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PROPERTIES AND DISTRIBUTION FUNCTIONS OF POLAR FLUIDS BY  
PERTURBATION THEORY

The University of Oklahoma

PH.D.

1980

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THE UNIVERSITY OF OKLAHOMA  
GRADUATE COLLEGE

PROPERTIES AND DISTRIBUTION FUNCTIONS OF POLAR  
FLUIDS BY PERTURBATION THEORY

A DISSERTATION  
SUBMITTED TO THE GRADUATE FACULTY  
in partial fulfillment of the requirements for the  
degree of  
DOCTORATE IN PHILOSOPHY

By  
EYNOLLAH ASSAD  
Norman, Oklahoma  
1980

PROPERTIES AND DISTRIBUTION FUNCTIONS OF POLAR  
FLUIDS BY PERTURBATION THEORY  
A DISSERTATION  
APPROVED FOR THE  
DEPARTMENT OF PHYSICS & ASTRONOMY

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#### ACKNOWLEDGEMENTS

I wish to thank Professor L. Lee for his encouragement and guidance, Professor Stanley Babb for his patience and understanding, and Professor Stephen Whitmore for his time and assistance.

I wish to especially thank my wife, Fatane and my parents without whose help I could not have succeeded.

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## ABSTRACT

It is proposed that perturbation theories and computer simulation be collectively applied to the study of static and dynamic properties of liquids and dense fluids composed of highly nonspherical and/or polar molecules. Perturbation theories rooted in classical statistical mechanics have, in recent years, proven successful in predicting the equilibrium properties of atomic fluids and molecular fluids composed of weakly anisotropic molecules. We propose to develop and test analogous theories for the thermodynamic and transport properties of fluids containing highly anisotropic molecules. New theories for equilibrium properties will be based on a first order expansion of the angular pair correlation function  $g(r\omega_1\omega_2)$ , or, equivalently, the function  $y(r\omega_1\omega_2)$ . Expansions about spherical and nonspherical reference fluids will be explored for static properties. A major strength of the proposed work is the concurrent use of computer simulation to guide and evaluate development of the theories. The investigation includes two different temperatures;  $T^*=1.294$  and  $T^*=1.277$ , with two quadrupole moments of  $Q^*=1$  and  $Q^*=0.5$ , respectively. The density is assumed to be  $\rho^*=0.85$ .

## CHAPTER I

### INTRODUCTION

Recently there have been several extensions to the polar system of the theoretical techniques that proved useful in predicting the thermodynamics of simpler fluids. Computational work (1-4) has allowed the evaluation of these theories for uncomplicated models consisting of dipoles embedded in otherwise spherical molecules.

The most surprising success among the theories was a special version of the thermodynamic perturbation theory (5), the so-called Padé approximation due to Stell, Rasaiah and others (6-8). Especially in the last decade there has been great enthusiasm among the laboratory experimentalists, statistical mechanical theorists and those who simulate fluids on the computer because of new developments in the study of the liquid and dense fluid states of matter with the equilibrium properties of "simple" fluids composed of atoms (Ar, Kr) or nearly spherical ( $\text{CH}_4$ ,  $\text{O}_2$ ) molecules.

The most important of these developments was the theoretical development of the perturbation concept which occurred in equilibrium statistical mechanics. This theory is based on the idea that fluids are related to each other and they differ by small amounts of their properties.

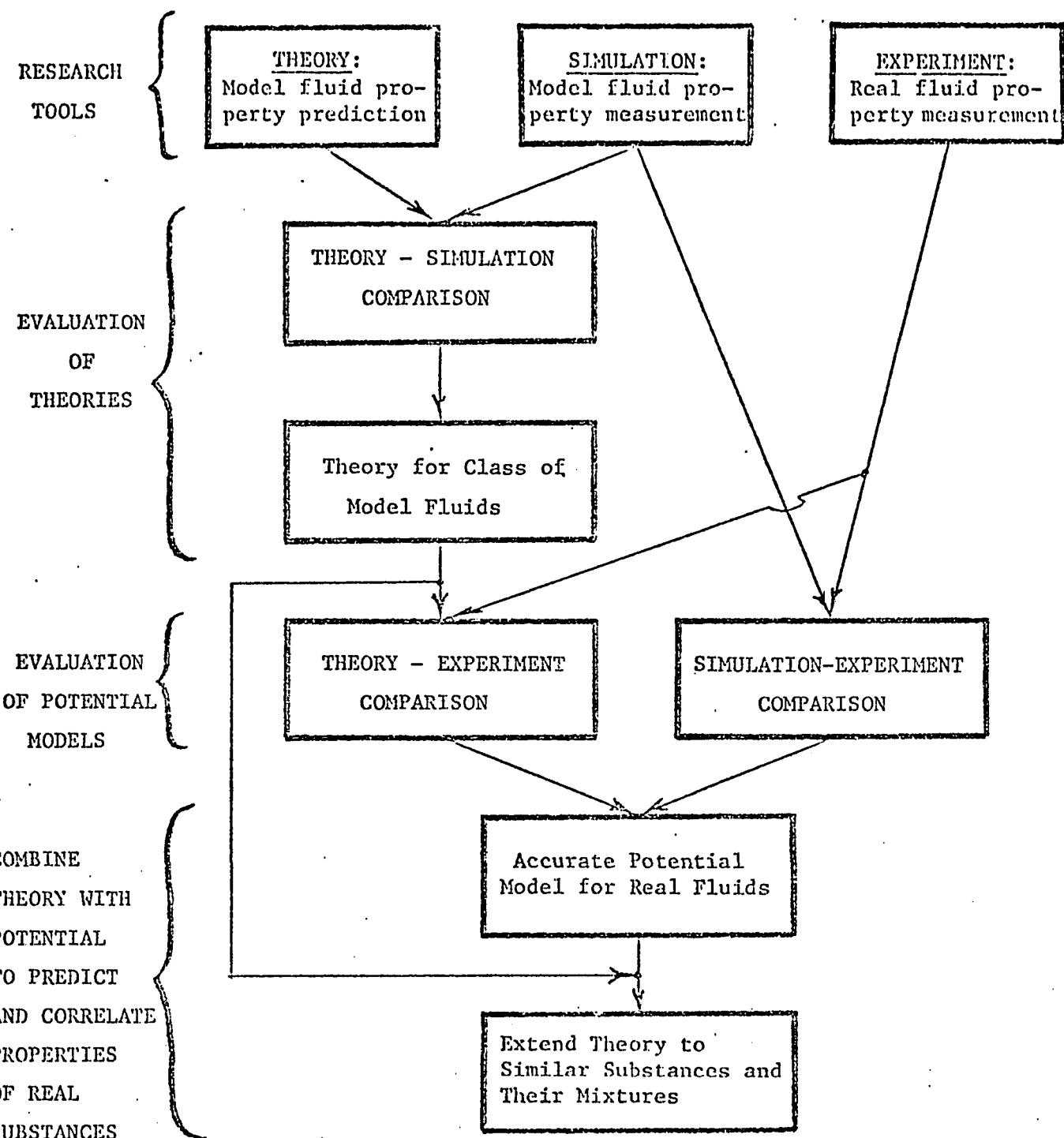
So if we know the properties of a simple fluid, then we can predict the properties of new fluids by treating the differences as perturbed parts. Now this perturbed part has to be calculated successfully. And this "reference" substance can be compared with the results obtained from simulation methods embodied in Monte Carlo and molecular dynamics (9-10) which are available to us.

Figure 1 shows the importance of simulation in providing data for model fluids. Thus, the accuracy of our predictions can be found out by comparing theoretical predictions with simulation results, and by comparing the simulation results with laboratory measurements we can learn about the accuracy of models with real fluids of interest.

The test would be on static properties of the class of model fluids composed of highly nonspherical and/or strongly polar molecules. The reasons can be explained from several viewpoints. From the theoretical point of view, there are presently very few satisfactory theories for predicting the static properties of such fluids. Not very much work has been done on developing perturbation expansions about nonspherical reference fluids which would be applicable to highly anisotropic molecules. There have been a number of thermodynamic perturbation theories developed in the last decade, but most of them are expansions of the Helmholtz free energy or the property of interest directly. But our approach to the equilibrium properties of molecular fluids will be via

second order perturbation theory for the angular pair correlation function  $g(r_{12}, \omega_1, \omega_2)$ . Since the expansion of  $g(r_{12}, \omega_1, \omega_2)$  is equivalent to the free energy expansion (11) and  $g(r_{12}, \omega_1, \omega_2)$  can be obtained by computer simulation via a spherical harmonic expansion in the molecular orientations  $\omega_1$  and  $\omega_2$  (11,13), other equilibrium properties may be obtained from standard formulas in statistical mechanics (14). The success of perturbation theories developed to date has been limited to atomic fluids or to those containing weakly anisotropic molecules for which a spherical reference fluid can be used.

Figure 1: STRATEGY FOR STUDY OF LIQUIDS AND MIXTURES



## CHAPTER II

### LITERATURE REVIEW

Although the perturbation expansion of the angular pair correlation function  $g(r_{12}, \omega_1, \omega_2)$  provides an alternate route to macroscopic thermodynamic properties (e.g., free energy, pressure, internal energy) of polar fluids having moderately strong multipolar strengths, expansion of  $g(r_{12}, \omega_1, \omega_2)$  has received much less attention than the free energy expansion.

Among those theories perhaps one of the best is the Padé extension to third order of the Pople expansion of the free energy due to Stell et al. (14,15); the idea here also is to find the perturbed part of the fluid of interest from the reference system.

$$v(x_i, x_j) = v_o(r) + \sum_k \lambda_k \omega(x_i, x_j); \quad \lambda_k \geq 0 \quad (2-1)$$

where  $v(x_i, x_j)$  is the orientational part of the pair potential for two molecules and  $v_o(r)$  is spherically symmetric reference system),  $\{\lambda_k\}$  are related to the dipole moment  $\mu$ , the quadrupole moment  $\sigma$ , and the octupole moment  $\phi$  ..... From recent evidence of computer simulation, the Padé expansion for the free energy in a quadrupole fluid up to moderate values of the quadrupole moment  $Q(\epsilon\sigma^5)^{-\frac{1}{2}}$  of  $O(1)$  (16) is promising. The Padé expansion for dipoles contains the correct low and high dipolar strength limits and Padé must be (almost)

correctly interpolating at moderate dipolar strengths, but the Pade' theory breaks down at high anisotropy in anisotropic overlap fluids (17,18).

One of the early expansions of  $g(r, \omega_1, \omega_2)$  was done by Gubbins and Gray. They wrote the first order expansion which is valid for a Pople reference and multipolar anisotropics (19).

$$g(r_{12}, \omega_1, \omega_2) = g_o(r)(1 - \exp(\beta u_a(r, \omega_1, \omega_2))) \quad (2-2)$$

where  $\beta = 1/RT$ ,  $u_a$  the reference fluid radial distribution function. This equation also breaks down for moderately strong anisotropics in that nonphysical, negative values of  $g(r_{12}, \omega_1, \omega_2)$  are obtained for certain pair orientations.

Perram and White also gave a first order theory of  $g(r_{12}, \omega_1, \omega_2)$  by using a reference potential of the log-exp mean type on  $u_a$ . They only tested the theory for the center pair correlation function  $g_c(r)$  (20). Henderson and Gray assumed the potential to consist of two parts:  $u_a + u_o$ , where  $u_a$  was quadrupole-quadrupole interaction and  $u_o$  was a reference potential and then they suggested a perturbation theory involving an expansion of the direct correlation function  $C(r, \omega_1, \omega_2)$  (22) in the form of  $C_a = C_1 + C_2 + C_3 + \dots$  where  $C_n$  includes all terms in  $C_a$  of order  $(u_a)^n$ .

They considered two possible closures of the Ornstein-Zernicke equation

$$C_a = -g_o u_a + h_o (h_a - C_a) \quad (2-3)$$

where

$$h_o = g_o - 1$$

(a mean field and a generalized mean field approximation) to obtain  $g_o(r, \omega_1, \omega_2)$  from  $C(r, \omega_1, \omega_2)$ . Anderson, Weeks and Chandler proposed the "blip function theory" for the calculation of radial distribution function and thermodynamic properties of dense fluids. It is written in a form which yields explicit expressions for both the angular and radial correlation of dense fluids of linear molecules (21).

Steel and Sandler compared this approximation with a variety of correlation functions obtained by numerically solving the extended Percus-Yevick equation for non-spherical molecules. They found that agreement is generally good at high densities, but becomes worse at lower densities because of the omission of the effects of attractive interactions in the blip function theory. Also they extended the blip function formula to  $g_c(r)$ . In addition, Gubbin et al. (25) and Sung Chandler (26) applied Schofield's temperature expansion for  $g(r)$  (27) to the first order and made approximations for higher order correlation functions. Although these theories are largely successful for predicting  $g_c(r)$  for moderately strong anisotropy, they are, for the most part, first order theories. Hence, they would not be expected to work for strongly anisotropic fluids. More importantly, knowledge of  $g_c(r)$  is not sufficient to enable calculation of all thermodynamic properties.

## CHAPTER III

### SPHERICAL HARMONIC EXPANSION

If we look at OZ equation

$$h(r_{12}, \omega_1, \omega_2) = c(r_{12}, \omega_1, \omega_2) + \rho \int dr_3 \langle c(r_1, \omega_1, \omega_3) h(r_{32}, \omega_3, \omega_2) \rangle_{\omega_3} \quad (3-1)$$

we record the analysis of (3-1) in spherical harmonics. We restrict considerations to linear molecules so that  $\omega = (\theta, \phi)$  denotes the orientation of the symmetry axis. Consider any function  $F(r, \omega_1, \omega_2)$  that is invariant under simultaneous rotation of  $r, \omega_1$  and  $\omega_2$ . We first Fourier transform the space variable according to

$$F(k, \omega_1, \omega_2) = \int dr \exp(ikr) f(r, \omega_1, \omega_2) \quad (3-2)$$

then the rotational invariance implies that  $F(r, \omega_1, \omega_2)$  and  $F(k, \omega_1, \omega_2)$  can be expanded in spherical harmonics

$$F(r, \omega_1, \omega_2) = \sum_l l_1 l_2 l_1 F(l_1 l_2 l_1, r) \psi_{l_1 l_2 l_1}(\omega_1, \omega_2, \omega_r) \quad (3-3)$$

$$F(k, \omega_1, \omega_2) = \sum_l l_1 l_2 l_1 F(l_1 l_2 l_1, k) \psi_{l_1 l_2 l_1}(\omega_1, \omega_2, \omega_k) \quad (3-4)$$

where  $r$  and  $k$  are the magnitudes of  $\vec{r}$  and  $\vec{k}$  and  $\omega_r$ .  $\omega_k$  denotes the orientation of  $r$  and  $k$  here

$$\psi_{l_1 l_2 l_1}(\omega_1 \omega_2 \omega) = m_1 \sum_m C(l_1 l_2 l_1, m_1 m_2 m) \cdot Y_{l_1}^{m_1}(\omega_1) Y_{l_2}^{m_2}(\omega_2) Y_l^{m^*}(\omega) \quad (3-5)$$

where  $Y_l^m(\theta, \phi)$  are spherical harmonics and given by

$$Y_1^m(\omega) = (-1)^m \left(\frac{2^{l+1}}{4\pi}\right) \cdot \frac{(l-m)!}{(l+m)!} P_1^m (c_o \theta) e^{im\phi} \quad (3-6)$$

and the Legendre polynomial  $P_1^m(x)$  is

$$P_1^m(x) = \frac{1}{2^l l!} (1-x^2)^{l/2} \frac{d^{m+1}}{dx^{m+1}} (x^2 - 1)^l$$

here  $-l \leq m \leq l; \quad l > 0$

(3-7)

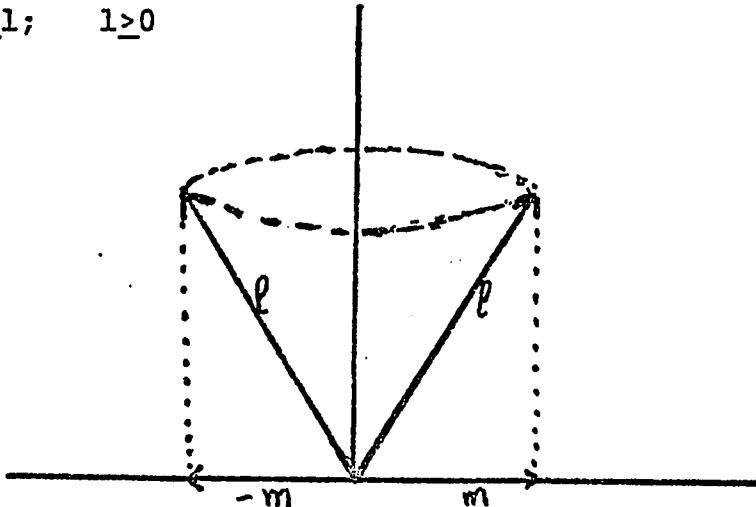


Figure 4

( $l$  is the normal angular moment and  $m$  is the azimuthal angular moment), and  $C(l_1 l_2 l, m_1, m_2, m)$  is Clebsch Gordon coefficient. The properties of  $C(l_1 l_2 l, m_1 m_2, m)$  are as follows:

$$\sum_l C(l_1 l_2 l, m_1, m-m_1, m) C(l_1 l_2 l', m'_1, m'-m'_1, m') = \delta_{m_1 m'_1} \delta_{mm'} \quad (3-8)$$

$$\sum_m C(l_1 l_2 l, m_1, m-m_1, m) C(l_1 l_2, l', m_1, m-m_1, m) = \delta_{ll'} \quad (3-9)$$

and

$$m = m_1 + m \quad (3-10)$$

and where  $\delta_{nk}$  is Kroncker delta function and is

$$\delta_{nk} = \begin{cases} 1 & n=k \\ 0 & n \neq k \end{cases} \quad (3-11)$$

Also here  $l_1, l_2, l$  must satisfy the triangle relation  $\Delta(l_1 l_2 l)$  which we can write as

$$|l_1 - l_2| \leq l \leq (l_1 + l_2) \quad (3-12)$$

Figure 4 shows the  $l$  and  $m$ . Further, parity considerations for linear molecules shows that  $F(l_1, l_2, r) = 0$  unless  $l_1 + l_2 + l$  is even.  $l_1$  and  $l_2$  must be drawn from the same set of values, as the symmetry of these functions under interchange of identical molecules implies

$$F(l_1 l_2 l, r) = (-1)^{l_1 + l_2} F(l_2, l_1, l, r) \quad (3-13)$$

## CHAPTER IV

### SPHERICAL HARMONIC EXPANSION OF THE ANGULAR PAIR CORRELATION FUNCTION

For fluid composed of axially symmetric molecules the angular pair correlation function  $g(r, \omega_1, \omega_2)$  can be expanded in terms of product of spherical harmonics of molecular orientation. Rotational invariance of  $g(r, \omega_1, \omega_2)$  implies that it depends on only  $(r, \omega_i)$  where  $\omega_i = (\phi_i, \psi_i)$  denotes the orientation of  $i^{\text{th}}$  molecule with respect to the intermolecular axis.

In this coordinate system, the harmonic series of  $g$  is

$$g(r, \omega'_1, \omega'_2) = 4\pi \sum_{l_1} \sum_{l_2} \sum_{m_1} g(l_1 l_2 m_1 r) Y_{l_1}^{m_1}(\omega'_1) Y_{l_2}^{-m_1}(\omega'_2) \quad (4-1)$$

( $\bar{m} = -m$ ). Only  $m_i$  and  $\bar{m}_i$  in any terms since the rotational invariance also implies that the  $\phi_i$  can appear only as  $\phi'_1 - \phi'_2$ . Putting the Polar axis in (3-3) in the direction of  $r$  so that  $\omega_i = \omega'_i$  and subsequently comparing to (4-1) we find

$$g(l_1 l_2 l, r) = 4\pi \left(\frac{4\pi}{2l+1}\right)^{1/2} \sum_{\bar{m}} C(l_1 l_2 l, m_1 \bar{m}_1 0) g(l_1 l_2 m_1, r) \quad (4-2)$$

so the desired harmonics in the space fixed axis can be evaluated from knowledge of the body fixed axis  $g(l_1 l_2 m_1, r)$ . Conversely, the  $g(l_1 l_2 m, r)$  may be found from the  $g(l_1 l_2 l, r)$  by

$$g(l_1 l_2 m r_{12}) = (4\pi)^{-3/2} \sum_l (2l+1)^{1/2} g(l_1 l_2 l, r_{12}) C(l_1 l_2 l, m_1 \bar{m}_1 0) \quad (4-3)$$

For homonuclear diatomic ( $O^2, N^2$ )  $l_1$  and  $l_2$  are both even. The validity of equations (4-3) and (4-2) can be proven by using equation (4-1) and substituting (4-2). In (4-3), we get

$$\begin{aligned} g(l_1 l_2 m_1 r) &= \sum_l \left( \frac{2}{4\pi} \frac{l+1}{l-1} \right)^{\frac{1}{2}} C(l_1 l_2 l, m_1 \bar{m}_1 o) \left( \frac{4\pi}{2} \frac{l}{l+1} \right)^{\frac{1}{2}} [\sum_{\bar{m}} C(l_1 l_2 l \bar{m}, \bar{m}) \cdot \\ &(g(l_1 l_2 m r)] = \sum_l \sum_{\bar{m}} C(l_1 l_2 l, m_1 \bar{m}_1 o) C(l_1 l_2 l, m \bar{m} o) \\ g(l_1 l_2 m r) &= \sum_{\bar{m}} g(l_1 l_2 m r) \sum_l C(l_1 l_2 l, m \bar{m} o) \cdot \\ &C(l_1 l_2 l, m \bar{m} o) \\ &= \sum_{\bar{m}} g(l_1 l_2 m r) \delta_{mm_1} \end{aligned} \quad (4-4)$$

With the help of orthogonality condition of spherical harmonic functions, equation (4-1) can be inverted to

$$g(l_1 l_2 m r) = 4\pi \int d\omega'_1 d\omega'_2 Y_{l_1}^{m_1*}(\omega_2) g(r \omega'_1 \omega'_2) \quad (4-5)$$

This integral can be evaluated by Monte Carlo techniques by taking an average of  $Y_{l_1}^{m_1*}(\omega'_1) Y_{l_2}^{m_2*}(\omega_2)$  over the shell between  $r$  and  $r + \Delta r$  in particular

$$g(l_1 l_2 m_1 r) = 4\pi g_{ooo}(r) \langle Y_{l_1}^{m_1}(\omega_1) Y_{l_2}^{m_2}(\omega_2) \rangle$$

where  $g_{ooo}$  is the center pair correlation function  $g_c$  and  $\langle \dots \dots \rangle$  represents an ensemble average over a spherical shell of radius  $r$  and thickness  $\Delta r$ . Several workers have used this approach in evaluating  $g(r, \omega_1, \omega_2)$  in order to study local structure in fluids composed of linear molecules (6, 7, 32, 33).

Typically, about thirty  $g(l_1 l_2 m, r)$  coefficients are evaluated and used to approximate the infinite series of equation (4-1). Convergence of the series appears to be satisfactory for this number of terms.

Figure 2 shows the  $g(l_1 l_2 m, r)$  coefficient obtained from a molecular dynamics simulation of a Lennard-Jones plus quadrupole fluid. Figure 3 shows a three dimensional surface of the  $g(r, \omega_1, \omega_2)$  function obtained by recombining the expansion of equation (4-1) with the molecular dynamics determined  $g(l_1 l_2 m | r)$ . In particular, Figure 3 demonstrates that the quadrupolar anisotropy forces molecular pairs into the "tee" orientation rather than parallel or related orientations in the shell of nearest neighbors.

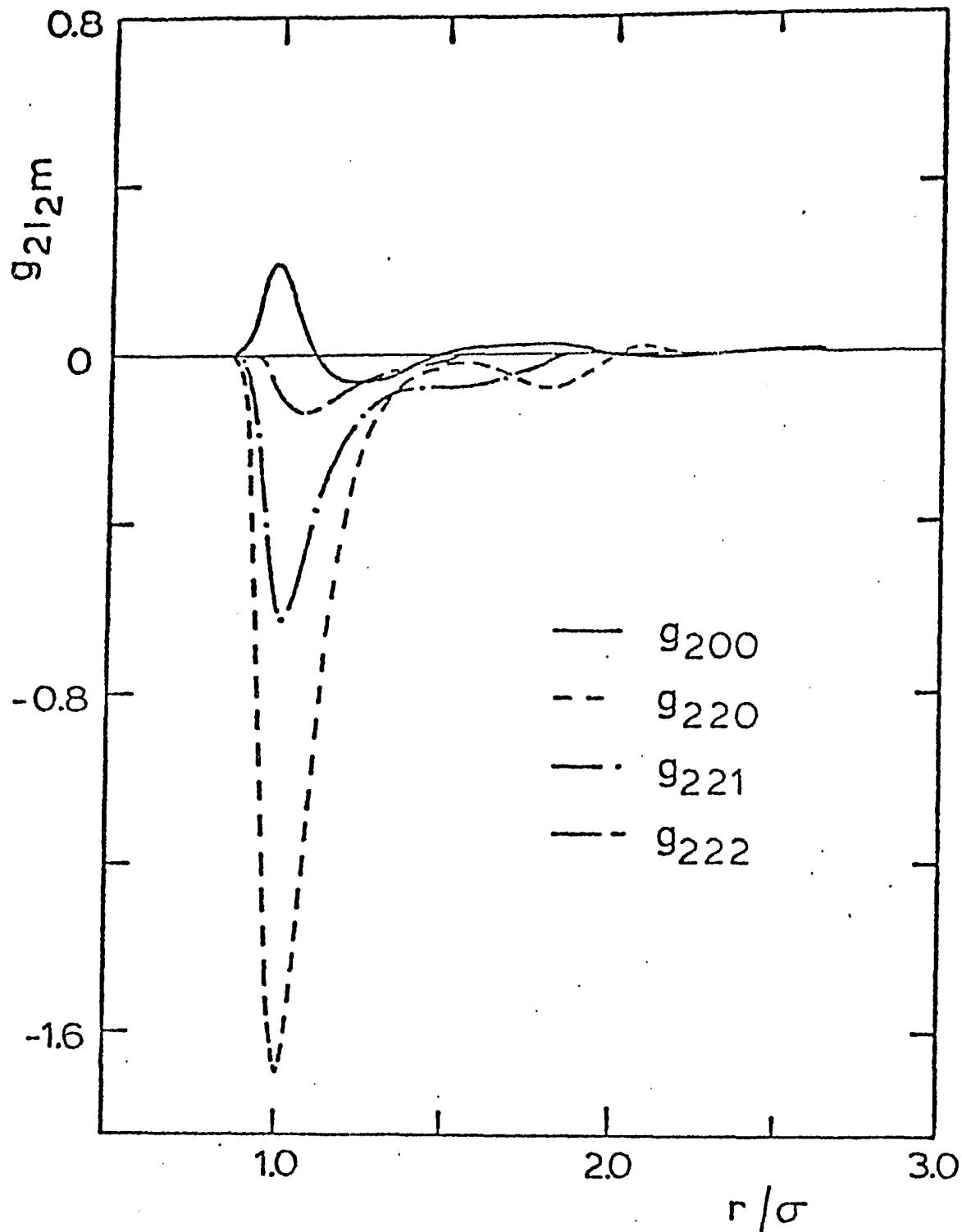


Figure 2 : The  $g_{2l2m}$  coefficients in the Spherical Harmonic

Expansion for the Angular Pair Correlation Function for  
a Lennard-Jones plus Quadrupole Fluid.  $KTe^{-1} = 1.277$ ,  
 $\rho\sigma^3 = .85$ ,  $Q(\sigma^5)^{-\frac{1}{2}} = 1.$  [2]

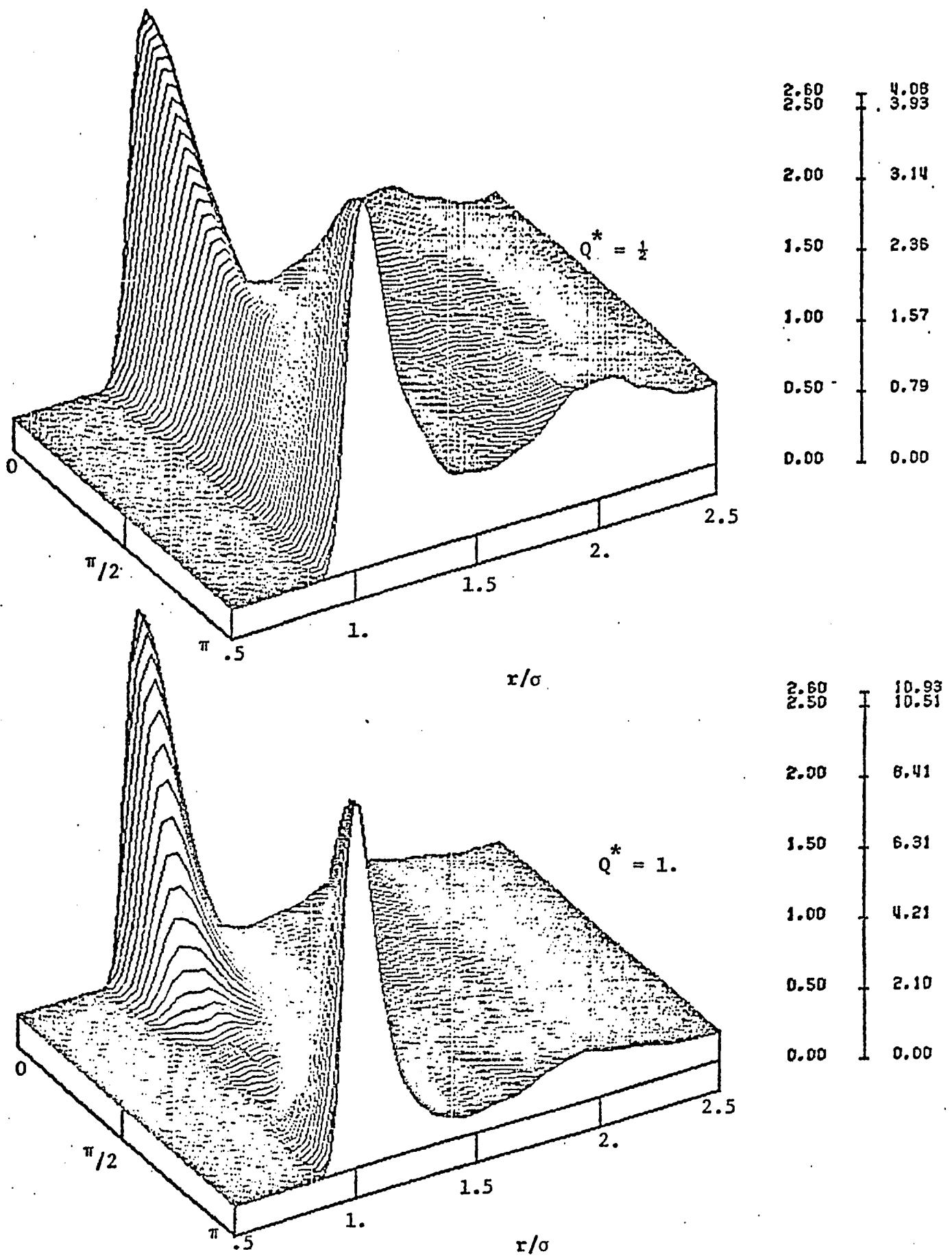


Figure 3 : Effect of the Quadrupole Moment on a Subspace of the Angular Pair Correlation Function for the fluid of Figure 1. Here  $\theta_1 = 90^\circ$ ,  $\phi$  undefined [7].

## CHAPTER V

### THEORY

Our basic approach is derived from the functional derivation formalism of Percus (126) and Lebowitz (124,125) for inhomogeneous systems. In this approach, the inhomogeneity arises due to the "turning on" of an external potential  $U(r)$ . With proper selection of the generating function, the Percus-Yevick (PY), Hypernetted Chain (HNC) and Yvon-Born-Green (YGB) internal equations were shown to be first order functional Taylor expansions in terms of the singlet density  $n_1$ . We first introduce the definitions of the important quantities. The partition function of the inhomogeneous system in a grand canonical ensemble is

$$\Xi(u) = \sum_{n \geq 0} (N!_1)^{-1} z^n / d\{n\} \exp\{-\beta V_n(\{n\}) - \beta \sum_i u(i)\} \quad (5-1)$$

the singlet density  $n_1(1/u)$  is

$$n_1(\frac{1}{u}) = \sum_{n \geq 1} \frac{z^n}{(n-1)!} / d\{n-1\} \exp\{-\beta V_n(\{n\}) - \beta \sum_i u(i)\} \quad (5-2)$$

Here  $\{n\}$  is the set of molecular configurations,  $\{1, 2, 3, 4, \dots, N\}$ , each number  $i$  represents the position  $r_i$  of the center of mass (com) and orientation (Euler angles)

$w_i$  of the  $i^{\text{th}}$  molecule,  $\beta$  is reciprocal temperature  $(KT)^{-1}$ ,  $K$  is the Boltzmann constant,  $z$  being the activity

$\exp(\beta\mu)/\lambda^3$ ,  $\mu$  the chemical potential,  $\lambda$  the deBroglie thermal wave length,  $V_N$  the  $N$  body potential assumed to be pairwise additive.

$$V(N) = \sum_i^N u(i,j) \quad (5-3)$$

where  $u$  is the full pair potential, which is separated into a reference potential  $U_0$  and a perturbation term  $u_p$

$$u(12) = u_0(12) + u_p(12) \quad (5-4)$$

where  $(12) = (r_{12}, \omega_1, \omega_2)$ .

We will talk about the potential in a separate section. To derive the perturbation equation in general, we adopt the generating functional of the PY type, i.e.,  $n_1(\frac{1}{u}) \exp(\beta u(1))$ . We note that from functional calculus, on  $\Xi$  we have the following relations

$$\frac{\partial \ell_n \Xi(u)}{\partial \exp -\beta u(1)} = (\exp(\beta u(1))) n_1(\frac{1}{u}) \quad (5-5)$$

$$\frac{\partial^2 \ell_n \Xi(u)}{\partial \exp -\beta u(1) \partial \exp -\beta u(2)} = \exp(\beta u(1)) \exp(\beta u(2)) \cdot \quad (5-6)$$

$$[n_2(\frac{12}{u}) - n_1(\frac{1}{u}) n_1(\frac{2}{u})]$$

$$\frac{\partial^3 \ell_n \Xi(u)}{\partial \exp -\beta u(1) \partial \exp(-\beta u(2)) \partial \exp(-\beta u(3))} = \exp \beta u(1) \exp \beta u(2) \cdot$$

$$\begin{aligned} & \exp \beta u(3) \left[ n_3(\frac{123}{u}) - n_2(\frac{12}{u}) - n_1(\frac{3}{u}) - n_2(\frac{23}{u}) n_1(\frac{1}{u}) \right. \\ & \left. - n_2(\frac{13}{u}) n_1(\frac{2}{u}) + 2 n_1(\frac{1}{u}) n_1(\frac{2}{u}) n_1(\frac{3}{u}) \right] \end{aligned} \quad (5-7)$$

Consider  $\eta_1 \exp(\beta u)$  as a functional of  $\exp(-\beta u)$  where  $\eta_1$  is the singlet density and expand to the second order in a functional Taylor series.

$$\begin{aligned}
 \eta_1 \left( \frac{1}{u} \right) e^{\beta u^o(1)} &= \eta_1^o \left( \frac{1}{u} \right) e^{\beta u^o(1)} + \int d_3 e^{\beta u^o(1)} e^{\beta u^o(3)} [\eta_2 \left( \frac{13}{u^o} \right) \\
 &\quad - \eta_1 \left( \frac{1}{u^o} \right) \eta_1 \left( \frac{3}{u^o} \right)] [e^{-\beta u(3)} - e^{-\beta u^o(3)}] + \frac{1}{2} \int d_3 d_4 e^{\beta u^o(1)} \cdot \\
 &\quad e^{\beta u^o(3)} e^{\beta u^o(4)} [\eta_3 \left( \frac{134}{u^o} \right) - \eta_2 \left( \frac{13}{u^o} \right) \eta_1 \left( \frac{4}{u^o} \right) + 2 \eta_1 \left( \frac{1}{u^o} \right) \cdot \\
 &\quad \eta_1 \left( \frac{3}{u^o} \right) \eta_1 \left( \frac{4}{u^o} \right) - \eta_2 \left( \frac{14}{u^o} \right) \eta_1 \left( \frac{3}{u^o} \right) - \eta_2 \left( \frac{34}{u^o} \right) \eta_1 \left( \frac{1}{u^o} \right)] \cdot \\
 &\quad [e^{-\beta u(3)} - e^{-\beta u^o(3)}] [e^{-\beta u(4)} - e^{-\beta u^o(4)}]
 \end{aligned} \tag{5-8}$$

This can be reduced to

$$\begin{aligned}
 Y(12) = Y^o(12) &\{ 1 + \rho \int d_3 \left[ \frac{g_3^o(132)}{g^o(12)} - g^o(32) [e^{-\beta u(32)} - 1] \right] \tag{5-9} \\
 &+ (\frac{1}{2}) \rho^2 \int d_3 d_4 \left[ \frac{g_4^o(1342)}{g^o(12)} - g_3^o(342) - \frac{g_3^o(132) g^o(42)}{g^o(12)} \right. \\
 &\quad \left. - \frac{g_3^o(142) g^o(32)}{g^o(12)} + 2 g^o(32) g^o(42) \right] [e^{-\beta u(32)} - 1]
 \end{aligned}$$

where we have introduced the Y-distribution function  $Y(12)$   $g(12) \exp \beta u(12)$ . The superscript o represents the reference potential  $u_o$ .

To make efficient numerical calculation, the higher order correlation functions  $g_3$ ,  $g_4$  must be approximated by the two body expressions. The Kirkwood superposition has been intensively used

$$g_3(123) = g(12)g(13)$$

$$\begin{aligned} g_4(1234) &= g_3(123)g_3(134)g_3(124)g_3(234)/(g(12) \\ &\quad \cdot g(13)g(14)g(23)g(24)g(34)) \end{aligned} \tag{5-11}$$

Jackson and Feenberg (131) gave a "convolution approximation"

$$\begin{aligned} g_3^{(c)}(123) &= 1 + h(12) + h(12)h(23) + \rho \int d4 h(14)h(24)h(34) \\ &\quad + h(23) + h(23)h(31) + h(31) + h(3)h(12) \end{aligned} \tag{5-11}$$

For convenience, we develop the perturbation equation with Kirkwood approximation and to the first order terms only.

A similar procedure can be used for the convolution approximation and to the second order terms, substituting the  $Y(123)=Y(12)Y(23)Y(13)$  into

$$Y(12) = Y^0(12)\{1 + \rho \int d3 \dots \dots \dots \dots \dots \tag{5-12}$$

By keeping only the first order perturbation, we have

$$Y(12) = Y^0(12)\{1 + \rho^{\Omega-1} \int d3 h(13)g^0(32)f(32) \tag{5-13}$$

$$\text{where } f(32) = \exp(-\beta u_p(32)) - 1. \tag{5-14}$$

$\Omega$  is the angle normalization constant, e.g.,  $\Omega=8\pi^2$  for the complete Euler angles of linear molecules.

Figure 5 depicts the geometric arrangement of molecules 1, 2, and 3 for a fluid. We have translational invariance; the change of origin to  $r_i$  gives Figure 5.2.

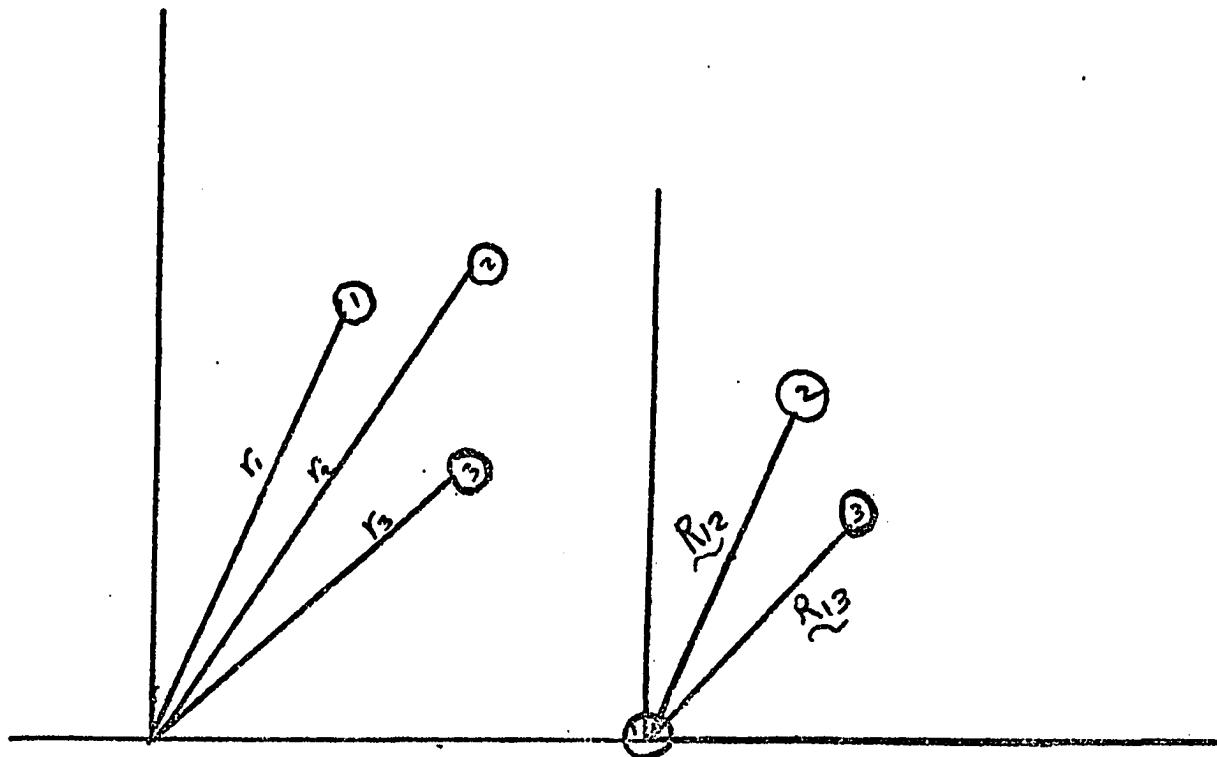


Figure 5.1

Figure 5.2

Equation (5-13) now can be written

$$\frac{Y(R_{12}\omega_1\omega_2)}{Y_0(R_{12})} = Y^O(R_{12}\omega_1\omega_2) \{ 1 + \rho \int dR_{13} h(R_{13}\omega_1\omega_3) g^O(R_{32}\omega_3\omega_2) f(R_{13}\omega_3\omega_2) \} \quad (5-15)$$

where  $\langle \dots \rangle$  is the angle average over  $\omega_3$ . To evaluate this equation, we expand the three functions  $Y/Y_0$ ,  $h$  and  $g^O$  into spherical harmonics

$$\frac{Y(R_{12}\omega_1\omega_2)}{Y_0(R_{12})} = \sum_{\substack{\bar{l}_1 \\ \bar{l}_2}} \sum_{\substack{\bar{m}_1 \\ \bar{m}_2}} \sum_{\substack{\bar{n}_1 \\ \bar{n}_2}} E_Y(\bar{l}_1 \bar{l}_2 \bar{l} \bar{n}_1 \bar{n}_2 R_{12}) C(\bar{l}_1 \bar{l}_2 \bar{l} \bar{m}_1 \bar{m}_2 \bar{m}) \cdot$$

$$D_{m_1 n_1}^{l_1 *}(\omega_1) D_{m_2 n_2}^{l_2 *}(\omega_2) Y_1^m(\Omega_{12}) \quad (5-16)$$

$$h(R_{13} \omega_3) = \sum_{l_1} \sum_{m_1} \sum_{n_1} E_Y(l_1 l_3 l, n_1 n_3 R_{13}) C(l_1 l_3 l, m_1 m_3 m)$$

$$\begin{matrix} l_1 \\ l_3 \\ l \end{matrix} \quad \begin{matrix} m_1 \\ m_3 \\ m \end{matrix} \quad \begin{matrix} n_1 \\ n_3 \\ \end{matrix}$$

$$D_{m_1 n_1}^{l_1}(\omega_1) D_{m_3 n_3}^{l_3}(\omega_3) Y_l^m(\Omega_{13}) \quad (5-17)$$

$$g^o(R_{23}\omega_2\omega_3) f(R_{23}\omega_2\omega_3) = \sum_{l_1} \sum_{m_3} \sum_{n_3} E_f(l'_1 l'_3 l'_2, n'_3 n'_2 R_{32})$$

$$\begin{matrix} l'_1 \\ l'_2 \\ l' \end{matrix} \quad \begin{matrix} m'_3 \\ m'_2 \\ m' \end{matrix} \quad \begin{matrix} n'_3 \\ n'_2 \\ \end{matrix}$$

$$C(l'_3 l'_2 l', m'_2 m'_3 m') D_{m'_3 n'_3}^{l'_3}(\omega_3) D_{m'_2 n'_2}^{l'_2}(\omega_2) Y_l^m(\Omega_{32}) \quad (5-18)$$

Here  $E$  are the coefficients of expansion,  $C(l_1 l_2 l m_1 m_2 m)$  the Clebsch-Gordon coefficients,  $D$  the rotational matrices (see Rose, M. E. "Elementary Theory of Angular Momentum", John Wiley, N.Y., 1957) and  $Y_l^m$  the spherical harmonic function.

## CHAPTER VI

### PROPERTIES OF SPHERICAL HARMONIC COEFFICIENTS

Spherical harmonic coefficients  $g(l_1 l_2 l r)$  not only give structural information via equation (4-1) but also are related to numerous equilibrium fluid properties via standard formulae in statistical mechanics (8). Thus, we derive the configurational internal energy in terms of the  $g(l_1 l_2 l, r)$  coefficients (17).

A number of these results are summarized in Table 1. In general, the relation in Table 1 involves infinite sums over the indices. However, in the special case of fluids which interact with a Lennard-Jones plus a multipotential, the relations reduce to sum over finite number of terms. For example, Table 2 gives the thermodynamic properties which are the specific equations for the properties of Table 1 for the Lennard-Jones plus quadrupole fluids.

In Table 2 the term  $J_n^{l_1 l_2 m}$  represents one dimensional integral over the coefficients  $g(l_1 l_2 l, r)$ .

$$J_n^{l_1 l_2 m} = \int_0^{\infty} dr r^{-(n-2)} g(l_1 l_2 m, r) \quad (6-1)$$

The results in Table 2 have been tested by computer simulation and found to give highly satisfactory agreement (7,17).

TABLE 1

Examples of Expressions for Equilibrium Properties in Terms of Coefficients  $u_{\ell_1 \ell_2 m}(r)$  and  $g_{\ell_1 \ell_2 m}(r)$  for Spherical Harmonic Expansions of the Intermolecular Potential  $u(r\omega_1 \omega_2)$  and Angular Pair Correlation Function  $g(r\omega_1 \omega_2)$

---

Configurational Internal Energy

$$U^C/N = \frac{1}{2} \rho \sum_{\ell_1 \ell_2 m} \int dr u_{\ell_1 \ell_2 m}(r) g_{\ell_1 \ell_2 m}(r)$$

Pressure

$$P(\rho kT)^{-1} = 1 - \rho (6kT)^{-1} \sum_{\ell_1 \ell_2 m} \int dr r \frac{du_{\ell_1 \ell_2 m}(r)}{dr} g_{\ell_1 \ell_2 m}(r)$$

Isothermal Compressibility

$$\chi = (\rho kT)^{-1} + (kT)^{-1} \int dr (g_{ooo}(r) - 1)$$

Mean Squared Torque

$$\langle \tau^2 \rangle = -\rho kT \sum_{\ell_1 \ell_2 m} \ell_1 (\ell_1 + 1) \int dr g_{\ell_1 \ell_2 m}(r) u_{\ell_1 \ell_2 m}(r)$$

Kirkwood Angular Correlation Parameters

$$G_L = (2L + 1)^{-1} \rho \sum_m (-1)^m \int dr g_{LLm}(r)$$

$G_0$ ,  $G_1$ ,  $G_2$  are related to the isothermal compressibility, dielectric constant, and Kerr constant, respectively.

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TABLE 2

Examples of Expressions for Equilibrium Properties of a Lennard-Jones plus Quadrupole Fluid in Terms of the  $\int_{\text{n}}^{L_1 L_2 m}$  Integrals of Equation (4)

---

Configurational Internal Energy

$$U^C/N = 8\pi\rho^* \epsilon [J_{12}^{000} - J_6^{000}] + U_{QQ}$$

$$U_{QQ} = \frac{12\pi}{5} \rho^* Q^* \epsilon [J_5^{220} + \frac{4}{3} J_5^{221} + \frac{1}{3} J_5^{222}]$$

$$\text{where } \rho^* = \rho\sigma^3 \text{ and } Q^* = Q(\epsilon\sigma^5)^{\frac{1}{2}}$$

Pressure

$$P(\rho kT)^{-1} = 1 - 16\pi\rho^* (kT)^{-1} [J_6^{000} - 2J_{12}^{000}] - \frac{5}{3}(kT)^{-1} U_{QQ}$$

Mean Squared Torque

$$\langle \tau^2 \rangle = -12\epsilon T U_{QQ}$$

Kirkwood Angular Correlation Functions

$$G_L = \frac{4\pi\rho^*}{2L+1} \sum_m (-1)^m J_0^{LLm}$$


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## CHAPTER VII

### POTENTIAL MODELS

Recently a new approach to the potential model has been developed. The potentials consist of two parts:  $u = u_o + u_a$ . The reference system is one in which the molecules interact with an isotropic pair potential  $u_o(R_{12})$  defined by

$$u_o(R_{12}) = \langle u(R_{12}\omega_1\omega_2) \rangle_{\omega_1\omega_2} \quad (7-1)$$

where  $u(R_{12},\omega_1,\omega_2)$  is the pair potential for the actual system.

This pair potential is  $U(R_{12},\omega_1\omega_2) = U(R_{12},\theta_1,\theta_2,\phi_{12})$  and the one which has been chosen for study here

$$u(R_{12},\theta_1,\theta_2,\phi_{12}) = 4\epsilon \left[ \left( \frac{\sigma}{R_{12}} \right)^{12} - \left( \frac{\sigma}{R_{12}} \right)^6 \right] + u_a \quad (7-2)$$

with the angle dependent part of the potential defined so that  $\langle u_{a\omega_1\omega_2} \rangle = 0$ ; thus for equation (7-1),  $u_o$  is the Lennard-Jones (12,6) potential. In general, there are several choices for  $u_a$  available. The choice depends on the system being investigated. These several parts are:

(1) Dipole-dipole part

$$u_{DD}(12) = - \frac{\mu^2}{R_{12}^2} [2C_1C_2 - S_1S_2C_{12}] \quad (7-3)$$

## (2) Dipole-quadrupole part

$$u_{DQ}(12) = \left(\frac{3}{2} \frac{Q \mu}{R_{12}^4}\right) [C_1(3C_2^2 - 1) - C_2(3C_1^2 - 1) - 2(C_2 - C_1) \cdot S_1 S_2 C_{12}] \quad (7-4)$$

## (3) Quadrupole-quadrupole interaction

$$u_{QQ}(12) = \left(\frac{3}{4} \frac{Q^2}{R_{12}^5}\right) [1 - 5C_1^2 - 5C_2^2 - 15C_1^2 C_2^2 + 2(S_1 S_2 C_{12} - 4C_1 C_2)^2] \quad (7-5)$$

## (4) Anisotropic overlap interaction

$$u_a = 4E \left(\frac{\delta}{R_{12}}\right)^{1/2} (3C_1^2 + 3C_2^2 - 2) \quad (7-6)$$

In these equations,  $S_1 = \sin \theta_1$ ,  $C_1 = \cos \theta_1$ ,  $C_{12} = \cos(\phi_1 - \phi_2)$  so  $C_{12} = \cos \phi_{12}$ .  $\mu$  and  $Q$  are dipole and quadrupole moments, respectively, and  $\delta$  is a dimensionless overlap constant ( $-0.25 < \delta < 0.5$ ).  $\theta_1, \theta_2, \phi_{12}$  are the usual orientation angles, referred to the line between molecular centers as polar axis. Figure 7.1 shows the orientation angles in the body fixed coordinate.

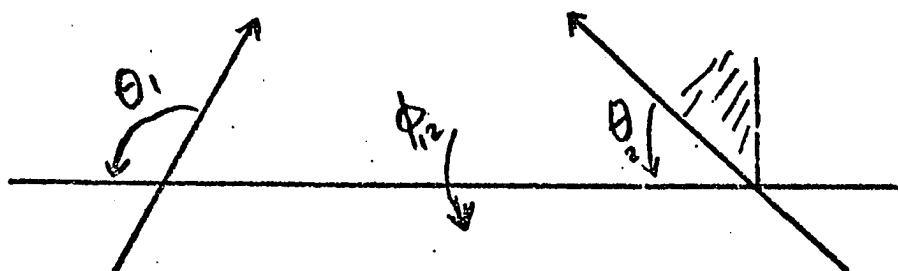


Figure 7.1: The Orientation Angles in the Body Fixed Coordinate

## Potential Model

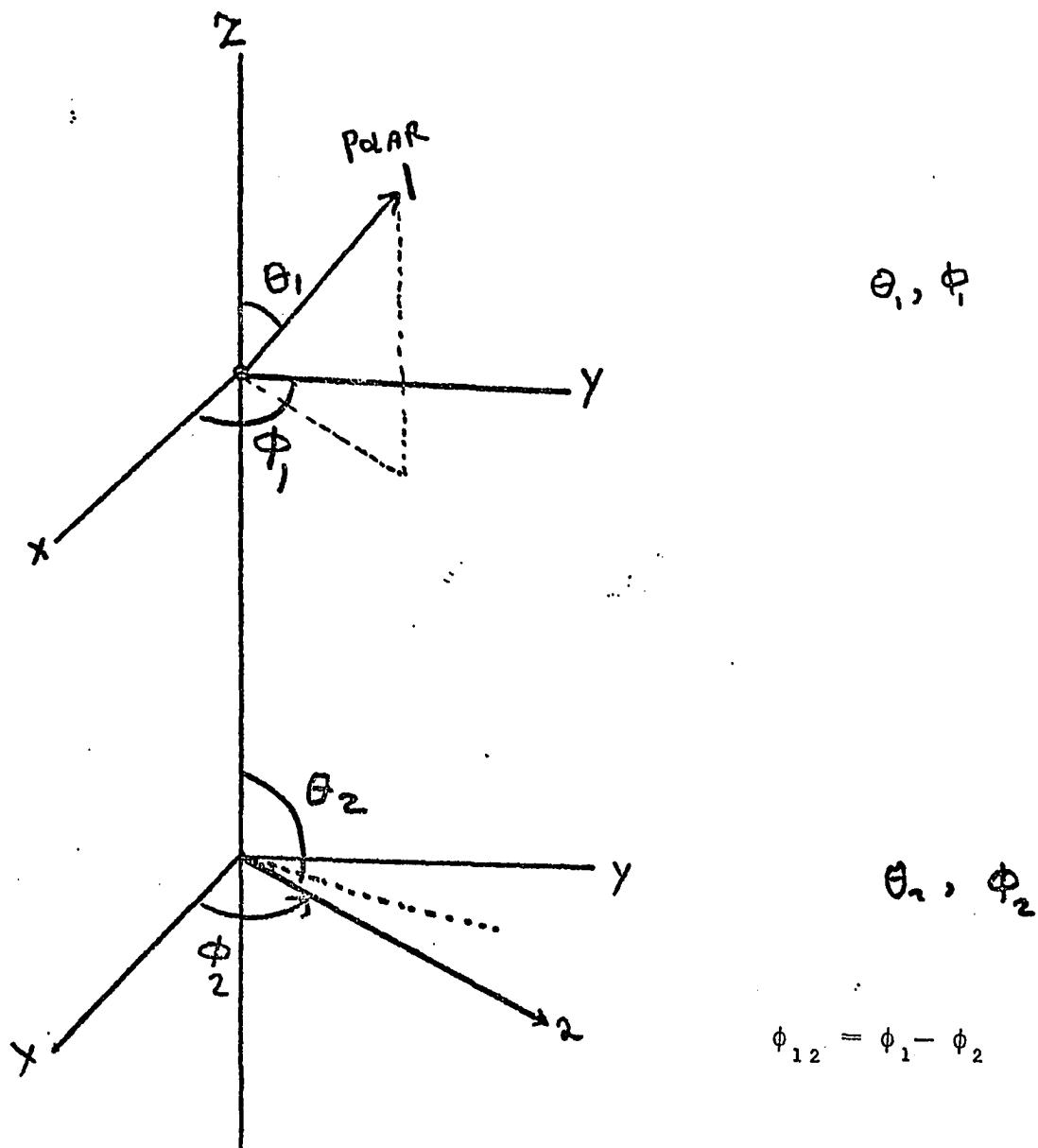


Figure 7.2

In our study, the perturbing potential part is the (quadrupole-quadrupole) interaction in the body fixed coordinate. We can expand this potential as

$$u_{QQ}(12) = 4\pi \sum_{m=-2}^2 (-1)^{(2+m)} x^{22m} Y_2^m(\omega_1) Y_2^{m*}(\omega_2) \quad (7-7)$$

where

$$x^{22-2} = x^{222} = \frac{1}{6} \cdot \frac{6}{5} \frac{Q^2}{R^5} = \frac{1}{5} \frac{Q^2}{R^5}$$

$$x^{22-1} = x^{221} = \frac{2}{3} x^{220}$$

$$x^{220} = \frac{6}{5} \frac{Q^2}{R^5} \quad (7-8)$$

And in the space fixed coordinate, we can expand

$$u_{QQ}(12) = \sum_{l=1}^2 \sum_{m_1=-2}^2 \sum_{m_2=-2}^2 f(221, R) C(221, m_1 m_2 m) Y_2^{m_1}(\omega_1) Y_2^{m_2}(\omega_2) \cdot Y_1^m(\Omega) \quad (7-9)$$

where  $l_1 + l_2 + l = \text{even}$  and  $l$  has the value of 0, 2, 4.

$$f(2, 2, l, R) = \frac{4\pi}{2^{l+1}} \sum_{m_1=-2}^2 C(2, 2, l, m_1, 0) 4\pi x^{22m} (-1)^{2+m} \quad (7-10)$$

Figure 7.2 shows  $\omega_1 = (\theta_1, \phi_1)$  and  $\omega_2 = (\theta_1, \phi_{12})$ .

Figure 7.3 shows the effect of the quadrupole moment on a subspace of angular pair correlation function.

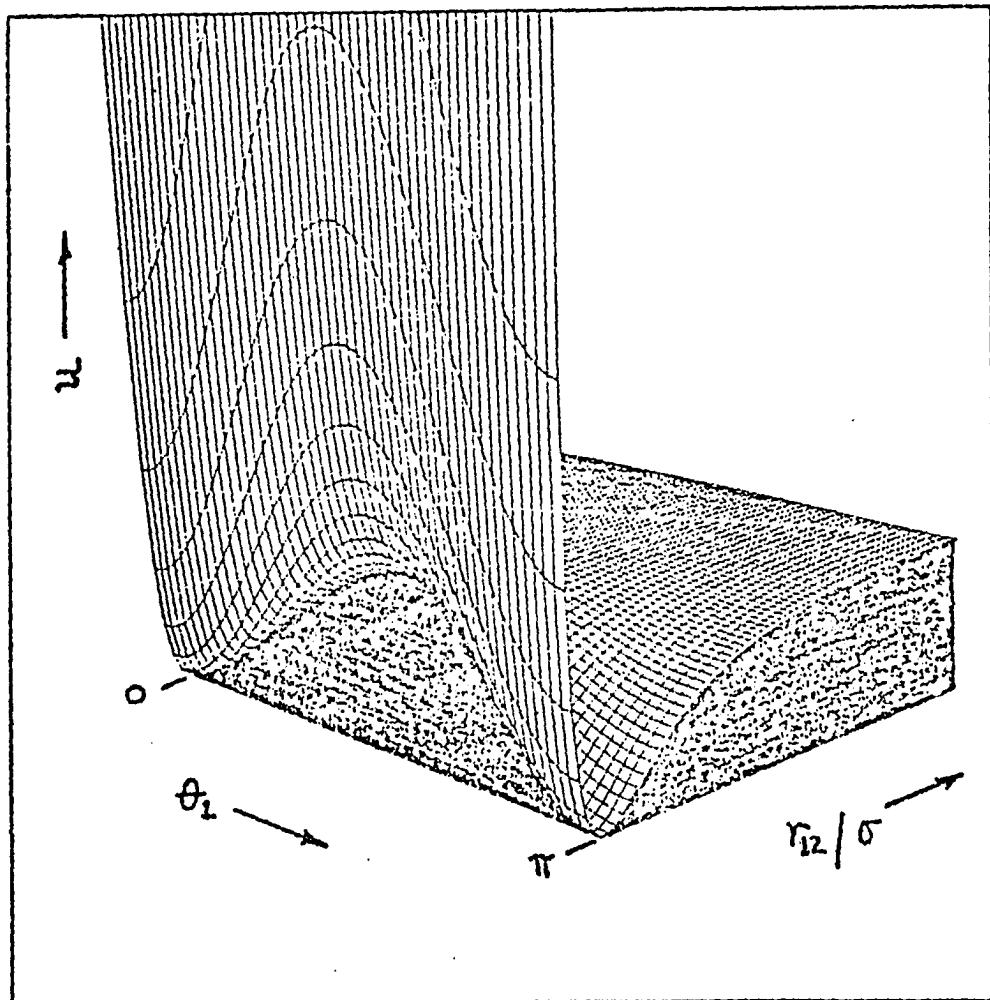


Figure 7.3 Surface of intermolecular pair potential  
 $u(r_{12}\theta_1\phi)$  for linear molecule modeled by Lennard-Jones  
 plus quadrupole interaction.  $Q/(\epsilon\sigma^5) = 1$ .  $\theta_2$  is fixed at  
 $\pi/2$  and  $\phi$  is undefined.

## CHAPTER VIII

### CALCULATION

As we mentioned before, we can write

$$\frac{Y(12)}{Y^O(12)} = 1 + \rho \Omega^{-1} \int d\Omega_3 h(13) g^O(32) f(32) \quad (8-1)$$

or

$$\frac{Y(12)}{Y^O(12)} = 1 + \rho \int dR_{13} \Omega_{13} \langle hg^O f \rangle_{w_3} R_{13}^2 \quad (8-2)$$

Now we can expand; in the space fixed spherical harmonic

$$\begin{aligned} \frac{Y}{Y^O} &= \sum_{\bar{l}_1} \sum_{\bar{m}_1} \sum_{\bar{n}_1} E_Y(\bar{l}_1 \bar{l}_2 \bar{l}, n_1, n_2, R_{12}) C(\bar{l}_1 \bar{l}_2 \bar{l}, \bar{m}_1 \bar{m}_2 \bar{m}) \cdot \\ &\quad \bar{l}_2 \bar{m}_2 \bar{n}_2 \qquad \qquad n_1 = n_2 = 0 \\ &\quad \bar{l} \bar{m} \\ \frac{Y}{I_1} \frac{\bar{m}_1^*}{I_1} (\omega_1) Y \frac{\bar{m}_2^*}{I_2} (\omega_2) Y \frac{\bar{m}}{I} (\Omega_{12}) & \end{aligned} \quad (8-3)$$

Expanding  $h$  (total correlation function)

$$\begin{aligned} h &= \sum_{l_1} \sum_{m_1} \sum_{n_1} E_h(l_1 l_3 l, n_1, n_3, R_{13}) C(l_1 l_3 l, m_1 m_3 m) \cdot \\ &\quad l_3 m_3 n_3 \qquad \qquad n_1 = n_3 = 0 \\ &\quad l m \\ \frac{Y}{l_1} \frac{m_1^*}{l_1} (\omega_1) Y \frac{m_3^*}{l_3} (\omega_3) Y \frac{m}{l} (\Omega_{13}) & \end{aligned} \quad (8-4)$$

Also, we expand  $f$  in the space fixed spherical harmonic

$$f = \sum_{l_1'} \sum_{m_1'} \sum_{n_1'} E_f(l_3', l_2', l', n_3, n_2) R_{32} C(l_3' l_2' l, m_3' m_2' m').$$

$\begin{matrix} l_1' & m_1' & n_1' \\ 3 & 3 & 3 \\ l_2' & m_2' & n_2' \\ 2 & 2 & 2 \\ l' & m' & \end{matrix}$

$$\sum_{l_3'}^{m_3'*} (\omega_3) Y_{l_3'}^{m_3'*} \sum_{l_2'}^{m_2'*} (\omega_2) Y_{l_2'}^{m_2'*} \sum_{l'}^m (\Omega_{32}) Y_{l'}^m \quad (8-5)$$

Furthermore, we can write

$$\langle hg^0 f \rangle_{\omega_3} = \sum \frac{(-1)^{m_1'}}{4\pi} \delta_{l_3 l_3'} \delta_{m_3 \bar{m}_3} E_h E_f(l_1 l_3 l m_1 m_3 m)$$

$$C(l_3 l_2 l m_3 m_2' m') Y_{l_1}^{m_1*} (\omega_1) Y_{l_2}^{m_2*} (\omega_2) Y_{l'}^{m_1'} (\Omega) g^0 \quad (8-6)$$

The terms are non zero only when  $l_3 = l_3'$ ,  $m_3 = m_3'$  and since

$$\frac{1}{\Omega} \int d\omega_3 Y_{l_3}^{m_3} (\omega_3) Y_{l_3'}^{m_3'*} (\omega_3) = \frac{1}{\Omega} \int d\omega_3 Y_{l_3}^{m_3*} (-1) Y_{l_3'}^{m_3' \bar{m}_3} \quad *8-7)$$

$$= \frac{1}{4\pi} (-1)^{m_3} \delta_{l_3 l_3'} \delta_{m_3 \bar{m}_3}$$

$$Y_1^{m*} (\theta, \phi) = (-1)^m \bar{Y}_1^{\bar{m}} (\theta, \phi)$$

Substituting in

$$\frac{Y}{Y^0} = 1 + \frac{1}{4\pi} \int dR_{13} d\Omega_{13} R_{13}^2 \sum_{l_1} \sum_{l_3} \sum_{m_1'} E_h(l_1 l_3 l R_{13}) C(l_1 l_3 l, m_1 m_3 m)$$

$\begin{matrix} l_3 & l_3 & m_2' \\ 1 & 1 & m' \end{matrix}$

$$Y_{l_1}^{m_1^*}(\omega_1) Y_{\ell}^{m}(\Omega_{13}) g_O \quad f(l_3 l_3' l', R_{32}) C(l_3 l_2' l_1', -m_3 m_2' m_1') . \quad (8-9)$$

$$Y_{l_2'}^{m_2^*}(\omega_2) Y_{l_1}^{m'}(\Omega_{32}) \quad (-1)^{m_3}$$

## CHAPTER IX

### MODEL

In the space fixed coordinate, the three molecules 1, 2, 3 can be shown as in Figure 9.1 with the polar coordinate of  $R_1, R_2, R_3$  and  $(\theta_1, \phi_1), (\theta_2, \phi_2)$ .

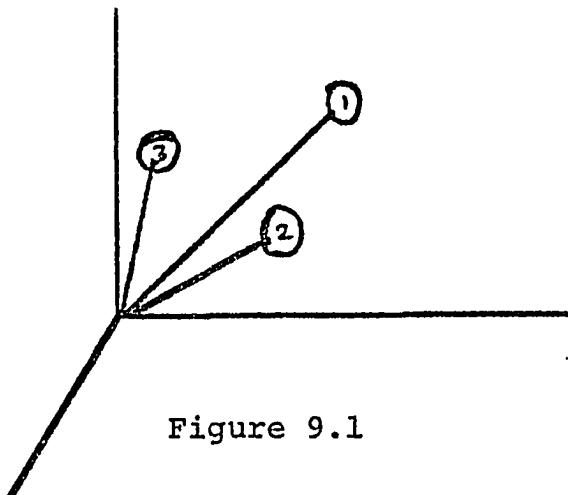


Figure 9.1

or we can simply fix our coordinates such that the origin is on molecule 1 and one axis passes through molecule 2. So Figure 9.1 can be simplified as Figure 9.2.

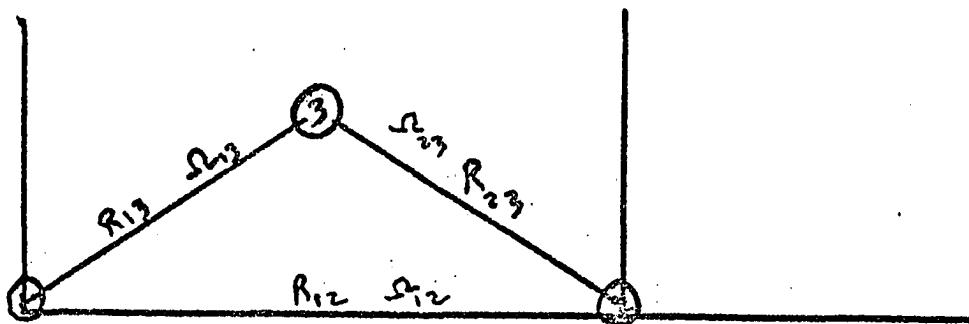


Figure 9.2

Since molecule 1 is always on the origin, new coordinate would be  $(R_{13}, \Omega_{13})$  and also  $(R_{12}, \Omega_{12})$ . Furthermore,  $(R_{12}, R_{13})$  remains constant and  $\Omega_{12}, R_{23}, \Omega_{23}$  would be the only variable factors. In other words, we have to calculate  $R_{23}, \Omega_{23}$  by knowing  $\Omega_{12}$  and the constants. Let us call  $\Omega_{12} = (\theta_2, \phi_2)$  and  $\Omega_{23} = (\theta_3, \phi_3)$ . Let us represent  $R_{23}, R_{13}, R_{12}$  by vector components so

$$\begin{aligned} \underline{R}_{23} &= (v_1, v_2, v_3) \\ \underline{R}_{13} &= (w_1, w_2, w_3) \end{aligned} \quad (9-1)$$

$$\begin{aligned} \underline{R}_{12} &= (u_1, u_2, u_3) \\ (w_1 - u_1) &= (v_1) \end{aligned}$$

$$\begin{aligned} \underline{R}_{23} &= (w_2 - u_2) = (v_2) \\ (w_3 - u_3) &= (v_3) \end{aligned} \quad (9-2)$$

but from the geometry we have

$$\begin{aligned} w_1 &= R_{13} \sin \theta_{13} \cos \phi_{13} \\ w_2 &= R_{13} \sin \theta_{13} \sin \phi_{13} \\ w_3 &= R_{13} \cos \theta_{13} \end{aligned} \quad (9-3)$$

also, by similarity, one can write

$$\begin{aligned} u_1 &= R_{12} \sin \theta_{12} \cos \phi_{12} \\ u_2 &= R_{12} \sin \theta_{12} \sin \phi_{12} \\ u_3 &= R_{12} \cos \theta_{12} \end{aligned} \quad (9-4)$$

and

$$\begin{aligned} \mathbf{v}_1 &= R_{23} \sin \theta_{23} \cos \phi_{23} \\ \mathbf{v}_2 &= R_{23} \sin \theta_{23} \sin \phi_{23} \\ \mathbf{v}_3 &= R_{23} \cos \theta_{23} \end{aligned} \quad (9-5)$$

From this last set of equations we can have

$$\begin{aligned} R_{23} &= \frac{w_1 - u_1}{\sin \theta_{23}} = \frac{w_3 - u_3}{\cos \theta_{23}} = \frac{R_{13} \cos \theta_{13} - R_{12} \cos \theta_{12}}{\cos \theta_{23}} \\ \cos \theta_{23} &= \frac{w_3 - u_3}{R_{23}} \end{aligned} \quad (9-6)$$

and

$$R_{23} = (w_1 - u_1)^2 + (w_2 - u_2)^2 + (w_3 - u_3)^2 \quad (9-7)$$

$$\cos \phi_{23} = \frac{w_1 - u_1}{R_{23} \sin \theta_{23}} \text{ or } \sin \phi_{23} = \frac{w_2 - u_2}{R_{23} \sin \theta_{23}} \quad (9-8)$$

$$R_{13}^2 = w_1^2 + w_2^2 + w_3^2$$

$$\cos \theta_{13} = \frac{w_2}{R_{13}} \quad (9-9)$$

$$\sin \theta_{13} = \frac{(w_1^2 + w_2^2)^{\frac{1}{2}}}{R_{13}}$$

$$\cos \phi_{13} = \frac{w_1}{(w_1^2 + w_2^2)^{\frac{1}{2}}} \quad (9-9)$$

$$\sin \theta_{13} \cos \theta_{13} = \frac{w_1}{R_{13}}$$

$$\sin \phi_{13} = \frac{w_2}{(w_1^2 + w_2^2)^{\frac{1}{2}}}$$

$$\text{so } \sin \theta_{13} \sin \theta_{13} = \frac{w_1}{R_{13}}$$

$$\text{but } R_{12}^2 = (w_1 - u_1)^2 + (w_2 - u_2)^2 + (w_3 - u_3)^2 \quad (9-10)$$

and also

$$\cos \theta_{12} = \frac{u_3}{R_{12}} \quad (9-11)$$

$$\sin \theta_{12} = \frac{(u_1^2 + u_2^2)^{\frac{1}{2}}}{R_{12}} \quad (9-12)$$

$$\cos \phi_{12} = \frac{u_1}{(u_1^2 + u_2^2)^{\frac{1}{2}}} \quad (9-13)$$

$$\sin \phi_{12} = \frac{u_2}{(u_1^2 + u_2^2)^{\frac{1}{2}}} \quad (9-14)$$

$$\sin \theta_{12} \cos \theta_{12} = \frac{u_2}{R_{12}} \quad (9-15)$$

$$\sin \theta_{12} \sin \phi_{12} = \frac{u_2}{R_{12}} \quad (9-16)$$

For  $\Omega_{23}$

$$\cos \theta_{23} = \frac{w_3 - u_3}{R_{23}} \quad (9-17)$$

$$\sin \theta_{23} = \frac{[(w_1 - u_1)^2 + (w_2 - u_2)^2]^{\frac{1}{2}}}{R_{23}} \quad (9-18)$$

$$\cos \phi_{23} = \frac{w_1 - u_1}{[(w_1^2 - u_1^2)^2 + (w_2 - u_2)^2]^{\frac{1}{2}}} \quad (9-19)$$

$$\sin \phi_{23} = \frac{w_2 - u_2}{[(w_1 - u_1)^2 + (w_2 - u_2)^2]^{\frac{1}{2}}} \quad (9-20)$$

And finally,

$$\sin \theta_{23} \cos \phi_{23} = \frac{w_1 - u_1}{R_{23}}$$

$$\sin \theta_{23} \sin \phi_{23} = \frac{w_1 - u_1}{R_{23}} \quad (9-21)$$

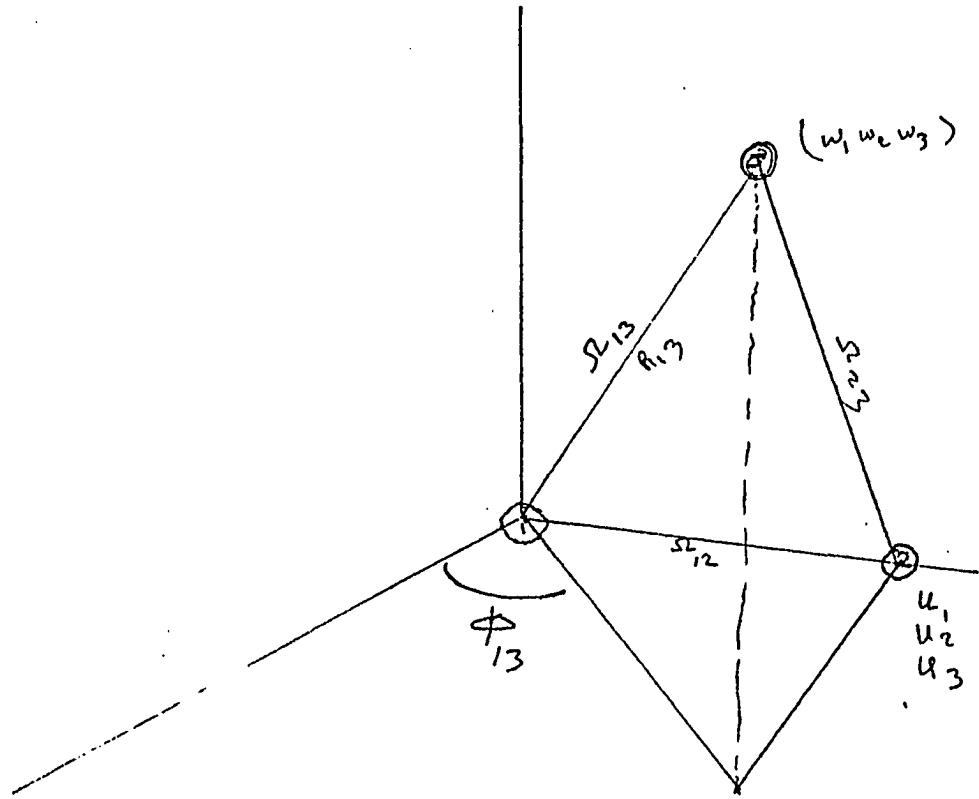


Fig 9-3

## CHAPTER X

### PROCEDURE

First we had to find  $F = \exp(-\beta u) - 1$ , so called Mayer function. The spherical harmonic expansion of  $F$  in the space fixed coordinate is

$$F(r, \omega_1 \omega_2) = \sum_{l_1} \sum_{m_1} E_f(l_1 l_2 l, r) C(l_1 l_2 l, m_1 m_2 m) Y_{l_1}^{m_1}(\omega_1) Y_{l_2}^{m_2}(\omega_2) Y_l^m(\Omega) \quad (10-1)$$

where  $E_f(l_1 l_2 l, r)$  are the expansion coefficients and by orthogonality condition of  $C(l_1 l_2 l, m_1 m_2 m)$  which has been mentioned before, and also because of the orthogonality of spherical harmonics function

$$\int d\omega_1 Y_l^m(\omega_1) Y_l^{m'}(\omega_1) = \delta_{ll} \delta_{mm'} \quad (10-2)$$

we can write

$$E_f(l_1 l_2 l, r) = \int_0^\pi d\theta_1 \int_0^\pi d\theta_2 \int_0^\pi d\theta_3 \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{2\pi} d\phi_3 F(r, \omega_1, \omega_2) \cdot \\ C(l_1 l_2 l, m_1 m_2 m) Y_{l_1}^{m_1}(\omega_1) Y_{l_2}^{m_2}(\omega_2) Y_l^m(\Omega) \quad (10-3)$$

This is a six fold integral.

Integration

Since this multidimensional integral cannot be evaluated by analytical means, numerical integration processes must be used. The methods generally employed are based on the product of Simpson's rule or the product of Gaussian formulae. While these formulae can be applied to give the value of a multiple integral correct to any accuracy, they are not the most economical formulae in the sense that a large number of integrand function evaluations is needed. Since these evaluations are the most time-consuming part of these computations, economy is an important factor. Integration formulae which are exact for multinomials over the entire region of integration are necessarily more economical than the product formulae and can be applied to the same accuracy by subdividing the integration region.

The construction of a satisfactory multidimensional non-product formula for a given hyper-region is a matter of some difficulty and is in fact a problem in the theory of algebraic equations.

The required formula must have positive weights associated with it in order to avoid loss of accuracy by cancellation and all the integration points must lie entirely within the integration region. Watts has given a general third degree formula valid for the hypercube of side 2 and centered on the origin. Let  $\Sigma_n(k)$  denote the point  $(\sigma_1^{(k)}, \sigma_2^{(k)}, \dots, \sigma_n^{(k)})$  of the n-dimensional Euclidean space,  $\sigma^n$ , for  $k=1, 2, \dots, 2n$

and

$$\sigma_{2S-1}^{(k)} = (2/3)^{\frac{1}{2}} \cos (2S-1) \frac{k\pi}{n}$$

$$\sigma_{2S}^{(k)} = (2/3)^{\frac{1}{2}} \sin (2S-1) \frac{k\pi}{n} \quad (10-4)$$

where  $S = 1, 2, \dots, n/2$ , and when  $n$  is odd  $\sigma_n^{(k)} = (-1)^k / \sqrt{3}$ .

If  $S_n$  be the hypercube of side 2 and the center at the origin an integral over  $S_n$  may be evaluated by the formula.

The weight for each point is  $1/2n$ . We also used shifting methods to get a more accurate answer. This method could cut the computer time, except that in this 6-fold integral  $f = \exp(-\beta u) - 1$ , where  $u$  itself must be expanded in the space fixed coordinate and again is 6-fold integral.

With shifting twice within 6 dimensions, we had to calculate one point  $(n+2)*(n+2)$  which is 64 summations and still the accuracy was poor. So what we decided was as follows. We calculated these coefficients of expansion in the body fixed, then transferred them to the space fixed coordinate by transformation formula (4-2). The spherical harmonic expansion of the Mayer function in the body fixed coordinate is

$$F(r_{12}, \omega_1, \omega_2) = 4\pi \sum_{l_1} \sum_{l_2} E_f(l_1 l_2 m_1 r_{12}) Y_{l_1}^{m_1}(\omega_1) Y_{l_2}^{-m_1}(\omega_2) \quad (10-5)$$

The  $E_f(l_1 l_2 m_1 r_{12})$  are the expansion coefficients to be determined. For molecules with a plane of symmetry perpendicular to the molecular axis, the  $E_f(l_1 l_2 m_1 r_{12})$  are zero unless

both  $l_1$  and  $l_2$  are even (homonuclear diatomics e.g.). For heteronuclear diatomics  $E_f(l_1 l_2 m_r)$  both odd and even values contribute. Multiplying both sides of (10-5) by  $Y_{l_1}^{m^*}(\omega_1)$   $Y_{l_2}^{\bar{m}^*}(\omega_2)$  where \* indicates complex conjugate and integrating over  $\omega_1$  and  $\omega_2$

$$\int d\omega_1 d\omega_2 g(r_{12}, \omega_1, \omega_2) Y_{l_1}^{m^*}(\omega_1) Y_{l_2}^{\bar{m}^*}(\omega_2) = 4\pi \sum_{l_1 l_2 m} E_g(l_1 l_2 m, r_{12}) \cdot (10-6)$$

$$\int d\omega_1 Y_{l_1}^m(\omega_1) Y_{l_1}^{m^*}(\omega_1) \int d\omega_2 Y_{l_2}^{\bar{m}}(\omega_2) Y_{l_2}^{\bar{m}^*}(\omega_2)$$

$$\text{but } \int d\omega Y_l^m(\omega) Y_{l'}^{m^*}(\omega) = \delta_{ll'} \delta_{mm'}$$

$$E_g(l_1 l_2 m, r_{12}) = \frac{1}{4\pi} \int d\omega_1 d\omega_2 Y_{l_2}^{\bar{m}^*}(\omega_2) g(r_{12}, \omega_1, \omega_2) \quad (10-7)$$

Dividing both sides of (10-7) by  $\int d\omega_1 d\omega_2 g(r_{12}, \omega_1, \omega_2)$

$$\frac{E_g(l_1 l_2 m, r_{12})}{\int d\omega_1 d\omega_2 g(r_{12}, \omega_1, \omega_2)} = \frac{1}{4\pi} \frac{\int d\omega_1 d\omega_2 Y_{l_1}^{m^*}(\omega_1) Y_{l_2}^{\bar{m}^*}(\omega_2) g(r_{12}, \omega_1, \omega_2)}{\int d\omega_1 d\omega_2 g(r_{12}, \omega_1, \omega_2)} \quad (10-8)$$

where  $\bar{m}_1 = -m_1$

and

$$Y_l^m(\omega) = Y_l^m(\theta, \phi) \quad (10-9)$$

We can write the formula for  $g(r_{12}, \omega_1, \omega_2)$  with the same form of expansion coefficient so we have equation (10-7) in terms of

$$g(r_{12}, \omega_1, \omega_2) = 4\pi \sum_{l_1 l_2 m} g(l_1 l_2 m, r_{12}) Y_{l_1}^m(\omega_1) Y_{l_2}^{\bar{m}}(\omega_2) \quad (10-10)$$

The average of a property  $X(r_{12}, \omega_1, \omega_2)$  in a spherical shell of radius between  $r_{12}$  and  $r_{12}+dr_{12}$  is defined by

$$\langle X(r_{12}, \omega_1, \omega_2) \rangle_{\text{shell}} = \frac{\int d\omega_1 d\omega_2 X(r_{12}, \omega_1, \omega_2) g(r_{12}, \omega_1, \omega_2)}{\int d\omega_1 d\omega_2 g(r_{12}, \omega_1, \omega_2)} \quad (10-11)$$

Further, from (10-7) we have

$$g_{ooo}(r_{12}) = \frac{1}{(4\pi)} \int d\omega_1 d\omega_2 g(r_{12}, \omega_1, \omega_2)$$

Combine (10-10) and (10-11) to give

$$Y(l_1 l_2 m, r_{12}) = 4\pi g_{ooo}(r_{12}) \langle Y_{l_1}^{m_1}(\omega_1) Y_{l_2}^{\bar{m}_2}(\omega_2) \rangle_{\text{shell}} \quad (10-12)$$

Note this  $g_{ooo}(r)$  is the same  $g_o(r)$  expression for all  $g(l_1, l_2, m, r_{12})$  having  $l_1$  and  $l_2$  even up to  $\{l_1, l_2, m\} = \{4, 4, 4\}$  and the additional terms with  $l_1$  and  $l_2$  even,  $m = 0$  to  $\{l_1 l_2 m = 800\}$  are tabulated in Appendix F of Reference 1.

We had used the data of  $g_{ooo}(r)$  from Verlet where  $r_{12}$  goes from 0.7 up to 3.2. (We found the  $g_{ooo}$  for higher  $r_{12}$  by extrapolation.)

### Calculation of $g_o F$

As mentioned before, to calculate the expansion coefficients of the  $F$  in the body fixed coordinate, we used Simpson's rule to solve multiple integral. Since  $\phi_2 = 0$

$$E_F(l_1 l_2 m, r_{12}) = \int d\theta_1 \int d\theta_2 \int d\theta_1 F(r_{12}, \omega_1, \omega_2) Y_{l_1}^{m_1}(\omega_1) Y_{l_2}^{\bar{m}_2}(\omega_2) \quad (10-13)$$

$$\omega_1 = (\theta_1, \phi_1), \omega_2 = (\theta_2, \phi)$$

$$\text{and } F(r_{12}, \omega_1, \omega_2) = \exp(-\beta U(r_{12}, \omega_1, \omega_2)) - 1$$

The potential  $U$  has been discussed before.  $\beta = 1/kT$  where  $k$  is the Boltzmann constant. This evaluation has been carried out for two different conditions:  $T^* = 1.277$  and  $Q = 0.5$ ;  $T^* = 1.294$  and  $Q = 1$ .  $Q$  is the quadrupole moment discussed in the potential chapter.

The  $Y_l^m(\omega)$  spherical harmonic functions have been calculated from  $l = 0$  to  $l = 4$  and in each calculation it has been fit in.

By finding the coefficients of  $E_f(l_1 l_2 m_1 r_{12})$  and using the conversion from body to spaced fixed coordinates, we could find the  $E_f(l_1 l_2 l, r_{12})$  where  $|l_1 - l_2| \leq l \leq (l_1 + l_2)$  and later we were able to find  $g^o(r_{12}) * E_f(l_1 l_2 l r_{12})$  for each  $r_{12}$  from 0.7 to 3.2 with  $dr = 0.04$  when  $g_o$  is at reduced density of 0.85. Pages 55 to 67 show some results for  $E_f$ .

After having reduced the  $g^o E_f(l_1 l_2 l, r_{12})$  for reduced density of 0.85 and two different temperatures and quadrupole moments, we are able to calculate the  $E_f(l_1 l_2 l, r_{12})$  and the space fixed spherical harmonic expansion coefficients of direct correlation function  $h(r_{12}, \omega_1, \omega_2)$ . Simulation results are given by Dr. Haile in his dissertation (7) for  $l_1 = 0$  to 4,  $l_2 = 0$  to 4 and  $m$  going from negative of  $l_1$  or  $l_2$  up  $l_1$  or  $l_2$  depending on which one is smaller.

Now since

$$g(l_1 l_2 \text{ m } R_{12}) = h(l_1 l_2 \text{ m } R_{12}) - 1$$

by transferring the body fixed to space fixed coordinates, we were able to calculate all  $E_g(l_1 l_2 l, r_{12})$  from  $l_1 = 0$  to 4 and  $l_2 = 0$  to 4 for  $r$  starting from 0.7 up to 3.2 with  $dr = 0.025$ . The only difference here is the  $dr$  in both coefficients of  $E_f(l_1 l_2 l, r_{12})$  and  $E_h(l_1 l_2 l \text{ m } r_{12})$  which we took care of by changing our  $dr$  from 0.04 to 0.025. In Table 1.2, some values for  $g(l_1 l_2 \text{ m } r_{12})$  and  $g(l_1 l_2 l \text{ m } r_{12})$ ,  $E_h(l_1 l_2 l \text{ m } r_{12})$  are given.

## CHAPTER XI

### CONVOLUTION PART B

We can arrange the formula for  $Y$  and write

$$\left( \frac{Y(12)}{Y_O(n)} - 1 \right) = \sum_{\substack{l_1 \\ l_2}} \sum_{\substack{\bar{m}_1 \\ \bar{m}_2 \\ m}} E_Y(\bar{l}_1 \bar{l}_2, \bar{l}, R_{12}) C(\bar{l}_1 \bar{l}_2, \bar{l}, \bar{m}_1, \bar{m}_2 m) \cdot$$

$$Y_{l_1}^{m_1}(\omega_1) Y_{l_2}^{m_2}(\omega_2) Y_l^{m^*}(\Omega_{12}) \quad (11-1)$$

Also for  $h(13)$ , we had

$$h(13) = \sum_{\substack{l_1 \\ l_3 \\ l}} \sum_{\substack{m_1 \\ m_3 \\ m}} E_h(l_1 l_2 l, R_{13}) C(l_1 l_3 l, m_1 m_3 m) Y_{l_1}^{m_1}(\omega_1) Y_{l_3}^{m_3}(\omega_2) \cdot$$

$$Y_l^m(\Omega_{13}) \quad (11-2)$$

For  $g^O F(23)$ , we can write

$$g^O F(23) = \sum_{\substack{l'_3 \\ l'_2 \\ l'}} \sum_{\substack{m'_3 \\ m'_2 \\ m'}} E_f(l'_3 l'_2 l', R_{32}) C(l'_3 l'_2, m'_3 m'_2 m') \cdot$$

$$Y_{l'_3}^{m'_3}(\omega_3) Y_{l'_2}^{m'_2}(\omega_2) Y_{l'}^{m'^*}(\Omega_{32}) \quad (11-3)$$

Now we substitute for  $g^O f(23)$  and  $h(13)$  to get  $\langle h^*(13) g^O f(23) \rangle_{\omega_3}$ .

By definition of  $\langle \dots \rangle_{\omega_3}$ , and using the orthogonality condition for both spherical harmonic functions and Clebsch Gordon coefficients, we have

$$\langle h^*(13) g^0 f(23) \rangle_{\omega_3} = \int d\omega_3 \sum_{l_1 l_3 l} \sum_{m_1 m_3 m} E_h(l_1 l_3 l, R_{13}) C(l_1 l_3 l, m_1 m_3 m)$$

$$Y_{l_1}^{m_1^*}(\omega_1) Y_{l_3}^{m_3^*}(\omega_3) Y_l^m(\Omega_{13}) \sum_{l'_3} \sum_{m'_3} E_f(l'_3 l'_2 l', R_{32}) \\ (11-4)$$

$$Y_{l'_2}^{m'_2}(\omega_2) Y_l^{m'}(\Omega_{32})$$

$$C(l'_3 l'_2 l' m'_3 m'_2) Y_{l'_3}^{m'_3}(\omega_3) Y_{l'_2}^{m'_2}(\omega_2) Y_l^{m'}(\Omega_{32})$$

Since

$$\int d\omega_3 Y_{l'_3}^{m'_3}(\omega_3) Y_{l_3}^{m_3}(\omega_3) = \frac{1}{4\pi} \delta_{l'_3 l_3} \delta_{m'_3 m_3} \quad (11-5)$$

The only non-zero term will be when  $l'_3 = l_3$  and  $m'_3 = m_3$  and the average will be

$$\langle h^*(13) g^0 f(23) \rangle_{\omega_3} = \frac{1}{4\pi} \sum_{l_1} \sum_{m_1} \sum_{l_2} \sum_{m_2} E_h(l_1 l_3 l, R_{12}) \\ (11-6)$$

$$E_f(l_3 l_2 l, R_{12}) C(l_3 l_2 l, m_3 m_2 m) C(l_1 l'_2 l' m_1 m'_2 m)$$

$$Y_{l_1}^{m_1^*}(\omega_1) Y_l^m(\Omega_{13}) Y_{l'_2}^{m'_2}(\omega_2) Y_l^{m'}(\Omega_{32})$$

Furthermore, we also had for

$$\frac{Y}{Y_0} = 1 + \iint dR_{13} d\Omega_{13} \langle h^* g^0 f \rangle_{\omega_3} R_{13}^2$$

or by substitution for

$$\langle h^* g^0 f \rangle_{\omega_3}$$

$$\frac{Y(12)}{Y_0} - 1 = \frac{\rho}{4\pi} \int dR_{13} R_{13}^2 d\theta_{13} \sin \theta_{13} d\phi_{13} \sum_{l_2} \sum_{l_1} \sum_{m_2} \sum_{m'} \\ l' l_3 m m' \\ l' l_3 m m' \\ l m m' \\ l m$$

$$E_h(l_1 l_3 l, R_{13}) C(l_1 l_3 l m_1 m_3 m) Y_{l_1}^{m_1^*}(\omega_1) Y_{l_2}^{m_2^*}(\omega_2) Y_l^m(\Omega_{12}) Y_{l_1}^{m'*}(\Omega_{32}) .$$

$$C(l_3 l_2 l' m_3 m_2 m') \quad (11-7)$$

From this equation one can have

$$\int d\omega_1 Y_{L_1}^{M_1}(\omega_1) \left( \frac{Y(12)}{Y_0(12)} - 1 \right) = \sum_{\bar{L}_2} \sum_{\bar{m}_2} E_Y(L_1 \bar{L}_2 \bar{L}, R_{12}) .$$

$$C(L_1 \bar{L}_2 \bar{L} \underline{M}_1 \bar{m}_2 \bar{m}) Y_{\bar{L}_2}^{\bar{m}_2}(\omega_2) Y_{\bar{L}}^{\bar{m}_2^*}(\Omega_{12}) (-1)^{M_1} \quad (11-8)$$

$$\bar{L}_1 = L_1$$

$$\bar{m}_1 = \underline{M}_1$$

where  $\underline{M}_1 = -M_1$ .

So one can substitute for the left hand side of (11-8) and integrate over  $d\omega_2$ . The non-zero terms occur when

$$l_1 = L_1$$

$$m_1 = M_1$$

$$\frac{\rho}{4\pi} \int dR_{13} R_{13}^2 d\theta_{13} \sin \theta_{13} d\phi \sum_{l'} \sum_{l_3} \sum_{m_3} \sum_{m'_2} E_h(L_1 l_3 l, R_{13}) \cdot \\ \begin{matrix} l' & l_3 & m_3 & m'_2 \\ l'_2 & l & m & m' \end{matrix}$$

$$C(L_1 l_3 M_1 m_3 m) Y_{l_2}^{m'_2}(\omega_2) Y_l^m(\Omega_{12}) Y_{l'}^{m'*}(\Omega_{32}) \quad (11-9)$$

Again we can multiply equation (11-8) by  $Y_{L_2}^{m*}(\omega_2)$  and integrate over  $d\omega_2$  to have

$$\int d\omega_2 Y_{L_2}^{M_2*}(\omega_2) \int d\omega_1 Y_{L_1}^{M_1}(\omega_1) \left( \frac{Y}{Y_0} - 1 \right) = \quad (11-10)$$

$$\sum_{\bar{l}} \sum_{\bar{m}} E_Y(L_1 L_2 \bar{l}, R_{12}) C(L_1 L_2 \bar{l}, M_1 M_2 \bar{m}) Y_{\bar{l}}^{m'*}(\Omega_{12}) (-1)^{M_1}$$

which gave us  $\bar{l}_2 = L_2$  and  $\bar{m}_2 = M_2$ . Substituting in (11-7) we have on the R.H.S.

$$\frac{\rho}{4\pi} \int dR_{13} R_{13} \sin \theta_{13} d\phi_{13} \sum_{\ell_3} \sum_{m_3} \sum_{\ell'} \sum_{m'} E_h(L_1 \ell_3 \ell, R_{13}) Y_{\ell}^m(\Omega_{13}) \cdot$$

$$Y_{\ell'}^{m'*}(\Omega_{32}) E_F(l_3 L_2 \ell', R_{32}) C(l_3 L_2 \ell', m_3 M_2 M') \quad (11-11)$$

where  $l'_2 = L_2$

$$m'_2 = M_2$$

Again on the R.H.S. of equation (11-10) we have

$$(-1)^{M_1} \sum_{\bar{L}} \sum_{\bar{L}} E_Y(L_1 L_2 \bar{L}, R_{12}) C(L_1 L_2 \bar{L}, M_1 M_2 \bar{m}) Y_{\bar{L}}^{\bar{m}*}(\Omega_{12}) \quad (11-12)$$

When we multiply both sides of equation (11-12) by  $Y_L^M(\Omega_{12})$  and integrate over  $\Omega_{12}$ , we have

$$\int d\Omega_{12} Y_L^M(\Omega_{12}) \int d\omega_2 Y_{L_2}^{M_2}(\omega_2) \int d\omega_1 Y_{L_1}^{M_1}(\omega_1) \left( \frac{Y}{Y_1} - 1 \right) = \\ (-1)^{M_1} E_Y(L_1 L_2 L, R_{12}) C(L_1 L_2 L, M_1 M_2 M) \quad (11-13)$$

where here again  $\bar{L} = L$  and  $\bar{m} = M$ . And from equation (11-11) the R.H.S. gives

$$\frac{\rho}{4\pi} \int dR_{13} R_{13} d\theta_{13} \sin \theta_{13} d\phi_{13} d\theta_{12} \sin \theta_{12} d\phi_{12} \cdot$$

$$\sum_{l_3} \sum_{m_3} \sum_{l'} \sum_{m'} E_h(L_1 l_3 l, R_{13}) E_f(l_3 L_2 l', R_{32}) \cdot$$

$$C(L_1 l_3 l, M_1 m_3 m) C(l_3 L_2 l', m_3 M_2 m') Y_l^m(\Omega_{13}) Y_{l'}^{m'}(\Omega_{32}) \cdot$$

$$Y_L^M(\Omega_{12}) \quad (11-14)$$

Finally, the result would be

$$(-1)^{M_1} E_Y(L_1 L_2 L, R_{12}) C(L_1 L_2 L, M_1 M_2 M) =$$

$$\frac{\rho}{\pi} \int dR_{13} R_{13} d\theta_{13} \sin \theta_{13} d\phi_{13} d\theta_{12} \sin \theta_{12} d\phi_{12} \cdot$$

$$\sum_{l_3} \sum_{m_3} \sum_{l'} \sum_{m'} E_h(l_1 l_3 l, R_{13}) E_f(l_2 L_3 l', R_{32}) \cdot$$

$$C(L_1 l_3 l, M_1 m_3 m) C(l_3 L_2 l', m_3 M_2 m') Y_l^m(\Omega_{13}) \cdot$$

$$Y_{l'}^{m'*}(\Omega_{32}) Y_L^M(\Omega_{12})$$

This is a 5 fold integral which must be computed by computer.

$$RHS = \sum_{\overline{l}} \sum_{\overline{m}} E_Y(l_1 L_2 \overline{l} R_{12}) C(L_1 L_2 \overline{l} M_1 M_2 \overline{m})$$

$$(-1)^{M_1} \frac{Y_l^m(\Omega_{12})}{l} = \sum_m \sum_{\overline{l}} E_Y(l_1 L_2 \overline{l} R_{12}) C(l_1 l M_1 M_2 \overline{m}) \cdot$$

$$(-1)^{M_1} \left( \frac{2l+1}{4\pi} \right)^{\frac{1}{2}} \left( \frac{(\overline{l}-\overline{m})!}{(\overline{l}+\overline{m})!} \right)^{\frac{1}{2}} e^{iM\phi} P_l^{\overline{m}}(\chi = \cos \theta)$$

$$= \frac{\rho}{4\pi} \int dR_{13} R_{13} d\theta_{13} \sin \theta_{13} d\phi_{13} \sum_{l_1} \sum_{m_1} E_h(l_1 l_2 l R_{13}) \cdot$$

$$l_1 m_1$$

$$l m$$

$$C(L_1 l_3 l M_1 m_3 m) E_f(l_3 L_2 l' R_{32}) C(l_3 L_2 l' m_3 M_2 m') \cdot$$

$$Y_l^m(\Omega_{13}) Y_{l'}^{m'*}(\Omega_{32}) (-1)^{M_1}$$

In the special case when

$$\phi_{12} = \frac{\pi}{2}, \quad \phi_{12} = 0$$

$$\theta_{12} = \frac{\pi}{2}, \quad \theta_{12} = 0$$

$$E_Y(l_1 l_2 m_1 R_{12}) = \frac{1}{4\pi} \sum_l \left(\frac{2}{4\pi}\right)^{\frac{l+1}{2}} E_Y(l_1 l_2 l, R_{12}).$$

$C(l_1 l_2 l m_1 \bar{m}_1 0)$

$$\text{since } Y_O^0 = \left(\frac{1}{4\pi}\right)^{\frac{l+1}{2}}$$

$$Y_1^{-1}\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = - \left(\frac{3}{8\pi}\right)^{\frac{l+1}{2}}$$

$$Y_1^m(\theta, \phi) = \left(\frac{2}{4\pi}\right)^{\frac{l+1}{2}} \frac{(l-m)!}{(l+m)!} e^{im\phi} P_1^m(x = \cos \theta)$$

where

$$P_1^m(x) = (-1)^m \sin \theta)^m \frac{1}{2^l l!} \left(\frac{d}{dx}\right)^{l+m} (x^2 - 1)^l$$

In the case when

$$\begin{cases} \phi_{12} = 0 \\ \theta_{12} = 0 \end{cases}$$

$$Y_{\bar{l}}^{\bar{m}}(\theta_{12}, \phi_{12}) = Y_{\bar{l}}^{\bar{m}}(0, 0) = \left(\frac{2}{4\pi}\right)^{\frac{\bar{l}+1}{2}} \frac{(\bar{l}-\bar{m})!}{(\bar{l}+\bar{m})!} . 1. 0$$

for  $\bar{m} \neq 0$

$$\text{For } \phi_{12} = 0 \quad \bar{m} = 0 \quad \theta_{12} = 0$$

$$M_1 = -M_2, C(L_1 L_2 \bar{\ell} M_1 M_2 0) = C(L_1 L_2 \bar{\ell} M_1 M_1 0)$$

But  $P_O(x) = 1,$

$$P_1(x) = x$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1) = 1 \quad \text{as } x = 1$$

$$P_3(x) = \dots$$

So

$$\text{RHS} + E_Y(l_1 l_2 \ell R_{12}) + E_Y(l_1 l_2 m_1 R_{12})$$

$$\frac{\Sigma}{m} \bar{m} = - \bar{I} + \bar{x} = 0 \text{ only}$$

RHS =  $4\pi E_Y(L_1 L_2 M_1 r_{12})$  for  $\phi_{12} = 0$  and  $\theta_{12} = 0$ , depending on which value of M was chosen.

## CHAPTER XII

### FOURIER TRANSFORMATION

In the convolution chapter we saw a five fold integral is existing which takes lot of computer time. So we decided to use Fourier transformation in order to cut the time.

Assume

$$B(12) = \frac{Y(12)}{Y_O(r_{12})} - 1 = \sum_{l'} \sum_{m'} E_Y(l' l'' l, r_{12}) C(l' l'', m' m'' m) \cdot \\ \cdot \sum_{l'' m''}^{} \sum_{l m}^{} Y_l^{m'}(\omega_1) Y_l^{m''}(\omega_2) Y_l^{m*}(\Omega_{12}) \quad (12-1)$$

Also

$$B(12) = \frac{Y(12)}{Y_O(r_{12})} - 1 = \rho \int d\Omega h(13) g_O f(32) \\ = \frac{\rho}{4\pi} \int d\Omega x_3 h(13) = g_O f(32) \quad (12-2)$$

And

$$\tilde{B}(k) = \sum_{l_1 l_2 l} \sum_{m_1} \sum_{m_2} \tilde{E}_Y(l_1 l_2 l, k) \sum_{m_1} C(l_1 l_2 l, m_1 m_2 m) Y_l^{m_1}(\theta_k \phi_k) Y_{l_2}^{m_2}(\omega_1) Y_{l_2}^{m_2}(\omega_2)$$

$$\tilde{E}_Y(l_1 l_2 l, k) = i^l (4\pi) \int_0^\infty dr r^2 J_l(kr) E_Y(l_1 l_2 l, r) \quad (12-3)$$

where  $J_l(kr)$  is the spherical Bessel function and  $\tilde{B}(k)$  is the Hankel transformation of  $B(12)$ .

Now

$$\tilde{\Sigma} E_Y C \frac{m_1}{l_1} \frac{m_2}{l_2} Y_l^{m*} = \frac{\rho}{4\pi} \{ (\tilde{\Sigma} E_h (l'_1 l'_2 l' k) C(l'_1 l'_2 l', m'_1, m'_2 m') \cdot$$

$$\frac{Y^{m'_1}}{l'_1} (\omega_1) \frac{Y^{m'_2}}{l'_2} (\omega_2) \frac{Y^{m'*}}{l'} ) (\Sigma E_f (l''_1 l''_2 l'', k) C(l''_1 l''_2 l'', k, m''_1 m''_2 m) \cdot$$

$$\frac{Y^{m''_2}}{l''_1} (\omega_3) \frac{Y^{m''_2}}{l''_2} * (\omega_2) \frac{Y^{m''}}{l''} ) \} \text{ implies } \begin{matrix} m'_1 = m''_1 \\ l'_2 = l''_1 \end{matrix}$$

also

$$\int dR_{12} \frac{ik \cdot (R_{13} - R_{32})}{l} = \int dR_{12} \frac{ik \cdot R_{13}}{l} - \frac{ik \cdot R_{32}}{l}$$

and

$$\int dR_{13} dR_{12} h(13) g^o f(32) \frac{ik \cdot R_{13}}{l} - \frac{ik \cdot R_{32}}{l} =$$

$$\int dR_{13} h(13) \frac{ik \cdot R_{13}}{l} \int dR_{32} g^o f(32) \frac{-ik \cdot R_{32}}{l}$$

$$= \tilde{\Sigma} E_h (l'_1 l'_2 l', k) C(l'_1 l'_2 l', m'_1 m'_2 m) Y_l^{m_1} (\omega_1) Y_l^{m_2} (\omega_3) Y_l^{m*} (\theta_k \phi_k) \cdot$$

$$\tilde{\Sigma} E_f (l''_1 l''_2 l'', k) C(l''_1 l''_2 l'', m''_1 m''_2 m'') \cdot$$

$$\frac{Y^{m''_1}}{l''_1} (\omega_3) Y_l^{m''_2} (\omega_2) Y_l^{m*} (\theta_k \phi_k)$$

where  $\frac{2_{111}}{1} \left( \frac{d}{dx} \right)_1 (x - 1) = 1$ ; for  $l = 1, 2, 3$ ,

$$y_0^1 = \left( \frac{2_{111}}{1+1} \right) \frac{1}{2_{111}} \left( \frac{d}{dx} \right)_1 (x^2 - 1)$$

If we look at  $y_1^1(\theta_k)$ , let  $\theta_k = 0$ , then  $y_1^1 \neq 0$  if  $a = 0$ .

$$C(n_1 n_{12}, v_1 - v_1 \alpha_2) (-1)^v y_{12}^{12}$$

$$\frac{d}{dt} \{ e^{\int_m n_1 l_1 k} C(n m_1 l_1, v_1 \alpha_1) y_{12}^{12} \}_{\alpha_1=0}^t (n_1 n_{12}, k).$$

$$e^{\int_m n_1 l_1 k} C(m n_1 l_1, k) C(m n_{12}, v_1 \alpha_2) y_{12}^{12}$$

then

$$y_{12}^{12}(w_2) = (-1)^{v_2 - v_2} y_{12}^{12}(w_2)$$

Since

$$n_2 = u \quad -v_2 = v$$

$$y_{12}^{12}(k) \} \text{ implies } m_1 = m \quad u_1 = u$$

$$y_{12}^{12}(k) e^{\int_m n_2 l_2, k} C(n_1 n_2 l_2, v_1 v_2 \alpha_2) y_{12}^{12}(w_2).$$

$$= \frac{d}{dt} \{ n_1 v_1 e^{\int_m (m_1 n_1 l_1, k)} C(m_1 n_1 l_1, v_1 v_1 \alpha_1) y_{12}^{12}(w_1) \}.$$

$$e^{\int_m n_1 l_1, k} C(m n_1 l_1, v_1 \alpha_1) y_{12}^{12}(w_1) y_{12}^{12}(w_2) y_{12}^{12}(k)$$

Also

so

$$Y_1^0 = \left(\frac{2}{4\pi} \frac{1+1}{1}\right)^{\frac{1}{2}}$$

Then

$$Y_1^\lambda(\theta_k \phi_k) = Y_1^\lambda(0, \phi_k) = \left(\frac{2}{4} \frac{1+1}{1}\right)^{\frac{1}{2}} \text{ when } \lambda = 0$$

$$\sum \tilde{E}_Y(m n l, k) C(m n l, \mu \nu o) \left(\frac{2}{4\pi} \frac{1+1}{1}\right)^{\frac{1}{2}} = 4\pi \tilde{E}_Y(m n \mu k)$$

where  $\tilde{E}_Y(mn\mu k)$  is in body-fixed coordinates.

$$4\pi \tilde{E}_Y(m n \mu k) = \rho \sum_m \tilde{E}_h(m n_l \mu k) \cdot$$

$$\{\tilde{E}_f(n_l n \mu k) * (-1)^\nu\} (4\pi)^2$$

or

$$\tilde{E}_Y(m n \mu k) = \rho \sum_n \tilde{E}_h(m n_l \mu k) \tilde{E}_f^*(n_l n \nu k) (-1)^\nu$$

so

$$\tilde{E}_Y(l l' m_k) = \rho \sum_{l''} \tilde{E}_h(l l'' m_k) (-1)^m \tilde{E}_f(l'' l' m_k)$$

where

$$\tilde{E}_f(m n l, k) = i^l (4\pi) \int_0^\infty dr r^2 J_l(kr) E_f(m n l, r)$$

$$\tilde{E}_f^*(m n l, k) = (-1)^l i^l (4\pi) \int_0^\infty dr r^2 J_l(kr) E_f^*(m n l, r)$$

and

$$\tilde{E}_f^*(m n l, k) = (-1)^l \tilde{E}_f(m n l, k)$$

## CHAPTER XIII

### COMPARISON

In order to compare our results with previous work, we chose Dr. Haile's (18) since we have  $E_Y(l_1 l_2 m r)$  and Dr. Haile's data are  $E_g(l_1 l_2 m r)$ , so we transferred our data as follows.

Since

$$\frac{Y(12)}{Y^o(r_{12})} - 1 = 4\pi \sum E_Y(l l' m r) Y_l^m(\omega_1) \bar{Y}_{l'}^{\bar{m}}(\omega_2) \quad (12-1)$$

or

$$\frac{g(12)e^{-\beta u_a}}{g^o(r_{12})} - 1 = 4\pi \sum E_Y(l l' m r) Y_l^m(\omega_1) \bar{Y}_{l'}^{\bar{m}}(\omega_2) \quad (12-2)$$

and

$$g(12) = g^o(r_{12}) (4\pi \sum E_Y(l l' m r) Y_l^m(\omega_1) \bar{Y}_{l'}^{\bar{m}}(\omega_2) e^{-\beta u_a}) + g^o(r_{12}) e^{\beta u_a}$$

Expanding  $e^{\beta u_a}$  in spherical harmonic, we have

$$g(12) = g^o(r_{12}) (4\pi \sum \sum E_Y(l l' m r) Y_l^m Y_{l'}^{\bar{m}}) (4\pi \sum \sum E_\mu(L L' M r) Y_L^M Y_{L'}^{\bar{M}}) + g^o(r_{12}) e^{\beta u_a}$$

Expand the left hand side

$$4\pi \sum \sum E_g(\lambda \lambda' \mu r) Y_\lambda^\mu Y_{\lambda'}^{\bar{\mu}} = \text{L.H.S.}$$

Multiply both sides by  $Y_A^{k*} Y_{A'}^{\bar{k}*}$ , and integrate over  $d\omega_1$  and  $d\omega_2$  we have

$$E_g(\Lambda \Lambda' k r) = g^O(r) (4\pi) \sum_{\substack{L \\ L'}} \sum_{\substack{M \\ M'}} \sum_{\substack{l \\ l'}} \sum_{\substack{m \\ m'}} E_\mu(L L' M m) \cdot$$

$$E_Y(l l' m r) \int d\omega_1 \int d\omega_2 Y_L^M(\omega_1) Y_{L'}^{M'}(\omega_2) Y_l^m(\omega_1) Y_{l'}^{m'}(\omega_2) \cdot$$

$$Y_\Lambda^{k*}(\omega_1) Y_{l'}^{k*}(\omega_2) + g^O E_\mu(\Lambda \Lambda' k r)$$

$$E_g(\Lambda \Lambda' k r) = g^O(r) \sum_{\substack{L \\ L'}} \sum_{\substack{M \\ M'}} \sum_{\substack{l \\ l'}} \sum_{\substack{m \\ m'}} E_\mu(L L' M \Lambda) E_Y(l l' m r) \cdot$$

$$\left[ \frac{(2 L+1)}{(2 \Lambda+1)} \frac{(2 L'+1)}{(2 \Lambda'+1)} \frac{(2 l'+1)}{(2 l+1)} \right]^{\frac{1}{2}} C(L l \Lambda, M m, k) \cdot$$

$$C(L l \Lambda, o o o) C(L' l' \Lambda', M' m' k) C(L' l' \Lambda', o o o)$$

$$+ g^O(r) E_\mu(\Lambda \Lambda' k r)$$

$$\text{Here } E_\mu(l l' m r) = E_f(l l' m r)$$

if  $l l' m \neq o o o$

and if  $l l' m = o o o$

$$E_f(ooor) = E_\mu(ooor) - 1.$$

In the coming pages we show our results for  $E_f(l_1 l_2 m r)$ ,

$E_Y(l_1 l_2 m r)$  and  $E_g(l_1 l_2 m r)$  at different temperatures and quadrupole moments. Conclusions are given on page 150.

DATA DOCUMENTS/NAME	
1	EFL-LP-M
2	ATL
3	LP
4	P
5	COMP=0
6	4
7	4
8	4
9	4
10	4
11	4
12	4
13	4
14	4
15	4
16	4
17	4
18	4
19	4
20	4
21	4
22	4
23	4
24	4
25	4
26	4
27	4
28	4
29	4
30	4
31	4
32	4
33	4
34	4
35	4
36	4
37	4
38	4
39	4
40	4
41	4
42	4
43	4
44	4
45	4
46	4
47	4
48	4
49	4
50	4

SCOTT REED

$\Delta e = 5$   
 $T = 1.27700 \times 10^{-6}$

	1	2	3	4	5	6	7
8	EF(L, L, M)	AT(L, L, M)	LP(L, L, M)	MP(L, L, M)	PF(L, L, M)	RF(L, L, M)	CF(L, L, M)
9	1	1	1	1	1	1	1
10	2	2	2	2	2	2	2
11	3	3	3	3	3	3	3
12	4	4	4	4	4	4	4
13	5	5	5	5	5	5	5
14	6	6	6	6	6	6	6
15	7	7	7	7	7	7	7
16	8	8	8	8	8	8	8
17	9	9	9	9	9	9	9
18	10	10	10	10	10	10	10
19	11	11	11	11	11	11	11
20	12	12	12	12	12	12	12
21	13	13	13	13	13	13	13
22	14	14	14	14	14	14	14
23	15	15	15	15	15	15	15
24	16	16	16	16	16	16	16
25	17	17	17	17	17	17	17
26	18	18	18	18	18	18	18
27	19	19	19	19	19	19	19
28	20	20	20	20	20	20	20
29	21	21	21	21	21	21	21
30	22	22	22	22	22	22	22
31	23	23	23	23	23	23	23
32	24	24	24	24	24	24	24
33	25	25	25	25	25	25	25
34	26	26	26	26	26	26	26
35	27	27	27	27	27	27	27
36	28	28	28	28	28	28	28
37	29	29	29	29	29	29	29
38	30	30	30	30	30	30	30
39	31	31	31	31	31	31	31
40	32	32	32	32	32	32	32
41	33	33	33	33	33	33	33
42	34	34	34	34	34	34	34
43	35	35	35	35	35	35	35
44	36	36	36	36	36	36	36
45	37	37	37	37	37	37	37
46	38	38	38	38	38	38	38
47	39	39	39	39	39	39	39
48	40	40	40	40	40	40	40
49	41	41	41	41	41	41	41
50	42	42	42	42	42	42	42

DATA FOR EQUATION 2

Q=5  
 3 T=1.27700 9 50000  
 4 NC1= 51, NC2= 191, N 101, HARC= 1000DE-0.  
 5  
 6  
 7  
 8  
 9

DATA DOCUMENTS/FILE	
10	1 EF(L,LF,M) AT LE 0,LPF=0,ME=0
11	1 1 R12E 0.8000 EEEFLP=0,533725
12	1 2 R12E 0.6400 EEEFLP=0,13493
13	1 3 R12E 0.5800 EEEFLP=0,21976
14	1 4 R12E 0.5200 EEEFLP=0,1411
15	1 5 R12E 0.4600 EEEFLP=0,4667
16	1 6 R12E 0.4000 EEEFLP=0,01
17	1 7 R12E 0.3500 EEEFLP=0,24476
18	1 8 R12E 0.3000 EEEFLP=0,17081
19	1 9 R12E 0.2500 EEEFLP=0,11909
20	1 10 R12E 0.2000 EEEFLP=0,64766
21	1 11 R12E 0.1500 EEEFLP=0,16451
22	1 12 R12E 0.1000 EEEFLP=0,10504
23	1 13 R12E 0.0500 EEEFLP=0,05292
24	1 14 R12E 0.0000 EEEFLP=0,00000
25	1 15 R12E 0.0000 EEEFLP=0,00000
26	1 16 R12E 0.0000 EEEFLP=0,00000
27	1 17 R12E 0.0000 EEEFLP=0,00000
28	1 18 R12E 0.0000 EEEFLP=0,00000
29	1 19 R12E 0.0000 EEEFLP=0,00000
30	1 20 R12E 0.0000 EEEFLP=0,00000
31	1 21 R12E 0.0000 EEEFLP=0,00000
32	1 22 R12E 0.0000 EEEFLP=0,00000
33	1 23 R12E 0.0000 EEEFLP=0,00000
34	1 24 R12E 0.0000 EEEFLP=0,00000
35	1 25 R12E 0.0000 EEEFLP=0,00000
36	1 26 R12E 0.0000 EEEFLP=0,00000
37	1 27 R12E 0.0000 EEEFLP=0,00000
38	1 28 R12E 0.0000 EEEFLP=0,00000
39	1 29 R12E 0.0000 EEEFLP=0,00000
40	1 30 R12E 0.0000 EEEFLP=0,00000
41	1 31 R12E 0.0000 EEEFLP=0,00000
42	1 32 R12E 0.0000 EEEFLP=0,00000
43	1 33 R12E 0.0000 EEEFLP=0,00000
44	1 34 R12E 0.0000 EEEFLP=0,00000
45	1 35 R12E 0.0000 EEEFLP=0,00000
46	1 36 R12E 0.0000 EEEFLP=0,00000
47	1 37 R12E 0.0000 EEEFLP=0,00000
48	1 38 R12E 0.0000 EEEFLP=0,00000
49	1 39 R12E 0.0000 EEEFLP=0,00000
50	1 40 R12E 0.0000 EEEFLP=0,00000

1 2 3 4 5  
1.27700 0.50000

$$EF(L, LP^M) \text{ AT } L = 6, LP = 2, M = 6, MP = 10$$

$\frac{1}{3} \cdot \frac{Q_1 - 5}{T-1} = 27700$       0.50000

	9	EF (L•R)	LF (M)	AT L	ELF (L•R)	LP (M)	MP = 0	NP = 0
1	1	R12	C12	0	R12	C12	0	0
2	2	R12	C12	0	R12	C12	0	0
3	3	R12	C12	0	R12	C12	0	0
4	4	R12	C12	0	R12	C12	0	0
5	5	R12	C12	0	R12	C12	0	0
6	6	R12	C12	0	R12	C12	0	0
7	7	R12	C12	0	R12	C12	0	0
8	8	R12	C12	0	R12	C12	0	0
9	9	R12	C12	0	R12	C12	0	0
10	10	R12	C12	0	R12	C12	0	0
11	11	R12	C12	0	R12	C12	0	0
12	12	R12	C12	0	R12	C12	0	0
13	13	R12	C12	0	R12	C12	0	0
14	14	R12	C12	0	R12	C12	0	0
15	15	R12	C12	0	R12	C12	0	0
16	16	R12	C12	0	R12	C12	0	0
17	17	R12	C12	0	R12	C12	0	0
18	18	R12	C12	0	R12	C12	0	0
19	19	R12	C12	0	R12	C12	0	0
20	20	R12	C12	0	R12	C12	0	0
21	21	R12	C12	0	R12	C12	0	0
22	22	R12	C12	0	R12	C12	0	0
23	23	R12	C12	0	R12	C12	0	0
24	24	R12	C12	0	R12	C12	0	0
25	25	R12	C12	0	R12	C12	0	0
26	26	R12	C12	0	R12	C12	0	0
27	27	R12	C12	0	R12	C12	0	0
28	28	R12	C12	0	R12	C12	0	0
29	29	R12	C12	0	R12	C12	0	0
30	30	R12	C12	0	R12	C12	0	0
31	31	R12	C12	0	R12	C12	0	0
32	32	R12	C12	0	R12	C12	0	0
33	33	R12	C12	0	R12	C12	0	0
34	34	R12	C12	0	R12	C12	0	0
35	35	R12	C12	0	R12	C12	0	0
36	36	R12	C12	0	R12	C12	0	0
37	37	R12	C12	0	R12	C12	0	0
38	38	R12	C12	0	R12	C12	0	0
39	39	R12	C12	0	R12	C12	0	0
40	40	R12	C12	0	R12	C12	0	0
41	41	R12	C12	0	R12	C12	0	0
42	42	R12	C12	0	R12	C12	0	0
43	43	R12	C12	0	R12	C12	0	0
44	44	R12	C12	0	R12	C12	0	0
45	45	R12	C12	0	R12	C12	0	0
46	46	R12	C12	0	R12	C12	0	0
47	47	R12	C12	0	R12	C12	0	0
48	48	R12	C12	0	R12	C12	0	0
49	49	R12	C12	0	R12	C12	0	0

$f_L = 1$

$T = 1.29400$ ,  $N_{C1} = 101$ ,  $N_{C2} = 101$ ,  $HARF = 10000E-01$ ,  $10000E-01$ ,  $10000E-01$

1 EFL(L,LP,M) AT L=

L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
LP	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
M	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

DATA FOR INTEGRATION OF EQUATIONS

$$T=1.29405 \cdot 10^{-6} \text{ sec}^{-1}$$

$$\text{EFC}(L_0(LP^M)) \text{ AT } L = 4 \cdot LP = 2^9 \cdot N = 512 \cdot N = 0$$

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卷二

T= 1.29400, NC1= 51, NC2= 101, N= 101. HAPC= .10000E-01 .10000E-01 .10000E-01

$R_1 = 1$

$T = 1 \cdot 29425$ ,  $X_C1 = 1 \cdot 30000$ ,  $X_C2 = 1 \cdot 161$ ,  $N = 161$ ,  $HAFC = .10000E-01$  •  $.10000E-01$  •  $.10000E-01$

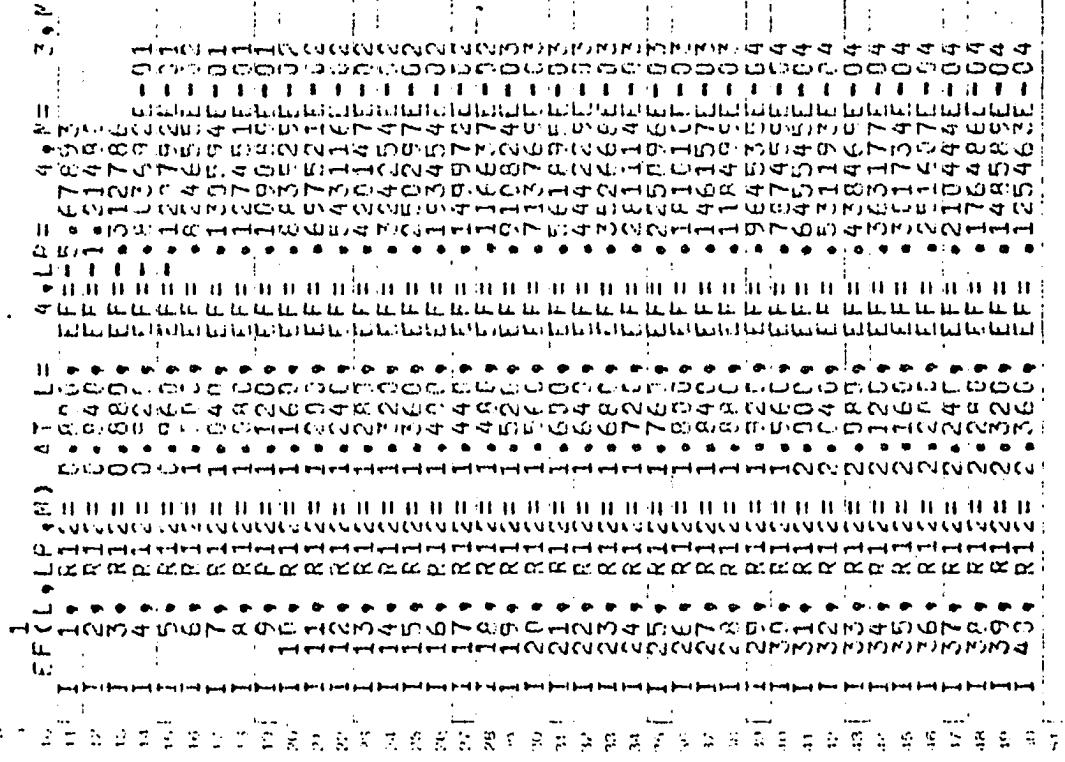


Fig. 10. Erf(x) vs x



*8*

2 T=1.2540C NC1=51. N 101. N 101. HAEC= .17000E-01 .10000E-01

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\* PLSA READING AT 3000



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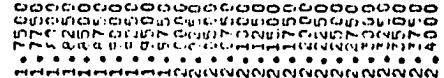
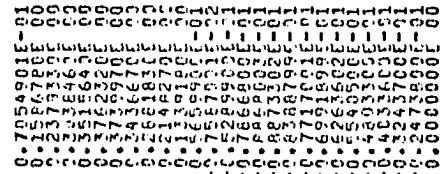
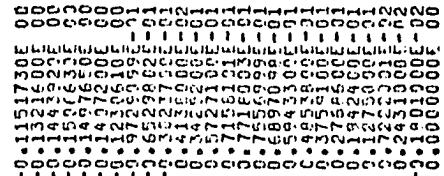
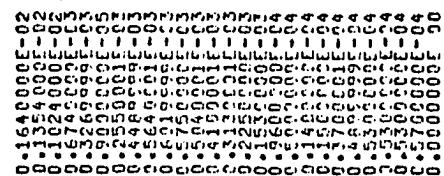
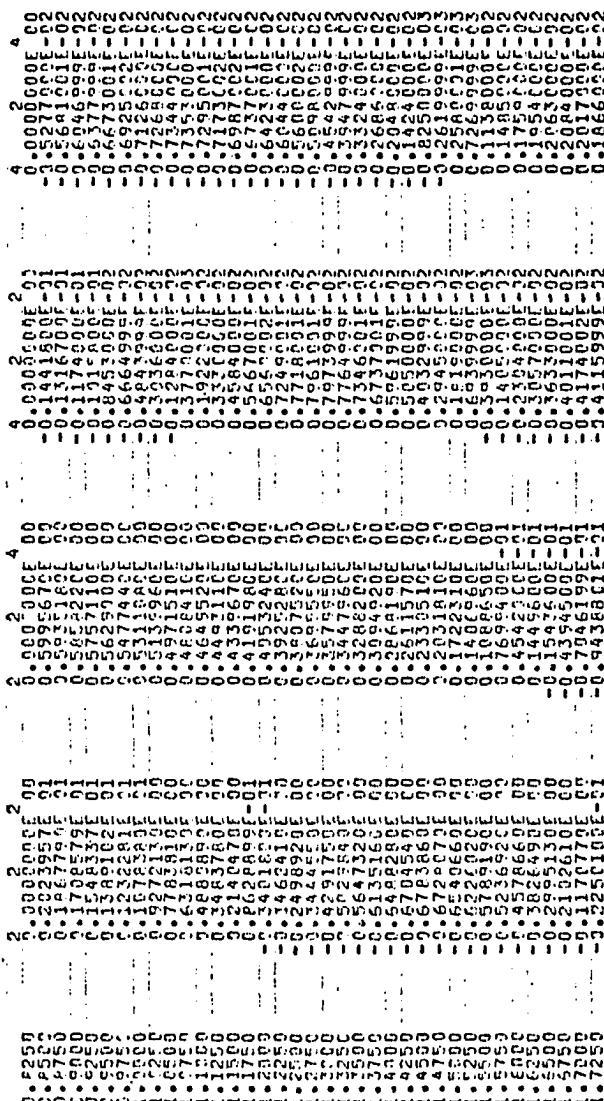
S.P.F.  
E.P.R. (etc.)

$$Q = 0.5$$

$G_f$   
 $E_f(\rho, \rho')$

$\Omega = .5$

$T = 1.277$



8.82500	4.000000E+00	6.00	4.000000E+00	0.000000E+00
8.85750	-0.10240000E-02	-0.2	-0.10240000E-02	-0.000000E+00
8.88000	-0.11620000E-02	-0.2	-0.11620000E-02	-0.000000E+00
8.89250	-0.13112000E-02	-0.2	-0.13112000E-02	-0.000000E+00
8.90500	-0.13888000E-02	-0.2	-0.13888000E-02	-0.000000E+00
8.91750	-0.14623000E-02	-0.2	-0.14623000E-02	-0.000000E+00
8.93000	-0.15323000E-02	-0.2	-0.15323000E-02	-0.000000E+00
8.94250	-0.16770000E-02	-0.2	-0.16770000E-02	-0.000000E+00
8.95500	-0.17860000E-02	-0.2	-0.17860000E-02	-0.000000E+00
8.96750	-0.20200000E-02	-0.2	-0.20200000E-02	-0.000000E+00
8.98000	-0.22420000E-02	-0.2	-0.22420000E-02	-0.000000E+00
8.99250	-0.25540000E-02	-0.2	-0.25540000E-02	-0.000000E+00
8.99500	-0.27110000E-02	-0.2	-0.27110000E-02	-0.000000E+00
8.99750	-0.27330000E-02	-0.2	-0.27330000E-02	-0.000000E+00
1.00000	-0.27330000E-02	-0.2	-0.27330000E-02	-0.000000E+00
1.00250	-0.27200000E-02	-0.2	-0.27200000E-02	-0.000000E+00
1.00500	-0.26750000E-02	-0.2	-0.26750000E-02	-0.000000E+00
1.00750	-0.26260000E-02	-0.2	-0.26260000E-02	-0.000000E+00
1.01000	-0.25710000E-02	-0.2	-0.25710000E-02	-0.000000E+00
1.01250	-0.24710000E-02	-0.2	-0.24710000E-02	-0.000000E+00
1.01500	-0.23380000E-02	-0.2	-0.23380000E-02	-0.000000E+00
1.01750	-0.21000000E-02	-0.2	-0.21000000E-02	-0.000000E+00
1.02000	-0.20060000E-02	-0.2	-0.20060000E-02	-0.000000E+00

S.F

 $E_5(l,l,l)$  $\alpha = 0.5$  $T = 1.277$ 

1.7500	-0.18180000E-22		-0.42000000E-04	-0.44999999E-04
1.7750	-0.16220000E-22		-0.30000000E-04	-0.15700000E-03
1.8000	-0.14260000E-02		-0.16666667E-03	-0.94000000E-04
1.8250	-0.12300000E-02		-0.13466667E-03	-0.16000000E-04
1.8500	-0.10340000E-02		-0.10466667E-03	-0.23100000E-04
1.8750	-0.89110000E-03		-0.74666667E-03	-0.32100000E-04
1.9000	-0.76100000E-03		-0.59666667E-03	-0.44999999E-04
1.9250	-0.62200000E-03		-0.49500000E-03	-0.63000000E-04
1.9500	-0.51200000E-03		-0.41200000E-03	-0.83000000E-04
1.9750	-0.41200000E-03		-0.34200000E-03	-0.10400000E-03
2.0000	-0.32200000E-03		-0.28200000E-03	-0.12600000E-03
2.0250	-0.24200000E-03		-0.23200000E-03	-0.15000000E-03
2.0500	-0.17200000E-03		-0.18200000E-03	-0.17600000E-03
2.0750	-0.12200000E-03		-0.14200000E-03	-0.20400000E-03
2.1000	-0.82200000E-04		-0.10200000E-03	-0.23400000E-03
2.1250	-0.52200000E-04		-0.72000000E-04	-0.26600000E-03
2.1500	-0.32200000E-04		-0.42000000E-04	-0.30000000E-03
2.1750	-0.20200000E-04		-0.22000000E-04	-0.33400000E-03
2.2000	-0.12200000E-04		-0.12000000E-04	-0.36400000E-03
2.2250	-0.72200000E-05		-0.60000000E-05	-0.39000000E-03
2.2500	-0.42200000E-05		-0.30000000E-05	-0.41600000E-03
2.2750	-0.24200000E-05		-0.15000000E-05	-0.44600000E-03
2.3000	-0.12200000E-05		-0.70000000E-06	-0.47799999E-03
2.3250	-0.62200000E-06		-0.35000000E-06	-0.44300000E-03
2.3500	-0.32200000E-06		-0.17500000E-06	-0.37599999E-03
2.3750	-0.17200000E-06		-0.87500000E-07	-0.17700000E-03
2.4000	-0.82200000E-07		-0.43750000E-07	-0.60000000E-04
2.4250	-0.42200000E-07		-0.21800000E-07	-0.48999999E-04
2.4500	-0.20200000E-07		-0.10900000E-07	-0.23999999E-04
2.4750	-0.10200000E-07		-0.54500000E-08	-0.62000000E-05
2.5000	-0.52200000E-08		-0.27200000E-08	-0.13400000E-05

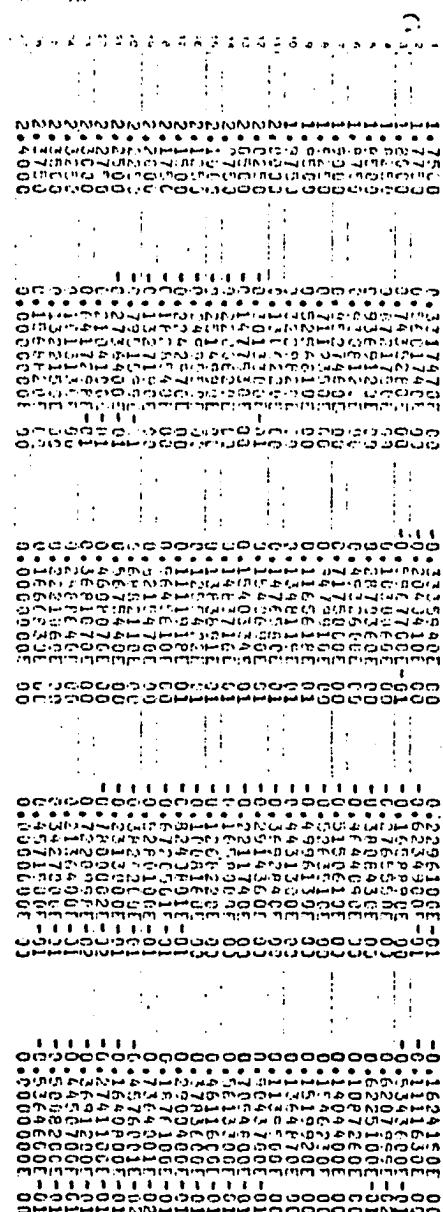
0.18900000E-03			
0.15700000E-03			
0.94000000E-04			
0.16000000E-04			
0.63000000E-04			
0.12900000E-03			
0.17200000E-03			
0.16500000E-03			
0.11600000E-03			
0.46000000E-04			
0.16000000E-03			
0.12500000E-03			
0.26700000E-03			
0.19600000E-03			
0.15700000E-03			
0.11900000E-03			
0.71000000E-04			
0.50000000E-04			
0.23999999E-04			
0.62000000E-05			
0.00000000E+00			

Ernesto Gómez

丁巳

סִפְרַתּוֹת

$$T = 1.294$$



SP fixed coordinate  
 $E_f(\ell, \ell_1, \ell_2)$   
 $\alpha = 1$   
 Tel. 1.294

$$S_f = f(\rho_1, \rho_2, \rho_3)$$

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$$E_{\text{ff}}(l_1 l_2 \dots l_r)$$

$$\alpha = 1$$

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$$E_f(\ell_1 \ell_2 \omega r)$$

一  
二  
三

T = 1.294

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1  
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	$E_F(l, l_1, r)$ $Q=1$ $T=1.2774$	$E_F(l, l_1, r)$ $Q=0.5$ $T=1.277$
0.82500	0.0000000E+00	0.0000000E+00
0.85000	0.113335000E-01	0.8999999E-04
0.87500	0.774480000E-01	0.1140000E-03
0.90000	0.59240001E-01	0.1400000E-03
0.92500	0.403970000E-01	0.1640000E-03
0.95000	0.136350000E-01	0.1820000E-03
1.00000	0.186350000E-01	0.1770000E-03
1.05000	0.606900000E-01	0.1460000E-03
1.07500	0.84480001E-01	0.1500000E-03
1.10000	0.105450000E-01	0.6993999E-04
1.12500	0.133849000E-01	0.1670000E-03
1.15000	0.155630000E-01	0.2659999E-03
1.17500	0.725780000E-01	0.3589999E-03
1.20000	0.162600000E-01	0.4420000E-03
1.22500	0.112200000E-01	0.5589999E-03
1.25000	0.151580000E-01	0.5769000E-03
1.27500	0.770500000E-02	0.2250000E-03
1.30000	0.273100000E-02	0.2300000E-03
1.32500	0.88440001E-02	0.2120000E-03
1.35000	0.151460000E-02	0.9099999E-04
1.37500	0.415590000E-02	0.1200000E-03
1.40000	0.936950000E-02	0.1360000E-03
1.42500	0.213390000E-01	0.2720000E-03
1.45000	0.274380000E-01	0.5080000E-03
1.47500	0.332600000E-01	0.3760000E-03
1.50000	0.382390000E-01	0.2540000E-03
1.52500	0.413459000E-01	0.1210000E-03
1.55000	0.458590000E-01	0.2199999E-03
1.57500	0.495750000E-01	0.1670000E-03
1.60000	0.532300000E-01	0.3060000E-03
1.62500	0.569500000E-01	0.5510000E-03
1.65000	0.604600000E-01	0.4260000E-03
1.67500	0.640390000E-01	0.3690000E-03
1.70000	0.35511030E-01	0.3690000E-03
1.72500	0.29057000E-01	0.2100000E-03

1.7500	0.2137000E-01	0.3064500E-01	-0.4230000E-03	0.3100000E-04
1.7750	0.1283893E-01	0.3074400E-01	-0.3100000E-03	0.1390000E-03
1.8000	0.3913399E-02	0.3282800E-01	-0.1830000E-03	0.2710000E-03
1.8250	0.1038000E-01	0.3988300E-01	-0.2000000E-03	0.3440000E-03
1.8500	0.1374000E-01	0.3217300E-01	0.3500000E-03	0.3519999E-03
1.8750	0.120487000E-01	0.3318900E-01	0.9800000E-03	0.3039999E-03
1.9000	0.3025000E-01	0.3446700E-01	0.1250000E-03	0.2170000E-03
1.9250	0.3734500E-01	0.3446300E-01	0.1290000E-03	0.1150000E-03
1.9500	0.4356300E-01	0.3337300E-01	0.4400000E-03	0.1700000E-03
1.9750	0.4684200E-01	0.4061200E-01	-0.1000000E-03	0.1310000E-03
2.0000	0.5082900E-01	0.636400E-01	0.6599999E-03	0.1120000E-03
2.0250	0.4826500E-01	0.2111100E-01	-0.1140000E-03	0.1780000E-03
2.0500	0.442675000E-01	0.1513500E-01	0.1510000E-03	0.1820000E-03
2.0750	0.3670000E-01	0.3444200E-02	-0.1620000E-03	0.1650000E-03
2.1000	0.3152000E-01	0.2417000E-02	-0.1460000E-03	0.1370000E-03
2.1250	0.2666700E-01	0.801400E-02	-0.1110000E-03	0.2100000E-04
2.1500	0.2224133E-01	0.1316E-01	0.1200000E-03	0.1330000E-03
2.1750	0.1762500E-01	0.176E-01	0.1000000E-03	0.1270000E-03
2.2000	0.1308600E-01	0.2146400E-01	0.8000000E-03	0.2360000E-03
2.2250	0.86220020E-01	0.2635300E-01	0.1500000E-03	0.2660000E-03
2.2500	0.4335301E-01	0.2722700E-01	0.2900000E-03	0.2810000E-03
2.2750	0.3242000E-01	0.2724500E-01	0.2790000E-03	0.2860000E-03
2.3000	0.0000000E-00	0.2635500E-01	0.4200000E-03	0.2860000E-03
2.3250	0.3360000E-01	0.2722700E-01	0.1500000E-03	0.2660000E-03
2.3500	0.3242000E-01	0.2635500E-01	0.2900000E-03	0.2810000E-03
2.3750	0.6248999E-02	0.0000000E-00	0.0000000E-03	0.0000000E-03
2.4000	0.0000000E-00	0.0000000E-00	0.0000000E-03	0.0000000E-03

Body fixed  
Eq (f. 12.11 r)

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T = 1.277

B.F  
 $E_J(\rho, \ell_2 \cup r)$

$\Omega = 0.5$

$T = 1.277$

1  
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2  
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1.7500	-9.2760E-04
1.7700	-2.2700E-04
1.7800	-2.2700E-04
1.7900	-2.2700E-04
1.8000	-2.2700E-04
1.8100	-2.2700E-04
1.8200	-2.2700E-04
1.8300	-2.2700E-04
1.8400	-2.2700E-04
1.8500	-2.2700E-04
1.8600	-2.2700E-04
1.8700	-2.2700E-04
1.8800	-2.2700E-04
1.8900	-2.2700E-04
1.9000	-2.2700E-04
1.9100	-2.2700E-04
1.9200	-2.2700E-04
1.9300	-2.2700E-04
1.9400	-2.2700E-04
1.9500	-2.2700E-04
1.9600	-2.2700E-04
1.9700	-2.2700E-04
1.9800	-2.2700E-04
1.9900	-2.2700E-04
2.0000	0.0000000E+00

6	0.0000000E+00
4	0.0000000E+00
B	0.0000000E+00
0	0.0000000E+00
-2	-2.2700E-04
-4	-2.2700E-04
-6	-2.2700E-04
-8	-2.2700E-04
-10	-2.2700E-04
-12	-2.2700E-04
-14	-2.2700E-04
-16	-2.2700E-04
-18	-2.2700E-04
-20	-2.2700E-04
-22	-2.2700E-04
-24	-2.2700E-04
-26	-2.2700E-04
-28	-2.2700E-04
-30	-2.2700E-04
-32	-2.2700E-04
-34	-2.2700E-04
-36	-2.2700E-04
-38	-2.2700E-04
-40	-2.2700E-04
-42	-2.2700E-04
-44	-2.2700E-04
-46	-2.2700E-04
-48	-2.2700E-04
-50	-2.2700E-04

4	0.0000000E+00
4	0.0000000E+00
1	0.0000000E+00
-2	-2.2700E-04
-4	-2.2700E-04
-6	-2.2700E-04
-8	-2.2700E-04
-10	-2.2700E-04
-12	-2.2700E-04
-14	-2.2700E-04
-16	-2.2700E-04
-18	-2.2700E-04
-20	-2.2700E-04
-22	-2.2700E-04
-24	-2.2700E-04
-26	-2.2700E-04
-28	-2.2700E-04
-30	-2.2700E-04
-32	-2.2700E-04
-34	-2.2700E-04
-36	-2.2700E-04
-38	-2.2700E-04
-40	-2.2700E-04
-42	-2.2700E-04
-44	-2.2700E-04
-46	-2.2700E-04
-48	-2.2700E-04
-50	-2.2700E-04

4	0.0000000E+00
4	0.0000000E+00
2	0.0000000E+00
-1	-2.2700E-04
-3	-2.2700E-04
-5	-2.2700E-04
-7	-2.2700E-04
-9	-2.2700E-04
-11	-2.2700E-04
-13	-2.2700E-04
-15	-2.2700E-04
-17	-2.2700E-04
-19	-2.2700E-04
-21	-2.2700E-04
-23	-2.2700E-04
-25	-2.2700E-04
-27	-2.2700E-04
-29	-2.2700E-04
-31	-2.2700E-04
-33	-2.2700E-04
-35	-2.2700E-04
-37	-2.2700E-04
-39	-2.2700E-04
-41	-2.2700E-04
-43	-2.2700E-04
-45	-2.2700E-04
-47	-2.2700E-04
-49	-2.2700E-04
-50	-2.2700E-04

-8	0.1600000E-04
-6	0.1600000E-04
-4	0.1600000E-04
-2	0.1600000E-04
0	0.1600000E-04
-1	-1.600000E-04
-3	-1.600000E-04
-5	-1.600000E-04
-7	-1.600000E-04
-9	-1.600000E-04
-11	-1.600000E-04
-13	-1.600000E-04
-15	-1.600000E-04
-17	-1.600000E-04
-19	-1.600000E-04
-21	-1.600000E-04
-23	-1.600000E-04
-25	-1.600000E-04
-27	-1.600000E-04
-29	-1.600000E-04
-31	-1.600000E-04
-33	-1.600000E-04
-35	-1.600000E-04
-37	-1.600000E-04
-39	-1.600000E-04
-41	-1.600000E-04
-43	-1.600000E-04
-45	-1.600000E-04
-47	-1.600000E-04
-49	-1.600000E-04
-50	-1.600000E-04

*B f*  
 $E_y(l, l_2 m \tau)$

$\alpha = 0.5$   
 $\tau = 1.277$

$B_f$   
 $\alpha = 0.$   
 $E_j(\lambda, \rho_2, m_r)$   
 $T = 1.294$

1  
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1  
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 4

BF

$E_g(l_1, l_2, m_r)$

$Q = L$

$T = 1.294$

1. 00  
2. 00  
3. 00  
4. 00

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51. 00  
52. 00

53. 00  
54. 00  
55. 00  
56. 00  
57. 00  
58. 00

$$B \cdot F$$

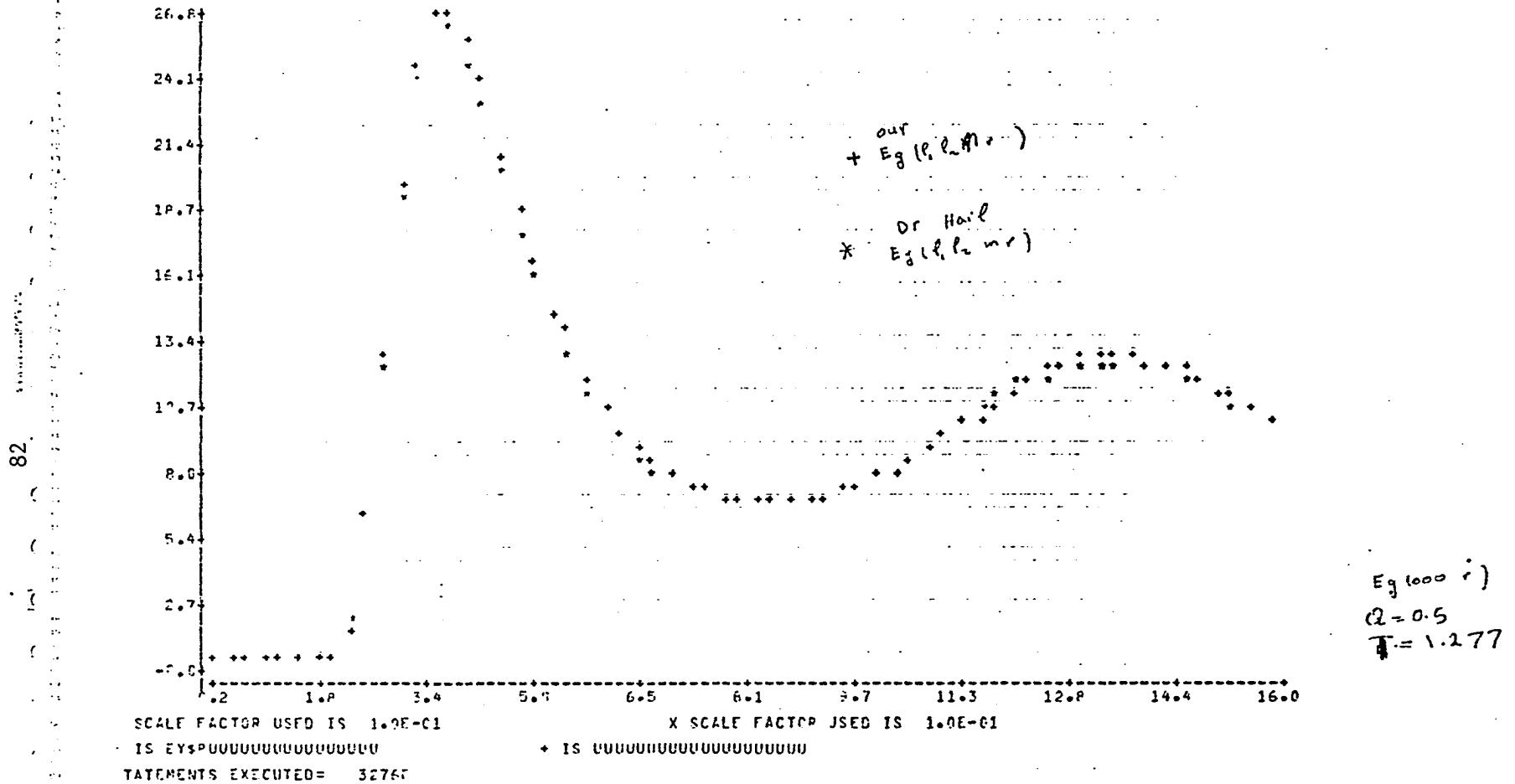
$$E_g(l_1, l_2) V(r)$$

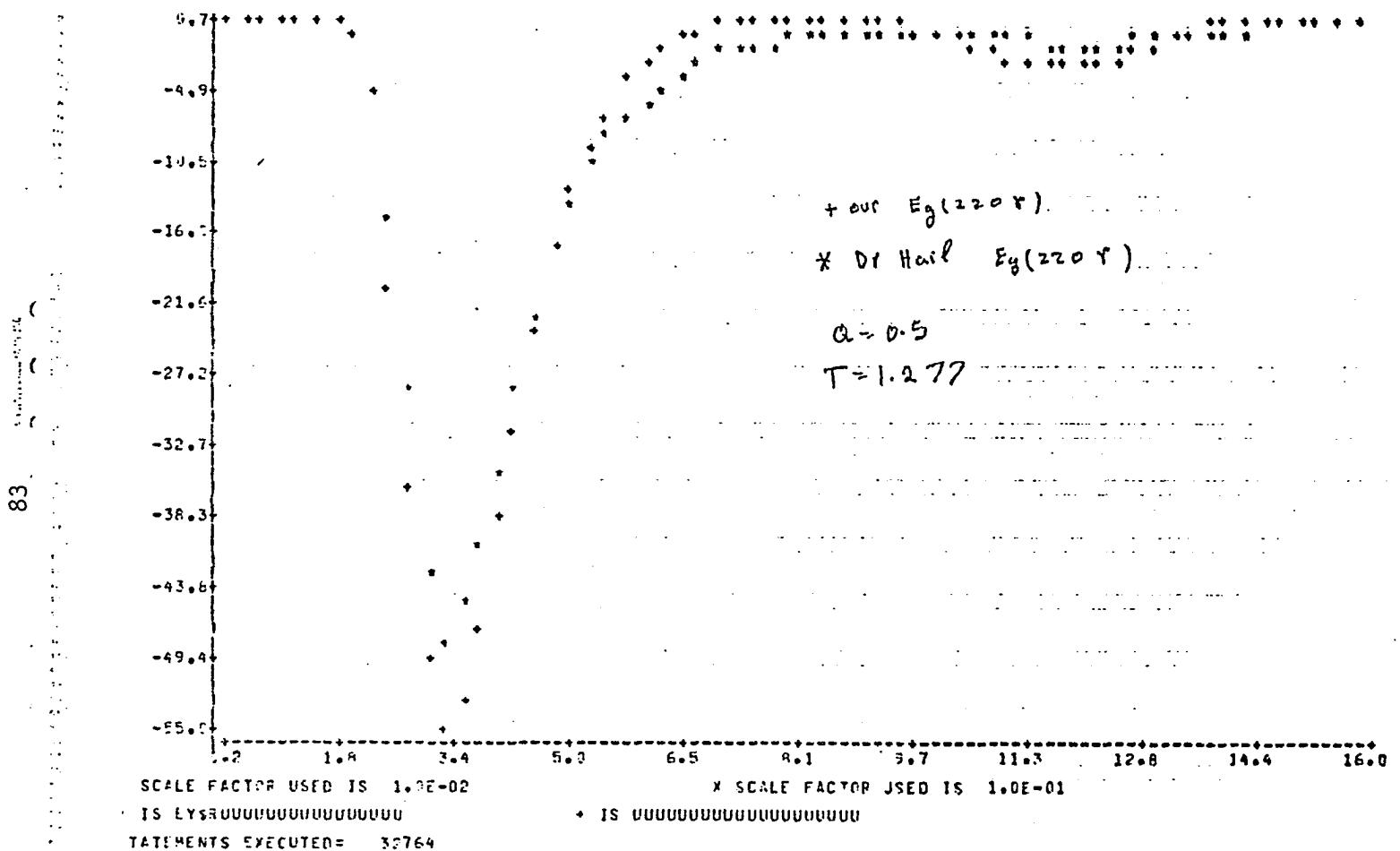
$$\Omega = 1$$

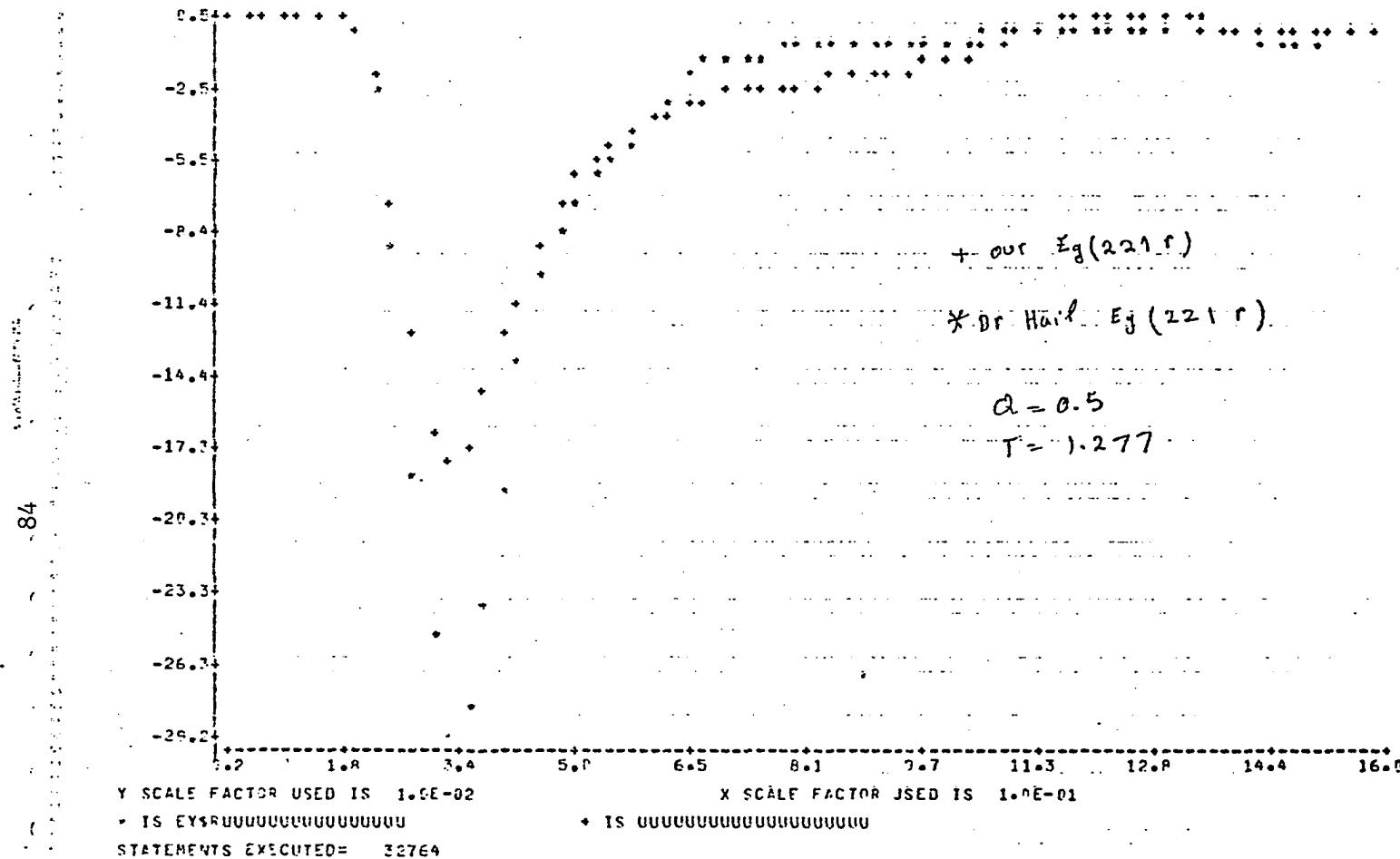
$$T = 1.294$$

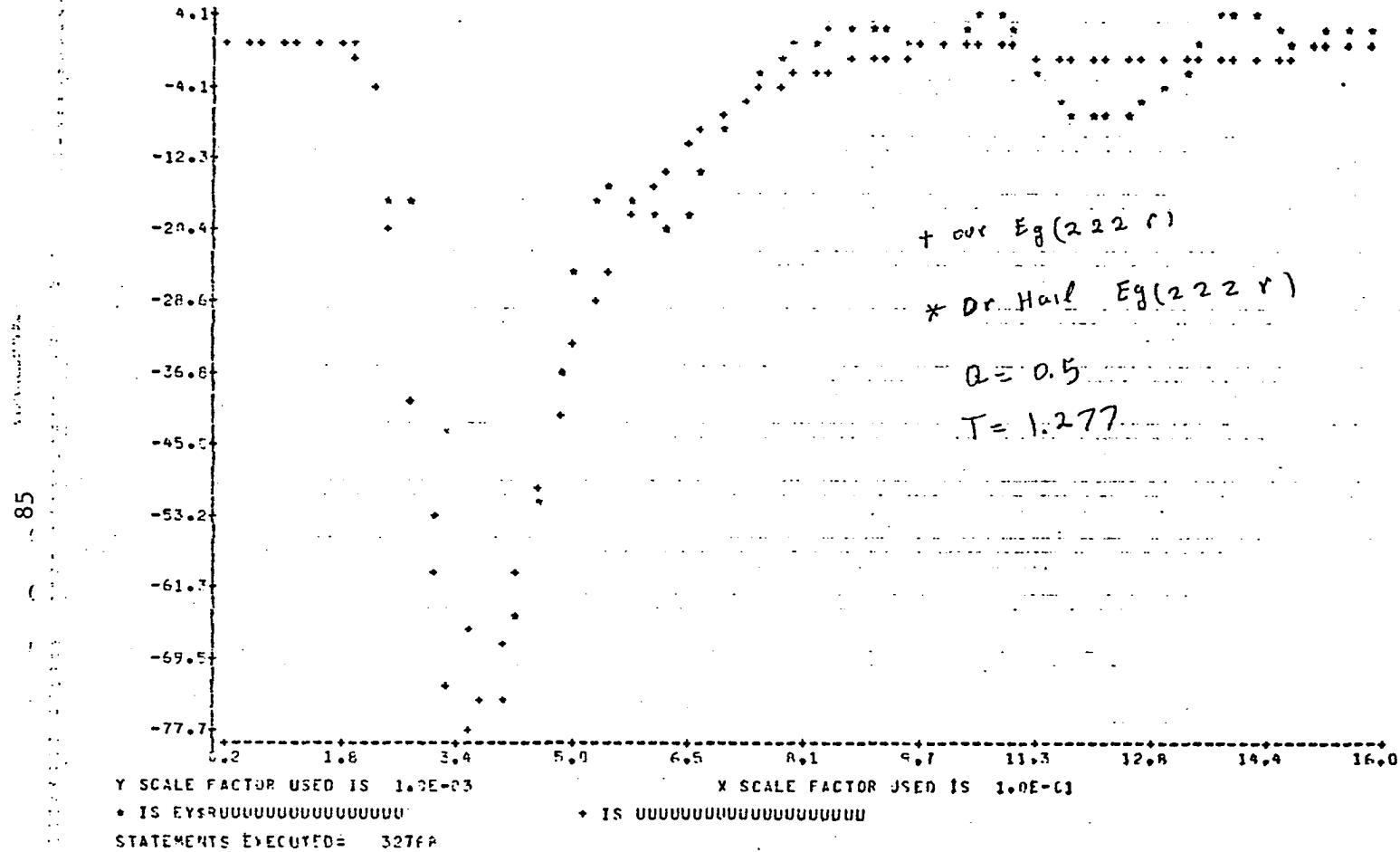
Q-1

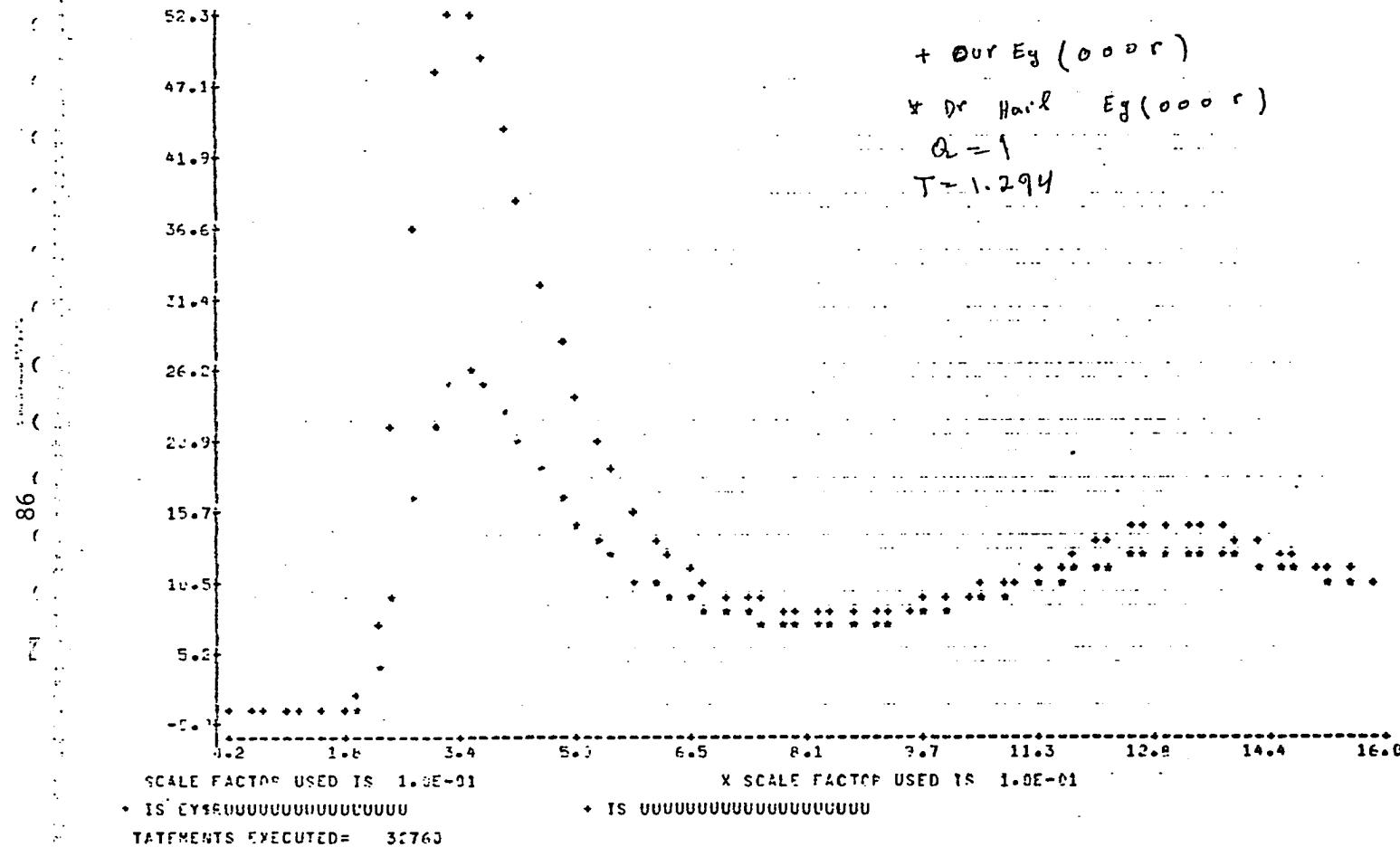
$$T = 1.294$$



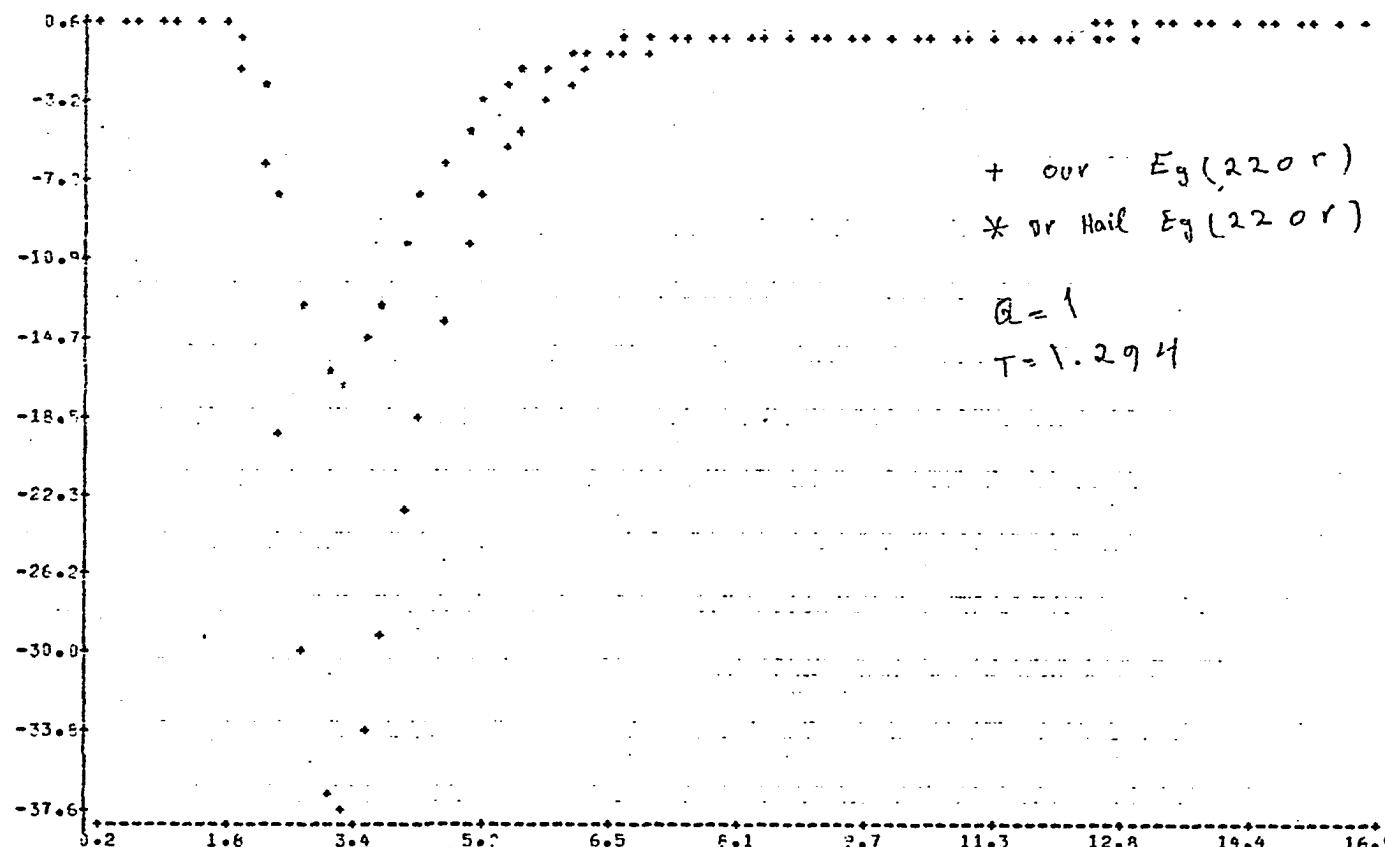


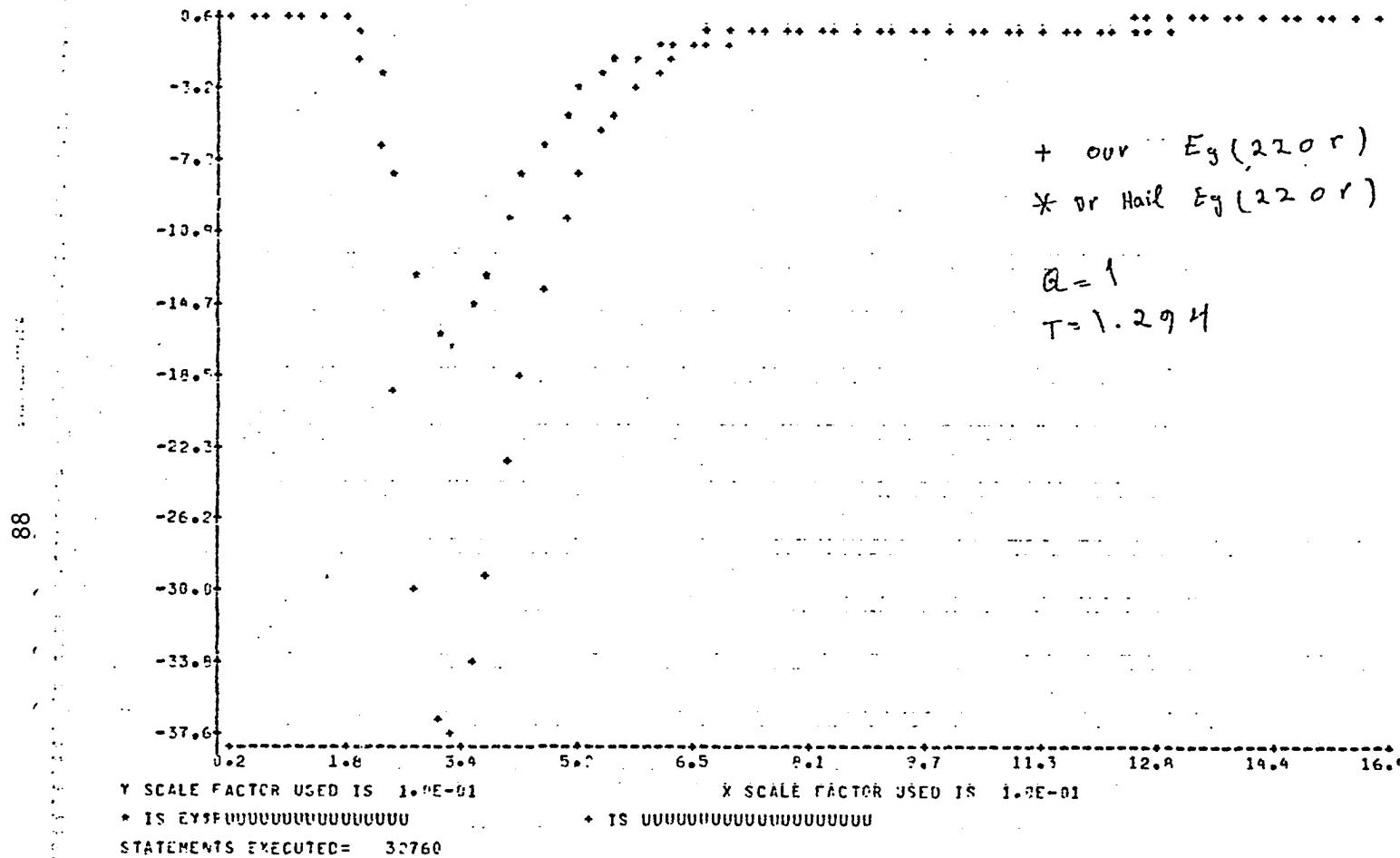


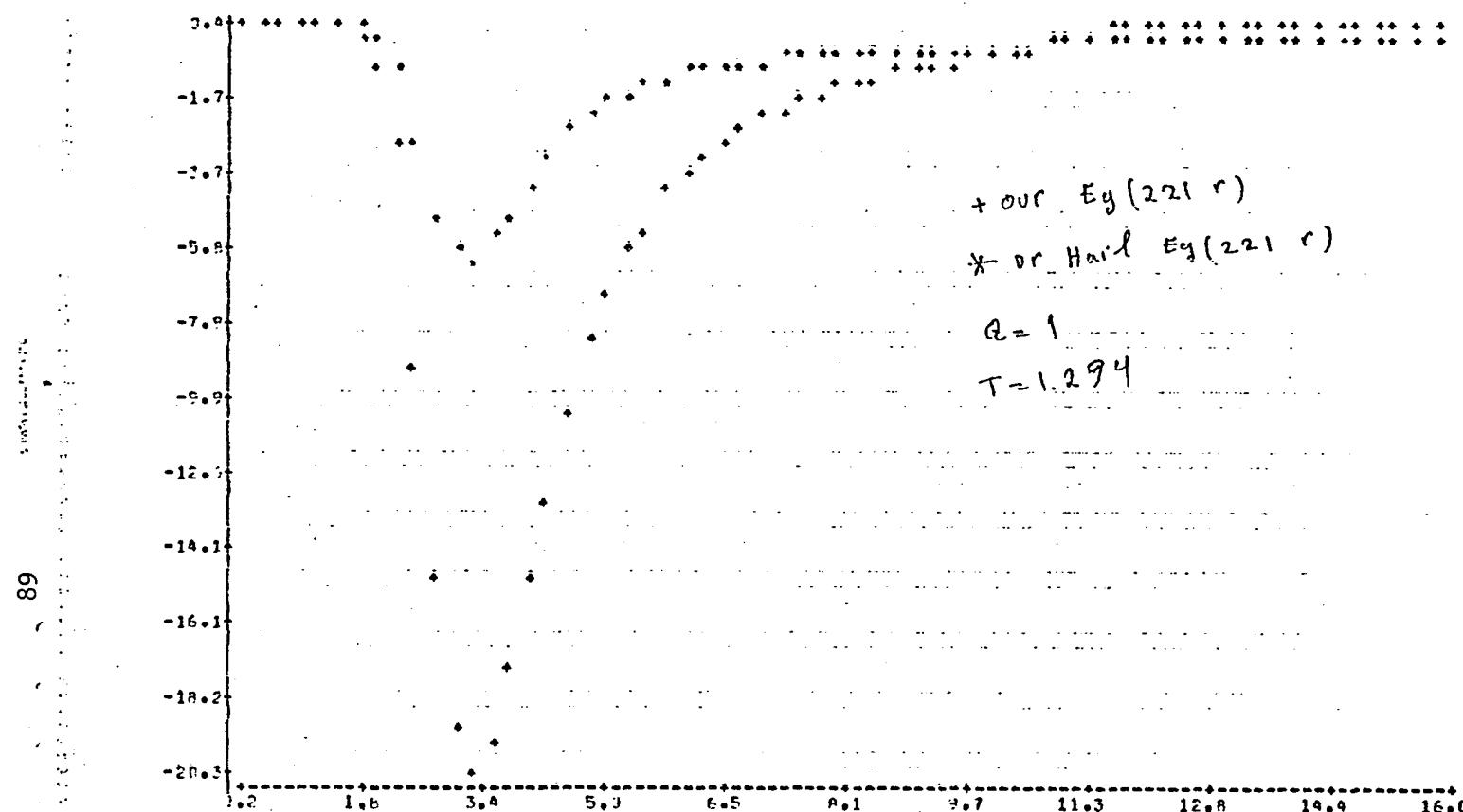




87







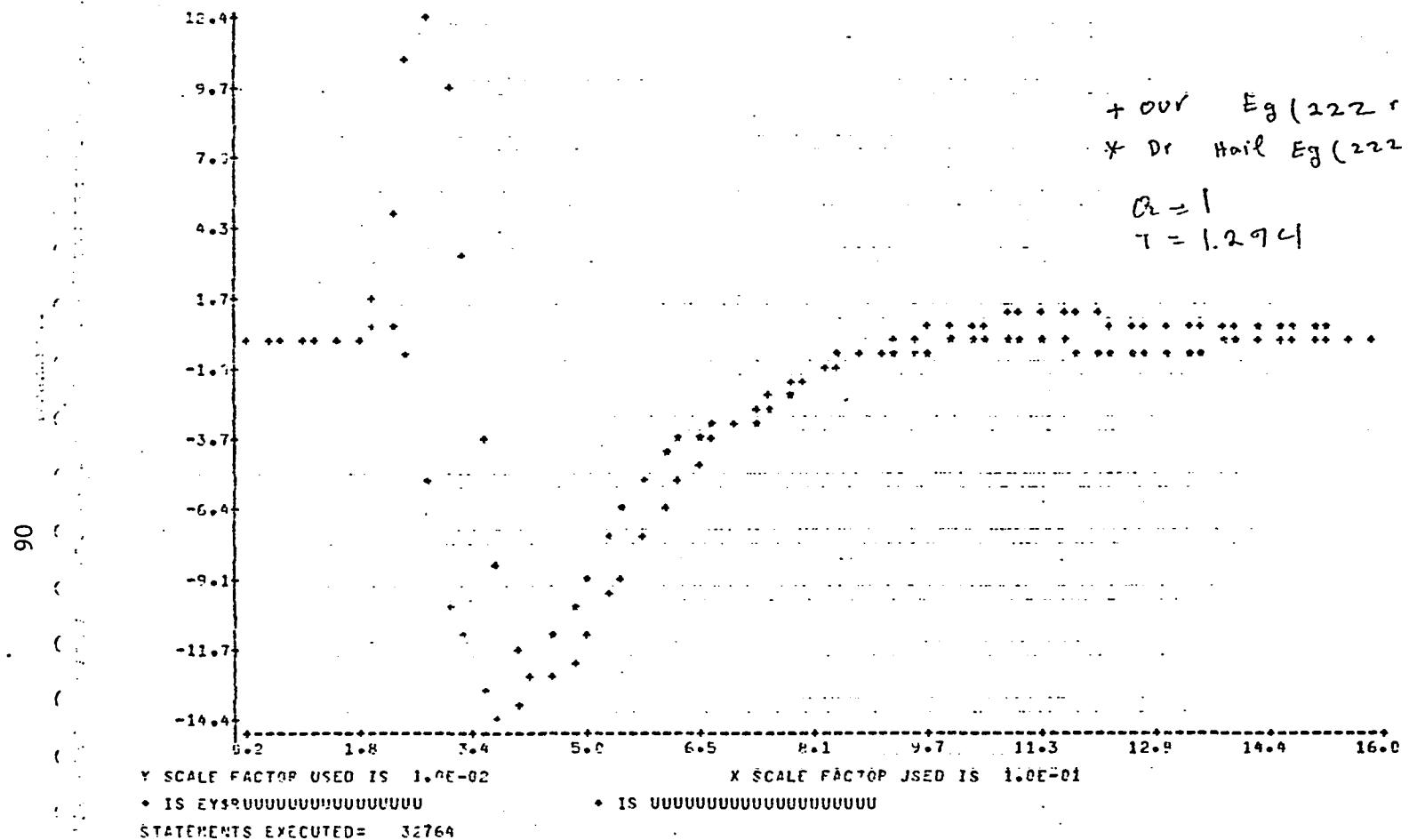
T SCALE FACTOR USED IS 1.0E-01

♦ IS EYRUUUUUUUUUUUUUUUUU

STATEMENTS EXECUTED= 32760

X SCALE FACTOR USED IS 1.0E-01

+ IS UUUUUUUUUUUUUUUUUUU



FORTRAN IV G1 RELEASE 2.0

MAIN

DATE = RJ213

03/14/12

PAGE 0001

0001 CALL CORPA  
0002 STOP  
0003 END

FORTRAN IV G1 RELEASE 2.0 COMPA DATE = 83213 03/14/12 PAGE 0001

```
601      SUBROUTINE COMPA
602      COMMON /CEYALL/ YINPUT(28,100)
603      COMMON /CG/ G(100)
604      COMMON /CH/ EH(100)
605      COMMON /CLP/LA,LB,LA,VB
606      COMMON /CFLST/X(100),Y(100,6)
607      DATA LAPL /*MAIL*/
608      FORMAT(16I5)
609      FORMAT(1D10.5)
610      FORMAT(1X, 1F10.4)
611      FORMAT(1F10.5,F10.5, 6F10.6)
612      FORMAT(1D8.3)
613      FORMAT(1X, 130(' '))
614      FORMAT(1H1)
615      NF=40
616      NG=60
617      NFE=64
618      NY=120
619      NC=60
620      NO=64
621      NC=64
622      NY=64
623      NFE=64
624      READ(5,6) NF0IS
625      WRITE(6,6) NF0IS
626      READ(5,6) (G(I)), I=1,NG
627      WRITE(6,6) (G(I)), I=1,NG
628      DO 1 INPUT=1,10
629      READ(5,6) K1,K2,K3
630      WRITE(6,6) K1,K2,K3
631      READ(5,6) (YINPUT(INPUT,I), I=1,NF)
632      WRITE(6,6) (YINPUT(INPUT,I), I=1,NF)
633      CONTINUE
634      DO 41 K=1,NF
635      YINPUT(1,K)=YINPUT(1,K) +1
636      CONTINUE
637      DO 456 INPUT=15,28
638      READ(5,6) K1,K2,K3
639      WRITE(6,6) K1,K2,K3
640      READ(5,6) (YINPUT(INPUT,I), I=2,NY)
641      WRITE(6,6) (YINPUT(INPUT,I), I=2,NY)
642      CONTINUE
643      DO 4931 IT=1,NFCIS
644      READ(5,6) LA, LAP, MA
645      WRITE(6,6) LA, LAP, MA
646      WRITE(6,6) MA
647      DO 48 NPOINT=1,NB
648      CALL HAILE(NPOINT)
649      CONTINUE
650      WRITE(6,6)
651      WRITE(6,6)
652      WRITE(7,6) LA, LAP, MA
653      WRITE(7,6)
654      WRITE(6,6) (EH(I), I=1,NP)
655      WRITE(7,6) (EH(I), I=1,NP)
```

FORTRAN IV G1 RELEASE 2.0 COMPILER DATE 8/31/13 PAGE 0002

```
0059      DO 472 I=1,100
          X(I) = (I-1)*0.025
472      Y(I,I)= EH(I,I)
          CALL PRPLOT(X, Y, 100, 6, 90, 6, LARL)
4801    CONTINUE
          RETURN
        END
```

FORTRAN IV-G1 RELEASE 2.0 TAILLE DATE 8/8/13 03/14/12 PAGE 0081

```
0001      SUBROUTINE HAILE (NPOINT)
0002      COMMON /CEYALE/ YINPUT(28,100)
0003      COMMON /CEH/ EH100
0004      COMMON /CG/ G100
0005      DIMENSION EU(7,7,7), EY(7,7,7)
0006      COMMON /CLA/LA, LAP, MA, NB
0007      FORMAT (RF10.5)
0008      60 FORMAT (16I5)
0009      61 FORMAT (3X, 15, F8.3,2X,6I5,7.9X,3G12.5)
0010      HAP=4A
0011      EU(1,1,1) = YINPUT(1,NPOINT)
0012      EY(1,1,1) = YINPUT(15,NPOINT)
0013      EU(3,1,1) = YINPUT(2,NPOINT)
0014      EY(3,1,1) = YINPUT(16,NPOINT)
0015      EU(5,1,1) = YINPUT(3,NPOINT)
0016      EY(5,1,1) = YINPUT(17,NPOINT)
0017      EU(5,3,1) = YINPUT(4,NPOINT)
0018      EY(5,3,1) = YINPUT(18,NPOINT)
0019      EU(3,3,2) = YINPUT(5,NPOINT)
0020      EY(3,3,2) = YINPUT(19,NPOINT)
0021      EU(3,3,3) = YINPUT(6,NPOINT)
0022      EY(3,3,3) = YINPUT(20,NPOINT)
0023      EU(5,3,1) = YINPUT(7,NPOINT)
0024      EY(5,3,1) = YINPUT(21,NPOINT)
0025      EU(5,3,2) = YINPUT(8,NPOINT)
0026      EY(5,3,2) = YINPUT(22,NPOINT)
0027      EU(5,3,3) = YINPUT(9,NPOINT)
0028      EY(5,3,3) = YINPUT(23,NPOINT)
0029      EU(5,5,1) = YINPUT(10,NPOINT)
0030      EY(5,5,1) = YINPUT(24,NPOINT)
0031      EU(5,5,2) = YINPUT(11,NPOINT)
0032      EY(5,5,2) = YINPUT(25,NPOINT)
0033      EU(5,5,3) = YINPUT(12,NPOINT)
0034      EY(5,5,3) = YINPUT(26,NPOINT)
0035      EU(5,5,4) = YINPUT(13,NPOINT)
0036      EY(5,5,4) = YINPUT(27,NPOINT)
0037      EU(5,5,5) = YINPUT(14,NPOINT)
0038      EY(5,5,5) = YINPUT(28,NPOINT)
0039      SUM=0
0040      DO 9 NLP=1,5,2
0041      LF=NLP-1
0042      DO 9 NLXP=1,5,2
0043      LXPNLXP=1
0044      SIZE=1
0045      DO 91 NL=NL+5,2
0046      L=NL-1
0047      TAILLE=1
0048      DO 92 NLX=NLXP,5,2
0049      LX=NLX-1
0050      LAGEUR=(2*L+1)*(2*LX+1)*(2*LXP+1)
0051      A=LAGEUR
0052      A=A/(2.*LA+1)/(2.*LAP+1)
0053      Q=SORT(A)
0054      MMAXMIN0(NL,NLP)
0055      HXMAX=MIN0(NLX,NLXP)
0056      PAYR2=14MAX-1
0057      MAXPX= 2+HXMAX-1
0058      DO 40 NM=1, MAXM
```



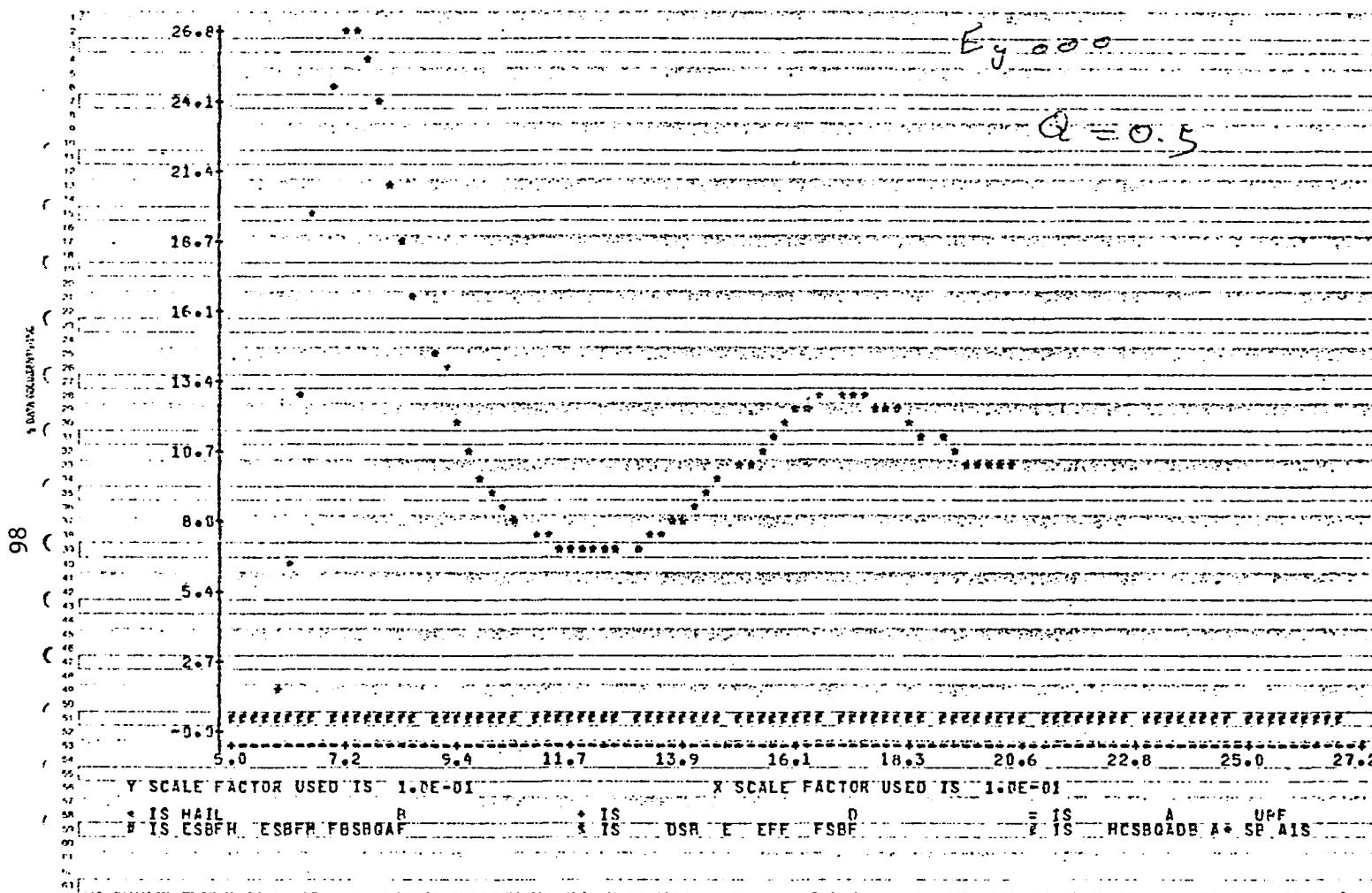


FORTRAN IV GI RELEASE 2.60 FACT DATE 5-B0213 03/14/12 PAGE 0001  
C001 FUNCTION FACT(N)

```

0001      FUNCTION FACT(N)
0002      FACT=1
0003      IF (N .LE. 0) GO TO 7
0004      U=1
0005      DO 4 I=1,N
0006      U=U*I
0007      FACT=U
0008      RETURN
0009      ENDF

```



سیده زین العابدین

9  
6

100

57.6

51.8

45.6

40.0

34.1

28.2

22.4

16.5

10.6

4.7

-1.2

Y SCALE FACTOR USED IS 1.0E-03

X SCALE FACTOR USED IS 1.0E-01

\* IS HAIL      B  
IS ESBFH ESBFR FSBFOAF

\* IS D  
IS DSB E EFF FSRF

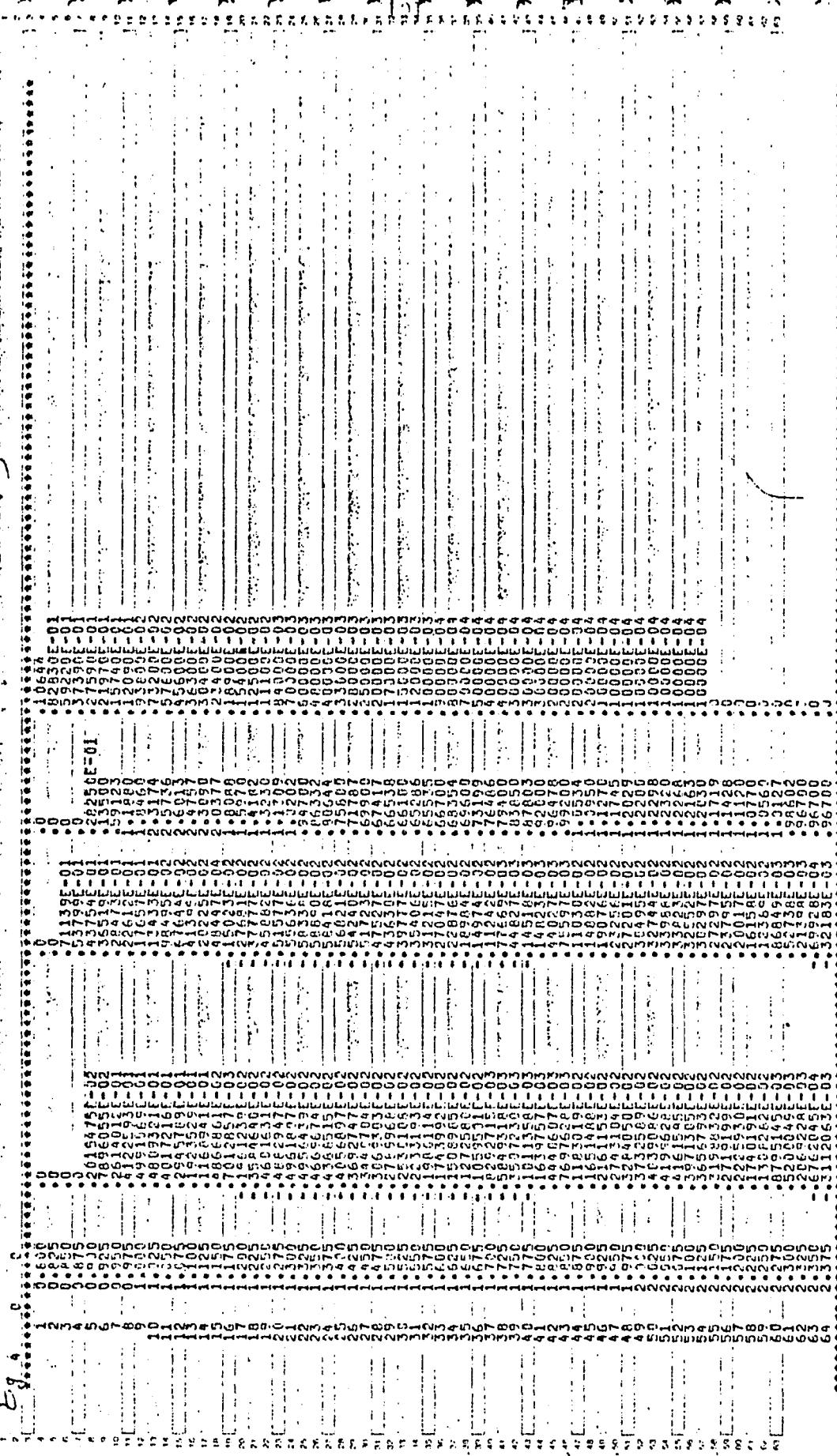
\* IS B URF  
IS HCSEGADBT A SB AIS

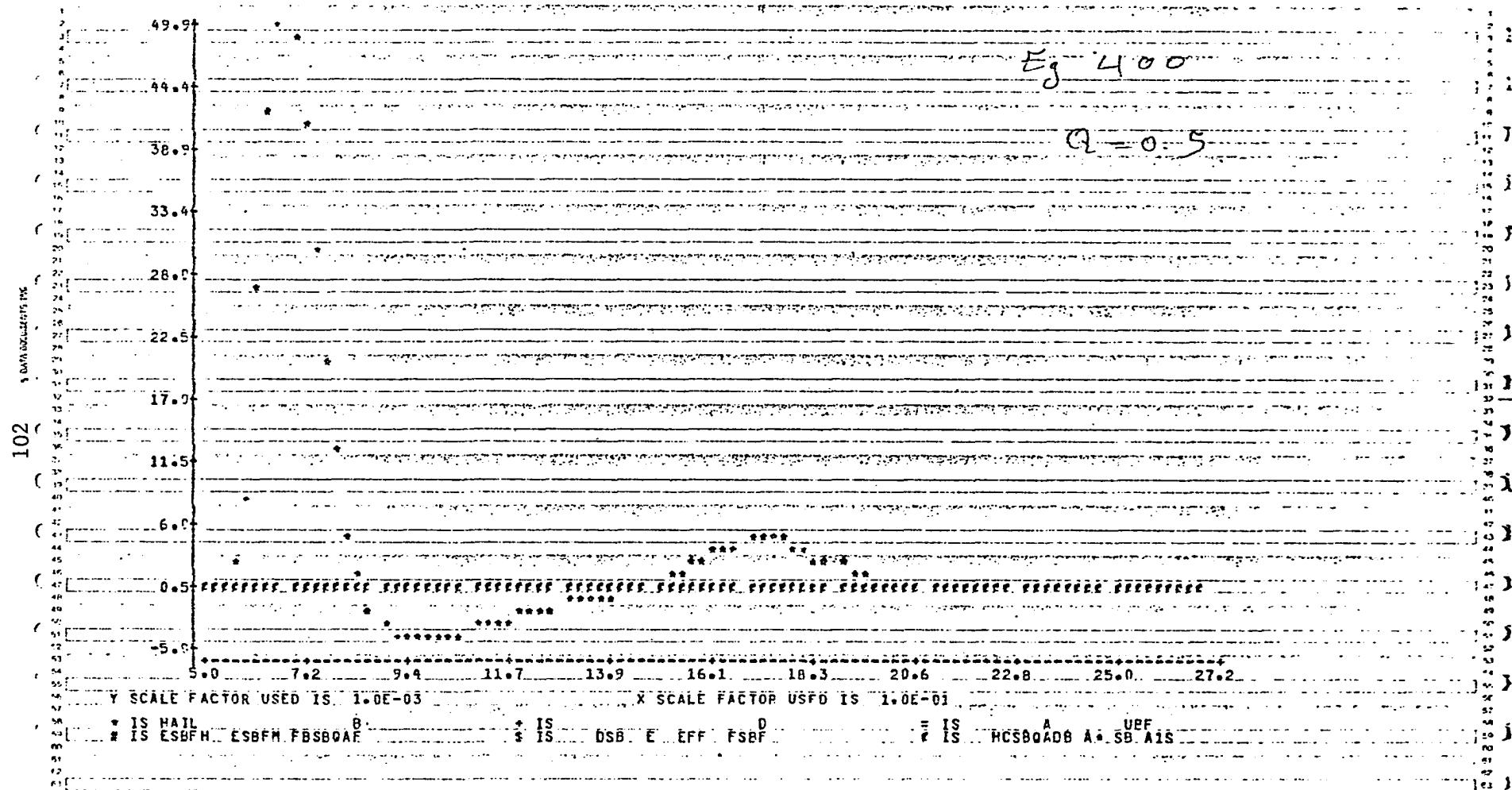
Eg 200

Q = 0.5

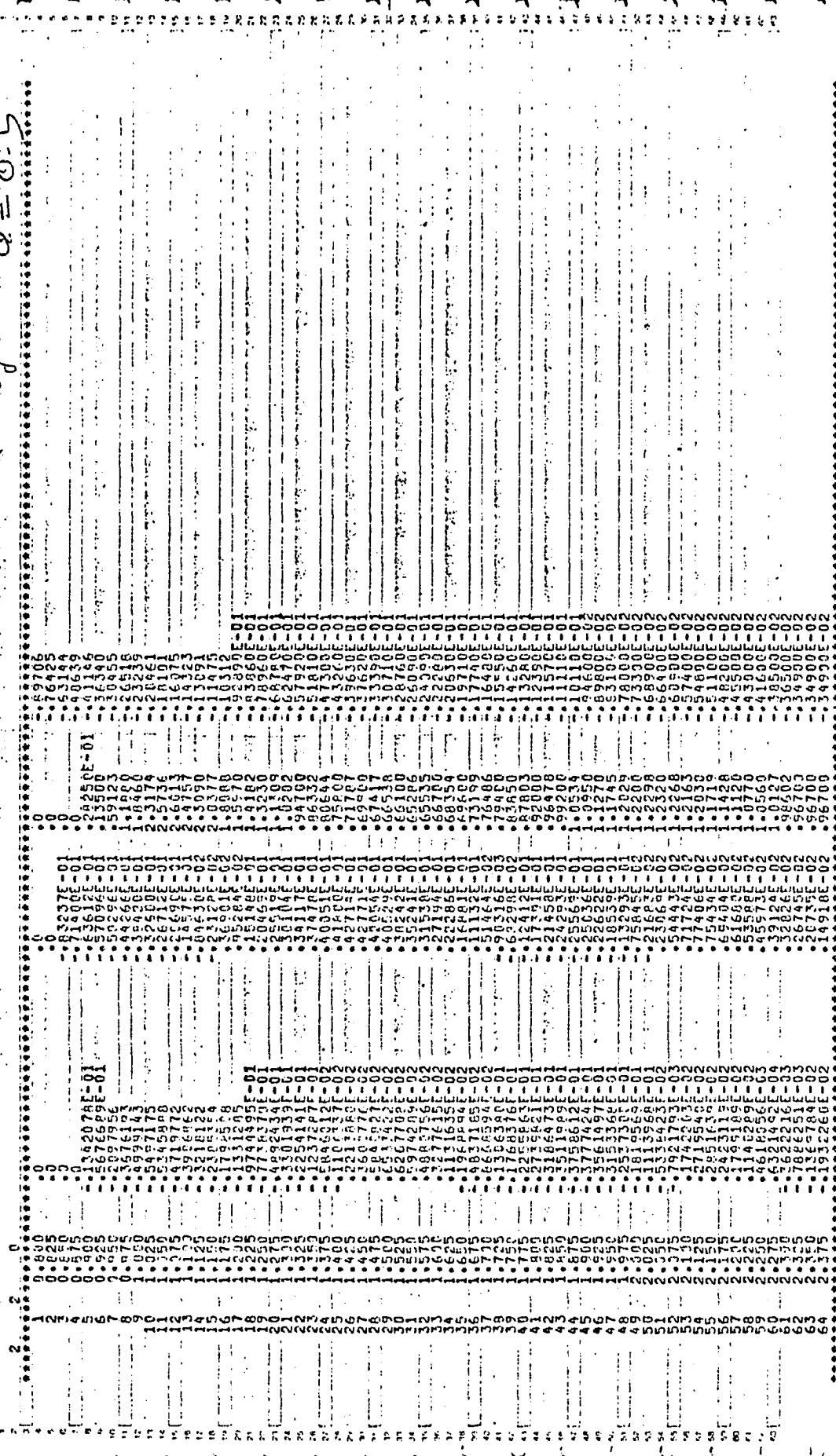
Eg.

$\Omega = 0.5$

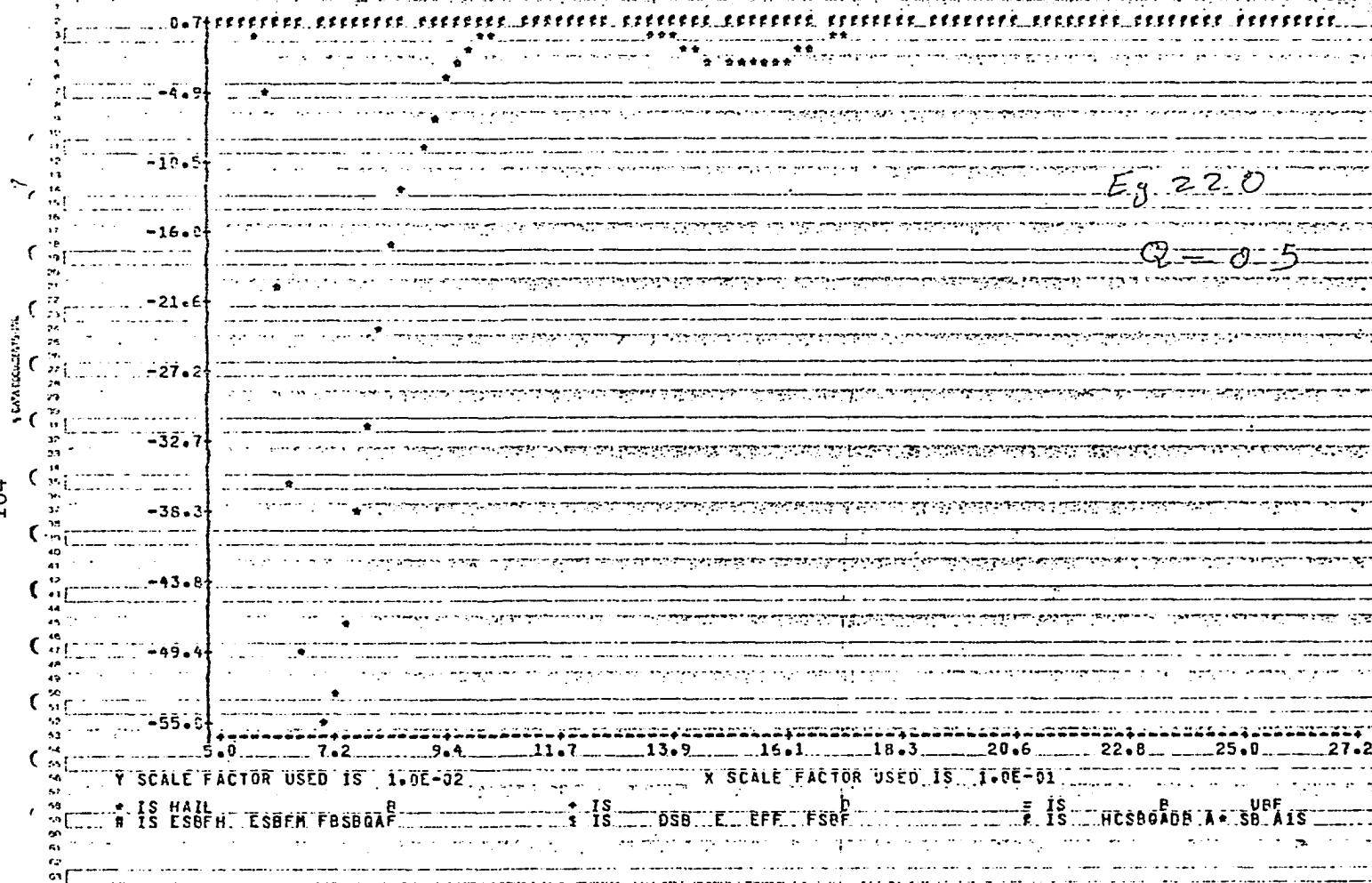


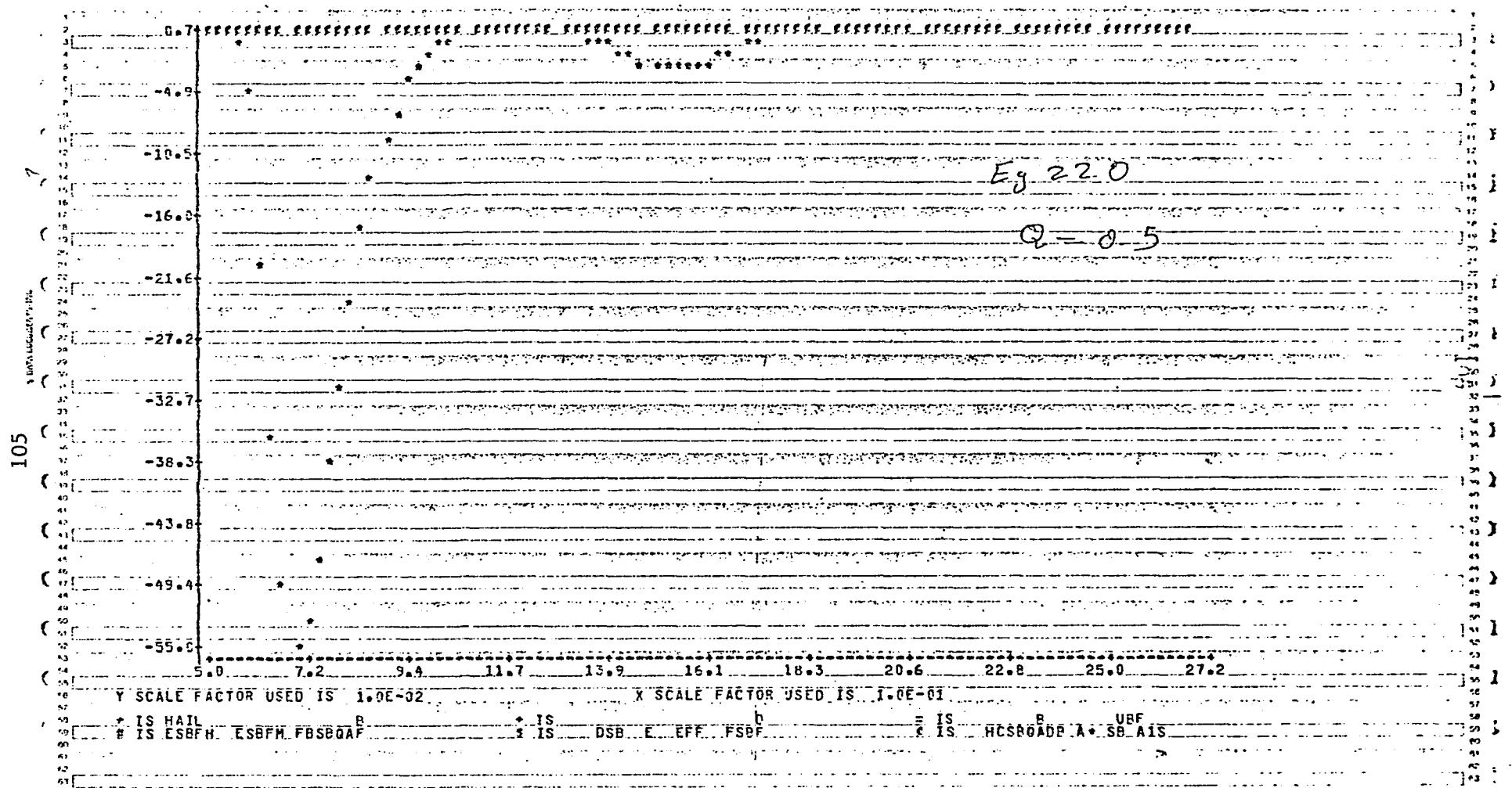


$Eg$        $\theta = 0.5$



104



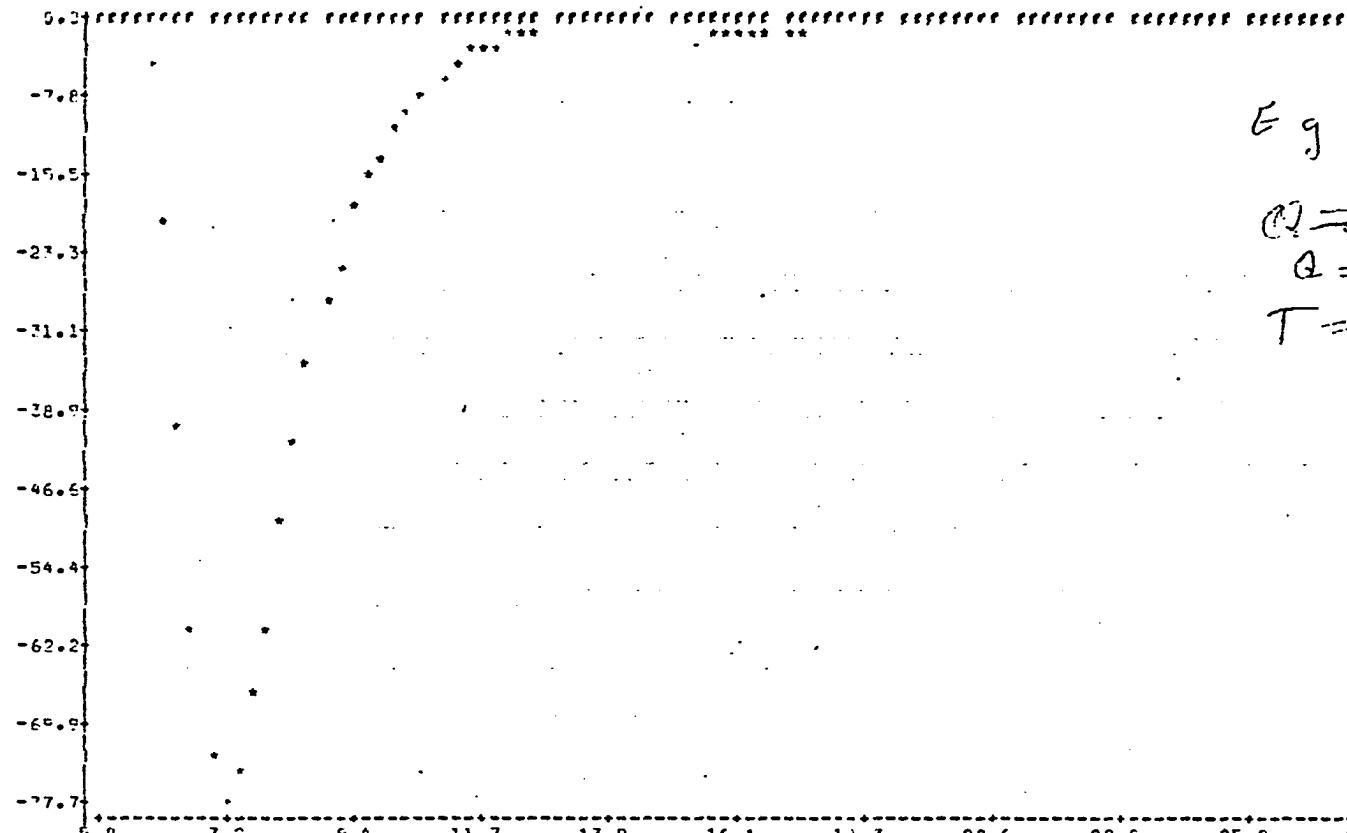


四

E5 - Q = 0.5

This figure displays a 2D cellular automaton simulation. The grid consists of 100 columns and 100 rows of binary digits (0s and 1s). The pattern is highly symmetric and exhibits a complex, fractal-like evolution over time. A central initial condition is visible at the top left. The entire pattern is contained within a dashed rectangular border.

107



\* IS HAIL  
# IS ESEFH ESBFM FBSQAF

+ IS DSP E EFF FSPP

= IS HCSBQADB A+ SR A1S

Eg 22.2.

$\bar{Q} = 0.5$   
 $Q = 0.5$

$T = 1.277$

E4.1 - E4.2 - E4.3

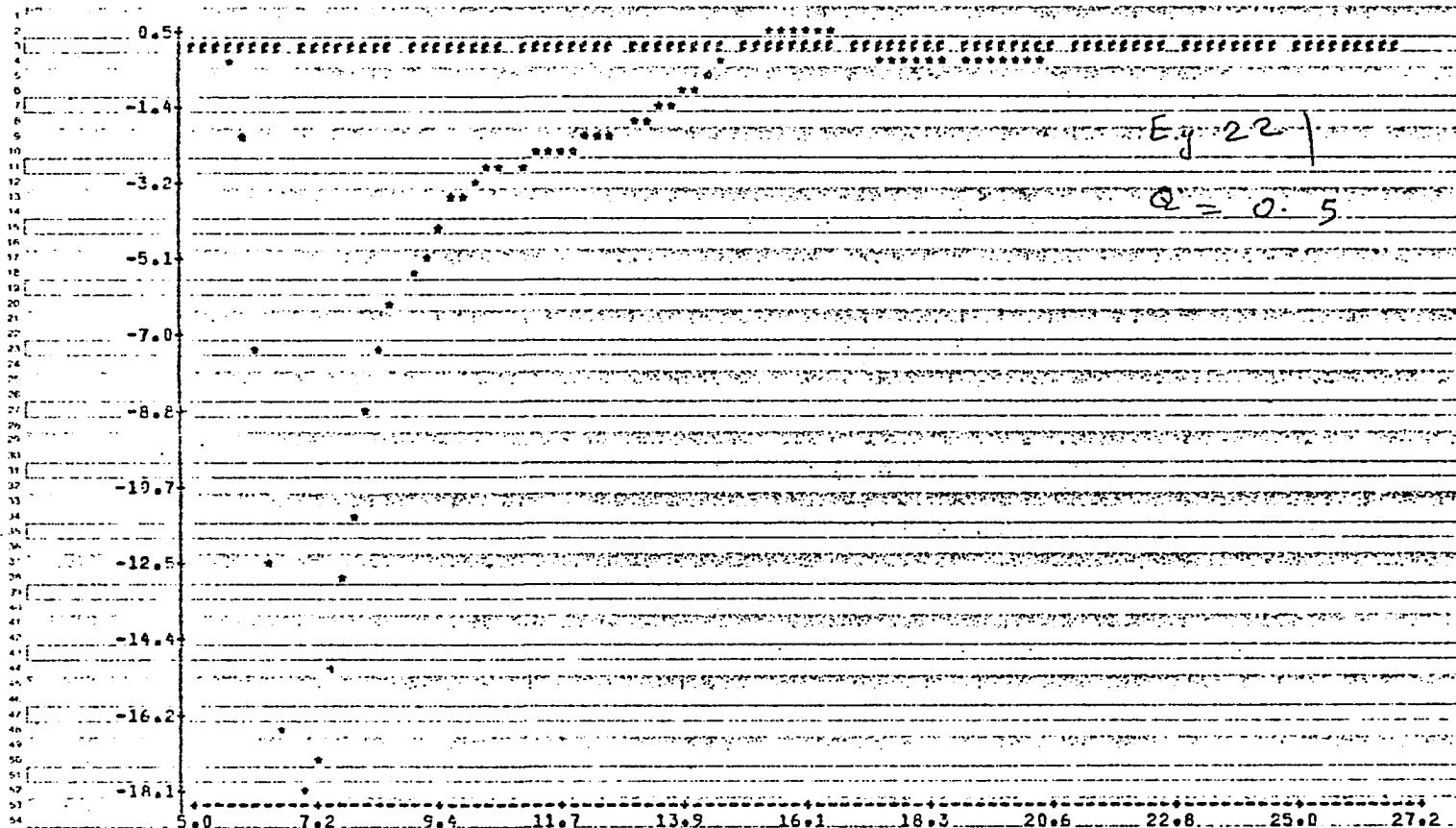
E4.2  
1  
2

01

02

1

109



Y SCALE FACTOR USED IS 1.0E-02

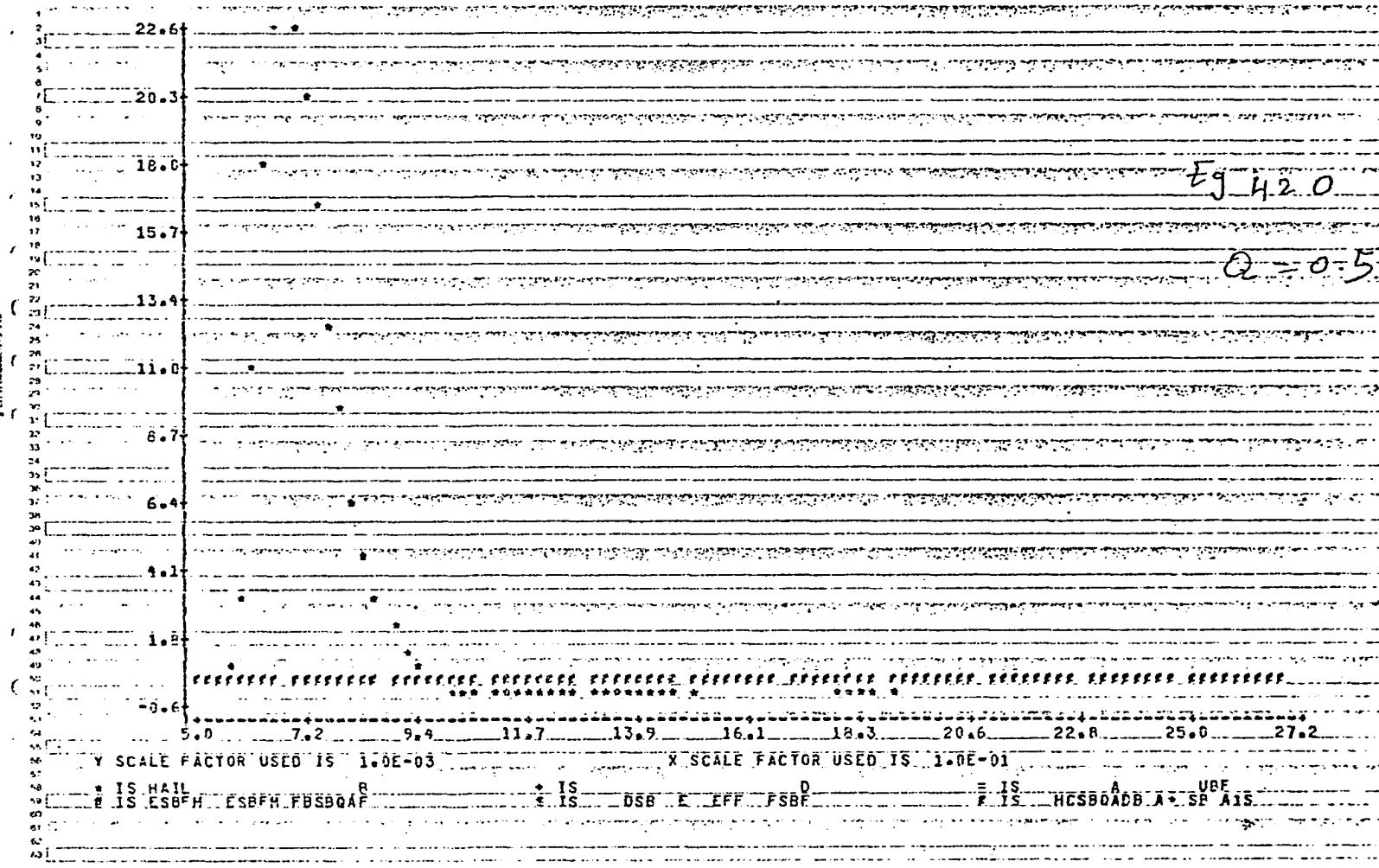
X SCALE FACTOR USED IS 1.0E-01

A IS MAIL B IS ESEFH C IS ESBFH D IS DSB E IS EFE F IS FSBF G IS HCSBDADE A IS SB AIS B IS UBF

Eg -  $a = 0.5$

59

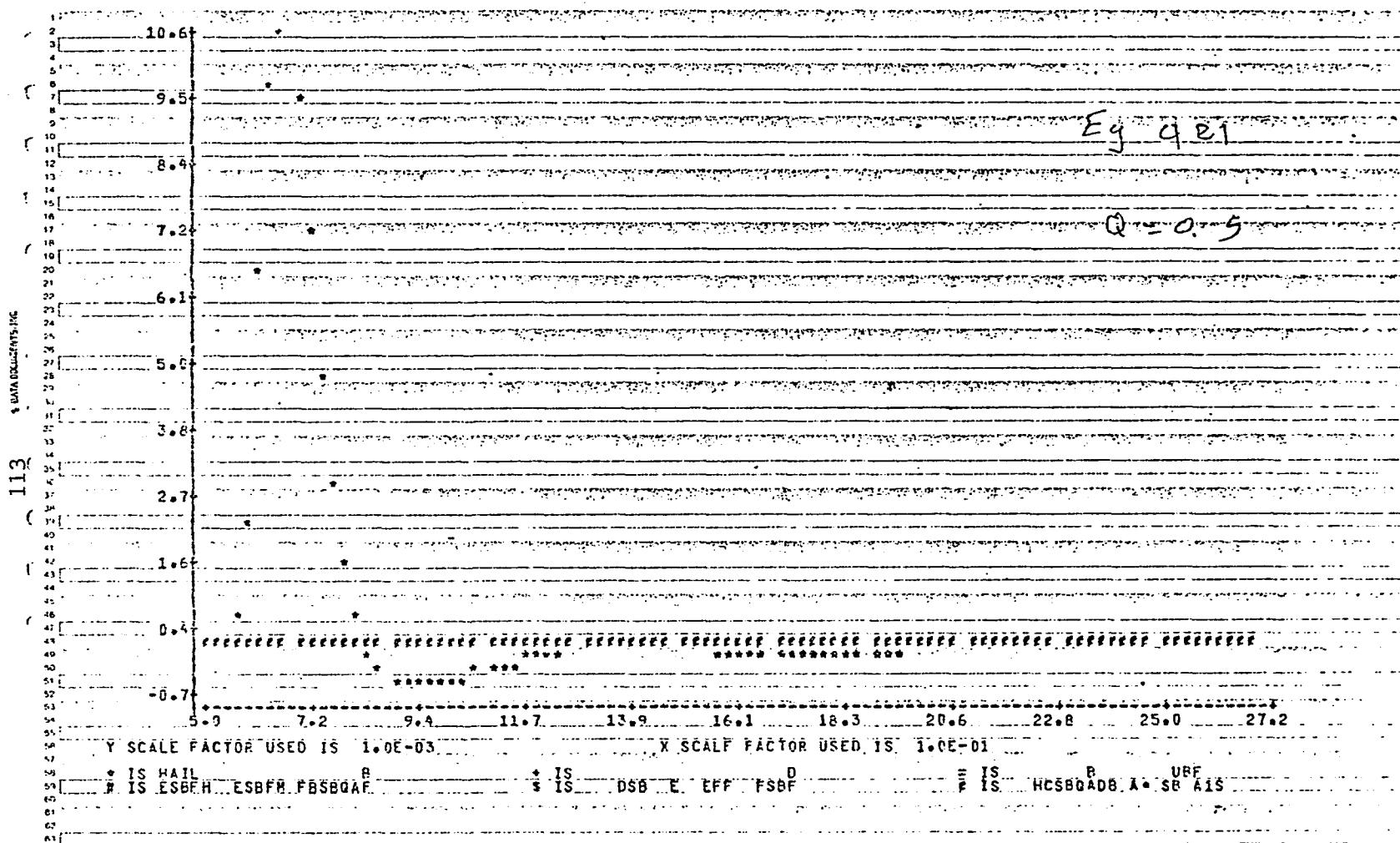
III 1. RADIATION



5. 4. 5

5

2 4 6



E<sub>9</sub> 4  
2

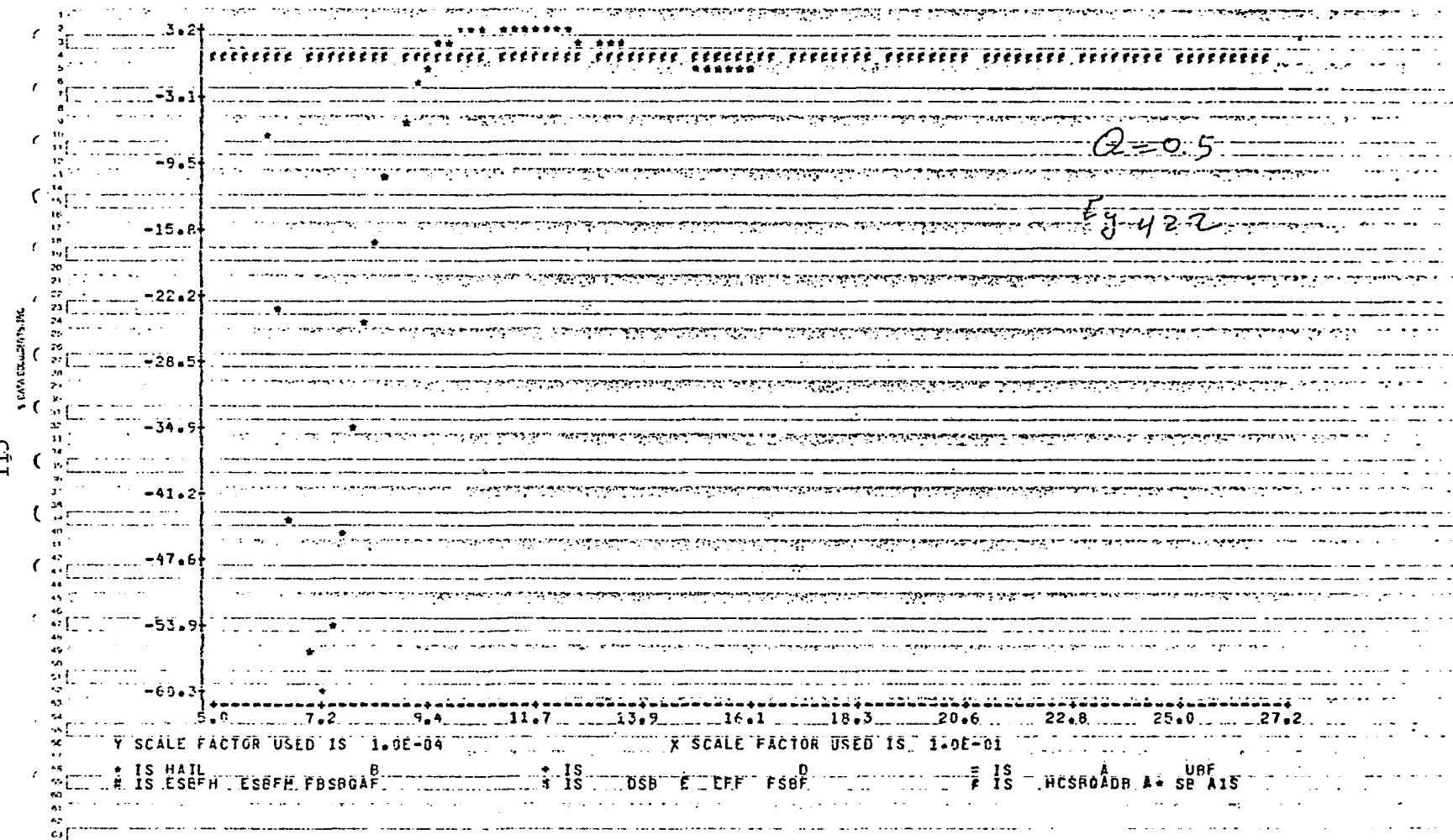
2

1

E<sub>9</sub> 5  
2  
1

1

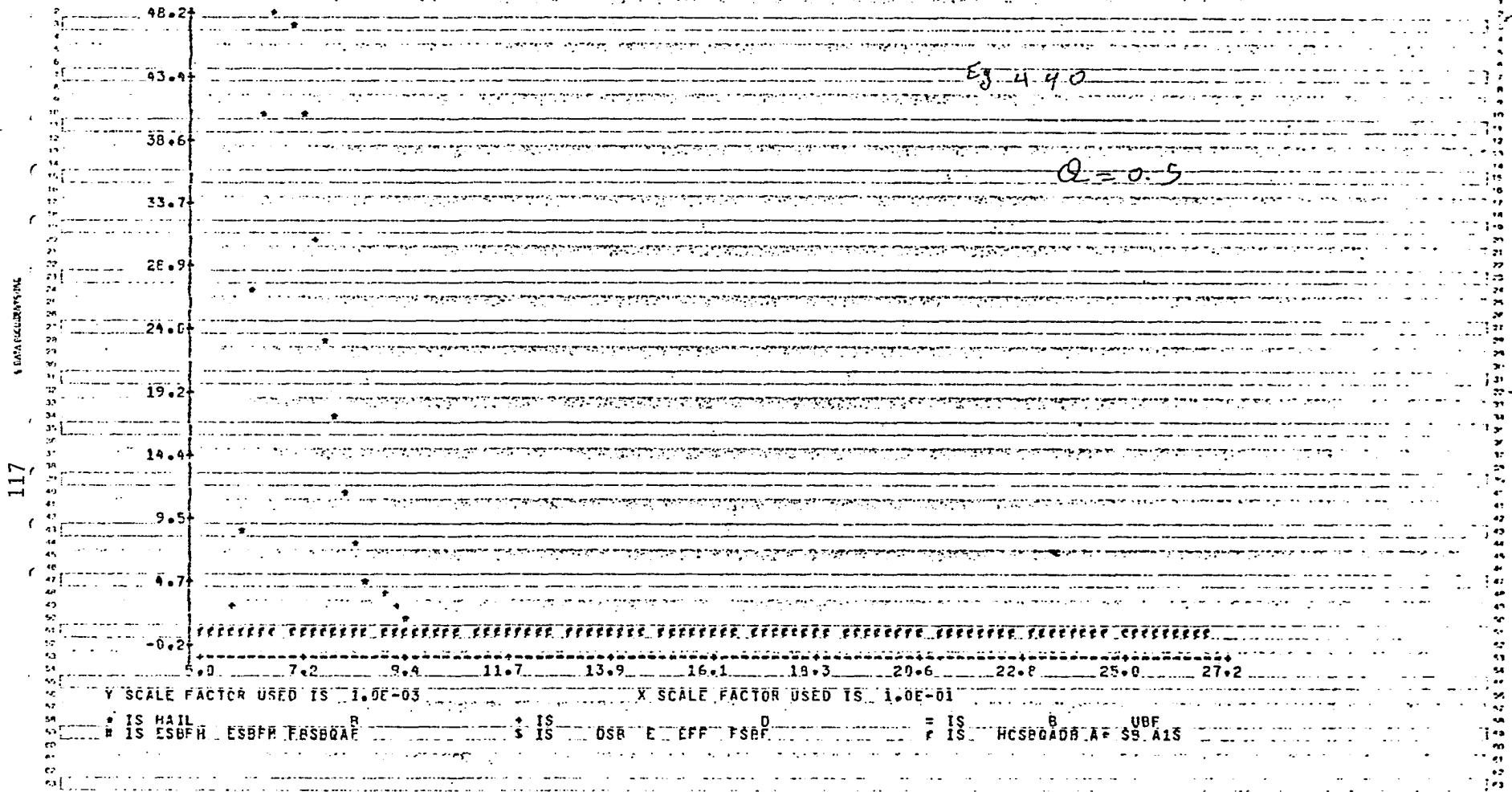
115



$E_g = 0.5$

$E_g$

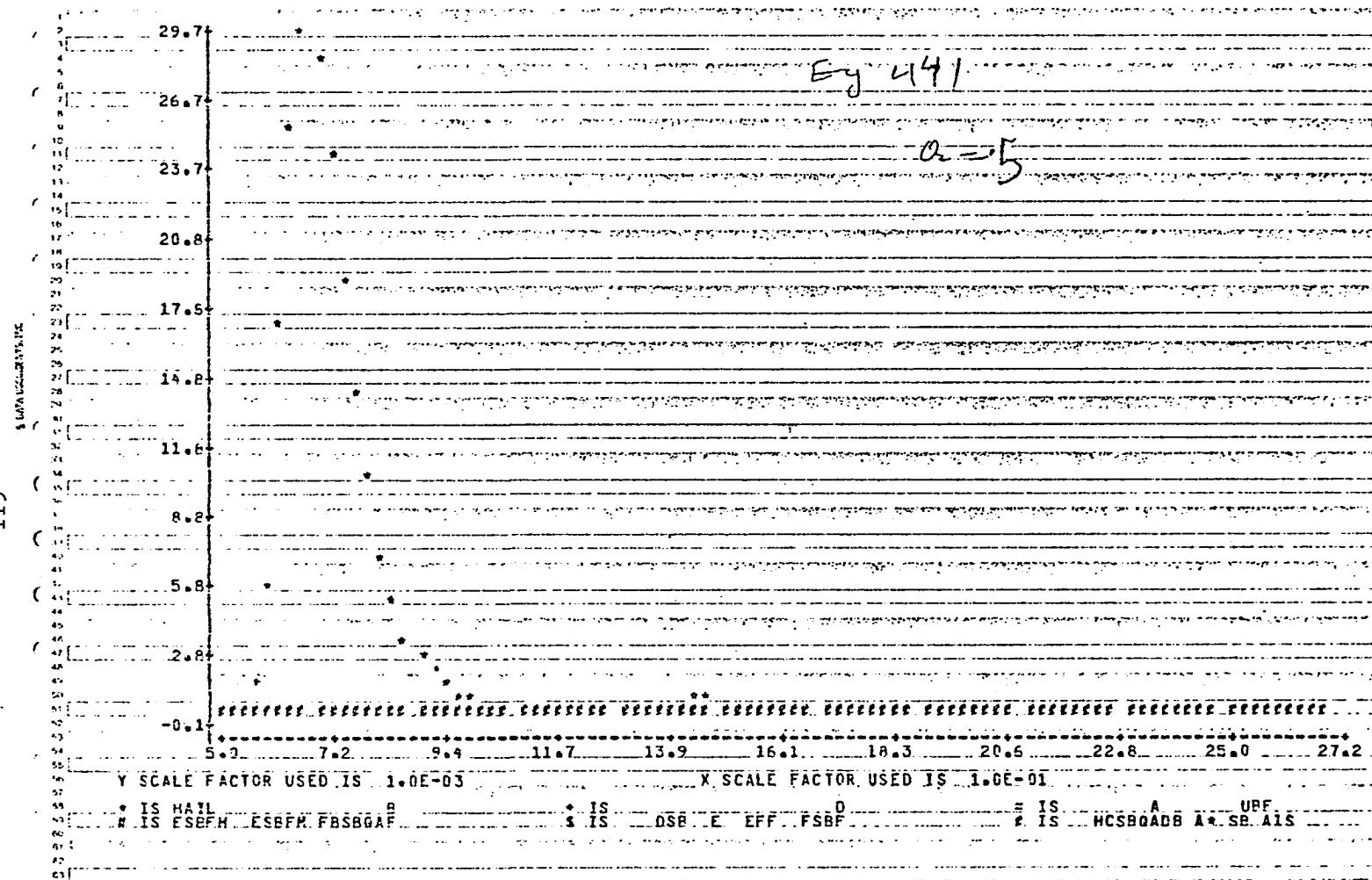
111



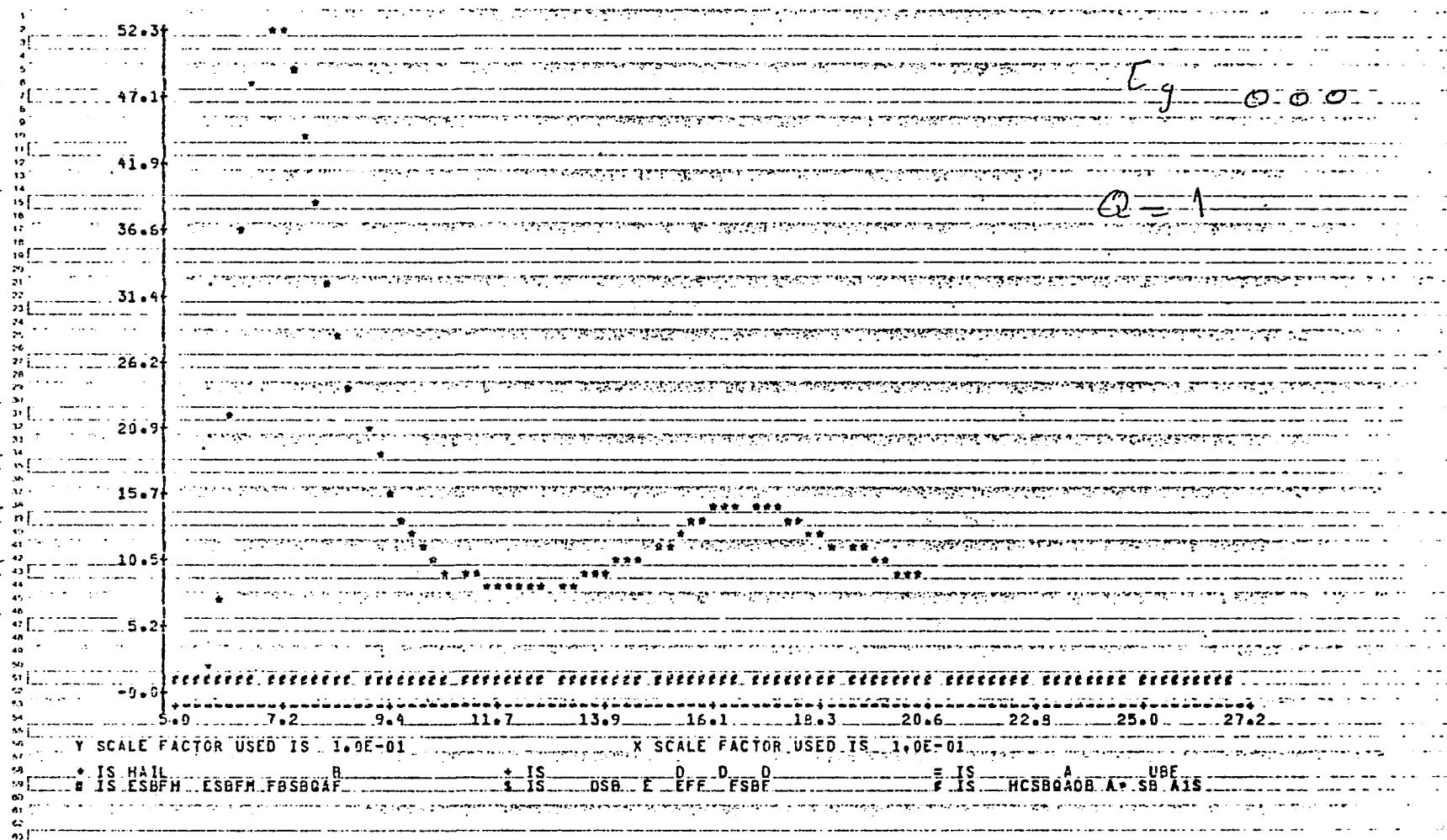
Ej	4	1	0	22320
1		0	17641	
2		-14158E-01	12662	
3		-18674E-01	53993E-01	
4		-11743E-01	256E-01	
5		-16116E-01	13500	
6		-820921E-02	159120	
7		-100000001	500420	
8		-100000001	100000001	
9		-100000001	100000001	
10		-100000001	100000001	
11		-100000001	100000001	
12		-100000001	100000001	
13		-100000001	100000001	
14		-100000001	100000001	
15		-100000001	100000001	
16		-100000001	100000001	
17		-100000001	100000001	
18		-100000001	100000001	
19		-100000001	100000001	
20		-100000001	100000001	
21		-100000001	100000001	
22		-100000001	100000001	
23		-100000001	100000001	
24		-100000001	100000001	
25		-100000001	100000001	
26		-100000001	100000001	
27		-100000001	100000001	
28		-100000001	100000001	
29		-100000001	100000001	
30		-100000001	100000001	
31		-100000001	100000001	
32		-100000001	100000001	
33		-100000001	100000001	
34		-100000001	100000001	
35		-100000001	100000001	
36		-100000001	100000001	
37		-100000001	100000001	
38		-100000001	100000001	
39		-100000001	100000001	
40		-100000001	100000001	
41		-100000001	100000001	
42		-100000001	100000001	
43		-100000001	100000001	
44		-100000001	100000001	
45		-100000001	100000001	
46		-100000001	100000001	
47		-100000001	100000001	
48		-100000001	100000001	
49		-100000001	100000001	
50		-100000001	100000001	
51		-100000001	100000001	
52		-100000001	100000001	
53		-100000001	100000001	
54		-100000001	100000001	
55		-100000001	100000001	
56		-100000001	100000001	
57		-100000001	100000001	
58		-100000001	100000001	
59		-100000001	100000001	
60		-100000001	100000001	
61		-100000001	100000001	
62		-100000001	100000001	
63		-100000001	100000001	
64		-100000001	100000001	
65		-100000001	100000001	
66		-100000001	100000001	
67		-100000001	100000001	
68		-100000001	100000001	
69		-100000001	100000001	
70		-100000001	100000001	
71		-100000001	100000001	
72		-100000001	100000001	
73		-100000001	100000001	
74		-100000001	100000001	
75		-100000001	100000001	
76		-100000001	100000001	
77		-100000001	100000001	
78		-100000001	100000001	
79		-100000001	100000001	
80		-100000001	100000001	
81		-100000001	100000001	
82		-100000001	100000001	
83		-100000001	100000001	
84		-100000001	100000001	
85		-100000001	100000001	
86		-100000001	100000001	
87		-100000001	100000001	
88		-100000001	100000001	
89		-100000001	100000001	
90		-100000001	100000001	
91		-100000001	100000001	
92		-100000001	100000001	
93		-100000001	100000001	
94		-100000001	100000001	
95		-100000001	100000001	
96		-100000001	100000001	
97		-100000001	100000001	
98		-100000001	100000001	
99		-100000001	100000001	
100		-100000001	100000001	
101		-100000001	100000001	
102		-100000001	100000001	
103		-100000001	100000001	
104		-100000001	100000001	
105		-100000001	100000001	
106		-100000001	100000001	
107		-100000001	100000001	
108		-100000001	100000001	
109		-100000001	100000001	
110		-100000001	100000001	
111		-100000001	100000001	
112		-100000001	100000001	
113		-100000001	100000001	
114		-100000001	100000001	
115		-100000001	100000001	
116		-100000001	100000001	
117		-100000001	100000001	
118		-100000001	100000001	

② -0.5

119



		Eg	Q = 1
1	0	0	0
2	124	0.825	34.672
3	0	0.825	35.637
4	0	0.850	35.602
5	0	0.875	35.5495
6	0	0.900	35.0576
7	0	0.925	35.2558
8	0	0.950	35.9747
9	0	0.975	36.2227
10	1	0.950	36.9710
11	1	0.975	37.7216
12	1	1.000	38.5491
13	1	1.025	39.4234
14	1	1.050	39.3308
15	1	1.075	39.2729
16	1	1.100	39.2068
17	1	1.125	39.1356
18	1	1.150	39.1082
19	1	1.175	39.0957
20	1	1.200	39.0723
21	1	1.225	39.0518
22	1	1.250	39.0322
23	1	1.275	39.0151
24	1	1.300	39.0008
25	1	1.325	39.0000
26	1	1.350	39.0000
27	1	1.375	39.0000
28	1	1.400	39.0000
29	1	1.425	39.0000
30	1	1.450	39.0000
31	1	1.475	39.0000
32	1	1.500	39.0000
33	1	1.525	39.0000
34	1	1.550	39.0000
35	1	1.575	39.0000
36	1	1.600	39.0000
37	1	1.625	39.0000
38	1	1.650	39.0000
39	1	1.675	39.0000
40	1	1.700	39.0000
41	1	1.725	39.0000
42	1	1.750	39.0000
43	1	1.775	39.0000
44	1	1.800	39.0000
45	1	1.825	39.0000
46	1	1.850	39.0000
47	1	1.875	39.0000
48	1	1.900	39.0000
49	1	1.925	39.0000
50	1	1.950	39.0000
51	1	1.975	39.0000
52	1	2.000	39.0000
53	1	2.025	39.0000
54	1	2.050	39.0000
55	1	2.075	39.0000
56	1	2.100	39.0000
57	1	2.125	39.0000
58	1	2.150	39.0000
59	1	2.175	39.0000
60	1	2.200	39.0000
61	1	2.225	39.0000
62	1	2.250	39.0000
63	1	2.275	39.0000
64	1	2.300	39.0000



		Ey	Q = 1
1	2	0	0
3	4	0	0
5	6	0	0
7	8	0	0
9	10	0	0
11	12	0	0
13	14	0	0
15	16	0	0
17	18	0	0
19	20	0	0
21	22	0	0
23	24	0	0
25	26	0	0
27	28	0	0
29	30	0	0
31	32	0	0
33	34	0	0
35	36	0	0
37	38	0	0
39	40	0	0
41	42	0	0
43	44	0	0
45	46	0	0
47	48	0	0
49	50	0	0
51	52	0	0
53	54	0	0
55	56	0	0
57	58	0	0
59	60	0	0
61	62	0	0
63	64	0	0

123

1 EQUATIONS INC.

10.8  
9.7  
8.6  
7.5  
6.4  
5.3  
4.1  
3.0  
1.9  
0.8  
-0.3

Y SCALE FACTOR USED IS 1.0E-01

X SCALE FACTOR USED IS 1.0E-01

\* IS HAIL \* IS DSB \* IS MCBOARD AFB  
\* IS ESBFH \* IS FBSBOAF \* IS E EFF D FSBF \* IS B UBF  
\* IS ESBFH \* IS FBSBOAF \* IS E EFF D FSBF \* IS B UBF  
\* IS ESBFH \* IS FBSBOAF \* IS E EFF D FSBF \* IS B UBF

Ey 200

a = 1

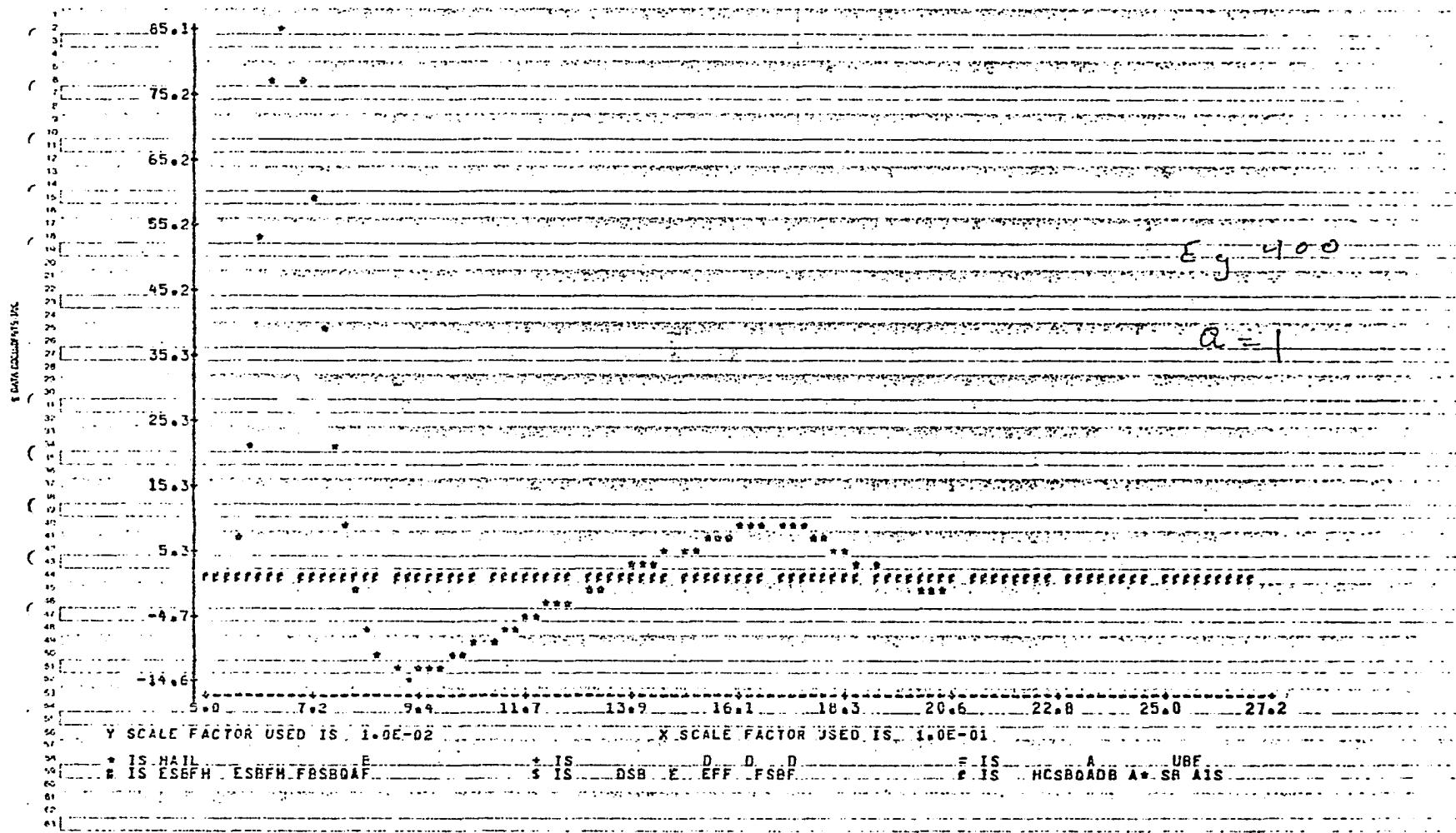
124

卷之三

Eq

E.g.

—



۱۱۲۶

E.

- 59 -

0.6 -3.2  
-7.0  
-10.9  
-14.7  
-16.5  
-22.3  
-26.2  
-30.0  
-33.6  
-37.6  
5.0 7.2 9.4 11.7 13.9 16.1 18.3 20.6 22.8 25.0 27.2

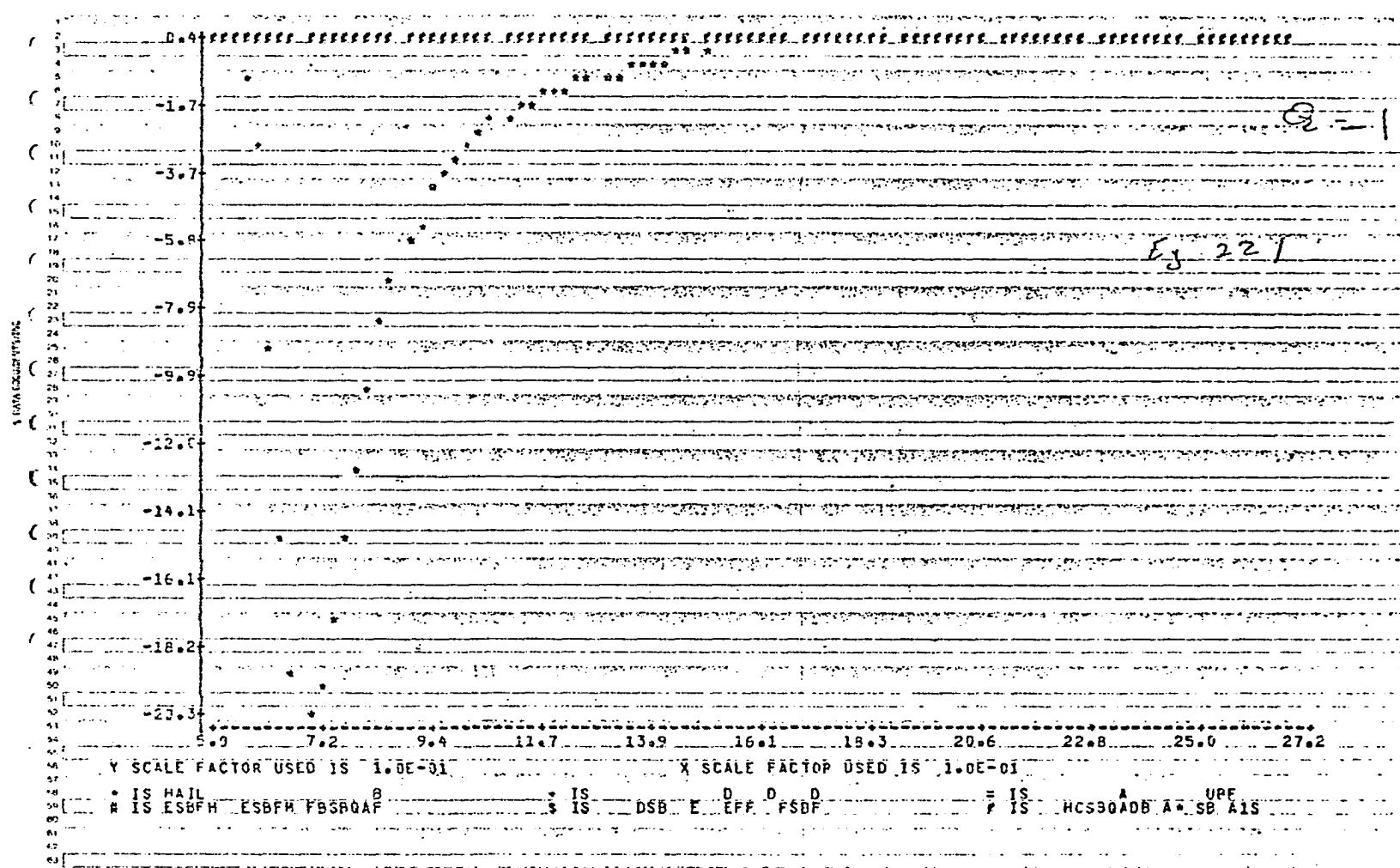
Eg-220

$\theta = 1$

Y SCALE FACTOR USED IS 1.0E-01 X SCALE FACTOR USED IS 1.0E-01

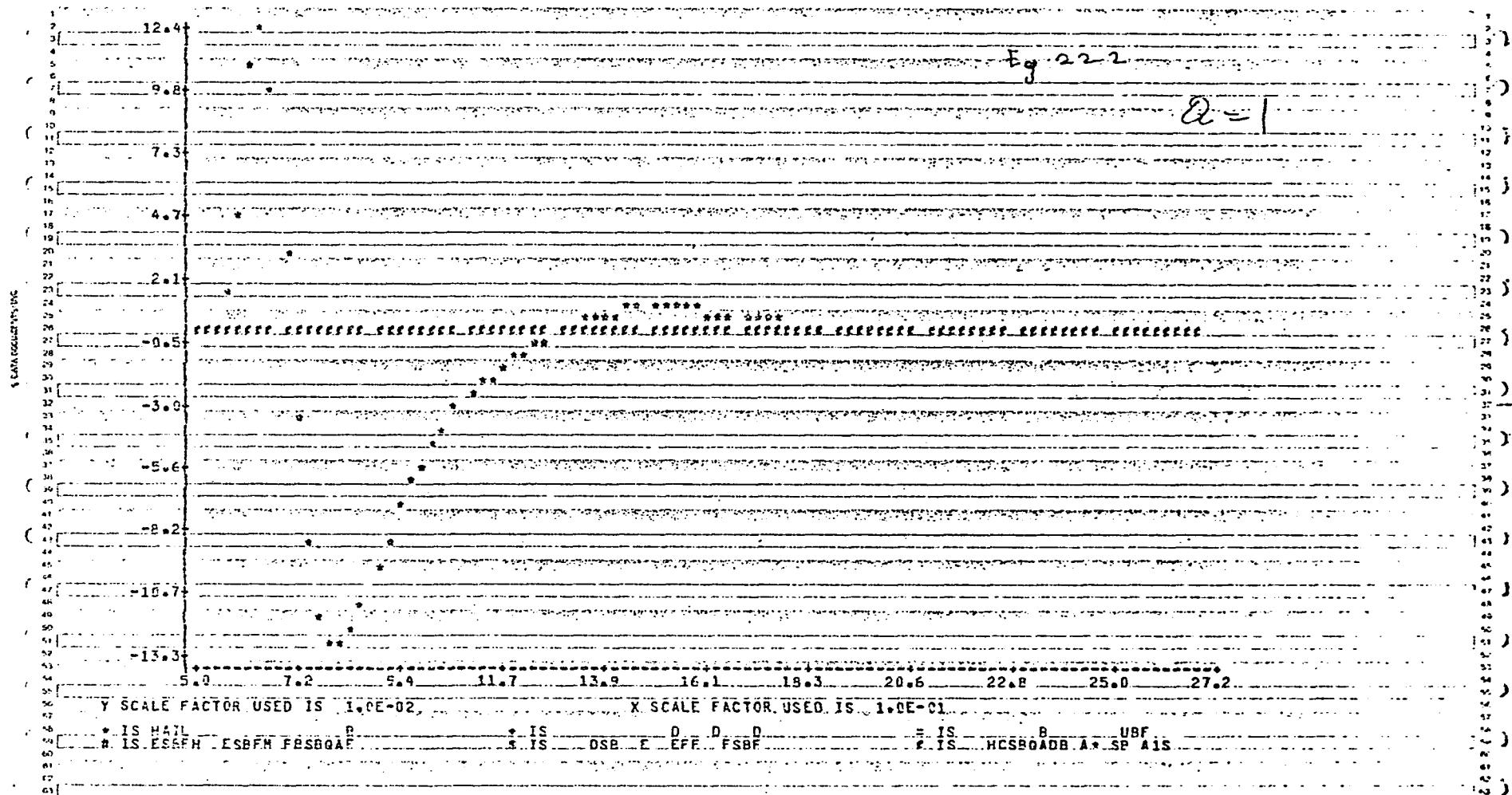
\* IS HAIL E IS DSB E EBF FSBF D D D IS UBF  
# IS ESBFH ESBFM ESBDAF \$ IS DSB E EBF FSBF % IS HCS3GADB A\* SB AIS





二二

卷之三

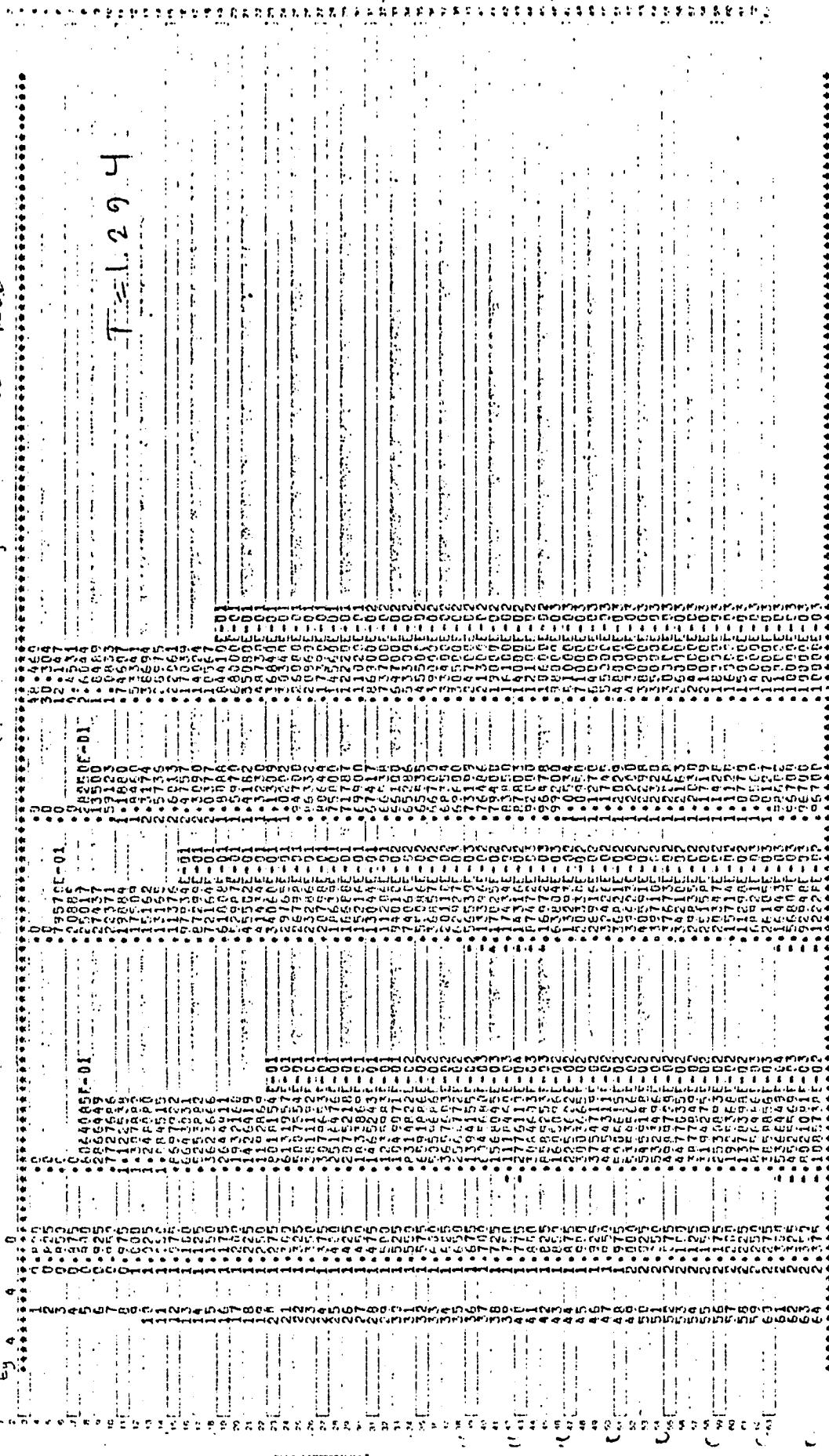


$\theta = 100^\circ$

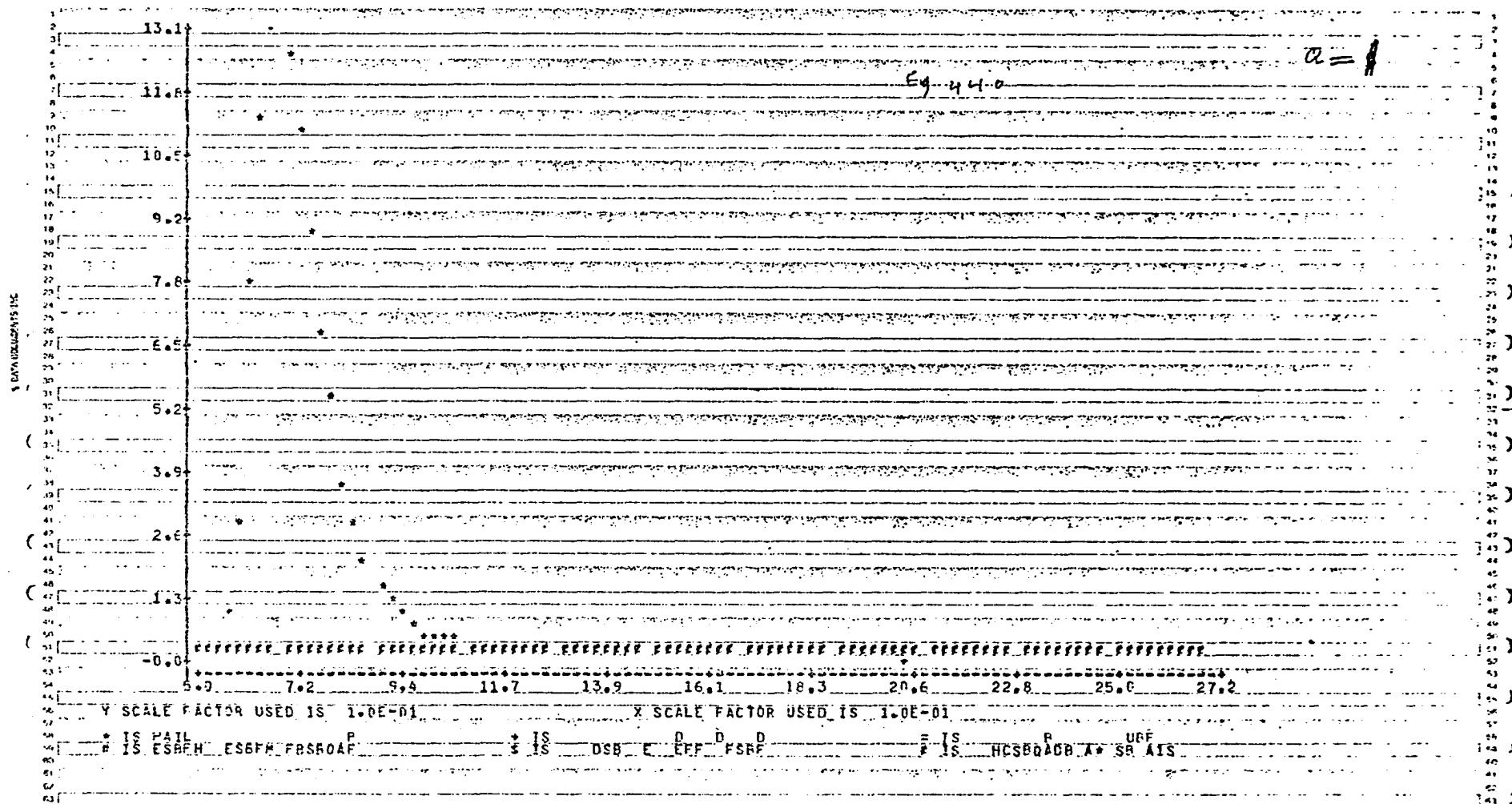
65

Eg 4

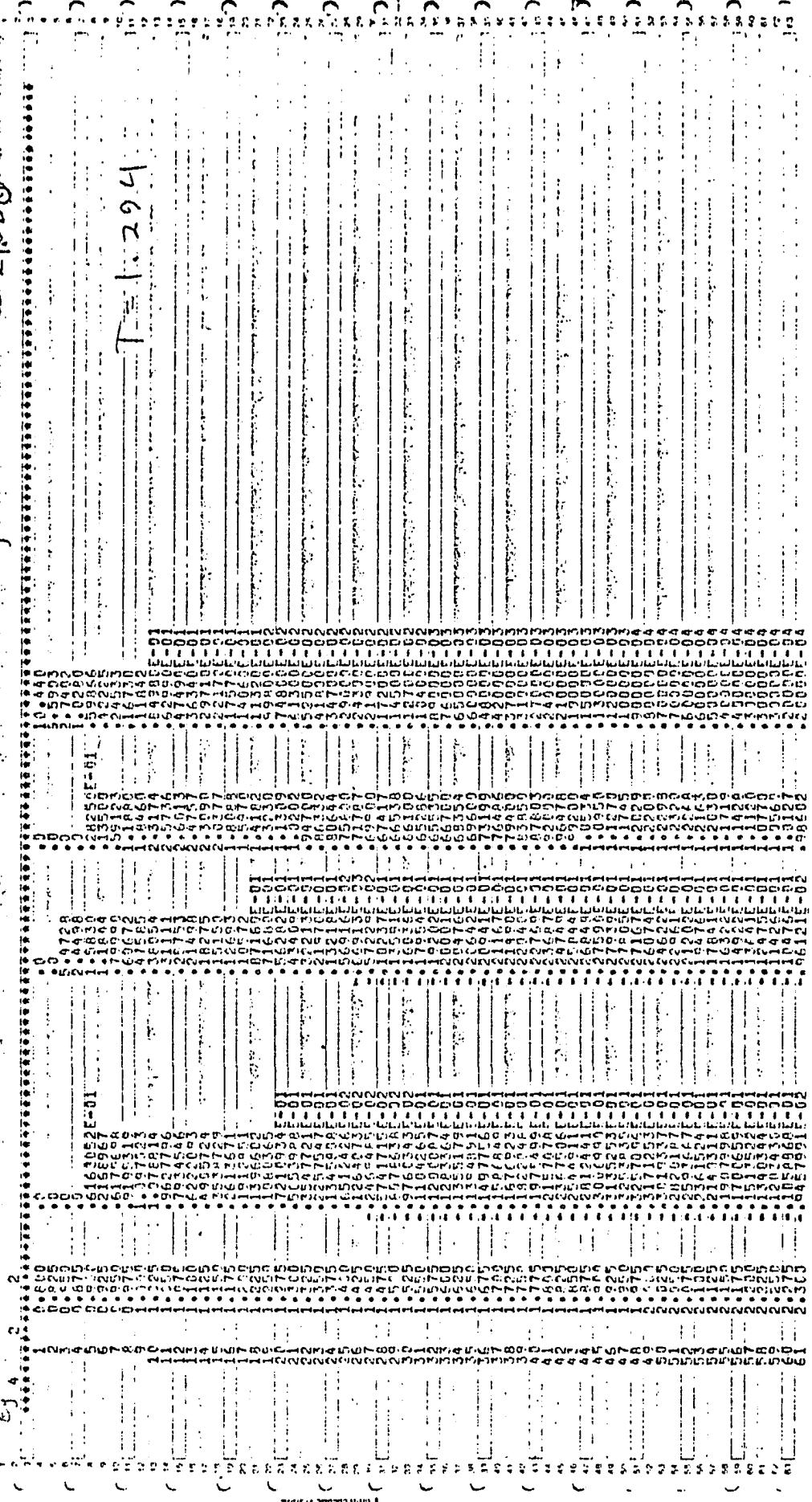
T = 1294



133



Eig  $\alpha = 1000$



135

100A DOCUMENTS INC.

11.6

9.8

8.7

7.6

5.5

5.3

4.2

3.1

1.8

0.8

-0.3

5.0

7.2

9.4

11.7

13.9

16.1

18.3

20.6

22.8

25.0

27.2

E9422

Q = 10

F = 1.294

Y SCALE FACTOR USED IS 1.0E-01

X SCALE FACTOR USED IS 1.0E-01

\* IS HAZE  
\* IS ESBFH ESBFM EBSBQAF

\* IS DSB E EFE FSBF

\* IS HCSBOADE A\*SB AIS

$E_g$

$\theta = 1$

$T = 1.294$

137

19.7

16.2

12.8

9.4

5.9

2.5

-1.0

-4.4

-7.8

-11.3

-14.7

-5.0

7.2

9.4

11.7

13.9

16.1

18.3

20.6

22.8

25.0

27.2

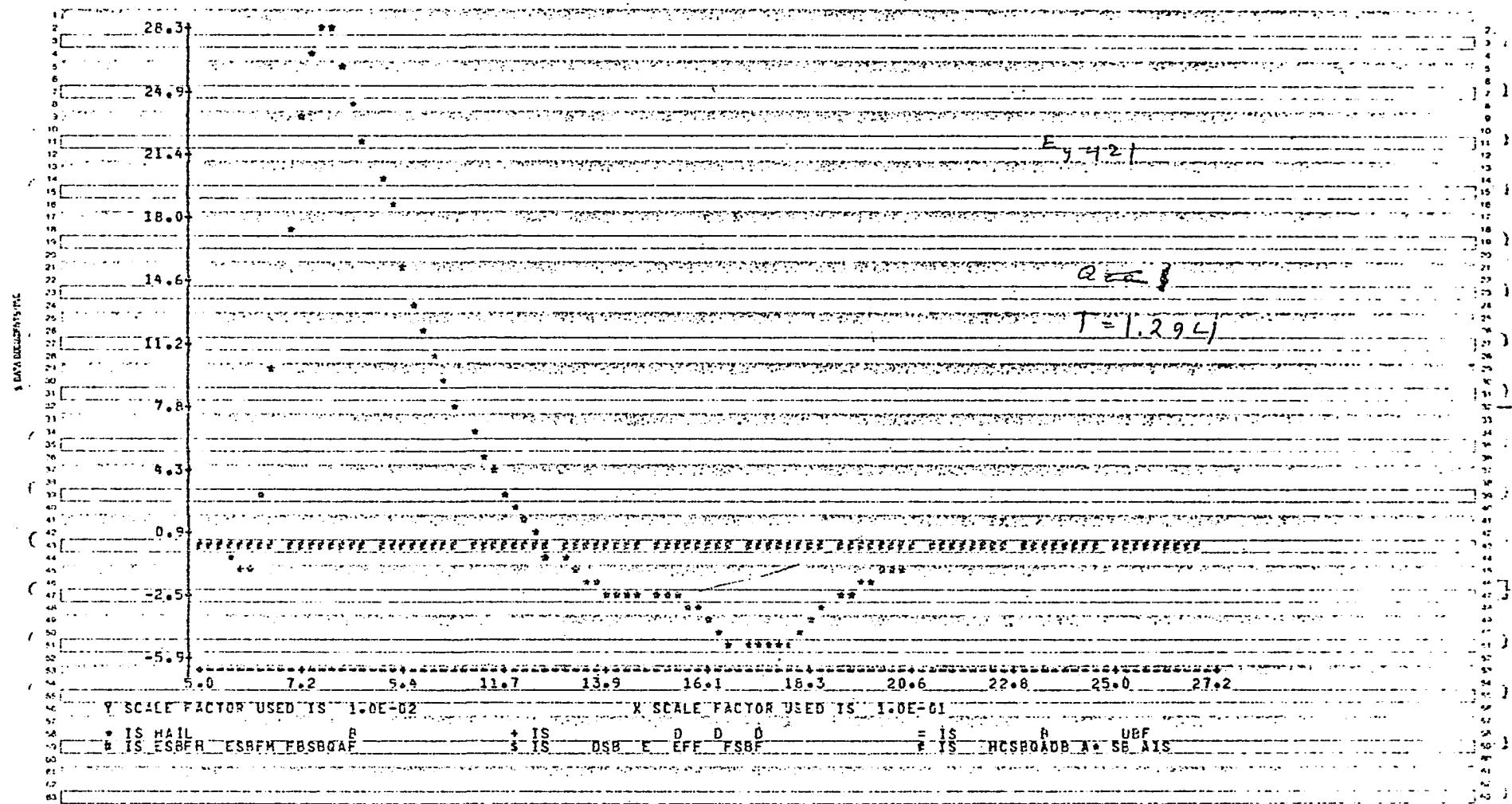
Y SCALE FACTOR USED IS 1.0E-02

X SCALE FACTOR USED IS 1.0E-01

IS HAIL  
IS ESEFH FRSBDAF

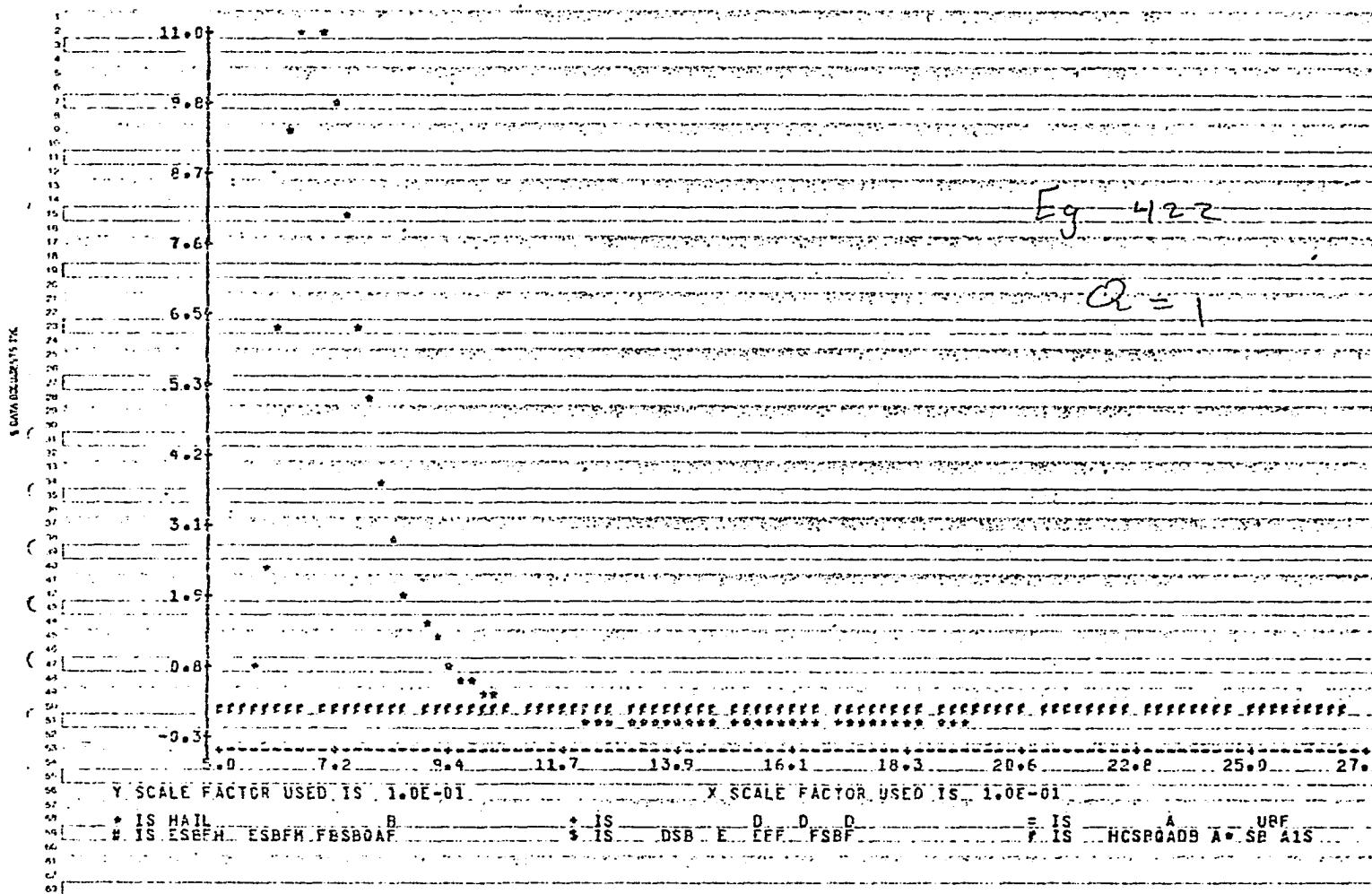
IS DSE EFF FSRF

IS A URF  
IS HCSPQADB AT SE AIS



三

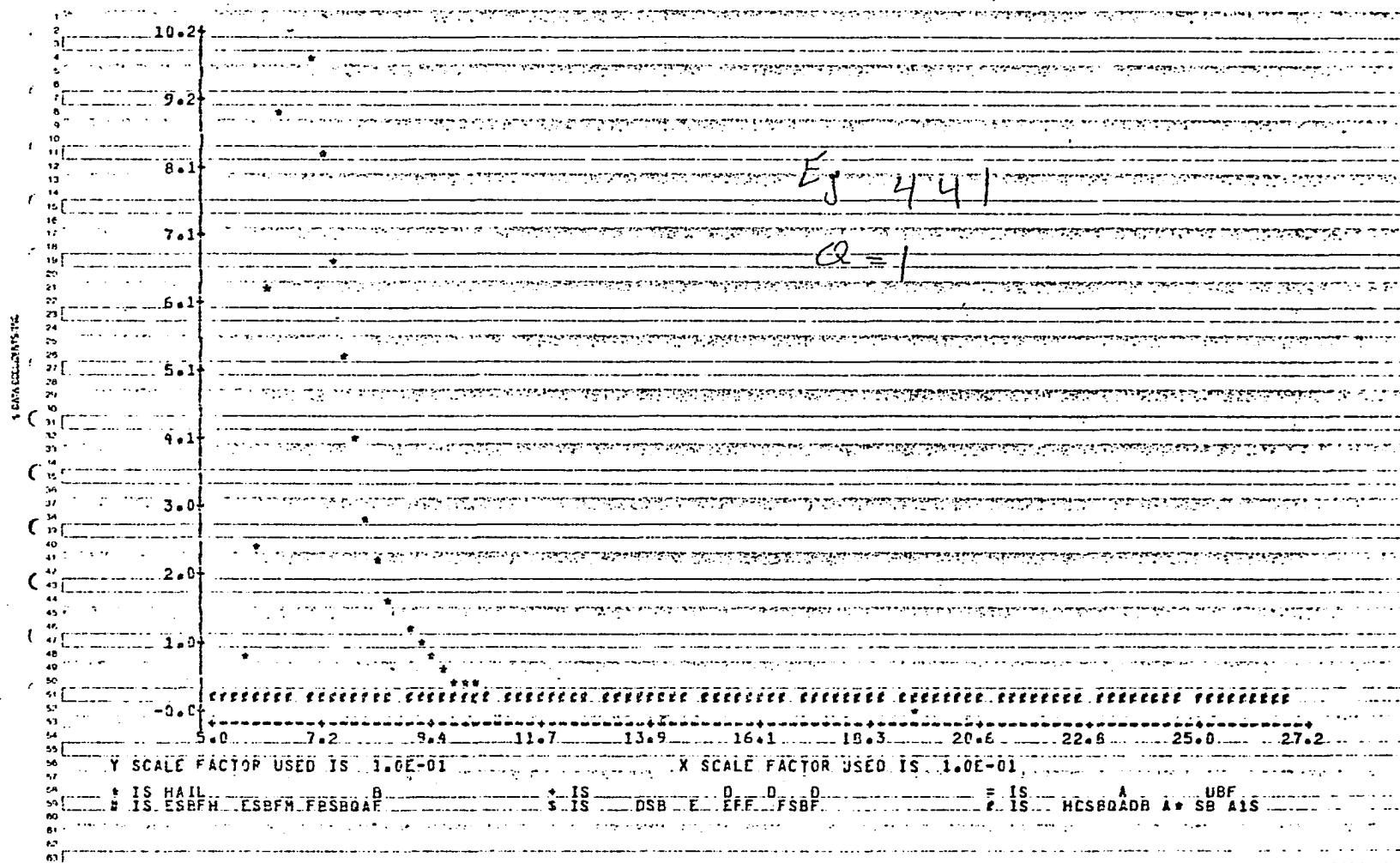
— १ —





Epi

Q = 1



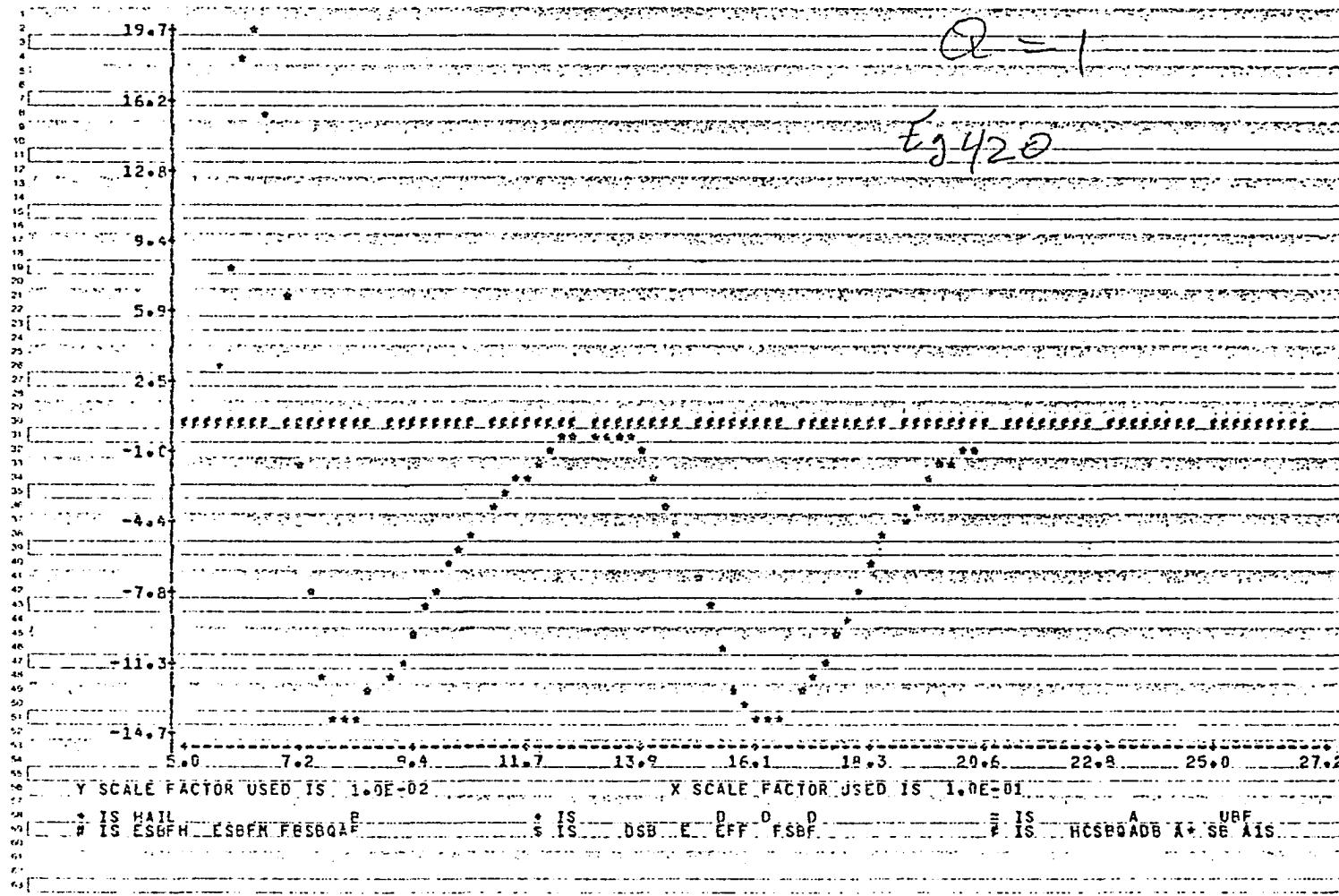
四

二  
三

63  
64

145

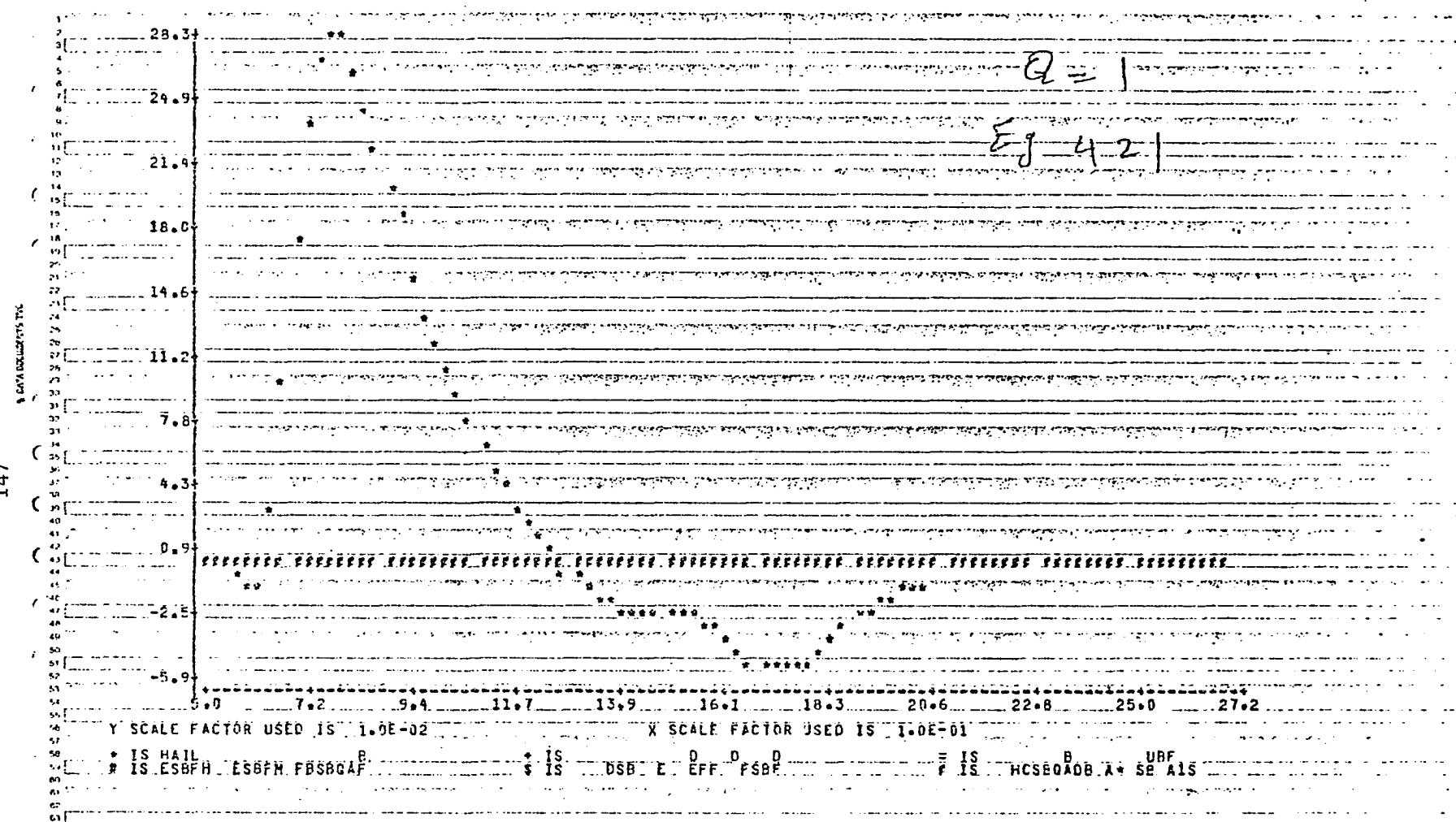
DATAACQUISITION INC.



卷之二

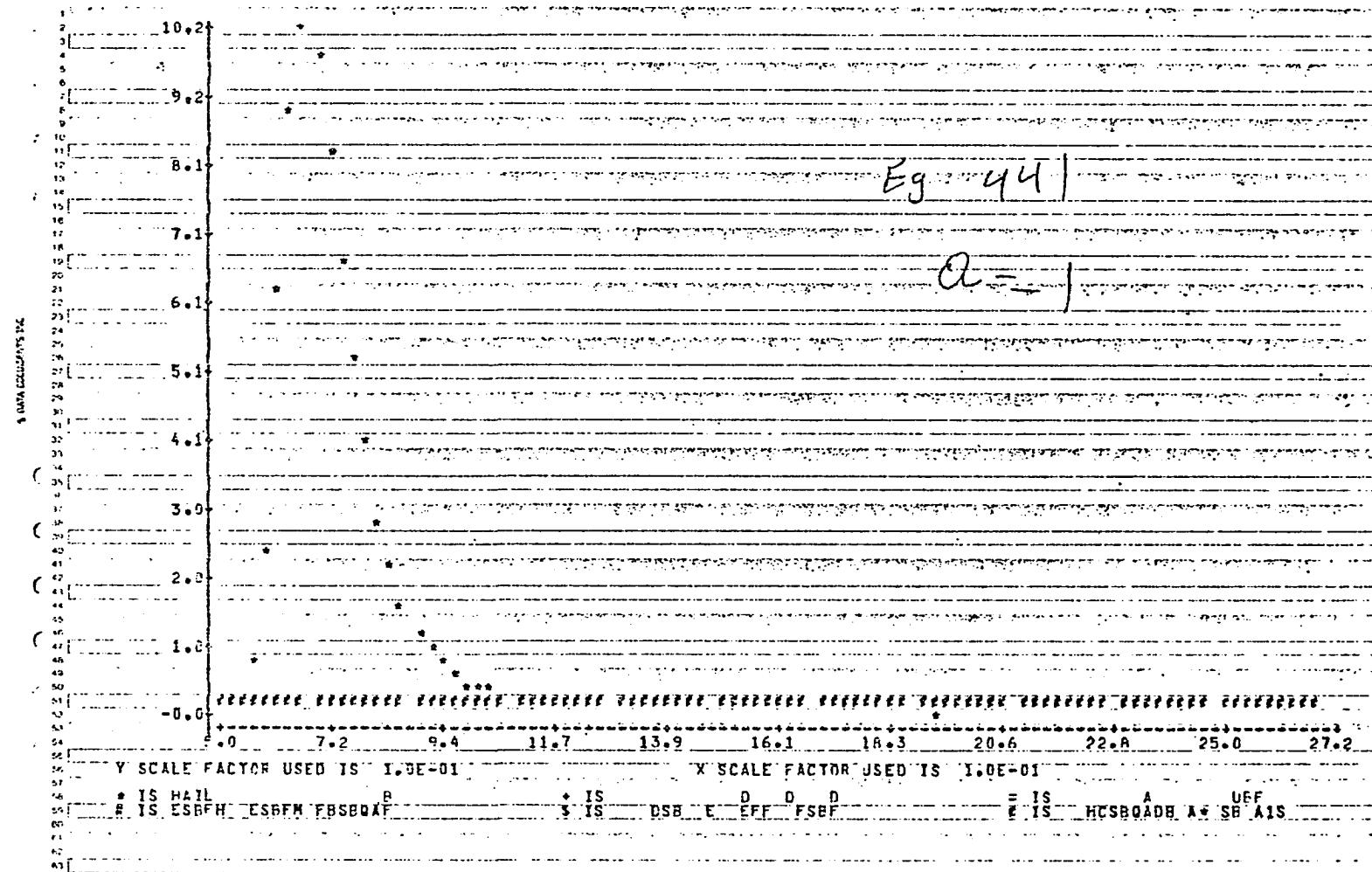
卷之二

147



5

$$-Q_2 = 1$$



## CONCLUSION

In this research we were looking at the first order perturbation. We looked at two cases of liquid states, one at weak quadrupole moment and the other at high. For low Q we got reasonable results. For high Q the first order perturbation appears to be inadequate.

We looked at Eg 220 and Eg 222 compared with simulation. The comparison in each case was fairly good, Eg 221 did not come as close; our results show more magnification than the simulation does.

Our theory can predict the SPH coefficients, at least qualitatively. The Lennard-Jones reference was found to be good for a low Q ( $Q \leq .5$ ) polar fluid, for high quadrupole fluid we need to use the second order theory or other references.

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