

UNIVERSITY OF OKLAHOMA
GRADUATE COLLEGE

PASSIVE ACTUATION OF A PLANETARY ROVER TO
ASSIST SANDY SLOPE TRAVERSE

A THESIS
SUBMITTED TO THE GRADUATE FACULTY
in partial fulfillment of the requirements for the
Degree of
MASTER OF SCIENCE

By
JEREMY DYLAN SMITH
Norman, Oklahoma
2016

PASSIVE ACTUATION OF A PLANETARY ROVER TO
ASSIST SANDY SLOPE TRAVERSE

A THESIS APPROVED FOR THE
SCHOOL OF AEROSPACE AND MECHANICAL ENGINEERING

BY

Dr. David Miller, Chair

Dr. Kuang-Hua Chang

Dr. Kurt Gramoll

© Copyright by JEREMY DYLAN SMITH 2016
All Rights Reserved.

DEDICATION

To my loving parents that have enabled this wonderful journey of inquiry, realization, and actualization. Without you, I'd just be me. I'm proud to be your son, and I strive every day to impress you and make you proud. I pray this work allows me to show you a small portion of the gratitude I have for your love and support.

Acknowledgments

First of all, I would like to thank Dr. Miller for this tutelage on this journey. The experience has been formative, enlightening, and beneficial.

I would like to acknowledge my fellow lab-mates Misters Ghazi, Nash, and Walker. Also to my fellow classmates who've enriched my experience both in undergraduate and graduate coursework. Thank you.

Dr. Brad Perry, you were a guiding light in my academic pursuits. I'm glad to be able to call you my friend.

Sophia Morren and the McNair Scholars Program. Thank you for the numerous types of support and sage advice offered throughout my academic experience at The University of Oklahoma.

Finally, I would like to thank the AME faculty at the University of Oklahoma for being such good teachers and professors. It is on your shoulders we students, as future engineers and the leaders of tomorrow, stand. Without your expertise and guidance, we would surely be lost.

Contents

1	Introduction	1
1.1	Unmanned Land-Based Exploration	1
1.2	Current Solar Rover 2 Locomotion	2
1.3	Hypothesis and Proposed Solution	5
1.4	Thesis Overview	8
1.4.1	Chapter 2: Related Work	8
1.4.2	Chapter 3: The Wheeled Problem	8
1.4.3	Chapter 4: Legged Solution Analysis	8
1.4.4	Chapter 5: Proposed Mechanical Design	9
1.4.5	Chapter 6: Computer Program Simulation	9
1.4.6	Chapter 7: Summary and Future Work	9
1.4.7	In summation	10
2	Related Work	11
2.1	Biomechanical Locomotion Analogue	12
2.2	Hybrid Mobility	14
2.2.1	WorkPartner Rolling	14
2.2.2	Marsokhod	15
2.2.3	Hybrid Legged Wheelchair	18
3	Wheeled Locomotion Analysis	23
3.1	Sojourner	24
3.2	Friction and Tractive Effort	24
3.2.1	Terrain Failure Criteria	25
3.2.2	Terramechanics of Wheeled Locomotion	28
4	Legged Locomotion Analysis	32
4.1	Biomechanical Analogue	32
4.2	Rigid Body Kinematics	37
4.2.1	Body Forces and Assumptions	37
4.2.2	Diagramming	38

5	Mechanical Design	53
5.1	System Design and Simulation Cases	54
5.2	Regolith Contact Foot Design	58
5.2.1	Terramechanic Theory	59
5.2.2	Grousers	60
5.2.3	Foot Design Solution	61
5.3	Hinge Design	67
5.3.1	SolidWorks Simulation Parameters	69
5.4	Actuator Thrust Rod Design	74
6	Program	78
6.1	Assumptions	79
6.1.1	Interfaces and Contacts	81
6.2	Parameter Definitions	81
6.3	Program Structure	83
6.3.1	Function Calls	85
6.4	Main Loop Sequence	85
6.4.1	Force Calculations	90
6.4.2	Settling Loop	91
6.4.3	Cycle Check	93
6.4.4	Tip Check	94
6.5	Beginning in static equilibrium	95
6.5.1	Case 1: Flat Traverse	95
6.5.2	Case 2: Five Degree Incline Traverse	99
6.5.3	Case 3: Thirty Degree Incline Traverse	101
6.5.4	Case 4: Forty-Five Degree Incline Traverse	107
7	Summary and Future Work	109
7.1	Future Work	110

List of Figures

1.1	Solar Rover 2 in Anza Borrego [18]	4
1.2	Proposed design cycle for flat traverse	7
1.3	Proposed design cycle for an inclined traverse	7
2.1	WorkPartner robot traversing soft, deep snow [26]	16
2.2	Marsokhod rover on sandy traverse [25]	17
2.3	Marsokhod “wheel-walking” (right to left traverse) [10]	19
2.4	Penn hybrid wheelchair exhibiting four-stage climbing sequence [11]	21
3.1	Experimental fixture demonstrating repose angle on cohesiveless tailings of JSC-1A lunar regolith [21]	26
3.2	Bekker terramechanic model for an individual wheel [3]	29
3.3	Better theory for wheels in tandem [3]	30
4.1	Kinematic behavior of peristaltic crawling [16]	34
4.2	Passive hinge collapse failure: obtuse collapse	36
4.3	Passive hinge collapse failure: acute collapse	36
4.4	Diagram of modified SR2, static on flat traverse	39
4.5	Diagram of SR2, actuated on flat traverse	41
4.6	Diagram of SR2 showing ideal passive hinge collapse behavior due to forward motion of the vehicle	42
4.7	Diagram of SR2 showing failure of passive hinge collapse behavior due to rearward motion of the vehicle	44
4.8	Diagram of SR2 elevated on inclined traverse showing failure of passive hinge collapse behavior due to rearward motion of the vehicle	45
4.9	Diagram of SR2 with high mounted actuator on flat traverse	46
4.10	Diagram of SR2 with high-mounted actuator elevated on inclined traverse	47
4.11	Diagram of SR2 with high-mounted actuator elevated on inclined traverse with a wheel torque applied to front driven wheels	48
4.12	Diagram of SR2 with high-mounted actuator on inclined traverse showing equivalence of passive actuator angle and traverse incline angle	51

5.1	SR2 Rover weight transfer on an inclined traverse	55
5.2	Vertical space interactions between components	57
5.3	Soil thrust for a solid foot and spaced grousers [4]	62
5.4	Regolith foot design, top view showing required tracked area for regolith safe working load	64
5.5	Regolith foot design, side and front views	65
5.6	Pressure force application (red arrows) to proposed foot	66
5.7	Regolith foot Von Mises stress plot (ksi)	66
5.8	Regolith foot stress detail view (ksi)	67
5.9	Regolith contact foot fatigue life	68
5.10	Mounting hinge safety factor plot for flat static force simulation, side view	70
5.11	Mounting hinge safety factor plot for inclined static force simula- tion, side view	71
5.12	Mounting hinge dimensions (inches), front view	72
5.13	Mounting hinge dimensions (inches), right view	73
5.14	Proposed Assembly: Diagram of SR2 on inclined traverse with grouser foot retracted	76
5.15	Proposed Assembly: Diagram of SR2 on inclined traverse with grouser foot penetrating terrain	76
6.1	Vehicle representation with physical parameters defined	84
6.2	Simplification of the system with relevant angles labeled	87
6.3	Simplification showing exaggerated forward pitching behavior	88
6.4	Simplification showing forces for static analysis	88
6.5	Simplification showing angle χ for settled condition	90
6.6	Inclining vehicle to show congruent angles ρ and χ	92
6.7	Settling loop showing kinematic behavior ρ and χ	93
6.8	Case 1: Flat rover and terrain orientation	96
6.9	Case 1 Result for X Motion	97
6.10	Case 1 Result for Y Motion	98
6.11	Case 1 Combined Motion	100
6.12	Case 2: Five degree rover and terrain orientation	101
6.13	Case 2: Five Degree Incline Result X Motion	102
6.14	Case 2: Five Degree Incline Result Y Motion	103
6.15	Case 3: Thirty degree rover and terrain orientation	104
6.16	Case 3: Thirty Degree Incline Result X Motion	105
6.17	Case 3: Thirty Degree Incline Result Y Motion	106

List of Tables

6.1	Vehicle Physical Parameter Definitions	82
6.2	Simulation Parameter Definitions	83
6.3	Function Calls and Descriptions	85

Abstract

This thesis introduces the design of a novel locomotive methodology. The problem being addressed is the traverse of unmanned locomotion over sandy inclined traverses. This is a special terramechanical issue regarding terrain or regolith that is non-cohesive in nature. The method uses a planetary exploration rover, Solar Rover 2 as its base.

The proposed solution methodology includes a passively-actuated leg affixed to the rover to assist in slope traversal. Proposed physical implementations are designed and virtual representations are created, studied, and simulated in SolidWorks. This solution is justified through the use of a simulation designed in MATLAB.

Chapter 1

Introduction

In the following work, we aim to explore and remedy traversal issues regarding wheeled vehicular mobility on certain terrains. We begin with an introduction chapter which will serve to first describe the nature of terrestrial exploration, including a discussion regarding the importance of having a reliable mobility scheme. To highlight specific problems we will introduce the locomotive scheme on a modern unmanned research vehicle. Thereafter we will present the main research inquiry and hypothesis, followed by an overview of each of the chapters in this work.

1.1 Unmanned Land-Based Exploration

The first physical contact mankind makes with extraterrestrial bodies is in the form of unmanned vehicles. Rovers explore the surface of these bodies while

conducting scientific experiments. Scientific objectives take place on-site and may be great distances apart, necessitating good mobility practices. If a rover were to be stuck, it would prevent conducting scientific research at different places on the planets. In such a scenario, science is limited to a single location, which defeats the purpose of having a mobile platform in the first place.

1.2 Current Solar Rover 2 Locomotion

Solar Rover 2 is a self-contained solar powered rover platform emphasizing drive-train simplicity and a high degree of mobility over rough terrain [18]. The Solar Rover 2 platform has a two-motor drive-train. Each of the motors power a pair of wheels which are mounted on either side of the rover. The rover utilizes a skid-steer driving methodology, which utilizes a differential velocity of the wheels on opposite sides of the vehicle to initiate a turn.

Solar Rover 2 (hereafter SR2) is a solar powered autonomous planetary rover which was designed and built at the Intelligent Robotics Laboratory at the University of Oklahoma in collaboration with Malin Space Science Systems. The primary objective for SR2 is to explore the feasibility of autonomous traverses exceeding a kilometer through a Mars-like terrain [14]. As such, it is outfitted with a solar powered control system, drive-train, and with a suspension designed for off-road capabilities. SR2 was designed with a rocker suspension. This type of suspension allows for ground force distribution through the four wheels. This

is accomplished by dictating the motion the wheels on the opposing sides. The wheels on the left side of the rover rotate with respect to the rover chassis, about the axle, equal and opposite the rotation of the wheels on the opposing side. This is possible by use of a compact differential mechanism.

When on a level surface, the passive nature of the suspension on Solar Rover 2 allows equal weight distribution to each of the four wheels. Tractive issues may arise when SR2 attempts to climb even meager sandy slopes. Traverse of an unmanned exploratory vehicle introduces risks not common to manned vehicles. The often expensive science platform must be piloted with extra care and caution, particularly if exploring an extraterrestrial body. Unmanned vehicles are most often not equipped to conduct self-repair in the event of equipment failure or traverse hindrance. In the event of granular or sandy slope traverse, an unmanned vehicle may easily become stuck.

Field testing was conducted on the long-range capabilities of Solar Rover 2 near the Salton Sea in southern California. At the time SR2 was utilizing dead reckoning, which would be degraded if the rover was allowed to slip [14]. Because of this, a conservative value for maximum traverse slope was set at fifteen percent (15%), even though the design is theoretically capable of slopes in excess of twenty-five degrees (equivalently, 46.6%) [14]. To improve the long-range accuracy of SR2, the rover's traverse capabilities may be improved. Figure 1.1 shows SR2 during field testing in the Salton Sea region.



Figure 1.1: Solar Rover 2 in Anza Borrego [18]

1.3 Hypothesis and Proposed Solution

The nature of unmanned rover exploration places heavy emphasis on the locomotion of the vehicle. Traverse issues commonly occur when vehicles are on sandy or soft terrain such as sand. In these conditions they may fail to have a solid tractive condition with the traverse media. To address rover mobility on sandy terrain, the maximum traverse angle the rover is capable of must be improved. The proposed solution to the traverse issue is a hybrid locomotion scheme made possible by the implementation of an actuated leg mechanism. The following contributions to this proposed solution include a computer simulation and system design.

The computer simulation is of the rover completing an inclined traverse. Included in this simulation, a physical system composed of an actuated leg, passively-mounted, is introduced. The leg would be mounted on a simple hinge, or pin joint, and the leg would be controlled only in the modes of its extension and retraction. The proposed implementation to Solar Rover 2 could greatly improve the vehicle's traverse capabilities. The passively-mounted actuator should affect vehicle dynamics to improve traverse capabilities up sandy inclines.

The proposed design of the hybrid locomotion scheme implements a leg which maintains a passive behavior, meaning the leg remains uncontrolled in a rotational sense. This simply means that it is pinned to the rear of the SR2 chassis; there should be no motivation regarding the angle of the actuator. The actua-

tor improves the tractive behavior of the rover when it lengthens. The proposed foot affixed to the end of the leg is to be designed in such a way that it provides a fixed anchoring point for the rover. This fixed point allows the rover to be elevated in the rear, favorably affecting the dynamic situation.

By increasing the angle of the rover with respect to gravity, the components of force on the vehicle due to gravity, which resist forward traverse, may be overcome.

Implementing an actuated foot on the rover, as shown in Figure 1.2, may alleviate tractive issues. The cycle initiates when the rover becomes stuck in some sand, Fig. 1.2 a. Lengthening of the leg instigates a forward velocity of the rover, represented by a red arrow in Fig. 1.2 b. As the actuator continues to lengthen, the rover chassis settles down to maintain wheeled contact with the traverse, Fig. 1.2 c. And once the actuator reaches maximum length, it retracts again to begin a new cycle, Fig. 1.2 d.

This proposed design may also be beneficial for inclined traverses on similar terrain. The same proposed behavior occurs on an inclined traverse, with some small caveats to be shown later. This proposed design behavior is shown in Figure 1.3, a. - d.

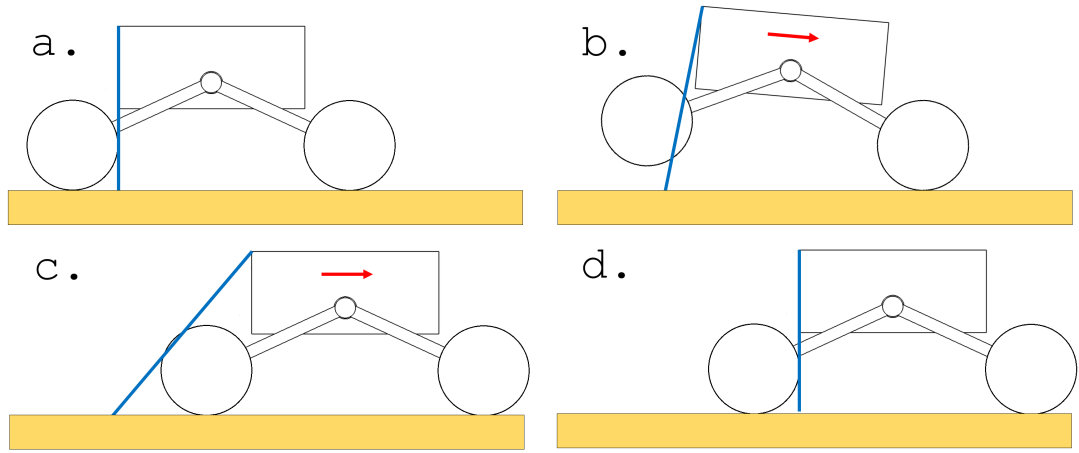


Figure 1.2: Proposed design cycle for flat traverse

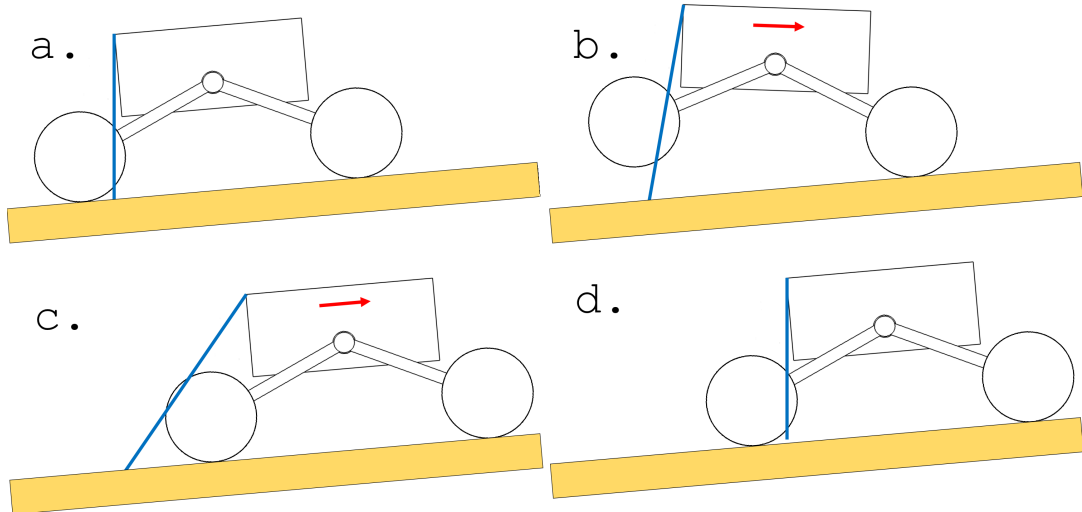


Figure 1.3: Proposed design cycle for an inclined traverse

1.4 Thesis Overview

1.4.1 Chapter 2: Related Work

Chapter 2 will review the background theories and relevant research related to the hybrid mobility rover. We will start by reviewing biological locomotion. Potential analogues to animal motion are introduced in Section 2.1. We will then review a number of hybrid locomotion implementations similar in scope to the proposed methodology.

1.4.2 Chapter 3: The Wheeled Problem

Chapter 3 sets the framework by which our mobility problem is defined. Wheeled, unmanned exploration rovers such as Solar Rover 2 (SR2) encounter terrain that is less than desirable for traverse. These terrains have a number of characteristic properties which can influence the relative ease or difficulty of traversal. This chapter will provide analysis which will focus on a particular case: when the path is composed of a sandy inclined traverse. Sandy and granular soils are special problems when compared with a common soil or traverse.

1.4.3 Chapter 4: Legged Solution Analysis

Chapter 4 evaluates the possible implementation of an actuated leg to assist sandy traverse. The leg combined with a default wheeled locomotion gives the rover a hybrid locomotion scheme. This proposed hybrid locomotion methodol-

ogy is explained, and reveals the physics and conditions required for successful inclined traverse.

1.4.4 Chapter 5: Proposed Mechanical Design

Chapter 5 This chapter gives the design of the physical implementation of the proposed actuated leg mechanism. This includes study, design, and analysis of three components. The components include the hinged actuator mounting clevis, the regolith contact foot, and the actuator thrust rod.

1.4.5 Chapter 6: Computer Program Simulation

Chapter 6 introduces and details the proposed solution simulation. The code language is MATLAB, using individually defined functions which are called as needed. Static physical parameters as well as variable vehicle and simulation parameters are taken as inputs. The primary output of the simulation reveals the motion of SR2 on an inclined traverse over time, utilizing actuation at the rear.

1.4.6 Chapter 7: Summary and Future Work

Analysis of the results of the simulation are presented here, including discovered limitations to the proposed methodology. The final chapter concludes with proposed opportunities to improve the simulation, design, and implementation.

1.4.7 In summation

The costly nature of an unmanned exploration vehicle makes it imperative that it not fail or become stranded during the mission. Unmanned rovers have become embedded in soft terrain numerous times, sometimes resulting in catastrophic failure of the machine. Implementing the proposed passive actuator design on the rover may improve rover locomotion on sandy inclined traverses. This implementation would ensure that should embedding events occur, they are recoverable.

Chapter 2

Related Work

Locomotion is a limiting factor for unmanned exploratory vehicles such as Solar Rover 2. While differential motion of the wheels allows for some terrain compliance, regolith traction remains a problem. The implementation of a passively-actuated foot designed for maximum traction may improve traversability of sandy, non-cohesive soils. In this chapter we will discuss works regarding land locomotion which are related to the proposed actuator solution. Discussion of land locomotion theory will be followed by biological and hybrid locomotion schemes which are similar to the proposed passive actuator solution.

The science of locomotion, mobility, and the mechanics involved have been explored for decades. The traversability of vehicles has been a priority from the first wheeled horse-drawn buggies to novel modes of transportation such as rail. Since the advent of linear actuators, a number of locomotive methods have been developed, including highly complex forms of legged locomotion.

Two great teachers have established the ground theory for land locomotion, Dr. Bekker and Mr. Rashevsky. Dr. Mieczyslaw Gregory Bekker established a number of locomotion standards and principles in his 1956 publication titled ‘Theory of Land Locomotion.’ In this tome, he theorizes that all land locomotion may be related to, and furthermore derived from, the dynamics of a flowing prism in a confined channel. He gives numerous analogues for locomotion, including a parallel drawn which describes a train-car rolling downhill as being mathematically similar to a column of flowing water [3]. Nicolas Rashevsky, pioneer of mathematical biology and mathematical biophysics [19], enlightened us the fact that all creatures with the exception of protozoans, move by a system of levers. The levers are the extremities protruding from the main body of the creature, and may be observed in numerous phyla [17].

2.1 Biomechanical Locomotion Analogue

Biological locomotion has evolved over generations to become very efficient, and is in many cases considered the epitome of locomotion. Animal locomotive forms have been studied and documented since the late 1940s. To this day, the natural world continues to inspire engineers to derive new forms of locomotion. Often, these methodologies mimic what nature has evolved over millions of evolutionary cycles. The peristaltic locomotion of animals can serve as a natural example of what might be accomplished by implementing a passively-actuated foot to the

rover.

A biological analogue to the proposed actuator implementation is peristaltic crawling. This form of locomotion is observed in the animal kingdom, particularly in the locomotion of worms, snails, and even some snakes [23]. Studies on the biological peristaltic locomotion scheme show that anisotropic friction is required to enable such a methodology [1]. On a multiple-segmented body, this difference in friction characteristics permits forward motion by inducing less resistance to motion on a forward-traveling segment than on an anchoring segment. The anchoring portion serves to constrain rearward motion, allowing for forward traverse [5]. Studies on worm locomotion have emphasized the increased contact patch area of the anchoring segment. In the region where the muscle is contracted, force is applied circumferentially to a worm burrow [22].

A similar locomotion method has been observed in the terrestrial traverse of the leech, which uses a set of suckers to vary the fixed locations [8]. Whether the anchoring portion is exactly stationary or relatively stationary further describes the peristaltic locomotion scheme. If the anchor is exactly fixed, or sticks, it is defined as stick-slip locomotion (SSL), meanwhile a difference in friction coefficients is characterized as slip locomotion (SL) [28]. These are important biomechanical analogues as the proposed solution behaves in a very similar manner. The proposed passively-actuated leg provides a fixed location at the rear to aid forward slope traverse in a SSL methodology.

2.2 Hybrid Mobility

Ever since all-terrain traverse has been studied, novel methods have been developed. Hybrid locomotion methods usually manifest by means of a primary locomotive methodology, with a secondary modality which is implemented for overcoming specific problems. In the case proposed in this work, the special circumstance is the traverse of a sandy slope with a wheeled primary locomotive method. In this section we will introduce some locomotion implementations including developments related to the proposed SR2 solution. These innovations include mechanisms which motivate the main body through peristaltic motion: the WorkPartner, Marsokhod rover, and a special wheelchair.

2.2.1 WorkPartner Rolking

An intuitive class of methods for locomotion combine both wheeled and legged motion. These schemes can be implemented at will between the two methods of locomotion. Hybrid locomotion of this sort shares the advantages of each locomotion method, walking and rolling [9]. One such development is called “Rolking” by its developers at the Helsinki University of Technology, integrated on a robot named WorkPartner.

WorkPartner greatly resembles a mythical centaur, as it is composed of a humanoid torso, head, arms and hands which are affixed to a wheeled, legged carriage-like body. Figure 2.1 shows the robot encountering a snowy traverse.

Similar to peristaltic locomotion, WorkPartner progresses difficult terrain when one leg or segment of the robot is motivated at a time. The remaining leg contacts act as anchoring points to maintain the chassis position. The rolking action is specifically defined as taking place when a legged wheel enters the transfer phase of legged motion. The wheel on the transferring leg is moved forward along the ground, it maintains contact under force control by adding a light torque to the wheel. Ultimately, each leg moves in turn, and the chassis mass is moved forward, advancing up the traverse [26].

This objective of traversal through a difficult terrain is similar to the issues faced by the SR2 Rover. WorkPartner has been shown experimentally to be capable of traversing sandy slopes using its rolking methodology [9]. The proposed solution to the traverse is also similar, excepting that the rover utilizes an extra appendage which assists the traverse. This differentiates the methods to solve the objective as the leg is not actively motivating individual wheels in a special manner to overcome obstacles.

2.2.2 Marsokhod

The Marsokhod exploratory rover built in St. Petersburg, Russia by Rover Company Ltd. is a six-wheeled rover platform composed of three axles with two wheels each. The links between each axled segment are actuated, allowing for the distance between each axle to be adjusted. The wheels are special in their cylindro-conical construction. The conical wheel design leaves nearly no chassis



Figure 2.1: WorkPartner robot traversing soft, deep snow [26]

exposed to drag or get caught on terrestrial anomalies. This feature has earned it the highest score for obstacle performance in a meta-analysis of wheels for lunar traverse [2]. The primary mode of locomotion for the Marsokhod is provided by typically-assumed wheeled motion. However, an interesting and novel behavior is initiated when difficult traverses are encountered, as in Figure 2.2.

The Marsokhod platform greatly improves traversal performance by resembling biological peristaltic motion. The peristaltic methodology is actualized by actively varying the wheelbase distance between the respective axles [12]. This allows four of the wheels to remain stationary, in a locked position with the terrain, while the third axle motivates up the problematic terrain. Similar to WorkPartner, the motivated wheels may turn while making progress. The remaining four wheels remain static, and theoretically utilize a static coefficient of friction at the wheel-regolith interface. The static coefficient of friction is typically larger than the dynamic coefficient [7]. Therefore, the four static wheels



Figure 2.2: Marsokhod rover on sandy traverse [25]

maintain purchase on the terrain to a greater extent. Meanwhile, the axle which will be traversing up the hill rotates its wheels as the actuated links between the axles change length. The Marsokhod rover method of hybrid locomotion has been referred to as “wheel walking” [10]. This terminology is also used by others to describe a sort of motion in which the drive axles can pivot about the chassis by a pinned joint. This sort of walking is typically actuated and controlled so that the walking is intentional, rather than being a simple reaction to the terrain. This behavior is exhibited in Figure 2.3.

Marsokhod’s hybrid traversal method is novel, and represents a solution similar to the proposed implementation on SR2. Marsokhod utilizes static wheels to increase purchase on the terrain, thus allowing for a relatively decreased friction force application to the traversing axle. Also, since only one segment of the rover is traversing at a time, the component of gravity resisting uphill traversal is reduced as the traversing mass is reduced. The proposed methodology for Solar Rover 2 traverse is differentiated from Maroskhod by the application of the anchoring member: SR2 is proposed to utilize an actuated leg with a well-designed contact foot to maintain a static purchase on the terrain, rather than relying on stalled wheels to provide multiple anchoring points.

2.2.3 Hybrid Legged Wheelchair

The hybrid wheelchair developed at the University of Pennsylvania combines both legged and wheeled locomotion, similar to SR2. Utilizing wheeled mo-

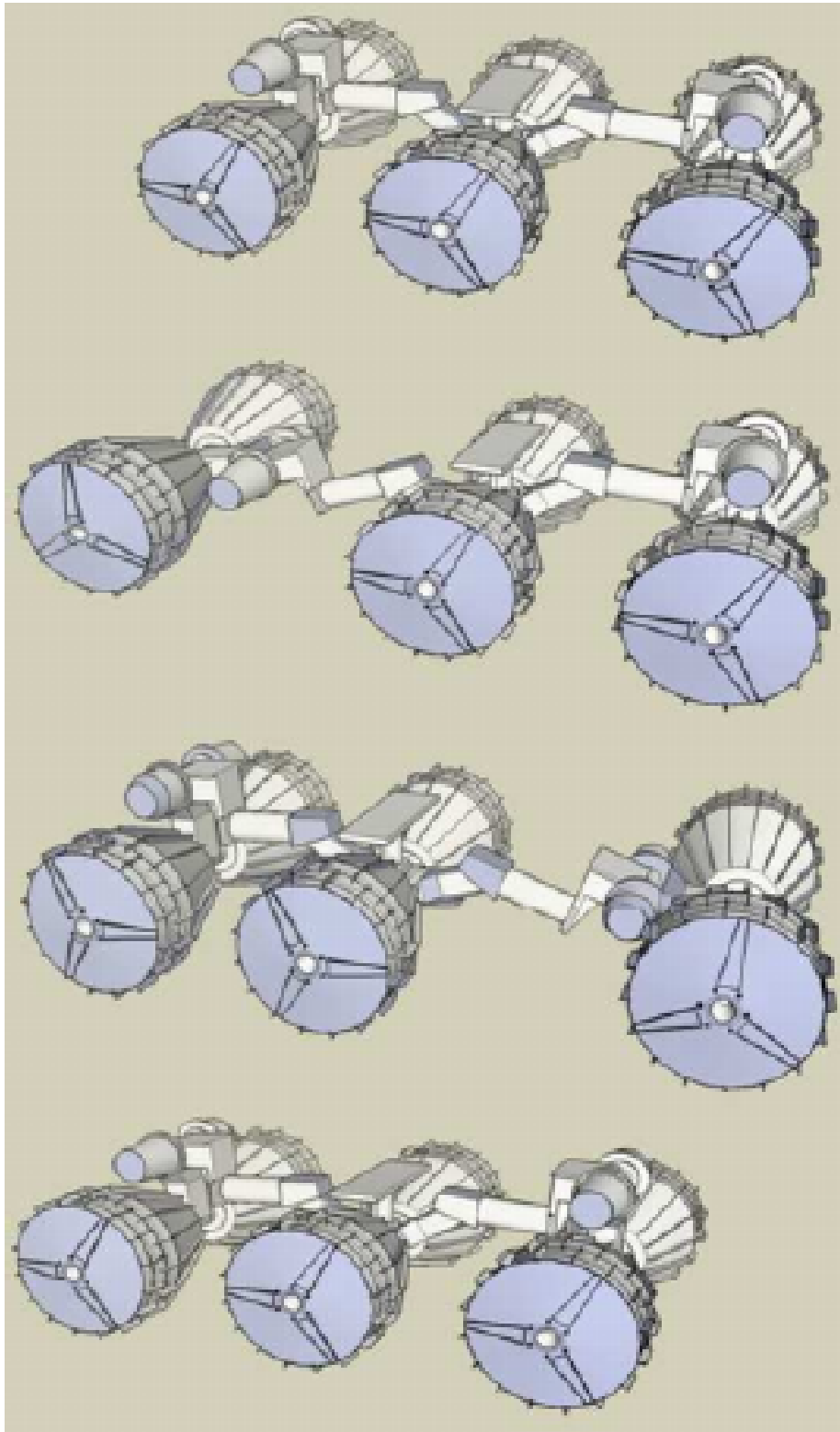


Figure 2.3: Marsokhod “wheel-walking” (right to left traverse) [10]

tion as a primary locomotion mode, it also has legs that enable active traction to overcome obstacles [11]. Locomotion is provided by two driven wheels, two passive caster wheels, and two articulated, two degree-of-freedom legs. The designers have also implemented redundancy measures in the actuator, and actively control and optimize the contact forces at both the feet and wheel. The implemented active traction scheme redistributes the contact forces on the wheelchair. The contact forces are distributed in such a way as to minimize the largest normalized ratio of tangential and normal forces among all the contacts. This implementation of hybrid mobility is shown in Figure 2.4.

An important characteristic of the hybrid wheelchair is that it is configured to be statically stable. The wheelchair legs provide tractive forces for stable support during the hybrid locomotion phase. Passive supports are provided by the wheels, providing better safety, less complexity, and less expense than traditional legged systems [11].

This hybrid wheelchair is similar to the proposed SR2 solution in that they both utilize a leg in combination with wheeled motion. The leg in both cases provides a positive purchase on the traverse in order to assist locomotion of the main chassis. Some notable differences occur between the two solutions as well.

The proposed SR2 solution utilizes a single leg which rotates passively, and is actuated. This is differentiated from the hybrid wheelchair in that the chair has two legs which do not lengthen linearly. The hybrid chair legs instead utilize two actively controlled and encoded revolute joints to drive the legs.

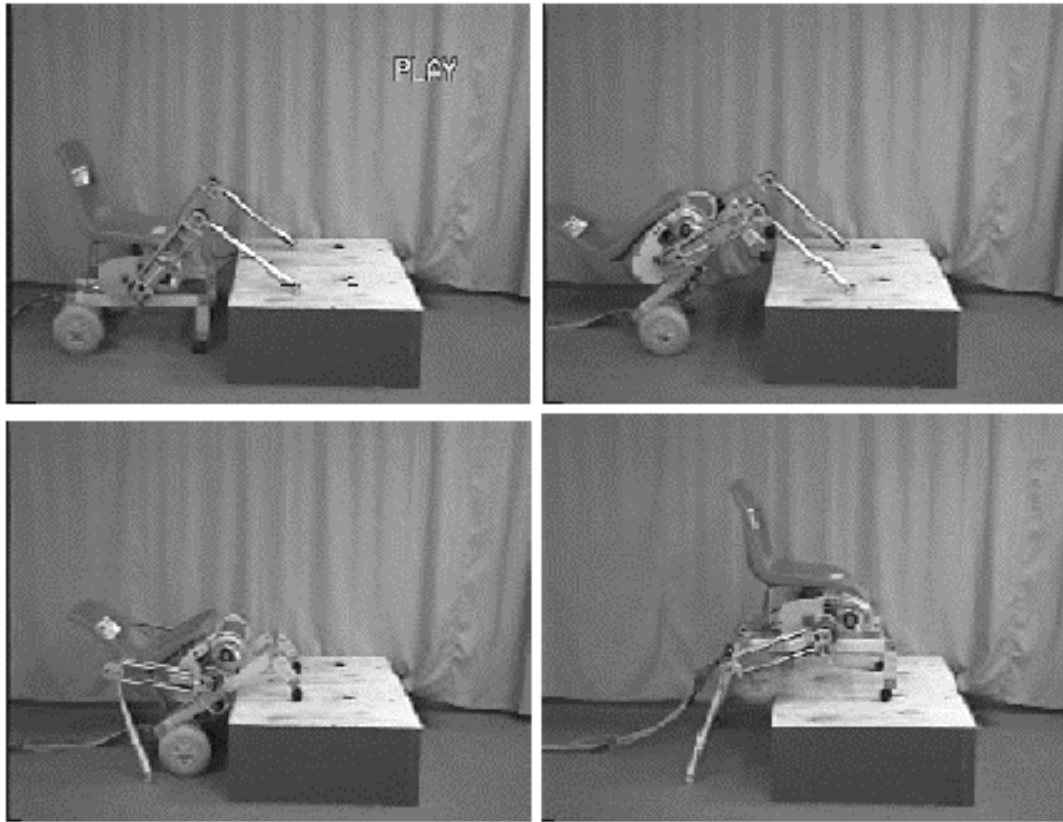


Figure 2.4: Penn hybrid wheelchair exhibiting four-stage climbing sequence [11]

Current methods for traverse include many novel implements and designs. Most of the solutions are created to solve specific problems such as climbing a curb or stairs. The Marsokhod and WorkPartner both employ novel, peristaltic-style methods which for traversing difficult terrain. Both methods utilize a static placement which is utilized as an anchor to support forward motion. In the next chapter, we will discuss the problem encountered with wheeled locomotion on a soft regolith.

Chapter 3

Wheeled Locomotion Analysis

Unmanned exploratory rovers are specialized equipment that have a very specific set of tools to perform scientific tasks. These mobile research platforms are able to conduct experiments and perform tasks at a number of locations according to their ability to traverse the regolith. Their ability to perform is greatly reduced when their mobility is restricted.

Wheeled locomotion is ideal on smooth and rigid terrain, and suffers on nonuniform and non-cohesive terrains. Unfortunately, these two characteristics are precisely what defines the terramechanics of extraterrestrial bodies. The rocks can vary greatly in size, and the regolith that has formed on the surface is typically fine-grained and non-cohesive. These terrain characteristics are also present here on Earth, and are often used to simulate traverses for planned exploratory rover missions.

3.1 Sojourner

An early example of wheeled traverse for a planetary rover is the Sojourner rover. This example was composed of six wheels in a rocker-bogie suspension. This allows for maximum compliance of an unsprung suspension to the terrain [18]. Sojourner was a special case as it was designed to stay in close proximity to the landing module it arrived on. It used the landing module to relay communications, so was not designed to venture very far. Still, there were a number of obstacles to overcome regarding the local traverse including large rocks and sandy traverses. Small platforms such as Sojourner can benefit from cube-square laws such as power and flotation while larger vehicles are able to overcome obstacles via mechanical advantage [15].

When traversing slopes composed of sandy and granular materials, tractive effort is often lost. Next we explore how the properties of the soil, in particularly the cohesiveness of the soil, affect traction.

3.2 Friction and Tractive Effort

The wheel-soil interface is a complicated yet well documented study which began nearly a century ago. Soil studies have been conducted to characterize and explain different soils, and how the sort of soil being traversed affects locomotion. Soils have characteristic properties which are typically parameterized experimentally. A common tool for soil characterization is a “soil bin.” A soil

bin is to a terrestrial locomotion engineer what a wind tunnel is to an aerospace engineer. In a soil bin the soil stress pattern can be found in a number of ways. The testing typically involves compressing the material and measuring loads at the instant the mass begins to flow [20].

3.2.1 Terrain Failure Criteria

Loose, granular soil mechanics are an important study as they make up the majority of the trafficable earth surface [3]. There are constituent factors regarding mathematical modeling of soils, including cohesive and frictional properties. A cohesive soil has interstitial bonds holding the individual grains together. These bonds hold the soil grain elements together and do not require pressure forces to keep them behaving as a solid. Conversely there is a frictional component to the soil behavior. Soil masses categorized as strictly “frictional” are dependent on interparticle friction to maintain form. Without a pressure force being imparted on these soils, they collapse into a cone characterized by their angle of repose.

The repose angle is defined when particles of the soil are dispersed in the air and a pile is formed. The steepest angle possible by the non-cohesive particles is called the angle of repose. This angle is the maximum that the loose, cohesiveless grains can maintain before gravity causes them to slip past one another. Figure 3.1 shows a characteristic repose angle for a lunar soil simulant, JSC-1A.

Repose angle behavior is important for traverse situations as this angle routinely defines the maximum grade a traversing vehicle will likely encounter.



Figure 3.1: Experimental fixture demonstrating repose angle on cohesiveless tailings of JSC-1A lunar regolith [21]

Characteristics of a sample can also affect regolith angle and behavior, particularly with very fine-grained dry materials. It is commonly known that fine particles or powders can behave very ‘cohesively’ when the particle size is reduced. An example of this would be the way baking flour can form standing formations when material is removed. Cohesive materials will typically give an avalanching behavior when collapsing, rather than flowing.

Extreme environments can also affect the way regolith and traverses behave. A study including the discrete element method (DEM) verified by experimentation has shown that gravity can affect the behavior of regolith. At increased gravity, up to $1200 g_0$, a very cohesive, micron-sized powder was shown to collapse and flow as a non-cohesive media [24]. This is of concern in our study here, as many environments requiring unmanned rover technology have gravity levels below our terrestrial level, g_0 . In this case the inverse would be of concern: an increased cohesiveness due to a reduced gravity effect.

A final consideration is load application. Applying a load to frictional

soils can improve traversability of frictional media such as sand. The shearing strength of sandy, frictional regolith increases with the load, a beneficial component to the proposed solution. This will be explored in the terramechanic analysis to come.

The primary equation governing traverse is a form of Coulomb's equation of shear failure as postulated by Mohr. The Mohr-Coulomb Equation (3.1) determines the failure condition of the traverse media by relating shear stress, τ , to normal stress, σ . The equation includes soil properties as two factors: cohesive and non-cohesive. A non-homogeneous traverse will have factors described by both constituent factors.

$$\tau = c + \sigma \tan \phi \quad (3.1)$$

where τ is the shearing stress of the soil, ϕ is the characteristic friction angle, or angle of repose, and c is called the cohesion coefficient. This equation can simplify depending on the primary constituents of the soil. For dry sands where the cohesion coefficient does not apply, ($c = 0$), the equation simplifies to Equation (3.2).

$$\tau = \sigma \tan \phi \quad (3.2)$$

It should be noted that any given traverse a plethora of soil factors may be encountered. The factors here are good for theoretical analysis in a controlled setting; in situ, a range of values will likely be encountered on any traverse. For-

tuitously, apart from soil properties, vehicle properties influence traversability.

3.2.2 Terramechanics of Wheeled Locomotion

Bekker describes drawbar pull as the ability of a vehicle to carry a load successfully. At a constant velocity, drawbar pull is the sum of external forces in the traverse direction.

$$DP = H - R \quad (3.3)$$

where DP is the drawbar pull, H is the vehicle thrust, R is resistance to forward motion. Positive drawbar pull is required for a successful traverse over any media or at any inclination angle. The two main components of forward motion resistance according to Bekker are bulldozing and wheel sinkage. The factors in determining resistance to forward motion for a deformable surface are illustrated in Figure 3.2.

Weight distribution effects

The distribution of the vehicle weight has an effect on the rolling resistance of the vehicle. According to Bekker, overloading the front axle with increases the rolling resistance [3]. This implies that our proposed mode of assisting traverse may impede progress, rather than support it. The increase of resistance is a product of the serial nature of wheeled traverse, where wheel sinkage and soil compaction are occurring for wheels in tandem. This behavior is specifically for

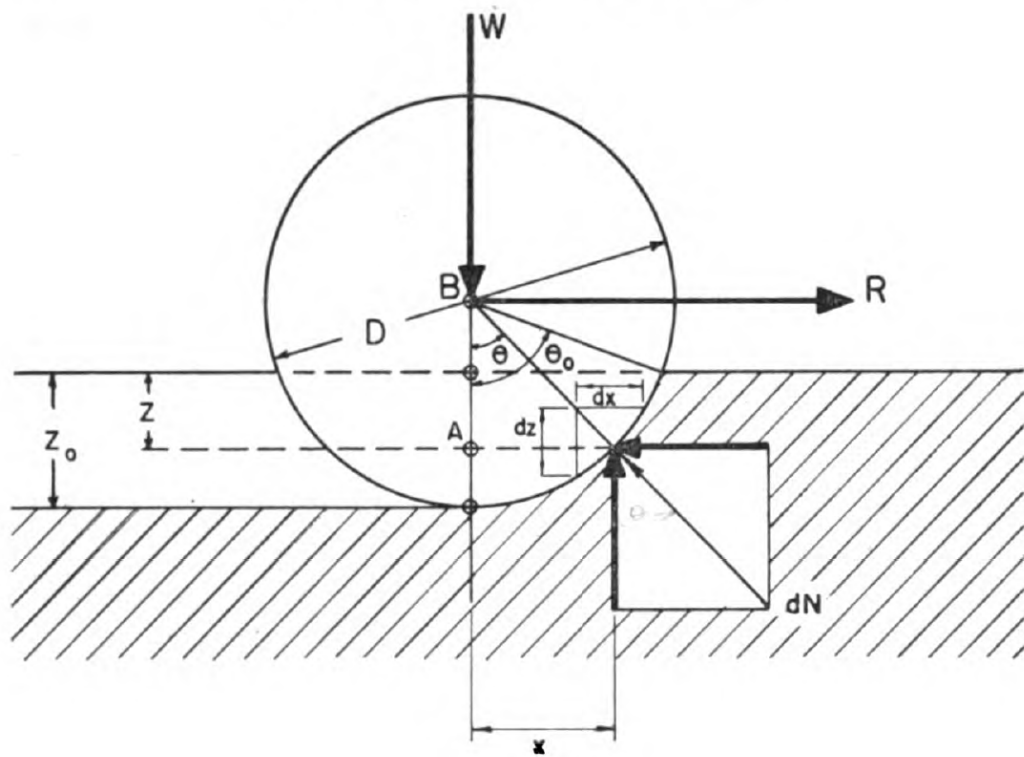


Figure 3.2: Bekker terramechanics model for an individual wheel [3]

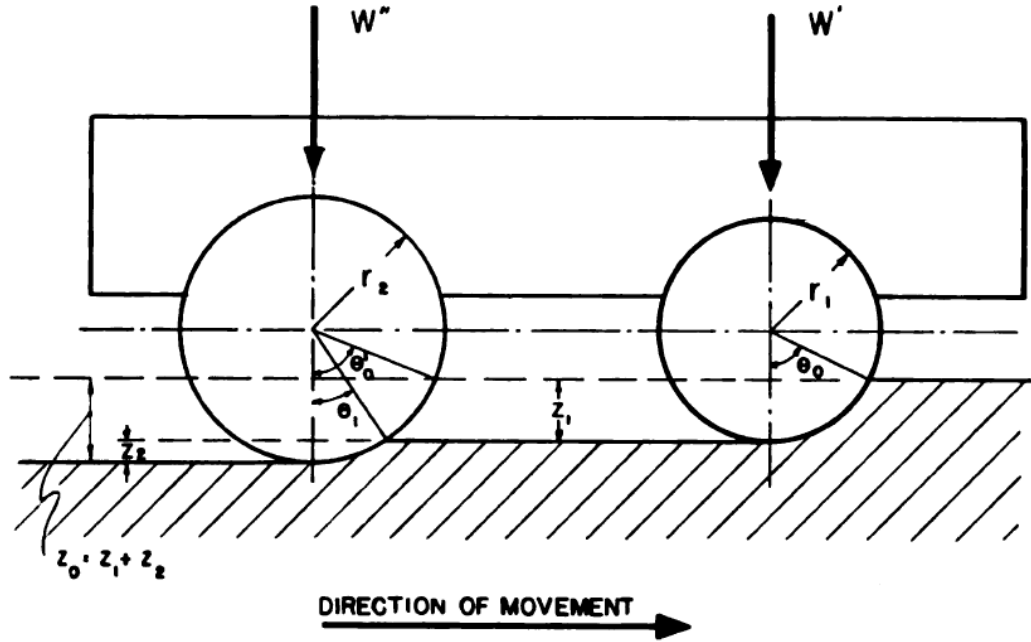


Figure 3.3: Better theory for wheels in tandem [3]

vehicles in soft ground, and is exhibited in Figure 3.3.

This conclusion is based on the terramechanics for two rigid wheels traversing a deformable soil, wherein the front wheel is performing an extra task of soil compaction. Bekker assumes that the soil compression increases proportionally with the depth of sinkage. In his equation, he assumes a factor called the compression mobility coefficient is equal to one-half, ($n = \frac{1}{2}$). Other factors in Equation (3.4) are the load W , wheel diameter D , width b , depth of sinkage z_0 , and coefficient of proportionality k .

$$W = \frac{bk\sqrt{Dz_0}}{3} z_0^n (3 - n) \quad (3.4)$$

If the compression mobility coefficient, n , is assumed to be a different value,

such as Gerstner's coefficient ($n = 1$), then overloading the front axle may in fact increase traversability [3]. This has important implications for the proposed solution, as this behavior may occur as a result of actuation at the rear of the SR2 vehicle. The weight distribution of the rover will also vary as the leg makes contact with the ground, distributing some of the weight.

In this chapter we introduced wheeled traverse theory. The wheel-soil interface is a complicated interaction upon which unmanned terrestrial exploration vehicles rely. If a costly asset is delivered to an exploration region with no possibility for external influence in the case of failure, it is imperative that the interface is considered. In this thesis we go on to explore the possibility of influencing the traverse of a planetary rover internally by utilizing a hybrid locomotion scheme. The hybrid locomotion scheme will be enacted when wheeled traverse alone is not sufficient. The scheme will be made possible by the implementation of a actuated leg-foot assembly which mounts to the rover chassis.

Chapter 4

Legged Locomotion Analysis

A hybrid mobility scheme utilizing peristaltic locomotion is presented as a feasible option to overcome sandy traverse. In this chapter, the implementation of a leg to behave as a device to accompany the vehicle's wheeled traverse is explored. In order to evaluate the feasibility of implementing a passively-actuated leg solution, a biological analogue is elaborated on, and the rigid body kinematics of the proposed solution are presented.

4.1 Biomechanical Analogue

The peristaltic locomotion cycle has been described by Quillin in experimental kinematics studies on the *Lumbricus Terrestris* worm. In this work, Quillin describes the peristaltic cycle as being divided into discrete strides, with a single stride being a complete cycle similar to two human steps [16]. This informs a

quasi-static methodology which we later adopt.

Quillin shows that three independent variables can be used to describe worm peristaltic locomotion: stride length, protrusion time, and stance time. In the study, flat ground was assumed and slipping was not described; two dissimilar factors from our proposed methodology. In Figure 4.1 these variables are shown along with some characteristics calculated from them.

In the proposed solution, the passive actuator leg at the rear is anchored, or fixed at a point along the soil interface. Motion takes place forward of the vehicle. Following successful movement of the main mass of the body, a forward anchoring point is established. In our study here, this is provided by the four rover wheels regaining secure footing, similar to the hybrid wheelchair functionality. Following this new purchase on the traverse, the rearward anchor point follows. As concluded in Section 2.1, there is differentiation in peristaltic motion between stick-slip locomotion (SSL) and slip locomotion (SL) [28]. The following free-body diagramming of the simulation will show the proposed methodology in the SSL paradigm.

The actuated leg will lift the rear of the vehicle, thus raising the center of gravity. By raising the rear of the rover, the force equilibrium state is changed, making forward incline traverse more favorable. The shifting center of gravity uses gravitational force to propel the vehicle forward via a gravity wrench. The gravity wrench is the moment of the gravitational force on the center of mass. This force motivates the passive, simple pinned joint to collapse.

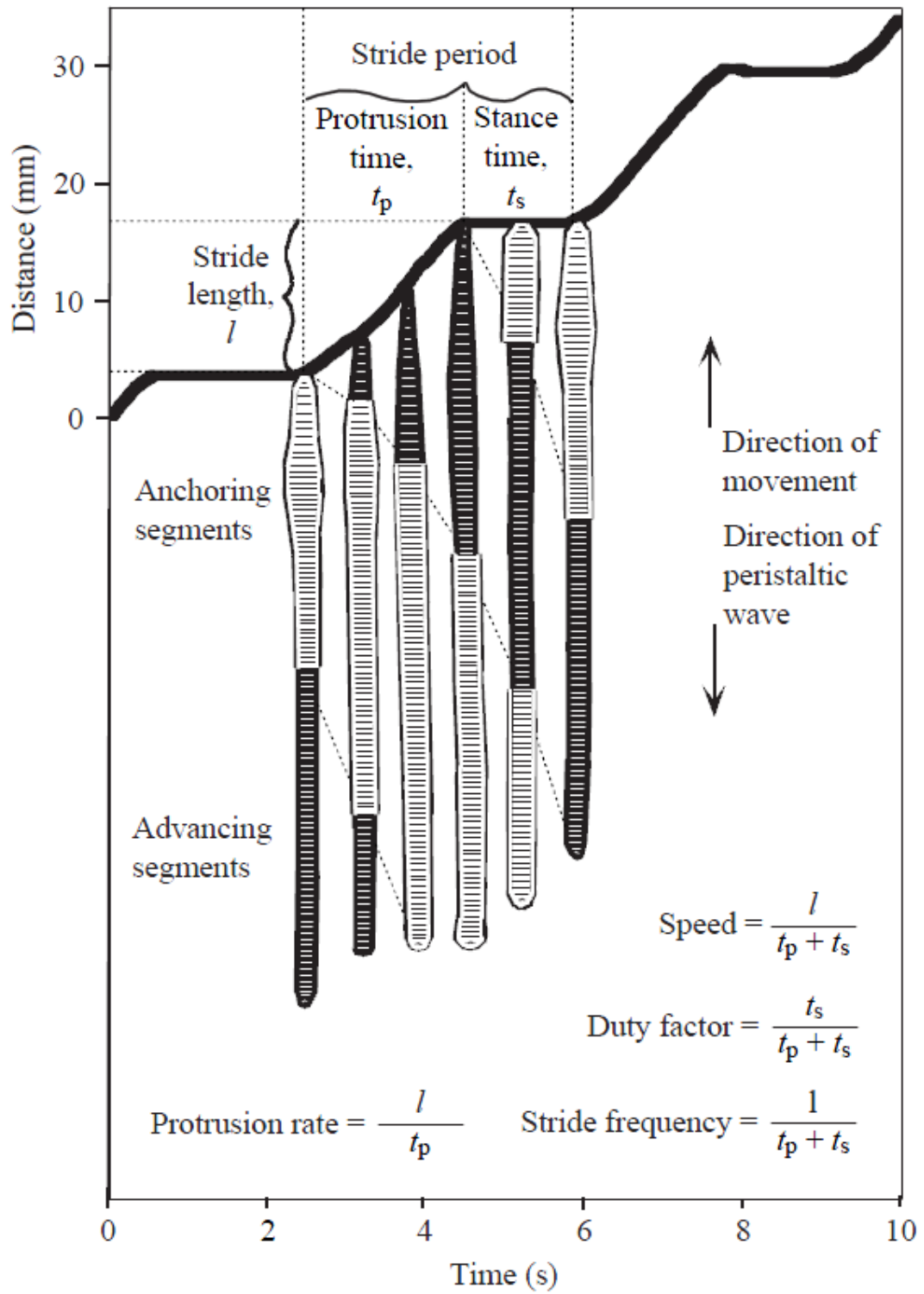


Figure 4.1: Kinematic behavior of peristaltic crawling [16]

A vertical actuation of the rear vehicle will be the means for hybrid mobility of the SR2 rover. This motion allows the rover to rely on wheeled motion when applicable, and offers the ability to conduct a novel method of locomotion on inclined slopes. The linear actuator will lift the rear end of the rover, providing a fixed contact anchoring point, with the traversal regolith. The wheels may either be powered or unpowered while the linear actuation is taking place, with advantages to each. Control of the proposed actuator solution includes the use of a passive hinge. With proper implementation, the hinge may prevent backward motion of the rover, while allowing forward motion to reach static equilibrium.

The collapsing behavior of the passive pin joint may be detrimental to the kinematic solution if it is allowed to collapse improperly. In this case, we must influence the force equilibrium on the vehicle to encourage desirable behavior. This behavior is specified by the angle between the actuator and the rover chassis. Assuming a fixed anchor at the actuator contact point, the pinned joint will collapse in a favorable way as in Figure 4.2: if the actuator angle becomes more obtuse. If the angle collapses in an acute manner, as in Figure 4.3 forward progress will not be made, and negative traverse may even occur. This would negatively impact the unmanned traverse, and may result in a catastrophic failure ending the mission. This proposed successive collapsing resembles peristaltic motion, and introduces a novel hybrid method of assisted locomotion. An in depth exploration of kinematic behaviors and force balancing for the proposed hybrid situation follows.

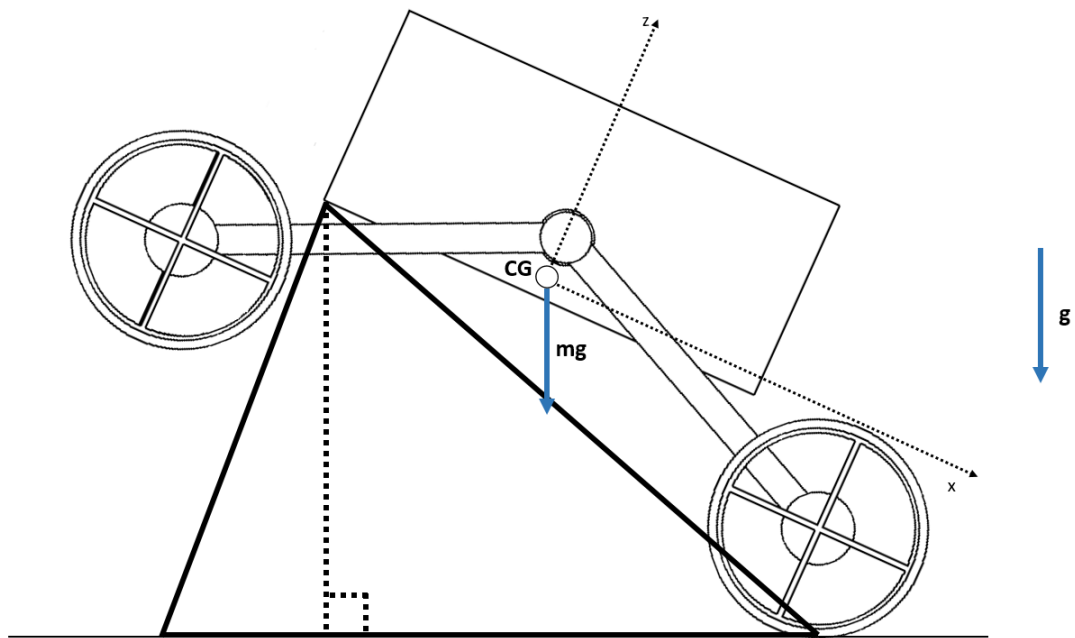


Figure 4.2: Passive hinge collapse failure: obtuse collapse

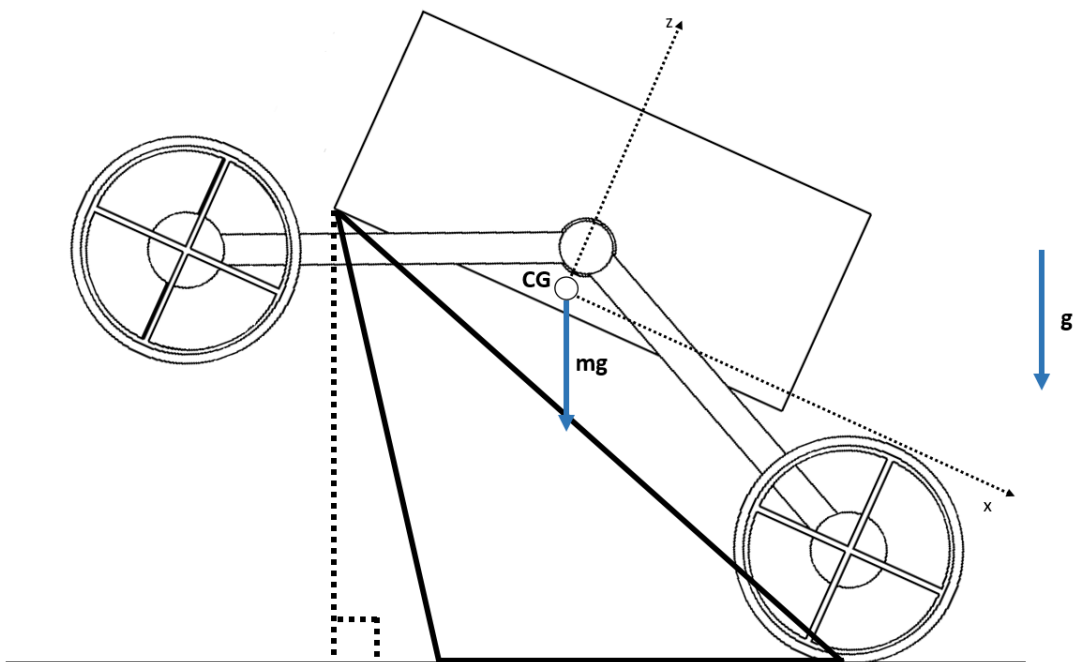


Figure 4.3: Passive hinge collapse failure: acute collapse

4.2 Rigid Body Kinematics

4.2.1 Body Forces and Assumptions

The analytical evaluation of a four-wheel driven vehicle on non-cohesive soil is very complex. The physics involved in such a simulation may be refined at numerous places, including the wheel-soil interface. These interactions require simplification to maintain the scope of the study, and may be further regarded in future work. To simplify the simulation, two major assumptions are made. The first assumption regards the terramechanical behavior at the wheel-terrain interface, and the second regarding system dynamics.

For the purpose of this study, the terramechanical interaction of the wheel and soil interface has been distilled. The work to follow assumes the regolith does not deform due to contact with the wheel. This assumption removes the possibility of the wheel sinking into the regolith, and allows for a quasi-static evaluation methodology.

The second assumption regards the dynamic situation of the rover. The lax velocity of the rover and magnitude of the forces involved allow for the neglecting of inertial forces. The relatively slow nature of traverse by a rover is not subject to many accelerations, and the few it is subject to are somewhat low in magnitude. Bekker points out that slow, deliberate locomotion cycle can sometimes allow for inertial forces to be neglected [3]. Furthermore, for animal locomotion studies if a small mass is assumed, the friction forces will dominate

the force of inertia [1]. A similar quasi-static, inertia-less methodology has been implemented in previous studies regarding peristaltic locomotion [27].

4.2.2 Diagramming

In the following section numerous diagrams will be given representing a modified SR2 rover. The diagrams present a narrative for leg implementation and proposed hybrid locomotion methodology. In the following, we will assume acceleration due to gravity on Earth. The gravity vector applied at the rover center of gravity will lie at the intersection of the x, or front plane, with the y or right plane. Rover longitudinal motion is in the x direction, and z is vertical motion, while lateral motion in the y is not considered.

The diagrams which follow assume a gravity-centric coordinate system. Most have been exaggerated to aid the discussion, affecting behavior of the front wheel. The diagrams will show behavior as if the front wheel were momentarily held fixed, and released following leg actuation. In the quasi-static simulation this takes place with very small incremental changes, and may not be otherwise visible or obvious to the observer.

Assumptions

Traverse of an unrestrained vehicle on a traverse is a very complicated endeavor. To make the simulation and evaluation manageable, we assume a simplified approach to the traverse. The simplified model consists of a straight-line longitu-

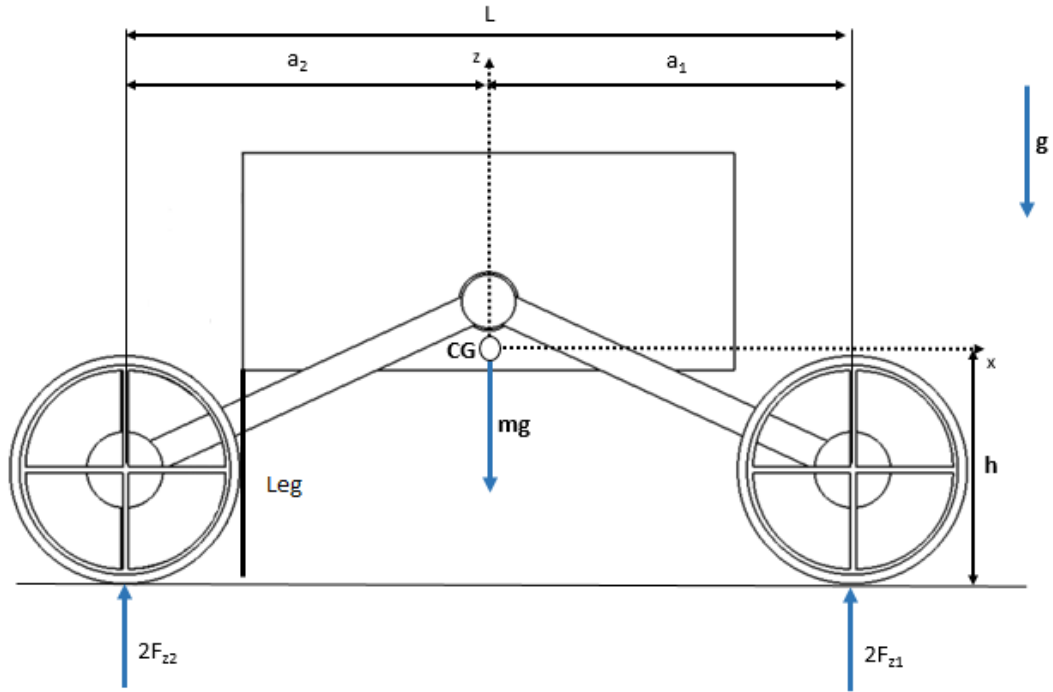


Figure 4.4: Diagram of modified SR2, static on flat traverse

dinal traverse and a laterally symmetric rover. These two simplifications allow for modeling of the forces on the left wheels will be equal to their counterpart on the right side. Therefore, a two-dimensional model of the rover and traverse model is possible, wherein the forces on left and right wheels may be combined into a single represented force.

Longitudinally, we assume the center of gravity of SR2 is at the geometric center of the traverse contact points. This feature of the rover design gives Equation (4.1), with variables illustrated in Figure 4.4.

$$L = a_1 + a_2 \quad (4.1)$$

where L is the wheelbase, or footprint of the rover, a_1 is the distance along x

axis from center of mass to front axle, and a_2 is the distance along x axis from center of mass to rear axle.

Flat Traverse Situation

When the rover is on a plane normal to the force of gravity, the forces are equally distributed on all four wheels. The component forces are only vertical in this state, and there are no forces in the horizontal directions. A simple free-body diagram is presented in Figure 4.4. The wheelbase of SR2 here is equal to the footprint made on the ground.

Assuming that the front wheel may rotate freely, if the rover is actuated at the rear, the force balance changes considerably. This elevation would come in the form of the proposed actuated leg, which maintains a fixed position at the rear. With this behavior, the rear of the rover incrementally raises. This causes a pitching behavior to occur about the lateral axis of the rover. Assuming no resistance from the wheel-terrain interface, the front wheels roll rearward to comply with the pitching of the rover. This is demonstrable and a right triangle is formed by ground plane and the leg perpendicular to it, as seen in Figure 4.5.

If the front wheels are held stationary, whether from resistance or positive wheel torque, the geometry of the situation is affected in another interesting way. Assuming the front wheels are fixed, the rover cannot travel backward when the overall wheelbase is increased. In this case the leg must tilt to compensate for the increased distance between the baseline ground contact points, as seen in

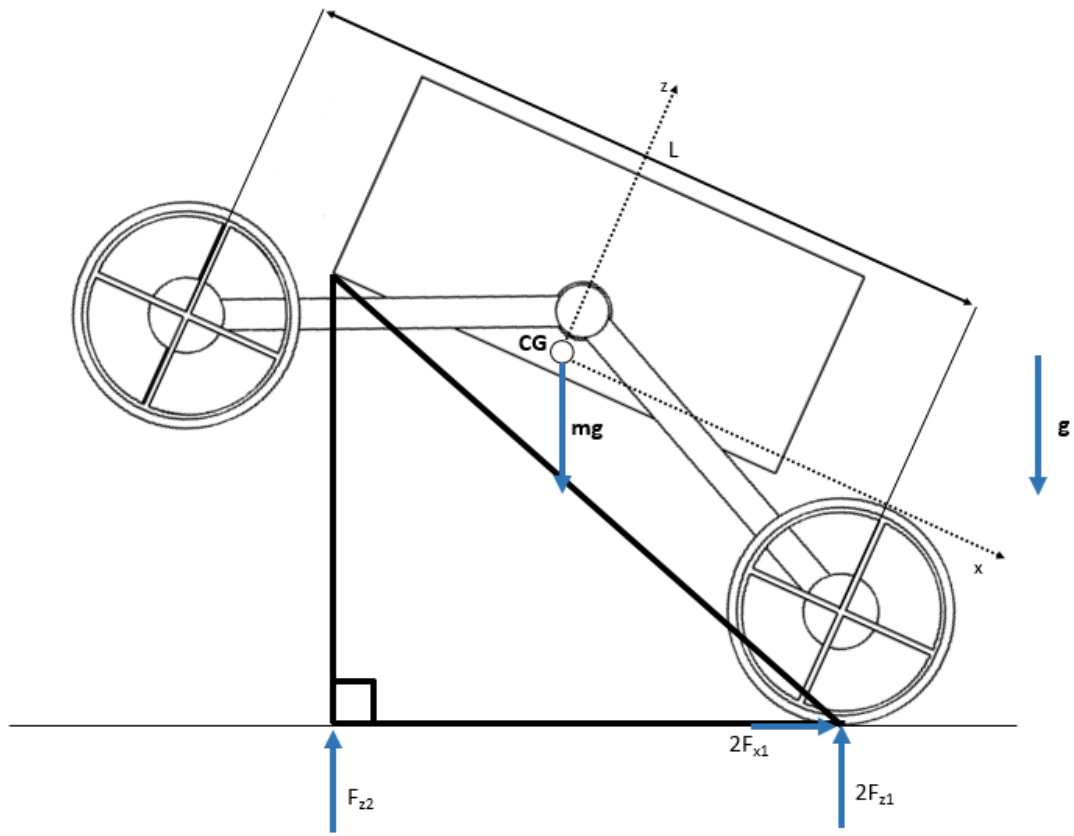


Figure 4.5: Diagram of SR2, actuated on flat traverse

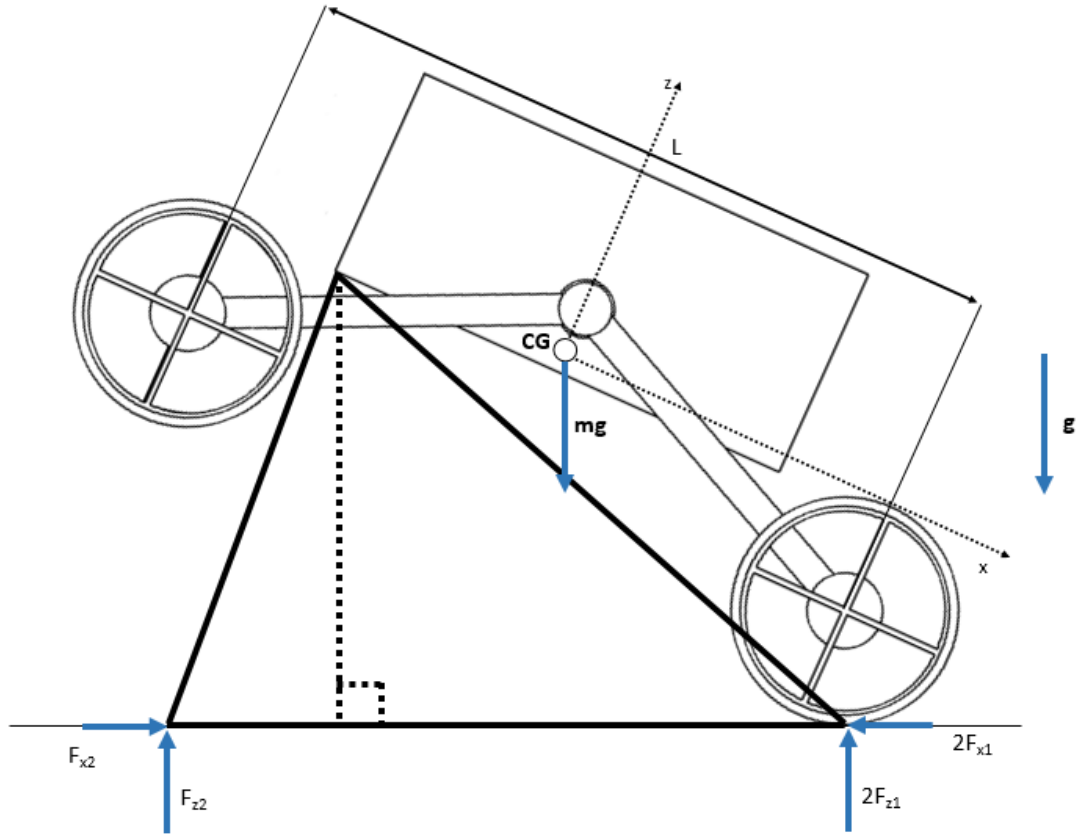


Figure 4.6: Diagram of SR2 showing ideal passive hinge collapse behavior due to forward motion of the vehicle

Figure 4.6.

Assuming the leg is fixed in the elevated rover state, the force of gravity varies the friction component on the front wheels. The downward force of gravity on the center of mass attempts to separate the ground contacts. This is due to the gravity force being transmitted through the non-vertical leg member. A horizontal component is created because the force of gravity is directed through a member that is not parallel to it. In this case static equilibrium is achieved

when Equation (4.2) is satisfied.

$$F_{x2} = 2F_{x1} \quad (4.2)$$

where F_{x1} is the x component of the friction force on a front wheel, and F_{x2} is the x component of the friction force on the leg at rear.

When the rover is on an inclined traverse, the forces at the center of gravity can vary according to the slope of the inclination.

Inclined Traverse Situation: Problems and Constraints

If the passive rover leg is made to rotate counter-clockwise, the solution can fail to achieve the desired goal of providing forward traverse. If the front wheel is allowed to roll rearward, the leg-ground contact will present below the rover chassis. Assuming the gravity vector is downward and free front wheels, this behavior will cause the hinge to collapse backward, rather than forward as desired. Figure 4.7 represents a snapshot shows the conditions of this undesirable behavior occurring in the event the front wheels are held fixed, and are released during the actuation cycle.

This rearward-traverse behavior becomes a serious issue when attempting to traverse inclined slopes. On an inclined slope, free-to-rotate front wheels will always be attempting to roll down the incline. This causes rearward motion of the rover chassis, resulting in a failed implementation, as seen in Figure 4.8.

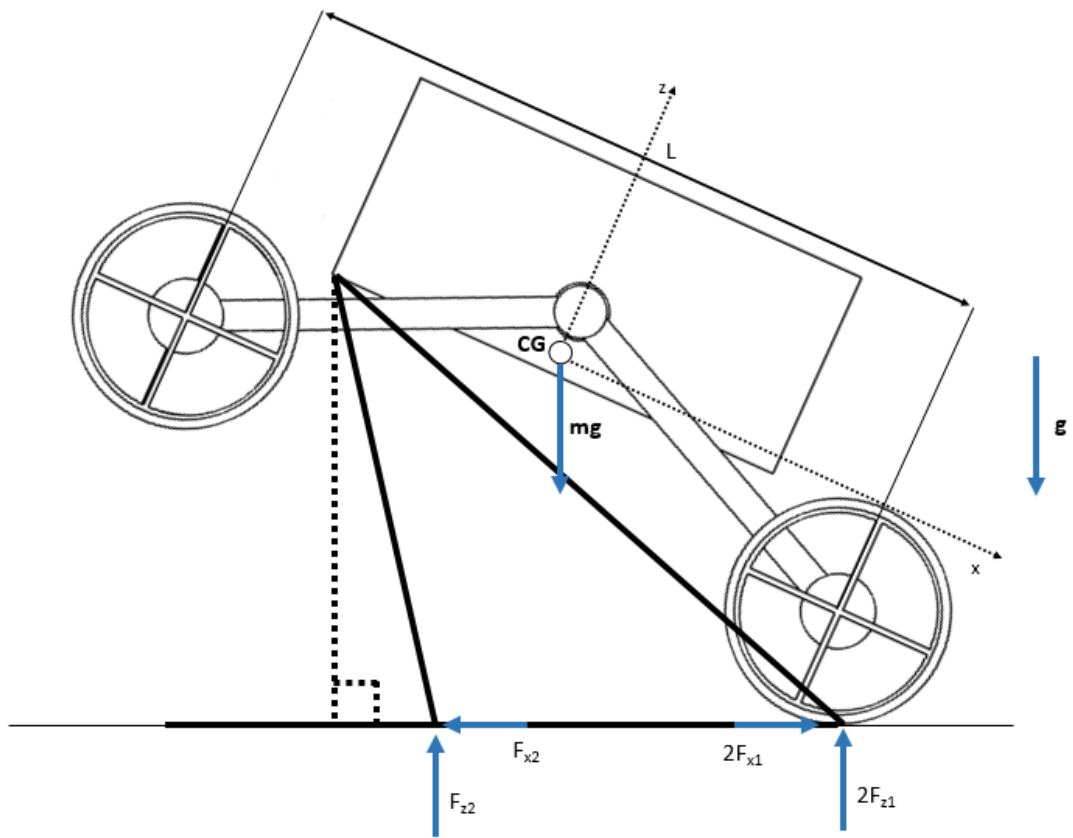


Figure 4.7: Diagram of SR2 showing failure of passive hinge collapse behavior due to rearward motion of the vehicle

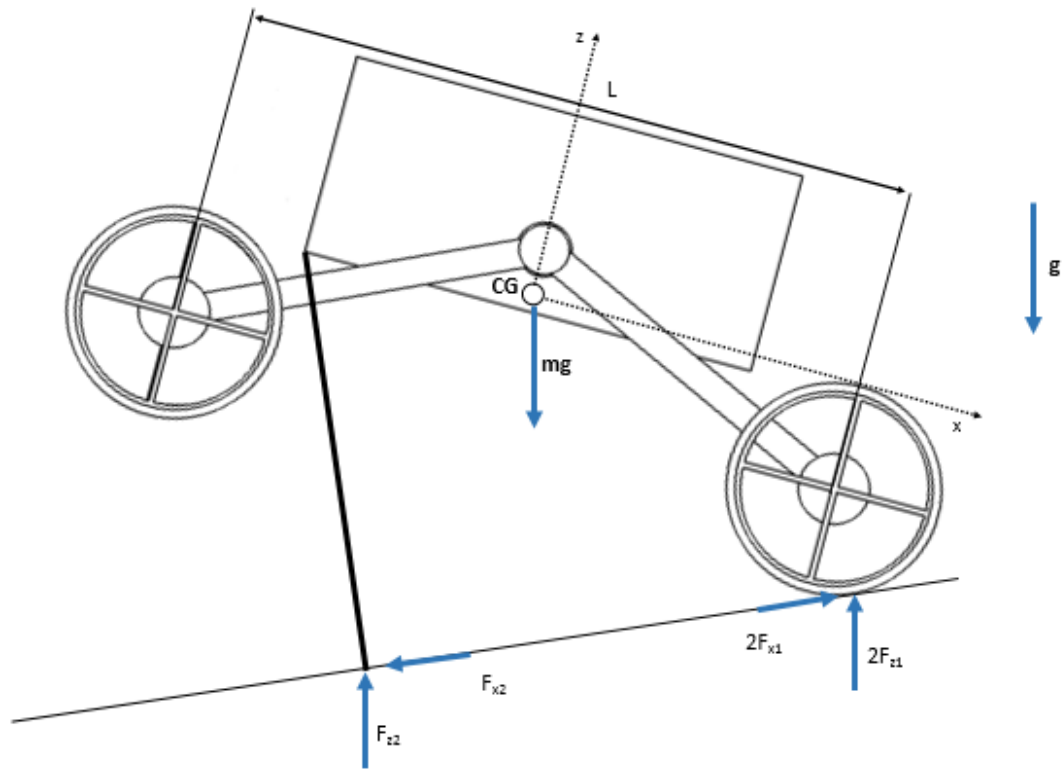


Figure 4.8: Diagram of SR2 elevated on inclined traverse showing failure of passive hinge collapse behavior due to rearward motion of the vehicle

Consideration must be given to prevent motion or rotation of the actuator which would allow the solution to fail in this manner.

With regard to actuator counter-clockwise rotation, a solution may lie in the location of actuator mounting to the vehicle chassis. By mounting the actuator at the top of the chassis, rather than the bottom, physical contact may prevent undesirable actuator results. A high actuator mount will enable the actuator body to contact the rover chassis if it begins to rotate counter-clockwise, preventing further undesirable motion. Such a mounting scheme on flat ground will appear as in Figure 4.9.

Figure 4.10 shows the rover with high mounting on a inclined traverse. When

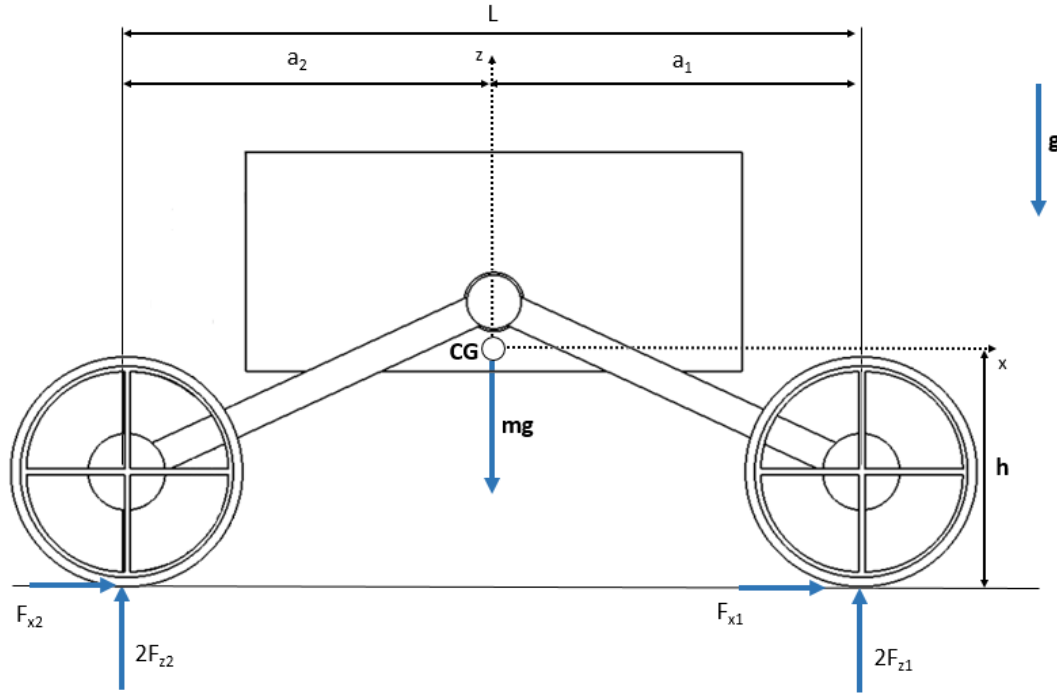


Figure 4.9: Diagram of SR2 with high mounted actuator on flat traverse

the rover is on an inclined plane, the free body diagram changes favorably. Applying static equilibrium equations, the force on the wheels changes according to Equation (4.3).

$$F_{x1} + F_{x2} = \frac{1}{2} m g \sin(\theta) \quad (4.3)$$

where m is the mass of the rover, g is the gravitational constant, and θ is the ground plane inclination angle.

The angle of planar inclination introduces a triangle into the static equilibrium with theta degrees between the hypotenuse and leg normal to the force of gravity. Assuming the actuator contact foot is fixed on the slope, the rover is prevented from rearward motion when lifted. This is due to a mechanical in-

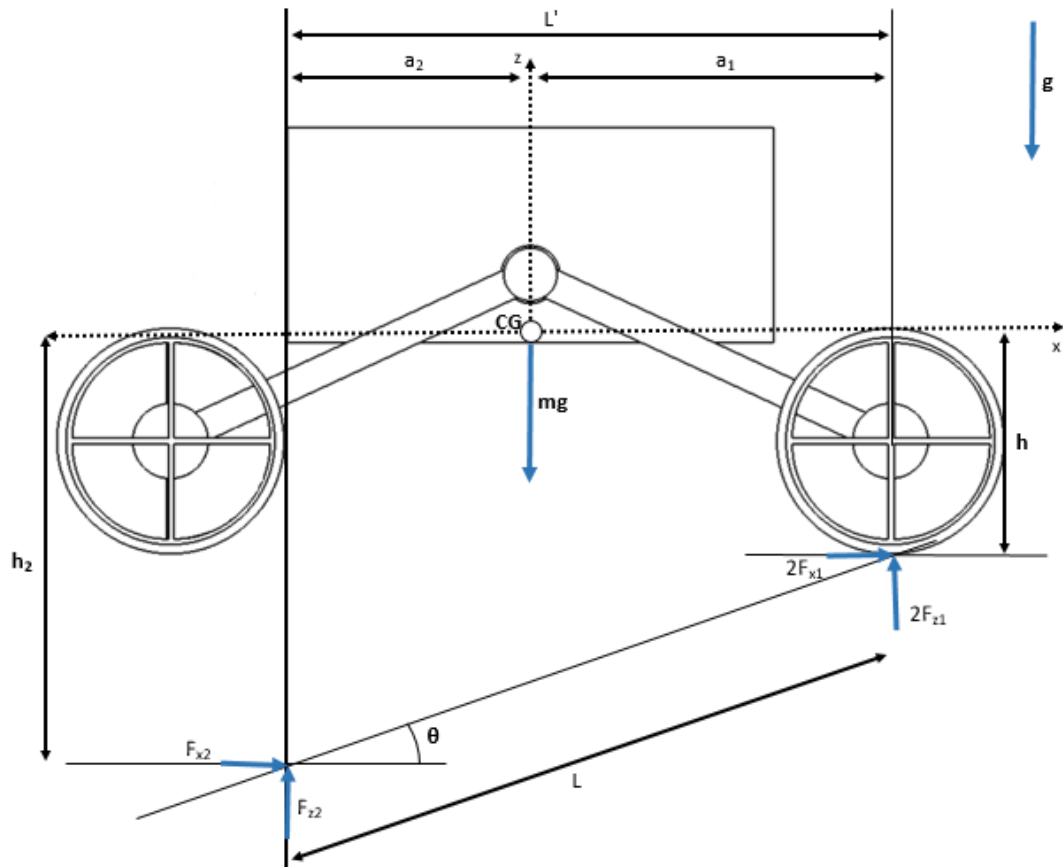


Figure 4.10: Diagram of SR2 with high-mounted actuator elevated on inclined traverse

interference of the actuator with the rover chassis. This functionality takes place because the passive pivot point is mounted to the top of the rover. If the hinge was below the rover, this interference would not exist and this methodology would not work.

To further mitigate the possibility of the actuator implementation failing, we can ensure wheeled locomotion is prescribed while the actuation is taking place. If the actuator were to operate as the front wheel is spinning, the front wheel would no longer act as an anchor, as in the crawling analog. Introducing

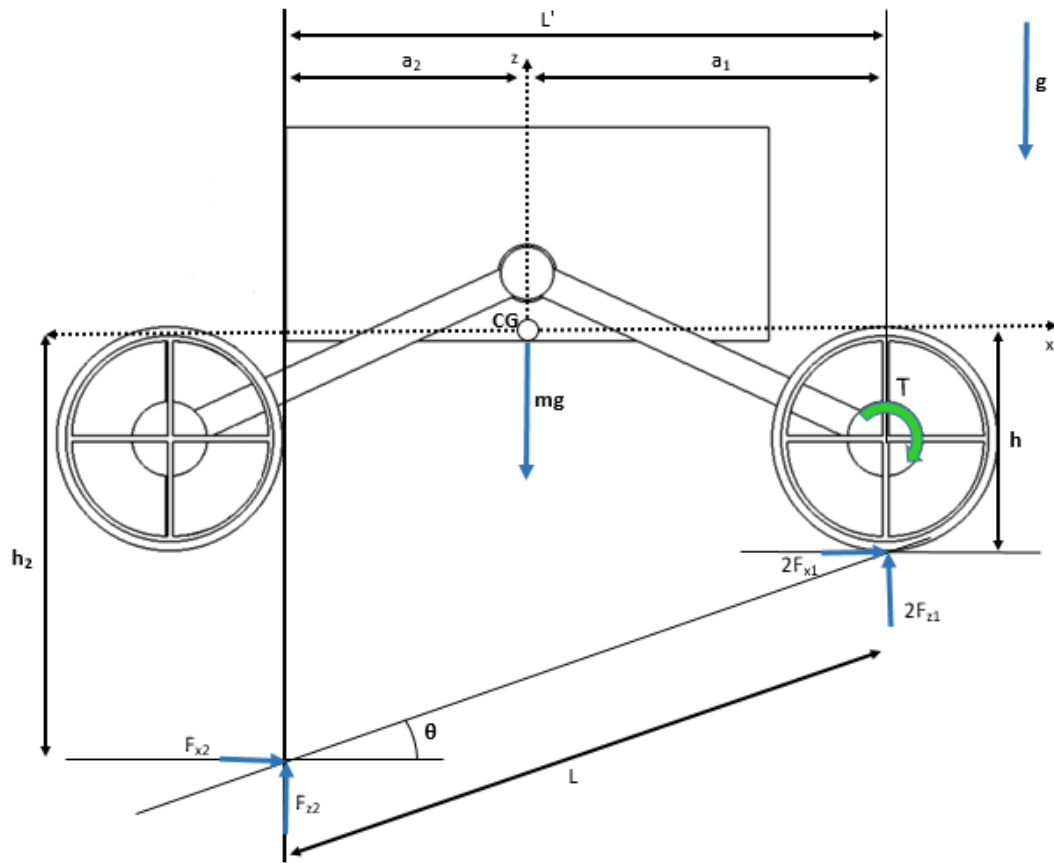


Figure 4.11: Diagram of SR2 with high-mounted actuator elevated on inclined traverse with a wheel torque applied to front driven wheels

a torque to the front wheel will allow the gravitational force to pull downward on the rover chassis at the center of gravity, while driving the front wheel up the slope. This locomotion would resemble peristaltic motion, and follows form as a hybrid scheme composed of both wheeled and actuated motion. This is illustrated in Figure 4.11.

By analyzing the diagram in Figure 4.11, some conclusions may be drawn. Elevation of the rear half of the rover will improve traction in the front. This occurs because of the weight redistribution from the rear of the rover to the front.

This increased downward force has a net positive effect on frictional traverse, neglecting bulldozing and sinkage effects.

Secondly, introduction of a rear actuation mechanism to lift the rear of the vehicle will decrease the rover attitude angle, thus forcing the front wheels forward. If this behavior occurs with little resistance at the front wheel or a driven front wheel, the vehicle may descend from its elevated position at a location higher on the slope than its initial location. This behavior assumes that the passive hinge of the actuator mount allows it to fall forward rather than backward. The wheel torque has been introduced in the example to draw SR2 forward, but what if the wheels spinning is not desirable? If the wheels are un-driven during the rear actuation, the rover must still be able to fall forward.

For the linear actuator leg to perform properly, the rover must be motivated up the sandy incline. To ensure this behavior, a relationship is established between the angle of inclination and the angle of the actuator with respect to the rover chassis. To accomplish this, we will set a reference plane normal to gravity. The inclined plane will be at θ degrees to the reference plane, and thus θ degrees to gravity. The rover is also given a plane, which is coincident to the rear chassis plane of SR2. The angle ϕ will be formed by the angle the linear actuator makes in reference to the rover plane. The gravitational vector remains vertical and downward in all cases. The linear actuator attitude vector is also downward, always parallel to the gravity vector.

Initially, while the rover is on a horizontal surface, $\theta = 0^\circ$, the angle the

linear actuator makes with the rover rear plane is also zero. As the reference angle increases, so too does the actuator angle, ϕ . This can be observed in Figure 4.12. Summing angle measurements with another degree value gives valuable analysis. The angle made between the vertical gravity vector and the rover coordinate plane x-axis, $\angle r$, is valuable here.

Theta is complimentary to gravity as $\theta + \angle r = 90^\circ$. Also, phi is complimentary to gravity as $\phi + \angle r = 90^\circ$. Therefore we derive (4.4) for this system.

$$\theta = \phi \tag{4.4}$$

Also note that when the actuator is extended, the dimensions of the traverse change. The rearmost contact is no longer the rear wheel in this case, but the actuator contact point. Further analysis of this condition and its influence on the system will be evaluated further in Chapter 5.

Assuming free-wheeling behavior of the front wheels, the proposed solution would not successfully motivate the rover forward. If the wheels are allowed to rotate without friction (either in the drivetrain, or induced from the soil-wheel interface), a forward wheel driving torque, nor a braking force, there will be not resistance to motion at the front wheel. In this scenario, the actuator will simply raise the vehicle body, with the front wheel traversing rearward until contact is made between the actuator and rover chasis.

In order to ensure uphill locomotion with the proposed solution, the linear

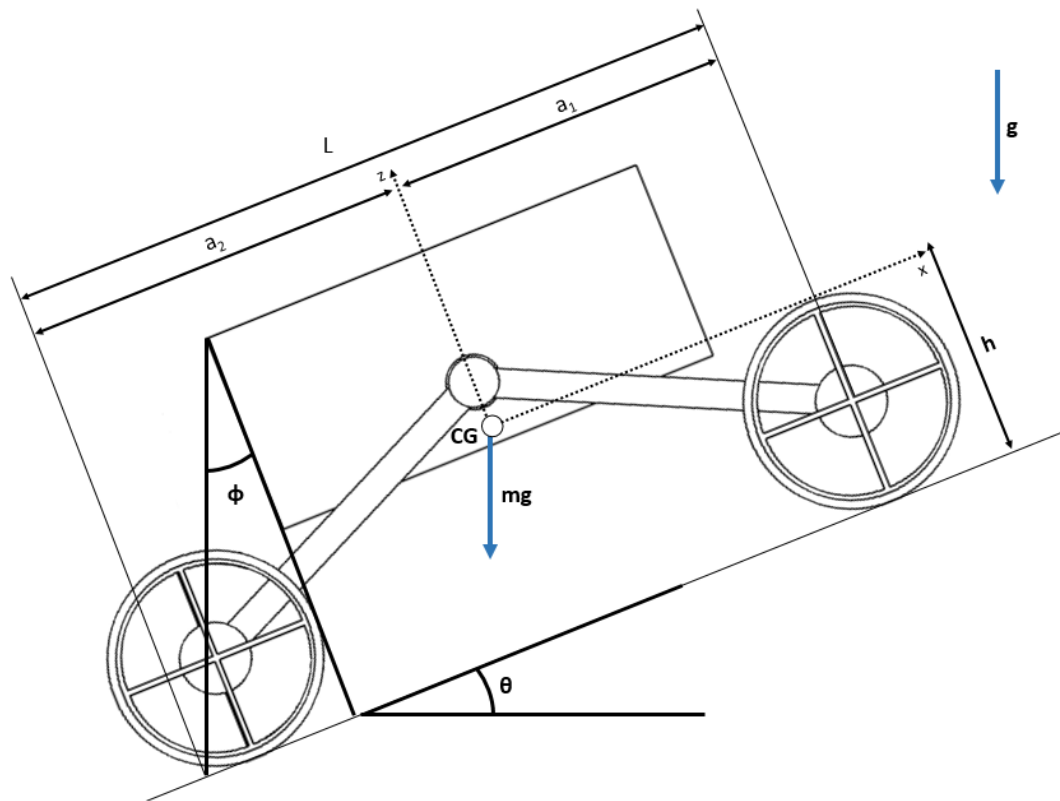


Figure 4.12: Diagram of SR2 with high-mounted actuator on inclined traverse showing equivalence of passive actuator angle and traverse incline angle

actuator must be influenced so that the angle ϕ does not remain perpendicular to gravity. This task could be maintained by applying forward wheel rotations to allow only for forward motion upon elevation of the rear of the rover.

This chapter served to introduce peristaltic locomotion theory and application for the proposed passive actuator solution. Diagrams were completed to show the special case of having a passive rotating actuator manifesting through a pinned joint. If measures are not taken to ensure proper function of the actuator the proposed solution could fail. In the next chapter we discuss the MATLAB simulation and program and its particulars.

Chapter 5

Mechanical Design

The investigation of a methodology for hybrid locomotion opens inquiry to how mechanical components should be best implemented. This system has many properties already defined in the SR2 design, but the new proposed system is without definition. In order to best assist the wheeled sandy traverse, a hybrid locomotion scheme is proposed. A virtual rendering of the proposed result including the new parts can be seen in Figure ??.

The proposed solution is a combination of multiple parts working together. These parts include the terrain contact foot and the actuator mount. The design objective for the contact foot is maximum tractive effort, or drawbar pull, on a sandy regolith. This study uses terramechanic theory to inform design decisions. Meanwhile, the actuator mount must be able to support the loads required for the proposed hybrid locomotion methodology. These designs should also focus on the minimization of dead weight.

5.1 System Design and Simulation Cases

Simulations for the system are completed for two cases: the rover on flat ground, and the rover traversing a dry sand dune. For the dry sand simulation, the slope of the traverse is set as the characteristic angle of repose for sand. This value is thirty degrees. Vehicle engineers routinely use an equivalent measure for inclined slopes called a grade. The formula for this conversion is presented in Equation 5.1. A thirty degree incline using this measure is a 58% grade.

$$Grade = \tan(SlopeAngle) * 100 \quad (5.1)$$

When traversing a thirty degree incline, weight transfer occurs to the rear of the vehicle. In the case of the system utilization, the distance from the rear contact to the rover center of gravity is changed. In this case, the rear contact is no longer the rearmost wheel pair, but the contacting foot. A representation of this situation can be found in Figure 5.1. The horizontal, top portion of the foot extends rearward, away from the rover chassis, and the grouser portion of the foot penetrates the terrain. Because of this, weight transfer mathematics were conducted to determine forces experienced by the system during utilization. The function to determine the rearward weight transfer on an incline is given in Equation 5.2.

$$W_{rear} = W \left(\frac{c}{L} + \frac{h}{L} \sin(\theta) \right) \quad (5.2)$$

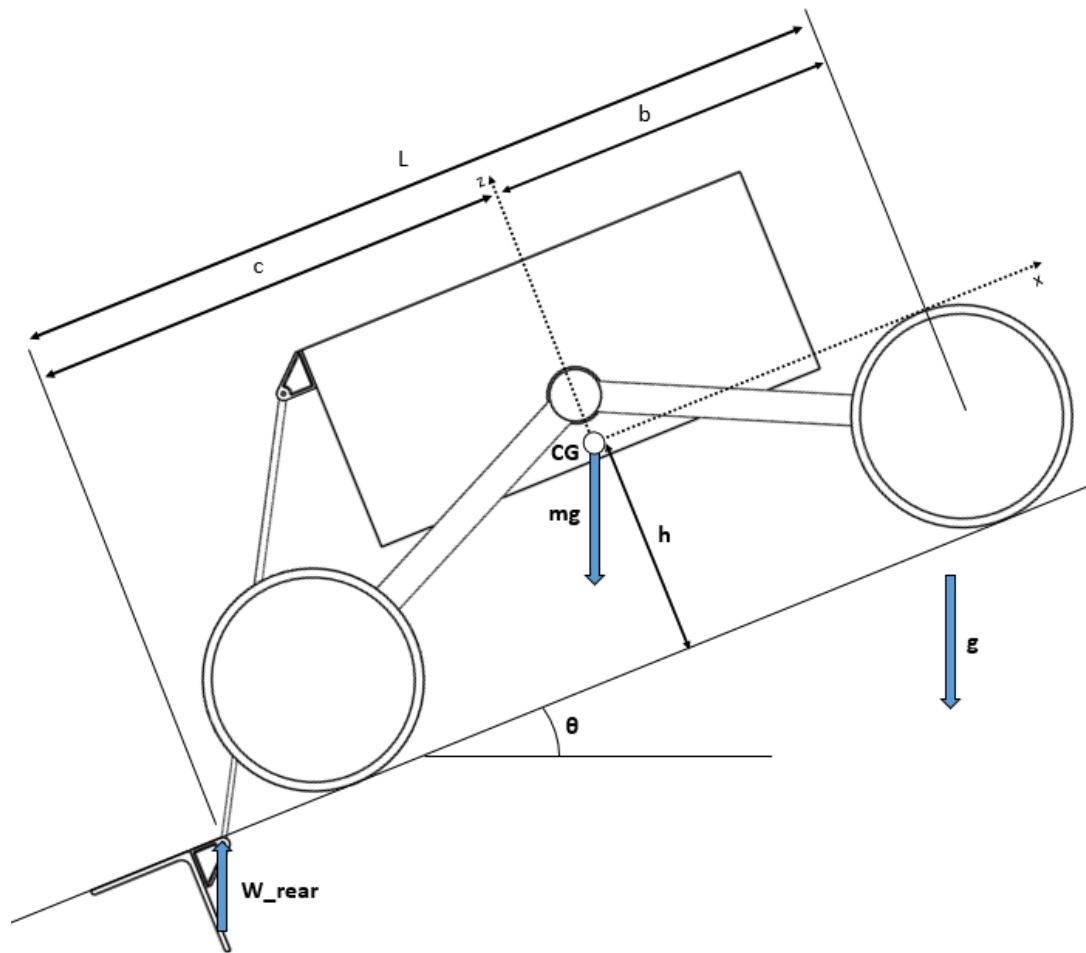


Figure 5.1: SR2 Rover weight transfer on an inclined traverse

where W_{rear} is the total weight at the rear contact point, W is the weight of the vehicle, c is the distance from the rear contact point to the vehicle center of gravity, L is the total distance between front and rear contact points, h is the height from the traverse to the vehicle center of gravity, and θ is the angle of the traverse in degrees.

The SR2 rover center of gravity position was found by analyzing a virtual model composed in SolidWorks. This model gives a center of gravity height value of 0.267 meters. A physical model of the SR2 rover was weighed in the Intelligent Robotics Laboratory in Oklahoma, and was found to weigh 16 kilograms. The thirty degree traverse case gives a rearward shift of the rover mass of 77.7%. Assuming Earth gravity, the proposed leg system must bear a weight of 12.43 kilograms.

The proposed leg system is mounted to the rear of the SR2 rover, at the top. Justification for mounting at the top can be found previously in Section 4.2.2. Due to this mounting, the depth of the grouser, the length of the actuator, along with their respective mounting hinges, must compete for vertical space. If the assembly is too long, it may be free of the traverse when fully retracted, dragging behind the rover. If too short, the proposed system will be inefficient, requiring numerous cycles per unit of forward distance traversed. So he have a hard height constraint as the minimum bound, and must optimize for maximum component lengths. The respective components and the vertical space required is shown in Figure 5.2.

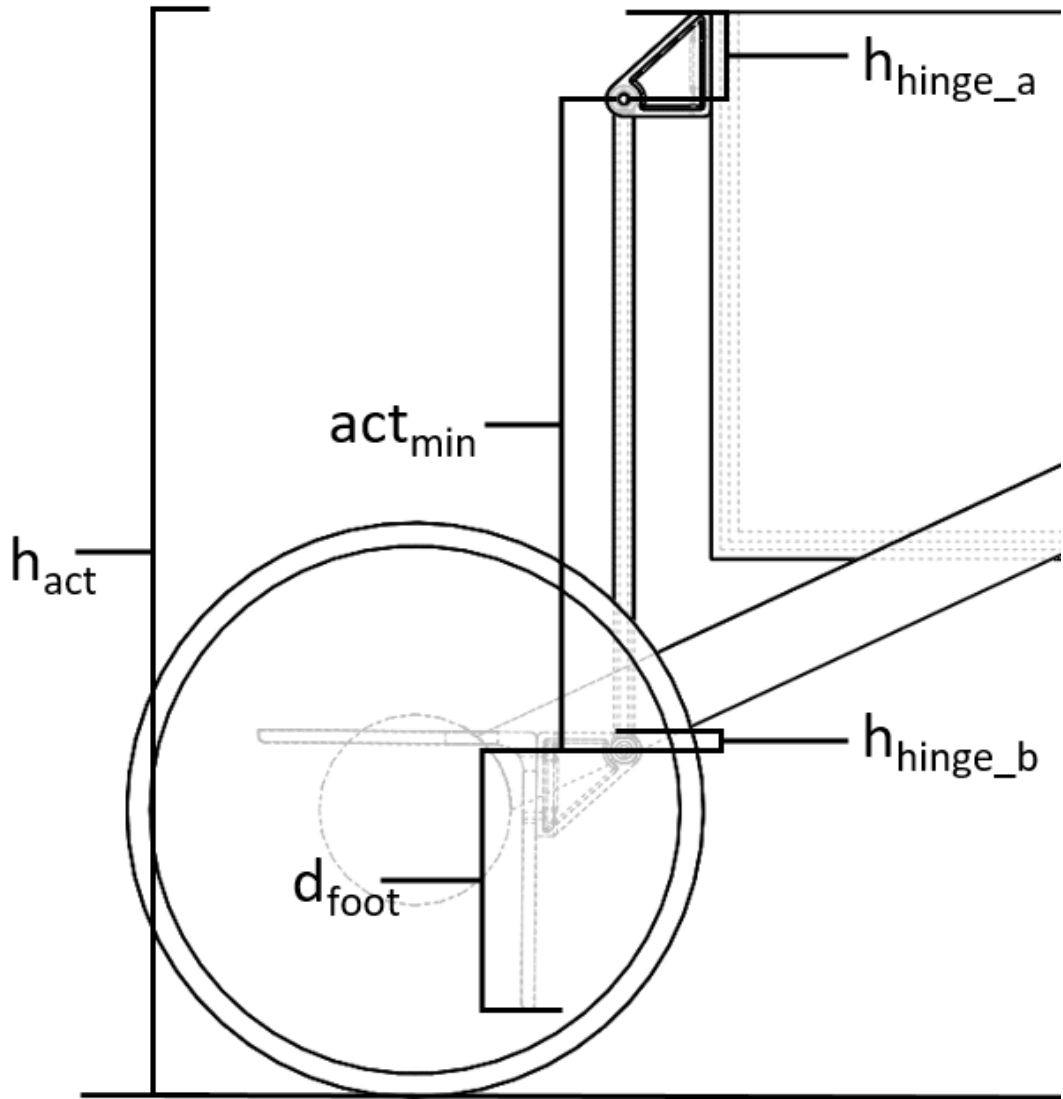


Figure 5.2: Vertical space interactions between components

In the figure, each component required height is designated by a bracketed region and a variable. It is useful to note that the height required of both of the respective hinges sum to the height of one whole hinge. Mathematically this becomes Equation 5.3.

$$h_{wholehinge} = h_{hinge_a} + h_{hinge_b} \quad (5.3)$$

Therefore, the height distribution constraint becomes as in Equation 5.4.

$$h_{act} < d_{foot} + act_{min} + h_{wholehinge} \quad (5.4)$$

Further discussion of the respective factors and their considerations for the height allowance follow.

5.2 Regolith Contact Foot Design

Of paramount importance to the functionality of the proposed solution is the interface contact between the actuator and the terrain. In reality a perfect bond with a traverse cannot occur, but we can attempt to achieve an optimal solution for our problem. In order to optimize the foot design we must first investigate terramechanics theory as the basis for achieving our design objectives.

5.2.1 Terramechanic Theory

The proposed solution is for a specific case: traverse over sandy terrain. This media has been described experimentally and mathematical formulae have been produced which define soils and regolith, as found in Section 3.2 of this work. Note that Equation 3.1 is similar in form, however considers shear failure of the regolith only, and does not consider parameters of the contact.

When dealing with off road vehicle locomotion, a number of criteria must be considered for successful traverse. In terramechanic theory, dry sand is considered a cohesiveless, or frictional regolith. The shearing strength of the soil is provided not by cohesive forces between the particles, but primarily through particle friction forces [3].

Increasing the weight has a positive effect on frictional soils, and should be increased so long as the regolith safe working load is not exceeded [4]. The safe working load is the maximum weight, W_{safe} , that enables a loaded area, A , to stay on the ground surface [4]. Equation 5.5, first introduced by Terzaghi, gives the formula for safe working load of a soil.

$$W_{safe} = A \left(cN_c + \gamma z N_q + \frac{1}{2} \gamma b N_\gamma \right) \quad (5.5)$$

Similar to soil equations discussed in Section 3.2, the formula consists of cohesive and frictional terms. Simplifying for cohesiveless soils, such as dry sand, the formula becomes Equation 5.6.

$$W_{safe} = \frac{1}{2}A\gamma bN \quad (5.6)$$

where A is the area loaded by the contact, b is the width of the contact, γ is the unit weight of the traverse media, and N is characteristic of the regolith and is a function of the fundamental angle of repose. Upon inspection it becomes apparent that for a fixed weight and regolith, the traversability of a vehicle can be improved by increasing the loaded area and the width of the contact.

5.2.2 Grousers

Grousers, spuds, and tread are features added to smooth tires, and can affect the maximum soil thrust for a given contact pair. Grousers can be observed in common tire tread design, as well as tank tread designs. The grouser is the portion of the tire or tread that is raised from the smooth surface. In effect, when these features contact the terrain, they dig in. This penetrative action allows for increased thrust loading by affecting the shearing stress of the traverse media. The addition of wheel features such as grousers introduce a factor, H' , to the equation for maximum thrust resulting in Equation (5.7).

$$H = W \tan \phi \left\{ 1 + 0.64 \left[\left(\frac{h}{b} \right) \cot^{-1} \left(\frac{h}{b} \right) \right] \right\} \quad (5.7)$$

where W is the weight, ϕ is the angle of repose, h is the height and depth of the grouser, and b is the width of the grouser.

In general, the addition of wheel features has little effect for conventional locomotion methods in frictional soils [4]. However, inspection of the constituent factors may enable an effective hybrid locomotion scheme. Increasing the factor $\frac{h}{b}$ in the equation can improve the traversability of the simulated rover over a sandy surface.

The height of the grouser is constrained in this problem as there is a limited amount of space to include both the actuator and the grouser when fully retracted. Analysis was conducted to determine the required depth of grouser, while minimizing the height. This maximizes the remainder, which allows for a longer stroke length of the actuator, and therefore more efficient movement per cycle.

Grousers have also been found to be more efficient when split. The theoretical finding was reinforced with experimental data, and the author concluded that spacing out a foot into multiple smaller segments achieves higher soil thrust for the weight of material [4]. An experimental result plot published in Bekker can be seen in Figure 5.3.

Utilizing all of the above theory and experimental results, a regolith contact foot was designed and iteratively amended until a good result was achieved.

5.2.3 Foot Design Solution

The above terramechanic theory, considerations, and design points are implemented into the foot design. Calculations for foot dimensions were completed,

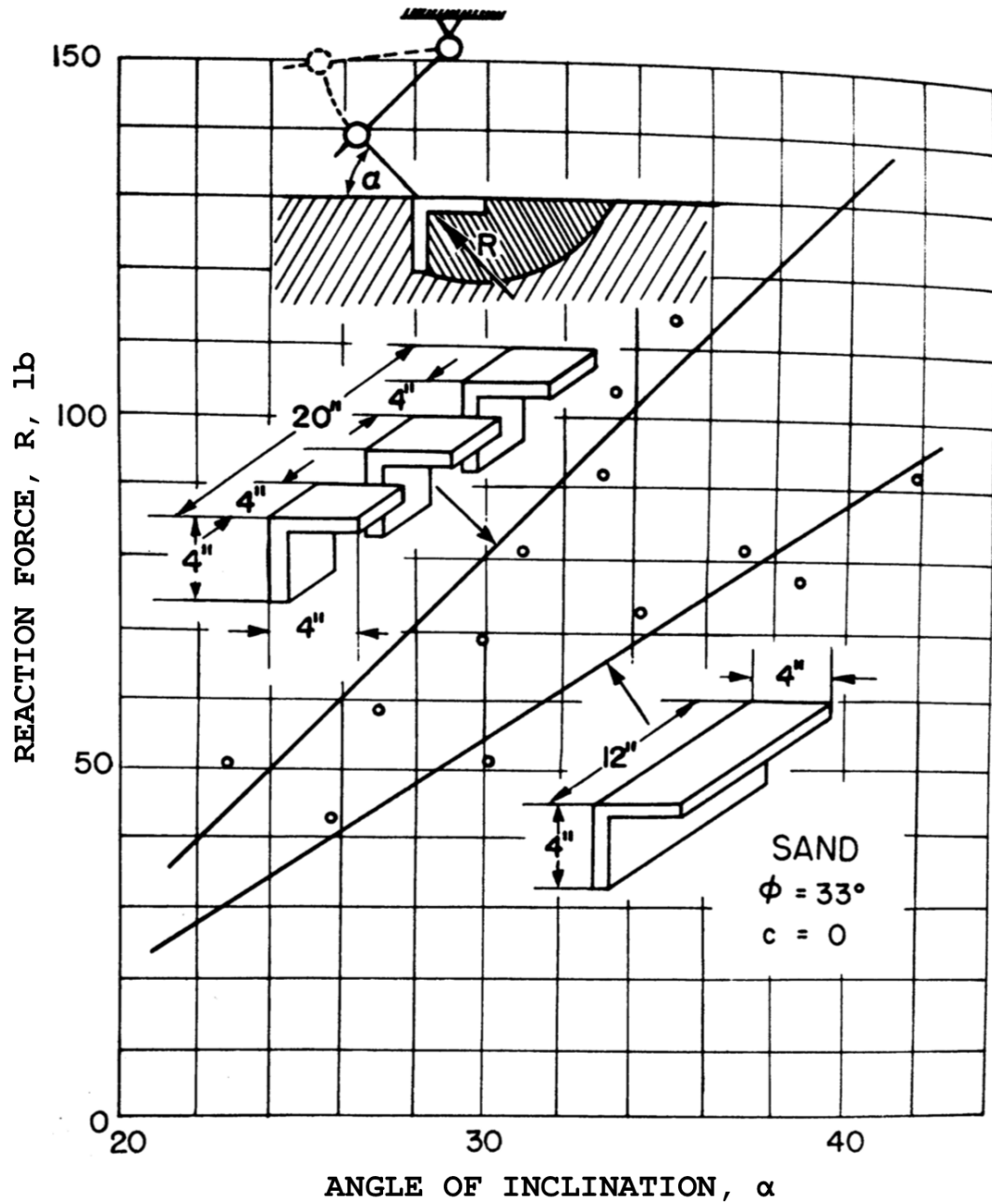


Figure 5.3: Soil thrust for a solid foot and spaced grousers [4]

and the values used are consistent with the values used for the simulation. The foot design resembles an upside-down capital letter “L.”

The top, horizontal portion of the foot is designed using terramechanic theory for terrain safe working load for a tracked vehicle. This utilizes the design case presented above, with a loading of 12.43kg. The vertical portion of the foot design is informed by theory for grousers, which can penetrate soils to improve tractive performance. These two analysis combine to ensure the foot does not sink into traverse beyond the top of the foot, and to quantify the maximum thrust load the foot can produce in dry sand.

Utilizing Bekker’s split grouser theory, the depth of the grouser is equal to the track length. For our design and analysis, this means that the horizontal portion of the foot and the vertical depth of the grouser are equal. In this case we have to consider this value critically as it also affects system performance. If the value is too large, the functionality of the leg will be limited due to the height constraints mentioned in Section 5.1. The safe working load parameters are those for dry sand, and the loading forces applied are determined from Equation (5.2).

The formula for safe working load was first solved considering only the horizontal contact with the traverse. Using Equation 5.6, a value of four inches for the track length with the design case of 12.43 kg gives a value of 4.77 inches for the track width. Splitting this width into three equal and equally-spaced sections as informed by Bekker gives an individual tang width of of 1.6 inches, and a total foot width of 7.96 inches. These dimensions can be seen in Figure 5.4.

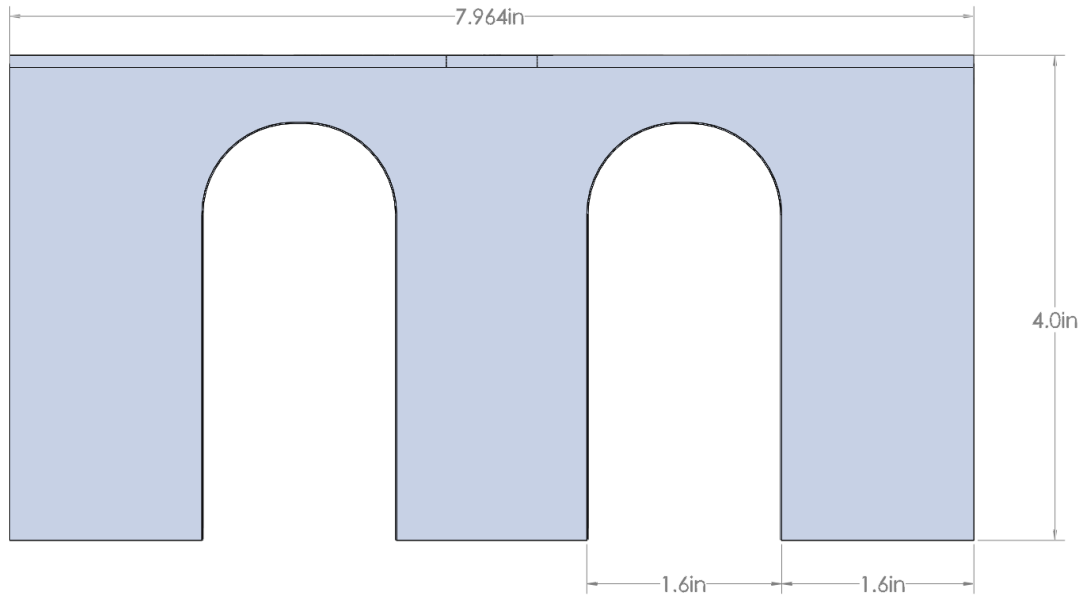


Figure 5.4: Regolith foot design, top view showing required tracked area for regolith safe working load

This analysis determined that a nineteen inch contact patch with the traverse would safely support the rover on an inclined traverse. Utilizing a value of four inches for the track length, nine inches remains for the actuator stroke. For this design, utilizing Equation 5.7, the split grouser will be able to apply a 31.6 kg thrust load to the sandy incline. This value which far exceeds the design case of 12.4 kg. Lastly, the track width value is back-solved to ensure safe working load on the terrain.

Using these results, a three-dimensional solid model was created in SolidWorks. Following parametric modeling, an iterative simulation process was used to relieve stress concentrations, improving fatigue life of the design. The final design takes the form shown in Figure 5.5. This figure is composed of two images, highlighting the dimensions relevant to the grouser Equation (5.7). The

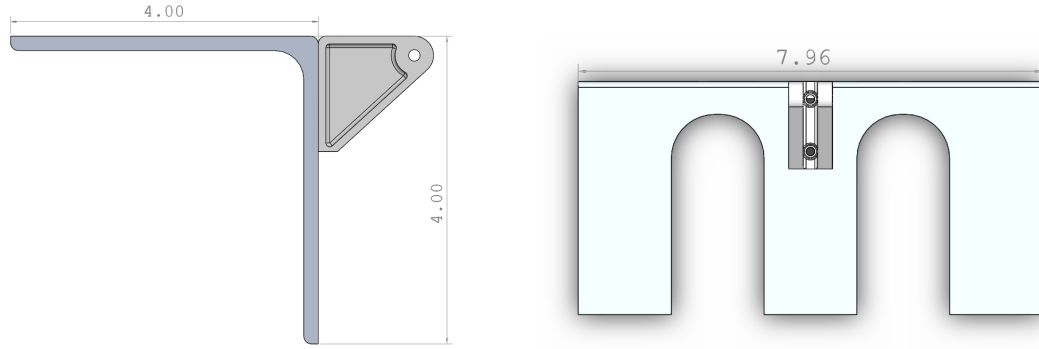


Figure 5.5: Regolith foot design, side and front views

leftmost image is a side view of the grouser, showing the depth of intended penetration into the terrain. The right image shows the calculated width of the proposed foot.

A simulation of the regolith soil pressure was conducted on the foot using values determined from Equation 5.6. The resulting safe working load value was divided by the contact area to determine the safe working pressure of the soil in pounds per square inch (psi). This pressure value, 1.433psi , was applied to the flat contacting faces of the foot. This load application is shown by the red arrows in Figure 5.6.

Maintaining a foot thickness and material of 0.375 inch 6061 Aluminum, fillets were added to relieve stress concentrations at the root of the tangs. Finite element analysis was conducted on the foot, and the resulting stresses are shown in Figure 5.7, with a detailed view of the maximum stress in Figure 5.8.

The design simulation done here has a safety factor of two on the applied pressure, which is an exaggerated yet prudent case. The fatigue is for a full loading cycle, including application and negative application, pulling the tangs.

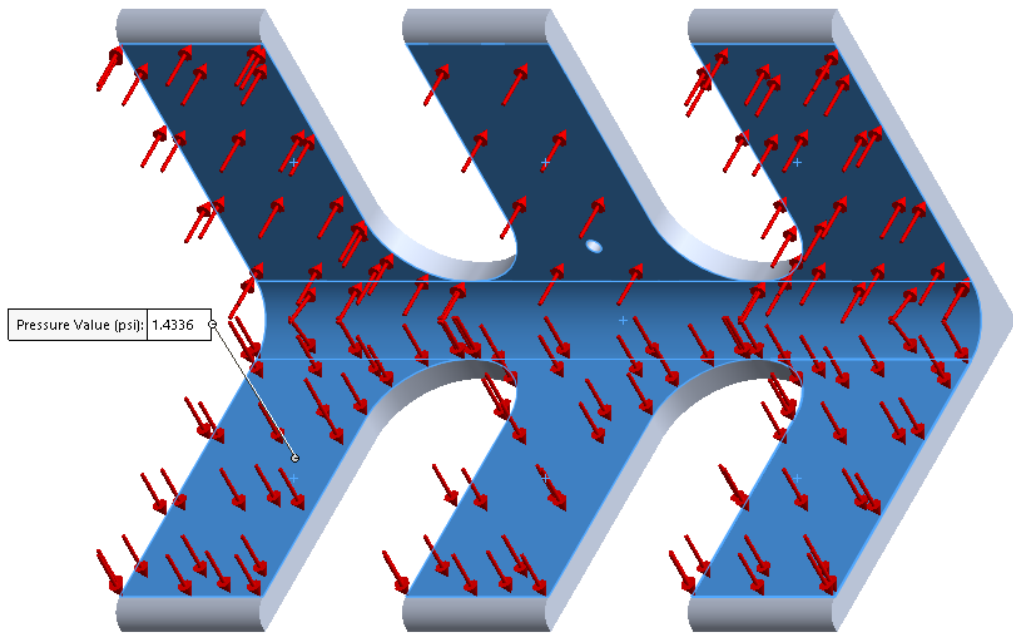


Figure 5.6: Pressure force application (red arrows) to proposed foot

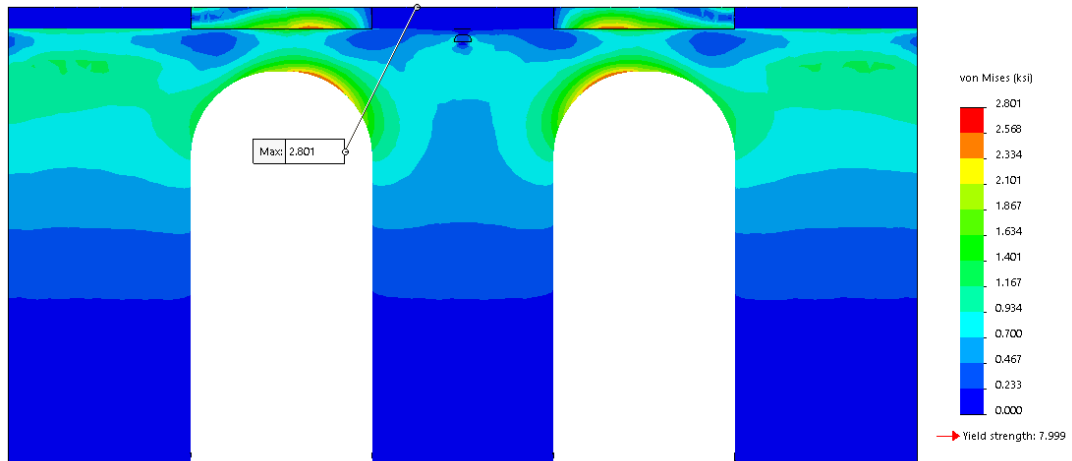


Figure 5.7: Regolith foot Von Mises stress plot (ksi)

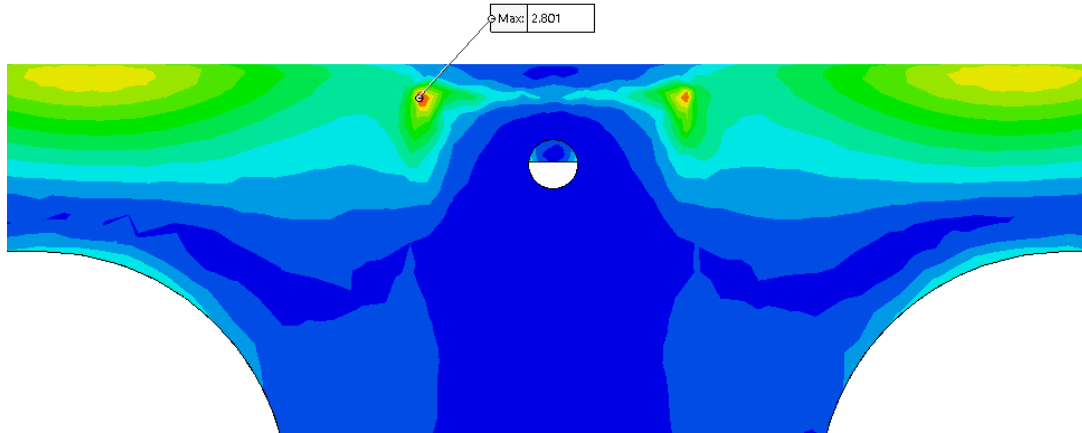


Figure 5.8: Regolith foot stress detail view (ksi)

This best represents the forces experienced by the foot, and the forces which may be experienced during foot withdrawal. The resulting design has a fatigue life of over three-thousand cycles. The minimum fatigue life for the proposed foot takes place at the fixed contact with the hinge it is mounted to. This stress concentration can be relieved by welding instead of using fasteners.

5.3 Hinge Design

The actuator and the foot both mount by means of a pair of hinges, acting as a revolute joints. This hinge can manifest in material form as a clevis, affixing the actuator as a simply supported beam by means of a smooth bolt. The clevis should have the capability to raise the weight of the rover on the thirty degree design case. Our design constraint here will be to design a clevis which has the capacity of the rover in standard earth gravity throughout the range of feasible angles. The extrema to the range of angles provide the design cases, and are

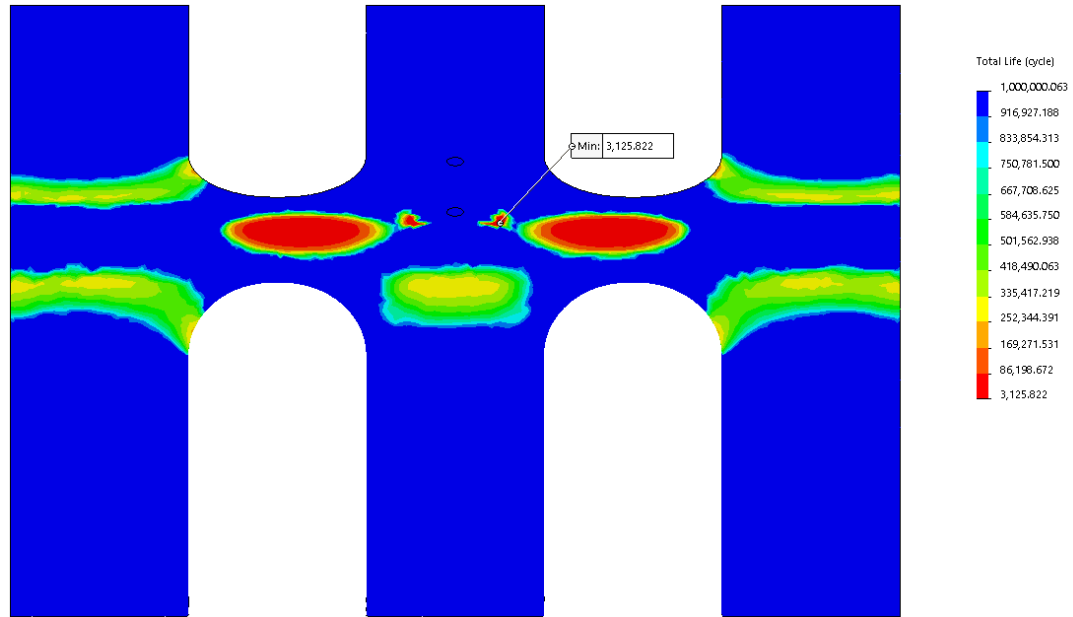


Figure 5.9: Regolith contact foot fatigue life

determined by the inclination angle of the traverse, as the actuator is passively mounted.

A fully defined part model was designed in SolidWorks software. The design is based off the actuator maximum simulated angle of inclination. Initial designs were for a solid clevis manufactured from 6061 series Aluminum. The design objective for the clevis was to minimize the weight of the part, while maintaining structural integrity by a safety factor of three.

Symmetric pockets were included in the clevis design utilizing parametric design principles. The distance from the pocket boundary to the edges of the sides were set as being identical, so that a change in this parameter would change all six values simultaneously. The pocketed features and extruded cuts were given fillets to represent true geometries created by a ball end mill cutter.

These fillets greatly reduce the presence of any stress concentrations in the part, and yield a more realistic simulation result. Without these geometry improvements, h-adaptive meshing would have converged to very small points which may represent erroneous values [6].

An iterative design study was completed for the part to determine the minimum material margin required between the pocket and edge of the part. Initial finite element analysis were verified and improved utilizing h-adaptive meshing for the SolidWorks simulations. The h-adaptive meshing varies the size, or h, of each individual element of the mesh, and greatly improves the precision of the model.

5.3.1 SolidWorks Simulation Parameters

The part design and simulation were conducted in SolidWorks. Numerous parameters were defined for the simulation including establishing fixtures and force applications.

Fixtures

The mounting of the clevis to the rover chassis is a flush mount. The mounting is simulated to be held in place by two bolted connectors. As such, in SolidWorks two separate connecting fixtures are required. The first is on the face which mounts to the SR2 chassis. This face has a constraining fixture that allows it to slide on that plane, but it cannot deflect to penetrate the mounting plane.

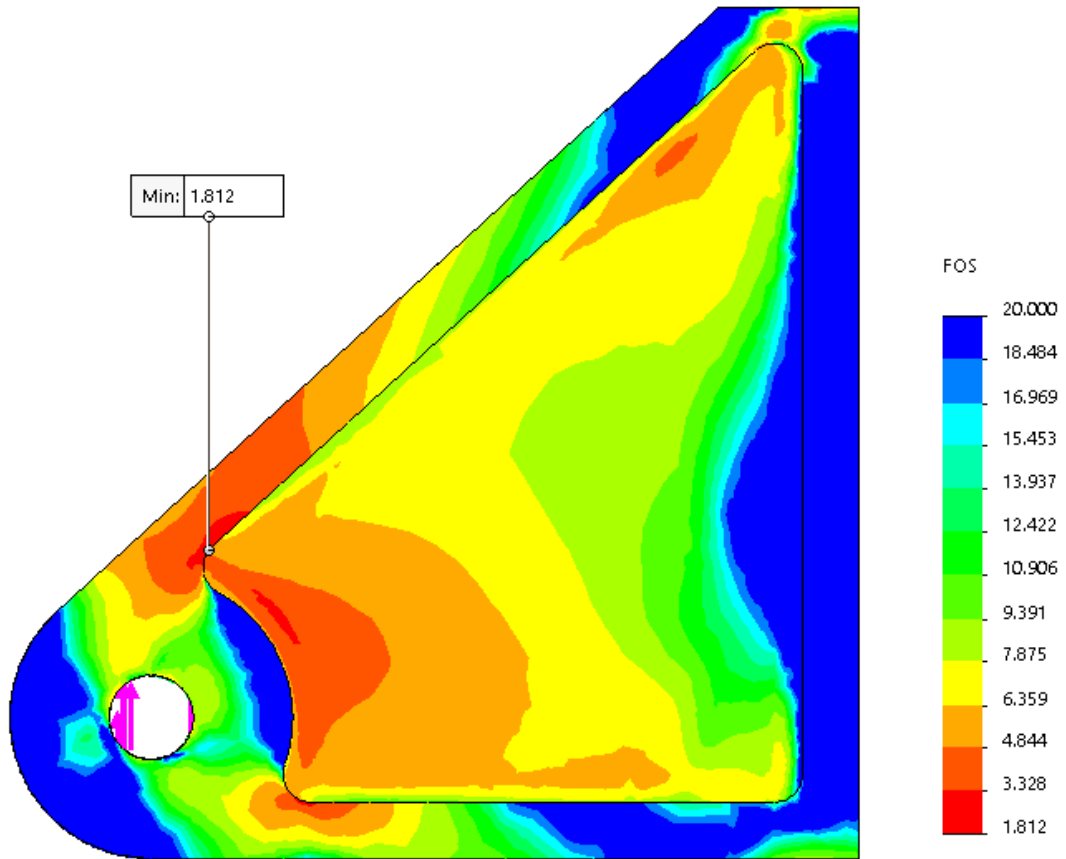


Figure 5.10: Mounting hinge safety factor plot for flat static force simulation, side view

Secondly, the two bolt holes are prescribed as fixed connections.

The simulations were composed for two above-stated cases which represent the extreme values of possible traverse grades: flat ground and an angle matching the top face of the clevis. The first presented result is for the application on a flat traverse. As seen in Figure 5.10, the dark blue region is nearly nonexistent. This is due to a simulated milling process in which excess material has been removed.

At the other end of the range of acceptable inclination values for traverse

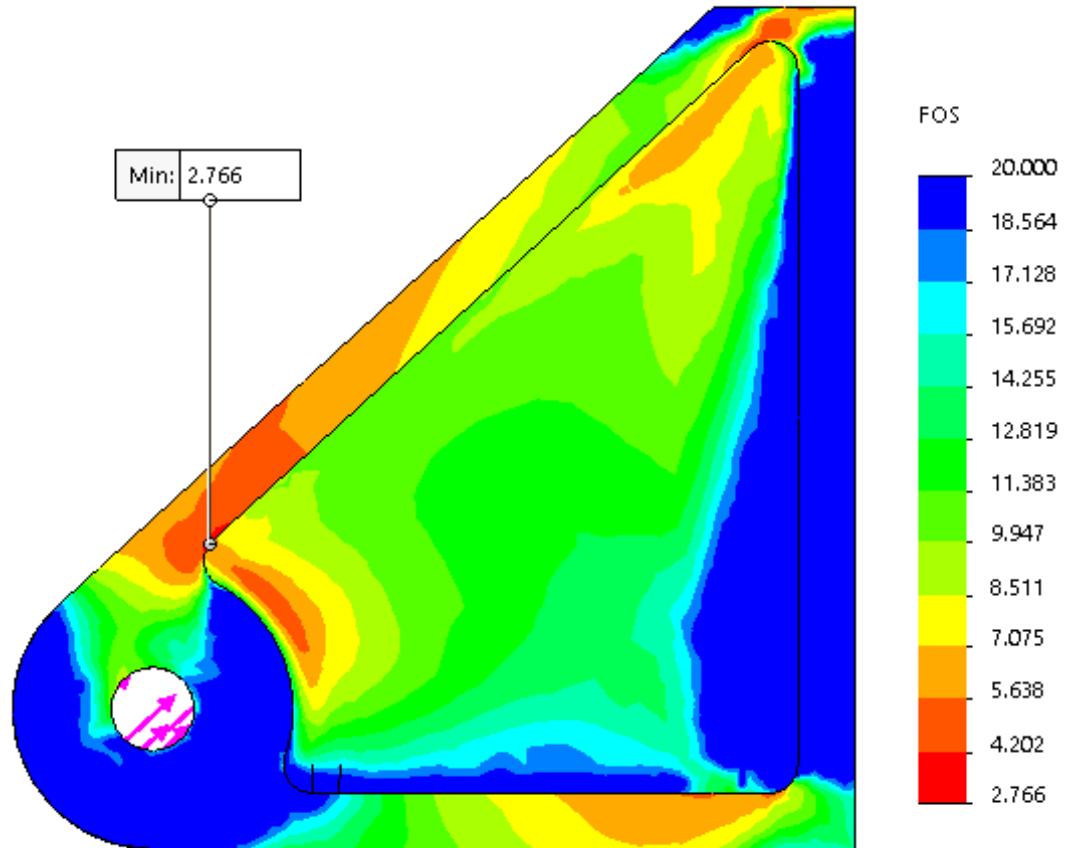


Figure 5.11: Mounting hinge safety factor plot for inclined static force simulation, side view

is the inclined simulation. These results are quite similar to the flat ground with regard to location. Upon close inspection the Factor of Safety for the inclined value is quite larger than of the flat ground simulation. This makes sense geometrically as the force due to the weight of the rover has different components in the inclined situation. The component force in simulation redistributes the stress from the hypotenuse of the triangle to the lower, horizontal leg. The force is dissipated with some small concentration represented in Figure 5.11 by the red regions.

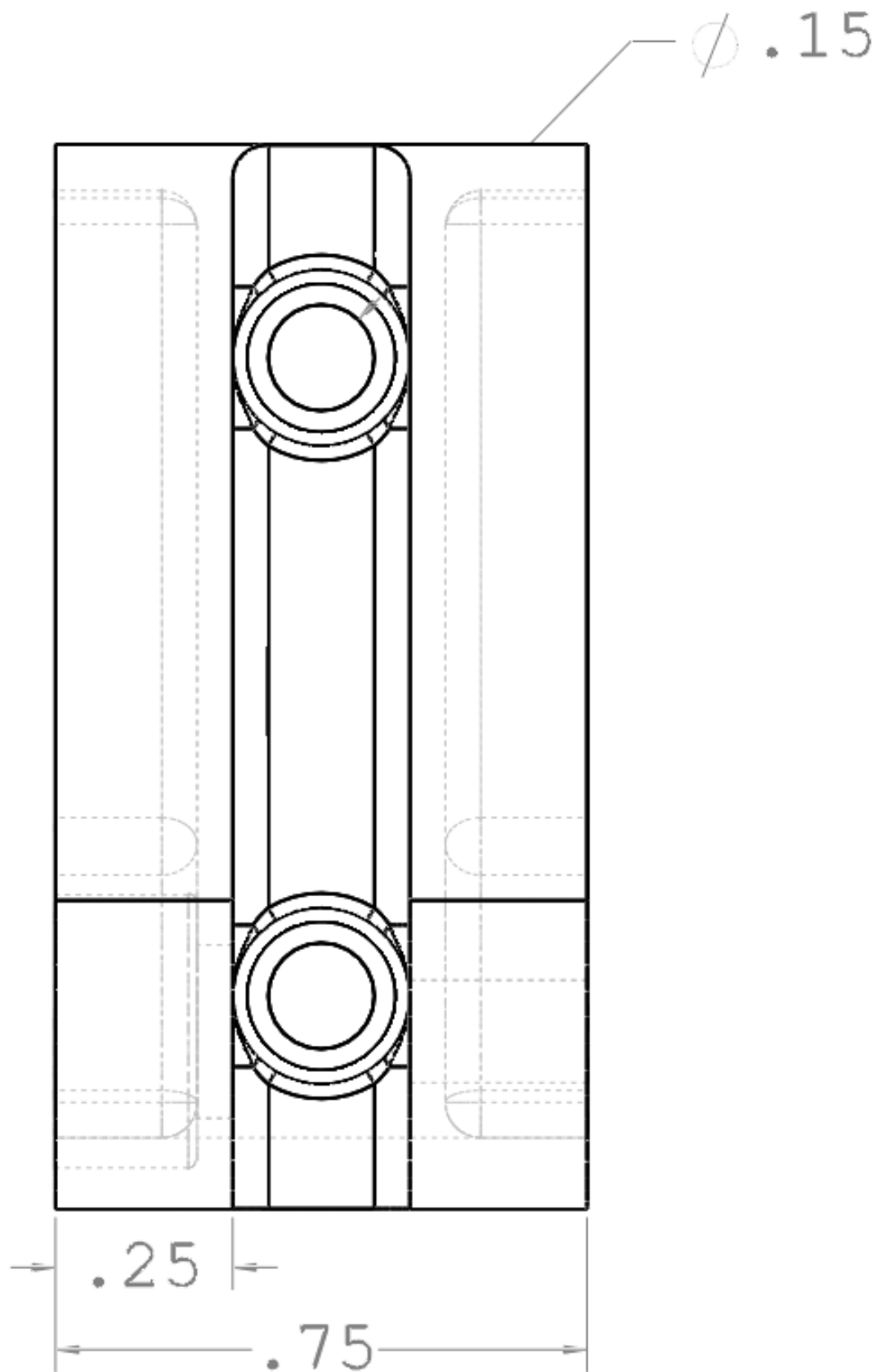


Figure 5.12: Mounting hinge dimensions (inches), front view

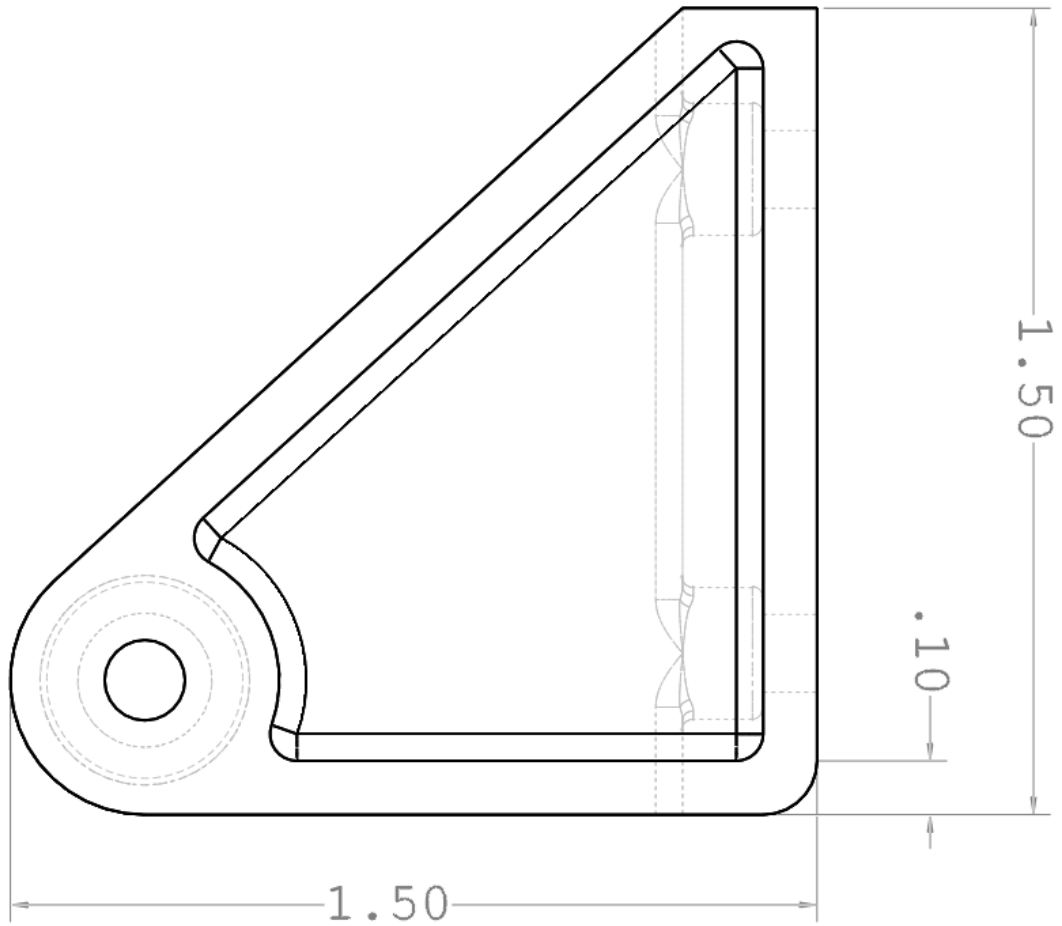


Figure 5.13: Mounting hinge dimensions (inches), right view

The final design of the hinge is designed to be milled from a 1.5” square section of 6061 Aluminum which is .75” thick. These dimensions are exhibited in front and right views of the hinge, Figures 5.12 and 5.13 respectively.

5.4 Actuator Thrust Rod Design

Actuators have characteristic thrust rod diameters which can influence the lifting capacity. To determine if the cylinder rod can handle the capacity of the rover, we use Euler’s equation for the buckling of a column. This equation determines the force required to place the column into a state of unstable equilibrium, upon which any lateral force applied to the column will cause a buckling of the rod and a failure of our system.

Euler’s Equation (5.8) was rearranged to solve for the area moment of inertia, which was used to determine the rod diameter. This formula requires the material property of the column. In this case, we select a stainless steel actuator rod for environmental resilience on Earth.

$$F_{buckling} = \frac{\pi^2 EI}{(KL)^2} \quad (5.8)$$

where I is the area moment of inertia of a solid circular cross section,

$$I = \frac{\pi d^4}{64} \quad (5.9)$$

Combined and rearranged to solve for the column diameter:

$$d = \left(\frac{64F_{buckling}(KL)^2}{\pi^3 E} \right)^{1/4} \quad (5.10)$$

where $F_{buckling}$ is the force required to put the column into a state of unstable equilibrium, E is the modulus of elasticity, L is the unsupported length of the column, and K is an effective length factor (in a doubly-pinned case, $K = 1$).

With these properties defined, we determine that the rod must have a minimum diameter of 4.13 centimeters to maintain stability when extended.

The length of the actuator was maximized in previous analysis of the system functionality and dimensions. The final length for the actuator minimum was nine inches. This will give another nine inches of actuation assuming the actuator can double in length, providing a good proportion of that actuation length to forward motivation, depending on the angle of inclination.

With these design results in mind, the proposed system is assembled virtually in SolidWorks. Representations of this model are provided in Figure 5.14 and Figure 5.15. The horizontal, top portion of the foot extends rearward, away from the rover chassis, and the grouser portion of the foot penetrates the terrain.

Figure 5.15 raises concerns regarding foot function. When initially contacting the traverse, the foot will not approach with the grouser tang first, rather the center of gravity of the foot-hinge assembly will lie below the actuator. As the actuator is extended, the forces will balance on the foot, distributed between

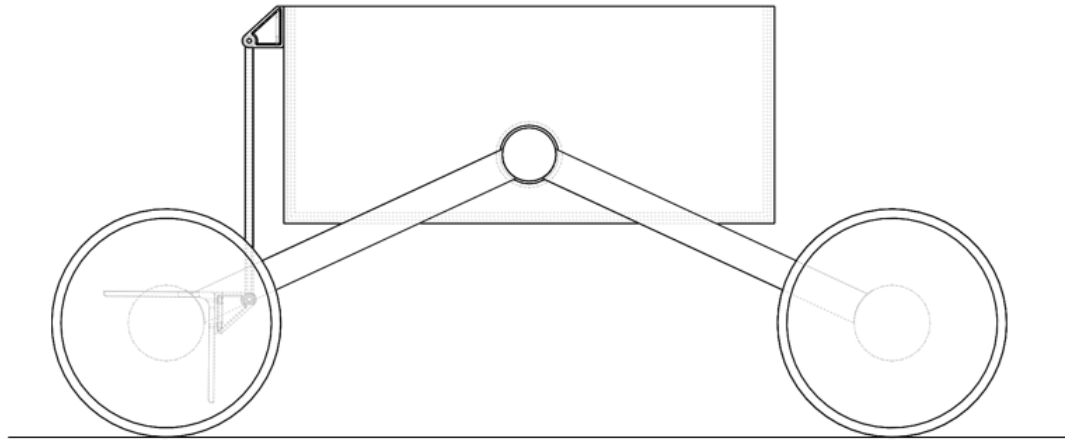


Figure 5.14: Proposed Assembly: Diagram of SR2 on inclined traverse with grouser foot retracted

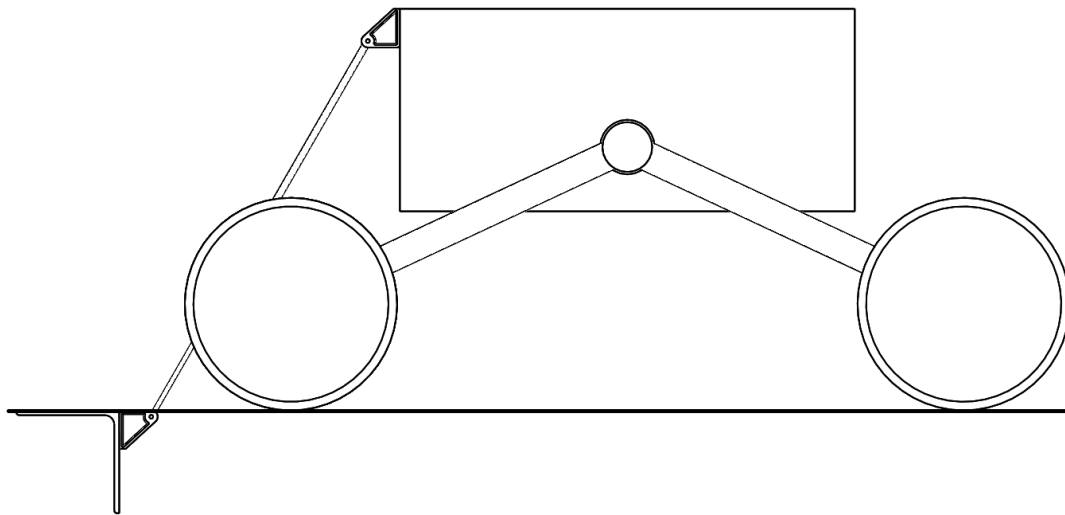


Figure 5.15: Proposed Assembly: Diagram of SR2 on inclined traverse with grouser foot penetrating terrain

the two point contacts of the foot, forming a sort of “A” shape with the terrain. The grouser portion needs to penetrate the traverse, while the horizontal portion maintains its angle with regard to the traverse. This issue and possible solutions are explored further in Chapter 7.

Chapter 6

Program

A computer program was created in MATLAB to define the hybrid mobility of the passively-actuated, legged rover. Physical parameters are set according to a physical rover in the Intelligent Robotics Lab, SR2. These physical specifications may vary according to the characteristics of any other known vehicle. Simulation parameters such as total time and actuation speed are easily defined at the top of the main program. Subroutines were called within the program main body. These subroutines perform specific tasks in the simulation, and are universal in that they are callable by any iteration or scenario of the main program.

The program written for the simulating the passive actuation process is pseudo-static. The simulation takes a prescribed change to the model, here lengthening of the actuator, and applies it as if the model were in static equilibrium. The actuator contact point is assumed fixed to the regolith. After the lengthening, the forces on the rover center of mass are solved for in the static

regime. Once solved for, the rover front wheel is released, allowing the body to settle dynamically. This process repeats iteratively, until the actuator reaches its maximum defined length. At this point, the extension portion of a cycle is complete. A retraction of the actuator then takes place, completing the cycle. A single simulation may be composed of a number of cycles within the defined simulation period.

6.1 Assumptions

Assumptions were made to simplify the model and calculations for this first analysis of the situation. These assumptions mitigate some extraneous equations that would further complicate the model with little improved precision. Continued study should improve the model, superseding these results with more accurate models.

The SR2 Rover is assumed to have both a rigid body and rigid wheels. In reality there is a mechanical differential in the SR2 rover, and the left and right wheels can move with regard to each other.

The rover simulation also assumes it can only move in plane, that is forward and vertically. There is no transverse motion to the rover causing dissimilar loads to the left and right sides. For the purposes here, the rover will be driving perfectly straight up the traverse. In reality, small divergences from the straight path will occur. These divergences are a product of the inherent instability of

the system, particularly on granular soils.

A massless actuator is assumed for the simulation. This allows the center of mass of the entire system including the SR2 rover to remain constant as the actuator changes position relative to the rover chassis position. If a massive actuator were included, the problem would be more challenging, and the result more accurate. A difference between the current work and one that included consideration for an actuator with mass would include a variation of the position of the center of gravity: the center of gravity of the system would vary as the leg and foot subsystem move with relation to the rover chassis. Depending on the relative mass of the actuator, the center of gravity of the system will vary. This variation will occur external to the proposed traverse methodology, but will affect the result.

In order to include a massive actuator in the simulation, the number of center of gravity (hereafter CG) calculations would need to increase. The CG would need to be calculated for every incremental change in actuator length and for every settling function call. These CG calculations occur in both reference frames, requiring a great number more calculations per cycle. This increased computational load would require more computational power. The simulation also assumes the extension and retraction speeds of the actuator are equal and constant throughout the cycle.

Out of plane tipping may occur due to the instability of the system when the actuator is extended. This is not considered here, but may occur when only

3 points of contact are made with the terrain. The assumption also allows for a great simplification of the terramechanical situation regarding slip. Due to the lack of lateral motion of the rover, we can assume there is no lateral slip at the wheel-soil interface.

6.1.1 Interfaces and Contacts

In the simulation, the rover is propelled forward, up the incline, solely due to actuator intervention. We are assuming no resistance to motion from the regolith in this case. The regolith may be viewed here as a simple non-deformable plane, represented two dimensionally as a line.

The interfaces are defined as having of infinite friction at all traverse contacts. Infinite friction provides a sticking contact so long as the velocity cone lies within ninety degrees of the friction cone [13]. In this study, the range of the actuated leg is limited to fall within these constraints. Furthermore we assume no slippage nor friction losses in the fifteen second simulation.

6.2 Parameter Definitions

Within the simulation environment, a number of definitions are required. The coordinate frame used by the simulation is consistent with the frame used in Section 4.2.2: longitudinal motion in the x direction, vertical motion in the z. Many variables are utilized in the simulation, most of which are user-specified.

These parameters are defined in the main program once. These definitions may then be used in all cases. It is possible to input values which are not feasible. A table of parameter names and descriptions is below.

Table 6.1: Vehicle Physical Parameter Definitions

Parameter	Description
m	Mass of vehicle (kilograms)
act_l	Length of the actuator (meters)
g	Gravity constant (meters per second squared)
I_{yy}	Moment of inertia of the vehicle, axis of rotation is normal to the right plane, through the CG (kilograms meters squared)
r	Wheel radius (meters)
L	Vehicle wheelbase (meters)
d	Distance from rear wheel contact point to actuator mounting location (meters)
j	Distance from actuator mounting location to front wheel contact point (meters)
m	Distance from front wheel contact point to vehicle center of mass location (meters)
o	Distance from rear wheel contact point to vehicle center of mass location (meters)
ν	Angular measurement between vectors J and n, between actuator mount and center of mass (degrees)
O	Angular measurement between vectors cg_y , the vehicle center of mass height (meters, rover frame) and o (degrees)
h_{act}	Height of the actuator mount (meters, rover frame)
$stroke$	Actuator stroke length (meters)
act_{min}	Actuator minimum length (meters)

An important user-specified parameter is the inclination angle of the traverse, taking the symbol θ . Also, the actuator speed is defined by the user. The units of the actuation speed are in meters per second. In order for actuation speed to not break the simulation, small values should be used so as to not impart large

inertial forces. Inertia is not accounted for in the program, but may be resolved by increasing the simulation resolution. Parameters relevant to the simulation and free-body diagram are tabulated below, followed by an illustration showing where the application of each parameter acts in Figure 6.1.

Table 6.2: Simulation Parameter Definitions

Parameter	Description
θ	Traverse angle of inclination, or characteristic angle of repose (degrees)
dt	Time step for a single loop (seconds)
$duration$	Total duration of a simulation run (seconds)
act_l	Length of the actuator
F_{x1}	Force in the x-direction at the front contact (rover frame)
F_{z1}	Force in the z-direction at the front contact (rover frame)
F_{x2}	Force in the x-direction at the rear contact (rover frame)
F_{z2}	Force in the z-direction at the rear contact (rover frame)

All of the parameters for the simulation and vehicle are input into the main program file. To vary these values the user should open the file and change the values in the initial sections of code.

6.3 Program Structure

The program is laid out in a very organized and logical structure. This organization was helpful not only for the author, but for future use and ease of interpretation of the code. The general structure of the MATLAB program is

1. Define adjustable simulation parameters
2. Define adjustable physical parameters
3. Matrix initialization of variables

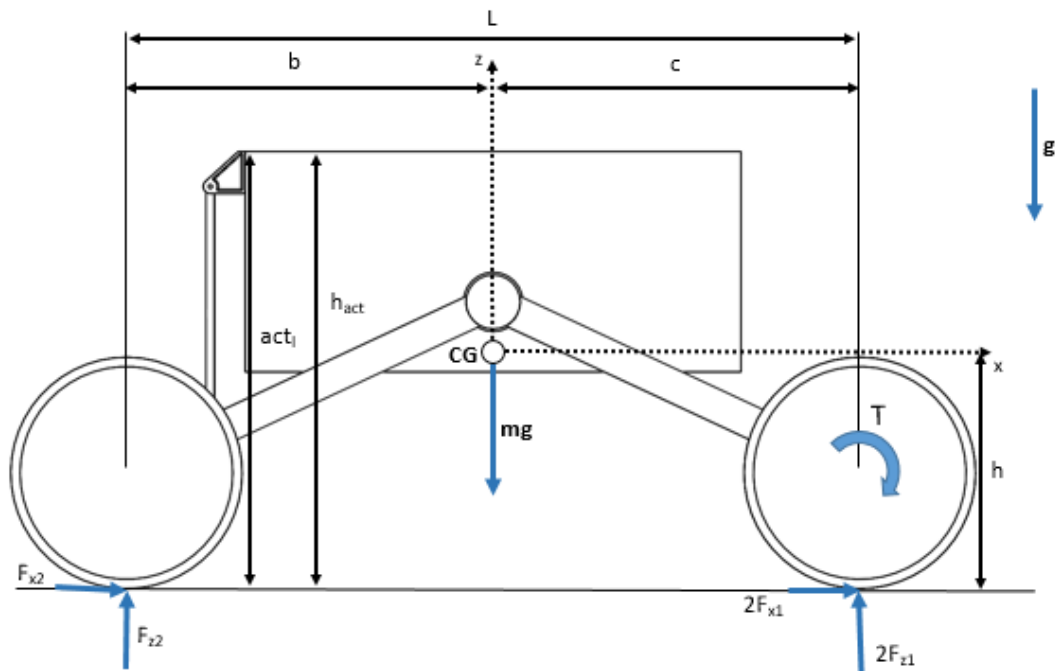


Figure 6.1: Vehicle representation with physical parameters defined

4. Define vehicle parameters and geometric properties
5. Define actuator parameters
6. Define all initial geometry values and cg locations
7. Fix front wheel location
8. Incrementally lengthen actuator, resulting in a rotation about front wheel contact point
9. Solve new geometric condition
10. Relate change of actuator mounting location to change in center of mass position
11. Solve for weight on ground contact points
12. Define forces relevant to the specific case
13. Begin settling loop, releasing fixed front wheel
14. Apply resultant force to find incremental center of mass position change
15. Solve for actuator mount location using change in center of mass location
16. Use new actuator location to solve new geometric situation

17. Loop back to fixing front wheel location

6.3.1 Function Calls

In the simulation program there are a number of functions which are called. These functions do not vary from case to case, only their inputs vary if necessary. The inputs are defined in the main program before the function is called. A table summarizing the functions is found in Table 6.3.

Table 6.3: Function Calls and Descriptions

Function	Description
Step1Forces	Solve for incremental actuator lengthening with fixed actuator contact and fixed front wheel location.
cycle_check	If actuator at max length, begin a new cycle
settle	Let the rover settle before next dt
tip_check	Check if SR2 falls backward when actuator is retracted for a new cycle

6.4 Main Loop Sequence

The ‘main’ loop of the program is where the function calls reside. These functions perform specific tasks at regularly defined intervals. The main loop utilizes a counter which maintains the relationship of the function with time. The initial value of the counter is correlated with the initial values of the rover defined position. Therefore, as the program runs, the main loop calls for the values of the simulation at iteration number two (2) until simulation completion. The main loop will exit, ending the simulation, in one of two ways: the counter reaches maximum duration, or the rover tips over backward on account of a very steep slope.

The first action that is taken by the simulation is an incremental increasing of the length of the actuator. The increment by which the length is increased is defined previously in the parameters of the simulation. The time increments for the simulation are so small that the incremental lengthening of the actuator is of a very small order. The increase in length of one of the legs of the simulation triangle puts the forces out of balance. This imbalance is due to the actuator contact at the rear being held constant.

In this simulation, a perfect bond at the actuator-ground interface is assumed. Therefore there is a static reaction force at the joint, but no dynamic behavior allowed. This assumption is dependent on the successful design of the contact foot in Chapter 5. Secondly, the following graphical representations of the rover are exaggerated. The lifting of the rover at the rear, in particular, has been overemphasized over to show the relationships between the rover entities.

The geometry of the changing rover may be represented as a triangle of linkages. A link is fixed with a hinged joint at the rear actuator contact point, and an identical joint at the front wheel contact. The rear link represents the actuator. The front link represents the distance between the actuator mount to the chassis and the front wheel contact. This may be represented as a simple link as we are assuming the rover to have a solid body, therefore this distance is fixed. The hypotenuse of the triangle connecting these two links is the distance between the actuator contact point at the rear, and the front wheel contact point up-slope at the bow. A representation of these links is provided in Figure 6.2.

Simulation of the incremental actuator lengthening assumes that the front wheel remains momentarily stationary. This momentary hold allows for the new static situation of the rover to be evaluated. With these assumptions, the geometry of the rover changes such that the wheelbase (distance between

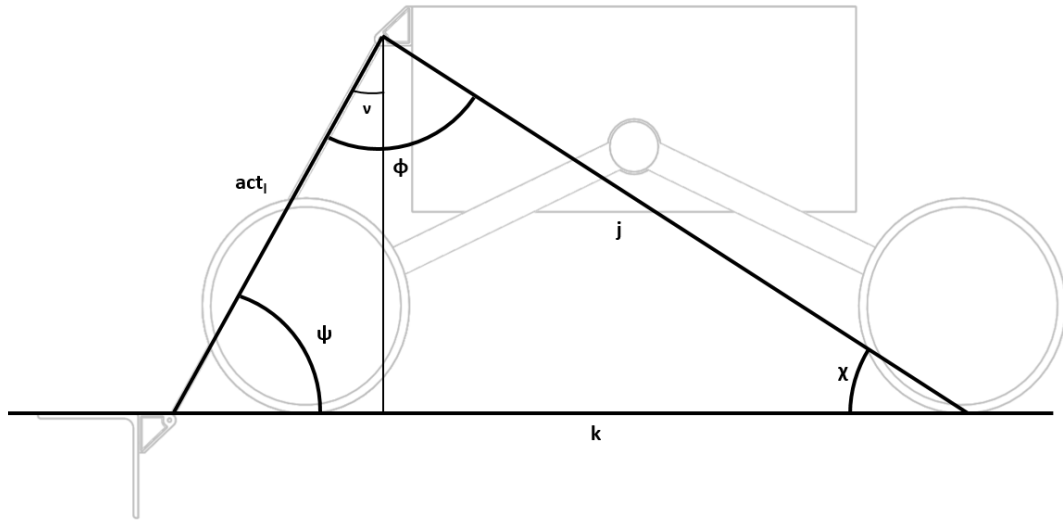


Figure 6.2: Simplification of the system with relevant angles labeled

the actuator contact point and front wheel contact point) remains constant. Therefore, a lengthening of the actuator requires the passive hinge of the rover to adjust accordingly, to comply with the changed geometry, as shown in Figure 6.3.

Due to the leg lengthening, the rover is forced to rotate, or pitch forward momentarily. This pitching about the lateral axis of the rover may remove the rear wheel contact with the ground, lifting the rear of the rover. Once lifting occurs, a function named *Step1Forces* is called which solves for forces for this system in equilibrium. The forces are statically balanced on the rover, and the resultant component accelerations and velocities are determined as in Figure 6.4. These values are returned from the function *Step1Forces* to be applied to the rover in following functions.

Following the force calculation for the incrementally extended leg, a determination is made regarding system cycle. Given the proposed methodology, the leg can only actuate a finite distance. When the maximum leg length is reached, the leg must be allowed to retract and begin a new cycle. The leg length is inspected by a function titled *cycle_check*.

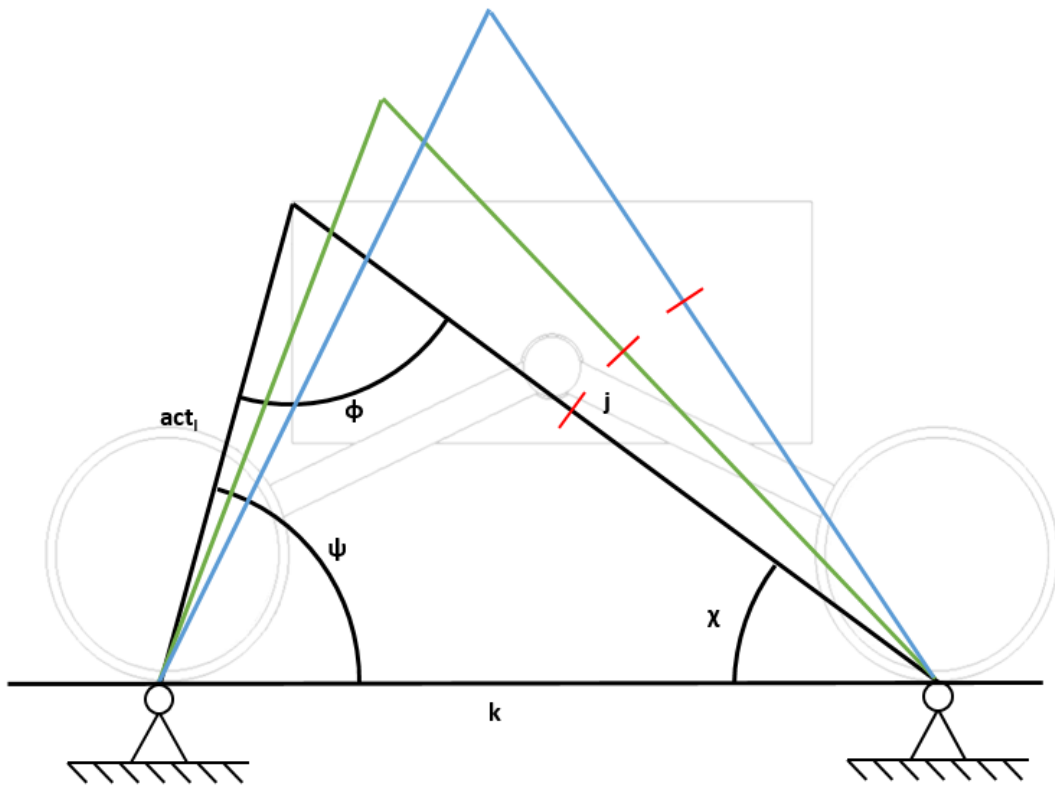


Figure 6.3: Simplification showing exaggerated forward pitching behavior

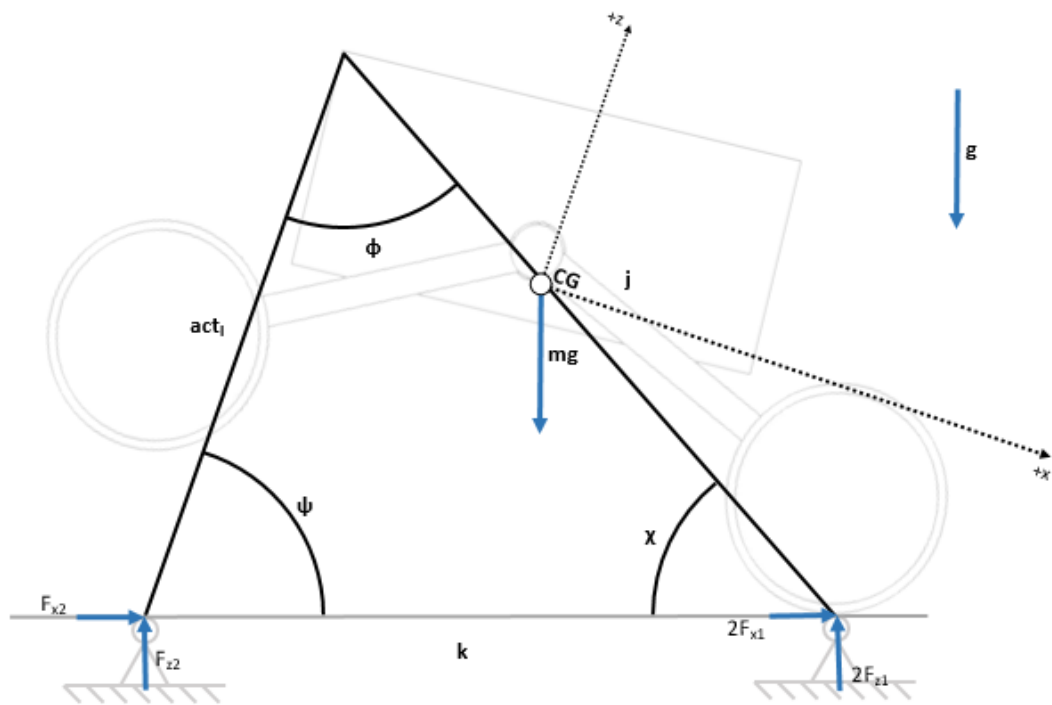


Figure 6.4: Simplification showing forces for static analysis

If the leg has reached maximum length, variables are changed accordingly. For a new cycle, the actuator length, act_l and distance between contact points, k , are reset to their initial values. The minimum length is determined to be the initial length of the previous cycle. This assumption is valid as the simulation takes place on a traverse of constant slope. Once minimum length is reached, a new cycle is set to begin, and the user is notified that a new cycle is occurring. Following this the value for $cycle$ is set to TRUE, and a counter, $cycle_count$ is incremented which keeps track of the number of cycles in the simulation.

The rover settling behavior is prescribed in another function call, $settle$. This utilizes the calculated component velocities and incrementally applies them to the center of gravity of the rover. Nested within this function is a loop which allows the rover to settle back down to equilibrium on the traverse. The loop for settling terminates if one of two conditions occur: either the angle χ defining the inclination angle of the rover with respect to the ground plane is within one-tenth of one percent (0.1%) of the initial value, or the maximum number of iterations allowable has been reached. A settled condition is shown in Figure 6.5. Following the settling sequence in this function, the values for specific points on the rover are determined geometrically. Lastly the new rover center of gravity values are determined in both the reference frame aligned with gravity and aligned with terrain.

Following exit from the $settle$ function, behavior is determined by whether a new cycle occurred or not. In either case, the system values for center of gravity are update. If this is an iteration upon which a new cycle occurs, the center of gravity coordinates for the rover in the system frame are held steady. This is due to the behavior assumed earlier in maintaining enough force to prevent backward motion of the rover. Also the actuator angle with regard to the rover

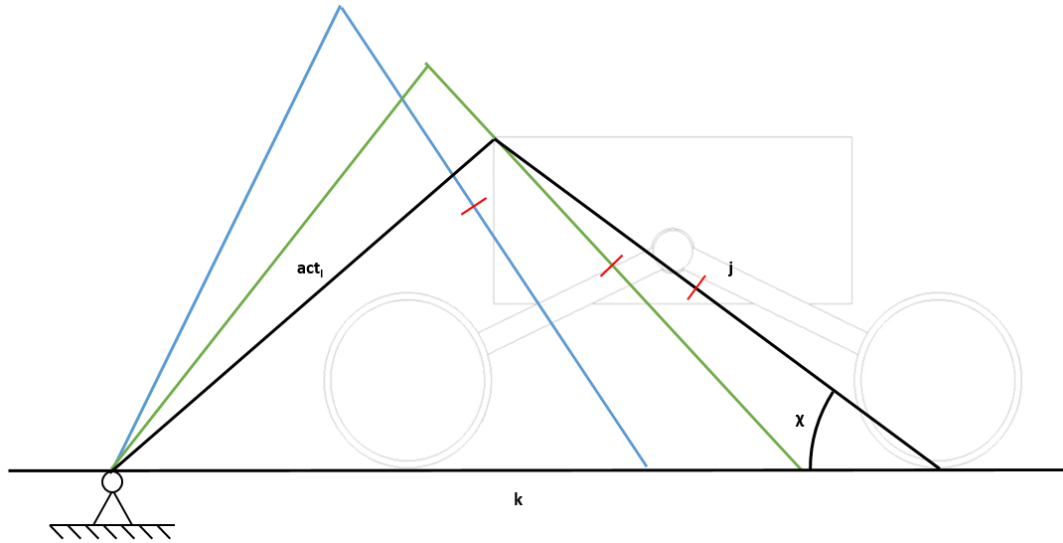


Figure 6.5: Simplification showing angle χ for settled condition

chassis, ρ , is reset to its initial value, and variable *cycle* is reset to *FALSE*. A function, *tip_check* is then called which checks to see if the rover is in such a state that it might tip over backward.

The check for falling over backward is a geometric formula that determines the position of the rover center of gravity with respect to its rear wheel contact point. Depending on this relationship, the rover may be stable, in unstable equilibrium, or tipping rearward over the back wheel.

If a new cycle is not occurring, *cycle* = *FALSE*, the motion of the rover center of gravity is updated as previously.

The resulting values for the system-level center of mass component positions with respect to time are plotted. These plots give lots of information about the motion of the rover, and about the hybrid mobility scheme.

6.4.1 Force Calculations

The force calculations for the rover are conducted for a pseudo-static simulation. The initial statement of the function sets the value of *k* to the value of the

previous k . This is necessary as the program scheme first inclines the rover, then allows the rover to settle back to equilibrium. In this statement, the value of k is held constant, which requires the distance from the actuator traverse contact point and the front wheel traverse contact point, to be constant. Due to the forcing of this variable, the lengthening of the actuator which was defined just before this function call, will force the rover to tip forward, rotating about the front wheel contact point. It is in this state that the forces are calculated.

The main triangle connecting the front wheel contact point, actuator contact, and actuator mounting point is solved for first. The changing actuator length varies all of these angles and lengths with the exception of j as stated above. Following this the center of gravity location is updated by the rigid body geometry definitions. The angle defining the attitude of the rover with respect to gravity, ρ is incremented by the same amount that the value for χ has changed, as they are congruent. Figure 6.6 demonstrates this behavior.

Once these values are updated, the forces are solved. The solution is found via static equilibrium laws. Once the values for all the forces are solved, the fixed front wheel is effectively released. Since the wheel is released, motion is now allowed for the rover, and accelerations and velocities are solved for in the x and y components. The force values are all solved for in the rover frame of reference.

6.4.2 Settling Loop

The settling loop subroutine is called to allow the rover to settle back to an equilibrium state following incremental actuator lengthening. This occurs by allowing the center of gravity position to change according to the component velocities of the CG. With each new iteration of the loop, the CG location is

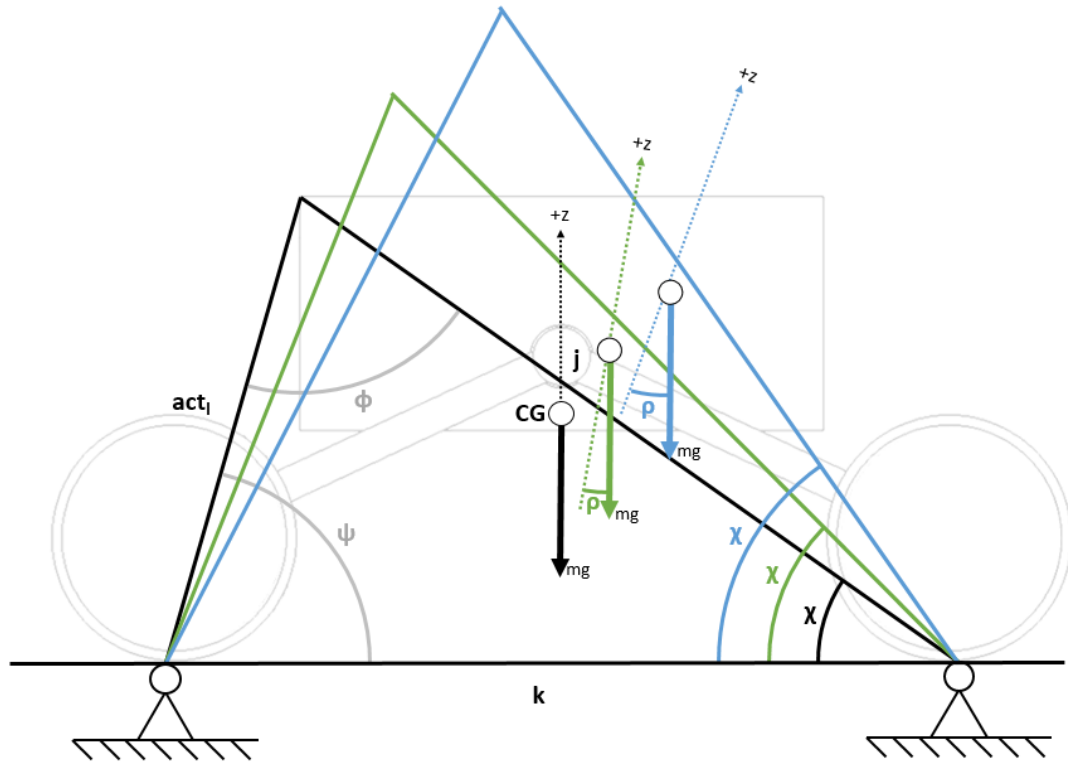


Figure 6.6: Inclining vehicle to show congruent angles ρ and χ

updated, and the position of the actuator is recalculated via angles χ and ρ . The angle ρ is the angle of the rover with regard to gravity, and χ is the angle defined by the line connecting the rover actuator mounting location to the front wheel contact point, and the line of the AlignedWithTerrain plane, as seen in Figure 6.7.

The condition to end the settling loop can be one of two things: maximum number of iterations for the settle loop is achieved, or χ settles to a value near the initial value. The approach of χ is given a small error margin of one-tenth of one percent (0.1%) of its value before incremental leg lengthening. This second condition is analogous to the center of mass reaching its initial y position on the AlignedWithTerrain frame, and therefore due to a rigid body assumption, the rear wheel making contact with the traverse. Once the rover has settled according to the above condition, the geometries are recalculated, including

6.4.4 Tip Check

Tipping is a concern when the actuator does not make contact with the terrain. This behavior is differentiated from a forward pitching about the lateral axis, which is the fundamental behavior required of the proposed solution. Tipping is defined here as occurring backward, opposite the intended traverse direction. In this phase of the simulation, the only contacts with the terrain are through the four wheels. If the vehicle center of gravity is allowed to pass rear of the rear wheel contact $x_{cg} < x_{rear_contact}$, the rover will tip over backward.

The simulation assumes a planar, two-dimensional model of the situation, and therefore does not consider falling out of plane. Out of plane motion may occur due to the instability of the system when the actuator is extended. In this situation, only three points of contact with the terrain; a much less stable situation than four or even five points of contact.

To check for this behavior in the simulation, values for the center of mass location and two angles O and θ are used. The location of the rear wheel is calculated in the `AlignedWithTerrain` frame, then is projected down onto the `AlignedWithGravity` frame of reference. The resultant x value is compared to the x value for the center of mass in the `AlignedWithGravity` frame of reference. If the x value for the center of mass is less than the rear wheel contact point, the rover is unstable and will tip over rearward. The user is notified of this behavior, and the program sends an error which breaks out of and ends the program.

6.5 Beginning in static equilibrium

We begin our analysis of a rover traverse simulation with the most computationally simple case. This first case involves motivation of the rover by the actuator only, although there is some passive participation by the rover wheels. The force provided by the wheels serves only to maintain the rover position on the traverse. In a Newtonian sense, the force provided by the wheels through the traverse-rover contact is equal and opposite to the force of gravity in the rover traverse direction, in the rover frame of reference. In this case, the actuation of the rover puts the rover out of static equilibrium, and an motion is required of SR2 to regain static equilibrium.

6.5.1 Case 1: Flat Traverse

This first case exhibit is with regard to the SR2 rover traverse on a flat traverse, `AlignedWithGravity`. In this orientation, shown in Figure 6.8, we expect the rover center of mass in the y direction to remain constant for both the frames of reference, as they are identical.

The plotted results are of the center of mass in both the x and y directions, which relate to the longitudinal motion of the rover and the vertical motion of the rover respectively. Each of these results is presented in two reference frames in the results figures.

The representations show motion of the center of gravity in two frames. The `AlignedWithGravity` frame projects the rover center of mass motion from the point of view of an observer which resides off of the `AlignedWithTerrain` traverse. The gravity vector here is parallel with the y axis of motion. The `AlignedWithTerrain` frame representations reside in the first row. This observation represents the motion of the center of mass of the rover with the sloped traverse inclined

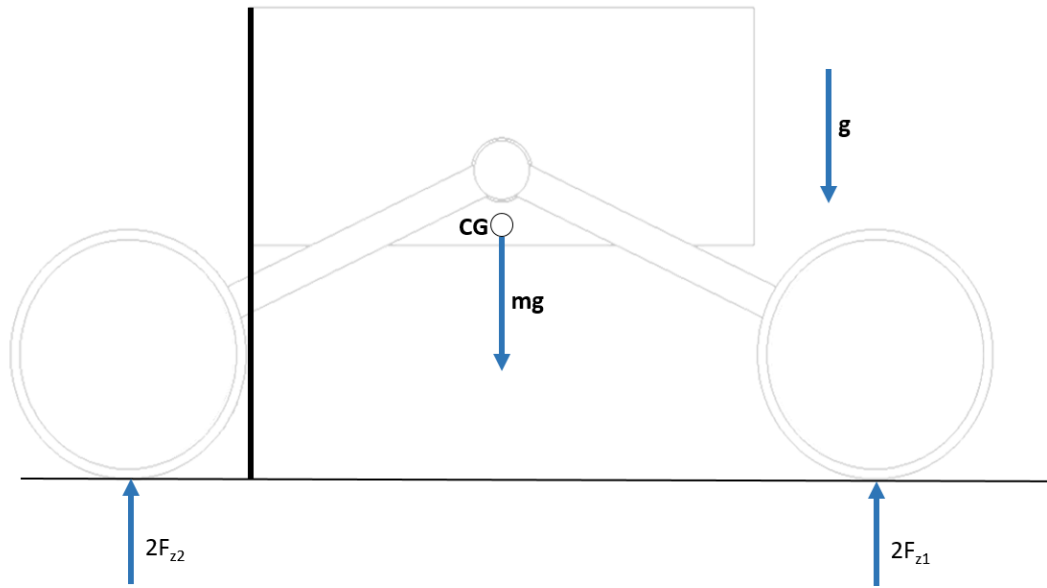


Figure 6.8: Case 1: Flat rover and terrain orientation

at the determined angle off the gravity vector.

The first result in Figure 6.9 shows the x motion of the center of gravity for each reference frame, while Figure 6.10 shows y coordinate motion.

The shape of the function defining motion is characterized by a varying slope. Initially the slope is more steep than in the middle of this case, followed by another steep portion which relaxes. This behavior is due to the nature of the legged motion from the actuator extension.

The dip that is seen at approximately twelve (12) seconds into the simulation is caused by the end of a cycle, and the beginning of a second cycle. At the beginning of each cycle, the actuator lengthening contributes a greater propulsion to the forward traverse than it does at the end of a cycle.

The values for longitudinal motion reveal the behavior provided by the rover actuator. The plot in Figure 6.9 are identical, which is to be expected. In this first example the AlignedWithTerrain frame is equivalent to the AlignedWithGravity reference frame, as $\theta = 0^\circ$.

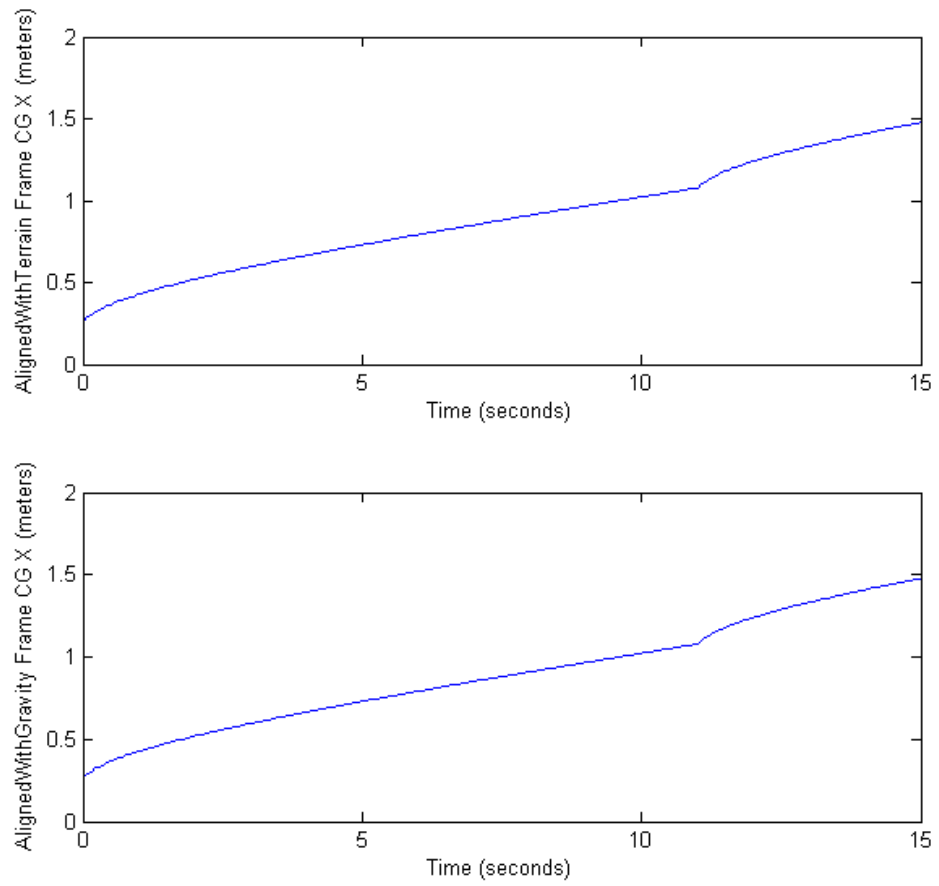


Figure 6.9: Case 1 Result for X Motion

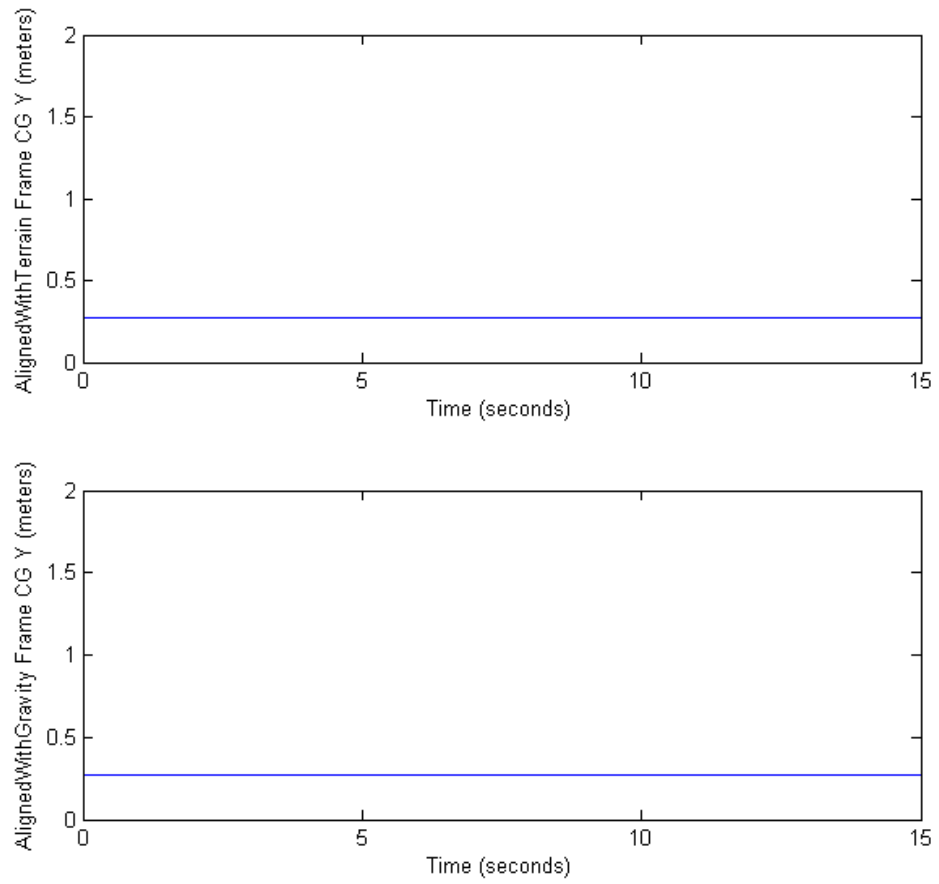


Figure 6.10: Case 1 Result for Y Motion

A notable observation is that the y values for the rover remain steady at their initial value. This result reveals that the rover fully settles to a resting position before continuing traverse for another incremental time step and actuator lengthening. Because there is no resistance to motion in either direction, there is no resistance to the rover moving forward the instant the actuation makes an incremental increase in length.

To verify the model, the AlignedWithTerrain frame y values are plotted as a function of the x values for the same reference frame in Figure 6.11. Similarly plotted are the AlignedWithGravity frame x position to the AlignedWithGravity frame y location. The anticipated result is verified, in that the behavior for each show straight, linear motion with no deviation on the domain excepting that within the range of acceptable error values previously defined.

6.5.2 Case 2: Five Degree Incline Traverse

To verify the simulation and ensure parameter definitions, we give it a small incline to climb and interpret the result. In this first case we give the simulation an inclination value of $\theta = 5^\circ$ in Figure 6.12.

As you can see in Figure 6.13, little noticeable variation occurs between the AlignedWithGravity and AlignedWithTerrain plots in the x direction. This is expected as the resulting values are very similar. These results are also similar to the result for the flat terrain shown in Figure 6.9. However, upon careful inspection the initial motion at the beginning of each cycle ($t = 0, 12$) is slightly less aggressive for the five degree incline.

It is useful to note however the variation that occurs in the y result, shown in Figure 6.14. In this plot we can see a noticeable increase in the position of the y center of gravity value over time. This behavior is predicted and acts as a

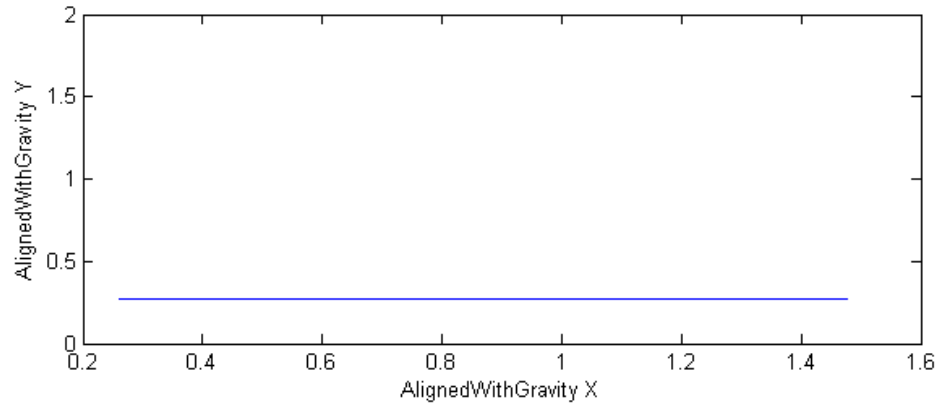
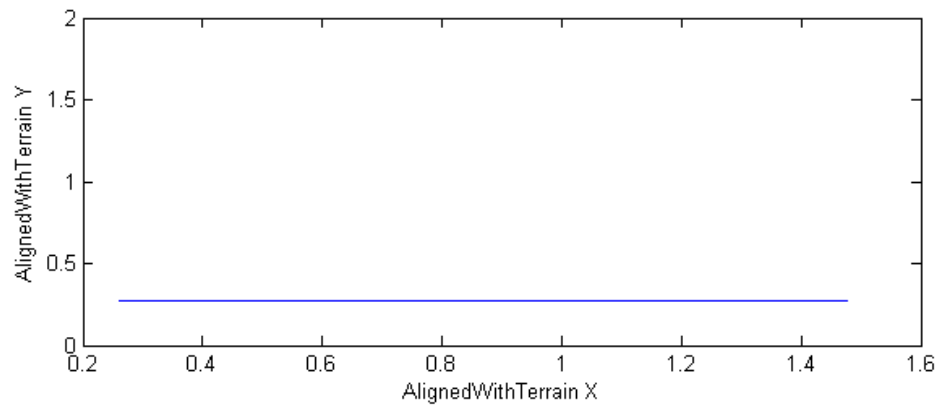


Figure 6.11: Case 1 Combined Motion

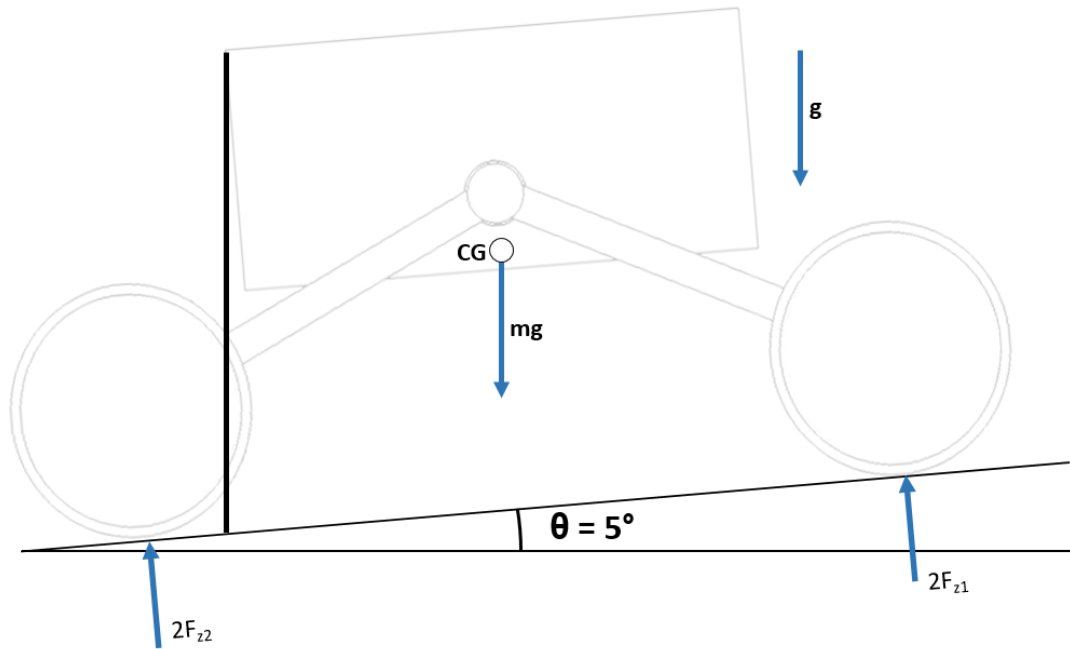


Figure 6.12: Case 2: Five degree rover and terrain orientation

verification of the result. As in the flat traverse case, the y values show that the rover completely settles before another incremental actuator lengthening cycle occurs.

6.5.3 Case 3: Thirty Degree Incline Traverse

This simulation stands as verification of the mechanical design case for dry sand, with a characteristic angle of repose of thirty degrees. The thirty degree results shows an even more severe slope traverse being successfully accomplished. This situation is shown in Figure 6.15. Note the red section at the tip of the actuator leg. This segment represents the extra length of the leg which must be utilized before terrain contact is made by the foot at the end of the actuator. Due to this extra length required, the progress of the rover is affected: cycle time is reduced due to fixed actuation speed, and incremental progress is reduced due to fixed stroke length of the actuator.

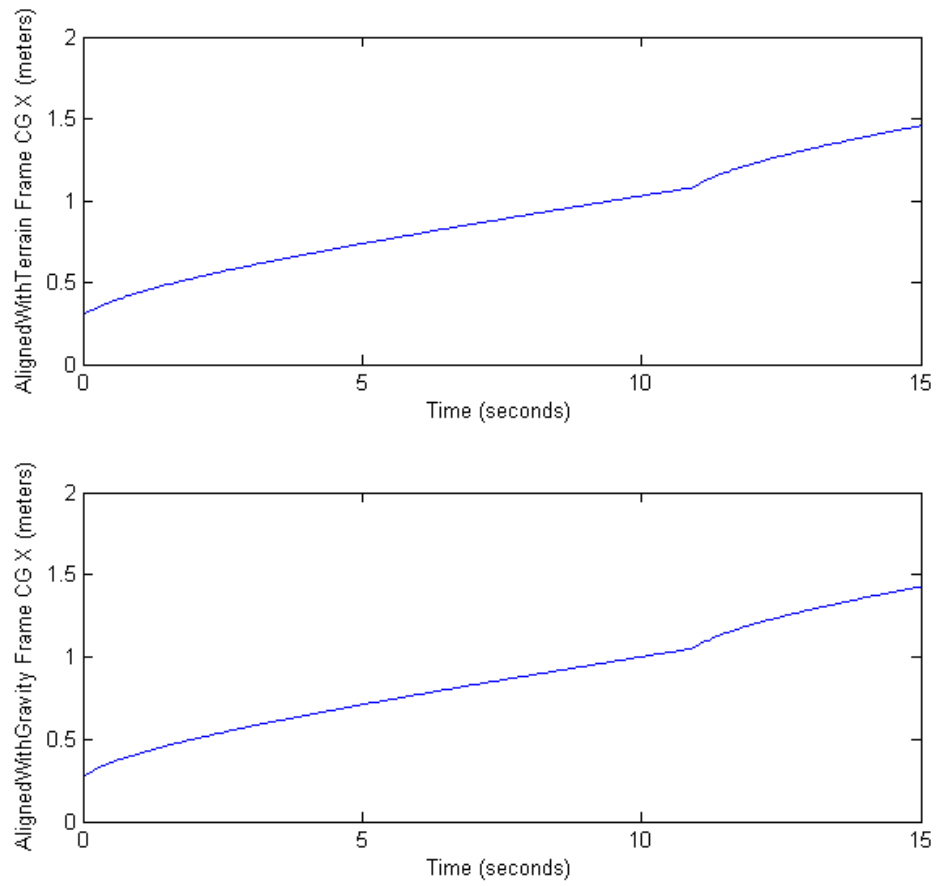


Figure 6.13: Case 2: Five Degree Incline Result X Motion

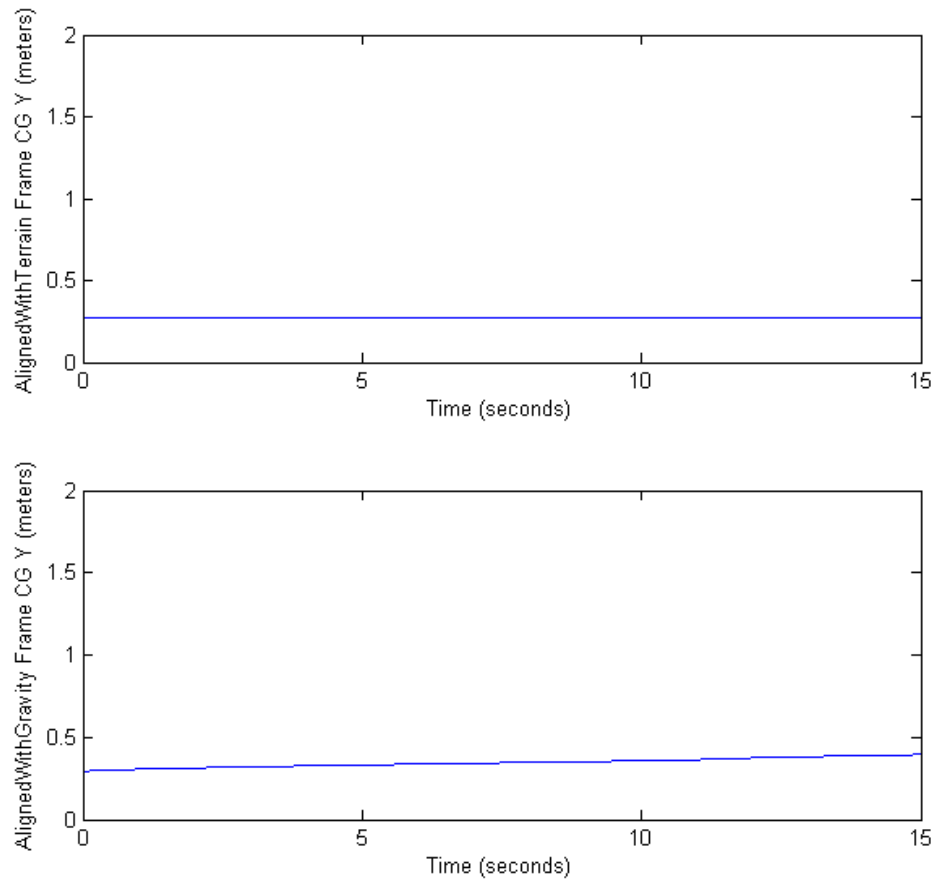


Figure 6.14: Case 2: Five Degree Incline Result Y Motion

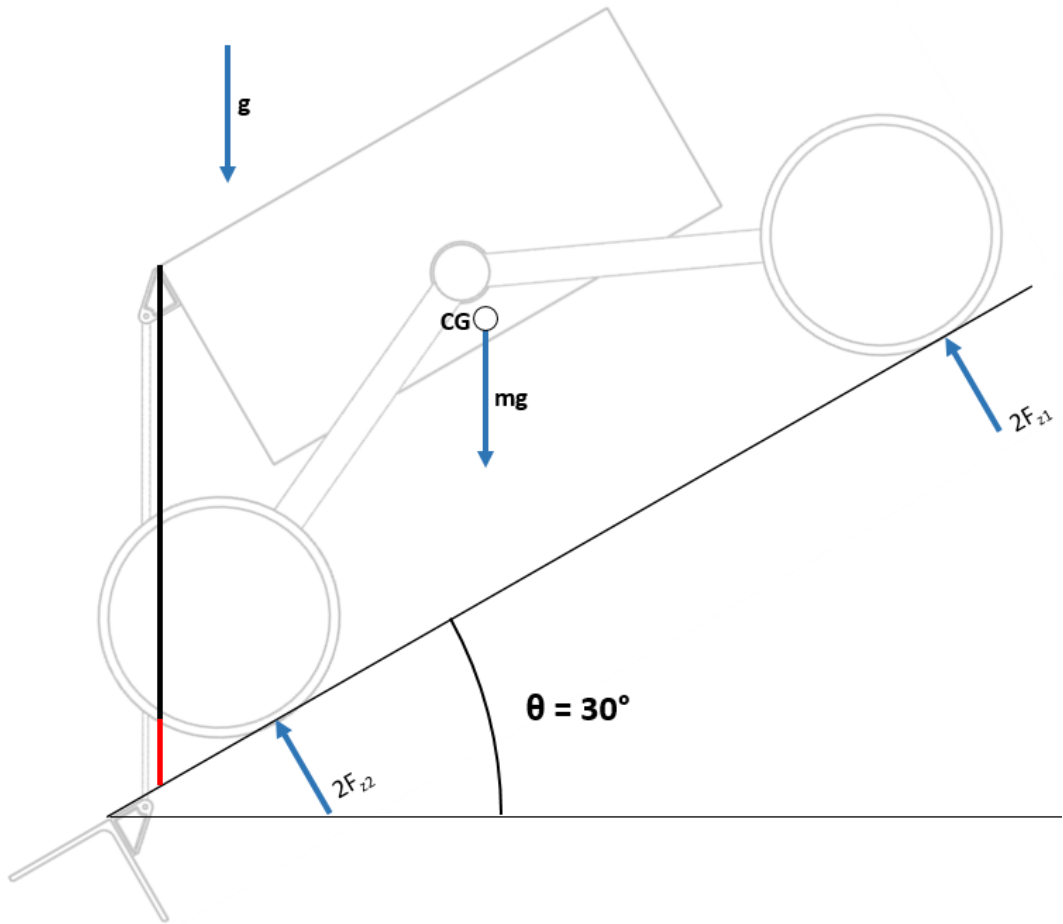


Figure 6.15: Case 3: Thirty degree rover and terrain orientation

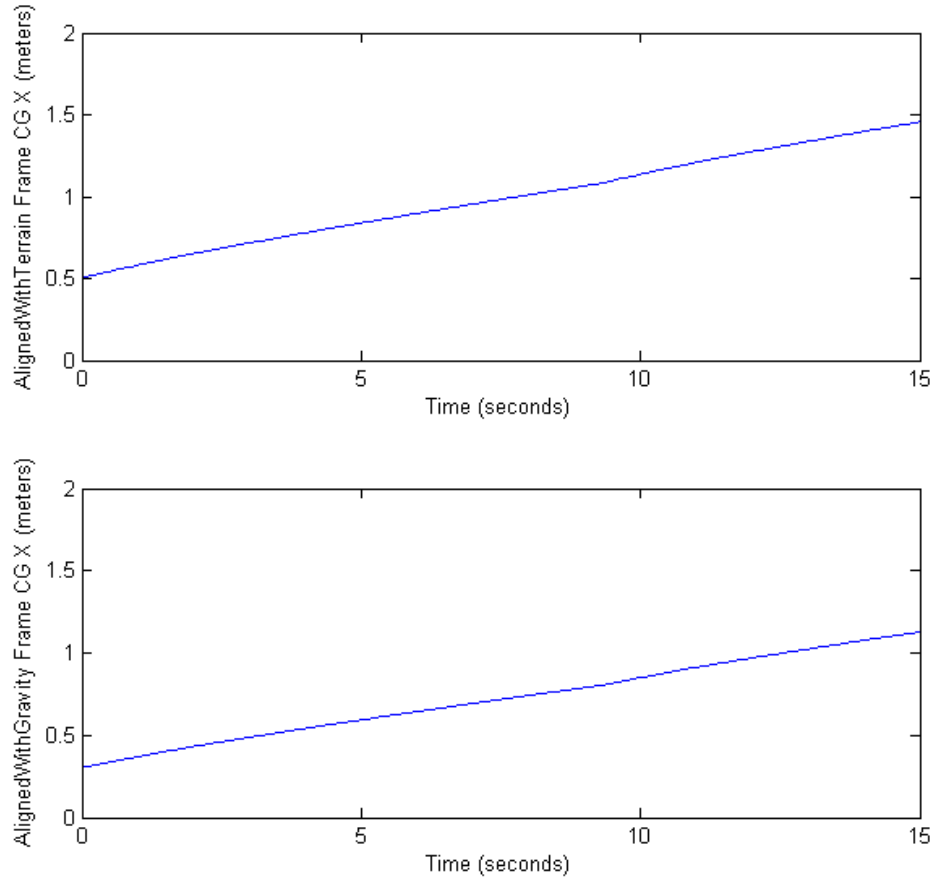


Figure 6.16: Case 3: Thirty Degree Incline Result X Motion

This simulation completes only one full cycle, which terminates near the end of the fifteen second simulation period. Unfortunately this behavior is not evident from examining the result plots in Figure 6.16, as the dipping behavior that occurs on steeper incline angles is much less pronounced.

Comparing the results for traverse at different inclination angles reveal an interesting behavior of the simulation, observed in the x motion plots. As the traverse inclination becomes more steep, the variation that occurs in these plots lessens, and the dipping behavior that occurs at the transition to a new cycle becomes much less pronounced. In order to explain this behavior, we need to

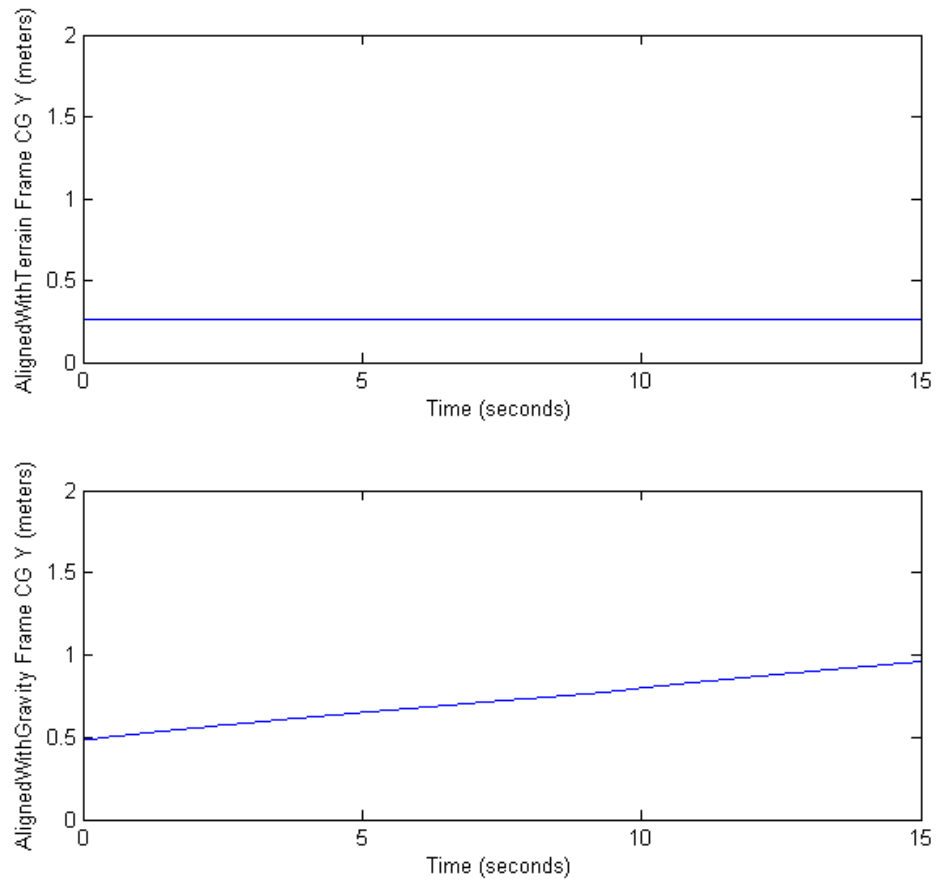


Figure 6.17: Case 3: Thirty Degree Incline Result Y Motion

mind the passive mounting of the actuator.

On steep inclines, the actuator must travel a larger distance before it makes contact with the terrain. This prevents full utilization of the actuator stroke distance, and also allows for more cycles per period as the cycling doesn't utilize the full stroke length. This effect works with another behavior of the actuator which varies with the terrain angle. As the angle increases, the initial angle of the actuator with regard to the terrain increases. Therefore at more steep angles, the incremental lengthening of the actuator as less of an effect on the center of gravity x motion in the reference frame aligned with the terrain. This can be explained mathematically using the simulation parameters in Equation (6.1), where maximum x motion per cycle is a function of the inclination angle θ .

$$dx = [l_{max} - (act_l - h_{act})]sin(\theta) \quad (6.1)$$

where dx is the maximum x motion possible per cycle, l_{max} is the maximum allowed actuator length, h_{act} is the y location of the actuator mount, and θ is the inclination angle.

6.5.4 Case 4: Forty-Five Degree Incline Traverse

Prescribing a forty-five degree incline for the slope of the traverse reveals an operational limit of the rover locomotion methodology. Without the rear foot contact, inclines approaching forty-five degrees will cause the rover to tip over the rear wheels. This is evident when attempting a simulation as the simulation gives an error. This error was programmed to break out of the program, immediately discontinuing the processing, while notifying the user of the problem. In this error, the angle is so great that the rover center of mass is rearward of the actuator contact point to the point of instability. As long as the rover is stable on

an incline the proposed system will improve stability and traversal capabilities.

In this chapter the MATLAB simulation was presented for a variety of cases with varying wheeled schemes. An actuator with perfect friction at the terrain contact was assumed. A notable result is that the SR2 rover fully settles to contact the terrain following each incremental actuator lengthening. This informs that in the tested cases with no wheel friction, the exaggerated situation shown in Figure 6.4 does not occur. In the following chapter design of the system will be presented in which this assumption was a main objective.

Chapter 7

Summary and Future Work

The costly nature of an unmanned exploration vehicle makes it imperative that the rover never become stranded during the mission. The proposed design gives a method to ensure the rover can escape dry sand entrapments should they occur. The proposed solution of a hybrid mobility system is proposed for implementation on Solar Rover 2 (SR2) to enable over otherwise impassible sandy inclined terrain. The system is composed of mechanical parts which are included in a system simulation.

A critical part is the rear-mounted actuator which is mounted to the rover via a passive hinge. As the traverse angle changes, the actuator remains normal to the gravity vector. This allows for extension, which elevates the rover at the rear. Rover elevation favorably changes the component forces at the terrain contacts, allowing for forward traverse up sandy inclines.

The solution simulation reveals the feasibility over the simulation terrain; testing the simulation shows some limitations.

The simulation of the solution shows that the passive actuator will fail if certain conditions are met. One such condition is when the inclination angle is greater than the angle which relates the center of mass to the rear wheel contact.

At these high angles of inclination the center of mass passes over the rearmost contact, allowing a state of unstable equilibrium to occur. This results in the rover tipping over backward, resulting in catastrophic failure.

The physical implementations have limits to their functionality as well. The aluminum contact foot is designed to last a finite number of cycles. These cycles are of full rover mass being applied, and include a safety factor, but the implementation may not meet the lifetime requirements of the whole rover system. This is to be determined based on the expected terrain and frequency of use of the assistive foot.

7.1 Future Work

The proposed solution here is a good starting point for further investigations into terramechanics and passive implementations. Future work opportunities are found regarding improvements to the simulation.

The simulation is based on a number of geometrical constraints and relationships. Within these some assumptions were made to simplify the model. For a more precise simulation, a number of factors should be considered including a varying wheelbase length. As the rover is elevated at the rear, the front wheel contact point varies about the cylindrical contact, effectively moving the front wheel slightly.

The implementation of a massive actuator assembly in the simulation may affect the results and should be considered. The introduction of actuator mounting clevis should implement friction or damping, which may affect the cycle speed of the solution.

The simulation condition requires any small improvement in forces permitting forward locomotion. This is only realistic if there is zero lag time between

terrain embedding discovery and the beginning of the solution cycle. The simulation would be greatly improved by including experimental terramechanics data for the SR2 regolith-wheel contact interface.

In the physical implementation, a mechanism should be in place which maintains a minimum angle of the actuator with respect to gravity. This should be done in order to ensure forward locomotion of the rover, regardless of the drawbar pull being applied by the front wheels during the duration of rearward elevation.

The foot mount to the actuator rod should be mindfully designed. This joint structure should limit the range of rotational motion of the foot. An ideal range of motion for the foot allows tilting upon withdrawal, while fixing the foot from rotating during the actuation phase. This could be accomplished by placing interferences on the hinges to prevent undesired ranges of motion. A minimum tilt must also be established, so that forward forces from the wheel need not pull the rover forward. This could be controlled by introducing a sort of cam mechanism into the hinge as well, which ensures some tilt on the system. The foot angle with the traverse should be maintained to ensure full utilization of the grouser theory soil thrust equation, which requires normal entry to the traverse. This might be possible by adding a mechanism that relates the angle of the foot to the initial attitude of the rover chassis, since the rover chassis begins each cycle consistently with regard to the traverse.

Bibliography

- [1] R McNeill Alexander. *Principles of animal locomotion*. Princeton University Press, 2003.
- [2] V. Asnani, D. Delap, and C. Creager. The development of wheels for the lunar roving vehicle. *Journal of Terramechanics*, 46(3):89–103, 2009.
- [3] M.G. Bekker. *Theory of Land Locomotion*. University of Michigan Press, Ann Arbor, 1956.
- [4] Mieczyslaw Gregory Bekker. *Off-the-road locomotion: research and development in terramechanics*. University of Michigan Press, 1960.
- [5] Alexander S Boxerbaum, Kendrick M Shaw, Hillel J Chiel, and Roger D Quinn. Continuous wave peristaltic motion in a robot. *The international journal of Robotics Research*, 31(3):302–318, 2012.
- [6] K.H. Chang. *Product performance evaluation using CAD/CAE*. Academic Press, Burlington, MA, 2013a.
- [7] Kurt Gramoll. *Multimedia Engineering Statics eBook*. University of Oklahoma, Norman, OK.
- [8] J Gray, HW Lissmann, and RJ Pumphrey. The mechanism of locomotion in the leech (*hirudo medicinalis* ray). *Journal of Experimental Biology*, 15(3):408–430, 1938.
- [9] A. Halme, I. Leppnen, J. Suomela, S. Ylnen, and I. Kettunen. Workpartner: Interactive human-like service robot for outdoor applications. *The International Journal of Robotics Research*, 22(7-8):627–640, 2003.
- [10] Z. Hu. Study and implementation of wheel walking for a mars rover. Master’s thesis, Helsinki University of Technology, Espoo, Finland, 2007.
- [11] V. Krovi and V. Kumar. Modeling and control of a hybrid locomotion system. *ASME Journal of Mechanical Design*, 121(3):448–455, 1999.
- [12] M. Lamboley, C. Proy, L. Rastel, T. Trong, A. Zashchirinski, and S. Buslaiev. Marsokhod: Autonomous navigation tests on a mars-like terrain. *Autonomous Robots*, 2(4):345–351, 1995.

- [13] Kevin M Lynch and Matthew T Mason. Pulling by pushing, slip with infinite friction, and perfectly rough surfaces. *The International journal of robotics research*, 14(2):174–183, 1995.
- [14] D.P. Miller, T. Hunt, M. Roman, S. Swindell, L. Tan, and A. Winterholler. Experiments with a long-range planetary rover. In *Proceedings of The 7th International Symposium on Artificial Intelligence. Robotics and Automation in Space*, May 2003.
- [15] P. Pirjanian S. Dubowsky K. Iagnemma V. Sujana P. Schenker, T. Huntsberger. Rovers for intelligent, agile traverse of challenging terrain. Technical report, NASA, 2003.
- [16] Kim J Quillin. Kinematic scaling of locomotion by hydrostatic animals: ontogeny of peristaltic crawling by the earthworm *lumbricus terrestris*. *Journal of Experimental Biology*, 202(6):661–674, 1999.
- [17] N. Rashevsky. *Mathematical Biophysics: Physico-Mathematical Foundations of Biology (second ed.)*. University of Chicago Press, Chicago, 1956.
- [18] M. Roman. Design and analysis of a four wheeled planetary rover. Master’s thesis, University of Oklahoma, Norman, Oklahoma, 2005.
- [19] M. Ruse. Review of: Evelyn fox keller, making sense of life: explaining biological development with models, metaphors, and machines. cambridge, ma: Harvard university press, 2002. *Annals of science*, 61(3):389, 2004.
- [20] JC Siemens and JA Weber. Soil bin for model studies on tillage tools & traction devices. *Journal of Terramechanics*, 1(2):56–67, 1964.
- [21] R. Sullivan, R. Anderson, J. Biesiadecki, T. Bond, and H. Stewart. Cohesions, friction angles, and other physical properties of martian regolith from mars exploration rover wheel trenches and wheel scuffs. *Journal of Geophysical Research: Planets*, 116(E2):n/a–n/a, 2011. E02006.
- [22] T. Iwanaga T. Nakamura. Locomotion strategy for a peristaltic crawling robot in a 2-dimensional space. In *International Conference on Robotics and Automation*.
- [23] Yoshimi Tanaka, Kentaro Ito, Toshiyuki Nakagaki, and Ryo Kobayashi. Mechanics of peristaltic locomotion and role of anchoring. *Journal of The Royal Society Interface*, page rsif20110339, 2011.
- [24] Otis R Walton, C Pamela De Moor, and Karam S Gill. Effects of gravity on cohesive behavior of fine powders: implications for processing lunar regolith. *Granular Matter*, 9(5):353–363, 2007.
- [25] T. Ylikorpi. Marsokhod - rover.

- [26] S. J. Ylonen and A. Halme. Workpartner ieee presentation.
- [27] David Zarrouk, Inna Sharf, and Moshe Shoham. Analysis of earthworm-like robotic locomotion on compliant surfaces. In *Robotics and Automation (ICRA), 2010 IEEE International Conference on*, pages 1574–1579. IEEE, 2010.
- [28] Xuance Zhou, Carmel Majidi, and Oliver M OReilly. Energy efficiency in friction-based locomotion mechanisms for soft and hard robots: Slower can be faster. *Nonlinear Dynamics*, 78(4):2811–2821, 2014.