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GRAPH-THEORETIC APPROACH TO THE ANALYSIS AND OPTIMAL DESIGN OF WATER DISTRIBUTION NETWORK

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GRAPH-THEORETIC APPROACH TO THE ANALYSIS AND OPTIMAL

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CHAPTER I

Introduction

The Surveys made by the World Health Organization for a selection of ninety-one developing countries in 1970, updated for selected countries in 1975, provide what appears to be the only sets of data as to the proportion of population having reasonable access to safe water supply. References (1) and (2) give detailed information. However, the overall picture for 1975 may be summarized as shown in Table I-1 below:

TABLE I-1. Estimated percent of population having reasonable access to safe and adequate water-1975.

Region	Rural	Urban
Americas	32	81
Eastern Mediterranean	16	80
Europe	63	81
South East Asia	19	70
Western Pacific	30	90

The overall estimate, with the exception of the Peoples Republic of China and a few other countries, indicates that about 12 percent of the rural population had "adequate" water supplies in 1970, and the level had reached 20 percent five years later.

In view of the fact that the incidence of water-borne disease is reduced considerably, if not eliminated altogether, thereby improving the health of all people through the use of increased quantities of water of a higher bacteriological quality for all uses, the Second United Nations Development Decade had set modest targets for improvement in community water supply, involving a doubling of the proportion of the world's people served with adequate water. The United Nations Conference on Human Settlements held in 1976 recommended a more ambitious goal. It proposed that

> safe water supply and hygienic waste disposal should receive priority with a view to achieving measurable qualitative and quantitative targets serving all the population by a certain date.

The conference also went on to urge member countries to "adopt programs with realistic standards for quality and quantity to provide water for urban and rural areas by 1990, if possible." The United Nations Water Conference held in 1977 reaffirmed the 1976 Habitat commitment, and recommended, among other things, the following (3):

1. that with a view to achieving these ends, the nations which need to develop their systems for providing drinking water and sanitation should prepare for 1980 programs and plans to provide coverage for populations and to expand and maintain existing systems; institutional development and human resources utilization; and identification of the resources which are found to be necessary;
2. that the United Nations agencies should co-ordinate their efforts to help Member States, when they so request, in the work of preperation referred to in subparagragh (1) above;
3. that in 1980 the national programs which have been implemented for that purpose, and that extent to which the countries concerned have succeeded in mobilizing local and national support should be reviewed

by an appropriate mechanism to be determined by the Economic and Social Council and based on the use of existing machinery with a view to attaining co-ordinated action toward agreed targets.

The estimated annual investment for the conventional water supply systems to meet targets set, namely, water for all urban and rural areas by 1990, if possible, ran into serveral billions of dollars. Clearly, the expenditure involved in such undertaking is so huge that alternative and cheaper means of providing wholesome water had to be devised. To this end, the World Health Organization International Reference Center for Community Water Supply at The Hague, in conjuction with the Dutch Government in 1975 mounted a program in selected developing countries to produce water relatively cheaply through the use of Slow Sand Filtration method for treating surface waters. Other projects being undertaken by the International Reference Center includes design of suitable hand pumps for use on wells.

Water production is only part of the story; the other part is getting the treated water to the consumer. It is widely believed that the most costly parts of most water supply systems are the facilities for water distribution. The facilities include pipe networks as well as the pumping units and storage tanks or reservoirs. No doubt, this complex system must be designed to satisfy a multitude of criteria imposed by many different water users, ranging from fire fighting, car washing and lawn sprinkling to the various industrial and domestic needs. Although water supply system, as a whole, undergoes an expansion process dictated by the increase in water consumption resulting from city growth, the distribution system appears to be the most dynamic component of the entire system. To design facilities to serve the various needs of consumers, in most cases with very limited resources (as is the case in developing countries), is a challenging goal. It stands to reason, therefore, that the economically efficient allocation of resources to water distribution facilities is unlikely without systematic, objective and computationally efficient design methodologies. With the increasing energy cost and world-wide inflation, it is no wonder that the optimal design of water distribution system has, quite recently, received a great deal of attention by a number of researchers some of whom will be discussed in this thesis. The discussion on the optimal design of water distribution network will be preceeded by a discussion of network analysis.

A Brief Review of Techniques Developed

Water Distribution System Analysis

The objective of water distribution analysis is to determine the pressure for each node and also the magnitude and the direction of flow in each pipe for given input, draw-offs, pipe sizes and pipe configurations. The oldest method for systematic solution of water distribution networks is the Hardy Cross method (5). The method consists of making assumptions, initially, of a particular flow distribution pattern throughout the grid and then determining the head loss that would occur between the source of supply and critical points in the system. Basic to the method are the principles which obey the following physical laws of the network:

1. the total flow reaching any junction of two or more pipes must equal the total flow leaving the junction;
2. the change in pressure between any two points in a closed network is the same by any and all paths connecting the points.

In addition to above conditions, a proper relation between head loss and flow must exist for each pipe.

For a pipe of specified diameter, length and roughness the frictional head loss is given by the expression:

$$h = kQ^m \qquad (1\text{-}1)$$

where

h = frictional headloss

Q = flow rate

k = constant

Based on condition (2) above, if the assumed flow pattern were correct the head loss around the circuit according Equation (1-1) would be

$$h = \sum kQ^m = 0 \qquad (1\text{-}2)$$

For an incorrect flow distribution pattern, the correction for each pipe would be:

$$Q = Q_o + \Delta Q \qquad (1\text{-}3)$$

where

Q = correct flow pattern

Q_o = assumed flow pattern

ΔQ = flow correction

Substituting Equation (1-3) into Equation (1-1) we have

$$kQ^m = k(Q_o + \Delta Q)^m$$

$$= k(Q_o{}^m + mQ^{m-1}\Delta Q + \ldots.) \qquad (1\text{-}4)$$

If Q_o is small compared with Q_o, the higher powers of ΔQ in Equation (1-4) can be neglected. Once again for all pipes in a loop or circuit,

$$\sum kQ^m = \sum kQ_o{}^m + m\Delta Q \sum kQ_o{}^{m-1} \qquad (1\text{-}5)$$

From Equations (1-2) and (1-5) and rearranging, we have

$$\Delta Q = -\frac{\sum kQ_o{}^m}{m\sum kQ_o{}^{m-1}}$$

$$= \frac{h_o}{m\sum(h_o/Q_o)} \qquad (1\text{-}6)$$

where

h_o = frictional headloss computed on basis of assumed flow, Q_o.

It must be cautioned here that in using Equation (1-6) care must be exercised with regard to sign of head loss in both the numerator and denominator. By convention, flow in the clockwise direction around a circuit is considered of one sign and flow in the counterclockwise direction is considered of the other sign. The summation of head losses in the denominator is computed using the absolute values of the individual head losses, whereas, the head losses in the numerator are computed with due regard to the signs. Thus the flow correction ΔQ has a single direction for all pipes in the circuit.

This method can be applied to solve the system of head equations or the flow equations. It is well suited for solution by hand, and is easily adapted for machine computations. With the advent of the computer, and as larger and more complex networks were analyzed, the Hardy Cross method was found to frequently converge too slowly if at all. To this end, a number of measures have been suggested and used with the view to improving its convergence characteristics. Various researchers including (6), (7), (8) and have written computer programs to perform the Hardy Cross analysis on a Digital Computer.

Long before digital computers were first used to perform network analysis, Camp and Hazen (12) adapted linear resistance direct current electric power computing boards to the balancing of water distribution networks. In 1943, Camp (13) reported on a similar alternating current linear resistance computing boards. These boards then became known as Analogue Computer. The principle under which the computing boards work is that the head loss in a water pipe can be expressed in a form similar or analogous to the voltage drop in an electric circuit. Thus, if the voltage drop, E, across a linear resistance, R, is given as :

$$E = IR \tag{1-7}$$

where

I = Current

the expression for the head loss, analogous to potential drop in a water pipeline may be reduced to

$$h = kQ^{m} \tag{1-8}$$

where

k = constant and is a resistance term embodying length diameter and friction coefficient of pipe

Q = flow rate analogous to current and

m = a power varying between 1.75 and 2.00 for turbulent flow.

To obtain an analogy, Equation (1-8) can be rewritten in the form of Equation (1-7) i.e. in linear form as follows:

$$h = (kQ^{m-1})\ Q$$

$$= k_o\ Q \qquad (1\text{-}9)$$

By successive adjustment of the linear electrical resistance, values R analogous to k_o, a final set of k_o values can be obtained which satisfy Equation (1-9) in the case of each and every pipe. The adjustments comprise successive changes in the settings of ordinary linear resistors in the direction indicated by the proceeding trial. Because the linear resistance electrical network is always in balance the algebraic sums of currents at junctions and voltage drops around loops are automatically zero. Nevertheless, for each pipe only one Q and only one associated value of k_o will satisfy simultaneously the prescribed k in kQ^{m-1}. The general feature of alternating current calibrating boards have been given by Reid and Wolferson (4).

A breakthrough in the alternating current computing boards came when McIlory (14) introduced a direct reading electric analyzer for pipeline networks. The outstanding feature of the McIlory Analyzer is a system of non-linear resistances called FLUISTORS. For a fluistor with a direct current supply

$$E = RI^m$$

where

E = potential drop

R = resistance

I = current and

m = a constant.

Measured voltages and currents can be read directly on a special multiplier meter scale in units of headloss and flow rate. One draw back is that each analyzer is tailored to customer's specifications. With tremendous improvements made in digital computer programming McIlory Analyzer is not much in use. Further details on the features of analogue computers used in water network analysis can be found in works by McPherson and Radzidul (15), McPherson (16) and Wood (17).

Other methods of mathematical solution of networks resulted from work on electrical networks. Warga (18) applied Duffin's (19) work on non-linear networks to distribution system. Warga proved the existance of a unique solution for the heads at the nodes under steady state flows in a network whose elements satisfy certain conditions. According to Warga, if the general law relating flow and headloss in any element is given by

$$Q_{ji} = f_{ji}(H_i - H_j)$$

then these conditions are that

(1) $f_{ji}(x) = -f_{ij}(-x)$,

(2) $f_{ji}(x)$ is continous for all x;

(3) for all couples (j,i), $f_{ji}(x)$ is nondecreasing as x increases;

(4) there exists a path between any two nodes in the network along which every element has an $f_{ji}(x)$ which increases as x increases and takes on all values. He discusses (17) two iterative procedures for solving a network which satisties the above conditions. The development of the procedures is in terms of Procedure I, "apparently new" and Procedure II, based on the Newton-Raphson method for iteration convergence. Procedure I always converges,

although slowly, from any starting assumption, whereas, Procedure II, converages rapidly from a reasonable assumption but may not converge at all if the initial assumption is unreasonable. However, these procedures permit an extremely simple program to be written for digital computers, with the minimum use of memory space.

Newton-Raphson Method

The Newton-Raphson technique is a root finding process which determines new improvements or correction to the values of the unknowns in each iteration. These corrections are computed from the linearized Taylor Series Expansion and evaluated at the present state of the solution. It will perhaps be helpful, at this stage, to consider the Taylor Series Expansion and how the Newton-Raphson technique is derived from it.

Suppose that an arbitrary non-linear function f(x) having derivatives of all orders is represented by a power series expansion of the form

$$f(x) = C_0+C_1(x-x_0)+C_2(x-x_0)^2+\ldots.+C_n(x-x_0)^n \tag{1-10}$$

where

x_0 is as shown in Fig. (1-1) below is the value being sought such that

$$f(x)\Big|_{x = x_0} = f(x_0) = 0 \tag{1-11}$$

Since a power series, by a theorem, may be differentiated term by term within its interval of convergence we have

$$f'(x) = C_1+2C_2(x-x_0)+3C_3(x-x_0)^2+\ldots.+nC_n(x-x_0)^{n-1}$$

$$f''(x) = 2!C_2+3!C_3(x-x_0)+\ldots.n(n-1)C_n(x-x_0)^{n-2}$$

$$f'''(x)= 3!C_3+\ldots.n(n-1)(n-2)C_n(x-x_0^{n-3}$$

f', f'' and f''' are respectively first, second and third order differentiation. In general $f^{(n-1)}(x) = (n-1)!C_{n-1}+n!C_n(x-x_0)+\ldots.$ (1-12)

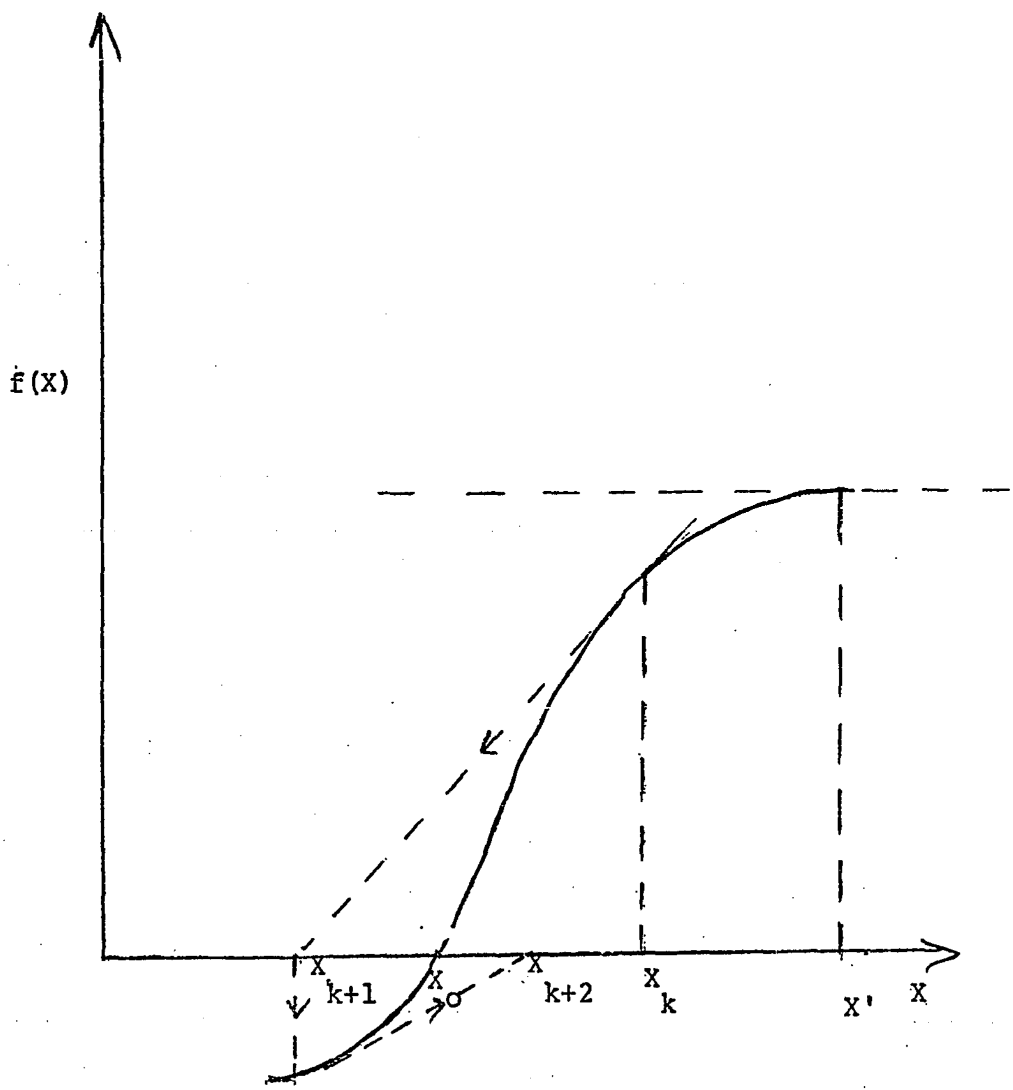


FIGURE 1-1: ILLUSTRATION OF THE NEWTON-RAPHSON METHOD.

If we substitute $x = x_o$ in these equations and also in Equation (1-10), we can solve for the C's as follows:

If $x = x_o$ and $f(x) = f(x_o) = C_o$

then

$f'(x_o) = C_1 \quad$ i.e. $C_1 = f'(a)$

$f''(x_o) = 2!C_2 \quad$ i.e. $C_2 = f''(x_o)/2!$

$f'''(x_o) = 3!C_3 \quad$ i.e. $C_3 = f'''(x_o)/3!$

$f^{(n-1)}(x_o) = (n-1)!C_{n-1} \quad$ i.e. $C_{n-1} = f^{(n-1)}(x_o)/(n-1)!$

Thus f(x) can be represented in a power series involving the differentials of f(x) itself namely:

$$f(x) = f(x_o)+f'(x_o)(x-x_o)+f''(x_o)(x-x_o)^2/2!+,\ldots,+f^{(n-1)}(x_o)(x-x_o)^{n-1}/(n-1)! \tag{1-13}$$

Equation (1-3) is what is known as the Taylor's Series.

The Newton-Raphson method ignores the second and the higher orders of differentiation. Thus Equation (1-13) becomes

$$f(x) = f(x_o)+f'(x_o)(x-x_o) \tag{1-14}$$

where

$x-x_o$ = the improvement or the correction to be applied.

From Equation (1-14) if $f(x) = 0$, then

$$x-x_o = -f(x_o)/f'(x_o).$$

From Fig. (1-1), at the k^{th} iteration, the approximation for x_o is denoted by x_k. The next approximation is given by

$$x_{k+1} = x_k + \Delta x_k$$

where

$$\Delta x_k = -f(x_k)/f'(x_k)$$

or

$$x_{k+1} = x_k - f(x_k)/f'(x_k). \tag{1-15}$$

in which $f'(x_k)$ or $df(x_k)/dx$ is the derivative of $f(x)$ at x_k. The equation for the k^{th} improvement x_k, can be expressed as:

$$f(x)=\frac{df}{dx}\Delta x = 0 \qquad (1\text{-}16)$$

In Equation (1-16) both f(x) and its derivative are evaluated using the present value of x. Since the second order differential was ignored in deriving the expression for the Newton-Raphson method, the error in the $(k+1)^{th}$ iterate, which is $(x-x_k)^2\ f''(x_k)/2f'(x_k)$, is seen to be proportional to the square of the error in the k^{th} iterate. Convergence of this type is referred to as quadratic convergence. This implies that each subsequent error reduction is proportional to the square of the previous error. Because the method requires an initial guess to the solution if this is say 20 per cent (i.e. 0.2) in error, successive iterations will produce errors of 4 per cent (i.e. 0.04), 1.6 per cent (i.e. 0.016), 0.026 per cent (0.00026), e.t.c.

To apply this method to the solution of a hydraulic network, if there are n equations to be satisfied:

$$\begin{aligned} f_1(x_1, \ldots\ldots\ldots, x_n) &= 0 \\ f_2(x_1, \ldots\ldots\ldots, x_n) &= 0 \\ &\vdots \\ f_n(x_1, \ldots\ldots\ldots, x_n) &= 0 \end{aligned}$$

and n unknowns $(x_1, \ldots\ldots\ldots, x_n)$ to be solved for, the set of n improvements or corrections $(x_1, \ldots\ldots\ldots, x_n)$ will be the solution of the set of n simultaneous linear equations of the form

$$f_j(x_1, \ldots\ldots\ldots, x_n) + \sum_{i=1}^{n} \frac{\partial f_i}{\partial x_i} \Delta x_i = 0 \qquad j = 1, \ldots\ldots, n \quad (1\text{-}17)$$

Thus the technique deals with the whole network at the same time, unlike Hardy Cross method, so that corrections are applied simultaneously to account for the joint interaction of all corrections. This method considers the effect of changing any one variable (e.g. pressure, resistance or demand at a node) on the entire network. This latent sensitivity information makes the Newton-Raphson method extremely useful for analysis and design purpose.

As mentioned earlier, Newton-Raphson method may encounter convergence problems. A number of researchers including Martin and Peters (20), Giudice (21), Pitchai (22), Jacoby and Twigg (23), Shamir and Howard (24) Epp and Fowler (25), Lam and Wolla (28) and Gagnon and Jacoby (31) have made substantial contributions to the efficient use of the Newton-Raphson method. For instance, Epp and Fowler (25) introduced a method of reducing core storage requirement by means of a Loop Defining Algorithm and a method of estimating initial flows that leads to fast convergence. Contributions made thus far have permitted Newton-Raphson method to be used to solve any of the three sets of equations describing flow in pipe networks. These sets of equation are obtained by considering

1. the flow rate in each pipe unknown,
2. the head at each junction unknown or
3. the corrective flow rate around each loop unknown.

Work by Shamir and Howard (24) made it possible for unknown consumptions and/or pipe resistances to be determined. Methods that allow network balancing and new pipe sizing to be accomplished without the need for repeated trials are described by Donachie (30). Other techniques developed are

1. relaxation based on repetitive solution of linear equation sets
2. general network analysis based on linear theory.

Linear Theory Method

The linear theory method, which bears close resemblance of the alternating current computing boards technique discussed earlier, has some distinct advantages over the Newton-Raphson or the Hardy Cross methods. First, it does not require an initialization of flows. Secondly, according to Wood and Charles (37) it always converges in a relatively few iterations. However, its use in solving head oriented equations or the corrective loop oriented equations is not recommended. Critics of this approach claim that large computer memory space is required for large networks.

Both Hardy Cross and Newton-Raphson methods can be applied to solve a set of equations based on either the node law (which states that the total flow reaching any junction of two or more pipes must equal the total flow leaving the junction) or the loop or circuit law (which states that the change in pressure between any two points in a closed network is the same by any and all paths connecting the points). The linear theory method, however, uses a combination of a set of equations obeying the node and circuit laws.

If the network comprises n nodes and e pipe elements then, from graph theory, the number of independent set of equations based on the node law is n-1. Since e is the total number of unknowns and is greater than n-1, a set of e-(n-1) or e-n+1 equations based on circuit laws are used in conjunction with set of n-1 equations to solve a set of equations in e unknowns simultaneously.

Symbolically, at each of the n-1 nodes, this relationship based on the node law holds:

$$Q_{in} = Q_{out} \qquad (1\text{-}18)$$

where

$$Q = \text{flow rate}$$

In addition, the set of e-n+1 loop equations are of the form

$$\sum h_l = 0 \tag{1-19}$$

where h_l = head loss in a pipe contained in a loop and is a function of the flow rate other things being held constant.

The head loss ,h_l, is given by the expression

$$h_{li} = k_i Q_i^m \qquad i = 1, \ldots\ldots, e\text{-}n\text{+}1 \tag{1-20}$$

where

h_{li} = head loss in loop i

k_i = a pipeline constant which is a function of pipe length, diameter and type of pipe material

m = constant with values ranging from 1.8 to 20.

From Equations (1-19) and (1-20), we have

$$\sum h_{li} = \sum k_i Q_i^m = 0 \qquad i = 1, \ldots\ldots. e\text{-}n\text{+}1 \tag{1-21}$$

It is seen that a set of equations indicated by Equation (1-21) are non-linear, whereas, those indicated by Equation (1-18) are linear.

By the linear theory, the e-n+1 non-linear loop equations are transformed into linear equations by approximating the head in each pipe by,

$$\begin{aligned} h_l &= k_i Q_{i_o}^{m-1} Q_i \\ &= k_i' Q_i \end{aligned} \tag{1-22}$$

where

Q_{io} = the approximate discharge in pipe i.

It is noted that Equation (1-22) is analogous to that of (1-9). Combining these so called artificial linear loop equations with n-1 node continuity equations provide a system of e linear equations which can be solved by linear algebra. The initial solution produces yet another approximate solution for the flows, and hence k'_i's. When Q_{io} approaches the actual Q_i, Equation (1-22) becomes an exact expression of the head loss.

Lam and Wolla (28), Kesavan and Chandrashekar (31), Hall (32) Chandrashekar and Steward (33) used the linear graph theory in the formulation of system equations. Hall (32) describes a geometric programming technique for the solution of the set of system equations obtained whereas the others solve the equations using Newton-Raphson method. Hall's approach considers optimal design (i.e. cost minimization) as well. A brief discussion of the linear graph theory is given in Appendix A.

To sum up, basically, about five different techniques have so far been developed for the analysis of hydraulic networks. These are:

1. Hardy Cross method;
2. Analogue Computer method based on the use of McIlory Analyzer principle;
3. Newton-Raphson method;
4. Linear Theory method and
5. Linear Graph Theory method.

Of these, Hardy Cross and Newton-Raphson methods are currently being used extensively for network analysis and design. The preponderance of previous work on distribution network analysis has centered on solving the non-linear equations that describe their hydraulic behaviour. In the next chapter, it will be shown how, based on linear graph theory, head loss and flow in each pipe are obtained without directly solving the non-linear terminal equations describing the relationship between flow and head loss.

Optimal Design of Hydraulic Networks

A hydraulic network problem may be considered a design problem if the diameter of each pipe in the network is unknown and is to be determined. In such a case, there usually exists a number of solutions which satisfy specified design criteria. In the wake of ever increasing energy costs coupled with world wide high inflation rate, it has become increasingly important (in the design of this nature) to find the solution that gives the least cost in terms of investment cost and operating cost. Compared to the development of techniques for hydraulic network analysis, optimal design of hydraulic networks began to receive some attention about mid 60's, which may be considered recently. Within a relatively short period of time a number of papers have been published on the subject. Some of the techniques developed to date will be described, briefly, next.

Towards the end of 1960's, the hydraulic network design practice was based, more or less, on arbitrary selection of pipe sizes and pump heads. The hydraulics of the network were then evaluated to determine if given requirements or constraints with respect to flow rates and pressures at various nodal points were satisfied. If this was not the case, then some of the pipe diameters were varied, and the pressures and flow rates re-evaluated. This process was repeated until a hydraulically acceptable network configuration with respect to pipe sizes was found. Since there usually exists a number of solutions which satisfy specified design criteria the cost of each hydraulically feasible alternative design was computed, and the design yielding the lowest cost was chosen. This design procedure, no doubt, was cumbersome, time consuming, uneconomical and not guaranteed to yield an optimal solution to the problem. Jacoby (39) and Karmeli, et al (40) finding this practice very unsatisfactorily contributed to the solution of the design problem by applying techniques which had been used very successfully in Operation Research field in optimization problem. They, respectively, used Non-Linear and Linear programming techniques. According to Phillips, et al (66), the term linear programming merely defines a particular class of programming that satisfy the following conditions:

1. The decision variables involved in the problem are non-negative (i.e., positive or zero).
2. The criterion for selecting the "best" values of the decision variable can be described by a linear function of these variables, that is, mathematical function involving only the first powers of the variables with no cross products. The criterion function is normally referred to as the objective function.
3. The operating rules governing the process (e.g., scarcity of resources) can be expressed as a set of linear equations or linear inequalities. This set is referred to as the constraint set.

Define the following generalized mathematical program:

$$\text{Maximize (or Minimize): } f(x) \qquad x \in E^n$$

$$\text{Subject to. : } H_j(x) = 0 \qquad j = 1,2,\ldots\ldots\ldots\ldots,M$$

$$G_k(x) = 0 \qquad k = 1,2,\ldots\ldots\ldots\ldots,\bar{M}$$

$$x = (x_1,x_2,\ldots\ldots\ldots\ldots,x_n).$$

From above, there are n decision variables defined by M equality constraints. and $\bar{M}$ inequality constraints.

f(x) which is referred to as the objective function can take the form:

$$Z = C_1x_1+C_2x_2+,\ldots\ldots\ldots\ldots,+C_nx_n.$$

$H_j(x)$ or $G_k(x)$ may take the form:

$$a_{11}x_1+a_{12}x_2+,\ldots\ldots\ldots\ldots,a_{1n}x_n = b_1$$

$$a_{21}x_1+a_{22}x_2+,\ldots\ldots\ldots\ldots,a_{2n}x_n = b_2$$

$$\vdots \qquad\qquad \vdots$$

$$a_{m1}x_1+a_{m2}x_2+,\ldots\ldots\ldots\ldots,a_{mn}x_n = b_m$$

$$x_1 = 0,\ x_2 = 0, \ldots\ldots\ldots\ldots, x_n = 0$$

$$b_1 = 0,\ b_2 = 0, \ldots\ldots\ldots\ldots, b_m = 0$$

If both the objective function and all constraints are linear, then we have a linear programming problem. If on the other hand, any component of f(x), $H_j(x)$ or $G_k(x)$ contains non-linear functions, then we have a non-linear programming problem. Any solution vector x which satisfies all sets of M equality constraints and $\bar{M}$ inequality constraints is called an admissible or feasible solution. A feature about this approach is that a particular set of solution variables which yield a minimizing value for f(x) is called the optimizing solution vector.

A number of techniques are available for solvimg linear programming problems once they have been expressed mathematically. Graphical solution is possible when the number of variables is not more than two. But a more common, all-purpose method is the simplex algorithm. An algorithm by definition, is a systematic method for testing various solutions; it guarantees that each successive solution will represent an improvement until the best solution is reached. Detailed information on these forms of optimization techniques (i.e. linear and non-linear programming) can be found in Phillips, et al (66).

In 1966 Pitchai (22) formulated the hydraulic design problem as a non-linear integer programming problem. Both the cost function and constraints were expressed in non-linear variables which were restricted to a set of discret values, hence, non-linear integer programming. Because at that time there were no known algorithms for direct computation of solutions to non-linear integer optimization problems, he overcame this difficulty by solving the problem by a random search technique. Cost of pipes and the annual cost of energy in operating the system were considered in the objective function to be minimized. Constraints which were stated as equalities included these:

minimum permissible pipe sizes; maximum permissible head loss along a specified path and operating pressures to coincide with pump characteristic curves. One aspect about the formulation procedure is that the constraint on maximum permissible head loss along a specified path was used to augment the objective function as a penalty function.

The solution process begins with an initial guess of each diameter in the network. These diameters serve as a so-called central design. Newton-Raphson method is then used to analyze the network for flows under one loading pattern. Using the cost function, the total cost for that particular design is computed. The next step is to generate a set of designs randomly about the central design. Then the corresponding total design cost in each design is computed and the best design among them is selected to serve as the central design of next random cost. Results obtained indicated that the system total cost decreased with the number of casts, but there is no proof that the global optimum is found.

In 1968, Jacoby (39) also used a non-linear integer programming technique in the optimal design of a hydraulic network having two loops, seven pipes, five consumption nodes and one pump supply node. Unlike Pitchai's approach, constraints were stated as inequalities; cost function and the constraints were combined to form a "merit function". Like Pitchai's approach, optimal solution was obtaind by a gradient-random search iteration method. Jacoby did not make guesses of each diameter in the network but assumed each diameter to be a continous variable. After the solution was obtained for each diameter, it was rounded off to the nearest integer size available on the market. If these round-offs resulted in infeasible solution, Hardy Cross method was applied to eliminate this infeasibility. Because the objective function has many local optima, the technique does not assure that the global optimum will be found. The author cautions that care should be exercised to avoid local minima.

In 1968, the same year that Jacoby came up with his method (39), Karmeli et al led the way in the use of linear programming method in hydraulic network optimal design. He considered a network without loops i.e., a simple branched network with only one source of supply. The decision variables were the piezometric head at the sources and the length of each pre-determined diameter to be allocated to each branch of the tree-shaped network. (The configuration of the network considered made it very easy for flows in each pipe to be computed directly once demands at the nodal points were known). The total length of each pipe and the minimum allowable piezometric head at each node formed the constraints. Subsequent contributions to the optimal network design problem which used linear-programming technique as a design tool are those of Gupta (41), Schaake and Lai (42), Kohlhaas and Mattern, (45) Kally (46), and Cembrowicz and Harrington (48).

An approach very similar to that of Karmeli, et al, was adopted by Gupta in 1969 in dealing with a simple branched network. The decision variables were taken as the lengths of pipe segments with a specified diameter since pipe resistance and pipe cost are linear functions of its length. He considered constraints on heads at nodes as linear inequalities imposed on the decision variables.

In the same year, Schaake and Lai (42) came up with a linear programming formulation which was based on two steps: Variable transformation, which linearizes the constraints, and a linear approximation of the objective function. The objective function was derived by considering one loading condition although extension to multiple loadings was developed. They considered flow as a constraint.

Two years later, Kohlhaas and Mattern, (45) observed that the application of the linear programming technique to a loop network resulted in "branch" or tree-like system because of the linear programming property that the solution of a problem of n decision variables and m constraints results in n-m+1 zero decision variables. According to them this presents no problem so long as additional pipes within the existing looped system are being optimized. However, they contended, 'the use of the linear programming for developing an optimal looped system in those areas not now served by existing pipes, requires an extension of existing water distribution system linear programming theory." To overcome this difficulty, they used separable programming to determine the optimal diameters, pumps, and reservoirs in a looped system in which all heads were given in advance.

Separable programming first introduced by C.F. Miller in 1963, is usually described as "probably the most useful non-linear programming technique." In separable programming, non-linear programming problems are solved by approximating the non-linear functions with piecewise linear functions and then solving the optimization problem through the use of a modified simplex algorithm of the linear programming, or in special cases, the ordinaty simplex algorithm. Basic assumption is that all functions in the problem must be separable. In other words, consider a function of two variables such as:-

$$f(x_1x_2) = x_1^3-2x_1^3+x_1+x_2^2-x_2.$$

This function is separable because it can be separated into two functions each a function of one variable:

where

$$f_1(x_1) = x_1^3 - 2x_1^2 + x_1$$

$$f_2(x_2) = x_2^2 - x_2.$$

The objective function they formulated was non-linear in the flow decision variables and costs of pipe, pumps, and reservoirs were considered; the constraints were linear.

In 1972, Kally (46) introduced a method which is equally applicable to tree-like and to looped network layouts. The method is based on a combination of two techniques: One is computation of the hydraulic data of the network by the Hardy Cross method, and the other is finding the optimum solution using linear programming technique. The objective function is linear in structure, whereas, constraints are inequalities in linear form. The length of a pipe is used as the decision variable.

Cembrowicz and Harrington (48) whilst working on the capital-cost minimization of hydraulic network developed a method for the evaluation of the global minimum of the non convex capital cost function based on a deterministic single load pattern and continious diameters. Results of their work was published in 1973. Using fundamental graph theory, the problem was decomposed into independent sets of convex functions subject to linear constraints. A standard algorithm was used in solving the transformed version whose variables are the flows and head losses in each pipe.

A year later Shamir (50) approached the optimal design and operation of water distribution systems as a linear programming program but the solution was based on a combination of the generalised reduced gradient method which draws on some of its variants used in optimal power flow solution. He considered the system being operated under one or several loading condition. In 1977, Alperovits and Shamir (55) improved upon the solution technique by using linear programing gradient (LPG) method. Briefly, the first step in developing the LPG method is to assume flow distribution in the network. This permits constraints on heads at nodes to be formulated. It must be pointed out that the cost of a pipeline is assumed to be linearly propotional to its length. Once the linear programming problem is formulated, it is then solved. If a vector of flows in all links, which satisfy continuity at all nodes, is denoted by Q, then for any Q the Optimal cost of the network is expressed as:

$$\text{Cost} = LP(Q)$$

where

LP implies that cost is the outcome of a linear program.

The next stage is to develop a method for systematically changing Q with the view to improving cost. If Q_p denotes the change in flow in path p. then

$$(\text{cost})/\ (\Delta Q_p) = (\text{cost})/\ b_p.\ b_p/\ (\Delta Q_p) = W_p.\ b_p/\ (\Delta Q_p)$$

where

b_p = known head difference between the end nodes of path p

W_p = value of the dual variable of the constraints for path p.W_p may be either positive or negative.

Apart from linear and non linear programming, dynamic programming is the other optimization technique which has been applied to hydraulic network optimization problem. Dynamic programming, the most complicated of the mathematical programming variants, is a technique which deals with the optimization of multistage decision processes. This technique was developed by Richard Bellman in early 1950's. In this technique, decision regarding a certain problem are typically optimized at subsequent stages, rather than simultaneously. This implies that the original decision problem is divided into small subproblems which can then be handled more efficiently from a computational view point. Some of the basic features of this technique are that:

1. at each stage the total decision process is related to its adjoining stages by a quantitative relationship called a transition function which can either reflect discrete quantities or continous quantities depending upon the mature of the problem;
2. given the current state, an optimal policy for the remaining stages in terms of a possible input state is independent of the policy adopted in previous stages and that the solution procedure always proceeds by finding the optimal policy for each possible input state at the present stage.

In 1971, Liang (43) applied this optimization technique to the optimal design of a single pumping source serial type water distribution system under a single loading pattern. Four years later, Fine (52) extended the technique to two cases involving (a) looped system and (b) serial system (both under single and multiple pumping sources systems) in the optimal planning and design of the water distribution systems. This technique has the advantage of being able to handle virtually any type of non-linear cost function very efficiently.

At the time "check design" approach was being used in the optimal design of hydraulic network, Tong, et al., (35) in 1960, introduced the concept of the equivalent pipe. By definition, an equivalent pipe is one in which the loss of head for a specified flow is the same as the loss in head of the system which it replaces. For any system, there are theoretically an infinite number of equivalent pipes. By this approach, equivalent lengths of pipes in a loop are balanced and proper sizes of pipes are obtained from pressure surfaces. They hold the view that for a given network, the total weight of pipe required is minimal when the total sum of the equivalent lengths of pipe in a loop is minimal. Further work on this line was carried out by Raman and Raman (36), and Deb and Sarkar (44).

Raman and Raman agreed with the equivalent pipe concept and went on further to demonstrate mathematically the necessary condition for minimum equivalent length. Deb and Sarkar, however, disagreed with this concept and pointed out that equivalent length is inversely proportional to the diameter raised to the power 4.87 and that by minimizing the equivalent length of pipes, the cost of network is increased. In 1971, they in turn, came up with investment minimization procedure based on both equivalent pipe and equivalent diameter concepts. The technique was applied to a branched network in which pressure surface profile i.e. heads must be known as well as head at the inlet. A method for determining the optimal pressure surface profile and inlet head was developed. The objective function includes the initial investment cost and reflects the cost of one loading condition to meet specified demands. Three years later and also in 1976, Deb (51,54) considered similar problem but this time he considered both the capital cost and operational cost. In both works, a discrete problem was treated as a continous one.

In 1973, Watanatada (49) tackled the least cost design of water distribution system problem by using the non-linear programming formulation technique. Mathematically, the problem is stated as follows:

Minimize: $C_T(D,H,Q)$

Subject to: $T_k\ (D,H,Q) = 0.,\qquad k = 1,\ldots\ldots,M$

$H_k = H_{Fk};\qquad k = 1,\ldots\ldots,M$

$Q_{Nk} = 0;\qquad k = 1,\ldots\ldots,MS$

$$D_p \geqq D_{min} \qquad p = 1,\ldots\ldots,P$$

where

C_T = total cost of the system and

$$C_T(D,H,Q) = \sum_{p=1}^{P} U_p L_p + \sum_{k=1}^{MS} S_k$$

P = total number of pipe in the network

U_p = Unit cost of pipe, p.

L_p = length of pipe, p.

MS = number of source nodes.

MC = number of consumption nodes.

M = MS+MC, the total number of nodes.

S_k = present value cost of supplying a quantity Q_{NK} at pressure H_k.

H_{Mk} = prescribed minimum allowable delivery pressure (consumption nodes only).

D_p = internal diameter of pipe,

D_{min} = minimum permissible pipe diameter

T_k = flow residual or sum of flows leaving the node.

$= Q_{Lp} - \sum Q_{Lp} + Q_{Nk}$.

$D = (D_1, D_2, \ldots, D_p)^T$, a column vector of P dimensions.

$H = (H_{12} H_{21}, \ldots, H_m)^T$, a column vector of M dimension.

$Q = (Q_{N1}, Q_{N2}, \ldots\ldots, Q_{Nm})^T$, a colum vector of MS dimensions ($T \equiv$ transpose).

Since the constraints stated above do not permit the design vectors, D,H and Q, to vary freely, the non-linear programming problem is referred to as constrained, otherwise, it is unconstrained. In his work, Watanatada employed a method developed by Box (67), to eliminate inequality constraints while the equality constraints were handled by the "gradient balancing" method of Haarhoff and Buys (68). The equality constraints are handled by the use of Lagrangian function.

Like others, the technique suffers from the fact that discrete problem is treated as continous since diameters obtained are rounded off to give sizes available on the market. However, the method is able to solve large networks say 50 nodes or more as well as several sources.

A method labelled as a "simplified optimization of water supply systems" was developed by Rasmusen (53) in 1976. The non-linear programming formulation was almost similar to that of Watanatada described previously. The difference lies in the method of solution. In Rasmusen's approach, the original problem is decomposed into two subproblems:

1. a hydraulic network flow problem which for any known values of pipe diameters and nodal consumption can be readily be solved; and
2. a diameter modification algorithm which attempts to modify the pipe diameters towards an optimal solution.

The decomposition, according to Rasmusen, implies that the hydraulic flow and diameter modifications must be solved iteratively until the optimal solution is found. The method considers discrete diameters available on the market but the diameter modifications are limited to one of three options.

The final hydraulic network optimization method to be discussed is the one proposed in 1972 by Barlow and Markland. The method optimizes the network by using the concept of cost-effictiveness as a criterion to decide between alternative possible changes to a tentative design. Cost-effectiveness is defined as friction head loss per unit cost. The concept may also be applied to produce an economic modification to an existing network.

Some Observations from above Discussion

In this chapter some of the techniques developed so far, both the analysis and the optimal design of a hydraulic network have been discussed. The treatment of the subject is by no means exhaustive. Nevertheless, it is believed that sufficient ground has been covered for some observations to be made.

Of the methods developed for hydraulic network analysis, Hardy Cross method and Newton-Raphson method are widely used. In spite of their wide-spread use, Hardy Cross method requires initial guesses of flow distribution which comply with the node law, whereas Newton-Raphson method requires initial guesses for either flow distribution or nodal heads. A big set back in these approaches is that a very bad initial estimates of either flow distribution pattern or heads at nodal points can lead to slow convergence or, in some cases, a situation where the successive trials to not converge and solution cannot be found. Hardy Cross method is known to converge much slower, if at all, than that of Newton-Raphson.

Quite apart from convergence problem encountered, both methods require large computer memory space for large networks. Attempts have been and are still being made to improve upon the efficiency of Newton-Raphson method by reducing memory space requirement. The use of a set of loop equation instead of a set of flow equations which obey the node law (thereby reducing the number of equations to be solved) has been suggested. To this end Epp and Fowler (25) have come up with a Loop Labelling Algorithms to enchance efficiency. Other researchers, for instance, Chandrashekar and Stewart (33) recommend the use of sparse matrix to reduce computer memory space requirement.

Techniques developed so far are aimed at solving for flows and heads in the network through iteration procedures. With the exception of the linear theory method, both the Hardy Cross and Newton-Raphson methods require initial estimates of either (1) flow distribution satisfying the node law or

(2) head at each nodal point.

In this thesis, linear graph theory is used in a method which solves a hydraulic network for head loss and flow through each link in the network in a straightforward fashion i.e., without iterative procedure. The technique developed does not require initialization of any form. Observations from discussion on techniques developed for optimal design of hydraulic network follows next.

From above it is apparent that the early attempt at optimal design of hydraulic network is what may be referred to as "check design." This is a procedure whereby, based on specified design criteria, the cost of each hydraulically feasible alternative design is computed and the design yielding the lowest cost in terms of either investment alone or investment and operation is chosen. This design procedure, which was adopted by Pitchai (18), Shamir an Howard (20), Epp and Fowler (22), and McCormick and Bellamy (30), no doubt, is cumbersome, uneconomical and not guaranteed to yield a global optimal solution to the problem.

The "equivalent pipe" method, which received some attention (35, 36, 44, 51, 54), lacks mathematical justification for cost equivalence between the actual and the equivalent pipes. The method also suffers from the fact that it cannot be applied to a system of multiple supply sources because a hydraulic pressure surface over the network must be created artificially even before the selection of pipe sizes can be made.

Linear and Non-linear programming techniques have been applied to both branched and looped network. Although these techniques are used extensively there are a number of drawbacks. Some of these are:

1. almost invariably, pipe diameter is considered as a continous variable. After a feasible solution has been obtained, the diameter of each link is rounded off to the next size corresponding to the one available on the market. Such round offs render the optimal solution obtained invalid. A common deficiency of these techniques is that a discrete problem is treated as a continous one.

2. the application of the linear programming method to a looped network may result in a "branch" or a tree-like network.

3. where link in a looped network is considered as a decision variable, the optimal solution gives a combination of pipes of different sizes for a link spanning two nodal points. In practice this is unacceptable.

4. in some cases the non-linear objective function or the non-linear constraints are approximated as linear functions through linear transformation. Clearly, an optimal solution obtained in this way is far from the global one.

Even though the dynamic programming technique has the capability of handling virtually any type of non-linear cost function very efficiently its use, so far, is limited to the optimal design of a serial type water distribution under single (43) or multiple pumping (52) condition. In 1975, Fine (52) extended the application to a single looped network under multiple loading condition. No successful attempt has been made to develop a model for an optimal design of a multi-looped network based on dynamic programming method.

The Layout of this Thesis

The foregoing part of this thesis has discussed the major drawbacks of some of the existing models for the analysis and optimal design of hydraulic network and has advocated the need for new models for both the analysis and the optimal design of hydraulic network.

Chapter II of the thesis gives a description of the theoretical derivation of the models based upon past efforts in these fields and the large body of analytical procedures and tools in the fields of network analysis and circuit theory. Chapter III describes the application of these models to the analyses and optimal designs of an example problem and an actual network. In the final chapter of the thesis an evaluation of the two models developed, in juxtaposition to the existing models, will be made. Certain recommendations for their future applications and developments will be made.

CHAPTER II

THE DEVELOPMENT OF THE ANALYSIS AND THE OPTIMIZATION MODELS.

Introduction

The purpose of this chapter is to develop mathematical models for the analysis and optimal design of a water distribution network based on the graph theory which is a branch of mathematics founded with Leonhard Euler's formulation and solution of the first graph theory problem in 1736. Over a century later James Clerk Maxwell and Gustav Robert Kirchhoff discovered certain basic principles of network analysis in the course of their studies of electrical circuits. Since then network analysts have relied very heavily on graph theory. Linear graph theory, therefore, provides comcepts and techniques which are often used in the process of formulating the set of equations necessary for the analysis of many types of physical systems (57). The optimal design of the network is based on the assumption that the flow through each pipe of the network is known. It is, therefore, considered appropriate to develop the model for the analysis of the network first and that of the optimal design next.

The Development of the Analysis Model

Generally, all systems (including water distribution system) are collections of parts or components designed to accomplish certain objectives.

Specifically, water distribution systems consist of water treatment plants, reservoirs and pumps on one hand and consumers, both commercial or domestic, on the other hand. They are interconnected by pipes of different size and type. Each of these parts, elements or components has certain properties inherent in its nature. Each system has a specific structure or interconnection pattern by which it constraints its components to perform only in certain ways. It is possible to identify the components of the system by their function in the total system and classify them in such a manner that each class of components have properties that are common to the class and differ from those of the other classes. Once the system components are thus classified it is possible to analyse the total system by techniques used for Network Analysis.

Network analysis is the study of connected lines and points. The lines, referred to as branches, can represent water mains or the generalized channels through which commodities flow, for example, roads, railroad tracks, airline routes or even powerlines and telephone wires. The points called the nodes, can represent communities, highway intersections, water reservoirs, railroad yards, airline terminals, power stations or telephone exchanges; in general, a node can represent any point where a flow originates, is relayed (or transmitted) or terminates. The natural limitations and capacities of the network's nodes and branches can be described by numbers. These numbers can be fixed, as in the case of the study of steady state or instantaneous flow conditions, or they can vary with time, as in the case of time variant flow conditions. In telecommunication networks, for instance, or even in transportation networks, the node numbers can even be random numbers whose values cannot be predicted precisely.

The fundamental objective of the network analysis technique is to relate mathematically the characteristics of a system to the characteristics of the component parts and their mode of interconnection. Regardless of what the pattern of interconnection is, a water distribution system has to obey certain fundamental physical laws whose roots are to be found in graph theory. These laws are: (1) the flow of any commodity or fluid, in this case water, into a junction must equal that out of this node. This is known as the node law. (2) the total headloss or propensity drop around a loop is zero. This is the circuit law. These two laws are respectively analogous to Kirchhoff's current and voltage laws. (3) each element of the system has a functional relationship between its pressure, voltage or propensity drop and flow rate. Some of the fundamental concepts of network analysis, which can also be considered as a systems analysis, will be presented here. This will be followed by a dissussion on how the method can be applied to a water distribution network.

The following are the steps to be followed in the analysis of a system by network analysis:

Step 1. The identification of the system component by purpose or function.

Step 2. The measurements on the components

Step 3. The determination of the terminal equations of the system components.

Step 4. The construction of the system graph.

Step 5. The establishment of a tree on the system graph.

Step 6. The formulation of the system equations.

Step 7. The numerical solution of equations.

The first step in the analysis of a system is the choice of units of the system components. This choice, generally, depends upon the type of the system being analyzed as well as the purpose of the system analysis. Thus, to the designer of a hydraulic solenoid valve, this component is a collection of valves, coils, pipes and pumps; to the designer of electronic amplifiers, the amplifier is a collection of resistors, inductors, capacitors, transitors and/or vacuum tubes.

The second step of the systems analysis is the mathematical description of the components selected in step 1. This description is connected with measurements on the components. For the purely physical systems (such as electrical, mechanical, heat transfer) these measurements are such that one is a "through" or "series" measurement represented by the letter Y and the other an "across" or "parallel" measurement represented by a letter X. For instance, in the electrical system X is voltage and Y is current; in the mechanical system X is a force and Y is displacement; in the thermal system X is temperature and Y is the heat flow and in the hydraulic system X is pressure and Y is fluid flow.

Once the components have been selected and the basic measurements chosen, it is possible to formulate mathematical equations describing the characteristics of the component in terms of the basic measurements. These equations are the terminal equations of the components and generally have the formula:

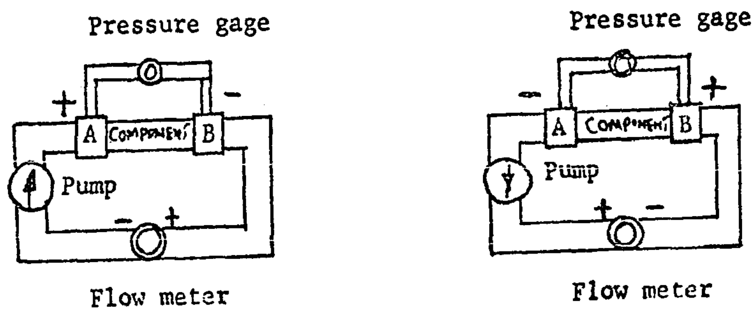
$$Y = kf(x)$$

where

k is a constant depending on component parameters and Y and X are fundamental measurements.

The next step of the analysis of the system is to formulate a mathematical equation describing quantitatively the interaction of the components of the system. The nature of the system is a factor in determining which technique is to be used to accomplish this. For instance, in some mechanical systems the system components are initially described by energy functions or the well known Lagrange's equations and the equations describing the component interconnection obtained by taking partial derivatives of the Lagrange equations. The application of the linear graph theory has been found to be a powerful tool in electrical network analysis.

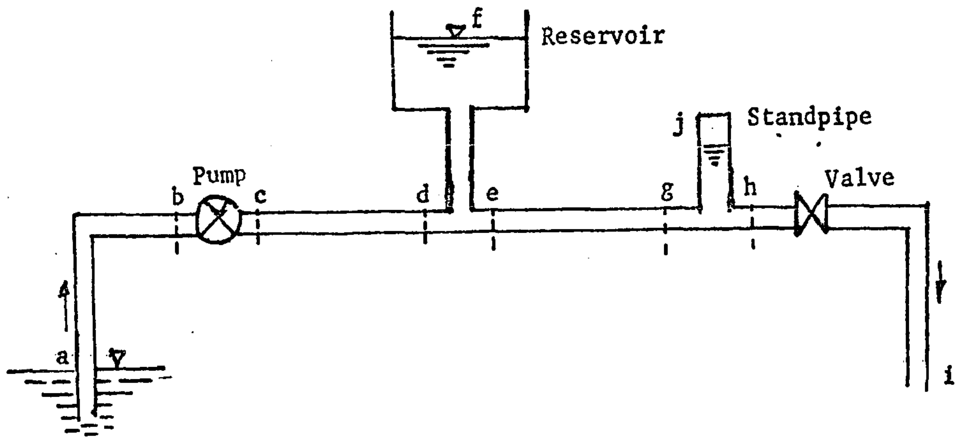
The use of linear graph theory requires that the physical system, in this case water distribution system, must be reduced to a system graph made up of line segments. To achieve this each component is represented by an oriented line segment called the component terminal graph. A pipe, for example, has two ends called the terminals, at which it can be connected to a network. Similarly, a pump operating under certain conditions, can be connected at two ends (suction and delivery), also known as terminals, to the network to deliver a certain flow through it. The inlet and the outlet ends of a reservoir or storage tank, valves and other hydraulic fixtures or equipment can be considered as terminals, Thus, a water distribution system is considered as a system of two-terminal components. Fig. 2-1 shows a schematic representation of a component and its corresponding component terminal graph. A system graph is obtained by uniting the vertices of the component terminal graphs in one-to-one correspondence with the union of the component terminals. The system graph for the hydraulic system of Fig. 2-2a is shown in Fig. 2-2b. It is seen that the system graph is simply a collection of the terminal graphs used in presenting the characteristics of the components. An important feature of the two-terminal component systems is that the system graph has a geometric appearance



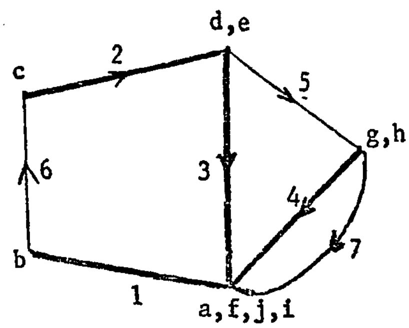
COMPONENT

TERMINAL GRAPH

FIGURE 2-1: SCHEMATIC REPRESENTATION OF A COMPONENT AND ITS CORRESPONDING TERMINAL GRAPH



(a) Hydraulic System



(b) System Graph

FIGURE 2-2: SCHEMATIC DIAGRAM OF A HYDRAULIC SYSTEM AND ITS CORRESPONDING SYSTEM GRAPH.

similar to the schematic diagram in Fig. 2-2a.

Linear graph theory provides concepts and techniques which are of help in the process of formulating the set of equations necessary for the analysis of many types of physical systems (57). The power of the linear graph technique lies in the ease and simplicity with which it describes the system's interconnections or formulates the system equation. The description of the system's equations is based upon two postulates. The first postulate is a generalization of Kirchhoff's Node Law and it is referred to as the "cut-set" or vertex postulate. Simply, this law states that the algebraic sum of the fundamental flow or through variable, Y, at any node in the system graph must equal zero. Mathematically, if the system graph contains e oriented elements and Y_j represents the fundamental flow variable of the j^{th} element, then the node law can be expressed as follows:

$$\sum_{j=1}^{e} a_j Y_j = 0$$

where

a_j = 0 if j^{th} element is not incident at k^{th} vertex

= 1 if j^{th} element is oriented away from k^{th} vertex

= -1 if j^{th} element is oriented toward k^{th} vertex.

The second postulate, called the "circuit" postulate, is a generalization of Kirchhoff's circuit law which states that if a linear graph contains e oriented elements and X_j represents the fundamental across variable of the j^{th} element then for the k^{th} circuit the following mathematical relationship holds:

$$\sum_{j=1}^{e} b_j X_j = 0$$

where

b_j = 0 if j^{th} element is not included in k^{th} circuit

= 1 if orientation of j^{th} element is same as orientation chosen for k^{th} circuit

$= -1$ if orientation of j^{th} element is opposite to that of k^{th} circuit.

The terminal characteristics of a two-terminal hydraulic or any other component are described completely when an equation is given which relates the X_j and Y_j variables. Such an equation, called a terminal equation, together with the two sets of equations obtained from the "cut-set" and "circuit" postulates, constitute the set of systems equations. The method of solution of these equations for the fundamental through and across variables, the Y's and the X's, depends upon the type of the system and the number of components. One of the main advantages of the linear graph technique is that the formulation procedure is independent of the numerical technique used in solving the resulting set of non linear equations. In other words, once the equations are formulated, a suitable numerical method for solution can be selected. Appendix A explains a basis for formulating the system equation for straighforward solution.

The above discussion has been quite general but it applies to all systems; electrical, mechanical, heat transfer, hydraulics or transportation. Discussion, centering mainly, on the application to a hydraulic system follows next.

The Selection of Components

It has been mentioned earlier on that water distribution systems consist of, among other things, water treatment plants, reservoirs and pumps on the one hand and consumers on the other hand. It was also stated that these are interconnected by pipes of different size and type and that each of these parts, elements or components has certain properties inherent in its nature.

This hydraulic system can thus be classified as a system consisting of a class of origin components (representing water treatment plants, reservoirs, storage tanks, pumps or generally supply or source areas), a class of destination components (representing all classes of consumers) and a class of transportation links (representing pipes or any conduit).

The Measurements on the Components

With reference to previous discussions, the two fundamental measurements on the components of the hydraulic system are the flow of water or any commodity through the components represented by Y, and the pressure causing this flow, also designated by the letter X.

The Component Terminal Equations

It is possible to establish, quantitatively, relationship between the X and the Y variables. That relationship is called the component terminal equation. The terminal equations for the three classes of components in the hydraulic system will be discussed below.

The Origin or Production (Source or Supply) Components

The cannonical formula for the origin component or production component can be written as

$$Y_{oi} = \text{known} \tag{2-1}$$

where

Y_{oi} = known flow from origin or supply point i.

This assumption is made because in water supply industry there are techniques for estimating or forecasting total water requirement for a given community or supply area. Total water requirement will, henceforth, be referred to as total water demand.

The Transportation Component

There are a number of ways in expressing the relationship between the X and the Y variables. One form is the Chezy equation which is expressed as

$$V = C\sqrt{R\ S} \qquad (2\text{-}2)$$

where

V = mean velocity, in feet per second

C = Chezy coefficient

R = hydraulic radius, feet. This is defined as the cross-sectional area of flow divided by the wetted or "frictional" perimeter.

S = slope

The value of C to be used in Eq. 2-2 for pipes of various construction, size, and shapes is difficult to ascertain. For this reason, this form of relationship is not used in hydraulic network analysis.

The Darcy-Weisbach equation is the other form of equation written to express relationship between the X and Y variables. The equation is generally written as:

$$h_1 = f\frac{LV^2}{D2g} \qquad (2\text{-}3)$$

where

h_1 = friction loss or across variable, feet

f = coefficient of friction

L = length of pipe, feet

D = diameter of pipe, feet

V = mean velocity, feet per second

= flow (Y-variable)/cross sectional area of pipe (A) g = acceleration due to gravity in feet per second.

Although the formula is fundamentally sound, independent of units used, and in excellent agreement with experimental measurements, the Darcy-Weisbach equation, according to Davis and Jeppson (65), is less used in practice than the Hazen-Williams empirical equation which will be given later. Eq. 2-3 is used less often because the friction factor, f, is generally unknown until a solution has been obtained. The use of this formula, therefore, requires the solution of the network by trial and error method.

Hazen-Williams equation developed in 1902 is the most commonly used for expressing the relationship between the through variable, Y, and the across variable, X. The formula is expressed as

$$Q_{ij} = 0.279 C_{ij} D^{2.63} (H_i - H_j)^{0.54} \frac{1}{L_{ij}^{0.54}} \qquad (2\text{-}4)$$

where

Q_{ij} = the flow or through variable, Y, from point i to point J, in m.g.d.

C_{ij} = Hazen-Williams coefficient for pipe ij

D_{ij} = diameter of pipe ij, in feet

L_{ij} = length of pipe ij, in feet

$H_i - H_j$ = across variable, X, or headloss, in feet

According to Davis and Jeppson (65), results obtained by using this form of relationship may be at great variance (20 per cent) with those obtained from the Darcy-Weisbach equation which is considered as more fundamentally sound. The discrepancy is quite evident for networks whose flows are sensitive to external conditions, networks with high velocities (or rough pipes), or networks with low velocities in some pipes. Because of the difficulty in using the Darcy-Weisbach equation, despite the fact that it applies for viscous flows such as oils and for gas flows such as natural gas and it is equally applicable in the English system of units (ES) or the International system (SI) without modification of coefficient, Hazen-Williams equation is currently being used extensively in expressing the relationship between the through variable, Y, and the across variable, X.

The Hazen-Williams coefficient, which is known for short as "C" value, is considered as the link resistance factor. It is a measure of the condition of the inside surface of a link or a pipe, in this case. A high value, usually, 140 or 150 depending on the pipe material, indicates that the pipe surface is relatively very smooth, whilst a low value of say 120 for a new pipe of another material indicates that the pipe surface is rough. It has been observed that the "C" value varies with the age of the pipe and that a reduction in the "C" value of the order of 20 percent is possible over a period of 10-15 years or more. Table 2-1 shows the variation of "C" values with pipe material and age. A high "C" value implies that the across variable is less than is the case when the "C" value is low for a given flow, pipe diameter and pipe length. The "C" value is reasonably constant over a wide range of flow conditions. This is probably one of the reasons for its widespread use. Unlike Hazen-Williams "C" value, the "f" value in the Darcy-Weisbach equation is sensitive to flow conditions which may be laminar or turbulent.

The Destination Areas

The destination areas will be considered as the draw-off or demand areas. On the micro level where the network is serving a village, a town or a city,

TABLE 2-1

VARIATION OF HAZEN-WILLIAMS "C" VALUE

WITH PIPE MATERIAL

PIPE MATERIAL	"C" VALUE
New cast iron	130
5-year-old cast iron	120
20-year-old cast iron	100
Average concrete	130
New welded steel	120
Asbestos cement	140
Steel, riveted, coated with coal tar	110
Wood stave	120
Cast iron lined with cement or bituminous enamel	140

the draw-off or demand areas will be assumed to be concentrated at pipe junctions or at nodal points of the network although consumers are tapped on the links or pipes. The assumption, no doubt, introduces errors on the network analysis. On the macro level where the network is serving a district or a regional water supply system, the destination areas will be taken as the villages, towns and cities considered in the network. Thus, villages, towns or cities will be regarded as nodal points and the pipes connecting the villages, towns or cities will represent network links or elements.

Two cases arise when modelling the destination areas. The first case is when the demand for the commodity (water in this case) at each destination or draw-off point is known. This is particularly the case for the network on the macro level. Here the cannonical formula for the destination areas is written as:

$$Y_{di} = \text{known}$$

where

Y_{di} = the demand for water at destination i.

The second case is where the demand for water cannot be specified directly and accurately. This is the case for the network on the micro level. Draw-offs from water distribution networks are time dependent: they vary from (1) hour to hour of the day, (2) day to day of the week, (3) week to week of the month and (4) month to month of the year. What seems to complicate the flow pattern in the water distribution network, just like any other physical system, is that as more and more consumers are added to the system in response to the completion of the construction of apartment complexies, housing estates, commercial centers, institutions and factories demand for water goes up accordingly.

The equation for the destination areas in this case is given as

$$Y_{di} = KA_{di}f(\Delta X_{di})$$

where

Y_{di} = the flow or demand at destination i

K = model calibration constant

A_{di} = the demand factor

ΔX_{di} = the propensity or desire for water or a commodity at node i.

It should be noted that the propensity or desire for water is different for water used for domestic, industrial and institutional uses.

From the expression just given above, both the demand, Y_{di} and the desire or propensity for water, X_i, are unknown. However, the demand factor can be constructed. The principle underlying the form or the structure of the demand factor of a destination is that water is transported to that draw-off point because of the demand for that commodity. The construction of the demand factor involves making a listing of the factors contributing to the demand for that commodity and the assessment of the contribution of each of these factors to the destinations draw-off pattern.

Basically, there are two approaches to the construction of the demand factor or attraction index in transportation system. One approach, useful in the absence of sufficient data, is to construct an approximate set of demand factors based on intuitive and subjective considerations. According to Gyamfi (60), these indices can be used to calibrate the model after which they are fine tuned by repeated adjustments to fit observed data. One disadvantage in this approach is that not much information is obtained on why an index has particular value. This is most undesirable because in the planning of new extensions to the network it is usually useful to know the contribution of each new draw-off point to the system.

The other approach is the use of powerful statistical tools and all available data to derive an accurate set of indices prior to the running of the model. The statistical tool normally used is the linear regression model. In this method, linear regression is performed on the factors considered important in the assessment of demand at a particular nodal point. For some demand patterns, such as the domestic water demand, these factors will be few in number and the linear regression can be performed very easily. In a very simple case, domestic water demand will be highly correlated to such factors as population characteristics and distribution, and the medium income of the population in the destination area (draw-off) area.

For aggregate water demand estimation or forecasting, variables contributing to the desire or propensity for water may be quite numerous. In this instance, it is absolutely necessary to reduce the large number of variables to a relatively small number of factors for regression. A very effective statistical tool for such a reduction is the factor analysis method. Various textbooks including (69) which treat this subject in detail are available, hence, it will not be considered in this thesis. Computer subroutine programs for performing such analysis is also available.

In brief, the objective of factor analysis is to resolve a set of variables linearly in terms of a small number of factors or categories. This is done by the analysis of the correlations among the large number of variables. The success of this method will be evident when it is possible to describe completely the original set of variables. The use of this method requires one to postulate the factors necessary to describe the demand pattern. The postulate is based on intutive or subjective judgement.

The Construction of the System Graph

The construction of the system graph follow procedures already explained. To recapitulate the procedure, the linear graph theory requires that the water distribution system be reduced to a system graph made up of line segments. To achieve this each component is represented by an oriented line segment referred to as the component terminal graph. Fig 2-1 shows a schematic representation of a component and its corresponding component terminal graph. A system graph is obtained by uniting the vertices of the component terminal graphs in one-to-one correspondence with the union of the component terminals. Fig. B-2 in Appendix B represents the linear graph of the physical system shown in Fig. B-1. The link from node 8 to node 1 is omitted for clarity. It is seen that the system graph is a collection of the terminal graphs used in presenting the characteristics of the components. The water distribution system is considered a two-terminal component system because each of the component (e.g. pipes, valves, reservoirs, pumps e.t.c.) has two terminals, namely inlet and outlet. An important feature of the two-terminal component system is that the system graph has geometric appearance similar to the schematic diagram of Fig. B-1.

It is worthy to note that all the draw-off points and the source or supply points are connected to a reference node 1 to form pseudo-loops. This form of arrangement is analogous to connecting to earth (i.e. grounding in electrical networks). In hydraulics, this point is taken as the datum and the elevation of the hydraulic grade-line at each nodal point is measured or described with respect to this point. The nodes are numbered consecutively with the flow in each pipe being in the direction of the node with a lower number to that with a higher number. While the order of numbering the nodes and links of the linear graph is immaterial for the solution of the system, there is some computational advantage to be gained from such numbering system.

The Formulation and Solution of the System Equations

Irrespective of the pattern or the complexity of the network, a water distribution system obeys certain fundamental physical laws. These are, first, the flow of a current, fluid or any commodity into a junction equals that which flows out of it. This is the "cut-set" postulate which is a generalization of Kirchhoff's current law. Secondly, the total headloss (in the case of water or any other fluid or gas) or the total voltage drop (in the case of an electrical circuit) around a loop is zero. This, it will be recalled, is the "circuit" postulate.

From the "cut-set" postulate, one set of equations can be written for the through variable, Y, at each vertex (node) of the system linear graph. Symbloically, this set of equations can be written as:

$$\sum_{j=1}^{e} a_j Y_j = 0$$

where all the symbols are as defined earlier.

The second set of equations involving the across variable, X, can be written for each circuit in the system linear graph. Once again, the symbolic expression for the set of equations is as follows:

$$\sum_{j=1}^{e} b_j X_j = 0$$

where all the symbols are as explained earlier. These two sets of equations together with the set of component terminal equations form the basis for the formulation and solution of the network.

Theoretically, it should be possible to solve these equations to obtain the X and the Y-variables of each component. Unfortunately, the system of equations, particularly those obtained through the use of the "cut-set" and the "circuit" postulates, are usually not all independent. The interdependence of a set of equations renders the system unsolvable since a singular matrix is obtained. A technique for formulating system equations that are all independent is the choice of a set of linear segments in the linear graph called the formulation tree. By definition, if a connected linear graph G contains V vertices, then the connected subgraph G, of G, containing all V vertices and no circuits is called a tree. Fig2-3 shows a simple linear graph with a possible tree drawn in bold lines. The line segments in the tree are called branches whilst the rest are called chords. Since each chord forms a fundamental circuit with a set of branches in a tree, there are as many fundamental circuits as there are chords. Each of the set of equations obtained from fundamental circuits through the use of the "circuit" postulate is independent. In Appendix A, the significance of a tree and other properties of linear graph are discussed in detail. The resulting sets of "cut-set" and "circuit" equations based on the formulation tree in Fig. 2-3 are shown below for the purpose of illustration.

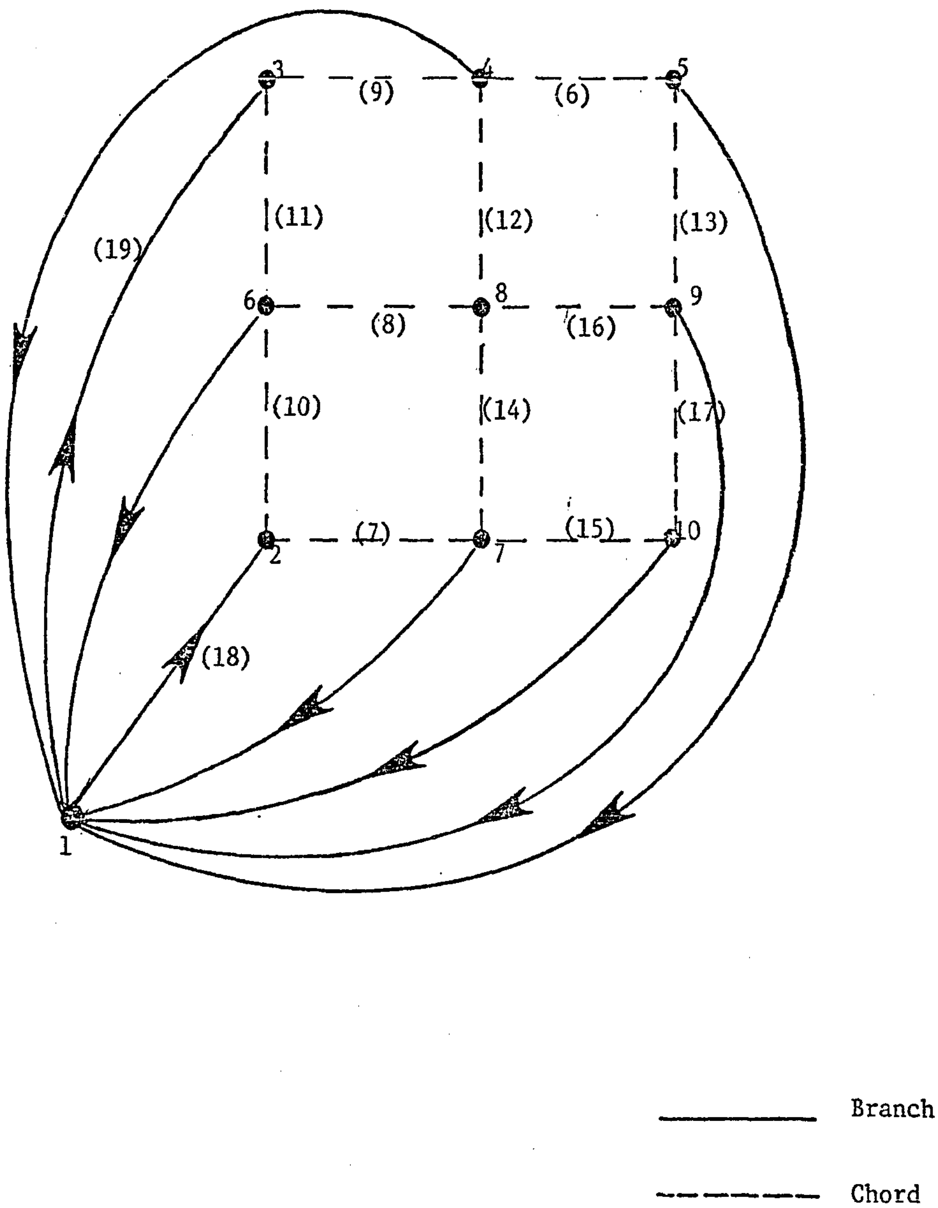


FIGURE. 2-3: SIMPLE LINEAR GRAPH OF A NETWORK WITH A POSSIBLE TREE DRAWN IN BOLD LINES.

"Cut-set" Equations

$$\begin{bmatrix} u & A \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \cdot \\ \cdot \\ \cdot \\ Y_{15} \\ Y_{16} \\ Y_{17} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Y_{18} \\ Y_{19} \end{bmatrix} = 0 \quad (2\text{-}5)$$

where

$$u = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and}$$

$$
A = \begin{bmatrix}
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
-1 & -1 & -1 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Circuit Equations

$$
\begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \\ \vdots \\ \Delta X_{15} \\ \Delta X_{16} \\ \Delta X_{17} \end{bmatrix} = \begin{bmatrix} u \\ \cdots\cdots \\ A' \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \\ \vdots \\ \Delta X_7 \\ \Delta X_8 \\ \Delta X_9 \end{bmatrix} \qquad (2\text{-}6)
$$

The Solution of the System Equations

The power of the linear graph theory lies in the fact that the formulation procedure is independent of the numerical technique used to solve the resulting set of equations which may be linear or non-linear. In other words, once the equations are formulated, a suitable numerical method for solution is adopted. The method employed in the solution of the systems equations depends on the explicit relationship between the fundamental variables, X, and Y, for the components as given by the component terminal equations. For an eletrical network, there is a linear relationship between the X and the Y-variables whereas in hydraulic network such a relationship is non-linear. Appendix A illustrates methods of solution for the case of a linear relationship between X and Y-variables and the case for any general non-linear relationship between them. For large networks the number of components increases, hence it becomes necessary to use shortcuts in solution of the system equations to reduce computer memory requirements. Some of the short-cuts are incorporated in the computer program, for the system's model and explained in detail in Appendix C.

The Development of the Optimization Model

As discussed in Chapter 1, a number of researchers have made various contributions to the optimization of water distribution system. Such contributions have focused mainly on either a branch system or a loop system. Techniques used, amongst others, are as follows:-

1. Linear programming
2. Non-linear programming
3. Non-linear integer programming
4. Dynamic programming and
5. Equivalent diameter concept.

Alperovits, E., and Shamir, V. (55) in 1977 categorized these techniques as:

1. Methods requiring the use of a network solver whereby, at each iteration of the optimization, one solves for the heads and flows in the network. The solution thus obtained is used in some procedure to modify the design.

2. Methods which do not require the use of a conventional network solver.

Examples of works which come under the first category are those of Jacoby (39), Kally (46), Watanatada (49), Shamir (50), and Rasmusen (53). According to Alperovits and Shamir, techniques used by Lai and Schaake (42), and Kohlhaas and Mattern (45) did not use a network solver, but both works treated the case in which the head distribution in the network is considered fixed. To overcome this shortcoming they proposed a linear programming gradient (LPG) method which incorporates the flow solution into the optimization procedure without making any assumptions about the hydraulic solution of the network. Like the other linear and non-linear techniques, the authors admit that their method can end up with "zero elements" (i.e., eliminate certain elements) thereby allowing some measure of selection between alternate system configurations. In other words, the optimal design of a looped network may end up as a branched or a tree configuration.

The New Approach

The method, presented here, is applicable to a branched or tree-like network and to looped network configurations. It is based on a combination of two techniques: one is the computation of flows through each link of the network based on known demand or draw-off at each nodal point and the other is the optimal design based on the dynamic programming approach developed by Rothfarb, B., et al (56) in 1968 whilst on a project supported by the Executive Office of the President, Office of Emergency Preparedness, Washington, D. C. The project, which was authorised by the Federal Power Commission was on "Optimal Design of Off-shore Natural-Gass Pipeline Systems." Techniques were developed for solving the following problems: (1) selection of pipe diameters in a specified pipeline network to minimize the sum of investment and operation cost; (2) selection of minimum-cost network structures, given gas-field locations and flow requirements; (3) optimal expansion of existing pipeline networks to include newly discovered gas fields. Techniques developed incorporated procedures for globally optimizing pipeline diameters for fixed tree structures and heuristic procedures for generating low-cost structures.

Network Analysis

The computation of flows through each link of the network (i.e. network analysis) may be performed either by the method described earlier or by any other well known method developed for the analysis of water distribution network. The objective here is to obtain, as accurately as possible, flows through each pipe at the required velocity, and satisfying draw-off requirements at specified nodal points. In short, the optimization technique is to be applied to a balanced network, primarily, in terms of flows and/or pressures. It is assumed that draw-off at each node is either known in advance or can be computed.

Optimal Design

Fig. 2-4 shows a typical tree-like configuration of an off-shore pipeline network. The techniques developed by Rothfarb et al (56) were applied to such network cinfiguration. It stands to reason, therefore, that if by some means a water distribution network can be reduced to such a configuration the technique developed can be applied. Fortunately enough, by means of linear graph theory, computer programs or algorithms for listing or obtaining spanning trees of a network is now available. It will be recalled that a tree is defined as a connected subgraph of a connected graph G containing all V vertices and no circuits. A tree T is said to be a spanning tree of a connected graph G if T is a subgraph of G and T contains all vertices of G. One such algorithm published in a textbook by Narsingh Deo (63) was used to obtain trees as shown in Figs. 2-5a, 2-5b, 2-5c and 2-5d. A line between two nodes is called a branch and the branches, forming the tree of a given network, are shown in solid lines; the dashed lines are chords.

The key to the solution of the design problem is the development of rules

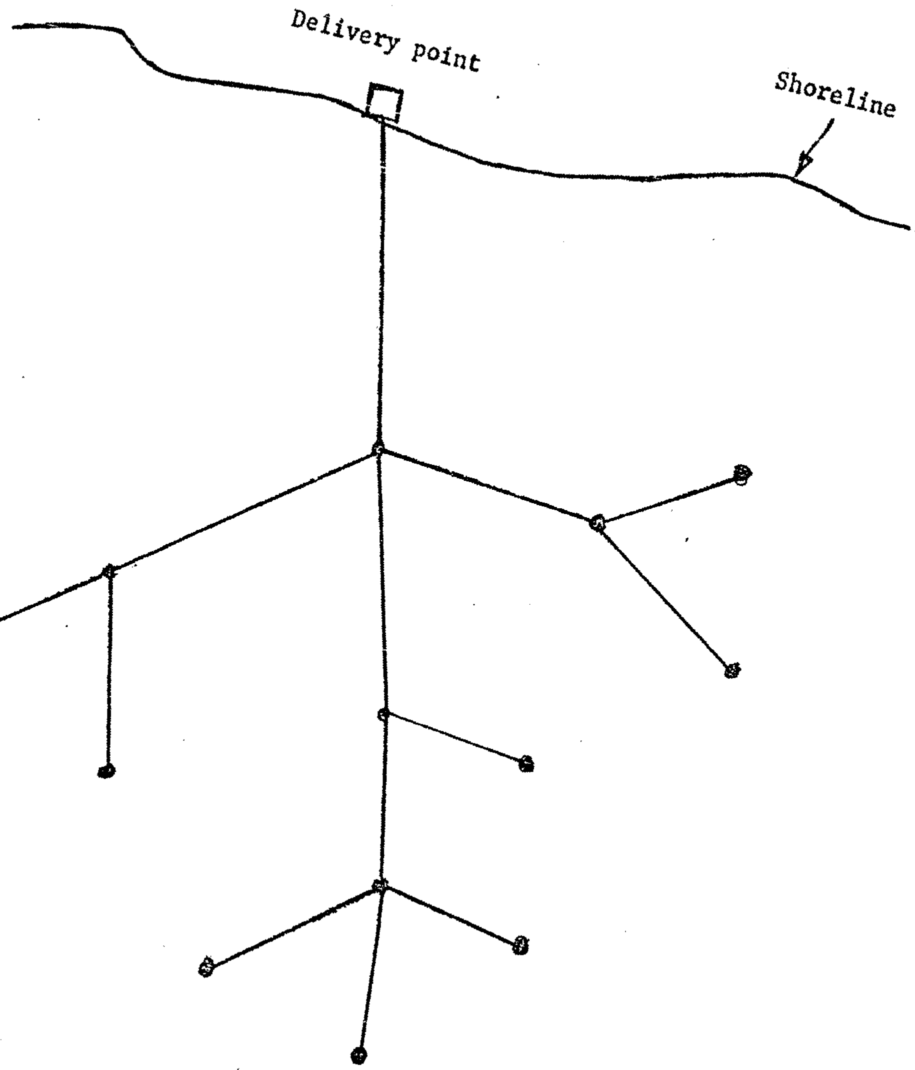


FIGURE 2-4: A TYPICAL TREE-LIKE CONFIGURATION OF AN OFF-SHORE PIPELINE NETWORK

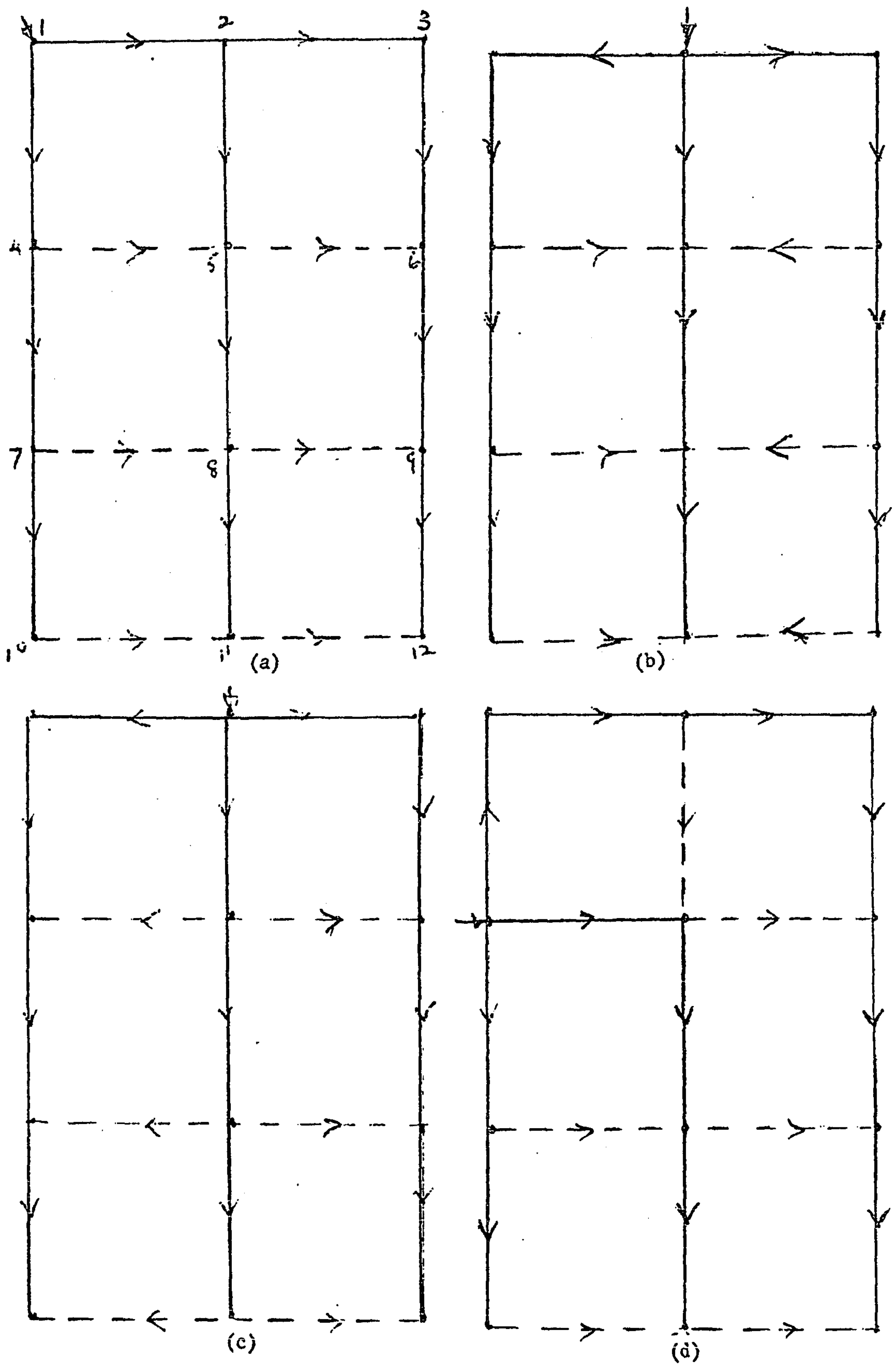


FIGURE 2-5: SPANNING TREES OF A 12 NODE 17 ELEMENT NETWORK.

to eliminate uneconomical diameter combinations in networks without enumeration. For example, given a tree containing n nodes and, therefore, n-1 branches (with, say, seven diameter choices for each branch to maintain the necessary velocity of flow in that branch) there are 7^{n-1} possible diameter assignments. Thus, for the 12-node tree shown in Fig. 2-5, there are 7^{11} diameter assignments. Out of these possible choices of diameters, the application of the rules should make it possible for one to find one choice that leads to the least expensive diameter combinations in terms of total investment and operating cost.

From the component terminal equation, which may be expressed as:

$$Q = 0.279CD^{2.63}\left[\frac{H_1-H_2}{L}\right]^{0.54} \qquad (2\text{-}7)$$

where Q = flow through a pipe or a branch in m.g.d.

C = Hazen-Williams "C" value

D = internal pipe diameter, in feet

L = length of pipe, in feet

H_1 = Outlet pressure, in feet.

since flow in each branch of a tree is known through a network analysis procedure, by specifying a diameter for a branch also specifies the head loss or pressure drop across that branch. It follows, therefore, that given a diameter assignment for all the branches, the node at which the pressure is greatest or least, as the case may be, can be determined. Thus, H_1 and H_2 are the only unspecified variables given in the component terminal equation shown above. It is assumed that flow through the network is due to pumping at a specified node. However, the technique being described is applicable also to the case whereby the supply is either by gravity or under pressure by pumping into more than one point on the network. There are major pressure constraints to be observed. First, the maximum allowable pressure in a pipeline should not exceed P_{max}, which is a specified constant. Secondly, water must be delivered to the most remote or the farthest nodal point on the network at a pressure greater than or equal to $P_{max,}$ which is another specified constant with $P_{min}<P_{max}$. Thirdly, the pressure available at each draw-off or demand point must be, at least, P_{min}. Since there is a minimum allowable node pressure $P_{min,}$ the pressure at the pump (or the elevation of the top water level in a storage tank) must be set high enough so that the necessary head loss or pressure drop can be maintained without falling below the pressure limit anywhere in the network. The path in the tree from the supply node (pumping unit(s) or elevated storage tank) to the node of the least pressure is called the Critical Path. The sum of the head loss or pressure drop along the critical path determines the pumping head, and hence, the pumping cost. It must be pointed out that even if the supply to the network is by gravity, in almost all cases, the elevated tank is fed by pumping, hence there is the need to reduce pumping cost to this storage tank.

The choice of diameters for some of the branches and leaving the diameters for the remaining branches unspecified is called a Partial Assignment. Given the very large number of possible diameter assignments, Rothfarb et al (56), developed methods that recognize partial assignments that cannot be in the optimal assignment. The elimination process was performed early, thus keeping the number of candidate partial assignments tractable. It is worthy to note that if one is to find a globally optimal assignment, one cannot discard any partial assignment that might possibly be part of the optimal assignment.

Elimination Process

The elimination process is in three steps, namely:

1. computation of vector PCOST and vector PSQ
2. parallel merge and
3. serial merge.

Step 1. Computations of vector PCOST and vector PSQ proceed as follows. From the result of the pipe network analysis performed earlier, flow through each branch is known. This then permits a range of flow velocities to be obtained for each branch from a given range of commercially available diameters. The range of flow velocities must be selected in advance to meet the minimum and the maximum velocity of flow through each branch depending upon the pipe material and the nature of the inside of the pipe. Once the upper and the lower limits of flow velocities are specified for each link, a range of diameters are then obtained. Rothfarb et al (56) considered seven different sizes of diameters for each branch. Irrespective of the number of different sizes of diameters considered, the elimination process remains the same; but it must be pointed out that the larger the number of diameters considered, the larger is the computer memory space requirement. However, it is computationally advantageous to consider the same number of diameters for each link regardless of sizes considered. The installed cost (pipe cost plus cost of excavation, laying and backfilling) of a branch of a given length is a function of the type of pipe and its diameter. This cost must be amortized over some time period to convert total capital cost to an annual cost. Any amortization scheme can be used in the design method. The pipe cost associated with the i^{th} smallest diameter choice for a given branch is defined as vector PCOST. Similarly, the head loss or the pressure drop across the branch arising from a choice of the i^{th} smallest diameter size for that branch is referred to as vector PSQ which is obtained from equation (2-7) by knowing the flow, Q, and D, for that branch. The values of the elements of PCOST are in increasing order whilst those of PSQ are in decreasing order. Fig. 2-6 shows this trend. The two vectors taken together is called a Branch List. Two techniques developed to be used together on a given tree to efficiently process these branch lists to obtain the optimal diameter assignment are (1) the parallel merge and (2) the serial merge. Detailed discussion of these techniques can be found in the reference cited. A brief discussion of these techniques follow

Step 2. The parallel merge is used on any set of branches that connect directly nodes of degree one to a common node. By definition, the degree of a node is the number of branches that are incident to the node. Branches b_1 and b_2 from the tree of Fig. 2-4 will be used to illustrate the procedure. Assume that the branch lists for these branches are as given below:

b_1: list:	PSQ	(153, 144, 125, 99, 87, 73, 64)
	PCOST	(26, 34, 46, 58, 72, 90, 116)
b_2 list:	PSQ	(183, 172, 151, 120, 108, 103, 100)
	PCOST	(12, 18, 28, 42, 60, 80, 112)

Each branch being merged has a column in the testing block tabulated below. If the index in a column is set to i then the PSQ and PCOST entries in that column are the i^{th} components of the list. Initially, the indices are set to 1.

	b_1	b_2
PSQ	153	183
PCOST	26	12
INDEX	1	1

The largest entry in the PSQ row of the testing block is located. In this

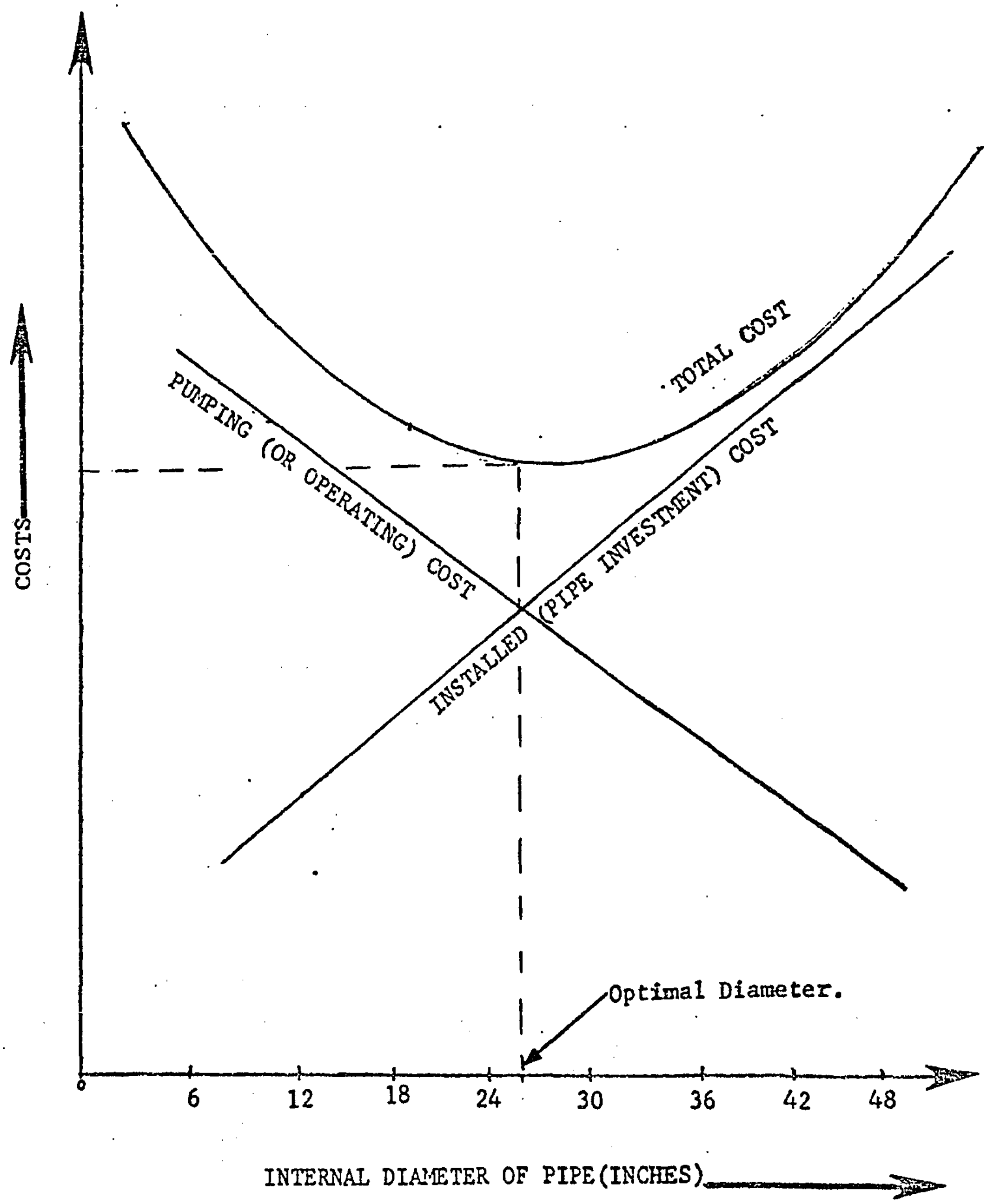


FIGURE 2-6: VARIATION OF PIPE COST AND PUMPING COST WITH PIPE DIAMETER

example, the largest entry is to be found in column b_2. What this means is that if the smallest pipe diameter is chosen for b_2, then b_1 cannot be on the critical path. Consequently, choosing other than the minimum diameter for b_1, when b_2 has the minimum diameter, will increase the total cost of pipes, but cannot reduce the pumping cost. If an optimal assignment has b_2 at its minimum diameter, then b_1 must also have minimum diameter. The largest PSQ entry and the sum of the PCOST entries of the testing block are entered on a new list. This entry on the new list corresponds to a partial assignment of minimum diameters to b_1 and b_2:

New list: PSQ (183)
PCOST (38).

The index in the b_2 column of the testing block is increased by 1 to give the table shown below

	b_1	b_2
PSQ	153	172
PCOST	26	18
INDEX	1	2

since no better choice of diameter for b_1 is possible with b_2 at this diameter. In the updated testing block, the new maximum PSQ entry is still in column b_2. This implies that, if b_2 has the second smallest diameter, it still will not pay to have b_1 at any diameter other than the smallest. The updated new list is shown below:

New list: PSQ (183, 172),
PCOST (38, 44).

The updated new list represents a partial assignment of the second smallest diameter to b_2 and the smallest diameter b_1. The index in the b_2 column is promoted once more to yield the updated testing block shown on the next page:

	b_1	b_2
PSQ	153	151
PCOST	26	28
INDEX	1	3

Now the largest entry in the PSQ row is in column b_1 for the first time. If an optimal assignment contains b_1 at its smallest diameter, it cannot contain b_2 at a larger diameter than the third smallest, or b_1 at a diameter other than the smallest. Updated new list is shown below:

New list: PSQ (183, 172, 153),
PCOST (38, 44, 54)

and the index in column b_1 is promoted to give the table shown below:

	b_1	b_2
PSQ	144	151
PCOST	34	28
INDEX	2	3

The process comes to an end when the largest entry of PSQ row of the testing block occurs in a column whose index has been promoted to 7 which is the number of different sizes of diameters being considered for each branch. Beyond this point, further promotion of the other indices would correspond to partial assignment of greater pipe cost and no possible savings in pumping cost. Tabulations given below show the remainder of the sequence of testing blocks and the complete resulting new list.

	b_1	b_2	b_1	b_2	b_1	b_2
PSQ	144	120	125	120	99	120
PCOST	34	42	46	42	58	42
INDEX	2	4	3	4	4	4

	b_1	b_2	b_1	b_2	b_1	b_2
PSQ	99	108	99	103	99	100
PCOST	58	60	58	80	58	112
INDEX	4	5	4	6	4	7

The final new list or equivalent branch list for b_1 and b_2 is as shown below:

PSQ (183, 172, 153, 151, 144, 125, 120, 108, 103, 100)

PCOST (38, 44, 54, 62, 76, 88, 100, 118, 138, 170)

This completes a parallel-merge of b_1 and b_2. It must be pointed out that each entry on the equivalent branch list represents an assignment of diameters to the branches b_1 and b_2. Furthermore, no other partial assignments for these branches need be considered. It is interesting to observe that the number of possible partial assignments for these two branches is $7^2=49$. However, the parallel-merge techniques will produce an equivalent branch list with at most 10 entries, one from the original testing block and one each from the testing blocks resulting from a maximum of 9 index promotions. The minimum number of components on a new list is equal to the number of different sizes of diameters from which a selection is to be made for each branch.

The concept of the equivalent branch is analogous to that of equivalent pipe. Equivalent branch, therefore, represents a branch connected between node 3 and a node consisting of a combination of nodes 1 and 2. Both PCOST and PSQ of the equivalent branch list are respectively in increasing and decreasing order so that no re-ordering of the list is necessary.

Steps 3. The serial-merge can be used on any two branches incident to a common node of degree two if at least one of the two branches is also incident to a node of degree one. The branches may be actual branches or equivalent branches. The objective of this technique is to combine, somehow, the list of the equivalent branch with the list of b_3 to produce a new equivalent branch list for b_1, b_2 and b_3 so as to retain as few partial assignments as possible without eliminating any partial assignments that can possibly be in the optimal assignment. Like the parallel-merge, a testing block will be used in illustrating the serial-merge. Data for the setting up of the testing block are obtained from the equivalent branch list for b_1 and b_2 and the following b_3 list:

b_3 list: PSQ (127, 119, 113, 94, 88, 81, 65),
PCOST (16, 24, 36, 52, 68, 86, 114).

	1	2	3	4	5	6	7
PSQ	310	302	296	277	271	261	248
PCOST	54	62	74	90	106	124	152
INDEX	1	1	1	1	1	1	1

The testing block required for a serial-merge is shown above. The i^{th} column corresponds to the i^{th} smallest diameter choice for b_3 and an index equal to j in a column corresponds to the j^{th} partial assignment of b_1 and b_2 in the equivalent branch list. It must be recalled that the PSQ and PCOST in a column are the corresponding headloss or pressure drop and pipe cost that would result from such a partial assignment of b_1, b_2 and b_3.

To begin with, the indices are all set to 1. The testing block given in Table 11-above, therefore, gives all the data for the partial assignments for every choice of diameter for b_3 with the partial assignment of b_1 and b_2 corresponding to the first component of the equivalent branch list. Thus, the PSQ entry in the i^{th} column of the initial testing block is the sum of the i^{th} PSQ component for b_3 and the first PSQ component on the equivalent branch list. Similarly, the i^{th} column PCOST entry is the sum of the i^{th} b_3 PCOST component and the first equivalent-branch PCOST component.

Next, the maximum entry in the PSQ row of the testing block is located. Initially, this will always occur in the first column. The PSQ and PCOST entries of this column become candidate components in a new equivalent branch list for b_1, b_2 and b_3. The index in the first column is then promoted to give result indicated in tabulation given below:

	1	2	3	4	5	6	7
PSQ	299	302	296	277	271	261	248
PCOST	60	62	74	90	106	124	152
INDEX	2	1	1	1	1	1	1

The largest PSQ entry is now in the second column whose PSQ and PCOST entries become the second component in the candidate list. The index of the second column is now increased to 2 to give a revised testing block shown below

	1	2	3	4	5	6	7
PSQ	299	280	296	277	271	261	248
PCOST	60	70	74	90	106	124	152
INDEX	2	2	1	1	1	1	1

The current candidate list is now this:

PSQ (310, 302, 299),

PCOST (54, 62, 60).

Each component on the candidate list corresponds to a partial assignment of b_1, b_2, and b_3. It is observed that the last PCOST component on the candidate list is smaller than the second PCOST component. According to Rothfarb et al (56), the partial assignment corresponding to the third component on the candidate list is always preferable to the partial assignment corresponding to the second component, since the former has a lower pipe cost and cannot result in a greater cost of compression. The second component is, therefore, eliminated from further consideration. The updated candidate list is:

Candidate list:	PSQ	(310, 299),
	PCOST	(54, 60).

Generally, when a new candidate is added to the list, all the candidates already on the list that have PCOST components not smaller than the latest entry are eliminated from further consideration. After each change, along procedure just described, the PCOST vector components in the candidate list will be in increasing order. The updating of the candidate list is, therefore, easy to perform.

As the serial-merge technique proceeds each of the 10 components of the equivalent branch list will form a candidate with each of the seven b_3 list components giving a total of 70 candidates to be processed. However, as can be seen, some of these candidates can be eliminated. For the example under consideration, only 16 candidates will be retained out of these 70 candidates. Candidates retained constitute an equivalent branch list for b_1, b_2 and b_3. In general only a small fraction of the possible candidates will be retained. An important point about this technique is that a greater percentage of the earlier and later candidates will generally be retained than those in the middle, so the power of the elimination procedure is not fully evident by the small example given.

By the use of both the parallel and serial-merge techniques, the entire tree can be processed to give a single equivalent branch list. The cost of the diameter assignment corresponding to each entry on this final list can then be evaluated by summing its pipe cost (PCOST) and the cost of pumping associated with the head loss or pressure drop (PSQ). The diameter assignment with the smallest cost is the optimal-diameter assignment. Pumping cost is obtained by first computing horse-power necessary to deliver total water required in the distribution system against total head loss or pressure drop PSQ, from this expression:

$$H.P = \frac{Qwh}{550\eta}$$

where

Q = flow delivered by the pump in cubic feet per second (cfs)

w = specific gravity of water,

h = total head, in feet,

η = pump efficiency.

Next, the pumping cost is obtained from this expression:

Pumping cost = 0.746xHPxIxT

where I = electric power cost per kilowatt hour,

T = number of hours of pumping.

In the development of the mathematical models for the analysis and optimal design of a water distribution network based on the graph theory this thesis has contributed in three ways to the present state-of-the-art of the analysis and optimal design of water distribution network. They are:

1. the solution of the set of non-linear component terminal equations, not by an iterative procedure but by the solution of simultaneous linear set of equations,
2. the introduction of the concept of water demand factor in the quantitative description of the various demand flows, at nodal points, and the propensity or the desire to use water,
3. the decomposition of a network to a tree-like structure, thereby, the entire network to be processed to give a single equivalent branch in the optimization process.

The next chapter of this thesis deals with the application of models developed to example problems and to an actual water distribution network. Examples of the decomposition of a network, with different flow patterns, into tree-like structures are also given.

CHAPTER III

THE APPLICATION OF MODELS DEVELOPED FOR THE ANALYSIS AND THE OPTIMAL DESIGN OF HYDRAULIC NETWORK

Introduction

The objective of this chapter is to apply the models developed to the analyses and optimal designs of an example problem and an actual network. The idea, here, is to demonstrate the validity of the models developed to a hypothetical case and also to a real one which, at this instance, is the Norman Oklahoma, water distribution network. It will be shown how the optimization model can be used to design an entirely new system by itself or to design an extension to the existing network. In the optimization model, the network is decomposed into first, a spanning tree and secondly into a number of branched-trees so that a modified form of the dynamic programming technique can be applied.

Application of Analysis Model

Figure (3-1) represents a physical system of pipes and junctions as a directed network. This is the example problem. Each pipe is assigned an arbitray positive direction with flow along the link in this direction being considered positive, while flow in the opposite direction is considered negative. By convention, therefore, a positive direction is given to a flow

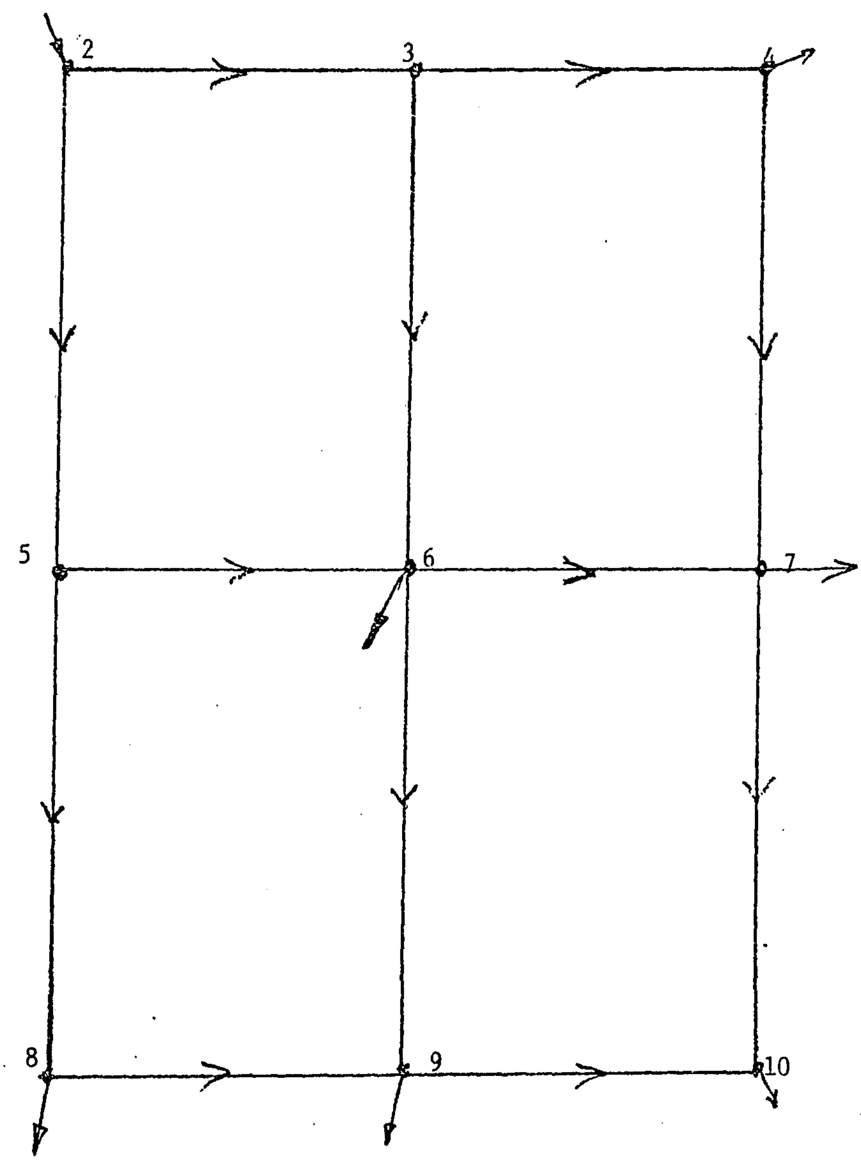


FIGURE 3-1: PHYSICAL SYSTEM OF PIPES AND JUNCTIONS AS A DIRECTED NETWORK.

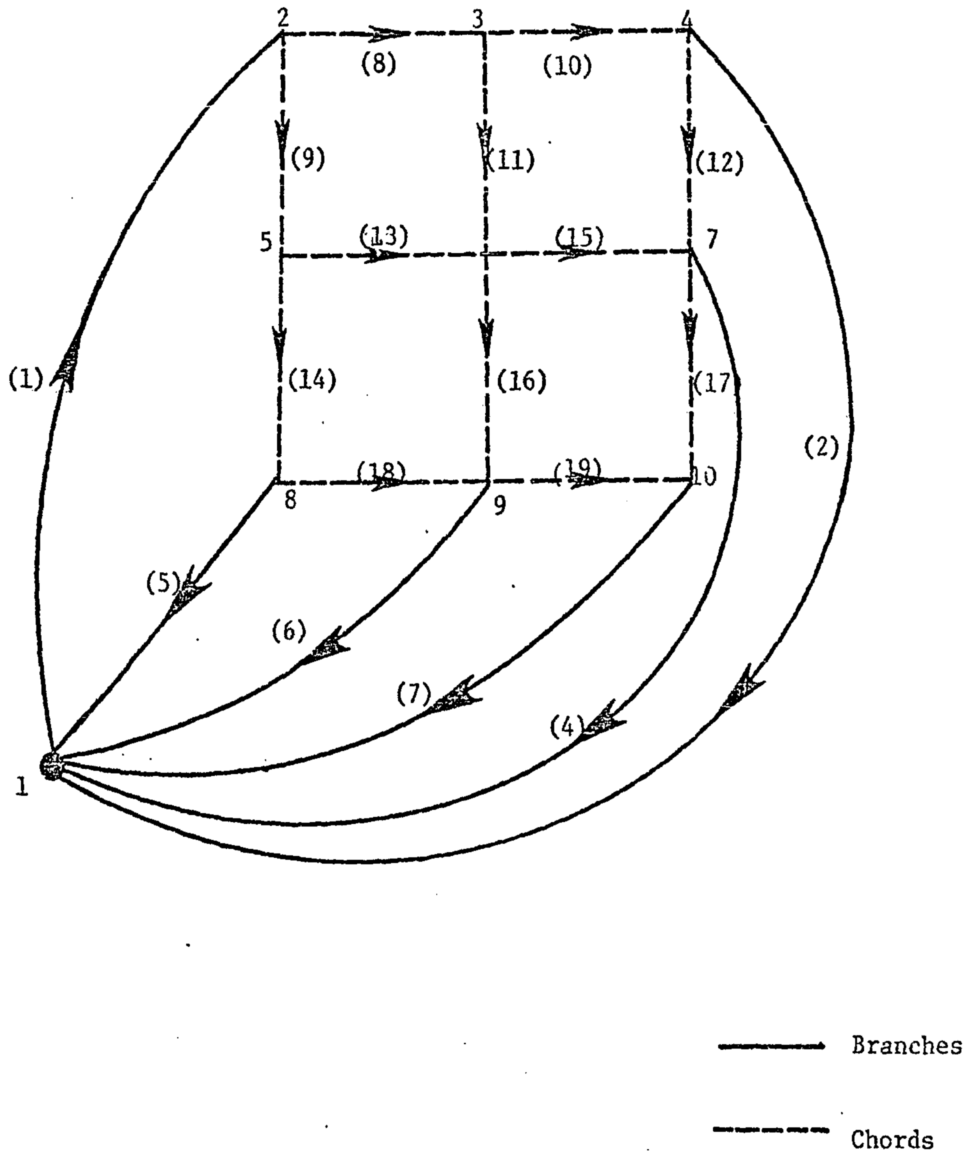


FIGURE 3-2: LINEAR GRAPH OF A PHISICAL SYSTEM SHOWN IN FIG. 3-1

direction oriented away from a node, while a negative direction is given to a flow direction oriented towards a node. Actual directions will become apparent on solving the network. Since this is an analysis problem, it is assumed that the size of each pipe in the network is known in advance.

To analyze this network using the linear graph theory approach, a linear graph of the physical system is to be drawn. Figure (3-2) represents the linear graph of the physical system shown in Figure (3-1). The link from node 6 to node 1 is omitted for clarity. The network shown comprises nine nodes and twelve elements or links. For ease in the formulation of the problem, a total of seven fictitious links and one fictitious node are added to the network. Each fictitious link is incident at the fictitious node which is numbered 1. Node 1 is, therefore, a reference node and it is analogous to grounding in an electrical circuit. The first six fictitious links or destination links represent outflow from the network. Consumptions or demands from the network are assumed to flow along such an element or link. The remaining one link originates from the reference node and is incident at the point of the network which serves as the source or the supply point. In the example, there is, thus a fixed rate of supply at node 2, and a fixed draw-off rate at nodes, 4, 6, 7, 8 9, and 10. The addition of fictitious links and node results in a total of nineteen links and ten nodes.

The cannonical forms of the various classes of components can now be formulated. The supply is assumed to be from a constant flow pump, thus, indicating one loading condition. The form of this component (also known as origin) equation is written as:

$$Y_{oi} = \text{known} \qquad i = 19$$

where

Y_{o_i} = known flow rate in m.g.d from origin or supply point i.

This assumption is made because in water supply industry, there are techniques available for estimating or forecasting total water requirement for a given community or supply area. Total water requirement will, henceforth, be referred to as total water demand.

For reasons given in chapter 2, the Hazen-Williams formula which expresses the non-linear, relationship between flow and head loss through a pipeline is used. The pipe flows are, therefore, modelled by the relationship

$$Y_i = k_i \Delta X_i \qquad i = 7, 8, 9, 10 \ldots\ldots, 18.$$

where

Y_i = flow on any link in m.g.d.

k_i = the resistance to flow on any link i.

$\Delta X_i = [H_i - H_j]^{0.54}$

$H_i - H_i$ = the head loss or pressure drop for flow on link i.

The resistance to flow on any link i is expressed as

$$k_i = 0.279 C_i D_i^{2.63} / L_i^{0.54}$$

where

C = Hazen-Williams friction coefficient for pipe i,

D_i = internal diameter of pipe i, in feet,

L_i = length of pipe i, in feet.

The use of ΔX_i transforms the non-linear flow equation to a linear one.

The demand regions or areas can be modelled by the equation

$$Y_d = k_d A_d \Delta X_d.$$

where

Y_d = demand flow in m.g.d. $d = 2,3,\ldots\ldots,7.$

k_d = model calibration factor,

A_d = demand index,

ΔX_d = (propensity or pressure drop)$^{0.54}$. ΔX_d is referred to as DELTAH in the computer program listings in Appendix C.

(Y_d may be known or unknown. k_d is determined from computer run but ΔX_d is unknown). The demand index, however, can be calculated. In this work A_d is assumed to be a function of the total demand, including wastage. It is taken to be linearly dependent on the population equivalent (in terms of demand) at respective demand node or region. Where draw-off or demand is known, one way or the other, the demand index is expressed as:

A_d = Nodal demand/Total demand.

This completes the first step in the solution of the problem.

The next step in the system solution is the selection of a tree in the linear graph. In this case, computer Program SYSFM which is described in Appendix C is used for this purpose. The program chooses a tree of the network made up of supply points, demand regions or areas and elements or links. It then formulates the cut-set (node equations) and, hence, the circuit or loop equations for the network in a form suitable for use in another program referred to as SYSAL. The demand, supply and link elements are respectively designated as D-element, O-element and L-element.

The input to the Program SYSFM is a coded map of the network being analyzed. Table 3-1 is such a map for the example problem. The coded map is derived from a linear graph of the network which is drawn initially in order to assign numbers to the nodes and elements. It is computationally advantageous to number the nodes consecutively with the number 1 assigned to the common node, as explained earlier. Further discussion on the input data is given in

TABLE 3-1

A CODED MAP OF AN EXAMPLE PROBLEM

TYPE OF ELEMENT	NUMBER ASSIGNED	FROM NODE	TO NODE
D	2	1	4
D	3	1	6
D	4	1	7
D	5	1	8
D	6	1	9
D	7	1	10
L	8	2	3
L	9	2	5
L	10	3	4
L	11	3	6
L	12	4	7
L	13	5	6
L	14	5	8
L	15	6	7
L	16	6	9
L	17	7	10
L	18	8	9
L	19	9	10
O	1	1	2

Appendix C.

By the use of Program SYSFM, all the demand links are selected as branches of the tree in addition to links 8, 10, and 13 which are re-assigned numbers 9, 7, 8, respectively. The remaining links, shown dotted in Figure (3-2), are referred to as chords. The equations in the Y-variables for the cut-set postulates for all elements can be written as:

$$\left[\begin{array}{c:c} U & A \end{array}\right] \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ \vdots \\ \vdots \\ Y_{16} \\ Y_{17} \\ Y_{18} \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} Y_{19} \end{bmatrix} = 0 \qquad (3\text{-}1)$$

where

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and}$$

$$
A = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

The equations for the X-variables around fundamental circuits of all elements i the graph in terms of elements in the tree are written next. These equations are:

$$
\begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \vdots \\ \Delta X_{17} \\ \Delta X_{18} \end{bmatrix} = \begin{bmatrix} U \\ \cdots \\ A' \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \vdots \\ \Delta X_8 \\ \Delta X_9 \end{bmatrix} \qquad (3\text{-}2)
$$

where

U = 9x9 unit matrix

$$A' = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

The component terminal equations explicit in the Y-variables are:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \\ \vdots \\ Y_{16} \\ Y_{17} \\ Y_{18} \end{bmatrix} = \begin{bmatrix} A'_1 & & & & & & & & & & \\ & A'_2 & & & & & & & & & \\ & & A'_3 & & & & & & & & \\ & & & A'_4 & & & & & & & \\ & & & & A'_5 & & & & & & \\ & & & & & A'_6 & & & & & \\ & & & & & & A'_7 & & & & \\ & & & & & & & A'_8 & & & \\ & & & & & & & & \ddots & & \\ & & & & & & & & & K_{16} & & \\ & & & & & & & & & & K_{17} & \\ & & & & & & & & & & & K_{18} \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \\ \Delta X_4 \\ \Delta X_5 \\ \Delta X_6 \\ \Delta X_7 \\ \Delta X_8 \\ \Delta X_9 \end{bmatrix}$$

where

$A'_1 = k_d A$ and $f(\Delta X_i)$ is taken as ΔX_i as explained earlier.

Substituting Equation (3-2) into Equation (3-3) and the result into Equation (3-1) we get:

$$\begin{bmatrix} A_1' & & & & & & & & \\ & A_2' & & & & & & & \\ & & A_3' & & & & & & \\ & & & A_4' & & & & & \\ & & & & A_5' & & & & \\ & & & & & \ddots & & & \\ & & & & & & K_{16} & & \\ & & & & & & & K_{17} & \\ & & & & & & & & K_{18} \end{bmatrix} \begin{bmatrix} U \\ \hline A \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \\ \Delta X_4 \\ \Delta X_5 \\ \Delta X_6 \\ \Delta X_7 \\ \Delta X_8 \\ \Delta X_9 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} Y_{19} \end{bmatrix} = 0 \qquad (3\text{-}4)$$

Replacing the A_i' and the K_i's by their respective numerical values and evaluating the matrix triple product we get:

$$\begin{bmatrix} 3.96 & -3.16 & -0.80 & 0.00 & 0.00 & 0.00 & 3.16 & -2.35 & 2.35 \\ -3.16 & 4.72 & -0.38 & -0.80 & -0.38 & 0.00 & -3.16 & 3.16 & -2.35 \\ -0.80 & -0.38 & 1.56 & 0.00 & 0.00 & -0.38 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.80 & 0.00 & 1.61 & -0.80 & 0.00 & 0.00 & -0.80 & 0.00 \\ 0.00 & -0.38 & 0.00 & -0.80 & 1.56 & -0.38 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -0.38 & 0.00 & -0.38 & 0.76 & 0.00 & 0.00 & 0.00 \\ 3.16 & -3.16 & 0.00 & 0.00 & 0.00 & 0.00 & 3.16 & -2.35 & 2.35 \\ -2.35 & 3.16 & 0.00 & -0.80 & 0.00 & 0.00 & -2.35 & 3.16 & -2.35 \\ 2.35 & -2.35 & 0.00 & 0.00 & 0.00 & 0.00 & 2.35 & -2.35 & 2.35 \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \\ \Delta X_4 \\ \Delta X_5 \\ \Delta X_6 \\ \Delta X_7 \\ \Delta X_8 \\ \Delta X_9 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} Y_{19} \end{bmatrix} = 0 \qquad (3\text{-}5)$$

It is seen that Equation (3-5) is a set of a simultaneous linear equations and can thus be solved by any suitable computer program. In this particular case, Gaussian Elimination procedure is used in the subroutine Gauss.

The solution of Equation (3-5) gives the branch flows. The chord flows are obtained by first substituting the branch pressures in Equation (3-2) for chord pressures and hence into Equation (3-3) for chord flows. Results of the element flows are shown in Table 3-2 from which it is seen that the demand flows sum up to the total supply flows.

The application of the analysis model to the Norman water distribution network follows the same procedure as described above. The difference lies in the size of the computer memory space requirement resulting from dealing with a larger network. The Norman network, as it existed in 1971, is as shown in Figure (3-3). Then, it comprised 140 pipe elements, 7 pumping source, and 5 reservoirs, including a proposed one. Other data, for example, pipe diameter length, "C" value, flow and flow direction, as indicated on Figure (3-3) are those compiled by the Pitometer Associates which analyzed the network at that time. 4 out of the 5 reservoirs are elevated. As can be seen, two of the elevated reservoirs were considered to be filling under the flow condition considered. Thus, these two reservoirs will be taken to be constituting demand nodes in addition to the other 81 demand or draw-off points, making a total of 83 demand nodes. Origin or supply nodes will be 10 in number.

As before, a linear graph of the network is drawn first. Figure (3-4) shows such a graph. It is seen that the addition of a reference node and fictitious links has increased the number of nodes and elements to 94 and 233 respectively. Fictitious links originating from the reference node are omitted for clarity. Next, a coded map of the network which serves as an input data to the Program SYSFM is prepared. Figure (C-1) shows such a map. This information is then fed into a computer for the selection of the formulation tree. A brief introduction to the linear graph theory is given in Appendix A. From this theory, if n is the number of nodes, then the number of branches in

TABLE 3-2: RESULTS OF THE DESTINATION ELEMENT FLOWS OF HYPOTHETICAL NETWORK

NODE	LINK FACTOR	DELTAH	HEAD LOSS (FT)	COMPUTED DEMAND (MGD)
1	0.0000	0.000	0.000	0.000
2	0.0000	3.374	9.485	0.000
3	0.1133	1.884	3.227	1.152
4	0.0000	2.222	4.381	0.000
5	0.1133	2.712	6.332	1.152
6	0.1417	4.225	14.379	1.440
7	0.1417	3.756	11.739	1.440
8	0.1417	4.225	14.379	1.440
9	0.3483	4.659	17.233	3.540
			TOTAL	10.164

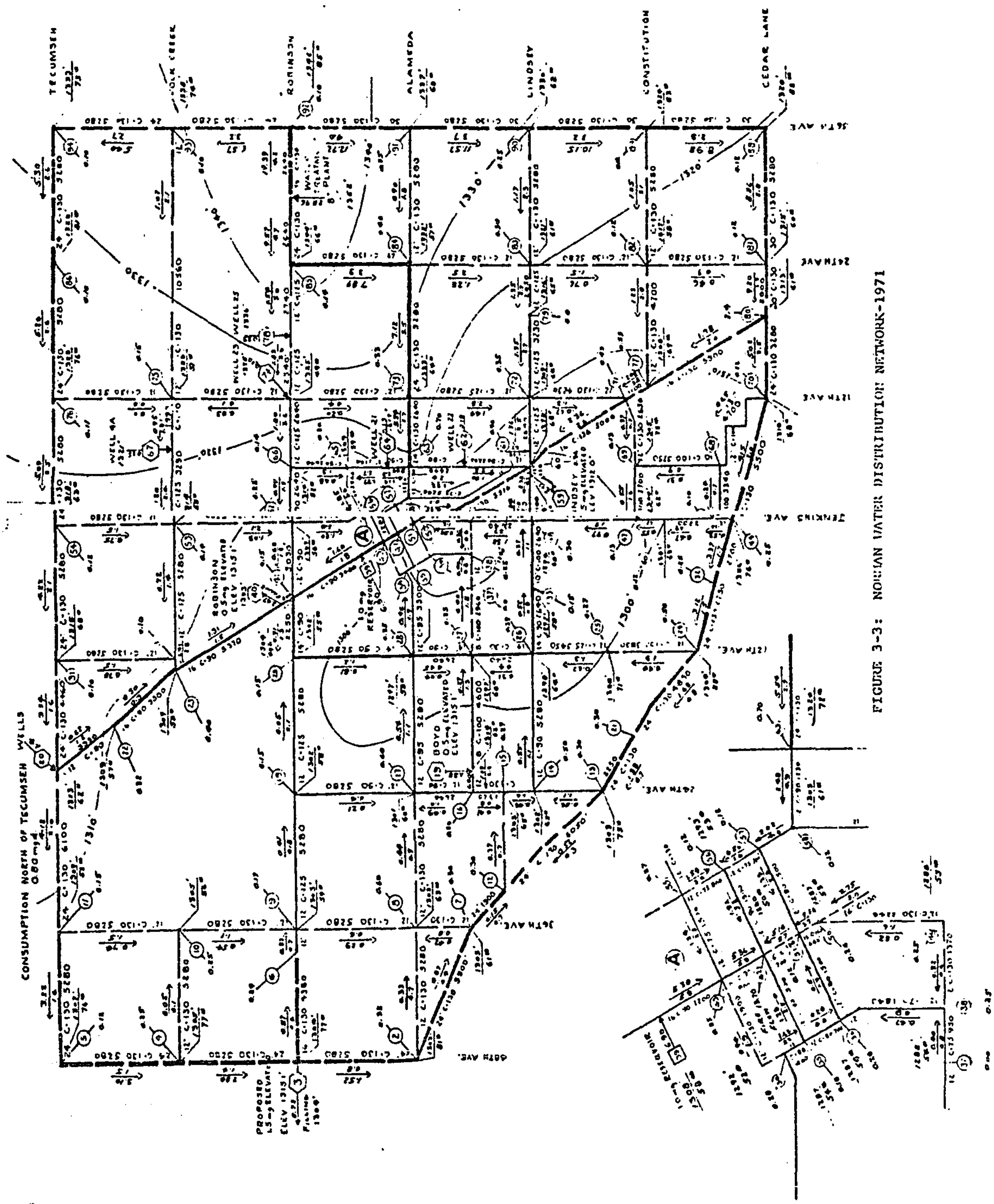


FIGURE 3-3: NORMAN WATER DISTRIBUTION NETWORK-1971

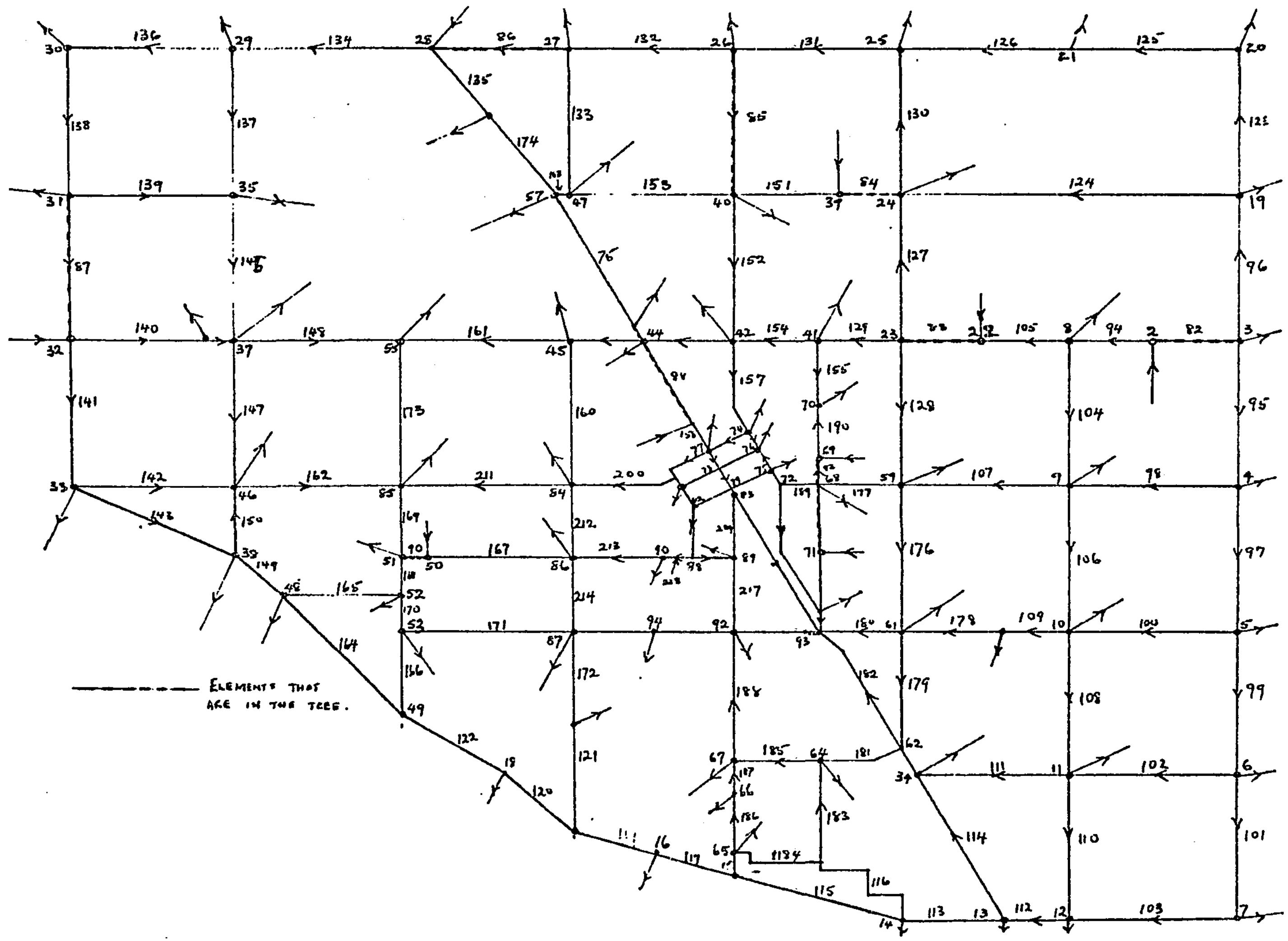


FIGURE 3-4: LINEAR GRAPH OF NORMAN WATER DISTRIBUTION NETWORK

the formulation tree is n-1. This means that the formulation tree for the Norman network will comprise 94-1 i.e. 93 branches. Since the Program SYSFM chooses all the 83 destination elements to form branches of the tree, it means that the remaining 10 branches are selected from the 140 pipe elements. Those pipe elements, 130 in all, not selected as branches become the chords of the formulation tree. In Figure (3-4), branches of the tree are as shown in bold lines. Fictitious links connecting the reference node, i.e. node 1, to either demand or supply points are omitted for clarity. The output from Program SYSFM is as indicated in Table C-1 of Appendix C. This serves as the input to Program SYSAL which formulates and solves the terminal equations. Solution of the terminal equations give values of estimated draw-offs at nodal points.

Application of the Optimization Model

Once again, an example problem will be used to illustrate the principle underlying the new approach to the optimal design of hydraulic network. Emphasis will placed on how networks with different flow patterns are decomposed into a number of parts with each part forming separate tree. The decomposition will result in a collection of trees referred to as forest. The technique will be extended to the Norman, Oklahoma, water distribution network. The new approach is in three phases; (1) the decomposition of the network into a spanning tree (2) the reduction of the spanning tree into a number of separate trees and (3) the application of the parallel merge and the serial merge techniques.

The decomposition of the network into a spanning tree is accomplished through the use of a computer program found on page 324 of Deo's book (63). The listings can be found in Appendix D. The input format for this program is referred to as the Two Linear Arrays. By this format, the network is represented by two arrays, say $F = (f_1, f_2, \ldots, f_e)$ and $H = (h_1, h_2, \ldots, h_e)$. In Figure (3-5a), the network is represented by the two arrays:

$F = (1,1,2,2,3,4,4,5,5,6,7,7,8,8,9,10,11)$,

$H = (2,4,3,5,6,5,7,6,8,9,8,10,9,11,12,11,12)$.

Each entry in these arrays is a vertex label. The i^{th} edge e_i is from vertex f_i to vertex h_i. At each stage in the algorithm a new edge is tested to see if either or both of its end vertices appear in any tree formed so far. Initially, there is no tree formed. The very first edge (f_1, h_1) considered will always occur in a spanning tree (or forest). Thus the spanning tree (or forest) generated by this algorithm is very much dependent on the ordering of the edges. The output of the program is in the form of ones and zeros in the corresponding positions of the two linear arrays as shown below for input array given above:

(1,1,1,1,1,0,1,0,1,1,0,1,0,1,1,0,0).

Each zero entry implies that the edge corresponding to that entry is considered as a chord. Figures (3-5a), (3-5b),(3-5c) and (3-5d) show the spanning trees of a 12 node 17 element network with different assumed flow patterns and supply points. Branches of the tree are shown in solid lines whilst chords are shown dotted. Figure (3-5d) appears to have a different tree from the others which all seem similar in spite of different flow patterns and supply points. In real terms, flow direction in each pipe element will be known following the network analysis by any method which must preceed the network optimization .

Having obtained a spanning tree (or forest) of the network, the next step in the optimization procedure is to reduce the spanning tree to a branched or a tree-link network to permit the two forms of optimization techniques discussed earlier (i.e. the parallel merge and the serial merge) to be applied.

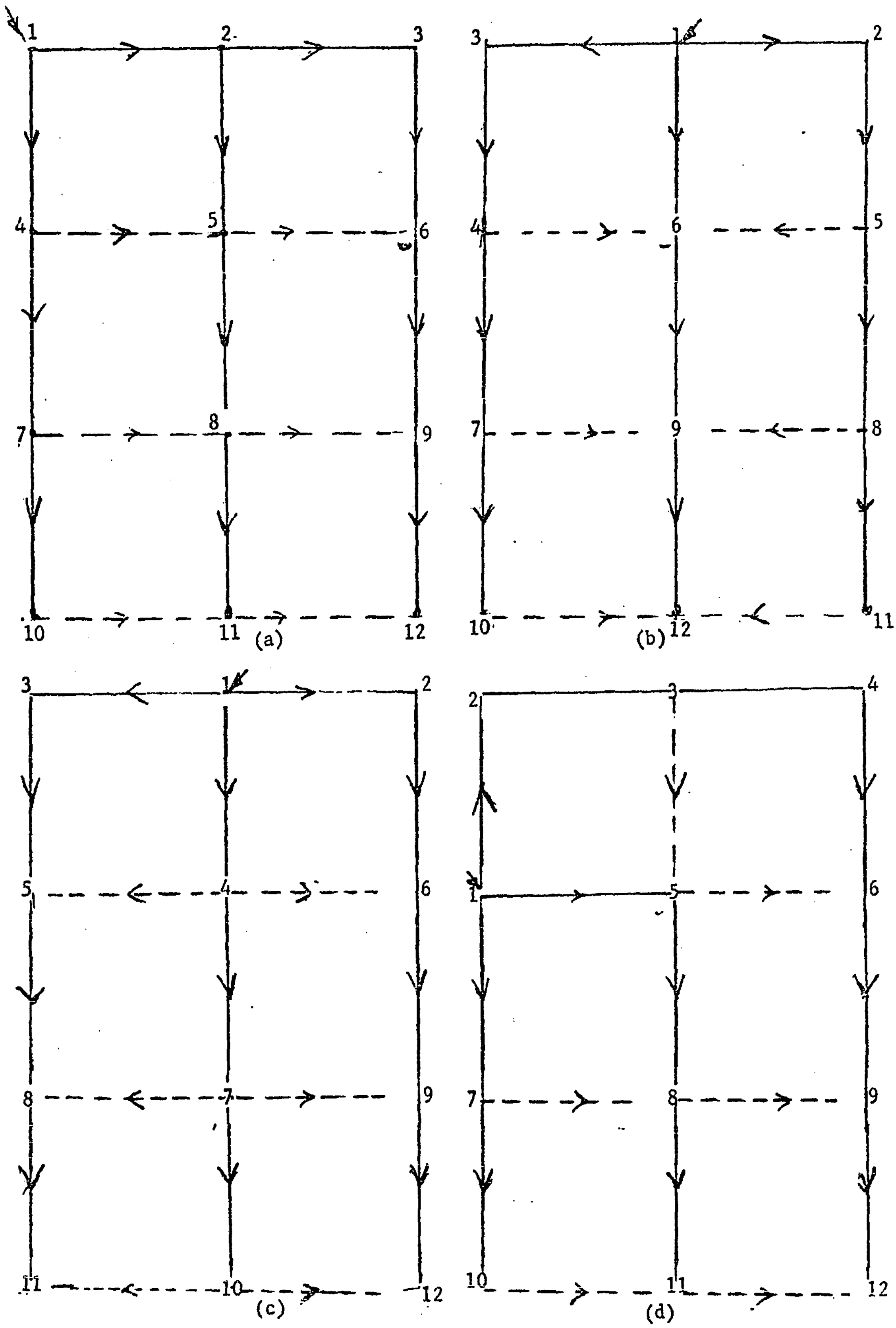


FIGURE 3-5: SPANNING TREES OF AN EXAMPLE PROBLEM.

The reduction is to be carried out taking account of the flow direction of each chord. For instance, if the flow direction on any given chord is from one node to another, this chord will be considered to be part of the node from which the flow along the chord originates. In other words, with reference to Figure (3-5a), three separate branched or tree-like network can be obtained: the first tree will consist of branches 1-4, 4-7, 7-10, chords 4-5, 7-8, and 10-11; the second tree comprises branches 1-2, 2-5, 5-8, 8-11, chords 5-6, 8-9, and 11-12; and the third part consists of branches 2-3, 3-6, 6-9, and 9-12. This done, the serial merge and the parallel merge techniques can be applied to one tree at a time commencing at each extreme point and working sequentially towards node 1, just as it is done in dynamic programming technique. The difference between this technique and that of dynamic programming is the absence of an objective function. This explains the reason for referring to the optimization model as a modified dynamic programming technique. Figure (3-6a) shows a 12 node, 17 element network of Figure (3-5a) reduced to three separate tree.

From this figure, it is noted that a serial merge will be performed on branches 7-10, and 10-11. Next, a parallel merge will be performed on the equivalent branch list obtained and the chord 7-8 and so on up to node 1. The second tree goes through this sequence; serial, parallel serial, parallel, and serial merges up to node 2. Finally, the third tree goes through three serial merges up to node 2. A parallel merge is then performed on equivalent branches obtained from the second and the third trees up to node 2. The resulting equivalent branch obtained is combined with branch 1-2 to form equivalent branch of the second and third trees. By combining the equivalent branch of the second and the third trees with that obtained from the first

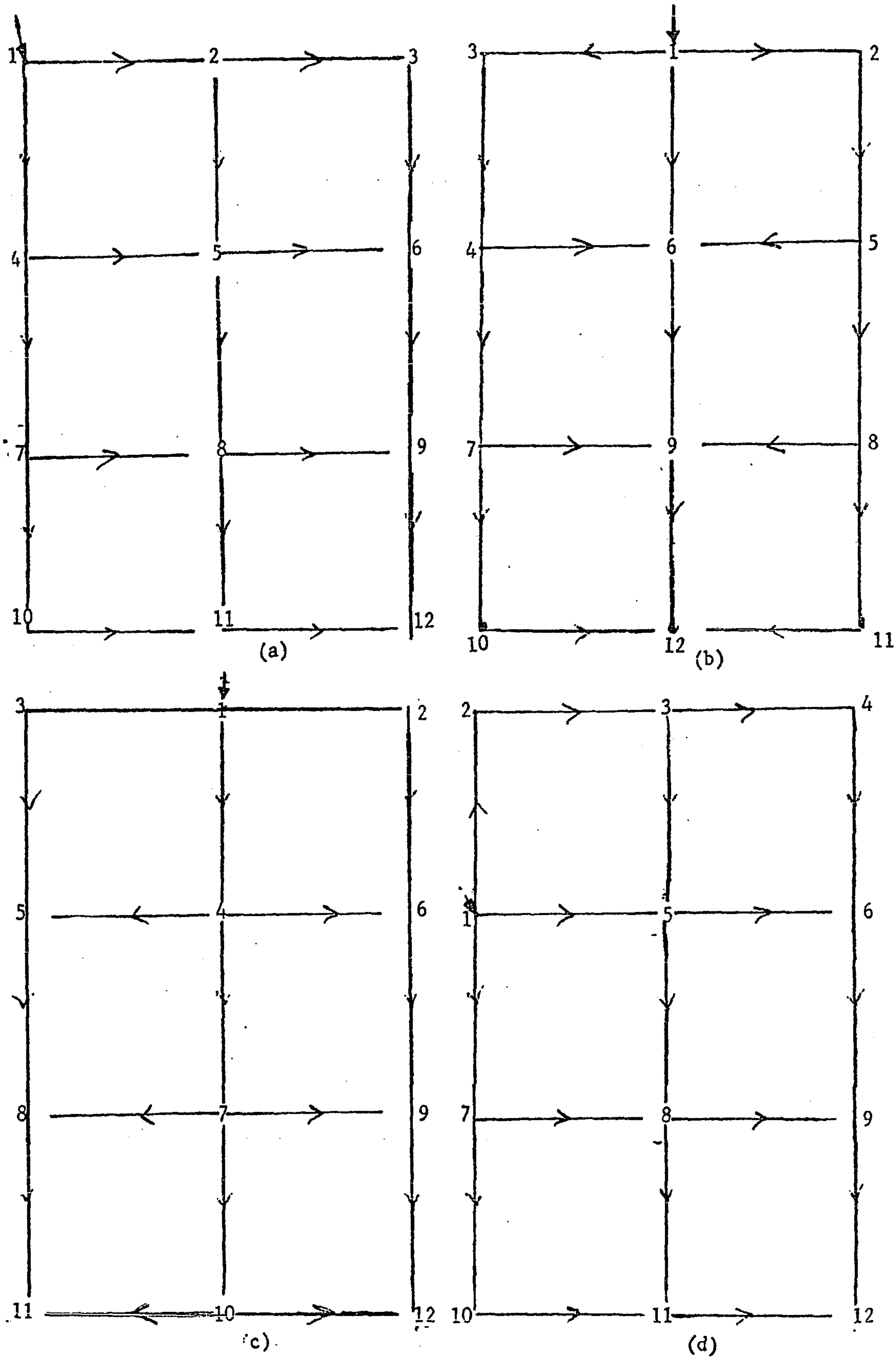


FIGURE 3-6: TREE-LIKE CONFIGURATIONS OF EXAMPLE PROBLEMS.

tree up till node 1 in a parallel merge gives a single equivalent branch for the entire network. The cost of the diameter assignment corresponding to each entry on this final list can then be evaluate by summing up its pipe cost (PCOST) and the cost of pumping associated with the head loss or pressure drop (PSQ). The diameter assignment with the smallest cost is the optimal diameter assignment. The optimization techniques described above were applied to (1) a hypothetical network shown in Figure (3-1), page 77, and (2) a portion of 1971 Norman Water Distribution Network including proposed extensions

Optimization of a Hypothetical Network

Two cases were considered; optimization of a hypothetical network without chords i.e. the spanning tree (see Figure (3-7a)) and then with chords (see Figure (3-7b)). In both Figures (3-7a) and Figures (3-7b) the numbering is done in such a way that the flow direction is from a node of a higher number to that of a lower number. The numbers in brackets refer to link numbers as used in the computer programs MERGEP AND MERGES. In Figure (3-7b) nodes 1 and 4, 2, and 6, 5 and 9, and 7 and 11 are, in fact, the same in each case but have been separated to give a tree-like configuration of the entire network based the spanning tree obtained for this network and the flow directien in each link obtained previously from a network analysis.

Input of the trees into the optimization models is in the form of Incidence Matrix. Incidence Matrices for Figures (3-7a) and (3-7b) are shown in Tables E-3a and E-3b respectively. For each case, three diameter selections, five diameter selection and seven diameter selection were . considered. Table E-1 in the Appendix E beginning at page 184 show all input data and computed head loss in feet (PSQ) and total installed cost in \$1,000 (PCOST) for each link and each diameter considered. Output of computer

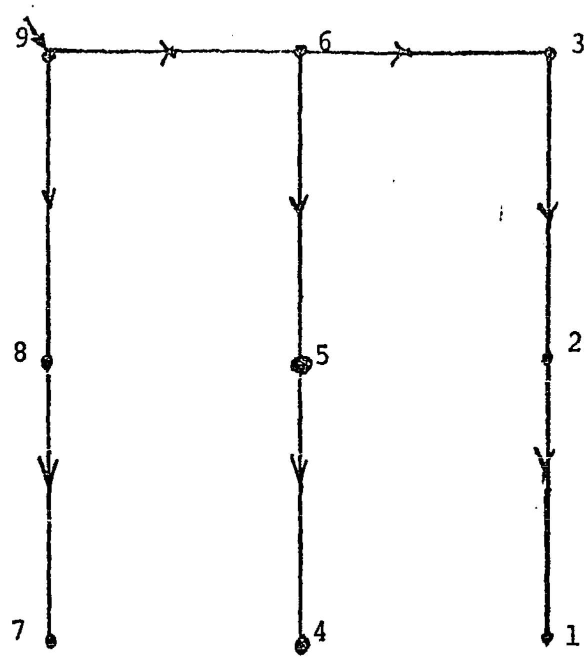


FIGURE (3-7a): SPANNING TREE OF EXAMPLE PROBLEM SHOWN IN FIGURE (3-1),PAGE 77

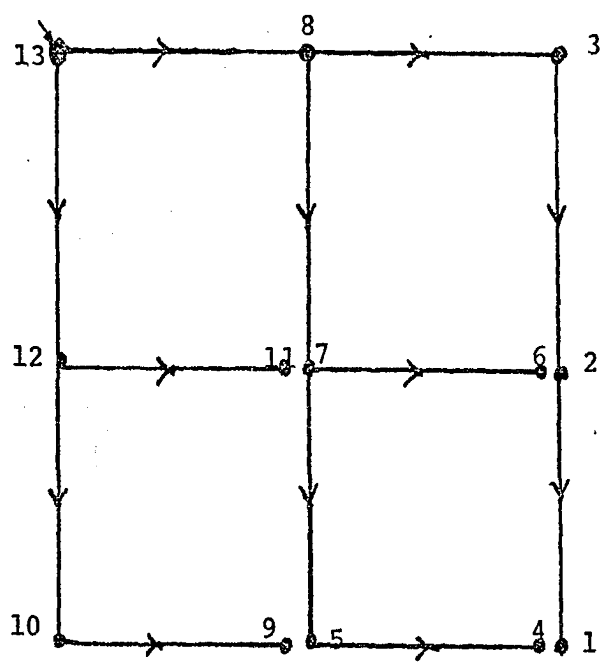


FIGURE (3-7b): TREE-LIKE CONFIGURATION OF ENTIRE NETWORK SHOWN IN FIGURE (3-1) programs MERGEP and MERGES are as shown in Table E-5 and Table E-6 in the Appendix E beginning at page 178.

The Application of Optimization Techniques to Existing Network Including Proposed Extensions

Methodology described above with reference to a hypothetical network can ba applied to any hydraulic distribution network or a gas distribution network. In dealing with an extension to an existing network a spanning tree is obtained for the ultimate network (which should comprise the existing network, the proposed network extension, the replacement and/or paralleling of one, two or more pipes) after a network analysis has been performed by any suitable method, for example, the Hardy Cross, to determine the flow pattern in the ultimate network.

The decomposition of the spanning tree into a number of separate trees follows procedures outlined earlier. This done, both the serial merge and the parallel merge techniques are applied to individual trees forming the spanning tree. There is one point to be noted here. In computing the PCOST and PSQ vectors, the internal diameters of pipes forming the existing network are condidered known and fixed, hence, no diameter ranges are considered for such pipes. However, for each pipe in the existing network being considered the same diameter must be specified for the number of different pipe sizes being considered in a given range.

In Figure (3-3) the dashed lines represent proposed expansion of the network. As mentioned earlier, the proposal was made in 1971 by a consulting Firm, namely, Pitometer Associates. The computer Program SPTREE was applied to this network to give a spanning tree of the network shown in Figure (3-8).

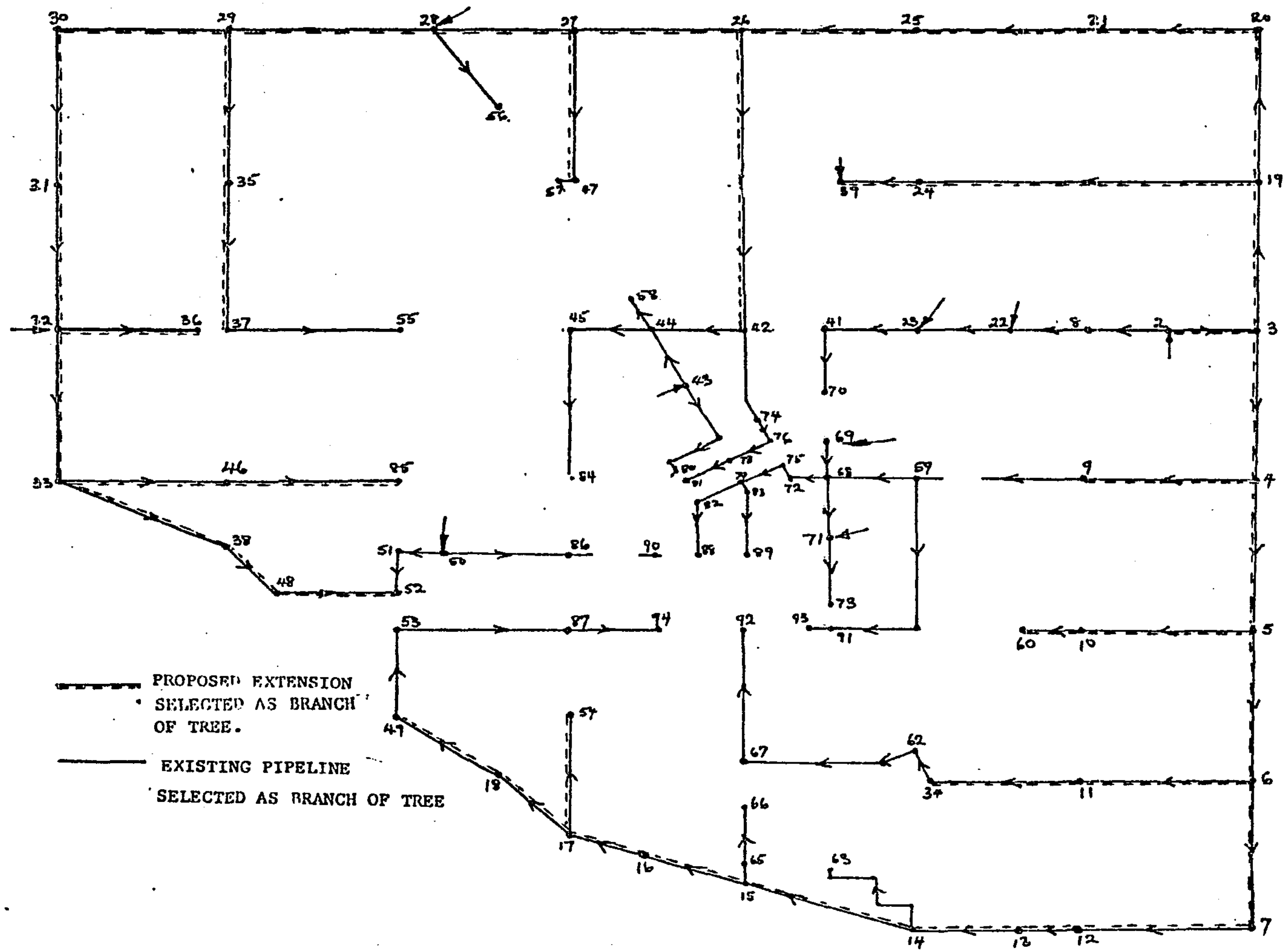


FIGURE 3-8: SPANNING TREE OF NORMAN WATER DISTRIBUTION NETWORK.

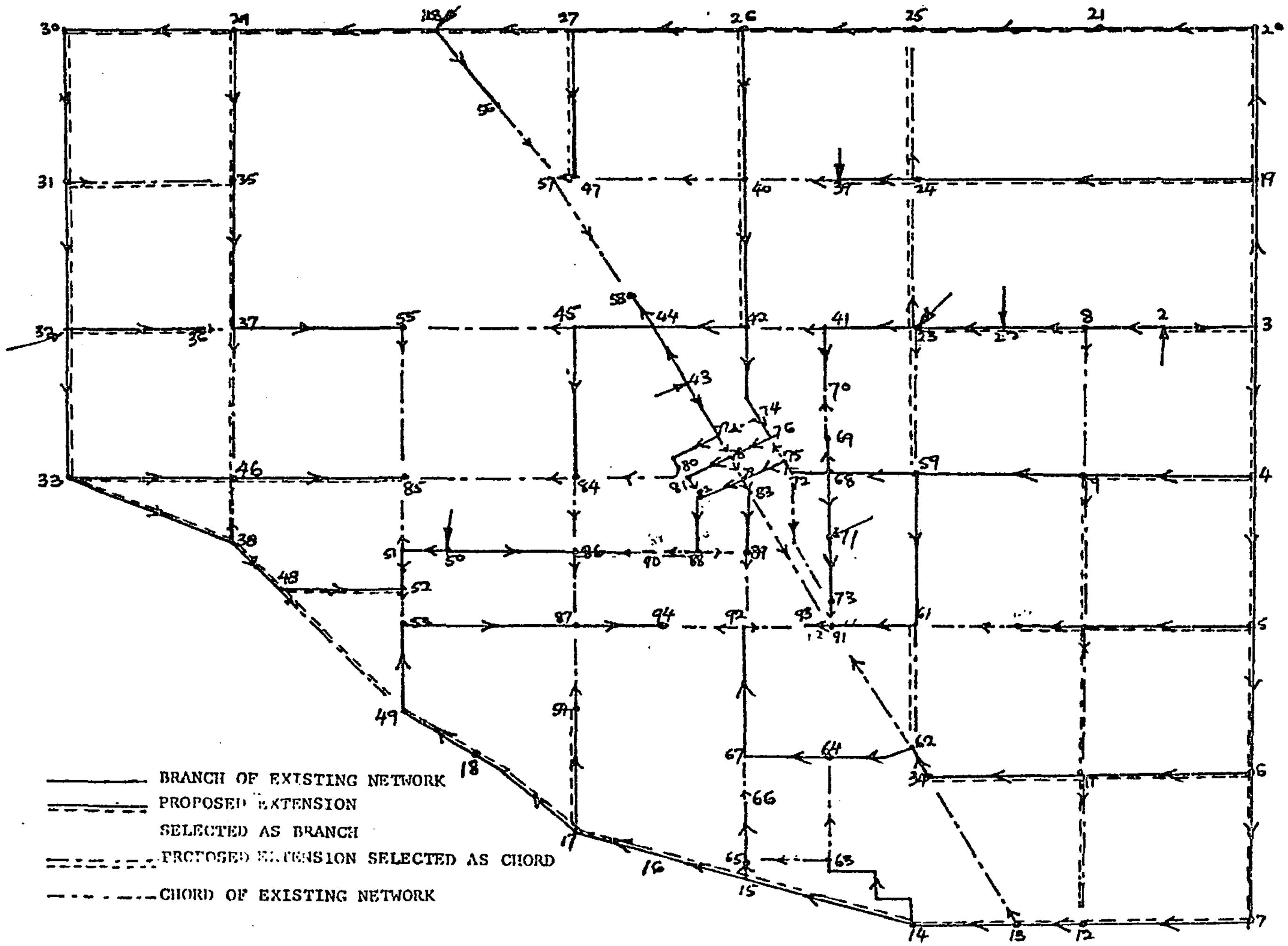


FIGURE 3-9: TREE-LIKE CONFIGURATION OF NORMAN WATER DISTRIBUTION NETWORK.

When chords are added to the spanning tree, a tree-like configuration of the Norman Water Distribution Network shown in Figure (3-9) is obtained. Bearing in mind that no pipe selections are made for pipes already existing in the network parallel merge and serial merge techniques are then applied to the entire network.

With reference to Figure (3-8) and Figure (3-9), the portion of the Norman Water Distribution Network to which the optimization techniques were applied comprises nodes 25, 20, 19, 3, 4, 5, 6, 7, 12, 13, 11, 10, 61, 59, 23, and 24. Like the hypothetical case, two cases were considered; optimization of the network without chords (i.e. the spanning tree) and the tree like configuration which includes chords. Figures (3-10a) and 3-10b) show the numbering system used in the programs MERGEP and MERGES. The numbering procedure followed is the same as described in the hypothetical case. Table E-2 in the Appendix E shown input data for the case of a 7-diameter selection. These cases are similar to those cases considered in the hypothetical case. Table 3-3 gives installed cost of various sizes of pipes considered. These costs are national (U.S.A) average costs for sanitary sewer used as a surogate-not a constraint on methodology. The author considers these costs to be approximate to those of water pipelines since the costs for the latter were not available to him. It must be borne in mind that these costs had to be used to demonstrate the procedure. Optimal sizes of pipes obtained through the use of these cost data will be different for a different set of cost data. As a final step, the designed network may be analysed once more for a check upon hydraulic balance. In this work final check did not produce change in hydraulic balance. Output of computer programs MERGEP and MERGEP and MERGES are shown in Table E-6 in the Appendix beginning at page 210.

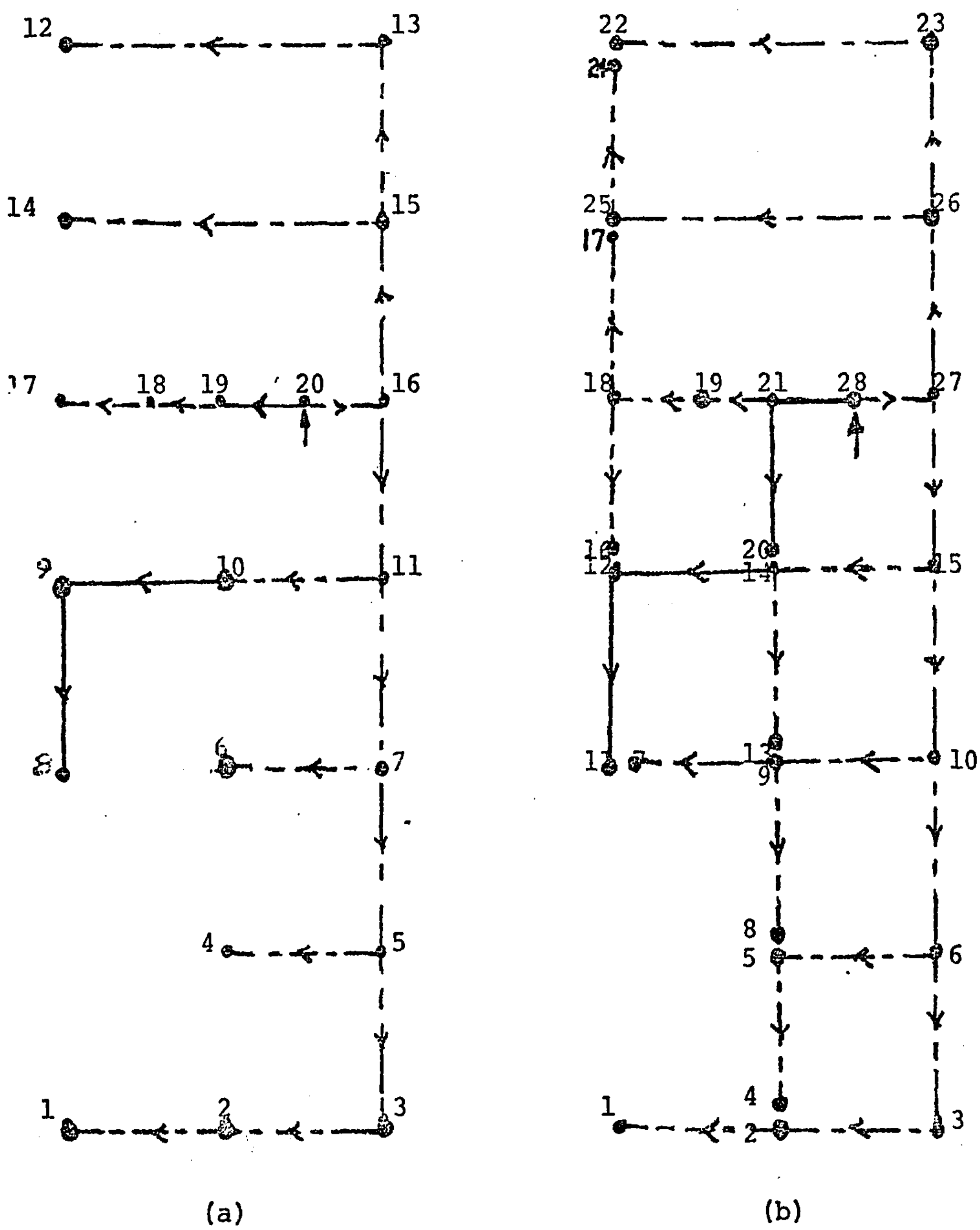


FIGURE 3-10: NUMBERING SYSTEM USED ON FIGURES 3-8 AND 3-9.

TABLE 3-3

INSTALLED COSTS PER LINEAR FOOT* OF VARIOUS SIZES OF PIPES CONSIDERED (January 1978 Dollars)

Pipe Diameter (inches)	Average Cost ($/foot)
6	24
8	43
10	47
12	59
15	73
18	94
21	118
24	124
27	136
30	178
36	215
42	250
48	302
54	337
60	418
66	445
72	483

* Includes associated appurtenances and non-construction costs. These are national (U.S.A) average costs for sanitary sewer. Source: reference (70) page 7-10.

CHAPTER IV

SUMMARY, CONCLUSION, AND RECOMMENDATION

Introduction

This chapter is intended to give a summary of a number of issues discussed in this thesis. One issue that motivated the author to work in this area is the fact that surveys conducted by the World Health Organization (1,2) for a selection of ninety-one developing countries in 1970, updated for selected countries in 1975 indicated that, with the exception of the Peoples Republic of China and a few other countries, it is estimated that about 20 per cent of the rural population had "adequate" water supplies in 1975. The picture is even very gloomy when viewed against the fact that, on the average, between 70 per cent and 80 per cent of the population in the Developing or Third World countries live in rural areas. It is not surprising, therefore, that the incidence of water borne diseases is roughly by the same proportion.
To reduce this alarming situation considerably, if not eliminated altogether, will require the use of increased quantities of water of a higher bacteriological quality for all uses. Because the provision and the distribution of wholesome water entails heavy expenditure, there is the need for the efficient utilization of financial resources. It is the opinion of the author that the analysis and optimal design of a water distribution network is a contribution to efforts being made worldwide to distribute, cheaply, wholesome water to the

needy. The treatment of this subject is by no means exhaustive. To this end, the chapter will conclude with recommendation on further research areas.

Summary

In chapter 1, a brief review of five techniques developed so far was given. From the literature review, it is obvious that past efforts in the development of techniques for hydraulic network analysis had concentrated on solving the set of non-linear equations expressing relationships between flow and head loss in a given pipe. Some of these techniques appear to be mathematically complicated. This thesis has demonstrated that by judicious application of the linear graph theory, the problem is very much simplified and that it is possible for flow along each element of the network and its corresponding head loss to be determined directly by non-iterative procedure. The key to the solution of the network analysis problem is the ability to express, mathematically, the demand or consumption at each node in terms of the "desire" or "propensity" to consume that commodity and the inclusion of a demand calibration factor to facilitate model calibration. Unlike the Hardy Cross method and the Newton-Raphson method, the analysis model developed does not require initialization. However, it is envisaged that the application of the analysis model to a very large network may result in problem with computer memory space as a result of the addition of fictitious links.

Techniques currently in use for the optimal design of a hydraulic network have been reviewed. Some of these techniques include what is termed a "check design" or a random search technique, non-linear programming, linear programming and dynamic programming. The use of the "check design" or the random search technique is cumbersome and time consuming, whereas, the linear programming technique invariably involves the linearization of the non-linear objective function. Thus, solution obtained by this approach is often far removed from the true optimum. Moreover, it has been noted that the use of linear programming method may result in branched or tree-like network. Dynamic programming method, which has the potential of coping with the solution of non-linear cost minimization problems, has been applied successfully to a single looped network, and branched or tree-like network. In chapter III, it has been demonstrated that a modified version of the dynamic programming method can be applied to multi-looped network.

Table 4-1 through Table 4-4 give a summary of results obtained through the use of optimization techniques developed in this thesis. In this work, the range of diameter selection for each link or pipe was selected around the pipe size used in the network analysis preceeding the optimization procedure. In other words, if commercially available pipe sizes are, say, 8", 10", 12", 15", 18", 21", 24", 27", and so on and if 15" pipe size is used for a particular link in the network to obtain a balanced flows then the range for a 3-diameter selection will be 12", 15", and 18"; that for a 5-diameter selection will be 10", 12", 15", 18", and 21" and so on. This needn't be the rule. The program user is free to consider any diameter ranges provided the lower and upper limits of flow velocities to be maintained in each pipe or link are satisfied. But it must be noted that in the serial merge of two links, the largest diameter size in each case is chosen. Therefore, if a 15" diameter pipe used in the network analysis is to be retained then the range for a 3-diameter selection for that particular link should be 10", 12", and 15", or for a 5-diameter selection, the range should be 6", 8", 10", 12", and 15". This observation does not apply to the optimal selection of pipes based on the parallel merge technique.

Comparison of results obtained for both the hypothetical case and a portion of the Norman Water Distribution Network indicates that, in each case, optimal pipe sizes selected for branches of the spanning trees obtained in the case where chords are not considered are almost the same as when chords are considered. However, diameters selected for the chords are invariably larger than what is referred to as the original diameter. Based on these observations a true optimum is obtained by considering the entire network (i.e. the spanning tree and the chord).

With reference to the total head loss and the total installed cost (see Table 4-2), it is deduced that, in the hypothetical case, by considering 5-diameter range and 7-diameter range in addition to the 3-diameter range, the reductions total head losses are 53.90 per cent and 81.06 per cent respectively. Total installed costs of pipeline, however, increase; the corresponding figures are 17.11 per cent and 45.41 per cent. For the portion of the Norman Distribution Network re-designed (see Table 4-4) and for the same cases considered, redutions in total head losses for the a 5-diameter range and a 7-diameter range in comparison to the 3-diameter range are 32.05 per cent and 54.30 per cent respectively. Corresponding figures for increases in total installed costs are 23.39 per cent and 50.14 per cent. It will be recalled that the horsepower developed by a pumping unit is given by the expression

Horse Power = Specific gravity of fluid x Total head loss
x Total fluid/550 x Pump efficiency.

Provided that (a) the specific gravity of fluid (b) the total flow or supply to the network and (c) pump efficiencies are held constant, then the per cent reductions in total head losses represent per cent reductions in horse power requirements which in turn reflect on per cent reductions in energy costs or pumping costs.

The use of PCOST and PSQ vectors in the partial assignment of diameters

TABLE 4-1: RESULTS OF OPTIMAL DESIGN OF A HYPOTHETICAL PROBLEM: CHORDS NOT CONSIDERED

LINK	ORIGINAL DIA.	3-DIAMETER SELECTIONS			5-DIAMETER SELECTIONS			7-DIAMETER SELECTIONS		
		OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)	OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)	OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)
1	12	15	8.51	292	18	3.50	376	21	1.65	472
2	15	18	4.67	376	21	2.20	472	24	1.15	496
3	21	24	2.58	496	27	1.45	544	30	0.87	712
4	12	15	4.91	292	18	2.02	376	21	0.95	472
5	15	12	15.50	236	15	5.23	292	15	5.23	292
6	21	24	4.97	496	27	2.80	544	30	1.67	712
7	15	18	10.06	376	21	4.75	472	24	2.48	496
8	24	21	12.70	472	24	6.63	496	27	3.74	544
			63.90	3,036		28.58	3,572		17.74	4,196

	5-DIAMETER	7-DIAMETER
Reduction in total head loss (%)	55.27	72.23
Increase in total installed cost (%)	17.35	38.21

Capitalized Cost=(O+M)/r + Capital Cost of pipeline

where O = Operating Cost.

M = Maintenance Cost

r = Interest rate.

If M is neglected, and operating (pumping) cost is assumed to be $100 for the 3-diameter selections, then capitalized cost are as follows:

Capitalized Cost: $4,702.67 $4,317.50 $4,658.83

r is assumed to be at the rate of 6 per cent per annum.

TABLE 4-2: RESULTS OF OPTIMAL DESIGN OF A HYPOTHETICAL PROBLEM: CHORDS CONSIDERED

LINK	ORIGINAL DIA.	3-DIAMETER SELECTIONS			5-DIAMETER SELECTIONS			7-DIAMETER SELECTIONS		
		OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)	OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)	OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)
1	12	15	8.51	292	18	3.50	376	21	1.65	472
2	15	18	4.67	376	21	2.20	472	24	1.15	496
3	21	24	2.58	496	27	1.45	544	30	0.87	712
4	12	15	7.23	292	18	2.97	376	21	1.40	472
5(4)	12	15	4.91	292	18	2.02	376	21	0.95	472
6	12	10	26.05	188	12	10.72	236	15	3.62	292
7(5)	15	15	5.23	292	15	5.23	242	18	2.15	376
8(6)	21	24	4.97	496	27	2.80	544	30	1.67	712
9	15	18	3.32	376	21	1.57	472	24	0.82	496
10(7)	15	18	10.06	376	18	10.06	376	24	2.48	496
11	15	12	37.70	236	15	12.72	292	18	5.23	376
12(8)	24	21	12.70	472	27	3.74	544	30	2.24	712
	TOTAL		127.93	4,184		58.98	4,900		24.23	6084

Reduction in total head loss (%) 53.90 81.06

Increase in total installed cost (%) 17.11 45.41

Capitalized Cost: $5,850.67 $5,668.33 $6,399.67

TABLE 4-3: RESULTS OF OPTIMAL DESIGN OF A PORTION OF NORMAN WATER DISTRIBUTION NETWORK (1971) INCLUDING PROPOSED EXTENSION: CHORDS NOT CONSIDERED

LINK	ORIGINAL DIA	3-DIAMETER SELECTIONS			5-DIAMETER SELECTIONS			7-DIAMETER SELECTIONS		
		OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)	OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000	OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)
1	30	36	0.49	281	42	0.23	327	48	0.12	395
2	30	36	1.88	1135	42	0.89	1320	48	0.46	1594
3	30	36	1.96	1135	42	0.92	1320	48	0.48	1594
4	12	12	7.80	311	15	2.63	385	18	1.08	496
5	30	36	2.49	1135	42	1.18	1320	48	0.61	1594
6	12	12	9.64	311	15	3.25	385	15	3.25	385
7	30	27	13.45	718	24	23.87	654	24	23.87	654
8	12	12	13.85	311	12	13.85	311	12	13.85	311
9	24	24	8.25	654	24	8.25	654	24	8.25	654
10	12	15	1.72	385	18	0.71	496	21	0.33	623
11	30	27	16.37	718	24	29.05	654	48	0.99	1594
12	24	27	59.41	14361	30	35.56	18796	36	14.63	22704
13	24	27	3.20	718	30	1.92	939	36	0.79	1135
14	12	12	155.90	6230	15	52.59	7708	18	21.64	9926
15	24	27	4.69	718	30	2.80	939	24	8.31	654
16	36	42	2.08	660	48	1.08	797	54	0.61	889
17	12	15	3.67	185	18	1.51	238	21	0.71	299
18	12	10	20.73	128	8	61.46	117	10	20.73	128
19	24	24	8.11	327	24	8.11	327	24	8.11	327
	TOTAL		335.69	30,421		249.86	37,360		128.82	45956

Reduction in total head loss (%) 25.57 61.63

Increase in total installed cost (%) 22.81 51.07

Capitalized Cost: $32,087.67 $38,600.50 $46,595.50

TABLE 4-4: RESULTS OF OPTIMAL DESIGN OF A PORTION OF NORMAN WATER DISTRIBUTION NETWORK (1971) INCLUDING PROPOSED EXTENSION: CHORDS CONSIDERED

LINK	ORIGINAL DIA.	3-DIAMETER SELECTIONS			5-DIAMETER SELECTIONS			7-DIAMETER SELECTIONS		
		OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)	OPTIMAL DIA (INS)	PSQ (FT)	PCOST (1000)	OPTIMAL DIA. (INS)	PSQ (FT)	PCOST ($1000)
1	30	36	0.49	281	42	0.23	327	48	0.12	395
2	30	36	1.87	1135	42	0.88	1320	48	0.46	1594
3	30	36	1.96	1135	42	0.92	1320	48	0.48	1594
4	12	15	0.53	385	18	0.22	496	21	0.10	623
5(4)	12	12	7.80	311	15	2.63	385	18	1.08	496
6(5)	30	36	2.49	1135	42	1.18	1320	48	0.61	1594
7	12	15	3.35	385	18	1.38	496	21	0.65	623
8	12	12	4.24	311	15	1.43	385	15	1.43	385
9(6)	12	12	9.64	311	15	3.25	385	18	1.34	496
10(7)	30	27	13.45	718	24	23.87	654	24	23.87	654
11(8)	12	12	13.85	311	12	13.85	311	12	13.85	311
12(9)	24	24	8.25	654	24	8.25	654	24	8.25	654
13	12	10	30.74	248	10	30.74	248	10	30.74	248
14(10)	12	15	1.72	385	18	0.71	496	21	0.33	623
15(11)	30	27	16.37	718	24	29.05	654	48	0.99	1594
16	12	10	0.63	248	12	0.26	311	15	0.09	385
17	12	15	0.28	385	18	0.12	496	21	0.05	623
18	12	15	3.67	185	18	1.51	238	21	0.71	299
19	12	12	8.53	161	10	20.73	128	10	20.73	128
20	24	24	11.17	654	24	11.17	654	24	11.17	654
21	24	24	8.11	327	24	8.11	327	24	8.11	327
22(12)	24	27	59.41	14361	30	35.56	18796	36	14.63	22704
23(13)	24	27	3.20	718	30	1.92	939	36	0.79	1135
24	12	15	0.13	385	18	0.05	496	21	0.03	623
25(14)	12	12	155.90	6230	15	52.59	7708	18	21.64	9926
26(15)	24	27	4.69	718	30	2.80	939	24	8.31	654
27(16)	36	42	2.08	660	48	1.08	797	54	0.61	889
TOTAL			374.55	33455		254.49	41280		171.17	50231

	5-DIAMETER	7-DIAMETER
Reduction in total head loss (%)	32.05	54.03
Increase in total installed cost (%)	23.39	50.14

Capitalized Cost: $35,121.67 $42,412.50 $50,997.17

is similar to the well known method used in the optimal design of a water transmission pipeline under pressure. As mentioned in Chapter II the optimization model is applicable to a network fed either under pressure or gravity or both.

Conclusion

In general, this thesis has contributed to the knowledge with respect to the analysis and optimal design of a hydraulic network in the following manner:

1. Through the application of Theorems in linear graph theory, it is possible to obtain a balance in a hydraulic network by non-iterative procedure and without initialization.

2. The analysis model allocates demands at various nodal points on the network very efficiently. This model has a tremendous potential in making full use of numerous water demand forecasting models already developed in the allocation of water on local district and regional levels.

3. The use of the spanning tree concept of PCOST and PSQ vectors simplifies the optimization procedure considerably. The technique developed in this thesis offers the network designer a wide range of choice in the optimal design of network based on commercially available pipe sizes. This is possible because the technique is very flexible.

4. Another aspect of the use of the spanning tree concept is that branches and chords determined, through the use of the spanning tree concept, can be considered as the primary mains and the secondary mains respectively and designed as such. Hitherto, the selections or the determinations of primary mains and secondary mains have been made in an arbitrary fashion.

5. By the development of simplified techniques for the analysis and the optimal design of both small and large distribution networks for water supply.

Although emphases have been placed on the use of models developed on hydraulic network, they are by no means restricted to this areaa alone. Both models can quite easily be applied to all other networks meant to transport either fluid or gaseous substances. In the analysis model, however, the only change necessary will be the inclusion of an appropriate non-linear relationship between the flow and head loss in a given conduit. In coming up with PCOST vector, all types of cost may be considered and then reduced to a uniform annual cost by the use of appropriate formulas such as the Capital Recovery Factor or the Present Worth Discount Factor.

Recommendation for Further Research and Application of the Models

In closure, the following recommendations for further research, development, and the application of the models developed are made:

1. The analysis model is very efficient in the allocation of demands at nodal points. Although the theory is very sound on the computation of chord flows, the Program SYSAL as it exists in its original form does not give balanced flows in the network. In this work, however, balanced flows were obtained for the hypothetical problem by making these three changes. First, the addition of fictitious destination links (with zero link value) to non-draw-off points, thus making the number of destination links equal to the number of nodes on the network, produced a cut-set matrix in the nodal form.

Secondly, the leading non-zero figure on each line of the circuit matrix obtained from the resulting cut-set matrix was put to zero to simplify the computation of chord from DELTAH values computed.

Thirdly, for the same reason given above, computer statements between lines 107 and 121, inclusive, had to be inserted. Such insertions automatically made DELTAH (or Head loss) on some links equal as a result of filling in the lower triangular matrix of matrix G. By this change it is possible to obtain balanced flows but sum of head losses in a closed loop is not necessarily zero. A way to get round this is to obtain field measurement of head loss for each second link appearing in each column of modified matrix G. The author considers this as acceptable in practice. Nevertheless, it is recommended that further work be done in this area to make the field measurements unnecessary. Such work should focus on reprogramming the part of Program SYSAL which affects matrix G.

2. The analysis model may be modified to handle continous flow conditions by making pressure and flow time dependent. This is analogous to an electric circuit with some inductance and capacitance elements in it. The analytical techniques for analyzing such systems are already well developed in the field of electrical circuit theory.

3. The program for the parallel merge and the Subroutine MERGES, which performs the serial merges, are still very crude and unsophisticated. The input to these programs is in the form of incidence matrix. In this thesis more emphasis was placed on making them work rather than on ingenuity of programming. Further work on this program is needed to make it more efficient. It should also be possible for coordinates of nodal points of a network to be fed into the program as input instead of the incidence matrix. Taking account of flow directions, improved program should select tree-like configuration of the network automatically by combining it with Program SPTREE.

4. The addition of fictitious links and reference node to the real network increases the number of pipes in the network by between 50 per cent and 65 per cent. This will be a limitation on the size of network to be analyzed.

5. The use of Hardy Cross Method of network analysis in conjunction with

the modified dynamic programming technique developed offers some advantages over existing optimization techniques which just balance hydraulics but do not optimize costs.

6. Finally, it is suggested that the maximum-flow minimum-cut theorem and vulnerability analysis or reliability analysis used extensively in Network Analysis Problems be extended to the analysis and the optimal design of hydraulic distribution network.

It is the hope of the author that this thesis has been successful in advocating the application of the Linear Graph Theory to the analysis and the optimal design of a hydraulic network. The important message being put across to all concerned is that the analysis and the optimal design of a distribution system for water supply is a network problem and should, therefore, be considered and approach as such by making full use of NETWORK ANALYSIS techniques.

BIBLIOGRAPHY

1. World Health Organization, "Community Water Supply Program; Progress Report by Director General," Geneva, 1972.

2. World Health Organization, "Community Water Supply and Wastewater Disposal, Mid-Decade Progress Report," Geneva, 1976.

3. United Nations Economic and Social Council, "Report of the United Nations Water Conference," E/C.7/L.58, 1977.

4. Reid G.W., and Wolfenson, L.B., "How Electric Calculators Solve Water Distribution Problems," Public Works, 84:10:78, 4 pages, Oct. 1953.

5. Gross, Hardy, "Analysis of Flow in Networks of Conduits or Conductors," Bulletin No. 286, University of Illinios Engineering Experimental Station, Urbana, Ill:, 1936.

6. Hoag, L.N., and Weinberg, G., "Pipeline Network Analysis by Electronic Digital Computer," Journal of the American Water Works Association, Vol. 49, 1957, pp. 517-524.

7. Graves, Q.B., and Branscome, D., "Digital Computers for Pipeline Network Analysis," Journal of the Sanitary Engineering Division, ASCE, Vol. 84, No. SA2, April, 1958, pp. 1608-1-1608-18.

8. Adams, R.W., "Distribution Analysis by Electronic Computer," Journal of the Institute of Water Engineers, Vol. 15, 1961 pp. 415-428.

9. Van den Berg, C., "An Iterative Method for Calculating the Pressure and Flow Distribution Pipeline Networks." Werktuig-en Scheepbouw,

Vol. 14, No. 12, 1963, pp. W203-W211.

10. Marlow, T.A., et al., "Improved Design of Fluid Networks with Computers," Journal of the Hydraulics Division, ASCE, Vol. 92, No. HY 4 Proc. Paper, July, 1966, pp. 43-61.

11. Dillingham, J.H., "Computer Analysis of Water Distribution Systems," Water and Sewage Works, Feb., 1967, pp. 43-45.

12. Camp, J.R., and Hazen, H.L., "Hydraulic Analysis of Water Distribution System by means of an Electric Network Analyzer," Journal of New England Water Works Association, 48:4 383-407, Dec., 1934.

13. Camp, T.R., "Hydraulics of Distribution Systems-same Recent Developments in Method of Analysis," Journal of New England Water Works Association 57: 334-358, Dec., 1943.

14. McIlroy, M.S., Direct Reading Electric Analyzer for Pipeline Networks," Journal of the American Water Works Association, Vol. 42, April, 1950, p. 347.

15. McPherson, M.B., and Radzidul, J.V., "Water Distribution Design and McIlroy Network Analyzer," Journal of the Hydraulic Division, ASCE, Vol. 84, No. HY 2, April, 1958, pp. 1588-1-1588-15.

16. McPherson, M.B., "Application of System Analyzer," Water and Sewage Work, Ref. No 1962, pp. R53-R67.

17. Wood, D.J., Analog Analysis of Water Distribution Networks," Transportation Engineering, Journal of the American Society of Civil Engineers, 17:TE2: 281, Paper 8112, May 1971.

18. Warga, J., "Determination of Steady State Flows and Currents in a Network," Proceedings, Instrument Society of America, Vol.9, pt. 5, 1954, Paper 54-43-4.

19. Duffin, R.J., "Nonlinear Networks," Bulletin of the American Mathematics Society, Vol. 53, 1947, pp. 963-971.

20. Martin, D.W., and Peters, G., "The Application of Newton's Method to Network Analysis by Digital Computer," Journal of the Institute of Water Engineers, Vol. 17, 1963, pp. 115-129.

21. Giudice, J.J., "Analysis of Pipe Networks Based on the Newton-Raphson Method," thesis presented to Massachusetts Institute of Technology, at Cambridge, Mass., in 1965, in partial fulfillment of the requirements for the degree of Master of Science.

22. Pitchai, R., "A Model for Designing Water Distribution Pipe Networks," thesis presented to Harvard University at Cambridge, Mass, in 1966, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

23. Jacoby, S.L.S. and Twigg, D.W., "Computer Solution to Network Problems," Proceedings of the National Symposium on the Analysis of Water Resources Systems-American Water Resources Association, Denver, Colorado, 1968.

24. Shamir, U. and Howard, C.D., "Water Distribution Systems Analysis," Journal of Hydraulic Division, Proceedings of the American Society of Civil Engineers, Jan., 1968, pp. 219-234.

25. Epp, R. and Fowler, A.G., "Efficient Code for Steady State flows in Networks," Journal of the Hydraulic Division, Proceeding of the American Society of Civil Engineers, Jan., 1970, pp. 43-56.

26. Zarghamee, M.S., "Mathematical Model for Water Distribution Systems," Journal of the Hydraulic Division-ASCE, 97:HYL:1, Jan,. 1971.

27. Lam, C.F., and Wolla, M.L., "Computer Analysis of Water Distribution Systems: Part 1-Formulation of Equations, Journal of the Hydraulic

Division-ASCE, Vol 98, No. HY 2, Feb., 1972.

28. Lam, C.F. and Wolla, M.L., "Computer Analysis of Water Distribution Systems,: Part 11-Numerical Solution, Journal of the Hydraulic Division-ASCE, 98:HY 3:447, Paper 8782, March, 1972.

29. Michel, H.L., and Wolfner, J.P., "Know what is Happening in your Water System," The American City, 87:6, June, 1972.

30. Donachie, R.P., "Digital Program for Water Network Analysis," Journal of the Hydraulic Division-ASCE, 100:HY 3:393, Paper 10424. March, 1974.

31. Gagnon, C.R. and Jacoby, S.L.S., "Computer Simulation of Water Distribution Networks," Transportation Engineering, Journal of the American Society of Civil Engineers, 101:TE 3:553 Paper 11515, August, 1975.

32. Kesavan, H.K., and Chandrashekar, M., "Graph-Theoretic Models for Pipe Network Analysis," Journal of the Hydraulic Division Proceedings of the American Society of Civil Engineers, Vol. 98, No. HY 2, Feb., 1972, pp. 345-363.

33. Hall, M.A., "Hydraulic Network Analysis using (Generalized) Geometric Programming, Network, 6:105 (1976).

34. Chandrashekar, M., and Stewart, K.H., "Sparsity Oriented Analysis of Large Pipe Networks," Journal of the Hydraulic Division, ASCE, 101:HY 4:341, April, 1975.

35. Tong, A.L., et al., "Analysis of Distribution Networks by Balancing Equivalent Pipe Lengths," Journal of the American Water Works Association, Vol. 53, No. 2, Feb., 1961, p. 192.

36. Raman, V., and Raman, S., "New Method of Solving Distribution System

Networks Based on Equivalent Pipe," Journal of the American Water Works Association, Bol, 58, No. 5, May, 1966, p. 615.

37. Wood, D.J., and Charles, C.O.A., "Hydraulic Network Analysis Using Linear Theory," Journal of the Hydraulics Division ASCE, 98, HY 7:1157-1170, July, 1972.

38. McCormick, M., and Bellany, B.E., "A Computer Program for the Analysis of Network of Pipes and Pumps," The Journal of the Institution of Engineers. Australia, March, 1968, pp. 51-58.

39. Jacoby, S.L.S., "Design of Optimal Hydraulic Networks," Journal of the Hydraulic Division, Proceedings of the American Society of Civil Engineers, Vol. 94, No. HY 3, May, 1958, pp. 641-661.

40. Darmeli, D., et al., "Design of Optimal Water Distribution Network," Journal of Pipeline Division, Proceeding of the American Society of Civil Engineers, Vol. 94, No. PL 1, October, 1968, pp. 1-9.

41. Gupta, Ishwar., "Linear Programming Analysis of a Water Supply System," A.I.I.E. Transactions, Vol. 1, No. 1, March, 1969, pp. 56-61.

42. Schaake, Jr. J.C., and Lai, D., "Linear Programming and Dynamic Programming Application to Water Distribution Network Design," Report No. 116, Hydrodynamics Laboratory, Department of Civil Engineering, M.I.T., Cambridge, July, 1969.

43. Liang, T., "Design of Conduit System by Dynamic Programming," Journal of Hydraulic Division, ASCE, Vol., 97, No. HY 3, Proc Paper 7988, March, 1971, pp. 383-393.

44. Deb, Arun K., and Sarkar, Asok K., "Optimization in Design of Hydraulic Network," Journal of the Samitary Engineering Division, Proceedings of the American Society of Civil Engineers, Vol. 97, No. SA 2,

April, 1971, pp. 141-159.

45. Kohlhaas, Charles A., and Mattern, David E., "An algorithm for Obtaining Optimal Looped Pipe Distribution Networks," Symposium on the Application of Computers to the Problems of Urban Society-Association of Computing Michinery, N.Y., 1971, pp. 138-151.

46. Kally, Elisha., "Computerized Planning of the Least Cost Water Distribution," Network, Water and Sewage Works, Reference No. 1972, pp. R-121-R-127.

47. Barlow, J. F., and Markland, E., "Computer Design of Pipe Networks," Institution of Civil Engineers, London, England, Vol. 53. Part 2, September, 1972, pp. 225-235.

48. Cembrowicz, Ralf G., and Harrington, Joseph J., "Capital-Cost Minimization of Hydraulic Network," Journal of the Hydraulic Division, Proceedings of the American Society of Civil Engineers, Vol. 99, No. HN 3, March, 1973, pp. 431-440.

49. Watanatada, Thawat., "Least Cost Design of Water Distribution Systems," Journal of the Hydraulic Division, Proceedings of the American Society of Civil Engineers, Vol. 99, No. HY 9, September, 1973, pp. 1497-1513.

50. Shamir, Uri., "Optimal Design and Operation of Water Distribution System," Water Resources Research, Vol. 10, No. 1, Fegruary, 1974, pp. 27-36.

51. Deb, Arun K., "Least Cost Design of Branched Pipe Network System," Journal of the Environmental Engineering Division, Vol. 100, No. EE 4 August, 1974, pp. 821-835.

52. Fine, Ta Kung., "Optimal Planning and Design of Water Distribution Systems Ph.D., Dissertation, University of Pittsburg, 1975.

53. Rasmusen, Haris Jørgen., "Simplified Optimization of Water Supply System," Journal of the Environmental Engineering Division, Proceedings of

American Society of Civil Engineers, Vol. 102, No. EE 2, April, 1976, pp. 313-327.

54. Deb, Arun K., "Optimization of Water Distribution Network Systems," Journal of the Environmental Engineering Division, Proceedings of the American Society of Civil Engineers, Vol. 102, No. EE 4, August, 1976, pp. 837-851.

55. Alperovits, E., and Shamir, U., "Design of Optimal Water Distribution Systems," Water Resources Research, Vol. 13, No. 6, December, 1977, pp. 885-900.

56. Rothfarb, B., et al., "Optimal Design of Off-shore Natural-Gas Pipeline Systems," Operations Research, Vol. 18, Nov-Dec., 1970, pp 992-1020.

57. Frank, Howard., and Frisch, Ivan T., "Network Analysis," Scientific American, Vol. 223, July, 1970, pp. 94-103.

58. Ellis, J.B., "The Description and Analysis of Socio-Economic System by Physical Systems Techniques," Ph.D. Thesis, Department of Electrical Engineering, Michigan State University, 1965.

59. Ellis, J.B., "A Systems Model for Recreational Travel in Ontario: A Progress Report," Report on Project W-6, Department of Electrical Engineering, University of Waterloo, Ontario, July, 1967.

60. Gyamfi Peter., "Allocation of Travel Demand to Low Density Areas," Ph. D. Dissertation, University of California, Berkely, 1970.

61. Davis, Allen L., and Jeppson, Rowland W., "Developing a Computer Program for Distribution System Analysis," Journal of the American Water Works Association, 67:7, July, 1975.

62. Koenig, Herman E., and Blackwell, William A., "Electromechanical System Theory," McGraw-Hill Book Company, Inc., New York, 1961.

63. Deo, Narshign., "Graph Theory with Applications to Engineering and Computer Science," Prentice-Hall Series in Automatic Computation, 1974.

64. Scheid, Francis., "Introduction to Computer Science," Schaum's Outline Series,"McGraw-Hill Book Company, 1970.

65. Jeppson, R.W., "Analysis of Flow in Pipe Networks," Ann Arbor Science, Ann Arbor, Michigan, 1976.

66. Don, T., Phillips, et al, "Operations Research," John Wiley and Sons, Inc., New York, 1976.

67. Box, M.J., "A Comparison of Several Current Optimization Methods and the use of Transformations in Constrained Problems," The Computer Journal, London, England, Vol. 9, 1966, pp. 67-77.

68. Haarhoff and Buys, J.D., "A New Method for Optimization of a Non-Linear Function Subject to Non-Linear Constraints," The Computer Journal, London, England, Vol. 13, No. 2, May, 1970, pp. 178-184.

69. Harmon, H.H. , "Modern Factor Analysis," University of Chicago Press, Chicago, 1962.

70. Dames and Moore, "Construction Costs for Municipal Wastewater Conveyance Systems: 1973-1977," Technical Report prepared for United States Environmental Protection Agency, Office of Water Program Operations, Washington, D.C., 20460, Document No. EPA 430/9-77-014 MCD-38, May, 1978.

APPENDIX A

Linear Graph Theory

The meaning of a linear graph is quite varied depending on the group the term is being addressed to. For instance, to the mathematician linear graph is simply a collection of line segments. To a system analyst the linear graph is considerably more. It forms a basis for combining the differential equations for the components to establish the mathematical description of the system. The idea of using graph theory for predicting the behavior of an electric network originated with G. Kirchhoff in 1847 and was improved upon J. C. Maxwell in 1892. A milestone in graph-theoretic analysis of electrical networks was achieved by W. S. Percival, when he extended the Kirchhoff and Maxwell methods to networks with active elements.

This section is not intended for a detailed treatise on linear graph theory. The subject is well treated in textbooks including these:

1. Koenig, H. E., and Blackwell, W. A. "Electromechanical Systems Theory", McGraw-Hill Book Co., Inc, New York, N.Y.,1961.

2. Seshu, S., and Reed, M. B., "Linear Graphs and Electrical Networks", Addison-Wesley, 1961.

A summary of the background needed to understand the application of the linear graph technique is presented below.

Definition 1-An oriented line segment (i.e. a line segment with a direction indicated on it) together with its two end points is called an edge (or an element).

Definition 2-The end point of an edge is called a vertex.

Definition 3-If there exists a path between every pair of vertices of a graph, then the graph is said to be a connected graph.

Definition 4-In the study of linear graphs it is convenient to refer to a subset of the edges of the graph. Such a subset is a subgraph of the graph.

Definition 5-A circuit is defined as a subgraph of G such that there are exactly two distinct paths between every pair of vertices of this subgraph.

Definition 6-A frequently used and simple concept of graph theory is that of a tree. A tree T of a connected graph G is defined as a set of edges which (1) is connected, (2) contains all the vertices of G, but (3) contains no circuits.

Definition 7-The edges of a connected graph G that are included in a given tree T are referred to as branches. Those edges of G which are not in T constitute a subgraph T, called the complement of T, or the cotree. The edges of the cotree are called chords (or links). The number of branches in a tree T of a connected graph G containing v vertices is v-1. Consequently, if G contains a total of e edges, then the total number of chords in the cotree is e-(v-1)=e-v+1.

One of the basic properties of a tree is that the addition of a chord between any two of its vertices establishes a circuit. Since, in a connected graph having v vertices and e edges, there are e-v+1 chords for any given tree T, a set of e-v+1 circuits is uniquely defined by the chords of T. In this set of e-v+1 circuits no two circuits are identical, since each contains a chord not included in the other.

Definition 8-The set of e-v+1 circuits formed by each of the defining chords of a given tree T is called a fundamental set of circuits or simply fundamental circuits.

Definition 9-The cut-set of a connected graph G is defined as a set of edges C having such properties that (1) when set C is deleted the graph is in exactly two parts and (2) no subset of C has the property 1.

Definition 10-The set of v-1 cut-sets corresponding to any one tree T in G is defined as a fundamental set of cut-set.

The Vertex and Circuit Postulates

Three sets of mathematical equations form the complete basis for formulating the mathematical equations describing the system as a whole regardless of its complexity. Two basic postulates together with the component terminal equations therefore, form the basis for the analysis of systems.

The Vertex or "Cut-Set" Postulate

The vertex or "cut set" postulate is a generalization of Kirchhoff's node law. It states that if the linear graph of a physical system contains e oriented elements, and Y_j represents the fundamental through variable of the j^{th} element; then at the k^{th} vertex of the graph

$$\sum_{k=1}^{e} a_j Y_j = 0$$

where

$a_j = 0$ if j^{th} element is not incident at k^{th} vertex

$= 1$ if j^{th} element is oriented away from k^{th} vertex

$= -1$ if j^{th} element is oriented toward k^{th} vertex.

The Circuit Postulate

The circuit postulate is a generalization of Kirchhoff's circuit law. It states that if the linear graph of a physical system contains e oriented elements, and if X_j represents the fundamental across variable of the j^{th} element; then for the k^{th} circuit

$$\sum_{k=1}^{e} b_j X_j = 0$$

where

$b_j = 0$ if j^{th} element is not included in k^{th} circuit

$= 1$ if orientation of j^{th} element is same as orientation chosen for k^{th} circuit

$= -1$ if orientation of j^{th} element is opposite to that of k^{th} circuit.

The vertex and circuit postulates imply that (1) one equation involving the through variables, Y's, can be written at each vertex of the linear graph and (2) one equation involving the across variables, X's can be written for each circuit. However, all these equations are not independent. To establish techniques for selecting convenient sets of independent circuit and vertex equations, a tree of the graph is chosen.

The Component Terminal Equation

The through and across terminal variables of each component are related by a mathematical equation called the terminal equation. The component terminal equations are determined from a study of the components in isolation.

Using Hazen-William formula for a pipeline we get

$$Q_{ij} = 0.27 \times C \times D_{ij}^{2.63} \times \left[H_i - H_j\right]^{0.54} \times \frac{1}{L_{ij}^{0.54}}$$

where

Q_{ij} = flow through variable for pipe ij, in m.g.d

C = Hazen-William's friction factor

D_{ij} = pipe deameter, in feet

L_{ij} = pipe length, in feet

H_i-H_j = pressure drop or propensity drop in feet.

The Fundamental Circuit Equation

The fundamental circuit equations are written symbolically as

$$\left[\begin{array}{c:c} B & U \end{array}\right]\begin{bmatrix} X_b \\ X_c \end{bmatrix} = 0$$

where

B = a coefficient matrix corresponding to the branches

U = a unit matrix corresponding to the chords

X_b = a column matrix of the branches

X_c = a column matrix of the chords

The Fundamental Cut-set Equations

The fundamental cut-set equations are written symbolically as

$$\left[\begin{array}{c:c} U & A \end{array}\right]\begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = 0$$

where

A = a coefficient matrix corresponding to the chords

U = a unit matrix corresponding to the branches

Y_b = a column matrix of the branches

Y_c = a column matrix of the chords.

It can be shown that

$$A = -B^T$$

where

B^T is a transpose of B. Thus when the cut-set and circuit coefficient matrices are written from the same formulation tree, the cut-set coefficient is the negative transpose of the circuit coefficient matrix. This is a very useful property from both computational and computer programming standpoints. Clearly it is unnecessary to formulate both system of equations, either set is sufficient.

The Formulation Technique

While the solution of the set of cut-set and circuit equations is independent of the formulation process, the formulation of the systems equations itself is dependent to some extent upon the form of the component terminal equations. For this reason it is sometimes preferable to separate the formulation and solution. Two methods of formulation are used depending upon the form of the terminal equations. These are branch formulation method and the chord formulation method.

The Branch Formulation Method

The branch formulation method is used when the terminal equations are are given explicitly in series variables. The method requires that the given pressures be placed in the branches while the given flows are placed in the chords of the formulation tree.

The cut-set equations thus become

$$\begin{bmatrix} u & o & a_{11} \\ o & u & a_{21} \end{bmatrix} \begin{bmatrix} Y_{b1} \\ Y_{b2} \\ Y_c \end{bmatrix} = 0 \qquad \text{(A-1)}$$

where

Y_{b1} = a column matrix of flows in branches whose pressures are specified

Y_{b2} = a column matrix of unknown flows

Y_c = a column matrix of chord flows.

The fundamantal circuit equations are written symbolically as

$$\begin{bmatrix} B_{11} & B_{12} & U \end{bmatrix} \begin{bmatrix} X_{b1} \\ X_{b2} \\ X_c \end{bmatrix} = 0 \qquad \text{(A-2)}$$

where

X_{b1} = a column matrix of known branch pressure

X_{b2} = a column marix of unknown branch pressure

X_c = a column matrix of chord pressures

The component terminal equations can be written symbolically as

$$\begin{bmatrix} Y_{b2} \\ Y_c \end{bmatrix} = \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} X_{b2} \\ X_c \end{bmatrix} \qquad \text{(A-3)}$$

where

W = coefficient matrix.

It is worthy to note that in the expressions given above, the specified across variables are always included in the tree and are written as the first set in the column matrices. There are three major steps in the partial solution of the three sets of equations.

Step 1. Since the terminal equations are explicit in the through variables, they can be substituted into the cut-set equations Eq. (A-3) into (A-2) .

$$\begin{bmatrix} u \\ o \end{bmatrix} \begin{bmatrix} Y_{b1} \end{bmatrix} + \begin{bmatrix} o & a_{11} \\ u & a_{21} \end{bmatrix} \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} X_{b2} \\ X_c \end{bmatrix} = 0 \qquad \text{(A-4)}$$

Step 2. From equation (A-1), express all across variables in equation (A-4) in terms of the across variables of the branches.

$$\begin{bmatrix} X_{b2} \\ X_c \end{bmatrix} = \begin{bmatrix} o & u \\ -B_{11} & -B_{12} \end{bmatrix} \begin{bmatrix} X_{b1} \\ X_{b2} \end{bmatrix} \qquad \text{(A-5)}$$

Step 3. Equation(A-5) can now be substituted into equation (A-4) to obtain the branch equations for the system.

$$\begin{bmatrix} u \\ 0 \end{bmatrix} \begin{bmatrix} Y_{b1} \end{bmatrix} + \begin{bmatrix} 0 & a_{11} \\ u & a_{21} \end{bmatrix} \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} 0 & u \\ -B_{11} & -B_{12} \end{bmatrix} \begin{bmatrix} X_{b1} \\ X_{b2} \end{bmatrix} = 0 \quad \text{(A-6)}$$

Note that equation (A-6) can be regarded as two sets of equations and written in the form

$$\begin{bmatrix} Y_{b1} \end{bmatrix} + \begin{bmatrix} 0 & a_{11} \end{bmatrix} \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} 0 & u \\ -B_{11} - & B_{12} \end{bmatrix} = 0 \qquad \text{(A-7)}$$

and

$$\begin{bmatrix} u & a_{21} \end{bmatrix} \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} 0 & u \\ -B_{11} & -B_{12} \end{bmatrix} \begin{bmatrix} X_{b1} \\ X_{b2} \end{bmatrix} = 0 \qquad \text{(A-8)}$$

Equation (A-8) is independent of Y_{b1} and represents a set of simultaneous equations equal in number to the variables contained in the column matrix X_{b2}. Since X_{b1} contains specified functions, it is convenient to write equation (A-8) in the form

$$\begin{bmatrix} u & a_{21} \end{bmatrix} \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} u \\ -B_{12} \end{bmatrix} \begin{bmatrix} X_{b2} \end{bmatrix} + \begin{bmatrix} U & a_{21} \end{bmatrix} \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} 0 \\ -B_{11} \end{bmatrix} \begin{bmatrix} X_{b1} \end{bmatrix} = 0$$

(A-9)

A solution of these simultaneous equations, showing the across variables in X_{b2} as explicit functions of Y_{b1}, constitutes a solution to the system. If a solution for Y_{b1} is required, it is obtained by substituting the solution for X_{b2} into equation (A-7)

Note that if n_x represents the number of elements for which the across variables are known and, v the number of vertices of the graph, then the number of simultaneous equations to be solved is $v-1-n_x$. Also since

$$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} -B_{11} & -B_{12} \end{bmatrix} \begin{bmatrix} -B'_{11} \\ -B'_{12} \end{bmatrix} \qquad \text{(A-10)}$$

or $a_{11} = -B'_{11}$ and $a_{21} = -B'_{12}$

then
$$\begin{bmatrix} 0 & u \\ B_{11} & -B_{12} \end{bmatrix} = \begin{bmatrix} 0 & a_{11} \\ u & a_{21} \end{bmatrix} \qquad \text{(A-11)}$$

Equation (A-11) has two important implications. First, it implies that the triple product in equation (A-6) is symmetric if W is symmetric. Secondly, equation (A-10) indicates that the first triple product in equation (A-9) is symmetric when W is symmetric. These properties lead to some computation advantage.

The Chord Formulation Method

The chord formulation method is used when the across variable are given as an explicit function of the through variables. In choosing a tree to use as a basis of formulation, the elements having specified through variables are always placed in the chord system and included as the last elements in the column matrix.

The circuit equations can then be written as

$$\begin{bmatrix} B_{11} & u & 0 \\ B_{21} & 0 & u \end{bmatrix} \begin{bmatrix} X_b \\ X_{c1} \\ X_{c2} \end{bmatrix} = 0 \qquad \text{(A-12)}$$

The cut-set equations can be written as

$$\begin{bmatrix} u & -B'_{11} & -B'_{21} \end{bmatrix} \begin{bmatrix} Y_b \\ Y_{c1} \\ Y_{c2} \end{bmatrix} = 0 \qquad \text{(A-13)}$$

where

Y_{c2} = Specified chord flows,

The terminal equation can be written as

$$\begin{bmatrix} X_b \\ X_{c1} \end{bmatrix} = \begin{bmatrix} Z \end{bmatrix} \begin{bmatrix} Y_b \\ Y_{c1} \end{bmatrix} \qquad \text{(A-14)}$$

The solution of equations (A-12), (A-13), (A-14) follow steps below:

Step 1. The terminal equations (A-14), are substituted into the circuit equations. The result is

$$\begin{bmatrix} B_{11} & u \\ B_{21} & 0 \end{bmatrix} \begin{bmatrix} Z \end{bmatrix} \begin{bmatrix} Y_b \\ Y_{c1} \end{bmatrix} + \begin{bmatrix} u \\ 0 \end{bmatrix} \begin{bmatrix} X_{c2} \end{bmatrix} = 0 \qquad \text{(A-15)}$$

Step 2. All the through variables in equation (A-15) are expressed in terms of the chord variables. This yields

$$\begin{bmatrix} Y_b \\ Y_{c1} \end{bmatrix} = \begin{bmatrix} B'_{11} & B'_{21} \\ u & 0 \end{bmatrix} \begin{bmatrix} Y_{c1} \\ Y_{c2} \end{bmatrix} \qquad \text{(A-16)}$$

Step 3. Equation (A-16) is substituted into equation (A-15) to give

$$\begin{bmatrix} B_{11} & u \\ B_{21} & 0 \end{bmatrix} \begin{bmatrix} Z \end{bmatrix} \begin{bmatrix} B'_{11} & B'_{21} \\ u & 0 \end{bmatrix} \begin{bmatrix} Y_{c1} \\ Y_{c2} \end{bmatrix} + \begin{bmatrix} u \\ 0 \end{bmatrix} \begin{bmatrix} X_{c2} \end{bmatrix} = 0$$

(A-17)

In general when n_y of the chord through variables are specified, the number of simultaneous equations required is $e-v-n_y+1$.

Summary of Branch and Chord Formulation

Up to this point the discussion has been limited to systems containing components having either specified through variables or specified across variables, not both. In the situations where both types of variables are specified grouping the variables into appropriate sets reduces the complexity of algebra considerably. The basic equations are written down as follows:

1. The cut-set equations

$$\begin{bmatrix} u & 0 & a_{11} & a_{12} \\ 0 & u & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} Y_{b1} \\ Y_{b2} \\ Y_{c1} \\ Y_{c2} \end{bmatrix} = 0 \tag{A-18}$$

2. The fundamental circuit equations

$$\begin{bmatrix} B_{11} & B_{12} & u & 0 \\ B_{21} & B_{22} & 0 & u \end{bmatrix} \begin{bmatrix} X_{b1} \\ X_{b2} \\ X_{c1} \\ X_{c2} \end{bmatrix} = 0 \tag{A-19}$$

where

X_{b1} = specified across variable

X_{b2} = specified through variable

3. The component terminal equations.

If the resulting equations are to be formulated in terms of the through variables, the component terminal equations must be explicit in the across variables

$$\begin{bmatrix} X_{b2} \\ X_{c1} \end{bmatrix} = \begin{bmatrix} Z \end{bmatrix} \begin{bmatrix} Y_{b2} \\ Y_{c1} \end{bmatrix} \tag{A-20}$$

If the system equations are to be formulated in terms of the across variables then the component equations must be explicit in the through variables

$$\begin{bmatrix} Y_{b2} \\ Y_{c2} \end{bmatrix} = \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} X_{b2} \\ X_{c1} \end{bmatrix} \qquad \text{(A-21)}$$

The procedure for solving the final equations is similar to that described for the branch and chord formulation techniques. The example problem in Appendix B illustrates the application of linear graph theory to a sample water distribution network.

Selecting the Formulation Tree

The tree used as a basis for writing the cut-set and circuit equations is selected such that all elements with specified across variables are included as branches and all elements with specified through variables are included as chords. If this is not possible, a complete solution cannot be obtained.

Relationship between Coefficient Matrices of the Cut-set and Circuit Equations.

It has been mentioned earlier that if the fundamental circuit and the cut-set equations are written from the same formulation tree,

$$\begin{bmatrix} B & u \end{bmatrix} \begin{bmatrix} X_b \\ X_c \end{bmatrix} = 0 \qquad \text{(A-22)}$$

$$\begin{bmatrix} u & A \end{bmatrix} \begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = 0 \qquad \text{(A-23)}$$

then $B = -A'$ (A-24)

where A' is the transpose of A.

Equations (A-22) and (A-23) can be written as

$$\begin{bmatrix} X_b \\ X_c \end{bmatrix} = \begin{bmatrix} u \\ -B \end{bmatrix} \begin{bmatrix} X_b \end{bmatrix} \quad \text{and} \qquad \text{(A-25)}$$

$$\begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = \begin{bmatrix} -A \\ u \end{bmatrix} \begin{bmatrix} Y_c \end{bmatrix} \qquad \text{(A-26)}$$

respectively.

By applying the identity in equation (A-24) equations (A-25) and (A-26) become

$$\begin{bmatrix} X_b \\ X_c \end{bmatrix} = \begin{bmatrix} u \\ A' \end{bmatrix} \begin{bmatrix} X_b \end{bmatrix} \text{ and} \qquad \text{(A-27)}$$

$$\begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = \begin{bmatrix} B' \\ u \end{bmatrix} \begin{bmatrix} Y_c \end{bmatrix} \qquad \text{(A-28)}$$

Equation (A-27) implies that all across variables of the system graph can be expressed as linear combinations of the across variables of the formulation tree by using the transpose of the cut-set matrix. From equation (A-28) we see that all through variables of the system graph may be expressed as linear combinations of the through variables of the complement of the formulation tree by using the transpose of the fundamental circuit matrix.

The Case of a Non-Linear Terminal Equations

The analysis given above can also be applied to the case of a non-linear terminal equation or a general functional ralationship of the type

$$Y_c = G \times f\,(\Delta X_c)$$

where

Y_c = the flow through a component

ΔX_c = the propensity or pressure drop across the component

G = a constant

The Circuit and Cut-set Equations

The circuit and cut-set equations do not depend on the characteristics of the components but only on their mode of interconnection. Hence, for non-linear case they remain the same as their linear case as shown below.

The cut-set equation is

$$\begin{bmatrix} Y_b \end{bmatrix} = \begin{bmatrix} \text{cut-set} \\ \text{matrix} \end{bmatrix} \begin{bmatrix} Y_c \end{bmatrix}$$

This is a set of n_b equations in n unknowns.

The circuit equations are

$$\begin{bmatrix} X_c \end{bmatrix} = \begin{bmatrix} \text{cut-set} \\ \text{matrix} \end{bmatrix} \begin{bmatrix} X_b \end{bmatrix}$$

This is a set of n_c equations of components is now written as

$$\begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} f(\Delta X_b) \\ f(\Delta X_c) \end{bmatrix}$$

This is a set of n equations in the 2n unknowns in equations above.

The number of unknowns to be determined by the three equations above is given by

$$N_u = n + n$$

while the number of independent equations is

$$\begin{aligned} N_1 &= n_b + n_c + n \\ &= 2n. \end{aligned}$$

It, therefore, follows that the system of equations can be solved algebraically. Once the equations are formulated, a suitable numerical method for the solution of the non-linear simultaneous equations can be chosen.

APPENDIX B

The Application of System Model

Figure (B-1) represents a physical system of pipes and junctions as a directed network. The pipes are referred to as elements (or links) and the junctions as vertices (or nodes). The links are numbered $d + 1, \ldots\ldots$ $\ldots\ldots, n$ while the nodes are numbered $2, 3, \ldots\ldots\ldots\ldots, m$. Each link is assigned an arbitrary positive direction with flow along the link in this direction being considered positive, while flow in the opposite direction is considered negative.

To analyse this network using the Graph Theoretic Approach, a linear graph of the physical system is to be drawn. Figure (B-2) represents the linear graph of the physical system shown in Figure (B-1). The link from node 8 to node 1 is omitted for clarity. For ease in the formulation of the problem a total of $d + 0$ fictitious links and one fictitious node are added to the network. Each fictitious link is incident at fictitious node which is numbered 1. Node 1 is, therefore, a reference node and it is analogous to grounding in an electrical network. The first d fictitious links or destination links represent outflow from the network. Consumptions or demands from the network are assumed to flow along such an element or link. The remaining 0 links originate from the reference node and are incident at the points of the network with known inflow rate. In the example, d = 5, m = 10, n = 17 and 0 = 2. From Figure (B-1), there is a fixed rate of supply at nodes 2 and 3, and a fixed draw-off rate at nodes 5, 7, 8, 9, and 10.

The cannonical forms of the various classes of components can now be formulated. The supply is assumed to be from constant flow pumps. The form of this component (also known as origin) equation is written as:

$$Y_i = \text{known} \qquad i = 18, 19.$$

where

Y_i = flow of water in m.g.d.

The pipe flows are modelled by the relationship

$$Y_i = k_i \, \Delta X_i \qquad i = 6, 7, 8, 9, 10, \ldots\ldots\ldots 17.$$

where

Y_i = flow on any link in m.g.d.

k_i = the resistance to flow on any link 1.

$\Delta X_i = \left[H_i - H_j\right]^{0.54}$

$H_i - H_j$ = the head loss or pressure drop for flow across link i.

<u>The use of ΔX_i, thus, transformed the non-linear flow equation to a linear equation</u>.

The demand regions can be modelled by the equation

$$Y_d = k_d A_d \Delta X_d$$

where

Y_d = demand flow, in m.g.d. $\qquad d = 1, 2, 3, 4, 5.$

k_d = model correction factor

A_d = demand index

ΔX_d = (pressure drop)$^{0.54}$

(Y_d may be known or unknown. K_d is determined from computer run. ΔX_d is unknown.) The demand index can be calculated. A_d in this case was assumed to be a function of the total demand, including wastage. It was taken as linearly dependent on the population equivalent (in terms of demand) at each demand node or region. Where draw-off or demand is known the demand index is expressed as:

$$A_d = \frac{\text{Nodal demand}}{\text{Total demand}} .$$

This completes the first step in the solution of the problem.

The next step in the system solution is the selection of a tree in the linear graph. In this case, links 1, 2, 3, 4, 5, 6, 7, 8, and 9 can be selected as the tree in Figure (B-2). The equations in the Y-variables for the cut-set postulates for all elements can be written as:

$$\left[\, u \;\vdots\; A \,\right] \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_{15} \\ Y_{16} \\ Y_{17} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Y_{18} \\ Y_{19} \end{bmatrix} = 0 \qquad \text{(B-1)}$$

where

$$
u = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \text{ and}
$$

$$
A = \begin{bmatrix}
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
-1 & -1 & -1 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

The equations for the X-variables around fundamental circuits of all elements in the graph in terms of elements in the tree are written next. These equations are:

$$
\begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \vdots \\ \Delta X_{16} \\ \Delta X_{17} \end{bmatrix} = \begin{bmatrix} u \\ \cdots\cdots \\ A' \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \vdots \\ \Delta X_8 \\ \Delta X_9 \end{bmatrix} \qquad \text{(B-II)}
$$

(Equation (B-II) is the application of equation (A-28) of Appendix A)

The component terminal equations explicit in the Y- variables are ;

$$
\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \\ \\ Y_{16} \\ Y_{17} \end{bmatrix} = \begin{bmatrix} A_1' & & & & & & & & & & \\ & A_2' & & & & & & & & & \\ & & A_3' & & & & & & & & \\ & & & A_4' & & & & & & & \\ & & & & A_5' & & & & & & \\ & & & & & K_6 & & & & & \\ & & & & & & K_7 & & & & \\ & & & & & & & K_8 & & & \\ & & & & & & & & \ddots & & \\ & & & & & & & & & K_{16} & \\ & & & & & & & & & & K_{17} \end{bmatrix} f \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \\ \Delta X_4 \\ \Delta X_5 \\ \Delta X_6 \\ \Delta X_7 \\ \Delta X_8 \\ \Delta X_9 \end{bmatrix} \qquad \text{(B-III)}
$$

where

$A_i' = k_d A_i$ and $f(\Delta Xi)$ is taken as ΔX_i as explained earlier.

Substituting equation (B-II) into equation (B-III) and the result into (B-1) we get ;

$$\begin{bmatrix} A_1' & & & & & & \\ & A_2' & & & & & \\ & & A_3' & & & & \\ & & & \ddots & & & \\ & & & & K_{16} & & \\ & & & & & K_{17} \end{bmatrix} \begin{bmatrix} U \\ \cdots\cdots \\ A \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \\ \vdots \\ \Delta X_8 \\ \Delta X_9 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} Y_{18} \\ \\ Y_{19} \end{bmatrix} = 0 \qquad \text{(B-IV)}$$

Replacing the A_i's and the k_i's by their respective numerical values and evaluating the matrix triple product we get:

$$\begin{bmatrix} 1.15 & -0.26 & -0.44 & -0.44 & 0.00 & 0.00 & 0.00 & 0.00 & 0.44 \\ 0.26 & 0.70 & 0.00 & 0.00 & -0.44 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.44 & 0.00 & 1.15 & -0.26 & 0.00 & -0.44 & 0.00 & 0.00 & -0.44 \\ -0.44 & 0.00 & -0.26 & 1.40 & -0.26 & 0.00 & -0.44 & 0.00 & 0.00 \\ 0.00 & -0.44 & 0.00 & -0.26 & 1.15 & 0.00 & 0.00 & -0.44 & 0.00 \\ 0.00 & 0.00 & -0.44 & 0.00 & 0.00 & 0.96 & -0.52 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -0.44 & 0.00 & -0.52 & 1.50 & -0.52 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -0.44 & 0.00 & -0.52 & 0.96 & 0.00 \\ 0.44 & 0.00 & -0.44 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.44 \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \\ \Delta X_4 \\ \Delta X_5 \\ \Delta X_6 \\ \Delta X_7 \\ \Delta X_8 \\ \Delta X_9 \end{bmatrix}$$

$$
+ \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} Y_{18} \\ Y_{19} \end{bmatrix} = 0 \qquad \text{(B-V)}
$$

For the small network, as it is in this example, it is quite easy to perform a matrix triple product in the conventional way. For much larger network this might require multiplying large matrices. This might be time consuming exercise even on a computer. However, it is possible to use time saving algorithms. Koenig and Blackwell (62) have suggested an algorithm which enables one to write equation (B-V) once the cut-set matrix A in equation (B-1) is written down. This is the algorithm used in the computer programme for the systems model listed in Appendix C.

It is seen that equation (B-V) is a set of simulataneous linear equations and can thus be solved by any suitable computer programme. In this particular case, Gaussian Elimination procedure was used in the subroutine Gauss.

The solution of equation (B-V) gives the branch flows. The chord flows are obtained by first substituting the branch pressures in equation (B-II) for chord pressures and hence into equation (B-III) for chord flows.

The results of the element flows are shown in Table F-1 from which it is seen that the demand flows sum up to the total supply flows.

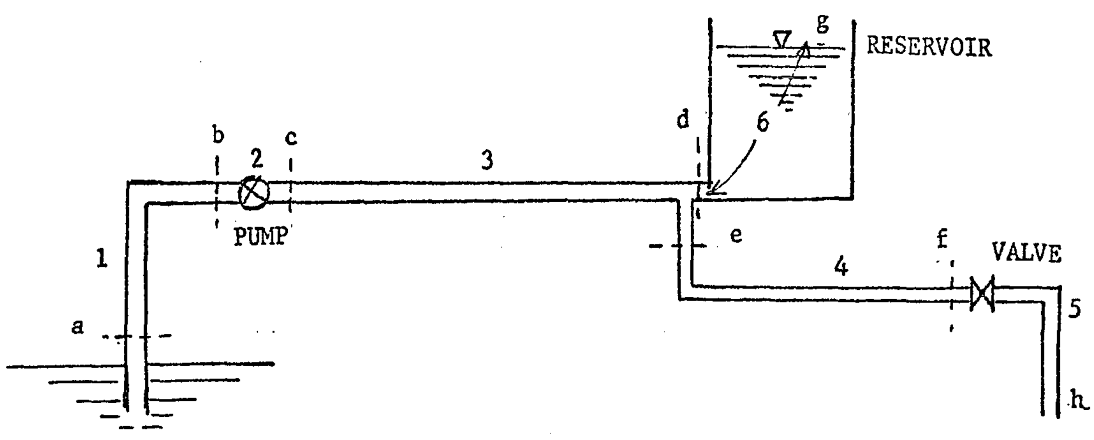


FIGURE B-1: SCHEMATIC DIAGRAM OF A PHYSICAL SYSTEM.

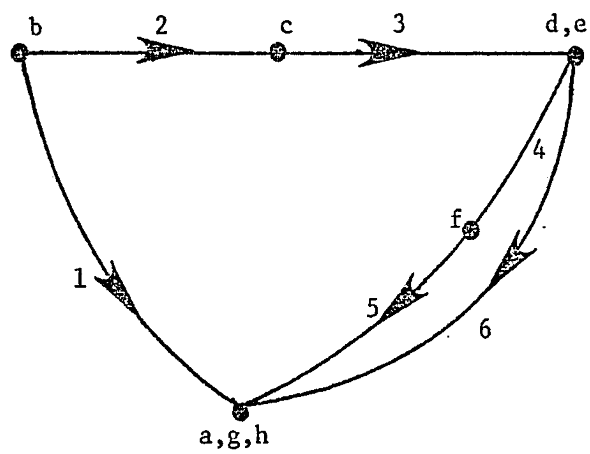


FIGURE B-2: LINEAR GRAPH OF A PHYSICAL SYSTEM SHOWN IN FIG. B-1.

APPENDIX C

A Computer Program for the Systems Model

Two main programs are used to solve the two main steps of the systems analysis. The first program codes the network and formulates the system equations while the second solves the system's equations for element flows and pressures. The two main programs are SYSFM and SYSAL.

Program SYSFM

Function of Program SYSFM

Program SYSFM chooses a tree of the network made up of origins (supply points), destinations (demand regions) and elements (pipes or any other appertunances). It then formulates the cut-set and hence, the circuit equations for the system in a form suitable for use in program SYSAL. The destination, origin and link elements are designated as D-element, O-element and L-element respectively.

Program Input

A coded map of the system or network being analyzed serves as the input to the program. The coded map is derived from a linear graph of the network which is drawn initially in order to assign numbers to the nodes and elements. It is computationally advantageous to number the nodes consecutively with the number 1 assigned to the common node (i.e. reference point) of all origin and destination elements. The first input card is the parameter card which stipulates the total number of elements (NE), the total number of nodes (N), the total number of origin (NØ) and the total number of destination (ND) elements in the linear graph of the network according to the format (4I5). This is followed by a set of data cards, each card being a coding of the individual elements of the graph. The form of the coding is the type of element, its assigned number and its origin and destination nodes. The direction of flow is assumed to be from the node with the smaller number to that with a higher number. The format in this case is (A1,3I5). The set of cards is arranged so that all D-elements are together as are all the 0 and L-elements. The three groups are placed in the order D-L-0. Figure C-1 shows the form of the input deck for the Norman, Oklahoma, Water Distribution Network.

Mechanics of Program SYSFM

Following the input of the linear graph of the network, the internal working of program SYSFM is as follows:

1. Choice of formulation tree and renumbering of elements.
2. Formulation of the circuit matrix.
3. Formulation of cut-set matrix
4. Output of the redesignated network and cut-set matrix.

The first step is the choice of the formulation tree. All the D-elements are chosen as part of this tree. A check is then made on the uncommon node of each of the D-elements to see if any of the L-elements incident on that node can be made part of the tree. If the above does not yield a complete tree then the program considers each node in turn and checks for L-elements incident on that node that can be made part of the tree. Once the complete tree has been selected the remaining elements are chosen as chords of the tree. Figure (C-3) is a listing of the cut-set program developed originally by L. A. White of the Department of Electrical Engineering in the University of Waterloo. According to J. B. Ellis (59) the FORTRAN IV computer program was written for the "express purpose of picking cut-sets of Linear graphs representing highway networks." The terminology used is peculiar to the branch equation formulation procedures of linear system theory.

The next step is the formulation of the cut-set matrix of the system equation. The program actually builds a circuit matrix and obtains the cut-set matrix from it by using the relationship

Cut-set matrix = Circuit matrix

In order to build the circuit matrix each chord in the graph is taken in turn and a circuit of branches formed for the chords.

Output of Program SYSFM

The first set of output consists of a printout of a list containing the type of element, the number the programmer assigned to it, the new number assigned to it by the program SYSFM which is the number used for the rest of the program. Table C-2 shows an example output for the Norman, Oklahoma, Water Distribution Network.

The second set of output is in a form suitable for use as part of the input to Program SYSAL. The first printout is a complete list of cut-set elements. This is followed by a list of the cut-set elements, excluding the origin elements. This is both printed and punched on cards to form the basis of formulating the matrix triple product in Program SYSAL. A list of the cut-sets, excluding the O and L-elements, is also printed and punched on cards, for use in Program SYSAL in quantifying the right hand side of the system equations.

Computer Program SYSAL

The function of Program SYSAL is to formulate the circuit and terminal equations of the system and solve the resulting systems equation resulting from all three sets of equations i.e. the cut-set equations, the circuit equations and the component terminal equations. Program SYSAL comes in two different forms. One form is based on the assumption of a linear relationship between element flows and pressure or propensity drops. This is called Program LSYSAL. The other form makes allowance for the use of a non-linear relationship between flow and pressure or propensity drop. This is called Program NSYSAL.

Mechanism of Program LSYSAL

Program LSYSAL attempts to formulate and solve a system of equations the form $\left[A\right]\left[W\right]\left[A\right]\left[Y_b\right] = \left[B\right]$ (C-1)

where

$\left[A\right]$ = the fundamental cut-set matrix

$\left[W\right]$ = the component terminal matrix

$\left[Y_b\right]$ = the vector product of the placement matrix and known chord flows

The usuallmethod of evaluation the triple matrix product $\left[A\right]\left[W\right]\left[A\right]$ is to pre-multiply $\left[W\right]$ by $\left[A\right]$ and then post-multiply the product by $\left[A\right]$. A might be of the order quite lengthy and tedious. Koenig and Blackwell (62) have suggested the algorithm described below upon which the program LSYSAL is based.

To illustrate the algorithm consider as an example the triple matrix product $\left[A\right]\left[W\right]\left[A\right]$ that expands into say

$$\underset{A}{\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}} \quad \underset{W}{\begin{bmatrix} K_2 & & & & \\ & K_3 & & & \\ & & K_4 & & \\ & & & W_5 & \\ & & & & W_6 \end{bmatrix}} \quad \underset{W}{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}}$$

It can be observed that since $\left[W\right]$ is diagonal when it is pre-multiplied by $\left[A\right]$ each term of $\left[A\right]$ is simply multiplied by the respective diagonal term in $\left[W\right]$. Thus $\left[A\right]\left[W\right]$ can be written as:

$$\left[A\right]\left[W\right] = \begin{array}{c} \begin{array}{ccccc} K_2 & K_3 & K_4 & W_5 & W_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \qquad \text{(C-11)}$$

Since the j^{th} column of $\left[A\right]$ is identical with the j^{th} row of $[A']$ the result of post-multiplying $\left[A\right]\left[W\right]$ by $\left[A\right]$ is obtained without the necessity of writing down $\left[A\right]$. Specifically the entry in (1,1) of $\left[A\right]\left[W\right]\left[A\right]$ is given by multiplying the first row of $\left[A\right]$ into itself with each term and the sum multiplied by its appropriate coefficient in $\left[W\right]$, the coefficient written above the columns in equation (C-11). For example, entry (1,1) is $0 \times 0 \times K_2 + (0 \times 0 \times K_3) + (0 \times 0 \times K_4) + (1 \times 1 \times W_5) + (1 \times 1 \times W_6) = W_5 + W_6$.

The coefficients in the (1,2), (1,3) and (1,4) positions are obtained by multiplying the first row of $\left[A\right]$ into the second, third and fourth rows respectively with each term in the sum multiplied by the coefficient written above the column of $\begin{bmatrix} A \end{bmatrix}$. The same procedure holds good for the entries in (2,2), (2,3) and (2,4). The result of the triple product is a symmetric matrix and the terms below the diagonal be obtained immediately.

In the computer program itself the input into LSYSAL of the matrix $\begin{bmatrix} A \end{bmatrix}$ is in the form.

Number of elements	Elements				
3	-11	-12	-13	0	0
3	-10	-14	-15	0	0
5	10	11	12	14	-16
3	13	16	-17	0	0

It must be noted that only the non-zero elements are input inoo the model. A - ve sign before the number of the element indicates a (-1) entry while its absence indicates a (+1) entry. The entry (1,1) of $\begin{bmatrix} A \end{bmatrix}\begin{bmatrix} W \end{bmatrix}\begin{bmatrix} A \end{bmatrix}'$ is obtained by the sum of ($1 \times 1 \times W_5$) and $1 \times 1 \times W_6$). Similarly, entry (2,2) is obtained from $(1 \times 1 \times K_2) + (1 \times 1 \times W_5) + (1 \times 1 \times W_6)$. For entries (1,2), (1,3) and (1,4) of $\begin{bmatrix} A \end{bmatrix}\begin{bmatrix} W \end{bmatrix}\begin{bmatrix} A \end{bmatrix}$ the program multiplies the common elements of Row 1 and Rows 2,3 and 4 respectively, multiplying each term in the sum by the coefficient in the $\begin{bmatrix} W \end{bmatrix}$ corresponding to the element number. Thus, for the (1,2) entry the result is ($1 \times 1 \times W_5$ + ($1 \times 1 \times W_6$), for the (1,3) entry ($1 \times 1 \times W_6$) and for the (1,4) entry Zero. The program uses the same procedure to obtain the (2,3) and (2,4) entries of $\begin{bmatrix} A \end{bmatrix}\begin{bmatrix} W \end{bmatrix}\begin{bmatrix} A \end{bmatrix}'$. The terms below the diagonal are obtained by symmetry.

In computing the vector B of equation (C-1) the computer uses a similar approach; only the non-zero elements of the placement matrix are considered in evaluating the product of the placement matrix and the origin flows. Once the matrix triple product is evaluated the next step in the solution of the set of the resulting simultaneous linear equations of the form

$$\begin{bmatrix} G \end{bmatrix} \begin{bmatrix} Y_b \end{bmatrix} = \begin{bmatrix} B \end{bmatrix} \qquad \text{(C-11)}$$

where G is the triple matrix product of rank equal to the number of branches or less and $\begin{bmatrix} Y_b \end{bmatrix}$ and $\begin{bmatrix} B \end{bmatrix}$ are as previously defined.

A number of methods are available for solving equation (C-111). The Gauss-Siedel iteration method which is the method commonly used in the past for small numbers of simultaneous equations is not suitable here for a number of reasons. The first is the large number of iterations required to achieve reasonable convergence to the solution. The second is the large numerical errors that occur with large sets of equations. A Gaussian Elimination procedure was used in the Subroutine Gauss.

Computation of the Chord Flows

Solution of above equations results in a list of branch flows or demand flows and the flows across the pipes in the formulation tree. Program LSYSAL next calculates the chord flows. It first computes the propensity drops or pressure drops across the links in terms of the drops across the elements of the tree. The flow through the chord elements are then calculated from the terminal equations. If a prior calibration of the model is required the program performs statistical analysis to evaluate the degree of fit of model predictions to observed data. The analysis includes the computation of individual departures between model predictions and observed flows for the elements, the accumulated error of prediction over the entire system and the standard deviation of predictions. A listing of Program LSYSAL with a sample output is shown in Appendix F and and Table F-1 respectively.

Program NSYSAL

Program NSYSAL is the version of Program SYSAL based on a general relationship between the flow through the elements and the propensity or pressure drops across them. It assumes terminal equations of the form

$$Y_i = K_i f(\triangle X_i)$$

Input to Program NSYAL is the same as that for Program LSYSAL and also comes from Program SYSFM but the former has to revert the cut-set equations into 0, 1 or Y format. The author did not find it necessary to use Program NSYSAL since the non-linear flow equations were linearized as explained in Appendix B page (138)

APPENDIX C

A COMPUTER PROGRAM FOR SYSFM MODEL

```
      C    PROGRAM SYSFM(INPUT,OUTPUT,PUNCH)
           DINENSION ID(972,4),NM(972,2),IBO(972),C(250,250),IB(250),
          1NB(250),IX(165)
           READ 1,NE,N,ND,NO
         1 FORMAT (4I5)
           READ 2,((ID(I,J),J=1,4),I=1,NE)
         2 FORMAT (A1,3I5)
           NB1=N-1
           NC1=NE-NB1
           DO 5 I=1,NB1
           DO 5 J=1,NC1
         5 C(I=J)=0.0
           CALL STCHAI(LE,1HD)
           I=1
         3 NM(I,1)=1
           NM(I,2)=ID(I,4)
           IBO(I)=ID(I,2)
           I=I+1
           IF(ID(I,1)-LE)4,3,4
      C    ADDITION OF BRANCHES TO D ELEMENTS.
         4 CALL STCHAR(LE,1HL)
           M=I
           K1=1
        16 J=NM(K1,2)
           K=M
        13 L=3
        10 IF(ID(K,L)-J)6,7,6
         6 IF(L-4)8,9,9
         8 L=4
           GO TO 10
         9 IF(K-NE)11,12,12
        11 K=K+1
           GO TO 13
        12 IF(K1-ND)14,15,15
        14 K1=K1+1
           GO TO 16
         7 IF (L-3)18,18,19
        18 JJ=ID(K,L+1)
           GO TO 20
        19 JJ=ID(K,L-1)
        20 J1=1
        27 J2=1
        24 IF(NM(J1,J2)-JJ)21,9,21
        21 IF(J2-2)22,23,23
        22 J2=2
           GO TO 24
        23 IF (J1-I+1)25,26,26
        25 J1=J1+1
```

```
      GO TO 27
   26 NM(I,1)=ID(K,3)
      NM(I,2)=ID(K,4)
      IBO(I)=ID(K,2)
      I=I+1
      GO TO 9
C *** ADDITION OF THE REMAINING BRANCHES.
   15 L=2
   40 K=1
C *** PROGRAM FOR CUT SETS OF NETWORKS.     ***
   37 J=3
   34 IF(ID(K,J)-L)30,31,30
   30 IF(J-4)32,33,33
   32 J=4
      GO TO 34
   33 IF(K-NE)35,36,36
   35 K=K+1
      GO TO 37
   36 IF(L-N)38,39,39
   38 L=L+1
      GO TO 40
   31 IF(ID(K,1).NE.DX)GO TO 33
      IF(ID(K,1).EQ.DX)GO TO 41
   41 IF(J-3)42,42,43
   42 JJ=ID(K,4)
      GO TO 44
   43 JJ=ID(K,3)
   44 J1=1
   51 J2=1
   48 IF(NM(J1,J2)-JJ)45,33,45
   45 IF(J2-2)46,47,47
   46 J2=2
      GO TO 4B
   47 IF(J1-I+1)49,50,50
   49 J1=J1+1
      GO TO 51
   50 NM(I,1)=ID(K,3)
      NM(I,2)=ID(K,4)
      IBO(I)=ID(K,2)
      I=I+1
      GO TO 33
C     ADDITION OF THE CHORDS.
   39 IC=I
      NB1=I-1
      L=ND
   55 K=1
   58 IF(ID(L,2)-IBO(K))53,54,53
   54 L=L+1
      GO TO 55
```

```
      53 IF(K-I+1)56,57,57
      56 K=K+1
         GO TO 58
      57 NM(I,1)=ID(L,3)
         NM(I,2)=ID(L,4)
         IBO(I)=ID(L,2)
         I=I+1
         IF(L-NE)54,60,60
C *** PRINT OUT OF LOD VRS NEW NUMBER.
      60 I=1
         PRINT 82
      82 FORMAT(1H1,15X,4HTYPE,8H OLD NO.,8H NEW NO./)
         PRINT 83
      83 FORMAT(5X,8HBRANCHES/)
      94 L=IBO(I)
      90 K=1
      93 IF(L-ID(K,2))91,92,91
      91 K=K+1
         GO TO 93
      92 PRINT 84,ID(K,1),ID(K,2),I
      84 FORMAT(17X,A1,4X,I3,5X,I3)
         IF(I-NB1)85,86,85
      86 PRINT 87
      87 FORMAT(5X,6HCHORDS/)
      85 IF(I-NE)88,89,89
      88 I=I+1
         GO TO 94
C *** FORMULATION OF THE CUT SET MATRIX.
      89 M=1
      81 J1=NM(IC,1)
         J2=NM(IC,2)
         K=1
      78 I=1
      69 J=1
      66 IF(NM(I,J)-J2)62,63,62
      62 IF(J-2)64,65,65
      64 J=2
         GO TO 66
      65 IF(I-NB1)67,68,68
      67 I=I+1
         GO TO 69
      68 K=K-1
         I=IB(K)
         J2=NB(K-1)
         C(I,M)=0.
         GO TO 65
      63 IF((K-1).LE.0)GO TO 70
         IF(I-IB(K-1))70,65,70
      70 IB(K)=I
```

```
      IF(J-1)71,71,72
   71 NB(K)=NM(I,2)
      IF(J2-NM(I,2))99,100,100
   99 IF(I-ND)74,74,73
  100 IF(I-ND)73,73,74
   73 C(I,M)=1.
      GO TO 75
   72 NB(K)=NM(I,1)
      IF(J2-NM(I,1))99,100,100
   74 C(I,M)=1.
   75 IF(J1-NB(K))76,77,76
   76 J2=NB(K)
      K=K+1
      GO TO 78
   77 IF(IC-NE)79,134,134
   79 IC=IC+1
      M=M+1
      GO TO 81
C *** PRINT OUT THE F- CUT SETS.
  134 L1=NE-NO
      L=1
  123 GO TO (80,124,125),L
   80 PRINT 114
  114 FORMAT(1H1,42HTHE FOLLOWING ARE THE FUNDAMENTAL CUT SETS/)
      PRINT 95
   95 FORMAT(15X,8HNO.  OF 8HDEFINING,10X,8HELEMENTS)
      PRINT 96
   96 FORMAT(4X,9HELEMENTS ,8H BRANCH )
      GO TO 126
  124 PRINT 127
  127 FORMAT(1H1,5X,8HCUT SET)
      GO TO 126
  125 PRINT 128
  128 FORMAT(1H1,5X,8HORIGINS )
  126 I=1
   97 K=1
      DO 108 LL=1,20
  108 IX(LL)=00000
      ISUM=0
      J=1
   98 IF(C(I,J))109,102,110
  110 IX(K)=J+NB1
      GO TO 115
  109 IX(K)=-(J+NB1)
  115 GO TO (101,116,117),L
  116 IF (J+NB1-L1)101,101,118
  117 IF(J+NB1-L1)118,118,101
  118 IX(K)=00000
      GO TO 102
  101 K=K+1
```

```
      ISUM=ISUM+1
  102 IF(J-NE+NB1)103,104,104
  103 J=J+1
      GO TO 98
  104 GO TO(129,130,130)L
  129 PRINT 105,ISUM,I(IX(K),K=1,20)
  105 FORMAT(7X,13,5X,13,5X,20I5)
      GO TO 111
  130 PRINT 112,ISUM,I(IX(K),K=1,20)
  112 FORMAT(13X,I3,1H+,I3,20I4)
      PUNCH 119,ISUM,I,(IX(K),K=1,16)
  119 FORMAT(I3,1H+,I3,16I4)
  111 IF(I-NB1)106,107,107
  106 I=I+1
      GO TO 97
  107 IF(L-3)121,122,122
      L=L+1
      GO TO 123
  122 CALL EXIT
      STOP
      END
```

FIGURE C-1: A CODED MAP OF NORMAN WATER

DISTRIBUTION NETWORK

233		94	83	10
D	1	1	3	
D	2	1	4	
D	3	1	5	
D	4	1	6	
D	5	1	7	
D	6	1	8	
D	7	1	9	
D	8	1	10	
D	9	1	11	
D	10	1	12	
D	11	1	13	
D	12	1	14	
D	13	1	15	
D	14	1	16	
D	15	1	17	
D	16	1	18	
D	17	1	19	
D	18	1	20	
D	19	1	21	
D	20	1	24	
D	21	1	25	
D	22	1	26	
D	23	1	27	
D	24	1	29	
D	25	1	30	
D	26	1	31	
D	27	1	33	
D	28	1	34	
D	29	1	35	
D	30	1	36	
D	31	1	37	
D	32	1	38	
D	33	1	40	
D	34	1	41	
D	35	1	42	
D	36	1	44	
D	37	1	45	
D	38	1	46	
D	39	1	47	
D	40	1	48	
D	41	1	49	
D	42	1	51	
D	43	1	52	
D	44	1	53	
D	45	1	54	
D	46	1	55	

FIGURE C-1 (Continued)

D	47	1	56
D	48	1	57
D	49	1	58
D	50	1	59
D	51	1	60
D	52	1	61
D	53	1	62
D	54	1	63
D	55	1	64
D	56	1	65
D	57	1	66
D	58	1	67
D	59	1	68
D	60	1	70
D	61	1	72
D	62	1	73
D	63	1	74
D	64	1	75
D	65	1	76
D	66	1	77
D	67	1	78
D	68	1	79
D	69	1	80
D	70	1	81
D	71	1	82
D	72	1	83
D	73	1	84
D	74	1	85
D	75	1	86
D	76	1	87
D	77	1	88
D	78	1	89
D	79	1	90
D	80	1	91
D	81	1	92
D	82	1	93
D	83	1	94
L	94	2	3
L	95	2	8
L	96	3	4
L	97	3	19
L	98	4	5
L	99	4	9
L	100	5	6
L	101	5	10
L	102	6	7
L	103	6	11
L	104	7	12
L	105	8	9
L	106	8	22

FIGURE C-1 (Continued)

L	107	9	10
L	108	9	59
L	109	10	11
L	110	10	60
L	111	11	12
L	112	11	34
L	113	12	13
L	114	13	14
L	115	13	34
L	116	14	15
L	117	14	63
L	118	15	16
L	119	15	65
L	120	16	17
L	121	17	18
L	122	17	54
L	123	18	49
L	124	19	20
L	125	19	24
L	126	20	21
L	127	21	25
L	128	22	23
L	129	23	24
L	130	23	59
L	131	23	41
L	132	24	39
L	133	24	25
L	134	25	26
L	135	26	27
L	136	26	40
L	137	27	28
L	138	27	47
L	139	28	29
L	140	28	56
L	141	29	30
L	142	29	35
L	143	30	31
L	144	31	35
L	145	31	32
L	146	32	36
L	147	32	33
L	148	33	46
L	149	33	38
L	150	34	62
L	151	35	37
L	152	36	37
L	153	37	46
L	154	37	55
L	155	38	48
L	156	38	46

FIGURE C-1 (Continued)

L	157	39	40
L	158	40	42
L	159	40	47
L	160	41	42
L	161	41	70
L	162	42	44
L	163	42	74
L	164	43	44
L	165	43	77
L	166	44	45
L	167	44	58
L	168	45	84
L	169	45	55
L	170	46	85
L	171	47	57
L	172	48	49
L	173	48	52
L	174	49	53
L	175	50	86
L	176	50	51
L	177	51	52
L	178	51	85
L	179	52	53
L	180	53	87
L	181	54	87
L	182	55	85
L	183	56	57
L	184	57	58
L	185	59	61
L	186	59	68
L	187	60	61
L	188	61	62
L	189	61	91
L	190	62	64
L	191	62	91
L	192	63	64
L	193	63	65
L	194	64	67
L	195	65	66
L	196	66	67
L	197	67	92
L	198	68	71
L	199	68	72
L	200	68	69
L	201	69	70
L	202	71	73
L	203	72	75
L	204	72	73
L	205	73	91

FIGURE C-1 (Continued)

L	206	74	76
L	207	74	77
L	208	75	76
L	209	75	79
L	210	76	78
L	211	77	80
L	212	77	78
L	213	78	81
L	214	78	79
L	215	79	82
L	216	79	83
L	217	80	84
L	218	80	81
L	219	81	82
L	220	82	88
L	221	83	89
L	222	83	91
L	223	84	85
L	224	84	86
L	225	86	90
L	226	86	87
L	227	87	94
L	228	88	90
L	229	88	89
L	230	89	92
L	231	91	93
L	232	92	93
L	233	92	94
O	84	1	2
O	85	1	22
O	86	1	23
O	87	1	28
O	88	1	32
O	89	1	39
O	90	1	43
O	91	1	50
O	92	1	69
O	93	1	71

TABLE C-1

OUTPUT OF PROGRAM SYSFM FOR THE
HYPOTHETICAL NETWORK

D	2	1	4
D	3	1	6
D	4	1	7
D	5	1	8
D	6	1	9
D	7	1	10
L	8	2	3
L	9	2	5
L	10	3	4
L	11	3	6
L	12	4	7
L	13	5	6
L	14	5	8
L	15	6	7
L	16	6	9
L	17	7	10
L	18	8	9
L	19	9	10
O	1	1	2

	TYPE	OLD NO.	NEW NO.
BRANCHES			
	D	2	1
	D	3	2
	D	4	3
	D	5	4
	D	6	5
	D	7	6
	L	10	7
	L	13	8
	L	8	9
CHORDS			
	L	9	10
	L	11	11
	L	12	12
	L	14	13
	L	15	14
	L	16	15
	L	17	16
	L	18	17
	L	19	18
	O	1	19

TABLE C-1 (Continued)

THE FOLLOWING ARE THE FUNDAMENTAL CUT-SETS

ELEMENTS	NO. BRANCH	OFDEFINING			ELEMENTS					
4	1	-10	-11	-12	19	0	0	0	0	0
5	2	10	11	-13	-14	-15	0	0	0	0
3	3	12	14	-16	0	0	0	0	0	0
2	4	13	-17	0	0	0	0	0	0	0
3	5	15	17	-18	0	0	0	0	0	0
2	6	16	18	0	0	0	0	0	0	0
3	7	-10	-11	19	0	0	0	0	0	0
2	8	10	-13	0	0	0	0	0	0	0
2	9	-10	19	0	0	0	0	0	0	0

CUT SETS

3+	1	-10	-11	-12	0	0	0	0	0	0	0	0	0
5+	2	10	11	-13	-14	-15	0	0	0	0	0	0	0
3+	3	12	14	-16	0	0	0	0	0	0	0	0	0
2+	4	13	-17	0	0	0	0	0	0	0	0	0	0
3+	5	15	17	-18	0	0	0	0	0	0	0	0	0
2+	6	16	18	0	0	0	0	0	0	0	0	0	0
2+	7	-10	-11	0	0	0	0	0	0	0	0	0	0
2+	8	10	-13	0	0	0	0	0	0	0	0	0	0
1+	9	-10	0	0	0	0	0	0	0	0	0	0	0

ORIGINS

1+	1	19	0	0	0	0	0	0	0	0	0	0	0
0+	2	0	0	0	0	0	0	0	0	0	0	0	0
0+	3	0	0	0	0	0	0	0	0	0	0	0	0
0+	4	0	0	0	0	0	0	0	0	0	0	0	0
0+	5	0	0	0	0	0	0	0	0	0	0	0	0
0+	6	0	0	0	0	0	0	0	0	0	0	0	0
1+	7	19	0	0	0	0	0	0	0	0	0	0	0
0+	8	0	0	0	0	0	0	0	0	0	0	0	0
1+	9	19	0	0	0	0	0	0	0	0	0	0	0

TABLE C-1 (Continued)

D	2	1	2
D	3	1	3
D	4	1	4
D	5	1	5
D	6	1	6
D	7	1	7
D	8	1	8
D	9	1	9
D	10	1	10
L	11	2	3
L	12	3	4
L	13	2	5
L	14	3	6
L	15	4	7
L	16	5	6
L	17	6	7
L	18	5	8
L	19	6	9
L	20	7	10
L	21	8	9
L	22	9	10
O	1	1	2

TYPE OLD NO. NEW NO.

BRANCHES

D	2	1
D	3	2
D	4	3
D	5	4
D	6	5
D	7	6
D	8	7
D	9	8
D	10	9

CHORDS

L	11	10
L	12	11
L	13	12
L	14	13
L	15	14
L	16	15
L	17	16
L	18	17
L	19	18
L	20	19
L	21	20
L	22	21
O	1	22

TABLE C-1 (Continued)

THE FOLLOWING ARE THE FUNDAMENTAL CUT SETS

ELEMENTS	NO. BRANCH	OFDEFINING			ELEMENTS					
3	1	-10	-12	22	0	0	0	0	0	0
3	2	10	-11	-13	0	0	0	0	0	0
2	3	11	-14	0	0	0	0	0	0	0
3	4	12	-15	-17	0	0	0	0	0	0
4	5	13	15	-16	-18	0	0	0	0	0
3	6	14	16	-19	0	0	0	0	0	0
2	7	17	-20	0	0	0	0	0	0	0
3	8	18	20	-21	0	0	0	0	0	0
2	9	19	21	0	0	0	0	0	0	0

CUT SETS

2+	1	-10	-12	0	0	0	0	0	0	0	0	0	0
3+	2	10	-11	-13	0	0	0	0	0	0	0	0	0
2+	3	11	-14	0	0	0	0	0	0	0	0	0	0
3+	4	12	-15	-17	0	0	0	0	0	0	0	0	0
4+	5	13	15	-16	-18	0	0	0	0	0	0	0	0
3+	6	14	16	-19	0	0	0	0	0	0	0	0	0
2+	7	17	-20	0	0	0	0	0	0	0	0	0	0
3+	8	18	20	-21	0	0	0	0	0	0	0	0	0
2+	9	19	21	0	0	0	0	0	0	0	0	0	0

ORIGINS

1+	1	22	0	0	0	0	0	0	0	0	0	0	0
0+	2	0	0	0	0	0	0	0	0	0	0	0	0
0+	3	0	0	0	0	0	0	0	0	0	0	0	0
0+	4	0	0	0	0	0	0	0	0	0	0	0	0
0+	5	0	0	0	0	0	0	0	0	0	0	0	0
0+	6	0	0	0	0	0	0	0	0	0	0	0	0
0+	7	0	0	0	0	0	0	0	0	0	0	0	0
0+	8	0	0	0	0	0	0	0	0	0	0	0	0
0+	9	0	0	0	0	0	0	0	0	0	0	0	0

TABLE C-2

OUTPUT OF PROGRAM SYSFM FOR THE NORMAN WATER DISTRIBUTION NETWORK

THE FOLLOWING ARE THE FUNDAMENTAL CUT SETS

ELEMENTS	NO. BRANCH	OFDEFINING			ELEMENTS						
4	1	-94	-95	-96	224	0	0	0	0	0	0
3	2	95	-97	-98	0	0	0	0	0	0	0
3	3	97	-99	-100	0	0	0	0	0	0	0
3	4	99	-101	-102	0	0	0	0	0	0	0
2	5	101	-103	0	0	0	0	0	0	0	0
4	6	94	-104	-126	225	0	0	0	0	0	0
4	7	98	104	-105	-106	0	0	0	0	0	0
4	8	100	105	-107	-108	0	0	0	0	0	0
4	9	102	107	-109	-110	0	0	0	0	0	0
3	10	103	109	-111	0	0	0	0	0	0	0
3	11	111	-112	-113	0	0	0	0	0	0	0
3	12	112	-114	-115	0	0	0	0	0	0	0
3	13	114	-116	-117	0	0	0	0	0	0	0
2	14	116	-118	0	0	0	0	0	0	0	0
3	15	118	-119	-120	0	0	0	0	0	0	0
2	16	119	-121	0	0	0	0	0	0	0	0
3	17	96	-122	-123	0	0	0	0	0	0	0
2	18	122	-124	0	0	0	0	0	0	0	0
2	19	124	-125	0	0	0	0	0	0	0	0
8	20	123	126	-127	-128	-129	-151	226	229	0	0
3	21	125	129	-130	0	0	0	0	0	0	0
3	22	130	-131	-132	0	0	0	0	0	0	0
5	23	131	-133	-134	-135	227	0	0	0	0	0
3	24	134	-136	-137	0	0	0	0	0	0	0
2	25	136	-138	0	0	0	0	0	0	0	0
5	26	138	-139	-140	-141	228	0	0	0	0	0
3	27	141	-142	-143	0	0	0	0	0	0	0
3	28	110	113	-144	0	0	0	0	0	0	0
3	29	137	139	-145	0	0	0	0	0	0	0
2	30	140	-146	0	0	0	0	0	0	0	0
4	31	145	146	-147	-148	0	0	0	0	0	0
3	32	143	-149	-150	0	0	0	0	0	0	0
4	33	132	151	-152	-153	0	0	0	0	0	0
3	34	128	-154	-155	0	0	0	0	0	0	0
4	35	152	154	-156	-157	0	0	0	0	0	0
5	36	156	-158	-159	-160	230	0	0	0	0	0
3	37	159	-161	-162	0	0	0	0	0	0	0
4	38	142	147	150	-163	0	0	0	0	0	0
3	39	133	153	-164	0	0	0	0	0	0	0
3	40	149	-165	-166	0	0	0	0	0	0	0
3	41	121	165	-167	0	0	0	0	0	0	0
4	42	-168	-169	-170	231	0	0	0	0	0	0
3	43	166	169	-171	0	0	0	0	0	0	0
3	44	167	171	-172	0	0	0	0	0	0	0

TABLE C-2 (Continued)

2	45	120	-173	0	0	0	0	0	0	0	0
3	46	148	162	-174	0	0	0	0	0	0	0
2	47	135	-175	0	0	0	0	0	0	0	0
3	48	164	175	-176	0	0	0	0	0	0	0
2	49	160	176	0	0	0	0	0	0	0	0
4	50	106	127	-177	-178	0	0	0	0	0	0
2	51	108	-179	0	0	0	0	0	0	0	0
4	52	177	179	-180	-181	0	0	0	0	0	0
4	53	144	180	-182	-183	0	0	0	0	0	0
3	54	115	-184	-185	0	0	0	0	0	0	0
3	55	182	184	-186	0	0	0	0	0	0	0
3	56	117	185	-187	0	0	0	0	0	0	0
2	57	187	-188	0	0	0	0	0	0	0	0
3	58	186	188	-189	0	0	0	0	0	0	0
6	59	178	-190	-191	-192	232	233	0	0	0	0
2	60	155	191	0	0	0	0	0	0	0	0
3	61	190	-193	-194	0	0	0	0	0	0	0
3	62	192	194	-195	0	0	0	0	0	0	0
3	63	157	-196	-197	0	0	0	0	0	0	0
3	64	193	-198	-199	0	0	0	0	0	0	0
3	65	196	198	-200	0	0	0	0	0	0	0
4	66	158	197	-201	-202	0	0	0	0	0	0
4	67	200	202	-203	-204	0	0	0	0	0	0
4	68	199	204	-205	-206	0	0	0	0	0	0
3	69	201	-207	-208	0	0	0	0	0	0	0
3	70	203	208	-209	0	0	0	0	0	0	0
3	71	205	209	-210	0	0	0	0	0	0	0
3	72	206	-211	-212	0	0	0	0	0	0	0
4	73	161	207	-213	-214	0	0	0	0	0	0
4	74	163	170	174	213	0	0	0	0	0	0
4	75	168	214	-215	-216	0	0	0	0	0	0
4	76	172	173	216	-217	0	0	0	0	0	0
3	77	210	-218	-219	0	0	0	0	0	0	0
3	78	211	219	-220	0	0	0	0	0	0	0
2	79	215	218	0	0	0	0	0	0	0	0
5	80	181	183	195	212	-221	0	0	0	0	0
4	81	189	220	-222	-223	0	0	0	0	0	0
2	82	221	222	0	0	0	0	0	0	0	0
2	83	217	223	0	0	0	0	0	0	0	0
2	84	-94	224	0	0	0	0	0	0	0	0
2	85	126	-225	0	0	0	0	0	0	0	0
4	86	126	-127	-128	226	0	0	0	0	0	0
2	87	151	-229	0	0	0	0	0	0	0	0
3	88	134	135	-227	0	0	0	0	0	0	0
3	89	140	141	-228	0	0	0	0	0	0	0
2	90	-158	230	0	0	0	0	0	0	0	0
2	91	-168	231	0	0	0	0	0	0	0	0
2	92	192	-233	0	0	0	0	0	0	0	0
2	93	191	-232	0	0	0	0	0	0	0	0

TABLE C-2 (Continued)

```
CUT SETS
        3+  1 -94 -95 -96   0   0   0   0   0   0   0   0   0   0
        3+  2  95 -97 -98   0   0   0   0   0   0   0   0   0   0
        3+  3  97 -99-100   0   0   0   0   0   0   0   0   0   0
        3+  4  99-101-102   0   0   0   0   0   0   0   0   0   0
        2+  5 101-103   0   0   0   0   0   0   0   0   0   0   0
        3+  6  94-104-126   0   0   0   0   0   0   0   0   0   0
        4+  7  98 104-105-106   0   0   0   0   0   0   0   0   0
        4+  8 100 105-107-108   0   0   0   0   0   0   0   0   0
        4+  9 102 107-109-110   0   0   0   0   0   0   0   0   0
        3+ 10 103 109-111   0   0   0   0   0   0   0   0   0   0
        3+ 11 111-112-113   0   0   0   0   0   0   0   0   0   0
        3+ 12 112-114-115   0   0   0   0   0   0   0   0   0   0
        3+ 13 114-116-117   0   0   0   0   0   0   0   0   0   0
        2+ 14 116-118   0   0   0   0   0   0   0   0   0   0   0
        3+ 15 118-119-120   0   0   0   0   0   0   0   0   0   0
        2+ 16 119-121   0   0   0   0   0   0   0   0   0   0   0
        3+ 17  96-122-123   0   0   0   0   0   0   0   0   0   0
        2+ 18 122-124   0   0   0   0   0   0   0   0   0   0   0
        2+ 19 124-125   0   0   0   0   0   0   0   0   0   0   0
        6+ 20 123 126-127-128-129-151   0   0   0   0   0   0   0
        3+ 21 125 129-130   0   0   0   0   0   0   0   0   0   0
        3+ 22 130-131-132   0   0   0   0   0   0   0   0   0   0
        4+ 23 131-133-134-135   0   0   0   0   0   0   0   0   0
        3+ 24 134-136-137   0   0   0   0   0   0   0   0   0   0
        2+ 25 136-138   0   0   0   0   0   0   0   0   0   0   0
        4+ 26 138-139-140-141   0   0   0   0   0   0   0   0   0
        3+ 27 141-142-143   0   0   0   0   0   0   0   0   0   0
        3+ 28 110 113-144   0   0   0   0   0   0   0   0   0   0
        3+ 29 137 139-145   0   0   0   0   0   0   0   0   0   0
        2+ 30 140-146   0   0   0   0   0   0   0   0   0   0   0
        4+ 31 145 146-147-148   0   0   0   0   0   0   0   0   0
        3+ 32 143-149-150   0   0   0   0   0   0   0   0   0   0
        4+ 33 132 151-152-153   0   0   0   0   0   0   0   0   0
        3+ 34 128-154-155   0   0   0   0   0   0   0   0   0   0
        4+ 35 152 154-156-157   0   0   0   0   0   0   0   0   0
        4+ 36 156-158-159-160   0   0   0   0   0   0   0   0   0
        3+ 37 159-161-162   0   0   0   0   0   0   0   0   0   0
        4+ 38 142 147 150-163   0   0   0   0   0   0   0   0   0
        3+ 39 133 153-164   0   0   0   0   0   0   0   0   0   0
        3+ 40 149-165-166   0   0   0   0   0   0   0   0   0   0
        3+ 41 121 165-167   0   0   0   0   0   0   0   0   0   0
        3+ 42-168-169-170   0   0   0   0   0   0   0   0   0   0
        3+ 43 166 169-171   0   0   0   0   0   0   0   0   0   0
        3+ 44 167 171-172   0   0   0   0   0   0   0   0   0   0
        2+ 45 120-173   0   0   0   0   0   0   0   0   0   0   0
        3+ 46 148 162-174   0   0   0   0   0   0   0   0   0   0
        2+ 47 135-175   0   0   0   0   0   0   0   0   0   0   0
```

TABLE C-2 (Continued)

```
3+ 48 164 175-176   0   0   0   0   0   0   0   0   0   0
2+ 49 160 176   0   0   0   0   0   0   0   0   0   0   0
4+ 50 106 127-177-178   0   0   0   0   0   0   0   0   0
2+ 51 108-179   0   0   0   0   0   0   0   0   0   0   0
4+ 52 177 179-180-181   0   0   0   0   0   0   0   0   0
4+ 53 144 180-182-183   0   0   0   0   0   0   0   0   0
3+ 54 115-184-185   0   0   0   0   0   0   0   0   0   0
3+ 55 182 184-186   0   0   0   0   0   0   0   0   0   0
3+ 56 117 185-187   0   0   0   0   0   0   0   0   0   0
2+ 57 187-188   0   0   0   0   0   0   0   0   0   0   0
3+ 58 186 188-189   0   0   0   0   0   0   0   0   0   0
4+ 59 178-190-191-192   0   0   0   0   0   0   0   0   0
2+ 60 155 191   0   0   0   0   0   0   0   0   0   0   0
3+ 61 190-193-194   0   0   0   0   0   0   0   0   0   0
3+ 62 192 194-195   0   0   0   0   0   0   0   0   0   0
3+ 63 157-196-197   0   0   0   0   0   0   0   0   0   0
3+ 64 193-198-199   0   0   0   0   0   0   0   0   0   0
3+ 65 196 198-200   0   0   0   0   0   0   0   0   0   0
4+ 66 158 197-201-202   0   0   0   0   0   0   0   0   0
4+ 67 200 202-203-204   0   0   0   0   0   0   0   0   0
4+ 68 199 204-205-206   0   0   0   0   0   0   0   0   0
3+ 69 201-207-208   0   0   0   0   0   0   0   0   0   0
3+ 70 203 208-209   0   0   0   0   0   0   0   0   0   0
3+ 71 205 209-210   0   0   0   0   0   0   0   0   0   0
3+ 72 206-211-212   0   0   0   0   0   0   0   0   0   0
4+ 73 161 207-213-214   0   0   0   0   0   0   0   0   0
4+ 74 163 170 174 213   0   0   0   0   0   0   0   0   0
4+ 75 168 214-215-216   0   0   0   0   0   0   0   0   0
4+ 76 172 173 216-217   0   0   0   0   0   0   0   0   0
3+ 77 210-218-219   0   0   0   0   0   0   0   0   0   0
3+ 78 211 219-220   0   0   0   0   0   0   0   0   0   0
2+ 79 215 218   0   0   0   0   0   0   0   0   0   0   0
5+ 80 181 183 195 212-221   0   0   0   0   0   0   0   0
4+ 81 189 220-222-223   0   0   0   0   0   0   0   0   0
2+ 82 221 222   0   0   0   0   0   0   0   0   0   0   0
2+ 83 217 223   0   0   0   0   0   0   0   0   0   0   0
1+ 84 -94   0   0   0   0   0   0   0   0   0   0   0   0
1+ 85 126   0   0   0   0   0   0   0   0   0   0   0   0
3+ 86 126-127-128   0   0   0   0   0   0   0   0   0   0
1+ 87 151   0   0   0   0   0   0   0   0   0   0   0   0
2+ 88 134 135   0   0   0   0   0   0   0   0   0   0   0
2+ 89 140 141   0   0   0   0   0   0   0   0   0   0   0
1+ 90-158   0   0   0   0   0   0   0   0   0   0   0   0
1+ 91-168   0   0   0   0   0   0   0   0   0   0   0   0
1+ 92 192   0   0   0   0   0   0   0   0   0   0   0   0
1+ 93 191   0   0   0   0   0   0   0   0   0   0   0   0
```

TABLE C-2 (Continued)

ORIGINS

1+	1	224	0	0	0	0	0	0	0	0	0	0	0	0
0+	2	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	3	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	4	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	5	0	0	0	0	0	0	0	0	0	0	0	0	0
1+	6	225	0	0	0	0	0	0	0	0	0	0	0	0
0+	7	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	8	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	9	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	10	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	11	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	12	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	13	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	14	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	15	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	16	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	17	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	18	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	19	0	0	0	0	0	0	0	0	0	0	0	0	0
2+	20	226	229	0	0	0	0	0	0	0	0	0	0	0
0+	21	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	22	0	0	0	0	0	0	0	0	0	0	0	0	0
1+	23	227	0	0	0	0	0	0	0	0	0	0	0	0
0+	24	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	25	0	0	0	0	0	0	0	0	0	0	0	0	0
1+	26	228	0	0	0	0	0	0	0	0	0	0	0	0
0+	27	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	28	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	29	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	30	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	31	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	32	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	33	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	34	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	35	0	0	0	0	0	0	0	0	0	0	0	0	0
1+	36	230	0	0	0	0	0	0	0	0	0	0	0	0
0+	37	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	38	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	39	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	40	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	41	0	0	0	0	0	0	0	0	0	0	0	0	0
1+	42	231	0	0	0	0	0	0	0	0	0	0	0	0
0+	43	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	44	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	45	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	46	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE C-2 (Concluded)

0+	47	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	48	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	49	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	50	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	51	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	52	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	53	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	54	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	55	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	56	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	57	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	58	0	0	0	0	0	0	0	0	0	0	0	0	0
2+	59	232	233	0	0	0	0	0	0	0	0	0	0	0
0+	60	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	61	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	62	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	63	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	64	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	65	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	66	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	67	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	68	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	69	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	70	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	71	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	72	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	73	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	74	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	75	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	76	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	77	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	78	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	79	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	80	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	81	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	82	0	0	0	0	0	0	0	0	0	0	0	0	0
0+	83	0	0	0	0	0	0	0	0	0	0	0	0	0
1+	84	224	0	0	0	0	0	0	0	0	0	0	0	0
1+	85	-225	0	0	0	0	0	0	0	0	0	0	0	0
1+	86	226	0	0	0	0	0	0	0	0	0	0	0	0
1+	87	-229	0	0	0	0	0	0	0	0	0	0	0	0
1+	88	-227	0	0	0	0	0	0	0	0	0	0	0	0
1+	89	-228	0	0	0	0	0	0	0	0	0	0	0	0
1+	90	230	0	0	0	0	0	0	0	0	0	0	0	0
1+	91	231	0	0	0	0	0	0	0	0	0	0	0	0
1+	92	-233	0	0	0	0	0	0	0	0	0	0	0	0
1+	93	-232	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX D

LISTINGS OF PROGRAM SPTREE

MAIN PROGRAM

```
      DIMENSION F (17),H(17),EDGE(34),V1(17),V2(17)
      INTEGER F,H,N,E,DDGE,C
      READ (5,9) F
    9 FORMAT (17I3)
      READ (5,91) H
   91 FORMAT (17I3)
      READ (5,92) N,E
   92 FORMAT (2I5)
      WRITE (6,11) F,H,N,E
   11 FORMAT (5X,17I3,//5X,17I3,//,5X,2(I3,2X))
      CALL SPTREE(F,H,N,E,EDGE,C)
      WRITE (6,30) (EDGE(K),K=1,E)
   30 FORMAT (///,5X,'THE SPANNING TREE IS',///,5X,17I3)
      STOP
      END
C *** PROGRAM : SPANNING TREE/FOREST.
      SUBROUTINE SPTREE(F,H,N,E,EDGE,C)
      INTEGER C,E,EDGE(E),F(E),H(E),VERTEX(34),VI,V2
      DO 4 L=1,N
    4 VERTEX(L)=0
      DO 6 L=1,E
    6 EDGE(L)=0
      C=0
      M=0
      K=0
   10 K=K+1
      V1=F(K)
      I=VERTEX(V1)
      IF(I.EQ.0)GO TO 39
      V2=H(K)
      J=VERTEXX(V2)
      I=(J.EQ.0)GO TO 36
      IF (I-J)21,50,18
   18 IJI=J
      J=I
      I=IJI
   21 DO 26 L=1,N
      IF (VERTEX (L)-J)26,23,25
   23 VERTEX(L)=I
      GO TO 26
   25 VERTEX(L)=VERTEX(L)-1
   26 CONTINUE
      DO 32 L=1,E
      IF (EDGE(L)-J)32,29,31
   29 EDGE(L)-I
      GO TO 32
```

```
31 EDGE(L)=EDGE(L)-1
32 CONTINUE
   C=C-1
   EDGE(K)=1
   GO TO 49
36 EDGE(K)=I
   VERTEX V2)=I
   GO TO 49
39 V2=H(K)
   J=VERTEX(V2)
   IF (J.EQ.0) GO TO 45
   EDGE(K)=J
   VERTEX(V1)=J
   GO TO 49
45 C=C+1
   EDGE(K)=C
   VERTEX(V1)=C
   VERTEX(V2)=C
49 M=M+1
50 IF(M.EQ.(N-1).OR.K.EQ.E)RETURN
   GO TO 10
   END
```

APPENDIX E

LISTINGS OF PROGRAM MERGEP AND SUBROUTINE MERGES

```
$JOB PAGES=80
      INTEGER PCOST,EPCOST,SPCOST,DIA,C,COST
      COMMON SPSQ(500),SPCOST(500),SPS(500),SPC(500),NI
      DIMENSION PSQ(35,7),PCOST(35,7),EQPSQ(35,150),EPCOST(35,150),IM(35
     1,35),EPSQ(150),IPCOST(150),MX(150),Q(35),LEN(35),C(35),COST(35,7),
     2DIA(35,7),HLOSS(35,7)
      DIMENSION ITEST2(150),KPLUSJ(150)
      NODES=28
      NE=27
      IDIAM=7
      IDIAM1=IDIAM+1
      NLESS1=NE
      DO 2 I=1,NE
      READ 1,Q(I),LEN(I),C(I)
    1 FORMAT (F5.2,2I10)
    2 CONTINUE
      DO 4 I=1,NE
      READ 3,(DIA(I,J),J=1,IDIAM)
    3 FORMAT(7I5)
    4 CONTINUE
      DO 10 I=1,NE
      READ 5,(COST(I,J),J=1,IDIAM)
    5 FORMAT(7I5)
   10 CONTINUE
      PRINT 315
  315 FORMAT ('1',////////////10X,'LINK',5X,'FLOW',5X,'LENGTH',7X,'C',6X
     1,'DIAMETER',2X,'UNIT COST',5X,'PSQ',5X,'PCOST'//)
      DO 20 I=1,NE
      DO 20 J=1,IDIAM
      HLOSS(I,J)=1910930.5*C(I)**1.85*LEN(I)/(C(I)**1.85*DIA(I,J)**4.87)
      PSQ(I,J)=HLOSS(I,J)
      PCOST(I,J)=COST(I,J)*LEN(I)/1000
      PRINT 15,I,Q(I),LEN(I),C(I),DIA(I,J),COST(I,J),PSQ(I,J),PCOST(I,J)
   15 FORMAT (/10X,I3,F10.2,4I10,F13.2,I8)
   20 CONTINUE
C
C *** READ INCIDENCE MATRIX. ***
C
      DO 30 I=1,NLESS1
      READ 25,(IM(I,J),J=1,NODES)
   25 FORMAT (28I2)
   30 CONTINUE
      PRINT 600
  600 FORMAT ('1',////////////5X,'OUTPUT OF SERIAL AND PARALLEL MERGES')
C
C *** COMPUTE SUMMATION OF EACH COLUMN IN INCIDENCE MATRIX. ***
C
      J=0
   35 ISUM=0
      J=J+1
      DO 40 I=1,NLESS1
      ISUM=IM(I,J)+ISUM
      IM(NODES,J)=ISUM
   40 CONTINUE
      IF (J.EQ.NODES) GO TO 45
      GO TO 35
C
C *** TEST FOR EITHER SERIES OR PARALLEL EXECUTION. ***
C
   45 JP=0
```

```
      JC=1
      IR=0
      MI=0
      IRLES2=2
   50 JC=JC+1
      JC1=JC+1
      IR=IR+1
      IR2=IR+2
      JCLES1=JC-1
      JCLES2=JC-2
      JCLES3=JC-3
      JPSG2=JC+2
      NODES1=NODES+1
      IDIFF=JC-IRLES2
      IF (IDIFF.EQ.0.OR.IDIFF.EQ.1) IRLES2=JC
      IF (JC.EQ.NODES.AND.IM(NODES,NODES).EQ.2) GO TO 65
      IF (JC.EQ.NODES1) GO TO 300
      IM(IR,NODES1)=0
      DO 60 J=IR2,NODES
      IF (IM(IR,J)) 60,60,55
   55 IRLES2=J
      JRLES2=J-2
   60 CONTINUE
      IF(IM(NODES,JCLES1).EQ.0.AND.IM(NODES,JC).EQ.1) JPLUSK=IDIAM
      PRINT 800,JC,IM(NODES,JC)
  800 FORMAT (//2I16)
      IF (JC.EQ.NODES) GO TO 300
   65 IF (IM(NODES,JC)-1) 140,70,150
   70 IF (IM(NODES,JCLES1).EQ.2.AND.IM(NODES,JC).EQ.1) GO TO 80
      IF (IM(NODES,JCLES1).EQ.1.AND.IM(NODES,JC).EQ.1) GO TO 80
      DO 75 JCL=1,JPLUSK
      EQPSQ(JCLES1,JCL)=PSQ(JCLES1,JCL)
      EPCOST(JCLES1,JCL)=PCOST(JCLES1,JCL)
   75 CONTINUE
   80 DO 85 L=1,JPLUSK
      DO 85 MS=1,IDIAM
      IL=L+MS-1
      SPSQ(IL)=EQPSQ(JCLES1,L)+PSQ(JC,MS)
      SPCOST(IL)=EPCOST(JCLES1,L)+PCOST(JC,MS)
   85 CONTINUE
      NI=IL
      CALL MERGES
C
C *** OUTPUT EQUIVALENT BRANCH LIST FOR A SERIAL MERGE. ***
C
      PRINT 90
   90 FORMAT (//15X,'EQUIVALENT BRANCH LIST FOR A SERIAL MERGE',//)
      PRINT 95
   95 FORMAT (//15X,'ITEM',13X,'SPSQ',13X,'SPCOST',/)
      MI=0
      DO 110 K=1,NI
      IF (SPCOST(K)) 100,110,100
  100 MI=MI+1
      SPSQ(MI)=SPSQ(K)
      SPCOST(MI)=SPCOST(K)
      PRINT 105,MI,SPSQ(MI),SPCOST(MI)
  105 FORMAT (/I18,F18.2,I18)
  110 CONTINUE
      IF (JF.EQ.0) GO TO 125
      DO 115 K=1,MI
```

```
      DO 155 JR=1,IDIAM
      EQPSQ(IRLES2,JR)=PSQ(JCLES2,JR)
      EPCOST(IRLES2,JR)=PCOST(JCLES2,JR)
      EQPSQ(JCLES1,JR)=PSQ(JCLES1,JR)
      EPCOST(JCLES1,JR)=PCOST(JCLES1,JR)
  155 CONTINUE
      ITEST2(JCLES1)=IDIAM
      IF (IM(NODES,JCLES1).EQ.2) IRLES2=JC
  160 IF (EQPSQ(IRLES2,J)-EQPSQ(JCLES1,K)) 175,240,180
  165 DO 170 JS=1,IDIAM
      EQPSQ(JCLES1,JS)=PSQ(JCLES1,JS)
      EPCOST(JCLES1,JS)=PCOST(JCLES1,JS)
  170 CONTINUE
      IF (IM(NODES,JCLES2).EQ.1.AND.IM(NODES,JC).EQ.2) IRLES2=JC
      ITEST2(JCLES1)=IDIAM
      GO TO 160
  175 EPSQ(L)=EQPSQ(JCLES1,K)
      IS=J-1
      IF (IS.EQ.0) IS=1
      IPCOST(L)=EPCOST(JCLES1,K)+EPCOST(IRLES2,IS)
      EQPSQ(JPSQ2,K)=EQPSQ(JCLES1,K)
      L=L+1
      K=K+1
      IF (K.GT.ITEST2(JCLES1)) GO TO 195
      GO TO 160
  180 EPSQ(L)=EQPSQ(IRLES2,J)
      IT=K-1
      IF (IT.EQ.0) IT=1
      IPCOST(L)=EPCOST(IRLES2,J)+EPCOST(JCLES1,IT)
      L=L+1
      J=J+1
      IF (J.GT.ITEST2(IRLES2)) GO TO 195
      GO TO 160
  185 J=J+1
      IF (J.EQ.1) MIN=1
      EPSQ(MIN)=EQPSQ(IRLES2,J)
      IPCOST(MIN)=EPCOST(IRLES2,J)
      IF (K.EQ.0) GO TO 190
      GO TO 200
  190 K=K+1
      IF (J.EQ.1.AND.K.EQ.1) MIN=2
      EPSQ(MIN)=EQPSQ(JCLES1,K)
      IPCOST(MIN)=EPCOST(JCLES1,K)
      EQPSQ(JPSQ2,K)=EQPSQ(JCLES1,K)
      IF (J.EQ.0) GO TO 185
      GO TO 200
  195 J=J-1
      K=K-1
      MIN=J+K+1
      IF (MIN.EQ.1) MIN=2
      IF (J.EQ.0) GO TO 185
      IF (K.EQ.0) GO TO 190
  200 JPLUSK=J+K
C
C *** OUTPUT EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE. ***
C
      PRINT 205
  205 FORMAT (//15X,'EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE',//)
      PRINT 210
  210 FORMAT (//15X,'ITEM',14X,'PSQ',14X,'PCOST',/)
```

```
      DO 220 L=1,JPLUSK
      EQPSQ(JCLES1,L)=EPSQ(L)
      EPCCST(JCLES1,L)=IPCOST(L)
      PRINT 215,L,EQPSQ(JCLES1,L),EPCCST(JCLES1,L)
  215 FORMAT (/I18,F18.2,I18)
  220 CONTINUE
      PRINT 225,J,IRLES2,K,JCLES1
  225 FORMAT (/4I10/)
      PRINT 230
  230 FORMAT(/6X,'J',7X,'PSQ1',11X,'K',9X,'PSQ2',11X,'IRLES2',8X,'JCLES1
     1',/)
      PRINT 235,J,EQPSQ(IRLES2,J),K,EQPSQ(JPSQ2,K),IRLES2,JCLES1
  235 FORMAT (/5X,I2,5X,F6.1,10X,I2,8X,F6.1,2I14/)
  240 PRINT 235,J,EQPSQ(IRLES2,J),K,EQPSQ(JPSQ2,K),IRLES2,JCLES1
      IF (JC.EQ.NCCES) GO TO 300
      GO TO 80
  245 M=1
      ITEST=IDIAM1
      DO 255 I=2,3
      MX(1)=PSQ(JCLES3,J)
      MX(2)=PSQ(JCLES2,K)
      MX(3)=PSQ(JCLES1,JKL)
      IF (MX(M)-MX(I)) 250,255,255
  250 M=I
  255 CONTINUE
      IF (M.EQ.1) GO TO 260
      IF (M.EQ.2) GO TO 265
      IF (M.EQ.3) GO TO 270
  260 EPSQ(L)=PSQ(JCLES3,J)
      K=K-1
      JKL=JKL-1
      IF (K.EQ.0) K=1
      IF (JKL.EQ.0) JKL=1
      IPCOST(L)=PCOST(JCLES3,J)+PCOST(JCLES2,K)+PCOST(JCLES1,JKL)
      L=L+1
      J=J+1
      IF (J.EQ.ITEST) GO TO 275
      GO TO 245
  265 EPSQ(L)=PSQ(JCLES2,K)
      J=J-1
      JKL=JKL-1
      IF (J.EQ.0) J=1
      IF (JKL.EQ.0) JKL=1
      IPCOST(L)=PCOST(JCLES2,K)+PCOST(JCLES1,JKL)+PCOST(JCLES3,J)
      L=L+1
      K=K+1
      IF (K.EQ.ITEST) GO TO 275
      GO TO 245
  270 EPSQ(L)=PSQ(JCLES1,JKL)
      J=J-1
      K=K-1
      IF (J.EQ.0) J=1
      IF (K.EQ.0) K=1
      IPCOST(L)=PCOST(JCLES1,JKL)+PCOST(JCLES3,J)+PCOST(JCLES2,K)
      L=L+1
      JKL=JKL+1
      IF (JKL.EQ.ITEST) GO TO 275
      GO TO 245
  275 J=J-1
      K=K-1
```

```
      JKL=JKL-1
      JPLUSK=J+K+JKL
      PRINT 205
      PRINT 210
      DO 285 L=1,JPLUSK
      EQPSQ(JCLES1,L)=EPSQ(L)
      EPCOST(JCLES1,L)=IPCOST(L)
      PRINT 280,L,EQPSQ(JCLES1,L),EPCOST(JCLES1,L)
  280 FORMAT (/I5,F10.2,I10/)
  285 CONTINUE
      PRINT 290
  290 FORMAT (/6X,'J',8X,'PSQ1',10X,'K',11X,'PSQ2',9X,'JKL',10X,'PSQ3')
      PRINT 295,J,PSQ(JCLES3,J),K,PSQ(JCLES2,K),JKL,PSQ(JCLES1,JKL)
  295 FORMAT (//5X,I2,5X,F6.1,10X,I2,8X,F6.1,9X,I3,8X,F6.1/)
      GO TO 80
  350 IRLES2=28
      GO TO 160
  300 PRINT 650
  650 FORMAT ('1',/////////////'INCIDENCE MATRIX'/////)
      PRINT 305,((IM(I,J),J=1,NODES),I=1,NLESS1)
  305 FORMAT (/28I3)
      PRINT 310,(IM(NODES,J),J=1,NODES)
  310 FORMAT (/28I3)
      STOP
      END
C
C *** MERGES PROGRAMME. ***
C
      SUBROUTINE MERGES
      INTEGER PSQ,PCOST,PS,PC
      COMMON PSQ(500),PCOST(500),PS(500),PC(500),N
      DO 5 I=1,N
      PS(I)=PSQ(I)
      PC(I)=PCOST(I)
    5 CONTINUE
C
C *** BEGIN ITH STEP OF SORTING PROCESS. ***
C
      NM1=N-1
      DO 20 I=1,NM1
      IP1=I+1
      M=I
C
C *** MAKE THE COMPARISONS. ***
C *** KEEP TRACK OF POSITION OF LARGEST NUMBER OF HEAD DIFFERENCE. ***
C
      DO 15 J=IP1,N
      IF (PSQ(M)-PSQ(J)) 10,15,15
   10 M=J
   15 CONTINUE
C
C *** SHIFT BOTH PSQ(M) AND PCOST(M) TO ITH PLACE. ***
C
      ITEMP=PCOST(I)
      PCOST(I)=PCOST(M)
      PCOST(M)=ITEMP
      ITEMP=PSQ(I)
      PSQ(I)=PSQ(M)
      PSQ(M)=ITEMP
```

```
   20 CONTINUE
C
C *** ITH STEP IS NOW FINISHED. ***
C
      DO 35 K=3,N
      L=K-1
      DO 30 I=1,L
      IF (PCOST(K-I)-PCOST(K)) 30,25,25
   25 PCOST(K-I)=0
      PSC(K-I)=0
   30 CONTINUE
   35 CONTINUE
C
C *** RETURN AND OUTPUT. ***
C
      RETURN
      END

$EXEC
```

TABLE E-1

INPUT DATA FOR 7-DIAMETER SELECTIONS FOR EACH LINK OF A HYPOTHETICAL NETWORK

LINK	FLOW (GPM)	LENGTH (FT)	C VALUE	DIAMETER (INS)	UNIT COST ($)	PSC	PCOST
1	1253	4000	100	6	24	737.71	96
1	1253	4000	100	8	43	181.73	172
1	1253	4000	100	10	47	61.20	188
1	1253	4000	100	12	59	25.23	236
1	1253	4000	100	15	73	8.51	292
1	1253	4000	100	18	94	3.50	376
1	1253	4000	100	21	118	1.65	472
2	1464	4000	100	8	43	242.37	172
2	1464	4000	100	10	47	81.76	188
2	1464	4000	100	12	59	33.64	236
2	1464	4000	100	15	73	11.35	292
2	1464	4000	100	18	94	4.67	376
2	1464	4000	100	21	118	2.20	472
2	1464	4000	100	24	124	1.15	496
3	2264	4000	100	12	59	75.37	236
3	2264	4000	100	15	73	25.42	292
3	2264	4000	100	18	94	10.46	376
3	2264	4000	100	21	118	4.94	472
3	2264	4000	100	24	124	2.56	496
3	2264	4000	100	27	136	1.45	544
3	2264	4000	100	30	178	0.87	712
4	1147	4000	100	6	24	626.43	96

TABLE E-1 (Continued)

4	1147	4000	100	8	43	154.32	172
4	1147	4000	100	10	47	52.06	188
4	1147	4000	100	12	59	21.42	236
4	1147	4000	100	15	73	7.23	292
4	1147	4000	100	18	94	2.57	376
4	1147	4000	100	21	118	1.40	472
5	931	4000	100	6	24	425.83	96
5	931	4000	100	8	43	104.50	172
5	931	4000	100	10	47	35.29	188
5	931	4000	100	12	59	14.56	236
5	931	4000	100	15	73	4.91	292
5	931	4000	100	18	94	2.02	376
5	931	4000	100	21	118	0.95	472
6	789	4000	100	6	24	313.52	96
6	789	4000	100	8	43	77.24	172
6	789	4000	100	10	47	26.05	188
6	789	4000	100	12	59	10.72	236
6	789	4000	100	15	73	3.62	292
6	789	4000	100	18	94	1.49	376
6	789	4000	100	21	118	0.70	472
7	963	4000	100	8	43	111.67	172
7	963	4000	100	10	47	37.67	188
7	963	4000	100	12	59	15.50	236
7	963	4000	100	15	73	5.23	292
7	963	4000	100	18	94	2.15	376
7	963	4000	100	21	118	1.02	472
7	963	4000	100	24	124	0.53	496
8	3227	4000	100	12	59	145.19	236
8	3227	4000	100	15	73	48.98	292
8	3227	4000	100	18	94	20.15	376

8	3227	4000	100	21	118	9.51	472
8	3227	4000	100	24	124	4.97	496
8	3227	4000	100	27	136	2.80	544
8	3227	4000	100	30	178	1.67	712
9	1217	4000	100	8	43	172.19	172
9	1217	4000	100	10	47	58.08	188
9	1217	4000	100	12	59	23.90	236
9	1217	4000	100	15	73	8.06	292
9	1217	4000	100	18	94	3.32	376
9	1217	4000	100	21	118	1.57	472
9	1217	4000	100	24	124	0.82	496
10	2217	4000	100	8	43	522.27	172
10	2217	4000	100	10	47	176.17	188
10	2217	4000	100	12	59	72.50	236
10	2217	4000	100	15	73	24.46	292
10	2217	4000	100	18	94	10.06	376
10	2217	4000	100	21	118	4.75	472
10	2217	4000	100	24	124	2.48	496
11	1557	4000	100	8	43	271.62	172
11	1557	4000	100	10	47	91.62	188
11	1557	4000	100	12	59	37.70	236
11	1557	4000	100	15	73	12.72	292
11	1557	4000	100	18	94	5.23	376
11	1557	4000	100	21	118	2.47	472
11	1557	4000	100	24	124	1.29	496
12	3773	4000	100	15	73	65.40	292
12	3773	4000	100	18	94	26.91	376
12	3773	4000	100	21	118	12.70	472
12	3773	4000	100	24	124	6.63	496
12	3773	4000	100	27	136	3.74	544

TABLE E-1 (Concluded)

12	3773	4000	100	30	178	2.24	712
12	3773	4000	100	36	215	0.92	860

TABLE E-2

INPUT DATA FOR 7-DIAMETER SELECTIONS FOR EACH LINK OF A PORTION OF NORMAN WATER DISTRIBUTION NETWORK, INCLUDING 1971 PROPOSED EXTENSIONS.

LINK	FLOW (MGD)	LENGTH (FT)	C VALUE	DIAMETER (INS)	UNIT COST $	PSC	FCOST
1	9.15	1311	130	21	118	6.72	154
1	9.15	1311	130	24	124	3.51	162
1	9.15	1311	130	27	136	1.98	178
1	9.15	1311	130	30	178	1.18	233
1	9.15	1311	130	36	215	0.49	281
1	9.15	1311	130	42	250	0.23	327
1	9.15	1311	130	48	302	0.12	395
2	8.92	5280	130	21	118	25.83	623
2	8.92	5280	130	24	124	13.48	654
2	8.92	5280	130	27	136	7.59	718
2	8.92	5280	130	30	178	4.55	939
2	8.92	5280	130	36	215	1.87	1135
2	8.92	5280	130	42	250	0.88	1320
2	8.92	5280	130	48	302	0.46	1594
3	9.14	5280	130	21	118	27.02	623
3	9.14	5280	130	24	124	14.10	654
3	9.14	5280	130	27	136	7.94	718
3	9.14	5280	130	30	178	4.76	939
3	9.14	5280	130	36	215	1.96	1135
3	9.14	5280	130	42	250	0.92	1320
3	9.14	5280	130	48	302	0.48	1594
4	0.45	5280	130	6	24	45.91	126

TABLE E-2 (Continued)

4	0.45	5280	130	8	43	11.31	227
4	0.45	5280	130	10	47	3.82	248
4	0.45	5280	130	12	59	1.57	311
4	0.45	5280	130	15	73	0.53	385
4	0.45	5280	130	18	94	0.22	496
4	0.45	5280	130	21	118	0.10	623
5	1.07	5280	130	6	24	227.55	126
5	1.07	5280	130	8	43	56.16	227
5	1.07	5280	130	10	47	18.94	248
5	1.07	5280	130	12	59	7.80	311
5	1.07	5280	130	15	73	2.63	385
5	1.07	5280	130	18	94	1.08	496
5	1.07	5280	130	21	118	0.51	623
6	10.42	5280	130	21	118	34.43	623
6	10.42	5280	130	24	124	17.97	654
6	10.42	5280	130	27	136	10.13	718
6	10.42	5280	130	30	178	6.06	939
6	10.42	5280	130	36	215	2.49	1135
6	10.42	5280	130	42	250	1.18	1320
6	10.42	5280	130	48	302	0.61	1594
7	1.22	5280	130	6	24	290.57	126
7	1.22	5280	130	8	43	71.58	227
7	1.22	5280	130	10	47	24.15	248
7	1.22	5280	130	12	59	9.94	311
7	1.22	5280	130	15	73	3.35	385
7	1.22	5280	130	18	94	1.38	496
7	1.22	5280	120	21	118	0.65	623
8	0.77	5280	130	6	24	124.02	126
8	0.77	5280	130	8	43	30.55	227
8	0.77	5280	130	10	47	10.31	248

8	0.77	5280	130	12	59	4.24	311
8	0.77	5280	130	15	73	1.43	385
8	0.77	5280	130	18	94	0.59	496
8	0.77	5280	130	21	118	0.28	623
9	1.20	5280	130	6	24	281.82	126
9	1.20	5280	130	8	43	69.43	227
9	1.20	5280	130	10	47	23.42	248
9	1.20	5280	130	12	59	9.64	311
9	1.20	5280	130	15	73	3.25	385
9	1.20	5280	130	18	94	1.34	496
9	1.20	5280	130	21	118	0.63	623
10	12.15	5280	130	21	118	45.74	623
10	12.15	5280	130	24	124	23.87	654
10	12.15	5280	130	27	136	13.49	718
10	12.15	5280	130	30	178	8.05	939
10	12.15	5280	130	36	215	3.31	1135
10	12.15	5280	130	42	250	1.56	1320
10	12.15	5280	130	48	302	0.82	1594
11	1.46	5280	130	12	59	13.85	311
11	1.46	5280	130	12	59	13.85	311
11	1.46	5280	130	12	59	13.85	311
11	1.46	5280	130	12	59	13.85	311
11	1.46	5280	130	12	59	13.85	311
11	1.46	5280	130	12	59	13.85	311
11	1.46	5280	130	12	59	13.85	311
12	6.84	5280	130	24	124	8.25	654
12	6.84	5280	130	24	124	8.25	654
12	6.84	5280	130	24	124	8.25	654
12	6.84	5280	130	24	124	8.25	654
12	6.84	5280	130	24	124	8.25	654

TABLE E-2 (Continued)

12	6.84	5280	130	24	124	8.25	654
12	6.84	5280	130	24	124	8.25	654
13	1.39	5280	130	6	24	369.88	126
13	1.39	5280	130	8	43	91.12	227
13	1.39	5280	130	10	47	30.74	248
13	1.39	5280	130	12	59	12.65	311
13	1.39	5280	130	15	73	4.27	385
13	1.39	5280	130	18	94	1.76	496
13	1.39	5280	130	21	118	0.83	623
14	0.85	5280	130	6	24	148.91	126
14	0.85	5280	130	8	43	36.68	227
14	0.85	5280	130	10	47	12.37	248
14	0.85	5280	130	12	59	5.09	311
14	0.85	5280	130	15	73	1.72	385
14	0.85	5280	130	18	94	0.71	496
14	0.85	5280	130	21	118	0.33	623
15	13.51	5280	130	21	118	55.67	623
15	13.51	5280	130	24	124	29.05	654
15	13.51	5280	130	27	136	16.37	718
15	13.51	5280	130	30	178	9.80	939
15	13.51	5280	130	36	215	4.03	1135
15	13.51	5280	130	42	250	1.90	1320
15	13.51	5280	130	48	302	0.99	1594
16	0.17	5280	130	6	24	7.58	126
16	0.17	5280	130	8	43	1.87	227
16	0.17	5280	130	10	47	0.63	248
16	0.17	5280	130	12	59	0.26	311
16	0.17	5280	130	15	73	0.09	385
16	0.17	5280	130	18	94	0.04	496
16	0.17	5280	130	21	118	0.02	623

TABLE E-2 (Continued)

17	0.32	5280	130	6	24	24.44	126
17	0.32	5280	130	8	43	6.02	227
17	0.32	5280	130	10	47	2.03	248
17	0.32	5280	130	12	59	0.84	311
17	0.32	5280	130	15	73	0.28	385
17	0.32	5280	130	18	94	0.12	496
17	0.32	5280	130	21	118	0.05	623
18	1.83	2540	125	6	24	318.22	60
18	1.83	2540	125	8	43	78.39	109
18	1.83	2540	125	10	47	26.44	119
18	1.83	2540	125	12	59	10.88	149
18	1.83	2540	125	15	73	3.67	185
18	1.83	2540	125	18	94	1.51	238
18	1.83	2540	125	21	118	0.71	299
19	1.54	2740	125	6	24	249.48	65
19	1.54	2740	125	8	43	61.46	117
19	1.54	2740	125	10	47	20.73	128
19	1.54	2740	125	12	59	8.53	161
19	1.54	2740	125	15	73	2.88	200
19	1.54	2740	125	18	94	1.18	257
19	1.54	2740	125	21	118	0.56	323
20	8.06	5280	130	24	124	11.17	654
20	8.06	5280	130	24	124	11.17	654
20	8.06	5280	130	24	124	11.17	654
20	8.06	5280	130	24	124	11.17	654
20	8.06	5280	130	24	124	11.17	654
20	8.06	5280	130	24	124	11.17	654
20	8.06	5280	130	24	124	11.17	654
21	9.86	2640	130	24	124	8.11	327
21	9.86	2640	130	24	124	8.11	327

TABLE E-2 (Continued)

21	9.86	2640	130	24	124	8.11	327
21	9.86	2640	130	24	124	8.11	327
21	9.86	2640	130	24	124	8.11	327
21	9.86	2640	130	24	124	8.11	327
21	9.86	2640	130	24	124	8.11	327
22	5.37	105600	130	15	73	1039.92	7708
22	5.37	105600	130	18	94	427.95	9926
22	5.37	105600	130	21	118	202.00	12460
22	5.37	105600	130	24	124	105.42	13094
22	5.37	105600	130	27	136	59.41	14361
22	5.37	105600	130	30	178	35.56	18796
22	5.37	105600	130	36	215	14.63	22704
23	5.59	5280	130	15	73	56.01	385
23	5.59	5280	130	18	94	23.05	496
23	5.59	5280	130	21	118	10.88	623
23	5.59	5280	130	24	124	5.68	654
23	5.59	5280	130	27	136	3.20	718
23	5.59	5280	130	30	178	1.92	939
23	5.59	5280	130	36	215	0.79	1135
24	0.21	5280	130	6	24	11.21	126
24	0.21	5280	130	8	43	2.76	227
24	0.21	5280	130	10	47	0.93	248
24	0.21	5280	130	12	59	0.38	311
24	0.21	5280	130	15	73	0.13	385
24	0.21	5280	130	18	94	0.05	496
24	0.21	5280	130	21	118	0.03	623
25	1.07	105600	130	6	24	4559.07	2534
25	1.07	105600	130	8	43	1123.11	4540
25	1.07	105600	130	10	47	378.65	4963
25	1.07	105600	130	12	59	155.90	6230

TABLE E-2 (Concluded)

25	1.07	105600	130	15	73	52.59	7708
25	1.07	105600	130	18	94	21.64	9926
25	1.07	105600	130	21	118	10.22	12460
26	6.87	5280	130	15	73	82.01	385
26	6.87	5280	130	18	94	33.75	496
26	6.87	5280	130	21	118	15.93	623
26	6.87	5280	130	24	124	8.31	654
26	6.87	5280	130	27	136	4.69	718
26	6.87	5280	130	30	178	2.60	939
26	6.87	5280	130	36	215	1.19	1135
27	20.59	2640	130	24	124	31.67	327
27	20.59	2640	130	27	136	17.85	359
27	20.59	2640	130	30	178	10.68	469
27	20.59	2640	130	36	215	4.40	567
27	20.59	2640	130	42	250	2.08	660
27	20.59	2640	130	48	302	1.08	797
27	20.59	2640	130	54	327	0.61	889

TABLE E-3

INCIDENCE MATRIX FOR THE HYPOTHETICAL NETWORK FOR SPANNING TREE AND TREE-LIKE CONFIGURATIONS

0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1

0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	1

TABLE E-4

INCIDENCE MATRIX FOR A PORTION OF NORMAN
WATER DISTRIBUTION NETWORK

0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

TABLE E-5

OUTPUT OF PROGRAMS MERGEP AND MERGES FOR HYPOTHETICAL NETWORK.

2 1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSG	SPCOST
1	980.08	268
2	424.10	344
3	303.67	360
4	267.60	408
5	250.88	464
6	245.87	548
7	244.02	644
8	83.41	660
9	35.30	708
10	13.00	764
11	6.32	848
12	3.86	944
13	2.80	968

3 1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

TABLE E-5 (Continued)

ITEM	SPSQ	SPCCST
1	1055.45	504
2	499.47	560
3	379.04	596
4	342.96	644
5	326.25	700
6	321.24	784
7	319.39	880
8	158.78	896
9	110.66	944
10	88.37	1000
11	81.69	1084
12	79.23	1180
13	78.17	1204
14	28.23	1260
15	13.27	1344
16	7.74	1440
17	5.38	1464
18	4.26	1512
19	3.67	1680

4 0

5 1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCCST
1	1052.26	192
2	580.15	268

TABLE E-5 (Continued)

3	477.88	264
4	447.25	332
5	433.05	388
6	428.80	472
7	427.23	568
8	106.31	644
9	36.79	660
10	15.97	708
11	6.32	764
12	3.43	848
13	2.36	944

6 0

7 2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSC	PCOST
1	1052.26	288
2	580.15	364
3	477.88	380
4	447.25	428
5	433.05	484
6	428.80	568
7	427.23	664
8	313.52	664
9	106.31	740
10	77.24	816
11	36.79	832

TABLE E-5 (Continued)

12	26.05	848
13	15.97	856
14	10.72	944
15	6.32	1000
16	3.62	1088
17	3.43	1140
18	2.36	1226

13 7 5 6

J	PSQ1	K	PSQ2	IRLES2	JCLES1
13	2.4	5	3.6	7	6
13	2.4	5	3.6	7	6

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	1163.93	460
2	691.82	536
3	589.55	552
4	558.92	600
5	544.72	656
6	540.47	740
7	425.19	836
8	217.98	912
9	188.91	988
10	148.46	1004
11	137.72	1020
12	127.64	1068
13	122.39	1116

TABLE E-5 (Continued)

14	117.99	1172
15	115.29	1228
16	115.09	1312
17	114.03	1408
18	40.03	1424
19	17.86	1472
20	7.59	1528
21	4.51	1612
22	3.37	1708
23	2.89	1732

8 2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSG	PCOST
1	1163.93	964
2	1055.45	964
3	691.82	1040
4	589.55	1056
5	558.92	1104
6	544.72	1160
7	540.47	1244
8	499.47	1320
9	425.19	1416
10	379.04	1432
11	342.96	1480
12	326.25	1536
13	321.24	1620
14	319.39	1716

TABLE E-5 (Continued)

15	217.98	1792
16	188.91	1868
17	158.76	1884
18	148.46	1900
19	137.72	1916
20	127.64	1964
21	122.39	2012
22	117.99	2068
23	115.29	2124
24	115.09	2208
25	114.03	2304
26	110.66	2352
27	88.37	2408
28	81.69	2452
29	79.23	2588
30	78.17	2612
31	40.03	2628
32	28.23	2684
33	17.86	2732
34	13.27	2816
35	7.74	2912
36	7.59	2968
37	5.38	2992
38	4.51	3076
39	4.26	3124
40	3.67	3292

19 8 21 7

J	PSQ1	K	PSQ2	IRLES2	JCLES1
19	3.7	21	4.5	8	7

TABLE E-5 (Continued)

19	3.7	21	4.5	8	7

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSC	SPCOST
1	1309.12	1200
2	1200.64	1200
3	837.01	1276
4	734.74	1292
5	704.11	1340
6	689.92	1396
7	685.66	1480
8	644.66	1556
9	570.38	1652
10	524.23	1668
11	488.16	1716
12	471.44	1772
13	466.43	1856
14	464.58	1952
15	363.17	2028
16	334.10	2104
17	303.97	2120
18	293.65	2136
19	282.91	2182
20	272.83	2200
21	267.58	2248
22	263.18	2304
23	260.48	2360

TABLE E-5 (Continued)

24	260.29	2444
25	259.22	2540
26	255.86	2588
27	233.56	2644
28	226.88	2728
29	224.42	2824
30	223.36	2848
31	185.22	2864
32	173.42	2920
33	163.05	2968
34	158.46	3052
35	152.93	3148
36	152.78	3204
37	150.57	3228
38	149.70	3312
39	149.45	3360
40	148.86	3528
41	52.65	3584
42	23.83	3668
43	13.19	3764
44	8.64	3788
45	6.47	3836
46	5.35	4004
9	0	
10	1	

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST

TABLE E-5 (Continued)

1	694.46	344
2	580.35	360
3	546.17	408
4	530.33	464
5	525.59	548
6	523.83	644
7	523.09	666
8	176.99	684
9	73.32	732
10	25.27	766
11	10.88	872
12	5.57	966
13	3.30	992

11 0

12 2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSQ	PCOST
1	694.46	516
2	580.35	532
3	546.17	580
4	530.33	636
5	525.59	720
6	523.83	816
7	523.09	840
8	271.62	840
9	176.99	856

TABLE E-5 (Continued)

10	91.62	872
11	73.32	920
12	37.70	968
13	25.27	1024
14	12.72	1080
15	10.88	1164
16	5.57	1260
17	5.23	1344
18	3.30	1368

13 12 5 11

J	PSQ1	K	PSQ2	IRLES2	JCLES1
13	3.3	5	5.2	12	11
13	3.3	5	5.2	12	11

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	759.86	808
2	645.75	824
3	611.57	872
4	595.73	928
5	590.99	1012
6	589.23	1108
7	337.02	1132
8	242.39	1148
9	157.02	1164
10	138.72	1212
11	103.11	1260

TABLE E-5 (Continued)

12	90.67	1316
13	78.12	1372
14	76.28	1456
15	70.97	1552
16	70.63	1636
17	68.70	1660
18	30.21	1744
19	16.00	1840
20	9.93	1864
21	7.03	1912
22	5.53	2080
23	4.22	2228

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSQ	PCOST
1	1309.12	2008
2	1200.64	2008
3	837.01	2064
4	759.86	2084
5	734.74	2100
6	704.11	2148
7	689.92	2204
8	685.66	2288
9	645.75	2304
10	644.66	2380
11	611.57	2426
12	595.73	2484
13	590.99	2568

TABLE E-5 (Continued)

14	589.23	2664
15	570.38	2760
16	524.23	2776
17	488.16	2824
18	471.44	2880
19	466.43	2964
20	464.58	3060
21	363.17	3136
22	337.02	3160
23	334.10	3236
24	303.97	3252
25	293.65	3268
26	282.91	3284
27	272.83	3332
28	267.58	3380
29	263.18	3436
30	260.48	3492
31	260.29	3576
32	259.22	3672
33	255.86	3720
34	242.39	3736
35	233.56	3792
36	226.88	3876
37	224.42	3972
38	223.36	3996
39	185.22	4012
40	173.42	4068
41	163.05	4116
42	158.46	4200
43	157.02	4216

TABLE E-5 (Concluded)

44	152.93	4312
45	152.78	4368
46	150.57	4392
47	149.70	4476
48	149.45	4524
49	148.86	4692
50	138.72	4740
51	103.11	4788
52	90.67	4844
53	78.12	4900
54	76.28	4984
55	70.97	5080
56	70.63	5164
57	68.70	5188
58	52.65	5244
59	30.21	5328
60	23.83	5412
61	16.00	5508
62	13.19	5604
63	9.93	5628
64	8.64	5652
65	7.03	5700
66	6.47	5748
67	5.53	5916
68	5.35	6084

46 13 22 12

J	PSQ1	K	PSQ2	IRLES2	JCLES1
46	5.3	22	5.5	13	12
46	5.3	22	5.5	13	12

TABLE E-6

OUTPUT OF PROGRAMS MERGEP AND MERGES FOR A PORTION OF NORMAN WATER DISTRIBUTION NETWORK

2 1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	32.55	777
2	29.33	785
3	27.80	801
4	27.01	856
5	26.31	904
6	26.06	950
7	25.55	1018
8	13.60	1049
9	7.71	1113
10	4.67	1334
11	1.99	1530
12	1.00	1719
13	0.58	1989

3 1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

TABLE E-6 (Continued)

ITEM	SPSQ	SPCOST
1	59.56	1400
2	56.39	1406
3	54.82	1424
4	54.03	1479
5	53.33	1527
6	53.07	1573
7	52.96	1641
8	40.61	1672
9	34.73	1736
10	31.68	1957
11	29.01	2153
12	28.02	2338
13	27.60	2612
14	14.68	2643
15	8.53	2707
16	5.34	2928
17	2.54	3124
18	1.50	3309
19	1.06	3583

4 0

5 1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	273.87	252
2	239.26	353

3	231.77	374
4	229.52	437
5	228.48	511
6	228.17	622
7	228.06	749
8	56.26	850
9	19.05	871
10	7.90	934
11	2.73	1008
12	1.18	1119
13	0.61	1246

6 2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSC	PCOST
1	273.87	1652
2	239.26	1753
3	231.77	1774
4	229.52	1837
5	228.48	1911
6	228.17	2022
7	228.06	2149
8	59.56	2149
9	56.35	2157
10	56.26	2258
11	54.82	2274
12	54.03	2329
13	53.33	2377

TABLE E-6 (Continued

14	53.07	2423
15	52.96	2491
16	40.61	2522
17	34.73	2566
18	31.68	2807
19	29.01	3003
20	28.02	3188
21	27.60	3462
22	19.05	3483
23	14.68	3514
24	8.53	3576
25	7.90	3641
26	5.34	3862
27	2.73	3936
28	2.54	4132
29	1.50	4317
30	1.18	4426
31	1.06	4702

19 6 12 5

J	PSQ1	K	PSQ2	IRLES2	JCLES1
19	1.1	12	1.2	6	5
19	1.1	12	1.2	6	5

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SFSQ	SPCOST
1	308.29	2275

TABLE E-6 (Continued)

2	273.69	2376
3	266.20	2397
4	263.95	2460
5	262.91	2534
6	262.60	2645
7	93.99	2772
8	90.78	2780
9	90.69	2881
10	89.25	2897
11	88.45	2952
12	87.76	3000
13	87.50	3046
14	87.39	3114
15	75.04	3145
16	69.16	3209
17	66.11	3430
18	63.44	3626
19	62.45	3811
20	62.03	4085
21	53.48	4106
22	49.11	4137
23	42.96	4201
24	42.33	4264
25	39.77	4485
26	37.16	4559
27	36.97	4755
28	35.93	4940
29	35.61	5081
30	35.49	5325
31	19.03	5386

TABLE E-6 (Continued)

32	11.19	5420
33	7.12	5641
34	3.56	5837
35	2.24	6022
36	1.68	6296

7	0
8	0
9	2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSQ	PCOST
1	290.57	252
2	124.02	252
3	71.58	353
4	30.55	454
5	24.15	475
6	10.31	496
7	9.94	559
8	4.24	622
9	3.35	696
10	1.43	770
11	1.38	881
12	0.65	1008

7 7 5 8

J	PSQ1	K	PSQ2	IRLES2	JCLES1
7	0.7	5	1.4	7	8

TABLE E-6 (Continued)

7	0.7	5	1.4	7	8

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCCST
1	572.39	378
2	405.84	378
3	353.40	479
4	312.37	580
5	305.97	601
6	292.13	622
7	291.76	685
8	286.06	748
9	285.17	822
10	283.25	896
11	283.20	1007
12	282.47	1134
13	70.08	1235
14	24.07	1256
15	10.29	1319
16	3.90	1393
17	1.99	1504
18	1.28	1631

10	2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSQ	PCCST

TABLE E-6 (Continued)

1	572.39	2653
2	405.84	2653
3	353.40	2754
4	312.37	2855
5	308.29	2855
6	305.97	2876
7	292.13	2897
8	291.76	2960
9	286.06	3023
10	285.17	3097
11	283.25	3171
12	283.20	3282
13	282.47	3409
14	273.69	3510
15	266.20	3531
16	263.95	3594
17	262.91	3668
18	262.60	3779
19	93.99	3906
20	90.78	3914
21	90.69	4015
22	89.25	4031
23	88.45	4086
24	87.76	4134
25	87.50	4180
26	87.39	4248
27	75.04	4279
28	70.08	4380
29	69.16	4444
30	66.11	4665

TABLE E-6 (Continued)

31	63.44	4861
32	62.45	5046
33	62.03	5320
34	53.48	5341
35	49.11	5372
36	42.96	5436
37	42.33	5499
38	39.77	5720
39	37.16	5794
40	36.97	5990
41	35.93	6175
42	35.61	6286
43	35.49	6560
44	24.07	6581
45	19.03	6612
46	11.19	6676
47	10.29	6739
48	7.12	6960
49	3.90	7034
50	3.56	7230
51	2.24	7415
52	1.99	7526
53	1.68	7800

36 10 17 9

J	PSQ1	K	PSQ2	IRLES2	JCLES1
36	1.7	17	2.0	10	9
36	1.7	17	2.0	10	9

TABLE E-6 (Continued)

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	618.14	3276
2	451.58	3276
3	399.15	3377
4	354.04	3478
5	351.71	3499
6	337.87	3520
7	337.50	3583
8	331.81	3646
9	330.92	3720
10	328.99	3794
11	328.94	3905
12	328.22	4032
13	319.44	4133
14	311.94	4154
15	309.70	4217
16	308.66	4291
17	308.35	4402
18	139.74	4529
19	136.52	4537
20	136.43	4638
21	134.99	4654
22	134.20	4709
23	133.50	4757
24	133.25	4803
25	133.14	4871
26	120.79	4902

TABLE E-6 (Continued)

27	115.82	5003
28	114.91	5067
29	111.86	5288
30	109.18	5484
31	108.19	5669
32	107.77	5943
33	99.22	5964
34	94.85	5995
35	88.70	6059
36	88.07	6122
37	85.51	6343
38	82.91	6417
39	82.71	6613
40	81.68	6798
41	81.36	6909
42	81.24	7183
43	69.81	7204
44	64.78	7235
45	56.93	7299
46	56.03	7362
47	52.87	7583
48	49.65	7657
49	49.30	7853
50	47.99	8038
51	47.73	8149
52	47.42	8423
53	25.55	8454
54	15.13	8518
55	9.73	8739
56	4.99	8935

TABLE E-6 (Continued)

57	3.24	9120
58	2.49	9394

11	0
12	1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSC	SPCCST
1	22.10	965

13	0
14	2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSC	PCCST
1	369.88	1091
2	91.12	1152
3	30.74	1213
4	22.10	1213

1 14 3 13

J	PSQ1	K	PSQ2	IRLES2	JCLES1
1	22.1	3	30.7	14	13
1	22.1	3	30.7	14	13

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

TABLE E-6 (Continued)

ITEM	SPSQ	SPCOST
1	518.79	1217
2	240.03	1316
3	171.01	1329
4	58.78	1440
5	34.47	1461
6	27.19	1524
7	23.82	1598
8	22.61	1709
9	22.43	1836

15 2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSQ	PCOST
1	618.14	4493
2	518.79	4493
3	451.56	4493
4	399.15	4594
5	354.04	4695
6	351.71	4716
7	337.87	4737
8	337.50	4800
9	331.61	4863
10	330.92	4937
11	328.99	5011
12	328.94	5122
13	328.22	5249

TABLE E-6 (Continued)

14	319.44	5350
15	311.94	5371
16	309.70	5434
17	308.66	5508
18	308.35	5619
19	240.03	5720
20	171.01	5741
21	139.74	5868
22	136.52	5876
23	136.43	5977
24	134.99	5993
25	134.20	6048
26	133.50	6098
27	133.25	6142
28	133.14	6210
29	120.79	6241
30	115.82	6342
31	114.91	6406
32	111.86	6627
33	109.18	6823
34	108.19	7008
35	107.77	7282
36	99.22	7303
37	94.85	7334
38	88.70	7398
39	88.07	7461
40	85.51	7682
41	82.91	7756
42	82.71	7952
43	81.68	8137

TABLE E-6 (Continued)

44	81.36	8248
45	81.24	8522
46	69.81	8543
47	64.78	8574
48	58.78	8675
49	56.93	8739
50	56.03	8802
51	52.87	9023
52	49.65	9057
53	49.30	9253
54	47.99	9478
55	47.73	9585
56	47.42	9863
57	34.47	9884
58	27.19	9947
59	25.55	9978
60	23.82	10052
61	22.81	10163
62	22.43	10290

53 15 9 14

J	PSQ1	K	PSQ2	IRLES2	JCLES1
53	25.6	9	22.4	15	14
53	25.6	9	22.4	15	14

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM SPSQ SPCOST

TABLE E-6 (Continued)

1	507.25	5116
2	454.81	5217
3	409.71	5318
4	407.38	5339
5	393.54	5360
6	393.17	5423
7	387.47	5486
8	386.58	5560
9	384.66	5634
10	384.61	5745
11	383.88	5872
12	375.10	5973
13	367.61	5994
14	365.36	6057
15	364.32	6131
16	364.01	6242
17	295.69	6343
18	226.67	6364
19	195.40	6491
20	192.19	6499
21	192.10	6600
22	190.66	6616
23	189.87	6671
24	189.17	6719
25	188.91	6765
26	188.80	6833
27	176.45	6864
28	171.49	6965
29	170.57	7029
30	167.52	7250

TABLE E-6 (Continued)

31	164.85	7446
32	163.66	7631
33	163.44	7905
34	154.89	7926
35	150.52	7957
36	144.37	8021
37	143.74	8084
38	141.18	8305
39	138.57	8379
40	138.36	8575
41	137.34	8760
42	137.03	8871
43	136.90	9145
44	125.48	9166
45	120.44	9197
46	114.45	9298
47	112.60	9362
48	111.70	9425
49	108.53	9646
50	105.31	9720
51	104.97	9916
52	103.65	10101
53	103.40	10212
54	103.09	10486
55	90.14	10507
56	82.86	10570
57	81.22	10601
58	79.48	10675
59	78.47	10786
60	78.10	10913

TABLE E-6 (Continued)

61	51.48	10944
62	38.80	11008
63	32.23	11229
64	26.47	11425
65	24.34	11610
66	23.43	11884

16	0
17	0
18	2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSQ	PCOST
1	24.44	252
2	7.58	252
3	6.02	353
4	2.03	374
5	1.87	475
6	0.84	538
7	0.63	559
8	0.28	633
9	0.26	696
10	0.12	807
11	0.09	881
12	0.05	1008

5	16	7	17		
J	PSQ1	K	PSQ2	IRLES2	JCLES1

5	0.1	7	0.1	16	17
5	0.1	7	0.1	16	17

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSG	SPCCST
1	342.66	312
2	325.81	312
3	324.24	413
4	320.25	434
5	320.09	535
6	319.06	598
7	318.85	619
8	318.51	693
9	318.48	756
10	318.34	867
11	318.31	941
12	318.28	1068
13	78.45	1117
14	26.50	1127
15	10.94	1157
16	3.73	1193
17	1.57	1246
18	0.77	1307

19 1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

TABLE E-6 (Continued

ITEM	SPSQ	SPCOST
1	592.13	377
2	575.28	377
3	573.72	478
4	569.73	499
5	569.57	600
6	568.53	663
7	568.33	684
8	567.98	758
9	567.96	821
10	567.82	932
11	567.79	1006
12	567.75	1133
13	327.92	1182
14	275.97	1192
15	260.41	1222
16	253.20	1256
17	251.04	1311
18	250.24	1372
19	62.23	1424
20	21.50	1435
21	9.30	1468
22	3.65	1507
23	1.95	1564
24	1.33	1630

20	0
21	2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

TABLE E-6 (Continued)

ITEM	PSC	PCOST
1	592.13	1031
2	575.28	1031
3	573.72	1132
4	569.73	1153
5	569.57	1254
6	568.53	1317
7	568.33	1338
8	567.98	1412
9	567.96	1475
10	567.82	1566
11	567.79	1660
12	567.75	1787
13	327.92	1836
14	275.97	1846
15	260.41	1876
16	253.20	1912
17	251.04	1965
18	250.24	2026
19	62.23	2078
20	21.50	2089
21	11.17	2089
22	11.17	2089
23	11.17	2089
24	11.17	2089
25	11.17	2089
26	11.17	2089
27	11.17	2089

TABLE E-6 (Continued)

20	21	7	20

J	PSQ1	K	PSQ2	IRLES2	JCLES1
20	21.5	7	11.2	21	20
20	21.5	7	11.2	21	20

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	600.25	1358
2	583.39	1358
3	581.83	1459
4	577.84	1480
5	577.68	1581
6	576.65	1644
7	576.44	1665
8	576.09	1739
9	576.07	1802
10	575.93	1913
11	575.90	1987
12	575.87	2114
13	336.04	2163
14	284.09	2173
15	268.52	2203
16	261.31	2239
17	259.15	2292
18	258.35	2353
19	70.34	2405
20	19.28	2416

22	0
23	1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	1095.93	8093
2	483.95	10311
3	258.01	12845
4	161.43	13479
5	115.41	14746
6	91.57	19181
7	70.64	23089
8	37.68	23200
9	25.51	23327
10	20.31	23358
11	17.83	23422
12	16.55	23643
13	15.42	23839

24	0
25	1

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	4570.27	2660
2	4561.62	2761

TABLE E-6 (Continued)

3	4560.00	2782
4	4559.45	2845
5	4559.20	2919
6	4559.12	3030
7	4559.09	3157
8	1123.14	5163
9	378.88	5566
10	155.93	6853
11	52.62	8331
12	21.67	10549
13	10.24	13083

26 2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSQ	PCOST
1	4570.27	10753
2	4561.82	10854
3	4560.00	10875
4	4559.45	10938
5	4559.20	11012
6	4559.12	11123
7	4559.09	11250
8	1123.14	13256
9	1095.93	13256
10	483.95	15474
11	378.88	15897
12	258.01	18431
13	161.43	19065

14	155.93	20332
15	115.41	21595
16	91.57	26034
17	70.64	29942
18	52.62	31420
19	37.68	31531
20	25.51	31658
21	21.67	33876
22	20.31	33907
23	17.83	33971
24	16.55	34192
25	15.42	34388

13 26 12 25

J	PSQ1	K	PSQ2	IRLES2	JCLES1
13	15.4	12	21.7	26	25
13	15.4	12	21.7	26	25

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	4652.29	11138
2	4643.84	11239
3	4642.01	11260
4	4641.46	11323
5	4641.21	11397
6	4641.13	11508
7	4641.10	11635

TABLE E-6 (Continued)

8	1177.94	13641
9	565.97	15859
10	460.89	16282
11	340.02	18816
12	243.44	19450
13	237.94	20717
14	197.42	21984
15	173.58	26419
16	152.65	30327
17	134.63	31805
18	119.70	31916
19	107.53	32043
20	103.68	34261
21	102.33	34292
22	99.85	34356
23	98.56	34577
24	97.44	34773
25	49.17	34884
26	31.35	35011
27	23.74	35042
28	20.11	35106
29	18.23	35327
30	16.58	35523

27 2

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSG	PCOST
1	4652.29	16254

TABLE E-6 (Continued)

2	4643.84	16355
3	4642.01	16376
4	4641.46	16439
5	4641.21	16513
6	4641.13	16624
7	4641.10	16751
8	1177.94	18757
9	565.97	20975
10	507.25	20975
11	460.89	21358
12	454.81	21459
13	409.71	21600
14	407.38	21621
15	393.54	21642
16	393.17	21705
17	387.47	21768
18	386.58	21842
19	384.66	21916
20	384.61	22027
21	383.88	22154
22	375.10	22255
23	367.61	22276
24	365.36	22339
25	364.32	22413
26	364.01	22524
27	340.02	25058
28	295.69	25159
29	243.44	25793
30	237.94	27060
31	226.67	27081

TABLE E-6 (Continued)

32	197.42	28348
33	195.40	28475
34	192.19	28483
35	192.10	28584
36	190.66	28600
37	189.87	28655
38	189.17	28703
39	188.91	28749
40	188.80	28817
41	176.45	28848
42	173.58	33283
43	171.49	33384
44	170.57	33448
45	167.52	33669
46	164.85	33865
47	163.86	34050
48	163.44	34324
49	154.89	34345
50	152.65	38253
51	150.52	38284
52	144.37	38348
53	143.74	38411
54	141.18	38632
55	138.57	38706
56	138.38	38902
57	137.34	39087
58	137.03	39198
59	136.90	39472
60	134.63	40950
61	125.48	40971

TABLE E-6 (Continued)

62	120.44	41002
63	119.70	41113
64	114.45	41214
65	112.60	41278
66	111.70	41341
67	108.53	41562
68	107.53	41689
69	105.31	41763
70	104.97	41959
71	103.68	44177
72	103.65	44362
73	103.40	44473
74	103.09	44747
75	102.33	44778
76	99.85	44842
77	98.56	45063
78	97.44	45259
79	90.14	45280
80	82.86	45343
81	81.22	45374
82	79.48	45448
83	78.47	45559
84	78.10	45686
85	51.48	45717
86	49.17	45828
87	38.80	45892
88	32.23	46113
89	31.35	46240
90	26.47	46436
91	24.34	46621

TABLE E-6 (Continued)

92	23.74	46652
93	23.43	46926

66 27 27 26

J	PSQ1	K	PSQ2	IRLES2	JCLES1
66	23.4	27	23.7	27	26
66	23.4	27	23.7	27	26

EQUIVALENT BRANCH LIST FOR A SERIAL MERGE

ITEM	SPSQ	SPCOST
1	4683.56	16581
2	4675.51	16662
3	4673.68	16703
4	4673.13	16766
5	4672.88	16840
6	4672.80	16951
7	4672.77	17076
8	1209.62	19084
9	538.92	21302
10	492.57	21725
11	486.48	21826
12	441.38	21927
13	439.05	21948
14	425.21	21969
15	424.84	22032
16	419.14	22055
17	418.25	22169
18	416.33	22243

TABLE E-6 (Continued)

19	416.28	22354
20	415.55	22481
21	406.78	22582
22	399.28	22603
23	397.04	22666
24	396.00	22740
25	395.68	22851
26	371.70	25385
27	327.36	25466
28	275.12	26120
29	269.62	27387
30	258.34	27408
31	229.10	28675
32	227.08	28802
33	223.86	28810
34	223.77	28911
35	222.33	28927
36	221.54	28982
37	220.84	29030
38	220.58	29076
39	220.47	29144
40	208.13	29175
41	205.25	33610
42	203.16	33711
43	202.24	33779
44	199.20	33956
45	196.52	34192
46	195.53	34377
47	195.11	34651
48	186.56	34672

TABLE E-6 (Continued)

49	184.33	38580
50	182.19	38611
51	176.04	38675
52	175.41	38738
53	172.85	38959
54	170.25	39033
55	170.05	39229
56	169.02	39414
57	168.70	39528
58	168.58	39799
59	166.30	41277
60	157.15	41298
61	152.11	41329
62	151.37	41440
63	146.12	41541
64	144.27	41605
65	143.37	41668
66	140.21	41889
67	139.20	42016
68	136.99	42090
69	136.64	42286
70	135.35	44504
71	135.32	44689
72	135.07	44800
73	134.76	45074
74	134.00	45105
75	131.52	45169
76	130.24	45390
77	129.11	45586
78	121.81	45607

TABLE E-6 (Continued)

79	114.53	45670
80	112.89	45701
81	111.16	45775
82	110.15	45886
83	109.77	46013
84	83.16	46044
85	80.85	46155
86	70.48	46219
87	63.91	46440
88	63.03	46567
89	58.14	46763
90	56.01	46948
91	55.41	46979
92	55.10	47253
93	41.27	47285
94	34.11	47395
95	27.82	47453
96	25.50	47586
97	24.51	47723
98	24.04	47815

EQUIVALENT BRANCH LIST FOR A PARALLEL MERGE

ITEM	PSG	PCOST
1	4683.96	17939
2	4675.51	18040
3	4673.66	18061
4	4673.13	18124
5	4672.88	18198

TABLE E-6 (Continued)

6	4672.80	18309
7	4672.77	18436
8	4570.27	18436
9	4561.82	18436
10	4560.00	18537
11	4559.45	18558
12	4559.20	18659
13	4559.12	18722
14	4559.09	18743
15	1209.62	20749
16	1123.14	20823
17	538.92	23041
18	492.57	23464
19	486.48	23565
20	441.38	23666
21	439.05	23687
22	425.21	23708
23	424.84	23771
24	419.14	23834
25	418.25	23908
26	416.33	23982
27	416.28	24093
28	415.55	24220
29	406.78	24321
30	399.28	24342
31	397.04	24405
32	396.00	24479
33	395.68	24590
34	378.88	24653
35	371.70	27187

TABLE E-6 (Continued)

36	327.36	27288
37	275.12	27922
38	269.62	29189
39	258.34	29210
40	229.10	30477
41	227.08	30604
42	223.86	30612
43	223.77	30713
44	222.33	30729
45	221.54	30784
46	220.84	30832
47	220.58	30878
48	220.47	30946
49	208.13	30977
50	205.25	35412
51	203.16	35513
52	202.24	35577
53	199.20	35798
54	196.52	35994
55	195.53	36179
56	195.11	36452
57	186.56	36474
58	184.33	40382
59	182.19	40413
60	176.04	40477
61	175.41	40540
62	172.85	40761
63	170.25	40835
64	170.05	41031
65	169.02	41216

TABLE E-6 (Continued)

66	168.70	41327
67	168.58	41601
68	166.30	43079
69	157.15	43100
70	155.93	43211
71	152.11	43242
72	151.37	43353
73	146.12	43454
74	144.27	43518
75	143.37	43581
76	140.21	43802
77	139.20	43929
78	136.99	44003
79	136.64	44199
80	135.35	46417
81	135.32	46602
82	135.07	46713
83	134.76	46987
84	134.00	47018
85	131.52	47082
86	130.24	47303
87	129.11	47499
88	121.81	47520
89	114.53	47583
90	112.89	47614
91	111.16	47688
92	110.15	47799
93	109.77	47926
94	83.16	47957
95	80.85	48068

TABLE E-6 (Concluded)

96	70.48	48132
97	63.91	48353
98	63.03	48480
99	58.14	48676
100	56.01	48861
101	55.41	48892
102	55.10	49166
103	52.62	49240
104	41.27	49272
105	34.11	49382
106	27.82	49480
107	25.50	49573
108	24.51	49710
109	24.04	49802

11 28 98 27

J	PSQ1	K	PSQ2	IRLES2	JCLES1
11	52.6	98	24.0	28	27
11	52.6	98	24.0	28	27

CPU TIME =.7.34secs

```
C$JOB TIME=(0,29)
C
C *** THIS IS PROGRAMME SYSAL. IT FORMULATES THE TERMINAL EQUATIONS OF
C    1THE SYSTEM AND SOLVES THE RESULTING SYSTEM EQUATIONS.
C
      INTEGER C,D,CD,CUTSET
      COMMON G(20,21),DI(20,1),XX(20),N,LLI
      DIMENSION KUTSET(15,20),E(35),FLOW(10),OD(15,20),D(15,20),CUTSET(2
     10,18),GK(30,15),DELTAH(30,1),HLOSS(30,1),DEAL(30,1),DL(30,15),DMAN
     2D(100),A(100)
      READ 1,N,NE,ND,NC
    1 FORMAT(4I5)
      NP=N-1
      MG=NP+1
      JG=NE-NC
      MP=NP
      ND1=ND+1
      NP1=JG-NP
C
C *** READ IN CUTSETS AND COMPUTE ENTRIES OF MATRIX G
C
      DO 93 I=1,NP
      READ 1003,(KUTSET(I,J),J=1,MG)
 1003 FORMAT(2X,I1,4X,13I4)
   93 CONTINUE
      SFLOW=0.
      DO 4100 K7=1,NC
      READ 910,FLOW(K7)
  910 FORMAT(F6.3)
      PRINT 9109,FLOW(K7)
 9109 FORMAT (///5X,F6.3/)
      SFLOW=SFLOW+FLOW(K7)
 4100 CONTINUE
      PRINT 9190,SFLOW
 9190 FORMAT (5X,///7H SFLOW=,F6.3///)
      DO 925 I=1,NP1
      READ 4321,(CUTSET(I,J),J=1,MP)
 4321 FORMAT(10I4)
  925 CONTINUE
      SUMA=0.
      PRINT 1916
 1916 FORMAT (5X,'NODE',5X,'DEMAND',///)
      DO 99 I=1,NC
      READ 1991,DMAND(I)
 1991 FORMAT(5X,F5.3)
      A(I)=DMAND(I)
      SUMA=SUMA+DMAND(I)
   99 CONTINUE
      DO 550 I3=1,ND
      E(I3)=A(I3)/SUMA
  550 CONTINUE
      E(1)=0.0
      E(2)=0.0
      E(4)=0.0
      PRINT 1376
 1376 FORMAT ('1',/////10X,'OUTPUT DEMAND FACTORS',/////)
      PRINT 3001,(I,E(I),I=1,ND)
 3001 FORMAT(5X,'DEMAND FACTOR AT NODE  ',I2, '    IS ',F7.4/)
      DO 911 I=ND1,JG
      READ 1001,C,DIA,L
```

```
 1001 FORMAT(I3,5X,F4.2,5X,I6)
  911 E(I)=0.279*C*DIA**2.63/L**0.54
      DO 97 I=1,NP
      READ 1007,(OD(I,J),J=1,NP)
 1007 FORMAT(2X,I1,4X,12I4)
   97 CONTINUE
      PRINT 5000,(I,E(I),I=ND1,JG)
 5000 FORMAT (4X,6H LINK ,I3,10H VALUE IS ,F7.4/)
C
C     BUILD DIAGONAL ELEMENT OF MATRIX G FROM ARRAY E
C
      DO 40 I=1,NP
      DO 20 L=1,NP
   20 G(I,L)=0.
      N1=KUTSET(I,1)+1
      DO 23 J=2,N1
      M=KUTSET(I,J)
      MM=-M
      IF(M)21,991,22
   21 G(I,I)=G(I,I)+E(MM)
      GO TO 23
   22 G(I,I)=G(I,I)+E(M)
   23 CONTINUE
C
C *** BUILD OFF-DIAGONAL ELEMENTS OF MATRIX. UPPER DIAGONAL ONLY.
C
      JJ=I+1
      DO 41 J=JJ,NP
      IF(JJ.GT.NP)GO TO 40
      N2=KUTSET(J,1)+1
      KSW=0
      DO 42 K=2,N1
      IF(JJ.GT.NP) GO TO 40
      DO 43 L=2,N2
      IF(JJ.GT.NP) GO TO 40
      IF(KUTSET(I,K)-KUTSET(J,L))30,32,30
   30 IF(KUTSET(I,K)+KUTSET(J,L))43,32,43
   32 KPRCD1=KUTSET(I,K)*KUTSET(J,L)
      IF(KSW)33,34,33
   33 IF(KPRCD1*KPRCD)992,992,35
   34 KSW=1
      KPRCD=KPRCD1
   35 M=KUTSET(I,K)
      IF(M)36,37,37
   36 M=-M
   37 IF(KPRCD)38,39,39
   38 G(I,J)=G(I,J)-E(M)
      GO TO 43
   39 G(I,J)=G(I,J)+E(M)
   43 CONTINUE
   42 CONTINUE
   41 CONTINUE
   40 CONTINUE
C
C *** FILL IN LOWER TRIANGLE OF MATRIX G
C
      DO 44 I=1,NP
      N1=I+1
      IF(N1.GT.NP) N1=NP
      DO 44 J=N1,NP
```

```
   44 G(J,I)=-G(I,J)
      PRINT 1367
 1367 FORMAT ('1',/////10X,'OUTPUT G_MATRIX',/////)
      PRINT 2001,((G(I,J),J=1,9),I=1,9)
 2001 FORMAT (///1X,9(9F9.4///1X))
      DO 1980 I=2,NP
      G(I,I)=0.0
 1980 CONTINUE
      I=1
 1985 RSUM=0.0
      I=I+1
      IF (I.GT.NP) GO TO 1960
      ILESS1=I-1
      DO 1958 J=1,ILESS1
      RSUM=G(I,J)+RSUM
      G(I,I)=RSUM
      G(I,J)=0.0
 1958 CONTINUE
      GO TO 1985
 1960 PRINT 2001,((G(I,J),J=1,9),I=1,9)
C
C *** SOLVE EQUATIONS FOR ESTIMATED DRAW-OFFS AT NODAL POINTS.
C
      N=NP
      NG=NP+1
      DO 70 I=1,NP
      DO 70 J=1,NP
      D(I,J)=0
   70 CONTINUE
      DO 4602 I=1,NP
      DO 4602 J=1,3
      D(I,J)=CD(I,J)
 4602 CONTINUE
      DO 28 I=1,NP
      G(I,NG)=0.0
      K6=D(I,2)
      IF(K6.EQ.0) GO TO 5806
 9992 NS=D(I,1)+1
      DO 912 J=2,NS
      M1=D(I,J)
      NM1=-M1
      IF(M1.GT.0) GO TO 914
  913 G(I,NG)=G(I,NG)+(-1.0*FLOW(NM1-JG))
      GO TO 912
  914 G(I,NG)=G(I,NG)+(1.0*FLOW(M1-JG))
  912 CONTINUE
 5806 CONTINUE
   28 CONTINUE
      DO 2500 I=1,NP
      DI(I,1)=DMAND(I)
      DI(1,1)=-10.164
 2500 CONTINUE
      N=NP
      CALL GAUSS
      DO 9500 I=1,NP
 9500 CONTINUE
      PRINT 3176
 3176 FORMAT ('1',/////10X,'OUTPUT BRANCH PRESSURES',/////)
      PRINT 1776
 1776 FORMAT (15X,'DELTAH',5X,'HEAD LOSS(FT)',///)
```

```
      DO 2600 IY=1,NP
      DELTAH(IY,1)=XX(IY)
      DELTAH(IY,1)=ABS(DELTAH(IY,1))
      HLOSS(IY,1)=DELTAH(IY,1)**1.85
      PRINT 1876,IY,XX(IY),HLOSS(IY,1)
 1876 FORMAT (5X,I2,F13.3,10X,F8.3/)
 2600 CONTINUE
      DO 111 KP=1,NP
      DO 111 IP=1,JG
      DL(IP,KP)=0.
  111 CONTINUE
      DO 4603 I=1,NP1
      DO 4603 J=1,NP
      DL(I,J)=CUTSET(I,J)
 4603 CONTINUE
      RUN=0.
      CONS1=1.0000
 8606 SUM=0.
      AP=0.
      DO 60 I=1,ND
      IF (E(I).EQ.0) GO TO 949
      DELTAH(I,1)=DMAND(I)/E(I)
      G(I,NG)=DELTAH(I,1)*E(I)*CONS1
      IF(G(I,NG))98,949,98
   98 AP=AP+1.
  949 CONTINUE
      G(1,NG)=0.0
      SUM=SUM+G(I,NG)
      PRINT 2000,I,G(I,NG)
 2000 FORMAT(10X,'DEMAND AT NODE ',I2,' IS ',F10.3/)
   60 CONTINUE
      IF(SUM.LT.0.0) SUM=-SUM
      FDIFF=SUM-SFLOW
      DO 58 I=1,NP1
      SUMB=0.
      DO 59 J=1,NP
   59 SUMB=SUMB+DL(I,J)*HLOSS(J,1)
      K=I+NP
      HLOSS(K,1)=ABS(SUMB)
      DELTAH(K,1)=HLOSS(K,1)**0.54
   58 CONTINUE
      DO 606 I=1,NP1
      K=I+NP
      GK(K,1)=DELTAH(K,1)*E(K)
  606 CONTINUE
      PRINT 1414
 1414 FORMAT ('1',/////10X,'OUTPUT ELEMENT FLOWS',//////)
      PRINT 6700
 6700 FORMAT (///5X,'LINK FLOWS(M.G.D.)',15X,'HEAD LOSS(FT)',///)
      DO 68 I=1,ND
      PRINT 6800,I,G(I,NG),I,HLOSS(I,1)
 6800 FORMAT (5X,2HY(,I3,1X,2H)=,F10.3,15X, 2HX(, I3, 1X,2H)=,F10.1/)
   68 CONTINUE
      DO 608 I=1,NP1
      K=I+NP
      PRINT 6800,K,GK(K,1),K,HLOSS(K,1)
  608 CONTINUE
C
C     ERROR ROUTINES
C
```

```
 9001 FCRMAT(15H  ERRCR CN CARC,I3,27H CF CUTSETS ELEMENT INDEX= ,I2,5H
     1IS ZERO.///)
 9002 FCRMAT(15H ERRCR CN CARCS,I3,31H  OF CUTSET,SIGNS DO NOT AGREE.//)
      DO 600 I=1,NO
      IF (E(I).EQ.0) GO TO 600
      G(I,NG)=DELTAH(I,1)/(CCNS1*E(I))
  600 CONTINUE
      RUN=RUN+1.
      PRINT 7705,CCNS1,RUN
 7705 FCRMAT (5X,6HCONS1=,F9.7,5X,16HTHE ABCVE IS RUN,F4.0)
      ERRCR=0.0005
      IF (FDIFF-ERRCR) 999,991,991
  991 CCNS1=SFLCW/SUM
      PRINT 9001,I,J
      IF (RUN.GT.1.0) GO TO 2010
      GC TC 8606
  999 CONS1=SFLOW/SUM
      IF (RUN.GT.1.0) GO TO 2010
      GO TO 8606
  992 PRINT 9002,I,J
 2010 CALL EXIT
      STCP
      ENC
C
C *** GAUSSIAN ELIMINATICN
C
      SUBRCUTINE GAUSS
      COMMON A(20,21),B(20),X(20),N,ILL
      ILL=0
C
C *** THE CASE N EQUALS ONE
C
      IF(N-1)4,1,4
    1 IF(A(1,1))2,3,2
    2 X(1)=B(1)/A(1,1)
      RETURN
    3 ILL=1
      RETURN
C
C *** THE GENERAL CASE,FINDING THE PIVOT
C
    4 NLESS1=N-1
      DC 13 I=1,NLESS1
      BIG=ABS(A(I,I))
      L=I
      IPLUS1=I+1
      DO 6 J=IPLUS1,N
      IF(ABS(A(J,I))-BIG)6,6,5
    5 BIG=ABS(A(J,I))
      L=J
    6 CCNTINUE
C
C *** INTERCHANGE IF NECESSARY
C
      IF(BIG)8,7,8
    7 ILL=1
      RETURN
    8 IF(L-I)9,11,9
    9 DC 10 J=1,N
```

```
            TEMP=A(L,J)
            A(L,J)=A(I,J)
         10 A(I,J)=TEMP
            TEMP=B(L)
            B(L)=B(I)
            B(I)=TEMP
      C
      C *** REDUCE COEFFICIENTS TO ZERO
      C
         11 DO 13 J=IPLUS1,N
            QUOT=A(J,I)/A(I,I)
            DO 12 K=IPLUS1,N
         12 A(J,K)=A(J,K)-QUOT*A(I,K)
         13 B(J)=B(J)-QUOT*B(I)
      C
      C *** THE BACK SUBSTITUTION STEP
      C
            IF(A(N,N))15,14,15
         14 ILL=1
            RETURN
         15 X(N)=B(N)/A(N,N)
            I=N-1
            IPLUS1=I+1
         16 SUM=0.
            IPLUS1=I+1
            DO 17 J=IPLUS1,N
         17 SUM=SUM+A(I,J)*X(J)
            X(I)=(B(I)-SUM)/A(I,I)
            I=I-1
            IF(I)18,18,16
         18 RETURN
            END

      $EXEC
```

TABLE F-1

OUTPUT OF PROGRAM SYSAL

```
DEMAND FACTOR AT NODE   1     IS   0.0000

DEMAND FACTOR AT NODE   2     IS   0.0000

DEMAND FACTOR AT NODE   3     IS   0.1133

DEMAND FACTOR AT NODE   4     IS   0.0000

DEMAND FACTOR AT NODE   5     IS   0.1133

DEMAND FACTOR AT NODE   6     IS   0.1417

DEMAND FACTOR AT NODE   7     IS   0.1417

DEMAND FACTOR AT NODE   8     IS   0.1417

DEMAND FACTOR AT NODE   9     IS   0.3483

LINK   10 VALUE IS   1.4636

LINK   11 VALUE IS   1.4636

LINK   12 VALUE IS   2.3517

LINK   13 VALUE IS   0.8043

LINK   14 VALUE IS   0.3799

LINK   15 VALUE IS   0.8043

LINK   16 VALUE IS   0.3799

LINK   17 VALUE IS   0.8043

LINK   18 VALUE IS   0.3799

LINK   19 VALUE IS   0.3799

LINK   20 VALUE IS   0.3799

LINK   21 VALUE IS   0.3799
```

TABLE F-1 (Continued)

	DELTAH	HEAD LOSS(FT)
1	-0.000	0.000
2	3.374	9.485
3	1.884	3.227
4	2.222	4.381
5	2.712	6.332
6	4.225	14.379
7	3.786	11.739
8	4.225	14.379
9	4.659	17.233

DEMAND AT NODE 1 IS 0.000

DEMAND AT NODE 2 IS 0.000

DEMAND AT NODE 3 IS 1.152

DEMAND AT NODE 4 IS 0.000

DEMAND AT NODE 5 IS 1.152

DEMAND AT NODE 6 IS 1.440

DEMAND AT NODE 7 IS 1.440

DEMAND AT NODE 8 IS 1.440

DEMAND AT NODE 9 IS 3.540

TABLE F-1 (Continued)

LINK FLOWS(M.G.D.)		HEAD LOSS(FT)	
Y(1)=	0.000	X(1)=	0.0
Y(2)=	0.000	X(2)=	9.5
Y(3)=	1.152	X(3)=	3.2
Y(4)=	0.000	X(4)=	4.4
Y(5)=	1.152	X(5)=	6.3
Y(6)=	1.440	X(6)=	14.4
Y(7)=	1.440	X(7)=	11.7
Y(8)=	1.440	X(8)=	14.4
Y(9)=	3.540	X(9)=	17.2
Y(10)=	4.932	X(10)=	9.5
Y(11)=	2.795	X(11)=	3.2
Y(12)=	5.222	X(12)=	4.4
Y(13)=	2.179	X(13)=	6.3
Y(14)=	1.603	X(14)=	14.4
Y(15)=	2.179	X(15)=	6.3
Y(16)=	1.603	X(16)=	14.4
Y(17)=	3.041	X(17)=	11.7
Y(18)=	1.603	X(18)=	14.4
Y(19)=	1.767	X(19)=	17.2
Y(20)=	1.603	X(20)=	14.4
Y(21)=	1.767	X(21)=	17.2

APPENDIX G

STORAGE RESIDENCY AND CPU TIME FOR VARIOUS PROGRAMS RAN
ON IBM COMPUTER MODEL 370/158

ITEM	PROGRAM	NUMBER OF NODES	STORAGE RESIDENCY	CPU TIME(SECS)
1.	SYSFM	10	34.3K	5.36
2.	SYSFM	93	156.1K	28.91
3.	SYSAL	10	4.2K	1.33
4.	OPTIMIZE	9	4.2K	1.33*
5.	OPTIMIZE	9	5.4K	1.69**
6.	OPTIMIZE	9	6.4K	2.00***
7.	OPTIMIZE	13	5.0K	1.59*
8.	OPTIMIZE	13	6.8K	2.13**
9.	OPTIMIZE	13	8.7K	2.74***
10.	OPTIMIZE	20	6.7K	2.11*
11.	OPTIMIZE	20	10.0K	3.22**
12.	OPTIMIZE	20	14.3K	4.48***
13.	OPTIMIZE	28	8.9K	2.81*
14.	OPTIMIZE	28	14.9K	4.68**
15.	OPTIMIZE	28	23.4K	7.34***

* 3 - Diameter selections.

** 5 - Diameter selections.

*** 7 - Diameter selections.

Program OPTIMIZE refers to PROGRAM MERGEP and SUBROUTINE MERGES.

APPENDIX H

DATA FOR HYPOTHETICAL NETWORK