FINANCIAL RATIOS AND FRAUD –
AN EXPLORATORY STUDY
USING CHAOS THEORY

By

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CHAPTER I

INTRODUCTION

Fraud in business is a matter of grave social and economic concern. "The U.S. Chamber of Commerce estimates that the annual cost of fraud exceeds $100 billion" [Glover and Aono 1995, 3]. Fraudulent financial reporting is a critical problem for external auditors, both because of the potential legal liability for failure to detect false financial statements and because of the damage to professional reputation that results from public dissatisfaction about undetected fraud. Current professional standards maintaining that the auditor is not responsible for detecting management fraud because of the inherent limitations of the audit process do not serve to prevent litigation against the auditor or significant payments by the auditor in cases of management fraud. Between 1990 and 1993, the (then) Big 6 alone paid out over $1 billion to settle cases related to fraud. According to a 1992 Big 6 joint statement entitled "The Litigation Crisis in the United States: Impact on the Accounting Profession," litigation expenses equal approximately 11 percent of audit revenues [Glover and Aono 1995]. The Private Securities Litigation Reform Act of 1995 was partly motivated by the expanding litigation risk faced by auditing practitioners. One provision of the Reform Act requires that audits include procedures designed to provide reasonable assurance of detecting illegal acts that would have a direct and material effect on the determination of financial statement amounts [King and Schwartz 1997].

Increasing pressure to reduce fraudulent financial reporting over the past 30 years has resulted in new laws, commission reports, and standards. Public concern for fraud detection began during the early 1970s with the famous Equity Funding case. Such concern eventually led to the Senate's Metcalf Commission and the American Institute of Certified Public Accountants' (AICPA) Cohen Commission. Various recommendations made by these commissions were eventually adopted by the Financial Accounting Standards Board (FASB) and the Auditing Standards Board (ASB). During the mid-1980s, the savings and loan debacle created a new wave of public concern. Congressional inquiry led to the formation of the Treadway Commission whose charge was to prescribe effective recommendations to guide the ASB's development of standards to help prevent and detect fraud.

In 1988, the ASB issued nine new "expectation gap" standards, Statement on Auditing Standards (SAS) 53 through 61. They were designed to (1) outline clearly the external auditor's role concerning fraud, (2) enhance overall audit procedures for detecting and preventing fraud, and (3) enhance communications between the auditor and management, the audit committee and the public [Glover and Aono 1995]. The SAS 53 was designed to narrow the gap between clients' expectations regarding the auditor's responsibility to detect fraud during an audit and what that responsibility actually is [Levy 1989]. However, in 1996, the U. S. General Accounting Office's (GAO) report *The Accounting Profession, Major Issues: Progress and Concerns* identified responsibilities for fraud as a major unresolved issue. The GAO contended that an expectation gap still exists for auditor responsibilities and performance related to detection of fraud [Carmichael 1997].

In 1996, the ASB issued SAS 82 *Consideration of Fraud in a Financial Statement Audit* in hopes of closing the seemingly ever-present expectation gap. The standard provides expanded operational guidance on the auditor's consideration of material fraud in conducting a financial statement audit. The revised standard makes it clear that the auditor's responsibility is framed by the concepts of materiality and reasonable assurance. The auditor should consider a compendium of risk factors or "red flags" [McConnell and Banks 1997]. While the standard clarifies the auditor's role, it does not increase the auditor's responsibility to detect fraud [Mancino 1997].

Concern for fraud detection is not limited to the U. S. financial reporting environment. In New Zealand, General Auditing Standard AS-210 requires the auditor to assess the risk of fraud and error at the planning stage of an audit. The auditor must design audit procedures to obtain reasonable assurance that material fraud or error is detected [Carslaw 1996]. In the United Kingdom, the purpose of SAS 110 *Fraud and Error* is to establish standards and provide guidance on the auditor's responsibility to consider fraud and error in an audit. It specifically requires auditors to take into account the risk that fraud may cause material misstatements in the financial statements. Warning signs that may indicate increased risk of fraud or error are also included in the standard [McAlpine 1995].

Users want auditors to assume more responsibility for the detection of fraud. A "clean" audit opinion is far from a warranty or certification that fraud has not occurred. An audit under generally accepted auditing standards (GAAS) is not a proper vehicle for fraud detection [Levy 1989]. Detection of fraud is not a primary objective of a financial statement audit and auditors are not particularly well trained to uncover sophisticated fraud [Johnston 1995]. There are a number of reasons auditors do not detect fraud. Some frauds are well beyond the scope of the audit. Some are too well concealed; others are too small. Frauds that tend to involve upper levels of management are not normally detected by traditional audit procedures [Wells 1990, 1992]. Two concepts that are integral to the GAAS audit make disclosure of certain frauds more difficult. They are the auditor's reliance on internal controls and the concept of materiality. The schemes that are most difficult to detect are those expressly designed to work within the framework of existing controls. There is no "cookbook" approach to detecting fraud [Levy 1985].

Although the number of fraudulent financial statements is small in relation to the number of audits performed, cost-effective methods are needed to improve their detection and deterrence. The combination of downward pressure on audit fees and demands that auditors take more responsibility for detecting misstatements in their clients' financial information has led auditors to seek audit procedures that are both efficient and effective. CPAs have increased their use of analytical procedures in an effort to improve audit efficiency and effectiveness. The Treadway Commission recommended that the ASB require the use of analytical review procedures (ARPs) on all audits to improve the detection of fraudulent financial reporting [Wheeler and Pany 1996]. However, the question still remains unresolved as to exactly what types of errors or irregularities are detected effectively through these procedures [Blocher 1992].

Despite the past efforts of researchers, there is a need for more detailed investigation of the precise cababilities of audit practice in detecting fraud [Humphrey et al 1993]. Better fraud detection depends on improved audit procedures that may result, in part, from expanded research on "red flag" conditions that indicate the potential for fraud [Elliott & Jacobson 1986].

Practical evidence is needed to better understand the error detection capabilities and cost savings potential of ARPs. Analysis of ratios of account balances is a widely applied attention-direction procedure. Yet little is known of the ability of ratio analysis to identify material monetary error in actual accounting data [Kinney 1987]. Failing to detect fraud when it occurs (i.e., Type II error) is much more costly to CPA firms than detecting fraud when it does not occur (i.e., Type I error). Type I errors lead to inefficient audits while Type II errors lead to potential litigation [Hansen et al 1996]. Additional research is needed in the areas of actual use of ARPs in the audit and the specific techniques being used in order to understand better how ARPs can be used best in the future [Tabor and Willis 1985].

In a discussion of Kinney's [1979] study, Lev [1979] posited that " . . . perhaps some information can be obtained from proven cases of errors or irregularities, such as bankruptcies and fraud" [Lev 1979, 167]. This comment was made more than twenty years ago, yet only a small number of studies have used such an approach. The Committee of Sponsoring Organizations (COSO) of the Treadway Commission sponsored a descriptive research study by Beasley et al [1999] that provides a comprehensive analysis of fraudulent financial reporting occurrences investigated by the SEC subsequent to the issuance of the 1987 Treadway Commission Report.[1]

In the Fall 1997 issue of *The Auditor's Report*, the American Accounting Association (AAA) encourages auditing practitioners and faculty to engage in research directed toward assisting auditors in preventing and detecting fraud. The report suggests examining data on prior fraud litigations to find "lessons" for auditors to follow. It posits the following question – "can analytical procedures be better used to detect fraud warning signals?" [Landsittel and Bedard 1997, 4]. SAS 82 has little guidance on the role of analytical procedures in fraud detection. Ratio analysis may serve such a purpose. Research in this area would have been helpful to the ASB Task Force in going beyond generic guidelines to offer more specific direction to auditors [Landsittel and Bedard 1997].

One can conclude from the above discussion that there is a strong need for auditing research approaches that enable the auditing practitioner to identify indications of potential fraud. This researcher chooses to accept the AAA's challenge and to pursue such exploration. The approach taken is not within the mainstream of current auditing research. Instead, this study will examine fraud within the context of chaos theory and its unique methodology. Financial statements are the product of a

[1] Findings from this study are discussed in Chapter IV.

dynamical system which has a feedback loop whereby the output of one period is the starting point for the subsequent period. According to chaos theory, such dynamical systems may be deterministic yet appear random and unpredictable. Prior research indicates that most dynamical systems are actually non-linear thereby rendering linear models ineffectual. Chaos theory methodology has various tools for measuring the non-linearity of a system. The objective of this research study is to use these tools to examine financial ratios of both fraudulent and non-fraudulent firms for evidence of non-linearity or the lack thereof. The findings will provide evidence as to the predictability of fraud and whether the pursuit of such should be based on a linear or a non-linear model. The development of such a model is beyond the scope of the current research study.

The remainder of this dissertation is organized as follows. Chapter II provides a review of the relevant research concerning ARPs and ratio analysis. Chapter III is an introduction to chaos theory and the terminology and concepts applicable to the ensuing research. A literature review of the use of chaos theory in the various research realms of the physical sciences, the social sciences and accounting is then presented. Chapter IV discusses various methodological issues, including sample selection, ratios chosen for inclusion in the study, and the unique tests that will be used to analyze the data. Chapter V presents the analysis and results of the empirical examinations performed for this study. Chapter VI presents the conclusions of the study, its inherent limitations and implications for future research.

CHAPTER II

LITERATURE REVIEW

Introduction

During the past thirty years, there has been wide-spread public concern for fraud detection. Both the government and the accounting profession have responded as evidenced by the creation of new laws, commission reports as well as accounting and auditing standards. The concern for fraud has also impacted accounting academicians, who have directed their research efforts accordingly. The purpose of this chapter is to review the relevant literature. The next section discusses analytical review procedures research, while the subsequent section explores the findings of ratio analysis research.

Analytical Review Procedures (ARPs)

ARPs have been posited to be a useful tool for identifying irregularities and/or fraud [Thornhill 1995]. ARPs is the name used for a variety of techniques the auditor can utilize to assess the risk of undetected error in financial records. These procedures involve the analysis of trends, ratios, and reasonableness tests derived from an entity's financial and operating data. Ratio analysis can identify and explain a company's financial strengths and weaknesses as well as changes in its long-term trends of financial position, results of operations and cash flows. The primary function of ratios is to act as indicators or red flags to point to areas of acceptable or unacceptable results or conditions. While these analytical procedures are well known and widely used, there is a general lack of understanding of how they are properly applied and how much reliance should be placed on them.

An outline is presented in Table 1 of the major empirical studies examining the use of ARPs for the detection of errors and/or irregularities in financial statements that were conducted during the period 1979 – 2000. Summarized conclusions include the following:

- Percent of errors identified by ARPs ranged from 15% to 50% [Blocher and Cooper 1988, Calderon and Green 1994, Hylas and Ashton 1982, Kreutzfeldt and Wallace 1986, Wright and Ashton 1989].
- Some studies found ARPs not effective when done at an aggregate level [Blocher 1992, Kinney 1987] while others found ARPs not effective when applied to quarterly data [Wheeler and Pany 1996].
- Wide range of ARPs is generally applied extensively and include both financial and operating data [Blocher 1992, Blocher and Cooper 1988, Daroca and Holder 1985, Holder 1983].
- There is a predominant use of simple, quantitative ARPs [Daroca and Holder 1985, Kinney 1979, Tabor and Willis 1985, Wright and Ashton 1989].

TABLE 1

ANALYTICAL REVIEW PROCEDURES RESEARCH

	Objectives of Study	Summary of Results
Bernardi [1994]	examined influence of client integrity and competence, auditor's prior belief of existence of fraud, auditor cognitive style on fraud detection	auditors insensitive to client integrity and competence data; prior expectations influence detection; managers detect fraud at a higher rate
Blocher [1992]	examined to what extent ARPs are used by various groups to detect management fraud, how effective ARPs are in detecting fraud, differences in usage of trend analysis, ratio analysis, and modeling among groups, experience, cognitive skills, differences in decision processes among groups	use of ARPs vary significantly; 4 of 24 fraud cases detected by ARPs; trend analysis used extensively; other ARPs used more extensively by external auditors and controllers; internal auditors observed more fraud cases but not detected by ARPs; external auditors used a risk-based approach; ARPs not expected to be effective when done at aggregate level
Blocher and Cooper [1988]	examined how ARPs are used and if effective in detecting materially seeded errors; used verbal protocal analysis	all auditors used ratio and trend analysis; detected <.5 of the errors; balance sheet relationships perform better
Calderon and Green [1994]	examined internal control, personnel most likely to commit fraud, personnel actions, initial signal of fraud, transaction cycle, business types	professional/managerial involved in 455 of cases; ARPs were initial signal in 15% of cases; fraud most prevalent in revenue/expenditure cycles
Coglitore and Berryman [1988]	identified cases where ARPs would have revealed unusual relationships or changes in relationships and lead to detection of material misstatement	properly employed ARPs would reveal a number of misstatements
Hylas and Ashton [1982]	examined initial event signaling error, cause of error, industry type, entity size	27.1% of errors signaled by ARPs; 10 errors thought intentional, 2 signaled by ARPs; 33% of errors caused by client personnel inexperience

TABLE 1

ANALYTICAL REVIEW PROCEDURES RESEARCH

	Objectives of Study	Summary of Results
Kaminski et al [1998]	examined 21 financial ratios of fraud versus non-fraud firms for the fraud year -/+ 3 years	certain financial ratios were significantly different; number of significant ratios increased in the fraud year + time period
Kinney [1987]	examined 3 investigation rules: simple percentage change, statistical standardized change, pattern analysis of cross-sectional changes	analysis more effective on disaggregated data; patterns of deviations from expectation over several related ratios help identify errors
Loebbecke et al [1989]	examined detailed information about one material irregularity selected by participant; determined presence of indicators per SAS 53	73% were management frauds; substantive tests were most effective at revealing irregularities; encountering a material irregularity was a rare event
Persons [1995]	examined variables for estimating models of fraudulent financial reporting, model estimation method, and assessment of models' predictive ability	fraud firms are smaller, have higher financial leverage, lower capital turnover, higher proportion of current assets than nonfraud firms; models tested outperform naïve strategy
Wheeler and Pany [1996]	examined effects of 8 common errors on 15 ARPs (8 ratios, 7 accounts); examined 6 models and five investigation rules	the ARPs did not signal very well applied in isolation to quarterly data; lowest error rates where substantive test would be direct recomputation
Wright and Ashton [1989]	examined types of errors, income effects, causes of errors, initial events identifying adjustments, internal control strength, ordering bias	16% of adjustments signaled by ARPs; ARPs signal more larger adjustments; 30% of adjustments signaled by ARPs were identified in review phase; 80% of adjustments signaled by ARPs resulted from simple comparison with prior year's balances

- Substantive tests are most effective at revealing irregularities [Loebbecke et al 1989].
- Prior period adjustments are a reasonable indicator of current year's error [Kinney 1979] and from 33% to 50% of errors caused by client personnel inexperience [Hylas and Ashton 1982, Kreutzfeldt and Wallace 1986].
- Only a few studies looked exclusively at detecting fraud [Bernardi 1994, Blocher 1992, Calderon and Green 1994, Loebbecke et al 1989, Persons 1995].

Much of this research addressed the effectiveness of analytical procedures. Three general approaches have been followed: ". . . (1) ex post analyses of errors discovered in actual audits, (2) application of analytical procedures to simulated accounting data seeded with errors, and (3) application of analytical procedures to actual accounting data seeded with errors" [Wheeler and Pany 1996, 559]. As the previous summary indicates, the conclusions reached by these approaches have been quite disparate.

Of the fraud studies, only two looked exclusively at financial ratios. Persons [1995] found that financial leverage, capital turnover and asset composition were significant factors influencing the likelihood of fraudulent reporting. Kaminski et al [2000] provides additional empirical support that some financial ratios for fraudulent firms are statistically different from those of non-fraudulent firms.[2] Such findings

[2] Further findings are discussed in Chapter IV.

indicate that further exploration of the differences in financial ratios of fraudulent versus non-fraudulent firms is warranted. This study undertakes such exploration and seeks additional empirical support of these differences.

Ratio Analysis

Whittington [1980] identified two principal uses of financial ratios. There is the traditional normative use of comparing a firm's ratio with a standard. There is also the positive use in estimating empirical relationships. Such relationships are then used for predictive purposes (e.g., forecasting future financial variables, predicting corporate failure).

In the past decades, financial ratios have been the topic of a number of empirical studies that can be grouped into four research streams. A brief discussion of these research streams follows and is summarized in Table 2.

One group of studies focuses on the development of empirically-based classifications or taxonomies of financial ratios. Such studies are also concerned with removing ratio redundancy by identifying a small critical set of independent financial ratios which contain most of the information in a more extensive set of ratios. Pinches et al [1973, 1975] used factor analysis to reduce 48 ratios to seven factor patterns. Their sample consisted of 221 industrial firms for the period 1951-1969. They also found that the composition of the groupings were reasonably stable over time. Similar studies were conducted by Stevens [1973], Libby [1975], Short [1978], Johnson [1979] and Laurant [1979]. Gombola and Ketz [1983] investigated the impact of cash

TABLE 2

RATIO ANALYSIS RESEARCH STREAMS

Author	Year	Objective
1. RATIO CLASSIFICATION STUDIES		
Gombola and Ketz	1983	investigated impact of cash flow ratios on factor patterns
Johnson	1979	examined cross-sectional stability of financial ratio patterns
Laurant	1979	investigated efficiency and effectiveness of financial ratio analysis
Libby	1975	reduced fourteen ratios to five with minimal loss of predictive ability
Pinches et al	1973 1975	used factor analysis to identify seven factor patterns and their stability over nineteen-year period
Stevens	1973	reduced twenty variables to six and accounted for 82% of the total variance
2. RATIO ANALYSIS AND BANKRUPTCY PREDICTION		
Altman	1968	identified statistically useful ratios for bankruptcy prediction
Altman et al	1977	developed Zeta analysis for bankruptcy prediction
Beaver	1966 1968	developed univariate model for bankruptcy prediction
Dambolena and Khoury	1980	developed discriminant function to classify failed vs non-failed firms with 78% accuracy five years prior to failure
Persons	1995	used step-wise logistic model to identify factors associated with fraudulent reporting

Author	Year	Objective
3. BEHAVIORAL DECISION MAKING		
Anderson	1988	examined differences between expert and novice analysts
Bouwman et al	1987	investigated behavior of chartered financial analysts and their use of ratio analysis
Choo	1989	examined differences between expert and novice analysts
Frishkoff et al	1985	compared findings of protocol analysis with results of questionnaire surveys on use of accounting data in financial analysis
Gibson	1983 1985 1987	examined useful financial ratios for accountants, bankers, and financial analysts
Shivaswamy and Matsumoto	1993	investigated correlation between behaviorally-useful and statistically-useful ratios
4. STATISTICAL LIMITATIONS OF RATIO ANALYSIS		
Deakin	1976	examined cross-sectional distributions of eleven ratios for normality
Ezzamel et al	1987	examined statistical properties of financial ratios
Frecka and Hopwood	1983	examined the effects of outliers on cross-sectional distributions of ratios
So	1987	proposed using non-normal stable Paretian distribution to describe financial ratios
Tippett	1990	examined two standard stochastic processes to model financial ratios and concluded that ratios are log-normally distributed and a non-linear function of time

flow measurement upon the classification patterns of financial ratios. They found that 40 ratios could be reduced to eight factors, seven of which were substantially similar to those found by Pinches et al [1973, 1975]. The cash flow ratios did load on a separate and distinct factor, capturing additional information not provided by the other ratio groups. For a comparison of individual ratios and their groupings as used in prior studies, see Chen and Shimerda [1981].

Another group of empirical studies utilizes ratios derived from the financial statements of failed and non-failed firms. The purposes of these studies are to obtain discriminant functions with the smallest classification error or logit functions with the best possible fit and to utilize such functions in the prediction of corporate failures. Beaver [1966, 1968] was among the first to use financial ratios to predict corporate failure. Using a paired-sample design with size and industry type as the bases for pairing, Beaver found overwhelming evidence of differences in the ratios of failed and non-failed firms. Using a univariate dichotomous classification technique to test the predictive power of ratios, Beaver found the cash flow to total debt ratio to be the best predictor of failure five years preceding failure. Altman [1968] introduced a multivariate approach to the prediction of failure. Using multiple discriminant analysis, a Z-score was computed and used as an indicator of corporate failure. This model was updated by Altman et al [1977] into Zeta analysis. Dambolena and Khoury [1980] developed a discriminant function that classified failed and non-failed firms with 78% accuracy five years prior to failure.

A third research direction is the behavioral aspects of decision making using financial ratios. Frishkoff et al [1984] focused on the use of accounting data in financial analysis and compared the findings of protocol analysis with results of questionnaire surveys. Bouwman et al [1987] investigated the behavior of chartered financial analysts and their use of ratio analysis. Differences between expert and novice analysts were found by Anderson [1988] and Choo [1989]. Gibson [1983, 1985, 1987] examined useful financial ratios for accountants, bankers and financial analysts. Shivaswamy and Matsumoto [1993] surveyed bankers and found no evidence of correlation between behaviorally-useful and statistically-useful ratios.

The fourth research stream examines statistical issues resulting from the various ratio studies. In applying the financial ratios, different studies employ different methodologies. Most of the statistical models employ parametric test procedures that assume a univariate or multivariate distribution that is not in agreement with the empirical evidence. Validity of such procedures partly depends on the nature of the underlying distribution of the data set. Both Horrigan [1965] and Mecimore [1968] looked at empirical distributions of financial ratios and found evidence of skewness. Expanding these studies, Deakin [1976] examined the cross-sectional distributions of 11 ratios for manufacturing firms over the period 1953 to 1972. He concluded that, except for the debt/total asset ratio, the normality assumption was not tenable. Frecka and Hopwood [1983] extended Deakin's study by examining the effects of outliers on cross-sectional distributional properties of selected financial ratios. Their analysis utilized the gamma distribution which is appropriate for skewed distributions, a condition evidenced by ratios in prior studies [Deakin 1976, Horrigan 1965, Mecimore 1968]. While ten of the 11 ratios departed from normality, upon deletion of the identified outliers, normality could not be rejected for almost one-half of the distributions. They found that the inclusion of outliers can produce a dramatic distortion in the shape of the distribution. They cautioned researchers in the use of linear statistical models derived from financial ratios. Discriminant models are sensitive to the presence of skewed data [Ezzamel et al 1987]. The parameter estimates can be severely impacted by the outlier observations and thereby potentially compromise the model's predictive ability.

So [1987] proposed using the non-normal stable Paretian distribution to describe financial ratios. The normal distribution is a special case of the stable Paretian family of distributions. The non-normal stable Paretian distribution is similar to the normal distribution except that the former has a fatter tail indicating that a greater probability of observations occur in the tail of the distribution [So 1987].

As financial ratios are constructed from two accounting variables, the joint distribution will depend on the behavior of both the numerator and the denominator and on the relationship between these two coordinates. An implicit assumption of ratio analysis is that of proportionality. It is expected that a proportionate relationship exists between the two variables used in the calculation of the ratio. If there is non-proportionality, then the distribution will be skewed. Ezzamel et al [1987] conducted a test on the same 11 ratios used in the Deakin [1976] study but used the non-normal stable asymmetric Paretian distribution. After removing the outliers, many of the distributions were found to still be non-normally and asymmetrically distributed. They concluded that non-proportionality probably explained why even after eliminating outliers normality could still not be achieved.

Tippett [1990] examined two standard stochastic processes to model financial ratios. First, he assumed that the financial aggregates from which the ratios are constructed are generated by geometric Brownian motion. He also used Lev's [1969] partial adjustment model which assumes that the ratio's underlying financial aggregates are generated by an elastic random walk. Both of these assumptions imply that the ratio will be lognormally distributed and a non-linear function of time. Based on his analysis, there are relatively few occasions on which the proportionality assumption can be justified. He concludes that normality will be the exception rather than the rule and that accounting ratios will be non-linear functions of time.

Given the abundance of empirical evidence refuting the basic assumptions of parametric-based methodologies, one can understand why the utilization of financial ratios for predictive purposes has had such limited success. Research findings suggest that financial statement data may be non-linear and that the use of linear models may be inappropriate. This study combines the findings of the ratio classification studies and the statistical limitations of ratio analysis and expands the research stream by exploring the nature of financial ratios for evidence of non-linearity. Such exploration is conducted using chaos theory and its unique methodology which has various tools for measuring the non-linearity of a system. The following chapter is an introduction to chaos theory and its applicability to the ensuing research.

CHAPTER III

CHAOS THEORY

Concepts

Chaos theory is the qualitative study of the behavior of deterministic non-linear dynamical systems [Kellert 1993]. Upon first examination, the behavior of the system appears irregular and seemingly random moment by moment. In actuality, the system is completely deterministic and defined by one or more equations. Discrete models are characterized by difference equations whereas continuous models are characterized by differential equations. The non-linearity of these equations usually renders a closed-form solution impossible.

In a discrete model, the value of a variable at time $t+1$ is related to its value at an earlier time:

$$X_{t+1} = f\,(X_t, X_{t-1}, X_{t-2}, \ldots).$$

If the function is non-linear, then the time series $(\ldots X_{t-1}, X_t, X_{t+1}, \ldots)$ may exhibit deterministic chaos. The logistic map[3] is probably the most well studied non-linear

[3] The term "map" is used rather than "function" because of the discrete way that difference equations associate paired data by jumping to the subsequent value without including any values in the continuous range between the points.

discrete model exhibiting chaos. May [1976] initially investigated the chaotic properties of the general logistic map:

$$X_{t+1} = a X_t (1 - X_t).$$

The above equation produces different results depending on your initial starting value X_0 and parameter value a. Holding the initial starting value X_0 constant and merely changing the parameter value a produces different values. For example, if a is between 0 and 4, then for any X_0 between 0 and 1, all subsequent X_t will be bounded between 0 and 1. Repeated iterations of the equation illustrate the dynamics of the non-linear system and result in different types of behavior.

An example of the logistic equation is given in Table 3A and its various figures. When $a = 2.5$ (Figure 1), iteration of the equation quickly converges to the fixed value 0.60. When $a = 3.2$ (Figure 2), iteration results in an oscillation between two values: 0.513045 and 0.799455. When $a = 3.5$ (Figure 3), iteration again results in an oscillation. This time, by X_{36}, the equation produces four values: 0.826941, 0.500884, 0.874997, 0.38282. When $a = 3.9$ (Figure 4), iteration results in values that appear to have no discernible pattern. While an infinite number of values are produced, they are bounded within the range $0 < X_t < 1$. When $a = 4.2$ (Table 3A), by X_6, the values are outside the previously bounded range and quickly approach negative infinity. Despite being a deterministic non-linear equation, prediction is not possible.

Being deterministic and being predictable are not the same thing. A system is called deterministic when its future states are completely fixed by its current state and

TABLE 3A - LOGISTIC MAP

	$a = 2.5$	$a = 3.2$	$a = 3.5$	$a = 3.9$	$a = 3.900001$	$a = 4.2$	$a = 3.9$
X_0	0.200000	0.200000	0.200000	0.200000	0.200000	0.200000	0.200001
X_1	0.400000	0.512000	0.560000	0.624000	0.624000	0.672000	0.624002
X_2	0.600000	0.799539	0.862400	0.915034	0.915034	0.925747	0.915031
X_3	0.600000	0.512884	0.415332	0.303214	0.303214	0.288705	0.303221
X_4	0.600000	0.799469	0.849910	0.823973	0.823973	0.862489	0.823984
X_5	0.600000	0.513019	0.446472	0.565661	0.565662	0.498128	0.565633
X_6	0.600000	0.799458	0.864971	0.958185	0.958186	1.049985	0.958200
X_7	0.600000	0.513040	0.408785	0.156258	0.156258	-0.220432	0.156206
X_8	0.600000	0.799456	0.845880	0.514181	0.514181	-1.129893	0.514042
X_9	0.600000	0.513044	0.456285	0.974216	0.974216	-10.107521	0.974231
X_{10}	0.600000	0.799456	0.868312	0.097966	0.097965	-471.531869	0.097909
X_{11}	0.600000	0.513044	0.400213	0.344638	0.344634	negative infinity	0.344460
X_{12}	0.600000	0.799455	0.840149	0.880864	0.880860	negative infinity	0.880648
X_{13}	0.600000	0.513044	0.470046	0.409276	0.409288	negative infinity	0.409917
X_{14}	0.600000	0.799455	0.871860	0.942900	0.942909	negative infinity	0.943352
X_{15}	0.600000	0.513045	0.391022	0.209975	0.209945	negative infinity	0.208412
X_{16}	0.600000	0.799455	0.833433	0.646953	0.646886	negative infinity	0.643409
X_{17}	0.600000	0.513045	0.485879	0.890778	0.890856	negative infinity	0.894793
X_{18}	0.600000	0.799455	0.874302	0.379440	0.379203	negative infinity	0.367142
X_{19}	0.600000	0.513045	0.384643	0.918314	0.918091	negative infinity	0.906160
X_{20}	0.600000	0.799455	0.828425	0.292551	0.293279	negative infinity	0.331634
X_{21}	0.600000	0.513045	0.497480	0.807164	0.808339	negative infinity	0.864446
X_{22}	0.600000	0.799455	0.874978	0.607036	0.604217	negative infinity	0.456999
X_{23}	0.600000	0.513045	0.382871	0.930319	0.932642	negative infinity	0.967789
X_{24}	0.600000	0.799455	0.826983	0.252821	0.245002	negative infinity	0.121578
X_{25}	0.600000	0.513045	0.500788	0.736720	0.721407	negative infinity	0.416508

Use table with Figures 1 - 6

	a = 2.5	a = 3.2	a = 3.5	a = 3.9	a = 3.900001	a = 4.2	a = 3.9
X_{26}	0.600000	0.799455	0.874998	0.756458	0.783818	negative infinity	0.947813
X_{27}	0.600000	0.513045	0.382818	0.718494	0.660845	negative infinity	0.192906
X_{28}	0.600000	0.799455	0.826939	0.788815	0.874103	negative infinity	0.607205
X_{29}	0.600000	0.513045	0.500887	0.649684	0.429184	negative infinity	0.930178
X_{30}	0.600000	0.799455	0.874997	0.887619	0.955442	negative infinity	0.253293
X_{31}	0.600000	0.513045	0.382820	0.389030	0.166032	negative infinity	0.737629
X_{32}	0.600000	0.799455	0.826941	0.926974	0.540016	negative infinity	0.754777
X_{33}	0.600000	0.513045	0.500884	0.264003	0.968755	negative infinity	0.721845
X_{34}	0.600000	0.799455	0.874997	0.757791	0.118047	negative infinity	0.783061
X_{35}	0.600000	0.513045	0.382820	0.715820	0.406037	negative infinity	0.662519
X_{36}	0.600000	0.799455	0.826941	0.793345	0.940567	negative infinity	0.871992
X_{37}	0.600000	0.513045	0.500884	0.639400	0.218012	negative infinity	0.435325
X_{38}	0.600000	0.799455	0.874997	0.899214	0.664884	negative infinity	0.958687
X_{39}	0.600000	0.513045	0.382820	0.353451	0.868973	negative infinity	0.154464
X_{40}	0.600000	0.799455	0.826941	0.891241	0.444051	negative infinity	0.509360
X_{41}	0.600000	0.513045	0.500884	0.378028	0.962792	negative infinity	0.974658
X_{42}	0.600000	0.799455	0.874997	0.916979	0.139711	negative infinity	0.096328
X_{43}	0.600000	0.513045	0.382820	0.296901	0.468748	negative infinity	0.339491
X_{44}	0.600000	0.799455	0.826941	0.814129	0.971191	negative infinity	0.874523
X_{45}	0.600000	0.513045	0.500884	0.590161	0.109117	negative infinity	0.427956
X_{46}	0.600000	0.799455	0.874997	0.943297	0.379122	negative infinity	0.954758
X_{47}	0.600000	0.513045	0.382820	0.208602	0.918016	negative infinity	0.168462
X_{48}	0.600000	0.799455	0.826941	0.643840	0.293525	negative infinity	0.546321
X_{49}	0.600000	0.513045	0.500884	0.894309	0.808735	negative infinity	0.966632
X_{50}	0.600000	0.799455	0.874997	0.368628	0.603262	negative infinity	0.125793
X_{51}	0.600000	0.513045	0.382820	0.907692	0.933415	negative infinity	0.428880
X_{52}	0.600000	0.799455	0.826941	0.326771	0.242392	negative infinity	0.955274

Use table with Figures 1 - 6

	a = 2.5	a = 3.2	a = 3.5	a = 3.9	a = 3.900001	a = 4.2	a = 3.9
X_{53}	0.600000	0.513045	0.500884	0.857968	0.716189	negative infinity	0.166631
X_{54}	0.600000	0.799455	0.874997	0.475251	0.792723	negative infinity	0.541575
X_{55}	0.600000	0.513045	0.382820	0.972611	0.640822	negative infinity	0.968259
X_{56}	0.600000	0.799455	0.826941	0.103891	0.897660	negative infinity	0.119861
X_{57}	0.600000	0.513045	0.500884	0.363081	0.358280	negative infinity	0.411427
X_{58}	0.600000	0.799455	0.874997	0.901887	0.896670	negative infinity	0.944404
X_{59}	0.600000	0.513045	0.382820	0.345098	0.361345	negative infinity	0.204770
X_{60}	0.600000	0.799455	0.826941	0.881421	0.900022	negative infinity	0.635073
X_{61}	0.600000	0.513045	0.500884	0.407620	0.350932	negative infinity	0.903845
X_{62}	0.600000	0.799455	0.874997	0.941717	0.888337	negative infinity	0.338945
X_{63}	0.600000	0.513045	0.382820	0.214054	0.386858	negative infinity	0.873839
X_{64}	0.600000	0.799455	0.826941	0.656117	0.925076	negative infinity	0.429953
X_{65}	0.600000	0.513045	0.500884	0.879947	0.270311	negative infinity	0.955865
X_{66}	0.600000	0.799455	0.874997	0.411997	0.769248	negative infinity	0.164531
X_{67}	0.600000	0.513045	0.382820	0.944796	0.692271	negative infinity	0.536097
X_{68}	0.600000	0.799455	0.826941	0.203410	0.830824	negative infinity	0.969918
X_{69}	0.600000	0.513045	0.500884	0.631934	0.548165	negative infinity	0.113789
X_{70}	0.600000	0.799455	0.874997	0.907114	0.965953	negative infinity	0.393281
X_{71}	0.600000	0.513045	0.382820	0.328607	0.128264	negative infinity	0.930583
X_{72}	0.600000	0.799455	0.826941	0.860435	0.436068	negative infinity	0.251933
X_{73}	0.600000	0.513045	0.500884	0.468337	0.959060	negative infinity	0.735005
X_{74}	0.600000	0.799455	0.874997	0.971090	0.153130	negative infinity	0.759613
X_{75}	0.600000	0.513045	0.382820	0.109489	0.505756	negative infinity	0.712144

Use table with Figures 1 - 6

FIGURE 1
LOGISTIC MAP ($a = 2.5$)

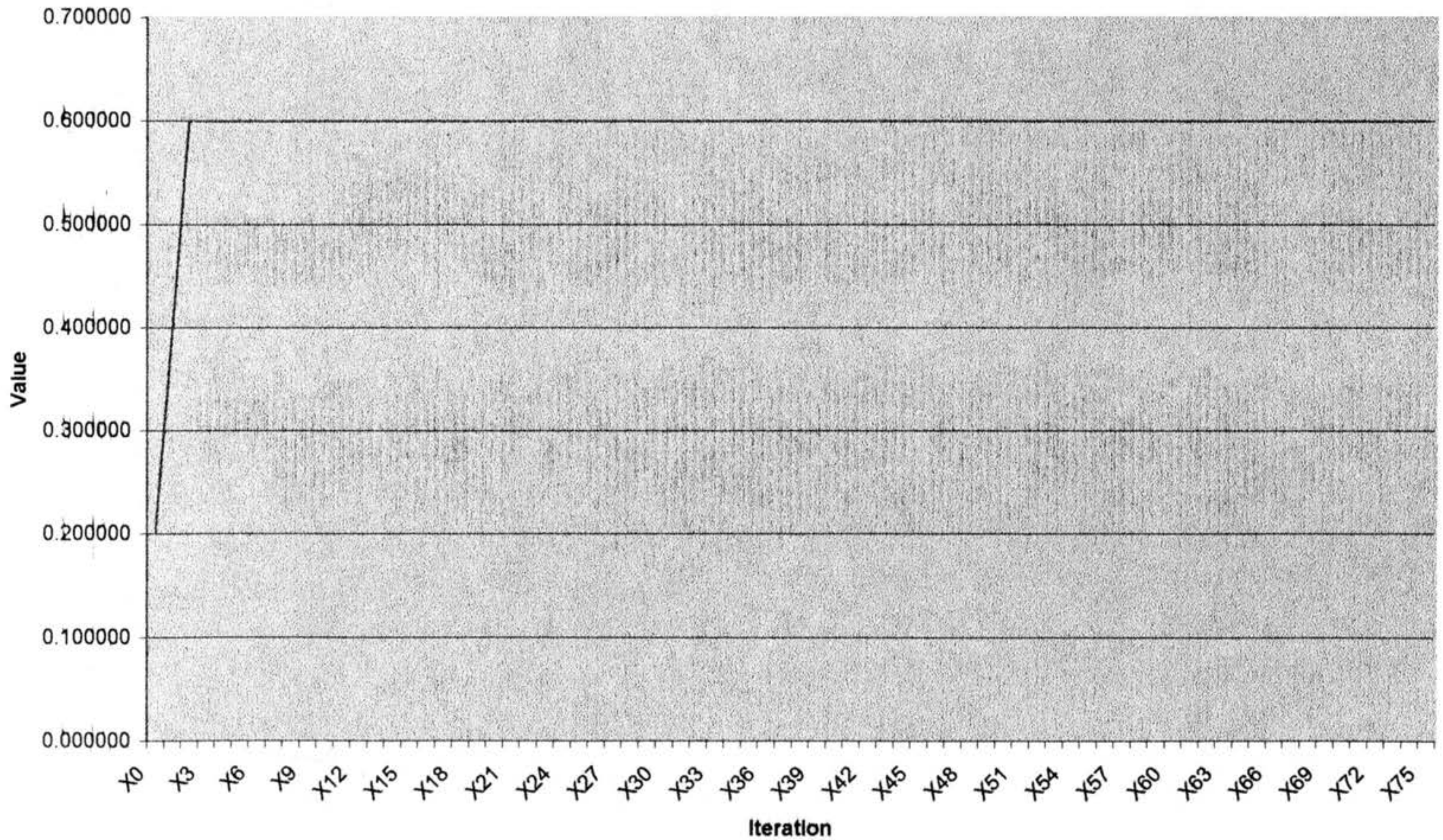


FIGURE 2
LOGISTIC MAP (a = 3.2)

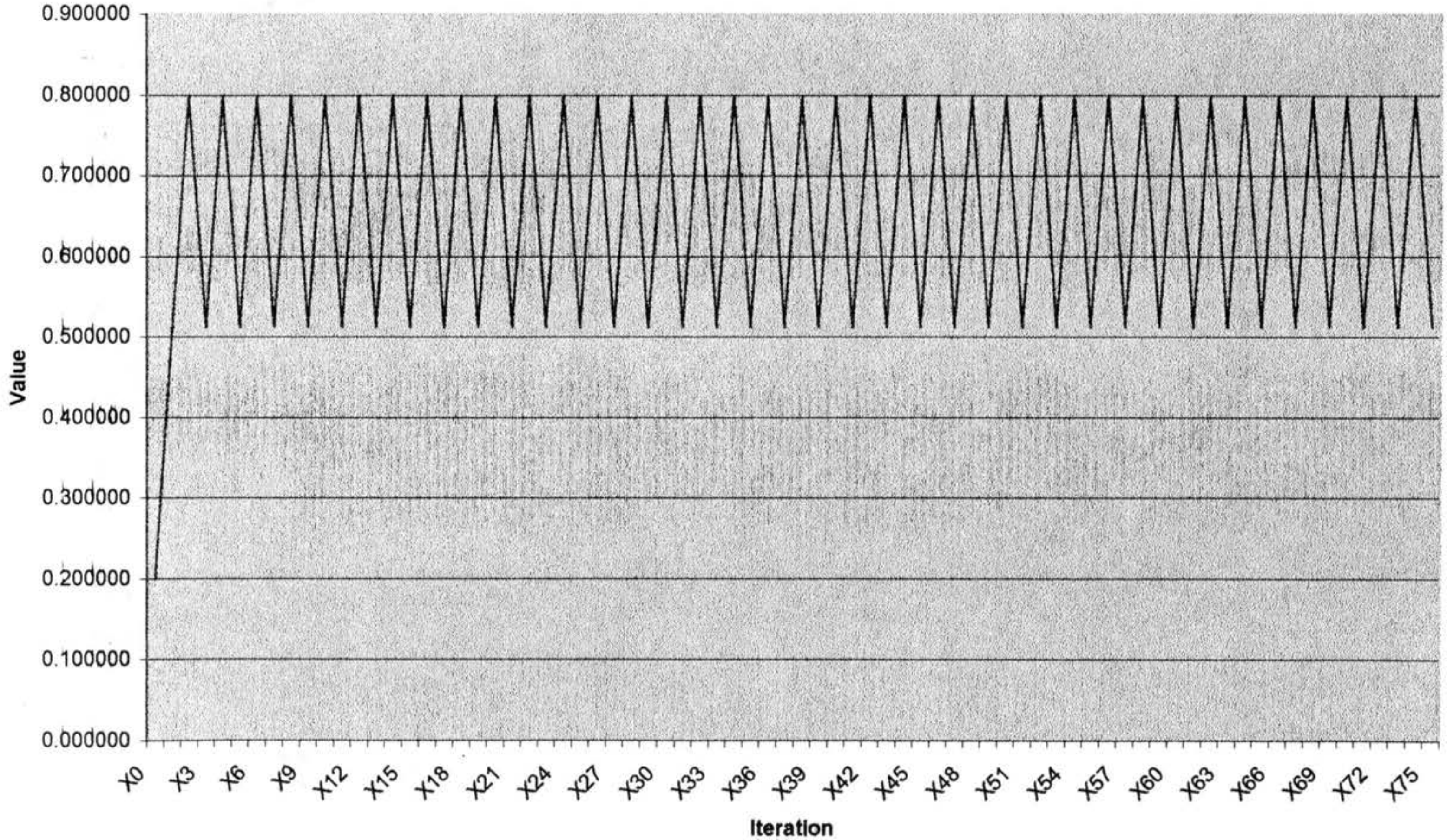


FIGURE 3
LOGISTIC MAP (a = 3.5)

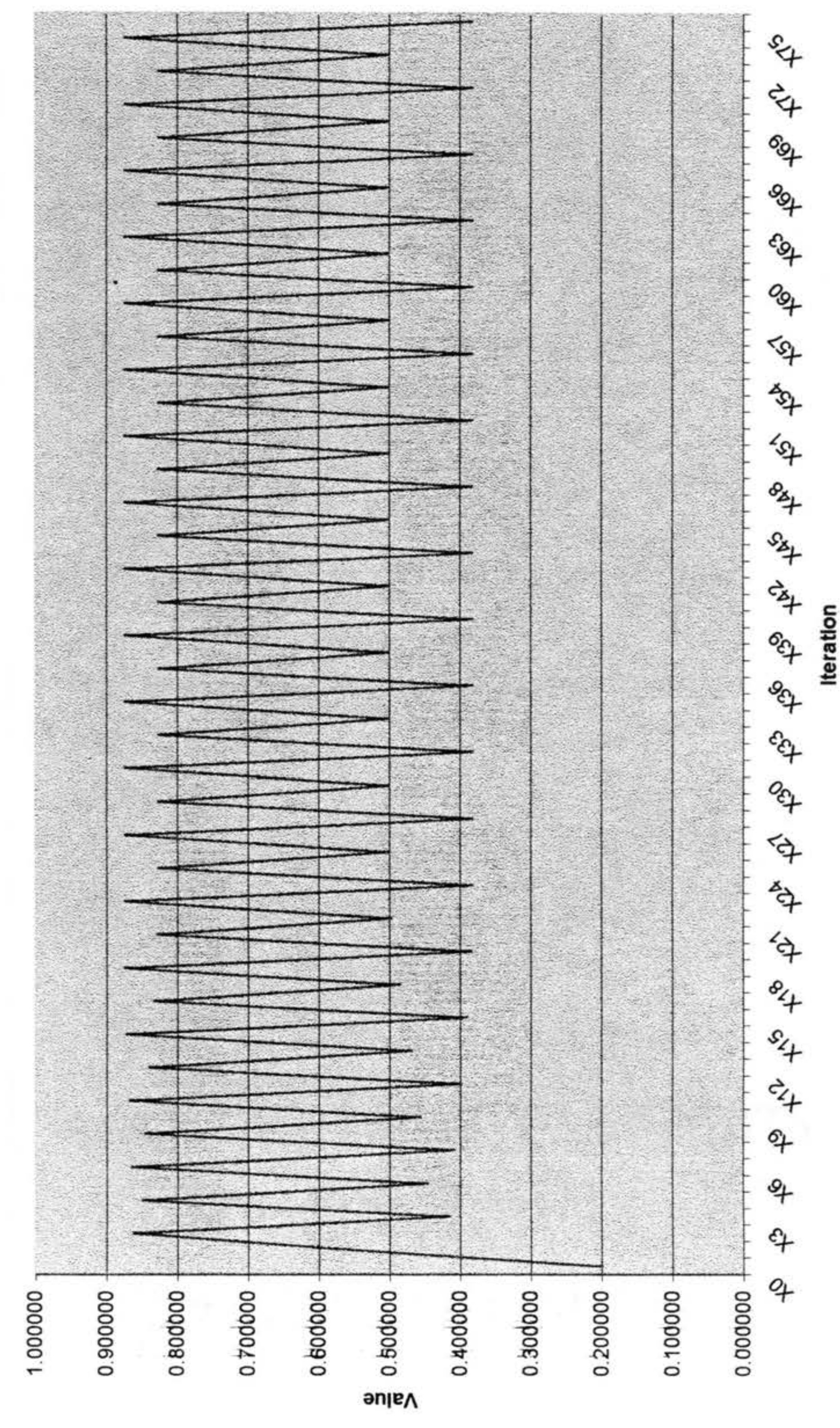
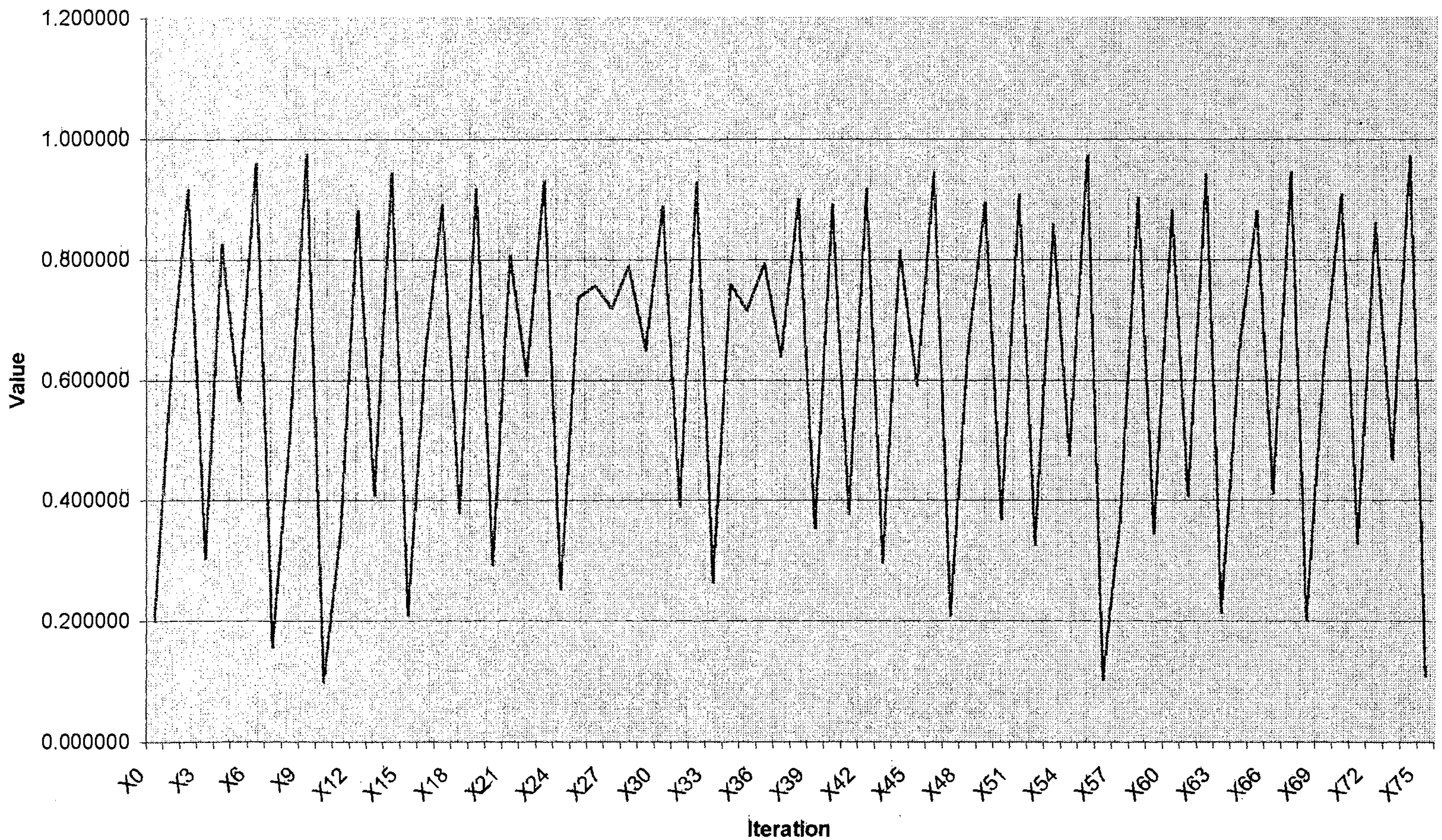


FIGURE 4
LOGISTIC MAP (*a* = 3.9)

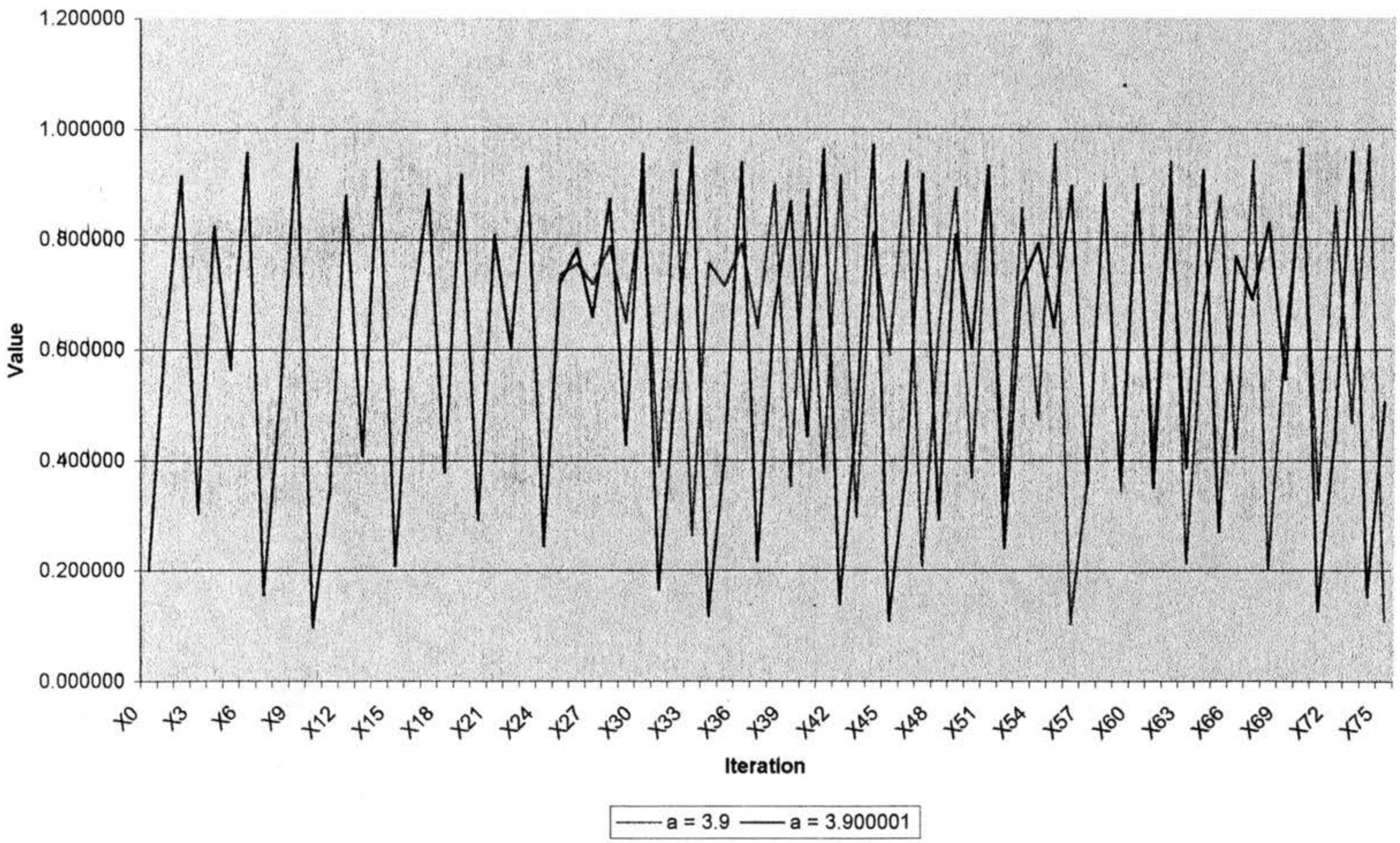


its rules of dynamical motion [Casti 1994]. As illustrated with the logistic map, order and chaos may appear within the same deterministic system and is clearly contradictory to the Newtonian world-view. It shatters the presumption that determinism necessarily implies predictability [Mouck 1998].

This theoretical impossibility of prediction is the result of three characteristics of the iterative process: (1) numbers extend to an infinite number of decimal places and must be rounded off in order to do practical calculations; (2) the outcome has a sensitive dependence upon the chosen parameters; and (3) in the chaotic range, the outcome has an extremely sensitive dependence on initial conditions (SDIC). SDIC means that small initial differences or fluctuations in variables may grow over time into very large differences.

The logistic map illustrates the concept of sensitive dependence on parameters. Once again, set $X_0 = .20$ (see Table 3A and Figure 5). When $a = 3.9$, iteration results in a seemingly random set of solutions bounded within the range $0 < X_t < 1$. What happens when we change the value of parameter a to 3.900001? Given such a miniscule change in the parameter value, one would expect a set of solutions very similar to those resulting when a was 3.9. Such is not the case. At $a = 3.900001$, the solution set is again infinite and seemingly random and bounded within the range $0 < X_t < 1$. Comparison of the two solution sets is similar only for the first 25 iterations or so. By X_{28}, the solution sets begin to diverge. Further iterations sometimes bring the solutions closer together (e.g., see X_{51}, X_{57}) only to become divergent once again (e.g., see X_{54}, X_{65}).

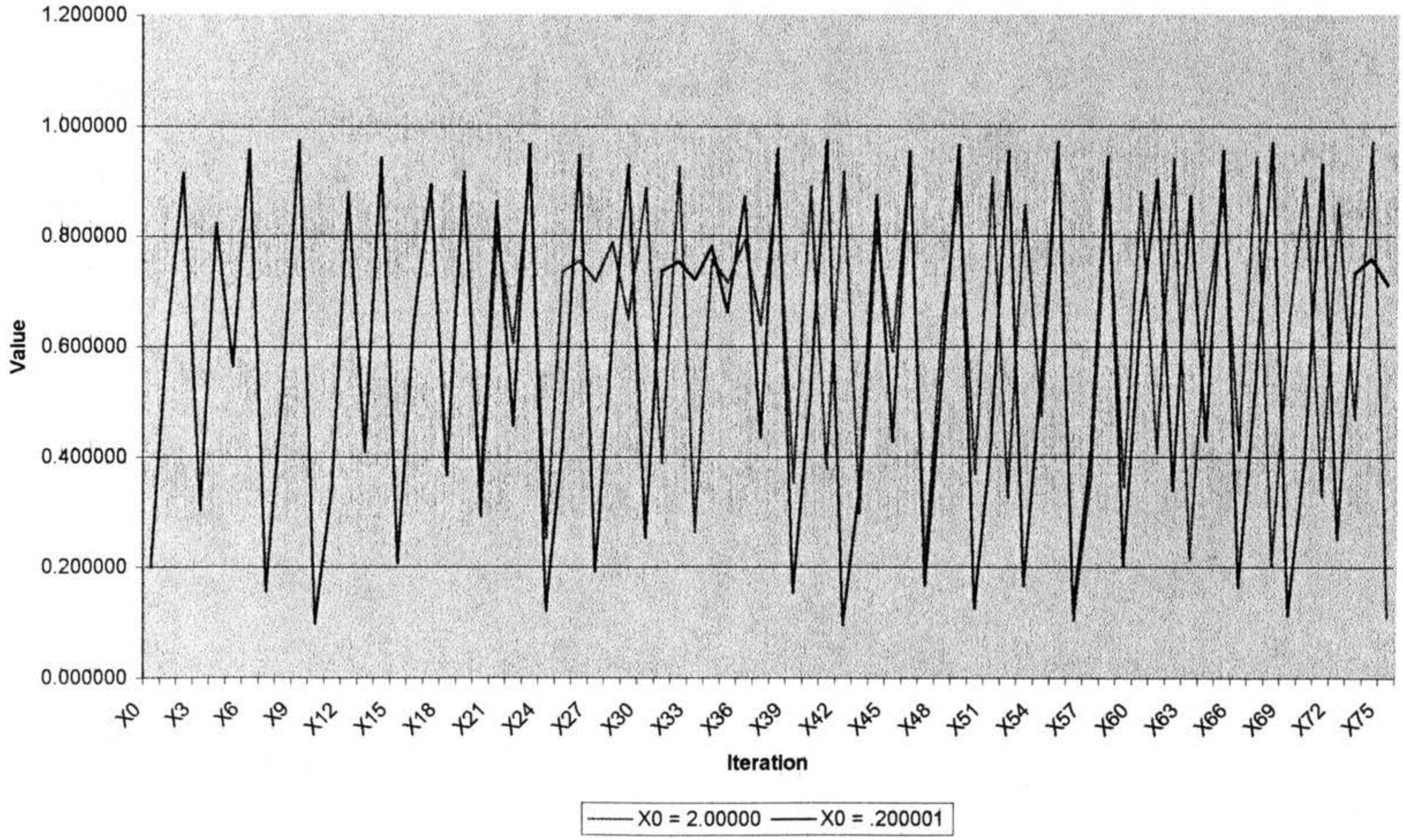
FIGURE 5
LOGISTIC MAP - SENSITIVE DEPENDENCE ON PARAMETER



Similarly, the logistic map illustrates the concept of SDIC. Set $a = 3.9$ (see Table 3A and Figure 6). When $X_0 = .200000$, iteration results in a seemingly random set of solutions bounded within the range $0 < X_t < 1$. What happens when we change the initial starting condition of X_0 to .200001? Again, this is a miniscule change in value and one would expect a similar set of solutions. However, this is not the case. At $X_0 = .200001$, the solution set is again infinite and seemingly random and bounded within the range $0 < X_t < 1$. The two solution sets are similar only for the first 20 iterations; the sets then begin to diverge. Some iterations bring the solutions closer together (e.g., see X_{34}, X_{56}) while others cause divergence (e.g., see X_{27}, X_{58}). Such SDIC means that effects may be wildly out of proportion to causes. A small cause such as a simple and seemingly insignificant rounding can have very major consequences. This is the nature of chaos. Small changes in initial conditions produce dramatically different evolutionary outcomes. A chaotic system is inherently unpredictable, not because its solution is seemingly random, but because one is unable to measure its initial state with absolute precision. While deterministic non-linear systems are highly predictable in theory (given infinite precision), they are extremely unpredictable in practice where precision is limited.

As a qualitative study, chaos theory investigates a system by asking about the general character of its long-term behavior rather than seeking to arrive at numerical predictions about its exact future state [Kellert 1993]. An important arena for understanding non-linear dynamical systems is phase space, a mathematically constructed conceptual space where each dimension corresponds to one variable of the

FIGURE 6
LOGISTIC MAP - SDIC



system. Every point in phase space represents a full description of the system in one of its possible states. Phase space is the space of the possible, containing not just the states that do occur but also those that might have occurred. Parameter space is similar but each dimension corresponds to a different parameter. A point in parameter space specifies the values of all the parameters of the system. For example, the logistic map has one variable X and one parameter a. Phase space is therefore the line of X values from 0 to 1 while parameter space is the line of a values from 0 to 4.

The evolution of the system manifests itself as the tracing out of a path or trajectory in phase space [Kellert 1993]. One can categorize the possible trajectories according to their shape resulting in a topological taxonomy. By linking topology and dynamical systems, phase space provides a way of turning numbers into pictures, abstracting all the essential information from the system and making a flexible road map to all of its possibilities. One can use the shape to visualize the whole range of behaviors of a system. If one can visualize the shape, one can gain understanding of the system. Traditional time-series and trajectories in phase space are two ways of displaying the same data and gaining a picture of a system's long-term behavior [Gleick 1987].

When a deterministic non-linear dynamical system is plotted in phase space, the resulting shape is called an attractor. In essence, the attractor is a graphical representation of an equilibrium state attainable by the system. Basically, there are three types of attractors. The simplest is the fixed point whereby the output of the system is a steady state. An example of a fixed point attractor was illustrated earlier with the logistic map when the parameter value was set at $a = 2.5$. Despite repeated iterations, the system is attracted to the solution 0.60. The equilibrium is a single value. The second type of attractor is the limit cycle whereby the trajectory repeats itself in a cyclic fashion. Once again, the limit cycle was demonstrated earlier with the logistic map. When $a = 3.2$, the system is attracted to a two-point equilibrium state in which the values alternate between 0.513045 and 0.799455. Similarly, when $a = 3.5$, the system is attracted to a different limit cycle consisting of a repeating sequence of four values: 0.826941, 0.500884, 0.874997, and 0.382820. As illustrated, limit cycles are dynamic equilibria being repeated with a regular periodicity. The third type of attractor is the strange attractor whereby the trajectory consists of aperiodic paths. The logistic map illustrated such a strange attractor. When $a = 3.9$, the system is attracted to infinite solutions all bounded within the range between 0 and 1. There are infinite equilibrium states, all confined within a region of phase space.

These various equilibria are completely independent of initial starting condition (i.e., the value of X_0) and depend only on the parameter value a. This is illustrated in Table 3B. Note that the parameter values for a are identical to those used previously (i.e., $a = 2.5$, 3.2, 3.5, 3.9, and 4.2). However, the initial starting condition X_0 is now .500000. When $a = 2.5$, the system is again attracted to the fixed point solution 0.60. The same two-point and four-point limit cycles described previously result when $a= 3.2$ and 3.5 respectively. Similarly, a strange attractor, still bound within the range $0 < X_t < 1$, results when $a = 3.9$.

TABLE 3B - LOGISTIC MAP

	a = 2.5	a = 3.2	a = 3.5	a = 3.9	a = 4.2
X_0	0.500000	0.500000	0.500000	0.500000	0.500000
X_1	0.625000	0.800000	0.875000	0.975000	1.050000
X_2	0.585938	0.512000	0.382813	0.095063	-0.220500
X_3	0.606537	0.799539	0.826935	0.335500	-1.130305
X_4	0.596625	0.512884	0.500898	0.869465	-10.113157
X_5	0.601659	0.799469	0.874997	0.442633	-472.034238
X_6	0.599164	0.513019	0.382820	0.962165	negative infinity
X_7	0.600416	0.799458	0.826941	0.141973	negative infinity
X_8	0.599791	0.513040	0.500884	0.475084	negative infinity
X_9	0.600104	0.799456	0.874997	0.972579	negative infinity
X_{10}	0.599948	0.513044	0.382820	0.104010	negative infinity
X_{11}	0.600026	0.799456	0.826941	0.363448	negative infinity
X_{12}	0.599987	0.513044	0.500884	0.902278	negative infinity
X_{13}	0.600007	0.799455	0.874997	0.343871	negative infinity
X_{14}	0.599997	0.513044	0.382820	0.879933	negative infinity
X_{15}	0.600002	0.799455	0.826941	0.412040	negative infinity
X_{16}	0.599999	0.513045	0.500884	0.944826	negative infinity
X_{17}	0.600000	0.799455	0.874997	0.203308	negative infinity
X_{18}	0.600000	0.513045	0.382820	0.631698	negative infinity
X_{19}	0.600000	0.799455	0.826941	0.907357	negative infinity
X_{20}	0.600000	0.513045	0.500884	0.327834	negative infinity
X_{21}	0.600000	0.799455	0.874997	0.859399	negative infinity
X_{22}	0.600000	0.513045	0.382820	0.471246	negative infinity
X_{23}	0.600000	0.799455	0.826941	0.971776	negative infinity
X_{24}	0.600000	0.513045	0.500884	0.106968	negative infinity
X_{25}	0.600000	0.799455	0.874997	0.372552	negative infinity
X_{26}	0.600000	0.513045	0.382820	0.911652	negative infinity
X_{27}	0.600000	0.799455	0.826941	0.314115	negative infinity
X_{28}	0.600000	0.513045	0.500884	0.840243	negative infinity
X_{29}	0.600000	0.799455	0.874997	0.523515	negative infinity
X_{30}	0.600000	0.513045	0.382820	0.972843	negative infinity
X_{31}	0.600000	0.799455	0.826941	0.103034	negative infinity
X_{32}	0.600000	0.513045	0.500884	0.360431	negative infinity
X_{33}	0.600000	0.799455	0.874997	0.899030	negative infinity
X_{34}	0.600000	0.513045	0.382820	0.354021	negative infinity
X_{35}	0.600000	0.799455	0.826941	0.891892	negative infinity
X_{36}	0.600000	0.513045	0.500884	0.376041	negative infinity
X_{37}	0.600000	0.799455	0.874997	0.915073	negative infinity
X_{38}	0.600000	0.513045	0.382820	0.303086	negative infinity
X_{39}	0.600000	0.799455	0.826941	0.823777	negative infinity

	a = 2.5	a = 3.2	a = 3.5	a = 3.9	a = 4.2
X_{40}	0.600000	0.513045	0.500884	0.566158	negative infinity
X_{41}	0.600000	0.799455	0.874997	0.957930	negative infinity
X_{42}	0.600000	0.513045	0.382820	0.157169	negative infinity
X_{43}	0.600000	0.799455	0.826941	0.516622	negative infinity
X_{44}	0.600000	0.513045	0.500884	0.973922	negative infinity
X_{45}	0.600000	0.799455	0.874997	0.099050	negative infinity
X_{46}	0.600000	0.513045	0.382820	0.348034	negative infinity
X_{47}	0.600000	0.799455	0.826941	0.884934	negative infinity
X_{48}	0.600000	0.513045	0.500884	0.397120	negative infinity
X_{49}	0.600000	0.799455	0.874997	0.933721	negative infinity
X_{50}	0.600000	0.513045	0.382820	0.241355	negative infinity
X_{51}	0.600000	0.799455	0.826941	0.714101	negative infinity
X_{52}	0.600000	0.513045	0.500884	0.796227	negative infinity
X_{53}	0.600000	0.799455	0.874997	0.632773	negative infinity
X_{54}	0.600000	0.513045	0.382820	0.906248	negative infinity
X_{55}	0.600000	0.799455	0.826941	0.331355	negative infinity
X_{56}	0.600000	0.513045	0.500884	0.864079	negative infinity
X_{57}	0.600000	0.799455	0.874997	0.458041	negative infinity
X_{58}	0.600000	0.513045	0.382820	0.968134	negative infinity
X_{59}	0.600000	0.799455	0.826941	0.120318	negative infinity
X_{60}	0.600000	0.513045	0.500884	0.412782	negative infinity
X_{61}	0.600000	0.799455	0.874997	0.945333	negative infinity
X_{62}	0.600000	0.513045	0.382820	0.201547	negative infinity
X_{63}	0.600000	0.799455	0.826941	0.627610	negative infinity
X_{64}	0.600000	0.513045	0.500884	0.911491	negative infinity
X_{65}	0.600000	0.799455	0.874997	0.314632	negative infinity
X_{66}	0.600000	0.513045	0.382820	0.840990	negative infinity
X_{67}	0.600000	0.799455	0.826941	0.521530	negative infinity
X_{68}	0.600000	0.513045	0.500884	0.973192	negative infinity
X_{69}	0.600000	0.799455	0.874997	0.101748	negative infinity
X_{70}	0.600000	0.513045	0.382820	0.356440	negative infinity
X_{71}	0.600000	0.799455	0.826941	0.894624	negative infinity
X_{72}	0.600000	0.513045	0.500884	0.367662	negative infinity
X_{73}	0.600000	0.799455	0.874997	0.906698	negative infinity
X_{74}	0.600000	0.513045	0.382820	0.329929	negative infinity
X_{75}	0.600000	0.799455	0.826941	0.862195	negative infinity

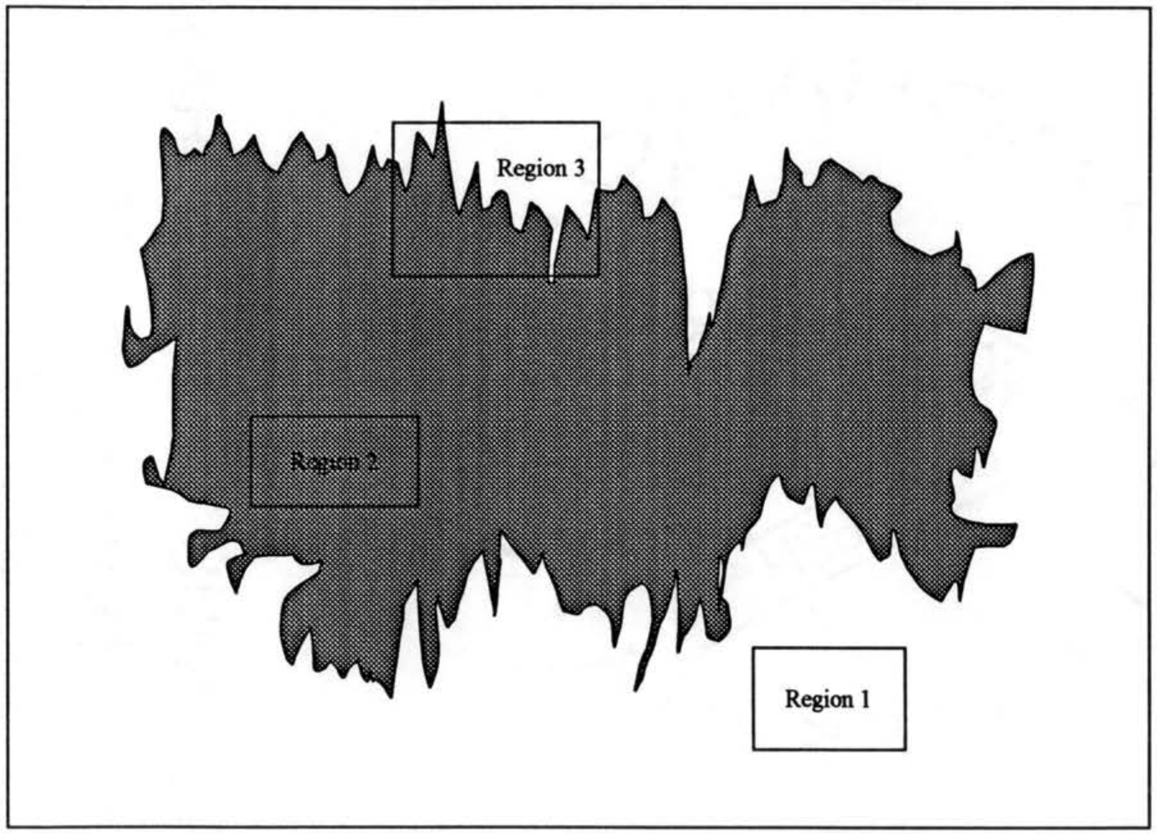
As illustrated with the logistic map, different parameter values can lead to totally different dynamics and attractors. Despite being a very simple deterministic non-linear system, different points in parameter space will have different attractors in phase space. Fixed points, various limit cycles and strange attractors coexist within the same dynamical system. One ends up with different long-term behaviors dependent upon the parameter value.

This dependence of long-term behavior on parameter values can be represented graphically by plotting points in parameter space. An example of a hypothetical deterministic non-linear chaotic system is shown in Figure 7. The two dimensions represent parameter space. White areas of the graph represent combinations which produce non-chaotic behavior (i.e., fixed point or periodic). Black areas of the graph represent combinations which produce chaotic behavior.

Gregersen and Sailer [1993] describe how various studies can be undertaken of such a system. As illustrated in Figure 7, a study conducted in Region 1 is well within the non-chaotic subset. In this situation, the inherent chaos within the system is relatively unimportant. Prediction should be feasible. Meanwhile, a study conducted in Region 2 is well within the chaotic subset. Such behavior is well-defined, even for small relative variations in the parameter values. The system would show clear evidence of chaos. A study conducted in Region 3 might include both chaotic and non-chaotic behavior. Analysis of such a system might produce mixed behavior that would appear random. It is not possible to predict which behavior would result.

FIGURE 7

HYPOTHETICAL DETERMINISTIC NON-LINEAR CHAOTIC SYSTEM



Most dynamic systems exhibit some degree of chaos over part of their domain [Gregersen and Sailer 1993]. It is imperative that one determines in which part of the domain the research study lies. If the study lies near the boundary, the standard statistical methods of prediction are not applicable. Analyzing chaotic systems using such methods will produce poor results. Identifying the system's behaviors is important because of their implications for model selection and the explanatory power of the model [Etheridge and Sriram 1993].

This research study is a first attempt to explore the dynamics of the financial accounting system to determine the behavior of financial statement ratios and identify in which part of the domain the ratios reside. The question is not simply whether or not chaos exists but also the degree to which chaos occurs and the degree to which such chaos is relevant to financial statement ratios and their ability to predict and/or detect fraud. The following section is a review of the application of chaos theory to the research realms of the physical sciences, the social sciences and accounting.

Literature Review

Physical Sciences

Chaos theory developed from research undertaken in the physical sciences. Initially scientists had an orderly and structured view of the physical world. Newtonian physics had postulated that physical systems were governed by fundamental laws of cause and effect. Nature was seen as orderly and the objective of all natural systems was to achieve equilibrium. Scientists discovered, however, that many physical systems did not behave in an orderly and linear fashion. Models based on such assumptions were ineffectual for the prediction of natural phenomena.

Some scientists began to look at the complex and disorderly behaviors from a different perspective. Henri Poincare studied the *n*-body problem and developed topological and geometrical techniques for examining the global structure of dynamic systems. In the 1920s, Fatou and Gaston worked with analytic maps while Birkhoff used iterative processes to analyze differential equations. Lorenz [1963] attempted to model the weather and discovered that there were patterns within the seemingly random output generated by his weather simulation program. He concluded that, due to sensitive dependence on initial conditions and the inability to conduct precise measurements to the most extreme level of detail, long-term weather prediction was impossible. May [1976] attempted to model the dynamics of population biology and found chaotic behavior in the iteration of the simple logistic equation. Feigenbaum [1978] studied the orderliness exhibited by a dynamic system on its way to chaos.

In the 1980s, with the advent of computer graphics, researchers revisited the work done by Fatou and Gaston and investigated the geometry of dynamic systems. Representations of chaotic behavior were pictured using fractal geometry. Examples include the Lorenz attractor (Figure 8) and the Mandelbrot Set, both of which are instantly recognized as symbols of chaos. For a more complete history of the development of chaos theory, please refer to Gleick [1987]. See Table 4 for a listing of selected studies in the physical sciences utilizing chaos theory.

FIGURE 8

LORENZ ATTRACTOR

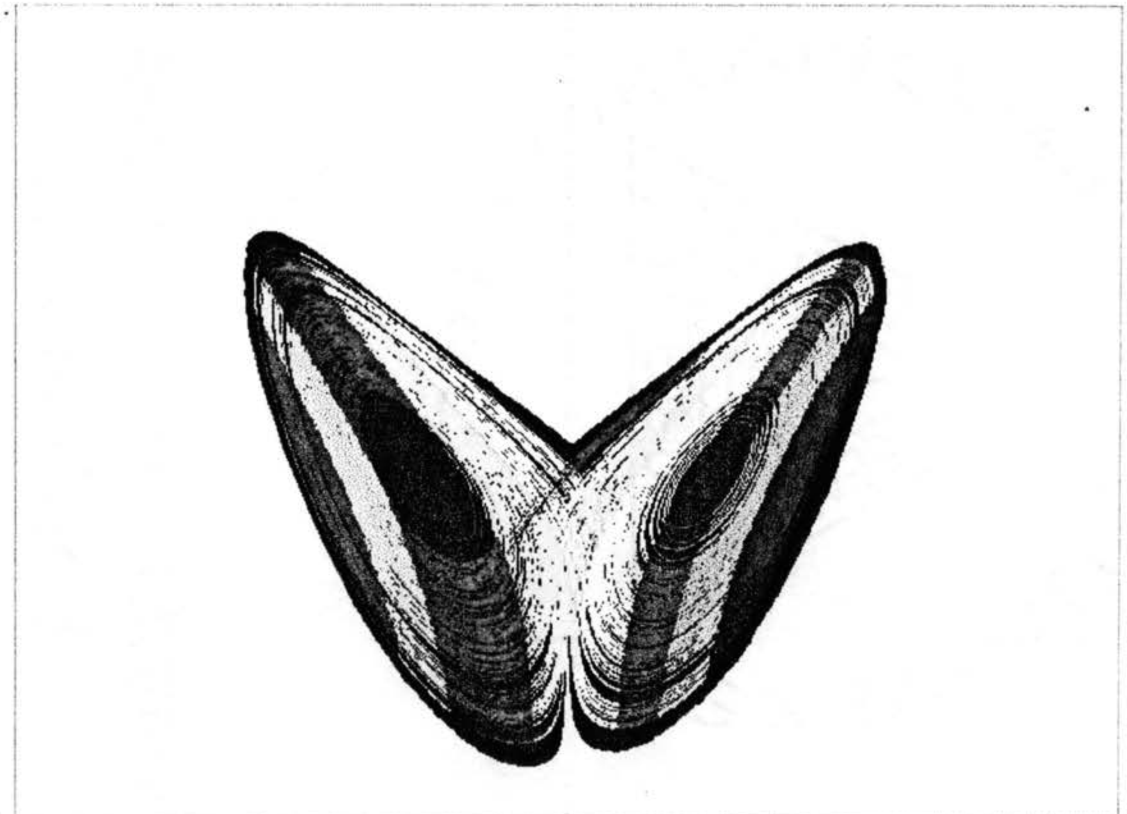


TABLE 4

CHAOS THEORY STUDIES – PHYSICAL SCIENCES

Author	Year	Area
Arun	1986	physiology
Berge et al	1984	physics
Eckman	1981	physics
Feigenbaum	1978	physics
Freeman	1991	physiology
Glass and Mackey	1988	physiology
Goldberger et al	1985	physiology
Li	1975	ecology
Lipsitz and Goldberger	1992	physiology
Lorenz	1963	meteorology
Lorenz	1984	meteorology
Mackey and Glass	1977	physiology
May	1974	ecology
Miles	1989	turbulent fluids
Peacocke	1982	biology
Pool	1988	physics
Prigogine	1980	biology
Prigogine	1980	physics
Ruelle	1983	physics
Steeb and Louw	1986	physics
Stewart	1989	physics
Swinney and Gollub	1975	physics
Wicken	1988	biology
Winfree	1987	physiology
Wisdom	1985	astronomy

Social Sciences

The nature of social science research is quite different than that of the physical sciences. The adaptation of theories developed in the physical sciences to the social sciences is often fraught with difficulties. The controlled laboratory conditions of physics are generally not available to social science researchers. Extraneous variables cannot always be removed or controlled. Comparing results of an occurrence to the results obtained from a control group may prove troublesome. Replications of an occurrence may not be possible. For these reasons, findings in the social sciences are often mixed and/or inconclusive. Some of the physical sciences not subject to the normal controlled laboratory conditions suffer these same limitations. Examples include meteorology and astronomy.

To ameliorate such limitations, much work has been done to develop methods more applicable to small, noisy data sets. Brock et al [1987] developed the BDS statistic[4] to look for evidence of non-linear structure in data. Wolf et al [1985] created an algorithm that estimates the Lyapunov exponents[5] from experimental data consisting of discrete measurements of a single time-series. A variation of this algorithm for use on small data sets was developed by Rosenstein et al [1993]. The use of these tools has expanded social science research utilizing chaos theory into such

[4] The BDS statistic examines the pattern of a series of data; information regarding the BDS statistic is discussed in Chapter IV.
[5] Lyapunov exponents measure the degree of SDIC; information regarding Lyapunov exponents is discussed in Chapter IV.

realms as economics, behavioral finance, management, psychology and sociology. See Table 5 for a listing of selected studies in the social sciences utilizing chaos theory. A brief discussion of some of this research follows.

Early research in economics can be grouped under two broad categories: (1) methods to detect chaos in economic systems; and (2) approaches to describe non-linear dynamics, including chaos [Etheridge and Sriram 1993]. Baumol and Benhabib [1989] provided examples of chaotic behavior resulting from economic activities. One such example involved advertising expenditures and its impact on profits. Chiarella [1990] discussed the applicability of non-linear theory to the study of economic dynamics. Denecker and Pelikan [1986] used chaos theory to explain the outcomes of competitive economies. Farmer [1986] examined deficits and economic cycles while Medio [1991] explored continuous time models. Rosser [1990] applied the concepts of chaos theory to classic Keynesian economics. While qualitative changes in a system's behavior are most evidenced via observation of its time-series, the applicability of chaos theory is not limited to longitudinal studies. Craig et al [1991] used a modified chaos technique for a cross-sectional study of marginal housing prices.

In the area of finance, Frank and Stengos [1988] conducted a study of gold and silver prices and found evidence of chaos. Larrain [1991] found non-linearities in Treasury bill rates. Thaler's [1993] book is a collection of works in behavioral finance with many studies exploring investor behavior. These studies report evidence that "irrational" investor behaviors such as overreactions, noise trading, and investment

TABLE 5

CHAOS THEORY STUDIES – SOCIAL SCIENCES

Author	Year	Author	Year
ECONOMICS		FINANCE	
Anderson et al	1988	Brock and Malliaris	1989
Arthur	1989	Frank and Stengos	1988
Baumol and Benhabib	1989	Larrain	1991
Chiarella	1990	Thaler	1993
Craig et al	1991		
Denecker and Pelikan	1986		
Farmer	1986	MANAGEMENT	
Grandmont and Malgrange	1986	Anderson and Sturis	1988
Holland	1988	Baburoglu	1988
Kelsey	1988	Bahlmann	1990
Mandelbrot	1963	Cheng and Van de Ven	1996
Medio	1991	Dooley	1997
Nijkamp and Reggiani	1991	Eisenhardt and Schoonhoven	1990
Rosser	1990	Feichtinger and Kopel	1993
Smale	1980	Goldstein	1994
		Kiel	1994
		Leifer	1989
PSYCHOLOGY		Mosekilde and Larsen	1988
Abraham et al	1990	Priesmeyer	1992
Barton	1994	Priesmeyer and Baik	1989
Guastello	1995	Rasmussen and Mosekilde	1988
Sterman	1988	Smilor and Feeser	1991
		Stacey	1992
		Sterman	1989
SOCIOLOGY		Thietart and Forgues	1995
Dendrinos and Sonis	1990		
Holland and Leinhardt	1977		

fads influence the financial markets thereby generating a significant portion of market volatility [Mouck 1998].

Management is another realm of social science where chaos theory has been extended. Priesmeyer and Baik [1989] used alternative forecasting techniques and found corporate performance to be non-linear. Mosekilde and Larson [1988] presented a chaotic inventory management model for production distribution and sales. Feichtinger and Kopel [1993] used a deterministic managerial decision rule to analyze the chaotic output of an iterative research and development model. Eisenhardt and Schoonhoven [1990] examined newly formed semi-conductor firms and found linkages to chaos. Leifer [1989] used a dissipative structure model to understand organizational transformation. Smilor and Feesor [1991] identified factors that can contribute to the chaotic behavior of entrepreneurial firms. Sterman [1988] used chaos theory to model management behavior while Holland and Leinhardt [1977] developed a dynamic model for social interaction.

Accounting

Within the realm of financial accounting, capital markets has been one of the most prominent areas of research. The efficient market hypothesis (EMH) is based on a linear paradigm and presumes that the market responds only upon the receipt of new information. Stock prices quickly adjust to a new equilibrium. The returns are independent (i.e., random variables), have a normal probability distribution and follow a random walk. According to portfolio theory, the distribution of returns is a function of the riskiness of a portfolio and is measured by its variance. The capital asset pricing model (CAPM) combined the EMH and portfolio theory into a model of investor behavior based on rational expectations in a general equilibrium framework.

The linear paradigm implies that returns should have approximately normal distributions and be independent. Many studies were conducted examining the distributional properties of capital markets data. Fama [1965] found that returns were negatively skewed with fatter tails and a higher peak (i.e., leptokurtotic) than that predicted by the normal distribution. Using daily Standard and Poor (S&P) index returns from 1928 through 1989, Turner and Weigel [1990] found similar results. Friedman and Laibson [1989] used quarterly S&P 500 returns from 1946 through 1988. In addition to being leptokurtotic, the authors noted that large movements were more often the result of crashes rather than rallies. Sterge [1989] studied financial futures prices of treasury bond, treasury note and Eurodollar contracts. He found the same leptokurtotic distribution and that very large price changes occurred two to three times as often as predicted by normality. Results from these studies show there is little basis to the assertion that the distribution of market returns is approximately normal. The findings also weaken the argument that stock price movements are the result of a random walk [Peters 1996].

Mandelbrot [1964] suggested that market returns follow a family of distributions called stable Paretian. Such distributions have high peaks at the mean and fat tails. They also exhibit two interesting characteristics. The first is called the "Joseph effect" and refers to the tendency of the distribution to be persistent, to follow trends and cycles. A persistent time-series will have a long memory. In other words, there is a long-term correlation between current events and future events. The second is called the "Noah effect" whereby the distribution is still persistent but subject to abrupt and discontinuous changes. The stock market crash of 1929 and the severe decline that occurred October 1987 provide evidence that large price changes can be discontinuous and very abrupt. Such characteristics are not applicable to normal distributions.

Using various chaos theory techniques, Peters [1996] examined the stock market, the treasury bond market and the currency exchange market. He found leptokurtotic distributions of persistent time-series characterized by long memory processes. Using U.S. stock returns, the long memory effect is approximately four years (i.e., the cycle time). The U.K. equity market has an eight-year cycle while Germany's is six years and Japan's is four years. There is a five-year cycle time for the treasury bond market. Peters [1996] found evidence that stock returns are the result of a biased random walk.

Numerous market anomalies have been found. For example, the January effect refers to the historical pattern that stock prices rise in the first few days of January. The small-firm effect is the tendency of small firms to outperform the stock market. Another effect is the tendency of portfolios of stocks with low price-earnings ratio to outperform portfolios of stocks with high price-earnings ratios. These effects have been shown to give statistically significant excess returns without an increase in volatility. Bernard [1993] examined anomalous findings in capital markets research.

Contrary to the EMH, information is not immediately reflected in prices and the capital markets appear to be non-linear dynamic systems that exhibit deterministic chaos. Hinich and Patterson [1985], Scheinkman and LeBaron [1989] and Willey [1992] examined non-linearity in daily stock returns and indices. Savit [1989] examined non-linearities in options prices while Hsieh [1989] and Sewell et al [1993] found non-linear dependence in foreign exchange rates. Freeman and Tse [1992] developed a non-linear model of security price responses to unexpected earnings. See Table 6 for a listing of selected capital markets studies utilizing chaos theory. Peters [1994, 1996] and Mouck [1998] have a more thorough discussion on the challenge to capital markets research generated by chaos theory.

Outside the area of capital markets, chaos theory has been little used in accounting research. Etheridge and Sriram [1993] discuss the implications of chaos theory and non-linear dynamics for accounting researchers. They suggest that chaos provides the theoretical framework and techniques to perform sensitivity analysis prior to model selection. For example, examination of the attractors would reveal the underlying dynamics of the time-series and indicate whether the series is stable, fluctuating (i.e., oscillating), or chaotic. Ignoring the underlying dynamics of the system would result in selecting models that do not robustly represent the system and therefore result in low explanatory power. They suggest that chaos techniques can be used in the following areas: (1) classification studies; (2) policy and planning studies; (3) budgeting studies; and (4) strategy studies. Lindsay and Campbell [1996]

TABLE 6

CHAOS THEORY STUDIES – CAPITAL MARKETS

Author	Year	Area
Aczel and Josephy	1991	examined foreign exchange rates
Ambrose et al	1993	examined long-term persistence in stock prices
Aydogan and Booth	1988	examined long-term persistence in stock prices
Bernard	1993	examined anomalous findings in capital markets research
Cheng et al	1992	examined unexpected earnings response regression model
Cochran et al	1993	examined predictability of foreign stock returns
Das and Lev	1994	examined returns/earnings relations
Freeman and Tse	1992	developed non-linear model of security price responses to unexpected earnings
Goetzmann	1993	examined long-term persistence in stock prices
Granger and Morgenstern	1964	performed spectral analysis on stock prices
Greene and Fielitz	1977	examined long-term dependence in stock returns
Hinich and Patterson	1985	examined non-linearity in daily stock returns
Hsieh	1989	examined non-linear dependence in daily foreign exchange rates

Author	Year	Area
Lo	1991	examined long-term memory in stock prices
Mandelbrot	1966	examined distributional properties of stock returns
Peters	1991	examined capital markets from a non-linear perspective
Savit	1989	examined non-linearities in options prices
Scheinkman and LeBaron	1989	examined non-linear dynamics in stock returns
Schwert	1989	examined business cycles and stock volatility
Sewell et al	1993	examined non-linearities in foreign capital markets
Willey	1992	examined non-lineary dynamics in daily stock indices

responded to Etheridge and Sriram's [1993] challenge and used non-linear dynamic methodology to develop a new model for bankruptcy prediction.

This research study similarly accepts the challenge and explores financial statement data of both fraudulent and non-fraudulent firms for evidence of non-linearity or the lack thereof. Examining the qualitative changes in financial ratios across time and identifying any differences in dynamics between fraudulent and non-fraudulent firms will provide evidence that could be used in subsequent research as a basis for the selection and construction of a fraud detection and/or prediction model. The following chapter presents the methodology employed in this exploratory study.

CHAPTER IV

METHODOLOGY

Introduction

Priesmeyer [1992] proposes that financial analysis requires more than examining the current relationships between various financial measures. Thus, an awareness of the chronological patterns in the financial relationships is necessary. Accordingly, this research study is a longitudinal examination of financial statement data for a sample of fraudulent and non-fraudulent firms. The variables used in this study are ratios computed from income statement and balance sheet data.

Sample Selection

Firms involved in fraudulent financial reporting were obtained from the SEC's Accounting and Auditing Enforcement Releases (AAERs) issued between 1982 and 1995. A firm reported in an AAER was included as a potential sample fraud firm if the SEC accused top management of reporting materially false and misleading financial statements. More specifically, the SEC alleged violation of Rule 10(b)-5 of the 1934 Securities Exchange Act. Rule 10(b)-5 requires the intent to deceive, manipulate or defraud. Upon finding sufficient evidence of fraud, the court entered final judgment of permanent injunction. For purposes of this study, the "fraud year" was defined as the first year for which the financial statement(s) included fraudulent data. In most instances, the actual discovery of the fraud occurred several years subsequent to the fraud year. A fraud firm was included in the sample if SEC 10Q reports were available for a minimum period of seven years inclusive of the fraud year. See Table 7 for a reconciliation of AAERs and the identification of the sample fraud firms.

TABLE 7

IDENTIFICATION OF FRAUD FIRMS

Accounting and Auditing Enforcement Releases (AAERs) #1 – 712 for the period 4/82 - 9/95	712
Less:	
• AAERs not referencing violation of Rule 10(b)-5	(423)
• AAERs affecting banks/insurance firms, CPA firms, registration statements, or fraud year(s) not identified	(161)
• AAERs expanding other AAERs (e.g., duplicate AAERs for same firm)	(49)
• SEC 10Q reports either not available or available for less than the minimum time period of seven years	(49)
Total number of fraud firms included in study	30

For each fraud firm included in the sample, financial statement data were collected for the entire period the firm is/was public, subject to data availability. Since federal legislation requires quarterly reporting, SEC 10Q reports were used for data collection. This allowed construction of a complete time-series of the financial statement data for the fraud firm for the greatest number of periods thereby significantly increasing the number of data points available for analysis. Note this time-series included the period before the occurrence of the fraud and the period subsequent to the occurrence of the fraud. While the sample firms had different pre-fraud and post-fraud periods, the time-series data captured any change in the dynamics of the financial statement data.

Using COMPUSTAT, each fraud firm was matched with a non-fraud firm based on the following requirements:

1. firm size – a non-fraud firm was considered similar if total assets were within +/- 40% of the total assets for the fraud firm in the year preceding the fraud year; if no matches were found, a non-fraud firm was considered similar if total sales were within +/- 40% of the total sales for the fraud firm in the year preceding the fraud year;
2. time period – firms identified in (1) above were reviewed to identify those non-fraud firms for which the 10Q reports were available for the same time period as the fraud firm;
3. industry – firms identified in (2) above were reviewed to identify a non-fraud firm within the same four-digit SIC as the fraud firm; the non-fraud firm chosen was the one with the closest total assets or total sales to the fraud firm; if no match was found using the four-digit SIC, then the three-digit codes were matched.

One inherent limitation of the above sample selection process involved the potential misclassification of a non-fraud firm. Financial statement fraud might have occurred but had not yet been detected. See Table 8 for a listing of the matched fraud and non-fraud firms.

Ratio Selection

The variables in this study were ratios computed from the sample firms' quarterly income statements and balance sheets. Ideally, the selection of financial ratios to be used for analysis should be based on theory and coupled with demonstrated empirical evidence of their usefulness. An acceptable theoretical foundation for the selection of ratios for decision making does not currently exist. As discussed in Chapter II, prior studies have produced scattered heterogeneous empirical evidence regarding ratio usefulness. To date, a complete set of useful ratios has not been identified.

For this research study, several considerations governed the process of ratio selection. The Committee of Sponsoring Organizations of the Treadway Commission (COSO) sponsored a research study by Beasley et al [1999] that provides a comprehensive analysis of fraudulent financial reporting occurrences investigated by the SEC subsequent to the issuance of the 1987 Treadway Commission Report. Beasley et al [1999] found that the two most common methods of fraudulent financial reporting were the improper recognition of revenue and the overstatement of assets (excluding accounts receivable due to revenue fraud). Fifty percent of the sample firms recorded revenues inappropriately, primarily by recording revenues prematurely or by creating fictitious revenue transactions. In addition, fifty percent of the sample

TABLE 8

MATCHED FRAUD / NON-FRAUD FIRMS

	Fraud Firms	Non-fraud Firms
1	Chronar Corp	Electric M&R
2	Comserve Corp	Rand Information Systems
3	Datapoint Corp (Data Point)	Cray Research
4	Digilog Inc	Boonton Electronics Corp
5	Earthworm (Tractor Co) Inc	Crown Zellerbach Corp
6	Electro-Catheter Corp	Bioresearch Medical Products
7	Flight International Group Inc	Offshore Logistics Inc
8	Information Solutions Inc	Hadron Inc
9	Kellett Corp	Goddard Industries
10	Levin Computer Corp (Levin International Corp)	Grantree Corp
11	Matrix Science Corp	Robinson-Nugent Inc
12	Miniscribe	Certron Corp
13	Oak Industries Inc	Burndy Corp
14	Poloron Products Inc	Lindal Cedar Homes Inc
15	Ramtek Corp	Scan Optics Inc
16	Rocky Mount Undergarment Co	FAB Industries
17	Stauffer Chemical Co	Big Three Industries Inc
18	Storage Technology Corp	Gould Inc
19	United States Surgical Corp	Cobe Laboratories Inc
20	Collins Industries Inc	Spartan Motors Inc
21	Fidelity Medical Inc	Biochem International Inc
22	Horizon Technology Inc	Communications Corp of America
23	Programming & Systems Inc	National Data Corp
24	MMI Medical Inc (R2 Scan Systems Inc)	OCG Technology
25	Star Technologies	Tandem Computers Inc
26	Telephone Specialists Inc	Coradian Corp
27	Video Station	Schwartz Bros Inc
28	Ocilla Industries Inc	Manufactured Homes Inc
29	Systems & Computer Technology Corp	Hogan Systems
30	United States Shoe Corp	Petrie Stores Liquidation

firms overstated assets by overvaluing existing assets, recording fictitious assets, or capitalizing items that should have been expensed. Even excluding the effects of misstating accounts receivable due to the revenue recognition frauds, the two most common misstated asset accounts were inventory and accounts receivable. Other frequently misstated accounts include property, plant and equipment and loans/notes receivable [Beasley et al 1999]. Financial ratios composed of such misstated accounts were selected for inclusion in this study.

Pinches et al [1973, 1975] used factor analysis to reduce 48 ratios to seven factor patterns. Gombola and Ketz [1983] investigated the impact of cash flow measurement upon the classification patterns of financial ratios and found that 40 ratios could be reduced to eight factors. The cash flow ratios loaded on a separate and distinct factor, while the other seven factors were substantially similar to those found by Pinches et al [1973, 1975].

Using the data from Kaminski et al [2000], a factor analysis was performed on 21 financial ratios from 76 matched fraud/non-fraud firms for the time period one year prior to the first occurrence of fraud. Results indicated a seven-factor solution. Despite using a different and smaller set of financial ratios (e.g., 21 versus 48) than those used in the early classification studies, the results were consistent with the findings of the prior studies of Pinches et al [1973, 1975] and Gombola and Ketz [1983]. Accordingly, the Pinches et al [1973, 1975] classifications were included in this study.

Because information overlaps individual ratios and results in ratio redundancy, only one or two ratios from each factor was needed to adequately represent that factor and at the same time be independent of the other factors thereby restricting multicollinearity. The resulting reduced set of financial ratios still provides for a representative and comprehensive analysis of the ratio factor patterns.

Another consideration in ratio selection was data availability. Quarterly financial statements are usually summarized reports and less detailed than annual financial statements. Accordingly, ratio choice was further limited to those ratios whose components were reported on the SEC 10Q financial statements. Given the time frame of this study, (i.e., mid-1970s thru mid-1990s) and the fact that the statement of cash flows was not a required financial statement until 1987, ratios based on cash flow data were not included. Lastly, the large number of tests that were performed on each ratio for each firm also limited the number of ratios used.

Given these considerations, a parsimonious yet comprehensive and representative selection of financial ratios was chosen for this study. See Table 9 for a listing of the selected ratios.

Tests

There are two basic methods for measuring chaos. The first method is to use the equations of a fully-specified model of the dynamic system. Iterations of the equations with various parameter and starting values are then analyzed. Very few systems have such models available. The second method is to use the data from the

TABLE 9

RATIOS

Classification		Ratio	COSO[1]	PMC[2]	KWG[3]
Return on Investment	R1	Net Income / Total Assets	*	*	*
	R2	Net Income / Sales	*	*	
Capital Intensiveness	R3	Sales / Total Assets	*	*	*
	R4	Fixed Assets / Total Assets	*		*
Inventory Intensiveness	R5	Inventory / Sales	*	*	*
	R6	Current Assets / Sales	*	*	
Financial Leverage	R7	Total Liabilities / Total Assets	*	*	*
Receivables Intensiveness	R8	Sales / Accounts Receivable	*	*	*
	R9	Accounts Receivable / Inventory	*	*	*
Short-Term Liquidity	R10	Current Assets / Current Liabilities	*	*	*
Cash Position[4]					

[1] Account or account category included in COSO report [Beasley et al 1999].
[2] Ratio had high factor loading in study by Pinches et al [1973, 1975].
[3] Ratio found significant in study by Kaminski et al [2000].
[4] Cash flow ratios were excluded in this study.

dynamic system to test whether a deterministic process exists. Presently, there are two approaches to the analysis of this data, the purposes of which are to gain additional descriptive insight about the dynamics of the system. The metric approach focuses on the distance between points on the attractor. The topological approach focuses on the organization of the attractor. Both approaches depend on the behaviors evidenced by the time-series data [Gilmore 1993, 1996]. Since there is no fully-specified model of the financial accounting system, this research study employed the second method.

Given that chaos theory is a relatively new field of research and its methodology is evolving as we speak, there exists no definitive set of procedures for analyzing a chaotic time-series. Several well-established tools have been identified in the literature, six of which were applicable to this study. The first five tools utilized metrics while the sixth one used the topological approach. A discussion of each of these tools follows and is summarized in Table 10.

The Hurst Exponent

One of the metric tools is the calculation of the Hurst exponent. H. E. Hurst, a hydrologist, discovered a methodology for distinguishing a random system from a non-random system. Speaking on a broad scale, a system is the result of a long stream of interconnected events. Where we are now is a result of where we have been in the past. Similarly, where we will be in the future is dependent upon where we are in the present [Peters 1996]. The dynamics of the system is captured in its time-series. One common type of time-series is a random walk, also known as Brownian motion. In

TABLE 10

METHODOLOGICAL TESTS

Approach	Tool	Benefits
Metric	Hurst exponent	Provides evidence of whether a time-series is random, periodic or chaotic
Metric	Lyapunov exponent	Provides evidence of type of attractor and degree of chaos exhibited by the system
Metric	Correlation dimension	Provides evidence of whether attractor is due to a random or chaotic process; also indicates number of variables necessary to model the system
Metric	BDS statistic	Provides evidence of whether the time-series is random (i.e., IID) or the result of non-linear dynamics
Metric	Shuffle test	Provides evidence of whether the time-series is random (i.e., IID) or the result of non-linear dynamics
Topological	Phase space map	Provides visual evidence of type of attractor

such a case, on average, the value of x moves away from its initial position by an amount proportional to the square root of time. Using each point in the time-series as an initial condition and plotting the root-mean-square displacement against time produces a curve, the slope of which is called the Hurst exponent [Sprott and Rowlands 1995]. As a nonparametric measure, no assumptions are made concerning the underlying distribution of the time-series (e.g., normally distributed).

In the case of a random walk, events are random and uncorrelated. A time-series may be antipersistent or mean-reverting (i.e., past trends tend to reverse in the future) and more volatile than a random series. Alternately, a time-series may be persistent and characterized by long-term memory effects (i.e., past trends persist into the future). What happens in the present has an impact on the future. Speaking in terms of chaos theory, there is a sensitive dependence on initial conditions (i.e., SDIC). Such a time-series is a biased random walk, also known as fractional Brownian motion.

The Hurst exponent can be used to classify a time-series. A random time-series should produce a Hurst exponent near zero whereas a periodic time-series should produce a Hurst exponent near one. If the Hurst exponent is neither near zero nor one, there is evidence that the time-series is chaotic.

In this study, the Hurst exponent was computed for each ratio for each of the fraud and non-fraud firms. The findings will provide evidence as to the behavior of the time-series and whether chaos is exhibited by the system. The findings will also indicate whether there are differences among firm type and/or financial ratios.

The Lyapunov Exponent

A second metric tool is the calculation of Lyapunov exponents. While chaotic systems are deterministic, they are only predictable for short periods of time due to their sensitive dependence on initial conditions (SDIC). Lyapunov exponents measure the degree of SDIC and tell us whether small changes in the initial values of the variables for the system produce different trajectories that are markedly divergent from the original trajectory. For chaotic systems, Lyapunov exponents tell us that the divergence from the original trajectory is very rapid. In essence, the divergence is exponential. Positive Lyapunov exponents indicate divergence from the original trajectory while negative values indicate convergence. A zero value indicates constancy.

The divergence and convergence of the trajectories is similar to the stretching and folding that results in the process of kneading dough [Peitgen et al 1992]. Start with two points on the dough that are close together. Next observe the distance between them as the dough is repeatedly stretched and folded. As the dough is being stretched, the points diverge from one another. A positive Lyapunov exponent measures stretching in phase space. When the dough is being folded, initial conditions that were distant may converge. A negative Lyapunov exponent measures folding in phase space. The stretching action represents SDIC while the folding action represents the presence of an attractor.

There are as many Lyapunov exponents as there are dimensions in the system. Each exponent describes divergence in a separate direction and is intuitively comparable to a separate direction along each axis in phase space [Brown 1995]. Lyapunov exponents offer a way to classify attractors. When dealing with one variable in one dimension, if the single Lyapunov exponent is zero, the system exhibits marginally stable behavior and the attractor is a fixed point. If the exponent's value is negative, the different trajectories converge and the attractor is periodic. If the exponent is positive, nearby trajectories diverge and the attractor is a strange attractor. The system is chaotic. The magnitude of the positive Lyapunov exponent determines the speed of divergence of the trajectory paths.

In this study, the largest Lyapunov exponent was computed for each ratio for each of the fraud and non-fraud firms. The findings will provide evidence as to the type of attractor as well as the degree of chaos exhibited by the system. The findings will also indicate whether there are differences among firm type and/or financial ratios.

The Correlation Dimension

Another numerical tool for measuring chaos is the correlation dimension. It is a measure of the spatial correlation of scatter points in m-dimensional space. It determines the relationship between each point in a time-series and all of the other points in the series [Grassberger and Procaccia 1983]. The correlation dimension indicates the dimension of the attractor. While a point has a dimension of zero, a line has dimension of one, and a plane has dimension of two, the correlation dimension for a chaotic attractor will be a non-integer. The issue is to determine whether an attractor that results from a seemingly random time-series is due to low-dimensional chaos or infinite-dimensional randomness. A two-dimensional phase space map of a time-series of uniformly distributed random numbers fills the plane. Continuing with spaces of higher dimension, the time-series continues to fill the available space. In the limit, random numbers are infinite-dimensional. This is a major difference between a chaotic system and a random system. In a chaotic system, the correlation measure converges to a fixed value despite being tested in higher and higher dimensions. In a truly random system, such convergence does not occur. The correlation dimension is also an indicator of the number of variables that are necessary to model the dynamic system.

Grassberger and Procaccia [1983] studied a measure called the correlation integral to test for evidence of chaos. The correlation integral essentially measures the frequency with which temporal patterns are repeated in time-series data and is obtained from the spatial correlations between points on the attractor. More technically, the correlation integral is a measure of spatial correlation of scatter points in *m*-dimensional space,

$$C_{m,T}(\varepsilon) = \sum_{t<s} I_\varepsilon\left(x_t^m, x_s^m\right) \times \left[2 / \left(T_m(T_m - 1)\right)\right],$$

where $T_m = T - (m-1)$,

$x_t^m = (x_t, \ldots, x_{t+m-1})$,

and $I_\varepsilon\left(x_t^m, x_s^m\right)$ is an indicator function which equals 1 if $\left\|x_t^m - x_s^m\right\| < \varepsilon$, and equals 0 otherwise.

Here $\{x_t\}$ is a scalar time-series under scrutiny for randomness. In order to use the equation to measure intertemporal local correlations and dependence, one embeds $\{x_t\}$ in an m-dimensional space by forming m-vectors $x_t^m = (x_t, \ldots, x_{t-m-1})$ starting at each date t.

For stochastic and deterministically chaotic systems, as $T \to \infty$,
$C_{m,T}(\varepsilon) \to C_m(\varepsilon) \equiv \text{Prob}\,\{\|x_t^m - x_s^m\| < \varepsilon\}$ w.p. 1, for almost all initial conditions.

The definition of correlation dimension in embedding dimension m is defined as:

$$d_m = \lim_{\varepsilon \to 0} \lim_{T \to \infty} \log [C_{m,T}(\varepsilon)] / \log(\varepsilon).$$

The correlation dimension itself is given by:

$$d = \lim_{m \to \infty} d_m.$$

A major difference between a chaotic process and a truly random process is that they both appear random to the naked eye and to standard linear time-series methods, yet the truly random process will have high to infinite dimension, whereas the chaotic process will have low dimension. According to Brock et al [1991], low dimensional chaos will have a correlation dimension substantially lower than 10, perhaps 5 or 6.

Since an independent and identically distributed stochastic process has correlation dimension of infinity, the correlation dimension can be used to distinguish deterministic chaos from truly random systems. If the time series is truly random, the slope d_m of log $C_m(\varepsilon)$ versus log (ε) will increase indefinitely as m is increased. If the time series is low dimensional, the slope d_m of log $C_m(\varepsilon)$ versus log (ε) increases at a rate slower than m. The specific value d_m^* where d_m stabilizes is then an estimate of the correlation dimension. If convergence does not occur, one can accept the null hypothesis that the time series is random.

In this study, the correlation dimension was computed for each ratio for each of the fraud and non-fraud firms. The resulting value will provide evidence as to whether the attractor of the time-series is due to randomness or due to a chaotic process. Comparisons can then be made between fraudulent and non-fraudulent firms as well as the various financial ratios.

The BDS Statistic

Brock et al [1987] developed the BDS statistic and used the correlation dimension to perform a statistical test of non-linear dynamics. The BDS test examines the pattern of a series of data. The ith observation is compared to the $(i + 1)$th observation. There is some positive probability that these observations are within a known distance d of each other. Next, a third observation is compared with the second and again there is a positive probability that it is within the same distance. This process is repeated for all observations in the time-series. The BDS statistic compares how often a series of data points are actually within the distance d of each other to the expected value if the series were independently and identically distributed (IID). In essence, it tests the null hypothesis that the time-series is IID (i.e., randomly distributed) by fitting a model to the data and then testing the estimated errors of the model [Craig et al 1991]. A significantly positive BDS statistic implies that points in an *m*-history space have a probability of clustering together more than what would be probable with truly random data. A random time-series would result in a large negative BDS statistic. The BDS test has been shown to have good power to detect non-IID behavior. The test has also been shown to be effective for small sample sizes.

In this study, the BDS statistic was computed for each ratio for each of the fraud and non-fraud firms. The statistic will provide direct evidence as to whether the time-series is random or the result of non-linear dynamics. The findings will also indicate whether there are differences among firm type and/or financial ratios.

Rejection of the null hypothesis of IID does not immediately lead to the conclusion of deterministic chaos. The statistic implies only non-IID behavior (i.e., non-linear dependence) which could be the result of various kinds of non-linear influences. Chaotic dynamics is one such influence but so is a non-linear stochastic process. Accordingly, the BDS statistic should be used in conjunction with other tests to provide more conclusive evidence of the presence of chaos. Such an approach is being utilized in this study through the use of multiple measures (e.g., Hurst exponents, Lyapunov exponents, correlation dimension, phase space maps).

The Shuffle Test

Another metric tool for investigating whether a time-series is random is the shuffle test [Theiler et al 1992]. Surrogate data is generated by taking the original time-series and randomly shuffling the sequence of the data. The surrogates have the same distributional characteristics as the original data, but the correlations in the original data that reflect the system dynamics have been destroyed by shuffling. A statistic of interest (e.g., the correlation dimension) is compared to the same statistic for the original data. If the results are the same, there is evidence that the original time-series is IID. If the results are different, there is evidence of non-linear structure in the original data. This test can be repeated for multiple shuffles. For example, performing the shuffle test 20 times and obtaining a lower correlation dimension on just one of the tests would give a non-parametric level of significance of 1/20 or p = .05. The result of this test would be the rejection of the null hypothesis of randomness at the 95% confidence level.

In this study, the shuffle test was performed for each ratio for each of the fraud and non-fraud firms. The shuffle test was conducted 20 times using the BDS statistic as the test statistic. The findings will provide additional evidence as to whether the time-series is random. Comparison of the findings among fraudulent and non-fraudulent firms and the various financial ratios can then be made.

Phase Space Map

In addition to the metrics described above, a topological approach focusing on the organization of the attractor can also be utilized. Through a phase space reconstruction of the shape of the attractor, additional descriptive insights about the dynamics of the system can be found. Phase space requires a dimension for each variable of the system. One quickly exceeds the capability to topologically construct the behavior of the system (e.g., drawing trajectories in the 4^{th} or 5^{th} dimension). The procedure therefore uses only one variable at a time and does not require *a priori* knowledge of the formal equation(s) that underlie the dynamic system. The procedure extracts the geometric features of the system's behavior and creates a simulated phase space of the one-dimensional time-series record [Kellert 1993]. The value of the variable is simply plotted against its lag. Using different values of lag length, the plot depicts the rotation of the attractor in its own phase space. This procedure has been shown to be effective even in the presence of substantial amounts of stochastic noise [Brown 1995]. This phase space map enables a visual inspection of the attractor and provides evidence as to the type of attractor (i.e., fixed point, limit cycle, strange). According to Kellert [1993], reconstruction of attractors is one of the most important methods for discovering and analyzing chaos. It enables the researcher to study a system's qualitative features without solving or even knowing the equations that govern the system.

A phase space map was constructed for each ratio for each of the fraud and non-fraud firms. Analysis of the maps should provide evidence as to whether there are differences in the attractors of the fraudulent versus non-fraudulent firms. Additionally, the maps should also indicate any differences in the attractors of the various financial ratios.

Each of the above procedures (i.e., Hurst exponent, Lyapunov exponents, correlation dimension, BDS statistic, shuffle test and phase space maps) was conducted using the software program *Chaos Data Analyzer – The Professional Version* [1995] (CDA). This program has been used in several published studies, including those applying chaos methodology to financial data (e.g., Lindsay and Campbell 1996, Dooley and Van de Ven 1999]. The results of these procedures will provide evidence of non-linear dynamics or the lack thereof in financial statement data. The findings will also indicate whether there are differences among firm type and/or financial ratios. Table 10 provides a summary of the methodological tests performed in this exploratory study. The following chapter presents the results of these tests.

CHAPTER V

RESULTS

Introduction

This research study was a longitudinal examination of financial statement data for 30 matched pairs of fraudulent and non-fraudulent firms. The variables used in this study were ten financial ratios computed from quarterly income statement and balance sheet data. Multiple tests of the ratio values were conducted to determine the behavior of the time-series. More specifically, the tests were conducted to provide evidence of random, periodic or chaotic behavior. This chapter presents the analysis and results of the empirical examination. It is organized as follows. The first section is a descriptive summary of the sample data that was used for the subsequent metric and topological tests. This is followed by a discussion of each measurement's sensitivity to sample size. The third section is a discussion of the grouping and reporting of the data for each measure. Next, the results of each of the measurement tests are then discussed. This is followed with a discussion of the performance of some additional analyses. Finally, there is a summary of the findings of this study.

Descriptive Summary of Sample Data

For each of the 30 fraud and non-fraud firms identified in Table 8, the ten financial ratios identified in Table 9 (i.e., R1 thru R10) were computed for the entire time period for which SEC 10Q financial statements were available. The minimum time period for inclusion in the study was seven years. This was to allow a minimum time-series of 28 data points for analysis for each ratio. The fraud firms had a mean sample size of 42.50 with a range of 28 to 80 time periods. The non-fraud firms had a larger mean sample size of 55.30 with a range of 30 to 89 data points. On average, the fraud firms reported fraudulent financial data over 55% of their time-series. Such fraudulent reporting ranged from 8% to 95%. These descriptive statistics are summarized in Table 11. Before proceeding with the results of the metric and topological tests, further discussion of sample size is warranted and is presented in the next section.

Measurement Sensitivity to Sample Size

Chaos theory developed from research undertaken in the physical sciences. The ability to control laboratory conditions and perform thousands of replications generated very large data sets. Early research in accounting utilizing chaos theory was primarily limited to the capital markets because thousands of data points were available for analysis. Most social science research is vastly different. Measurements are usually discrete rather than continuous, plus the data sets are much smaller and more prone to the inclusion of noise. Methods have been developed to account for

TABLE 11

SUMMARY OF DESCRIPTIVE STATISTICS

	Fraud Firms	Non-Fraud Firms
Sample Size		
Mean	42.50	55.30
Range	28 - 80	30 - 89
<u>% Fraudulent</u>		
Mean	0.55	
Range	.08 - .95	

such conditions and chaos theory research utilizing these methods has expanded into such realms as management, psychology and sociology. Rather than thousands of data points, this research uses sample sizes of fewer than 100 or even 50 data points. Despite the new methods, the smaller data sets produce measured results with reduced predictive power as would be expected.

To determine the sensitivity of sample size on the measurements utilized in this study, some preliminary tests and analyses were performed. The software program *Chaos Data Analyzer – The Professional Version* [1995] (CDA) includes sample data files of known random, periodic and chaotic time-series. The Hurst exponent, Lyapunov exponent, correlation dimension and BDS statistic were computed for each type of time-series (i.e., random, periodic and chaotic) for various sample sizes ranging from 2,000 data points to only 50 data points. A summary of the test results is presented in Table 12. A discussion of the analysis of each of the measures follows.

TABLE 12

MEASUREMENT SENSITIVITY TO SAMPLE SIZE

	N	Hurst Exponent	Lyapunov Exponent	Correlation Dimension	BDS Statistic
Random	2000	0.0026	0.895+/-0.031	4.407+/-0.048	(16.8631)
Random	200	(0.0041)	0.835+/-0.110	4.804+/-0.234	(0.5038)
Random	100	(0.0109)	0.698+/-0.222	5.006+/-2.393	(1.3327)
Random	50	0.0070	0.549+/-0.255	7.398+/-7.398	(2.5105)
Periodic	2000	0.7216	(0.001)+/-0.026	0.894+/-.0624	0.0699
Periodic	200	0.9634	(0.003)+/-0.082	1.451+/-0.501	0.0521
Periodic	100	0.9152	(0.003)+/-0.143	2.146+/-0.460	0.0713
Periodic	50	0.8049	(0.015)+/-0.188	7.398+/-7.398	0.2371
Chaos	2000	0.4382	0.075+/-0.034	2.001+/-0.103	0.3339
Chaos	200	0.6019	0.127+/-0.101	2.742+/-0.649	0.0950
Chaos	100	0.3833	0.261+/-0.121	2.975+/-0.957	0.1430
Chaos	50	0.2660	0.412+/-0.210	4.247+/-3.473	0.0598

For the Hurst exponent, the random time-series generated a near zero value regardless of sample size. The periodic time-series consistently resulted in a large value, though there was more variability in the range of values. The chaotic time-series produced middle values, but once again, the range of values was larger. As calculated by the software program CDA, it can be concluded that the Hurst exponent is a reliable measure even for small sample sizes.

For the Lyapunov exponent, both the random and chaotic time-series generated a range of positive values. However, the values for the random time-series were significantly larger. The periodic time-series resulted in a value consistent with zero regardless of sample size. While the Lyapunov exponent appeared somewhat sensitive to sample size, it is still a fairly reliable measure for discerning the type of time-series.

For the correlation dimension, each type of time-series produced a wide range of values that appears very sensitive to sample size. One would expect a large correlation dimension for a random series but the values ranged from 4.407 for the large data set to 7.396 for the smallest data set. The sensitivity to sample size is even more evident with the periodic series where the range varied from 0.894 to 7.398. With small data sets, it was not possible to distinguish a periodic time-series from a purely random one. For the chaotic series, the correlation dimension for the smallest sample size was 4.247. This value is very close to the 4.407 value for the largest sample size of the random series. Once again, using the correlation dimension as a measurement parameter, it was not possible to discern a chaotic time-series from a random one. In addition, note should be made of the substantial increase in the uncertainty of the mean as the sample size decreased. As calculated by the software program CDA, this measure is extremely sensitive to sample size. Accordingly, the correlation dimension is deemed an unreliable measure for the purposes of this study.

For the BDS statistic, the random time-series generated negative values regardless of sample size. Both the periodic and chaotic series produced positive values irrespective of sample size. It can be concluded that the BDS statistic has good power to detect non-IID behavior, even for small data sets such as those used in this study.

To summarize, given the results of the preliminary analysis described above, the Hurst exponent, Lyapunov exponent and BDS statistic are reliable measures for this study's limited sample size. The next section discusses the grouping and reporting of the data for each of the appropriate measures.

Data Grouping and Measurements

Using the software program CDA, six tests [i.e., (1) Hurst exponent; (2) Lyapunov exponent; (3) correlation dimension; (4) BDS statistic; (5) shuffle test; and (6) phase space maps] were conducted on the time-series of each of the financial ratios. More specifically, for each ratio for each firm, the following measures were computed: the Hurst exponent, Lyapunov exponent, correlation dimension and BDS statistic. For each of these measures, a mean value was computed for the fraud firm sample and the non-fraud firm sample for each ratio. This grouping by firm-type (i.e., fraud vs non-fraud) remained constant through subsequent analyses. Further sub-groupings are discussed below.

To determine if the time-series was sensitive to sample size, the data for each ratio was split into two groups – low sample size N and high sample size N. Given that the sample sizes fluctuated for each ratio depending upon the availability of company data, the split was determined by looking for a natural break for each ratio. Accordingly, the split between low sample size N and high sample size N varied between 47 and 50. Mean values for each of the measures were then computed for each group for each ratio.

To determine if the time-series was sensitive to the percent of time the fraud firm data was fraudulent, the data for each ratio was again split into two groups – low % F and high % F. Since the range of percent of time fraudulent varied so significantly from a low of 8% to a high of 95%, those fraud firms with less than 50% of the data prior to the first occurrence of fraud were placed in the low % F group. The remaining fraud firms were placed in the high % F group. Mean values for each of the measures were then computed for each group for each ratio.

Given the extensive use of mean values and the fact that the uncertainty in each mean is strongly dominated by the spread in the distribution, estimated errors of the means were also computed. For each of the mean values discussed above, the uncertainty of the means was reported as the mean value +/- the estimated error. The reporting of the uncertainty of the means strengthens the subsequent conclusions for the various quantitative measures. The following sections discuss the results of each of the measurement tests.

The Hurst Exponent

The Hurst exponent is a nonparametric measure that can be used to classify a time-series. A random time-series is subject to a normal Gaussian distribution where the average displacement never gets very large because the mean is fixed. The Hurst exponent is therefore near zero. Straight-line motion would have a Hurst exponent equal to one. A periodic time-series would therefore have a large Hurst exponent very near one. For a chaotic time-series, the Hurst exponent is dependent upon the attractor and would result in a value that is neither near zero nor one.

A summary of the Hurst exponent mean values is presented in Table 13. Ranges for the mean Hurst exponent for the various groupings are summarized below:

	F	N
Total	.0277 to .4250	.0280 to .3927
Low N	.0497 to .4385	.0714 to .4320
High N	(.0326) to .4420	.0028 to .3700
Low % F	.0085 to .3814	
High % F	.0405 to .4600	

The data indicates that R4, R7, R9, and R10 are the result of a chaotic time-series. There is no evidence that any of the ratios are from a periodic time-series. The ratios are consistent regardless of firm-type. The means for each ratio for both the sample size sub-grouping and the % fraudulent sub-grouping are consistent with the

TABLE 13

HURST EXPONENT – MEAN VALUES

	F	mean	est error	low N	est error	hi N	est error	low %F	est error	hi %F	est error
R1-NI/TA	F	0.0277	.0277+/-.0201	0.0497	.0497+/-.0171	(0.0326)	(.0326)+/-.0563	0.0085	.0085+/-.0464	0.0405	.0405+/-.0142
R2-NI/S	F	0.0845	.0845+/-.0308	0.1119	.1119+/-.0384	0.0092	.0092+/-.0381	0.0496	.0496+/-.0433	0.1078	.1078+/-.0425
R3-S/TA	F	0.1711	.1711+/-.0404	0.1860	.1860+/-.0449	0.1302	.1302+/-.0913	0.1493	.1493+/-.0854	0.1857	.1857+/-.0381
R4-FA/TA	F	0.3948	.3948+/-.0354	0.3776	.3776+/-.0450	0.4420	.4420+/-.0487	0.3814	.3814+/-.0686	0.4037	.4037+/-.0391
R5-INV/S	F	0.1787	.1787+/-.0328	0.1929	.1929+/-.0347	0.1401	.1401+/-.0810	0.2147	.2147+/-.0596	0.1477	.1477+/-.0335
R6-CA/S	F	0.1440	.1440+/-.0285	0.1657	.1657+/-.0255	0.0844	.0844+/-.0810	0.1338	.1338+/-.0571	0.1508	.1508+/-.0300
R7-TL/TA	F	0.4250	.4250+/-.0348	0.4385	.4385+/-.0394	0.3878	.3878+/-.0756	0.3725	.3725+/-.0675	0.4600	.4600+/-.0359
R8-S/AR	F	0.1751	.1751+/-.0337	0.1481	.1481+/-.0364	0.2496	.2496+/-.0747	0.1826	.1826+/-.0557	0.1702	.1702+/-.0434
R9-AR/INV	F	0.2535	.2535+/-.0272	0.2379	.2379+/-.0324	0.2848	.2848+/-.0508	0.2614	.2614+/-.0376	0.2481	.2481+/-.0389
R10-CA/CL	F	0.3336	.3336+/-.0258	0.3501	.3501+/-.0332	0.2882	.2882+/-.0286	0.2999	.2999+/-.0443	0.3561	.3561+/-.0311

R1-NI/TA	N	0.0280	.0280+/-.0268	0.0714	.0714+/-.0359	0.0028	.0028+/-.0363
R2-NI/S	N	0.0678	.0678+/-.0200	0.0983	.0983+/-.0394	0.0554	.0554+/-.0233
R3-S/TA	N	0.1242	.1242+/-.0281	0.1513	.1513+/-.0407	0.1084	.1084+/-.0379
R4-FA/TA	N	0.3927	.3927+/-.0347	0.4320	.4320+/-.0518	0.3700	.3700+/-.0460
R5-INV/S	N	0.1392	.1392+/-.0408	0.1722	.1722+/-.0645	0.1227	.1227+/-.0529
R6-CA/S	N	0.1090	.1090+/-.0268	0.1091	.1091+/-.0305	0.1090	.1090+/-.0382
R7-TL/TA	N	0.3546	.3546+/-.0390	0.4124	.4124+/-.0691	0.3212	.3212+/-.0466
R8-S/AR	N	0.1269	.1269+/-.0268	0.1510	.1510+/-.0427	0.1129	.1129+/-.0348
R9-AR/INV	N	0.1921	.1921+/-.0323	0.2413	.2413+/-.0641	0.1632	.1632+/-.0346
R10-CA/CL	N	0.3023	.3023+/-.0275	0.3056	.3056+/-.0398	0.3004	.3004+/-.4296

means from the full data set. The ratios do not appear to be sensitive to sample size or percent of time fraudulent.

The Lyapunov Exponent

The Lyapunov exponents measure the degree of SDIC and indicate whether small changes in the values of the variables for the system produce different trajectories that are markedly divergent from the original trajectory. Lyapunov exponents provide evidence as to the type of attractor produced by the dynamic system. If the Lyapunov exponent is zero, the system exhibits marginally stable behavior and has a periodic attractor. A negative exponent indicates converging trajectories and a fixed-point attractor. If the exponent is positive, trajectories diverge, indicating a strange attractor and therefore a chaotic system.

It should be noted that the software program CDA reports the Lyapunov exponent as a range. For example, the Lyapunov exponent for R1 for a sample firm might be 0.465+/-0.170. Such a range reports the estimated error of the mean value. The raw data produced ranges that were very disparate among the individual firms and their various ratios. It was deemed necessary to determine if the uncertainty of the ranges would adversely affect the mean values. Such could be determined if the mean were weighted inversely by the uncertainty of the range. Accordingly, for this measure, weighted means were calculated in addition to simple means. Such calculations were performed for the total data set grouped by firm type as well as the sample size sub-grouping.

A comparison of the weighted means and the simple means is presented in Table 14. As evidenced by the table, the results were very similar for both the fraud and non-fraud total data set and the sample size groupings. The uncertainty of the ranges did not have a material impact on the mean values. Given such results, it was deemed appropriate to use the simple means for analysis purposes.

A summary of the Lyapunov exponent values is presented in Table 15. Ranges for the mean Lyapunov exponent for the various groupings are summarized below:

	F	N
Total	.2333 to .3674	.2676 to .3758
Low N	.1971 to .4017	.1567 to .4453
High N	.2564 to .3799	.2923 to .3491
Low % F	.1983 to .3653	
High % F	.2567 to .3979	

The mean Lyapunov exponent is positive for all ratios tested, thereby eliminating the possibility of a periodic attractor. Since none of the positive values are very large, the ratios appear to be the product of a chaotic system. Doing a firm-type comparison, all of the Lyapunov exponents are relatively close except for R4. This ratio's Lyapunov exponent is .2333 for the fraud firms and .3587 for the non-fraud firms. The measure indicates that for this ratio, on average, the non-fraud firms seem to exhibit more chaos. Doing a low/high sample size comparison, the ratios are relatively close except for R2, R4, R7, and R10. No differences were found for the low/high % fraudulent grouping. It should be noted that the ranges for the Lyapunov

TABLE 14

LYAPUNOV EXPONENT – WEIGHTED vs SIMPLE MEANS

		R1 NI/TA weighted mean	R1 NI/TA simple mean	R2 NI/S weighted mean	R2 NI/S simple mean	R3 S/TA weighted mean	R3 S/TA simple mean	R4 FA/TA weighted mean	R4 FA/TA simple mean	R5 INV/S weighted mean	R5 INV/S simple mean
F	lo N	0.3788	0.3685	0.3881	0.4017	0.3477	0.3684	0.2773	0.1971	0.2369	0.2500
	hi N	0.3405	0.3585	0.2513	0.2564	0.3737	0.3646	0.3107	0.3330	0.3055	0.3123
	total	0.3670	0.3658	0.3459	0.3630	0.3559	0.3674	0.2905	0.2333	0.2599	0.2678
N	lo N	0.3536	0.4220	0.3950	0.4453	0.3395	0.3679	0.4180	0.3992	0.2970	0.3001
	hi N	0.3150	0.3491	0.2952	0.3039	0.3086	0.3216	0.3255	0.3353	0.3124	0.3260
	total	0.3262	0.3758	0.3231	0.3510	0.3180	0.3386	0.3519	0.3587	0.3081	0.3174

		R6 CA/S weighted mean	R6 CA/S simple mean	R7 TL/TA weighted mean	R7 TL/TA simple mean	R8 S/AR weighted mean	R8 S/AR simple mean	R9 AR/INV weighted mean	R9 AR/INV simple mean	R10 CA/CL weighted mean	R10 CA/CL simple mean
F	lo N	0.2492	0.2524	0.3960	0.3936	0.2992	0.2980	0.2305	0.2801	0.2632	0.2527
	hi N	0.2982	0.3120	0.2675	0.2699	0.2899	0.3069	0.3529	0.3731	0.3840	0.3799
	total	0.2663	0.2683	0.3565	0.3606	0.2954	0.3003	0.2964	0.3111	0.3074	0.2866
N	lo N	0.2177	0.2184	0.2875	0.3016	0.2710	0.2695	0.2202	0.2335	0.1766	0.1567
	hi N	0.3014	0.2923	0.2836	0.2996	0.2983	0.3204	0.3294	0.3374	0.3366	0.3327
	total	0.2775	0.2676	0.2849	0.3004	0.2906	0.3017	0.2968	0.2989	0.2850	0.2682

TABLE 15

LYAPUNOV EXPONENT – MEAN VALUES

	F	mean	est error	low N	est error	hi N	est error	low %F	est error	hi %F	est error
R1-NI/TA	F	0.3658	.3658+/-.0226	0.3685	.3685+/-.0236	0.3585	.3585+/-.0576	0.3177	.3177+/-.0276	0.3979	.3979+/-.0312
R2-NI/S	F	0.3630	.3630+/-.0311	0.4017	.4017+/-.0318	0.2564	.2564+/-.0666	0.3653	.3653+/-.0647	0.3614	.3614+/-.0306
R3-S/TA	F	0.3674	.3674+/-.0246	0.3684	.3684+/-.0284	0.3646	.3646+/-.0524	0.3354	.3354+/-.0470	0.3887	.3887+/-.0265
R4-FA/TA	F	0.2333	.2333+/-.0504	0.1971	.1971+/-.0648	0.3330	.3330+/-.0536	0.1983	.1983+/-.0746	0.2567	.2567+/-.0688
R5-INV/S	F	0.2678	.2678+/-.0291	0.2500	.2500+/-.0344	0.3123	.3123+/-.0575	0.2305	.2305+/-.0373	0.2957	.2957+/-.0427
R6-CA/S	F	0.2683	.2683+/-.0358	0.2524	.2524+/-.0440	0.3120	.3120+/-.0592	0.2135	.2135+/-.0633	0.3048	.3048+/-.0413
R7-TL/TA	F	0.3606	.3606+/-.0333	0.3936	.3936+/-.0417	0.2699	.2699+/-.0352	0.3637	.3637+/-.0588	0.3586	.3586+/-.0408
R8-S/AR	F	0.3003	.3003+/-.0322	0.2980	.2980+/-.0402	0.3069	.3069+/-.0529	0.2488	.2488+/-.0369	0.3347	.3347+/-.0468
R9-AR/INV	F	0.3111	.3111+/-.0374	0.2801	.2801+/-.0449	0.3731	.3731+/-.0687	0.2836	.2836+/-.0585	0.3299	.3299+/-.0494
R10-CA/CL	F	0.2866	.2866+/-.0238	0.2527	.2527+/-.0284	0.3799	.3799+/-.0213	0.2891	.2891+/-.0289	0.2850	.2850+/-.0354
R1-NI/TA	N	0.3758	.3758+/-.0382	0.4220	.4220+/-.0758	0.3491	.3491+/-.0419				
R2-NI/S	N	0.3510	.3510+/-.0351	0.4453	.4453+/-.0804	0.3039	.3039+/-.0306				
R3-S/TA	N	0.3386	.3386+/-.0222	0.3679	.3679+/-.0354	0.3216	.3216+/-.0284				
R4-FA/TA	N	0.3587	.3587+/-.0246	0.3992	.3992+/-.0449	0.3353	.3353+/-.0284				
R5-INV/S	N	0.3174	.3174+/-.0333	0.3001	.3001+/-.0733	0.3260	.3260+/-.0365				
R6-CA/S	N	0.2878	.2676+/-.0180	0.2184	.2184+/-.0181	0.2923	.2923+/-.0238				
R7-TL/TA	N	0.3004	.3004+/-.0190	0.3016	.3016+/-.0327	0.2996	.2996+/-.0240				
R8-S/AR	N	0.3017	.3017+/-.0242	0.2695	.2695+/-.0193	0.3204	.3204+/-.0362				
R9-AR/INV	N	0.2989	.2989+/-.0347	0.2335	.2335+/-.0478	0.3374	.3374+/-.0471				
R10-CA/CL	N	0.2682	.2682+/-.0310	0.1567	.1567+/-.0461	0.3327	.3327+/-.4315				

exponent are narrower for the larger data sets indicating some sensitivity of this measure to sample size.

Correlation Dimension

The correlation dimension determines the relationship between points in a time-series. The measure indicates the dimension of the attractor and determines whether the attractor is the result of a random or chaotic process. Random numbers are infinite-dimensional whereas in a chaotic system, the correlation measure converges to a fixed value.

Similar to the Lyapunov exponent, the software program CDA reports the correlation dimension as a range. For example, the correlation dimension for R1 for a sample firm might be 4.737+/-3.096. For the same reasons cited with the previous measure, weighted means were calculated in addition to simple means for the correlation dimension. Such calculations were performed for the total data set grouped by firm type as well as the sample size sub-grouping.

A comparison of the weighted means and the simple means is presented in Table 16. As evidenced by the table, unlike the Lyapunov exponent, the results are quite disparate for both the total data set and the various groupings. Given the findings of the preliminary analysis regarding the correlation dimension's sensitivity to sample size described previously, such results are not surprising. This provides additional evidence of the unreliability of this measure for this study. However, prior

TABLE 16

CORRELATION DIMENSION – WEIGHTED vs SIMPLE MEANS

		R1 NI/TA weighted mean	R1 NI/TA simple mean	R2 NI/S weighted mean	R2 NI/S simple mean	R3 S/TA weighted mean	R3 S/TA simple mean	R4 FA/TA weighted mean	R4 FA/TA simple mean	R5 INV/S weighted mean	R5 INV/S simple mean
F	lo N	4.0596	6.4642	4.5319	6.3660	6.1331	7.0046	6.0818	7.0041	4.7303	6.2827
	hi N	5.1584	4.3119	3.5433	4.3988	4.3478	6.0609	4.6739	5.8303	5.5201	4.4650
	total	4.3042	5.8265	4.1470	5.8234	5.4516	6.7443	5.6443	6.6429	4.9717	5.6768
N	lo N	5.6895	6.3514	5.2590	6.5398	3.9413	6.7137	5.0992	6.0923	4.2155	6.9544
	hi N	5.1524	5.3844	5.1120	5.0738	4.6013	4.8335	4.2330	5.3305	4.3563	5.1985
	total	5.3688	5.6953	5.1494	5.5287	4.2959	5.4818	4.5174	5.5754	4.2878	5.8063

		R6 CA/S weighted mean	R6 CA/S simple mean	R7 TL/TA weighted mean	R7 TL/TA simple mean	R8 S/AR weighted mean	R8 S/AR simple mean	R9 AR/INV weighted mean	R9 AR/INV simple mean	R10 CA/CL weighted mean	R10 CA/CL simple mean
F	lo N	4.2653	6.8441	5.1379	6.7677	5.5352	6.6492	4.0651	6.5776	5.2707	6.5943
	hi N	4.4138	5.0530	3.9832	5.0674	4.3781	5.5148	6.0084	5.1881	6.1911	5.0419
	total	4.3496	6.3324	4.5207	6.2639	5.0158	6.3131	4.5514	6.1330	5.6241	6.1661
N	lo N	4.8901	7.2726	5.2885	6.7311	4.4881	6.6270	4.6179	6.8339	4.7499	6.1698
	hi N	4.5161	5.0493	5.2529	5.3279	4.9327	4.9189	5.1791	5.8792	4.8350	5.8161
	total	4.6584	5.8160	5.2641	5.7789	4.7115	5.5079	4.9260	6.2464	4.7965	5.9298

to reaching a final conclusion on the applicability/dependability of this measure, further analysis was conducted using the simple means.

A summary of the correlation dimension values is presented in Table 17. Ranges for the mean correlation dimension for the various groupings are also summarized below:

	F	N
Total	5.6768 to 6.7442	5.4818 to 6.2464
Low N	6.2827 to 7.0046	6.0923 to 7.2726
High N	4.3119 to 6.0609	4.8335 to 5.8792
Low % F	5.0544 to 7.2677	
High % F	5.8111 to 6.9146	

The mean correlation dimension is large for all ratios tested regardless of firm type, sample size and % fraudulent groupings. Such findings would normally indicate the ratios are the product of a random series. In addition, there are lower values for the high sample size grouping and this is consistent across all ratios. However, given the inconclusive results of the preliminary analysis, the limited sample size of this study, and the disparate uncertainty of the means, the correlation dimension is deemed an unreliable measure. Accordingly, for the purposes of this study, the correlation dimension is an inappropriate measure and is excluded.

BDS Statistic

The BDS statistic tests the null hypothesis that the time-series is IID. A random time-series would result in a negative BDS statistic. Meanwhile, a positive

TABLE 17

CORRELATION DIMENSION – MEAN VALUES

	F	mean	est error	low N	est error	hi N	est error	low %F	est error	hi %F	est error
R1-NI/TA	F	5.6265	5.8265+/-.4644	6.4642	6.4642+/-.5869	4.3119	4.3119+/-.6261	5.6566	5.6566+/-.4731	5.9624	5.9624+/-.7301
R2-NI/S	F	5.8233	5.8233+/-.3730	6.3660	6.3660+/-.4212	4.3988	4.3988+/-.6301	5.0544	5.0544+/-.5117	6.3661	6.3661+/-.5122
R3-S/TA	F	6.7442	6.7442+/-.3361	7.0046	7.0046+/-.3985	6.0609	6.0609+/-.6364	6.4655	6.4655+/-.7595	6.9146	6.9146+/-.2437
R4-FA/TA	F	6.6429	6.6429+/-.4966	7.0041	7.0041+/-.6841	5.8303	5.8303+/-.4010	7.2677	7.2677+/-.1303	6.1074	6.1074+/-.7617
R5-INV/S	F	5.6768	5.6768+/-.6101	6.2827	6.2827+/-.7960	4.4650	4.4650+/-.8526	5.4888	5.4888+/-1.0186	5.8111	5.8111+/-.7803
R6-CA/S	F	6.3324	6.3324+/-.4243	6.8441	6.8441+/-.5434	5.0530	5.0530+/-.5032	6.0387	6.0387+/-.5718	6.5526	6.5526+/-.6145
R7-TL/TA	F	6.2639	6.2639+/-.4470	6.7677	6.7677+/-.5925	5.0674	5.0674+/-.4791	6.1167	6.1167+/-.3955	6.3817	6.3817+/-.7174
R8-S/AR	F	6.3131	6.3131+/-.4580	6.6492	6.6492+/-.6162	5.5148	5.5148+/-.4711	6.1954	6.1954+/-.4856	6.4072	6.4072+/-.7081
R9-AR/INV	F	6.1330	6.1330+/-.5575	6.5776	6.5776+/-.5604	5.1881	5.1881+/-1.0556	6.0922	6.0922+/-.9765	6.1601	6.1601+/-.7369
R10-CA/CL	F	6.1660	6.1660+/-.3417	6.5943	6.5943+/-.3962	5.0419	5.0419+/-.5944	6.2967	6.2967+/-.3985	6.0738	6.0738+/-.5094
R1-NI/TA	N	5.6953	5.6953+/-.3516	6.3514	6.3514+/-.9348	5.3840	5.3844+/-.2506				
R2-NI/S	N	5.5287	5.5287+/-.3533	6.5398	6.5398+/-.7644	5.0738	5.0738+/-.3745				
R3-S/TA	N	5.4818	5.4818+/-.3581	6.7137	6.7137+/-.7277	4.8335	4.8335+/-.3572				
R4-FA/TA	N	5.5754	5.5754+/-.3753	6.0923	6.0923+/-.9456	5.3305	5.3305+/-.3109				
R5-INV/S	N	5.8063	5.8063+/-.4785	6.9544	6.9544+/-.3459	5.1985	5.1985+/-.5855				
R6-CA/S	N	5.8160	5.8160+/-.3974	7.2726	7.2726+/-.1254	5.0493	5.0493+/-.5009				
R7-TL/TA	N	5.7789	5.7789+/-.3966	6.7311	6.7311+/-.9639	5.3279	5.3279+/-.3582				
R8-S/AR	N	5.5079	5.5079+/-.3316	6.6270	6.6270+/-.7425	4.9189	4.9189+/-.2902				
R9-AR/INV	N	6.2464	6.2464+/-.4711	6.8339	6.8339+/-.3097	5.8792	5.8792+/-.6500				
R10-CA/CL	N	5.9298	5.9298+/-.3681	6.1698	6.1698+/-.9460	5.8161	5.8161+/-.4918				

BDS statistic implies non-IID behavior (i.e., non-linear dependence) which may or may not be the result of a chaotic process.

A summary of the BDS statistic values is presented in Table 18. Ranges for the mean BDS statistic for the various groupings are summarized below:

	F	N
Total	(.3997) to .2054	(.4104) to .2208
Low N	(.6505) to .1832	(.9074) to .1438
High N	(.2360) to .3965	(.3692) to .2704
Low % F	(.4604) to .1785	
High % F	(.3556) to .2415	

For the full data set, all ratios are negative except for R4, R7, and R10 indicating that these ratios are the result of a non-IID time-series. Doing a firm-type comparison, there are wide fluctuations in R2 and R4. The BDS statistic for the low sample size group is negative for all ratios except R4, R7, and R10. For the sample size groupings, the results are quite disparate, both within the grouping and among firm type. For the fraud firms, the BDS statistic is positive for all ratios except for R5 and R9. For the non-fraud firms, the BDS statistic is positive only for R3, R4, R7, and R10. Similar disparity is found for the low/high % fraudulent grouping. Most of the ratios have wide fluctuations among the various groupings. While the results of the sub-groupings are inconclusive, the results of the total data set appear reliable and are consistent with the findings of prior measures. To summarize, it appears that R4, R7, and R10 are the result of a non-linear time-series.

TABLE 18

BDS STATISTIC – MEAN VALUES

	F	mean	est error	low N	est error	hi N	est error	low %F	est error	hi %F	est error
R1-NI/TA	F	(0.3213)	(.3213)+/-.1116	(0.5706)	(.5706)+/-.1333	0.1773	.1773+/-.1069	(0.3413)	(.3413)+/-.1464	(0.3265)	(.3265)+/-.1602
R2-NI/S	F	(0.0399)	(.0399)+/-.0898	(0.0990)	(.0990)+/-.1075	0.0612	.0612+/-.1605	0.1785	.1785+/-.0723	(0.1462)	(.1462)+/-.1286
R3-S/TA	F	(0.1494)	(.1494)+/-.1266	(0.3649)	(.3649)+/-.1706	0.2200	.2200+/-.1110	0.1588	.1588+/-.0970	(0.2916)	(.2916)+/-.1856
R4-FA/TA	F	0.0253	.0253+/-.1270	(0.2716)	(.2716)+/-.1792	0.3965	.3965+/-.0385	(0.0402)	(.0402)+/-.1622	0.0909	.0909+/-.1942
R5-INV/S	F	(0.2323)	(.2323)+/-.1336	(0.2341)	(.2341)+/-.1633	(0.2295)	(.2295)+/-.2500	(0.1832)	(.1832)+/-.1941	(0.2691)	(.2691)+/-.1880
R6-CA/S	F	(0.1262)	(.1262)+/-.1018	(0.3378)	(.3378)+/-.1311	0.1911	.1911+/-.1231	(0.0365)	(.0365)+/-.1259	(0.1861)	(.1861)+/-.1492
R7-TL/TA	F	0.2054	.2054+/-.0868	0.1832	.1832+/-.1113	0.2529	.2529+/-.1314	0.1620	.1620+/-.1480	0.2415	.2415+/-.1083
R8-S/AR	F	(0.3997)	(.3997)+/-.1486	(0.6505)	(.6506)+/-.1949	0.1436	.1436+/-.0951	(0.4604)	(.4604)+/-.2181	(0.3556)	(.3556)+/-.2059
R9-AR/INV	F	(0.1711)	(.1711)+/-.1234	(0.1256)	(.1256)+/-.1105	(0.2360)	(.2360)+/-.2859	(0.4445)	(.4445)+/-.2696	0.0204	.0204+/-.0845
R10-CA/CL	F	0.0265	.0265+/-.0895	0.0349	.0349+/-.1119	0.0098	.0098+/-.1559	0.1651	.1651+/-.0635	(0.0428)	(.0428)+/-.1369

R1-NI/TA	N	(0.3017)	(.3017)+/-.1374	(0.7735)	(.7735)+/-.2613	(0.1182)	(.1182)+/-.1543
R2-NI/S	N	(0.3448)	(.3448)+/-.1137	(0.9074)	(.9074)+/-.2021	(0.0217)	(.0217)+/-.0826
R3-S/TA	N	(0.1301)	(.1301)+/-.1034	(0.7646)	(.7646)+/-.2564	0.0109	.0109+/-.1056
R4-FA/TA	N	0.2200	.2200+/-.0653	0.1438	.1438+/-.0906	0.2468	.2468+/-.0845
R5-INV/S	N	(0.3112)	(.3112)+/-.1663	(0.7930)	(.7930)+/-.3892	(0.1185)	(.1185)+/-.1623
R6-CA/S	N	(0.1178)	(.1178)+/-.1625	(0.4565)	(.4565)+/-.2966	(0.0049)	(.0049)+/-.1967
R7-TL/TA	N	0.2208	.2208+/-.0928	0.1073	.1073+/-.2297	0.2704	.2704+/-.0720
R8-S/AR	N	(0.4104)	(.4104)+/-.1784	(0.4651)	(.4651)+/-.2959	(0.3692)	(.3692)+/-.2288
R9-AR/INV	N	(0.1983)	(.1983)+/-.1494	(0.0809)	(.0809)+/-.0957	(0.2372)	(.2372)+/-.2207
R10-CA/CL	N	0.1859	.1859+/-.1121	(0.0768)	(.0768)+/-.2820	0.1747	.1747+/-.4091

Shuffle Test

The shuffle test is another metric tool for investigating whether a time-series is random. Shuffling the sequence of the data destroys the dynamics within the system but produces surrogates with the same distributional characteristics as the original data. Comparisons can then be made of a test statistic for the original data with the test statistic for the shuffled data. If the results are the same, there is evidence that the original time-series is IID.

Using the BDS statistic as the test statistic, the shuffle test was performed 20 times for each ratio for each of the fraud and non-fraud firms. A two-tailed *t*-test was then performed for each ratio. This tested the null hypothesis that the original BDS statistic calculated for that ratio for a given firm was equal to the mean BDS statistic for the shuffled data. The alternate hypothesis was that the original BDS statistic was not equal to (i.e., two-tailed) the mean BDS statistic for the shuffled data. Alpha was set at both the 5% and 10% levels. If the null hypothesis is rejected (i.e., the BDS statistics are different), there is evidence of non-linear structure in the time-series. Conversely, failing to reject the null hypothesis (i.e., the BDS statistics are the same) provides evidence that the time-series is IID.

A summary of the results of the shuffle test is presented in Table 19. On numerous occasions, an original BDS statistic value of 0 was calculated. Since the BDS statistic should be positive for a non-IID time-series and negative for an IID time-series, a value of 0 was deemed indeterminate. Subsequent *t*-tests performed on such ratios were similarly classified as indeterminate. Given that there are 30 fraud

TABLE 19

SHUFFLE TEST

	F				N			
	indeterminate	fail to reject = IID	%	reject = non-IID	indeterminate	fail to reject = IID	%	reject = non-IID
R1	6/30	4/24	0.1667	0.8333	5/30	4/25	0.1600	0.8400
R2	9/30	4/21	0.1905	0.8095	8/30	5/22	0.2273	0.7727
R3	11/30	3/19	0.1579	0.8421	8/30	2/22	0.0909	0.9091
R4	12/30	1/18	0.0556	0.9444	7/30	0/23	0.0000	1.0000
R5	10/30	6/20	0.3000	0.7000	8/30	4/22	0.1818	0.8182
R6	10/30	4/20	0.2000	0.8000	10/30	1/20	0.0500	0.9500
R7	8/30	0/22	0.0000	1.0000	7/30	0/23	0.0000	1.0000
R8	11/30	4/19	0.2105	0.7895	7/30	1/23	0.0435	0.9565
R9	13/30	1/17	0.0588	0.9412	12/30	1/18	0.0556	0.9444
R10	12/30	0/18	0.0000	1.0000	5/30	0/25	0.0000	1.0000

and 30 non-fraud firms, the number of indeterminate firms for each ratio is indicated. The table then indicates the number of "fail to reject" firms followed by its percentage. Finally, the percentage of "reject" firms is reported.

Per the results presented in Table 19, the null hypothesis is consistently rejected for all ratios thereby indicating a non-IID time-series. For the fraud firms, R4, R7, R9, and R10 have the highest rejection percentages. For the non-fraud firms, R4, R6, R7, R8, R9, and R10 have the highest rejection percentages. These results are consistent with the findings of the other measures. Rejecting the null hypothesis of IID only implies non-IID behavior (i.e., non-linear dependence) which could be the result of chaos or other non-linear influences.

Phase Space Maps

There are two approaches to the analysis of data from a dynamic system. The first approach utilizes metrics and focuses on the distance between points on the attractor. Each of the measures discussed previously employed metrics. However, additional descriptive insight concerning system dynamics can be obtained by taking a topological approach and focusing on the organization of the attractor. Through the construction of phase space maps, the geometric features of a one-variable time-series record can be observed.

Using the software program CDA, phase space maps were plotted in two-dimensions. The time derivative was plotted versus the original value at each data point.

To determine the sensitivity of sample size on the phase space maps, preliminary tests and analyses were performed on known random, periodic and chaotic time-series. Figure 9 consists of two phase space maps of a random time-series with a sample size of 200 data points. The first map was created from the original time-series while the second map was created from a random shuffle of the data. Figure 10 consists of similar maps but the sample size was only 50 data points. As evidenced by the figures, random data appears to fill the plane without any discernable pattern and produces similar maps for both the original and shuffled data. This is true for both the large and small data sets.

Similar phase space maps were constructed for a periodic time-series and is presented in Figures 11 and 12. The phase space map of the shuffled data is vastly different than that of the original data and closely resembles the maps of the random time-series. Such a map is a visual representation of how the shuffle test destroys the dynamics occurring within the original time-series.

The phase space maps for a chaotic time-series is presented in Figures 13 and 14. The map of the original data depicts a strange attractor with a discernable pattern. This pattern is still in evidence even with the small data set. The maps of the shuffled data are again vastly different and resemble the maps of the random time-series.

The conclusions reached by this analysis include the following. First, a phase space map of a random time-series will fill the plane without any discernable pattern. Similar findings result from a phase space mapping of the shuffled data. Second, a phase space mapping of a periodic or chaotic time-series will not fill the plane; the

FIGURE 9

PHASE SPACE MAP

RANDOM TIME-SERIES (N = 200)

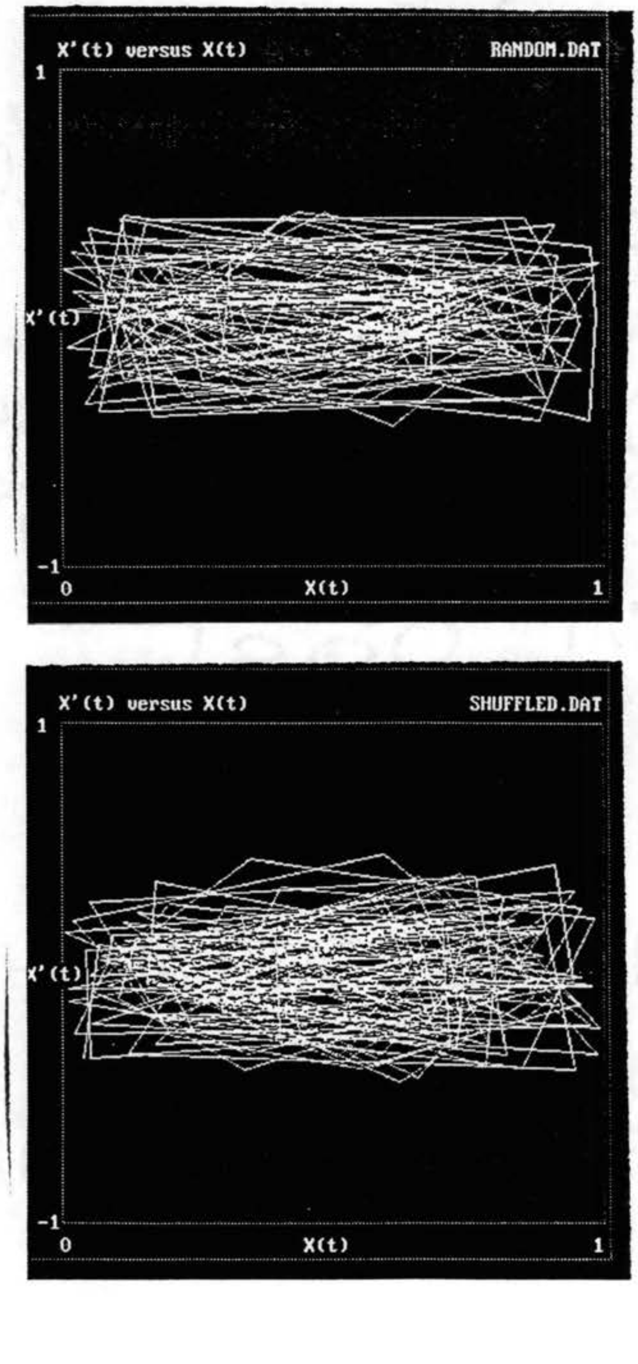
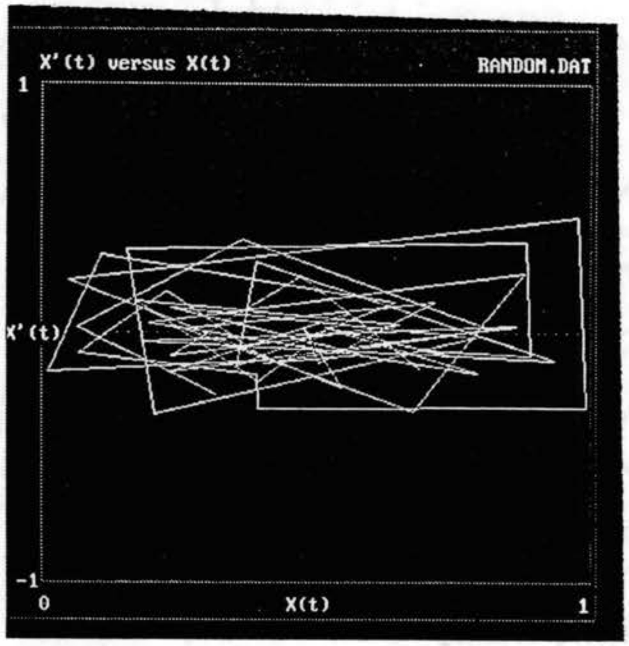


FIGURE 10

PHASE SPACE MAP

RANDOM TIME-SERIES (N = 50)



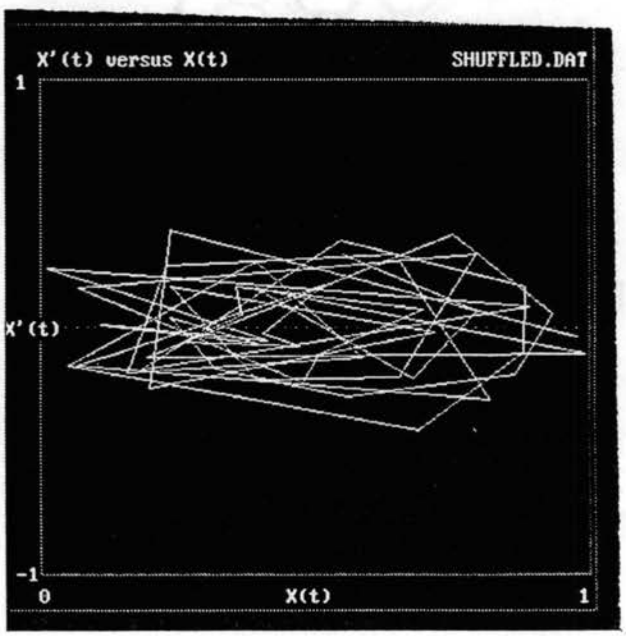


FIGURE 11

PHASE SPACE MAP

PERIODIC TIME-SERIES (N = 200)

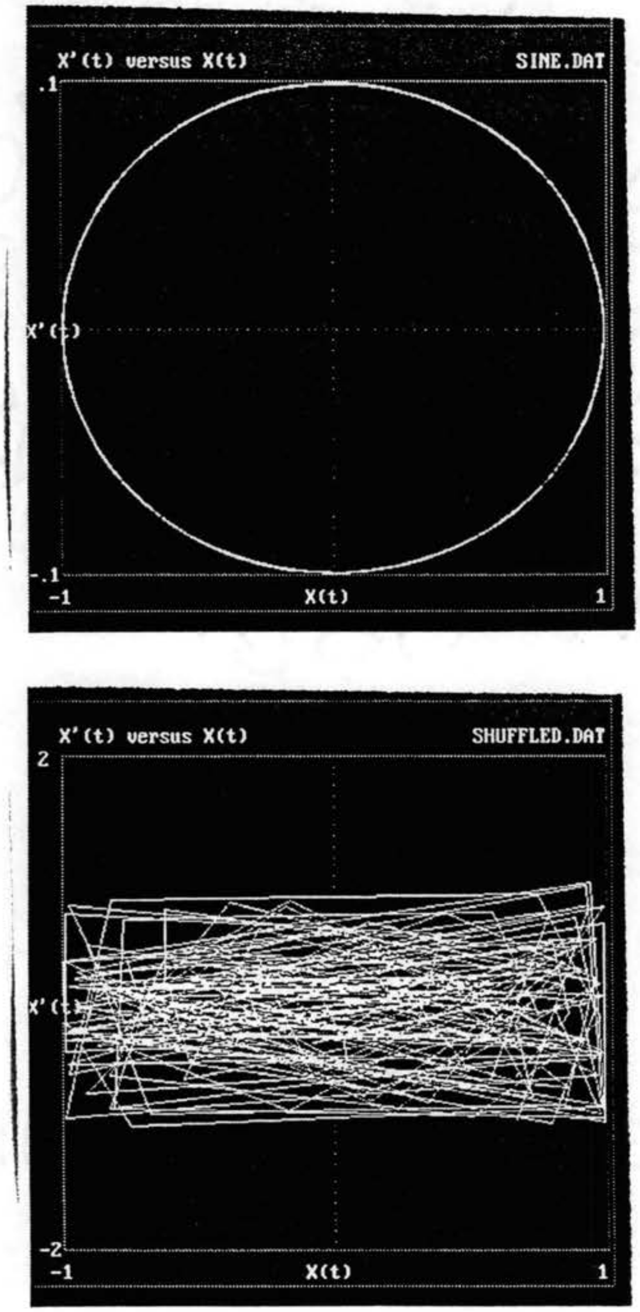


FIGURE 12

PHASE SPACE MAP

PERIODIC TIME-SERIES (N = 50)

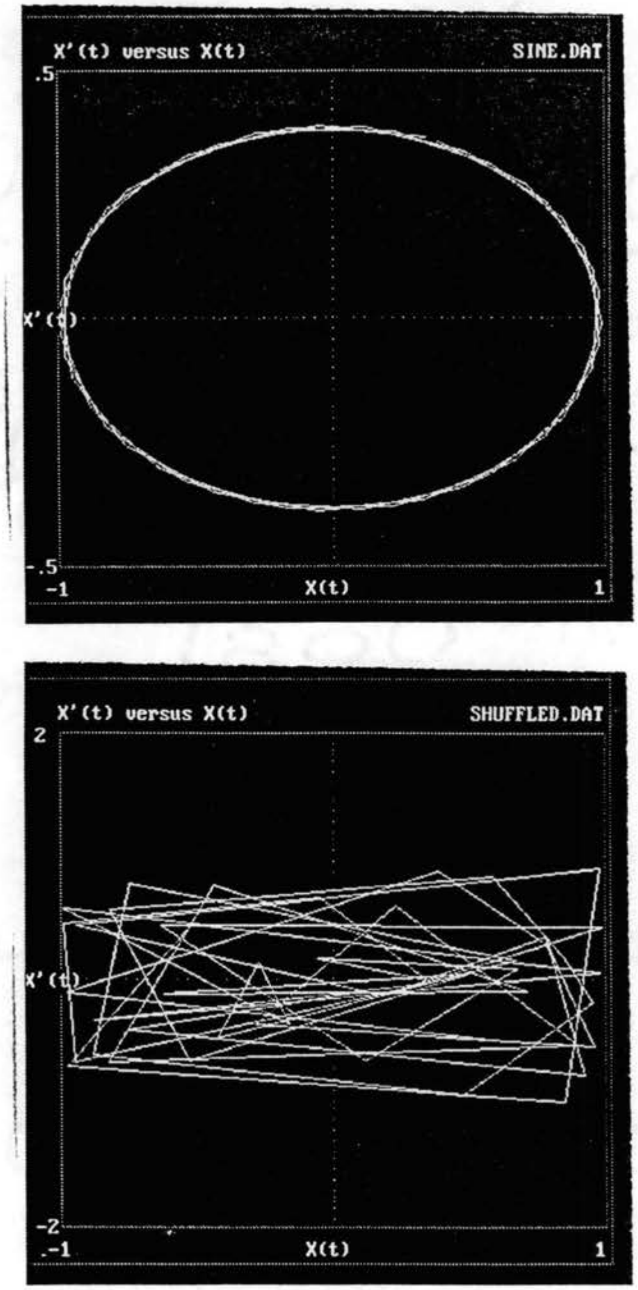
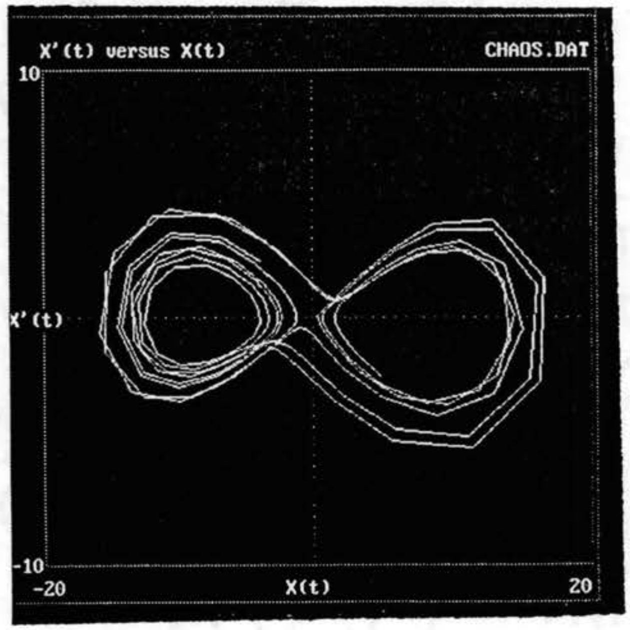


FIGURE 13

PHASE SPACE MAP

CHAOTIC TIME-SERIES (N = 200)



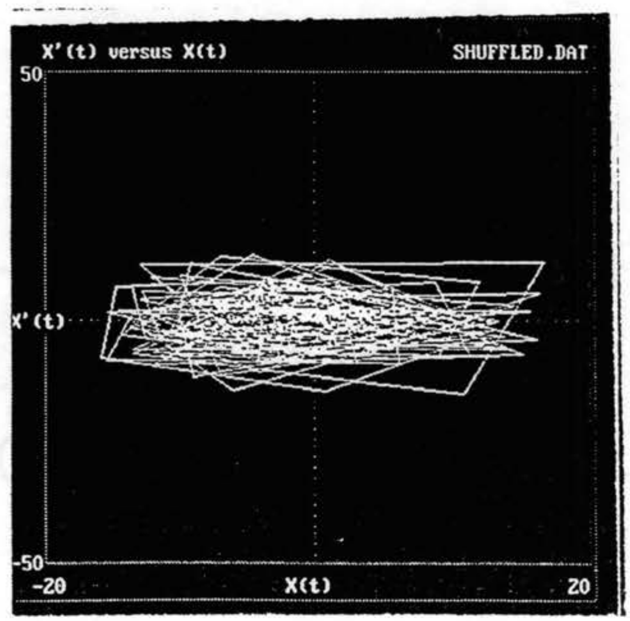
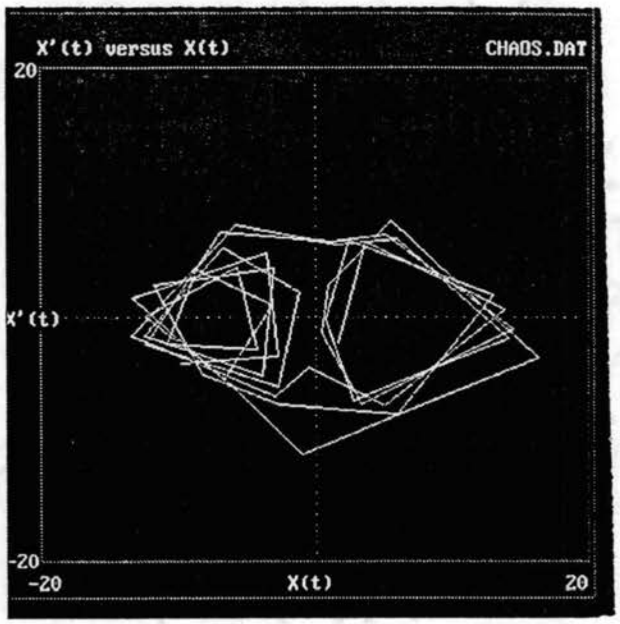
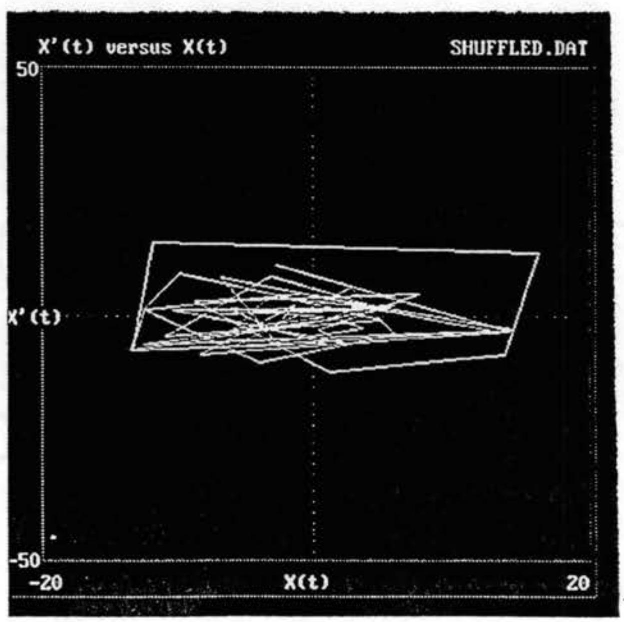


FIGURE 14

PHASE SPACE MAP

CHAOTIC TIME-SERIES (N = 50)





mapping depicts an attractor with a discernable pattern. Finally, shuffling the data of a periodic or chaotic time-series will destroy the original pattern and result in a map that fills the plane without any pattern.

Two phase space maps were constructed for each fraud and non-fraud firm for each of the ten ratios. One map was created using the original data. The second map was created from a random selection of one of the 20 shuffles performed for the shuffle test described in a previous section. Given that this study had a total of 60 firms, each with ten ratios and each ratio having two maps, this procedure resulted in 1,200 maps requiring analysis.

A map was coded as "random" if the points comprising the map appeared to fill the plane and/or the shuffled map was very similar to the original despite the various shuffles. An "**R**" was used to indicate a map that clearly appeared random, while a "r" was used for those maps that still appeared random but were less clearly discerned. A map was coded as "chaotic" if there appeared to be some type of attractor or grouping of points and/or the shuffled map was vastly different from the original. Similarly, a "C" was used to indicate a map that clearly appeared chaotic, while a "c" was used for those maps that still appeared chaotic but were less clearly discerned. The sample sizes ranged from 28 to 80 data points for the fraud firms and 30 to 89 data points for the non-fraud firms. The firms with the smaller sample sizes produced less definitive phase space maps and were generally coded with an "r" or a "c". Given the exploratory nature of this study and the small data sets employed, the above determinations were very subjective and were based upon the preliminary analysis of the maps of the random, periodic and chaotic time-series described previously.

Ranked in order of decreasing randomness, the original map may be coded "**R**", "r", "c", or "**C**". The shuffle should destroy the dynamics within the system and result in a "random" code. A shuffled map was given a code of "**R**" only if the original map was clearly random and the shuffled map appeared identical. All other shuffled maps received a code of "r". The resulting possible classifications were "**RR**," "rr," "**C**r," and "cr". The first letter indicates the classification of the original map while the second letter indicates the classification of the shuffled map. An example of a ratio with phase space maps resulting in an "**RR**" classification is presented in Figure 15. Both the original map and the shuffled map have no discernable pattern and fill the plane. An example of a ratio with phase space maps resulting in an "rr" classification is presented in Figure 16. Both maps appear to be the result of a random time-series, though this conclusion is more subjective than that reached in the prior classification. An example of a ratio with phase space maps resulting in an "**C**r" classification is presented in Figure 17. The original map appears to be of a strange attractor with a pattern or grouping of points thereby indicating a chaotic time-series. The shuffled map has no pattern and appears to fill the plane. Finally, an example of a ratio with phase space maps resulting in a "cr" classification is presented in Figure 18. Once again, the original map appears to have a grouping of points but this conclusion is more subjective than that reached in the prior classification. The shuffled map has no pattern and appears random.

FIGURE 15

PHASE SPACE MAP

SAMPLE "**RR**" CLASSIFICATION

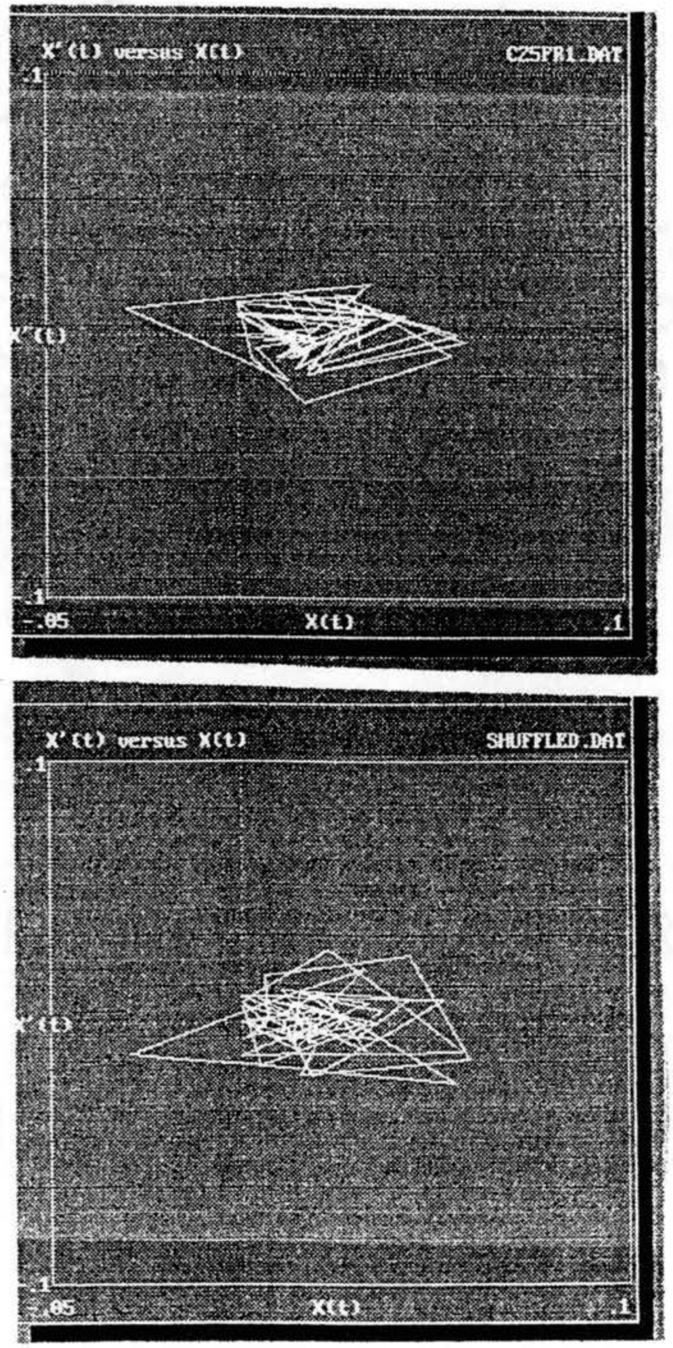


FIGURE 16

PHASE SPACE MAP

SAMPLE "rr" CLASSIFICATION

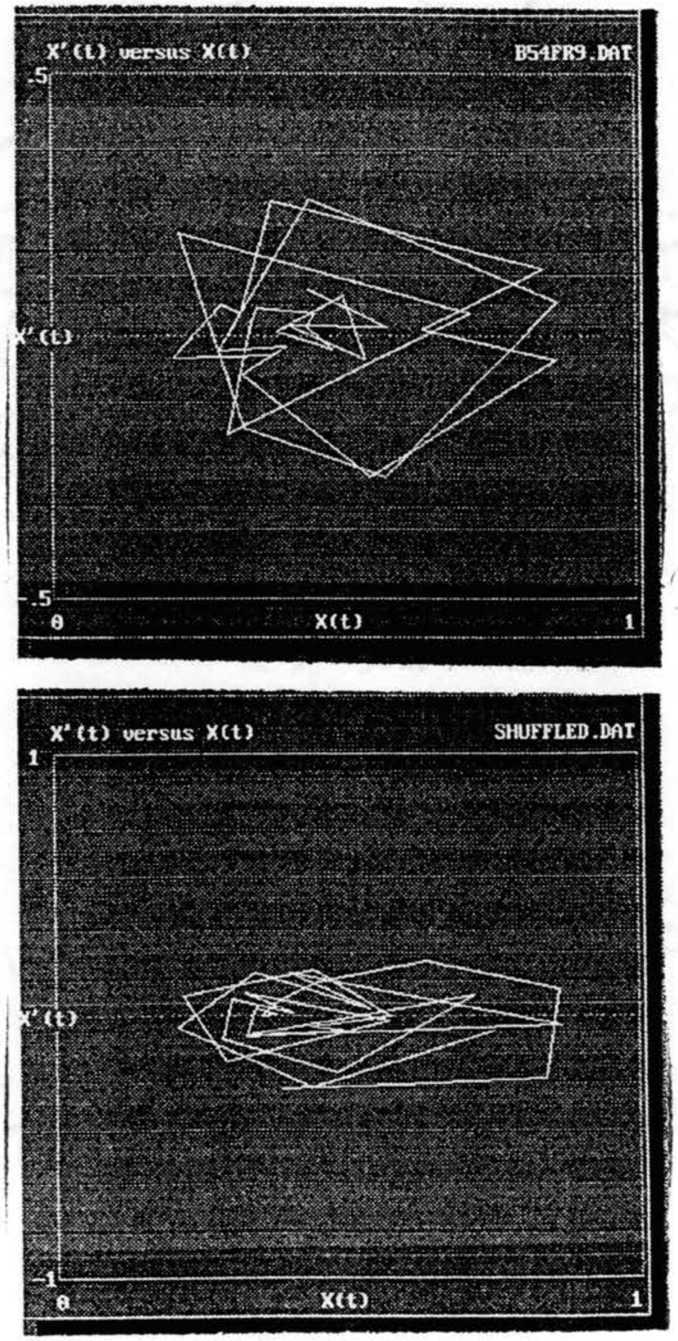
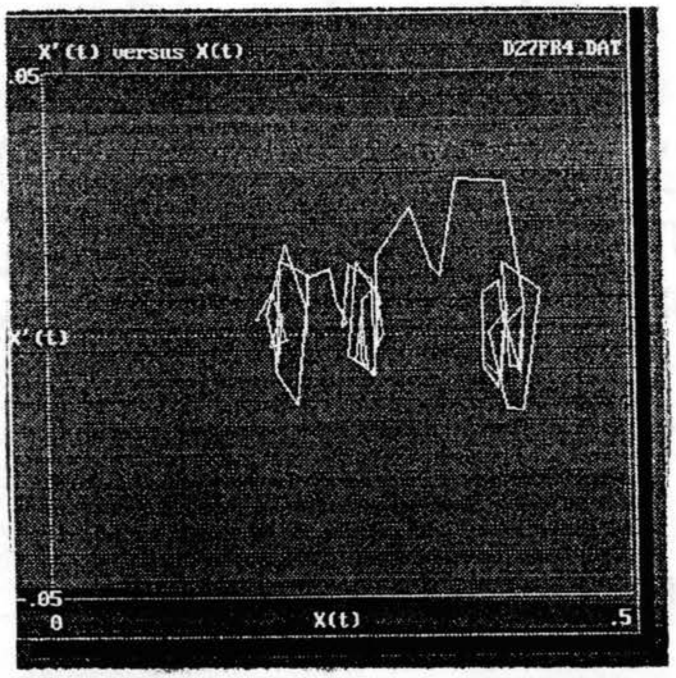


FIGURE 17

PHASE SPACE MAP

SAMPLE "**Cr**" CLASSIFICATION



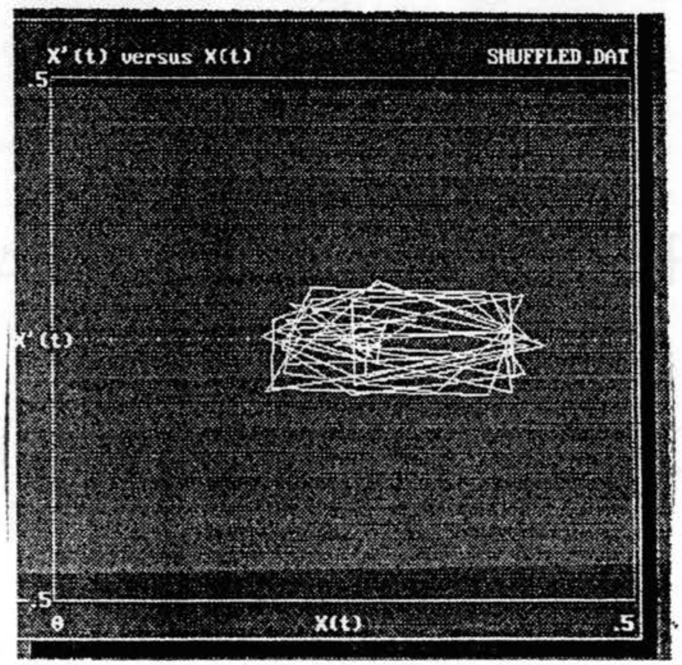
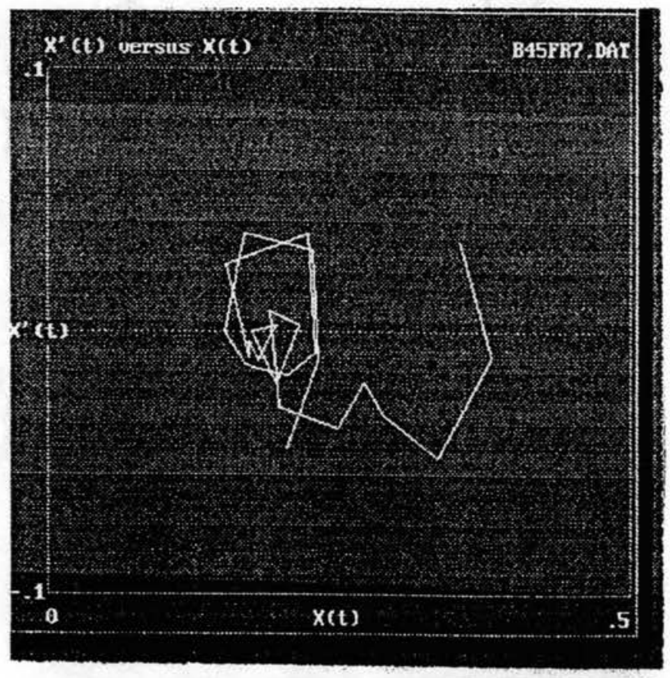
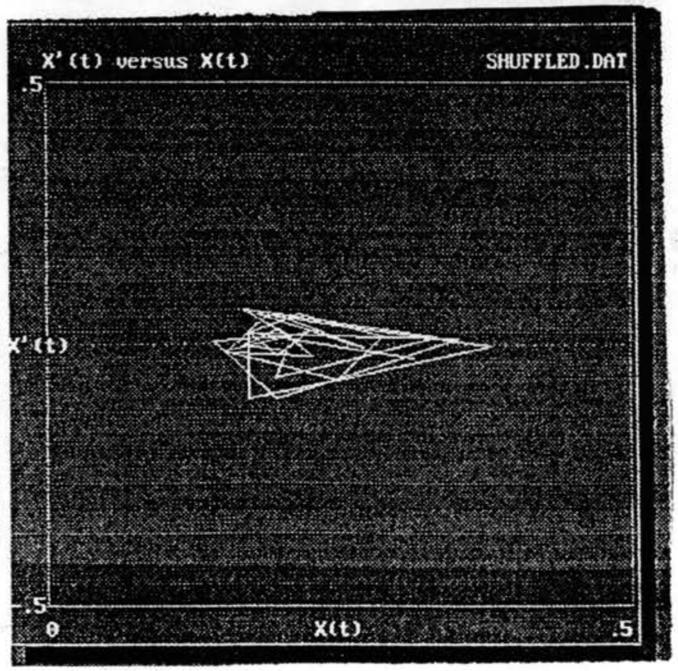


FIGURE 18

PHASE SPACE MAP

SAMPLE "cr" CLASSIFICATION





A summary of the phase space map classifications is presented in Table 20. Percentages for each classification for each ratio are reported, first for the fraud firm grouping, next for the non-fraud firm grouping and finally for the total data set. The highest percentages of "Cr" and "cr" classifications occur for R4, R7, and R10. R9 appears borderline. These findings hold true for the fraud firm and non-fraud firm groupings as well as for the total data set. Given the subjective nature of the classifications, the usefulness of the conclusions is limited. However, the results are consistent with the findings for the previously reported measures.

Additional Analyses

Since the previously described analysis of the various measures utilized mean values, additional analyses were performed using the raw ratio data organized simply by company. The objective was to see if there were any patterns evident in the raw data. Given the unreliability of the correlation dimension for the purposes of this study, such analysis was only conducted on the Hurst exponent, Lyapunov exponent and BDS statistic. A summary of the findings is presented in Table 21. Prior to a discussion of the findings, an explanation of the organization and reporting of the data is necessary.

The first column lists the code number for each matched firm pair. The "difference" row lists those ratios where the measure's value differences were larger between firm type. For the Hurst exponent, the listed ratios are those that appeared chaotic. For the Lyapunov exponent, since almost all ratios appeared chaotic, the

TABLE 20

PHASE SPACE MAPS

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
F										
RR	0.9000	0.8667	0.6667	0.2667	0.6429	0.7333	0.1667	0.6000	0.4074	0.2333
rr	0.1000	0.1000	0.1333	0.1667	0.1429	0.2000	0.0667	0.2333	0.1852	0.3000
Cr			0.1000	0.3667	0.0357		0.3667		0.1481	0.0667
cr		0.0333	0.1000	0.2000	0.1786	0.0667	0.4000	0.1667	0.2593	0.4000
N										
RR	0.8667	0.9667	0.8000	0.3333	0.6296	0.8000	0.2000	0.7667	0.5556	0.2333
rr	0.0667		0.0667	0.0667	0.1111	0.1333	0.0667	0.1667	0.1852	0.2000
Cr				0.3667	0.1111		0.4000		0.1481	0.3000
cr	0.0667	0.0333	0.1333	0.2333	0.1481	0.0667	0.3333	0.0667	0.1111	0.2667
Total										
RR	0.8833	0.9167	0.7333	0.3000	0.6364	0.7667	0.1833	0.6833	0.4815	0.2333
rr	0.0833	0.0500	0.1000	0.1167	0.1273	0.1667	0.0667	0.2000	0.1852	0.2500
Cr			0.0500	0.3667	0.0727		0.3833		0.1481	0.1833
cr	0.0333	0.0333	0.1167	0.2167	0.1636	0.0667	0.3667	0.1167	0.1852	0.3333

TABLE 21

COMPANY DATA

	Hurst exponent	Lyapunov exponent	BDS statistic
B12F	R4,R7,R8,R10	R6 neg	R5,R9
B12N	R4,R7,R9,R0		R4,R7,R9,R10
difference	R4,R7,R9,R10	R2,R7,R8	
B16F	R4,R7,R10		R3,R6,R7,R10
B16N	R2,R4,R10		R4,R6,R10
difference	R2,R7,R10	R4,R6,R8	
B22F	R4,R7,R9		R1,R3,R4,R6,R7,R8,R10
B22N	R4,R10	R8	R5,R8,R9,R10
difference	R4,R7,R9	R1,R2,R3,R5,R7,R9,R10	
B24F	R3,R4,R7,R10	R8	R7
B24N	R4,R5,R6,R7		R1,R3,R4,R7,R10
difference	R4,R5,R7	R2,R4,R5,R8,R9	
B25F	R4,R7,R9,R10	R7	R7,R10
B25N	R4,R5,R9,R10		R7,R10
difference	R4,R5,R7,R10	R1,R2,R7,R8,R9,R10	
B26F	R3,R4,R7,R8,R9,R10		all
B26N	R3,R4,R7	R10	R7,R10
difference	R2,R3,R4,R5,R6,R8,R9,R10	R2,R3,R4,R10	
B32F	R3,R5,R7,R8,R9,R10		R10
B32N	R3,R5,R6,R8,R10		all except R8
difference	R6,R7,R9	R1,R2,R3,R5,R6,R7,R10	
B38F	R4,R7,R9,R10		R9,R10
B38N	R4,R7,R10	R5	all
difference	R3,R4,R9,R10	R2,R3,R5,R7,R9	
B41F	R4,R6,R10	R10	R4
B41N	R4,R7,R10		R7,R10
difference	R4,R7,R10	R6,R7,R10	
B42F	R4,R7,R10		R2,R6,R7
B42N	R3,R4,R5,R7,R9	R2	
difference	R3,R5,R8,R9,R10	R1,R2,R3,R4,R5,R10	
B45F	R4,R7,R9,R10	R8	R3,R5,R7,R9
B45N	R4,R5,R7,R10		R3,R4,R5,R6,R7
difference	R5,R6,R9	R1,R2,R3,R6,R7	
B48F	R7,R9,R10		R1,R2,R5
B48N	R4,R7,R10		R3,R4,R7,R9,R10
difference	R4,R7	R4,R7,R8	
B52F	R4,R7,R9,R10		R1,R2,R4,R7,R9
B52N	R4,R5,R7,R10		R1,R2,R3,R4,R7
difference	R1,R4,R10	R3,R6,R10	
B54F	R4,R7,R10		R1,R7,R10
B54N	R4,R7,R10	R1	R4
difference	R4,R7,R10	R1,R4,R7,R9,R10	
B57F	R4,R7,R10		R1,R2,R4,R10
B57N	R3,R4,R7,R9,R10		R1,R2,R4,R7,R8,R9
difference	R3,R6,R9,R10	R3,R9	

TABLE 21

COMPANY DATA

	Hurst exponent	Lyapunov exponent	BDS statistic
B59F	R3,R4,R7,R9,R10		R4,R5,R8,R9
B59N	random		R3,R4,R5,R6,R7,R10
difference	R3,R4,R7,R8,R9,R10	R1,R5,R9	
B61F	R5,R6,R8,R10	R4	R1,R2,R3,R5,R6,R7
B61N	R4,R8		all
difference	R3,R4,R7,R8,R9,R10	R1,R3,R4,R6,R9	
B62F	R4,R7,R9		all
B62N	R9		R3,R6,R7,R8,R9
difference	R4,R7,R8	R1,R2,R5,R9,R10	
B69F	R3,R4,R5,R6,R8,R9,R10		all R1,R2
B69N	R3,R4,R5,R8,R9,R10		R1,R3,R4
difference	R1,R3,R5,R6,R7,R8,R9	R1	
C09F	R4,R7,R9		R7,R10
C09N	R1,R2,R4,R10	R1	R4,R7,R10
difference	R1,R2,R7,R9	R1,R3,R10	
C16F	R2,R5,R6,R9		R2
C16N	R4,R7,R9,R10	R10	R2,R6,R7,R8,R10
difference	R2,R3,R4,R5,R6,R7	R2,R4,R6,R7,R9,R10	
C17F	R3,R4,R7	R5	R4,R7,R10
C17N	R4,R7,R8,R10		R1,R4,R5,R6,R8,R9
difference	R3,R6,R7,R8,R10	R2,R4,R5	
C25F	R4,R8,R10		R1,R2,R3,R4,R5,R6,R8
C25N	R4,R7,R10		R4,R6,R8,R10
difference	R4,R7,R10	R1,R2,R7,R8,R10	
C27F	R3,R4,R5,R6,R9	R6	R2,R4,R6,R7,R9
C27N	R7,R10		R1,R3,R4,R6,R7,R8,R10
difference	R3,R4,R5,R6,R8,R9,R10	R1,R2,R5,R6,R7	
C31F	R3,R4,R7,R8,R10	R5,R6	R3,R4,R5,R8
C31N	R4,R5,R9		R1,R2,R4,R5,R10
difference	R3,R6,R7,R8,R10	R1,R2,R3,R5,R6,R9	
C32F	R4,R7,R9,R10		R5
C32N	R4,R7,R10	R10	R10
difference		R2,R4,R6,R8,R9,R10	
C34F	R2,R4,R7,R8,R10		none
C34N	R4,R7,R9,R10		R5,R7,R8,R9
difference	R2,R7,R8,R9	R2,R4,R7,R9,R10	
D21F	R4,R7,R10		none
D21N	R4,R7,R8,R9		R7
difference	R3,R7,R10	R4,R5,R6,R7,R10	
D24F	R4,R7,R10		R2,R7,R8,R10
D24N	R4,R7,R10		R6
difference	R2,R4	R1,R4,R6,R7	
D27F	R4,R10		R4,R6,R8,R9,R10
D27N	R4,R7,R8		all
difference	R6,R7,R10	R1,R5	

listed ratios are those that appeared non-chaotic. For the BDS statistic, the listed ratios have positive values which indicate non-IID behavior. Since many of the ratios had a BDS statistic value of "0" which was previously deemed indeterminate, a comparison of the values between firm type was not meaningful. Accordingly, there is no data in the "difference" row for this measure.

For the Hurst exponent, R4, R7, and R10 are consistently chaotic. This is true for both the fraud and non-fraud firms. R9 is borderline and more chaotic for the fraud firms. Comparing the ratios by firm type, R4, R7 and R10 are consistently different. These differences however are not consistently larger or smaller. There does not appear to be a pattern for the percent of time fraudulent grouping. These findings are consistent with the results from the mean data.

For the Lyapunov exponent, the 30 fraud firms each had 10 ratios producing 300 values of which only 17 values did not appear chaotic. For the non-fraud firms, only 10 of the 300 ratios did not appear chaotic. These findings are consistent with the findings of the mean data given previously. Comparing the ratios by firm type, except for R8, all ratios are consistently different. Similar to the Hurst exponent, these differences are not consistently larger or smaller. For the percent of time fraudulent grouping, there does not appear to be any pattern.

For the BDS statistic, R4, R7, and R10 are consistently positive thereby indicating evidence of non-linear structure in the data. Most of the other ratios show evidence of non-linear dependence but not as consistently. There does not appear to be any pattern for the percent of time fraudulent grouping. All of these findings are consistent with the results of the mean data reported previously. Given that many of the ratios had a "0" value for this measure, the findings are not as reliable as the results of the prior measures.

In summary, the results of these additional analyses of the individual company data are consistent with the findings of the mean data. No additional or conflicting patterns are evident.

Summary of Findings

The Hurst exponent provides evidence of whether a time-series is random, periodic or chaotic. Of the ten ratios analyzed in this study, R4, R7, R9 and R10 appear to be the result of a chaotic time-series. The ratios were consistent regardless of firm type and were not sensitive to sample size or percentage of time fraudulent.

The Lyapunov exponent provides evidence of the type of attractor as well as the degree of chaos exhibited by the system. The mean Lyapunov exponent was positive for all ratios tested indicating that the ratios appear to be the product of a chaotic system. On average, for R4, the non-fraud firms exhibited a higher degree of chaos than the fraud firms.

The correlation dimension provides evidence of whether the attractor is due to a random or chaotic process. Given the results of the preliminary analysis regarding this measure's sensitivity to sample size and the disparate results of the weighted means/simple means comparisons, this measure was deemed unreliable for the purposes of this study. Accordingly, no conclusions were drawn.

The BDS statistic provides evidence of whether a time-series is random (i.e., IID) or the result of non-linear dynamics. Of the ten ratios analyzed in this study, R4, R7, and R10 appear to be the result of a non-IID time-series. Such non-IID behavior could result from chaotic dynamics or from other non-linear influences.

Since the BDS statistic was used as the test statistic, the shuffle test provides further evidence of whether the time-series is random (i.e., IID) or the result of a non-IID time-series. For the fraud firms, R4, R7, R9, and R10 appear to be the result of a non-IID time-series. For the non-fraud firms, R6 and R8 also appear to be products of non-linear dynamics. Once again, chaos or other non-linear influences could produce such dependence.

Finally, the phase space maps provide evidence of the type of attractor for the ratios for each of the fraud and non-fraud firms. R4, R7, and R10 appear to have strange attractors. R9 has similar though less conclusive results.

Combining the results of the multiple metric and topological tests can be summarily condensed as follows. Of the ratios tested, R4, R7 and R10 were consistently strong indicators of chaos. R9 also indicated a chaotic time-series but not as consistently. None of the ratios appeared periodic. Each of these findings was true for both the fraud firms and the non-fraud firms. The various measures were more consistent/definitive with the larger data sets. Grouping the fraud firms by the percentage of time fraudulent did not cause a difference in the measures. Given the limited sample size for this study and the sensitivity of the measures to sample size, the low/high sample size sub-grouping did not provide meaningful results. The following chapter presents the conclusions, limitations and implications for future research of the above findings.

CHAPTER VI

CONCLUSION

Introduction

Fraud in business is a matter of grave social and economic concern. Both the government and the accounting profession have responded as evidenced by the creation of new laws, commission reports as well as accounting and auditing standards. In 1997, the American Accounting Association (AAA) formally posited the following question: "can analytical procedures be better used to detect fraud?" [Landsittel and Bedard 1997, 4]. This researcher accepted the AAA's challenge and explored fraud within the context of chaos theory. Financial statements are the product of a dynamical system. According to chaos theory, such dynamical systems are deterministic, yet unpredictable and usually non-linear. Chaos theory methodology has various tools for measuring the non-linearity of a system.

This research study explored financial statement data of both fraudulent and non-fraudulent firms for evidence of non-linearity. A longitudinal examination of ten financial ratios computed from quarterly income statement and balance sheet data was conducted for 30 matched pairs of fraudulent and non-fraudulent firms. Using chaos theory methodology, multiple metric tests were performed on the ratio values to determine the behavior of the time-series (i.e., random, periodic, or chaotic). Topological tests were also conducted to provide further evidence. This chapter presents the conclusions reached by the study's findings. Limitations of the study and implications for future research are also discussed.

Conclusion

The combined results of the multiple metric and topological tests indicate that R4, R7, and R10 were consistently strong indicators of chaos. R9 also indicated a chaotic time-series but less consistently. Each of these ratios is comprised of balance sheet account(s) and/or categories. R4 is the ratio of fixed assets to total assets, a capital intensiveness measure. R7 is the ratio of total liabilities to total assets, a measure of financial leverage. R9 is the ratio of accounts receivable to inventory and indicates the intensiveness of receivables. R10 is the ratio of current assets to current liabilities, a prominent measure of short-term liquidity. These findings indicate that ratios comprised of financial data as reported on the balance sheet are the result of non-linear chaotic dynamics. The remaining tested ratios were comprised of income statement amounts (e.g., R2 is the ratio of net income to sales) or a combination of income statement and balance sheet amounts (e.g., R3 is the ratio of sales to total assets, R5 is the ratio of inventory to sales).

None of the tested ratios appeared periodic. This finding is somewhat surprising for the income statement data. Given that quarterly income statements were the data source, and most firms have a cyclical nature to their operations, one would intuitively expect evidence of periodicity. The sample was comprised of a wide range of industry classifications, each with their own unique operating cycle. When combined together, the periodic patterns may dilute and/or confound each other. Another explanation could be the small sample size. A minimum of 28 data points was used for each ratio, representing seven annual cycles. The fraud firms had a mean sample size of 42.50 or ten annual cycles while the non-fraud firms had a mean sample size of 55.30 or 13 annual cycles. A longer time-series may be necessary to adequately capture the cyclical nature of the firm's operations.

This study found none of the tested ratios exhibited stable or periodic behavior. However, four ratios (i.e., R4, R7, R9, and R10) were consistently strong indicators of chaos indicating that these ratios are less affected by noise. The parameter values coupling these ratios are probably larger than the other parameters. Recall Figure 7 which depicted a hypothetical deterministic non-linear chaotic system. If one was to presume the figure was a depiction of the financial reporting system, this study would be represented in Region 2. Financial ratios appear to be part of the chaotic subset of the system.

The above findings were true for both the fraud and non-fraud firms. One cannot use the ratios to differentiate firm type. This result is consistent with prior research findings of the limited ability of financial ratios to detect fraudulent firms [Kaminski et al 2000, Persons 1995]. Beasley et al [1999] found that fraudulent firms overstate asset accounts, particularly inventory, accounts receivable and fixed assets.

Despite the use of these accounts in R4, R7, R9, and R10, there was not a distinction between fraud versus non-fraud firms.

Contributions of this research include the following. First and foremost, this study has employed chaos theory within the realm of accounting and auditing research. Up to this point, chaos theory has only been applied to the area of capital markets and stock returns. This study has expanded the ratio analysis research stream by exploring the dynamics of the financial accounting system and determined the behavior of financial statement ratios. This study found evidence of non-linearity in ratios comprised of balance sheet accounts/categories. Such ratios exhibit chaos thereby severely limiting their utility in predictive models. Through an exploration of the underlying dynamics of financial statement data, this study also provided evidence that the use of linear models derived from financial ratios based on balance sheet data is inappropriate. Such models do not robustly represent the system and result in low explanatory power. This study examined the qualitative changes in financial ratios across time and found no differences in dynamics between fraudulent and non-fraudulent firms, thereby providing further evidence of the limited ability of financial ratios to detect fraudulent firms.

Limitations

As with all empirical investigations, there are limitations to this study. Most of these limitations are directly linked to chaos theory and its unique methodology. Chaos theory is a relatively new field of research and there is no comprehensive theory of all chaotic phenomena. Instead, there exists a cluster of theoretical models, mathematical procedures and experimental techniques, none of which are definitive for analyzing a chaotic time-series. Most of the metric tests are actually the product of graphical techniques and only provide clues of non-linear dynamics. Confidence limits and probability values cannot be assigned to these measures. The BDS statistic is an exception and is a well-founded statistical test of non-linear dynamics. The conclusions reached by topological tests such as phase space maps are very subjective and open to various interpretations. To mitigate these limitations, this study utilized different diagnostics to triangulate the empirical evidence and thereby increase confidence in the findings.

Non-linear dynamic systems are characterized by long memory processes. To perform a proper analysis, the crucial element is more time, not necessarily more data points. Wolf et al [1985] recommend that ten cycles are necessary. In this study, there was no known cycle time. To mitigate this unknown, the approach taken was to obtain financial statement data for the entire period for which the firm is/was public, within data availability and cost constraints. The measures used in this study were more consistent and definitive with the larger data sets.

A limitation of the sample selection process involved the potential misclassification of a non-fraud firm. Financial statement fraud might have occurred but has yet to be detected and subjected to SEC investigation.

At the present time, an acceptable theoretical foundation for the selection of financial ratios for decision making does not exist. The parsimonious set of ratios selected for inclusion in the study was based upon scattered heterogeneous empirical evidence and logical inferences of accounts most likely involved in fraudulent financial reporting. Different results might ensue if different ratios were selected for examination.

Implications for Future Research

This study found that ratios comprised of financial data as reported on quarterly balance sheets are the result of non-linear chaotic dynamics. The utilization of such ratios for predictive purposes will have very limited success. Additional investigation of predictive models comprised of financial ratios is warranted. Models that incorporate ratios computed from balance sheet amounts should be revised to exclude such ratios. Tests can then be performed comparing the original model with the revised model and evaluating their respective effectiveness.

The findings in this study can be used in subsequent research as a basis for model selection. Models that include financial ratios comprised of balance sheet amounts should be non-linear.

While the ratios examined in this study found no differences in the dynamics of fraudulent and non-fraudulent firms, the ratios may have other utility. Financial analysts, investors, creditors, internal and external auditors employ ratio analysis for very disparate purposes. These additional findings about the dynamics underlying such ratios may have implications for such users.

Additional exploration of balance sheet and income statement data seems warranted. Rather than using ratios, one could use the raw financial data to examine the dynamics of the time-series to see if the system exhibits stable, periodic or chaotic behavior. One could combine the results of this study and the Beasley et al [1999] study and examine accounts most susceptible to fraud (e.g., accounts receivable, inventory). Comparisons could then be made between fraudulent and non-fraudulent firms to see if there is a difference between firm type.

One of the limitations discussed previously was the limited time frame and data points available for analysis. Longitudinal studies of firms with 30, 40 or more years of quarterly data can be conducted. Analysis can be performed on either ratio data or the raw financial statement amounts or both to again examine the dynamics of the financial reporting system. Stability in dynamics or the lack thereof would provide additional evidence of the appropriateness of the use of such data in model selection.

Non-linearity in feedback processes serves to regulate and control [Gleick 1987]. Goldberger [1990] discussed the use of non-linear dynamic models to predict cardiac events such as myocardial infarction. He argues that healthy systems exhibit more chaos than unhealthy systems. Perhaps the same is true of firms. A fraudulent firm can be considered unhealthy, whereby its behavior may change from chaotic to random. Recall Figure 7 which depicted a hypothetical deterministic non-linear chaotic system. Subsequent to the occurrence of fraud, a firm may move to a different region of parameter space. Such a shift may have implications in model selection and the resulting explanatory power of the model. Alternately, there could be a subtle change in behavior such as different types of chaos (e.g., Hurst exponents, BDS statistics, attractors). Such a change in behavior means there has been a change in some of the parameter values. Inherent noise comes from variation in the parameter values. There will be fluctuations (i.e., noise) around the nominal behavior. While this study did not find any dramatic change in behavior for the fraud firms, subtle changes in behavior may have resulted and warrant further investigation.

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