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SYNERGISTIC ARGUMENTATION IN A PROBLEM-CENTERED LEARNING ENVIRONMENT

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ABSTRACT

This research looks at argumentation in the whole-class sharing portion of a second grade problem-centered classroom learning environment in which making sense of mathematical ideas was the accepted norm. This research analyzes nine episodes that identify various aspects, or characteristics, of argumentation. Videotapes, field notes, and interviews were used to observe and analyze student interactions and associated learning proclivities in the classroom episodes. The students openly shared their strategies and solutions for solving mathematical tasks. The teacher as argumentation facilitator was interviewed after each observation period. From my observations of the whole-class sharing time, argumentation emerged as the major ingredient in augmenting the students' process of mathematical sense making. The argumentation, which was inherent within this learning environment, provided the catalyst that helped students to effectively engage in, and to clarify and refine, their own mathematical thinking.

CHAPTER ONE

INTRODUCTION

Over much of our history in the United States the traditional/conventional method for teaching mathematics in our public schools has been one in which the teacher is the authoritarian/disciplinarian and the students are his/her subjects in learning. The teacher makes the assignments, gives examples of how to work problems, assigns homework, tests, grades, etc. Traditionally, much of the mental activity requires rote memorization of rules for basic facts. The basic mechanics of the way of doing mathematics is usually learned before concepts are understood. Many students struggle with mathematics because it is dry, boring, unexciting, or frustrating. On the other hand, those who love it are the ones who have succeeded in understanding its concepts and the importance of its applications to real life. As a result, there have been concerns and debates about teaching methods, classroom environment, and how students learn mathematics.

Several significant recommendations from the National Council of Teachers of Mathematics (NCTM) have been suggested as integral parts of the mathematics curriculum that will help students learn mathematics with understanding. Included in their book, *Principles and Standards for School Mathematics* (NCTM, 2000), are some recommendations which indicate that students need to be actively involved in learning, communicating mathematical ideas, problem solving, and reasoning. As students interact in the classroom and try to make sense of mathematical ideas, communication and working together emerge as critical factors in enhancing the students' learning (Cohen, 1996; Hiebert et al., 1997; Corwin, Storeygard, & Price, 1996).

A variety of terms such as dialog, discourse, discussion, open communication, talking, arguing, and exploring have been used by researchers and educators in describing an interactive classroom learning environment. In this study I have focused on the whole-class discussion portion of a problem-centered learning environment, which uses argumentation as the major medium for mathematical sense making and understanding.

I will explain the term "synergistic" argumentation and show its function in a community of learners where the goal is mathematical sense making. I will show how synergistic argumentation enhances the learners' ways of knowing as they openly and interactively search for viable solutions.

Organization of the Dissertation

My dissertation contains five chapters. Chapter One explains the Purpose of the Study and Rationale for doing the research. The background and Literature Review for the study are provided in Chapter Two. Chapter Three explains my research Methodology and describes the Setting of the research site. The Research Findings and Summary are provided in Chapter Four. The last chapter, Chapter Five, discusses Synergistic Argumentation, its Learning Implications, and suggestions for Further Research.

Purpose Of The Study

Seeking better and more effective ways for students to learn mathematics has been an issue of growing intensity in the United States in recent years. The need for reform has become more apparent from several areas of our society. The state and federal governments, for instance, have expressed concern with sub-standard levels of mathematical achievement as a nation when measured by industrial needs, and when compared to some of the other industrial nations in the Western World. The Third International Mathematical and Science Study (TIMSS) added to this concern when their report findings showed that U. S. students fell behind their counterparts in many countries (Smith, 1999). These results have been generating debates concerning the improvement of education in the United States.

Government has often spent increased amounts of money on education in efforts to raise learning standards, but usually to no avail. Generally, there has been little correlation between increased spending and increased student achievement. In some areas our public schools have two standards of achievement, one standard for one ethnic group, and another standard for another ethnic group. In some areas the overcrowding of our classrooms, and various social pressures, have pressured school officials to "just get them (students) through school," to "just get them graduated." This has tended to lower achievement levels.

In 1983, the National Commission on Excellence in education published a report stating that rising mediocrity in schools threatens the future of our nation. It also stated that America's slippage was due to poor educational achievement and that national standards must be raised for all students. In that same year another report, *Educating Americans for the Twenty-First Century*, published findings showing that the old educational "basics" were not adequate for the "age of technology" (Smith, 1999). Socially, over the past thirty years there has been a profound negative change in this nation's structure concerning marriage and family standards, which has had and is having tremendous adverse affects on children. This has increased the number of social problems in children, which leave them angry, insecure, undisciplined, and/or disruptive, all of which make teaching and learning in the classroom very difficult at times. Add to these difficulties the more recent incidents of school violence. All of these factors, and more, have raised the levels of concern for our public school systems among parents, teachers, educators and government officials.

Currently our government and other organizations desiring to improve achievement have developed goals and standards to be reached by all students. An example is "Goals 2000" which was proposed at the 1989 Education Summit. The proposal was in response to a perceived educational crisis, and it embraced a national policy containing six national educational goals. One such goal stated that all students would master challenging content in the traditional subject-matter areas (Smith, 1999). In order to reach these goals there are published lists of standards and skills that every teacher must teach. These standards-based reform movements presuppose that all students need to learn more. The word "all" is used to include any socially underrepresented groups of students. Learning "more" usually means that teachers need to cover extra amounts of material with the students. Meanwhile, the students are supposed to be gaining an "in-depth" understanding in the use of mathematical principles. Concurrent with the government's and the public's concerns with raising learning standards in general, there exists throughout the country one common emphasized theme, the need to search for ways to improve the teaching and learning of mathematics.

Recognizing current education failures though, the public tends to think that many schools have fallen down on their jobs. The expected role of school is to educate. Does that mean that the teacher "indoctrinates" the students with his/her knowledge and procedures that have been handed down from "higher ups"? Or does it mean that teachers are to provide the opportunities for students to discover and learn about their world around them and to make sense of it?

For the past few years a "traditional" view of mathematics has been in place and is described thusly. "Students' brains can be filled with knowledge given by the teacher. Subject matter has been broken down into separate little parts and given to students to memorize and regurgitate." And yet, while the public frets about low test scores, the education system seems to think that teachers should do more of the same thing; tell students what they need to know. Even as far back as the fifties, Dewey (1959/1967) said that an authoritarian principle and the consequences that flow from it would not be effective. Teachers busily just transmitting fixed immutable subject matter are not good enough. Dewey (1959/1967) stated that this type of totalitarian view of education perverts and destroys the foundations of a democratic society. He stresses that a democratic society needs an educational system where the processes of high level moral-intellectual development is always in practice.

In an effort to encourage research and reform the National Council of Teachers of Mathematics (NCTM) has set up guidelines (standards) for teaching. In the beginning of their book, *Principles and Standards for School Mathematics* (NCTM, 2000), there are several statements that are important, but I consider the following two statements as the most important for my research:

> Effective mathematics teaching requires: (a) Understanding what students know and need to learn, then challenging and supporting them to learn it well, and (b)

> that students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. (pp. 16, 20)

In other words, effective education should be about students cooperatively, freely, and openly engaged in treating past ideas and past heritage as foundational springboards for developing better means and methods for the further enrichment of life. This view leads to the idea that students need to be given opportunities and tasks that engage them in experiences and interactive communication, which in turn promotes thinking, problem-solving, reasoning, and reflecting. Said another way, it implies that the greater the student's experiences the greater the opportunities for learning.

If teachers are to teach what students need to learn by challenging and supporting them, then information should be available on what children are capable of knowing. I believe that curriculum developers and teachers tend to sell short students' capabilities, as well as teachers' capabilities. Curriculums need to include appropriate types of problems that are worthwhile challenges. If students are to learn with understanding, then educators should be aware of how all students learn. This idea leads to more questions. How can learning for understanding effectively be characterized? What instructional approaches promote mathematical understanding for all students? What is an appropriate role of the teacher? How can the instructor provide and ensure ample opportunities for the learning of mathematics with understanding? What information and strategies are necessary for students to become active learners? (Confrey, 2000).

Goal Of The Study

Researchers and reformers have been seeking answers to the above questions in order to find alternative ways to help support students' mathematical learning. Attempts have been made to change the participation structure within mathematics classrooms. Instruction has become organized so that students are actively engaged in doing mathematics and communicating their mathematical ideas. NCTM (2000) calls for students to communicate their mathematics. Communication in the classroom has emerged as a critical factor in enhancing students' mathematical learning (Corwin, Storeygard, & Price, 1996). Communication in the classroom enables students to explain their thinking and to challenge others as well as informing the teacher of the students' progress.

With the above questions in mind, and as I observed the interactive communication taking place in a second-grade problem-centered classroom, I became aware that the term, argumentation, might best describe the type of communication I was observing in that particular learning environment.

The goal of my study is to more clearly define and understand the important role of **argumentation** in students' constructions of mathematical meanings during the **whole-class discussion** portion of the problem-centered learning classroom setting.

Focus Question

What aspects of argumentation help support students' mathematical thinking and understanding within a problem-centered learning environment?

In conjunction with this, I will explore and describe:

1. The socio-mathematical norms that emerge.
2. The social dynamics of the class (teacher and student role).
3. How learning opportunities occur (identification of potential learning opportunities during mathematical discussions).

Rationale For The Study

As schools began to grow rapidly in the industrial revolution period, the overarching mindset of the public focused on the demands for product, efficiency, and standardization. Thus, the factory mass-production model for schools was created. It was in the searching for ways to handle the rapid changes taking place, and for ways to meet the subsequent demands in America for educated workers, that educators decided that the public schools could best be run similarly to factories. That is, the curricula could be standardized and efficiently taught to all students at the same time, and then turn out in-mass productive adults. The mathematics curricula for schools were based on the assumption that mathematical understanding could be developed through mechanical preciseness and rigorous memorization, the assumption being that the main characteristic of mathematical understanding was correctness. Thus, little effort was put forth to teach students how to think in terms of understanding the principles behind mathematical construction.

Then in later times during the sixties and seventies the American school system found itself caught in the middle of social and moral upheaval. **First**, the "hippie movement," for example, represented the open rebellion against long standing moral issues and social establishment. **Second**, the ten-year Viet Nam War, 1965-1975, was a very unpopular war amidst a divided nation and divided government leadership. Over 50,000 American soldiers lost their lives in a failed American victory (Casualties, 2000). **Third**, the Martin Luther King peaceful marches and related government-forced school integration labeled, "the civil rights movement," fanned the flames of many racial conflicts and riots. The government forced-bussing school policy forced students to be bussed to non-neighborhood schools, which put tremendous stress on family stability. For schools this was a time of major transition, stress, and disorder for parents, students, teachers, and school officials. Needless to say, the academic achievement level of learning declined (Encyclopedia, 2002). The development of "New Math" came about as an attempt to compensate for the low achievement in mathematics (World Book, 2002). **Fourth**, as a result of the above, the business world and the military were forced to hire and use an increased number of graduating young adults who lacked basic computational skills to meet demand (Kieran, 1994). As a result, much of the focus in mathematics was placed on performance and getting right answers.

Over the past decade reformers have brought about an emergence of a different mathematical perspective. Emphasis has shifted from student performance to student competence (Wheatley, 1991). They have been (a) **thinking** more about the nature of teaching, learning, and problem-solving, (b) **looking** at the ways in which children construct their mathematics, (c) **asking** the meaning of "to develop deeper understanding," and (d) **studying** the interactions in the learning environment. These emerging perspectives tend to focus on understanding in terms of spectrums rather than in linear terms of right or wrong. An alternative way to view mathematical learning is as a "whole" process, not just as outcomes. In this realm the focus has shifted from individual learning to mutual learning that occurs for all individuals in the social situation of the whole-classroom discussion environment. This focus implies that students should become more active participants in their own learning and that the teacher's role must shift from telling to modeling and facilitating, thus creating opportunities for students to restructure their ideas at a higher level (Wheatley, 1991). More involvement and more participation by students bring about more interaction, and thus, more communication/dialogue within the classroom. This increased interaction and mathematical negotiation among students, and between students and teacher tends to develop taken-to-be-shared mathematical understandings (Cobb et al., 1991), which emerge through open and interactive communication. In the following quotes Piaget (1965/1995) points out the importance of communication to learning advances:

> A central feature of social experience is human communication, which leads to an exchange of thought. Yet not all communication is successful. As a consequence, criteria are necessary to identify the minimum conditions, which must be satisfied for attempted, or intended, communication to actually succeed. (1965, p. 11)

> An educational advance, or even a modest exchange of thought, requires individuals to think initially in terms of the culturally transmitted values, rules, concepts, and signs at their disposal, and then to re-think them using their own intellectual resources. (p. 12)

Basic communication is made possible through the use of a common language. We see from the above quotes that there are two important areas of communication. One is **social** and the other is **educational** (for our purposes educational means mathematical). Social communication can be defined as exchanges of basic thoughts, information, and feelings. It may or may not lead to understanding and agreement. Mathematical communication can be defined as exchanges of basic mathematical ideas, rules, concepts, signs, values, systems, research, etc.

The recent shift from a teacher-centered classroom toward a child-centered classroom has brought with it a shift in how we picture and understand the role of communication. Classroom communication has shifted from the traditional teacher-to-student (one-way limited dialogue) to open **interactive** communication between all students and teacher combined. Thus, it is important to recognize the discovery that an effective learning process involves all students, and that students learn through interactive communication with each other and with the teacher. Hence, this process might be labeled "maximum communication for maximum learning."

Concurrently with the above shift there has been an emergence of differing perspectives as to how best to learn mathematics. This has resulted in at least two general camps of thinking: traditional classrooms versus non-traditional classrooms. At this juncture I am not holding up one way as better than another. There have been many changes throughout history that have brought about changes in education. However, some researchers and educators continue to look for alternative ways that might help students learn mathematics with understanding more effectively. Both of the above two groups differ in their approaches to mathematical learning processes, as noted by Piaget (1965/1995) in the following quote:

> Thus at one pole is the educator who has access to the available knowledge in some society, where this knowledge is set in over-arching systems, the connections within which produce intellectual and pedagogical difficulties. This knowledge is also enmeshed in systems of belief and ideology. At the other pole is the individual learner, who is a bundle of intellectual and affective propensities and powers. A rationally successful exchange should lead to truth rather than to

> conformity.... Intellectual activity requires the individual to think through and to re-think with, collectively transmitted concepts rather than to be the passive recipient of the legacies of past generations...(pp. 14-15).

In all quests for developing ways for promoting student learning and academic growth there have always been, and still are, many different mindsets and beliefs about how to best teach children and how children learn best. The ongoing issue is directly related to finding a functional classroom setting that provides the most favorable student environment conducive to the teaching and learning of all children, in all subject areas, but particularly in mathematics. One study conducted by Wood and Sellers (1997) compared students in a problem-centered learning environment to those in a traditional textbook classroom. Their research confirmed that talking and working together improves learning. Another study conducted by Cohen (1996) found that students working cooperatively and talking in the classroom led to improved performance.

Frequently these ideas of talking and working together (communicating) in the classroom are accepted in other subjects of study, but not in mathematics. Generally, teachers permit students to discuss books they have read to decipher meanings and to gain a better understanding of the vocabulary words as well as what the author was trying to convey. In the subject of social studies students are permitted to role-play to learn about various aspects of our American heritage. Usually students are permitted to work in groups and to openly and interactively discuss their readings or assignments. But in the majority of mathematics classrooms, it is back to the "old grind." The teacher delivers the information and the students sit quietly at their desks while trying to work the problems to find the right answers. Once the students finish their work, then the teacher evaluates the students' knowledge by noting how many right answers there are on their papers.

As standards were developed for helping create successful environments that promoted improved learning, communication and working together became important issues. In recent years education reformers and researchers have been focusing on types of classroom environments in which students openly communicate and interact in small groups. The 1989 National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for the Teaching and Learning of Mathematics* included the following statement:

> [s]mall-group work, large group discussions, and the presentation of individual and group reports, both written and oral, create an environment in which students can practice and refine their growing ability to communicate mathematical thought processes and strategies (p. 78).

Continuing to encourage learning enhancement NCTM (2000) in *Principles and Standards for School Mathematics* once again supports reform movement toward the learning of important "mathematical ideas with understanding." With this goal in mind there is an emphasis on communication/discussion, worthwhile tasks, and learning through problem solving in the mathematics classroom environment.

A Teacher's Struggle

One study (McClain & Cobb, 2001) illustrates a teacher's struggle to "figure out" communication in her classroom. The teacher in this study was dissatisfied with traditional methods of teaching and decided to group her students allowing them to go to centers set up in the classroom that focused on mathematical concepts. She discovered that this situation did not encourage whole-class discussion, which she felt was important. So students then were encouraged to share their solution methods for various mathematical tasks. While this was taking place, the teacher did not attempt to indicate which solutions were valued nor did she contrast any of their solutions. During these whole-class discussions, students usually repeated solutions that had already been stated, or offered ideas that did not contribute to the mathematical agenda. The teacher felt that if she interjected her thoughts about the solutions she would be judging the worth of the students' contributions, thus, going against her educational philosophy. So, she began accepting all students' contributions equally because she did not want to act as the mathematical authority in the classroom. She noted that the students would wait their turn to share, but this waiting gave their minds time to wander, and it did not contribute to their intellectual development. Therefore, she concluded that students just taking turns and sharing solutions are not productive situations for in-depth mathematical learning. As she tried various teaching practices searching for ways to improve education and support the students' mathematical development, she became aware of the need for greater student interaction. And she found that the negotiation of socio-mathematical norms is important if in-depth mathematical learning is to take place. Throughout the study the teacher was continually reorganizing her thoughts relative to what it means to support children's mathematical learning, she was gaining a better understanding of the role of negotiation and communication.

The above study illustrates one teacher's struggle of going from one end of the teaching spectrum to the other, trying to develop an approach that would benefit all students' mathematical development. Even before her research study, she was aware that talking was important but did not have a clear idea or understanding of what kind of talk would be beneficial. This study is an example of limited perspectives that are prevalent in many classrooms today. Some teachers think that as long as students are talking together, they are promoting learning. This study showed that talking and taking turns in themselves did not contribute to the students' mathematical development, but that open interactive communication and negotiation of socio-mathematical norms support students' mathematical development and intellectual autonomy (McClain & Cobb, 2001).

NCTM (2000) states that students need to be actively involved in learning, communicating their mathematical ideas. If students are to become more actively involved in their own learning, then more communication within the classroom is necessary. Through the students' and teacher's open and interactive communication mathematical understandings emerge (Cobb et al., 1991). Numerous research studies, including the position statement in the NCTM document, support and promote communication in the classroom. All these statements reiterate that communication and working together emerge as critical factors enhancing students' learning as they interact in the classroom and try to make sense of the mathematics.

Researchers have attempted to describe and use numerous and various aspects of communication. For example, research has identified and labeled a type of communication in the classroom as "discourse." Sfard (2000) in her article, *On Reform Movement and the Limits of Mathematical Discourse,* says that learning can be viewed as becoming a participant in a certain kind of discourse. She then defines discourse as having a broad meaning that refers to the totality of communicative activities within a community. Thus her description is general and broad, as is most of research literature referencing the various aspects of communication in classroom learning environments. And teachers, researchers, and students all seem to have differing concepts of communication and the various types of communication. Many of these concepts are often used loosely and improperly applied. Some think that as long as students are talking that that is good enough.

But I ask, "Does the quality of the talk matter? What exactly is this communication we talk about? What is effective communication? What are the aspects and types of communication? What sort of communication helps support students' mathematical thinking and understanding within the classroom-learning environment"?

While many researchers and educators say that they support open and interactive communication in the classroom, it is my observation that they have not yet fully analyzed and clearly defined communication with its multifaceted applications within the problem-centered learning environment. Research and language usage has referred to many types of communication. The various terms researchers have used all fall under the broad term, communication. The word communication encompasses a broad spectrum of words to describe specific aspects within the communication process. Discourse has been mentioned as a type of communication. Argumentation is another type of communication mentioned by researchers.

In an effort to help make more sense from these various research terms and to more adequately describe the "talk" that takes place within the problem-centered mathematics classroom I am offering a few definitions as a quick-look source to show general differences between these various types of communication. These definitions are from the following sources: *The Random House Thesaurus College Edition*, 1984;

Webster's New Collegiate Dictionary, 1974; *The Emergence of Mathematical Meaning–Interaction in Classroom Cultures* (Glossary), (Cobb & Bauersfeld, 1995).

Definitions

Communication

Thesaurus: 1. Exchanging information, expressing feelings, rapport, liaison, and conversation. Ant. Uncommunicative, untalkative, quiet, reserved, introverted, unsociable, secretive, guarded, and uninformative.

Dictionary: 1. An act or instance of transmitting. 2. Information communicated. A verbal or written message. 3. Process by which information is exchanged between individuals through a common system of symbols, signs, or behavior.

Discourse

Thesaurus: 1. Conversation, talk, intercourse, converse, discussion, colloquy, dialogue, chat, gab. 2. Lecture, address, speech, sermon, oration, dialogue, formal discussion, harangue, diatribe, essay, dissertation, treatise.

Dictionary: 1. To express oneself in oral discourse.2. Talk, converse, verbal interchange of ideas, conversation. 3. Formal and orderly and extended expression of thought on a subject. 4. Connected speech or writing.

Discursive

Thesaurus: digressive, rambling, roundabout, wandering, meandering circuitous, diffuse, long-winded.

Dictionary: To run about. 1. Passing from one topic to another, digressive. 2. Marked by analytical reasoning.

Argue

Thesaurus: 1. Reason, contend, maintain, assert, claim, hold, plead, expostulate, remonstrate. 2. Quarrel, dispute, debate, bicker, quibble, wrangle.

Dictionary: 1. To accuse, reason, to make clear, to give reasons for or against something. 2. To contend or disagree in words, dispute. 3. Try to prove, consider the pros and cons, discuss.

Argumentation

Dictionary: 1. The act or process of forming reasons and of drawing conclusions and applying them to a case in discussion. 2. Debate, discussion.

Glossary definition from *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures* (Cobb & Bauersfeld, 1995): *Argumentation:* (a) A primarily social process in which cooperating individuals try to adjust their interpretations and interactions by verbally presenting rationales for their actions. (b) The techniques or methods used to establish the validity or claim of a statement. During an argumentation, if one participant explains a solution, the implicit message is that the claim is valid. A successful argumentation refurbishes a challenged claim into a consensurable or acceptable one for all participants.

Collective Argumentation

(Glossary from the above publication): A process of argumentation involving several people who are conjointly engaged.

Definition Analysis

An analysis of the above terms, discourse, discursive, argue, argumentation reveal broad meanings, overlapping usages, and multiple applications. Yet all are somehow interrelated to communication.

From the above definitions I note that **communication** is a very general term encompassing every conceivable transaction relating to sending or receiving information whether it be verbal, written, signal, or electronic and recognized by seeing, hearing, or feeling. Therefore, communication would not be an appropriately definitive word to describe the interactions of mathematical students in a problem-centered classroom. Communication seems to be the root term, the "umbrella" made up of the various types of communication.

What about **discourse**? Discourse is a little more specific in that it usually refers to a specific kind of academic conversation, lecture, speech, discussion, dialogue, or writing. Although researchers readily use it to describe the open interactions/transactions of a problem-centered classroom, it seems to fall short in appropriately describing such a classroom learning environment.

Next is **discursive**. Although related to discourse, discursive seems to imply a more negative type of communication. On the positive side it does make application to analytical reasoning, which certainly can apply to academia. But it does not fully describe the problem-centered classroom environment.

The word **argue** first brings to mind negative scenes of heated intense destructive communication such as quarrels, accusations, or angry disagreements. However, when classroom students agree to abide by positive social norms such as mutual acceptance and respect for each other, then the idea of arguing becomes positive in an environment of interactive sharing, discussing, listening, exploring, and constructing. In this light the word argue may come closest to describing appropriately and adequately a problem-centered classroom learning environment.

Thus, **argumentation** with its positive implications in social/mathematical communication and learning by and for all students should perhaps be used to describe the problem-centered classroom interactions.

Summary

The previous pages give some insight into several past events and practices that have adversely affected the schools and our educational systems. Subsequently, reformers have been searching for alternative ways to enhance mathematical learning. Many teachers are searching for ways to help children learn with understanding. The classroom environment and its importance in providing positive communication settings for learning has become one area of focus. Teachers and reformers have found that communication and working together (Wood & Sellers, 1997; Cohen, 1996) do enhance learning. But the struggle is focused on determining the type of communication to use and how to set up the classroom to promote the necessary and appropriate type of communication.

> A real exchange of thought is liberating, permitting the individual to re-cast available knowledge into valid forms of new knowledge which is manifest both in the continual adaptation to new circumstances which are never identical and in the growth of the human powers required in their coordination (Piaget, 1965, p. 14).

So what are the qualities or aspects of communication that best support children's mathematical learning? In an attempt to answer that question I will discuss various aspects of argumentation as observed during the whole-class discussion time in a second-grade mathematics classroom. The following quote was taken from the teacher of this second-grade classroom who has worked with the teachers in her school for many years and has listened to their comments and concerns connected to communication within the classroom.

> [t]eachers are having hard times in their classrooms because they have an authority. They don't feel like they are having rich conversations that need to go on about the mathematical ideas, and I thought that was interesting because that is the hard thing to work out and you have to have that kind of climate in your room all day long. I mean you can do it to a certain point but really for it to work it needs to be the social norms in the classroom. This is the way it is, it's not that anybody in here [in her classroom] is the expert.

Through argumentation students exchange thoughts, laying out ideas for all to make sense of. As the conversation meanders in and around ideas while students are arguing, they construct new knowledge and new understandings emerge. Thus communicating in the classroom is important. Therefore, my focus is on identifying and defining an appropriate type of communication, "synergistic argumentation," that will more adequately describe the whole-class discussion portion of a problem-centered learning environment.

Chapter two explores literature that discusses arguing and learning in a problem-centered learning environment.

CHAPTER TWO

BACKGROUND

In this chapter I give background information, which explains and supports mathematical learning in a problem-centered learning environment. There are several discussed aspects that are important to the learning process in general. **First**, I discuss the social environment and its related impact on learning, gaining knowledge, understanding, making connections, and academic growth in general. **Second**, I describe a problem-centered learning environment in contrast to the traditional classroom environment only to help the reader understand problem-centered learning and the differences. **Third**, I describe various dynamic aspects of a problem-centered learning environment, each with its own unique impact on mathematical learning.

Social and Academic Growth

Fostering student social and academic growth is a primary goal in education. Growth is the creation of meaning and addition of value to our experiences. One's educational experience can cultivate growth. Growth is brought about through the proper cultivation of self, because growth comes from within (Dewey, 1959/1967). According to Caine and Caine (1997), genuine growth requires change, and if change is to take place, then there must be empathy, congruence, and respect. This growth cannot be forced from the outside, and yet stimulation and guidance by the teacher can foster students' growth and understanding of their world. In order to actively foster growth, the learning environment becomes very important. There must be interaction between student, teacher, and subject matter. Each student should be able to utilize the ideas in his/her own experiences, thus creating meaning and understanding. Children are biological beings and not purely mechanical things—their learning involves attending to the "whole child." Learning takes time and cannot be hastened by forcing it. Simply telling the children the truth about something cannot make them understand it (Duckworth, 1996). The challenge then is in fitting the curriculum to the learner, rather than fitting the learner to the curriculum.

Learning

Learning involves children constructing meaning and resolving conflicts for themselves. Learning occurs as students construct meaning for their experiences, as they act and interact with the world around them, actively trying to resolve conflicts while engaging in purposeful activity (Wood & Sellers, 1996). Simply watching children reveals that they are active, constantly looking for, playing with, exploring, and discovering. They are not passive. They are in a constant state of exploration and excitement. They are not possessors of answers up front but are continually seeking and searching. The discovery and exploration brings conflict and disequilibrium into the child's awareness. It is the child's own effort to resolve conflict that brings him/her to a different level of understanding. The child constructs knowledge from within as s/he interacts with the environment (Kamii, 2000). It is paramount that teachers provide opportunities for the children to reach distant goals while, at the same time, allowing them the freedom to follow their own unique paths. Children create meaning through their own thinking and reasoning (Wood, 1999). Glasersfeld (1995) states that knowledge is an active construction through the senses or by way of communication by a cognizing agent. He also states that the function of cognition is towards viability. Glasersfeld's idea is in sharp contrast to the view that learning is the acquisition of what is already written in textbooks, and, that it is taught as a finished product. Dewey (1959/1967) states that it is important to find conditions in which learning occurs naturally and comes as a result of doing things. Thus, teaching begins not just by showing respect for children, but also by recognizing and promoting their innate curiosities and capacities to learn.

Mathematics learning is a constructive, problem-solving process in which experience, activity, and communication are essential (Cobb, Yackel, & Wood, 1993). Therefore, the development of students' reasoning and sense-making processes cannot be separated from their participation in the interactive constitution of taken-as-shared mathematical meanings (Yackel & Cobb, 1996). It is essential to provide classroom experiences that encourage children to make sense of important mathematical ideas for themselves through interaction with their world and each other.

Knowledge and Understanding

As humans, we do not passively encounter knowledge. According to Duckworth (1996) there are three types of knowledge: perceptual, action, and conceptual. **Perceptual** is how one sees things and how that seeing connects with what was previously seen. It is in the ways in which we perceive our world. **Action** is what one does and seeing how that relates to previous actions. **Conceptual** makes links among words, thoughts, and/or formulas. The interplay among these three determines a student's understanding and could lead to what Caine & Caine (1997) refer to as dynamical knowledge. True dynamical knowledge is not possible in a classroom where lessons, body, and mind are thought of as separate, fragmented parts. Nor is it in a classroom where students are viewed as being separate from each other. If teachers are not concerned with the whole student, then learning and understanding will be shallow. Caine & Caine (1997) say that if teachers focus on surface knowledge (product of rote learning) the students will only memorize and not have understanding. Classroom teachers need to move from knowledge deprived of meaning to dynamical learning rich in meaning. This type of learning develops as students make sense of their world and have personal meaning attached. Understanding is wholly the active power of the child. Logical structures, such as procedures, alone do not give rise to understanding and learning (Kamii, 1982).

Connections

By constructing their thoughts in various ways students are mentally relating and making sense of things for themselves. If students explain constructive connections they have made, then for teachers and other students to understand those connections, the teacher and students must be able to communicate in the same common language. For example, we must be able to go through the same thought processes ourselves to make those same connections. Making connections must be personal elaborations, but sometimes a student is simply not capable of making those connections. S/he is incapable of seeing the same mathematical relationships that some other student, or teacher, is trying to point out. In this light the teacher must go beyond just simply tailoring narrow exercises for the students. S/he must provide situations in which children can work at various levels and can come to know their world in new ways, and to seek out ways, acknowledge, and take advantage of all the pathways people might take to their own understanding. Emphasis needs to be placed on tasks which are authentic and multi-faceted, and which call for problem solving and critical thinking. This is not always easy, but is much more interesting and profitable than prescribed or predetermined problems from a curriculum written at a distance from the student. When children explore together and clarify meanings, they develop new and deeper mathematical concepts, thereby enhancing academic development as well as social relationships.

Problem-Centered Learning Environment

Different Than The Traditional Classroom

There is a striking difference between the problem-centered learning classroom environment and the historically traditional classroom environment. In a traditional classroom setting the teacher gives the directions and procedures to follow, and the children carry them out quietly and individually. The teacher asks questions and students give answers when called on. The student responses are usually short and based on what they think the teacher expects as the answer. In this environment the teacher is the one who has access to available knowledge, the one who knows where this knowledge is set in over-arching systems, and who is aware of the connections which produce intellectual and pedagogical difficulties. This perception of knowledge is also enmeshed in systems of belief, and metaphors similar to, "school as a factory." Students go to school. Students sit in classrooms all day listening to teaching and following teaching instructions. Students are given two recess breaks, plus a lunch break, each day. Students then go home.

Question! Why is it that in the world outside of school the students work together, play together, join teams, etc., but in most schools the students are expected to learn individually in relative isolation?

Problem-Centered Learning

Contrasting with the traditional classroom environment is the problem-centered learning environment in which every individual learner becomes a bundle of intellectual and affective propensities and powers. The teacher and the students create this learning environment in which open dialogue promotes understanding. The primary goal of rationally successful communication exchanges is to pursue and find viability, and is not primarily focused on the pursuit of conformity. Progressive intellectual activity requires the individual to think (and re-think) through negotiations of ideas for confirmation or rejection, rather than just becoming a passive recipient of legacies of past generations. But this achievement of viability is possible only if the human mind uses it's capacity to think autonomously, to act on the basis of reasons rather than through occurrences and causes, and to engage in reasoning on the basis of formal rather than on merely functional factors (Piaget, 1965/1995). In such a problem-centered learning environment the students can work with their peers, provide directions, follow them, and ask and answer questions. In this way students can practice academic discourse (special and various ways of communicating and talking in school) that aids in the students' social and intellectual development.

The dynamic culture of problem-centered learning in a mathematics classroom provides an important positive influence on the nature of mathematics learning for elementary students. Mathematics learning becomes a constructive, **problem-solving** process in which experience, activity, and communication are essential. This problem-centered learning approach to mathematics education advocates a shift of emphasis away from rote procedures and toward the development of higher order thinking. It acknowledges that favorable conditions for learning exist when students are faced with a task for which no known procedure is available. That is when learners find themselves in a problematic situation (Murray, Olivier, & Human, 1998). Problem solving/doing mathematics is not mere memorization of teacher prescribed algorithms. Rather, by analyzing and understanding the essential features of the learning environment and related processes, teachers are able to create a mathematical learning environment for all students and to define what it means to "teach for understanding" (Cobb, Yackel, & Wood, 1992). In this problem-centered environment students not only learn what mathematics is all about, but they learn what it means to do mathematics with understanding. Students are expected to spend time solving problems, developing solutions, and reflecting on their thinking. Wheatley (1991) identified problem-centered learning as an instructional model that supports the idea of mathematics as problem solving.

A problem-centered learning classroom involves three components: **tasks, groups,** and **sharing**. First, the teacher, or students, pose a task (mathematical problem). Then the students individually spend a little time reading the proposed task and clarifying for themselves the question/s to be answered. Second, the students work in small collaborative groups for approximately 20-25 minutes. During this time each individual group works independently to solve the task. Third, finally all the groups come together and share in a whole-class open discussion for approximately 20-25 minutes. They discuss their solutions and strategies with each other and with the teacher (Wood, 1993, Wheatley & Reynolds, 1999).

Tasks

The task proposed to the students is problematic but doable. The tasks focus on key mathematical concepts that will guide the students to construct effective ways of thinking about mathematics. These tasks are developed from the children's activities. The tasks are not the same as those used in traditional classrooms. Unlike the traditional classroom, the tasks may not be able to be solved in an hour time period. Nor do they focus only on one mathematical concept. As the students attempt to resolve problematic situations, several mathematical ideas are developed and learning opportunities are then created. The students, when working on the tasks, develop their own methods, but expect the processing to take time, and to be puzzling. The teacher does not limit the students' mathematical thinking activity by giving them specific procedures to follow, nor does s/he expect them to act in accordance with her/his own ways of thinking.

Small Groups

One way to encourage the development of mathematical understanding and to have students engage in meaningful mathematics is through small group interaction. In finding ways to solve the task at hand the students work in homogenous pairs, which enables them to participate equally and to communicate openly and interactively. They are involved in explicit negotiation, not in the imposition of meanings, as they engage in mathematical activity. In this sense the students struggle with the mathematics. There are no definitions of words or procedures given. Through the students' communication they discuss the task and their ideas trying to come to an understanding of the task and the mathematics involved. The viability of their answers evolve from their discussion, sharing of ideas, and navigating through their perspectives in order to come to an agreed upon solution. Each individual constructs his/her mathematics based on experience and interpretation. Thus, the students' mathematical meanings are formed as part of their interactions. Working with someone else is a way of opening up each person's thought processes. While working collaboratively student pairs must come to an agreement on a solution. In pairs students attempt to solve mathematical problems and, at the same time, solve the social problem of working productively together. When students think about each other's reasoning, they become aware of conflict between their own thinking and that of their partner's thinking. As students become aware of this contradiction and struggle to resolve it, they arrive at a new dimension of understanding (Azmitia, 1993). Also, during this pair collaboration students learn to rely on themselves and one another instead of waiting their turn to ask the teacher.

The dialogue in collaborative pairs is mathematical in nature because both students focus on discussing the mathematical activity therein trying to gain understanding and to develop mathematical meaning for themselves as well as for each other. The communication is based on the shared mathematical activity. The primary goal for the pairs is that both students in each pair arrive at an agreed upon solution that they personally understand, can make sense of, and can explain and justify.

During this small-group time, the teacher interacts with the pairs during their activities in order to make informed decisions about appropriate ways to orchestrate the subsequent whole-class discussion. The teacher makes decisions about which solutions and/or ways of reasoning should be highlighted during this subsequent next portion of the mathematics class; thus the need to become aware of the students' diverse strategies. The teacher becomes aware of each person's thinking by listening, observing, and interacting with each pair.

Different Than Cooperative Groups

This approach of working in homogeneous pairs (students who will challenge each other) is in stark contrast to cooperative groups (focus is on assigned roles). In cooperative groups students have assigned differentiated roles and the goal is to complete an activity as a group. Each student is assigned a different role, such as, facilitator, reporter, materials manager, etc. These roles are procedural rather than substantive. And these roles are to be rotated so that each student has a turn. Again the group members must focus on the new and possibly different role assigned and figure out what to do and how the group needs that role to be fulfilled. Cohen (1996) in her article, *A Sociologist Looks at Talking and Working Together in a Mathematics Classroom*, noted some teacher-sourced negatives to cooperative group learning: failure to delegate, failure to permit interaction or discovery, frequent interruption of the groups, telling them better ways to accomplish tasks, becoming the rescuer of any group in trouble, giving out too many detailed procedures, and hovering over groups. Thus, mathematical activity may or may not be the primary focus of the students' cooperation or the teacher's. For instance, students may be focused on a social way to organize the classroom. The goal is not only to complete an activity consisting of a large number of problems, but also to have correct answers. The dialogue in the cooperative groups tends to be primarily about their assigned roles and not primarily mathematical.

For example, recalling a recent event makes me question even more the use of teacher delegated roles. While attending a teacher workshop, we teachers were put into groups and asked to solve some mathematical problems. One teacher announced that she would be the recorder and the rest of the group would tell her what to write. Someone asked her if that meant she did not have to help the group or do any mathematical thinking. She replied, "Right! I am only the recorder and it is my job to write." Although this response came from an adult, I wonder whether or not children would have the same response about the role of recorder?

There may or may not be a goal for each individual to make sense of the problems given because the focus is on getting right answers. While the cooperative group tends to focus on group scores based on cooperation, the collaborative pair is a way for each individual to make sense of the mathematical activity. The roles assigned to the cooperative group members may hinder the teacher in assessing individuals fairly. One such role is the reporter. That person's job is to express to the class or teacher the answers the group came up with. Is the reporter well equipped to represent his/her group? After hearing the reporter the teacher then gives a group score based on what the reporter shared, not what the other individuals share. Therefore, the teacher does not have an opportunity to hear from each individual, thus, s/he may not know whether or not a student has made sense of the mathematics or whatever task was assigned. Cohen (1996) reported that there are inequalities when using cooperative groups. She states that status problems occur when some students dominate the group discussion, monopolize the manipulative materials, and/or take over the work and decision making of the group. Within the same group other students fail to participate or, if they do attempt to make a contribution, are ignored. In this type of setting students tend to perceive each other as high or low academically, which leads to an established ranking order based on academic and peer status. When a student perceives himself/herself as more mathematically knowledgeable than another student, thus judging the other student to be less knowledgeable, the following often occurs: the "more" knowledgeable tries to help the "less" knowledgeable to understand. This results in a clear imbalance of "power." The "less" mathematically knowledgeable student tends to accept without question what the "more" mathematically knowledgeable student says. Therefore, neither student is mentally challenged. According to Yackel (1995) these kinds of situations are not productive for either student. When one student in the group is perceived as the "authority" learning opportunities diminish and inequality increases. But, in a problem-centered learning environment the students are in homogeneous pairs, which eliminates status problems and inequalities.

Whole-Class Sharing

After all the small-groups have developed their own mathematical solutions, all the students gather with the teacher in whole-class discussion. Each group openly shares, explains, and justifies its strategies to the whole class, including the teacher, for discussion. During this interactive communication each student is listening to the various strategies, reflecting on his/her own mathematical construction, re-thinking his/her approach, comparing solutions, commenting, explaining, negotiating, questioning, and clarifying. During the whole-class sharing, the students' methods for solving problems are the topic of discussion. In this setting the students' responses are their own and do not need to conform to a particular predetermined procedure (Wood, Cobb, & Yackel, 1993). The sharing of ideas provides an exchange of viewpoints and presentation of a variety of methods. In this kind of discourse all students are learning how to think, talk, and make sense of mathematical ideas. Thus, participation in this kind of mental-processing allows for the construction of mathematical understanding in all participants. Likewise, it can be said about whole-class discussions: the greater the interaction among all students the greater the mathematical learning for all students. The over-all purpose of the whole-class discussion then, is to maximize learning for all students through interactive discourse.

Explain and Justify

Yackel and Cobb (1996) indicated that a good way to enhance mathematical learning is to have children explain and justify their solutions and ideas. The teacher and the students interactively constitute explanations and justifications. An explanation is identified as a **rationale** offered voluntarily to clarify aspects of the mathematical thinking that might not be obvious to others, whereas a **justification** is an attempt to explicate one's thinking when challenged by other participants. Duckworth (1996) said that another part of explaining is that all students accept the responsibility of making sure they understand one another. In this setting, an atmosphere of mutual trust and respect exists, creating a freedom and willingness to share ideas through explanations and justifications.

Social Norms

Humans, by nature, have developed various ways of social communicational acting and interacting with each other. For example, they communicate by making sounds, by making motions, or by writing or drawing. Most social groups develop guideline rules for behavior applicable to all individuals in the group. These rules may be explicitly communicated to the group, or they may be implicitly (implied, inferred indirectly) communicated. These rules of behavior developed and accepted by the group are called "social norms." They are notions collectively set up by the whole group, not those set up by one individual. In other words, there is no set predetermined pattern of interaction. Through the interactions the norms are negotiated which make up the social reality of the classroom. For the most part, the teacher and students are not consciously aware of the patterns.

This same concept of social norms applies to problem-centered mathematical classrooms in which students develop sets of routines and expectations. These become rules and guidelines, which students agree to abide by in the functioning classroom. Students actively participating in a mathematical classroom become aware of not only their own actions, but also of the actions of the other students around them. Each individual automatically compares his/her actions with the other students in the classroom, thereby checking, comparing, and mentally measuring, his/her own self-worth socially. As we look at a group of students in the classroom we see a multiplicity of individuals as they each interact socially with each other. But if we focus on the individual too much, we loose sight of the social. If we focus on the social aspects too much, we loose sight of the individual. Therefore, we are looking at learning as negotiation of meaning in social interactions (Yackel & Cobb, 1996). These norms, which define the classroom participation structure, are not specific to mathematics, but can apply to any subject. This construction of norms is essential for setting the learning environment. The children in a problem-centered learning environment accept responsibility for their own learning and for their conduct (Wood, 1993). Students' mathematical learning is influenced by both the mathematical practices and the social norms implicitly and explicitly negotiated by the learning community. Social acceptance and interaction is viewed as important for autonomous mathematical development.

This is a cyclical process (back and forth communication) in which each student makes sense of his/her own actions by adapting to the other students' actions and expectations. The negotiation of the social norms between teacher and students encourages the doing of mathematics and the expression of mathematical ideas. From this perspective students learn important mathematical ideas, as well as how and when to participate in class (Wood, 1999). In this situation the students also learn to solve problems, agree on answers, and respect each other's ideas.

Socio-Mathematical Norms

There is an implicit relationship between the negotiation of classroom social norms, socio-mathematical norms, and students' mathematical development. Socio-mathematical norms are different from social norms. Socio-mathematical norms apply to the subject area of mathematics. These norms are negotiated in the interactions of students and teacher as they attempt to interpret and solve mathematical problems. In problem-centered classrooms socio-mathematical norms are interactively constituted and focus on aspects of mathematics discussion specific to students' mathematical activities. These norms include the social aspects of the classroom, but are different than general social norms, because they pertain particularly to the students' mathematical activity (Yackel, 2000). These norms determine what counts as a mathematical justification and explanation and what counts as a mathematically different strategy. These norms are not obligations or rules that the students must fulfill, but they deal with the process of making a mathematical contribution. They aid in the students' development of interpretations and solutions for mathematical problems. An example of what counts as mathematically different is when the students share and explain how they arrived at a common solution but their strategies were different. Discussing the different solutions may lead to the discussion of mathematically significant issues. Situations in which students attempt to resolve conflicting points of view through negotiation of strategies give rise to learning opportunities. In this situation students not only talk about mathematics, but talk about talking about mathematics as they reflect on each other's explanations. Thus, thinking is valued more than merely getting the right answers (Wood, 1993).

Teachers, as well as students, are provided learning opportunities in situations involving the negotiation of socio-mathematical norms. While children explain their solutions, the teacher is listening and trying to understand students' reasoning. Based on what s/he hears, the teacher is able to select appropriate and challenging tasks, which will further students' mathematical thinking. Also, the teacher's responses are based on the students' thinking. In this way the negotiation of socio-mathematical norms gives rise to learning opportunities for the teachers (Yackel & Cobb, 1996) as well as for learners. As teachers attempt to make sense of the children's various mathematical explanations and solutions, they are able to recognize and use learning opportunities that come from listening to the conversations.

Mathematical Communication and Meanings

The process of working together to try to understand and make sense of the mathematics involves developing mathematical communication among all participants, which allows them to freely and comfortably express ideas based on shared mathematical activities. This process also creates a situation for interactive talking, listening, and negotiating. Through this interactive process of negotiating, mathematical meanings are considered to be "taken-to-be-shared" (Voigt, 1994). Voigt emphasizes that if the students' background knowledge differs from the subject matter, then negotiations of meaning are necessary. As negotiation continues what is meant by taken-to-be-shared communication emerges. It is not only an individual construction, but it includes meanings that emerge from the interactions of the students. When students talk directly with each other, they are sharing in the construction of community as well as building mathematical knowledge. Therefore, the social aspects, as well as individual constructions, are necessary for mathematical development (Cobb, Boufi, McClain, & Whitenack, 1997).

It should be noted in the literature two phrases have been used to describe the collective sense-making process, taken-to-be-shared and taken-as-shared. Both terms seem to indicate the same general idea; as students openly interact and discuss mathematical tasks, they must negotiate meanings. The students do not have the same knowledge or understandings of words or math concepts. Through their explanations and justifications there arises a common language of ideas that are shared, thus providing a basis for further communication. This interactive (circular) process dissolves misunderstandings between the students' language and mathematical ideas. In other words, students mutually adjust their ideas and activities with other students' ideas and activities, while continuing to focus on the mathematical tasks.

Ideas are "placed out in the open" for all to try to make sense of and understand. It is through this process of looking at ideas objectively that students can discuss tasks from similar perspectives and understandings. Taken-to-be-shared and taken-as-shared do not mean that the students have the same exact knowledge but they have a "common ground" from which new understandings can emerge.

Mathematical meanings are also under continuous negotiation, but this does not mean that students and the teacher share the same knowledge. As students work collaboratively to complete an assigned task, mathematical meanings that arise between them are formed as part of their interactions with their partners. The teacher and students negotiate mathematical meanings as they do problem solving and talk about mathematics (Cobb, Yackel, & Wood, 1992). In this context the negotiation of socio-mathematical norms defines what kind of talk is valued in the classroom (Kazemi, 1998). The negotiation among teacher and students can encourage the doing of mathematics by enabling students to investigate all the mathematical possibilities of a task, to develop their own methods for solving a task, to reflect on their ideas, and to explain and justify their thinking and solutions to the class. As a result the classroom becomes a mathematical community where the development of mathematical ideas is valued (Reynolds & Wheatley, 1996).

Collaboration

Mevarech (1999) argues that collaboration allows children to learn through opportunities to explain, justify, and listen to one another's ideas. Further, he contends that explanations are one of the best means for elaborating information and making connections. These conditions help students construct rich networks of meaning. When students work with peers who are in various stages of mastering a task, mutual reasoning and resolution of conflicts are likely to occur, which in turn facilitate learning. As students share their explanations they must actively communicate with each other and the teacher. In order for that communication to be successful they must negotiate meanings, not just recite facts. As they negotiate they adjust their interactions by presenting rationales for their strategies. Working together gives rise to learning opportunities as students express their thoughts while attempting to understand their partner's ideas. Sometimes, however, mathematical communication does break down when students are talking "past" one another (not listening), or are failing to understand the other's perspective.

Reflective Discourse

Teachers can aid in developing mathematical communication through "reflective discourse" (Cobb, Boufi, McClain, & Whitenack, 1997). Reflective discourse is a sociological construct by which a mathematical action becomes an entity, which can be manipulated. Thus reflective discourse is characterized by changing what the teacher and students do in action into objects of discussion. The students lay their ideas out on the table, so to speak, for all, including themselves, to question and make sense of. They externalize their mental processes as they create representations, such as, math sentences, pictures, graphs, etc., to express their internal experiences. Thus, the representations become the objects of discussion. The students are not necessarily looking for right answers when they reflect, but the reflection allows for each one to think about its meaning and application. It allows for students to hear and see how other students give meaning to the task in their quest for sense making and finding solutions. By comparing their answers and determining whether their solutions are similar, or different, students are provided with a mechanism through which they can monitor their own thinking concerning their previous activity. This reflective discourse enables students to develop mathematically as they "think about" another student's perspective, and as they "think about" questions or issues rather than solving a specific problem. In order to do this, students learn to distance themselves from the activity, and to look at it objectively in order to extend their own interpretation as well as make sense of the other's activity. Through reflection, discussions, group interactions, and reasoning, mathematical concepts emerge. As students hear others' explanations, reflect on what is said, use what they understand, and inform their own thinking, they are aided in the development of mathematical reasoning.

Teacher As Facilitator

An essential element in effective whole-class discussions is the teacher who must facilitate student interactive discourse and help direct it toward mathematical learning and understanding. The teacher observes, listens, and asks relative questions. S/he may repeat something that has been said for student reflection, or verbally highlight a student's strategy for constructive thinking, or provide more information for building cognitive models of students' thinking (Wood, 1993). Therefore, the teacher's role is not to direct the students to a predetermined mathematical procedure or solution.

In reflective discourse, the role of the teacher as an "interactive" facilitator/participant becomes important. Throughout the mathematical discussion the teacher repeatedly asks students to explain their thinking. When students give their reasons, the teacher does not evaluate their validity but asks the class what they think and solicits other arguments. In other words, s/he is not checking for right or wrong answers. Nor does s/he respond to their answers with value-laden words, such as, "good." S/he listens intently to try to understand the students' mathematical thinking. By listening to the students the teacher can reflect on responses and can select appropriate tasks, which further the students' mathematical thinking and development. It is crucial that the teacher does not limit the students' thinking, but rather allows the process of negotiation and discussion to aid in the development of mathematical concepts. This discussion/dialogue involves developing mathematical communication about shared mathematical activities. The teacher listens to discussions in order to support students' learning while they discuss their mathematical thinking. S/he promotes mathematical discourse through the assignment of appropriate tasks, and provides guidance throughout the process of learning mathematics. As facilitator, the teacher is an important part of the classroom culture. It has been found that when teachers help students build on their thinking, that student achievement in problem solving and conceptual understanding is increased (Kazemi, 1998).

Whenever the teacher realizes there is a conflict s/he interjects questions, which create situations that help the children engage reflectively in their thinking. Throughout the lessons, the teacher emphasizes the importance of communicating mathematical ideas. S/he encourages each individual to make sense of the ideas and to ask for clarification whenever needed. For these discussions she creates the time and space necessary for children to experience participation. At the same time, by her demeanor she shows her expectations for how the children should interact and speak to one another. Since mathematical thinking and knowing are most important, the teacher creates and supports a context for open argument. And the children respond with positive expectations and respectful open communication. As the students argue and try to make sense of their mathematical thinking, they become intrinsic learners. The teacher is an active participant who creates situations in which mentally active students reflect back on their thinking. Since students are viewed as active constructors of their own knowledge within a problem-centered learning environment, the following is implied for the teacher:

1. Teachers should provide students with instructional activities that will give rise to problematic situations.

2. Children's actions, which are logical to them, should be viewed as rational by the teacher.

3. Teachers should recognize that what seem like errors and confusions, are children's expressions of their current understandings.

4. Teachers should realize that substantive learning occurs in periods of conflict, confusion, surprise, over long periods of time, and during social interactions (Wood, 1993).

Thus, the teacher is responsible for knowing the students and the subject matter well enough to be able to select activities that lend themselves to an organization in which all students have the opportunity to contribute. Such activities allow for participation by all students. The activity is the main focus (Dewey, 1938/1997).

Mathematical Community

Students build ideas based on prior experiences, the meanings they make of a situation, their knowledge about a topic, the connections they make to other areas, and more. Ideas are constructed, developed, matched with language, stored, reorganized, and retrieved. Sense is made of them, then lost, then regained. In order to build solid mathematical concepts, students need to reorganize their levels of understanding and develop multiple routes to their knowledge over time. Just as students develop spoken and written language facility along different timetables, they construct mathematical ideas at very different rates.

Shared talk is at the heart of many communities (Corwin, Storeygard, & Price, 1996). In a mathematical community, students use language as one avenue to communicate mathematical ideas. Their talk expresses, clarifies, makes assertions or conjectures, proposes and defends solutions, or conveys their observations or questions. These activities are all part of doing mathematics and all involve intensive communication and social interaction. In a community, students are not afraid to take risks or to make mistakes, because the community recognizes trial and error as a part of the contribution to collective knowledge. Therefore, interaction is seen as a way to encourage mathematical understanding, reasoning, and communication while engaging students in meaningful mathematics. Communication by all students is essential so that they can express their ideas and reflect on their thinking, and so that teachers can understand each child's thinking. Thus, a social community emerges in which all can engage in mathematical discussions. The learning opportunities for the entire class increase when all students' ideas can become part of the discourse (Heibert, Carpenter, Fennema, Fuson, Wearne, Murry, et al, 1997). Thus, an active intellectual community emerges in which students openly share their experiences and interactively construct mathematical meanings.

Discourse

Cazden (1998) defines discourse in two ways: (a) as conversation and (b) various ways of understanding. In both instances Cazden says the students talk to one another instead of the teacher doing all of the talking. Through peer interaction the students have an opportunity to talk about school subjects. Thus the students are developing socially as well as academically. The National Council of Teachers of Mathematics (NCTM) in their *Principles and Standards for School Mathematics* (2000) discusses various characteristics of the discursive process, namely ways of representing, thinking, agreeing, and disagreeing. Students should be able to develop the ability to analyze controversial statements, to search out relevant information, to test evidence and make conclusions based on evidence, to recognize underlying assumptions, to draw and criticize inferences, and to perceive reservations to inferences in argument. In a culture that demands student understanding, teaching is more than merely informing or demonstrating for pupils. Teachers must enable pupils to create meaning through their own thinking and reasoning (Wood, 1999). As students are learning to resolve their conflicts by verbalizing their thoughts in dialogue they are simultaneously attempting to interpret and make sense of others' verbalizations. This discoursing can help students clarify their own understandings by talking, by re-conceptualizing their mathematical constructions, and by making sense of other students' ideas (Yackel, Cobb, & Wood, 1993). Teachers and students facilitate mathematical discourse by questioning and challenging each other to defend answers and explanations. In this way students have an opportunity to talk about what they hear and understand and learn from one another. It is not just a matter of taking turns in a discussion. Their discussions relate to the mathematics at hand. Their commenting leads to understanding and/or clarification. Constructive interactions occur when both the sharer and the receiver are actively involved in communicating, which in turn helps students make sense of the mathematics.

Argumentation

Argumentation is more than talking about differences and misunderstandings, and it is more than seeing mathematics from a different perspective. It involves exploring together and continually clarifying meanings for each other. This helps to lay the groundwork for children's construction of mathematics. One aspect of argumentation is dialogue, not only among students, but also between students and teacher where the teacher is not a constant intermediary. Cazden (1998) portrays classrooms as complex systems in which there needs to be a great deal of shared experiences and conversations. It is through the discussions concerning what was done, what was observed, what was argued about, what was understood, that ideas multiply and lead to other/further explorations. Argumentation is a multifaceted term, which describes the total interaction when students listen actively, present their ideas and solutions, defend them in the face of questions, and question other students' ideas. These constructive interactions encourage students to deal with incongruities, to reevaluate their solutions, to elaborate, to clarify, and to reorganize their own mathematical thinking. In this way the focus is on what is most likely to facilitate understanding, rather than on personal issues. Cazden (1998) also states that our underlying goal must be to communicate, to understand, and to be understood. Thus, variations of discourse are necessary. Teachers need to plan more deliberately for the many purposes of talk within the classroom. Teachers need to create the best environments, both physical and interpersonal, for their students.

Azmitia (1993) found that involvement in arguing increases learning. It is difficult to change peoples' beliefs or what they think simply by telling them or showing them something different. But students' active involvement in the argumentation process enhances their individual learning and their cognitive capabilities, which influence the process of argumentation. This is a circular back-and-forth process in which one aspect influences the other, and vice versa. One cannot change a person's thoughts by trying to excise or replace them with other thoughts. Students' and teacher need to try to understand each other's thoughts, and to work from there. This means having to articulate personal thoughts in a way so others, as well as self, can see where the thoughts are in conflict. Resolutions may not be resolved in a simple one step explanation from the teacher. Complexity exists when students are presenting their thoughts (Duckworth, 1996). Disputes/challenges, which are statements or questions of disagreement about a given explanation, can lead to modifications, retractions, corrections and/or a whole new set of ideas. This is a process whereby solutions are eventually supported and agreed upon. The agreed upon solution is derived through convincing explanations and justifications.

Wood (1999) acknowledges general patterns found in students' mathematical argumentations. The solution is presented with an explanation. Someone challenges the solution presented and may or may not tell why s/he disagrees. The student who offered the solution gives a justification for the previous explanation. The student who challenged may accept the explanation or disagree giving a rationale for his/her position. The student presenting the solution offers further justification, which could be with modifications, or with the acceptance of the challenger's statement. As this process continues other listeners may interject thoughts in an attempt to resolve the contradiction. These statements given throughout the argument may lead to new thoughts and other mathematical ideas. This exchange continues until a solution is agreed upon or is postponed to add needed time for mutual reflection. Through the process of argumentation the students work together to create a single over-all solution by mutual agreement based on the justifications and explanations that emerged and were honored and accepted by all participants. Thus, the collective learning of a class can be documented to show changes that take place over time. In mathematics classrooms argumentation makes great cognitive demands on all students, individually as well as on the whole group, in their comprehension and development of convincing explanations and justifications.

The emphasis on the task and the depth of argumentation are reflected in the level of classroom interactions. During classroom interactions the importance of argument, and its positive effects on learning, become apparent as the teacher reflects back on the mathematical conversations. In the mathematics classroom, which encompasses arguing, learning opportunities emerge for the students and teacher, and are only limited by communication inabilities. As students verbally present their various solutions, they learn to skillfully give explanations, justify their solutions, and give themselves and others new food for thought. The students reflect on and compare solutions while discussing and questioning. In this way their attempts to develop logical reasoning and to communicate explanations give rise to the generation of fluency in the growing use of everyday language, as well as developing mathematical concepts.

In the context of the Wood's (1999) paper, argument was defined as a discursive exchange among participants for the purpose of convincing others through the use of certain modes of thought (p. 172). The author defines argumentation as an interactive process of knowing how and when to participate in the exchange (p. 172).

Voigt (1994) defines argument and argumentation as an exchange for the purpose of convincing others. From the previous statement, look at the word, exchange. Does that mean that a child gives an idea and another takes that idea and gives his/her idea away?

Must a student have an idea to give away before taking an idea? Are students able to exchange whole ideas or parts of ideas? In a mathematics classroom, ideas should be made "public domain" for each person to reflect on and to think about while s/he is trying to make sense of a mathematical idea, solution, or strategy. For example, a student pondering how to solve a task might have a strategy that will work but will have an incorrect solution. Another student may identify the error, but as they both discuss the problem they are able to use the strategy to solve the mathematical task. The student with the right strategy does not give up his idea or exchange it for another idea. In this way these two have shared their mathematical meanings.

When a student has as his purpose to convince others that he is right this may imply that he does not listen to the ideas of other students, and that he will not reason about his own thoughts, but insists that he must convince the others that they must agree with him. Under these conditions, how can argumentation become an interactive process? Is it possible for the argumentation to benefit that individual also? Obviously the student who puts forth his own solution but closes his mind to other possibilities will benefit little from argumentation because he will not interact with the group. The rest of the group, however, may benefit by including the closed-minded student's solution in their argumentations. For argumentation to have maximum benefit to all students all students must feel free to participate openly in the interactions. It is in this socio-mathematic environment that provides maximum learning by all students. As students give ideas, these ideas become publicly seen and heard by all to try to make sense of and understand how the other students are thinking. Through this sharing of meanings, students develop mathematical reasoning and have opportunities to reflect upon and to reorganize their levels of understanding.

Respect

When partners in collaborative small-groups and whole-class sessions negotiate socio-mathematical norms, the interactions and conversations are more involved and more in depth because of respect and a sense of mutuality. The negotiation and maintenance of the socio-mathematical norms and the development of respect for thoughts and for each other serve to provide opportunities for greater cognitive growth. In turn, students are more willing to express themselves and challenge each other. Negotiation helps to establish an atmosphere of respect in which students can work together to solve tasks or reach their goals. Working together collaboratively also enhances cognitive growth because students are more willing to tackle challenging tasks. The learning environment, which contains mutual respect, also gives opportunities for students to critique each other's ideas and to pursue mutually agreeable resolutions. In the classroom where respect is foremost, the students' arguing and questioning are not about personalities, but about their search for appropriate solutions, which do not hurt their relationships.

Argumentation Patterns

Krummheuer (1995) has identified patterns in argumentation. He states:

> Conceptual mathematical learning has as much to do with the development of an everyday language platform for indicating differences in individual interpretations as it does with the establishment of different formally valid mathematical

> argumentations. Learning therefore appears to be indirectly related to interactions within a classroom culture of argumentizing (p. 267).

Krummheuer explains that there are four parts (data, conclusion, warrant, and backing) to an argument and that each part plays a different role. These four parts are not used to identify different components of interactions, nor are they predetermined. But they are a function of the total interaction emerging from the students' negotiations. As students try to explain and justify their solutions they present the solutions (conclusions) from undoubted facts (the **data** the students use in developing their own solutions). From the data the students use they form **conclusions** (solutions), then they give **warrants**, which certify their argument or explain why the data support the conclusion. And finally, they provide **backing** which are their primary strategies supporting the argument and giving it validity. This complex process is a result of the student's interactions in listening, questioning, and discussing their ideas. This is not necessarily a linear process that is cut and dry. The four components may be interwoven, may overlap, may be used repeatedly in different orders, but they are not necessarily formal logical reasoning.

Listening

In a problem-centered classroom where argumentation has been negotiated as a norm, students listen eagerly to each other while they share. They know that each will have opportunities to comment and/or question what the others have done or discovered. Davis (1994) in his article, *Mathematics Teaching: Moving from Telling to Listening*, states that when students are conversing, it is obvious they are listening because their actions become bodily interactions. They move close to one another, focus on what is being said, regardless of the noise around them, and may become animated. Thus, listening is not silent or motionless. It is active. In a traditional classroom listening is perceived as being still and quietly looking at the teacher. By contrast, listening in a learning environment is intensely interactive. Energy is spent in this environment because students are mentally and physically active. As students listen they question, challenge, smile, frown, etc. Their actions confirm that they are listening and learning from the interactions. Argumentation that involves active listening incorporates the insights of each other while moving toward a consensus. This processing is more than just taken-to-be-shared. It allows a collective consciousness to emerge. The students' thoughts are "on the table" so to speak for all to see and discuss. The meanings may or may not be shared because of individual backgrounds and experiences. But by having open and interactive discussion the classroom community is aware of the overall thoughts and solutions. Therefore, the students and teacher become aware as a mathematical community.

Active listening is not passive. It involves giving attention to and concentrating on what someone is saying. Someone may relate passive to a picture of a student sitting in a desk looking at the teacher. But that picture could represent an active listener giving attention to what the teacher is saying, or, it could represent a passive non-focused mind, depending on the inner mental disposition of the student. Students who are not really listening are prone to look bored, or they may ask irrelevant questions, or give irrelevant answers to questions when asked. Students who are participants in the interactivity are prone to ignore a student who is not listening. A student who is listening may be working on something that seems to be irrelevant or doing other things such as using manipulatives, drawing, etc. S/he may not be sitting still in a desk with her/his eyes on the speaker. As each person listens actively (internally), s/he becomes aware of each other's thoughts and tries to make sense of those thoughts.

Davis (1994) states that listening is something we enter into with our minds. Listening is put into gear as we interact with others in order to understand their points of view, their meanings (solutions), and how those meanings impact our points of view. Listening involves our whole being. Our senses are attuned to the immediate conversation as well as on the subtexts, such as the other persons' mannerisms, tones, etc. Listening brings into our view the collective weight of our experiences and compares them with our currently emerging understandings. Listening is not just hearing, but is something we do that is intentional and attentive. Listening is vitally important in argumentation because it aids the teacher and students in their understanding of each other's solutions, explanations, justifications, and questions. As they become aware of each other's perspectives listening enables them to ask appropriate and constructive questions that help to deepen their own and each other's mathematical understandings.

A good example of a student not listening is taken from a second grade mathematics classroom. Her name is Miriam (Episode #8). When she shares her solution, she states it matter-of-factly and gives very little explanation. When other students ask questions or probe for clarification she usually repeats what she originally said or will bluntly say, "That's what I have", or "That's what I did." She does not look back at the original task to show a relationship between her solution and what the task asks. Nor does she acknowledge any other student's solution. She seems to be "closed-minded." This example also shows how a student may be "in" the interaction but not part <u>of</u> the active interaction.

Teacher Role

Teachers' actions are what encourage or discourage students to think, question, solve problems, and discuss their ideas, strategies and solutions. Teachers are responsible for creating intellectual environments where mathematical thinking is expected and expressed. Effective teaching involves observing students, active listening, establishing mathematical goals, and using that information to make good instructional decisions, which allow students to become mathematical thinkers. The article, *Creating Context for Argument in Mathematics Class* by Terry Wood, (1999) illustrates the importance of the teacher's role, and how s/he encourages mathematical thinking within an environment necessary for mathematical thinking to occur. The author states that students are involved in learning what others expect of them in terms of participation as well as learning the content of mathematical lessons.

In order to examine students' mathematical thinking and find ways of knowing, educators must accept that there are different ways in which students discover, understand, get to know their world, and come to know the various ways reasoning strategies are used to make sense of mathematics. Therefore, in a student-centered classroom the teacher places emphasis on the students ways of knowing and the strategies they use to arrive at mathematical solutions. In this particular mathematics classroom, arguments are used to observe and analyze students' mathematical thinking and learning. In order for these discussions to take place a different type of context is created for learning, different from traditional teacher-centered classrooms.

Sociological Perspective

From the sociological viewpoint, students need to adapt to social existence. In order to adapt, students must become aware of behaviors around them as well as their own behaviors. Through this awareness the students are able to form patterns of interaction, routines, and come to know the expectations for their own behaviors. Throughout this awareness processing they examine and evaluate the appropriateness of their own behaviors. The explanations of students' behaviors when they participate during interactive situations are influenced by their awareness of social norms, what is expected of them, and what their responsibilities are. From these perspectives, students learn how and when to participate in class, and they learn important mathematical ideas.

Emotion

With new technologies scientists have been able to study the different parts of the brain and the role each part plays in learning. The hippocampus and the amygdala are two brain structures that are crucial to learning and memory. The hippocampus stores factual information, both trivial and important. Sprenger (1999) compares the hippocampus to a filing cabinet. Its function is to catalog and store factual information. It acts as a relay system to retrieve or store the information from the other parts of the brain. Without the hippocampus, students cannot form new long-term factual memories (Sprenger, 1999).

Another important structure is the amygdala. This structure stores and retrieves emotional information. The amygdala determines if the incoming information is emotionally important for long-term storage. This brain part is very sensitive and relevant in every transmission of information. Reactions to situations are drastically affected by the amygdala's response. A strong positive response will store information into long-term memory. The storing of information in long-term storage is beneficial for all students. These reactions can and do come from students' responses to the environment around them. Creating safe environments in which children can learn is very important. Safe environments are those in which students themselves feel safe, meaning they feel cared for, accepted, and respected for who they are along with their ideas. It is important that students not only know that the classroom is physically safe, but they need to know that it is emotionally safe. It must be emotionally safe for maximum argumentation and maximum learning to take place. When the hippocampus and amygdala are working together cooperatively (without one overriding the other) the students' can explore and make sense of their world, thus increasing memories.

Emotion helps us to focus the mind and set priorities. For example, our logical side says, "set a goal." But our emotions allow us to become passionate enough to act on the actual goal. Some students are fearful of being embarrassed, fearful of failing in front of their peers, or fearful of being bullied. Their brains treat these emotions as threats to their well-being. The amygdala is heavily involved in direct emotions and the amygdala exerts tremendous influence on the cortex, even overriding the cortex (Jenson, 1998). The flow of information back and forth from the amygdala to the cortex contains all our thoughts, biases, ideas, and arguments. The amygdala gives us the capacity for creative play, imagination, and key decision-making. Teachers who help students feel good about learning successes through their classroom experiences, friendships, and celebrations are doing the very things the students' brains crave (Jenson, 1998).

Emotions are a distillation of learned wisdom from life's experiences. The critical survival lessons of life are emotionally hardwired into our DNA. Emotions are related to every experience we encounter and serve as a crucial source of information for learning. Students who feel tentative or afraid to speak in front of their peers are that way for a "logical" reason. Maybe their experiences have caused such negative social dispositions. Maybe they have not yet been exposed to enough positive (feeling good about self) experiences. Thus, they may shut down and not participate in the verbal sharing of ideas or mathematical solutions. The stress chemicals involved shut down preventing the brain from making logical connections. The student then shifts from a higher level of thought and engagement to a lower level of thought and engagement, thus not as involved mentally.

Making daily decisions always involves our emotions to some greater or lesser extent. Appropriate emotions speed up decision-making. Emotions also help us to make better quality, value-based decisions. Emotions activate and chemically stimulate our brain, which helps us remember things better. Thus, triggering emotions randomly is counterproductive. Engaging appropriate emotions within the learning environment has powerful effects on students' learning. Research indicates that when emotions are engaged in the learning process, memories are much more likely to be recalled and accuracy goes up.

Jenson (1998) suggests several ways to increase learning while engaging emotions; one of those ways is through argumentation. In an atmosphere of care and respect students will willingly and openly participate in mathematical argumentations. In this type of environment the students feel comfortable enough and secure enough to display their thoughts to the whole group. At the same time they are challenged and are actively thinking about other students' ideas. Thus the complex process of mathematical thinking continues through communication.

Research Focus

Argumentation helps students to develop greater insights and deeper mathematical understandings. Effective listening is an important ingredient in argumentation, and is the key to deeper sense-making. Students' abilities to listen, to attend to, and to interpret what is said are paramount. Conversation is a continuous flow of ideas put forth to be discussed and to provide new levels of understanding, particularly so in the exchange of mathematical ideas for greater mathematical learning.

My research focused on identifying elements of this discursive process, as demonstrated in a second grade mathematics classroom and modeled in a problem-centered learning environment where students were expected to engage in negotiating meaning through argumentation as a fundamental aspect of their mathematics learning. My goal was to elaborate the ways in which this discursive process supported (or otherwise) the students' mathematical learning. In answering this I provide a critical analysis of this process, thus contributing to the knowledge of teaching and learning of mathematics. The method for analysis will be discussed in chapter three (next chapter).

CHAPTER THREE

METHODOLOGY

Teaching Experiment

To frame my research I employed Cobb and Steffes' (1983) teaching experiment methodology to answer the questions that are the focus of this research: "What are the aspects/characteristics of argumentation and what is the role of argumentation during the whole-class discussion time"? The teaching experiment method assumes that learning can be characterized as both a process of active individual construction and a socially situated process. Therefore, students' mathematical development is related to their participation in communities of practice, such as those constituted by the teacher and students in the classroom. This methodology is a way of organizing and analyzing the messiness and complexity of the classroom. It also allows for analysis during and after the session being investigated. Cobb (2000) states that one of the primary aims of this analysis is to place the events of the classroom in a broader theoretical context. The primary goal of the researcher is to establish a living model of the students' mathematical activity embedded in the social interactions of the classroom. Also, analyzing after the session relates to the development of models of teaching that are compatible with the constructivist view. The teaching experiment provides a way of exploring the prospects and possibilities for reform at the classroom level.

A teaching experiment methodology allows for collaboration with a teacher. Through the process of collaboration, teacher and researcher negotiate an effective basis for communication and a common purpose. Before the teaching experiment takes place the teacher and researcher negotiate several additional things, such as students' mathematical activities, video recorded interviews, and their general views about schooling. Through the negotiation process the goal is that the teacher and researcher will develop taken-as-shared understandings concerning the teaching experiment and situation under investigation (Cobb, 2000).

Explaining students' mathematical activity is a primary concern of the teaching experiment. Cobb and Steffe (1983) believe that the researcher must observe learning firsthand in order to form an accurate explanation of students' mathematical activity. The teaching experiment consists of a series of teaching episodes and individual interviews with students that cover an extended period of time. The interviews investigate the steps students take when constructing a mathematical concept. In a clinical interview, mathematical knowledge can be traced back to less abstract concepts and operations. By using the clinical interview, a researcher can describe structural patterns students may learn from the experience gained through interaction with their environment (Cobb & Steffe, 1983).

This methodology allows for the investigation of the social context of development. Students participating in a classroom culture influence the goals they attempt to achieve and what it means to know and do mathematics in school. Thus, the researcher might analyze the teacher's and the students' negotiations, standards of mathematical argumentation, or a particular practice. The teaching experiment also investigates the teacher's activity. It investigates the development of instructional sequences and activities as well. It is not about replicating instructional innovations by ensuring that they are enacted in precisely the same way in different classrooms, but to develop ways of analyzing innovations that make their realization in different classrooms similar.

Researcher as Facilitator Participant Observer

In this design, the researcher is a facilitator participant observer, working with the teacher and students, listening to their interactions and ideas (Steffe & Thompson, 2000). In so doing, I observed the mathematics lesson at least one day each week throughout the school year. I also monitored and questioned students where appropriate, helping them clarify thinking. I took extensive field notes during each observation period, supplemented by transcribed video recordings of these lessons. I interviewed the teacher, Kathleen, for approximately 45 minutes immediately after each observed lesson and transcribed those texts. This provided a time for Kathleen to reflect and explain why certain actions were taken. Moreover, to explore students' mathematical development, I interviewed and video recorded six key informants (students) several times during the year and transcribed all inquiries. The six key informants were representative of the small-groups formed. They also represented various ethnic groups and both genders. *The classroom teacher and I chose these six key informants after listening to the students' explanations of their solutions and strategies. We looked for students with varied ways of solving tasks; sophisticated methods as well as different and immature methods in order to get a broad spectrum of ideas to consider.* Key informant interviews, observations of students in their small-group problem solving, and teacher interviews were used to provide a variety of explanations and interpretations of the whole-class events. The individuals' and groups' mathematical activities are interdependent. Therefore, students' participation in the small groups and whole-class discussions gave insights into their mathematical development, and supplemented and helped when I interpreted the whole-class discussion in greater depth. I gave particular attention to the mathematical activity during the whole-class discussions in which argumentation was prevalent in students' constructions of mathematics.

Method of Analysis

A constant-comparison method of analysis guided the socio-mathematical interchanges that lead to individual mathematical constructions (Strauss & Corbin, 1998). This process was nonlinear because more than one pattern may be identified at the same time and certain relationships may exist among new and old patterns. After each class session two assistants and I examined the whole class data, which I first separated into specific event sections as frames in which to focus subsequent classroom observations, interactions, and interviews. Independently, we coded and categorized each of the other three data sets within each event, looking for regularities and patterns in the ways students and teacher or students and students mathematically interacted within and then across sets. Using the constant comparative method and more than one analyst's coding furthered the study's trustworthiness and fidelity by considering the classroom in its natural state without a prior presupposition (Schwandt, 2001).

Along with new and emerging perspectives in education to promote mathematical understanding, several researchers in general have analyzed the whole learning environment where students and teacher interact while working on worthwhile tasks. Measuring only the regurgitation of facts does not necessarily give informed decisions about the learning of mathematics. Analyzing and investigating the ways of thinking of students, groups, teachers, classrooms, programs, or schools involves systems that are complex, dynamic, and continually adapting, and they cannot be reduced to simple checklists. Therefore, I felt it necessary to include the description of the classroom setting and each part of the problem-centered learning environment. The whole-class discussion was videotaped while students were discussing their solutions and strategies. Field notes were also taken during this time. The videotape data and field notes were used to examine the argumentation and interactions that transpired during the whole-class discussion portion. The other data sets from the interviews and collaborative pairs were used to further interpret my understanding of the episodes used in chapter four.

Background Setting

I conducted my research in a second grade mathematics classroom. This was a low to middle income suburban public school and a site for the district's emotionally handicapped program. Of the eighteen students, eight were girls, and ten were boys. The students were from various backgrounds and had varying abilities and disabilities.

The classroom was set up so that students could easily work together and have access to many different kinds of manipulatives. The desks were touching side to side, arranged in a u-shape with a few desks in the center but still touching the desks forming the u. With this arrangement all students could easily see the whiteboard and all students were visible to the teacher, Kathleen (pseudo-name). The u-shaped arrangement left room in the back for the students to sit on the floor in a circle during whole class sharing time.

The problem-centered mathematics lessons usually lasted one hour, beginning with a short whole group activity, a brief explanation of the problem to solve, paired or individual collaboration, and whole-group discussion at the end. The instructional activities emphasized tasks wherein Kathleen did not give students specific procedures for finding a solution. Instead, she encouraged them to complete the tasks in ways that made sense to them (Wheatley, 1991; Wheatley & Reynolds, 1999).

Although I discuss Kathleen's and the students' experiences separately, the classroom in this study (comprising teacher, students, and artifacts) was like others—a self–organizing entity and a complex adaptive system (Varela, Thompson & Roche, 1993). All elements were interwoven because educator and students negotiated the classroom environment as they attempted to make sense of and respond to each other's actions.

When I investigated the functioning of Kathleen's classroom I focused on the socio-mathematical rather than social norms, the latter being the roles played and expected in a complex classroom environment. The social norm could be the same for all subjects, the guide for the whole day, and/or the general behaviors expected within this classroom (Cobb & Yackel, 1996). The socio-mathematical ones were those behaviors that were pertinent only to the students' mathematical activities. In short, a social norm was an explanation of a solution, but the understanding of what counted as an acceptable mathematical explanation was socio-mathematical (Yackel, 2000).

Teacher's Role

Acutely aware of fostering socio-mathematical thinking by building on prior knowledge, Kathleen used activities which were based on the students' previous thinking and which would promote an understanding of mathematical relationships. When students would share their strategies, Kathleen would ask, "Is there another way"? After a few students share their thinking she again would ask, "Are there any other ways"? She often asked the students if the strategies made sense to them. While they were sharing answers, she or the students would record their solutions and strategies on the board. As she listened and made sense of their mathematical thinking, the accumulated information enabled her to devise developmentally appropriate lessons throughout the year.

Often times she would use an opening activity designed to foster mathematical thinking and encourage students to relate those ideas to the day's task. As the students discussed the opening activity, which usually lasted 5-8 minutes, she encouraged them to share their different ideas. Kathleen stated her purpose in using an opening activity as follows:

> I think it is a good way to get kids to get their minds engaged, to be thoughtful, to become engaged with ideas, and to activate conversation in response. An opening activity is a good warm-up exercise for what we will be doing for the rest of the lesson—getting them to talk to each other.

After the opening activity Kathleen would introduce the task they would be working on for that day or longer. Since she had listened to the students in small group interactions and during whole-class sharing time Kathleen was able to select an activity to enhance their mathematical thinking and sense making. Rather than finding a different task for each student she always chose one for the entire class that all the children could complete in a variety of ways, depending on each one's level of sophistication. She usually displayed the task on the overhead or read it out loud before they worked in their collaborative pairs.

Small-Group Collaboration

While students worked on the assigned task, Kathleen walked around observing their work, taking notes, and asking questions as she learned about the mathematical thinking of individual students. She did not look for right or wrong answers but for a variety of strategies being used by the students. As stated and demonstrated, Kathleen's intention was to be aware of methods that students used so she could be prepared to orchestrate the whole-class discussion by discussing all of them. Explaining her routine she stated:

> In order to get students to talk I walk around and ask them lots of questions relating to how they are able to see certain aspects of the problem. I will ask some pair partners (from among the small-group pairs of students) if a certain aspect of the problem makes sense to them, and/or pull another couple sitting close by. I am trying to see what they are making sense of as well as what is disturbing their thinking, or what may facilitate them in taking another step.

In so doing, Kathleen deftly monitored and encouraged each student's participation. She was almost always able to understand their thinking processes and to determine when assistance was necessary. Sometimes she helped youngsters in their development of mathematical meanings, aided them in making sense of, or helped them reflect on their solutions in order for others to understand their explanations. Usually, her facilitation came in the form of posing provocative questions and initiating dialogue. Other times she encouraged children to work cooperatively or to listen to one another's explanations. Kathleen's role among her students was to help them construct and develop their mathematical thinking, whether in small-group collaborations or in individual deliberations.

Whole-Class Sharing

In contrast to the traditional lecture-oriented type classroom where students wait for and depend on the teacher's answers or explanations, Kathleen's classroom provided open discussion in which teacher and students listened, responded, verbalized, visualized, and interacted with various explanations. Kathleen encouraged the exchange of ideas among all the students. She expected them to explain and justify their solutions during this time, while at the same time maintaining respect for each other's ideas. Students were taught to respect one another, but since the emphasis throughout the school day in this classroom was on making sense, ideas were the most important topics of concern, focus, and discussion. Students' minor misbehaviors, such as picking on one another, were ignored. Kathleen would ask such questions as, "Did anyone solve it a different way? Does anyone else agree with this idea? Why or why not? Does anyone have a question or comment about this? Did anyone else start that way and it didn't work for you"? She expected the students to listen to one another, make sense of the ideas, and ask questions to those who were presenting their solutions. Thus, Kathleen modeled appropriate types of inquiries, which the students could ask one another duing the whole class discussion, thereby deepening their mathematical thinking.

Discussion

Kathleen's actions encouraged interactive mathematical argumentation among her students. They were not merely giving correct answers to procedures, but were offering explanations for and clarifications of the strategies involved in finding solutions. This processing helped students sharpen their thinking and make conceptual connections from one problem to another. Students would use these connections then in explaining, asking questions, or in arguing a solution. The conversations also taught students to develop a language for expressing mathematical ideas more clearly. With Kathleen's patient insistence, as the students attended to each other's ideas and expressed their own, they learned to listen, paraphrase, question, and interpret. The classroom goal was to make sense of tasks, negotiate meaning, resolve any conflicting ideas, and strengthen students' reasoning abilities. In the process, they learned mathematics through interactive communication. As they learned how to talk to one another they learned to communicate mathematically (NCTM, 2000).

Kathleen was aware of her important role in determining the extent and quality of students' mathematical understanding and knowledge. However, she did not perform this duty through the traditional means of clearly explaining procedures and providing students with adequate time to practice, but rather by promoting curiosity and intellectual autonomy (Kamii, 1982). She enhanced learning through interaction in an open risk-free environment, while encouraging students to take responsibility for making sense of the various mathematical tasks. To this end Kathleen expected and encouraged respect for each other's right to speak, and made allowances for the youngsters' mistakes.

In order to develop this type of atmosphere, Kathleen and her students negotiated and renegotiated the classrooms' socio-mathematical norms. When students had disagreements with each other about an answer, Kathleen expected them to resolve the issue mathematically in their own way. The negotiation of these norms was an ongoing process, occurring with each task, or perturbation. Because Kathleen was more interested in the students' reasoning strategies than in managing their behaviors, she was able to successively mentor and model positive learning behavior.

Because Kathleen was interested in the students' thinking and developing autonomy she chose tasks that led to conflict and discussion. In order to promote group understanding of individual children's thought processes she used their own words when clarifying, asking questions, or repeating so that all could hear (Cobb, Wood, & Yackel, 1991). In this way the students hear their original solutions or responses and reflect and reorganize their thoughts so that others can better understand each one's perspective (NCTM, 2000).

As a researcher/participant I tried to be a good communicator, empathize with the participants, establish rapport, ask appropriate questions, and listen intently. I looked for patterns that would help answer the questions in this study. The data collection and analysis occurred in alternating sequence beginning with the first interview and/or observation. This led to more observations and interviews followed by more analysis as the cycle continued.

In chapter four I present in a specific manner some of the data that were collected. There are nine episodes representing the whole-class discussion time of this particular second-grade mathematics classroom. Interwoven in the episodes are remarks and summary statements helping the reader to make sense of what was happening; the task the students were working on, and the communication that was taking place.

CHAPTER FOUR

RESEARCH FINDINGS

In this section I will describe nine episodes (scenarios) from my observations of a problem-centered second grade classroom learning environment in which argumentation is an essential component. I will describe a classroom designed for maximum mathematical learning for all students; one that promotes and encourages students to engage in open argumentation.

The classroom is structured with four time periods: (a) the classroom morning meeting, (b) class opener, (c) students working in collaborative pairs, and (d) the whole-class discussion. Argumentation is the type of communication used in all four time periods.

The various aspects of these nine episodes help to illustrate how the teacher as facilitator/participant provides for students opportunities that help them figure out what it means to "argue", and how "maximum" argumentation provides maximum learning, sense making, and understanding for all students. These episodes illustrate how Kathleen's students come to accept responsibility for their own learning by getting interactively involved in each other's ideas. This particular learning environment gives students opportunities to think about one another's perspectives as well as reflect on their own. It helps students to become active participants in argumentation by listening and talking with and among each other.

First, the classroom **morning meeting** at the start of each day shows how students and teacher socially open up, listen to, and interact with each other in sharing and caring. Students feel socially secure and relaxed in an interactive class relationship of mutual acceptance and respect for each other. It is in this morning meeting that the whole class is involved in negotiating social norms (behavior guidelines) for the day.

Second is the **class opener,** which is a short mathematical warm-up activity designed by the teacher to stimulate open interactive thinking, thus promoting argumentation. This open interaction with one another becomes routine as an established social norm throughout the whole day. The emphasis is always on mathematical learning. These first two scenarios, the morning meeting and class opener, are vital to enhancing and preparing the students for argumentation during the mathematics lesson. Both scenarios precipitate an atmosphere where the students can negotiate norms and share ideas in a relaxed and safe environment.

Third, after the class opener the students move into **collaborative pairs** (small groups) during which time each student with his/her partner works on the task (problem) assigned by the teacher. The teacher walks around observing each group, and sometimes asking questions to facilitate dialogue. She also is monitoring their activity in order to make informed decisions about appropriate ways to orchestrate the whole-class sharing time. From these observations and interactions she can encourage students to share that have a different way of solving the task. Even though my research does not focus on the collaborative pairs, this is an important component that facilitates and encourages argumentation in the whole-class sharing time. The students on a more individual level negotiate with a partner and solve problems. This helps students not only make sense of the task but to feel confident enough to share later during the whole-class discussion. And when they share their ideas they are not alone, but have a partner that can help explain and/or illustrate their solution and strategy.

Then last, all groups and teacher come together in a **whole-class** discussion/argumentation interactively listening to, and openly discussing, each other's ideas, constructions, disagreements, and solutions. Argumentation enhances student participation, spontaneous interaction, and mathematical learning. This whole-class argumentation is the primary focus of my research and from which most of the scenarios are taken.

Throughout these episodes the teacher facilitated learning by: (a) holding the students accountable for their own learning and participation, (b) giving them opportunities that elicited participation in argumentation, and (c) providing tasks and opportunities that elicited students' explanations and justifications. Therefore, the intellectual autonomy rests with the students as they negotiate differences and work toward consensus (Kamii, 2000).

Each episode, individually illustrates how it supports successful whole-class discussion where students are engaged in open and interactive argumentation in order to make sense of mathematical ideas. While a key characteristic of the whole-class argumentation is recognized and identified for each episode, it should be noted that each episode contains a multiplicity of argumentation characteristics, which are not necessarily restricted to any one particular episode.

As seen in each episode, these interactive experiences in which children are involved lead students to become **autonomous** learners and problem-solvers. In each episodic argumentation process the teacher encourages the sharing of ideas in order to give students opportunities to become autonomous learners.

Kamii (2000) defines autonomy as "the ability to be self-governing in the intellectual realm, the ability to make judgments for one-self, searches and questions for one-self (not just accepting without checking), puts things in proper relationships" (p. 57). According to Piaget (1965/1995), the exchange of points of view with others is indispensable for children's development of logic (reasoning). He states that social interaction is essential for children's development of logic.

Exchanging points of view is important because these exchanges help students to see things from others' perspectives, giving them a common language to be able to communicate effectively. Looking at other perspectives helps them to make sense of their own reasoning by reflecting on their own thoughts in comparison to others. This whole-class back-and-forth interactive argumentation process provides all students opportunities to explain and justify their own reasoning, strategies, and constructions. It is during this process that erroneous reasoning and erroneous conclusions are sifted out and discarded, ultimately leaving an agreed-to correct solution. When all students participate in argumentation all students learn how to think and logically reason, thereby gaining mathematical understanding.

My observations in the problem-centered learning environment show that all nine scenarios illustrate the importance of argumentation in mathematical learning. A different aspect of argumentation is identified in each scenario as listed below. Each episode is identified by name.

Name	Characteristic
1. Morning Meeting	Social adjustments
2. Class Opener	Confidence building
3. Trapezoid Triangle	Teacher/facilitator
4. Balance Task	Minority and mutual acceptance
5. Red/Yellow Flowers	Open interaction
6. Red/Green Suckers	Incubation
7. Valentine Task	Sense making
8. Loop, Whorl, Arch	Listening for understanding
9. Class Arguing	Related to learning

Through the various episodes I worked to illustrate many of the characteristics of argumentation, such as interactive and open communication, reflection, listening, meaning-making, and adjustment of intentions. In doing so I noted how well argumentation promoted mathematical learning for all students in Kathleen's particular problem-centered classroom learning environment. As a starting point in my identification of argumentation characteristics I began my observations during the first part of the students' day when they began negotiating norms. Throughout my observations I identified key characteristics, which emerged in the various episodes.

Successful classroom argumentation is not restricted to certain types of students like smart, handsome, one race, one color, blonde hair, poor, rich, etc., but is mutually inclusive of all students of every type. Argumentation focuses on mathematical constructions, not on personality destructions. All of the following episodes show that argumentation provides potential opportunities for maximized effective learning by all students, because all students interactively openly participate in the thinking, listening, reasoning, reflecting, and discovering processes.

In the following episodes the names of the students and teacher have been changed to protect their identities.

Observations of a Second Grade Classroom

(Episode #1)

Morning Meetings – *Social Adjustments*

First, I want to describe a very important tool used by Kathleen to set the stage (the mood) for the rest of the day in terms of acceptance, attitudes, and behavior. That tool is the first meeting of the day, the morning meeting. The morning meetings are held in the back of the classroom, which is arranged so that the students sit in a circle and talk about something important to them. Kathleen said that she started this because several years ago she asked students to tell about a time in their lives when someone "really listened" to them. Not one child could share a time when someone had listened to him/her. According to Davis (1997) the importance of listening to learners has been under appreciated. Thus, Kathleen has tried to incorporate listening (which has enhanced argumentation) as one priority throughout the school day.

During these meetings the students and Kathleen listen very attentively to each other as they bring interactive dialogue into meaningful argumentation. Kathleen genuinely cares for all her students. And the students also care for each other as is evidenced in their comments and in their listening. Kathleen is the living model for providing a positive atmosphere in which to build these caring relationships among her students. In these morning meetings the whole class is involved in negotiating the social norms (the rules for behavior) constructed with an atmosphere of caring for, empathizing with, and respecting one another. Students do not judge anyone, but accept each other as having equal value. In this sense there emerges a rich and dynamic learning community consisting of active caring, respecting, and eager learning. The above-negotiated social norms carry over into and throughout the whole day in all Kathleen's subjects taught that day. They are not limited to just the mathematics class. Also, the morning meetings reflect the same depth of argumentation and listening that takes place within the mathematics classroom.

Noddings (1992) states the following:

1. Students will not succeed academically if they are not cared about.
2. Children learn in "communion," and they listen to people who matter to them and to whom they matter.
3. Caring relations can prepare children for an initial receptivity to all sorts of experiences and subject matter, including mathematics.
4. Dialogue taking place while learning in communion connects children to each other, and it provides knowledge about each other that forms a foundation for caring (p. 23).
5. Schools cannot achieve high academic goals successfully without providing care and continuity for all students. (p.14)

According to Noddings (1992) and Jenson (1998) a safe and caring environment must be provided for students if they are to use their full potential and capabilities, and to make new constructions and connections. If students are not provided safe, risk-free environments in which to learn, or they learn "helplessness" (Jenson, 1998, p. 57). Their brains will not be able to reason, problem-solve, or think straight. This is because in a threatening setting or one that is perceived as threatening causes our glands to release cortisol. This causes the large muscles to tense up, blood pressure increases, and the immune system is depressed, any and all of which hinder students' learning.

In search of a way for students to feel cared for, listened to, and a way to handle differences, Kathleen began using morning meetings. During the morning meetings Kathleen and the students negotiate necessary steps to be taken to resolve differences and maintain respect for one another. Following is an actual case of friction between students. It is used to show how negotiation, care, and respect are maintained.

Sue had a problem with Zena and Grover. After Sue presented her problem and explained that her feelings were hurt, Kathleen asked Zena and Grover what they thought. After the three of them talked a little about the situation, Kathleen asked the class if they could think of ways to resolve the situation, or help Sue. A couple of students asked Sue more questions concerning why her feelings had been hurt. Then they asked Zena why she did what she had done. Zena replied that she was mad at Sue and would stay mad. Kathleen asked Zena if she was using revenge. The reply was `yes'. Some of the other students offered their thoughts about revenge and how it did not resolve anything. In this way the students are also contributing to ongoing constitution of the social norms in this environment. Not once did the students attack one another. Instead, Zena merely and openly expressed her anger toward Sue about her action. Kathleen did not step in with an authoritative response or solution. She listened and questioned as an equal participant who truly cared about her students. Through collaborative discussion the issue was resolved and the students put it behind them.

Concerning the use of morning meetings Kathleen states:

> Well, the morning meetings are another time when I am listening to them [students] and they are listening to each other. And they are also working on solving problems. And I think it has really helped just to start discourse and conversations with each other. So you have another time during the day, which is an important thing when they see themselves as problem solvers; sometimes you

[teacher] have to get away from where they [students] think that you [teacher] are going to solve their problems for them.

A key interactive component in Kathleen's classroom is listening. Listening emerges as a very important interwoven part of their everyday routine. Students listen to one another's ideas in morning meetings, during class openers, when working in collaborative pairs, and when sharing their solutions and strategies during the whole-class discussion time.

Every morning Kathleen and students re-negotiate the class social norms for that particular day. Various aspects and needs of their "community" life may change on a daily basis. For instance, if a student comes into the meeting in a bad mood, or with some particular problem, which will affect the whole class, the whole class will re-negotiate and adjust its behavior (norm) for that day according to needs. Now, for the most part, negotiation of social norms has been taken care of for the day. The students are now free to focus on mathematical concepts, and to openly participate in related activities without feeling inferior, threatened, or embarrassed, but feeling respected and cared for.

The Math Class (later in the day)

When students finished their morning meeting they were genuine learners who were prepared and willing to engage each other in open argument and to make sense of things in the classroom, not only in the subject of mathematics, but in all other subjects taught by Kathleen that day. All this is because they enjoy a sense of equal worth and security. At no time does Kathleen exhibit an authoritarian role. Rather, she exhibits the level of humility and equal worth with the students. In her role as facilitator to learning she may ask such questions as, "How do you think the problem can be resolved? What do you think needs to be done"? In this way she challenges her students to take responsibility for their own actions and learning. Here the children learn that they are responsible for inquiry in their lives and they determine what goals are important and the ways in which they can be met. This process will continuously be played out over the course of their lifetime.

Because of Kathleen's positive disposition among the students, they students respond with visible eagerness to take on their responsibilities for learning mathematics. Students consider each other as having equal-value roles to play. They pay attention to and enjoy participating in the challenges of their task at hand. Teacher and students stay busy observing, questioning, listening, and interacting in the discussions taking place. All related comments are welcomed. The community is safe and the students are respectful of all ideas and thoughts. There are no vendettas from anyone.

Agenda Book

In Kathleen's classroom there is an agenda book (spiral notebook), which serves an interesting purpose. During the morning meetings the students are free to either openly share something they have done, or something from the agenda book, keeping in mind that in this classroom, learning is the main focus. If a problem occurs between students during the day, they can write it in the agenda book. Unless the problem is serious, Kathleen and the class will address the issue the next morning at the morning meeting, thus continuing with the subject matter at hand. So rather than bother Kathleen during class time the students learn to write down the problem in the agenda book for later discussion. The students are comfortable with this and generally do not talk about it to each other, but go back to working on their task.

Kathleen has commented that many times she has seen students write in the agenda book during the lessons. At the end of the day she reads through the agenda book and finds that the problem has been resolved and crossed out before the next morning's meeting.

This kind of respect of students by the teacher preserves and maintains an uninterrupted total classroom-learning environment. There are exceptions on occasions, however. There may be times when the classroom needs to be interrupted to deal with an immediate problem, or when time-out is needed to renegotiate norms. But the agenda book is used as an avenue for students' voices to be heard in a classroom environment where learning is the focus.

In some settings students are seen as separate and unique individuals who are to "be taught," as if the knowledge to be given is an objective entity to be acquired outside the student. The teacher is thought to be the beholder and giver of that knowledge. The children's world is viewed as distinct from the "real" or "school" world. Each child is viewed as distinct from others while knowledge is reduced to merely data, or "sanitized" information to be taken in by each student. And the data taken in is thought to carry the same meaning for all students. Thus, all students should know the same amount and kind of information. In contrast with this is the idea that students can be autonomous learners and are in a constant flux with each other and their environment. Therefore, learning is seen as a process of organizing and reorganizing one's world of experience, meaning that the learner and everything with which it is associated are inextricably interwoven. The focus of inquiry is on relations that bind people, objects, and places together in action (Davis, 1997). The morning meeting is a setting in which students and teacher come together and the primary focus is to negotiate norms, to listen, and to work out problems or differences, thereby establishing a thoughtful and positive social community where individuals' learning is enhanced as they engage in everyday activities. (Glassman, 2001). As a result of the morning meetings a positive atmosphere of equity and worth is created. In essence the "relationships" are fostered and mutually adjusted, which leaves ample time for everyone to focus on the learning of meaningful mathematics. (This is not to say that renegotiation does not take place any other time. Negotiation takes place throughout the day as needed.) The negotiation of norms, then, provides for the resulting unity and harmony of eager students freely working and learning together, as opposed to students who reluctantly work together because they know they have to.

During another interview, Kathleen was asked why she thought the meetings were important. She replied:

> [w]anting kids to really make sense out of things and trying to empower children...a way for kids to work out ways of solving their own problems in the classroom and ways for kids to talk and discuss ideas with each other and to have discourse with each other and work through that.

Once the teacher has stepped out of the role of the only "authority" in the classroom, students have the freedom to depend on each other, negotiate with each other, and support and encourage one another. The students do not stand in line at the teacher's desk waiting for an answer, nor are they sitting with their hands raised waiting restlessly for an answer. They are working and talking together and making sense of the mathematics as well as social aspects of life. Thereby, the teacher is giving students opportunities to become problem solvers and autonomous learners. This does not mean that the teacher is a "fly on the wall." Kathleen is very interactive with the students in all aspects of their learning throughout the day. She is a facilitator/participant and an authority only when necessary.

This episode is important to the later whole-class discussion because it sets the "ground rules" for the community for the day. This is a time when the teacher listens to and cares for all of her students, and promotes respectful argumentation. This is a time in which the argumentation aids in supporting autonomy among and for each student. If students are comfortable making decisions, expressing themselves, and listening to one another socially, then the whole-class mathematics discussion is enhanced because students are willing to share, feel comfortable enough to take risks, and help one another. The morning meeting is a time as throughout the day that many voices are heard through the students and teacher interactions, and students can explicate their own thinking. This time of day is important because through teacher and student interactions a viable basis for communication is established. The morning meeting is a time for personal sharing of issues pertinent to each child giving learning and understanding the focus for the rest of the day. Thus, the community environment carries through into the mathematical discussions as well.

(Episode #2)

Class Openers – *Confidence Building*

As part of students' mathematical development, it is important for them to develop thinking strategies. Too much reliance on mechanical counting and counting on, rather than reliance on number patterns can be detrimental to significant learning. Standard mechanical learning procedures become rote, boring, and they interfere with the analytical creative constructions of number. One particular activity, "spot-the-dot," helps children to form images of dot patterns from which they can construct number. By forming mental images children can construct numbers in meaningful ways and use powerful thinking strategies for adding and subtracting (Wheatley & Reynolds, 1999).

Constructing ten as an abstract unit (Cobb & Wheatley, 1988) is an important aspect of children's mathematical development in this classroom. Kathleen provides opportunities for her students to think of the number ten as a unit as well as ten separate things simultaneously, an abstract unit. Students can develop powerful strategies for adding and subtracting larger numbers once they have constructed ten as a unit. Keeping this in mind, Kathleen would use a number of different mathematical opening exercises, which would help foster mathematical thinking, and encourage students to relate those ideas to that day's task. The task she used for this episode was the above mentioned "spot-the-dot." Kathleen arranged pennies on the overhead without the students seeing them. She then showed the pennies for approximately three seconds. The students were instructed not to shout out what they had seen but to wait so everyone would have the opportunity to make sense of what they saw. Kathleen then asked students to share what they had seen.

When asked about her purpose for using this opening activity, Kathleen responded:

> I think the openers are good ways to warm up, to get kids to get their minds engaged to be thoughtful, [and into] the engagement of ideas [and into] the conversation that comes about because of them. They [opening activities] are a good way to warm up to watch what we will be doing for the rest of the lesson; getting them to talk to each other to express their points of view and how that helps them to see things in a different light. It does get the conversation flowing. Just for kids to get that flow and energy and ideas going is a real productive thing to be engaged in.

During this short time period of the mathematics lesson, Kathleen did let as many students as possible share their ideas. As soon as the opener had been shown, the students would quickly raise their hands wanting to share their ideas. When there was a lull in the conversation she would ask if the students had another way of looking at it, or another idea. This usually gave a couple more students an opportunity to share. More importantly this helped students focus on different ways of thinking. In this kind of group setting students felt safe and relaxed enough to be willing and ready to openly share their ideas. This non-threatening environment helped build confidence in students and enabled them to better participate in argumentation.

This particular "spot-the-dot" activity encourages students to think of objects as collections rather than things to be counted individually (Wheatley & Reynolds, 1999). The dots are shown briefly so the students cannot count. This fosters the building of mental images instead of simply relying on counting. And it enables them to develop thinking strategies. The goal is for students to develop mental images associated with numbers and then be able to use these images in doing mathematics. This particular opening task reveals the students' various ways of thinking and looking at objects. It encourages them to share without the threat of being wrong or of making a mistake.

Spot-the-Dot #1

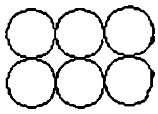

Figure 1 Spot-the-Dot

Kathleen showed the arrangement as seen in Figure 1.

Kathleen: "What do you see?

Class: (shouts) 6.

Even though six is the "right" answer, Kathleen expects the students to share their strategies. In doing this she re-emphasizes the fact that students' thinking is important, which is important later for the whole-class discussion.

Kathleen: Miriam, what did you see and how?

Miriam: 6, I know 3 + 3 = 6. 3 is one part.

Kathleen: 2 sets of 3?

Miriam: Makes 6.

Kathleen: Does anyone see it in a different way?

Here Kathleen is opening discussion up for different ideas and not just one "right" answer from an "authoritative math person."

Sue: 2…

Jan: (interrupting Sue) 4 and 2.

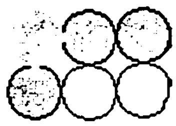

Figure 2 Spot-the-dot

Figure 2 has the same arrangement as Figure 1, but Figure 2 illustrates Jan's way of thinking.

Kathleen: Where did you see 4?

Asking Jan to clarify her thinking models for the students that it is appropriate to ask questions when the ideas are not clearly explained or if one does not understand. Just acceptance of an answer does not mean that two people (Jan and the teacher) or the class is thinking the same thing. There are many different ways to see 4 in this dot arrangement. This gives the students the opportunity to explore others' thinking as well as helping them to understand the various strategies. It also emphasizes that there is not always just one way of thinking.

Jan: Look at top, one below, and these 2.

Jan saw a group of four on the left with two dots on the bottom right. When she said, "Look at top" she meant the top three dots using one dot on the left below them making her group of four (see Figure 2). She did not see her four being the same as the domino arrangement of four, instead a row of 3 and one more dot making her 4. Then there were 2 dots on the bottom row left.

Spot-the-Dot #2

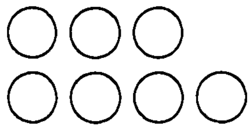

Figure 3 Spot-the-dot

Kathleen then used the arrangement as shown in Figure 3.

Kathleen: Let me give you another one.

Class: (hands go up)

Kathleen: Here is a second look.

Brett: 7, 4 on bottom and 3 on top. I know 3 + 3 = 6 and add one more. (He is relating to prior knowledge.)

Kathleen: "Were you not sure about 3 + 4?

Brett: No, I knew it.

Ron: 2 and 4.

Kathleen: Where do you see 4?

Ron: 3 on top with one by it and saw other 3.

Trying to make sense Kathleen asked, "You saw this three and this four (pointing to the dots)? What happened to the two"? Her question was for him to be able to reflect on what he said and be able to make sense of it himself as well as clarifying his meaning for Kathleen. He saw the same three dots on the top row and three dots on the bottom row, but with one extra dot on the bottom row. So as Kathleen asked the question she pointed to the top row of three and the bottom row of four. She is also modeling that sense making is important. If one does not understand, it is okay (and encouraged) to ask questions. And it is expected to be able to explain one's thinking.

Ron: 3 and 4…

Jan: I looked for 3 and 3 but knew 4.

Miriam: 3 at top, 2 at bottom and 2 would be 7.

Kathleen: What about the way Miriam did it? (no one responded verbally, only a few nods.)

Kathleen only moved to the next image when there was a lull in the conversation, meaning there were no other ideas, the students started repeating their visualizations, or all number combinations had been described.

Spot-the-Dot #3

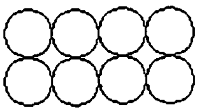

Figure 4 Spot-the-dot

Kathleen then showed another arrangement of dots as seen in Figure 4.

Kathleen: Another one.

Class: 4 and 4 is 8.

Sue: Two 4s is 8, I saw 4, then 5, 6, 7, 8. I knew 8.

Zena: 4 sets of 2 and I knew that was 8.

Kathleen: 4 sets of 2

Nick: I knew because you showed us before.

Nick is referring to a class opener they had done a few days before, "What's my rule"? (Wheatley & Reynolds, 1999). He made a connection between this task and the previous task; made sense of. For this activity the teacher thinks of a rule such as 2n. She asks for students to give her a number. After students take turns giving her a number, she says, "My rule gives you this number." For example, if a student said, "Three" then the rule (using 2n) gives six. The students try to figure out the rule. Kathleen writes on chart paper their numbers followed by an arrow pointing to the number the rule gives, $2 \rightarrow 4$. The chart paper is left hanging on the wall for a few days. On this particular day Kathleen's rule was to double the number. Written on the chart paper with a list of numbers followed by arrows and numbers was $4 \rightarrow 8$. The numbers to the left of the arrows are numbers the students had chosen. Thus, Kathleen had not specifically "showed" him but it had been discussed and written down during a previous class opener, "What's my rule"? Nick reflected back to a previous task and was able to make connections to and meaning of the spot-the-dot activity. Using prior knowledge helps make new constructions and provides opportunities for new levels of understanding to emerge.

Jan: Twos, (she proceeds to count by twos) 2, 4, 6, 8.

Brad: I first saw 4 on bottom and 4 on top.

This opening activity provided an opportunity for students to hear, see, and talk about a few basic math facts, as well as to think about collections of numbers. The students were exposed to several different ways of looking at a number. While using the spot-the-dot activity the students were also given an opportunity to think multiplicatively, four sets of two, and exposed to looking at numbers in groups.

As each spot-the-dot was displayed the students gave several different ways of seeing the dots. The students freely gave their explanations without any prompting from Kathleen. A couple of times she did repeat the explanation or would ask a question for clarification. The students did not repeat any ways of seeing the patterns. Students were expected to listen to ways being shared and share different ways rather than just "taking turns" to say what they saw without reference to what had already been shared. (The idea of what is a mathematically different solution had been previously negotiated and is enacted throughout their mathematical discussions.) Students would respond when there was a quiet space, or when Kathleen pointed to them because their hands were raised. She did not offer right or wrong as a response to the students nor did she reveal her way of seeing the pattern. She did not ask any leading questions trying to help them to get the "right" number. All students were involved in hearing all the solutions, thus, expanding their ways of seeing arrangements of objects and numbers. At the same time the students were also hearing the various addends for certain numbers. By constructing each number in a variety of ways and becoming aware of their resulting associated facts, students experience meaningful learning, which is more beneficial to students than just memorization of facts (Kamii, 2000).

Throughout the activities the students were expected to make sense of what they saw, and to explain their thinking. They did not wait to be called on but felt comfortable enough to express their various thoughts. This type of opening activity helps the students to feel comfortable in openly sharing their ideas, and to feel secure in knowing they will no be facing possible embarrassment of being wrong. This type of activity prepares for and promotes students' mathematical focus and argumentation later on in the whole-class discussion.

As students listen to the discussion they are able to contribute to the discussion by giving a mathematically different solution. While listening to other students' ideas, individual students reflect on their own personal thinking then decide when and what to share. By Kathleen asking the students if anyone saw it a different way, she is encouraging them to mentally compare their thinking to ideas that have already been shared. The comparing of solutions provides opportunities for the students' activity to extend beyond just listening to others' explanations. They must make sense of and identify similarities and differences in the solutions and their own thinking. Listening to and comparing of ideas has the potential to contribute to children's mathematical learning (Yackel & Cobb, 1996). It is important that as students actively listen they become aware. Listening is an important component in the aiding of mathematical discussions because it gives rise to, and opportunities for, students to present mathematically different solutions. Thus, during the later whole-class discussion students listen carefully to make sense of others' solutions and can contribute further to the mathematical discussions and mathematical constructions safe in their environment where ideas are valued by all.

(Episode #3)

Trapezoid Triangle – *Teacher/Facilitator*

Task: Using only the trapezoid blocks: What is the smallest triangle you can make? What is the largest triangle you can make?

Kathleen had asked the students to find the smallest triangle possible and to find the largest triangle possible using trapezoid blocks. (See page 186 in Appendix C.) The students explored this for several days discussing the smallest triangle and the different ways they made the triangle and the patterns they had found. The students worked in groups putting their trapezoid blocks together to try to make the largest trapezoid triangle. After listening to the students' ideas she gave them dot paper so they could draw their trapezoid triangle. The students taped their dot papers together so they could make larger triangles. They were exploring their ideas to find ways of making the largest trapezoid triangle. Also they drew their trapezoid triangles onto paper so the other students could visually see their ideas. The drawings also provided a means for discussing and making sense of the different ideas. On this particular day the students had taped several pieces of dot paper together in order to draw their larger triangles. (See page 187 in Appendix C.) In this way they could see and discuss the triangles and patterns they had found.

This episode is one in which Kathleen initiated and took more of the lead in the discussion. In many of the whole-class discussions, the students do most of the talking and questioning. There is equal student participation without any one specific leader. Kathleen asks key questions at appropriate times to facilitate the discussion. In this whole-class sharing time she was trying to get the students to argue. She also asked particular types of questions which would challenge the students mentally and promote argumentation. But in this instance, the students were quieter than usual and tended to wait for Kathleen to keep asking questions. Thus, on this particular day my observations proved to be most interesting to me, because I was able to focus more than usual on Kathleen's questioning techniques, and on her role of negotiating and modeling argumentation.

The students had previously been working on drawing the trapezoid triangle on large pieces of dot paper. Kathleen now asked them to come sit on the floor in the back of the room to discuss and show their triangles. Karen mentioned that starting with the outline was easy.

Kathleen: How many of you drew the outline first? (Several students raised their hands.) How many of you did not draw the outline first?

Note here that Kathleen began with a question based on one of the students' comments. She does this regularly.

Cort: (Talking very softly explaining that doing the outline first was easier.)

Kathleen: So you thought doing the outline was easy?

By repeating in question form Kathleen was giving Cort the opportunity to elaborate and/or reflect.

Cort: Yeah, but I wanted to do it the hard way.

Kathleen: Okay! Who thought drawing the outline was easier? (She looks around at raised hands.) Tina, why do you think it's easier that way?

Tina: Because then you know when to fill in...see like...(pointing to her trapezoid triangle) well, if you didn't you wouldn't know where to fill in and how big it is.

Kathleen: So how many of you agree with Tina that it helps you to know where to fill things in?

Up to this point in the discussions Kathleen has not asked Karen to explain why she thought starting with the outline was easier. Time is allowed for students to reflect on their own solutions while listening to and making sense of someone else's. Karen and the others also have time to compare the different reasons for starting with the outline, and how that action helped them construct the large trapezoid triangle. By asking how many agree with Tina the students were given another opportunity to respond in sharing their ideas and to emphasize strategies. At the same time the students can compare others' strategies of making the outline to their own strategies.

It is possible for teachers to intervene/question in ways that stimulate and push students' thinking forward while at the same time promoting students' autonomy. Kathleen is good at doing that.

Kathleen: Okay! Was there anyone that found drawing the outline did not help you (looking around)? Why does it not help you Mary?

This question brings to light any contrasting strategies for discussion, and sets up an atmosphere in which students feel that it is okay to have different solutions, and to be able to indulge in open argumentation.

Mary: Because hum…if I draw outline when I draw in there might be problems, there might…they might be too big or they might be too small, some may be wrong.

Kathleen: Does anyone have anything to say to Mary about what she just said? (no comments from students). Okay! Who can tell me about how you are doing yours and what your plans are?

Cort: I have something to say about Mary…never mind.

Karen: It's about Mary.

Kathleen: Everyone! We are looking at Mary's.

Here Kathleen is refocusing the whole class back on Mary's idea.

Karen: Which ones did you start on?

Mary: First we started making the top. Then we started by adding on.

Kathleen: Is this the same pattern you used last week? And you just started adding on? (Noting that Mary and Edith started this last week.) What did you add on to it?

Mary: Markers and papers to make it bigger, and kept adding to make it bigger and bigger, the biggest.

Notice that Mary is responding but she is not focusing on the patterns, just that they need more paper to make it bigger.

Kathleen: Look at Mary's and Edith's. They think theirs will work.

Mary: We have some little triangles like this. (There was no focusing on the pattern).

Kathleen: Look at Mary's and Edith's. They think theirs will be the biggest. What do you think? Will their plan work?

Class: (Some said yes and some said no.)

Kathleen: Why not Tina?

Tina: Cause mine will.

Kathleen: Let me ask you something. Is it possible for yours to work and Edith's to work, also? Or do you think there is one way? What do you think, Janie?

Here is a good example of how the teacher, focusing on continuing the argumentation, provides a new possibility for the students to think about.

Janie: (Gets paper, makes this triangle here, makes the middle, and keeps adding on so that it gets wider and wider.) wider and wider (Using more paper but making the triangle pattern Janie points to the small three trapezoid triangles at the top of her larger trapezoid triangle. She pointed to a triangle in the middle of her trapezoid triangle. Then with her arms stretched over the two sides of the triangle she shows how it is getting wider if she drew the two sides longer.)

Kathleen: How did you do your lines there on the outside?

Janie: Followed the dots.

Kathleen: How did you follow the dots?

Mack: Why are you doing your lines like that?

Kathleen: Look at Lucy's and Karen's. Do you have any suggestions?

Class: It looks like a pumpkin. It is!

Kathleen: Well, they started out with...(students interrupt). Lucy, what would you like to say about yours?

After hearing ideas from the other students, Kathleen has brought the discussion back to the triangle of partners, Karen and Lucy, giving them opportunity to join into the discussion about their drawing. Kathleen had started discussion with Karen's statement (above) and now has brought the focus back on their work.

In this whole-class situation the students were not initiating any discussion, so Kathleen took the lead. She asked questions that would elicit responses, and would provide opportunities for the students to mentally and orally **compare among themselves each other's ideas and strategies.** This is an important part of her classroom argumentation, that is, comparison/contrast. Her questions set the tone for the students to be thinking about their solutions and strategies in relation to others' solutions and strategies. She has also modeled that thinking is important and that it is okay to have differences to discuss. It is important that students learn to communicate mathematically. Kathleen's questions give them opportunities to share their mathematical thinking as well as to make sense of each other's. Her questioning also models for the students what are appropriate questions and how to think, rather than what to think.

Not shown during this discussion is what the students are visibly doing. For example, after Kathleen asked questions the students would go over and look at each other's triangles. They asked questions and compared their own drawings to those of the other students. When Kathleen noticed students becoming interested in another student's work, she infuses the system by asking questions about that person's drawing in order to stimulate the students to respond in interactive discussions (argumentation).

Learning with understanding by all students happens when teachers specifically attend to creating a classroom environment that takes into account the uniqueness of each individual, and attends to critical dimensions of learning characteristics of individuals. This means that each child has the opportunity to engage in and reflect on tasks that are mathematically problematic in a social community where his/her thinking is discussed and valued. This is what Kathleen has in mind in the above scenario when she is trying to bring all the students into a discussion/argument.

(Episode #4)

Balance Task – *Minority and Mutual Acceptance*

Task:

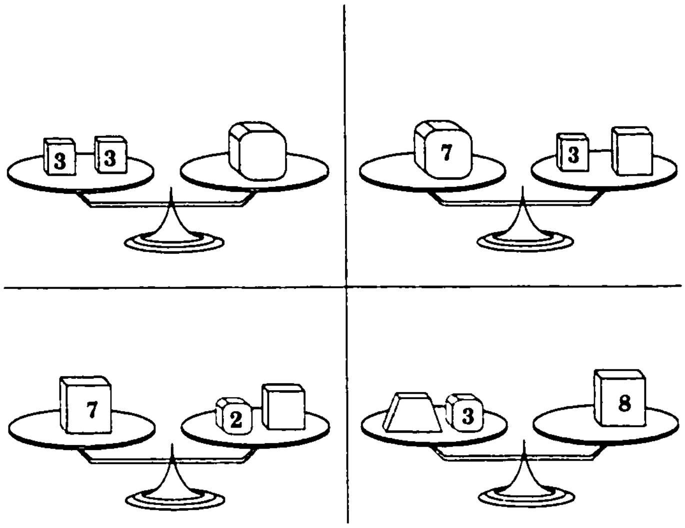

Figure 5 Balance task

Having listened to the students during the small-group interactions and during the whole-class sharing time, Kathleen was able to select further activities to enhance their mathematical thinking and sense making. Rather than finding a different task for each student she always chose one for the entire class, one that all the students could complete in a variety of ways, depending on each one's level of sophistication. As this was the first day she had used the balance problem she displayed the task on the overhead projector and asked the students what they knew about balances, being careful not to impose particular analysis methods on them.

The children's discussion at the beginning of the task centered on the meaning of the word "balance" with the students noticing that each balance was level. Kathleen explained that their goal for the day was to find numbers that would make each balance level, that is, one side is equal to the other. Then she asked if there were any questions. There were none, so the students went to work finding solutions for the balance. First, they negotiated what was intended in the assigned task. As noted by several students the focus was to "find numbers that make it balance," rather than focusing on establishing procedures to use in finding these numbers. When asked about this later Kathleen commented that she attempted to take herself out of that position in which students look to her for knowing how to do the task and as the authority who has the knowledge to give.

Unlike the traditional lecture-oriented classroom where students wait for the teacher's answers or explanations, the whole-class discussion provided the teacher and students opportunities to listen to each other's explanations and then to make interactive responses. Kathleen encouraged the exchange of ideas among all the students. As in other settings she expected them to explain and justify their solutions during this time while maintaining respect for each other's ideas. For instance, she asked questions such as, "Does anyone else agree with this idea? Why or why not? Did anyone solve it in a different way? Does anyone have a question or comment about this"? She expected the students to listen to one another, make sense of the ideas, and ask questions of those who were presenting their solutions. Thus, Kathleen modeled appropriate types of inquiries that the students could ask one another during the whole-class discussion, thereby deepening their mathematical thinking. Throughout the interactions the primary focus is always upon the construction of mathematics.

The balance task is one activity that helps the students to deepen their mathematical thinking by allowing them to give meaning to their experiences, instead of following set procedures (Wheatley & Reynolds, 1999). The balance tasks help students conceptualize addition and subtraction. While students are playing they experience attempts at trying to keep themselves balanced, walking on a curb, standing on one leg, etc. By presenting activities in a balance format students can easily give meaning to these activities. The balance tasks must be interpreted and given meaning. And because they do not lend themselves to mechanical responses, they encourage meaning making (Wheatley & Reynolds, 1999). The balance task does not contain operation instructions, so the students must decide how to act, what operation to perform, and how to think about the task.

The students had been given a sheet with four balance tasks on it (see Figure 4). The first three tasks that were discussed did not bring about argumentation, but just the sharing of solutions and justifications. But when the students began to discuss the last balance assignment on their sheet, argumentation emerged.

In that task this particular balance contained a large square on the left pan and a small and a medium square in the right pan, illustrated as (7 / 2, _) for this paper.

Ron: It's 9…7 + 2 = 9.

Nick: It goes over there (pointing that the 2 goes in the right pan).

Class: (shouting) It's 5! It's 5!

In the meantime Brett and Nick draw an equal sign over the fulcrum between the seven and two. Neither student explained the equal sign.

Zena/Grover: It is 5….see (showing fingers) 2 + 5 = 7.

The students used finger patterns to try to explain this solution to Ron so he could make sense of it.

Miriam and Jan also showed 2 + 5 = 7 by drawing tally marks on the board.

Miriam: You know 5. Five is one more than 6, and 2 is 7.

Ron/Max: Maybe it is minus. (They write 9 – 2 = 7 on the board.)

Nick: It's the same, 9 – 2 = 7, it's just backwards. (Nick compares their original solution to this one.)

Zena/Grover: It's 7 ...see (They move closer to Ron and show him their fingers.) 2 + 7 = 9.

Ron: It is 9! (He stated matter-of-factly and confidently.)

All the solutions were written on the board. The students were trying to help Ron to understand that five needed to go in the blank square. It seems as though they realized that the number in the big square on the left had to equal the two squares on the right. Ron seemed to disregard the position of the fulcrum and used the numbers he saw, 7 and 2. The other students knew that the one square on the left of the fulcrum must be equal to the two squares on the right. Ron saw 7 and 2 equals something while the other students saw 2 plus something equals 7 (even though the numbers were in reversed order). Ron perceived the two numbers as "countable items" (Steffe, 1983). That is Ron saw 7 and 2 and physically counted them. He used tally marks to justify his solution. Throughout the argumentation process the students did not argue against Ron personally. They were discussing ideas and strategies. Ron (sitting in his chair) at one point crossed his arms, stretched out his legs, crossed them and said, "It is 9!" He did not seem to mind that others disagreed with his idea. The other students did not put him down for having a different answer. His whole body language showed that he knew he was right and that it did not matter that others disagreed. Obviously Ron has shown confidence and did not feel threatened.

Ron was the only African American in the class. But at no time did he seem to be uncomfortable, and the other students did not treat him differently. They all interacted and put their ideas out in the open for discussion. Each comment was focused on mathematics, and not on personal attacks, nor on disrespectful remarks. In this environment all students are on equal ground as they try to make sense of the activities. "Equal ground" does not mean equal as in having identical knowledge or understanding, but all students have the same regard and respect for each other as they work together.

As illustrated in this episode not all students are at the same level of understanding. According to Steffe (1983) there are five major periods in the construction of counting. They are perceptual, figural, motoric, verbal, and abstract unit items. These periods involving counting indicate changes within children. "The range extends from needing overt physical action to the capacity to count "abstract unit items" according to Steffe (1983). At this level students are able to intentionally make records of their counting acts and can construct part-whole relationships. One possible explanation for Ron's answer may be that he is a counter of perceptual items. He was not one of my key informants so I do not have any further information about his thinking. Because it seems that he is in this level of construction, he is not in a position to make sense of the other students' reasoning and explanations. This was not a problematic task for him at the time and the students were unable to problematize it for him.

The students did not say anything about the position of the blocks on the balance. Nor did they try to reason that the fulcrum was where the equal sign would go. They did not use their balance to help justify their answers, but used only their fingers, drawings, or reasoning alone. At this point the students were not able to understand from a different perspective and explain so there would be understanding. Interpreting from their perspective does help build confidence in themselves and their ideas. The confidence gained enables the students to be able to openly share ideas and explore different ideas. Obviously, Ron has shown confidence and did not feel threatened.

This whole-class discussion ended abruptly because the students had to go to special events such as art or music. While they were standing in line some of them continued to discuss the task, in contrast with traditional math classes—when the bell rings for dismissal, students usually stop discussing math. Kathleen has commented that she knows she has had a good mathematics class when the students are still arguing about it on the way to their special class.

Kathleen's students accept responsibility for discussing and making sense of the ideas themselves, thus, she does not have to take the traditional role of leading the discussion. The students are the ones asking the most questions and explaining as they see fit. They accept their responsibility for making sense of their world. She does not need to call on them or to interrupt their discussion. Accepting this responsibility provides them the opportunities to gain confidence, which is an essential aspect of argumentation.

(See Appendix D for the mathematical thought involved in the Balance Task, p.205).

(Episode #5)

Red/Yellow Flowers – *Open Interaction*

The task: There are 8 red flowers and 4 yellow flowers. How many more red flowers are there than yellow flowers? (In the following figures the red flowers are shaded.)

This particular task was given within the first four weeks of school when the students and Kathleen are normally still negotiating what it means to do mathematics in the class, which can be different from their first grade experiences. Kathleen read the math task for the day and asked the students to think about it by themselves first and then get with their partner to see if they can agree upon a solution. Students can write it out, draw a picture, or use whatever materials lying around the classroom they need to solve the problem.

Students began to work on their assignment. Most of them responded by picking up the paper with the task written on it and going to their desks to work. Dana worked on the board. Nick asked Kathleen for help, and she has him read the problem to her, then asks him what he thinks, thus reinforcing his responsibility for making sense of the task.

Brett:	(Finishes his problem and wants to explain it into the camera. He explains that 4 + 4 = 8 and 8 – 4 = 4. So his answer is 4.)
Researcher:	What do you mean 4?
Brett:	4 more red flowers than yellow flowers.
Researcher:	(Notices that Jan has finished, then moves the camera to Jan's paper where she has written 4 + 4 = 8, then 4.)
Jan:	There are 8 red flowers and 4 yellow flowers, and I know 4 + 4 = 8, so that makes 2, I mean 4.
Researcher:	4 what?
Jan:	4 more flowers.

Researcher: 4 more flowers?

Jan: Red. (Jan originally had 12 on her paper, but changed it to 4.)

The atmosphere in this classroom is non-threatening and students know sense-making is important. Because they feel comfortable and cared for students are willing to share their strategies and solutions. They did not ask if they had the right answer or for help, but shared their thinking.

Kathleen: (Working with Sue and noticing surrounding answers.) Some people wrote down 7. Why did you write 7? (Asks several girls what they wrote down.)

Miriam: Some have 12, some have 8, some have 7, and some have 4.

Kathleen stimulates thinking and open argumentation among students as they complete the task in their pairs by asking them certain questions and by stating some observations, rather than by telling them the correct answer. She is thus preparing the scene for the whole-class discussion, highlighting the different answers that students have, and challenging them to think about their reasons, to be prepared to justify their answers. Kathleen then calls the class to sit on the floor at the front of the room close to the white board and to be ready to discuss their answers. Even though some students are at the white board and some are still at their desks, she instructs that those not finished can continue working at their desks, and that all others are to come sit on the floor. This is in stark contrast to traditional classrooms where all the students would be called together, finished or not. In order for students to make sense of the mathematics, they are given time to work on their thoughts. The students at their desks continue working while listening to the discussion-taking place. As they finished, the students joined the whole-class discussion. In fact, Zena was one at her desk but quickly joins in the discussion, as shown in the following dialogue.

Some students state that the answer is 7 while others say that it is 12, and others say that it is 4.

Kathleen: Is it possible to have all these answers and for all of them to be correct, or is there just one answer?

This is an example of the teacher asking important key questions encouraging students to think about, and talk about, how they got their answers. She also challenges them to think about whether it is reasonable and when it is reasonable to have several answers for a task.

Dana: Writes on the white board, "8 + 4 = 4."

As she writes she is verbally saying 8 minus 4 equals 4. She does not realize that what she wrote (+) is different from what she is saying.

Zena: (to Dana) It is take away.

Dana is normally a very quiet child and does not usually share except when she has been called on. She has begun to feel more comfortable in this environment and more willing to take verbal risks. As Zena is thinking about the math, she is focusing on making sense of the explanations of the other students. At this point of interaction Dana becomes the audience for the other students as they try to help her understand the conflict between what she wrote and what she said.

Dana still has not realized that there is something wrong. She looks at her paper and does not change her equation. She apparently has not listened to Zena. Apparently, this is a case of "**talking past**" one another. Dana isn't listening and Zena isn't explaining in a way that makes sense to Dana. In the meantime most of the students are sitting on the floor with their hands raised to speak. Sue tries to explain that what Dana has on the board should be subtraction, because the answer "goes down."

Zena: (Re-phrasing so Dana can make sense of Zena's earlier statement.) There is a plus sign.

Dana: "8 take away 4 is 4". (She points to the math sentence $8 + 4 = 4$ on the board).

Ron: (Understands what the conflict is about and tries to help Dana understand what Zena is telling Dana.) "You still have plus."

Dana finally reflecting on and making sense of the other comments, notices she has a plus sign and changes the sign to minus.

Here each student listens to the argumentation and joins in when necessary to further the sense-making. During all this argumentation Kathleen is listening and allowing the students to interactively participate. She does not say anything as long as the students are continuing the discussion about the task. This is a good example of the teacher acting out the role of facilitator, which promotes greater open argumentation among the students. This also shows the students accepting responsibility for their learning as well as a community of learners helping one another.

The conversation continues with Jay saying that he does not agree that the answer is 4. Sue walks over to Ron and Jay and tries to explain to them that the answer is 4. She counts on her fingers in an effort to try to explain her thinking and also in an attempt to help them understand why the answer is 4.

Then all of a sudden all the students are talking at the same time giving their answers and reasons for them. It becomes difficult to make out which specific students are saying what. So Kathleen steps in and says, "We need to be respectful of one another and take turns." The class then quiets down. Note that the teacher simply reminds the students of one of the rules (social norms) they have previously agreed to.

Miriam: (Reads the problem again.)

Jan: I thought 8 + 4 = 12. (This is NOT the answer she had on her paper earlier.) I think it is adding not take away.

Kathleen keeps working as facilitator promoting continued open argumentation among the students. Sometimes she asks the students why they think the way they do, and sometimes she repeats their thoughts or strategies for reflecting.

Miriam: It is take away.

Students: No it says "more" so it is not take away.

Kathleen: Can you prove to them that it is take away?

By asking her this, Kathleen is encouraging Miriam to re-think her strategy, and to be able to openly explain the process to the other students. Kathleen is also encouraging Miriam, to accept responsibility as a member of the community to help others understand. Kathleen is asking Miriam to explain her reasoning, thus giving a different perspective and opening the floor for more discussion and the comparing of solutions. Kathleen continues to ask other students, also, for their thoughts, and then for them to be able to explain their answers and strategies. As a result, the whole class now responds, each student trying to add his/her reasoning of why it is take away. In an ongoing effort to maintain orderly discussion, sometimes Kathleen refocuses the whole group by calling on one of the students to give his/her explanation for his/her answer to the problem. Refocusing makes it possible for all students to hear and make sense of individual strategies. Refocusing also gives students opportunity to compare the strategies presented. Thus, she again uses a technique that she uses often to bring the class back to orderly argumentation.

Gay Arla: It is like a graph. (She drew one column of 8 circles then next to it a column of 4 circles, and she drew a line where the 2 columns were even to show that there were 4 circles left in the first column below the line.) So the answer is 4.

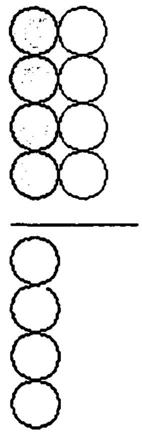

Figure 6 Gay Arla's drawing

Kathleen: Jay, does that make sense to you?

Kathleen focuses on one particular student who disagreed. Notice she asked him if it made sense. As students (and teacher) listen, trying to make sense of a statement, they must first construct what they think the other person is trying to say.

Jay: It means plus.

Ron: It means plus.

Some students in the group are now saying that sometimes the word "more" means take away, and that sometimes it means plus. Other students said that the word "more" meant you had to add. Brad stated, "My teacher last year told me that if you saw the word more, it means you add." They remembered that their teacher from last year had told them that the word more meant to add. Kathleen is focused on the students' understanding and making sense of the problem themselves, not on trying to have them follow pre-given definitions or procedures. This is an example illustrating that language (or giving a specific word) does not transmit an appropriate meaning, but leaves the students confused. In these types of situations, (the previous first grade class/traditional classroom), children learn by routine the appropriate word or procedure to use but, their mathematical constructions are left unsupported (Cobb, 1995). At times students must re-think their explanations so they can communicate their ideas effectively. Discussing their explanations and justifications stimulate the construction of logic as they try to communicate their ideas consistently and coherently (Kamii, 2000).

At this point note that Jay and Ron are not listening to, nor are they considering what Gay Arla had said. They are not referencing Gay Arla's drawing. They are sticking with their own ideas. In successful argumentation listening to others, even when ideas are different, is an important aspect in promoting student mathematical learning and making sense.

Ron: 8 + 4 = 12. You start with 8 and add up 4 and that equals12.

Here Ron's justification is to use counting on as a strategy. He is not really attending to the fact that others are arguing that this is a "take away" problem, not add to.

From her observations Kathleen now states that the class is strongly divided into two sides, the take away side, and the plus side. Note that throughout whole-class discussions she generally makes a point to call on students who have not spoken up much. So now she asks some of them what they think, and what side they are on. By doing this she draws more students into active discussion, which helps them to continue to pay attention. This also gives her insight into the students' thinking. She believes that when all students are drawn into active argumentation mathematical learning is maximized for all students.

Brett: I'm on the take away side. (He drew 4 yellow flowers in a row and then 8 red flowers in 2 rows of 4.) 8 of these...how much more red than yellow? So it means 4 more red than yellow (as he points to the extra row of red).

Figure 7 below shows Brett's drawing.

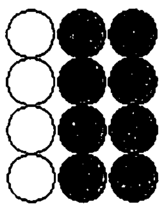

Figure 7 Brett's drawing

Brad: I think 12. More means add.

Note that he did not do the same thing as Gay Arla, but gave a different justification aiding in sense making, as another way to support subtraction. Others did not make sense of the first drawing so he drew something different and re-explained his solution. (Also, the class has already negotiated what counts as mathematically different.) In interactive argumentation all students are exposed to many different views as they learn how to think mathematically. Sometimes the correct answer comes to mind after a long period of time spent in reflecting on and processing mathematical constructions.

Meanwhile Kathleen continued to call on students who have not participated much asking what they think. The students continue to discuss among themselves the various strategies shown, and they continue to offer comments once in awhile. By calling on reluctant students Kathleen is letting them know that the sharing of their thinking is valued and benefits the whole-class discussion and argumentation.

Brett: $4 + 4 = 8 - 4$. This is one row, this is one row, and this row is left. (Using his drawing (Figure 7) he is re-examining and explaining a little differently in an effort to try to help others understand his thinking.)

Brett has obviously been listening and interacting for the benefit of all students. He is explaining that one row of yellow flowers matches one row of red flowers and that there is still one row of red flowers left. Thus the 4 (yellow row) + 4 (red row) equals 8 but there are still 4 red flowers left without a matching row.

Kathleen: Do these strategies make sense to anyone else?

The above question is an example of how Kathleen continues to facilitate argumentation for maximum learning with understanding.

Nick: (Tries to explain using, What's My Rule? 'Wheatley & Reynolds', 1999.) He writes: $8 \Rightarrow 4$ and $4 \Rightarrow 8$.) (He asks the class,) "If you give me 8, my rule gives you 4 and if you give me 4 my rule gives you 8. Which one has the better advantage"? (No one responded to his strategy.) The answer is 4.

"What's My Rule"? is a class opener Kathleen uses. The teacher thinks of a rule and asks students to give her a number. A student will call out a number then the teacher tells the students the number given by that rule. The students continue giving numbers until they think they know the rule. Nick is trying to help the students realize that the answer is 4, not 12, by giving yet, another different explanation. This was not related (from the students' perspective) to what the students were doing or thinking. Thus, they did not respond but continued the open discussion on the meaning of "more." Notice that several students, as part of their own listening, reconstruct a different explanation to try to help other students understand the solutions presented. The students themselves try to understand the other students' perspectives, re-think their solution, compare them, and then give a different solution that might be understood. When students realize that their interpretation might not be useful, they seek arguments that will be accepted (understood) by others. As members of this community they all attempt to help one another (and themselves) make sense of the mathematics as well as the language they use

Ron: (Wants to justify his own answer by doing tally marks. He draws 8 marks then 4 more marks to show that 8 + 4 = 12. See Figure 8 below.) It doesn't say take away.

|||||||| ||||

Figure 8 Tally marks

Kathleen: (Repeats his comment.) It doesn't say take away?

A student: It doesn't say plus.

At this point the students seem to have reached an impasse. And when one student asks if she could use the white board, Kathleen takes the opportunity to end the whole-class discussion by saying "it is a good idea for everyone to get with their partner and use the white board to try to come up with a way in which they can explain their strategy to the class; a way that makes sense to each one." Thus, the whole class discussion stops and the students return to working with their partners to try to sort out their thinking based on the conflict that has emerged. She wants all students to think about their own reasoning and thought processes, and to be able to explain them to the class so the rest of the class will understand. She is giving them time to reflect.

Jan comes back to the camera and shows her white board drawings (see Figure 9). She has a column of eight circles next to a column of four circles and draws a line under the four circles showing that there are four circles left under the line, which show the answer is four. She has changed her answer again. Figure 9 below shows Jan's white board drawing.

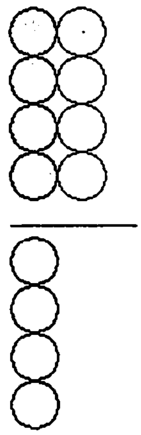

Figure 9 Jan's drawing

Kathleen: Jan. Does this make sense? (Jan nods.)

The discussion has provided an opportunity for Jan to think about these ideas more deeply and reflect on an interpretation that makes sense to her in light of others' ideas.

Kathleen walks around listening and talking with each group of two students. While in one group she will ask, "Does any other group's strategy make sense to you?" Thus, she is encouraging more discussion and comparison of solutions. She is continually urging groups to observe each other, thereby giving to each other broad exposure to the many ways of thinking. In this way she encourages all students to ultimately make sense of the task.

Note that Kathleen very seldom interjects her own thoughts, nor does she have to ask students to explain their ideas, because students do that automatically and voluntarily. In fact during this task Kathleen was not given much opportunity to talk or ask questions. The students were continually responding to each other without Kathleen calling on them. They took and accepted the responsibility for learning. At one point during all of the interaction one student commented, "Mrs.____, you like for us to argue." That action of making sense has already been negotiated between students and teacher and set up as a classroom norm to abide by. In the first grade students were used to verbalizing their answers and strategies, but not necessarily making sense of anyone else's ideas. Therefore, in response to the students explaining their ideas, she would ask whether or not those ideas make sense? Kathleen continues to help students focus on making sense of their experiences and at the same time add more depth to the discussion. Throughout the argumentation she would also ask students to compare and discuss their strategies with one another.

(Episode #6)

Red/Green Suckers - *Incubation*

Task: Brian had 9 green suckers and 5 red suckers. How many more green suckers than red suckers did he have?

Previously in the year the students were given a "flower" task involving eight red flowers and four yellow flowers (see Episode #5). The students were to find out how many more red flowers there were than yellow flowers. The class argued whether the answer was four or twelve. Several students commented that they had been told from their first grade teacher that the word "more" meant to add. So half of the class thought the answer was twelve because they focused on the word more in a procedural way rather than on making sense of the task. Throughout the discussion many shared their strategies, but the class remained divided half and half. When Kathleen realized the students were at an impasse, she asked them to get their white boards and reflect on their solutions and strategies. She asked them to think of ways that would help students with different answers understand their own answers. She asked questions, which would encourage the students to make mental comparisons of others' strategies and ideas, as well as to re-think their own ideas. As teacher/facilitator of mathematical classroom learning she did not ask questions, which would interfere with open argumentation, but promoted maximum argumentation because she knew that maximized argumentation relates to maximized learning.

For five months following this "flower" task, the teacher gave the class various tasks that generated reflection, thereby helping them to focus on the ideas, which would help them think about the mathematics involved in this task. Now, five months later, she gives them a task very similar to the flower task as follows.

Kathleen: Look at the first problem.

Miriam: (Reads the first problem orally, then starts drawing on the board.)

Gay Arla: (Drawing on the board and draws 9 suckers.)

Kathleen: Okay! Are you showing how you got your answer?

Miriam: (Points to the 4 un-circled suckers on the board, but does not verbalize an answer.) See Figure 10.

Kathleen: How many of you got 4? (Most all of the students raised their hands.)

Miriam: We also got 4.

Kathleen: Are you explaining how you got 4?

Miriam: I put there are 5 red and if you add 4 more that is 9. There are 4 more. That means there are 4 more green suckers than red suckers.

Gay Arla: (Has been drawing her 9 suckers on the board during the interaction. She now points to her 9 suckers and circles 5 of them.) See Figure 10.

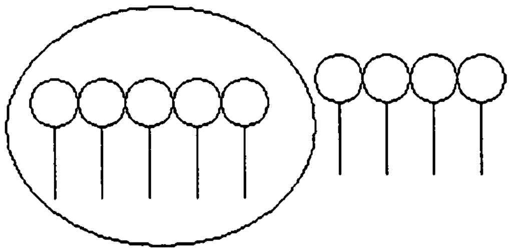

Figure 10 Red/Green Suckers

Miriam: (Underlines the 4 left.)

Gay Arla: There are 4 left over.

Kathleen: Does anyone have something you need to say about this? (no one had anything) Does anyone agree with Miriam and Gay Arla?

Class: (All together) Yes!

This episode is the first time since September that Kathleen uses the word "more" in a mathematical problem as it was used in the flower problem. In contrast to five months earlier there is now no controversy about the word more, or about the answer. The students are able to understand what the task is asking of them, and they are able to complete the mathematics. Specifically, the students are no longer hung up on the word "more", but are making sense of the mathematics involved. Normally at the end of the day Kathleen reflects on that day's mathematics, developing tasks that would help further the students thinking. Then the next day or so the students would do a related task. Through the discussion Kathleen knew what to plan next. However, after the flowers task for various reasons, Kathleen did not revisit this for five months. Although they completed balance tasks and did various related tasks, "more" was not specifically addressed until now.

Kathleen: Grover, you had a different way. Could you show us? That would be interesting.

Grover: (Draws a hand with 5 fingers + a hand with 4 fingers = 9.) I know 5 + 4 = 9. If you take away (crosses off the 5 hand) 5 that is 4.

Kathleen: Any other comments? (no comments)

During small-group collaboration it is Kathleen's style to walk around and interact with each group. By doing this she becomes aware of the various strategies the students are using. The above red/green task interaction illustrates how Kathleen stays familiar with their strategies by asking Grover to share his method. During the whole-class discussions she is prepared to call on students, who are not prone to volunteer, to share their strategies with the class. Her motive is to give the whole class greater exposure to the various differing strategies and methods, which in turn will bring about greater argumentation for learning and sense making among all students. Throughout the whole-class discussion she gives students time to make comments about any of the various ideas. If they do not understand a strategy (even if they have the same answer) they can ask questions.

In contrast to the above, in traditional classrooms the primary concept of learning is that the teacher must first show the students how to work the problem (correct procedures) by working an example. It is assumed that students do not have the means of solving the problems themselves. The succession of events in the traditional classroom is as follows. First, the teacher gives instructional directions. Second, the students work the problems to get correct answers. Third, the teacher grades the students' answers. This kind of teacher-student-teacher pattern restricts students' mathematical activity to that of responding to the teacher's questions and expectations. This traditional classroom environment in which the teacher is the director and controller of student responses does not encourage argumentation. Therefore, interactive open communication needed for the negotiation of meanings cannot take place because opportunities for students to interactively engage in mathematical activity are not created. In this traditional setting the teacher asks evaluative questions to quickly know whether, or not, the students are understanding what s/he had intended. The teacher's focus is in listening for correct answers. If a student gives the correct answer s/he assumes that mutual understanding exists. If there is an incorrect response, the teacher attempts to reduce the ambiguity by correcting the student's thinking. If a student cannot parrot back the answers, the teacher proceeds to explain the "right" answer and, many times, the "right" procedure. Then the teacher often assigns homework practice problems expecting the student to catch on. In doing so, the teacher provides little opportunity for students to engage in mathematical activity (Wood, Cobb, & Yackel, 1993). This traditional view represents primarily one-way communication from teacher to student. In this setting, the nature of mathematics is not open to much student questioning. Instead, the goal is for students to accept and understand the mathematics, which the teacher already knows and deems important.

Sometimes even in nontraditional classrooms teachers may tend to feel that it is their job to "correct' a student's response or to "clear-up" any confusion. Sometimes students develop incorrect or incomplete constructions that are only temporary departures in the process of forming productive conceptualizations. On other occasions students may make constructions, which threaten to stymie their mathematical development. Teachers do need to become aware of this dilemma and to be sensitive to know when a student's construction might be either a "sidetrack" (departure from) or a "bridge" (connector to) the next idea (Webster, 1974). A sidetrack could restrict students' thinking, thereby preventing them from moving beyond their current mathematical thinking. While it is important to allow students opportunities for making errors in order for mathematical learning to occur, a danger exists when an incorrect idea emerges. Thus, sometimes a teacher's dilemma is in trying to maintain a balance between the "sidetrack" and the "bridge."

Along these same lines of thought in the previous "flower" episode the class was divided in half at the end of the discussion. Kathleen gave them time to work with their partners and to reflect on their thinking. She did not come back to this task until the "sucker" task, which was months later. But before the "sucker" task, Brad (and several other key informants) was interviewed. When asked about the flowers problem, Brad responded that the answer was four. Previously, he had adamantly argued that the answer was twelve. He now knew the answer was four and gave a reasonable explanation for his solution.

The "red/green suckers" and the "flower" episodes challenge the traditional view that teachers need to "correct" students' work. Thus, it might not always be advantageous to give answers to tasks immediately. The tendency is for most teachers to do evaluative listening for a pre-determined purpose of checking for right or wrong answers (Davis, 1999). They listen with the intent to devise more problems for the students to practice on in order to "correct" students.

In the interim between the two episodes from September to February Kathleen continued to provide complex tasks which gave students opportunities to grapple and deal with mathematical ideas and meanings, rather than having them focus on specific words or specific procedures. Thus, "more" was no longer an issue because the focus continued to be on meanings, which the students were in the process of constructing. The new aspects of this task that are understood are not found in the solutions, but lie in reflective thinking (Russell, 1999).

(Episode #7)

Valentine Task – *Sense Making*

The Task (Task #2, see page 188-190 in Appendix C): **Dawn made 23 valentines using purple, pink, and red colors. She made two more pink valentines than red valentines. She made three less purple valentines than red valentines. How many purple valentines did she make? How many pink valentines did she make? How many red valentines did she make?**

On this particular day Kathleen had given the students a paper containing three tasks but I will only focus on the second of the tasks. Because the priority is to understand and make sense of mathematics, students are not given worksheets filled with practice problems. This allows time for discussion of tasks rather than time spent in checking the correctness of lots of answers. In these types of tasks all students are given time to make sense of important mathematics through opportunities for open and interactive discussions. The goal is not for students to complete a certain number of problems but to learn with understanding (Wheatley & Reynolds, 1999). During-small group work, while Kathleen was walking around interacting with the collaborative pairs, Miriam commented that the second problem (involving valentines) was hard. Kathleen asked if she had found one that challenged her?

During the subsequent whole-class episode there were several intercom interruptions because it was picture-taking day. Groups of students were continually going out and coming in when it was their turn for pictures. Therefore, there was a tremendous amount of movement and noise. However, regardless of the interruptions the whole-class discussion continued. Each time a student came back in s/he picked up and continued immediately with the conversation. Kathleen did find it necessary to tell a couple of them which problem the class was working on, and she very quickly and briefly summarized what solutions had been written on the board.

Tina: (Putting her answers on the board she writes: purple-3 pink- 20 red- 18.)

Notice Tina does have two more pink valentines than red. But she only has three purple even though it was stated in the task that there were three fewer purple valentines than red valentines.

Brett: (Moves up beside Tina to draw, to explain, and to question some of her work.) I think that's 41. 20 + 18 = 38 + 3 = 41. (Brett continues drawing on the board so he can explain his solution: purple- 5 pink - 10 red- 8.)

He understands that the total has to be 23 but Tina does not appear to have coordinated that aspect of the task in her solution. After adding her numbers, he uses Tina's three addends from her solution to begin his explanation of why he disagrees with her answer. In this way he is trying to understand her perspective and re-explain "in" her perspective. Both are listening but are at an impasse because they are not at the same level of mathematical constructions. Their listening in this situation, though, is in stark contrast to what we see later when Miriam just does not listen but stubbornly sticks to her own solution in the Loop, Whorl, Arch task (Episode #8).

Earlier in the Flowers task (Episode #5) the students were also listening, but had also reached an impasse because they were at different levels of mathematical constructions and understandings. However, later during the Sucker task (Episode #6,) the students were not only listening but were also at the same level of mathematical constructions. Thus, there were no conflicts.

Tina: I don't see how you get that...that all adds up to 23. (She points to his solution and then looks at him.) (Note that 23 is the total number of valentines.)

Brett: (Looks at Tina, but doesn't respond, and keeps looking at her.)

Kathleen: Did you hear her question? How would you answer her?

Kathleen makes sure Brett knows that the question is directed to him. But she does not step in as the authority when students are discussing their ideas. She leaves it up to the students to make sense of what is going on. She provides opportunities for them to accept responsibility for their own learning, as well as becoming autonomous learners.

Brett: How could you not plus anything and it adds up to 23? When it said 2 left...that made 2 more pink than red. 2 more pink than red. 8 to 10 is 2. Purple is three less but must add to other numbers to equal 23 (using fingers for Tina to see why he used 5 and 8).

Tina: But you put 5 more.

On the white board is Purple-5, Pink-10, Red-8. Tina is possibly comparing only the 5 and 10 and does not see 5 and 8. The task asked them to find three less purple than red.

Brett: (Stops and quietly re-reads the paper and then looks at the two solutions on the board; his and Tina's.)

Tina: There is 5, and 10.

Brett: 8 (Pointing to red). (Using fingers he points and counts to show pink.) 9, 10 (showing two fingers).

Tina: See how on the paper....

Brett: (Looks at paper while listening to Tina read and then he re-explains.) Purple is three less so that's 5 and 10 and 8 is 18...19, 20, 21, 22, 23 is 5 (verbally counting on his fingers showing that 18 + 5 gives 23.)

Note that Brett comes back to the paper to help Tina refocus. In order for them to make sense of the task, they must re-focus on the mathematics and the language. They are not arguing at each other personally but are trying to understand differences in their solutions, and what the task is really asking them to do. At no time does he put her down nor does she get mad and give up. Neither one demonstrates a disrespectful attitude toward the other. The hesitancy in their responses signifies (along with body language) that they are perplexed, yet they continue trying to make sense of the mathematics and each other's thinking.

Miriam: I think it's 9, 10, 4. (She again interjects, but does not explain). I don't agree. I think it is 9, 10, 4. I think it is 9, 10, 4.

Miriam wants to put her solution up on the board. She does not make comments about anyone else's work or explanations. She just wants her solution written on the board. She keeps talking out while standing close to the board. Meanwhile Brett and Tina are still trying to make sense of the problem with each other. They focus on each other, ignoring all the interruptions of the intercom, students coming and going, and irrelevant statements. Brett and Tina are prime examples of active listening. They are fully engrossed in solving this task. Their body language and verbalizations indicate that they are not aware of everything else going on around them. They are fully attending to one another's ideas.

Brett: (Questions her about 41 and tells her that if she pluses, then it is 41. I could not clearly make out the exact words Brett used to explain his thinking, but this is the jest of what he said.)

Tina: It says nothing on the paper about plus up.

Brett: (Tries to give her an example of why he thinks it is plus up. Again I could not clearly make out his exact words.)

It does not appear that Tina has made sense of the question asked in the problem nor does she seem to understand the relationship of more, less, and the total being 23. She has tried to use different aspects of the task individually but is not able to coordinate their various relationships. Brett has made sense of the task and is trying to help Tina understand the task. He uses her solution in his explanation and then tries using his when she does not understand. An impasse exists when students cannot explain their reasoning or when they are arguing past one another. Here one student understands and is trying to help the other student, but neither has been able to communicate in the language of the other to get them past this conflict. Both were listening and questioning with a true willingness of trying to understand. Throughout the argumentation, both Tina and Brett displayed an interest in trying to understand and solve the task. Is this task and the flower task possibly challenging the traditional notion that mathematics can be resolved in a one 50-minute period? Is it necessary to give students time to process information?

After this conversation it was time for the students to go to their special classes; art, music, etc. Another student, Jan, walked over to Tina and tried to help her understand the task. She showed Tina her paper with her solution and tried to share her reasoning. Throughout all the previous commotion Jan was "paying attention" and making sense of the argument and the mathematics. We know this by her action of trying to help Tina. Even though the room was full of noise and continuous movement, students could "pay attention."

Throughout the year there were several occasions when students would try to help other students, not in the sense of showing them a procedure or how to get the right answer, but by helping them to make sense of the mathematics involved in the task. In this learning community making sense of the mathematics is important and all the students share that responsibility equally. Students in this learning environment are in the habit of analyzing, rethinking, facing one another's questions, and learning from erroneous thinking. These are all vital to constructive and interactive argumentation.

(Episode #8)

Loop, Whorl, Arch – *Listening for Understanding*

Task [rephrased for clarity]: **Mrs. Stolt's class of eighteen students decided to figure out which thumbprint pattern each student had. From research the class discovered that there exists three basic fingerprint patterns: the loop, the whorl, and the arch. After all students' thumbprints were recorded, they discovered that there are two more loop thumbprints than whorl thumbprints. There were five less arch thumbprints than whorl thumbprints. How many loop thumbprints were there? How many whorl thumbprints were there? How many arch thumbprints were there?**

This scenario contrasts the stance of two people's interaction patterns during discourse. The contrast between the two students shows what happens when one person talks past another person verses when a person is truly engaged in discourse and arguing. Even though I focus on two specific students' dialogue I do not mean to imply that the other students were not involved. All the students were involved.

Brett: (Wrote on the board 9, 7, 2.) 9 + 7 = 16, 9 and 7 are like our class then add 2 more. I knew because on paper it said 3 less ...and 5 more than whorl is 2.

Brett related the addends to the number of students in the class.

Jan: (Was putting her problem on the board.)

Miriam: (Said that she was helping Jan with it so they were drawing 18 tally marks and counting each one individually and writing the number under the mark. They wrote on the board 8 loops, 4 whorls, and 6 arches.)

Kathleen: Boys and girls, Miriam, came up with the same answer as Jan, so let's listen to Miriam.

Miriam: (interrupting Kathleen) I'll count them all first. Kind of well... there are 18...6 arches because 2 more loops than arches so 6 here and 8 here then whorl added 4 more.

Ron: I disagree. 9 + 3 + 6 = 18.

Both solutions, Miriam's and Ron's, equal 18. It is not clear whether they had made sense of the task because they have given numbers, but with no explanations. Even though Ryan disagreed he did not offer an explanation at this point.

Kathleen: (Suddenly, a number of students were disagreeing.) That's interesting.

Miriam: And so, then, I tried well there are 18 tally marks. The reason I came up with 6 arches, because there are 2 more loops than arches. 2 more arches makes 6 here and 8 here. 2 more is 8. That's how I got loop and arch. For whorl I just added 4 more.

Miriam ignores Ron's statement. From her perspective, she has demonstrated her solution method and sees no need to respond to Ron. From a "turn-taking" perspective all she needs to do is state her solution without listening to others.

Jay: I disagree. It is 9, 3, 6 and that's 18.

Jay and Ron saw conflict between Miriam's solution and theirs. Without teacher intervention they are accepting responsibility for making sense of the mathematics for themselves and for helping others. The socio-mathematical norm established was to give explanations and justifications. For those who disagreed, they were also to give explanations or ask questions. Thus, it is not clear if the two boys had made sense of theirs and Miriam's solutions, or if they did not have the language to ask her questions at this time.

Miriam: Can you prove it up here?

Brett: (Reads part of the problem over again.) Because it says 2 more loops than whorls, not arches.

Miriam: (Has her back to Brett the whole time he is talking, and she is doodling on the white board.) more loops than whorls...

Kathleen: Did you hear what he said, Miriam? (Miriam is still not looking at Brett)

Kathleen interjects with a question so that discussion will continue. She does not try to clear up the confusion. It is the students' responsibility to make sense of the mathematics. By doing this she continues to support "making sense", and is not necessarily concerned with getting the "right" answer.

Miriam: No!

Brett: (Waits for Miriam to turn around. When she does he then rereads the problem and looks at her while talking.) 2 more loops than whorls….

Miriam: What!!! (really loud)

Brett: On the paper it says there are 2 more loops than whorls.

Brett rereads the task to bring into focus the question asked. By doing this, he gives Miriam, and the rest of the class, a chance to hear and reflect on what is being asked.

Miriam: There is!! On my paper there is 2 more arches than whorls. There is because 2 more equals 6 so there is!

Brett: Loops than whorls?

Again Brett tries to use another approach to help Miriam see the conflict. This time he repeats two significant words in the question instead of the whole question, thus keying in on the specific part Miriam needs to look at. He is again bringing the focus back on the task, hoping she will realize she has arches, not loops.

Miriam: Same thing, that's the same thing! (a bit irritated)

Brett: 5 more.

Miriam: 2 more that's it, that' s the same!

Brett: 5 less arches than whorls and that's 2 more. (Keeps trying to refer to the paper with it in his hand. He was trying to repeat part of the problem.)

Brett now uses Miriam's words to argue from her point of view. He is trying to help her understand her reasoning in comparison to what the task asks. Miriam becomes defensive and does not acknowledge his solution. She did not ask him how he got 5.

Miriam: That's what I have! (She emphatically repeats several times, "That's what I have," but doesn't read the paper or ask him any questions.) That's the same!

Nick: There are 5 less arches than whorls.

Miriam: You can't prove it with what the paper says!

Nick: Yours looks like more arches than whorls.

Miriam: There is 3 more on my paper.

Nick: It says less.

As students listen to the argument, they try to follow along and make sense of the discussion. Several students try to aid in the sense making process. As each student understands what is being discussed, s/he interjects with important ideas.

Miriam: Can't prove it with the paper. (She is standing next to her answer on the board and does not move away from it. Other students come up to the board as they try to explain or question her, and then go sit down. But she stays up the whole time of the conversations.)

It is not clear if Miriam's solution has meaning for her or if she was just reciting number facts that equal 18.

Ron: 9 loops, 8 whorls, 1 arch. (The class ignores his answer because Miriam interrupts.)

Ron originally said 9, 3, 6. Now he is changing his answer to 9, 8, 1. In both situations the numbers add up to 18. There is no further information about his thinking.

Miriam: Gay Arla knows answer so she needs to come up here.

Kathleen: Maybe we need to leave this.

Miriam: I want to settle this now! (Arms are crossed stiffly in front talking to Nick as Kathleen continues.) Prove it, not on paper but on the board!

This episode provides insight into the contrast between merely sharing ideas verses engaging in argumentation, thus deepening our understanding not only of the importance of socio-mathematical norms, but also in particular, the importance of argumentation within the mathematics classroom, and how it facilitates students' mathematical development.

In the literature two terms, taken-to-be-shared and taken-as-shared have been used to describe the collective sense-making process. From my perspective in this particular second grade classroom, a difference between the two terms has emerged. In this classroom the ideas are laid out for everyone to try to make sense of. In this way the ideas are meant to-be-shared with all learners. This does not necessarily mean that the ideas will be shared. If a student is not listening or does not understand, how can the ideas be shared? For example in this episode, Brett and Miriam are discussing their strategies. His are laid out to-be-shared. But, at no time did Miriam share his ideas. She could not make sense of them and they had no common ground from which to communicate. Thus, the argumentation process was hindered because understanding did not emerge. In other episodes when students discussed ideas that had been laid out they negotiated meanings, the ideas and meanings that emerged were taken-as-shared. The students made sense of and discussed ideas, thus arriving at an agreed upon solution.

Note the following comparisons:

Brett: arguing	Miriam not arguing
Looks at person talking	Does not have eye contact
Rereads problem	Ignores written problem
Questions answer	Gives definitive answer
Open minded	Closed minded
Positive body language	Negative body language
Uses other person's words	Repeats her own words
Mutual adjusting	No adjusting

The teacher's responsibility is to facilitate and assist where needed in the enhancement of interactive collaboration and argumentation. During the time of this episode, Miriam could not appreciate the differences in the interpretation of the solutions, thereby causing an impasse. In order to help the discussion move forward Kathleen gave students time to think and reflect about their solutions. If she had intervened, the students would have seen her as the authority, would have accepted her solution, and would not have continued to make sense of the mathematics. By giving students time to resolve the tension concerning their obligations to try to understand each other's explanations, the teacher, personally, is providing opportunities to construct meaningful mathematics.

In this whole-class argumentation students were attempting to accommodate each other in listening for understanding, to properly orient each other's mathematical constructions and ideas for meaningful learning, and to resolve conflicting solutions. Such mutually interactive and continually adjusting discourse can be labeled **"synergistic argumentation."** (see chapter 5 conclusions)

(Episode #9)

Class Arguing – *Related to Learning*

This episode is used to illustrate the importance of arguing as an integral part of this particular mathematics classroom. Before, Kathleen has not talked about arguing even though it has been negotiated. On this day, she purposefully chose to engage the class in a discussion about arguing. But notice, Kathleen begins her questions based on a comment from a student.

Kathleen: Last time Betty you brought up the fact about arguing. Why is it arguing? How does it help you?

Betty: umm… It gives you questions you try to solve them.

Kathleen: Okay, you try to solve it. How does arguing help, or what do you think about the arguing we do?

Cort: Ummmm… How does it help you?

Kathleen: Yes.

Cort: Some people have different problems and different answers….

Kathleen: Do you know what Cort is saying?

Class: No.

Kathleen: Okay Cort, say it louder.

Cort: Some people's answers might be wrong and some people might say these wrong, so they say something else like, one time we're doing shapes on the paper for one of (Mrs. __) lessons. You have to argue over everything because some people said some shapes count or don't count. Sometimes people know the answer and other people don't, and **they don't want to get theirs wrong.**

Research shows that students who feel embarrassed or threatened by being wrong will not perform well academically. Jenson (1998) says that we all respond differently to potential threats. Even a simple threat of having the wrong answer can cause the brain to trigger an imbalance of chemicals. Once a situation is perceived as a threat, even being embarrassed in front of a peer, for example, our bodies throw up defense mechanisms. These defense mechanisms are great for survival but lousy for learning (Jenson, 1998). Students in both traditional and nontraditional classrooms share answers on the chalkboard, and/or orally, both of which are in the public domain. Students traditionally know that if they put up or say a wrong answer it is embarrassing and sometimes other students make fun of them. Students want to succeed and feel good about themselves and what they have done. If students are called on for answers in an environment that is not risk-free, they feel embarrassed and will not perform their full potential. That is why Cort emphasized that students did not want to get wrong answers.

I interviewed several third-grade students who had been in Kathleen's class the previous year. One student (who is in third-grade but was in Kathleen's class last year) remarked that having right and wrong answers in this second-grade class was good. A safe environment in which to argue seems to be important not only to students currently in this classroom but also to students who had been in this classroom in previous years. Arguing against right and wrong answers helped her to learn. She was not afraid or embarrassed to give solutions because the class would talk about them and then they would figure out the answers. She did not argue in any her other classes, only in the second-grade class. Another third-grade student commented that while he was in first-grade he usually got his math wrong. That is why he never got up to the board in other classrooms. While in Kathleen's room, he participated in arguing and going to the board did not bother him. He also said that when people argued he would think harder. Last year's second-grade students' statements agreed with Cort's statement that arguing does help people learn because people do not want to get the wrong answers.

The conversation about arguing continues.

Kathleen:	What about when somebody has a wrong answer, or a different one than you are thinking about? Does it help you to listen to their ideas?
Class:	Yes!
Kathleen:	Why does it help? Tina, why does it help?
Tina:	Because **arguing helps you know more things** like, shapes don't have to have corners and sides to actually be a shape. (She is talking about a previous class setting where they were talking about shapes.)
Kathleen:	Yeah! We had a big argument about that didn't we?
Tina:	I know corner, and I know, actually, because everyone was arguing, everyone knows it. It doesn't have to have corner so arguing lets people tell you.
Kathleen:	When we were arguing about the shapes, Bill did you hear that?
Bill:	Yes.
Kathleen:	What did you think?
Bill:	More... What I think it does, you have more people arguing the more easier it is to find out because you have this amount of people saying this is right and this amount of people saying this is wrong. You really go over it for a long time and the biggest group wins.

In this second-grade classroom, it is not just one person putting up problems on the board with the teacher saying yes or no to the answer and giving procedures. All the students discuss and try to make sense of the mathematics. As students put up their solutions and discuss them, they end up working more math problems. Again a student from last year said, "You get more problems because you have to think in your head how to explain so all can agree on a solution." She stated that this way of arguing about tasks gets you "thinking a lot."

Learners actively construct meanings for mathematical ideas and relationships. The sense that learners make of their experiences is strongly influenced by interaction with others. Through these discussions students think about their solution as well as other students' solutions. They are trying to make sense of all the solutions, whether right or wrong. If a solution is wrong students must figure out why and give a viable explanation of why and then be able (possibly) to give a correct solution. Therefore, it does take time as Bill says, "You go over it for a long time" for them to make sense of the mathematics.

Kathleen: I want to ask you something... you notice I don't go over here and go, okay this is class, this is what a shape is, and this is a shape, and there is no arguing....

Class: Yeah! Because **arguing it helps us work it out ourselves...kids can figure it out... teachers are not supposed to tell us the answers.**

Kathleen: Do you think it's better when you figure it out yourself?

Class: Yeah!

Kathleen: Why do you think its better?

Class: It's better because you get smarter and you won't have to ask or....

Kathleen: Maggie, what do you guys think about all this?

Maggie: I think you shouldn't tell us. You have to let us figure what it is, that helps.

Kathleen: Why does that help Maggie?

Maggie: **We learn how to do things and we remember when you let us figure it out. That is what helps us learn.**

Five students from two years ago were recently interviewed. When asked about Kathleen's room they said they remembered arguing during math class. They also remembered several of the tasks she had given them and how the class came to an agreed upon solution. They said that math was easier then because they could figure it out. These students said that their teachers now let them discuss sometimes, but for the most part they are told what to do. Math for them is harder without the arguing. They do not see this year's teacher taking part in their limited discussion whereas they saw Kathleen taking part in all of their discussions and arguing. Kathleen let them figure it out but this year's teacher doesn't. These five students were not necessarily the "top" math students and yet they all remembered positive mathematical learning experiences in this second-grade environment where arguing was a negotiated and accepted norm.

Kathleen: Do you remember better if I go, okay this is the way it is? This is how you do it?

Class: No, we have to learn we have to do it ourselves. We wouldn't have to go to school if you told us things our parents could tell us. Then we wouldn't be learning.

Kathleen: Why do you think you learn better if you figure it out yourself?

Mack: **You figure it out yourself you remember it better** than if you tell us what it is.

Cort: You have teacher to tell you the answer because... well cause she is smarter than you.

Kathleen: Cort, your jacket is green! (he is wearing a red and black jacket) I like your green jacket!

Class: It's not green!

Kathleen: What if a teacher tells you something that doesn't make sense? Is it okay to argue? Respectfully?

Class: Respectfully, yes. Don't be disrespectful.

Mary: It's not green!

Kathleen: Is it important for kids to figure things out themselves?

Mary: YES!!! My mama always told me if you want something done you have to do it yourself.

Kathleen: You want something done...you gotta do it yourself. But how does... how the arguing [help] when we all get together and share ideas? You know how we are doing that?

Class: Take turns. Let people share.

Kathleen: What if you disagree?

Class: Do it nice. Yeah, nice.

Kathleen: That is a good idea to say it respectfully... because important issue [is] respectful. Do you think if someone gave an idea and another person went duh! That's dumb! Is that respectful?

Class: No! You don't say that's wrong.

Kathleen: Have you ever thought you had an idea this is the way it was, and then someone showed something else [and] it made you think about it?

Cort: When Mary said that thing about the shape it **made me change** my answer.

Kathleen provides opportunities for the students to engage in reflective thinking. She is providing students opportunities for learning how to think and investigate for themselves. When students openly share, interact, talk about, and reflect on their ideas they are involved in what is termed, argumentation. Argumentation promotes mathematical constructions from which comes learning with understanding. Dewey (1910/1991) states reflective thinking in another way. He states that "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends, constitutes reflective thinking" (p 6). In other words students must learn to continually review and think about their constructions in the light of new data as well as old data to determine what is valid and what is not valid. Kathleen helps them do this by leaving solutions up on the board to be discussed, asking questions which compares/contrasts a solution to other solutions, and asking students if it makes sense to them. In this way students are thinking about thinking.

Kathleen: Did you hear that Mary? Say it again Cort.

Cort: When you got that thing about the circle it made me change my answer.

Cort changed his answer because he had made sense of the arguments. Through interactive discussions and arguments students do make sense of the mathematics and will change their answers. The students feel comfortable enough in sharing mathematics and know it is acceptable to change answers once you've made sense of it. These attitudes are not written on poster board but are negotiated, and students know what is acceptable.

Kathleen: You saw something a different way; almost like a bird's eye view. So you think arguing is an important thing to do in school?

Class: YES!!! In a respectful way!

There is a distinct kind of discussion in the episode above. The discussion does not focus on mathematical concepts or strategies, but focuses on ways of working together. This has been referred to as talking about talking about mathematics, as opposed to just talking about mathematics (Yackel, 2000). When the discussion is talking about talking about mathematics, the subject of discussion is primarily social in nature;

how to act and contribute. It is not specifically about how to make mathematical constructions. This primarily social aspect illustrates how social norms (rules for behavior, but not generally written down) are peculiar to each classroom.

In the above discussion Kathleen and the students have taken time-out from doing mathematics for the specific purpose of letting the students openly express their understanding and importance of arguing within a mathematical classroom context. As you can see by the student responses, they identified the type of open interactive arguing that is acceptable, and they expressed its use in the classroom as very beneficial to learning. Thus, in Kathleen's mathematics classroom arguing is routine, but essential, and very important in its contribution to student learning.

Additionally, the above teacher/student discussion helps both teacher and students to identify and to become more keenly aware of expectations in the classroom environment. By their participation in the discussion the students and the teacher interactively construct meaning to arguing, and reason for arguing. By its usage it becomes a social norm in their classroom environment. Furthermore, from the discussion we can see the development of taken-to-be-shared understanding about arguing. The students themselves agree that arguing is good in the mathematics classroom and must be done respectfully.

We can identify and list several aspects resulting from students arguing in the classroom:

1. The students construct meanings.
2. They have heard their own thinking as well as that of other students.

3. They ultimately end up with the "right" answers without the teacher telling them.
4. They remember the information better.
5. They experience greater learning.

According to a Caine & Caine (1994) and Sprenger (1999), brain research indicates that arguing is helpful to students in their development of mathematical intelligence, and in becoming autonomous (independent thinkers). This is because the students are focused on making sense of the mathematical constructions, rather than on completing the task to get the "right" answer, which is provided by the teacher. Saying it another way; they are focused on the mathematics involved, rather than on an apprehensive game of school in search for the answer by which they either pass or fail.

Summary of Episodes

These episodes illustrate the in depth meaningful argumentation that can take place when sense making is the focus of the learning environment. Said another way, focusing on sense making in the classroom infuses argumentation. At the same time argumentation infuses sense making. These episodes show that in order to accomplish individual and collective sense making, it is necessary for argumentation to be part of the classroom environment. Argumentation allows for all voices to be heard whether they agree or disagree. Even when students had the same answer or agreed on the same answer, Kathleen continued to ask students if their solutions and/or strategies made sense to them. She also continued to ask if there were other ways of solving the task. Sense-making is emphasized and is important whether students agree or disagree with the solution. In this way Kathleen provided opportunities for all students to make sure they personally understood and focused on the different ways of thinking.

In disagreements the children's methods are laid out for all to see, hear, and think about. The students as well as the teacher ask questions about or challenge the solutions and/or the strategies presented. The points of disagreement may vary. For instance students may have the wrong strategy and the wrong answer, students may have a viable strategy but the incorrect solution, or they may have a flawed strategy and a correct solution. In times of disagreement students must reflect on their thinking, compare their thinking to others' thinking, and then give counter arguments that make sense. Counter arguments can be in the form of a different strategy, in discussing meaning of words, or about the numbers involved. In order for this type of argumentation to be effective students must actively listen. Arguing, reflecting, and listening of the magnitude demonstrated in these episodes occur only in a problem-centered learning environment where making sense of the mathematics is the focus and where caring for all students is demonstrated. The environment must be one where everyone feels cared for, respected, and equal (no separate authority, either student or teacher).

Every episode showed students eager, willing, and volunteering to contribute to making sense of the mathematics as individuals and as a mathematical community. The environment of this classroom was one of caring and respecting. The morning meetings provided opportunities for the teacher and students to negotiate the social norms of the classroom. These unwritten rules of behavior, such as listening, set the tone for the rest of the day. This was a time when students could discuss problems between students and/or personal individual problems. Discussing common problems during a social setting such as the morning meeting is a good place for children to think about the problems. The teacher encourages the students' to accept the responsibility to resolve their problems by not intervening as the authority. By giving children opportunities to come up with solutions to their own problems, they are empowered to follow through with their ideas. These meetings are good for students' social development as well as their development of logic (Kamii, 2000).

The class openers set the tone for sense making. The teacher provided opportunities when students could openly share their thinking about mathematics without the fear of being right or wrong. It also gave the teacher an opportunity to "see" the students thinking. In this way it puts all students on equal ground with no one being the math authority. This is a time when the teacher facilitated the discussion letting students know that thinking is valued.

The trapezoid triangle task emphasized the teacher's style of questioning. To promote a sense of worth and a non-threatening environment her way of questioning and the questions themselves were very important.

Episodes three through eight highlighted students arguing in order to make sense of the mathematics during whole-class discussion time. Students focused on the mathematics or tried to understand the mathematics. At no time did arguing become a personal vendetta against another person. Students represented various cultural and ethnic backgrounds but all readily participated in the arguing about mathematics as they tried to make sense of the tasks given.

When Kathleen asked the students about arguing they all said it was important and it helped them learn better. Students actually changed their answers, not based on a math authority's answer, but because a different method shown made sense to them. The students also said that they remember things longer when they argue about them.

In this particular classroom argumentation played an important role and through arguing students were learning significant mathematics. The success of argumentation depends on how, why, and when should argumentation be used in the classroom. It also depends on the teacher's understanding of its role and how comfortable s/he is with argumentation. It is not enough to say the teachers are to be facilitators, nor is it enough to say that students are to be actively involved in their learning. Argumentation does away with non-interactive descriptions like "learning procedures, memorization of facts, and being taught math." These descriptions are replaced with "intellectual autonomous learners making sense of mathematics." When the focus changes from testing as goal to sense making, then argumentation promotes autonomy and aids in all students learning meaningful mathematics.

In chapter five I will use data from this chapter to draw conclusions and highlight parts of the episodes to support my conclusions about argumentation.

CHAPTER 5

RESEARCH ANALYSIS

From my research of Kathleen's second-grade problem-centered mathematics classroom, argumentation emerged as a form of communication that is a positive and effective way for students to construct meaningful mathematics with enhanced learning. Each characteristic identified in the previous episodes help support the argumentation process. Students were able to discuss issues important to them while other students listened. As a group they tried to help one another solve any personal problems as well as mathematical problems. Through the open and interactive discussions that took place the students and teacher built a community where argumentation was an accepted and expected form of communication. But the term argumentation alone does not completely capture the complexity and richness of what was happening within this classroom. While conducting research I looked for applicable descriptions that would adequately and uniquely define Kathleen's open interactive classroom discussions as I observed them. I found that literature uses various related descriptions, to name a few, such as communication, dialogue, discourse, argue, and argumentation. The literature seemed to use broad and general definitions of these terms. Additionally I researched *Webster's New Collegiate Dictionary* definitions of these words and I found that its definitions also did not adequately describe the dynamic and interactive learning process that is a daily routine in Kathleen's classroom. Some of the words contain aspects/characteristics that other words do not contain while some aspects are described in more than one word. These dictionary terms are representative of similar terms used in the literature and are helpful to show similarities and differences.

Please refer to the following list:

Communication: an act of transmitting, verbal or written.

Sfard (2000) uses one definition of communication in her research to mean interactively exchanging ideas. NCTM (2000) states that students should be involved in mathematical communication. In both instances communication does not specifically indicate the type of communication taking place in Kathleen's mathematics classroom. The word in general might be appropriate for describing a traditional classroom but not for my research site. The use of the word communication applied to the problem-centered classroom is just too generic, non-descriptive, and non-applicable.

Discourse: to express oneself in oral discourse, talk, converse, and verbal exchange of ideas.

Although more descriptive than communication, discourse falls short of an applicable description for Kathleen's open and interactive class discussions and problem-solving.

Argue: to accuse, reason, to make clear, to give reasons, contend, try to prove.

In Kathleen's classroom there were no accusations. Students tried to help one another understand various mathematical strategies and ways of thinking. Arguing more closely describes what happens in Kathleen's classroom. It is similar in description to argumentation, but still does not fully describe the interactive communication process.

Argumentation: the act or process of forming reasons and drawing conclusions and applying them to a case in discussion, to debate and discuss.

In the second-grade classroom students worked together discussing ideas. As they presented their solutions they gave reasons for them. The whole-class discussion was dynamic, interactive, and involved all students and the teacher. Kathleen was a vital facilitator/participant in the interactions and throughout the whole-class discussion time.

Argumentation definition by Cobb (1995): (a) a primarily social process in which cooperating individuals try to adjust their interpretations and interactions by verbally presenting rationales for their actions. (b) The techniques or methods used to establish the validity or claim of a statement. During argumentation, if one participant explains a solution, the implicit message is that the claim is valid. A successful argumentation refurbishes a challenged claim into a consensurable or acceptable one for all participants (p. 293). This definition may or may not include the teacher.

As students **interact** in the classroom and try to make sense of mathematical ideas, **communication** and **working together** emerge as critical factors in **enhancing the students' learning** (Cohen, 1996; Hiebert et al., 1997; Corwin, Storeygard, & Price, 1996; Azmita, 1993). Problem-centered learning is an approach that supports the constructivists' orientation (Wheatley, 1991). The components of problem-centered learning are tasks, groups, and sharing, all of which promote opportunities for interaction. The emphasis of my research effort was to identify, document, ad clarify the dynamic and complex role of argumentation in students' construction of mathematical meaning during the whole-class discussion portion of the problem-centered learning environment. This was done by observing Kathleen's second-grade mathematics classroom. What I have observed is that argumentation encompasses a totally interactive communicative process that enhances learning. This term is my preference for and comes closest to describing Kathleen's classroom interactions. However, argumentation alone did not completely fulfill my search for a more complete and precise term to describe the interactive learning that was taking place in Kathleen's classroom. Thus, a problem-centered learning setting that includes argumentation becomes a rich and dynamic environment for students' meaning making.

Synergistic Argumentation

Therefore, I propose the term *synergistic argumentation* as a more appropriate and consistent term to describe this interactive open classroom learning discussion. Synergistic argumentation describes the unique discourse of arguing-for-sense-making used in this particular mathematics problem-centered learning environment.

It can be defined as cooperative and interactive communication in which the sum-total learning by the whole interactive and dynamic group of students and teacher is greater than the sum-total learning derived by summing together the learning from each individual student making up the whole group.

To put it another way, the sum total learning (understanding) by a number of students and teacher interacting as a group is greater than the sum total learning (understanding) of those same students acting independently.

Multiple Facets Of Synergistic Argumentation

In an effort to further define synergistic argumentation I submit the following descriptive facets: dialogue, discourse, arguing, interactive conversation, questioning, explaining, justifying, reflective thinking, exploring, transactional dialogue, discussing, analytical reasoning, rationalizing, verbal exchange, talking, expressing, orally communicating, cooperating interactively, working together, interactive learning, deciphering, and hermeneutical listening.

In summary, various research sources indicate that students in a classroom that provides for whole-class open interaction are higher achievers (Wood & Sellers, 1996; Cohen, 1996) than those students in classrooms with no, or limited, student interaction. Also, classrooms where students are afforded the opportunities to engage in whole-class interactions provide opportunities for students to develop intellectual autonomy. Synergistic argumentation emerged as a critical factor in the second-grade students' mathematical learning.

Several key components emerged as critical dynamic and complex aspects of synergistic argumentation. Without any one of these aspects there is no synergism. All of these are interwoven and make up the spirit of the complex and dynamic learning environment. (Even though these aspects are presented in a linear fashion they are all interwoven and emerging throughout the whole-class discussion portion. These are not explicit rules the teacher taught the children nor are they specific steps the teacher followed. These were negotiated and re-negotiated throughout the day.)

- Arguing respectfully enhances learning.
- Argumentation that focuses on mathematics enhances learning.
- Providing opportunities for students to argue appears to be crucial for maximized mathematical learning for all students.
- The social dynamics create a learning environment where argumentation is accepted and expected.
- Second-grade students are capable of arguing in a logical manner to make sense of the mathematics.

Each characteristic identified in the previous nine episodes (Chapter 4) infuses the synergistic argumentation process in this second-grade mathematics learning environment. These characteristics along with the aspects above were very much a part of and interwoven throughout all the lessons. Individual students shared and discussed issues that were important to them while other students actively listened with respect. They had opportunities to gain confidence in themselves and their ideas as they openly shared and discussed their ideas. They openly and interactively discussed solutions and strategies while trying to make sense of the mathematics and their world. They were active listeners who questioned and challenged one another's ideas. These opportunities that were provided throughout the day as well as during the mathematics lesson enabled the students to participate in synergistic argumentation.

Discussion of Key Aspects

- *Arguing Respectfully Enhances Learning.*

In Episode #1 (morning meeting/social adjustments) the students were trying to help three other students resolve conflict. Sue had a problem with Zena and Grover. Sue said her feelings had been hurt. Several students asked Sue questions about why her feelings were hurt. Questions were asked of Zena and Grover also. The students listened intently. At no time did the students make negative remarks toward one another.

In Episode #5 (flower task/open interaction) the students were excitedly sharing their solutions and several of them started talking at the same time. It was difficult to understand what they were saying. Kathleen stepped in and said, "We need to be respectful of one another and take turns." At this point the students began to listen to each other's mathematical ideas.

In Episode #9 (class arguing/related to learning) the students were discussing arguing itself. A couple of times in the discussion the students brought up the fact that if students are going to argue then it must be done respectfully. Class, "...be respectful." Mack "Don't be disrespectful." Class, "Yes [arguing is important]...in a respectful way."

Summary of Arguing Respectfully.

Students effectively argue but not about a person. The focus of the discussion is on the ideas or actions and not the person involved. This carries over into the mathematics classroom. Students argue about the mathematics and not the students themselves. At no time did they make negative and personal statements but commented only about the mathematical ideas that were laid out to be discussed and made sense of. For example in the fourth episode the students were arguing the mathematics, not at Ron personally. There were no disrespectful remarks because the focus is making sense of the mathematics as an individual and as a community. Thus, synergistic argumentation enables students to make sense of the mathematics as they interactively respect the ideas of others.

- *Argumentation That Focuses on the Mathematics Enhances Learning.*

In Episode #5 (flower task/open interaction) the students were trying to figure out how many more red flowers than yellow flowers. Jan was unsure of her initial answer, $8 - 4 = 4$. After hearing a few arguments she thought $8 + 4 = 12$, and she orally stated that when Brett and Gay Arla presented their solutions. But she did not have a drawing on her paper or a justification to support her verbal statement. At the end of the discussion time she had reworked her solution with a new strategy. She had circles drawn on her paper and pointed to them as she explained, "$8 - 4 = 4$." When asked if it made sense to her she said, "Yes." Jan, as she reflected on the arguments presented, changed her interpretation of the task, thus her answer changed and she was able to justify it.

Dana had written on the board, 8 + 4 = 4 but verbally said, "Eight minus four equals four." Zena explained that she had plus and Dana eventually realized what she had done and changed it.

In Episode #8 (loop, whorl, arch/listening for understanding) students where deciding how many of each kind of thumbprint. Miriam had 6 arches, 8 loops, and 4 whorls. Brett gave a justification for his solution and questioned Miriam about her solution. Nick understood both and tried to aid in their making sense of the problem and each other's solution.

In Episode #9 (class arguing/related to learning) the students were discussing arguing in class. Tina said, "...it helps you know things." Bill said, "...you really know what it is..." Several students said, "We learn by doing it ourselves." Maggie said, "We remember...we figure it out..." Mack said, "...figure it out, you remember better." Cort said, "...helps me to know...I changed my answer..."

With the focus being on the mathematics the students are comfortable sharing their ideas. Through the sharing and discussing of ideas the students are able to make sense of the mathematics. Without the fear of someone making fun of a wrong answer or being called names the students take risks, questioning their ideas as well as other students' ideas. Throughout all the episodes respect for each other and each other's ideas emerged as a key component in synergistic argumentation.

Summary of Argumentation that Focuses on the Mathematics.

In order to argue respectfully and about the mathematics involved, the students must be actively listening. The students and teacher are jointly interacting and exploring mathematical concepts. Kathleen's questions and the students' questions are based on what they heard other students say. This listening is not toward a prescribed set of learning outcomes but more of an invitation for the students, individually and collectively, to make sense of the mathematics. Kathleen and her students interact in each other's thoughts, question understanding, and genuinely listen. With this type of learning the students can effectively argue about the mathematics and not the person. Also, this deepens their understanding. During the second episode the students had given the "right" answer several times but Kathleen continued to ask for different ways of seeing the dots. Also, in the third episode Kathleen asked one student how she began the trapezoid triangle but let another student who started it the same way give the explanation. Then Kathleen posed the question, "Is that how you did it"? Now the students who started this way were mentally brought into comparing their methods. The students, as they think of questions, voluntarily ask other students. Thus the conversation continues, not coming to a dead end simply because the right answer or textbook procedure had been given. This was also evident in episode six. The students had solved the sucker task but Kathleen asked Grover to show the class his method, even though the class had the right answer. Thus, looking at and listening to the different ways of thinking about tasks are important in this learning environment, and from which respectful argumentation emerges.

The students throughout these discussions were focused on trying to make sense of the task for themselves and for the community of learners within this environment. Jill is representative of many students in math classes, unsure of what to do and how to explain their answers. Since the whole-class discussion was on mathematics, Jan and other students were able to hear several mathematical ideas. Not only did she hear the right answer but at least three different mathematical justifications for that answer. She and the other students heard justifications for a different answer also. They were able to compare the justifications for both answers and better able to make sense of the task. One of those justifications made sense to her and she was able then to understand the task and confidently defend her answer. Thus, through synergistic argumentation the focus was on the mathematics and that provided opportunities for students to make sense of their mathematics.

- *Providing Opportunities for Students to Argue Appears to be Crucial for Maximized Mathematical Learning for all Students.*

In Episode #6 (sucker task/incubation) the students were asked to find out how many more green suckers than red suckers. The students had previously listened to a similar task (flower) and had spent the whole discussion time arguing about the answer. Different students presented solutions and justifications for the two answers given. But they had come to an impasse. However, in this episode the students had all made sense of it and knew the answer without any discussion.

In Episode #9 (class arguing/related to learning) the students were asked, "What about arguing"? During the discussion all of the students gave ideas why arguing was necessary. Their comments were that it helps them. "Arguing helps you to know more things. Arguing helps us figure it out ourselves. We learn to do things and remember. You remember better. It helps to change your answer."

Summary of Providing Opportunities for Students to Argue.

In order for Kathleen to be able to provide opportunities for students to argue synergistically, she needed to listen to their discussions during the small group and whole-class sharing. Listening to students is not only a critical factor in students' success in mathematics learning but is also an important component to effective mathematics instruction. Kathleen chose tasks based on the students' arguments, questions, misunderstandings, curiosities, etc. She listened to find out where the students were mathematically and what types of tasks would further the students' mathematical thinking. This type of listening was not for the purpose of finding out who had the right or wrong answers but focused on deepening her understanding of the children's mathematics. Davis (1994) identifies this type of listening as hermeneutical. While participating in hermeneutical listening the students and Kathleen are oriented towards sense-making, which further stimulates the mathematical conversation. She is not just interested in following the school's curriculum or "teaching" students the procedure that leads to the right answer. When Kathleen was asked about the choosing of tasks she responded:

> I'm walking around the groups listening to ideas in the way they are doing their problems...I look for tasks to help them...what their needs are. I have to pay attention to what they are thinking and what our choices of needs are and what tasks can engage them in productive argument.

Kathleen is focused on what will further the students mathematical thinking as well as what types of tasks will help them make sense of the mathematics at the same time. Sense making is based on the students' mental constructs that are based on meaningful mathematical activity, not on memorizing procedures. Focusing on meaningful mathematics emerged as a key aspect of synergistic argumentation.

- *The Total Social Interactive Dynamics Create a Learning Environment Where Argumentation is Accepted and Expected.*

In Episode #1 (morning meeting/social adjustments) the students use this time to discuss anything in the agenda book or any personal relationship problems. Students and teacher sit in a circle actively listening to each other trying to offer appropriate solutions to any discourse or problems. The teacher and students discuss with care and respect for one another, thus creating a relaxed warm and caring atmosphere.

In Episode #2 (class opener/confidence building) the students express what they see and how they see the dots arranged. Kathleen said that this was a great way to get the children engaged, get their minds warmed up, and to see things in a different way.

In Episode #3 (trapezoid triangle/teacher/facilitator) the students were asked to find the smallest and largest triangle using trapezoids. Kathleen purposefully asks students questions to get the discourse going.

In Episode #4 (balance task/minority and mutual acceptance) the students were asked to solve four problems. The students took the lead in the discussion after Ron gave a solution and a strategy that the class knew was not right. Several students challenged Ron and tried explaining their strategies to Ron. They wanted Ron to understand. Not only is it important for the individual to understand, but it is important for the whole community to understand as well. Ron was not offended or hurt by their mathematical arguments. He sat back and listened to them and then stated his original solution.

In Episode #9 (class arguing/related to learning) the students were discussing arguing in the classroom. Several students made the following comments: Tina said, "...it helps you know things." Bill said, "...you really know what it is..." Several students said, "We learn by doing it ourselves." Maggie said, "We remember...we figure it out..."

Mack said, "...figure it out, you remember better." Cort said, "...helps me to know...I changed my answer..."

Summary of Total Interactive Social Dynamics.

Throughout several episodes, students accepted the responsibility to volunteer to join in the arguments, argue their strategies, and make sense of the mathematics. One side note is that Kathleen could join in any of the collaborative pair's discussion without the students feeling uncomfortable or becoming quiet. Due to the openness and the goal of sense making in the morning meetings and the class openers, students did not change their actions when Kathleen joined in any of the groups' discussions. They accepted her as a facilitator/participant and she accepted them arguing as they tried to make sense of the mathematics.

The students are not only trying to make sense of the mathematics but are trying to negotiate a common language from which to operate. Words alone many times have multiple meanings. Students are trying to figure out the meanings of words as well as what other students are trying to mean. By participating in synergistic argumentation students can sort out the implicit meanings, which gives opportunities to further their mathematical thinking. Understanding what someone else has said implies that through an exchange of language a compatible context was developed which further aids in the present conversation and sense making process as well as emergence of new ideas (Glasersfeld, 1996). This means, as students communicate through argumentation, they make their thoughts and perturbations topics of conversation, thus creating a unique learning opportunity for all involved. The student with understanding has an opportunity to extend his/her thoughts in order to make sense of the other person's ideas, while at the same time trying to formulate a meaningful argument for his/her own personal thoughts. At the same time, the other students become aware of perturbations and possible alternative interpretations. The negotiation of mathematical meanings enable students to make connections from their individual mathematical constructions with the constructions made by the class. As students explain and justify their solutions and strategies while listening to questions and challenges, opportunities emerge for the teacher to facilitate mathematical understandings. This is not creating a situation where the teacher "tells" the student the answer or procedure but gives the teacher opportunities to ask key questions to help further the student's thinking. The argumentation process lets the teacher "see" student's thinking which enables the teacher to choose appropriate tasks for the next day that will continue their mathematical thinking (Wood, Cobb, & Yackel, 1993). Synergistic argumentation about students' solutions and strategies provides opportunities that enable the teacher to facilitate the development of mathematical meanings.

- *The Negotiation of Socio-Mathematical Norms Makes it Possible for Students to Construct Meaningful Arguments.*

In Episode #2 (class opener/confidence building) the students are asked to tell what they see and how they see it. Kathleen continually asks, "Does anyone see it in a different way"? "Does anyone have a different solution and/or strategy"? She has several students share their ways of seeing. She does this throughout all of her mathematics lessons.

In Episode #3 (trapezoid triangle/teacher/facilitator) the class was asked to make the smallest and largest triangle they could, using only trapezoids. Kathleen hears Karen's idea and asks another student who did it the same way to explain how she made the triangle. Kathleen also asked students who had started the same way as these two students but found it did not work to explain their strategies. Thus, the students were comparing their thinking to others' thinking, helping them to make sense of the ideas. Throughout all of Kathleen's mathematics classes she continually asks students to explain and justify their solutions and strategies. Then she asks the class if they agree or disagree and why. She also asks them if any of the strategies make sense to them.

Summary of Negotiation of Socio-Mathematical Norms.

Even as students share their solutions and methods they are not necessarily having the same experience, nor do they have the same meaning of words. Through the process of justifying and explaining their ideas, they are negotiating meanings of words. This gives them a common basis from which to discuss and argue. Argumentation then becomes a catalyst by which their thoughts become compatible, enabling them to continue conversing. Thus, the notion of sharing ideas does not mean sameness. That is why it is important to provide students opportunities to share their methods, and argumentation provides that.

Voigt (1994) emphasizes that one can never be sure that two people are thinking the same if they collaborate without conflict. Many times authors of textbooks assume that mathematical objects have unambiguous meanings. But students with differing mathematical constructions may not be able to make sense of the mathematics. Thus, through the process of negotiation, mathematical meanings and understandings emerge. Mathematics is not a fixed body of knowledge, but through argumentation becomes a topic of discourse from which meanings emerge. Therefore, synergistic argumentation is a way to "see" the students' thinking.

Tasks in a problem-centered learning environment enable students to give viable meanings to the mathematical concepts they are constructing. These tasks keep students engaged for a lengthy period of time. Embedded in these tasks are several mathematical concepts to be dealt with at once. Unlike doing mathematics from a traditional textbook where the tasks are sanitized portions of a concept to be practiced in isolation, students in a problem-centered learning environment negotiate meanings of these tasks and may develop different strategies for solving the tasks (Reynolds & Wheatley, 1996).

The negotiation of socio-mathematical meanings is more than students taking turns to explain their answers. It is the opportunity for everyone to make sense of the mathematics through interactive discussion. If students just take turns giving an answer, then there would be no need for comments or discussion. Student A would respond, then student B and so on. There would not necessarily be any interaction between and among students, thus mathematical conversations would not emerge. In McClain and Cobb's (2001) article, *An Analysis of Development of Socio-Mathematical Norms in One First-Grade Classroom*, they describe a situation where the teacher let all the students in turn give an answer and a method for finding the answer. During the whole-class discussion the students were not engaged mathematically, but were waiting their turn to talk. There was no shared discussion of important mathematics and no indication of which solutions and methods were of value. Thus, without open comparison of the students' thoughts, the whole-class gathering became a series of disjointed "turn-taking." Some students repeated what others had said and some explanations did not contribute to the mathematical agenda. Students were unaware of which solutions and methods were mathematically significant. Therefore, the students were not engaged in the mathematics and the discussion did not contribute to their mathematical welfare. Without the negotiations during mathematical discussions there are no rich mathematical conversations or ideas to continue with the next day. This kind of whole-class discussion contrasts with the second-grade problem-centered classroom in which synergistic argumentation promotes opportunities for mathematical meanings and understandings to emerge.

- *Second-Grade Students are Capable of Arguing to Make Sense of the Mathematics.*

In Episode #4 (balance task/minority and mutual acceptance) the task was (7 / 2 ___). Ron saw the 7 and 2 and just added them to get 9. He ignored the placement of the numbers. Three students developed mathematical arguments to try to help him understand the task. Zena used her fingers to show that you start with 2 and count on until 7, which uses 5 fingers. So the answer should be 5. Miriam made the statement, "You know 5. Five is one more than 6 and 2 is 7." Nick tried to explain that the 2 was on the right side of the balance and 7 was on the left, so 2 and 5 go together.

In Episode #5 (flower task/open interaction) the problem was to figure out how many more red flowers there are than yellow flowers. One student, Gay Arla, said. "It was like a graph." She drew two columns side by side: a column of 8 circles signifying the red flowers and a column of 4 circles signifying yellow flowers. She drew a line under the fourth circle in each column. There were 4 circles left over under the line. So the answer is 4. Ron offered a counter-argument stating, "You start with 8 and count up 4 making 12." He used tally marks to demonstrate his solution. Brett gave a different justification for his solution of 4. He drew two rows of red flowers with 4 in each row.

Under that he drew a third row of yellow flowers with 4 in it. He said that the one row of yellow and one row of red matched and that left 4.

In Episode #7 (Valentine task/sense-making) the students were asked to find how many purple, pink, and red Valentines. Brett commented about Tina's solution and then said that 8 to 10 is two and you must add numbers to get 25. He re-read the task and tries to re-explain so Tina could make sense of it. "Purple is 3 less so that's 5 and 10 and 8 is 18." Now showing with his fingers he counted out loud 19, 20, 21, 22, 23, 24, 25. He had 5 fingers up so "18 plus 5 equals 23."

In Episode #8 (loop, whorl, arch/listening for understanding) the students were trying to figure out how many students had loop, whorl, or arch thumbprints. Miriam had on the board 6 arches, 8 loops, and 4 whorls. Nick said, "There are 5 less arches than whorls. Your [answer] looks like more arches than whorls."

These students were making sense of their own mathematics, making sense of other students' solutions, and providing arguments for their justifications when explaining and/or trying to help others understand.

Summary of Second-Graders are Capable of Arguing.

In this problem-centered learning environment these second-grade students are expected to make sense of the mathematics. The types of tasks used in this learning environment lend themselves to discussion as students work to solve them in collaborative pairs and then explain and justify their ideas in a whole-class sharing time. The argumentation that naturally emerges in the whole-class sharing time provides opportunities for children to reflect on their own methods while comparing them to other methods. Through the arguing and listening students have opportunities to re-think and re-evaluate their own thinking. These types of activities that promote arguing give opportunities for students to try one another's problems, give alternative methods, and give constructive feedback, all of which help them to refine their thinking.

Students are given opportunities to present and defend their thinking to the class. They present their solutions and strategies in a logical manner for others to be able to follow. The use of the word logical here is different from what most adults would think; proof, formal reasoning, or analytical. Children's mathematical knowledge and thinking is different from adults. Krummheuer (1995) states that primary children's mathematical knowledge is at an empirical level. The mathematics that children talk about focuses on experientially real mathematical objects (p. 236). He also says that primary children do not draw analytical conclusions (from adult's perspective). Krummheuer (1995) does say that children are capable of constructing substantial arguments (p. 236). Students listen to one another's solutions trying to understand them, then ask questions, give constructive feedback, and refine their original thoughts. Wheatley (1992) states that discourse promotes reflection. Reflection is a way for students to strengthen their mathematical reasoning abilities and develop autonomy.

Students construct mathematics with understanding when they have been given challenging tasks for which their own methods for solving are inefficient. Dewey (1959/1967) states that any significant problem involves conditions which, for the time being, contradict each other. Solutions only come when one gets away from the meaning of the terms...seeing from another perspective, and hence in a new light (p. 91). This gives rise to learning opportunities and encourages students to reflect on their activity as well as activities of others. When students become conscious of their organizing activity, they can effectively change or adjust their actions. Students thinking about their own thinking develop intellectual autonomy.

So, are second-grade students capable of participating in synergistic argumentation in order to make sense of the mathematics? The answer is a resounding "Yes"!

Common Characteristics in the Episodes and Synergistic Argumentation

There are important common characteristics that are interwoven throughout all of the episodes, in synergistic argumentation, and are essential aspects of the learning process. These essential aspects promote enhanced learning opportunities.

Listening

Kathleen does not seem to be doing most of the talking throughout these episodes. The students do most of the talking. Their attention is not on the teacher and the teacher's attention is not on the students per se. Both are focused on the mathematical discussions at hand. The teacher and students become participants in the listening and have a desire to hear the explanations and justifications being presented. As the participants listen they question and/or challenge each other, move closer to one another, and become immersed in the argument. The listening participants discuss and arrive at a collective agreement about solutions and/or strategies, thus deeper mathematical understandings emerge.

Listening is something the teacher and students enter into. Each brings meaning and tries to make sense of other people's meanings. Entering into listening enables participants to attend not only to the context of the argument, but also to the subtexts, such as tone and mannerisms. The participants listening interpret the interactions and desires of others to understand them.

Davis (1994) states that listeners must be oriented toward gaining a fuller understanding, searching for viability of interpretations, and not just the "right" answers. Thus, listening becomes a means of probing and checking emergent understandings. Listening becomes a dynamic and continuous process. The probing and questioning of mathematical ideas solicits more intentional listening, and the listening solicits more probing and questioning of mathematical ideas. The emergence of mathematical ideas and listening are interdependent. Listening provides students opportunities to construct new mathematical understandings while the emergence of new mathematical understandings provides opportunities for students to listen. Davis (1997) states the quality of the students' explanations and justifications seem to be closely related to how well the teacher listens. Teacher, students, mathematical ideas, and listening create a dynamic and complex entity where one person's idea is intertwined with another person's idea.

Reflection

Another characteristic is reflection. Because sense making is so important in this environment, sometimes Kathleen will ask students to go back with their partners and reflect on what their strategies and solutions were; trying to make sense of them and re-explain them so other students can understand their way of thinking. The students are given time to step back and think about their discussion and arguments that took place during the whole-class sharing time. This looking back does not mean they re-experience the exact same previous experience but re-create it in their minds. They take previous experiences and look at them as objects to be discussed. Thus, the ideas are not the original ones but are now topics to be thought about. In this circular process students are recalling experiences, thinking about them, and what picture emerges through this looking back process becomes the content for the next experience. Glasersfeld (1995) uses the word re-presentation to describe the bringing of a prior experience into consciousness. It is the recollection of figurative material that made up the experience (p. 95). Re-presentations are created internally by the self and require some sensory material for its execution. Stepping back and reflecting on actions or thought is similar to an instant replay. One chooses which aspects of previous experiences upon which to focus. Instead of being physically involved in the action, s/he is watching from a distance, more objectively. As one watches the replay, new things are noticed and new insights are gained pertaining to the activity. After watching the replay one may better know how to deal with the next similar situation, or what to say or ask that would be meaningful and aid in understanding. Watching the replay, one can compare hers/his experience to other experiences as well. Through the discussion of the replay the student's ideas may be modified. Part of wisdom and reasoning comes from stepping back and looking at situations in order to look at things objectively to learn how to improve upon future situations. Brett is a good example of a student that steps back and looks at the replay. In both episodes, where he is arguing with Miriam and Tina, he takes their thoughts in, compares them to his, and tries to make suggestions or ask questions that will perturb the other students thinking or help them to make sense of what he has made sense of. He looks at the thoughts objectively trying to say things in a way that would makes sense to them from their perspective.

Learning opportunities emerge for children as they attempt to resolve conflicts and as they build on each other's activity as they meaningfully interpret each other's actions and comments (Yackel, Cobb, & Wood, 1993). One way to resolve conflict is through argumentation. The argumentation process during the whole-class sharing time is enhanced because the students had been given opportunities during the pair collaboration time to think through their solutions and strategies. The students come together during the whole-class portion to listen to other students' solutions and strategies and try to make sense of those also. Students explain to each other their ideas and compare strategies when each student is ready. Thus, students flow in and out of the conversation as they make sense of what is being discussed or if they have questions about different ideas. This activity of arguing during the whole-class portion has the potential to facilitate children's mathematical learning in a way that may not be possible in a traditional setting.

Implications

My research focused on one particular mathematics classroom. Even though this research focused on one classroom, it does have implications for enhancing mathematical learning in other classrooms as well. **The second-grade students were willing and capable of participating in synergistic argumentation to make sense of their mathematics. This could possibly mean that students of other ages might well could benefit from participation in synergistic argumentation.** The students' informal knowledge provided a foundation for their development of their understanding of mathematics through argumentation. In the absence of predetermined standard mathematical procedures and instructions the students readily used all opportunities to busily construct meaningful solutions, strategies, and lines of reasoning.

The present study provides rationales for using synergistic argumentation in the whole-class discussion portion of a problem-centered learning environment. In this type of environment, students and teacher continually interact and communicate about their ideas with one another. As students discuss and challenge each other's ideas they continually revisit those ideas and deepen their own understanding. Conflicts often arise when discussing ideas. In a problem-centered learning environment the students are expected to make sense of and resolve them. Classrooms in which students discuss their personally constructed mathematical meanings reveal higher reasoning abilities than those in the traditional classrooms (Wood, 1995). This is because for students to give an additional explanation requires them to think from another student's perspective.

Synergistic argumentation provides opportunities for all students to learn meaningful mathematics.

This study has provided rationales for students and teacher to become active listeners. This involves listening with the intent to understand. During the whole-class sharing time students do not just sit and wait their turn to share. Rather, they become mentally involved and attend carefully to others' explanations. Listening with the intent to understand creates a context in which the students are reflecting on their own methods while attempting to understand the other methods. The use of problematic tasks provides students opportunities to explain and justify their thinking. The students are not given procedures to follow in order to solve the tasks. The students develop their own personal strategies for solving the tasks. Since there are no predetermined and uniformed procedures given, students share their ideas during the whole-class discussion. They listen to and attempt to make sense of the explanations of others. As students give their solutions the teacher does not evaluate them as right or wrong. It is up to the listening community of learners to ask questions, to agree or disagree, or to make challenges, thus, soliciting other arguments. As children solve the problematic tasks and resolve any conflicting ideas, a new level of understanding emerges. In this learning environment, active listeners become aware of interpretations from other students and continually try to adjust their thoughts and questions until understanding is reached among the students. This implies that active listening provides opportunities for students to construct meaningful mathematics.

From my study I imply that synergistic argumentation provides maximum learning for all students. But for it to function and provide maximum mathematical learning the classroom environment must be socially conducive and safe for students to openly interact in mathematical constructions. To say it another way, **in order to incorporate synergistic argumentation within the classroom environment, teachers must create a socio-mathematical atmosphere that is safe and at the same time challenging.** The best learning takes place when students experience low threat and high challenge. Caine and Caine (1997) indicate that the students need an environment where the mind-set is one of "relaxed alertness" (p. 123). "Relaxed alertness" can best be understood by using the description of a runner. While a runner is running at high speed, he challenges himself to beat previous records, and yet at the same time establishes a rhythm for his body that is comfortable. While in rhythm the runner still notices others around him and can change his pace accordingly and establishes another rhythm. In learning environments students can continually be challenged while at the same time feel safe enough to discuss ideas and take risks. The research site for this study promoted an atmosphere of "relaxed alertness."

Kamii (2000) argues that the nature of the learning environment in mathematics classrooms greatly influences children's interest and motivation for learning. Environments that promote autonomous thinking and resolution of conflicts create opportunities for children to construct personally meaningful mathematics. Duckworth (1996) states, "...the development of intelligence is a matter of having wonderful ideas and feeling confident enough to try them out..." This implies emergence of wonderful mathematical ideas and confidence can be fostered in an atmosphere of "relaxed alertness" while students are engaged in synergistic argumentation.

Further Research

Reflecting back on my research several ideas for further research became apparent. One is the question, ***What does it mean for students to attend?*** In a traditional classroom the students generally sit quietly in their desks and wait for the teacher's instructions. They hear the procedures to be mimicked and then follow those procedures on their worksheets. Students were considered to be "on task" if they were sitting quietly at their desks.

Throughout the course of my research within the second-grade classroom, I noticed that students were continually moving around and seemed to be doing other things than mathematics. I thought students were "off task" or that there was no way they could be listening, but suddenly they would make a significant contribution to the discussion or argument. Sometimes a student would seemingly be "off task" while another student was asking questions about someone else's solution. The student I thought was "off task" would respond out-of-the-blue with, "I know what they mean," and then explain the same strategy in a different way in which the other students could understand. During the trapezoid triangle whole-class discussion two boys who were not in the circle, but in close proximity and were talking and moving around on the floor. After one student explained how she started the triangle, one of the boys went over to where she was and asked key questions that aided in her ability to better explain her strategy. Therefore, I think it necessary to do further research on the idea of attending.

A <u>second</u> interesting phenomenon that continued to emerge as I conducted my research observations involved two similar tasks which were five months apart, Episodes #5 and #6. I refer to this phenomenon as "**incubation**" because it is related to the five-month time elapse between the two tasks. The first task, Episode #5, when given created a lot of conflict and several students were unable to solve it. One of those students was Brad. Five months later the similar task, Episode #6, was given and the students were able to solve it without any conflict or questions. Brad had no problem working the task.

Note, in contrast to Kathleen's classroom is the traditional classroom. In a traditional classroom if students were unable to work the problems one day the teacher would usually address those problems with the students the following day rather than let significant time elapse. But in Kathleen's classroom I observed several times throughout my research that she might give a task that created conflict and went unresolved that day. For whatever the reason she was not always able to come back to it the next day or even in the same week. So what took place in Brad's mathematical thinking during those five months that ultimately provided the solution when confronted again with the similar task?

I think this challenges the notion of bringing closure to lessons and warrants further research.

A third area that I think warrants further research emerged during my observations of the second-grade classroom. Several times the teacher and students, because of time limits, were unable to finish a discussion about a mathematical task and the various strategies that had been presented. Later, maybe days or weeks, Kathleen would ask the students, **"Remember when..."**? Interestingly those time lapses did not produce memory lapses. Students and Kathleen were able to pick up the discussion right where they had left off just as if the prior discussion had just taken place. The students did not forget. They were able to immediately carry on a mathematical discussion without having to be reminded (or without reviewing) what had previously taken place.

What does this mean for educators who think students forget over the summer or during a break? Is it necessary to spend the first six weeks of school reviewing what students did before? Maybe the types of tasks or the goal of the teacher needs to be investigated?

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APPENDIX A

The following are the IRB forms that students and parents signed.

Date:

Dear Parent/Guardian of

As a research assistant for Dr. Anne Reynolds, in the College of Education, University of Oklahoma, Norman campus I, Darlinda Cassel, am presently conducting research on how children learn mathematics. As a part of this research I would like permission to interview your child **during the 2000-2001 school year.** Questions asked during this interview are designed to explore how students construct meaning for various mathematical ideas. The interview is no way intended to evaluate correctness of responses but rather to explore how a student is thinking mathematically. The interview will be arranged in cooperation with your child's teacher at a time that will best fit your child's classroom routine with as little disruption as possible. Your child's responses will not be used in any way for grading purposes in her/his classroom. Rather it is hoped that by encouraging your child to explain how she/he is thinking about the tasks used in the interview a deeper understanding of how students learn mathematics will be developed. To allow for later analysis by myself as researcher the interview will be video recorded.

By using the video tape, I would like to communicate the results of this research effectively within the mathematics education community. I would like **your permission and your child's assent to show selected portions of the videotaped interview.** Your child's name will not be used and tapes will be edited to preserve the student's anonymity whenever possible. Please let me emphasize that the use of these tapes is for professional purposes only. No public showing will be considered.

If for any reason you have reservations about this please call Dr. Anne Reynolds at 325-0445 or me at 771-5484. **If you have any questions regarding your rights as a research participant please call the Office of Research Administration at (405) 325-4757.**

Sincerely,

Darlinda Cassel

Date________________________________

Signature____

ASSENT FORM

For research being conducted under the auspices of the University of Oklahoma Norman Campus

APPROVAL FORM FOR THE INTERVIEW

I give permission form my child, ______________________________, to be interviewed as described above.

Signed: ____ Parent/Guardian

Date: __________________

APPROVAL FORM FOR USE OF VIDEOTAPE

I give permission for a videotape to be made of my child,

___________________________,
during interviews to be used as described above. I understand that I may withdraw this permission at anytime.

Signed: ____ Parent/Guardian

Date: __________________

If you have questions regarding your rights as a research participant please call the Office of Research Administration at (405) 325-4757.

Date:

(Parent/Guardian: Please read this to or with your child as needed)

Dear

As a graduate student at the University of Oklahoma I am interested in how students learn mathematics. To help me understand this better I would like to talk with you individually. The questions that I would like to ask you will help me to see how you are thinking about your mathematics. I do not want to see if your answers are right or wrong, so the ideas you share with me will not be used in grading you. Instead, I want to understand what you are thinking about the problem. Then I can share these ideas with other teachers so that they can help students like you learn mathematics. I will arrange to do this at a time that best fits other things you are doing in the classroom. Also, so that I can better remember what you share with me I want to video record what you say and do as you answer the questions.

Sometimes as I talk with teachers about the way students are thinking about mathematics, it is helpful if they see what you are doing on tape. So, I would like your permission to show parts of the tape, when I think it might help teachers to understand your ideas better. I will not use your real name when I do this. Instead I will ask you to make up a name for me to use.

If you have any questions about this, please talk with Dr. Reynolds or me when we visit your classroom or call us at 325-0445.

Sincerely,

Darlinda Cassel

ASSENT FORM

For research being conducted under the auspices of the University of Oklahoma-Norman Campus

APPROVAL FORM FOR INTERVIEW

I, ______________________________________, I agree to being interviewed as described in the letter I have just read or had read to me.

Signed: ______________________________ Student

Date: _____________

APPROVAL FORM FOR USE OF VIDEOTAPE

I, ____, agree that videotapes made can be used to help teachers to understand better how students think about mathematics. I understand that if I later change my mind about the tapes being shown to other teachers I can withdraw this permission.

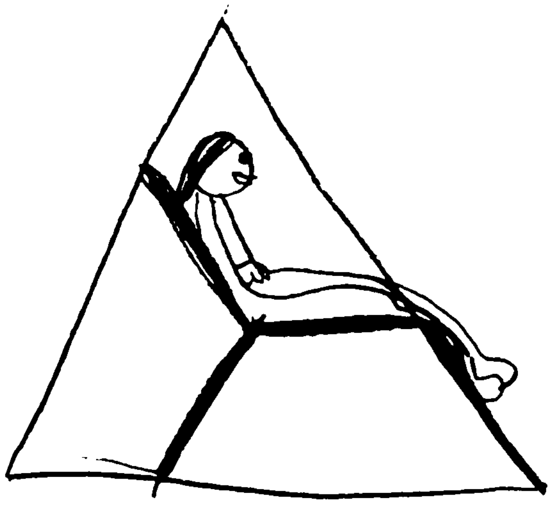
Signed: __________________________________ Student

Date: _______________

If you have questions concerning your rights as a research participant please call the Office of Research Administration at (405) 325-4757.

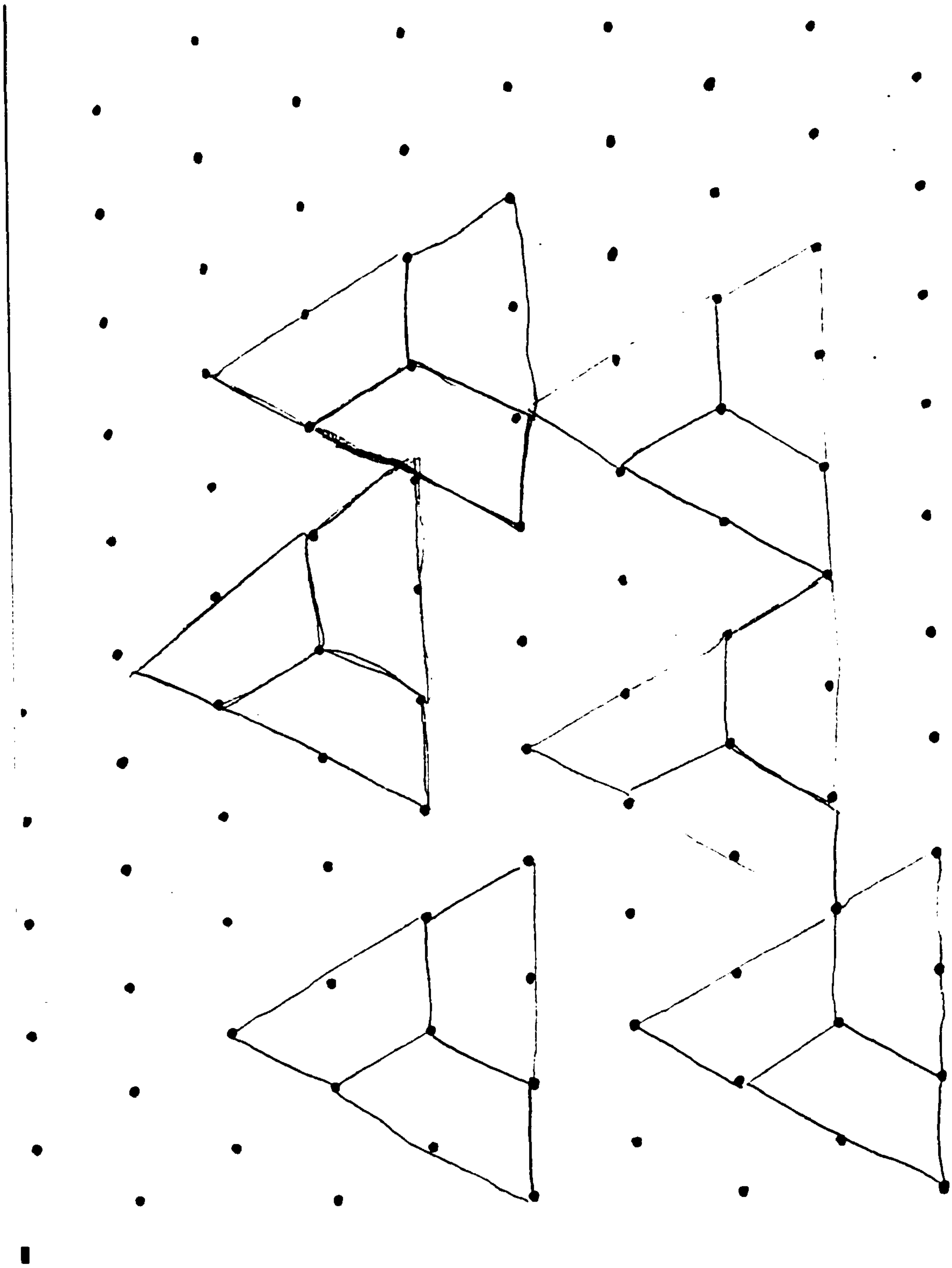
APPENDIX B

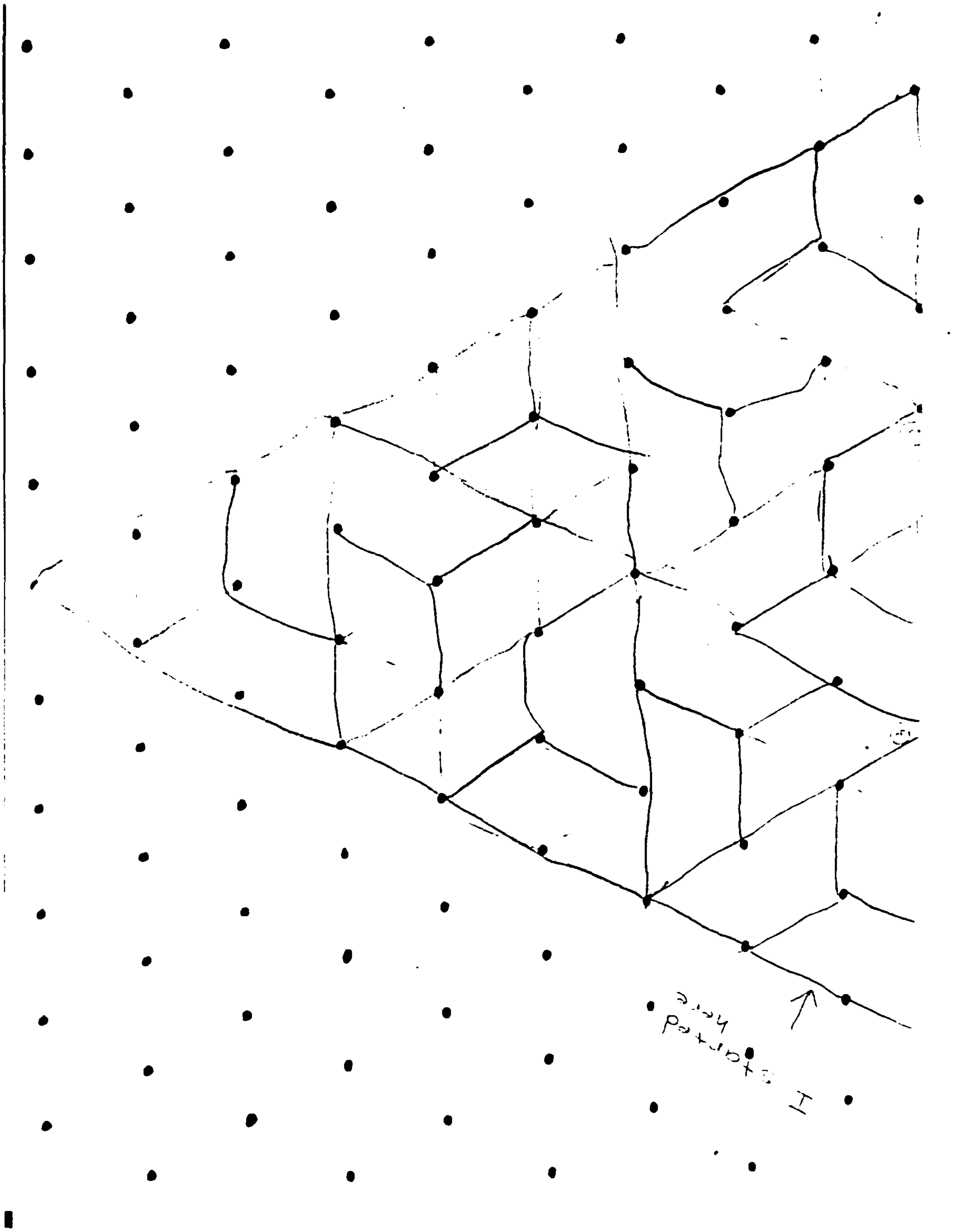
I mentioned that the students try to help each other understand their ways of thinking. One day Kathleen had as a class opener a "Quick Look" activity. She showed a picture on the overhead for three seconds, covered it up, and asked the students what they saw. The students discussed the various ways of seeing. On this particular day, Kathleen displayed the trapezoid triangle as the quick look object. Several students commented that they saw a triangle with three lines in it. One student said that she saw the lines inside like a chair. The other students could not see that so she went up and drew this for them so they could understand from her perspective. Helping others make sense of their ideas is a norm in this classroom.



APPENDIX C

The following pages are copies of the students' work. The first two pages are the trapezoid triangles (Episode #3). The first one is the three trapezoid triangle and the second one is an example of the students trying to make a larger trapezoid triangle. The next three pages are the sucker and valentines tasks (Episode #6 and #7 respectively). The last two pages are the flowers task (Episode #5). If you look at the last page, Jan had several erased answers before deciding on the final answer.




I started here

Name______________________ Date______________

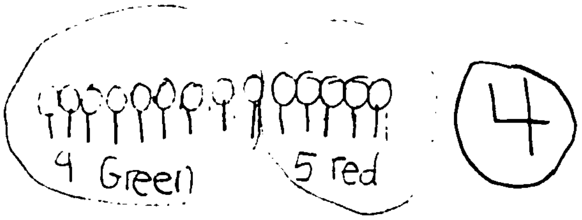
Brian had 9 green suckers and 5 red suckers. How many more green suckers did he have than red suckers?

ther are 4 More 9 Suckers

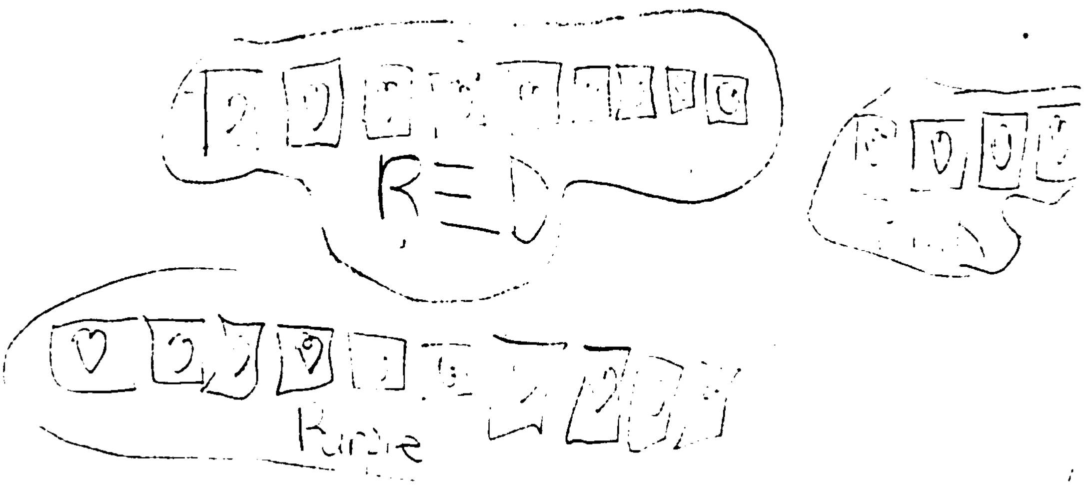
Dawn made 23 Valentines using purple, pink and red colors. She made two more pink Valentines than red Valentines. She made three less purple Valentines than red Valentines. How many purple Valentines did she make? 9 How many pink Valentines did she make? 10 How many red Valentines did she make? 4

Name____________________ Date______________

Brian had 9 green suckers and 5 red suckers. How many more green suckers did he have than red suckers?

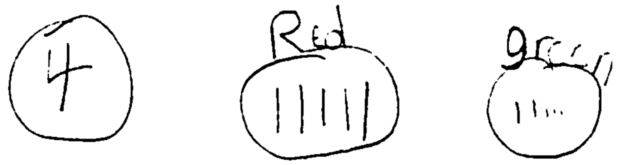


Dawn made 23 Valentines using purple, pink and red colors. She made two more pink Valentines than red Valentines. She made three less purple Valentines than red Valentines. How many purple Valentines did she make? __1__ How many pink Valentines did she make? __10__ How many red Valentines did she make? __4__

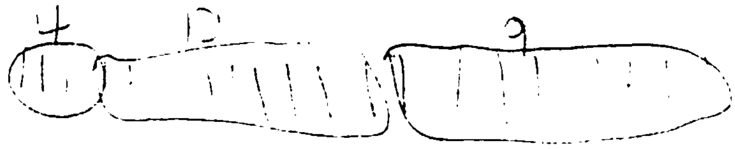


Name____________________ Date__________

Brian had 9 green suckers and 5 red suckers. How many more green suckers did he have than red suckers?



Dawn made 23 Valentines using purple, pink and red colors. She made two more pink Valentines than red Valentines. She made three less purple Valentines than red Valentines. How many purple Valentines did she make? 9 How many pink Valentines did she make? 10 How many red Valentines did she make? 4



Name ______ Date ________

There are 8 red flowers and 4 yellow flowers. How many more red flowers are there than yellow flowers?

4

I know that 4+4 = 8 so that would make 4

Name ______ Date________

There are 8 red flowers and 4 yellow flowers. How many more red flowers are there than yellow flowers?

|||| ||||||||

?

12

I useed a talleymarks

APPENDIX D

Mathematical Thought

Sense making was and is a very important goal for all students in this particular second-grade learning environment. The teacher tries to choose tasks that will promote new ways of thinking, creating thinking strategies, and constructing ten as a unit. One activity that encourages students to develop thinking strategies for addition and subtraction is the balance task (Episode #4). My dissertation focused on the whole-class discussion but I think it is important to note here that the students' sharing of solutions was only a part of their mathematical activities. During the collaborative pair work, students interacted with each other for 20 minutes trying to put numbers into relationships and trying to figure out all the different possible combinations for these numbers. The balance task activity sheet is divided into four sections with one balance in each section. At first glance one might wonder, "Why would second-graders be given only four problems to do"? Looking at the example below (see figure 11), try to think of all the possible number combinations (they are numerous) while keeping in mind that children have not constructed the same mathematical knowledge as adults.

(8 , 9 / ___)   (10 , ___ / 17)

(___ / 7 , 9)   (9 , 9 / ___)

Balance task

In pairs the students discuss each other's ideas thus "doubling" in effect their work and then they come together as a whole-class to discuss everybody's ideas. During the whole-class sharing time they are trying to make sense of all the other solutions presented while at the same time trying to formulate appropriate questions and challenges. The students in small and whole-group discussions are making comparative judgments about **various** solutions. They must decide if their solution is different in order to present it. Therefore, they have done much more "math" than students who have been given a worksheet of twenty practice problems. These second-grade students' computations are embedded in their problem-solving, not to mention their sense making.