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SMILE CONSISTENT VOLATILTY

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NOMENCLATURE

ATM	at the money option
ITM	in the money option
OTM	out of the money option

Chapter 1

1. Introduction

Financial markets for equities, interest rates, and commodities have contingent claims or options that provide rights to the holders. Although the development of valuation models for contingent claims in equities and interest rates is extensive, this is not the case for commodity contingent claims. This research focuses on the natural gas and crude oil commodity markets. A valuation framework for energy contingent claims is presented that extends current methods by introducing an alternative volatility function for the price process.

Valuing energy contingent claims requires the specification of a pricing framework. Two frameworks currently exist for developing valuation models. The first method is to specify the spot process for an asset and derive contingent claims. The Classic Option Pricing Models of Black and Scholes (1973) and Black (1976) originate from this framework. The second method is to specify a forward rate or curve process and derive the spot process and contingent claims from this specification. Heath, Jarrow, and Morton (1990) developed their interest rate model using this framework. Commodity valuation models utilize both frameworks. Gibson and Schwartz (1990) and Schwartz (1997) develop energy pricing using a spot process specification. Clewlow and Strickland (1997 and 1999) and Cortazar and Schwartz (1994) utilize forward curve developments. The research presented here extends the second framework and presents an alternative specification for the volatility component of the forward process.

The Classic Option Pricing Models (COPM) developed by Black and Scholes (1973) and Black (1976) assumes that volatility of the returns is constant for the underlying asset. The forward curve models have a term structure of volatilities with constant volatility across strike prices. Implied volatility is defined as the standard deviation of returns that equates the market price of an option with its option premium for a given asset and strike price. The assumption of constant volatility means that the implied volatility should be equal across all strike prices for a given time to expiration. Volatilities implied by these option pricing models display consistent deviations from this assumption. This abnormality is called the "volatility smile," or "volatility skew."[1] This empirical relationship between implied volatility and the option's moneyness is illustrated in Figure 1. Panel A illustrates a volatility smile that is skewed to the right with out of the money (OTM) call and in the money (ITM) puts having higher volatilities, whereas Panel B is a left skewed volatility smile. Panels A and B are common volatility smiles in crude oil and natural gas commodities. Panel C is a sneer that is representative of an equity's leverage effects as the stock price decreases. The combination of volatility skew and volatility term structure together form what is termed the "Volatility Surface."

Volatility surfaces are a common feature in commodity markets. Research devoted to developing pricing models that are "smile" consistent is limited. Deterministic Volatility Models (DVM) are smile consistent pricing models that specify a deterministic volatility function incorporating time and price (strike) components. Dupire (1994), Rubinstein (1994) and Kani, Derman, and Kamal (1996) provide the seminal work in this area. The Heath, Jarrow, and Morton (1990) and Clewlow and

[1] For example, Bates (1991) finds a negative skew or "volatility sneer" in the S&P 500 index options.

Strickland (1999) model's specification of the volatility term structure is also a deterministic volatility function. These types of models specify a deterministic function for volatility and use implied volatilities from traded options for estimating the parameters of the pricing model. The first objective of this research is to present an alternative specification for the volatility function for the underlying price process that is consistent with observed volatility surfaces which is termed the Smile Consistent Volatility Function (SCVF).

The miss-specification of the pricing model for contingent claims can lead to serious hedging errors. The SCVF requires testing hedging effectiveness to determine the validity of the model. Hedging performance is the second objective of this study. The hedging performance will be tested against two benchmarks. The SCVF contains the Clewlow and Strickland (1999) model as a subset, which makes it a logical benchmark. The other benchmark is the COPM that is industry standard for determining hedging parameters. These two benchmarks have closed form solutions whereas the SCVF is tractable, but at this time, a closed form solution is not currently available. Alternatives to the closed form solution are the numerical methods of utilizing trees or finite difference methods.

The SCVF is implemented using numerical methods that will produce metrics for evaluating the model against two benchmarks. The numerical method is Implied Volatility Tree (Derman, Kani, and Chriss, 1996 and Chriss, 1996 and 1997, IVT). The SCVF parameters are estimated daily for the period 1/1/1995-12/31/99. Weekly hedge portfolios including transaction costs will be constructed using the SCVF and the benchmarks with a six month hedge window for natural gas and a three month hedge window for crude oil. The hedge positions will be static and the performance is evaluated on the ability of the SCVF and the benchmarks to hedge the price risk of the portfolio. The evaluation criteria will be the non-parametric squared rank test for equal variance and the Diebold and Mariano (1995) hedge statistic.

The SCVF provides a way to estimate the influences of a proposed set of fundamental economic factors and seasonal components on volatility. The SCVF, by design, has estimated parameters that describe the volatility surface. These parameters are estimated each trading day for the period previously specified, creating a data series of parameters to test the magnitude of supply, demand, and market factor influences on volatility.

The third objective is to estimate the relationship of fundamental economic factors with the volatility structure. The levels and changes in fundamental factors can be regressed on the time series of parameters estimated for the SCVF, as Nandi (1995), Bakshi, Cao, and Chen (1997), and Pena, Rubio, and Serna (1999) performed in their research. Fundamental factors for this research are supply and demand measures for natural gas and crude oil found in commodity inventory levels, trader's positions, and seasonality of volatility. Hypotheses relating these factors to changes in volatility surfaces are tested.

The objectives of this research are:

1. To derive a deterministic, smile consistent volatility function,
2. To determine the test hedging performance of the smile consistent model relative to benchmark models,
3. To estimate the effect of analyze economic factors on the volatility surface through the parameters of the SCVF.

The contributions of this research to contingent claim valuation is in three main areas. With the development and testing of the proposed SCVF, current research in the pricing specification of forward curve commodity models and deterministic volatility models is extended. The proposed model is internally consistent with the time varying term structure of volatility as found in the interest rate literature which to date has not been shown for commodity markets. Second, this research will provide a better understanding of changes in volatility from fundamental economic factors, market liquidity factors, and seasonal components.

1.1. Background of energy markets

This research focuses on natural gas and crude oil commodity markets and the specific commodities in these markets are Light Sweet crude oil and Henry Hub natural gas. Crude oil is the primary input to refining heating oil and unleaded gasoline, making these products and their markets inter-related. Natural gas is a primary fuel for electric power generation, fuel for heating businesses and homes, and raw inputs in the chemical industry. The market microstructure of these markets contains trading mechanisms that will affect valuation and parameter estimation.

The depth of the traded contracts is an important consideration. All futures trade on monthly cycles, but individual commodities vary on the length of time covered. Crude oil contracts extend out to 30 months and natural gas contracts to 36 months. Options trade 12 consecutive months out for both crude oil and natural gas options. June and December contracts extend the trading horizon to 36 months. These long term options will have higher trading volumes and will be utilized more for hedging and speculating on long term positions. The off month options in the shorter term will have lower trading volumes. Table I[2] lists requirements for future and option contracts on these commodities.

Crude oil and natural gas have external factors of supply and demand. Seasonal patterns are easily seen in these commodities. Natural gas traditionally has a forward price curve that is in both contango and backwardation. Contango is when longer time to expiration forwards exceeds nearer term forwards, and backwardation is when longer term forwards are less than near term contracts. The natural gas forward contracts in the summer months exhibit lower prices than the winter months (contango) and winter months exceed the next summer forwards (backwardation). Early summer months typically are the lowest prices and December and January are the highest prices. The natural gas term structure is divided into two seasons: injection and withdrawal. The injection season is April through October and the withdrawal season in November through March. Since the deregulation of electricity markets, the natural gas forward market has exhibited a double humped market with July and August forward prices having higher prices than the later injection season months. These price patterns are attributable to weather factors. Summer months, primarily July and August, have the highest extreme temperatures and the winter months of December, January, and February, have the lowest extreme temperatures. Crude oil has less of a seasonal pattern than natural gas, but it is also heavily driven by the supply and demand factors. Refined products from crude oil are needed throughout the year and as supplies and demand for these products change, the price and volatility of crude oil changes.

The volatility term structure of crude oil and natural gas options exhibit an exponentially damped volatility term structure where near term volatilities are greater

[2] Additional information can be found at www.nymex.com.

than longer term volatilities. Figures 2-5 show the volatility term structure of crude oil (level), log-returns of crude oil, natural gas (level), and log-returns of natural gas futures contracts by expiry and promptness.[3] Crude oil's volatility term structure is almost linear with slight concavity in the levels, while the log-returns of crude oil display a negative exponential form. Seasonality is not observed in either case. Natural gas is quite different. The term structure of the levels displays the higher volatilities for near expiry futures with lower volatilities as expiry progress. The log-returns have the negative exponential form and strong seasonality.

Two features affecting the analysis include options using spot and forward information. First, crude oil spot transactions are available for delivery no later than 20 days from the initiation of the transaction. This type of transaction becomes a forward contract; therefore, the spot commodity for crude oil is a non-traded asset. Past research on this commodity uses the nearest to delivery future as the spot price because of this feature of the spot market (Gibson and Schwartz, 1990, and Schwartz, 1997). Second, natural gas forwards require delivery at a uniform rate of flow for the entire contract month on a daily basis. This means the price of the forward contract, the underlying asset for the option, is an index price or a basket of daily forwards with the strike set at the beginning of the month. Daily forwards and forwards on the index do not exist making these types of contracts non-traded assets like the spot for crude oil.

[3] Promptness is defined as the succeeding active contracts by time to expiration. For example, in January the prompt month is February, prompt 2 is March, and so forth.

Chapter 2

2. Literature review

The following literature review elucidates the importance of volatility in option pricing, which is the premise for this research. This review presents a brief background of the Classical Option Pricing Models (COPM) by Black and Scholes (1973) and Black (1976). Regularity conditions for the drift and volatility functions of option pricing models are also addressed. The COPM is derived based on assumptions of the dynamic process of the underlying asset. A violation of an assumptions means the COPM is miss-pricing the contingent claim. Alternative option pricing models attempt to correct for these errors. The remaining sections discuss the alternative option pricing models including benefits and weaknesses of each model. These sections include material relating to characteristics other than the volatility component to provide a complete coverage of asset pricing models and their relationships.

2.1. Theory: Black-Scholes and the Classic Option Pricing Model

Development of an asset pricing model, for single and multi-factor models, begins with a description of an assets' price evolution. A generalized price process is

$$dF/F = \mu(F,t)dt + \sigma(F,t)dz, \tag{1}$$

where $\mu(F,t)$ is the drift of the diffusion process, $\sigma(F,t)$ is the volatility function, and dz is an increment of Brownian motion. The COPM for options on equity assets has the following assumptions:

1. $\mu(F,t)$ and $\sigma(F,t)$ are constant,
2. short sales with full access to proceeds are permitted,
3. frictionless markets,

4. a constant risk free asset,
5. no payouts or dividends by the underlying asset, and
6. trading is continuous.

Derivation of alternative valuation models for other classes of assets like commodities and interest rates relax these assumptions with re-specification of the drift (mean) function, volatility function, or additional random components (multi-factors).

Baxter and Rennie (1998) provide the following regularity conditions for the volatility and drift functions of single and multi-factor models. The regularity conditions ensure the drift and volatility functions are well behaved or tractable. These regularity conditions are as follows:

1. for each T, the process $\mu(t,T)$ and $\sigma(t,T)$ are previsible and their integrals $\int_0^T \sigma^2(t,T)dt$ and $\int_0^T |\alpha(t,T)|dt$ are finite;
2. the initial forward curve is deterministic and satisfies the condition that $\int_0^T |f(0,u)|du < \infty$;
3. the drift α has finite integral $\int_0^T \int_0^u |\alpha(t,T)|dtdu$;
4. each volatility, σ, has finite expectation $\mathbf{E}\int_0^T \left| \int_0^u \sigma^2(t,T)dW(t) \right| du$.

The progression of option pricing developments follows re-specifying the drift or volatility functions to match market characteristics or overcome a violation of the COPM assumptions.

2.2. Development of options pricing models

The COPM was developed under the previously mentioned assumptions for valuing equities. Asset or market characteristic that differs or violates these assumption cause miss-pricing of contingent claim. Alternative models were developed to value derivatives on assets to incorporate different characteristics. Single-factor equity models correct for a violation of the no-payout/dividend assumption, where single-factor interest rate models are correcting for time-varying characteristics of the drift and volatility functions of the diffusion equation. Extending these single factor models to multi-factor models includes additional state variables and alternative specifications of the in the drift and volatility functions.

2.2.1. Single factor models

Single factor models seminal extensions were the incorporation of dividend yields for equities (Merton, 1973) and a foreign risk free rate for currency options (Garman and Kohlhagen, 1983). Together these developments led to option pricing models for pricing derivatives on commodities and other assets with cost of carry or convenience yields (Gibson and Schwartz, 1991 and Brennan and Schwartz, 1985). Cost of carry and convenience yield is the gains attributable to owning the physical asset verses a financial derivative where storage, transportation, and insurance are costs incurred against the value of the physical asset. These single factor models for contingent claims of equities, currencies, and commodities have alternative drift specifications and the assumption of constant volatility. These specifications are shown in Table II.

Interest rate models of the short rate illustrate alternative specifications for both the drift and volatility functions of the diffusion process in contrast to the previously discussed single factor models where only the drift function was modified. The Ho and Lee (1986) model has the short rate with drift, θ, and constant volatility and this specification is equivalent to the COPM for assets. The Vasicek (1977) model drift function is $\theta - \alpha r_t$, where α and θ are constant, and the volatility function is constant. The Vasicek model is an Ornstein-Uhlenbeck stochastic process with mean reversion properties. The Cox, Ingersoll, and Ross (1985, CIR) model has the drift function of the

Vasicek model, but α and θ are deterministic functions of time and their volatility function varies with time and interest rate levels as $\sigma_t\sqrt{r_t}$. The Black-Karasinski (1991) model is the Vasicek model in log-space with the volatility function and drift parameters, α and θ, being deterministic functions of time. All of these models are contained within the set of models called Constant Elasticity of Variance models (CEV) where the volatility function is $\sigma_t r_t^{\beta}$.[4] Table II provides a summary of these models.

All of the interest rate models, except for the Cox-Ingersoll-Ross model have the volatility defined as functions of time only, and there are tractable solutions for these models. The Cox-Ingersoll-Ross model contains a derivative with respect to time in its DVF specification and has no analytical solution but is numerically tractable (Baxter and Rennie, 1998). These models can be expressed in terms of the HJM multi-factor model discussed in the following section where the drift and volatility functions are specified in terms of the forward curve.

2.2.1.1. Derivation

The models discussed above utilize assumptions of the COPM. Embedded in the derivations of these models is that agents have a constant relative risk aversion (CRRA) utility function, which the lognormal distribution is a member. This allows one to derive these models in a "Black-Scholes" world where the distribution of returns for the underlying asset are log normally distributed. CIR utilizes a consumption based market approach to derive their interest rate model that allows them to specify the market price of risk.[5]

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[4] See Baxter and Rennie (1998) for detailed derivation of these interest rate models.
[5] The market price of will be discussed in the following section.

2.2.1.2. Parameters

There is only one parameter for the single factor equity, currency, and commodity models that is unknown: the volatility parameter. Estimation methods for this parameter include using historical data, time series methods, and loss functions. The loss function estimation method uses traded options to fit the model price to a market price by inverting the COPM to solve for the unknown volatility parameter termed the implied volatility. Interest rate models have the additional mean reversion and long run mean parameters. These models utilize the same methods where the loss function is the prevalent choice to fit traded bond prices and solve for the model parameters.

2.2.1.3. Benefits and weaknesses

Single factor models are easy to understand, implement, and estimated parameters are reproducible. The weakness of these models is that empirical data shows that several assumptions are violated. Violated assumptions include non-normality in the return distributions like leptokurtosis and additional state variables like stochastic volatility. These issues with single factor models have led to the development of multi-factor models to address non-normality of return distributions and include additional stochastic state variables.

2.2.2. Multi-factor models

Multi-factor models were developed to account for the excess kurtosis and skewness, or incorporate additional market factors as state variables. Multi-factor models incorporating additional state variables have multiple stochastic or random components. The random components include forward rates or prices, and additional stochastic components that are incorporated in the drift and volatility functions. Stochastic jumps can also be included. Multi-factor models can be grouped under two categories: Single-factor extensions and forward models.

There are three prevalent specifications with stochastic components in the drift or volatility functions that are extensions of single factor models. Two specify random variables in the drift function and one in the volatility function. Common random components are stochastic interest rates, cost of carry (dividend yield), volatility[6], and jump processes specified as a system of stochastic differential equations (Wiggins, 1987, Scott, 1987, Hull and White, 1987, Heston, 1993, Stein and Stein, 1991, Bakshi, Cao, and Chen, 1997, Bates, 1991 and 1996, Cortazar and Schwartz, 1994, and Schwartz, 1997). The forward models have the forward prices or rates specified as a system of stochastic differential equations (Clewlow and Strickland, 1999, and Heath, Jarrow, and Morton, 1990).

A generalized specification for a multi-factor model is

$$\begin{aligned} dF/F &= \mu(F,t,r,\delta)dt + \sum_{i=1}^{k} \sigma_i(F,t)^{\gamma} dz_i + J_t dq \\ d\sigma_{F,t} &= f(\sigma_{F,t})dt + g(\sigma_{F,t})dz_v \qquad \forall i = 1 \ldots k \\ d\delta &= f(\delta_t)dt + g(\sigma_\delta)dz_\delta \\ dr &= f(r_t)dt + g(\sigma_r)dz_r \\ \ln(1+J_t) &\sim N\left(\ln[1+\mu_J] - \tfrac{1}{2}\sigma_J, \sigma_J\right) \\ \rho_{s,q}dt &= dz_s dz_q \qquad \forall s,q \in (1 \ldots k, v, \delta, r) \end{aligned} , \qquad (2)$$

1. $\mu(*)$, $\sigma(*)^{\gamma}$, $f(*)$, and $g(*)$ are drift and volatility functions,
2. δ is the cost of carry,
3. r is the instantaneous risk free interest rate,
4. μ_j is the jump mean,
5. $\rho_{s,q}$ is the correlation between the brownian process,
6. σ_j is the standard deviation of the jump, and

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[6] Discrete time series (GARCH) option pricing models of Duan (1995), Heston and Nandi (1997), Duan and Wei (1999), Garcia and Renault (1998), Kallsen and Taqqu (1998), and Sabatini and Linton (1998) can be included in this category because in the continuous limit, a GARCH model is equivalent to stochastic volatility.

[7] This is the commodity return variance conditioned on no jumps occurring,

7. J_t is the magnitude of the jump.[8]

This generalized specification includes multi-factor spot and forward models with stochastic volatility, stochastic convenience yield (dividends), stochastic interest rates, and jump diffusion components. The functions $\mu(*)$, $\sigma(*)$, $f(*)$, and $g(*)$ are varied and subject to the regularity conditions 1-4 presented in Section 2.1. A common specification for $\mu(*)$ and $f(*)$ is an *Ornstein-Uhlenback* (OU) drift, where $\sigma(*)$ is commonly specified as a square root process of CIR, while $g(*)$ is assumed constant. Table III provides a summary of multi-factor models and specifications of the drift and volatility functions.

2.2.2.1. Derivation

Multi-factor models require the basic assumptions of continuous trading, perfect markets, and no borrowing restrictions for deriving a valuation model. Multi-factor models for interest rates, stochastic volatility, and stochastic convenience yield have components that are not traded assets. Non-traded assets introduce a market price of risk that requires a change of numeraire for risk neutral valuation and assumptions about the preference structure of agents.[9] The specified assumptions allow deriving an option pricing formula in a risk neutral environment utilizing Ito's Lemma and the Feynman-Kac formula resulting in a COPM type solution.[10] Single factor extensions specify the risk adjusted spot price process and derive the forward specification. Multi-factor models of forward curves specify the risk adjusted forward price process and derive the spot

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[8] The jump in this specification from BCC is assumed log-normally distributed, but a Poison process can be utilized as did Merton (1976).

[9] Utility functions with properties that are tractable fall under the class of Hyperbolic Relative Risk Aversion functions that include constant relative risk aversion, isoelastic, and lognormal utility functions (Merton, 1990).

[10] See Duffie (1996), Baxter and Rennie (1998), Heston (1993), Heston and Nandi (1998), and Bakshi, Cao, and Chen (1997) for details of this type of derivations.

specification. Each of these types of models must incorporate the correlation structure in the system of equations shown in (2).

Multi-factor models contain correlations between random components that affect the complexity of the solution and parameter estimation. Stochastic volatility models provide a good example of how restricting the correlation can change the solution. Models of Hull and White (1987, HW) and Stein and Stein (1991, SS) assume the correlation to be zero, allowing for tractable solutions for the option pricing formulas. This solution results in the valuation being the integral of COPM contingent claim over the distribution of volatilities. Johnson and Shanno (1987) assumed the correlation as nonzero, but require two options with different maturities to create a hedged portfolio for deriving a solution. These two stochastic volatility models demonstrate how restrictions or assumptions of a parameter can change the results.

2.2.2.2. Parameters

The parameters of multi-factor models are estimated several ways: historical data, time series estimation, and loss functions. A loss function method and time series estimations are the prevalent techniques. Time series estimation includes GARCH, EGARCH, and GARCH in mean techniques. Bakshi, Cao, and Chen (1997) use the loss function with traded options. Stein (1987) use time series estimation with data of the underlying asset and Schwartz (1997) uses the advanced time series estimation Kalman Filter technique with spot price data. Clewlow and Strickland (1999) use Hull and White's (1987) numerical methods to approximate the parameters with forward price data. Selecting a parameter estimation method is usually dictated by the availability of derivative prices for implementing a loss function and historical prices of the underlying asset for time series or historical estimation techniques.

2.2.2.3. Benefits and weaknesses

Multi-factor models are complex and difficult to implement, because the number of parameters increases the complexity and difficulty of estimation (Brooks, 1993, Derman and Kani, 1994, and Dumas, Fleming, and Whaley, 1998). This is seen in equation (2). Market characteristics such as depth and liquidity impact the choice of estimation technique because depth and liquidity of derivatives or underlying assets provide data for estimating parameters. The limited availability of prices causes parameter estimation to be difficult. Models that require the estimation of the unobservable market price of risk parameter also have difficulties because of this additional parameter. Finally, the parameters are sensitive to outliers.

The empirical findings suggest mixed results for these models. The correlation of the volatility and commodity level is found in the price distributions and effect pricing ITM options relativity to OTM options (Heston, 1993, Bates, 1996, and Stein and Stein, 1991). HW find that correlation of volatility with the price seems to explain the smile effect whereas, Bates (1996) finds that a stochastic volatility model does not capture the smile effect; and a jump diffusion model can only capture the volatility smile effect with implausible parameters relative to their time series counterparts.

These multi-factor models have not been able to capture the behavior of the smile even though Ball and Roma (1994), HW, and Taylor and Xu (1994) show that the quadratic smile exists under stochastic volatility models. Wiggins (1987) explains that shocks increase OTM and ITM implied volatilities in his study of the S&P 500 options. The effects of shocks on the OTM and ITM options are smile effects relative to the ATM volatility resulting in a negative linear skew. Figure 1, panel C shows the negative skew.

2.2.3. Deterministic volatility models

The literature to date has classified deterministic volatility models (DVM) as empirical anomalies. This is justified since many of the specifications of DVM are ad hoc or exercises in data mining. The main disadvantage for DVM is that theoretical justification is incomplete. Theoretical justification can be found in the development of multi-factor models as Baxter and Rennie (1996) provided with their single factor interest models. This is shown in Table II in the DVF column. Originally, Breeden and Litzenberger (1978) showed that the risk neutral conditional density function of the terminal distribution of the underlying asset could be specified as a function of the strike. Dupire (1994), using this result, develops a continuous time DVM. Rubinstein (1994) and Derman and Kani (1994, DK) extend Dupire's work and utilize binomial tree methods to implement valuation models. This research addresses the limitations of past DVM research to bridge the empirical results of volatility with financial theory.

The specification of DVM in equities, commodities and other derivatives to date has been ad hoc but some do exhibit theoretical justification without completely linking the empirical with financial theory. The following sections will attempt to overcome this limitation.

2.2.3.1. Multi factor models

The Heath, Jarrow, and Morton (1990, HJM) type multi-factor models of interest rates and the commodity model of Clewlow and Strickland (1999) are DVM. The specification of the volatility function as an exponentially damped volatility structure is deterministic. This feature provides the major link from empirical analysis to theoretical specification. Baxter and Rennie (1996) show how to specify the single factor interest rate models as multi-factor forward rate models in a deterministic environment. The volatility function for short term interest rate models is

$$\sigma(t,T) = \sigma_0 r_t^{\gamma}\, e^{(-\kappa(T-t))}. \tag{3}$$

HJM class of multi-factor interest rate and the commodity models of Clewlow and Strickland (1999) and Schwartz (1997) commodity model specify $\sigma(F,t,T)$ as only time dependent as

$$\sigma(t,T) = \sigma_0\, e^{(-\kappa(T-t))}. \tag{4}$$

In addition, Clewlow and Strickland highlight the work of Schwartz (1997) and demonstrate that Schwartz's two-factor model with stochastic convenience yield is recovered by choosing the appropriate DVM. Clewlow and Strickland (1999) show that the volatility function for Schwartz's models contains two components, which are

$$\begin{aligned} \sigma_s(t,T) &= \sigma_s - \rho_{s,\delta}\sigma_\delta\left(\tfrac{(1-\exp(-\kappa(T-t)))}{\kappa}\right), \text{ and} \\ \sigma_\delta(t,T) &= -\sigma_\delta\sqrt{1-\rho^2}\left(\tfrac{(1-\exp(-\kappa(T-t)))}{\kappa}\right) \end{aligned}, \tag{5}$$

where $\sigma_s(t,T)$ is the time varying volatility for the spot, $\sigma_\delta(t,T)$ is the time varying volatility for convenience yield, σ_s is the fundamental spot volatility, and σ_δ is the fundamental volatility of convenience yield. The key feature of these models is the specification of a volatility function that allows for deriving a closed form pricing solution. The majority of the literature specific to DVM has not exhibited the empirical and theoretical ties that this section has shown.

2.2.3.2. Smile consistent option pricing

Deterministic Volatility Model (DVM) research has addressed methods to capture the volatility skew and term structure and provide procedures for valuing options. The deterministic volatility assumption does not rule out stochastic volatility or that volatility estimates contain jump components, only that the information embedded in the volatility smile and term structure dominates the stochastic components.[11] This alternative approach uses traded options to extract the volatility surface that is then used for valuation and risk management. DVM's require a Deterministic Volatility Function (DVF) for volatility that is a function of time and *moneyness* as $\sigma=f(T\text{-}t, K,F)$. Moneyness is defined as the ratio of the strike price divided by the underlying asset price (K/F)[12] and is a distance measure of how far an option is ITM or OTM.

2.2.3.3. Derivation

Deterministic volatility models are derived based on the *effective theory of volatility* where the local volatility surface is a deterministic function based on a spectrum of options and futures prices. This allows making inferences about the terminal distribution of the underlying asset (Derman and Kani, 1994). Based on Derman and Kani (1994 and 1997), the relationship between implied volatility and deterministic volatility functions for a futures contract can be derived. The forward (Fokker-Plank) equation for a forward based on (1) is

$$\frac{\partial C}{\partial t}-\frac{1}{2}\frac{\partial^2 C}{\partial K^2}\sigma_K^2 K^2 = rC\,, \tag{6}$$

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[11] The distinction between stochastic volatility and deterministic assumes that the random error of the DVM has an expectation of zero. That is the same as the random component of a stochastic volatility model, but the DVM has the time varying volatility embedded in a static volatility surface. Stochastic volatility models will allow the volatility to be random. Rosenberg (2000) and Derman and Kani (1997) develop a Dynamic Implied Volatility Function where the volatility surface is modeled as a function of one or more stochastic state variables, which extends DVM to stochastic volatility models. This extension has the stochastic volatility being a function of strike and price level where each combination of strike and price is a state variable.

[12] Alternative definitions for moneyness is centering around zero that is accomplished by subtracting one from the ratio and as specified by Hull (2003) $1/\sqrt{T}\ \ln\left(\frac{K}{F}\right)$.

where C is the option price, r is the risk free interest rate, K is the strike price for the option, $\partial C / \partial t$ is the partial derivative of the option with respect to time, $\partial^2 C / \partial K^2$ is the second partial derivative of the option with respect to strike price, and σ_K is the volatility function. COPM assume σ_K is constant, whereas CS and HJM assume an exponentially damped volatility structure. This is analogous to a static forward rate curve defining the effective theory of interest rates (Derman and Kani, 1997). The solution to (5) for a given strike, K, is

$$\sigma^2_{K,T} = 2\frac{\partial C_{K,T}/\partial t - rC_{K,T}}{\partial^2 C_{K,T}/\partial K^2 \, K^2} \qquad (7)$$
$$\sigma^2_{K,T} = E^{(K,t)}\{\sigma^2(T) \mid F_t = K\}$$

This local variance is an unbiased, risk adjusted expectation of future realized variance at time T (Derman and Kani, 1997 and Fleming, 1998). The volatility $\sigma(F,t)$ is a function of the underlying futures price, F_t, and time t, and $\sigma(K,t)$ is a function of the strike and time. The effective theory of volatility is defined by (7) with a non-random local volatility surface. The effective theory of volatility is equivalent to the effective interest rate theory used in modeling bond price dynamics (Derman and Kani, 1997).

2.2.3.4. Summary of implied volatility lattice methods

There are three types of implied volatility lattice methods. Dupire (1994) and Derman and Kani (1994, DK) pioneered the implied volatility tree (IVT); Rubinstein (1994) developed the implied binomial tree (IBT); and Clewlow and Grimwood (1997) utilized finite difference methods as an alternative to the IVT tree process. The differences in the IBT and IVT techniques are in the assumptions and application under investigation, the finite difference method is equivalent to the IVT. The IVT produces the skewed terminal distribution implied by the market whereas, the IBT restrict the results to an *a-priori* specified terminal distribution. Derman, Kani, and Zou (1996); Derman and Kani (1997), Derman, Kani, and Chriss (1996) and Chriss (1997) extend DK to include trinomial trees and American options. This research utilizes binomial methods to implement IVT because the IBT method's a-priori restriction on the terminal distribution.

Implied volatility trees are implemented in the same manner as forward rate models of interest rates. Initially, a set of options are used as inputs with a varied cross-section of strike prices and maturities.[13] A method for translating between price space and volatility space is needed for the set of options. Additionally, an interpolating function for the volatility surface of these options is required. Finally, IVT uses forward induction to build the tree structure.

2.2.3.5. Assumptions of lattice methods

All of the methods previously discussed assume that markets are efficient with no arbitrage conditions and option pricing models like the COPM are misspecified (Chriss, 1997). The IVT of a number of researchers were developed under similar assumptions with technical conditions to ensure the IVT correctly prices the contingent claim (Derman and Kani, 1994, Derman, Kani, and Zou, 1996, Chriss, 1996 and 1997, Clewlow and Grimwood, 1997, and Derman, Kani, and Chriss, 1996). The assumptions and technical conditions are

Assumption:

Markets are efficient with no arbitrage opportunities (Derman and Kani, 1994, Derman, Kani, and Chriss, 1996, Clewlow and Grimwood, 1997, and Finucane and Thomas, 1997);

[13] Rubinstein (1994) Implied Binomial Trees specify only a cross section of options with different strike prices for a given time to maturity and a pre-determined terminal distribution.

Technical Conditions:

1. The commodity follows a binomial or trinomial process (Rubinstein, 1994),
2. The tree is recombining (Rubinstein, 1994),
3. Negative transition probabilities do not exist (Barle and Cakici, 1995),
4. The branching process is risk neutral at each time step (Barle and Cakici, 1995),
5. The discount rate is constant over the time to expiration or is exogenous (Rubinstein, 1994), and
6. The volatility at each node, as specified by the volatility function, dominates other stochastic or jump components of volatility (Derman, Kani and Zou, 1997).

Derman and Kani (1994) specify the no arbitrage condition as the forward price equal to the future value of the current commodity level: $F = s * \exp(r * Dt)$. Technical condition 2 may be relaxed where a non-recombining tree is used, but this author is not aware of any research implementing an IVT in this manner.

2.2.3.6. Parameters

DVM is an attempt to capture the leptokurtosis in the terminal distribution of the underlying asset using implied volatilities estimated from traded options. Constructing an implied volatility tree requires a technique for interpolating volatilities for different strike prices of non-traded options. There are many methods of interpolation. A simple method for interpolating is ordinary least squares (OLS) or other econometric technique (Pena, Rubio, and Serna, 1999, Dumas, Fleming, and Whaley, 1998, and Ncube, 1996). Numerical methods for interpolation are cubic, bicubic splines, and Edgeworth Expansions (Bates, 1991, Chriss, 1997, and Clewlow and Grimwood, 1999, Derman, Kani and Zou, 1996). Brown and Randall (1999), Chriss and Morokiff (1999), and Demeterfi, Derman, Kani, and Zou (1999) specify subjective volatility functions based on pragmatic observations of markets. Numerical methods can exactly fit the volatility structure but do not allow for interpretation or analysis of the estimation results. In contrast, regression and econometric techniques allow for interpretation of the estimation results. Finally, the subjective models are valid only in their specific application. Derman (1999) provides the only volatility specification that relates financial theory with empirical observations and this is important to development of DVM's.

The numerical methods provide an algorithm for interpolating volatility for use in the IVT procedures. The other methods specify the volatility as a function of the ATM volatility, moneyness, strike, time, or delta. This function is normally a linear relation that may contain quadratic terms in strike or time.[14] These specifications will be discussed more in the following chapter. A general volatility function can be defined as

$$\sigma_{F,T} = f(\sigma_{atm}, T, K, K^2, ..., \Delta). \tag{8}$$

The ATM volatility is the intercept and the other terms scale this value up or down depending on the observed volatility skews.

2.2.3.7. Benefits and weaknesses

A deterministic volatility function will incorporate leptokurtosis in the terminal distribution of prices than are observed in the markets. This is exhibited in Figure 6. In the CRR binomial tree with constant volatility is depicted in Figure 7 Panel A, and a skewed tree utilizing DVM is depicted in Figure 7 Panel B. This will allow the pricing of options and calculation of risk management metrics that account for the skew.

Past empirical research is multifarious concerning DVM's benefits and weaknesses. Bates (1996) compares deterministic volatility with stochastic volatility and jump diffusion models, and finds that over a short time to expiration, deterministic volatility out performs the stochastic model. Bates finds that with longer expiry options,

[14] Brown and Randall (1999) use a non-linear functional form containing hyperbolic tangent and secant functions.

the stochastic component dominates. Ncube (1996) uses panel data econometric procedures to estimate deterministic volatility functions in FTSE 100 index options and finds volatility estimated with this methodology outperforms implied volatility estimates in minimizing pricing errors. Dumas, Fleming, and Whaley (1998) in their research conclude the more flexible a deterministic model is, the more reliably it estimates the volatility structure. Also, the longer the forecast period, the less accurate the predictive capabilities the model becomes relative to the COPM model. They also find that the hedging performance of the COPM is superior relative to their deterministic volatility model. Buraschi and Jackwerth (1999) find that DVM's are inconsistent with the no arbitrage assumption, but are consistent with stochastic volatility models for ATM options. Additionally, they find that options at strike levels away from the money are driven by an additional factor related clientele utility considerations.

Hull (2000) provides an interesting comment about DVM and volatility surfaces. He notes that traders do price options using volatility surfaces and the COPM. This relegates the COPM to an interpolation formula that translates volatility to the correct market prices. Changing the pricing model changes the volatility surface, but not the option price. Rebonato (1999) is even more critical of both DVM and the COPM. He states "the smiley implied volatility is the wrong number to put in the wrong formula to obtain the right price." Both authors are critical of the DVM to correct the problems associated with the violation of COPM assumptions, but even with their criticism, both state that these types of models may be applicable to interest rates, commodities, and other assets.

Deterministic volatility models have their detractors, but traders do price options using these methods as Hull (2000) states. Under these conditions, research of a DVM that is grounded in financial theory will benefit the financial industry and extend the current research in commodities as suggested by Hull and Rebonato.

Chapter 3

3. Theoretical framework

The proposed smile consistent volatility model (SCVF) will be derived in this section, and the framework for implementing the model will be discussed. The COPM and the Clewlow and Strickland (CS) model are benchmarks for the SCVF. The COPM is used in the industry as the standard model for risk management, and for this reason is a benchmark. The CS model is a common alternative and this model nests within the proposed SCVF. The following sections present these models, the methods for estimating the parameters of the three models, and valuation techniques for these models.

3.1. Smile consistent volatility model

3.1.1. Derivation of smile consistent deterministic volatility function

A deterministic volatility function (DVF) must satisfy the regularity conditions in section 2 and link the DVF with implied volatility in a parameterization that incorporates moneyness and time to expiration. Derman (1999) and Derman, Kamal, and Zou (1996) are the only authors that attempt to link the volatility function of the diffusion equation for the underlying asset with implied volatilities. Dumas, Fleming, and Whaley (1998, DFW), Pena, Rubio, and Serna (1999, PRS) parameterize the DVF correctly but these authors introduce specification error in their models. DFW assume that the implied volatility and local volatility are equivalent measure of the DVF. This is not a valid assumption (Chriss, 1997). PRS apply their log-transformation on the moneyness variable and not the time to expiration variable. Authors applying the DVM in practical examples parameterize implied volatility with ad hoc specifications that is only applicable in their specific applications (Brown and Randall, 1999, Chriss and Morokiff, 1999, and Demeterfi, Derman, Kamal, and Zou, 1999).

The goal is to obtain a DVF that is internally consistent with multi-factor models. Term structure models have a volatility specification that is time and price level dependent. A deterministic volatility function should have the same structure to account for price level as well. Derivation of the DVF begins by specifying the relationship of the underlying price volatility with the instantaneous volatility, price level, and time to expiry. The chosen specification for this research uses a quadratic price level variable to capture non-linear skewness in the implied volatilities. The specification in log-space is

$$\sigma_{F,T} = \sigma_0 + \beta_1 F + \gamma T + \beta_2 F^2 . \tag{9}$$

This equation is a linear because of the log transformation. This is an extension of Derman (1999) with the addition of the time and quadratic price level terms. The last term adjusts the underlying volatility for price level that will capture the quadratic (skewed) volatility smile, thereby allowing for OTM (ITM) calls (puts) to have higher volatilities and ITM (OTM) calls (puts) to have lower volatilities. The first three terms on the right hand side of (8) are based on Ball and Roma's (1994) and Taylor and Xu's (1994) theoretical results that show stochastic volatility can be a function of price level and time. The proposed DVF follows Rebonato's "Floating Smile" and Derman's "Sticky Implied Tree Smile" rules discussed in Chapter 2.

The implied volatility, $\Sigma_{K,T}$, is the average over all possible price paths to finish ITM for a call option. The implied volatility is found by integrating over the time to expiration and the price/strike level in log space as

$$\Sigma_{K,T} = \frac{1}{T(K-F)} \int_0^T \int_F^K \sigma(f,t)\,df\,dt \,. \tag{10}$$

By substituting (9) for $\sigma(f,t)$, this equation becomes

$$\Sigma_{K,T} = \frac{1}{T(K-F)} \int_0^T \int_F^K \left\{ \sigma_0 + \beta_1 f + \gamma t + \beta_2 f^2 \right\} df\,dt \,. \tag{11}$$

The solution to this double integral is

$$\Sigma_{K,T} = \sigma_0 + \frac{\beta_1}{2}\left(K+F\right) + \frac{\gamma}{2}T + \frac{\beta_2}{3}\left(K^2 + KF + F^2\right). \tag{12}$$

Equation (9) is substituted for the variable σ_0, resulting in the DVF in log-space for implied volatility

$$\Sigma_{K,T} = \sigma_{F,T} + \tfrac{\beta_1}{2}(K-F) - \tfrac{\gamma}{2}T + \tfrac{\beta_2}{3}\left(K^2 + KF - 2F^2\right). \tag{13}$$

The third term in (5) increases volatility as the strike increases (decreases), and as the underlying price decreases (increases). Rewriting (5) and transforming from log-space, we obtain

$$\Sigma'_{K,T} = \sigma'_{F,T} \exp\left\{ \alpha_1 F(\tfrac{K}{F} - 1) - \kappa\left(T-t\right) + \alpha_2 F^2 \left(\left({}^{K}\!/\!_{F} \right)^2 + {}^{K}\!/\!_{F} - 2 \right) \right\}, \tag{14}$$

where α_1 is $\beta_1/2$, α_2 is $\beta_2/3$, and κ is $\gamma/2$. The variables Σ' and σ' are the transformations of Σ and σ from log-space. Equation (6) reduces to an internally consistent volatility structure (Clewlow and Strickland, 1999, Schwartz, 1997, and Ritchken, 1996) by restricting α_1 and α_2 equal to 0 or when the strike equals the forward, (ATM or $K=F$) as

$$\Sigma'_{K,T} = \sigma'_{F,T} \exp\left\{ -\kappa\left(T-t\right) \right\}. \tag{15}$$

This structure assumes no skewness and the implied volatility is equal to the forward volatility. Equation (7) is a mean-reverting model that is an exponentially dampened volatility structure (Ritchken, 1996). This structure is the volatility structure of the HJM class of term structure models and the multi-factor model of Clewlow and Strickland (1999) and Schwartz (1997) for price process with Σ and σ replaced by the forward volatilities and the long run mean of volatility respectively.

The estimated parameters α_1, α_2, and β allow interpolating or predicting volatilities for options that are not traded. This method for predicting volatilities is needed for building implied volatility trees. The following section provides details of the IVT method chosen for this research.

3.1.2. Model

3.1.2.1. IVT implementation

This section presents the construction and implementation of an IVT. The description presented here summarizes methods originally derived by Dupire (1994) and Derman and Kani (1994) with extensions by Chriss (1996 and 1997). The notation used in this procedure is listed in Table IV (Haug, 1998).

The first step in constructing an IVT requires a set of input options across multiple strikes and times to expiration. These options are inputs to build a volatility surface using a chosen interpolation technique. At this point, the interpolation technique and construction of the volatility surface is assumed to be completed, and building the IVT can begin. The outcome will be a binomial tree that incorporates the information from the implied volatility surface or a "skewed" binomial tree.

The IVT begins at time step n and requires the calculation of n+2 commodity prices and n+1 transition probabilities or 2n+3 parameters, where 2n+2 are known at time step n. These known parameters are n+1 forward prices and n+1 option prices expiring at time T_{n+1}. The transition probabilities, $u_{\text{prob},n,i}$, are risk neutral probabilities for transitioning from node *(n,i)* to *(n+1,i+1)*. The forward prices follow the no arbitrage condition that makes the tree risk neutral:

$$F_{n,i} = u_{prob,n,i} S_{n+1,i} + (1 - u_{prob,n,i}) S_{n+1,i+1}. \tag{16}$$

The n+1 options defined as $c_{n,i}$ and $p_{n,i}$ are theoretical options calculated with volatilities defined by the volatility surface under COPM assumptions.[15] The market value of a put, $p_{n,i}$, with strike price corresponding to the current commodity level $S_{j-1,k}$ and expiring and time j is summed over all nodes at the time of expiration multiplied by the Arrow-Debreu prices. The market value of this option at time j is

$$p_{j,K} = e^{-r\Delta t} \sum_{k=0}^{j} \lambda_{j-1,k} \{u_{prob,j-1,k} \max(K - S_{j,k+1}) + (1 - u_{prob,j-1,k}) \max(K - S_{j,k})\}. \tag{17}$$
$$\text{where } \quad K = S_{j-1,k}$$

The market value of a call, $c_{n,i}$, is defined in a similar manner. The Arrow-Debreu prices are state prices that have a cash flow of \$1.00 in period n and state i but 0 elsewhere. The Arrow-Debreu prices are determined for each period n+1 at period n for all states i as follows

$$\lambda_{n+1,i} = \begin{cases} u_{n,n} \lambda_{n,n} e^{-rt} & \text{if } i = n+1 \\ \left(u_{n,i-1} \lambda_{n,-1} + (1 - u_{n,i}) \lambda_{n,i}\right) e^{-rt} & \text{if } 1 \le i \le n \\ (1 - u_{n,0}) \lambda_{n,0} e^{-rt} & \text{if } i = 0 \end{cases} \tag{18}$$

Forward induction is used to construct the IVT at each node. The value of $p_{j,K}$ at node j-$1,k$ can be determined from (9) and by recognizing that at nodes above j (k>j) the value a put is zero and below j (k<j) the value is positive, therefore the value of a put, $v^{Put}_{j-1,k}$ is

$$\begin{aligned} v_{j-1,k} &= e^{-r\Delta t} \{u_{j-1,k} (K - S_{j,k+1}) + (1 - u_{j-1,k})(K - S_{j,k})\} \\ &= \sum_{k=0}^{j} \{\lambda_{j-1,k} (e^{-r\Delta t} K - S_{j-1,k})\} \end{aligned}. \tag{19}$$

[15] These options could use the Cox, Ross, and Rubinstein (1979) or As Chriss (1997) points out, a closed form model like Black (1976).

Using (9) and solving, the put value is

$$v_{j-1,j}^{put} = \frac{p_{j,K} - \sum_{k=0}^{j-1}\{\lambda_{j-1,k}(e^{-r\Delta t}K - S_{j-1,k})\}}{\lambda_{j-1,j}}. \tag{20}$$

A similar definition for the tree value of the call, $v^{call}_{j-1,k}$ is

$$v_{j-1,j}^{call} = \frac{c_{j,K} - \sum_{k=0}^{j-1}\{\lambda_{j-1,k}(S_{j-1,k} - e^{-r\Delta t}K)\}}{\lambda_{j-1,j}}. \tag{21}$$

The up and down transition probabilities are respectively

$$u_{prob} = \frac{e^{r\Delta t}K - S_d}{S_u - S_d} \quad \text{, and } d_{prob} = \frac{S_u - e^{r\Delta t}K}{S_u - S_d}. \tag{22}$$

The CRR binomial tree provides that $S_u=K*u$ and $S_d=K*d$ where u and d are the up and down transition jumps and $d=1/u$. Substituting these relationships in d_{prob} above, the up transition probability becomes

$$u = \frac{v_{j-1,i} + K}{e^{-r\Delta t}K - v_{j-1,i}}. \tag{23}$$

This function of this equation is to determine the up and down commodity levels respectively based on the choice of centering conditions.[16] The choice for centering is the spot as in Derman and Kani (1994) and Chriss (1996 and 1997) where

$$\begin{aligned} S_u &= Ku = S_{j-1,i}u \\ S_d &= K/u = S_{j-1,i}/u \end{aligned}. \tag{24}$$

[16] A second choice is centering at the forward with S_u and S_d are

$$S_u = e^{r\Delta t}Ku = e^{r\Delta t}S_{j-1,i}u \text{ and } S_d = e^{r\Delta t}K/u = e^{r\Delta t}S_{j-1,i}/u.$$

The differences in these equations and the results in DK come from the choice of the strike set at the current commodity level. The results of DK are obtained by replacing K with the forward relationship, $F=e^{r\Delta t}S$, and $v_{j-1,i}$ with the above equations.

The implied local volatility at each node in the tree is the logarithmic spacing of Derman and Kani (1994) calculated as

$$\sigma_{n,i} = \frac{1}{\sqrt{\Delta t}} \sqrt{u_{prob,n,i}(1 - u_{prob,n,i})} \ln\left(\frac{S_{n+1,i+1}}{S_{n+1,i}} \right). \quad (25)$$

A problem that occurs with this methodology is that a probability is not in the range $0<u_{prob}<1$. This happens with the forward price is not between S_u and S_d or a violation of no arbitrage conditions occurs. There are several alternatives for correcting this type of problem. One alternative, presented by Chriss (1997), is applicable to the futures options. This method is to ensure that proper spacing of the futures and current nodes through the local volatility. To ensure proper spacing a new S'_u or S'_d need to be calculated. Defined these as

$$S'_u = S_{j,i+1} = \frac{S_{j,i} S_{j-1,i+1}}{S_{j-1,i}} \text{ and } S'_d = S_{j,i} = \frac{S_{j,i+1} S_{j-1,i-1}}{S_{j-1,i}}. \quad (26)$$

The second method is to grow the tree at the forward curve thereby displacing the time step to ensure maintaining the no arbitrage condition (Derman, Kani, and Chriss, 1996). , Trinomial trees are another solution.[17] Chriss (1997) method is implemented in this research. The basic algorithm for constructing an IVT is present in Table V.

The IVT produces a price, Arrow-Debreu, up probability, down probability, and local volatility tree. The price and Arrow-Debreu trees will value vanilla options and exotic options. This tree also determines hedge parameters.

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[17] Trinomial trees are an option as seen in Derman, Kani, and Chriss (1996), Haug (1998), and Clewlow and Grimwood (1997).

3.1.3. Hedge parameters

The delta of an option is the change of the option with respect to the change in the underlying asses or $\partial c / \partial F$. The IVT price tree determines the delta of the option following the methods of standard binomial trees (Hull, 2000) for small changes in the underlying as

$$\Delta = \frac{c_{1,1} - c_{1,0}}{F_{1,1} - F_{1,0}}, \tag{27}$$

where, $c_{i,j}$ is the value of the option at node i,j and $F_{i,j}$ is the underlying asset price at node i,j.

3.2. Classic option pricing model

3.2.1. Model

The Classic Option Pricing Model (COPM) of Black (1976) is

$$\text{c}(\text{F}, r, t, K; \sigma) = e^{-r(\Delta t)}\left[FN(d_1) - KN(d_2)\right], \tag{28}$$

where F is the maturity of the forward, Δt is the maturity of the option, K is the strike price,

$$d_1 = \frac{\ln(F / K) + \frac{1}{2}\Delta t \sigma^2)}{\sigma\sqrt{\Delta t}}, \text{ and}$$
$$d_2 = d_1 - \sigma\sqrt{\Delta t}. \tag{29}$$

The value of the put is

$$\text{p}(\text{F}, \sigma, r, t, K) = e^{-r(T-t)}\left[KN(-d_2) - FN(-d_1)\right], \tag{30}$$

3.2.2. Parameter estimation

The COPM only unknown parameter is σ. Equation (20) or (22) is used to estimates this parameter by inverting the equation and using a traded option price. The

Newton-Raphson technique is the method for finding the solution. This provides an implied volatility for each expiration and strike combination of forwards.

3.2.3. Hedge parameters

The delta of the COPM is required to conduct the hedging performance for this study. The delta of the option is the first derivative with respect to the price. The delta for a call and a put respectively is

$$\frac{\partial c}{\partial F} = \mathrm{e}^{-rT} N\left(d_1\right) \text{ and } \frac{\partial p}{\partial F} = \mathrm{e}^{-rT}\left[N\left(d_1\right)-1\right]. \tag{31}$$

ATM implied volatilities estimated in the previous section determine the hedge parameters.

3.3. Clewlow and Strickland commodity model

3.3.1. Model

Clewlow and Strickland (1999, CS) derive an option pricing formula for pricing options of forwards and futures. The solution for valuing a European option is

$$c\left(t, F(t,s); K, T, s; \sigma_0, \gamma\right) = e^{-r(T-t)}\left[F(t,s)N(d_1) - KN(d_2)\right] \tag{32}$$

where F is the maturity of the forward, T-t is the maturity of the option, K is the strike price,

$$\begin{aligned} d_1 &= \left[\ln\left(\tfrac{F(t,s)}{K}\right) + \tfrac{1}{2}w^2\right] \Big/ \sqrt{w^2}, \text{ and} \\ d_2 &= d_1 - \sqrt{w}. \end{aligned} \tag{33}$$

The value of the put can be valued using put-call parity. The volatility of the forward price returns from t to T is integrated to determine w^2 as

$$\begin{aligned} w^2(t,T,s) &= \int_t^T \sigma_0^2 e^{-2\gamma(s-u)} du \\ &= \tfrac{\sigma_0^2}{2\gamma}\left[e^{-2\gamma(s-T)} - e^{-2\gamma(s-t)}\right] \end{aligned}. \tag{34}$$

3.3.2. Parameter estimation

The parameters σ_o and γ requires estimation. The loss function, using near-the-money options with different expirations as inputs, accomplishes this task. The objective is to find the value of the parameter set that minimizes the loss function. The loss function with the market price of the option, c^*, and (24), is

$$\begin{aligned} &Min\ \left(c^* - c(t, F(t,s); K, T, s : \sigma_0, \gamma)\right)^2 \\ &s.t.\ \sigma_0 > 0 \end{aligned} . \tag{35}$$

Parameters σ_0 and γ, are estimated using traded options, both puts and call, that are within 10% of the ATM option. These parameters provide the term structure of volatilities.

3.3.3. Hedge parameters

The Delta of the CS model is the first derivative with respect to the price, as with the COPM. The delta for a call and a put respectively is

$$\frac{\partial c}{\partial F} = \mathrm{e}^{-rT} N(d_1) \text{ and } \frac{\partial p}{\partial F} = \mathrm{e}^{-rT}\left[N(d_1) - 1\right]. \tag{36}$$

Modifying the volatility parameter of the COPM Delta calculation allow for determining the hedge parameters. This modification uses the volatility calculated with (26).

Chapter 4

4. Model evaluation

The evaluation of the smile consistent volatility functions (SCVF) is against two benchmarks, Classic option pricing model (COPM) and the Clewlow and Strickland (1999, CS) commodity model. The following sections describe the data used in implementing the SCVF and benchmark models, a discussion of the SCVF parameter estimates based on the procedures outlined in section 3, a comparison of pricing errors of the SCVF verses the benchmark models, and a hedging application for evaluating the performance of the SCVF verses the benchmark models.

4.1. Data requirements

Volatility of a price (return) is a measure of the dispersion of an asset price (return) about its mean level over a fixed time interval (Abken and Nandi, 1996). The normal definition of volatility is the standard deviation of the price series. In option pricing models, expected price volatility is required. Options and futures prices are used to determine market's expected volatilities required in this study. The expected volatility is implied volatility estimated by procedures discussed in section 3.

4.1.1. Options and Futures Data

The data includes call and put options for crude oil and natural gas futures for the years 1995 through 1999. This research uses all traded options across strike prices and expiration months with premiums that are greater than zero. The data is from the New York Mercantile Exchange and is composed of futures and options on futures settlement prices, open interest, and transacted volumes. The implied volatilities are calculated for each option every trading day to expiration using COPM.[18]

The focus of this study is on medium and long time to expiration options. Options with a short time to expiration are excluded due to liquidity effects and the extreme measures of volatility in the short expiry options (Taylor and Xu, 1994, Dumas, Fleming, and Whaley, 1998, DFW, and Bakshi, Cao, and Chen, 1997, BCC). All options with less than 10 days to expiration are excluded in this study.

Liquidity of an option, time to maturity, and moneyness influence implied volatility. These characteristics cause option prices to be highly sensitive to changes in volatility. Previous studies focus on short to medium term options ranging from 30–60 days to expiration (Hamid, 1998, Fofana and Brorsen, 1998, Resnick, Sheikh, and Song, 1993, BCC, Ncube, 1996, and DFW). Other studies analyzed longer term options over 90 days to expiration (Engle, Kane, and Noh, 1997, Gesser and Poncet, 1997, Heynen, Kemna, and Vorst, 1994). The diversity of estimation periods used in option research depends on whether the purpose of the study is the development of a pricing model, development of a volatility model, or research on informational characteristics of a pricing structure. Pricing models require liquid options for estimating and testing the pricing model's performance, whereas volatility models and informational analysis require longer expiration times to test the volatility estimates and the model's performance. This research is a combination of these three research areas.

[18] Deterministic or stochastic time varying interest rates or the daily U.S. Treasury Bill Yields could be used for the implied volatility calculations. This adds complexity to the model and the sensitivity of the implied volatilities to changes in interest rates is small.

The studies previously mentioned exclude options that are deep ITM or OTM due to liquidity concerns and sensitivity to volatility. The moneyness range varies from as small as ±6% to ±25%, again depending on the purpose of the study. Building implied trees and estimating DVF require a broad range of options. Implied trees need to build a term structure volatility relationship for estimating future implied option prices. Excluding deep ITM or OTM options excludes useful market information imbedded in these options about the evolution of volatility over time. This study has transaction volume for each option and will use zero transaction volume as the exclusion rule for illiquid options. This paper addresses the behavior of volatility over time and will not exclude options based liquidity as measure by the transaction volume not moneyness.

4.1.2. Interest Rates

Each option requires a risk free rate of interest as an input to the option pricing model. Time to expiration of most energy options is less than 18 months. This research assumes a risk free asset exists, which are US Treasury Bills (T-Bill) and US Treasury Notes (T-Note). The mid-price is used in this analysis. The US Government treasury price data is obtained from the Wall Street Journal and Bloomberg. Each option is matched with a T-Bill or T-Note by expiration dates where the selected T-Bill or T-Note matures at or after the expiration of the option. Options that expire less a year have a matching T-Bill for each week. Options with expiry over 1 year have no more than one week of time differences in T-note maturity and option expiry.

T-Notes are quoted in yields and T-Bills are quoted in prices. T-Bills require conversion to a yield. T-Bills are quoted using the banker discount method based on 360 days per year. The discount yield is $y_D = D*360 / F*t$ where D is the quoted discount yield, F is the face amount, and t is the time from the settlement date to the maturity date. This yield is converted to a Bond Equivalent Yield (*BEY*) that will account for reinvestment opportunities and 365 compounding days. The *BEY* depends on whether there are more or less than 182 days to maturity. The formulas for converting the discount yield to *BEY* are[19]

$$
\begin{aligned}
&t \le 182: \\
&\qquad BEY = \frac{365 * y_D}{360 - t * y_D} \\
&t > 182: \\
&\qquad BEY = \frac{-\frac{2*t}{365} + 2\sqrt{\left(\frac{t}{365}\right)^2 - \left(\frac{2*t}{365} - 1\right)\left(1 - \frac{1}{P}\right)}}{\frac{2*t}{365} - 1}
\end{aligned}
\tag{37}
$$

Bid and Asked yields are calculated, and the average of these yields determines the mid yield for each T-Bill. T-Notes are quoted in BEY and the mid yield is used in this research where the definition of the mid yield is the average of the bid and ask yields.

4.2. Smile consistent volatility model

4.2.1. Smile consistent volatility model parameter estimation

The derived deterministic volatility functions (DVF) from Section 3 are equations (4) and (5). Equation (4) is the first model which contains the unknown expected volatility parameter, $\sigma_{F,T}$, and equation (5) is the second model where the fundamental volatility, σ_o is an estimated parameter. The models are

$$\text{Model 1: ATM: } \Sigma_{K,T} = \sigma_{F,T} + \frac{\beta_1}{2}(K - F) - \frac{\gamma}{2}T + \frac{\beta_2}{3}\left(K^2 + KF - 2F^2\right). \tag{38}$$

$$\text{Model 2: Sigma0: } \Sigma_{K,T} = \sigma_0 + \frac{\beta_1}{2}(K + F) + \frac{\gamma}{2}T + \frac{\beta_2}{3}\left(K^2 + KF + F^2\right). \tag{39}$$

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19 See Fabozzi, Frank J. and T. Dessa Fabozzi (1989) Bond Markets, Analysis and Strategies, Prentice Hall, Englewood Cliffs, NJ 1989, Page 86-87 for more details on bond values and yield calculations.

These equations estimate the volatilities used in the implied volatility tree (IVT) algorithm. The variable $\sigma'_{F,T}$ of the underlying price is a function of the instantaneous volatility that is unobservable. This research will replace $\sigma'_{F,T}$ with the ATM implied volatility, $\sigma'_{ATM,T}$. This allows a cross section of options with the same maturity to be a function of the ATM volatility and the last three terms on the right hand side of Model 1. This is consistent with Derman (1999), Brown and Randall (1999), Chriss and Morokoff (1999), and Demeterfi, Derman, Kamal, and Zou (1999). Model 2 removes the dependency of the volatility surface on the ATM volatility for a given expiration, and provides an estimate of the fundamental instantaneous volatility for all expirations.

The DVF is estimated daily to obtain parameter estimates used to construct the volatility surface. The parameter estimation will utilize the cross section of traded options implied volatilities and the associated exogenous variables to estimate the two relationships above daily. The estimation technique will be ordinary least square (OLS).

4.2.1.1. Smile Consistent Volatility Model Selection Criteria

There are two critical aspects of the data to consider in the DVF parameter estimation stage: stale prices and put-call parity violations exhibited in different implied volatilities for puts and call for the same strike price. The two option data series contain all traded options that have open interest on the exchange. Some option prices are stale prices, which are determined by zero traded volume for an option at each strike price on each day. These stale prices may affect the parameter estimates. Calls and puts for the same strike price have different implied volatilities. This can be seen in Figure 8 for crude oil and Figure 9 for natural gas. This can cause convergence problems and affect the parameter estimates. Past research uses the prompt forward month as the spot price due the characteristic of the spot discussed above (Schwartz, 1999, Gibson and Schwartz, 1991, and Brennan and Schwartz, 1985). The prompt contract exhibits unstable skews due to erroneous volatilities for deep ITM and OTM options. The prompt contract causes convergence problems and dominates the parameter estimates because of the instability in the volatility. The research objective of this is estimating the medium and long term price structure, so excluding the prompt contract is evaluated. There are eight scenarios for each commodity for stale prices, put/call parity violations, and prompt contract instability. The eight scenarios are:

1. stale prices with all puts and calls with the prompt month,
2. stale prices with all puts and calls without the prompt month,
3. stale prices with only OTM puts and call with the prompt month,
4. stale prices with only OTM puts and call without the prompt month,
5. excluding stale prices with all puts and calls with the prompt month,
6. excluding stale prices with all puts and calls without the prompt month,
7. excluding stale prices with OTM puts and calls with the prompt month, and
8. excluding stale prices with OTM puts and calls without the prompt month.

Evaluation of the exclusion rules uses the adjusted-R^2 of the models. This model selection rule is chosen because additional variables can increase the value of the measure without accounting for the decrease in degrees of freedom. The purpose of the DVF is for prediction and smoothing of the volatility surface so the last two problems are of minimal concern. The adjusted-R^2 from Green (1997) is

$$\overline{R}^2 = 1 - \frac{n-1}{n-k}(1 - R^2), \tag{40}$$

where n is the number of observations, k is the number of exogenous variables, and R^2 is the R-squared of the regression.

4.2.1.2. SCVF Model Selection Results

The model selection criteria is presented in Table VI, which shows the mean, standard deviation and ranges of the adjusted-R^2 each model. The highest adjusted-R^2 is for the exclusion of stale prices with small differences when the ITM money calls and puts are excluded, or scenario 6. The adjusted-R^2 for crude oil Model 1 is 94.9 for both cases that use OTM only calls and puts and uses all moneyness of calls and puts. The adjusted-R^2 for Model 2 is 71.0 and 69.8 for each moneyness case respectively. The adjusted-R^2 for the natural gas Model 1 is 93.7 and 94.2 for each moneyness case and for Model 2 is 71.5 and 70.03. The exclusion of the ITM calls and puts makes little difference for crude oil or natural gas where stale prices do influence the models explanatory power. Therefore, the model selected for further investigation is scenario 6 above using all call and put options while excluding stale prices and the prompt contract month. The negative mean adjusted-R^2 can be explained by the fact that this model does not contain an intercept (Green, 1997).

The univariate statistics for the parameters of Model 1 and Model 2 are presented in Table VII for crude oil and Table VIII for natural gas. The models differ in the resulting number of daily parameter sets because of convergence problems for 3 days in natural gas and this commodity is missing 16 days of option data in March of 1998 and July of 1995. The detail of each daily set of parameters is not presented here because there are 1252 daily estimates for crude oil and 1234 for natural gas. The univariate statistics for the number of cross section observations used to estimate the daily DVF models is presented in Table IX of scenario 6 for the entire sample and for each year. This shows that the mean number of observations increased each year increasing the ability of the DVF to "fit" the volatility surface. The minimum and maximum values provide the range of daily cross sections. These also increased over the sample period. The average daily observations for crude oil were 63 and natural gas were 43. The minimum for crude oil and natural gas were 20 and 4 respectively with the maximums of 149 and 119.

Probability plots show that the errors are symmetric with longs ends in both tails (Curtis, 1999). This means that there is a loss of efficiency estimating the models (Green, 1997). Extreme observations dominate these errors. The dominance of extreme observations is seen with model 2 in the mean value of the parameter *sigma0* and the differences in the *gamma* parameter of model 1 and 2. The mean value of *sigma0* is 1.03e24 for crude oil and 2.91e32 for natural gas with interquartile ranges of 419.3 and 0.4 respectively. The mean values for *sigma0* are unreasonable values while the interquartile ranges are acceptable. The values of gamma change magnitude and sign drastically between the two models. Each of these facets could cause problems with the model evaluation stage.

Model 1 is selected over Model 2 based on the adjusted-R^2 statistic better accuracy of model 1. These two models differ in their specification and so each model is compared against the benchmarks in the analysis of the pricing errors and hedging application.

4.2.1.3. Implied Volatility Tree Results

Implementation of the implied volatility tree (IVT) is as described in Chapter 3. IVTs are built for each commodity and contract per trade day of the data. The number of time steps is selected as 30 for each IVT after evaluating estimated terminal distributions for 30, 40, and 50 time steps, and finding there was not significant differences in the distributions. This test was conducted on a subset of sample of days because of computational limitations.[20] The computational limitations also required the implementation of the Bjerksund and Stensland (1993) American option approximation instead of a CRR tree during construction of the IVT that Chriss (1997) suggested as an alternative. Volatility is capped at 1e[100].

4.3. Pricing errors

The COPM and CS models pricing errors are evaluated against the two models of the proposed SCVF. The pricing error is defined as the actual price of the option less the model predicted price. This research utilizes two nonparametric tests to evaluate the pricing errors. Nonparametric test are chosen because no distributional assumptions are made about the data and therefore, are not sensitive to violations of non-normality as the F-Test is (Conover, 1980). The Wilcoxon Sign Rank Test evaluates the expected values of the pricing errors and the Squared Ranks Test for Equal Variances evaluate the variances of the pricing errors (Conover, 1980).

The Wilcoxon Sign Rank Test has the following one tailed hypothesis:

$$H_o: E(X) \geq E(Y),$$
$$H_a: E(X) < E(Y).$$

The null hypothesis, H_0, is rejected at significance level α (.05) if the test statistic, T_1 is greater than the critical value $\omega_{1-\alpha}$. The test statistic, T_1 is

$$T_1 = \sum_{i=1}^{n} R_i \Big/ \left(\sum_{i=1}^{n} R_i^2\right)^{1/2}, \tag{41}$$

where R_i is

$$R_i = \begin{cases} +Rank(D_i) & \text{if } D_i \geq 0 \\ -Rank(D_i) & \text{if } D_i < 0 \end{cases}, \tag{42}$$

[20] It took 3 days/model, on a Dell 8300 with a 3GHz processor to build each model's IVT using 30 time steps.

and $D_i = |Y_i - X_i|$. The variable Y is the benchmark model's pricing error and X is the proposed model's pricing error. This test compares the ranks of the differences of the pricing errors of the benchmark and the alternative model.

The Squared Rank Test for Equal Variance has the following one tailed hypothesis:

H_0: X and Y are identically distributed, except for possible different means,
H_a: VAR(X) < VAR(Y).

The null hypothesis, H_0, is rejected at significance level α (.05) if the test statistic, T_1 is less than the critical value ω_α. The test statistic, T_1 for samples that have ties in the rank orders is

$$T_x = \frac{T - n\overline{R}^2}{\left[\frac{nm}{N(N-1)}\sum_{i=1}^{N} R_i^4 - \frac{nm}{N-1}\left(\overline{R}^2\right)^2\right]^{1/2}}, \tag{43}$$

$$T = \sum_{i=1}^{n} R(U_i), \tag{44}$$

$$\overline{R}^2 = \frac{1}{N}\left\{\sum_{i=1}^{n} R(U_i)^2 + \sum_{i=1}^{m} R(V_i)^2\right\}, \text{ and} \tag{45}$$

$$\sum_{i=1}^{N} R_i^4 = \sum_{i=1}^{n} R(U_i)^4 + \sum_{i=1}^{m} R(V_i)^4 . \tag{46}$$

The variables U and V are the absolute deviations of the pricing errors from their means, as follows:

$$U_i = |X_i - \mu_x| \text{ and } V_i = |Y_i - \mu_Y| . \tag{47}$$

R(*) is the rank function and N=n+m. This statistic tests whether the alternative model's variance of pricing error is less than the benchmarks variance of pricing error.

4.3.1. Pricing error results

The univariate statistics of the pricing errors for natural gas and crude oil are presented in Tables X-XV. These results show that the average pricing error for the COPM using the ATM volatility and the SCVF models consistently under values options for natural gas (the average pricing error is positive), while the CS model consistently over values options for natural gas (the average pricing error is negative). This result is also found for the crude oil put options, but the ITM call options exhibit instances where all the models over value options. The ranges of pricing errors, as shown by the minimum and maximum values, show that the COPM has the smallest range for both natural gas and crude oil. The SCVF models produce the smaller range of pricing errors for OTM and across the traded options only class of natural gas, while the CS model has a smaller range of pricing errors for stale options and ITM options. The SCVF model 2 consistently produces smaller ranges of pricing errors across all option classes verses the CS model. The SCVF model 1 exhibits over valuation of extreme magnitudes for call options that occur on 14 days of the 1252 trading days in the sample. The elimination of these days causes the results of the ranges of the SCVF model 1 pricing errors to become consistent with model 2. The standard deviations of pricing errors for both natural gas and crude oil exhibit the same results, with the COPM have the smallest standard deviation of pricing errors followed both SCVF models, when adjusting for the 14 extreme days for crude oil, and finally the CS model.

The Wilcoxon sign rank statistic tests the hypothesis that the mean pricing error of the alternative is greater than or equal to the mean pricing error of the benchmark is shown in Tables XVI-XVIII. All cases except the COPM verses the CS models reject the null hypothesis. The test of the COPM verses the CS model is complicated by the pricing error results that showed the CS model consistently over priced options resulting in a negative pricing error. The sign of the pricing errors affects this test statistic when the pricing errors are of opposite sign. This is a deficiency of this test and a reason why the squared rank test of equal variances is employed.

The squared rank test for equal variances tests the null hypothesis that the two sets for pricing errors are identically distributed. The test results are also shown in Tables XVI-XVII. The null hypothesis that the COPM and the alternative model have identical variances is not rejected for any of the alternative models, but is for the CS model.

The Wilcoxon test indicates the mean pricing error of the SCVF models equivalent or greater than the mean of the COPM benchmark. The Squared rank test implies that the COPM and SCVF variance of pricing errors are equivalent or the simple model is as good as or better than the SCVF.

4.4. Hedging application

The hedging parameters from the IVT provide a method for evaluating the performance of the proposed volatility specification that prices options under a skew. This study will undertake hedging performance based on four practical situations. Crude oil and natural gas have four natural hedging requirements. First, a producer of crude oil and natural gas is long the commodities. The producer has a break-even price for the commodity and can hedge this price or buy insurance by purchasing puts. Second, if a refinery is short crude oil to process into refined products. The refiner can purchase forwards to hedge the required capacity or purchase calls to hedge. The refiner also has a break-even price for the cost of crude oil. Crude oil prices above this price will incur losses to the refinery. Last, a local distribution company (LDC) or gas utility is required to provide gas service to its customers. This is a short position in natural gas and a hedge for gas would be the purchase of forwards or calls to cover the load. This case is not as clear as the other two cases. The LDC can pass through gas cost to the customers, so one would think that the cost of gas is immaterial, but customers are often given the choice to purchase their gas at a fixed price over specified period of time. This exposes the LDC to price risk that sets a maximum break-even cost for gas, and therefore calls can provide insurance.

All the above cases are real situations. The range of operating costs for domestic crude oil production is between $12 to $18 per barrel and a similar dollar per mmbtu value for natural gas. This will allow for establishing a floor price for crude oil and natural gas with the ability to purchase OTM puts and track the hedge portfolio. Similarly, a refinery has a break-even cost for crude oil. The LDC will establish a summer price and winter price for gas based on the average price from the forward curve at the initiation of the evaluation period. The transaction costs for futures are $100 per contract, or $0.01/mmbtu for natural gas, and $0.10/barrel for crude oil. The transactions costs for options are $3.50 per contract, or $.00035/mmbtu for natural gas, and $0.0035 for crude oil.[21] The assumption of credit quality counterparties or letters of credit will mitigate margin requirements.

The quantities to be hedged are monthly quantities of crude oil and natural gas. The production cases will monthly production in the amounts of 10,000 barrels of crude oil and 1,000,000 mmbtu of natural gas. The refiner will require a monthly quantity of 10,000 barrels of crude oil, and the LDC will have a 1,000,000 mmbtu load requirement monthly.

Hedge portfolios are constructed weekly for 3 month hedge horizon for crude oil and 6 month horizon for natural gas. The portfolios remained static for the hedge

[21] The transction cost estimates were provided by the Williams Energy, Marketing, ang Trading Natural Gas, Crude, and Refined products trading desk.

window. Hedge portfolios use deltas from the IVT, COPM model, and the CS model. There are 260 hedge portfolios for the two crude oil applications, 255 portfolios for the natural gas producer application, and 256 portfolios for the natural gas LDC application.

4.4.1. Hedging application evaluation

Evaluating the hedging application utilizes the squared rank test for equal variance and Deibold and Mariano (1995, DM) test as evaluation criteria. These tests are employed because of limitations of other statistics like F-test and T-test when disturbances of the errors violate normality (Deibold and Mariano, 1995 and Green, 1997). The DM test is used when the forecast errors of the hedging application contain autocorrelation.

The DM statistic uses a pair of forecasting models with a step ahead forecast error of ($e_{1,t}$, $e_{2,t}$) for sample size t. The null hypothesis of the expected pricing performance for an error function g(*)=$e_{i,t}$ is $E[g(e_{1,t})-g(e_{2,t})]=0, \text{for } t=1,...,n$. The error functions can be a quadratic loss functions or other functions that measure the dispersion of the errors. This research is using the squared error function or $g(e)=e^2$ or $d_t = g(e_{1,t})-g(e_{2,t}), \text{for } t=1,...,n$. The DM is

$$S_{DM} = \bar{d} \Big/ \sqrt{\hat{V}(\bar{d})}, \qquad (48)$$

where the mean and variance are defined asymptotically as

$$\bar{d} = n^{-1}\sum_{t=1}^{n} d_t \text{and } V(\bar{d}) \approx n^{-1}\left[\gamma_0 + 2\sum_{k=1}^{h-1}\gamma_k\right]. \qquad (49)$$

The step ahead forecast length is h, and the autocovariance is

$$\gamma_k = n^{-1}\sum\nolimits_{t=k+1}^{n}\left(d_t-\bar{d}\right)\left(d_{t-k}-\bar{d}\right). \qquad (50)$$

The DM statistic has an asymptotical standard normal distribution. This test relies on approximately unbiased estimator of the variance of the sample mean of the loss differential (Harvey, Leybourne, and Newbold, 1997, HLN). HLN suggest an MDM for the variance of S_{DM} which is

$$S^{*}_{DM} = S_{DM}\left[\frac{n+1-2h+n^{-1}h(h-1)}{n}\right]^{1/2}. \tag{51}$$

This modified DM statistic is distributed students-t with *n-1* degrees of freedom. The hedging application uses a three month hedge window for crude oil and six month window for natural gas. The step ahead forecast length, h, for crude oil is three months and six months for natural gas.

4.4.2. Hedging application results

The results of hedging applications based on the squared rank tests for equal variance are presented in Table XVIII and the MDM tests are presented in Table XIX. The squared rank test for equal variance tests show that all models have lower portfolio variances than the unhedged portfolio. The CS model verses the COPM benchmark for both crude oil and natural gas fail to reject equal portfolio variances for the consumption scenario. The CS model verses the both models 1 and 2 fails to reject the equal portfolio variances for natural gas for the producer scenario. The SCVF models exhibit that their portfolio variances are lower than the benchmark models in the other hedging applications. The possible portfolios for the hedging applications have a maximum of 256 portfolios and a minimum of 139. The differences in the numbers of observations are because options that are required to hedge the downside or upside risks do not exist or are illiquid in the sample.

The MDM test is employed due the autocorrelation in the hedging errors because the portfolios' ability to maintain minimum variance is based on the hedging performance over the hedge window. The MDM tests indicate that the SCVF models and the benchmark COPM have smaller variance than the CS benchmark. The hedging performances of crude oil for both the consumer and producer scenarios do not exhibit any differences. The hedging performance for natural gas is mixed with variations of the SCVF verses the COPM being equivalent for consumer model 2 and the producer for both models 1 and 2.

The hedging performance exhibited statistical differences based on the squared rank test statistics when analyzing the minimum portfolio variation at the end of the hedge period. The proposed SCVF does not out perform the COPM when accounting for autocorrelation of the hedge variances based on the MDM statistics. Autocorrelation of the hedges is an important property of financial risk management indicating that the MDM tests have more power than the squared rank tests.

Chapter 5

5. Factors of volatility

There are economic factors and risk management characteristics of commodity markets that affect volatility through changes in price levels. Economic factors of supply and demand distort the true volatility of an option resulting in the volatility smile (Sullivan, 1993). Specifically crude oil and natural gas are physical commodities with storage and transportation issues affecting the commodity's price volatility. Risk management and speculative activities affect market depth and liquidity. Gesser and Poncet (1997) show that market microstructure characteristics of participants' behavior in trading and hedging activities effect volatility in equity and index markets. Murphy (1994) states that volatility is effectively a price where actual volatility depending on the willingness of buyers to absorb risk and sellers to lay off risk. This is consistent with Natenberg, Meisner, and Boyle (1990) that stated option traders are trying to pick the appropriate volatility to hedge their risks. Murphy (1994) further explains that the skewness is a result of unexpected movements in price, dynamic hedging of risk and liquidity effects of near-the-money options.

In this study, variables used for analyzing financial components of volatility structures listed as "positions" contained in the Commitment of Traders Report published by the Commodity Futures Trading Commission. Variables used for analyzing supply and demand components are inventory statistics published by both the American Petroleum Institute and American Gas Association. Additional factors analyzed are a market momentum variable and seasonal effects. These data sources and characteristics are discussed in the following sections in more detail.

5.1. Data requirements

Supply and demand variables are inventory statistics compiled by the American Petroleum Institute (API) and American Gas Association (AGA). Risk management variables are contained in the Commodity Futures Trading Commission (CFTC) *Commitment of Traders Report.* The timing of the release of the information from these agencies is important for market participants.

API and AGA statistics release their data on Wednesday of each week.[22] The weekly inventories of crude oil products for the United States provide data on changes in supply that will affect price volatility. The AGA inventory report includes available storage capacity and changes in storage inventories. Both API and AGA provide true measures of supply and demand for these commodities.[23]

The Commitment of Traders Report is available for the financial instruments on the NYMEX. The CFTC compiles this report, which contains open interest, reportable positions, nonreportable position data, and spread position data for commercial and non-commercial traders. A commercial trader is a party that transacts in derivative securities for hedging purposes, and a non-commercial trader is a speculator that is transacting for reasons other that hedging. Reportable positions are for holdings equal to or exceeding the minimum reporting level established by the CFTC (www.cftc.gov) and exchanges.

[22] The Energy Information Administration replaced the AGA weekly inventory report in April of 2002 and changed the release date to Thursday. This change of responsibilities was due to irregularities in the data reported found in 2000-2001 to AGA and the potential liability of the AGA. The actual AGA data is no longer available though the AGA will provide their corrected historical data.

[23] Williams Energy, Marketing, and Trading thankfully provided this data.

Nonreportable positions are the differences between open interest and reportable positions. These show the long and short positions of traders and how long or short the market is. A spread position is the short (long) position that covers a long (short) position of a non-commercial trader. The position of a trader can indicate the supply and demand for a commodity in the future. Long (short) positions in the financial instruments indicate future supply shortages (surpluses) of the underlying commodity or demand increases (decreases). These are variables used for analyzing liquidity concerns (Murphy, 1994) and supply and demand considerations (Sullivan, 1993). Trader's commitments are a proxy for the market participant's hedging activities that Gesser and Poncet (1997) discussed.

The market momentum variables show current trends in the market. The changes in the supply and demand views of market participants affect these trends. The long term momentum variable is a 60-day moving average of historical prices. The short term momentum variable is a 20-day moving average. These provide insights about changes to underlying economic factors and price levels. The slope or changes in these moving averages provide indications of market volatility levels. Flat moving averages indicate that volatility is relatively time invariant, while steep slopes indicate increasing volatility that varies with time. In addition, a common technical buy (sell) rule is when a short term moving average crosses above (below) a longer term moving average indicating a buy (sell) of the financial instrument. These moving averages indicate whether a market is trending up or down and provide insights on volatility. The changes in the moving average provide inferences about the magnitude of the change in the volatility or direction of volatility change. A generalized n-period moving average for price (MA_n) is calculated as follows

$$MA_n = \sum_{i=1}^{n} F_i \Big/ n \,, \tag{52}$$

where F is the commodity price and n is the number of periods.

5.2. Methodology

The DVF's daily parameters from models 1 and 2 can be interpreted as volatility skew and term structure components where B_1 and B_2 are the skew component and γ is the term structure component. The skew component B_1 is the slope of the skew and B_2 is the rate of change in the slope of the skew for changes in moneyness. The term structure component, γ, causes the volatility surface to vary in time as the yield curve of interest changes with time. Changes in the magnitude of these variables change the shape of the volatility surface causing portfolio values to change and hedge ratios to change. Portfolio values and hedge ratios changing are key concerns for trading and risk management activities.

The component estimates, B_1, B_2, and γ are regressed on the market momentum variable, the liquidity variables from the CFTC, seasonal components, and economic supply and demand variables from the API and AGA. The regression equations for Natural Gas and Crude Oil are

$$\begin{aligned}
&\text{Natural Gas: } Parameter_k = a_{1,k} AGAEast + a_{2,k} AGA\Pr od + \\
&\quad a_{3,k} ShortTermMA / LongTermMA + a_{4,k} HedgeLong\% + \\
&\quad a_{5,k} HedgeShort\% + a_{6,k} Spread\% + \sum_1^2 b_{i,k} Season_{i,k} + error_k \\
&\text{Crude Oil: } Parameter_k = a_{1,k} \ln(TotalDomestic) + a_{2,k} ln(Imports) \\
&\quad a_{3,k} ShortTermMA / LongTermMA + a_{4,k} HedgeLong\% + \\
&\quad a_{5,k} HedgeShort\% + a_{6,k} Spread\% + \sum_1^4 b_{i,k} Season_{i,k} + error_k
\end{aligned} \,, \tag{53}$$

where $a_{i,k}$ and $b_{i,k}$ are parameters to be estimated and $error_k$ is the regression error. The variables specific to natural gas are AGA inventories for the percentage of the in U.S. production region (AGAProd) and percent of the eastern U.S. consuming region (AGAEast). The seasonal dummy variables for natural gas are injection season: April through October ($Season_{1,k}$), and withdrawal season: November through March ($Season_{2,k}$). The variables specific to crude oil are the API inventory variables, which are the natural log of total domestic inventory and natural log of total imports. The seasonal dummy variables for crude oil are constructed from refineries product cycles, where the products are unleaded gasoline and heating oil. The unleaded season is spring and summer ($Season_{1,k}$ and $Season_{2,k}$), and for is heating oil season during fall and winter ($Season_{3,k}$ and $Season_{4,k}$). The market variables for depth and liquidity for both commodities are the CFTC's commitment of trader's positions: percent of long hedgers defined as long reportable positions divided by open interest (HedgeLong%), percent of short hedgers defined as short reportable positions divided by open interest (HedgeShort%), percent of Spreads defined as Spread positions divided by open interest (Spread%). The market momentum variable is the ratio of the 20-day short term moving average divided by the 60-day moving average of spot prices (ShortTermMA/LongTermMA).

The endogenous variables, β_1, β_2, γ, are from the daily parameter estimates from the DVF models 1 and 2, and σ_0 from model 2, discussed previously, are regressed on the exogenous variables to determine how market factors impact volatility. An important aspect of the variables from models 1 and 2 are the correlations of these variables within each model, which is shown in Table XX. Notice that β_1 and β_2 are significantly negatively correlated for both models and 60% correlated with σ_0 for natural gas. This correlation of parameters implies that contemporaneously correlation is an issue that influences the analytical results. Seemingly unrelated regression (SUR) is utilized to estimate the regression equations for model 1 and model 2 parameters. The estimation method for SUR is

$$\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} = \begin{bmatrix} X_1 & 0 & 0 \\ 0 & X_2 & 0 \\ 0 & 0 & X_3 \end{bmatrix} + \begin{bmatrix} e_1 \\ . \\ . \\ . \\ e_n \end{bmatrix}. \tag{54}$$

$y_1 \ldots y_k$ is $\beta_{1,1} \ldots, \beta_{1,k}$, $y_1 \ldots y_l$ is $\beta_{2,1} \ldots, \beta_{2,l}$, $y_1 \ldots y_m$ is $\gamma_{1,1} \ldots, \gamma_{1,m}$ for model 1 where k+l+m=n, and X_i is the exogenous variable in (2). Model 2 has the additional component $y_1 \ldots y_s$ is $\sigma_{0,1} \ldots, \sigma_{0,s}$ and k+l+m+s=n with X_i is the exogenous variable in (2), but is excluded from this factor analysis due to the instability of this variable. The test for contemporaneous correlation of the SUR model a Breusch-Pagan test for contemporaneous correlation and the test statistic with v=3 equations is

$$\lambda = n\sum_{j=2}^{v}\sum_{i=1}^{j-1}\left[\sigma_{i,j}^2 \big/ \sigma_{j,j}\sigma_{i,i}\right] \sim \chi\left(v(v-1)\big/2\right). \tag{55}$$

Autocorrelation in the errors is tested in addition to contemporaneous correlation across the equations. The Durbin-Watson test statistic for both positive and negative correlation is

$$d = \frac{\sum_{t=2}^{T}\left(e_t - e_{t-1}\right)^2}{\sum_{t=2}^{T} e_t^t}. \tag{56}$$

This test statistic is based on the hypothesis that the errors are an autoregressive process of order one, AR(1). The AR(1) process is

$$e_t = \rho e_{t-1} + \varepsilon_t, \tag{57}$$

where the null hypothesis is $\rho=0$. Values of d close to two indicate no autocorrelation in the errors.

5.3. Factors of volatility results

The analysis of volatility factors of crude oil and natural gas models 1 and 2 contains the tests for contemporaneous correlation, tests for autocorrelation in the errors, regression results, and seasonality tests. The test for contemporaneous correlation and the covariance matrix of residuals for (2) are presented in Table XXI. The Breusch-Pagan test statistics are highly significant for both commodities for each model. Crude oil is 1309.4 and 1251.9 for models 1 and 2 respectively, and natural gas is 1217.0 and 1284.9. SUR regressions can improve efficiency. The final Durbin-Watson statistics are presented in Table XXII. The Durbin-Watson statistics are all close to two after adjusting for autocorrelation. Crude oil required three moving average terms in the skew equations for models 1 and 2, and required four moving average terms only for the term structure equation, γ, equation. Natural gas exhibited autocorrelation in the term structure equation for model 1 where a single moving average term corrected the autocorrelation. There where two moving average terms required for the skew equations for model 2 and three terms for the term structure equation. The moving average terms that adjust for autocorrelation are all statistically significant and negative, as shown in Table XXIII for crude oil and Table XXIV for natural gas. The sign of the moving average terms indicates strong negative autocorrelation in the data.

Final parameter estimates for the factors of volatility are presented in Table XXIII for crude oil and Table XXIV for natural gas. A goal of this research is to investigate factors that influence volatility. The supply and demand factors for crude are the natural log of total domestic production and natural log total imports and for natural gas are the percentage of the production zone storage levels and eastern consuming region storage levels.

The sign of the volatility skew and term structure components is important for interpreting the impact of the volatility factors. The average values of these components are shown in Table VIII for crude oil and Table IX for natural gas. Crude oil and natural gas have the sign of second skew component, β_2, and the term structure component, γ[24], as positive for both models. The first skew component, β_1, is negative in model 1 for both commodities, while in model 2 it is negative for crude oil and positive for natural gas. A direct relationship of a volatility factor with a skew or term structure component is when both the component estimate and the volatility factor parameter have the same sign. For example, a volatility factor's parameter estimate that is positive and a skew component that is also positive causes an increase in the skew for increases in a volatility factor.

The significant volatility factors of crude oil of the skew components for model 1 are natural log of imports, the hedge long percent, hedge short percent, and the spread percent variable. The positive parameter values for imports, hedge long percent, and spread percent imply a direct relationship with the volatility. This direct relationship

[24] This component is negative in the Tables VIII and IX for model 1, but the equation for estimating in also negative resulting in a positive value for γ.

causes the skew to increase as one of these variables increase. The value of the hedge short parameter is negative causing skew to decrease with increases in this factor. The relationships reverse with respect to the second skew component for each of the variables. The negative values for imports, hedge long percent, and spread percent have the curvature of the skew decreasing as these variables increase.

The significant volatility factors of crude oil in model 2 with the two skew components are the same as for model 1 except the domestic production parameter becomes significant. The volatility factors sign reverses with respect to the skew components with the sign of the parameters consistent with the previous results. Positive parameters values for the first skew component imply a direct relationship and negative parameters an inverse relationship with respect to changes in the fundamental factors.

The term structure equation for crude oil of model 1 has two significant parameters, which are the hedge short percentage, and the natural log of domestic production. The hedge short percentage is negative implying an inverse relationship with the volatility term structure. The domestic production parameter is positive therefore having a direct relationship with the volatility term structure. The time effects decrease as the hedge short positions increase, and increase as domestic production increases. The term structure for crude oil of model 2 has hedge short percentage, the natural log of imports, the ratio of the short term to the long term moving average prices or momentum ratio, and the hedge long percentage parameters significant. The hedge short percentage and the momentum parameters are directly related to the volatility term structure, while the imports and hedge long percentage inversely related to the volatility term structure.

The first skew component of natural gas in model 1 has a positive parameter value for the production percentage implying that as this variable increases the skew increases. Model 2 of natural gas has the production percentage negative with the first skew component while it is positive for the second skew component implying that as this variable increases the slope of the skew decreases while the curvature increases. The east percentage is positive for the first skew component and negative for the second causing the slope and curvature to decrease as this variable changes.

The volatility term structure component for model 1 has the momentum ratio and the hedge long percentage both with negative parameters implying that as these variables increase the time effect decrease. The spread percentage is positive for model 2 for the volatility term structure component implying a direct relationship with the volatility term structure meaning time effects increase as production inventory increases. The east percentage, momentum, and hedge long percentage are inversely related to the volatility term structure of natural gas implying that as these variables increase, the time effects diminish.

Tests for seasonal effects in the parameters are presented in Table XXV. The seasonal effects for the volatility skew and term structure of crude oil skew in both models is for the summer or unleaded season as seen in the Panel A of Table XXV. The differences in the magnitude of these parameters is small supporting only slight seasonality across the skew and term structure for crude oil as empirically observed in Figures 1 and 2. Natural gas has no seasonal components for the skew parameters as shown in Panel B, but has differing injection and withdrawal seasonal effects for the term structure as shown in section Gamma. This supports empirical observations from Figures

4 and 6 that there exists seasonality in the term structure of natural gas. Different seasonal effects are evident for the skew and term structure components for both crude oil and natural gas in both models, though only marginally for crude oil's model 2, as shown in section Across Equation of Panels A and B.

5.3.1. Day of the week analysis

The API, AGA, and CFTC statistics are released on different days during the week. The analysis of effects by API and AGA inventory release date has CFTC information that is from the previous two weeks. The timing of the information releases is a characteristic directing an analysis of volatility factors by weekday classifications. The analysis by day of week controls for the nonsynchronous release of information by using only information available from the previous release date.

Tests for contemporaneous correlation and autocorrelation are shown in Table XXVI for weekday effects. Contemporaneous correlation is present in both commodities. The first skew variable, β_1, and the second skew variable, β_2, are negatively correlated as previously shown. The term structure variable, γ, exhibits economically insignificant correlation with the other two variables for crude oil, while being economically significant for natural gas as shown in Table XXVII panels A and B. The Durbin-Watson statistics are presented in Table XXVIII and show that autocorrelation can be controlled.

The inventories statistics from the API and the AGA are released on each Wednesday for the previous week, while the CFTC releases their information for the previous week on Friday. The sign convention for the skew parameters is consistent with previous results where the sign of the skew curvature component is the opposite of the skew slope component.

The weekday analysis of crude oil's significant factors of volatility from the API and CFTC are shown in Table XXIX The API variables are significant on the days around the Wednesday release date. This result implies the market starts interpreting the API statistics the day before the release using proprietary forecasting models and continues updating their forecasts after the release of the data. The natural log of imports is significant on Tuesday and Thursday for both skew variables for both models, and is significant for the term structure equation for model 2. Imports are the swing component of total supply for the United States. Decreases (increases) in domestic production cause imports to increase (decrease) depending on the demand for crude oil. Imported levels of crude oil tend to dominate the effects of domestic production on the volatility skew and term structure. The CFTC variables hedge long percent, hedge short percent, and spread percent all are significant during the week for both models. The spread percent is important for the skew components but not the volatility term structure. These variables provide market information of liquidity and depth, additionally market intelligence is gained because these variables are the registered hedgers. The amount short (long) the market is will shift the skew and term structure of volatility.

The weekday analysis of natural gas' significant factors of volatility from the AGA and CFTC are shown in Table XXX. The AGA inventory variables are significant for the skew components on Tuesday in models 1 and 2. This is the day before the AGA releases inventory numbers for the previous week. Traders typically begin resetting their positions and speculating on Tuesday based on proprietary forecasts of the AGA numbers the following day. The results also show that on Wednesday the eastern inventory values significantly impact the volatility term structure. Changes in these numbers cause the volatility term structure to shift based on the demand forecasts and natural gas storage levels for following months. An increase (decrease) in eastern consumption results in stockouts (surpluses) in inventories of storage in succeeding months. Companies will have to cover the stockouts or liquidate the surpluses. The Friday significance of these variables allows firms time to analyze the results of their fundamental models, decide on alternative actions, and implement them. The momentum ratio is significant for natural gas across most weekdays for the term structure equation of both models, showing that trends in the market are important. Natural gas has a cyclic behavior and trends up and down during each year. The magnitude of these trends is important. The trend, along with the inventory levels, causes the term structure of volatility to change. The CFTC variables hedge long and hedge short percentages are important for the volatility term structure. These variables are providing liquidity, depth, and market intelligence for natural gas consistent with the results for crude oil.

The Wald tests of equal seasonal components by day of the week are presented in Table XXXI for natural gas and Table XXXII for crude oil. The weekday analysis for seasonal effects is consistent with previous results. Crude oil has a summer or unleaded seasonal aspect for the skew and natural gas has no seasonality components in the skew. Crude oil has summer seasonality in the volatility term structure and natural gas also exhibited seasonality for this component.

5.3.2. Elasticity of Factors

The two classes of volatility factors are supply and demand economic factors and risk management factors. This research showed that both sets of factors contribute, in varying degrees of magnitude, to explaining the skew and term structure of volatility. The elasticity of a factor is the marginal contribution of the factor to changes in the skew and term structure components. Elasticity is defined as the percentage change of the dependent variable divided by the percentage change in the independent variable (Pindyck and Rubinfeld, 1991) as

$$E_i = \frac{\Delta Y_i / Y_i}{\Delta X_i / X_i} = B_i * \bar{X}_i / \bar{Y}, \tag{58}$$

where Y_i is the skew or term structure parameter, X_i is an independent factor, and B_i is a skew or term structure component. The average, $\bar{X}_j$, of the skew and term structure parameter values are shown in Tables VII and VIII for crude oil and natural gas.

The elasticities of the significant factors for natural gas and crude oil are presented in Table XXXIII. The absolute magnitude of an elasticity provides a ranking for the marginal contribution of a volatility factor, and the marginal contribution of a factor can be used to determine the average change in the skew and term structure components.

The elasticities for the economic factors for the skew and term structure components of crude oil for models 1 and 2 provide the largest marginal contribution to the volatility components followed by the risk management factors. Natural gas is consistent with crude oil for the skew components, where the economic factors provide the largest marginal contributions for models 1 and 2 followed by the risk management factors. The term structure component for natural gas has the hedge long percentage have the largest marginal contribution.

These elasticities provide prediction capabilities or sensitivity analysis of the changes in the volatility surface. For example, the natural log of total imports of crude oil for model 2 has elasticities that imply the slope component of the skew decreases

18.43%, the curvature component decreases 19.28%, and the term structure component decreases 4.4% for a one percent increase in the factor. Using the average components from Table VII, the slope component of the skew would change from -0.571 to -0.466, the curvature component would change from 0.015 to 0.012, and the term structure component would change from 1.117 to 1.068. A one percent change in the spread percentage, the smallest elasticity for model 2 of crude oil, has the slope component changing to -.568, the curvature component to .015, and the term structure changing to 1.118. The impact of a one percent change in the spread percentage is minimal when compared to the one percent change in the natural log of total imports. This example shows the magnitude of an elasticity is important for the volatility surface and can be applied to all the volatility factors.

Chapter 6

6. Summary

An alternative specification, termed the "Smile Consistent Volatility Function" (SCVF), of the volatility function for valuing financial derivatives was presented. This SCVF was applied to energy commodities natural gas and crude oil. The SCVF is consistent with the volatility skew, "Volatility Smile," and term structure empirically observed in market data. The three parameters of the SCVF can be interpreted as a slope component, a curvature component, and a term structure component. This SCVF extends past research in Deterministic Volatility Models (DVM). The objectives of this research were to derive the relationship of the SCVF with implied volatilities, evaluate pricing and hedging performance of the SCVF against two benchmarks, and evaluate the power of market factors in explaining volatility. The parameters of the SCVF were estimated using ordinary least squares. The estimated SCVF parameters were used to construct Implied volatility trees (IVT) for each trading day. The IVT for each trading day was used to determine the model prices for traded options and the option's hedge parameter, delta. The time series of SCVF parameters were regressed on economic, risk management, and seasonal factors to examine factors affecting volatility.

Implementing the IVT's required two steps. Estimating the SCVF parameters was the first step, and constructing the IVT was the second step. Two important results emerged during the SCVF parameter estimation phase. First, the residual errors were not normal and attempts to overcome this were unsuccessful. Second, estimating the SCVF parameters and constructing the IVT on a daily basis was computationally expensive requiring 3 weeks at maximum CPU computer usage and several hundred gigabytes of dedicated storage. The second aspect is important because past research has not addressed the computing requirements for this type of procedure.

The one problem that exists with building IVT is invalid transition probabilities that are less than zero or greater than one. This research found the frequency of invalid probabilities increased the further away from the money an option was and the shorter the step size of the IVT. The trade off is that accuracy is improved using shorter step lengths, but efficiency decreases because of the need to correct the increased occurrence of invalid probabilities.

The second objective was to evaluate the SCVF pricing performance by comparing pricing errors of the model verses market prices of options. The hedging performance of the SCVF was evaluated by comparing the volatility of SCVF hedge portfolios verses benchmark hedge portfolios. The SCVF was compared to two benchmarks, the industry standard classic option pricing model (COPM) and the Clewlow and Strickland commodity model (CS). Nonparametric tests of the difference in the mean pricing errors and difference in the variance of the pricing errors both indicate the SCVF model is equivalent to the COPM benchmark, or the simple model is as good as or better than the SCVF. Hedge performance of the SCVF was different than the benchmark, based on the squared rank test. The test revealed the SCVF had lower volatility of the hedge position value at the end of the hedge period. The proposed SCVF does not out perform the COPM when accounting for autocorrelation of the hedge variances based on the MDM statistics. In summary, the SCVF performed marginally better than the CS model in both pricing and hedging performance, but the SCVF was insignificantly different from the COPM.

The analysis of factors of volatility provided insights on changes in the skew and term structure of volatility. The three endogenous variables were contemporaneously correlated, requiring concurrent estimation of the parameters of the factors of volatility using Seemingly Unrelated Regressions. The regression errors exhibited autocorrelation, which required the addition of moving average terms to correct. The analysis of volatility factors produced an estimated relationship that has a sign relationship between the factors and the skew parameters. The SUR estimated coefficients reveal the sign convention that a positive (negative) parameter for the first skew variable of a factor would have a negative (positive) parameter for the second skew variable for the same factor. This implies the skew is increasing (decreasing) at a decreasing (increasing) rate. This is consistent with the quadratic skew structure that is observed empirically. The analysis of economic and risk management factors showed that these factors impact the skew and term structure of both crude oil and natural gas. The seasonality analysis showed that the skew and term structure of crude oil volatility has a seasonal component for the unleaded season. The estimated coefficients of the seasonal components indicate that the changes of downstream products to unleaded gasoline during the production cycle marginally impact the volatility structure. The term structure of natural gas volatility has a seasonal component. The estimated coefficients of the seasonal components indicate different volatility regimes during the two natural gas seasons.

The elasticities of natural gas volatility skew component with respect to supply and demand factors were the largest, and risk management factors were smaller. These estimated elasticities indicate that supply and demand factors contribute more to changes in natural gas volatility skew than risk management factors. The elasticities for the natural gas volatility term structure with respect to the risk management factor, hedge long percent, was the largest. This estimated elasticity indicates that the hedge long percent dominates changes in the natural gas volatility term structure. The elasticities of crude oil volatility skew and term structure with respect to supply and demand factors were the largest, and risk management factors were smaller. These estimated elasticities indicate that supply and demand factors contribute more to changes in crude oil volatility than risk management factors. The magnitude of an estimated elasticity's marginal contribution to changes in volatility allow for predicting changes in the volatility surface using fundamental forecasts of volatility factors.

6.1. Conclusions

This research has investigated a Deterministic Volatility Function (DVF) that is consistent with the volatility smile (SCVF) exhibited by empirical data in crude oil and natural gas markets. Key findings are that a Classic Option Pricing model pricing and hedging performance are comparable to SCVF pricing and hedging performance. The SCVF exhibits instability in the parameter estimates that are unreasonable in magnitude. The SCVF may be over parameterized and increases the complexity of the valuation process (Brooks, 1993, Derman and Kani, 1994, and Dumas, Fleming, and Whaley, 1998). Implementing the implied volatility tree is computationally expensive with respect to time and computer memory to manage the data and resulting output.

The DVF provided a means to analyze factors that explain volatility. American Petroleum Institute and American Gas Association weekly inventory statistics are statistically related to shifts in the volatility skew and term structure. Commodity Futures Trading Commission statistics of market participants "positions" are also important explanatory variables of changes in the volatility surface. The magnitude of an elasticity of a volatility factor is useful in measuring marginal contribution of the factor to changes in volatility, and this can aid in calibrating fundamental forecasting models by determining the volatility of the terminal price distribution utilizing the factor elasticities.

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Table I. Description of futures and options contracts of the crude oil and natural gas commodities traded on the New York Mercantile Exchange.

Trading Unit/ Trading Months		Trading Times/ Contract Quote	Trading Expiration/ Option Strikes		Special Delivery Requirements
Options	**Futures**		**Options**	**Futures**	
		Light Sweet Crude Oil			
Crude Futures	1000 BBLSs	Open outcry scheduled 9:45am - 3:10pm with after hours trading 4:00pm-8:00am Monday-Thursday and Sunday 7:00pm-8:00am.	3 days before the futures expiration. Exercise of the option is any day up to the last trading day by 5:30pm or 45 minutes after close whichever is later.	3rd Business day prior to the 25th of the month.	
12 consecutive months plus months 18, 24, 36 on a June-December Cycle.	30 consecutive months, plus months 36, 48, 60, and 72.	$ and ¢/BBLS	0.50 above and below the at-the-money strike price and 2.50 above the highest an lowest existing strike price for a total of 61.		
		Henry Hub Natural Gas			
Natural Gas Futures	10000 MMBtu	Same as Crude Oil	Immediate day before the futures and early exercise is the same as Crude Oil.	3rd business day preceding the start of the delivery month	Uniform rate of flow delivery over the month
12 consecutive months plus months 15, 18, 21, 24, 27, 39, 33, 36 on a June-December cycle.	36 consecutive months	$ and ¢/MMBtu	Strike prices in 0.05 increments above and below the at-the-money strike price, and 0.25 increments above and below the highest and lowest s strike prices for a total of 81 options in the nearby 3 month contracts and 61 options in the other delivery months.		

Notes: Bbls - Barrels or 42,000 US Gallons and MMBTU- Million British thermal.

Table II Options pricing model diffusion equation parameter comparison

Model	Drift: $\mu(F,t)$	Volatility: $\sigma(F,t)$	DVF: $\sigma(F,t)$
Assets			
Black and Scholes	r	σ	
Merton	$r-q$	σ	
Garman and Kohlhagen	$r-r_f$	σ	
Black		σ	
Cost of Carry	$r-c$	σ	
Interest Rate			
Ho and Lee	θ	σ	$\sigma_t^2(T-t)$
Vasicek	$\theta-\alpha r_t$	σ	$\sigma e^{-\alpha(T-t)}$
Cox-Ingersoll-Ross	$\theta_t-\alpha_t r_t$	$\sigma_t\sqrt{r_t}$	$\sigma_t\sqrt{r_t}\,\partial B/\partial t\,(t,T)$ [25]
Black- Karasinski	$\theta_t-\alpha_t r_t$	σ_t	

[25] $B(t,T)$ is the solution to the Riccati differential equation.

Table III. Multi-factor model drift and volatility function summary.

Model/Process	Drift: $\mu(F,t,r,\delta)$	Volatility: $\sigma_i(F,t)$	Volatility Drift: $f(\sigma_{F,t})$	Volatility of Volatility: $g(\sigma_{F,t})$	Convenience Yield Drift: $f(\delta_t)$	Volatility of Convenience Yield: $g(\sigma_{F,t})$
Spot Process for Equities and Commodities						
BCC	$[r_t - \chi_r \mu_J]$	$\sqrt{\sigma}$ where γ=1/2	$[\theta_v - \kappa_v \sigma_{F,t}]$	$\sigma_v \sqrt{\sigma_{F,t}}$		
Schwartz			Constant σ_v		$[\kappa - \lambda_\delta - \delta]$	Constant σ_δ
Forward Curve Process for Interest Rates and Commodities						
Clewlow and Strickland		$\sigma e^{(-\alpha(T-t))}$				
Heath, Jarrow, and Morton		$\sigma e^{(-\alpha(T-t))}$				
Stochastic Volatility Process						
Hull and White and Wiggins		$\sigma_i(F,t)$	Constant	Constant		
Scott, Johnson and Shanno, Heston, and Ball and Roma		$\sigma_i(F,t)$	Constant and OU process	$\sigma_v \sqrt{\sigma_{F,t}}$		

The parameters are as previously defined and κ_i is speed of adjustment, θ_v long run mean of volatility, σ_v is volatility of volatility, λ_i is the market price of risk for parameter i, and χ is the magnitude of the jump component.

Table IV. Implied volatility tree procedure description of notation.

Variable	Description
n	time step,
i	position node in tree starting at 0,
$S_{n,i}$	current commodity price at time step n and node i,
$F_{n,i}$	forward commodity price at time step n and node i,
$c_{n,K}$	market value of an European call option with strike, K and expiring at time n,
$C_{n,K}$	market value of an American call option with strike, K and expiring at time n,
$p_{n,K}$	market value of an European put option with strike, K and expiring at time n,
$P_{n,K}$	market value of an American put option with strike, K and expiring at time n,
$\lambda_{n,i}$	Arrow-Debreu price at time step n and node i,
$u_{prob.n,i}$	up transition probability,
$d_{prob.n,i}$	down transition probability,
$v^{put}_{j-1,k}$	value of $p_{n,K}$ at time $j-1$ at node $(j-1,k)$,
$v^{call}_{j-1,k}$	value of $c_{n,K}$ at time $j-1$ at node $(j-1,k)$,
$v_{j-1,k}$	reduced form of tree value of call or put at time $j-1$.

Table V. Implied volatility tree building algorithm.

Step		Description
1		Initialization of inputs
	A	Estimate SCVF parameters from input option prices and implied volatilities.
	B	Input initial commodity level for node (0,0).
	C	Set initial Arrow-Debreu price to 0.
2		Set the centering condition and begin building the next time step outward from the middle node.
3		Determine the market price of an option at the next time step.
4		Determine the tree value of the option from STEP 2 for node (n,i).
5		Determine the up and down commodity levels for the nodes branching from current node.
6		Determine the up and down transition probabilities.
7		Compute Arrow Debreu prices over next time step.
8		Do STEPS 2-7 until the tree is constructed.
9		Calculate the local volatility tree using.
10		Output desired results of implied commodity tree, transition probabilities, Arrow-Debreu prices, and Local volatility tree.

Table VI. Adjusted-R^2 Statistics for Model 1 (ATM) and Model 2 (Sigma0) estimated daily from 1/1/1995-12/31/1995.

Commodity	Model	Prompt	Call-Put Flag	Traded Options Only	MEAN	STD	MAX	MIN
Crude Oil	Model 1: ATM	1	OFF	NO	0.371	2.028	0.942	-42.149
				YES	0.905	0.262	0.998	-4.105
			ON	NO	0.281	2.441	0.932	-49.850
				YES	0.931	0.109	0.997	-1.091
		2	OFF	NO	0.799	0.097	0.950	0.430
				YES	**0.949**	**0.055**	0.999	0.439
			ON	NO	0.793	0.102	0.947	0.439
				YES	**0.949**	**0.055**	0.999	0.622
	Model 2: SIG0	1	OFF	NO	0.474	0.193	0.859	-0.560
				YES	0.297	0.740	0.961	-15.394
			ON	NO	0.526	0.170	0.890	-0.042
				YES	0.414	0.354	0.952	-3.306
		2	OFF	NO	0.668	0.116	0.899	0.314
				YES	**0.698**	**0.153**	0.970	-0.036
			ON	NO	0.692	0.126	0.934	0.342
				YES	**0.710**	**0.152**	0.972	0.056
Natural Gas	Model 1: ATM	1	OFF	NO	-4.447	132.719	0.995	-4397.111
				YES	0.923	0.417	1.000	-7.784
			ON	NO	-3.271	90.077	0.998	-2893.468
				YES	0.938	0.567	1.000	-17.874
		2	OFF	NO	0.888	0.125	0.995	0.045
				YES	**0.942**	**0.120**	1.000	-0.834
			ON	NO	0.872	0.208	0.999	-1.650
				YES	**0.937**	**0.136**	1.000	-0.880
	Model 2: SIG0	1	OFF	NO	0.411	0.278	0.936	-1.203
				YES	0.479	0.795	0.983	-13.417
			ON	NO	0.432	0.255	0.942	-0.788
				YES	0.529	0.443	0.996	-4.931
		2	OFF	NO	0.527	0.242	0.927	-0.043
				YES	**0.703**	**0.270**	1.000	-0.960
			ON	NO	0.540	0.232	0.927	0.023
				YES	**0.715**	**0.250**	0.999	-0.386

The data filter method with the largest R^2 is indicated by bold text.

Table VII Deterministic volatility function parameters: univariate statistics for Crude Oil of daily estimated models from 1/1/1995-12/31/1995.

Model 1 (ATM):Beta1 — Basic Statistical Measures

Location		Variability	
Mean	-0.600	Std Deviation	0.663
Median	-0.353	Variance	0.439
		Range	4.375
		Interquartile Range	0.644

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic	p Value	
Kolmogorov-Smirnov D	0.182	Pr > D	<0.010

Model 2 (Sigma0):Beta1 — Basic Statistical Measures

Location		Variability	
Mean	-0.571	Std Deviation	0.839
Median	-0.315	Variance	0.703
		Range	9.119
		Interquartile Range	0.808

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic	p Value	
Kolmogorov-Smirnov D	0.150	Pr > D	<0.010

Model 1 (ATM): Beta2 — Basic Statistical Measures

Location		Variability	
Mean	0.016	Std Deviation	0.018
Median	0.009	Variance	0.000
		Range	0.121
		Interquartile Range	0.016

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic	p Value	
Kolmogorov-Smirnov D	0.182	Pr > D	<0.010

Model 2 (Sigma0):Beta2 — Basic Statistical Measures

Location		Variability	
Mean	0.015	Std Deviation	0.023
Median	0.008	Variance	0.001
		Range	0.257
		Interquartile Range	0.020

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic	p Value	
Kolmogorov-Smirnov D	0.165	Pr > D	<0.010

Model 1 (ATM):Gamma — Basic Statistical Measures

Location		Variability	
Mean	-0.037	Std Deviation	0.068
Median	-0.028	Variance	0.005
		Range	0.689
		Interquartile Range	0.077

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic	p Value	
Kolmogorov-Smirnov D	0.074	Pr > D	<0.010

Model 2 (Sigma0):Gamma — Basic Statistical Measures

Location		Variability	
Mean	1.117	Std Deviation	0.590
Median	0.989	Variance	0.348
		Range	3.483
		Interquartile Range	0.710

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic	p Value	
Kolmogorov-Smirnov D	0.042	Pr > D	<0.010

Model 2 (Sigma0):Sigma0 — Basic Statistical Measures

Location		Variability	
Mean	1.031E+24	Std Deviation	3.649E+25
Median	3.222	Variance	1.331E+51
	0.000	Range	1.291E+27
		Interquartile Range	419.3

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic	p Value	
Kolmogorov-Smirnov D	0.510	Pr > D	<0.010

Table VIII Deterministic volatility function parameters: univariate statistics for Natual Gas of daily estimated models from 1/1/1995-12/31/1995.

Model 1 (ATM):Beta1
Basic Statistical Measures

Location		Variability	
Mean	-0.483	Std Deviation	5.531
Median	-0.373	Variance	30.595
Mode		Range	189.467
		Interquartile Range	1.427

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic		p Value
Kolmogorov-Smirnov D	0.317	Pr > D	<0.010

Model 1 (ATM): Beta2
Basic Statistical Measures

Location		Variability	
Mean	0.126	Std Deviation	1.511
Median	0.094	Variance	2.284
		Range	52.860
		Interquartile Range	0.304

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic		p Value
Kolmogorov-Smirnov D	0.335	Pr > D	<0.010

Model 1 (ATM):Gamma
Basic Statistical Measures

Location		Variability	
Mean	-0.019	Std Deviation	0.099
Median	-0.015	Variance	0.010
		Range	2.687
		Interquartile Range	0.055

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic		p Value
Kolmogorov-Smirnov D	0.181	Pr > D	<0.010

Model 2 (Sigma0):Beta1
Basic Statistical Measures

Location		Variability	
Mean	-0.117	Std Deviation	7.840
Median	0.388	Variance	61.470
		Range	184.811
		Interquartile Range	2.285

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic		p Value
Kolmogorov-Smirnov D	0.263	Pr > D	<0.010

Model 2 (Sigma0):Beta2
Basic Statistical Measures

Location		Variability	
Mean	0.033	Std Deviation	1.658
Median	-0.015	Variance	2.750
		Range	30.858
		Interquartile Range	0.436

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic		p Value
Kolmogorov-Smirnov D	0.286	Pr > D	<0.010

Model 2 (Sigma0):Gamma
Basic Statistical Measures

Location		Variability	
Mean	1.677	Std Deviation	1.333
Median	1.648	Variance	1.778
		Range	15.576
		Interquartile Range	1.633

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic		p Value
Kolmogorov-Smirnov D	0.046	Pr > D	<0.010

Model 2 (Sigma0):Sigma0
Basic Statistical Measures

Location		Variability	
Mean	6.977E+62	Std Deviation	2.451E+64
Median	0.091	Variance	6.007E+128
		Range	8.609E+65
		Interquartile Range	0.3

Goodness-of-Fit Tests for Normal Distribution

Test	Statistic		p Value
Kolmogorov-Smirnov D	0.511	Pr > D	<0.010

Table IX. Univariate statistics of the number of observations for each cross section DVF estimation.

Panel A: Crude Oil											
YEAR	N	MEAN	STD	MIN	MAX	YEAR	N	MEAN	STD	MIN	MAX
Model 1: ATM						**Model 2: SIGMA0**					
ALL	1252	63	23	20	149	ALL	1252	63	23	20	149
1995	250	41	9	20	65	1995	250	41	9	20	65
1996	251	60	17	23	121	1996	251	60	17	23	121
1997	252	57	14	25	100	1997	252	57	14	25	100
1998	251	73	21	36	142	1998	251	73	21	36	142
1999	248	86	22	42	149	1999	248	86	22	42	149
Panel B: Natural Gas											
YEAR	N	MEAN	STD	MIN	MAX	YEAR	N	MEAN	STD	MIN	MAX
Model 1: ATM						**Model 2: SIGMA0**					
ALL	1234	43	21	4	119	ALL	1234	43	21	4	119
1995	244	21	10	4	80	1995	244	21	10	4	80
1996	251	37	17	9	108	1996	251	37	17	9	108
1997	248	48	22	7	119	1997	248	48	22	7	119
1998	243	51	16	13	106	1998	243	51	16	13	106
1999	248	55	18	16	118	1999	248	55	18	16	118

Table X. Natural gas options statistics of model pricing errors for all options.

Call-Put Flag	Moneyness	Traded Options Only	Data	COPM: ATM	CS	Model 1: ATM	Model 2: Sigma0
All Options	All Options	All Options	Average	0.002	-0.645	0.060	0.090
			Standard Deviation	0.013	0.213	0.056	0.077
			Minimum	-2.023	-1.964	-2.031	-2.031
			Maximum	0.383	2.674	3.958	3.958
			NOBS	379919	379919	379919	379919
		Options Not Traded	Average	0.002	-0.635	0.061	0.090
			Standard Deviation	0.013	0.201	0.058	0.078
			Minimum	-2.023	-1.964	-2.031	-2.031
			Maximum	0.383	2.674	3.958	3.958
			NOBS	305441	305441	305441	305441
		Options Traded Only	Average	0.002	-0.686	0.057	0.092
			Standard Deviation	0.012	0.255	0.046	0.073
			Minimum	-0.435	-1.386	-0.355	-0.356
			Maximum	0.294	1.960	1.492	1.492
			NOBS	74478	74478	74478	74478
	ITM	All Options	Average	0.001	-0.660	0.059	0.094
			Standard Deviation	0.016	0.207	0.076	0.093
			Minimum	-2.023	-1.964	-2.031	-2.031
			Maximum	0.383	1.408	3.958	3.958
			NOBS	126516	126516	126516	126516
		Options Not Traded	Average	0.001	-0.653	0.059	0.093
			Standard Deviation	0.016	0.199	0.078	0.093
			Minimum	-2.023	-1.964	-2.031	-2.031
			Maximum	0.383	1.408	3.958	3.958
			NOBS	109847	109847	109847	109847
		Options Traded Only	Average	0.000	-0.704	0.059	0.105
			Standard Deviation	0.016	0.246	0.066	0.090
			Minimum	-0.435	-1.386	-0.355	-0.356
			Maximum	0.294	0.773	1.492	1.492
			NOBS	16669	16669	16669	16669
	OTM	All Options	Average	0.002	-0.638	0.061	0.088
			Standard Deviation	0.011	0.216	0.043	0.067
			Minimum	-0.250	-1.310	-0.225	-0.251
			Maximum	0.273	2.674	0.462	0.572
			NOBS	253403	253403	253403	253403
		Options Not Traded	Average	0.002	-0.626	0.062	0.088
			Standard Deviation	0.011	0.201	0.044	0.067
			Minimum	-0.250	-1.285	-0.225	-0.251
			Maximum	0.273	2.674	0.451	0.572
			NOBS	195594	195594	195594	195594
		Options Traded Only	Average	0.002	-0.681	0.057	0.088
			Standard Deviation	0.011	0.258	0.039	0.067
			Minimum	-0.249	-1.310	-0.116	-0.121
			Maximum	0.157	1.960	0.462	0.504
			NOBS	57809	57809	57809	57809

Pricing errors are the actual prices less the model's predicted prices where negative (positive) average pricing errors indicate the model over (under) estimates an option's value. Options that are classified as not traded have no transaction volume for the day the price is recorded.

Table XI. Natural gas options statistics of model pricing errors for call options only.

Call-Put Flag	Moneyness	Traded Options Only	Data	COPM: ATM	CS	Model 1: ATM	Model 2: Sigma0
Calls	All Options	All Options	Average	0.003	-0.656	0.048	0.081
			Standard Deviation	0.014	0.231	0.052	0.074
			Minimum	-2.023	-1.964	-2.031	-2.031
			Maximum	0.383	2.674	2.092	2.092
			NOBS	204092	204092	204092	204092
		Options Not Traded	Average	0.003	-0.647	0.046	0.078
			Standard Deviation	0.014	0.215	0.054	0.075
			Minimum	-2.023	-1.964	-2.031	-2.031
			Maximum	0.383	2.674	2.092	2.092
			NOBS	160417	160417	160417	160417
		Options Traded Only	Average	0.003	-0.689	0.053	0.092
			Standard Deviation	0.013	0.278	0.043	0.073
			Minimum	-0.249	-1.329	-0.173	-0.173
			Maximum	0.294	1.960	1.492	1.492
			NOBS	43675	43675	43675	43675
	ITM	All Options	Average	0.000	-0.654	0.039	0.070
			Standard Deviation	0.017	0.207	0.071	0.086
			Minimum	-2.023	-1.964	-2.031	-2.031
			Maximum	0.383	0.556	2.092	2.092
			NOBS	72404	72404	72404	72404
		Options Not Traded	Average	0.000	-0.647	0.037	0.065
			Standard Deviation	0.018	0.199	0.072	0.084
			Minimum	-2.023	-1.964	-2.031	-2.031
			Maximum	0.383	0.548	2.092	2.092
			NOBS	62760	62760	62760	62760
		Options Traded Only	Average	0.000	-0.697	0.052	0.097
			Standard Deviation	0.012	0.249	0.066	0.089
			Minimum	-0.165	-1.329	-0.173	-0.173
			Maximum	0.294	0.556	1.492	1.492
			NOBS	9644	9644	9644	9644
	OTM	All Options	Average	0.005	-0.657	0.053	0.087
			Standard Deviation	0.012	0.242	0.036	0.067
			Minimum	-0.250	-1.310	-0.225	-0.251
			Maximum	0.273	2.674	0.358	0.572
			NOBS	131688	131688	131688	131688
		Options Not Traded	Average	0.005	-0.646	0.052	0.086
			Standard Deviation	0.011	0.224	0.037	0.067
			Minimum	-0.250	-1.285	-0.225	-0.251
			Maximum	0.273	2.674	0.343	0.572
			NOBS	97657	97657	97657	97657
		Options Traded Only	Average	0.004	-0.687	0.053	0.090
			Standard Deviation	0.013	0.286	0.035	0.067
			Minimum	-0.249	-1.310	-0.116	-0.121
			Maximum	0.157	1.960	0.358	0.504
			NOBS	34031	34031	34031	34031

Pricing errors are the actual prices less the model's predicted prices where negative (positive) average pricing errors indicate the model over (under) estimates an option's value. Options that are classified as not traded have no transaction volume for the day the price is recorded.

Table XII. Natural gas options statistics of model pricing errors for put options.

Call-Put Flag	Moneyness	Traded Options Only	Data	COPM: ATM	CS	Model 1: ATM	Model 2: Sigma0
Puts	All Options	All Options	Average	0.000	-0.633	0.075	0.101
			Standard Deviation	0.011	0.191	0.058	0.078
			Minimum	-0.435	-1.386	-0.355	-0.356
			Maximum	0.183	1.408	3.958	3.958
			NOBS	175827	175827	175827	175827
		Options Not Traded	Average	0.000	-0.623	0.078	0.103
			Standard Deviation	0.011	0.183	0.059	0.079
			Minimum	-0.344	-1.347	-0.334	-0.336
			Maximum	0.183	1.408	3.958	3.958
			NOBS	145024	145024	145024	145024
		Options Traded Only	Average	-0.001	-0.682	0.063	0.092
			Standard Deviation	0.011	0.218	0.049	0.073
			Minimum	-0.435	-1.386	-0.355	-0.356
			Maximum	0.138	0.773	0.929	0.929
			NOBS	30803	30803	30803	30803
	ITM	All Options	Average	0.002	-0.667	0.087	0.127
			Standard Deviation	0.014	0.206	0.075	0.092
			Minimum	-0.435	-1.386	-0.355	-0.356
			Maximum	0.162	1.408	3.958	3.958
			NOBS	54112	54112	54112	54112
		Options Not Traded	Average	0.002	-0.660	0.089	0.129
			Standard Deviation	0.013	0.199	0.076	0.093
			Minimum	-0.344	-1.347	-0.334	-0.336
			Maximum	0.162	1.408	3.958	3.958
			NOBS	47087	47087	47087	47087
		Options Traded Only	Average	-0.001	-0.714	0.069	0.115
			Standard Deviation	0.019	0.242	0.066	0.089
			Minimum	-0.435	-1.386	-0.355	-0.356
			Maximum	0.092	0.773	0.929	0.929
			NOBS	7025	7025	7025	7025
	OTM	All Options	Average	-0.001	-0.618	0.070	0.089
			Standard Deviation	0.008	0.182	0.047	0.068
			Minimum	-0.107	-1.298	0.002	-0.055
			Maximum	0.183	0.416	0.462	0.439
			NOBS	121715	121715	121715	121715
		Options Not Traded	Average	-0.001	-0.605	0.072	0.090
			Standard Deviation	0.009	0.172	0.048	0.068
			Minimum	-0.099	-1.209	0.002	-0.055
			Maximum	0.183	0.322	0.451	0.439
			NOBS	97937	97937	97937	97937
		Options Traded Only	Average	0.000	-0.673	0.062	0.085
			Standard Deviation	0.006	0.210	0.043	0.067
			Minimum	-0.107	-1.298	0.002	-0.042
			Maximum	0.138	0.416	0.462	0.427
			NOBS	23778	23778	23778	23778

Pricing errors are the actual prices less the model's predicted prices where negative (positive) average pricing errors indicate the model over (under) estimates an option's value. Options that are classified as not traded have no transaction volume for the day the price is recorded.

Table XIII. Crude oil options statistics of model pricing errors for all options.

Call-Put Flag	Moneyness	Traded Options Only	Data	COPM: ATM	CS	Model 1: ATM	Model 2: Sigma0
All Options	All Options	All Options	Average	0.001	-5.539	0.201	0.364
			Standard Deviation	0.337	1.332	4.611	0.605
			Minimum	-9.550	-13.435	-800.184	-9.714
			Maximum	7.630	17.422	12.700	12.700
			NOBS	322734	322734	322734	322734
		Options Traded Only	Average	0.018	-6.120	0.252	0.394
			Standard Deviation	0.154	1.371	0.699	0.378
			Minimum	-5.910	-12.041	-187.718	-6.140
			Maximum	7.483	13.208	7.990	7.990
			NOBS	97036	97036	97036	97036
		Options Not Traded	Average	-0.007	-5.289	0.179	0.351
			Standard Deviation	0.389	1.233	5.494	0.679
			Minimum	-9.550	-13.435	-800.184	-9.714
			Maximum	7.630	17.422	12.700	12.700
			NOBS	225698	225698	225698	225698
	ITM	All Options	Average	-0.055	-5.671	0.185	0.346
			Standard Deviation	0.511	1.370	3.958	0.845
			Minimum	-9.550	-13.435	-800.184	-9.714
			Maximum	1.948	5.075	12.700	12.700
			NOBS	117803	117803	117803	117803
		Options Traded Only	Average	-0.010	-6.345	0.229	0.400
			Standard Deviation	0.213	1.378	0.462	0.489
			Minimum	-5.910	-12.041	-26.002	-6.140
			Maximum	1.948	2.299	7.400	7.400
			NOBS	25597	25597	25597	25597
		Options Not Traded	Average	-0.068	-5.484	0.173	0.330
			Standard Deviation	0.565	1.308	4.467	0.919
			Minimum	-9.550	-13.435	-800.184	-9.714
			Maximum	1.240	5.075	12.700	12.700
			NOBS	92206	92206	92206	92206
	OTM	All Options	Average	0.033	-5.463	0.210	0.375
			Standard Deviation	0.160	1.303	4.947	0.406
			Minimum	-0.900	-10.871	-790.617	-2.403
			Maximum	7.630	17.422	8.010	8.010
			NOBS	204931	204931	204931	204931
		Options Traded Only	Average	0.027	-6.040	0.261	0.392
			Standard Deviation	0.125	1.359	0.766	0.330
			Minimum	-0.328	-10.871	-187.718	-1.979
			Maximum	7.483	13.208	7.990	7.990
			NOBS	71439	71439	71439	71439
		Options Not Traded	Average	0.036	-5.154	0.184	0.366
			Standard Deviation	0.176	1.160	6.104	0.441
			Minimum	-0.900	-10.771	-790.617	-2.403
			Maximum	7.630	17.422	8.010	8.010
			NOBS	133492	133492	133492	133492

Pricing errors are the actual prices less the model's predicted prices where negative (positive) average pricing errors indicate the model over (under) estimates an option's value. Options that are classified as not traded have no transaction volume for the day the price is recorded.

Table XIV. Crude oil options statistics of model pricing errors for call options.

Call-Put Flag	Moneyness	Traded Options Only	Data	COPM: ATM	CS	Model 1: ATM	Model 2: Sigma0
Calls	All Options	All Options	Average	-0.011	-5.597	0.005	0.205
			Standard Deviation	0.441	1.353	6.039	0.637
			Minimum	-9.550	-13.435	-800.184	-9.714
			Maximum	7.630	17.422	12.700	12.700
			NOBS	187104	187104	187104	187104
		Options Traded Only	Average	0.020	-6.176	0.194	0.349
			Standard Deviation	0.198	1.378	0.873	0.376
			Minimum	-5.910	-12.041	-187.718	-6.140
			Maximum	7.483	13.208	7.990	7.990
			NOBS	57984	57984	57984	57984
		Options Not Traded	Average	-0.024	-5.337	-0.080	0.141
			Standard Deviation	0.513	1.258	7.244	0.715
			Minimum	-9.550	-13.435	-800.184	-9.714
			Maximum	7.630	17.422	12.700	12.700
			NOBS	129120	129120	129120	129120
	ITM	All Options	Average	-0.119	-5.608	-0.188	-0.030
			Standard Deviation	0.690	1.432	5.362	0.886
			Minimum	-9.550	-13.435	-800.184	-9.714
			Maximum	1.948	1.020	12.700	12.700
			NOBS	63091	63091	63091	63091
		Options Traded Only	Average	-0.023	-6.318	0.104	0.250
			Standard Deviation	0.276	1.376	0.490	0.460
			Minimum	-5.910	-12.041	-26.002	-6.140
			Maximum	1.948	0.463	6.740	6.740
			NOBS	15087	15087	15087	15087
		Options Not Traded	Average	-0.149	-5.385	-0.280	-0.118
			Standard Deviation	0.774	1.375	6.139	0.965
			Minimum	-9.550	-13.435	-800.184	-9.714
			Maximum	1.040	1.020	12.700	12.700
			NOBS	48004	48004	48004	48004
	OTM	All Options	Average	0.045	-5.591	0.103	0.325
			Standard Deviation	0.204	1.312	6.353	0.414
			Minimum	-0.483	-10.871	-790.617	-2.403
			Maximum	7.630	17.422	8.010	8.010
			NOBS	124013	124013	124013	124013
		Options Traded Only	Average	0.035	-6.126	0.226	0.384
			Standard Deviation	0.159	1.376	0.971	0.334
			Minimum	-0.277	-10.871	-187.718	-1.979
			Maximum	7.483	13.208	7.990	7.990
			NOBS	42897	42897	42897	42897
		Options Not Traded	Average	0.050	-5.308	0.038	0.294
			Standard Deviation	0.223	1.183	7.823	0.447
			Minimum	-0.483	-10.771	-790.617	-2.403
			Maximum	7.630	17.422	8.010	8.010
			NOBS	81116	81116	81116	81116

Pricing errors are the actual prices less the model's predicted prices where negative (positive) average pricing errors indicate the model over (under) estimates an option's value. Options that are classified as not traded have no transaction volume for the day the price is recorded.

Table XV. Crude oil options statistics of model pricing errors for put options.

Call-Put Flag	Moneyness	Traded Options Only	Data	COPM: ATM	CS	Model 1: ATM	Model 2: Sigma0
Puts	All Options	All Options	Average	0.016	-5.459	0.472	0.583
			Standard Deviation	0.038	1.297	0.392	0.476
			Minimum	-4.090	-10.897	-3.180	-3.176
			Maximum	1.240	5.075	8.470	8.470
			NOBS	135630	135630	135630	135630
		Options Traded Only	Average	0.014	-6.038	0.339	0.460
			Standard Deviation	0.024	1.356	0.264	0.372
			Minimum	-0.610	-10.876	-0.080	-0.030
			Maximum	0.528	2.299	7.400	7.400
			NOBS	39052	39052	39052	39052
		Options Not Traded	Average	0.017	-5.225	0.526	0.633
			Standard Deviation	0.043	1.195	0.421	0.504
			Minimum	-4.090	-10.897	-3.180	-3.176
			Maximum	1.240	5.075	8.470	8.470
			NOBS	96578	96578	96578	96578
	ITM	All Options	Average	0.018	-5.743	0.616	0.778
			Standard Deviation	0.046	1.292	0.468	0.532
			Minimum	-4.090	-10.897	-3.180	-3.176
			Maximum	1.240	5.075	8.470	8.470
			NOBS	54712	54712	54712	54712
		Options Traded Only	Average	0.009	-6.384	0.408	0.615
			Standard Deviation	0.026	1.381	0.349	0.447
			Minimum	-0.610	-10.876	-0.080	-0.030
			Maximum	0.528	2.299	7.400	7.400
			NOBS	10510	10510	10510	10510
		Options Not Traded	Average	0.020	-5.591	0.666	0.817
			Standard Deviation	0.049	1.221	0.479	0.544
			Minimum	-4.090	-10.897	-3.180	-3.176
			Maximum	1.240	5.075	8.470	8.470
			NOBS	44202	44202	44202	44202
	OTM	All Options	Average	0.015	-5.267	0.375	0.452
			Standard Deviation	0.032	1.264	0.293	0.382
			Minimum	-0.900	-10.797	0.010	0.020
			Maximum	0.335	0.807	1.835	2.267
			NOBS	80918	80918	80918	80918
		Options Traded Only	Average	0.016	-5.910	0.313	0.403
			Standard Deviation	0.023	1.323	0.220	0.322
			Minimum	-0.328	-10.797	0.010	0.020
			Maximum	0.278	0.565	1.831	2.173
			NOBS	28542	28542	28542	28542
		Options Not Traded	Average	0.014	-4.916	0.409	0.478
			Standard Deviation	0.036	1.080	0.321	0.408
			Minimum	-0.900	-9.149	0.020	0.020
			Maximum	0.335	0.807	1.835	2.267
			NOBS	52376	52376	52376	52376

Pricing errors are the actual prices less the model's predicted prices where negative (positive) average pricing errors indicate the model over (under) estimates an option's value. Options that are classified as not traded have no transaction volume for the day the price is recorded.

Table XVI. Natural gas options pricing error nonparametric tests of alternative models verses benchmark.

		ALL					
		ALL		ITM		OTM	
Method	_NAME_	ALL	Traded Options Only	ALL	Traded Options Only	ALL	Traded Options Only
Wilcoxon Sign Rank Test	COPM vs Model 1: ATM	**-519.55**	**-235.74**	**-283.34**	**-110.59**	**-434.79**	**-208.14**
	COPM vs Model 2: Sigma0	**-523.14**	**-235.88**	**-291.46**	**-110.97**	**-434.44**	**-208.14**
	COPM vs CS	529.50	233.08	307.05	111.33	431.34	204.76
	CS vs Model 1: ATM	**-531.18**	**-234.54**	**-307.57**	**-111.60**	**-433.06**	**-206.28**
	CS vs Model 2: Sigma0	**-531.51**	**-234.84**	**-307.70**	**-111.71**	**-433.36**	**-206.55**
Squared Rank Test for Variance	COPM vs Model 1: ATM	**542.50**	**235.02**	**316.69**	**113.50**	**442.22**	**206.19**
	COPM vs Model 2: Sigma0	**622.82**	**276.75**	**365.41**	**132.77**	**503.93**	**241.97**
	COPM vs CS	**690.75**	**321.67**	**389.92**	**150.45**	**568.90**	**284.44**
	CS vs Model 1: ATM	-480.70	-253.55	-246.51	-113.11	-412.26	-226.71
	CS vs Model 2: Sigma0	-302.40	-174.05	-137.55	-72.74	-269.20	-159.36
NOBS	m	379919	74478	126516	16669	253403	57809
	n	379919	74478	126516	16669	253403	57809
		Calls					
		ALL		ITM		OTM	
Method	_NAME_	ALL	Traded Options Only	ALL	Traded Options Only	ALL	Traded Options Only
Wilcoxon Sign Rank Test	COPM vs Model 1: ATM	**-367.98**	**-180.13**	**-191.70**	**-83.16**	**-312.45**	**-159.65**
	COPM vs Model 2: Sigma0	**-375.02**	**-180.41**	**-206.19**	**-83.82**	**-312.17**	**-159.67**
	COPM vs CS	386.30	177.12	232.57	84.71	308.57	155.60
	CS vs Model 1: ATM	**-387.98**	**-178.74**	**-232.87**	**-84.93**	**-310.48**	**-157.30**
	CS vs Model 2: Sigma0	**-388.40**	**-179.11**	**-232.98**	**-85.04**	**-310.90**	**-157.66**
Squared Rank Test for Variance	COPM vs Model 1: ATM	**364.24**	**168.81**	**223.42**	**86.11**	**294.66**	**147.06**
	COPM vs Model 2: Sigma0	**449.31**	**209.33**	**272.56**	**102.55**	**361.24**	**183.06**
	COPM vs CS	**514.82**	**248.09**	**305.23**	**116.67**	**416.82**	**219.32**
	CS vs Model 1: ATM	-392.08	-204.14	-222.33	-92.02	-328.48	-182.33
	CS vs Model 2: Sigma0	-248.25	-140.05	-138.33	-59.86	-207.88	-127.04
NOBS	m	204092	43675	72404	9644	131688	34031
	n	204092	43675	72404	9644	131688	34031
		Puts					
		ALL		ITM		OTM	
Method	_NAME_	ALL	Traded Options Only	ALL	Traded Options Only	ALL	Traded Options Only
Wilcoxon Sign Rank Test	COPM vs Model 1: ATM	**-363.14**	**-152.00**	**-201.46**	**-72.59**	**-302.14**	**-133.54**
	COPM vs Model 2: Sigma0	**-363.06**	**-151.93**	**-201.43**	**-72.52**	**-302.06**	**-133.52**
	COPM vs CS	362.32	151.66	200.56	72.25	301.88	133.37
	CS vs Model 1: ATM	**-362.80**	**-151.87**	**-201.05**	**-72.42**	**-302.08**	**-133.50**
	CS vs Model 2: Sigma0	**-362.86**	**-151.91**	**-201.12**	**-72.46**	**-302.11**	**-133.53**
Squared Rank Test for Variance	COPM vs Model 1: ATM	**390.36**	**163.31**	**213.65**	**73.38**	**328.65**	**146.76**
	COPM vs Model 2: Sigma0	**432.76**	**181.80**	**236.11**	**84.11**	**356.27**	**160.18**
	COPM vs CS	**462.03**	**205.00**	**254.22**	**95.34**	**388.85**	**181.75**
	CS vs Model 1: ATM	-300.24	-151.71	-157.87	-68.26	-256.53	-134.91
	CS vs Model 2: Sigma0	-174.22	-102.78	-83.55	-43.14	-165.52	-95.42
NOBS	m	175827	30803	54112	7025	121715	23778
	n	175827	30803	54112	7025	121715	23778

Wilcoxon Sign Rank Test tests the null hypothesis that the benchmark model's (COPM or CS) pricing error is less than or equal to the alternative model's pricing error at the critical value (alpha=.05) of 1.64. Squared Rank Test for Equal Variances tests the null hypothesis that the benchmark model's (COPM or CS) variance of pricing errors and the alternative model's variance of pricing errors are identically distributed at the critical value (alpha=.05) of -1.64. Bold indicates failure to reject the null hypothesis at the 5% level

Table XVII. Crude oil options pricing error nonparametric tests of alternative models verses benchmark.

		ALL					
		ALL		ITM		OTM	
Method	_NAME_	ALL	Traded Options Only	ALL	Traded Options Only	ALL	Traded Options Only
Wilcoxon Sign Rank Test	COPM vs Model 1: ATM	**-386.7**	**-256.7**	**-180.8**	**-120.0**	**-358.9**	**-226.8**
	COPM vs Model 2: Sigma0	**-409.2**	**-260.6**	**-205.8**	**-126.0**	**-367.8**	**-228.2**
	COPM vs CS	491.9	269.7	297.2	138.6	391.9	231.4
	CS vs Model 1: ATM	**-491.2**	**-269.7**	**-297.0**	**-138.5**	**-391.3**	**-231.4**
	CS vs Model 2: Sigma0	**-491.9**	**-269.7**	**-297.2**	**-138.6**	**-391.9**	**-231.4**
Squared Rank Test for Variance	COPM vs Model 1: ATM	**443.9**	**267.5**	**242.4**	**128.3**	**374.2**	**234.0**
	COPM vs Model 2: Sigma0	**545.6**	**320.5**	**295.5**	**158.7**	**464.0**	**277.1**
	COPM vs CS	**660.9**	**385.9**	**367.5**	**194.8**	**546.6**	**333.1**
	CS vs Model 1: ATM	-476.1	-322.9	-232.8	-158.8	-427.0	-282.6
	CS vs Model 2: Sigma0	-391.1	-264.2	-186.5	-129.9	-347.9	-230.1
NOBS	m	322734.0	97036.0	117803.0	25597.0	204931.0	71439.0
	n	322734.0	97036.0	117803.0	25597.0	204931.0	71439.0
		Calls					
		ALL		ITM		OTM	
Method	_NAME_	ALL	Traded Options Only	ALL	Traded Options Only	ALL	Traded Options Only
Wilcoxon Sign Rank Test	COPM vs Model 1: ATM	**-211.9**	**-189.9**	**-2.9**	**-75.1**	**-259.0**	**-173.0**
	COPM vs Model 2: Sigma0	**-249.5**	**-195.9**	**-51.4**	**-85.7**	**-272.2**	**-175.1**
	COPM vs CS	374.5	208.5	217.5	106.4	304.8	179.3
	CS vs Model 1: ATM	**-373.6**	**-208.4**	**-217.2**	**-106.3**	**-304.0**	**-179.2**
	CS vs Model 2: Sigma0	**-374.5**	**-208.5**	**-217.5**	**-106.4**	**-304.8**	**-179.3**
Squared Rank Test for Variance	COPM vs Model 1: ATM	**319.3**	**188.1**	**128.1**	**83.0**	**260.0**	**164.4**
	COPM vs Model 2: Sigma0	**348.3**	**241.2**	**108.7**	**112.4**	**343.6**	**210.9**
	COPM vs CS	**498.9**	**298.2**	**256.1**	**149.8**	**424.0**	**257.8**
	CS vs Model 1: ATM	-370.3	-258.2	-189.7	-129.3	-337.8	-225.7
	CS vs Model 2: Sigma0	-336.4	-208.6	-183.0	-107.1	-280.7	-178.8
NOBS	m	187104.0	57984.0	63091.0	15087.0	124013.0	42897.0
	n	187104.0	57984.0	63091.0	15087.0	124013.0	42897.0
		Puts					
		ALL		ITM		OTM	
Method	_NAME_	ALL	Traded Options Only	ALL	Traded Options Only	ALL	Traded Options Only
Wilcoxon Sign Rank Test	COPM vs Model 1: ATM	**-318.9**	**-171.1**	**-202.6**	**-88.8**	**-246.4**	**-146.3**
	COPM vs Model 2: Sigma0	**-318.9**	**-171.1**	**-202.6**	**-88.8**	**-246.4**	**-146.3**
	COPM vs CS	318.9	171.1	202.6	88.8	246.4	146.3
	CS vs Model 1: ATM	**-318.9**	**-171.1**	**-202.6**	**-88.8**	**-246.4**	**-146.3**
	CS vs Model 2: Sigma0	**-318.9**	**-171.1**	**-202.6**	**-88.8**	**-246.4**	**-146.3**
Squared Rank Test for Variance	COPM vs Model 1: ATM	**355.6**	**191.2**	**227.0**	**99.0**	**277.6**	**165.4**
	COPM vs Model 2: Sigma0	**388.2**	**211.6**	**241.4**	**108.7**	**297.8**	**179.3**
	COPM vs CS	**428.2**	**245.3**	**266.6**	**126.5**	**341.5**	**210.8**
	CS vs Model 1: ATM	-297.0	-200.1	-167.6	-100.9	-254.9	-172.6
	CS vs Model 2: Sigma0	-229.4	-162.3	-132.2	-82.0	-210.5	-144.8
NOBS	m	135630.0	39052.0	54712.0	10510.0	80918.0	28542.0
	n	135630.0	39052.0	54712.0	10510.0	80918.0	28542.0

Wilcoxon Sign Rank Test tests the null hypothesis that the benchmark model's (COPM or CS) pricing error is less than or equal to the alternative model's pricing error at the critical value (alpha=.05) of 1.64. Squared Rank Test for Equal Variances tests the null hypothesis that the benchmark model's (COPM or CS) variance of pricing errors and the alternative model's variance of pricing errors are identically distributed at the critical value (alpha=.05) of -1.64. Bold indicates failure to reject the null hypothesis at the 5% level

Table XVIII. Squared rank test for equal variance for unhedged portfolio, COPM, Clewlow and Strickland model, Model 1: ATM, and Model 2: Sigma0.

Commodity	Case	Model	Forecast Error Model	Statistic Value
Crude Oil	Consumer	Model 1: ATM	Unhedged-COPM	16.754
			Unhedged-CS	5.443
			Unhedged-IVT	21.167
			COPM-IVT	18.286
			COPM-CS	**-14.808**
			CS-IVT	20.831
			NOBS	245
		Model 2: Sigma0	Unhedged-COPM	7.083
			Unhedged-CS	7.069
			Unhedged-IVT	19.251
			COPM-IVT	17.853
			COPM-CS	-0.045
			CS-IVT	17.879
			NOBS	211
	Producer	Model 1: ATM	Unhedged-COPM	2.500
			Unhedged-CS	7.736
			Unhedged-IVT	18.270
			COPM-IVT	17.828
			COPM-CS	5.667
			CS-IVT	16.382
			NOBS	192
		Model 2: Sigma0	Unhedged-COPM	0.027
			Unhedged-CS	6.928
			Unhedged-IVT	15.551
			COPM-IVT	15.352
			COPM-CS	6.607
			CS-IVT	13.715
			NOBS	139
Natural Gas	Consumer	Model 1: ATM	Unhedged-COPM	22.444
			Unhedged-CS	10.504
			Unhedged-IVT	20.803
			COPM-IVT	**-21.295**
			COPM-CS	**-22.221**
			CS-IVT	19.252
			NOBS	255
		Model 2: Sigma0	Unhedged-COPM	14.719
			Unhedged-CS	9.266
			Unhedged-IVT	21.528
			COPM-IVT	18.521
			COPM-CS	**-9.431**
			CS-IVT	20.683
			NOBS	253
	Producer	Model 1: ATM	Unhedged-COPM	17.130
			Unhedged-CS	17.921
			Unhedged-IVT	21.651
			COPM-IVT	20.751
			COPM-CS	15.196
			CS-IVT	**-8.102**
			NOBS	256
		Model 2: Sigma0	Unhedged-COPM	12.271
			Unhedged-CS	21.407
			Unhedged-IVT	17.613
			COPM-IVT	16.090
			COPM-CS	20.804
			CS-IVT	**-5.362**
			NOBS	253

Squared Rank Test for Equal Variances tests the null hypothesis that the benchmark model's (COPM or CS) variance of hedging errors and the alternative model's variance of hedging errors are identically distributed at the critical value (alpha=.05) of -1.64. Bold indicates failure to reject the null hypothesis at the 5% level

Table XIX. Modified Diebold and Mariano statistics for hedging performance of the COPM, Clewlow and Strickland model, Model 1: ATM, and Model 2: Sigma0.

Commodity	Case	Model	Forecast Error Model	MDM Statistic
Crude Oil	Consumer	Model 1: ATM	COPM-CS	15.340
			COPM-Model 1: ATM	**-5.677**
			CS-Model 1: ATM	**-5.678**
		Model 2: Sigma0	COPM-CS	29.003
			COPM-Model 2: Sigma0	**-9.268**
			CS-Model 2: Sigma0	**-9.268**
	Producer	Model 1: ATM	COPM-CS	**-1.770**
			COPM-Model 1: ATM	**-4.773**
			CS-Model 1: ATM	**-4.773**
		Model 2: Sigma0	COPM-CS	**-6.470**
			COPM-Model 2: Sigma0	**-5.528**
			CS-Model 2: Sigma0	**-5.528**
Natural Gas	Consumer	Model 1: ATM	COPM-CS	0.978
			COPM-Model 1: ATM	0.978
			CS-Model 1: ATM	**-13.654**
		Model 2: Sigma0	COPM-CS	18.295
			COPM-Model 2: Sigma0	**-12.473**
			CS-Model 2: Sigma0	**-12.473**
	Producer	Model 1: ATM	COPM-CS	-0.978
			COPM-Model 1: ATM	**-7.760**
			CS-Model 1: ATM	0.968
		Model 2: Sigma0	COPM-CS	-0.979
			COPM-Model 2: Sigma0	**-9.165**
			CS-Model 2: Sigma0	0.965

Modified Diebold and Mariano test has the null hypothesis that the differences of the squared hedging errors of benchmark (COPM or CS) and the alternative models are equal to zero at the critical value (alpha=.05) of -1.651. Bold indicates failure to reject the null hypothesis at the 5% level

Table XX. Parameter correlations for Models 1 and 2.

Model 1: ATM								
	Commodity: Crude Oil				Commodity: Natural Gas			
Parameter	Beta1	Beta2	Gamma		Beta1	Beta2	Gamma	
Beta1	1.000	-0.996	-0.082		1.000	-0.997	0.017	
Beta2	-0.996	1.000	0.128		-0.997	1.000	-0.015	
Gamma	-0.082	0.128	1.000		0.017	-0.015	1.000	
Model 2: Sigma0								
Parameter	Beta1	Beta2	Gamma	Sigma0	Beta1	Beta2	Gamma	Sigma0
Beta1	1.000	-0.997	-0.298	-0.224	1.000	-0.996	-0.139	-0.605
Beta2	-0.997	1.000	0.294	0.228	-0.996	1.000	0.147	0.604
Gamma	-0.298	0.294	1.000	0.001	-0.139	0.147	1.000	0.051
Sigma0	-0.224	0.228	0.001	1.000	-0.605	0.604	0.051	1.000

Table XXI. Tests for contemporaneous correlation for crude oil and natural gas models 1 and 2.

Panel A: Crude Oil				
	n	v	Statistic	Pr > ChiSq
Model 1: ATM	1231	3	**1310.748**	1.000
Model 2:Sigma0	1231	3	**1251.056**	1.000
Covariance of Residuals				
	Beta1	Beta2	Gamma	
Model 1: ATM				
Beta1	0.231	-0.006	-0.005	
Beta2	-0.006	0.000	0.000	
Gamma	-0.005	0.000	0.003	
Model 2:Sigma0				
Beta1	0.464	-0.013	-0.031	
Beta2	-0.013	0.000	0.001	
Gamma	-0.031	0.001	0.188	
Panel B: Natural Gas				
	n	v	Statistic	Pr > ChiSq
Model 1: ATM	1222	3	**1216.973**	1.000
Model 2:Sigma0	1220	3	**1284.906**	1.000
Covariance of Residuals				
	Beta1	Beta2	Gamma	
Model 1: ATM				
Beta1	30.390	-8.292	0.013	
Beta2	-8.292	2.274	-0.003	
Gamma	0.013	-0.003	0.010	
Model 2:Sigma0				
Beta1	57.797	-15.514	-1.308	
Beta2	-15.514	4.196	0.344	
Gamma	-1.308	0.344	0.951	
Bold indicates significant at the 5% level.				

Table XXII. Durbin-Watson statistics for residual autocorrelation for crude oil and natural gas models 1 and 2.

Equation	Order	DW	Pr < DW	Pr > DW
Panel A: Crude Oil				
Model 1: ATM				
Beta1	1	1.962	0.188	0.812
Beta2	1	1.946	0.122	0.878
Gamma	1	1.925	0.062	0.938
Model 2:Sigma0				
Beta1	1	2.012	0.495	0.505
Beta2	1	2.008	0.468	0.532
Gamma	1	1.924	0.061	0.939
Panel B: Natural Gas				
Model 1: ATM				
Beta1	1	1.947	0.130	0.870
Beta2	1	1.956	0.168	0.832
Gamma	1	2.009	0.483	0.517
Model 2:Sigma0				
Beta1	1	1.797	0.000	1.000
Beta2	1	1.802	0.000	1.000
Gamma	1	1.923	0.063	0.937

Table XXIII. SUR parameter estimates for crude oil models 1 and 2.

Parameter	Model 1: ATM Estimate	Model 1: ATM Std Err	Model 1: ATM t Value	Model 1: ATM Pr > |t|	Model 2:Sigma0 Estimate	Model 2:Sigma0 Std Err	Model 2:Sigma0 t Value	Model 2:Sigma0 Pr > |t|
Beta1								
Ln(Total Domestic)	0.819	0.615	1.333	0.183	**1.680**	0.790	2.128	**0.034**
Ln(Total Imports)	**0.800**	0.223	3.586	**0.000**	**1.174**	0.299	3.928	**0.000**
ShortTermMA/LongTermMA	-0.260	0.596	-0.437	0.662	-0.499	0.767	-0.651	0.515
HedgeLong%	**8.876**	1.430	6.209	**0.000**	**10.856**	1.859	5.839	**0.000**
HedgeShort%	**-2.274**	1.279	-1.778	**0.076**	**-3.637**	1.666	-2.183	**0.029**
Spread%	**7.346**	1.617	4.542	**0.000**	**5.742**	2.077	2.764	**0.006**
Spring	**-23.488**	7.581	-3.098	**0.002**	**-37.800**	9.738	-3.882	**0.000**
Summer	**-23.643**	7.582	-3.118	**0.002**	**-38.098**	9.740	-3.912	**0.000**
Fall	**-23.453**	7.563	-3.101	**0.002**	**-37.799**	9.716	-3.891	**0.000**
Winter	**-23.495**	7.566	-3.105	**0.002**	**-37.931**	9.719	-3.903	**0.000**
MA(e_{-1}) Beta1	**-0.464**	0.019	-24.538	**0.000**	**-0.338**	0.020	-17.234	**0.000**
MA(e_{-2}) Beta1	**-0.289**	0.019	-15.269	**0.000**	**-0.237**	0.020	-12.092	**0.000**
Beta2								
Ln(Total Domestic)	-0.006	0.017	-0.328	0.743	-0.028	0.022	-1.305	0.192
Ln(Total Imports)	**-0.022**	0.006	-3.555	**0.000**	**-0.033**	0.008	-3.989	**0.000**
ShortTermMA/LongTermMA	0.002	0.016	0.150	0.880	0.013	0.021	0.616	0.538
HedgeLong%	**-0.230**	0.039	-5.826	**0.000**	**-0.282**	0.051	-5.523	**0.000**
HedgeShort%	0.052	0.035	1.476	0.140	**0.081**	0.046	1.782	**0.075**
Spread%	**-0.209**	0.045	-4.669	**0.000**	**-0.166**	0.057	-2.910	**0.004**
Spring	**0.432**	0.209	2.062	**0.039**	**0.818**	0.267	3.063	**0.002**
Summer	**0.436**	0.209	2.081	**0.038**	**0.826**	0.267	3.093	**0.002**
Fall	**0.431**	0.209	2.066	**0.039**	**0.819**	0.267	3.073	**0.002**
Winter	**0.432**	0.209	2.069	**0.039**	**0.823**	0.267	3.085	**0.002**
MA(e_{-1}) Beta2	**-0.466**	0.019	-24.679	**0.000**	**-0.334**	0.020	-17.051	**0.000**
MA(e_{-2}) Beta2	**-0.290**	0.019	-15.359	**0.000**	**-0.235**	0.020	-11.992	**0.000**
Gamma								
Ln(Total Domestic)	**0.530**	0.072	7.390	**0.000**	-0.361	0.666	-0.541	0.588
Ln(Total Imports)	0.018	0.025	0.712	0.476	**-0.548**	0.219	-2.496	**0.013**
ShortTermMA/LongTermMA	-0.013	0.069	-0.182	0.856	**1.879**	0.642	2.928	**0.003**
HedgeLong%	0.231	0.164	1.406	0.160	**-3.927**	1.484	-2.646	**0.008**
HedgeShort%	**-0.452**	0.147	-3.077	**0.002**	**2.940**	1.321	2.226	**0.026**
Spread%	0.271	0.189	1.437	0.151	1.895	1.756	1.079	0.281
Spring	**-6.736**	0.885	-7.610	**0.000**	9.248	8.228	1.124	0.261
Summer	**-6.749**	0.885	-7.625	**0.000**	9.510	8.229	1.156	0.248
Fall	**-6.721**	0.883	-7.612	**0.000**	9.367	8.209	1.141	0.254
Winter	**-6.736**	0.883	-7.625	**0.000**	9.335	8.212	1.137	0.256
MA(e_{-1}) Gamma	**-0.295**	0.027	-11.055	**0.000**	**-0.407**	0.028	-14.542	**0.000**
MA(e_{-2}) Gamma	**-0.207**	0.028	-7.524	**0.000**	**-0.347**	0.030	-11.704	**0.000**
MA(e_{-3}) Gamma	**-0.161**	0.027	-5.863	**0.000**	**-0.246**	0.030	-8.300	**0.000**
MA(e_{-4}) Gamma	**-0.115**	0.027	-4.327	**0.000**	**-0.170**	0.028	-6.047	**0.000**

Bold indicates significance at the 10% level

Table XXIV. SUR parameter estimates for natural gas models 1 and 2.

	Model 1: ATM				Model 2:Sigma0			
Parameter	Estimate	Std Err	t Value	Pr > |t|	Estimate	Std Err	t Value	Pr > |t|
Beta1								
ΔAGAProd%	**3.919**	1.935	2.026	**0.043**	**-7.008**	3.324	-2.109	**0.035**
ΔAGAEast%	-1.276	1.532	-0.833	0.405	**5.856**	2.632	2.225	**0.026**
ShortTermMA/LongTermMA	-0.021	1.582	-0.013	0.990	1.768	2.703	0.654	0.513
HedgeLong%	-6.169	6.978	-0.884	0.377	-1.730	11.873	-0.146	0.884
HedgeShort%	14.355	11.703	1.227	0.220	-1.967	19.526	-0.101	0.920
Spread%	-9.054	6.446	-1.405	0.160	15.910	11.029	1.443	0.149
Injection: April-October	-9.424	7.287	-1.293	0.196	0.574	12.160	0.047	0.962
Withdrawal: November-March	-8.802	7.275	-1.210	0.227	1.345	12.138	0.111	0.912
MA(e_{-1}) Beta1					**-0.152**	0.020	-7.578	**0.000**
MA(e_{-2}) Beta1					**-0.105**	0.020	-5.243	**0.000**
Beta2								
ΔAGAProd%	-0.778	0.529	-1.469	0.142	**1.591**	0.905	1.757	**0.079**
ΔAGAEast%	0.224	0.419	0.534	0.594	**-1.192**	0.717	-1.662	**0.097**
ShortTermMA/LongTermMA	-0.052	0.433	-0.119	0.905	-0.365	0.736	-0.496	0.620
HedgeLong%	1.903	1.909	0.997	0.319	0.321	3.233	0.099	0.921
HedgeShort%	-4.354	3.202	-1.360	0.174	0.125	5.313	0.024	0.981
Spread%	2.015	1.764	1.143	0.253	-3.653	3.004	-1.216	0.224
Injection: April-October	2.743	1.993	1.376	0.169	0.093	3.309	0.028	0.978
Withdrawal: November-March	2.574	1.990	1.293	0.196	-0.012	3.303	-0.003	0.997
MA(e_{-1}) Beta2					**-0.164**	0.020	-8.129	**0.000**
MA(e_{-2}) Beta2					**-0.107**	0.020	-5.354	**0.000**
Gamma								
ΔAGAProd%	-0.032	0.038	-0.820	0.413	0.617	0.627	0.983	0.326
ΔAGAEast%	0.000	0.030	0.013	0.990	**-1.082**	0.497	-2.178	**0.030**
ShortTermMA/LongTermMA	**-0.067**	0.031	-2.124	**0.034**	**-1.975**	0.504	-3.919	**0.000**
HedgeLong%	**-0.227**	0.138	-1.643	**0.101**	**-10.990**	2.182	-5.037	**0.000**
HedgeShort%	0.079	0.230	0.344	0.731	4.053	3.408	1.189	0.235
Spread%	-0.016	0.128	-0.126	0.900	**4.450**	2.060	2.160	**0.031**
Injection: April-October	0.167	0.143	1.166	0.244	**7.889**	2.129	3.706	**0.000**
Withdrawal: November-March	0.173	0.143	1.208	0.227	**8.942**	2.124	4.209	**0.000**
MA(e_{-1}) Gamma	**-0.116**	0.028	-4.077	**0.000**	**-0.152**	0.020	-7.578	**0.000**
MA(e_{-2}) Gamma					**-0.376**	0.028	-13.626	**0.000**
MA(e_{-3}) Gamma					**-0.288**	0.028	-10.120	**0.000**
MA(e_{-4}) Gamma					**-0.195**	0.028	-7.032	**0.000**
Bold indicates significance at the 10% level.								

Table XXV. Wald Tests for seasonal components of crude oil and natural gas models 1 and 2.

Panel A: Crude Oil				
	Model 1: ATM		Model 2:Sigma0	
Test	Statistic	Pr > ChiSq	Statistic	Pr > ChiSq
Beta1				
Spring=Fall	0.257	0.612	0.000	0.991
All Season Equal	9.081	0.028	**15.812**	**0.001**
Summer Different	0.412	0.814	2.750	0.253
Beta2				
Spring=Fall	0.001	0.971	0.121	0.728
All Season Equal	6.735	0.081	**14.227**	**0.003**
Summer Different	0.222	0.895	3.040	0.219
Gamma				
Spring=Fall	2.913	0.088	2.419	0.120
All Season Equal	**12.091**	**0.007**	**13.355**	**0.004**
Summer Different	3.872	0.144	2.604	0.272
Across Equations				
All Springs Equal	**74.610**	**0.000**	**15.454**	**0.000**
All Summers Equal	**75.037**	**0.000**	**15.731**	**0.000**
All Falls Equal	**74.668**	**0.000**	**15.548**	**0.000**
All Winters Equal	**74.927**	**0.000**	**15.634**	**0.000**
Panel B: Natural Gas				
	Model 1: ATM		Model 2:Sigma0	
Test	Statistic	Pr > ChiSq	Statistic	Pr > ChiSq
Beta1				
Injection=Withdrawal	3.352	0.067	1.765	0.184
Beta2				
Injection=Withdrawal	3.321	0.068	0.436	0.509
Gamma				
Injection=Withdrawal	0.708	0.400	94.939	0.000
Across Equations				
All Injections Equal	1.769	0.413	13.190	0.001
All Withdrawals Equal	1.571	0.456	16.880	0.000

Table XXVI. Tests for contemporaneous correlation for crude oil and natural gas models 1 and 2 by day of week.

Panel A: Crude Oil				
Model 1: ATM				
Weekday	n	v	Statistic	Pr > ChiSq
Monday	237	3	255.648	1.000
Tuesday	258	3	276.215	1.000
Wednesday	257	3	264.092	1.000
Thursday	251	3	265.537	1.000
Friday	243	3	263.390	1.000
Model 2:Sigma0				
Weekday	n	v	Statistic	Pr > ChiSq
Monday	237	3	253.524	1.000
Tuesday	258	3	257.724	1.000
Wednesday	257	3	279.746	1.000
Thursday	251	3	263.036	1.000
Friday	243	3	242.945	1.000
Panel B: Natural Gas				
Model 1: ATM				
Weekday	n	v	Statistic	Pr > ChiSq
Monday	234	3	237.259	1.000
Tuesday	234	3	245.448	1.000
Wednesday	257	3	314.263	1.000
Thursday	257	3	314.715	1.000
Friday	253	3	302.650	1.000
Model 2:Sigma0				
Weekday	n	v	Statistic	Pr > ChiSq
Monday	253	3	245.131	1.000
Tuesday	247	3	262.150	1.000
Wednesday	247	3	272.165	1.000
Thursday	240	3	306.509	1.000
Friday	240	3	236.764	1.000

Table XXVII. Covariance matrix of residuals for tests of contemporaneous correlation for crude oil and natural gas models 1 and 2 by day of week.

Panel A: Crude Oil							
	Beta1	Beta2	Gamma		Beta1	Beta2	Gamma
Model 1: ATM				**Model 2:Sigma0**			
Monday				Monday			
Beta1	0.207	-0.006	-0.005	**Beta1**	0.597	-0.016	-0.073
Beta2	-0.006	0.000	0.000	**Beta2**	-0.016	0.000	0.002
Gamma	-0.005	0.000	0.003	**Gamma**	-0.073	0.002	0.240
Tuesday				Tuesday			
Beta1	0.195	-0.005	-0.005	**Beta1**	0.361	-0.010	0.013
Beta2	-0.005	0.000	0.000	**Beta2**	-0.010	0.000	0.000
Gamma	-0.005	0.000	0.003	**Gamma**	0.013	0.000	0.204
Wednesday				Wednesday			
Beta1	0.245	-0.007	-0.003	**Beta1**	0.436	-0.012	-0.070
Beta2	-0.007	0.000	0.000	**Beta2**	-0.012	0.000	0.002
Gamma	-0.003	0.000	0.003	**Gamma**	-0.070	0.002	0.230
Thursday				Thursday			
Beta1	0.189	-0.005	-0.004	**Beta1**	0.365	-0.010	-0.044
Beta2	-0.005	0.000	0.000	**Beta2**	-0.010	0.000	0.001
Gamma	-0.004	0.000	0.004	**Gamma**	-0.044	0.001	0.191
Friday				Friday			
Beta1	0.295	-0.008	-0.007	**Beta1**	0.477	-0.013	-0.020
Beta2	-0.008	0.000	0.000	**Beta2**	-0.013	0.000	0.000
Gamma	-0.007	0.000	0.004	**Gamma**	-0.020	0.000	0.246
Panel B: Natural Gas							
	Beta1	Beta2	Gamma		Beta1	Beta2	Gamma
Model 1: ATM				**Model 2:Sigma0**			
Monday				Monday			
Beta1	110.525	-30.638	0.071	**Beta1**	15.116	-3.983	0.329
Beta2	-30.638	8.503	-0.020	**Beta2**	-3.983	1.099	-0.117
Gamma	0.071	-0.020	0.006	**Gamma**	0.329	-0.117	1.403
Tuesday				Tuesday			
Beta1	7.695	-1.876	-0.039	**Beta1**	61.488	-16.889	-1.469
Beta2	-1.876	0.467	0.011	**Beta2**	-16.889	4.668	0.403
Gamma	-0.039	0.011	0.007	**Gamma**	-1.469	0.403	1.035
Wednesday				Wednesday			
Beta1	14.987	-3.971	0.205	**Beta1**	142.236	-38.287	-3.161
Beta2	-3.971	1.060	-0.055	**Beta2**	-38.287	10.333	0.844
Gamma	0.205	-0.055	0.025	**Gamma**	-3.161	0.844	1.331
Thursday				Thursday			
Beta1	15.272	-4.178	-0.113	**Beta1**	64.004	-17.206	-3.729
Beta2	-4.178	1.152	0.031	**Beta2**	-17.206	4.652	1.039
Gamma	-0.113	0.031	0.007	**Gamma**	-3.729	1.039	1.589
Friday				Friday			
Beta1	8.378	-2.200	-0.058	**Beta1**	24.814	-6.431	0.079
Beta2	-2.200	0.588	0.017	**Beta2**	-6.431	1.691	-0.032
Gamma	-0.058	0.017	0.004	**Gamma**	0.079	-0.032	0.942

Table XXVIII. Durbin-Watson statistics for residual autocorrelation for crude oil and natural gas models 1 and 2 by weekday.

Panel A: Crude Oil							
		Model 1: ATM			Model 2:Sigma0		
Equation	Order	DW	Pr < DW	Pr > DW	DW	Pr < DW	Pr > DW
		Monday					
Beta1	1	1.843	0.067	0.933	2.095	0.656	0.344
Beta2	1	1.853	0.078	0.922	2.089	0.638	0.362
Gamma	1	1.841	0.056	0.944	1.877	0.101	0.899
		Tuesday					
Beta1	1	2.067	0.604	0.396	2.056	0.570	0.430
Beta2	1	2.057	0.574	0.426	2.054	0.562	0.438
Gamma	1	1.989	0.317	0.683	1.954	0.265	0.735
		Wednesday					
Beta1	1	2.013	0.435	0.565	1.846	0.062	0.938
Beta2	1	1.995	0.378	0.622	1.843	0.059	0.941
Gamma	1	1.922	0.159	0.841	1.872	0.085	0.915
		Thursday					
Beta1	1	1.988	0.343	0.657	1.968	0.280	0.720
Beta2	1	1.973	0.301	0.699	1.969	0.282	0.718
Gamma	1	1.863	0.074	0.926	1.914	0.164	0.836
		Friday					
Beta1	1	1.918	0.173	0.827	2.064	0.571	0.429
Beta2	1	1.886	0.117	0.883	2.048	0.518	0.482
Gamma	1	1.924	0.163	0.837	1.903	0.147	0.853

Panel B: Natural Gas							
		Model 1: ATM			Model 2:Sigma0		
Equation	Order	DW	Pr < DW	Pr > DW	DW	Pr < DW	Pr > DW
		Monday					
Beta1	1	2.136	0.733	0.267	1.997	0.328	0.672
Beta2	1	2.123	0.699	0.301	1.929	0.167	0.833
Gamma	1	1.960	0.232	0.768	1.990	0.325	0.675
		Tuesday					
Beta1	1	2.083	0.602	0.398	2.004	0.355	0.645
Beta2	1	2.133	0.747	0.253	2.022	0.409	0.591
Gamma	1	2.019	0.406	0.594	2.016	0.405	0.595
		Wednesday					
Beta1	1	2.098	0.645	0.355	2.237	0.934	0.066
Beta2	1	2.066	0.547	0.453	2.236	0.932	0.068
Gamma	1	2.081	0.592	0.408	1.934	0.183	0.817
		Thursday					
Beta1	1	1.921	0.149	0.851	2.151	0.781	0.219
Beta2	1	1.922	0.151	0.849	2.151	0.781	0.219
Gamma	1	2.034	0.434	0.566	2.005	0.365	0.635
		Friday					
Beta1	1	1.903	0.129	0.871	2.212	0.889	0.111
Beta2	1	1.924	0.166	0.834	2.231	0.914	0.086
Gamma	1	2.061	0.520	0.480	1.981	0.293	0.707

Table XXIX. Weekday analysis of the signs for crude oil's significant factors of volatility from API and CFTC.

	Model 1: ATM					Model 2:Sigma0				
Parameter	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Beta1										
Ln(Total Domestic)										
Ln(Total Imports)				+			+		+	
ShortTermMA / LongTermMA										+
HedgeLong%	+	+	+		+	+	+	+		
HedgeShort%		−		+						
Spread%	+	+	+		+		+	+	+	+
Beta2										
Ln(Total Domestic)										
Ln(Total Imports)		−		−			−		−	
ShortTermMA / LongTermMA										
HedgeLong%	−	−	−		-	−	−	−		−
HedgeShort%				−					-	
Spread%	−	−	−	−	−		−	−		−
Gamma										
Ln(Total Domestic)	+	+	+	+	+					
Ln(Total Imports)								−	−	−
ShortTermMA / LongTermMA								+		
HedgeLong%							−			
HedgeShort%						+	+		+	
Spread%					+					

Table XXX. Weekday analysis of the sign for natural gas' significant factors of volatility from AGA and CFTC.

	Model 1: ATM					Model 2:Sigma0				
Parameter	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Beta1										
AGAProd%		+					–			
AGAEast%						+	+			
ShortTermMA / LongTermMA										
HedgeLong%										
HedgeShort%		+								
Spread%										
Beta2										
AGAProd%										
AGAEast%										
ShortTermMA / LongTermMA										
HedgeLong%										
HedgeShort%		–								
Spread%										
Gamma										
AGAProd%					–					
AGAEast%								–		–
ShortTermMA / LongTermMA	–				–	–	–	–	–	–
HedgeLong%						–	–	–	–	–
HedgeShort%		+								
Spread%										

Table XXXI. Wald Tests for seasonal components by day of week for crude oil models 1 and 2.

Test	Model 1: ATM Statistic	Model 1: ATM Pr > ChiSq	Model 2:Sigma0 Statistic	Model 2:Sigma0 Pr > ChiSq
	Monday		Monday	
	Beta1			
Spring=Fall	0.025	0.874	1.145	0.285
All Seasons Equal	0.560	0.905	1.610	0.657
Summer Different	0.252	0.882	1.242	0.537
	Beta2			
Spring=Fall	0.000	0.989	1.370	0.242
All Seasons Equal	0.350	0.950	1.767	0.622
Summer Different	0.137	0.934	1.473	0.479
	Gamma			
Spring=Fall	0.922	0.337	0.043	0.836
All Seasons Equal	2.486	0.478	0.977	0.807
Summer Different	1.569	0.456	0.090	0.956
	Across Equations			
All Springs Equal	**9.915**	**0.007**	2.189	0.335
All Summers Equal	**9.981**	**0.007**	2.224	0.329
All Falls Equal	**10.010**	**0.007**	2.227	0.328
All Winters Equal	**9.930**	**0.007**	2.219	0.330
	Tuesday		Tuesday	
	Beta1			
Spring=Fall	0.220	0.639	0.095	0.758
All Seasons Equal	1.303	0.728	1.780	0.619
Summer Different	1.300	0.522	1.209	0.546
	Beta2			
Spring=Fall	0.111	0.739	0.238	0.626
All Seasons Equal	1.308	0.727	1.928	0.588
Summer Different	1.307	0.520	1.473	0.479
	Gamma			
Spring=Fall	0.366	0.545	0.071	0.790
All Seasons Equal	2.731	0.435	0.396	0.941
Summer Different	0.671	0.715	0.272	0.873
	Across Equations			
All Springs Equal	**19.013**	**0.000**	3.424	0.181
All Summers Equal	**19.090**	**0.000**	3.462	0.177
All Falls Equal	**19.029**	**0.000**	3.451	0.178
All Winters Equal	**19.111**	**0.000**	3.475	0.176
	Wednesday		Wednesday	
	Beta1			
Spring=Fall	0.538	0.463	2.212	0.137
All Seasons Equal	2.175	0.537	4.886	0.180
Summer Different	1.593	0.451	3.749	0.153
	Beta2			
Spring=Fall	0.406	0.524	2.072	0.150
All Seasons Equal	1.917	0.590	4.628	0.201
Summer Different	1.425	0.490	3.588	0.166
	Gamma			
Spring=Fall	0.522	0.470	0.790	0.374
All Seasons Equal	3.967	0.265	**7.888**	**0.048**
Summer Different	1.371	0.504	0.814	0.666
	Across Equations			
All Springs Equal	**11.840**	**0.003**	3.131	0.209
All Summers Equal	**11.912**	**0.003**	3.129	0.209
All Falls Equal	**11.827**	**0.003**	3.087	0.214
All Winters Equal	**11.911**	**0.003**	3.142	0.208

Table continued on next page.

Test	Model 1: ATM Statistic	Model 1: ATM Pr > ChiSq	Model 2:Sigma0 Statistic	Model 2:Sigma0 Pr > ChiSq
	Thursday		Thursday	
	Beta1			
Spring=Fall	0.079	0.778	0.000	0.994
All Seasons Equal	7.246	0.064	2.722	0.437
Summer Different	0.296	0.862	0.199	0.906
	Beta2			
Spring=Fall	0.026	0.872	0.002	0.967
All Seasons Equal	6.228	0.101	2.643	0.450
Summer Different	0.173	0.917	0.252	0.882
	Gamma			
Spring=Fall	0.005	0.945	1.516	0.218
All Seasons Equal	2.668	0.446	7.818	0.050
Summer Different	2.379	0.304	1.979	0.372
	Across Equations			
All Springs Equal	4.525	0.104	0.429	0.807
All Summers Equal	4.589	0.101	0.456	0.796
All Falls Equal	4.539	0.103	0.437	0.804
All Winters Equal	4.576	0.101	0.440	0.803
	Friday		Friday	
	Beta1			
Spring=Fall	0.000	0.989	0.072	0.789
All Seasons Equal	0.570	0.903	4.585	0.205
Summer Different	0.465	0.792	4.584	0.101
	Beta2			
Spring=Fall	0.038	0.846	0.174	0.676
All Seasons Equal	0.735	0.865	5.034	0.169
Summer Different	0.534	0.766	5.024	0.081
	Gamma			
Spring=Fall	2.462	0.117	0.048	0.827
All Seasons Equal	4.440	0.218	2.743	0.433
Summer Different	2.512	0.285	2.434	0.296
	Across Equations			
All Springs Equal	**16.584**	**0.000**	3.232	0.199
All Summers Equal	**16.600**	**0.000**	3.237	0.198
All Falls Equal	**16.543**	**0.000**	3.251	0.197
All Winters Equal	**16.613**	**0.000**	3.204	0.201

Table XXXII. Wald Tests for seasonal components by day of week for natural gas models 1 and 2.

Test	Model 1: ATM Statistic	Model 1: ATM Pr > ChiSq	Model 2:Sigma0 Statistic	Model 2:Sigma0 Pr > ChiSq
	Monday		Monday	
		Beta1		
Injection=Withdrawal	1.705	0.192	2.544	0.111
		Beta2		
Injection=Withdrawal	1.669	0.196	0.733	0.392
		Gamma		
Injection=Withdrawal	0.013	0.908	**36.528**	**0.000**
		Across Equations		
All Injections Equal	0.488	0.784	1.605	0.448
All Withdrawals Equal	0.407	0.816	2.426	0.297
	Tuesday		Tuesday	
		Beta1		
Injection=Withdrawal	1.201	0.273	0.008	0.927
		Beta2		
Injection=Withdrawal	1.310	0.252	0.119	0.730
		Gamma		
Injection=Withdrawal	**4.195**	**0.041**	**55.741**	**0.000**
		Across Equations		
All Injections Equal	**8.122**	**0.017**	**5.361**	**0.069**
All Withdrawals Equal	**7.514**	**0.023**	**7.066**	**0.029**
	Wednesday		Wednesday	
		Beta1		
Injection=Withdrawal	0.096	0.756	1.077	0.299
		Beta2		
Injection=Withdrawal	0.105	0.745	0.637	0.425
		Gamma		
Injection=Withdrawal	0.161	0.688	**37.220**	**0.000**
		Across Equations		
All Injections Equal	0.197	0.906	2.632	0.268
All Withdrawals Equal	0.180	0.914	3.729	0.155
	Thursday		Thursday	
		Beta1		
Injection=Withdrawal	0.094	0.759	0.003	0.953
		Beta2		
Injection=Withdrawal	0.082	0.775	0.057	0.811
		Gamma		
Injection=Withdrawal	0.970	0.325	**25.106**	**0.000**
		Across Equations		
All Injections Equal	0.988	0.610	**5.114**	**0.078**
All Withdrawals Equal	1.028	0.598	**6.243**	**0.044**
	Friday		Friday	
		Beta1		
Injection=Withdrawal	0.336	0.562	1.175	0.278
		Beta2		
Injection=Withdrawal	0.441	0.506	0.521	0.470
		Gamma		
Injection=Withdrawal	0.011	0.917	**37.297**	**0.000**
		Across Equations		
All Injections Equal	1.032	0.597	**10.883**	**0.004**
All Withdrawals Equal	0.981	0.612	**12.751**	**0.002**

Table XXXIII. Elasticity measures of volatility factors for crude oil and natural gas.

Panel A: Crude Oil			
Model 1: ATM			
	Beta1	**Beta2**	**Gamma**
Ln(Total Domestic)			-183.90
Ln(Total Imports)	-11.97	-12.06	
HedgeLong%	-11.72	-11.18	
HedgeShort%	3.02		-9.87
Spread%	-0.69	-0.73	
ShortTermMA/LongTermMA			
Model 2:Sigma0			
	Beta1	**Beta2**	**Gamma**
Ln(Total Domestic)	-37.26		
Ln(Total Imports)	-18.43	-19.28	-4.40
HedgeLong%	-15.04	-14.65	-2.78
HedgeShort%	5.07	4.26	2.10
Spread%	-0.57	-0.62	0.10
ShortTermMA/LongTermMA			1.69
Panel B: Natural Gas			
Model 1: ATM			
	Beta1	**Beta2**	**Gamma**
AGAProd%	-4.96		
AGAEast%			
HedgeLong%			-8.96
HedgeShort%			
Spread%			
ShortTermMA/LongTermMA			-3.57
Model 2:Sigma0			
	Beta1	**Beta2**	**Gamma**
AGAProd%	36.58	11.07	
AGAEast%	-30.03	-8.15	-0.39
HedgeLong%			-4.87
HedgeShort%			
Spread%			0.13
ShortTermMA/LongTermMA			-1.19

Figures and Diagrams

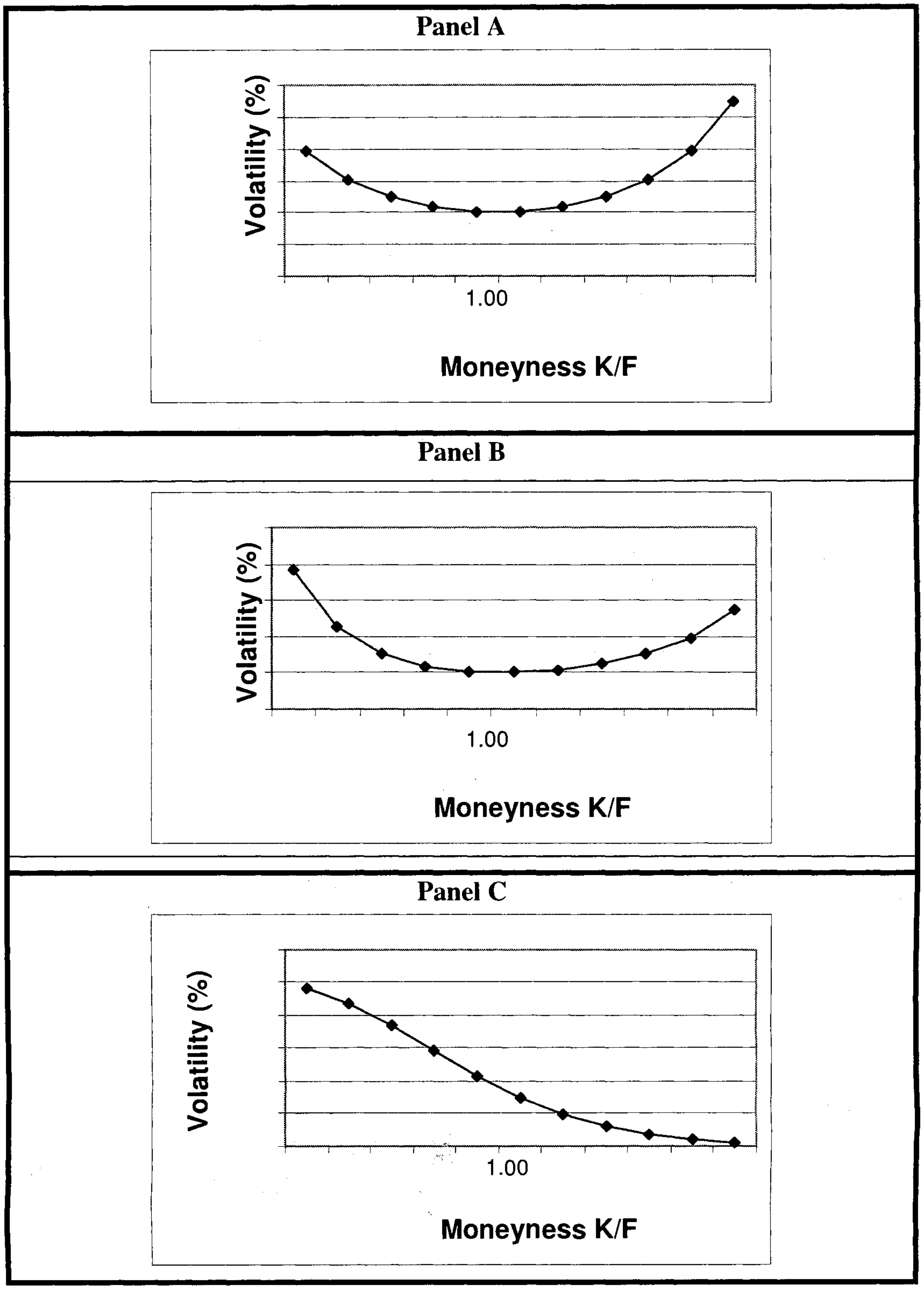

Figure 1. Volatility smiles for left skew, right skew, and volatility sneer.

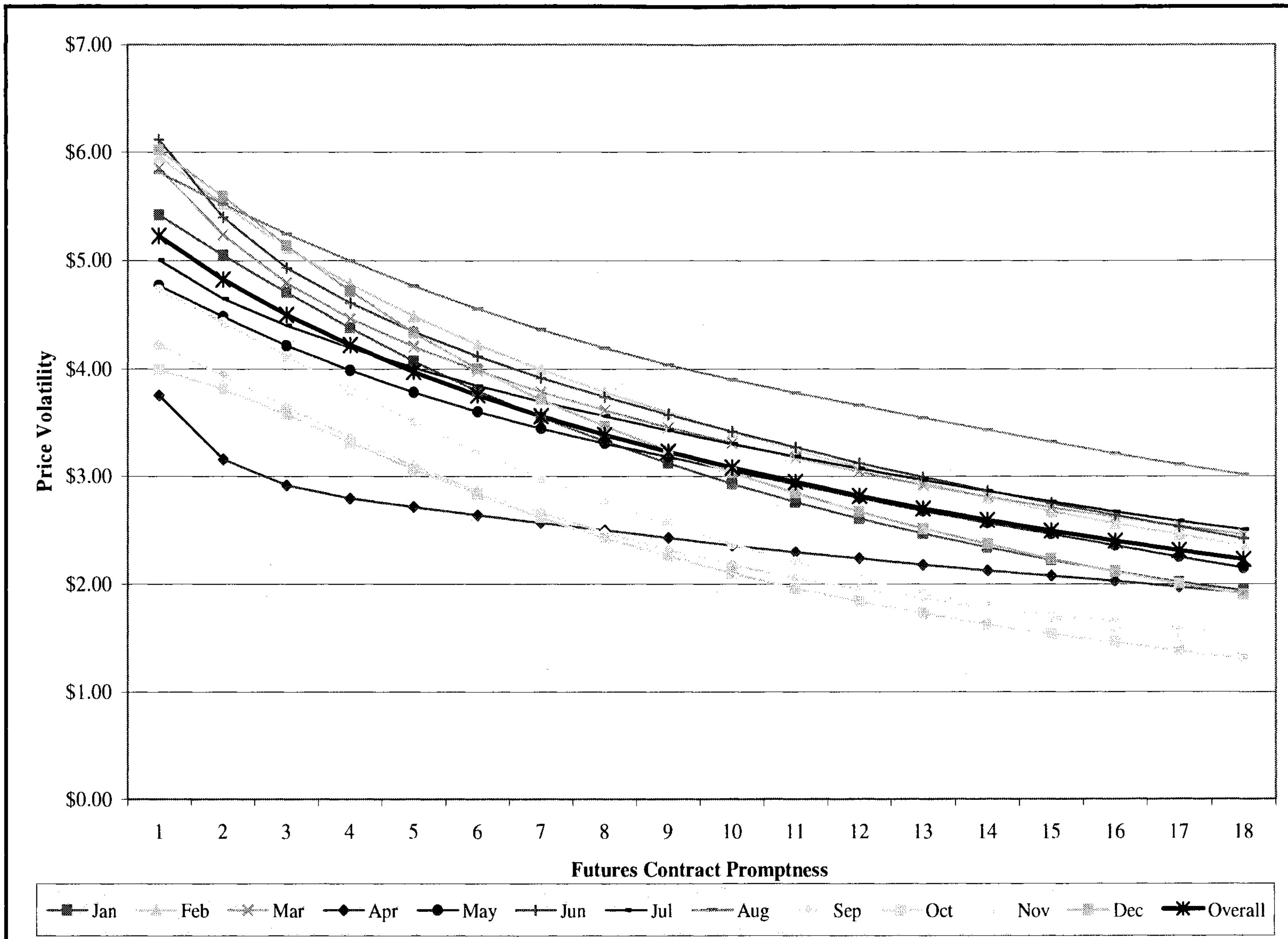

Figure 2. NYMEX crude oil futures volatility term structure by month and promptness: 1/1995 - 12/1999

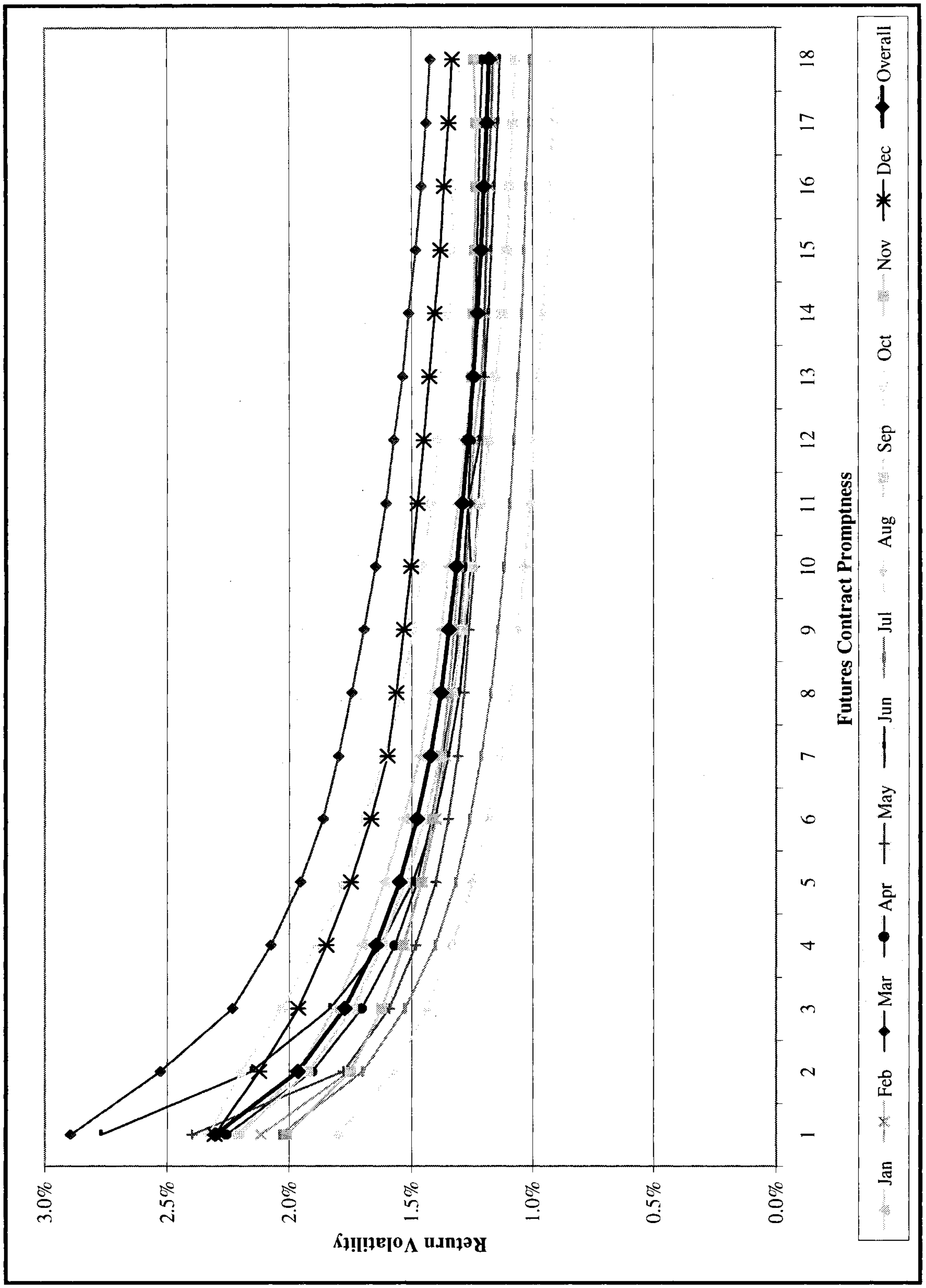

Figure 3. NYMEX log returns of crude oil futures volatility term structure by month and promptness: 1/1996 - 12/1999

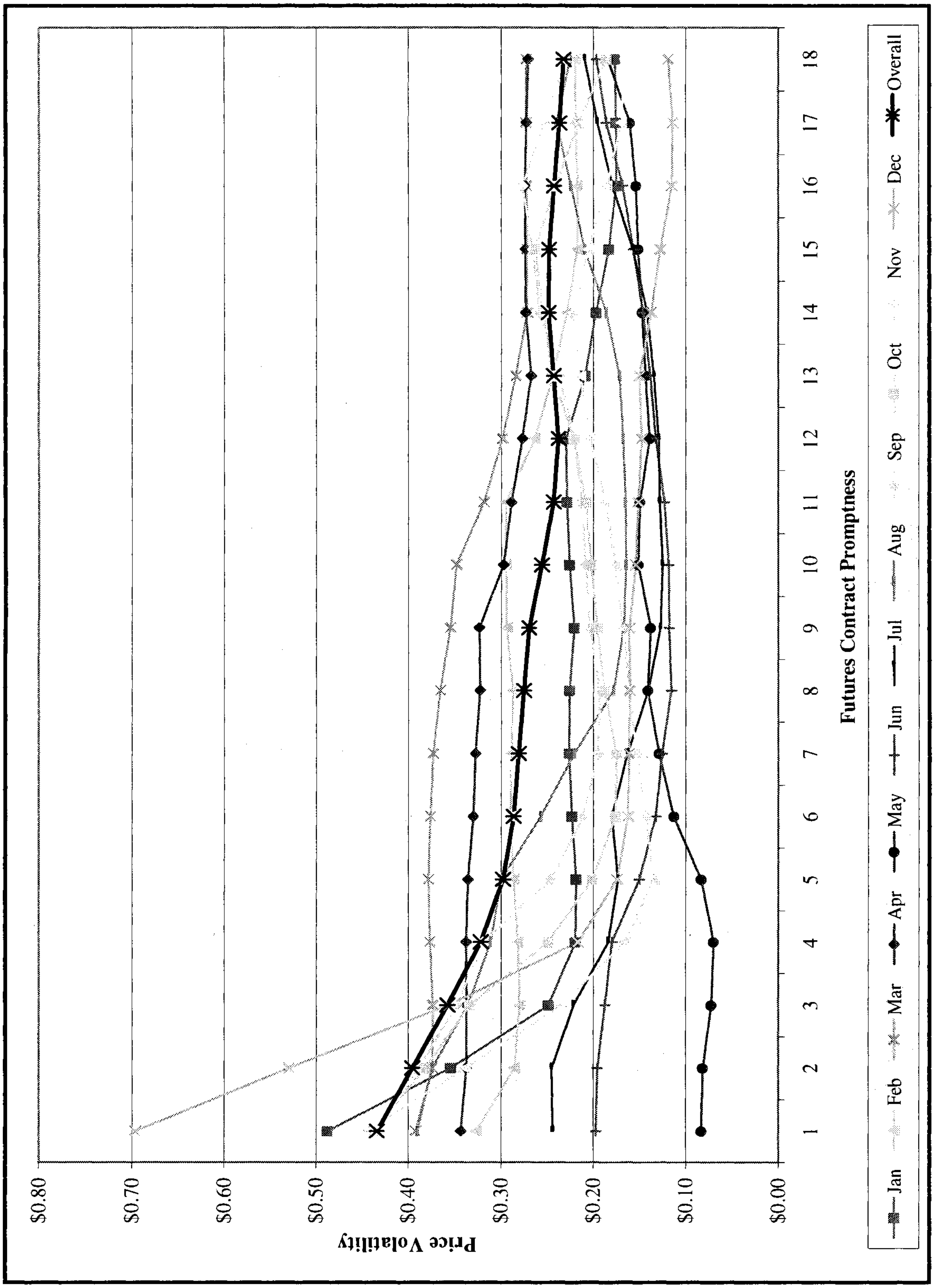

Figure 4. NYMEX natural gas futures volatility term structure by month and promptness: 1/1995 - 12/1999

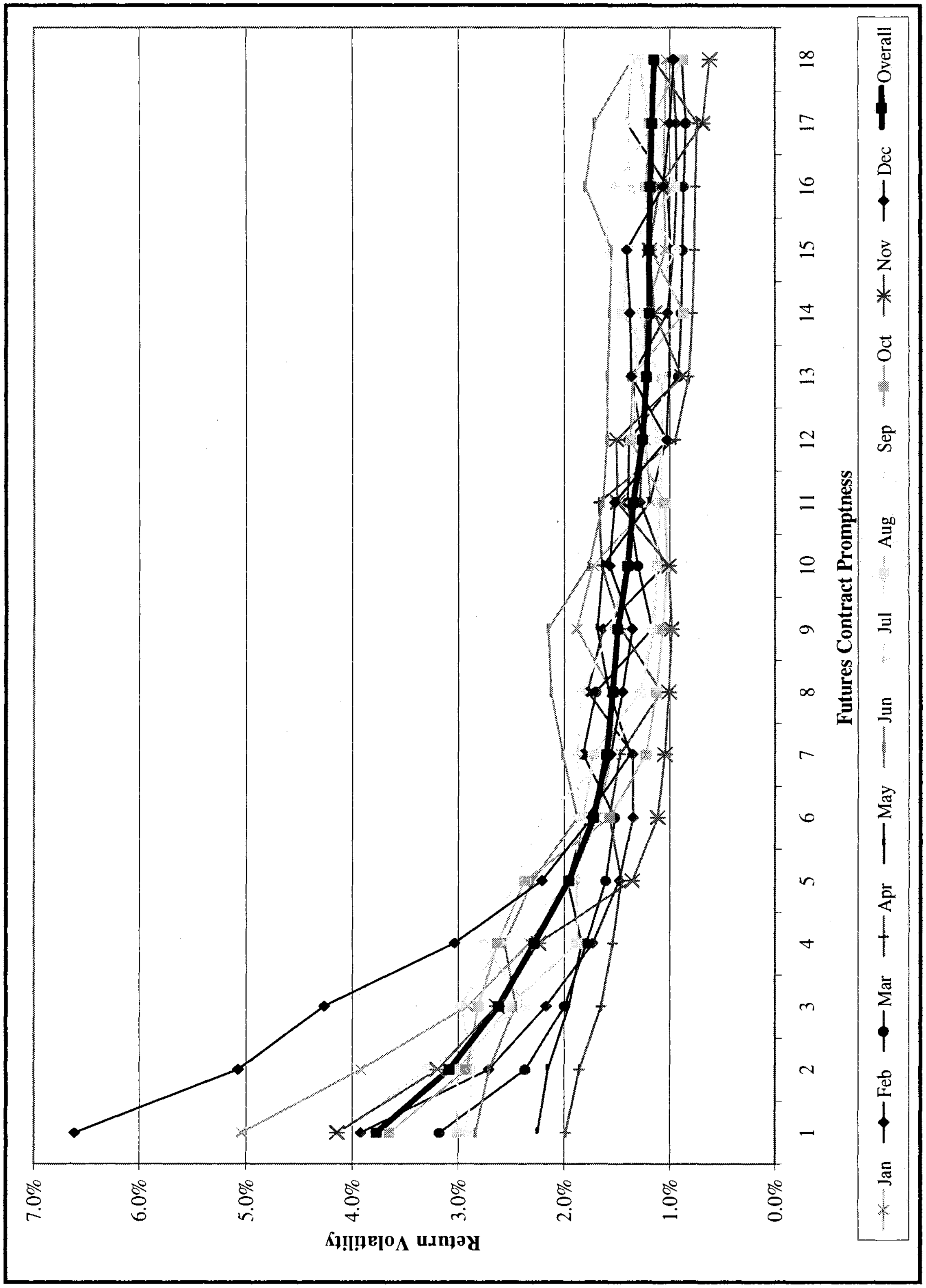

Figure 5. NYMEX log returns of natural gas futures volatility term structure by month and promptness: 1/1995 - 12/1999

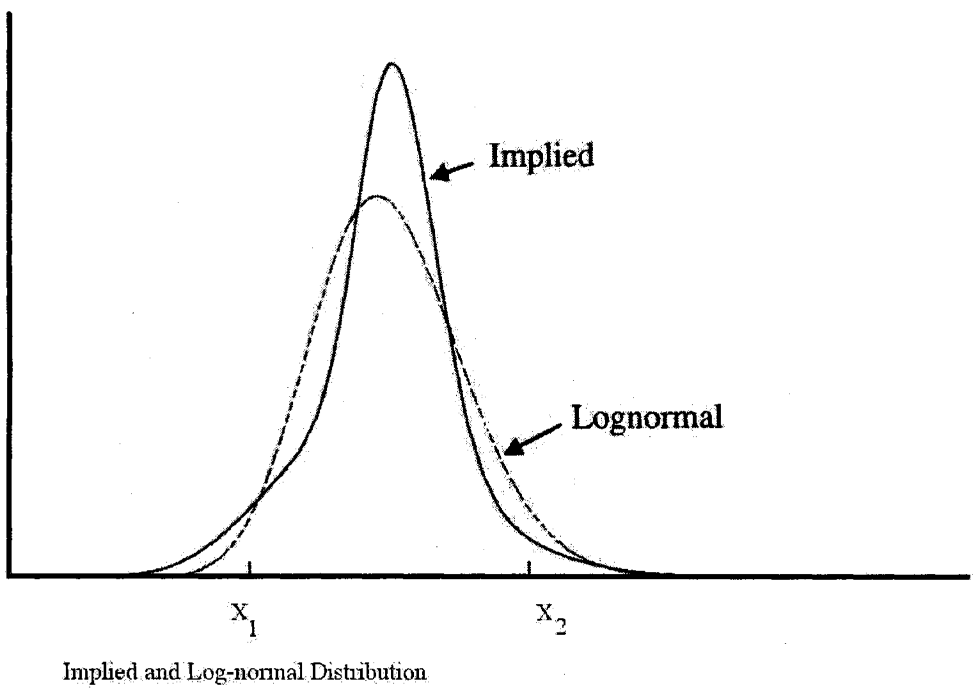

Figure 6. Implied verse lognormal distribution, reproduce from Hull (2000).

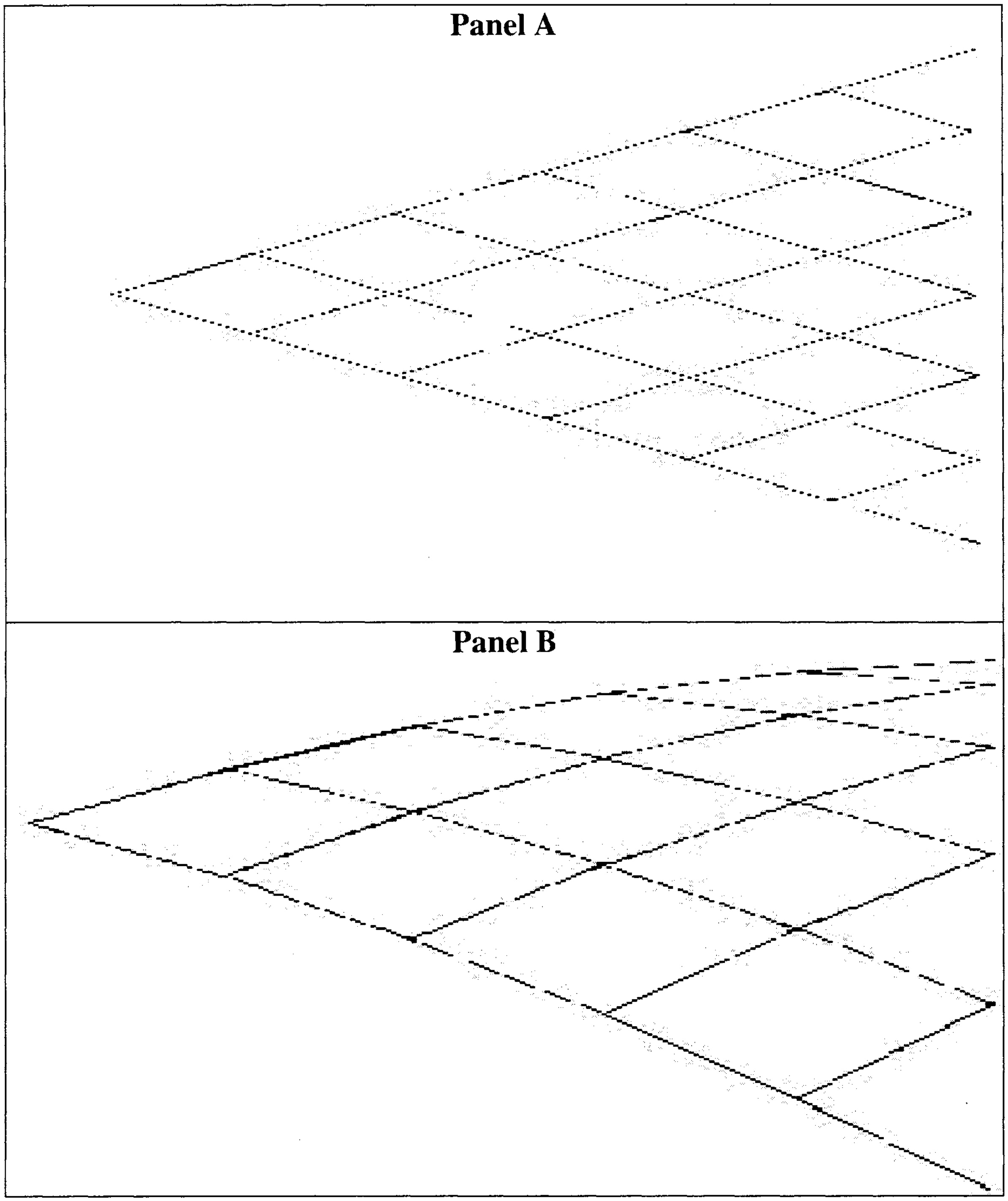

Figure 7. Cox, Ross, and Rubenstein constant volatility binomial tree and skewed binomial tree.

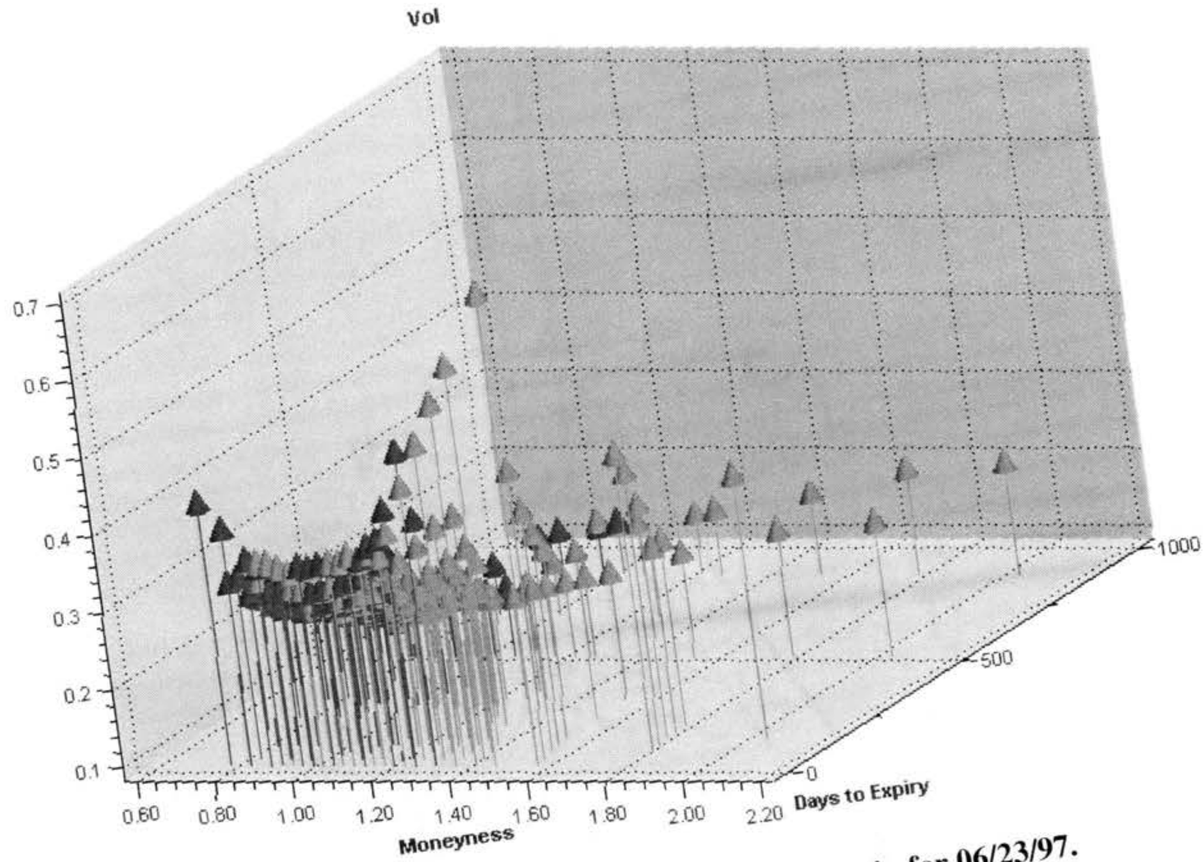

Figure 8. Implied volatilities of crude oil option (LO) chain for 06/23/97.

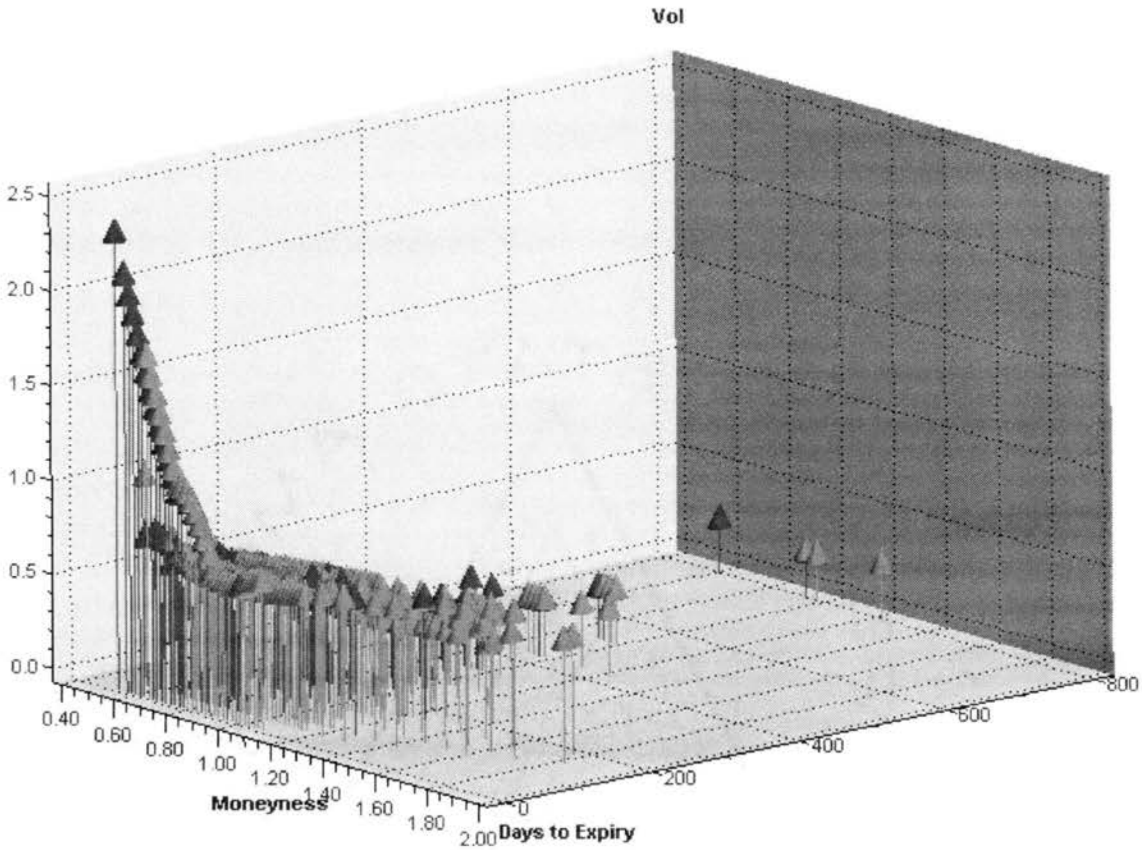

Figure 9. Implied volatilities of natural gas option (LO) chain for 09/22/97.

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