INFORMATION TO USERS

This reproduction was made from a copy of a document sent to us for microfilming. While the most advanced technology has been used to photograph and reproduce this document, the quality of the reproduction is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help clarify markings or notations which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure complete continuity.

2. When an image on the film is obliterated with a round black mark, it is an indication of either blurred copy because of movement during exposure, duplicate copy, or copyrighted materials that should not have been filmed. For blurred pages, a good image of the page can be found in the adjacent frame. If copyrighted materials were deleted, a target note will appear listing the pages in the adjacent frame.

3. When a map, drawing or chart, etc., is part of the material being photographed, a definite method of "sectioning" the material has been followed. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.

4. For illustrations that cannot be satisfactorily reproduced by xerographic means, photographic prints can be purchased at additional cost and inserted into your xerographic copy. These prints are available upon request from the Dissertations Customer Services Department.

5. Some pages in any document may have indistinct print. In all cases the best available copy has been filmed.

University Microfilms International

300 N. Zeeb Road
Ann Arbor, MI 48106

8224189

Choi, Jai Young

VARIABLE RETURNS TO SCALE AND THE PURE THEORY OF INTERNATIONAL TRADE

The University of Oklahoma Ph.D. 1982

University Microfilms International 300 N. Zeeb Road, Ann Arbor, MI 48106

THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

VARIABLE RETURNS TO SCALE AND THE PURE THEORY OF INTERNATIONAL TRADE

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

JAI YOUNG CHOI

Norman, Oklahoma

1982

VARIABLE RETURNS TO SCALE AND THE PURE THEORY OF INTERNATIONAL TRADE

APPROVED BY

DISSERTATION COMMITTEE

ACKNOWLEDGEMENTS

I wish to express my deepest gratitude to Dr. Eden S. H. Yu and Dr. Chong K. Liew for their unforgettable support and guidance throughout my graduate studies. To them I am genuinely indebted.

Appreciation is extended to the other members of my dissertation committee, Dr. James E. Hibdon, Dr. Alexander J. Kondonassis, Dr. Terry D. Robertson and Dr. Ed F. Crim Jr.

Unconditional love and commitment of my family, especially my mother, Bong K. Lee, my wife, Kun-Woo, and my daughter, Me-Hee will be always remembered.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

VARIABLE RETURNS TO SCALE AND THE PURE THEORY OF INTERNATIONAL TRADE

CHAPTER I

INTRODUCTION TO VARIABLE RETURNS TO SCALE

I. INTRODUCTION

It is generally agreed among economists that the assumptions underlying the conventional theory of international trade delineate a mythical world. Supporting the argument are the assumptions of constant returns to scale, perfect competition in product and factor markets and full employment of resources. The decades of the sixties and seventies have witnessed a propagation of studies that investigate the implication of relaxing those special assumptions in the context of the pure theory of international trade. The issue of returns to scale, however, has attracted less attention than the others mainly because the compatibility of increasing returns to scale with perfectly competitive equilibrium has remained unresolved issue.

Recently, trade theorists have shown that perfect competition can prevail under conditions of increasing returns to scale if the economies of scale are external to individual firm and internal to industry and the competitive output is efficient if the externalities are output-generated (Kemp, 1955). Based on this concept, a group of leading trade economists has analyzed the validity of some fundamental theorems of international trade including the Rybczynski theorem, the Stolper-Samuelson theorem and price-output response, etc. (Herberg and Kemp, 1969; Jones, 1968). Several additional efforts have been subsequently made to reestablish other theoretical principles governing international trade. However, those are limited in numbers and there is still ample room for improvement and investigation.

The present work deals with some previously unexplored but important aspects of international trade under the assumption of variable returns to scale. The topics under investigation include the gains from trade, the theory of customs union, the theory of nominal tariffs and economic expansion. The variable returns to scale are assumed to take a specific form, namely that economies of scale are external to individual firm and internal to industry and the externalities are output-generated.

The next section of this chapter is devoted to the review of the related literature to obtain a better understanding of the current knowledge regarding this topic. The last section of this chapter presents the analytical framework that will be utilized throughout this study. The standard model of general equilibrium is extended to include the variable returns to scale. The method is mainly derived from the constructions devised by Kemp (1969) and Jones (1968). Although some extensions and modifications are made for rigorous demonstration, no other results are reached in this section.

In Chapter 2, the gains from trade theorems are examined under the condition of variable returns to scale. The compensation principle is adopted as the welfare criterion. The topics discussed are the non-optimality of free trade, the first-best-policy maximizing social welfare, free trade versus no trade, welfare implication of some protection measures including tariffs, production subsidies and consumption taxes and the welfare rankings among those policy instruments.

Chapter 3 is devoted to the examination of the welfare consequences of discriminatory tariffs under the setting of variable returns to scale. The traditional customs unions theory associated with the names of Viner, Lipsey and Gehrels are reconsidered under the new assumption. In particular, the welfare effects of such discriminatory tariffs are analyzed by differentiating two types of trade creation and trade diversion according to the manner in which trade is being created and diverted.

Chapter 4 deals with the positive aspects of the nominal tariffs in the presence of variable returns to scale. The two country, two commodity and two factor model provided by Jones (1969) and Batra (1973) is extended to incorporate variable retuns to scale. Under such an extended setting, the effects of tariffs on the terms of trade and domestic price ratios are analyzed. Additionally, the necessary conditions for Metzler paradox and the optimum tariff are derived.

In Chapter 5, the implication of economic expansion for output levels, social welfare and the terms of trade are examined under the assumption of variable returns to scale. As in the conventional analysis, two sources of economic expansions are identified, technical progress and factor accumulation. To analyze the effects of technical progress on the outputs of commodities, Jones (1969) model is extended. The Prebisch hypothesis regarding the secular deterioration of the terms of trade in underdeveloped countries is reconsidered in the new dimension.

Chapter 6, the final chapter, presents concluding remarks.

II. REVIEW OF LITERATURE

Increasing or decreasing returns to scale in production are indisputable phenomena that characterize the real world. As such, they have been recognized as one of the principal determinants of international trade, since Adam Smith's discovery of the division of labor as the elemental source of the wealth of nations (Chipman, 1965). Nonetheless, they have never played a major role in both the classical and the modern theory of international trade, chiefly because until the 1930's the compatibility of increasing returns to scale with perfectly competitive equilibrium has been a matter of intense debate without apparent conclusions (Chipman, 1970).

Increasing or decreasing returns to scale are usually attributable to certain economies or diseconomies that are reflected in production costs. They may be internal or external to the firm. If the economies of scale are internal to the firm, perfect competition breaks down because one firm would eventually supply the whole industry output. Therefore, the debates have been centered on the type of economies of scale that are external to the firm.[1] Chipman (1965), in his illuminating survey article, describes it as follows:

> It is probably correct to say that economies of scale tend to be ignored in theoretical models not so much on empirical grounds as for the simple reason that the critical difficulties are considerable, and it is not generally agreed how they can be incorporated into a model of general equilibrium or whether they are at all compatible with the assumption of perfect competition.

Some efforts, however, have been made recently to investigate the implication of variable returns to scale in the context of the pure theory of international trade by a group of trade theorists who believe that perfect competition can prevail under conditions of increasing returns, if the economies of scale are external to individual firms.[2] Kemp (1955) has shown that perfect competition and static increasing returns can be reconciled by the introduction of external economies and the competitive output is efficient if the externalities are output-generated. Based on the

---

[1] This type of externalities may be generated for the following reasons: learning by doing, fuller utilization of production capacity and improvement in the quality of labor and management. Professor J.E. Hibdon noted the importance of information sharing.

[2] This concept was originally introduced by Marshall (1879, 1890) and refined by Edgeworth (1905), Knight (1924, 1925), Meade (1952, 1955) and Kemp (1955), among others.

concept, Jones (1969) has formulated a model of variable returns to scale (VRS) and examined the validity of the Rybczynski and Stolper-Samuelson theorems in the presence of variable returns to scale. Allowing for nonhomothetic industry production functions, he has concluded that both theorems based on the assumption of constant returns to scale do not carry over directly to the case of variable returns to scale whereby the degrees of externalities and the correspondence between average and marginal factor intensities play a critical role.

Herberg and Kemp (1969) have derived the locus of production possibility frontier and analyzed price-output response under the assumption of variable returns to scale. Utilizing homothetic industry production functions, they have shown that a) the locus of production possibility frontier is strictly concave to the origin near the increasing returns to scale (IRS) axis and strictly convex to the origin near the decreasing returns to scale (DRS) axis and b) the output of a commodity responds perversely (positively) to an increase in its relative price if it displays IRS (DRS) in the neighborhood of its zero output.

Recently, Mayer (1974) has reconsidered the Rybczynski and Stolper-Samuelson theorems by introducing a dynamic stability condition and argued that a) both the Rybczynski and the Stolper-Samuelson theorems are valid if the system is stable and industry production functions are homothetic, b) if only the stability condition is met, the Rybczynski theorem carries over while the Stolper-Samuelson theorem requires the additional condition that average and marginal factor intensities are the same and c) price-output response is positive in a stable system.

Mayer's results have been challenged by Panagaria (1980) on the ground that the significance of the results based on comparative statics is weakened. Eaton and Panagaria (1979), based on a alternative stability criterion, have shown that a perverse price-output response in the presence of IRS implies a stable internal production equilibrium if and only if the Marshall-Lerner condition is not met. Since this result is derived from one factor model under specially defined stability criterion, the absence of factor substitution and the uniqueness of the stability criterion significantly reduce its generality. Recently, Panagaria (1981) has analyzed the patterns of specialization based on IRS in one industry and DRS in the other using the two commodity and one factor model and shown that a) a small country will never specialize in the DRS industry if it ever specializes in production and b) a small country will never specialize in production if an internal production equilibrium exists. Although these results are derived from the one factor model with constant elasticities of returns to scale, they are general because the asymptotic properties of the production possibility frontier always hold.[1] Note that the studies introduced until now are concerned

---

[1]Elasticity of returns to scale is discusssed in the next section. For a detailed discussion of asymptotic properties of production possibility curve, see Herberg and Kemp (1969).

with the production side of general equilibrium. Melvin (1969) has discussed the increasing returns to scale as a possible determinant of international trade by introducing the demand side of general equilibrium. Kemp and Negishi (1970), using the revealed preference arguments, have proposed the sufficient conditions under which the opening of trade and the improvement in the terms of trade are not harmful in the presence of variable returns to scale, production subsidies and consumption taxes. This subject has been recently reexamined by Eaton and Panagaria (1979) in terms of Pareto welfare criterion. It is, however, noteworthy that both studies still do not relax the assumption of constant returns to scale in their discussions of production subsidies and consumption taxes.

Through the brief survey of the notable contributions to the subject, it has been observed that a) the fundamental theorems constituting the skeleton of the conventional theory of international trade have been analyzed under variable returns to scale, b) the theorems known for constant returns to scale case do not generally carry over to the case of variable returns to scale and c) although there are some works which introduce the demand side of general equilibrium, those are limited in numbers and still ample room for improvement and extension.

In concluding this section, Chipman (1965) should be quoted once again: "this is a poor reason for excluding them (economies of scale) from consideration is evident, especially if it is true that they constitute one of the principal sources of international trade".

III. THE ANALYTICAL FRAMEWORK

This section presents the analytical framework and explores the implication of variable returns to scale for the production side of general equilibrium. The standard two-commodity two-factor model of international trade is extended to the variable returns to scale model using the production functions devised by Herberg and Kemp (1969). The results obtained in this section will be utilized throughout subsequent chapters. Although the validity of Euler's exhaustion theorem for such production functions and the method of deriving the internal production equilibrium condition along with its generalization to the multi-commodity and multi-factor case seem to be new, no other new results are reached in this section.

Assumptions and the Model

The standard two commodity and two factor model of international trade utilizes the following six assumptions as its format.

(1) There are two commodities, X_1 and X_2, respectively produced by Industry 1 and Industry 2 using the factors of production, labor (L) and capital (K), which are indispensable to production.

(2) A firm's production function is subject to constant returns to scale and the marginal productivity of each input is positive but diminishing.

(3) Factors of production are perfectly mobile between sectors within an economy but perfectly immobile among economies.
(4) Factor prices are perfectly flexible such that full-employment is always maintained.
(5) Each commodity is characterized by a different factor intensity, which is non-reversible among commodities.
(6) Perfect competition prevails both in product and factor markets.

The main concern of this research is to relax assumption (2) by introducing the economies of scale. To make the system compatible with assumption (6), it is assumed that the economies of scale are external to individual firm and internal to industry and they are output-generated. Moreover, the optimal factor ratio is independent of the industry's output at a given wage-rental ratio. The development of the model proceeds with the specification of production functions satisfying such properties.[1]

$$x_i = g_i(X_i)F_i(c_i,l_i) \qquad i = 1,2 \tag{1.1}$$

$$X_i = g_i(X_i)F_i(K_i,L_i) \qquad i = 1,2 \tag{1.2}$$

where x_i is the output of a typical firm in industry i and c_i and l_i are capital and labor employed by it. X_i is the output of industry i and K_i and L_i are its employment of capital and labor. g_i describes the role of externality and is a positive

---

[1] Examples: Meade (1952), in his famous example of apple blossoms and honey production, has assumed that the output of honey depends on the output of apples. Herberg and Kemp (1969) have begun with a production function of the more general form, $X_i=g_i(X_1,X_2)F_i(K_i,L_i)$ i=1,2.

function defined on $(0,\infty)$ and F_i is homogeneous of degree one. (output) elasticity of returns to scale of the ith industry, e_i, defined on $(-\infty,1)$ may be written

$$e_i = (dg_i/dX_i)F_i = (dg_i/dX_i)(X_i/g_i) \qquad i=1,2 \qquad (1.3)$$

Note that $e_i \geq 0$ for increasing (constant) returns to scale industry and $e_i < 0$ for decreasing returns to scale industry. Total differentiation of (1.2) yields

$$(1-e_i)dX_i = g_i(F_{Ki}dK_i + F_{Li}dL_i) \qquad i=1,2 \qquad (1.4)$$

where F_{Ki} and F_{Li} are respectively the first partial derivatives of F_i with respect to capital and labor. Since economies of scale are external to individual firm and internal to industry, each factor is paied the value of its marginal product to the individual firm not the value of its marginal product to the industry.

$$w = p_i g_i F_{li} = p_i g_i F_{Li} = p_i g_i (f_i - k_i f_i') \qquad i=1,2 \qquad (1.5)$$

$$r = p_i g_i F_{ci} = p_i g_i F_{Ki} = p_i g_i f_i'(k_i) \qquad i=1,2 \qquad (1.6)$$

where a prime indicates the first partial derivative and f_i is F_i/L_i and k_i and p_i respectively stand for capital-labor ratio and price of the ith commodity. In the absence of external economies, private marginal product $(g_i F_{ji})$ of factor j (j=K,L) is smaller than the social marginal product of it $(g_i F_{ji}/(1-e_i))$. In other words, private marginal cost exceeds social marinal cost. It is noteworthy that industry production function (1.2) satisfies Euler's exhaustion theorem regardless of the degree of returns to scale.[1] With full-employment of factors of production.

$$L = L_1 + L_2 \tag{1.7}$$

$$K = K_1 + K_2 \tag{1.8}$$

where L and K stand for fixed supplies of labor and capital. Differentiation of (1.7) and (1.8) yields

$$dL_1 = -dL_2 \tag{1.9}$$

$$dK_1 = -dK_2 \tag{1.10}$$

Production Possibility Frontier

The shape of production possibility curve has been derived by Herberg and Kemp (1969) under variable returns to scale. It is, however, worth reconsidering for the attainment of a more rigorous understanding of the effects of returns to scale on the shape of production possibility curve as aeparated from that of factor intensities. From (1.4), we get the expression for dX_1/dX_2.

$$\frac{dX_1}{dX_2} = \frac{(1-e_2)g_1(F_{K1}dK_1+F_{L1}dL_1)}{(1-e_1)g_2(F_{K2}dK_2+F_{L2}dL_2)} \tag{1.11}$$

Profit maximization conditions, (1.5) and (1.6), yields

$$F_{L1} = \frac{F_{L2}}{F_{K2}} F_{K1} \tag{1.12}$$

Substituting (1.9), (1.10) and (1.12) in (1.11), we obtain

---

[1] F_i is homogeneous of degree one by assumption. Thus, it holds $(tX_i/g_i)=F_i(tK_i,tL_i)$. Differentiating with respect to t and substituting t=1, we obtain $(X_i/g_i)=F_{Ki}K_i+F_{Li}L_i$. Substitution of (1.5) and (1.6) yields $p_iX_i= rK_i+ wL_i$.

$$\frac{dX_1}{dX_2} = - \frac{(1-e_2)g_1F_{K1}}{(1-e_1)g_2F_{K2}} = - \frac{(1-e_2)g_1f_1'(k_1)}{(1-e_1)g_2f_2'(k_2)} \tag{1.13}$$

Equation (1.13) explicitly shows that the shape of production possibility curve is influenced by two forces, elasticities of returns to scale and factor intensities. Under constant returns to scale where g_is are constant and e_is are zero, only factor intensities affect the shape of production possibility schedule. In this case, production possibility curve is linear if there is only one factor of production or if the factor intensities of the two commodities are the same. Under variable returns to scale, however, production possibility curve is not linear for both of the above cases. To assess the effect of returns to scale as seperated from that of factor intensities, assume that the factor intensities of the two commodities are the same so that $k_1=k_2=k$, where k denotes the overall capital-labor ratio (K/L). Then $f_1'(k_1)/f_2'(k_2)$ in (1.13) becomes some positive constant (C). Following Panagaria (1981), assume further that g_i has a form, $X_i^{a_i}$, so that $e_i=a_i$. Then equation (1.13) can be reexpressed as,

$$\frac{dX_1}{dX_2} = -C\left(\frac{1-a_2}{1-a_1}\right)X_1^{a_1}\ X_2^{-a_2} \tag{1.14}$$

Differentiation of (1.14) with respect to X_2 yields

$$\frac{d^2X_1}{dX_2^2} = C\left(\frac{1-a_2}{1-a_1}\right)X_1^{(2a_1-1)}X_2^{-(a_2+1)}\{a_2(1-a_1)X_1^{(1-a_1)}+$$

$$Ca_1(1-a_2)X_2^{(1-a_2)}\} \tag{1.15}$$

The curvature of production possibility frontier can be inferred from (1.14) and (1.15). (1.14) shows that the slope of production possibility curve is negative while (1.15) indicates that if Industry 1 exhibits IRS ($a_1>0$) and Industry 2 DRS ($a_2<0$), $d^2X_1/dX_2^2>0$ in the neighborhood of $X_1=0$ and $d^2X_1/dX_2^2<0$ in the neighborhood of $X_2=0$. Furthermore, there is only one inflexion point at $d^2X_1/dX_2^2=0$. This result is exactly identical to Panagaria's (1981) result derived from one factor model. Figures 1a, 1b and 1c depict three different shapes of production possibility curve based on the results obtained above. In the general setting where both factor intensities and elasticities of returns to scale are variable, the shape of production possibility curve is determined by e_i, $\partial e_i/\partial X_i$ and k_i. As a result, there may exist multiple inflexion points but the neighborhood properties remain unchanged.[1] To generalize, increasing returns to scale tend to make production possibility curve bowed-in toward the origin while different factor intensities tend to make it bowed-out from the origin.[2] Therefore, the final shape is determined by the relative strength of the two forces.

Internal Production Equilibrium

Kemp (1955, 1969) has shown that in the presence of

---

[1]For neighborhood properties of production possibility curve, see Herberg and Kemp (1969).

[2]The effect of factor intensities on the shape of production possibility curve is not discussed here because it is not the main concern. However, by using Edgeworth box diagram and Savosnick's (1958) technique of deriving production possibility curve from a contract curve, it can be easily demonstrated that different factor intensities tend to make the curve bowed-out from the origin.

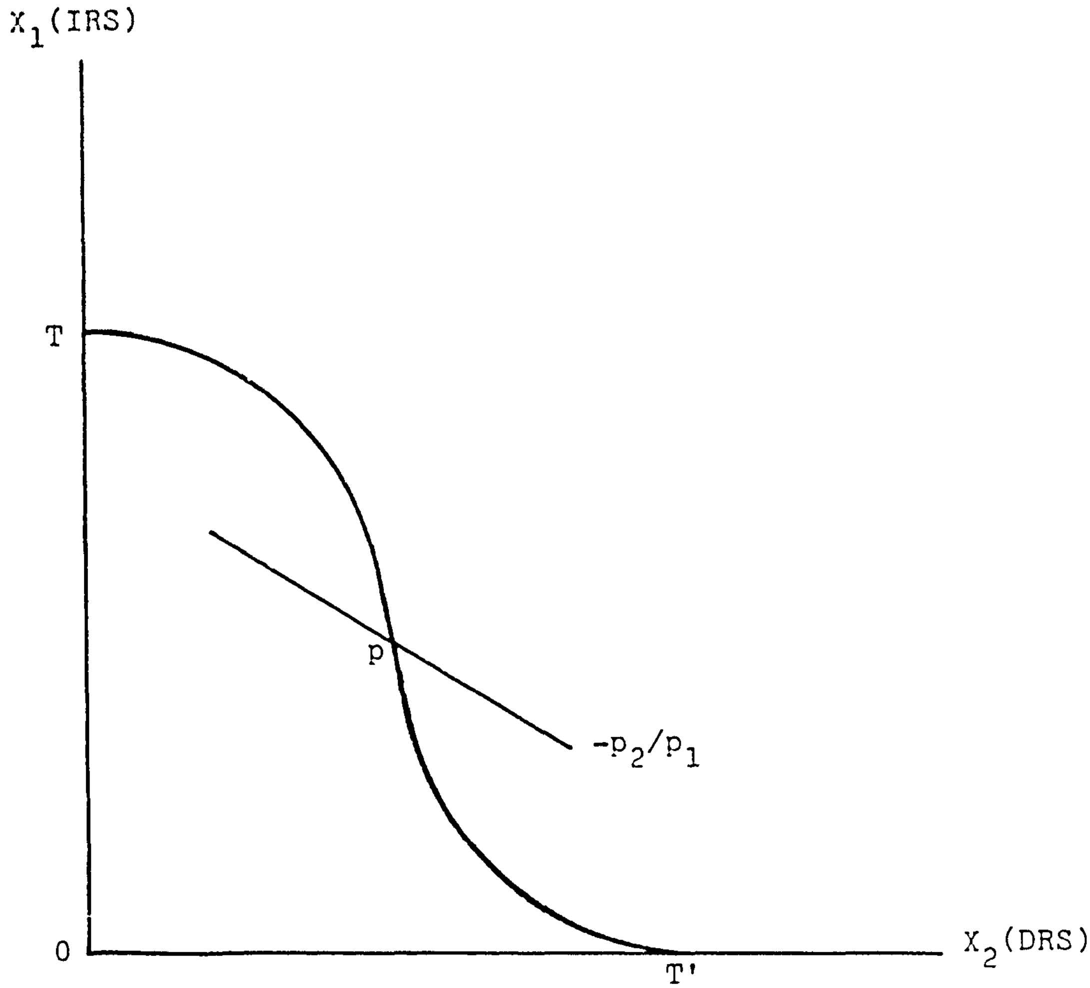
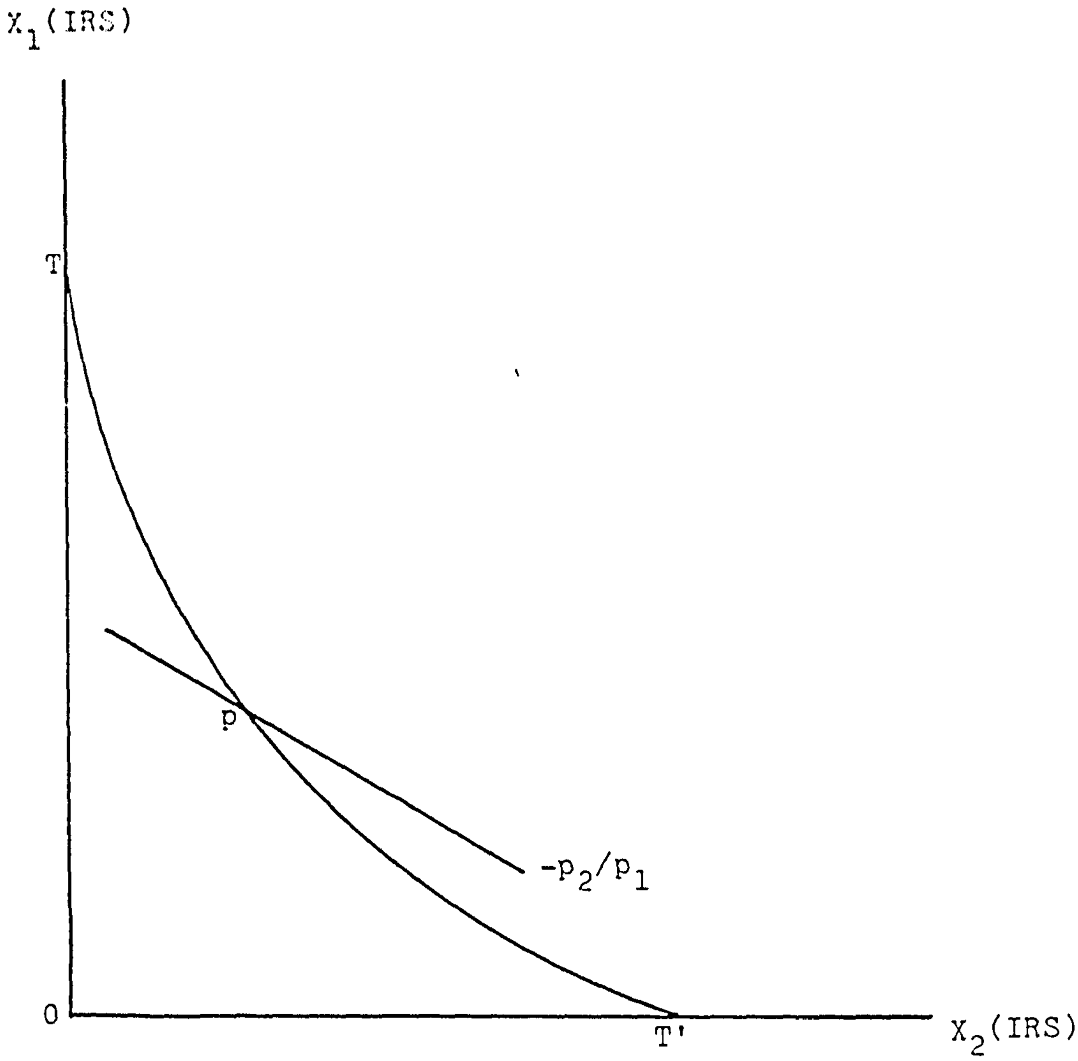
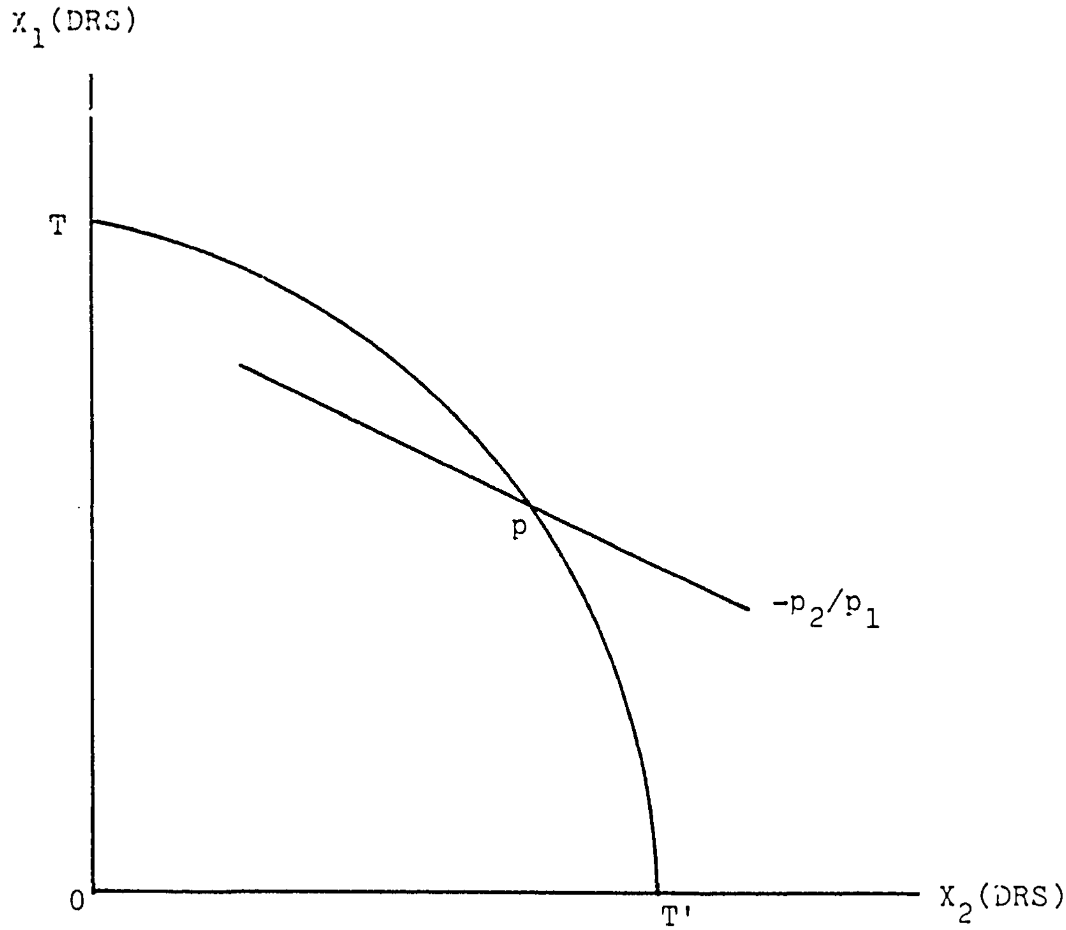


Figure 1a



$e_1 > e_2$

Figure 1b



$e_1 > e_2$

Figure 1c

external economies production may take place inside production possibility curve but a special type of externalities, output-generated economies, cannot prevent a competitive economy from producing on the production possibility curve. The condition for internal production equilibrium is readily established by substituting (1.6) in (1.13).

$$\frac{dX_1}{dX_2} = -\left(\frac{1-e_2}{1-e_1}\right)\frac{p_2}{p_1}$$

or $$p_1(1-e_1)dX_1 + p_2(1-e_2)dX_2 = 0 \qquad (1.16)$$

A glance at (1.16) reveals that the marginal rate of transformation is not equal to $-(p_2/p_1)$ unless $e_1=e_2$. That is, price line cuts the production possibility curve unless the elasticities of returns to scale of the two industries are the same. In Figures 1a, 1b and 1c, price line $(-p_2/p_1)$ is flatter than the slope of the transformation curve at the point of production equilibrium (p) since $e_1 > e_2$. The converse is true if $e_1 < e_2$.

The relationship (1.16) can be generalized to multi-commodity and multi-factor cases. For simplicity's sake, let us consider the three commodity (X_1, X_2 and X_3) and three factor (K_i, L_i and R_i) case where R_i denotes the land employed by Industry i. Then equation (1.4) becomes

$$(1-e_i)dX_i = g_i(F_{Ki}dK_i+F_{Li}dL_i+F_{Ri}dR_i) \quad i=1,2,3 \qquad (1.17)$$

Similarly, total differentiation of full-employment condition yields

$$dK_1+dK_2+dK_3 = dL_1+dL_2+dL_3 = dR_1+dR_2+dR_3 = 0 \qquad (1.18)$$

In addition to (1.5) and (1.6), we have another profit maximization condition, $z=p_i g_i F_{Ri}$ (i=1,2,3), where z is the rental price of the land. Then the expression for $(1-e_1)dX_1$ is

$$(1-e_1)dX_1 = g_1(F_{K1}dK_1+F_{L1}dL_1+F_{R1}dR_1)$$
$$= -(p_2/p_1)g_2\{F_{K2}(dK_2+dK_3)+F_{L2}(dL_2+dL_3)+F_{R2}(dR_2+dR_3)\}$$
$$= -(p_2/p_1)\{(1-e_2)dX_2+(p_3/p_2)(1-e_3)dX_3\}$$

Hence,

$$p_1(1-e_1)dX_1 + p_2(1-e_2)dX_2 + p_3(1-e_3)dX_3 = 0 \qquad (1.17)$$

The extension to the larger numbers of commodities and factors essentially follows the same procedure.

This section has been concerned with the implication of variable returns to scale for the production side of general equilibrium. In the following chapter, we introduce the demand side of general equilibrium and examine the gains from trade theorems in the presence of variable returns to scale.

CHAPTER II

GAINS FROM TRADE UNDER VARIABLE RETURNS TO SCALE

The theory of gains from trade has a long chronicle that dates back to Adam Smith, who directed his criticism against the mercantilist doctrine of protection. Embedded in his principle of absolute advantage is the idea of increased opportunities for division of labor and specialization provided by international trade. Smith's idea was inherited by David Ricardo who advocated the principle of comparative advantage which has remained unchallenged until today. However, the two principles are based on quite restrictive assumpions, particularly, the labor theory of value and constant returns to scale. Modern general equilibrium theory developed by Heckscher and Ohlin largely takes the gains from trade for granted and describes why and how trade takes place. In the general equilibrium theory, the neoclassical production function replaces the classical production function so that factor substitution is critical in explaining why and how trade occurs. But the assumption of constant returns to scale is still retained. Recently, several trade theorists have investigated the welfare consequences of international trade under variable returns to scale. Kemp and Negishi (1970), using the revealed preference arguments, have proposed the sufficient conditions under which the opening of trade and improvement in the terms of trade are not harmful in the presence of variable returns to scale, production subsidies and consumption taxes. Recently, the same subject has been reconsidered by Eaton and Panagaria (1979) in terms of social utility function. It is noteworthy that both studies still do not relax the assumption of constant returns to scale in their discussion of production subsidies and consumption taxes.

This chapter is concerned with the welfare consequences of international trade and some protection measures in the presence of variable returns to scale. The topics discussed are the non-optimality of free trade, free trade versus no trade, the first-best-policy maximizing social welfare, welfare implication of tariffs, production subsidies and consumption taxes and welfare rankings among those policy instruments.

The country under analysis is assumed to be a small country that is incapable of influencing the world prices by manipulating its volume of trade. For analytical purposes, the compensation principle is adopted as the welfare criterion. In addition, all goods are assumed to be non-inferior.

The demand side of the model is represented by the social utility function (U) which is dependent on the consumption demand for the two commodities (D_1 and D_2)

$$U = U(D_1, D_2) \tag{2.1}$$

where $U_i > 0$ and $U_{ii} < 0$ for $i=1,2$. An economy's budget constraint stipulates that the value of production is matched by the value of consumption in terms of world prices.

$$X_1 + pX_2 = D_1 + pD_2 \tag{2.2}$$

where p $(=p_2/p_1)$ is the world price of the second commodity in terms of the first.

I. THE NON-OPTIMALITY OF FREE TRADE

From the outset, it will be demonstrated that the optimality of free trade does not necessarily hold in the presence of variable returns to scale, where the criterion of the optimality is the maximization of social welfare. Following Samuelson's (1939) definition, free trade is defined as a situation in which the domestic and world prices of all trade goods are the same, assuming the absence of frictional costs such as transportation costs.

Differentiating (2.1), we obtain

$$dU = U_1(dD_1 + \frac{U_2}{U_1} dD_2) \tag{2.3}$$

To maximize utility, consumers equate the marginal rate of substitution to the relative price of the two commodities $(U_2/U_1 = p)$. Therefore, (2.13) becomes

$$\frac{dU}{U_1} = dD_1 + pdD_2 \tag{2.4}$$

Total differentiation of (2.2) yields

$$dX_1 + pdX_2 = dD_1 + pdD_2 \tag{2.5}$$

Substituting (1.16) and (2.5) in (2.4), we obtain

$$\frac{dU}{U_1} = (\frac{e_1 - e_2}{1 - e_2})dX_1 = (\frac{e_2 - e_1}{1 - e_1})pdX_2 \gtrless 0 \tag{2.6}$$

Notice that the necessary condition required for free trade to be the optimal policy is $dU/U_1=0$. The sufficient condition requires the additional condition, $d^2U < 0$. It is obvious from (2.6) that the necessary condition is satisfied if and only if $e_1=e_2$. Under constant returns to scale, the optimality of free trade holds since $e_1=e_2=0$. This implies that the optimality of free trade does not necessarily require the assumption of constant returns to scale. If the industries in the economy are operating under identical returns to scale, free trade is the optimal policy as long as the second-order condition is met. Expression (2.6), however, is not zero if $e_1 \neq e_2$. Hence, we can state the following proposition.

Proposition 2.1: Free trade is not the optimal policy if industry production functions are subject to divergent returns to scale.

II. FREE TRADE VERSUS NO TRADE

The case of non-optimality of free trade in the presence of variable returns to scale can be easily demonstrated by geometrically comparing the two extreme cases, free trade and no trade. The production possibility curves in Figures 1a, 1b and 1c are reproduced in Figures 2a, 2b and 2c and denoted by TT'. Consider that the economy is initially in autarky situation, where DP indicates the domestic price ratio, s self-sufficiency equilibrium and U_0 social welfare level. Assume further that the elasticity of returns to scale of the

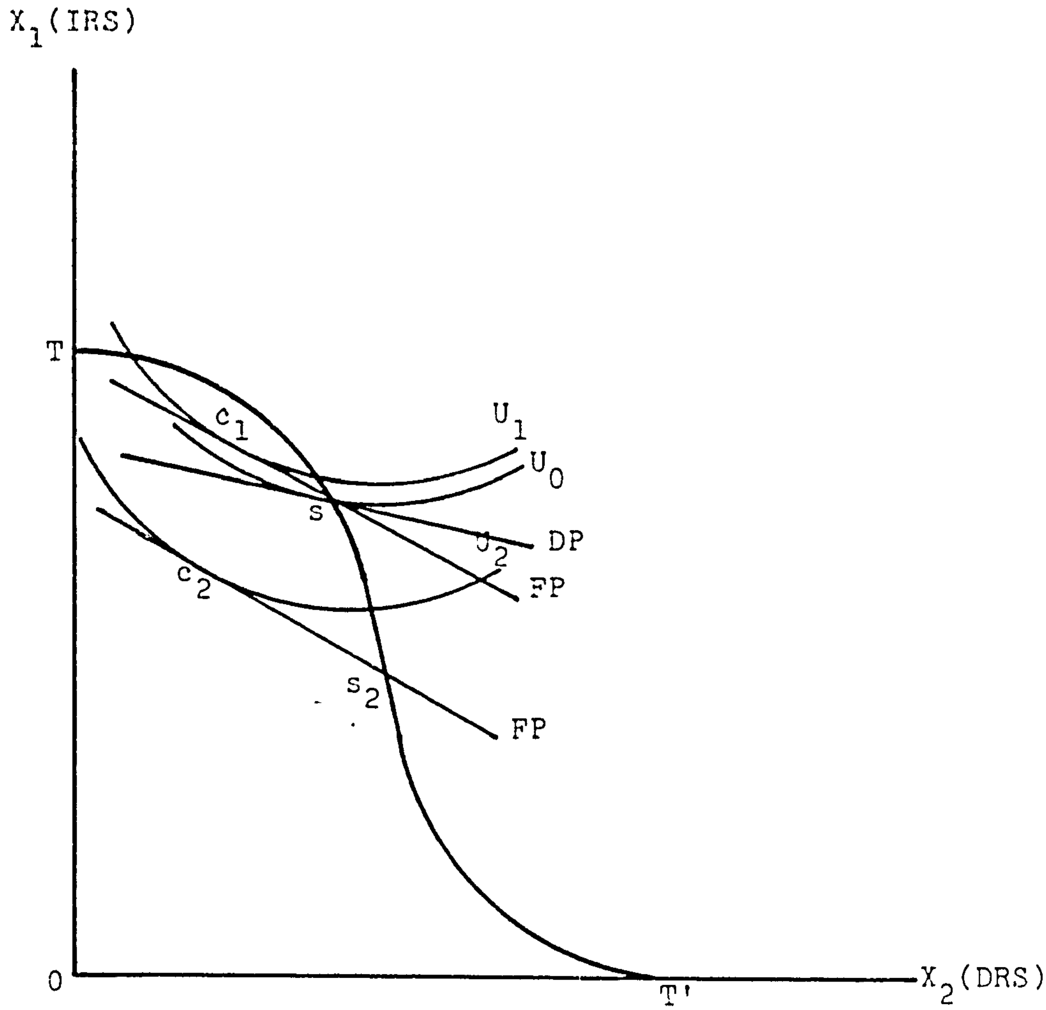
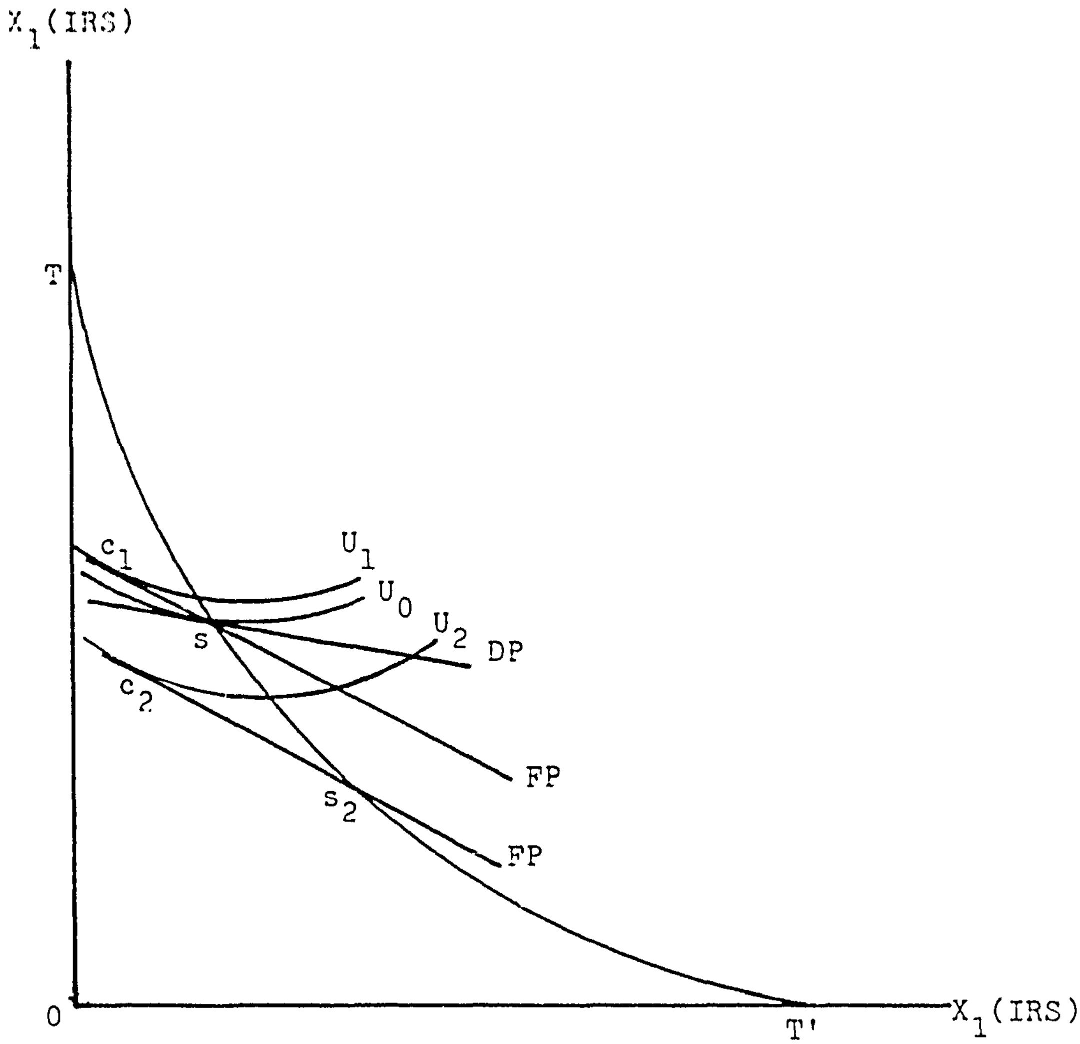
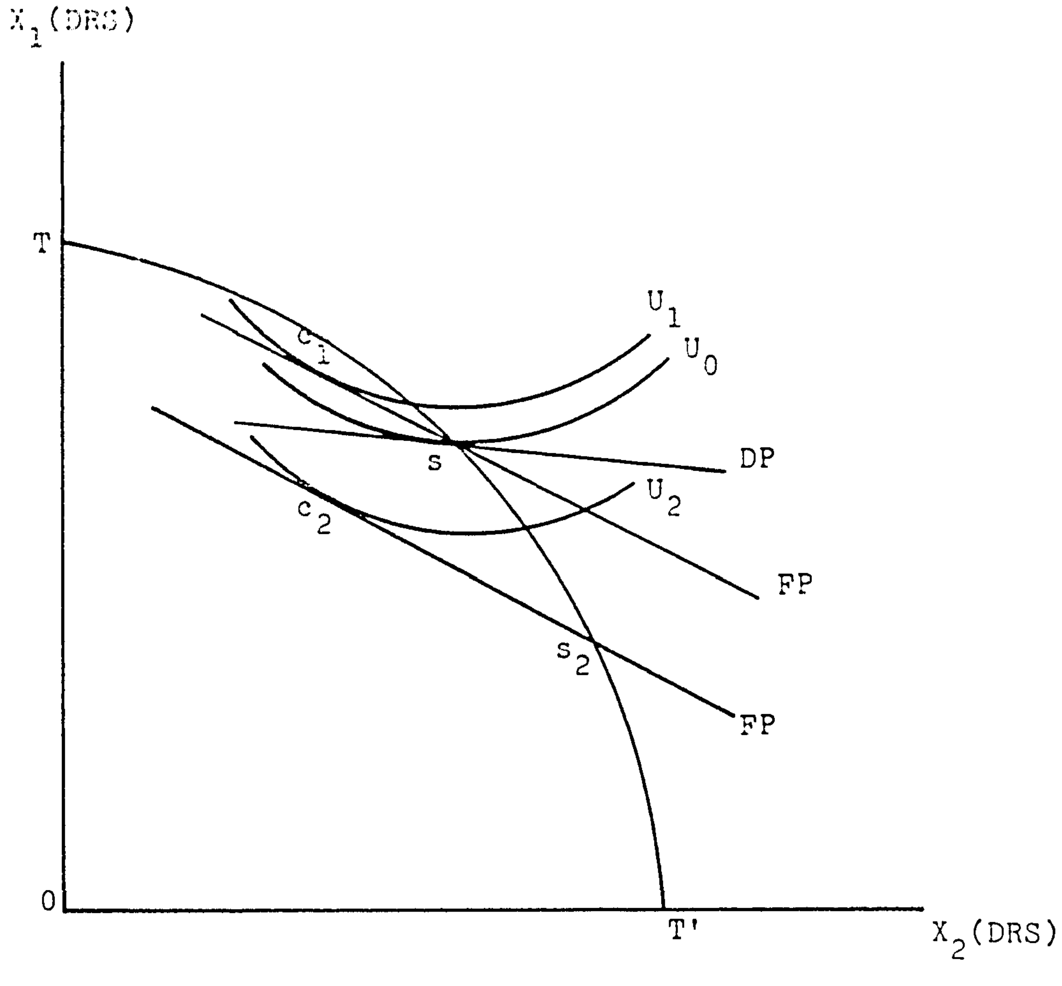


Figure 2a



$e_1 > e_2$

Figure 2b



$e_1 > e_2$

Figure 2c

first industry is greater than that of the second industry ($e_1 > e_2$); price line is flatter than the rate of transformation at the production point. Let FP indicate the exogenously given international price ratio which is steeper than DP. Assuming a non-negative price-output response, production (consumption) now occurs at s_2 (c_2) and the social welfare is U_2, which lies below U_0. Hence, free trade may be inferior to no trade.

The economic explanation of this result is as follows. In this example, we have assumed that Industry 1 is operating under greater elasticity of returns to scale than Industry 2 and the international price ratio favors X_2 sector compared with the autarky price ratio. If production were held at the autarky equilibrium point s, welfare clearly would be increased due to the consumption gain associated with the opening of trade. The consumption gain is given by the improvement in welfare from U_0 to U_1. However, the consumption gain may be outweighed by the production loss which occurs due to the change in the relative price of commodities as the home country switches from no trade to free trade. Since international price favors the second industry, the transfer of factors from the higher returns to scale sector (X_1) to the lower returns to scale sector (X_2) brings about the productivity loss to the economy. The production loss is given by the deterioration of welfare from U_1 to U_2. If the production loss is greater than the consumption gain, free trade is inferior to no trade.

III. THE OPTIMAL POLICY

If free trade is not the optimal policy in the presence of variable returns to scale, what is the optimal policy? Equation (2.6) provides the answer. (2.6) shows that $dU/U_1 \neq 0$ if $e_1 \neq e_2$. Clearly, the best policy is one that makes $dU/U_1 > 0$ if $e_1 \neq e_2$. Such a policy requires $dX_2 \gtrless 0$ (and $dX_1 \lessgtr 0$) if $e_1 \lessgtr e_2$. Outputs of X_1 and X_2 can be increased or decreased by a policy of production tax-cum-subsidy. The reason why this should be achieved by a production tax-cum-subsidy rather than by other alternatives, such as tariffs, is that it can increase the productivity by transferring factors from the lower returns to scale sector to the higher returns to scale sector without creating a divergence between the international price ratio and the marginal rate of substitution. Such a policy takes the production (consumption) point from s (c) to s' (c') in Figures 2d, 2e and 2f. At the new production point, the world price ratio, FP', parallel to FP, is tangential to the production possibility curve (TT') unless both Industry 1 and Industry 2 are operating under increasing returns to scale so that the production possibility curve is strictly convex to the origin. If the production possibility curve is strictly convex to the origin, a policy of tax-cum-subsidy results in the specialization in production of the commodity whose elasticity of returns to scale is greater than the other. This case is geometrically described in Figure 2e.

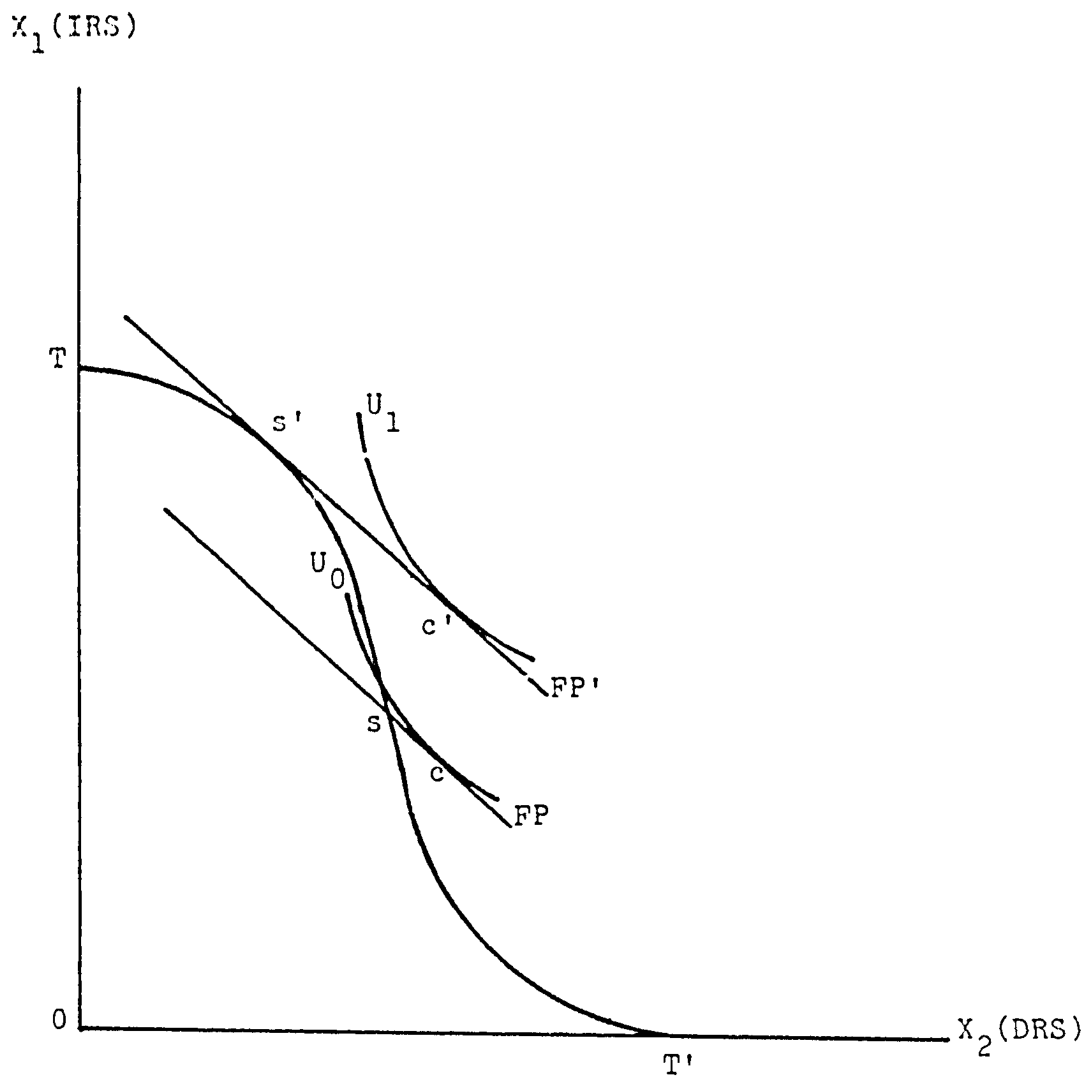
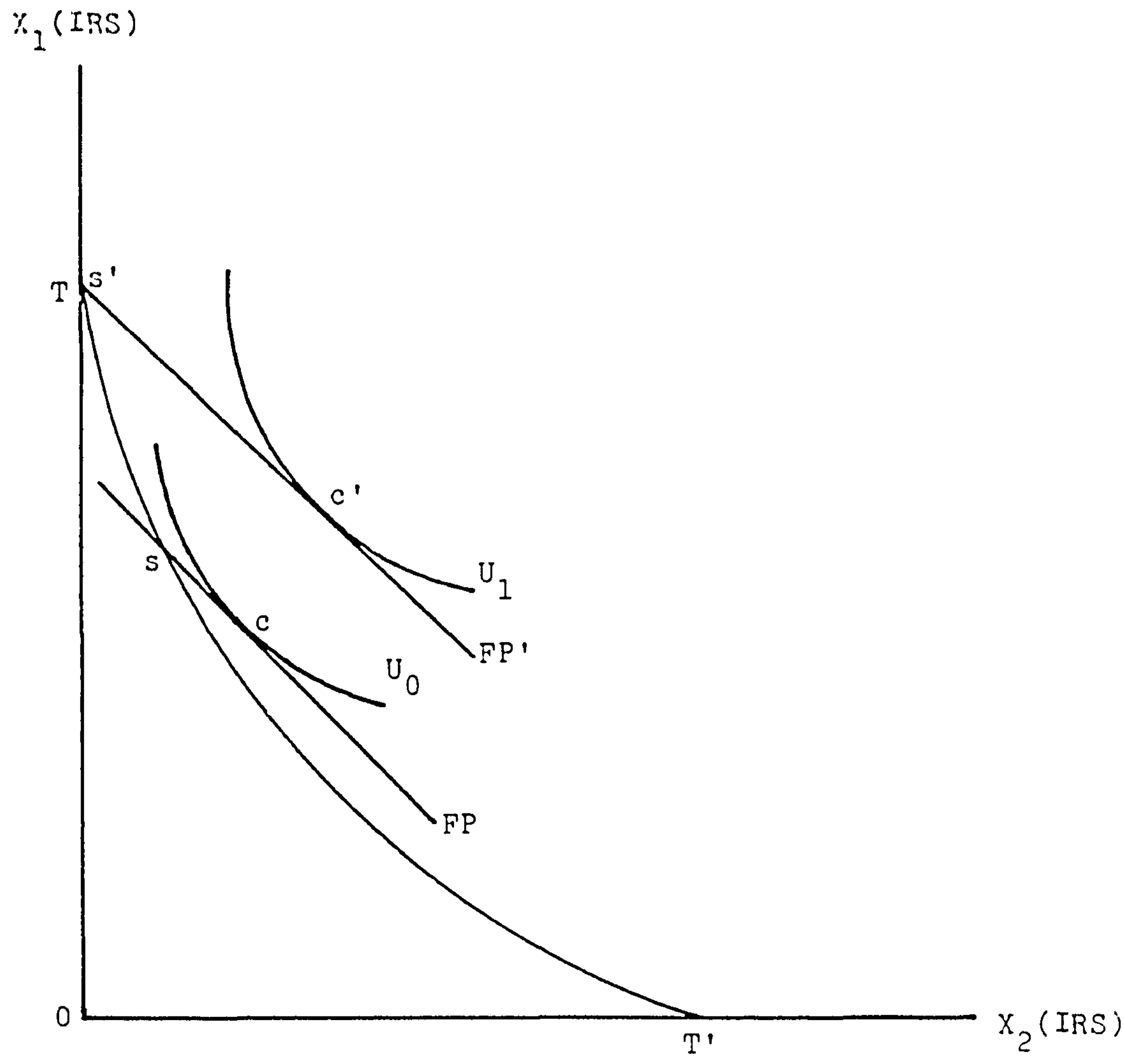
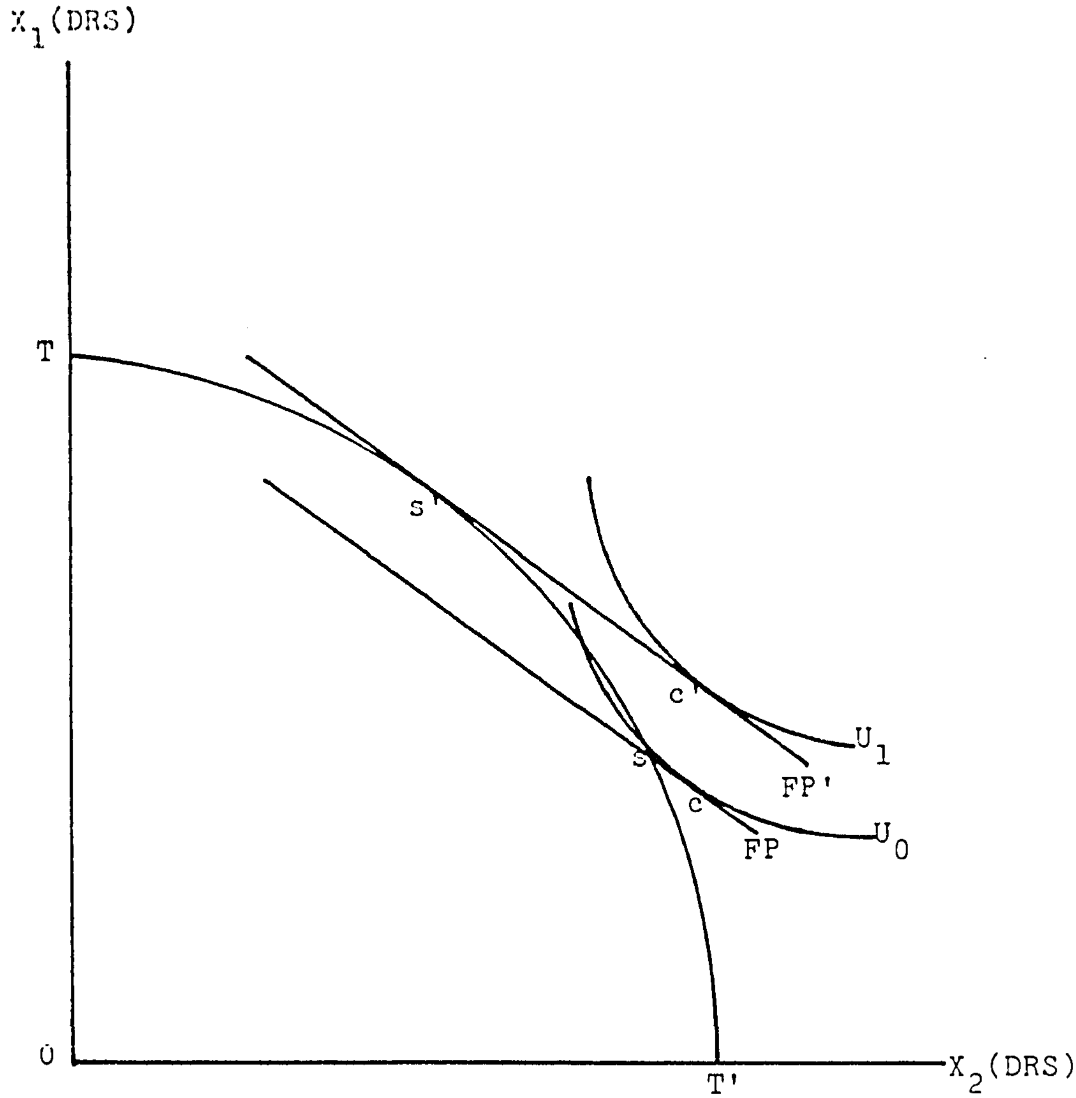


Figure 2d



$e_1 > e_2$

Figure 2e



$e_1 > e_2$

Figure 2f

Proposition 2.2: <u>If industry production functions are subject to different returns to scale, the welfare can be maximized by introducing, in addition to free trade, a policy of production tax-cum-subsidy such that the output of the industry with greater (smaller) elasticity of returns to scale is at the maximum (minimum).</u>

IV. WELFARE IMPLICATIONS OF SOME PROTECTION MEASURES

In the previous section, it has been shown that in the presence of variable returns to scale free trade may be inferior to no trade. No trade, however, is a special case of restricted trade. This section deals with the welfare consequences of restricted trade, which may be affected by the introduction of non-prohibitive tariffs, production subsidies or consumption taxes in the presence of variable returns to scale.

For analytical purposes, let the first and the second commodities be the exportables and the importables respectively.

$$D_1 = X_1 - E_1 \tag{2.7}$$

$$D_2 = X_2 + E_2 \tag{2.8}$$

The balance of payment equilibrium requires that the value of exports is matched by the value of imports in terms of foreign prices.

$$E_1 = pE_2 \tag{2.9}$$

Note that the balance of payments equilibrium condition, (2.9), is consistent with the economy's budget constraint, (2.2).

Tariffs

Suppose that the home country imposes a non-prohibitive tariff (t) on the imports of the second commodity. It should be noted that a tariff under small country assumption changes the price ratio facing both producers and consumers. Then the local relative price of the second commodity in terms of the first commodity becomes

$$p_h = p(1+t) \tag{2.10}$$

To maximize utility, consumers equate the marginal rate of substitution to the tariff inclusive price ratio ($U_2/U_1 = p_h$). It has been previously shown that production equilibrium occurs when

$$\frac{dX_1}{dX_2} = -\left(\frac{1-e_2}{1-e_1}\right)p_h \tag{2.11}$$

Totally differentiating (2.1), (2.7) and (2.8) with respect to t and after a simple arrangement, we obtain

$$\frac{1}{U_1}\frac{dU}{dt} = \frac{dX_1}{dt} - \frac{dE_1}{dt} + p_h\left(\frac{dX_2}{dt} + \frac{dE_2}{dt}\right) \tag{2.12}$$

By differentiating (2.9) and (2.10) and using (2.11) and (2.12) we derive

$$\frac{1}{U_1}\frac{dU}{dt} = pp_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{dX_2}{dp_h} + p^2t\frac{dE_2}{dp_h} \tag{2.13}$$

---

* The value of consumption in domestic price is equal to the value of consumption in domestic price plus the tariff revenue: $D_1 + p_h dD_2 = X_1 + p_h X_2 + tpE_2$. This is consistent with (2.2) and (2.9).

Equation (2.13) furnishes the key expression for determining the effect of a tariff on the welfare of the home country. Under constant returns to scale ($e_1=e_2=0$), (2.13) reduces to $p^2t(dE_2/dp_h)$, which is necessarily negative in the absence of inferior goods, so that a higher rate of tariff results in the lower level of social welfare. Furthermore, free trade turns out to be the optimal policy because $dU/dt=0$ if $t=0$. These results, however, no longer hold under variable returns to scale due to variable elasticities of returns to scale and ambiguous price-output response.

With regard to the price-output response, Herberg and Kemp (1969) have shown that the output of a commodity responds perversely (positively) to an increase in its relative price if it displays increasing (decreasing) returns to scale in the neighborhood of its zero output and under variable returns to scale the response of output to a small change in price cannot be inferred from the local curvature of production possibility frontier. Recently, Mayer (1974) has considered price-output response in connection with dynamic stability condition. Assuming the Marshallian adjustment process (quantities respond to excess demand prices) in product market and the Walrasian adjustment process (prices respond to excess demand) in the factor market, Mayer has shown that under variable returns to scale the output of a given commodity responds positively to an increase in its relative price if the system is dynamically stable.

To control the variety of possible outcomes, the following analysis will be restricted to the dynamically stable system in Mayer's sense, i.e. $dX_2/dp_h > 0$. In the absence of inferior goods, dE_2/dp_h is then negative. From (2.13), it is clear that $dU/dt < 0$ if $e_1 \geq e_2$. Hence, the following proposition can be stated.

Proposition 2.3: <u>If the elasticity of returns to scale of the exportable industry is equal to or greater than that of the importable industry, a higher rate of tariff results in the lower level of social welfare given the positive price-output response.</u>

But if $e_1 < e_2$, $dU/dt \gtrless 0$.

Proposition 2.4: <u>If the elasticity of returns to scale of the exportable industry is smaller than that of the importable industry, social welfare and the tariff rate are not uniquely related.</u>

The economic explanation of these results are as follows. Suppose that the exportable industry is operating under greater elasticity of returns to scale than the importable industry. If the output of a commodity responds positively to an increase in its relative price, a higher tariff increases (decreases) the output of the importable (exportable) commodity. This results in the productivity loss to the economy since factors are transferred from the higher returns to scale sector to the lower returns to scale sector. Furthermore, a higher tariff generates higher consumption distortion than the lower tariff. In this case, the total welfare effect of a higher tariff is unambiguously negative. Hence, a higher tariff is inferior to a lower tariff.

But if the exportable industry is operating under smaller elasticity of returns to scale than the importable industry, a higher tariff brings about the productivity gain to the economy because factors are transferred from the lower returns to scale sector to the higher retunrs to scale sector. A higher tariff, however, imposes a higher consumption loss Therefore, the total effect on the welfare depends on the relative strength of these two opposite forces. If the production gain outweighs (underweighs) the consumption loss, a higher tariff is better (worse) than a lower tariff.

Production Subsidies

Suppose the home country grants a production subsidy (s) to the importable industry for protection purposes. A production subsidy under small country assumption changes the relative commodity prices to the producers but not to the consumers. The local relative price of the second commodity to the producers is

$$p_s = p(1+s) \tag{2.14}$$

Hence, production equilibrium occurs when

$$\frac{dX_1}{dX_2} = -\left(\frac{1-e_2}{1-e_1}\right)p_s \tag{2.15}$$

But the marginal rate of substitution between the first and the second commodity reflects the world price ($U_2/U_1 = p$).

Total differentiation of (2.1), (2.7) and (2.8) and appropriate substitution yields

$$\frac{1}{U_1}\frac{dU}{ds} = \frac{dX_1}{ds} - \frac{dE_1}{ds} + p(\frac{dX_2}{ds} + \frac{dE_2}{ds}) \tag{2.16}$$

Differentiating (2.9) and (2.14) and using (2.15) and (2.16), we obtain

$$\frac{1}{U_1}\frac{dU}{ds} = pp_s(\frac{e_2-e_1}{1-e_1})\frac{dX_2}{dp_s} - p^2s\frac{dX_2}{dp_s} \tag{2.17}$$

The first term on the right hand side indicates the returns to scale effect on production, where as the second term captures the production loss due to subsidy. Under constant returns to scale, (2.17) reduces to $-p^2s(dX_2/dp_s)$, which is always negative. Under variable returns to scale, however, it stands in need of revision due to variable elasticities of returns to scale. In the dynamically stable system ($dX_2/dp_s>0$), $dU/ds< 0$ if $e_1 \geq e_2$. Hence, the following proposition can be deduced.

Proposition 2.5: <u>If the elasticity of returns to scale of the exportable industry is equal to or greater than that of the import-competing industry, a higher production subsidy results in the lower level of social welfare given the positive price-output response.</u>

Equation (2.17) may be reexpressed to analyze the alternative case.

$$\frac{1}{U_1}\frac{dU}{ds} = p^2\{\frac{e_2-e_1-s(1-e_2)}{1-e_1}\}\frac{dX_2}{ds} \tag{2.18}$$

It is clear from (2.18) that $dU/ds > 0$ if $s < (e_2-e_1)/(1-e_2)$. Since $s > 0$, $e_1 < e_2$ if $s < (e_2-e_1)/(1-e_1)$.

Proposition 2.6: <u>If the elasticity of returns to scale of the exportable industry is smaller than that of the import-competing industry, a higher production subsidy results in the higher level of social welfare given the positive price-output response.</u>

This result can be easily explained in comparison with a tariff. We have previously shown that a tariff creates both consumption distortion and production distortion (or gain). But a production subsidy does not affect the consumption side. Therefore, the total welfare effect is solely determined by the effect on the production side, which is equivalent to the production effect of a tariff.

Consumption Taxes

The imposition of a consumption tax (c) on the imports of the second commodity increases its relative price to the consumers but not to the producers. The local relative price facing consumers is now given by

$$p_c = p(1+c) \tag{2.19}$$

Since the relative price of the commodities to the producers remains unaltered, the condition for production equilibrium is

$$\frac{dX_1}{dX_2} = -\left(\frac{1-e_2}{1-e_1}\right)p \tag{2.20}$$

Totally differentiating (2.1), (2.7), (2.8) and (2.9) and using (2.20), we obtain

$$\frac{1}{U_1}\frac{dU}{dc} = p^2 c \frac{dD_2}{dp_c} \tag{2.21}$$

which is necessarily negative in the absence of inferior goods.

Proposition 2.7: <u>A higher consumption tax results in the lower level of social welfare regardless of the retunrs to scale of the exportable and the importable industries.</u>

The standard result based on the assumption of constant returns to scale carries over to the case of variable returns to scale. The reason is that a consumption tax introduces consumption distortion by creating a divergence between the world prices and the marginal rate of substitution of the consumers without affecting the production side.

V. WELFARE COMPARISONS OF TARIFFS, SUSIDIES AND TAXES

From the practical point of view, the welfare comparisons among different policy measures are important. It is relatively simple matter now to compare the welfare effects of tariffs with those of production subsidies and consumption taxes using the information acquired in the previous section.

Tariffs versus Production Subsidies

To compare the welfare effects of the two policy instruments, we must first define the equivalence criterion. Adopting the Corden's (1957) criterion of equivalence, let the equivalence between the tariff and the production subsidy be defined by

$$t = s,\ dt = ds \text{ and } \frac{dX_2}{dt} = \frac{dX_2}{ds}$$

That is, the welfare effects resulting from those rates of the two policy instruments which generate an equal change in output of the importable commodity are compared.

Subtracting (2.17) from (2.13), we obtain

$$-\frac{1}{U_1}\left(\frac{dU}{dt} - \frac{dU}{ds}\right) = p^2 t \frac{dD_2}{dp_h} \tag{2.22}$$

which is always negative in the absence of inferior goods.

Proposition 2.8: <u>A tariff is inferior to an equivalent production subsidy regardless of the returns to scale of the exportable and the importable industries.</u>

The reason is that the effects on the production side of an equivalent tariff and production subsidy are the same, whereas only the tariff creates consumption distortion such that the foreign price ratio diverges from the marginal rate of substitution of the domestic consumers.

Tariffs versus Consumption Taxes

We need again to define the equivalence criterion of the two policy measures. Adopting the Bhagwati and Srinivasan's(1969) criterion of equivalence, let the equivalence between the tariff and consumption taxes be defined by

$$t\left(\frac{dD_2}{dt}\right) = c\left(\frac{dD_2}{dc}\right)$$

That is, the objective of the policy maker is to set the consumption of the importable commodity at a certain level. Subtracting (2.21) from (2.13), we obtain

$$-\frac{1}{U_1}\left(\frac{dU}{dt} - \frac{dU}{dc}\right) = p^2\left\{\frac{e_2-e_1-t(1-e_2)}{1-e_1}\right\}\frac{dX_2}{dp_h} \tag{2.23}$$

In the dynamically stable system ($dX_2/dp_h > 0$), the sign of (2.23) is negative if $e_1 \geq e_2$. But (2.23) is positive if $t < (e_2-e_1)/(1-e_2)$, which implies that $e_1 < e_2$. Hence, the following proposition is immediate.

Proposition 2.9: <u>If the elasticity of returns to scale of the exportable industry is equal to or greater than (smaller than) that of the importable industry, a tariff is superior (inferior) to an equivalent consumption tax given the price-output response is positive.</u>

As is well-known, a tariff affects both the production and consumption while a consumption tax affects the consumption only. The consumption distortions created by an equivalent tariff and consumption tax are identical; the tariff creates a further effect on the production, which is identical to the effect of an equivalent production subsidy. Comparing Proposition (2.9) with Proposition (2.5) and (2.6), it is clear that the condition under which a tariff is inferior (superior) to an equivalent consumption tax is the same condition under which a higher production subsidy lowers (improves) social welfare.

In this chapter, we have examined the traditional gains from trade theorems in the one country open-economy model allowing variable returns to scale. In particular, some sections have been devoted to the examination of the welfare consequences of the tariffs which are non-discrimiatory in nature. The following chapter explores the welfare implications of discriminatory tariffs, the customs union, under the assumption of variable returns to scale.

CHAPTER III

VARIABLE RETURNS TO SCALE AND CUSTOMS UNION THEORY

In his pioneering contribution to the theory of customs unions, Viner (1950) has demonstrated that the customs unions may be welfare-decreasing based on the now familiar concepts of trade creation and trade diversion. According to Viner, trade creating customs union is good and tends to increase welfare while trade-diverting customs union is bad and tends to decrease welfare. Viner's proposition was subsequently challenged by Lipsey (1957, 1960) and Gehrels (1956) on the ground that Viner has ignored the inter-commodity substitution in consumption. Lipsey and Gehrels have advanced the proposition such that trade diversion does not necessarily decrease welfare as Viner had thought. On the contrary, trade diversion may be welfare-increasing if the consumption effects are taken into consideration. The Lipsey-Gehrels arguments have been generalized by Melvin (1969) and Bhagwati (1971) to the case of concave production possibility frontier exhibiting increasing opportunity costs. According to Melvin-Bhagwati arguments, in addition to the consumers' gain obtained by inter-commodity consumption substitution, there is also a producers' gain obtained by adjusting the production up to the point at which the marginal rate of transformation is equated with the barter terms of trade. Johnson (1974) and Chacholiades (1978) have pointed out that the disputes over welfare-increasing trade diversion are a semantic problem which arises from a definition of trade diversion.

In addition to the static effects of the customs unions, Balassa (1961), Scitovsky (1958), Leibenstein (1956), Corden (1972) and Chacholiades (1978) have discussed the dynamic effects of customs unions including economies of scale, technical change, increased competition and changes in investment pattern.

This chapter examines the traditional customs unions theory under the setting of variable returns to scale. As in the foregoing analyses, it is assumed that increasing or decreasing returns to scale are caused by output-generated economies or diseconomies of scale that are external to individual firm and internal to industry. The problem we consider is particularly important since the creation (contraction) of a market resulting from the formation of customs union leads to a greater (lesser) degree of specialization, which changes production costs for the following reasons: fuller utilization of plant capacity, learning by doing, development of a pool of skilled labor and management (Chacholiades, 1978).

I. ASSUMPTIONS AND THE MODEL

As in the standard customs unions theory, we employ the three country, two commodity and two factor model with the modification allowing the production functions which are subject to variable returns to scale. Suppose that the world consists of three countries; the home country A and its potential union partners, B and C. All three countries produce two commodities, X_1 and X_2, using the factors of production, capital and labor, which are indispensable to production. Furthermore, A is the highest-cost and C is the lowest-cost producer of X_2. Countries B and C are similar, but different from A and hence do not trade each other. In addition, A is a small country, a price taker, so that if A engages in trade it exports X_1 to B and C but imports X_2 from B or C but not from both.

The demand side of the model is represented by a concave utility function,

$$U = U(D_1, D_2) \tag{3.1}$$

where D_1 and D_2 are the consumption demand for the two commodities in the home country and that $U_i > 0$ and $U_{ii} < 0$ for i=1,2.

Let X_1 be the exportable commodity and X_2 the importable commodity.

$$D_1 = X_1 - E_1 \tag{3.2}$$

$$D_2 = X_2 + E_2 \tag{3.3}$$

where E_1 and E_2 stand for the export of X_1 and the import of X_2, respectively.

Assuming the balance of payments equilibrium is always maintained

$$E_1 = pE_2 \tag{3.4}$$

where p $(=p_2/p_1)$ is the world price of the second commodity in terms of the first. A tariff in the case of a small country changes the domestic price ratio facing both producers and consumers. The domestic relative price of the second commodity in terms of the first becomes

$$p_h = p(1+t)$$

We have shown in Chapter I that production equilibrium occurs when

$$\frac{dX_1}{dX_2} = -\left(\frac{1-e_2}{1-e_1}\right) p_h \qquad (3.5)$$

where dX_1/dX_2 is the marginal rate of transformation between the two commodities and e_i the output elasticity of returns to scale of Industry i.

The model consisting of (3.1)-(3.5) will be utilized to investigate the welfare consequences of trade-creating and trade-diverting customs unions in the presence of variable returns to scale.

II. ANALYSIS

To analyze the welfare implications of a customs union under variable returns to scale, we follow the procedures developed by Batra (1973) and later extended by Yu (1981, 1982) to the case of factor market imperfections and regid wage economy. Differentiating social utility function (3.1) and utilizing the consumer equilibrium condition, $U_2/U_1=p_h$, we obtain

$$\frac{dU}{U_1} = dD_1 + p_h dD_2 \tag{3.6}$$

Totally differentiating (3.2)-(3.4) and using (3.5) and (3.6), we get

$$\frac{dU}{U_1} = \left(\frac{e_2-e_1}{1-e_1}\right)p_h dX_2 + ptdE_2 - E_2 dp \tag{3.7}$$

Since import is a function of the tariff and the terms of trade, $E_2=E_2(t,p)$ and $dE_2=(\partial E_2/\partial t)dt+(\partial E_2/\partial p)dp$. Substituting in (3.7) to obtain

$$\frac{dU}{U_1} = \left(\frac{e_2-e_1}{1-e_1}\right)p_h dX_2 + pt\frac{\partial E_2}{\partial t}\,dt + \left(pt\frac{\partial E_2}{\partial t} - E_2\right)dp \tag{3.8}$$

The first term of the right hand side captures the welfare effect of variable returns to scale and the second (third) term indicates the effect of an exogenously changed tariff rate (terms of trade).

Since X_2 depends on t and p, $X_2=X_2(t,p)$ and $dX_2=(\partial X_2/\partial t)dt+(\partial X_2/\partial p)dp$. Substituting in (3.8) to get

$$\frac{dU}{U_1} = \left[\left(\frac{e_2-e_1}{1-e_1}\right)p_h\frac{\partial X_2}{\partial t}+pt\frac{\partial E_2}{\partial t}\right]dt + \left[\left(\frac{e_2-e_1}{1-e_1}\right)p_h\frac{\partial X_2}{\partial p}+pt\frac{\partial E_2}{\partial p} - E_2\right]dp \tag{3.9}$$

Taking partial derivatives of $p_h=p(1+t)$ with respect to t and p, we obtain $\partial p_h/\partial t=p$ and $\partial p_h/\partial p=(1+t)$. substituting in (3.9), we get the final expression

$$\frac{dU}{U_1} = \left.\frac{dU}{dt}\right|_{dp=0} dt + \left.\frac{dU}{dp}\right|_{dt=0} dp$$

$$= \left[pp_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{\partial X_2}{\partial p_h}+p^2t\frac{\partial E_2}{\partial p_h}\right]dt + (1+t)\left[p_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{\partial X_2}{\partial p_h}+pt\frac{\partial E_2}{\partial p_h} - \frac{E_2}{(1+t)}\right]dp \tag{3.10}$$

Equation (3.10) is the key expression for determining the welfare effects of forming a customs union. The first (second) term of the right hand side is the change in welfare as a result of a change in the tariff rate (terms of trade), given that terms of trade (tariffs) remain constant.

Suppose that the home country is initially under autarky due to prohibitive tariffs. As in the standard customs unions theory, we define trade creation as the home country's switch of its consumption of the importable commoditiy from a higher-cost producer to a lower-cost producer and trade diversion as that from a lower-cost producer to a higher-cost producer. Yu (1981) has refined the traditional definition by differentiating two types of trade creation and trade diversion according to the manner in which trade is being created or diverted. According to his definition,

Definition 1: Trade creation I refers to A's switch of its consumption of X_2 from domestic (highest-cost) producers to C's (lowest-cost) producers.

Definition 2: Trade diversion I refers to A's switch of its consumption of X_2 from C's producers to B's producers by discriminatorily abolishing tariffs on B only.

Definition 3: Trade creation II refers to A's switch of its consumption of X_2 from B's producers to C's producers.

Definition 4: Trade diversion II refers to A's switch of its consumption of X_2 from C's producers to B's producers by discriminatorily levying a tariff only against C.

Notice that the traditional literature of customs unions theory is mostly concerned with trade creation I and trade diversion I. It will, however, be shown that trade creation and trade diversion of different types have different implications for the home country's welfare.

Under trade creation I, A switches its consumption of X_2 from domestic producers to C's producers by reducing its tariffs against B and C such that A trades with C only. As a consequence, A's domestic price ratio decreases but A faces the same foreign price ratio as before which is given by C, dp=0. Further, the reduction in A's tariff implies dt < 0. Since dp=0, (3.10) reduces to

$$\frac{dU}{U_1} = \left[pp_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{\partial X_2}{\partial p_h} + p^2 t\frac{\partial E_2}{\partial p_h}\right]dt \qquad (3.11)$$

The first terms of the right hand side captures the production effect of trade creation I via variable returns to scale while the second term includes both production (direct) and consumption effects. Consider the dynamically stable system in which the output of a commodity reponds positively to an increase in its relative price, $\partial X_2/\partial p_h > 0$. Then $\partial E_2/\partial p_h < 0$ in the absence of inferior goods. If the industries in the economy operate under identical returns to scale ($e_1=e_2$) with constant returns to scale as its special case, (3.11) reduces $pt(\partial E_2/\partial p_h)dt$, which is necessarily positive. This is the standard result obtained by many authors including Batra (1973). This result is retained if $e_1 > e_2$ since dU/U_1 is still positive. Hence, we can now state the following proposition.

Proposition 3.1: <u>If the elasticity of returns to scale of the exportable industry is equal to or greater than that of the importable industry, trade creation I is always welfare-improving given the positive price-output response.</u>

But if $e_1 < e_2$, $dU/U_1 \gtrless 0$; the standard result breaks down.

Proposition 3.2: <u>If the elasticity of returns to scale of the exportable industry is smaller than that of the importable industry, trade creation I may be welfare-reducing.</u>

Under trade diversion I, A completely rmoves tariffs against B (dt<0) such that A now trades with B at B's terms of trade. Hence, A's terms of trade become unfavorable, $dp > 0$. The welfare effects of trade diversion I are given by (3.10). As we discussed the economic meanings of the terms in the first bracket, it suffices to note the following. The first term in the second bracket $(1+t)p_h(\frac{e_2-e_1}{1-e_1})(\partial X_2/\partial p_h)dp$ captures the production effect of a change in the terms of trade via variable returns to scale, the second term the terms of trade effect on production (direct) and consumption and the third term the terms of trade effect via change in the value of import. Suppose $e_1=e_2$ with constant returns to scale $(e_1=e_2=0)$ as a special case. Then (3.10) reduces to

$$\frac{dU}{U_1} = p^2t\frac{\partial E_2}{\partial p_h}\,dt + (1+t)(pt\frac{\partial E_2}{\partial p_h} - \frac{E_2}{1+t})\,dp$$

$$= \frac{dU}{dt}\bigg|_{dp=0} dt + \frac{dU}{dp}\bigg|_{dt=0} dp \qquad (3.12)$$

In this case, $dU/dt_{\,dp=0}dt > 0$ and $dU/dp_{dt=0}dp < 0$. It is clear that $dU/U_1 \gtrless 0$, depending on the rlative strength of these opposite two forces. It is worth noting that (3.12) reveals both the consumption and the production gains, which are respectively emphasized by Lipsey-Gehrels and Melvin-Bhagwati. From (3.12), it is obvious that the traditional result, welfare-improving tariff effect and welfare-reducing terms of trade effect of trade diversion I, is retained if $e_1 > e_2$. But if $e_1 < e_2$, $dU/dt_{\,dp=0}\,dt \gtrless 0$ and $dU/dp_{\,dt=0}\,dp \gtrless 0$, i.e. a lower tariff may be welfare-decreasing and the deterioration in the terms of trade may be welfare-increasing. In any case, the welfare effect of trade diversion I is ambiguous.

Proposition 3.3: <u>In the presence of variable returns to scale, trade diversion I may be welfare-improving.</u>

Under trade creation II, A completely removes its tariffs against C so that now A engages in trade with C only. Thus A's domestic price ratio, p_h decreases to C's terms of trade. In addition, A experiences an exogenous improvement in its terms of trade, $dp < 0$. Since A had previously engaged in free trade with B under trade diversion I, there is no change in the tariff rate, $dt=0$. Therefore, (3.10) reduces to

$$\frac{dU}{U_1} = (1+t)\left[p_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{\partial X_2}{\partial p_h} + pt\frac{\partial E_2}{\partial p_h} - \frac{E_2}{1+t}\right]dp \qquad (3.13)$$

As before, consider first the case in which $e_1=e_2$. Then (3.13) reduces again to

$$\frac{dU}{U_1} = (1+t)\left(pt\frac{\partial E_2}{\partial p_h} - \frac{E_2}{1+t}\right) dp$$

which is necessarily positive if $\partial X_2/\partial p_h > 0$, i.e. trade diversion II improves social welfare. It is clear from (3.13) that

$dU/U_1 > 0$ if $e_1 \geq e_2$. Hence, the following proposition can be deduced.

Proposition 3.4: <u>If the elasticity of returns to scale of the exportable industry is equal to or greater than that of the importable industry, trade creation II is welfare-improving given the positive price-output response.</u>

But if $e_1 < e_2$, $dU/U_1 \gtrless 0$.

Proposition 3.5: <u>If the elasticity of returns to scale of the exportable industry is smaller than that of the importable industry, trade creation II may be welfare-reducing.</u>

Finally, consider the welfare effects of trade diversion II under which A imposes a discriminatory tariff against imports from C. Consequently, A engages in trade with B only at B's terms of trade. Since there is no change in the tariff rate imposed against imports from B, dt=0. But A's switch of its consumption of X_2 from C to B deteriorates A's terms of trade, dp> 0. The welfare effect of trade diversion II is given by (3.13). If $e_1 \geq e_2$, $dU/U_1 < 0$.

Proposition 3.6: <u>If the elasticity of returns to scale of the exportable industry is equal to or greater than that of the importable industry, trade diversion II decreases social welfare given positive price-output response.</u>

But if $e_1 < e_2$, $dU/U_1 \gtrless 0$.

Proposition 3.7: <u>If the elasticity of returns to scale of the exportable industry is smaller than that of the importable industry, trade diversion II may be welfare-improving.</u>

The equations for determining the welfare effects of trade creation I, trade creation II, trade diversion I and trade diversion II are presented in Table 3.1 and the analyses of those equations in Table 3.2 for ease of comparison.

Until now, we have employed a one-country open economy model, based on the small country assumption. In the next chapter, we extend the analysis to a two-country model, where either country's actions are substantial enough to influence the world prices. On the basis of such an extended model, the implications of variable returns to scale are explored in the context of the positive aspects of tariffs.

TABLE 3.1

Welfare Effects of Trade Creation and Trade Diversion under Variable Returns to Scale

Types of Trade Creation & Diversion	Welfare Effects
Trade Creation I ($dp = 0$, $dt < 0$)	$\frac{dU}{U_1} = \left[pp_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{\partial X_2}{\partial p_h}+p^2t\frac{\partial E_2}{\partial p_h}\right]dt$
Trade Diversion I ($dp > 0$, $dt < 0$)	$\frac{dU}{U_1} = \left[pp_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{\partial X_2}{\partial p_h}+p^2t\frac{\partial E_2}{\partial p_h}\right]dt + (1+t)\left[p_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{\partial X_2}{\partial p_h}+pt\frac{\partial E_2}{\partial p_h} - \frac{E_2}{1+t}\right]dp$
Trade Creation II ($dp < 0$, $dt = 0$)	$\frac{dU}{U_1} = (1+t)\left[p_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{\partial X_2}{\partial p_h}+pt\frac{\partial E_2}{\partial p_h} - \frac{E_2}{1+t}\right]dp$
Trade Diversion II ($dp > 0$, $dt = 0$)	$\frac{dU}{U_1} = (1+t)\left[p_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{\partial X_2}{\partial p_h}+pt\frac{\partial E_2}{\partial p_h} - \frac{E_2}{1+t}\right]dp$

TABLE 3.2

Analysis of the Welfare Effects of Trade Creation and Trade Diversion

Types	Returns to Scale	Tariff Effects	Terms of Trade Effects	Total Effects
Trade Creation I	$e_1 \geq e_2$	+	0	+
	$e_1 < e_2$	?	0	?
Trade Diversion I	$e_1 \geq e_2$	+	-	?
	$e_1 < e_2$	?	?	?
Trade Creation II	$e_1 \geq e_2$	0	+	+
	$e_1 < e_2$	0	?	?
Trade Diversion II	$e_1 \geq e_2$	0	-	-
	$e_1 < e_2$	0	?	?

Note: The price-output response of a commodity is assumed to be positive.

CHAPTER IV

VARIABLE RETURNS TO SCALE AND THE THEORY OF NOMINAL TARIFFS

The theory of nominal tariffs shares the same historical root with the theory of gains from trade in that both of them branched out from the classical controversy over free trade and protectionism. In spite of the enormous contributions made toward the theory of tariffs, notably by Mill (1909), Graham (1925) and Marshall (1949), among others, the development of the modern theory is greatly indebted to two pioneering works, one by Stolper-Samuelson (1941) and the other by Metzler (1949).

Stolper-Samuelson, in their 1941 work, have investigated the effects of a tariff on the factor rewards, which combined with the previously developed Heckscher (1919) and Ohlin (1933) theorem, has earned a insurmountable position in the modern theory of nominal tariffs. However, the Stolper-Samuelson theorem is based on rather stringent assumptions, particularly the assumptions of a small country and constant returns to scale. In the small country framework, a country's external terms of trade are unaffected by a tariff and hence the domestic price of the importable commodity is increased by the size of the tariff, which Batra (1973) has called the normative aspects of tariffs.

It was not, however, until the publication of Metzler's 1949 article that the positive aspects of tariffs attracted the attention of trade theorists. In this prominent work, Metzler has examined the effects of a tariff on the factor rewards in the presence of monopoly power of the tariff imposing country. He argued that if the tariff imposing country is large enough to influence the world prices, the domestic price of the importable commodity is influenced by two conflicting forces. The tariff tends to raise it directly while the resultant decrease in domestic import demand tends to lower it indirectly by lowering the international price. If the latter is stronger than the former, the domestic price of the imported good falls as a result of the tariff; this is the so called Metzler paradox.

Recently, Metzler's result has been challenged by Södestern and Vind (1968, 1969) on the ground that Metzler has posed the problem artificially. According to their arguments, if tariff income is spent in the same manner as all other incomes, Metzler's result cannot be produced; a tariff will always turn the terms of trade in favor of a tariff imposing country and increase the domestic price of imports. But Jones (1969) has quickly defended Metzler's result by demonstrating the fallacious nature of their criticism based on his well-established general equilibrium model.

Until recently, however, few efforts have been made to investigate the implication of variable returns to scale in the context of the positive aspects of tariffs. Exceptions are the works by Matthews (1950) and Kemp (1969), in which they have diagrammatically derived the offer curve assuming that industry production functions are subject to identical increasing returns to scale. In addition, Kemp has considered the stability of international equilibrium. It will be shown, however, that the shape of their offer curve is the one of several possible shapes that can be derived under such assumption.

This chapter deals with the positive aspects of tariffs in the presence of variable returns to scale. For analytical purposes, we employ the two-country, two-commodity and two-factor model, which was originally provided by Jones (1969) and later elaborated by Batra (1973), and then extend the model by incorporating variable returns to scale. Under such an extended setting, the effects of tariffs on the terms of trade and domestic price ratios are examined. Additionally, the necessary conditions for the Metzler paradox and the optimum tariff are derived.

I. JONES-BATRA MODEL

For analytical convenience, we follow the mathematical procedures developed by Batra (1973, Chapter 5). It is assumed that the world consists of two countries, the home country and the rest of the world. Both countries produce two commodities, X_1 and X_2, using the factors of production, labor and capital. Either country is large enough to influence the international prices by manipulating its volume of trade.

In addition, the home country exports (imports) the first (second) commodity while the foreign country exports (imports) the second (first) commodity.

$$E_2 = D_2 - X_2 \tag{4.1}$$

$$E_{1f} = D_{1f} - X_{1f} \tag{4.2}$$

where E_{1f}, D_{1f} and X_{1f} are respectively the foreign country's import, total domestic demand and output of the first commodity. Notice that previous notations are preserved unless specified otherwise. Let t_h (t_f) be the ad valorem tariff imposed on the importable commodity by the home (foreign) country and let $T_h=(1+t_h)$ and $T_f=(1+t_f)$.

At a constant rate of tariff, a change in the terms of trade affects the domestic demand for the importables by changing the domestic price ratio, which in turn gives rise to substitution and income effects. In addition, a change in the terms of trade shifts the production point along the production possibility schedule by changing the domestic relative price of the imports. Hence, we can write

$$E_2 = D_2(p_h,Y) - X_2(p_h) \tag{4.3}$$

$$E_{1f} = D_{1f}(p_f,Y_f) - X_{1f}(p_f) \tag{4.4}$$

Where Y (Y_f) is the national income of the home (foreign) country, $p_h=p(1+t_h)$ and $p_f=p/(1+t_f)$ are respectively the tariff inclusive domestic price ratio of the home and foreign countries. The change in real income is of concern. If the change in social welfare is an index of the change in real income,

$$\frac{dU}{U_1} = dY = dD_1 + p_h dD_2 \qquad (4.5)$$

Similarly,

$$dY_f = dD_{1f} + p_f dD_{2f} \qquad (4.6)$$

The tariff income of the home and the foreign countries are given by

$$p_h E_2 - pE_2 = t_h pE_2 = (T_h - 1)pE_2 \qquad (4.7)$$

$$p_{1f}E_{1f} - p_1 E_{1f} = (T_f - 1)p_{1f}E_{1f}/T_f = (T_f - 1)E_{1f}/T_f \qquad (4.8)$$

Note that the last term of the right hand side of (4.8) is expressed in terms of the first commodity. Following Jones (1969) and Södestern and Vind (1968), assume that the governments of the two countries give the tariff revenue back to the private sectors in lump-sum fashion so that the tariff incomes are spent in the same manner as all other incomes.[1] Then the value of consumption in domestic price is equal to the value of production plus the tariff revenue.

$$D_1 + p_h D_2 = X_1 + p_h X_2 + (T_h - 1)pE_2 \qquad (4.9)$$

$$D_{1f} + p_f D_{2f} = X_{1f} + p_f X_{2f} + (T_f - 1)E_{1f}/T_f \qquad (4.10)$$

Assuming the balance of payments is always at equilibrium

$$E_1 = pE_2 \qquad (4.11)$$

$$E_{1f} = pE_{2f} \qquad (4.12)$$

Differentiating (4.3), we obtain

$$dE_2 = (\partial D_2/\partial p_h)dp_h + (\partial D_2/\partial Y)dY - (\partial X_2/\partial p_h)dp_h$$

---

[1]The effects of tariffs on the terms of trade and domestic prices are different depending on how tariff revenue is disposed by government. For related discussions, see Metzler (1949), Södestern and Vind (1968) and Jones (1969).

Dividing by E_2 and after some manipulation, we get

$$E_2^{*}=-e_h p_h^{*}+(m_h/p_h E_2)dY-s_h p_h^{*} \qquad (4.13)$$

where an asterisk denotes the relative rate of change, $e_h=-(p_h/E_2)(\partial D_2/\partial p_h)$ describes the consumption substitution as a result of change in p_h for a given income, $m_h=(\partial D_2/\partial Y)p_h$ the marginal propensity to consume X_2, $s_h=(p_h/E_2)(\partial X_2/\partial p_h)$ the substitution in production in response to a change in p_h. In the absence of inferior goods, $e_h>0$ and $0<m_h<1$.

II. TARIFFS, IMPORT DEMAND AND THE TERMS OF TRADE

The Jones-Batra system consisting of (4.1)-(4.13) is now extended to examine the effects of a tariff on import demand and the terms of trade in the presence of variable returns to scale. To accomplish this, we must first obtain the expression for dY. Differentiating (4.9) and solving for $dD_1+p_h dD_2$, we obtain

$$dY=-E_2dp_h+(T_h-1)pdE_2+(T_h-1)E_2dp+pE_2dT_h+dX_1+p_hdX_2 \qquad (4.14)$$

Under constant returns to scale, the last two terms of the right hand side vanishes from the expression since production equilibrium occurs when $dX_1+p_hdX_2=0$. This standard result is, however, in need of revision under variable returns to scale such that

$$\frac{dX_1}{dX_2} = -\left(\frac{1-e_2}{1-e_1}\right)p_h$$

Hence, we rewrite (4.14)

$$dY=-E_2dp_h+(T_h-1)pdE_2+(T_h-1)E_2dp+pE_2dT_h+\left(\frac{e_2-e_1}{1-e_1}\right)p_hdX_2 \qquad (4.15)$$

Differentiating $p_h = pT_h$ to obtain (4.16) and (4.17)

$$dp_h = T_h dp + p dT_h \qquad (4.16)$$

$$p_h^* = p^* + T_h^* \qquad (4.17)$$

By substituting (4.16) in (4.15), we get the simplified expression for dY

$$dY = -E_2 dp + (T_h - 1) p dE_2 + \left(\frac{e_2 - e_1}{1 - e_1}\right) p_h dX_2 \qquad (4.18)$$

The first term of the right hand side captures the change in national income due to a changed terms of trade, the second term the income effect of a change in import demand via tariff revenue and the last term the returns to scale effect on real income.

By substituting (4.17) and (4.18) in (4.13), we obtain

$$E_2^* = -a_h p^* - A_h T_h^* \qquad (4.19)$$

where a_h is the home country's terms of trade elasticity of import demand and A_h the tariff elasticity of import demand; their expressions are respectively given by[1]

$$a_h = \frac{e_h + s_h\{1 - m_h(e_2 - e_1)/(1 - e_1)\} + m_h/T_h}{1 - m_h t_h/T_h} \qquad (4.20)$$

$$A_h = \frac{e_h + s_h\{1 - m_h(e_2 - e_1)/(1 - e_1)\}}{1 - m_h t_h/T_h} \qquad (4.21)$$

Expression (4.19) furnishes the two factors that affect the demand for imports, the terms of trade and the tariff rate. To be specific, a_h (A_h) is the rate of change in import demand due to a change in the terms of trade (tariff rate) for a

---

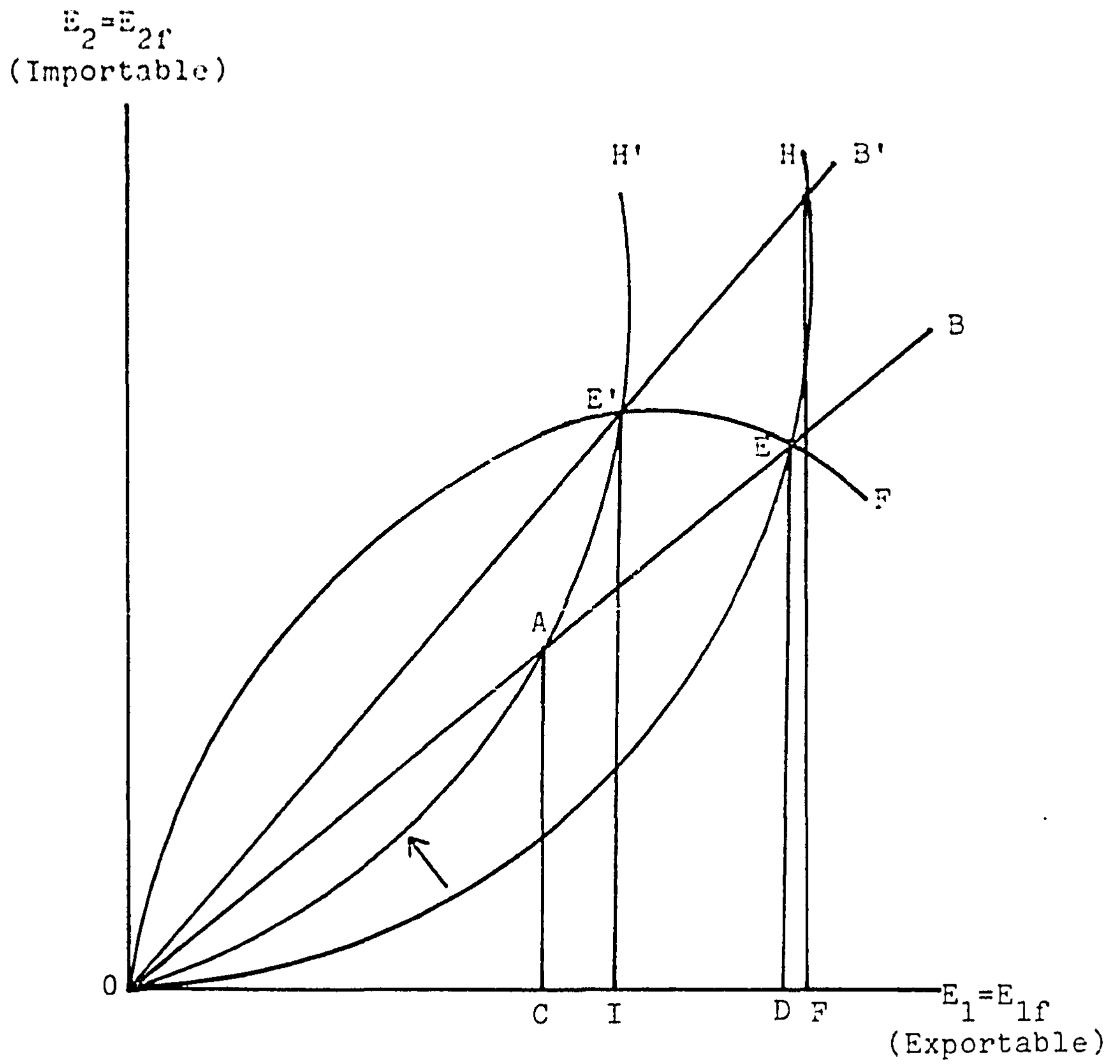
[1]For detailed mathematical derivation, see the Appendix.

given tariff rate (terms of trade), that moves along (shifts) the offer curve. Since $t_h/T_h < 1$, and in the absence of inferior goods $0 < m_h < 1$, the denominator of (4.20) and (4.21) is positive. Hence, the shape of the offer curve is determined by the sign of the numerator of (4.20) and the direction of its shift is decided by the sign of the numerator of (4.21). It is noteworthy that $1-m_h t_h/T_h$, which Jones called "the Keynsian type of multiplier", is unaffected by variable returns to scale. Under variable returns to scale, however, e_i (i=1,2) plays an important role for determining the demand for import, in addition to e_h, s_h and m_h. Suppose that the industries in the economy operates under identical returns to scale ($e_1=e_2$) with constant returns to scale as a special case. Then $m_h(e_2-e_1)/(1-e_1)$ vanishes from (4.20) and (4.21) so that we reach the Jones-Batra result. As in the foregoing analyses, let us confine the analysis to the dynamically stable system, in which the output of a commodity responds positively to an increase in its relative price (Mayer, 1974). Then a_h and A_h are both positive since $s_h > 0$. This implies that an improvement in the terms of trade at a constant tariff increases the demand for imports, i.e. the offer curve is bowed-out toward the axis of the exportable commodity. Furthermore, an increase in the tariff rate for given terms of trade results in the lower import demand, i.e. the offer curve shifts toward the axis of the importable commodity. This standard result is retained if $e_1 > e_2$ since a_h and A_h are still positive.

Proposition 4.1: <u>If the elasticity of returns to scale of the exportable industry is equal to or greater than that of the importable industry, an improvement in the terms of trade (a higher tariff) for a given tariff (terms of trade) increases (decreases) the demand for imports, given the positive price-output response.</u>

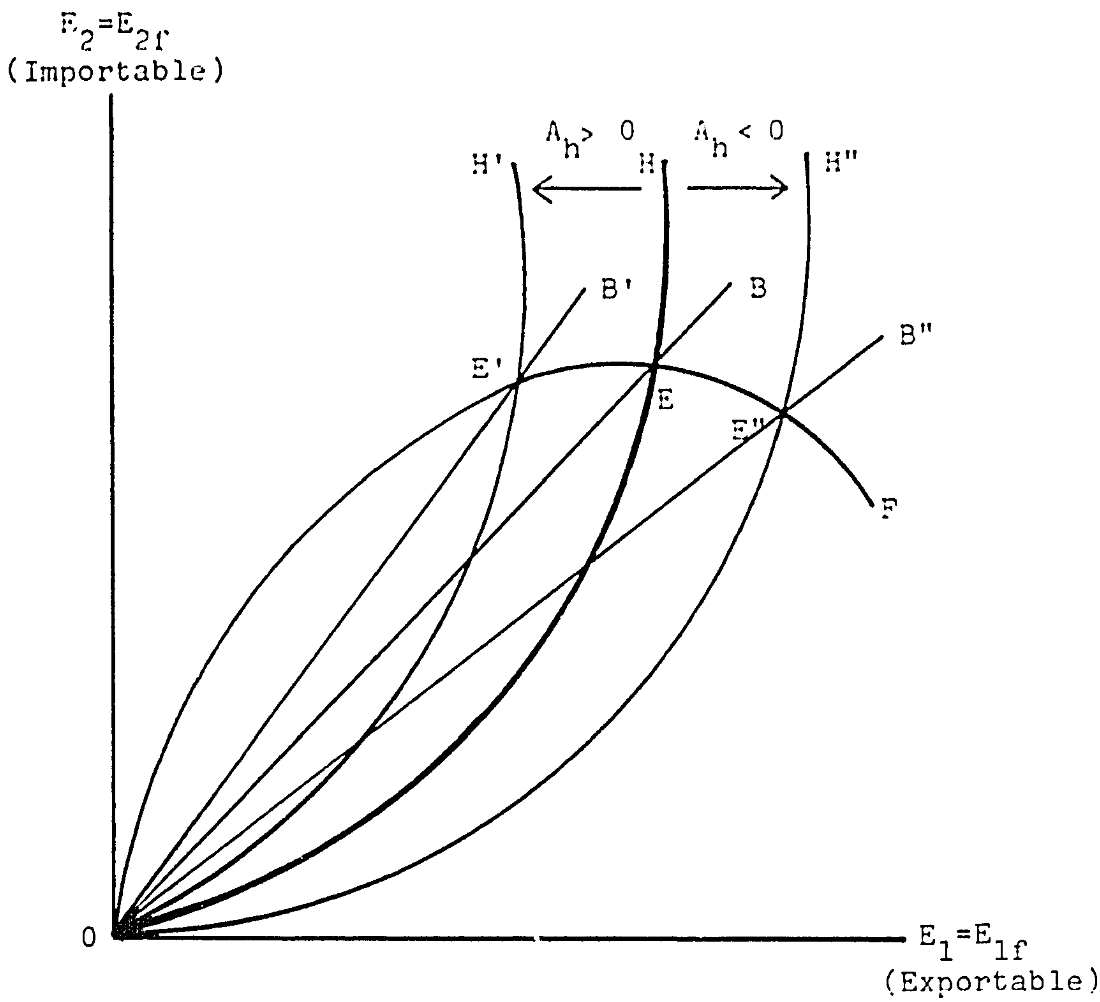
This is geometrically described in Figure 4a. For the terms of trade OB, the home country's import demand is DE. If the terms of trade are improved to OB', the demand for imports is increased to FG provided the terms of trade are constant; the offer curve of the home country is bowed-out to the axis of the exportable commodity. A higher tariff shifts the offer curve from OH to OH', so that the demand for imports falls from DE to CA (FG to IE') for the terms of trade OB (OB').

But if $e_1 < e_2$, the signs of a_h and A_h are not clear-cut. Notice that $a_h > 0$ if $A_h > 0$ but the converse is not true, i.e. $A_h \gtrless 0$ if $a_h < 0$. The reason is that a_h has an additional positive term, m_h/T_h, compared with A_h. This implies that if the elasticity of returns to scale of the exportable industry is smaller than that of the importable industry and the offer curve has a normal shape, an increase in the tariff may or may not shift the offer curve and the direction of the shift is not determinate. This is depicted in Figure 4b. The home offer curves (OH, OH' and OH") have the normal shapes. A higher tariff may shift the original offer curve, OH, to OH' (OH") if $A_h > 0$ ($A_h < 0$), but if $A_h = 0$, it does not shift the offer curve.



$e_1 \geq e_2$

Figure 4a



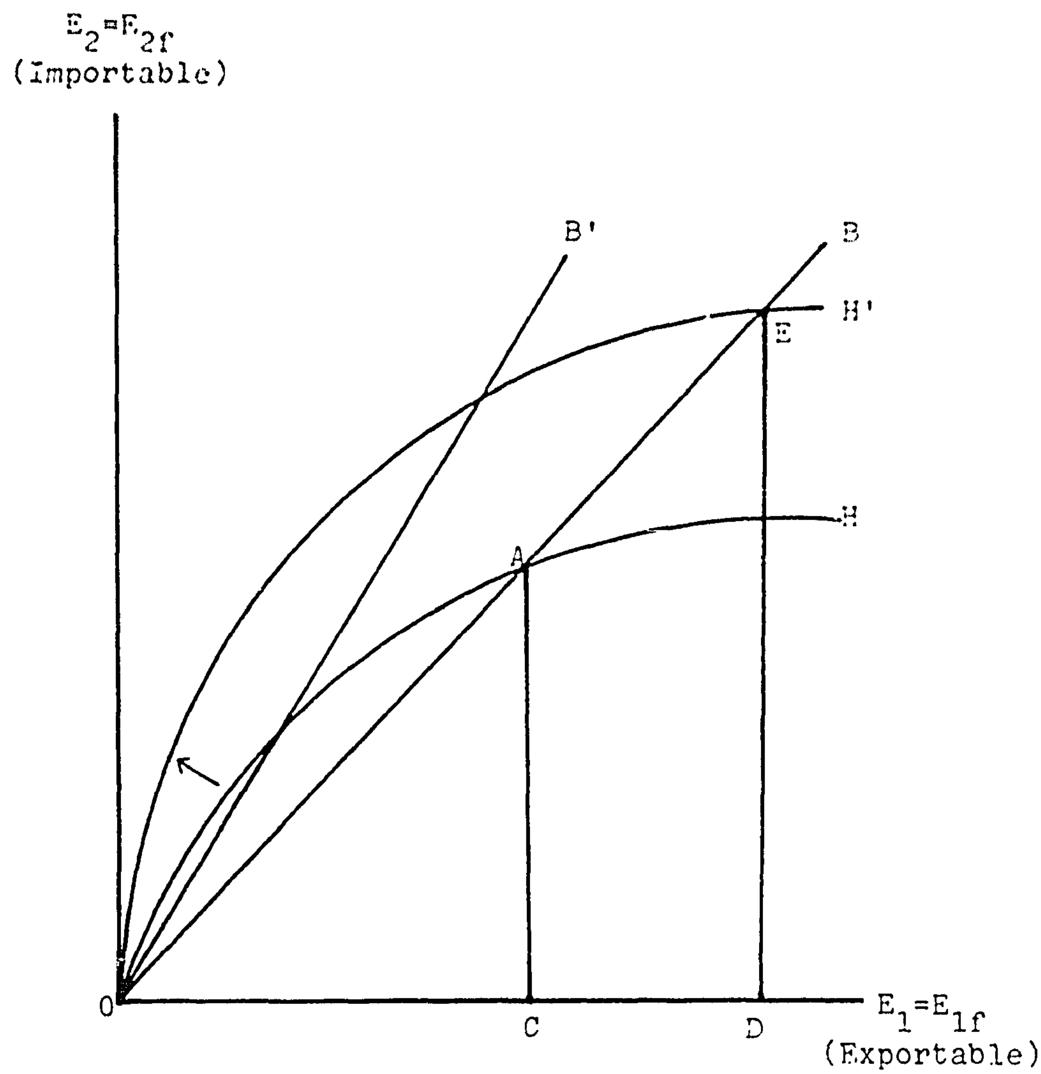
$e_1 < e_2$ and $a_h > 0$

Figure 4b

Other interesting cases that may occur when $e_1 < e_2$ include that a_h and A_h are both negative. It should be noted that $A_h < 0$ if $a_h < 0$ but the converse is not true, i.e. $a_h \lesseqgtr 0$ if $A_h < 0$. This implies that if the offer curve is bowed-out toward the axis of the importable commodity, a higher tariff always shifts the curve toward the axis of the importable commodity. In Figure 4c, the offer curves of the home country, OH and OH' are bowed out toward E_2 axis and a higher tariff shifts the curve from OH to OH'. As a result, the demand for imports increases from AC to ED, given the terms of trade OB. The following proposition symmarizes these results.

Proposition 4.2: If the elasticity of returns to scale of the exportable industry is smaller than that of the importable industry, an improvement in the terms of trade (a higher tariff) for a given tariff (terms of trade) may decrease (increase) the demand for imports.

Matthews (1950) and Kemp (1969, Chapter 8), assuming that identical increasing returns to scale prevail among industries, have diagrammatically derived the offer curve that has a normal shape in the range of incomplete specialization. Using (4.20), however, it can be easily demonstrated that under their assumption the offer cuve may have other shapes. Matthews and Kemp assumed that production possibility curve is strictly convex to the origin as in Figure 1b of Chaper I. Since $e_1 = e_2 > 0$, the price line is tangent to the production possibility curve if the production is of incomplete specialization. As the production possibility frontier is smooth,



$e_1 < e_2$ and $a_h < 0$

Figure 4c

the output of a commodity responds negatively to an increase in its relative price, implying that the system is dynamically unstable in light of the Mayer's stability criterion. For $e_1=e_2$, $m_h(e_2-e_1)/(1-e_1)$ vanishes from (4.20) and hence $a_h \gtreqless 0$ if $|e_h+m_h/T_h| \gtreqless |s_h|$, i.e. the offer curve may be bowed-in, bowed-out or horizontal toward the axis of the exportable commodity.

It is now appropriate to consider the effects of a tariff on the terms of trade. By following the similar procedure, we can derive the expression for the rate of change in foreign import demand, E^*_{1f}.

$$E^*_{1f}= a_f p^* - A_f T^*_f \tag{4.22}$$

where a_f is the foreign country's terms of trade elasticity of import demand and A_f the tariff elasticity of foreign import demand, whose expressions are given by

$$a_f= \frac{e_f+s_f\{1-m_f(e_{2f}-e_{1f})/(1-e_{1f})\}+m_f/T_f}{1-m_f t_f/T_f} \tag{4.23}$$

$$A_f= \frac{e_f+s_f\{1-m_f(e_{2f}-e_{1f})/(1-e_{1f})\}}{1-m_f t_f/T_f} \tag{4.24}$$

Using (4.19),(4.22) and $E^*_{1f}= p^* + E^*_2$, we obtain

$$p^* = \frac{A_f T^*_f - A_h T^*_h}{a_f+a_h-1} \tag{4.25}$$

According to Marshall-Lerner condition, the denominator of (4.25) should be positive to ensure the stability of the foreign exchange market. We have previously shown that $A_h > 0$ if $e_1 \geq e_2$. In this case, an increase in the tariff by the home country ($T_h^* > 0$) for the given foreign tariff ($T_f^* = 0$). results in an improvement in the terms of trade. This is the standard result obtained by Metzler (1949), Jones (1969) and Batra (1973). Thus the following proposition can be stated.

Proposition 4.3: If the elasticity of returns to scale of the exportable industry is equal to or greater than that of the importable industry, an increase in the tariff results in the improvement in the terms of trade of the tariff imposing country in the dynamically stable system.

But if $e_1 < e_2$, the sign of A_h can be either positive or negative.

Proposition 4.4: If the elasticity of returns to scale of the exportable industry is smaller than that of the importable industry, an increase in the tariff may deteriorate the terms of trade of the tatiff imposing country.

This situation is geometrically depicted in Figure 4b, where OH, OH' and OH" are the offer cuves of the home country and OF the foreign offer curve. If $A_h < 0$, the introduction of the tariff by the home country shifts its offer curve from OH to OH", and as a result the terms of trade deteriorate from OB to OB".

III. TARIFFS AND THE DOMESTIC PRICE RATIO

In the previous section, it has been shown that if industry production functions are characterized by divergent returns to scale, an increase in the tariff rate may improve, deteriorate or may not affect the terms of trade of the tariff imposing country. In this section, we are interested in examining the effects of the tariffs on the domestic price ratio in the presence of variable returns to scale. To accomplish this, we need to obtain the expression for p_h^*. Since $p_h^* = p^* + T_h^*$, we derive (4.25) by adding T_h^* to either side of (4.25).

$$p_h^* = \frac{A_f T_f^* + T_h^*(a_f + a_h - 1 - A_h)}{a_f + a_h - 1} \qquad (4.26)$$

For the given foreign tariff ($t_f^*=0$), the tariff elasticity of domestic price ratio is equal to

$$\frac{p_h^*}{T_h^*} = 1 - \frac{A_h}{a_f + a_h - 1} \qquad (4.27)$$

With the foreign market stability ($a_f+a_h-1> 0$), it is clear from (4.25) and (4.26)

$$\frac{p_h^*}{T_h^*} \gtrless 1 \text{ and } \frac{p^*}{T_h^*} \gtrless 0, \text{ if } A_h \lesseqgtr 0 \qquad (4.28)$$

We may now consider the following four possible cases. The first is the case in which the terms of trade are unaffected by the tariff and hence the domestic price ratio of the importable commodity is increased by the tariff rate, $p_h^*/T_h^*=1$ and $p^*/T_h^*=0$. In the small country case, this is undoubtedly true since the foreign elasticity of import demand facing the home country is perfectly elastic, $a_f=\infty$. Hence, the second term of the right hand side of (4.28) vanishes. This may also occur, however, in the large country case if industy production functions are subject to varying returns to scale. The sufficient condition is readily established from (4.21), i.e. $e_h = -s_h\{1-m_h(e_2-e_1)/(1-e_1)\}$. Given positive price-output response, it is clear that this may happen only if $e_1 < e_2$. Second is the case where $p_h^*/T_h^* > 1$ $(p^*/T_h^* > 0)$, that is, the terms of trade are deteriorated as a result of a tariff and hence the demestic price ratio of the importables is increased by more than the tariff rate. From (4.23), it is clear that $p_h^*/T_h^* > 1$ only if $e_h < -s_h\{1-m_h(e_2-e_1)/(1-e_1)\}$. Note that this condition is satisfied only if $e_1 < e_2$ but the converse is not true. Third is the case in which an increase in the tariff rate increases the domestic relative price of the imports but by less than the size of the tariff, $0 < p_h^*/T_h^* < 1$ $(-1 < p^*/T_h^* < 0)$. This occurs when $0 < A_h/(a_f+a_h-1) < 1$. Using (4.21), we obtain the lower boundary condition, $e_h > -s_h\{1-m_h(e_2-e_1)/(1-e_1)\}$. Note that the lower boundary condition is satisfied if $e_1 > e_2$ and may be satisfied if $e_1 < e_2$. To derive the upper boundary condition, we substitute (4.20) and (4.21) in $A_h/(a_f+a_h-1) < 1$ and obtain $a_f+(m_h/T_h)/(1-m_h t_h/T_h) > 1$. If initial free trade is assumed ($t_h = 0$, $T_h = 0$), this expression is reduced to $a_f+m_h > 1$, i.e. the foreign country's terms of trade elasticity of import demand plus the home marginal propensity to consume the importables are greater than the unity. By substituting (4.23) in $a_f+m_h < 1$, we derive the weaker upper boundary condition, $e_f+s_f\{1-m_f(e_{2f}-e_{1f})/(1-e_{1f})\}+m_f+m_h > 1$. Finally, consider the case in which a higher tariff results in the lower domestic price ratio of the importable commodity, $p_h^*/T_h^* < 0$ $(p^*/T_h^* < -1)$.

This is the Metzler paradox and occurs when $A_h/(a_f+a_h-1)>1$. Using the same procedure that we have used to derive the upper boundary condition for the third case, we obtain the necessary condition for the Metzler paradox:

$a_f+(m_h/T_h)/(1-m_h t_h/T_h)< 1$

If initial free trade is assumed, (4.29) becomes $a_h+m_h<1$, that is, the foreign country's terms of trade elasticity of import demand plus the home marginal propensity to consume the importables are less than the unity. Weaker necessary condition can be derived by substituting (4.23) in $a_f+m_h<1$, i.e. $e_f+s_f\{1-m_f(e_{2f}-e_{1f})/(1-e_{1f})\}+m_f+m_h< 1$. It is interesting that the returns to scale of the home industries do not play any role in determining the necessary condition for the Metzler paradox, whereas those of the foreign industries do affect the necessary condition by influencing on the shape of the forign offer curve.

IV. THE OPTIMUM TARIFF

Under constant returns to scale, if the tariff imposing country is large enough to influence the world prices, its welfare is subject to two conflicting forces. A higher tariff tends to raise the welfare by improving the terms of trade; at the same time, it also tends to deteriorate the welfare by increasing the levels of production and consumption distortions, Hence, there is a unique tariff rate that maximizes social welfare, which is given by $1/(a_f-1)$.[1]

---

[1] For discussions on the optimum tariff, see, for example, Jones (1969) and Batra (1973).

Based on the previous analyses, however, we can conjecture that under variable returns to scale many more complications should be involved for deriving the optimum tariff: first, the effect of a tariff on the terms of trade are not certain; second, the effect of a tariff on the production is not clear-cut (see Chapter II).

The necessary condition for the optimum tariff, however, can be derived by setting dY in (4.18) equal to zero. As a preliminary step, Differentiate (4.11) and reexpress it by using (4.19) and (4.22) to obtain (4.29).

$$E_2^* = E_1 - p^* = a_f p^* - A_f T_f^* - p^* = -a_h p^* - A_h T_h^* \tag{4.29}$$

Solving for T_h^*, we obtain

$$T_h^* = -\frac{p^*}{A_h}(a_f + a_h - 1) + \frac{A_f}{A_h} T_f^* \tag{4.30}$$

Substituting (4.22) and (4.30) in (4.18), we derive

$$\begin{aligned} dY &= (T_h - 1)dE_{1f} - T_h E_2 dp + \left(\frac{e_2 - e_1}{1 - e_1}\right) p_h dX_2 \\ &= (T_h - 1)dE_{1f} - T_h E_2 dp + \left(\frac{e_2 - e_1}{1 - e_1}\right) s_h E_{1f} T_h (p^* + T_h^*) \\ &= E_{1f} p^* \left[t_h \left(a_f - 1 + \frac{e_2 - e_1}{1 - e_1} s_h \frac{A_h - a_f - a_h + 1}{A_h}\right) - \left(1 - \frac{e_2 - e_1}{1 - e_1} s_h \frac{A_h - a_f - a_h + 1}{A_h}\right) \right] \\ &\quad - E_{1f}\left(t_h A_f - \frac{e_2 - e_1}{1 - e_1} s_h T_h \frac{A_f}{A_h}\right) T_f^* \end{aligned} \tag{4.31}$$

Using (4.20) and (4.21), we reexpress (4.31)

$$\begin{aligned} dY &= \frac{E_{1f} p^*}{e_h + s_h\{1 - m_h(e_2 - e_1)/(1 - e_1)\}} \left[t_h (a_f - 1)\left(e_h + s_h - \frac{e_2 - e_1}{1 - e_1} s_h\right) \right. \\ &\quad \left. - \left\{e_h + s_h + \frac{e_2 - e_1}{1 - e_1} s_h (a_f - 1)\right\} \right] - E_{1f}\left(t_h A_f - \frac{e_2 - e_1}{1 - e_1} s_h T_h \frac{A_f}{A_h}\right) T_f^* \end{aligned} \tag{4.32}$$

For the given foreign tariff ($t_f^*=0$), (4.32) can be reduced by using (4.21)

$$dY=\frac{E_{1f}p^*}{A_h(1-m_ht_h/T_h)}\left[t_h(a_f-1)(e_h+s_h-\frac{e_2-e_1}{1-e_1}s_h)\right.$$

$$\left.-\{e_h+s_h+\frac{e_2-e_1}{1-e_1}s_h(a_f-1)\}\right] \qquad (4.33)$$

From (4.25), $p^*=0$ if $A_h=0$ given $t_f^*=0$. In this case, dY is indeterminate since $E_{1f}p^*/A_h(1-m_ht_h/T_h)=0/0$. The reason is that a tariff is impotent to the demand for imports and the terms of trade. Hence, the optimum tariff does not exist. But if $A_h\neq 0$, the necessary condition for the optimum tariff can be obtained by setting the terms in the bracket of (4.33) equal to zero.[1]

$$t_o=\frac{1}{a_f-1}+(1+\frac{1}{a_f-1})\frac{(e_2-e_1)s_h}{(1-e_1)e_h+(1-e_2)s_h} \qquad (4.34)$$

The sufficient condition requires $d^2Y<0$ in addition to (4.34) If the industry production functions are subject to identical returns to scale($e_1=e_2=0$) with constant returns to scale as a special case, (4.34) reduces to $1/(a_f-1)$. In the small country case($a_f=\infty$), (4.34) becomes

$$t_o=\frac{(e_2-e_1)s_h}{(1-e_1)e_h+(1-e_2)s_h} \qquad (4.35)$$

It should be noted that (4.35) can be directly derived from (2.13) in Chaper II by equating $(1/U_1)(dU/dt)$ equal to zero.[2]

---

[1]A more detailed mathematical procedure for the optimum tariff is provided in the Appendix.

[2]See the next page for the proof.

If we assume both the small country and the constant returns to scale, $t_o=0$, that is, free trade is the best policy. Under variable returns to scale, not only the foreign country's terms of trade elasticity of import demand (a_f) but also the domestic price elasticity of import demand (e_h), price-output response and the elasticities of returns to scale of the home industries play important role for determining the optimum tariff.

---

[2]The effect of a tariff on social welfare was given by (2.13) in Chapter II.

$$\frac{1}{U_1}\frac{dU}{dt} = pp_h\left(\frac{e_2-e_1}{1-e_1}\right)\frac{dX_2}{dp_h} + p^2 t\frac{dE_2}{dp_h} \qquad (2.13)$$

Using the relationship, $dE_2=dD_2-dX_2$, we reexpress (2.13) as follows:

$$\frac{1}{U_1}\frac{dU}{dt} = p^2\ \frac{e_2-e_1-t(1-e_2)}{1-e_1}\ \frac{dX_2}{dp_h} + t\frac{dD_2}{dp_h} \qquad (1)$$

As defined earlier, $e_h=-(p_h/E_2)(dD_2/dp_h)$ and $s_h=(p_h/E_2)(dX_2/dp_h)$. Using these and after some mathematical manipulation, we obtain

$$\frac{1}{U_1}\frac{dU}{dt} = \frac{p\{(e_2-e_1)s_h-t(1-e_2)s_h-t(1-e_1)e_h\}}{(1+t)(1-e_2)} \qquad (2)$$

By equating $(1/U_1)(dU/dt)$ to zero, we obtain the optimum tariff for the small country.

$$t_o = \frac{(e_2-e_1)s_h}{(1-e_1)e_h+(1-e_2)s_h}$$

CHAPER V

VARIABLE RETURNS TO SCALE
AND THE THEORY OF ECONOMIC EXPANSION

In his 1894 article, Edgeworth suggested for the first time the possibility that an expanding economy might be worse off after growth than before, if the deterioration in the terms of trade outweighs the output gain as a result of growth. The possibility of immiserizing growth was resurrected in the late 1940s and the early 1950s by the problem of persistent balance of trade deficits in the Western European countries, particularly Great Britain. Hicks (1953) has tried to explain 'the dollar problem' in connection with the nature of economic growth in the United States. He argued that technological progress in the United States tended to be import-biased such that productivity increases were concentrated in the import-competing industries. The issue of immiserizing growth has been raised again by the Prebisch (1959) hypothesis, which ascribes the secular deterioration in the terms of trade of the developing countries to the monopolistic elements of the developed Western economies. In addition, many other important contributions have been made to establish the modern theory of economic expansion, notably by Rybczynski (1955), Bhagwati (1958), Johnson (1959) and Findlay and Grubert (1959), among others.

Recently, several trade theorists have investigated the implication of variable returns to scale for some growth-related trade theorems. Jones (1969) has examined the validity of the Rybczynski theorem in the presence of variable returns to scale. Allowing for non-homothetic production functions, he has concluded that the Rybczynski theorem based on the assumption of constant returns to scale does not carry over to the case of variable returns to scale in a straightforward manner, whereby the degrees of externalities and the correspondence between average and marginal factor intensities play a critical role. Later, Mayer (1974) has reconsidered the Rybczynski theorem by introducing a dynamic stability condition and argued that the Rybczynski theorem is valid if the system is dynamically stable. Kemp and Negishi (1970), using the revealed preference arguments, have proposed the sufficient condition under which the improvement in the terms of trade is not harmful in the presence of variable returns to scale. Recently, the same subject has been reconsidered by Eaton and Panagaria (1979) in terms of social utility function.

This chapter explores the implication of economic expansion for the output levels, social welfare and the terms of trade under the assumption of variable returns to scale. As in the conventional analysis, two sources of economic expansion are identified, technical progress and factor accumulation. In Section I, we extend the Jones model by incorporating technical progress. Based on the extended model. Section II examines the effects of economic expansion on the output levels and social welfare of a small country, which is a price taker in the international market. Also, the Prebisch hypothesis (1959) regarding the secular deterioration in the terms of trade of the developing countries is reconsidered in the new dimension. Finally, in Section III, we analyze the effect of economic exapansion on the terms of trade in a large country framework.

I. ASSUMPTIONS AND THE MODEL

Consider an economy in which there are two industries producing commodities, X_1 and X_2, using capital (K) and labor (L). The production function of an individual firm is affected by external economies (diseconomies) that are output generated, but the externalities are internal to industry. The industry production functions satisfying these conditions may be written

$$X_i = g_i(X_i)F_i(K_i,L_i,t_i) = G_i(K_i,L_i,t_i) \quad i=1,2 \qquad (5.1)$$

where t_i denotes the state of technology of the ith industry. F_i is homogeneous of degree one in K_i and L_i and defined on the positive quadrant and the origin. g_i describes the role of externalities and is a positive function defined on $(0,\infty)$. It should be noted that these properties are satisfied if and only if $X_i=G_i(K_i,L_i,t_i)$ is a homothetic function.[1]

[1]For a related discussion, see Herberg and Kemp (1969).

The Extended Jones Model

It is now time to extend the Jones model (1969) to the case allowing technical progress, based on industry production functions as described in (5.1). The previous notations will be preserved unless specified otherwise. With full-employment of factors of production, we can write

$$C_{L1}X_1 + C_{L2}X_2 = L \tag{5.2}$$

$$C_{K1}X_1 + C_{K2}X_2 = K \tag{5.3}$$

where C_{ij} is the quantity of the ith factor used to produce one unit of the jth commodity, i=L,K, j=1 2. For example, $C_{L1} = L_1/X_1$ and $C_{K2}=K_2/X_2$. Under perfect competition, the price of each commodity equals to the unit cost. Hence,

$$C_{L1}w + C_{K1}r = p_1 \tag{5.4}$$

$$C_{L2}w + C_{K2}r = p_2 \tag{5.5}$$

Differentiating (5.2)-(5.5), we obtain

$$\lambda_{L1}X_1^* + \lambda_{L2}X_2^* = L^* - (\lambda_{L1}C_{L1}^* + \lambda_{L2}C_{L2}^*) \tag{5.6}$$

$$\lambda_{K1}X_1^* + \lambda_{K2}X_2^* = K^* - (\lambda_{K1}C_{K1}^* + \lambda_{K2}C_{K2}^*) \tag{5.7}$$

$$\theta_{L1}w^* + \theta_{K1}r^* = p_1^* - (\theta_{L1}C_{L1}^* + \theta_{K1}C_{K1}^*) \tag{5.8}$$

$$\theta_{L2}w^* + \theta_{K2}r^* = p_2^* - (\theta_{L2}C_{L2}^* + \theta_{K2}C_{K2}^*) \tag{5.9}$$

where an asterisk indicates the rate of change. λ_{ij} is the proportion of the ith factor employed in the jth industry whereas θ_{ij} is the share of the ith factor in the total value of the jth commodity (i=L,K, j=1,2). For example, $\lambda_{L1}=L_1/L$ and $\theta_{L1}=wL_1/p_1X_1$. By definition, we can derive the following relationships:

$$\lambda_{i1} + \lambda_{i2} = 1 \qquad i=L,K \qquad (5.10)$$

$$\theta_{Lj} + \theta_{Kj} = 1 \qquad j=1,2 \qquad (5.11)$$

Let $[\lambda]$ and $[\theta]$ be the matrices of λ and θ coefficients, that is,

$$[\lambda] = \begin{bmatrix} \lambda_{L1} & \lambda_{L2} \\ \lambda_{K1} & \lambda_{K2} \end{bmatrix} \qquad [\theta] = \begin{bmatrix} \theta_{L1} & \theta_{K1} \\ \theta_{L2} & \theta_{K2} \end{bmatrix}$$

The determinants of $[\lambda]$ and $[\theta]$ are

$$|\lambda| = \lambda_{L1}\lambda_{K2} - \lambda_{K1}\lambda_{L2} = \lambda_{L1} - \lambda_{K1} = \lambda_{K2} - \lambda_{L2} = L_1L_2(k_2-k_1)/LK$$

$$|\theta| = \theta_{L1}\theta_{K2} - \theta_{K1}\theta_{L2} = \theta_{L1} - \theta_{L2} = \theta_{K2} - \theta_{K1} = \frac{wrL_1L_2(k_2-k_1)}{P_1X_1P_2X_2} \qquad (5.12)$$

If $k_2 \gtrless k_1$, $|\lambda|$ and $|\theta|$ are both positive (negative). Note that $|\lambda|$ and $|\theta|$ always have the same sign so that $|\lambda||\theta| > 0$.

In the presence of technical progress and external economies (output-generated), each input-output coefficient is a function of wage-rental ratio (ω), technical improvement (t_j) and output levels; that is,

$$C_{ij} = C_{ij}(\omega, t_j, X_j) \qquad i=L,K,\ j=1,2 \qquad (5.13)$$

Total differentiation of (5.13) yields

$$C^*_{ij} = A^*_{ij} - B^*_{ij} - R_{ij}X^*_j \qquad i=L,K,\ j=1,2 \qquad (5.14)$$

where $A^*_{ij}=(1/C_{ij})(\partial C_{ij}/\partial\omega)$ is the change in input-output coefficient due to a change in wage-rental ratio, $B^*_{ij}=(-1/C_{ij})\cdot(\partial C_{ij}/\partial t_j)$ is the change in C_{ij} due to technical progress in the jth industry and $R_{ij}=-(X_j/C_{ij})(\partial C_{ij}/\partial X_j)$ is the change in C_{ij} due to a change in the output of the jth industry.

Define $R_j = \theta_{Lj}R_{Lj} + \theta_{Kj}R_{Kj}$ $(j=1,2)$, which measures the degree of external economies or diseconomies that influence the firms in industry j. $R_j \geq 0$ for increasing (constant) returns to scale and $R_j < 0$ for decreasing returns to scale. For homothetic production functions, $R_j = R_{Lj} = R_{Kj}$ $(j=1,2)$.[1] Note that industry production functions, $X_j = G_j(K_j, L_j, t_j)$ for $j=1,2$, possess such a property since they are homothetic.

The elasticity of factor substitution may be defined as

$$\sigma_j = \frac{A^*_{Kj} - A^*_{Lj}}{w^* - r^*} \qquad j=1,2 \tag{5.15}$$

For given factor prices and level of externalities, the unit cost of the jth commodity is minimized if

$$wC_{Lj}A^*_{Lj} + rC_{Kj}A^*_{Kj} = 0 \qquad j=1,2 \tag{5.16}$$

Dividing by p_j, we obtain

$$\theta_{L1}A^*_{L1} + \theta_{K1}A^*_{K1} = 0 \tag{5.16}$$

$$\theta_{L2}A^*_{L2} + \theta_{K2}A^*_{K2} = 0 \tag{5.17}$$

We can solve (5.16) and (5.17) by using (5.11) and (5.15) to obatin

$$A^*_{Lj} = -\theta_{Kj}\sigma_j(w^* - r^*) \qquad i=1,2 \tag{5.18}$$

$$A^*_{Kj} = \theta_{Lj}\sigma_j(w^* - r^*) \qquad i=1,2 \tag{5.19}$$

Substituting (5.18) and (5.19) in (5.14), we get

$$C^*_{L1} = -\theta_{K1}\sigma_1(w^* - r^*) - B^*_{L1} - R_1X^*_1 \tag{5.19}$$

---

[1]For a related discussion, see Mayer (1974).

$$C^*_{L2} = -\theta_{K2}\sigma_2(w^* - r^*) - B^*_{L2} - R_2 X^*_2 \qquad (5.21)$$

$$C^*_{K1} = \theta_{L1}\sigma_1(w^* - r^*) - B^*_{K1} - R_1 X^*_1 \qquad (5.22)$$

$$C^*_{K2} = \theta_{L2}\sigma_2(w^* - r^*) - B^*_{K2} - R_2 X^*_2 \qquad (5.23)$$

Substitution of (5.20)-(5.23) in (5.6)-(5.9) yields

$$\lambda'_{L1}X^*_1 + \lambda'_{L2}X^*_2 = L^* + H_L + \delta_L(w^* - r^*) \qquad (5.24)$$

$$\lambda'_{K1}X^*_1 + \lambda'_{K2}X^*_2 = K^* + H_K - \delta_K(w^* - r^*) \qquad (5.25)$$

$$\theta_{L1}w^* + \theta_{K1}r^* = p^*_1 + H_1 + R_1 X^*_1 \qquad (5.26)$$

$$\theta_{L2}w^* + \theta_{K2}r^* = p^*_2 + H_2 + R_2 X^*_2 \qquad (5.27)$$

where

$$\lambda'_{ij} = \lambda_{ij}(1 - R_{ij}) \qquad i = L, K, \; j = 1, 2$$

$$H_L = \lambda_{L1}B^*_{L1} + \lambda_{L2}B^*_{L2}$$

$$H_K = \lambda_{K1}B^*_{K1} + \lambda_{K2}B^*_{K2}$$

$$H_1 = \theta_{L1}B^*_{L1} + \theta_{K1}B^*_{K1}$$

$$H_2 = \theta_{L2}B^*_{L2} + \theta_{K2}B^*_{K2}$$

$$\delta_L = \lambda_{L1}\theta_{K1}\sigma_1 + \lambda_{L2}\theta_{K2}\sigma_2$$

$$\delta_K = \lambda_{K1}\theta_{L1}\sigma_1 + \lambda_{K2}\theta_{L2}\sigma_2$$

H_i represents the reduction in the requirement of the ith factor as a result of technical improvement in both industries, H_j the reduction in the capital and labor costs of producing one unit of the jth commodity consequent upon the technical advance in the jth industry, δ_i the change in the use of the ith factor per unit of output that occurs in both industries due to the change in wage-rental ratio.

Let $[\lambda']$ denote the matrix of λ' coefficients

$$[\lambda'] = \begin{bmatrix} \lambda'_{L1} & \lambda'_{L2} \\ \lambda'_{K1} & \lambda'_{K2} \end{bmatrix} = \begin{bmatrix} \lambda_{L1}(1-R_1) & \lambda_{L2}(1-R_2) \\ \lambda_{K1}(1-R_1) & \lambda_{K2}(1-R_2) \end{bmatrix}$$

Jones has assumed that at constant factor prices the expansion of any industry results in an increased demand for each factor of production. This implies that each λ'_{ij} is positive and $R_{ij}=R_j< 1$. If production functions are homothetic, $|\lambda'|$ and $|\lambda|$ always have the same sign since

$$|\lambda'|=(1-R_1)(1-R_2)|\lambda|=(1-R_1)(1-R_2)L_1L_2(k_2-k_1)/LK \qquad (5.28)$$

Therefore,

$$\text{sign } |\lambda'||\lambda|=\text{sign } |\lambda'||\theta|=\text{sign } |\lambda||\theta| > 0 \qquad (5.29)$$

With factors of production constant ($L^*=0$, $K^*=0$), solve (5.24) and (5.25) to obtain

$$X_1^* = \frac{\lambda'_{L2}H_L - \lambda'_{L2}H_K + u_1(w^*-r^*)}{|\lambda'|} \qquad (5.30)$$

$$X_2^* = \frac{\lambda'_{L1}H_K - \lambda'_{K1}H_L + u_2(w^*-r^*)}{|\lambda'|} \qquad (5.31)$$

where

$$u_1 = \lambda'_{K2}\delta_L + \lambda'_{L2}\delta_K$$

$$u_2 = \lambda'_{K1}\delta_L + \lambda'_{L1}\delta_K$$

u_j ($j=1,2$) is the percentage change in the output of the jth commodity as a result of a change in wage-rental ratio when factor endowments and state of tecnology are constant.

Subtracting (5.27) from (5.26), we obtain

$$w^* - r^* = \frac{1}{|\theta|}(p_1^*-p_2^*)+\frac{1}{|\theta|}(H_1-H_2)+\frac{1}{|\theta|}(R_1X_1^*-R_2X_2^*) \qquad (5.32)$$

Substitution of (5.32) in (5.24) and (5.25) yields

$$\lambda''_{L1}X_1^* + \lambda''_{L2}X_2^* = L^* + H_L + \frac{\delta_L}{|\theta|}(p_1^* - p_2^*) + \frac{\delta_L}{|\theta|}(H_1 - H_2) \tag{5.33}$$

$$\lambda''_{K1}X_1^* + \lambda''_{K2}X_2^* = K^* + H_K + \frac{\delta_K}{|\theta|}(p_1^* - p_2^*) - \frac{\delta_K}{|\theta|}(H_1 - H_2) \tag{5.34}$$

where

$$[\lambda''] = \begin{bmatrix} \lambda'_{L1} - \frac{R_1\delta_L}{|\theta|} & \lambda'_{L2} + \frac{R_2\delta_L}{|\theta|} \\ \lambda'_{K1} + \frac{R_1\delta_K}{|\theta|} & \lambda'_{K2} - \frac{R_2\delta_K}{|\theta|} \end{bmatrix}$$

At this point, Jones has made another assumption that at constant commodity prices the expansion of any industry increases the demand for each factor of production. This implies that each λ''_{ij} is positive. Solving the determinant of $[\lambda'']$, we obtain

$$|\lambda''| = \alpha|\lambda'| = \alpha(1-R_1)(1-R_2)|\lambda|$$

where

$$\alpha = 1 - \left(\frac{R_1u_1 + R_2u_2}{|\lambda'||\theta|}\right) \tag{5.35}$$

It has been shown by several authors that α is an instrumental factor for determining whether the conventional theorems based on the assumption of constant returns to scale carry over to the case of variable returns to scale (Jones, 1969; Kemp, 1969). Now, we can consider the following two cases:

Case 1: The industry production functions of the two industries exhibit decreasing or constant returns to scale, i.e. $R_j \geq 0$, $j=1,2$. Then it is obvious $\alpha > 0$. Hence,

$$\text{sign}|\lambda''||\lambda| = \text{sign}|\lambda''||\lambda| = \text{sign}|\lambda''||\lambda| > 0$$

Case 2: At least one of the two production functions exhibits increasing returns to scale.

In this case, α can be either positive or negative. Thus,

$$\text{sign}|\lambda''||\lambda|=\text{sign}|\lambda''||\lambda|=\text{sign}|\lambda''||\theta| \gtreqless 0$$

Mayer (1974), allowing for non-homothetic production functions, has demonstrated that if the system is dynamically stable $\beta=|\lambda''||\theta|=\alpha|\lambda''||\theta|> 0$. Since $|\lambda'||\theta|>0$ under homothetic production functions, this implies, $\alpha>0$ if the system is stable.

II. THE EFFECTS OF ECONOMIC EXPANSION ON THE SMALL ECONOMY

The analysis of the previous section provides us with the necessary tool kits to examine the effects of economic expansion on the output levels and the welfare of a small country. The main sources of economic expansion are technical progress and factor accumumation. We begin with technical progress.

Technical Progress

To accomplish this, the effects of technical advance on the outputs of the two commodities must be first determined. Suppose that technical progress occurs only in the first industry, i.e. $B^*_{i2}=0$ for i=L,K. Under small country assumption, $p^*_1=p^*_2=0$. If the endowments of the two factors remain constant ($L^*=K^*=0$), (5.33) and (5.34) reduces to

$$\lambda''_{L1}X^*_1+\lambda''_{L2}X^*_2 = H_L+\frac{\delta_L H_1}{|\theta|} \qquad (5.36)$$

$$\lambda''_{K1}X_1^* + \lambda''_{K2}X_2^* = H_K - \frac{\delta_K H_1}{|\theta|} \tag{5.37}$$

Solving for X_1^* and X_2^*, we obtain

$$X_1^* = \frac{(\lambda''_{K2}H_L - \lambda''_{L2}H_K)|\theta| + H_1(\lambda''_{K2}\delta_L + \lambda''_{L2}\delta_K)}{|\lambda''||\theta|} \tag{5.38}$$

$$X_2^* = \frac{(\lambda''_{L1}H_K - \lambda''_{K1}H_L)|\theta| - H_1(\lambda''_{L1}\delta_K + \lambda_{K1}\delta_L)}{|\lambda''||\theta|} \tag{5.39}$$

In the absence of technical improvement in the second industry, $H_L = \lambda_{L1}B_{L1}^*$ and $H_K = \lambda_{K1}B_{K1}^*$. Hence, (5.37) and (5.38) become

$$X_1^* = \frac{(\lambda''_{K2}\lambda_{L1}B_{L1}^* - \lambda''_{L2}\lambda_{K1}B_{K1}^*)|\theta| + H_1(\lambda''_{K2}\delta_L + \lambda''_{L2}\delta_K)}{|\lambda''||\theta|} \tag{5.40}$$

$$X_2^* = \frac{(\lambda''_{L1}\lambda_{K1}B_{K1}^* - \lambda''_{K1}\lambda_{L1}B_{L1}^*)|\theta| - H_1(\lambda''_{L1}\delta_K + \lambda''_{K1}\delta_L)}{|\lambda''||\theta|} \tag{5.41}$$

We now consider three different types of technical improvement; Hicks neutral, intensive-factor-saving and intensive-factor-using technical progresses.

i. Hicks Neutral Technical Progress: In the case of Hicks neutral technical progress, $B_{L1}^* = B_{K1}^*$. Utilize this and rearrange (5.40) and (5.41) to obtain

$$X_1^* = \frac{B_{L1}^*}{(1-R_1)} + \frac{\{B_{L1}^*R_1 + H_1(1-R_1)\}(\lambda''_{L2}\delta_K + \lambda''_{K2}\delta_L)}{(1-R_1)|\lambda''||\theta|} \tag{5.42}$$

$$X_2^* = \frac{-\{B_{L1}^*R_1 + H_1(1-R_1)\}(\lambda''_{K1}\delta_L + \lambda''_{L1}\delta_K)}{(1-R_1)|\lambda''||\theta|} \tag{5.43}$$

If the system is stable, $\alpha > 0$ and hence the denominators of (5.42) and (5.43) are positive. It is obvious then that $X_1^* > 0$ and $X_2^* < 0$ if $R_1 \geq 0$. This is the standard result obtained by many authors including Corden (1956), Bhagwati (1959) and Batra (1973). But if $R_1 < 0$, $X_1^* \gtrless 0$ and $X_2^* \lesseqgtr 0$, that is, the standard result based on the assumption of constant returns to scale breaks down. It is easy to show that $X_1^* \gtrless 0$ and $X_2^* > 0$ if the system is unstable ($\alpha < 0$) and $R_1 \geq 0$. But if $\alpha < 0$ and $R_1 < 0$, $X_1^* \gtrless 0$ and $X_2^* \lessgtr 0$. Thus, we conclude that neutral technical progress is ultra-biased if it occurs in increasing or constant returns to scale industry in a stable system.

ii. Intensive-Factor-Saving Technical Progress:

Suppose that X_1 and X_2 are respectively the capital and the labor intensive commodities ($|\theta| < 0$). If the tecnical improvement of the first industry is capital saving type, $B_{K1}^* > B_{L1}^*$ since it tends to save more capital than labor. Utilizing the relationships, $\lambda'_{ij} = (1-R_j)\lambda_{ij}$ and $\lambda''_{ij} = \lambda'_{ij} \pm (R_j\delta_i/|\theta|)$ for i=L,K and j=1,2, (5.40) and (5.41) can be reexpressed as

$$X_1^* = \frac{(\lambda''_{L1}\lambda''_{K2}B_{L1}^* - \lambda''_{K1}\lambda''_{L2}B_{K1}^*)|\theta| + R_1(\lambda''_{L2}B_{K1}^*\delta_K + \lambda''_{K2}B_{L1}^*\delta_L)}{(1-R_1)|\lambda''||\theta|}$$

$$\frac{+H_1(1-R_1)(\lambda''_{L2}\delta_K + \lambda''_{K2}\delta_L)}{(1-R_1)|\lambda''||\theta|} \tag{5.44}$$

$$X_2^* = \frac{\lambda''_{L1}\lambda''_{K2}(B_{K1}^* - B_{L1}^*)|\theta| - R_1(\lambda''_{K1}B_{L1}^*\delta_L + \lambda''_{L1}B_{K1}^*\delta_K)}{(1-R_1)|\lambda''||\theta|}$$

$$\frac{-H_1(1-R_1)(\lambda''_{K1}\delta_L + \lambda''_{L1}\delta_K)}{(1-R_1)|\lambda''||\theta|} \tag{5.45}$$

In the dynamically stable system, the denominators of (5.44) and (5.45) are positive (Note that $|\lambda''|$ and $|\theta|$ have the same sign if $\alpha > 0$). The first terms of (5.44) and (5.45) may be considered to be additional in comparison with (5.42) and (5.43).

It is evident from (5.44) that with $R_1 \geq 0$, the sum of the second and the third terms is positive. But the first term needs a careful examination. Since $|\lambda''|<0$ $(\lambda''_{L1}\lambda''_{K2}-\lambda''_{K1}\lambda''_{L2}< 0)$ and $B^*_{K1}>B^*_{L1}$, $\lambda''_{L1}\lambda''_{K2}B^*_{L1}-\lambda''_{K1}\lambda''_{L2}B^*_{K1}< 0$. Hence, with $|\theta|<0$, the first term of (5.44) is positive; that is, the increase in the output of X_1 is greater than that in neutral improvement. With $R_1 \geq 0$, it is clear that $X^*_2< 0$. Furthermore, the output of X_2 declines more than in neutral technical progress due to the negative first term. If the production function of the first industry exhibits decreasing returns to scale, $R_1< 0$, the signs of X^*_1 and X^*_2 are both ambiguous. As in the neutral technical progress, $X^*_1 \gtrless 0$ and $X^*_2> 0$ if $\alpha< 0$ and $R_1 \geq 0$. But if $\alpha< 0$ and $R_1< 0$, $X^*_1 \gtrless 0$ and $X^*_2 \gtrless 0$.

iii. Intensive-Factor-Using Technical Progress: If the technical improvement in the first industry is intensive factor using, $B^*_{L1}>B^*_{K1}$, the signs of X^*_1 and X^*_2 are not categorical. With $B^*_{L1}>B^*_{K1}$ and $|\theta|< 0$, the first term of (5.44) may be either positive or negative. Also with $B^*_{L1}>B^*_{K1}$ and $|\theta|< 0$, the first term of (5.45) is positve (negative) but the third term is negative (positive) if $\alpha>0$ $(\alpha<0)$ while the sign of the second term depends on the sign of R_1. Hence, the signs of X^*_1 and X^*_2 are not clear regardless of the signs of α and R_1.

Based on the above results, it can be concluded that under variable returns to scale the effects of techncial progress on the output of the commodities depend on the following three factors: 1) type of technical progress, 2) system stability and 3) returns to scale of the industry in which technical improvement occurs. The effects of technical progress on the outputs are summarized in Table 5.1.

Previous analysis provides enough informations to investigate the impact of technical progress on the welfare of a small country. The traditional notion of immiserizing growth is that a country might be worse-off after growth than before if the deterioration in the terms of trade outweighs the output gain as a result of growth. If the terms of trade are constant, however, the change in real income is the same as the change in output. Is it still possible that the real income decreases as a result of economic expansion so that immiserizing growth occurs? This question may be answered in the context of variable returns to scale and technical improvement. As in the previous analysis, assume that technical progress takes place in the first industry only. Then, industry production functions, (5.1), may be written as

$$X_1 = g_1(X_1)F_1(K_1, L_1, t_1) \tag{5.46}$$

$$X_2 = g_2(X_2)F_2(K_2, L_2) \tag{5.47}$$

Totally differentiating (5.46) and (5.47) and after a simple mathematical manipulation, we obtain

$$\frac{dX_1}{dX_2} = \frac{(1-e_2)g_1(F_{K1}dK_1 + F_{L1}dL_1 + F_{t1}dt_1)}{(1-e_1)g_2(F_{K2}dK_2 + F_{L2}dL_2)} \tag{5.48}$$

Since each factor is paid the value of its marginal product to the firm not the value of its marginal product to the industry, we can write (see Chapter I)

TABLE 5.1

THE EFFECTS OF TECHNICAL PROGRESS ON THE OUTPUTS
UNDER VARIABLE RETURNS TO SCALE

Type of Technical Progress	System / Returns to Scale / Output	Stable ($\alpha > 0$)		Unstable ($\alpha < 0$)	
		Increasing or Constant ($R_1 \geq 0$)	Decreasing ($R_1 < 0$)	Increasing or Constant ($R_1 \geq 0$)	Decreasing ($R_1 < 0$)
Neutral	X_1^*	+	?	?	?
	X_2^*	−	?	+	?
Intensive Factor Saving	X_1^*	+	?	?	?
	X_2^*	−	?	+	?
Intensive Factor Using	X_1^*	?	?	?	?
	X_2^*	?	?	?	?

Note: Technical progress is assumed to take place in the first industry only.

$$w = g_1F_{L1} = pg_2F_{L2} \tag{5.49}$$

$$r = g_1F_{K1} = pg_2F_{K2} \tag{5.50}$$

Substituting (5.49) and (5.50) in (5.48), we get

$$\frac{dX_1}{dX_2} = -\beta p \quad \text{where } \beta=\left(\frac{rdK_1+wdL_1+g_1Ft_1dt_1}{rdK_1+wdL_1}\right)\left(\frac{1-e_2}{1-e_1}\right) > 0 \tag{5.51}$$

Note that $g_1Ft_1dt_1$ represents the shift factor, which appears due to technical progress. It is clear that $\beta > 1$ if $e_1 \geq e_2$ and $\beta \gtrless 1$ if $e_1 < e_2$. Let U be a quasi-concave social utility function which is dependent on the consumption of the two commodities, with their demand indicated by D_1 and D_2,

$$U = U(D_1,D_2) \tag{5.52}$$

where $U_i > 0$ and $U_{ii} < 0$, $i=1,2$. An economy's budget constraint stipulates that the value of production is matched by the value of consumption in foreign prices.

$$X_1 + pX_2 = D_1 + pD_2 \tag{5.53}$$

Differentiating (5.52) with respect to t_1, the state of technology of the first industry, we obtain

$$\frac{dU}{dt_1} = U_1\left(\frac{dD_1}{dt_1} + \frac{U_2}{U_1}\frac{dD_2}{dt_1}\right) \tag{5.54}$$

To maximize utility, consumers equate the marginal rate of substitution to the relative price of the commodities ($U_2/U_1 = p_2/p_1 = p$). Hence,

$$\frac{1}{U_1}\frac{dU}{dt_1} = \frac{dD_1}{dt_1} + p\frac{dD_2}{dt_1} \tag{5.55}$$

Differentiating (5.53) with respect to t_1 and substituting in (5.55), we get

$$\frac{1}{U_1}\frac{dU}{dt_1} = \frac{dX_1}{dt_1} + p\frac{dX_2}{dt_1} \tag{5.56}$$

Substitution of (5.51) in (5.56) yields the final expression,

$$\frac{1}{U_1}\frac{dU}{dt_1} = \left(\frac{\beta - 1}{\beta}\right)\frac{dX_1}{dt_1} \qquad (5.57)$$

Equation (5.57) furnishes the key expression that indicates the welfare change as a consequence of technical progress in the first industry. It is obvious that normal growth occurs if $(\beta-1)/\beta$ and dX_1/dt_1 have the same sign. Immiserizing growth occurs when the two terms are of opposite signs, i.e. $(\beta-1)/\beta \gtrless 1$ and $dX_1/dt_1 \lessgtr 0$. Figures 5a-5c geometrically depict the results obtained above. Figure 5a describes normal growth. TT and TT' represent the pre-growth and post-growth production possibility schedules. If $\beta > 1$ ($e_1 > e_2$), the price lines, FP and FP', are flatter than the slopes of production possibility curves at the production points. p_0, c_0 and U_0 indicate the pre-growth production point, the consumption point and the level of social welfare. Technical progress in the first industry shifts the production possibility curve from TT to TT' and increases the output of X_1. After growth, production occurs at p_1, consumption at c_1 and the social welfare is U_1. Since $U_1 > U_0$, welfare increases as a result of growth.

Figure 5b presents a case of immiserizing growth. The pre-growth and post-growth production possibility curves are given by TT and TT'. If $\beta<1$ ($e_1 < e_2$), price lines are steeper than the slopes of TT and TT' at the production points. The pre-growth production and consumption points are respectively given by p_0 and c_0 and social welfare by U_0. Technical progress in the first industry increases the output of X_1. After growth, production takes place at p_1, consumption at c_1 and the social welfare is U_1. Immiserizing growth occurs since $U_1 < U_0$.

Another case of immiserizing growth is depicted in Figure 5c, namely, a perverse output response to technical progress. All notations are the same with those in Figures 5a and 5b, but $\beta > 1$ ($e_1 > e_2$) and $dX_1/dt_1 < 0$. Post-growth welfare (U_1) is lower than pre-growth welfare (U_0). Therefore, immiserizing growth occurs.

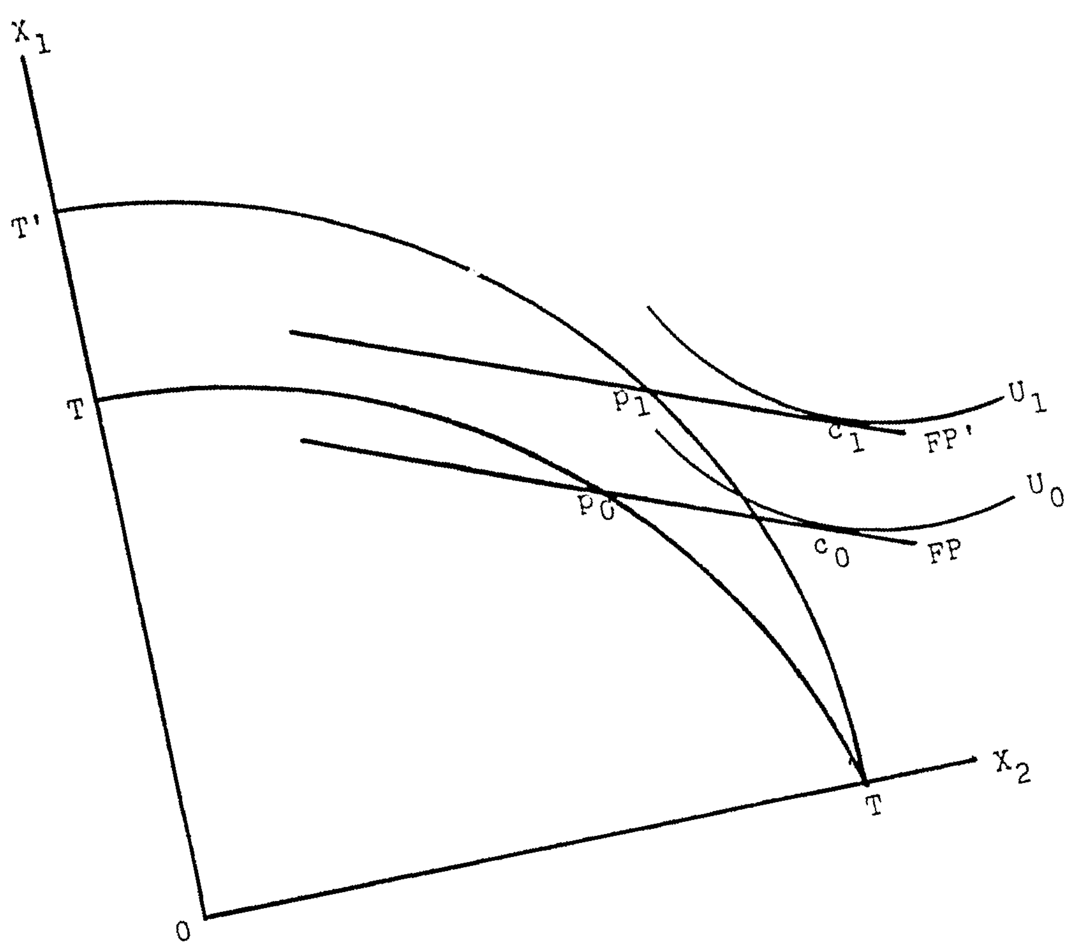
The outputs responses to technical progress in Table 5.1 can be utilized for the detailed treatment of the welfare consequences of technical improvement. Equation (5.57) combined with Table 5.1 presents Table 5.2. Due to the variety of outcomes, verbal explanations will not be provided. Table 5.2, however, explicitly shows that the welfare consequences of technical progress are dependent on the following factors: 1) system stability, 2) type of technical improvement, 3) returns to scale of the industry in which technical change takes place, 4) the difference in the returns to scale between industries and 5) shifting factor of the transformation curve.

Factor Accumulation

The Rybczynski theorem states that if one of the factors increases while the other is constant, the output of the commodity using the increased factors intensively increases and that of the other decreases, provided that commodity and factor prices remain constant.

Jones (1969) has analyzed the validity of the theorem

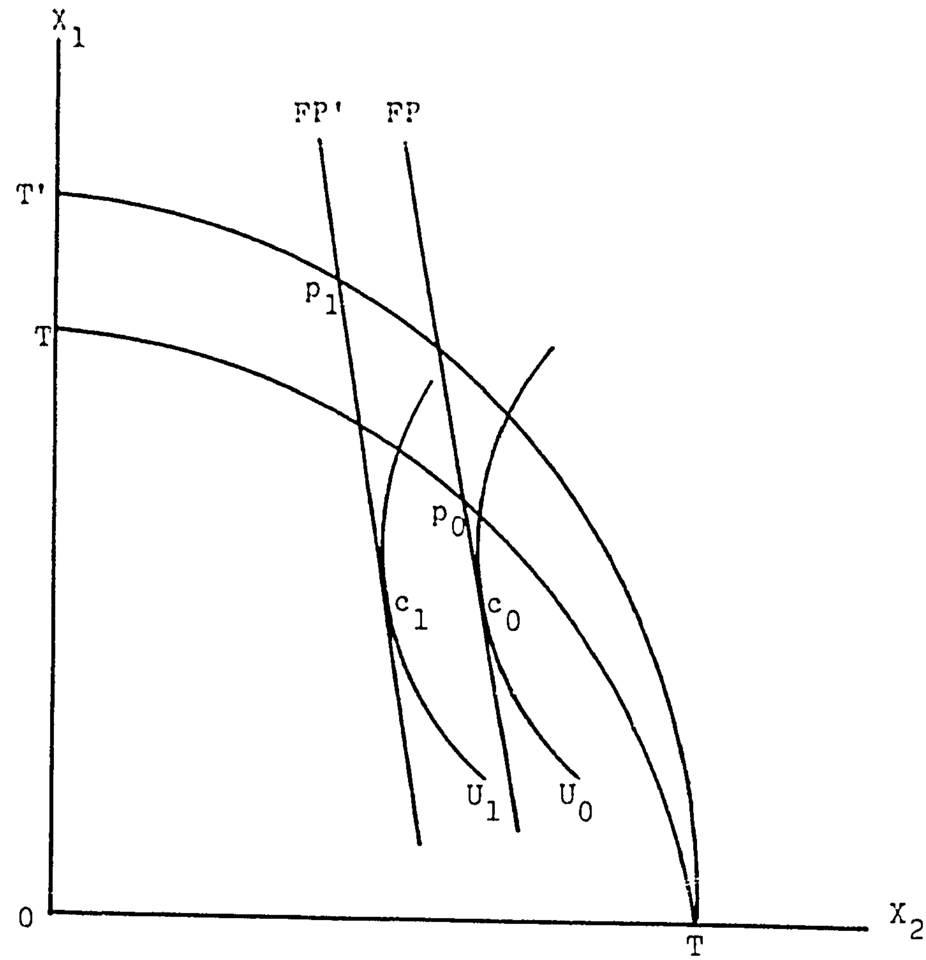
NORMAL GROWTH



$\beta > 1$ ($e_1 > e_2$) and $dX_1/dt_1 > 0$

Figure 5a

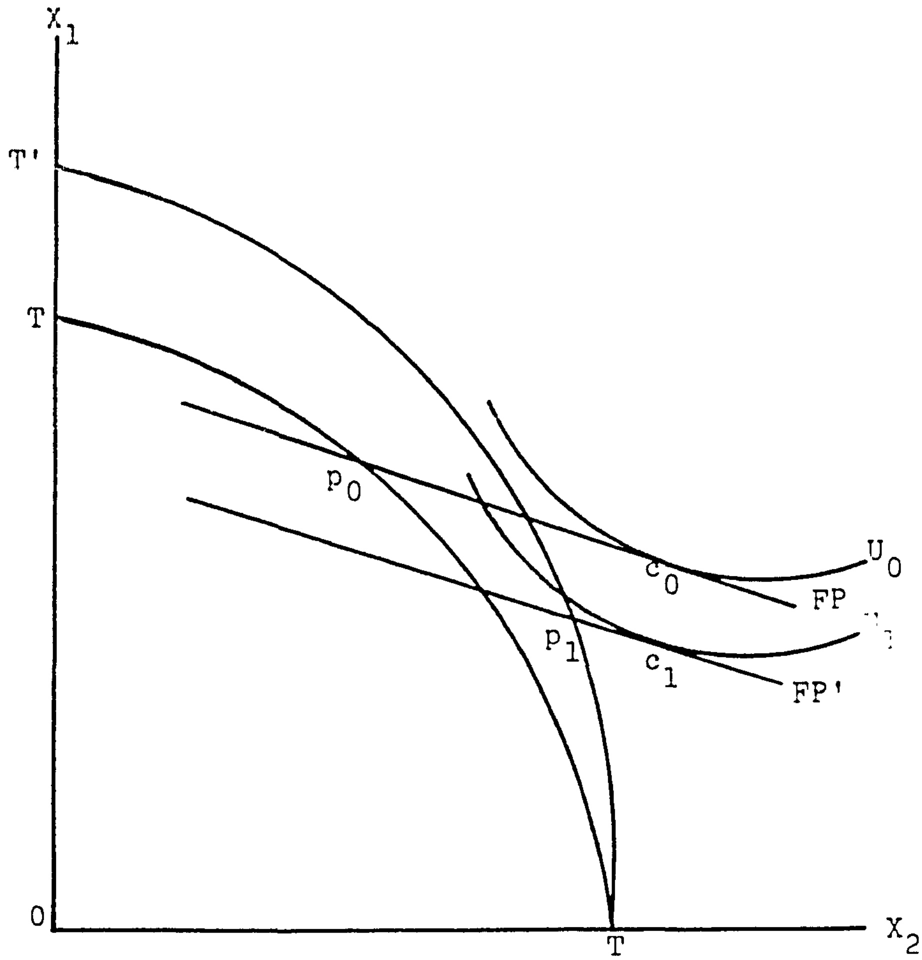
IMMISERIZING GROWTH



$\beta<1$ $(e_1<e_2)$ and $dX_1/dt>0$

Figure 5b

IMMISERIZING GROWTH



$\beta>1$ $(e_1>e_2)$ and $dX_1/dX_2<0$

Figure 5c

Table 5.2

THE EFFECTS OF TECHNICAL PROGRESS ON WELFARE
UNDER VARIABLE RETURNS TO SCALE

Type of Technical Progress	System / Returns to Scale	Stable($\alpha > 0$)		Unstable($\alpha < 0$)	
		Increasing or Constant ($R_1 \geq 0$)	Decreasing ($R_1 < 0$)	Increasing or Constant ($R_1 \geq 0$)	Decreasing ($R_1 < 0$)
Neutral	$\beta > 1$	+	?	?	?
	$\beta < 1$	-	?	?	?
Intensive Factor Saving	$\beta > 1$	+	?	?	?
	$\beta < 1$	-	?	?	?
Intensive Factor Using	$\beta > 1$	?	?	?	?
	$\beta < 1$	?	?	?	?

Note: Technical progress is assumed to take place in the first industry only.

under variable returns to scale. However, a brief discussion will be provided.

For constant commodity prices and technology, (5.33) and (5.34) reduce to

$$\lambda''_{L1}X_1^* + \lambda''_{L2}X_2^* = L^* \tag{5.58}$$

$$\lambda''_{K1}X_1^* + \lambda''_{K2}X_2^* = K^* \tag{5.59}$$

Solving for X_1^* and X_2^*, we obtain

$$X_1^* = \frac{\lambda''_{K2}L^* - \lambda''_{L2}K^*}{|\lambda''|} \tag{5.60}$$

$$X_2^* = \frac{\lambda''_{L1}K^* - \lambda''_{K1}L^*}{|\lambda''|} \tag{5.61}$$

With constant capital endowment,

$$\frac{X_1^*}{L^*} = \frac{\lambda''_{K2}}{|\lambda''|} = \frac{\lambda''_{K2}}{\alpha(1-R_1)(1-R_2)|\lambda|} \tag{5.62}$$

$$\frac{X_2^*}{L^*} = \frac{-\lambda''_{K1}}{|\lambda''|} = \frac{-\lambda''_{K1}}{\alpha(1-R_1)(1-R_2)|\lambda|} \tag{5.63}$$

Similarly, holding labor constant,

$$\frac{X_1^*}{K^*} = \frac{-\lambda''_{L2}}{|\lambda''|} = \frac{-\lambda''_{L2}}{\alpha(1-R_1)(1-R_2)|\lambda|} \tag{5.64}$$

$$\frac{X_2^*}{K^*} = \frac{\lambda''_{L1}}{|\lambda''|} = \frac{\lambda''_{L1}}{\alpha(1-R_1)(1-R_2)|\lambda|} \tag{5.65}$$

Remember that $R_j < 1$ $(j=1,2)$ and $\lambda''_{ij} > 0$ $(i=L,K;\ j=1,2)$ by assumption. In the dynamically stable system $(\alpha > 0)$, it is clear that $X_1^*/L^* \gtrless 0$ and $X_2^*/L^* \lessgtr 0$ if $k_1 \lessgtr k_2$ and $X_1^*/K^* \lessgtr 0$ and $X_2^*/K^* \gtrless 0$ if $k_1 \lessgtr k_2$; that is, the Rybczynski theorem carries over to the case of variable returns to scale. In the unstable system $(\alpha<0)$, however, all the signs are reversed and hence the Rybczynski theorem breaks down. These results are summarized in Table 5.3.

The Prebisch-Singer Thesis

The conventional notion of immiserizing growth signifies that an expanding economy might be worse off after growth than before if the deterioration in the terms of trade outweighs the output gain as a result of growth. Many authors, based on the standard model, have proved the validity of the notion. For example, Batra (1973) has shown that the growth in the output of a country is the same as the growth in its real income if the terms of trade remain constant and the former (latter) exceeds the latter (former) if the terms of trade deteriorate (improve). In other words, the necessary condition for immiserizing growth is a deterioration in the terms of trade.

In their controversial theses, prebisch and singer blamed the strong monopolistic elements of the western economies for suppressing the relative prices of exportables from underdeveloped countries. They argued that the chronic poverty in the developing countries is the result of the secular deterioration in their terms of trade. Recently, Bhagwati (1968), Johnson (1970) and Batra (1973) have shown that the terms of trade are not directly related to the rate of economic growth, if the factor markets are distortionary.

In this chapter, it was demonstrated that in the

Table 5.3

THE EFFECTS OF FACTOR ACCUMULATION ON OUTPUTS UNDER VARIABLE RETURND TO SCALE

Type of Factor Accumulation	Output \ Factor Intensity \ System	Stable($\alpha > 0$) $k_1 > k_2$	Stable($\alpha > 0$) $k_1 < k_2$	Unstable($\alpha < 0$) $k_1 > k_2$	Unstable($\alpha < 0$) $k_1 < k_2$
Labor Growth	X_1^*	-	+	+	-
	X_2^*	+	-	-	+
Capital Accumulation	X_1^*	+	-	-	+
	X_2^*	-	+	+	-

Note: Production functions are assumed to be homothetic.

presence of variable returns to scale, growth may be immiserizing even if the terms of trade are constant. In light of this finding, it should be necessary to examine the economies of scale of the exporting and the importing sectors of the developing countries before involving in such controversy.

III. ECONOMIC EXPANSION AND THE TERMS OF TRADE

The previous sections have been concerned with a small country which is a price taker in the international market. This section deals with a large country whose terms of trade are variable. For complete analysis, the following two questions should be answered: 1) how does growth affect the terms of trade? and 2) what is the welfare effect of a changed terms of trade? The answer to the latter has been recently provided by Eaton and Panagaria (1979). Thus, the following discussion will be confined to the former question.

Suppose that only the home country experiences growth while the foreign country remains stationary. Let the first commodity be the exportable and the second commodity the importable.

$$E_1 = X_1 - D_1 \tag{5.66}$$

$$E_2 = D_2 - X_2 \tag{5.67}$$

The import demand of each country depends on the terms of trade (p) and economic growth (G). Since growth occurs only in the home country, we can write the budget constraint as,

$$E_1(p) - pE_2(p,G) = 0 \tag{5.68}$$

Differentiating (5.68) and solving for $\partial E_2/\partial G$, we obtain

$$\frac{\partial E_2}{\partial G} = \left(\frac{p}{E_1}\ \frac{\partial E_1}{\partial p} - \frac{p}{E_2}\ \frac{\partial E_2}{\partial p} - 1\right)\frac{E_2}{p}\ \frac{dp}{dG} = (a_f+a_h-1)\frac{E_2}{p}\ \frac{dp}{dG} \tag{5.69}$$

where $a_f=(p/E_1)(\partial E_1/\partial p)$ is the elasticity of foreign import demand and $a_h=-(p/E_2)(\partial E_2/\partial p)$ the elasticity of home import demand. Solving for dp/dG, we get

$$\frac{dp}{dG} = \frac{p}{(a_f+a_h-1)}\ \frac{\partial E_2}{\partial G} \tag{5.70}$$

(5.70) indicates the effect of economic growth on the terms of trade. By the Marshall-Lerner stability condition, $a_f+a_h-1 > 0$. Hence, the terms of trade of the home country is a negative function of its import demand, i.e. $dp/dG \gtrless 0$ if $\partial E_2/\partial G \gtrless 0$. Since consumption demand is a function of price and income while production is a function of price and growth, we can write (5.67) as,

$$E_2 = D_2(p,Y)-X_2(p,G) \tag{5.71}$$

where $Y=X_1+pX_2$ represents the national income. By differentiating (5.71) with respect to G, holding price constant, we obtain

$$\frac{\partial E_2}{\partial G} = \frac{\partial D_2}{\partial Y}\ \frac{\partial Y}{\partial G} - \frac{\partial X_2}{\partial G} = m_h\frac{\partial Y}{\partial G} - \frac{\partial X_2}{\partial G} \tag{5.72}$$

where $m_h=\partial D_2/\partial Y$ is the home marginal propensity to consume the importables. In the absence of inferior goods, $0< m_h< 1$. Partial differentiation of $Y=X_1+pX_2$ with respect to G yields

$$\frac{\partial Y}{\partial G} = \frac{\partial X_1}{\partial G} + p\frac{\partial X_2}{\partial G} \tag{5.73}$$

In chapter I, it has been shown that production equilibrium occurs when

$$\frac{\partial X_1}{\partial X_2} = -(\frac{1-e_2}{1-e_1})p \tag{5.74}$$

Using (5.74), (5.73) can be reexpressed as

$$\frac{\partial Y}{\partial G} = (\frac{e_2-e_1}{1-e_1})p\frac{\partial X_2}{\partial G} \tag{5.75}$$

Substituting (5.75) in (5.72), we obtain

$$\frac{\partial E_2}{\partial G} = m_h\left[p(\frac{e_2-e_1}{1-e_1})-1\right]\frac{dX_2}{dG} \tag{5.76}$$

(5.76) furnishes the expression for determining the effects of the output change on the demand for imports as a result of growth. Substituting (5.76) in (5.70), we get the final expression,

$$\frac{dp}{dG} = \frac{[\{m_h(e_2-e_1)/(1-e_1)\}-1]}{(a_f+a_h-1)} \cdot \frac{dX_2}{dG} \tag{5.77}$$

(5.77) indicates the effect of economic expansion on the terms of trade. Note that in (5.77) the commodity units were chosen so that p is initially equal to unity. It is clear that the sign of dp/dG is determined by the sign of $m_h(e_2-e_1)/(1-e_1)-1$ and dX_2/dG. If the exportable and import competing industries operate under identical returns to scale ($e_1=e_2$), with constant returns to scale as a special case, growth results in the improvement (deterioration) in the terms of trade provided that it increases (decreases) the output of the importables. This result remains unchanged if the elasticity of returns to scale of the exportable industry is greater than that of the importable industry, $e_1 > e_2$, since $m_h(e_2-e_1)/(1-e_1)-1$ is still negative. But if the elasticity of returns to scale of the importable industry is greater than that of the exportable industry, $e_1 < e_2$, growth and the terms of trade are not uniquely related.

CHAPTER VI

CONCLUSIONS

Increasing or decreasing returns to scale in production are indisputable phenomena that characterize the real world. They are usually attributable to certain economies or diseconomies that are reflected in production costs. Recently, trade theorists have shown that perfect competition can be reconciled with static increasing returns if the economies of scale are external to individual firm and the competitive output is efficient when the externalities are output generated. The present work dealt with some previously unexplored but important topics of international trade in light of this type of economies of scale.

Chapter I, an introduction to variable returns to scale, was concerned with the production side of general equilibrium. Although the validity of Euler's exhaustion theorem for the industry production functions with above mentioned properties and the method of deriving the internal production equilibrium condition along with its generalization to the multi-commodity and muti-factor case seem to be new, no other new results are reached in this chapter.

In Chapter II, traditional gains from trade theorems were examined under variable returns to scale. For a small economy, the optimality of free trade breaks down if industry production functions are subject to divergent returns to scale. Furthermore, social welfare can be maximized by introducing, in addition to free trade, a policy of tax-cum-subsidy such that the output of the industry exhibiting greater (smaller) returns to scale is pushed to the maximum (minimum). In addition, the introduction of tariffs and production subsidies may be either harmful or beneficial whereas the imposition of consumption taxes is always harmful. A production subsidy is superior to an equivalent rate of tariff while a consumption tax may be either superior or inferior to an equivalent tariff.

Chapter III was devoted to the examination of welfare consequences of forming customs unions in a framework allowing variable returns to scale. Trade creation and trade diversion were differentiated according to the manner in which trade is being created or diverted. Under variable returns to scale, trade creation I and trade creation II, as defined in the text, may be welfare-decreasing, while trade diversion I and trade diversion II may be welfare-increasing. Furthermore, an improvement in the terms of trade or a reduction in the rate of tariff may result in welfare loss. The crucial factors determining the welfare change in the presence of trade creation and trade diversion are 1) types of trade creation and trade diversion, 2) system stability and 3) the difference in the returns to scale between industries.

Chapter IV analyzed the effects of a tariff on the import demand, the terms of trade and domestic price ratio in a large country framework. In the presence of variable returns to scale, an improvement in the terms of trade for a given tariff may decrease the demand for imports while a higher tariff for given terms of trade may increase it. An increase in the rate of tariff may worsen the terms of trade and hence raise the domestic price ratio of the importable commodity by more than the tariff rate. The necessary condition for Metzler paradox based on the assumption of constant returns to scale carries over to the case of variable returns to scale: the sum of the foreign country's terms of trade elasticity of import demand and the home marginal propensity to consume the importables is less than unity. Under variable returns to scale, not only the foreign country's terms of trade elasticity of import demand but also the domestic price elasticity of the demand for the importable commodity, price-output response and the elasticity of returns to scale of the home industries are the deterministic factors for the optimum tariff.

Chapter V explored the implication of economic expansion for output levels, the terms of trade and social welfare. As in the conventional analysis, two sources of economic expansion are identified, technical progress and factor accumulation. In the presence of variable returns to scale, neutral and/or intensive-factor-using technical progress may decrease the output of the industry in which technical progress occurs while it may increase the output of the other industry. Moreover, economic expansion generated by technical progress may result in immiserizing growth, even if the terms of trade remain unchanged. If the elasticity of returns to scale of the exportable industry is equal to or greater than that of the importable industry, growth improves the terms of trade provided that it increases the output of the importables. However, if the elasticity of returns to scale of the importable industry is greater than that of the exportable industry, growth and the terms of trade are not uniquely related.

This dissertation should be concluded with some notes and suggestions. Some of the mathematical results in the present work may be demonstrated by alternative mathematical procedures or geometrical techniques. For example, the optimum tariff can be derived using the utility function approach within variable returns to scale framework. The effects of economic expansion on the outputs can be depicted using the factor endowment vector in the input space diagram. Finally, the present work suggests a few potential areas to be investigated in a similar framework involving economies of scale, such as intermediate product, factor market distortion, effective protection, economic integration and stochastic environment.

BIBLIOGRAPHY

Arndt, S. W. "On Discriminatory vs Non-Preferential Tariff Policies.", Economic Journal, December, 1968, pp. 971-979.

Balassa, B. The Theory of Economic Integration. Richard D. Irwin Inc., Homwood, Ill. 1961.

Balassa, B. "Trade Creation and Trade Diversion in the European Common Market.", Economic Journal, June, 1967, pp. 1-17.

Batra, R. N. "Protection and Real Wages under Conditions of Variable Returns to Scale.", Oxford Economic Papers, 1968, pp. 353-360.

Batra, R. N. Studies in the Pure Theory of International Trade. New York: St. Martin Press, 1973.

Batra, R. N. and Scully, G. W. "The Theory of Wage Differential: Welfare and Immiserizing Growth.", Journal of International Economics, May, 1971, pp. 241-247.

Bhagwati, J. N. "Immiserizing Growth: A Geometrical Note.", Review of Economic Studies, June, 1958, pp. 201-205.

Bhagwati, J. N. "The Gains from Trade Once Again.", Oxford Economic Papers, July, 1968, pp. 137-148.

Bhagwati, J. N. "Distortions and Immiserizing Growth: A Generalization.", Review of Economic Studies, October, 1968, pp. 481-485.

Bhagwati, J. N. "Trade Diverting Customs Unions and Welfare Improvement: A Clarification.", Economic Journal, September, 1971, pp. 580-587.

Bhagwati, J. N. and Srinivasan, T. N. "Optimal Intervention to Achieve Non-Economic Objectives.", Review of Economic Studies, January, 1969, pp. 27-38.

Chacholiades, M. International Trade Theory and Policy. McGraw-Hill, Inc., 1978.

Chipman, J. S. "A Survey of the Theory of International Trade: Part 2. The Neo-Classical Theory.", Econometrica, October, 1965, pp. 685-760.

Chipman, J. S. "External Economies of Scale and Competitive Equilibrium.", Quarterly Journal of Economics, August, 1970, pp. 347-385.

Corden, W. M. "Economic Expansion and International Trade: A Geometrical Approach.", Oxford Economic Papers, June, 1956, pp. 223-228.

Corden, W. M. "Tariffs, Subsidies and the Terms of Trade.", Economica, August, 1957, pp. 235-242.

Corden, W. M. Recent Developments in the Theory of International Trade. International Finance Section, Department of Economics, Princeton University, Princeton, N.J. Chapter V.

Edgeworth, F. Y. "The Theory of International Values I." Economic Journal, March, 1894, pp. 35-50.

Edgeworth, F. Y. "On a Point in the Pure Theory of International Trade.", Economic Journal, March, 1899, pp. 125-128.

Edgeworth, F.Y. "Review of a Geometrical Political Economy by Henry Cunyngham.", Economic Journal, March, 1905, pp.61-71.

Eaton, J. and Panagaria, A. "Gains from Trade under Variable Returns to Scale, Commodity Taxation, Tariffs and Factor Market Distortions.", Journal of International Economics, 1979, pp. 481-501.

Findlay, R. and Grubert, H. "Factor Intensity, Technological Progress and the Terms of Trade.", Oxford Economic Papers, February, 1959, pp. 111-121.

Gehrels, F. "Customs Unions from a Single Country viewpoint.", Review of Economic Studies, January 1956, pp. 61-64.

Graham, F. D. "Some Aspects of Protection Further Considered.", Quarterly Journal of Economics, February, 1925, pp. 324-330.

Graham, F. D. The Theory of International Values. Princeton, N.J.: Princeton University Press, 1948.

Hazari, B. R. The Pure Theory of International Trade and Distortions. Halsted Press, a Division of John Wiley and Sons, Inc., New York, 1978.

Heckscher, E. "The Effects of Foreign Trade on the Distribution of Income.", Economisk Tidskrift, vol.21, 1919, pp. 1-32. Reprinted in AEA Readings in the Theory of International Trade. Richard D. Irwin, Inc., Homewood, Ill. 1950.

Herberg, H. and Kemp, M. C. "Some Implication of Variable Returns to Scale.", Canadian Journal of Economics and Political Science, August, 1969, pp. 401-415

Johnson, H. G. International Trade and Economic Growth. Georgy Allen and Unwin, Ltd., London, 1958.

Johnson, H. G. "A Note on Distortions and the Rate of Growth of an Open Economy.", Economic Journal, December, 1970.

Johnson, H. G. "Trade Diverting Customs Unions: A Comment.", Economic Journal, September, 1974, pp. 618-621.

Johnson, H. G. "A Note on Welfare-Increasing Trade Diversion.", Canadian Journal of Economics and Political Science, February, 1975, pp.117-123.

Jones, R. W. "Variable Returns to Scale in General Equilibrium Theory.", International Economic Review, October, 1968, pp. 261-272.

Jones, R. W. "Tariffs and Trade in General Equilibrium: Comment.", American Economic Review, June, 1969, pp. 418-424.

Kemp, M. C. "The Efficiency of Competition as an Allocator of Resources: I. External Economies ofProduction.", Canadian Journal of Economics and Political Science, February, 1955, pp. 30-42.

Kemp, M. C. The Pure Theory of International Trade and Investment. Englewood Cliffs, N.J.: Prentice Hall Inc., 1969, Chapter 8.

Kemp, M. C. and Negishi. T. "Variable Returns to Scale, Commodity Taxes, Factor Market Distortions and

and Their Implications for Trade Gains.", Swedish Journal of Economics, 1970, pp. 1-11.

Knight, F. H. "Some Fallacies in the Interpretation of Social Cost.", Quarterly Journal of Economics, August, 1924, pp. 582-606.

Knight, F. H. "On Decreasing Cost and Comparative Cost. A Rejoinder.", Quarterly Journal of Economics, February, 1925, pp. 331-333.

Leibenstein, H. "Allocative Efficiency versus 'X Efficiency'.", American Economic Review, June, 1966, pp. 392-415.

Lerner, A. P. "The Symmetry between Import and Export Taxes.", Economica, August, 1936, pp. 306-313.

Lipsey, R. "The Theory of Customs Unions: Trade Diversion and Welfare.", Economica, February, 1957, pp. 40-46.

Lipsey, R. "Mr. Gehrels on Customs Unions.", Review of Economic Studies, 1957, pp. 211-214.

Lipsey, R. "The Theory of Customs Unions: A General Survey.", Economic Journal, September, 1960, pp. 496-513.

Marshall, A. The Pure Theory of Foreign Trade. London: London School of Economics and Political Science, 1949.

Matthews, R. C. O. "Reciprocal Demand and Increasing Returns.", Review of Economic Studies, 1950, pp. 149-158.

Mayer, W. "Variable Returns to Scale in General Equilibrium Theory: Comment.", International Economic Review, 1974, pp. 225-235.

Meade, J. E. A Geometry of International Trade. Georgy Allen and Unwin, Ltd., London, Chapter 1-4.

Meade, J. E. "External Economies and Diseconomies in a Competitive Situation.", Economic Journal, March, 1952, pp. 54-67.

Meade, J. E. The Theory of International Economic Policy, Vol. II, Trade and Welfare, London: Oxford University press, 1955.

Melvin, J. R. "Comments on the Theory of Customs Unions." Manchester School od Economic and Social Studies, June, 1969, pp. 161-168.

Melvin, J. R. "Increasing Returns to Scale as Determinants of Trade.", Canadian Journal of Economics, August, 1969, pp. 389-402.

Metzler, L. A. "Tariffs, the Terms of Trade and the Distribution of National Income.", Journal of Political Economy, February, 1949, pp. 1-29.

Metzler, L. A. "Tariffs, International Demand and Domestic Prices.", Journal of Political Economy, August, 1949, pp. 345-351.

Mill, J. S. Principle of Political Economy. London: Longmans, Green, 1969.

Minabe, N. "The Stolper-Samuelson Theorem under Conditions of Variable Returns to Scale.", Oxford Economic Papers, July, 1966, pp. 204-212.

Mishan, E. J. "The Welfare Gains of Trade-Diverting Customs Union Reinterpreted.", Economic Journal, September, 1966, pp. 669-672

Ohlin, B. Interregional and International Trade. Harvard University Press, Cambridge, Mass., 1933.

Panagaria, A. "Variable Returns to Scale in General Equilibrium Theory Once Again.", Journal of International Economics, 1980, pp. 499-526.

Panagaria, A. "Variable Returns to Scale in Production and Patterns of Specialization.", American Economic Review, March, 1981, pp. 221-230.

Prebisch, R. "Commercial Policy in the Underdeveloped Countries.", American Economic Review, May, 1951, pp. 251-273.

Rybczynski, T. M. "Factor Endowment and Relative Commodity Prices.", Economica, November, 1955, pp. 336=341.

Samuelson, P. A. "The Gains from International Trade.", Canadian Journal of Economics and Political Science, May, 1939, pp. 195-205.

Samuelson, P. A. "Social Indifference Curves.", Quarterly Journal of Economics, February, 1956, pp. 1-21.

Savosnick, K. M. "The Box Diagram and the Production Possibility Curve.", Ekonomisk Tidskrift, September, 1958, pp. 183-197.

Scitovsky, T. Economic Theory and Western European Integration. Stanford University Press, Stanford, Cal., 1958.

Södestern, B. and Vind, K. "Tariffs and Trade in General Equilibrium.", American Economic Review, June, 1968, pp. 394-408.

Södestern, B. and Vind, K. "Tariffs and Trade in General Equilibrium: Reply.", American Economic Review, June, 1969, pp. 424-426.

Tinbergen, J. International Economic Integration. Elsevier Publishing Company, Amsterdam, 1965.

Viner, J. The Customs Union Issue. Carnegie Endowment for International Peace, New York, 1950.

Yu, E. S. H. "Trade Diversion, Trade Creation and Factor Market Imperfections.", Review of World Economics (Weltwirtschaftliches Archiv), 1981, pp. 546-561.

Yu, E. S. H. "Unemployment and the Theory of Customs Union.", Economic Journal, June, 1982 (in process).

APPENDIX

This appendix contains a mathematical derivation of some equations in Chapter IV that require lengthy calculation.

1. Equation (4.19):

Substituting (4.17) and (4.18) in (4.13), we obtain

$$E_2^* = -e_h p_h^* + (m_h/p_h E_2)dY - s_h p_h^* = -e_h(p^* + T_h^*) + (m_h/p_h E_2)\{-E_2 dp$$
$$+(T_h - 1)pdE_2 + \frac{e_2 - e_1}{1-e_1} p_h dX_2\} - s_h(p^* + T_h^*) = -e_h p^* - e_h T_h^* - (m_h/T_h)p^*$$
$$+(m_h/T_h)t_h E_2^* + m_h \frac{e_2 - e_1}{1-e_1} s_h(p^* + T_h^*) - s_h p^* - s_h T_h^* \qquad (1)$$

Hence,

$$E_2^*(1 - \frac{m_h t_h}{T_h}) = -(e_h + \frac{m_h}{T_h} - m_h \frac{e_2 - e_1}{1-e_1} s_h + s_h)p^* - (e_h + s_h - m_h \frac{e_2 - e_1}{1-e_1} s_h)T_h^*$$

Solving for E_2^*, we obtain

$$E_2^* = \frac{-[e_h + s_h\{1 - m_h(e_2 - e_1)/(1-e_1)\} + m_h/T_h]p^* - [e_h + s_h\{1-}{1 - m_h t_h/T_h}$$
$$\frac{m_h(e_2 - e_1)/(1-e_1)\}]T_h^*}{1 - m_h t_h/T_h} \qquad (4.19)$$

2. The Optimum Tariff

I) Equation (4.31):

From (4.22),

$$dE_{1f} = (a_f p^* - A_f T_f^*)E_{1f} \qquad (2)$$

From (4.30),

$$dT_h = \{-(p^*/A_h)(a_f + a_h - 1) + (A_f/A_h)T_f^*\}T_h \qquad (3)$$

Let $(e_2-e_1)/(1-e_1)=B$.

Substituting (2) and (3) in (4.18), we obtain

$$dY=(T_h-1)dE_{1f}-T_hE_2dp+Bp_hdX_2=E_{1f}t_ha_fp^*-E_{1f}t_hA_fT_f^*-T_hE_{1f}p^*+$$
$$Bs_hE_{1f}T_h(p^*+T_h^*)=E_{1f}t_ha_fp^*-E_{1f}t_hA_fT_f^*-t_hE_{1f}p^*-E_{1f}p^*+$$
$$Bs_hE_{1f}T_hp^*+Bs_hE_{1f}dT_h=E_{1f}t_ha_fp^*-E_{1f}t_hA_fT_f^*-t_hE_{1f}p^*+$$
$$Bs_hE_{1f}T_hp^*+Bs_hE_{1f}(A_f/A_h)T_hT_f^*-Bs_hE_{1f}(p^*/A_h)T_h(a_f+a_h-1)$$
$$= E_{1f}p^*\left[t_h\{a_f-1+Bs_h(A_h-a_f-a_h+1)/A_h\}-\{1-Bs_h(A_h-a_f-a_h+1)/A_h\}\right]$$
$$-E_{1f}(t_hA_f-Bs_hT_hA_f/A_h)T_f^* \qquad (4.31)$$

II) Equation (4.32):

As a preliminarly step, solve the term, $Bs_h(A_h-a_f-a_h+1)/A_h$, by substituting (4.20) and (4.21) to get

$$Bs_h(A_h-a_f-a_h+1)/A_h=-Bs_h\{m_h+(a_f-1)(T_h-m_ht_h)\}/\{e_h+s_h(1-m_hB)\} \qquad (4)$$

Substituing (4) in (4.31), we derive

$$dY=E_{1f}p^*\left[t_h(a_f-1)+Bs_h\{(A_h-a_f-a_h+1)/A_h\}(t_h+1)-1\right]-E_{1f}(t_hA_f-$$
$$Bs_hT_hA_f/A_h)T_f^*=E_{1f}p^*\left[t_h(a_f-1)+T_h(-Bs_h)\ \frac{m_h+(a_f-1)(T_h-m_ht_h)}{e_h+s_h(1-m_hB)}\right.$$
$$\left.-1\right]-E_{1f}(t_hA_f-Bs_hT_hA_f/A_h)T_f^* =E_{1f}p^*\left[t_h(a_f-1)\{1+\frac{Bs_hm_h-Bs_h}{e_h+s_h(1-m_hB)}\}\right.$$
$$\left.-\frac{Bs_h\{m_h+(a_f-1)\}}{e_h+s_h(1-m_hB)}\ -1\right]-E_{1f}(t_hA_f-Bs_hT_hA_f/A_h)T_f^*=E_{1f}p^*\left[t_h(a_f-1)\cdot\right.$$
$$\left.\frac{e_h+s_h-Bs_h}{e_h+s_h-(1-m_hB)}\right]-\ \frac{Bs_h\{m_h+(a_f-1)\}+e_h+s_h(1-m_hB)\}}{e_h+s_h(1-m_hB)}\ -E_{1f}(t_hA_f-$$
$$Bs_hT_hA_f/A_h)T_f^*=\ \frac{E_{1f}p^*}{e_h+s_h(1-m_hB)}\left[t_h(a_f-1)(e_h+s_h-Bs_h)-\{e_h+\right.$$
$$\left.s_h+Bs_h(a_f-1)\}\right]\ -\ E_{1f}(t_hA_f-Bs_hT_hA_f/A_h)T_f^* \qquad (4.32)$$