMAT(I5,3F10.4)
 0087                 IF(CODE(M,K) .EQ. 1)GO TO 131
 0088                 READ(5,129)RL2(M,K),WR2(M,K)
 0089            129  FORMAT(2F10.4)
 0090                 QSTRM2(M,K)=QSTRM1(M,K)
 0091                 GO TO 415
 0092            131  RL2(M,K)=0.0
 0093                 WR2(M,K)=0.0
 0094                 QSTRM2(M,K)=0.0
 0095            415  WRITE(6,17)M,K,CODE(M,K),RL1(M,K),WR1(M,K),QSTRM1(M,K)
                    & ,RL2(M,K),WR2(M,K),QSTRM2(M,K)
 0096             17  FORMAT(5X,I5,3X,I5,3X,I5,3X,3F7.2,4X,3F7.2)
 0097             15  CONTINUE
```

```
0098              45    CONTINUE
                C
                C.....CALCULATE THE DIMENSIONLESS CONSTANTS
                C
0099                    WRITE(6,98)
0100              98    FORMAT(1H1,//,30X,'CALCULATED RESULTS',/30X,'-------'
                     &  ,'-----------',//10X,'REGION',7X,'LAMDA',5X,
                     &  'DIMENSIONLESS TIME FACTOR',/10X,'------',5X,'-------',
                     &  3X,'-------------------------',//)
0101                    RHOCB=DENCB*CCB
0102                    TDFAC=4.0*CBK/(DENCB*CCB*THICK**2)
0103                    DELTST=TST-TI
0104                    DO 94 I=1,2
0105                    RHOCR(I)=DENR(I)*CR(I)*(1-POR(I))+(POR(I)*DENW2*CW*
                     &  AVSATW(I))+(POR(I)*DENO2*CO*AVSATO(I))+(POR(I)*DENST
                     &  *STLAT*AVSATG(I)/DELTST)
0106                    LAMDA(I)=RHOCB/RHOCR(I)
0107              94    CONTINUE
0108                    DO 99 I=1,2
0109                    WRITE(6,96)I,LAMDA(I),TDFAC
0110              96    FORMAT(12X,I2,6X,E10.4,7X,E10.4)
0111              99    CONTINUE
                C
                C...INITIALIZE ARRAYS
0112                    DO 122 M=1,NPROD
0113                    CUMREC(M)=0.0
0114                    NTEMP=NSTLNE(M)
0115                    DO 123 K=1,NTEMP
0116                    OILREC(M,K)=(1.0/5.615)*0.0
0117                    ENDTM1(M,K)=0.0
0118                    ENDTM2(M,K)=0.0
                C
                C CONVERT THE INJECTION RATE FROM  BARRELS/DAY
                C (WATER EQUIVALENT) TO CUBIC FT PER HOUR
                C
0119                    QSTRM1(M,K)=QSTRM1(M,K)*5.615*(64.5/DENST)/24.0
0120                    QSTRM2(M,K)=QSTRM2(M,K)*5.615*(64.5/DENST)/24.0
0121             123    CONTINUE
0122             122    CONTINUE
                C
                C.....FOR EACH STREAMTUBE,CALCULATE THE TIMES TO TRAVERSE THE
                C......FIRSTND SECOND REGIONS AND STORE IN ENDTM1 AND ENDTM2
                C......RESPECTIVELY
                C...TTDI=DIMENSIONLESS TIME VALUES OBTAINED DURING ITERATION
                C.....TDI IS AN INITIAL ARBITRARY GUESS
                C......TTDI ARE IMPROVED ITERATES.
                C
                C...ASSUME AN INITIAL TIME GUESS OF T=1.0
0123                    TIME=1.0
0124                    TDI=TIME*TDFAC
0125                    ITER=30
0126             126    DO 124 M=1,NPROD
0127                    NTEMP=NSTLNE(M)
0128                    DO 125 K=1,NTEMP
0129                    FA(M,K)=X*DENST*(STLAT+(CW*DELTST))+(1.0-X)*DENW2
                     &  *CW*DELTST
0130                    INJECA(M,K)=FA(M,K)*QSTRM1(M,K)*THICK/(WR1(M,K)
                     &  *LAMDA(1)*4.0*CBK*DELTST)
```

```
0131                VA(M,K)=FA(M,K)*QSTRM1(M,K)*LAMDA(1)/(WR1(M,K)*THICK*
                  &  RHOCB*DELTST)
              C
              C  SINCE FORTGCLG COMPILER DOES NOT ALLOW ARRAY NAMES IN THE
              C  STATEMENT FUNCTION PARAMETER LIST, REASSINGN ARRAYS TO
              C  VARIABLES
              C
0132                LAMDA1=LAMDA(1)
0133                INJEC1=INJECA(M,K)
0134                INJEC3=VA(M,K)
0135           102  TTDI=0.0
0136                TTDI=RL1(M,K)/VTD(LAMDA1,TDI,INJEC3)
              C     OO=VTD(LAMDA1,TDI,INJEC3)
              C     WRITE(6,723)FA(M,K),LAMDA1,QSTRM1(M,K),INJEC1,WR1(M,K)
              C    &  ,INJEC3,TTDI,OO
              C 723  FORMAT(//10X,'FA(M,K)=',F12.4,/10X,'LAMDA1=',F10.4,/10X,
              C    &  'QSTRM1=',F12.8,/10X,'INJEC1=',F12.8,/10X,'WR1(M,K)=',
              C    &  F12.8,/10X,'INJEC3=',F12.8,/10X,'TTDI=',E12.4,/10X,
              C    &  'OO=',F12.8//)
0137                TTDI=TTDI*TDFAC
0138                DO 116 J=1,ITER
0139                IF(TTDI .LT. 10.0)GO TO 724
0140                WRITE(6,505)
0141           505  FORMAT(//10X,'VELOCITY WILL BE ZERO BEFORE TUBE END'//)
0142                GO TO 304
0143           724  DELTD(J)=-1.0*(ZTD(LAMDA1,TTDI,INJEC1)-RL1(M,K))/
                  &  VTD(LAMDA1,TTDI,INJEC3)
0144           725  DELTD(J)=DELTD(J)*TDFAC
0145                TTDI=TTDI+DELTD(J)
0146                IF(ABS(DELTD(J)/TTDI) .LT. EPS) GO TO 118
0147           116  CONTINUE
0148                WRITE(6,117)
0149           117  FORMAT(//2X,'NO CONVERGENCE'//)
0150                GO TO 304
0151           118  ENDTM1(M,K)=TTDI
0152                TZTD1(M,K)=ZTD(LAMDA1,TTDI,INJEC1)
0153           324  IF(CODE(M,K) .LE. 1)GO TO 125
              C...CALCULATE THE TIME TO TRAVERSE THE SECOND REGION(ENDTM2)
0154                FB(M,K)=X*DENST*(STLAT+(CW*DELTST))+(1.0-X)*DENW2*CW*
                  &  DELTST
0155                INJECB(M,K)=FB(M,K)*QSTRM2(M,K)*THICK/(WR2(M,K)*
                  &  LAMDA(2)*4.0*CBK*DELTST)
0156                VB(M,K)=FB(M,K)*QSTRM2(M,K)*LAMDA(2)/(WR2(M,K)*
                  &  THICK*RHCCB*DELTST)
0157                LAMDA2=LAMDA(2)
0158                INJEC2=INJECB(M,K)
0159                INJEC4=VB(M,K)
0160                TDEL=RL2(M,K)/VTD(LAMDA2,TTDI,INJEC4)
0161                TDEL=TDEL*TDFAC
0162                TTDI=ENDTM1(M,K)+TDEL
0163                TINIT=ENDTM1(M,K)
0164                DO 662 J=1,ITER
0165                IF(TTDI .LT. 10.0)GO TO 726
0166                WRITE(6,506)
0167           506  FORMAT(//10X,'VELOCITY WILL BE ZERO BEFORE TUBE END'//)
0168                GO TO 304
0169           726  DELTD(J)=-1.0*(ZTD(LAMDA2,TTDI,INJEC2)
                  &  -RL2(M,K))/VTD(LAMDA2,TTDI,INJEC4)
```

```
      727  TTDI=TTDI+DELTD(J)
           IF(ABS(DELTD(J)/TTDI) .LT. EPS)GO TO 663
      662  CONTINUE
           WRITE(6,664)
      664  FORMAT(//2X,'NO CONVERGENCE',//)
           GO TO 304
      663  ENDTM2(M,K)=ENDTM1(M,K)+TTDI
           TZTD2(M,K)=ZTD(LAMDA2,TTDI,INJEC2)
      125  CONTINUE
      124  CONTINUE
           WRITE(6,36)
       36  FORMAT(1H1////10X,'CALCULATED BREAKTHROUGH TIMES(HOURS)'
         &  ' FOR EACH STREAMTUBE',/)
           WRITE(6,149)
      149  FORMAT(//,6X,'PROD. NO.',3X,'STREAMLINE NO.',3X,'CODE',
         &  3X,'ENDTIME(1)',4X,'ENDTIME(2)'/)
           DO 134 M=1,NPROD
           NTEMP=NSTLNE(M)
           DO 135 K=1,NTEMP
           END1(M,K)=ENDTM1(M,K)/TDFAC
           END2(M,K)=ENDTM2(M,K)/TDFAC
      136  WRITE(6,137)M,K,CODE(M,K),END1(M,K),END2(M,K)
      137 FORMAT(4X,I5,12X,I5,6X,I5,4X,2(F10.4,3X))
      135  CONTINUE
      134  CONTINUE
     C
     C...CALCULATE THE VELOCITY,DISTANCE,AND OIL RECOVERY AT
     C...VARIOUS TIMES
           READ(5,14)TIME
       14  FORMAT(F10.2)
           CTIME=TIME/20.0
           DO 923 KK=1,20
           TIME=CTIME*KK
           TIMED=TIME*TDFAC
           WRITE(6,323)TIME,TIMED
      323  FORMAT(1H1,///25X,'REAL TIME(HOURS)= ',F10.4,//
         &  25X,'DIMENSIONLESS TIME=',F10.4,//2X,'WELL',2X,
         &  'STREAMLINE',2X,'NO. OF REGIONS',2X,'RECOVERY',5X,
         &  'WELL TOTAL',5X,'RESERVOIR TOTAL'//)
           TOTREC(KK)=0.0
           DO 432 M=1,NPROD
           NTEMP=NSTLNE(M)
           CUMREC(M)=0.0
           DO 433 K=1,NTEMP
           IF(TIMED .GT. ENDTM1(M,K))GO TO 260
           TDDEL=TIMED
           INJEC1=INJECA(M,K)
           DIST1(M,K)=ZTD(LAMDA1,TDDEL,INJEC1)
           DIST2(M,K)=0.0
           OILREC(M,K)=(1.0/5.615)*DIST1(M,K)*WR1(M,K)
         &  *THICK*POR(1)*(SOI(1)-AVSATO(1))
           CUMREC(M)=CUMREC(M)+OILREC(M,K)
           GO TO 777
     C...CALCULATE THE BREAKTHROUGH RECOVERY IN FIRST REGION
      260  TDDEL=ENDTM1(M,K)
           INJEC1=INJECA(M,K)
           DIST1(M,K)=RL1(M,K)
           OILREC(M,K)=(1.0/5.615)*DIST1(M,K)*WR1(M,K)
```

```
                  &  *THICK*POR(1)*(SOI(1)-AVSATO(1))
0218                 IF(CODE(M,K) .GT. 1.0)GO TO 602
0219                 DIST2(M,K)=0.0
0220                 CUMREC(M)=CUMREC(M)+OILREC(M,K)
0221                 GO TO 777
0222          602    TEMREC=OILREC(M,K)
0223                 IF(TIMED .GT. ENDTM2(M,K))GO TO 665
0224                 TIME2=TIMED
0225                 TINIT=ENDTM1(M,K)
0226                 INJEC2=INJECB(M,K)
             C...CALCULATE THE RECOVERY FROM SECOND REGION AND ADD TO
             C...FIRST
0227                 DIST2(M,K)=ZTD(LAMDA2,TIME2,INJEC2)
                  &  -ZTD(LAMDA2,TINIT,INJEC2)
0228                 OILREC(M,K)=(1.0/5.615)*DIST2(M,K)*WR2(M,K)
                  &  *THICK*POR(2)*(SOI(2)-AVSATO(2))+TEMREC
0229                 CUMREC(M)=CUMREC(M)+OILREC(M,K)
0230                 GO TO 777
             C... THERE IS BREAKTHROUGH IN THE SECOND REGION OF A TWO
             C...PERMEABILITY REGION STREAMTUBE. CALCULATE RECOVERY .
             C...FROM BOTH REGIONS
0231          665    TIME2=ENDTM2(M,K)
0232                 INJEC2=INJECB(M,K)
0233                 DIST2(M,K)=RL2(M,K)
0234                 OILREC(M,K)=(1.0/5.615)*DIST2(M,K)*WR2(M,K)
                  &  *THICK*POR(2)*(SOI(2)-AVSATO(2))+TEMREC
0235                 CUMREC(M)=CUMREC(M)+OILREC(M,K)
0236          777    WRITE(6,307)M,K,CODE(M,K),OILREC(M,K)
0237          307    FORMAT(I5,I8,I13,6X,E13.4)
0238          433     CONTINUE
0239                 TOTREC(KK)=TOTREC(KK)+CUMREC(M)
0240                 WRITE(6,671)CUMREC(M)
0241          671    FORMAT(45X,E13.4)
0242          432     CONTINUE
0243                 WRITE(6,672)TOTREC(KK)
0244          672    FORMAT(60X,E13.4)
0245          923    CONTINUE
0246                 WRITE(6,675)
0247          675    FORMAT(1H1,///,17X,'PREDICTED RECOVERY',//,10X,
                  &  'TIME(DAYS)',10X,'RECOVERY(BBLS)'//)
0248                 DO 674 KK=1,20
0249                 TIME=CTIME*KK
0250                 RTIME(KK)=CTIME*KK
             C...CONVERT THE TIME TO DAYS AND RECOVERY TO BARRELLS
0251                 RTIME(KK)=RTIME(KK)/24.0
0252                 TOTREC(KK)=TOTREC(KK)/5.615
0253                 WRITE(6,676)RTIME(KK),TOTREC(KK)
0254          676    FORMAT(9X,F10.4,10X,E13.4)
0255          674    CONTINUE
             C
             C....PLOT THE CALCULATED RESULTS
             C
0256                 CALL PLOT(1.0,1.0,-3)
             C NP=NO OF INTERVALS OF THE SCALED PLOT
             C  K=EVERY K-TH DATA WILL BE USED TO GENERATE THE SCALE
0257                 NP=20
0258                 K=1
0259                 AXLEN=4.0
```

```
0260            CALL SCALE(RTIME,AXLEN,NP,K)
0261            XS=0.0
0262            YS=0.0
0263            D=0.0
0264            CALL AXIS(XS,YS,'TIME(DAYS)',-10,AXLEN,D,RTIME(21),
              & RTIME(22))
0265            NP=20
0266            K=1
0267            AXLEN=4.0
0268            CALL SCALE(TOTREC,AXLEN,NP,K)
0269            D=90.0
0270            CALL AXIS(XS,YS,'CUMULATIVE OIL RECOVERY(BBLS)',23,
              & AXLEN,D,TOTREC(21),TOTREC(22))
0271            CALL PLOT(0.0,0.0,3)
0272            DO 32 K=1,20
0273            CALL PLOT(RTIME(K)/RTIME(22),TOTREC(K)/TOTREC(22),2)
0274       32   CONTINUE
0275            CALL PLOT(0.0,0.0,999)
0276      304   STOP
0277            END
```

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