COMPUTATION OF RADIATION CONFIGURATION FACTORS BY CONTOUR INTEGRATION

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FACTORS BY CONTOUR INTEGRATION

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CHAPTER I

INTRODUCTION

It is quite a simple matter to obtain the net exchange of thermal radiation between two black surfaces separated by nonabsorbing media once several factors are known, i.e., the temperatures of the surfaces under consideration, and the geometric relationship of the surfaces. All bodies at temperatures above absolute zero will continuously radiate heat to their surroundings, even though they may at the same time be absorbing more heat than they emit. The net exchange between a black body and its surroundings is merely the difference in the energy radiated from the body and the energy received from the surroundings.

In order to determine the net energy exchange between two bodies, the shape and orientation of the bodies must be considered. The object of this study was to provide a rapid and accurate determination of the geometric relationship between various surfaces. This relationship is referred to as the configuration factor. The configuration factor from surface A_1 to surface A_2, written as F_{1-2}, is defined as the fraction of radiant flux leaving surface A_1 directly incident on surface A_2.

To obtain a mathematical expression for the geometrical relationship between two surfaces, consider a small element of surface dA_1 on A_1 (see Fig. 1). If a hemisphere is placed over dA_1 with dA_1 at the center, the hemisphere will intercept all of the radiation beams emitted by the

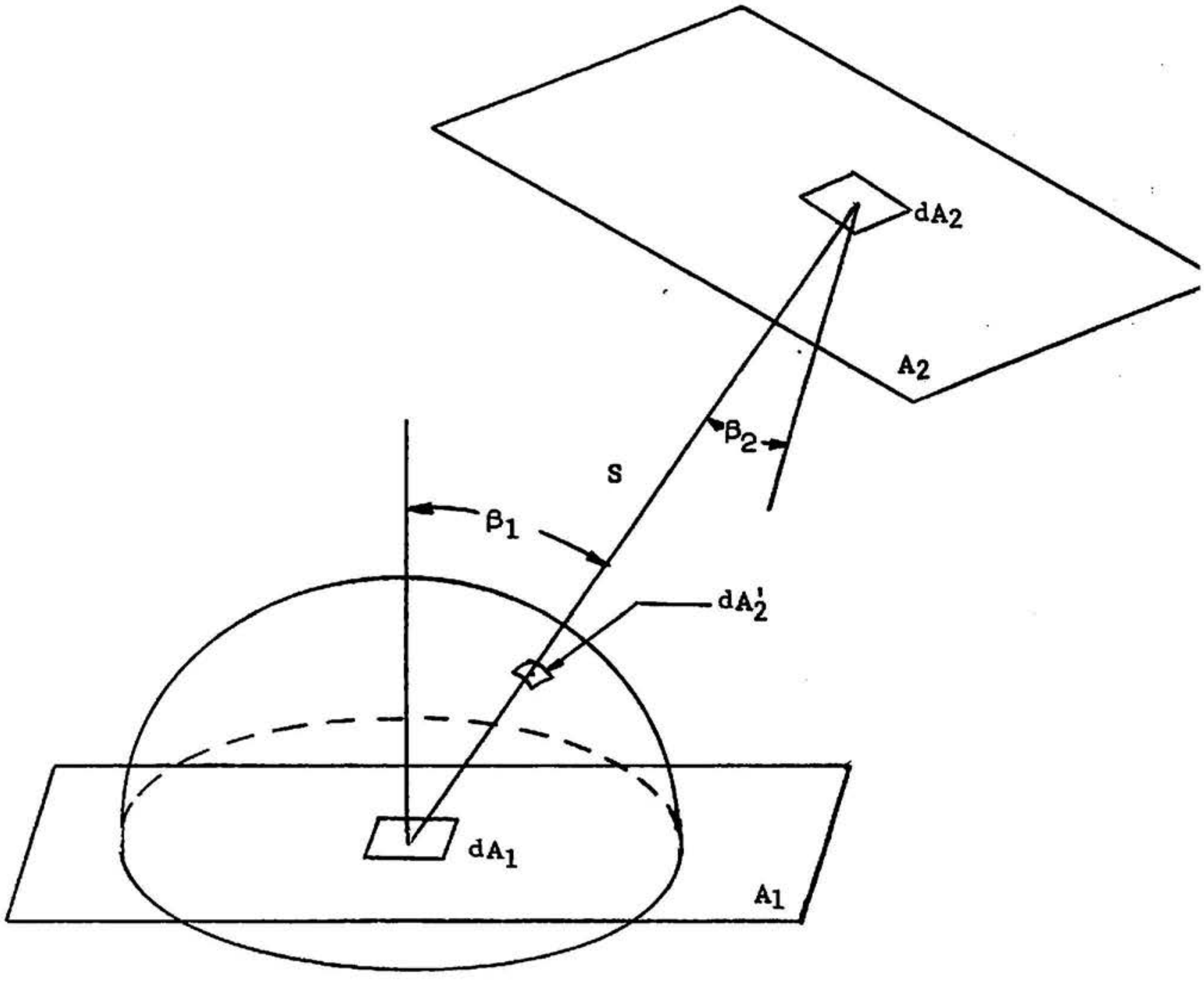

Fig. 1. Geometric Illustration of Configuration Factor

area dA_1. A point directly above dA_1 on the hemisphere will see dA_1 without distortion, but any other point on the hemisphere will see the projected area of dA_1, i.e., $dA_1 \cos \beta_1$ where β_1 is the angle between the normal to dA_1 and the line connecting the center of dA_1 with the point on the hemisphere. The radiant energy emitted from dA_1 per unit of time can be determined from the definition of radiation intensity. Radiation intensity, I, is defined as the radiant energy emitted by a surface per unit solid angle, per unit time, and per unit area of emitting surface perpendicular to the direction of the ray. The energy emitted from dA_1 reaching an area dA_2' on the hemisphere is then

$$dq_{1-H} = I \cos \beta_1 \, dA_1 \, d\omega_{1-H} \tag{1.1}$$

where dq_{1-H} = radiant energy emitted from dA_1 per unit time

I = intensity of radiation as defined above

β_1 = angle between ray and normal to dA_1

$d\omega_{1-H} = \frac{dA_2'}{r^2}$ solid angle subtended at dA_1 by dA_2' on the hemisphere

r = radius of hemisphere

If surface A_1 is assumed to be a diffuse emitter, where the intensity, I, is independent of the direction of the ray, the total energy emitted per unit time by A_1 will be

$$Q_{1-H} = I \int_H \cos \beta_1 \cdot d\omega_{1-H} \, dA_1 \tag{1.2}$$

where the integration is performed over the hemisphere. If dA_2 is take as a surface element on A_2, the subtended solid angle $d\omega_{1-2}$ is the projected area of dA_2 in the direction of the incident radiation divided by the distance between dA_1 and dA_2 squared or

$$dq_{1-2} = \frac{I \cos \beta_1 \cos \beta_2 \, dA_1 \, dA_2}{S^2} \tag{1.3}$$

where, β_2 = angle between normal to dA_2 and incident radiat

S = distance between dA_1 and dA_2.

Integrating equation (1.3) over both surfaces, the total energy p unit time leaving surface A_1 and reaching surface A_2 is

$$Q_{1-2} = I \int_{A_1} \int_{A_2} \frac{\cos \beta_1 \cos \beta_2 \, dA_1 \, dA_2}{S^2} \tag{1.4}$$

From the definition of the configuration factor and equations (1.2) a (1.4) there is obtained,

$$F_{1-2} = \frac{I \int_{A_1} \int_{A_2} \frac{\cos \beta_1 \cos \beta_2 \, dA_2}{S^2}}{I \int \cos \beta_1 \, dA_1 \, d\omega_{1-H}} \tag{1.5}$$

The denominator of equation (1.5) integrated over a hemisphere yields $\pi A_1 I$, so the mathematical expression for the configuration factor fo surfaces A_1 and A_2 becomes:

$$F_{1-2} = \frac{1}{\pi A_1} \int_{A_1} \int_{A_2} \frac{\cos \beta_1 \cos \beta_2 \, dA_1 \, dA_2}{S^2} \tag{1.6}$$

Equation (1.6) has been evaluated for a number of configurations, how ever, for many geometrical relationships and for curved surfaces, the analysis becomes quite complex and tedious. Integration of equation (1.6) can be accomplished by dividing the surfaces into small sub-are and numerically evaluating the double integral obtained. The method in this study to obtain the configuration factor was obtained from a approach outlined in reference (1). The method involves subdivision of one of the surfaces into small areas and subsequent computation of irradiation of the second surface by the subarea. This computation involves the evaluation of a contour integral, and the procedure is repeated for each small area, resulting in the average configuration factor from all of the small areas of the subdivided surface to the second surface.

By the use of the reciprocal relationship,

$$A_1 F_{1-2} = A_2 F_{2-1} \tag{1.7}$$

where,

A_1 = Area of surface 1

A_2 = Area of surface 2

F_{2-1} = Configuration factor from surface 2 to surface 1

the configuration factor for surface 2 to surface 1 can be obtained if the areas of the surfaces in question and the configuration factor for surface 1 to surface 2 are known.

The results of this study are in the form of four electronic computer programs in the Fortran language. The programs were written for the IBM 650 digital computer, but with small additions, they can be made compatible with IBM 704 Fortran. By simply specifying the geometric descriptions required to define the surfaces as the input of the programs a configuration factor will be obtained that is reasonably accurate for engineering applications. A provision has also been provided in the program to increase the accuracy of the result as required.

CHAPTER II

EXISTING METHODS FOR COMPUTATION OF RADIATION CONFIGURATION FACTORS

There are several methods by which configuration factors can be computed, however, they mainly are limited to specialized shapes and geometrical relationships, and may involve making some simplifying assumptions. Some of the methods and approaches are discussed here along with their limitations.

As previously mentioned, equation (1.6) can be evaluated numerically by subdividing the surfaces involved into small areas and evaluating the double integral obtained. This method involves considerable calculations, and it is impracticable for any but very simple surfaces. D. C. Hamilton and W. R. Morgan have developed the configuration factor equations for several geometrical relationships in reference (2). A series of curves and tables are presented as the results of numerical evaluation of the equations. The configuration factors are given as functions of dimensionless ratios of the describi geometrical parameters of the configurations. Results are tabulated for planes intersecting at various angles, and for configurations invo ing plane, line and point sources. Configuration factors are also giv for cylinders with point and line sources. The information presented c be very useful in certain cases, but due to the mathematical complexit involved in obtaining the configuration factors, the geometrical relat ionships presented are limited to specialized cases.

The tabulated results in reference (2) can be extended to a certain extent to cover more general geometrical configurations by the use of geometric flux algebra. With the aid of several basic rules of flux algebra, Hamilton and Morgan show how the configuration factors of a nonintersecting and nonparallel segments of planes can be expanded as functions of the configuration factors for intersecting planes. By dividing the planes into pairs of areas with known configuration factors the desired configuration factor can be found arithmetically from the known factors. This method is limited to isothermal surfaces, also the geometrical relationship must be reducible to known relationships. The procedure involves, in some cases, the squaring and adding of numbers differing by several orders in magnitude which in turn are obtained from a graph. The error in the final result may therefore be many times the error in reading the curves.

William H. McAdams, in reference (3), pp. 66-68, develops a method by which the configuration factor can be evaluated directly for some classes of irregular surfaces. Areas of infinite extent in one direction generated by a straight line moving always parallel to itself, will have identical cross sections on planes normal to the infinite dimension. On one of these cross sections he constructs lines representing tangents between pairs of points, reducing the surface into an equivalent simple enclosure. From a simple relationship between the lines drawn to reduce the complex surface to the equivalent simple surface he obtains the configuration factor.

If one of the surfaces is small in relation to the other and can considered a point source, the unit sphere method can be used. A hem: phere of unit radius is constructed about the point source, and the projection of the second surface is obtained on the surface of the hemisphere. This projection is then transferred to the base of the hemisphere. The configuration factor is then the projected area on tl base of the hemisphere, divided by the area of the base, or π. The u sphere method is useful for simple geometrical configurations, but in many cases the method does not lend itself readily to numerical calculations.

Other methods of obtaining the configuration factor exist, emplo ing photography, mechanical integrators, or some type of optical projection. These methods require specialized equipment or models, and can be time consuming.

CHAPTER III

CONTOUR INTEGRATION

As was pointed out in the preceding chapter, no readily availal method exists for obtaining the configuration factors for all types (surfaces and which does not involve extensive computation or error-inducing simplifying approximations. In many cases difficulty arise: from the evaluation of the double integral (1.6). It is possible through the use of vector calculus to replace the double integral of (1.6) with a single integral, saving a considerable amount of labor.

In reference (1), the author describes a method whereby the substitution is made possible. It has been developed for the calculation of illumination from light sources, but it can be readily adapted for radiation heat transfer calculations.[1] In order to demonstrate the vector relationships of thermal radiation, consider a point source, S, placed at the origin of a system of co-ordinates (se Fig. 2). If the intensity of the source in the direction of point P is I, then the irradiation of a surface dA_2 parallel to the x-y plane from a point source is

$$G' = I \cos \theta_z \, d\omega = I \cos \theta_z \, dA_2 / r^2 \tag{3.1}$$

where I = intensity of source along r

r = distance from source to point P

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[1] See Appendix B.

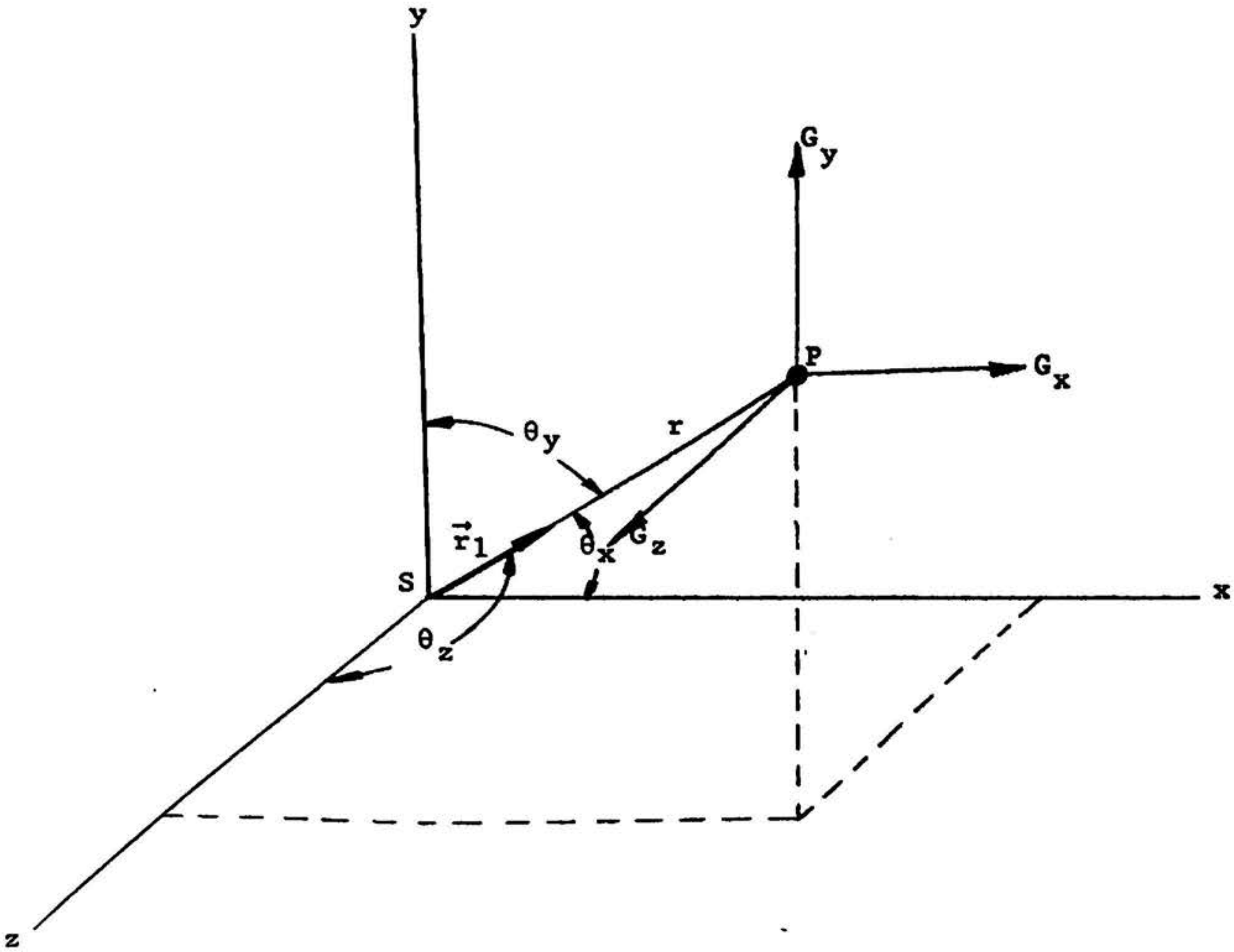

Fig. 2. Illustration of Vector Concepts for Radiation

θz = angle from normal to dA_2 to z axis

dA_2 = area of surface dA_2.

Since this is the irradiation of a surface whose normal is z direction, it will be denoted as Gz. Using the above reasoni values of the irradiation on surfaces at P whose normals are in y, and z direction are, respectively,

$$\vec{G}x' = \frac{IdA_2}{r^2} \cos \theta x$$

$$\vec{G}y' = \frac{IdA_2}{r^2} \cos \theta y$$

$$\vec{G}z' = \frac{IdA_2}{r^2} \cos \theta z$$

where θx, θy, θz are the angles between r and the three co-ordi axes. The coefficient of the cosine term, $\frac{IdA_2}{r^2}$, is the same in three equations, and is equal to the irradiation of a surface pe pendicular to r. It follows that $\vec{G}x$, $\vec{G}y$, and $\vec{G}z$ are components vector whose magnitude is $\frac{IdA_2}{r^2}$ and whose direction is from S to along r. In vector notation, the irradiation at P is

$$\vec{G}' = G'_x i + G'_y j + G'_z k$$

If $\vec{r}_1$ is a unit vector along r,

$$\vec{G}' = \vec{r}_1 \frac{IdA_2}{r^2}$$

If S is a surface rather than a point source, each element of th surface provides an irradiation vector $d\vec{G}$ at point P, and the tc irradiation vector at P due to the entire source is the vector sum of all the component vectors. For a single area element dA_1 of a surfac source A_1,

$$d\vec{G} = \frac{I \cos\varphi \, dA_2 \, dA_1}{r^2 \, dA_2} \vec{r}_1 \tag{3.7}$$

where $\vec{r}_1$ is a unit vector pointing from the particular element toward and $\varphi$is the angle between the normal to dA_1 and r. For the entire su face source,

$$\vec{G} = I\int_s \vec{r}_1 \frac{\cos\varphi \, dA_1}{r^2} \tag{3.8}$$

Equation (3.8) gives the value of the irradiation on a plane normal t r at point P. The irradiation on any other plane at point P is obtai by multiplying the absolute value of $\vec{G}$, or $|\vec{G}|$ by the cosine of the a between $\vec{G}$ and the normal to the plane.

Using the vector relationship in (3.8), it is now possible to ob an expression for the configuration factor for an arbitrary geometric relationship. The mathematical expression for the dot product betwee two vectors is,

$$\vec{r}_1 \cdot \vec{n}_1 = |\vec{r}_1| \; |\vec{n}_1| \cos\varphi \tag{3.9}$$

where $|\vec{r}_1|$ = absolute value of vector $\vec{r}_1$

$|\vec{n}_1|$ = absolute value of vector $\vec{n}_1$ normal to surface S

φ = angle between $\vec{r}_1$ and $\vec{n}_1$

If both $\vec{r}_1$ and $\vec{n}_1$ are unit vectors, (3.9) becomes

$$\vec{r}_1 \cdot \vec{n}_1 = \cos\varphi \tag{3.1}$$

Using the relationship (3.10) in equation (3.8) there is obtaine

$$\vec{G} = I \int_s \frac{\vec{r}_1}{r^2} \left(\vec{r}_1 \cdot \vec{n}_1\right) dA_1$$

What is now desired is the reduction of the surface integral (3. a more easily evaluated line integral around the boundary or con of the surface S. This can be done with the use of Stokes' theo Stokes' theorem relates the surface integral of the curl of a ve quantity to the line integral of the quantity. Stated mathemati

$$\int_s \vec{n}_1 \cdot \text{curl } \vec{A} \, d\tau = \int_c \vec{A} \cdot d\vec{s}$$

where $\vec{A}$ = vector point function

$\vec{n}_1$ = unit normal to surface S

$d\vec{s}$ = an element of the contour of surface S

$d\tau$ = an element of area of surface S

If both sides of equation (3.11) are multiplied by an arbitrary vector $\vec{N}$, it becomes

$$\vec{N} \cdot \vec{G} = I \int_s \frac{\vec{N} \cdot \vec{r}_1}{r^2} \left(\vec{r}_1 \cdot \vec{n}_1\right) dA_1$$

Since dot multiplication is associative, i.e., $(t\vec{A}) \cdot \vec{B} = t(\vec{A} \cdot \vec{B})$, equation (3.13) can be rearranged as

$$\vec{N} \cdot \vec{G} = I \int_s \vec{n}_1 \cdot \left[\frac{\vec{r}_1}{\vec{r}^2} \left(\vec{N} \cdot \vec{r}_1\right)\right] dA_1$$

It can be shown[1] that

$$\frac{\vec{r}_1}{r^2}\left(\vec{N}\cdot\vec{r}_1\right) = \frac{1}{2}\,\text{curl}\left(\frac{\vec{r}_1 \times \vec{N}}{r}\right) \qquad (3.1$$

Using this relation, equation (3.14) becomes

$$\vec{N}\cdot\vec{G} = I\int_s \vec{n}_1\cdot\left[\frac{1}{2}\,\text{curl}\left(\frac{\vec{r}_1 \times \vec{N}}{r}\right)\right] dA \qquad (3.1$$

Using Stokes theorem with $\left(\frac{\vec{r}_1 \times \vec{N}}{r}\right)$ substituted for $\vec{A}$ in equation (3

equation (3.16) becomes

$$\vec{N}\cdot\vec{G} = \frac{I}{2}\int_c \left(\frac{\vec{r}_1 \times \vec{N}}{r}\right)\cdot d\vec{s} \qquad (3.1$$

The scalar triple product in (3.17) can be commuted cyclically witho

altering the sign, and since $\vec{N}$ is constant with respect to the inte-

gration, (it is an arbitrary unit vector) equation (3.17) becomes

$$\vec{N}\cdot\vec{G} = \vec{N}\cdot\frac{I}{2}\int_c \left(\frac{d\vec{s} \times \vec{r}_1}{r}\right) \qquad (3.1$$

or, it follows,

$$\vec{G} = \frac{I}{2}\int_c \frac{d\vec{s} \times \vec{r}_1}{r} \qquad (3.1$$

The mathematical expression of the cross product in (3.19) is

$$\left|\frac{d\vec{s} \times \vec{r}_1}{r}\right| = \frac{1}{r}\sin\theta\; ds \qquad (3.2$$

where θ is the angle between vector $d\vec{s}$ and unit vector $\vec{r}_1$. Equation

(3.20) gives the absolute magnitude of the cross product of the vect

[1] See Appendix A.

$\vec{ds}$ and $\vec{r}_1$. Setting this magnitude equal to $|d\vec{\alpha}|$ and substituting (3.20),

$$\vec{G} = \frac{I}{2} \int_c d\vec{\alpha}$$

where $d\vec{\alpha}$ is a vector whose magnitude is equal to the angle inter by $\vec{ds}$ from point P and whose direction is perpendicular to the p and $\vec{ds}$ (see Fig. 3). The vector $d\vec{\alpha}$ points in the direction of of a right handed screw turned from $\vec{ds}$ to $\vec{r}_1$. Equation (3.21) g irradiation at a point P due to a surface source S in terms of a gral taken around the contour of the source. For surfaces with gonal boundaries, the $d\vec{\alpha}$ vectors along one side of the figure ar collinear and add directly.

Consider a polygon source ABCDE (Fig. 4). In order to co the irradiation at point P due to ABCDE using contour integrati proceed as follows. From equation (3.21), the contribution to diation at P due to side AB is

$$\vec{\Delta}_1 = \frac{I}{2} \int_{AB} d\vec{\alpha}$$

Since $d\vec{\alpha}$ is a vector perpendicular to the plane formed by $\vec{ds}$ an and all vectors $d\vec{\alpha}$ are collinear due to the straight line bound integration indicated in equation (3.22) becomes an ordinary sc Thus

$$\vec{\Delta}_1 = \vec{\alpha}_1 \frac{I}{2} \int_o^{\varphi} d\alpha$$

(3.23) becomes, after integration,

$$\vec{\Delta}_1 = \vec{\alpha}_1 \frac{I}{2} \varphi_1$$

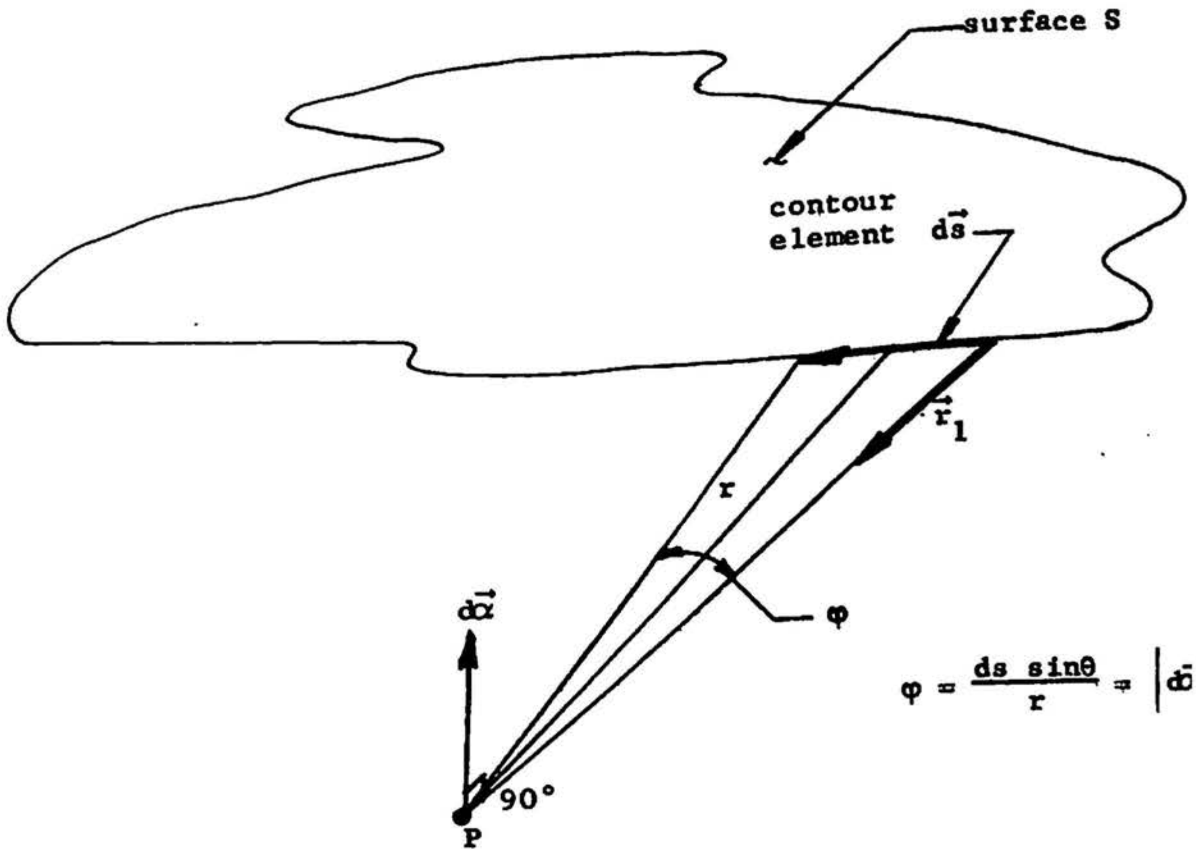

Fig. 3. Contour Integration for Surface S

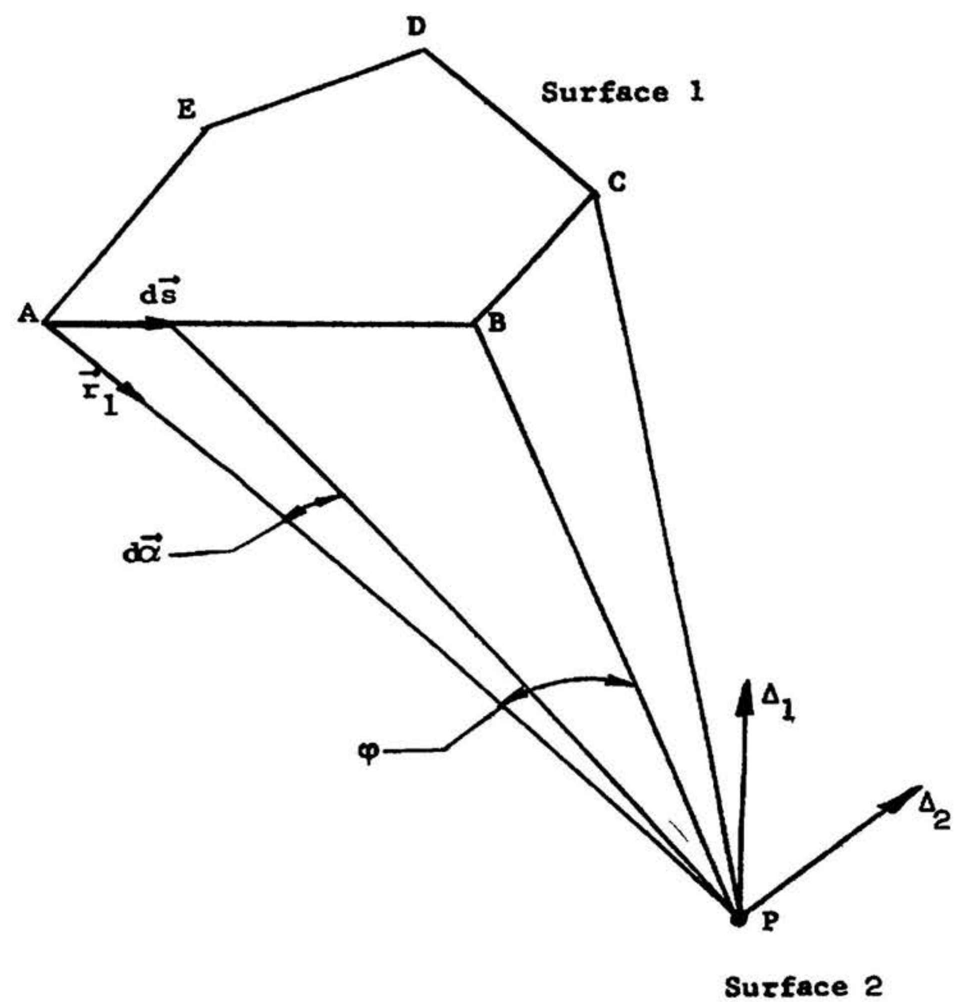

Fig. 4. Contour Integration for Polygon Source

where $\vec{\alpha}_1$ is a unit vector perpendicular to the plane formed by c
and φ_1 is the angle subtended at P by side A-B of the polygon.
the contribution to the irradiation at point P due to side BC is

$$\vec{\Delta}_2 = \vec{\alpha}_2 \frac{I}{2} \varphi_2$$

where $\vec{\alpha}_2$ is a unit vector perpendicular to plane PBC and φ_2 is
angle intercepted at P by side BC. For a polygon of n sides, tl
irradiation at P is

$$\vec{G} = \frac{I}{2} \sum_{i=1}^{i=n} \vec{\alpha}_i \varphi_i$$

where $\vec{\alpha}_i$ is a unit vector normal to the i th plane. The direct:
be always the direction dictated by the right hand rule for vec
products. The use of equation (3.26) eliminates the need to in
to obtain the irradiation at a point due to a source having a p
boundary.

Obtaining the configuration factor using equation (3.26) is
matter. Equation (3.26) gives the irradiation at point P due t
gonal source, and from the definition of the configuration fact
simply becomes necessary to divide the total irradiation at poi
the total flux emitted by the source. As in the denominator of
(1.5), the total energy emitted by the source per unit time is :
dividing this into the total flux reaching point P, the result

$$F_{1-2} = \frac{1}{2\pi} \sum_{i=1}^{i=n} \vec{\alpha}_i \varphi_i \frac{dA_2}{A_1}$$

This equation is the basis of the method used to calculate conf: factors for several configurations in this report. As will be (the next chapters, the method is readily adaptable for electron: computers and gives excellent results.

CHAPTER IV

ADAPTION OF CONTOUR INTEGRATION TO AN

ELECTRONIC DIGITAL COMPUTER

The method outlined in the previous chapter lends itself rea computer calculation. The four computer programs presented in th port all use as a basis of calculation the formula (3.27). The t the programs is fairly simple. Formula (3.26) expresses the irra at a point due to a polygonal source. Since the configuration fa between two surfaces is usually desired, rather than between a po a surface, it simply becomes necessary to obtain the irradiation sufficient number of points on one of the surfaces and add the re vectorially.

In each program, the surfaces are specified as to shape, si location by co-ordinates on a cartesian system. The program div of the surfaces into small subarea elements, and obtains the co-c nates of the center of each subarea. The center point of each su is considered as a point being irradiated by the second surface. tour integration theory is then applied to each center point in the results are added vectorially. The final result is then the irradiation by the second surface on the surface defined by the points of the subareas, or for a sufficient number of points the is the total irradiation by the second surface on the first surf

Once the co-ordinates of the center points are known, it is tively simple to apply the contour integration theory using the known procedures of analytic geometry. For example, consider a center point P on the subdivided surface. Assume also that the surface is a polygon ABCDE of five sides (see Fig. 4). To deter irradiation at P using contour integration, according to formula the contribution side AB of the second surface makes to the irra is simply the angle subtended by AB at P multiplied by the unit to the plane PBA. In order to find the unit normal to the plane by P, B and A, the equation of the plane passing through the co-nates of P, B, and A is calculated. This is easily accomplished the equation of a plane is of the form

$$A\,x + B\,y + C\,z + 1 = 0,$$

we can substitute the co-ordinates of each point for x, y, and z result is three equations with three unknowns A, B and C. The c cients of x, y and z in (4.1) are the direction cosines of a nor the plane from the origin. To obtain an expression for the unit to the plane PBA, it is necessary to use the relationship

$$\vec{n}_1 = \frac{1}{\sqrt{A^2 + B^2 + C^2}} \left(Ai + Bj + Ck\right)$$

where $\vec{n}_1$ is the unit normal to plane PBA.

To obtain the angle subtended by AB at P, the direction cos the lines PA and PB are determined from the relations

$$\cos\alpha = \frac{X2 - X1}{d}$$

$$\cos\beta = \frac{Y2 - Y1}{d}$$

$$\cos\gamma = \frac{Z2 - Z1}{d}$$

where $$d = \sqrt{\left(X2 - X1\right)^2 + \left(Y2 - Y1\right)^2 + \left(Z2 - Z1\right)^2}$$

X1, Y1, Z1 = co-ordinates of end of line at P

X2, Y2, Z2 = co-ordinates of end of line at plane.

The cosine of the angle between the two lines can be determ the relationship

$$\cos \varphi = \cos \alpha_A \cos \alpha_B + \cos \beta_A \cos \beta_B + \cos \gamma_A \cos \gamma_B$$

The angle φ is then obtained from the arc cosine relationshi tiplying the vector $\vec{n}_1$ by the scaler φ, the contribution to the tion at P due to side AB is obtained. The process is repeated BC, CD, DE, and EA of the polygon. The equations of the planes PDE, and PEA are obtained, the unit normals to each plane are ca and multiplied by the angles subtended by each of the sides of tl and the results - the x, y and z components of the irradiation v added to obtain the total irradiation at point P. The whole pro is then repeated for another point on the surface. The subseque are those defined by the co-ordinates of the center points of th on the surface. The result obtained for each point P is added v Since each point P represents a small area rather than a point, irradiation of the second surface by the first surface is obtain multiplying the vector result obtained above by the cosine of th between the irradiation vector and the normal to the second surf

The procedure is fairly straightforward in cases where ev on one of the surfaces can "see" every point on the second sur complication arises when one or both of the surfaces are curve portions of the surface exist that can see only a part or none other surface. In other words, each point on a curved surface horizon, and the second surface can either be totally above or the horizon, or only a portion of the second surface may be ab horizon. If the second surface lies totally below the horizon no radiant energy can reach the point from the surface, and co no radiant energy leaving the point can reach the surface. In of a surface lying partly below the horizon of a point, only t from the portion of the surface above the horizon is considere

In view of this, the programs dealing with curved surface horizon decision made for each subarea center point. This dec termines the position of the second surface relative to the ho the point in question. In cases where the second surface lies above the horizon, the co-ordinates of the points of intersect horizon and the surface must be calculated. Considering the s surface to be the polygon ABCD, the point P will be irradiate portion of ABCD that appears above the horizon. This new poly defined by the co-ordinates of the intersection points of the and ABCD. Even though the original polygon was four-sided, th polygon can be either three-, four-, or five-sided, depending section points.

A thorough discussion of each of the programs follow in the ing chapters.

CHAPTER V

PLANE-TO-PLANE PROGRAM

In order to utilize the digital computer to calculate a cc ation factor using the contour integration theory, it is necess present the calculations in a logical sequence and in a format ables the computer to perform the necessary manipulations. The whereby this was accomplished utilized the Fortran system. Thi allowed the program to be written in symbols and algebraic equa closely resembling conventional mathematical formulas. The con translated this program into a form usable for actual calculati the Fortran program as written will be discussed since the logi followed readily.

The plane-to-plane program was too large for the storage c of the IBM 650 and it appears here in two parts. This does not way affect the logic of the program. The splitting of the prog accomplished by using the same dimension statement for both par program. The computer then reserves the same storage area for scripted variables appearing in the dimension statement in both As a result of this, the information calculated in the first pa program is saved within the computer in the proper place, and i necessary to read in any new information or data before startin second part of the program. Care had to be taken that the firs subscripted variable appearing in the dimension statement was the variables used in the second part of the program. The fir variable also had to be larger than fourteen. This had to be the computer will use the first fourteen spaces of the dimens for temporary storage while reading the program on the drum. stroys any information which may have been stored in those lo

The data input to the program consists of the x, y and z of the corner points of the planes. The only limitation for of the two planes is that it is necessary for one of the plan tally above the horizon of the other plane. The necessary pr required is, in order, the problem number, N, and the co-ordi plane A followed by co-ordinates of plane B. The co-ordinate cyclic or ordered around the plane (see Fig. 5).

Plane B should be the smaller of the two planes, as it wi vided into a number of subareas. The smaller the plane is, th accurately it can be represented by a given number of points. of N will determine the number of subdivisions for plane B. I be divided into N squared subareas.

The flow chart for the plane-to-plane program appears in The actual Fortran program statements appear in Table 1. The lated result is in the form of the product of the area of plan the configuration factor between plane A and plane B. The des figuration factor can be found easily by the use of the recipr lationship in equation (1.7).

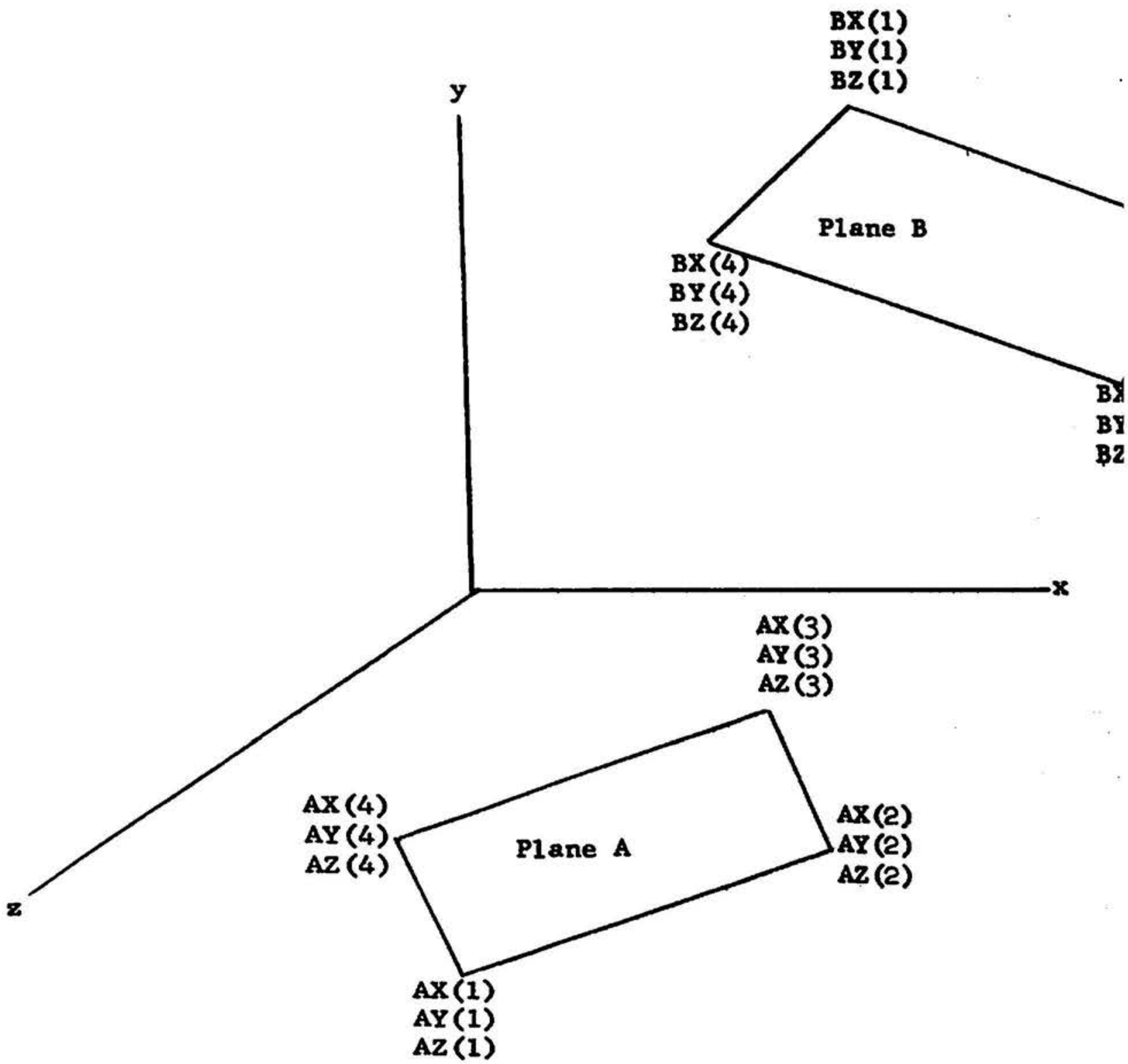

Fig. 5. Plane-to-Plane Configuration

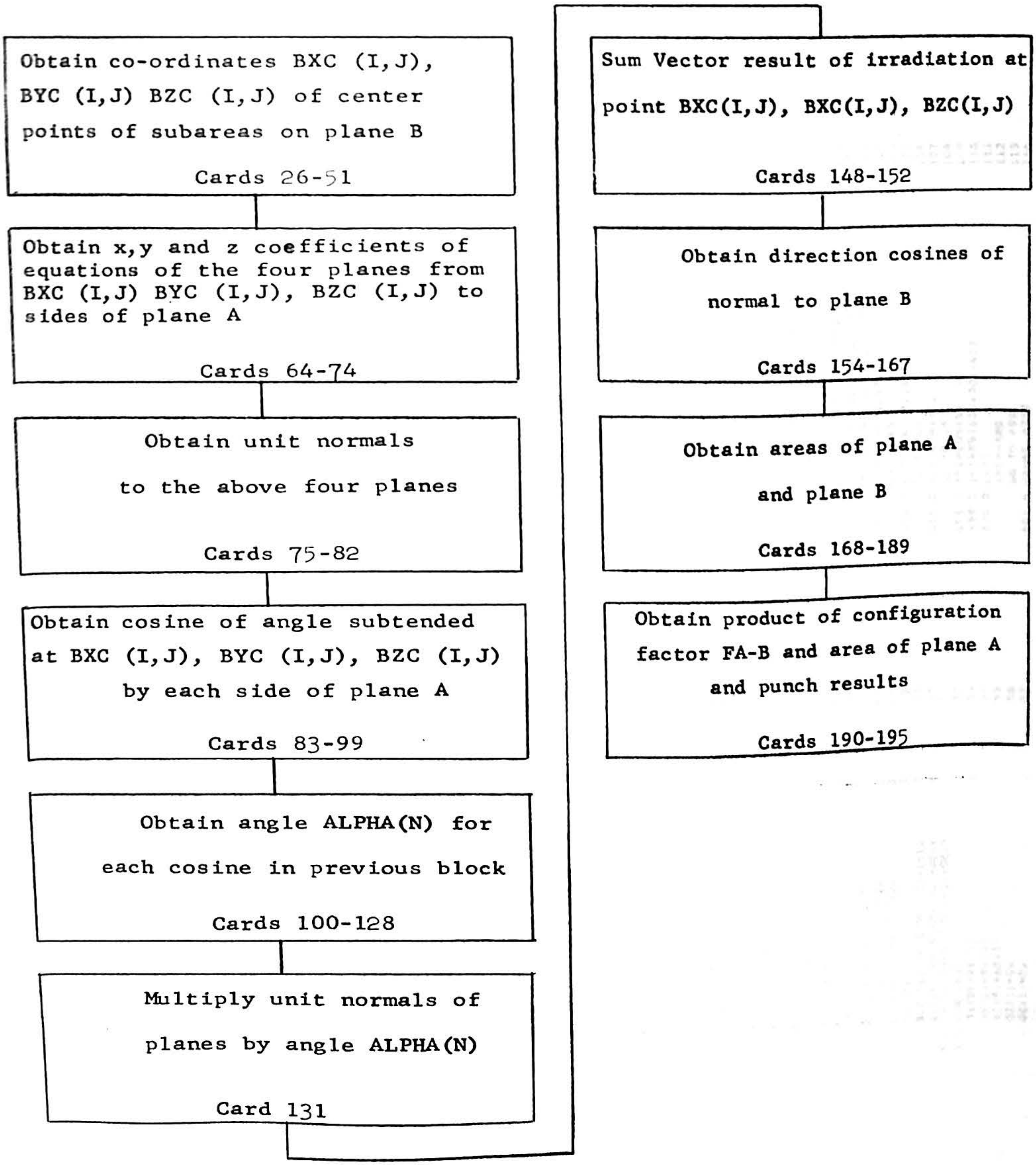

Fig. 6. Block Diagram of Plane-to-Plane Program

TABLE I

PLANE-TO-PLANE PROGRAM

```
C 0000 0 RADIATION CONFIGURATION FACTO
C 0000 0 RS FOR FLAT RECTANGULAR PLANES
         DIMENSIONR(18),
       1                    AX(5),AY(5),
       2 AZ(5),BX(4),BY(4),BZ(4),
       3 CSTH(3,5),COS(4),ALPHA(4)
         DIMENSIONBXC(10,10),BYC(10,10)
       1 ,BZC(10,10),G1(1),G2(1),G3(1),
       2 E(4,3),PRONO(1),N1(1)
         READ,PRONO(1),N1(1),AX(1),
       1 AY(1),AZ(1),AX(2),AY(2)
         READ,AZ(2),AX(3),AY(3),AZ(3),
       1 AX(4),AY(4),AZ(4)
         READ,BX(1),BY(1),BZ(1),BX(2),
       1 BY(2),BZ(2),BX(3)
         READ,BY(3),BZ(3),BX(4),BY(4),
       1 BZ(4)
C 0000 0 INCREM. MID POINT COORDS
         S=N1(1)
         N1=N1(1)
         A0=1.5707879
         A1=-.21412453
         A2=.08466649
         A3=-.03575663
         A4=.00864884
         DELXR=(BX(2)-BX(1))/S
         XR=BX(1)-DELXR/2.
         DELXL=(BX(3)-BX(4))/S
         XL=BX(4)-DELXL/2.
         DELYR=(BY(2)-BY(1))/S
         YR=BY(1)-DELYR/2.
         DELYL=(BY(3)-BY(4))/S
         YL=BY(4)-DELYL/2.
         DELZR=(BZ(2)-BZ(1))/S
         ZR=BZ(1)-DELZR/2.
         DELZL=(BZ(3)-BZ(4))/S
         ZL=BZ(4)-DELZL/2.
         DELX=(XL-XR)/S
         DELY=(YL-YR)/S
         DELZ=(ZL-ZR)/S
         DO 6 M=1,N1
         XR=XR+DELXR
         YR=YR+DELYR
         ZR=ZR+DELZR
         BXC(M,1)=XR+DELX/2.
         BYC(M,1)=YR+DELY/2.
         BZC(M,1)=ZR+DELZ/2.
         DO5J=2,N1
         BZC(M,J)=BZC(M,J-1)+DELZ
         BXC(M,J)=BXC(M,J-1)+DELX
     5 0 BYC(M,J)=BYC(M,J-1)+DELY
```

```
         G3(1)=0.
         DO13I=1,N1
         DO13J=1,N1
         BXC=BXC(I,J)
         BYC=BYC(I,J)
         BZC=BZC(I,J)
         DO7M=1,4
         E(M,1)=BYC      *(AZ(M)-AZ(M+1)
       1 )+AY(M)*(AZ(M+1)-BZC      )+AY(
       2 M+1)*(BZC      -AZ(M))
         E(M,2)=BZC      *(AX(M)-AX(M+1)
       1 )+AZ(M)*(AX(M+1)-BXC      )+AZ(
       2 M+1)*(BXC      -AX(M))
         E(M,3)=BXC      *(AY(M)-AY(M+1)
       1 )+AX(M)*(AY(M+1)-BYC      )+AX(
       2 M+1)*(BYC      -AY(M))
     7 C CONTINUE
         DO8M=1,4
         ESQD=E(M,1)*E(M,1)+E(M,2)*E(M,
       1 2)+E(M,3)*E(M,3)
         R(M)=SQRTF(ESQD)
     8 0 CONTINUE
         DO9M=1,4
         DO9N=1,3
     9 0 E(M,N)=E(M,N)/R(M)
         DO10N=1,4
         DELX=BXC      -AX(N)
         DELY=BYC      -AY(N)
         DELZ=BZC      -AZ(N)
         R1SQD=DELX*DELX+DELY*DELY+DELZ
       1 *DELZ
         R1=SQRTF(R1SQD)
         CSTH(1,N)=DELX/R1
         CSTH(2,N)=DELY/R1
    10 0 CSTH(3,N)=DELZ/R1
         CSTH(1,5)=CSTH(1,1)
         CSTH(2,5)=CSTH(2,1)
         CSTH(3,5)=CSTH(3,1)
         DO 650 N=1,4
         COS(N)=CSTH(1,N)*CSTH(1,N+1)+
       1 CSTH(2,N)*CSTH(2,N+1)+CSTH(3,N
       2 )*CSTH(3,N+1)
         IF(COS(N))651,652,653
   651 0 IF(COS(N)+1.)658,654,659
   653 0 IF(COS(N)-1.)659,655,657
   652 0 ALPHA(N)=1.57079
         GO TO 650
   654 0 ALPHA(N)=3.14159
         GO TO 650
   655 0 ALPHA(N)=0.
         GO TO 650
```

TABLE I (Continued)

```
802 0 LL=0
803 0 PHI=ARG*(ARG*(ARG*(A4*ARG+
803 1  A3)+A2)+A1)+A0
      PHI2=SQRTF(1.-ARG)
      ANGLE=PHI*PHI2
      IF(LL-1)810,815,820
815 0 ANGLE=3.14159-ANGLE
810 0 ALPHA(N)=ANGLE
      GO TO 650
820 0 PAUSE 9877
658 0 IF(COS(N)+1.0001)656,654,654
657 0 IF(COS(N)-1.0001)655,655,656
656 0  PAUSE 1111
650 0 CONTINUE
      DO12M=1,4
      DO12N=1,3
 12 0 E(M,N)=E(M,N)*ALPHA(M)
      G1(1)=      G1(1)   +E(1,1)+E
    1 (2,1)+E(3,1)+E(4,1)
      G2(1)=G2(1)          +E(1,2)+E
    1 (2,2)+E(3,2)+E(4,2)
      G3(1)=G3(1)          +E(1,3)+E
    1 (2,3)+E(3,3)+E(4,3)
 13 0 CONTINUE
      END
C 0000 0 START SECONDPROGRAM
      DIMENSIONR(18),
    1                  AX(5),AY(5),
    2 AZ(5),BX(4),BY(4),BZ(4),
    3 CSTH(3,5),COS(4),ALPHA(4)
      DIMENSIONBXC(10,10),BYC(10,10)
    1 ,BZC(10,10),G1(1),G2(1),G3(1),
    2 E(4,3),PRONO(1),N1(1)
      S=N1(1)
      S1=SQRTF(G1(1)*G1(1)+G2(1)*
    1 G2(1)+G3(1)*G3(1))
      CSBTX=G1(1)/S1
      CSBTY=G2(1)/S1
      CSBTZ=G3(1)/S1
C 0000 0 AREA OF PLANES CALCULATION
      D1=BY(1)*(BZ(2)-BZ(3))+BY(2)*(
    1 BZ(3)-BZ(1))+BY(3)*(BZ(1)-BZ(2
    2 ))
      D2=BZ(1)*(BX(2)-BX(3))+BZ(2)*(
    1 BX(3)-BX(1))+BZ(3)*(BX(1)-BX(2
    2 ))
      D3=BX(1)*(BY(2)-BY(3))+BX(2)*(
    1 BY(3)-BY(1))+BX(3)*(BY(1)-BY(2
    2 ))
      RSQD=D1*D1+D2*D2+D3*D3
      R=SQRTF(RSQD)
      CSALX=D1/R
      SIDE1=SQRTF(DELX*DELX+DELY*DEL
    1 Y+DELZ*DELZ)
      DELX=BX(3)-BX(2)
      DELY=BY(3)-BY(2)
      DELZ=BZ(3)-BZ(2)
      SIDE2=SQRTF(DELX*DELX+DELY*DEL
    1 Y+DELZ*DELZ)
      AREAB=SIDE1*SIDE2
      DELX=AX(2)-AX(1)
      DELY=AY(2)-AY(1)
      DELZ=AZ(2)-AZ(1)
      SIDE1=SQRTF(DELX*DELX+DELY*DEL
    1 Y+DELZ*DELZ)
      DELX=AX(3)-AX(2)
      DELY=AY(3)-AY(2)
      DELZ=AZ(3)-AZ(2)
      SIDE2=SQRTF(DELX*DELX+DELY*DEL
    1 Y+DELZ*DELZ)
      AREAA=SIDE1*SIDE2
      COS1=CSALX*CSBTX+CSALY*CSBTY+C
    1 SALZ*CSBTZ
      GAB=(S1*COS1)/(6.28318)
      AAGAB=GAB*AREAB/( S*S)
      PUNCH,PRONO(1),AREAA,AREAB,
    1 AAGAB
      END
```

CHAPTER VI

CYLINDER-TO-PLANE PROGRAM

The basic method for calculating the configuration factor for a cylinder to a plane differs little from the method used in the plane-to-plane program. As was pointed out in the derivation of equation (3.27), the summation replaces the integral sign only when the source is a polygon. Because of this, the cylinder is the surface to be subdivided and represented by the center points of the subareas. One of the differences between the cylinder-to-plane program and the plane-to-plane program is of course the method by which the subareas are obtained along with their respective center point co-ordinates.

The biggest difference, however, is represented by the addition of a horizon decision loop. As was mentioned previously, there may be points on a curved surface which cannot "see" any or all of the second surface. Before equation (3.27) can be evaluated for any particular subarea center point, a horizon decision is made for the point. If the plane lies totally above the horizon for the point, the contour integration is performed as in the preceding program. If the plane lies totally below the horizon for the point, the program will then skip the contour integration for that point, and progress to the next point on the cylinder and repeat the horizon decision until a point is found from which the plane can be seen. If only a portion of the plane can be seen, it will be in the form of a three-, four-, or five-sided polygon. The corner points of the polygon will be represented by the co-ordinates of the original corners appearing above the horizon and the co-ordinates of the intersection points between the horizon and the plane. The latter co-ordinates are calculated within the horizon decision loop. The contour integration is then performed for the new polygon representing the portion of the plane appearing above the horizon.

The horizon decision is performed by rotating the y axis about the z axis to the point being considered. The co-ordinates of the corner points of the plane are then calculated for the new rotated axis. Since the point in question now lies on the y axis, it simply becomes necessary to compare the radius of the cylinder with the rotated co-ordinates of the plane. Any corner point of the plane whose ordinate has a value less than the radius of the cylinder lies below the horizon of the point being considered on the cylinder. When the corner point lies below the horizon, the co-ordinates of the intersection point between the horizon and the plane is then calculated. The co-ordinates so calculated are then transformed back to the original axis location before the rotation was made. The contour integration is then performed with relation to the original axes.

As in the plane-to-plane program, the irradiation vector must be multiplied by the normal to the subarea. The normal to the subarea is calculated from the gradient of the cylinder evaluated at the center of the subarea in question.

The cylinder to plane program will calculate configuration factors for full cylinders, or for any segment of a cylindrical surface, to a plane located anywhere outside the cylindrical surface. The necessary data input are the co-ordinates and angles necessary to describe the geometrical relationship of the cylinder and plane. The axis of the cylinder is located along the z axis (see Fig. 7). The angular size of the cylinder is defined by angles Thet 1 and Thet 2 in degrees. Taking the y axis as zero degrees and proceeding clockwise, the angle (Thet 2 - Thet 1) must define the angular segment of the cylinder. The length of the cylinder is designated by the co-ordinates ZL and ZR of its end points. The radius of the cylinder is designated by R. The plane is defined by the x, y, and z co-ordinates of the corner points, designated as AX(1), AY(1), AZ(1), AX(2), AY(2), AZ(2), etc. The subscripts of the corner points must be cyclic or ordered around the plane.

A block diagram of the cylinder to plane program along with the Fortran program appears on the following pages.

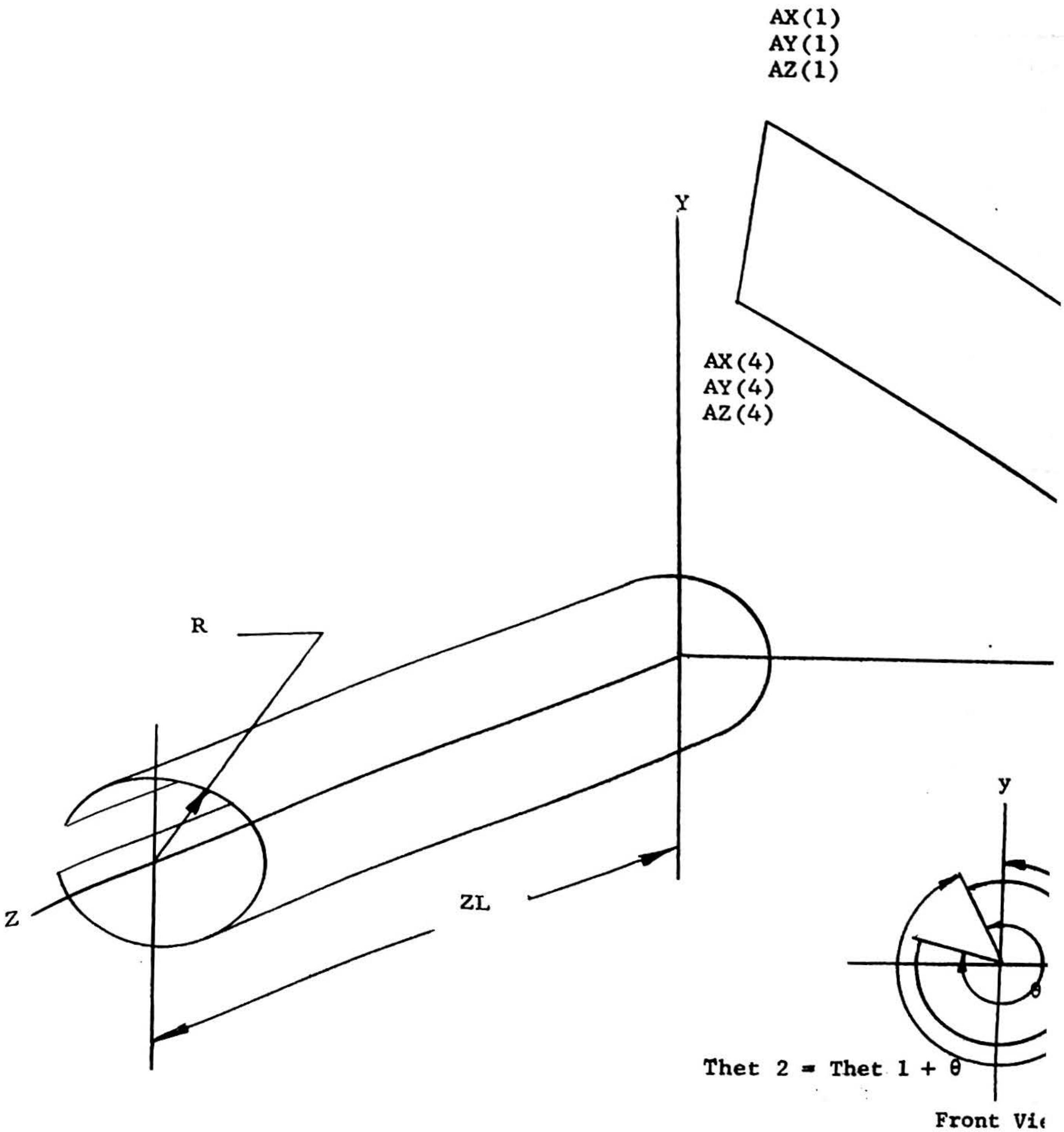

Fig. 7. Cylinder-to-Plane Configuration

Fig. 8. Block Diagram of Cylinder-to-Plane Program

TABLE II

CYLINDER-TO-PLANE PROGRAM

```
C 0000 0 CONFIG FAC CYLINDER TO PLANE
         DIMENSIONAR(5 ),AX(5),AY(5),AZ
       1 (5),AXR(5),AYR(5),CSTHX(6),
       2 CSTHY(6),CSTHZ(6),CX(6),CY(6)
         DIMENSION CZ(6),E(5,3),ALPHA(5
       1 ),CSS(5)
     1 0 READ,PRONO,J2,     AX(1),
     1 1 AY(1),AZ(1),AX(2)
         READ,AY(2),AZ(2),AX(3),AY(3),
       1 AZ(3),AX(4),AY(4)
         READ,AZ(4),THET1,THET2,
       1 ZR,ZL,R
         AN=J2
         A0=1.5707879
         A1=-.21412453
         A2=.08466649
         A3=-.03575663
         A4=.00864884
C 0000 0 OBTAIN COORD MD PTS
         DELTH=((THET2-THET1)/AN)
       1 *0.0174533
         THET3=THET1*0.0174533-
       1 DELTH/2.0
C 0000 0 DETERMIN OF HORIZ INTERSECT
         G1=0.
         G2=0.
         G3=0.
         AX(5)=AX(1)
         AY(5)=AY(1)
         AZ(5)=AZ(1)
         DELZ1=(ZL-ZR)/AN
         BZ=ZR-DELZ1/2.
         CON1=BZ
   500 0 DO 672 M=1,J2
         THET3=THET3+DELTH
         NA=1
         ARGM=THET3
   220 0 CON4=ARGM
         ARGM=ARGM-1.570788
         IF(ARGM)202,202,204
   204 0 NA=NA+1
         GO TO 220
   202 0 ARGM=CON4
         VAR1=SINF(ARGM)
         VAR2=COSF(ARGM)
         GO TO (229,230,231,232,229,
       1 230,231,232,229,230),NA
   229 0 SNALR=VAR1
         CSALR=VAR2
         GO TO 250
   230 0 SNALR=VAR2
         CSALR=-VAR1
         GO TO 250
   231 0 SNALR=-VAR1
         CSALR=-VAR2
         GO TO 250
   232 0 SNALR=-VAR2
         CSALR=VAR1
   250 0 DO 510 N=1,4
         AXR(N)=AX(N)*CSALR-AY(N)*SNALR
   510 0 AYR(N)=AY(N)*CSALR+AX(N)*SNALR
         AXR(5)=AXR(1)
         AYR(5)=AYR(1)
         BX=R*SNALR
         BY=R*CSALR
         BZ=CON1
   600 0 DO 671 M1=1,J2
         BZ=BZ+DELZ1
         TMP1=0.
         TMP2=0.
         TMP3=0.
         K=0
         DO 350 N=1,4
         IF(AYR(N)-R)310,320,330
   310 0 IF(AYR(N+1)-R)350,350,311
   311 0 K=K+1
         LA=0
   312 0 XR=(R-AYR(N+1))*(AXR(N)-AXR
   312 1 (N+1))/(AYR(N)-AYR(N+1))
   312 2 +AXR(N+1)
         K1=K+LA
         CX(K1)=XR*CSALR+R*SNALR
         CY(K1)=R*CSALR-XR*SNALR
         CZ(K1)=(R-AYR(N+1))*(AZ(N)-AZ
       1 (N+1))/(AYR(N)-AYR(N+1))
       2 +AZ(N+1)
         IF(LA-1)350,334,313
   313 0 PAUSE7777
   320 0 IF(AYR(N+1)-R)321,322,323
   321 0 K=K+1
         GO TO 350
   322 0 K=K+1
         CX(K)=AX(N)
         CY(K)=AY(N)
         CZ(K)=AZ(N)
         GO TO 350
   323 0 K=K+1
         CX(K)=AX(N)
         CY(K)=AY(N)
         CZ(K)=AZ(N)
         CX(K+1)=AX(N+1)
         CY(K+1)=AY(N+1)
         CZ(K+1)=AZ(N+1)
         GO TO 350
   330 0 K=K+1
         IF(AYR(N+1)-R)331,332,332
   331 0 CX(K)=AX(N)
         CY(K)=AY(N)
         CZ(K)=AZ(N)
         LA=1
         GO TO 312
   334 0 K=K+1
         GO TO 350
   332 0 CX(K)=AX(N)
```

TABLE II (Continued)

```
          CY(K)=AY(N)
          CZ(K)=AZ(N)
          CX(K+1)=AX(N+1)
          CY(K+1)=AY(N+1)
          CZ(K+1)=AZ(N+1)
   350 0  CONTINUE
          IF(K-2)671,671,620
   620 0  CX(K+1)=CX(1)
          CY(K+1)=CY(1)
          CZ(K+1)=CZ(1)
          DO 640 I=1,K
          E(I,1)=BY*(CZ(I)-CZ(I+1) )+CY
       1  (I)*(CZ(I+1)-BZ)+CY(I+1)
       2  *(BZ-CZ(I))
          E(I,2)=BZ    *(CX(I)-CX(I+1))
       1  +CZ(I)*(CX(I+1)-BX)+CZ(I+1)*
       2  (BX-CX(I))
          E(I,3)=BX*(CY(I)-CY(I+1))+CX(I
       1  )*(CY(I+1)-BY)+CX(I+1)*( BY-
       2  CY(I))
          AR(I)=SQRTF(E(I,1)*E(I,1)+E(I
       1  ,2)*E(I,2)+E(I,3)*E(I,3))
          DO 630 J=1,3
   630 0  E(I,J)=E(I,J)/AR(I)
          DELX=BX-CX(I)
          DELY=BY-CY(I)
          DELZ=BZ-CZ(I)
          R1=SQRTF(DELX*DELX+DELY*DELY+
       1  DELZ*DELZ)
          CSTHX(I)=DELX/R1
          CSTHY(I)=DELY/R1
   640 0  CSTHZ(I)=DELZ/R1
          CSTHX(K+1)=CSTHX(1)
          CSTHY(K+1)=CSTHY(1)
          CSTHZ(K+1)=CSTHZ(1)
          DO 6501=1,K
          CSS(I)=CSTHX(I)*CSTHX(I+1)+
       1  CSTHY(I)*CSTHY(I+1)+CSTHZ(I)*
       2  CSTHZ(I+1)
          IF(CSS(I))651,652,653
   651 0  IF(CSS(I)+1.)658,654,659
   653 0  IF(CSS(I)-1.)659,655,657
   652 0  ALPHA(I)=1.57079
          GO TO 650
   654 0  ALPHA(I)=3.14159
          GO TO 650
   655 0  ALPHA(I)=0.
          GO TO 650
   659 0  ARG=CSS(I)
   799 0  IF(ARG)800,801,802
   800 0  LL=1
          ARG=-ARG
          GO TO 803
   801 0  PAUSE 9777
   802 0  LL=0
   803 0  PHI=ARG*(ARG*(ARG*(A4*ARG+
   803 1   A3)+A2)+A1)+A0
          PHI2=SQRTF(1.-ARG)
          ANGLE=PHI*PHI2
          IF(LL-1)810,815,820
   815 0  ANGLE=3.14159-ANGLE
   810 0  ALPHA(I)=ANGLE
          GO TO 650
   820 0  PAUSE 9877
   658 0  IF(CSS(I)+1.0001)656,654,654
   657 0  IF(CSS(I)-1.0001)655,655,656
   656 0   PAUSE 1111
   650 0  CONTINUE
          DO 660 I=1,K
          DO 660 J=1,3
   660 0  E(I,J)=E(I,J)*ALPHA(I)
          DO 665 I=1,K
          TMP1=E(I,1)+TMP1
          TMP2=E(I,2)+TMP2
   665 0  TMP3=E(I,3)+TMP3
          TEMP=SQRTF((TMP1*TMP1+TMP2*TMP
       1  2+TMP3*TMP3)*(BX*BX+BY*BY))
          CSGMA=(TMP1*BX+TMP2*BY )/TEMP
          G1=TMP1*CSGMA+G1
          G2=TMP2*CSGMA+G2
          G3=TMP3*CSGMA+G3
   671 0  CONTINUE
   672 0  CONTINUE
          S1=SQRTF(G1*G1+G2*G2+G3*G3)
C 0000 0  AREA OF PLANES CALCULATION
          DELX=AX(2)-AX(1)
          DELY=AY(2)-AY(1)
          DELZ=AZ(2)-AZ(1)
          SIDE1=SQRTF(DELX*DELX+DELY*DEL
       1  Y+DELZ*DELZ)
          DELX=AX(3)-AX(2)
          DELY=AY(3)-AY(2)
          DELZ=AZ(3)-AZ(2)
          SIDE2=SQRTF(DELX*DELX+DELY*DEL
       1  Y+DELZ*DELZ)
          AREAA=SIDE1*SIDE2
C 0000 0  AREA OF CYLINDER
          DELZ=ZL-ZR
          AREAB=6.28318*R*DELZ*
       1  (THET2-THET1)/360.
          AAGAB=S1*AREAB/(AN*AN*6.283185
       1  )
          PUNCH,PRONO   ,AREAA,AREAB,
       1  AAGAB
          GO TO 1
          END
```

CHAPTER VII

SPHERE-TO-PLANE PROGRAM

As in the previous two programs, the basic method used to calculate the configuration factor is the same. This program will calculate the configuration factor from a sphere to a plane or from any portion of a spherical surface that can be defined by the method used in the program. The center of the sphere is located at the origin. Taking the y axis as positive and proceeding clockwise, a spherical segment is designated using the angles Alph 1 and Alph 2. The angle (Alph 2 - Alph 1) must define the angular segment of the sphere (see Fig. 9). The portion of the spherical segment is defined by the coordinates of the edges.

The plane is defined by the x, y, and z co-ordinates of the corner points exactly the same way as in the cylinder-to-plane program. As in the previous programs, the accuracy of the results can be varied by specifying a greater value for J2.

The basic difference between the cylinder-to-plane program and the sphere-to-plane program is in the axis rotation procedure. All points on the surface of a cylinder along a line parallel to the axis of the cylinder have the same horizon plane. Because of this, a horizon decision need only be made for one point on the line. In the case of a

sphere, each and every point on the surface of the sphere h
plane. In addition, if the axis is to be rotated to a point
face, it must be rotated through two angles in order to use
transformation relationships to calculate the new co-ordina
corner points of the plane. After the z axis is rotated th
two angles to the point in question, the z co-ordinates of
points of the plane are compared with the radius of the sph
previous program to determine which corner points are above
Plane and horizon intersections are calculated when they ex
co-ordinates of the intersections are then transformed back
inal axes. The contour integration is then performed with
original axes.

A block diagram of the sphere-to-plane program, along
Fortran program, follows.

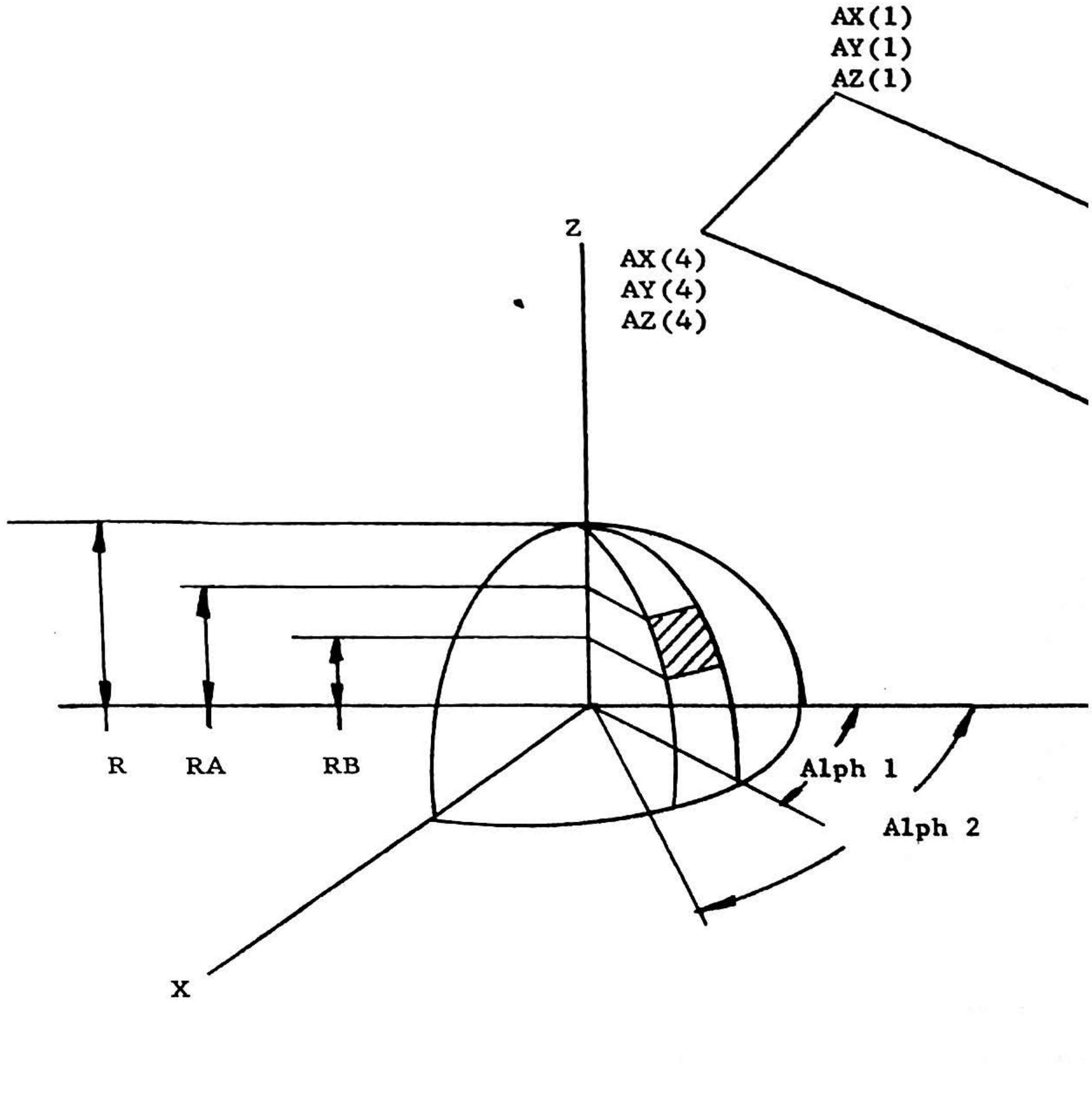

Fig. 9. Sphere-to-Plane Configuration

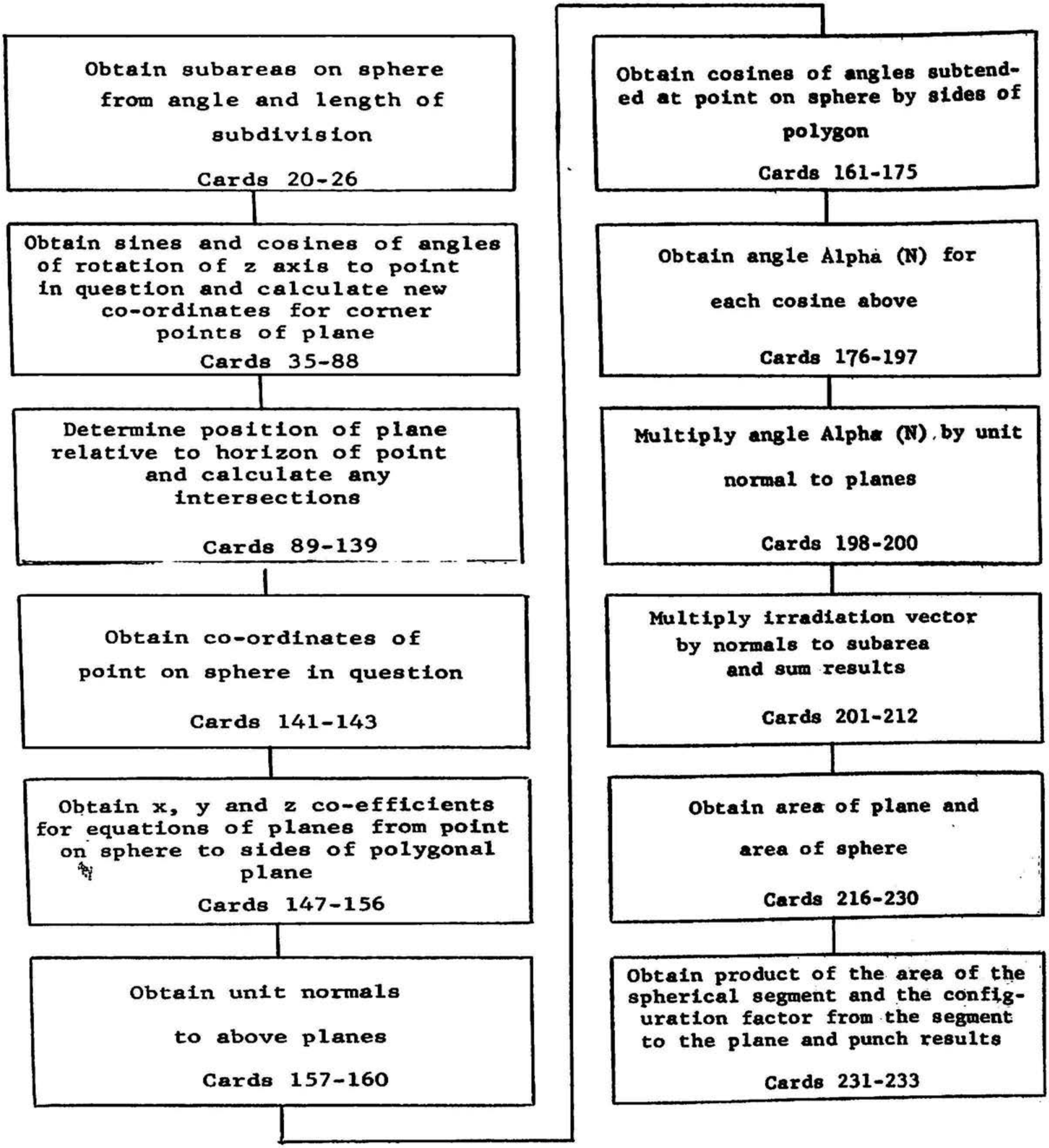

Fig. 10. Block Diagram of Sphere-to-Plane Program

TABLE III

SPHERE-TO-PLANE PROGRAM

```
C 0000 0 RADIATION CONFIG HEMI PLANE
         DIMENSIONAR(5 ),AX(5),AY(5),AZ
       1 (5),AYR(4),CSTHX(6),CSTHY(6),
       2 CSTHZ(6)
         DIMENSION AXRR(5),AYRR(5),AZ
       1 RR(5),CX(6),CY(6),CZ(6),E(5,3)
       2 ,CSS(5),ALPHA(5)
     1 0 READ,PRONO,J2,    AX(1),
     1 1 AY(1),AZ(1),AX(2)
         READ,AY(2),AZ(2),AX(3),AY(3),
       1 AZ(3),AX(4),AY(4)
         READ,AZ(4),ALPH1,ALPH2,R,RA,RB
         AN=J2
         A0=1.5707879
         A1=-.21412453
         A2=.08466649
         A3=-.03575663
         A4=.00864884
C 0000 0 OBTAIN COORD MD PTS
         DELAL=((ALPH2-ALPH1)/AN)
       1 *0.0174533
         ALPH3=ALPH1*0.0174533-
       1 DELAL/2.0
         DELR=(RA-RB)/AN
         RC=RB-DELR/2.
         CON1=RC
C 0000 0 DETERMIN OF HORIZ INTERSECT
         G1=0.
         G2=0.
         G3=0.
         AX(5)=AX(1)
         AY(5)=AY(1)
         AZ(5)=AZ(1)
   500 0 DO 672 M=1,J2
         ALPH3=ALPH3+DELAL
         ARGM=ALPH3
         LN=1
         GO TO 200
   201 0 SNALR=VAR1
         CSALR=VAR2
   505 0 DO510N=1,4
         AXRR(N)=AX(N)*CSALR-AY(N)*
       1 SNALR
         AXRR(5)=AXRR(1)
   510 0 AYR(N)=AY(N)*CSALR+AX(N)*SNALR
         RC=CON1
   600 0 DO 671 N1=1,J2
         RC=RC+DELR
         ARG=RC/R
[illegible]

   204 0 NA=NA+1
         GO TO 220
   202 0 ARGM=CON4
   210 0 VAR3=SINF(ARGM)
         VAR4=COSF(ARGM)
         GO TO (229,230,231,232,229,
       1 230,231,232,229,230),NA
   229 0 VAR1=VAR3
         VAR2=VAR4
         GO TO 250
   230 0 VAR1=VAR4
         VAR2=-VAR3
         GO TO 250
   231 0 VAR1=-VAR3
         VAR2=-VAR4
         GO TO 250
   232 0 VAR1=-VAR4
         VAR2=VAR3
   250 0 GO TO (201,280),LN
   280 0 SNTHR=VAR1
         CSTHR=VAR2
         TMP1=0.
         TMP2=0.
         TMP3=0.
   601 0 DO 610 J=1,4
         AYRR(J)=AYR(J)*CSTHR-AZ(J)*
       1 SNTHR
         AYRR(5)=AYRR(1)
   610 0 AZRR(J)=AZ(J)*CSTHR+
   610 1 AYR(J)*SNTHR
         AZRR(5)=AZRR(1)
         K=0
         DO 350 N=1,4
         IF(AZRR(N)-R)310,320,330
   310 0 IF(AZRR(N+1)-R)350,350,311
   311 0 K=K+1
         LA=0
   312 0 YRR=(R-AZRR(N+1))*(AYRR(N)-
   312 1 AYRR(N+1))/(AZRR(N)-AZRR(N+1))
   312 2 +AYRR(N+1)
         XRR=(R-AZRR(N+1))*(AXRR(N)-
       1 AXRR(N+1))/(AZRR(N)-AZRR(N+1))
       2 +AXRR(N+1)
         YR=YRR*CSTHR+R*SNTHR
         K1=K+LA
         CX(K1)=XRR*CSALR+YR*SNALR
         CY(K1)=YR*CSALR-XRR*SNALR
         CZ(K1)=R*CSTHR-YRR*SNTHR
         IF(LA-1)350,334,313
   313 0 PAUSE77777
```

```
      GO TO 350
323 0 K=K+1
      CX(K)=AX(N)
      CY(K)=AY(N)
      CZ(K)=AZ(N)
      CX(K+1)=AX(N+1)
      CY(K+1)=AY(N+1)
      CZ(K+1)=AZ(N+1)
      GO TO 350
330 0 K=K+1
      IF(AZRR(N+1)-R)331,332,332
331 0 CX(K)=AX(N)
      CY(K)=AY(N)
      CZ(K)=AZ(N)
      LA=1
      GO TO 312
334 0 K=K+1
      GO TO 350
332 0 CX(K)=AX(N)
      CY(K)=AY(N)
      CZ(K)=AZ(N)
      CX(K+1)=AX(N+1)
      CY(K+1)=AY(N+1)
      CZ(K+1)=AZ(N+1)
350 0 CONTINUE
      IF(K-2)671,671,620
620 0 BY=R*SNTHR*CSALR
      BX=R*SNTHR*SNALR
      BZ=R*CSTHR
      CX(K+1)=CX(1)
      CY(K+1)=CY(1)
      CZ(K+1)=CZ(1)
      DO 640 I=1,K
      E(I,1)=BY*(CZ(I)-CZ(I+1) )+CY
    1 (I)*(CZ(I+1)-BZ)+CY(I+1)*(BZ-
    2 CZ(I))
      E(I,2)=BZ*(CX(I)-CX(I+1))+CZ(
    1 I)*(CX(I+1)-BX)+CZ(I+1)*(BX-
    2 CX(I))
      E(I,3)=BX*(CY(I)-CY(I+1))+CX(I
    1 )*(CY(I+1)-BY)+CX(I+1)*( BY-
    2 CY(I))
      AR(I)=SQRTF(E(I,1)*E(I,1)+E(I
    1 ,2)*E(I,2)+E(I,3)*E(I,3))
      DO 630 J=1,3
630 0 E(I,J)=E(I,J)/AR(I)
      DELX=BX-CX(I)
      DELY=BY-CY(I)
      DELZ=BZ-CZ(I)
```

```
      DO 650I=1,K
      CSS(I)=CSTHX(I)*CSTHX(I+1)+
    1 CSTHY(I)*CSTHY(I+1)+CSTHZ(I)*
    2 CSTHZ(I+1)
      IF(CSS(I))651,652,653
651 0 IF(CSS(I)+1.)658,654,659
653 0 IF(CSS(I)-1.)659,655,657
652 0 ALPHA(I)=1.57079
      GO TO 650
654 0 ALPHA(I)=3.14159
      GO TO 650
655 0 ALPHA(I)=0.
      GO TO 650
659 0 ARG=CSS(I)
      LM=1
799 0 IF(ARG)800,802,802
800 0 LL=1
      ARG=-ARG
      GO TO 803
802 0 LL=0
803 0 PHI=ARG*(ARG*(ARG*(A4*ARG+
803 1  A3)+A2)+A1)+A0
      PHI2=SQRTF(1.-ARG)
      ANGLE=PHI*PHI2
      IF(LL-1)810,815,820
815 0 ANGLE=3.14159-ANGLE
      GO TO 810
820 0 PAUSE 9877
810 0 IF(LM-1)840,850,860
840 0 PAUSE 9887
850 0 ALPHA(I)=ANGLE
      GO TO 650
658 0 IF(CSS(I)+1.0001)656,654,654
657 0 IF(CSS(I)-1.0001)655,655,656
656 0  PAUSE 1111
650 0 CONTINUE
      DO 660 I=1,K
      DO 660 J=1,3
660 0 E(I,J)=E(I,J)*ALPHA(I)
      DO 665 I=1,K
      TMP1=E(I,1)+TMP1
      TMP2=E(I,2)+TMP2
665 0 TMP3=E(I,3)+TMP3
      TEMP=SQRTF((TMP1*TMP1+TMP2*
    1 TMP2+TMP3*TMP3)*(BX*BX+BY*BY
    2 +BZ*BZ))
      CSGMA=(TMP1*BX+TMP2*BY
    1 +TMP3*BZ)/TEMP
      G1=TMP1*CSGMA+G1
```

TABLE III (Continued)

```
      DELZ=AZ(2)-AZ(1)
      SIDE1=SQRTF(DELX*DELX+DELY*DEL
     1Y+DELZ*DELZ)
      DELX=AX(3)-AX(2)
      DELY=AY(3)-AY(2)
      DELZ=AZ(3)-AZ(2)
      SIDE2=SQRTF(DELX*DELX+DELY*DEL
     1Y+DELZ*DELZ)
      AREAA=SIDE1*SIDE2
C 0000 0 AREA OF HEMISPHERE
      AREAB=.017453*R*(ALPH2-ALPH1)
     1*(RA-RB)
      AAGAB=S1*AREAB/(AN*AN*6.28318)
      PUNCH,PRONO   ,AREAA,AREAB,
     1AAGAB
      GO TO 1
      END
```

CHAPTER VIII

CONE-TO-PLANE PROGRAM

The cone in this program can be a full cone, a frustrum, ment of a frustrum of a cone. The axis of the cone is on the the segment of the cone is defined, as in the previous progra angle (Alph 2-Alph 1). The intersection of the conical surfa y axis is designated as AH, and the height of the cone is des BH. The base of the cone is always on the x-y plane. The ra base is designated as R (see Fig. 12). The plane is defined, previous programs, by the cyclic co-ordinates of the corner p cone-to-plane program differs somewhat from the preceding two for curved surfaces. One of the differences arises due to th each subdivision on the cone does not have the same area. In ceding programs, the dA term in equation (3.27) was removed : summation since the surface was divided into equal subareas. conical surface, however, each subdivision becomes smaller in the program proceeds toward the apex of the cone. As a resu term cannot be removed from the summation in equation (3.27) area must be calculated for each subdivision. In addition to method used to calculate the co-ordinates of the center point sub-areas, a major difference occurs in the horizon decision Since the conical surface does not present a constant radius which to compare the co-ordinates of the corner points of the vector method was used to determine the plane's relative posit horizon of the point.

The vector method is accomplished by first calculating th nates of the intersection point of a normal to the conical su the origin. A unit normal to the conical surface from the or then obtained. The x axis is then rotated so that the point on the conical surface is contained in the x-y plane. The x, co-ordinates of the corner points of the plane are then calcu respect to the new axes. A vector from the origin to each co is then obtained, and from the dot product of each vector so and the unit normal to the conical surface, a horizon decisio Figure 11 shows a projection of plane ABCD on rotated x'-y' p point in question will be on line GH and its horizon is the p taining line GH perpendicular to the page.

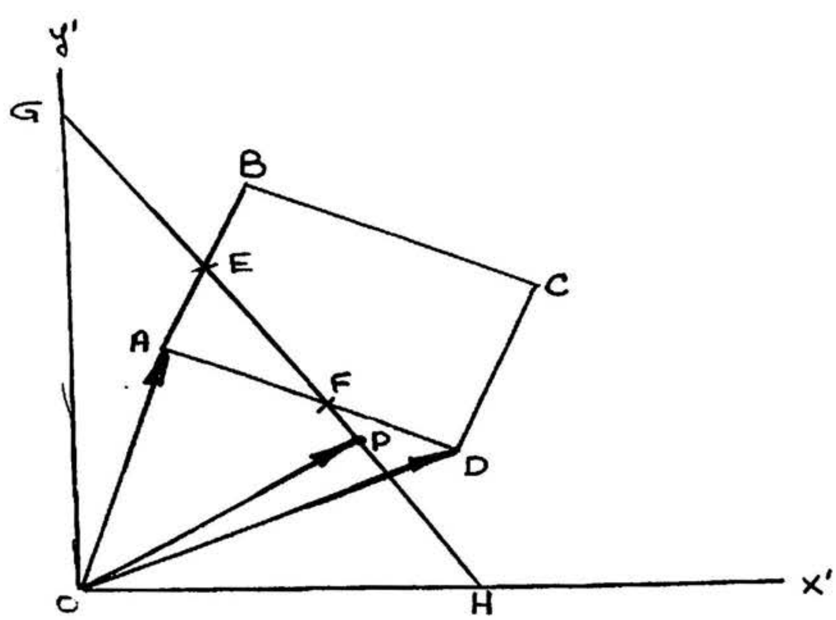

Fig. 11. Horizon Decision for Cone

If vector $\overrightarrow{OD}$ is dotted with a unit vector along OP, the the result will be greater than the magnitude of the unit ve therefore, point D of the plane appears above the horizon of vector $\overrightarrow{OA}$ is dotted with the unit vector along OP, the magnit result will be less than the magnitude of unit vector along of the plane will be below the horizon of point P. The co-o the intersection F will then be calculated. The process is all four points ABCD of the plane, and the polygon EBCDF is of ABCD that point P actually sees. The contour integration carried out for this polygon with reference to the original

The block diagram and the Fortran program for cone-to-p uration factor follows.

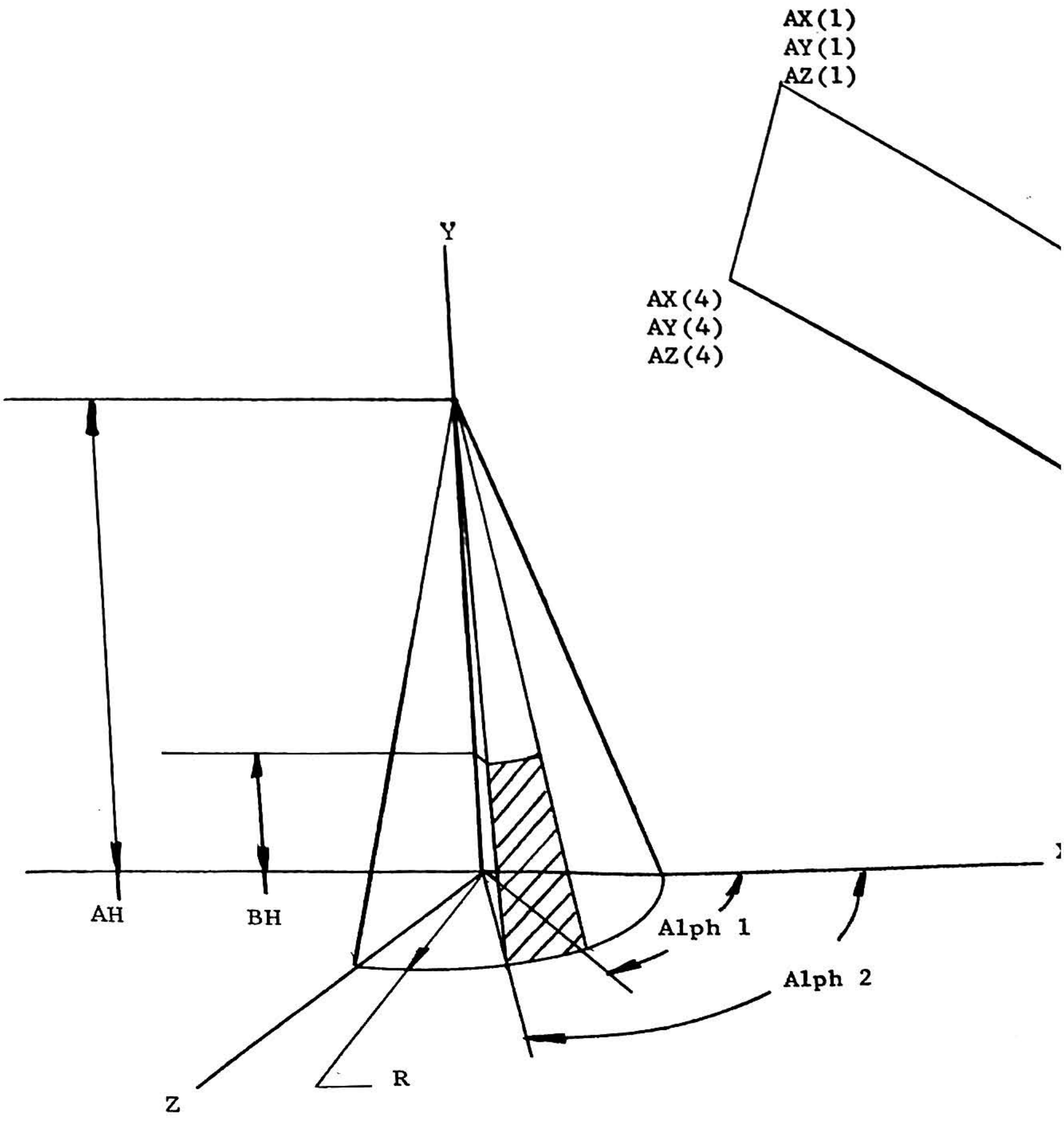

Fig. 12. Cone-to-Plane Configuration

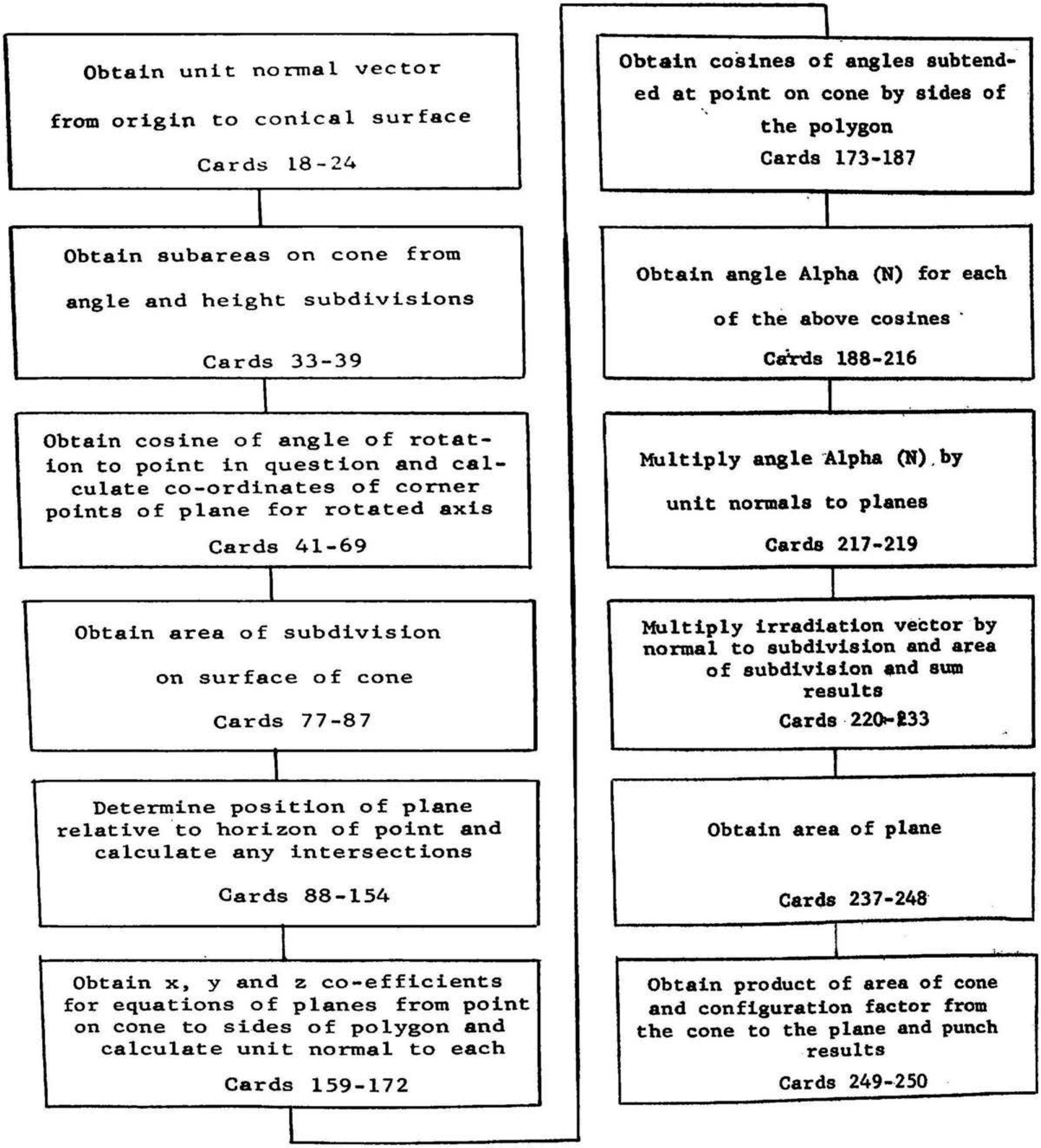

Fig. 13. Block Diagram of Cone-to-Plane Program

TABLE IV

CONE-TO-PLANE PROGRAM

```
C 0000 0 RAD CONFIG FAC CONE TO PLANE
         DIMENSIONAR(5),AX(5),AY(5),
       1 AZ(5),CSTHX(6),CSTHY(6),CSTHZ
       2 (6),CSS(6),AXR(5),AZR(5)
         DIMENSIONCX(6),CY(6),CZ(6),
       1 E(5,3),ALPHA(5)
     1 0 READ,PRONO,J2,LOC,AX(1),
     1 1 AY(1),AZ(1),AX(2)
         READ,AY(2),AZ(2),AX(3),AY(3),
       1 AZ(3),AX(4),AY(4)
         READ,AZ(4),ALPH1,ALPH2,R,AH,BH
         AN=J2
         A0=1.5707879
         A1=-.21412453
         A2=.08466649
         A3=-.03575663
         A4=.00864884
C 0000 0 OBTAIN UNIT NORMAL VECTOR
         PROD=AH*R/(R*R+AH*AH)
         XV=PROD*AH
         YV=PROD*R
         AMOD=SQRTF(XV*XV+YV*YV)
         ABAR=XV/AMOD
         BBAR=YV/AMOD
C 0000 0 DETERMINATION OF HORIZON INTER
         G1=0.
         G2=0.
         G3=0.
         AREAB=0.
         AX(5)=AX(1)
         AY(5)=AY(1)
         AZ(5)=AZ(1)
         DELAL=((ALPH2-ALPH1)/AN)*
       1 0.0174533
         ALPH3=ALPH1*0.0174533-
       1 DELAL/2.0
         DELH=BH/AN
         TEMP1=R/H
         ANT1=0.5*DELAL
   500 0 DO 672 M=1,J2
         ALPH3=ALPH3+DELAL
         ARGM=ALPH3
         NA=1
   220 0 CON4=ARGM
         ARGM=ARGM-1.5707879
         IF(ARGM)202,202,204
   204 0 NA=NA+1
         GO TO 220
   202 0 ARGM=CON4
         CSALR=-VAR1
         GO TO 250
   231 0 SNALR=-VAR1
         CSALR=-VAR2
         GO TO 250
   232 0 SNALR=-VAR2
         CSALR=VAR1
   250 0 DO 510 N=1,4
         AZR(N)=AZ(N)*CSALR+AX(N)*SNALR
   510 0 AXR(N)=AX(N)*CSALR-AZ(N)*SNALR
         AXR(5)=AXR(1)
         AZR(5)=AZR(1)
         RB=R
         AH2=0.
         BYC=-DELH/2.
   600 0 DO 671 N1=1,J2
         TMP1=0.
         TMP2=0.
         TMP3=0.
         RB1=RB
         BYC=BYC+DELH
         AH2=AH2+DELH
         RC=R-TEMP1*BYC
         RB=R-TEMP1*AH2
         ANT2=RB1+RB
         ANT3=SQRTF(DELH*DELH+(RB1-RB)*
       1 (RB1-RB))
         DAREA=ANT1*ANT2*ANT3
         BX=RC*CSALR
         BZ=RC*SNALR
         K=0
         DO 350 N=1,4
         DOT=ABAR*AXR(N)+BBAR*AY(N)
         IF(DOT) 310,302,302
   302 0 IF(DOT-AMOD)310,320,330
   310 0 DOT=ABAR*AXR(N+1)+BBAR*AY(N+1)
         IF(DOT)350,311,311
   311 0 IF(DOT-AMOD)350,350,312
   312 0 K=K+1
         LB=0
   313 0 K1=K+LB
         TEMP2=AXR(N+1)-AXR(N)
         SLP2=AH/R
         IF(TEMP2)400,401,400
   400 0 SLP1=(AY(N+1)-AY(N))/TEMP2
         XR=(AH+AXR(N)*SLP1-AY(N))/
       1 (SLP1+SLP2)
         YR=AH-SLP2*XR
         ZR=((XR-AXR(N))*(AZR(N+1)-
```

TABLE IV (Continued)

```
      CY(K1)=YR
      CZ(K1)=ZR*CSALR-XR*SNALR
316 0 IF(LB-1)350,333,317
317 0 PAUSE 77777
320 0 DOT=ABAR*AXR(N+1)+BBAR*AY(N+1)
      IF(DOT)322,321,321
321 0 IF(DOT-AMOD)322,323,325
322 0 K=K+1
      GO TO 350
323 0 LA=0
324 0 K=K+1
      CX(K)=AX(N)
      CY(K)=AY(N)
      CZ(K)=AZ(N)
      IF(LA-1)350,327,326
326 0 PAUSE 87777
327 0 CX(K+1)=AX(N+1)
      CY(K+1)=AY(N+1)
      CZ(K+1)=AZ(N+1)
      GO TO 350
325 0 LA=1
      GO TO 324
330 0 K=K+1
      DOT=ABAR*AXR(N+1)+BBAR*AY(N+1)
      IF(DOT)331,332,332
332 0 IF(DOT-AMOD)331,334,334
331 0 CX(K)=AX(N)
      CY(K)=AY(N)
      CZ(K)=AZ(N)
      LB=1
      GO TO 313
333 0 K=K+1
      GO TO 350
334 0 CX(K)=AX(N)
      CY(K)=AY(N)
      CZ(K)=AZ(N)
      CX(K+1)=AX(N+1)
      CY(K+1)=AY(N+1)
      CZ(K+1)=AZ(N+1)
350 0 CONTINUE
      IF(K-2)670,670,620
620 0 CX(K+1)=CX(1)
      CY(K+1)=CY(1)
      CZ(K+1)=CZ(1)
      DO 640 I=1,K
      E(I,1)=BYC*(CZ(I)-CZ(I+1) )
    1 +CY(I)*(CZ(I+1)-BZ)+CY(I+1)*
    2 (BZ-CZ(I))
      E(I,2)=BZ*(CX(I)-CX(I+1))
    1 +CZ(I)*(CX(I+1)-BX) +CZ(I+1)*
```

```
630 0 E(I,J)=E(I,J)/'R(I)
      DELX=BX-CX(I)
      DELY=BYC-CY(I)
      DELZ=BZ-CZ(I)
      R1=SQRTF(DELX*DELX+DELY*DELY
    1 +DELZ*DELZ   )
      CSTHX(I)=DELX/R1
      CSTHY(I)=DELY/R1
640 0 CSTHZ(I)=DELZ/R1
      CSTHX(K+1)=CSTHX(1)
      CSTHY(K+1)=CSTHY(1)
      CSTHZ(K+1)=CSTHZ(1)
      DO 650 I=1,K
      CSS(I)=CSTHX(I)*CSTHX(I+1)+
    1 CSTHY(I)*CSTHY(I+1)+CSTHZ(I)*
    2 CSTHZ(I+1)
      IF(CSS(I))651,652,653
651 0 IF(CSS(I)+1.)658,654,659
653 0 IF(CSS(I)-1.)659,655,657
652 0 ALPHA(I)=1.57079
      GO TO 650
654 0 ALPHA(I)=3.14159
      GO TO 650
655 0 ALPHA(I)=0.
      GO TO 650
659 0 ARG=CSS(I)
799 0 IF(ARG)800,801,802
800 0 LL=1
      ARG=-ARG
      GO TO 803
801 0 PAUSE 9777
802 0 LL=0
803 0 PHI=ARG*(ARG*(ARG*(A4*ARG+
803 1  A3)+A2)+A1)+A0
      PHI2=SQRTF(1.-ARG)
      ANGLE=PHI*PHI2
      IF(LL-1)810,815,820
815 0 ANGLE=3.14159-ANGLE
810 0 ALPHA(I)=ANGLE
      GO TO 650
820 0 PAUSE 9877
658 0 IF(CSS(I)+1.0001)656,654,654
657 0 IF(CSS(I)-1.0001)655,655,656
656 0  PAUSE 1111
650 0 CONTINUE
      DO 660 I=1,K
      DO 660 J=1,3
660 0 E(I,J)=E(I,J)*ALPHA(I)
      DO 665 I=1,K
      TMP1=E(I,1)+TMP1
```

TABLE IV (Continued)

```
            CSGMA=CON1+R*BYC/TEMP3
            G1=(TMP1*CSGMA)*DAREA+G1
            G2=TMP2*CSGMA*DAREA+G2
            G3=TMP3*CSGMA*DAREA+G3
   670 0    AREAB=AREAB+DAREA
   671 0    CONTINUE
   672 0    CONTINUE
            S1=SQRTF(G1*G1+G2*G2+G3*G3)
C 0000 0    AREA OF PLANE
            DELX=AX(2)-AX(1)
            DELY=AY(2)-AY(1)
            DELZ=AZ(2)-AZ(1)
            SIDE1=SQRTF(DELX*DELX+DELY*
          1 DELY+DELZ*DELZ)
            DELX=AX(3)-AX(2)
            DELY=AY(3)-AY(2)
            DELZ=AZ(3)-AZ(2)
            SIDE2=SQRTF(DELX*DELX+DELY*
          1 DELY+DELZ*DELZ)
            AREAA=SIDE1*SIDE2
            AAGAB=S1/6.28318
```

CHAPTER IX

SUMMARY AND CONCLUSIONS

The purpose of this study was to provide the means for d the radiation configuration factor for various types of surfa using contour integration theory, it was possible to eliminat dable task of evaluating the double integral in equation (1.6 a configuration factor for two surfaces. With the use of vec equation (1.6) can be transformed into an easily evaluated co gral.

The computer programs that were developed using the cont gration theory provided results with good accuracy. The dif sults obtained were checked with values obtained by the use integral in equation (1.6). In reference (2) the author pre of tables and graphs giving configuration factors for variou relationships obtained through the use of the integral in (1. figuration factors were calculated with the programs present report and checked with the results listed in the above refe results were in agreement for the configurations calculated. to-plane program was checked by comparing values obtained fo figuration factors of planes intersecting at finite angles, lel planes. The cylinder-to-plane program was checked also with a configuration given for a line source parallel to a cylinder of equal length. Since the program will also calculate a configuration factor for a segment of a cylindrical surface, a configuration factor was calculated for a thin (approximately ten degrees) segment of a cylinder and a plane. The result was compared with the configuration factor obtained from the plane-to-plane program for a narrow strip and a larger parallel plane. The surfaces were so devised that the only difference in the surfaces for both programs was the slight curvature in the cylindrical segment. The results from the two programs compared favorably. The sphere-to-plane program was checked in the same way as the previous two programs. Since the tabulated configuration factors for spheres and planes in reference (2) was very limited, the program was further checked by describing a narrow strip on the surface of a sphere of large radius irradiating a parallel plane. Again the geometrical relationship between the spherical strip and the plane approximated the narrow strip and larger plane of the plane-to-plane program. The results again compared satisfactorily with the results of the previous two programs. The sphere-to-plane program was checked further by describing a full sphere and a plane in such a way that the plane represented one side of a cubical box enclosing the sphere. The answer to this particular configuration is known from logical considerations. If a sphere is located in the center of a cubical box, the energy reaching any side of the box would be exactly one-sixth of the total energy leaving the sphere, since all of the energy leaving the sphere will be intercepted equally on all sides of the enclosure. Even though the sphere was approximated by only one hundred points, (the parameter J2 was given a value of ten) half of which cannot "see" the plane, the resulting configuration factor was very close to one-sixth. The cone-to-plane program was checked by describing a very tall cone with a small base. A frustrum of the cone closely approximated the cylinder used in the cylinder-to-plane program. The results obtained for the configuration compared very favorably with the data listed in reference (2) and the results obtained from the cylinder-to-plane program. The program was checked further by describing a narrow strip on the conical surface and a larger plane, the configuration approximating those used in checking the previous three programs. As was expected, the result was very close to being the same as for the similar configuration in the cylinder-to-plane program and compared favorably with the results obtained from the plane-to-plane and sphere-to-plane programs for that configuration.

For comparision purposes, some of the results obtained with the programs are presented in Table V, along with values obtained from reference (2). The parameter J2 is also listed. It was found during the program evaluation that a much larger value of J2 had a comparatively small effect when the surface that is subdivided is small in comparison to the other surface. The surfaces that are subdivided, as mentioned previously, are the B plane in the plane-to-plane program and the curved surfaces in the remaining programs. This fact can be utilized to save computer time when possible.

TABLE V

COMPUTER PROGRAM RESULTS

Description of Configuration	J2	F_{1-2}	Desired Result	Source of Result
1. Two planes each 30 by 30 intersecting at angle of:				
30°	3	.63968		
	6	.62579	.6202	Ref. 2
2. 60°	3	.37542		
	6	.37255	.3712	"
3. 90°	3	.19918		
	6	.19983	.20004	"
4. 120°	3	.08493		
	6	.08615	.08700	"
5. 150°	3	.02666		
	6	.02112	.02151	"
6. Two parallel 30 by 30 planes 30 units apart	3	.20326		
	6	.20006	.19982	"
7. Narrow strip 0.16 by 30 and parallel plane 30 by 60	3	.48721	Not available	
8. Narrow strip 0.16 by 30 and plane 30 by 60 perpendicular to one end	3	.21648	Not available	

TABLE V (Continued)

Description of Configuration	J2	F_{1-2}	Desired Result
9. Full cylinder and narrow parallel plane	10	.19480	0.200
10. Narrow cylindrical segment and parallel plane 30 by 60	3	.47099	.48721
11. Narrow cylindrical segment and plane 30 by 60 perpendicular to one end	3	.21460	.21648
12. Full sphere and one side of cubical enclosure	10	.16454	.16667
13. Narrow spherical strip and parallel 30 by 60 plane	3	.47052	.47099
14. Narrow spherical strip and 30 by 60 plane perpendicular to one end	3	.21566	.21460
15. Narrow conical strip and parallel 30 by 60 plane	3	.47102	.47099
16. Narrow conical strip and 30 by 60 plane perpendicular to one end	3	.21062	.21460
17. Frustrum of full cone and narrow parallel strip	10	.19602	.19480

CHAPTER X

RECOMMENDATIONS FOR FUTURE STUDY

The programs presented in this report detail a method by which the configuration factor can be calculated between two surfaces with an electronic computer. In the last three programs, curved surfaces were presented containing areas that could not "see" the second surface, and a horizon decision had to be made for each point on the curved surface. If only a portion of the plane could be seen, it was a fairly simple matter to calculate the intersection points. If the second surface is another curved surface rather than a flat plane, complications rapidly become apparent. For example, consider a simplified case of two cylinders with parallel axes. It becomes more complicated to obtain the visible portion of the second cylinder from any given point on the first cylinder. In addition, the given point no longer "sees" a polygonal surface. The ends of the cylinder will be seen as a portion of an ellipse or as a full ellipse. The irradiation vectors at the given point will no longer be collinear. The integral in equation (3.22) is no longer an ordinary scalar one. A program was developed for two parallel cylinders with the above factors considered, but it exceeded the capacity of the computer and could not be checked. The program evaluated the integral in (3.22) numerically, and used the vector method in the horizon decision. By the use of the vector horizon decision method and a numerical method of evaluating the integral (3.22), the theory of contour integration can be extended to develop programs covering a large amount of surfaces more complicated than developed in this report.

The programs are limited to calculating the configuration factor for diffuse surfaces where the intensity is independent of the angle from normal, more commonly referred to as Lambert radiators. Many engineering materials do not radiate as Lambert radiators. This fact can be taken into consideration in the calculation of the configuration factor by a modification to the programs. If the intensity can be expressed as a function to the angle from the normal to the surface, it would be possible to incorporate the necessary changes in the programs to accommodate non-Lambertian radiators.

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APPENDIX A

VECTOR IDENTITY

The vector identity used in Chapter III in the mathematical derivation of the contour integral (footnote 1) will now be shown to be true. The identity is

$$\frac{\vec{r}_1}{r^2}\left(\vec{N} \bullet \vec{r}_1\right) = \frac{1}{2}\,\text{curl}\left(\frac{\vec{r}_1}{r} \times \vec{N}\right) \tag{A.1}$$

where $\vec{r}_1$ = unit vector along r

r = magnitude of vector r

$\vec{N}$ = arbitrary unit vector

The expansion of the vector cross product in equation (A-1) yields

$$\frac{\vec{r}_1}{r} \times \vec{N} = \frac{1}{r}\left(r_y N_z - r_z N_y\right)i + \frac{1}{r}\left(r_z N_x - r_x N_z\right)j + \frac{1}{r}\left(r_x N_y - r_y N_x\right)k \tag{A.2}$$

Where the subscripts denote the x, y, and z components of the respective unit vectors. Unit vector $\vec{r}_1$ can be written as

$$\vec{r}_1 = r_x i + r_y j + r_z k$$

or

$$\vec{r}_1 = \frac{x}{r}i + \frac{y}{r}j + \frac{z}{r}k \tag{A.3}$$

where x, y, and z are the components of vector r. Using the relation (A.3) in equation (A.2) and obtaining the curl indicated in (A.1), the result is

$$\text{curl}\left(\frac{\vec{r}_1}{r}\times\vec{N}\right) = \frac{\partial}{\partial y}\left(\frac{x}{r^2}Ny - \frac{y}{r^2}Nx\right) - \frac{\partial}{\partial z}\left(\frac{z}{r^2}Nx - \frac{x}{r^2}Nz\right) i$$

$$+ \frac{\partial}{\partial z}\left(\frac{y}{r^2}Nz - \frac{z}{r^2}Ny\right) - \frac{\partial}{\partial x}\left(\frac{x}{r^2}Ny - \frac{y}{r^2}Nx\right) j$$

$$+ \frac{\partial}{\partial x}\left(\frac{z}{r^2}Nx - \frac{x}{r^2}Nz\right) - \frac{\partial}{\partial y}\left(\frac{y}{r^2}Nz - \frac{z}{r^2}Ny\right) k \qquad (A.4)$$

Since $\vec{N}$ was defined as an arbitrary unit vector it is constant with respect to the differentiation. Performing the differentiation in equation (A.4) and simplifying, the result is

$$\frac{1}{2}\,\text{curl}\left(\frac{\vec{r}_1}{r}\times\vec{N}\right) = -\left(\frac{x^2Nx}{r^4} + \frac{xyNy}{r^4} + \frac{xzNz}{r^4}\right) i$$

$$-\left(\frac{xyNx}{r^4} + \frac{y^2Ny}{r^4} + \frac{yzNz}{r^4}\right) j$$

$$-\left(\frac{xzNx}{r^4} + \frac{yzNy}{r^4} + \frac{z^2Nz}{r^4}\right) k \qquad (A.5)$$

The left hand side of equation (A.1) when expanded yields

$$\frac{\vec{r}_1}{r^2}\left(\vec{N}\cdot\vec{r}_1\right) = \frac{1}{r^2}\left(\frac{x}{r}\,i + \frac{y}{r}\,j + \frac{z}{r}\,k\right)\left(Nx\frac{x}{r} + Ny\frac{y}{r} + Nz\frac{z}{r}\right) \qquad (A.6)$$

or

$$\frac{r_1}{r^2}\left(N\bullet r_1\right) = \frac{1}{r^4}\left(x^2Nx + xyNy + xzNz\right) i$$

$$+ \frac{1}{r^4}\left(xyNx + y^2Ny + yzNz\right) j$$

$$+ \frac{1}{r^4}\left(xzNx + yz + z^2Nz\right) k \qquad (A.7)$$

The minus sign in equation (A.) will disappear due to the direction taken for vector $\vec{r}_1$. Vector $\vec{r}_1$ was taken as a vector pointing from the variable point on the surface S toward the fixed point P rather than in the usual opposite sense.

APPENDIX B

CORRELATION BETWEEN RADIATION AND ILLUMINATION

Much of the theory involved in the derivation of the contour integration method of calculating configuration factors has its background in illuminating engineering. Since light is merely radiant energy with wave lengths in the visible portion of the frequency spectrum, the theoretical considerations are identical. The only difference in the energy flux considered in the field of illumination and the energy flux considered in the field of radiation heat transfer is the wave length, or the frequency range in which the radiant energy lies.

An illuminating engineer is more concerned with the visual effect produced when a ray of energy strikes a surface, whereas a heat transfer engineer would be interested in the temperature effect due to the ray. As a result of this difference in interest, the units and definitions used in the two fields are not, in most cases, directly applicable to both fields.

A ray of radiant energy incident on a surface appears the same to the surface regardless of whether or not it is in the visible frequency range. The only difference can occur in the magnitude of the effect on the surface. An illuminating engineer is interested in the luminous flux rather than the radiant flux striking the surface. Luminous flux is only that part of the radiant flux that invokes a sensation to the eye. Since the eye can detect a difference in brightness, color, and saturation, or paleness of the color, an illuminating engineer needs three quantities with which to calculate the visual effects in which he is interested. In contrast, the heat transfer engineer, in most cases, is mainly interested in only one quantity - the total energy absorbed by the surface. The tools of the illuminating engineer - the equations and mathematical formulas - are more often expressed in terms of luminous flux or photometric quantities. A correlation exists, therefore, between the quantities associated with heat transfer calculations and illumination calculations. The range of wavelengths considered for heat transfer calculations is much greater than the visible spectrum. With the exception of luminous efficiencies and some specialized quantities existing in one field only, the correspondence between the quantities used in the two fields is presented in Table VI.

TABLE VI

CORRELATION BETWEEN RADIATION AND ILLUMINATION QUANTITIES

HEAT TRANSFER QUANTITY			PHOTOMETRIC QUANTITY		
Quantity	Symbol	Unit	Quantity	Symbol	Unit
Radiant Energy	Q	BTU	Light	Q	Lumen-Sec
Radiant Flux	Φ	$\frac{BTU}{Hr}$	Luminous Flux	F	Lumen
Total Emissive Power	E	$\frac{BTU}{Hr.\ Sq.\ Ft.}$	Luminosity	L	$\frac{Lumen}{Sq.\ Ft.}$
Irradiation	G	$\frac{BTU}{Hr.\ Sq.\ Ft.}$	Illumination	E	$\frac{Lumen}{Sq.\ Ft.}$
Intensity	I	$\frac{BTU}{Hr.\ Sq.\ Ft.-\ Steradian}$	Intensity	I	$\frac{Lumen}{Steradian}$

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