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UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

**EMERGENT MATHEMATICS CURRICULUM:
A CASE STUDY OF TWO TEACHERS**

A Dissertation

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

Doctor of Philosophy

By

STACY REEDER

Norman, OK

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**EMERGENT MATHEMATICS CURRICULUM:
A CASE STUDY OF TWO TEACHERS**

**A Dissertation APPROVED FOR THE
DEPARTMENT OF INSTRUCTIONAL LEADERSHIP AND ACADEMIC
CURRICULUM**

BY

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ABSTRACT

The purpose of this case study was to explore the pedagogic practices of two purposively selected teachers and their implementation of an emergent mathematics curriculum. The study aimed to provide insight into what pedagogic practices these teachers include in their implementation of an emergent mathematics curriculum and to understand how these teachers not only envisioned curriculum alternatives and enacted something different in their classrooms but also how they addressed the many constraints and limitations often facing teachers in their attempts to create something different with and for their students. This case study, focusing on an elementary teacher and a middle school mathematics teacher, sheds light on the envisioning of curriculum alternatives for mathematics education amidst the many constraints of current and traditional schooling practices. The analysis of both these teachers' stories reveals that the role of mentoring and mutual inquiry is key in their transformation. Support in the form of continual and ongoing collaboration and reflection helped to create a process of evolution in both these teachers' pedagogic practices and orientations about teaching and learning. Both these teachers' orientations about teaching and learning have evolved from a focus on what teaching practices they might adopt or problems they might choose to involve their students into a focus entirely on student learning. Rather than considering what they might give to the students or present to the students they are participant learners with and along side their students.

CHAPTER I

INTRODUCTION

There are those who see no alternative between forcing the child from without, or leaving him entirely alone. Seeing no alternative, some choose one mode, some another. Both fall into the same fundamental error. Both fail to see that development is a definite process, having its own law which can be fulfilled only when adequate and normal conditions are provided (Dewey, 1902, p. 195).

Envisioning alternatives for education is a difficult and complex task often met with protest from outside forces and influences and sometimes even stifled by our own inability to see beyond the current and traditional ideas about education. Imagining alternatives different than those described by Dewey above ones in which we create and develop those conditions optimal for student learning, calls on us to question our own educational experiences and to consider the relationship between curriculum, learning, and development. Our ability to critically question current education practices, constraints and limitations, and our own ideas about curriculum, learning, and development are inextricably linked with a desire for change. While recognizing a need for change emerges from our ability to question not only the views and ideas of others but our own as well, a desire for change and transformation creates in turn a challenge to understand and address those things that constrain and limit alternatives in education.

Fleener (2002) suggests that in “exploring the possibilities of and creating curriculum futures, we must address our own boogie men; those ideas, practices, and goals that have constrained our ability to change, adapt or create a new reality for schooling” (p. 12). There are several important questions that arise regarding the

constraints and limitations we face in education: Are these 'boogie men' the same for all educators across all school districts and classrooms or do they vary dependent on some situations? Are some of those things that constrain and limit our ability to create alternatives in education more readily identifiable and universally imposed from the outside as opposed to those less easily identified and named, those that may be inherent to our own beliefs, values, and ideas and implicitly accepted in societal institutions? Of those that are more readily identifiable, and perhaps even consistent across a variety of teaching situations and scenarios, how do some educators challenge those constraints and limitations, finding ways to address them and create alternatives for education and curriculum? Among those that are more subtle and tacitly accepted, what can be done to challenge thinking, foster questioning, and create a sense of freedom to envision change?

Understanding the relationship between the child, the curriculum, society, and the teacher is crucial if we want to create a new reality for schooling and curriculum alternatives. Dewey places the child, the curriculum, and society in dialectic with one another. The three are in a constant and continual tug and pull, back and forth between needs, expectations and often ideas about the needs of the other. The teacher is at the center of this interactive system in the sense that the teacher is in dialectic with all three. Teachers are in a constant state of reciprocal interaction between the child, the curriculum, and society, continually trying to establish and maintain balance between the needs and expectations of all three, recognizing there exist constraints and limitations in all three directions. With the new emphasis on holism that postmodernism supports, a new sense of educational order and systemic relations can emerge, helping us to better understand and examine the recursive interaction between teachers and learning and

teachers and students, the curriculum, and society. Questioning that leads to a better understanding of these dialectic relationships can culminate in our ability to create new realities for curriculum and schooling.

Focusing on the dialectic relationship among teachers and students, the curriculum, society, and learning, from a postmodern framework, we can begin to identify and understand the complexity of those things that challenge our ability to envision change. Critically questioning those constraints and limitations we face as educators in creating alternatives for curriculum and schooling will allow us to consider possibilities for change.

There are some teachers who have experienced transformation; they have envisioned curriculum and relationship alternatives and have implemented them in their classrooms despite the sometimes obvious but more often tacit challenges to transformation in education and the mathematics classroom.

PURPOSE OF STUDY

To begin to understand how teachers can create curriculum alternatives in their classrooms with and for their students amidst the many constraints of schooling, I focused the current study on two teachers and their reflective accounts of how their pedagogic practices have evolved and how they have developed as teachers; their “stories.” The purpose of this case study was to explore the pedagogic practices of two purposively selected teachers and their implementation of an emergent mathematics curriculum. The study aimed to provide insight into what pedagogic practices these teachers include in their implementation of an emergent mathematics curriculum and to

understand how these teachers not only envisioned curriculum alternatives and achieved something different in their classrooms but also how they addressed the many constraints and limitations often facing teachers as they attempt to create something different with and for their students. By providing descriptive detail of the teachers' reflective accounts of the factors they believe to have effected their development as constructivist teachers, I hoped to provide information that might assist other teachers as they struggle to envision transformation in their classrooms and work for education reform. This study seeks to aid others in comprehending and addressing the challenges inherent in envisioning possibilities for and enacting an emergent curriculum in mathematics. An emergent mathematics curriculum will be seen to evolve from constructivist teaching and learning practices and understandings. The broader goal of this study is to shed light on the envisioning of curriculum alternatives for mathematics education amidst the constraints of current schooling practices. Case study research was conducted with two purposively selected teachers who have successfully enacted alternative approaches to the teaching and learning of mathematics in their classrooms.

GUIDING QUESTIONS

The study seeks to address the following questions:

1. What pedagogic practices do these teachers include in their implementation of an emergent mathematics curriculum?
2. What factors do these teachers consider to have influenced their commitment to an emergent mathematics curriculum?

3. **What do these teachers identify as constraining and limiting their options for creating an emergent mathematics curriculum? How have they addressed those limits and constraints?**

RATIONALE

In working with preservice and in-service teachers it is very difficult to talk about the complexities of classroom interaction without reducing them to a set of steps or a plan all teachers can follow to make this or that happen. It is equally challenging to share with others from our own experience and from the literature the profound impact that creating an environment within mathematics classrooms where students are truly engaged in meaningful problem solving will have on student understanding and learning without questions arising to the effect of “how can this be done” and “how can we do these kinds of things if our administrators or students’ parents expect us to do something else.” Teachers often make comments suggesting that it is difficult to think about mathematics teaching and learning in a different way because all their own personal experiences as mathematics learners have been primarily one specific way; the teacher at the overhead demonstrating a procedure to do the problems, worksheets or problem sets from textbooks, and a great deal of memorization. Additionally, preservice teachers are often very concerned with the many constraints, including curricular materials, parents, their colleagues, and administrators they believe they will encounter while they teach mathematics in particular.

Several semesters ago I taught a course entitled Problem Centered Learning, N-8 for the first time and remember clearly a student saying to me, and to the class, after we

had spent a considerable amount of time discussing a chapter by Constance Kamii from her book *Young Children Continue to Reinvent Arithmetic* and how students learn mathematics, that she already knew that she would be taught a lot of this “stuff” in college about “new” hands-on ways of teaching and learning. She boldly proclaimed “my sister went through this program a few years ago and now she is an elementary teacher, and students might learn better this way, but the reality is you can’t do any of this stuff – not in the real world.” I cannot recall my response but I certainly remember that I was struck by how challenging reform and change is and will always be in education, particularly mathematics education. We not only face outside constraints but the challenge of addressing our own beliefs about teaching and learning and the ability to truly believe something different is possible and right.

The National Council of Teachers of Mathematics (NCTM) was established in 1920 with a desire and purpose to help foster reform in mathematics education. Since its inception there have been constant and continual calls for reform and change in school mathematics. From the NCTM’s birth out of the adversity of declining high school enrollment and pressure to decrease the amount of time spent on mathematics in the elementary classroom in the 20’s to their stance against the so called ‘back-to-basic’ movements of the 70’s, and their current stand against the “mathematically correct” movement of the 90’s the discussion of issues in mathematics education has “ebbed and flowed” (Kilpatrick & Stanic, 1995).

Still working for reform in school mathematics in the 80’s, “problem solving” was brought to the forefront as a major focus by the NCTM with their statement that “problem solving be the focus of school mathematics in the 1980’s” (NCTM, 1980, p. 1).

In their *Curriculum and Evaluation Standards for School Mathematics* (1989), the NCTM emphasized problem solving as their first standard at every grade level.

“Becoming a mathematical problem solver” is also emphasized as one of the NCTM’s five general goals for all students. They state

the development of each student’s ability to solve problems is essential if he or she is to be a productive citizen...To develop such abilities, students need to work on problems that may take hours, days, and even weeks to solve. Although some may be relatively simple exercises to be accomplished independently, others should involve small groups or an entire class working cooperatively. Some problems also should be open-ended with no right answer, and others need to be formulated. (1989, p. 6)

The notion of students being “mathematically empowered” was also introduced by the NCTM in the late 80’s and is dependant upon the kinds of experiences students have in class with mathematics and with mathematical problem solving.

Mathematical power includes the ability to explore, conjecture, and reason logically; to solve nonroutine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions. Student’s flexibility, perseverance, interest, curiosity, and inventiveness also affect the realization of mathematical power. (NCTM, 1991, p.

1)

The development of mathematical power for all students, central to the NCTM's *Curriculum and Evaluation Standards*, marks a shift toward a more balanced approach to mathematics teaching; one that emphasizes student problem solving and student mathematical understanding.

More recently in the *Principal and Standards for School Mathematics* published in 2000, in a continued effort to promote reform for school mathematics, the NCTM states a vision for school mathematics asking teachers of mathematics to “imagine a classroom, a school, or a school district where all students have access to high-quality, engaging instruction” (p. 3). Further describing, the NCTM continues:

There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding... They [students] draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with skilled guidance of their teachers. Orally and in writing, students

communicate their ideas and results effectively. They value mathematics and engage actively in learning it. (2000, p. 3)

This vision for school mathematics is by the NCTM's admission challenging and "highly ambitious" requiring "solid mathematics curricula, competent and knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms with ready access to technology, and a commitment to both equity and excellence" (2000, p. 3).

Constituting the vision for school mathematics described by the NCTM is their *Principles and Standards for School Mathematics* (2000). Stopping short of prescribing a particular "recipe" for effective mathematics teaching, the NCTM support six *Principles* that address overarching themes: *Equity, Curriculum, Teaching, Learning, Assessment, and Technology* and ten *Standards* – *Content* and *Process* represent strands that should run through all grade levels. Together, the Content Standards: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis, and Probability, describe explicitly the content that students should learn, and the Process Standards: Problem Solving, Reasoning and Proof, Communication, Connections, and Representation, which discuss ways of using and developing the content knowledge, provide a "connected body of mathematical understanding and competencies – a comprehensive foundation recommended for all students, rather than a menu from which to make curricular choices" (p. 29).

In an effort to help teachers envision new realities and understandings for school mathematics, the NCTM discusses their six principles separately while emphasizing that their power "derives from their interaction in the thinking of the educator," that while

each is important and not unique to mathematics education per se they are inextricably intertwined. The NCTM, describing the Equity principle, states that “excellence in mathematics education requires equity – high expectations and strong support for all students” (p. 12). The notion of equity implies explicitly that “mathematics can and must be learned by all students”, a belief that runs in contrast to pervasive societal beliefs, particularly in North America, that mathematics is only for a few; “that only some students are capable of learning mathematics” (p.12 – 13). The Curriculum principal defines curriculum as “more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades” (p.14). The NCTM’s stance that a school mathematics curriculum determines greatly what mathematics students have the opportunity to learn conjoined with their belief that what mathematics students learn is inextricably tied to how they learn them, further intertwines the Curriculum and Teaching principals.

Recognizing the complexity of teaching mathematics well, the NCTM states “there are no easy recipes” and that there is “no right way” to teach mathematics. However, founded in the belief that there are certainly more effective ways to teach mathematics, the NCTM does address in its *Professional Standards for Teaching Mathematics* (1991) six standards for the teaching of mathematics: Worthwhile mathematical tasks; the teacher’s role in discourse; the student’s role in discourse; tools for enhancing discourse; the learning environment; and the analysis of teaching and learning. (p.17). These six standards are supported as the main issues or areas that effective teachers need to address as they make curricular decisions based on reflection

and in their continual efforts to improve, working to understand mathematics, students as learners, and pedagogical practices.

Learning with understanding is described as “essential” in the Learning principal. “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, 2000, p. 20). Citing the work of Bransford, Brown, and Cocking (1999), who conducted research regarding students’ conceptual understanding, the NCTM states that conceptual understanding is an important component of proficiency, along with procedural facility and factual knowledge. Focusing on learning with understanding for all students and emphasizing the relationship between mathematics curriculum and teaching, the NCTM has supported reform in school mathematics. Reform that requires a new vision of school mathematics, one in which mathematics educators must consider what mathematics is and what it means to know mathematics.

Despite the longevity and purpose of the NCTM’s efforts, change has seemingly not been affected for school mathematics in this country. Kilpatrick and Stanic (1995) state that

reform may be too strong a word to characterize developments in mathematics education at any point over the past century. Professional and public discussion of issues in mathematics education ebbs and flows. School mathematics continually changes, but it has yet to achieve a form substantially different from that being established in the closing years of the last century. (p.14)

This criticism is supported by the results of the Third International Mathematics and Science Study (TIMSS) that suggested students of mathematics in the U.S. were

achieving at levels far below the international average (Smith, 1999). Schmidt, McKnight, and Raizen (1996), in their curriculum studies portion of the TIMSS data, called *Splintered Vision*, that also criticizes school mathematics in the U.S., state that the mathematics curricula in the U.S. has been described as being a “mile wide and an inch deep” and further claim that the teaching methods of U.S. teachers are superficial. Based on both the curriculum analyses and video studies of instruction in three countries collected as part of TIMMS, Schmidt et al. (1996) claims that U.S. teachers, typically relying on a textbook driven curriculum, give “little sustained attention to any one aspect of a content area” (North Central Regional Educational Laboratory, 1997, p. 1). Rather than allowing time for students to work in groups and engage in problem solving, U.S. teachers tend to lecture and that the most frequent activities in mathematics classes still remain teacher presentation and demonstration of specific procedures followed by students working individually on worksheets and problem sets from the textbook.

Kilpatrick and Stanic (1995) suggest that despite the NCTM’s efforts for change and improvement in the U.S. for school mathematics, “true reform” has not been realized and that “true reform ... may require not doing something better but something different” (p.15). In order to envision and develop curriculum alternatives, ideas about curriculum that are qualitatively and substantially different, we may be required to critically question and challenge our traditional ideas about what is mathematics and what it means to know mathematics.

Since the NCTM contends that what mathematics students learn is fundamentally tied to how they are taught it is important to examine what factors impact the way teachers choose to teach. Is it primarily teacher beliefs about the nature of mathematics,

their beliefs about how students learn mathematics, or what it means to know mathematics? Are teachers constrained by outside forces such as administrators, parents, their colleagues, or the resources that are available to them? Recognizing the relationship between how teachers believe students learn, teacher's ideas about the nature of mathematics, social and political constraints that influence instructional decisions, and how teachers teach mathematics may be key in bringing about "transformative change"; change that goes beyond the back and forth cycles described by Kilpatrick and Stanic (1995) above. Unless we change the way we think about what it means for students to know mathematics our attempts to consider alternatives for curriculum and pedagogy for school mathematics may continue to be an effort to do what we have always done rather than something qualitatively different.

It is very difficult to discuss a different vision for school mathematics with preservice teachers or teachers at varying points in their professional development. Preservice teachers come to methods and curriculum courses with ideas about how and what should be taught, ideas about what constraints will face them in their future classrooms, their own fears and frustrations or feelings of disempowerment with mathematics, and many years of experiences with mathematics teaching and learning that are traditional in nature, contrary to the vision for school mathematics described by NCTM. Likewise, in-service teachers often have very salient beliefs about how and what mathematics should be taught and reject change despite their feeling that their students do not seem to be learning what it is they have taught. The challenges that face reform for mathematics education are evidenced by the lack of change in school mathematics over the last century. There is a need to hear from teachers who have envisioned and enacted

curriculum alternatives.

In the following chapter, I will provide an overview of literature that discusses student learning and current research on various aspects of mathematics teaching and curriculum.

CHAPTER II

RELATED LITERATURE

Due in part to the continuing and ongoing calls for reform in school mathematics, research in the area of mathematics education has focused on a variety of aspects of mathematics teaching and learning. The following review of literature will provide an overview of research that addresses the relationship between teachers' beliefs about how students learn and their pedagogic practices. Additionally a constructivist perspective of what is learning and knowledge, and problem centered learning will be discussed.

Teacher Beliefs

Teacher preparation programs often focus on theories of teaching and learning, curricular issues, and working with preservice teachers to develop pedagogical knowledge. Many times each content area has a course or two of its own where content and content specific pedagogical knowledge can be developed and better understood (Shulman, 1986). Regardless of how these courses are taught and what preservice teachers take with them from their "methods" courses, their content specific knowledge and pedagogical knowledge when translated into practice, is filtered through their beliefs about the nature of learning and the nature of the specific content area. Swafford (1995) suggests that

[t]eachers' content and pedagogical knowledge and their knowledge of students play crucial roles in determining what goes on in the mathematics classroom. But knowledge alone is not the determining factor. Knowledge is colored by beliefs.

(p. 164)

Many researchers have argued that it is these teacher held beliefs about how children learn and what it means to know mathematics that have a strong impact on the choice of actions and behaviors related to their instructional approaches in mathematics and the type of environment they foster in their classroom (Enochs, Smith, & Huinker, 2000; Ernest, 1989; Foss & Kleinsasser, 1997; Raymond, 1997; Smith, 1996). Although research has shown a consistent relationship between teachers' instructional practices and their beliefs about the nature of mathematics (Thompson, 1984) and also that a relationship exists between teachers' beliefs about teaching and their beliefs about learning (Cobb, Wood, & Yackel, 1990), understanding the impact teachers' beliefs have on their pedagogic practices in the mathematics classroom is a complex and challenging endeavor (Ernest, 1989; Pajares, 1992).

Since, according to the NCTM (1989, 1991), what a student learns is dependent on how it is learned, student learning then, is affected or even possibly constrained by the pedagogic choices of the teacher, which are influenced by the teacher's beliefs. For example, Raymond (1997) suggests that if a teacher views mathematics as a collection of facts, procedures, and skills that are static and fixed his/her teaching practices are likely to reflect this viewpoint. Teacher's who hold these beliefs will be more likely to view the role of the learner as passive receiver and their role as teacher as one who demonstrates and lectures to transmit mathematical knowledge. The teacher acts as the holder of "right answers" wherein students are encouraged to memorize and practice certain skills for mastery. On the other hand, according to Raymond (1997), a teacher who believes differently about mathematics, that it is relationships, patterns, problem driven, or a way

of thinking and reasoning may also think very differently about the way in which his/her classroom will operate. This teacher may plan more flexible lessons for problem solving and mathematical discourse, placing more value on process than product. Students are viewed as active participants in this classroom, working together in collaborative groups wherein they validate and challenge one another's thinking. The students are encouraged to reflect on their own thinking since the teacher believes learning to be the active construction of knowledge as a learner interacts with his/her environment. This second example provided by Raymond (1997) is that of a teacher whose orientation or framework is constructivism. Operating from a constructivist orientation, the teacher views the learners as actively participating in the construction of their own knowledge as they interact within their environment rather than beings that passively receive information that is communicated to them.

A Constructivist Orientation

Constructivism is viewed by most as a radical departure from behaviorist notions about learning and knowledge. From a constructivist perspective, as elaborated on by Piaget, learners are in a constant state of flux and dynamic interaction with their environment, continually creating cognitive structures that will help them make sense of what they perceive. Knowledge, then, grows out of a learner's activity through a process of construction and reorganization as the learner makes adaptations to neutralize perturbations that arise through interactions with their world (Steffe, 1990). In this way knowledge is not independent from the learner but rather constructed by the learner through purposeful interactions with his/her environment. Piaget's ideas have been brought forward in the area of mathematics education by many including Ernst von

Glaserfeld, a follower of Piaget who coined the term “radical constructivism.”

Radical constructivism attempts to provide insights and explanations about the nature of knowledge claiming that humans actively create knowledge from interaction between their existing knowledge and new experiences and ideas they encounter that cause perturbation or challenge to their already existing knowledge. Radical constructivism, while not a teaching approach or model, does provide a theory about how learners come to know that is in stark contrast to more positivist and behaviorist ways of thinking about knowledge and the subsequent teaching approaches that emerge from those beliefs.

More traditional ways of thinking about learning view knowledge as a set of “objective truths” that exist outside the knower. Traditional models of teaching and pedagogic practices that reflect this belief about learning and knowing operate from a perspective that it is the job of teachers to convey knowledge directly to students. Every subject matter has its own set of these “objective truths” that must be conveyed to students with the belief that knowledge only increases with the acquiring of these “truths”. Freire (1970) claims that traditional models of education based on a belief that knowledge is a set of truths outside the knower have become

... an art of depositing, in which the students are the depositories and the teacher is the depositor. Instead of communicating, the teacher issues communiqués and makes deposits which the students patiently receive, memorize, and repeat. This is the “banking” concept of education, in which the scope of action allowed to the students extends only as far as receiving, filing, and storing the deposits. (p. 58)

Rejecting the notion that knowledge is independent of the knower and is something that

can be transmitted from one person to another, radical constructivism, according to Airasian and Walsh (1997), is based on the view that

... knowledge is produced by the knower from existing beliefs and experiences. All knowledge is constructed and consists of what individuals create and express. Since individuals make their own meaning from their beliefs and experiences, all knowledge is tentative, subjective, and personal. Knowledge is viewed not as a set of universal "truths," but as a set of "working hypotheses." (p. 445)

Von Glassersfeld also states "knowledge cannot simply be transmitted ready-made ... from the teacher to the student but has to be actively built up by each learner in his or her own mind (1989, p.1). Further elaborating, in his book *Radical Constructivism*, von Glasersfeld states:

Radical constructivism starts from the assumption that knowledge, no matter how it is defined, is in the heads of the person ... What we make of experience constitutes the only world we consciously live in... But all kinds of experiences are essentially subjective, and though I may find reasons to believe that my experience may not be unlike yours, I have no way of knowing that it is the same. (1995, p. 1)

Constructivist approaches, contrasting with other theories of learning where knowledge is considered predetermined and independent of the mind, view learning as a very personal matter. "Learning is accomplished by constructing and elaborating schemes based on experiences" (Wheatley, 1991, p. 12). This process of constructing, reconstructing, throwing out of old schemes based on new experiences occurs for each learner.

At one time we believe we have something figured out. But if we are reflective

and inquiring, it is likely that we will encounter events which call into question our conceptualizations and we will be forced to reorganize our ideas. This reorganization may require throwing out much of what we have constructed and reconstructing our schemes of knowledge. (Wheatley, 1991, p. 12)

This view of knowledge and learning supports a need for educators to foster an environment within their classroom wherein students are continually working to reconcile their existing knowledge with new experiences rather than teachers attempting to convey “bits” of information or “objective truths” to students.

While learning occurs, from a constructivist perspective, as individuals interact with their environment and learning is seen as a personal matter, it cannot be ignored that a major aspect of the learner’s interaction within her environment is social. Wheatley (1991) believes that “because constructivists state that each person constructs knowledge for him/herself and, in fact, construct their own reality, they have been accused of ignoring the role of social interaction in learning” (p. 13). Expanding the notion of learning slightly from radical constructivism, research on role of social interactions in student learning suggests that individuals “coconstruct” knowledge (Bauersfeld, 1988; Wheatley, 1991; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). Bauersfeld (1988) states that “learning is characterized by the subjective reconstruction of societal means and models through the negotiation of meaning in social interaction” (p. 39). Since knowledge is both individually and socially constructed and the learner cannot be viewed as acting independently of his/her social or cultural context, the meanings that learners make are unique to them. Thus, there is the potential for as many perspectives to be developed as there are learners, emphasizing again that learning is a personal matter and

that knowledge does not exist outside the mind of the individual.

Recognizing that learning is both an individual and a social activity, a classroom environment should include opportunities for students to talk and interact with one another in such a way as for meaning to be negotiated and a consensus reached. Having conducted research in mathematics education from a constructivist perspective on learning, Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, and Perlwitz (1991) state that “mathematics learning is an interactive, as well as an individual constructive activity” (p. 6). They continue by discussing the social nature of learning:

In the course of classroom social interactions, the teacher and students mutually construct taken-to-be-shared mathematical interpretations and understandings. This taken-to-be-shared mathematical knowledge, itself the product of prior interactive negotiations, both makes possible communication about mathematics and serves to constrain individual students’ mathematical activity. In other words, students, in the course of their individual cognitive development, actively participate in the classroom community’s negotiation and institutionalization of mathematical meanings and, practices. (p. 6)

Cobb, Yackel and Wood (1993) also describe mathematics learning as a constructive, problem solving process in which purposeful activity and communication are necessary. The learner’s development of sense making processes and mathematical reasoning cannot be separated from their interactive participation in an environment where mathematical meanings are negotiated and developed (Yackel & Cobb, 1996). When learning is believed by teachers to be a constructive process, by which the learner interacts with the environment individually and socially, that there are not pre-existing “truths” outside the

knower, what happens in the classroom should be different than traditional pedagogic practices. However, recognizing that the relationship between teachers' beliefs and their pedagogic practices may not be aligned, Swafford (1995) states that

... a teacher may believe in a problem-solving approach to teaching but find that the demands or expectations of the school to improve standardized test scores or to cover the district syllabus forces him or her to teach in a manner inconsistent with this belief. However, a teacher who believes that teaching is mostly telling is unlikely to forsake that belief for a problem-solving approach. (p. 165)

When mathematics learning is viewed as learner activity rather than an independent set of "truths" this leads to consideration of different pedagogical and educational practices.

Wheatley (1991) states:

Rather than identifying the set of skills to be gotten in children's heads, attention shifts to establishing learning environments conducive to children constructing their mathematics... in social setting. Since there is a social dimension of knowledge, this learning environment necessarily includes children talking mathematics... with each other. Such learning environments provide opportunities for children to share their ideas with peers, both in small groups and within the society of the classroom. (p. 12)

A problem centered learning approach has been suggested as an instructional approach that can support a constructivist orientation to teaching (Wheatley, 1991; Wheatley & Reynolds, 1999).

Problem Centered Learning

Problem centered learning is an instructional model that supports student construction of meaningful mathematics. This model involves three main elements: tasks, collaboration, and presentations and it is from these elements that the teacher's three roles emerge (Wheatley, 1991; Wheatley & Reynolds, 1999). The first aspect of the teacher's role in problem centered learning is to choose tasks that are potentially meaningful to all the students in the class, i.e., tasks that may be problematic for students. In a problem centered learning classroom, class typically begins with a task or investigation that is chosen by the teacher, or often by a student. The students, usually divided into small collaborative groups of two to three students of similar capabilities, are engaged in the mathematical investigation or problem solving task. While students are working together the teacher's role shifts to that of facilitating the interaction in the classroom by listening carefully to the groups while they work and to providing thoughtful questions for them to consider as they attempt to find solutions to the problem posed. The teacher also fosters the idea that collaboration in solving the problems is a goal. Finally, the teacher facilitates a whole class discussion wherein the main goal of the discussion is validation and justification of thinking. Students often share their methods for solving the problem with the class or present their ways of thinking about the task. The teacher remains neutral in these discussions, allowing for the class to "...serve as a community of validators" (Wheatley & Reynolds, 1999). The teacher makes every effort to remain nonjudgmental during this time of discussion and to encourage students to express their mathematical ideas with a goal that the class as a whole comes to consensus regarding the validity of solutions.

Worthwhile tasks. Within a problem centered learning environment the choosing or selecting of mathematical tasks or problems that are worthwhile is essential. For more than two decades problem solving has been a major concern in reform efforts and a focus of much research in mathematics education and beyond reform efforts, problem solving has always been associated with and a part of school mathematics in some form or another (Stanic & Kilpatrick, 1988). When the role of problem solving is considered in school mathematics it must be recognized that “chances are that any two people talking about mathematics problem solving are not talking about the same thing” (Wilson, Fernandez, & Hadaway, 1993, p. 57).

To many mathematically literate people, mathematics is synonymous with solving problems – doing word problems, creating patterns, interpreting figures, developing geometric constructions, proving theorems, and so forth. On the other hand, persons not enthralled with mathematics may describe any mathematics activity as problem solving (Wilson et al., 1993, p. 57).

The term problem solving means different things to different people and has been used in research to mean a variety of different things (Polya, 1957, 1980; Schoenfeld, 1992; Stanic & Kilpatrick, 1989, Henningsen & Stein, 1997; Fernandez, Hadaway, & Wilson, 1994; Wheatley, 1991; Wheatley & Reynolds, 1999). Students refer to the worksheets or sets of problems they are assigned from textbooks as problem solving. Likewise, teachers often refer to these practice sets as students solving a page of problems when in fact students are simply applying a procedure or algorithm previously demonstrated and provided by the teacher, taking cues from the teacher and the textbook as to what they should do. In many cases problem solving is set aside as a separate unit reserved only for

certain times or given to those students who are deemed to need the extra challenge.

Although there are many different definitions of problem solving, in a problem centered learning environment problem solving is more than the memorization of procedures to be applied to sets of rote problems or the few word problems at the end of each section in the textbook.

In a problem centered learning environment, according to Wheatley (1991) and Wheatley and Reynolds (1999), worthwhile mathematical tasks or problems should be ones that are both accessible to students, potentially meaningful to students, and should lead somewhere mathematically. Wheatley and Reynolds (1999) state that such non-routine problems, mathematical tasks and investigations should have these characteristics:

- Be potentially meaningful to students
- Be problem based
- Be replete with patterns
- Encourage students to make decisions
- Lead somewhere mathematically; and
- Promote discussion and communication (p. 29).

In a classroom where non-routine problem solving experiences are not viewed as a separate activity from “other” learning, where students are encouraged to be creative in their problem solving approaches and feel free to explore a variety of possible solutions students can construct their own mathematics (Cobb, Yackel, & Wood, 1993; von Glassersfeld, 1989; Wheatley, 1991; Wheatley & Reynolds, 1999; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990).

Collaboration and Discourse. In a problem centered learning environment students should be encouraged and allowed to work together. Establishing sociomathematical norms that support student collaboration and mathematical discourse is necessary (Cobb, 1995) in creating a classroom environment where students can become risk takers with their own inventiveness, creativity, and ideas in order to construct meaningful mathematics. Wheatley and Reynolds (1999) state:

If students interpret the environment as a recitation, then the way they act will reflect that interpretation and they will see themselves in an evaluative position. In contrast we can negotiate a classroom environment which is interpreted by students as a sense making place where their ideas are valued and listened to. When classrooms are seen as learning places rather than work places the dynamics can foster learning and risk taking. (p. 31)

The dynamic culture of the mathematics classroom has a major impact on the mathematics that is learned and the teacher plays an important role in establishing and helping to develop and negotiate a learning environment that supports his/her students becoming mathematically empowered (Cobb, Yackel, Merkel, Wheatley, 1988). Lappan and Briars (1995) emphasize that establishing social norms for mathematics classes, working to create an environment in which students can become mathematically empowered, is not an easy task. They state that when

...[t]he teacher works to set... expectations for both the work of students and the ways in which students are to engage in discussions with each other and the teacher and class together. Changing classroom norms is very hard. Students are resistant. They know what mathematics class should be like and even if they do

not enjoy mathematics, they will clamor for more directions, less thinking on their own, and for task completion to be the goal of the class. (p. 148)

In addition to establishing norms in the mathematics classroom that supports students working to solve problems in their own way rather than procedures being legislated by the teacher, students in a problem centered learning environment should collaborate, working together to solve problems.

Johnson and Johnson (1985) established that students can benefit greatly from working together while problem solving. While some models support the idea of “cooperative” learning groups where students work together in groups of four or five with each student having an assigned role perhaps, a problem centered model encourages students working together in small collaborative groups of two or three wherein the students are able to challenge one another’s thinking. Wheatley and Reynolds (1999) state that

[t]he evidence is strong that pairs should be formed of students who will challenge each other. It is the resolving of the perturbations resulting from disagreements that produces learning. Thus we should attempt to find pairings of individuals who will challenge each other’s thinking in their attempt to give meaning to their mathematical experiences. (p. 31)

Working in homogeneous pairs, students can participate equally while engaging in meaningful mathematics. Rather than being concerned with a particular assigned social role as part of the cooperative learning group (Cohen, 1996), students can communicate openly and be engaged in the negotiation of mathematical meanings. Together students can discuss their ideas and attempt to come to an understanding of both the task and the

mathematics involved in determining a solution to the task. As students work together to come to an agreement regarding the solution to the task they must think about each other's reasoning and work to resolve any conflicts that might exist between each other's ways of understanding. As a result of students interactively work together as collaborative pairs they can learn to rely on themselves rather than deferring to the teacher for answers to their questions. Thus the role of the teacher becomes that of listening, observing, and interacting with each pair as he/she considers questions to ask, subsequent tasks that could be introduced, and how to facilitate the whole-class discussion regarding the task that may follow.

Once all small groups have worked to determine their own mathematical solutions, the whole class may engage in a discussion with the teacher. In the whole-class discussion each group can share their solutions, explain their thinking and strategies for their particular approach to the problem or task, and justify those strategies to the whole class. During these "presentations" each student, including the teacher, listens to the strategies, reflectively comparing the approach and solution to his/her own approach. Students ask questions, comment, and discuss the strategies being presented. In this way, students are not encouraged to conform to a particular procedure (Wood, Cobb, & Yackel, 1993) but rather serve as an intellectual community of validators; the class, rather than the teacher alone, determines what solutions are valid (Wheatley & Reynolds, 1999).

While problem centered learning can be perceived, and it has been by many, particularly when translated into practice, as simply another technique or method of teaching, it can evolve as an orientation or a way of seeing and perceiving learning and classroom dynamics for some teachers. This study will focus on two such teachers. The

following chapter (Chapter III) will present the research design and methodology along with the selection of cases and the background setting for the study. Each participant's case and the telling of their "stories" will be presented in Chapters IV and V; the Case of Wesley will be presented in Chapter IV and the Case of Kathleen will be presented in Chapter V. In the final chapter the notion of problem centered learning as a way of seeing or perceiving the dynamics of teaching and learning will be elaborated on as the two cases are examined and analyzed as a whole.

CHAPTER III

RESEARCH DESIGN AND METHOD OF INVESTIGATION

In this chapter, the research design and methodology used in the study will be discussed. Elaboration on the purposeful selection of the two cases examined for this qualitative case study research will be provided as well as the interpretive and constant comparative method of data analysis employed. My role as researcher, data collection procedures, method of data analysis, and a description of each of the classrooms in which the study took place will be discussed in order to establish an understanding of the guidelines adopted for the study.

RESEARCH PERSPECTIVE

One of the difficult tasks for education researchers interested in exploring complex relationships between aspects of education that have traditionally been separated is discussing and examining these relationships in such a way as not to reduce them, thereby fragmenting the holism of their interactions and interdependencies. I believe that it can be easily said that the dynamics of curriculum, learning, and development and students, teachers, and classroom communities are like those of complex self-organizing systems; however, one of the challenges lies in whether it can be shown that curriculum, learning, and development are in relationship, a complex system displaying the properties of self-regulation and self-organization. How can we, for example, effectively capture the complex dynamics of a classroom, the interaction between students and students, and

teachers and students, without reducing them to a series of linear events? Can the interactions of small group problem solving be explored and then represented so as not to lose the synergistic interplay that occurred? These challenges, among many, have compelled some researchers to consider asking questions and to search for new ways to answer those questions from a postmodern perspective. The questions we now face are whether or not postmodernism provides us with more than a source for new metaphors for education and can we legitimately research from a postmodern perspective. Can chaos and complexity sciences as suggested by Doll (1993), offer a new way to view learning, social organizations and interactions, and inform research paradigms and practices?

Examining issues, in particular those of schooling, from a postmodern perspective is a difficult task. Order, control, grades, and a set curriculum are based on assumptions so endemic to modernist thought that it is difficult for us to consider these issues in a qualitatively different way. However, the emphasis on relations that postmodernism provides can aid in our understanding the complex integrated connections between and among those values and ideas that constrain and limit our ability to change. Postmodern thinking can provide new perspectives from which to consider alternative realities for schooling and a framework for envisioning curriculum futures. The exploration of relationships associated with the dynamics of a classroom such as the role of the teacher, the role of the students, interaction and discourse among the students with their peers and with their teacher, the classroom environment, the problems posed, and the establishing of sociomathematical norms within the context of two particular teachers' classrooms seemed most appropriately served by qualitative research methods rather than

quantitative methodologies. The use of qualitative methods of inquiry, in particular, narrative inquiry, in exploring the experiences of these teachers and their envisioning of change and their implementation of problem centered learning environments reflects my perspective as researcher in trying to capture and maintain in the telling of these teachers' stories the complexity I believe to be inherent in classroom interaction and development.

The present case study employed a narrative inquiry approach in exploring and investigating the two teachers' experiences as they envisioned and implemented curriculum alternatives in their classrooms. An interpretive and constant comparative model guided data collection and analysis. The characteristics of case study research, narrative inquiry, along with interpretive and constant comparison methods of data analysis will be discussed to establish their relevance in this study.

METHODOLOGY

The reasons for choosing one methodology over another are fundamentally connected to the nature of the subject or subjects to be studied along with the goals and purpose of the research. Since the purpose of this study includes understanding the development and experiences of two teachers and their current pedagogic practices from their own perspectives and exploring what factors they believe to have influenced their envisioning of curriculum alternatives, a narrative inquiry approach was taken to explore these two similar but separate cases. Schubert (1992) suggests that dialogue among teachers willing to share their stories is a powerful way to add to our knowledge of teaching and learning and a narrative inquiry approach for a study of this nature

recognizes the wholeness of experience as the two teachers selected for this research relate their stories.

Clandinin and Connelly (1999) state that narrative inquiry attempts to understand experience and that experience, taken as Dewey discusses it, “is both personal and social” (p. 2).

Both the personal and the social are always present. People are individuals and need to be understood as such, but they cannot be understood only as individuals. They are always in relation, always in a social context. The term *experience* helps us think through such matters as an individual child’s learning while also understanding that learning takes place with other children, with a teacher, in a classroom, in a community, and so on. (p. 2)

They continue by stating “education and educational studies are a form of experience,” so experience is what we study “and we study it narratively because narrative thinking is a key form of experience and a key way of writing and thinking about it” (p. 18).

Stake (1975) contends that case study research is concerned with discerning and pursuing understanding of issues intrinsic to the case itself and that “case study is not a methodological choice, but a choice of object to be studied” (Stake, 1995, p. 236). Studying particular cases allows the qualitative researcher to “... seek out both what is common and what is particular about the case...” (1995, p. 238). The study of a case allows the researcher to shed light on the particular phenomenon and to explore the complexities connecting ordinary practice in natural habitats to the abstractions and concerns of diverse academic disciplines” (1995, p. 239). Case study research, seeking to answer “how” or “why” questions, places the case itself as the main focus. A narrative

inquiry approach, considering the teacher's stories as part of the case, provides a way to capture the wholeness of the their experiences in their telling, as Clandinin and Connelly (1999) describe it as "filled with narrative fragments, enacted storied moments of time and space, and reflected upon and understood in terms of narrative unities and discontinuities" (p. 17).

Since qualitative research can be quite descriptive it is important to consider ways in which to organize the data and information so that people who have not had the opportunity to observe the phenomenon (in this case the classroom or the teacher's pedagogic practices) can make sense of it. An interpretive approach to organizing the data supports a "process of examining data closely in order to find constructs, themes, and patterns that can be used to describe and explain the phenomenon being studied" (Gall, Borg, & Gall, 1996, p. 562). This interpretational approach combined with a method of constant comparison of data can be used to guide both data collection and analysis of data throughout the study. A constant comparison method of data analysis is often associated with grounded theory approaches since the categories emerge from the data and are therefore "grounded" in the data rather than pre-determined prior to data collection. Gall et al., (1996) state that researchers using a constant comparison process of category development "derive their categories directly from their data rather than from theories developed by other researchers. In other words, the categories are 'grounded' in the particular set of data collected." (pp. 564-565). These categories and themes developed from the data are then used to describe and explain the phenomenon. Using an interpretive method for coding data, the researcher codes segments of the data recognizing that particular segments of the data can be coded in a variety of ways. Once

initial data are coded all other data is coded by a method of constantly and continually comparing data segments within and across categories that emerge. Gall et al., (1996) suggest that

... the researcher carefully examines and then re-examines all the data that have been collected. As this process continues, certain features of the phenomena are likely to become salient. The researcher should then develop an understanding of these features by themselves and in relation to each other. In other words, the analysis should account for as much as possible of the phenomenon being studied. An interpretation or criticism that fits some of the data should not be contradicted by other data. (p. 571)

SELECTION OF THE CASES

A particular case can be chosen because it is “thought to be instrumentally useful in furthering understanding of a particular problem, issue, concept, and so on” (Schwandt, 1997, p. 13). Two teachers, in particular from different school districts, one an elementary teacher and the other a middle school mathematics teacher, were purposively selected as the cases for study in this research. The middle school teacher, who will be called Wesley for this study, teaches seventh grade mathematics, pre-algebra and algebra I in a middle to upper income suburban public school in the Southwest region of the United States. The elementary grade teacher, who will be called Kathleen, teaches second grade in a lower to middle income suburban public school, in the same state as Wesley in the Southwest region of the United States. Both teachers are known

throughout their state as innovative teachers and have received some notoriety as a result of their innovative approaches and nontraditional teaching. While some may criticize and challenge their non-traditional approaches to teaching mathematics, which includes problem centered learning and not using textbooks regularly, many praise their classroom approaches to teaching as being among the best and most effective in their districts and state.

Having met Kathleen several years ago in a graduate course we took together entitled, Problem Centered Learning, I can recall thinking as I listened to her talk about experiences she had with her students, how I would love for my own children to have the opportunity to spend a year in her class. As my coursework in graduate school continued I did not have any other classes with Kathleen. I did however come to know her better and learn more about her and her classroom through other ongoing research projects in her classroom focusing on children's mathematical thinking and also through several acquaintances we had in common. As I learned more about the types of experiences that Kathleen tried to provide for her students and reflecting on her comments from our class together that developing a sense of community and the children's mathematical thinking were among the most important goals in her classroom, I found myself, curious about how Kathleen got to this point? My students in undergraduate mathematics methods and curriculum courses, when I shared with them the kinds of things that were happening in Kathleen's room with second grade students asked similar questions about Kathleen. What support did she have along the way? What experiences did she have that brought her to this enactment of something so seemingly different from a traditional classroom approach? They would often comment that she must be in a school that does things

differently or have a very unusual principal.

Likewise, I have pondered these questions about Wesley for many years. I first heard of Wesley during my first year as a classroom teacher in 1993. I began my teaching career as a junior high mathematics teacher, teaching eighth and ninth grade mathematics. As soon as I received the approval of my principal I enrolled my students in a Stock Market Game that was supported by grant monies at one of our state's regional universities. Our team met at lunch and greatly enjoyed the freedom they had in making choices about which stock and how many shares of each stock to choose. Without a classroom computer we relied on my husband's day old Wall Street Journals and local newspapers to follow and track our stocks for the duration of the eight week competition period. We were quite surprised that we had won the competition during the fall semester. Encouraged by our success, I shared with the students about another stock market game that a local newspaper hosted every year. The school supported us financially to enter this contest and just as we had done in the prior stock market game we followed our stocks for the period of the contest and waited for the final results to be published in the newspaper. In reviewing the results of the contest, we of course noted that we did not place, but that in the top ten groups about four of them were from the same school district with the same teacher as a sponsor. That sponsor was Wesley.

Over the next few years of my teaching career, I changed schools and continued to enter teams in the stock market game, bringing in speakers from banks to discuss with them strategies and what certain terms like "S & P 500" meant. Each year when the results were published the same thing occurred, several teams from this particular school with Wesley as the sponsor always placed in the top ten. Having taken notice of his

name, I had followed Wesley's career in the papers, as he won State Teacher of the Year and then a Presidential Award for Teaching, wondering along the way what it was about him and his teaching that was so different. Quotes from him in the newspapers indicated that he did not use the textbook as the primary source for his curriculum but rather used a problem solving approach with his students. In the midst of my own attempts to implement change in my classroom, and to develop an environment supportive of students' construction of meaningful mathematics, I felt the constraints of having fellow colleagues that supported more traditional approaches, parents, administrators, and the lack of computers and technology in my classroom that was so needed, and wondered if teachers like Wesley taught in schools without these same problems and limitations, imagining that the "grass is greener" in other places. In addition to my knowing who Wesley was from a distance, so to speak, I had the opportunity to meet him three years ago through the placing of a secondary education graduate student from our university in his classroom for his internship. It was through my association with this student and his university supervisor that I came to realize what kinds of things were happening in Wesley's room and the importance of hearing from teachers like him about how they have envisioned curriculum alternatives and implemented them despite the many constraints most teachers face.

I met with both Kathleen and Wesley and discussed the goals of my research and asked if they would be willing to share their stories and ideas about curriculum, the constraints they face, and mathematics learning. For several years now, both Kathleen and Wesley have been students of problem centered learning methodology and research and have worked fervently to enact this pedagogic approach in their classrooms. Each

have spent a tremendous amount of time thinking about, developing, and trying to understand how students learn mathematics, what it means to know mathematics and how their answers to these questions impact their pedagogic practices. Kathleen and Wesley were purposively selected for this study not because they have been touted as excellent teachers but rather because they are doing something different; they have not only envisioned curriculum alternatives but they have implemented them effectively in their classrooms.

DATA COLLECTION

My role as the researcher in the present study involved interviewing with the two selected teachers who had adopted and implemented an approach to mathematics curriculum grounded in problem centered learning (Wheatley, 1991; Wheatley & Reynolds, 1999) and constructivism (Ernest, 1996; von Glassersfeld, 1995) and observing and analyzing both of their classrooms and their pedagogic practices for several months during a school year.

Kathleen's Classroom

Research has been ongoing in Kathleen's classroom for six years. My involvement in her room was as part of a research team, whose members have been conducting research in her room for this extended period of time (see Cassel, 2002 and Geoghegan, 1998). This research team consisted of a university professor and graduate research assistants over the six years. I visited her classroom on a bi-weekly basis for one semester, observing her and her students during their mathematics lesson that

typically took place for one hour the same time each day. Other members of the research team visited her room more frequently, sometimes more than once per week. During each of my visits to her classroom, Kathleen would, following the mathematics lesson, use her planning or release time to discuss the mathematics lesson, her understanding of what her students made sense of and what they might still be grappling with, any interesting questions that arose, and what she wanted to involve the students in next. These informal discussions served as debriefing and brainstorming sessions between the teacher and those observing the class. All the sessions I observed and participated in for the current study were audio-taped and later transcribed to be included as data for this study. Handouts provided to the students, field notes, written observations, and video-taped recordings of classroom interactions and lessons that were later transcribed also formed the data for this study.

Additionally, I conducted three formal audio-taped interviews privately with Kathleen teacher to provide an opportunity for her to specifically tell her “story”; those experiences she believed to have influenced her development as a teacher. The interview guide for the first interview is provided in Appendix C. The first interview was transcribed and coded. It was through a process of coding each interview that the questions for the subsequent interviews emerged. The final two interviews focused on possible themes that had emerged from the data previously collected and coded and also on aspects of Kathleen’s story and experiences that I felt needed further elaboration in order for others to make sense of them. Any issues from the classroom observations that needed further elaboration or explanation also a part the later interviews as well. Following my first interview with Kathleen she also provided her teaching philosophy in

writing along with a reflective paper that she had written several years earlier about some of her experiences as a teacher. These documents became an important part of the data for this study.

Wesley's Classroom

The process of my data collection for Wesley's classroom and about Wesley involved my attending two of the six seventh grade classes he teaches each day. I observed one of Wesley's pre-algebra classes and his algebra I class and stayed for his planning period following the pre-algebra class each time to conduct interviews on a regular basis for six months of a school year. Field notes, written observations, video-tape recordings of the classroom interactions and lessons, audio-taped recordings of student small group interactions, and audio-taped teacher interviews formed the data from Wesley's classroom. In addition, handouts provided to students, problem solving packets given to the students in each class, in-class written assessments, the course syllabus, and other school documents were collected as data.

Each day, immediately following my observation of the two 55 minute class sessions, the teacher and I spent another hour during his release time in post observation reflective discourse, analyzing each lesson, student discourse and interaction that had occurred during the lessons, and examining related issues. The teacher's reflective account of how he developed as a teacher was also collected during this time prompted by interview questions (the guide for the first interview is provided in Appendix C) and my expressing a desire to hear his "story."

DATA ANALYSIS

The data collection for this study generated a great number of pages of transcripts from both the formal and informal interviews of both the teachers, field notes, transcripts of video-taped classroom interactions, and documents collected from the teachers for coding and analysis. Both the audio-taped interviews and the video-tapes of the classroom interactions were transcribed following each classroom visit. I carefully read the transcripts and coded them for key words, and ideas, looking for themes or categories to emerge. Since one of the goals of the study was the telling of the two teachers' stories, as I read and reread the interview transcripts I used my own judgment to determine if certain aspects of the teachers' experiences needed to be further elaborated so other people reading about them might have a an understanding of them. In this way the data pertaining to the teachers' stories were organized using an interpretational approach which is a "process of examining data closely in order to find constructs, themes, and patterns that can be used to describe and explain the phenomenon being studied" (Gall et al., 1996, p. 562). A reflective or interpretational approach to data analysis is described by Gall et al. (1996) as involving "a decision by the researcher to rely on intuition and personal judgment to analyze the data rather than on technical procedures involving an explicit category classification system" (p. 570).

The classroom observation data, including field notes and the transcripts of the classroom video-tapes, and teacher written documents were coded and considered from an interpretational approach as well, but a method of constant comparison also guided the analysis of this data. Data were coded and as certain patterns or themes became apparent,

new segments of data were continually and constantly compared within and across these themes and categories (Strauss & Corbin, 1998). This constant and continual comparison of data and emergence of themes guided the interviews with each of the teachers subsequent to the initial interviews conducted.

CLASSROOM SETTINGS

Kathleen's Classroom

For Kathleen's case I conducted my research in her second grade mathematics classroom. This elementary school was a low to middle income suburban public school and served as a site for the district's emotionally handicapped program. In Kathleen's class there were eighteen students, eight of whom were girls and ten were boys. The students were from various backgrounds and had varying abilities and disabilities.

The classroom was arranged so that students could easily work together, they had access to a variety of manipulatives, and the students were free to move around the room as they desired. The student desks were most commonly arranged in small clusters so they were touching side to side. Although the class might choose to rearrange their desks into a large U-shape or sometimes into groups of four, the desks were always arranged so the whiteboard at the front of the room was visible to all students. Additionally, Kathleen and her students specifically arranged the desks so as to leave space in the back of the room for students to sit on the floor in a circle during their whole class sharing time.

The problem centered mathematics lessons usually lasted one hour each day beginning with a short whole group activity, often times a calculator activity or an over-

head warm-up activity. This was typically followed with a brief explanation of the problem students were to consider that day; this explanation often took place in the back of the room with students sitting on the floor. The students would then work with their “math partners,” or often in larger groups on the problem or task for that day and then a whole-group discussion conducted at the back of the room ended each daily mathematics lesson. Kathleen encouraged students to solve problems and complete the tasks in ways that made sense to them rather than providing them with specific procedures for determining the solution (Wheatley, 1991; Wheatley & Reynolds, 1999).

Wesley's Classroom

To conduct research for Wesley's case I attended two of his seventh grade classes, one pre-algebra class and an algebra I class. This middle school was a middle to upper income suburban public school. The first class each day was the algebra I class which was comprised of twenty-four students, thirteen of whom were boys and eleven were girls. The second period class was the pre-algebra class which was comprised of twenty-seven students, fifteen of whom were girls and twelve were boys. The students in both these classes would not be considered widely diverse in their abilities since for this district students are placed in these classes based on their standardized test scores and previous grades. Additionally, the racial background of the students in both these classes would not be considered diverse with only two or three students in each class being non-Caucasian. It should also be noted that an Asian-American fifth grade girl is one of the students in the algebra I course.

The classroom was arranged with approximately thirty student desks divided into six rows that faced the front of the room where a white board was hung on the wall with

two overhead projectors and screens. Along one side of the room there were seven computers placed on built-in desks with seven accompanying chairs. On the other side of the room was the teacher's desk with a computer and storage, all of which were built in and attached to the wall. Along the back wall of the room there were windows that extended the full length of the room under which were storage cabinets without doors that held a variety of manipulatives and materials for student access and use. On the surface of the cabinets there were containers with a variety of mathematics games, challenge puzzles, and transparency sheets with pens for student use. A Texas Instrument graphing calculator was attached with Velcro to each of the student desks for the students to use at their discretion. The walls of the room were decorated with a variety of posters, some motivational in nature and others presenting mathematical ideas, and with the many newspaper articles and awards that Wesley had won. The tops of the cabinets were lined with the many trophies won in the stock market game.

Each of the class periods lasted fifty-five minutes and typically began with either a warm-up activity and discussion or some discussion regarding homework assignments from the previous night. Occasionally, the class period would begin with discussion about an assessment they had on the previous day. Regardless of the focus of the initial discussion in class, this time typically lasted about ten minutes until there was a shift of activity to the problem or task for that day. Following the opening discussion students were often given a choice of two or three tasks or problems that they could address during class. Students often voted on which problem they would consider first; this would also be the problem they would try to discuss prior to the end of class if they were ready. Many times the students would ask if they could continue to look at a problem

they had first looked at maybe the day before or in some cases maybe a week or longer before. There would be some discussion about this and most often Wesley would agree that they could use their class time to continue looking at that problem if most everyone agreed. During these problem solving sessions students worked with their “partner.” The seating arrangement changed often in both these classes, based on a rotation of students through the seats, so no student was always in the front or likewise in the back of the room. Students, very quickly would move their chairs to face their partner or move across the room to an open seat near their partner. Also during the problem solving sessions students would often discuss their thinking with members of other groups and move around the room freely, to and from the white board “showing” another student what they were thinking or to retrieve manipulatives or a transparency sheet and pen. There were other times, however when the desks remained in rows and Wesley presented the problem solving tasks at the overhead. During these times, students would often be using their calculators or manipulatives to explore certain ideas or problems. Also, even when the desks remained in rows with Wesley primarily in the front of the room, students were free to work with one another and to discuss with those around them their thinking and ideas about the problem. Most usually following problem solving where students had worked with their partners, but not always, class ended with student presentations of their solutions using the overhead and their transparency sheet. During this time the students presenting would discuss ways in which they thought about the problem that perhaps did not arrive them at a solution and then also ways in which they felt they had derived a solution for the problem. The students not presenting, those at their seats, would raise their hands and wait to be called on by those presenting to ask questions

about the solution being presented or to share perhaps a different way they had in looking at the problem. Often this time would result in only two or three groups presenting their ideas with several students going to the board to explain other ideas they had regarding the problem. There were many days that the “end of class” discussion did not occur at the end of class but rather the next day or perhaps even a week later but they always came back to each problem for a time of discussion. All the instructional activities emphasized student thinking, whether a warm-up activity, a problem solving task, or problem solving specifically with manipulatives, Wesley encouraged the students to complete the tasks in ways that made sense to them rather than providing them with specific procedures or algorithms for finding a solution (Wheatley, 1991; Wheatley & Reynolds, 1999).

In the following chapters I will focus on each teacher as an individual case highlighting their reflective accounts of their stories of vision and change along with the factors they believed to have influenced their desire and ability to envision and enact change. Additionally, woven throughout their stories are their ways of addressing and dealing with the constraints and limitations they feel have been an issue during their transformation as teachers and those that will continually remain when one is attempting to achieve something different with schooling. Chapter IV will focus on Wesley’s story and Chapter V will focus on Kathleen’s story.

In the final chapter I will discuss several significant themes that emerged when the pedagogic practices of both the teachers were examined as a whole. Likewise, themes that emerged as the issues of constraints and limitations arose in their stories of seeking change and ultimately their transformation as teachers will be discussed.

CHAPTER IV

THE CASE OF WESLEY

In this chapter the case of Wesley will be presented. His reflective account of his teaching experiences and the factors he believes to have affected his vision and desire for change will be discussed. The sections of this chapter will be presented largely in Wesley's words in an attempt to reflect his perception of what factors and experiences are important in understanding his transformation as a teacher, his pedagogical practices, and how he has addressed constraints and limitations in his envisioning and enacting an emergent mathematics curriculum. In the section focusing primarily on Wesley's current pedagogical practices, classroom scenarios, student interactions, and tasks presented in class obtained via audiotape and videotape of classroom sessions and field notes will be included. In this presentation and analysis of Wesley's pedagogic practices the classroom scenarios, student interactions, and tasks will be inserted only when it is believed necessary to capture a complete understanding of what happens in Wesley's classroom daily and over a period of time. Much of Wesley's Teacher Story is presented as a sequence of events, using dates to order and represent certain experiences, for example. This is done to capture Wesley's reflective account of how transformation took place in his life as a teacher, to show where he has been, has gone, and is hoping to go. This is not an attempt on my part or his to imply that any of these events and experiences stand alone or in isolation, occurring one after the other, creating change or the opportunity for change, but rather that in reflection these experiences seem to stand out as

some of the most significant. This chapter will specifically address the following focus questions of the study:

1. What pedagogic practices does this teacher include in his implementation of an emergent mathematics curriculum?
2. What factors does this teacher consider to have influenced his commitment to an emergent mathematics curriculum?
3. What does this teacher identify as constraining and limiting his options for creating an emergent mathematics curriculum? How has he addressed those limits and constraints?

WESLEY'S TEACHER STORY

Background

Wesley's story with mathematics education begins with the early interactions with and the influence of his family. Having been reared in a Southwestern state, Wesley lived in a small town where his father worked in the "oil patch" as a diesel mechanic. His father expressed that "he knew that the folks that made money were the chemical and petroleum engineers." Since Wesley was "strong in mathematics" in high school his father encouraged him to pursue an engineering degree. Wesley says of his father, "I think he thought, if he's (Wesley) gonna do anything and has got a petroleum engineering degree he can make some money instead of being greasy with his hands."

Wesley began his college career at a regional state university, with his father's advice and goals for him in mind, working toward a degree in engineering where his

strengths in mathematics could serve him well. His progress toward earning a degree was quickly interrupted by an illness that required him to withdraw from all his classes and return home for the semester. While at home, Wesley's college career was further delayed by a draft notice to join the U.S. forces in the Vietnam War. Following spending time in the National Guard, Wesley returned to college thinking at this point more about a degree in education. After much consideration and re-evaluation of what he wanted to do in life he decided to pursue a degree in health and physical education and later graduated with minors in mathematics and science.

Wesley's first teaching position was as a health and physical science teacher at a suburban elementary school with coaching responsibilities at one of the district's middle schools. He completed his fourth year teaching in that district when his wife's career relocated them to another school district in the same state. It was this relocation and a search for teaching positions that lead him back to mathematics, middle school mathematics in particular. Wesley comments on his move into middle school mathematics:

I didn't want to coach anymore and the only jobs open were math and people noticed that I had a math endorsement so I kind of got over here by accident... I didn't think I was going to teach middle school mathematics, I didn't even think I was going to teach but I think I've got a lot of gifts that lend themselves to teaching adolescents and I think that has really helped me because I have some gifts that I know about like patience that I think you've got to have... patience and a good sense of humor are gifts you've got to have to work with this age level, but it wasn't anything I had deliberately planned to go in and wanted to go

teach middle school math or even mathematics but it kind of worked out and I've been very pleased that I've been able to get into this area. [Interview data]

Wesley is now completing his twentieth year teaching middle school mathematics in the same district where he "accidentally" ended up.

Starting Out In Middle School Mathematics

Being new to the school district, Wesley, now a sixth and seventh grade mathematics teacher, did not have a classroom of his own; he was a "traveling" teacher moving to a different classroom to teach each period. While this was difficult with regards to organization and time it provided the opportunity to observe a variety of teachers and their differing styles of teaching and interacting with their students.

I was new to this district so the principal put me as a traveling teacher so I went from five different classrooms and taught 6th and 7th grade, five different rooms between basically two buildings ... I did see a teacher Jamie (another mathematics teacher), who I started out the day with. ... Jamie was very nurturing. There was one person who was very, very organized and very behaviorist inclined, just everything was control, I finished up the day with her class and started out the day with Jamie and you know I thought that's the one (Jamie) I want to try to model. [Interview data]

The middle school where Wesley began teaching in 1982 was operating in many ways as a junior high with strong departmental control in each of the content areas. Wesley recalls,

I think the thing if you go back to 1982, there was a strong departmental focus within the middle school here ... the department head called the department

together at a luncheon, ... the 7th grade teachers a week or two later sat down with their lesson plan books. We decided we need to do these lessons and we all got sort of lock-step and decided we need to do these lessons on a certain day and test on a certain day and decided there would be no pens, I'm talking about ink pens, you wouldn't allow it, I mean that was a zero. [Interview data]

He elaborates,

[e]verybody was just pretty much doing by the book. Jamie showed me where the teacher's edition was, what lessons to do, and kind of just come in and we'll do these lessons and she showed me that she did her grading on a 1/3, 2/3; 2/3 percentage on your tests and 1/3 on your homework. She was a little more lenient, some of those things have influenced me throughout because some of those people back then gave no extra credit, not extra credit, but they never gave kids a break. [Interview data]

Wesley describes his pedagogical methods as being teacher centered and a traditional type of approach during his first several years of teaching mathematics. "It was I'll explain or demonstrate from the overhead ... the students would do problem sets and then they would come in the next day, we would grade that." It was not until a change in building administrators in the late 1980's that Wesley was encouraged to try something that varied from his traditional approach.

Trying New Things

Around 1986 or 1987 the district where Wesley teaches experienced a change in superintendents. With a new superintendent whose focus was on "changing the direction of the district" and "innovation" several new building administrators were hired including

a new principal at the middle school. This new principal, Wesley recalls, “was talking about cooperative learning and said to me ‘when you get done with the book maybe you could go back and teach it without using the book, teach it with manipulatives, maybe some cooperative learning’.” Describing himself as a “risk taker” Wesley believes that he was approached and encouraged by administration to try some new teaching approaches because he was team leader and because he was willing and open to consider new things. “So I started messing around with looking for some stuff I could do, some stuff we could do with cooperative problem solving and letting kids talk.” Wesley continues, referring to urges by his principal to explore cooperative learning and the use of manipulatives with his students, “that kind of started this whole, my getting into this, my messing around with different things.” It was these beginning experiences, trying some new and different things, letting his students talk and him listening to them, that created a sense of wanting or searching for more in Wesley. During the last few years of the 1980’s, Wesley describes himself as “searching.” “My ideas about mathematics hadn’t changed necessarily at this point, the late 80’s era but I was searching.” It was during this time that Wesley began looking at and considering the newly released National Council of Teachers of Mathematics Standards that was published in 1989 and began talking with and working with elementary teachers, “trying to get some ideas from those folks.”

Transformation Begins

During the summer of 1991, the school district’s new superintendent, having been there only one year, brought in Allan, a university professors whose primary research

focus was children's mathematical thinking, for an elementary and middle school mathematics inservice workshop. Wesley recalls this workshop:

I think it was mandatory, anyway they bring in all middle school teachers to basically work with Allan one afternoon, maybe one day, in the summer and Allan went around and he had everybody answer the question "what do you value for your middle school mathematics." I can't remember exactly, but something like that. Anyway, he had us sitting in a circle and I was the last one so I sat there and took notes, writing down what I thought sounded good and had this answer all fabricated. "My goal for my students is to have them do the four basic operations, add, subtract, multiply, and divide with fractions, decimals, percents, and whole numbers and be very proficient at that." [Interview data]

Wesley also points out that something significant did happen that day. "What happened that day that was different is that Allan started me thinking about mathematics, and what it is, what is mathematics, and so that's been an ongoing thing." While believing that the questions asked of him that day impacted him greatly, Wesley does not feel that it was this in isolation that began a process of change in the way he now teaches mathematics.

I think I was impacted some by a middle school conference I attended with my principal. Some of the people there were talking about a learning pyramid that says something like you learn only 10% of what you read and then more if you have to teach it to somebody. I can remember wondering, "well what about if you interact with others to help each other." So I know that I was really beginning to question some things and beginning to see the benefits of students working together to learn mathematics at that time. [Interview data]

Wesley also recalls that as he has reflectively considered how and perhaps why he has changed so significantly as a teacher that there were many things that impacted him, influenced his thinking, or helped him become more open to other ways of teaching mathematics.

When I look back I have really asked those questions like “what’s going on with children” and “what are we doing to children with traditional approaches” but I don’t think I was really dissatisfied with what I was doing in my classroom, you know when my approach was more traditional, I just know that I was open to trying things and seemed to be searching for something and I started to explore things. However, I do remember working with a teacher when I was an intern in college and she was Teacher of the Year and a Presidential Award winner in Science. She did a lot of student group work and it is hard to know if those experiences have an impact on you because you discount them and some of those maybe stuck with me. She did a unit outdoors with her students, planted a garden, sowed seeds and all that, so you wonder if some of those things weren’t already planted that we just don’t give credit for. I can remember thinking when I worked in her room that I wanted to interact with students the way she did. [Interview data]

As Wesley continued to explore and implement new things in his classroom, such as students working in groups more often and using manipulatives during the more frequent problem solving opportunities he was providing for them he in turn was afforded more opportunities to listen to his students. It was this listening to his students, trying to

understand and make sense of what they were understanding that infused his desire to continue to provide more of these kinds of opportunities as a part of his teaching.

Although Wesley was trying some new things in his classroom and beginning to change his approach to teaching in the late 1980's there had not been at that time a radical shift or change in his way of thinking about mathematics that he believes to be instrumental in his transformation and change as a teacher. Referring again to his first workshop session with Allan he says:

I think what Allan did back in '91 for me was posing that question "what is mathematics" – it started me thinking. Until that point I had never thought about what mathematics was except basically nothing more than addition, subtraction, multiplication, and division of fractions, decimals, and whole numbers for me as a middle school teacher. So I think this is what really started it. That's when perturbation really started for me. He really caused some disturbance in my world with that question. [Interview data]

Allan offered a definition of mathematics as the study of patterns and relationships in his discussion with the teachers and further elaborated on this idea in an article he provided for the teachers at that workshop. While this definition of mathematics is a part of Wesley's belief system and a common part of the discussions in his classroom today it was not something that he readily embraced or even fully made sense of initially.

Although, after that first meeting he had created some disturbance in my world, I don't know at what point that became adapted as part of my vocabulary and part of my belief system, I wouldn't know exactly when that transformation really happened and if you were to place it at a certain time – I don't know. I do know

that by the time I wrote the presidential award application up about four years later I can imagine that I had come to a pretty powerful belief that mathematics is the study of patterns and relationships. I certainly believe it is something that needs to be brought to the forefront and looked at in the classroom so I try to make it not a constant but something we continually think about you know “what is math?” to keep that at the forefront. [Interview data]

During the school year, following that first meeting with Allan, Wesley began to wrestle with the idea of mathematics as the study of patterns and relationships implementing more problem solving opportunities for his students and having them work in groups. Based on some discussion regarding cooperative groups during the summer workshop and in reading the article Allan had given him Wesley began to have his students work in smaller groups than he had been doing in previous years.

Slavin, Johnson, and Johnson, I had been reading a lot of their stuff that said, have students work in groups of four or five and you would assign roles and all that. That is pretty much what I had been doing. Well Allan came along and he said you know two or three people will work well together – and that made sense to me. [Interview data]

It was also during this year, as Wesley attempted to continue to transform his pedagogical approach to teaching mathematics that he began to face one of many constraints and frustrations inherent in attempting alternative approaches to teaching.

So at that time the first obstacles I had were “where are the problems?” ... So I started trying to find some problems and he (Allan) had given us some stuff but a lot of them were kind of elementary. ... Those were some problems to begin with.

What problems do you use, which is continually something we are trying to evaluate. Pace? Where do you get the problems? ... So I started at that point trying to find monies to buy books. I just started looking everywhere to find as much as I could to pick from. [Interview data]

Wesley began trying problems from the variety of sources he was collecting including Dale Seymour TOPS cards and the problems that Allan had left for them. Since he had no way of contacting Allan to ask new questions that were arising as he implemented the problems in ways that made sense to him at the time he really began a process of trial and error with the problems he had.

I think a set of the problems that Allan had sent us were designed for 9th grade non-college bound students and they were divided into I think five sections of fifty problems each. So you would do a couple of weeks of problem solving and then you could get into whole numbers and integers and then you could do another section of the problems for a couple of weeks and then rational numbers, sequences, and patterns and so on. He had the whole thing sort of laid out there.

Well I did those fifty problems in two weeks no problem. I assigned some as homework, we did some in class and it didn't really go that well. [Interview data]

Struggling with where to find problems and just how a focus primarily on problem solving might actually play out in the classroom, Wesley also found that other issues and constraints or frustrations arose.

Determining just how he wanted to implement problem solving and whether the problems he selected were worthwhile and accessible to his students is also an issue that concerns Wesley still today but certainly more so years ago. Additionally, since he was

no longer following the curriculum as laid out in the textbook and his focus was shifting away from skills and procedures to students' mathematical thinking questions about how to assess students arose.

How do you assess student learning? This was one of the big, big issues, and really still is, but in the beginning this was particularly difficult because of the issue of grades and then parent concerns. I think I was just stumbling in the dark at that point. I mean one of the early problems I did was 48 fence posts 5 meters apart (see figure 1). What is the area bounded by the fence and my pre-algebra class they were 3 or 4 days on that and could not solve it. So it was trying to find tasks also that would be accessible. [Interview data]

Figure 1. The fence post problem.

*I built a square fence using 48 posts. I place the posts 5 meters apart.
What is the area of the field bounded by the fence?*

Over the next six or seven years Allan continued to be brought back to Wesley's school district to conduct mathematics inservice workshops for both middle school and elementary school teachers. During these years the process of change and transformation in Wesley's ways of thinking about mathematics teaching and learning continued as did the emergence of new issues to be addressed and a variety of constraints that had to be considered. In his attempt to make sense of mathematics as the study of patterns and relationships and what this means for mathematics teaching and learning, and in dealing

with issues of assessment, homework, finding accessible and worthwhile tasks, and parent concerns, Wesley began to develop what he now describes as a “mentoring” relationship with Allan.

So when Allan would be back, when the district started bringing him back every year, I’d get with him and take some time, he’d spend an hour with me and I’d ask some questions. ...The early questions where tasks: where do you get them? What tasks are appropriate and do they lead somewhere mathematically and how do you assess this thing? Homework, so what do you do for homework, how can it be meaningful, these are always things we have looked at and I guess will always be an issue. [Interview data]

Wesley continues,

Allan would come back to town and I would have questions for him and we would talk about assessment and you know that is still something we wrestle with in playing the game with administrative issues and grades and parental expectations and the children have expectations. ... At some point along they way, maybe after about 5 or 6 years, our district didn’t do the workshops with Allan anymore. That was a very sad day here. Anyway, by then I had e-mail and we were talking some via e-mail through the year and that has just continued. Now we e-mail often, talk on the phone some, and I send him stuff to look at for us to talk about. He sends me things to read and so on, you know. It has been crucial to have someone to collaborate with and to run ideas by and to really hear from him about his experiences and his research. We also see each other from

time to time at a conference for a presentation and we are working on some things right now with the middle school activities. [Interview data]

In addition to developing this mentoring relationship with Allan, one that is characterized by the sharing of common beliefs, goals, and collaboration, Wesley has developed and fostered collaborative relationships within his building with two other mathematics teachers over the past few years.

Working Collaboratively With Other Teachers

Not to the exclusion of the many conversations that Wesley and Allan have had concerning the issues and constraints inherent in implementing a more problem solving approach, one that is radically different than following the textbook and assigning practice sets for homework, the opportunity to collaborate and have discussions at his school site with other teachers regarding issues relating to parents and administration has been very beneficial for Wesley. Over about the last three years Wesley has been able to collaborate, discuss, and plan with two other seventh grade mathematics teachers in his building. Referring to them as “the three amigos,” Wesley says that they try to meet at least once a week to plan lessons.

See I don’t see lesson planning as what goes on the paper, in fact I don’t do that much any more, put things down like this on Monday, this on Tuesday, and so on. What we do is try to get together and talk about how our students have responded to certain tasks, and we’re talking all the time, its not that we just wait until Friday or whatever. We are thinking about and talking about these things constantly. What’s been good about having these guys to talk to is that one of us can try one thing in one class and something else in the other and see how it went. We can

also really talk about how best to work with the parents and communicate with the parents. [Interview data]

Wesley talks more about lesson planning and how his thinking about planning has evolved as part of this process of change.

I used to do more of laying out what was going to happen on this day, that day, and so on and now I use some software called Inspiration if I do any of that because we don't really have to send them to the office each week. ... I think where we've got to go from my perspective or my lens in thinking about and looking at curriculum is this bigger picture, bigger idea, a global perspective.

While I don't really go back and write things down about well this went well or this didn't work, I guess I just see that as part of it and I am always reflecting, even if I don't write it down I am always thinking about it in that way. ... One time ... Allan was talking about assessment and all of this stuff that gets in the way of student understanding,[he said] "I see a lot of people going home with an arm load of this stuff to grade and what I would like to see is teachers leaving the building and going you know what good tasks can I give my children"... So when I think about planning I think about it in that way and am always trying to think bigger picture, bigger ideas. I think the time when I can get together with the other two guys and we can really talk about tasks and what our students have been doing with them, that's really great, really powerful lesson planning. [Interview data]

Keeping the "bigger picture" in mind, the development of an orientation that allows Wesley to think about his curriculum in terms of "bigger ideas," creates a conflict with

his pedagogical approach and more traditional ways of thinking about lesson plans and using textbooks wherein the curriculum is reduced to concepts that are seemingly unrelated and introduced in a linear and lock-step fashion. While parents and students may not have a sense of Wesley's envisioning of the curriculum from a broader perspective they certainly recognize that what happens in his classroom is different than other mathematics classrooms.

Working collaboratively with the other two teachers has not only provided opportunities for them to plan lessons and develop tasks for their students but it also has helped in the working with and communicating with parents.

I think one of the things that parents have said is that we don't do any homework, and this is really one of the things I heard several years ago and not so much now, but we do it, its just not what they have come to expect. That is something that we still do deal with right now is the amount of homework that they get, the type of homework and then there are some parents that do question the methodology we embrace. I think what we try to do, the big thing is, if you can try to impress upon parents that you want their child to be able to compute effectively and efficiently then talk about decimals, fractions, and percents for a regular math class and for a pre-algebra class that is a heavy focus and is going to continue to be all year long and you know on back to school night I talk about those things... You know it has helped with the three of us, I don't know if it is the united front thing or if we have just gotten better about what we say but we really thought about what we wanted to tell parents on back to school night this year. And also, the things I have been spending so much time on, those math squares, two-ways, and so on

are really helping as powerful things to send home with the kids and we can talk about them the next day. So it is a format that the students can get used to and their parents can get used to rather than maybe some different problem every night like I had done back at first. [Interview data]

Working with and communicating with parents has been an issue throughout this process of transformation for Wesley and the collaboration with his other two colleagues has seemed to be beneficial in many ways. However, it is difficult to separate whether expectations by parents have changed over time since it may be more “known” that some of the mathematics classes are different in the seventh grade or if Wesley’s recognition for excellence in teaching affords him a certain amount of latitude with some parents that may not have been there several years earlier. Wesley also comments and recognizes with regard to the frustrations and constraints in dealing with parent expectations that parents and students always have and always will have certain expectations about classroom practices regardless of what teachers do noting that this constraint is one that most teachers face not just those trying to implement change.

WESLEY’S CURRENT PEDAGOGIC PRACTICES

Rather than being the end product of his attempts to change from one pedagogic approach to another, what happens in Wesley’s classroom today is the expression of his beliefs about mathematics teaching and learning wherein a way of communicating with his students about mathematics is the learning environment in his classroom. While Wesley states that he follows a problem centered learning model what he does with his

students is more than a model or a set of steps that someone else could follow to create the same thing, it is an orientation about what is mathematics and what it means to teach and learn mathematics, one that views the classroom as a community and the teacher as co-learner. Wesley believes that

... if you are going to have a problem centered learning classroom then you also probably need to be one that is solving problems and enjoys solving and so it can't be done unless you yourself are looking at problems for relationships and for patterns and looking at the richness of activities trying to see those connections that exist in these kinds of tasks we propose. ... I also think that maybe Cobb, Wood, and Yackle, that group with Bauersfield would look at the classroom as a community, not the students interacting, but it is a classroom where there are adults and students interacting and ideas are valued over methods and right answers. ... I really think that is an issue for some folks who have not taken that step or have not had the opportunity to see what could really be going on with learning. ... You know until you really embrace it you know it has taken me a really long time to get to where I am at in looking at things from a more global perspective – looking at the bigger picture. [Interview data]

While Wesley certainly feels he is in a different place in his thinking about mathematics with entirely different pedagogic practices than a decade ago he sees himself as still working to make sense of things as a teacher, a problem solver, and a learner along side of and with his students.

I think I continue to look more and continue to appreciate more and I am always looking for the patterns that exist... See this relationship is something that is new

for me, I've had that as part of my vocabulary but I am really now seeing the relationships that exist. Okay mathematics is the study of patterns and I know what patterns are but what of these relationships. I mean I'm evolving in my understanding even 10 years later just seeing those relationships. I learn from the students and their different ways of looking at things and thinking about things continually and that's really exciting. [Interview data]

Working with students to develop a community of learners is something that Wesley has been doing for some time now but with each new year and with each class establishing relationships built on trust and respect and the valuing of one another's thinking must be deliberately fostered. The students come to Wesley's class as seventh graders with seven years of school mathematics experiences. They have been participating in mathematics classes where most of their work has been done independently, where practice, drill, and speed have indicated competency and understanding, and in the words of one of Wesley's students from a few years ago, "where textbook is king." It is a challenge to work with the students to establish a community, one in which there is trust between students and their peers and their teacher and a willingness to share ideas about problems and problem solving. Wesley begins the very first day, in his first communications with his students letting them know that "this mathematics class will be different than the others you've had." He keeps no secrets from the students regarding his beliefs about mathematics, how those beliefs inform his selection of tasks, and that what he values in his class is their ideas and their thinking. A syllabus is handed out to each student at the beginning of the school year for them and their parents to read that states, "this course will differ from the conventional approach in

both method and content.” It goes further to discuss that students will develop their own methods for solving problems, listen to one another, work together, and participate in whole class discussions. He also points out in this syllabus that while there is a grade scale, he will also consider the students’ initiative, attitude, cooperation, participation, and persistence, those things that are not easily measured with exams and assignments when making final determinations regarding grades. Wesley further reiterates these ideas in each of the problem solving books he has compiled with tasks for students to use in lieu of their textbook on a daily basis. The first page of each of the several problem solving booklets that Wesley has developed for his students contains much of what the syllabus states but includes such things as “What is expected in the math class?” as a title section. The list below this title includes:

- Expect to be puzzled or stumped.
- Expect to work together.
- Expect to negotiate your methods and understanding with others.
- Expect to explain your thinking to others.
- Expect a task/problem to require investigation.
- Expect a task/problem to require time.
- Expect the possibility of more than one solution or answer. [Problem solving booklet]

On this page as well, Wesley explicitly outlines the students’ obligations in class and the expectation that they work together as a community of problem solvers, that each student’s thinking and ideas are as important and valued as his with statements such as “If you have a question, ask your teammate. Do not ask your teacher” and “listen

carefully” to your teammate. (See Appendix D for copies of the syllabus for both the seventh grade algebra I and pre-algebra classes and the first page insert for Wesley’s problem solving booklets.)

In addition to the syllabus and the initial communications with his students Wesley expresses to his students his expectations about their participation, sharing of their ideas, and their collaboration as an essential part of helping everyone in the class develop understanding. Through his own participation in problem solving with the students and in his efforts to explicitly express his expectations to his students, Wesley deliberately fosters relationships based on trust and caring wherein the students are comfortable to express their ideas, validate one another’s thinking, and challenge one another’s ideas. As you enter Wesley’s classroom a sign hangs next to the whiteboard at the front of the room that reads “Our Objective for Today’s Class” underlined at the top with “To think, reason, problem solve, and to express our ideas effectively” written below. In every communication with his students, both oral and written, Wesley expresses his valuing of the students’ ideas and participation in the classroom community in an effort to establish with his students the sociomathematical norms of their group which include actively listening to each another, expressing your thinking and reasoning, and exploring and investigating mathematical ideas. In his algebra I class during the third week of the school year Wesley discussed with his students his belief that “all mathematics is image based.” He continued by talking with them about how this belief plays a role in many of the problems and tasks he selects for them. Following this discussion he put what he calls “the lawnmower problem” on the overhead (see figure 2) and asked the students to look at it for a few minutes and if they had any questions before

they began to work with their partners. One boy in the class asked, “when we work with our partners and we present our solutions do we just tell our answers or what?” Wesley responded to the student and the entire class with another question “what do you think?” After a moment when no one in the class responded Wesley said, “I’m not about right answers – I’m about good thinking. You have some obligations during the whole class discussions.” Writing on the board (see figure 3) he continued:

... you need to agree, disagree, don’t understand, you need to listen intently, and we’ll try to come to some sort of consensus. Ideally we will look at problems and then discuss them at the end of class but that is what I’d like to happen and it will rarely happen that neatly. Also, not everyone can present their ideas and their findings and that doesn’t mean that I don’t value or appreciate your work its just that depending on how things go and where they are going we may not get there.

[Interview data]

Following these comments the students moved their desks around to work with their partner who was the person setting across from them in the next row of seats. Students moved throughout the room freely to retrieve items they felt were needed to solve the problem and transparency sheets and pens to record their work for the whole class discussion and presentations. While Wesley was still addressing some questions from students during the third week of school regarding the expectations about specific aspects of what was valued in his room his students were already working together collaboratively with ease and moving freely about the room gathering what they needed and talking with other groups about the problem they were attempting to solve. After only a few class periods, the sociomathematical norms of this classroom were becoming

well established and a sense of community was being developed as all students were engaged in collaborative problem solving and the sharing of one another's ideas.

Figure 2. The lawn mower problem.

At a clearance sale, a gas-powered lawn mower was discounted 32%. The lawn mower was sold for \$117.81, including a sales tax of 5% of the sale price. What was the original price of the mower?

Figure 3. Whole class discussion items written on the board.

<u>W</u> <u>C</u> <u>D</u>	Whole Class Discussion
	1. A D DU -- agree, disagree, don't understand
	2. listen intently
	3. consensus

Encouraging students to share their ideas, listening to one another, and encouraging them come to consensus are essential parts of establishing a community of problem solvers from Wesley's perspective. Listening to students for Wesley not only played a major role in his transformation as a teacher but is now a natural and inherent part of his interaction with his students.

One of the things that was really important and still is, I just don't think about it as much anymore because I just do it, I think, is listening to the students. Back when I was really trying things out and searching for problems it was really

helpful and important to listen to the students while they worked on problems. I would try to understand what they were understanding and what they didn't have figured out. [Interview data]

For Wesley this kind of listening, listening to determine what the students are making sense of, is more than just listening for assessment purposes, for asking questions, or for even selecting the next task. It is listening to participate, to be engaged in the problem solving and in the community with his students. Wesley believes that this kind of listening, an active or engaged listening, is essential for his students to fully participate in the community he strives to develop in his classroom. Class opener activities, such as “math squares”, “two-ways”, and “guess my number” (see Appendix D), that require students to share their thinking and solution strategies with the class, are designed and selected to work in concert with collaborative group work and whole class discussions to not only provide students the opportunity to construct mathematical ideas but also to involve students in listening to one another's ideas in order to support or likewise challenge and question one another's thinking. As an example of students listening to one another and questioning one another's solutions the following beginning of class discussion stemmed from two math squares problems (see figure 4 and figure 5). The math squares problems were assigned as homework problems (see Appendix D for a copy of the homework) the previous night. The math squares problems require students to either determine the total of all four boxes or determine the number for a box that is left blank based on the total of all four boxes and the three other numbers. With the math squares tasks no operation is indicated so students are free to determine how they should solve the problem and the numbers are selected for these tasks in such a way as to

provide “rich opportunities for students to use tens and refine their thinking strategies” (Wheatley and Reynolds, 1999).

Figure 4. The first math square presented on the overhead as a class opener.

175	150
350	25

Figure 5. The second math square presented on the overhead as a class opener.

175	
24	50

300

Kelly: (Looking at the first math square.) I am taking the 175 and the 25. Make it 200.

Wesley: Why would you want to add them that way?

Kelly: They are easy to add that way and then you can put the 150 and the 350 together and then just put the 2 of them together. You get 700.

Wesley: Why is she adding it that way? Can anyone explain it?

(Class laughs.)

Holly: I can explain it. You add the 25 to the 75 and get 100. Add the other 100 that's 200. You take the 50 from the 150 add it to 350 that's 400. 400 and 200 that's 600 plus the 100 is 700.

Wesley: Does anyone have any other comments on that first one? (Pausing for a moment.) Ok let's look at number 2. What might you do on number 2? Yes, Taylor.

Taylor: Well I took 175 and 24 but I made the 24, 25 and you get 200 then subtract 1 so 199 and I added 50 to that.

Mitchell: (Jumps in.) So you are making the 24, 25 (pointing to the 24). You're just rounding it up so you can add it to the 175 easier?

Taylor: Yes. So then add the 50 to that and get 249. So we need 51 in that box.

Wesley: Ok. I see where you are getting that. You are using adding up or adding on. You had 249 and needed 300 (Writes on the board $249 \rightarrow 300$.) and you added up to get 300, that is a strategy. (Several students have their hands raised.) Lisa.

Lisa: I did the same thing as Taylor, rounded the 24 to 25 got 250 take away 1 but then I subtracted 300 take away 249.

Wesley: Ok. Did you do that subtraction on paper.

Lisa: No.

Wesley: How did you do it?

Lisa: In my head.

Several other students shared their strategies for determining the solution to the math square in figure 5 before the class moved on to work with their partners for problem solving.

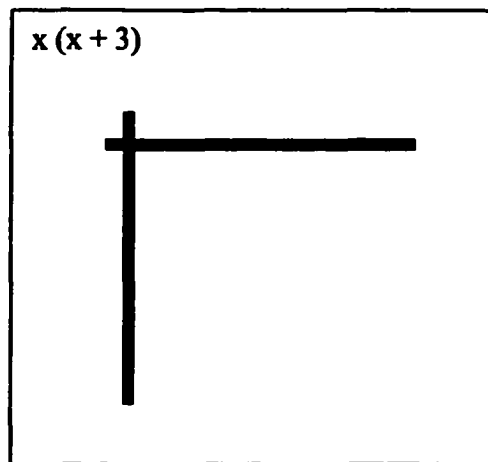
In addition to the use of class openers that help to create an environment wherein students work together, listen to one another, and challenge and support one another's thinking, Wesley combines the use of collaborative problem solving with students working in pairs with investigations using manipulatives in which the class moves back and forth from students exploring the problems individually to whole class discussion throughout the class period. The following vignette is one taken from Wesley's algebra I class as they work with Algebra Lab Gear (Creative Publications) to explore multiplying polynomials. This vignette highlights the involvement of each student as the whole class works together for understanding as they investigate, explore, make conjectures, and validate one another's thinking. Throughout this vignette the students make connections between what they are exploring and their previous knowledge and in their attempts to make sense of what they are thinking and to explore conjectures they are making, they challenge and perturb one another's thinking. Additionally, this vignette highlights Wesley's intuition regarding task selection and questioning. His listening to the students enables him to further perturb their thinking through questioning and task selection.

Wesley: **Let's get our materials out if we can by, let's say 20 after.**

Students: **We have a minute 20 seconds (looking at the digital clock on the wall).**

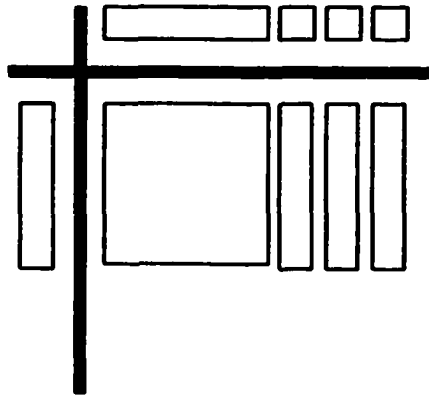
With the desks in rows, 4 or 5 students at the back of the room form a chain to distribute the buckets of Algebra Lab Gear. Two students towards the front of the room made sure that everyone had a bucket. For about the next 3 minutes the room was very noisy as students remove the pieces from their buckets and organize them into piles. During this time Wesley displayed the first problem for them to consider using the overhead projector. See figure 6.

Figure 6. The first problem used with Algebra Lab Gear and a representation of the frame used with Algebra Lab Gear.



Wesley: Ok. Here's the problem. Can you model that problem?

Figure 7. One student's use of the Algebra Lab Gear at her desk.



Students worked at their desks. Many of them compared what they had with the students around them for several minutes before Wesley asked if everyone was ready to look at this problem and if anyone wanted to share what they had.

Wesley: Brenda is going to present what she has.

Brenda goes to the front of the room and uses the second overhead projector to model her solution for the class. The solution she presented is the same as the one in figure 7.

Wesley: (Asking the class while Brenda remains at the overhead.) The x in this problem can be classified as what?

Class: (In unison.) A monomial.

Wesley: The $x + 3$ can be classified as what?

Class: (In unison.) A binomial.

Brenda then begins to explain her solution to the class pointing to each piece of Lab Gear as she talks.

Brenda: Here are my x 's (pointing to the rectangles on the outside of the frame) and x times x is x squared (pointing to the square inside the frame) then x times 3 (pointing to the small squares on the outside of the frame that represent single units) gives 3 x 's. So my product is $x^2 + 3x$.

Lisa: It could be $x^2 + 5x - 3x$.

Wesley writes on the board $x(x + 3) = x^2 + 3x$ and then below it $x^2 + 5x - 3x$. Several students raise their hands at this point.

Melissa: How did you get that Lisa?

Lisa: Well it is the same thing.

Melissa: I see that it is the same but there is no 5 in the problem.

Jason: Can't we also call the x and the $x + 3$ factors.

Wesley: What do you guys think?

Class: (Several students say yes and others show agreement in other ways.)

Wesley: (As he writes on the board. See figure 8.) If you all agree with Jason that the x and the $x + 3$ are factors, can we also classify the $x^2 + 3x$ as a product?

Figure 8. Example of what Wesley wrote on the board as he talked with the students.

The diagram shows the equation $x(x + 3) = x^2 + 3x$ enclosed in a rectangular box. Below the equation, there are three arrows pointing upwards. The first arrow points to the x in $x(x + 3)$ and is labeled "factor". The second arrow points to the $(x + 3)$ in $x(x + 3)$ and is also labeled "factor". The third arrow points to the $x^2 + 3x$ on the right side of the equation and is labeled "product".

Wesley: Can we classify this product?

Tim: Yes, it's a binomial.

Wesley: Why?

Tim: Because it is the sum of 2 monomials.

Wesley: What about Lisa's $x^2 + 5x - 3x$?

Tiffany: It's a binomial it's just not simplified.

Wesley: What do you mean?

Tiffany: We can subtract the $5x$ and the $2x$.

Wesley: And get back to the other one?

Class: (In unison.) Yes.

Wesley: So we might want to talk about these, the $5x$ and $2x$ are like terms. Are there any disagreements here or any questions on this problem? (Several students raising their hands.) Yes, Alicia.

Alicia: I disagree with Lisa's answer because it really has nothing to do with the real problem because you could put the x^2 plus anything minus anything as long as you get $3x$ when you subtract.

Wesley: Sure, I see where you are coming from. Let's look at another problem.

Lisa raises her hand and talks more about different ways the problem could be represented and even goes to the board to add parenthesis in different places throughout the problem. This leads the class to a discussion about the distributive property.

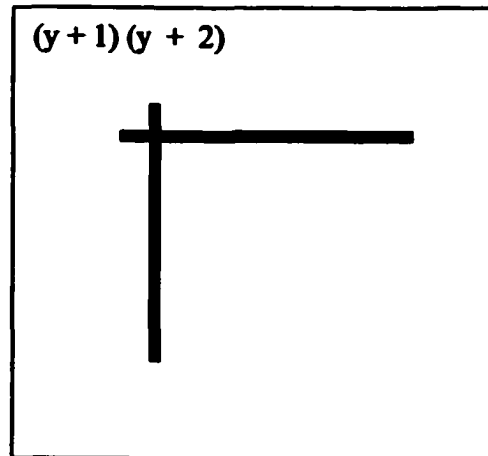
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Wesley places another problem on the overhead for students to model at their desks using their Algebra Lab Gear. See figure 9.

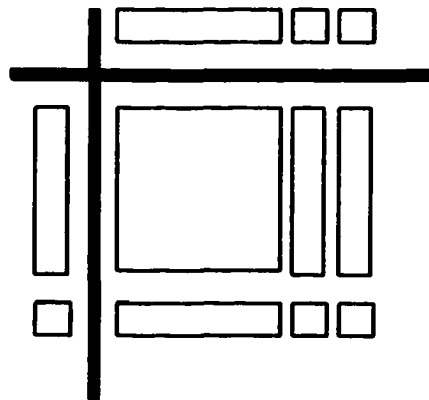
Figure 9. The second problem used with Algebra Lab Gear and a representation of the frame used with Algebra Lab Gear.



Wesley: Let's everyone look at this problem for a few minutes. (Wesley moved around the room looking at the students' representations and asking a few questions.)

Several students had constructed different representations at their desks with their manipulatives. Jenny was asked to present hers to the class. See figure 10 for Jenny's representation.

Figure 10. Jenny's model of the problem that she presented at the overhead.



Jenny: (Explains her solution as she points to each piece.) Well, this is the $y + 2$ and this is the $y + 1$ (pointing to the pieces outside the frame). Here is the y^2 (pointing to the square inside the frame) and then $3y$ (pointing to the 3 rectangles in the frame) and then 2.

Wesley: What is your product Jenny? Can you write it on the board for us?

Jenny: $y^2 + 3y + 2$ (she writes it on the board).

Wesley: Can we classify these factors?

Kevin: The $y + 1$ is a binomial and the $y + 2$ is a binomial.

Wesley: So we have a binomial times a binomial and we get a ...?

Jenny: A trinomial.

(David raises his hand.)

Wesley: Yes, David.

David: You can kind of split it into groups you can take the y times the y and get y^2 and then the y times $+2$ and get $2y$ and then the y times the 1 and get $1y$ and then the 1 times the 2 and get 2. If you add up what you can you get the same answer.

Wesley: So you think maybe there is some grouping we can do to get the product and maybe not use these tools (referring to the Lab Gear).

David: Maybe. It works on this one. I'll have to test it on some other ones but the distributive property worked on the last one.

Wesley: What we are wanting to do with you guys is maybe. We want to take ... I think it is very important that we look at mathematics (he writes on the board "All mathematics is imaged based"). Some of us believe that all mathematics is image based and what we want to do is move some of you from here (writes on the board "CONCRETE" and "ABSTRACT") concrete to maybe abstract so we don't necessarily need those tools.

(Alicia raises her hand.)

Wesley: Alicia.

Alicia: Well I want to go back to that short cut David found.

Wesley: We'll talk about the short cut in a minute. I'm kind of going to put you on hold for just a minute and there is a short cut and mathematicians have a name for it and usually that is how it is taught but I feel very strongly that we need to have a visual representation of it. Maybe we can call the short cut the "David method" others have come up with a short cut and they named it and you may have the same exact one.

At this point there is a frenzy of activity as several students are testing David's method at their seats creating problems and checking them. Some discussion ensues among the students about their impromptu investigation.

Wesley: I really think some of you are seeing some things here and I don't want to say don't go there yet but I do want us to look at some other things and we'll get there. (Realizing the students were not going to let this go.) Let's go ahead and check this with our other examples.

Lisa came to the board to show her short cut method with $(y + 1)(y + 2)$. Many students disagree and offer other ways of looking at the problem.

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Jessica: Lets put 5 in for y and see what we get (writing on the board as she talks $(y + 1)(y + 2) = y^2 + 3y + 2 = 25 + 15 + 2 = 42$). Putting the 5 in we get 25 and 15 and 2 equals 42. If you put the 5 in early (shows on board $(5 + 1)(5 + 2)$) you get 6 times 7 is 42 (writes on board $(6)(7) = 42$) we also get 42.

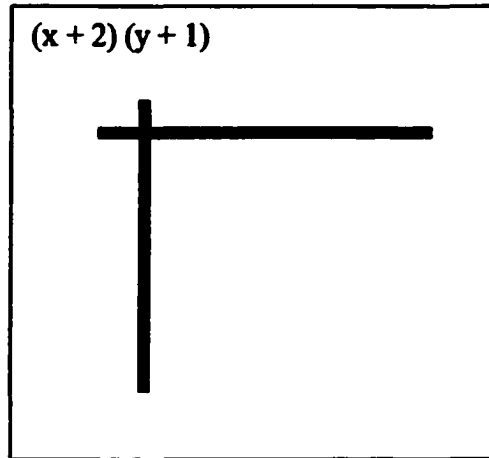
Wesley: We are getting the same thing in both places. Interesting (pause). Let's go back to this other problem for a second.

Jessica: Sorry, I got us off track.

Wesley: No, No. This is good. I've never really thought about these this way before so I have really been listening in on this.

Wesley put another problem on the overhead at this point for the students to model with their Lab Gear. See figure 11.

Figure 11. The third problem used with Algebra Lab Gear and a representation of the frame used with Algebra Lab Gear.



Lisa: I have something to say about my short cut method from before. It doesn't work on this problem. You have to have the same variable in both binomials. (Paused looking at Wesley.) You knew that!

Wesley: I did?

Lisa: Yes, because you are smiling!

(Class laughing.)

Lisa: I think really weird compared to most people, not weird just different.

Wesley: I like weird and different. It's good.

Jimmy: This is the same problem as the last one. It just has an x in one of them instead of a y.

During this time students are actively working at their desks and many of them have their hands raised.

Kelsey: David's method works on this one too it just doesn't have any like terms in it.

At this point the class period was close to ending so students put their manipulatives away and planned to continue their discussion the following day. Wesley's algebra I class continued to investigate with their Algebra Lab Gear for a couple of days as students explored and tested their conjectures about ways to solve problems involving multiplying polynomials.

While Wesley's pre-algebra class may use different manipulatives than the algebra I class at different times during the year and consider different problems, the investigations, explorations, and conversations are of the same nature in both classes and the previously presented vignette. Wesley's combining of whole class investigations and what he calls "a kind of Socratic questioning" with student small group problem solving, presentations, and whole class discussions, and class opening activities that require students to communicate their thinking and ideas with one another creates a community of learners who participate in an ongoing conversation that emerges as the curriculum. Reflectively considering what has happened in Wesley's class over a period of time, trying to determine what might have "caused" students to come to understand something is perhaps not possible. The complex emergence of conversation and understanding that occurs over time with each individual student and the class as a whole is not something that is pre-planned or determined ahead of time. Wesley has developed an orientation about the teaching and learning of mathematics wherein based on his intuitions about student learning and tasks, listening to his students, and his participation in problem

solving with his students he makes determinations or selections about tasks and investigations that may infuse the conversation.

CHAPTER V

THE CASE OF KATHLEEN

In this chapter the case of Kathleen will be presented. Her reflective account of her teaching experiences and the factors she believes to have affected her vision and desire for change will be discussed. The sections of this chapter will be presented largely in Kathleen's words in an attempt to reflect her perception of what factors and experiences are important in understanding her transformation as a teacher, her pedagogical practices, and how she has addressed constraints and limitations in her envisioning and enacting an emergent mathematics curriculum. In the section focusing primarily on Kathleen's current pedagogical practices, classroom scenarios, student interactions, and tasks presented in class obtained via audiotape and videotape of classroom sessions and field notes will be included. In this presentation and analysis of Kathleen's pedagogic practices the classroom scenarios, student interactions, and tasks will be inserted only when it is believed necessary to capture a complete understanding of what happens in Kathleen's classroom daily and over a period of time. Much of Kathleen's Teacher Story is presented as a sequence of events and experiences with the use of dates occasionally to order several of Kathleen's experiences. This is done to capture Kathleen's reflective account of how transformation took place in her life as a teacher, to show where she has been, has gone, and is hoping to go. This is not an attempt on my part or hers to imply that any of these events and experiences stand alone or in isolation, occurring one after the other creating change or the opportunity for change, but rather that in reflection these experiences seem to stand out as some of the

most significant. This chapter will specifically address the following focus questions of the study:

1. What pedagogic practices does this teacher include in her implementation of an emergent mathematics curriculum?
2. What factors does this teacher consider to have influenced her commitment to an emergent mathematics curriculum?
3. What does this teacher identify as constraining and limiting her options for creating an emergent mathematics curriculum? How has she addressed those limits and constraints?

KATHLEEN'S TEACHER STORY

Background

The flexibility and respect for diversity of ideas and thinking that have become hallmarks of Kathleen's working with students is something that may have its roots in Kathleen's experiences as a child. Kathleen's father was a "Protestant from Georgia" and a sergeant in the military who married an "Irish Catholic from New Jersey." While growing up Kathleen's family moved around a lot and the flexibility she developed as a child may play a role in the kind of teacher she is today.

We traveled a lot when my dad was in the military. We spent five and a half years in Germany during the time we were moving around. We had to be flexible, going from one place to another. I think there are a lot of drawbacks to

moving like that but there are also a lot of benefits too because you do become very flexible. [Interview data]

Kathleen also experienced issues related to diversity as she grew up being raised in a home with parents from different backgrounds. “I don’t think my parents ever had to deal with any controversy related to their differences since they were in the military but my mom’s family was certainly more traditional so they might have had some problems with it.” Kathleen is the middle child of three children with a sister two years older and a brother eight years younger. Since her brother was so many years younger than she and her sister, they played the role of “mother” to him. “We were like mothers to him. Really my sister was more like that than me but we really were like a whole different family for him but it wasn’t like brother and sister when we played.”

Kathleen’s nurturing relationship with younger children extended beyond her brother as she has “always loved children and loved being with children.” “I love kids. I used to love to baby sit. I know I had friends that wouldn’t really want to do it but I loved it and just wanted to be around kids.” It was this love of being around children and her brother’s problems with reading that influenced her decision to become a teacher and more specifically a reading specialist. “I focused on reading as a teacher I think because my brother had so much trouble with reading. He had to do eye exercises and everything. It was really hard for him.” Kathleen graduated from a small regional state university in a southwestern state with an elementary education degree and three years later completed a Masters of Science in Reading degree. She spent her first two years as a reading specialist in a rural school teaching second through sixth graders. Following that second year she moved to a suburban area where she now lives and taught seventh through tenth

grade students in a reading lab for the next four and a half years in a nearby town until the birth of her son. After staying home with her son for two and a half years she wanted to return to teaching. She was interested in looking for an elementary position at this time since she had a young child. "I guess I wanted to get in at an elementary school since I had a young child because I wanted to see what was going on." She was hired as an elementary teacher in a suburban school district and is completing her twenty first year of teaching at the same site. "I began here as a second grade teacher. I taught 5 years in second grade and then went to first grade for about 8 years and then have been back with second grade for 8 years now. I also taught one year of third grade in there as well." Kathleen believes the varied experiences she had with teaching different ages, especially her work with older children in reading, to be relevant to the kind of teacher she is today.

Searching for Something that Meshed

Reflection, "risk taking", and searching for curricular materials and a teaching approach that would help create a better learning environment for her students have seemingly always been a part of Kathleen's teaching practice. When she returned to teaching as an elementary teacher following the birth of her son she worked to try to individualize instruction in her classroom. It was at this point that she began to notice the discrepancies between what she was doing in reading and language arts and mathematics.

At first I tried to individualize instruction with my classroom. In reading I felt this to be much more beneficial than round robin reading groups which I have always found boring. However, it was hard to get to everyone and I experimented with different ways of doing reading groups. I had a lot of confidence in doing things in reading due to my experience as a reading specialist. However, math

was a different story. I felt very limited in math. I tried to go to workshops and read articles and books that might help me. For a while, I let them move on in their math book but this proved to be overwhelming because I discovered I was having to explain procedures over and over to students. Reluctantly, I went back to whole class instruction. [Reflective paper]

By the time Kathleen's son began kindergarten she was reassigned to teach a class of third graders who were considered the "high" group in mathematics. Since Kathleen felt that most of the students in this class had a good understanding of mathematics she felt a certain amount of pressure to keep them busy. That year she recalls placing a great deal of focus on drill and speed with math facts thinking this would challenge her students and allow them to become faster with their facts and would additionally help them stay busy and out of trouble. It was during this year that Kathleen's son and his experiences with school began to have an impact on her own teaching approach.

One day Kathleen's son's kindergarten teacher remarked that her son had been "as slow as molasses today." Since Kathleen herself was placing value and importance on speed and efficiency in her classroom she was struck by initial feelings of disappointment by her son's teacher's comments thinking to herself sadly "he's going to be one of those." As a teacher she knew what her son's teacher was alluding to. She herself had been frustrated through the years with students who did not, could not, or chose not to make it through the assignments and worksheets with speed and ease. Kathleen recalls that the words of her son's kindergarten teacher were not unlike those of his other teachers in the years to come since her son, as Kathleen describes him, "is one of those kids that doesn't fit into that box." As a caring, thoughtful, and reflective parent

and teacher, Kathleen spent a great deal of time working with and talking with her son about ways to “pay better attention” and focus on his work at school. She believes it was talking with her son, listening to him and trying to see things from his perspective, “through the eyes of a child” that she realized there had to be a better way to teach.

I think that it does help having your own child but not only having my own child but having the child that I had, the one that did not fit into that “box.” I think it would be different for me if I had one that fit into that box then maybe he would have not helped me to see so dynamically the way things should be different... He really helped me to see it from a child’s eyes, you know as much as we can. I had never really worked with a child one on one before. Maybe when I worked in my remedial reading room a lot of that was one on one but as far as small children he really helped me think about getting away from the idea that you just stand up there and disperse all knowledge and they listen. [Interview data]

Having a child that she could work one on one with, one who forced her to listen to him, created a situation for Kathleen in which she began to question the kinds of assignments she gave her students and what it means for students to be attending to a task.

I can remember my son, when I would say “u uh, honey, I’m listening to you,” he would take my face and say “no you’re not” and I mean he really did force me to look at it from his point of view because he would bring stuff home from school and he would be so upset about having to do it and I could see how worthless it was and I would think “I’m sending home the same stuff with my students.” ... My son was also one of those kids that could sit there in class, not be disruptive,

he was never a discipline problem, look like he was listening and paying attention, and his mind would be somewhere else. [Interview data]

Understanding this about her son, Kathleen began to question what it means for children to attend to a task, what does it mean for children to “pay attention?” Kathleen continues with another experience related to her son that stands out to her as one that had a great impact on the way she thinks about how children learn and the way she teaches today.

I was working with my son with his math when he was in about fourth or fifth grade or so and we were working on the procedure for dividing fractions which is very difficult and I was getting upset at him. It was at the end of the day and I was tired but I looked at him and I said, “if you would just listen, you never listen!” And he looked up at me and said, “how can I listen to something that doesn’t make sense.” And I just thought, you know that is one of those moments that you remember that sticks out. It really did hit me when he said that about how many of my other kids think the same thing because it doesn’t make sense to them. You kind of think for the kids it may be like Charlie Brown’s teacher up there “wa, wah, wah, wa, wah” (laughing). [Interview data]

As a result of listening to her son Kathleen began to try to listen to her students more and requested that she be moved to first grade with the hope that she could learn more about how children learn.

With this move to first grade Kathleen continued to try to listen to her students more and began to look for ways to make learning in her classroom more meaningful. She began to try to use more manipulatives and games in mathematics, attended workshops on Math Their Way, and found that eventually she was using her mathematics

textbook less frequently than she had in years past. Still struggling with ways her students could learn their math facts without drilling them, one of her students introduced her to “touch math.” “I felt like at the time that touch math was the perfect solution for the students to learn their facts without my having to drill them which I knew really didn’t work and most of them hated it.” It was around this same time that Kathleen also began to learn about a Whole Language approach to language arts and literature.

When I heard about Whole Language and began to learn about it I really felt like it could help create a learning environment that I had been looking for and working for with my students. My principal was very supportive with us getting to go to workshops and learn more about it and a group of us (referring to other teachers in her building) began to read books by Routman, Atwell, and Donald Graves to learn more and we formed a group to talk about ways we could make our classrooms more child centered.... I know we ordered a lot of authentic literature at that time.... With a Whole Language approach there was more kid watching, listening to your students, so I did that and I took notes on how the students were doing and we even created portfolios to see how the students would progress over time with their writing. [Interview data]

As Kathleen’s classroom became more “child centered” she became more aware and concerned with the differences she perceived between what she was doing in language arts and mathematics with her students.

...But even with this I really thought, you know what I’m doing in math just doesn’t mesh with what I was doing in language arts and reading.... If you came into my room during the language arts and reading time there were more child

centered activities, more authentic activities, but if you came in later during math time it was the same old quiet and worksheets with the desks in rows. ... So as that started, as our school started to pursue teaching it with the language arts, it was more child centered and it was more listening to our children and doing a lot of kid watching and trying to make your room more child centered...the problem was that you would be doing all this neat stuff in the language arts for children and then when it was the 45 minutes for math you were still handing the worksheets out and math is the quiet time. ...I was looking at one lesson plan book and sure enough they had all this innovative stuff with examples for language arts and then they had 45 minutes and it said page so and so out of you math book and I didn't feel like it meshed. It just didn't go with what I was doing with the rest of the day... but it was still procedures though. It was still the idea of telling them how to do something. [Interview data]

Over the next couple of years Kathleen continued to learn more about and pursue a Whole Language approach for language arts in her classroom trying to make connections between the literature she was using and her objectives in mathematics. Her goal was to have a more holistic approach with her students. While pursuing Whole Language had been a goal for the building site where Kathleen worked and was something being supported by the administration there were eventually some teachers that resisted changing to this approach to teaching.

It was during Kathleen's thirteenth year of teaching at her school when the turmoil surrounding Whole Language and Outcome Based Education finally escalated and Kathleen was faced with having to stand up for what she believed to be best for

children. Although Kathleen was already a reflective teacher who was working for transformation and change in her classroom, trying to find an approach that was more holistic and child centered and trying to create an environment in which her students' learning was the focus, it was being forced to make a stand at this time that she believes was a turning point for her in her teaching.

When our school went through all this turmoil with Whole Language and OBE, there were parents involved. I think they were put in the middle in some ways.

There was stuff in the paper and it was really hard you know knowing that parents were upset and people you worked with that were your friends were so upset and divided over something and so many things were being said about our principal. So I felt like I had to stand up for what she was doing and what we were trying to do. [Interview data]

...I think this was a real turning point for me to quietly submit to what others wanted or to stand up for what I believed to be right. [Reflective paper]

In addition to the turmoil that Kathleen was experiencing at work each day and being forced to stand up for what she believed to be right she believes that her supportive friend and colleague moving to a different school to teach was also a catalyst in her searching further for ways to teach that made sense.

When all this was going on at our school ... I had a very good friend who is a wonderful teacher and she taught the same grade level I did and we really could work together and we supported each other. We'd go racing into each other's rooms "can you believe this!" It was like we were on the same wavelength you know we really could. I really felt like I had support from her and from my

principal.... Well this really good friend of mine ended up moving to another school for a year And during that year I was so depressed because I didn't feel like I had anyone who could give me that support and I think you really need that. I have been very fortunate because I have had a lot of support with reflecting because I think the reflection is so essential. [Interview data]

It was during this same year that Kathleen decided to follow her first graders to second grade. Although she had taught second grade before she was looking forward to continuing with the same group of students in developing a more holistic approach to teaching.

Following the same group of students to the next grade also became an experience that Kathleen believes made her question some of the things she had been doing with her students in mathematics.

I had tried to find ways to make the literature connect to second grade math objectives. I still used touch math but started to notice how dependent the students were on using it and how confusing it was to them as we started regrouping. I started using Kathy Richardson's Developing Number Concepts, A Collection of Math Lessons, 1-3 by Marilyn Burns and Bonnie Tank, and Young Children Continue to Reinvent Arithmetic –2nd Grade by Constance Kamii.

[Reflective paper]

When I went from first to second, you know especially with the same kids you taught in first, you realize that just because you are standing up there and teaching it – it doesn't mean that they are learning it. I really am thankful for that experience of moving up with my kids and then also for working with older kids.

I think that sometimes teachers who teach in the same grade level for a long time they are like in this box and they think that they've taught it and that the kids know it. [Interview data]

The following summer Kathleen had the opportunity to attend a workshop being offered for teachers who would be having preservice teachers in their classrooms in the fall as interns. The workshop was offered at a large state university in the same town where Kathleen lives and she felt she needed to attend because she "needed to grow." She believes that this is when she really found what she had been searching for.

...That summer they were having one of those seminars for interns and no one else from my school was going but I just felt like I needed to go to that. I felt like I needed that to have something to grow. I had this void and so I went and it was with two of the university professors and I will never forget when one of them got up there and she started drawing like a time line or a continuum and she was showing from behaviorism to constructivism and I was like, I went "this is what I want – that is it!".. I think I said something like "I am so excited that I can come here and you can help me get from here to there" and she [the professor] said "and you can help us." It was kind of like I felt this bond because I will always remember that moment – oh my gosh! It was like everything I really believed about how children learn because the other just went against my grain. [Interview data]

Continued Transformation

While feeling a definite connection with what she had heard in those initial presentations at the seminar what Kathleen heard from another professor regarding

mathematics teaching and children left her with many questions. Amy, a university professor whose primary research focus is on children's mathematical thinking, also spoke to the group of teachers attending the seminar that summer. Kathleen remembers the presentation:

Amy started talking about touch math and how debilitating it is and I knew that she was right because I had done touch math with my first graders teaching first grade for 8 years and then I moved to second grade and when I started re-grouping I thought "oh, this didn't work." I knew this didn't make sense so I knew she was right but I felt like "ok, if you don't teach them touch math" because I had done that to get away from all those drill sheets "so how can they learn their number facts?" So I said, "what do you do?" And she went through some things that she had with shapes and I said "ok" thinking "I'll try to do some of those things.... During the fall I had an early childhood intern and she did some of the Quick Draw and I thought it was really neat. They were really talking with each other. I think it is a really good way to get them communicating and to see things from a different point of view, that someone else can see something in a different way. [Interview data]

Kathleen worked with the intern during that fall semester to implement and try some of the things that Amy had presented at the seminar during the summer but was not able to figure out how to implement all of it. It was a subsequent per chance meeting with Amy at a Phi Delta Kappan meeting where Amy was speaking that began what would become a mentor relationship and friendship, developing over the following years, that Kathleen believes to be of critical importance in helping her transform and evolve as a teacher in

her ways of thinking about how children learn mathematics and in her pedagogic approach. At this meeting Kathleen approached Amy to ask about the materials she had given her during the summer seminar.

...Amy was speaking and when it was over I went up to her and said “you know I’ve been able to watch my intern do Quick Draw and that’s been really great with the kids but I can’t really figure out how to do all this other stuff.” And she said, “well, why don’t I just come by.” ... So she just came by one day and then she started coming once a week and she was so great because I would do things like, because back then I was into a lot of stuff like “I like the way Mary said ...”, you know, trying to do positive stuff and she would just kind of casually bring me an article on it you know. Not saying you shouldn’t do that so now if I do it at least I know I’m not saying that there are not times when I do just like Alphonse Kohn says you know there are times when you are drawing on something that may not be the best but I try not to do that because it’s not good and I try to do other things.

[Interview data]

After the intern completed her work in the fall semester Kathleen continued to try to implement many of the things that Amy had presented at the seminar and things that they would discuss during Amy’s weekly visits. Although reflection was already a part of Kathleen’s pedagogic practice and she was focused on listening to her students and trying to make sense of their thinking she believes that having someone to talk with about things that had occurred in the classroom and how the student’s had responded to particular tasks was beneficial and helped to further shift her focus to the student’s thinking.

Well I had already been using an audio-cassette recorder to record my students and then I would go home and transcribe it. This was before Amy came to my class and I would listen to it and I started having this awareness that I really wasn't listening to my kids. You know we say the kids don't listen but how do we model that to them. So I had really tried to learn how to listen because it's like when you are teaching you are in this different dimension you know you are up there, standing up there giving out all this information and there is no way you can really listen unless you work at it. Listening to those tapes was really an awakening for me because we, teachers make all these assumptions about what the kids know and we are really wrong a lot of the time ... So after that meeting Amy started coming once a week and then over the summer. ... It was really different. She was really looking for something different when she came to my room, something different than anyone else had that had ever been in my room. You know its just like we really connected. She wasn't looking at "opening" and "closing" and lessons in that way. She wasn't looking for it they were all sitting in rows. But anyway she just kept coming and we kept doing more and more and talking more and more and then the next year she asked if her graduate assistant (Jeff) could come to my room as well. [Interview data]

Working with Amy and her graduate students helped Kathleen evolve further and get closer to what she had been trying to develop over the years with her students in mathematics, something that "meshed" with what she was doing in language arts and throughout the rest of her classroom activities, something in mathematics that supported her constructivist orientation to teaching. As she began to allow her students to

communicate more in mathematics and as they worked together on tasks this provided Kathleen the opportunity to step away from the center, shifting the focus to her students and her being able to listen to them as they worked. Having Amy in her classroom to discuss things with on a weekly basis, and now Jeff in her room several times during the week, provided Kathleen with support and an opportunity to reflect and discuss the happenings of classroom activities with others whose perspectives on teaching and learning were similar to hers. This support, Kathleen believes, was truly instrumental in the transformation she has made in her pedagogic practices.

So all of this was pretty intense and what Jeff did was help me with the idea of building a community and that you just can't have them doing whatever. You know trying to find some sort of balance between them doing whatever they want and then you being in control all the time. You have to have some boundaries.

He was really more into the community, the whole day... really into the kids with their problems and it was good, a different perspective. It really did help to have to sit and talk with Jeff. Also he would read the same things I was reading. I was reading a lot of Alphonse Kohn then and I was taking classes then ... that would help a lot – I would read a book and he would read it or vice versa. It just really made a tremendous difference that reflection and really talking about things with him and with Amy. [Interview data]

In addition to having the opportunity to talk with Amy and her students on a regular and consistent basis Kathleen continued to pursue learning more about constructivism through meeting on a regular basis with a cohort of teachers at the university.

The other thing that helped me a lot during that year ... I was doing those constructivist meetings – we had a group that met so I was getting a lot of support from that end and that really helped me. That’s what’s great about everyone who comes in here – everyone brings a different perspective. So that’s just it and we kept going and now she’s a wonderful friend and I can’t imagine without her.

[Interview data]

Kathleen’s relationship with Amy has continued to develop as a friendship, one in which they share ideas and continue to discuss what is going on with children’s mathematical thinking as part of ongoing research in Kathleen’s classroom. These ongoing discussions have resulted in collaborative inquiry projects with Amy, her students, and Kathleen. Their research has been presented both nationally and locally and published in a variety of journals. The visits to Kathleen’s classroom are ongoing and on a regular weekly basis and also include other graduate students observing and interacting with Kathleen and her students. Kathleen believes the discussions that follow the math lessons to be extremely important for her growth as a teacher and in her continued and ongoing reflection. Focusing on student interactions and student thinking, these conversations serve as debriefing and reflection for Kathleen, a time in which she can share her thoughts with others and listen to different perspectives about tasks and perhaps what questions and tasks might be good to provide further perturbation for the students’ thinking. While there is often more than one person visiting and observing in Kathleen’s room during the course of any given week there is a relationship between Kathleen and those observing that revolves around and extends beyond communication about her student’s thinking and their evolving mathematical understanding. Kathleen possesses an openness in which she

shares her ideas, her classroom, and her students, with others, including her friend and mentor Amy, who in turn share their thoughts and provide validation and support for what Kathleen is doing.

The constraints and limitations Kathleen has faced during her tenure as a teacher have included working with and communicating with parents and her colleagues regarding the uniqueness of what happens in her classroom. Additionally, as Kathleen worked to enact an alternative in her classroom, working to develop something better for and with her students she had to work through her own frustrations, knowing at times what she was trying was not “meshing.” Kathleen’s openness extends to her communications with parents about what she is trying to do in her classroom. She shares with parents that she focuses on developing a community of learners who will be engaged in problem solving. Her goals and beliefs are expressed in her “parent letter” that is sent home at the beginning of the year with each child (see Appendix E for a copy of Kathleen’s parent letter) and again as she talks with parents on parent night and during parent teacher conferences. She believes that listening to parents’ concerns and communicating with them to be of key importance.

I send home parent letters explaining the room because they are not getting all the worksheets and busy work and they need to understand and I use key words in there you know like “we are studying our phonics.” I try to use words I know the parents are comfortable with.... I think as far as dealing with parents you really have to listen, listen to everything they have to say.... I think we should always listen to parents because those are their children and we do need to listen to them and there are some issues that you may have to compromise on like I really

wouldn't do that weekly spelling work because I don't feel like they learn that way but it is kind of one of those things that is an expectation and we have tried to talk about it as a school but sometimes there are those things. [Interview data]

There have been times however when Kathleen's listening to and talking with parents has not alleviated their concerns or helped them to see past their expectations about what "should" be happening in her classroom and the kind of homework they expect to be coming home. In these cases Kathleen has simply invited the parent to spend a few days in her classroom. On a couple of occasions the parents have accepted that invitation and have spent a day or two in her room with their child and their concerns were assuaged.

My not focusing on drill and practice is another thing that parents are sometimes concerned about. Especially when they have maybe had an older child in another teacher's room and they can remember that child doing spelling lists each week and math worksheets but I just feel like I am not going to send home that busy work, I just won't do it. [Interview data]

There have been times when Kathleen really had to "fight" for how she believes children learn in her school and with her colleagues. Beyond the one very public concern regarding what her school was doing with Whole Language and the stance Kathleen had to make at that time for what she believed, she often has to deal with comments and criticisms from her colleagues.

You know sometimes you wish you could see ahead and know everything before you jump in and before I started this whole mess because as this progressed as I started trying more things I realized if I didn't fight for how I believe kids need to be taught or the learning environment it might not be around later on... you have

to justify things to parents and how can you really justify everything... I remember one time another teacher came in my room during one of the class meetings and he said "I think it is great what you are doing Kathleen but I can't believe you are getting away with it." [Interview data]

Kathleen feels that while there have been times during "this process" that she has had to stand up for what she believes she has not really faced many tremendous obstacles. Since she has taught in the same building for more than twenty years she recognizes that "you kind of get a reputation for doing things a certain way and people talk, you know someone will put all their children in a class because they know what you are doing in the room and then they will tell a neighbor, so that has helps."

KATHLEEN'S CURRENT PEDAGOGIC PRACTICES

Kathleen believes that she found what she was searching for, something that would "mesh" or that would fit for the teaching of mathematics and would align with how she believes children learn when she began to learn about a problem centered learning approach for teaching mathematics. She had always believed that learning and teaching should be focused on the individual but as her belief that students learn through constructing meaning for themselves through collaboration developed she recognized that her teaching approach should foster an environment that reflected her beliefs about learning. As her classroom became more "child-centered" through the use of a "whole language" approach to language arts it became obvious to Kathleen that something needed to change in her teaching of mathematics, she was just somewhat unclear about

how this could happen in a way that reconciled with her beliefs. With a problem centered learning approach Kathleen found something that would allow her and her students the space to continue to explore things from their own perspectives just as they were doing in language arts and with their writing during the rest of their school day – it “meshed” in this way. A problem centered learning approach provided Kathleen a way to continue to “invite” her students to participate and explore things from their own perspective in mathematics rather than her showing them or guiding them to methods and procedures that she felt were best.

Kathleen follows a problem centered learning model for the teaching of mathematics, in that most every day the daily math lesson begins with a class opener and then a problem, task, or question for students to investigate or solve, the students then work together in collaborative pairs, and this is followed by a whole class discussion. However, Kathleen’s focus on her student’s communication and argumentation as a way in which they can construct mathematical meaning and become problem solvers extends beyond problem centered learning as an instructional model and beyond mathematics. Kathleen comments about how problem solving is not just about mathematics for her:

I guess what I’m trying to say why the math and the problem solving and all of it makes so much sense to me instead of kids just following procedures is because I see the need for these kids to be able to solve problems and the emotional need for it. ... I tell that story about my student from when I was a reading specialist in secondary school coming back to me, calling me saying that she wanted to thank me because I had talked her out of committing suicide twice. You know I can’t remember us ever talking about anything like suicide but I do remember that she

would talk to me and she was troubled butyou know she was not a problem solver, she thought that some people's lives were perfect and some people had problems. So I think it is being able to value that teachers have a huge responsibility and a large part of that is having to deal with the emotional stuff, you just can't teach out of a textbook. I think you really have to value that.

[Interview data]

For Kathleen, her current pedagogic practices regarding mathematics are an expression of her beliefs about problem solving and her understanding that she is working with children whose ideas and opinions need to be valued and heard. This belief extends beyond mathematics and emerges as a hallmark of the norms that are established as part of Kathleen's classroom community. Kathleen sees an inextricable connection between what her students do in mathematics and the kind of communication that is fostered during the morning meetings that are a part of every school day in her classroom.

Each day in Kathleen's classroom the day begins with a classroom meeting and is a time when the students focus on problems that might have arisen the day before and work together to establish understanding about how they would like certain problems to be handled. Often times issues are raised in the morning meetings from the agenda book, a book that is available to students throughout the day where they can write down things they would like to address with the class or record a problem they have had with another student. The goal of the morning meeting is for students to be heard and to solve their own problems in their own way through talking with one another, arguing, and then ultimately attempting to resolve conflict. The morning meetings create an opportunity at the beginning of each day for students to be actively engaged in problem solving,

questioning and validating one another's ideas and thoughts, and to be an active part in an ongoing conversation that is the essence of their community. During the classroom meetings Kathleen listens intently to each student and together they work to establish social norms for their community that revolve around respect and caring for one another. Kathleen believes that the kind of problem solving and communication that happens during the morning meetings permeates throughout the whole day and plays a particular role in the student's communication and collaboration during their math lesson.

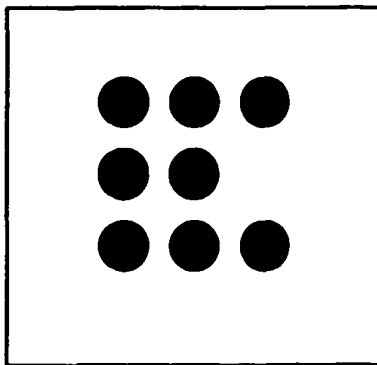
I have had people ask me, like when we've been at a conference or something, if I would like to be responsible for maybe just the mathematics at my school, like the second grade mathematics teacher, and I say "no!" ... I couldn't imagine doing the math without the morning meetings and vice versa. That's where we really can talk about how we are working together and they have to resolve some of their problems which is problem solving. [Interview data]

Kathleen's use of morning meetings in this way works in concert with the kind of collaboration that her students engage in throughout the day so what emerges is a dynamic learning community wherein the students are actively engaged in their own learning, caring towards one another, and are respectful of others' ways of understanding.

Kathleen begins each daily mathematics lesson with a class opener. The mathematics lesson does not follow directly after the morning meeting in the course of the day but the class opener is designed in such a way as to bring the class together again for discussion and the sharing of one another's ways of thinking. The classroom vignette below is an example of one of the class openers that Kathleen uses and the student discussion that arises from such activities. One of Kathleen's broader goals for her

students is for them to construct units, that is, for them to begin to see numbers in groups rather than continue to count using one-to-one correspondence. The following class opener is designed in such a way to promote students perhaps beginning to see numbers as units, groups of numbers, rather than simply counting one-by-one. Kathleen places pennies on the overhead projector while the projector is still turned off. Once the pennies are in place, the overhead is turned on so students can see the pennies projected on the screen but for only a few seconds (see figure 12).

Figure 12. The first arrangement of pennies Kathleen used as a class opener.



Kathleen turned the overhead projector on for a few seconds to display the arrangement of pennies as seen in figure 12. Students had been previously directed that for this activity they were to determine how many pennies were displayed and to share how they determined their answer. Once the overhead was turned off students began to raise their hands to share their answers.

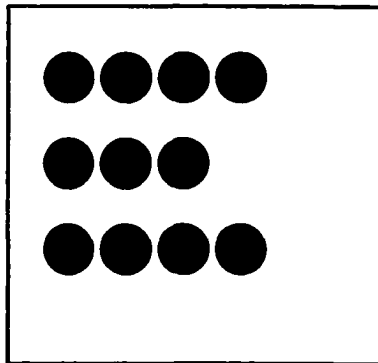
Kevin: I saw 8. 3 on the top, 2 in the middle and 3 on the bottom.

Billy: I saw and "E".

Shelly: I saw 8 like Kevin did and 2 and 3 and 3 makes 8.

This discussion continued for a few more minutes and during this time the students were listening to one another. Several students were sharing their ideas with one another at their seats and many students just automatically called out "8" when they saw the pennies displayed. The classroom was alive with activity and while students were not sitting quietly waiting their turn, Kathleen would direct them to listen to whoever might be sharing his/her thinking at the time. Kathleen decided after some discussion to display another arrangement of pennies (see figure 13).

Figure 13. The second arrangement of pennies Kathleen used.



Melanie: (Calling out.) That's another "E".

Kathleen: How did you see that Melanie?

Melanie: I saw 4 on the top and then 4 on the bottom and then, uh, 3 on the bottom.

Kelly: (Jumping in.) If you take those 2 on the outside away it looks like a plane.

Todd: It looks like school bread.

Kathleen: Show me how.

Todd comes to the overhead and shows the class by blocking out certain pennies with his hand how he sees the arrangement as “school bread.”

Melissa: It looks like a sandwich with a bite out. (Melissa moves up to the overhead to show what she is talking about.)

This activity continued for a few minutes with students sharing their ideas and their different ways of seeing and thinking about the arrangement of pennies before Kathleen asked them to move to the back of the room to talk about their task for the day.

Kathleen’s selection of a variety of classroom openers including “what’s my rule,” “ten frames,” and “balance tasks” taken from *Coming To Know Number* (Wheatley and Reynolds, 1999) all provide opportunities for students to construct mathematical meaning and place primary focus on the students’ mathematical thinking rather than predetermined procedures or methods.

Following the class opener, most everyday, Kathleen has the students move to the back of the room where they sit in a circle and discuss what it is they will be working on during their mathematics time. During this time Kathleen may present an entirely new and different task to the students for them to begin thinking about with their partner or, as on most days, she will talk with them about a task or problem that has been ongoing for several days. Based on her listening to the students, working with them, and trying to understand what it is they are making sense of, Kathleen makes determinations or

selections about what questions may be presented to the students or extensions to the previous days work that might continue to perturb the student's thinking. On some occasions this time is used to briefly discuss what the student's had talked about the previous day during their whole class discussion in order for them to then get back with their partner and continue working on a task that may take several days or even weeks for them to completely resolve.

The tasks Kathleen chooses are ones she believes will allow the students opportunities to work together, discussing and arguing about what might be the best way to investigate or solve the particular problem or problems. Kathleen says that she is not happy with what happens during the mathematics lesson unless the students "have really argued about something." When this happens, in the way that Kathleen is referring to, the students leave after the mathematics lesson to go to music or P.E. class still talking about their task. Arguing in the sense that Kathleen is referring to it is a way of communicating for her students. When they argue they share their ideas with one another, challenge one another's thinking, validate one another's thinking and are provided opportunities to "make sense" of mathematics. Kathleen's students are encouraged to participate in an ongoing conversation that is their mathematics curriculum. To communicate and argue in such a way the students are actively listening to one another and are engaged in reconciling their ways of thinking with others in the class. While the tasks that Kathleen presents to the students may vary from her using children's literature, such as *Measuring Penny* by Loreen Leedy and asking the students questions about how old they think the book is if it was published in 1997 and having them find ten things around the room to measure using non-standard units of measure to

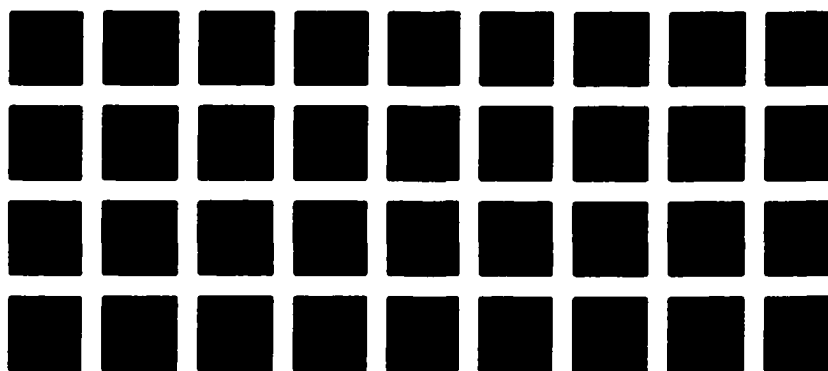
the “Trapezoid task” wherein students spend several weeks constructing varying sizes of triangles from trapezoids (see Cassel, 2000), communication and argumentation permeate all that happens in her classroom. The following classroom vignette is an example of the kinds of non-routine mathematics tasks that Kathleen presents to her students, their working together collaboratively, and the whole class discussion that follows such activities.

This activity followed the students having worked on a problem involving a quilt that was designed as a 6 by 6 array. The students had spent several days working on how to arrange the 36 pieces of the quilt in such a way as “to cover grandpa’s feet” as was part of a children’s book that Kathleen had read to them. Kathleen selected this particular task as a way to continue to perturb her students’ thinking and to provide the continued opportunities to construct ideas about grouping numbers. Additionally, by continuing with another activity using 36 tiles the situation created in this problem gave students the opportunity to make connections between what they were trying to do with this task and their work with the quilt problem. The students’ papers and work with the quilt problem was now displayed on a bulletin board at the front of the room and was readily accessible to the students. The students were asked, during their whole class discussion time following the class opener, to work with their partner using 36 tiles (one-inch squares) and make as many different rectangles as they could come up with. Specifically Kathleen said, “make as many different rectangles as you can that are filled in with 36 tiles. You will need to decide some way to record what you find.” Once a couple of questions about the task were answered students immediately moved to somewhere in the room they and their partner wanted to work. The following is an example of the

discussion that occurred between two boys as they worked together to construct a variety of “filled in” rectangles on the floor at the front of the room.

Josh and Billy had an array of four by nine tiles laid out on the floor (see figure 14).

Figure 14. The four by nine array that Josh and Billy had constructed.



Both boys were seated across from one another each facing a side of the array with nine tiles.

Josh: (Looking at the array) We have nine fours. (Pointing to each row of four and counting each one until he gets to nine.)

Billy: (Agreeing with Josh) I'll put in on the paper. (He proceeds to record their array on paper by drawing rows of squares stopping from time to time to count what he has.)

As Josh worked on representing the array on paper, Josh crawled around on the floor so that he was now looking at the array from a side that had four tiles.

Josh: Hey! (He started counting the tiles.) We have four nines too. We need to draw (pause)

Billy: (Looking up from his work) You didn't move anything did you?

Josh: No!

Both boys paused for a few minutes and then began to discuss if they needed to draw a four by nine array on another sheet of paper.

Josh: I didn't move anything. It's the same thing. Four nines and nine fours. (By now he is counting the tiles again to make sure that there are 36 tiles when they are counted from the where he is now sitting. Billy is finishing the representation on paper.)

Billy: I don't think we have to draw another one because (holding up the paper) we can just hold up our paper and say we have one that is nine fours and then turn it like this (turning the paper) and say here's another one that is four nines.

Josh: But is that the same one?

At this point they decide to leave that issue behind and to try to make another rectangle.

Around the room all the other students are engaged in constructing and recording on paper their triangles. It is very loud in the room as the students are talking with one another and several pairs of students have moved back and forth to the front of the room to look at their quilt patterns.

This kind of engaged activity and collaboration happens daily in Kathleen's classroom. Following students working together on these types of activities is a whole class discussion wherein students may present their solutions and ideas or they may discuss what they are thinking about a particular task or problem. The whole class discussion takes place after students have worked together for about twenty minutes at the back of the room where students gather in a circle on the floor. Much of what happens during the whole class discussion is probably not best described as a discussion but rather more often as an argument between students expressing their ideas and

questioning one another. On the day the students worked on the rectangle task described previously, one pair of girls volunteered to share what they had found during their investigation. While they shared that they had found a two by eighteen rectangle many students burst out saying things like “we didn’t get that, “ “I don’t think that is a rectangle,” and “do you really have thirty-six there.” Both the girls presented then were challenged to defend what they had found and continued to talk about how they thought that what they had was a triangle. The whole class discussion time is not marked by what could be called a conclusion or even by consensus on most days but the students understand that anything discussed or left unresolved can be discussed at a later time. Many days they leave the room for music or P.E. following the whole class discussion still arguing about what was said or presented in the whole class discussion and as far as Kathleen is concerned “that’s when I know it was a good math lesson.”

While described as Kathleen’s current pedagogic practices what Kathleen does in her classroom with her students may not be best discussed as a method or approach but rather as the expression of her orientation about teaching and learning. As her focus has shifted entirely to learning she has moved away from a role as guide or teacher to that of learner along side and with her students and while she does make selections and determinations about certain tasks and questions to present to the solutions it is no longer done as an attempt to try to get the students to a certain defined point or to try to help them understand something predetermined. Rather the selections and determinations about questions to ask the students and tasks for them to explore that she makes are made to infuse the conversation that is ongoing in her classroom and to continually perturb her

students' thinking so they can construct mathematical meaning and make sense of something in their own way.

CHAPTER VI

ANALYSIS AND DISCUSSION

Dewey (1902) states “ there are those who see no alternative between forcing the child from without, or leaving him entirely alone. Seeing no alternative, some choose one mode, some another” (p. 195). Both Kathleen and Wesley have “seen” an alternative. They have envisioned and enacted something alternative in their classrooms, something that is far from the more common traditional methods in mathematics teaching of “forcing the child from without” and yet neither is it “leaving him entirely alone.” Kathleen’s and Wesley’s pedagogic practices have metamorphosed to become something very different and beautiful from where they began. What they do with their students emerges from their orientation about how students learn, an orientation wherein they view themselves as learners along side their students participating in an ongoing conversation, focused entirely on learning. Both their individual stories of transformation are imbued with their desire to understand how children learn and to develop pedagogic practices that would align with those beliefs.

Case study research is designed to answer “how” and “why” questions particular to the case or cases considered. However, in trying to explicitly answer “how” and “why” questions related to any case we run the risk of reducing the complexity of intertwining experiences, events, and individual beliefs and orientations implying that others could reproduce certain behaviors having the same consequences. Much like trying to determine what thread begins or completes an intricate woven tapestry; which one completes the picture, which one could be removed without changing what makes it

inherently and qualitatively different. Some events and experiences, even in retrospect, cannot be fully disentangled and understood, much less reproduced. The discussion in this chapter is certainly not an attempt to disentangle every aspect of these two cases but rather a bringing together of the stories of these two teachers focusing on some common themes that became apparent as both cases were considered as a whole.

The role of mentoring and reflection in the cases of these two teachers' change and transformation will be discussed as well as how both teachers addressed the constraints and limitations they encountered as their teaching practices and orientations evolved. Finally, their enactment of an emergent mathematics curriculum with their students and the inherent role of listening as part of a curriculum that emerges as ongoing conversation, as "that which is among them," (Davis, 1994, p. 269) will be discussed.

Dealing with Constraints and Limitations

Constraints and limitations are an inherent part of education. Often times these constraints are obvious or more "known" obstacles like budget short falls at a state or local level or being reassigned to a grade level that you are not comfortable teaching in a district with oversized classes. However, many times constraints come as consequences of obvious factors like the lack of support from administration and parents; others as in relationship with the more tacit constraints of our own beliefs about how children learn, what children are capable of doing, and what we are willing to do as teachers. Parent, student, and administrative expectations and concerns and outside forces through legislation always have been, and always will be a part of education; they are a part of the nature of schooling and the teaching profession. Teachers face constraints and limitations regarding what they are trying to do from many different directions every year

and every day in their classrooms. Kathleen and Wesley are no different in that regard. What is unique about both these teachers is the way they view obstacles and challenges they face and the way in which they in turn face those challenges.

An important goal of this study was to understand how these teachers, both of whom have envisioned and enacted something different in their classrooms, have addressed constraints and limitations throughout their tenure as teachers. What became a valuable part of understanding how these two teachers have transformed was recognizing that they do not always view what others would see as obstacles or “things” in the way of change or alternatives as limiting or constraining but rather as a natural part of “the game of school” (Wesley, Interview data). As both teachers shared their stories for this study neither one focused on problems or constraints they had faced along the way. Only when purposively asked about challenges they might have faced or obstacles they may have encountered did they share such accounts. Both these teachers had a desire to know and were in search of something or a way to create a better learning environment for their students. In this quest to know they did encounter situations and issues they had to address but they did so without allowing those things to become obstacles to their own transformation.

Both Kathleen and Wesley faced the same kinds of outside forced changes as most teachers generally do. While Wesley teaches in a more affluent district in the state than Kathleen, having a great deal of support with regard to materials and supplies for his classroom, he has had to address very defined expectations from parents and students alike. Kathleen, on the other hand, deals with a very diverse group of students on a daily basis, many of whom have tremendous emotional needs. She too, has had to address

parent concerns as she works to implement change in her teaching approaches. Both teachers have addressed the concerns regarding their alternative approaches to teaching over the past ten years with honesty and open communication. Kathleen and Wesley both share stories of talking with parents, even inviting them into her classroom in Kathleen's case, and attempting on their part to inform parents at the beginning of the year with letters, or a syllabus, that what happens in their rooms is different than other classrooms. In both cases, these teachers have addressed the expectations of others, whether it be administration, colleagues, parents, or students openly, sharing with them that what they are doing comes from a belief that children learn by constructing ideas for themselves and that problem solving and communicating with one another are important for that to happen. Kathleen believes that "listening to parents is key ... they really need to be heard" which is no different than she believes for her students and Wesley shares that he tries to express to parents his desire for their children to be successful, "its just that we are going about it a little bit differently." Both these teachers have transformed their beliefs and practices and have met with obstacles along the way. Each teacher has dealt with those constraints in his or her own way; they have not let constraints be a deterrent in their doing what they believe to be right. Kathleen shares that she believed at one point several years ago that if she "didn't stand up for what was right this kind of teaching environment might not be there later." The one thing that is salient throughout both of these teachers' stories is that their own beliefs about what was or was not possible was not an obstacle for their own transformative change.

Reflection

The case study of these teachers reveals that they were willing to change and possessed a desire for change. In Kathleen's case she was actively seeking something that she felt was better for her students through seemingly most of her career. Reflection is and was a part of both teacher's natural behavior and response to new ideas and things that might happen with their students. Dewey suggests that reflection should be a continual and important part of what happens in teaching; that students and teachers alike should be engaged in continually reflecting on their beliefs and questioning what they *think* to be true in order to develop "habits of the mind" (1991/1910, p. 28). He states that what constitutes reflective thought is "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends" (p. 6). The kind of change that has occurred in both these teachers' beliefs and practice has not come about through a process of simply trying "new" things as they come along to see if they "work" but rather through a process of careful consideration and reflection: as part of an ongoing quest to know and to change. While Wesley says that he doesn't "write down" his reflective thoughts or record them in any way regularly, although he has at times, he is continually reflecting and thinking about what happens or has happened in his classroom with a particular task or conversation that occurred with the students. When he met Allan for the first time at the problem centered learning workshop the question "what is mathematics" is something that he has reflected on and considered since. Kathleen states, "reflection is the main thing with me and it is the main thing in my classroom, I want the students to reflect on what they are thinking as well."

Kathleen and Wesley both made decisions about wanting change and then through reflective inquiry and actively seeking a “better way” they created a process of evolution in their way of thinking about teaching and learning and simultaneously their teaching practices that emerge from those beliefs. This process of actively reflecting on practice in order to better understand what students are making sense of, to make determinations about how to adjust or change practice, and to create a more democratic classroom has been defined by some as praxis. Schwandt (1997) states that “praxis is ... directed at a specific end but its aim is not to produce an object but to realize some morally worthwhile object” (p. 124). Carr (1995) defines the “good” as

[t]he “good” for the sake of which a practice [praxis] is pursued cannot be “made,” it can only be “done.” “Practice” is a form of “doing action” precisely because its end can only be realized through action and can only exist in the action itself. (p. 68)

Praxis then is not reflection for the purpose of a fixed end result. The ends are continually being revised and “played out” differently “as the goods internal to a practice are pursued” (Schwandt, 1997, p. 124). The reflective practice of both Kathleen and Wesley is continual and ongoing. What “plays out” as practice in their classrooms is continually different and being revised as the dynamics of the students personalities interact and change and as the students explore and pursue of variety of problems engaged in reflection themselves. Through praxis, actively reflecting on their students’ understandings and their own practices, Kathleen and Wesley began a process of change and evolution with what they do with their students that is continually ongoing.

Teacher Change

Much of the literature on teacher change suggests that change does not happen easily or readily for most teachers and that it is often stifled when teachers are working for change in isolation (Allison, Cristol, El-Amin, Garling, Hammond, & Pissano, 1988; Walmsley & Adams, 1993). In addition when attempts to change practice are not based in theory and not visualized in the classroom environment the difficulty can be compounded (Ridley, 1990). Weaver and Henke (1992) further suggest that for change to be successful it has to begin with an individual decision on the part of the teacher, it cannot come as something forced from without. Finally, Connelly and Clandinin (1990) contend that most teachers resist extensive change particularly when they find themselves alone in the endeavor. This research, along with Lortie's (1975) suggestion that teacher beliefs and perspectives are formed early in a teacher's career imply that that teacher change is seemingly improbable, particularly after the teacher has advanced a few years into his/her career. However, research on the role of mentoring in teacher change suggests otherwise; that support in teacher change efforts is key.

Change in our beliefs and the practices that emerge from those beliefs can be a difficult endeavor and one that is more challenging to sustain over time. The literature that considers teacher change focusing on the role of mentoring suggests that when teachers make a decision about changing or possess a desire to change and their efforts toward change are supported, change is possible and sustainable. Hudson (1999) states "change must be an individual decision, but it is sustained through support" (p. 17). She also suggests that support may come from teacher's taking college classes or from their administrators and peers. This idea is further elaborated in the work of Poetter,

McKamey, Ritter, and Tisdell (1999) who suggests the role of mentoring is significant in the process of change. Their research focused on mentors who were conducting research in collaboration with classroom teachers. The results of their study indicate that the amount of time spent in the classroom with the classroom teacher required to conduct research with and about the teacher had an impact on the teacher's ability to create change and sustain it. While they believe that mentors working in the classroom can play a very important role in change efforts they discuss what they call a "mentoring mosaic" which is made up of a network of "secondary" mentors. Head, Reiman, Thies-Sprinthall (1992) discuss this network of "secondary" mentors further by stating:

[O]ne's mentoring mosaic can have a great variety and be comprised of a network of secondary mentors: events, situations, and circumstances of life; books one reads; or crises one faces. (p. 15-16).

The notion of a "mentoring mosaic" expands the traditional idea or assumption that mentoring is a relationship wherein "*expert* knowledge flows only from veteran to novice teachers concerning planning curriculum, teaching, and managing the classroom" (Poetter et al., 1999, p. 106) to that of mentoring as a great variety of experiences, people, or events in one's life. Poetter et al. (1999) further states, "the development of a sound teaching style doesn't depend on adopting someone else's successful or appealing style by mimicking, even modeling." Mentoring should provide a safe environment for a teacher's own style and persona to develop and be enhanced. This should involve opportunities for deep reflection, before, during, and after teaching with the mentor and perhaps others. A shift is needed away from the conception of mentoring as relating "expert" knowledge (Poetter et al., 1999). When mentoring is viewed as the transfer of

skills from an “expert” teacher to a “novice” teacher there are problems. Recognizing that people learn by constructing meaning for themselves implicates the idea of a “transfer” of knowledge as not truly possible. Likewise when one does in fact attempt to simply emulate another’s skills or teaching practices their own inventiveness may be hampered. Mentoring as the transfer of “expert” knowledge from one person to the other or when one attempts to mimic another’s methods, is referred to by Eisner (1994) as the tension between automaticity and inventiveness.

[I]f responses are too automatic or routine, if they become too reflexive, the teacher’s ability to invent is hampered. Teaching becomes a series of routine responses rather than an opportunity for ingenuity. (Eisner, 1994, p. 155)

The success of mentoring in teacher change efforts needs to be much about sharing; sharing in a quest for change on both the part of the mentor and the teacher, and perhaps in the sharing of inquiry based research projects (O’Loughlin, 1992; Poetter et al., 1999).

Mutual inquiry and collaboration can provide opportunities for teachers to engage in reflective practices and critical questioning wherein they are actively participating in a process of their own transformation. O’Loughlin (1992) asks the question “how might we enable teachers to experience emancipatory knowledge construction so that they can engage their students in these processes?” In attempting to answer this question he looked to the work of Goodman (1992) who illustrated that in order for emancipatory change to occur there is a need to work with teachers in their schools over an extended period of time. However, simply spending time in a teacher’s classroom is not going to bring about change but rather developing a mentoring, sharing relationship through the process of inquiry. This suggestion echoes that of Poetter et al. (1999), that it is not only

the amount of time but time engaged in mutual inquiry. This mutual inquiry regarding what is happening in the classroom can create a cycle of reflection and practice.

Henderson (2001) discusses what he calls “craft reflection” which he defines as “a teacher’s thinking during recurring cycles of instructional study, application, observation, and reflection” (p. 17). He continues by adding, “craft reflections occur in a context of continuous study on the ‘how-to’s’ of teaching.” What is suggested by O’Loughlin (1992) is something different than “craft reflection”; it is the kind of critical reflection that looks beyond the “how-to’s” of teaching to one’s own beliefs, an emancipatory reflection and inquiry. This is the kind of inquiry and reflection that Wesley and Kathleen were engaged in over time with both Allan and Amy as their relationships developed. Asking questions about the nature of mathematics and how children learn provided opportunities for Wesley and Kathleen to, as suggested by O’Loughlin (1992), “...construct a pedagogical environment in which teachers can experience the power of constructing critical knowledge for themselves” (p. 338).

Embodying a desire for change is certainly a part of both these teacher’s stories. Searching for something, believing that there is a better way to teach mathematics, and being willing to “take risks” for both of these teachers were really the “initial conditions” of their transformation. A willingness and openness to change, not believing that what they were doing in their classrooms to be “the only way” or even “the best way” of approaching instruction created for them the possibility for change. While Kathleen believed that “there had to be a better way” than what she was attempting to do with her students, Wesley initially did not feel frustrated by his teaching approaches. However, his willingness to try things and listen to ideas that “made sense to him” allowed him to

be “perturbed” or for “a disturbance to occur in his world.” In addition to a disposition for change both teachers believe that they had a great deal of support through the mentoring relationships they developed with both Allan and Amy. The support that was consistent and ongoing for several years was characterized by critical reflection and inquiry rather than simply a focus on “teaching methods” or attempts to mimic another’s teaching approaches.

Emergent Mathematics Curriculum

Both Kathleen and Wesley convey that initially when they began trying to change what they were doing in their classrooms in order to provide their students with more meaningful learning opportunities they were implementing new approaches, “taking risks,” and trying out problems with the students. Initially, the focus in their desire for change was on what they could do as the teacher; how they could change, what they could find and give to the students to create an environment more conducive to how they believed children learn. As this process of trying things and searching for things unfolded they were afforded opportunities to listen to their students more. As they began to appreciate what could be gained in listening to their students and in turn encouraging their students to listen to one another a change slowly occurred in their focus. This was a shift in their focus to student learning entirely rather than teaching methods or approaches. This shift marked a change in their orientation.

Kathleen had been thinking about student thinking and creating an environment in her classroom wherein students could approach things in their own way for most of her career. It was just that she could not quite determine how what was happening with her “whole language” approach to language arts with her students could happen for her

students with mathematics. It was this frustration, in not finding something that would “mesh,” that infused her determination to continue seeking and trying until she found what was “right.” It was in this quest for change that she met Amy and thus began to learn about a problem centered learning approach to teaching mathematics. For Wesley, he too was experimenting with some new things in his classroom, encouraging his students to work together more and providing them opportunities to engage in cooperative learning when he was introduced to a problem centered learning approach. For both these teachers what began as a quest to find ways to involve their students in more problem solving, in Kathleen’s words, “to make her classroom more child centered,” and their eventual transitioning to a curriculum based in a problem centered learning approach to teaching has over time evolved as an orientation about teaching: An orientation in which they view themselves as learners along side their students participating in an ongoing conversation in which meanings are negotiated, a conversation that emerges as the curriculum.

What emerges as the curriculum in both these classes is a conversation wherein meanings are negotiated. The conversation is infused by the selection of worthwhile tasks, questioning that continues to perturb student thinking, and the students’ participation through active inquiry and argumentation (Cassel, 2002). Engaging oneself in this kind of mathematics conversation with students requires reflection and a different kind of listening. Rather than listening, as one would who is trying to “guide” or direct students to some predetermined method or solution path, for “correct reasoning”, or listening to correct “incorrect reasoning,” both these teachers listen for the purpose of joining in the conversation. They listen so they too can interact and be part of the

conversation and so they can perhaps add to the conversation in the sense that based on their intuition about certain tasks and student approaches they may be able to further perturb student thinking through questioning and presenting other possibly worthwhile tasks. As both teachers interact with their students it is surprisingly noticeable that neither one of them seems to be saying much at all. They do not teach by telling, they teach by listening (Davis, 1997). The classroom is a place wherein the students are doing much of the talking in order to make sense of their mathematics rather than the teaching being the primary speaker. Both teachers communicate with their students by listening and interacting with them as part of a conversation rather than the “one-way” communication that is characteristic of a traditional approach to teaching in which the teacher “tells” the students what they need to know. Kathleen and Wesley listen to their students with enthusiasm and can often be heard responding to students with “I really hadn’t thought of it that way, that’s interesting.” Both teachers place a value on listening; it is what they do. They encourage their students to listen in the same way so they can fully participate in the conversation as it unfolds, challenging one another’s thinking and providing validation for each other’s ideas. Davis (1997) describes the kind of listening that is critical to an emergent curriculum as “hermeneutical listening.”

The Role of Listening. Carpenter and Fenema (1992) suggest that listening to their students may be a critical behavior for teachers wanting to develop more effective mathematics instruction. Davis (1997) further suggests that “attentiveness to how mathematics teachers listen may be a worthwhile route to pursue as we seek to understand and, consequently, to help teachers better understand their practice” (p. 356).

Davis (1994) believes that listening is not just hearing what someone has said but rather an engaged physical activity.

When two persons converse, for example, it can be *seen* that they are listening to each other as the actions of their bodies become bodily *interactions*. They lean toward and reach out for each other, momentarily unaware that they are violating the Western taboos on proximity, touch, and extended eye contact. They seem to focus in a way that suggests they are oblivious to the noise around them; they attend to each word and to each action as though nothing of importance had occurred prior to the discussion and nothing of importance awaits them at its end. They are unconcerned that their voices are perhaps too loud, their bodies too animated. (p. 268).

So listening is neither “motionless or silent” (p. 269). Listening, as Davis describes it, is an activity in which one is engaged and participating. Traditional approaches to the teaching of mathematics are often centered around the teacher’s telling and presenting predetermined methods and procedures to an audience of quiet, motionless students who are assumed to be attending simply because they are quiet and motionless. Davis claims that this assumption cannot be made, that it is something other than the motionless silence of students that can make us aware that they are listening.

In the classroom ... as the novel is read or as the mathematical principle emerges, the teacher knows the students are listening not because they have ceased to move but because a certain rhythm or harmony is established – there is an awareness that each is immersed in and conducted by the same subject matter. The gazes are

**fixed not on the teacher nor on one another, but on that which is among them
(1994, p.269)**

Both Kathleen and Wesley do not make the assumption that students are attending to a task or engaged in learning unless the students are actively communicating, asking questions, and challenging one another's thinking. Kathleen has considered what it means for students to attend to a particular task and the assumptions teachers make about students. She states,

you know sometimes I can have a kid practically standing on his head back there and one crawling around on the floor and you would think they aren't listening and then out of the blue they will say something or jump in to tell you what they were thinking – it's amazing. [Interview data]

Likewise, Wesley has addressed this issue and discusses it openly with his students. During the early part of the school year a student asked Wesley if they (the class) could go back to the "bridge problem." He responded by saying,

**we probably won't look at that today but really I can't keep you from thinking about it, you can be thinking about it all day today in class and I wouldn't know, just like, Justin back there, can be sitting very quietly and look like he is listening and he may be working on some problem from yesterday in his head, so while I am saying, no, as a class we aren't really going to look at that problem today because I think we may need to continue with the problems we looked at yesterday, I can't assume you won't be thinking about the "bridge" problem.
[Classroom observation]**

Kathleen's room is alive with movement, noise, and activity, children up moving freely around the room and working with their partner wherever they feel is comfortable because she is not bound by assumptions about what listening "looks like." Rather she has developed an awareness or intuition about how children learn and construct things for themselves and that it is "messy." While Wesley's room is more "structured" than Kathleen in the sense that you are not likely to find his middle school students lying on the floor as they work together, there exists a freedom in which the students can learn in their own way. Engaged listening, which implies questioning, responding, querying, and a focus on "what is among them" is fostered in both classrooms (Davis, 1994).

Davis extends his consideration of listening as a lens from which teacher practice can be reinterpreted with his discussion of evaluative, interpretive, and hermeneutical listening. Evaluative listening "tends to forget its own responsibility in interactions (1997, p. 360). This type of listening occurs when the teacher is not listening *to* the students and the listening is focused on the elicitation of correct responses or "guiding" the students to a pre-determined method or solution path. Interpretive listening is described by Davis as listening *to* what learners are saying and perhaps even trying to understand what they are making sense of but still "listening *for* particular responses" (1997, p. 363). The kinds of questions that arise from this type of listening may allow for a greater range of responses than those accompanying evaluative listening but they may not foster much diversity. Hermeneutical listening is the third type of listening that Davis (1997) describes. It is the

sort of listening [that] is an imaginative participation in the formation and transformation of experience. Hermeneutical listening demands the willingness to

interrogate the taken for granted and the prejudices that frame our perceptions and actions. (p. 369-370)

Hermeneutical listening occurs when the teacher becomes “a participant in the exploration of ... mathematics” (p.369). A teacher listening hermeneutically frees herself to participate with her students in the negotiated and evolving nature of interaction rather than simply listening to “take” the students to a certain predetermined point. The alternatives, evaluative and interpretive listening, engage the teacher in attempting to draw students into “*front-end* mathematics” where the “mathematics concepts [are] treated as though they ... are independent of learners and their experience” (p. 339).

Kathleen and Wesley are both participants in exploring mathematics with their students. Hermeneutical listening is part of their orientation about teaching and learning. Both view teaching not as the act of telling and learning as a not as a sequence of actions. They both understand and view learning “in terms of an ongoing ... dance” (Davis, Sumara, and Kieren, 1996, p.153), wherein “individual and collective meanings are seen to evolve in the course of classroom interactions” (Cobb, Jaworski, and Presmeg, 1996, p. 15). Teacher and students are involved in the unfolding of the curriculum as knowledge emerges as meaning as their focus shifts from teaching or the learning of “things” to “that which is among them.”

Changing Our Orientation

If we are to participate with our students in learning, imagining and embracing the possibility that mathematics is subject to change, that it is not fixed and static, we may need a change of perception or a change of orientation. Rorty (1989) suggests that if we hope ever to effect qualitative change in our orientation and action in education we will

need to develop new ways of talking about education, develop new language or a new vocabulary. Fleener (2002) states that Wittgenstein's notion of language-games can provide valuable insights into the complexity of meaning-making in social settings and classroom dynamics. Furthermore, by exploring the language-games of classroom life we can perhaps begin to understand the relationships between teaching, learning, and mathematics. Fleener (2002) discusses language games:

Language-games as a philosophical technique explore the use to which language is put. Thus, rather than answering philosophical questions such as "What is knowledge?" or "What is mathematics?" Wittgenstein used language-games to reveal there is no "thing" which answers ways in which we use language.

"Game" for example can refer to board games, games of strategy, games of strength, games played with pieces, games played by people, and so on. (p. 130)

How we use language often results in our collapsing of concepts into "things." Fleener uses the example of time to illustrate this.

[W]e think of time as an actual entity...[it]is reflected in the language we use.

"How much time did it take to get there?" "What is your hourly wage?" All treat time as a measurable, quantifiable thing. "Telling time," an early concept taught in schools, emphasizes and passes on to our children a mechanical understanding of the quantification of time. It suggests we can point to time the same way we point to a clock....The language-game of time includes all of the ways we discuss time. Our concept of time revealed through these games is the family resemblance of the ways we discuss time in the language-games we play. (p. 131)

Fleener and Reeder (in press) claim that:

Our typical language games in mathematics classrooms treat mathematics like a foreign language and mathematics learning as an intricate game of translating the teacher's or the textbook usage by students who often find the process both unenlightening and discouraging. (p. 3)

Our language-games in mathematics education constrain not only how we can think about mathematics, reducing it to a set of predetermined procedures and "things," but also our teaching of mathematics as the telling and dispensing of these "things."

Considering the language-games of mathematics education, much like Davis' (1997) listening as a framework for interpreting teacher practice, may help us to have "a change of aspect, to see classroom mathematics learning differently" (Fleener & Reeder, in press, p. 3). Wittgenstein (1953) refers to this change of aspect as changing our way of seeing the world. "Changing aspect, and thus, changing or world-picture, entails changing language games and changing, not how we see, but how we see-as" (Fleener & Reeder, p. 3).

When the language-games of both Wesley's and Kathleen's classrooms are explored it becomes apparent that communication is the medium from which meaning emerges (Sfard, 2000). Mathematical meaning emerges in their classrooms from a rhythm of discourse communication. Communication in each of their classrooms is a process that includes "listening as well as verbal and non-verbal behaviors and denies that meaning is something bartered, exchanged, or passed on from one person to another (Fleener, Carter, and Reeder, in press, p. 4).

Kathleen and Wesley have experienced a change in their orientation, a way of seeing-as or an orientation about mathematics teaching and learning that focuses entirely

on learning. While their individual processes of change initially included searching for teaching approaches that would allow their students to collaboratively work together and explore problems from their own individual perspectives, both teachers have transformed their ways of thinking about mathematics learning and teaching. Their understandings of and the use of worthwhile mathematical tasks, problem centered learning, and hermeneutical listening have evolved over time. Problem centered learning has become for both of them a way of seeing the dynamics of teaching and learning as the unfolding of conversation infused and enriched by worthwhile mathematical tasks and the interactive nature of hermeneutical listening rather than a methodology.

Implications

The cases of both these teachers suggest some possible implications for teacher preparation and ongoing professional development. Teacher education programs should consider inviting preservice and in-service teachers into discussion concerning the “big” ideas and issues that are a part of their practice - to discuss and ask questions such as what are our beliefs about the nature of mathematics and how does our answer to that question affect the teaching and learning of mathematics. Rather than solely focusing on a particular learning theory, subject matter knowledge, and specific teaching approaches, teacher education programs should invite teachers to listen to their students, to participate in mathematics learning with their students. As Davis (1997) suggests, preservice and in-service teachers should perhaps focus less on “anticipated routes” and place value on “paths laid in walking” so the “desire for certainty” may be “dissolved in a recognition of inevitable ambiguity” (p. 354).

The importance of strong mentor relationships is also evidenced as important to teacher transformation in both these cases. Kathleen and Wesley were both on a quest to know which brought about significant transformation and change in who they are as teachers. Their quest to know and desire for change exemplifies what Davis (1997) refers to as dissolving a desire for certainty and a “recognition of inevitable ambiguity” (p. 354). In their desire for change and searching for a way in which they could create a better learning environment for their students they developed relationships that would become an important part of their transformations. The mentoring relationships in both Kathleen’s and Wesley’s stories are not those characterized by the “transfer” of expert knowledge from one to another but rather by the developing of collaborative relationships built around mutual inquiry and reflection. An important component in both these cases is the teacher’s involvement in asking questions and reflection as a part of their relationship with their mentor. Both teachers comment on the importance of having someone to talk with, share ideas with, and someone to reflect with about the happenings of the classroom. In this way, the suggestion that the kind of mentoring that may be instrumental in teacher change, as was indicated by both these cases, is mentoring that is about building collaborative, sharing relationships over time. It is also important to understand that these relationships were not established by the matching of an “expert” teacher with a “novice” teacher by an administrator or university supervisor but rather Kathleen and Wesley sought out and developed these relationships in their own searching and quest to know.

Possibilities for Further Research

While this research focused on two different teachers from different school districts and communities they both described their teaching situations similarly as being supportive. They both identify the support from the strong collaborative mentor relationships they developed with Allan and Amy as being key in their transformation. This kind of collaborative mentoring and support, characterized by mutual inquiry with someone from outside the school may not be possible for all teachers. Further research should examine how teacher change survives or flounders in an environment less supportive of change efforts. Likewise, research that would explore the possibility for teachers to provide support for one another through the developing of collaborative teacher communities may also warranted. For example, in Wesley's teacher story he describes his working with two other teachers to reflect on how their students approach various problems, lesson planning, sharing of ideas, and the developing of worthwhile tasks for their students. He believes this collaborative relationship to be important in sustaining what he is trying to do with his students but there is also an element of his mentoring these teachers in their mutual collaboration and inquiry. Exploring whether it is possible to affect and sustain change through communities of teachers, such as the one described by Wesley, whose relationships are characterized by collaboration, the sharing of common beliefs, and mutual inquiry is important.

Additionally, central to the efforts and desire for change in school mathematics is student learning. Further research related to student learning in classroom cultures such as these described by Kathleen and Wesley is needed. The students in both these classrooms are engaged in reflective thinking, listening to one another, presenting and

justifying their thinking and reasoning, and creative thinking. While many of the aspects of student learning in inquiry based classrooms have been explored there is still much to be considered. For example, the role creative thinking plays in mathematics learning and the implications for a teacher's pedagogic practice when he or she encourages creative thinking in a mathematics classroom, may warrant further study. In both these cases the teachers are not merely enacting a "constructivist" teaching approach; choosing tasks and investigations that will allow their students opportunities to construct a particular and perhaps predetermined concept. They are participating in learning and the making of meaning with their students. What they are engaging in with their students has "emancipatory potential" (O'Loughlin, 1992) for the students as their questions and ways of exploring certain problems become part of the conversation and the direction or directions that are followed and "hashed out." Student learning in these types of possibly transformative and empowering classroom environments needs further study.

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APPENDIX A

CONSENT AND ASSENT FORMS

The following pages are the consent and assent forms signed by Kathleen, her students, and their parents.

Teacher Informed Consent Form

Dear (Teacher) _____,

As a doctoral student in the College of Education at the University of Oklahoma, I am planning a research project entitled **Emergent Mathematics: A Case Study of Two Teachers** which is being conducted under the auspices of the University of Oklahoma, Norman Campus. Dr. M. Jayne Fleener from the College of Education is the chair of my dissertation committee and is helping to assist and guide my work. As part of my research I would like permission to interview you and to observe in your classroom five to six times during this fall semester as part of my data collection process.

I would like to interview you initially in order to gather some basic background information and will plan to audio tape this interview for the purpose of accuracy in my data. In addition, I will be asking you to provide me with some written items throughout the semester which may include answering some questions as they arise (we may want to do this via e-mail for your convenience), lesson plans and/or reflections on what happened in the classroom, and any handouts or activity sheets you provide for the students. Throughout the data collection process, it may also be necessary for us to have several informal interviews which would only be done at times best suited for you and your schedule. Lastly, I am planning a final interview once all data is collected to provide you an opportunity to clarify and/or add to my findings regarding your pedagogic practices and the classroom observations. The interviews, both initial and final will be scheduled at your convenience and we can negotiate the form and length of the other items mentioned.

I would like to visit your classroom on a bi-weekly basis up to five or six times during the fall semester. During this time I plan to observe you and your students during daily classroom interactions. I anticipate that this experience will not cause any harm or disruption whatsoever for you or any of your students. To allow for later analysis by myself as researcher I would also like to videotape two or more of the class sessions on the days that I am there to observe. If at any time any of the students wish to stop the observations and the videotaping they may do so. All students, and their parents, will be aware that they may cease participating at any time and without penalty. Similarly you may withdraw at any time from any or all parts of this project. None of your students' or your name will be used in the data collection process. Please let me emphasize that the use of these tapes and data from the observations are for professional purposes only. No public showing of the videos will be considered and strict confidentiality of all participants will be observed.

If for any reason you have any reservations about this please call me at (405) 325-1498. You may also contact the Institutional Review Board for the University of Oklahoma, Norman Campus at (405) 325-4757. I am excited about learning from you and your students this fall and look forward to receiving your approval form. I have enclosed a stamped addressed envelope for you to use in returning the form and have provided an additional copy for your records.

Sincerely,

Stacy Reeder
Doctoral Student, Mathematics Education

Teacher's Consent Form for Interviews and Observations

I, _____, agree to participate in interviews and allow my class to be observed as described in the attached letter.

Signed: _____ Classroom Teacher

Date: _____

Teacher's Consent Form for use of Videotape

I, _____, agree that videotapes for the purpose of data collection can be made in my classroom while my students work on mathematical activities. I understand that if I later change my mind about the videotaping I can withdraw this permission.

Signed: _____ Classroom Teacher

Date: _____

Student Informed Assent Form

(Parent/Guardian: Would you please read this to or with your child as needed)

Dear _____,

As a mathematics teacher working at the University of Oklahoma I am interested in how teachers can better help students learn mathematics. I am planning a research project entitled **Emergent Mathematics: A Case Study of Two Teachers** which is being conducted under the auspices of the University of Oklahoma, Norman Campus. For this project I would like to investigate the pedagogic practices of your mathematics teacher, in other words, the experiences and activities your teacher involves you in. I will be visiting your class five to six times to observe your teacher as she teaches and the activities that you and your classmates work on. I will take notes while I am visiting your class and may find it necessary to video tape a few of your class sessions so I can make sure my research is accurate and complete.

The observations and videotaping I plan to do will focus mainly on your teacher, her interaction with you and your classmates, and also on some of the activities you work on during class. I don't think that my observations or the videotaping of your mathematics class will interrupt the things you usually do in class. I will not use your real name, your teacher's real name, or the name of your school or town when I do this. Instead I will ask you to make up a name for me to use in the place of your own.

Even though I don't think my visits to your classroom this fall will change any of the normal activities your teacher plans, I do want you to know that your agreeing to participate in the videotaping is your choice. If at any time during my visits to your class you wish to stop being taped all you have to do is say so. There is no penalty connected to withdrawing.

I look forward to visiting in your class this fall to observe and learn from you and your teacher. If you have any questions about this please let me know when I visit your class or call me at (405) 325-1498. You may also contact the Institutional Review Board for the University of Oklahoma, Norman Campus at (405) 325-4757.

Sincerely,

Stacy Reeder
Doctoral Student, Mathematics Education

Approval Form for the use of Videotape

I, _____, agree to be videotaped and observed in my classroom while I work on mathematics activities. I understand that if I later change my mind about being taped I can withdraw this permission.

Signed: _____, Student Date: _____

Parent Informed Consent Form

Dear Parent/Guardian of _____,

As a doctoral student in the College of Education at the University of Oklahoma, I am planning a research project entitled *Emergent Mathematics: A Case Study of Two Teachers* which is being conducted under the auspices of the University of Oklahoma, Norman Campus. Dr. M. Jayne Fleener from the College of Education is the chair of my dissertation committee and is helping to assist and guide my work. As part of my research I would like permission to videotape your child's class two or more times during the fall semester. The focus of my research is your child's teacher and the kinds of activities she involves the students in while they learn mathematics. I will be observing the mathematics class five or six times this fall and feel it is important for the purposes of accuracy and my own understanding that I videotape some of the class sessions.

I anticipate that my visits to the classroom will in no way interrupt the normal activities of your child's class. I also see no foreseeable risks for your child if you choose to allow my videotaping of his/her mathematics activities. While I believe your child's participation in this project will greatly help educators improve mathematics instruction for elementary students, your child's participation is completely voluntary. If at any time your child wishes to not be videotaped, all he/she has to do is say so. Your child has the right to withdraw from participation and there is no penalty connected to withdrawing from participating.

Strict confidentiality of your child's participation will be observed. Your child's name will not be used and the tape will be edited to preserve his/her anonymity whenever possible. Likewise, your child's teacher's name will not be used nor will the name of the school or location. Please let me emphasize that the use of the tape and data from the observations are for professional purposes only. No public showing of the video will be considered.

I am excited about working with your child's teacher and learning from her and the activities her class is involved in and am hopeful that the understanding I gain will be of great benefit for other mathematics teachers. If for any reason you have reservations about these requests please call me at (405) 325-1498. You may also contact the Institutional Review Board for the University of Oklahoma, Norman Campus at (405) 325-4757. I look forward to receiving your approval form and have enclosed a stamped addressed envelope for you to use in returning your and your child's forms. I have also provided an additional copy for your records.

Sincerely,

Stacy Reeder
Doctoral Student, Mathematics Education

Approval Form for the use of Videotape

I give permission for my child, _____, to be observed and videotaped as described above as he/she works on mathematical activities. I understand that I may withdraw this permission at any time.

Signed: _____, Parent/Gaurdian Date: _____

APPENDIX B

CONSENT AND ASSENT FORMS

The following pages are the consent and assent forms signed by Wesley, his students, and their parents.

Teacher Informed Consent Form

Dear (Teacher) _____,

As a doctoral student in the College of Education at the University of Oklahoma, I am planning a research project entitled **Emergent Mathematics: A Case Study of Two Teachers** which is being conducted under the auspices of the University of Oklahoma, Norman Campus. Dr. M. Jayne Fleener from the College of Education is the chair of my dissertation committee and is helping to assist and guide my work. As part of my research I would like permission to interview you and to observe in your classroom five to six times during this fall semester as part of my data collection process.

I would like to interview you initially in order to gather some basic background information and will plan to audio tape this interview for the purpose of accuracy in my data. In addition, I will be asking you to provide me with some written items throughout the semester which may include answering some questions as they arise (we may want to do this via e-mail for your convenience), lesson plans and/or reflections on what happened in the classroom, and any handouts or activity sheets you provide for the students. Throughout the data collection process, it may also be necessary for us to have several informal interviews which would only be done at times best suited for you and your schedule. Lastly, I am planning a final interview once all data is collected to provide you an opportunity to clarify and/or add to my findings regarding your pedagogic practices and the classroom observations. The interviews, both initial and final will be scheduled at your convenience and we can negotiate the form and length of the other items mentioned.

I would like to visit your classroom on a bi-weekly basis up to five or six times during the fall semester. During this time I plan to observe you and your students during daily classroom interactions. I anticipate that this experience will not cause any harm or disruption whatsoever for you or any of your students. To allow for later analysis by myself as researcher I would also like to videotape two or more of the class sessions on the days that I am there to observe. If at any time any of the students wish to stop the observations and the videotaping they may do so. All students, and their parents, will be aware that they may cease participating at any time and without penalty. Similarly you may withdraw at any time from any or all parts of this project.

Also, in order to communicate the results of this research more effectively within the mathematics education community I would like your permission to show selected portions of the videotapes. None of your students' or your name will be used and tapes will be edited to preserve complete anonymity whenever possible. Please let me emphasize that the use of these tapes and data from the observations are for professional purposes only. No public showing of the videos will be considered and strict confidentiality of all participants will be observed.

If for any reason you have any reservations about this please call me at (405) 325-1498. You may also contact the Institutional Review Board for the University of Oklahoma, Norman Campus at (405) 325-4757. I am excited about learning from you and your students this fall and look forward to receiving your approval form. I have enclosed a stamped addressed envelope for you to use in returning the form and have provided an additional copy for your records.

Sincerely,

Stacy Reeder
Doctoral Student, Mathematics Education

Teacher's Consent Form for Interviews and Observations

I, _____, agree to participate in interviews and allow my class to be observed as described in the attached letter.

Signed: _____ Classroom Teacher

Date: _____

Teacher's Consent Form for use of Videotape

I, _____, agree that videotapes of me can be used to help teachers better understand how students learn mathematics. I understand that if I later change my mind about the tapes being shown to other teachers I can withdraw this permission.

Signed: _____ Classroom Teacher

Date: _____

Student Informed Assent Form

Dear _____,

As a mathematics teacher working at the University of Oklahoma I am interested in how teachers can better help students learn mathematics. I am planning a research project entitled **Emergent Mathematics: A Case Study of Two Teachers** which is being conducted under the auspices of the University of Oklahoma, Norman Campus. For this project I would like to investigate the pedagogic practices of your mathematics teacher, in other words, the experiences and activities your teacher involves you in. In order to do this I will be visiting your class five or six times this fall. When I visit your class I will be observing your teacher as he teaches as well as the activities that you and your classmates work on. I will take notes while I am visiting your class and may find it necessary to video tape a few of your class sessions so I can make sure my research is accurate and complete.

The observations and videotaping I plan to do will focus mainly on your teacher, his interaction with you and your classmates, and also on some of the activities you work on during class. I do not anticipate that my observations or the videotaping of your mathematics class will interrupt the things you usually do in class. When I finish my time observing your class it is my hope that the things I learn will be beneficial for other mathematics teachers to learn about and understand. Sometimes, when I talk with other teachers about activities students can be involved in to help them learn mathematics, it is helpful if I can show them examples. So I would like permission to show parts of the videotape when I think it might help teachers to understand better. I will not use your real name, your teacher's real name, or the name of your school or town when I do this. Instead I will ask you to make up a name for me to use in the place of your own.

Although I do not anticipate that my visits to your classroom this fall will interrupt any of the normal activities your teacher plans, I do want you to know that your agreeing to participate in the videotaping is voluntary. If at any time during my visits to your class you wish to stop being taped all you have to do is say so. There is no penalty connected to withdrawing.

I look forward to visiting in your class this fall to observe and learn from you and your teacher. If you have any questions about this please let me know when I visit your class or call me at (405) 325-1498. You may also contact the Institutional Review Board for the University of Oklahoma, Norman Campus at (405) 325-4757.

Sincerely,

Stacy Reeder
Doctoral Student, Mathematics Education

Approval Form for the use of Videotape

I, _____, agree that the videotape can be used to help teachers to understand how students can better learn mathematics. I understand that if I later change my mind about the tape being shown to other teachers I can withdraw this permission.

Signed: _____, Student Date: _____

Parent Informed Consent Form

Dear Parent/Guardian of _____,

As a doctoral student in the College of Education at the University of Oklahoma, I am planning a research project entitled Emergent Mathematics: A Case Study of Two Teachers which is being conducted under the auspices of the University of Oklahoma, Norman Campus. Dr. M. Jayne Fleener from the College of Education is the chair of my dissertation committee and is helping to assist and guide my work. As part of my research I would like permission to videotape your child's mathematics class two or more times during the fall semester. The focus of my research is your child's teacher and the kinds of activities he involves the students in while they learn mathematics. I will be observing the mathematics class five or six times this fall and feel it is important for the purposes of accuracy and my own understanding that I videotape some of the class sessions.

I anticipate that my visits to the classroom will in no way interrupt the normal activities of your child's mathematics class. I also see no foreseeable risks for your child if you choose to allow my videotaping of his/her mathematics class activities. While I believe your child's participation in this project will greatly help educators improve mathematics instruction for middle school students, your child's participation is completely voluntary. If at any time your child wishes to not be videotaped, all he/she has to do is say so. Your child has the right to withdraw from participation and there is no penalty connected to withdrawing from participating.

Also, in order to communicate the results of this research more effectively within the mathematics education community I would like your child's and your permission to show selected portions of the videotape. Your child's name will not be used and the tape will be edited to preserve his/her anonymity whenever possible. Likewise, your child's teacher's name will not be used nor will the name of the school or location. Please let me emphasize that the use of the tape and data from the observations are for professional purposes only. No public showing of the video will be considered. Strict confidentiality of your child's participation will be observed.

I am excited about working with your child's mathematics teacher and learning from him and the activities his classes are involved in and am hopeful that the understanding I gain will be of great benefit for other mathematics teachers. If for any reason you have reservations about these requests please call me at (405) 325-1498. You may also contact the Institutional Review Board for the University of Oklahoma, Norman Campus at (405) 325-4757. I look forward to receiving your approval form and have enclosed a stamped addressed envelope for you to use in returning your and your child's forms. I have also provided an additional copy for your records.

Sincerely,

Stacy Reeder
Doctoral Student, Mathematics Education

Approval Form for the use of Videotape

I give permission for a videotape made of my child, _____, to be used to help teachers to understand how students can better learn mathematics. I understand that I may withdraw this permission at any time.

Signed: _____, Parent/Guardian Date: _____

APPENDIX C

INITIAL INTERVIEW GUIDE FOR BOTH PARTICIPANT TEACHERS

The following page is a copy of the initial interview guide for interviewing both participant teachers.

Initial Interview Guide

- 1. Is there a pseudonym you would like me to use for you in this study?**
- 2. Do you have any questions about this study?**
- 3. Tell me about your teacher preparation (college education)? When? Where?**
- 4. How many years have you been teaching? Where? How many years at each place? What were your teaching assignments?**
- 5. Why did you want to become a teacher? A mathematics teacher?**
- 6. Have those reasons changed through your experience as a teacher?**
- 7. Tell me about some of the awards and recognitions you have received as a teacher.**
- 8. Going back to your teacher education (college preparation), does what you learned then in your methods classes support or align with what you do with you students now? Did it when you began your career as a teacher?**
- 9. Tell me about what you do with your students (describe your pedagogic practices).**

APPENDIX D

ARTIFACTS PERTAINING TO WESLEY'S CASE

The following pages include copies of Wesley's class syllabi, the first two pages of the problem solving booklet he developed for his students, and examples of class opener activities.

7TH GRADE ALGEBRA

Course Syllabus

2000-2001 School Year

Instructor: [REDACTED]

Office: [REDACTED] Middle School, [REDACTED]
[REDACTED]

E-mail: [REDACTED]

Office Hours: MTWTF 9:35 -11:20 AM

Assistance: Students who are in need of additional help should make arrangements to meet with the instructor before or after school. There is no charge for this service.

Classroom Structure: This course will differ from the conventional approach in both method and content. This class will use a **Problem-Centered Learning** approach. Problem-Centered Learning is an instructional strategy that involves students in the learning process by having them work together in small groups on problematic activities. These tasks encourage students to ask questions and make decisions, thereby promoting discussion and communication within the group. Students use problem-solving strategies to develop their own methods to solve these problems. After the groups have completed the problem, the class discusses the problem. Students explain how they solved the problem and their solution methods to the class. As different solutions are presented, the class now has the opportunity to discuss the various solutions presented. Students listen to other class members' explanations, indicate if they agree, disagree, or if they do not understand. The goal of the class is to work toward total agreement. Whole class discussion allows students to discover that there is more than one way to solve a problem. Students are exposed to many different methods and strategies by listening to peers describing other solutions.

Goals: The main goal of this algebra class will focus on developing an understanding of patterns, relations, and functions with an emphasis of *thinking in variables*. Additional goals of this class are to help students develop mathematical reasoning, problem solving, an understanding of fundamental algebraic concepts, and the ability to effectively use calculators and computers.

Grading and Evaluation: The student's academic performance will be based on homework, tests, and classroom performance. The following scale will determine the final letter grades:

98%-100%= A+	83%-87% = B	70%-72% = C-	0% -59% = F
93%-97% = A	80%-82% = B-	68%-69% = D+	
90%-92% = A-	78%-79% = C+	63%-67% = D	
88%-89% = B+	73%-77% = C	60%-62% = D-	

In addition to scholarship, such factors as initiative, attitude, cooperation, persistence, participation, and individual improvement will be a factor in the final professional judgment of the instructor in assigning final letter grades.

Classroom Environment: This mathematics classroom will have a business-like atmosphere in which the study of mathematics has first priority. Collaboration (working and talking) with your assigned partner is necessary and encouraged, but you should not distract or disturb the other classmates in the classroom with unnecessary disruptions. The Team Discipline Plan will be used to address the unwillingness or failure of a student to observe these expectations.

Student Handbook: Information regarding conduct, student behavior, student appearance, gum chewing, etc., is contained in the [REDACTED] Middle School Student Handbook. You are responsible for reading and understanding the rules and information contained in this document. You should also conduct yourself accordingly not only in the mathematics classroom but all during the school day.

Tardy: You are to be sitting in your assigned seat when class is scheduled to begin. You will be considered tardy if you are not, and discipline will be administered according to the Student Handbook.

Start of Class: The beginning of class is not a time to visit. After being seated, you should prepare yourself for the beginning of class and the **Start of Class Activity**.

Textbooks: *Glencoe Algebra*, the APSW (Algebra Problem Solving Workbook) and the MSPSW (Middle School Mathematics Problem Solving Workbook).

Classroom Materials: The following materials should be brought to class daily:

- Pencil.
- Notebook paper: standard-sized lined notebook paper is preferred.
- Your textbook and APSW or MSPSW workbooks.

Calculator: Calculators are available in the classroom for each student. This class will use the Texas Instruments Model 83+ Graphing Calculator.

The Instructor and his Philosophy Statement: Mr. [REDACTED] as he is more commonly known, is a native Oklahoman. He graduated from [REDACTED] High School in 1970 and also attended public schools in both [REDACTED]. He graduated from [REDACTED] University in 1978 with a Bachelor of Science Degree in Education. That same year he married his wife [REDACTED], and also began his teaching career with the

████████ School District. He became a ██████████ in August of 1982, and this school year marks his 19th year in the ██████████ Public Schools teaching mathematics to middle school students. Mr. ██████ firmly believes that *all students can enjoy mathematics and achieve success.*

7TH GRADE PRE-ALGEBRA

Course Syllabus

2000-2001 School Year

Instructor: [REDACTED]

Office: [REDACTED] Middle School, [REDACTED]
[REDACTED]

E-mail: [REDACTED] **Office Hours:** MTWTF 9:35 -11:20 AM

Assistance: Students who are in need of additional help should make arrangements to meet with the instructor before or after school. There is no charge for this service.

Classroom Structure: This course will differ from the conventional approach in both method and content. This class will use a Problem-Centered Learning approach. Problem-Centered Learning is an instructional strategy that involves students in the learning process by having them work together in small groups on problematic activities. These tasks encourage students to ask questions and make decisions, thereby promoting discussion and communication within the group. Students use problem-solving strategies to develop their own methods to solve these problems. After the groups have completed the problem, the class discusses the problem. Students explain how they solved the problem and their solution methods to the class. As different solutions are presented, the class now has the opportunity to discuss the various solutions presented. Students listen to other class members' explanations, indicate if they agree, disagree, or if they do not understand. The goal of the class is to work toward total agreement. Whole class discussion allows students to discover that there is more than one way to solve a problem. Students are exposed to many different methods and strategies by listening to peers describing other solutions.

Goals: The main focus of the class will be to prepare students for the [REDACTED] High School Algebra 1 course. Additional goals of this class are to help students develop mathematical reasoning, problem solving, an understanding of fundamental-number concepts, and the ability to effectively use calculators and computers. Moreover, it is the expectation of the instructor that each student will develop a depth of understanding while also acquiring a certain level of fluency and facility in each of the content strands listed below:

7th Grade Pre-Algebra:

- Mental Arithmetic and Estimation
- Problem Solving & Reasoning

- Spatial Visualization
- Whole numbers and integers
- Rational number concepts
- Percents
- Ratio & Proportions
- Data Analysis, Probability & Statistics
- Geometry, Measurement & Algebra

Grading and Evaluation: The student's academic performance will be based on homework, tests, and classroom performance. The following scale will determine the final letter grades:

98%-100% = A+	83%-87% = B	70%-72% = C-	0% -59% = F
93%-97% = A	80%-82% = B-	68%-69% = D+	
90%-92% = A-	78%-79% = C+	63%-67% = D	
88%-89% = B+	73%-77% = C	60%-62% = D-	

In addition to scholarship, such factors as initiative, attitude, cooperation, persistence, participation, and individual improvement will be a factor in the final professional judgment of the instructor in assigning final letter grades.

Classroom Environment: This mathematics classroom will have a business-like atmosphere in which the study of mathematics has first priority. Collaboration (working and talking) with your assigned partner is necessary and encouraged, but you should not distract or disturb the other classmates in the classroom with unnecessary disruptions. The Team Discipline Plan will be used to address the unwillingness or failure of a student to observe these expectations.

Student Handbook: Information regarding conduct, student behavior, student appearance, gum chewing, etc., is contained in the [REDACTED] Middle School Student Handbook. You are responsible for reading and understanding the rules and information contained in this document. You should also conduct yourself accordingly not only in the mathematics classroom but all during the school day.

Tardy: You are to be sitting in your assigned seat when class is scheduled to begin. You will be considered tardy if you are not, and discipline will be administered according to the Student Handbook.

Start of Class: The beginning of class is not a time to visit. After being seated, you should prepare yourself for the beginning of class and the Start of Class Activity.

Textbooks: *Glencoe Pre-Algebra*, the APSW (Algebra Problem Solving Workbook) and the MSPSW (Middle School Mathematics Problem Solving Workbook).

Classroom Materials: The following materials should be brought to class daily:

- Pencil.
- Notebook paper: Standard-sized lined notebook paper is preferred.
- Pre-Algebra textbook and APSW or MSPSW workbooks.

Calculator: Calculators are available in the classroom for each student. This class will use the Texas Instruments Model 83+ Graphing Calculator.

The Instructor and his Philosophy Statement: Mr. [REDACTED] as he is more commonly known, is a native Oklahoman. He graduated from [REDACTED] High School in 1970 and also attended public schools in both [REDACTED]. He graduated from [REDACTED] University in 1978 with a Bachelor of Science Degree in Education. That same year he married his wife [REDACTED], and also began his teaching career with the [REDACTED] School District. He became a [REDACTED] in August of 1982, and this school year marks his 19th year in the [REDACTED] Public Schools teaching mathematics to middle school students. Mr. [REDACTED] firmly believes that *all students can enjoy mathematics and achieve success.*

A HELPFUL GUIDE TO SOLVING PROBLEMS

ALWAYS REMEMBER THESE FUNDAMENTALS:

- Mathematics is the activity of constructing patterns and relationships, NOT a collection of procedures to be memorized and practiced.
- Asking questions is more important than answering them.
- In this mathematics class, mistakes are viewed as opportunities to learn.
- Successful problem solvers are resourceful and persistent in their search for a solution.
- The first key to success in mathematics is to ask the right question.
- The best way to explain it is to do it.
- Imagination is more important than knowledge.
- Learning without thinking is useless.
- Happy is the person who gets to know the reasons for things.
- The task is not so much to see what no one has yet seen; but to think.

TO UNDERSTAND A TASK, MAKE SURE TO:

- Read the problem carefully.
- Decide what you're looking for.
- Find the important information.

TO DEVELOP A PLAN OF ATTACK, USE THESE:

- Guess and check.
- Draw a picture.
- Look for a pattern.
- Make a model.
- Act it out.
- Use easier numbers.
- Write a number sentence.
- Make an organized list.
- Make a table or chart.
- Use logic.
- Work backwards.

BEFORE YOU COMPLETE YOUR WORK, ANSWER THE FOLLOWING QUESTIONS:

- Have I used all the important information?
- Did I check my arithmetic?
- Does my answer make sense?
- Did I write my answer in a complete sentence?
- Does my partner fully understand and agree with the solution?

WHAT IS VALUED IN THIS MATH CLASS?

- Powerful thinking.
- Tenacious behavior.
- An eagerness to learn.
- Persistence.
- Confidence.
- Cooperation.
- Communication.
- Competence.

WHAT IS EXPECTED IN THIS MATH CLASS?

- Expect to be puzzled or stumped.
- Expect to work together.
- Expect to negotiate your methods and understanding with others.
- Expect to explain your thinking to others.
- Expect a task/problem to require investigation.
- Expect a task/problem to require time.
- Expect the possibility of more than one solution or answer.

CLASSROOM DISCUSSION OBLIGATIONS

- Listen intently to other class member's explanations.
- Indicate agreement, disagreement, or your failure to understand.
- In the case of conflicting interpretations, work toward a consensus.

GROUP WORK PROCEDURES

- You are responsible for your own work and also for the results of your group.
- If you have a question, ask your teammate. Do not ask your teacher.
- You must be willing to help if your partner asks for help.

- If no one in your group can answer your question, agree on a consensus question. Have the group spokesperson raise their hand and ask for help from the teacher.
- Work only with the members of your own group. Do not talk or ask questions of the members of other groups.
- Stay focused and work only on your assigned task. If you complete your assigned task, start another task/problem that both you and your teammate agree upon.

EFFECTIVE GROUP MEMBERS

- Cooperate by working together with their partner.
- Ask questions. They also listen carefully to their partner.
- Show an interest in what the other person is thinking.
- Will not criticize another group members' question or ideas.
- Are flexible and willing to change.
- Will make sure that his/her partner understands the solution.
- Agree to a group solution/s for each problem.
- Share the leadership of the group and other job responsibilities.
- Make sure that everyone participates and that no one dominates.
- Proceed at a comfortable pace.
- Smile and enjoy mathematics.

Math Squares

1.

175	150
250	25

2.

175	
24	50

300

3.

425	
175	300

1000

4.

53	600
42	

1000

5.

325	
57	325

1000

6.

475	125
327	

1000

7.

14	29
41	6

8.

97	38
	13

250

9.

146	
337	304

1250

APPENDIX E
ARTIFACTS PERTAINING TO KATHLEEN'S CASE

The following pages include a copy of Kathleen's parent letter sent home to each parent at the beginning of the school year.

**Second Grade
[REDACTED] Elementary School
Parent Information Packet
2001 - 2002 School Year**

[REDACTED]

Dear Parents,

What an honor it is for me to have your children in my classroom. I look forward to getting to know you better as we become partners in helping your children on their journey in becoming lifelong learners. Hopefully, this packet will provide you with more information about me and our second grade curriculum.

This is my 28th year of teaching and 21st year at [REDACTED]. Before coming to [REDACTED], I was a reading specialist for elementary and secondary students. As a classroom teacher I have taught 1st, 2nd and 3rd grade. I received my Bachelor of Science degree in Elementary Education in 1973 and my Master of Science in Reading in 1976. I have twenty hours above my M.S. degree (12 of which is in mathematics education).

One of my main goals for my students is that they become problem solvers. As I present tasks for students to solve, they begin to investigate, find patterns and discuss different solutions that make sense to them. Through this process, they develop a strong foundation of skills. As their reasoning ability becomes more sophisticated, so does their confidence as lifelong learners.

Respectfully,
[REDACTED]

General Information:

School Times

School is in session from 8:00 A.M. to 2:45 P.M. Students are allowed into the classrooms at 7:50 A.M. If they come before that time, they should go to the cafeteria where a teacher is on duty.

Communication

Communication between home and school is vital to your child's success in school. If you need to get in touch with me, please call the school office and leave a message at [REDACTED], send a note with your child or call me at home after school at [REDACTED]. You may also reach me through e-mail ([REDACTED]) and I will get in touch with you as soon as possible. Please be sure to check homework folders on Monday and Thursday folders for any communications from me or the school.

Volunteers

You are always welcomed into our classroom either as an observer or volunteer. Please be sure to check in at the office first to receive a name tag. If you would like to volunteer please let me know.

Book Orders

Book club orders are an inexpensive source of children's books and a wonderful way to build your child's library. Please send the money and order form to school in an envelope labeled with your child's name. Checks should be made out to the book clubs.

Check In/Out

In order to provide a safe environment for our students, all visitors (including parents) must check in through the office before coming in our classroom. All students must be checked in or out through the office before coming to class late or leaving the school grounds early.

Thursday Folders

Every child will bring home a school folder on Thursday. This folder contains school work and school/district correspondences. The folders are to be signed and returned the next day.

Breakfast

We now have a breakfast program at Jackson. Student meals are \$.75.

Lunch

Lunch money can be prepaid or paid when students go through the line. A student lunch is \$1.65. Our lunch is scheduled from 11:30 to 11:50. This is followed by recess which ends at 12:15.

Snacks

A full day of school can be a long day for a second grader. Healthy snacks can provide the needed break and energy to allow our students to achieve their best work. Each month I will send home a calendar with the date your child is assigned to bring snack. If this is a hardship for anyone, I will provide the snack.

Birthdays

Students and parents often like to bring simple treats to celebrate a child's birthday. If you would like to do this, please send a note or leave a message a day or two ahead of time. Please do not send invitations to school to be handed out. This often causes hurt feelings.

Homework

Homework is sent home on Monday. It is due back on Friday with the Thursday folder. I will record the minutes read and return the form in the next week's folder. If you would like more forms at home, just let me know.

Schedule

Morning:

Reading

We start the day off with reading. I will usually call the students to the back and read them a book that goes along with the unit we are studying. This is followed by a short, whole class lesson on strategies such as story prediction, character studies, plot analysis, story sequence and word analysis. After this the students have an independent or small group reading time where they may select books according to their interest and reading ability. I have individual or small group conferences at this time. These conferences give me opportunities to help students individually with their reading. As I listen to them, I record any areas that I need to work with them such as phonetic sounds, certain word families, watching for punctuation or remembering what they have read. This is followed by a short mini-lesson where we work together on those areas. Keeping records of these conferences helps keep track of my students' progress.

Class Meeting

We will try to have a class meeting everyday. We usually start out saying any announcements that we want to share with the class. Then we discuss problems or ideas that have been written in the "Agenda Book". The emphasis is on how we can help and encourage each other instead of blaming. It is a time we work on respecting each other and our different points of view. I have found this to be a powerful way for children to

develop problem solving skills. This program is called Positive Discipline. I have a few parent books if anyone is interested in finding out more about it.

Spelling I use a program called Spell It Myself. Using the premise that children should first learn to spell the words they use most frequently, Spell It Myself employs a level system based upon word frequency. Words that children use most frequently in their writing are Level 1 words. These are words like go, come, get, at, of, the, be, can, do, so and very. The 100 most frequently used words comprise over 50% of the words used in children's writing. Once children have learned to spell these words, they become significantly more independent in their writing. More difficult and less frequently used words like oyster and explosion appear at Level 7.

Your child will be pretested to determine at which level he or she should begin. At certain times throughout the year, your child will be tested to see whether he or she has achieved level mastery and is ready to move on to the next level.

One of the great features of Spell It Myself is that it is based upon mastery. A student competes against him/herself and no one else. Thus, the student can monitor his/her own progress.

Students will begin each week with a personal spelling list. I will be working each day with every student on spelling. For homework, all students will be expected to know their words for an individual test each Friday.

Writing, Penmanship

Students act as real authors by writing about what is personally meaningful to them. Skills and strategies are discussed during language lessons to help students with their writings. Students will work on the writing process of writing a rough draft, revising, editing and publishing their stories.

We do have a penmanship book this year that will be used to practice how we form our letters.

Mathematics

For mathematics I will be using Investigations and Coming to Know Number. ■

■ She and her graduate assistant, ■ will be in our mathematics classes frequently. We will be video taping students in class discussions and individually to see how they are making sense of mathematics. I have worked with Amy for the past seven years and have found having her knowledge of how children learn mathematics extremely helpful in helping design meaningful lessons for my students.

Encouraging children to value mathematics, become confident in their own ability, become problem solvers, learn to communicate and reason mathematics are standards set by the National Council of Teachers which are goals that I want for all of my students.

We usually start math with a warm up activity like Quick Draw or What's My Rule? where students have to listen to each other's ideas and express different ways that

they saw the problem. Then I give them a problem to solve. After allowing them some time to work on their problem individually, they get with their partner and exchange ideas. During this time I am observing different strategies that children are using and how they are organizing their thoughts. I choose different partners to discuss with the class how they worked their problem. The students listen to different students and decide what makes sense to them. This helps me determine what task I need to choose for the next day to help students become more efficient problem solvers.

Computation skills are closely monitored and activities used are designed to strengthen student's ability to not just memorize number facts but use meaningful strategies that will build a foundation or mathematic memory that will serve them in the years to come.

A rubric that I will be using with the students to keep track of the strategies that they use to solve numeric problems is:

Counting all

Counting on

Counting on but organizing numbers to find answer

Beginning to make numeric units to solve problems

Using sophisticated strategies to make numeric units in solving problems

Is able to explain how they solved problems to others.

I will be discussing this more in depth at our first nine weeks parent-teacher conference.

Calendar, money, measurement, geometric and numeric skills will be incorporated into the various tasks presented to the students.

Computers

The Macintosh and Dell computers in our classroom will be used on a regular basis by each student for word processing and reinforcement and development of math, language and thinking skills. In addition, we have access to Internet and e-mail capabilities on our classroom computer and in the computer lab located in Jackson's library. Every Wednesday morning we are scheduled to go to the computer lab where we will work on various computer skills and programs.

Afternoon:

Rotation

Social Studies On Tuesday [REDACTED] will come into our room to teach a social studies lesson.

Science On Wednesday, [REDACTED] will come into our room to teach a science lesson using the SCIS science program required by the district. In the fall we will be working on objects, their different properties and how they interact with each other. In the spring we will be doing life cycles of frogs, butterflies and mealworms.

Art I will be teaching art for the rotation using objectives determined by our district.

P.E. and Music

Second graders participate in physical education and music classes every day on a rotation basis. On even numbered days we will go to P.E. On odd numbered days we will go to music.

Centers and Skills for Growing

Centers will be set up for students to use focusing on math, reading, listening, art and science activities. This is also a time we might have a lesson using the district's **Skills for Growing**.

Clean-up

Each student has a job they are responsible for in helping keep our room clean.

Library, Enrichment Classes, Outdoor Classroom

Trips to the library, outdoor classroom, guidance classes and enrichment classes will occur usually during the morning.

Evaluation Practices

Students will be evaluated on the basis of participation, tests and other formal assessments, daily work, teacher observation and homework. The students will also periodically self-evaluate their own work.

The grading scale is as follows:

E Excellent
VG Very Good
S Satisfactory
NI Needs Improvement