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OPTIMIZATION TECHNIQUES FOR A MULTI-DIMENSIONAL DRILLING MODEL

The University of Oklahoma PH.D. 1980

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OPTIMIZATION TECHNIQUES FOR A MULTI-DIMENSIONAL DRILLING MODEL

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OPTIMIZATION TECHNIQUES FOR A MULTI-DIMENSIONAL

DRILLING MODEL

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ABSTRACT

A new non-linear, unconstrained, mathematical model for the drilling process has been developed. The number of parameters to optimize are six, namely, the weight on bit, the rotary speed, the drilling fluid volumetric flow rate, the jet nozzle diameter, and the drilling fluid density and viscosity.

The method of Hooke and Jeeves using Fibonacci line search has been used to solve the non-linear, multi-variable, unconstrained, objective function. A comparison between the Hooke and Jeeves searching method and other optimization techniques, such as discrete Hooke and Jeeves and Rosenbrock methods is given. Also the effects of changing one decision variable on the drilling cost are studied for all the six variables.

It is concluded that the new developed drilling model gives better results than the Galle and Woods model. Therefore, the drilling companies could save thousands of dollars on one well and perhaps millions on one field. The optimum solution, using the accelerated Hooke and Jeeves search method, is more economical and realistic than solutions produced by other optimization techniques.

CHAPTER 1

INTRODUCTION

There are several factors which relate directly and indirectly to the drilling rate and cost. Some of these parameters can be altered and some others cannot. The alterable factors that may be controlled are: drilling fluid properties, hydraulics, bit type and size, weight on bit, and rotary speed. The unalterable factors are: weather and location, rig flexibility, bottom hole temperature, round-trip time, rock properties, depth, formation to be drilled, characteristic hole problems, and crew efficiency.

Drilling optimization is the technique which is used to minimize the cost of drilling. In past studies, only two variables were considered in developing the mathematical models. These variables are weight on bit and rotary speed. Other factors were considered to be well chosen.

In this work, other factors like drilling fluid properties, hydraulics, bit type and size, formation to be drilled, and differential pressure between drilling fluid column pressure and formation pore pressure are considered in the development of the new mathematical drilling model.

CHAPTER 2

LITERATURE REVIEW

In 1969, Lummus [31] outlined all the factors to be considered in drilling optimization. He classified the drilling variables as alterable or unalterable. The alterable variables are:

1. Drilling fluid: (a) drilling fluid weight, (b) solids content, (c) viscosity, (d) fluid loss, (e) drilling fluid type
2. Hydraulics: (a) pump pressure, (b) jet velocity, (c) circulating rate, and (d) annular velocity
3. Bit type
4. Weight on-bit
5. Rotary speed

The unalterable variables are:

1. Weather
2. Location
3. Rig conditions
4. Corrosive bore-hole gases
5. Bottom-hole temperature
6. Round-trip time
7. Rock properties

8. Characteristic hole problems
9. Water available
10. Formation to be drilled
11. Crew efficiency
12. Depth

In considering which variables to choose for mathematical optimization, Lummus said that experience and research suggest six variables: four alterable and two unalterable. These are:

1. Drilling fluid
2. Hydraulics
3. Bit type and size
4. Weight on bit-rotary speed
5. Formation to be drilled
6. Depth

Each of the above variables have been studied extensively in the field and in the laboratory.

Drilling Fluid:

The properties of the circulating fluid that have been found to affect drilling rate are as follows:

Drilling Fluid Weight:

It has been shown [41] that drilling rate decreases as the drilling fluid pressure increases, and that the decrease is actually more correctly attributed to the excess of the hydrostatic pressure over the formation pressure [13]. The reason for the decrease is thought to be due to the fact that compression of the rock makes it harder for the bit to break up the rock. In order to keep the penetration rate at a ressonable level, the pressure and hence, the drilling fluid weight, should be kept as low as possible while allowing for the highest formation pressure to be encountered.

Solid Content:

Eckel [15] pointed out both solids type and amount affect viscosity and reduce drilling rate. He stated that solids content does not independently affect drilling rate. Excluding pure water, the best drilling fluid is a non-dispersed fluid having a total clay-solid content of no more than 4% and having drilled solids to bentonite ratio of less than 2:1. Laboratory drilling tests [31, 34] showed that the particle size, as well as total colloidal size, has an important effect on drilling efficiency.

Lummus [32] stated that air or gas is a higher penetration rate drilling fluid than water or oil. He showed that as the percentage of clay increases, the penetration rate is reduced. This effect is not totally dependent upon the total solid content of the water, due to the nature of the particle size distribution of the solids making up the drilling fluid. It was found that colloidal size particles which are less than 1μ in size, have 12 times more effect on drilling rate than particles coarser than 1μ.

Drilling Fluid Viscosity:

Moore [35] pointed out that an increased drilling fluid viscosity results in slower rate of penetration. In the case of normal drilling fluids, the solids content and viscosity are strongly interdependent and hence, it might be thought that the viscosity was not an independent parameter. Also high viscosity could be achieved by using viscous glycerine. He suggested that the high viscosity resulted in lower fluid velocity in the vicinity of the cuttings at the bottom of the hole and hence, the cuttings are not removed efficiently.

Eckel [15] studied the effect of viscosity on drilling rate using several fluids and found that the drilling rate decreases as the kinematic viscosity of the fluid increases.

Eckel [16] showed that in order to minimize the reduction in the drilling rate, the viscosity should be raised by increasing the yield point (Y_p) to plastic viscosity (P_v) ratio. His work illustrates the effect of shear rate on viscosity for different drilling fluids all with the same apparent viscosity; the high shear-rate viscosity reduces drilling rate.

Murphy [38] also showed the effect of viscosity on drilling rate. He indicated that when the viscosity exceeds 40 centipoise, an additional increase will have very little effect on drilling rate.

Fluid Loss:

Drillers noted that drilling rates were decreased when the filtration rates were decreased. Moore [36] stated that this reduction in drilling rate was due more because of the materials added to reduce filtration rate than because of the filteration rate reduction.

When low solids drilling fluids were introduced it was not uncommon to have an API filtration rate of 10 cubic centimeters and a high initial loss of fluid, called spurt loss. The spurt loss was simply the loss of fluid necessary to form a filter cake of solids. As a result, drilling rates with the low solids drilling fluid with an API water loss of 10 cubic centimeters may be substantially higher than a higher solids drilling fluid having an API water loss of 20 cubic centimeters.

Oil Content:

Moore [35] said that shale drills much more quickly when oil is added to the drilling fluid. This effect probably results from the oil preventing "balling-up" of the bit. The economic advantage of adding oil is not always clear. The loss rate must be strictly controlled. Otherwise, the cost of the oil used may well be higher than the cost of saving it.

Murphy [38] explained the change in drilling rate to additional amounts of oil to a drilling fluid system. The drilling rate increases as the oil percentage increases.

The reason for this increase has been assumed to be better bit cleaning while drilling shales. Murphy said that the amount of oil required for a particular drilling fluid system will vary widely; he suggested that a maximum of 3% oil will give best results.

Hydraulics:

The circulating medium does not destroy rock, it clears away the rock destroyed by the bit. In accomplishing this, its functions are: (1) to remove the cuttings from the bottom of the hole rapidly to prevent recuttings, (2) to clean the cutters so that the teeth are free to penetrate the rock, and (3) to carry the cuttings away from the bit so as not to interfere with bit life [47]. Consequently, there is a critical hydraulic horsepower for each weight on bit in a specific formation which provides adequate bottom-hole scavaging for maximum efficiency [47]. Speer [47] concluded that penetration rate under any specific condition varies linearly with pump hydraulic horsepower. Cleaning of the hole is primarily a function of circulating volume. An understanding of annular rising velocity, total circulating rate, and nozzle fluid velocity is needed to know the cleaning action.

Bit hydraulic horsepower is a function of the circulation rate and the pressure drop across the bit which is proportional to the nozzle fluid velocity. Circulation rate must be considered as the upward fluid velocity has to exceed the cutting's slip velocity so that the cuttings are removed from the hole [24].

Edwards [17] mentioned that hydraulics programs are designed for a given rig to give maximum bit horsepower or impact force with the available pump input horsepower at maximum surface pressure. The thought is that since actual requirements are unknown, the more horsepower the better. Edwards concluded that at low weight, the penetration rate is the same for all horsepower levels. At the lowest level, bit balling is evident as weight is increased. The intermediate level allows the drilling rate to increase with higher weights and the high level of hydraulics to a still higher weight.

In their works, Eckel [15] and Walker [49], studied the combined effect of viscosity and hydraulics. They found that the Reynold's number controlled the combined effect of fluid properties and hydraulics on rate of penetration. For many tests they found that any change that increased the Reynold's number caused a corresponding increase in the drilling rate. Eckel [16] suggested that, rate of penetration—other conditions being constant, can be described as an exponential function of a Reynold's number.

Walker [49] pointed out that the viscosity is calculated at the shear rate which is directly related to the fluid velocity through the nozzle and inversely related to the nozzle diameter.

Murphy [38] presented two approaches to optimize hydraulic conditions. One method is to provide maximum hydraulic horsepower at the bit. The basis for this is that maximum hydraulic horsepower will produce maximum work and do a better cleaning. The other approach is to maximize cross flow which is the product of flow rate and nozzle velocity.

Lummus [31] mentioned that optimum hydraulics is the proper balance of the hydraulic elements that will adequately clean the bit and bore hole with minimum horsepower. The elements are flow rate, pump pressure, the flow rate-pump horsepower relationship, and the drilling fluid. These elements have to work in the proper ratios to achieve optimum hydraulics.

Lummus said that a successful hydraulics programs can be prepared by first considering two factors: bit cleaning and hole cleaning. Adequate jet velocity and fluid impact toward the formation are required for bit cleaning. The most important aspect of hole cleaning is having a mud with sufficient yield value to lift cuttings from the hole. An adequate annular velocity depends upon hole size and the yield value of the mud system. These values should be adjusted together to keep: (a) the yields value as low as possible to facilitate settling of small cuttings in the surface pits, (b) annular velocity and cutting transport rate reasonably close in value, and (c) annular flow pattern neither in extreme turbulence nor in total plug flow.

Bit Type:

The bit selection is, naturally, very important and depends mainly on the degree of hardness of the formation being drilled and the mode of failure of the rock formation.

Type of bit refers to: (1) number and length of teeth; (2) number of cutter elements; and (3) circulation pattern (jet or regular bit). Selection of tooth style depends primarily on the type of formation to be penetrated. It has been shown that the three-cone is the best overall choice of bit type [47].

The jet-bit drilling rate is used as the unit of formation hardness. Speer [47] concluded that: (1) jet bits perform appreciably better than regular bits in very hard formations; (2) little advantage is obtained with jet bits in the medium-hard formations; and (3) the jet bit's advantages increases with softness of formations from medium hard to the very soft. A comprehensive bit-correlation chart, continually updated to include new bits is the starting point in selecting the proper bits for drilling a well.

Lummus [31, 32] said it is also important for the engineer to have both qualitative and quantitative descriptions of bit wear from at least two nearby control wells in order to do a good job of selecting bits for the proposed well.

Weight-RPM:

The effects of weight and speed have been extensively investigated but it would appear from the variety of expressions that have been suggested that the process involved is complicated and requires careful analysis.

Bielstein and Cannon [5] were among the early investigators and they noted that drilling rate appeared to vary approximately linearly with weight and somewhat less than linearly with rotary speed.

Moore [35] suggested an analytical method for finding the optimum weight and speed. He suggested that the drilling rate is directly proportional to the weight on bit and to the rotary speed. Bit life limited by tooth wear was considered but no formula was given so that optimization under this limiting condition was not considered. He suggested that the bearing life is inversely proportional to the weight on bit and the rotary speed.

All the early investigations showed that the drilling rate is related to the weight on bit and the rotary speed through a special function. Maurer [33] suggested that the function is equal to the rotary speed times the square of the weight on bit with the instantaneous removal of all the cuttings. Also, the drilling rate is inversely proportional to the square of the hole diameter and the square of the formation drillability strength. In this way Maurer related penetration rate to weight per inch of bit diameter and made the formation drillability constant independent of bit size.

Outmans [42] derived a drilling rate equation which described the rate in terms of weight, rotary speed and hydraulic horsepower at the bit. The resulting equation contains several unknown constants which have to be estimated from previous experience for wells drilled in the same area. The simplifications that arise when weight or rotary speed is the only variable give a general insight into field results. He pointed out the danger in examining one variable at a time as appears to have been done by most experimentalists.

Cunningham [11] noted that, even when a homogeneous formation was being drilled, the proportionality constant of the drilling rate equation appeared to vary with the age of the bit.

Galle and Woods [21] introduced the concept of bit dullness. They said that the drilling rate is proportional to the weight on bit-rotary speed and inversely proportional to the bit tooth dullness. They also mentioned that the dullness rate is proportional to the weight-rotary speed and to the percentage wear in the tooth of the bit. Finally, they developed the bearing rate equation which is a function of the weight on bit and the rotary speed along with the fluid condition. They made the following assumptions: (1) diamond bits are excluded from their analysis, (2) bit life is limited by bearing failure or tooth wear or a combination of these factors, (3) circulating hydraulics are adequate and do not limit drilling rate, (4) the drilling rate is a function of only bit weight, rotary speed and degree of tooth dullness, and (5) within the range of rotary speed specified there are no restrictions brought about a prime-mover performance.

Edwards [17] gave another expression for the drilling rate in any interval. He related the drilling rate is related to: the drillability constant which is determined by the formation bit type and mud properties, the weight on bit, the rotary speed and inversely related to the rate function of tooth height. The rate of tooth wear suggested by Edwards is a function of weight, rotary speed, and bit condition.

In 1972, Wilson and Bentsen [55] investigated various optimization procedures which could be used, in conjunction with the selected mathematical model, to achieve the reduction of the drilling cost of a well. They restricted the number of parameters to be optimized to the weight on bit and the rotary speed. Other factors, such as mud properties and bit type, were assumed to have been properly selected. Within these limits, they developed three methods of varying complexity. The first method seeks to minimize the cost per foot drilled during a bit run. The second method minimizes the cost of a selected interval, and the third method minimizes the cost over a series of intervals.

Reed [44] used a Monte Carlo approach for variable weight on bit-rotary speed optimal drilling problems to get the least cost per foot. He concluded that variable weight-speed optimization offers very little advantage over the simpler constant weight-speed.

Cunningham [12] suggested an empirical equation for the drilling rate. He included in his equation the effect of the drilling strength of the formation and the differential pressure between the drilling fluid and formation pressure at the bit on the drilling rate along with the effect of weight on bit and rotary speed.

CHAPTER 3

A DRILLING MODEL USED BY INDUSTRY

3.1 Introduction

There are several drilling models which have been developed during the past twenty years. The Galle and Woods [21] model has been considered the best available model by the industry. Most drilling companies use this model to optimize their drilling programs.

The main disadvantage of this model is that the drilling rate is a function of bit weight, rotary speed and bit condition only. The differential pressure at the bit, fluid properties and circulating are all assumed adequate and do not limit the drilling rate. Another disadvantage of the Galle and Woods model is that diamond bits are excluded from their analysis.

The aim of this chapter is to show the development of the Galle and Woods model and how to use it to optimize the drilling operations. In the later chapters the development of a new mathematical drilling model, the procedures to optimize drilling operations and the comparisons with the Galle and Woods model will be shown.

3.2 Development of the Model

Galle and Woods [21, 22] gave the following basic equations:

Drilling rate:

$$\frac{dF}{dT} = C_f \frac{W^k N}{a^P} \tag{3-1}$$

Rate of tooth drilling:

$$\frac{dD}{dT} = \left(\frac{1}{A_f}\right) \frac{R}{am} \tag{3-2}$$

Rate of bearing wear:

$$\frac{dB_x}{dT} = \frac{N}{SL} \tag{3-3}$$

where,

F = distance drilled by bit, ft

T = rotating time, hours

C_f = formation drillability factor

W = bit weight, 1000 lb

H = bit or hole diameter, inches

$\bar{W}$ = equivalent 7.875" bit weight = $\frac{7.875\ W}{H}$

K = weight exponent

N = rotary speed, rpm

D = normalized tooth wear

$a = 0.928125\ D^2 + 6.0\ D + 1$

$$P = \begin{cases} 1.0 \text{ for flat-crested wear} \\ 0.5 \text{ for self-sharpening wear} \\ 0.0 \text{ for button bits} \end{cases}$$

A_f = formation abrasiveness factor

R = function which relates rotary speed to the rate of tooth wear

m = function which relates bit weight to the rate of tooth wear

S = drilling fluid factor

L = function which relates bit weight to the rate of bearing wear

$= 21340./(1 + 0.03\,\bar{w})^{3.23}$

B_x = fraction of total life expended

The form of the functions in (3-1) is such that the drilling rate increases as higher weights and rotary speeds are applied, and decreases as the bit dulls. The drilling rate is also proportional to the formation drillability parameter C_f which includes the effects of bit type, hydraulics, drilling fluids and the formation. Formations of a very soft nature, for which the penetration rate is not a linear function of weight, are covered, to a certain extent, by the use of the exponent k. The value of 0.6 for k was found to be the best compromise in these cases [24].

From (3-2), the rate of dulling increases as higher weights and rotary speeds are applied. The rate of dulling decreases as the dullness increases. This happens because of the conical shape of the bit tooth results in a larger area being available as the tooth wears. As in the drilling rate equation, where C_f was affected by down-hole conditions, A_f

in this equation includes the effect of bit type, hydraulics, drilling fluid and formation and will be altered if any of these factors are adjusted.

From (3-3), the rate of bearing wear decreases as greater weight or speed is used. Again it should be noted that the parameter S is a function of the bit type, mud and hydraulics, and will change if the hole conditions or bit type are altered. In the case of sealed-bearing type of bit the parameter for a particular bit type should always be the same. In practice, however, some variation may be expected because the bit action, and hence the forces on the bearing is affected by down-hole conditions.

Galle and Woods also defined several functions:

$$U = 714.19 \int_0^D a \, dD \tag{3-4}$$

$$V = 714.19 \int_0^D a^{\frac{1}{2}} \, dD \tag{3-5}$$

$$a = 0.928125 \, D^2 + 6.0 \, D + 1.0 \tag{3-6}$$

$$R = N + 0.00004348 \, N^3 \tag{3-7}$$

$$m = 1359.1 - 714.19 \log \bar{w} \tag{3-8}$$

For calculation purposes, all functions of bit-weight are normalized to a 7 7/8 inch bit size by:

$$\bar{w} = \frac{7.875 \, W}{H} \tag{3-9}$$

$$\bar{m} = \frac{m}{714.19} \tag{3-10}$$

$$L = \text{tabluated function of } \bar{w}$$
$$= 21340./(1 + 0.03\,\bar{w})^{3.23}$$

Solving (3-1) and (3-2) for time and equating:

$$dT = \frac{a^P}{C_f W^k N}\, dF$$

$$dT = A_f \frac{am}{R}\, dD$$

and $$\frac{dF}{dD} = \frac{C_f A_f W^k Nm}{R}\, a^{(1-p)}$$

By normalizing for 7 7/8 inch bit size and solving for F:

$$F = \frac{C_f A_f (\bar{w})^k N\bar{m}}{R} \left(714.19 \int_0^D a^{(1-p)}\, dD \right)$$

By setting the portion in brackets equal to Z (which will be a function that relates tooth dullness D to tooth life as a function of tooth wear type P, the equation for footage becomes

$$\bar{F} = C_f \frac{A_f (\bar{w})^k N\bar{m}}{R} Z \qquad (3\text{-}11)$$

Since Z is a function of a and P, Z will vary as P varies:

$$Z = \begin{cases} 714.19 \int_0^D dD = 714.19D, & \text{when } P = 1.0 \\ 714.19 \int_0^D a^{\frac{1}{2}} dD = V, & \text{when } P = 0.5 \\ 714.19 \int_0^D a\, dD = U, & \text{when } P = 0.0 \end{cases}$$

From (3-2) by normalizing to 7 7/8 inch bit size and solving for T:

$$T = A_f \frac{\bar{m}U}{R} \tag{3-12}$$

Also, rotating time, T, can be described as a function of bearing wear from equation (3-3):

$$T = \frac{SLB_x}{N} \tag{3-13}$$

Equations (3-11), (3-12), and (3-13) can then be rearranged to solve for the three formation parameters which control bit life:

Drillability Factor

$$D_f = C_f\ A_f = \frac{FR}{\bar{m}(\bar{w})^k NZ} \tag{3-14}$$

Abrasiveness Factor

$$A_f = \frac{TR}{\bar{m}U} \tag{3-15}$$

Bearing Wear Factor

$$B_f = S = \frac{TN}{B_x L} \tag{3-16}$$

When the rotary speed function, N, is being replaced by a function N^r, where r describes the effect of rotary speed on penetration rate.

Thus, (3-14) will be:

$$D_f = C_f\ A_f = \frac{FR}{\bar{m}(\bar{w})^k (N^r) Z} \tag{3-17}$$

where, Z relates bit tooth dullness to bit tooth life and is a function of $a^{(1-P)}$

The basic cost equation is given by:

$$\text{Cost/Foot} = \frac{C_B + C_R\ (T_t + T)}{F} \qquad (3\text{-}18)$$

where,

C_B = Bit cost, \$

C_R = Rig cost, \$/hr

T_t = Trip time, hr

T = Rotating time, hr

F = Footage, ft

Since the basic cost equation is primarily a dT/dF function, equations (3-15), (3-16), and (3-17) relating the function of weight, speed and bit condition to formation factors are rewritten so as to allow time and footage to be calculated by:

$$T = A_f\ U\bar{m}/R \qquad (3\text{-}19)$$

$$T = B_f\ B_x\ L/N \qquad (3\text{-}20)$$

$$F = D_f\ Z(\bar{w})^k\ N^r\ \bar{m}/R \qquad (3\text{-}21)$$

It is now possible to expand the cost equation to show cost/foot in terms of all variables by:

$$\text{cost/foot} = \frac{C_B + C_R\ (T_t + A_f\ U\bar{m}/R)}{D_f\ Z(\bar{w})^k\ (N^r)\bar{m}/R} \qquad (3\text{-}22)$$

3.2.1 Calculation Procedure (Tooth Type Bits)

Using (3-22), it is possible to calculate a cost per foot for a given weight and speed, providing values can be determined for the various functions. Of the many functions, several have known values for a given well. Therefore, in the cost equation, only the parameters, U, Z, $\bar{m}$, R $\bar{w}$ and N are not known.

Since $\bar{m}$ and R are functions of weight and speed respectively, it follows that if U and Z can be determined, a cost/foot for any weight-speed combination can be calculated. A value for U can be found from (3-19) and (3-20).

$$U = B_x B_f L R/A_f \bar{m} N \tag{3-23}$$

U can be determined for a given weight-speed combination by assuming a value for B_x (bearing wear), along with values of B_f and A_f from previous drilling data. Since U and Z are both functions of tooth dullness D, once U has been calculated, the corresponding Z can be determined.

Once U and Z have been established for a given weight-speed combination, the calculation of the related cost/foot is simply one of arithmetic. By repeating this calculation for a series of weight-speed combinations, a cost grid can be developed. The weight-speed combination which corresponds to the lowest cost/food is the optimum weight and speed for the interval in question.

3.2.2 Calculation Procedure (Insert-Type Bits)

For tungsten carbide insert-type bits, a somewhat different situation exists. Since the tungsten carbide inserts do not wear appreciably, the tooth dullness factor, D, remains constant, and bit life is determined only by bearing life. This eliminates U and Z in the equation and the cost per foot formula becomes:

$$\text{Cost/foot} = \frac{C_B + C_R\ (T_t + B_f\ L\ B_x/N)}{C_f\ \bar{w}^k\ N^r\ B_f\ L\ B_x} \tag{3-24}$$

Since insert bits do not experience noticeable tooth wear, and, therefore are never pulled for tooth wear, (3-24) can be solved for any pre-selected value of bearing wear on which it might be desired to pull the bit. The procedure for determining the optimum weight-speed is the same as with tooth type bits, i.e., solving (3-24) for a range of weights and speeds.

Tables 1-6 in Appendix D simplify calculations necessary to compute equations (3-15), (3-16), (3-17), (3-22), and (3-24).

The first step in determining an optimum speed schedule is to develop required input data. Table (3-1) illustrates the input data that is required to complete the calculation. Basically, the data can be split into two separate categories, namely operational information and formation parameters.

Table 3-1

REQUIRED INPUT DATA

Optimized Drilling Program

Operational Information

Bit Cost	C_B	$/Bit
Rig Cost	C_R	$/Hour
Trip Time Factor	t	Hours/Foot
Bit Size	d	Inches
Minimum Bit Weight	W_{min}	Pounds
Maximum Bit Weight	W_{max}	Pounds
Minimum Rotary Speed	N_{min}	RPM
Maximum Rotary Speed	N_{max}	RPM

Drilling Parameters

Formation Abrasiveness	A_f
Formation Drillability	D_f
Bearing Wear Factor	B_f
Tooth Wear Factor	P
Weight Exponent	K
Speed Exponent	r

CHAPTER 4

AN OPTIMIZATION DRILLING MODEL

4.1 Introduction

In past researches, only two variables were considered in developing optimization models. These variables are weight on bit and rotary speed. Other factors were considered to be well chosen. One of these models was presented by Galle and Woods [21].

In addition to the weight on bit and rotary speed, other factors like mud properties (i.e., density and viscosity), hydraulics, bit type and size, formation to be drilled, and differential pressure are included in the new drilling model. These variables make the model more practical and realistic. Development of this drilling model will be shown in this chapter, which consists of two cases. The first case is the bearing-wear limited and the other one is the tooth-wear limited.

4.2 Model Development

The mathematical drilling model is a series of interrelated equations which accept relevant drilling variables and realistically predict, among other factors, drilling rate and cost.

The model referred to in this work consists basically of three relations:

1. Rate of penetration equation,
2. Rate of dulling equation, and
3. Bearing life equation.

Rate of penetration is determined by:

1. Weight on bit
2. Rotary speed
3. Bit type and size
4. Nozzles size and number
5. Drilling mud density
6. Drilling mud viscosity
7. Volumetric flow rate
8. Depth
9. Formation drillability
10. Tooth dullness, which varies during the bit run and has its own equation.

The drilling rate equation presented by Young [54] is:

$$\frac{dF}{dT} = \frac{C_f\ W\ N^z}{(1 + C_2 D)} \tag{4-1}$$

In order to derive the drilling rate equation the following parameters are defined:

q = volumetric flow rate, gallon/min

ρ = drilling fluid density, lb/gal.

d_n = jet-nozzle diameter, inch

μ = drilling fluid viscosity, C.p.

P_m = drilling fluid pressure, psi

P_f = formation pressure, psi

ΔP = differential pressure, 10^3 psi

$= P_m - P_f$

$$= 0.052(\rho)(\text{depth}) - P_f \tag{4-2}$$

$$\text{Reynold's number } (R_e) = \frac{kq\rho}{d_n \mu} \tag{4-3}$$

(i) For bit with one Jet-nozzle;

$$R_e = 379.11 \frac{q\rho}{d_n \mu} \tag{4-4}$$

(ii) For bit with two-equal jet nozzle;

$$R_e = 189.56 \frac{q\rho}{d_n \mu} \tag{4-5}$$

(iii) For bit with three-equal jet nozzles;

$$R_e = 126.37 \frac{q\rho}{d_n \mu} \tag{4-6}$$

(iv) For bit with three-unequal size jet nozzles;

$$R_e = 379.11 \frac{q \quad \rho}{(d_{n1} + d_{n2} + d_{n3})} \tag{4-6a}$$

The differential pressure at the bit ΔP is inversely proportional to the drilling rate [11, 12, 13]. The combined effect of drilling fluid properties and hydraulics relate to the drilling rate through a Reynold's number function [15, 16, 49].

By including the effects of differential pressure, drilling fluid properties, and hydraulics into (4-1), the new drilling rate equation is:

$$\frac{dF}{dT} = \frac{C_f\ W^y N^z}{(1 + C_2 D)} \cdot \frac{1}{(1+\Delta p^x)} \cdot \log \frac{K\dot{q}\rho}{d_n \mu} \qquad (4\text{-}7)$$

The rate of dulling is determined by:

(a) Weight on bit

(b) Rotary speed

(c) Bit size and type

(d) Formation abrasiveness

(e) Tooth dullness

Galle and Woods [21] represented the tooth-wear rate by (3-2) which is:

$$\frac{dD}{dT} = \frac{1}{A_f} \cdot \frac{R}{am}$$

where a, R, and m were defined by (3-6), (3-7), and (3-8), respectively.

The bearing life equation includes the following variables:

(a) Bit weight

(b) Rotary speed

(c) Bearing-wear constant, which varies with the drilling fluid composition, solids content, and bit size and type.

Young [54] gave the bearing-wear rate equation which is:

$$\frac{dB}{dT} = \frac{N\ W^{cn}}{b} \tag{4-8}$$

where:

B = Bearing-wear fraction of the bit

cn = weight exponent in the bearing-wear equation

b = bearing-wear constant

The cost per foot for a single bit run is given by the following equation:

$$CPF = \frac{C_B + C_R\ (T_t + T_c + T)}{F} \tag{4-9}$$

where:

C_B = bit cost, \$

C_R = rig cost, \$/hr

T_t = trip time, hr

T_c = connection time, hr

T = rotating time, hr

F = feet drilled, ft

The optimization problem consists of finding the values of the variables that are corresponding to a minimum value for CPF subject to the three constraints. Thus, the problem will be:

Minimize CPF (\$/ft)

Subject to:

$$\frac{dF}{dT} = \frac{C_f\ W^Y N^z}{(1+C_2 D)} \cdot \frac{1}{(1 + \Delta p^x)} \cdot \log\ (\frac{kq\rho}{d_n \mu})$$

$$\frac{dD}{dT} = \frac{1}{A_f}\ (\frac{R}{am})$$

$$\frac{dB}{dT} = \frac{N\ W^{cn}}{b}$$

At the initial condition (T = 0);

tooth-dullness D = 0.0 and bearing-wear B = 0.0

When the drilling time T = T, two cases are considered;

1. Bearing-wear limited; where the bearing completely damaged B = 1.0 and the tooth dullness $D < 1.0$
2. Tooth-wear limited; where the teeth are completely damaged D = 1.0 and the bearing-wear $B < 1.0$

In order to find the value of the drilling time, T, and the total footage, F, in the cost per foot equation (4-9), the previous three differential equations should be integrated and solved simultaneously. Two cases will be considered which are: bearing-wear limited and tooth-wear limited.

4.3 Bearing-Wear Limitation

The life of the bit in this case is limited by the bearing failure. Let B be the independent variable in place of t. Then, two cases will be studied, these are:

4.3.1 A case where the variables N, W, p, q and μ are not constant over the entire bit run:

For n bearing-wear increments, the constants values of the rotary speed, weight on bit, mud density, flow rate, and mud viscosity during the i th increment are: N_i, W_i, ρ_i, q_i and μ_i, respectively.

From (4-8),

$$B = \frac{NW^{cn}}{b} T$$

or

$$T = \frac{bB}{NW^{cn}}$$

The total time per bit run, T, for n bearing-wear increments is:

$$T = \sum_{i=1}^{n} b \frac{\Delta B_i}{N_{i+1} W_{i+1}^{cn}} \tag{4-10}$$

where;

ΔB = change in bit bearing-wear

$\Delta B_i = B_{i+1} - B_i$, $B_1 = 0.0$ and $B_{n+1} = 1.0$ for i=1, ..., n.

From (3-2), (3-6), (3-10) and (4-8);

$$(0.928125\ D^2 + 6.0\ D + 1.0)dD = \left(\frac{1}{714.19}\right) \cdot \frac{1}{A_f} \cdot \frac{R}{\bar{m}} \cdot \frac{b}{NW^{cn}}\ dB$$

Integrate the above equation;

$$0.309375\ D^3 + 3D^2 + D = \frac{R}{714.19\ A_f\bar{m}} \left(\frac{b}{NW^{cn}}\right) B \tag{4-11}$$

Let $G = \frac{A_f\bar{m}}{R}$, and $E = \frac{bB}{714.19\ GNW^{cn}}$

Therefore (4-11) can be written as,

$$0.309375\ D^3 + 3D^2 + D = E \tag{4-12}$$

and at (i + 1) increment;

$$D_{i+1} = \frac{E_{i+1}}{0.309375 D_{i+1}^2 + 3D_{i+1} + 1.0} \qquad (4-13)$$

Equation (4-13) can be solved by trial and error to get the value of the tooth dullness D_{i+1}, which is correspondent to the value of E_{i+1} (for i = 1, 2, . . . , n).

Where;

$$D_1 = 0$$

$$D_{n+1} = D_f = D \text{ when } B = 1.0$$

$$E_{i+1} = \frac{b\, B_{i+1}}{714.19\, G_{i+1}\, N_{i+1}\, W_{i+1}^{cn}} \qquad (4-14)$$

$$E_1 = 0, \text{ since } B_1 = 0$$

$$G_{i+1} = \frac{A_f \bar{m}_{i+1}}{R_{i+1}}$$

$$\bar{m}_{i+1} = \frac{1}{714.19}(1359.1 - 714.19 \text{ Log } \bar{w}_{i+1})$$

$$\bar{w}_{i+1} = \frac{7.875\, W_{i+1}}{H}$$

$$R_{i+1} = N_{i+1} + 4.348 \times 10^{-5}\, N_{i+1}^3$$

Now two cases will be studied which are:

First: When Tooth-Wear Constant, $C_2 \neq 0.0$

From (4-7), (3-2) and (3-10),

$$dF = \frac{C_f W^Y N^Z}{(1+C_2 D)(1+\Delta p^x)} \cdot \log\left(\frac{kq\rho}{d_n \mu}\right) \frac{A_f \bar{m}}{R} (714.19 \text{ a dB}) \qquad (4-15)$$

For simplicity let:

$$(1) \quad M = C_f W^Y N^Z \tag{4-16}$$

$$M_i = M(W_i, N_i)$$

$$(2) \quad Q = \log\left(\frac{kq\rho}{d_n \mu}\right) \tag{4-17}$$

$$Q_i = Q(q_i,\ \rho_i,\ \mu_i)$$

$$(3) \quad V = \frac{1}{1+\Delta p^X} \tag{4-18}$$

$$V_i = V(\rho_i)$$

$$(4) \quad G = \frac{A_f \bar{m}}{R} \tag{4-19}$$

$$G_i = G(W_i,\ N_i)$$

Substitute (4-16, 17, 18, 19) in (4-15);

$$dF = (714.19)\ GMQV\ \frac{a}{1+C_2D}\ dD \tag{4-20}$$

and from (3-6), then

$$dF = 714.19\ GMQV\left(\frac{0.928125\ D^2 + 6.0\ D + 1.0}{1 + C_2D}\right)\ dD \tag{4-21}$$

Integrating (4-21) between two incremental tooth dullness,

$$F = 714.19\ GMQV \left\{ 0.928125\left[\frac{(1+C_2D_{i+1})^2}{2\ C_2^3} - \frac{2\ (1+C_2D_{i+1})}{C_2^3} + \frac{1}{C_2^3}\ \ln (1+C_2D_{i+1})\right] - 0.928125\left[\frac{(1+C_2D_i)^2}{2\ C_2^3} - \frac{2(1+C_2D_i)}{C_2^3}\right.\right.$$

$$+ \frac{1}{C_2^3} \ln(1+C_2D_i)\Big] + 6.0 \left[\frac{D_{i+1}}{C_2} - \frac{1}{C_2^2} \ln (1+C_2D_{i+1})\right]$$

$$- 6.0\left[\frac{D_i}{C_2} - \frac{1}{C_2^2} \ln(1+C_2D_i)\right] + \frac{1}{C_2} \ln (1+C_2D_{i+1})$$

$$- \frac{1}{C_2} \ln (1 + C_2D_i)\Bigg\} \qquad (4\text{-}22)$$

For n bearing-wear increments the total footage will be:

$$F = 714.19 \sum_{i=1}^{n} G_{i+1} M_{i+1} Q_{i+1} V_{i+1} \Bigg\{0.928125\Bigg[\frac{1}{C_2^3} \ln\left(\frac{1+C_2D_{i+1}}{1+C_2D_i}\right) +$$

$$\left(\frac{(1+C_2D_{i+1})^2 - (1+C_2D_i)^2}{2\, C_2^3}\right) - \left(\frac{2(1+C_2D_{i+1})-2(1+C_2D_i)}{C_2^3}\right)\Bigg] +$$

$$6.0 \left[\left(\frac{D_{i+1}-D_i}{C_2}\right) - \frac{1}{C_2^2} \cdot \ln\left(\frac{1+C_2D_{i+1}}{1+C_2D_i}\right)\right] +$$

$$\frac{1}{C_2} \cdot \ln \left(\frac{1+C_2D_{i+1}}{1+C_2D_i}\right)\Bigg\} \qquad (4\text{-}23)$$

Equation (4-23) is the general form of the total footage, F, for bearing-wear limited case when $C_2 \neq 0.0$ and the variables N, W, q, ρ, and μ are changing with time.

Second: When Tooth-Wear Constant, $C_2 = 0.0$

This is a case that implies that the initial drilling rate (new-bit) is equal to the final drilling rate (used bit). This can usually happen for insert bits and diamond bits.

From Eq. (4-7), (4-16), (4-17), and (4-18),

$$dF = MQV \left(\frac{1}{1+C_2D}\right) d\tau$$

For $C_2 = 0$

$$dF = MQV\ dT \tag{4-24}$$

From (3-2) and (3-10), then

$$dT = 714.19\ Ga\ dD \tag{4-25}$$

Substitute (4-25) into (4-24)

$$dF = GMQV\ (714.19\ a\ dD) \tag{4-26}$$

Integrating (4-26)

$$F = 714.19\ GMQV \left(0.309375D^3 + 3D^2 + D\right)_{D_i}^{D_{i+1}} \tag{4-27}$$

For n bearing-wear increments; the total footage will be:

$$F = 714.19 \sum_{i=1}^{n} G_{i+1}\ M_{i+1}\ Q_{i+1}\ V_{i+1} \left[0.309375(D_{i+1}^3 - D_i^3) + 3(D_{i+1}^2 - D_i^2) + \Delta D_i\right] \tag{4-28}$$

<u>4.3.2 A case when all the six variables (N, W, q, ρ, d_n & μ) are held constant over the entire bit run</u>:

For a constant variable (4-10) is written as

$$T = \frac{b}{NW^{cn}} \sum_{i=1}^{n} \Delta B_i$$

but $\sum_{i=1}^{n} \Delta B_i = B_f = 1.0$ for bearing-wear limited and for variables are held constant.

Thus;

$$T = \frac{b}{NW^{cn}} \tag{4-29}$$

From (4-13),

$$D_f = \frac{E_f}{0.309375\ D_f^2 + 3D_f + 1.0} \tag{4-30}$$

Where;

$$D_f = D\ (B = 1.0)$$

$$D_f = \frac{b\ B_f}{714.19\ GNW^{cn}} = \frac{b}{714.19\ GNW^{cn}} \tag{4-31}$$

Therefore;

$$E_f = \frac{T}{714.19G} \tag{4-32}$$

Two cases will be studied which are:

First: When Tooth-Wear Constant, $C_2 \neq 0.0$

From (4-21),

$$dF = 714.19\ GMQV\left(\frac{0.928125D^2 + 6.0D + 1.0}{1 + C_2D}\right)\ dD$$

Integrate the above equation for D from 0.0 to D_f.

The total footage will be:

$$F = 714.19\ GMQV \left\{ 0.928125 \left[\frac{(1+C_2D_f)^2}{2\ C_2^3} - \frac{2(1+C_2D_f)}{C_2^3} + \frac{1}{C_2^3} \ln (1+C_2D_f) + \frac{3}{2\ C_2^3} \right] + 6.0 \left[\frac{D_f}{C_2} - \frac{1}{C_2^2} \ln(1+C_2D_f) \right] \frac{1}{C_2} \ln(1+C_2D_f) \right\} \tag{4-33}$$

Second: When Tooth-Wear Constant, $C_2 = 0.0$

From (4-7), (4-16), (4-17);

$$dF = MQV\left(\frac{1}{1+C_2D}\right)\ dT$$

For $C_2 = 0.0$

$dF = MQV\ dT$

From (3-2) and (3-10), then

$dF = GMQV\ (714.19a)\ dD$

but $a = 0.928125\ D^2 + 6D+1.0$

Integrating for D from 0.0 to D_f.

Therefore the total footage, F is:

$$F = 714.19\ GMQV\ (0.309375\ D_f^3+3D_f^2+D_f) \tag{4-34}$$

4.4 Tooth-Wear Limitation

The life of the bit in this case is limited by the tooth-wearing failure. Let D be the independent variable in place of t. Two cases will be studied, these are:

4.4.1 A case where the variables N, W, ρ, q, and μ are not constant over the entire bit run:

For n tooth-wear increments, the constant value of the rotary speed, weight on bit, mid density, flow rate, and mud viscosity during the i th increment are N_i, W_i, ρ_i, q_i, and μ_i, respectively.

From (3-2), (3-6), and (3.10),

$$(0.928125\ D^2 + 6.0D + 1.0)\ dD = \frac{1}{714.19G}\ dT$$

Integrating the above equation;

$$T = 714.19\ G \left(0.309375\ D^3 + 3.0D + D\right)_{D_i}^{D_{i+1}}$$

At i th increment, where i = 1, ..., n;

$$T_{i+1} = 714.19\ G_{i+1} \left[0.309375\ (D_{i+1}^3-D_i^3) + 3.0(D_{i+1}^2-D_i^2) + (D_{i+1}-D_i)\right] \tag{4-35}$$

For n tooth-wear increments, the total rotating time is;

$$T = \sum_{i=1}^{n} T_{i+1} = 714.19 \sum_{i=1}^{n} G_{i+1} \Big[0.309375 (D_{i+1}^3 - D_i^3)$$

$$+ 3.0 (D_{i+1}^2 - D_i^2) + \Delta D_i \Big] \qquad (4\text{-}36)$$

where;

$D_1 = 0$

$D_{n+1} = D_f = 1.0$

$T_1 = 0$

Integrating (4-8)

$$B_{i+1} = \frac{N_{i+1} W_{i+1}^{cn}}{b} T_{i+1} \qquad (4\text{-}37)$$

where;

$B_1 = 0$

$B_{n+1} = B_f = B \ (D = 1.0)$

Therefore;

$$\text{Final bearing-wear } (B_f) = \frac{1}{b} \sum_{i=1}^{n} N_{i+1} W_{i+1}^{cn} T_{i+1} \qquad (4\text{-}38)$$

Two cases will be studied which are:

<u>First</u>: When Tooth-Wear Constant, $C_2 \neq 0.0$

The total footage is the same as in Section 4.3 (i.e., bearing-wear limitation case), which is given by:

$$F = 714.19 \sum_{i=1}^{n} G_{i+1} M_{i+1} Q_{i+1} V_{i+1} \Bigg\{ 0.928125$$

$$\Bigg[\frac{1}{C_2^3} \ln \left(\frac{1+C_2 D_{i+1}}{1+C_2 D_i} \right) + \left(\frac{(1+C_2 D_{i+1})^2 - (1+C_2 D_i)^2}{2\, C_2^3} \right)$$

$$- \left(\frac{2(1+C_2D_{i+1}) - 2(1+C_2D_i)}{C_2^3}\right)\Bigg] + 6.0\left[\frac{\Delta D_i}{C_2} - \frac{1}{C_2^2}\ln\right.$$

$$\left.\left(\frac{1+C_2D_{i+1}}{1+C_2D_i}\right)\right] + \frac{1}{C_2}\ln\left(\frac{1+C_2D_{i+1}}{1+C_2D_i}\right)\Bigg\} \qquad (4\text{-}39)$$

Second: When Tooth-Wear Constant, $C_2 = 0.0$

When $C_2 = 0.0$, the total footage, F, is the same as in Section 4.3 (i.e.: bearing-wear limitation case), which is given by:

$$F = 714.19 \sum_{i=1}^{n} G_{i+1}\, M_{i+1}\, Q_{i+1}\, V_{i+1} \Big[0.309375(D_{i+1}^3 - D_i^3) + 3.0(D_{i+1}^2 - D_i^2) + \Delta D_i\Big] \qquad (4\text{-}40)$$

4.4.2 A case when all the six variables (N, W, ρ, q, d_n & μ) are held constant over the entire bit run:

From (4-36),

$$T = 714.19G \sum_{i=1}^{n} \Big[0.309375\ (D_{i+1}^3 - D_i^3) + 3.0(D_{i+1}^2 - D_i^2) + \Delta D_i\Big] \qquad (4\text{-}41)$$

but for constant variables over the entire bit run;

$$\sum_{i=1}^{n} \Delta D_i = D_f = 1.0$$

$$\sum_{i=1}^{n} D_{i+1}^2 - D_i^2 = D_f = 1.0$$

$$\sum_{i=1}^{n} D_{i+1}^3 - D_i^3 = D_f = 1.0$$

Thus; the final (total) rotating time, T is:

$$T = 3077.71 \ G \tag{4-42}$$

From (4-38), the final (total) bearing-wear, B_f is:

$$B_f = \frac{1}{b} (NW^{cn}) T \tag{4-43}$$

Two cases will be studied which are:

<u>First</u>: When Tooth Wear Constant, $C_2 \neq 0.0$

From (4-21),

$$dF = GMQV \ 714.19 \ \left(\frac{0.928125 \ D^2 + 6.0 \ D + 1.0}{1 + C_2 D}\right) dD$$

Integrating the above equation between D = 0.0 to D_f in the same way as in Section 4.3. Therefore;

$$F = 714.19 \ GMQV \left\{ 0.928125 \left[\frac{(1+C_2D_f)^2}{2C_2^3} - 2\frac{(1+C_2D_f)}{C_2^3} + \frac{1}{C_2^3} \ln (1+C_2D_f) + \frac{3.0}{2C_2^3} \right] + 6.0 \left[\frac{D_f}{C_2} - \frac{1}{C_2^2} \ln (1+C_2D_f) \right] + \frac{1}{C_2} \ln (1+C_2D_f) \right\}$$

$D_f = 1.0$ for tooth-wear limited case. Therefore;

$$F = 714.19 \ GMQV \left\{ 0.928125 \left[\frac{(1+C_2)^2}{2C_2^3} - 2\frac{(1+C_2)}{C_2^3} + \frac{1}{C_2^3} \ln (1+C_2) + \frac{3.0}{2C_2^3} \right] + 6.0 \left[\frac{1}{C_2} - \frac{1}{C_2^2} \ln (1+C_2) \right] + \frac{1}{C_2} \ln (1+C_2) \right\} \tag{4-44}$$

<u>Second</u>: When Tooth-Wear Constant, $C_2 = 0.0$

From (4-21) for $C_2 = 0.0$,

$$dF = 714.19\ GMQV\ (0.928125\ D^2 + 6.0\ D + 1.0)\ dD$$

Integrating the above equation for D from 0.0 to D_f;

$$F = 714.19\ GMQV\ (0.309375\ D_f^3 + D_f^2 + D_f)$$

$D_f = 1.0$ for tooth-wear limited case. Therefore;

$$F = 3078\ GMQV \qquad (4\text{-}45)$$

Now the optimization problem consists of finding the values of the variables that are corresponding to a minimum value for CPF without constraints. Thus, the problem will be:

$$\text{Minimize} \quad CPF = \frac{C_B + C_R\ (T_t + T_c + T)}{F}$$

Where the variables T and F can be represented by different equations as follows:

For Bearing-wear limitation:

(i) the six parameters are not constant;

T is given by (4-10)

F is given by (4-23) when $C_2 \neq 0.0$

F is given by (4-28) when $C_2 = 0.0$

(ii) the six parameters are held constant;

T is given by (4-29)

F is given by (4-33) when $C_2 \neq 0.0$

F is given by (4-34) when $C_2 = 0.0$

For Tooth-Wear Limitation:

(i) the six parameters are not constant;

T is given by (4-36)

F is given by (4-39) when $C_2 \neq 0.0$

F is given by (4-40) when $C_2 = 0.0$

(ii) the six parameters are held constant;

T is given by (4-42)

F is given by (4-44) when $C_2 \neq 0.0$

F is given by (4-45) when $C_2 = 0.0$

CHAPTER 5

DESCRIPTION OF THE OPTIMIZATION PROCEDURE

5.1 Introduction

There are several techniques available which can be used to find the optimum solution for a certain non-linear problems. The techniques are dependent on the type of the problem itself, the number of decision variables, the degree of complexity of the objective function, and whether the problem is constrained or unconstrained.

The basic concepts of the non-linear programming techniques and the descriptions of the search methods with the emphasis on the Hooke and Jeeves method will be seen in this chapter.

5.2 Basic Concepts

Basically an optimization problem consists of: (1) a decision variables which are the actual field variables to be optimized, (2) an objective function which is a mathematical function involving the decision variable, and (3) a set of constraints which can be represented as an equation or inequalities.

For a non-linear problem, the objective function is a non-linear function of the decision variables and the constraints could be linear or non-linear equations and inequalities. A general example of non-linear problem is:

Optimize f(X)

Subject to

$$H_j\ (X) = 0 \qquad j = 1, 2, \ldots, M$$

$$G_k\ (X) \leq 0 \qquad k = 1, 2, \ldots, \bar{M}$$

$$X = (x_1, x_2, \ldots, x_n)$$

Where f, G, ..., $G_{\bar{M}}$, H_1, ... H_M are functions defined on E^n, X is a subset of E^n. A feasible solution to the non-linear problem is the solution vector X which satisfies all sets of the constraints. The local optimum solution is one which yields to a local minimun (or maximum) value for f (X) and the global optimum solution is the best optimizing solution.

5.3 A Search Method

The mathematical drilling model which has been developed in Chapter 4 is a non-linear programming problem due to the non-linearity of the objective function. The decision variables are: the rotary speed N, weight on bit W, drilling fluid density ρ, volumetric flow rate q, jet nozzle diameter d_n, and fluid viscosity μ. The problem will be to minimize the drilling cost (CPF in $/foot) or to maximize the total footage drilled by the bit F.

The unconstrained optimization techniques using derivatives have been eliminated from this study due to the complexity in deriving the gradient and the Hessian matrix of the objective function.

The method of Hooke and Jeeves [9] has been modified to accommodate an acceleration technique using a Fibonacci line search and have been selected for solving the unconstrained multidimensional non-linear drilling model.

The Fibonacci search algorithm is very effective in dealing with univariate non-linear functions that are assumed to be unimodal. Generally, the univariate search methods can be used in multi-variable optimization through successive perturbations of each decision variable. For an N-variables optimization problem, the procedure is to fix N-1 variables at a selective value, and search over the $N^{\underline{th}}$ decision variable a maximizing (or minimizing) solution is found with respect to that one variable. The procedure is then repeated by choosing one of the original fixed N-1 variables as a decision variable and finding a new optimal solution. The procedure is repeated until no change in any variable will bring about an improvement in the current value of the objective function. The Fibonacci line search method has been explained in Appendix C.

The Hooke-Jeeves method performs two types of search. The first is an exploratory search which serves to establish a direction of improvement, and the second is a pattern search which extracts the current solution vector to another point in the solution space. Figure (5-1) shows the first two iterations of Hooke and Jeeves method. By knowing the starting point X_1, the exploratory search along the coordinate directions produces the point X_2. Then the pattern search along the direction (X_2-X_1) produces the point Y. From point Y exploratory search starting again along the coordinate directions which produce point X_3. The next pattern search is along the direction (X_3-X_2), yielding Y', thus the process is repeated. The coordinate directions are designated by d_1, d_2 . . ., d_n, where n is the number of decision variables. Therefore, we have six coordinate directions (d_1. . . .d_6) according to the mathematical drilling model. The algorithm of the method Hooke and Jeeves using line search is presented in Appendix C.

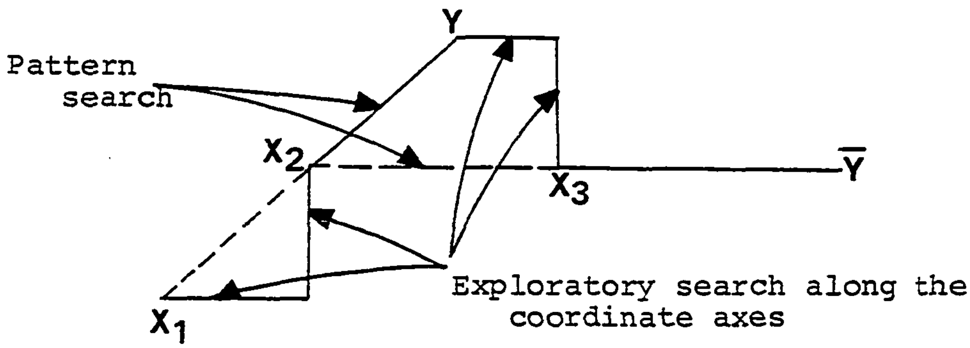


Figure (5-1). Illustration of the Method of Hooke and Jeeves.

CHAPTER 6

APPLICATIONS OF THE OPTIMIZATION MODEL

6.1 Introduction

Mainly, this optimization technique has been designed to be applicable in the oil fields. To verify the applicability of this technique, a set of bit records for different wells in different locations has been provided by the "Security Division-Dresser Industries, Inc." These data are given in Appendix D [Table (D-12)].

Well number 1, which was drilled in Caddo County, Oklahoma, has been chosen to verify the results of this research. All the required information are plainly shown in the Security bit record [Table (D-12)]. The depth of this well is 10,050 feet, which took eleven bit runs to reach.

The types of formations which are encountered during the drilling operations are: soft, medium, hard, and extra hard. The rock bits are classified in the light of the formation's type to be drilled. Table (D-7) shows all the bits types, which are recommended for the corresponding formation type, according to Security company classification.

Table (D-8) explains the Security rock bit comparison chart. This chart is used to find the Hughes, Reed, or Smith rock bits, which are equivalent to the Security rock bits for the different types of formation.

The two major bits classifications are the steel milled tooth rock bits and the tungsten carbide insert bits. Each of these classifications contains several types of rock bits according to the type of formation.

Table (D-9) shows the price of the rock bits, which is released by the Security company, effective May 1, 1979. The price (in U.S. dollars) depends on the type and the size of the bit itself.

In this section, the optimization is performed for well number 1.

The specification of the mud pump, which is given in the Security bit record, is used to find the volumetric flow rate of the drilling fluid (q in gallon/minute). These specifications are: the linear size D, the pump speed N, and the mud pressure. The type of the pump is a duplex one, National of model K-380. Tables (D-10) and (D-11) show the specification of four manufacturer pumps companies. The maximum discharge pressure, the stroke length S, the input horsepower required, and the maximum pump speed can all be determined through tables (D-10) and (D-11) according to the type of the pump. The volumetric flow rate of the drilling fluid in gallon/minute can be calculated through the following equation:

$$q = 0.00679\ SN\ (2D)^2\ e \qquad (6\text{-}1)$$

where;

q = Volumetric flow rate, gal/min

S = Stroke length, inch

N = Pump speed, spm

D = Linear size, inch

e = Volumetric efficiency (commonly taken as 90% for power or 85% for steam pump)

From (6-1) the mud pump, which is used for well no. 1, is operated at volumetric flow rate q = 269 gal/min, while the maximum flow rate attainable from this pump is 362 gal/min.

The drilling cost in dollars per foot can be calculated using (4-9). The cost of the bit C_B can be determined from Table (D-9), while the rig cost C_R can be determined by knowing the daily renting cost of the rig. This daily renting cost is changing with depth, location, contractor company, and the rig facilities required. The approximate daily cost of the rig is reported by the contractor is:

For rig of 7,000 ft depth = 3,600 $/day

For rig of 10,000 ft depth = 4,000 $/day

For rig of 15,000 ft depth = 4,600 $/day

For well no. 1, the rig cost will be 191.667 $/hr. The connection time T_C is usually about 5-8 minute/connection. The average connection length is about 90 feet. Finally the trip time T_t is about 1.5 hours/1,000 feet depth.

6.2 Analysis of Drilling Condition in Practice

Well No. 1 drilled in Caddo County, Oklahoma, is selected to do all the analysis on. All the required information are given in Security bit record [Table (D-12)].

Table (6-1) shows the bit size and type, the footage drilled and time required, the weight on bit, the rotary speed, the mud properties (density and viscosity), the volumetric flow rate, the bit cost, the rig cost, the trip and connection time, and finally the drilling cost for the eleven bit runs (intervals). Usually, the drilling companies use the Galle and Woods model to find the best value for the drilling parameters. As mentioned before, this model considers only the weight on bit and the rotary speed. Therefore, the values of weight on bit W and rotary speed N shown in Table (6-1) are considered to be the best values to give the maximum penetration rate or the minimum drilling cost according to Galle and Woods model.

The type of the bit is selected according to the type of the formation to be drilled. Table (6-2) shows the type of the formation which has been drilled at each interval by the corresponding bit.

The driller kept the same flow rate value of 269 gal/min throughout all the runs. This is his best guess for the necessary flow rate which can clean the bit and the formation beneath it and can also carry out the cuttings up to the surface.

Table 6-1

Well #1 (Field Data)

Run No.	Bit Size, inch	Bit Type	Jets-Size 32" dn_1	dn_2	dn_3	Interval, ft From	To	F, ft	T, hrs	W 1000 lb	N rpm	q gal/min	ρ lb/gal	μ C.P.	Dull Condition 1/8 T	B	G	C_B $	C_R $/hr	T_T hrs	T_C hrs	CPF $/ft	R	Drilling Cost Per Bit Run $
1	12¼	OSC3A	12	12	12	94	1040	946	10	20	120	269	8.33	1.0	2	2	1	1071	191.667	1.56	1.25	3.73	94.6	3,528.80
2	7 7/8	X3A	10	10	10	1040	2692	1652	285	35	"	"	"	"	3	2	1	661	"	4.04	3.24	4.55	57.9	7,516.60
3	"	"	10	10	10	2692	3376	684	16	"	"	"	9.1	10	"	"	"	661	"	5.06	4.06	8.01	42.7	5,478.84
4	"	J22	9	9	10	3376	5631	2255	96.25	"	54	"	"	"	2	SF	"	1845	"	8.45	6.78	10.29	23.4	23,201.95
5	"	F3	11	11	11	5631	7329	1698	114	"	"	"	9.2	"	6	"	1	"	"	10.99	8.82	16.19	14.8	27,490.62
6	"	J33	"	"	"	7329	7921	592	54.75	"	"	"	9.3	"	8	SE	0 1/8	"	"	11.88	9.53	27.77	10.8	16,439.84
7	"	J44	"	"	"	7921	8458	537	56	37.5	"	"	"	"	7	"	1	"	"	12.69	10.18	31.59	9.5	5,101.60
8	"	"	"	"	"	8458	9281	823	90.5	"	"	"	"	13	8	"	0 1/8	"	"	13.92	11.17	29.16	9.1	23,998.68
9	"	F4	"	"	"	9281	9397	116	54.25	"	"	"	"	"	5	"	1	"	"	14.10	11.31	147.53	2.1	17,113.48
10	"	J44	"	"	"	9397	9717	320	52.25	"	"	"	10	15	0	"	1	"	"	14.58	11.70	52.00	6.1	16,896.00
11	"	J55R	"	"	"	9717	10,005	288	56	40	50	"	9.8	"	1	"	1	"	"	15.03	12.04	61.68	5.1	17,761.84

Total drilling cost of well #1 164,532

Table 6-2

Well No. 1; Formation and Bit Types

Run No.	Interval ft	Bit Type	Formation Type	Company
1	1,040	OSC-3A (Milled Tooth) Standard	Soft formation	Hughes
2	2,692	X 3A (Milled Tooth) Sealed bearing	"	"
3	3,376	X 3A (Milled Tooth) Sealed bearing	"	"
4	5,631	J-22 (Insert) Friction-bearing	Medium-soft formation	"
5	7,329	F-3 (Insert) Friction-bearing	"	Smith
6	7,921	J-33 (Insert) Friction-bearing	"	"
7	8,458	J-44 (Insert) Friction-bearing	Medium formation	"
8	9,281	J-44 (Insert) Friction-bearing	"	"
9	9,397	F-4 (Insert) Friction-bearing	"	"
10	9,717	J-44 (Insert) Friction-bearing	"	"
11	10,005	J-55R (Insert) Friction-bearing	"	"

The pressure gradient for an Oklahoma formation is about 0.433 psi/ft. Usually, the drillers use water, which has a density of 8.33 lb/gal, as drilling fluid for the shallow zones. As the depth of the formation increases, the density of the drilling fluid is increased in order to control the down hole formation pressure. From Table (6-1) the density has been increased from fresh water of 8.33 lb/gal for bit runs one and two, to a dense mud of more than 9.0 lb/gal for bit runs 3 through 11.

The trip time and connection time increased as the depth of the formation drilled increases.

The drilling cost in dollars per foot has been calculated for each interval using equation (4-9). These are the best drilling costs the driller can get according to Galle and Woods model in addition to crew experience and efficiency.

The drilling data that are given in Table (6-1) can be used to find the corresponding constants which are used to solve the new non-linear drilling model. Appendix B shows the mathematical procedures to calculate these constants. In the next section the effect of using the optimization algorithm on the drilling parameters of the new model will be given. Also the difference in the drilling rate and drilling cost will be considered.

6.3 Analysis of Drilling Condition in Practice (Optimized)

The method of Hooke and Jeeves using Fibonacci line-search has been used to find the optimum solution for the newly developed non-linear mathematical drilling model which has been derived in Chapter 4. The best combination of the six decision variables leads to the maximum penetration rate and then to the minimum drilling cost.

Table (6-3) shows the necessary input data, which are required to perform the optimization procedure, for the eleven bit runs of well number one. Determination of the constants given in Table (6-3) are explained in Appendix B. Table (6-4) shows all the results of the optimum solution for the eleven bit runs.

The values of the rotary speed N from the optimum solution are higher than the values used in the field by the Security company for the eleven bit runs. This increase in the rotary speed helps the bit to drill faster and gives a higher penetration rate especially in the soft and medium soft formations. However, there are some factors which limit the values of the rotary speed such as: the failure that occurs in the bit (either in the teeth or in the bearing), the other six decision variables, and the size of the rotary table to offer such a speed. The values of the rotary speed from the optimum solution are within the practical and reasonable limits.

The weight on bit should be enough to break down the formation. A very high weight on the bit may cause an early

Table 6-3

Well #1 (Required Input Data)

Run No.	Bit Size	Bit Type	* b	* C_2	* Z	* y	* A_f	* C_f	Depth ft	TT hrs	TC hrs	CR $/hr	CB $
1	12¼	OSC3A	429,325	1.412	0.587	0.906	7.772	0.087	1,040	1.56	1.25	191.667	1,071
2	7 7/8	X3A	2,832,619	1.34	0.696	1.259	26.794	0.0059	2,692	4.04	3.24	"	661
3	"	X3A	1,590,242	1.58	0.549	1.182	15.042	0.0147	3,376	5.06	4.06	"	"
4	"	J22	1,076,209	1.47	0.692	1.277	52.045	0.0043	5,631	8.45	6.78	"	1,845
5	"	F3	1,274,679	2.27	0.646	1.466	10.621	0.0034	7,329	10.99	8.82	"	"
6	"	J33	1,224,362	2.148	0.640	1.388	3.039	0.0038	7,921	11.88	9.53	"	"
7	"	J44	1,388,860	1.263	0.736	1.166	4.329	0.0031	8,458	12.69	10.18	"	"
8	"	J44	1,244,498	1.088	0.731	1.040	5.485	0.0049	9,281	13.92	11.17	"	"
9	"	F4	1,345,459	1.067	0.703	0.956	7.569	0.0014	9,397	14.10	11.31	"	"
10	"	J44	1,295,857	0	0.718	0.947	9,747	0.0023	9,717	14.58	11.70	"	"
11	"	J55R	1,862,301	1.020	0.78	0.911	113.3	0.0017	10,005	15.01	12.04	"	"

*Determination of these constants are well explained in Appendix B.

Table 6-4

Well #1 (Optimized)

Run No.	Bit Size	Bit Type	N rpm	W 1000lb	q gal/min	ρ lb/gal	μ c.p.	dn, 1/32 dn_1	dn_2	dn_3	Dull Condition D_f	B_f	C_B $	C_R $/hr	T hrs	F ft	CPF $/ft	R ft/hr	Drilling Cost Per Bit Run $
1	12¼	OSC3A	124.65	50	350	8.34	1.00	8	8	8	0.411	1.0	1071	191.667	9.742	2339.14	1.406	240.11	1405.76
2	7 7/8	X3A	200	50	350	8.34	1.00	8	8	8	1.00	0.766	661	"	30.711	3443.13	2.307	112.02	3012.82
3	"	X3A	148.113	50	350	9.00	10.0	8	8	8	0.966	1.00	661	191.667	30.368	1650.26	4.987	54.34	3411.11
4	"	J22	132.97	50	350	9.00	10.0	8	8	8	0.347	1.00	1845	"	22.891	1362.31	6.718	59.51	15149.09
5	"	F3	104.97	50	350	9.0	10.0	8	8	8	0.889	1.00	"	"	34.346	1328.50	9.201	38.60	15623.30
6	"	J33	58	42.5	255	9.0	10.0	9	9	9	1.00	0.5071	"	"	38.633	590.160	22.626	15.28	13394.59
7	"	J44	70.5	50	290.5	9.0	14.5	8	8	8	1.0	0.5691	"	"	31.726	441.51	27.078	13.92	14970.49
8	"	J44	90.6	50	297.5	9.0	20.0	8	8	8	1.0	0.7216	"	"	28.072	447.977	26.864	15.958	22109.07
9	"	F4	90.48	50	297.47	9.0	20.0	8	8	8	1.0	0.9210	"	"	38.737	112.550	126.63	2.91	14573.08
10	"	J44	195	45	350	14.5	19.5	8	8	8	1.0	0.10	"	"	22.014	2502.326	4.4364	113.67	1419.65
11	"	J55R	200.0	48.61	350.0	10	15.0	8	8	8	0.392	1.00	"	"	27.476	220.637	55.725	8.03	16040.80
																Total drilling cost of well #1			121,918

failure of the bit, while a low weight on the bit may cause the bit to drill in the same vicinity without any progress down hole. The values of the weight on bit W from the optimum solution for the eleven bit runs are higher than the values used in the field by Security company (from 35,000 to 50,000 lb) in order to get better penetration rate. But these values are within the practical average range, which are attainable in the field.

The optimum solution gave a higher value for the weight on bit and rotary speed than those used by Security company, and leads to a faster penetration of the formation to be drilled. Therefore, a sufficient volumetric flow rate of the drilling fluid is required in order to clean the bit teeth and cones along with the formation underneath and also to carry out the cuttings which have been generated. For these reasons, the values of q are increased in the optimum solution (from 269 to 300 gal/min). These values should not be higher than the maximum rate attainable from the pump which depends on the type and size of the pump used in the field during the drilling operation.

There is a small difference in the drilling fluid properties (i.e., density ρ and viscosity μ) between those obtained from the optimum solution and those which have been already used by Security company for all the eleven bit runs.

Selecting the proper type of bit will improve the penetration rate. For all the eleven intervals, the bits are chosen to have a three jet nozzle. The jet flow from these nozzles is very helpful in the drilling operation. According to the new drilling model, the optimization technique decreases the size of these nozzles in order to help improve the penetration rate by cleaning the bit teeth and exerting a high pressure flow jet of the drilling fluid onto the formation under action. Practically, the size of the jet nozzles are controlled by the contractor limits. A jet nozzle size equal to 8/32" is the smallest size that can be used in the field in order to avoid plugging the nozzle by the drilling fluid additives and solids content. Also the optimum solution shows that a bit with three equal jet nozzles will give better results.

Failure of bits to do their jobs is either due to tooth dullness or due to bearing wear, whichever takes place first. Table (6-4) shows that either the final bearing wear parameter B_f or the final tooth dullness D_f is equal to unity. If the parameter $B_f = 1.0$, this means that the bearing of the bit is completely damaged and if the parameter $D_f = 1.0$ which means that the teeth of the bit are worn out.

The total footage F which is drilled by one bit and the total time required to drill it have been computed and tabulated in Table (6-4). These F and T values are corresponding to the optimum solution (i.e., best combination of the six decision variables) which lead to the best penetration rate and then to the best drilling cost. For the first interval the solution of the new model shows that one bit is more than enough to drill the footage of this interval drilled by the Security company. This is true also for intervals two and three. But for interval four (and some others) the optimum solution shows that the footage drilled by one bit is not enough for this interval. Thus, another bit should be run to complete the remaining footage. These two cases are shown in the comparison Table (6-5).

The optimum solution of the six decision variables lead to a better penetration rate R. It is clearly shown in Table (6-5) that R from the optimum solution is much better than R which was reached by the Security company. Since the drilling cost CPF is inversely proportional to the drilling rate R, therefore, the drilling cost will decrease noticeably. The last column of Table (6-5) shows the percentage improvement in the drilling cost after using the optimization technique to control the drilling parameters.

The total drilling cost of well number one as drilled by the Security company is equal to 164,532 dollars, while the total drilling cost of the same well after using the optimization technique is equal to 121,918 dollars. Thus, this new model helps in saving about 42,614 dollars just in one well. Therefore, the next wells to drill in the same area where well No. 1 has been drilled should be optimized by this technique for a greater saving. The method will be then applied for other wells and fields in different areas.

Table 6-5

Well #1 (Comparison)

Run No.	Bit Size	Bit Type	Optimized Well: N, rpm	W 1000 lb	q gal/min	ρ lb/gal	μ c.p.	dn, 1/32: dn_1	dn_2	dn_3	F feet	T hrs	CPF $/ft	Un-Optimized Well: N rpm	W 10^3 lb	q gal/min	ρ lb/gal	μ c.p.	dn_1	dn_2	dn_3	F ft	T hrs	CPF $/ft	% Improvement in CPF
1	12¼	OSC3A	174.65	50	350	8.34	1.00	8	8	8	2319.14	9.742	1.485	120	20	269	8.33	1.0	12	12	12	946	10	3.73	60.19
2	7 7/8	X3A	200.0	"	"	8.34	1.00	8	8	8	3443.13	30.711	2.307	"	35	"	"	1.0	10	10	10	1652	28.5	4.55	49.30
3	"	X3A	148.113	"	"	9.00	10.0	8	8	8	1650.26	30.368	4.987	"	"	"	9.1	10	10	10	10	684	16	8.01	37.74
4	"	J22	132.97	"	"	"	"	8	8	8	1362.31	22.891	6.718	54	"	"	"	"	9	9	10	2255	96.25	10.29	34.71
5	"	F3	104.97	"	"	"	"	8	8	8	1378.50	34.346	9.201	"	"	"	9.2	"	11	11	11	1698	114	16.19	43.17
6	"	J33	58	42.5	255	"	10.0	9	9	9	592.41	38.633	22.549	"	"	"	9.3	"	"	"	"	592	50.75	27.77	18.80
7	"	J44	70.5	50	290.5	"	14.5	8	8	8	441.51	31.726	27.878	"	37.5	"	"	"	"	"	"	537	56	31.59	11.75
8	"	J44	90.5	"	297.5	"	20.0	8	8	8	447	28.072	26.92	"	"	"	"	13	"	"	"	823	90.5	29.16	7.67
9	"	F4	90.48	"	297.5	"	20.0	8	8	8	112.30	38.737	125.912	"	"	"	"	"	"	"	"	116	54.25	147.63	14.65
10	"	J44	195.0	45.0	350.0	14.5	19.50	8	8	8	2502.32	22.01	4.436	"	"	"	10	15	"	"	"	320	52.25	52.8	91.59
11	"	J55R	200.0	48.61	350.0	10	15.0	8	8	8	220.637	27.416	55.725	50	40	"	9.8	"	"	"	"	288	56	61.68	9.65

Figure (6-1) shows the optimum time T* to pull the bit out of the hole for bit run number one. As shown in Table (6-4) the bearing of the bit is completely worn out while about half of the bit teeth are damaged at the optimum time. Figure (6-2) shows the optimum time T* for bit run number two at which the bit teeth are completely worn out and about 75 percent of the bit bearing is damaged. The optimum time T* yields a maximum footage drilled by the bit and a minimum cost of drilling. These figures can be repeated for the other nine bit runs, which show the footage drilled by each bit and the drilling cost when the bits are to be pulled out of the hole before reaching the optimum time.

Figure 6-1

Optimum Time for Bit Run #1

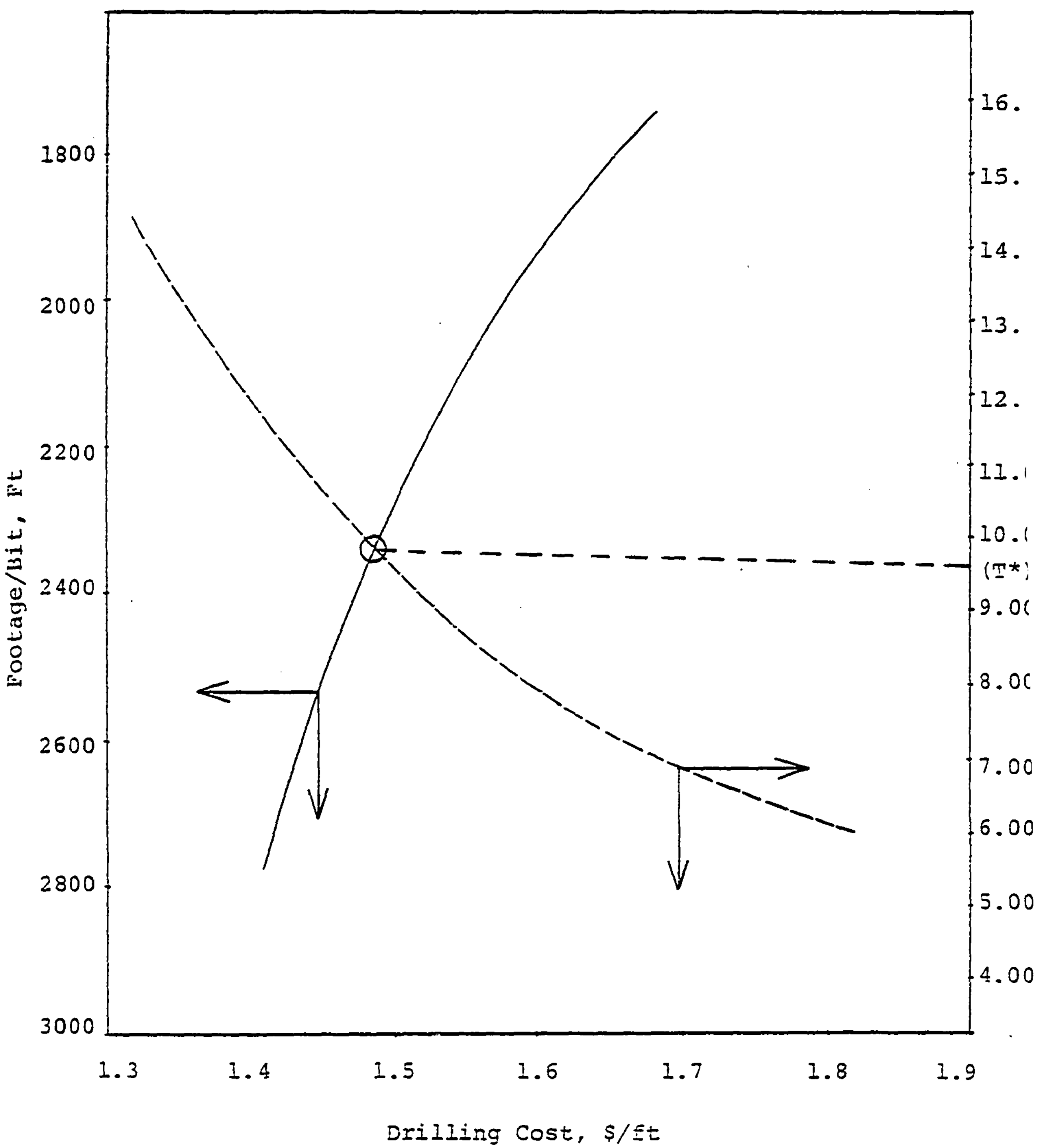
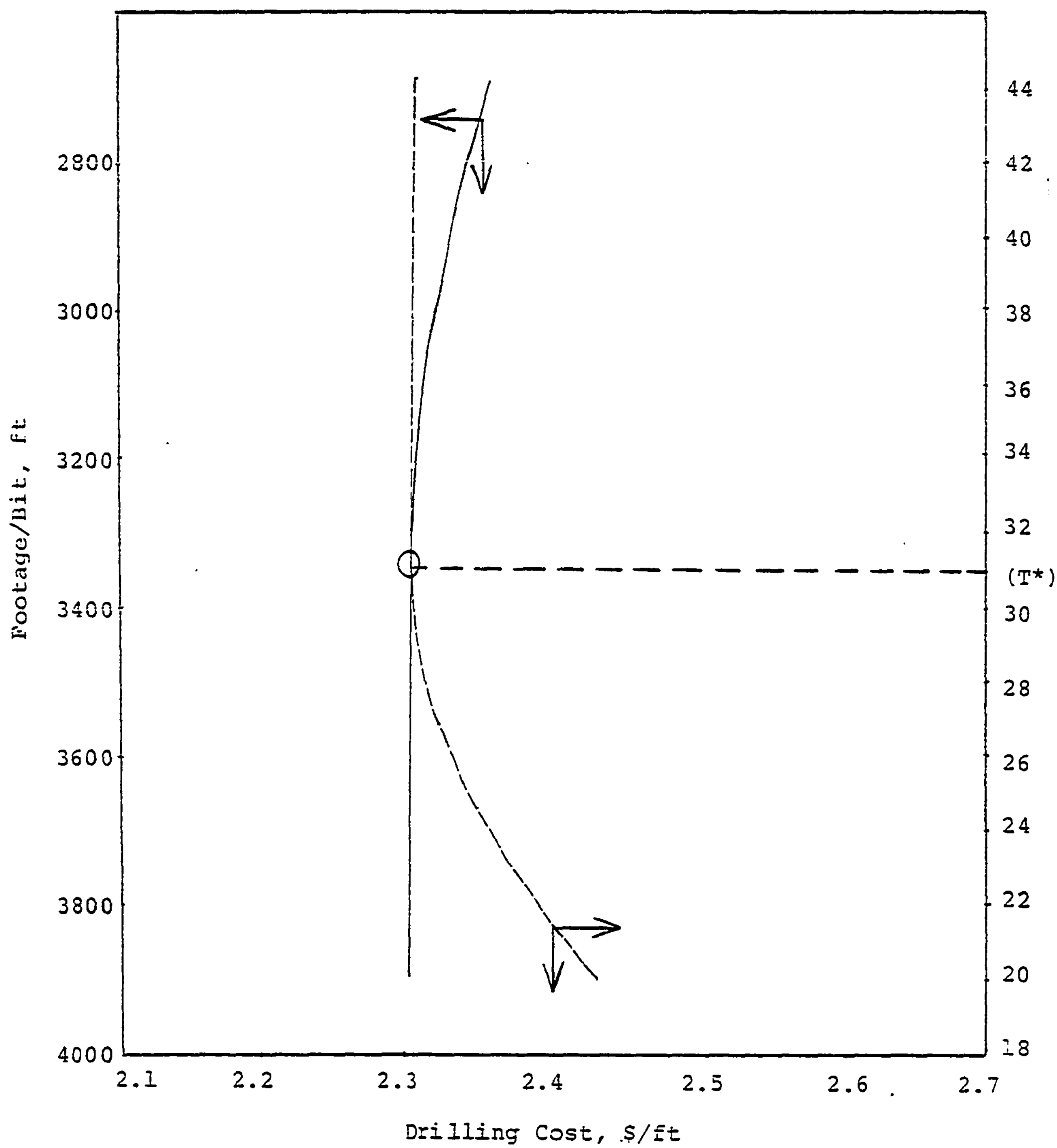


Figure 6-2

Optimum Time for Bit Run #2



CHAPTER 7

SENSITIVITY ANALYSIS OF THE MODEL

7.1 Introduction

All the sensitivity analysis shown in this chapter have been done for bit run number one of well number one. These analysis will be the same for the rest of the ten bit runs of well number one. The effect of changing one of the six decision variables on the penetration rate and the drilling cost will be studied in this chapter. These effects are: the effect of the drilling fluid properties, the effect of the drilling fluid volumetric flow rate, the effect of the weight on bit, the effect of the rotary speed, and the effect of the jet-nozzle diameter. Also the comparison between the Hooke and Jeeves search technique and other optimization techniques will be presented.

7.2 Effect of Drilling Fluid Properties on Penetration Rate and Drilling Cost

The drilling fluid density has a direct effect on the rate of penetration. The best rate of penetration can be attained by using air which has a low density. Drilling with water gives better penetration rate than drilling with mud.

The drilling fluid density is related to the penetration rate mainly through the effect of the differential pressure which is the difference between the mud column pressure and the formation pressure. As the differential pressure positively increases the penetration rate decreases. The penetration rate increases if the formation pressure is greater than the fluid column pressure. One must make sure not to let the formation pressure become greater than the fluid column pressure in order to avoid a possible blowout. So the least we can do, is to equalize the column pressure with the formation pressure (i.e., zero differential pressure) so that we could get the best drilling rate. Figure (7-1) shows the effect of the increase in the drilling fluid density from fresh water of 8.34 lb/gal to a dense mud of about 15 lb/gal, on the penetration rate and the effect of the increase in density on the drilling cost while all the other decision variables are kept constant for run number 1. These data agree with Eckel [15] and Kock [28].

Figures (7-2) shows the effect of a positive increase in the differential pressure on the penetration rate and the drilling cost for run number one. It is clear that the drilling rate drops sharply as the fluid density increases or when the differential pressure increases positively. This inverse relation between the differential pressure and the drilling rate has been mentioned and discussed very well by many authors [8, 12, 13, 15, & 41]. On the other hand, the

Figure 7-1

Density vs Drilling Rate and Cost

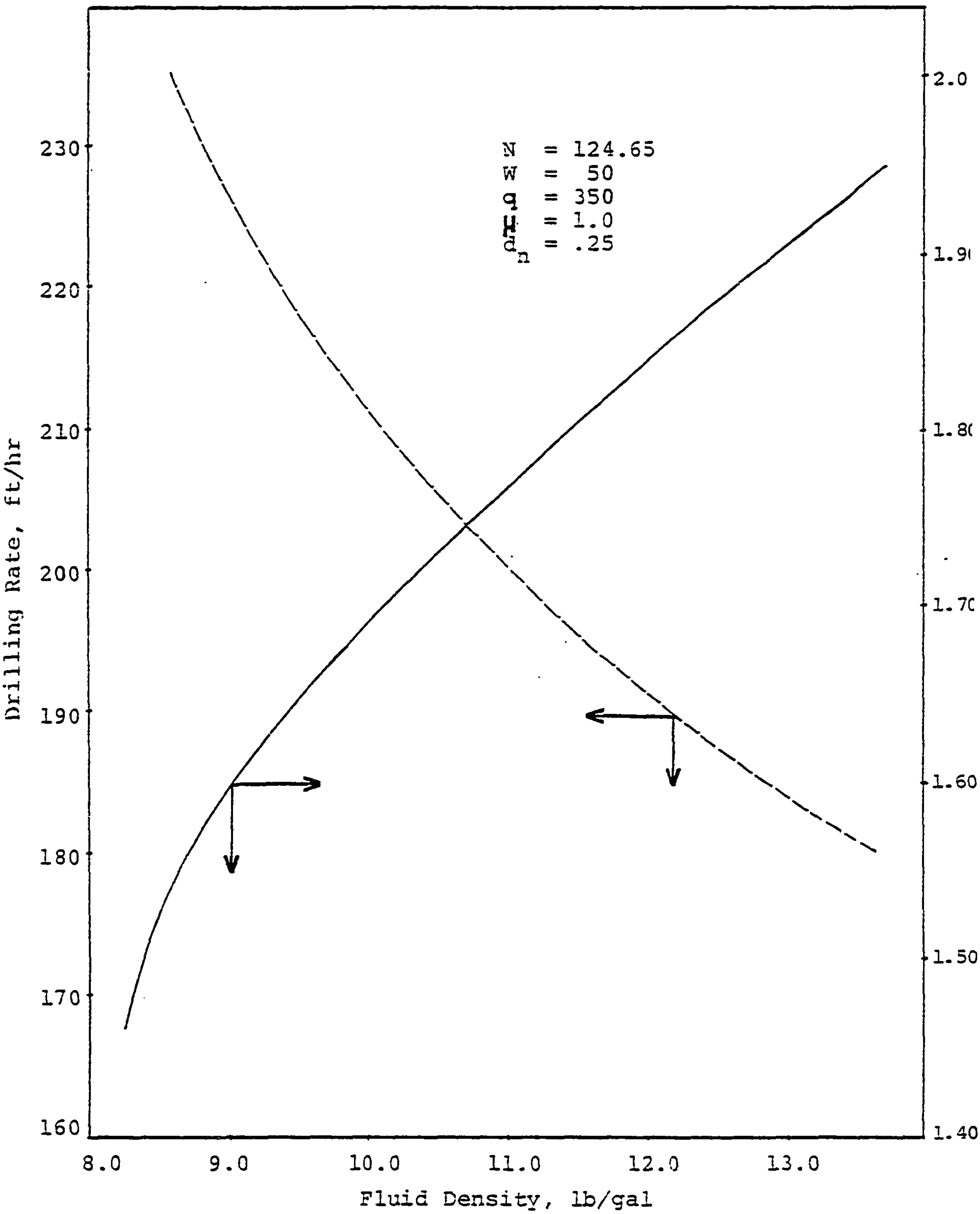
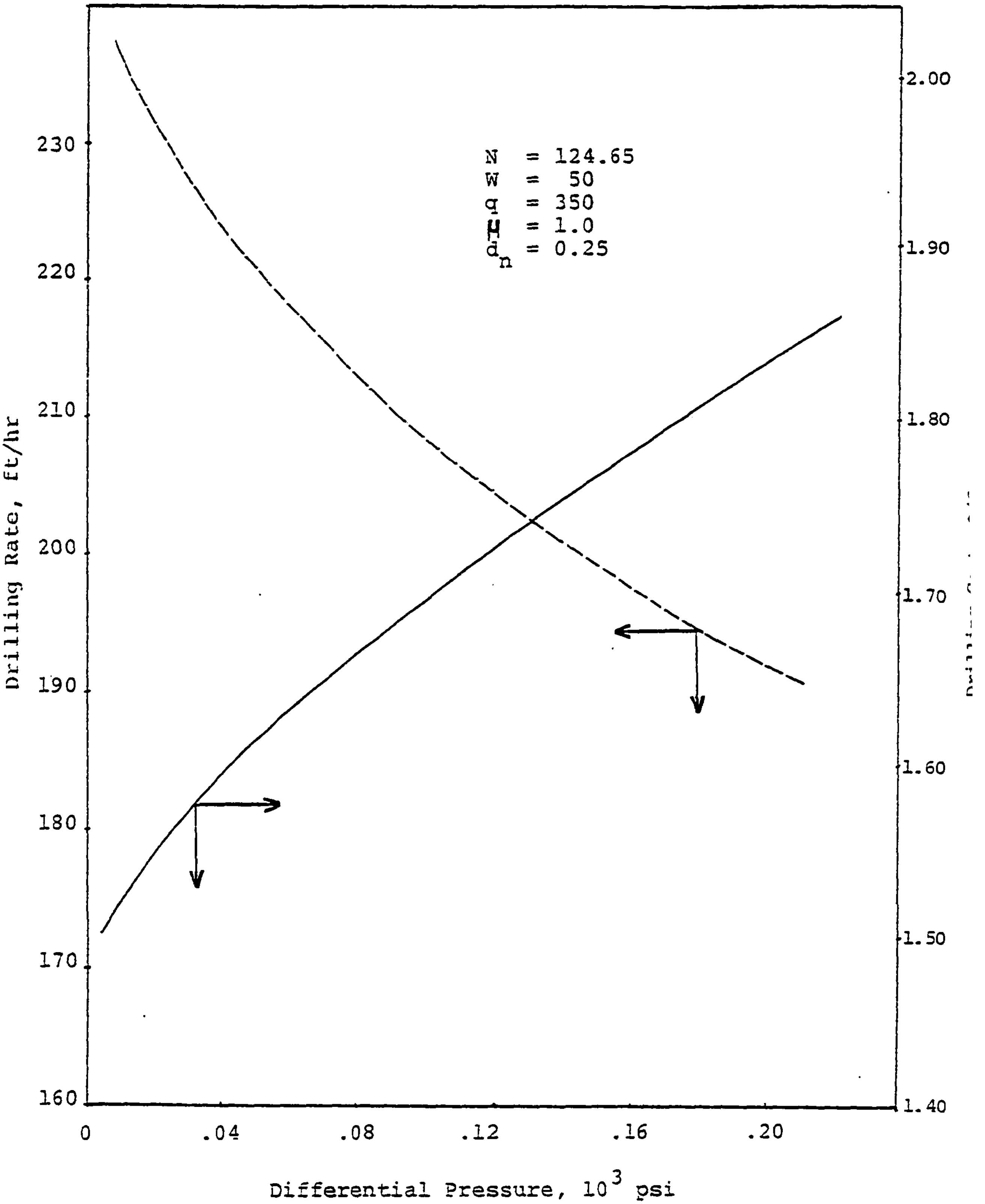


Figure 7-2

Differential Pressure vs Drilling Rate and Cost

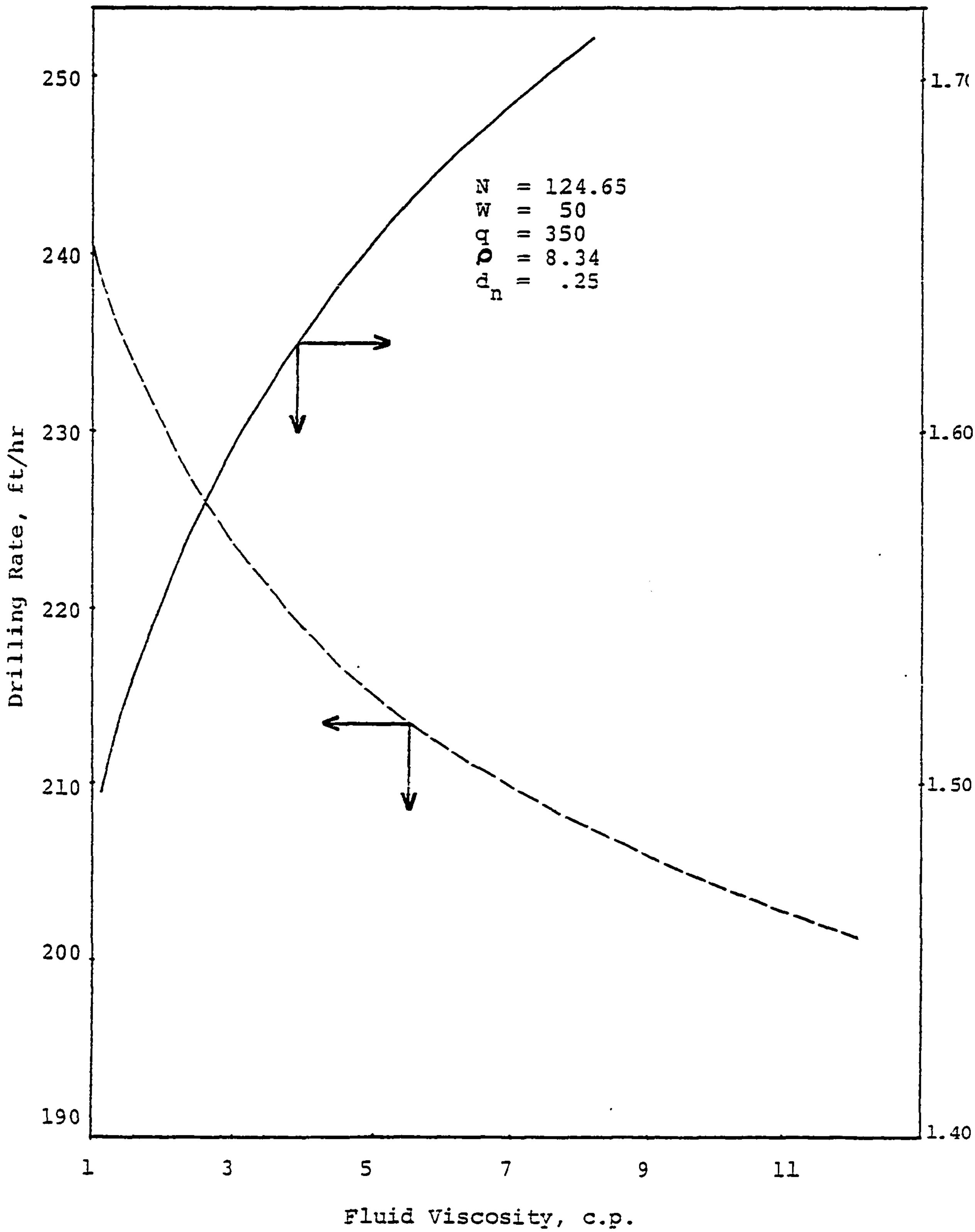


drilling cost in $/ft increases as the drilling fluid density increases or when the differential pressure increases positively. This cost-pressure relation is shown in Figure (7-2).

The fluid viscosity has an inverse effect on the penetration rate. As the viscosity of the drilling fluid increases from fresh water to a viscous of drilling mud, the rate of penetration drops quickly. This relation between the viscosity and the drilling rate agrees with the work of Eckel [15], Lummus [31], Walker [49] and Kock [28]. The effect of viscosity on the penetration rate and drilling cost is shown in Figures (7-3).

Figure 7-3

Fluid Viscosity vs Drilling Rate and Cost



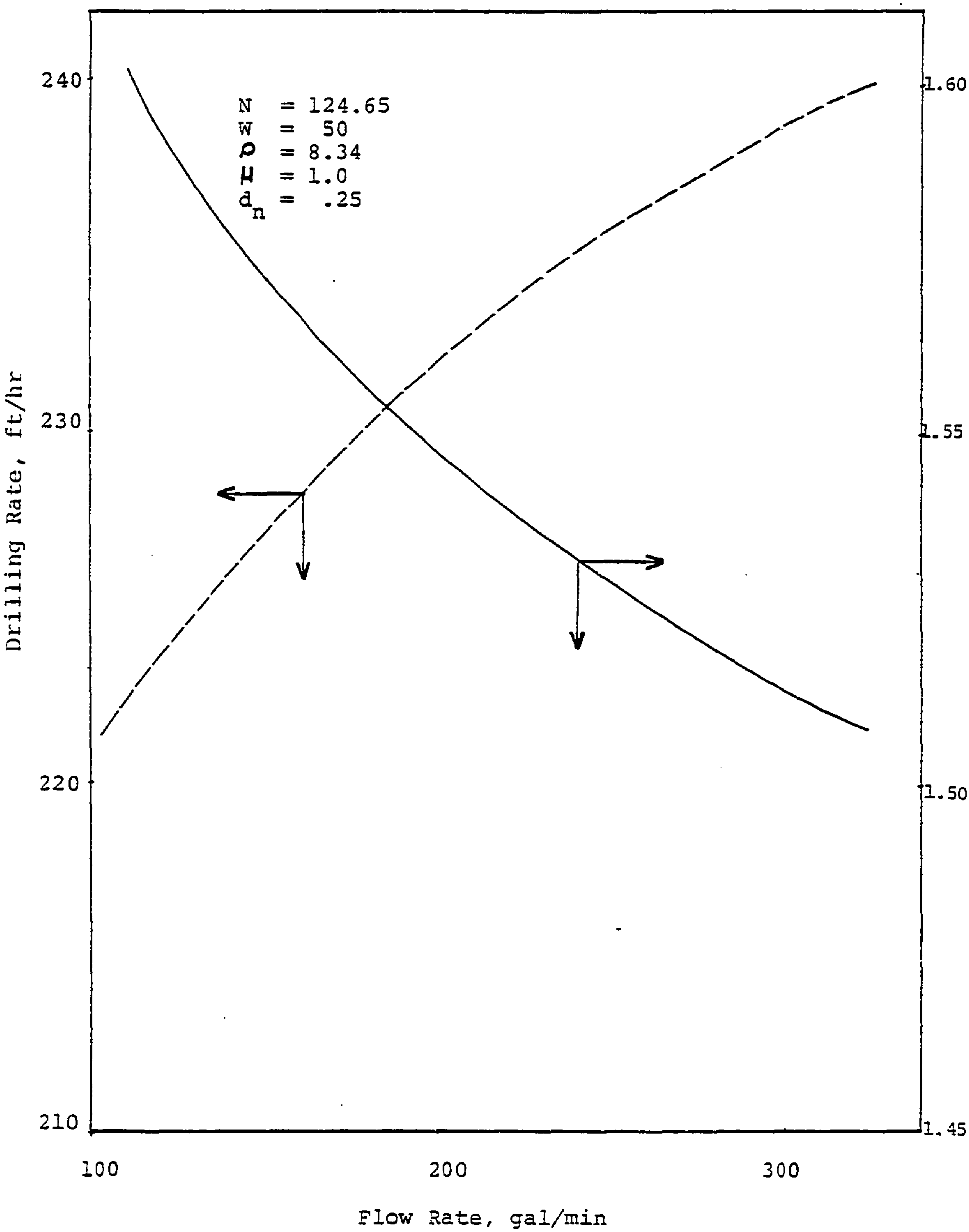
7.3 Effect of the Volumetric Flow Rate on the Penetration Rate and the Drilling Cost

The volumetric flow rate of the drilling fluid is related to the penetration rate through the Reynold's number equation. This relation is a reflection of the relation between the drilling rate and the hydraulics. As the flow rate increases, the penetration rate increases. This increase in the penetration rate is due to good and rapid cleaning of the formation beneath the rock bit and also to good cleaning and lubrication of the bit's teeth and cones. Good and rapid cleaning of the formation prevents the accumulation of the cuttings beneath the bit's teeth and also prevents the process of drilling and grinding these cuttings again and again.

In the oil fields, drillers choices are determined by the type of pumps available to them. Accordingly, the drillers cannot raise the volumetric flow rate above the maximum flow rate of the pump used. This maximum value depends on the size and type of the pump. Figure (7-4) shows the effect of the increase in the volumetric flow rate on the penetration rate and the drilling cost. It is clear, therefore, that the drilling cost ($/ft) drops as the flow rate increases. The flow rate—penetration rate and the flow rate—drilling cost relationships agree with the work of Eckel [16], Murphy [38] and Eckel [15].

Figure 7-4

Flow Rate vs Drilling Rate and Cost



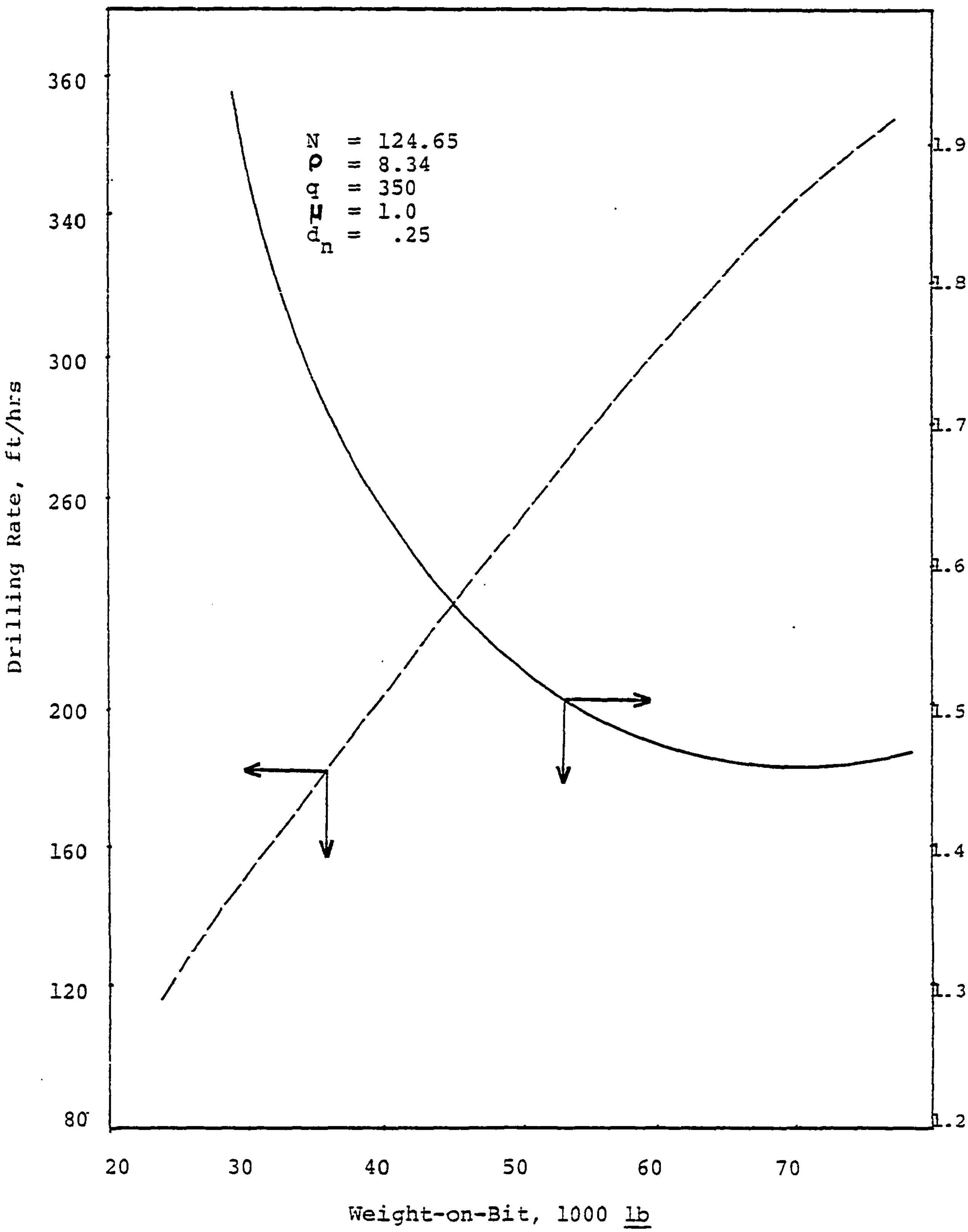
7.4 Effect of Weight-on-Bit on the Penetration Rate and the Drilling Cost

Figure (7-5) shows the effect of increasing the bit weight on the penetration rate and the drilling cost while keeping all the other variables unchanged. These relationships agree with that of Edwards [17], Speer [47] and Feenstra [19]. If weight on bit is increased, drilling rate increases until a rate is reached at which hydraulics are not sufficient to remove generated cuttings. This point is referred to as the "flounder" or "ball-up" point. Further weight increases may actually result in a reduction in drilling rate.

At low bit weight, the cost per foot decreases until some minimum value is reached. More weight increases drives costs up. This minimum value is dependent on the nature of formation drilled, and the values of the other decision variables. There is a combination of weight on bit and all the other decision variables that yields a lower cost than any other combination which is the optimum solution for the non-linear programming.

Figure 7-5

Weight on Bit vs Drilling Rate and Cost



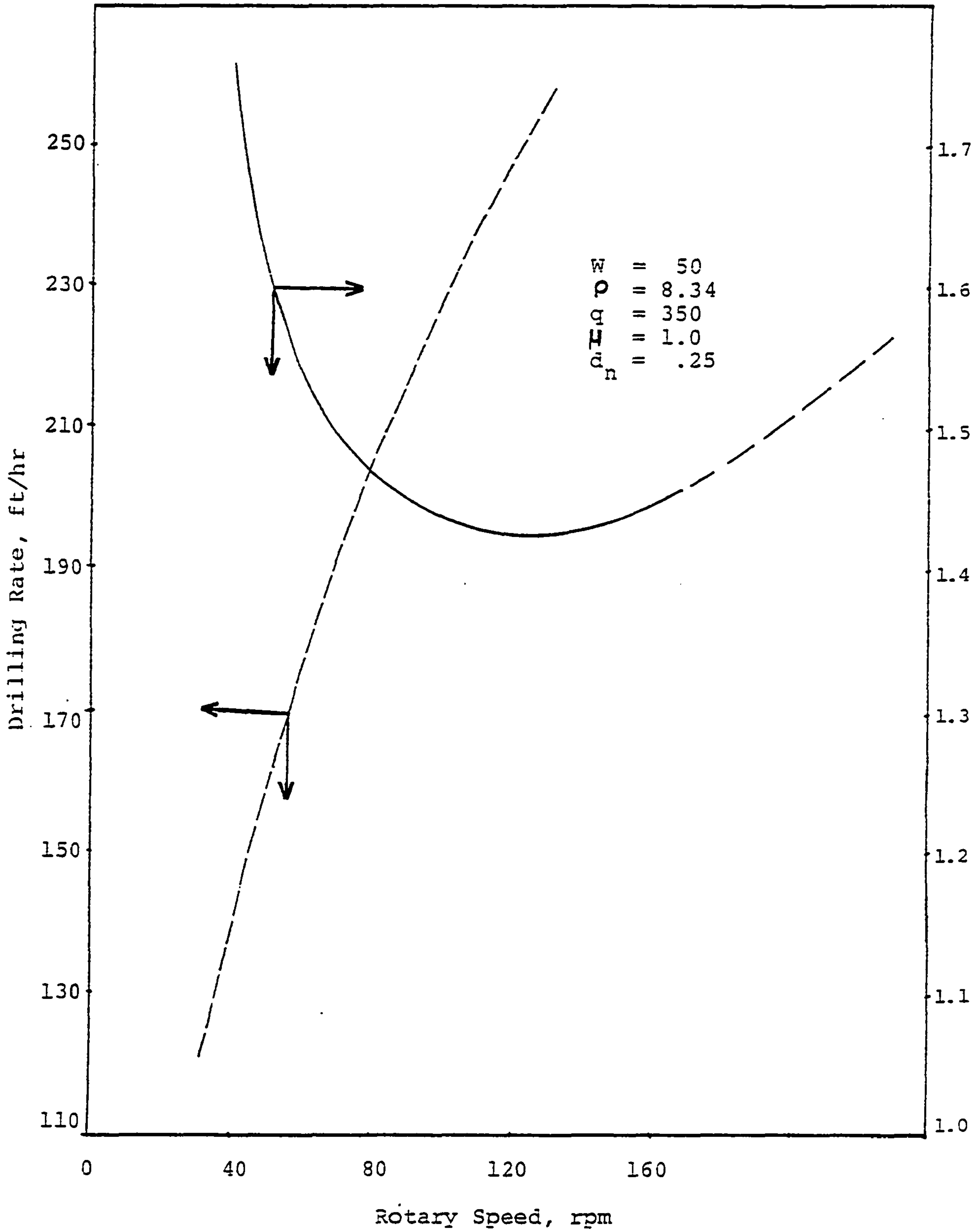
7.5 Effect of Rotary Speed on the Penetration Rate and the Drilling Cost

Figure (7-6) shows the effect of the increase in the rotary speed on the penetration rate and the drilling cost per foot while keeping all the other decision variables unchanged. These effects agree with results in [17, 19, 47]. If the rotary speed is increased, the drilling rate increases. The response of drilling rate to the increase in rotary speed is less than linear as shown in Figure (7-6). This response will vary according to formation type.

At low rotary speed, the cost per foot decreases until some minimum value is reached. Further rotary speed increases cause costs to go up. This minimum value is dependent on the type of formation drilled, and the values of other decision variables. There is a combination of rotary speed and all the other decision variables that yields a lower cost than any other combination which is the optimum solution for the non-linear programming.

Figure 7-6

Rotating Speed vs Drilling Rate and Cost



7.6 Effect of Jet-Nozzle Diameter on the Penetration Rate and the Drilling Cost

Figure (7-7) shows the effect of the increase in the size of the jet-nozzle diameter on the penetration rate and the drilling cost. This relation agrees with Eckel [16]. In this study, a bit with three equal jet-nozzles have been used. If the jet-nozzle diameter is increased, the drilling rate decreases. On the other hand, if the jet nozzle diameter is increased, the drilling cost increases. The jet flow of the drilling fluid through the bit jet-nozzles, is very helpful in the drilling process due to jet pressure exerted into the formation, and to the jet flow which clean and carry the cuttings away as soon as they were generated.

The effect of changing the number of jet-nozzles of the bit on the drilling cost and rate is considered in this section together with the effect of unequal size jet-nozzles. Table (7-1) shows all above mentioned effects for run number one. The improvement in the drilling cost increases as the number of the jet-nozzles decreases. Thus, the best drilling cost one could get is through the use of a bit with one jet-nozzle which is case (5) in Table (7-1).

Figure 7-7

Nozzle Diameter vs. Drilling Rate and Cost

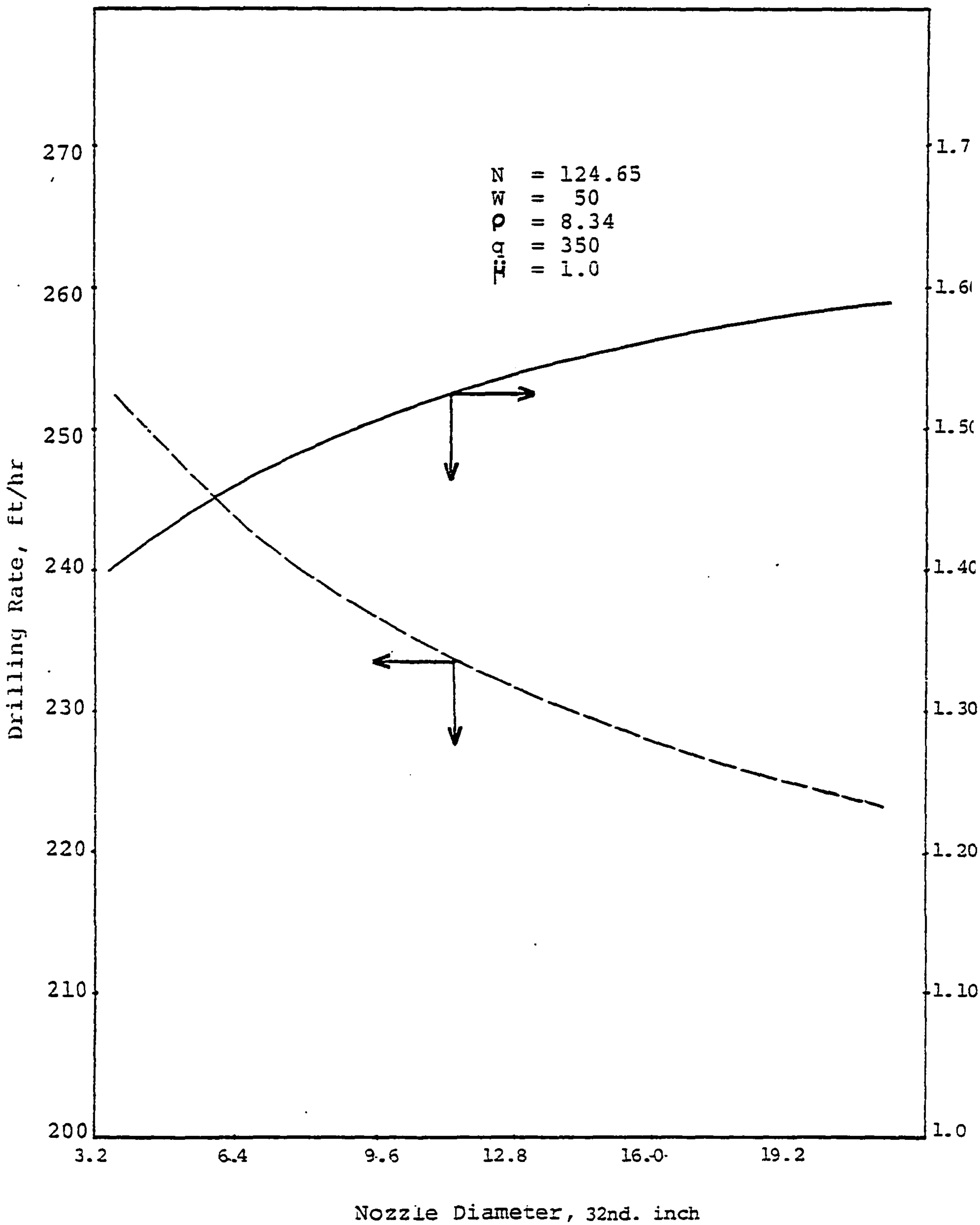


Table (7-1)

Effect of Changing the Number of Jet-Nozzles of the Bit

Run #1	Jet-Nozzles Size, 1/32			Footage	Drilling Rate	CPF	% Change in CPF
	dn_1	dn_2	dn_3	ft	ft/hr	\$/ft	over case (1)
Case (1): 3-Equal Jet Nozzles $Re = 126.371 \frac{q\rho}{dn\mu}$	8	8	8	2339.14	240.11	1.486	-
Case (2): 3-Unequal Jet Nozzles $Re = 379.11 \frac{q\rho}{\mu(dn_1+dn_2+dn_3)}$	8	9	8	2332.3	239.41	1.491	0.335*
Case (3): 2-Equal Jet Nozzles $Re = 189.56 \frac{q\rho}{dn\mu}$	8	8	-	2405.94	246.97	1.445	2.759**
Case (4): 2-Unequal Jet Nozzles $Re = 379.11 \frac{q\rho}{(dn_1+dn_2)\mu}$	8	9	-	2387.0	245.03	1.457	1.952**
Case (5): One Jet Nozzle $Re = 379.11 \frac{q\rho}{dn\mu}$	8	-	-	2520.1	258.68	1.380	7.133**

* disimprovement in CPF.
** improvement in CPF.

7.7 Comparison of the Hooke and Jeeves Search Method with Other Optimization Methods

The comparison in this section is between the method of Hooke and Jeeves using Fibonacci line search and the following two methods:

1. Hooke and Jeeves [9, 23] method which is usually used to find the optimum solution for a multi-variable, unconstrained non-linear function without using derivatives.
2. The Rosenbrock method [9, 46] which is usually used to find the optimum solution for a multi-variable, unconstrained non-linear function without using derivatives.

In the previous two methods, the procedure assumes a unimodal function; therefore, several sets of starting values for the independent variables should be used if it is known that more than one minimum (maximum) exists or if the shape of the surface is unknown.

In evaluating most of the non-linear optimization techniques available, it would be hard to say which is better than the other. There are several factors which should be taken into account while evaluating each technique. These factors are: the objective function, the initial values, the number of function evaluations required to reach the optimum solution, the CPU time on computer, and finally the optimum solution of the problem itself. Table (7-2) shows the difference in the optimum solution which is reached by the three different techniques, and also shows how much improvement in

Table (7-2)

Comparison Table for Run No. 1

Optimization Method — Notes

* Security CPF = 3.73 \$/ft (from table 6-1).
* Same termination factor for all the three methods.
* Same initial step size for each of the decision variables.

Starting Points

N rpm	W 10^3 lb	ρ lb/gal	q gal/min	dn_1 inch (dn, 1/32")	dn_2 inch (dn, 1/32")	dn_3 inch (dn, 1/32")	μ c.p.
53.0	37.5	9.0	250.0	11	11	11	7.0

Optimization Method	N rpm	W 10^3 lb	ρ lb/gal	q gal/min	dn_1 inch	dn_2 inch	dn_3 inch	μ c.p.	F ft	T hrs	D_f	CPF \$/ft	# of Function Evaluations	Computer Time, sec.	% Improvement cost/ft Over Security CPF
Rosenbrock Method	63.05	37.7	8.99	250.2	11	11	11	6.99	3121.17	34.962	.351	2.663	70	0:07.35	28.606
Discrete Hooke and Jeeves Method	71.19	57.29	8.314	269.76	8	8	8	5.019	2505.487	13.904	.326	1.706	1007	0:14.83	54.26
Method of Hooke and Jeeves Using Fibonacci Line Search	124.65	50.0	8.34	350.0	8	8	8	1.0	2339.14	9.742	0.411	1.486	84	24:29.32	60.016

cost/foot we can get by switching from the Rosenbrock or Hooke and Jeeves methods to the method of Hooke and Jeeves using Fibonacci line search. Usually, the search method is faster and more effective than the discrete method, due to its ability to change the search directions faster.

Since the method of Hooke and Jeeves using Fibonacci line search gives better drilling cost results compared with the other two methods, it has been selected as the solution procedure to solve the non-linear mathematical drilling model.

Table (7-3) shows the effect of changing the starting points for run number one. The starting points have direct effect on arriving to an optimum solution, on the required computer time, and on the number of function evaluations. For the Rosenbrock and discrete Hooke and Jeeves methods (if the starting points are chosen far away from the optimum solution, such as the sets No. one and three in Table (7-3), a longer computer time would be required to reach a solution, which is actually not the optimum solution). For the method of Hooke and Jeeves, using Fibonacci line search, the optimum solution has been reached through different starting points, but with varying computer time. For the starting points #2, which is close to optimum, the optimum solution could be reached in the shortest possible computer time.

Table (7-3)

Effect of Changing the Starting Points

Starting Points Sets No.	Optimization Method	N rpm	W 10^3 lb	ρ lb/gal	q gal/min	3-equal size nozzles dn_{inch}	μ c.p.	F ft	T hrs	D_f	CPF $/ft	No. of Function Evaluations	Computer Time Required
(1)													
N=53	Rosenbrock	53.05	37.7	8.99	250.2	11	6.99	3121.17	34.96	.351	2.663	70	07.35
W=37.5													
ρ =9	Discrete	71.19	59.29	8.34	269.76	8	5.019	2505.48	13.904	.326	1.706	1007	14.83
q=250	H & J												
dn=.344	Search	124.64	50.0	8.34	350.0	8	1.0	2339.14	9.742	.411	1.486	84	24:29.32
μ =7.0	H & J												
(2)													
N=120	Rosenbrock	120.20	20.20	8.34	269.20	12	.99	3453.8	39.343	.606	2.649	80	8.01
W=20													
ρ =8.34	Discrete	126.1	28.80	8.340	277.8	9	.120	2378.577	22.028	0.482	2.452	463	14.06
q=269	H & J												
dn=.375	Search	126.43	50.0	8.34	350.0	8	1.00	2320.98	9.604	.414	1.487	49	8:59.67
μ =1.0	H & J												
(3)													
N=160	Rosenbrock	160.12	40.20	12.99	200.20	14	4.99	1281.73	10.52	.508	2.829	80	0:7.91
W=40													
ρ =13	Discrete	163.399	43.799	11.279	217.19	13	3.279	1410.69	9.064	.500	2.372	1008	19.91
q=200	H & J												
dn=.75	Search	126.43	50.0	8.34	350.0	8	1.0	2320.99	9.604	.414	1.487	98	7:35.64
μ =5.0	H & J												

CHAPTER 8

SUMMARY, CONCLUSIONS, AND FUTURE WORK

8.1 Summary

The drilling models developed in the past years by so many drilling experts are restricted by different limits and assumptions. They related the penetration rate only to the weight on bit and rotary speed assuming that all the other factors are well chosen. This is the main reason for the development of a new mathematical non-linear drilling model which includes the effect of six variables on the penetration rate and, then, on the drilling cost. Beside the weight on bit and rotary speed, the other four variables are: the drilling fluid density, the drilling fluid viscosity, the drilling fluid rate, and the jet-nozzle diameter.

Many optimization techniques have been tested in order to find the best solution for the drilling model (i.e., the best combination of the six decision variables to give maximum penetration rate or minimum drilling cost). Among these optimization techniques, the method of Hooke and Jeeves using Fibonacci line search gives the best solution for the non-linear drilling model.

8.2 Conclusions

1. For the first time a new drilling mathematical model has been developed that reflects the effect of the following variables: weight on bit, rotary speed, bit type and size, drilling fluid properties, hydraulics, differential pressure at the bit nozzles, formation to be drilled, round trip time, and connection time on the drilling rate and, then, on the drilling cost.

2. From a practical viewpoint there are several restrictions which limit the feasible region of the objective function, such as the maximum rotary speed obtained from the draw-work, the maximum weight exerted on the bit, the maximum volumetric flow rate attained from the pump, and the sizes of the drilling bit jet nozzles.

3. With the above mentioned restrictions, the optimum solution to the non-linear multi-variable drilling cost function (i.e., minimizing the drilling cost in $/ft) using the Hooke and Jeeves search method, is more economical and realistic than solutions offered by other optimization techniques.

4. The main application of this new technique of optimization is mainly in the oil and gas fields. After drilling the first test well in a certain area of the oil or gas field, the next wells to be drilled in the same area should be optimized in order to get the minimum drilling cost.

5. The drilling companies could save thousands of dollars on one well and perhaps millions on one field.

6. Basically, the new mathematical drilling model consists of two cases: the first is the case when the decision variables change with time, and the second is the case when all the decision variables are held constant over the entire bit run.

7. Uusally, there is a lack of field data available for the case when the decision variables varied with time. Therefore, there is a need for field data which gives the number of increments for each bit run, the values of the decision variables for each increment, the change in the bit teeth and bearing at each increment, and finally the total footage drilled and time required for each increment.

8. The bit records data offered by the Security company, are for the case where the decision variables are held constant.

9. For run number one (the formation is soft), it was found that the second case of optimization (i.e., when the decision variables are held constant over the entire bit run) offered very little advantage over the first case of optimization (i.e., when the decision variables are varying with time). Therefore, for bit run No. 1, it is preferable to hold all the decision variables unchanged over the entire bit run.

8.3 Future Work

It is recommended that a drilling company should perform certain tests which are necessary for the optimization procedure during the drilling operations, from which the rotary speed exponent Z, the weight exponent Y, and other constants (as shown in Appendix B) can be determined.

It is also recommended for future work that experimental work, using a simulated drilling rig, should be conducted in order to study the effect of some additional controllable variables (such as the down-hole temperature and the hole problems) on the drilling rate and drilling cost. The change in the down-hole temperature, expecially in geothermal wells, has a direct effect on the drilling fluid properties (such as density, viscosity, and gel-strength) and on the drilling bit and pipes. Therefore, the temperature of the well relates to the drilling rate and drilling cost.

Nomenclature

A_f = formation abrasiveness factor

a = function of tooth dullness D

= $0.928125\ D^2 + 6.0\ D + 1.0$

b = bearing-wear constant

B = bearing-wear fraction of the bit

B_f = final bearing-wear

C_f = formation drillability factor

C_2 = tooth-wear parameter

C_B = bit cost, \$

C_R = rig cost, \$/hr

CPF = drilling cost, \$/ft

C_n = weight exponent in the bearing-wear equation

D = tooth-wear fraction

D_f = final tooth-wear

d_n = jet-nozzle diameter, inch

$E = \dfrac{b\ B}{714.19\ GNW^{cn}}$

F = total depth drilled by bit, ft

$G = A_f\ \bar{m}/R$

H = hole or bit diameter, inch

K = Reynold's number constant

$M = C_f\ W^Y\ N^z$

$m = 1359.1 - 714.19 \log \bar{w}$

$\bar{m} = \dfrac{m}{714.19}$

N = rotary speed, rpm

n = increments number per bit run, $i = 1, \ldots, n$

P_m = drilling fluid pressure, psi

= 0.052 x depth x density

P_f = formation pressure, psi

Q = $\text{Log}\left(\frac{kq\rho}{dn\mu}\right)$

q = volumetric flow rate, gallon/min

R = rotary speed function

= $N + 0.00004348$

R_e = Reynold's number, fraction = $kq\rho/dn\mu$

T = rotating time for the bit, hrs

T_c = connection time, hrs

T_t = trip time, hrs

U = $714.19 \int_0^{D_f} a\, dD$

V = $\frac{1}{1 + \Delta p^x}$

W = weight on bit, 10^3 lb

$\bar{W}$ = equivalent 7 7/8 inch bit weight = 7.875 W/H

x = differential pressure exponent

y = weight exponent in the drilling rate equation

z = speed exponent in the drilling rate equation

Latine Letters

Δp = differential pressure, 10^3 psi

ρ = drilling fluid density, lb/gal

μ = drilling fluid viscosity, c.p.

ζ = length of uncertainty

ϵ = step size

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APPENDIX A

EXAMPLE ON GALLE AND WOODS TECHNIQUE

Using the information shown in Table (D-1), determine the best constant weight and rotary speed.

<u>Step 1</u>: From the bit record determine the formation factors using the equations (example shown for bit 9).

1. Formation Abrasiveness: $A_f = FR/\bar{m}U$ (A-1)

   T = rotating hours = 12.7

   N = rotary speed = 140

   R = from Table (D-3) = 259

   $\bar{w}$ = weight on bit = 45 (1000 lbs.)

   $\bar{m}$ = from Table (D-4) = .249

   D = 4/8

   U = from Table (D-2) = 920

   A_f = (12.7)(259) ÷ (.249 x 563)

   A_f = 14.4

2. Drillability: $D_f = FR/\bar{m}(\bar{w})^k(N^r)Z$ (A-2)

   F = bit footage = 368

   R = from Table (D-3) = 259

   $\bar{m}$ = from Table (D-4) = 0.249

   $\bar{w}$ = bit weight = 45(100 lbs.)

   K = weight exponent = 1.0

   $\bar{w}^k$ = from Table (D-5) = 45

   N = rotary speed = 140

   r = speed exponent = 0.6

   N^r = from Table (D-6) = 19.4

   Z = from Table (D-2) = 563
   (for p = 0.5 - self-sharpening tooth wear)

$D_f = (368 \times 259) \div (.249 \times 45 \times 19.4 \times 563)$

$D_f = 0.779$

3. Bearing Factor: $B_f = TN/B_xL$ (A-3)

T = rotating time = 12.7

N = rotary speed = 140

B_x = bearing condition = .75

L = from Table (D-4) = 1288

$B_f = (12.7 \times 140) \div (.75 \times 1288)$

$B_f = 1.84$

Formation Factors

Bit No.	A_f	D_f	B_f
9	14.4	.779	1.84
10	12.2	.617	1.97
11	13.6	.611	2.03
12	15.0	.599	1.91
13	14.3	.665	1.83
Average for interval			
	13.9	0.654	1.92

Step 2: Develop input data for cost/foot equation.

C_B = bit cost = \$210

C_R = rig cost = \$100/hour

T_t = trip time (based on average for interval) = 4.7 hrs

A_f = abrasiveness factor = 13.9

D_f = drillability factor = 0.654

B_f = bearing factor = 1.92

P = tooth wear factor = 0.5 (self-sharpening)

K = weight exponent = 1.0

r = speed exponent = 0.6

$\bar{W}_{max}$ = maximum weight = 70 (1000 lbs.)

$\bar{W}_{min}$ = minimum weight = 0

N_{max} = maximum speed = 175 rpm

N_{min} = minimum speed = 100 rpm

Step 3: Determine optimum weight-speed

For the weight and speed limitations given, an applicable grid might be N- 100, 125, 150, 175 and $\bar{W}$- 40, 50, 60, 70. Therefore, a total of sixteen calculations will be made. This is accomplished using the equation (example for $\bar{W}$- 40 and N- 100):

$$\text{cost/foot} = \frac{C_B + C_R(T_t + A_f\, u\, \bar{m}/R)}{D_f\, Z(\bar{W})^K (N^r)\bar{m}/R} = \frac{C_B + C_R\,(T_t + T)}{F} \qquad \text{(A-4)}$$

CB = \$210 (given)

CR = \$100/hour (given)

T_t = 4.75 hours (given)

To solve the equation, it is first necessary to determine if the bit life is dependent on tooth wear or bearing wear. This is done by assuming bearing wear at 100% and solving for U (tooth dullness factor) using the equation:

$$U = B_x \, B_f \, LR/A_f \, \bar{m} \, N \quad \text{(from equations 3-15 \& 3-16)} \qquad \text{(A-5)}$$

where;

B_x - bearing wear = 1.0

B_f - bearing factor = 1.92

L - from Table (D-4) = 1578 (for $\bar{w}$ = 40)

R - from Table (D-3) = 143 (for N = 100)

A_f - abrasiveness = 13.9

$\bar{m}$ - from Table (D-4) = 0.300 (for $\bar{w}$ = 40)

N - rotary speed = 100

U = (1.0 x 1.92 x 1578 x 143) ÷ (13.9 x .300 x 100)

U = 1039

When bearings are worn, the tooth will be 54% gone [from Table (D-2)].

Since U is less than 3078 (for D = 1.0), we know that at 40,000 lbs. and 100 RPM the bearings will wear out before the teeth. It is now possible to calculate the estimated rotating hours T for this weight speed combination using the formula:

$$T = A_f \frac{U\bar{m}}{R} \qquad \text{(A-6)}$$

Where,

A_f = 13.9

U = 1039

$\bar{m}$ = 0.300

R = 143

Thus, T = (13.9 x 1039 x .300 ÷ (143)

= 30.3 rotating hours.

Estimated footage can also be calculated for this weight and speed using the formula:

$$F_f = \frac{D_f \ Z \ \bar{w}^k \ N^r \ \bar{m}}{R} \tag{A-7}$$

where

D_f = 0.654

Z = 620 from Table (D-2) for 0.54 tooth wear

$\bar{w}^k$ = 40 from Table (D-5) for $\bar{w}$ = 40

N^r = 15.9 from Table (D-6) for N = 100

$\bar{m}$ = 0.300 from Table (D-4) for $\bar{w}$ = 40

R = 143 from Table (D-3) for N = 100

Thus;

F = (0.654 x 620 x 40 x 15.9 x 0.30) ÷ (143)

= 541 feet

Based on the mathematical model presented, it has been determined that in the interval in question, for a weight of 40,000 lbs. and a rotary speed of 100 RPM, a bit should theoretically drill 541 feet in 30.3 hours. At this point, the bit condition should show the bearings 100 percent worn, and the tooth structure 54 percent gone.

Cost/foot for this interval can be calculated by the formula:

$$\text{cost/foot} = \frac{C_B + C_R\ (T_t + T)}{F}$$

Thus,

$$\text{cost/ foot} = \frac{210 + 100(4.75 + 30.3)}{541}$$

$$= 6.87\ \$/\text{foot}$$

Repeating this calculation for the other 15 combinations of weight and speed, the following cost grid is determined:

Bit Weight		Rotary Speed 100	125	150	175
40,000					
	cost/ft	6.83	6.53	6.32	6.23
	T	30.3	24.2	20.2	17.3
	F	541	476	428	388
50,000					
	cost/ft	5.94	5.78	5.67	5.68
	T	20.4	16.3	13.6	11.7
	F	459	401	361	327
60,000					
	cost/ft	5.65	5.62	5.60	5.65
	T	14.2	11.4	9.50	8.1
	F	373	325	292	265
70,000					
	cost/ft	6.06	6.12	6.25	6.38
	T	10.0	8.0	6.7	5.7
	F	278	243	217	197

Based on this grid, the optimum or best weight/speed would be: Bit weight = 60,000 lbs.

Rotary speed = 150 RPM

APPENDIX B

DETERMINATION OF THE CONSTANTS

(1) Bearing-wear constant (b):

This constant is varied with the drilling fluid properties composition, solids content, and bit size and type. It can be calculated from the following equation:

$$b = \frac{N\ W^{cn}\ T}{B_f} \tag{B-1}$$

where,

T = total rotating time for the bit, hrs.

B_f = final bearing-wear for the bit

N = rotary speed, rpm

w = weight on bit, 10^3 lb

cn = bit weight exponent

(2) Tooth-wear constant (C_2):

This constant has historical value and is used to show the magnitude of the penetration rate reduction due to bit-tooth wear. A soft formation bit has higher value than hard formation bits due to a decrease in scraping action as the tooth dulls. The crushing action of the bit is not effected that much, so, a hard-formation bit would have a low value of C_2, which can be calculated from the following equation for a homogenous formation:

$$C_2 = \frac{R_o - R_f}{R_f\ D_f} \tag{B-2}$$

where,

R_o = drilling-rate for new bit, ft/hr

R_f = final drilling rate, ft/hr

D_f = final tooth-dullness

(3) Rotary Speed Exponent (Z):

This is the rotary speed exponent in the drilling rate equation. From previous laboratory and field tests, it was found that Z is always < 1.0, and is approximately equaled to 0.6 for very soft formation and about 0.85 for harder formations. It can be determined from the following equation:

$$Z = \frac{\log (R_1/R_2)}{\log (N_1/N_2)} \qquad \text{(B-3)}$$

where;

N_1 , N_2 = Rotary speeds at constant W, ρ, q, d_n and μ, rpm.

R_1 , R_2 = Drilling rate at N_1 and N_2, respectively; ft/hr.

(4) Weight exponent (Y):

This is the weight exponent in the drilling rate equation. It can be determined in the same way as Z was determined, but in this case the weight on bit is varied while all the other variables are held constant.

$$Y = \frac{\log (R_1/R_2)}{\log (W_1/W_2)} \qquad \text{(B-4)}$$

where;

W_1, W_2 = weight on bit at constant N, ρ, q, d_n and μ, 10^3 lbs.

R_1, R_2 = drilling rates, ft/hr at W_1 and W_2 respectivel

(5) Weight exponent in bearing-wear equation (cn):

This weight exponent, cn, relates bearing wear rate to bit weight, and has determined experimentally. A value of 1.5 was observed for common drilling fluids [57].

(6) Differential-pressure exponent (X):

This exponent, X, relates the drilling rate to the pressure differential at the bit. Experimental work [11, 12, 13] showed that X = 0.75.

(7) Formation abrasiveness parameter (A_f):

A_f is decreased with increase of formation abrasiveness. A_f can be determined from the following equation:

$$\frac{dD}{dT} = \frac{(R)}{A_f\ a\bar{m}} = \frac{R}{714.19\ A_f\ a\bar{m}}$$

$$\int_0^{D_f} 714.19\ a\ dD = \int_0^{T} \frac{R}{A_f\ \bar{m}}\ dT$$

$$U = \int_0^{D_f} 714.19\ a\ dD \qquad \text{(by definition)}$$

$$U = \frac{R}{A_f \bar{m}}\ T$$

Thus;

$$A_f = \frac{RT}{u\bar{m}} \tag{B-5}$$

where;

$$U = \int_0^{D_f} 714.19\ a\ dD = 714.19\ (0.309375\ D_f^3 + 3D_f^2 + D_f) \tag{B-6}$$

$$a = 0.928125\ D^2 + 6.0\ D + 1.0$$

$$R = N + 0.0000438\ N^3$$

$$\bar{m} = \frac{1}{714.19}\ (1359.1 - 714.19\ \log\ \bar{w})$$

$$\bar{w} = \frac{7.875\ w}{H}$$

H = hole or bit size, inch

T = total rotating time per bit

U = can be determined by using Table (D-2), which is the solution to (B-6).

(8) Formation drillability factor (C_f):

This factor reflects a formation's relative resistance to the drilling. Hard formations have low drillabilities, and soft formations (shales) have high values of C_f. This factor is constant, which would not change the calculated optimum variables under consideration.

Formation drillability factor is calculated from the drilling rate equation as following:

$$C_f = \frac{F(1 + C_2 \ D_f)(1 + \Delta P^x)}{T(W^y)(N^z) \ \log(\frac{kq\rho}{d_n \mu})} \qquad (B-7)$$

where;

F: is the total footage drilled by the bit, ft.

T: is the total time needed to drill F, hrs.

D_f: is the final tooth dullness of the bit.

Δp: is the differential pressure, 1000 psi.

W: weight on bit, 1000 lb.

N: rotary speed, rpm

q: volumetric flow rate, gal/min.

ρ: drilling fluid density, lb/gal.

μ: drilling fluid viscosity, c.p.

d_n: jet-nozzle diameter, inch

K: Reynold's number constant.

Determination of the Constants for Well #1

The field data which is presented in Security bit record [Table (E-12)] can be used to calculate the necessary constants for the optimization technique.

(1) Bearing-wear constant (b):

Table B-1

Bearing-Wear Constant

Run No.	Bit Size inch	Bit Type	W Wt. on bit 1000 lb	N RPM	F Feet	T hours	B_f	$b = \frac{NW^{cn}T}{B_f}$
1	12¼	OSC-3A	20	120	946	10	0.25	429,325
2	7 7/8	X3A	35	"	1652	28.5	0.25	2,832,619
3	"	"	"	"	684	16.0	0.25	1,590,242
4	"	J22	"	54	2255	96.25	1.0	1,076,209
5	"	F-3	"	"	1698	114	1.0	1,274,679
6	"	J-33	"	"	592	54.75	0.50	1,244,362
7	"	J-44	37.5	"	537	56	"	1,388,860
8	"	"	"	"	823	90.5	"	2,244,498
9	"	F-4	"	"	116	54.25	"	1,345,459
10	"	J-44	"	"	320	52.25	"	1,295,857
11	"	J-55R	48	50	288	56.0	"	1,862,301

(2) Tooth-Wear Constant, (C_2):

Table B-2

Tooth-wear Constant

Run No.	Bit Size	Bit Type	R_f ft/hr	R_o ft/hr	D_f	$C_2 = \frac{R_o - R_f}{R_f\ D_f}$
1	12½	OSC3A	94.6	128.0	.25	1.412
2	7 7/8	X3A	57.9	87.0	.375	1.340
3	"	"	42.7	68.0	.375	1.580
4	"	J22	23.4	32.0	.25	1.470
5	"	F3	14.8	40.0	.750	2.270
6	"	J33	10.8	34.0	1.0	2.148
7	"	J44	9.5	20.0	.875	1.263
8	"	"	9.1	19.0	1.0	1.088
9	"	F4	2.1	3.50	.625	1.067
10	"	J44	6.1	6.10	0	0
11	"	J55R	5.1	5.75	.125	1.020

(3) Rotary-Speed Exponent (Z):

Table B-3

Rotary-Speed Exponent

Run No.	Bit Size	Bit Type	N_1 rpm	R_1 ft/hr	N_2 rpm	R_2 ft/hr	$Z=\frac{\log(R_1/R_2)}{\log(N_1/N_2)}$
1	12¼	OSC3A	120	94.6	100	85	0.587
2	7 7/8	X3A	"	57.9	"	51	0.696
3	"	X3A	"	42.7	"	38	0.549
4	"	J22	"	23.4	"	28	0.692
5	"	F3	54	14.8	70	17.5	0.646
6	"	J33	"	10.8	"	12.75	0.640
7	"	J44	"	9.5	"	11.50	0.736
8	"	J44	"	9.1	"	11.0	0.731
9	"	F-4	"	2.1	"	2.52	0.703
10	"	J44	"	6.1	"	7.35	0.718
11	"	J-55R	50	5.1	75	7.0	0.780

(4) Weight Exponent (Y):

Table B-4

Weight Exponent

Run No.	Bit Size	Bit Type	W_1 1000 lb	R_1 ft/hr	W_2 1000 lb	R_2 ft/hr	$Y = \frac{\log(R_1/R_2)}{\log(W_1/W_2)}$
1	12¼	OSC3A	20	94.6	25	115.8	0.906
2	7 7/8	X3A	35	57.9	40	68.5	1.259
3	"	X3A	"	42.7	"	50	1.182
4	"	J22	"	23.4	"	27.75	1.277
5	"	F3	"	14.8	"	18	1.466
6	"	J-33	"	10.8	"	13	1.388
7	"	J-44	37.5	9.5	45	11.75	1.166
8	"	J-44	"	9.1	"	11.0	1.040
9	"	F4	"	2.1	"	2.5	0.956
10	"	J-44	"	6.1	"	7.25	0.947
11	"	J-55R	48	5.1	60	6.25	0.911

(5) <u>Formation Abrasiveness (A_f):</u>

Table B-5

Formation Abrasiveness

Run No.	Bit Size	Bit Type	N rpm	R	w 1000lb	$\bar{w}$ 1000lb	$\bar{m}$	T hrs	D_f	U	$A_f = \frac{R\,T}{U\,\bar{m}}$
1	12¼	OSC3A	120	195	20	12.857	.794	10	.25	316	7.772
2	7 7/8	X3A	"	"	35	35	.357	28.5	.375	581	26.794
3	"	"	"	"	"	"	"	16	"	"	15.042
4	"	J22	54	61	"	"	"	96.25	.25	316	52.045
5	"	F3	"	"	"	"	"	114	.750	1834	10.621
6	"	J33	"	"	"	"	"	54.75	1.0	3078	3.039
7	"	J44	"	"	37.5	37.5	.327	56	.875	2413	4.329
8	"	"	"	"	"	"	"	90.5	1.0	3078	5.485
9	"	F4	"	"	"	"	"	54.25	.625	1337	7.569
10	"	J44	"	"	"	"	"	52.25	0	1.0	9747
11	"	J55R	50	55	48	48	.221	56	.125	123	113.3

(6) Formation Drillability Factor (C_f):

Table (B-6)

Formation Drillability Factor

Run No.	Bit Size	Bit Type	F ft	T hrs	C_2	D_f	W 1000lb	Y	N rpm	Z	P 1000psi	q gal/min	ρ lb/gal	μ c.p.	dn, 1/32" d_1	d_2	d_3	C_f (Eq. B-7)
1	12¼	OSC3A	946	10	1.412	.25	20	0.906	120	0.587	0	269	8.33	1	12	12	12	0.087
2	7 7/8	X-3A	1652	28.5	1.340	.375	35	1.259	"	0.696	0	"	"	1	10	10	10	0.0059
3	"	"	684	16	1.580	.375	"	1.182	"	0.549	.136	"	9.1	10	9	9	10	0.0147
4	"	J22	2255	96.25	1.470	.25	"	1.277	54	0.692	.226	"	"	10	11	11	11	0.0043
5	"	F3	1698	114	2.27	.750	"	1.466	"	0.646	.333	"	9.2	10		"		0.0034
6	"	J33	592	54.75	2.148	1.0	"	1.388	"	.640	.401	"	9.3	10		"		0.0038
7	"	J44	537	56	1.263	.875	37.5	1.166	"	.736	.428	"	"	10		"		0.0031
8	"	"	823	90.5	1.088	1.0	"	1.040	"	.731	.470	"	"	13		"		0.0049
9	"	F4	116	54.25	1.067	.625	"	0.956	"	.703	.470	"	"	13		"		0.0014
10	"	J44	320	52.25	0	0	"	0.947	"	.718	.845	"	10	15		"		0.0023
11	"	J55R	288	56	1.020	.125	48	0.911	50	.780	.766	"	9.8	15		"		0.0017

APPENDIX C

ALGORITHM FOR THE METHOD OF HOOKE AND JEEVES USING FIBONACCI LINE SEARCH

(C-1) Fibonacci-Search Algorithm

The following algorithm is an outline of the Fibonacci method for minimizing a strictly quasiconvex function over the interval $[a_1, b_1]$. In the algorithm, ζ is the length of uncertainty, ε is step size, and n is the number of observations (such that $F_n > \frac{b_1 - a_1}{l}$).

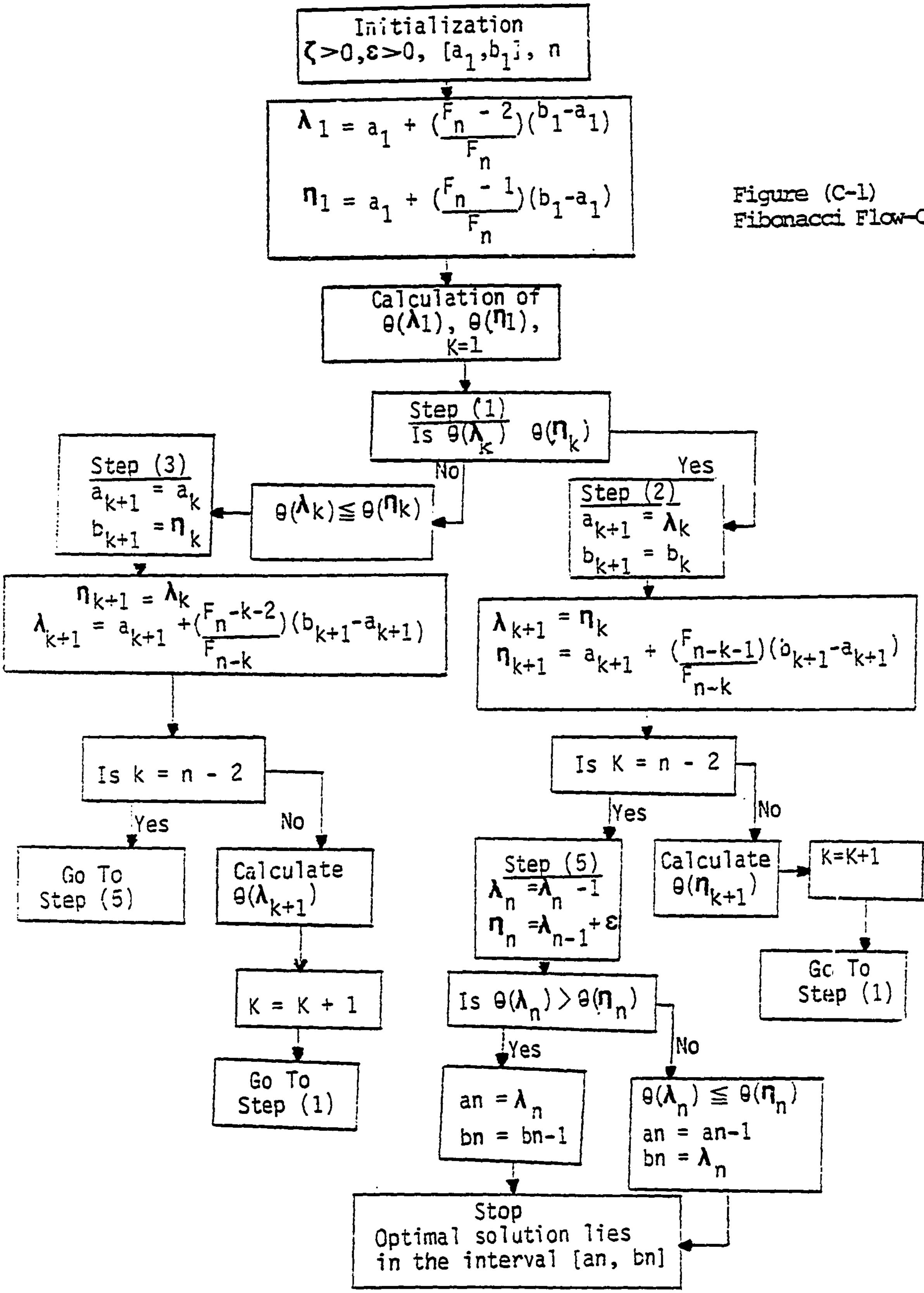

Figure (C-1)
Fibonacci Flow-Chart

(C-2) Hooke and Jeeves Algorithm

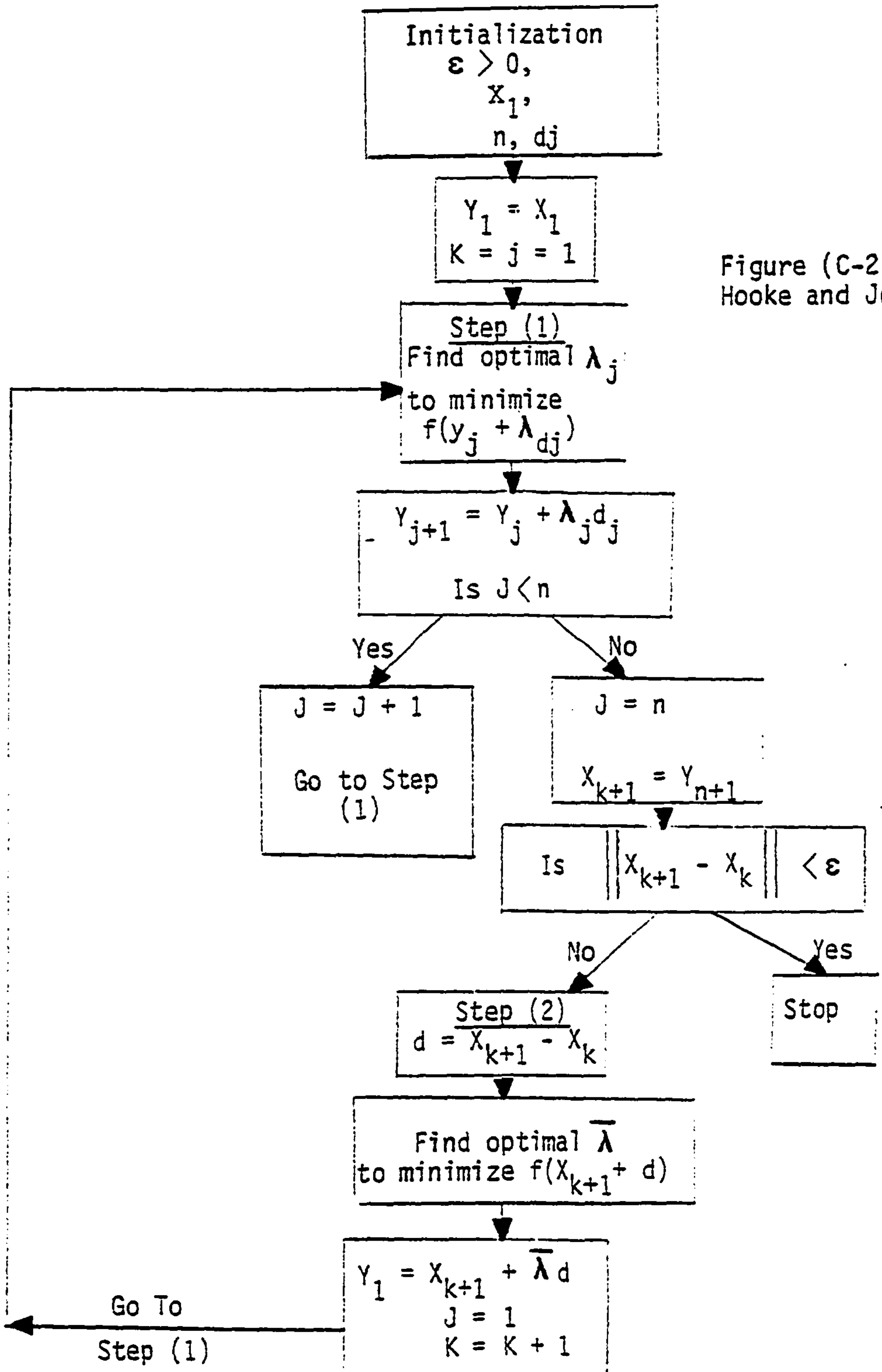

Figure (C-2)
Hooke and Jeeves Flow-Char

APPENDIX D

TABLES LISTING

Table D-1

Approximate Values of K and r

Formation Hardness Bity Types	Weight Exponent k	Speed Exponent r
Soft:		
S-3, S-4 (or equivalent)	0.95	0.7
Medium:		
M4N, M4L (or equivalent)	1.00	0.6
Hard:		
H7, H7U (or equivalent)	1.05	0.5

Table D-2

D Versus U and Z

D	U	Z when P = 0	0.5	1.0
1/8	123	123	105	89
2/8	316	316	236	179
3/8	581	581	389	268
4/8	920	920	563	357
5/8	1337	1337	756	446
6/8	1834	1834	967	536
7/8	2413	2413	1194	625
8/8	3078	3078	1437	714

Table D-3

N Versus R

N	R	N	R	N	R	N	R	N	R
10	10	50	55	90	122	130	226	190	488
15	15	55	62	95	132	135	242	200	548
20	20	60	69	100	143	140	259	225	720
25	26	65	77	105	155	145	278	250	929
30	31	70	85	110	168	150	297	275	1179
35	37	75	93	115	181	160	338	300	1474
40	43	80	102	120	195	170	384	350	2214
45	49	85	112	125	210	180	434	400	3183

Table D-4

$\bar{w}$ versus $\bar{m}$ and L

$\bar{w}$	$\bar{m}$	L	$\bar{w}$	$\bar{m}$	L	$\bar{w}$	$\bar{m}$	L
15	.726	6240	37	.334	1800	59	.132	766
16	.698	5840	38	.323	1725	60	.124	739
17	.672	5440	39	.311	1650	61	.117	714
18	.647	5080	40	.300	1578	62	.110	689
19	.624	4750	41	.290	1515	63	.103	665
20	.601	4439	42	.279	1460	64	.096	642
21	.580	4170	43	.269	1400	65	.090	620
22	.560	3920	44	.259	1340	66	.083	599
23	.541	3680	45	.249	1288	67	.076	578
24	.522	3470	46	.240	1240	68	.070	558
25	.505	3270	47	.230	1195	69	.064	538
26	.488	3080	48	.221	1150	70	.057	520
27	.471	2910	49	.212	1105	71	.051	502
28	.455	2770	50	.204	1063	72	.045	484
29	.440	2630	51	.195	1025	73	.039	467
30	.425	2496	52	.186	988	74	.033	450
31	.411	2370	53	.178	953	75	.027	434
32	.397	2260	54	.170	918	76	.022	418
33	.384	2160	55	.162	884	77	.016	403
34	.371	2060	56	.154	853	78	.010	388
35	.358	1963	57	.147	823	79	.005	373
36	.346	1880	58	.139	794			

Table D-5

$\bar{W}$ Versus $\bar{W}^k$

k

0.60	0.70	0.80	0.90	0.95	1.00	1.05	1.10	1.20
4.0	5.0	6.3	7.9	8.9	10.0	11.12	12.6	15.8
4.4	5.7	7.3	9.4	10.3	12.0	13.6	15.4	19.8
4.4	6.3	8.2	10.8	12.2	14.0	16.0	18.2	23.7
5.3	7.0	9.2	12.2	13.8	16.0	18.4	21.1	27.9
5.7	7.6	10.1	13.5	15.6	18.0	20.8	24.1	32.1
6.0	8.1	11.1	14.8	17.2	20.0	23.2	27.0	36.4
6.4	8.7	11.9	16.2	18.9	22.0	25.6	29.9	40.8
6.7	9.2	12.7	17.4	20.5	24.0	28.2	33.0	45.2
7.1	9.6	13.6	18.8	22.1	26.0	30.7	36.0	50.0
7.4	10.3	14.4	20.0	23.7	28.0	33.1	39.1	54.6
7.7	10.8	15.2	21.3	25.3	30.0	35.7	42.1	59.2
8.0	11.3	16.0	22.6	27.0	32.0	38.1	45.2	64.1
8.3	11.8	16.8	23.9	28.5	34.0	40.6	48.5	68.8
8.6	12.3	17.5	25.1	30.0	36.0	43.2	51.6	73.9
8.9	12.8	18.4	26.5	31.7	38.0	45.9	54.7	78.7
9.2	13.2	19.1	27.6	33.2	40.0	48.1	58.0	83.9
9.4	13.6	19.9	28.9	34.8	42.0	50.3	61.1	88.5
9.7	14.1	20.6	30.0	36.3	44.0	53.5	64.1	93.8
10.0	14.6	21.5	31.3	38.0	46.0	55.7	67.2	98.3
10.2	15.0	22.1	32.5	39.4	48.0	58.1	70.4	104
10.4	15.5	22.9	33.9	41.1	50.0	61.0	74.0	109
10.7	15.9	23.6	35.0	42.8	52.0	63.6	87.2	115
11.0	16.3	24.3	36.1	44.1	54.0	66.0	80.5	120
11.2	16.7	25.1	37.5	45.9	56.0	68.4	83.9	125
11.4	17.2	25.7	38.8	47.4	58.0	70.9	87.0	131
11.7	17.6	26.5	39.9	48.9	60.0	73.6	90.2	136
11.9	18.0	27.2	41.0	50.4	62.0	76.2	93.8	141
12.1	18.4	27.8	42.2	51.9	64.0	78.7	96.5	147
12.4	18.8	28.5	43.4	53.4	66.0	81.4	100	153
12.6	19.2	29.2	44.7	54.9	68.0	84.0	104	159
12.8	19.6	29.9	45.9	56.5	70.0	86.5	107	165
13.0	19.9	30.7	46.9	58.0	72.0	88.9	111	170
13.2	20.3	31.3	48.1	59.6	74.0	91.7	115	176
13.4	20.7	32.0	49.2	61.1	76.0	94.2	118	181
13.7	21.1	32.7	50.4	62.6	78.0	96.9	121	187

Table D-6

N Versus N^r

N	0.4	0.45	0.5	0.55	0.60	0.65	0.7	0.75	0.8
20	3.31	3.85	4.47	5.19	6.03	7.01	8.14	9.45	11.0
25	3.62	4.26	5.00	5.88	6.90	8.12	9.52	10.6	13.2
30	3.90	4.62	5.48	6.49	7.70	9.12	10.8	12.8	15.2
35	4.15	4.95	5.92	7.08	8.43	10.1	12.0	14.4	17.2
40	4.37	5.26	6.32	7.61	9.15	11.0	13.2	15.9	19.1
45	4.58	5.55	6.70	8.10	9.60	11.8	14.4	17.4	21.0
50	4.78	5.81	7.07	8.60	10.5	12.7	15.5	18.8	22.9
55	4.96	6.07	7.41	9.08	11.1	13.5	16.5	20.2	24.6
60	5.14	6.31	7.74	9.51	11.6	14.3	17.5	21.6	26.5
65	5.31	6.54	8.06	9.92	12.2	15.1	18.6	22.9	28.2
70	5.47	6.77	8.37	10.3	12.8	15.8	19.6	24.2	29.9
75	5.63	6.98	8.68	10.8	13.4	16.5	20.5	25.5	31.6
80	5.77	7.18	8.94	11.1	13.9	17.3	21.5	26.8	33.3
85	5.91	7.39	9.22	11.5	14.4	18.0	22.4	28.0	35.0
90	6.05	7.58	9.49	11.9	14.9	18.7	23.3	29.2	36.6
95	6.18	7.77	9.74	12.2	15.4	19.3	24.2	30.4	38.1
100	6.31	7.94	10.0	12.6	15.9	20.0	25.2	31.6	39.8
105	6.43	8.11	10.2	12.9	16.3	20.6	26.0	32.8	41.3
110	6.55	8.29	10.5	13.3	16.8	21.2	26.9	34.0	42.9
115	6.67	8.46	10.7	13.6	17.2	21.8	27.7	35.1	44.5
120	6.79	8.62	10.9	13.9	17.7	22.5	28.5	36.3	46.1
125	6.90	8.79	11.2	14.2	18.1	23.1	29.4	37.4	47.6
130	7.01	8.94	11.4	14.5	18.6	23.7	30.2	38.5	49.1
135	7.11	9.09	11.6	14.8	19.0	24.2	31.0	39.6	50.6
140	7.22	9.24	11.8	15.1	19.4	24.8	31.8	40.7	52.1
145	7.32	9.39	12.0	15.4	19.8	25.4	32.5	41.8	53.6
150	7.42	9.53	12.2	15.7	20.2	26.0	33.3	42.9	55.1
155	7.52	9.67	12.4	16.0	20.6	26.5	34.1	44.0	56.6
160	7.61	9.81	12.6	16.3	21.0	27.1	34.9	45.0	58.0
165	7.71	9.95	12.8	16.6	21.4	27.6	35.7	46.0	59.5
170	7.80	10.1	13.0	16.8	21.8	28.2	36.4	47.0	60.9
175	7.89	10.2	13.2	17.1	22.2	28.7	37.1	48.0	62.3
180	7.98	10.4	13.4	17.4	22.6	29.2	37.8	49.0	63.7
185	8.08	10.5	13.6	17.6	22.9	29.8	38.6	50.0	65.1
190	8.18	10.6	13.8	17.9	23.3	30.3	39.4	51.0	66.5

Table D-7

Formation and Bit Classifications

ROCK BIT CLASSIFICATION	BIT TYPE	FORMATION	CUTTING STRUCTURE	OFFSET & PIN ANGLE	BEARING SIZE & CONE SHELL THICKNESS	CUTTING ACTION: Chipping Crushing	CUTTING ACTION: Gouging Scraping
STEEL TOOTH BITS							
Soft Formation	S3S S3 S3T	Soft formations having low compressive strength and high drillability (soft shales, clays, red beds, salt, soft limestone, unconsolidated formations, etc.)	Widely spaced, long teeth for deep penetration; use of removal and interruption design for efficient cleaning and less steel on bottom resulting in fast penetration rates.	Maximum offset; pin angle designed for twisting, tearing, gouging action and fast penetration in soft formations.	Generally, smaller bearings, thinner cone shell to allow for longer teeth that result in faster penetration rates.		
Medium Soft Formation	S4 S4T S4TG S6	Soft to medium formations or soft interspersed with harder streaks (firm, unconsolidated, or sandy shales, red beds, salt, anhydrite, soft limestone, etc.)					
Medium Hard Formation	M4N M4NG M4L M4LG	Medium to medium hard formations (harder shales, sandy shales, shales alternating with streaks of sand and limestone, etc.)	Medium length teeth, closer spacing for combined penetration, greater strength and resistance to breakage. Some types with removals and interruptions for less steel on bottom and faster penetration rates.	Medium offset and pin angle for combined scraping-gouging action and chipping-crushing action.	Medium bearings and cone shell thickness. Medium heavy weights with medium length teeth.		
Hard Formation	H7 H7G H7T H7TG H7U H7UG H7SG HCG	Medium hard abrasive to hard formations (high compressive strength rock, dolomite, hard limestone, hard slaty shale, etc.)	Short, stubby teeth closely spaced for crushing action with maximum resistance to breakage.	Little offset for chipping-crushing action with no gouging on hard formation or formations with high compressive strength.	Large bearings and thick cone shells, for heavy weights, necessary to overcome harder formations.		
CARBIDE INSERT BITS							
Medium Soft Formation	S84 S86 S88	Soft, unconsolidated, low compressive strength, high drillability, clays, shales, salts, etc. of considerable interval.	Soft Insert: Maximum extension of tooth shaped inserts.	Soft insert provides scraping-crushing action.	Soft insert type provides thinner shell section and smaller bearing.	Primarily gouging, scraping—with a minimal chipping, crushing requirement.	
Medium Formation	M84 M88 M89T	Softer segment of hard formations (lime, dolomite, and hard sandy shale)	Medium Insert: Medium extension of Wedge Shaped Insert.	Medium insert—slight scraping with crushing action.	Medium insert type provides thicker shell sections for strength.	Mostly chipping, gouging with some crushing action.	
Medium Hard Formation	M89	Medium segment of hard formations (chert, granite, basalt, quartzitic formations)				Primarily crushing with some scraping action.	
Hard to Extremely Hard Formation	H88 H99 H100	Hardest of hard abrasive formations (quartzite and hard quartzite sands)	Hard Insert: Minimum extension, Conical shaped inserts with maximum strength.	Hard insert—crushing action.	Hard insert type provides large bearings with thick shell sections.	Crushing and fracturing action only	

Table D-8

Security Rock Bit Comparison Chart

SECURITY ROCK BIT COMPARISON CHART

Series	Classification	Types	SECURITY Standard (1)	SECURITY T Gauge (2)	SECURITY Gauge Insert (3)	SECURITY Sealed Bearing (4)	SECURITY Sealed Bearing & Gauge (5)	SECURITY Friction Sealed Bearing (6)	SECURITY Friction Bearing & Gauge (7)	SECURITY Directional (8)	HUGHES Standard (1)	HUGHES T Gauge (2)	HUGHES Gauge Insert (3)	HUGHES Sealed Bearing (4)	HUGHES Sealed Bearing & Gauge (5)	HUGHES Friction Sealed Bearing (6)	HUGHES Friction Bearing & Gauge (7)	REED Standard (1)	REED T Gauge (2)	REED Gauge Insert (3)	REED Sealed Bearing (4)	REED Sealed Bearing & Gauge (5)	REED Friction Sealed Bearing (6)	REED Friction Bearing & Gauge (7)	SMITH Standard (1)	SMITH T Gauge (2)	SMITH Gauge Insert (3)	SMITH Sealed Bearing (4)	SMITH Sealed Bearing & Gauge (5)	SMITH Friction Sealed Bearing (6)	SMITH Friction Bearing & Gauge (7)
1	MILLED TOOTH SOFT	1	S3S			S33S		S33SF		S3SJ	OSC-3A			X3A				Y11			S11		F11		DS			SDS			
		2	S3	S3T		S33				S3JO	OSC-3			X3		J3		Y12	Y12T		S12		F12		DT	DTT		SDT			
		3	S4	S4T		S44		S44F			OSC-1G	C1C	ODG	X1G	XDG	J4		Y13	Y13T		S13	S13G			DG	DGT	DGH	SDG	SDGH		
		4								DS/ DSS	OSC												F14		K2		K2H				
2	MILLED TOOTH MEDIUM	1	M4N			M44N		M44NF			OWV/ OW4		ODV/ OD4	XV	SDV			Y21		Y21G	S21	S21G	F21		V1		V1H				
		2	M4								WO							Y22					F22		V2		V2H	SV	SVH		
		3	M4L			M44L		M44LF		DM/ DMM	OWC			XC				Y23		Y23G	S23	S23G			T2		T2H	ST2			
		4																													
3	MILLED TOOTH HARD	1	H7	H7T		H77					W7*		WD7	X7	XD7	J7		Y31		Y31G	S31	S31G		F31G	L4		L4H	SL4	SL4H		
		2									W7R-2							Y32		Y32G											
		3			H7SG		H77S											Y33													
		4				H77C		H77CF			WR		WDR	XWR		J8	J08	Y34		Y34G	S34	S34G	F34	F34G	WC		WCH	SWC	SWCH	FWC	
4																															
5	INSERT SOFT	1					S84		S84F								J22							F52/ FP52					2JS		F2
		2					S86		S86F								J33					S53		F53/ FP53					3JS		F3
		3			S8		S88		S88F	OS88																					
		4																						F54/ FP54							
6	INSERT MEDIUM	1							M84F				A44		X44		J44												4JS		F4/ F45
		2			M8		M88		M88F/ M89TF						X55R		J55R					S62		F62/ FP62					47JS		F47
		3							M89F				A55		X55		J55					S63		F63/ FP63					5JS		F5
		4																				S64		F64/ FP64					57JS		F57
7	INSERT HARD	1															J77														
		2			H8		H88		H88F				A88		RG7X		J88			Y72		S72		F72/ FP72					6JS		F6
		3																		Y73		S73		F73/ FP73					7JS		F7
		4			H9		H99		H99F		R1				RG1X		J99			Y74		S74		F74/ FP74					8JS		F8
8	INSERT EXTRA HARD	1																													
		2																													
		3			H10		H100		H100F						RG20X					Y83		S83		F83/ FP83					9JS		F9
		4																													

NOTE: Bit classifications are general and are to be used only as simple guides. All bit types will drill effectively in formations other than those specified. This chart shows the relationship between the specific bit types.

Table D-9

Price List Rock Bits

STEEL MILLED TOOTH BITS

BIT SIZE RANGE	STANDARD SIZES*	NON-SEALED		SEALED BEARING		SEALED JOURNAL BEARING		DEVIATION CONTROL	
		Jet or Regular	Gauge Protection (G or SG)	Jet Circulation	Gauge Protection (G or SG)	Jet Circulation	Gauge Projection (G or SG)	Non-Sealed	Sealed
4½ — 4¾	4¾	371.00				525.00			
5⅝ — 6¼	5⅞, 6, 6⅛, 6¼	483.00	533.00			679.00			
6½ — 6¾	6½, 6¾	504.00	580.00			700.00			
6⅞ — 7⅞	7⅞	577.00	664.00	661.00	760.00	705.00	825.00	750.00	1,050.00
8⅜ — 9	8⅜, 8½, 8¾	632.00	735.00	754.00	867.00	806.00	945.00	835.00	1,130.00
9½ — 9⅞	9½, 9⅞	753.00	875.00	901.00	1,045.00	1,067.00	1,245.00	1,085.00	
10⅝ — 11	10⅝, 11	918.00		1,102.00		1,260.00	1,475.00		
12 — 12¼	12¼	1,071.00	1,250.00	1,262.00	1,460.00	1,497.00	1,750.00	1,520.00	1,860.00
13¾ — 15	13½, 13¾, 14¾	1,845.00							
17½ — 18½	17½	3,033.00	3,488.00	3,640.00					

TUNGSTEN CARBIDE INSERT BITS

BIT SIZE RANGE	STANDARD SIZES*	NON-SEALED ROLLER BEARING	SEALED ROLLER BEARING	SEALED JOURNAL BEARING	DEVIATION CONTROL	
					Sealed Roller Bearing	Sealed Journal Bearing
4½ — 4¾	4¾			1,880.00		
5⅝ — 6¼	5⅞, 6, 6⅛, 6¼	1,671.00	1,840.00	2,090.00		
6½ — 6¾	6½, 6¾	1,760.00	1,940.00	2,300.00		
7⅝ — 7⅞	7⅞	1,845.00	2,030.00	2,555.00	2,588.00	
8⅜ — 9	8⅜, 8½, 8¾	2,080.00	2,335.00	2,955.00	3,076.00	
9½ — 9⅞	9½, 9⅞	2,345.00	2,974.00	3,775.00		
10½ — 11	10⅝, 11	2,696.00	3,308.00	4,415.00		
12 — 12¼	12¼	3,522.00	4,572.00	5,720.00		6,000.00
13½ — 15	13¾, 14¾	5,800.00	6,875.00	9,075.00		
17½	17½	8,000.00	9,560.00	11,840.00		

*NON-STANDARD BITS ARE PRICED 25% ABOVE STANDARD BITS IN THE SAME SIZE RANGE.

PRICING POLICY

1. These prices apply to the United States (excluding Alaska) and direct export shipments from the United States and United Kingdom. Prices for Alaska, Canada and purchases from local stock in International areas are published separately and are available on request.
2. All sales are subject to Standard Terms of Sale and Rental for Security Rock Bits and Drilling Tools.
3. Domestic prices are F.O.B. Dallas, Texas *(see opposite page for weights and rates).*
4. Export prices are F.A.S. Houston, Texas or United Kingdom port and include packaging for export shipment.
5. Terms of payment are net 30 days, from date of invoice.
6. Any tax or levy imposed by city, county, state or other Governmental bodies, is added to prices quoted.
7. Prices are subject to change without notice.

TABLE D-10

OPERATING DATA FOR DUPLEX PUMPS

MANUFACTURER	MODEL	STROKE LENGTH (in.)	INPUT H.P. REQ'D.	PUMP SPEED (SPM)	MAXIMUM DISCHARGE PRESSURE (psi) USING LINER SIZE SHOWN												
					5"	5-¼"	5-½"	5-¾"	6"	6-¼"	6-½"	6-¾"	7"	7-¼"	7-½"	7-¾"	8"
NATIONAL	C-150-B	12	220	70	1205	1085	985	895	820	750	690	640	595	550			
	K-380	14	380	70	2100	1875	1675	1520	1370	1255	1145	1055	970	900	835		
	H-1250	16	1250	65					4135	3765	3445	3165	2915	2700	2505	2335	
	N-1100	16	1100	65					3640	3305	3025	2785	2565	2375			
OILWELL	12-PLD	12	100	70	540	485	440	400	365								
	214-P	14	350	70	1700	1525	1375	1250	1140	1050	960	890	820	765			
	816-P	16	700	65			2735		2235		1875	1725	1593	1478		1280	1197
	218-P	18	500	65	2040				1370		1155	1085	985	915			
GARDNER DENVER	GXN	14	500	70	2435		1974		1633		1377	1271	1177	1094			
	GXP	16	700	70	3060		2470		2040		1712	1578	1460	1357		1171	
	GXR	18	1000	60					3113	2815	2578	2373	2194	2035	1903	1172	
	GXH	18	1250	60					3942		3281	3035	2793	2580	2400	2232	
EMSCO	D-300	14	300	70	1430	1280	1162	1060	965	886	815	754	698	650	602		
	D-375	14	375	70	1777	1600	1415	1318	1156	1104	1018	939	871	810	744		
	D-1000	18	1000	60				3480	3153	2871	2635	2418	2229	2068	1917	1782	1669
	D-1250	18	1250	60					4144	3758	3432	3141	2891	2667	2471		

TABLE D-11

OPERATING DATA FOR TRIPLEX PUMPS

MANUFACTURER	MODEL	STROKE LENGTH (in.)	INPUT H.P. REQ'D.	MAX PUMP SPEED (SPM)	MAXIMUM DISCHARGE PRESSURE (psi) USING LINER SIZE SHOWN																
					3"	3-1/4"	3-1/2"	3-3/4"	4"	4-1/4"	4-1/2"	4-3/4"	5"	5-1/4"	5-1/2"	5-3/4"	6"	6-1/4"	6-1/2"	6-3/4"	7"
NATIONAL	8P-80	8-1/2	800	175					5005	4505	4020	3605	3265	2950	2690	2460	2260				
	10 P-130	10	1300	150									5245	4765	4335	3965	3645				
	12P-160	12	1600	125											6335	4880	4485	4130	3820	3540	3295
OILWELL	560 PT	8	560	175					3780		2990		2420		2000	1830	1680		1430		
	1400 PT	10	1400	150									6000		4714		3960		3300		
	1700 PT	12	1700	150									5000		4714		3960		3300		
GARDNER DENVER	PY-7	7	500	160							3150		2550		2110		1770		1510		1300
	PJ-8	8	275	175	3118	2657	2200		1753		1386		1122								
	PA-8	8	310	175					2000		1650										
	PZ-9	9	1000	147							5340		4330		3570		3000				
EMSCO	F-350	7	350	175	4800	4100	3535	3080	2705	2390	2135										
	F-750	8	750	175					5075	4485	4000	3590	3230	2035	2675	2450	2250				
	F-1300	12	1300	120									6460	4950	4516	4126	3791	3494	3260	2997	2789
	F-1000	12	1600	120											5558	5078	4665	4289	4012	3688	3423
IDECO	T-380	12	380	70									1406	1269	1146	1042	955	874	803	741	687
	T-440	12	440	65					2960		2260		1785	1600	1445	1300	1190	1090	1000	920	863

TABLE D-12

BIT RECORD FOR WELL # 1

CONTRACTOR: E. W. Moran RIG NO: # 4
COMPANY: El Paso Nat Gas FIELD: Binger
WELL NAME: Minnie Shaw WELL NO: # 1
STATE: Okla COUNTY: Caddo
SEC: [illegible] T'SHIP: 9N RANGE: 10W

DRAWWORKS: Wilson
PUMP MAKE 1: Gardner
PUMP MAKE 2: Tri State

DC: 6 1/2 x 2 1/4 x
DRILL PIPE: 4 1/2
T.J. TYPE: XH

SPUD: 6 / 24 / 77
U.S.: 6 / 26 / 77
INTER: / /
T.D.: 8 / 6 / 77
TOTAL DAYS: 44
TOTAL ROT HRS: 626 3/4

TP DRILLERS: Harrion Johnson TP; Harry Mitchell; I D Beggs TP
WATER SOURCE:
FUEL SOURCE:

RUN NO	SIZE	TYPE	SERIAL NO	JETS 1	JETS 2	JETS 3	DEPTH OUT	FEET	HOURS	FEET PER HOUR	WT. 1000 LBS.	R.P.M.	PUMP PRESS	PUMP LINER	PUMP SPM	MUD WT	MUD W.L.	MUD FV	MUD PV/YP	MUD % SOL	VER DEV	DULL COND T	DULL COND B	DULL COND G	DULL COND NO	REMARKS	DATE	DEPTH
1	12 1/4	OSC3AJ	DL448	12	12	12	1040	946	10	9	40	120	1200	5 1/2	52	Water						2	2	I	N	Fishing	7/8	5631
2	7 7/8	X3A	DM338	10	10	10	2692	1652	28 1/2	57	35	120	1600	5 1/2	52							3	2	I	N		7/16	
3	7 7/8	X3A	DM356	10	10	10	5376	684	16	42	35	120	1600	5 1/2	52	9.1	NC	29	NC	.52		3	2	I	N		7/24	5631
4	7 7/8	J22	DF421	9	9	10	5631	2255	96 1/4	23	35	54	1600	5 1/2	52							2	F	I	N			
5	7 7/8	F3	103KE	11	11	11	7329	1678	114	14	–	–	–	–	–	9.2	18	40	11/3	6		6	SE	L	1/16	Con cut		
6	7 7/8	J33	631	–	–	–	7921	592	54 1/4	10	–	–	–	–	–	9.3	13.6	40	14/7	6.5		8	SE	C	1/16	BT		
7	7 7/8	J44	332	–	–	–	8458	537	56	9	35/40	–	–	–	–							2	SE	L	1/8	BT		
8	7 7/8	J44	241	–	–	–	9281	823	92 1/2	9	–	–	–	–	–	9.3	11	40	13/7	6.5		8	SE	C	1/16	BT		
9	7 7/8	F4	444	–	–	–	9397	116	53 3/4	2	–	–	–	–	–							5	SE	1		BT		
10	7 7/8	J44	444	–	–	–	9717	320	52 1/4	6	–	–	–	–	–	10	11.5	52	15/12	11		0	SE	L	N		8/1	9703
11	7 7/8	J55R	420	–	–	–	10005	288	54 3/4	5	48	50	–	–	–	9.8	8	50	15/15	11		L	SE	1	N		8/4	10047

DENOTE BY INI-NO (L)-LIGHT (M)-MEDIUM (H)-HEAVY ROUNDING OF GAGE

SUBJECT TO PROVISIONS ON REVERSE SIDE