SYMMETRICAL COMPONENT EQUATIONS FOR SYSTEM FAULTS
SUPPLIED BY A LOADED ALTERNATOR

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INTRODUCTION

Since the publication of C. L. Fortescue's classic paper[1] on Symmetrical Components in 1918, the electrical industry has gradually accorded the Method of Symmetrical Components a more and more important place as an electical tool; taking it from the laboratory and putting it to work.[2] Power companies have expressed their confidence in this new tool by their large investments in calculating boards, based on the theory of symmetrical components, which enable engineers to calculate accurately, in a fraction of the time formerly required, the currents that will flow under certain conditions of fault.

In the analytical solution of systems containing an alternator supplying an unbalanced load, it has been conventional practice[3] to assume the alternator to be carrying no load at the time of fault, and then to set up a different equivalent circuit connecting each of the three symmetrical components for each type of fault. W. E. Slemmer shows several examples[4] using this method in his thesis published in 1934. During the same year however, E. M. Sabbagh

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[1] C.L. Fortescue, Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks, pp. 1027-1140.

[2] H.A. Travers and W. W. Parker, An Alternating-current Calculating Board, pp. 266-270. H.R. Searing and R. E. Powers, Sequence Principles Used for Network Relaying, pp. 649-697.

[3] C. F. Wagner and R. D. Evans, Symmetrical Components, par. 24.

[4] W. E. Slemmer, Symmetrical Components, p. 59

published his Ph. D. dissertation, "Unbalance in Alternating-current Rotating Machines." In this dissertation general expressions were developed for the steady state currents in a wye connected alternator supplying power to unbalanced loads.[5]

It is the purpose of this thesis to present in as simple a manner as possible the principles involved and the steps covered in developing these general expressions, and to show by means of examples their use and how the results obtained by their use compare with the results obtained by other methods. It is also shown that the classical types of faults usually considered are but special cases of a general solution.

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[5] E. M. Sabbagh, Unbalance in Alternating-current Rotating Machines, p. 21.

FUNDAMENTAL PRINCIPLES

There are methods available for solving balanced polyphase circuits by replacing the mutual reactance between phases by equivalent self-impedances and solving one of the phases. A method which would allow a similar simplification in the solution of unbalanced loads on a symmetrical system, which would simplify and in some cases permit the solution of otherwise unsolvable cases of unbalanced polyphase circuits, would be a great improvement over the simultaneous solution otherwise employed. The method of symmetrical components allows any three unbalanced three phase voltages[6] to be resolved into three systems of balanced or symmetrical components.[7] If the system containing the three unbalanced voltages is itself symmetrical, the voltages and currents of different sequences do not react upon each other.[8] This allows each of the balanced systems or components to be treated separately just as balanced polyphase problems and greatly simplifies their solution.

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6 For polyphase systems of more than three phases, see C. L. Fortescue, op. cit. p. 1130.

7 In the usual sense of the word, the zero-sequence component is not a symmetrical component but a uniphase component. If the three zero-sequence phase components of current in the three phases are thought of as a single current in three parallel branches, the zero-sequence current might be called a single-phase current.

8 Symmetrical circuit conditions, i.e., systems whose constants are the same viewed from any phase, except for fault and load, will be assumed throughout this thesis. Under these conditions the three sequences are independent, i.e., the current of one sequence produces impedance drops of that sequence only. (See C. F. Wagner and R. D. Evans, Symmetrical Components, op. cit. pp. 374-375.)

These three components will be discussed very briefly.[9] Since each of the three sequences are independent of each other, they may each be shown as a symmetrical system of vectors. The three systems of balanced vectors that give the original voltage vectors when added together, as in Fig. 2, are called positive-, negative- and zero-sequence components.

In Fig. 1(a) is shown the positive system of vectors for a three-phase system, of which E_{a1}, E_{b1} and E_{c1} are the positive-sequence line-to-neutral voltages of phases (a), (b) and (c), respectively. Each of the positive-sequence components are of equal magnitude, and are separated by a phase angle of 120 degrees. The developed instantaneous values of the vectors, as projected on the X-axis, are shown in Fig. 1(b).[11] The positive-sequence component is the only component found in balanced systems, and so it is the one considered in ordinary balanced system calculations. It is considered positive because its order of maxima occurs in the sequence[12] (abc) which is the same order as that used in the original vectors,[13] E_a, E_b, and E_c (Fig. 2).

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[9] For a more elaborate and rigorous proof of the symmetrical component theory, see C. L. Fortescue, op. cit. p. 1027.

[10] Throughout this paper the subscripts o, 1, and 2 will be used to denote the zero-, positive-, and negative-sequence components, respectively.

[11] The conventional rotation of vectors in the counter-clockwise direction is assumed throughout this thesis.

[12] Note that the sequence has no relation to the direction of rotation of the vectors themselves.

[13] O. G. C. Dahl, Electric Circuits - Theory and Application, Vol. 1, p. 78.

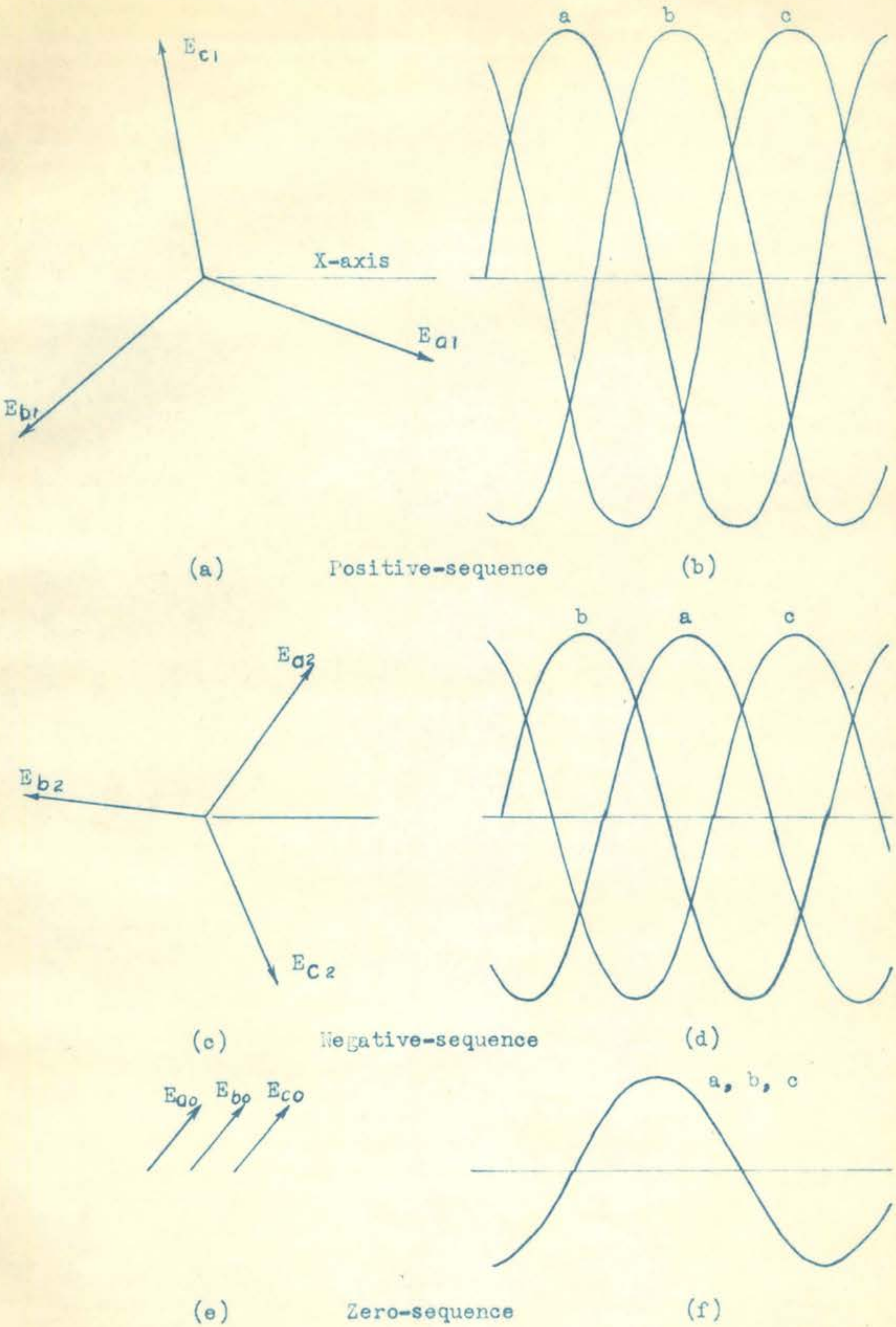

Fig. 1. The Three Sequence Components

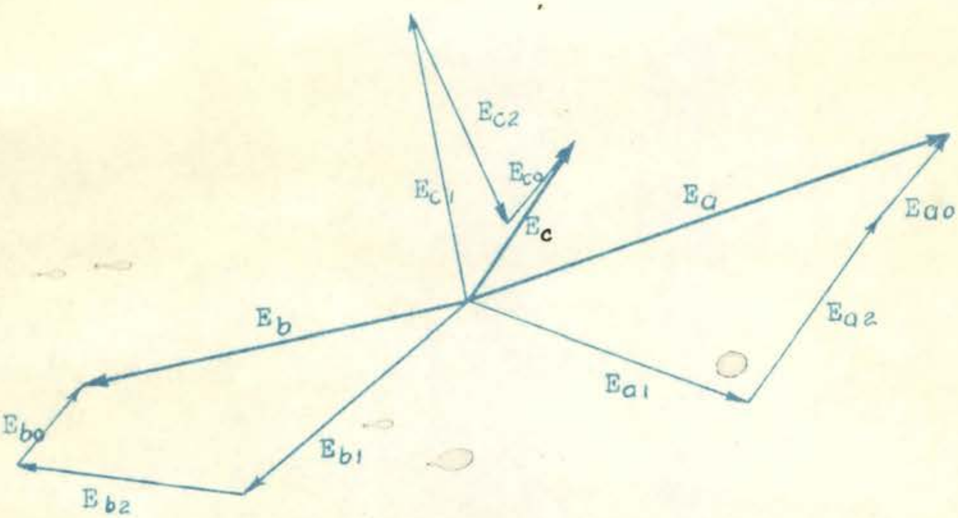

Fig. 2. The summation of the three sequence components to form the original unbalanced vector system.

In the negative-sequence system, Fig 1(c), the vectors E_{a2}, E_{b2}, and E_{c2} revolve in the same direction as in the positive-sequence system; but it will be noted, the maxima of the phase components build up in the reverse order, that is (bac). One might obtain a better physical picture by considering the rotating magnetic fields set up if the phases (a), (b), and (c) were connected to an induction motor. If the positive-sequence system (abc) produces a field revolving in one direction, by interchanging any two connections, the field rotation will reverse. The relative phase positions of the voltages are then similar to those of the negative-sequence system (bac).

These two sequences, however, are not enough to completely replace the original vectors in some cases. Since both the positive- and negative-sequence systems are symmetrical, their sums must be equal to zero. Thus when using a delta or ungrounded wye system, these two sequences would be sufficient for the line voltages; but with a grounded wye system, the sum of the three-line current vectors may not be zero. For the more general case Fortescue introduced a third component, called the zero-sequence component because the phase angles were zero, Fig. 1(e). These three zero-sequence components are in phase and of equal magnitude.

There are then, three sets of symmetrical vectors which can be combined in a graphical manner to form a set of unbalanced voltages, as shown in Fig. 2, or the operation may be reversed to resolve a system of unbalanced voltages, for example those in Fig. 2, into three sets of symmetrical components, as in Fig. 1(a), (b), and (c). From then on, since in symmetrical circuits currents and voltages of different sequences do not react upon each other, the sequences may be treated separately as balanced polyphase problems and solved on a single-phase basis.

Resolution of Three Unbalanced Three-Phase Vectors into their Symmetrical Components

Since the three phase-components of a sequence system are balanced, one phase-component differs from either of the other two only by being displaced 120 degrees. Therefore, it is possible to express either of the other two components in terms of one phase component and an angle of rotation. Thus,

$$E_{c1} = e^{j120°} E_{a1} \quad \text{and } E_{b1} = e^{j240°} E_{a1}$$

Since $e^{j120°}$ is used often, it is convenient to let the letter (a) be a vector operator such that

$$a = e^{j120°} = -0.5 + j0.866$$

and

$$a^2 = e^{j240°} = -0.5 - j0.866 \qquad (1)$$

From Fig. 1(a)

$$E_{b1} = a^2 E_{a1}, \text{ and } E_{c1} = aE_{a1} \qquad (2)$$

From Fig. 1(c)

$$E_{b2} = aE_{a2} \text{ and } E_{c2} = a^2 E_{a2} \qquad (3)$$

and from Fig. 1(e) since there is zero phase angle between the three vectors

$$E_{a0} = E_{b0} = E_{c0} \qquad (4)$$

From the previous discussion and from Fig. 2, it is apparent that each of the original voltage vectors is equal to the vector sum of the three sequence components. That is

$$E_a = E_{a0} + E_{a1} + E_{a2} \tag{5}$$

$$E_b = E_{b0} + E_{b1} + E_{b2} \tag{6}$$

$$E_c = E_{c0} + E_{c1} + E_{c2} \tag{7}$$

Substituting the equivalent values as found in equations 2, 3, and 4 in equations 5, 6, and 7 gives

$$E_a = E_{a0} + E_{a1} + E_{a2} \tag{8}$$

$$E_b = E_{a0} + a^2E_{a1} + aE_{a2} \tag{9}$$

$$E_c = E_{a0} + aE_{a1} + a^2E_{a2} \tag{10}$$

Thereby expressing each of the original three-phase vectors in terms of phase (a).

Solving equations 8, 9, and 10 for E_{a0} gives

$$E_a + E_b + E_c = (1 + a^2 + a)E_{a1} + (1 + a + a^2)E_{a2} + 3E_{a0}$$

But from equation 1

$$a = -0.5 + j0.866 \text{ and } a^2 = -0.5 - j0.866$$

Therefore

$$1 + a^2 + a = 1 - 0.5 + j0.866 - 0.5 - j0.866 = 0$$

then

$$3E_{a0} = E_a + E_b + E_c$$

or

$$E_{a0} = 1/3(E_a + E_b + E_c) \tag{11}$$

Solving equations 8, 9, and 10 for E_{a1}

$$E_a = E_{a0} + E_{a1} + E_{a2} \tag{8}$$

Multiplying equation 9 by (a) gives

$$aE_b = aE_{a0} + a^3E_{a1} + a^2E_{a2} \tag{12}$$

and multiplying equation 10 by (a^2) gives

$$a^2E_c = a^2E_{a0} + a^3E_{a1} + a^4E_{a2} \tag{13}$$

Adding equations 8, 12, and 13 and remembering that $a^3 = 1$ and that $a^4 = a$, then

$$E_a + aE_b + a^2E_c = 3E_{a1} + (1 + a + a^2)E_{a0} + (1 + a^2 + a)E_{a2}$$

The last two terms being equal to zero

$$E_{a1} = 1/3(E_a + aE_b + a^2E_c) \tag{14}$$

Likewise, solving equations 8, 12, and 13 for E_{a2} gives

$$E_{a2} = 1/3(E_a + a^2E_b + aE_c) \tag{15}$$

These three equations, 11, 14, and 15, are the fundamental equations expressing each of the phase (a) sequence voltages in terms of the three line-to-neutral voltages. Expressions for the other phase sequences of voltage can be obtained by substituting equations 11, 14, and 15 in equations 2 and 3, page 8.

FAULTS ON A LOADED ALTERNATOR

The method of symmetrical components is widely used in determining the system performance of a network; for example, the current values for investigating mechanical and thermal limitations of equipment on a transmission line under conditions of unsymmetrical faults. Perhaps its most valuable use is for determining the various values of sequence currents and voltages throughout the system so that protective and control relays can be accurately set to maintain system stability. Voltage regulators operating from a positive-sequence voltage selector network will give a much closer regulation[14] under certain conditions than when using the regular line voltage. Likewise, protective relays excited by negative-sequence current[15] may be used to protect alternators from heavy single-phase fault or load currents. As interconnected transmission networks become more and more complex, better methods of relay protection are necessary, which require more accurate mechanisms and closer relay settings. In order to take full advantage of automatic relay protection, it is necessary that the protective relays be set accurately to obtain the necessary selectivity to give the proper sequence of operations in case of faults. Because of the instability effects the faults may

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[14] W. E. Slemmer, op. cit., p. 16.

[15] John Henderson, Automatic Protective Gear, p. 142. Also H. R. Searing and R. E. Powers, Sequence Principles used for Network Relaying, pp. 694-697.

have on the remainder of the system, it is necessary, not only to investigate the faults that will produce the maximum fault current, but also to investigate other types of faults.

The values of the constants that are ordinarily used in determining line regulation and current distribution under normal operating conditions are not usually the same values as those that determine the amount of fault current and degree of unbalance in line currents and voltages that occur under conditions of fault. The constants involved are usually found experimentally[16] or are determined from the physical characteristics[17] of the network. When a line in a system fed by an alternator faults, the currents that flow into the fault depend upon the characteristics of the generators, both synchronous and induction if any, and the load involved as well as the fault conditions.

In a network supplied by a single alternator, or one in which the voltages of the alternators may be assumed to be in phase, the network can usually be reduced to an equivalent single line circuit[18] consisting of a generator supplying a single load. To further simplify the solution the generator is usually considered to be operating at no load or feeding a balanced load.[19] The sequence networks from which equivalent

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[16] W. V. Lyon, Applications of The Method of Symmetrical Components, pp. 452-469.

[17] C. F. Wagner and R. D. Evans, op. cit. pp. 136-220.

[18] Ibid. pp. 53-63.

[19] Ibid. p. 53.

single line circuits can be set up for each of the three sequences may be quite different in appearance, as only the branches of the networks in which currents of that particular sequence flow are included in each network. These equivalent circuits may then be so interconnected that they meet the terminal conditions[20] of the fault. For each type of fault there will be a different plan of interconnection of one or more of the equivalent sequence circuits.

In assuming the alternator to be supplying a balanced load at the time of fault, an appreciable error is often introduced, especially if the alternator has a high negative-sequence impedance. This difference, which ranges from 2 to 24 per cent, in the majority of cases does not exceed 15 per cent.[21] The methods,[22] that were available before Sabbagh's solution was published, were so complicated and tedious that this refinement was usually neglected. Assuming the constants of a machine to be real constants, Sabbagh developed general equations[23] for the steady state currents and voltages in a wye-connected alternator supplying power to three lumped (wye) loads. This solution not only offers a useful refinement and simplification in the solution of faults involving a loaded generator, but also shows that the various types of faults

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20 Ibid. p. 30-37.

21 E. M. Sabbagh, op. cit. p. 6.

22 C. F. Wagner and S. H. Wright, Calculation of Short Circuits on Power Systems.

23 E. M. Sabbagh, op. cit. pp. 21-24.

solved by other methods (assuming no load on the alternator) and published previous to E. M. Sabbagh's dissertation are but special cases of a general problem.[24]

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[24] E. M. Sabbagh, op. cit., p. 43.

DEVELOPMENT OF GENERAL EQUATION

FOR AN ALTERNATOR CARRYING A THREE PHASE LOAD

Part I. Derivation of expressions for sequence components of load voltages in terms of load sequence impedances and sequence currents.

Let the alternator in Fig. 3 feed three lumped loads, Z'_a, Z'_b and Z'_c, connected in wye.[25] If I_a, I_b and I_c are the three line currents flowing in Z'_a, Z'_b and Z'_c respectively, then

$$V_a = I_a Z'_a \qquad (16)$$

$$V_b = I_b Z'_b \qquad (17)$$

$$V_c = I_c Z'_c \qquad (18)$$

where V_a, V_b and V_c are the three-phase load voltages. From equations 8, 9 and 10 page 9, dropping the phase (a) subscript,[26] it is found that

$$V_a = V_0 + V_1 + V_2 \qquad (19)$$

$$V_b = V_0 + a^2V_1 + aV_2 \qquad (20)$$

$$V_c = V_0 + aV_1 + a^2V_2 \qquad (21)$$

and likewise

$$I_a = I_0 + I_1 + I_2 \qquad (22)$$

$$I_b = I_0 + a^2I_1 + aI_2 \qquad (23)$$

$$I_c = I_0 + aI_1 + a^2I_2 \qquad (24)$$

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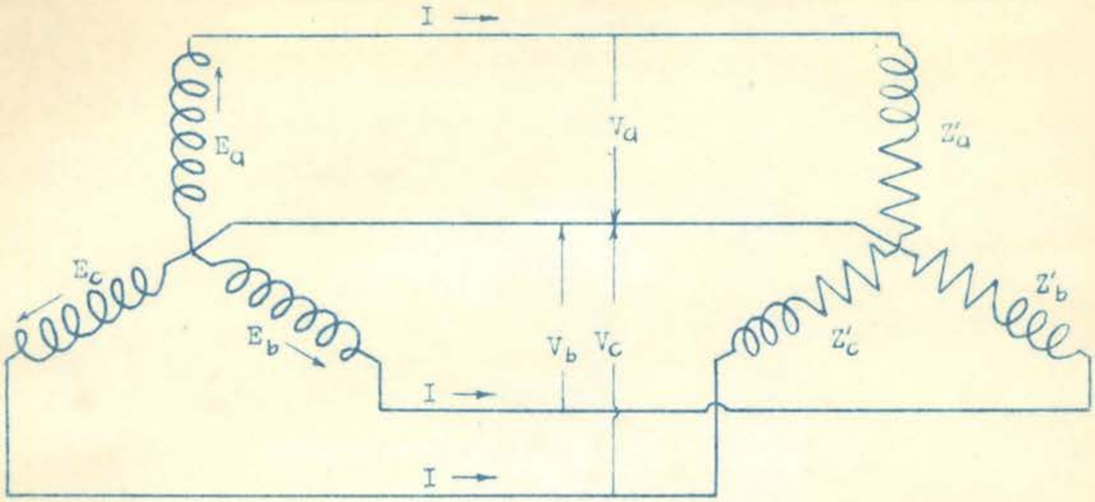
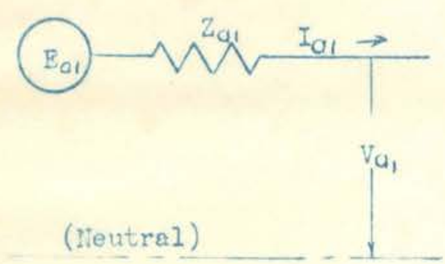
25 This may be considered a fairly general case since any network with its loads can be reduced to an equivalent wye load supplied by a single generator, provided the generators may be replaced by a single generating unit.

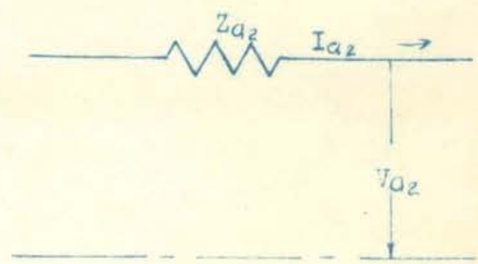
26 The phase (a) will be used through out this section and therefore the subscript (a) may be dropped.



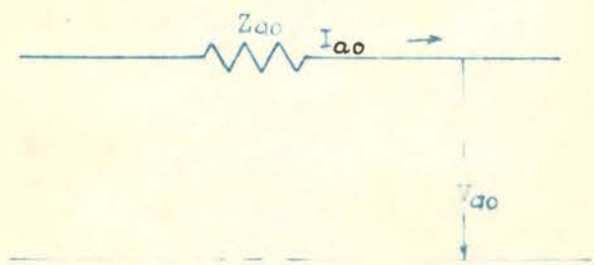
(a) Schematic diagram for generator and load.



(b) Positive Sequence



(c) Negative Sequence



(d) Zero Sequence

Fig. 3. Loaded generator with its single line sequence circuits.

Substituting these values in equations 16, 17, and 18 gives

$$V_0 + V_1 + V_2 = (I_0 + I_1 + I_2)Z'_a \tag{25}$$

$$V_0 + a^2V_1 + aV_2 = (I_0 + a^2I_1 + aI_2)Z'_b \tag{26}$$

$$V_0 + aV_1 + a^2V_2 = (I_0 + aI_1 + a^2I_2)Z'_c \tag{27}$$

Solving[27] equations 25, 26 and 27 simultaneously for V_0, V_1, and V_2 gives

$$V_0 = I_0Z'_0 + I_1Z'_2 + I_2Z'_1 \text{ [28]} \tag{28}$$

$$V_1 = I_0Z'_1 + I_1Z'_0 + I_2Z'_2 \tag{29}$$

and

$$V_2 = I_0Z'_2 + I_1Z'_1 + I_2Z'_0 \tag{30}$$

where

V_0 is the zero-sequence load voltage

V_1 is the positive-sequence load voltage

V_2 is the negative-sequence load voltage

I_0 is the zero-sequence load current

I_1 is the positive-sequence load current

I_2 is the negative-sequence load current

$$Z'_0 = 1/3(Z'_a + Z'_b + Z'_c) \tag{31}$$

$$Z'_1 = 1/3(Z'_a + aZ'_b + a^2Z'_c) \tag{32}$$

and

$$Z'_2 = 1/3(Z'_a + a^2Z'_b + aZ'_c) \text{ [29]} \tag{33}$$

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27 The complete solution is shown in Appendix A, page 33.

28 Currents and impedances of different sequences react in that part of the circuit which is unsymmetrical. (See footnote 8, also, C. F. Wagner and R. L. Evans, op. cit., pp. 162.)

29 Equations 31, 32, and 33 may be considered as expressions for the zero-, positive-, and negative-sequence impedances of the load.

Part II. Derivation of expressions for sequence components of terminal voltages in terms of alternator sequence impedances and sequence line currents.

One of the fundamentals of symmetrical components is the independence of sequences.[30] This means that the sequence components of current in a symmetrical network do not react on one another. When a voltage of a given sequence is applied to a piece of apparatus a current of the same sequence flows limited only by the impedance of that particular sequence. Since the impedances offered to the different sequences may vary with the sequence, it is often desirable, for analytical purposes, to consider each of the sequences as forming an independent circuit only retaining the parts of the original circuit in which the currents of that particular sequence flow.

By re-drawing the generator end of phase (a), Fig. 3(a), as a single line diagram for each of the three sequences, Figures 3(b), (c) and (d) are obtained. In the positive-sequence circuit (Fig. 3(b)) E_1[30] is the positive-sequence internal voltage generated by phase (a) of the alternator. Since the alternator phases are built as symmetrical as possible and since only positive-sequence currents and voltages are generated in a synchronous machine, E_1 is simply the open circuit phase voltage. Z_1 is the positive-sequence impedance of the alternator. I_1 is the positive-sequence current flowing in phase (a) of the alternator and into the line. V_1,

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30 See footnote 8, p. 3.

the terminal voltage, is equal to the generated voltage minus the (I Z) drop in the alternator windings or

$$V_1 = E_1 - I_1 Z_1 \qquad (34)$$

Since a synchronous machine does not generate a negative- or zero-sequence voltage[31] there is no source of this voltage shown in either the negative-or zero-sequence circuits, Fig. 3(c) and (d). The source of the negative and zero-sequence voltages is conventionally explained as being set up by the flow of the fault current. The flow of this fault current causes currents of all three sequences to flow in the faulted line.[32] These sequence currents give rise to voltage drops of the corresponding sequences in the symmetrical portions of the circuit. These voltages are in general a maximum at the point of fault and decrease in value as the neutral bus is approached. The negative-sequence terminal phase voltage, in Fig. 3, is determined from

$$V_2 = 0 - I_2 Z_2 \qquad (35)$$

and likewise the zero-sequence terminal phase voltage

$$V_0 = 0 - I_0 Z_0 \qquad (36)$$

Congining equations 34, 35, and 36 with equations 28, 29, and 30, gives

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31 C. F. Wagner and R. D. Evans, op. cit. p. 28.

32 Ibid. p. 33. also W. V. Lyon, op. cit. p. 483. (Although the above explanation works out beautifully, mathematically, a good clear cut physical explanation of the origin of the negative and zero sequence voltages and currents seems to be lacking in so many references, that in the opinion of the writer, such an explanation would be of much value for the student, especially.)

$$- I_0 Z_0 = I_0 Z_0' + I_1 Z_2' + I_2 Z_1' \tag{37}$$

$$- I_2 Z_2 = I_0 Z_2' + I_1 Z_1' + I_2 Z_0 \tag{38}$$

$$E_a - I_1 Z_1 = I_0 Z_1' + I_1 Z_0' + I_2 Z_2' \tag{39}$$

These three equations, 37, 38, and 39, express equality between the three sequence values of phase (a) voltage derived from the alternator end and from the load end of the original circuit, Fig. 5(a).

Combining and re-arranging the above equations

$$I_0(Z_0 + Z_0') + I_2 Z_1' + I_1 Z_2' = 0 \tag{40}$$

$$I_0 Z_2' + I_2(Z_0' + Z_2) + I_1 Z_1' = 0 \tag{41}$$

$$I_0 Z_1' + I_2 Z_2' + I_1(Z_0' + Z_1) = E_a \tag{42}$$

The solution[33] of equations 40, 41, and 42 for I_0, I_2, and I_1, gives

$$I_0 = \frac{E_a}{\Delta}\left[Z_1'^2 - Z_2'(Z_0' + Z_2)\right] \tag{43}$$

$$I_2 = \frac{E_a}{\Delta}\left[Z_2'^2 - Z_1'(Z_0 + Z_0')\right] \tag{44}$$

$$I_1 = \frac{E_a}{\Delta}(Z_0 Z_0' + Z_0 Z_2 + Z_0'^2 + Z_0' Z_2 - Z_1' Z_2') \tag{45}$$

Where

$$\Delta = Z_a' Z_b' Z_c' + 1/3(Z_a' Z_b' + Z_b' Z_c' + Z_a' Z_c')(Z_0 + Z_1 + Z_2) + 1/3(Z_a' + Z_b' + Z_c')(Z_0 Z_2 + Z_1 Z_2 + Z_0 Z_1) + Z_0 Z_1 Z_2 \tag{46}$$ [34]

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[33] The complete solution is shown in Appendix B, page 35.

[34] Attention is called to the marked symmetry of the expression for Δ.

By combining the sequence values of current into line values, the following general expressions for I_a, I_b, and I_c are obtained.

$$I_a = I_0 + I_1 + I_2 = \frac{E_a}{\Delta}(Z_1'^2 + Z_2'^2 + Z_0'^2 + Z_0Z_0' + Z_0Z_2 + Z_0'Z_2 - Z_2'Z_2 - Z_2'Z_0' - Z_1'Z_0 - Z_1'Z_0' - Z_2'Z_1') \tag{47}$$

$$I_b = I_0 + a^2I_1 + aI_2 = \frac{E_a}{\Delta}(Z_1'^2 - Z_2'Z_0' + a^2Z_0Z_0' - Z_2'Z_2 + a^2Z_0Z_2 + a^2Z_0'^2 + a^2Z_0'Z_2 - a^2Z_2'Z_1' + aZ_2'^2 - aZ_1'Z_0 - aZ_1'Z_0') \tag{48}$$

$$I_c = I_0 + aI_1 + a^2I_2 = \frac{E_a}{\Delta}(Z_1'^2 - Z_2'Z_0' - Z_2'Z_2 + aZ_0Z_0' + aZ_0Z_2 + aZ_0'^2 + aZ_0'Z_2 - aZ_2'Z_1' + a^2Z_2'^2 - a^2Z_1'Z_0 - a^2Z_1'Z_0') \tag{49}$$

The terminal voltages will, then, be

$$V_a = I_aZ_a \tag{50}$$

$$V_b = I_bZ_b \tag{51}$$

$$V_c = I_cZ_c \tag{52}$$

EXAMPLES OF FAULT CALCULATIONS USING GENERAL EQUATIONS

Example I. Assuming the load on the alternator in Fig. 3(a) to be balanced, i.e., $Z'_a = Z'_b = Z'_c = Z$, determine the line currents.

Substituting these values of load sequence impedances in equations 31, 32, and 33, page 17

$$Z'_0 = 1/3(Z + Z + Z) = Z$$

$$Z'_1 = 1/3(1 + a + a^2)Z = 0$$

$$Z'_2 = 1/3(1 + a^2 + a)Z = 0$$

and substituting in the general equations, 46, 47, 48, and 49, page 21

$$I_a = \frac{E_a(Z^2 + Z_0 Z + Z_0 Z_2 + Z Z_2)}{Z^3 + 1/3(Z^2 + Z^2 + Z^2)(Z_2 + Z_1 + Z_0) + ZZ_0Z_2 + ZZ_1Z_2 + ZZ_0Z_1 + Z_0Z_1Z_2}$$

factoring

$$I_a = \frac{E_a(Z + Z_0)(Z + Z_2)}{(Z + Z_0)(Z + Z_2)(Z + Z_1)} = \frac{E_a}{Z + Z_1}$$

Similarly

$$I_b = \frac{a^2 E_a}{Z + Z_1} \qquad \text{and } I_c = \frac{aE_a}{Z + Z_1}$$

as expected, since $E_a = E_b = E_c$ That is, the line current equals the generated voltage divided by the generator plus the positive sequence load impedances.[35]

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[35] $Z_1 = Z_a = Z_b = Z_c$ since only positive-sequence voltages are generated by the alternator and balanced loads were assumed.

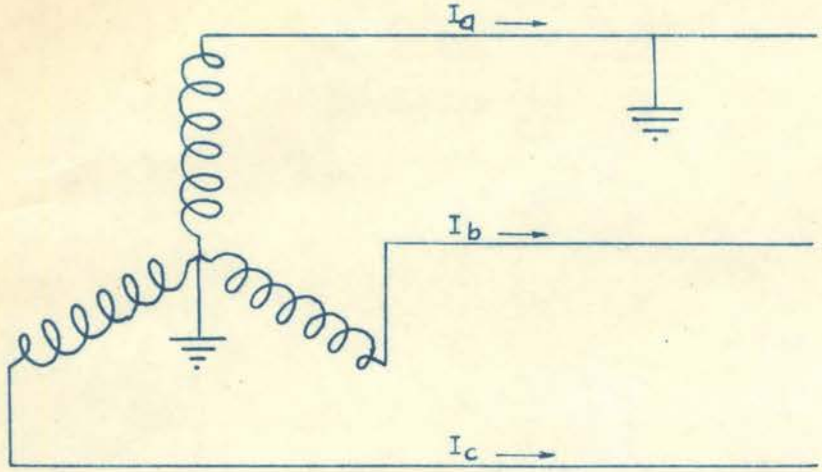

Fig. 4. Single Line-to-ground Fault. (Alternator not loaded)

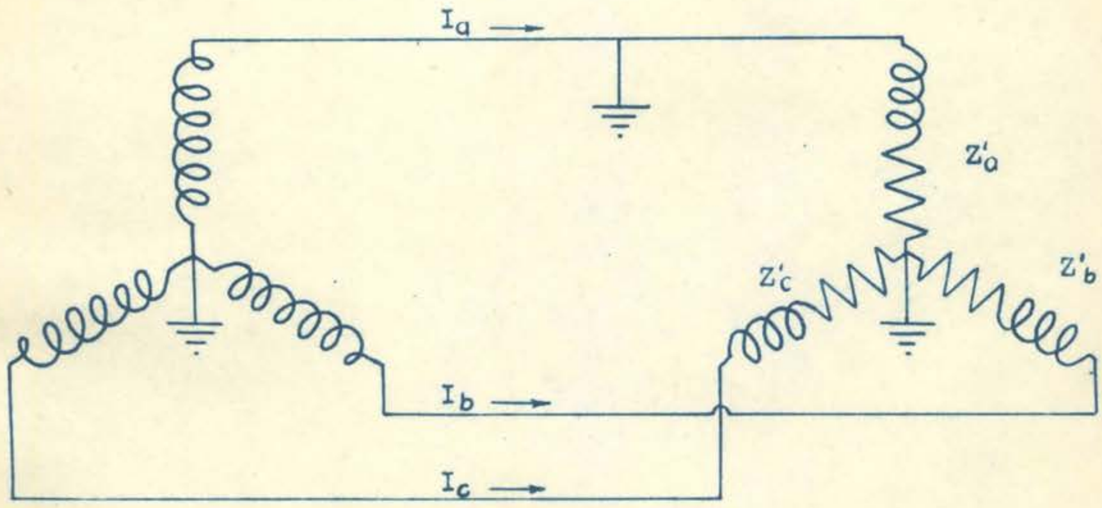

Fig. 5. Single Line-to-ground Fault. (Alternator loaded)

Example 2.

Single Line-to-ground Fault (Alternator not loaded)

Let phase (a) of a wye-connected alternator with grounded neutral be faulted to ground as shown in Fig. 4, page 23.

In the general equations, 47, 48, and 49, page 21

$Z_a = 0$

$Z_b = Z_c = Z = \infty$

$$Z_0' = 1/3(0 + 2Z) \text{ and } Z_0'^2 = 4Z^2/9 \qquad (53)$$

$$Z_1' = 1/3(0 + aZ + a^2Z) = -Z/3 \quad (\text{Since } a^2 + a\ 1 = 0)$$

$$Z_1'^2 = Z^2/9 \qquad (54)$$

$$Z_2' = 1/3(0 + a^2Z + aZ) = -Z/3$$

$$Z_2'^2 = Z^2/9 \qquad (55)$$

$$Z_2'Z_0' = -2Z^2/9 \qquad (56)$$

$$Z_1'Z_0' = -2Z^2/9 \qquad (57)$$

$$Z_2'Z_1' = Z^2/9 \qquad (58)$$

Substituting the values as expressed by equations 53, 54, 55, 56, 57, and 58 into the general equation 47 and into equation 46 for Δ gives

$$I_a = E_a \frac{1/9(Z^2 + Z^2 + 4Z^2 + 4Z^2 - Z^2) + Z_0Z_0' + Z_0Z_2 + Z_0'Z_2 - Z_1'Z_0}{Z^2/3(Z_0 + Z_1 + Z_2) + Z_0'(Z_0Z_2 + Z_1Z_2 + Z_0Z_1) + Z_0Z_1Z_2}$$

$$= E_a \frac{Z^2 + Z_0Z_0' + Z_0Z_2 + Z_0'Z_1 - Z_2'Z_0}{Z^2/3(Z_0 + Z_1 + Z_2) + Z_0'(Z_0Z_2 + Z_1Z_2 + Z_0Z_1) + Z_0Z_1Z_2}$$

Dividing the numerator and denominator by Z^2

$$I_a = E_a \frac{1 + 1/Z^2(Z_0Z_0' + Z_0Z_2 + Z_0'Z_2 - Z_1'Z_0)}{1/3(Z_0 + Z_1 + Z_2) + Z_0'/Z^2(Z_0Z_2 + Z_1Z_2 + Z_0Z_1) + 1/Z^2(Z_0Z_1Z_2)}$$

and setting $Z = \infty$

$$I = E \frac{1}{1/3(Z_0 + Z_1 + Z_2)} = E_a \frac{3}{Z_0 + Z_1 + Z_2} \qquad (59)$$

Making similar substitutions in equation 48 and remembering that the denominator, Δ, will be the same for each of the general equations

$$I_b = E_a \frac{1/9(3 + 3a^2 + 3a)Z^2 - Z_2'Z_2 + a^2 Z_0 Z_0' + a^2 Z_0 Z_2 +}{\Delta}$$

$$\frac{a^2 Z_0' Z_2 - a Z_0 Z_1'}{\Delta}$$

Dividing the numerator and denominator by Z^2 and setting $Z^2 = \infty$ and noting that as before $\Delta = 1/3(Z_0 + Z_1 + Z_2)$

$$I_b = E_a \frac{1/3(1 + a^2 + a)}{1/3(Z_0 + Z_1 + Z_2)}$$

But

$$1 + a^2 + a = 0$$

Therefore

$$I_b = 0$$

and similarly

$$I_c = 0$$

These results are in agreement with results found by other methods.[36]

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36 C. F. Wagner and R. D. Evans, op. cit. p. 44.

Example 3.

Single Line-to-ground Fault (Alternator loaded)

Let phase (a) of a wye-connected alternator, with grounded neutral and feeding a grounded neutral wye load, be faulted to ground as shown in Fig. 5, page 23.

Since phase (a) is shorted to ground,

$$Z_a' = 0$$

The equations, 31, 32, and 33, now become

$$Z_0' = 1/3(Z_b' + Z_c')$$

$$Z = 1/3(aZ_b' + a^2 Z_c')$$

$$Z = 1/3(a^2 Z_b' + aZ_c')$$

The expression for Δ becomes

$$\Delta = 1/3(Z_b' Z_c')(Z_0 + Z_1 + Z_2) + Z_0'(Z_1 Z_2 + Z_1 Z_2 + Z_0 Z_1) + Z_0 Z_1 Z_2$$

The general expressions for the line currents and phase voltages remain unchanged, as given on page 21.

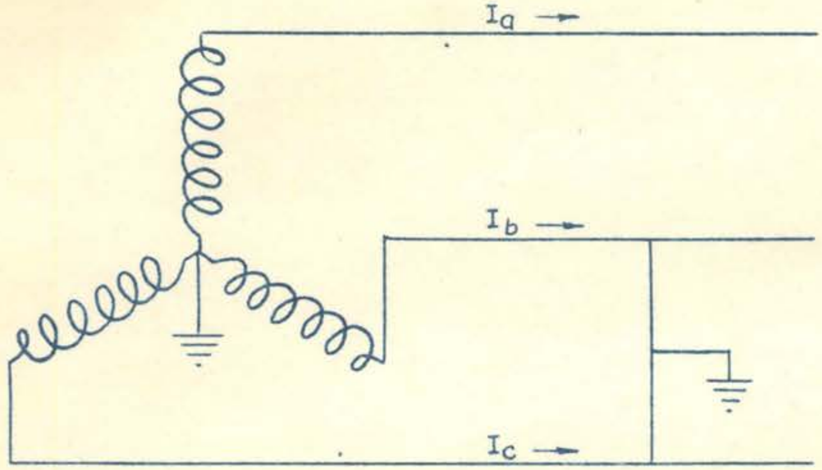

Fig. 6. Double Line-to-ground Fault. (Alternator not loaded)

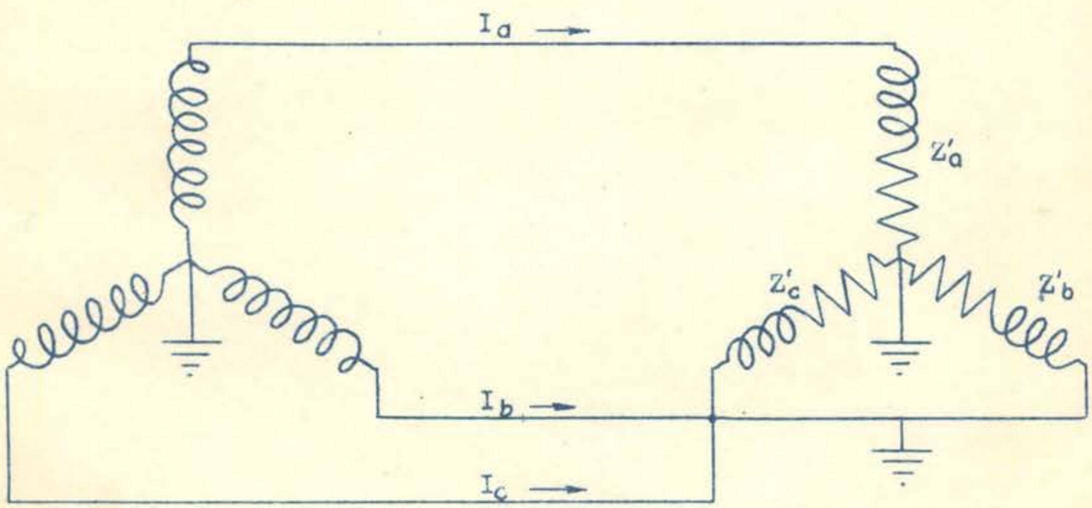

Fig. 7. Double Line-to-ground Fault. (Alternator loaded)

Example 4.

Double Line-to-ground Fault. (Alternator not loaded)

Let phases (b) and (c) of a grounded neutral wye connected alternator be faulted to ground as shown in Fig. 6, page 27.

Since phases (b) and (c) are shorted to ground

$$Z_b = Z_c = 0$$

$$Z_a = \infty$$

The equations, 31, 32, and 33, now become

$$Z_0' = Z_1' = Z_2' = Z_a'/3$$

and

$$\Delta = Z_a/3(Z_0Z_2 + Z_1Z_2 + Z_0Z_1) + Z_1Z_2Z_0$$

Substituting in general equation, 47

$$I_a = \frac{E}{\Delta}(Z_a'^2/9 + Z_a'^2/9 + Z_a'^2/9 + Z_0Z_a'/3 + Z_0Z_2 + Z_2Z_a'/3 -$$

$$Z_2Z_a'/3 - Z_a'^2/9 - Z_0Z_a'/3 - Z_a'^2/9 - Z_a'^2/9)$$

$$= \frac{E_a3(Z_0Z_2)}{Z_a'(Z_0Z_2 + Z_1Z_2 + Z_0Z_1) + 3Z_0Z_1Z_2}$$

Dividing both numerator and denominator by Z and setting $Z_a' = \infty$

$$I_a = 0$$

Similarly

$$I_b = \frac{E_a}{\Delta}(Z_a'^2/9 - Z_a'^2/9 - Z_2Z_a'/3 + a^2Z_0Z_a'/3 + a^2Z_0Z_2 +$$

$$a^2Z_a'^2/9 + a^2Z_2Z_a'/3 - a^2Z_a'^2/9 + aZ_1^2/9 - aZ_0Z_a'/3 -$$

$$aZ_a^2/9)$$

$$= E_a\frac{Z_a'(-Z_2 + a^2Z_0 + a^2Z_2 - aZ_0) + 3a^2Z_0Z_2}{Z_a'(Z_0Z_2 + Z_1Z_2 + Z_0Z_1) + 3Z_0Z_1Z_2}$$

Dividing both numerator and denominator by Z'_a, setting $Z'_a = \infty$, and collecting terms gives

$$I_b = E_a \frac{Z\,(a^2 - 1) + Z\,(a^2 - a)}{Z_0 Z_2 + Z_1 Z_2 + Z_0 Z_1}$$

By a similar process

$$I_c = E_a \frac{Z_2(a - 1) + Z_0(a - a^2)}{Z_0 Z_2 + Z_1 Z_2 + Z_0 Z_1}$$

These results are in agreement with results found by other methods.[37]

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[37] Ibid., p. 46.

Example 5.

Double Line-to-ground Fault (Alternator loaded)

Let phases (b) and (c) of a grounded neutral wye connected alternator, feeding a grounded neutral wye load, be faulted to ground as shown in Fig. 7, page 27. Since phases (b) and (c) are shorted to ground

$$Z_b' = Z_c' = 0$$

The equations 31, 32 and 33, now become

$$Z_0' = 1/3(Z_a')$$

$$Z_1' = 1/3(Z_a')$$

$$Z_2' = 1/3(Z_a')$$

and the expression for Δ becomes

$$\Delta = 1/3\left[Z_a'(Z_0 Z_2 + Z_1 Z_2 + Z_0 Z_1)\right] + Z_0 Z_1 Z_2$$

The general expression for I_a, 46, becomes

$$I_a = \frac{E_a}{\Delta}(Z_a'^2/9 + Z_a'^2/9 + Z_a'^2/9 + Z_0 Z_a'/3 + Z_2 Z_0 + Z_2 Z_a'/3 -$$

$$Z_0 Z_a'/3 - Z_a'^2/9 - Z_2 Z_a'/3 - Z_a'^2/9 - Z_a'^2/9)$$

$$= E_a \frac{3\, Z_0 Z_2}{Z_a' Z_0 Z_2 + Z_a' Z_1 Z_2 + Z_a' Z_1 Z_0 + 3 Z_0 Z_1 Z_2}$$

Similarly

$$I_b = \frac{E_a}{\Delta}(Z_a'^2/9 - Z_a'^2/9 - Z_2 Z_a'/3 + a^2 Z_0 Z_a'/3 + a^2 Z_0 Z_2 +$$

$$a^2 Z_a'^2/9 + a^2 Z_2 Z_a'/3 - a^2 Z_a'^2/9 + a Z_a'^2/9 - a Z_a' Z_0/3 -$$

$$a Z_a'^2/9)$$

$$= E_a \frac{Z_a' Z_2(a^2 - 1) + Z_a' Z_0(a^2 - a) + 3\, a^2 Z_0 Z_2}{Z_a' Z_0 Z_2 + Z_a' Z_1 Z_2 + Z_a' Z_1 Z_0 + 3\, Z_0 Z_1 Z_2}$$

By a similar process, it is found that

$$I_c = E_a \frac{Z_a' Z_2(a - 1) + Z_a' Z_0(a - a^2) + 3\, a\, Z_0 Z_2}{Z_a' Z_0 Z_2 + Z_a' Z_1 Z_2 + Z_a' Z_1 Z_0 + 3\, Z_0 Z_1 Z_2}$$

ADVANTAGES OF USING THE GENERAL EQUATIONS

When the fault is a very simple case, such as a three phase fault, conditions are still symmetrical and conventional methods of solution can be used. If the fault is supplied by an alternator operating at no load, the well known symmetrical component formulas can be used advantageously. However, in a case where the fault or simultaneous faults are supplied by a loaded alternator, the above mentioned methods become inadequate and the general equations developed in this thesis offer the only simple and direct solution to the problem known to the author.

The generality of this solution is more apparent when one considers that if the open circuit voltage of the alternator and the impedances of the load, generator, fault, etc., are known, the general expressions for the currents in the three phases of the alternator and line can be evaluated.

SUMMARY

A general method of solution for faults on a loaded alternator has been developed by use of the fundamental principles of symmetrical components. Several examples were solved by means of the general equations showing that they can be used when assuming either balanced or unbalanced load conditions. The various types of faults on alternators with balanced loads, usually treated in texts and references, are but special cases of a general problem.

Had a more detailed treatment been advisable, problems could have been solved quantitatively showing the variations in the results secured when the load on the alternator was considered. However, attention has been directed to the advantages of using the general solutions outlined by E. M. Sabbagh and developed in detail in this thesis.

APPENDIX A

The solution for V_0, V_1, and V_2 from the equations taken from page 17 follows:

$$V_0 + V_1 + V_2 = (I_0 + I_1 + I_2)Z'_a \tag{25}$$

$$V_0 + a^2V_1 + aV_2 = (I_0 + a^2I_1 + aI_2)Z'_b \tag{26}$$

$$V_0 + aV_1 + a^2V_2 = (I_0 + aI_1 + a^2I_2)Z'_c \tag{27}$$

Using determinants:

Let the coefficients of the unknowns, V_0, V_1, and V_2 be as follows:

$$\begin{array}{llll} a_1 = 1 & b_1 = 1 & c_1 = 1 & d_1 = (I_0 + I_1 + I_2)Z'_a \\ a_2 = 1 & b_2 = a^2 & c_2 = a & d_2 = (I_0 + a^2I_1 + aI_2)Z'_b \\ a_3 = 1 & b_3 = a & c_3 = a^2 & d_3 = (I_0 + aI_1 + a^2I_2)Z'_c \end{array}$$

$$V_0 = \frac{\begin{matrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{matrix}}{\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}}$$

$$= \frac{d_1(b_2c_3 - b_3c_2) - b_1(d_2c_3 - c_2d_3) + c_1(d_2b_3 - b_2d_3)}{a_1(b_2c_3 - b_2c_3) - b_1(a_2c_3 - a_3c_c) + c_1(a_2b_3 - b_2a_3)}$$

Substituting

$$V_0 = \frac{(a - a^2)(I_0 + I_1 + I_2)Z'_a + (a - a^2)(I_0 + a^2I_1 + aI_2)Z'_b +}{3(a - a\ )}$$

$$\frac{(a - a^2)(I_0 + aI_1 + a^2I_2)Z'_c}{3(a - a^2)}$$

$$V_0 = 1/3\left[Z'_a(I_0 + I_1 + I_2) + Z'_b(I_0 + a^2I_1 + aI_2) + Z'_c(I_0 + aI_1 + a^2I_2)\right]$$

$$= 1/3\left[I_0(Z'_a + Z'_b + Z'_c) + I_1(Z'_a + a^2Z'_b + aZ'_c) + I_2(Z'_a + aZ'_b + a^2Z'_c)\right]$$

Letting

$$Z'_0 = 1/3(Z'_a + Z'_b + Z'_c)$$

$$Z'_1 = 1/3(Z'_a + aZ'_b + a^2Z'_c)$$

$$Z'_2 = 1/3(Z'_a + a^2Z'_b + aZ'_c)$$

be the zero, positive, and negative sequence impedances, respectively

Then

$$V_0 = I_0Z'_0 + I_1Z'_2 + I_2Z'_1 \qquad (28)$$

and similarly

$$V_1 = I_0Z'_1 + I_2Z'_2 + I_1Z'_0 \qquad (29)$$

and

$$V_2 = I_0Z'_2 + I_2Z'_0 + I_1Z'_1 \qquad (30)$$

APPENDIX B

The solution for I_o, I_2 and I_1 from these equations taken from page 20 follows:

$$I_o(Z_o + Z'_o) + I_2(Z'_1) + I_1(Z'_2) = 0 \qquad (40)$$

$$I_o(Z'_2) + I_2(Z'_o + Z_2) + I_1(Z'_1) = 0 \qquad (41)$$

$$I_o(Z'_1) + I_2(Z'_2) + I_1(Z'_o + Z_1) = E_a \qquad (42)$$

Let the coefficients of the unknowns, I_o, I_1 and I_2 be as follows:

$$a_1 = Z_o + Z'_o \qquad b_1 = Z'_1 \qquad c_1 = Z'_2 \qquad d_1 = 0$$

$$a_2 = Z'_2 \qquad b_2 = Z'_o + Z_2 \qquad c_2 = Z'_1 \qquad d_2 = 0$$

$$a_3 = Z'_1 \qquad b_3 = Z'_2 \qquad c_3 = Z'_o + Z_1 \qquad d_3 = E_a$$

(continued on the right)

Then using determinants

$$I_o = \frac{\begin{matrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{matrix}}{\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}}$$

Expanding

$$I_o = \frac{d_1(b_2c_3 - b_3c_2) - b_1(d_2c_3 - c_2d_3) + c_1(d_2b_3 - b_2d_3)}{a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - b_2a_3)}$$

Substituting

$$I_o = \frac{-Z'_1(-Z'_1E_a) + Z'_2(-Z'_o - Z_2)E_a}{(Z_o + Z'_o)(Z'_o + Z_2)(Z'_o + Z_1) - (Z_o + Z'_o)Z'_1Z'_2 - Z'_1Z'_2(Z'_o + Z_1) + Z'^3_1 + Z'^3_2 - Z'_2Z'_1(Z'_o + Z_2)} = \frac{E_a[Z'^2_1 - Z'_2(Z'_o + Z_2)]}{\Delta}$$

Letting Δ = the denominator and expanding

$$\Delta = Z_oZ'^2_o \;\; Z_oZ'_oZ_1 + Z_oZ'_oZ_2 + Z_oZ_1Z_2 + Z'^3_o + Z'^2_oZ_1 + Z'^2_oZ_2 + Z'_oZ_1Z_2 - Z_oZ'_1Z'_2 - Z'_oZ'_1Z'_2 - Z'_oZ'_1Z'_2 - Z_1Z'_1Z'_2 + Z'^3_1 + Z'^3_2 - Z'_oZ'_1Z'_2 - Z_2Z'_1Z'_2$$

Re-arranging and separating into members so that $\Delta = A + B + C$ where

$$A = Z'^3_o + Z'^3_1 + Z'^3_2 - 3(Z'_oZ'_1Z'_2), \quad B = Z_oZ'^2_o + Z_2Z'^2_o + Z_1Z'^2_o - Z_oZ'_1Z'_2 - Z_2Z'_1Z'_2 - Z_1Z'_1Z'_2 \text{ and } C = Z'_oZ_oZ_2 + Z'_oZ_oZ_1 + Z'_oZ_2Z_1 + Z_oZ_1Z_2$$

Substituting $Z'_o = 1/3(Z'_a + Z'_b + Z'_c)$, $Z'_1 = 1/3(Z'_a + aZ'_b + a^2Z'_c)$ and $Z'_2 = 1/3(Z'_a + a^2Z'_b + aZ'_c)$

$$Z'^3_o = 1/27\left[Z'^3_a + Z'^3_b + Z'^3_c + 3(Z'^2_aZ'_b + Z'^2_aZ'_c + Z'^2_bZ'_a + Z'^2_bZ'_c + Z'^2_cZ'_a + Z'^2_cZ_2) + 6Z_aZ_bZ_c\right]$$

$$Z'^3_1 = 1/27\left[Z'^3_a + Z'^3_b + Z'^3_c + 3(aZ'^2_aZ'_b + a^2Z'^2_aZ'_c + a^2Z'^2_bZ'_a + aZ'^2_bZ'_c + a^2Z'^2_cZ'_b + aZ'^2_cZ'_a) + 6Z'_aZ'_bZ'_c\right]$$

$$Z'^3_2 = 1/27\left[Z'^3_a + Z'^3_b + Z'^3_c + 3(a^2Z'^2_aZ'_b + aZ'^2_aZ'_c + aZ'^2_bZ'_a + a^2Z'^2_bZ'_c + aZ'^2_cZ'_b + a^2Z'^2_cZ'_a) + 6Z'_aZ'_bZ'_c\right]$$

$$-3(Z'_oZ'_1Z'_2) = -1/9(Z'^3_a + Z'^3_b + Z'^3_c) + (Z'_aZ'_bZ'_c)(3a + 3a^2) \qquad \text{Since } (a + a^2 + 1) = 0$$

Adding

$$A = 1/9\left[(Z'^3_a + Z'^3_b + Z'^3_c) - (Z'^3_a + Z'^3_b + Z'^3_c)\right] + 2/3(Z'_aZ'_bZ'_c) - 1/3(a + a^2)(Z'_aZ'_bZ'_c) = 1/3(2 - a - a^2)(Z'_aZ'_bZ'_c) = Z'_aZ'_bZ'_c$$

Likewise

$$B = 1/9\Big\{Z_o\left[Z'^2_a + Z'^2_b + Z'^2_c + 2(Z_aZ_b + Z_aZ_c + Z_bZ_c) - (Z'^2_a + Z'^2_b + Z'^2_c) - (a + a^2)(Z'_aZ'_b + Z'_aZ'_c + Z'_bZ'_c)\right] + Z_23(Z'_aZ'_b + Z'_aZ'_c + Z'_bZ'_c) -$$
$$(Z_1 + Z_2)(Z'^2_a + Z'^2_b + Z'^2_c + (a + a^2)(Z'_aZ'_b + Z'_aZ'_c + Z'_bZ'_c) + 3Z_1(Z'_aZ'_b + Z'_aZ'_c + Z'_bZ'_c)\Big\}$$

$$B = 1/9(Z_o + Z_1 + Z_2)(Z'_aZ'_b + Z'_bZ'_c + Z'_aZ'_c)(2 - a - a^2) = 1/3(Z_o + Z_1 + Z_2)(Z'_aZ'_b + Z'_bZ'_c + Z'_aZ'_c)$$

$$C = 1/3(Z'_a + Z'_b + Z'_c)(Z_oZ_2 + Z_1Z_o + Z_1Z_2) + Z_oZ_1Z_2$$

and finally

$$\Delta = A + B + C = Z'_aZ'_bZ'_c + 1/3(Z_o + Z_1 + Z_2)(Z'_aZ'_b + Z'_bZ'_c + Z'_aZ'_c) + 1/3(Z'_a + Z'_b + Z'_c)(Z_oZ_2 + Z_oZ_1 + Z_1Z_2) + Z_oZ_1Z_2$$

Therefore

$$I_o = \frac{E_a}{\Delta}\left[(Z'^2_1 - Z'_2(Z'_o + Z_2)\right]$$

(The expression for Δ which is the common denominator for the three unknowns now being known.)

Similarly

$$I_2 = \frac{E_a}{\Delta}\left[(Z'^2_2 \;\; Z'_1(Z_o + Z'_o)\right]$$

and

$$I_1 = \frac{E_a}{\Delta}(Z_oZ'_o + Z_oZ_2 + Z'^2_o + Z'_oZ_2 - Z'_2Z'_1)$$

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