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## THE UNIVERSITY OF OKLAHOMA <br> GRADUATE COLLEGE

## DEVELOPMENT AND PROPERTIES OF A SELF - BURSTING PELLET AS AGGLOMERATED FROM COAL FINES BY USE OF AN ORGANIC BINDER

## A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

# in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY 

By<br>JOHN DAVID MORRIS<br>Norman, Oklahoma

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# DEVELOPMENT AND PROPERTIES OF A SELF - BURSTING PELLET AS AGGLOMERATED FROM COAL FINES <br> BY USE OF AN ORGANIC BINDER 

## A DISSERTATION

APPROVED FOR THE SCHOOL OF PETROLEUM AND GEOLOGICAL ENGINEERING

APPROVED BY


## ABSTRACT

In all aspects of the mining, processing, and handling of coal, fine particles are generated that are in many cases wasted. These fines can be agglomerated into strong and durable pellets by use of an organic binder which eliminates the need for grinding at the coal burning power plant. When heated, the water in the binder vaporizes, causing a rapid pressure build-up inside the pellet and degredation of the binder, resulting in the return of the coal to fine sized particles.

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# DEVELOPMENT AND PROPERTIES OF A SELF-BURSTING PELLET AS AGGLOMERATED FROM COAL FINES BY USE OF <br> AN ORGANIC BINDER 

## CHAPTER I

## BACKGROUND AND LITERATURE SEARCH

With the development of more mechanized systems for the mining, preparation and transportation of coal has come an increase in the amount of fine particles produced and subsequently wasted. It has been estimated that if these fines are not recaptured, up to $30 \%$ of some coals could be lost.

Continuous mining methods result in an extraordinary amount of coal fines which are washed out in the process of coal preparation. Tailing ponds receive much of the waste as the dust settles out in still water. $(9,44)$ Coal seams at great depth are now accessible, but due to the friable nature of such coal, waste problems are compounded. Mechanical crushing is frequently employed to clean undesirable minerals and ash from the coal, but again, much fine coal is lost in the
process. Some is lost due to the inability of froth floatation or gravity separation techniques to recover the finer fractions, some is blown away. Most must be disposed of in some way by the operation (47).

Further losses are experienced in the transportation of coal. Open - top railroad car losses are normally about $2 \%$ of the total weight of the coal shipped. Similar losses are known in truck transportation(33). Conveyor belts also inflict heavy damage to the rather fragile coal and the fines may be lost.

But concurrent with the increased production of and potential loss of coal fines has come an increased need to utilize all available coal. The price has increased as have mining costs, and excessive waste is uneconomical. Additionally, as the supply of liquid petroleum is dwinding, greater attention is being focused on coal. Furthermore, stricter environmental controls have been enacted, and polluted mine water, wind losses, burning of impure coal etc. are no longer acceptable.

It might be thought that the existence of coal fines would be a boon to the industry rather than a problem, since it is well known that nearly all coal-fired electrical plants must grind their coal to minus 200 mesh before injecting it into the furnace to gain maximum burning efficiency. By acquiring coal fines directly from the mine the grinding step could be eliminated. Unfortunately, unless the electric
plant is near enough to the mine, cleaning operation or major pipeline, so that the fines can be pumped in slurry form, there is no economical way to transport them $(17,45)$.

## The Taconite Industry

A solution proposed for the production of iron from low-grade iron ores (taconite) has recently been applied to the coal industry. By adding a bentonitic binder to the fines produced by the crushing and separating of the iron ore, and rotating the mixture in a balling drum, small pellets can be produced which are sufficiently strong and elastic to allow proper handling and transportation to the blast furnace. (See for example 5,41).

Unfortunately, the presence of the clay decreases the thermal efficiency of the process and except in rare cases contributes to the slag and ash produced, causing some to investigate the use of organic binders instead of the inorganic bentonite $(21,24)$.

At the present time some in the coal industry are attempting to agglomerate coal fines using the experience gained in the taconite industry, producing coal pellets which have moderately high physical characteristics (10,16,52,54). However, since the pellet is to be burned, the clay binder increases the ash content to unacceptably high levels, and the pellets so produced are in many cases too weak to be transported if the water content is greater than $3 \%(21,52)$.

## Organic Binders

Numerous organic binders have been investigated from lignin to various hydrocarbon compounds (most notably coal tar pitch, fuel 011 and kerosene). As might be expected several problems have arisen to dampen the enthusiasm of the investigators. When heated, such mixtures frequently give off unacceptably high levels of noxious gasses. If the pellets encounter heat before they are burned, they may "melt" into a sticky paste, making handing nearly impossible. Furthermore the excessive cost of such petroleum binders drives up the cost of the coal product.

Probably the most serious drawback to the use of these organic additives is that they are all rather slow-burning, much slower than that of pulverized coal. In some cases the burning of the binder actually masks the heat value of the coal. It is this problem of the low-efficiency burning process of coal pellets as presently produced to which this work is primarily addressed.

## Self - Bursting Pellets

Preliminary research with an oil-in-water emulsion containing hydrocarbon solvents has proved interesting. Pellets with this emulsion as the binder exhibited excellent physical characteristics. They had acceptably high compressive strength, were somewhat elastic, and were rather resistant to alteration by water once dried (21).

In addition, the use of an increased percentage of water in the binder provides a solution to the slow burning aspect of the previously produced pellets, and eliminates the need for regrinding the coal pellets at the power plant as well. As the pellet is introduced into the furnace or preliminary heating chamber, the heat vaporizes the moisture inside the pellet causing both a rapid internal pressure build-up and degradation of the binder, reducing its compressive strength to a very low level. Confining or encasing the pellet maintains the pressure until the pellet actually bursts or the confining environment loads the pellet with loads it can no longer withstand. The pellet explodes or disintegrates, and reduces to powder once again, allowing the efficient combustion of pulverized coal to control.

## Coal Used In Study

The coal selected for laboratory agglomeration was studied solely for its ease of access to this investigator, not for any other property. The principles derived herein are sufficiently general, and although it is thought that they are adaptable to nearly every coal, each coal reacts differently and must be investigated separately for particular responses. For comparison purposes the coal used in this study is known as the Stigler coal, and was surface mined in south central Haskell County, Oklahoma. Of low to medium volatile rank, this bituminous coal is typically of low sulfur and ash content and is used for electrical power generation and coke manufacture.

Much of the pyritic sulfur present in fresh samples of the coal was removed in the grinding and sieving process. The chemical make-up of the coal varies from mine to mine as can be seen in Table One, with the range of values given in parentheses (18, 32).

TABLE 1
AVERAGE CHEMICAL ANALYSIS OF STIGLER COAL, HASKELL COUNTY, OKLAHOMA

| Rank | Bituminous; low to medium volatile <br> $(24.2$ to $27.4 \%$ volatile matter) |
| :--- | :--- |
| Sulfur | Low; Average $=1.5 \%$ (range 0.4 to 5.2\%) |
| Ash | Low; Average $=4.4 \%$ (range 4.2 to 5.1\%) |
| BTU | Average $=14,400$ Btu/1b (range 13,869 to <br> $15,030 \mathrm{Btu} / 16)$ |
| Fixed Carbon | Average $=67.0 \%$ (range 63.2 to 70.1\%) |
| Ash Softening <br> Temperature | $2,180^{\circ} \mathrm{F}$ |

## LABORATORY EXPERIMENTATION

The production of suitable pellets from coal fines is at least a four stage process; mixing, pelletizing, warming and coating. The coal fines themselves can be acquired in many ways, whether as waste from a mine operation, product of a separator, or as in this case, raw coal ground, dried and passed through a 200 mesh sieve. Each of the remaining four operations can likewise be carried out in an almost limitless number of ways.

The mixing stage consists of adding to a known quantity of dried fines a suitable amount of binder of known concentration. Catalysts or additional water may be deemed beneficial. This could be a one step or several step process, as determined necessary for the particular coal used and characteristics desired. Typical binders are bentonite, coal tar, oil, hydrocarbon derivatives, waxes, cross-linked polymers, organic polymers and many other organic derivatives. A two-armed, kneading type mixer is best suited for mixing such a combination, with some provision to scrape the bottom and sides of


Figure 1. The Agglomeration Drum


Figure 2. Typical Pellets as Agglomerated
the container to insure complete spreading. In this paper a linear linking wax derivative was chosen as a binder, and a rotating, variable speed mixer was used.

Once the binder is evenly spread throughout the fines, the entire amount of moist material can be placed in the agglomerating or balling drum. The huge rotary kilns in use by the taconite industry can be approximated on a laboratory scale by a rotating pan driven by compressed air, maintained at constant revolution by a regulator. Both the rpm and the tilt of the pan or drum as well as the temperature applied, can be adjusted to achieve a variety of energy levels within the drum. The constant rolling action combined with the agglomerating nature of a proper binder transforms moist fines into pellets, with physical characteristics such as size, strength and stability depending on the energy level. The accompanying picture is of the balling apparatus used in this experiment.

It may be beneficial to warm the fresh pellets shortly after agglomeration to increase the initial strength. This can be done in any oven or hot air grate, but the warming temperature should have as its goal the drying out of the surface of the pellets and curing of the binder. A temperature slightly above the boiling point of water is sufficient, while higher temperatures may tend to devolatilize the coal itself, thereby lowering its BTU content, as well as weakening the pellet. A temperature of $120^{\circ} \mathrm{C}$ was chosen for the follow-
ing experiment.
The coating of the pellets is a crucial step for two reasons. If it is suspected that the pellets may be subjected to water action (rain while stockpiled, on a railroad car, etc) before reaching the user, an insoluble film must protect the pellet from being dissolved or altered by the wa-: , ter. Few potential binders would be unaffected, but effective coating removes the problem. The waxy binder used herein is water soluble and uncoated pellets have absolutely no strength when soaked in water. In addition, the film must be sufficiently strong to aid in entrapment of internal pressures which develop in the pellet when heated, aiding its bursting and return to its powdered state. Several cross-linking polymers, rubber derivatives, and organic materials fulfill these needs although none have been developed specifically for such purposes, and it is reasonable to assume that future polymers will perform even better. A cross-linking organic polymer was used in this experiment to coat the pellets.

## Factorial Design

Obviously many factors are involved in the production of such coal pellets. Many have already been briefly discussed, others are rather obvious. As can be seen from the following list they fall into three broad categories, but the list could well be much longer.

## Preparation

Type binder
Concentration of binder
Amount of birder
Initial moisture content
Additives to water (salt content, pH, etc)
Method of mixing
Order of mixing
Temperature of mixing
Size of sample
Catalysts
Surrounding conditions (temperature and humidity)

## Agglomeration

Revolutions per minute
Time of agglomeration
Tilt of drum
Variable rom
Variable tilt
Temperature
Curing
Prior gel time
Warming time
Warming temperature
Aging time
Aging temperature
Coating

It would be impractical to thoroughly. study the effects of all these factors as well as their interactions. A well-known statistical method known as the method of factorial design is therefore employed to provide a look at selected factors. Commonly three of the most important factors are selected in this method and varied in a controlled way allowing the effect of each factor to be determined with the same accuracy as if only one factor were varied at a time. In addition the interaction between the selected factors can also be determined.

The three factors which have been selected for study are 1) concentration of binder (from the preparation group) 2) rpm and 3) temperature (unrelated factors from the agglomeration group). Simultaneously, three of the factors from the curing group are also studied including warming, aging and coating, although not in the statistical matrix.

In this factorial design method, each of the three selected factors are assigned upper and lower values, and each combination of factors is then tested experimentally. In this way a great deal of information can be obtained from only $2^{n}=2^{3}=8$ trials. In Table 2 , the high and low values of each factor are represented as + or - respectively, and the possible combinations are listed.

TABLE 2
SYMBOLS FOR TREATMENT COMBINATIONS

| $\begin{array}{r} \text { Trial } \\ \text { No. } \end{array}$ | Symbols for Combination | A <br> Concentration Of Binder, \% $\begin{aligned} & \text { High }=25 \\ & \text { Low }=15 \\ & \hline \end{aligned}$ | $B$ RPM High $=50$ Low $=30$ | Temperature, ${ }^{\circ} \mathrm{C}$ $\begin{aligned} & \text { High }=90 \\ & \text { Low }=\text { room } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | (1) | - | - | - |
| 2 | a | + | - | - |
| 3 | b | - | $+$ | - |
| 4 | $a b$ | $+$ | $+$ | - |
| 5 | c | - | - | + |
| 6 | ac | $+$ | - | + |
| 7 | bc | - | $+$ | + |
| 8 | $a b c$ | $+$ | + | + |

In order to eliminate repetitive error, the sequence of trials was run in a random order, and a replicate series was also carried out, (again in a random order) designated as trials 1, 2, 3. . . . 8, and $1 r, 2 r, 3 r$. . . $8 r$ respectively.

## Responses

The resulting pellets were first evaluated for total yield of pellets caught on a \#8 sieve. In each case, all of the fines agglomerated into pellets of one size or another, those passing the sieve were discarded. In an industrial situation they would probably be returned to the drum for further agglomeration.

To determine the average stability of the pellets the standard 18" drop test was performed on raw pellets and on others warmed at $120^{\circ} \mathrm{C}$ for 15 minutes. The drop test consists of dropping the pellet onto a steel plate from a height of $18^{\prime \prime}$ until the pellet.breaks into smaller fragments. A value of 8 to 10 was considered satisfactory although the test was continued if necessary up to a maximum of 25 , at which time a subjective judgement was made based on the elasticity of the pellet. Such pellets were then given an index of either 25 or elastic. All results were the average of the response of several pellets.

The average crushing strength of both raw and warmed pellets was evaluated simply as the vertical load necessary to crush a single pellet. The raw pellets straight from the
agglomerating drum seldom exhibited any crushing strength, but were rather plastic. Warming for 15 minutes produced a thin, frail shell which was able to withstand a slight load. Aging increased the strength dramatically.

An effort to determine the average resistance to alteration by water consisted of soaking both raw and warmed pellets in water for 15 minutes, draining and drying them. Seldom did pellets remain after such treatment unless coated with an impermeable shell. When coated pellets were so tested, very little alteration occurred.

All three of the foregoing tests were run on both raw and warmed pellets. Two days later, the same tests were repeated on both raw and warmed "aged" pellets and coated pellets. In each test the set of individual pellets selected for investigation represented the entire yield in size and shape.

Results

The results of all these trials are tabulated in Tables 3 and 4.

TABLE 3
RESULTS OF THE PRIMARY RUN


TABLE 4
RESULTS OF THE REPLICATE RUN


## Interpretation of the Results

The effects of the three major factors studied (le.concentration of binder, rpm, and temperature) can be statistically quantified. Likewise the interactions between the factors can be quantified (15).

Using the symbolic expressions introduced in Table 2, we find the effect of factor $A$, (i.e.concentration of binder) by averaging the responses corresponding to all trials containing 'a' (i.e. at the higher level of factor $A$ ) and all those not containing 'a' (i.e. at the lower level of factor $A$ ) and determining the difference. Specifically the main effect of A on the average responses is found by:

Average yield at higher level of $A$

$$
=1 / 4(a+a b+a c+a b c)
$$

Average yield at lower level of $A$

$$
=-1 / 4(1+b+c+b c)
$$

Therefore the main effect of $A$ can be rearranged as follows:

$$
A=1 / 4(a+a b+a c+a b c-1-b-c-b c)
$$

Likewise,

$$
\begin{aligned}
& B=1 / 4(b+a b+b c+a b c-1-a-c-a c) \\
& C=1 / 4(c+a c+b c+a b c-1-a-b-a b)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\mathrm{AB} & =1 / 4(1+a b+c+a b c-a-b-a c-b c) \\
\mathrm{AC} & =1 / 4(1+b+a c+a b c-a-c-a b-b c) \\
\mathrm{BC} & =1 / 4(1+a+b c+a b c-b-c-a b-a c) \\
\mathrm{ABC} & =1 / 4(a+b+c+a b c-1-a b-a c-b c)
\end{aligned}
$$

Due to the fact that the sample is small, a determination of the standard deviation may not be accurate. However by using the method of pooled variance we can approximate its square by summing the squares of the differences between the primary and replicate responses and dividing by 2 and multiplying by the inverse of the degree of freedom (in this case 8 , after losing 8 out of 16 due to averaging).

$$
S p^{2}=\frac{\sum}{2} d i^{2}\left(\frac{1}{d f}\right)
$$

The variance of the effect, $V(E 1)$, is

$$
V(E i)=\frac{S p^{2}}{4}
$$

For a confidence interval of $95 \%$ and a degree of freedom of 8 , the 't' distribution ratio is 2.31, which implies that for a $95 \%$ confidence interval
$\mathrm{Ei} \pm t \sqrt{\mathrm{~V}(\mathrm{Ei})}$
$=E i \pm 2.3 \cdot \sqrt{V(E i)}$
The system of 7 effects has been calculated for each of the responses which yield meaningful information. They are reproduced in Tables 5-16 below.

Note that there are no statistical studies for the responses of the Drop Index for both aged raw and aged warm states. After two days of aging nearly all pellets were elastic, with drop indices >25. No study is possible.

Likewise no study for the crushing strength of raw pellets is possible. All were rather plastic and supported essentially no load.

## TABLE 5

CALCULATION OF EFFECTS OF FACTORS ON AVERAGE PELLET YIELD

| Symbol | $\#$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v 3}$ | $d=Y_{1}-Y_{2}$ | $a^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 69 | 67 | 68 | 2 | 4 |
| $a$ | 4 | 58 | 53 | 55.5 | 5 | 25 |
| $b$ | 6 | 64 | 65 | 64.5 | -1 | 1 |
| $a b$ | 2 | 45 | 56 | 50.5 | -11 | 121 |
| $c$ | 7 | 85 | 78 | 81.5 | 7 | 49 |
| $a c$ | 3 | 71 | 56 | 63.5 | 15 | 225 |
| $b:$ | 5 | 60 | 60 | 60 | 0 | 0 |
| $a b c$ | 1 | 50 | 54 | 56.5 | 5 | 25 |
|  |  |  | $A v g=62.5$ |  | $\sum=450$ |  |

$$
\begin{aligned}
& S p^{2}=\frac{\sum d_{i}^{2}}{2}\left(\frac{I}{d f}\right) \\
& =\frac{450}{2}\left(\frac{1}{8}\right)=28.12 \\
& V\left(E_{1}\right)=\frac{S_{0}^{2}}{4}=7.03 \\
& t_{\delta, 0.05}^{2}=2.31 \\
& t \sqrt{V\left(E_{1}\right)}=2.31 \sqrt{7.03} \\
& =6.12
\end{aligned}
$$

$$
\begin{aligned}
& \therefore=1 / 4(-48)=-12.00 \pm 6.12 \\
& B=1 / 4(-37)=-9.25 \pm 6.12 \\
& C=1 / 4(23)=5.75 \pm 6.12 \\
& \therefore B=1 / 4(13)=3.25 \pm 6.12 \\
& \therefore C=1 / 4(5)=1.25 \pm 6.12 \\
& B C=1 / 4(-20)=-5.0 \pm 6.12 \\
& A B C=1 / 4(16)=4.0 \pm 6.12
\end{aligned}
$$

TABLE 6
CALCULATION CF EFFECTS CF FACTORS ON THE
AVERAGE DROP INDEX OF RAW PELLETS

| Symbol | $\frac{11}{7}$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v g}$ | $d=Y_{1}-Y_{2}$ | $a^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 22 | 14 | 18 | 8 | 64 |
| $a$ | 4 | 25 | 18 | 21.5 | 7 | 49 |
| $b$ | 5 | 25 | 23 | 24 | 2 | 4 |
| $a b$ | 2 | 25 | 25 | 25 | 0 | 0 |
| $c$ | 7 | 12 | 10 | 11 | 2 | 4 |
| $a c$ | 3 | 21 | 25 | 23 | -4 | 16 |
| $b a$ | 5 | 24 | 12 | 18 | 12 | 144 |
| $a b c$ | 1 | 25 | 25 | 25 | 0 | 0 |
|  |  |  | Avg $=20.65$ |  |  | $\sum=281$ |

$$
\begin{aligned}
& \mathrm{Su}^{2}=\frac{\sum a_{i}^{2}}{2}\left(\frac{1}{d^{2}}\right) \\
& \quad=\frac{281}{2}\left(\frac{1}{8}\right)=17.56 \\
& \mathrm{~V}\left(E_{1}\right)=\frac{S_{0}^{2}}{4}=4.39 \\
& \therefore 8,0.05=2.31 \\
& i \sqrt{V\left(E_{1}\right)}=2.31 \sqrt{4.39} \\
& \quad=4.85
\end{aligned}
$$

CALCULATION OF EFFECTS OF FACTORS ON THE
AVERAGE DROP INDEX OF WARNED PELLETS

| Symbol | $\#$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v g}$ | $A=V_{1}-V_{2}$ | $a^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 3 | 20 | 11.5 | -17 | 289 |
| $a$ | 4 | 12 | 16 | 14 | -4 | 16 |
| $b$ | 6 | 7 | 7 | 7 | 0 | 0 |
| $a b$ | 2 | 25 | 22 | 23.5 | 3 | 9 |
| $c$ | 7 | 9 | 6 | 7.5 | 3 | 9 |
| $a c$ | 3 | 19 | 11 | 15 | 8 | 64 |
| $b ?$ | 5 | 13 | 19 | 16 | -6 | 36 |
| $a b c$ | 1 | 8 | 20 | 14 | -12 | 144 |
|  |  |  | $4 v g=13.56$ |  | $\sum=567$ |  |

$$
\begin{aligned}
& S p^{2}=\frac{\sum}{2} E_{i}^{2}\left(\frac{I}{d f}\right) \\
& =\frac{567}{2}\left(\frac{I}{8}\right)=35.44 \\
& T\left(E_{i}\right)=\frac{S_{0}^{2}}{4}=8.86 \\
& t_{0}, 0.05=2.31 \\
& t \sqrt{V\left(E_{i}\right)}=2.31 \sqrt{8.86} \\
& =6.88
\end{aligned}
$$

$\therefore=1 / 4(24.5)=6.12 \pm 6.88$
$3=1 / 4(12.5)=3.12 \pm 6.88$
$C=1 / 4(-3.5)=0.88 \pm 6.88$
$\therefore=1 / 4(4.5)=1.12 \pm 6.88$
$\therefore C=1 / 4(-13.5)=-3.38 \pm 6.88$
$3 C=1,4(2.5)=0.62 \pm 6.88$
$A B C=1 / 4(-23.5)=-5.88 \pm 6.88$

CALCULATION OF EFFECTS OF FACTORS ON THE AVERAGE CRUSHING STRENGTH OF WARNED PELLETS

| Symbol | $\#$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v 5}$ | $d=v_{1}-Y_{2}$ | $d^{2}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 0.2 | 0.6 | 0.4 | -.4 | .16 |  |  |  |  |  |
| $a$ | 4 | 0.6 | 0.3 | 0.45 | .3 | .09 |  |  |  |  |  |
| $b$ | 6 | 0.3 | 0.2 | 0.25 | .1 | .01 |  |  |  |  |  |
| $a b$ | 2 | 0.9 | 0.5 | 0.7 | .4 | .16 |  |  |  |  |  |
| $c$ | 7 | 0.3 | 0.2 | 0.25 | .1 | .01 |  |  |  |  |  |
| $a c$ | 3 | 0.4 | 0.3 | 0.35 | .1 | .01 |  |  |  |  |  |
| $b=$ | 5 | 0.3 | 0.5 | 0.4 | -.2 | .04 |  |  |  |  |  |
| $a b c$ | 1 | 0.2 | 0.6 | 0.4 | -.4 | .16 |  |  |  |  |  |
|  |  |  | $A v g=0.4$ |  |  |  |  |  |  |  | $\sum=.64$ |

$$
\begin{aligned}
& {S p^{2}}_{2}=\frac{\sum}{2} a_{1}^{2}\left(\frac{1}{d f}\right) \\
& =\frac{.64}{2}\left(\frac{1}{8}\right)=.04 \\
& V\left(\Xi_{1}\right)=\frac{s_{n}^{2}}{4}=0.01 \\
& t_{6,0.05}=2.31 \\
& E \sqrt{V\left(E_{1}\right)}=2.31 \sqrt{0.1} \\
& =0.23
\end{aligned}
$$

## TABLE 9

CALCULATION OF EFFECTS OF FACTORS ON THE AVERAGE CRUSHING STRENGTH OF AGED, RAW PELLETS


$$
\begin{aligned}
& S p^{2}=\frac{\sum d_{i}^{2}}{2}\left(\frac{i}{d f}\right) \\
& =\frac{649}{2}\left(\frac{1}{8}\right)=0.41 \\
& T\left(\Xi_{i}\right)=\frac{S_{0}^{2}}{4}=0.10 \\
& \pm 8,0.05=2.31 \\
& \pm \sqrt{V\left(E_{i}\right)}=2.3 I \sqrt{0.1} \\
& =0.73
\end{aligned}
$$

CALCULATION OF EFFECTS OF FACTORS ON THE AVERAGE CRUSHING STRENGTH OF AGED, WARMED PELLETS

| Symbol | $\frac{\pi}{7}$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v z}$ | $d=\underline{v}_{1}-\underline{v}_{2}$ | $a^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 0.93 | 1.25 | 1.03 | -0.32 | 0.10 |
| $a$ | 4 | 2.07 | 1.57 | 1.82 | 0.5 | 0.25 |
| $b$ | 5 | 2.05 | 1.71 | 1.88 | 0.34 | 0.12 |
| $a b$ | 2 | 2.36 | 2.98 | 2.97 | -0.02 | 0.00 |
| $c$ | 7 | 1.74 | 2.07 | 1.90 | -0.33 | 0.11 |
| $a c$ | 3 | 2.35 | 1.91 | 2.13 | 0.44 | 0.19 |
| $b=$ | 5 | 1.96 | 1.58 | 1.77 | 0.38 | 0.14 |
| $a b c$ | 1 | 1.55 | 2.12 | 1.84 | -0.57 | 0.32 |
|  |  |  | $A v g=1.32$ | $\sum=1.23$ |  |  |

$$
\begin{aligned}
& S p^{2}=\frac{\sum a_{i}^{2}}{2}\left(\frac{I}{i f}\right) \\
& =\frac{1.23}{2}\left(\frac{1}{8}\right)=0.08 \\
& \therefore=1 / 4(2.12)=0.53 \pm 0.33 \\
& B=1 / 4(1.52)=0.38 \pm 0.33 \\
& C=1 / 4(-0.12)=-0.03 \pm 0.33 \\
& \bar{v}\left(E_{i}\right)=\frac{S_{0}^{2}}{4}=0.02 \\
& \text { t?, } 0.05=2.31 \\
& i \sqrt{V\left(E_{1}\right)}=2.3 I \sqrt{0.02} \\
& =0.33 \\
& \therefore B=1 / 4(0.20)=0.05 \pm 0.33 \\
& \therefore C=1 / 4(-1.52)=-0.38 \pm 0.33 \\
& 3 C=1 / 4(-2.36)=-0.59 \pm 0.33 \\
& A B C=1 / 4(-0.52)=-0.13 \pm 0.33
\end{aligned}
$$

CALCULATION OF EFFECTS OF FACTORS ON THE AVERAGE RESISTANCE TO WATER DAMAGE OF RAW PELLETS

| Symbol | $\frac{1}{i}$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v g}$ | $d=Y_{1}-Y_{2}$ | $d^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\varepsilon$ | 0 | 10 | 5 | 10 | 100 |
| $a$ | 4 | 10 | 10 | 10 | 0 | 0 |
| $b$ | 6 | 20 | 15 | 17.5 | 5 | 25 |
| $a b$ | 2 | 10 | 10 | 10 | 0 | 0 |
| $c$ | 7 | 25 | 10 | 17.5 | 15 | 225 |
| $a c$ | 3 | 0 | 10 | 5 | 10 | 100 |
| $b s$ | 5 | 10 | 10 | 10 | 0 | 0 |
| $a b c$ | 1 | 0 | 15 | 7.5 | 15 | 225 |
|  |  |  | $A v g=10.3$ |  | $\sum=675$ |  |

$$
\begin{aligned}
& s_{p^{2}}^{2}=\frac{\sum}{2} A_{1}^{2}\left(\frac{I}{d f}\right) \\
& =\frac{675}{2}\left(\frac{I}{8}\right)=42.19 \\
& V\left(\Xi_{1}\right)=\frac{S_{0}^{2}}{4}=10.55 \\
& 亡 3,0.05=2.31 \\
& t \sqrt{V\left(E_{1}\right)}=2.31 \sqrt{10.55} \\
& \quad=7.50
\end{aligned}
$$

$$
i=1 / 4(-17.5)=-4.38 \pm 7.50
$$

$$
3=1 / 4(7.5)=1.88 \pm 7.50
$$

$$
c=1 / 4(-2.5)=-0.62 \pm 7.50
$$

$$
\therefore B=1 / 4(-2.5)=-0.62 \pm 7.50
$$

$$
\therefore C=1 / 4(-12.5)=-3.12 \pm 7.50
$$

CALCULATION OF EFFECTS OF FACTORS ON THE AVERAGE RESISTANCE TO WATER DAMAGE OF WARMED PELLETS

| Symbol | $\#$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v 5}$ | $d=Y_{1}-V_{2}$ | $a^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 0 | 40 | 20 | 40 | 1600 |
| $a$ | 4 | 25 | 30 | 27.5 | 5 | 25 |
| $b$ | 6 | 0 | 15 | 7.5 | 15 | 225 |
| $a b$ | 2 | 10 | 20 | 15 | 10 | 100 |
| $c$ | 7 | 20 | 20 | 20 | 0 | 0 |
| $a c$ | 3 | 10 | 10 | 10 | 0 | 0 |
| $b a$ | 5 | 10 | 30 | 20 | 20 | 400 |
| $a b c$ | 1 | 10 | 10 | 10 | 0 | 0 |
|  |  |  | $4 v g=16.25$ |  |  | $\sum=2350$ |

$$
\begin{aligned}
& S_{p^{2}}=\frac{\sum d_{i}^{2}}{2}\left(\frac{1}{d f}\right) \\
& =\frac{2350}{2}\left(\frac{1}{8}\right)=146.88 \\
& V\left(\Xi_{1}\right)=\frac{s_{0}^{2}}{4}=36.72 \\
& t_{0}^{2}, 0.05=2.31 \\
& E \sqrt{V\left(E_{1}\right)}=2.31 \sqrt{36.72} \\
& =14.00
\end{aligned}
$$

$$
\therefore=1 / 4(-5.00)=-1.25 \pm 14.00
$$

$$
B=1 / 4(-25.00)=-6.25 \pm 14.00
$$

$$
c=1 / 4(-10.00)=-2.50 \pm 14.00
$$

$$
\therefore B=1 / 4(0)=0 \pm 14.00
$$

$$
\therefore C=1 / 4(-35.00)=-8.75 \pm 14.00
$$

$$
B C=1 / 4(25.00)=6.25 \pm 14.00
$$

$$
\therefore B C=1 / 4(0)=0 \pm 14.00
$$

CALCUIATION OF EFFECTS OF FACTORS ON THE AVERAGE
RESISTANCE TO WATER DAMAGE OF AGED, RAW PELIETS

| Symbol | $\#$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v 5}$ | $d=Y_{1}-Y_{2}$ | $a^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 0 | 0 | 0 | 0 | 0 |
| $a$ | 4 | 10 | 0 | 5 | 10 | 100 |
| $b$ | 5 | 0 | 0 | 0 | 0 | 0 |
| $a b$ | 2 | 5 | 0 | 2.5 | 5 | 25 |
| $c$ | 7 | 15 | 0 | 7.5 | 15 | 225 |
| $a c$ | 3 | 0 | 0 | 0 | 0 | 0 |
| $b 3$ | 5 | 0 | 30 | 15 | 30 | $Y 00$ |
| $a b c$ | 1 | 0 | 0 | 0 | 0 | 0 |
|  |  |  | $A v g=3.75$ |  |  | $\sum=1250$ |

$$
V\left(\Xi_{1}\right)=\frac{S_{0}^{2}}{4}=12.53 \quad \therefore B=1 / 4(-10)=-2.50 \pm 10.21
$$

$$
\pm C=1 / 4(-30)=-7.5 \pm 10.21
$$

$$
B C=1 / 4(10)=2.5 \pm 10.21
$$

$$
A B C=1 / 4(-5)=-1.25 \pm 10.21
$$

$$
\begin{aligned}
& S p^{2}=\frac{\sum d_{i}^{2}}{2}\left(\frac{1}{d f}\right) \\
& =\frac{1250}{2}\left(\frac{1}{8}\right)=78.12
\end{aligned}
$$

CALCULATION OF EFFECTS OF FACTORS ON THE AVERAGE RESISTANCE TO WATER DAMAGE OF AGED, WARMED PELLETS


$$
\begin{aligned}
& S D^{2}=\frac{\sum i_{1}^{2}}{2}\left(\frac{I}{a f}\right) \\
& =\frac{2754}{2}\left(\frac{1}{8}\right)=172.12 \\
& F\left(\Sigma_{\perp}\right)=\frac{S_{r}^{2}}{4}=43.03 \\
& 48,0.05=2.31 \\
& \pm \sqrt{V\left(E_{1}\right)}=2.31 \sqrt{43.03} \\
& =15.15
\end{aligned}
$$

CALCULATION OF EFFECTS OF FACTORS ON THE AVERAGE RESISTANCE
TO WATER DAMAGE OF AGED, RAW, COATED PELLETS

| Symbol | $\overline{7}$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v}$ | $d=Y_{1}-Y_{2}$ | $a^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 90 | 95 | 92.5 | 5 | 25 |
| $a$ | 4 | 100 | 100 | 100 | 0 | 0 |
| $b$ | 6 | 100 | 100 | 100 | 0 | 0 |
| $a b$ | 2 | 100 | 100 | 100 | 0 | 0 |
| $c$ | 7 | 100 | 100 | 100 | 0 | 0 |
| $a c$ | 3 | 85 | 100 | 92.5 | 15 | 225 |
| $b c$ | 5 | 85 | 100 | 92.5 | 15 | 225 |
| $a b c$ | 1 | 90 | 100 | 95 | 10 | 100 |
|  |  |  | $1 v g=96.6$ |  | $\sum=575$ |  |

$$
\begin{aligned}
& S p^{2}=\frac{\sum d_{i}^{2}}{2}\left(\frac{i}{d_{i}}\right) \\
& =\frac{575}{2}\left(\frac{1}{8}\right)=35.94 \\
& \begin{aligned}
& v\left(\Sigma_{1}\right)=\frac{S_{0}{ }^{2}}{4}=8.08 \\
& t 8,0.05=2.31 \\
& t \sqrt{v\left(E_{1}\right)}=2.31 \sqrt{8.38} \\
&=6.92
\end{aligned}
\end{aligned}
$$

CALCULATION OF EFFECTS OF FACTORS ON THE AVERAGE RESISTANCE TO WATER DAMAGE OF AGED, WARMED, COATED PELLETS

| Symbol | $\#$ | $Y_{1}$ | $Y_{2}$ | $Y_{a v}$ | $d=Y_{1}-Y_{2}$ | $d^{2}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 90 | 95 | 92.5 | 5 | 25 |  |  |  |  |  |
| $a$ | 4 | 90 | 100 | 95 | 10 | 100 |  |  |  |  |  |
| $b$ | 6 | 100 | 100 | 100 | 0 | 0 |  |  |  |  |  |
| $a b$ | 2 | 70 | 100 | 85 | 30 | 900 |  |  |  |  |  |
| $c$ | 7 | 95 | 100 | 97.5 | 5 | 25 |  |  |  |  |  |
| $a c$ | 3 | 95 | 95 | 95 | 0 | 0 |  |  |  |  |  |
| $b s$ | $E$ | 100 | 95 | 97.5 | 5 | 25 |  |  |  |  |  |
| $a b c$ | 1 | 90 | 90 | 90 | 0 | 0 |  |  |  |  |  |
|  |  |  | $A v g=94.1$ |  |  |  |  |  |  |  | $\sum=1075$ |

$$
\begin{aligned}
& =\frac{1075}{2}\left(\frac{1}{8}\right)=67.19 \\
& V\left(Z_{1}\right)=\frac{S_{0}{ }^{2}}{4}=16.80 \\
& \text { ᄂ8,0.05 = } 2.31 \\
& \div \sqrt{V\left(E_{1}\right)}=2.31 \sqrt{16.80} \\
& =9.47
\end{aligned}
$$

## Statistical Observations

In an experimert such as this, with so many factors to control and unknowns to approximate, many sources of error can exist. It is thought that subsequent research can build on this work eliminating more of the experimental and systematic errors. Some cautions and recommendations follow, but given the data as collected and calculated, some very interesting observations can be made, keeping in mind that the variances of the effects are in almost every case far too large for certainty. (These large variances reemphasize the need for more carefully controlled experiments with more sensitive equipment than was available for this study).

## Yield

On first glance the extremely negative effect of binder concentration ( $A=-12.00 \pm 6.12$ ) might seem anomalous, but a higher concentration implies less water available for wetting. Evidently the binder at higher concentrations tends to remain immobile, not spreading out and agglomerating other particles and small pellets. Better mixing techniques, alternate concentrations or use of a different binder may improve yield.

The highly negative effect of $\mathrm{rpm}(\mathrm{B}=-9.25 \pm 6.12)$ is more predictable. The greater the energy available to the process in the agglomeration drum, the stronger and more spherical the pellets, with the material more consolidated.

However, weaker pellets are destroyed, and there is less chance for each pellet to "grow".

Increased heat of agglomeration aided in the spreading of the binder, $(C=5.75 \pm 6.12)$. On the other hand, the interaction of temperature and rpm produced a large negative effect, ( $B C=-5.0 \pm 6.12$ ). This high state of energy tends both to destroy the pellets and evaporate the water in the binder.

The negative effects of $A$ and $B$ are overcome in the interactions, $(A B=3.25 \pm 6.12 ; A C=1.25 \pm 6.12 ;$ and $\mathrm{ABC}=4.00 \pm 6.12)$

It is worth noting that the highest average yield was produced at low binder concentration, low rpm, and high temperature.

Drop Index Raw

The effect of both $A$ and $B$ on raw drop index was strongIy positive $(A=5.88 \pm 4.85 ; B=4.62 \pm 4.85)$ and furthermore, whenever either $A$ or $B$ was high, the drop index was high, while when both were low, the index was low ( $1=18$, ${ }^{\prime} c^{\prime}=11$, compared to the overall index of 20.7).

Correspondingly, the effect of $C$ was negative ( $C=-2.88$
$\pm 4.85)$, but this was overcome in the interaction with $A$ $(A C=3.62 \pm 4.85)$

Drop Index Warmed
After warming the pellets for 15 minutes at $120^{\circ} \mathrm{C}$ the effects of $A$ and $B$ remained strongly positive, ( $A=6.12 \pm 6.88$; $B=3.12 \pm 6.88)$. Interestingly, even though the effect of $C$ was also positive ( $C=0.88 \pm 6.88$ ) the interaction of all three was strongly negative ( $A B C=-5.88 \pm 6.88$ ), presumably because the individual indices of ' $b$ ' and ' c ' were both quite low.

## Crushing Strength Warmed

The crushing strength of raw unaged pellets was essentially zero. Warming for 15 minutes sufficiently dried the shell to generate a weak crushing resistance. The individual effects were all quite small, with only A being much greater than $50 \%$ of $t \sqrt{V\left(E_{1}\right)}= \pm 0.23$. Consequently, very little can be deduced from the effects. It. is worth noting that trial $A B$ produced reasonably strong pellets.

After warming, the smaller pellets exhibited a higher crushing strength than did the larger ones. The surface drying included a greater percentage of the total volume of the smaller pellets, producing a greater crushing strength.

Crushing Strength; Aged, Raw
Again, the variance of the effects is quite large, in all but one case larger than the effects. The effect of binder concentration was highly positive, ( $A=1.97 \pm 0.73$ ) with the four highest responses resulting when the higher concentration was used. None of the interactions with A were strong however.

All of the effects were positive except $C(C=-0.37 \pm 0.73)$ which was slightly negative.

## Crushing Strength; Aged, Warmed

Binder concentration once again exhibited the strongest positive effect $(A=0.53 \pm 0.33)$, while $C$, alone or in any interaction produced negative effects.

## All Rain Wash Tests

There seems to be very little correlation or prediction of effects from one response to another, perhaps due to the subjective and error-prone nature of this test. No doubt better results could be gained by using a binder that is not water soluble.

The effects, variances and differences of effects between raw and warmed states are tabulated in Table 17 . Of greater possible interest is Table 18, which shows the average values of the effects of the 12 meaningful tests, and the number of times an effect greatly affected the outcome.

The effect $A$, binder concentration, was strongly positive 5 and strongly negative 4 times out of the 12 . In addition its average effect was far greater than any other. This effect must be a part of any future follow-up to this research. Of the other two primary effects, $B$ and $C$ had strong influences 5 and 4 times respectively.

TABLE 17 RESULTS AND EFFECTS

|  |  | Drop Index |  |  |  |  |  | Crushing Strength，Kg |  |  |  |  |  | Rain Wash， 8 retention |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { 触 } \\ & 2 \end{aligned}$ | $\begin{array}{r} \text { 貝 } \\ 3 \\ 3 \end{array}$ | $\begin{gathered} \\ 4 \\ 2-3 \end{gathered}$ | $\begin{aligned} & \text { 触 } \\ & \text { 世 } \\ & \$ \\ & \$ \\ & 5 \end{aligned}$ | $\begin{gathered} \text { 星 } \\ \text { 采 } \\ \text { 先 } \end{gathered}$ <br> 6 | 5－6 | 8 | $\begin{aligned} & \text { 冒 } \\ & 9 \end{aligned}$ | 10， |  |  |  | $\begin{aligned} & \text { 触 } \\ & 14 \end{aligned}$ | $\begin{aligned} & \text { 夏 } \\ & 15 \end{aligned}$ |  |  |  | 19． 17－18 |  |  | 22， |
| A | －12．00 | 5.88 | 6.12 | －0．74 | － | － | 1 | $\cdots$ | 0.15 | ／ | 1.97 | 0.53 | 1.44 | －4．38 | －1．25 | －3．13 | －3．75 | －8．50 | 4.75 | 0.62 | －5．62 | 6.25 |
| B | －9．25 | 4.62 | 3.12 | 1.5 | － | － | 1 | － | 0.08 | ／ | 0.59 | 0.38 | 0.21 | 1.88 | －6．25 | 8.13 | 1.25 | －4．75 | 6.00 | 0.62 | －1．88 | 2.50 |
| $\omega \quad c$ | 5.75 | －2．88 | 0.88 | －3．76 | － | － | 1 | － | －0．1 | ／ | 0.37 | －0．03 | 0.40 | －0．62 | －2．50 | 1.88 | 3.75 | －7．25 | 11.00 | －3．12 | 1.88 | －5．00 |
| AB | 3.25 | －1．88 | 1.12 | －3．00 | － | － | 1 | － | 0.08 | ， | 0.09 | 0.05 | 0.04 | －0．62 | 0 | －0．62 | －2．50 | 2.25 | －4．75 | 0.62 | －5．62 | 6.24 |
| ${ }^{\text {A }}$ | 1.25 | 3.62 | －3．38 | 7.00 | － | － | ＇ | － | －0．1 | ／ | 0.19 | －0．38 | 0.57 | －3．12 | －8．75 | 5.63 | －7．50 | 4.75 | －12．25 | －3．12 | 0.62 | －3．74 |
| BC | －5．00 | －0．12 | 0.62 | －0．74 | － | － | 1 | － | 0.02 | ／ | 0.29 | －0．59 | 0.88 | －4．38 | 6.25 | －10．63 | 2.5 | 1.00 | 1.50 | －3．12 | －0．62 | －2．50 |
| ABC | 4.00 | －0．62 | －5．88 | 14.26 | － | － | 1 | － | －0．12 | ／ | 0.03 | －0．13 | 0.16 | 5.62 | 0 | 5.62 | －1．25 | －3．50 | 2.00 | 4.38 | 3.12 | 1.26 |
| $t \sqrt{v\left(E_{i}\right)}$ | $\pm 6.12$ | ＋4．85 | $\pm 6.88$ | ／ | － | － | ／ | － | $\pm 0.23$ | 1 | $\pm 0.73$ | $\pm 0.33$ | ／ | $\pm 7.50$ | ＋14．00 | ／ | $\pm 10.21$ | $\underline{+15.15}$ | 1 | $\pm 6.92$ | ＋9．47． | ／ |

TABLE 18
AVERAGE VALUES OF EFFECTS

|  | Algebraic | Absolute | $\neq$ Strongly ( + ) | \# Strongly ( - ) |
| :---: | :---: | :---: | :---: | :---: |
| $A$, | -1.69 | 4.23 | 5 | 4 |
| $B$, | -0.80 | 2.89 | 3 | 2 |
| $C$, | -0.32 | 2.43 | 1 | 3 |
| $A B$, | -0.26 | 1.51 | - | 1 |
| $A C$, | -1.33 | 3.06 | 1 | 4 |
| $B C$, | -0.26 | 2.04 | 1 | 4 |
| $A B C$, | 0.47 | 2.39 | 3 | 1 |

TABLE 19

|  | EFFECT OF WARMING |  |  |
| :---: | :---: | :---: | :---: |
|  | Raw | Warmed | $\%$ Change |
| DI Fresh | 20.7 | 13.56 | $-34 \%$ |
| Aged | e | e | $0 \%$ |
| CS Fresh | call 0.1 | 0.4 | $+300 \%$ |
| Aged | 3.15 | 1.92 | $-39 \%$ |
| RW Fresh | 10.3 | 16.25 | $+58 \%$ |
| Aged | 3.75 | 8.0 | $+113 \%$ |
| Coating | 96.6 | 94.1 | $-3 \%$ |

## Warming

Even though the effect of warming was not a part of the statistical array, its significant role is easily discernible, as is seen in Table 19. Warming the raw pellets for 15 minutes immediately after their production increased their crushing strength markedly. However, at the same time, warming made the pellets more brittle, resulting in a much lower drop index.

Raw and warmed pellets from each trial were kept separate for two days and the tests were repeated. By this time, the binder had cured to where both types were nearly elastic, with very high drop indices; no difference remained between raw and warmed. However, the warmed pellets did not develop the high crushing strengths that the unwarmed ones gained with age. In the long run, the initial warming proved quite detrimental to the crushing strength. Warming may have somewhat improved a pellet's resistance to water, but given the nature of the data, it is difficult to tell.

## Aging

Aging is by far the single most important parameter studied as can be seen in Table 20. While being detrimental to the resistance to water damage in uncoated pellets, aging dramatically increased the crushing strength and elasticity of both raw and warmed pellets. By the end of two days the pellets were quite stable and easily transportable.

Coating
As seen in Table 21 , coating did not appreciably alter

TABLE 20
EFFECT OF AGING

|  | Fresh | Aged | $\%$ Change |
| :--- | :---: | :---: | :---: |
| DI Raw | 20.70 | 25 | $+21 \%$ |
| DI Warmed | 13.56 | 25 | $+84 \%$ |
| CS Raw | call 0.10 | 3.15 | $+3.050 \%$ |
| CS Warmed | 0.40 | 1.92 | $+380 \%$ |
| RW Raw | 10.3 | 3.75 | $-64 \%$ |
| RW Warmed | 16.25 | 8.0 | $-51 \%$ |

TABLE 21

|  | EFFECT OF COATING |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Aged | Avg | Coated | $\%$ Change |
| DI Raw | 25 | 25 | 25 | $0 \%$ |
| DI Warmed | 25 |  |  |  |
| CS Raw | 3.15 |  | 2.53 | 1.82 |
| CS Warmed | 1.92 |  | $-28 \%$ |  |
| RW Raw | 3.75 |  | 96.6 | $+2,476 \%$ |
| RW Warmed | 8.0 |  | 94.1 | $+1,076 \%$ |

either the drop index or the crushing strength of the pellets. But great improvement was seen in the rainwash test of both raw and warmed pellets. Uncoated pellets were virtually destroyed by the water action, but once coated, there was essentially no damage. It is thought that much of the reported minor loss in coated pellets was due to mechanical damage not water damage, and responses much nearer $100 \%$ would be more accurate.

## General Errors

There are several potential sources of error that should be discussed, one being the number of variables investigated. Of the three controlled variables, one turned out to be very important, the others less so. Three more variables were studied as responses, but not in the statistical matrix. All proved to be highly important. No doubt other variables, which were not varied in a controlled way, play a potentially more important role, such as moisture content of coal batch, temperature and humidity of surroundings, type binder, mixing style, curing time and temperature etc. Without further study, it is impossible to tell.

The levels, (high and low), of the variables may have been chosen improperly. Only if the variation is nearly linear between the two levels can such an analysis represent the true situation. If the response oscillates between the two levels, a multi-level experiment would reveal it, while a two level investigation would not.

In this experiment, 100 gms of coal fines were agglomerated with 25 ml of binder, and the yield of pellets caught on a \# 8 sieve averaged 62.5 gms . A series of 14 tests were run on small samples from each trial, and by the end of the tests, few pellets remained. In order to increase the accuracy of the results larger samples should be used which would necessitate the use of a larger agglomerating drum. At least 20 pellets, representing the spectrum of sizes of the pellets, should be tested and their responses averaged to obtain the average response for each of the 14 tests. Testing such a large number of pellets was impossible on this scale, but future work should eliminate this possible source of error.

## Experimental Observations

Discussed briefly below are several observations, insights and warnings gained in the experimental process. Hopefully, these will allow future research to proceed more smoothly and accurately.

In the preparation of the binder, great care is needed. Small amounts of binder must be mixed with the proper amount of water, and care must be taken to insure precision. Heating and agitation aids in dissolving the binder material, but continued application may cause evaporation of the water, changing the concentration.

A variable speed mixer is not adequate for mixing the binder and coal. A two-armed kneading type mixer is preferred
with provision for scraping the sides and bottom of the mixing bowl. Adding the binder to the coal slowly, perhaps as a spray would help eliminate the formation of large "blobs", which are difficult to break up. Total mixing is of paramount importance in these controlled trials. It was noticed that if small balls remain after mixing, spreading of the binder and agglomeration of fines into pellets is hindered. The balls tend to become pellets, and the finer fractions have little ability to agglomerate in the presence of balls which contain a large amount of binder. All of the coal agglomerated to some degree however, even that which did not become part of a larger pellet. After 5 minutes of agglomeration, none of the coal was "dirty". That portion which passed a $\neq 8$ sieve was discarded, although it would probably be rerouted back into the drum for more growth in an industrial cycle. After each trial, the drum was cleaned to remove remaining material sticking to the inside. Sticking was minimal, but it was removed to assure the integrity of each trial. Such residue, if allowed to remain would discourage further sticking in less precise processes.

The heat applied during half of the trials was in the form of hot air, approximately $95^{\circ} \mathrm{C}$, blown onto the back of the drum. The air could not of course be blown onto the coal fines or pellets, but in this manner the agglomerating environment was heated, and the coal was heated when in contact with the drum. Pellets so produced were moderately warm to the touch, but not hot.

Raw pellets, when first produced, were very nearly plastic. They possessed essentially no crushing strength, and the normal elevated drop index probably represented their cohesiveness rather than a strength property. Both the drop index and the crushing strength determinations were rather subjective, although an effort was made to standardize the tests.

Due to the small yields, only 5 pellets normally represented the trial for each of the eight drop index and crushing strength tests with a larger amount being used for the rainwash and coating tests. A larger number of pellets available for testing would have probably yielded less scatter in the responses.

Once dried at $110^{\circ} \mathrm{C}$ for 15 minutes, the drop and crushing tests were noticeably size dependent. Smaller pellets dried faster and developed a larger crushing strength, but the dried pellets, which had not had time to cure thoroughly became less cohesive, resulting in a decrease in the drop index. Larger pellets were less affected by 15 minutes of drying. On the other hand, once aged at room temperature in an open container for 2 days, sufficient time for the binder to cure, the larger pellets gained a large crushing resistance, whereas the smaller ones increased much less. Pellets which had been initially warmed to gain some crushing resistance did not gain the high crushing strengths after two days that the non-warmed pellets obtained.

The rainwash tests on uncoated pellets were nearly meaningless. The binder used in this experiment was a water soluble polymer and once immersed in water, pellets made from it readily dissolved. In this modified test a known weight of pellets were allowed to soak for 15 minutes, after which the water was drained off. The remaining pellets and saturated fines were heated in an oven until all water had evaporated. Any pellet or portion of pellets retaining strength was weighed, and the percentage of original weight was recorded. In no case did a single pellet remain intact throughout the process. In all likelihood, although this was not tested, no pellet retained any strength while saturated. Some strength returned with drying, and no doubt much of the recorded percentage consists of rehardened material. But even with this optimistic process, in which any hardened piece was acceptable, the greatest amount remaining was $27 \%$. A more realistic test would yield percentages much closer to zero. No doubt subsequent work can identify binders which will make uncoated pelletsless susceptible to water damage.

When the pellets were protected by an insoluble film coating however, virtually no water damage was noted when the same test was performed. The pellets were coated with a crosslinked polymer, which when it dries forms an unbroken film on the surface to which it is applied (in appearance much like the familiar Saran Wrap). The coating was accomplished with the polymer freshly mixed, soon after an insolubilizing catalyst
was added. The pellets were submersed in the fresh mixture before it hardened into a gel, withdrawn and placed on a drying plate. The entire plate was then heated at $120^{\circ} \mathrm{C}$. to facilitate the drying of the film. Once removed, the pellets exhibited very little change in strength and stability characteristics while the resistance to water damage increased dramatically.

A problem existed in the removal of the coated pellets from the drying plate during preliminary experiments which was only partially solved, and its effect bears to a minor degree on these trials. The film is designed to adhere to smooth surfaces, and consequently the pellets coated with unhardened polymer stick fast to the plate as they dry. The problem was alleviated somewhat by pre-coating the plate with the same film. This film, once dried, is designed to be able to be peeled from the plate with care. Pellets covered with a fresh batch of the same, still liquid polymer were placed on the film covered drying plate. Once the shell hardened, the plate film was peeled up bringing the pellets with it. Less damage was thus done to the pellets themselves, and it is thought that better technique would completely eliminate this hazard to the pellet. It is also likely that much of the damage to the coated pellets in the rain wash test was due to the mechanical breaking of the shell on removal from the plate, as no damage was noted to any pellet while being soaked, drained, or dried. Clearly the coating removed the danger of axtensive
water damage, as the lowest percentage retained in the coated tests was $85 \%$, compared to the highest percentage retained in the uncoated tests of $27 \%$.

Self-Bursting Pellets
Protection from potential water damage is only one of the shell's two functions. It must also aid in the bursting process. Once the pellet is introduced into the presence of heat, gasses of devolatilization are given off and the moisture in the binder and coal undergoes a phase change, causing pressure to build up internally. Much of this pressurized steam and gas would be vented into the heating chamber or furnace unless the shell is intact and can maintain it. Once the internal pressure exceeds the tensile strength of the shell, the pellet will burst and be reduced to fine sized particles similar to those normally introduced into a coal burning furnace.

The pellets as produced in this experiment burst in the presence of high heat of several kinds, from an open flame to hot, convective gasses to a radiant heat furnace. Larger pellets, (diameter=1/2") take correspondingly longer to burst than do smaller ones, but all pellets burst within two seconds.

Explosions are a function of both rapid expansion and confinement, and both are needed in this case. A pellet resting on a grate or plate heated from below will generally not burst, since the pressures tend to dissipate prior to high inter-
nal pressure build-up. For individual tests it was found that holding a pellet in the hot environment with a pair of tongs or tweezers provided sufficient confinement for bursting. This confinement is thought to be much more easily developed on an industrial scale than for a single pellet. Recommendations are presented in Chapter 6 .

CHAPTER III

## FORMUIATION OF EQUATIONS DESCRIBING <br> SELF-BURSTING PELIET

Several factors work together to yield a tendency for the coal pellet to burst when suddenly heated by an external source. It is well known that coal when heated undergoes devolatilization of certain fractions. Gases are evolved and will leave the coal (as in a coke oven) if not maintained internally. Due to the impermeable shell and confinement of the pellets as proposed, the gases cannot easily escape. Their sudden appearance and expansion when heated rapidly causes an internal pressure buildup, primarily caused by the difference in diffusivities in the outer, reacted portion as compared to the inner core, not yet affected by the external heat (33).

As the coal devolatilizes pores develop which can cause unrestrained coal to swell. In fact coals must have this property if they are to be used for coking. Depending on the nature of the confinement, the tendency to swell will either increase the internal pressure (if the shell is rigid or packing is tight) or decrease the internal pressure (if the shell or packing will allow the volume increase) (56).

Probably the most important factor involved in internal pressure build-up is due to the phase change of the moisture inside the pellet from liquid to vapor. The intense heating conditions to which the pellet is subjected would first vaporize the moisture near the pellet surface. The large volume change of the moisture produces an intense and nearly instantaneous pressure build-up.

Perhaps other factors could be identified which contribute to the internal pressure build-up. It is likely however, that the phase change is the most significant, and it is this factor which is modeled in this paper. Incorporation of all effects into one comprehensive model is left to subsequent work.

Description of the Physical Model
As can be seen in Figure 3, the pellet has a radius of $r_{0}$, and is introduced suddenly into a furnace, the gases of which are of uniform temperature $T^{\infty}$, (a convective, wellstirred fluid). Pressure is atmospheric both inside and outside the pellet at $t=0$.

Figure 3. The Coordinate System


The convective heat affects only the pellet surface at $t=0$, but the effects migrate inward with increasing time. The problem is readily seen as a stefan problem in spherical coordinates (a moving boundary problem due to phase change). Numerous models have been proposed which deal with similar problems (see for example 14, 39), but most notably Saito and Seki's work describing pressure build-up in a moist sand mold when molten metal is introduced. (40)

## Assumptions

The present theoretical model which conforms to the physical model of Chapter II is based on the following assumptions:

1. No liquid water movement is present in the pellet. The water is held in place as a constituent of the binder (a cross-linked polymer, waxy derivative, or other material), which serves to reduce the permeability to a very low level. The water disassociates from the polymer only as it vaporizes.
2. Air and other gas in the pellet have no significant effect on the internal heat transfer, since their enthalpies are significantly smaller than that of water vapor. (This assumption ignores the gases of devolatilization of the coal).
3. Laminar flow is assumed, i.e. Darcy's Law is valid for the vapor flow through the pellet. The permeability of the pellet is so low that high velocity flow is impossible, as demonstrated by the fact that the high velocity terms in
the conservation of momentum equation are less than $1 \%$ of the Darcy term.
4. Thermal conductivity, diffusivity, heat capacity, porosity and mobility are constant in each region. It is well known that thermal conductivity and diffusivity do vary with temperature, but at the range of temperatures employed in this research, the change is slight. Furthermore, the specific heat and density of the solid are only slightly dependent in pressure and temperature. The use of constant porosity and permeability values ignores the possibility of pellet swelling.
5. Thermal equilibrium between the solid particles and the moisture at every point in the inner, wet region is assumed. This is reasonable since the temperature change in the wet region takes place comparatively slowly.
6. The critical build-up of pressure is at the moving front due to the vapor generation, and is caused by the difference in permeabilities across the boundary. The presence of an impermeable film on the pellet surface is not considered, although it would serve to accelerate the build-up as it entraps the generated steam.
7. Although it is known that as coal is heated it both devolatilizes and swells, both effects are assumed to be negligible (or if preferred to cancel one another). This is done in order to isolate the more important effect of the phase change.

## Applicable Equations

From the foregoing assumptions the following equations can be introduced. (All equations are fully derived in Appendix 2).

Mass Transfer in Region One

$$
\begin{equation*}
\frac{\delta \varepsilon_{1}}{\delta t}=\frac{M_{1}}{\phi r_{1}{ }^{2}} \frac{\delta}{\delta r_{1}}\left(r_{1}{ }^{2} \varepsilon_{1} \frac{\delta p_{1}}{\delta r_{1}}\right)\left(S \leq r_{1} \leq r_{0}\right) \tag{1}
\end{equation*}
$$

It is worth pointing out here that equation one includes an expression of conservation of momentum, as demonstrated in Appendix Two.

Mass Transfer in Region Two
By assumptions 1 and 5 there is no mass transfer in Region Two.

Mass Balance at the Moving Boundary

$$
\begin{align*}
& \dot{S}\left(\varepsilon_{2}-\varepsilon_{1}\right)=\frac{M_{1}}{\phi} \quad \varepsilon_{1} \frac{\delta p_{1}}{\delta r_{1}} \quad r_{2}, r_{1}=S \tag{2}
\end{align*}
$$

## Energy Transfer in Region One

Solid

$$
\begin{equation*}
\rho_{s} C_{S}(1-\phi) \frac{\delta T_{s}}{\delta t}=\frac{k_{S}(1-\phi)}{r_{1}{ }^{2}} \frac{\delta}{\delta r}\left(r_{1}{ }^{2} \frac{\delta T_{S}}{\delta r}\right)+H F\left(T_{g}-T_{S}\right) \tag{3}
\end{equation*}
$$

Fluid

$$
\begin{align*}
\rho_{g} C_{g}\left(\frac{\delta T_{g}}{\delta t}\right. & \left.+\frac{V}{\phi} \frac{\delta T_{g}}{\delta r_{1}}\right)=\frac{k_{g}}{r_{1}^{2}} \frac{\delta}{\delta r_{1}}\left(r_{1}^{2} \frac{\delta T_{q}}{\delta r_{1}}\right)+\frac{\delta P_{1}}{\delta t} \\
& +\frac{H F}{\phi}\left(T_{s}-T_{g}\right) \tag{4}
\end{align*}
$$

## Energy Transfer in Region Two

Since there is no mass transfer in region two, energy transfer is by conduction only.

$$
\begin{equation*}
\rho_{p} C_{p} \frac{\delta T_{2}}{\delta t}=\frac{k_{p}}{r_{2}{ }^{2}} \frac{\delta}{\delta r_{2}}\left(r_{2}^{2} \frac{\delta T_{2}}{\delta r_{2}}\right), 0 \leq r_{2} \leq s \tag{5}
\end{equation*}
$$

VI Energy Balance at Moving Boundary
Region $V_{2}, \varepsilon_{2}$
$\left(\varepsilon_{1} h_{1}-\varepsilon_{2} h_{2}\right) \dot{s}=-k_{1} \frac{\delta T_{1}}{\delta r_{1}}+\left(k_{2} \frac{\delta T_{2}}{\delta r_{2}}\right)-\frac{M_{1}}{\phi}\left(\varepsilon_{1} h_{1} \frac{\delta D_{1}}{\delta r_{1}}\right)$,

$$
\begin{equation*}
r_{1}, r_{2}=s \tag{6}
\end{equation*}
$$

## Initial and Boundary Conditions

The initial and boundary conditions relevant to the
present problem are:

$$
\begin{equation*}
@=r_{0}: T_{1}=T_{\infty} ; \frac{j p_{1}}{r}=0 \tag{7}
\end{equation*}
$$

$$
\begin{align*}
& @ r=0: \frac{\delta p_{2}}{\delta r}=0 ; \frac{\delta T_{2}}{\delta r}=0  \tag{8}\\
& @ t=0: T_{1}=T_{2}=T i ; P_{1}=P_{2}=P i  \tag{9}\\
& @ r=r_{0}: k_{1} \frac{\delta T}{\delta r}\left(r_{0} ; t\right)=\lambda\left(T_{\infty}-T\right) \tag{10}
\end{align*}
$$

## Simplification of Equations

The solution of such a set of intricate equations is Obviously quite difficult. Analytical solution is unreasonable and numerical solution requires an advanced knowledge of numerical methods as well as a sophisticated computer with massive storage space. It is suggested that these equations be solved in the future by one with such knowledge and access to such a computer, but they are not solved hereir. However, acceptable results can be obtained by simplifying and non-dimensionalizing the equations.

By assuming that all energy transfer is by conduction through the pellet matrix only, (i.e., that the liquid or vapor content does not participate in the conduction), energy transfer is the same in both regions. Equations (3) and (5) therefore, can be rewritten as follows:

$$
\begin{equation*}
\mathbf{C}_{\mathbf{s}} \rho_{\mathbf{s}} \frac{\delta T}{\delta t}=\frac{\mathbf{k}_{\mathbf{s}}}{\mathbf{r}^{2}} \frac{\delta}{\delta r}\left(r^{2} \frac{\delta T}{\delta t}\right) \quad 0<r<r_{0} \tag{11}
\end{equation*}
$$

Which can also be expressed as

$$
\begin{equation*}
\frac{\delta T}{\delta t}=\alpha\left(\frac{\delta^{2} T}{\delta r \delta}+\frac{2}{r} \frac{\delta T}{\delta r}\right) \quad 0<r<r_{0} \tag{12}
\end{equation*}
$$

In non-dimensional form,

$$
\begin{equation*}
\frac{\delta T^{*}}{\delta t^{*}}=\frac{\delta^{2} \mathrm{~T}^{*}}{\delta r^{* 2}}+\frac{2}{r^{*}} \frac{\delta T^{*}}{\delta r^{*}} 0<r^{*}<r_{0}{ }^{*} \tag{12a}
\end{equation*}
$$

As discussed in Appendix $B$, the energy balance equation at the moving boundary can be expressed as

$$
\left(h_{2}-h_{1}\right)(\text { mass })=q_{1}-q_{2}=-k_{s}\left(\frac{\delta T}{\delta r}\left|s+-\frac{\delta T}{\delta r}\right| s-\right)
$$

Here, at the evaporation front, the difference in enthalpies is known as the latent heat of vaporization as is denoted by L. The mass crossing the moving boundary is $\varepsilon_{2} \dot{S}$, leaving

$$
\begin{equation*}
\varepsilon_{2} L \dot{S}=-k_{S}\left(\frac{\delta T}{\delta r}\left|S+-\frac{\delta T}{\delta r}\right| S-\right) \tag{13}
\end{equation*}
$$

Which, in dimensionless form is

$$
\begin{equation*}
\dot{S} *=N\left(\frac{\delta T^{*}}{\delta r^{*}}\left|S^{*+}-\frac{\delta T^{*}}{\delta r^{*}}\right| S^{*-}\right) \tag{13a}
\end{equation*}
$$

The solution of equations (12a) and (13a) by the finite difference method yields the temperature distribution throughout the pellet when the following initial and boundary conditions are applied.

$$
\begin{equation*}
T(r, 0)=T_{i}, \quad 0<r<r \quad, T_{i}<T_{E} \tag{14}
\end{equation*}
$$

$$
\begin{align*}
& T(S, t)=T_{E}  \tag{15}\\
& \frac{\delta T}{\delta r}(0, t)=0  \tag{16}\\
& k_{S} \frac{\delta T}{\delta t}\left(r_{0}, t\right)=\lambda\left(T_{\infty}-T\right) \tag{17}
\end{align*}
$$

Equations (14-17) can be expressed in non-dimensional form as follows.

$$
\begin{align*}
& T^{*}\left(r^{*}, 0\right)=T_{i}^{*}  \tag{14a}\\
& T^{*}\left(S^{*}, t^{*}\right)=T_{E}^{*}  \tag{15a}\\
& \frac{\delta T^{*}}{\delta r^{*}}\left(0, t^{*}\right)=0  \tag{16a}\\
& \frac{\delta T^{*}}{\delta r^{*}}=-G T^{*} \tag{l7a}
\end{align*}
$$

Ignoring all effects of the shell in this simplified treatment, the pressure distribution can be found once the temperature distribution is established. The continuity equation for region 1 still holds:

$$
\begin{equation*}
\phi \frac{\delta \varepsilon_{1}}{\delta t}=\frac{M_{j}}{r^{2}} \frac{\delta}{\delta r}\left(r^{2} \varepsilon_{1} \frac{\delta p}{\delta r}\right) \quad S \leq r<r_{0} \tag{18}
\end{equation*}
$$

In its non-dimensional form

$$
\begin{equation*}
\frac{1}{M_{1}^{*}} \frac{\delta \varepsilon_{1}^{*}}{\delta t^{*}}=\frac{1}{r^{*} z} \frac{\delta}{\delta r^{*}}\left(r^{* 2} \varepsilon_{1}^{*} \frac{\delta p^{*}}{\delta r^{*}}\right) \quad S^{*} \leq r^{*}<1 \tag{18a}
\end{equation*}
$$

In this case $\frac{1}{M_{1}{ }^{*}}$ is quite small, and the left hand side will be neglected for convience. Therefore since

$$
\begin{align*}
& \frac{\delta}{\delta r^{*}}\left(r^{*^{2}} \varepsilon_{1}^{*} \frac{\delta p^{*}}{\delta r^{*}}\right)=0 \\
& r^{*^{2}} \varepsilon_{j}^{*} \frac{\delta p^{*}}{\delta r^{*}}=f\left(t^{*}\right) \tag{19}
\end{align*}
$$

only. We assume that the basic interaction between the pressure and the temperature and moisture takes place at the moving boundary due to the phase change. We know that by virtue of the ideal gas law

$$
\begin{equation*}
p=R_{0} \varepsilon_{1} T \tag{20}
\end{equation*}
$$

Which in non dimensional form is

$$
\begin{equation*}
\mathrm{p}^{*}=\varepsilon_{1}^{*}\left(1-\mathrm{D} \mathrm{~T}^{*}\right) \tag{20a}
\end{equation*}
$$

Since no mass is lost or stored, the continuity equation at the moving boundary can be written

$$
\begin{equation*}
-\varepsilon_{i} v_{1}(S, t)=\varepsilon_{2} \frac{d S}{d t} \tag{21}
\end{equation*}
$$

In dimensionless form

$$
\begin{equation*}
\mathrm{A} \dot{\mathrm{~S}}^{*}=\varepsilon_{1}^{*} \frac{\delta \mathrm{p}^{*}}{\delta \mathrm{r}^{*}} \tag{2la}
\end{equation*}
$$

From equations ( 19 and 21a) we see that at the
moving boundary (i.e. $r^{*}=s *$ )

$$
\begin{equation*}
r^{*^{2}} \varepsilon_{1}^{*} \frac{\delta p^{*}}{\delta t^{*}}=f(t)=S^{*^{2}}\left(A \dot{S}^{*}\right) \tag{22}
\end{equation*}
$$

Substituting for $\varepsilon_{1}^{*}$ accoridng to equation (17a)

$$
\begin{equation*}
r^{*^{2}} p^{*} \frac{\delta p^{*}}{\delta t^{*}}=A S^{*^{2}} \frac{\delta S^{*}}{\delta t^{*}}\left(I-D T^{*}\right) \tag{23}
\end{equation*}
$$

We know the boundary condition

$$
\begin{equation*}
p\left(r_{0}, t\right)=p_{i} \tag{24}
\end{equation*}
$$

Which implies

$$
\begin{equation*}
p^{*}\left(1, t^{*}\right)=1 \tag{24a}
\end{equation*}
$$

Using this information we can integrate equation (20) to obtain an expression for $p^{*}$.

$$
\begin{equation*}
p *^{2}=1+A S^{*^{2}} \frac{\delta S^{*}}{\delta t^{*}} \int \frac{\left[1-D T^{*}\left(r_{1}^{*} t^{*}\right)\right]}{r^{*^{2}}} d r^{*} \tag{25}
\end{equation*}
$$

remembering that $S^{*}, \frac{\delta S^{*}}{\delta t^{*}}$, and $T^{*}$ are all known from the determination of the temperature distribution, equations (9a) through 14a) above.

## SOLUTION AND INTERPRETATION OF EQUATIONS

Solution of the simplified set of equations was accomplished by a combination of the finite element method and the finite difference method. Since the finite element method is best suited for spatial relationships and the finite difference for temporal relationships the two methods are used accordingly. Numerous other schemes exist in the literature for solution of such a system of equations (34) but the combination method employed here was developed for its accuracy.

Scheme of Solution

The first step in the solution is to develop the temperature profile as given by equation 12 and the initial and boundary conditions 13a through 17.a. Once the temperature profile is known and the velocity of the moving boundary is calculated, the pressure at the moving boundary can be found from equation 25 and the equation of state 20 .

Both temperature and pressure profiles are important in understanding the nature of the self-bursting phenomenon since the evaporation of the water due to rising temperature
causes both a rapid pressure build-up and deterioration of the binder to the point where its compressive strength is quite small. The return of the pellet to fine particles is due to both of these factors, as well as the others listed in Chapter Three.

The values of the dimensionless constants appearing in the governing equations were calculated using the following values for the necessary parameters. Some of these are standard or tabulated values, some are calculated, and others are assumed or measured.

$$
\begin{aligned}
& \mathrm{c}_{\mathrm{S}}=1.297 \times 10^{3} \mathrm{~J} / \mathrm{kg}^{\circ} \mathrm{K} \\
& \mathrm{~K}=10 \mathrm{md}=9.87 \times 10^{-15} \mathrm{~m}^{2} \\
& \mathrm{k}_{\mathrm{S}}=0.1159 \mathrm{~J} / \mathrm{sm}^{\circ} \mathrm{K} \\
& \mathrm{~L}=2.26 \times 10^{6} \mathrm{~J} / \mathrm{kg} \\
& \mathrm{P}_{\mathrm{a}}=12 \mathrm{~atm}=1 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2} \\
& \mathrm{R}_{\mathrm{O}}=8.314 \mathrm{~J} / \mathrm{g} \mathrm{~mole}{ }^{\circ} \mathrm{K} \\
& \mathrm{r}_{\mathrm{O}}=0.002 \mathrm{~m} \\
& \mathrm{~T}_{\mathrm{E}}=100^{\circ} \mathrm{C}=373^{\circ} \mathrm{K} \\
& \mathrm{~T}_{\mathrm{I}}=20^{\circ} \mathrm{C}=293^{\circ} \mathrm{K} \\
& \mathrm{~T}=600^{\circ} \mathrm{C}=873^{\circ} \mathrm{K} \\
& \mathrm{~W}_{\mathrm{m}}=0.25 \\
& \alpha=1.29 \times 10^{-7} \mathrm{~m}^{2} / \mathrm{s} \\
& \lambda=234.2 \mathrm{~J} / \mathrm{sm}^{2} 0_{\mathrm{K}} \\
& \mu=1.429 \times 10^{-5} \mathrm{Ns} / \mathrm{m}^{2} \\
& \rho_{\mathrm{P}}=736.8 \mathrm{~kg} / \mathrm{m}^{3} \\
& \mathrm{O}=1000 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

$$
\begin{aligned}
& \phi=0.20 \\
& \varepsilon_{m}=\phi W_{m} \rho_{m}=50 \mathrm{~kg} / \mathrm{m}^{3} \\
& \text { The values of the constants as calculated follow. } \\
& D=\frac{T_{\infty}-T E}{T \infty}=0.517,0.617,0.682 \\
& G=\frac{m_{0} \lambda}{k}=4.043 \\
& A=\frac{\varepsilon_{m} R_{0} T_{\infty \rho} \mu \alpha}{K} P_{A}^{2} \\
& N=\frac{k_{p}\left(T_{\infty}-T_{E}\right)}{\varepsilon_{m}^{L \alpha}}=.00176
\end{aligned}
$$

In the derivation of equation 22 it was assumed that $\frac{1}{M_{1}{ }^{*}}$ was very small.


Graphical Presentation of Results

According to the major simplifying assumption proposed in Chapter three, conduction is due to and through the solid coal particles only. Neither the vapor in region one nor the moisture in region two participates in the conduction process. If this were the only factor influencing the temperature profile of the pellet when heated, the plot of temperature versus location would be as presented in Figure 4.

However, another factor influences the temperature profile. The phase change of liquid to moisture which occurs at the moving front uses up a significant amount of the conducted energy, and retards the temperature rise at any point in the

wet region. Instead of a change in temperature the energy is expended in changing the physical state of the liquid (with a resultant change in volume and pressure) and is termed the "Iatent heat of vaporization". See Figure 5 .

Typical temperature histories at specific points in the pellet are shown in Figure 6 . Temperature at any point increases rather slowly at first when some distance behind the evaporation boundary. As the boundary nears the point in question the temperature rises rapidly to the evaporation temperature. Here, however, the temperature remains nearly constant while all the liquid is restructured into vapor, after which normal conduction occurs. At $r=r_{\rho} / 2$, the evaporation temperature is reached in 0.75 seconds, given an ambient heating temperature of $400^{\circ} \mathrm{C}$.

The rising temperature decreases the compressive strength of the pellet as the water entrained in the binder is vaporized. No longer is the matrix rigid and the pellet is no more able to Withstand any external loading, including even the weight of 2 or 3 pellets stacked on top of it. The pellet automatically reduces to dust with any disturbance.

Pressure build-up internally contributes to the process. Under certain conditions the pressure difference across the boundary and the resultant mass transfer so disrupts the matrix that it bursts into small fragments. Figure 7 shows the pressure generated at the moving boundary as the boundary travels toward the center. As in this typical figure, for every set of parameters used the maximum pressure developed occurred when


[^0]

Figure 6 . Temperature histories during firing.


Figure 7 - Pressure history at the moving
front.
the moving boundary was in the range $0.5 \leq S^{*} \leq 0.6$, as shown in Figure 8 although the numeric values varied. If the pellet were to burst at this point it is conceivable (as is borne out in experiments) that the unaffected core would remain intact. But if so, it would likewise be directly heated and disintegrated soon afterwards.

## Interpretation of Results

The remaining graphs Figures 9 through 46 show the result of varying combinations of the dimensionless constants N, G, A, and D. These combinations represent various design considerations and pellet characteristics as values for $k_{s}$, $r_{0}, T_{\infty}, \alpha, \lambda, \varepsilon_{m}, c_{s}$ and $\rho_{s}$ are changed.

From these dimensionless graphs and associated data the following worthwhile conclusions can be drawn. Many are consistent with simple logic, others expected from the equations. Unfortunately, experimental verification was beyond the scope of this investigation, but all of the conclusions agree qualitatively with conclusions drawn from previous studies involving similar problems.

1) It is worth pointing out that the time for the sirface to reach evaporation temperature is about $10 \%$ of the total drying time.
2) Decreasing drying time improves the efficiency of the operation. Drying time decreases with:
a) higher initial temperature, $T_{i}$. However, the


Figure 8 . Pressure generated at the moving boundary.
range of possible initial temperatures is low, and effects are minimal.
b) higher ambient temperature $T \infty$. This parameter is one of the easiest to adjust in an industrial situation, and its effects are large.
c) higher thermal conductivities, $k_{s}$ and thermal diffusivities, $\alpha$. These parameters hasten the conduction of energy to the interior.
d) decreased pellet radius, $r_{0}$.
e) decreased initial water content $w_{m}$ or $\varepsilon_{m}$. With less water to evaporate, the conduction is less retarded and the velocity of the moving boundary increases. However, with lower quantities of water being evaporated, the pressures generated at the moving front will be less.
3) Schulz, et al., (42) experimentally investigated the conditions under which iron ore pellet decrepitation occurs during drying. Their conclusions were confirmed in a qualitative sense by this analysis. Iron ore pellet decrepitation has been shown to be related to high internal pressure gradients, (19and on the assumption that high pressure gradients are associated with coal pellet bursting, the mathematical predictions contained in this study support all of Schulz's findings. The internal pressure gradient
a) is inversely proportional to pellet radius, $r_{0}$. Since by assumption $T_{E}=100^{\circ} \mathrm{C}$, and pressure is
highly temperature dependent, under the same heating condition ( $T \infty$ ) the gradient would decrease as the radius increases.
b) increases with the ambient temperature.
c) increases, but only mildly so with the initial water content $\varepsilon_{\mathrm{m}}$.
d) decreases with an increase in porosity, $\phi$.
e) decreases with an increase in pellet permeability, K.

It is left for future researchers to determine the instant of pellet bursting experimentally and mathematically. Here it has been shown that given the assumed, measured and tabulated values for the terms above, the temperature profile and internal pressure build-up are sufficiently high to cause the pellet to return to fine size particles, thus eliminating the grinding stage of coal preparation at an electrical plant.

VALUES FOR THE VARIABLES AND DIMENSIONLESS
CONSTANTS USED IN SOLUTION OF EQUATIONS

| Case No. | Variables | D | G | A | N |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{T}_{\infty}=500^{\circ} \mathrm{C}$ | . 517 | 4.04 | 6.00 | 3.18 |
| 2 | $\mathrm{T}_{\infty}=600^{\circ} \mathrm{C}$ | . 572 | 4.04 | 6.78 | 3.98 |
| 3 | $\mathrm{T}_{\infty}=700^{\circ} \mathrm{C}$ | . 617 | 4.04 | 7.55 | 4.78 |
| 4 | $\mathrm{k}=.1159 . \mathrm{J} / \mathrm{sm}^{0} \mathrm{~K}$ |  |  | e 2 |  |
| 5 | $\mathbf{k}=.1739 \mathrm{~J} / \mathrm{sm}^{\circ} \mathrm{K}$ | . 572 | 2.695 | 6.78 | 5.97 |
| 6 | $\mathrm{k}=.2318 . \mathrm{J} / \mathrm{sm}^{\circ} \mathrm{K}$ | . 572 | 2.02 | 6.78 | 7.96 |
| 7 | $r_{0}=.002 \mathrm{~m}$ |  |  | e 2 |  |
| 8 | $r_{0}=.0035 \mathrm{~m}$ | . 572 | 7.07 | 6.78 | 3.98 |
| 9 | $r_{0}=.005 \mathrm{~m}$ | . 572 | 10.10 | 6.78 | 3.98 |
| 10 | $\varepsilon_{2}=50 \mathrm{~kg} / \mathrm{m}^{3}$ |  |  | e 2 |  |
| 11 | $\varepsilon_{2}=100 \mathrm{~kg} / \mathrm{m}^{3}$ | . 572 | 4.04 | 13.56 | 1.99 |
| 12 | $\varepsilon_{2}=150 \mathrm{~kg} / \mathrm{m}^{3}$ | . 572 | 4.04 | 20.34 | 1.33 |
| 13 | $\phi=.1$ | . 572 | 4.04 | 3.39 | 7.96 |
| 14 | $\phi=.2$ |  |  | e 2 |  |
| 15 | $\phi=.3$ | . 572 | 4.04 | 10.17 | 2.65 |
| 16 | $\mathrm{K}=9.87 \times 10^{-15}$ |  |  | se 2 |  |
| 17 | $K=19.74 \times 10^{-15}$ | . 572 | 4.04 | 3.39 | 3.98 |
| 18 | $K=29.61 \times 10^{-15}$ | . 572 | 4.04 | 2.26 | 3.98 |
| 19 | $\alpha=1.29 \times 10^{-7}$ |  |  | se 2 |  |
| 20 | $\alpha=1.94 \times 10^{-7}$ | .572 | 4.04 | 10.17 | 2.65 |
| 21 | $\alpha=2.58 \times 10^{-7}$ | . 572 | 4.04 | 13.56 | 1.99 |
| 22 | Iron Ore | . 572 | 7.30 | 476 | 15.64 |



Figure 9. The temperature profile at intervals during the heating process for case 1 in which $T_{\infty}=500^{\circ} \mathrm{C}$.


Figure 10. The temperature profile at intervals during the heating process for case 2 in which $T_{\infty}=600^{\circ} \mathrm{C}$.




Figure 13. The temperature profile at intervals during the heating process for case 5 in which $\mathrm{k}=.1739 \mathrm{~J} / \mathrm{sm} \mathrm{K}$.



Figure 15. The temperature profile at intervals during the heating process for case 7 in which $r_{0}=.002 \mathrm{~m}$.


Figure 16. The temperature profile at intervals during the heating process for case 8 in which $r_{0}=.0035 \mathrm{~m}$.


Figure 17. The temperature profile at intervals during the heating process for case 9 in which $r_{0}=.005 \mathrm{~m}$.


Figure 18. The temperature profile at intervals during the heating process for case 10 in which $\varepsilon_{2}=50 \mathrm{kE} / \mathrm{m}^{3}$.


Figure 19. The temperature profile at intervals during the heating process for case 11 in which $\varepsilon_{2}=100 \mathrm{~kg} / \mathrm{m}^{3}$.


Figure 20. The temperature profile at intervals during the heating process for case 12 in which $\varepsilon_{2}=150 \mathrm{~kg} / \mathrm{m}^{3}$.


Figure 21. The temperature profile at intervals during the heating process for case 13 in which $\phi=0.1$.


Figure 22. The temperature profile at intervals during the heating process for case 14 in which $\phi=0.2$.


Figure 23. The temperature profile at intervals during the heating process for case 15 in which $\phi=0.3$.


Figure 24. The temperature profile at intervals during the heating process for case 16 in which $K=9.87 \times 10^{-15 \mathrm{~m}^{2}}$ 。


Figure 25. The temperature profile at intervals during the heatifg process for case 17 in which $K=19.74 \times 10^{-15} \mathrm{~m}^{2}$ 。


Figure 26. The temperature profile at intervals during the heatipg process for case 18 in which $K=29.61 \times 10^{-15 \mathrm{~m}^{2}}$ 。


Figure 27. The temperature profile at intervals during the heating process for case 19 in which $\alpha=1.20 \times 10^{-7} \mathrm{~m}^{2} / \mathrm{s}$.


Figure 28. The temperature profile at intervals during the heating process for case 20 in which $\alpha=1.94 \times 10^{-7} \mathrm{~m}^{2} / \mathrm{s}$.


Figure 29. The temperature profile at intervals during the heating process for case 21 in which $\alpha=.2 .58 \times 10^{-7} \mathrm{~m}^{2} / \mathrm{s}$.


Figure 30. The temperature profile at intervals during the heating process for case 22 , a typical iron cre pellet.


Figure 31. Variation in the temperature histories at $r=r_{0} / 2$ of heated coal pellets due to changes in ambient temperature. Cases 1, 2, and 3.


Figure 32 . Variation in the temperature histories at $r=r_{0} / 2$ of heated coal pellets due to changes in thermal conductivity. Cases 4,5, and 6.


Figure 33. Variation in the temperature histories at $r=r_{0} / 2$ of heated coal pellets due to changes in pellet radius. Cases 7, 8, and 9.


Figure 34. Variation in the temperature histories at $r=r_{0} / 2$ of heated coal pellets due to changes in mass density of water. Cases 10,11 , and 12.


Figure 35 . Variation in the temperature histories at $m=r_{0} / 2$ of heated coal pellets due to changes in pellet porosity. Cases 13, 14, and 15.


Figure 36. Variation in the temperature histories at $r=r_{0} / 2$ of heated coal pellets due to changes in permeability in outer zone. Cases 16, 17, and 18.



Figure 38 . The temperature history at $r=r_{0} / 2$ for a typical coal pellet compared to that of an iron ore pellet. Cases 2 and 22.


Figure 39 . Variation in pressure generated at the moving boundary due to changes in ambient temperature. Cases 1 , 2 and 3.


Figure 40. Variation in pressure generated at the moving boundary due to changes in thermal conductivity. Cases 4,5 and 6 .


Figure 41. Variation in pressure generated at the moving boundary due to changes in pellet radius. Cases 7, 8 and 9.

##  <br> DIMENSIONLESS BOUNDARY POSITION, $S^{*}$

Figure 42. Variation in pressure generated at the moving boundary due to changes in mass density of water. Cases 10, 11 and 12.


Figure 43. Variation in pressure generated at the moving boundary due to changes in pellet porosity. Cases 13, 14 and 15.


Figure 44. Variation in pressure generated at the moving boundary due to changes in permeability in outer zone. Cases 16, 17 and 18. 108


Figure 45 . Variation in pressure generated at the moving boundary due to changes in thermal diffusivity. Cases 19, 20 and 21.


## CHAPTER V

## CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

In previous sections of this paper certain conclusions have been drawn and recommendations for future use or study have been made. These will be combined in this chapter for easy reference and will in most cases be expanded and supplemented.

## Experimental

The experimental portion of this research has served to show that a self-bursting pellet with acceptably high strength and stability characteristics can be agglomerated with relative ease. Left for future investigators are many unanswered questions concerning proper binder, conditions of agglomeration, evaluation schemes, etc.

It was pointed out that the experimental apparatus used was not optimum. The most urgent need is for a two-armed, kneading type mixer for mixing the coal and the binder with provision for scraping the bottom and the sides of the nonsticking. mixing bowl. Such a mixer would spread the binder much more evenly throughout the fines and eliminate the "globs"
of moist material lef't by the variable speed mixer as used.
Testing for the resultant crushing strength could also be improved on. In this experiment the load was applied manually to the pellet atop a spring loaded weighing scale, and the "weight" of the load recorded. Obviously more consistent and accurate measurements could result from lateral confinement and controlled loading, but such technique was beyond the scope of this investigation.

A much larger sample size would necessitate a larger agglomerating drum, but would allow a larger number of pellets to be tested for each response. This would presumably reduce the scatter in the data, nullifying the impact of anomalous readings.

Subsequent research could investigate many different binders, and isolate those which would produce acceptable pellets. All potential binders must not only produce strong, stable pellets but must burn without releasing harmful gases. These investigations could center their effects on linear linking wax derivatives such as in this paper, cross linking organic polymers, rigid gels, etc. Since it has been shown feasible that returning the pellet to dust at the plant can be accomplished without the use of a shell, it would be advantageous to eliminate the shell. This could be accomplished by using a binder that is insoluble in water, possessing long storage ife despite potential harsh conditions that may be encountered before reaching the plant. Eliminating the shell would drastically
reduce the cost of pellet production, enhancing the marginal economic picture of this research.

The environment of agglomeration must be better controlled in order to isolate the effects of binder concentration, energy levels, etc. It is suspected that the relative humidity and the temperature of the laboratory air have a significant effect on the growth of the pellets. On an industrial scale these and other factors could not be well controlled, but their known levels could be compensated for by altering binder concentration, energy of mixing, etc.

In order to gain the most information from future experiments they should be carried out at multiple levels, not just the high/low scheme employed here. A five-level response investigation would be more able to determine the optimum levels of the parameters investigated. Without such a multi-level scheme it is also impossible to discover an oscillating response, since a two-level experiment assumes a linear variation between the extreme levels.

It must be remembered that each coal acts and reacts differently. Before this method can be applied to any industrial situation the coal to be used must be completely studied and understood. The five-level investigation proposed above would establish a proper technique for the necessary industrial investigation.

## Mathematical

The equations developed in this study model only the effect of the phase change of the water in the binder to steam when the pellet is heated. This is thought to be the most significant factor in the internal pressure build-up, but it is recognized that other factors, i.e. the formation of gases of devolatilization, and the swelling of the coal, are not insignificant. Indeed high speed photographic studies have shown that powdered coal devolatilizes rapidly and to a great extent when suddenly introduced into a very hot environment. As the coal devolatilizes pores open to allow the gases to escape, causing the coal to swell. If such studies apply to agglomerated coal fines, then the presence of steam would accelerate the process. Certainly these factors are interrelated and a true modeling of the internal pressure build-up should include them all. This has not been done; it awaits further studies.

However, the equations as developed do provide a look at the important phase change phenomenon, and show that pressure builds up rapidly and extensively, causing the confined pellet to burst.

As was discussed in Chapter Three, solution of the initial set of equations was likewise deferred to subsequent investigations and a simplified although adequate set of equations was solved to yield meaningful results. It is suggested that one with an advanced knowledge of numerical methods and access to a large computer solve these equations to gain a more complete
understanding of the reaction.
It was found in the experimental stage of this study that a confining shell was not necessary to maintain the internal pressures inside the pellet for bursting to occur, and the shell was not included in the model. (Its presence however would accentuate the process). The effect of confining the pellet mechanically was sufficient to produce bursting.

Chapter Four detailed the internal pressure and temperature gradients as determined from the derived equations. It is reasonable to assume that the gradients accurately represent the true situation, since the liquid-steam phase change is well known and studied and the equations are built on this knowledge and reasonable assumptions. It would be enlightening to test the results experimentally in an attempt to verify them. The temperature profile could be measured directly with sensors placed inside the heated pellet, while the pressures could be gained by inference.

## Industrial

As future research solves the remaining problems, an economic study would be necessary to determine this method's industrial feasibility. Elimination of the need for a protective shell greatly improves the outlook. Most importantly, the elimination of the expensive grinding stage at the electrical plant makes the moderately costly agglomeration of waste fines attractive.

This, of course, is accomplished by the bursting of the
heated pellet. The confining of individual pellets while heating is much more easily accomplished on an industrial scale than in the laboratory. Numerous schemes could be proposed to return the pellets to fine size particles such as these that are discussed below.

If large numbers of pellets were rapidly introduced into a very hot, convective furnace, it is possible that sufficient confinement may be provided by the rising gases and nearby exploding pellets to allow bursting. However, this has not been tested, and could only be tested on an industrial scale. In this researcher's opinion successful bursting with this method is not a foregone conclusion, and other methods must be developed.

Sufficient confinement could be achieved by redesign of feed line nozzles. Developed for injecting powdered coal into the furnace, present nozzles would be incapable of handling pellets and redesign is necessary under any condition. By replacing the jet with a constricting nozzle which would either be pre-heated or simply heated by the nearby furnace, pellets would readily burst as they neared the point of injection. After being reduced to fines once again, the coal could be injected as before and no redesign of the furnace is necessary.

Even as the pressure builds up inside the pellet the compressive strength of the reacted portion decreases. In a pre-heating chamber in the feed line moderate external loading would serve both to confine the exploding pellets and jostle them
enough to insure complete destruction. Metallic balls mixed with the pellets which could be removed magnetically before leaving the chamber would provide the load necessary. Other foreign objects, such as larger glass balls, which could be removed easily by a strainer would accomplish the same purpose.

A heating chamber constructed of ceramic material, glass or other substance not affected by microwave radiation would allow bursting to occur by another method. Both coal parti-. cles and binder could be heated rapidly and effectively by low level microwave radiation, again causing phase-change of the moisture in a confined environment. Individual pellets heated in a microwave oven completely lose their compressive strength and the confinement and loading of a packed bed would insure bursting.

## Summary

The feasibility of the manufacture of a self-bursting coal pellet has been demonstrated. The coal fines agglomerated from waste material are utilized in such a way that the normal grinding of all coal at a power plant is eliminated. The pellet returns to fine particles as a combined result of internal pressure build up, volume change and decrease of compressive strength.

## APPENDIX A

## NOMENCLATURE

## Parameters and Variables

```
c = specific heat (J/kg O
F = specific internal area per unit volume (m}/\mp@subsup{m}{}{2}
h = enthalpy (J/kg)
H = heat transfer coefficient (J/sm2 O
k = thermal conductivity ( }\textrm{J}/\textrm{sm}\mp@subsup{}{}{\circ}\textrm{K}
K = permeability of dry region (m
L = latent heat of vaporization (J/kg)
m = mass (kg)
M = mobility of moist pellet = K\phi/\mu, (M }\mp@subsup{M}{}{4}/\textrm{sN}
p = pressure (N/m
q= flow (m}/\textrm{m}
r = radial distance from pellet center (m)
ro = pellet radius (m)
RO}= ideal gas constant (J/g mole OK)
S = distance of moving front from center (m)
s}=ds/d
t = time (s)
T = temperature ( }\mp@subsup{}{}{\circ}\textrm{K}
v = Darcy velocity (m/s)
```

```
\(\mathrm{w}=\) water saturation
\(\alpha=\) thermal diffusivity of moist pellet ( \(\mathrm{m}^{2} / \mathrm{s}\) )
\(\beta=\) inertial resistance coefficient ( \(\mathrm{m}^{-1}\) )
\(\varepsilon=\) mass density of water ( \(\mathrm{kg} / \mathrm{m}^{3}\) )
\(\lambda=\) surface heat transfer coefficient ( \(\mathrm{J} / \mathrm{sm}^{2} \mathrm{O}_{\mathrm{K}}\) )
\(\mu=\) absolute viscosity ( \(\mathrm{Ns} / \mathrm{m}^{2}\) )
\(\rho=\) density \(\left(\mathrm{kg} / \mathrm{m}^{3}\right)\)
\(\phi=\) porosity of pellet \(\left(\mathrm{m}^{3} / \mathrm{m}^{3}\right)\)
\(\Phi=\) viscous dissipation function ( \(N / \mathrm{m}^{2} \mathrm{~S}\) )

\section*{Dinensionless Constants}
\[
A=\frac{\varepsilon_{2} R_{0} T_{\infty} \mu \alpha}{K P_{A}^{2}}
\]
\[
D=\frac{T_{\infty}-T_{E}}{T_{\infty}}
\]
\[
G=\frac{r_{0}^{\lambda}}{k}
\]
\[
N=\frac{k\left(T \quad-T_{E}\right)}{\varepsilon_{2} L \alpha}
\]
\[
M^{*}=\frac{K P_{i} C_{1}{ }^{\rho} p_{p}}{\mu k}
\]
```

p* = dimensionless pressure (= p/pi)
r* = dimensionless radial distance (= r/ro)
S* = dimensionless position of moving boundary
(=S/ro)
t* = dimensionless time (= \alphat/r (ro )
T* = dimensionless temperature (= ( }\mp@subsup{T}{\infty}{}-T)/(\mp@subsup{T}{0}{}-\mp@subsup{T}{E}{})
\varepsilon* = dimension\ess mass density (= \varepsilon/\varepsilon ( )

```

\section*{Suffixes}
```

O = heating surface
1 = dry region
2 = wet region
\infty = external conditions
g = water vapor
i = initial state
m = liquid
p = pellet
s = solid
A = atmospheric
E = evaporation

```

\section*{APPENDIX B}

\section*{Derivation of Equations}

\section*{Mass Transfer in Region One}

The continuity equation for compressible fluid flow
is
\[
\frac{\delta \rho}{\delta t}=-\operatorname{div} \cdot(\rho \overrightarrow{\mathrm{V}})
\]

But for flow in porous media
\[
\frac{\delta \rho \phi w}{\delta t}=-\operatorname{div} \cdot \rho \phi w \vec{v}
\]
or \(\quad \frac{\delta \varepsilon}{\partial t}=-\operatorname{div} \cdot \varepsilon \vec{V}\)
where \(\varepsilon=\rho \phi w\)

From Darcy's Law we know
\[
\vec{V} \quad=\frac{-K}{\mu} \quad \frac{\Delta P}{\mathbb{L}}=\frac{-K}{\mu} \quad \frac{\delta p}{\delta r}
\]
which yields in spherical coordinates
\[
\frac{\delta \varepsilon}{\delta t}=\frac{K}{\mu}\left[\operatorname{div} \varepsilon \frac{\delta p}{\delta r}\right]=\frac{K}{\mu}\left[\frac{1}{r^{2}} \frac{\delta}{\delta r} \quad r^{2} \varepsilon \frac{\delta p}{\delta r}\right]
\]

Since by definition, \(M=\frac{K \phi}{\mu}\)
\[
\begin{equation*}
\frac{\delta \varepsilon_{1}}{\delta t}=\frac{M}{\phi r_{1}{ }^{2}} \frac{\delta}{\delta r_{1}}\left(r_{1}{ }^{2} \varepsilon_{1} \frac{\delta p_{1}}{\delta r_{1}}\right) \quad s \leq r_{1} \leq r_{0} \tag{1}
\end{equation*}
\]

\section*{Mass Balance at the Moving Boundary}


By conservation
\[
\varepsilon_{1}\left(V_{1}-\dot{S}\right)=\varepsilon_{2}(V-\dot{S}) \quad @ r_{1}=r_{2}=s
\]

However, by assumptions 1 and \(5, \mathrm{~V}_{2}=0\)
\[
\begin{aligned}
& \varepsilon_{1}\left(V_{1}-\dot{S}\right)=\varepsilon_{2}(-\dot{S}) \\
& \dot{S}\left(\varepsilon_{1}-\varepsilon_{2}\right)=\varepsilon_{1} V_{1}
\end{aligned}
\]

By Darcy's Law \(V_{2}=-\frac{K}{\mu} \frac{\delta p}{\delta r}\)
and as before \(M=\frac{K}{\mu} \phi\)
\[
\begin{equation*}
\dot{S}\left(\varepsilon_{2}-\varepsilon_{1}\right)_{\left(r_{1}, \quad r_{2}=S\right)}=\frac{M_{1}}{\phi} \varepsilon_{1} \frac{p}{r} \tag{2}
\end{equation*}
\]

Energy Transfer in Region One
The equation for conservation of energy of a fluid moving through a porous media has been developed and consists of two parts, that of the solid and that of the fluid, in this case water vapor. (36)

Solid
\[
\begin{equation*}
\rho_{s} C_{s}(1-\phi) \frac{\delta T}{\delta t}=\frac{k_{s}(1-\phi)}{r_{1}{ }^{2}} \frac{\delta}{\delta r_{1}}\left(r_{1}{ }^{2} \frac{\delta T}{\delta r}\right)+H F \quad(T g-T s) \tag{3}
\end{equation*}
\]

Fluid
\[
\begin{aligned}
\rho_{g} C_{g}\left(\frac{\delta T_{g}}{\delta t}\right. & \left.+\frac{V}{\phi} \frac{\delta T_{g}}{\delta r_{1}}\right)=\frac{k_{g}}{r_{1}{ }^{2}} \frac{\delta}{\delta r_{1}}\left(r_{1}{ }^{2} \frac{\delta T_{g}}{\delta r_{1}}\right)+\frac{\delta P_{1}}{\delta t}+\frac{V}{\phi} \frac{\delta P}{\delta r} \\
& +\mu \Phi+\frac{H F}{\phi}\left(T_{s}-T_{g}\right)
\end{aligned}
\]
where \(\mu \Phi=\) the energy dissipated per unit volume
\[
\begin{aligned}
& =\frac{V}{\phi}\left(\frac{\mu}{K}\right) V+\beta \rho_{g} v^{2} \\
& \beta=\text { internal resistance coefficient. }
\end{aligned}
\]

However, at low velocity flow for which Darcy's Law applies, the \(\beta\) term \(=0\), and the energy dissipated per unit volume is equal in magnitude and opposite in sign to the term \(\frac{V}{\phi} \frac{\delta p}{\delta r}\). The remaining terms are applicable to this nroblem.
\[
\begin{align*}
\rho_{g} C_{g}\left(\frac{\delta T_{g}}{\delta t}\right. & \left.+\frac{V}{\phi} \frac{\delta T_{g}}{\delta r_{1}}\right)=\frac{k_{g}}{r_{1}{ }^{2}} \frac{\delta}{\delta r_{1}}\left(r_{1}^{2} \frac{\delta T_{g}}{\delta r_{1}}\right)+\frac{\delta P_{1}}{\delta t} \\
& +\frac{H F}{\phi}\left(T_{s}-T_{g}\right) \tag{4}
\end{align*}
\]

\section*{Energy Transferred in Region Two}

Since there is no mass transfer in region two, and the solid and liquid are in thermal equilibrium energy transfer is by conduction through the moist pillet only. The heat conduction equation in spherical coordinates is
\[
\begin{equation*}
\rho_{p} C_{p} \frac{\delta T_{2}}{\delta t}=\frac{k_{p}}{r_{2}^{2}} \quad \frac{\delta}{\delta r_{2}} \quad\left(r_{2}^{2} \quad \frac{\delta T_{2}}{\delta r_{2}}\right), 0 \leq r_{2} \leq S \tag{5}
\end{equation*}
\]

Energy Balance at Moving Boundary


The energy balance equation is
\[
(\text { mass })\left(h_{1}+\left(\frac{\mathrm{v}-\dot{\mathrm{S}}}{2}\right)^{2}\right)+q_{1}=(\text { mass })\left(h_{2}+\left(\frac{\mathrm{v}+\dot{\mathrm{S}}}{2}\right)^{2}\right)+q_{2}
\]

However, for any flow less than supersonic flow, the velocity term is much less than the enthalpy term and can be neglected, which leaves the relationship
\[
\left(h_{2}-h_{1}\right)(\text { mass })=q_{1}-q_{2}
\]

As in the derivation of equation 2 , the continuity equation for mass transfer at the moving boundary is
\[
\varepsilon_{1}\left(V_{1}-\dot{S}\right)=\varepsilon_{2}\left(V_{2}-\dot{S}\right)
\]
but by assumptions 1 and \(5, V_{2}=0\)
\[
\varepsilon_{1} V_{1}-\varepsilon_{1} \dot{S}=-\varepsilon_{2} \dot{S}
\]

Substituting
\[
\left(h_{2}-h_{1}\right)\left(\varepsilon_{1} V_{1}-\varepsilon_{1} \dot{S}\right)=q_{1}-q_{2}
\]

\section*{Expanding}
\[
\left(h_{2}-h_{1}\right)\left(\varepsilon_{1} V_{1}\right)-\varepsilon_{1} h_{2} \dot{s}+\varepsilon_{1} h_{1} \dot{S}=q_{1}-q_{2}
\]

For manipulation purposes add adn subtract \(\varepsilon_{2} h_{2} \dot{S}\)
\[
\left(h_{2}-h_{1}\right)\left(\varepsilon_{1} V_{1}\right)-\varepsilon_{1} h_{2} \dot{S}+\varepsilon_{1} h_{1} \dot{S}+\varepsilon_{2} h_{2} \dot{S}-\varepsilon_{2} h_{2} \dot{S}=q_{1}-q_{2}
\]
and simplify
\[
\varepsilon_{1} V_{1}\left(h_{2}-h_{1}\right)+h_{2} \dot{s}\left(\varepsilon_{2}-\varepsilon_{1}\right)+\dot{s}\left(\varepsilon_{1} h_{1}-\varepsilon_{2} h_{2}\right)=q_{1}-q_{2}
\]

Since from before
\[
\begin{aligned}
& \varepsilon_{1} \mathrm{~V}_{1}-\varepsilon_{1} \dot{S}=-\varepsilon_{2} \dot{S} \\
& \left(\varepsilon_{2}-\varepsilon_{1}\right) \dot{S}=-\varepsilon_{1} \mathrm{v}_{1}
\end{aligned}
\]

Sụbstituting, expanding and simplifying
\[
\begin{aligned}
& \varepsilon_{1} V_{1}\left(h_{2}-h_{1}\right)+h_{2}\left(-\varepsilon_{1} V_{1}\right)+\dot{S}\left(\varepsilon_{1} h_{1}-\varepsilon_{2} h_{2}\right)=q_{1}-q_{2} \\
& \varepsilon_{1} V_{1} h_{2}-\varepsilon_{1} V_{1} h_{2}-\varepsilon_{1} V_{1} h_{1}+\dot{s}\left(\varepsilon_{1} h_{1}-\varepsilon_{2} h_{2}\right)=q_{1}-q_{2} \\
& \left(\varepsilon_{1} h_{1}-\varepsilon_{2} h_{2}\right) \dot{s}=q_{1}-q_{2}+\varepsilon_{1} V_{1} h_{1}
\end{aligned}
\]

From the general Fourier equation
\[
q_{1}=-k \nabla T=-k_{1} \frac{\delta T_{1}}{\delta r_{1}} \text { and } q_{2}=-k_{2} \frac{\delta T_{2}}{\delta r_{2}}
\]

Also from Darcy's Law as before
\[
\nabla_{1}=-\frac{K}{\mu} \quad \frac{\delta P_{1}}{\delta r_{1}}=-\frac{M_{1}}{\bar{\phi}} \frac{\delta P_{1}}{\delta r_{1}}
\]

Substituting yields
\[
\begin{equation*}
\left(\varepsilon_{1} h_{1}-\varepsilon_{2} h_{2}\right) \dot{S}=-k_{1} \frac{\delta T_{1}}{\delta r_{1}}+k_{2} \frac{\delta T_{2}}{\delta r_{2}}-\frac{M_{1}}{\phi} \varepsilon_{1} h_{1} \frac{\delta p_{1}}{\delta r_{1}} \tag{6}
\end{equation*}
\]
where \(\quad r_{1}, r_{2}=S\)

\section*{APPENDIX C}

COMPUTER PROGRAM

\section*{LZT}


CALL PLOT(5..0..-3)

CALL GFAPH(IT.JH.0..:
CALL
stop
STOP
END

\begin{tabular}{|c|c|c|c|c|}
\hline forthan ev gi & nelease & 2.0 LLSTIF DATE \(=80269\) & 11/44/38 & Pagit 0001 \\
\hline OVO: & & Suthuutine elsitifielx.si.s 2 atif) & & \\
\hline & c & & & \\
\hline 0002 & & DIMENSION St(2,2).S2(2,2), ELX 2 , ELF(2) & & \\
\hline & C & & & \\
\hline 0003 & & \(\cdot \mathrm{DX}=\mathrm{ELX}(2)-E L X(1)\) & & \\
\hline 0004 & & Si(tol) \(=0 \times 13\). & & \\
\hline 0005 & & Si(2.2)= \(51(1.1)\) & & \\
\hline 0006 & & Sille2) \(=0 \times 16\). & & \\
\hline 0007 & & Si(2.1)= Si(1.2) & & \\
\hline & c & & & \\
\hline 0008 & & \(C=E L X(2) / E L X(1)\) & & \\
\hline 0009 & &  & & \\
\hline 0010 & & S2(t.2) \(=(-1 .-(E L X(2) * A L U G \& C) / 0 X-1) * 2..) /\) DX & & \\
\hline 0011 & &  & & \\
\hline 0012 & &  & & \\
\hline & c & & & \\
\hline 0013 & & FETURN & & \\
\hline 0019 & & END & & \\
\hline
\end{tabular}

SUBRUUTINE FORCESGSZ.T.GF:NNMI
    C OIMENSION GS2(101.101).T1103),GF(101)

0003


0004
0006
0009
DC \(1001=1\), ANM
\(f=0\).
DC 101 LII.ANM

\(100 \mathrm{GF}(1)=\)
    RE TURN
    END
```

        FURTGAN IV GI HELEASE 2.0
    ```
0001
```

0001
SUHRUUTINE PHESUR (P,NEM,A,D,T,X,TE,OT,XF,XB,SOF.SOD,NF,NG.ISTUP,
SUHRUUTINE PHESUR (P,NEM,A,D,T,X,TE,OT,XF,XB,SOF.SOD,NF,NG.ISTUP,
\varepsilon
\varepsilon
1S.IENU.OX.G.AN.NP,IT,TF.TG.KAIS
1S.IENU.OX.G.AN.NP,IT,TF.TG.KAIS
0 0 0 2
0 0 0 2
C
C
dimENSIUN T(103) ox{103)
dimENSIUN T(103) ox{103)
FIND FRUNT
FIND FRUNT
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0025
NP=0
NP=0
P=0.
P=0.
IS=0
IS=0
OC 1O I={,NEM
OC 1O I={,NEM
TL= T(1)
TL= T(1)
jk= T(1*)!
jk= T(1*)!
EL= ABS(IL-TEJPTE
EL= ABS(IL-TEJPTE
ER= ABS(TR-:E)/TE
ER= ABS(TR-:E)/TE
LF(EL-LV.2 eE-3) TL=A.E-S*TE
LF(EL-LV.2 eE-3) TL=A.E-S*TE
IF(ENOLT-1.E-3) TR=1.E-3+YE
IF(ENOLT-1.E-3) TR=1.E-3+YE
IFITL.EG.TE: GO TO II
IFITL.EG.TE: GO TO II
IF(TH.EG.JE) GL TO I2
IF(TH.EG.JE) GL TO I2
IFITL.GT.TE. AND -TR.LT.TEIGL TU IJ
IFITL.GT.TE. AND -TR.LT.TEIGL TU IJ
xf=x(1)
xf=x(1)
11}\textrm{Xf}=\textrm{Cl
11}\textrm{Xf}=\textrm{Cl
IG=
IG=
co ra 5000
co ra 5000
12 xi= x(1+1)
12 xi= x(1+1)
xi:= x(1+i)
xi:= x(1+i)
1S= 1+
1S= 1+
GC TO 5000
GC TO 5000
13 XF= X(I)+(X(I+I)-X(I))*(tE-T(I))/(t(I+I)-T(I))
13 XF= X(I)+(X(I+I)-X(I))*(tE-T(I))/(t(I+I)-T(I))
15=1
15=1
1G=
1G=
5000 NP= NP+1
5000 NP= NP+1
10 CCNTINUE
10 CCNTINUE
1F( IS.EO.O) KAI= KAItI
1F( IS.EO.O) KAI= KAItI
IF( IS.EQ.O) GU TU 4000
IF( IS.EQ.O) GU TU 4000
IFINP.GI.II GC TC 5001
IFINP.GI.II GC TC 5001
ISTUP= 0
ISTUP= 0
IEND= IENODI
IEND= IENODI
IF(IEND.GT.1) GU TO 2000
IF(IEND.GT.1) GU TO 2000
CL= (TGIS+IS-T(ISi)/OX
CL= (TGIS+IS-T(ISi)/OX
OR= O.
OR= O.
GO TO 300
GO TO 300
2000 CCNTINUE
2000 CCNTINUE
DL= (T(NF)-T(NF-1))MDX
DL= (T(NF)-T(NF-1))MDX
CK= (t(NF+1)-T(NF)IMOX
CK= (t(NF+1)-T(NF)IMOX
3000 SD= AN*OR
3000 SD= AN*OR
TH=OT*IT
TH=OT*IT
SLU= (XF-XE)/(JF-TB)
SLU= (XF-XE)/(JF-TB)
IFIIEND.EG.1) GO TO 5002
IFIIEND.EG.1) GO TO 5002
It(SOD.LT.SO) Gu TO 5col
It(SOD.LT.SO) Gu TO 5col
S002 CCAIANUE
S002 CCAIANUE
KAL= KAI+I
KAL= KAI+I
xE= XF
xE= XF
H0= TF
H0= TF
DS= SD*DT
DS= SD*DT
xfl= XF-DX
xfl= XF-DX
IFIXN-LT.O.D ISICP=1

```
    IFIXN-LT.O.D ISICP=1
```


## ஏモT



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$\begin{array}{ll}=0.05000 t+00 & X N=0.62500 E+00\end{array}$ ${ }_{x}$
$\begin{array}{ll}=0.65000 E+00 & X N=c .62500 E+00\end{array}$
 $\begin{array}{ll}0.65000 E+00 & X N=0.62500 E+00 \\ 0.65000 E+00 & X N=0.62500 E+00\end{array}$

 $\mathrm{x}=0.57500 \mathrm{E} .00$ $X_{N}=0.57500 E+00$

 $x \mathrm{~N}=\mathrm{c} .550 \mathrm{coc}+00$ xN＝ $0.55000 \mathrm{E}+00$ $X N=0.55000 E+00$ 0
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$\times$ XA $=C .500 C C E+00$ 0
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4
$x$ $x_{n}=0.50000 E * 00$ 0
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0
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0
0
11
2
$\times$ $x_{N}=0.50 c c o e r 00$ $X N=0.50000 \mathrm{~F}+00$ $x A=0.50000 \mathrm{E} 00$ $\mathrm{XN}=0.50000 E+00$ 0
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in
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5
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0
0
11
2 $\mathrm{XN}=0.4500 \mathrm{CE}+00$ $X N=C .45000 E+00$
 01 $\mathrm{O}_{10}^{\times 13}$ $0^{0} \times$ $0.600 \quad$ PRESSLRE $=0.18600 \mathrm{E}+01$
SDO $=-0.50000 E * O 1 \quad$ XF $=0.60000 \mathrm{e} \cdot 00$
 $0.575 \quad$ PRESSUHE $=0.18095 E+01$
SDU $=0.10000 E+02$



 $S U D=-0.20000 E+02 \quad x F=0.5250 C E+00 \quad x(J=0.575 C O E+00$
 $02 \quad \mathrm{HF}=0.5250 \mathrm{CE}+00$ RESSUFE $=0.16393 E+01$
$2 X=0.5250$ CE 000 PRES XVRE $=0.17593 E+01$
$02 X_{F}=0.5250 \mathrm{cE}+00$ $02 \quad \mathrm{XF}=0.5250 \mathrm{CE}+00$
PRESSURE $=0.17625 \mathrm{E}+01$
01 XF $=0.52500 \mathrm{E}+00$ PRESSURE $=0.18407 E+01$
OI XF $=0.32500 E * 00$


 $0.475 \quad$ PRES SUHE $=0.16124 E+01$
SOD $=-\mathrm{C} .20000 \mathrm{O}+02 \quad \mathrm{XF}=0.47500 \mathrm{E}+00$


 $0.475 \quad$ PRESSUIAL $=0.18211 L+01$
SDD $=-0.666 G U E+01 \quad K F=0.4750 \mathrm{CL}+00$,
 $0.475 \quad$ PRESSURL $=0.1885 I L+C!$
$S O D=-0.20001 E 402 \quad X F=0.450 J 0 E t 00$ $0.450 \quad$ PFE $S S U R E=0.17 E 61 E+01$
SUC $=-0.1000 O E+02 \quad X F=0.450001+00$


$0.1048 A E+01$
$0.10344 E+01$
$0.1212 A E+01$
$0.96595 E+00$
$0.82391 E+00$
$0.67788 E+00$
$0.51957 E+00$
$0.35 S O S E+00$

$15=75$ TIME $=0.93750 E-01$

SOD=-0.40000E+01 XF = U.40000E +00 XR=0.45000Et00 XN=0.42500E*00
 SCO $=-0.20000 E+02 \quad X f=0.40000 E+00 \quad X H=0.45000 E * 00 \quad X N=0.42500 E+00$
 $0.400 \quad$ PRESSURE $=0.17195 E+01$
SDD $=-C .10000 E+02 \quad X F=0.4000 G E+00 \quad X A=0.450 .40 E+0 U \quad X N=0.42500 E+00$



 $X N=0.37500 E+00$ $X N=0.3750 C E+00$ $X N=0.3750 C E+00$ $X N=0.35000 E * 00$ $x n=0.35000 E+00$




 0
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$\vdots$
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4


$\mathrm{XN}=\mathrm{C} 3000 \mathrm{CL}+$.
 $\begin{array}{ll}\text { SDF }=-0.54185 E+01 \\ 1 & 0.10525 E+01 \\ 6 & 0.10467 E+01 \\ 11 & 0.10304 E+01 \\ 16 & 0.10085 E+01 \\ 21 & 0.93749 E+00 \\ 26 & 0.79557 E+00 \\ 31 & 0.64715 E+00 \\ 36 & 0.42737 E+00 \\ 41 & 0.32311 E+00 \\ S D=-0.54125 E+01\end{array}$








 $S D=-0.46277 E+01$
$15=16 \quad X F=$ $15=16 \quad$ XF $=$
$S 0=-0.37507 E+01$
$15=$
$16 \quad$ XF $=$













 IS $=14 \quad$ XF $=$
$s u=-0.18853 E .01$ 11 "
 $0.400 \quad$ PRESSURE $=0.17221 E+01$
$S D=-0.42453 E+01 \quad S D D=-0.66067 E+01 \quad X F=0.40000 E+00 \quad 1, ~$ $0.400 \quad$ PRESSURE $=0.879 .3 E+01$
$S D D=-0.66667 E+01 \quad X F=0.40000 E+00 \quad X L$ 0.400 PRESSURE $=0.17959 E+01$ 0.400 PFESSURE $=0.18530 E+01$ $S U D=-0.20000 E+02 \quad X F=0.3750 C E+00$ SDO $=-C_{1} 10000 E+02 \quad x F=0.37500 C+00 \quad x G=0.40000 E+00$
 SOD=-0.50000E*OS $\quad x=0.37500 E * 00 \quad X I S=0.40000 E+00$ $0.375 \quad$ PRESSURE $=0.17710 E+01$
$S C D=-0.40000 E * O 1 \quad X F=0.37500 E * U 0 \quad X U=0.375 C O E+C O$ SDD $=-\mathbf{C . 4 0 0 0 0 E + 0 1} \quad \mathrm{KF}=0.325 \mathrm{CCE}+00 \quad \mathrm{KII}=0.3750 \mathrm{CE} * 00$ $x B=0.37500 E+00$
$x B=0.37500 E+00$ $\begin{array}{cc}0.325 \quad \text { PRESSURE }=0.17869 E+0 \\ O D & =0.2 C O O O E+02 \quad X F\end{array}$ $0.325 \quad$ PRESSURE $=0.1563 \mathrm{EE}+01$
$-0.13333 E+02 \quad K F=0.32: 300 E+00$
 $13=0.37500 E+00$
$x B=0.375005+00$ $0.325 \quad$ PRESSURE $=0.16755 E+01$
$=-0.800 U 0 E * O I \quad X F=$ $U=0.375 C C E+00$
$=0.37500 E+00$
 $x=0.32530 \varepsilon * 00$ 0.s.s500E:0 $=4.32500 \mathrm{E}+00$

0.10008 E - 01
0.99977 EOO $0.99977 E+00$
$0.886 A C E+00$
$0.771 G E E 00$
$00+325 E S 9^{\circ} 0$
$00+319121^{\circ} 0$ 08
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$t \%=108 \quad \operatorname{timE}=0.13500 E * 00$ SUF $=-0.51403 E 001 \quad S 0 B=-0.54505 E+01 \quad$ NF $=$

 $0.200 \quad$ PRESSURE $=0.12509 E+01$
$S D U=-C .20000 E+02 \quad X F=0.2000 C E+00 \quad X J=0.25000 E+00 \quad X N=0.22500 E+00$
 SDU=-0.10000E $+02 \quad X F=0.20000 E+00 \quad X B=0.25000 E+00 \quad X N=0.22 E C C E+00$ $\times N=0.22500 E * 00$ $x A=0.22500 E 400$
 $X N=0.17 \leqslant 00 E 400$

 XN $=0.17500 E+00$ KN $=0.15000 \mathrm{~L}+00$ $X N=0.15000 E+00$ $X N=0.15000 E+00$ $X_{N}=0.1500 C L+00$ $X N=C .15000 E: 00$ $x N=0.150001:+00$

PRESSUFE $=0.14807 C+01 \quad X U=0.1250 U E+00 \quad X N=0.10000 L+00$
$X B=0.12500 E+00 \quad X N=0.10000 E+00$
$X N=0.10000 E+00$
$x N=0.10000 E+00$
$x A=0.10000 E+00$
XN $=0.50000 E-01$
$X N=0.50000 E-01$
XN $=$ C.5000cE-01
$\begin{array}{cc}\text { SOD } & =0.13333 E+02 \quad X F=0.25000 E-01 \\ 0.025 & \text { PRESSURE }=0.11976 E+02\end{array} \quad X B=0.75000 E-01 \quad X A=0.5000 c E-01$

 $15=136 \quad 1 /$ ME $=0.17000 E 600$

$2 \mathrm{NH}=2$




$0.25000 \mathrm{E}-01 \quad \times N=-0.37253 \mathrm{E}-07$
11
4
4
$S D B=-0.10974 E+02 \quad \mathrm{NF}$
-
7 $\quad$ M
13


SUF $=-0.13766 E+02$

## SUF 0 $0.80000 E+01$ $0.86300 E+00$



$x[$

0.025 PRESSURE $=0.0$

$S O D=-0.100$ OUE $+02 \quad$ XF $=0.25000 E-01 \quad x L=0.25000 E-C 1$
$0.17625 E+00$
2

SD=-0. $10000 E+02 \quad x F=0.25 U J U E-01 \quad X B=0.25000 E-01 \quad X N=-0.37253 E-07$

SUD $=-0.10000 \mathrm{E}+02 \quad \mathrm{XF}=0.25000 \mathrm{~L}-\mathrm{OL} \quad \mathrm{X}_{1}=0.25000 \mathrm{E}-01 \quad \mathrm{XN}=-0.37253 \mathrm{~F}-01$

 $15=$
$S U=-0.1370 C E+02$
KF $=$



IJ= $140 \quad$ T1ME $=0.18250 E \& 00$
$2 \mathrm{HH}=2$


$\begin{array}{rl}1 & 0.7 E 377 E+00 \\ 6 & 0.74 B 41 E+00 \\ 11 & 0.72158 E+00 \\ 16 & 0.65372 E+00 \\ 21 & 0.57150 E+00 \\ 26 & 0.47973 E+00 \\ 31 & 0.34232 E+00 \\ 36 & 0.28319 E+00 \\ 41 & 0.1 E 644 E * 00\end{array}$
 $0.18875 E+00$
2
  $0.70400 E+00$
$0 . E 714 S E+00$ $0 . E 714 S E+00$
$0.616 H J F+00$
$0.54574 E+00$ $0.54574 E+00$
$0.46323 E+00$
$0.373120 F 400$ 0
0
0

$\frac{11}{2}$

$0.24000 E-01 \quad \mathrm{XN}=-0.372 \mathrm{E} 3 \mathrm{E}-07$

0. 250 U0E-01
0
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$n$
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0

## $0.25000 E-01$

$X N=-0.37253 E-07$

$\downarrow$


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## $\varepsilon ゅ \tau$



It $=106$ tIME＝ 0.2325 SuEt0u

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$$
\begin{aligned}
\text { SDD }= & -0.10000 E \bullet 02 \quad x F=0.25000 E-01 \\
& 0.025 \quad \text { PRESSURE }=0.0
\end{aligned}
$$ $X N=-0.37253 E-07$

$$
\begin{aligned}
& \text { NNNミNNNNMN } \\
& \begin{array}{cc}
2 & 0.56574 E * 00 \\
7 & 0.54269 E * 00 \\
12 & 0.51251 E+00 \\
17 & 0.46700 E * 00 \\
22 & 0.40907 E+00 \\
27 & 0.34221 E+00 \\
32 & 0.27015 E * 00 \\
37 & 0.19663 E+00
\end{array}
\end{aligned}
$$

$\begin{array}{ll}33 & 0.2554 E E * 00 \\ 38 & 0.18207 E * 00\end{array}$

0.53109 E 4
0.53109 L
－9：
$90 \%$ or
$=11 \times 10-300052$
 ．25000E－01 $\quad$ N $N=-0.37253 E-07$ 0.0
-2500 $\begin{array}{ll}0.25000 E-01 & X N=-0.37253 E-07 \\ 0.25000 E-01 & X N=-0.37253 E-07 \\ 0.25000 E-01 & X N=-0.372 E 3 E-07\end{array}$ ．25000E－CI $\quad$（NO－C． 372 E 3E－07
$0.23875 E+00$
－

II＝ 191 IIME＝
－ $3292 \mathrm{Et00}$ $0.53292 E+00$
$0.51722 E+00$

$0.43907 E+00$
$0.36153 E+00$
0
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$00+3 F 1942 \cdot 0$

$N F=$
－vNM $S D F=-0.13766 E+02$
$2 \mathrm{NH}=$

$$
\begin{array}{rl}
2 & 0.53376 E+00 \\
7 & 0.52170 E+00 \\
12 & 0.45287 E+00 \\
17 & 0.44932 E+00 \\
22 & 0.39379 E+00 \\
27 & 0.3395 E E+00 \\
32 & 0.26028 E+00 \\
37 & 0.18949 E+00
\end{array}
$$

 SD $=-0.13766 E+02$
$1 S=0 \quad 0 \quad$ $\begin{array}{cc}1 S= & X F= \\ S D=-0.13700 E+02\end{array}$


 $S D=-0.83766 E+02$
IS $=0 \quad 0 \quad \mathrm{XF}=$

$\begin{array}{ll}1 S=196 & \mathrm{KF}= \\ I \mathrm{I}=1 \mathrm{ME}=\end{array}$

4500E＋00

2
$\mathbf{N H}=$

## $0.5122 E E+00$

 $0.5122 E E+00$$0.437 J E E * 00$
$0.46 C 7 E E 00$

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33 SDF $=-0.13766 E+02$
 $S D D=-0.10000 E+02 \quad x F=0.2 E C C C E-C i \quad x H=0.25000 E-01$
$\times N=-0.37 \check{5} 3 E-07$
$\times A=-0.37253 \mathrm{E}-07$
$x n=-0.37253 E-07$

$$
\text { Scg }=-0 \cdot 10974 E+02 \quad \mathrm{NF}:=
$$

$\rightarrow 0 \pm 0$ が心が
$\qquad$ 02
$\times F=$
02 $\begin{aligned} & S U=-0.1376 E E+02 \\ & 1 S= 0\end{aligned}$ KF $\begin{array}{cc}1 & 0.5052 Y E+00 \\ 11 & 0.4 E 07 E E+00 \\ 16 & 0.44171 E+00 \\ 21 & 0.350526 .+00 \\ 26 & 0.33025 L+00 \\ 38 & 0.26431 E+00 \\ 36 & 0.19018 E+00 \\ 41 & 0.12924 E+00 \\ S U=-0.1376 E E+02 \\ 1 S= & 0\end{array}$
$0.5082 A E+00$
$0.486 G 1 E+00$
$0.450 \leq R E+00$
$0.40159 F+00$
$0.342 R 7 t+00$
$0.27779 F+00$
$0.209 A 2 F+00$
$0.1423 E F+00$

## ヘOtoncon <br> 7


$5 t+00$
$7 t+00$
$7 t+00$

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[^0]:    Figure 5 . Temperature profiles at various times during the heating process. Time increases from lower right to upper left.

