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Fast and accurate calculation of the soil temperature distribution around ground heat exchanger based on a Response Factor model

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ABSTRACT

Ground heat exchanger (GHE) is an important component of ground coupled heat pump system (GCHP). To calculate the soil temperature around GHE accurately and fast, a refined response factor model (RF model) is proposed. It combines the heat transfer inside and outside the U pipe through the temperature of pipe wall and the heat flux of U pipe. For the RF model, after calculating the response factors by CFD simulation, the soil temperature can be calculated by the deduced analytical equations. The sandbox experiment is built up to validate the the RF model. Based on the experiment, this case is also studied by the numerical simulation and the RF model. Results show that the soil temperature differences between the RF model and the experiment are only -0.21 ℃ ~0.69 ℃ at the 96th time step. The relative errors of the soil temperatures between RF model and numerical simulation at the 1800th time step are only 1.86%~3.94%. RF model consumes 30% time of the numerical simulation for the soil temperature calculation with 1800 time steps and consumes only 1% time of the numerical simulation for that with 350400 time steps. Therefore, the RF model is accurate and fast to calculate the soil temperature around the GHE with fluid inside.

1 INTRODUCTION

1.1 Background

Ground-coupled heat pump (Sanner et al. 2003, Pawel 2004, Lund et al. 2011, Mustafa and Hikmet 2004) is more and more popular in the world because it's a clean and efficient technology for heating and cooling. The ground heat exchanger (GHE) is an important component in this system whose heat exchanging performance has greatly influenced the system design and operation (Florides and Kalogirou 2007, Yang et al. 2010).

The method of calculating the soil temperature around GHE and the heat exchanged by it is important to the design and the performance improvement of GHE. There are three main kinds of GHE models now: the analytical solution, the numerical solution, and the g-function model. The analytical solution (Zeng et al. 2003, Diao et al. 2004, Li and Lai 2013) needs many assumptions to simplify the problem and sacrifices its accuracy. The numerical solution (Lee and Lam 2008, Cui et al. 2008, Congedo et al. 2012) consumes long time because of its complexity. The g-function model (Yavuzturk and Spitler 1999, Yavuzturk et al. 1999, Yavuzturk 1999, Li et al. 2014) combines the analytical and numerical solution while ignores some important details, such as: the shape of U pipe and the non-uniform soil property. Calculating the soil temperature distribution accurately as well as at a fast speed needs to be studied further.

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1.2 Response factors

In our previous work, RF model (You et al. 2016) based on the response factors was proposed to calculate the soil temperature accurately and fast. Response factors represent the contribution of heat sources to the temperature variation of soil points, of which the definition is shown as Equation 1. It is equal to the excess soil temperature variation divided by average soil temperature variation caused by the heat flux at the initial time step. Therefore, the response factor is dimensionless and tends to be 1 when time becomes long. Physically, it means the heat pulse at the initial time step spreads to every corners of the soil heat retainer homogeneously at last and every soil points have the same temperature variation. The response factor is determined by the distance between the heat source and the soil points, having no relationship with the heat flux.

$$Y_{n,p}(j\Delta\tau) = \frac{\theta_p(j\Delta\tau)}{Q_n(0)\cdot\Delta\tau/(\sum_{k=1}^{K}\rho_k c_k V_k)} \quad (1)$$

$$\theta_p(j\Delta\tau) = T_p(j\Delta\tau) - T_{\text{initial}}$$

where, $Y_{n,p}(j\Delta\tau)$ is the response factor of soil point p to the heat pulse of the nth heat source at the jth time step; $\theta_p(j\Delta\tau)$ is the excess temperature of point p at the jth time step, [℃]; $T_p(j\Delta\tau)$ is the temperature of point p at the jth time step, [℃]; $T_{initial}$ is the initial soil temperature, [℃]; $Q_n(0)$ is the heat flux released by the nth heat source at the initial time steps, [W]; ρ_k is the density of the kth material in the soil heat retainer, [kg/m^3]; c_k is the specific heat capacity of the kth material in the soil heat retainer, [J/(kg·℃)]; V_k is the volume of the kth material in the soil heat retainer, [m^3]. Different materials in the soil heat retainer can be the grout, concrete, different soil layers.

Based on the definition of response factors, the soil temperature can be calculated by the accumulated contribution of heat fluxes at different time steps. The calculating method is shown in Equation 2, which is the accumulation of the heat fluxes timing the corresponding response factors during the period and divided by the soil heat capacity.

$$\theta_p(j\Delta\tau) = \sum_{n=1}^{N}\sum_{i=0}^{j} Q_n(i\Delta\tau)\cdot\Delta\tau\cdot Y_{n,p}[(j-i)\Delta\tau]/(\sum_{k=1}^{K}\rho_k c_k V_k) \quad (2)$$

where, $\theta_p(j\Delta\tau)$ is the excess temperature of soil point p at the jth time step, [℃]; *n* is the number of the different heat sources.

The RF model is the combination of the numerical simulation and the analytical solution. As shown in Equation 1, the response factors are based on the soil temperature under a heat pulse. These soil temperature can be calculated by the numerical simulation, like FLUENT solver, during the limited period. All the specific parameters, like the different soil thermal conductivities, different geometries of pipes, different borehole grouting materials, borehole group field can be accounted for at this stage. Different parameters contribute to different response factors. After the calculation of response factors, the soil temperature under any flux at any time can be calculated based on the analytical equations, like Equation 2.

1.3 The purpose of this paper

In the previous work (You et al. 2016), the definition of response factors is suitable to the heat sources with the known heat flux. However, for the GHE with the fluid inside the U pipe, the known variables are usually the inlet water temperature and the mass flow rate. Therefore, the RF model is refined to combine the heat transfer inside and outside the pipe in this paper. The principle of RF model is illustrated in detail. A case is studied to show the calculating procedure and the sandbox experiment is built up to validate the model. At last, the calculating speed and accuracy of RF model are compared with that of the numerical simulation.

2 PRINCIPLE OF RF MODEL

With the known inlet temperature and flow rate of U pipe, the heat flux of U pipe should be calculated in advance for the soil temperature calculation in equation 2. Thus, the heat transfer of U pipe is regarded as two parts: heat transfer outside the pipe and heat transfer inside the pipe. The excess temperature of pipewall and the heat flux of the pipe are used to connect them together. The heat transfer outside can be expressed by the response factors of the pipewall. The heat transfer inside can be expressed by the heat balance law. This is the principle of RF model and is illustrated in detail in this section.

2.1 Heat transfer outside the pipe

Based on the definition of response factors, the excess temperature of pipewall can be calculated by Equation 3. For the excess temperature of pipewall at j^{th} time step, it is determined by the accumulated contribution of heat fluxes during $0\sim j^{th}$ time steps. Taking the contribution of heat flux at i^{th} time step, its temperature contribution is the heat flux at i^{th} time step timing the response factors at the $(j-i)^{th}$ time step, because it takes (j-i) time steps to get the influence of the heat flux, and then divided by the soil heat capacity.

$$\theta_{wall}(j\Delta\tau) = \sum_{i=0}^{j-1} Q_s(i\Delta\tau)\cdot\Delta\tau\cdot Y_{s,wall}[(j-i)\Delta\tau]/(\sum_{k=1}^{K}\rho_k c_k V_k) + Q_s(j\Delta\tau)\cdot\Delta\tau\cdot Y_{s,wall}(0)/(\sum_{k=1}^{K}\rho_k c_k V_k) \quad (3)$$

where, $\theta_{wall}(j\Delta\tau)$ is the excess temperature of pipewall at the j^{th} time step, [℃]; $Q_s(i\Delta\tau)$ is the heat flux released by the U pipe at the i^{th} time steps, [W]; $Y_{s,wall}[(j-i)\Delta\tau]$ is the response factor of pipewall to the U pipe at the $(j-i)^{th}$ time step.

2.2 Heat transfer inside the pipe

The heat flux of the pipe exchanged to the soil can be calculated by the heat capacity of the fluid timing the temperature difference of the inlet and outlet fluid, which is shown in Equation 4.

$$Q_s(j\Delta\tau) = c_f m_f[\theta_{in}(j\Delta\tau) - \theta_{out}(j\Delta\tau)] \quad (4)$$

where, c_f is the specific heat capacity of the fluid, [J/(kg·℃)]; m_f is the mass flow rate of the fluid, [kg/s]; $\theta_{in}(j\Delta\tau)$ is the inlet fluid temperature of U pipe at the j^{th} time step, [℃]; $\theta_{out}(j\Delta\tau)$ is the outlet fluid temperature of U pipe at the j^{th} time step, [℃].

The heat flux also can be calculated by the heat convection between the fluid and the pipewall, as shown in Equation 5. The convective heat transfer coefficient can be calculated by Equation 6.

$$Q_s(j\Delta\tau) = h_f F_{pipe}[\theta_f(j\Delta\tau) - \theta_{wall}(j\Delta\tau)]$$
$$\theta_f(j\Delta\tau) = \frac{1}{2}[\theta_{in}(j\Delta\tau) + \theta_{out}(j\Delta\tau)] \quad (5)$$

$$Nu = 0.023\cdot \mathrm{Re}_f^{0.8}\cdot \mathrm{Pr}_f^{0.3} \qquad \mathrm{Re} > 10000$$
$$Nu = 0.116\cdot(\mathrm{Re}_f^{2/3} - 125)\cdot \mathrm{Pr}_f^{1/3}\cdot(1+(\frac{d_{pipe}}{l_{pipe}})^{2/3}) \qquad 2200 < \mathrm{Re} < 10000 \quad (6)$$
$$Nu = 1.86\cdot(\mathrm{Re}_f\cdot \mathrm{Pr}_f\cdot\frac{d_{pipe}}{l_{pipe}})^{1/3} \qquad \mathrm{Re} < 2200, \mathrm{Pr} > 0.6$$

where h_f is the convective heat transfer coefficient of the fluid,[W/(m² ·℃)]; F_{pipe} is the area of U pipe, [m²]; $\theta_f(j\Delta\tau)$ is the average fluid temperature inside U pipe at the j^{th} time step, [℃]; d_{pipe} is the inner diameter of the U pipe,[m]; l_{pipe} is the length of the U pipe, [m].

2.3 Heat connection of inside and outside of the pipe

Conbining Equation 3~5, once the inlet temperature is known, the heat flux of U pipe, the outlet temperature and the pipewall temperature can be calculated by the matrix, as shown in Equation 7. As a consequence, as long as the response factors are calculated by numerical simulation, the heat transfer process can be demonstrated by Equation 7 and then the soil temperature distribution can be calculated by Equation 2. Since the soil thermal conductivity and the geometry of pipes are accounted for already in the numerical simulation, they have influence on the response factors. In the Equation 7, there is no need to consider them again.

$$\begin{pmatrix} Q_s(j\Delta\tau) \\ \theta_{out}(j\Delta\tau) \\ \theta_{wall}(j\Delta\tau) \end{pmatrix} = \begin{pmatrix} 1 & c_f m_f & 0 \\ 1 & -\frac{1}{2} h_f F_{tube} & h_f F_{tube} \\ -\Delta\tau \cdot Y_{s,wall}(0)/(\sum_{k=1}^{K} \rho_k c_k V_k) & 0 & 1 \end{pmatrix}^{-1} \times \begin{pmatrix} c_f m_f \cdot \theta_{in}(j\Delta\tau) \\ \frac{1}{2} h_f F_{tube} \cdot \theta_{in}(j\Delta\tau) \\ \sum_{i=0}^{j-1} Q_s(i\Delta\tau) \cdot \Delta\tau \cdot Y_{s,wall}[(j-i)\Delta\tau]/(\sum_{k=1}^{K} \rho_k c_k V_k) \end{pmatrix} \quad (7)$$

Since the response factors has no relationship with the heat flux, when calculating it, the pipe can be considered as a solid without fluid inside and it releases the heat pulse at the initial time step. In this way, the calculating speed of numerical simulation for response factors can be greatly increased. Besides, the response factors tend to be 1 when time becomes longer. As a consequence, for a long term soil temperature calculation, only the response factors at the initial time steps needs to be simulated and those at the following time steps can be assumed as 1. The simulated time steps of real response factors are determined by the demand of accuracy.

3 CASE STUDIES

3.1 Sandbox experiment

To validate the RF model, the sandbox experiment is built up, which is composed of a sandbox placed with thermocouples, data acquisition system, a device providing constant temperature water, water pump and a flow meter, as shown in Figure 1.

Figure 1 Sandbox experiment

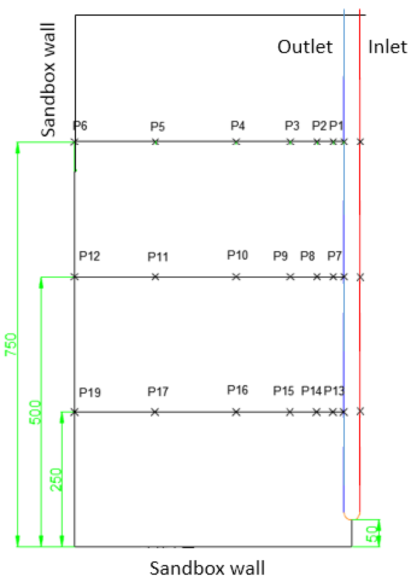
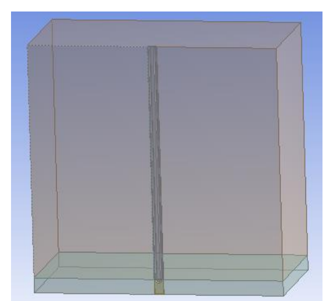
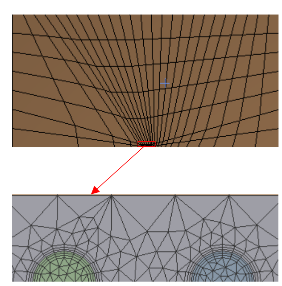


Figure 2 Side view of thermocouples placement within sandbox



(a) Geometry



(b) Meshing

Figure 3 The studied case in ANSYS

The sandbox is 1m×1m×1m with a U-pipe placed at the center of the sandbox and is filled with sand. The diameter and length of U pipe are 8mm and 950mm. The thermal properties of the sand are tested by the transient plane source method. The specific heat capacity, density and heat conductivity of the sand are respectively 757J/(kg·K), 1255kg/m^3, and 0.22655W/(m·K). 18 thermocouples are placed to test the soil temperatures, as shown in Figure 2. The sandbox is wrapped by the thermal insulation material with the thickness about 100mm. So, the sandbox wall is regarded as adiabatic. The initial excess soil temperature is homogeneous and considered as 0. In the experiment, the water flows inside the U pipe at the speed of 1.725m/s and it heats the sand for 24 hours. The excess temperature of the U pipe inlet keeps constant at about 14 ℃. The data acquiring system records every 10 seconds.

3.2 Case design

To compare the calculation accuracy and speed of RF model to those of the experiment and numerical simulation, the model of the case based on the sand experiment is built in ANSYS. ANSYS is a general purpose finite element modeling package for numerically solving a wide variety of mechanical problems, including the fluid and heat transfer. The geometry and meshing of the case for CFD simulation are shown in Figure 3 respectively. As the sandbox is symmetric, the built geometry is half of it to reduce the mesh number and increase the calculation speed. The size, material and boundary conditions of the case for CFD simulation are kept the same with the sandbox experiment. There are 640,000 meshes and the size of simulating time step is 900s.

In the RF model, the excess soil temperatures under the initial heat pulse are simulated by the ANSYS model to further calculate the response factors. Besides, the soil temperature calculated by the numerical simulation is also based on this ANSYS model.

4 RESULTS AND ANALYSES

As for the studied case, the calculation results of RF model are demonstrated in this section. First, the accuracy of RF model is verified by the numerical simulation and validated by the sandbox experiment. Then, for the calculation of soil temperatures in a long term, when the time tends to be infinite, the response factors can be approximate to 1 to save the calculation time. The accuracy of RF model with the approximate response factors are compared to that with the real response factors. At last, the long-term soil temperatures calculated by RF model with approximate response factors are compared with those by the numerical simulation.

4.1 Validation by the numerical simulation and sandbox experiment

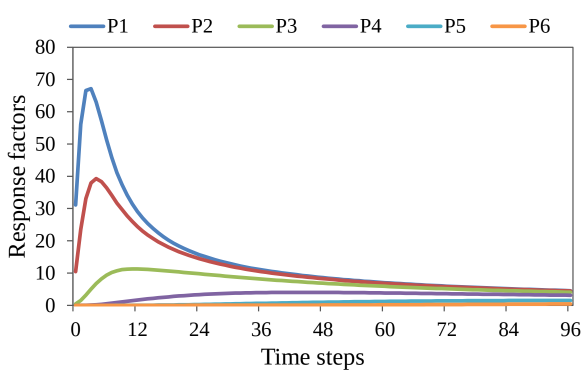


Figure 4 The response factors of P1~P6 during the initial 96 time steps

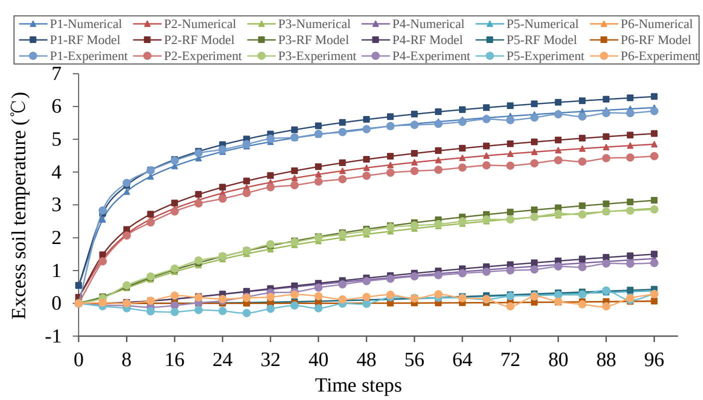


Figure 5 The excess soil temperature comparison of P1~P6 calculated by three different methods

To calculate the soil temperatures by RF model, the response factors at different time steps should be calculated in advance. The response factors of P1~P6 during the initial 96 time steps are shown in Figure 4. For the points next to the U pipe, the response factors first increase rapidly and then decrease, like that the response factor of P1 peaks at 67.16 at the 3rd time step. For the points far from the U pipe, the response factors keep increasing slowly, like the P6. The response factors of P1~P6 at the 96th time steps range from 0.42~4.53.

Based on the response factors, the soil temperatures of P1~P6 of RF model are calculated and compared with those of the numerical simulation and sandbox experiment, which is shown in Figure 5. Due to the heat of constant inlet temperature, the soil temperatures of P1~P6 increase. The soil temperature differences of P1~P6 between the RF model and the numerical simulation are 0.01 ℃ ~0.34 ℃ at the 96th time step. And, the soil temperature differences of P1~P6 between the RF model and the experiment are -0.21 ℃ ~0.69 ℃ at the 96th time step. When the meshing of model in CFD simulation becomes finer, the accuracy of RF model can be furtherly improved.

4.2 The accuracy of RF model with the approximate response factors

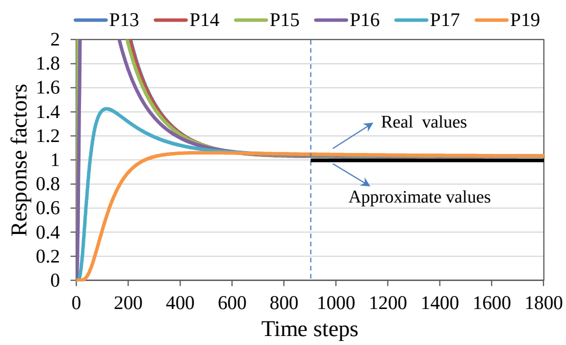


Figure 6 The the real and approximate response factors of P13~P19

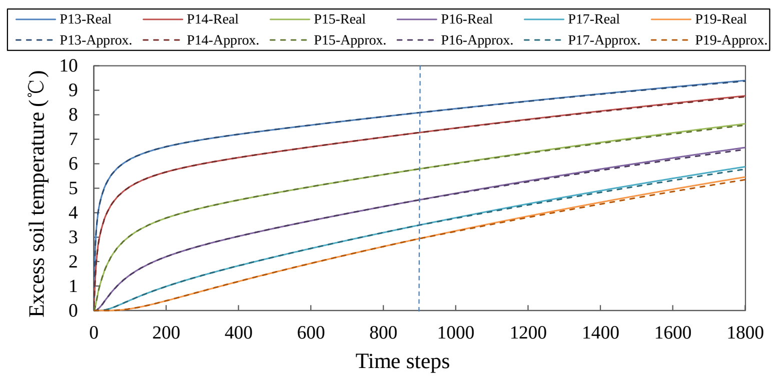


Figure 7 The soil temperature calculated by RF model based on the real and approximate response factors

Taking P13~P19 as examples, the soil temperatures during 1800 time steps are calculated based on RF model. The response factors of P13~P19 during 0~1800 time steps are calculated under the initial heat pulse, as shown in Figure 6. The maximum response factors of P13, P14, P15 and P16 are respectively 67.20 at the 3rd time step, 39.29 at the 4th time step, 11.26 at the 11th time step and 3.82 at the 41th time step. All the response factors at the 900th and the 1800th time steps are respectively 1.030~1.048 and 1.030~1.035, which are the real values. It shows that when the time becomes longer, the variation of response factor becomes gentle and its value tends to be 1. To save the calculation time, the response factors of P13~P19 during 900~1800 time steps are assumed 1, which are the approximate values. So, the CFD simulation of response factors only takes 900 time steps, saving half of the calculation time steps.

The soil temperatures of P13~P19 calculated by the RF model with the real and approximate response factors are illustrated in Figure 7. The approximation of response factors only cause very small errors of soil temperatures. The relative errors at P13 are less than 0.27%.

4.3 Soil temperature calculation in long term

Because of the good accuracy, the approximate response factors are used by RF model to calculate the soil temperature of P13~P19. The calculation results are compared with those by numerical simulation, which are shown in Figure 8. The temperature difference between RF model and numerical simulation are only less than 0.24 ℃. The relative errors of the soil temperatures between RF model and numerical simulation at the 1800th time step are 1.86%~3.94%, they show the RF model has a very good accuracy.

The calculating time consumption between RF model and numerical simulation is compared in Table 1. For RF model, it takes 38min on CFD simulation for response factors and 67 seconds for the following soil temperature calculation. Therefore, the total time consumption of RF model is about 39min and 7sec, while that of the numerical simulation is about 120min. The RF model only consumes 30% time of the numerical simulation. When the soil temperature calculation becomes much longer to 10 years, the time consumption of RF model is about 255min22sec, but the time consumption of numerical simulation increases greatly to 23360min. The RF model only consumes 1% time of the numerical simulation. Consequently, the advantage of RF model on time saving becomes more obvious for a long-term soil temperature calculation.

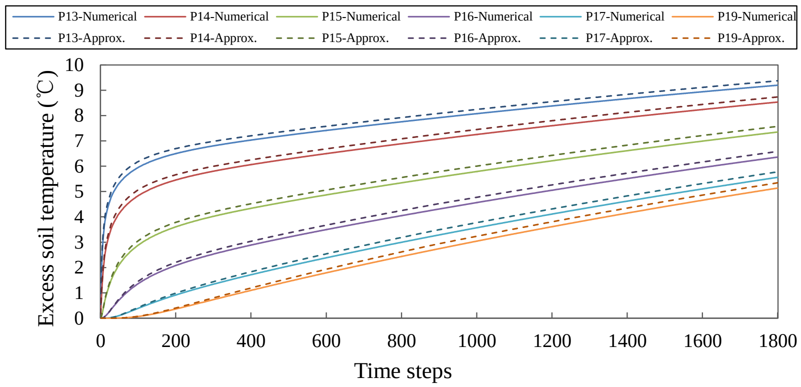


Figure 8 Soil temperatures calculated by numerical simulation and RF model

Table 1 Time consumption between RF model and numerical simulation

Time steps	Time (Day)	Calculating time consumption	
		RF model with approximate response factors	Numerical simulation
1800	19	39min7sec	120min
350400	3650	255min22sec	23360min

5 CONCLUSION

To improve the calculating speed and accuracy of soil temperature around the GHE, the refined RF model based on the response factors is proposed. It combines the heat transfer inside and outside the U pipe through the pipe wall temperature and the heat flux of U pipe. To validate the the RF model, the sandbox experiment is built up and the model of CFD simulation is also established based on it. The results and analyses are as follows:

1. The soil temperatures of P1~P6 during 96 time steps calculated by RF model have small temperature difference with those by the numerical simulation and the experiment. The soil temperature differences between the RF model and the numerical simulation are 0.01 ℃ ~0.34 ℃ at the 96^{th} time step. And, those between the RF model and the experiment are -0.21 ℃ ~0.69 ℃ at the 96^{th} time step.

2. The RF model with approximate response factors has the nearly the same accuracy with that with the real response factors. Their relative errors of soil temperature at P13 are less than 0.27%.

3. The RF model has a good accuracy by the verification with the numerical simulation. The relative errors of the soil temperatures between RF model and numerical simulation at the 1800^{th} time step are 1.86%~3.94%. What's more, RF model has a fast speed than the numerical simulation, consuming 30% time of the numerical simulation for the soil temperature calculation with 1800 time steps and consuming only 1% time of the numerical simulation for the soil temperature calculation with 350400 time steps.

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NOMENCLATURE

ρ	=	density (kg/m³)
θ	=	excess temperature (°C)
c	=	specific heat capacity (J/(kg· ℃))
d	=	inner diameter of the U pipe (m)
F	=	pipe area (m²)
h	=	convective heat transfer coefficient (W/(m²· ℃))
l	=	length of the U pipe (m)
m	=	mass flow rate of the fluid (kg/s)
Q	=	heat flux (W)
Y	=	response factor
V	=	Volume (m³)

Subscripts

f	=	fluid
in	=	inlet of U pipe
k	=	number of the soil layers
n	=	number of the different heat sources
out	=	*outlet of U pipe*
p	=	*soil point p*
wall	=	*pipe wall*

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