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# A FORMALISM FOR THE SYNTACTIC DESCRIPTION AND RECOGNITION OF TWO DIMENSIONAL PATTERNS 

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## THE UNIVERSITY OF OKLAHOMA <br> GRADUATE COLIEGE

## A FORMAIISM FOR THE SYNTACTIC DESCRIPTION

 AND RECOGNITION OF TWO DIMENSIONAL PATTERNSA DISSERTATION<br>SUBMITTED TO THE GRADUATE FACUITY<br>In partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

BY
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A FORMAIISM FOR THE SYNTACTIC DESCRIPTION AND RECOGNITION OF TWO DIMENSIONAL PATTERNS


DISSERTATION COMMITTEE

## To

## Sri Venkateswara

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#### Abstract

A new formalism is proposed for the syntactic description and recognition of two-dimensional patterns. The recognition problem is treated comprehensively from scanning and primitive identification all the way to recognition of patteris syntactically.

The vehicle of grammars coupled with ideas of static and dynamic chaining of pattern primitives are used for describing arbitrary two-dimensional patterns. Primitives in the grammar are unquantized vector entities. The grammar derives a "pattern form," while parametrization of the pattern form yields specific patterns.

The concept of imaginary parsing is advanced for recognition of partially obscured patterns.


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## CHAPTER I

## OVERVIEW OF PROBLEM


#### Abstract

1.0 Introduction

The past two decades have witnessed a tremendous growth in the body of knowledge in pattern recognition. This growth has been spurred on by the evolution of the computer during the same period. As in other areas of study, the use of computers as a tool has opened up exciting new possibilities for the processing of two-dimensional patterns, which would otherwise be impossible. Of the three approaches to pattern recognition, the heuristic approach is perhaps the most efficient, but the least amenable to generalization (38). Since the solutions are problem knowledge-based, the solution to one problem is not readily translated to other areas of study. The statistical approach accounts for the major portion of work done to date in pattern recognition. In this approach, recognition classes are first established from known samples. Subsequently, the objective is to identify an unknown sample with one of the classes previously established. The literature is very extensive in this area and one may refer to


the bibliography for reference information. The third approach - namely, the syntactic approach, is relatively a recent entry to the field of pattern recognition compared to the other two. In this study, this is the area of prime interest.

The interest in syntactic pattern recognition is a natural outgrowth of the highly successful use of the syntactic technique in processing computer languages. A formidable body of knowledge has developed in the field of formal language theory and one can almost trace its development paralleling that of the computer. The main difficulty in applying the principles of syntactic techniques of language theory to two-dimensional pattern recognition is that, in languages, the only relationship between the primitives is one of concatenation, while in the case of two-dimen-. sional patterns, spatial relationships, size and shades of color play an important role.

Some of the earlier works in syntactic pattern recognition overcame the above difficulty by defining the primitives in such a manner as to reduce a two-dimensional pattern to a one-dimensional string. This approach was successfully demonstrated in the chromosome analysis by Ledley et al. (21). Narasimhan $(26,27,28)$ has used this approach for analysis of Fortran characters and hand-printed English characters. Shaw (40) has formulated the "Picture Description

Language," which is similar in concept and has used this idea for the analysis of Bubble Chamber pictures, description of Alphabetic characters and recognition of line patterns. This approach of reducing a two-dimensional pattern to a one-dimensional string has produced a number of publications in the syntactic analysis of wave forms. Stockman et al. (42), Pavlidis (31), Horowitz (18), Udupa et al (44), have carried out syntactic analysis of wave forms by reducing them to one dimensional string.

Researchers in syntactic pattern recognition have sought to overcome the limitations of representing twodimensional patterns as one-dimensional strings. This led to the development of formalisms that took into account spatial relationships. The most noteworthy among these are the "Plex Grammar" by Feder (9), "Web Grammar" by Pfaltz and Rosenfeld (35) and "Tree Grammar" by Brainerd (4). In these works, the relationship between the primitives is more involved than mere concatenation. Productions in some of the grammars are depicted pictorially with additional qualifications that specify the constraints that must be adhered to in the rewriting process. Anderson (2) has made use of coordinates in the syntactic recognition of two-dimensional handprinted mathematics. Rosenfeld (39) has extended the concept of string grammars to array grammars in which the syntactic entities are elements in a two-dimensional array.

In reviewing these formalisms, there are several interesting and important questions that seem to remain unanswered. The first and most basic question that comes to one's mind is - "How are the primitives in the various schemes identified?" It appears that syntactic analysis of a pattern can be carried out after the primitives are identified and "handed over" to the various schemes. This is done almost without an exception by the various statistical methods (13).

The second point to be made is that there is no uniformity in the manner in which the primitives are chosen in the various formalisms. This makes it very difficult to use the solution of one problem area in another - at least in part, if not in whole.

Thirdly, almost without an exception, none of these formalisms address the question of partial patterns. It is not clear as to how to cope with situations wherein a pattern desired to be recognized is partially hidden or obscured by others that overlap them.

### 1.1 Objectives of Study

The formalism developed in this study endeavors to address the important questions raised above. The objectives of this study are fourfold. The first of these is to develop a framework in which provision is made for syntactically identifying the pattern primitives. The second
objective is to seek to formulate the pattern primitives in such a manner as to provide a more uniform basis for their definition from one problem area to another. The third objective is to address the issue of how partial patterns may be recognized within this framework. The last, but not the least important of the objectives, is to verify the ideas advanced through computer implementation and draw conclusions as to the validity of this formalism.

### 1.2 The Two-Dimensional Pattern

If a two-dimensional pattern is placed under a microscope, without a doubt one will observe that the pattern is nothing more than a collection of fine points, each with an associated color of some intensity. In this sense, one might call these fine points the "universal primitives" in terms of which every two-aimensional pattern is composed. However, one might additionally observe that it is not the microscopic points themselves that are perceived by the human eye, but a collection of such points, which, taken together, forms the meaningful pattern. Therefore, for purposes of this study, collection of such points is considered for pattern primitives rather than the microscopic points themselves.

Intuitively speaking, the microscopic points can combine to form one of three entities in any arbitrary twodimensional pattern. They are - dots, lines and areas. No effort is made in this work to precisely define each of
these entities, for the simple reason that what constitutes a dot in one application can turn out to be an area in another: Associated with each of these is a color of a given intensity. The color may be uniform throughout or vary within each of the above three entities. It is the arbitrary combination of these dots, lines and areas that makes an arbitrary twodimensional pattern. Specific combination of these imparts meaning to the two-dimensional pattern that we are conditioned by experience and learning to recognize. Additionally, it must be noted that in a real two-dimensional pattern, there are situations in which the variation in colors and intensity levels are such that the demarcation between these entities is not clear. This study assumes that a two-dimensional pattern is composed of an arbitrary combination of dots, lines and areas, distinguishable from one another by their associated color functions. The inability to distinguish one entity from another on the basis of color; while a limitation of this formalism, is common to most pattern recognition systems.

The principal dilemma in using linguistic ideas for description of two-dimensional patterns is, that the primitives in languages have only identity and their position within a string of these primitives, while pattern primitives not only have identity, but magnitude as well as direction or spatial position. This factor needs to be remembered in developing a syntactic formalism for patterns if it
has to be a viable one. This requirement seems to be naturally met by choosing vectors as the basis for describing two-dimensional patterns. Vectors are entities that may be represented symbolically, which have magnitude as well as associated direction. These symbols may be manipulated using the linguistic approach to define a "pattern form," while the final pattern may be derived by parametrization of the attributes of this pattern form.

### 1.3 The Two-Dimensional Pattern Representation Space

The system proposed to be used for representing the pattern is the familiar $x-y$ coordinate system. The pattern is assumed to lie entirely within the top right-hand quadrant of this $x-y$ coordinate system. It is bounded on the left by the $y$-axis; the bottom by the $x$-axis; to the right and top by the outer periphery of the pattern. The: lengths defining the boundary of the pattern space are finite. Within this space, a vector of unit length $\mathrm{V}_{0}$ is considered to lie along the $+x$-axis, starting at the origin and moving away from it. Its rotation counter-clockwise is considered positive and clockwise, negative. Its movement away from the origin is considered positive and towards it, negative. This is illustrated in Figure 1.1. From here on, the term "pattern space" refers to the physical pattern representation space.


Figure 1.1 The Two-dimensional Pattern Space

## CHAPTER II

## SYNTACTIC PATTERN DESCRIPTION

### 2.0 Introduction to Pattern Description

The logical first step in the recognition of twodimensional patterns is their description. The basic building block in this formalism is the unit vector. By means of operations on the unit vector and the transformation of the resulting vectors thereof, the entities of dots, lines and areas are derived. Within this formalism, the concept of chaining, coupled with the vehicle of grammars, enable the description of arbitrary two-dimensional pattern forms in terms of these three entities. Specific patterns are obtained through parametrization of the pattern forms. These ideas are covered in detail in the following sections of this chapter.
2.1 First Set of Operations on Unit Vector

Two operators on the unit vector are defined.

1. Length modifying operator, denoted by "q."
2. Rotation modifying operator, denoted by " $\theta$."

By temporarily shifting the origin to a point within the pattern space, any arbitrary vector within this space may be represented by the letter "V" as

$$
\begin{array}{ll}
v=q \theta V_{0^{\prime}} \quad & 0 \leq q \leq q_{\max } \\
& 0 \leq \theta \leq 2 \pi \tag{2.1}
\end{array}
$$

The pattern entities of dots, lines and areas are derived from vectors in a unified manner. The first set of operations deal with the derivation of dots from vectors to accomplish the first of these objectives.

If the vector is treated as a non-terminal entity, one of three alternatives is possible.

1. Replace the vector by a point or dot at the arrowhead of the vector.
2. Replace the vector by a point at the starting point of the vector.
3. Replace the vector by a null value.

It is possible to show that the above three entities are obtained through the operator $q$; but for the present study the above statements suffice. The above operations may be represented symbolically as shown below.

$$
\begin{align*}
& \left(q \ominus \mathrm{~V}_{0}\right):=\square  \tag{2.2}\\
& \left(q \ominus \mathrm{~V}_{0}\right):=\square  \tag{2.3}\\
& \left(q \ominus \mathrm{~V}_{0}\right):=\square \tag{2.4}
\end{align*}
$$

Since $q \quad \theta \mathrm{~V}_{0}$ represents an arbitrary vector V , the above can simply be written as

$$
\begin{equation*}
v::=\square * \tag{2.5}
\end{equation*}
$$

$\mathrm{V}::=$ 田 *
$\mathrm{V}::=\Phi$

### 2.2 Chaining

The concept of chaining is an important one in this formalism. Chaining allows the derivation of complex patterns from simpler ones.

Two types of chaining are defined.

1. Static chaining
2. Dynamic chaining

In static chaining, the point at which chaining occurs remains fixed, while in dynamic chaining, this shifts to the end point of the newly chained vector. This concept is best illustrated by the graphic example as seen in Figure 2.1.

It should be noted that in static chaining, the order in which the vectors are chosen is not important, while in

```
The symbols chosen reflect the idea of looking at a point
when it is located at the tip of the arrow (2.2) and (2.5)
or at its tail (2.3) and (2.6).
```





$a=$ CHAINING POINT BEFORE $z=$ CHAINING POINT AFTER


Figure 2.1 Examples of static and dynamic chaining. dynamic chaining, the order determines the final pattern form derived. Two distinguished symbols are introduced at this time. They are the letters "S" and "D." $S$ stands for a statically chained sub-pattern, while $D$ stands for a dynamically chained sub-pattern. Further, symbols enclosed within parenthesis indicate statically chained pattern elements and square brackets are used to indicate dynamic chaining. Entities within these delimeters are taken in a strictly left-to-right sequence in the chaining process. To keep the number of brackets used to a minimum, the following expressions are equivalent and the simplest expressions are used.

```
1. (V(V)) = (V) (V) = (VV) static
2. [V[V]] = [V][V] = [VV] dynamic
```


### 2.3 Dot Grammar

The operations on unit vector described in section 2.1 together with chaining described in section 2.2 enable the description of any arbitrary collection of points or "dots" within the two-dimensional space chosen. This grammar is, therefore, referred to as the "dot grammar."

Dot Grammar

1. $P:=:=[Q P]|(Q P)|[Q] P|(Q) P| Q P|[Q]|(Q) \mid Q$
2. $Q:==S \mid D$
3. $D::=[V] D \mid[V]$
4. $S:=(\mathrm{V}) \mathrm{S} \mid \mathrm{V})$
5. $V::=f_{1}(h) \square\left|f_{2}(h) \square\right| \Phi$

Discussion
Productions 1 and 2 state that a dot pattern is derived from an arbitrary combination of statically and dynamically chained vectors. Production 3 indicates that a dynamically chained dot pattern is derived from an arbitrary number of vectors dynamically chained together. Please note that the vectors are enclosed in square brackets, implying dynamic chaining. Production 4 is a repeat of production 3, but for
statically chained vectors, as indicated by round brackets. Production 5 shows the reduction of a vector to a point at its tip ( $\square$ ) with an associated color function $f_{I}(h)$ or at its tail ( $⿴$ ) with its associated color function $f_{2}(h)$ or a null value $\Phi$. The above grammar merely derives the form of the dot pattern. Parametrization of this form yields specific patterns.

### 2.4 Dot Grammar Examples

## Example \#1

Consider the following derivation of the dot grammar.

$$
\begin{aligned}
P & \Rightarrow Q P \Rightarrow D P \Rightarrow\left[V_{a}\right] P \Rightarrow\left[\square_{a}\right] P \Rightarrow\left[Q_{a}\right] Q \\
& \Rightarrow\left[\square_{a}\right] S \Rightarrow\left[\square_{a}\right]\left(V_{1}\right) S \Rightarrow\left[\square_{a}\right]\left(\square_{1}\right) S \Rightarrow> \\
& \Rightarrow\left[\square_{a}\right]\left(\square_{1}\right)\left(V_{2}\right) S \Rightarrow>\Rightarrow\left[\square_{a}\right]\left(\square_{1} \square_{2} \ldots \square_{8}\right)(2.10)
\end{aligned}
$$

Derivation 2.10 represents all dot pattern forms that comprise eight points $\left(\square_{1} \square_{2} \cdots\left(\square_{8}\right)\right.$ about a point $\left[\square_{a}\right]$ which is located away from the origin.

Case \#1 Consider the following parametrization of the above derivation. Since $\mathrm{V}_{0}$ is unit length, it is omitted.

$$
\begin{aligned}
\text { Since } v_{i}= & q_{i} \theta_{i}, \text { with } q_{i}=q_{i+1}=d \quad i=1,3,5,7 \\
& \text { and } \theta_{i}=(i-1) \times \Pi / 4 ; \theta_{1}=0 \quad i=1,2, \ldots 8
\end{aligned}
$$

and $v_{a}=q_{a} \theta_{a}$, with $q_{a}=c$ and $\theta_{a}=\Pi / 4$ yields the dot pattern show in Figure 2.2.


Figure 2.2 Dot pattern example \#1, case \#l

Case \#2 Consider the following parametrization of the same derivation in (2.10).

$$
\begin{array}{ll}
q_{i}=d-(d / 8) i ; & q_{1}=d \\
\theta_{i}=(i-1) \times H / 4 ; & \sigma_{1}=0
\end{array}
$$

and $q_{\dot{a}}=c$ and $\theta_{a}=\mathbb{E} / 4$ yields the dot pattern shown in Figure 2.3.

## Example \#2

Consider the following derivation of the dot grammar.

$$
\begin{aligned}
P & \Rightarrow Q P \Rightarrow D P \Rightarrow\left[V_{1}\right] P \Rightarrow\left[\square_{1}\right] Q P \Rightarrow\left[\square_{1}\right] D P \\
& \Rightarrow\left[\square_{1}\right]\left[V_{2}\right] D P \Rightarrow\left[\square_{1}\right]\left[\square_{2}\right] D P \Rightarrow\left[\square_{1}\right]\left[Q_{2} \square_{3} \square_{4}\right] P
\end{aligned}
$$



Figure 2.3 Dot pattern example \#1, case \#2
$\Rightarrow\left[\square_{1}\right]\left[\square_{2} \square_{3} \square_{4}\right] \quad Q P=>\Rightarrow\left[\square_{1}\right]\left[\square_{2} \square_{3} \square_{4}\right]\left[\square_{5}\right]\left[\square_{6} \square_{7}\left[\square_{8}\right]\right.$ (2.11)

The pattern form represented by the above derivation is a set of dynamically chained points, but grouped together as shown for convenience.

Case \#l Consider the following parametrization of the derivation in (2.11)

Since $v_{i}=q_{i} \theta_{i}$
a. $\quad q_{1}=a_{1} ; \theta_{1}=\pi / 6$
b. $\quad q_{2}=q_{3}=q_{4}=q_{6}=q_{7}=q_{8}=d$ $\theta_{2}=\theta_{3}=\theta_{4}=\theta_{6}=\theta_{7}=\theta_{8}=\Pi / 4$
c. $\quad q_{5}=d_{2} ; \theta_{5}=3 \pi / 4$

This yields two parallel sets of 4 dots each as shown in Figure 2.4.


Figure 2.4 Dot pattern example \#2, case \#1.
Case \#2 For the same derivation as in (2.11) consider the following parametrization.
a. $q_{1}=d_{1} ; \theta_{1}=\Pi / 6$
b. $q_{2}=q_{3}=q_{4}=d_{2}$
$\dot{\theta}_{2}=\Pi / 6 ; \theta_{3}=5 \pi / 6 ; \theta_{4}=3 \Pi / 2$
c. $q_{5}=d_{3} ; \theta_{5}=\Pi / 2$
d. $q_{6}=a_{7}=q_{8}=d_{4}$
$\theta_{6}=\pi / 6 ; \theta_{7}=5 \pi / 6 ; \theta_{8}=3 \pi / 2$
This yields a dot pattern, with the dots located at
the corners of two equilateral triangles as shown in Figure 2.5.


Figure 2.5 Dot pattern example \#2, case \#2.
2.5 Classification of Line Patterns

A line pattern may be thought of as being made up of "Iine-components." In this formalism, two types of linecomponents are defined.

1. Straight lines.
2. Curves.

Even though a straight line is a special case of a curve, it is given special consideration because of the importance it plays in two-dimensional line patterns. Further,
straight line patterns may be described in terms of vectors themselves as the primitives.

A straight "line-component" is defined to be that part of a curve wherein the slope remains constant for all points within that line-component.

A curve "Iine-component" is defined to be that part of a curve wherein there is a continuous change of slope along the points that lie on that line-component. Figure 2.6 illustrates some line patterns composed of straight and curve line-components.

## 2. 6 Straight Line Grammar

If the dot grammar presented in section 2.3 is stripped of those productions that replace the vectors by dots, then the grammar that results describes all arbitrary straight line pattern forms and is presented below for completeness. Discussion of each production is dispensed with, however.

Straight line grammar

$$
\begin{align*}
& \text { 1. } \mathrm{D}::=[Q P]|(Q P)|[Q] P|(\Omega) P| Q P|[Q]|(Q) \mid Q \\
& \text { 2. } Q::=S \mid D \\
& \text { 3. } D::=\left[V^{\prime}\right] D \mid\left[V^{\prime}\right] \\
& \text { 4. } S::=\left(V^{\prime}\right) S \mid\left(V^{\prime}\right) \\
& \text { 5. } \quad V^{\prime}::=£(h) V \mid \Phi \tag{2.12}
\end{align*}
$$



## STRAIGHT LINE

 PATTERN

## STRAIGHT LINE \& CURVE PATTERN



CURVE PATTERN

Figure 2.6 Examples of straight line and curve patterns
2.7 Straight Line Grammar Eramoles

## Example \#1

Consider the following derivation of the straight line grammar.

$$
\begin{align*}
P & \Rightarrow Q P \Rightarrow D P \Rightarrow\left[V_{a}\right] P \Rightarrow\left[V_{a}\right] Q P \Rightarrow\left[V_{a}\right] S P \Rightarrow\left[V_{a}\right]\left(V_{1} V_{2} V_{3}\right) P \Rightarrow\left(V_{1} V_{2} V_{3}\right) Q \Rightarrow \\
& \Rightarrow\left[V_{a}\right]\left(V_{1} V_{2} V_{3}\right) D \Rightarrow\left[V_{a}^{\prime}\right]\left(V_{1} V_{2} V_{3}\right)\left[V_{2}^{\prime} V_{3}^{\prime} V_{4}^{\prime} V_{5}^{\prime} V_{6}^{\prime}\right]
\end{align*}
$$

The above derivation represents a line pattern form composed of a vector dynamically chained to the origin followed by three statically chained vectors and five dynamically chained vectors.

For one parametrization (not shown), the line pattern obtained may be as shown in Figure 2.7.


Figure 2.7 Straight line pattern, example \#l.

## Example \#2

This example presents a more interesting linepattern, which is encountered in several areas of study and application. Consider the following derivation of the straight line grammar.

$$
\begin{array}{r}
P \Rightarrow Q \Rightarrow D_{0} \Rightarrow\left[V_{1}\right] D_{1} \Rightarrow\left[V_{1} V_{2}\right] D_{2}^{-} \Rightarrow\left[V_{1} V_{2} V_{3}\right] D_{3} \Rightarrow\left[V_{1} V_{2} V_{3} V_{4}\right] \\
(2.14)
\end{array}
$$

Please Note. Subscripts are associated with the distinguished symbol D to show sub-patterns. Choosing the following parameters we have -

$$
\begin{aligned}
q_{1} & =q_{2}=q_{3}=q_{4}=1 \text { (unit length) } \\
\text { and } \theta_{1} & =0 ; \theta_{2}=-\Pi / 2 ; \theta_{3}=0 ; \theta_{4}=+\Pi / 2,
\end{aligned}
$$

The sub-patterns of the above derivation are as shown in Figure 2.8a.


Figure 2.8a

By repeating the above process, square wave patterns of unit lengthare obtained as shown in Figure 2.8b.


Figure 2.8b
By varying only the value chosen for $q_{1}$, the patterns shown in Figure 2.8c are obtained.

- In general, the derivation given in this example, with $\theta_{i}, \theta_{i+2}=0 ; \theta_{i+1}=-\pi / 2 ; \theta_{i+3}=+\pi / 2$ and the appropriate choice of values for $q$ all arbitrary square wave patterns are defined as shown in Figure 2.8 .


### 2.8 Second Set of Operations On Unit Vector

Consider the following derivation of the straight line grammar.

$$
P=Q=S=\left(V_{1}\right) S=\left(V_{1} V_{2}\right)
$$

Since $V_{i}=q_{i} \Theta_{i} V_{0}$, the above derivation may be graphically


Figure 2.8c


Figure 2.8d
represented as shown in Figure 2.9. with 1 and 2 labelling the end points of the two vectors. For small change in the values of $q$ and $\theta$ between vectors $V_{1}$ and $V_{2}$, they constitute what might be termed an "incremental vector pair."


Figure 2.9 Incremental vector pair for curves

The letter "v" is used for incremental vector pairs. In instances where $\theta_{2}=\theta_{1} \pm \delta \theta_{1}$ and $q_{2}=q_{1} \pm \delta q_{1}$, the two vectors may be replaced by the resultant vector. The above operation on the incremental vector pair is shown symbolically as follows.

$$
\begin{equation*}
\{\ddot{v}(1,2)\}::=\left(v_{1} v_{2}\right) \tag{2.15}
\end{equation*}
$$

In the above notation, the left part comprises the two vectors $v_{1}$ and $v_{2}$ and is shown in parentheses implying static chaining. The right part is shown in chain brackets and means that it is a resultant vector. Its subscripts indicate that the resultant vector originates at point 1 and
ends at point 2. Further, chaining with the resultant vector, if any, should occur with 2 as the starting point. The general form of the above operation may be symbolically represented as shown below.

$$
\begin{equation*}
\left\{v_{(i, i+2)}\right\}::=\left\{v_{(i, i+1)}\right\}\left(v_{i+2}\right) \tag{2.16}
\end{equation*}
$$

The significance of this operation on the vectors must be obvious. It enables the syntactic description of curves in this formalism, as presented in the following section.

### 2.9 Classification of Curves

A curve may be thought of as being made up of "curvecomponents." A distinguished symbol, which is the letter "C," is introduced at this time and stands for a curvecomponent.

A curve-component is defined to be that portion of a curve between two "curve-points." If vectors are drawn from the origin to points on a curve, then for a curve component the following conditions hold.

1. The sign of the $x$ and $y$ component change of incremental vector pairs for all points on the curve-component is consistent.
2. The sign of the slope change along the curvecomponent is consistent.

A curve-point is reached when either of the above conditions is not met. This idea is best illustrated by a
figure as shown in Figure 2.10.


Figure 2.10 Illustration of curve-components

The origin is shown shifted to a point 0 within the pattern space. From this new origin, vectors $o a, o b$ and oc are drawn to points $a, b$ and $c$ on the circumference of the circle. In order to find the curve points, the following steps are performed.

First, find the x and y components of these vectors. Then $\Delta x$ between oa and ob is - and $\Delta y$ between oa and ob is +. Likewise, $\Delta x$ is - and $\Delta y$ is + between $o b$ and oc.

Next draw tangents to points $a, b$ and $c$ on the curve.

These give the slope along the curve. The sign of slope change is + between $a$ and $b$ (being counter-clockwise). The same is true between points $b$ and $c$.

These two conditions hold good for all points between $p$ and $q$ on the curve. However, beyond $p$ and $q$ the above conditions don't hold. Therefore, pq constitutes a curvecomponent and $p$ and $q$ are curve-points. Similarly, qr., rs. and $s p$ are curve-components.

In light of the distinction already made between straight lines and curves, there are only 4 curve component types possible. These correspond to the types indicated in Figure 2.12 when a curve is traversed around its periphery in a counter-clockwise direction and their negative counterparts when traversed in a clockwise direction.

In this formalism, all curve patterns are described in terms of these four curve-component types. One should be careful not to misread this statement to mean that a curve is made up of an arbitrary combination of arcs of a circle, but go back to the definition of the curve components.

### 2.10 Curve Grammar

The following grammar, referred to as "curve grammar" describes all curve patterns as arbitrary combination of curve component types 1 - 4 as defined in section 2.9 .

## Curve Grammar

1. $P \quad:==[Q P]|(Q P)|[Q] P|(Q) P| Q P|[Q]|(Q) \mid Q$
2. $Q \quad:==S \mid D$
3. $D \quad:==\left[C_{i}\right] D \mid\left[C_{i}\right] \quad i=1,2,3,4$
4. $\mathrm{S} \quad::=\left(C_{i}\right) S \mid\left(C_{i}\right)$
5. $C_{i} \quad::=f(h)\left\{v_{(1, n)}\right\} \mid f(h)\left\{v_{(1,2)}\right\}$
6. $\left\{v_{(1, n)}\right\}::=\left\{v_{(1, n-1)}\right\}\left(v_{n}\right)$
7. $\left\{v_{(1,2)}\right\}:=\left(v_{1} v_{2}\right)$

## Discussion

Productions 1 and 2 state that a curve pattern is an arbitrary combination of statically and dynamically chained curve sub-patterns. Production 3 shows a dynamically chained curve pattern to be an arbitrary number of curve component types 1,2,3 and 4 dynamically chained together. Production 4 is the same as production 3, except that it applies to statically chained curve patterns. Production 5 states that a curve component is an elemental curve (the right alternative) or a series of elemental curves (the left alternative), each with an associated color function $f(h)$. Production 6 shows the curve to be composed of a series of elemental curves. Production 7 shows an elemental curve to be the resultant of an incremental vector pair that are chained together statically. As before, this grammar derives the form of curve patterns. Specific patterns are derived through parametrization of the form obtained.

### 2.11 Curve Grammar Examples

## Example \#1

Consider the following derivation of the curve grammar given in section 2.10.

$$
\begin{aligned}
P & \Rightarrow Q \Rightarrow D \Rightarrow\left[C_{1}\right] D \Rightarrow \Rightarrow\left[c_{1} c_{2} c_{3} c_{4}\right] \\
C_{1} & \Rightarrow\left\{v_{(1, n)}\right\} \\
& \Rightarrow\left\{v_{(1, n-1)}\right\}\left(v_{n}\right) \\
& \Rightarrow\left\{v_{(1, n-2)}\right\}\left(v_{n-1}\right)\left(v_{n}\right) \\
& \left.\Rightarrow \Rightarrow \Rightarrow v_{1} v_{2} v_{3} \ldots v_{n}\right)
\end{aligned}
$$

Consider the following parameters.
Choose $q_{1}=q_{2}=\ldots=q_{n}=1$ (unit length) and

$$
\delta \theta=+(\pi / 2) / n .
$$

The pattern obtained is shown in Figure 2.11. (The derivation for curve components 2,3 and 4 are not shown.)


Figure 2.11 Curve grammar example \#1

Example \#2 Use the same derivation as before, but choose the following parameters

$$
q_{i+1}=q_{i}-(d / 2 \Pi) \times \delta \theta
$$

The pattern obtained is a spiral as shown in Figure 2.12 in which the radius decreases by "d" units per revolution.


Figure 2.12 Curve grammar example \#2

### 2.12 Third Set of Operations on Unit Vector

The ideas expressed in this section for areas are analogous to those already expressed for curves in section 2.8. Consider the following derivation of the straight line grammar.

$$
P \Rightarrow Q \Rightarrow s \Rightarrow\left(V_{1}\right) s \Rightarrow\left(V_{1} V_{2}\right)
$$

The graphical representation for the above derivation is as shown in Figure 2.13.


Figure 2.13 Incremental vector pair for areas
For small increments in the value of $q$ and $\theta$, the vectors $V_{1}$ and $V_{2}$ are replaced by the incremental vector pair $\mathrm{v}_{1}$ and $\mathrm{v}_{2}$. They may be joined together by the resultant vector, $\mathrm{V}_{(1,2)}$ thereby forming an elemental area. A new distinguished symbol, " $\Delta$," is introduced at this point to represent this elemental area. The operation on the incremental vector pair described above may be symbolically expressed as follows.

$$
\begin{equation*}
\{\Delta(1,2)\}::=\left(v_{1} v_{2}\right) \tag{2.18}
\end{equation*}
$$

As with curve-components, where more than one incremental vector pair is involved, the above expression may be written in the general form as

$$
\begin{equation*}
\{\Delta(i, i+2)\}::=\{\Delta(i, i+1)\} \quad\left(v_{i+2}\right) \tag{2.19}
\end{equation*}
$$

### 2.13 Area Grammar

In order to retain the consistency in the formalism developed, it is logical to think of an area as being made up of "area-components," similar to a curve being made up of curve-components.

An area-component of an area is one that is subtended by a portion of the area periphery at the origin. It seems a natural extension of the ideas expressed thus far to associate an area-component with the curve type that subtends the area at the origin, i.e., the total area is composed of individual area-components, which are identified by the curve points on the periphery of the area. There are, therefore, four area-components corresponding to the four curve-component types as defined earlier.

Additionally, it must be recognized that part of an area periphery may be a straight line. Even though only one straight line-component type was defined in the grammars presented so far, it is logical to define four types of straight lines, corresponding to the four curve-component types identified. Accordingly, vectors are defined as follows.

```
Vi
where }\quad\mp@subsup{V}{i}{}=\mp@subsup{q}{i}{}\mp@subsup{0}{i}{
    such that 0 \leq q}\mp@subsup{|}{i}{}\leq\mp@subsup{q}{max}{
    and }\quad\mp@subsup{0}{i}{}\mathrm{ is defined as shown.
```

| Value of i | Range of $\theta$ |
| :---: | :--- |
| 1 | $\pi / 2<\theta \leq \pi$ |
| 2 | $\Pi<\theta \leq 3 \pi / 2$ |
| 3 | $3 \pi / 2<\theta \leq 0$ |
| 4 | $0<\theta \leq \pi / 2$ |

The above definition adds four more types to the possible types of area-components making it a total of eight. The letter "A" is used as the distinguished symbol to identify an area-component. The following "area grammar" describes all arbitrary area patterns in terms of the eight area-components.

Area-Grammar

1. $P::=[Q P]|(Q P)|[\Omega] P|(Q) P| Q P|[Q]|(Q) \mid Q$
2. $Q:=: S \mid D$
3. $D::=\left[A_{i}\right] D \mid\left[A_{i}\right] \quad i=1-8$
4. $S:=\left(A_{i}\right) S \mid\left(A_{i}\right)$
5. $A::=f(h)\{\Delta(1, n)\} \mid f(h)\{\Delta(1,2)\}$
6. $\left\{\Delta_{(1, n)}\right\}::=\left\{\Delta_{(1, n-1)}\right\}\left(v_{n}\right)$
7. $\left\{\Delta_{(1,2)}\right\}::=\left(v_{1} v_{2}\right)$

Since this grammar is analogous to the curve grammar, discussion by each production is omitted.

### 2.14 Area Grammar Examples

Example \#I
Consider the following derivation of the area grammar
presented in section 2.13.

$$
\begin{aligned}
P \Rightarrow Q & \Rightarrow D \Rightarrow\left[A_{1}\right] D \\
& \Rightarrow[\{\Delta(1, n)\}] D \\
& \Rightarrow\left[\{\Delta(1, n-1)\}\left(v_{n}\right)\right] D \\
& \Rightarrow \Rightarrow \Rightarrow \\
& \Rightarrow\left[\left(v_{1} v_{2} \ldots v_{n}\right) A_{2} A_{3} A_{4}\right]
\end{aligned}
$$

Choosing $q_{1}=q_{2}=\ldots q_{n}=1$ and $\delta \theta=+(\pi / 2) / n$ for the parameters, the area (shaded quadrant) shown in Figure 2.14 is obtained for the derivation shown. Derivations for $A_{2}$, $A_{3}$ and $A_{4}$ are omitted, which constitute the unshaded portion of the circle.


Figure 2.14 Area grammar example \#l.
Example \#2
Consider the following derivation.

$$
\begin{aligned}
P & \Rightarrow \Rightarrow \ldots\left[A_{1} A_{1}^{\prime}\right] \\
& \Rightarrow \quad\left[\{\Delta(1, n)\} A_{1}^{\prime}\right] \\
& \Rightarrow \quad\left[\left(v_{1} v_{2} \ldots v_{n} l A_{1}^{\prime}\right]\right.
\end{aligned}
$$

As before, choosing $q_{1}=q_{2}=\ldots q_{3}=1$
and $\delta \theta=+(\pi / 2) / n$, for all the vectors
generated thus far, the pattern obtained is the top righthand quadrant in Figure 2.15. In generating the quadrant, the periphery of the area is traversed counter-clockwise and the area computed is positive. Now the rest of the derivation is shown.

$$
\begin{aligned}
& \Rightarrow \quad\left[\left(v_{1} v_{2} \ldots v_{n}\right) A^{\prime} \cdot\right] \text { from before } \\
& \left.\Rightarrow \quad\left[v_{1} v_{2} \ldots v_{n}\right)\left(v_{1}^{\prime} \ldots v_{n}^{\prime}\right)\right]
\end{aligned}
$$



Fig. 2.15 Area grammar example \#2

For the second set of vectors derived, for each $\delta 0$ choose $q$ such that ba is a straight line. The area generated is the unshaded triangle in the upper right-hand quadrant. Since the periphery of this part of the area is traversed clockwise while generating it, this area is negative and gets subtracted from the area for the first part of the derivation, leaving the shaded segment shown in Figure 2.15.

### 2.15 Two-Dimensional Pattern Grammar

The gramar presented below describes any arbitrary two-dimensional pattern in terms of dot, curve, straight line and area-components as the primitives. It is given the acronym "D-L-A" grammar and stands for dots, lines and areas grammar.

The D-L-A grammar is a consolidation of the individual grammars presented in the preceding sections. There is a logical subdivision of this grammar into two parts, The first part, referred to as the "Macro" grammar, describes the pattern in terms of the primitives mentioned above. It is important to recognize, that the macro grammar and its parametrization would vary from one problem to another, but the framework would remain the same and cut across problem areas. The second part, referred to as the "Micro" gramar, is problem independent and shows how the primitives may be syntactically described for all problem areas.

## D-L-A Grammar For Two-Dimensional Patterns

$$
\begin{array}{lll}
\text { 1. } P::=[Q P]|(Q P)|[Q] P|(Q) P| Q P|[Q]|(Q) \mid Q & : \\
\text { 2. } Q::=S \mid D & \text { macro } \\
\text { 3. } D::=[T] D \mid[T] & \text { grammar } \\
\text { 4. } S::=(T) S \mid(T) & \\
\text { 5. } T::=W\left|f(h) c_{i}\right| f(h) A & \begin{cases}j=1-8 \\
i=1-4\end{cases} &
\end{array}
$$

The following defined for each $V_{i}, C_{i}$, and $A_{j}$.
7. $v::=f\left(h\left\{\left\{v_{(1, n)}\right\} \mid f(h)\left\{v_{(1,2)}\right\}\right.\right.$

9. $\left\{\mathrm{v}_{(1,2)}\right\}::=\left(\mathrm{v}_{1} \mathrm{v}_{2}\right)$
10. $\left.C::=f(h)\left\{v_{(1, n)}\right\} \mid f(h){ }^{\{ } V_{(1,2)}\right\}$
11. $\left\{v_{(1, n)}\right\}:=:=\left\{v_{(1, n-1)}\right\}\left(v_{n}\right)$
12. $\left\{\mathrm{v}_{(1,2)}\right\}::=\left(\mathrm{v}_{1} \mathrm{v}_{2}\right)$
13. $\left.A \quad:=:=f(h){ }_{\{\Delta}^{(1, n)}\right\} \mid f(h)\{\Delta(1,2)\}$
14. $\left\{\Delta_{(1, n)}\right\}::=\{\Delta(1, n-1)\}\left\{v_{n}\right\} \quad$ Micro
15. $\left\{\Delta_{(1,2)}::=\left(v_{1} v_{2}\right)\right.$ Grammar

The following constraints apply to productions 5 and 6 and relate to straight line and curve-component primitives that use the subscript i.


The area-component primitives shown in production 5 map into straight line-component and curve-component primitives as given below.

$$
\begin{array}{ll}
A_{j}=V_{i} & j=5,8 ; i=1,4 \\
A_{j}=C_{i} & j=1,4 ; i=1,4 \tag{2.22}
\end{array}
$$

Since this grammar is a consolidation of the individual grammars presented earlier, discussion by each production is omitted.

## CHAPTER III

## SYNTACTIC PATTERN RECOGNITION

### 3.0 Concept of A Two-Dimensional Pattern Recognizer

The proverb "a picture paints a thousand words" is familiar to most of us. In line with this maxim, the concept of the recognizer within this formalism is best presented in the form of a picture as seen in Figure 3.1. It is hoped that the levity of this characterization will not detract from the seriousness of this study.

The recognition scheme may be thought of as being composed of three distinct logical steps. In step 1 , the scanner receives impulses of reflected light from the twodimensional pattern to be recognized and converts them into signals that can be processed by the recognizer. Step 2 accepts the signals from the scanner and converts them into dot, straight line, curve and area-component primitives in terms of which the pattern is defined. This step is given the name "Micromaton," since it is an automaton that is driven by the micro grammar component of the D-L-A grammar. Step 3 uses these primitives for carrying out the recognition and is named "macromaton." This phase may be thought


Fig. 3.1 The two-dimensional pattern recognition system
of as being driven by the macro grammar component of the D-I-A grammar.

### 3.1 Scanner Construction and Function

The method of scanning the pattern to provide input to the recognizer must be compatible with the formalism proposed. A conceptual diagram of the scanner is presented in Figure 3.2.


Fig. 3.2 Scanner assembly

The scanner consists of an eye that is capable of focussing its sight on a point in the pattern mounted below. The head, housing the eye, is attached to a radial arm that can move back and forth in a guide. The guide itself is mounted on a pillar in such a way that it can rotate about the center-line of the pillar.

From the arrangement just described, it is clear that with the radial motion of the head coupled with the rotation of the guide about the center-line of the pillar, the eye can effectively cover a circular area, whose center is the center-line of the pillar and whose radius is the maximum outward extension of the radial arm. The two-dimensional pattern to be recognized is mounted on the base of the scanner assembly as shown in Figure 3.3.

The input to the eye is a beam of light reflected oif the pattern. At the issuance of a signal, the scanner samples the input to the eye, which consists of the color (h) and its associated intensity level (g). The sole purpose of the scanner is to sample this input from the pattern, make certain decisions and put out data, which in essence represent the pattern. This then forms the input to the recognizer.

In what follows, the arm-eye assembly is referred to as the "sweep-vector." The location of the eye along the arm measures the distance of points on the pattern from the


Fig. 3.3 Pattern location in scanner assembly
origin, while the position of the arm measures the angle of the line joining the point and the origin with the + x-axis. The arm-eye assembly therefore gives the vectors defining the points on the pattern as it sweeps across the pattern. Hence the name. Further, the movement of the eye radially outward for any given angular position of the arm is referred to as a "probe." In practice, it is convenient to think of the
scanner as a selfecontained unit that performs its function with little or no external intervention. The only outside intervention is in the form of a set of parameters supplied to it that govern its actions and are as follows.

1. Incremental distance along the probe at which the pattern is to be sampled.
2. Angular increment of the arm from one probe to the next.
3. A set of color functions that differentiate between those aspects of the pattern that are of interest from those that are not. Also, for each color of interest, a set of colors and threshold intensities, which may be considered the same as the color of interest.

### 3.2 Scanner Structure

The structure of the scanner may be formally described by a deterministic finite transducer. It is defined by an 8-tuple-
$T=(Q, \Sigma, F(C), S, F, I, \Delta, \delta)$

Where
$Q=\operatorname{Set}$ of states $\{S, A, D, F\}$ where $S=$ Start state

A = Active state
$D=$ dormant state
F = Final state
$\Sigma=$ Domain of the scanner
$=$ Set of inputs $\left(q_{i}, \theta, h_{i} g_{i}\right) \quad i=1,2 \ldots n$
where $q_{i}=$ distance of point sampled from ori-
$g$ in with $q_{0}=0$ and $q_{n}=q_{\max }$ for any
probe. $\left(q_{i+1}-q_{i}\right)=d$ the distance
between samples along the probe.
$\theta=$ angle of the arm with the $+x$ axis
with $\theta_{\text {min }}=0$ and $\theta_{\text {max }}=\pi / 2$.
$h_{i}=$ color of point sampled
$g_{i}=$ intensity of $h_{i}$
$F(c)=\left\{f_{i}(c)\right\} \cup\left\{f_{0}(c)\right\}$
where $\left\{f_{i}(c)\right\}=$ set of color functions defined
for the active state, where each
$f_{i}(c)$ is defined as
$=\left\{c_{i_{1}},\left(c_{i_{1}}, t_{i_{1}}\right),\left(c_{i_{2}}, t_{i_{2}}\right), \ldots\right.$.
$\left.\ldots\left(c_{i_{n}}, t_{i_{n}}\right)\right\}$
In the above set, $c_{i}$ denotes the color and
( $c_{i_{j}}, t_{i_{j}}$ ) denote the colors and threshold inten-
sity values assumed to represent the same value
as $c_{i}$. Each $f_{i}(c)$ is distinct from the other.
$\left\{f_{0}(c)\right\}$ is the complement of $\left\{f_{i}(c)\right\}$ and is
defined for the dormant state for all colors
of no interest. Its form is the same as shown
for $\left\{f_{i}(c)\right\}$.

S, FعQ as described above
I = Set of internal configurations of three types identified by $G H$ and $J$ where $G=\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,\left(n_{1} n_{2}\right), v\right)$ $H=\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta,\left(n_{1} n_{2}\right), J\right)$ $J=(\Phi, \Phi, \Phi, \Phi, Ј \mathbf{J})$ Type $G$ is defined for a point $q_{i}$ and angle $\theta$ with an associated color function $f_{i}(c)$ while type $H$ is defined for a series of consecutive points starting at $q_{i}$ and ending at $q_{j}$ with a probe angle of $\theta$ and color function $f_{i}(c)$. $q_{m}=$ maximum length of probe to the pattern boundary, $n_{1}$ indicates the "transition from" color, while $n_{2}$ indicates the "transition to" color during the scanning process. This is used to indicate the overlap of primitives and is defined as shown. $n_{1}, n_{2} \dot{\varepsilon}\{0,1\}$ where $0=$ transition from or to the background color which is not of interest. $1=$ transition from or to a color of interest. $J$ assumes values from the positive index set and identifies the probe number. Type $J$ is a dummy type defined to indicate the end of scan to the Hicromaton and only..has the probe number $J$ 'in it.

```
\Delta = Range of the scanner
    = Set of outputs that is the same as one of the
        three internal types G, H or J defined above.
    \delta= Mapping from finite subsets of Q x \ x I* xF(c)
        to finite subsets of Q x I* x |* and specified
        as shown, for any }0\mathrm{ .
        1. }\delta(S,(\mp@subsup{q}{i}{},0,\mp@subsup{h}{i}{\prime}\mp@subsup{g}{i}{}),\Phi,\mp@subsup{f}{0}{\prime}(c)
        = (D,\Phi,\Phi)
```

    2. \(\delta\left(S,\left(q_{i}, \theta, h_{i} g_{i}\right), \Phi, f_{i}(c)\right)\)
        \(=\left(A,\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0 ; 0), J\right), \Phi\right) \mid\)
        \(=\left(S, \Phi,\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0,0), J^{\prime}\right) q_{i}=q_{\max }\right.\)
    3. \(\delta\left(D,\left(q_{i}, \theta, h_{i} g_{i}\right), \Phi, f_{0}(c)\right)\)
        \(=(D, \Phi, \Phi) \mid\)
        \(=\left(S, \Phi,\left(\Phi, \Phi, \Phi, \Phi, J^{\prime}\right) q_{i}=q_{\text {max }}\right.\)
    4. \(\delta\left(D,\left(q_{i}, \theta, h_{i} g_{i}\right), \Phi_{i} f_{i}(c)\right)\)
        \(=\left(A_{1}\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0,0), J\right), \Phi\right) \mid\)
        \(\left(S, \Phi,\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0,0), J^{\prime}\right)\right) q_{i}=q_{\max }\)
    5. \(\delta\left(A,\left(q_{i}, \theta, h_{j} g_{j}\right),\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0,0), J\right), f_{i}(c)\right)\)
    \(=\left\langle A,\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta,(0,0), J\right), \Phi\right) \mid\)
        \(\left(S, \Phi,\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta,(0,0), J^{\prime}\right)\right) q_{j}=q_{\max }\)
    6. \(\delta\left(A,\left(q_{j}, \theta, h_{j} g_{j}\right),\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0,0), J\right), f_{0}(c)\right)\)
        \(=\left(D, \Phi,\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0,0), J\right)\right) \mid\)
        \(\left(S, \Phi,\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0,0), J^{\prime}\right)\right) q_{j}=q_{\text {max }}\)
    7. \(\delta\left(A,\left(q_{j}, \theta, h_{j} g_{j}\right),\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0,0), J\right), f_{k}(c)\right)\)
        \(=\left(A,\left(\left(q_{j}, \Phi\right), f_{k}(c), \theta,(1,0), J\right)\right.\),
    $$
\begin{aligned}
& \left.\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(0,1), J\right)\right) \mid \\
& \left(S, \Phi,\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,(1,0), J\right),\right. \\
& \left.\left(\left(q_{j}, \Phi\right), f_{k}(c), \theta_{i}(0,1), J^{\prime}\right)\right) q_{j}=q_{\max } \\
& \text { 8. } \left.\quad \delta\left(A,\left(q_{\ell}, \theta, h_{\ell}{ }_{\ell}\right),\left(q_{i}, q_{j}\right), f_{i}(c), \theta,(0,0), J\right), f_{i}(c)\right) \\
& =\left(A,\left(\left(q_{i}, q_{\ell}\right), f_{i}(c), \theta,(0,0), J\right) \mid\right. \\
& \left(S, \Phi,\left(\left(q_{i}, q_{\ell}\right), f_{i}(c), \theta,(0,0), J^{\prime}\right)\right) q_{\ell}=q_{\text {max }} \\
& \text { 9. } \delta\left(A,\left(q_{\ell}, \theta_{,} h_{\ell}{ }_{\ell}\right),\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta ;(0,0), J\right), f_{0}(c)\right) \\
& =\left(D, \Phi,\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta,(0,0), J\right)\right. \\
& \left(S, \Phi,\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta,(0,0), J^{\prime}\right) q_{i}=q_{\max }\right. \\
& \text { 10. } \delta\left(A,\left(q_{\ell}, \theta, h_{\ell} g_{\ell}\right),\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta,(0,0), J\right), f_{k}(c)\right) \\
& =\left(A,\left(\left(q_{\ell}, \Phi\right), \Phi_{\ell}(c), \theta,(1,0), J\right),\right. \\
& \left.\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta,(0,1), J\right)\right) \mid \\
& \left(S, \Phi,\left\{\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta,(0,1), J^{\prime}\right),\right.\right. \\
& \left.\left.\left(\left(q_{\ell}, \Phi\right), f_{\ell}(c), \theta,(1,0), J^{\prime}\right)\right\}\right) q_{\ell}=q_{\max }
\end{aligned}
$$

In all the above mappings, when $q=q_{\text {max }}$, replace $S$ by $F$ if $\theta=\pi / 2$; output type $J$, with $J!=J^{\prime \prime}$ and stop.

Discussion of Mappings. Note. Each mapping is explained below by its number. In the above mappings, J' indicates end of probe.

1. Move from start state to dormant state if the input color and intensity level are defined by a color function that is not of interest.
2. Move from start state to active state if input color and intensity are defined by a color function of interest. Store type $G$ of internal configuration.

However, if this happened at the end of a probe, output type $G$ and return to start state.
3. Stay in the dormant state as long as input color and intensity levels are defined by a color function that is not of interest. If end of probe is reached, revert to start state and output a null value.
4. Move from dormant state to active state if input color and intensity are defined by a color function of interest, store type $G$ internal configuration. If this happens at the end of a probe, output type $G$ and revert to start state.
5. Stay in the active state as long as the input color and intensity is defined by the same color function. However, go from internal configuration type $G$ to type $H$. Output type $H$ and return to start state if end of probe is reached.
6. Move from active state to dormant state if input color and intensity are defined by a color function not of interest; output type G. If end of prove is reached, output type $G$ and return to start state.
7. Stay in the active state if input color and intensity level are defined by a color function different to the current one, with type $G$ internal configuration for the new color. Indicate

> color transition $n_{2}=1$ for current configuration and $n_{1}=1$ for the new color. Output type $G$ and active state for the current color. However, if end of probe is reached, output type $G$ and start state for the current color and new color with the same values for $n_{1}$ and $n_{2}$; return to startstate. $$
8,9 \text { and } 10 \text { are repetitions of } 5,6 \text { and } 7,
$$ but type $H$ is considered instead of type $G$.

### 3.3 Micromaton Structure

The function of the Micromaton is tomap the set of vectors with their associated colors received from the scanner, to a set of dot, straight line, curve and area-component primitives. The domain of the Micromaton is the range of the scanner. The range of the Micromaton is the set of dot, straight line, curve and area component primitives that comprise the pattern. Figure 2.3 presents the logical components of the Micromaton.

The Micromaton is, therefore, in a formal sense a transducer. It is characterized as a series of automata which are driven by a conmon set of states. In some states, it acts as the controlling element performing its overall function; in others; it acts in the capacity of one of its logical components to accomplish specific functions. In its overall function, it has one set of input (the data from the scanner) and one set of output (the primitives formed,output to the Macromaton). In each of its component states, it has


Fig. 3.4 Micromaton structure
a set of input and output which are internal to it.
This section presents the description of its overall function. A recap of the discussion on the Micromaton is presented in section 3.8 with a state transition diagram. In order to gain a general understanding of the Micromaton
action, it might be advantageous to review section 3.8 before proceeding further.

The structure of the Micromaton is described by a deterministic finite transducer. It is a l0-tuple

$$
T=\left(Q, \Sigma, \lambda, D^{\prime}, N, O, \Delta, S, F, \delta\right) \text { where }
$$

$Q=$ Set of common states
$\left\{S_{1}, S_{2}, S_{2}^{\prime}, S_{3}, S_{4}, W, E, Z\right\} d e f i n e d$ as shown.
$S_{\text {I }}=$ New primitive creation state
$S_{2}=$ Connectivity recognition state
$S_{2}^{1}=$ Connectivity recognition return state
$S_{3}=$ Reduction state
$\mathrm{S}_{4}=$ Output state
W = Wait state
E = End State
$z=$ Terminate state
In the above, $W, E$ and $Z$ are the controling
states. The rest are component states.
$\Sigma=$ Input received from the scanner
Type $G, H$ or $J$ as described in section 3.2
$\lambda=$ Primitive set in Micromaton internal storage.
$(\mathrm{P},(\Omega, \Psi))$ in which
$\mathrm{P}=$ Set of primitives.
$=\left\{p_{i}\right\}=\left\{p_{d}\right\} \quad U^{\prime}\left\{p_{\ell}\right\}$
$p_{d}$ above represent the unlinked dot primitives and $p_{\ell}$ the linked straight, line; curve anü area-component primitives. Each $p_{i}$ above is either $p_{0}$ or $p_{N} \cdot p_{0}$ is a primitive which is overlapped with others while $P_{N}$ are the non-overlapped primitives.
$\Omega=$ Set of pointers to all the primitives that are "active" and eligible for reduction.
$=\{r\}$ where each $r$ points to one $p_{i} \varepsilon P$. All $r$ are linked together in $\Omega$.
$\Psi=$ Set of pointers to all primitives that are "inactive" and not eligible for reduction.
$=\left\{a, l_{i}, c_{i}, a_{j}\right\}$ in which $i=1-4 ; j=1,8$ and defined as shown.
$\mathrm{d}=\left\{r_{\mathrm{d}}\right\}$ where each $r_{\mathrm{d}}$ points to a dot primitive peP. All $r_{d}$ are linked tom gether.
$\ell_{i}=\left\{r_{\ell_{i}}\right\}$ where each $r_{\ell_{i}}$ points to a straight line primitive $p \varepsilon P$ of type $i$. All $r_{\ell_{i}}$ are linked together. $c_{i}=r_{c_{i}}$ where each $r_{c_{i}}$ points to a curve component primitive peP of type i.. All $r_{c_{i}}$ are linked together.
$a_{j}=r_{a_{j}}$ where each $r_{a_{j}}$ points to an area component primitive pep of type $j$. All $r_{a_{j}}$ are linked together.
The following notations are introduced here and have the meaning shown.

| Notation | Meaning |
| :---: | :---: |
| $\Omega<=r_{p_{i}}$ | Add a pointer to the list of pointers to the active primitive set pointing to primitive $p_{i} \varepsilon P$. |
| $\Omega=>r_{p_{i}}$ | Remove a pointer from the list of pointers to the active primitive set pointing to primitive $p_{i} \varepsilon$. |
| $\Psi<=r_{p_{i}}$ | Add a pointer to the list of pointers to the inactive primitive set pointing to primitive $p_{i} \varepsilon P$. The list corresponds to the type of $p_{i}$. |
| $\Psi \Rightarrow r_{p_{i}}$ | Remove a pointer from the list of pointers to the inactive primitive set pointing to primitive $p_{i} \varepsilon P$. The list corresponds to the type of $p_{i}$. |
| $\mathrm{P}=\mathrm{P} \dot{\cup}\left\{\mathrm{p}_{\mathrm{d}_{\mathrm{i}}} \neq \mathrm{p}_{\ell_{4}} \ddagger \mathrm{p}_{\mathrm{d}_{j}}\right\}$ |  |
| (Cther notations |  |
| similar to the one correspond to dot primitives for $q_{i}$ and |  |
| above are easily understood and not | $q_{i}$ of $\Sigma$ and $p_{\ell_{4}}$ is a straight line |

## Meaning

explicitly shown here.)
primitive of type 4 corresponding to
$q_{i}, q_{j} . .$. doubly link the primitives as shown by the arrows.
$D^{\prime}=$ Distance criterion.
$=\{\Phi, 0,1\}$ and defined as shown.
$D=\Phi$ undefined (don't care)
$D=0$ if $\left(q_{j}-q_{i}\right)>d$
$D=1$ if $\left(q_{j}-q_{i}\right) \leq d$
$D=1$ if $\left(q_{i}-\Phi\right)$
In the above, $q_{i}, q_{j}$ are elements in $G$ and $H$ of $\Sigma$
and $d$ is the connectivity distance used in scanning
the pattern.
$\mathrm{N}=$ New primitive creation index.
$\{\Phi, 1,2,3\}$ and defined as shown.
$\Phi=$ Undefined (don't care)
$I=$ Create new primitive corresponding to $q_{i}$ of $\Sigma$.
$2=$ Create new primitive corresponding to $q_{j}$ of $\Sigma$.
$3=$ Create new primitive corresponding to $q_{i}$ and
$q_{j}$ of $\Sigma$.
$4=$ Create new primitive corresponding to $q_{i}$ and
$q_{j}$ and a straight line primitive corresponding
to $q_{i}, q_{j}$.
In the above $q_{i}$ and $q_{j}$ are elements in $G$ and $H$ of
$\Sigma$.
$0=$ Output internal to the Micromaton.
$\left(\lambda, \Sigma, N, S^{\prime}\right)$ in which
$\lambda, \Sigma$ and $N$ are as defined. $S^{\prime}$ is the state to which return should be made upon completion of the state being entered. S' $\varepsilon Q$ or $\Phi, \Phi$ being undefined.
$\Delta=$ The final output of the Micromaton.
$\lambda^{\prime}=(P,(\Phi, \Psi))$ in which $P$ and $\Psi$ are as defined.
$S=$ Start state of Micromaton.
$=W \varepsilon Q$ as defined
$F=$ Final state of Micromaton.
$=Z \varepsilon Q$ as defined
$\delta=$ Mapping from finite subsets of $Q \times \Sigma^{*} \times D^{\prime *} \times \lambda^{*}$ to finite subsets of $Q \times O^{*} X \Delta^{*}$ and is defined as shown.

1. $\delta(W, \Phi, \Phi, \Phi)=(W, \Phi, \Phi)$
2. $\delta(W, \Phi, \Phi, \Omega)=(W, \Phi, \Phi)$
3. $\delta(W, G, 1, \Phi)=\left(S_{1},(\Phi, G, I, W), \Phi\right)$
4. $\delta\left(W_{i}, \because, 1, \Phi\right)=\left(S_{1},(\Phi, H, 3, W), \Phi\right)$
5. $\delta(W, E, O, \Phi)=\left(S_{1},(\Phi, H, 4, W), \Phi\right)$
6. $\delta(T, J, 0, \Omega)=\left(S_{4},(\lambda, \Phi, \Phi, W), \Phi\right)$
7. $\delta(W, G, \Phi, \Omega)=\left(S_{2},(\lambda, G, \Phi, \Phi), \Phi\right)$
8. $\delta(W, H, \Phi, \Omega)=\left(S_{2},(\lambda, H, \Phi, \Phi), \Phi\right)$
9. $\delta\left(W, J^{\prime \prime}, \Phi, \Omega\right)=\left(S_{4},(\lambda, \Phi, \Phi, E), \Phi\right)$
10. $\delta(E, \Phi, \Phi, \Omega)=\left(Z, \Phi, \lambda^{\prime}\right)$

Discussion of mappings.

1. Continue in the wait state as long as there is no input from scanner and the active primitive set is empty.
2. Same as 1 except, the active primitive set is not empty.
3. For type $G$ input with the active primitive set empty, move to new primitive create state; request new primitive creation for $q_{i}$ and return to wait staice.
4. Same as 3 except, request new primitive creation for $q_{i}$ and $q_{j}$.
5. Same as 4 with the addition of request for creation of a straight line primitive if points $q_{i}$ and $q_{j}$ are farther apart than $d$.
6. When there is a type $J$ input from the scanner, move to output state with a request to return to wait state. This is also done in cases of input types $G$ and $H$ where $J$ is $J$ 'indicating the end of probe. This has not been explicitly shown.
7. With active primitive set not empty and an input of type $G$, move to connectivity check state. Pass primitive set and input type G.
8. Same as 7 except type $H$ is involved.
9. When the scanner has reached its terminating
state, pass primitive set and enter output state, requesting return to end state.
10. In the end state, output primitive set representing the pattern and terminate.

### 3.4 New Primitive Creation Function

This component of the Micromaton creates new primitives when an input from the scanner is not connected to any of the currently active primitives in Micromaton's temporary storage.

This function is characterized by a deterministic finite automaton. It is a 6-tuple -

```
T = (Q',I,O,S,F,\delta) where
    Q' = Set of states
        Q'\subseteqQ defined in section 3.3.
    I = Input to this component and internal to
        Micromaton.
        = (\lambda, \Sigma,N,S'). This is the output 0 defined
        in section 3.3.
        O = Output of this component; internal to Micro-
        maton.
        = \lambda as defined in section 3.3.
        S = Start state of this component.
        = S S & Q as defined in section 3.3.
```

F = Final states of this component:
$=\left\{W, S_{2}\right\} \subseteq Q$ as defined in section 3.3.
$\delta=$ Mapping from finite subsets of $Q \times I$ to finite subsets of $Q \times O$ and is defined as shown.

1. $\delta\left(\dot{S}_{1},(\Phi, G, I, W)\right)$

$$
=\left(W,\left(P=\left\{p_{d_{i}}\right\}, \Omega=r_{p_{d_{i}}}\right)\right)
$$

2. $\delta\left(S_{1},(\Phi, H, 2, W)\right)$

$$
=\left(W,\left(P=\left\{p_{d_{i}} \neq p_{d_{j}}\right\}, \Omega=r_{p_{d_{i}}}<=r_{p_{d_{j}}}\right)\right)
$$

3. $\delta\left(\mathrm{S}_{1},(\Phi, \mathrm{H}, 3, \mathrm{~W})\right)$

$$
=\left(W_{r}\left(P=\left\{p_{d_{i}} \neq p_{\ell_{4}} \neq p_{d_{j}}\right\}, \Omega=r_{p_{d_{i}}}<=r_{p_{d_{j}}}, \psi<=r_{p_{\ell_{4}}}\right)\right)
$$

4. $\delta\left(S_{1},\left(\lambda, G, I, S_{2}\right)\right)$

$$
=\left(S_{2},\left(P=P \cup\left\{p_{d_{i}}\right\}, \Omega<=r_{p_{d_{i}}}\right)\right)
$$

5. $\delta\left(S_{1},\left(\lambda, H, 2, S_{2}\right)\right)$

$$
=\left(S_{2},\left(P=P \cup\left\{p_{d_{i}} \neq p_{d_{j}}\right\}, \Omega \varepsilon=r_{p_{d_{i}}}<=r_{p_{d_{j}}}\right)\right)
$$

6. $\delta\left(S_{1},\left(\lambda, H, 3, S_{2}\right)\right)$

$$
=\left(S_{2},\left(P=P \cup\left\{p_{d_{i}} \neq p_{\ell_{4}} \neq p_{d_{j}}\right\}, \Omega<=r_{p_{d_{i}}}<=r_{p_{j}}, \Psi<=r_{p_{\ell}}\right)\right)
$$

## Discussion of Mappings

1. Create a new dot primitive corresponding to $q_{i}$ of G. Make this dot primitive the primitive set. Create a pointer to this primitive and make this the new active primitive pointer list.
2. Same as l except, two dot primitives are created corresponding to $q_{i}$ and $q_{j}$ of $H$. The two dot primitives are linked together pointing to each other to show connectivity. Pointers corresponding to these two primitives are added to the active primitive pointer list as in 1.
3. Here a straight line primitive is involved in addition to the two dot primitives. The two dot primitives are linked to the straight line primitive. The primitive set and pointers to the active primitive set are created as described for mapping 2. The straight line primitive type 4 is added to the appropriate list of pointers in the inactive primitive set. In all three mappings above, return is made to the wait state after this state.

4, 5 and 6 are a restatement of 1, 2 and 3, except that the primitive set is not empty and request is made for return to the connectivity check state.

### 3.5 Connectivity Recognition Function

Under this scheme, the concept of connectivity needs special mention. In an $x-y$ raster scanning scheme, connectivity is established by virtue of adjacency in a two-dimensional matrix, provided the squares of the matrix are color compatible. This is illustrated in Figure 3.5.


Fig. 3.5 Connectivity of points in raster scanning

In Figure 3.5, those squares numbered 1 through 8 are connected to square 0 if they are color compatible. In the present formalism, the idea of connectivity is different in view of the method by which the pattern is scanned. Here, the notion of connectivity is in the form of a circle that surrounds the point in question as shown in Figure 3.6. The connectivity distance "d" may be arbitrarily chosen and it determines how finely or coarsely a pattern is scanned. The end points of the vectors are said to be connected if the length of the resultant vector is $\leq d$ and the points are color compatible, otherwise not.

The connectivity recognizer is a deterministic finite automaton and is an ll-tuple -

$$
T=\left(Q^{\prime}, I, D^{\prime}, K, C, \gamma, O_{S}, O, S, F, \delta\right) \text { where }
$$



Fig. 3.6 Connectivity of points in radial scanning $Q^{\prime}=$ Set of states $Q^{\prime} \subseteq Q$ defined in section 3.3 . $I=$ Input to this component and internal to Micromaton.
$=(\lambda, \Sigma, N, S)$. This is the output $O$ defined in section 3.3.
$D^{\prime}=$ Distance criterion
$=\{\Phi, 0,1\}$ as defined in section 3.3
K = Connectivity check outcome
$=\left(k_{1}, k_{2}\right), k_{1}, k_{2} \varepsilon\{0,1\}$ and defined as shown. $k_{1}=$ Outcome of connectivity check between $q_{i}$ of $\Sigma$ and the last vector defining the primitive $p$ in the active primitive set. $k_{2}=$ Same as $k_{1}$ and applies to $q_{j}$ of $\Sigma$.

```
C = Count of the number of primitives that satis-
    fied the connectivity criterion.
    = ( }\mp@subsup{c}{1}{},\mp@subsup{c}{2}{})\mathrm{ where
        c}\mp@subsup{]}{1}{\prime}=\mathrm{ Number of primitives p in the active primi-
        tive set for which connectivity criterion
                was satisfied for }\mp@subsup{q}{i}{}\mathrm{ of }\Sigma\mathrm{ .
        c
        At start c}\mp@subsup{c}{1}{}=\mp@subsup{c}{2}{}=0
\gamma = Next selection function for the next primitive
        to be checked in the active primitive set for
        connectivity and defined as shown.
        \gamma(\Omega)=The primitive p pointed to by ri+1; the
                current value in \Omega being ri.
        \gamma(\Omega)=\Phi when there are no more pointers in the
        list representing the active primitive
        set:
    O
        = {0,1,2} and defined as shown.
        O = Hold output
        I = Output O
        2 = Output O
O = Output internal to the Micromaton defined in
        two forms corresponding to two states that are
        entered from the connectivity check state.
        O_ = (\lambda, \Sigma,N,S'). This is the same as O in
```

section 3.3 passed to the new primitive creation state. $O_{2}=\left(P_{R^{\prime}} \Sigma, S!\right)$ where $P_{R}=$ Set of primitives that need to undergo reduction with the input $\Sigma$ from the scanner.
$=\left\{\left(p_{i} K_{i}\right)\right\}$ in which $p_{i}$ is an active primitive and $K_{i}$ is the connectivity established for $p_{i}$ with type $G$ or type $H$ input from scanner. $\Sigma=$ Input from scanner. $S^{\prime}=$ State to which return is to be made after completion of the state being entered.
s = Start State
$=S_{2} \varepsilon Q$ as defined in section 3.3
F = Final state
$=\left\{S_{3}, W\right\} \subseteq Q$ as defined in section 3.3
$\delta=$ Mapping from finite subsets of QxI*x K*x C* $\mathrm{X}^{\prime *}$ to finite subsets of $Q \times I * \times C \times O_{S} \times O_{1} \times O_{2}$ and specified as shown.

1. $\delta\left(S_{2},(\mathrm{p}, \mathrm{G}),(0, \Phi),(0,0), \Phi\right)$

$$
=\left(S_{2}, \gamma(\Omega),(0,0), 0, \Phi, \Phi\right)
$$

2. $\delta\left(S_{2},(p, H),(0,0),(0,0), \Phi\right)$
$=\left(S_{2}, \gamma(\Omega),(0,0), 0, \Phi, \Phi\right)$
3. $\delta\left(S_{2},(p, G),(1, \Phi),\left(c_{1}, c_{2}\right), \Phi\right)$

$$
=\left(S_{2}, \gamma(\Omega),\left(c_{1}=c_{1}+1, c_{2}\right), 0, \Phi,\left(P_{R}=P_{R} \cup(p, K), G, W\right)\right)
$$

4. $\delta\left(S_{2},(p, H),(1,0),\left(c_{1}, c_{2}\right), \Phi\right)$

$$
=\left(S_{2}, \gamma(\Omega),\left(c_{1}=c_{1}+1, c_{2}\right), 0, \Phi,\left(P_{R}=P_{R} \cup(p, K), H, W\right)\right)
$$

5. $\delta\left(S_{2},(\mathrm{p}, \mathrm{H}),(0,1),\left(\mathrm{c}_{1}, \mathrm{c}_{2}\right), \Phi\right)$

$$
=\left(S_{2}, \gamma(\Omega),\left(c_{1}, c_{2}=c_{2}+1\right), 0, \Phi,\left(P_{R}=P_{R} \cup(p, K), H, W\right)\right)
$$

ह. $\delta\left(S_{2},(p, H),(1, I),\left(c_{1}, c_{2}\right), \Phi\right)$

$$
=\left(S_{2}, \gamma(\Omega),\left(c_{1}=c_{1}+1, c_{2}=c_{2}+1\right), 0, \Phi,\left(P_{R}=P_{R} \cup(p, K) H, W\right)\right)
$$

7. $\delta\left(S_{2},(\Phi, G), \Phi,(0,0), \Phi\right)$

$$
=\left(S_{1}, \Phi, \Phi, I,(\lambda, G, I, W), \Phi\right)
$$

8. $\delta\left(S_{2},(\Phi, H), \Phi,(0,0), y\right)$

$$
=S_{1}, \Phi, \Phi, 1,(\Omega, H, 3,(\mathbb{W}), \Phi)
$$

9. $\delta\left(S_{2},(\Phi, H), \Phi,(0,0), 0\right)$

$$
=\left(S_{1}, \Phi, \Phi, I,(\Omega, H, \cdot, W), \Phi\right)
$$

10. $\delta\left(S_{2},(\Phi, H), \Phi,\left(C_{1}, 0\right), \Phi\right)$

$$
=\left(S_{1}, \Phi, \Phi, I,\left(\Omega, H, 2, S_{2}^{\prime}\right), O_{2}\right) .
$$

11. $\delta\left(S_{2},(\Phi, H), \Phi,\left(0, C_{2}\right), \Phi\right)$

$$
=\left(S_{1}, \Phi, \Phi, I,\left(\Omega, H, I, S_{2}^{1}\right), O_{2}\right)
$$

12. $\delta\left(S_{2}^{\prime},(\Phi, \Phi), \Phi, \Phi, \Phi\right)$

$$
\begin{equation*}
=\left(S_{3}, \Phi, \Phi, 2, \Phi, O_{2}\right) \tag{3.4}
\end{equation*}
$$

Discussion of mappings.

1. With type $G$ input, if connectivity is not established with the current primitive, select the next active primitive.
2. Same as 1 , but type $H$ is shown.
3. If connectivity is established for type $G$ input with an active primitive, add to it the set of primitives to be reduced. Add 1 to the number of primitives satisfying connectivity check for $q_{i}$. (No output is made at this point.)
4., 5 and 6 are a repeat of 3 but for type $H$ input and different combinations of connectivity for $q_{i}$ and $q_{j}$
4. If all active primitives have been checked for connectivity and none of them are connected to type $G$ input, then request primitive creation for $q_{i}$ and return to wait state.

8 and 9 are a restatement of 7 , but for type $H$ input and different values of distance criteria.

10 and ll. Here, either $q_{i}$ or $q_{j}$ is connected to one or more primitives in the active set for input type $H$, but not both. Therefore, create ä new primitive for $q_{i}$ or $q_{j}$ for which connectivity is not established and return made to the connectivity recognition state in preparation for entering the reduction state.
12. Pass the set of primitives for which connectivity is established, to the reduction state, requesting return to the wait state.

## 3. 6 Micromaton Reduction Function

A few preliminary comments are in order before describing the reduction function of the Micromaton. The recognition of curve-components needs special mention. The reduction function needs to know when to stop reduction of the input from the scanner with one curve component type and start reduction with another type. Consider the circle in Figure 3.7 with the origin located within the circle.


Fig. 3.7 Origin within the closed curve

As the sweep-vector rotates in a counter-clockwise direction, if the change in its $x$ and $y$ components where it intersects the curve-component type 1 are computed, the relations $\Delta x<0$ and $\Delta y>0$ hold. Similarly, for curve-component type $2 \Delta x<0$ and $\Delta y<0$; for curve-component type $3 \Delta x>0$ and $\Delta y<0$ and for curve-component type $4 \Delta x>0$ and $\Delta y>0$. In all these cases,
the periphery of the circle is traversed in a counter-clockwise direction by the sweep-vector, where it makes contact with it.

The above relations don't hold if the origin is shifted away from within the circle as shown in Figure 3.8.


Fig. 3.8 Origin outside the closed curve

As the sweep-vector rotates counter-clockwise, it makes contact with the circle at point $F$ and leaves contact at point E. The arc FABE is intersected by the sweep-vector in a counter-clockwise direction, while, the arc FDCE is intersected in a clockwise direction along the periphery of the circle. This results in curve-components $C_{4}(F A), C_{1}(A B)$, $C_{2}(B E)$ for arc $F A B E$ and $-C_{4}(F D),-C_{3}(D C)$ and $-C_{2}(C E)$ for arc FDCE.

If the $\Delta x$ and $\Delta y$ for the various curve-components are computed, the results summarized in Table 3.1 is obtained.

Table 3.1
Table of curve-component types with origin outside closed curve.

| No. | Arc | Curve-component | $\Delta x$ | $\Delta y$ | Sense of <br> Rotation |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | FA | $C_{4}$ | $C_{1}$ | + | + |
| 2 | AB | $C_{2}$ | - | + | + |
| 3 | BE | $-C_{4}$ | - | - | + |
| 4 | FD | $-C_{3}$ | - | - | + |
| 5 | DC | $-C_{2}$ | - | + | - |
| 6 | CE |  | + | + | - |

From an examination of the above table it is clear that, for correctly identifying the curve-component types, the direction in which the sweep-vector intersects the periphery of the curve needs to be considered in addition to the sign of the change in the values of the $x$ and $y$ components at the points of intersection. Table 3.1 may be restated to aid the reduction function in correctly identifying the curvecomponent types. This is shown in Table 3.2. Because of the manner in which area-components are defined in relation to straight line and curve-components, the above discussion holds for them as well.

The reduction process makes use of two types of signals from the scanner. They are -

Table 3.2
Table of relations for deducing curve-component types

| No. | $\Delta x$ | $\Delta y$ | Sense of <br> Rotation | Curve-Component <br> Type |
| :---: | :---: | :---: | :---: | :---: |
| 1 | - | + | + | $C_{1}$ |
| 2 | - | + | - | $C_{3}$ |
| 3 | + | + | + | $C_{4}$ |
| 4 | + | + | - | $C_{2}$ |
| 5 | - | - | - | $C_{4}$ |
| 6 | - | - | + | $C_{2}$ |

1. Type G given by $\left(\left(q_{i}, \Phi\right), f_{i}(c), \theta,\left(n_{1}, n_{2}\right), J\right)$ and
2. Type $H$ given by $\left(\left(q_{i}, q_{j}\right), f_{i}(c), \theta,\left(n_{1}, n_{2}\right), J\right)$ (3.5) Type $G$ may be rewritten as $\left(\left(q_{i}, \theta_{j}\right), f(c),\left(n_{1}, n_{2}\right), J\right)$ which in turn may be rewritten as $\left(v_{i}, f(c),\left(n_{1}, n_{2}\right), J\right)$ since there is only one $q_{i}$ to be dealt with. Type $H$ may similarly be written as $\left(\left(v_{i}, v_{j}\right), f(c),\left(n_{1}, n_{2}\right), J\right)$.

In general, type $G$ input defines dots and lines, whereas, type $H$ defines areas. (One exception is the case where type $H$ defines a straight line that coincides with the sweep-vector.) A sequence of type $G$ and type $H$ inputs defining portions of a curve and an area are illustrated in Figure 3.9.

The reduction process makes use of incremental vector pairs defined earlier. It computes the $\Delta x$ and $\Delta y$ between the vector pairs as well as the sign of the angle change between


Fig. 3.9 $\begin{aligned} & \text { Input types defining portions of curve and } \\ & \text { area }\end{aligned}$
consecutive resultant vectors. It compares the computed values from the previous step with those of the present step in its reduction decision. Additionally, it computes the cumulative value of the resultant vector length as well as the area subtended by the area-component at the origin. Even though length and area do not play any role in the reduction process, they are passed as data that characterize the primitive, to the Macromaton.

Let $v_{1}, v_{2}, \ldots, v_{n}$ be the consecutive vectors making up the incremental vector pairs that need to be reduced by the reduction process. The following sets are defined.

1. Set of incremental vector pairs

$$
=\left\{\left(v_{i}, v_{i+1}\right)\right\}=\left\{\left(\Phi, v_{1}\right),\left(v_{1}, v_{2}\right), \ldots,\left(v_{n}, v_{n+1}=\Phi\right)\right\}
$$

2. Set of changes in $x$ and $y$ components of
incremental vector pairs
$=\left(\Delta x y_{i}\right)=\left\{\Delta x y_{1}=\Phi, \Delta x y_{2}, \Delta x y_{3}, \ldots, \Delta x y_{n+1}=\Phi\right\}$
3. Set of resultant vectors
$=\left\{R_{i}\right\}=\left\{R_{1}=\Phi, R_{2}, R_{3}, \ldots, R_{n+1}=\Phi\right\}$
4. Set of angles of resultant vectors witi2 $+x$-axis
$=\left\{S_{i}\right\}=\left\{S_{1}=\Phi, S_{2}, S_{3}, \ldots, S_{n+1}=\Phi\right\}$
5. Set of sign change between consecutive resultant vectors

$$
\begin{equation*}
=\left\{\Delta S_{i}\right\}=\left\{\Delta S_{1}=\Phi, \Delta S_{2}=\Phi, \Delta S, \therefore, \Delta S_{n+1}=\Phi\right\} \tag{3.6}
\end{equation*}
$$

$\Delta x y_{i}$ defined in 2 above actually contains a pair, i.e., $\Delta x$ and $\Delta y$. Each may assume any value from the set $\{<0,=0,>0\}$. In particular, the following combinations of $\Delta x$ and $\Delta y$ are defined as follows.

$$
\begin{align*}
& \alpha=-\Delta x,+\Delta y \text { (i.e. } \Delta x<0 \text { and } \Delta y>0) \\
& \beta=-\Delta x,-\Delta y \\
& \gamma=+\Delta x,+\Delta y \tag{3.7}
\end{align*}
$$

The reduction process itself may be described by a deterministic finite automaton. To keep the discussion simple, only the mappings are described. The mappings use the symbols and their associated meanings as shown below.

$$
\begin{aligned}
& P=\text { Primitive type set } \\
& =\left\{D, L_{i}, C_{i}, A_{j}\right\} \\
& \text { where } D=\text { Dot primitive } \\
& \qquad \begin{aligned}
L_{i} & =\text { straight line } \quad i=1,4 \\
C_{i} & =\text { curve-component } \quad i=1,4
\end{aligned}
\end{aligned}
$$

```
    A
VP = Set of incremental vector pairs- Item l
\Deltaxy = Set of x-y component changes - Item 2 (3.6)
    R = Set of resultant vectors - Item 3
    S = Set of resultant vector angles - Item4
    \DeltaS = Set of resultant vector slope changes
                                    Item 5 (3.6)
    I = Type G or. H input from Scanner See
    (3.5)
    O = Set of intermediate results describing the
        primitive reduced up to this step.
        The intermediate primitive is described by
        the following elements.
        1. (v}\mp@subsup{|}{1}{},\mp@subsup{v}{i}{})\mathrm{ where }\mp@subsup{v}{1}{}\mathrm{ defines the starting point
        of primitive and }\mp@subsup{v}{i}{}\mathrm{ its current ending point
        2. f(c) the color function of this primitive
        3. L' showing the link of this primitive with
        the one that precedes or follows it.
        4. R' showing the cumulative length of the
        resultant vectors defining this primitive.
        5. A' showing the cumulative area subtended by
        this primitive at the origin.
        6. t the tag number identifying the probe num-
        ber for which the primitive underwent
        reduction.
        7. # the reduction id. that signifies whether
        the primitive is to be reduced with }\mp@subsup{q}{i}{}\mathrm{ or
```

$q_{j}$ of scanner input $\Sigma$.
Please Note. In order to keep the mappings from getting cumbersome, only element number 1 of the intermediate result above is shown in the mappings that follow.
$\delta=$ Mapping from finite subsets of

into finite subsets of
$\mathrm{P} \times \mathrm{O} \times \mathrm{VP} \times \Delta \mathrm{xy} \times \Delta \mathrm{S} \times \mathrm{O}^{*}$
and is defined as follows.

1. $\delta\left(D,\left(v_{1}, \Phi\right),\left(\Phi V_{1}\right)\left(V_{1} V_{2}\right), \Phi, \Delta x Y_{2}, \Phi, \Phi, G\right)$

$$
=\left(D,\left(v_{1} v_{2}\right),\left(v_{1} v_{2}\right), \Delta x y_{2}, \Phi, \Phi\right)
$$

2. $\delta\left(D,\left(v_{1} v_{2}\right),\left(v_{1} v_{2}\right)\left(v_{2} v_{3}\right), \Delta x y_{2}, \Delta x y_{3}, \Phi, \Delta S_{3}, G\right)$

$$
=\left(I,\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right), \Delta x y_{3}, \Delta S_{3}, \Phi\right)
$$

$$
\text { if } \Delta x y_{2}=\Delta x y_{3} \text { and } \Delta S_{3}=0
$$

3. $\quad=\left(C_{1},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right), \Delta x y_{3}, \Delta S_{3}, \Phi\right)$ if $\Delta x y_{2}=\Delta x y_{3}=\alpha$ and $\Delta S_{3}=+$
4. $\quad=\left(C_{2},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right), \Delta x y_{3}, \Delta S_{3}, \Phi\right)$ if $\Delta x y_{2}=\Delta x y_{3}=\beta$ and $\Delta S=+$
5. $\quad=\left(C_{4},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right), \Delta x y_{3}, \Delta S_{3}, \Phi\right)$ if $\Delta x y_{2}=\Delta x y_{3}=\beta$ and $\Delta S_{3}=-$
6. $\quad=\left(C_{3},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right), \Delta x y_{3}, \Delta S_{3}, \Phi\right)$ if $\Delta x y_{2}=\Delta x y_{3}=\alpha$ and $\Delta S_{3}=-$
7. $=\left(C_{4},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right), \Delta x y_{3}, \Delta S_{3}, \Phi\right)$ if $\Delta x y_{2}=\Delta x y_{3}=\gamma$ and $\Delta S_{3}=+$
8. $\quad=\left(C_{2},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right), \Delta x Y_{3}, \Delta S_{3}, \Phi\right)$ if $\Delta x y_{2}=\Delta x y_{3}=\gamma$ and $\Delta S_{3}=-$
9.     - 14. are repetition of mappings 3-8 for input
type H. The states shown on the right should be replaced as follows. $C_{1}=A_{1} ; C_{2}=A_{2}$;

$$
C_{3}=A_{3} ; C_{4}=A_{4} .
$$

15. $\delta\left(L_{1},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right)\left(v_{i} v_{i+1}\right), \Delta x y_{i}, \Delta x y_{i+1}, \Delta S_{i}, \Delta S_{i+1} G\right)$

$$
\begin{aligned}
& =\left(L,\left(v_{1} v_{i}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Delta S_{i+1}, \Phi\right) \\
& \quad \text { for } i=3,4, \ldots, m ; m \neq n \\
& \quad \text { and } \Delta x y_{i}=\Delta x y_{i+1} \text { and } \Delta S_{i}=\Delta S_{i+1}=0
\end{aligned}
$$

16. $=\left(L, \Phi, \Phi, \Phi, \Phi,\left(L,\left(v_{1} V_{n}\right)\right)\right)$
for $i=n$
and $\Delta x y_{i-1}=\Delta x y_{i}$
and $\Delta S_{i}=0$
17. 

$$
\begin{gathered}
=\left(D,\left(v_{i}, v_{i+1}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Phi,\left(L_{1},\left(v_{1}, v_{i}\right)\right)\right) \\
\quad \text { for } i=3,4, \ldots, m ; m \neq n \\
\quad \text { and } \Delta x y_{i}=\Delta x y_{i+1} \text { and } \Delta S_{i}=\Delta S_{i+1}=0
\end{gathered}
$$

18. $\delta\left(C_{1},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right)\left(v_{i} v_{i+1}\right), \Delta x y_{i}, \Delta x y_{i+1}, \Delta S_{i}, \Delta S_{i+1}, G\right)$

$$
\begin{aligned}
& =\left(C_{1},\left(v_{1} v_{i}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Delta S_{i+1}, \Phi\right) \\
& \quad \text { for } i=3,4, \ldots, m ; m \neq n \\
& \quad \text { and } \Delta x y_{i}=\Delta x y_{i+1}=\alpha \text { and } \Delta S_{i}=\Delta S_{i+1}=+
\end{aligned}
$$

19. $\quad=\left(C_{1^{\prime}}, \Phi, \Phi, \Phi,\left(C_{1},\left(V_{I^{\prime}} V_{n}\right)\right)\right)$
for $i=n$
and $\Delta x y_{i-I}=\Delta x y_{i}=\alpha$
and $\Delta S_{i}=+$
20. $=\left(D,\left(v_{i}, v_{i+1}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Phi,\left(C_{1},\left(v_{1}, v_{i}\right)\right)\right)$
for $i=3,4, \ldots, m ; m \neq n$
and $\Delta x y_{i}=\alpha \neq \Delta x y_{i+1}$
or $\Delta S_{i}=+\neq \Delta S_{i+1}$
set $i=1$ for dot primitive part
21. $\delta\left(\mathrm{C}_{2},\left(\mathrm{v}_{1} \mathrm{v}_{3}\right),\left(\mathrm{v}_{2} \mathrm{v}_{3}\right)\left(\mathrm{v}_{\mathrm{i}} \mathrm{v}_{\mathrm{i}+1}\right), \Delta \mathrm{XY} \mathrm{I}_{\mathrm{i}}, \Delta \mathrm{XY} \mathrm{Y}_{\mathrm{i}+1}, \Delta \mathrm{~S}_{\mathrm{i}}, \Delta \mathrm{S}_{\mathrm{i}+1}, \mathrm{G}\right)$
$=\left(C_{2},\left(v_{1} v_{i}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Delta S_{i+1}, \Phi\right)$
for $i=3,4, \ldots, m ; m \neq n$
and $\Delta x y_{i}=\Delta x y_{i+1}=\beta$ and $\Delta S_{i}=\Delta S_{i+1}=+$
22. $=\left(C_{2}, \Phi, \Phi, \Phi, \Phi,\left(C_{2},\left(v_{1} v_{n}\right)\right)\right)$
for $\mathrm{i}=\mathrm{n}$
and $\Delta x y_{i-1}=\Delta x y_{i}=\beta$
and $\Delta S_{i}=+$
23. $=\left(D,\left(v_{i}, v_{i+1}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Phi,\left(C_{2},\left(v_{I}, v_{i}\right)\right)\right)$
for $i=3,4, \ldots, m ; m \neq n$
and $\Delta x y_{i}=\beta \neq \Delta x y_{i+1}$
or $\Delta S_{i}=+\neq \Delta S_{i+l}$
set $i=1$ for dot primitive part
24. $\delta\left(C_{4},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right)\left(v_{i} v_{i+1}\right), \Delta x y_{i}, \Delta x y_{i+1}, \Delta S_{i}, \Delta S_{i+1}, G\right)$
$=\left(C_{4},\left(v_{1} v_{i}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Delta S_{i+1}, \Phi\right)$
for $i=3,4, \ldots, m ; m \neq n$
and $\Delta x y_{i}=\Delta x y_{i+1}=\beta$ and $\Delta S_{i}=\Delta S_{i+1}=-$
25. $=\left(C_{4}, \Phi, \Phi, \Phi, \Phi,\left(C_{4},\left(v_{1} v_{n}\right)\right)\right)$
for $i=n$
and $\Delta x y_{i-1}=\Delta x y_{i}=\beta$
and $\Delta S_{i}=-$
26. $=\left(D,\left(v_{i}, v_{i+1}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Phi,\left(C_{4},\left(v_{1}, v_{i}\right)\right)\right)$
for $i=3,4, \ldots, m ; m \neq n$
and $\Delta x y_{i}=\beta \neq \Delta x y_{i+1}$
or $\Delta S_{i}=-\neq \Delta S_{i+1}$
set $i=1$ for dot primitive part
$27 . \delta\left(C_{3},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right)\left(v_{i} v_{i+1}\right), \Delta x y_{i}, \Delta x y_{i+1}, \Delta S_{i}, \Delta S_{i+1}, G\right)$

$$
=\left(C_{3},\left(v_{1} v_{i}\right),\left(r_{i} v_{i+1}\right), \Delta x y_{i+1}, \Delta S_{i+1}, \Phi\right)
$$

for $i=3,4, \ldots, m ; m \neq n$
and $\Delta x y_{i}=\Delta x y_{i+1}=\alpha$ and $\Delta S_{i}=\Delta S_{i+1}=-$
28. $=\left(C_{3}, \Phi, \Phi, \Phi, \Phi,\left(C_{3},\left(v_{1} v_{n}\right)\right)\right)$
for $i=n$
and $\Delta x y_{i-1}=\Delta x y_{i}=\alpha$
and $\Delta S_{i}=-$
29. $=\left(D,\left(v_{i}, v_{i+1}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Phi,\left(C_{2},\left(v_{1}, v_{i}\right)\right)\right)$
for $i=3,4, \ldots, m ; m \neq n$
.and $\Delta x y_{i}=\alpha \neq \Delta x y_{i+1}$
or $\Delta S_{i}=-\because \neq \Delta S_{i+1}$
set $i=1$ for dot primitive part
30. $\delta\left(C_{4},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right)\left(v_{i} v_{i+1}\right), \Delta x y_{i}, \Delta x y_{i+1}, \Delta S_{i}, \Delta S_{i+1}, G\right)$

$$
\begin{aligned}
& =\left(C_{4},\left(v_{1} v_{i}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Delta S_{i+1}, \Phi\right) \\
& \quad \text { for } i=3,4, \ldots, m ; m \neq n \\
& \quad \text { and } i x y_{i}=\Delta x y_{i+1}=\gamma \text { and } \Delta S_{i}=\Delta S_{i+1}=+
\end{aligned}
$$

31. $=\left(C_{4}, \Phi, \Phi, \Phi, \Phi,\left(C_{4^{\prime}}\left(v_{I} v_{n}\right)\right)\right)$
for $i=n$
and $\Delta x y_{i-1}=\Delta x y_{i}=\gamma$
and $\Delta S_{i}=+$
32. $=\left(D,\left(v_{i} v_{i+1}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Phi,\left(C_{4^{\prime}}\left(v_{i} v_{i}\right)\right)\right)$
for $i=3,4, \ldots, m ; m \neq n$
and $\Delta x y_{i}=\gamma \neq \Delta x y_{i+1}$
or $\Delta S_{i}=+\neq \Delta S_{i+1}$
set $i=1$ for dot primitive part
33. $\delta\left(C_{2},\left(v_{1} v_{3}\right),\left(v_{2} v_{3}\right)\left(v_{i} v_{i+1}\right), \Delta x y_{i}, \Delta x y_{i+1}, \Delta S_{i}, \Delta S_{i+1}, G\right)$
$=\left(C_{2},\left(v_{1} v_{i}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Delta S_{i+1}, \Phi\right)$
for $i=3,4, \ldots, m ; m \neq n$
and $\Delta x y_{i}=\Delta x y_{i+1}=\gamma$ and $\Delta S_{i}=\Delta S_{i+1}=-$
34. $=\left(C_{2}, \Phi, \Phi, \Phi, \Phi,\left(C_{2},\left(v_{1} v_{n}\right)\right)\right)$
for $i=n$
and $\Delta x y_{i-1}=\Delta x y_{i}=\gamma$
and $\Delta S_{i}=-$
35. $=\left(D_{1}\left(v_{i} v_{i+1}\right),\left(v_{i} v_{i+1}\right), \Delta x y_{i+1}, \Phi,\left(C_{2},\left(v_{1}, v_{i}\right)\right)\right)$
for $i=3,4, \ldots, m ; m \neq n$
and $\Delta x y_{i}=\gamma \neq \Delta x y_{i+1}$
or $\Delta S_{j}=-\neq \Delta S_{i+1}$
set $i=1$ for dot primitive part
36.     - 52. are repetition of mappings 18-35 for input type $H$, in which $v_{j}$ is considered instead of $v_{i}$. The states shown on the right should be replaced as follows.

Please Note. $A_{I}=C_{1} ; A_{2}=C_{2} ; A_{3}=C_{3} ; A_{4}=C_{4}$.
In the mappings shown, only one type is shown for the straight line primitive. It should be replaced by the appropriate type based on the $\Delta x$ and $\Delta y$ values for the primitive.

Discussion of Mappings.

1. A dot primitive $\left(v_{1}\right)$ and a vector defining a connected point $\left(v_{2}\right)$ are reduced to a dot primitive.
2. A dot primitive defined by two points ( $v_{1}$ and $v_{2}$ ) are reduced with a third point $\left(v_{3}\right)$ to form a straight line if the change in the $x$ and $y$ components are consistent and there is no change in the angle between the Jesultant vectors.
3. If $\Delta x<0$ and $\Delta y>0$ and there is a positive change in slope then the three points $\left(v_{1}, v_{2}\right.$ and $\mathrm{v}_{3}$ ) are reduced to form a curve-component type 1.
4.     - 8. do the same as 3 for different combinations of $\Delta x$ and $\Delta y$ and sign of slope changes to define curve-component types 2,3 and 4.
1.     - 14. do the same as 3-8 for input type from the scanner. This results in area-components, rather than curve-components.
1. Shows the reductions for a straight line. A series of incremental vector pairs are reduced to a straight line primitive if the $\Delta x$ and $\Delta y$ are consistent and change in the angle between
the resultant vectors is zero.
2. If all vector pairs are exhausted, then the final state is entered and the primitive is a straight line.
3. If before all vector pairs are exhausted, a curve point is reached (which is determined by the $\Delta x$ and $\Delta y$ and the sign of the angle change between the resultant vectors), a new dot primitive is spawned at the curve point and further reductions with the straight line is ceased.
4.     - 35. express the same principles as discussed in 15, 16 and 17 for a straight line, but pertain to curve-components.
1.     - 52. are a repetition of 18 - 35 , but for type H input from the scanner and pertain to areacomponents.

Please note. In the above discussion, two elements have not been shown, though important. The first one pertains to the color transition values obtained from the scanner. When a new dot primitive is created, it assumes the color transition value of the scanner input corresponding to $q_{i}$ or $q_{j}$ for which the dot primitive is created. From then on, reduction with that primitive is carried out only if this color transition value is compatible. If this changes, the reduction for the primitive is terminated and a new dot primitive for the new value is started. This ensures that the primitives are separated into those that overlap with
others and those that do not. The second point to be made is that a primitive can undergo reduction with either $q_{i}$ or $q_{j}$, but not both. Under certain conditions, connectivity for a primitive is established with both $q_{i}$ and $q_{j}$. In order to keep the reduction proceeding properly, each primitive is assigned a \#. This is the reduction identification number, depending upon whether the primitive is to undergo reduction with $q_{i}$ or $q_{j}$. These are shown in the mappings that follow.

The reduction function makes use of the reduction process described above to decide if the input from the scanner is to be reduced with the current primitive or a new primitive is to be created. The reduction function is characterized by a deterministic finite automaton. It is described by an 8-tuple -
$T=\left(Q^{\prime}, I, \gamma, \xi, O, S, F, \delta\right)$ where
$Q^{\prime} \subseteq Q$ the set of states as defined in section 3.3 $I=T h e$ input to this state and internal to the Micromaton.
$=\left(P_{R}, \Sigma, S^{\prime}\right)$ as defined by the output 0 of the connectivity check state in section 3.5. $\gamma=$ The next primitive to be reduced and defined as follows.
if the ith member of $P_{R}$ is $\left(p_{i}, K_{i}\right)$ then the next one to be tried for reduction is ( $p_{i+1}, K_{i+1}$ ). The notation used to indicate the above is $\gamma\left(P_{R}\right)$.
$p_{i}$ is of the form ( $p_{i}, \#$ ) where \# refers to whether the primitive undergoes reduction with $q_{i}$ or $q_{j}$. \# assumes values from the set $\{0,1,2\}$. A value of 0 or 1 signify that it can undergo reduction with $q_{i} . \quad 2$ indicates reduction with $q_{j}$. $\xi=$ Outcome of the reduction process
$=\{0,1\}$ where
0 = No new primitive was created, but current primitive was reduced.

1 = Reduction with current primitive was terminated and new dot primitive was formed. $0=$ Output of the reduction state and is internal to the Micromaton.
$=(P, \Omega)$ which are elements of $\lambda$ as defined in section 3.3.
$S=$ Start state
$=S_{3} \varepsilon Q$ as defined in section 3.3.
$F=$ Final state
$=W \varepsilon Q$ as defined in section 3.3.
$\delta=$ Mappings from finite subsets of $Q \times I * x \xi^{*}$ to
finite subsets of $Q \times I^{*} x$ O and defined as shown.

1. $\delta\left(S_{3},\left(\left(\left(p_{i}, 0\right),(1,0)\right), \Sigma, W\right), 0\right)$
$=\left(S_{3}, \gamma\left(P_{R}\right),(P, \Omega)\right)$
2. $\left.\delta\left(S_{3},\left(\left(p_{i}, I\right),(1,0)\right), \Sigma, W\right), 0\right)$
$=\left(S_{3}, \gamma\left(P_{R}\right),(R, \Omega)\right)$
3. $\delta\left(S_{3},\left(\left(\left(p_{i}, 2\right),(0,1)\right), \Sigma, W\right), 0\right)$
$=\left(S_{3}, \gamma\left(P_{R}\right),(P, \Omega)\right)$
4. $\left.\delta\left(S_{3},\left(\left(p_{i}, 0\right),(1,1)\right), \Sigma, W\right), \Phi\right)$
$=\left(S_{3},\left(\left(p_{i}, 1\right),(1,0)\right),\left(P=P \cup\left\{\left(p_{d}, 2\right)\right\} \neq\left(p_{i}, 1\right) \Omega<=r_{p_{d}}\right)\right.$
5. $\left.\delta\left(S_{3},\left(\left(p_{i}, 1\right),(1,0)\right), \Sigma, W\right), 1\right)$
$=\left(S_{3}, \gamma\left(P_{R}\right),\left(P=P \cup\left\{\left(p_{d}, 1\right)\right\} \nrightarrow\left(p_{i}, 1\right), \Omega<=r_{p_{d}}\right)\right)$
6. $\left.\delta\left(S_{3},\left(\left(p_{i}, 2\right),(0,1)\right), \Sigma, W\right), 1\right)$
$=\left(S_{3}, \gamma\left(P_{R}\right),\left(P=P \cup\left\{\left(p_{d}, 2\right)\right\} \nleftarrow\left(p_{i}, 2\right), \Omega<=r_{p_{d}}\right)\right)$
7. $\delta\left(S_{3}, \Phi, \Phi\right)=(W, \Phi,(P, \Omega))$

Discussion of mappings.

1. If a primitive reduction id. is zero and connectivity is established with $q_{i}$ and the reduction process does not terminate reduction of the current primitive, get the next primitive that has to be reduced.
2. and 3. are a restatement of 1 except that the reduction id is 1 or 2 and connectivity is established with $q_{i}$ or $q_{j}$ and reduction is not terminated with the current primitive.
3. If the primitive reduction id. is zero and connectivity is established with $q_{i}$ and $q_{j}$, create a new-dot primitive with a reduction id. of 2 corresponding to $q_{j}$, linkit to the current primitive and addit to the primitive set.

Add a pointer to it in the active primitive pointer list. Change the current primitive's reduction id. to $I$ and carry our reduction with $q_{i}$.
5. and 6. are mappings in which the reduction process terminates reduction with the current primitive and starts a new dot primitive. Mapping 5 pertains to reduction with $q_{i}$ and 6 pertains to $q_{j}$. In each case, assign the new dot primitive the same reduction id. as the one for which reduction is terminated, link the two and add the new one to the primitive set. Add a pointer to the active primitive list for the new primitive.
7. When all primitives are reduced, return is made to the wait state.

### 3.7 Micromaton Output Function

The role of the output function component is to look at every primitive in the Micromaton temporary storage and output those that are eligible. The candidates for output are those that have a tag number identity less than the current probe number.

The output function is described by a deterministic
finite automaton. It is an 8-tuple -
$T=\left(Q^{i}, I, R, \gamma, O, S, F, \delta\right)$ where
$Q^{\prime}=$ Set of states
$Q^{\prime} \subseteq Q$ defined in section 3.3.
I = Input to this component and internal to Micromaton.
$=(\lambda, \Sigma, N, S)$. This is the output $O$ defined in section 3.3.

The active primitive set pointed to by $\Omega$ is represented here as $\left\{\left(p_{i}, t_{i}\right)\right\}$ where $p_{i}$ is the primitive and $t_{i}$ is its associated tag number. $t$ represents the probe number for which $p_{i}$ underwent reduction last or when it was created.
$R=$ Set of relational values between $J$ (the current probe number from scanner input $\Sigma$ ) and $t_{i}$ of $p_{i}$.
$=\{0,1\}$ and is defined as shown.
$R=0$ if $t_{i}=J$
$R=1$ if $t_{i}<J$
$\gamma=$ Next choice function as defined in section 3.5.
O = Output internal to the Micromaton
$=(\Omega, \Psi)$ which are elements of $\lambda$ as defined.
$\delta=$ Mapping from finite subsets of $Q \times I X R$ to finite subsets of $Q \times I * \times O$, and defined as shown.

1. $\delta\left(S_{4},\left(p_{i}, W\right), 0\right)=\left(S_{4},(\gamma(\Omega), W),(\Omega, \Psi)\right)$
2. $\delta\left(S_{4},\left(p_{i}, W\right), 1\right)=\left(S_{4},(\gamma(\Omega), W),\left(\Omega \Rightarrow r_{p_{i}}, \psi<=r_{p_{i}}\right)\right)$
3. $\delta\left(S_{4},(\Phi, W), I\right)=(W,(\Phi, W),(\Omega, \Psi))$
4. $\delta\left(S_{4},\left(p_{i}, E\right), \Phi\right)=\left(S_{4},(\gamma(\Omega), E),\left(\Omega \Rightarrow r_{p_{i}}, \Psi \ll r_{p_{i}}\right)\right)$
5. $\delta\left(S_{4},(\Phi, E), \Phi\right)=\left(E_{,}(\Phi, E),(\Omega=\Phi, \Psi)\right)$
(3.10)

Discussion of mappings.

1. If an active primitive's tag number is equal to the current probe number, select the next primitive from the active set for examination.
2. If an active primitive's tag number is less than the current probe number, remove the pointer to it from the active list and add it to the inactive set, the list chosen depending upon the primitive type.
3. If all active primitives have been checked, return, to wait state if that is the return state that is requested.
4. If return to end state is requested, it signals the completion of input from the scanner. Remove pointers from the active primitive list and add them to the inactive primitive list, the list chosen based on the primitive type.
5. Return to end state after setting the active primitive pointer list to a null value.

### 3.8 Micromaton State Transitions

The discussion of the Micromaton is concluded with a brief review of its function in converting the input data from the scanner to a set of primitives suitable for use by
the Macromaton. This is best presented in the form of a state transition diagram as seen in Figure 3.10.


Fig. 3.10 Micromaton state transition diagram.

After initialization, the Micromaton moves from the start state to a wait state. If no input is available from the scanner, it continues in this state. When an input is received, if the active primitive set is empty, it moves to the new primitive creation state. After creating the necessary primitives and adding them to the active primitive set, it returns to the wait state. However, if the active primitive set is not empty, it moves to the connectivity recognition state. In this state, connectivity is determined for the existing active primitives with the scanner input. If no connectivity is established for the input signal, it moves to the new primitive creation state. After creating the required new primitives and adding then to the active primitive set, it returns to the wait state. If Connectivity is partially established for the input signal, it first moves to the new primitive creation state. After creating the required new primitives and adding them to the active primitive set, it returns to the connectivity recognition state before going to the reduction function state.

In the reduction function state, reduction of the input signal is achieved with the primitives for which connectivity is established. If the end of a probe is not reached, the Micromaton returns to the wait state. When the end of a probe is reached, it moves to the output processor state. Here, all primitives whose tag numbers
are less than the current scanner probe number, are removed from the active primitive set and added to the inactive primitive set. The inactive primitive set consists of primitives that are fully identified, such that they do not undergo any further reduction with input from the scanner. The inactive primitive set forms the output of the Micromaton. If the end of scan is not reached, it returns to the wait state. When scanning is complete, it moves to the output processor state and outputs all the remaining active primitives as described above. The Micromaton then moves to the final or end state.

This completes the discussion of the Micromaton phase of the recognizer.

### 3.9 Introduction to Macromaton

In a general sense, a two-dimensional pattern consists of all the objects within the two-dimensional pattern space. The total pattern may be viewed as being made up of subpatterns or shapes, some of which are of interest, while others are not. In what follows, the term "pattern recognition" refers to the recognition of sub-patterns or individual shapes within the two-dimensional pattern space. Instead of referring to them as individual shapes or subpatterns, they are referred to as patterns. The collection of all patterns within the two-dimensional space is referred to as the "scene."

The set of primitives identified by the Micromaton is hereafter referred to as the "source" primitive set. This represents the domain of the Macromaton. The set of possible primitives comprising the patterns of interest is referred to as the "target" primitive set and represents the range of the Macromaton. The process of identifying the subset of the source primitive set as a subset of the target primitive set is referred to as "parsing" in the spirit of formal language theory. The function of the Macromaton is, therefore, to partially map the source primitive set into the target primitive set.

Before proceeding with the description of the parsing process, a few preliminary comments are in order. A pattern form defined using the D-I-A grammar has an implicit orientation associated with it. This is illustrated in Figure 3.11. Starting at a point "a," the area pattern form $P=\left[A_{1} A_{2} A_{6} A_{3} A_{4} A_{8}\right]$ is shown in solid line while the pattern form $P=\left[A_{4}^{\prime} A_{1}^{\prime} A_{5}^{\prime} A_{2}^{\prime} A_{3}^{\prime} A_{7}^{\prime}\right]$ is shown in dotted line. The dotted pattern is obtained by rotating the solid pattern by $90^{\circ}$ clockwise, keeping the point "a" fixed.


Figure 3.11 Pattern representation due to orientation.

In a general recognition problem, it is necessary to recognize both of the patterns shown in Figure 3.11 as the same. Therefore, if the $D-I-A$ grammar is used for defining the pattern form, certain additional considerations are required to identify a pattern as being the same for different orientations. The pattern form derived by using the $D-I-A$ grammar may be referred to as a "cannonical" one. The objective is to go from the cannonical pattern form to a "free-form," which represents the cannonical pattern form for all orientations.

Consider Figure $3.12 a$ showing three representations of curve-component type 1. \#1 spans the whole range (90 ) , while, $\ddagger 2$ and \#3 span approrimately $30^{\circ}$ at either end. Figure 3.12b shows the same figure rotated counter-clockwise by $45^{\circ}$.


Figure 3.12 Effect of rotation on primitive type.

Comparing Figure 3.12a and Figure 3.12b, the following observations may be made. \#l started as curve-component type 1 and ended up as curve-component type 2 . \#2 started and ended the same; as type 1. \#3 started as type 1 and ended as type 2. If Figure 3.12 a had been rotated clockwise" by $45^{\circ}$, the following results would have. been obtained.

Curve\# Curve-component type
Start End

| 1 | 1 | 1 and 4 |
| :--- | :--- | :---: |
| 2 | 1 | 4 |
| 3 | 1 | 1 |

Based on the above, when a segment of a curve which constitutes a curve-component is rotated, one of the following takes place.

1. It retains its curve-component type.
2. It straddles its type and one of its adjacent types.

It is worthy of note that a curve-component, when rotated, does not straddle more than one of its adjacent types. Using the above facts, a cannonical pattern form representation may be transformed into a free-form by choosing columns 1 and 2 or columns 2 and 3 of (3.7) as alternates in the parsing process.

$$
\begin{align*}
& \text { COLUMN \# } \\
& 1 \quad 2 \quad 3 \\
& C_{1}=C_{4} C_{1} \quad\left|C_{1}\right| \quad C_{1} C_{2} \\
& c_{2}=c_{1} C_{2}\left|c_{2}\right| \quad c_{2} C_{3} \\
& c_{3}=c_{2} c_{3}\left|c_{3}\right| \quad c_{3} c_{4} \\
& C_{4}=C_{3} C_{4}\left|C_{4}\right| \quad C_{4} C_{1} \\
& A_{1}=A_{4} A_{1} \quad\left|A_{1}\right| \quad A_{1} A_{2} \\
& A_{2}=A_{1} A_{2} \quad\left|A_{2}\right| \quad A_{2} A_{3} \\
& A_{3}=A_{2} A_{3}\left|A_{3}\right| \quad A_{3} A_{4} \\
& \dot{A}_{4}=A_{3} A_{4}\left|A_{4}\right| \quad A_{4} A_{1} \tag{3.7}
\end{align*}
$$

### 3.10 Partial Patterns

One of the more difficult problems associated with pattern recognition has to do with patterns that are partially obscured by others overlaying them. For purposes of this study, patterns are grouped into three classes as shown in Figure 3.13. Figure 3.13a shows two patterns that do not overlap one another. This is referred to as "class A" patterns. In Figure 3.13b, the circle overlays and obscures part of a rectangle. This is referred to as "class $\mathrm{B}^{\prime \prime}$ patterns. In Figure 3.13c the circle overlays and bifurcates the rectangle into two parts. This is referred to as "class C" patterns.

This study deals with the recognition of class A and class $B$ patterns. Class $C$ problem is beyond the scope of


Figure 3.13a Class A patterns (non-overlapped)


Figure 3.13b Class B patterns (overlapped, non-bifurcated)


Figure 3.13c Class $C$ patterns (overlapped, bifurcated) Figure 3.13 Classes of two-dimensional patterns.
this work and has not been dealt with. From hereon, the term "non-overlapped patterns" implies class A type and "overlapped patterns" implies class B type as described above.

Consider the scene in Figure 3.14. It shows two circles overlapped as shown, each circle being an area pattern.


Figure 3.14 Overlapped patterns

Circle a is composed of the dynamic chain $\left[p_{1} p_{2} p_{3} p_{4} p_{5} p_{6}\right]$ while circle $b$ is composed of the dynamic chain [ $p_{1}^{\prime} p_{2}^{\prime} p_{3}^{\prime} p_{4}^{\prime} p_{5}^{\prime} p_{6}^{\prime}$ ], each $p_{i}, p_{i}^{\prime} \varepsilon\left\{A_{1}, A_{2}, A_{3}, A_{4}\right\}$ where each $A_{i}$ is an area component. The primitives $p_{2}$ and $p_{3}$ of circle a are overlapped with primitives $p_{5}^{\prime}$ and $p_{4}^{\prime}$ of circle $b$. In order to correctly determine that the two patterns are circles, it is necessary to recognize that primitives $p_{2}$ and $p_{3}$ are valid primitives that should participate in the parse for circle a, while,
primitives $p_{3}^{\prime}$ and $p_{4}^{\prime}$ for circle $b$ should not. Otherwise, the parse for circle $b$ will be erroneously rejected as being invalid for a circle from the chain of primitives shown above. One method of accomplishing this is to first find the full circle a and mark the primitives $p_{5}^{\prime}$ and $p_{4}$ of circle $b$ that are overlapped with the primitives $p_{2}$ and $p_{3}$ of circle a as "unusable" in the parse for circle b.

### 3.11 Macromaton Structure

The function of the Macromaton is to map a subset of the source primitive set into a subset of the target primitive set. In this process, the Macromaton subdivides the source primitive set into four disjoint subsets as follows.
I. A subset that corresponds to fully visible target patterns.
2. A subset that corresponds to partially visible target patterns.
3. A subset for which it is not possible to uniquely identify the target pattern.
4. A subset for which mapping into the chosen target pattern set does not exist.

Figure 3.15 shows the logical components of the Macromaton.
The treatment of the Macromaton is analogous to that of the Micromaton. It is characterized as a series of automata driven by a common set of states. In some it performs specific functions, while in others it acts as the controling element, directing the action of its components. A recap of


Figure 3.15 Macromaton structure
the discussion on the Macromaton is presented in section 3.16 with a state transition diagram. It is worthwhile to review section 3.16 before proceeding with the rest of this chapter.

The Macromaton has two controlling states. The first one is the state in which the patterns which are fully visible are recognized. The second controlling state is for recognition of partial patterns.

The following symbols and their associated meanings are defined for the Macromaton.

```
Q = Common set of states
```



```
            S I = Full pattern recognition state
            S'' = Return state in full pattern recognition
            S
            S}\mp@subsup{2}{}{\prime}=\mathrm{ Return state l in partial pattern recognition
            S'" = Return state 2 in partial pattern recognition
            S
            S
            E = End state
            Z = Termination state
                :In the above, {S }\mp@subsup{I}{1}{},\mp@subsup{S}{1}{\prime},\cdot\mp@subsup{S}{2}{},\mp@subsup{S}{2}{\prime},\mp@subsup{S}{2}{\prime\prime}\cdot\mathbb{E},Z}\mathrm{ are
                the controlling states; }\mp@subsup{S}{3}{}\mathrm{ and }\mp@subsup{S}{4}{}\mathrm{ are compo-
                nent states.
            \tau = Set of target patterns to be recognized consti-
                tuting one of the inputs to the Macromaton.
            = {T i
            In this set, Tl is the most complex pattern com-
            prising the largest number of primitives and m}\mp@subsup{T}{n}{
```

is the simplest with the least number.
$\left\{T_{i}\right\}$ is therefore a set of hierarchical patterns arranged in order of diminishing complexity. Each $T_{i}$ is defined as shown below.

$$
\begin{aligned}
T_{i} & =\left(G_{i}, \mu_{i}, t_{d_{i}}\right) \text { where } \\
G_{i} & =D-I-A \text { grammar that defines pattern } T_{i} \text { by }
\end{aligned}
$$ the set $\left\{t_{i}\right\}$, each $t_{i}$ being a target primitive type corresponding to one of the types output by the Micromaton. $\mu_{i}=$ Parametrization of $G_{i}$ and defined for each $t_{i} \varepsilon T_{i}$ as shown.

$\left\{o_{j}, a_{j k}, v_{j k}, r_{j k}, e_{j k}\right\}$ in which the symbols have the following meaning. $o_{j}=j t h$ offset, where $o \varepsilon I$ the index set. It specifies the offset primitive from the current one with which parametrization compliance is required. $j=$ 1,2,......m.
$a_{j k}=$ Attributes $k=1,2, \ldots, n$ corresponding to each offset $o_{j}$ that is to be checked.

$$
v_{j k}=\text { Value of } a_{j k}
$$

$$
r_{j k}=\text { Relation to be used in checking values }
$$ of attributes between $t_{i}$ and $t_{i+o_{j}}$.

$e_{j k}=$ Error tolerance in the above parameter compliance check.

```
\(t_{d_{i}} \varepsilon\left\{t_{i}\right\}\) and represents a distinguished target primitive that must be present in the parse if the pattern is fully visible.
\(\lambda=\) Input to the Macromaton. This constitutes the set of primitives output by the Micromaton.
\(=(P, \Psi)\) shown as \(\lambda^{\prime}=(P,(\Phi, \Psi))\) in section 3.3. \(\gamma=\) Next selection function.
For the choice of the next target pattern to be parsed, it is defined as shown.
\(\gamma\left(T_{i}\right)=T_{i+1}\) for \(l \leq i \leq n-1\)
\(\gamma\left(T_{n}\right)=\Phi\) where only \(n\) patterns are defined.
\(\Delta=\) Set of outputs of the Macromaton.
\(=\left\{p_{i}\right\}, i=1,2,3,4\) where
\(\rho_{1}=\) Primitive subset corresponding to fully visible patterns.
\(\rho_{2}=\) Primitive subset corresponding to partially visible patterns.
\(\rho_{3}=\) Primitive subset corresponding to patterns that cannot be uniquely identified.
\(\rho_{4}=\) Primitive subset for which no mapping exists into the chosen target pattern set.
Each \(p\) is of the form \(\{(L,\{p\})\}\) where \(L\) corresponds to the index of \(T\) the target pattern recognized. In the case of \(\rho_{3}\) and \(\rho_{4}, L=\Phi\).
```


### 3.12 Full Pattern Recognition Function

The structure of the full pattern recognition function is described by a deterministic finite automoton. It is an 8-tuple -
$R_{1}=\left(Q^{\prime}, S^{\prime}, F^{\prime}, \tau, \lambda, \gamma, 0, \delta\right)$ where
$Q^{\prime}=\left\{S_{1}, S_{1}^{\prime}, S_{2}, S_{3}, S^{\prime}, F^{\prime}\right\}$ in which
$\left\{S_{1}, S_{1}^{\prime}, S_{2}, S_{3}\right\} \subseteq Q$ as defined in section 3.11 .
$S^{\prime}=$ The start state of full pattern recognition
$F^{\prime}=S_{2} \varepsilon Q$ the final state as defined.
$\tau=$ The target pattern set as defined in section
3.11.
$\lambda=$ Input from Micromaton
as defined in section 3.11 .
$\gamma=$ Next selection function for full pattern
recognition as shown.
$\gamma_{\mathrm{T}}=$ Next target pattern selection function as
defined in section 3.11 .
$\gamma_{p}=$ The next distinguished source primitive
selection function as defined below.
Let $p_{d_{i}}$ be a source primitive $\varepsilon P$, of the
same type as $t_{d_{i}}$ the distinguished target
primitive that must be present in the
pattern for it to be fully visible. If
$\mathrm{p}_{\mathrm{d}}$ exists, then there is a pointer to it in one of the pointer lists of $\Psi$. $\gamma_{P}\left((P, \Psi), P_{d_{i}}\right)$ returns the primitive pointed to by the next pointer in the same list as the pointer to $\mathrm{p}_{\mathrm{d}_{i}}$ of $\psi$. If all pointers in this list are exhausted, then $\gamma_{P}\left((P, \Psi), P_{d_{i}}\right)=\Phi . \quad \gamma_{P}\left((P \Psi), P_{\dot{d}_{i}}\right)$ is abbreviated as $\gamma_{P}(P)$. $0=$ Output of this state, internal to the Macromaton, going to the parsing function. It is defined as shown.
$=\left(L, T_{i},\left((P, \Psi), p_{d_{i}}\right), S_{R}\right)$ where $L$ is the index of $T_{i}$, $T,\left((P, \Psi), P_{d_{i}}\right)$ are as defined and $S_{R}$ is the state to return to after completion of the state being entered. $\delta=$ Mapping from finite subsets of $0 \times \tau \times \lambda$ to finite subsets of $Q \times \tau \times \lambda \times O^{*}$ and is defined as shown.

1. $\delta\left(S^{\prime}, \tau, \lambda\right)$

$$
=\left(S_{1}, T_{1}, \lambda, \Phi\right)
$$

2. $\delta\left(S_{1}, T_{i},\left((P, \Psi), \underline{p}_{\alpha_{i}}\right)\right)$
$=\left(S_{3},\left(T_{i}, t_{d_{i}}\right),\left((P, \Psi), p_{d_{i}}\right),\left(L_{1},\left(T_{i}, t_{d_{i}}\right),\left((P, \Psi), p_{d_{i}}\right), S_{1}{ }^{\prime}\right)\right.$
3. $\delta\left(S_{1}{ }^{\prime}, T_{i},(P, \Psi)\right)$

$$
=\left(S_{1}, T_{i}, \gamma_{P}(P), \Phi\right)
$$

4. $\delta\left(S_{1}, T_{i},((P, \Psi), \Phi)\right)$

$$
=\left(S_{1}, \gamma_{T}\left(T_{i}\right),(P, \Psi), \Phi\right)
$$

5. $\delta\left(S_{I}, \Phi,(P, \Psi)\right)$

$$
=\left(S_{2}, \tau,(P, \Psi), \Phi\right)
$$

## Discussion of Mappings

1. From the start state, enter full pattern recognition state by picking the first target pattern for recognition.
2. If there is a source primitive of the same type as the distinguished target primitive for the pattern chosen: pass the target pattern and the source primitive set to the parsing function; go from full pattern recognition state to parsing function state, requesting return to full pattern recognition return state.
3. Upon return to the full pattern recognition state, get the next source primitive of the same type as the distinguished target primitive. Retain the same target pattern for re-try.
4. If there are no more source primitives of the same type as the distinguished target primitive for the pattern chosen, get the next target pattern for parse.
5. If all target patterns have been tried for full pattern recognition, enter the partial pattern recognition state.

### 3.13 Partial Pattern Recognition Function

This function is defined by a deterministic finite automaton which is a 12 -tuple as shown.

$$
\begin{aligned}
R_{2}= & \left(Q^{\prime}, S^{\prime}, F^{\prime}, \tau, \lambda, K, \gamma, I, \eta, \omega, O, \delta\right) \text { where } \\
Q^{\prime}= & \left\{S_{2}, S_{2}^{\prime}, S_{2}^{\prime \prime}, S_{3}, S_{4}, E, Z, S^{\prime}, F^{\prime}\right\} \text { in which } \\
& \left\{S_{2}, S_{2}^{\prime}, S_{2}^{\prime \prime}, S_{3}, S_{4}, E, Z\right\} \subseteq Q \text { as defined } \\
& \text { in section } 3.11 . \\
S^{\prime}= & S_{2^{2}} \varepsilon Q \text { the start state as defined. } \\
F^{\prime}= & Z \varepsilon Q \text { the final state as defined. } \\
\tau= & \text { The target pattern set as defined in section } \\
& 3.11 .
\end{aligned}
$$

$\lambda=$ Input from Micromaton as defined in section 3.11 .
$K=$ Relational value of good parse primitive count corresponding to a starting source primitive.

$$
=\left\{k_{i}\right\}, i=1,6 \text { defined as shown. }
$$

$k_{1}$ : Old count $=$ New count $=0$ $k_{2}$ : Highest old count $=$ New count $\neq 0$ $k_{3}$ : Highest old count $>$ New count $k_{4}$ : Highest old count < New count $k_{5}$ : Old count $=\Phi$; New count $=0$ $k_{6}$ : Undefined (don't care) $\gamma=$ Next selection function as shown below.

```
\gammaT = Next target pattern selection function as
    defined in section 3.11.
\gammas}= Next source primitive selection function
    defined as follows.
    The very first source primitive p geP
        selected for partial parse is the one
        pointed to by the first pointer r of the
        first pointer list in \Psi. The next source
        primitive }\mp@subsup{P}{s}{}\mathrm{ selected for partial parse and
        denoted by the symbol }\mp@subsup{\gamma}{S}{}(P:\Psi) is the on
        pointed to by the next pointer in the list
        to which r belongs. If there are no more in
        the same list, selection is maüe fron the
        next available list in \psi. }\mp@subsup{\gamma}{S}{}(P,\Psi)=\Phi\mathrm{ if no
        more source primitives can be selected using
        \Psi.
\mp@subsup{\gamma}{t}{}}=\mathrm{ Next target primitive selection function
        defined as follows.
        In the grammar defining the target pattern,
        t ps corresponds to the same primitive type
        as the source primitive p p
        reduction can be started. }\mp@subsup{\gamma}{t}{}(T)=t i
        such a target primitive exists in the gram-
        mar;
        Y}(T)=\Phi if no such target primitive exist
        in the pattern definition.
```


same as shown for $I$.
$0=$ The output of this state and internal to the Macromaton.

This is specified in one of three forms.
The first form goes to the parsing function and the next two go to the output function.
$O_{1}=\left(L, T_{i},\left((P, \Psi), D_{S}\right), S_{R}\right)$ where
I is the index of $T_{i}$,
$T_{i},\left((P, \Psi), P_{S}\right)$ are as defined and $S_{R}$ is the state to return to after completion of the state being entered.
$O_{2}=\left(\left((P, \Psi), P_{S}\right), S_{R}\right)$ where
Pifir $_{S}$ are as defined and $S_{R}$ ja the
return state.
$O_{3}=\left(s, C,(L, \beta), \beta^{\prime}, S_{R}\right)$ where the symbols have the same meaning as shown for $I$ above.
$\delta=$ Mapping from finite subsets of
Q $\mathrm{x} \tau^{*} \mathrm{x} \lambda^{*} \mathrm{x} \mathrm{K}^{*} \mathrm{x}$ I* to finite subsets of
Q $\mathrm{x} \tau \mathrm{x} \lambda^{*} \mathrm{x} \eta^{*} \mathrm{x} \omega^{*} \mathrm{x} \mathrm{O}^{*}$ and is defined as
shown.

1. $\delta\left(S_{2},\left(T_{1}, \Phi\right),\left((P, \Psi), P_{S}\right), \Phi, \Phi\right)$
$=\left(S_{2}, \gamma_{T}\left(T_{I}\right),\left((P, \Psi), D_{S}\right), \Phi, \Phi, \Phi\right)$
2. $\delta\left(S_{2}, \Phi,\left((P, \Psi), P_{s}\right), k_{1}, I\right)$
$=\left(S_{\Delta}, \tau,\left((P, \Psi), D_{S}\right), \Phi, \Phi,\left(\left((P, \Psi), P_{S}\right), S_{2}{ }^{\prime \prime}\right)\right)$
3. $\delta\left(S_{2}{ }^{\prime \prime}, \tau,(P, \Psi), \Phi, \Phi\right)$

$$
=\left(S_{2}, T_{I}, \gamma_{s}(P, \Psi), \Phi, \Phi, \Phi\right)
$$

4. $\delta\left(S_{2},\left(T_{i}, t_{S_{i}}\right) r\left((P, \Psi), P_{s}\right), \Phi, \Phi\right)$

$$
=\left(S_{3},\left(T_{i}, t_{s_{i}}\right),\left((P, \Psi), p_{s}\right), \Phi, \Phi,\left(\left(T_{i}, t_{s_{i}}\right),\left((P, \Psi), \mathrm{p}_{s}\right), s_{2}\right)\right)
$$

5. $\quad \delta\left(S_{2},\left(T_{i}, \Phi\right),\left((P, \Psi), P_{S}\right), \Phi, \Phi\right)$

$$
=\left(S_{2}, \gamma_{T}\left(T_{i}\right),\left((P, \Psi), p_{S}\right), \Phi, \Phi, \Phi\right)
$$

6. $\delta\left(S_{2}, \Phi,\left((P, \Psi), P_{S}\right), \Phi, \Phi\right)$

$$
\left.=S_{4}, \tau,\left((P, \Psi), p_{s}\right), \Phi, \omega,\left(2, \omega, s_{2}^{\prime \prime}\right)\right)
$$

7. $\delta\left(S_{2}, \Phi,\left((P, \Psi), P_{S}\right), \Phi, \Phi\right)$

$$
=\left(S_{4}, \tau,\left((P, \Psi), P_{S}\right), \eta, \Phi,\left(3, \eta, S_{2} "\right)\right)
$$

8. $\delta\left(S_{2},\left(T_{i}, t_{S_{i}}\right),\left((P, \Psi), D_{S}\right), k_{I^{\prime}}, I\right)$

$$
=\left(S_{2}, \gamma_{T}\left(\bar{I}_{i}\right),\left((P, \Psi), P_{s} j, \Psi, \Psi_{,} \overline{( }\left((P, \Psi), p_{s} j, S_{2}{ }^{W}\right)\right)\right.
$$

9. $\delta\left(S_{2}^{\prime},\left(T_{i}, t_{S_{i}}\right),\left((P, \Psi), P_{S}\right), k_{\Delta}\left(I, C,(L, \beta), \beta^{\prime}, S_{1}{ }^{\prime}\right)\right)$

$$
=\left(S_{2}, \gamma_{T}\left(T_{i}\right),\left((P, \Psi), P_{S}\right),(n=\Phi),(\omega=I), \Phi\right)
$$

10. $\delta\left(S_{2}^{\prime},\left(T_{i^{\prime}}, t_{S_{i}}\right),\left((P, \Psi), P_{S}\right), k_{3},\left(I, C,(L, B), B^{\prime}, S_{1}^{\prime}\right)\right)$

$$
=\left(S_{2}, \gamma_{T}\left(T_{i}\right),\left((P, \Psi), P_{s}\right), \eta, \omega, \Phi\right)
$$

11. $\delta\left(S_{2}^{\prime},\left(T_{i}, t_{S_{i}}\right),\left((P, \Psi), p_{s}\right), k_{2},\left(1, C,(L, \beta), \beta^{\prime}, S_{1}^{\prime}\right)\right)$

$$
=\left(S_{2}, \gamma_{T}\left(T_{i}\right),\left((P, \Psi), P_{S}\right),\left(\eta=\eta \cup \beta^{\prime} \cup \beta^{\prime} \varepsilon \omega\right),(\omega=\Phi), \Phi\right)
$$

12. $\delta\left(S_{2}{ }^{\prime}, T_{1},(P, \Phi), \Phi, \Phi\right)$

$$
=\left(S_{4}, \tau,(P, \Phi), \Phi, \Phi,(4,(P, \Phi), E)\right)
$$

13. $\delta(E, \tau, \Phi, \Phi, \Phi)$

$$
=(Z, \tau, \Phi, \Phi, \Phi, \Phi)
$$

## Discussion of Mappings

1. Corresponding to the first source primitive selected using the pointer list set $\Psi$, if the first target pattern has no primitive of the same type that can undergo reduction, get the next target pattern.
2. If none of the target patterns defined have a primitive of the same type that can undergo reduction as the source primitive chosen, output this source primitive to the output function, requesting return to the partial pattern recognition state. $S_{2}{ }^{\prime \prime}$.
3. Upon return to patial pattem recognition state from the output function, get the next source primitive available and start again with pattern

4. For any target pattern, if a target primitive of the same type as the source primitive chosen can be reduced, pass the target pattern and the source primitive set to the parser; enter the parsing function state requesting return to the partial pattern recognition state $S_{2}$ '
5. This is a repeat of mapping $I$ but pertains to any pattern in the target pattern set.
6. If all the target patterns defined have been checked and the partial pattern recognized is
non-null, pass the partial pattern recognized $\omega$ to the output function, to be added to the set of partial patterns recognized. Request return to partial pattern recognition state $S_{2}^{\prime \prime}$.
7. If all the target patterns defined have been checked and the set corresponding to the nonunique patterns $\eta$ is non-null, pass $\eta$ to the output function, to be added to the set of nonunique patterns. Request return to partial pattern recognition state $S_{2}{ }^{\prime \prime}$.
8. If none of the target patterns defined have a good parse corresponding to the source primitive chosen, output this source primitive to the output function, requesting return to the partial pattern recognition state $S_{2}{ }^{\prime \prime}$.
9. Upon return from the parsing function state, if the new primitive count for the current pattern is greater than the highest old primitive count, set the current partial pattern recognized $\omega$ to the input; set the non-unique pattern primitive set $\eta$ to null. Get the next target pattern and retry.
10. Upon return from the parsing function state, if the new primitive count for the current pattern is less than the highest old primitive count, get the next target pattern and retry.
11. Upon return from the parsing function state, if the new primitive count for the current pattern is equal to the highest old primitive count, add the primitives to be removed (from the input) to the non-unique pattern primitive set. To this add the primitives to be removed from the partial pattern set $\omega$. Set the partial pattern set to null, get the next target pattern and retry.
12. When no more starting source primitives are available, enter the output state; indicate addition of the remaining primitives to the set of uncecogrized primitives and request return to the end state.
13. Enter termination state and stop.

### 3.14 Parsing Function

The structure of the parsing function for a general pattern is a non-deterministic finite automaton. It is a 19-tuple specified as follows.
$R=\left(Q^{\prime}, S^{\prime}, F^{\prime}, I, \gamma, F, K, K^{\prime}, C, R^{\prime}, \xi, B, B^{\prime}, W, N, \alpha, \alpha^{\prime}, O, \delta\right)$ where

$$
\begin{gathered}
\Omega=\left\{S_{1}^{\prime}, S_{2}^{\prime}, S_{3}, S_{4}, S^{\prime}, F^{\prime}, A_{1}, A_{2}, A_{3}, A_{4}\right\} \text { in which } \\
\\
\left\{S_{1}^{\prime}, S_{2}^{\prime}, S_{3}, S_{4}\right\} \subseteq Q \text { as defined in section } \\
3.11 \text { and }
\end{gathered}
$$

```
    {A1, A}\mp@subsup{A}{2}{},\mp@subsup{A}{3}{},\mp@subsup{A}{4}{}}\mathrm{ are states internal to the
        parsing function.
S' = S S & Q the start state as defined.
F' = {S', ,S ', ,S4}
\gamma = Next selection function, It is defined for the
    parsing function as follows.
\gamma(T) = Next target primitive selected for parse.
        = t\varepsilon{ {ti}
        In the grammar defining the target pattern, t
        corresponds to the next target pattern primi-
        tive expected in the parse sequence.
        Since the parse can be started from any primi-
        tive in the pattern definition, parse proceeds
        in a forward sequence or a backward sequence
        from the starting point. When parse proceeds
        in a forward sequence and all target pattern
        primitives have been parsed, the parse reverts
        to the starting point and parse is resumed in
        the backward sequence.
        Y(T) = \Phi when all target pattern primitives have been
        tried.
    \gamma(P,\Psi) = Next source primitive available for parse.
        This is defined in one of two forms.
        \gamma(P,\Psi) = Next p pointed to by a pointer in }
        for dot primitives and statically chained
        straight line, curve and area-component
```

primitives.
$\gamma(\mathrm{P}, \Psi)=$ Link $p \in P$ for dynamically chained primitives.
$\gamma(P, \Psi)=\Phi$ if no more source primitives are available for parse.
$\gamma(P, \Psi)$ is abbreviated as $\gamma(P)$.
$\gamma(\mu)=$ The next parametrization of $p$ to be tried. (Refer to section 3.11 for symbols.) If subscript $k>n \quad$ then $j=j+1$
$\gamma(\mu)=\Phi$ if $j=m \quad$ and $k=n$.
$I=$ Input internal to this state of the Macromaton and defined as shown. This is the output from the full or partial pattern recognition states.
$=\left(L, T_{i},((P, \Psi), P), S_{R}\right)$ where
L is the index of $T_{i}$,
$T_{i},((P, \Psi), p)$ are as defined and $S_{R}$ is the state to return to after completion of the state being entered. In the mappings this is also shown as $\left((T, t),\left((P, \Psi), S_{R}\right)\right.$.
$F=$ Outcome of form check where the source primitive type is compared to the target primitive type. $=\{\Phi, 0,1\}$ where $\Phi=$ Undefined (don't care) $0=$ Failure $1=$ Success
$K=$ Parametrization outcome defined as shown.
$=\{\Phi, 0,1,2\}$ where
$\Phi=$ Undefined (don't care)
$0=$ Parametrization failed
1 = Parametrization succeeded for all values that had to be checked.

2 = Parametrization succeeded for all the ones checked, but not all of them were checked. The manner in which the above values are arrived at is shown below.
(For symbols $p_{f}, p_{I}$ and $\beta$ used here, refer to output 0 later in this section. Symbols a,v,r,o are elements of $\mu_{i}$ as described in section 3.11.)
Let $p_{f}=p$ where $p_{f} \varepsilon \beta$.
and $p^{\prime} \varepsilon \beta$, the offset primitive from $p$ corresponding to offset $o_{j}$.
$K\left(p_{j k}\right)=1 \quad$ if $\left|r_{j k}\left(a v_{j k} A^{\prime} v^{\prime}{ }_{j k}\right)\right| \leq e_{j k}$
$k\left(p_{j k}\right)=0 \quad$ if $\left|r_{j k}\left(a v_{j k}, a^{\prime} v^{\prime}{ }_{j k}\right)\right|>e_{j k}$
or $p$ or $p^{\prime}=p_{I}$.
The above expressions paraphrased have the following meaning. Get the kth attribute value corresponding to the $j$ th offset for $p$ given by $a v_{j k}$ and the kth attribute corresponding to $p^{\prime}$ given by $a^{\prime} v^{\prime} j k$. Get their relational value $V=\left(r_{j k}\left(a v_{j k} a^{\prime} v^{\prime}{ }_{j k}\right)\right.$. If the absolute value of $V$ is less than or equal to the error tolerance specified, $e_{j k^{\prime}}$ then the parametrization outcome $K=1$.

```
If it is greater than e jk or p or 'p' is an ima-
ginary primitive }\mp@subsup{P}{I}{}\mathrm{ , then the parametrization
outcome K = 0.
C' = The outcome of final parametrization at the end
of the parse. This represents the parameters that
could not be verified while parsing was in progress.
={\Phi,(\mp@subsup{c}{1}{},\mp@subsup{c}{2}{})} defined as shown.
    \Phi = Undefined (don't care)
    c
        O = All remaining parameters did not comply.
        l = All remaining parameters complied
        c
                corresponding to the remaining parameters.
    C = Cumulative good parse primitive count.
R' = Outcome of current primitive reduction.
    = {0,1} where
    0 = Failure
    I = Success
\xi = Synchronization value
    = {¢:0,i} where
        = Undefined (don't care)
    0 = Value of }\xi\mathrm{ when the source and target
        primitives are in synchronization.
    l = Value assigned to }\xi\mathrm{ when an "unusable" or over-
        lapped primitive is encountered in a dynamic
        chain. This primitive does not participate in
```

the parse, and makes the source and target primitives get out of synchronization. $B=$ Backtrack function.
$=\{\Phi, 1\}$ where
$\Phi \equiv$ Undefined (don't care)
l = Backtrack

This is a stack function that shows how to recover from a parse that fails. It maintains pointers into the parse lists of $\alpha$ and $\alpha^{\prime}$. Associated with each pointer is the type of chain that is being parsed, i.e., whether it is a static chain or dynamic chain. When parse fails at any point, the parse reverts back to the most recent pointer with which the static chain type is associated. If no such pointer exists, or the start of the parse is reverted to, the entire parse fails.
$B^{\prime}=$ Outcome of Backtrack
$=\{\Phi, 0,1\}$ where
$\Phi=$ Undefined (don't care)
$0 .=$ failure
$1 .=$ Success
$W=$ Chaining status of parse
$=\{\Phi, S, D\}$ where
$\Phi=$ Undefined (don't care)
$s=$ Static chaining
$D=$ Dynamic chaining
$\mathrm{N}=$ Next mapping to be tried
$N \varepsilon\{i\}$ where each $i$ corresponds to a number associated with each mapping shown.
$\alpha=$ Set of primitives participating in the parse.
This is the same as $\beta$ of 0 shown below.
$\alpha^{\prime}=$ Set of primitives to be removed from the primiset. This is the same as $\mathrm{B}^{\prime}$ of O as shown below. $0=$ Output of the parsing function.
$\left(S, C,(L, B), \beta^{\prime}, S_{R}\right)$
This goes to the output function as well as the partial pattern recognition function. s has the following meaning. When return is made to the partial pattern recognition state, s refers to the success (1) or failure (0) of the parse. However, when the output state is entered during full pattern recognition,s refers to the output set $\rho$ to which the successful primitives are to be added.

C is the cumulative good parse primitive count.
L is the index of the target pattern being parsed.
$\beta$ is the parse of pattern corresponding to $L$ and is defined as follows.
$=\left\{p_{f}, \mu_{f}\right\}$ where
$p_{f}$ is the primitive parsed for form and $\mu_{f}$ is
its associated parametrization as defined in the target pattern definition. Each $\mathrm{p}_{\mathrm{f}}$ is of the form $p_{i}$ or $p_{I}$ where $p_{i} \varepsilon P$ and $p_{I}$ is primitive imagined to exist in the parse sequence as dictated by $G_{i} \cdot \mu_{f}$ assumes one of two values - $\mu_{i}$ the parametrization corresponding to $p_{i}$ as defined by $G_{i}$ or $\Phi$ corresponding to $p_{I}$. The form of $\mu_{i}$ is as defined in section 3.11.
$\beta^{\prime}=\left\{p_{j}\right\}$ the set of primitives corresponding to pattern $T_{i}$ to be removed from the primitive set where $p_{j} \varepsilon$ P. Each $p_{j}$ is one of the forms
 overlapped primitives and $p_{u}$ are marked as "unusable." In the mappings, $p_{\text {ou }}$ refers to a primitive that is either overlapped or unusable; $p_{\text {on }}$ refers to a primitive that is either overlapped or non-overlapped.
$\delta=$ Mapping from finite subsets of

to finite subsets of
Q x I x $\xi \mathrm{x} C \mathrm{x} \alpha \mathrm{x} \alpha^{\prime} \mathrm{x} B \mathrm{x} O \mathrm{x} \mathrm{N}$ and is defined as shown.

## Full Pattern Parse Mappings

1. $\delta\left(S_{3} ;\left((t, \mu), p, S_{1}^{\prime}\right), 1, \Phi, \Phi, \Phi, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{3},\left(\gamma(T), \gamma(P), S_{1}^{\prime}\right), 0,0, \alpha=\{p\}, \alpha^{\prime}=\{p\}, \Phi, \Phi,\{2,3\}\right)
$$

2. $\delta\left(S_{3}^{\prime},\left((t, \mu), P, S_{1}^{\prime}\right), \Phi, 1, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{3},\left((t, \gamma(\mu)), p, S_{1}^{\prime}\right), 0, C, \alpha, \alpha^{\prime}, \Phi, \Phi,\{2,3,4,5\}\right)
$$

3. $\delta\left(S_{3},\left((t, \mu), \mathrm{P}, \mathrm{S}_{1}^{\prime}\right), \Phi, 0, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{1}^{\prime},\left(T, \lambda, S_{1}^{\prime}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

4. $\delta\left(S_{3},\left((t, \Phi), p, S_{1}^{\prime}\right), \Phi, 1,1,0, \Phi, \Phi, \Phi\right)$

$$
\left.=\left(A_{1}, \gamma(T), \gamma(P), S_{1}^{\prime}\right), 0, C=C+1,(\alpha=\alpha \cup p),\left(\alpha^{\prime}=\alpha^{\prime} \cup p\right), \Phi, \Phi,\{2,3\}\right)
$$

5. $\delta\left(S_{3},\left((t, \Phi), p, S_{1}^{\prime}\right), \Phi, 2,1,0, \Phi, \Phi, \Phi\right)$

$$
=\left(A_{1^{\prime}}\left(\gamma(T), \gamma(P), S_{1}^{\prime}\right), 0, C,(\alpha=\alpha \cup p),\left(\alpha^{\prime}=\alpha^{\prime} \cup p\right), \Phi, \Phi,\{2,3\}\right)
$$

6. $\delta\left(A_{1},\left((t, \mu), P, S_{1}^{\prime}\right), 1, \Phi, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{3},\left((t, \mu), p, S_{1}^{j}\right), 0, C, \alpha, \alpha^{\prime}, \Phi, \Phi,\{2,3\}\right)
$$

7. $\delta\left(A_{1},\left((t, \mu), P, S_{1}^{\prime}\right), 0, \Phi, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(A_{2},\left((t, \mu), \gamma(P), S_{1}^{1}\right), 0, C, \alpha, \alpha^{1}, I, \Phi, 8\right)
$$

8. $\delta\left(A_{2},\left((t, \mu), P, S_{1}^{\prime}\right), \Phi, \Phi, \Phi, 0,1, \Phi, \Phi\right)$

$$
=\left(A_{1},\left((t, \mu), p, S_{1}^{1}\right), 0, C, \alpha, \alpha^{\prime}, \Phi, \Phi,\{6,7\}\right)
$$

9. $\delta\left(A_{2},\left((t, \mu), \Phi, S_{1}^{\prime}\right), \Phi, \Phi, \Phi, 0,1, \Phi, \Phi\right)$

$$
=\left(S_{I}^{\prime},\left(T, \lambda, S_{I}^{\prime}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

10. $\delta\left(A_{2},\left((t, \mu), p, S_{1}^{i}\right), \Phi, \Phi, \Phi, 0,0, \Phi, \Phi\right)$

$$
=\left(S_{1}^{j},\left(T, \lambda, S_{1}^{\prime}\right), 0,0, \Phi, \Phi, \Phi, \Phi, \Phi\right)
$$

11. $\delta\left(S_{3},\left(\Phi, R, S_{1}^{\prime}\right), \Phi, \Phi, \Phi, 0, \Phi,\left(1, C^{\prime}\right), S\right)$

$$
=\left(S_{4},\left(T, \lambda, S_{1}^{\prime}\right), \Phi, C=C+C^{\prime}, \alpha, \alpha^{\prime}, \phi,\left(1, C,(L, \beta=\alpha), \beta^{\prime}=\alpha^{\prime}, S^{\prime}\right), \Phi\right)
$$

12. $\delta\left(S_{3},\left(\Phi, D, S_{l}^{\prime}\right), \Phi, \Phi, \Phi, 0, \Phi,\left(0, C^{\prime}\right), S\right)$

$$
=\left(S_{1}^{\prime},\left(T, \lambda, S_{1}^{\prime}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

13. $\delta\left(S_{3},\left(\Phi, \mathrm{P}, \mathrm{S}_{1}\right), \Phi, \Phi, \Phi, 0, \Phi,(0, \mathrm{C}), \mathrm{D}\right)$

$$
=\left(S_{1}^{1},\left(T, \lambda, S_{1}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

14. $\delta\left(S_{3},\left((t, \mu), \Phi, S_{1}^{\prime}\right), \Phi, \Phi, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{1}^{\prime},\left(T, \lambda, S_{1}^{\prime}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

## Discussion of Full Pattern Parse Mappings

1. Start full pattern recognition with this mapping. When parsing function is entered, one source and target primitive type compliance is ensured. Save the source primitive in the set of good parse primitives and the set of primitives and the set of primitives to be removed. (The parametrization from the target pattern definition is saved in the parse primitive set. This is not shown in the mappings.) Set synchronization value to 0 and the cumulative primitive count to 0 . Get the next source and target primitives.
2. Check parametrization. If it is satisfied, get the next parameter to check.
3. If parametrization fails, fail recognition and return to full pattern recognition state.
4. If all parameters for the current primitive are checked and they are satisfied, add 1 to the primitive count. Add the source primitive to the parse primitive set and the set of primitives to be removed. Get the next source and target primitives.
5. If all parameters cannot be checked, but the ones checked are satisfied, add the source primitive to
the parse primitive set and the set of primitives to be removed. Get the next source and target primitives for parse.
6. If the source and target primitive types match, return to check the parametrization as shown in mapping 2.
7. If the check for form fails, backtrack. Get the rext source primitive to try for parse at that point in the parse.
8. If backtrack is successful, try for form check again.
9. If there are no more source primitives that can be tried for parse at this point, fail recognition and return to the full pattern recognition state.
10. If backtrack fails, return to the full pattern recognition state signalling failure.

1l. If all the target primitives are found and the final parametrization is successful, add the number of primitives from this check to the cumulative primitive count. Move to the output state and pass the cumulative primitive count, the parse primitive set, the set of primitives to be removed and signal successful recognition. Request return to the full pattern recognition state $S_{1}$ '.
12. If final parametrization fails, return to full pattern recognition state signalling recognition
failure.
13. If target pattern primitives comprising the full pattern are exhausted (which says that the whole pattern has been parsed) at the time of reducing a dynamic chain and there are more source primitives in the dynamic chain, discard the parse. Return to full pattern recognition state with a recognition failure.
14. If there are more target primitives to be parsed, but no more source primitives to be tried, fail parse. Revert to the full pattern recognition state.

## Partial Pattern Parse Mappings

1. $\delta\left(S_{3},\left((t, \mu), p, S_{2}^{\prime}\right), 1, \Phi, \Phi, \Phi, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{3^{\prime}}\left(\gamma(T), \gamma(P), S_{1}^{\prime}\right), 0,0, \alpha=\{p\}, \quad \alpha^{\prime}=\{p\}, \Phi, \Phi,\{2,3\}\right)
$$

2. $\delta\left(S_{3},\left((t, \mu), p, S_{2}^{\prime}\right), \Phi, 1, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{3},\left((t, \gamma(\mu)), p, S_{1}^{\prime}\right), 0, C, \alpha, \alpha_{1}^{\prime} \Phi, \Phi,\{2,3,4,5\}\right)
$$

3. $\delta\left(S_{3},\left((t, \mu), p, S_{2}^{\prime}\right), \Phi, 0, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{2}^{\prime},\left(T, \lambda, S_{2}^{\prime}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

4. $\delta\left(S_{3},\left((t, \Phi), p, S_{2}^{\prime}\right), \Phi, 1,1,0, \Phi, \Phi, \Phi\right)$

$$
=\left(A_{3 r}\left(\gamma(T), \gamma(P), S_{2}^{j}\right), 0, C=C+1,(\alpha=\alpha \cup p),\left(\alpha^{\prime}=\alpha^{\prime} \cup p\right), \Phi_{r}, \Phi_{f}\{2,3\}\right)
$$

5. $\delta\left(S_{3},\left((t, \Phi), P, S_{2}^{\prime}\right), \Phi, 2,1,0, \Phi, \Phi, \Phi\right)$

$$
\left.=\left(A_{3^{r}} \gamma(T), \gamma(P), S_{2}^{\prime}\right), 0, C,(\alpha=\alpha \cup p),\left(\alpha^{\prime}=\alpha^{\prime} \cup p\right), \Phi, \Phi_{r}\{2,3\}\right)
$$

6. $\delta\left(A_{3},\left((t, \mu), p, S_{2}^{\prime}\right), 1, \Phi, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{3},\left((t, \mu), p, S_{2}^{\prime}\right), 0, C, \alpha, \alpha^{\prime}, \Phi, \Phi,\{2,3\}\right)
$$

7. $\delta\left(A_{3},\left((t, \mu), P, S_{2}^{\prime}\right), 0, \Phi, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(A_{4},\left((t, \mu), \gamma(P), S_{2}^{\prime}\right), 0, C, \alpha, \alpha^{\prime}, 1, \Phi, 8\right)
$$

8. $\delta\left(A_{4},\left((t, \mu), P, S_{2}^{\prime}\right), \Phi, \Phi, \Phi, 0,1, \Phi, \Phi\right)$

$$
=\left(A_{3},\left((t, \mu), p, S_{2}^{\prime}\right), 0, C, \alpha, \alpha, \Phi, \Phi,\{6,7\}\right)
$$

9. $\delta\left(A_{4},\left((t, \mu), \Phi, S_{2}^{\prime}\right), \Phi, \Phi, \Phi, 0,1, \Phi, \Phi\right)$

$$
\left.=S_{2}^{\prime},\left(T, \lambda, S_{2}^{\prime}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

10. $\delta\left(A_{4},\left((t, \mu), p, S_{2}^{\prime}\right), \Phi, \Phi, \Phi, 0,0, \Phi, \Phi\right)$

$$
\left.=S_{2}^{\prime},\left(T, \lambda, S_{2}^{\prime}\right), 0,0, \Phi, \Phi, \Phi, \Phi, \Phi\right)
$$

11. $\delta\left(S_{3},\left(\Phi, P, S_{2}^{\prime}\right), \Phi, \Phi, \Phi, 0, \Phi,\left(1, C^{\prime}\right), S\right)$
$=\left(S_{2^{\prime}}^{\prime}\left(T, \lambda, S_{2}^{\prime}\right), \Phi, C=C+C^{\prime}, \alpha, \alpha^{\prime}, \Phi,\left(1, C,(L, \beta=\alpha), \beta^{\prime}=\alpha^{\prime}, S_{2}^{\prime}\right), \Phi\right)$
12. $\delta\left(S_{3}\left(\Phi, \mathrm{P}, S_{2}^{\prime}\right), \Phi, \Phi, \Phi, 0, \Phi,\left(0, C^{\prime}\right), S\right)$

$$
=\left(S_{2}^{\prime},\left(T, \lambda, S_{2}^{\prime}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

13. $\delta\left(S_{3},\left(\Phi, D, S_{2}^{r}\right), \Phi, \Phi, \Phi, 0, \Phi,(0, C), D\right)$

$$
=\left(S_{2}^{\prime},\left(T, \lambda, S_{2}^{\prime}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

14. $\delta\left(S_{3},\left((t, \mu), \Phi, S_{2}^{\prime}\right), \Phi, \Phi, \Phi, 0, \Phi, \Phi, \Phi\right)$

$$
=\left(S_{2}^{\prime},\left(T, \lambda, S_{2}^{\prime}\right), \Phi, 0, \alpha=\Phi, \alpha^{\prime}=\Phi, \Phi,(0,0, \Phi, \Phi, \Phi), \Phi\right)
$$

15. $\delta\left(A_{3},\left((t, \mu), \mathrm{P}_{\text {ou }}, S_{2}^{\prime}\right), 0, \Phi, \Phi, 0, \Phi, \Phi, D\right)$

$$
=\left(A_{4},\left((t, \mu), \gamma(P), S_{2}^{\prime}\right), 1, C, \alpha, \alpha^{\prime}=\alpha^{\prime} \cup p_{o u}, \Phi, \Phi, 16\right)
$$

16. $\delta\left(A_{4},\left((t, \mu), \mathrm{P}_{\text {ou }}, S_{2}^{\prime}\right), 0, \Phi, \Phi, 1, \Phi, \Phi, D\right)$

$$
=\left(A_{4},\left((t, \mu), \gamma(P), S_{2}^{\prime}\right), 1, C_{r} \alpha, \alpha^{\prime}=\alpha^{\prime} \cup p_{\text {Ou }}, \Phi, \Phi,\{16,17\}\right)
$$

17. $\delta\left(\mathrm{A}_{4},\left((t, \mu), \mathrm{P}, \mathrm{S}_{2}^{\prime}\right), 0, \Phi, \Phi, 1, \Phi, \Phi, \mathrm{D}\right)$

$$
=\left(A_{4},\left(\gamma(T), p, S_{2}^{\prime}\right), 1, C, \alpha=\alpha \cup p_{I}, \alpha^{\prime}, \Phi, \Phi,\{17,18\}\right)
$$

18. $\delta\left(A_{4},\left((t, \mu), p_{\text {on }} S_{2}^{\prime}\right), 1, \Phi, \Phi, 1, \Phi, \Phi, D\right)$

$$
=\left(S_{3},\left((t, \mu), p, S_{2}^{\prime}\right), 0, C, \alpha, \alpha^{\prime}, \Phi, \Phi,\{2,3\}\right)
$$

19. $\delta\left(A_{4},\left(\Phi, P, S_{2}^{\prime}\right), 0, \Phi, \Phi, l, \Phi, \Phi, D\right)$

$$
=\left(A_{3},\left((t, \mu), \gamma(P), S_{2}^{\prime}\right), 0, C, \alpha, \alpha^{\prime}, 1, \Phi, 8\right)
$$

## Discussion of Partial Pattern Parse Mappings

Mappings 1 through 14 are a repetition of the mappings shown for the full pattern parse but pertain to the partial pattern recognition state.
15. If the source and target primitive types do not match and the source primitive is an overlapped or unusable one in a dynamic chain, do the following. Add the current source primitive to the set of primitives to be removed, set the synchronization value to 1 , implying that the source and target primitive sequences are out of synchronization. Get the next source primitive for parse.
16. As long as source primitives in a dynamic chain are overlapped or unusable and the type does not match the target primitive type expected, add the source primitive to the set of primitives to be removed. Get the next source primitive.
17. If the synchronization value is 1, a source primitive in a dynamic chain is not overlapped or unusable and the source and target primitive types to not match, do the following. Add an imaginary primitive corresponding to the target primitive type expected (along with the parametrization values) to the parse set $\beta$, but not to
the set $\beta^{\prime}$, the primitives to be removed. Get the next target pattern primitive.
18. If the source primitive is overlapped or nonoverlapped (but not unusable) and the source and target primitive types match when the synchronization value is 1 , reset this value to 0 and revert to normal parsing as specified in mapping 2.
19. If the synchronization value is 1 and all the target primitives have been checked, backtrack and try again.

### 3.15 Output Function

This function of the Macromaton is described by a deterministic finite automaton. It is a 7-tuple and is specified as follows:

$$
\begin{aligned}
R_{4}= & \left(Q^{\prime}, \lambda, I, \Delta, S^{\prime}, F^{\prime}, \delta\right) \text { where } \\
Q^{\prime}= & \left\{S_{1}^{\prime}, S_{2}^{\prime}, S_{2}^{\prime \prime}, S_{4}, S^{\prime}, F^{\prime}\right\} \text { in which } \\
& \left\{S_{1}^{\prime}, S_{2}^{\prime}, S_{2}^{\prime \prime}\right\} \subseteq Q \text { as defined in section } 3.11 . \\
S^{\prime}= & S_{4} \varepsilon Q \text { the start state as defined. } \\
F^{\prime}= & \left\{S_{1}^{\prime}, S_{2}^{\prime}, S_{2}^{\prime \prime}\right\} \varepsilon Q \text { the final states as defined. } \\
\lambda= & \text { The primitive set } \\
= & (P, \Psi) \text { as defined } \\
I= & \text { Input to this state and internal to the Mac- } \\
& \text { romaton. It has one of two forms. } \\
& \left.I_{1}=((P, \Psi), P), S_{R}\right) \text { as defined by } 0 \text { in section }
\end{aligned}
$$

3.13 for partial pattern recognition.
$I_{2}=\left(s, C,(L, \beta), \beta^{\prime}, S_{R}\right)$ as defined by $O_{3}$ in section 3.13 for partial pattern recognition and $O$ in section 3.14 for parsing function. In the mappings, only $\left\{p_{i}\right\} \subseteq \beta=\left\{p_{f}\right\}$ is shown. (These are the non-imaginary primitives.) $\beta^{\prime}$ is shown as $\left\{p_{j}\right\}$.
$\Delta=$ The output of the Macromaton as defined in section 3.11.
$=\left\{p_{i}\right\}, i=1,4$.
$\delta=$ Mapping from finite subsets of $Q \times \lambda \times$ I to finite subsets of $Q \times \lambda \times \Delta$ and defined as shown.

1. $\delta\left(S_{4},(P, \Psi),\left(\left((P, \Psi), P_{s}\right), S_{2}^{\prime \prime}\right)\right)$

$$
=\left(S_{2}^{"},\left(P, \Psi \ll r_{p_{s}}\right), \Phi\right)
$$

2. $\delta\left(S_{4},(P, \Psi),\left(1, C,\left(L,\left\{p_{i}\right\}\right),\left\{p_{j}\right\}, S_{1}^{\prime}\right)\right)$

$$
=\left(s_{1^{\prime}}^{\prime}\left(P-\left\{p_{j}\right\} ; P<=\left\{p_{0}\right\} \subseteq\left\{p_{i}\right\} \psi=>\left\{x_{p_{j}}\right\}\right), \rho_{1}=\rho_{1} \cup\left(L,\left\{p_{j}\right\}\right)\right)
$$

3. $\delta\left(S_{4},(P, \Psi),\left(2, C,\left(L,\left\{p_{i}\right\}\right),\left\{p_{j}\right\}, S_{2}^{\prime}\right)\right)$

$$
=\left(S_{2}^{\prime},\left(P-\left\{p_{j}\right\} ; P<=\left\{p_{0}\right\} \subseteq\left\{p_{i}\right\}, \Psi \Rightarrow\left\{r_{p_{j}}\right\}\right), p_{2}=\rho_{2} \cup\left(L,\left\{p_{j}\right\}\right)\right)
$$

4. $\delta\left(S_{4},(P, \Psi),\left(3,\left(\Phi,\left\{p_{i}\right\}\right),\left\{p_{j}\right\}, S_{2}^{i}\right)\right.$

$$
=\left(S_{2}^{\prime},\left(P-\left\{p_{j}\right\} ; \Psi \Rightarrow\left\{r_{p_{j}}\right\}\right), \rho_{3}=\rho_{3} \cup\left\{p_{j}\right\}\right)
$$

5. $\delta\left(S_{4},(P, \Psi)_{r}(4,(\Phi, P), \Phi, E)\right)$

$$
=\left(E,(P, \Psi=\Phi), \rho_{4}=P\right)
$$

## Discussion of Mappings

1. If a type 1 input is received, mark the pointer corresponding to the primitive for which parse was unsuccessful in the appropriate pointer list. Return to the partial pattern recognition state.
2. From the input, add the index and the set of primitives to be removed, to the set of patterns fully recognized. Remove these primitives from the primitive set and their pointers from the pointer list set. The primitives left over are marked unusable corresponding to the overlapped primitives removed. Return to the full pattern recognition state.
3. This is a repeat of 3 , but for partial patterns. Ai completion, return to partial pattern recognition.
4. Add the primitives to be removed to the set where recognition cannot be uniquely determined. Remove these primitives from the primitive set and their pointers from the pointer list set. Revert to partial pattern recognition.
5. When return to end state is requested, make all the remaining primitives the set for which no mapping into the chosen target pattern set exists. Return to the end state.

### 3.16 Imaginary Parsing Scheme

This section concludes the discussion on the Macromaton function. Figure 3.16 shows the state transition diagram of the Macromaton.


Figure 3.16 Macromaton State Transitions The following discussion is aimed at providing an
overall understanding of the Macromaton function. The parsing scheme outlined is referred to as the "imaginary" parsing scheme. In a pattern parsing scheme wherein patterns overlap each other obscuring some of the primitives, it is not possible to get a complete parse for the pattern. Any attempt to deduce a pattern from only a part of it implies that the missing parts are "imagined" to be present. Hence the name. This scheme is general in scope and works equally well for class $A$ and class $B$ patterns. The following technique describes the imaginary parsing scheme.

The patterns of interest are defined individually using the $D-L-A$ grammar. The order in which the patterns are defined has a hierarchy; the most complex pattern being the first and the simplest the last. The complexity of a pattern is determined by the number of primitives that make up the pattern.

To begin with, all patterns that parse fully are found and removed from the primitive set, starting with the first pattern defined (the most complex) down to the last (the simplest). As the source primitives comprising each target pattern is removed from the source primitive set, the primitives that are left behind and overlapped with the primitives removed, are marked "unusable." Unusable primitives are not eligible to participate in any parse. Consequently they cannot be picked as the starting primitive of a parse. After all the patterns fully visible are found and
removed, recognition of partial patterns is attempted. For any given starting source primitive, parse of each pattern is attempted starting from the most complex, proceeding down to the simplest. Corresponding to this starting source primitive, every possible starting point in each pattern is tried for a parse. Out of all these tries, the parse yielding the greatest number of primitives identifies the partial pattern recognized. This is done as described below.

Starting from the first pointer in the first pointer list, source primitives are selected from the primitive set one at a time as the starting point of a parse. Corresponding to che chosen starting source primitive, the target pattern is searched to see if such a primitive type exists where reduction can be started. If such a primitive exists, parse is tried; if not the next pattern is selected. Once parsing is started, it proceeds by looking for a match between the target primitive and source primitive types and parameter compliance between the primitives in the parse. In dynamic chains, primitives that are marked unusable or those that are overlapped and do not conform to the expected form do not participate in the parse. These are added to the set of primitives to be removed, but not to the parse set. Because of these intervening primitives, if an expected primitive is not found in the parse sequence, an imaginary primitive is filled in and the parse continued until the whole pattern is parsed or the parse fails. The
imaginary primitives are not eligible to participate in the parametrization and therefore do not contribute to the count of good primitives in the parse. This is repeated for every possible starting point in the current pattern and the count of the greatest number of good parse primitives is obtained.

The above process is repeated for each pattern defined. The pattern yielding the highest good parse primitive count is identified as the partial pattern. As in the case of full pattern recognition, the source primitives partici. pating in the partial pattern parse are removed from the primitive set and the corresponding overlapped primitives left behind are marked unusable.

If two or more patterns have the same good parse primitive count, then it is not possible to uniquely determine which partial pattern is represented by the parse. In this case, these primitives are added to the set of primitives for which unique recognition is not possible.

If a starting primitive does not yield a good parse for any of the patterns defined, the pointer to that starting primitive is marked. Marked pointers in the pointer list are ineligible to be used to pick a starting source primitive for parse.

The above process is repeated for every possible starting source primitive. Parsing terminates when there are no pointers in any of the pointer lists, using which
starting primitive can be obtained. The primitives left over are identified as the set for which no target pattern definitions exist.

## VERIFICATION OF FORMAIISM

### 4.0 Introductory Remarks

Computer implementation of concepts discussed was carried out to test the validity of the formalism. Two sets of scenes were defined - one with non-overlapped and the other with overlapped patterns. Scanning and recognition were performed using the principles outlined in the first three chapters. The recognized patterns were displayed on an IBM 3279 color console using "GDDM" software. The scenes defined, data structures and algorithms used in the recognition and the results obtained are presented in the following sections of this chapter.

### 4.1 Definition of Patterns for Recognition

A set of geometric patterns consisting of circles, squares, rectangles, equilateral triangles and right angle triangles of different colors were used as the basic componerts of the scenes defined. Patterns from this set were arbitrarily selected and each assigned an arbitrary color. These were then arbitrarily positioned and rotated within the pattern space. The resulting scenes were drawn to scale
on paper.
A computer program was written to convert the above scenes to data that could be used by the scanner simulation program. The program was designed to handle up to seven patterns in each scene definition. Of the two sets of scenes defined, the first set consisted of three scenes comprising class A patterns in which none of the shapes overlapped each other. The second set consisted of five class B patterns in which overlapping occurred. The following values obtained from the drawings formed the input to the scene definition program.

1. Vectorial distance $\left(\mathrm{V}_{0}\right)$ to the starting point from the origin - a corner in the case of a straightsided figure and the center in case of a circle.
2. Angle $\left(\theta_{0}^{\circ}\right)$ made by the vector $v_{0}$ with the +x-axis.
3. Length (Ll) of first side in case of a straightsided figure; radius ( R ) in case of a circle.
4. Angle $\left(\theta_{\mathrm{H}}^{\mathrm{H}}\right)$ made by the first side Ll with the $+\mathrm{x}-$ axis in the case of straight-sided figures.
5. Length (L2) of the second side in the case of straight-sided figures.
6. Rotation of remaining sides with respect to the first side in the case of straight-sided figures. ( 0 = clockwise; 1 = counter-clockwise.)
7. Color \# of figure.

The smallest side (or radius was limited to l" for all patterns defined. The program converted the angles
from degrees to radians and output the information to a dataset. This then formed the input to the scanner simulation program. The set of patterns defined is shown in Figures 4.1 through 4.8. The symbols used for table headings in these figures is the same as shown in parentheses in the above description. The starting point for the definition of each of these shapes is marked by the symbol " $x$ " in these figures. Samples of output from the scene definition program is included in Appenaix A.

### 4.2 Simulation of Pattern Scanning

A scanner simulation program was written to convert the defined scenes to a set of values (as outlined in the formalism), which formed the input to the recognition program. Based on the smallest dimension chosen for the pattern definitions (1"), the connectivity distance was arbitrarily selected as 0.1". The sampling distance along the probe was set at $0.025^{\prime \prime}$, which amounted to a redundancy factor of 4 in relation to the connectivity distance. The incremental probe angle de was arrived at as follows. As a first approximation, the expression $r_{\max } \mathrm{d} \theta=$ connectivity distance was used, where $r_{\max }$ is the maximum probe length of the pattern. Based on an $8-1 / 2^{\prime \prime} \times 11 "$ pattern space, $r_{\max }=$ $\sqrt{\left(8.5^{2}+11^{2}\right)}=13.9^{\prime \prime}$. Since the connectivity distance was chosen as $0.1^{\prime \prime}, d \theta=.007$ radians or $0.4^{\circ}$. With a redundancy factor of 10 , the incremental probe angle was set at 0.04 .

The scanner simulation program performed the following function. For a given point along the probe, the program


| No. | Type | $v_{0}$ | $\theta_{0}^{0}$ | $R / L I$ | $\theta_{\mathrm{H}}^{0}$ | $I 2$ | Rotation | Color |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I. | Circle | 5.5 | 15.0 | 1.0 |  |  |  | 1 |
| 2. | Eq.Tri. | 3.0 | 30.0 | 1.5 | 0 | 1.5 | 1 | 3 |
| 3. | Square | 5.0 | 55.0 | 3.0 | 0 | 3.0 | 1 | 2 |
| 4. | Rectangle | 3.0 | 60.0 | 1.0 | 0 | 1.5 | 1 | 4 |
| 5. | Rt.Tri. | 5.5 | 70.0 | 1.0 | 0 | 2.0 | 1 | 5 |

Fig. 4.l Scene \#1 - Non-Overlapped Patterns

| No. | Type | $v_{0}$ | $\theta_{0}^{0}$ | $R / L l$ | $\theta_{H}^{O}$ | L2 | Rotation | Color |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Eq.Tri. | 5.0 | 15.0 | 1.0 | 15.0 | 1.0 | 0 | 1 |
| 2. | Rectangle | 3.0 | 30.0 | 2.5 | 20.0 | 1.0 | 1 | 2 |
| 3. | Square | 7.0 | 45.0 | 1.0 | 30.0 | 1.0 | 0 | 3 |
| 4. | Rt.Tri. | 5.0 | 60.0 | 2.0 | 20.0 | 2.0 | 0 | 4 |
| 5. | Circle | 8.0 | 75.0 | 1.0 |  |  |  | 5 |

Fig. 4.2 Scene \#2 - Non-Overlapped Patterns


| No. | Type | $\mathrm{v}_{\mathrm{O}}$ | $\theta_{0}^{0}$ | $\mathrm{R} / \mathrm{LI}$ | $\theta_{\mathrm{H}}^{0}$ | L 2 | Rotation | Color |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Circle | 2.5 | 30.0 | 1.0 |  |  |  | 2 |
| 2. | Rectangle | 4.0 | 15.0 | 3.0 | 30.0 | 1.0 | 1 | 1 |
| 3. | Rectangle | 4.0 | 60.0 | 1.5 | 30.0 | 1.0 | 1 | 5 |
| 4. | Circle | 7.0 | 45.0 | 1.5 |  |  |  | 3 |
| 5. | Square | 9.0 | 52.0 | 1.5 | 60.0 | 1.5 | 1 | 4 |
| 6. | Rt.Tri. | 9.0 | 75.0 | 2.0 | 10.0 | 4.0 | 0 | 1 |
| 7. | Eq.Tri. | 7.0 | 85.0 | 1.5 | 50.0 | 1.5 | 1 | 2 |

Fig. 4.3 Scene \#3 - Non-Overlapped Patterns


| No. | Type | $\nu_{0}$ | $\theta_{0}^{0}$ | $\mathrm{R} / \mathrm{Ll}$ | $\theta_{\mathrm{H}}^{0}$ | L 2 | Rotation | Color |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Eq.Tri. | 6.3 | 55.5 | 1.0 | 0 | 1.0 | 1 | 5 |
| 2. | Rt.Tri. | 4.7 | 58.0 | 2.0 | 0 | 2.0 | 1 | 4 |
| 3. | Rectangle | 4.2 | 61.5 | 3.0 | 0 | 2.5 | 1 | 3 |
| 4. | Square | 3.5 | 62.5 | 3.8 | 0 | 3.8 | 1 | 2 |
| 5. | Circle | 6.1 | 55.0 | 3.0 |  |  |  | 1 |

Fig. 4.4 Scene \#4 - Overlapped Patterns


| No. | Type | $v_{0}$ | $\theta_{0}^{\circ}$ | $\mathrm{R} / \mathrm{LI}$ | $\theta_{\mathrm{H}}^{\circ}$ | L 2 | Rotation | Color |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Circle | 5.8 | 72.0 | 1.0 |  |  |  | 1 |
| 2. | Eq.Tri. | 5.5 | 64.0 | 1.0 | 0 | 1.0 | 1 | 2 |
| 3. | Rt.Tri. | 5.1 | 54.0 | 2.0 | 0 | 2.0 | 1 | 3 |
| 4. | Rectangle | 3.9 | 59.0 | 2.0 | 0 | 1.0 | 1 | 4 |
| 5. | Square | 2.2 | 63.0 | 2.0 | 0 | 2.0 | 1 | 5 |

Fig. 4.5 Scene \#5 - Overlapped Patterns


| No. | Type | $v_{0}$ | $\theta_{0}^{0}$ | $R / L 1$ | $\theta_{H}^{0}$ | $L 2$ | Rotation | Color |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Circle | 5.8 | 40.0 | 1.0 |  |  |  | 1 |
| 2. | Eq.Tri. | 3.0 | 20.0 | 1.5 | 20.0 | 1.5 | 1 | 2 |
| 3. | Rt.Tri. | 3.0 | 20.0 | 2.0 | 20.0 | 2.5 | 1 | 3 |
| 4. | Rectangle | 3.0 | 20.0 | 3.0 | 20.0 | 3.5 | 1 | 4 |
| 5. | Square | 3.0 | 20.0 | 4.0 | 20.0 | 4.0 | 1 | 5 |

Fig. 4.6 Scene \#6 - Overlapped Patterns


| No. | Type | $v_{0}$ | $\theta_{0}^{0}$ | $R / L 1$ | $\theta_{H}^{0}$ | L2 | Rotation | Color |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Rt.Tri. | 7.1 | 58.0 | 3.0 | 26.5 | 1.0 | 1 | 5 |
| 2. | Circle | 9.0 | 58.0 | 1.0 |  |  |  | 6 |
| 3. | Rectangle | 2.8 | 60.0 | 2.6 | 0 | 1.0 | 1 | 1 |
| 4. | Eq.Tri. | 3.4 | 50.0 | 3.5 | 20.0 | 3.5 | 1 | 2 |
| 5. | Eq.Tri. | 3.2 | 72.0 | 3.0 | 25.0 | 3.0 | 1 | 3 |
| 6. | Square | 4.8 | 48.0 | 3.0 | 12.0 | 3.0 | 1 | 4 |

Fig. 4.7 Scene \#7 - Overlapped Patterns


| No. | Type | $v_{0}$ | $\theta_{0}^{0}$ | $R / L 1$ | $\theta_{\mathrm{H}}^{0}$ | L2 | Rotation | Color |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Circle | 8.1 | 80.0 | 1.0 |  |  |  | 1 |
| 2. | Circle | 9.0 | 60.0 | 1.0 |  |  |  | 2 |
| 3. | Circle | 7.0 | 60.0 | 1.5 |  |  |  | 3 |
| 4. | Circle | 5.5 | 75.0 | 1.0 |  |  |  | 4 |
| 5. | Circle | 5.0 | 40.0 | 1.0 |  |  |  | 5 |
| 6. | Circle | 5.0 | 45.0 | 2.0 |  |  |  | 6 |

Fig. 4.8 Scene \#8 - Overlapped Patterns
determined whether the point lay within each of the patterns comprising the scene. The color chosen depended on the color of the pattern enclosing the point. In the case of overlapped patterns, the color chosen was that of the first pattern in the pattern definition sequence. Therefore, for overlapped patterns, the first pattern defined was the topmost, while the last pattern defined was the bottommost as would be seen visually. The sampling point was incremented by the sampling distance and the above process was repeated until the end of probe was reached. No output was made until there was a change from a color of interest to the background color or to another color of interest. When this occurred, the following values were output for the color of interest.

1. Distance along probe for start of color.
2. Distance along probe for end of color.
3. Angle made by probe with the +x-axis.
4. Color identification number.
5. Previous scanner color state. (0 = Transition from background color. l $=$ Transition from another color.)
6. Next scanner color state. (0 = Transition to background color. I = Transition to another color.)
7. Current probe number.

The probe angle was next incremented by do as defined earlier and the process repeated for a new probe until the
probe angle was $\Pi / 2$. For this probe angle, when the probe length was ll", scanning was terminated. Samples of scanner output is included in Appendix B.

### 4.3 Data Structures for Primitive Set Representation

Conforming to the requirements of the formalism, a twolevel data structure was used for each primitive in the set of primitives constituting the pattern. One level defined the primitive set $P$ as described in the formalism. It linked together all primitives that constituted any given pattern (such as circle, rectangle, etc., overlapped or not) in a doubly-linked list as shown in Figure 4.9.


Figure 4.9 Doubly-linked list of pattern primitives

The second level of data structure corresponded to the pointer seis $\Omega$ and $\Psi$. Pointers to all primitives in an "active" state of reduction were linked together in a doublylinked list, forming the pointer list $\Omega$. Once reduction was completed for a given primitive, the pointer to it was removed from the list $\Omega$, its type determined and added to the appropriate "inactive" primitive pointer list set $\Psi$. The lists in $\Psi$ were also doubly-linked. This is shown in Figure 4.10.

Samples of output of the primitive set at the end of the primitive formation phase are included in Appendix $C$.


Figure 4.10 Active and inactive primitive pointer lists.

### 4.4 Data Structure for Target Pattern Representation

The target pattern definitions included the primitive types that made up the form of each pattern and the associated parameters that needed to be satisfied. In the pattern definition sequence, the patterns were defined in the order by the decreasing number of primitives comprising the pattern. The following sequence was used - circle, square, rectangle, equilateral triangle, right angle triangle. Table 4.1 shows the type of parametrization used for the patterns chosen.

Table 4.l Parametrization Table

| Entity | Attribute within entity | Relation | Offset | Tolerance |
| :---: | :---: | :---: | :---: | :---: |
| (A) | (B) | (C) | (D) | (E) |
| $\begin{aligned} & 1= \\ & \text { Primitive } \\ & \text { Attribute } \end{aligned}$ | $\begin{aligned} & 1=\text { Length } \\ & 2=\text { Angle } \end{aligned}$ | $\begin{aligned} 1 & =\text { equal } \\ 2= & \text { not } \\ & \text { equal } \end{aligned}$ | $+/$ - integer showing the primitive with which this | difference acceptable |
| $2=$ <br> Computed value | $\begin{aligned} & 1=\text { Radius } \\ & 2=\text { Center } \end{aligned}$ | $\begin{aligned} & 3<\pi / 2 \\ & 4=\Pi / 2 \\ & 5=\Pi / 3 \end{aligned}$ | be satisfied |  |

The data structure used for definition of these target patterns whose recognition was desired is shown in Figure 4.11. It shows not only the form of the pattern, but its parametrization as well. Details for the rectangle pattern are presented in Figure 4.11.

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Figure 4.11 Data structure for rectangle target pattern

### 4.5 Algorithms

Algorithms used in the recognition scheme are presented in this section. The set of algorithms are presented as a series of "procedures." Each procedure has a set of input and output shown, if applicable. The input are the values that must be available to the procedure upon entry. The output are the values returned or passed by the procedure.

### 4.5.1 Procedure: Scan

Input: 1. Pattern to be scanned
2. Sampling-distance along probe
3. Incremental-probe-angle

Output: 1. Start-distance of color (QI)
2. End-distance of color (QJ)
3. Probe-angle with +x-axis ( $\theta$ )
4. Color-id. (Scolor)
5. Previous-scanner-color-state (PSCS)
6. Next-scanner-color-state (NSCS)
7. Probe-number (PN)

Begin
Set Probe-angle $=0$
Probe-distance $=0$
Probe-number $=0$
Do Until Probe-angle $=\Pi / 2$ and
Probe-distance $=Y_{\text {max }}$
Probe-distance $=0$

```
Probe-angle = Probe-angle
    + Incremental-probe-angle
Probe-number = Probe-number + l
Current-color = Background-color
Compute Probe-length
Do Until Probe-end
    Probe-distance = probe-distance
        + Sampling-distance
    New-color = Background-color
    Do for each pattern defined I,
    If point at Probe-distance is contained by
        pattern (I)
    Then point(I) = 1
    Else point(I) = 0
    End
    Do for each pattern defined I,
    If point(I) = 1
    Then Do
        New-color = color(I)
        stop further checks
        End
    End
    If Current-color f New-color
    Then Do
        If Current-color = Background-color
        Then Do
```

```
Current-color \(=\) New-color
Start-distance \(=\) Probe-distance
End-Distance \(=0\)
```

End
Else Do
If New-color = Background-color
Then Next-scanner-color-state $=0$
Else Next-scanner-color-state $=1$
End-distance $=$ Probe-distance

- Sampling-distance

Output for Current-color
Start-distance
End-distance
Probe-angle
Color-id.
Previous-scanner-color-state
Next-scanner-color-state
Probe-number
If New-color $=$ Background-color
Then Previous-scanner-color-state $=0$
Else Previous-scanner-color-state $=1$
Set Current-color $=$ New-color
If Current-color $\neq$ Background-color
Then Do
Start-distance = Probe-distance
End-distance $=0$
End

End Do Until
End Do Until
End Scan

### 4.5.2 Procedure: Recognize

Output: 1. Output-all-flag
Begin
Initialize
Call Form-Primitives
Do While primitive set is not null or
Inactive-primitive-pointer-lists is not null
Do Until No-more-full-patterns
Call Find-full-pattern
End Do Until
Do Until No-more-partial-patterns
Call Find-partial-pattern
End Do Until
Set Output-all-flag = 1
Call Output
End Do While
End Recognize
4.5.3 Procedure: Form-Primitives

Input: 1. Input set from scanner, each input consisting of the following elements.
a. Start-distance of color (QI)
b. End-distance of color (QJ)
c. Probe-angle with +x-axis ( $\theta$ )
d. Color-id. (Scolor)
e. Previous-scanner-color-state (PSCS)
f. Next-scanner-color-state (NSCS)
g. Probe-number (PN)
2. Active-primitive-pointer-list
3. QI-number from Connectivity-Check Procedure
4. QJ-number - Do -
5. QI-twin from Connectivity-Check Procedure
6. QJ-twin - Do -
7. Reduction primitive set (RPS) from Connec-tivity-Check Procedure for current scanner input.

Output: 1. One scanner input consisting of the elements shown above.
2. QI-number showing the number of primitives to be reduced with $Q I$ of current scanner input
3. QJ-number showing the number of primitives to be reduced with QJ of current scanner input
4. Reduction primitive set (RPS)
5. Primitive-set
6. Final-output-flag

Begin

Do Until No-more-scanner-input
Get scanner input
Using Active-primitive-pointer-list
If active primitive set is empty
Then Do
Set QI-number $=0$ QJ-number $=0$

Call Create-new-primitive
End
Else Do
Call Connectivity-check
If $Q I$-number $=0$ or $Q J$-number $=0$ and $Q J \neq 0$

Then Call Create-new-primitive
If $Q I$-number $\neq 0$ or QJ-number $\neq 0$

Then Call Reduce
If Previous-probe \# $\neq$ Probe-number
Then Do
Set Previous-probe \# = Probe-number
Call Output-Primitives
End
End Do Until
Set Final-output-flag =1
Call Output-Primitives
End Form-Primitives

### 4.5.4 Procedure: Connectivity-Check

Input: 1. Current input from scanner as shown in section 4.5 .3
2. Active-primitive-pointer-list
3. Primitive-set
4. Connectivity-distance

Output: 1. Number of primitives to be reduced with QI of current scanner input (QI-number)
2. Number of primitives to be reduced with $Q J$ of current scanner input ( $Q J$-number)
3. Reduction prinitive set (RPS) comprising primitives that satisfy connectivity check with current scanner input
4. QI-twin
5. QJ-twin

Begin
Set QI-number $=0$
QJ-number $=0$
QI-save = QI-twin
QJ-save $=$ QJ-twin
(QI-twin and QJ-twin are from previous scanner input)

QI-twin $=0$
QJ-twin $=0$
Do While Active-primitive-pointer not null
Using Active-primitive-pointer-list

Get next Active-primitive-pointer
Using Active-primitive-pointer
Get Primitive
If Primitive-color $=$ Scolor
Then Do
Set QI-flag = 0

$$
Q J-f l a g=0
$$

Check distance between QI and New-vector of primitive

If distance $\leq$ Connectivity-distance
Then QI-flag = 1
I毛 $Q J \neq 0$
Then Do
Check distance between $Q J$ and New-vector of primitive
If distance $\leq$ Connectivity-distance
Then QJ-flag $=1$
End
If QI-flag $=1$ and
$Q J-f l a g=1$ and
primitive Reduction-id $=0$
Then Do
$Q I$-number $=Q I-$ number +1
$Q J$-number $=Q J$-number +1
Set primitive Reduction-id $=1$
Get next available primitive cell

Create new primitive for $Q J$ and set its Reduction-id $=2$
(For details on creating a new primitive for $Q J$ refer to Create-new-primitive Procedure)

Link primitive and new primitive to each other in the Linkl fields Add pointer to the Active-primitive-pointer-list for the new primitive; mark it ineligible for connectivitycheck on current probe

End
If QI-flag = 1 and
primitive Reduction-id $=1$
Then Do
RPS (I) = primitive \#
$I=I+I$
QI-number $=$ QI-number +1
QI-twin = primitive \#
End
If $Q J-f l a g=I$ and
primitive reduction-id $=2$
Then Do
$\operatorname{RPS}(I)=$ primitive \#
$I=I+I$
QJ-number $=$ QJ-number +1
QJ-twin = primitive \#

End
End Do While
If QI-twin $\neq 0$ and Previous-scanner-color-state $=1$ and

QJ-Save $\neq 0$
Then Do
Save QJ-save as overlapped primitive in QI-twin Save QI-twin as overlapped primitive in QJ-save End

End Connectivity-check

### 4.5.5 Procedure: Create-New-Primitive

Input: 1. Current input from scanner consisting of the elements shown in section 4.5.3
2. Number of primitives to be reduced with $Q I$ of current scanner input (QI-number)
3. Number of primitives to be reduced with QJ of current scanner input (QJ-number)
4. Active-primitive-pointer-list
5. Inactive-primitive-pointer-lists
6. Primitive-set
7. Connectivity-distance

Output: 1. New primitive (s)
Begin
If $Q I$-number $=0$
Then Do
Get next available primitive cell

Set the following values for new primitive QI \# = New primitive \#

Tag \# = Probe-number
Color = Scolor
Color-state $=$ Previous-scanner-color-state
Start-vector $=$ QI
Start-angle $=$ Probe-angle
End-vector $=$ QI
End-angle = Probe-angle
New-vector $=$ QI
New-angle $=$ Probe-angle
I王 $Q J \neq 0$
Then Reduction-id $=1$
Else Reduction-id $=0$
Add pointer to Active-primitive-pointer-list
for new primitive
End
If $Q J$-number $=0$ and
$Q J \neq 0$
Then Do
Get next available primitive cell
Set the following values for new primitive -
QJ \# = New primitive \#
Tag \# = Probe-number
Color $=$ Scolor
Color-state $=$ Next-scanner-color-state

```
            Start-vector = QJ
            Start-angle = Probe-angle
            End-vector = QJ
            End-angle = Probe-angle
            New-vector = QJ
            New-angle = Probe-angle
            Reduction-id = 2
            Add pointer to Active-primitive-pointer-list for
        new primitive
            End
If QJ f 0
    If (QJ - QI) \leq Connectivity-distance
    Then Do
    Set Linkl(QI #) = QJ #
        Linkl(QJ #) = QI #
        End
Else Do
    Get next available primitive cell
    Set the following values for new primitive -
        Tag # = Probe-number
        Primitive-type = 4
        Color = Scolor
        Start-vector = QI
        Start-angle = Probe-angle
        End-vector = QJ
        End-angle = Probe-angle
        Linkl = QI #
```

$$
\begin{aligned}
& \text { Link2 = QJ \# } \\
& \text { Linkl (QI \#) = New primitive \# } \\
& \text { Linkl }(Q J \#) \text { = New primitive \# } \\
& \text { Add pointer to Inactive-primitive-pointer- } \\
& \quad \text { list type } 8 \text { for new primitive }
\end{aligned}
$$

## End

End Create-new-primitive

### 4.5.6 Procedure: Reduce

Input: 1. Current input from scanner as described in section 4.5.3
2. Reduction primitive set (RPS)
3. Primitive-set
4. QI-twin
5. QJ-twin

Output: l. New primitive (if warranted)

Begin
Set $I=1$
Do Until No-more-reduction-primitives
Current-primitive $=$ RPS (I)
I = I + I
If Current-primitive Reduction-id = 1
Then Vector $=$ QI
Else Vector $=$ QJ
For Current-primitive
If Old-vector $=0$
Then Do

$$
\begin{aligned}
& \text { Old-vector }=\text { New-vector } \\
& \text { Old-angle }=\text { New-angle } \\
& \text { New-vector }=\text { Vector } \\
& \text { New-angle }=\text { Probe-angle } \\
& \text { End }
\end{aligned}
$$

## Else Do

Compute the following values -
$\mathrm{dxl}=$ Algebraic change between the values of
Start-vector and End-vector in the $x-$ direction
dyI = - Do - in the $y$-direction
Anglel = Angle of line joining Start-vector and
End-vector (defining the primitive) with the x-axis
dx2 = Algebraic change between the values of
End-vector and Old-vector in the x-direction
dy2 = $\quad$ - Do - in the y-direction
Angle2 $=$ Angle of line joining End-vector and Old-vector (defining the primitive) with the $x$-axis

Rotationl $=$ Sense of Rotation between the two vectors for which Anglel and Angle2 were computed
dx3 = Algebraic change between the values of Old-vector and New-vector in the x-direction
dy3 $=\quad$ - Do - in the $y$-direction
Angle3 $=$ Angle of line joining Old-vector and New-vector (defining the primitive) with the x -axis

Rotation2 $=$ Sense of Rotation between the two vectors for which Angle2 and Angle3 were computed

If ( $d x 1, d x 2, d x 3$ ) and ( $d y 1, d y 2, d y 3$ ) are sign consistent

And
(Anglel $=$ Angle2 $=$ Angle3)
Or
If ( $d x 1, d x 2, d x 3$ ) and ( $d y 1, d y 2, d y 3$ )
are sign consistent
And
Sense of (Rotationl $=$ Rotation 2)
Or
If Reduction-id = 1 and
Color-state $=$ Previous-scanner:color-state
Or
If Reduction-id $=2$ and
Color-state $=$ Next-scanner-color-state
Then Do
Set the following values for Current-primitive
Tag \# = Probe-number
End-vector $=$ Old-vector

$$
\begin{aligned}
& \text { End-angle }=\text { Old-angle } \\
& \text { Old-vector }=\text { New-vector } \\
& \text { Old-angle }=\text { New-angle } \\
& \text { New-vector }=\text { Vector } \\
& \text { New-angle }=\text { Probe-angle } \\
& \text { Cumulative-angle = Cumulative-angle } \\
& + \text { Angle of Line joining Start and } \\
& \text { End-vectors with the x-axis } \\
& \text { Cumulative-total = Cumulative-total }+1 \\
& \text { If (Anglel }=\text { Angle2 }=\text { Angle3) } \\
& \text { Then Rotation }=0 \\
& \text { Else Rotation }=\text { Rotationl } \\
& \text { If } Q I-t w i n \neq 0 \text { and } \\
& \text { QJ-twin } \neq 0 \\
& \text { If Reduction-id }=1 \\
& \text { Then Frimitive-twin }=\text { QJ-twin } \\
& \text { Else Primitive-twin }=\text { QI-twin } \\
& \text { End } \\
& \text { Else Do } \\
& \text { Set } Z=\text { Current-primitive \# } \\
& \text { Get next available primitive cell } \\
& \text { Set the following values for the } \\
& \text { new primitive from the current primitive- } \\
& \text { Color }=\text { Color ( } \mathrm{Z} \text { ) } \\
& \text { If Reduction-id( } Z \text { ) }=1 \\
& \text { Then Color-state }=
\end{aligned}
$$

## Previous-scanner-color-state <br> Else Color-state $=$ Next-scanner-color-state

Tag \# = Probe-number
Start-vector = Old-vector (Z)
Start-angle = Old-angle (Z)
End-vector $=$ New-vector (Z)
End-angle $=$ New-angle (Z)
Reduction-id $=$ Reduction-id(Z)
Primitive-twin $=$ Primitive-twin (Z)
Linkl $=\mathrm{z}$
Link2 (Z) = New primitive \#
Primitive-twin(Z) $=0$
Add pointer to the Active-primitive-pointer-list for the new primitive End

End Do Until
End Reduce

### 4.5.7 Procedure: Output-primitives

Input: 1. Current input from scanner as shown in section 4.5.3
2. Primitive-set
3. Active-primitive-pointer-list
4. Final-output-flag
5. Inactive-primitive-pointer-lists

Output: 1. Inactive-primitive-pointer-lists

Begin

Do While Active-primitive pointer not null Using Active-primitive-pointer-list

Get next Active-primitive-pointer
Using Active-primitive-pointer
Get Primitive
If (Probe-number - primitive tag \#) $>0$ or
Final-output-flag = 1
Then Do
Compute for current active primitive -
Chord length between Start and End-vectors defining the primitive

Angle that the chord makes with the $+x$-axis
Average-angle $=$ Cumulative-angle/Cumulative-total
If Average-angle $\neq$ Chord-angle
Then set primitive type = curve
Else set primitive type $=$ straight-line
If Primitive-type $=$ straight-line
Then Do
If Reduction-id $=1$
Then Do

```
Vsave = Start-vector
Asave = Start-angle
Lsave = Linkl
Start-vector = End-vector
Start-angle = End-angle
Linkl = Link2
End-vector = Vsave
```

```
End-angle = Asave
Link2 = Lsave
Area = - Area
```

End
Compute $\mathrm{dx}=$ Algebraic change between End-vector and Start-vector in the $x$-direction $\mathrm{dy}=-\mathrm{Do}$ - in the y -direction If $d x<0$ and $d y>0$

Then primitive type $=1$

## Else

If $d x<0$ and $d y<0$
Then primitive type $=2$

## Else

If $d x>0$ and $d y<0$
Then primitive type $=3$

## Else

If $d x>0$ and $d y>0$
Then primitive type $=4$
Remove pointer to primitive from
Active-primitive-pointer-list
Add pointer to Inactive-primitive-pointer-
list for primitive by primitive type
End
Else Do
Compute dx = Algebraic change between

```
            End-vector and Start-vector
            in the x-direction
                dy = - Do - in the y-direction
If dx < 0 and dy > 0 And Rotation > 0
Then primitive type = 5
Else
If dx < 0 and dy < 0 And Rotation > 0
Then primitive type = 6
Else
If dx < 0 and dy < 0 And Rotation < 0
Then primitive type = 8
Else
If dx < 0 and dy > 0 And Rotation < 0
Then primitive type = 7
Else
If dx > 0 and dy > 0 And Rotation < 0
Then primitive type = 6
Else
If dx > 0 and dy > 0 And Rotation > 0
Then primitive type = 8
Compute radius of primitive
If Reduction-id = 1
Then Do
```

```
Vsave = Start-vector
```

Vsave = Start-vector
Asave = Start-angle
Asave = Start-angle
Lsave = Linkl

```
Lsave = Linkl
```

$$
\begin{aligned}
& \text { Start-vector = End vector } \\
& \text { Start-angle = End-angle } \\
& \text { Linkl = Link2 . } \\
& \text { End-vector = Vsave } \\
& \text { End-angle - Asave } \\
& \text { Link2 = Lsave } \\
& \text { Area = - Area } \\
& \text { End }
\end{aligned}
$$

Remove pointer to primitive from
Active-primitive-pointer-list Add pointer to Inactive-primitive-pointer-
list for primitive by primitive type
End
If Final-output-flag = 1
Then Do (for each Primitive I in the
Primitive-set)
NI = Twin-primitive \# of primitive
If $N 1 \neq 0$
Then Do
$\mathrm{N} 2=$ Twin-primitive \# (vi)
If $\mathrm{N} 2=\mathrm{I}$
Ther Do
Compute
Chord-length connecting Endvectors of primitives N2
and I
If Chord-length $\leq$

Connectivity-distance
Then Do
Link2 (I) $=$ N2
Link2 (N2) $=1$
End
Else Do
If Reduction-id (I) $=1$
Then M1 = I
Else M1 = N2
If M1 = I
Then $\mathrm{M} 2=\mathrm{N} 2$
Else M2 = I
Get next available
primitive cell
Set the following values
for new primitive -
Color $=$ Color (M1)
Type $=2$
Start-vector $=$ Endvector (M2)

Start-angle = End-angle
(M2)
End-vector = Start-vector
(M1)
End-angle $=$ Start-angle
(M1)

> Set Links as follows Linkl (M1) = New prim\#
> Link2 (M2) = New prim\#
> Linkl (New prim\#) = M2
> Link2 (New prim\#) = M1
> Add pointer to Inactive primitive-pointer-list type6 for new primitive

End
End Output-Primitives

### 4.5.8 Procedure: Find-Full-Pattern

Input: 1. Primitive-set
2. Inactive-primitive-pointer-list
3. Target pattern definitions
4. Form-found-flag
5. Recognition-flag
6. Output-primitive-set
7. Parse-primitive-set

Output: 1. Primitive-set
2. Form-found-flag
3. Recognition-flag
4. Target pattern definition
5. Output-primitive-set
6. Recognition-type
7. Starting-source-primitive
8. Output-primitive-set
9. Parse-primitive-set
10. Output-set

Begin
Set Recognition-type $=$ Full
Do Until No-more-target-patterns
Get next target pattern
Set Form-found-flag $=$ No
, Recognition-flag = Fail
Rotation-exhausted-flag $=$ No
Do While Rotation-exhausted-flag = No

```
Do Until No-more-starting-source-primitives or
                                    Form-found-flag = Yes
    Set Distinguished-primitive-type =
                                    First Primitive type in target
                                    pattern definition
    Using Inactive-primitive-pointer-list
        Get next Inactive-primitive-pointer
    Using Inactive-primitive-pointer
    Get Starting-source-primitive
    Call Parse-For-Form
    End Do until
        If Form-found-flag = Yes
        Then Do
            Call Check-Parameters
                If Recognition-flag = Success
```

Then Do
Set Output-set $=1$
Call Output
End
Else Do
If Form-found-flag $=$ No Or
Recognition-flag = Fail
Then Do
If Rotate-pattern $=$ No
Then Rotation-exhaustedflag $=$ Yes

Else Do
Rotate target pattern
Set Form-found-flag $=$
No
Recognition-flag = Fail
End
End Do while
End Do Until
End Find-Full-Pattern
4.5.9 Procedure: Find-Partial-Pattern

Input: 1. Primitive-set
2. Inactive-primitive-pointer-lists
3. Target pattern definitions
4. Form-found-flag
5. Recognition-flag
6. Good-primitive-count
7. Parse-primitive-set
8. Output-primitive-set
Output: 1. Primitive-set
2. Form-found-flag
3. Recognition-flag
4. Target pattern definition
5. Output-primitive-set
6. Recognition-type
7. Starting-source-primitive
8. Good-primitive-count
9. Output-primitive-set
10. Parse-primitive-set
Begin
Set Recognition-type = Partial
Do Until No-more-starting-source-primitives
Using Inactive-primitive-pointer-lists
Get next Inactive-primitive-pointer
Using Inactive-primitive-pointer
Get Starting-source-primitive
Set Target-pattern \# = 0
Non-unique-set $=$ Null
Partial-set = Null
Previous-primitive-count $=0$
Do Until No-more-target-patterns
Get next target pattern
Set Form-found-flag $=$ No
Recognition-flag = FailRotation-exhausted-flag $=$ No
Do While Rotation-exhausted-flag = No
Call Parse-for-form
If Form-found-flag $=$ Yes
Then Do
Set good-primitive-count $=0$
Call check-parameters
If good-primitive-count >previous-primitive-count
Then Do
Set Partial-set=Output-primitive- set

            Partial-pattern \# = Pattern \#
    
            Partial-parse-set = Parse-
    
                Primitive-set
    
            Non-Unique-set \(=\mathrm{Nu}\) ll
                                    End
                                    Else If Good-primitive-count \(=\)
                    Previous-primitive-count
            Then Do
            Set Non-Unique-set \(=\)
                    Partial-set
                                    Add to Non-Unique-set the
                                    Output-primitive-set
                                    Set Partial-set = Null
    Partial-Parse-set = Null

End
Else If Rotate-pattern = No Or
Rotations-exhausted
Then Set Rotations-exhausted-flag = Yes

Else Rotate pattern
End Do While
End Do Until
If Previous-primitive-count $=0$
Then Mark pointer to current source primitive Else Do

If Non-Unique-set $\neq$ Null
Then Do
Set Output-set $=3$
Output-primitive-set $=$ non-Unique-set

End
Else Do
Set Output-set $=2$
Parse-primitive-set = Partial parse-set

Output-primitive-set = Partial
set
End
Call Output
End Do Until
End Find-Partial-Pattern
4.5.10 Procedure: Parse-For-Form
Input: 1. Primitive-set
2. Target pattern to be parsed
3. Starting-source-primitive
4. Recognition-type
5. Form-found-flag
Output: 1. Output-primitive-set
2. Parse-primitive-set
Begin
If Recognition-type = Partial
Then Do
Search target pattern form for same type as
starting-source-primitive
If primitive type not found
Then Return to calling procedure
Else Set TPTR = Pointer to target pattern primitiveof same type as starting-source-primitive
End
Else Set TPTR = pointer to first target patternprimitiveTSAVE $=$ TPTRSave Starting-source-primitive in
Output-primitive-set ..... and

```
    parse-primitive-set
Set N = Starting-source-primitive #
    NSAVE = N
Do Until check = complete
    Check = Forward
    Increment = +]
    Sync-flag = Off
    Do While NSAVE }\not=\textrm{N}\mathrm{ N or
    N F 0
    If Check = Forward
    Then N = Link2(N)
    Else IN = LINKI(N)
    If Source-primitive is marked "Unusable" and
    Recognition-type = Full
    Then Return to calling procedure
    If Source-primitive-type = Target primitive type
        pointed to by TPTR or (TPTR + Increment)
    Then Save Source-primitive in
        Output-primitive-set and
        Parse-primitive-set
    Else Do
    If Recognition-type = Full
        Then Return to calling procedure
        Else If Source-primitive is "Overlapped"
            Then Do
                Set Sync-flag = On
```

Save Source-primitive in
Output-primitive-set
If Check $=$ forward
Then $N=\operatorname{Link} 2(\mathbb{N})$
Else $N=\operatorname{Linkl}(\mathbb{N})$
End
Else Do
Set PTR = Current target primitive pointer

Do While PTR $\neq$ TSAVE or
Target primitive type
same as Source-primitive
type
If Target primitive (PTR) not same type as sourceprimitive type

Then Do
Save Imaginary primitive the same type as target primitive in Parse-primitive-set Set $P T R=(P T R+$ Increment)

End
End Do while
If $\operatorname{PTR}=$ TSAVE

Then Return to calling procedure
Else Do
Save Source-primitive in
Output-primitive-set and Parse-primitive-set

Set Sync-flag = Off $\mathrm{TPTR}=\mathrm{PTR}$

If Check = Forward
Then $N=$ Link2 ( $N$ )
Else $N=\operatorname{Linkl}(N)$
End
End Do While
If $N=$ NSAVE or
TPTR = TSAVE
Then Do
Set check $=$ complete
Form-found-flag $=$ Yes
Return to calling procedure
End
Else Do
II $N=0$
Then Do
Set Check = Backward
NEND $=\mathrm{N}$
$\mathrm{N}=\operatorname{Linkl}(\mathrm{NSAVE})$
NSAVE $=$ NEND

```
TPTR = TSAVE
```

End
End Do until
End Parse-For-Form
4.5.11 Procedure: Check - Parameters

Input: 1. Parse-primitive-set
2. Recognition-type
3. Recognition-flag

Output: 1. Good-primitive-count
2. Recognition-flag

Begin
Set Good-primitive-count $=0$
If Recognition-type = Partial
Then Do
For each non-Imaginary primitive in the parse primitive set, extrapolate straight-line Primitives End

Do Until No-more-Parse-primitives
Do Until No-more-parameters-to-check
Get offset
If Offset primitive not imaginary and
Parse-primitive not imaginary
Then Do
Set value $I=$ value of attribute of parse-primitive
value 2 = value of attribute of offset-primitive

Find Difference = |Valuel - Value2|
If Difference > Error tolerance
Then Do
Set Recognition-flag = Fail
Return to calling procedure
End
End
End Do Until
Set Good-primitive-count $=$
Good-primitive-count +1
Total $=$ Total +1
End Do Until
Set Recognition-flag $=$ Success
End Check-Parameters

### 4.5.12 Procedure: Output

Input: 1. Primitive-set
2. Inactive-primitive-pointer-lists
3. Output-primitive-set
4. Parse-primitive-set
5. Recognition-type
6. Output-all-flag
7. Output-set

Output: 1. Full pattern primitive set
2. Partial pattern primitive set
3. Non-unique primitive set
4. Unrecognized primitive set

## Begin

If Output-all-flag $\neq$ Yes
Then Do
If Output-set $=1$ or 2
Then Do

## Begin

For each primitive in the Parse-primitive-
set that is non-imaginary and overlapped
Get corresponding overlapped primitive(s)
in the Primitive-set
Mark overlapped primitive(s) "unusable"
Remove pointers to Output-primitive-set
from Inactive-primitive-pointer-lists
End
If Output-set $=1$
Then ADD Output-primitive-set to Full pattern primitive set

Else $A D D$ Output-primitive-set to Partial pattern primitive set

End
Else Add Output-primitive-set to
Non-unique pattern primitive set
End
Else Add remaining primitives to
Unrecognized pattern primitive set
End
End Output

### 4.6 Discussion of Results

The results obtained from the recognition program were displayed on an IBM 3279 color console, using IBM's "GDDM" software. Color pictures obtained from these displays are shown in color plates 1 and 2. The recognition output from the program showing the primitives of each pattern identified in a given scene is presented in Appendix D.

Out of a total of 44 patterns defined in the 8 scenes, one partial pattern was misrecognized. Two others which were fully visible, though correctly recognized, were identified as partially visibie. The misrecognition occurred on the partially visible equilateral triangle (Pattern \#2) in Figure 4.5. This was due to an error in identifying the primitive type. In Appendix C, for scene 5, the circled primitive, \#23 shows a type 2 , but should have been type 3 . In the other two instances, the first relates to the square (pattern \# 5) in Figure 4.3. The program failed to generate the closing side which coincided with the sweep-vector. The second one relates to one of the circles (pattern \#2) in Figure 4.8. In Appendix $C$ for scene 4.8, the circled primitive, \#22 shows a radius of 1.8 but should have been close to 1.0 .

The straight sided figures were drawn from the primitives identified by the recognition program with no smoothing, while circles were drawn using the radius and center


SCENE \#1


SCENE \#3
\#3


SCENE \#2


SCENE \#4


SCENE \#5


SCENE \#7


SCENE \#6


SCENE \#8
identified by the program. The following observations of the color pictures are worthy of note. In some of the straight sided figures, the horizontal edges have a stairstep appearance. This is especially pronounced in scene \#5. In scene \#6, a dark area at the right top corner of the green rectangle has been left unfilled. This area belongs to the outer blue square. These are minor problems with the recognition program itself and have no bearing on the validity of the formalism.

One final observation is in order. Referring to Figure 4.8 and its corresponding color picture in color plate 2 , the view of the circles in Figure 4.8 differs from the corresponding ones in the color picture. This is due to the "overcoat"/"undercoat" feature of the "GDDM" software and the sequence in which the circles are drawn. Since this does not relate directly to this study, no further discussion is made regarding this.

### 4.7 Space-Time Complexity of Recognition Algorithm

The space requirements of the recognition algorithms is of the order $N$ where $N$ is the number of primitives comprising the scene. Since the source data from the scanner is not retained, but only its reduced version in terms of the primitives, the memory requirements are small compared to matrix manipulation schemes by an order $N$. In the worst case, the memory requirements are of the order $N^{2}$ for a pattern that consisted of nothing but dots.

The time requirements is addressed in two parts - one for formation of primitives and the other for carrying out the recognition.

During the primitive formation phase, the time require... ments are governed by two factors - the number of primitives in the active primitive set and the number of input received from the scanner. For each input from the scanner, the active primitive set has to be searched once for establish ing connectivity. If there are $N$ active primitives and $N$ input is received per probe, then the time requirement per probe is $N^{2}$. If there are $N$ probes in scanning the pattern, then the time requirement is of the order $N^{3}$.

During the recognition phase, the time requirements depend on the number of patterns defined, the number of primitives in the pattern and the number of primitives in the inactive primitive set. The worst case occurs in the recognition of partial patterns. For each starting primitive in the inactive primitive set, a target pattern is searched for a corresponding primitive type for starting the parse. Since recognition is not carried out until each pattern is checked, if there are $N$ primitives in each of the $N$ patterns, the time required for each starting primitive is $\mathrm{N}^{2}$. With $\mathbb{N}$ primitives in the inactive primitive set for starting the parse, the time requirement is of the order of $\mathrm{N}^{3}$.

## CHAPTER V

## CONCLUSION

### 5.0 Review of Current Work

The details presented in the preceding four chapters fulfill the fourfold objectives of this study as stated in section 1.1. The significant contributions of this work may be summarized as follows. Of Primary importance is the approach to the problem of syntactic pattern recognition as presented in Figure 3.1. The conprehensive treatment of the two-dimensional pattern recognition problem starting from the method of scanning the pattern, followed by the syntactic identification of the pattern primitives, all the way to the syntactic recognition of the desired patterns is important as well.

One of the significant outcomes of this work is that the structure of a two-dimensional pattern is nothing more than a static and dynamic combination of the pattern primitives. The choice of primitive types as described, allow a more uniform basis for choosing pattern primitives from one problem area to another. Perhaps one of the more significant contributions of this work is the approach outlined for the
recognition of partially obscured patterns. The implementation of these ideas for recognition of geometric patterns gives credibility to the viability of the proposed formalism.

### 5.1 Suggestions for Further Work

The scope of the two-dimensional pattern recognition problem is too broad indeed for its complete solution in a study such as this. The value of this work may be enhanced by addressing and pursuing several areas of work, some of which is described below.

One of the areas needing further work is the determination of the optimum scanning parameters in terms of the sampling-distance and the incremental probe angle. In the implementation presented in Chapter IV, it appears that the redundancy in sampling is very high in relation to the con-nectivity-distance chosen. The problem of connectivity not being met presented itself in those instances where an edge was almost coincident with the probe angle, but was slightly off. Since this implementation was not aimed at efficiency, a "brute force" approach was taken to ensure that points sampled in a pattern met the connectivity criterion.

A more basic question to be answered, is the technique for arriving at the connectivity-distance to be used, given the pattern parameters in a scene.

Another area of work involves the study of the organization of data and the structures to be used for improving
the efficiency of the algorithms. As an instance, the following may be pointed out. Instead of having a single active primitive pointer list, if one list was maintained for each color function, then only that list needs to be searched corresponding to the color function of the scanner input.

Additional work is required in the formalism developed in two important areas. The first deals with the problem of noise and missing input values from the scanner. This would necessitate the incorporation of smoothing techniques as another module to the formalism outlined. The second involves the extension of the formalism from two to threedimensional patterns.

In this study, if the scanner is able to distinguish one object from another, then the objects are said to have different colors. What physical condition must prevail to realize this in the real world was of no concern to the present study but warrants further elaboration where required.

This system is not an intelligent system. It is not designed to synthesize patterns from the scanner's input, not already specified in its set of grammars. The possibilities exist for such generalizations, but have not been pursued.

It is hoped that this work has made some innovative contributions to the area of syntactic two-dimensional pattern recognition. The generalizations presented would enable the use of this system to a wide ranging field of applications, all the way from recognition of medical laboratory specimens to vision systems for robots.

## BIBLIOGRAPHY

1: Aho, A. V. and J. D. Ullman, The Theory of Parsing, Translation and Compiling, Vol. 1: Prentice-Hall; N.J. , 1972.
2. Anderson, R. H., "Syntax Directed Recognition of Handprinted Two-Dimensional Mathematics," Ph.D. Dissertation, Div. of Engr. and Appl. Physics, Harvard Univ., Cambridge, MA, 1968.
3. Bajcsy, R. and A. Tidhar, "Using a Structured World Model in Flexible Recognition of Two Dimensional Patterns," Pattern Recognition, Vol. 9, pp. 1-10, 1977.
4. Brainerd, W. S., "Tree Generating Regular Systems," Information and Control 14, pp. 217-231, 1969.
5. Cohen, M. and G. T. Toussaint, "On the Detection of Structures in Noisy Pictures," Pattern Recognition, Vol. 9, pp. 95-98, 1977.
6. Dacey, M. F., "The Syntax of a Triangle and Some Other Figures," Pattern Recognition 2, pp. 11-3l, Jan. 1970.
7. Dacey, M. F., "POLY: A Two Dimensional Language for a Class of Polygons," IBID 3, pp. 197-208, July 1971.
8. De Mori, R. et al., "A Syntactic Procedure for the Recognition of Glottal Pulses in Continuous Speech," Pattern Recognition, Vol. 9, pp. 181-189, 1977.
9. Feder, J.; "Plex Languages", Information Science 3, pp. 225241, 1971.
10. Feder, J.,"Languages of Encoded Line Patterns," Information and Control 13, pp. 230-244, 1968.
11. Fu, K. S., Syntactic Methods in Pattern Recognition, Academic Press, New York, 1974.
12. Fu, K. S. and B. K. Bhargava, "Tree Systems for Syntactic Pattern Recognition," IEEE Trans. on Computers, Vol. C-22, pp. 1087-1099, 1973.
13. Fu, K. S. and A. Rosenfeld, "Pattern Recognition and Image Processing," IEEE Trans. on Computers, Vol. C-25, No. 12 , Dec. 1976.
14. Gips, J., "A Syntax-Directed Program that Performs a Three-Dimensional Perceptual Task," Pattern Recognition, Vol. 6, pp. 189-199, 1974.
15. Grenander, U., Pattern Synthesis, Vol. I, SpringerVerlag, New York, 1976.
16. Granander, U., Pattern Analysis, Vol. II, SpringerVerlag, New York, 1976.
17. Grusser, O. and R. Klinke, Editors, Pattern Recognition in Biological and Technical Systems, Springer-Verlag, New York, 1971.
18. Horowitz, S., "A Syntactic Algorithm for Peak Detection in Waveforms with Applications to Cardiography, Comm. of the ACM, Vol. 18, No. 5, pp. 281-285, May 1975.
19. Kanal, L., "Patterns in Pattern Recognition: 1968-1974," IEEE Transactions on Information Theory, Vol. IT-20, No. 6, Nov. 1374.
20. Kanal, L. and B. Chandrashekharan, "On Linguistic, Statistical and Mixed Models for Pattern Recognition," Frontiers of Pattern Recognition, pp. 163-171, Editor: Satoshi Watanabe, Honolulu Conference, Jan. 18-20, 1971.
21. Ledley, R. S., et al., "FIDAC: Film Input to Digital Automatic Computer and Associated Syntax Directed Pattern Recognition Programming System," J. Tippett, et al., Editors, Chapter 33, pp. 591-614, MIT Press, Cambridge, MA, 1965.
22. Lee, H. C. and K. S. Fu, "A Stochastic Syntax Analysis Procedure and its Application to Pattern Classification," IEEE Transaction on Computers, C-2l, pp. 660-666, 1972.
23. Meninga, L. D., "A Syntax-directed Approach to Pattern Recognition and Description," Fall Joint Computer Conference, Vol. $38, \mathrm{pp} .145-151,1971$.
24. Moayer, B. and K. S. Fu, "A Syntactic Approach to Fingerprint Pattern Recognition," Proc. of the Int. Joint Conf. on Pattern Recognition, pp. 423432, 1973.
25. Muchnik, I., "Simulation of Process of Forming the Language for Description and Analysis of the Forms of Image," Pattern Recognition, Vol. 4, pp. 101-140, 1972.
26. Narasimhan, R., "On the Description, Generation and Recognition of Classes of Pictures," Automatic Interpretation and Classification of Images, Editor: A. Grasselli, Academic Press, NY, 1969.
27. Narasimhan, R., "Labelling Schemata and Syntactic Description of Pictures," Information and Control, pp. 151-179, Vol. 7, 1964.
28. Narasimhan, R. and V. S. Reddy, "A Syntax Directed Recognition Scheme for Hand Printed English Letters," Pattern Recognition, Vol. 3. pp. 345-361, Nov. 1971.
29. Nilsson, N. J., Problem Solving Methods in Artificial Intelligence, McGraw-Hill, New York, 1971.
30. Nitzan, D. and C. A. Rosen, "Programagle Industrial Automation," IEEE Trans. on Computers, Vol. C-25, No. 12, pp. 1259-1269, Dec. 1976.
31. Pavlidis, T., "Linguistic Analysis of Waveforms," Software Engineering, Vol. 4, J. T. Tou, (Ed.), pp. 203-225, Acadernic Press, NY, 1971.
32. Pavlidis, T. and F. Ali, "A Hierarchical Syntactic Shape Analyzer," IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. PAMI-1, No. l, Jã. 1979.
33. Pavlidis, "Hierarchies in Structural Pattern Recognition, Proc. of the IEEE, pp. 737-744, Vol. 67, No. 5, May 1979.
34. Pavlidis, Structural Pattern Recognition, SpringerVerlag, NY, 1977.
35. Pfaltz, J. L. and A. Rosenfeld, "Web Grammars," Proc. of the International Joint Conference on Artificial Intelligence, Washington DC, pp. 609-619; May, 1969.
36. Preston, K., et al., "Basics of Cellular Logic with Some Applications in Medical Image Processing," Proc. of the IEEE, Vol. 67, No. 5, pp. 826-856, May 1979.
37. Rosenfeld, A. and M. Thurston, "Edge and Curve Detection for Visual Scene Analysis," IEEE Trans. on Computers, Vol. C-20, No. 5, pp. 561-569, May 1971.
38. Rosenfeld, A., "Frontiers of Pattern Recognition," Editor: Satoshi Watanabe, Honolulu Conf., Jan. 1820, 1971.
39. Rosenfeld, A., "Isotonic Grammars, Parallel Grammars and Picture Grammars," Editors: Metzer and Mitchie, Machine Intelligence VI, Univ. Press, Edinburgh, pp. 281-294, 1971.
40. Shaw, A. C., "A Formal Picture Description Scheme as a Basis for Picture Processing Systems," Information and Control 14, pp. 9-52, 1969.
41. Stallings, W. W., "Recognition of Printed Chinese Characters by Automatic Pattern Analysis," Computer Graphics and Image Processing l, pp. 47-65, 1972.
42. Stockman, G., et al., "Structural Pattern Recognition of Carotid Pulse Waves Using a General Waveform Parsing System," Comm. of the ACM, Vol. 19, No. 12, pp. 688695, Dec. 1976.
43. Swain, P. H. and K. S. Fu, "Stochastic Programmed Grammars for Syncactic Pattern Recognition," Pattern Recognition, Vol. 4, pp. 83-100, 1972.
44. Udupa, J. K. and I. S. N. Murthy, "Syntactic Approach to ECG Rhythm Analysis," IEEE Trans. on Biomedical Engineering, Vol. BME-27, No. 7, pp. 370-375, 1980.
45. Uhr, L., Pattern Recognition, Learning and Thought, Prentice-Hall, 1973.
46. Ullmann, J. D., Pattern Recognition Techniques, Butterworth, London, 1973.
47. Williams, K. I., "A Multidimensional Approach to Syntactic Pattern Recognition," Pattern Recognition, Vol. 7, pp. 125-137, 1975.
\% \% \% \% TSO FOREGROUND HARDCOPY $\%$ \%rir DSNAME=USYSOO2. INPUT.LIST

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115.500.261.000.261.0011
235.000.783.000.003.0012
343.000.521.500.001.5013
433.001.051.000.001.5014
555.501.222.001.571.0015
145.000.261.000.261.0001
233.000.522.500.351.0012
337.000.791.000.521.0003
455.001.052.000.352.0004
518.001.311.000.01.0005
134.000.263.000.521.0011
212.500.521.000.0 1.0012
317.000.791.500.0 1.5013
439.000.911.501.051.5014
534.001.051.500.521.0015
659.001.312.000.174.0001
747.001.481.500.871.5012
```

<--- C O L U M N S ---> 000000000111111111122222 123456789012345678901234
146.350 .971 .000 .01 .0015
254.701 .012 .000 .02 .0014
334.251 .073 .000 .02 .5013 433.501 .093 .800 .03 .8012
516.100 .963 .000 .03 .0001
115.801 .261 .000 .01 .0001
245.551 .121 .000 .01 .0012
355.150 .942 .000 .02 .0013
433.951 .032 .000 .01 .0014
532.251 .102 .000 .02 .0015
115.800 .701 .000 .01 .0001
243.000 .351 .500 .351 .5012
353.000.352.000.352.5013
433.000 .353 .000 .353 .5014
533.000 .354 .000 .354 .0015

SCENE 4 DEFINITION

SCENE 5 DEFINITION

SCENE 6 DEFINITION

```
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00000000011111111111122222
123456789012345678901234
157.101.013.000.461.0016 SCENE 7 DEFINITION
219.001.011.000.01.0015
332.801.052.600.0 1.0011
443.400.873.500.353.5012
543.201.263.000.443.0013
634.800.843.000.213.0014
118.001.401.000.0 1.0001
219.001.051.000.0 1.0002
317.001.051.500.0 1.5003
415.501.311.000.0 1.0004
515.000.701.000.0 1.0005
615.000.792.000.02.0006
COLUMNS DESCRIPTION
1 INPUT NUMBER
2 PATTERN TYPE 1 = CIRCLE
                                    2 = SQUARE
                                    3 = RECTANGLE
                                    4 = EQUILATERAL TRIANGLE
                            5 = RIGHT ANGLE TRIANGLE
3-6 LENGTH TO STARTING POINT FROM ORIGIN
7-10 ANGLE OF STARTING POINT
11 - 14 LENGTH OF FIRST SIDE / RADIUS
15-18 ANGLE OF FIRST SIDE / RADIUS WITH +X AXIS
```

COLUMNS DESCRIPTION (CONTD.)

19-22 LENGTH OF SECOND SIDE
23 ROTATION OF SUBSEQUENT SIDES WITH FIRST SIDE ( $0=$ CLOCKWISE; 1 = COUNTER-CLOCKWISE )
24
COLOR OF PATTERN
\% $\%$ \% $\%$ TSO FOREGROUND HARDCOPY \% \% \%
USNAME=USYSOO2.SCAN6.LIST

03.02504 .47490 .35007621585
04.49994 .97490 .35007631585
04.99995 .99990 .35007641585
06.02496 .99990 .35007650585
03.02504 .47490 .35067621586
$04.49994 .97490 .35067631 \quad 586$
04.99995 .99990 .35067641586
06.02496 .99990 .35067650586
03.02504 .47490 .35127521587
04.49994 .97490 .35127531587
04.99995 .99990 .35127541587
$06.02496 .99990 .35127550 \quad 587$
03.02504 .47490 .35187521588
04.49994 .97490 .35187531588
04.99995 .99990 .35187541588
$06.02496 .99990 .35187550 \quad 588$
$03.02504 .47490 .35247421 \quad 589$
04.49994 .97490 .35247431589
04.99995 .99990 .35247441589
06.02496 .99990 .35247450589
03.02504 .47490 .35307421590
$04.49994 .97490 .35307431 \quad 590$
04.99995 .99990 .35307441590
06.02496 .99990 .35307450590
03.02504 .47490 .35367321591
04.49994 .97490 .35367331591
$04.99995 .99990 .35367341 \quad 591$
$06.02496 .99990 .35367350 \quad 591$
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$06.02496 .99990 .35427350 \quad 592$
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$04.49994 .97490 .35487231 \quad 593$
04.99995 .99990 .35487241593
06.02496 .99990 .35487250593
03.02504 .47490 .35547121594
04.49994 .97490 .35547131594
04.99995 .99990 .35547141594
$06.02496 .99990 .35547150 \quad 594$
03.02504 .47490 .35607121595
04.49994 .97490 .35607131595
04.99995 .99990 .35607141595

```
04.9999 0.0 1.27417450 2127
04.9999 0.0 1.27477350 2128
04.9999 0.0 1.27537250 2129
04.9999 0.0 1.27597050 2130
04.9999 0.0 1.27656950 2131
04.9999 0.0 1.27716850 2132
40.0 0.0 1.57078000 2623
COL
DESCRIPTION
    1 NEXI STATE
2-8 START OF COLOR (QI)
9-15 END OF COLOR (QJ)
16-23 PROBE ANGLE (RADIANS)
24 COLOR ID.
25 NEXT SCANNER COLOR STATE
(0 = TRANSITION TO BACKGROUND COLOR
    1 = TRANSITION TO ANOTHER COLOR )
26-33 CURRENT PROBE NUMBER
```








| \% CELL | 11 \% | PRIMNO | 11 LINK1 | 0 | LINK2 | 17 | COLOR | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \% CELL \# | 17 \%\% | PRIMNO | 17 LINK1 | 11 | LINK2 | 22 | COLOR | 4 |
| \% CELL \# | 22 \% | PRIMNO | 22 LINK1 | 17 | LINK2 | 0 | COLOR | 5 |
| CURVE TYPE 1 PRIMITIVE CHAIN CONSISTS OF : |  |  |  |  |  |  |  |  |
| \% CELL \# | $3 \%$ | PRIMNO | 3 LINK1 | 0 | LINK2 | 0 | COLOR | 1 |
| CURVE TYPE 2 PRIMITIVE CHAIN CONSISTS OF : |  |  |  |  |  |  |  |  |
| \% CELL \# | $4 \%$. | PRIMNO | 4 LINK1 | 0 | LINK2 | 0 | COLOR | 1 |
| CURVE TYPE 3 PRIMITIVE CHAIN CONSISTS OF : |  |  |  |  |  |  |  |  |
| \% CELL 非 | $1 \%$ | PRIMNO | 1 LINK1 | 0 | LINK2 | 0 | COLOR | 1 |
| CURVE TYPE 4 PRIMITIVE CHAIN CONSISTS OF : |  |  |  |  |  |  |  |  |
| \% CELL $\ddagger$ | $2 \%$ | PRIMNO | 2 LINK1 | 0 | LINK2 | 0 | COLOR | 1 |

##  <br> 


PRIM\# 2 TYPE 0 ALT.TYPE 0 COLOR 1 TAG\# FROM 113 TO 162

START VECTOR => LENGTH 5.12 ANGLE (DEG) 3.85 LINK1 1 END VECTOR => LENGTH 5.12 ANGLE (DEG) 3.88 LINK2 3 TWIN POINTER \# 0 REDUCTION ID. 2.0 PRIMITIVE VALUES => LENGTH 0.0 AREA 0.0 ROTATION 0.0
AVG.SLOPE 0.0 COMPUTED SLOPE 1.64 CHORD 0.00 RADIUS $\quad 0.0 \quad$ CENTER CO-ORD $X=0.0 \quad Y=0.0$ $\begin{array}{lcccccccc}\text { NON-OVERLAP COUNT }= & 50 \text { OVERLAP } & \text { COUNT }= & 0 & & & \\ \text { OVERLAP PRIMITIVES } & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}$


PRIM非 4 TYPE 1 ALT. TYPE 5 COLOR 1 TAG\# FROM 334 TO 425
START VECTOR $\Rightarrow$ LENGTH 5.87 ANGLE (DEG) 10.06 LINK1 3
END VECTOR $\Rightarrow$ LENGTH 5.05 ANGLE (DEG) 14.56 LINK2 1
TWIN POINTER \# 1 REDUCTION ID. 2.0
PRTMITIVE VALUES $=>$ LENGIH 0.9 AREA 1.161 ROTATION 0.0028
AVG.SLOPE 2.89 COMPUTED SLOPE 2.88 CHORD 0.93
RADIUS 0.0 CENTER CO-ORD $X=0.0 \quad Y=0.0$
NON-OVERLAP COUNT= 91 OVERLAP COUNT= 0





＜＜＜INACTIVE PRIMITIVE POINTER LISTS＞＞＞

| NPRMHD | 0 NPRMLT | 23 |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| NL9HD | 9 NL9LST | 9 |  |  |  |
| NDOUTH | 2 NDOUTL | 17 |  |  |  |
| NLIHD | 4 NL1LST | 15 | NL2HD | 12 | NL2LST |
| NL3HD | 1. NL3LST | 10 | 18 |  |  |
| NL4HD | 3 NL4LST | 20 |  |  |  |
| NC3HD | 21 | NC1LST | 21 | NC2HD | 23 |
| NC2LST | 23 |  |  |  |  |
| NXTCEL | 22 | NC3LST | 22 | NC4HD | 19 NC4LST |
| 19 |  |  |  |  |  |


| UNDETERMINED（TYPE 9）CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL \＃ | $9 \%$ | PRIMNO | 9 LINK1 | 0 LINK2 |  | COLOR | 3 |
| DOT PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |
| \％CELL 非 | $2 \%$ | PRIMNO | 2 LINK1 | 0 LINK2 | 8 | 8 COLOR | 1 |
| \％CELL 非 | $8 \%$ | PRIMNO | 8 LINK1 | 2 LINK2 | 17 | COLOR | 3 |
| \％CELL \＃ | $17 \%$ | PRIMNO | 17 LINK1 | 8 LINK2 |  | O COLOR | 2 |


ST．LINE TYPE 2 PRIMITIVE CHAIN CONSISTS OF ：

|  | CELL $\#$ | 12 | $\%$ | PRIMNO | 12 | LINK1 | 0 |
| :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: |
| LINK2 | 16 | COLOR | 2 |  |  |  |  |
| $\%$ | CELL $\#$ | 16 | $\%$ | PRIMNO | 16 | LINK1 | 12 |
| LINK2 | 18 | COLOR | 3 |  |  |  |  |
| $\%$ | CELL 非 | 18 | $\%$ | PRIMNO | 18 | LINK1 | 16 LINK2 | 0 COLOR 4

ST．LINE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL 非 | 1 | \％\％ | PRIMNO | 1 | LINK1 | 0 | LINK2 | 7 | COLOR | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL \＃ | 7 | \％\％ | PRIMNO | 7 | LINK1 | 1 | LINK2 | 10 | COLOR | 2 |
| \％CELL 非 | 10 | \％\％ | PRIMNO |  | LINK1 | 7 | LINK2 | 0 | COLOR | 3 |

ST．LINE TYPE 4 PRIMITIVE CHAIN CONSISTS OF ：
\％CELL 非 3 \％PRIMNO 3 LINK1 0 LINK2 5 COLOR 1


PRIM\# 3 TYPE 7 ALT. TYPE 3 COLOR 2 TAG\# FROM 230 TO 1395
START VECTOR $\Rightarrow$ LENGTH 1.70 ANGLE (DEG) 46.50 LINK1 14
END VECTOR $\Rightarrow$ LENGTH 2.20 ANGLE (DEG) 6.35 LINK2 1
TWIN POINTER \# 0 REDUCTION ID. 1.0
PRIMITIVE VALUES => LENGTH 1.7 area -0.931 ROTATION -0.0016
AVG.SLOPE 2.69 COMPUTED SLOPE 5.51 CHORD 1.42
RADIUS $\quad 1.03$ CENTER CO-ORD $X=2.55 Y=0.53$
NON-OVERLAP COUNT= 1165 OVERLAP COUNT= 0
OVERLAP PRIMITIVES $0 \begin{array}{llllllll} & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}$
PRIM\# 4 TYPE 3 ALT.TYPE 7 COLOR 1 TAG非 FROM 435 TO 856
START VECTOR => LENGTH 3.88 ANGLE (DEG) 29.37 LINK1 0
END VECTOR => LENGTH 4.00 ANGLE (DEG) 14.91 LINK2 5
TWIN POINTER \# 0 REDUCTION ID. 1.0
PRIMITIVE VALUES $\Rightarrow$ LENGTH 1.1 AREA -1.928 ROTATION 0.0000
AVG.SLOPE 2.05 COMPUTED SLOPE 5.22 CHORD 1.00
RADIUS $\quad 0.0 \quad$ CENTER CO-ORD $X=0.0 \quad Y=0.0$
NON-OVERLAP COUNT $=421$ OVERLAP COUNT= 0





PRIM非 18 TYPE 4 ALT.TYPE 8 COLOR 4 TAG\# FROM 1520 TO 1595
START VECTOR $\Rightarrow$ LENGTH 9.02 ANGLE (DEG) 52.17 LINK1 17
END VECTOR => LENGTH 10.47 ANGLE (DEG) 53.37 LINK2 20
TWIN POINTER \# 0 REDUCTION ID. 2.0
PRIMITIVE VALUES => LENGTH 1.5 AREA 0.929 ROTATION 0.0039
AVG.SLOPE 1.05 COMPUTED SLOPE 1.06 CHORD 1.46

RADIUS $\quad 0.0 \quad$ CENTER CO-ORD $X=0.0 \quad \mathrm{Y}=0.0$
NON-OVERLAP COUNT= 75 OVERLAP COUNT= 0
OVERLAP PRIMITIVES $0 \begin{array}{llllllll} & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}$
PRIM\# 19 TYPE 6 ALT.TYPE 2 COLOR 3 TAG\# FROM 1572 TO 1679
START VECTOR => LENGTH 8.15 ANGLE (DEG) 52.55 LINKI 12 END VECTOR => LENGTH 6.87 ANGLE (DEG) 57.63 LINK2 21




PRIM非 33 TYPE 3 ALT.TYPE 7 COLOR 2 TAG非 FROM 2516 TO 2596
START VECTOR $\Rightarrow$ LENGTH 8.37 ANGLE (DEG) 89.10 LINK1 32
END VECTOR $\Rightarrow$ LENGTH 7.02 ANGLE (DEG) 84.67 LINK2 31 TWIN POINTER \# 32 REDUCTION ID. 1.0
PRIMITIVE VALUES $\Rightarrow$ LENGTH 1.5 AREA -2.241 ROTATION -0.0019
AVG.SLOPE 1.94 COMPUTED SLOPE 5.07 CHORD 1.47

RADIUS 0.0 CENTER CO-ORD $X=0.0 \quad Y=0.0$


ST．LINE TYPE 2 PRIMITIVE CHAIN CONSISTS OF ：

|  | CELL 非 | 34 | \％\％ | PRIMNO |  | LINK1 |  | LINK2 | 22 | COLOR | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％ | CELL 非 | 22 | \％\％ | PRIMNO | 22 | LINK1 | 34 | LINK2 | 26 | COLOR | 4 |
| $\%$ | CELL \＃ | 26 | \％\％ | PRIMNO | 26 | LINK1 | 22 | LINK2 | 30 | COLOR | 5 |
| \％ | CELL \＃ | 30 | \％\％ | PRIMNO | 30 | LINK1 | 26 | LINK2 | 0 | COLOR | 1 |
| ST．LINE TYPE 3 PRIMITIVE CHAIN CONSISTS OF： |  |  |  |  |  |  |  |  |  |  |  |
| \％ | CELL \＃ | 4 | \％\％ | PRIMNO |  | LINK1 |  | LINK2 | 17 | COLOR | 1 |
| \％ | CELL \＃ | 17 | \％\％ | PRIMNO | 17 | LINKI |  | LINK2 | 23 | COLOR | 4 |
| \％ | CELL 非 | 23 | \％\％ | PRIMNO | 23 | LINK1 | 17 | LINK2 | 33 | COLOR | 5 |
| \％ | CELL \＃ | 33 | \％\％ | PRIMNO | 33 | LINK1 | 23 | LINK2 | 0 | COLOR | 2 |
| ST．LINE TYPE 4 PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |  |  |  |  |
| \％ | CELL \＃ | 1 | \％\％ | PRIMNO |  | LINK1 |  | LINK2 | 6 | COLOR | 2 |
| \％ | CELL \＃ | 6 | \％\％ | PRIMNO |  | LINK1 |  | LINK2 | 15 | COLOR | 1 |
| \％ | CELL \＃ | 15 | \％\％ | PRIMNO | 15 | LINK1 |  | 6 LINK2 | 18 | COLOR | 5 |
| \％ | CELL \＃ | 18 | \％\％ | PRIMNO | 18 | LINK1 | 15 | LINK2 | 24 | COLOR | 4 |
| \％ | CELL \＃ | 24 | \％\％ | PRIMNO | 24 | LINK1 | 18 | LINK2 | 31 | COLOR | 1 |
| \％ | CELL \＃ | 31 | \％\％ | PRIMNO | 31 | LINK1 | 24 | LINK2 | 0 | COLOR | 2 |

CURVE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL \＃ |  | \％\％ | PRIMNO | 7 | LINKI | 0 | LINK2 | 12 | COLOR | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL \＃ | 12 | \％\％ | PRIMNO | 12 | LINK1 | 7 | LINK2 | 0 | COLOR | 3 |
| CURVE TYPE 2 PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |  |  |  |
| \％CELL 非 | 13 | \％\％ | PRIMNO | 13 | LINK1 | 0 | LINK2 | 14 | COLOR | 2 |
| \％CELL 非 | 14 | \％\％ | PRIMNO | 14 | LINK1 | 13 | LINK2 | 19 | COLOR | 2 |
| \％CELL 非 | 19 | \％\％ | PRIMNO | 19 | LINK1 | 14 | LINK2 | 21 | COLOR | 3 |
| \％CELL \＃ | 21 | \％ | PRIMNO | 21 | LINK1 | 19 | LINK2 | 0 | COLOR | 3 |

CURVE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ：

| $\%$ | CELL $⿰ ⿰ 三 丨 ⿰ 丨 三$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |

CURVE TYPE 4 PRIMITIVE CHAIN CONSISTS OF ：

| \％ | CELL 非 | 2 | \％\％ | PRIMNO | 2 | LINK1 | 0 | LINK2 | 9 | COLOR | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％ | CELL 非 | 9 | \％\％ | nRIMNO | 9 | LINK1 | 2 | LINK2 | 10 | COLOR | 3 |
| \％ | CELL 非 | 10 | \％\％ | PRIMNO | 10 | LINK1 | 9 | LINK2 | 0 | COLOR | 3 |








PRIM\# 27 TYPE 3 ALT.TYPE 7 COLOR 1 TAG\# FROM 1538 TO 1629 START VECTOR $\Rightarrow$ LENGTH 8.50 ANGLE (DEG) 54.54 LINK1 29 END VECTOR => LENGTH 8.77 ANGLE (DEG) 51.79 LINK2 6 TWIN POINTER \# 0 REDUCTION ID. 1.0 PRIMITIVE VALUES $=>$ LENGTH 0.5 AREA - 1.789 ROTATION -0.0001 AVG.SLOPE 3.07 COMPUTED SLOPE 6.23 CHORD 0.50 RADIUS $\quad 0.0$ CENTER CO-ORD $X=0.0 \quad Y=0.0$ NON-OVERLAP COUNT $=0$ OVERLAP COUNT $=92$

OVERLAP PRIMITIVES 26 |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

PRIM非 28 TYPE 1 ALT.TYPE 5 COLOR 2 TAG\# FROM 1630 TO 2235
START VECTOR => LENGTH 8.47 ANGLE (DEG) 54.54 LINK1 26 END VECTOR $\Rightarrow$ LENGTH 7.07 ANGLE (DEG) 76.71 LINK2 38 TWIN POINTER \# 38 REDUCTION ID. 2.0
PRIMITIVE VALUES $\Rightarrow$ LENGTH 3.4 AREA 11.304 ROTATION -0.0065 AVG.SLOPE 3.14 COMPUTED SLOPE 3.15 CHORD 3.29 RADIUS $\quad 0.0 \quad$ CENTER CO-ORD $X=0.0 \quad Y=0.0$ NON-OVERLAP COUNT $=0$ OVERLAP COUNT $=606$ OVERLAP PRIMITIVES $29 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$




UNDETERMINED（TYPE 9）CHAIN CONSISTS OF ：

| $\%$ CELL 非 | 16 | $\%$ | PRIMNO | 16 | LINK1 | 0 | LINK2 |
| :--- | :--- | :--- | :--- | :--- | :--- | ---: | :--- | 21 COLOR 4

DOT PRIMITIVE CHAIN CONSISTS OF ：
\％CELL \＃ $20 \%$ PRIMNO 20 LINK1 0 LINK2 0 COLOR 3

ST．LINE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL \＃ | 3 | \％\％ | PRIMNO | 3 | LINK1 | 0 | LINK2 | 8 | COLOR | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL 北 | 8 | \％\％ | PRIMNO | 8 | LINK1 | 3 | LINK2 | 13 | COLOR | 2 |
| \％CELL \＃ | 13 | \％\％ | PRIMNO | 13 | LINK1 | 8 | LINK2 | 17 | COLOR | 3 |
| \％CELL 非 | 17 | \％\％ | PRIMNO | 17 | LINK1 | 13 | LINK2 | 18 | COLOR | 4 |
| \％CELL \＃ | 18 | \％\％ | PRIMNO | 18 | LINK1 | 17 | LINK2 | 23 | COLOR | 3 |


| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $23 \%$ | PRIMNO | 23 LINK1 | 18 LINK2 | 25 | COLOR | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL 非 | $25 \%$ | PRIMNO | 25 LINK1 | 23 LINK2 | 26 | COLOR | 3 |
| \％CELL 非 | $26 \% \%$ | PRIMNO | 26 LINK1 | 25 LINK2 | 28 | COLOR | 2 |
| \％CELL 非 | $28 \%$ | PRIMNO | 28 LINK1 | 26 LINK2 | 0 | COLOR | 2 |
| ST．LINE TYPE 2 PRIMITIVE CHAIN CONSISTS OF |  |  |  |  |  |  |  |
| \％CELL \＃ | $6 \%$ | PRIMNO | 6 LINK1 | 0 LINK2 | 10 | COLOR | 1 |
| \％CELL 非 | $10 \%$ | PRIMNO | 10 LINK1 | 6 LINK2 | 33 | COLuR | 2 |
| \％CELL 非 | $33 \%$ | PRIMNO | 33 LINK1 | 10 LINK2 | 35 | COLOR | 4 |
| \％CELL 非 | ． $35 \%$ | PRIMNO | 35 LINK1 | 33 LINK2 | 38 | COLOR | 3 |
| \％CELL 非 | $38 \%$ | PRIMNO | 38 LINK1 | 35 LINK2 | 31. | COLOR | 2 |
| \％CELL \＃ | $31 \%$ | PRIMNO | 31 LINK1 | 38 LINK2 | 0 | COLOR | 5 |
| ST．LINE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |
| \％CELL 非 | $4 \% \%$ | PRIMNO | 4 LINK1 | 0 LINK2 | 9 | COLOR | 2 |
| \％CELL 非 | $9 \% \%$ | PRIMNO | 9 LINK1 | 4 LINK2 | 14 | COLOR | 3 |
| \％CELL 非 | $14 \%$ | PRIMNO | 14 LINK1 | 9 LINK2 | 15 | COLOR | 4 |
| \％CELL \＃ | $15 \%$ | PRIMNO | 15 LINK1 | 14 LINK2 | 19 | COLOR | 3 |
| \％CELL \＃ | $19 \%$ | PRIMNO | 19 LINK1 | 15 LINK2 | 22 | COLOR | 5 |
| \％CELL 非 | 22 \％ | PRIMNO | 22 LINK1 | 19 LINK2 | 24 | COLOR | 3 |
| \％CELL 非 | 24 \％ | PRIMNO | 24 LINK1 | 22 LINK2 | 27 | COLOR | 2 |
| \％CELL 非 | 27 \％\％ | PRIMNO | 27 LINK1 | 24 LINK2 | 29 | COLOR | 1 |
| \％CELL 非 | $29 \%$ | PRIMNO | 29 LINK1 | 27 LINK2 | 36 | COLOR | 1 |
| \％CELL 非 | $36 \%$ | PRIMNO | 36 LINK1 | 29 LINK2 | 0 | COLOR | 2 |
| ST．LINE TYPE 4 PRIMITIVE CHAIN CONSISTS OF： |  |  |  |  |  |  |  |
| \％CELL \＃ | $5 \%$ | PRIMNO | 5 LINK1 | 0 LINK2 | 11 | COLOR | 2 |
| \％CELL \＃ | $11 \%$ | PRIMNO | 11 LINK1 | 5 LINK2 | 32 | COLOR | 3 |
| \％CELL \＃ | $32 \%$ | PRIMNO | 32 LINK1 | 11 LINK2 | 34 | COLOR | 3 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $34 \%$ | PRIMNO | 34 LINK1 | 32 LINK2 | 37 | COLOR | 2 |
| \％CELL 非 | $37 \%$ | PRIMNO | 37 LINK1 | 34 LINK2 | 30 | COLOR | 1 |
| \％CELL 非 | $30 \%$ | PRIMNO | 30 LINK1 | 37 LINK2 | 0 | COLOR | 3 |

CURVE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ：
\％CELL 非 $12 \%$ PRIMNO 12 LINKI 0 LINK2 0 COLOR 1

CURVE TYPE 2 PRIMITIVE CHAIN CONSISTS OF ：
\％CELL \＃ $39 \%$ PRIMNO 39 LINK1 0 LINK2 0 COLOR 1

CURVE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ：
\％CELL \＃ $7 \%$ PRIMNO 7 LINK1 0 LINK2 0 COLOR 1

## CURVE TYPE 4 PRIMITIVE CHAIN CONSISTS OF :

| \% CELL 非 | 1 | \%\% | PRIMNO | 1 | LINK1 | 0 | LINK2 | 2 | COLOR | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \% CELL 非 | 2 | \%\% | PRIMNO | 2 | LINK1 | 1 | LINK2 | 0 | COLOR | 1 |

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PRIM非（23）TYPE（2）ALT．TYPE 6 COLOR 2 TAG非 FROM 1730 TO 1849
START VECTOR $\Rightarrow$ LENGTH 5.65 ANGLE（DEG） 62.26 LINK1 32
END VECTOR＝＞LENGTH 5．82 ANGLE（DEG） 59.18 LINK2 18
TWIN POINTER $\#$ O REDUCTION ID． 1.0
PRIMITIVE VALUES $\Rightarrow$ LENGTH 0.4 AREA -0.890 ROTATION 0.0043
$\begin{array}{lllll}\text { AVG．SLOPE } & 3.09 & \text { COMPUTED SLOPE } 0.00 & \text { CHORD } & 0.36\end{array}$
RADIUS 66.48 CENTER CO－ORD $X=61.55 Y=-1.52$
NON－OVERLAP COUNT＝ 85 OVERLAP COUNT＝ 0
OVERLAP PRIMITIVES $\begin{array}{lllllllll}21 & 26 & 29 & 27 & 0 & 0 & 0 & 0\end{array}$
PRIM\＃ 24 TYPE 2 ALT．TYPE 6 COLOR 4 TAG非 FROM 1760 TO 1844





| NC3HD NXTCEL | 39 NC3LST <br> 42 LSTCEL | $\begin{array}{ll} \mathrm{T} & 39 \mathrm{~N} \\ \mathrm{~L} & 41 \end{array}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UNDETERMINED（TYPE 9）CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |  |  |
| \％CELL | $2 \%$ | PrimNo |  | LINK1 | 0 | LINK2 |  | COLOR | 5 |
| dot primitive chain consists of ： |  |  |  |  |  |  |  |  |  |
| \％CELL \＃ | $8 \%$ | PRIMNO |  | LINK1 |  | LINK2 | 11 | COLOR | 3 |
| \％CELL 非 | $11 \%$ | PRIMNO |  | LINK1 |  | LINK2 | 20 | COLOR | 5 |
| \％CELL 非 | $20 \%$ | PRIMNO |  | LINK1 |  | LINK2 | 31 | COLOR | 2 |
| \％CELL 非 | $31 \%$ | PRIMNO |  | LINK1 |  | LINK2 |  | COLOR | 4 |

St．LINE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL \＃ | 5 | $\% \%$ | PRIMNO | 5 | LINK1 | 0 | LINK2 | 13 | COLOR |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | 13 | \％ | PRIMNO | 13 | LINK1 | 5 | LINK2 | 14 | COLOR |  |
| \％CELL 非 | 14 | \％ | PRIMNO | 14 | LINK1 | 13 | LINK2 | 22 | COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | 22 | \％\％ | PRIMNO | 22 | LINK1 | 14 | LINK2 | 26 | COLOR | 2 |
| \％CELL \＃ | 26 | \％$\%$ | PRIMNO | 26 | LINK1 | 22 | LINK2 | 27 | COLOR | 4 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | 27 | \％ | PRIMNO | 27 | LINK1 | 26 | LINK2 | 40 | COLOR | 2 |
| \％CELL 非 | 40 | \％\％ | PRIMNO | 40 | LINK1 | 27 | LINK2 |  | COLOR |  |


| \％CELL \＃ | $15 \%$ | PRIMNO | 15 LINK1 | 0 LINK2 | 23 COLOR |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一 灬$ | $23 \%$ | PRIMNO | 23 LINK1 | 15 LINK2 | 24 COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $24 \%$ | PRIMNO | 24 LINK1 | 23 LINK2 | 33 COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $33 \%$ | PRIMNO | 33 LINK1 | 24 LINK2 | 32 COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $32 \%$ | PRIMNO | 32 LINK1 | 33 LINK2 | 35 COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一 灬$ | $35 \%$ | PRIMNO | 35 LINK1 | 32 LINK2 | 37 COLOR | 2 |
| \％CELL 非 | $37 \%$ | PRIMNO | 37 LINK1 | 35 LINK2 | 38 COLOR |  |
| \％CELL \＃ | $38 \%$ | PRIMNO | 38 LINK | 37 LINK2 | O COLOR |  |

ST．LINE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $1 \%$ | PRIMNO | 1 | LINK1 | 0 | LINK2 | 4 | COLOR | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL | $4 \%$ | PRIMNO | 4 | LINK1 | 1 | LINK2 | 6 | COLOR | 3 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $6 \%$ | PRIMNO | 6 | LINK1 | 4 | LINK2 | 10 | color | 4 |
| \％CELL \＃ | $10 \%$ | Primo | 10 | LINK1 | 6 | LINK2 | 12 | COLOR | 3 |
| \％CELL 非 | $12 \%$ | PRIMNO | 12 | LINK1 | 10 | LINK2 | 18 | COLOR | 4 |
| \％CELL | $18 \%$ | PRIMNO | 18 | LINK1 | 12 | LINK2 | 19 | COLOR |  |
| \％CELL 非 | $19 \%$ | PRIMNO | 19 | LINK1 | 18 | LINK2 | 30 | COLOR | 3 |
| \％CELL 非 | 30\％ | PRIMNO | 30 | LINK1 | 19 | LINK2 |  | COLOR |  |

ST．LINE TYPE 4 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL | $3 \%$ | PRIMNO | 3 LINK1 | 0 LINK2 | 7 COLOR | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL \＃ | $7 \%$ | PRIMNO | 7 LINK1 | 3 LINK2 | 25 COLOR | 4 |
| \％CELL 非 | $25 \%$ | PRIMNO | 25 LINK1 | 7 LINK2 | 29 COLOR | 5 |
| \％CELL 非 | $29 \%$ | PRIMNO | 29 LINK1 | 25 LINK2 | 16 COLOR | 1 |
| \％CELL \＃ | $16 \%$ | PRIMNO | 16 LINK1 | 29 LINK2 | 0 COLOR | 4 |
| CURVE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |
| \％CELL \＃ | 9\％\％ | PRIMNO | 9 LINK1 | 0 LINK2 | 17 COLOR | 4 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $17 \%$ | PRIMNO | 17 LINK1 | 9 LINK2 | 21 COLOR | 3 |
| \％CELL 非 | $21 \%$ | PRIMNO | 21 LINR1 | 17 LINK2 | 34 COLOR | 4 |
| \％CELL 非 | $34 \%$ | PRIMNO | 34 LINK1 | 21 LINK2 | 36 COLOR | 5 |
| \％CELL 非 | $36 \%$ | PRIMNO | 36 LINK1 | 34 LINK2 | 0 COLOR | 1 |
| CURVE TYPE 2 PRIMITIVE Chain Consists of ： |  |  |  |  |  |  |
| \％CELL \＃ | $41 \%$ | PRIMNO | 41 LINK1 | 0 LINK2 | 0 COLOR | 1 |
| Curve type 3 primitive chain consists of ： |  |  |  |  |  |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $39 \%$ | PRIMNO | 39 LINK1 | 0 LINK2 | 0 COLOR | 1 |
| CURVE TYPE 4 PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |
| \％CELL 非 | $28 \%$ | PRIMNO | 28 LINK1 | 0 LINK2 | 0 COLOR | 1 |

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PRIM非 32 TYPE I ALT．TYPE 5 COLOR 4 TAG非 FROM 1329 TO 1508
START VECTOR $\Rightarrow$ LENGTH 6.65 ANGLE（DEG） 45.61 LINK1 31
END VECTOR $\Rightarrow$ LENGTH 6．92 ANGLE（DEG） 50.39 LINK2 39
TWIN POINTER \＃ 0 REDUCTION ID． 2.0
PRIMITIVE VALUES $\Rightarrow$ LENGTH 0.7 AREA 1.875 ROTATION－0．0003
AVG．SLOPE 1．99 COMPUTED SLOPE 1.96 CHORD 0.63
RADIUS $\quad 0.0 \quad$ CENTER CO－ORD $X=0.0 \quad Y=0.0$
NON－OVERLAP COUNT＝ 0 OVERLAP COUNT＝ 180

PRIM非 33 TYPE 6 ALT．TYPE 2 COLOR 1 TAG\＃FROM 1417 TO 1457
START VECTOR $\Rightarrow$ LENGTH 6．45 ANGLE（DEG） 47.23 LINK1 26
END VECTOR $\Rightarrow$ LENGTH 5．77 ANGLE（DEG） 50.01 LINK2 36
TWIN POINTER 非 36 REDUCTION ID． 2.0
PRIMITIVE VALUES $\Rightarrow$ LENGTH 0.8 AREA 0.936 ROTATION 0.0076
AVG．SLOPE 2．81 COMPUTED SLOPE 3.58 CHORD 0.74
RADIUS 1．09 CENTER CO－ORD $X=5.77 Y=0.68$


＜＜＜．INACTIVE PRIMITIVE POINTER LISTS＞＞＞

| NPRMHD | 0 | NPRMLT | 42 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NL9HD | 0 | NL9LST | 39 |  |  |  |  |
| NDOUTH | 21 | NDOUTL | 41 |  |  |  |  |
| NL1HD | 2 | NL1LST | 32 | NL2HD | 1 | NL2LST | 39 |
| NL3HD | 4 | NL3LST | 42 | NL4HD | 3 | NL4LST | 38 |
| NC1HD | 24 | NC1LST | 26 | NC2HD | 33 | NC2LST | 36 |
| NC3HD | 25 | NC3LST | 31 | NC4HD | 14 | NC4LST | 23 |
| NXTCEL | 43 | LSTCEL | 42 |  |  |  |  |

UNDETERMINED（TYPE 9）CHAIN CONSISTS OF ：

DOT PRIMITIVE CHAIN CONSISTS OF ：
$\left.\begin{array}{lllllllll} & \text { \％CELL \＃} & 21 & \% & \text { PRIMNO } & 21 & \text { LINK1 } & 0 & \text { LINK2 }\end{array}\right) 22$ COLOR 4

ST．LINE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL 非 | $2 \%$ | PRIMNO | 2 | LINKI | 0 | LINK2 | 5 | COLOR | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL \＃ | $5 \%$ | PRIMNO | 5 | LINK1 | 2 | LINK2 | 8 | COLOR | 3 |
| \％CELL \＃ | $8 \%$ | PRIMNO | 8 | LINK1 | 5 | IINK2 | 11 | COLOR | 4 |
| \％CELL 非 | $11 \%$ | PRIMNO | 11 | LINKI | 8 | LINK2 | 32 | COLOR | 5 |
| \％CELL \＃ | $32 \%$ | PRIMNO | 32 | LINK1 | 11 | LINK2 | 0 | COLOR | 4 |

ST．LINE TYPE 2 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL \＃ | 1 | \％\％ | PRIMNO | 1 | LINK1 | 0 | LINK2 | 15 | COLOR | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL 非 | 15 | \％\％ | PRIMNO | 15 | LINK1 | 1 | LINK2 | 20 | COLOR | 2 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | 20 | \％\％ | PRIMNO | 20 | LINK1 | 15 | LINK2 | 37 | COLOR | 4 |
| \％CELL \＃ | 37 | \％\％ | PRIMNO | 37 | LINK1 | 20 | LINK2 | 39 | COLOR | 5 |
| \％CELL \＃ | 39 | \％\％ | PRIMNO | 39 | LINK1 | 37 | LINK2 | 0 | COLOR | 4 |

ST．LINE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ：

| \％ | CELL \＃ | 4 | \％\％ | PRIMNO |  | LINK1 | 0 | LIN：2． | 7 | COLOR | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％ | CELL \＃ | 7 | \％\％ | PRIMNO | 7 | LINK1 | 4 | LINK2 | 10 | COLOR | 4 |
| \％ | CELL \＃ | 10 | \％\％ | PRIMNO | 10 | LINK1 | 7 | LINK2 | 13 | COLOR | 5 |
| \％ | CELL \＃ | 13 | \％\％ | PRIMNO | 13 | LINK1 | 10 | LINK2 | 40 | COLOR | 3 |


| \％CELL 非 | 40 | $\%$ | PRIMNO | 40 | LINK1 | 13 | LINK2 | 42 | COLOR |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| CELL |  |  |  |  |  |  |  |  |  |


| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $3 \%$ | PRIMNO | 3 | LINK1 | 0 | LINK2 | 6 | COLOR | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $6 \%$ | PRIMNO | 6 | LINK1 | 3 | LINK2 | 9 | COLOR | 3 |
| \％CELL 非 | $9 \%$ | PRIMNO | 9 | LINK1 | 6 | LINK2 | 12 | COLOR | 4 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $12 \%$ | PRIMNO | 12 | LINK1 | 9 | LINK2 | 16 | COLOR | 5 |
| \％CELL \＃ | $16 \%$ | PRIMNO | 16 | LINK1 | 12 | LINK2 | 38 | color | 3 |
| \％CELL ${ }^{\text {P }}$ | $38 \%$ | PRIMNO | 38 | LINK1 |  | LINK2 |  | COLOR |  |

CURVE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $24 \%$ | PRIMNO | 24 LINK1 | 0 LINK2 | 26 COLOR | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $26 \%$ | PRIMNO | 26 LINK1 | 24 LINK2 | 0 COLOR | 1 |

CURVE TYPE 2 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL | 33 | $\%$ | PRIMNO | 33 | LINK1 | 0 |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- |
| \％ |  |  |  |  |  |  |

CURVE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL \＃ | 25 | $\%$ | PRIMNO | 25 | LINK1 | 0 | LINK2 | 27 |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| COLOR | 1 |  |  |  |  |  |  |  |
| \％CELL | \＃ | 27 | $\%$ | PRIMNO | 27 | LINK1 | 25 | LINK2 |
| \％CELL | 31 | COLOR | 5 |  |  |  |  |  |

CURVE TYPE 4 PRIMITIVE ChAIN CONSISTS OF ：

| \％CELL \＃ | $14 \%$ | PRIMNO | 14 LINK1 | 0 LINK2 | 17 COLOR | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL 非 | $17 \%$ | PRIMNO | 17 LINK1 | 14 LINK2 | 18 COLOR | 4 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $18 \%$ | PRIMNO | 18 LINK1 | 17 LINK2 | 19 COLOR． |  |
| \％CELL 非 | $19 \%$ | PRIMNO | 19 LINK1 | 18 LINK2 | 23 COLOR | 1 |
| \％CELL 非 | $23 \%$ | PRIMNO | 23 LINK1 | 19 LINK2 | 0 COLOR | 5 |

 $======$ OUTPUT FOR SCENE 7 =e====












START VECTOR $\Rightarrow$ LENGTH 3．23 ANGLE（DEG） 71.84 LINK1 55 END VECTOR $\Rightarrow$ LENGTH 3．52 ANGLE（DEG） 67.00 LINK2 51 TWIN POINTER \＃ 0 REDUCTION ID． 1.0 primitive values $\Rightarrow$ LengTh 0.4 area－0．479 rotation－0．0007 AVG．SLOPE 2.69 COMPUTED SLOPE 0.45 CHORD 0.41 RADIUS $\quad 0.0 \quad$ CENTER CO－ORD $X=0.0 \quad Y=0.0$ NON－OVERLAP COUNT＝ 134 OVERLAP COUNT＝ 0 OVERLAP PRIMITIVES $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$
PRIM非 54 TYPE 0 ALT．TYPE O COLOR 4 TAG非 FROM 2038 TO 2038 START VECTOR $\Rightarrow$ LENGTH 7．00 ANGLE（DEG） 68.20 LINK1 46 END VECTOR $\Rightarrow$ LENGTH 7．00 ANGLE（DEG） 68.24 LINK2 43 TWIN POINTER 非 43 REDUCTION ID． 2.0 PRIMITIVE VALUES $\Rightarrow$ LENGTH 0.0 AREA 0.015 ROTATION 0.0 AVG．SLOPE 0．0 COMPUTED SLOPE 2.76 CHORD 0.00 RADIUS $\quad 0.0$ CENTER CO－ORD $X=0.0 \quad \mathrm{Y}=0.0$ $\begin{array}{lccccclll}\text { NON－OVERLAP COUNT }= & 0 & \text { OYERLAP } & \text { COUNT }= & 0 & & \\ \text { OVERLAP PRIMITIVES } & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{array}$
PRIM\# 55 TYPE 2 ALT.TYPE 6 COLOR 3 TAG非 FROM 2142 TO 2286
PRIM\# 55 TYPE 2 ALT.TYPE 6 COLOR 3 TAG非 FROM 2142 TO 2286
START VECTOR => LENGTH 6.15 ANGLE (DEG) 78.46 LINK1 42
START VECTOR => LENGTH 6.15 ANGLE (DEG) 78.46 LINK1 42
END VECTOR => LENGTH 3.23 ANGLE (DEG) 71.84 LINK2 53
END VECTOR => LENGTH 3.23 ANGLE (DEG) 71.84 LINK2 53
TWIN POINTER 非 42 REDUCTION ID. 1.0
TWIN POINTER 非 42 REDUCTION ID. 1.0
PRIMITIVE VALUES => LENGTH 2.9 AREA -1.091 ROTATION -0.0032
PRIMITIVE VALUES => LENGTH 2.9 AREA -1.091 ROTATION -0.0032
AVG.SLOPE 1.51 COMPUTED SLOPE 4.64 CHORD 2.97
AVG.SLOPE 1.51 COMPUTED SLOPE 4.64 CHORD 2.97
RADIUS 0.0 CENTER CO-ORD X= 0.0 Y=0.0
RADIUS 0.0 CENTER CO-ORD X= 0.0 Y=0.0
NON-OVERLAP COUNT= 144 OVERLAP COUNT= 0
NON-OVERLAP COUNT= 144 OVERLAP COUNT= 0
OVERLAP PRIMITIVES 0
OVERLAP PRIMITIVES 0
NPRMHD 0 NPRMLT 55
NL9HD 2 NL9LST 44
NDOUTH 1 NDOUTL 54
NL1HD $\quad 6$ NL1LST $\quad 42$ NL2HD 34 NL2LST 55
$\begin{array}{ll}\text { NL3HD } & 3 \text { NL3LST } \\ 43 & \text { NLLHD }\end{array} 4$ NL4LST 53
NC1HD 8 NC1LST 25 NC2HD 38 NC2LST 38
NC3HD 13 NC3LST 35 NC4HD 37 NC4LST 37
NXTCEL 56 LSTCEL 55

UNDETERMINED（TYPE 9）CHAIN CONSISTS OF ：

| \％CELL ${ }^{\text {F }}$ | $2 \%$ | PRIMNO | 2 | LINK1 | 0 | LINK2 |  | COLOR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $17 \%$ | PRIMNO | 17 | LINK1 | 2 | LINK2 | 44 | COLOR |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $44 \%$ | PRIMNO | 44 | LINK1 | 17 | LINK2 |  | COLOR |

DOT PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL \＃ | 1 | \％\％ | PRIMNO |  | LINK1 | 0 | LINK2 | 9 | COLOR |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | 9 | \％ | PRIMNO | 9 | LINK1 | 1 | LINK2 | 41 | COLOR |  |
| \％CELL | 41 | \％ | PRIMNO | 41 | LINK1 | 9 | LINK2 | 48 | COLOR |  |
| \％CELL | 48 | \％\％ | PRIMNO | 48 | LiNK1 | 41 | LINK2 | 50 | COLOR | 6 |
| \％CELL 非 | 50 | \％ | PRIMNO | 50 | LINK1 | 48 | LINK2 | 54 | COLOR | 3 |
| \％CELL \＃ | 54 | \％\％ | PRIMNO | 54 | LINK1 | 50 | LINK2 |  | COLOR | 4 |

ST．LINE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL $\#$ | $6 \%$ | PRIMNO | 6 LINKI | 0 LINK2 | 10 COLOR | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $10 \%$ | PRIMNO | 10 LINK1 | 6 LINK2 | 14 COLOR | 2 |
| \％CELL 非 | $14 \%$ | PRIMNO | 14 LINK1 | 10 LINK2 | 26 COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $26 \%$ | PRIMNO | 26 LINK1 | 14 LINK2 | 32 COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $32 \%$ | PRIMNO | 32 LINK1 | 26 LINK2 | 36 COLOR | 4 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一 灬$ | $36 \%$ | PRIMNO | 36 LINK1 | 32 LINK2 | 42 COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $42 \%$ | PRIMNO | 42 LINK1 | 36 LINK2 | 0 COLOR | 3 |


| \％CELL 非 | $34 \%$ | PRIMNO | 34 | LINK1 | 0 | LINK2 | 52 | COLOR | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $52 \%$ | PRIMNO | 52 | LINK1 | 34 | LINK2 | 18 | COLOR |  |
| \％CELL 非 | $18 \%$ | PRIMNO | 18 | LINK1 | 52 | LINK2 | 22 | COLOR | 6 |
| \％CELL | $22 \%$ | PRIMNO | 22 | LINK1 | 18 | LINK2 | 23 | COLOR | 6 |
| \％CELL $\#$ | $23 \%$ | PRIMNO | 23 | LINK1 | 22 | LINK2 | 24 | COLOR | 5 |
| \％CELL | $24 \%$ | PRIMNO | 24 | LINK1 | 23 | LINK2 | 27 | COLOR | 4 |
| \％CELL | $27 \%$ | PRIMNO | 27 | LINK1 | 24 | LINK2 | 30 | COLOR | 2 |
| \％CELL | $30 \%$ | PRIMNO | 30 | LINK1 | 27 | LINK2 | 33 | COLOR | 2 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $33 \%$ | PRIMNO | 33 | LINK1 | 30 | LINK2 | 39 | COLOR | 6 |
| \％CELL \＃ | $39 \%$ | PRIMNO | 39 | LINK1 | 33 | LINK2 | 45 | COLOR | 2 |
| \％CELL | $45 \%$ | PRIMNO | 45 | LINK1 | 39 | LINK2 | 46 | COLOR | 2 |
| \％CELL ${ }^{\text {P }}$ | $46 \%$ | PRIMNO | 46 | LINK1 | 45 | LINK2 | 47 | COLOR | 4 |
| \％CELL ${ }^{\text {P }}$ | $47 \%$ | PRIMNO | 47 | LINK1 |  | LINK2 | 55 | COLOR | 5 |
| \％CELL 非 | $55 \%$ | PRIMNO | 55 | LINK1 | 47 | LINK2 |  | COLOR |  |

ST．LINE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ：

| CELL \＃ | 3 | \％\％ | PRIMNO | 3 | LINK1 |  | NK2 |  | OLOR |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | 11 | \％ | PRIMNO | 11 | LINK1． | 3 | LINK2 | 15 | COLOR |  |
| \％CELL 非 | 15 | \％ | PRIMNO | 15 | LINK1 | 11 | LINK2 | 31 | COLOR |  |
| \％CELL \＃ | 31 | $\%$ | PRIMNO | 31 | LINK1 | 15 | LInk | 40 | col |  |


| \％CELL \＃ | $40 \% \%$ | PRIMNO | 40 LINK1 | 31 LINK2 |  | COLOR | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL 非 | $43 \%$ | PRIMNO | 43 LINK1 | 40 LINK2 | 0 | COLOR | 4 |
| ST．LINE TYPE 4 PRIMITIVE CHAIN CONSISTS OF： |  |  |  |  |  |  |  |
| \％CELL 非 | $4 \%$ | PRIMNO | 4 LINK1 | 0 LINK2 | 51 | COLOR | 1 |
| \％CELL 非 | $51 \%$ | PRIMNO | 51 LINK1 | 4 LINK2 | 5 | COLOR | 3 |
| \％CELL 非 | $5 \%$ | PRIMNO | 5 LINK1 | 51 LINK2 | 7 | COLOR | 4 |
| \％CELL 非 | $7 \% \%$ | PRIMNO | 7 LINK1 | 5 LINK2 | 16 | COLOR | 2 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 彡$ | $16 \%$ | PRIMNO | 16 LINK1 | 7 LINK2 | 19 | COLOR | 6 |
| \％CELL 非 | $19 \%$ | PRIMNO | 19 LINK1 | 16 LINK2 | 20 | COLOR | 6 |
| \％CELL 非 | $20 \%$ | PRIMNO | 20 LINK1 | 19 LINK2 | 21 | COLOR | 5 |
| \％CELL 非 | 21 \％ | PRIMNO | 21 LINK1 | 20 LINKŻ | 28 | COLOR | 5 |
| \％CELL \＃ | $28 \%$ | PRIMNO | 28 LINK1 | 21 LINK2 | 29 | COLOR | 3 |
| \％CELL \＃ | $29 \%$ | PRIMNO | 29 LINK1 | 28 LINK2 | 49 | COLOR | 3 |
| \％CELL 非 | $49 \%$ | PRIMNO | 49 LINK1 | 29 LINK2 | 53 | COLOR | 4 |
| \％CELL 非 | $53 \%$ | PRIMNO | 53 LINK1 | 49 LINK2 | 0 | COLOR | 3 |
| CURVE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $8 \%$ | PRIMNO | 8 LINK1 | 0 LINK2 | 12 | COLOR | 2 |
| \％CELL \＃ | $12 \%$ | PRIMNO | 12 LINK1 | 8 LINK2 | 25 | COLOR | 1 |
| \％CELL \＃ | $25 \%$ | PRIMNO | 25 LINK1 | 12 LINK2 | 0 | COLOR | 5 |
| CURVE TYPE 2 PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一 1$ | $38 \%$ | PRIMNO | 38 LINK1 | 0 LINK2 | 0 | COLOR | 5 |
| CURVE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |
| \％CELL 非 | $13 \%$ | PRIMNO | 13 LINK1 | 0 LINK2 | 35 | COLOR | 2 |
| \％CELL \＃ | $35 \%$ | PRIMNO | 35 LINK1 | 13 LINK2 | 0 | COLOR | 5 |
| CURVE TYPE 4 PRIMITIVE CHAIN CONSISTS OF ： |  |  |  |  |  |  |  |
| \％CELL \＃ | $37 \%$ | PRIMNO | 37 LINK1 | 0 LINK2 | 0 | COLOR | 3 |

##  <br> $=E===$ OUTPUT FOR SCENE 8 = $====$ 









PRIM非 31 TYPE 8 ALT．TYPE 4 COLOR 6 TAG\＃FROM 1882 TO 2005 START VECTOR＝＞LENGTH 5．35 ANGLE（DEG） 64.60 LINK1 0 END VECTOR＝＞LENGTH 4．65 ANGLE（DEG） 68.78 LINK2 41 TWIN POINTER 非 0 REDUCTION ID． 2.0
PRIMITIVE VALUES＝＞LENGTH 0.8 AREA 0.801 ROTATION－ 0.0017 AVG．SLOPE 2．32 COMPUTED SLOPE 3.82 CHORD 0.79 RADIUS $\quad 1.07$ CENTER CO－ORD $X=5.52 Y=1.32$ NON－OVERLAP COUNT＝O OVERLAP COUNT＝ 123

OVERLAP PRIMITIVES 32 0 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

PRIM非 32 TYPE 8 ALT．TYPE 4 COLOR 4 TAG非 FROM 1882 TO 2014
START VECTOR＝＞LENGTH 4.67 ANGLE（DEG） 68.75 LINK1 42
END VECTOR＝＞LENGTH 5．37 ANGLE（DEG） 64.60 LINK2． 33
TWIN POINTER $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ R REDUCTION ID． 1.0
PRIMITIVE VALUES $\Rightarrow$ LENGTH 0.8 AREA -0.810 ROTATION -0.0022
AVG．SLOPE 2.32 COMPUTED SLOPE 0.68 CHORD 0.79
RADIUS $\quad 1.13$ CENTER CO－ORD $X=5.59 Y=1.33$
NON－OVERLAP COUNT＝O OVERLAP COUNT＝ 121
OVERLAP PRIMITIVES $3100 \begin{array}{lllllll}0 & 0 & 0 & 0 & 0 & 0\end{array}$
PRIM非 33 TYPE 4 ALT．TYPE 8 COLOR 4 TAG非 FROM 1882 TO 1938
START VECTOR＝＞LENGTH 5．45 ANGLE（DEG） 64.60 LINK1 32
END VECTOR＝＞LENGTH 5．57 ANGLE（DEG） 64.84 LINK2 38
TWIN POINTER \＃ 0 REDUCTION ID． 2.0
PRIMITIVE VALUES＝＞LENGTH 0.1 AREA 0.0 ROTATION 0.0
$\begin{array}{llllll}\text { AVG．SLOPE } \quad 0.0 & \text { COMPUTED SLOPE } & 1.31 \text { CHORD } & 0.13\end{array}$
RADIUS 0.0 CENTER CO－ORD $X=0.0 \quad Y=0.0$






| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $36 \%$ | PRIMNO | 36 LINK1 | 0 LINK2 | 0 COLOR | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| St．line type 4 PRIMITIVE Chain consists of ： |  |  |  |  |  |  |
| \％CELL ${ }^{\text {吅 }}$ | $26 \%$ | PRIMNO | 26 LINK1 | 0 LINK2 | 33 COLOR | 2 |
| \％CELL \＃ | $33 \%$ | PRIMNO | 33 LINK1 | 26 LINK2 | 42 COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $42 \%$ | PRIMNO | 42 LINK1 | 33 LINK2 | 46 COLOR |  |
| \％CELL \＃ | $46 \%$ | PRIMNO | 46 LINK1 | 42 LINK2 | 0 COLOR |  |

## CURVE TYPE 1 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL \＃ | $\%$ | PRIMNO | 8 | LINK1 |  | LINK2 | 0 | COLOR |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL | $10 \%$ | PRIMNO | 10 | LINK1 | 8 | LINK2 | 11 | COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $11 \%$ | PRIMNO | 11 | LINK1 | 10 | LINK2 | 20 | COLOR |  |
| \％CELL \＃ | $20 \%$ | PRIMNO | 20 | LINK1 | 11 | LINK2 | 24 | COLOR |  |
| CELL 非 | $24 \%$ | PRIMNO | 24 | LINK1 | 20 | LINK2 | 27 | COLOR |  |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | $27 \%$ | PRIMNO | 27 | LINK1 | 24 | LINK2 | 30 | COLOR |  |
| CELL 非 | $30 \%$ | PRIMNO | 30 | LINK1 | 27 | LINK2 | 38 | COLOR |  |
| CELL 非 | $38 \%$ | PRIMNO | 38 | LINK1 | 30 | LINK2 |  | COLOR |  |
| \％CELL 非 | $44 \%$ | PRIMNO | 44 | LINK1 | 38 | LINK2 |  | COLOR |  |
| \％CELL \＃ | $49 \%$ | PRIMNO | 49 | LINK1 | 44 | LINK2 |  | COLOR |  |

CURVE TYPE 2 primitive chain consists of ：

| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

CURVE TYPE 3 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL $\#$ | 3 | \％ | PRIMNO | 3 | LINK1 | 0 | LINK2 | 9 | COLOR | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL ${ }^{\text {F }}$ | 9 | \％\％ | primmo | 9 | LINK1 | 3 | LINK2 | 12 | COLOR | 5 |
| CELL 非 | 12 | \％ | PRIMNO | 12 | LINK1 | 9 | LINK2 | 23 | COLOR | 6 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | 23 | \％ | PRIMNO | 23 | LINK1 | 12 | LINK2 | 29 | COLOR | 3 |
| \％CELL \＃ | 29 | \％ | PRIMNO | 29 | LINK1 | 23 | LINK2 | 47 | COLOR | 2 |
| \％CELL \＃ | 47 | \％ | PRIMNO | 47 | LINK1 | 29 | LINK2 |  | COLOR | 4 |
| \％CELL \＃ | 51 | \％ | PRIMNO | 51 | LINK1 | 47 | LINK2 |  | COLOR | 1 |

CURVE TYPE 4 PRIMITIVE CHAIN CONSISTS OF ：

| \％CELL \＃ | 1 | \％\％ | PRIMNO | 1 | LINK1 | 0 | LINK2 | 2 | COLOR | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \％CELL \＃ | 2 | \％\％ | PRIMNO | 2 | LINK1 | 1 | LINK2 | 4 | COLOR | 6 |
| \％CELL 非 | 4 | \％\％ | PRIMNO | 4 | LINKI | 2 | LINK2 | 5 | COLOR | 6 |
| \％CELL \＃ | 5 | \％\％ | PRIMNO | 5 | LINK1 | 4 | LINK2 | 6 | COLOR | 5 |
| \％CELL \＃ | 6 | \％\％ | PRIMNO | 6 | LINK1 | 5 | LINK2 | 7 | COLOR | 5 |
| \％CELL 非 | 7 | \％\％ | PRIMNO | 7 | LINK1 | 6 | LINK2 | 13 | COLOR | 6 |
| \％CELL \＃ | 13 | \％\％ | PRIMNO | 13 | LINK1 | 7 | LINK2 | 14 | COLOR | 6 |
| \％CELL $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ | 14 | \％\％ | PRIMNO | 14 | LINK1 | 13 | LINK2 | 15 | COLOR | 3 |
| \％CELL \＃ | 15 | \％\％ | PRIMNO | 15 | LINK1 | 14 | LINK2 | 21 | COLOR | 3 |
| \％CELL \＃ | 21 | \％\％ | PRIMNO | 21 | LINKI | 15 | LINK2 | 22 | COLOR | 2 |
| \％CELL \＃ | 22 | \％\％ | PRIMNO | 22 | LINKI | 21 | LINK2 | 31 | COLOR | 2 |
| \％CELL \＃ | 31 | \％\％ | PRIMNO | 31 | LINK1 | 22 | LINK？ | 32 | COLOR | 6 |
| \％CELL \＃ | 32 | \％\％ | PRIMNO | 32 | LINK1 | 31 | LINK2 | 45 | COLOR |  |
| \％CELL \＃ | 45 | \％\％ | PRIMNO | 45 | LINK1 | 32 | LINK2 | 0 | COLOR | 1 |

APPENDIX D

```
PATTERN TYPE = 1 COLOR = 1
    FULL PRIMITIVE 非 = 3
    FULL PRIMITIVE ## = 8
    FULL PRIMITIVE 非 = 4
    FULL PRIMITIVE 非 = 1
    FULL PRIMITIVE # = 2
    PATTERN TYPE =2 COLOR = 2
    FULL PRIMITIVE # = 14
    FULL PRIMITIVE 非 = 13
    FULL PRIMITIVE ## = 9
    FULL PRIMITIVE # = 10
    FULL PRIMITIVE 非 = 11
    PATTERN TYPE = = COLOR = 4
    FULL PRIMITIVE # = 18
    FULL PRIMITIVE # = 19
    FULL PRIMITIVE·非 = 15
    FULL PRIMITIVE 非 = 16
    FULL PRIMITIVE ## = 17
PATTERN TYPE = 4 COLOR = 3
FULL PRIMITIVE 非=7
FULL PRIMITIVE ## = 12
FULL PRIMITIVE 非 = 5
FULL PRIMITIVE ##=6
PATTERN TYPE = 5 COLOR = 5
FULL PRIMITIVE 非=22
FULL PRIMITIVE ## = 23
FULL PRIMITIVE.非 = 20
```

```
PATTERN TYPE = = 1 COLOR = 5
FULL PRIMITIVE # = 21
FULL PRIMITIVE # = 23
FULL PRIMITIVE # = 22
FULL PRIMITIVE # = 19
FULL PRIMITIVE ## = 20
PATTERN TYPE = 2 COLOR = 3
FULL PRIMITIVE 非 = 13
FULL PRIMITIVE ## = 16
FULL PRIMITIVE 非 = 10
FULL PRIMITIVE # = 9
FULL PRIMITIVE # = 11
PATTERN TYPE = 3 COLOR = 2
FULL PRIMITIVE # = 6
FULL PRIMITIVE # = 12
FULL PRIMITIVE ## = 7
FULL PRIMITIVE # = 5
PATTERN TYPE = 4 COLOR = 1
FULL PRIMITIVE # = 1
FULL PRIMITIVE 非 = 3
FULL PRIMITIVE 非 = 4
PATTERN TYPE = 5 COLOR = 4
FULL PRIMITIVE ## = 18
FULL PRIMITIVE # = 14
FULL PRIMITIVE # = 15
```


MACROMATON OUTPUT FOR SCENE 非 3 ．．．．．．．


```
PATTERN TYPE = 1 COLOR = 2
FULL PRIMITIVE # = 7
FULL PRIMITIVE # = 13
FULL PRIMITIVE # = 14
FULL PRIMITIVE 非 = 3
FULL PRIMITIVE ## = 1
FULL PRIMITIVE # = 2
PATTERN TYPE = 1 COLOR = 3
FULL PRIMITIYE # # = 12
FULL PRIMITIVE # = 19
FULL PRIMITIVE 非 = 21
FULL PRIMITIVE # = 11
FULL PRIMITIVE 非 = 9
FULL PRIMITIVE # = 10
PATTERN TYPE = 3 COLOR = 1
FULL PRIMITIVE # = 4
FULL PRIMITIVE # = 5
FULL PRIMITIVE 非 = 6
FULL PRIMITIVE # = 8
FULL PRIMITIVE # = 34
PATTERN TYPE = 3 COLOR = 5
FULL PRIMITIVE # = 23
FULL PRIMITIVE # = 15
FULL PRIMITIVE # = 16
FULL PRIMITIVE 非 = 26
PATTERN TYPE = 4 COLOR = 2
FULL PRIMITIVE # = 33
FULL PRIMITIVE ## = 31
FULL PRIMITIVE # = 32
```

```
PATTERN TYPE = 5 COLOR = 1
FULL PRIMITIVE 非 = 30
FULL PRIMITIVE 非 = 24
FULL PRIMITIVE ## = 25
PATTERN TYPE = 2 COLOR = 4
PART *** PRIM # = = 17
PART *** PRIM ## = 18
PART *i*% PRIM # = 20
```

...... MACROMATON OUTPUT FOR SCENE \# 4


```
PATTERN TYPE = 1 COLOR = 1
FULL PRIMITIVE 非 = 7
FULLL PRIMITIVE ## = 1
FULL PRIMITIVE # = 2
FULL PRIMITIVE # = 12
FULL PRIMITIVE # = 39
PATTERN TYPE = 2 COLOR = 2
FULL PRIMITIVE # = 26
FULL PRIMITIVE # = 28
FULL PRIMITIVE # = 38
FULL PRIMITIVE # = 36
FULL PRIMITIVE # = 4
FULL PRIMITIVE # = 5
PATTERN TYPE = 3 COLOR = 3
FULL PRIMITIVE # = 25
FULL PRIMITIVE # = 35
FULL PRIMITIVE # = 9
FULL PRIMITIVE ## = 11
PATTERN TYPE = 4 COLOR = 5
FULL PRIMITIVE # = 23
FULL PRIMITIVE # = 31
FULL PRIMITIVE ## = 19
FULL PRIMITIVE # = 21
PATTERN TYPE = 5 COLOR = 4
FULL PRIMITIVE # = 14
FULL PRIMITIVE # = 16
FULL PRIMITIVE # = 17
FULL PRIMITIVE 非 = 33
```


．．．．．．．MACROMATON OUTPUT FOR SCENE \＃ 5 ．．．．．．


| PATTERN TYPE | $=$ | COLOR $=1$ |
| :---: | :---: | :---: |
| FULL PRIMITIVE 非 | $=36$ |  |
| FULL PRIMITIVE 非 | $=41$ |  |
| FULL PRIMITIVE \＃\＃ | $=39$ |  |
| FULL PRIMITIVE 非 | $=28$ |  |
| FULL PRIMITIVE \＃ | $=29$ |  |
| PATTERN TYPE | $=3$ | COLOR $=4$ |
| FULL PRIMITIVE \＃ | $=13$ |  |
| FULL PRIMITIVE 非 | $=16$ |  |
| FULL PRIMITIVE \＃ | $=21$ |  |
| FULL PRIMITIVE \＃ | $=26$ |  |
| FULL PRIMITIVE \＃ | $=37$ |  |
| FULL PRIMITIVE \＃ | $=24$ |  |
| FULL PRIMITIVE 非 | $=12$ |  |
| FULL PRIMITIVE \＃ | $=6$ |  |
| FULL PRIMITIVE 非 | 7 |  |
| FULL PRIMITIVE \＃ | $=9$ |  |
| PATTERN TYPE | $=5$ | COLOR $=3$ |
| PART 施施 PRIM \＃ | $=15$ |  |
| PART \％\％$\%$ PRIM 非 | $=10$ |  |
| PART \％$\%$ \％PRIM 非 | $=4$ |  |
| PART $\%$ \％${ }^{\text {PRIM }}$ P非 | $=5$ |  |
| PART \％ri＊PRIM 非 | $=30$ |  |
| PART \％ $2 \%$ PRIM 非 | $=19$ |  |
| PART＊\％PRIM \＃ | $=17$ |  |
| PATTERN TYPE | $=3$ | COLOR $=2$ |
| PART＊＊＊PRIM 非 | $=22$ |  |
| PART \％\％\％PRIM 非 | $=27$ |  |
| PART＊ $2 \dot{\text { r }}$ PRIM 非 | $=35$ |  |
|  | $=32$ |  |
| PART $\%$ \％$\%$ PRIM \＃ | $=23$ |  |
| PART \％$\%$ PRIM \＃ | $=18$ |  |


| PATTERN | YPE |  |  |
| :---: | :---: | :---: | :---: |
| RT *** | PRIM |  | 40 |
| RT \% | PRIM |  | 33 |
| PART , | PRIM |  |  |
| PART *** | PRIM |  | 2 |
| PART *** | PRIM |  | 3 |
| PART * | PRIM |  | 14 |
| PART | PRIM |  | 25 |
| PART **** | PRIM |  | 34 |
| Part \%* | PRIM |  | 38 |


| PATTERN TYPE | $=1$ | COLOR $=1$ |
| :---: | :---: | :---: |
| FULL PRIMITIVE 非 | $=26$ |  |
| FULL PRIMITIVE 非 | $=33$ |  |
| FULL PRIMITIVE 非 | $=36$ |  |
| FULL PRIMITIVE 非 | $=25$ |  |
| FULL PRIMITIVE 非 | $=18$ |  |
| FULL PRIMITIVE 非 | $=19$ |  |
| PATTERN TYPE | $=4$ | COLOR $=2$ |
| FULL PRIMITIVE 非 | $=3$ |  |
| FULL PRIMITIVE 非 | $=2$ |  |
| FULL PRIMITIVE 非 | $=15$ |  |
| FULL PRIMITIVE \＃ | $=1$ |  |
| PATTERN TYPE | $=5$ | COLOR $=3$ |
| PART ${ }^{*} \%$ \％PRIM | $=5$ |  |
| PART ${ }^{\circ}{ }^{\text {cte }}$ PRIM 非 | $=13$ |  |
| PART \％＊＊＊PRIM 非 | $=14$ |  |
| PART \％${ }^{\text {cte }}$ PRIM 非 | $=16$ |  |
| PART ${ }^{\text {\％}}$ \％$\%$ PRIM 非 | $=4$ |  |
| PART ricte PRIM 非 | $=6$ |  |
| PATTERN TYPE | $=3$ | COLOR $=4$ |
| PART \％${ }^{\text {cter }}$ PRIM 非 | $=20$ |  |
| PART＊＊＊PRIM 非 | $=17$ |  |
| PART＊＊＊\％PRIM 非 | $=24$ |  |
| PART \％＊＊PRIM 非 | $=35$ |  |
| PART＊＊＊＊PRIM \＃ | $=34$ |  |
| PART＊＊＊＊PRIM 非 | $=31$ |  |
| PART 2 \％＊PRIM \＃ | 32 |  |
|  | $=39$ |  |
|  | $=40$ |  |
|  | $=7$ |  |
| PART＊＊＊PRIM \＃ | $=9$ |  |
| PART \％$\%$ \％PRIM \＃ | $=8$ |  |


 MACROMATON OUTPUT FOR SCENE $\# 7$




..... MACROMATON OUTPUT FOR SCENE 非 8 ......




1
2
3
4
5

CIRCLE
SQUARE RECTANGLE EQUILATERAL TRIANGLE RIGHT ANGLE TRIANGLE

## A.PPENDIX E

## PROOF OF THE RECOGNIZER'S CORRECTNESS

The symbols used in the proof have the following meaning. $P$ represents the source pattern recognized while p represents the source primitives such that $p \varepsilon$. The target pattern set $\tau$ consists of individual target patterns $T$ to be recognized while $t$ represents the target primitive such that $t \varepsilon T$ and $T \varepsilon \tau, \hat{P}$ is used in place of $P$ where partial patterns are involved.

1. Assume that the recognizer recognizes a pattern $P=T$. In addition assure that for a fully visible pattern $F=T$, for any $t_{i}, t_{j} \varepsilon \tau$, at least one $t_{i} \varepsilon T_{i}$ which is distinct from all $t_{j} \varepsilon T \in \tau$.
2. For partially obscured patterns, assume that $\left\{t_{i}\right\}$ $=\left\{p_{i}\right\} 口 \hat{P}_{1}$ and $\left\{t_{j}\right\}=\left\{p_{j}\right\} \subseteq \hat{P}_{2}$. Further assume that if $\left|\left\{t_{i}\right\}\right|>\left|\left\{t_{j}\right\}\right|$ matching $\hat{p}$, then $\left\{t_{i}\right\} \subseteq$ $T_{i}$ is picked in preference to $\left\{t_{i}\right\} \subseteq T_{j}$.
3. Suppose the recognizer ristakes $D=T_{I} \varepsilon \tau$ for $P=T_{2} \in \tau$ and all $t_{i} \in T_{1}$ are fully visible. Then $\exists t_{j} \varepsilon T_{2}$ such that $t_{j}=t_{i} \varepsilon T_{1}, \forall t_{i}$. This is a contradiction since $\mathrm{I}_{\mathrm{E}} \in \mathrm{T}_{2}$ such that $t_{f} \notin \mathrm{~T}_{1}$.
4. Given that $\hat{P}=P$ is recognized as a partial pattern. Suppose some $\left\{t_{i}\right\} \subseteq T_{1}$ matches some $\left\{p_{j}\right\} \underline{\hat{p}}$ and some $\left\{t_{k}\right\} \cup T_{2}$ matches some $\left\{p_{\ell}\right\}$ 氖. Then $\left|\left\{p_{j}\right\}\right|=\left|\left\{p_{\ell}\right\}\right|$ or $\mid\left\{p_{j} j\left|\neq\left|\left\{\hat{p}_{\ell}\right\}\right|\right.\right.$
5. If $\left|\left\{p_{j}\right\}\right|=\left|\left\{p_{\ell}\right\}\right|$ then the recognizer has no way
of determining whether $\hat{\mathrm{p}}$ is $\mathrm{T}_{1}$ or $\mathrm{T}_{2}$.
6. Assume that $\left|\left\{p_{j}\right\}\right|>\left|\left\{p_{\ell}\right\}\right|$ and the recognizer mistakes $\left\{p_{j}\right\}$ for $T_{2}$. This is a contradiction of the assumption in 2. Q.E.D.
