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THRUST REQUIREMENTS FOR DYNAMIC POSITIONING OF
A SEMI-SUBMERSIBLE DRILLING VESSEL.

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THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

THRUST REQUIREMENTS FOR DYNAMIC POSITIONING OF A SEMI-SUBMERSIBLE DRILLING VESSEL

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements

for the degree of

DOCTOR OF ENGINEERING

By

Terry Don Petty

Norman, Oklahoma

1975

THRUST REQUIREMENTS FOR DYNAMIC
POSITIONING OF A SEMI-SUBMERSIBLE
DRILLING VESSEL

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ABSTRACT

This study was concerned with determining the thrust forces necessary for maintaining a free-floating drilling vessel in position while drilling a well in deep water. Drilling operations require that the vessel be held within a given radius of the sea-bed position of the well. Typical mooring systems for maintaining the position of the vessel consist of eight chains held by anchors. The weight and cost of these mooring systems imposes a limit to their use of about 1200 to 1500 feet water depth.

The semi-submersible drilling vessel is treated as a rigid body with six degrees of freedom constrained by an arbitrarily symmetric mooring system of anchor chains and fixed anchor points. The primary forces acting on the vessel are buoyancy, wind, ocean currents and waves. As a result of the exciting forces and subsequent motion, the inertia forces due to the mass of the vessel and the added mass of the displaced seawater, drag, damping effects, and perturbations of the mooring line forces must be considered. Second order terms of the wave theory are retained to improve the accuracy in relatively short wave lengths. Further, the rotations, roll, yaw, and pitch are presented in terms of the Euler angles to retain the non-linear effect of small angle theory. The transcendental catenary equations which

are basic when describing mooring systems are approximated with a high degree of accuracy by Chebyshev polynominal series. The equations of motion based on Newton's Second Law result in a set of six second order differential equations of the ordinary type which are non-linear. This system of equations is solved numerically which simultaneously give the instantaneous mooring line forces. The horizontal components of the mooring forces are summed and equated to a thrust vector whose magnitude and direction are the desired result for a given sea and wind condition. The motion response is confirmed by testing in a model basin.

ACKNOWLEDGEMENT

The author wishes to express his appreciation to Dr. A. W. McCray, who served as advisor in this research, for his continued support, interest and guidance in the completion of this effort.

Appreciation is extended to the Shell Oil Company for their fellowship during my tenure as a graduate student.

The semi-submersible vessel used in this analysis is the D/B Ocean Prospecter owned by the Ocean Drilling and Exploration Co.. In particular, my appreciation is expressed to Mr. A. J. Laborde and Mr. G. H. Troxell, Jr. and to ODECO for granting permission to model the vessel, furnishing the necessary physical data, and considerable financial assistance.

The theoretical reviews and comments by Dr. Bruce J. Muga, Duke University and Dr. R. O. Reid, Texas A&M University have been invaluable.

Finally, I would like to thank my wife Katie and my sons, Mike, Shannon, John and Bruce who made the many sacrafices required to achieve this academic goal.

Terry Don Petty

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CHAPTER I

INTRODUCTION

The art of drilling and producing reserves from structures located offshore has progressed at a spectacular pace. In 1949 there was one mobile unit capable of drilling in twenty feet water depths [17]. Presently, this number exceeds 200; the development of which has required an investment in excess of a billion dollars. Many of these units are capable of conventional drilling with subsea equipment in water depths of 1200 to 1500 feet. One such vessel, a dynamically positioned ship has successfully cored the ocean floors in water depths ranging up to 20,000 feet.

The trend in new construction, however, has been toward the semi-submersible drilling vessel. The vessel shown in Figure 1 is a recent design; however, many of the features shown are typical of most semi-submersibles. This trend is due primarily to the fact that the industry is actively exploring for reserves in world-wide areas where the environmental conditions of the seas and prevailing wind conditions are adverse to what has been known as normal operating conditions. This type of vessel has proven to be less susceptible to the excitation of the seas which results in motion with smaller amplitudes. Therefore, the normal operating range is extended to somewhat more adverse conditions than that experienced by drilling ships.

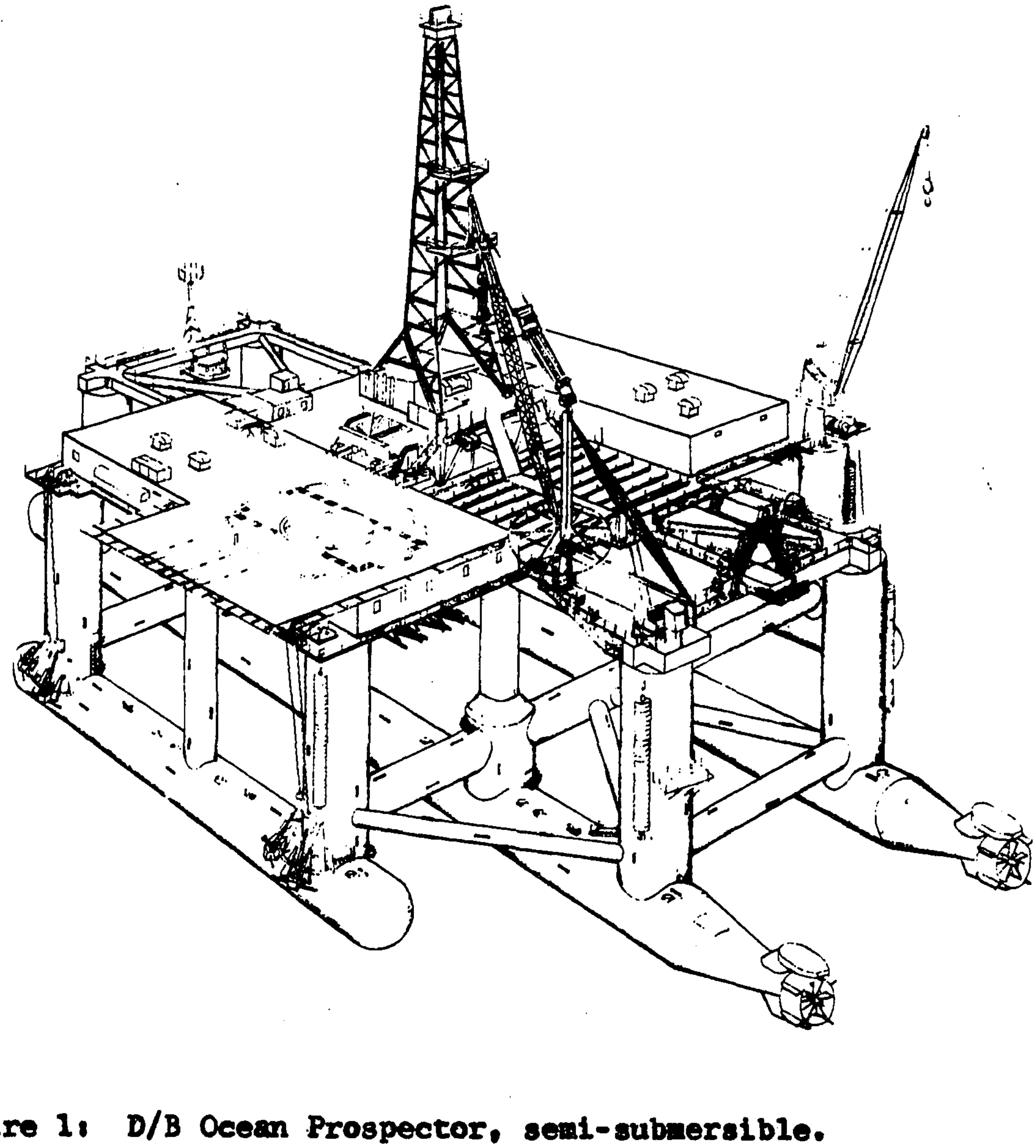


Figure 1: D/B Ocean Prospector, semi-submersible.

In these areas, the integrity of the mooring system and its capability to maintain the vessel on station have been the predominant factors in loss of operating time. This loss of time which increases the cost of the well may be categorized as follows:

a) Time that the rig is not operating due to precautionary decisions.

b) Time lost due to repair of the mooring system and damage to subsea equipment which may have been caused by failure of a mooring line or an anchor slipping.

In general, the people responsible for drilling the well give preference to (a) rather than take the chance of excessive damage to equipment, most of which are not off-the-shelf items. This philosphy has proven to be a valid and less costly one for present-day designs mainly because the repair must be done in calmer seas when drilling efficiency is at a maximum. On the other hand, it is not an optimal situation when a drilling operation of which costs may exceed $50,000.00 daily may be shut-down solely on the decisions of people whose judgement and past experience may be questionable.

A third category which contributes to the cost of a well is the setting and retrieving of anchors; again, this must be done in a sea that is relatively calm. Much time has been lost just waiting. In many areas, the seabed is too hard for the anchors to provide sufficient holding power. Typical remedies for such situations have been to set anchors in series (one or two a distance behind the first) or to drill in permanent type anchors or piles. These alternatives have taken expensive time --- days, weeks and even months.

Most engineers agree that the design limit of a mooring system has been reached in water depths of 1200 to 1500 feet. And this limit would be in a "mill-pond" compared to conditions in the offshore areas of western Canada, the North Sea, the Grand Banks and many others. This is primarily because of just the shear bulk and weight of equipment required. To drill in deeper depths, the conventional mooring system must be replaced in total with some other method for maintaining the drilling vessel on station. To drill in those areas with extreme and predominatly adverse sea conditions and intermediate water depths the mooring systems need an assist to keep the vessel within a reasonable tolerance of location in order that drilling may continue at an economical pace.

An approach to this problem is the concept of dynamic-positioning, a mechanical system of thrusters which could provide the same restoration force as a system of mooring lines. This idea is not new. In the 1950's, small coring vessels were so equipped to secure cores from the ocean depths. In 1963 the National Science Foundation proposed the design and construction of the "Mohole Platform". This vessel was a semi-submersible type to be equipped with dynamic positioning to maintain station in the open ocean for long periods of time for drilling through the earth's crust. This idea was abandoned in 1968 in favor of coring a series of much shallower holes without means for re-entry. A re-entry technique has since been developed. This program which uses a ship with dynamic positioning has been regarded by many as being totally successful.

The level of sophistication of such a system could vary widely, ranging from sight and manual control to a fully computerized automatic control system complete with acoustic transmitters and receivers that permits continuous monitoring of the vessels relative position.

The incentive and motivation for the engineer to use such a system for positioning should be easy to see. Such a mechanical system could very well eliminate some of the human factors of decision making as to when a temporary well abandonment should be initiated. The capability of self-propulsion permits self-setting and retrieval of anchors in much rougher seas than those that permit an anchor-handling crew and workboat to work. In fact, if this requirement is eliminated, one of the chief functions of the supply vessel is also eliminated. This appears to be a step in the right direction in reducing the overall cost of drilling offshore. The above is in addition to having the capability of getting the job done. Then too, for a system that is something less than adequate in many areas, a mooring system of anchors, chains, windlasses, lockers, etc. is expensive. Depending on the length and size of chains and anchors, investments easily exceed a million dollars to outfit a single vessel.

By the same token the concept of dynamic positioning is not without problems. The technology is neither fully developed nor proven in the field. It was mentioned earlier that ships have been used for coring in deep water with similar equipment. One must realize the two essential differences in this type of operation as compared to drilling an oilwell:

a) In deep water, say in excess of 3,000 feet, the response time for the system to react to an excitation is not nearly as critical as in shallower depths. The response time in depths of 400 to 600 feet and shallower must be relatively fast and above all ---reliable. The critical nature is due to the angle at which the marine riser deviates from vertical at the wellhead [33]. Ships which core at extreme depths do not have this restriction. It is easy to see that in deeper water the horizontal displacement may be a much greater distance without causing too severe a dogleg as the drillstring enters the wellbore at the seabed.

b) A ship without the restriction of the marine riser, the associated choke and kill lines and other paraphernalia that connects to the subsea system may change its heading in order that its resistance to the elements is at a minimum. A change in heading allows full use of the main propulsion which in most cases would be much more than adequate.

This leads to the question of interest in this study. How much thrust is required to keep a semi-submersible on station when subjected to the elements? This problem was encountered in designing the "Mohole Platform" and was resolved by model testing techniques [25]. The approach taken here will be to model the motion of the vessel constrained by a mooring system analytically.

Statement of the Problem

In order to determine the thrust needed to keep the semi-submersible within a given radius of the well, the equivalent restoration forces of the mooring system must be resolved. The total thrust required may then be equated to the vector sum of the horizontal components of the moorings. The tensions that arise in the mooring lines are directly coupled to the displacements of the vessel as functions of time. In essence, a set of differential equations must be formulated that will simultaneously yield the motion and the mooring restoring forces.

Formulating the Equations of Motion

The ordinary differential equations that describe the vessel motion are formulated on the basis of Newton's Second Law, i.e., equating to the motions the sum of the forces (and moments) acting on the body. The total force is the sum of the forces arising from dynamic body motions, damping, hydrostatic restoring, excitation, and mooring restoration.

The force exerted by the vessel in accelerating the water particles gives rise to equal and opposite forces by the water acting on the vessel. These forces arise from the dynamic body motions which at the same time displaces the surrounding water to a new position [27]. The mass of the water displaced is usually expressed as the added mass.

The damping forces involve the dissipation of energy in the form of a drag force and results from the relative velocity of a body to the velocity of the surrounding fluid. The most significant types are thought to be dynamic damping and skin friction. The latter will be assumed negligible.

The hydrostatic restoring forces arise from continuous change in the wetted lengths of the vertical columns. These forces act in the vertical direction only.

The excitation forces are due to the waves, wind and current. Those forces due to waves result from integrating the water pressure gradients over the wetted body. Their magnitude will depend on the many parameters of wave theory and the theory itself. Wind forces act on the exposed areas above the water and will be a function of the vessels position, wind direction and wind velocity. Current may be treated either as a constant or as a variable and may be added directly to the relative velocity term and will considerably affect the damping and displacement mechanisms.

The restoring forces that arise in the mooring system constitute a major problem within itself. The differential equations as given by Pugsley [35] and Routh [37] that describe a chain suspended between two points have been determined as being intractable. However, in static equilibrium a flexible chain is known to hang in the form of a catenary which is described by a transcendental equation. A new equation, anchor tension as a function of displacement will be derived in terms of a Chebyshev polynominal expansion. This equation which gives the tension explicitly may be included in the equations of motion.

Once the forces have been determined, the moments around the vessel's center of gravity will be given by multiplying the total force component on a particular member by the appropiate moment arm.

CHAPTER II

LITERATURE REVIEW

Extensive literature is available on the motion of ships at sea, wave theory, and hydrodynamics. The existing state of the art and its shortcomings up to 1961 along with a huge bibliography is presented in a monograph by Korvin-Kroukovsky [22]. A recently published text by Muga and Wilson [28] serves to update the fore-mentioned work with an attempt to bridge the gap between present-day design practices and available analytical techniques. These references which contain many important contributions are generally available and will not be explicitly discussed here.

The magnitude of research works pertaining directly to the subject of motion and station-keeping of a semi-submersible drilling vessel is somewhat limited. This is primarily due to the fact that the semi-submersible is a relatively new concept which was introduced by the conversion of the submersible unit "Blue Water I" in 1961. This was followed by successive construction of new units by various drilling contractors. However, the mainstream of published research in the area was undertaken 1963 by the National Science Foundation with its proposed "Mohole Platform" which was later abandoned. Much of the literature available now is a direct consequence of the "Mohole" study.

The following review is an attempt to summarize the contributions of experimental and theoretical research which relates to the present investigation. Brief summaries of five motion studies, three reports pertaining to mooring systems, and one on maneuvering and station-keeping are presented in chronological sequence.

Research of Pugsley (December 1949)

The general equations of motion (three) for a uniform chain suspended between two points are derived but not solved. These partial differential equations which are non-linear are found to be somewhat intractable in their general form. A simplification is made whose basis is on theory presented in an early dynamics text by Routh [37] who adopts a condition more amendable to solution. This condition assumes that a chain hangs in the form of a cycloid rather than a catenary. In this case, solutions as stated by Pugsley are possible only when the chain hangs with a dip that is small compared with its span. Within these limitations, theoretical results for the first three natural modes are resolved and compared with small scale experiments with good agreement.

Research of Bain (December 1963)

This report is just one of a series which investigates the motion response of the proposed "Mohole Platform". The equations of motion for this particular vessel are derived in detail with linear assumptions and an excitation provided by a simple deep-water wave train whose direction of progagation is arbitrary. Bain gives two analyses: 1) one set of equations include the non-linear dynamic damping, and 2) the second analysis is a simplification of the first with additional emphasis on linearity. The results of the simplified analysis are presented together with extensive test-tank data with resonable agreement with exceptions for high-frequency waves.

This semi-submersible was designed for dynamic positioning and for drilling in ultra-deep water. However, the equations of motion include neither the restoring forces of the positioning system nor other effects such as wind and current.

Research of Gormally and Pringle (September 1966)

This report bears on the static and dynamic response of suspended arrays and their mooring systems to varying currents and initial conditions. Bipod and tripod mooring systems are examined in some detail. It is shown that a considerable increase in mooring rigidity may be obtained by using a neutrally buoyant cable which reduces the sag in the individual anchor lines. Numerical calculations are carried out for the motion of tetrahedral and spherical bodies submerged beneath the sea surface and subjected to steady current loadings.

Research of McClure and Hove (December 1966)

The mathematical representation of motion for the "Mohole

Platform" moving over the surface of the sea under the control of an automatic positioning system is presented. The authors conclude that the equations describing thrust variations are functions of a) relative velocity, b) the angle of the thrust vector to flow direction, and c) the angle of hull relative to flow direction (where the configuration of the hulls pertains only to the "Mohole Platform"). An equation for maximum thrust required for a given sea and wind condition is given in terms of the above parameters. Empirical coefficients were determined from towing and self-propelled maneuvering tests of a scale model in addition to wind tunnel tests.

Further, a brief non-technical description is given for the proposed positioning computer and motor control systems.

Research of Mercier (October 1968)

A procedure is presented for the prediction of the longitudinal motions in regular head seas of ocean platforms supported by slender vertical floats. Experimental information describing the hydrodynamic forces acting on the floats is used to solve the equations of motion for harmonic oscillation in heave, surge, and pitch. These hydrodynamic forces are obtained from tests conducted on a single isolated float. They are expressed as two separable parts, one due to the disturbance imposed by the waves acting as though the vehicle were completely restrained and one due to the motion of the vehicle as though there was no wave disturbance. Mercier found that the measured motion of a free four-float vehicle compares favorably with the motion computed under the above procedure.

Research of Burke (May 1969)

An analytical model is developed to compute the motions of semi-submersible drilling vessels in waves for a variety of configurations. The model is derived from a linear representation of motions, simple ocean waves, and forces. The vessel is represented as a rigid space frame composed of an arbitrary number of cylindrical members with diameters, lengths and orientations equally arbitrary. Forces derived from a linear anchoring system and hydrodynamic principles are superimposed in the equations of motion. These equations are solved numerically for motion in six degrees of freedom for a simple sinusoidal wave-train with given amplitude, period, direction of propagation and water depth. Results from the analysis of three semi-submersibles are compared favorably with results of available model test data.

Research of Muga (May 1969)

This paper presents a linear theory of motion for a rigid ship-shape vessel with a linear spread-type mooring system which is applied to a construction barge moored in the open ocean. Water level variations at three locations, ship rotations, accelerations, mooring forces, and wind velocity were measured in a Neumann Sea State 2 and 3.

Three recordings representing nearly beam-on, quartering, and stern-on seas were analyzed using time-series techniques to provide amplitude-response operators for all of the ship's motions and mooring forces. The amplitude-response operators were computed from a linear theory based on the slender-body approximation. The coupled equations of motion for the six degree of freedom system are derived and solved analytically. The results in the form of complex-response operators are compared with those obtained from actual measurement. General agreement is found with exception to the yaw motion. In addition, discrepancies were found to exist for certain headings and wavelength combinations which are explained by the presence of cross-coupling terms and empirical values of damping coefficients. Muga suggests that linear theory is adequate for sea states of up to 3.

Research of Pangalila and Martin (May 1969)

For a semi-submersible platform, a comparison is made between results obtained from measurements at sea and those predicted from model basin tests. The different techniques used for model testing are discussed. Data pertaining to "Anchor Tension Versus Surge Motion" are illustrated for different chain weights, number of mooring lines, water depths, and pretensions. Based on this information, the authors conclude that heavier chain is most effective for shallow water use while relatively light-weight chain is most satisfactory for use in deeper water.

Research of Wang (June 1969)

A theoretical method developed by Wang is presented to derive and solve the equations of motion of a semi-submersible type platform constructed from assemblies of vertical and horizontal cylinders. The platform is assumed to act as a rigid body responding to six degrees of freedom in simple sinusoidal deep-water waves progressing from an arbitrary direction. The solutions to the equations of motion are used to construct transfer functions or response ratios. With knowledge of the response ratios, the motion responses and the forces exerted on the structure for a given sea state are computed according to the linear superposition principle. The platform in this study is free to drift; i.e, the constraints of a mooring system are not included.

Wang's approach is very similar to that just presented by Muga.

CHAPTER III

THEORETICAL CONSIDERATIONS

The "Literature Review" in Chapter II indicates a strong emphasis on simple wave theory and linearity for deriving the equations of motion. General agreement with experimental data has been found insofar as to when the testing procedures had been held rigidly within the bounds of critical theoretical assumptions. This study will also have certain simplifying assumptions which has been regarded as standard procedure. However, its general course will tend in the opposite direction → higher-order wave theory and non-linearity.

Simple wave theory is based on the premise that motions are sufficiently small to allow the free surface boundary conditions to be linearized (this will be pointed out in the development which soon follows). In particular, terms involving the wave amplitude to the second and higher orders have been considered negligible. If the wave amplitude is large, the small amplitude considerions are not valid as discussed by Dean and Eagleson [5], and in any theory it is necessary to retain higher-order terms to obtain an accurate representation of the wave motion. In many applications offshore, it is the extreme non-linear waves which possess a wave height-wave length ratio that cannot be considered as small and which are of primary importance.

The second and higher corrections are well developed in texts [5,21,42] and other literature [39,40] and will be given only token coverage here. A full derivation of the second-order wave theory applied in this study is presented in Appendix A.

Potential Theory of Waves with Finite Amplitude

The theory of two-dimensional gravity waves on the surface of water is usually known as "potential theory of surface waves". The wave crests are of infinite length, uniformly spaced and parallel to each other, and are advancing in a normal direction at a certain celerity C. With infinitely long and uniform wave crests, the various functional relationships involved in the wave description remain unchanged for any change of position along the direction parallel to the wave crests. Only the horizontal distances x measured in the direction of propagation, and the vertical distances y have any effect on these functions. Such waves are referred to as being two-dimensional. Since the wave form advances at a constant celerity C, the wave height, and all other functions involved, in particular the pressure depend on time t as well as the x and y distances; that is, all relationships have the general form F(x,y,t).

Seawater is assumed to be inviscid and incompressible which gives two definitions

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad \text{(Irrotationality)}\ , \qquad \text{EQ. (1)}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(Continuity)}\ . \qquad \text{EQ. (2)}$$

Such motion is called "potential" and is characterized by the existence of a velocity potential Φ such that the horizontal component of water velocity may be defined by

$$u = -\frac{\partial \Phi}{\partial x}\ , \qquad \text{EQ. (3)}$$

and the vertical component by

$$v = -\frac{\partial \Phi}{\partial y}\ . \qquad \text{EQ. (4)}$$

Substituting Equations (3 and 4) into Equation (2) yields the two-dimensional Laplacian which must be satisfied for any form of potential flow,

$$\nabla^2 \Phi = 0\ . \qquad \text{EQ. (5)}$$

In order to obtain an explicit expression for pressure, one must integrate the Eulerian equations of motion

$$-\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\ , \qquad \text{EQ. (6)}$$

and

$$-\frac{1}{\rho}\frac{\partial P}{\partial y} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\ , \qquad \text{EQ. (7)}$$

to obtain the integrated form of the Bernoulli equation in terms of pressure

$$P = -\rho g y + \rho\frac{\partial \Phi}{\partial t} - \frac{\rho}{2}(u^2 + v^2). \qquad \text{EQ. (8)}$$

It is at this point that second order theory begins to deviate from simple wave theory. The small amplitude wave theory is based upon the assumption that all motions are so small that the quadratic terms u^2 and v^2 in Equation (8) are negligible. The complication comes from the fact that Equation (8) is a non-linear boundary condition; i.e., the pressure intensity at the wave free surface ($y=\eta$) is a constant. The differential equation (the Laplacian) remains the same; but the retention of higher order terms to be satisfied at the boundaries does indeed complicate the solution.

Formulation and the Solution to the Boundary Value Problem

Since the wave form propagates at a constant celerity C without change in shape, one may render the wave train stationary by superimposing a uniform current velocity in the opposite direction, therefore effecting steady motion with respect to a stationary frame of reference. This is illustrated in Figure 2.

The differential equation as given by Equation(5) has the following boundary conditions:

1) At the seabed, $y = -h$, flow does not occur, i.e.

$$v = -\frac{\partial\Phi}{\partial y} = 0 \text{ at } y = -h\ . \qquad \text{EQ. (9)}$$

2) At the free surface, $y = \eta$, a dynamic condition is the requirement that the total energy along the free surface must be a constant, Q. This condition is derived from Equation (8) by letting the pressure

be constant at the free surface, then

$$\eta + \frac{1}{2g}[(u-C)^2 + v^2] = Q \, . \qquad \text{EQ. (10)}$$

3) The final boundary condition which must be satisfied is a kinematic condition which requires that no fluid be transported across the stationary free surface. This condition is formulated by requiring the the resulting velocity vector to be everywhere tangent to the free surface, i.e. ,

$$\frac{\partial \eta}{\partial x} = \frac{v}{u-C} \, , \text{ at } y = \eta . \qquad \text{EQ. (11)}$$

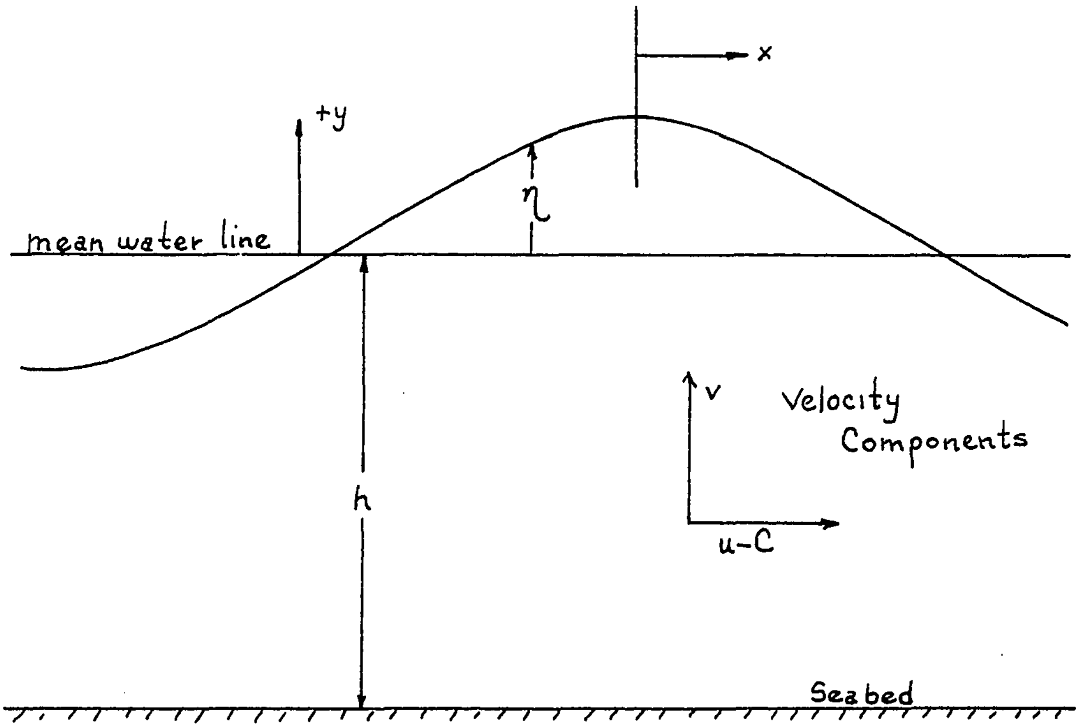


Figure 2: Stationary Wave System.

A perturbation technique illustrated by Dean and Eagleson [6] is used to solve the two dimensional boundary-value problem and is given full detail in Appendix A. The results are expanded to three dimensions by letting the horizontal x distance become $x\cos\alpha + z\sin\alpha$ where α is the angle of incidence. This is illustrated in Figure 3.

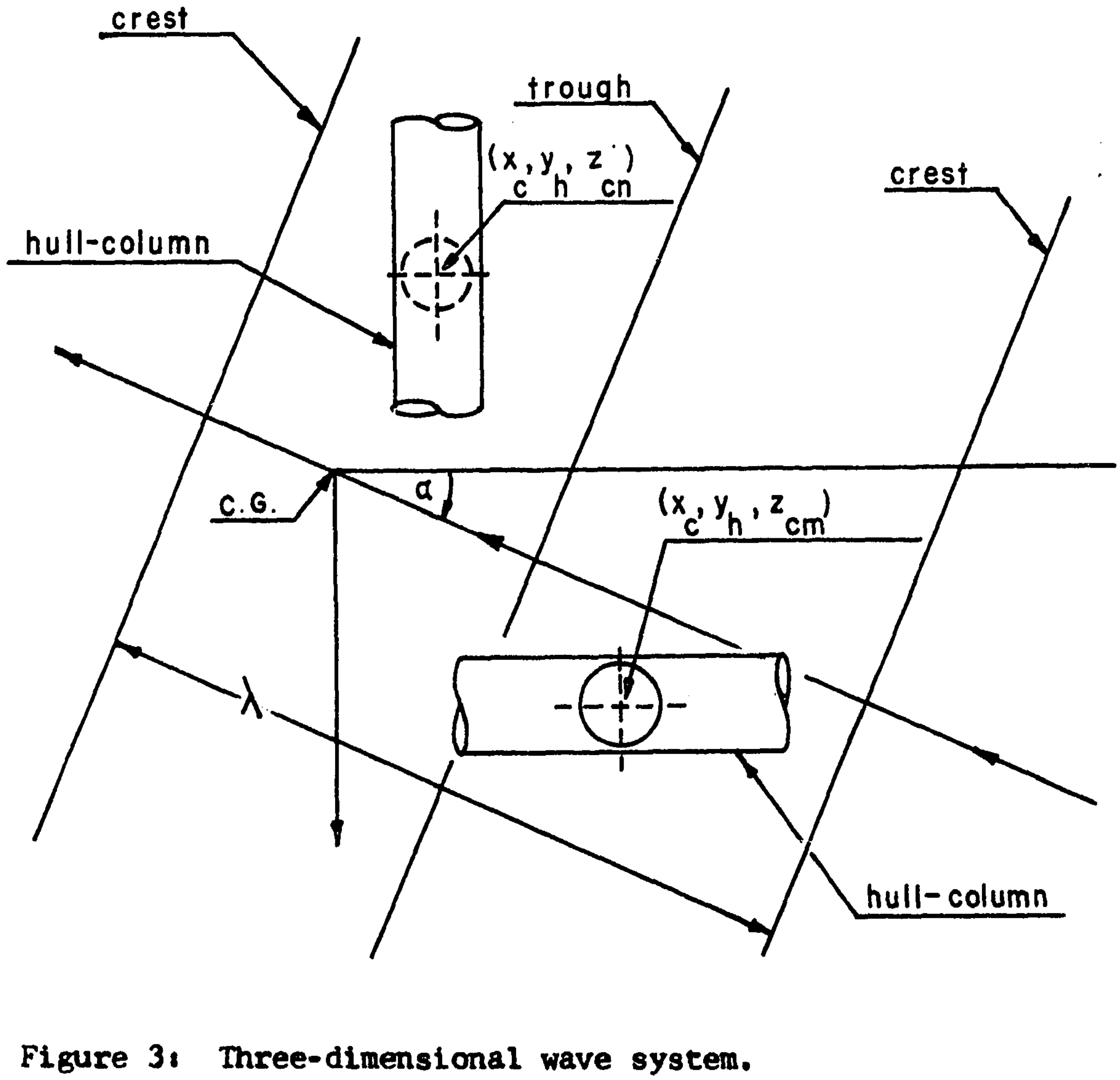


Figure 3: Three-dimensional wave system.

Note that the wave train in the above figure is shown to be propagating in a general negative direction. Thus, the results of the boundary-value problem are modified by changing the radial frequency σ to $-\sigma$.

A typical hull-column segment of the vessel is shown in Figure 4. The origin of the coordinate axis is placed at the center of gravity by shifting the origin (which was at the mean free surface) downward a distance b.

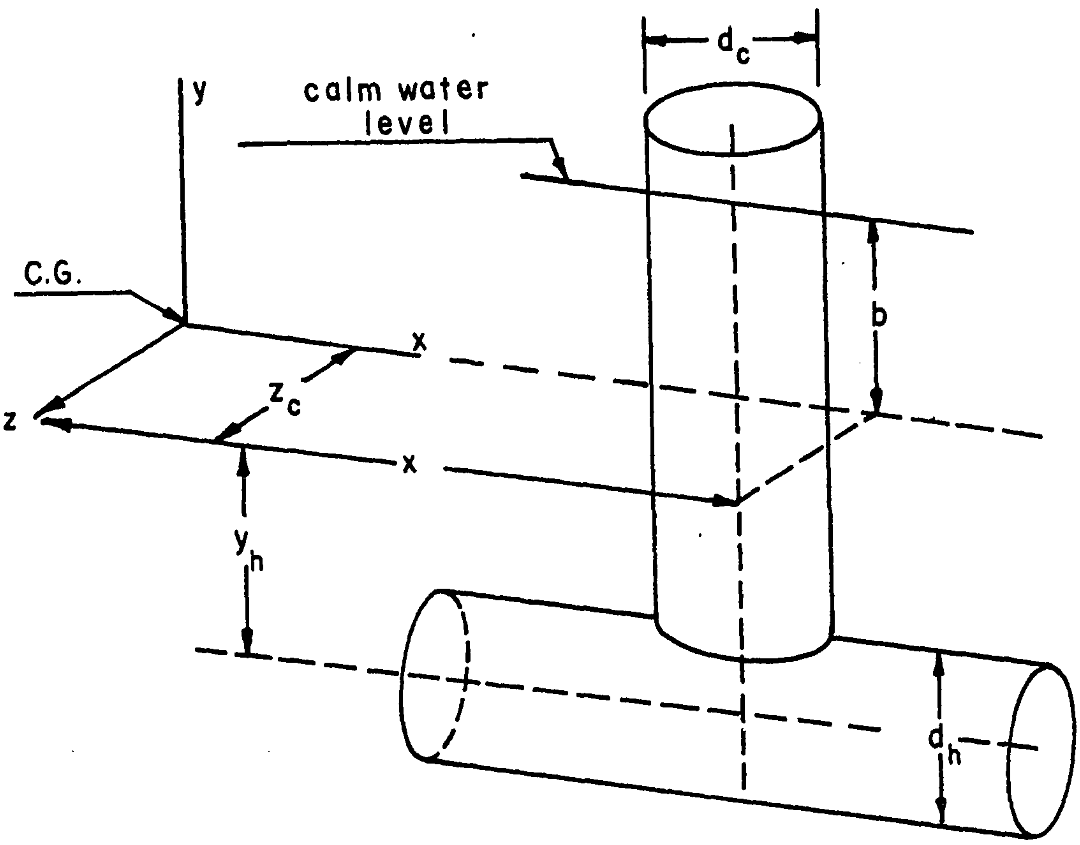


Figure 4: Typical hull-column segment.

The required results of the boundary-value problem modified to fit the situation are summarized as follows:

<u>Velocity Potential</u>

$$\Phi = aC\frac{\cosh k(h+y-b)}{\sinh kh}\sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+ \frac{3}{4}\left(\frac{\pi a}{\lambda}\right)^2 C\frac{\cosh 2k(h+y-b)}{\sinh^4 kh}\sin 2[k(x\cos\alpha + z\sin\alpha) + \sigma t] ,$$

EQ. (12)

<u>Wave Profile</u>

$$\eta = a\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+ \frac{1}{2}\,\frac{\pi a^2}{\lambda}\,\frac{\cosh kh}{\sinh^4 kh}(\cosh 2kh+2)\cos 2[k(x\cos\alpha + z\sin\alpha) + \sigma t],$$

EQ. (13)

<u>Particle Velocities</u>

<u>x - direction</u>

$$u = -\frac{2\pi a}{\lambda}C\cos\alpha\frac{\cosh k(h+y-b)}{\sinh kh}\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$-3\left(\frac{\pi a}{\lambda}\right)^2 C\cos\alpha\frac{\cosh 2k(h+y-b)}{\sinh^4 kh}\cos 2[k(x\cos\alpha + z\sin\alpha) + \sigma t] ,$$

EQ. (14)

<u>y-direction</u>

$$v = -\frac{2\pi a}{\lambda}C\frac{\sinh k(h+y-b)}{\sinh kh}\sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$-3\left(\frac{\pi a}{\lambda}\right)^2 C\frac{\sinh 2k(h+y-b)}{\sinh^4 kh}\sin 2[k(x\cos\alpha + z\sin\alpha) + \sigma t] ,$$

EQ. (15)

<u>z-direction</u>

$$w = -\frac{2\pi a}{\lambda}C\sin\alpha\frac{\cosh k(h+y-b)}{\sinh kh}\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$-3\left(\frac{\pi a}{\lambda}\right)^2 C\sin x\,\frac{\cosh 2k(h+y-b)}{\sinh^4 kh}\cos 2[k(x\cos x + z\sin x) + \sigma t]\,.$$

EQ. (16)

<u>Particle Accelerations</u>

<u>x-direction</u>

$$\dot{u} = \frac{\delta u}{\delta t} + u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} + w\frac{\delta u}{\delta z}\,,$$ EQ. (17)

$$\dot{v} = \frac{\delta v}{\delta t} + u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y} + w\frac{\delta v}{\delta z}\,,$$ EQ. (18)

$$\dot{w} = \frac{\delta w}{\delta t} + u\frac{\delta w}{\delta x} + v\frac{\delta w}{\delta y} + w\frac{\delta w}{\delta z}\,,$$ EQ. (19)

It is noteworthy that the particle accelerations including the convective terms are derived directly from the pressure gradient relations developed in Equations (6) and (7). Proceeding, it is verified that the above relations for particle accelerations can be expressed in terms of pressure gradients, or

$$\dot{u} = -\frac{1}{\rho}\frac{\delta p}{\delta x}\,,$$ EQ. (17a)

$$\dot{v} = -\frac{1}{\rho}\frac{\delta p}{\delta y}\,,$$ EQ. (18a)

$$\dot{w} = -\frac{1}{\rho}\frac{\delta p}{\delta z}\,.$$ EQ. (19a)

The pressure gradients which follow are fully developed in Appendix B, beginning with page 162.

Pressure Gradients

x-direction

$$\frac{\partial P}{\partial x} = -(A_1 + A_5)k\cos\alpha\cosh k(h+y-b)\sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$-2A_2k\cos\alpha\cosh 2k(h+y-b)\sin 2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$+2A_3k\cos\alpha\sin 2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$+4A_4k\cos\alpha\sin 4[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$+6A_5k\cos\alpha\cosh k(h+y-b)\cos^2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$\text{times } \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$+2A_5k\cos\alpha\sinh^2 k(h+y-b)\cosh k(h+y-b)$$
$$\text{times } \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t] \;,$$

EQ. (20)

y-direction

$$\frac{\partial P}{\partial y} = (A_1+A_5)k\sinh k(h+y-b)\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$+2A_2k\sinh 2k(h+y-b)\cos 2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$-2A_3k\sinh 2k(h+y-b)$$
$$-4A_4k\sinh 4k(h+y-b)$$
$$+2A_5k\sinh k(h+y-b)\sin^2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$\text{times } \cos[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$
$$-6A_5k\sinh k(h+y-b)\cosh^2 k(h+y-b)$$
$$\text{times } \cos[k(x\cos\alpha + z\sin a) + \sigma t] \;,$$

EQ. (21)

z-direction

$$\frac{\partial P}{\partial z} = -(A_1+ A_5)k\sin\alpha\cosh k(h+y-b)\sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$-2A_2k\sin\alpha\cosh2k(h+y-b)\sin2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+2A_3k\sin\alpha\sin2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+4A_4k\sin\alpha\sin4[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+6A_5k\sin\alpha\cosh k(h+y-b)\cos^2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$\text{times } \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+2A_5k\sin\alpha\sinh^2k(h+y-b)\cosh k(h+y-b)$$

$$\text{times } \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t] ,$$

EQ. (22)

The coefficients A_1, A_2, A_3, A_4, and A_5 are defined in Appendix B, page .

Equations of Motion

The semi-submersible drilling vessel shown in Figure 1 will be treated as a rigid space frame whose distances between members are constrained to remain absolutely fixed. This is to say that displacements due to elastic deformations can be safely neglected in deriving the equations of motion.

The general motion of a rigid body can be defined by six scalar differential equations corresponding to the six degrees of freedom, three linear and three rotational degrees. Nara [32] gives the equations of motion based on Newton's Second Law for a rigid body in their most general form as

$$M\ddot{X} = \sum \text{Forces}_x \ , \qquad \text{EQ. (23)}$$

$$M\ddot{Y} = \sum \text{Forces}_y \ , \qquad \text{EQ. (24)}$$

$$M\ddot{Z} = \sum \text{Forces}_z \ , \qquad \text{EQ. (25)}$$

$$\dot{H}'_x - \dot{\sigma}_z H'_y + \dot{\sigma}_y H'_z = \sum \text{Moments}_x \ , \qquad \text{EQ. (26)}$$

$$\dot{H}'_y - \dot{\sigma}_x H'_z + \dot{\sigma}_z H'_x = \sum \text{Moments}_y \ , \qquad \text{EQ. (27)}$$

$$\dot{H}'_z - \dot{\sigma}_y H'_x + \dot{\sigma}_x H'_y = \sum \text{Moments}_z \ , \qquad \text{EQ. (28)}$$

where

$$H'_x = I_{xx}\dot{\sigma}_x - I_{xy}\dot{\sigma}_y - I_{xz}\dot{\sigma}_z \ , \qquad \text{EQ. (26a)}$$

$$H'_y = -I_{xy}\dot{\sigma}_x + I_{yy}\dot{\sigma}_y - I_{yz}\dot{\sigma}_z \ , \qquad \text{EQ. (27a)}$$

$$H'_z = -I_{xz}\dot{\sigma}_x - I_{yz}\dot{\sigma}_y + I_{zz}\dot{\sigma}_z \ , \qquad \text{EQ. (28a)}$$

and

$$\dot{H}'_x = I_{xx}\ddot{\sigma}_x - I_{xy}\ddot{\sigma}_y - I_{xz}\ddot{\sigma}_z \ , \qquad \text{EQ. (26b)}$$

$$\dot{H}'_y = -I_{xy}\ddot{\sigma}_x + I_{yy}\ddot{\sigma}_y - I_{yz}\ddot{\sigma}_z \ , \qquad \text{EQ. (27b)}$$

$$\dot{H}'_z = -I_{xz}\ddot{\sigma}_x - I_{yz}\ddot{\sigma}_y + I_{zz}\ddot{\sigma}_z \ . \qquad \text{EQ. (28b)}$$

where Equations (23,24, and 25) define the linear displacements of the mass center; and Equations (26,27 and 28) define the rotational motion.

The following sections will derive the terms that are included in above equations which will be summarily rewritten at the end of this chapter.

Rigid Body Rotation

In order to avoid the problem of dealing with the inertia terms which would be variables for a fixed coordinate system, it is convenient to define a stationary frame of reference XYZ located at the center of gravity at time t = o (these axes define the initial position) and to rigidly attach a second set of axes xyz with the origin at the center of gravity that is allowed to rotate. In essence, the rigid body will have no motion with reference to the rotating frame. The two systems of coordinates are illustrated in Figure 5.

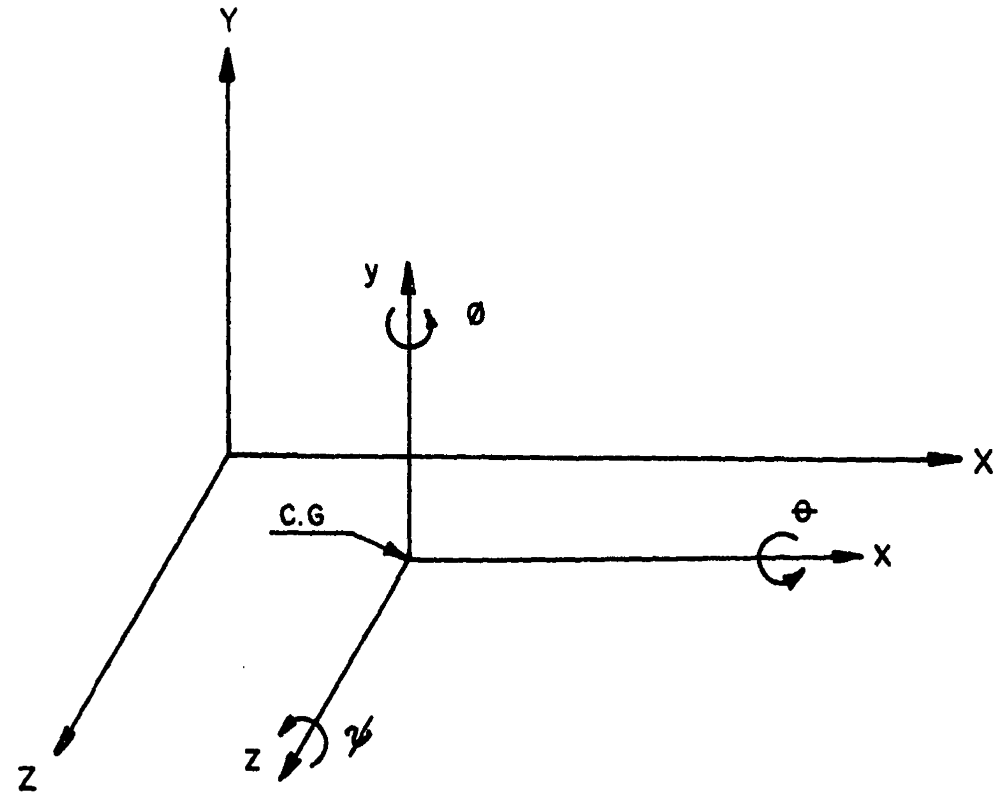


Figure 5: Stationary (XYZ) and Rotating (xyz) Coordinate System.

In deriving the velocity and acceleration terms that are to be later substituted into the equations of motion, it is desirable to retain the full non-linear effect of rotation. Past research has assumed small values for the roll (θ), pitch

(ψ) and yaw (ϕ) which linearizes the differential equations. It is agreed that the small angle assumptions are probably valid as long as the vessel remains at its drilling draft for which it was designed to operate (partially submerged as shown in Figure 1), however, this is a very definitely invalid assumption when the vessel is at transit draft where only the lower hulls are partially submerged. Then too, in this' study the non-linear constraints of the mooring system are' definitely affected by even small rotations in yaw.

Since the frame of reference adopted for the equations of motion are rigidly fixed to the vessel and moves with it, the position and orientation of the vessel must be relative to the stationary axes XYZ. Also, since this problem deals with finite rotations superposition of the angular displacements are no longer valid. Goldstein [12] gives some insight to this problem. Suppose $\bar{A}$ and $\bar{B}$ are two vectors associated with transformations A and B. To qualify as vectors they must be commutative in addition:

$$\bar{A} + \bar{B} = \bar{B} + \bar{A}.$$

But, the addition of two rotations, i.e., one rotation performed after another corresponds to the product $\overline{AB}$ of the two matrices. However, matrix multiplication is not commutative, $\overline{AB} \neq \overline{BA}$, and hence $\overline{AB}$ are not commutative in addition and cannot be accepted as vectors. By necessity, the concept of finite rotations must be compromised.

The problem of determining the position and orientation of the vessel relative to the stationary axes at any time Δt is solved by deriving a general rotation matrix. The elements of which will be the direction cosines of the rotating axes relative to the stationary axes. This derivation will be the product of three successive orthogonal rotations in a specific order which proves to be extremely important.

The first rotation (pitch) is counterclockwise about the z-axis.

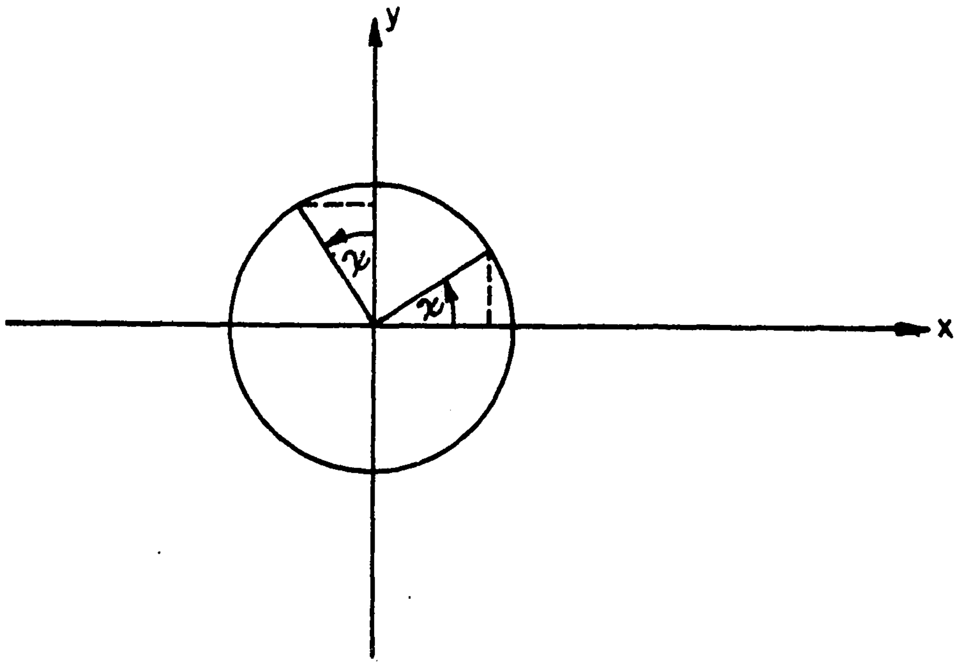


Figure 6: Counterclockwise Rotation About the z-axis.

The transformation matrix which defines the orientation is wirtten as

$$\lambda_\psi = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} . \qquad \text{EQ. (29)}$$

Here, an important point that should be observed in the above rotation and those that follow is that for orthogonality to be preserved, the determinant of Equation (29) must equal +1 or -1. Further, for the rotation to be "proper" the determinant must equal +1. A determinant whose value is -1 reveals an "improper" rotation even though it may be orthogonal.

A second rotation (roll) about the x-axis is shown in Figure 7.

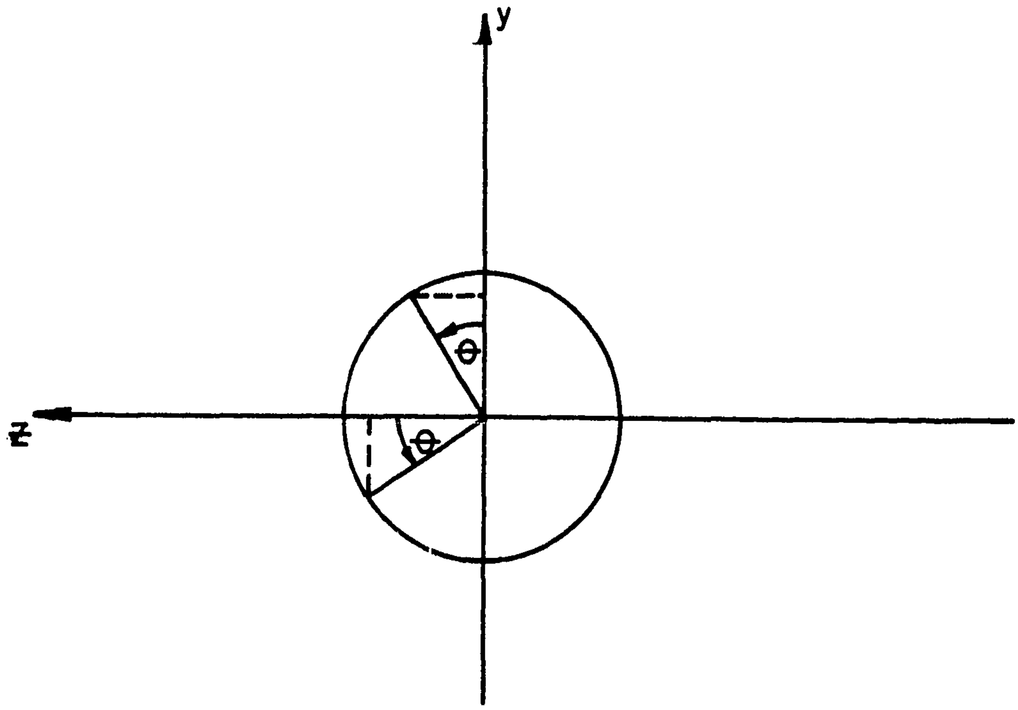


Figure 7: Counterclockwise Rotation About the x-axis.

The transformation matrix is thus,

$$\lambda_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} . \qquad \text{EQ. (30)}$$

The third rotation (yaw) about the y-axis as shown in Figure 8 has the transformation

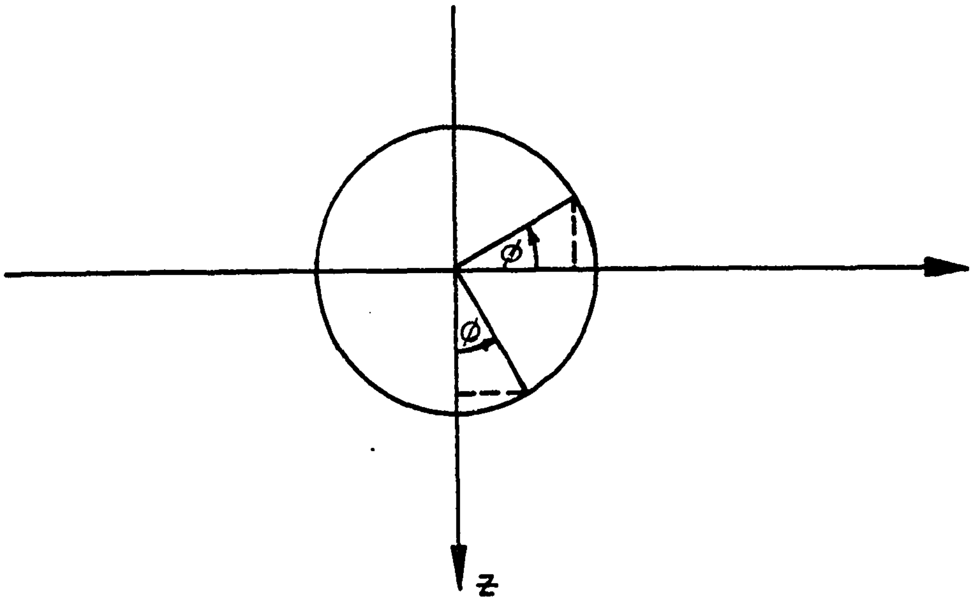


Figure 8: Counterclockwise Rotation About the y-axis.

$$\lambda_{\emptyset} = \begin{pmatrix} \cos\emptyset & 0 & -\sin\emptyset \\ 0 & 1 & 0 \\ \sin\emptyset & 0 & \cos\emptyset \end{pmatrix}, \qquad \text{EQ. (31)}$$

$$\lambda^T = [\lambda_{\phi}\,\lambda_{\theta}\,\lambda_{\psi}]^T. \qquad \text{EQ. (32)}$$

Therefore, with angular displacements of pitch, roll and yaw the new position of an arbitrary point within the rigid body relative to the stationary axes is given by

$$X = \lambda_{11}\, x + \lambda_{21}\, y + \lambda_{31}\, z, \qquad \text{EQ. (33)}$$

$$Y = \lambda_{12} x + \lambda_{22} y + \lambda_{32}\, z, \qquad \text{EQ. (34)}$$

$$Z = \lambda_{13}\ x + \lambda_{23}\ y + \lambda_{33} z , \qquad \text{EQ. (35)}$$

where the small (xyz) indicate the coordinates of a point on the rotating axes. As mentioned earlier, the elements of the general rotation matrix are the nine direction cosines. These terms which are somewhat lengthy are given in Appendix B, page 139. The correctness of this transformation has been verified by the rule that the sum of the direction cosines squared must be unity,

$$\begin{aligned} \lambda_{11}^2 + \lambda_{12}^2 + \lambda_{13}^2 &= 1 , \\ \lambda_{21}^2 + \lambda_{22}^2 + \lambda_{23}^2 &= 1 , \\ \lambda_{31}^2 + \lambda_{32}^2 + \lambda_{33}^2 &= 1 . \end{aligned} \qquad \text{EQ. (36)}$$

A second test given by Goodman and Warner [10] specifies that

$$\begin{aligned} \lambda_{11}\lambda_{21} + \lambda_{12}\lambda_{22} + \lambda_{13}\lambda_{23} &= 0 , \\ \lambda_{11}\lambda_{31} + \lambda_{12}\lambda_{32} + \lambda_{13}\lambda_{33} &= 0 , \\ \lambda_{21}\lambda_{31} + \lambda_{22}\lambda_{32} + \lambda_{23}\lambda_{33} &= 0 . \end{aligned} \qquad \text{EQ. (37)}$$

In addition, to preserve the right-handed nature of the co-ordinate system, one must satisfy the relations,

$$\begin{aligned} \lambda_{31} &= \lambda_{12}\ \lambda_{23} - \lambda_{13}\ \lambda_{22} , \\ \lambda_{32} &= \lambda_{13}\ \lambda_{21} - \lambda_{11}\ \lambda_{23} , \\ \lambda_{33} &= \lambda_{11}\ \lambda_{22} - \lambda_{12}\ \lambda_{21} . \end{aligned} \qquad \text{EQ. (37a)}$$

The direction cosines of the general rotation matrix does in fact satisfy Equations (36, 37, and 37a).

This derivation has been similar to that for Eulerian angles found in the texts [11, 13, 24, 36]

The total angular displacement along the axes XYZ is

$$\sigma = \lambda_\phi^T \lambda_\theta^T \lambda_\psi^T \begin{pmatrix} 0 \\ 0 \\ \psi \end{pmatrix} + \lambda_\phi^T \lambda_\theta^T \begin{pmatrix} \theta \\ 0 \\ 0 \end{pmatrix} + \lambda_\phi^T \begin{pmatrix} 0 \\ \phi \\ 0 \end{pmatrix} . \qquad \text{EQ. (38)}$$

The angles of rotation in terms of the Euler angles and with respect to XYZ are now written as

$$\sigma_x = \psi \sin\phi \cos\theta + \theta \cos\phi , \qquad \text{EQ. (38a)}$$

$$\sigma_y = \phi - \psi \sin\theta , \qquad \text{EQ. (38b)}$$

$$\sigma_z = \psi \cos\phi \cos\theta - \theta \sin\phi . \qquad \text{EQ. (38c)}$$

The inverse which is with respect to xyz follows

$$\bar{\sigma}_x = \theta \cos\psi + \phi \sin\psi \cos\theta , \qquad \text{EQ. (39a)}$$

$$\bar{\sigma}_y = \phi \cos\psi \cos\theta - \theta \sin\psi , \qquad \text{EQ. (39b)}$$

$$\bar{\sigma}_z = \psi - \phi \sin\theta . \qquad \text{EQ. (39c)}$$

Angular velocities in both axis systems are obtained by the simple division of Δt. One should note that for the special case of $\theta = \phi = \psi = 0$, the angular velocities reduce to

$$\dot{\bar{\sigma}}_x = \dot{\sigma}_x = \dot{\theta} , \qquad \text{EQ. (40a)}$$

$$\dot{\bar{\sigma}}_y = \sigma'_y = \dot{\phi} \ , \qquad \text{EQ. (40b)}$$

$$\dot{\bar{\sigma}}_z = \sigma'_z = \dot{\psi} \ , \qquad \text{EQ. (40c)}$$

which is a condition that must be satisfied.

The angular accelerations are taken as the total time derivatives. For the XYZ system the acceleration components are

$$\ddot{\sigma}_x = \ddot{\psi}\sin\phi\cos\theta + \dot{\psi}\dot{\phi}\cos\phi\cos\theta - \dot{\psi}\dot{\theta}\sin\phi\sin\theta + \ddot{\theta}\cos\phi - \dot{\theta}\dot{\phi}\sin\phi \ , \qquad \text{EQ. (41)}$$

$$\ddot{\sigma}_y = -\ddot{\psi}\sin\theta - \dot{\psi}\dot{\theta}\cos\theta + \ddot{\phi} \ , \qquad \text{EQ. (42)}$$

$$\ddot{\sigma}_z = \ddot{\psi}\cos\phi\cos\theta - \dot{\psi}\dot{\phi}\sin\phi\cos\theta - \dot{\psi}\dot{\theta}\cos\phi\sin\theta - \ddot{\theta}\sin\phi - \dot{\theta}\dot{\phi}\cos\phi \ . \qquad \text{EQ. (43)}$$

Derivation of Forces

The theory is clarified somewhat by the top view of the superstructure in Figure 9. Each of the horizontal members are divided symetrically about its respective column. The dashed columns are ficticious (columns with zero diameters) and serve only as a convenient computational scheme. The dashed lines depict relatively small diameter members which are not included in this analysis. The hull-column segment shown earlier in Figure 4 is considered typical for those which the hull lies paralled to the x-axis.

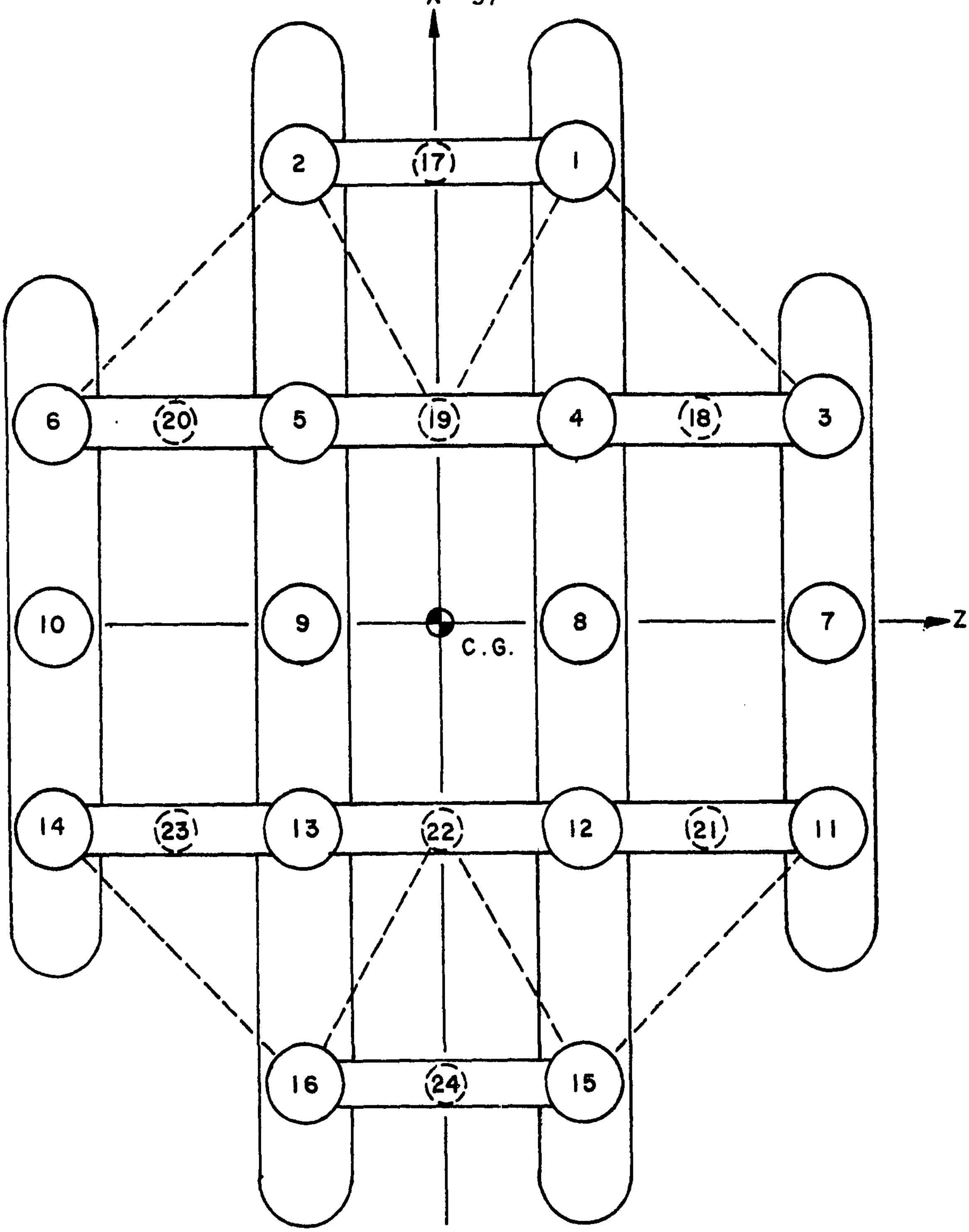


Figure 9: Top View of Superstructure

The forces to be derived in the following analysis are force perturbations from the case in which the vessel floats in a calm sea. Three critical assumptions which greatly simplifies the force derivations merits mention now. It is assumed that the presence of the body in the sea does not alter the wave form in any manner; and, the physical dimensions of the members are sufficiently small compared to the wave length such that the motion of the sea may be considered uniform across a column or hull diameter. It is further assumed that forces on the hull sections are concentrated at the intersection of the hull-column centerlines; whereas, the horizontal force on a column is concentrated at the midpoint of its instantaneous wetted length. These assumptions appear to be the most critical. Other simplifications will be pointed out as the analysis proceeds.

The derivations that follow are somewhat repetitious; for example some of the hull forces in the x and z direction differ only by a trigometric function. However, some forces due to oblique flow are not applied to the hulls laying parallel to the z-direction because they do not possess an exposed end. It is primarily for this reason that all forces will be given consideration even though some will be left which contains an integral to be evaluated. The full detail of the integrations are found in Appendix B. Those forces which will be considered in the given order are as follows:

a) Forces due to interaction between the vessel and the sea.
b) Forces due to varying bouyancy.
c) Hull forces due to oblique flow.
d) Mooring line forces.

e) Wind forces.

Forces Due to Interaction of Added Mass

The interaction between the sea and the vessel involve the concepts of added mass, drag, and pressure. In fluid dynamic systems where forms accelerate relative to the fluid, Milne-Thompson [27] indicates that the presence of the surrounding sea effectively increases the mass of the moving form by an amount ΔM proportionately equal to the mass of fluid displaced. The ΔM will be referred to as the added mass and is defined by

$$\Delta M = C_M(\text{body volume}) ,$$

where C_M is an added mass coefficient. The force exerted on the form opposite in direction of motion is proportional to its relative acceleration through the sea. This is basically Newton's Second Law, $F = ma$.

The general equation of the acceleration vector of a point P on the vessel due to motion in a calm sea is well known from vector mechanics as

$$\ddot{\bar{r}} = \ddot{\bar{R}} + (\ddot{\bar{\sigma}} x \bar{\rho}) + \dot{\bar{\sigma}} x (\dot{\bar{\sigma}} x \bar{\rho}) + a + 2\dot{\bar{\sigma}} x \bar{V} , \qquad \text{EQ. (44)}$$

where the terms are identified as follows:

$\ddot{\bar{R}}$ = acceleration of the moving origin with respect to XYZ.

$(\ddot{\bar{\sigma}} x \bar{\rho})$ = tangential acceleration of P considered as fixed in the xyz.

$\dot{\bar{\sigma}} x (\dot{\bar{\sigma}} x \bar{\rho})$ = normal acceleration of P considered fixed in xyz.

$\bar{a}$ = acceleration of P as measured in the moving system ,

$2\dot{\bar{\sigma}}x\bar{v}$ = Coriolis component of acceleration due to the motion on a rotating path.

The last two terms $\bar{a}$ and $2\dot{\bar{\sigma}}x\bar{v}$ will be considered negligible. Added to Equation (44) will be the acceleration of the water particles averaged over a length of the moving surface. This gives the acceleration of a moving body relative to the sea,

$$\dot{\bar{V}} = \ddot{\bar{R}} + (\ddot{\bar{\sigma}}x\bar{\varrho}) + \dot{\bar{\sigma}}x(\dot{\bar{\sigma}}x\bar{\varrho}) - \frac{1}{k}\int_{\xi_1}^{\xi_2} \dot{\bar{B}}d\xi \quad . \qquad \text{EQ. (45)}$$

Note the minus sign prior to the integral; this is due to the previously defined negative celerity C which is included in the particle acceleration $\dot{\bar{E}}$. Note also, Equation (45) is defined as a vector. The required angular velocities and acceleration components have been derived, Equations (38, 39, 40,41, 42 and 43) respectively. With these, the tangential acceleration components are determined by the components of the definition

$$(\ddot{\bar{\sigma}}x\bar{\varrho}) = \begin{pmatrix} i & j & k \\ \ddot{\sigma}_x & \ddot{\sigma}_y & \ddot{\sigma}_z \\ x & y & z \end{pmatrix} \qquad \text{EQ. (46)}$$

which yield

$$(\ddot{\bar{\sigma}}x\bar{\varrho})_x = (z\ \ddot{\sigma}_y - y\ \ddot{\sigma}_z)\ i\ , \qquad \text{EQ. (47)}$$

$$(\ddot{\bar{\sigma}}x\bar{\varrho})_y = (x\ \ddot{\sigma}_z - z\ \ddot{\sigma}_x)\ j\ , \qquad \text{EQ. (48)}$$

$$(\ddot{\bar{\sigma}}x\bar{\varrho})_z = (y\ \ddot{\sigma}_x - x\ \ddot{\sigma}_y)\ k\ , \qquad \text{EQ. (49)}$$

Likewise, the components of the normal acceleration are obtained from

$$\dot{\bar{\sigma}} x (\dot{\bar{\sigma}} x \bar{Q}) = \begin{pmatrix} i & j & k \\ \dot{\sigma}_x & \dot{\sigma}_y & \dot{\sigma}_z \\ [z\,\dot{\sigma}_y - y\,\dot{\sigma}_z] & [x\,\dot{\sigma}_z - z\,\dot{\sigma}_x] & [y\,\dot{\sigma}_x - x\,\dot{\sigma}_y] \end{pmatrix}$$

EQ. (50)

which give

$$[\, \dot{\bar{\sigma}} x (\dot{\bar{\sigma}} x \bar{Q})\,]_x = [\, \dot{\sigma}_y (y\dot{\sigma}_x - x\dot{\sigma}_y) - \dot{\sigma}_z (x\dot{\sigma}_z - z\dot{\sigma}_x)\,] , \qquad \text{EQ. (51)}$$

$$[\, \dot{\bar{\sigma}} x (\dot{\bar{\sigma}} x \bar{Q})\,]_y = [\, \dot{\sigma}_z (z\dot{\sigma}_y - y\dot{\sigma}_z) - \dot{\sigma}_x (y\dot{\sigma}_x - x\dot{\sigma}_y)\,] , \qquad \text{EQ. (52)}$$

$$[\, \dot{\bar{\sigma}} x (\dot{\bar{\sigma}} x \bar{Q})\,]_z = [\, \dot{\sigma}_x (x\dot{\sigma}_z - z\dot{\sigma}_x) - \dot{\sigma}_y (z\dot{\sigma}_y - y\dot{\sigma}_z)\,] . \qquad \text{EQ. (53)}$$

For motion in the horizontal directions, the hull-column segments are treated separately. And, for the hull sections only the forces due to the motion in the direction normal to the hull's longitudinal axis are considered. The forces on the hulls are assumed to be concentrated at a point where the centerlines of the hull and column intersect; whereas, for column motion normal to its longitudinal axis, the force is assumed to be concentrated at the mid-point of its instantaneous wetted-length.

First, considering the hull forces in the z-direction, only those which lay parallel to the x-axis are eligible.

The theory is complete for the hull force exerted by the relative acceleration of the added mass and may be written directly as

$$F_{haz} = -\Delta M_{hz}\dot{\bar{V}}_{hz} \quad , \qquad \text{EQ. (54)}$$

where

$$\dot{\bar{V}}_{hz} = \ddot{Z} + y_h\ddot{\sigma}_x - z_c\dot{\sigma}_y(z_c\dot{\sigma}_y - y_h\dot{\sigma}_z) + \frac{1}{L_h}\int_{x_c - \frac{1}{2}L_h}^{x_c + \frac{1}{2}L_h}[x\dot{\sigma}_x\dot{\sigma}_z - x\ddot{\sigma}_y - \dot{w}(x, y_h, z_c)]dx \quad . \qquad \text{EQ. (55)}$$

The horizontal forces on the column are are determined similarly except that the wetted length in which the force is distributed is a variable due to waves and the motion of the vessel itself. The approach taken is to evaluate the integrals with limits that correspond to a calm sea and then multiply by the following exposure correction factor

$$S(t) = \frac{\begin{array}{c}\text{wetted length}\\ \text{in calm sea}\end{array} + \begin{array}{c}\text{wave}\\ \text{amplitude}\end{array} - \begin{array}{c}\text{vertical}\\ \text{displacement}\end{array}}{\text{wetted length in calm sea}} \; . \qquad \text{EQ. (56)}$$

This is the proportion of instantaneous wetted length to wetted length in a calm sea as a function of time. This correction will be applied to all horizontal column forces. The force exerted on the column by the added mass acceleration may now be written as

$$F_{cza} = -S(t)\Delta M_{cz}\dot{\bar{V}}_{cz} \quad , \qquad \text{EQ. (57)}$$

where

$$\dot{\bar{V}}_{cz} = \ddot{Z} - x_c\ddot{\sigma}_y - z_c\dot{\sigma}_y^2 + \dot{\sigma}_x(x_c\dot{\sigma}_z - z_c\dot{\sigma}_x)$$
$$+ \frac{1}{L_c}\int_{y_h+\frac{1}{2}d_h}^{b} [y\ddot{\sigma}_x + y\dot{\sigma}_y\dot{\sigma}_z - \dot{w}(x_c, y, z_c)]dy, \qquad \text{EQ. (58)}$$

and

$$L_c = b - y_h - \frac{1}{2}d_h \; . \qquad \text{EQ. (59)}$$

Likewise, with appropiate components of tangential and normal acceleration the forces in the x-direction are given by

$$F_{hxa} = -\Delta M_{hx}\dot{\bar{V}}_{hx} \quad , \qquad \text{EQ. (60)}$$

and

$$F_{cxa} = -S(t)\Delta M_{cx}\dot{\bar{V}}_{cx} \quad , \qquad \text{EQ. (61)}$$

where

$$\dot{\bar{V}}_{hx} = \ddot{X} - y_h\ddot{\sigma}_z - x_c\dot{\sigma}_z^2 + \dot{\sigma}_y(y_h\dot{\sigma}_x - x_c\dot{\sigma}_y)$$
$$+ \frac{1}{L_h}\int_{z_c-\frac{1}{2}L_h}^{z_c+\frac{1}{2}L_h} [z\ddot{\sigma}_y + z\dot{\sigma}_z\dot{\sigma}_x - \dot{u}(x_c, y_h, z)]dz \; , \qquad \text{EQ. (62)}$$

and

$$\dot{\bar{V}}_{cx} = \ddot{X} + z_c\ddot{\sigma}_y - x_c\dot{\sigma}_y^2 - \dot{\sigma}_z(x_c\dot{\sigma}_z - z_c\dot{\sigma}_x)$$

$$- \frac{1}{L_a}\int_{y_h + \frac{1}{2}d_h}^{b} [y\, \ddot{\sigma}_z - y\, \dot{\sigma}_x\, \dot{\sigma}_y + \dot{u}(x_c, y, z_c)]dy \quad . \qquad \text{EQ. (63)}$$

For motion in the vertical direction, only the hull is considered to have an added mass. In this direction all hulls (1-24) are considered. The corresponding force due to the interaction is

$$F_{ya} = - \Delta M_{hy} \dot{\bar{V}}_{hy} \quad , \qquad \text{EQ. (63)}$$

where

$$\dot{\bar{V}}_{hy} = \ddot{Y} - z_c \ddot{\sigma}_x + x_c \ddot{\sigma}_z - y_h(\dot{\sigma}_z^2 + \dot{\sigma}_x^2)$$

$$+ \dot{\sigma}_y(z_c \dot{\sigma}_z + x_c \dot{\sigma}_x) - \frac{\delta_{jm}}{L_h}\int_{x_c - \frac{1}{2}L_h}^{x_c + \frac{1}{2}L_h} \dot{v}(x, y_h, z_c)dx \qquad \text{EQ. (64)}$$

$$- \frac{\delta_{jn}}{L_h}\int_{z_c - \frac{1}{2}L_h}^{z_c + \frac{1}{2}L_h} \dot{v}(x_c, y_h, z)dz \quad ,$$

and

$$\delta_{jm} = \begin{cases} 1 & j = m,\ m = 1\text{-}16 \\ 0 & j \neq m \end{cases} \qquad \delta_{jn} = \begin{cases} 1\ , & j = n,\ n = 17\text{-}24 \\ 0\ , & j \neq n \end{cases} .$$

Drag Forces Due to Interaction

The concept of an opposing force due to fluid flow past a body is fundamental to fluid mechanics. This force is commonly referred to as dynamic or square-law damping and is given by

$$F_d = -\frac{1}{2}\varrho C_D A\bar{V}|\bar{V}| \;, \qquad \text{EQ. (65)}$$

where A is the projected area normal to the direction of flow and C_D is a non-dimensional drag coefficient. The absolute value signs are to insure that the resulting force will be opposite in direction of the relative velocity $\bar{V}$.

The components of relative velocity $\bar{V}$ are found from the vector equation

$$\bar{V} = \dot{\bar{R}} + (\dot{\bar{\sigma}} \times \bar{\varrho}) - \frac{1}{k}\int_{\xi_1}^{\xi_2} \bar{B}\, d\xi \;, \qquad \text{EQ. (66)}$$

where

$\dot{R}$ = the velocity of the moving origin (C.G.) referred to XYZ.

$(\dot{\bar{\sigma}} \times \bar{\varrho})$ = the time derivative of $\bar{\varrho}$, the position vector to a point P within the moving system, due to the rotation of xyz.

$\frac{1}{k}\int_{\xi_1}^{\xi_2} \bar{B}\, d\xi$ = the water particle velocity averaged over a length and is referred to XYZ.

The components of the product are obtained from the definition

$$(\dot{\bar{\sigma}} \times \bar{\varrho}) = \begin{pmatrix} i & j & k \\ \dot{\sigma}_x & \dot{\sigma}_y & \dot{\sigma}_z \\ x & y & z \end{pmatrix} , \qquad \text{EQ. (67)}$$

which yield the desired components

$$(\dot{\bar{\sigma}} \times \bar{\rho})_x = z\dot{\sigma}_y - y\dot{\sigma}_z \quad , \qquad \text{EQ. (68)}$$

$$(\dot{\bar{\sigma}} \times \bar{\rho})_y = x\dot{\sigma}_z - z\dot{\sigma}_x \quad . \qquad \text{EQ. (69)}$$

$$(\dot{\bar{\sigma}} \times \bar{\rho})_z = y\dot{\sigma}_x - x\dot{\sigma}_y \quad , \qquad \text{EQ. (70)}$$

By making the appropiate substitutions, the drag forces are written as

$$F_{czd} = -S(t)D_{cz}\bar{V}_{cz}|\bar{V}_{cz}| \quad , \qquad \text{EQ. (71)}$$

$$F_{hzd} = -D_{hz}\bar{V}_{hz}|\bar{V}_{hz}| \quad , \qquad \text{EQ. (72)}$$

$$F_{cxd} = -S(t)D_{cx}\bar{V}_{cx}|\bar{V}_{cx}| \quad , \qquad \text{EQ. (73)}$$

$$F_{hxd} = -D_{hx}\bar{V}_{hx}|\bar{V}_{hx}| \quad , \qquad \text{EQ. (74)}$$

$$F_{hyd} = -D_{hy}\bar{V}_{hy}|\bar{V}_{hy}| \quad , \qquad \text{EQ. (75)}$$

where

$$\bar{V}_{cz} = \dot{Z} - x_c\dot{\sigma}_y + \frac{1}{L_c}\int_{y_h + \frac{1}{2}d_h}^{b} [y\,\dot{\sigma}_x - w(x_c, y, z_c)]dy \quad , \qquad \text{EQ. (76)}$$

$$\bar{V}_{hz} = \dot{Z} + y_h\,\dot{\sigma}_x - \frac{1}{L_h}\int_{x_c - \frac{1}{2}L_h}^{x_c + \frac{1}{2}L_h} [x\,\dot{\sigma}_y + w(x, y_h, z_c)]dx \quad , \qquad \text{EQ. (77)}$$

$$\bar{V}_{cx} = \dot{X} + z_c\,\dot{\sigma}_y - \frac{1}{L_c}\int_{y_h + \frac{1}{2}d_h}^{b} [y\,\dot{\sigma}_z + u(x_c, y, z_c)]dy \quad , \qquad \text{EQ. (78)}$$

$$\bar{V}_{hx} = \dot{X} - y_h\dot{\sigma}_z + \frac{1}{L_h}\int_{z_c - \frac{1}{2}L_h}^{z_c + \frac{1}{2}L_h} [z\dot{\sigma}_y - u(x_c, y_h, z)]dz \ , \qquad \text{EQ. (79)}$$

$$\bar{V}_{hy} = \dot{Y} + x_c\dot{\sigma}_z - z_c\dot{\sigma}_x$$

$$- \frac{\delta_{jm}}{L_h}\int_{x_c - \frac{1}{2}L_h}^{x_c + \frac{1}{2}L_h} v(x, y_h, z_c)dx - \frac{\delta_{jn}}{L_h}\int_{z_c - \frac{1}{2}L_h}^{z_c + \frac{1}{2}L_h} v(x_c, y_h, z)dz \ , \qquad \text{EQ. (80)}$$

and

$$\delta_{jm} = \begin{cases} 1 \ , & j=m, \ m=1\text{-}16 \\ 0 \ , & j=m \end{cases} \ , \qquad \delta_{jn} = \begin{cases} 1 \ , & j=n, \ n=17\text{-}24 \\ 0 \ , & j=n \end{cases} .$$

Note once again that the resulting drag forces on the columns are corrected by the exposure factor S(t).

<u>Wave Damping Forces</u>

Havelock [15,16] and Ursell [43,44] have thoroughly investigated the resistance and wave-making phenomena of oscillating cylinders. The damping force of the wave is similar in appearance to a linearized drag force, being proportional to the relative velocity and has often been confused with a viscous drag. But the fluid has been defined as inviscid which in essence neglects forces of a viscous nature.

Following the work of Havelock and Ursell, Wang [46] has termed the damping of the wave on a vertical cylinder as negligibly small compared to the forces that arise on a submerged cylindrical hull. This damping force was found to be a function of the oscillating frequency, depth below the free surface, mass, the radius of the submerged cylinder, and directly proportional to the vessel velocity when multiplied by a damping parameter. The wave damping forces associated with this phenomena are defined as

$$F_{dmx} = \beta(\sigma, y_h/r)m\sigma V_{hx} \; , \qquad \text{EQ. (81a)}$$

$$F_{dmy} = \beta(\sigma, y_h/r)m\sigma V_{hy} \; , \qquad \text{EQ. (81b)}$$

$$F_{dmz} = \beta(\sigma, y_h/r)m\sigma V_{hz} \; , \qquad \text{EQ. (81c)}$$

where $\beta(\sigma, y_h/r)$ is a damping parameter taken from experimental and analytical data given in Appendix E page 219; m is the displaced mass of the member, σ is the oscillating frequency and V is the vessel velocity.

<u>Forces Due to Pressure</u>

The pressure gradients are obtained by differentiating the Bernoulli equation and substituting the velocity potential and velocity terms, or vice-versa. The latter method is used to derive the gradients in Appendix B, p. 168 and will not be repeated here. It is worth mentioning that the equation so derived for pressure, Equation (B.73), does not conform to that given by Muga and Wilson [30] for second order theory. One might suspect that Muga and Wilson have omitted terms that appear to be negligible; for example, terms divided by $(\sinh kh)^8$. Certainly, this would be a valid omission for a range of the argument kh which is defined by

$$kh = \frac{2\pi h}{\lambda} .$$

In essence, whether the term may be neglected or not depends on the ratio of water depth to wave length, h/λ. For shallow water with a long swell, this term would remain relatively insignificant. In any case, Equation (B.73) with the higher-order terms included is used to determine the pressure gradients required by Equation (81).

Since the hulls and columns are of constant diameter the integration may be carried out over the length only. For a column, this integration is in the form

$$F_{c\xi p} = -S(t)\,\frac{\pi d_c^2}{4}\int_{y_h+\frac{1}{2}d_h}^{b}\frac{\partial P}{\partial \xi}\,dy \;.$$

The hull-pressure forces are obtained similarly without the exposure factor S(t). These forces are summarized as follows

$$F_{czp} = -\,\frac{S(t)\pi d_c^2}{4}\int_{y_h+\frac{1}{2}d_h}^{b}\frac{\partial P}{\partial z}\,dy \,, \qquad \text{EQ. (82)}$$

$$F_{hzp} = -\,\frac{\pi d_h^2}{4}\,\delta_{jm}\int_{x_c-\frac{1}{2}L_h}^{x_c+\frac{1}{2}L_h}\frac{\partial P}{\partial z}\,dx \,, \qquad \text{EQ. (83)}$$

$$F_{cxp} = - \frac{S(t)\pi d_h^2}{4} \int_{y_h + \frac{1}{2}d_h}^{b} \frac{\partial P}{\partial x}\, dy \ , \qquad \text{EQ. (84)}$$

$$F_{hxp} = - \frac{\pi d_h^2}{4} \delta_{jn} \int_{z_c - \frac{1}{2}L_h}^{z_c + \frac{1}{2}L_h} \frac{\partial P}{\partial x}\, dz \ , \qquad \text{EQ. (85)}$$

$$F_{yp} = - \frac{\pi d_h^2}{4} \delta_{jm} \int_{x_c - \frac{1}{2}L_h}^{x_c + \frac{1}{2}L_h} \frac{\partial P}{\partial y}\, dx - \frac{S(t)\pi d_c^2}{4} \int_{y_h + \frac{1}{2}d_h}^{b} \frac{\partial P}{\partial y}\, dy$$

$$- \frac{\pi d_h^2}{4} \delta_{jm} \int_{z_c - \frac{1}{2}L_h}^{z_c + \frac{1}{2}L_h} \frac{\partial P}{\partial y}\, dz \ , \qquad \text{EQ. (86)}$$

where

$$\delta_{jm} = \begin{cases} 1 \ , & j = m, \ m = 1\text{-}16 \\ 0 \ , & j \neq m \end{cases} \ , \quad \delta_{jn} = \begin{cases} 1 \ , & j = n, \ n = 17\text{-}24 \\ 0 \ , & j \neq n \end{cases} \ .$$

This completes the derivation of forces due to the interaction of the vessel and sea.

Force Perturbations Due to the Rate of Change of Momentum

Newton's Second Law is actally defined by

$$F = \frac{d}{dt}(mv) \ ,$$

which equates a force to the time rate of change in momentum and when expanded becomes

$$F = m\frac{dV}{dt} + V\frac{dm}{dt} \ . \qquad \text{EQ. (87)}$$

The first term, force due to linear acceleration has been accounted for already; therefore, the force in question becomes

$$F_m = \bar{V}\frac{dm}{dt} \ , \qquad \text{EQ. (88)}$$

The existence of the mass derivative dm/dt is due to the continuous change of the column's wetted length in calculating the movement of the displaced mass of fluid. Differentiating Equation (B.43) with respect to time, the instantaneous wetted length of a column, and multiplying by its cross-section will produce the above derivative. After this operation the force components are given by

$$F_{czm} = - \frac{\pi \rho d_c^2 \bar{V}_{cz}}{4} \frac{d}{dt} \Delta W_L \ , \qquad \text{EQ. (89)}$$

$$F_{cxm} = - \frac{\pi \rho d_c^2 \bar{V}_{cx}}{4} \frac{d}{dt} \Delta W_L \ , \qquad \text{EQ. (90)}$$

$$F_{cym} = - \frac{\pi \rho d_c^2 \bar{V}_{hy}}{4} \frac{d}{dt} \Delta W_L \ , \qquad \text{EQ. (91)}$$

where

$$\frac{d}{dt}\Delta W_L = \dot{\eta}(x_c, b, z_c) - (\dot{Y} - z_c\cos\theta\dot{\theta}$$
$$- x_c\sin\theta\dot{\theta}\sin\psi + x_c\cos\theta\cos\psi\dot{\psi}$$
$$- y_h\sin\theta\dot{\theta}\cos\psi - y_h\cos\theta\sin\psi\dot{\psi} \,. \qquad \text{EQ. (92)}$$

In a similar manner, the hydrostatic bouyancy is perturbed due to the simultaneous change in bouyancy with a change in the column wetted length. This perturbation is given as

$$F_{cb} = -\rho g\frac{\pi d_c^2}{4}\Delta W_L \,. \qquad \text{EQ. (93)}$$

<u>End Effect Due to Oblique Flow</u>

Consider a long cylinder moving at a velocity V at an angle of incidence α to a uniform stream as shown in Figure 9.

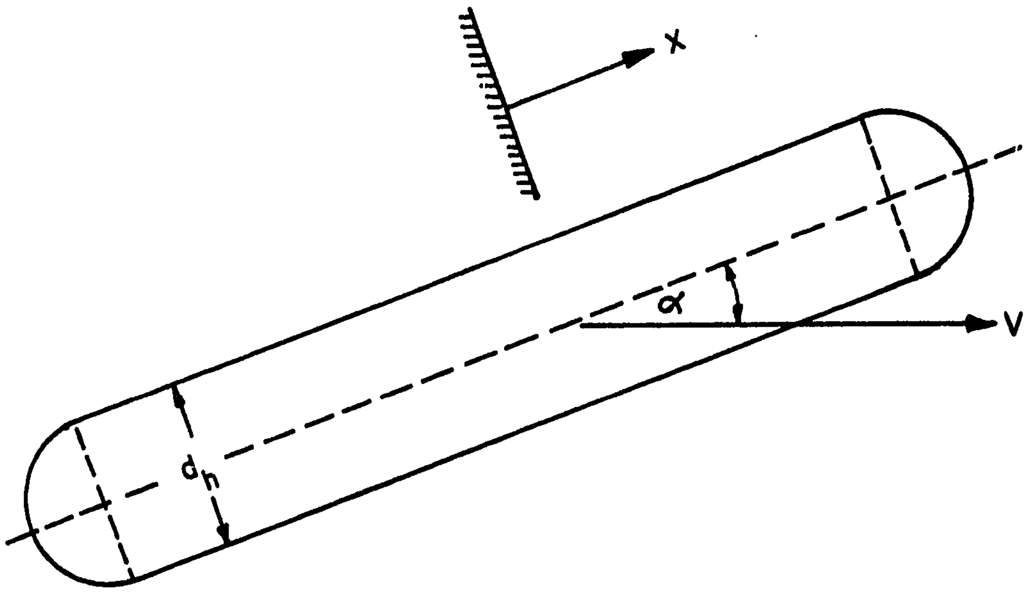


Figure 10: Hull motion in oblique flow.

It is given by Munk [31] and Bain [1] that the nose and tail experience a lateral force per unit length

$$\frac{dF}{dx} = \frac{\varrho V^2}{2}\frac{dS}{dx}\sin 2\alpha \ , \qquad \text{EQ. (94)}$$

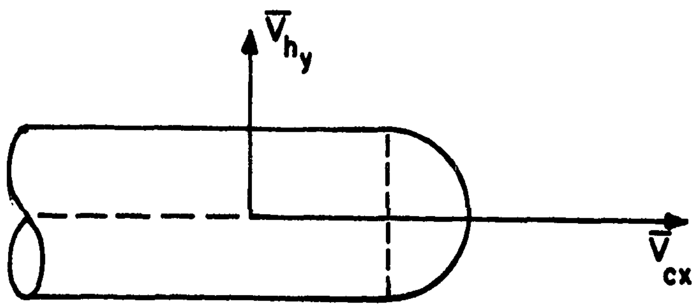
where S is the area of a general cross-section. If Equation (94) is rearranged to read

$$dF = \varrho\, dS V\sin\alpha V\cos\alpha \ , \qquad \text{EQ. (95)}$$

and integrated from the beginning of each end section (where $S = \pi d_h^2/4$) to the end (where $S = 0$), the force will be given by

$$F = \pm \frac{\pi \varrho d_h^2}{4} V\cos\alpha V\sin\alpha \begin{cases} + \text{ fore} \\ - \text{ aft} \end{cases} . \qquad \text{EQ. (96)}$$

Examining the nose section,



it is observed that $+\bar{V}_{hy}$ (or $+\bar{V}_{hz}$) is analogous to $-V\sin\alpha$ and that $+\bar{V}_{cx}$ corresponds to $V\cos\alpha$ in Equation (96). Thus, the force perturbations on the end of those hull forms affected are

$$F_{hyo} = \frac{\pi \rho d_h^2}{4} \begin{cases} -\bar{V}_{hyj}\bar{V}_{cxj} & , j=1,2,3,6 \\ 0 & , j\neq 1,2,3,6,11,14,15,16 \, , \\ +\bar{V}_{hyj}\bar{V}_{cxj} & , j=11,14,15,16 \end{cases}$$

EQ. (97)

and

$$F_{hzo} = \frac{\pi \rho d_h^2}{4} \begin{cases} -\bar{V}_{hzj}\bar{V}_{cxj} & , j=1,2,3,6 \\ 0 & , j\neq 1,2,3,6,11,14,15,16 \, , \, . \\ +\bar{V}_{hzj}\bar{V}_{cxj} & , j=11,14,15,16 \end{cases}$$

EQ. (98)

Restoration Forces of the Mooring System

Vessels of this type are moored by a system of mooring lines, usually eight or more, symetrically spread over 360 degrees. The mooring lines in this analysis are made of a very heavy chain which may be idealized as a completely flexible, inextensible and uniformly heavy string whose shape may be described by the catenary equations.

Routh [37] derived the governing differential equations of motion for a heavy string supported at the ends but found them to be intractable for an analytical solution. His approach, and later Pugsley [35], to simplify the system was to assume a stretched-out catenary is closely approximated by a cycloid which is solvable. Later Saxon and Cahn [38] and Goodey [9] linearize the equations of motion to obtain an asymtotic solution. This work was to describe the vibration of a string hanging between two points, a catenary. The problem one encounters in a mooring line is basically the same; except, the point from which it hangs is in motion and the arclength is some function of time. This problem is yet to be solved.

The approach taken in this analysis will be to neglect the inertia effects of the chain's motion and to work with

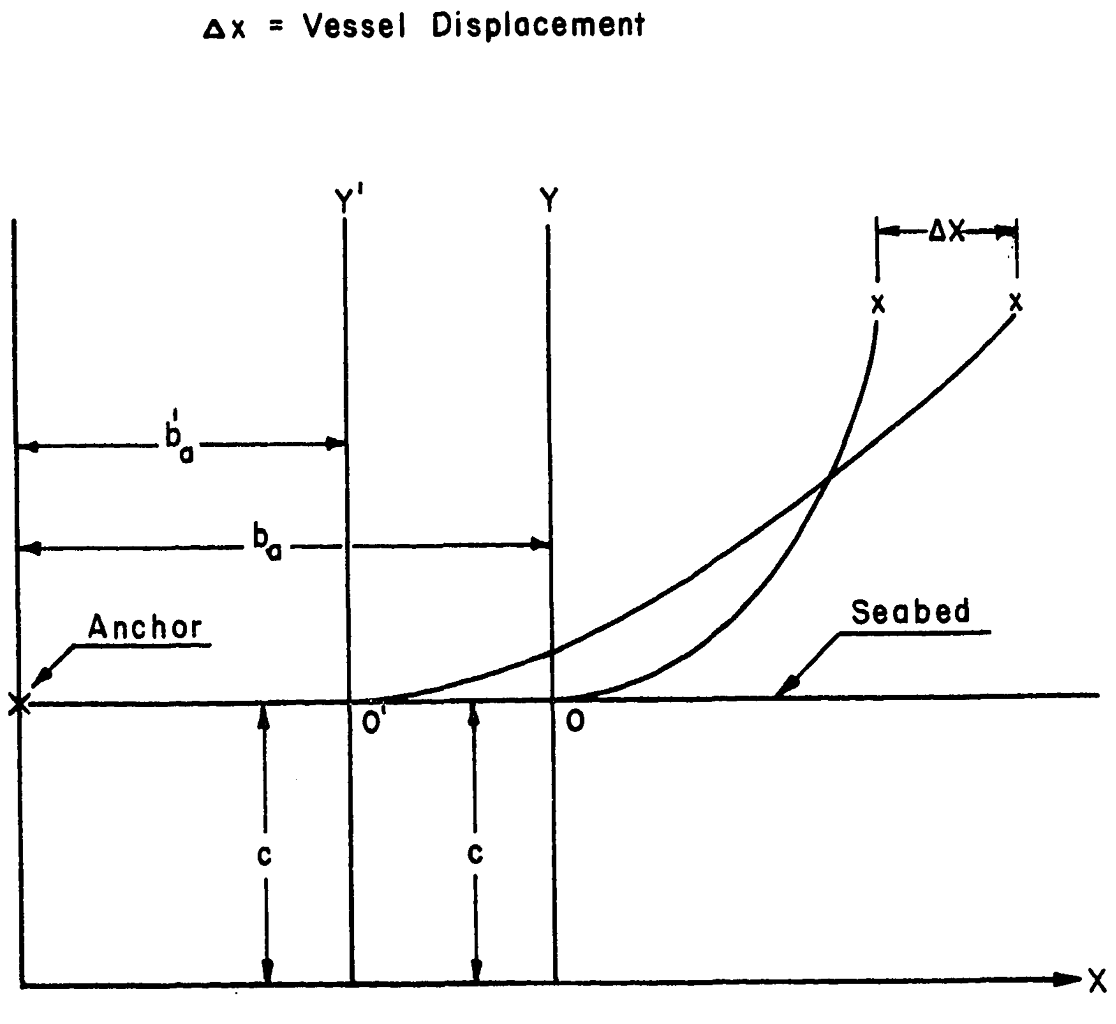


Figure 11: Typical mooring line in static equilibrium.

the equations of static equilibrium. Consider the right-hand side of a catenary shown in Figure 10 as being in static equilibrium. The curve which forms the catenary is well-known to be

$$y = c \cosh \frac{x}{c}, \qquad \text{EQ. (99)}$$

and the tangential tension along the chain is

$$F = W_c y, \qquad \text{EQ. (100)}$$

where W_c is the effective weight per unit length. Note that a portion of the total chain length lies flat on the seabed. The tension is found readily assuming something is known about the system. But, let the end point be displaced to a new equilibrium position at $x + \Delta x$, then the solution for tension is by trial and error. The displacement causes two things to happen that are difficult to handle:

1) It changes the arclength by lifting a portion of the chain from the seabed, thus shifting the origin an unknown distance to the left.
2) The increase in tension simultaneously affects the parameter "c" defined by the ratio

   $$c = \frac{H_a}{W_c},$$

   where H_a is the chain tension at the origin.

It is desirable that the forces that arise in the mooring system due to the motion of the vessel be in an explicit form that can be evaluated quickly. To achieve this point, a separate problem will be formulated to give the tension in each line at the vessel as a function of distance from the anchor point which is stationary.

Here, some of the parameters of the system are defined:

Total length, each chain = 5000 ft.

Effective unit weight = 91.4 lb./ft.

Water depth = 1000 ft.

The arclength with the chain completely slack corresponds to the water depth. Calculating the tension at this point, the slack distance, and successively increasing the arclength by equal increments to the total length, data points for tension at the vessel as a function of distance from the anchor are found. The variation in chain tension due to vertical displacements is assumed to be negligible. The computer program in Appendix C makes this calculation for 417 data points. Selected data is plotted in Figure 12 for a tension range of approximately 100,000 lbs. to 1,200,000 lbs.

These data points are fitted by "least-squares" to a finite series of Chebyshev polynominals which results in an explicit equation for the tension F_m at the vessel as a function of vessel displacement in the form

$$F_m(x) = \sum B_n P_{n-1}(x) \quad , \qquad \text{EQ. (101)}$$

where the polynominal is calculated in the form of its

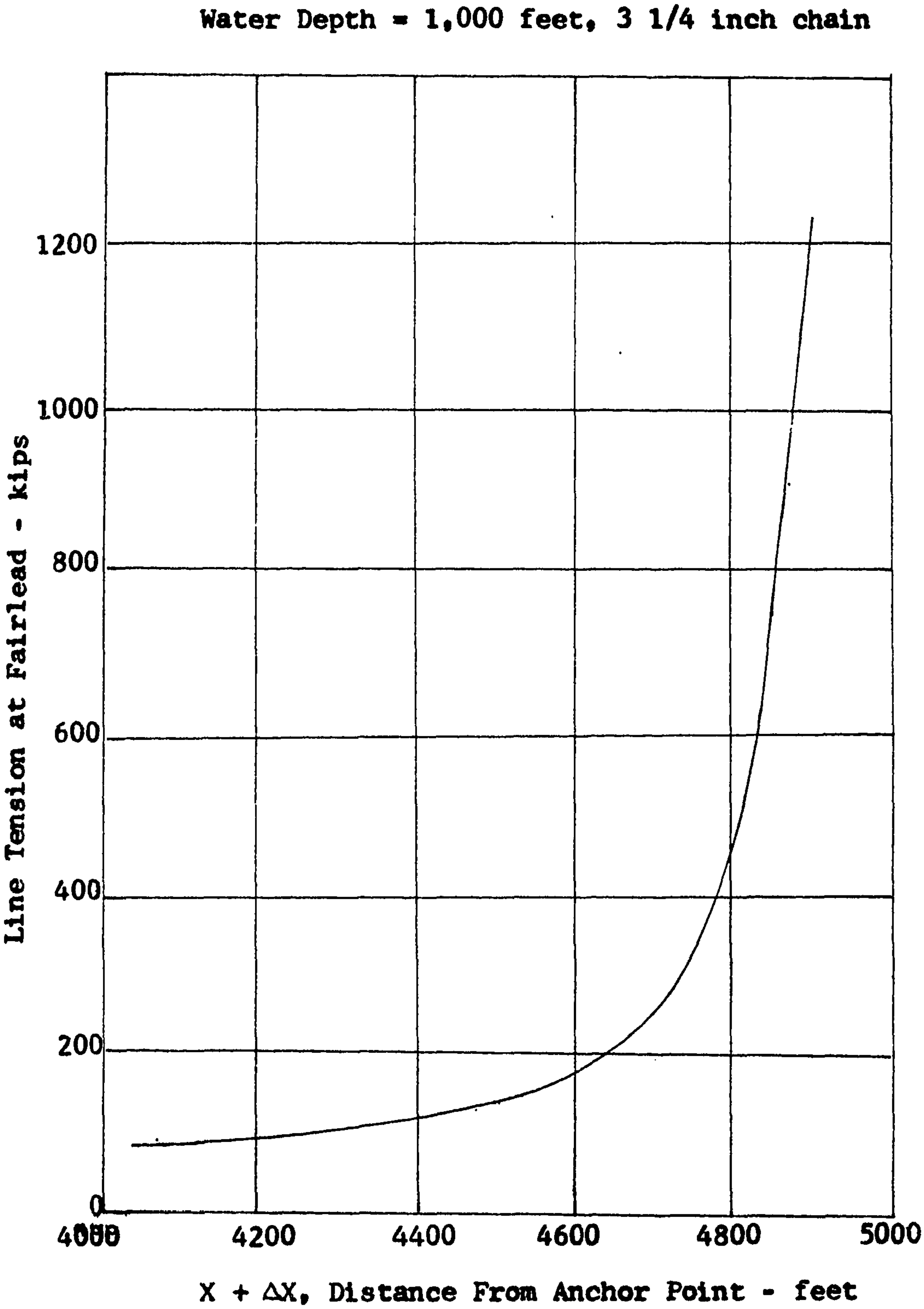


Figure 12: Tension at the fairlead as a function of distance from the anchor point.

TABLE 1

MOORING LINE DATA

Water Depth = 1000 feet   3 1/4 inch chain at 105.11 lbs/ft.

X+ ΔX feet		Anchor Tension lbs.	Percent Error
4032	(completely slack)	91434	.0127
4070		92217	.0344
4118		93779	-.0012
4173		96171	.0061
4202		97797	-.0108
4252		101130	.0071
4294		104560	.0071
4330		108100	-.0058
4375		113600	-.0022
4430		122280	.0064
4471		130440	-.0023
4514		141220	-.0036
4559		156200	.0040
4601		174830	.0012
4644		200520	-.0028
4687		236300	.0016
4730		288020	.0006
4773		367980	-.0011
4816		501950	.0010
4859		749470	-.0002
4898		1203500	.0011

Chebyshev expansion

$$\sum B_n P_{n-1}(x) = B_1 P_0(x) + B_2 P_1(x) + \ldots + B_n P_{n-1}(x) \ . \qquad \text{EQ. (102)}$$

The detail of the curve-fitting technique and the solution for the coefficients are given in Appendix C.

A relative error of the fitted curve was calculated for each of the data points to determine the accuracy where

$$\text{Error} = \frac{F_D - F_c}{F_D} \times 100 \ . \qquad \text{EQ. (103)}$$

The relative error in the fitted curve is less than 0.01 percent in the above tension range with eighteen terms; this could be reduced by using additional terms in the polynominal expansion during the "least squares" process.

As mentioned earlier Equation (101) gives the tangential tension at the vessel. In order to separate this force into its component parts, an identical curve fitting procedure is used to determine the corresponding angle that the force deviates from horizontal. This angle is defined by

$$\delta = \arctan\left[\sum C_n P_{n-1}(x_D)\right] \ . \qquad \text{EQ. (104)}$$

The horizontal and vertical restoration forces of the mooring system are now written as

$$F_{hm}(x_D) = \pm\cos\delta \sum B_n P_{n-1}(x_D) \ , \qquad \text{EQ. (105)}$$

and

$$F_{vm}(x_D) = -\sin\delta \sum B_n P_{n-1}(x_D) \ . \qquad \text{EQ. (105a)}$$

The sign of Equation (105) depends on the location of the particular mooring line as shown in Figure 13.

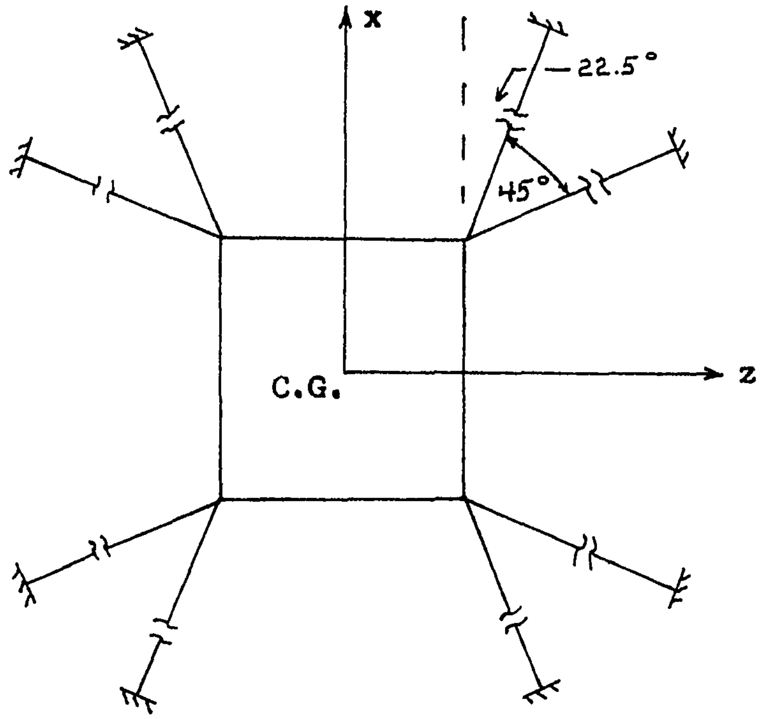


Figure 13: Top view and orientation of mooring system.

To utilize the above equations for chain tension, initial distances from each anchor point must be specified which gives the tensions at calm conditions.

Wind Forces

The general wind force equation appropiate to bodies exposed to a uniform wind velocity V_w is

$$F_w = 0.0038AV_w^2 \quad , \qquad \text{EQ. (106)}$$

where the units of V_w is in knots and the area of flat surface A is in $(\text{feet})^2$. The constant, .0038, contains the dimensionless drag coefficient $C_D = 1.0$.

The wind velocities used in this analysis together with the wave amplitudes correspond to the Beaufort Scale of which an abbreviated summary is given below.

Beaufort Scale

Beaufort Force	Description	Average Wind Velocity-knots	Average Wave Height-feet
0	Calm	0	0
1	Light Airs	2	0.05
2	Light Breeze	5	0.18
3	Gentle Breeze	8.5	0.6
4	Moderate Breeze	13.5	1.8
5	Fresh Breeze	19	4.3
6	Strong Breeze	24.5	8.2
7	Moderate Gale	30.5	14.0
8	Fresh Gale	37	23.0
9	Strong Gale	44	36.0
10	Whole Gale	51.5	52.0
11	Storm	59.5	73.0
12	Hurricane	>64	>80.0

Basic data and calculated areas have been furnished by the vessel owner; this data is included in Appendix D. From this data the applied wind forces in the negative x and z directions are found to be

$$F_{wx} = -69.3056(V_w \cos\alpha \cos\psi)^2 , \qquad \text{EQ. (107)}$$

and

$$F_{wz} = -73.4496(V_w \sin\alpha\cos\theta)^2 \quad . \qquad \text{EQ. (108)}$$

These forces are applied to the vessel as in a calm sea. The affect of roll and pitch are included; however, no attempt is made to account for the change in the exposed area of the columns due to motion and wave action.

Summary of Forces

The forces on each of the hull-column segments are linearly superimposed to give a total force. These terms are summarized as follows:

Vertical Force

$$F_y = -\Delta M_{hy}\dot{\bar{V}}_{hy} \qquad \text{(Added Mass)}$$

$$-D_{hy}\bar{V}_{hy}|\bar{V}_{hy}| - C_{hy}V_{hy} \qquad \text{(Drag and Damping)}$$

$$-\frac{\pi d_h^2}{4}\,\delta jm\int_{x_c - \frac{1}{2}L_h}^{x_c + \frac{1}{2}L_h} \frac{\delta P}{\delta y}\,dx \qquad \text{(Pressure)}$$

$$\frac{-S(t)\pi d_c^2}{4}\int_{y_h + \frac{1}{2}d_h}^{b} \frac{\delta P}{\delta y}dy \qquad \text{(Pressure)}$$

$$-\frac{\pi d_h^2}{4}\,\delta jn\int_{z_c - \frac{1}{2}L_h}^{z_c + \frac{1}{2}L_h} \frac{\delta P}{\delta y}dz \qquad \text{(Pressure)}$$

$$+\frac{\pi\varrho d_c^2}{4}\begin{cases} -\bar{V}_{hyj}\bar{V}_{cxj}, & j = 1,2,3,6 \\ 0 \quad , & j \neq 1,2,3,6,11,14,15,16 \\ +\bar{V}_{hyj}\bar{V}_{cxj}, & j = 11,14,15,16 \end{cases}$$ (Oblique Flow)

$$-\frac{\pi\varrho g d_c^2}{4}\Delta W_L$$ (Change in bouyancy)

$$-\sin\delta\ (\delta_{jk})\sum B_n P_{n-1}(X_D)\ .$$ (Mooring lines)

EQ. (109)

Horizontal Forces

$$F_{cx} = -S(t)\Delta M_{cx}\dot{\bar{V}}_{cx}$$ (Added mass)

$$-S(t)D_{cx}\bar{V}_{cx}|\bar{V}_{cx}| - C_{cx}S_j(t)V_{cx}$$ (Drag and Damping)

$$-S(t)\frac{\pi d_c^2}{4}\ \delta_{jn}\int_{y_h+\frac{1}{2}d_h}^{b}\frac{\delta P}{\delta x}dy$$ (Pressure)

$$-\frac{\pi\varrho d_c^2}{4}\bar{V}_{cx}\frac{d}{dt}\Delta W_L$$ (Change in momentum)

$$\pm\cos\delta\sum B_n P_{n-1}(X_D)\begin{cases} + \ , & j = 3,6 \\ 0 \ , & j \neq 3,6,11,14 \\ - \ , & j = 11,14 \end{cases}$$ (Mooring lines)

EQ. (110)

$$F_{hx} = -\ \Delta M_{hx}\dot{\bar{V}}_{hx}$$ (Added mass)

$$-\ D_{hx}\bar{V}_{hx}|\bar{V}_{hx}| - C_{hx}\bar{V}_{hx}$$ (Drag and Damping)

$$-\frac{\pi d_h^2}{4}\,\delta_{jn}\int_{z_c - \frac{1}{2}L_h}^{z_c + \frac{1}{2}L_h} \frac{\delta P}{\delta x}dz \quad , \quad \text{(Pressure)}$$

EQ. (111)

$$F_{cz} = -S(t)\triangle M_{cz}\dot{\bar{V}}_{cz} \quad \text{(Added mass)}$$

$$-D_{cz}\bar{V}_{cz}|\bar{V}_{cz}| - C_{cz}S_j(t)V_{cz} \quad \text{(Drag and Damping)}$$

$$-\frac{\pi d_c^2}{4}\,\delta_{jm}\int_{y_h + \frac{1}{2}d_h}^{b} \frac{\delta P}{\delta z}dy \quad \text{(Pressure)}$$

$$-\frac{\pi \rho d_c^2}{4}\,\bar{V}_{cz}\,\frac{d}{dt}(\triangle W_L) \quad \text{(Change in momentum)}$$

$$\pm\cos\delta \sum B_n P_{n-1}(X_D) \begin{cases} + \,, & j = 3,11 \\ - \,, & j = 6,14 \end{cases} \quad , \quad \text{(Mooring lines)}$$

EQ. (112)

$$F_{hz} = -\triangle M_{hz}\dot{\bar{V}}_{hz} \quad \text{(Added mass)}$$

$$-D_{hz}\bar{V}_{hz}|\bar{V}_{hz}| - C_{hz}V_{hz} \quad \text{(Drag and Damping)}$$

$$-\frac{\pi d_h^2}{4}\,\delta_{jm}\int_{x_c - \frac{1}{2}L_h}^{x_c + \frac{1}{2}L_h} \frac{\delta P}{\delta z}dx \quad \text{(Pressure)}$$

$$+\frac{\pi \varrho d_h^2}{4} \begin{cases} -\bar{V}_{hzj}\bar{V}_{cxj}, & j = 1,2,3,6 \quad \text{(Oblique Flow)} \\ 0 & j = 1,2,3,6,11,14,15,16 \\ +\bar{V}_{hzy}\bar{V}_{cxj}, & j = 11,14,15,16 \end{cases} ,$$

EQ. (113)

where

$$\delta_{jk} = \begin{cases} 1, & j = k,\ k = 3,6,11,14 \\ 0, & j \neq k \end{cases} ,$$

$$\delta_{jm} = \begin{cases} 1, & j = m,\ m = 1\text{-}17 \\ 0, & j \neq m \end{cases} ,$$

$$\delta_{jn} = \begin{cases} 1, & j = n,\ n = 17\text{-}24 \\ 0, & j \neq n \end{cases} .$$

The uniform wind force which is applied to the vessel in its entirety completes the equations of motion for linear displacement.

$$M\ddot{X} = \sum_{j=1}^{24} [F_{cxj}+F_{hxj}] - 69.3056(V_w \cos\alpha \cos\psi)^2,$$

EQ. (114)

$$M\ddot{Y} = \sum_{j=1}^{24} F_{yj} ,$$

EQ. (115)

$$M\ddot{Z} = \sum_{j=1}^{24} [F_{czj}+F_{hzj}] - 73.4496(V_w \sin\alpha \cos\theta)^2.$$

EQ. (116)

Equations of Rotational Motion

Roll Moment

The moments around the center of gravity are determined by multiplying by the appropiate moment arm. It was previously specified that horizontal and vertical forces acting on the hull are concentrated at the intersection of the hull-column centerline; whereas, forces which act horizontally on the columns were assumed to be concentrated at the midpoint of the wetted length. An exception to this is the horizontal components of anchor line tension. In calm water, these forces act at the same elevation as the center of gravity for the particular vessel modeled.

For the roll moment consider the schematic shown in Figure 14.

The figure depicts only the perturbation forces and the anchor line force. For the moment equations only the hydrostatic buoyancy force must be added to the vertical force perturbation F_{yj}. This term is equivalent to the mass of displaced seawater and is given as

$$+ \frac{\pi\rho}{4}[d_c^2(b-y_h- \tfrac{1}{2}d_h) + d_h^2L_h] \text{ .}$$

The total roll moment contains the effect of wind which has been determined from furnished data in Appendix D as

$$M_W(\theta) = -6411.55(V_W\sin\alpha\cos\theta)^2. \qquad \text{EQ. (117)}$$

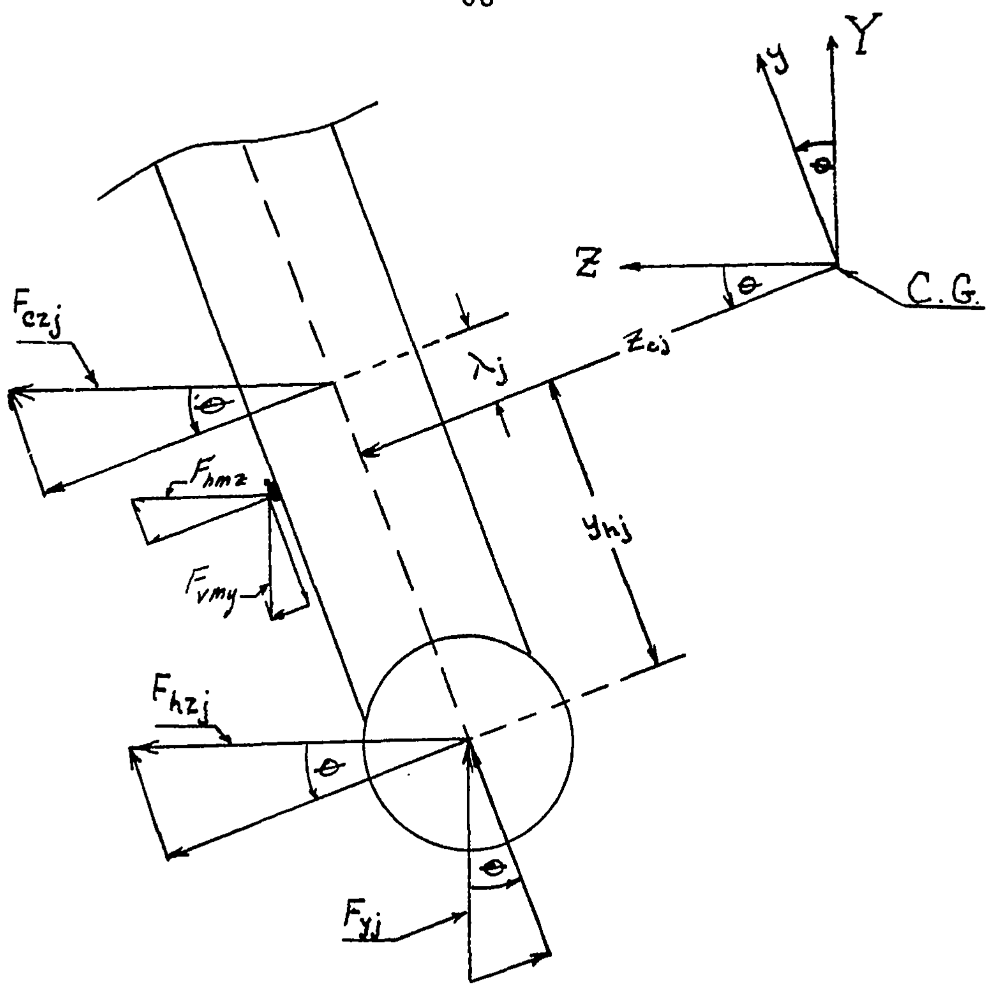


Figure 14: Hull-column in large roll.

The total moment is then corrected for the position change for the center of buoyancy. The total buoyant force is equal and opposite in direction of the weight of the vessel acting through the center of gravity. In roll and pitch the total buoyant force moves to a new line of action where it exerts a restoring moment on the vessel for positive stability. The metacentric height GM which is the vertical distance between the center of gravity and the center of buoyancy in calm water must be determined by static inclination tests on the vessel. This bit of data has also been furnished by the vessel owner. The restoring roll moment due to this concept is well-known to be

$$-M_g GM_t \sin\theta$$

and is illustrated in Appendix B, p. 181.

With these additions the total roll moment is observed to be

$$\begin{aligned} M(\theta) = \sum_{j=1}^{24} \Big[& -F_{yj}(y_{hj}\sin\theta + z_{cj}\cos\theta) + F_{hzj}(y_{hj}\cos\theta - z_{cj}\sin\theta) \\ & + F_{czj}(\lambda_j\cos\theta - z_{cj}\sin\theta) \\ & - F_{hmzj}(z_c \pm d_{cj})\sin\theta - F_{vmj}(z_{cj} \pm d_{cj})\cos\theta \Big] \\ & - M_g GM_t \sin\theta - 6411.55(V_W \sin\alpha\cos\theta)^2 , \end{aligned}$$

EQ. (118)

where λ_j designates the vertical distance from the z-axis to the mid-point of the column's wetted length. The sign preceding d_c in the mooring line moment is taken as the same as the z_c coordinate.

Included in Equation (118) is the damping moment of the added inertia of the displaced water since the force terms consider the added mass. Similarly, added inertia will be included in the yaw and pitch moments which follow.

Yaw Moment

Since only horizontal forces produce a yaw moment the hull and column forces are added to give a composite F_{xj} and F_{zj} which is shown in Figure 15.

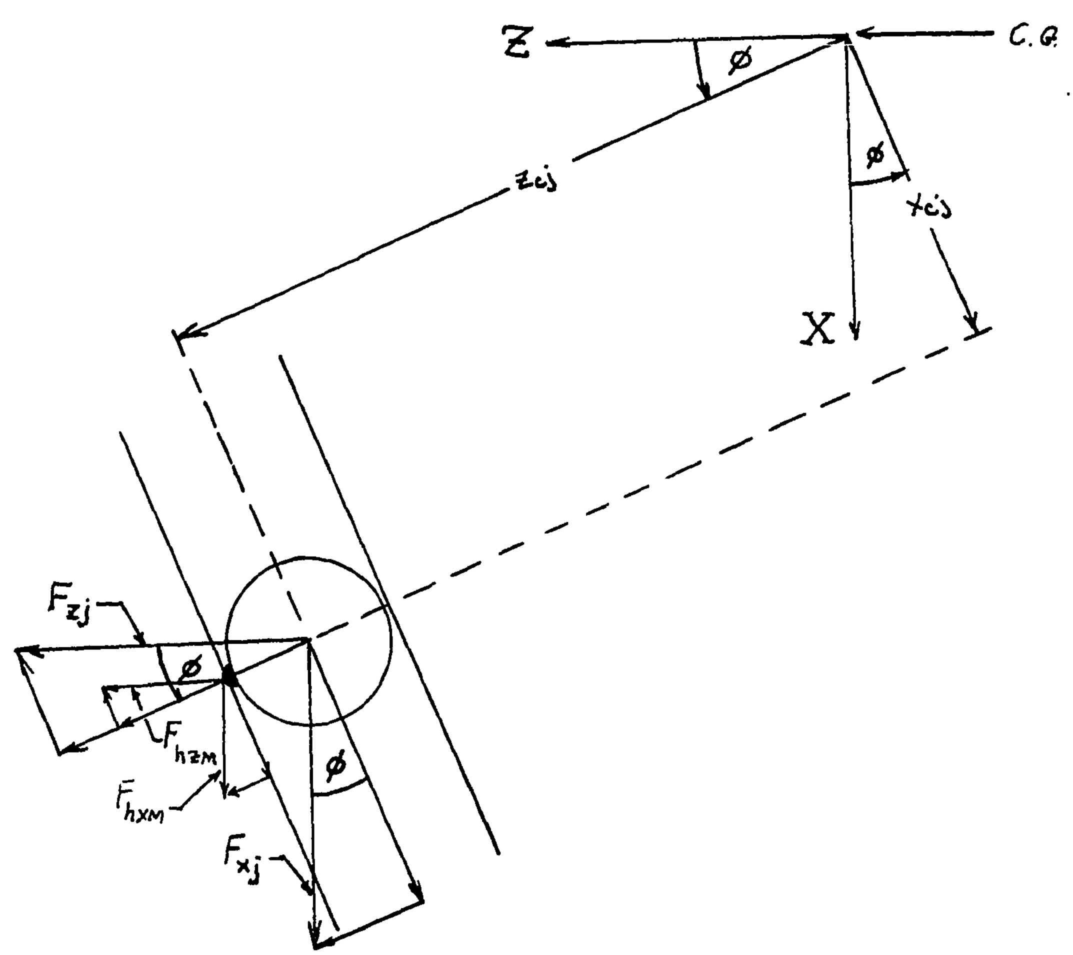


Figure 15: Typical hull-column with large yaw angle.

The moment equation for yaw is observed to be

$$M(\phi) = \sum \Big\{ F_{xj}(z_{cj}\cos\phi - x_{cj}\sin\phi) - F_{zj}(x_{cj}\cos\phi + z_{cj}\sin\phi)$$
$$+ F_{hmxj}[(z_{cj} \pm d_{cj})\cos\phi - (x_{cj} \pm d_{cj})\sin\phi]$$
$$- F_{hmzj}[(x_{cj} \pm d_{cj})\cos\phi + (z_{cj} \pm d_{cj})\sin\phi] \Big\} . \quad \text{EQ. (119)}$$

The wind forces would impose a minor effect on the yaw moment; this is omitted due to insufficient data that is needed to give the position of the surface areas relative to the horizontal plane.

<u>Pitch Moment</u>

The pitch moment is found in much the same manner as the roll moment. Consider the typical hull-column segment illustrated in Figure 16 which exhibits a positive counter-clockwise rotation about the z-axis.

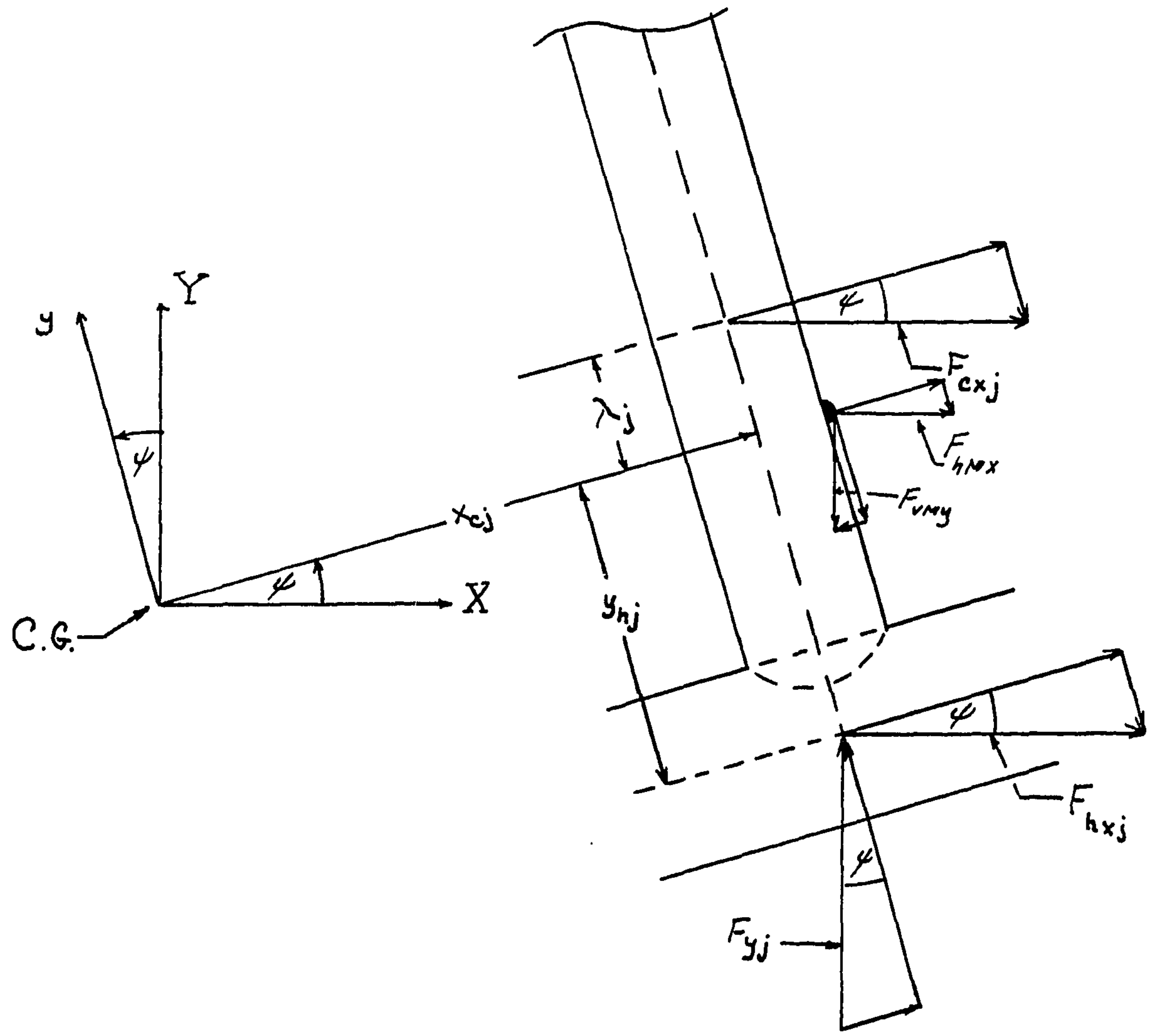


Figure 16: Typical hull-column with large pitch angle.

The total pitch moment is readily observed to be

$$
\begin{aligned}
M(\psi) = \sum \big[& F_{yj}(x_{cj}\cos\psi - y_{hj}\sin\psi) \\
& - F_{hxj}(x_{cj}\sin\psi + y_{hj}\cos\psi) \\
& - F_{cxj}(\lambda_j\cos\psi + x_{cj}\sin\psi) \\
& - F_{hmxj}(x_{cj} \pm d_c)\sin\psi \\
& - F_{vmxj}(x_{cj} \pm d_c)\sin\psi \big] \\
& + 6967.07(V_W\cos\alpha\cos\psi)^2 \\
& - M_g GM_L \sin\psi \; . \qquad \text{EQ. (120)}
\end{aligned}
$$

Again, the last term is the correction due to the position change in the center of buoyancy.

<u>Summary of Equations of Motion</u>

The development of the forces, moments, angular velocities and accelerations that enter into the general equations of motion for a rigid body excited by a wave train with an accompanying wind force is now complete. These equations are rewritten in their final form as

$$M\ddot{X} = \sum_{j=1}^{24} [F_{cxj} + F_{hxj}] + F_{wx} \; , \qquad \text{EQ. (121)}$$

$$M\ddot{Y} = \sum_{j=1}^{24} F_{yj} \; , \qquad \text{EQ. (122)}$$

$$M\ddot{Z} = \sum_{j=1}^{24} [F_{czj} + F_{hzj}] + F_{wz} \; , \qquad \text{EQ. (123)}$$

$$\dot{H}_x' - \dot{\sigma}_z H_y' + \dot{\sigma}_y H_z' = M(\theta) \; , \qquad \text{EQ. (124)}$$

$$\dot{H}_y' - \dot{\sigma}_x H_z' + \dot{\sigma}_z H_x' = M(\phi) \; , \qquad \text{EQ. (125)}$$

$$\dot{H}_z' - \dot{\sigma}_y H_x' + \dot{\sigma}_x H_y' = M(\psi) \; . \qquad \text{EQ. (126)}$$

Thrust Requirements

The objective of this study, to determine the required thrust of a mechanical system to maintain the vessel within a reasonable tolarance of location, may be reached by solving the above set of ordinary differential equations. The solution for the linear and angular displacements will simultaneously yield the forces that arise in the mooring system which keeps the vessel on station. It should be recognized that only the horizontal components of the mooring line forces contribute to station keeping. The required thrust is equated to the maximum of the resultant sum

$$T_r = \max[(\sum F_{hmx})^2 + (\sum F_{hmz})^2]^{\frac{1}{2}} , \qquad \text{EQ. (127)}$$

in the direction

$$\Upsilon = \arctan[\sum F_{hmz} / \sum F_{hmx}] \; , \qquad \text{EQ. (128)}$$

where Υ is considered positive when clockwise from the positive x-axis.

The thrust moment which is equally important in order to keep the vessel oriented properly is taken directly from the yaw moment, Equation (119)

$$M(T_r) = \sum \Big\{ F_{hmxj}[(z_{cj} \pm d_{cj})\cos\phi - (x_{cj} \pm d_{cj})\sin\phi] - F_{hmzj}[(x_{cj} \pm d_{cj})\cos\phi + (z_{cj} \pm d_{cj})\sin\phi] \Big\} \; . \qquad \text{EQ. (129)}$$

CHAPTER IV

SOLVING THE EQUATIONS OF MOTION

The six equations of motion are second-order ordinary differential equations which contain many facets of non-linearity. No attempt is made to simplify the equations to obtain an analytical solution. This would be opposed to one of the minor objectives of this study, the retention of the non-linear features. It is indeed fortunate that all second-derivative terms are linear with respect to themselves. This permits a straight-forward numerical integration by solving the linear matrix equation

$$[A][\ddot{x}] = b$$

at each time step.

The equations of motion derived in Chapter III are rearranged where the left-hand side of the system contain only terms involving a second derivative. This involves transposing the second derivative terms in the "added mass". Likewise, certain terms in the angular momentum equations contain first-order terms alone and are transposed to the right-hand side. The system of six second-order equations is solved by reducing each to two first-order differential equations. This reduction is deduced by letting

$$x = y_1$$

$$y_1' = \frac{dx}{dt} = f_1(y_1) = y_2$$

$$y_2' = \frac{d^2x}{dt^2} = f\ (y_1, y_2, \ldots, y_{12}, t)$$

$$y = y_3$$

$$y_3' = \frac{dy}{dt} = f_3(y_3) = y_4$$

$$y_4' = \frac{d^2y}{dt^2} = f_4(y_1, y_2, \ldots, y_{12}, t)$$

$$z = y_5$$

$$y_5' = \frac{dz}{dt} = f_5(y_5) = y_6$$

$$y_6' = \frac{d^2z}{dt^2} = f_6(y_1, y_2, \ldots, y_{12}, t)$$

$$\theta = y_7$$

$$y_7' = \frac{d\theta}{dt} = f_7(y_7) = y_8$$

$$y_8' = \frac{d^2\theta}{dt^2} = f_8(y_1, y_2, \ldots, y_{12}, t)$$

$$\phi = y_9$$

$$y_9' = \frac{d\phi}{dt} = f_9(y_9) = y_{10}$$

$$y_{10}' = \frac{d^2\phi}{dt^2} = f_{10}(y_1, y_2, \ldots, y_{12}, t)$$

$$\psi = y_{11}$$

$$y_{11}' = \frac{d\psi}{dt} = f_{11}(y_{11}) = y_{12}$$

$$y_{12}' = \frac{d^2\psi}{dt^2} = f_{12}(y_1, y_2, \ldots, y_{12}, t)$$

with the vector of given initial values

$$Y_o = \begin{Bmatrix} y_1\ (0) \\ y_2\ (0) \\ \cdot \\ \cdot \\ \cdot \\ y_{12}\ (0) \end{Bmatrix} .$$

"PROSPCTR" —— Computer Program

The program "Prospctr" is written in single-precision Fortran IV to solve the system of twelve first-order differential equations. The program is divided into the following parts:

a) MAIN

b) Subroutine PHPCG

c) Subroutine FCT

d) Subroutine RHSV

e) Subroutine DISPL

f) Subroutine FOXYZ

g) Function P

h) Subroutine AMAT

i) Subroutine SIMQX

j) Subroutine OUTP

k) Subroutine PLOTR

MAIN

The "MAIN"-line reads data and calculates numerous constants that pertain to a particular hull or column. The constants are arranged as vectors containing twenty-four elements; however, for the ficticious columns (17-24) the elements are zero. Once, the data is set up, the integration routine PHPCG is called to integrate the equations of motion. Integration will cease and return to MAIN after a specified number of wave cycles. The MAIN will then return to read an additional set of data or end on the error message "END OF DATA SET UNIT 5".

SUBROUTINE PHPCG

The integration subroutine is taken directly from the IBM Scientific Subroutine Package [18]. This subroutine uses Hamming's modified predictor-corrector method to obtain an approximate solution to a general system of first-order ordinary differential equations with given initial values. It is a stable fourth-order integration procedure that requires the evaluation of the right-hand side of the system only two times per time step. This feature offers considerable savings in computation time as compared with other methods of the same order of accuracy. However, the method is not self-starting; that is, the functional values at a single previous point are not enough to get the functional values ahead. To obtain the starting values, a special Runge-Kutta procedure followed by one iteration step is added to the predictor-corrector method. At the end of the integration PHPCG returns to MAIN.

SUBROUTINE FCT

The subroutine "FCT" has two functions:

1). The constants calculated in MAIN must be updated with each time step since the angle of incidence is a function of yaw.

2). To calculate the vector of derivatives in the matrix equation when called for by the integration subroutine "PHPCG".

Values of the derivative vector are obtained by calling the Subroutines RSHV, AMAT, and SIMQX. RSHV calculates the elements in the right-hand side vector and AMAT calculates the elements of the coefficient matrix. Then Gauss-elimination (SIMQX) is applied to solve the system of linear equations. The vector of derivatives are returned to the integration subroutine PHPCG.

SUBROUTINE RHSV

In essence, RHSV (abbreviated as right-hand side vector) calculates the forces and moments involved in the equations of motion less those terms of "added mass" that contain second-order derivatives (these are transposed to the left-hand side).

The forces that arise in the mooring lines are determined by calling the subroutines DISPL and FOXYZ which returns these values to RHSV.

RHSV returns the right-hand side vector to FCT.

SUBROUTINE DISPL

The only purpose of this subroutine is to determine the horizontal displacement (X and Z directions) of those columns in which mooring lines are attached. The transformation given by the general rotation matrix is used to give the added displacement due to rotation. DISPL returns the displacements to RHSV.

SUBROUTINE FOXYZ

The horizontal and vertical components of anchor line tensions are calculated from the displacement values determined by DISPL with a given initial distance from the anchor point. The coefficients calculated by computer program in Appendix C are expressed explicitly in this subroutine. The tensions are then calculated by the Chebyshev polynominal expansion where the value of each polynominal in the expansion is given by the Function P. Values for anchor tension are returned to RHSV.

Function P

For each term in the Chebyshev expansion a calculation is made using explicit polynominal equations derived from the Chebyshev recurrance relation. This function subprogram accomodates FOXYZ up to eighteen terms.

SUBROUTINE AMAT

The purpose of this subroutine is to calculate the elements of the coefficient matrix as called upon by FCT. The elements are the coefficients of all second-order derivatives in the equations of motion including those second-derivative terms inbedded in the "added-mass" which have been transposed to the left-hand side. This two-dimensional array is returned to FCT.

SUBROUTINE SIMQX

After the system of linear equations are set up by the subroutines RHSV and AMAT, FCT calls upon this subroutine to solve for the unknowns, a vector of first-order derivatives. This subroutine is identical to SIMQ in the IBM Scientific Subroutine Package [9]. The solution is obtained by Gauss-elimination using the largest pivotal divisor. The right-hand side vector is replaced by the solution vector within SIMQX before return to FCT.

SUBROUTINE OUTP

As the name infers, OUTP is called upon by PHPCG to print the results of integration at each time-step.

Verification of the Computer Program

The computer program for solving the six non-linear differential equations produces a solution for the six degrees of freedom (surge, heave, sway, roll, yaw, and pitch). The theoretical results are plotted versus experimental results of model tests for head and beam seas for a thirty feet high regular wave train (see Figures 17, 18, 19, and 20).

The model tests were conducted without any attempt to model the mooring system. Only slack restraining lines were used to eliminate the model drift. Further, wind or current forces were not considered in the model testing.

The computer program was idealized to suit these conditions; i.e., all mooring, wind, and current forces were set equal to zero. No attempt has been made to match the model test results by adjustment of the added mass and/or damping coefficients.

Comparison of results of model testing versus the analytical approach, one finds that the comparison is very close and well within the inherent basin and scaling error associated with model testing.

Wind tunnel testing, although very desirable could not be arranged for this study.

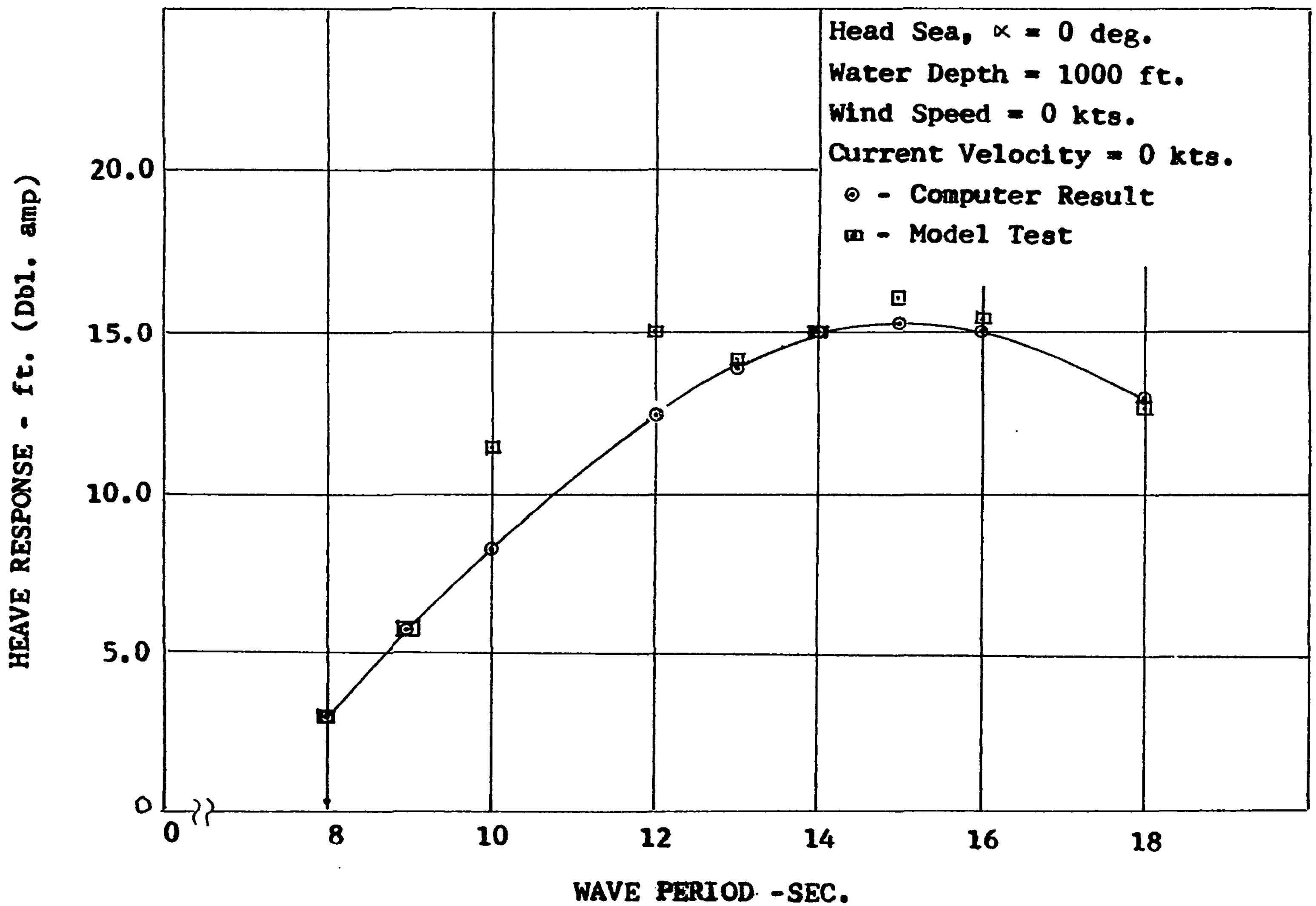


Figure 17: HEAVE RESPONSE VS WAVE PERIOD, HEAD SEAS

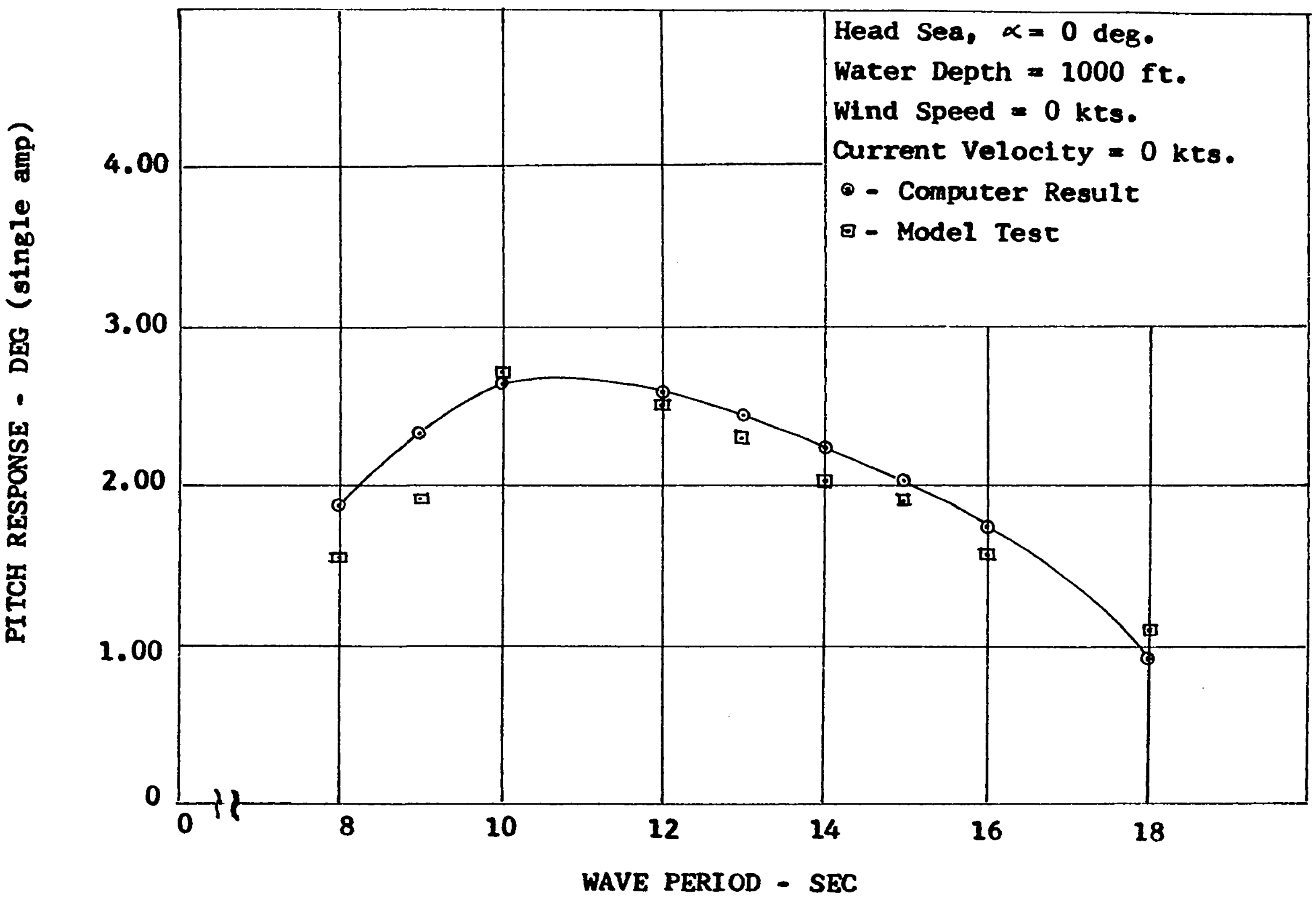


Figure 18: PITCH RESPONSE VS WAVE PERIOD, HEAD SEAS

TABLE 2a

MOTION RESPONSE

MODEL TEST - COMPUTER RESULTS

Water Depth = 1000 ft.

Wind Speed = 0 kts.

Current Velocity = 0 kts.

Head Sea, α = 0 degrees

Wave Amplitude = 15 ft.

Anchor Chain Tension = 0 lbs.

Period sec	Wave Length -ft.	Heave - ft.		Pitch - Deg.	
		Model	Computer	Model	Computer
8	328	3.0	3.0	1.56	1.89
9	415	5.7	5.7	1.92	2.34
10	512	11.4	8.1	2.73	2.67
12	737	15.0	12.3	2.55	2.61
13	865	14.1	13.8	2.32	2.46
14	1000	15.0	15.0	2.04	2.25
15	1152	16.2	15.3	1.92	2.04
16	1310	15.6	15.0	1.59	1.74
18	1700	12.6	12.9	1.11	0.91

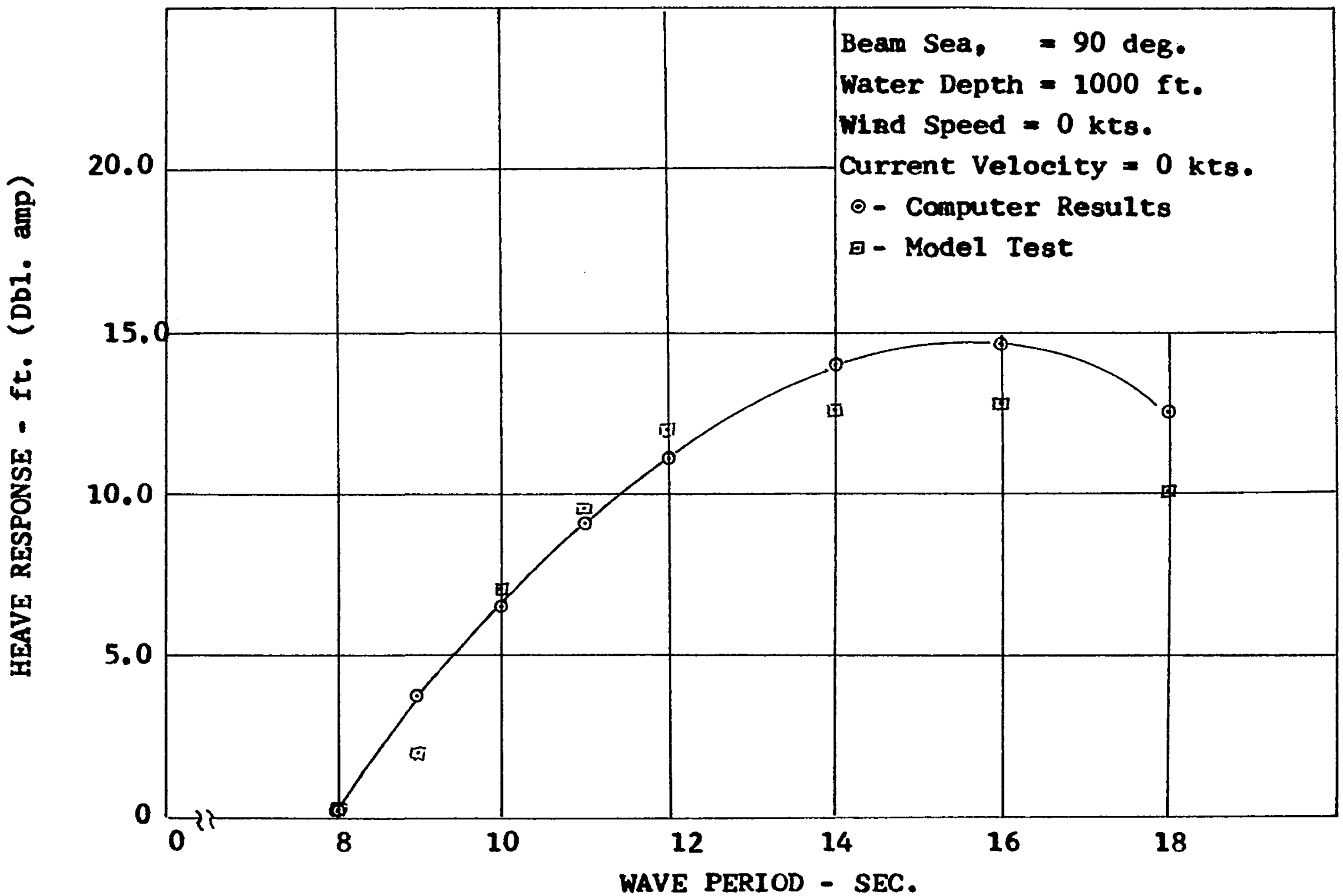


Figure 19: HEAVE RESPONSE VS WAVE PERIOD, BEAM SEAS

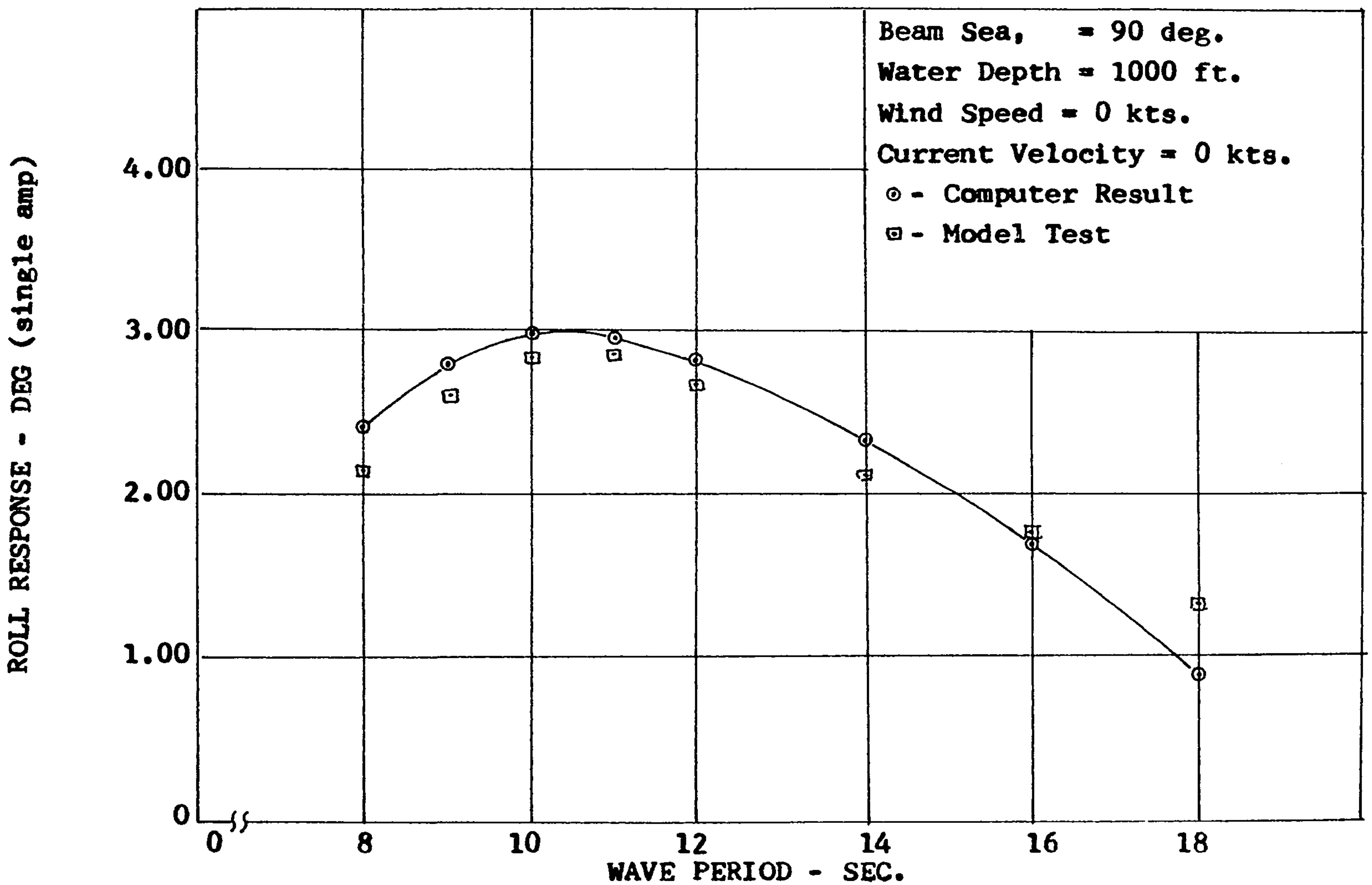


Figure 20: ROLL RESPONSE VS WAVE PERIOD, BEAM SEAS

TABLE 2b

MOTION RESPONSE

MODEL TEST - COMPUTER RESULTS

Water Depth = 1000 ft.

Wind Speed = 0 kts.

Current Velocity = 0 kts.

Beam Sea α = 90 degrees.

Wave Amplitude = 15 ft.

Anchor Chain Tension = 0 lbs.

Period Sec	Wave Length -ft.	Heave - ft.		Roll - Deg.	
		Model	Computer	Model	Computer
8	328	0.3	0.3	2.14	2.43
9	415	2.1	3.9	2.61	2.78
10	512	7.2	6.3	2.85	3.06
11	620	9.6	9.3	2.85	2.96
12	737	12.0	11.3	2.67	2.82
14	1000	12.6	14.0	2.10	2.34
16	1310	12.9	14.7	1.77	1.65
18	1700	10.2	12.5	1.32	0.88

CONCLUSION

Chapter V

Determination of Thrust Requirements

The differential equations of motion for six degrees of freedom have been formulated and a method of solution has been presented. Determination of the thrust needed to maintain the vessel within a pre-determined radius of the well and under a given weather and sea condition requires instantaneous magnitudes and directions of the resultant force vectors at the fairleads associated with the mooring lines. Equating the vector sum of the horizontal components of the mooring line force vectors to a single thrust vector with its magnitude and direction is basic to the subject of this entire study. Equating the vector sum of the mooring line components to the thrust vector is built into the computer program and its value and direction is also determined as a function of time. Although the instantaneous value of the thrust vector is required, the general subject of this study was defined to determine the maximum thrust required to maintain horizontal displacement within a predetermined radius over a broad range of weather and sea conditions which could be further defined as the upper extremities of conditions that the vessel could be expected to continue normal drilling operations. This range is chosen as Beaufort Force 7 thru 10.

Simple economics dictate that vessel heading should be such that resistance to the elements be a minimum. Arbitrarily, Beaufort 10 was used for this determination. Thrust requirements were then determined by incrementing the approaching weather and sea direction from a head sea (zero degrees) to a beam sea (ninety degrees). All acting forces, wind, current, and waves were assumed to be acting in the same direction. The resulting data is shown in Figure 19 which clearly defines a heading of thirty degrees counterclockwise away from the weather as being optimum. It should be noted that this optimum applies only for the particular vessel used as an example in this study. The bottom portion of the curve gives the minimum values resulting from vessel oscillation. The heading of thirty degrees is used in all subsequent calculations.

The next step is then to determine the thrust required to obtain a steady-state displacement. A maximum displacement equal to five percent of the water depth is generally considered the limit of efficient drilling operations. In order to determine the thrust required to stay within this limit, the pre-tensions in the mooring lines were incremented and the displacement was determined under steady-state conditions upon integration of the differential equations. Obviously, larger displacements than desired result with

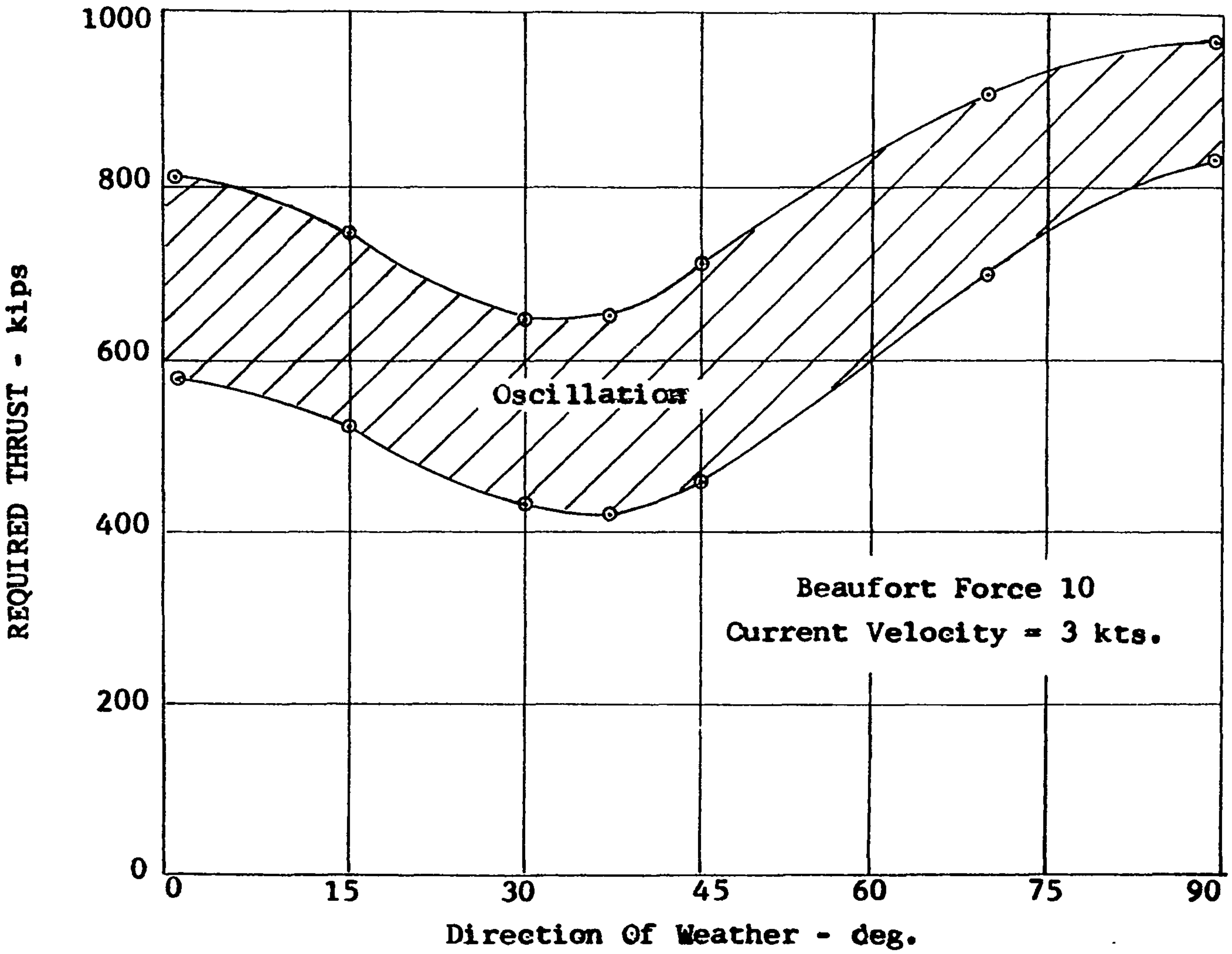


Figure 21: MINIMUM RESISTANCE DETERMINATION

Table 3a

Linear Displacements

Water Depth = 1000 ft.

Beaufort Force 10

Current Vel. = 3 kts.

Pre-Tensions = 299 kips (All lines)

Weather Direction (deg.)	Surge Min/Max (ft.)	Sway Min/Max (ft.)	Displ. Min/Man (ft.)
1.0	-79.7/-100.4	— — —	79.7/100.4
15.0	-74.4/-94.2	-5.1/-14.4	74.5/94.2
30.0	-58.0/-75.6	-23.2/-40.7	62.5/85.9
37.5	-44.2/-59.0	-41.7/-62.9	60.6/86.2
45.0	-30.6/-44.3	-56.7/-81.1	64.4/92.4
70.0	-2.5/-8.9	-92.1/-108.3	92.1/108.7
89.0	— — —	-103.6/-113.8	103.6/113.8

Table 3b

Restorations Requirements

Weather Direction (deg.)	Mooring Line Tensions Min/Max (kips)	Thrust Min/Max (kips)	Thrust Moment Min/Max (kips)
1.0	482/572	519/810	-58/+257
15.0	470/570	531/746	-827/+2718
30.0	390/461	437/645	-2967/+1282
37.5	423/518	420/656	-2691/-4732
45.0	449/568	454/713	-1287/-4836
70.0	534/633	703/907	-800/-4427
89.0	581/640	833/969	-785/-1923

low pre-tensions. This observation should have been anticipated by noting the soft spring constant at low tensions in the curve shown in Figure 11. The resulting data is plotted in Figure 22 for Beaufort Force 7, 8, 9 and 10. It is noted that the plot approaches linearity and for practical purposes the extrapolation to the five percent line is of sufficient accuracy. The thrust values of the intersecting points are then plotted against the Beaufort Scale as shown in Figure 23 which is the desired result. Other than the maximum and minimum values of thrust, it is noteworthy to observe the effects of the oscillating forces and vessel inertia associated with waves as opposed to the uniform effects of current and wind forces. The current and wind forces dominate the relatively low amplitudes of oscillation on the lower portion of the Beaufort Scale.

The theoretical approach to determine the thrust requirements is relatively simple compared to tank and wind tunnel testing of models and is believed to be at least as accurate as the experimental approach. The small cost of computer solutions by the method presented compared to model testing is most significant.

Although, the subsequent conversion of pounds of thrust to horsepower is not presented, this necessary portion is straight forward and is available directly from equipment manufacturers.

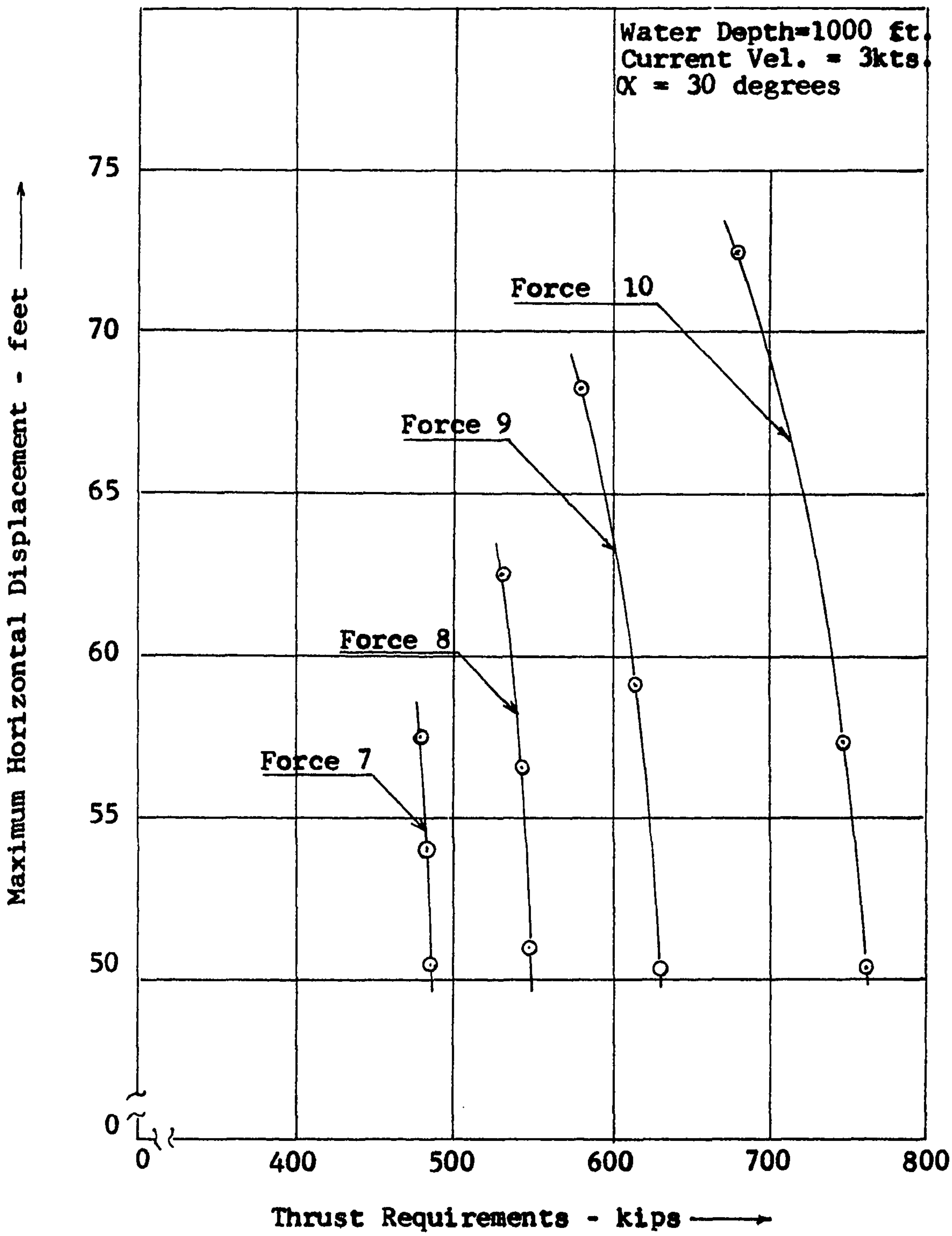


Figure 22: Thrust Requirements to Maintain 5 percent Horizontal Displacement.

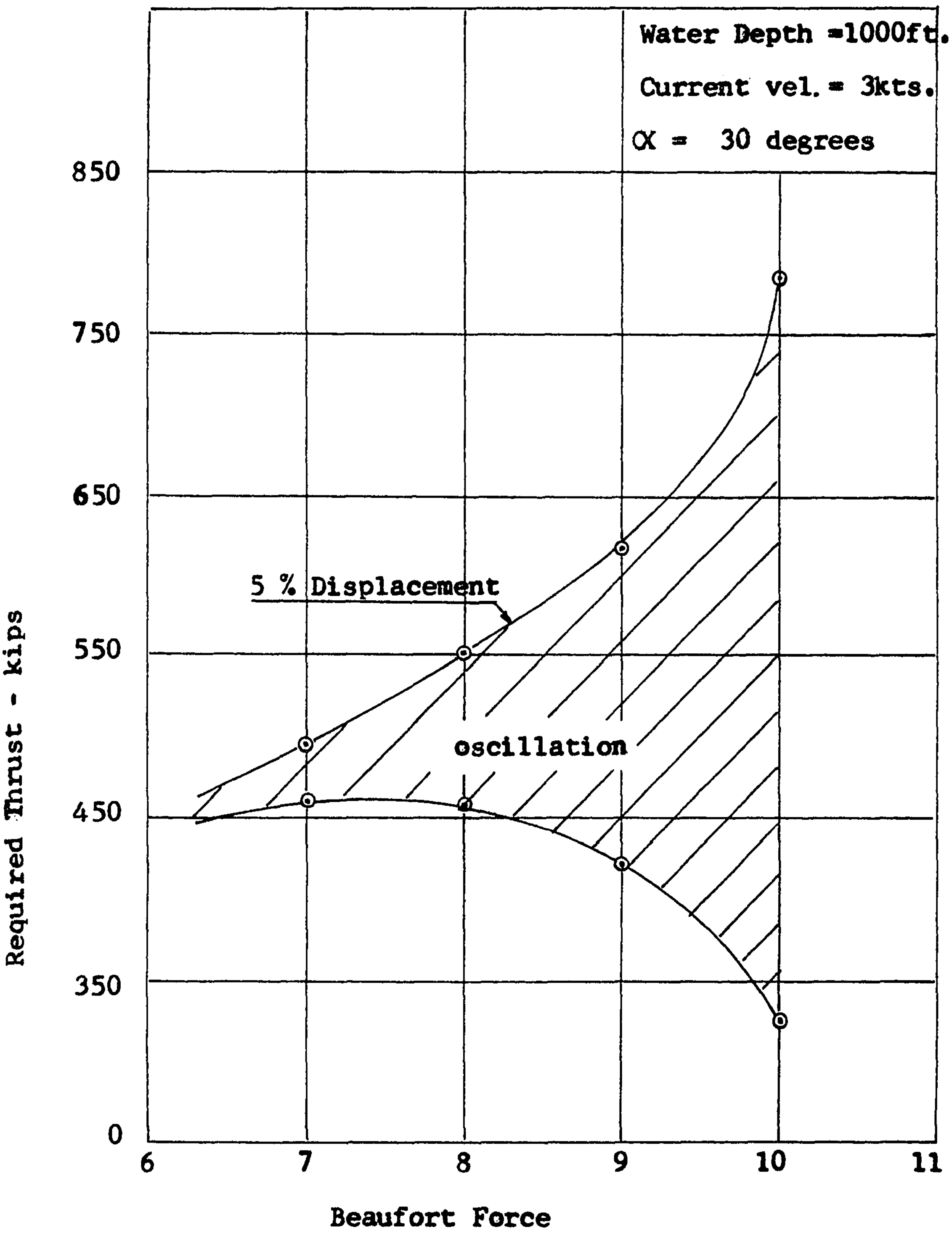


Figure 23: Thrust Requirements for 5 percent Maximum Displacement.

Table 4a: Thrust Requirements to Maintain Five percent Horizontal Displacement

x = 30 degrees
Current Vel. = 3 kts.
Water Depth = 1000 ft.

Beaufort Force	Initial Chain Tension - kips	Surge ft.	Sway ft.	Displacement ft.	Thrust kips	Thrust at 5% kips
7	330	-46.5	-34.0	57.6	481	489
7	362	-40.6	-30.1	50.5	488	
8	330	-49.4	-38.3	62.5	533	550
8	380	-40.2	-31.6	51.1	549	
9	330	-51.7	-44.6	68.3	582	632
9	410	-37.7	-33.4	50.4	631	
10	339	-61.9	-37.7	72.5	680	
10	410	-38.9	-42.1	57.3	744	762
10	450	-34.2	-37.0	50.34	761	

Table 4b: Minimum Results from Oscillation

7	362	-39.1	-28.7	48.5	462	
8	380	-35.6	-27.1	44.6	460	
9	410	-28.2	-22.6	36.1	420	
10	450	-17.6	-15.7	23.6	326	

In order to achieve the end result of being able to calculate the thrust requirements, two major problems were overcome whose solutions have other valuable applications.

The first of these problems was the polynominal series approximation to the catenary which was shown to be quite accurate. The explicit equations of the polynominals save iteration on the transendental functions of the catenary resulting in savings of considerable computation. This is especially true in working with a system of catenaries applicable to the spread mooring system. It was found initially that iteration in each of the eight mooring lines at each time step during the numerical integration of the equations of motion was completely impractical.

The second problem was formulating the equations of motion of the rigid body utilizing the Euler angles and second order wave theory; both tended to seriously complicate the differential equations. The resulting motion response program is shown to give good data compared to model tests. The motion response program has the desirable feature in that it may be used during the design phase to minimize vessel motion. Optimization of vessel motion by model testing alone is cost prohibitive.

The same program used to determine thrust requirements may also be utilized for mooring analyses of conventional semi-submersible vessels and determination of maximum water depth capability of the mooring system.

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NOMENCLATURE

a = Wave amplitude, ft

b = Vertical distance from center of gravity to mean water level, ft

b_a = Length of mooring line laying on seabed, ft

b_n = Coefficient, Chebyshev polynominal expansion

c = Catenary parameter, ft

C = Wave celerity, ft/sec.

C.G. = Coordinate position, center of gravity

C_D = Drag coefficient, dimensionless

C_M = Added mass coefficient, dimensionless

C_n = Coefficient, Chebyshev polynominal expansion

d_c = Column diameter, ft.

d_h = Hull diameter, ft.

D_{cx}, D_{cz} = Composite drag coefficients for a column, $(lb\text{-}sec^2)/ft^2$

$D_{hx}, D_{hy} D_{hz}$ = Composite drag coefficients for a hull, $(lb\text{-}sec^2)/ft^2$

F_a = Acceleration force of added mass, lb.

F_{cb} = Vertical column force due to change in buoyancy, lb.

F_{cx}, F_{cz} = Total hydrodynamic column force in horizontal directions, lb.

$F_{cxm}, F_{cym}, F_{czm}$ = Change in column momentum, lb.

F_{cxp}, F_{czp} = Wave pressure column force in horizontal directions, lb.

F_d = Drag force, lb.

F_{hx}, F_{hz} = Total hydrodynamic horizontal hull force, lb.

F_{hyo}, F_{hzo} = Hull end forces due to oblique flow, lb.

F_{hm}, F_{vm} = Components of mooring line force, lb.

F_{hxp}, F_{hzp} = Wave pressure hull force, lb.

F_{wx}, F_{wz} = Horizontal wind force, lb.

F_y = Total hydrodynamic vertical force, lb.

F_{yp} = Wave pressure vertical force on hull column, lb.

g = Acceleration of gravity, ft./sec^2

GM_L = Longitudinal metacentric height, ft.

GM_t = Transverse metacentric height, ft.

h = Water depth, ft.

H = Wave height, trough to crest, ft.

H_a = Horizontal force at anchor, lb.

I_{xx}, I_{yy}, I_{zz} = Principle mass moments of inertia lb-ft-sec^2

I_{xy}, I_{xz}, I_{yz} = Cross products of mass moments of inertia, lb-ft-sec^2

k = Wave number, ft^{-1}

k_{ξ} = Coefficient of accession to inertia, dimensionless

L_c = Column length, ft.

L_h = Hull length, ft.

M = Mass of vessel, $(lb\text{-}sec^2)/ft.$

$M(T_r)$ = Thrust moment, lb-ft.

$M(\theta), M(\phi), M(\psi)$ = Total moments in roll, pitch and yaw, lb-ft.

$M_w(\theta)$ = Transverse wind moment, lb-ft.

$M_w(\psi)$ = Longitudinal wind moment, lb-ft.

P = Wave pressure, psi

$P_{n-1}(x_D)$ = Chebyshev polynominal of order n-1

Q = Constant (Bernoulli equation), ft.

s = Catenary arclength, ft.

S_T = Total mooring line length, ft.

$S(t)$ = Exposure factor, column wetted length, dimensionless

t = Time, sec

T_r = Thrust requirement, lb.

u, v, w = Water particle velocities in x,y,z directions respectively, ft/sec.

$\bar{V}$ = Relative velocity of hull-column segment, ft/sec.

V_w = Wind velocity, ft/sec.

W = Total effective weight of catenary

w_c = Effective weight per unit length of mooring line, lb.

W_L = Column wetted-length, ft.

x, y, z = Coordinate axis of rotating system, ft.

X, Y, Z = Coordinate axis of stationary system, ft.

X_D = Horizontal distance from anchor point to the vessel, ft.

M_{cx} M_{cz} = Composite added column mass, sec^{-1}

M_{hx}, M_{hy}

M_{hz} = Composite added mass of hull, sec^{-1}

β = Angle of wave incidence, rad

γ = Direction of thrust, rad

δ = Tangent angle of mooring line, rad

δ_{ij} = Kronecker delta, dimensionless

λ = Wave length, ft.

λ_{ij} = Direction cosines, dimensionless

σ = Wave frequency, sec^{-1}

$\sigma_x, \sigma_y, \sigma_z$ = Angular displacements, rad

θ, ϕ, ψ = Euler angles, roll, yaw, pitch, rad^{-1}

Φ = Velocity potential, ft^2/sec

η = Elevation of wave profile, ft.

ϱ = Mass density, (lb-sec^2)/ft

π = 3.14159

$\beta(\sigma, y_h/r)$ = Damping parameter, dimensionless.

ϵ = Perturbation parameter, dimensionless.

APPENDIX A

APPENDIX A

Conservation of Mass

Consider the volume element and the coordinate system in Figure A.1.

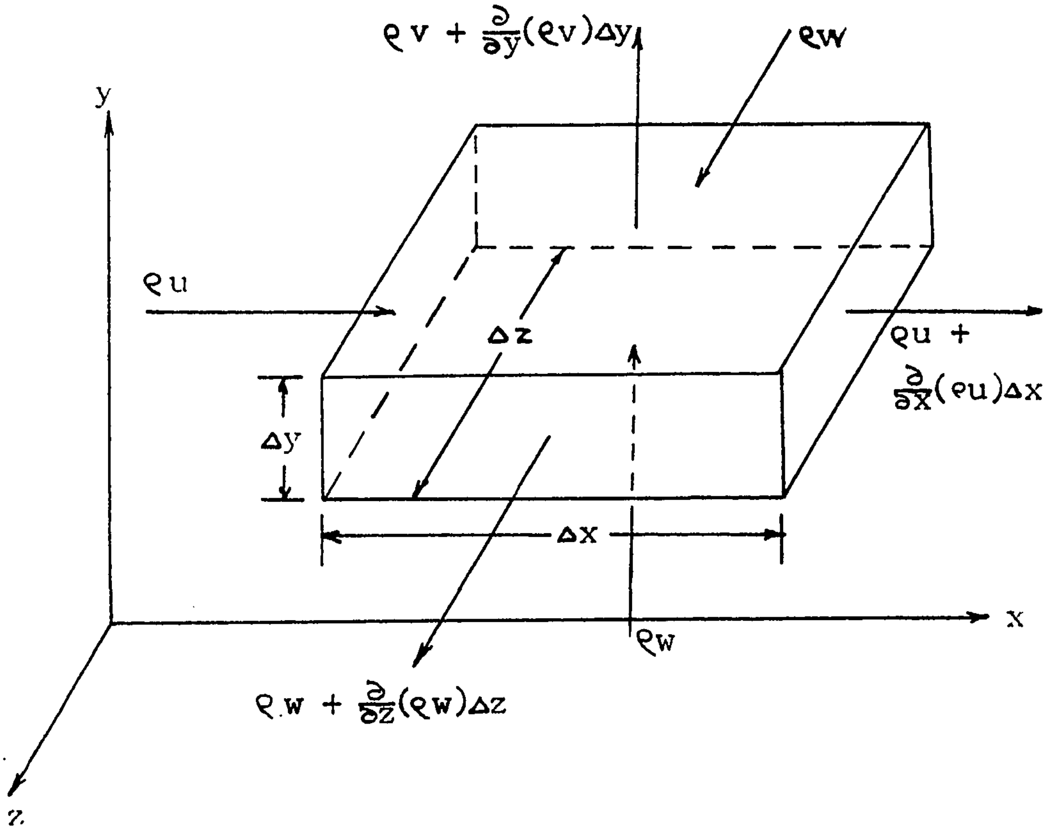


Figure A.1: Fluid flow through a volume element.

The net "flow of fluid mass across the element's boundaries in time Δt is

$$-[\frac{\delta}{\delta x}(\varrho u) + \frac{\delta}{\delta y}(\varrho v) + \frac{\delta}{\delta z}(\varrho w)]\Delta x \, \Delta y \, \Delta z \, ,$$

at time t, the mass present within the element is

$$\rho \Delta x \; \Delta y \; \Delta z \; ,$$

then at time t + Δt, the mass present will be

$$\rho \Delta x \; \Delta y \; \Delta z + \frac{\delta}{\delta t}(\rho \Delta x \Delta y \; \Delta z) \; \Delta t$$

where the second and higher order terms of the Taylor series are truncated. The net increase in mass in a time increment Δt is

$$\frac{\delta \rho}{\delta t} \Delta x \Delta y \Delta z \Delta t \; .$$

Since mass is neither created nor destroyed this must be equal to the inflow of mass across the boundaries

$$\frac{\delta \rho}{\delta t} \Delta x \Delta y \Delta z \Delta t = -\left[\frac{\delta(\rho u)}{\delta x} + \frac{\delta(\rho v)}{\delta y} + \frac{\delta(\rho w)}{\delta z}\right] \Delta x \; \Delta y \; \Delta z \; \Delta t$$

which reduces to

$$\frac{\delta \rho}{\delta t} = -\left[\frac{\delta}{\delta x}(\rho u) + \frac{\delta}{\delta y}(\rho v) + \frac{\delta}{\delta z}(\rho w)\right] .$$

EQ. (A.1)

Expanding the right-hand side and transposing yields

$$\frac{\delta \rho}{\delta t} + u\frac{\delta \rho}{\delta x} + v\frac{\delta \rho}{\delta y} + w\frac{\delta \rho}{\delta z} + \rho\left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} \; \frac{\delta w}{\delta z}\right) .$$

EQ. (A.2)

It is further assumed that seawater, the fluid of interest, is incompressible which reduces Equation (A.2) to

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0 \; .$$

EQ. (A.3)

This result is the well-known continuity equation for an incompressible fluid in steady or unsteady flow.

Equations of Motion for a Fluid Particle

The equations of motion for a fluid particle will be derived on the basis of Newton's "Second Law",

Forces = (mass)(acceleration).

This study assumes that seawater is a frictionless fluid which implies a zero viscosity and is incapable of sustaining a shear force. With this assumption the only forces acting on a fluid element are 1) forces due to fluid pressure acting normal to the element face and 2) the body forces which is the force of gravity acting in the negative vertical only.

Consider the fluid element in Figure (A.2).

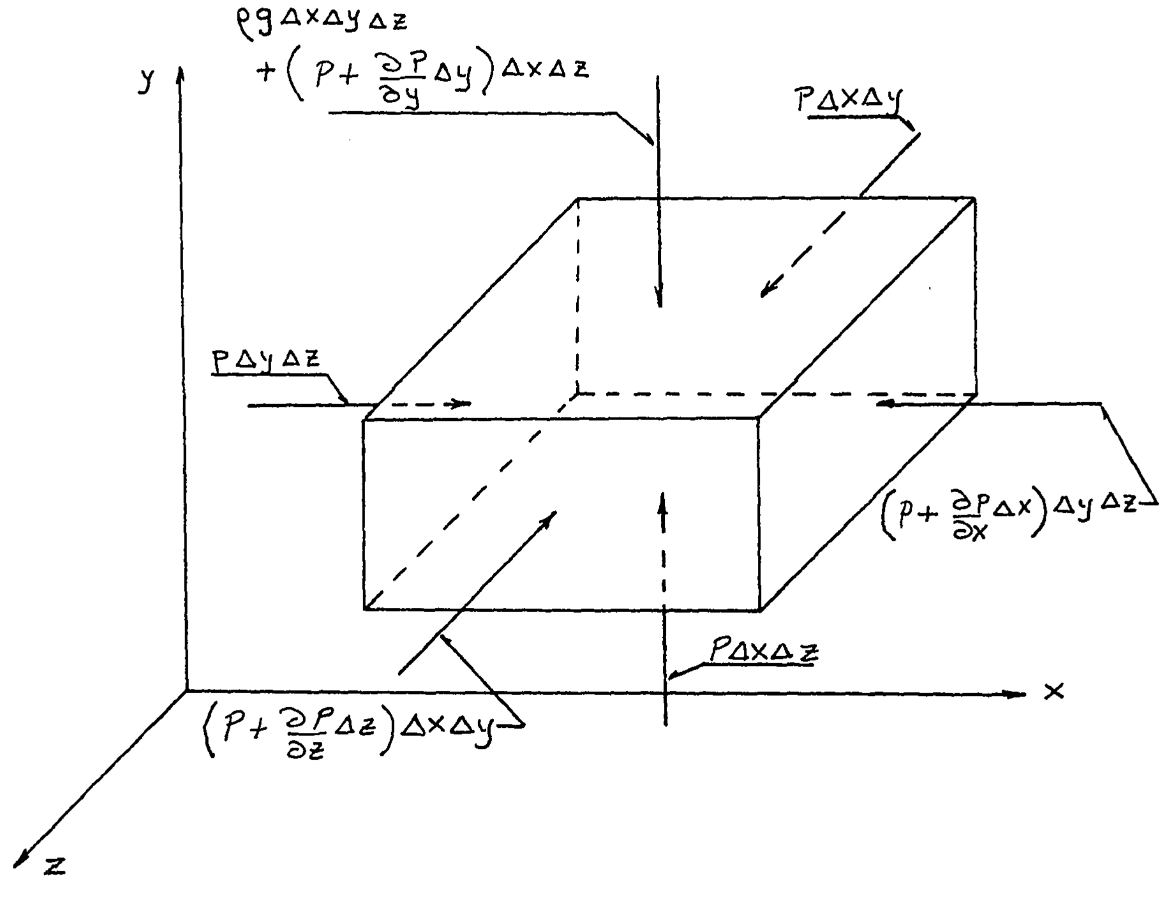


Figure A.2: Forces acting on a fluid element.

Summing forces in each direction and setting the sum equal to the mass-acceleration product gives

$$\varrho \Delta y \Delta z - (p + \frac{\delta p}{\delta x}\Delta x)\Delta y \Delta z = \varrho \Delta x \Delta y \Delta z(\frac{\delta u}{\delta t} + u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} + w\frac{\delta u}{\delta z}),$$

EQ. (A.4)

$$\varrho \Delta x \Delta y - (p + \frac{\delta p}{\delta z} z)\Delta x \Delta y = \varrho \Delta x \Delta y \Delta z(\frac{\delta w}{\delta t} + u\frac{\delta w}{\delta x} + v\frac{\delta w}{\delta y} + w\frac{\delta w}{\delta z}),$$

EQ. (A.5)

$$\varrho \Delta x \Delta z - (p + \frac{\delta p}{\delta y}\Delta y)\Delta x \Delta z - g\varrho \Delta x \Delta y \Delta z =$$
$$\varrho \Delta x \Delta y \Delta z(\frac{\delta v}{\delta t} + u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y} + w\frac{\delta v}{\delta z}) .$$ EQ. (A.6)

These equations are easily simplified to read

$$-\frac{1}{\varrho}\frac{\delta p}{\delta x} = \frac{\delta u}{\delta t} + u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} + w\frac{\delta u}{\delta z} ,$$ EQ. (A.7)

$$-\frac{1}{\varrho}\frac{\delta p}{\delta z} = \frac{\delta w}{\delta t} + u\frac{\delta w}{\delta x} + v\frac{\delta w}{\delta y} + w\frac{\delta w}{\delta z} ,$$ EQ. (A.8)

$$-g - \frac{1}{\varrho}\frac{\delta p}{\delta y} = \frac{\delta v}{\delta t} + u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y} + w\frac{\delta v}{\delta z} .$$ EQ. (A.9)

The last three equations comprise the equations of motion for a fluid particle in the three coordinate directions.

An Irrotational Fluid

Further development of the motion of fluid particles is to be simplified by assuming that the fluid is irrotational by the previous definition of a frictionless fluid; i.e., if the fluid cannot sustain a shear force which imparts a moment around the particles center of gravity the particle will not rotate. This is expressed as

$$\sigma_{xy} = \frac{1}{2}(\frac{\delta v}{\delta x} - \frac{\delta u}{\delta y}) ,$$ EQ. (A.10)

$$\sigma_{xy} = \frac{1}{2}(\frac{\delta w}{\delta x} - \frac{\delta u}{\delta z}) ,$$ EQ. (A.11)

$$\sigma_{yz} = \frac{1}{2}(\frac{\delta w}{\delta y} - \frac{\delta v}{\delta z}) .$$ EQ. (A.12)

where σ_{xy}, σ_{xz}, and σ_{yz} are the angular velocities in their respective planes. Clearly, if the fluid is taken to be without rotation

$$\frac{\delta v}{\delta x} = \frac{\delta u}{\delta y} \,, \qquad \text{EQ. (A.13)}$$

$$\frac{\delta w}{\delta x} = \frac{\delta u}{\delta z} \,, \qquad \text{EQ. (A.14)}$$

$$\frac{\delta w}{\delta y} = \frac{\delta v}{\delta z} \,. \qquad \text{EQ. (A.15)}$$

The assumption of an irrotational fluid is fundamental to the existence of a velocity potential where the field of flow can be represented by a scalar quanity Φ. The velocities of the fluid particle in the coordinate directions are defined as

$$u = -\frac{\delta\Phi}{\delta x} \,, \qquad \text{EQ. (A.16)}$$

$$v = -\frac{\delta\Phi}{\delta y} \,, \qquad \text{EQ. (A.17)}$$

$$w = -\frac{\delta\Phi}{\delta z} \,. \qquad \text{EQ. (A.18)}$$

Laplace's Equation

Thus far, we have an incompressible, irrotational fluid moving in three-dimensional space. It is desirable to utilize the existence of the velocity potentials by making the substitution of Equations (A.16, A.17 and A.18) into the continuity equation (A.3),

$$\frac{\delta}{\delta x}\left(-\frac{\delta\Phi}{\delta x}\right) + \frac{\delta}{\delta y}\left(-\frac{\delta\Phi}{\delta y}\right) + \frac{\delta}{\delta z}\left(-\frac{\delta\Phi}{\delta z}\right) = 0 \qquad \text{EQ. (A.19)}$$

which yields the three-dimensional Laplace equation for fluid flow

$$\nabla^2\Phi = 0 \,, \qquad \text{EQ. (A.20)}$$

where the del operator is given as

$$\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2} \qquad \text{EQ. (A.21)}$$

Bernoulli Equation

By taking a time derivative of the velocity potentials

$$-\frac{\delta u}{\delta t} = \frac{\delta^2 \Phi}{\delta x \delta t}, \qquad \text{EQ. (A.22)}$$

$$-\frac{\delta v}{\delta t} = \frac{\delta^2 \Phi}{\delta y \delta t}, \qquad \text{EQ. (A.23)}$$

$$-\frac{\delta w}{\delta t} = \frac{\delta^2 \Phi}{\delta z \delta t}, \qquad \text{EQ. (A.24)}$$

the equations of motion (A.7, A.8 and A.9) may be written as

$$-\frac{1}{\rho}\frac{\delta p}{\delta x} = \frac{\delta^2 \Phi}{\delta x \delta t} + u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} + w\frac{\delta u}{\delta z}, \qquad \text{EQ. (A.25)}$$

$$-\frac{1}{\rho}\frac{\delta p}{\delta z} = -\frac{\delta^2 \Phi}{\delta z \delta t} + u\frac{\delta w}{\delta x} + v\frac{\delta w}{\delta y} + w\frac{\delta w}{\delta z}, \qquad \text{EQ. (A.26)}$$

$$-g - \frac{1}{\rho}\frac{\delta p}{\delta y} = -\frac{\delta^2 \Phi}{\delta y \delta t} + u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y} + w\frac{\delta v}{\delta z}. \qquad \text{EQ. (A.27)}$$

Observing that

$$u\frac{\delta u}{\delta x} = \frac{1}{2}\frac{\delta}{\delta x}(u^2), \qquad \text{EQ. (A.28)}$$

$$w\frac{\delta w}{\delta z} = \frac{1}{2}\frac{\delta}{\delta z}(w^2), \qquad \text{EQ. (A.29)}$$

$$v\frac{\delta v}{\delta y} = \frac{1}{2}\frac{\delta}{\delta y}(v^2), \qquad \text{EQ. (A.30)}$$

and since ρ is constant for an incompressible fluid one may write

$$\frac{1}{\rho}\frac{\delta p}{\delta x} = \frac{\delta}{\delta x}\left(\frac{p}{\rho}\right), \qquad \text{EQ. (A.31)}$$

$$\frac{1}{\rho}\frac{\delta p}{\delta z} = \frac{\delta}{\delta z}\left(\frac{p}{\rho}\right), \qquad \text{EQ. (A.32)}$$

$$\frac{1}{\rho}\frac{\delta p}{\delta y} = \frac{\delta}{\delta y}\left(\frac{p}{\rho}\right), \qquad \text{EQ. (A.32a)}$$

and Equations (A.25, A.26 and A.27) may be rewritten as

$$\frac{\partial}{\partial x}\left[-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}u^2\right] + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0, \qquad \text{EQ. (A.33)}$$

$$\frac{\partial}{\partial z}\left[-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}v^2\right] + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = 0, \qquad \text{EQ. (A.34)}$$

$$\frac{\partial}{\partial y}\left[-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}w^2\right] + u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} + g = 0, \qquad \text{EQ. (A.35)}$$

Now introducing the concept that the fluid is irrotational and by Equations (A.13, A.14 and A.15) the last set of equations reduce to

$$\frac{\partial}{\partial x}\left[-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}(u^2 + v^2 + w^2)\right] = 0, \qquad \text{EQ. (A.36)}$$

$$\frac{\partial}{\partial z}\left[-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}(u^2 + v^2 + w^2)\right] = 0, \qquad \text{EQ. (A.37)}$$

$$\frac{\partial}{\partial y}\left[-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}(u^2 + v^2 + w^2) + gy\right] = 0. \qquad \text{EQ. (A.38)}$$

This set is integrated directly to give

$$-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}(u^2 + v^2 + w^2) = f_1(y,z,t)\ , \qquad \text{EQ. (A.39)}$$

$$-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}(u^2 + v^2 + w^2) = f_2(x,y,t)\ , \qquad \text{EQ. (A.40)}$$

$$-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}(u^2 + v^2 + w^2) + gy = f_3(x,z,t)\ . \qquad \text{EQ. (A.41)}$$

Observing Equations (A.39 and A.40), one should note that

$$f_1(y,z,t) = f_2(x.y.t)\ . \qquad \text{EQ. (A.42)}$$

Subtracting Equation (A.41 from A.40) gives

$$gy = f_3(x,z,t) - f_2(x,y,t)\ . \qquad \text{EQ. (A.43)}$$

Observe further that neither g nor y may be a function of either x or z. This indicates that

$$f_3(x,z,t) = f_3(t) \qquad \text{EQ. (A.44)}$$

a function of time alone which gives

$$f_3(y,t) = f_3(t) - gy \; . \qquad \text{EQ. (A.45)}$$

Substitution of Equations (A.44 and A. 45) into Equations (A.40 and A.41) respectively, one will see that the resulting equations are equivalent and read as

$$- \frac{\partial\Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}(u^2 + v^2 + w^2) = f_3(t) \; . \qquad \text{EQ. (A.46)}$$

For steady fluid flow that does not change with respect to time at a fixed point

$$f_3(t) = \text{constant}, \qquad \text{EQ. (A.47)}$$

an arbitrary constant that may be incorporated into Φ without change in generality. This final result is

$$- \frac{\partial\Phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}(u^2 + v^2 + w^2) + gy = 0 \; . \qquad \text{EQ. (A.48)}$$

With this change the horizontal component of water velocity is now u-C. This system of stationary waves is shown in Figure A.3.

The differential equation applicable to this system has been shown to be the familiar Laplace equation. In two-dimensions this may be defined as

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0. \qquad \text{EQ. (A.49)}$$

The boundary condition at the seabed, $y = - h$ is that fluid flow does not occur across this surface,

$$v = -\frac{\partial\Phi}{\partial y} = 0 \text{ at } y = - h \; . \qquad \text{EQ. (A.50)}$$

At the free surface, $y = \eta$, there are two boundary conditions to be satisfied. The first is a dynamic requirement that the total energy along the free surface be a constant, say Q.

Formulating the Boundary Value Problem for Finite Amplitude Wave Theory of Second Order.

The development thus far has been three-dimensional. This is to be reduced to two-dimensions by assuming that the wave crests and troughs are parallel and infinite in length. This assumption is indeed valid in view of the fact that our interest lies only for a crest length just greater than the maximum dimension of the vessel. It is also convenient to assume that the wave form travels without change in shape and propagates with a constant celerity C. Further, in order that the motion be steady with respect to a reference system moving with celerity C, a uniform current -C is superimposed effecting steady motion with respect to a stationary reference system.

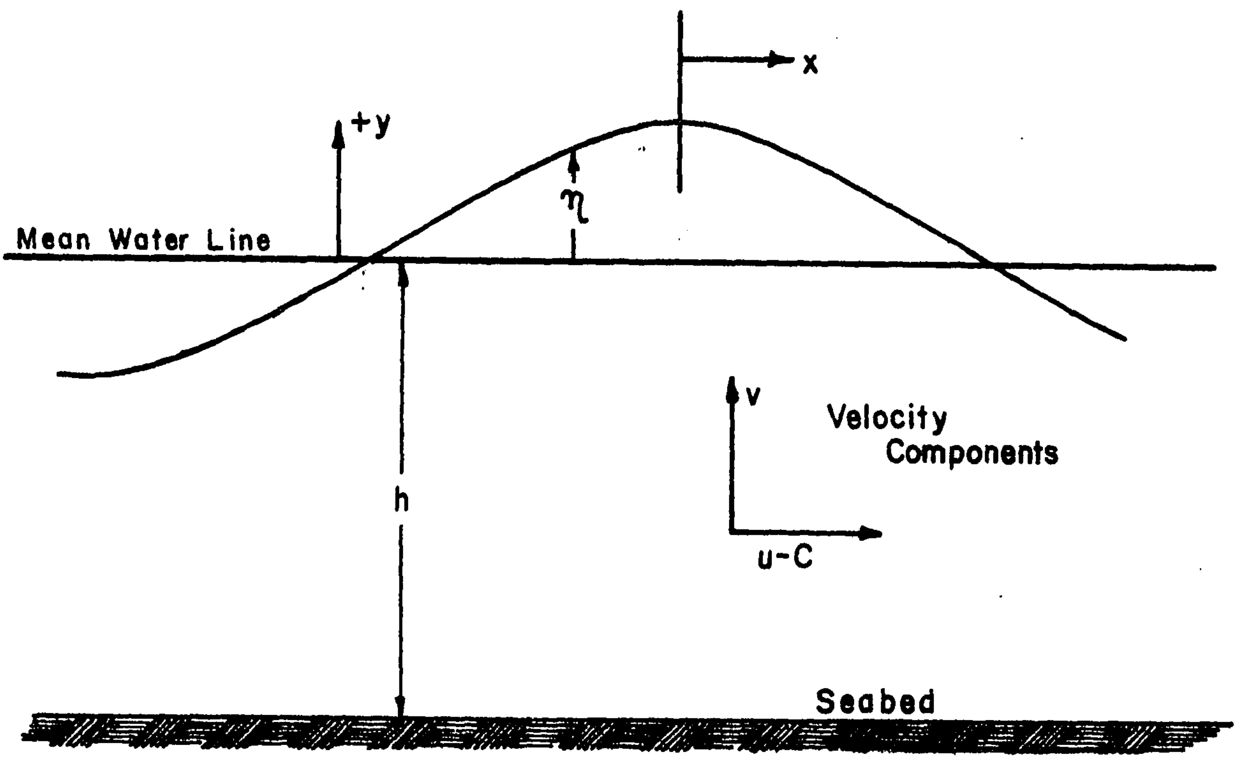


Figure A.3: Stationary Wave System.

Making these substitutions the steady-state form of Bernoulli's equation (A.48) becomes

$$\eta + \frac{1}{2g}[(u-C)^2 + v^2] = Q. \qquad \text{EQ. (A.51)}$$

The second condition at the free surface to be satisfied is that no fluid be transported across the free surface, i.e., the resultant velocity vector at the free surface must be everywhere tangent to that surface. This condition may be expressed as

$$\frac{\partial \eta}{\partial x} = \frac{v}{u-C} . \qquad \text{EQ. (A.52)}$$

A solution to this boundary value problem is to be derived by a perturbation procedure primarily due to the non-linear terms $(u-C)^2$ and v^2 in Equation (A.51). The solutions for Φ, η, and C are assumed to have the form

$$\Phi = \sum_{n=1} \epsilon^n \Phi_n , \qquad \text{EQ. (A.53)}$$

$$\eta = \sum_{n=1}^{\infty} \epsilon^n \eta_n , \qquad \text{EQ. (A.54)}$$

and

$$C = \epsilon^0 C_0 + \sum_{n=1}^{\infty} \epsilon^n C_n \qquad \text{EQ. (A.55)}$$

where ϵ is a small parameter to be determined. Because the wave free surface is an unknown, it will prove advantageous to define an expression for a general function f(x,y) evaluated on $y = \eta$ as

$$f(x;\eta) = \sum_{m=0}^{\infty} \frac{\eta^m}{m!} \left(\frac{\partial^m f}{\partial y^m}\right)_{y=0} \qquad \text{EQ. (A.56)}$$

which is nothing more than a Taylor series. Further, a general function $f(x,y)$ may be defined as

$$f(x,y) = \sum_{n=1}^{\infty} \epsilon^n f_n(x,y) \; . \qquad \text{EQ. (A.57)}$$

Substituting Equation (A.57) into (A.56) and collecting like powers of ϵ leads to an expression for $f(x,\eta)$ correct to the second order which is

$$f(x,\eta) = \epsilon f_1 + \epsilon^2 (f_2 + \eta_1 \frac{\partial f_1}{\partial y}) \text{ at } y = 0. \qquad \text{EQ. (A.58)}$$

Now substituting the assumed solution for Φ, Equation (A.53) into the Laplace equation (A.49) gives

$$\frac{\partial^2}{\partial x^2}[\sum_{n=1}^{\infty} \epsilon^n \Phi_n] + \frac{\partial^2}{\partial y^2}[\sum_{n=1}^{\infty} \epsilon^n \Phi_n] = 0 \; , \qquad \text{EQ. (A.59)}$$

this is expanded as

$$\epsilon^1 \frac{\partial^2 \Phi}{\partial x^2} + \epsilon^2 \frac{\partial^2 \Phi}{\partial x^2} + \ldots + \epsilon^1 \frac{\partial^2 \Phi}{\partial y^2} + \epsilon^2 \frac{\partial^2 \Phi}{\partial y^2} + \ldots = 0. \qquad \text{EQ. (A.60)}$$

Collecting like powers of ϵ leads to

$$\epsilon [\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}] + \epsilon^2 [\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}] + \ldots + \epsilon^n [\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}] = 0 \cdot$$

$$\text{EQ. (A.61)}$$

Since ϵ is unequal to zero, it is permissible to set the coefficients of ϵ and powers of ϵ^2 to zero. The set of differential equations to be satisfied now is

$$\nabla^2 \Phi_n = 0, \quad n = 1,2,\ldots \; . \qquad \text{EQ. (A.62)}$$

The new boundary conditions at the seabed are

$$\frac{\partial \Phi_n}{\partial y} = 0 \text{ at } y = -h. \qquad \text{EQ. (A.63)}$$

The new dynamic boundary condition at the free surface ($y=\eta$) is found by expanding the term $(u-C)^2$ in Equation (A.51)

$$\eta + \frac{1}{2g}\left[u^2 - 2uC + C^2 + v^2\right] = Q\,, \qquad \text{EQ. (A.64)}$$

and substituting the equivalent terms for u and v giving

$$\eta - Q + \frac{1}{2g}\left[\left(\frac{\partial\Phi}{\partial x}\right)^2 - 2C\frac{\partial\Phi}{\partial x} + C^2 + \left(\frac{\partial\Phi}{\partial y}\right)^2\right] = 0, \qquad \text{EQ. (A.65)}$$

where the unknown constant Q must be expressed in the form

$$Q = \epsilon^0 Q_0 + \sum_{n=1}^{\infty}\epsilon^n Q_n \;. \qquad \text{EQ. (A.66)}$$

The second free surface condition, Equation (A.52)

$$\frac{\partial\eta}{\partial x}(u - C) = v \text{ at } y = \eta\;, \qquad \text{EQ. (A.67)}$$

may now be written as

$$\left(\frac{\partial\eta}{\partial x}\right)\left(\frac{\partial\Phi}{\partial x}\right) + C\frac{\partial\eta}{\partial x} = \frac{\partial\Phi}{\partial y} \text{ at } y = \eta. \qquad \text{EQ. (A.68)}$$

Substituting the series definitions of the unknowns Φ, η, C, and Q into Equation (A.65),

$$\sum_{n=1}^{\infty}\epsilon^n\eta_n - \left(Q_0 + \sum_{n=1}^{\infty}\epsilon^n Q_n\right)$$
$$+\frac{1}{2g}\left\{\left[\frac{\partial}{\partial x}\left(\sum_{n=1}^{\infty}\epsilon^n\Phi_n\right)\right]^2 - 2\left[\frac{\partial}{\partial x}\left(\sum_{n=1}^{\infty}\epsilon^n\Phi_n\right)\right]\left[C_0 + \sum_{n=1}^{\infty}\epsilon^n C_n\right]\right.$$
$$\left. +\left(\epsilon^0 C_0 + \sum_{n=1}\epsilon^n C_n\right)^2 + \left[\frac{\partial}{\partial y}\left(\sum_{n=1}^{\infty}\epsilon^n\Phi_n\right)\right]^2\right\} = 0, \text{ at } y = \eta\;, \qquad \text{EQ. (A.69)}$$

expanding and gathering the coefficients of like powers of gives this boundary condition in terms of the perturbation parameter

$$\epsilon^0\left[\frac{C_0^2}{2g} - Q_0\right] + \epsilon\left[\eta_1 - Q_1 + \frac{C_0}{g}\frac{\partial\Phi_1}{\partial x} + \frac{C_0 C_1}{g}\right]$$
$$+\epsilon^2\left\{\eta_2 - Q_2 + \frac{C_0}{g}\frac{\partial\Phi_2}{\partial x} + \frac{1}{2g}\left[\left(\frac{\partial\Phi_1}{\partial x}\right)^2 + \left(\frac{\partial\Phi_1}{\partial y}\right)^2\right.\right.$$
$$\left.\left. + 2C_1\frac{\partial\Phi_1}{\partial x} + 2C_0\eta_1\frac{\partial^2\Phi_1}{\partial x\partial z} + C_1^2 + 2C_0C_2\right]\right.$$
$$+ O(\epsilon^3) = 0 \text{ at } y = \eta\;. \qquad \text{EQ. (A.70)}$$

Similarly, the second free surface boundary condition given by Equation (A.68) becomes

$$\epsilon\,(C_o\frac{\partial\eta_1}{\partial x} - \frac{\partial\Phi_1}{\partial y}) + \epsilon^2(C_o\frac{\partial\eta_2}{\partial x} + \frac{\partial\eta_1}{\partial x}\frac{\partial\Phi_1}{\partial x}$$
$$+ C_1\frac{\partial\eta_1}{\partial x} - \frac{\partial\Phi_2}{\partial x} - \eta_1\frac{\partial^2\Phi_1}{\partial y^2}) = 0 \text{ at } y = \eta. \qquad \text{EQ. (A.71)}$$

From the zeroth order term in Equation (A.70), one observes that

$$Q_o = \frac{Co^2}{2g}\,. \qquad \text{EQ. (A.72)}$$

Next the coefficients of the first and second powers of in Equations (A.62), (A.63), (A.70) and (A.71) will be grouped and solved for the first and second order approximations respectively.

<u>First Order Wave Theory</u>

The coefficients of ϵ are

$$\nabla^2\Phi_1 = 0\,, \qquad \text{EQ. (A.73)}$$

$$\frac{\partial\Phi_1}{\partial y} = 0 \text{ @ } y = -h\,, \qquad \text{EQ. (A.74)}$$

$$\eta_1 - Q_1 + \frac{Co}{g}\frac{\partial\Phi_1}{\partial x} + \frac{CoC1}{g} = 0 \text{ @ } y = 0\,, \qquad \text{EQ. (A.75)}$$

and

$$C_o\frac{\partial\eta_1}{\partial x} - \frac{\partial\Phi_1}{\partial y} = 0 \text{ @ } y = 0\,. \qquad \text{EQ. (A.76)}$$

Separation of variables [4,41] may be used to solve Equation (A.73). Assume a solution in the form

$$\Phi_1(x,y) = X(x)\;Y(y)\,, \qquad \text{EQ. (A.77)}$$

and taking derivatives gives

$$X''Y + XY'' = 0\,, \qquad \text{EQ. (A.78)}$$

separation of variables gives two differential equations

$$X'' + k^2 X = 0 , \qquad \text{EQ. (A.79)}$$

and

$$Y'' - k^2 Y = 0 . \qquad \text{EQ. (A.80)}$$

The equations are readily solved and substituted into Equation (A.77) as

$$\Phi_1 = -A_1 \cosh[k(h + y)]\sin kx , \qquad \text{EQ. (A.81)}$$

where k is defined as the wave number and is equal to

$$k = \frac{2\pi}{\lambda} . \qquad \text{EQ. (A.82)}$$

One should note that since the wave form is constant regardless of the celerity, it can be seen from Equation (A.75) that

$$Q_1 = \frac{CoC}{g} . \qquad \text{EQ. (A.83)}$$

Omitting these terms from Equation (A.75) and differentiating with respect to x gives

$$\frac{C_o^2}{g} \frac{\partial^2 \Phi_1}{\partial x^2} + \frac{\partial \eta_1}{\partial x} = 0 , \; y = 0 . \qquad \text{EQ. (A.84)}$$

Now using this result in combination with Equation (A.76), the term ($\partial \eta_1 / \partial x$) may be eliminated to give

$$\frac{C_o^2}{g} \frac{\partial^2 \Phi_1}{\partial x^2} + \frac{\partial \Phi_1}{\partial y} + 0 . \qquad \text{EQ. (A.85)}$$

Substituting the solution for Φ_1, into the above result leads to

$$\frac{C_o^2}{g} A_1 k^2 \cosh(kh)\sin kx - A_1 k \sinh(kh)\sin kx = 0 , \qquad \text{EQ. (A.86)}$$

or

$$C_o^2 = \frac{g}{k} \tanh(kh) . \qquad \text{EQ. (A.87)}$$

This is the first order approximation for the wave celerity

C_o.

The expression for η_1 may be found by substituting the solution for Φ_1, into Equation (A.75) which follows

$$\eta_1 = \frac{C_o}{g} A_1 k\cosh(kh)\cos kx \quad , \qquad \text{EQ. (A.88)}$$

at which the wave crest is located a x = 0. This solution alone is usually referred to as the small amplitude or Airy wave theory.

Second Order Approximation

Again referring to Equations (A.62), (A.63), (A.70) and (A.71), the coefficients of ϵ^2 set equal to zero are

$$\nabla^2\Phi_2 = 0 \ , \qquad \text{EQ. (A.89)}$$

$$\frac{\partial\Phi_2}{\partial y} = 0 \quad \text{at } y = -h \ , \qquad \text{EQ. (A.90)}$$

$$\eta_2 - Q_2 + \frac{C_o}{g}\frac{\partial\Phi_2}{\partial x} + \frac{1}{2g}\Big[\Big(\frac{\partial\Phi_1}{\partial x}\Big)^2 + \Big(\frac{\partial\Phi_1}{\partial y}\Big)^2 + 2C_1\frac{\partial\Phi_1}{\partial x}$$

$$+ 2C_o\frac{\partial^2\Phi_1}{\partial x\partial y}\eta_1 + C_1^2 + 2C_oC_2\Big] = 0 \text{ at } y = 0 \ , \qquad \text{EQ. (A.91)}$$

and

$$C_o\frac{\partial^2\eta_2}{\partial x} + \frac{\partial\eta_1}{\partial x}\frac{\partial\Phi_1}{\partial x} + C_1\frac{\partial\eta_1}{\partial x} - \frac{\partial\Phi_2}{\partial y} - \frac{\partial^2\Phi_1}{\partial z^2}\eta_1 = 0 \text{ at } y = 0 \ . \qquad \text{EQ. (A.92)}$$

Again, by separation of variables, a second order solution periodic in x and satisfying Equations (A.89) and (A.90) is easily found to be

$$\Phi_2 = -A_2 \cosh[2k(h+y)]\sin 2kx \ . \qquad \text{EQ. (A.93)}$$

The second order celerity C is obtained by taking the partial derivative of Equation (A.91) with respect to x

$$\frac{\partial \eta_2}{\partial x} + \frac{C_o}{g}\frac{\partial^2 \Phi}{\partial x^2} + \frac{1}{2g}\left[2\frac{\partial \Phi_1}{\partial x}\frac{\partial^2 \Phi}{\partial x^2} + 2\frac{\partial \Phi_1}{\partial y}\frac{\partial^2 \Phi_1}{\partial x \partial y}\right.$$

$$\left. + 2C_1\frac{\partial^2 \Phi_1}{\partial x^2} + 2C_o\eta_1\frac{\partial^3 \Phi_1}{\partial x \partial x \partial y} + 2C_o\frac{\partial^2 \Phi_1}{\partial x \partial y}\frac{\partial \eta_1}{\partial x}\right] = 0 \text{ at } y = 0 \text{ .}$$

Solving Equation (A.92) for $(\partial \eta_2/\partial x)$ and substituting into Equation (A.94) yields

$$-\frac{1}{C_o}\frac{\partial \Phi_2}{\partial x} - \frac{C_o}{g}\frac{\partial^2 \Phi_2}{\partial x^2}$$

$$= \frac{1}{g}\frac{\partial \Phi_1}{\partial x}\frac{\partial^2 \Phi_1}{\partial x^2} + \frac{1}{g}\frac{\partial \Phi_1}{\partial y}\frac{\partial^2 \Phi_1}{\partial x \partial y} + \frac{C}{g}\frac{\partial^2 \Phi_1}{\partial x^2}$$

$$+ \frac{C_o}{g}\frac{\partial^2 \Phi_1}{\partial x \partial y}\frac{\partial \eta_1}{\partial x} + \frac{C_o}{g}\frac{\partial^3 \Phi_1}{\partial x \partial x \partial y} - \frac{1}{C_o}\frac{\partial \Phi_1}{\partial x}\frac{\partial \eta_1}{\partial x}$$

$$- \frac{C}{C_o}\frac{\partial \eta_1}{\partial x} + \frac{\eta_1}{C_o}\frac{\partial^2 \Phi_1}{\partial y^2} \text{ at } y = 0 \text{ .} \qquad \text{EQ. (A.95)}$$

Substituting the appropriate forms of η_1, Φ_1 and Φ_2 into Equation (A.95) and simplifying the result by using Equation (A.87) leads to

$$A_2 \sinh^2 kh \sin 2kx$$

$$= \frac{3}{8}\frac{A_1{}^2 k}{C_o}\sin 2kx - \frac{1}{2}\frac{C_1}{C_o}\cosh kh \sin kx \text{ .} \qquad \text{EQ. (A.96)}$$

Equating coefficients of sinkx and sin2kx gives

$$A_2 = \frac{3A_1^2 k}{8C_o \sinh^2 kh} \text{ ,} \qquad \text{EQ. (A.97)}$$

and

$$C_1 = 0 \text{ .} \qquad \text{EQ. (A.98)}$$

Substituting A_2 into Equation (A.93) gives the second order term for the velocity potential as

$$\Phi_2 = -\frac{3A_1^2 k}{8C_o \sinh^2 kh}\cosh[2k(h+y)]\sin 2kx \text{ .} \qquad \text{EQ. (A.99)}$$

The second order term for the wave form η_2 is determined from its free surface boundary condition, Equation (A.91), by making the necessary substitutions at $z = 0$. This result after much simplification is

$$\eta_2 = Q_2 - \frac{A_1^2k^2}{4g} - \frac{C_0C_2}{g}$$

$$+ \frac{A_1^2k^2}{4g}\left[\frac{2\cosh^2kh + \cosh 2kh\cosh^2kh}{\sinh^2kh}\right]\cos 2kx \; . \qquad \text{EQ. (A.100)}$$

Similar to the first order theory one sees that

$$Q_2 = \frac{A_1^2k^2}{4g} + \frac{C_0C_2}{g} \; . \qquad \text{EQ. (A.101)}$$

The only remaining unknown in the solution now is the perturbation parameter ϵ. This may be specified by setting the maximum value of the first order wave amplitude equal to a, giving

$$a \cos kx = \epsilon\frac{kA_1C_0}{g}\cosh kh\cos kx \quad , \qquad \text{EQ. (A.102)}$$

which gives

$$\epsilon = \frac{ag}{kA_1C_0\cosh kh} \; . \qquad \text{EQ. (A.103)}$$

Now substituting the appropiate terms into Equations (A.53), (A.54) and (A.55) gives the desired solutions for velocity potential,

$$\Phi = -\frac{H}{2}C\frac{\cosh k(h+y)}{\sinh kh}\sin(kx - \sigma t)$$

$$-\frac{3}{16}\frac{H^2\pi}{T}\frac{\cosh 2k(h+y)}{\sinh^4kh}\sin 2(kx - \sigma t) \; , \qquad \text{EQ. (A.104)}$$

the wave form,

$$\eta = \frac{H}{2}\cos(kx - \sigma t)$$

$$+ \frac{\pi}{8}\frac{H^2}{\lambda}\frac{\cosh kh}{\sinh^3kh}(\cos 2kh + 2)\cos 2(kx - \sigma t) \; , \qquad \text{EQ. (A.105)}$$

and the celerity

$$C_o = \left(\frac{g}{k} \tan kh\right)^{1/2} . \qquad \text{EQ. (A.106)}$$

Note that the celerity in Equation (A.106) is identical to that of first order, Equation (A.87) since C_1 proved to be zero.

By previous definition the water particle velocities in the x and y directions are the partial derivatives of the velocity potential. These terms may now be expressed explicity as

$$u = \frac{\pi H}{\lambda} C \frac{\cosh k(h+y)}{\sinh kh} \cos(kx - \sigma t)$$

$$+ \frac{3}{4} \left(\frac{\pi H}{\lambda}\right)^2 C \frac{\cosh 2k(h+y)}{\sinh^4 kh} \cos 2(kx - \sigma t) , \qquad \text{EQ. (A.107)}$$

and

$$v = \frac{\pi H}{\lambda} C \frac{\sinh k(h+y)}{\sinh kh} \sin(kx - \sigma t)$$

$$+ \frac{3}{4} \left(\frac{\pi H}{\lambda}\right)^2 C \frac{\sinh 2k(h+y)}{\sinh^4 kh} \sin 2(kx - \sigma t) . \qquad \text{EQ. (A.108)}$$

This completes the development of wave theory of second order used in the equations of motion developed in another section.

APPENDIX B

RIGID BODY EQUATIONS OF MOTION

Three Dimensional Wave Theory

The final results of the second-order wave theory developed in Appendix A is to be expanded to give the second

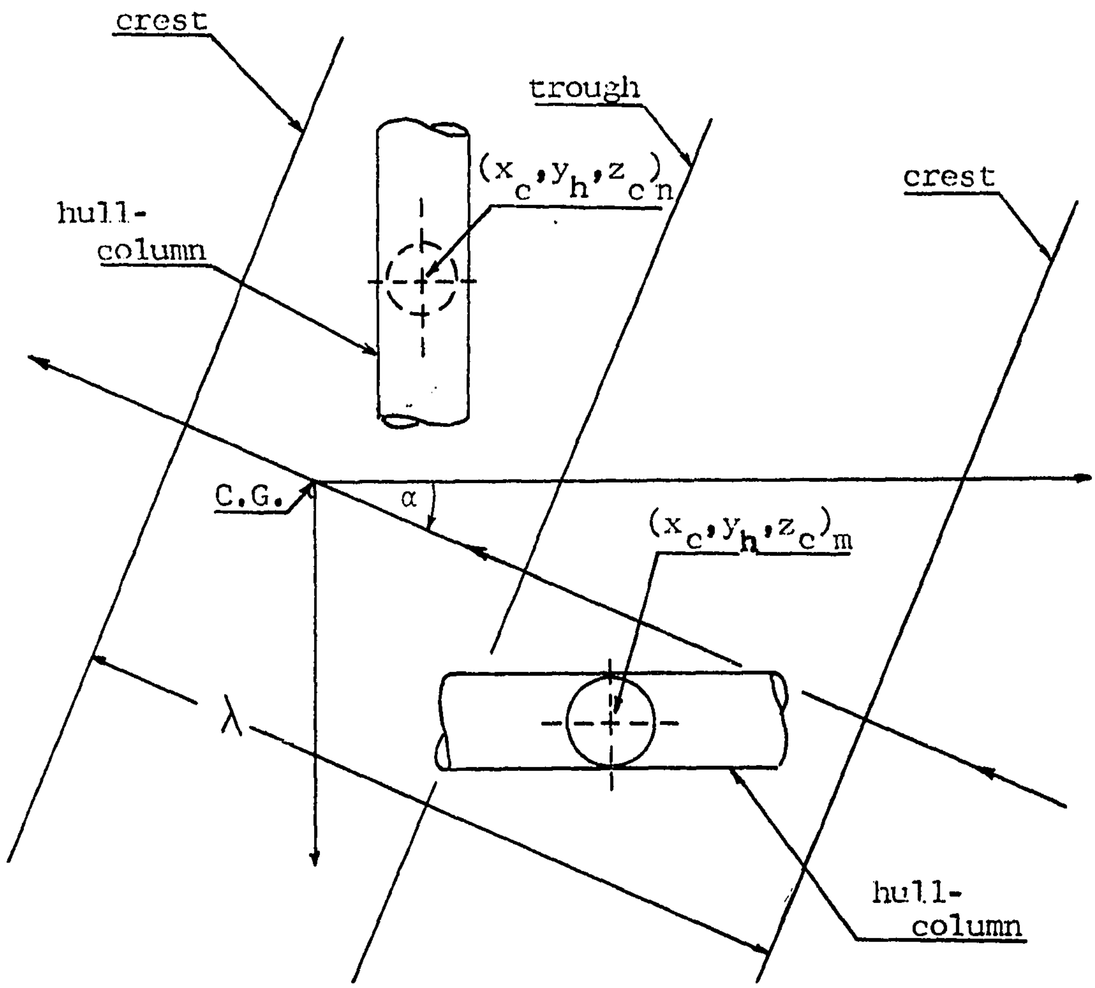


Figure B-1: Typical hull-column sections in a wave train.

dimension z in the horizontal plane. Consider the figure **above** which depicts a wave train whose direction of propagation is at an arbitrary angle α relative to the coordinates of the vessel. The hull-column sections are typical. Note that the wave train is travelling in a general negative direction relative to x and z. By replacing x in the two-dimensional theory of Equations (A.105), (A.107), and (A.108) with

$x\cos\alpha + z\sin\alpha$

and changing the sign preceding σt and the sign of the celerity c, the new equation for the wave form is

$$\eta_x = \frac{H}{2}\cos\alpha\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t] +$$

$$\frac{\pi}{8}\frac{H^2}{\lambda}\cos\alpha\frac{\cosh kh}{\sinh^3 kh}(\cosh 2kh + 2)\cos[2k(x\cos\alpha + z\sin\alpha) + 2\sigma t];$$

EQ. (B.1)

$$\eta_y = \frac{H}{2}\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t] +$$

$$\frac{\pi}{8}\frac{H^2}{\lambda}\frac{\cosh kh}{\sinh^3 kh}(\cosh 2kh + 2)\cos[2k(x\cos\alpha + z\sin\alpha) + \sigma t],$$

EQ. (B.2)

$$\eta_z = \frac{H}{2}\sin\alpha\cos[k(x_c\cos\alpha + z_c\sin\alpha) + \sigma t]$$

$$\frac{\pi}{8}\frac{H^2}{\lambda}\sin\alpha\frac{\cosh kh}{\sinh^3 kh}(\cosh 2kh+2)\cos[2k(x_c\cos\alpha + z_c\sin\alpha) + 2\sigma t].$$

EQ. (B.3)

Similarly, the particle velocities in the x and y directions may be rewritten as

$$u = -\left(\frac{\pi H}{\lambda}\right) C\cos\alpha \frac{\cosh k(h+y-b)}{\sinh kh} \cos[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$- \frac{3}{4}\left(\frac{\pi H}{\lambda}\right)^2 C\cos\alpha \frac{\cosh 2k(h+y-b)}{\sinh^4 kh} \cos[2k(x\cos\alpha + z\sin\alpha) + 2\sigma t] ,$$

EQ. (B.4)

$$v = -\left(\frac{\pi H}{\lambda}\right) C \frac{\sinh k(h+y-b)}{\sinh kh} \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$- \frac{3}{4}\left(\frac{\pi H}{\lambda}\right)^2 C \frac{\sinh 2k(h+y-b)}{\sinh^4 kh} \sin[2k(x\cos\alpha + z\sin\alpha) + 2\sigma t],$$

EQ. (B.5)

and in the z direction as

$$w = -\left(\frac{\pi H}{\lambda}\right) C\sin\alpha \frac{\cosh k(h+y-b)}{\sinh kh} \cos[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$- \frac{3}{4}\left(\frac{\pi H}{\lambda}\right)^2 C\sin\alpha \frac{\cosh 2k(h+y-b)}{\sinh^4 kh} \cos[2k(x\cos\alpha + z\sin\alpha) + \sigma t].$$

EQ. (B.6)

The particle accelerations are obtained by differentiating with respect to time t which will be indicated by a dot above the symbol

$$\dot{u} = \left(\frac{\pi H}{\lambda}\right) \sigma C\cos\alpha \frac{\cosh k(h+y-b)}{\sinh kh} \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+ \frac{3}{2}\left(\frac{\pi H}{\lambda}\right)^2 \sigma C\cos\alpha \frac{\cosh 2k(h+y-b)}{\sinh^4 kh} \sin[2k(x\cos\alpha + z\sin\alpha) + \sigma t],$$

EQ. (B.7)

$$\dot{v} = -\left(\frac{\pi H}{\lambda}\right) \sigma C \frac{\sinh k(h+y-b)}{\sinh kh} \cos[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$- \frac{3}{2}\left(\frac{\pi H}{\lambda}\right)^2 \sigma C \frac{\sinh 2k(h+y-b)}{\sinh^4 kh} \cos[2k(x\cos\alpha + z\sin\alpha) + 2\sigma t] ,$$

EQ. (B.8)

and

$$\dot{w} = \left(\frac{\pi H}{\lambda}\right) \sigma C\sin\alpha \frac{\cosh k(h+y-b)}{\sinh kh} \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+ \frac{3}{2}\left(\frac{\pi H}{\lambda}\right)^2 \sigma C \sin\alpha \frac{\cosh 2k(h+y-b)}{\sinh^4 kh} \sin[2k(x\cos\alpha + z\sin\alpha) + 2\sigma t] \;.$$

EQ. (B.9)

Rigid Body Rotation

The vessel is treated as a rigid body free to rotate about its center of gravity. This system requires six degrees of freedom to define the vessels position in space. The six coordinates may be further defined as three being displacements of the center of gravity relative to a stationary coordinate system, and three rotations in a moving coordinate system rigidly attached to the vessel as shown in Figure B.2.

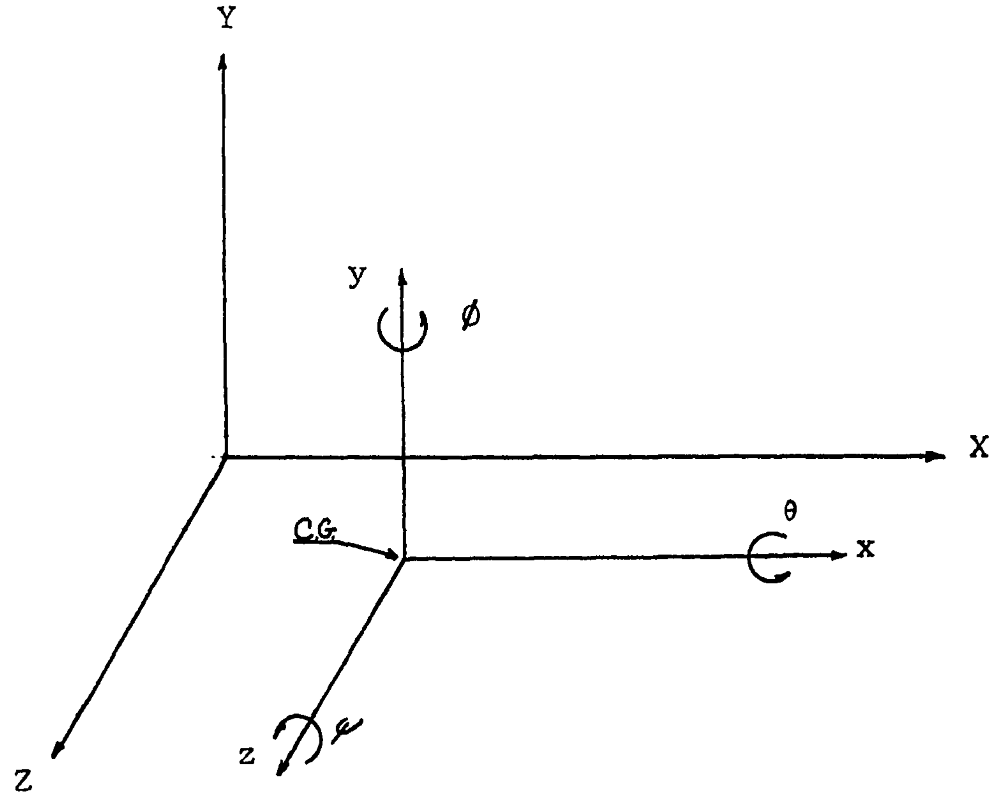


Figure B.2: Six degree of freedom coordinate system.

The angular velocities and accelerations associated with the roll (θ), pitch (ψ), and yaw (ϕ) are to be derived by first deriving a general rotation matrix. First rotate z-axis counterclockwise on a unit circle as shown in Figure B.3.

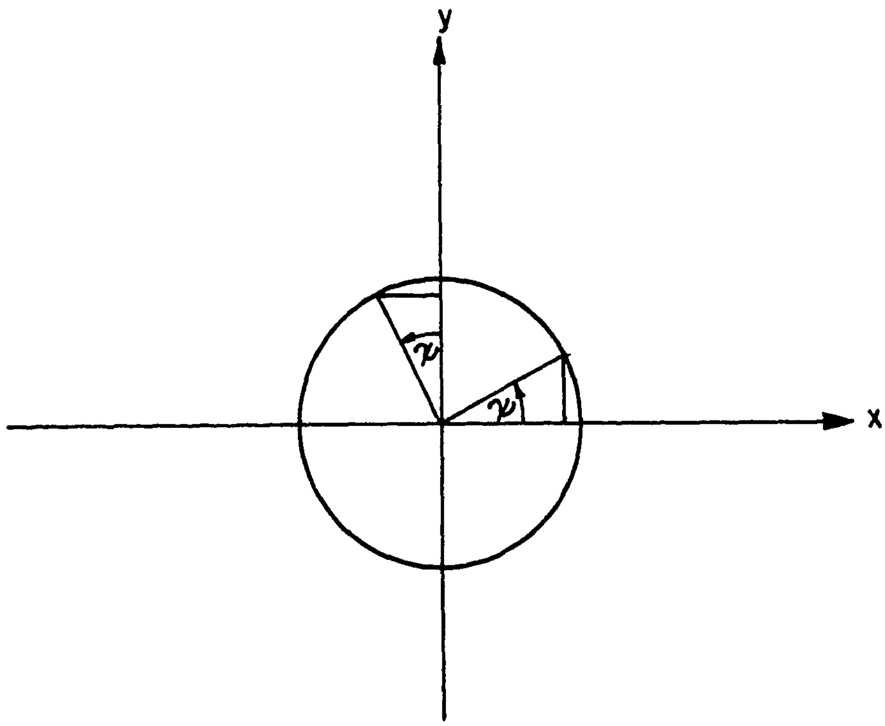


Figure B.3: Rotation around the z-axis.

The transformation matrix for this rotation is

$$\lambda_{\psi} = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} . \qquad \text{EQ. (B.10)}$$

Second, revolve the unit circle counterclockwise through an angle θ around the x-axis as in Figure B.4.

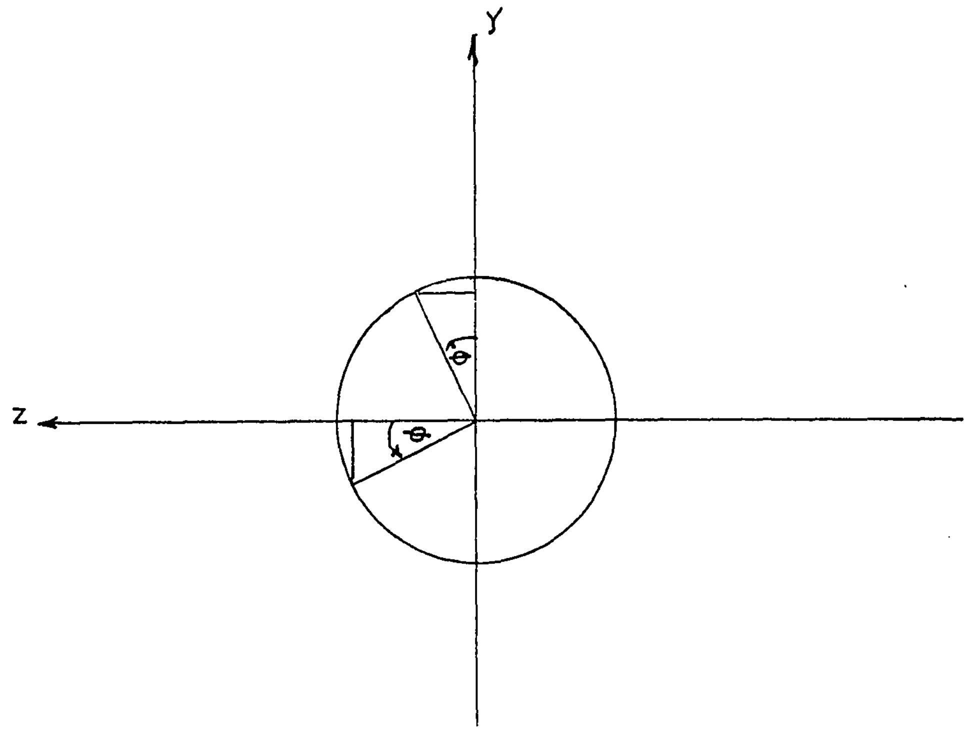


Figure B.4: Rotation around the x-axis.

The transformation matrix is easily seen to be

$$\lambda_{\theta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} . \qquad \text{EQ. (B.11)}$$

Finally, rotate the unit circle in the xy plane counterclockwise thru an angle ϕ as in Figure B.5. The transformation matrix is

$$\lambda_{\phi} = \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix} . \qquad \text{EQ. (B.12)}$$

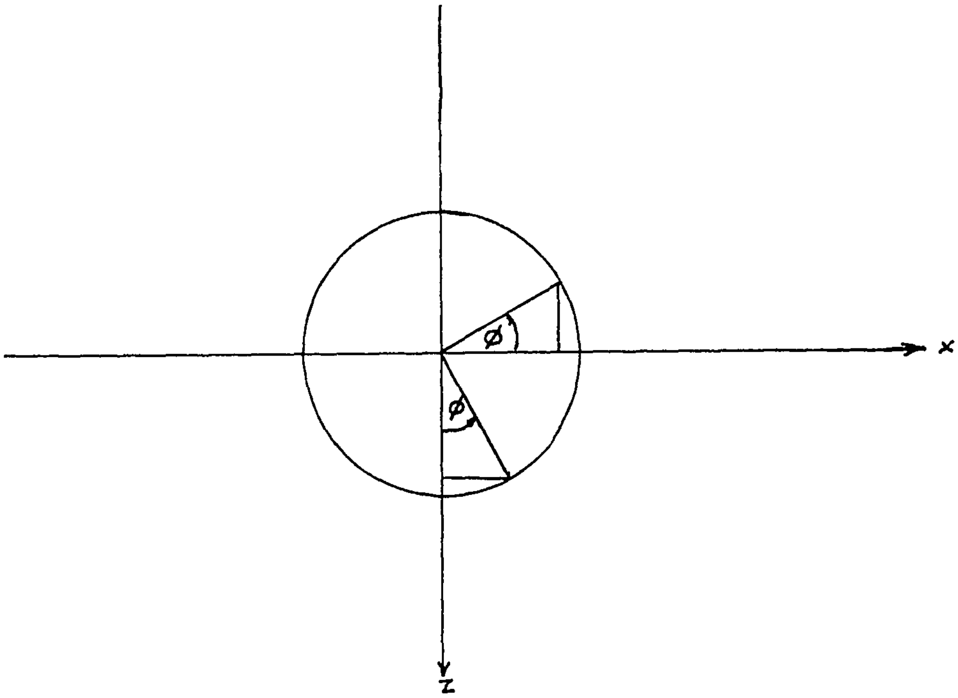


Figure B.5: Rotation around the y-axis.

The complete transformation to define the position in space of a point on a rigid body relative to its original position in the xyz system is given as the product of the individual rotation matrices

$$\lambda = \lambda_{\psi}\,\lambda_{\theta}\,\lambda_{\phi} \; ; \qquad \text{EQ. (B.13)}$$

whereas, the transpose λ^T will give the position relative to the XYZ stationary system. The elements of the resulting matrix are the direction cosines of a position vector to a point and the original coordinate axes. This development results in the following elements:

$\lambda_{11} = \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi$,

$\lambda_{12} = \cos\phi\sin\psi - \sin\phi\sin\theta\cos\psi$,

$\lambda_{13} = \sin\phi\cos\theta$,

$\lambda_{21} = -\cos\theta\sin\psi$,

$\lambda_{22} = \cos\theta\cos\psi$,

$\lambda_{23} = \sin\theta$,

$\lambda_{31} = -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi$,

$\lambda_{32} = -\sin\phi\sin\psi - \cos\phi\sin\theta\cos\psi$,

$\lambda_{33} = \cos\phi\cos\theta$.

One should note that all coordinates of the vessel relative to the rotating axis system remain constant; however, when the restoring forces of the mooring system are to be derived, the displacements relative to the stationary X Y Z system will be needed. These displacements which will include the displacement due to rotation may be written now for use later as

$$X = \lambda_{11} x_c + \lambda_{21} y_h + \lambda_{31} z_c , \qquad \text{EQ. (B.14)}$$

$$Y = \lambda_{12} x_c + \lambda_{22} y_h + \lambda_{32} z_c , \qquad \text{EQ. (B.15)}$$

$$Z = \lambda_{13} x_c + \lambda_{23} y_h + \lambda_{33} z_c . \qquad \text{EQ. (B.16)}$$

The components of angular velocities in the x, y, and z

Directions (of rotating system) due to each of the rotations may be chosen directly from the rotation matrix. The components of these angular displacements with respect to X Y Z are:

$$\bar{\sigma} = \lambda_\phi^T\ \lambda_\theta^T\ \lambda_\psi^T \begin{pmatrix} 0 \\ 0 \\ \psi \end{pmatrix} + \lambda_\theta^T\ \lambda_\psi^T \begin{pmatrix} \theta \\ 0 \\ 0 \end{pmatrix} + \lambda_\phi^T \begin{pmatrix} 0 \\ \phi \\ 0 \end{pmatrix} , \qquad \text{EQ. (B.17)}$$

which reduces to

$$\bar{\sigma} = \begin{pmatrix} -\psi\sin\phi\cos\theta & & \theta\cos\phi & & 0 \\ \psi\sin\theta & + & 0 & + & \phi \\ \psi\cos\phi\cos\theta & & \theta\sin\phi & & 0 \end{pmatrix} , \qquad \text{EQ. (B.18)}$$

which may be written in the classical form known as the Euler angles:

$$\sigma_x = -\psi\sin\phi\cos\theta + \theta\cos\phi \qquad \text{(roll)} , \qquad \text{EQ. (B.19a)}$$

$$\sigma_y = \psi\sin\theta + \phi \qquad \text{(yaw)} , \qquad \text{EQ. (B.19b)}$$

$$\sigma_z = \psi\cos\phi\cos\theta + \theta\sin\phi \qquad \text{(pitch)} . \qquad \text{EQ. (B.19c)}$$

The angular velocities along the rotating axes (x,y,z) are determined by the simple division of Δt or

$$\dot{\sigma}_x = -\dot{\psi}\sin\phi\cos\theta + \dot{\theta}\cos\phi , \qquad \text{EQ. (B.20)}$$

$$\dot{\sigma}_y = \dot{\psi}\sin\theta + \dot{\phi} , \qquad \text{EQ. (B.21)}$$

$$\dot{\sigma}_z = \dot{\psi}\cos\phi\cos\theta + \dot{\theta}\sin\phi . \qquad \text{EQ. (B.22)}$$

By taking derivatives with respect to time the angular accelerations are formed to be as follows:

$$\ddot{\sigma}_x = -\ddot{\psi}\sin\phi\cos\theta - \dot{\psi}\dot{\phi}\cos\phi\cos\theta + \dot{\psi}\dot{\theta}\sin\phi\sin\theta + \ddot{\theta}\cos\phi - \dot{\theta}\dot{\phi}\sin\phi \,, \qquad \text{EQ. (B.23)}$$

$$\ddot{\sigma}_y = \ddot{\psi} + \dot{\psi}\dot{\theta}\cos\theta + \ddot{\phi} \,, \qquad \text{EQ. (B.24)}$$

$$\ddot{\sigma}_z = \ddot{\psi}\cos\phi\cos\theta - \dot{\psi}\dot{\phi}\sin\phi\cos\theta - \dot{\psi}\dot{\theta}\cos\phi\sin\theta + \ddot{\theta}\sin\phi + \dot{\theta}\dot{\phi}\cos\phi \,. \qquad \text{EQ. (B.25)}$$

It is emphasized that roll, yaw, and pitch are not defined as θ, ϕ, and ψ except in the most simplified case case where

$\sigma_x = \theta$ when $\psi = \phi = 0$,

$\sigma_y = \phi$ when $\theta = \psi = 0$,

$\sigma_z = \psi$ when $\phi = \theta = 0$.

This is a condition that must be satisfied. However, the Euler angles θ, ϕ and ψ are defined as dependent variables in the equations of motion rather than σ_x, σ_y, and σ_z. The angular displacements are calculated directly from the solution by integration.

Equations of Motion

The six scalar equations of motion based on Newton's Second Law are now written in their most general form:

$$M\ddot{X} = \Sigma \text{ Forces}_x \,, \qquad \text{EQ. (B.26)}$$

$$M\ddot{Y} = \Sigma \text{ Forces}_y \,, \qquad \text{EQ. (B.27)}$$

$$M\ddot{Z} = \Sigma \text{ Forces}_z \,, \qquad \text{EQ. (B.28)}$$

$$\dot{H}'_x - \dot{\sigma}_z H'_y + \dot{\sigma}_y H'_z = \Sigma \text{ Moments}_x \,, \qquad \text{EQ. (B.29)}$$

$$\dot{H}'_y - \dot{\sigma}_x H'_z + \dot{\sigma}_z H'_x = \Sigma \text{ Moments}_y \,, \qquad \text{EQ. (B.30)}$$

$$\dot{H}'_z - \dot{\sigma}_y H'_x + \dot{\sigma}_x H'_y = \Sigma \text{ Moments}_z \,, \qquad \text{EQ. (B.31)}$$

where

$$H'_x = I_{xx}\,\dot{\sigma}_x - I_{xy}\,\dot{\sigma}_y - I_{xz}\,\dot{\sigma}_z \,, \qquad \text{EQ. (B.29a)}$$

$$H'_y = -I_{xy}\,\dot{\sigma}_x + I_{yy}\,\dot{\sigma}_y - I_{yz}\,\dot{\sigma}_z \,, \qquad \text{EQ. (B.30a)}$$

$$H'_z = -I_{xz}\,\dot{\sigma}_x - I_{yz}\,\dot{\sigma}_y + I_{zz}\,\dot{\sigma}_z \,, \qquad \text{EQ. (B.31a)}$$

and

$$\dot{H}'_x = I_{xx}\,\ddot{\sigma}_x - I_{xy}\,\ddot{\sigma}_y - I_{xz}\,\ddot{\sigma}_z \qquad \text{EQ. (B.29b)}$$

$$\dot{H}'_y = -I_{xy}\,\ddot{\sigma}_x + I_{yy}\,\ddot{\sigma}_y - I_{yz}\,\ddot{\sigma}_z \qquad \text{EQ. (B.30b)}$$

$$\dot{H}'_z = -I_{xz}\,\ddot{\sigma}_x - I_{yz}\,\ddot{\sigma}_y + I_{zz}\,\ddot{\sigma}_z \qquad \text{EQ. (B.31b)}$$

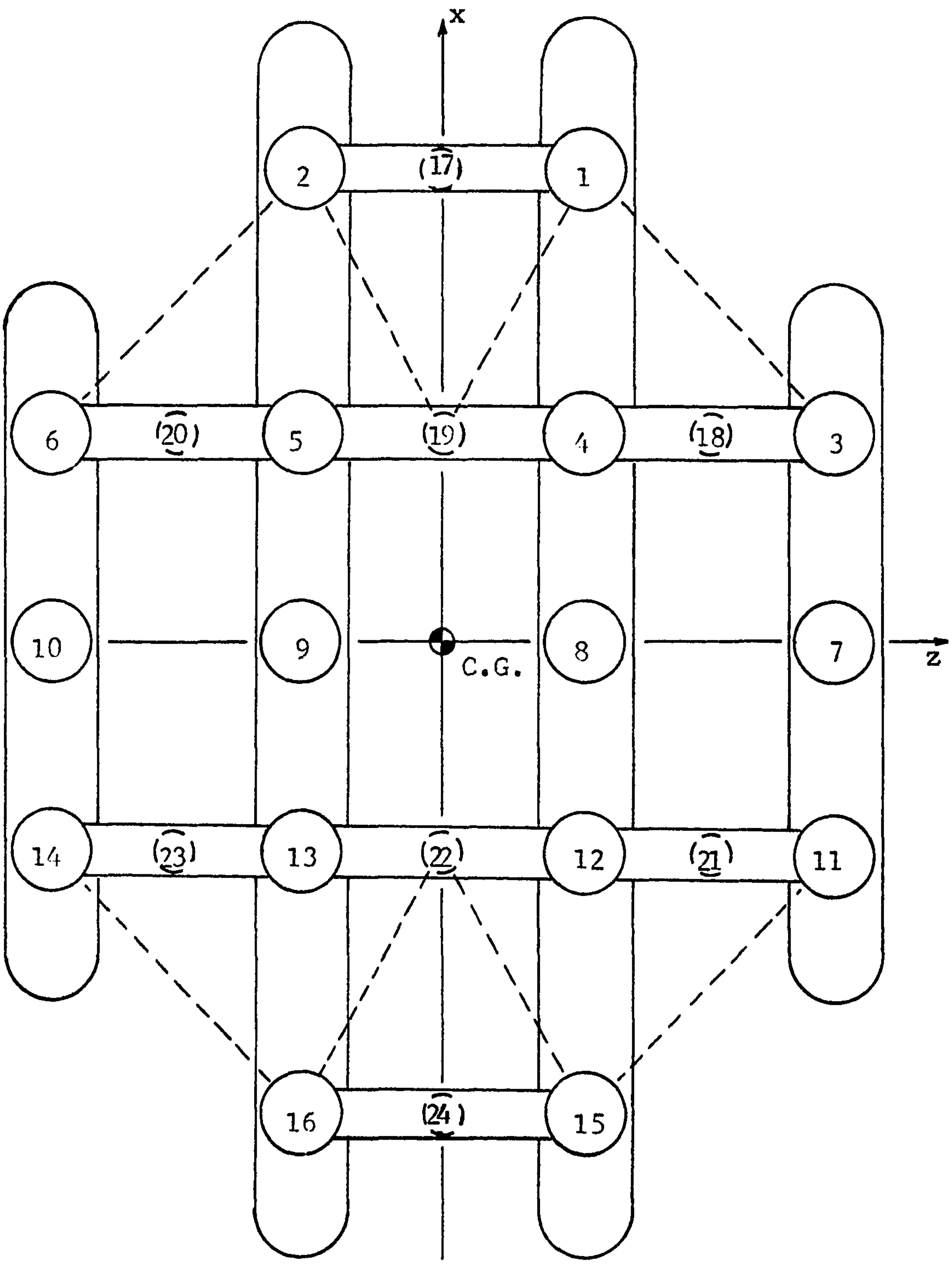


Figure B.6: Top view of superstructure.

Before going further, the theory will be somewhat clarified if a plan is given for dividing the vessel into segments which are numbered. This plan is shown in Figure B.6. Each of the horizontal members are subdivided symetrically about its respective column. The dashed columns are fictitious (columns with zero diameter) and serve only as a convenient computational scheme. The dashed lines depict relatively small diameter members which are not included in this analysis. A typical hull-column segment is shown in Figure B.7.

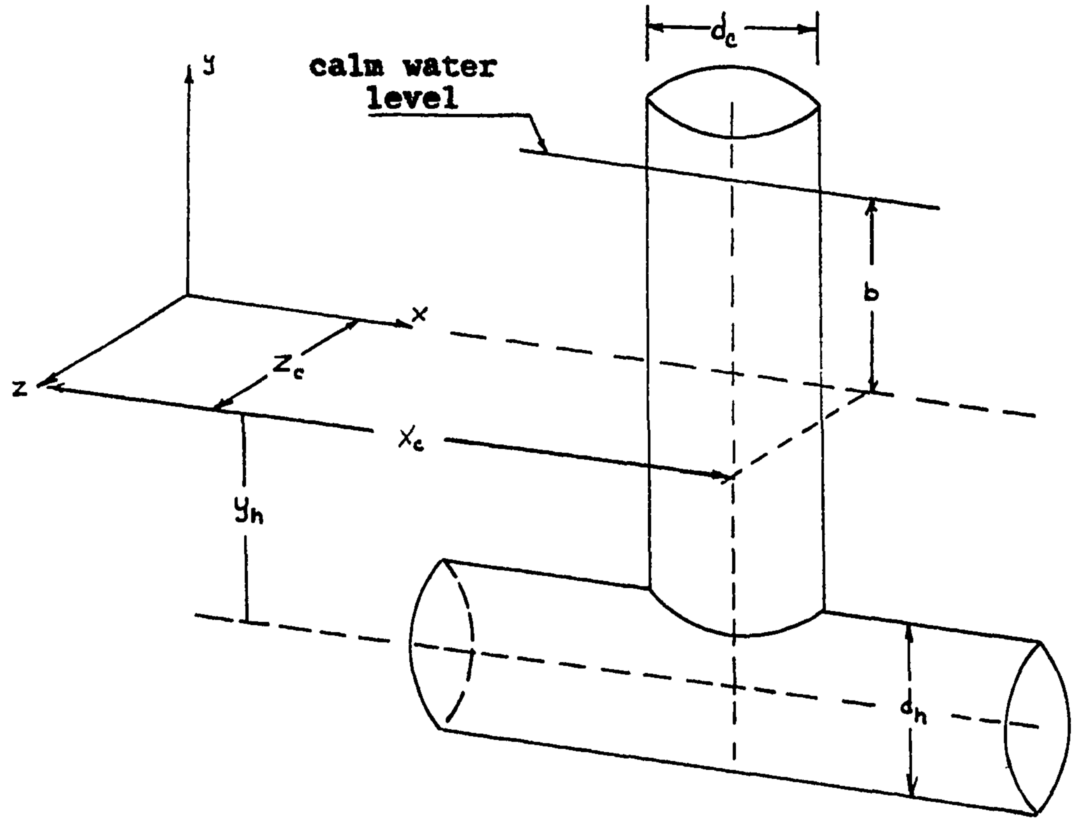


Figure B.7: Typical hull-column segment.

The interaction between the sea and the vessel involves the concepts of added mass and drag. In fluid dynamic systems where forms accelerate relative to the fluid, it is well founded that there will be a force exerted on the form proportional to the relative acceleration. The concept of drag forces proportional to the relative velocity squared are even more familiar.

The relative acceleration of the added mass in an arbitrary direction ξ is formed by the following component parts:

$\ddot{\xi}$ = Acceleration of the moving origin with respect to the stationary axis system.

$(\ddot{\sigma}\times\bar{\rho})_\xi$ = Tangential acceleration of a point P fixed in space within the rotating system xyz.

$\dot{\sigma}\times(\dot{\sigma}\times\bar{\rho})_\xi$ = Normal acceleration of a point P similarly fixed in the rotating system.

$\int_{\xi_1}^{\xi_2} \ddot{\beta}\, d\xi$ = Acceleration of the water particles integrated over the moving surface.

It will be best if all components of the relative tangential and normal accelerations are defined now. By definition tangential acceleration is given by the cross-product in matrix form

$$(\ddot{\sigma}\times\bar{\rho}) = \begin{pmatrix} i & j & k \\ \ddot{\sigma}_x & \ddot{\sigma}_y & \ddot{\sigma}_z \\ x & y & z \end{pmatrix} \qquad \text{EQ. (B.32)}$$

whose components are

$$(\ddot{\sigma}\times\underset{\sim}{r})_x = (z\ddot{\sigma}_y - y\ddot{\sigma}_z)i \quad , \qquad \text{EQ. (B.33)}$$

$$(\ddot{\sigma}\times\underset{\sim}{r})_y = (x\ddot{\sigma}_z - z\ddot{\sigma}_x)j \quad , \qquad \text{EQ. (B.34)}$$

$$(\ddot{\sigma}\times\underset{\sim}{r})_z = (y\ddot{\sigma}_x - x\ddot{\sigma}_y)k \quad . \qquad \text{EQ. (B.35)}$$

Whereas, the components of normal acceleration may be obtained from the definition

$$\dot{\sigma}\times(\dot{\sigma}\times\underset{\sim}{r}) = \begin{pmatrix} i & j & k \\ \dot{\sigma}_x & \dot{\sigma}_y & \dot{\sigma}_z \\ [z\dot{\sigma}_y - y\dot{\sigma}_z] & [x\dot{\sigma}_z - z\dot{\sigma}_x] & [y\dot{\sigma}_x - x\dot{\sigma}_y] \end{pmatrix} \qquad \text{EQ. (B.36)}$$

which yields

$$[\dot{\sigma}\times(\dot{\sigma}\times\underset{\sim}{r})]_x = \dot{\sigma}_y(y\dot{\sigma}_x - x\dot{\sigma}_y) - \dot{\sigma}_z(x\dot{\sigma}_z - z\dot{\sigma}_x) \quad , \qquad \text{EQ. (B.37)}$$

$$[\dot{\sigma}\times(\dot{\sigma}\times\underset{\sim}{r})]_y = \dot{\sigma}_z(z\dot{\sigma}_y - y\dot{\sigma}_z) - \dot{\sigma}_x(y\dot{\sigma}_x - x\dot{\sigma}_y) \quad , \qquad \text{EQ. (B.38)}$$

$$[\dot{\sigma}\times(\dot{\sigma}\times\underset{\sim}{r})]_z = \dot{\sigma}_x(x\dot{\sigma}_z - z\dot{\sigma}_x) - \dot{\sigma}_y(z\dot{\sigma}_y - y\dot{\sigma}_z) \quad . \qquad \text{EQ. (B.39)}$$

where $\dot{\sigma}_x$, $\dot{\sigma}_y$, and $\dot{\sigma}_z$ are given by Equations (B.20), (B.21), (B.22) and $\ddot{\sigma}_x$, $\ddot{\sigma}_y$, and $\ddot{\sigma}_z$ are defined by Equations (B.23), (B.24), and (B.25) respectively.

Relative Acceleration of Added Mass in z Direction

For motion in the x and z directions the hull-column sections are treated separately. Further, since the hull motion does not produce a change in the displaced volume (being fully submerged at all times), for added mass and drag considerations only motion in a direction normal to its longitudinal axis will be included. The motion of a column whose submergence varies with time must include the affect of a continuous change in bouyancy due to the wave form and vertical displacement.

For accelerated motion normal to a hull's longitudinal axis, note from Figure B.6 that only numbers 1 thru 16 are considered for the z direction. These will be given the subscript m. Thus, the relative acceleration of the added mass integrated over the hull length may be written as

$$\dot{V}_{hz} = \ddot{Z} + y_h\ddot{\sigma}_x - \frac{1}{L_{hm}}\int_{x_c-\frac{1}{2}L_{hm}}^{x_c+\frac{1}{2}L_{hm}} x\ddot{\sigma}_y dx + \frac{1}{L_{hm}}\int_{x_c-\frac{1}{2}L_{hm}}^{x_c+\frac{1}{2}L_{hm}} x\dot{\sigma}_x\dot{\sigma}_z dx$$

$$-z_c\dot{\sigma}_x^2 - \dot{\sigma}_y(z_c\dot{\sigma}_y - y_h\dot{\sigma}_z) - \frac{1}{L_{hm}}\int_{x_c-\frac{1}{2}L_{hm}}^{x_c+\frac{1}{2}L_{hm}} \dot{w}(x,y_h,z_c)dx$$

which simplifies to

$$\dot{V}_{hz} = \ddot{Z} + y_h(\ddot{\sigma}_x + \dot{\sigma}_y\dot{\sigma}_z) - x_c(\ddot{\sigma}_y - \dot{\sigma}_x\dot{\sigma}_z) - z_c(\dot{\sigma}_x^2 + \dot{\sigma}_y^2)$$

$$+\left(\frac{1}{L_{hm}}\right)\left(\frac{\pi H}{\lambda}\right)\left(\frac{\sigma C}{k}\right) \tan\alpha \; \frac{\cosh k(h+y_h-b)}{\sinh kh}$$

$$\text{times} \left(\cos[k(x_c+\tfrac{1}{2}L_{hm})\cos\alpha + kz_c\sin\alpha + \sigma t] - \cos[k(x_c-\tfrac{1}{2}L_{hm})\cos\alpha + kz_c\sin\alpha + \sigma t]\right)$$

$$+\frac{3}{2}\left(\frac{1}{L_{hm}}\right)\left(\frac{\pi H}{\lambda}\right)^2\left(\frac{\sigma C}{2k}\right) \tan\alpha \; \frac{\cosh 2k(h+y_h-b)}{\sinh^4 kh}$$

$$\text{times} \left(\cos[2k(x_c + \tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t] - \cos[2k(x_c - \tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t]\right) . \quad \text{EQ. (B.40)}$$

The force exerted by the relative acceleration of the added mass of a hull section moving in the z direction is

$$F_{hza} = -C_m \rho \frac{\pi d_{hm}^2}{4} L_{hm} \dot{\bar{V}}_{hzm} \qquad \text{EQ. (B.41)}$$

where C_m is an empirical added mass coefficient. Note that this term is negative which indicates that the force is applied as a damping term.

The force exerted on a column by the relative acceleration of the column's added mass is derived in a similar manner with the exception that provision must be made to obtain the instantaneous wetted length of the column corrected for the effects roll, pitch, yaw, vertical displacement and a time-varying wave height against the column. The wetted length of the column in calm water, as seen from Figure B.7 is

$$L_c = b - y_h - \tfrac{1}{2}d_h , \qquad \text{EQ. (B.42)}$$

where y_h is a negative coordinate. The effect of roll and pitch can be treated separately and superimposed only for small angles. For larger angles, one must return to the results of the rotation matrix. However, this is convenient since the position of the wave form is relative to the stationary system of axes. The transpose of the rotation matrix λ gave the new position of a hull-column intersection relative to a set of stationary axes due to body rotations. Using these relations, the instantaneous change in wetted length of a column is

$$\Delta W_L = \eta_y(x_c, b, z_c) - Y - \Delta y_h \qquad \text{EQ. (B.43)}$$

where the argument of η_y is given by Equations (B. 2). Since the constants associated with the added mass of a column and the drag (to be discussed) are based on the wetted length in a calm sea, Equation (B.43) must be used as part of an exposure factor which is defined as

$$S(t) = 1 - \frac{Y + \Delta y_h - \eta_y(x_c, b, z_c)}{b - y_h - .5d_h} \qquad \text{EQ. (B.44)}$$

The above exposure factor is the proportion of instantaneous wetted length of a column to wetted length in a calm sea.

The relative acceleration of a column's added mass in the z direction is similar to that of a hull except the integration must be over the wetted length of the column. This term can be written as

$$\dot{V}_{cz} = \ddot{Z} - x_c\dot{\sigma}_y + \frac{1}{b - y_h - \frac{1}{2}d_h}\int_{y_h + \frac{1}{2}d_h}^{b} y\dot{\sigma}_x dy + \sigma_x(x_c\sigma_z - z_c\sigma_x)$$

$$-z_c\sigma_y^2 + \frac{1}{b - y_h - \frac{1}{2}d_h}\int_{y_h + \frac{1}{2}d_h}^{b} [y\sigma_y\sigma_z - \dot{w}(x_c, y, z_c)]dy$$

which reduces to

$$\dot{\bar{V}}_{cz} = \ddot{Z} - x_c(\dot{\sigma}_y - \sigma_x\sigma_z) - z_c(\sigma_x^2 + \sigma_y^2)$$

$$+ \frac{1}{2}(y_h + \frac{1}{2}d_h + b)(\dot{\sigma}_x + \sigma_y\sigma_z)$$

$$+ \left(\frac{1}{y_h + \frac{1}{2}d_h - b}\right)\left(\frac{\pi H}{k}\right)\sigma C\frac{\sin\alpha}{\sinh kh}\sin[k(x_c\cos\alpha + z_c\sin\alpha) + \sigma t]$$

$$\text{times } [\sinh k(h + y_h + \tfrac{1}{2}d_h - b) - \sinh kh]$$

$$+ \left(\frac{3}{2}\right)\left(\frac{1}{y_h + \frac{1}{2}d_h - b}\right)\left(\frac{\pi H}{k}\right)^2\sigma C\frac{\sin\alpha}{\sinh^4 kh}\sin[2k(x_c\cos\alpha + z_c\sin\alpha) + 2\sigma t]$$

$$\text{times } [\sinh 2k(h + y_h + \tfrac{1}{2}d_h - b) - \sinh 2kh]. \qquad \text{EQ. (B.45)}$$

The force exerted by the relative acceleration of the column's added mass may now be written as

$$F_{cza} = -S(t)C_m\left(\frac{\pi\rho d_c^2}{4}\right)(b - y_h - \tfrac{1}{2}d_h)\ \dot{\bar{V}}_{cz}\ . \qquad \text{EQ. (B.46)}$$

Relative Acceleration of Added Mass in X Direction

The derivation of relative acceleration in the x direction is very similar to that for z, except 1) the components of tangential and normal acceleration must be in the x direction given by Equations (B.34) and (B.38), 2) the integrations over the hull lengths must be carried out with respect to z since the hulls No. 17 through No. 24 (these hulls will be subscripted with an n) lie parallel to the z axis, and 3) components of water particle acceleration must be given in the x direction as defined by Equation (B.7). With these exceptions the relative acceleration of the hull forms in the x direction is written directly,

$$\dot{\bar{V}}_{hx} = \ddot{X} - y_h\ddot{\sigma}_z + \frac{1}{L_{hn}}\int_{z_c-\frac{1}{2}L_{hn}}^{z_c+\frac{1}{2}L_{hn}} z\ddot{\sigma}_y\,dz + \dot{\sigma}_y(y_h\dot{\sigma}_x - x_c\dot{\sigma}_y) - x_c\dot{\sigma}_z^2$$

$$+ \frac{1}{L_{hn}}\int_{z_c-\frac{1}{2}L_{hn}}^{z_c+\frac{1}{2}L_{hn}} [z\dot{\sigma}_z\dot{\sigma}_x - \dot{u}(x_c, y_n, z)]dz$$

which reduces to

$$\dot{\bar{V}}_{hx} = \ddot{X} - y_h(\ddot{\sigma}_z - \dot{\sigma}_y\dot{\sigma}_x) + z_c(\ddot{\sigma}_y + \dot{\sigma}_z\dot{\sigma}_x) - {}_c(\dot{\sigma}_y^2 + \dot{\sigma}_z^2)$$

$$+\left(\frac{1}{L_{hn}}\right)\left(\frac{\pi H}{\lambda}\right)\left(\frac{\sigma C}{k}\right)\cot\alpha\frac{\cosh k(h+y_h-b)}{\sinh kh}$$

$$\text{times}\ \Big(\cos[kx_c\cos\alpha + k(z_c + \tfrac{1}{2}L_{hn})\sin\alpha + k\sigma t] - \cos[2kx_c\cos\alpha + 2k(z_c - \tfrac{1}{2}L_{hn})\sin\alpha + 2k\sigma t]\Big)$$

$$+\frac{3}{2}\left(\frac{1}{L_{hn}}\right)\left(\frac{\pi H}{\lambda}\right)\left(\frac{\sigma C}{2k}\right)\cot\alpha\frac{\cosh 2k(h+y_h-b)}{\sinh^4 kh}$$

$$\text{times}\ \Big(\cos[2kx_c\cos\alpha + 2k(z_c + \tfrac{1}{2}L_{hn})\sin\alpha + 2k\sigma t] - \cos[2kx_c\cos\alpha + 2k(z_c - \tfrac{1}{2}L_{hn})\sin\alpha + 2k\sigma t]\Big)\,.$$

EQ. (B.47)

The force exerted by the hull's added mass for motion in the x direction is

$$F_{hxa} = -C_m\left(\frac{\pi \rho d_{hn}^2}{4}\right)L_{hn}\dot{\bar{V}}_{hx}\,. \qquad \text{EQ. (B.48)}$$

Similarly, the relative acceleration of a column's added mass in the x direction may be written as

$$\dot{\bar{V}}_{cx} = \ddot{X} + z_c\ddot{\sigma}_y - \left(\frac{1}{b-y_h-\frac{1}{2}d_h}\right)\int_{y_h+\frac{1}{2}d_h}^{b} y\ddot{\sigma}_z dy - \dot{\sigma}_z(x_c\dot{\sigma}_z - z_c\dot{\sigma}_x)$$

$$- x_c\dot{\sigma}_y^2 + \left(\frac{1}{b-y_h-\frac{1}{2}d_h}\right)\int_{y_h+\frac{1}{2}d_h}^{b} y\dot{\sigma}_x\dot{\sigma}_y dy - \left(\frac{1}{b-y_h-\frac{1}{2}d_h}\right)\int_{y_h+\frac{1}{2}d_h}^{b} \dot{u}(x_c,y,z_c)dy ,$$

which reduces to

$$\dot{\bar{V}}_{cx} = \ddot{X} + z_c(\ddot{\sigma}_y + \dot{\sigma}_x\dot{\sigma}_z) - x_c(\dot{\sigma}_y^2 + \dot{\sigma}_z^2) + \frac{1}{2}(y_h + \frac{1}{2}d_h + b)(\dot{\sigma}_x\dot{\sigma}_y - \ddot{\sigma}_y)$$

$$+\left(\frac{1}{b-y_h-\frac{1}{2}d_h}\right)\left(\frac{\pi H}{k\lambda}\right)\frac{\sigma C\cos\alpha}{\sinh kh}\sin[k(x_c\cos\alpha + z_c\sin\alpha) + \sigma t]$$

$$\text{times } [\sinh k(h+y_h+\tfrac{1}{2}d_h-b) - \sinh kh]$$

$$+\frac{3}{2}\left(\frac{1}{b-y_h-\frac{1}{2}d_h}\right)\left(\frac{\pi H}{\lambda}\right)^2 \frac{\sigma C\cos\alpha}{2k\sinh^4 kh}\sin[2k(x_c\cos\alpha + z_c\sin\alpha) + 2\sigma t]$$

$$\text{times } [\sinh 2k(h+y_h+\tfrac{1}{2}d_h-b) - \sinh 2kh] . \qquad \text{EQ. (B.49)}$$

The force exerted on the column due to the above relative acceleration of the column's added mass corrected for the instantaneous wetted length may be written as

$$F_{cxa} = -S(t)C_m\left(\frac{\pi\rho d_c^2}{4}\right)(b-y_h-\tfrac{1}{2}d_h)\dot{\bar{V}}_{cx} . \qquad \text{EQ. (B.50)}$$

Relative Acceleration of Added Mass in y Direction

For motion in the vertical direction, the added mass of all hull forms (No. 1 through No. 24) must be considered. The The relative acceleration of a hull's added mass in the vertical direction is derived by the method used in the x and z directions. This term is written as

$$\dot{\bar{V}}_{hy} = \ddot{Y} - z_{cm}\ddot{\sigma}_x + \frac{1}{L_{hm}}\int_{x_c-\frac{1}{2}L_{hm}}^{x_c+\frac{1}{2}L_{hm}} [x\ddot{\sigma}_z - \dot{v}(x,y_h,z_c)]dx$$

$$+ x_{cn}\ddot{\sigma}_z - \frac{1}{L_{hn}}\int_{z_c-\frac{1}{2}L_{hn}}^{z_c+\frac{1}{2}L_{hn}} [z\ddot{\sigma}_x + \dot{v}(x_c,y_h,z)]dz \quad ,$$

which upon integration over the appropriate lengths of the hull forms reduces to

$$\dot{\bar{V}}_{hy} = \ddot{Y} - z_c\ddot{\sigma}_x + x_c\ddot{\sigma}_z + (\frac{1}{L_{hm}})(\frac{\pi H}{\lambda})(\frac{\sigma C}{k\cos\alpha})\frac{\sinh k(h+y_h-b}{\sinh kh}$$

$$\text{times} \quad \Big(\sin[k(x_c+\tfrac{1}{2}L_{hm})\cos\alpha + kz_c\sin\alpha + \sigma t] - \sin[k(x_c-\tfrac{1}{2}L_{hm})\cos\alpha + kz_c\sin\alpha + \sigma t]\Big)$$

$$+\frac{3}{2}(\frac{1}{L_{hm}})(\frac{\pi H}{\lambda})^2(\frac{\sigma C}{2k\cos\alpha})\frac{\sinh 2k(h+y_h-b)}{\sinh^4 kh}$$

$$\text{times} \quad \Big(\sin[2k(x_c+\tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t] - \sin[2k(x_c-\tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t]\Big)$$

$$+(\frac{1}{L_{hn}})(\frac{\pi H}{\lambda})(\frac{\sigma C}{k\sin\alpha})\frac{\sinh k(h+y_h-b)}{\sinh kh}$$

$$\text{times} \quad \Big(\sin[kx_c\cos\alpha + k(z_c+\tfrac{1}{2}L_{hn})\sin\alpha + \sigma t] - \sin[kx_c\cos\alpha + k(z_c-\tfrac{1}{2}L_{hn})\sin\alpha + \sigma t]\Big)$$

$$+\frac{3}{2}(\frac{1}{L_{hn}})(\frac{\pi H}{\lambda})^2(\frac{\sigma C}{2k\sin\alpha})\frac{\sinh 2k(h+y_h-b)}{\sinh^4 kh}$$

$$\text{times} \quad \Big(\sin[2kx_c\cos\alpha + 2k(z_c+\tfrac{1}{2}L_{hn})\sin\alpha + 2\sigma t] - \sin[2kx_c\cos\alpha + 2k(z_c-\tfrac{1}{2}L_{hn})\sin\alpha + 2\sigma t]\Big) \; .$$

EQ. (B.51)

where $1 <> m < 16$, and $17 < n < 24$. The force related to Equation (B.51) is simplified to read

$$F_{hya} = -C_m\left(\frac{\pi \rho d_h^2}{4}\right) L_h \dot{\bar{V}}_{hy} \; . \qquad \text{EQ. (B.52)}$$

This completes the derivation of forces associated with the relative accelerations of added masses due to the vessel's motion. It is to be emphasized that the acceleration terms 1) translation, 2) tangential, and 3) normal are with respect to the motions perpendicular to a longitudinal axis, neglecting those terms perpendicular to the transverse axes which are considered very small relative to the terms just derived. In addition, the "Coriolis" terms which would consider the acceleration on a rotating path is considered relatively small; they too are excluded.

Drag Forces Due to Motion

The second type of force due to interaction of vessel and sea that is considered in this analysis is the familiar term called "drag force" given by

$$F_d = -\tfrac{1}{2}\rho C_D A \bar{V} |\bar{V}| \; . \qquad \text{EQ. (B.53)}$$

Many authors refer to Equation (B.53) as "dynamic damping" or "square-law damping". The symbol A represents the projected area normal to the direction of motion. The $\bar{V}$'s are defined as the velocity of the surrounding fluid relative to cylindrical form moving in a direction normal to its longitudinal axis. The components of relative velocities of a hull and column moving in the three coordinate directions will be derived in much the same way as the derivations of the relative accelerations. In general, the relative velocity is written as

$$\bar{V}_\varsigma = \dot{\xi} + (\dot{\bar{\sigma}} x \bar{\varrho})_\varsigma - \int_{\xi_1}^{\xi_2} \beta \, d\xi \quad , \qquad \text{EQ. (B.54)}$$

where the components of the cross-product $(\bar{\sigma} x \bar{\varrho})$ are chosen from the definition

$$(\dot{\bar{\sigma}} x \bar{\varrho}) = \begin{pmatrix} i & j & k \\ \dot{\sigma}_x & \dot{\sigma}_y & \dot{\sigma}_z \\ x & y & z \end{pmatrix} \quad . \qquad \text{EQ. (B.55)}$$

The terms incorporated into Equation (B.54) involve:

1) $\dot{\xi}$ = velocity of a hull or column with respect to the stationary coordinate system XYZ,
2) $(\dot{\bar{\sigma}} x \bar{\varrho})_\varsigma$ = the velocity component due to rotation of a rigid body,
3) $\int \beta \, d\xi$ = the velocity component of the water particles integrated over the appropriate surface and direction.

Relative Velocity in Z Direction

The relative velocity of a column in the z direction may be expressed by substituting the corresponding terms in Equation (B.54),

$$\bar{V}_{cz} = \dot{z} - x_c \dot{\sigma}_y + \frac{1}{L_c} \int_{y_h + \frac{1}{2} d_h}^{b} [y \dot{\sigma}_x - w(x_c, y, z_c)] dy \quad ,$$

which after integration becomes

$$\overline{V}_{cz} = \dot{z} - x_c\dot{\sigma}_y + \frac{1}{2}(y_h + \frac{1}{2}d_h + b)\dot{\sigma}_x$$

$$-\left(\frac{1}{b-y_h-\frac{1}{2}d_h}\right)\left(\frac{\pi H}{\lambda}\right)\left(\frac{C\sin\alpha}{k\sinh kh}\right)\cos[k(x_c\cos\alpha + z_c\sin\alpha) + \sigma t]$$

$$\text{times} \quad [\sinh k(h+y_h+\frac{1}{2}d_h-b) - \sinh kh]$$

$$-\frac{3}{4}\left(\frac{1}{b-y_h-\frac{1}{2}d_h}\right)\left(\frac{\pi H}{\lambda}\right)^2\left(\frac{C\sin\alpha}{2k\sinh^4 kh}\right)\cos[2k(x_c\cos\alpha + z_c\sin\alpha) + 2\sigma t]$$

$$\text{times} \quad [\sinh 2k(h+y_h+\frac{1}{2}d_h-b) - \sinh 2kh] \;. \qquad \text{EQ. (B.56)}$$

The drag force on a column moving in the z direction corrected for the instantaneous wetted length according to Equation (B.53) is

$$F_{czd} = -\frac{1}{2}S(t)\, C_D d_c(b-y_h-\frac{1}{2}d_h)\overline{V}_{cz}|\overline{V}_{cz}| \;. \qquad \text{EQ. (B.57)}$$

Likewise, the relative velocity of those hulls positioned perpendicular to the z direction (Nos. 1 - 16 and subscripted m) may be written as

$$\overline{V}_{hz} = \dot{z} + y_h\dot{\sigma}_x - \frac{1}{L_{hm}}\int_{x_c-\frac{1}{2}L_{hm}}^{x_c+\frac{1}{2}L_{hm}} [x\dot{\sigma}_y + w(x,y_h,z_c)]dx \;,$$

and after integration becomes

$$\overline{V}_{hz} = \dot{z} + y_h\dot{\sigma}_x - x_c\dot{\sigma}_y + \left(\frac{\pi H}{\lambda}\right)\left(\frac{C}{L_{hm}k}\right)\tan\alpha\frac{\cosh k(h+y_h-b)}{\sinh kh}$$

$$\text{times} \quad \Big(\sin[k(x_c + \frac{1}{2}L_{hm})\cos\alpha + kz_c\sin\alpha + \sigma t]$$

$$-\sin[k(x_c - \frac{1}{2}L_{hm})\cos\alpha + kz_c\sin\alpha + \sigma t]\Big)$$

$$+\frac{3}{4}\left(\frac{\pi H}{\lambda}\right)^2\left(\frac{C}{2L_{hm}k}\right)\tan\alpha\frac{\cosh 2k(h+y_h-b)}{\sinh^4 kh}$$

$$\text{times} \quad \Big(\sin[2k(x_c + \tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t]$$

$$-\sin[2k(x_c - \tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t]\Big).$$

EQ. (B.58)

The drag force on a hull whose longitudinal axis is normal to the z direction may be written as

$$F_{hzd} = -\frac{1}{2}\rho C_D d_h L_{hm} \overline{V}_{hz} |\overline{V}_{hz}| \;. \qquad \text{EQ. (B.59)}$$

<u>Relative Velocity in x Direction</u>

The derivation of relative velocity in the x direction is identical in the method after choosing the correct terms. For column motion

$$\overline{V}_{cx} = \dot{X} + z_c\dot{\sigma}_y - \left(\frac{1}{b - y_h - \frac{1}{2}d_h}\right)\int_{y_h + \frac{1}{2}d_h}^{b} [y\dot{\sigma}_z + u(x_c, y, z_c)]dy \;,$$

which is evaluated as

$$\overline{V}_{cx} = \dot{X} + z_c\dot{\sigma}_y - \frac{1}{2}(y_h + b + \tfrac{1}{2}d_h)\dot{\sigma}_z$$

$$-\left(\frac{1}{b - y_h - \frac{1}{2}d_h}\right)\left(\frac{\pi H}{\lambda}\right)\left(\frac{C\cos\alpha}{k\sinh kh}\right)\cos[k(x_c\cos\alpha + z_c\sin\alpha) + \sigma t]$$

$$\text{times} \quad [\sinh k(h + y_h + \tfrac{1}{2}d_h - b) - \sinh kh]$$

$$-\frac{3}{4}\left(\frac{1}{b - y_h - \frac{1}{2}d_h}\right)\left(\frac{\pi H}{\lambda}\right)^2\left(\frac{C\cos\alpha}{2k\sinh^4 kh}\right)\cos[2k(x_c\cos\alpha + z_c\sin\alpha) + 2\sigma t]$$

$$\text{times} \quad [\sinh 2k(h + y_h + \tfrac{1}{2}d_h - b) - \sinh 2kh] \;. \qquad \text{EQ. (B.60)}$$

The drag force corresponding to this motion is

$$F_{cxd} = -\frac{1}{2}S(t)\rho C_D d_c(b - y_h - \tfrac{1}{2}d_h)\overline{V}_{cx}|\overline{V}_{cx}| \,. \qquad \text{EQ. (B.61)}$$

$$\text{times} \quad \Big(\sin[2k(x_c + \tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t]$$
$$-\sin[2k(x_c - \tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t]\Big). \qquad \text{EQ. (B.58)}$$

The drag force on a hull whose longitudinal axis is normal to the z direction may be written as

$$F_{hzd} = -\frac{1}{2}\rho C_D d_h L_{hm} \overline{V}_{hz} |\overline{V}_{hz}| \;. \qquad \text{EQ. (B.59)}$$

Relative Velocity in x Direction

The derivation of relative velocity in the x direction is identical in the method after choosing the correct terms. For column motion

$$\overline{V}_{cx} = \dot{X} + z_c\dot{\sigma}_y - \left(\frac{1}{b - y_h - \frac{1}{2}d_h}\right)\int_{y_h + \frac{1}{2}d_h}^{b} [y\dot{\sigma}_z + u(x_c, y, z_c)]dy \;,$$

which is evaluated as

$$\overline{V}_{cx} = \dot{X} + z_c\dot{\sigma}_y - \tfrac{1}{2}(y_h + b + \tfrac{1}{2}d_h)\dot{\sigma}_z$$

$$-\left(\frac{1}{b - y_h - \frac{1}{2}d_h}\right)\left(\frac{\pi H}{\lambda}\right)\left(\frac{C\cos\alpha}{k\sinh kh}\right)\cos[k(x_c\cos\alpha + z_c\sin\alpha) + \sigma t]$$

$$\text{times} \quad [\sinh k(h + y_h + \tfrac{1}{2}d_h - b) - \sinh kh]$$

$$-\frac{3}{4}\left(\frac{1}{b - y_h - \frac{1}{2}d_h}\right)\left(\frac{\pi H}{\lambda}\right)^2\left(\frac{C\cos\alpha}{2k\sinh^4 kh}\right)\cos[2k(x_c\cos\alpha + z_c\sin\alpha) + 2\sigma t]$$

$$\text{times} \quad [\sinh 2k(h + y_h + \tfrac{1}{2}d_h - b) - \sinh 2kh] \;. \qquad \text{EQ. (B.60)}$$

The drag force corresponding to this motion is

$$F_{cxd} = -\tfrac{1}{2}S(t)\rho C_D d_c(b - y_h - \tfrac{1}{2}d_h)\overline{V}_{cx}|\overline{V}_{cx}| \,. \qquad \text{EQ. (B.61)}$$

The relative velocity of those hull forms which lie normal to the x direction (No's. 17 - 24 and subscripted with an n) may be written as

$$\bar{V}_{hx} = \dot{X} - y_h\dot{\sigma}_z + \frac{1}{L_{hn}}\int_{z_c-\frac{1}{2}L_{hn}}^{z_c+\frac{1}{2}L_{hn}} [z\dot{\sigma}_y - u(x_c, y_h, z)]dz \quad ,$$

which is integrated to give

$$\bar{V}_{hx} = \dot{X} - y_h\dot{\sigma}_z + z_c\dot{\sigma}_y$$

$$+\left(\frac{\pi H}{\lambda}\right)\left(\frac{C}{kL_{hn}}\right)\cot\alpha\frac{\cosh k(h+y_h-b)}{\sinh kh}$$

$$\text{times}\quad \Big(\sin[kx_c\cos\alpha + k(z_c+\tfrac{1}{2}L_{hn})\sin\alpha + \sigma t]$$

$$-\sin[kx_c\cos\alpha + k(z_c-\tfrac{1}{2}L_{hn})\sin\alpha + \sigma t]\Big)$$

$$+\frac{3}{4}\left(\frac{\pi H}{\lambda}\right)^2\left(\frac{C}{2kL_{hn}}\right)\cot\alpha\frac{\cosh 2k(h+y_h-b)}{\sinh^4 kh}$$

$$\text{times}\quad \Big(\sin[2kx_c\cos\alpha + 2k(z_c+\tfrac{1}{2}L_{hn})\sin\alpha + 2\sigma t]$$

$$-\sin[2kx_c\cos\alpha + 2k(z_c-\tfrac{1}{2}L_{hn})\sin\alpha + 2\sigma t]\Big) \quad .$$

EQ. (B.62)

Whereas, the drag force on the hull is

$$F_{hxd} = -\tfrac{1}{2}\rho C_D d_h L_{hn}\bar{V}_{hx}|\bar{V}_{hx}| \quad . \qquad \text{EQ. (B.63)}$$

<u>Relative Velocity in y Direction</u>

As in deriving the relative acceleration in the y direction, all of the hull forms must be included. Again, those hulls positioned parallel to the x-axis will be subscripted with an m, and those parallel to the z-axis will be subscripted with an n. The relative velocity in the vertical direction is given as

$$\bar{V}_{hy} = Y - (z_c\dot{\sigma}_x)_m + \frac{1}{L_{hm}}\int_{x_c-\frac{1}{2}L_{hm}}^{x_c+\frac{1}{2}L_{hm}} [x\dot{\sigma}_z - v(x,y_h,z_c)]dx$$

$$+ (x_c\dot{\sigma}_z)_n - \frac{1}{L_{hn}}\int_{z_c-\frac{1}{2}L_{hn}}^{z_c+\frac{1}{2}L_{hn}} [z\dot{\sigma}_x + v(x_c,y_h,z)]dz \quad ,$$

which is evaluated as

$$\bar{V}_{hy} = Y + x_c\dot{\sigma}_z - z_c\dot{\sigma}_x$$

$$-\left(\frac{\pi H}{\lambda}\right)\left(\frac{C}{kL_{hm}\cos\alpha}\right)\frac{\sinh k(h+y_h-b)}{\sinh kh}$$

$$\text{times}\quad \Big(\cos[k(x_c+\tfrac{1}{2}L_{hm})\cos\alpha + kz_c\sin\alpha + \sigma t]$$

$$-\cos[k(x_c-\tfrac{1}{2}L_{hm})\cos\alpha + kz_c\sin\alpha + \sigma t]\Big)$$

$$-\frac{3}{4}\left(\frac{\pi H}{\lambda}\right)^2\left(\frac{C}{2kL_{hm}\cos\alpha}\right)\frac{\sinh 2k(h+y_h-b)}{\sinh^4 kh}$$

$$\text{times}\quad \Big(\cos[2k(x_c+\tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t]$$

$$-\cos[2k(x_c-\tfrac{1}{2}L_{hm})\cos\alpha + 2kz_c\sin\alpha + 2\sigma t]\Big)$$

$$-\left(\frac{\pi H}{\lambda}\right)\left(\frac{C}{kL_{hn}\sin\alpha}\right)\frac{\sinh k(h+y_h-b)}{\sinh kh}$$

$$\text{times}\quad \Big(\cos[kx_c\cos\alpha + k(z_c+\tfrac{1}{2}L_{hn})\sin\alpha + \sigma t]$$

$$-\cos[kx_c\cos\alpha + k(z_c-\tfrac{1}{2}L_{hn})\sin\alpha + \sigma t]\Big)$$

$$-\frac{3}{4}\left(\frac{\pi H}{\lambda}\right)^2\left(\frac{C}{2kL_{hn}\sin\alpha}\right)\frac{\sinh 2k(h+y_h-b)}{\sinh^2 kh}$$

$$\text{times}\ \Big(\cos[2kx_c\cos\alpha + 2k(z_c+\tfrac{1}{2}L_{hn})\sin\alpha + 2\sigma t]$$

$$-\cos[2kx_c\cos\alpha + 2k(z_c-\tfrac{1}{2}L_{hn})\sin\alpha + 2\sigma t]\Big)\ ,$$

EQ. (B.64)

This result is used to determine the component of drag force on a hull moving in the vertical direction as

$$F_{hyd} = -\tfrac{1}{2}\rho C_D d_h L_h \bar{V}_{hy}\left|\bar{V}_{hy}\right| \ . \qquad \text{EQ. (B.65)}$$

End Effect Due to Oblique Flow

The third type of force due to interaction concerns the force applied to certain hull forms that are discontinuous; i.e., those hull segments that form the ends (fore and aft) of the vessel. Observing Figure B.6, it is seen that these forces will be limited to hull No's. 1, 2, 3, 6, 11, 14, 15, and 16. Consider a hull whose longitudinal axis lies parallel to the x direction which is moving at an angle of incidence to a uniform stream as shown in Figure B.8. It has been verified that the fore and aft ends experience a lateral differential force per unit length given by

$$dF = \tfrac{1}{2}\rho V^2 \sin 2\alpha\, dS\ , \qquad \text{EQ. (B.66)}$$

where S is the area of a general cross-section. This equation can be rearranged to read

$$dF = \rho\, dS\, V\sin\alpha\, V\cos\alpha\ ,$$

and integrated from the beginning of each section (where $S = \pi d_h^2/4$) to the end (where S = 0). This result is

$$F = \pm \frac{\pi \rho d_h^2}{4} V\sin\alpha V\cos\alpha \begin{cases} + \text{ fore} \\ - \text{ aft} \end{cases} .$$

EQ. (B.67)

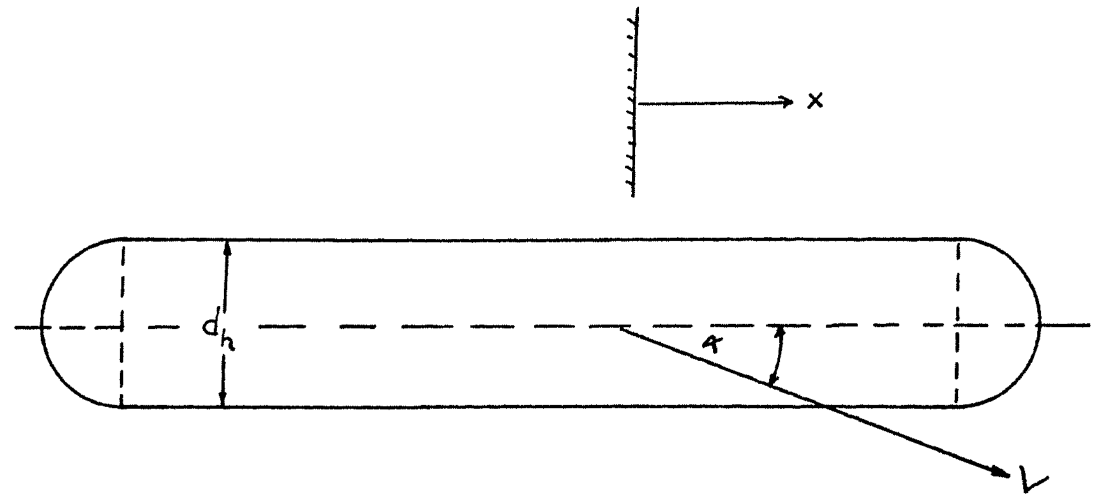
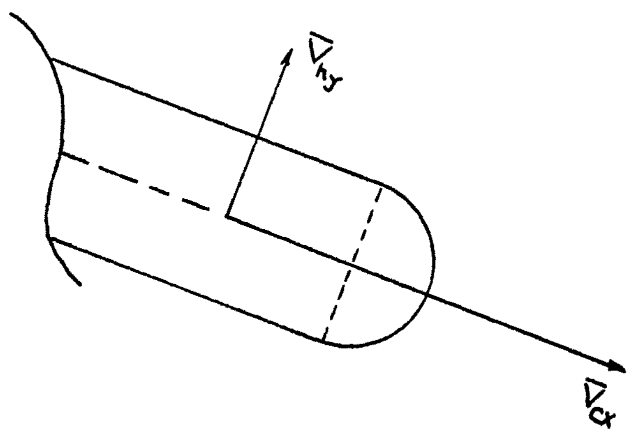


Figure B.8: Hull motion in oblique flow.

Examining the nose section,



it is observed that $+\overline{V}_{hy}$ or $+\overline{V}_{hz}$ is analogous to $-V\sin\alpha$ and $+\overline{V}_{cx}$ corresponds to $+V\cos\alpha$ in Equation (B.67). Therefore the forces exerted on the ends due to oblique flow may be written as

$$F_{hyo} = \frac{\pi\rho d_h^2}{4}\begin{cases} -\overline{V}_{hyj}\overline{V}_{cxj}, & j = 1,2,3,6 \\ 0 & , j \neq 1,2,3,6,11,14,15,16 \\ +\overline{V}_{hyj}\overline{V}_{cxj}, & j = 11,14,15,16 \end{cases} , \qquad \text{EQ. (B.68)}$$

and

$$F_{hzo} = \frac{\pi\rho d_h^2}{4}\begin{cases} -\overline{V}_{hzj}\overline{V}_{cxj}, & j = 1,2,3,6 \\ 0 & , j \neq 1,2,3,6,11,14,15,16 \\ +\overline{V}_{hzj}\overline{V}_{cxj}, & j = 11,14,15,16 \end{cases} . \qquad \text{EQ. (B.69)}$$

Force Perturbations Due to Non-uniform Pressure

In Appendix A, Bernoulli's equation was derived in terms of pressure. Omitting the hydrostatic pressure this relation is rewritten as

$$\frac{P}{\rho} = \frac{\partial\Phi}{\partial t} - \frac{1}{2}(u^2 + v^2 + w^2) . \qquad \text{EQ. (B.70)}$$

To use Equation (B.70) effectively the required substitutions for $\partial\Phi/\partial t$ and the velocities must be made and then reduced to an integrable form of pressure gradients rather than pressure. When the pressure is caused by velocities and accelerations in the fluid the resulting force is the pressure integrated over a surface; or by the Divergence theorem, pressure gradient integrated over a volume which will be the approach taken here.

Differentiating Φ, Equation (A.104) gives

$$\frac{\partial\Phi}{\partial t} = aC\sigma\frac{\cosh k(h+y-b)}{\sinh kh}\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+ \frac{3}{2}\frac{\pi a^2}{\lambda}C\sigma\frac{\cosh 2k(h+y-b)}{\sinh^4 kh}\cos 2[k(x\cos\alpha + z\sin\alpha) + \sigma t],$$

EQ. (B.72a)

squaring the velocity terms and abreviating by letting

$\cosh 1 = \cosh k(h+y-b)$,

$\cosh 2 = \cosh 2k(h+y-b)$, etc.

and

$B1 = k(x\cos\alpha + z\sin\alpha) + \sigma t$,

$B2 = k(x\cos\alpha + z\sin\alpha) + \sigma t$, etc.

leads to

$$u^2 = \left(\frac{aCk\cos\alpha}{\sinh kh}\right)^2\cosh^2 1\cos^2 B1$$

$$+3\left(\frac{aCk\cos\alpha}{\sinh kh}\right)\left(\frac{\pi a^2Ck\cos\alpha}{\lambda\sinh^4 kh}\right)\cosh 1\cosh 2\cos B1\cos B2$$

$$+\frac{9}{4}\left(\frac{\pi a^2Ck\cos\alpha}{\lambda\sinh^4 kh}\right)^2\cosh^2 2\cos^2 B2,$$ EQ. (B.72b)

$$v^2 = \left(\frac{aCk}{\sinh kh}\right)^2\sinh^2 1\sin^2 B1$$

$$+3\left(\frac{aCk}{\sinh kh}\right)\left(\frac{\pi a^2Ck}{\lambda\sinh^4 kh}\right)\sinh 1\sinh 2\sin B1\sin B2$$

$$+\frac{9}{4}\left(\frac{\pi a^2Ck}{\lambda\sinh^4 kh}\right)^2\sinh^2 2\sin^2 B2,$$ EQ. (B.72c)

$$w^2 = \left(\frac{aCk\sin\alpha}{\sinh kh}\right)^2\cosh^2 1\cos^2 B1$$

$$+3\left(\frac{aCk\sin\alpha}{\sinh kh}\right)\left(\frac{\pi a^2Ck\sin\alpha}{\lambda\sinh^4 kh}\right)\cosh 1\cosh 2\cos B1\cos B2$$

$$+ \frac{9}{4}\left(\frac{\pi a^2 Ck\sin\alpha}{\sinh^4 kh}\right)^2 \cosh^2 2\cos^2 B2 \ . \qquad \text{EQ. (B.72d)}$$

Substituting Equations (B.73a, B.73b, B.73c and B.73d) into Equation (B.72) reads

$$P = \rho aC\sigma \frac{\cosh 1}{\sinh kh}\cos B1 + \frac{3}{2}\frac{\rho\pi a^2 C\sigma}{\lambda}\frac{\cosh 2}{\sinh^4 kh}\cos B2$$

$$-\frac{\rho}{2}\left\{ \left[\left(\frac{aCk\cos\alpha}{\sinh kh}\right)^2 + \left(\frac{aCk\sin\alpha}{\sinh kh}\right)^2\right]\cosh^2 1\cos^2 B1\right.$$

$$+\left[3\left(\frac{aCk\cos\alpha}{\sinh kh}\right)\left(\frac{\pi a^2 Ck\cos\alpha}{\lambda\sinh^4 kh}\right) + 3\left(\frac{aCk\sin\alpha}{\sinh kh}\right)\left(\frac{\pi a^2 Ck\cos\alpha}{\lambda\sinh^4 kh}\right)\right]$$

times cosh1cosh2cosB1cosB2

$$+\left[\frac{9}{4}\left(\frac{\pi a^2 Ck\cos\alpha}{\lambda\ \sinh^4 kh}\right)^2 + \frac{9}{4}\left(\frac{\pi a^2 Ck\sin\alpha}{\lambda\ \sinh^4 kh}\right)\right]\cosh^2 2\cos^2 B2$$

$$+\left(\frac{aCk}{\sinh kh}\right)^2 \sinh^2 1\sin^2 B1$$

$$+ 3\left(\frac{aCk}{\sinh kh}\right)\left(\frac{a^2 Ck}{\lambda\sinh^4 kh}\right)\sinh 1\sinh 2\sin B1\sin B2$$

$$\left. + \frac{9}{4}\left(\frac{\pi a^2 Ck}{\lambda\sinh^4 kh}\right)^2 \sinh^2 2\sin^2 B2\right\}. \qquad \text{EQ. (B.72e)}$$

By making use of the identity

$$\cos^2\xi + \sin^2\xi = 1$$

and letting the constants in front of each term become a single constant, Equation (B.72e) simplifies to

$$P = A_1\cosh 1\cos B1 + A_2\cosh 2\cos B2$$

$$-A_3(\cosh^2 1\cos^2 B1 + \sinh^2 1\sin^2 B1)$$

$$-A_4(\cosh^2 1\cos^2 B2 + \sinh^2 2\sin^2 B2)$$

$$-A_5(\cosh 1\cosh 2\cos B1\cos B2$$

$$+\sinh 1\sinh 2\sin B1\sin B2)\ . \qquad \text{EQ. (B.72f)}$$

Again using the identities

$$\sin^2 B1 = 1-\cos^2 B1\ ,$$

$$\sin^2 B2 = 1-\cos^2 B2\ ,$$

Equation (B.72f) is replaced by

$$P = A_1\cosh 1\cos B1 + A_2\cosh 2\cos B2$$

$$-A_3(\cosh^2 1\cos^2 B1 + \sinh^2 1 - \sinh^2 1\cos^2 B1)$$

$$-A_4(\cosh^2 2\cos^2 B2 + \sinh^2 2 - \sinh^2 2\cos^2 B2)$$

$$-A_5(\cosh 1\cosh 2\cos B1\cos B2 + \sinh 1\sinh 2\sin B1\sin B2)\ ,$$

EQ. (B.72g)

which by utilizing the relation

$$\cosh^2\xi - \sinh^2\xi = 1\ ,$$

reduces to

$$P = A_1\cosh 1\cos B1 + A_2\cosh 2\cos B2$$

$$-A_3(\cos^2 B1 + \sinh^2 1) - A_4(\cos^2 B2 + \sinh^2 B2)$$

$$-A_5(\cosh 1\cosh 2\cos B1\cos B2 + \sinh 1\sinh 2\sin B1\sin B2)\ .$$

EQ. (B.72h)

Further use of the identities

$$\cos^2\xi = \frac{1}{2} + \frac{1}{2}\cos 2\xi\ ,$$

$$\sinh^2\xi = \frac{1}{2}\cosh 2\xi - \frac{1}{2}\ ,$$

Equation (B.72h) transforms to

$$P = A_1\cosh 1\cos B1 + A2\cosh 2\cos B2$$
$$- \tfrac{1}{2}A_3(\cos B2 + \cosh 2)$$
$$- \tfrac{1}{2}A_4(\cos B4 + \cosh 4)$$
$$- A_5(\cosh 1\cosh 2\cos B1\cos B2 + \sinh 1\sinh 2\sin B1\sin B2)\,.$$

EQ. (B.72i)

Working with only the coefficient of A_5 and using the hyperbolic identities

$$\cosh 2 = 1 + 2\sinh^2 1\,;$$
$$\sinh 2 = 2\sinh 1\cosh 1\,,$$

this term becomes

$$\mathrm{coef}(A_5) = \cosh 1(1 + 2\sinh^2 1)\cos B1\cos B2$$
$$+2\sinh^2 1\cosh 1\sin B1\sin B2\,, \qquad \text{EQ. (B.72j)}$$

which is rearranged to read

$$\mathrm{coef}(A_5) = \cosh 1\cos B1\cos B2$$
$$+2\sinh^2 1\cosh 1(\cos B1\cos B2 + \sin B1\sin B2)\,.$$

EQ. (B.72k)

The factor in the second term corresponds to the cosine double angle formula

$$\cos(\xi - \zeta) = \cos\xi\cos\zeta + \sin\xi\sin\zeta\ ,$$

where

$$\xi - \zeta = B1-B2 = -B1\ ,$$

which for the cosine the entire factor reduces to just cosB1 which gives

$$coef(A_5) = \cosh 1 \cos B1 \cos B2 + 2\sinh^2 1 \cosh 1 \cos B1 \ . \qquad \text{EQ. (B.72l)}$$

Now replacing cosB2 by its equivalent $1-2\sin^2 B1$ the coefficient is in the form

$$coef(A_5) = \cosh 1 \cos B1 - 2\cosh 1 \cos B1 \sin^2 B1 + 2\sinh^2 B1 \cosh 1 \cos B1 \ , \qquad \text{EQ. (B.72m)}$$

which is a desirable form for differentiation and integration. Substituting $coef(A_5)$ back into Equation (B.72i) and letting the factor 1/2 become part of A_3 and A_4, the final form of Bernoulli's equation for pressure is

$$\begin{aligned}
P = {} & A_1 \cosh k(h+y-b)\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& + A_2 \cosh 2k(h+y-b)\cos 2[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& - A_3 \cos 2[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& - A_3 \cosh 2k(h+y-b) \\
& - A_3 \cosh 2k(h+y-b) \\
& - A_4 \cos 4[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& - A_4 \cosh 4k(h+y-b) \\
& - A_5 \cosh k(h+y-b)\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& + 2A_5 \cosh k(h+y-b)\sin[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& \text{times } \cos[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& - 2A_5 \sinh k(h+y-b)\cosh k(h+y-b) \\
& \text{times } \cos[k(x\cos\alpha + z\sin\alpha) + \sigma t] \ ,
\end{aligned} \qquad \text{EQ. (B.73)}$$

where the coefficients are given as

$$A_1 = \frac{a\rho C\sigma}{\sinh kh} , \qquad \text{EQ. (B.74)}$$

$$A_2 = 1.5 \frac{\pi a^2 C\rho\sigma}{\sinh^4 kh} , \qquad \text{EQ. (B.75)}$$

$$A_3 = \frac{\rho}{2}\left(\frac{aCk}{\sinh kh}\right)^2 , \qquad \text{EQ. (B.76)}$$

$$A_4 = \frac{\rho g}{4}\left(\frac{\pi a^2 Ck}{\lambda \sinh^4 kh}\right)^2 , \qquad \text{EQ. (B.77)}$$

$$A_5 = \frac{3\pi a^3 C^2 k^2 \rho}{\lambda \sinh^5 kh} . \qquad \text{EQ. (B.78)}$$

The desired pressure gradients are found readily by differentiating as follows :

$$\begin{aligned}
\frac{\partial P}{\partial x} = & -(A_1 + A_5)k\cos\alpha\cosh k(h+y-b)\sin[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& -2A_2 k\cos\alpha\cosh 2k(h+y-b)\sin 2[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& +2A_3 k\cos\alpha\sin 2[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& +4A_4 k\cos\alpha\sin 4[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& +6A_5 k\cos\alpha\cosh k(h+y-b)\cos^2[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& \text{times } \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& +2A_5 k\cos\alpha\sinh^2 k(h+y-b)\cosh k(h+y-b) \\
& \text{times } \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t] , \qquad \text{EQ. (B.79)}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial P}{\partial y} = & (A_1 + A_5)k\sinh k(h+y-b)\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& +2A_2 k\sinh 2k(h+y-b)\cos 2[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& -2A_3 k\sinh 2k(h+y-b) \\
& -4A_4 k\sinh 4k(h+y-b) \\
& +2A_5 k\sinh k(h+y-b)\sin^2[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& \text{times } \cos[k(x\cos\alpha + z\sin\alpha) + \sigma t] \\
& -6A_5 \sinh k(h+y-b)\cosh^2 k(h+y-b)\cos[k(x\cos\alpha + z\sin\alpha) + \sigma t] ,
\end{aligned}$$

EQ. (B.80)

$$\frac{\partial P}{\partial z} = -(A_1 + A_5)k\sin\alpha\cosh k(h+y-b)\sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$-2A_2 k\sin\alpha\cosh 2k(h+y-b)\sin 2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+2A_3 k\sin\alpha\sin 2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+4A_4 k\sin\alpha\sin 4[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+6A_5 k\sin\alpha\cosh k(h+y-b)\cos^2[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$\text{times } \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t]$$

$$+2A_5 k\sin\alpha\sinh^2 k(h+y-b)\cosh k(h+y-b)$$

$$\text{times } \sin[k(x\cos\alpha + z\sin\alpha) + \sigma t] \;. \qquad \text{EQ. (B.81)}$$

Integrating the above pressure gradients over appropriate volumes leads to the following forces due to pressure:

$$F_{cxp} = -\frac{\pi d_c^2}{4}\Big\{ -(A_1+A_5)\cos\alpha[\sinh kh - \sinh k(h+y_h+\tfrac{1}{2}d_h-b)]$$

$$\text{times } \sin B_1(t)$$

$$-A_2\cos\alpha[\sinh 2kh - \sinh 2k(h+y_h+\tfrac{1}{2}d_h-b)]$$

$$\text{times } \sin B_{21}(t)$$

$$+2A_3(b-y_h-\tfrac{1}{2}d_h)k\cos\alpha\sin B_{21}(t)$$

$$+4A_3(b-y_h-\tfrac{1}{2}d_h)k\cos\alpha\sin B_{41}(t)$$

$$+6A_5\cos\alpha[\sinh kh - \sinh k(h+y_h+\tfrac{1}{2}d_h-b)]$$

$$\text{times } \cos^2 B_1(t)\, \mathrm{SIN}\, B_1(t)$$

$$+\frac{2}{3}A_5\cos\alpha[\sinh^3 kh - \sinh^3 k(h+y_h+\tfrac{1}{2}d_h-b)]\sin B_1(t)\Big\},$$

EQ. (B.82)

$$F_{hxp} = -\frac{\pi d_h^2}{2}\Big\{(A_1 + A_5)\cot\alpha\cosh k(h+y_h-b)[\cos B_4(t)-\cos B_5(t)]$$

$$+A_2\cot\alpha\cosh 2k(h+y_h-b)[\cos B_{24}(t)-\cos B_{25}(t)]$$

$$-A_3\cot\alpha[\cos B_{24}(t)-\cos B_{25}(t)]$$

$$-A_4\cot\alpha[\cos B_{44}(t)-\cos B_{45}(t)]$$

$$-2A_5\cot\alpha\cosh k(h+y_h-b)[\cos^3 B_4(t)-\cos^3 B_5(t)]$$

$$-2A_5\cot\alpha\sinh^2 k(h+y_h-b)\cosh k(h+y_h-b)$$

$$\text{times } [\cos B_4(t)-\cos B_5(t)]\Big\} . \qquad \text{EQ. (B.83)}$$

$$F_{yp} = F_{yp1} + F_{yp2} + F_{yp3} ,$$

$$= -\frac{\pi d_h^2}{4}\int_{x_c-\frac{1}{2}L_h}^{x_c+\frac{1}{2}L_h}\frac{\partial P}{\partial y}dx - \frac{\pi d_c^2}{4}\int_{y_h+\frac{1}{2}d_h}^{b}\frac{\partial P}{\partial y}dy - \frac{\pi d_h^2}{4}\int_{z_c-\frac{1}{2}L_h}^{z_c+\frac{1}{2}L_h}\frac{\partial P}{\partial y}\,dz ,$$

EQ. (B.84)

$$F_{yp1} = -\frac{\pi d_h^2}{4}\Big\{\frac{(A_1+A_5)}{\cos\alpha}\sinh k(h+y_h-b)[\sin B_2(t)-\sin B_3(t)]$$

$$+\frac{A_2}{\cos\alpha}\sinh 2k(h+y_h-b)[\sin B_{22}(t)-\sin B_{23}(t)]$$

$$-\frac{2A_3}{\cos\alpha}L_h\sinh 2k(h+y_h-b)$$

$$-\frac{4A_4}{\cos\alpha}L_h\sinh 4k(h+y_h-b)$$

$$+\frac{2}{3}\frac{A_5}{\cos\alpha}\sinh k(h-y_h-b)[\sin^3 B_2(t)-\sin^3 B_3(t)]$$

$$- \frac{6A_5}{\cos\alpha}\sinh k(h+y_h-b)\cosh k(h+y_h-b)[\sin B_2(t)-\sin B_3(t)]\Big\},$$

EQ. (B.85)

$$F_{y_{p2}} = -\frac{\pi d_c^2}{4}\Big\{(A_1+A_5)[\cosh kh-\cosh k(h+y_h+\tfrac{1}{2}d_h-b)]\cos B_1(t)$$
$$+A_2[\cosh 2kh-\cosh 2k(h+y_h+\tfrac{1}{2}d_h-b)]\cos B_{22}(t)$$
$$-A_3[\cosh 2kh-\cosh 2k(h+y_h+\tfrac{1}{2}d_h-b)]$$
$$-A_4[\cosh 4kh-\cosh 4k(h+y_h+\tfrac{1}{2}d_h-b)]$$
$$+2A_5[\cosh kh-\cosh k(h+y_h+\tfrac{1}{2}d_h-b)]\sin^2 B_1(t)\cos B_1(t)$$
$$-\frac{2A_5}{3}[\cosh^3 kh-\cosh^3 k(h+y_h+\tfrac{1}{2}d_h-b)]\cos B_1(t)\Big\},$$

EQ. (B.86)

$$F_{y_{p3}} = -\frac{\pi d_h^2}{4}\Big\{\frac{(A_1+A_5)}{\sin\alpha}\sinh k(h+y_h-b)[\sin B_2(t)-\sin B_3(t)]$$
$$+\frac{A_2}{\sin\alpha}\sinh 2k(h+y_h-b)[\sin B_{22}(t)-\sin B_{23}(t)]$$
$$-\frac{2A_3L_h}{\sin\alpha}\sinh 2k(h+y_h-b)$$
$$-\frac{4A_4L_h}{\sin\alpha}\sinh 4k(h+y_h-b)$$
$$+\frac{2}{3}\frac{A_5}{\sin\alpha}\sinh k(h+y_h-b)[\sin^3 B_2(t)-\sin^3 B_3(t)]$$
$$-\frac{6A_5}{\sin\alpha}\sinh k(h+y_h-b)\cosh^2 k(h+y_h-b)$$
$$\text{times } [\sin B_2(t)-\sin B_3(t)]\Big\},$$

EQ. (B.87)

$$F_{czp} = -\frac{\pi d_c^2}{2}\Big\{-(A_1+A_5)\sin\alpha[\sinh kh-\sinh k(h+y_h+\tfrac{1}{2}d_h-b)]\sin B_1(t)$$
$$-A_2\sin\alpha[\sinh 2kh-\sinh 2k(h+y_h+\tfrac{1}{2}d_h-b)]\sin B_{21}(t)$$

$$+2A_3 k\sin\alpha(b-y_h-\tfrac{1}{2}d_h)\sin B_{21}(t)$$

$$+4A_4 k\sin\alpha(b-y_h-\tfrac{1}{2}d_h)\sin B_{41}(t)$$

$$+6A_5\sin\alpha[\sinh kh-\sinh k(h+y_h+\tfrac{1}{2}d_h-b)]\cos^3 B_1(t)\sin B_1(t)$$

$$+\tfrac{2}{3}A_5\sin\alpha[\sinh^3 kh-\sinh^3 k(h+y_h+\tfrac{1}{2}d_h-b)]\sin B_1(t)\Big\},$$

EQ. (B.88)

$$F_{hzp} = -\frac{\pi d_h^2}{4}\Big\{(A_1+A_5)\tan\alpha\cosh k(h+y_h-b)[\cos B_2(t)-\cos B_3(t)]$$

$$+A_2\tan\alpha\cosh 2k(h+y_h-b)[\cos B_{22}(t)-\cos B_{23}(t)]$$

$$-A_3\tan\alpha[\cos B_{22}(t)-\cos B_{23}(t)]$$

$$-A_4\tan\alpha[\cos B_{42}(t)-\cos B_{43}(t)]$$

$$-\frac{2}{3}A_5\tan\alpha\cosh k(h+y_h-b)[\cos^3 B_2(t)-\cos^3 B_3(t)]$$

$$-2A_5\tan\alpha\sinh^2 k(h+y_h-b)\cosh k(h+y-b)$$
$$\text{times } [\cos B_2(t)-\cos B_3(t)]\Big\},$$

EQ. (B.89)

where

$B_1(t) = k(x_c\cos\alpha + z_c\sin\alpha) + \sigma t$, EQ. (B.90)

$B_{21}(t) = 2B_1(t)$, EQ. (B.91)

$B_{41}(t) = 4B_1(t)$, EQ. (B.92)

$B_2(t) = k(x_c+\tfrac{1}{2}L_h)\cos\alpha + kz_c\sin\alpha + \sigma t$, EQ. (B.93)

$B_{22}(t) = 2B_2(t)$, EQ. (B.94)

$$B_3(t) = k(x_c - \tfrac{1}{2}L_h)\cos\alpha + kz_c\sin\alpha + \sigma t \,, \qquad \text{EQ. (B.95)}$$

$$B_{23}(t) = 2B_3(t) \,, \qquad \text{EQ. (B.96)}$$

$$B_4(t) = kx_c\cos\alpha + k(z_c + \tfrac{1}{2}L_h)\sin\alpha + \sigma t \,, \qquad \text{EQ. (B.97)}$$

$$B_{24}(t) = 2B_4(t) \,, \qquad \text{EQ. (B.98)}$$

$$B_{44}(t) = 4B_4(t) \,, \qquad \text{EQ. (B.99)}$$

$$B_5(t) = kx_c\cos\alpha + k(z_c - \tfrac{1}{2}L_h)\sin\alpha + \sigma t \,, \qquad \text{EQ. (B.100)}$$

$$B_{25}(t) = 2B_5(t) \,, \qquad \text{EQ. (B.101)}$$

$$B_{45}(t) = 4B_5(t) \,. \qquad \text{EQ. (B.102)}$$

Once again, the integrations over the column length was as if it were in a calm sea. The force perturbations on the columns will be corrected for the instantaneous wetted length by the exposure factor $S(t)$ in the equations of motion.

Forces Due to Change in Momentum

The time-varying wetted length of a column eith accompanying motion produces a rate of change in momentum.

$$F = \frac{d}{dt}(mV) \,,$$

or

$$F = V\,\frac{dM}{dt} + m\,\frac{dV}{dt} \,,$$

but, the second term is the linear acceleration which has been accounted for already. Therefore, the component forces are

$$F_{cxm} = -\bar{V}_{cx} \frac{dm}{dt} , \qquad \text{EQ. (B.103)}$$

$$F_{cym} = -\bar{V}_{hy} \frac{dm}{dt} , \qquad \text{EQ. (B.104)}$$

$$F_{czm} = -\bar{V}_{cz} \frac{dm}{dt} . \qquad \text{EQ. (B.105)}$$

The instantaneous change in wetted length of a column has been defined by Equation (B.43). Differentiating $\triangle W_L$ with respect to time and multiplying by the column cross-section will produce the mass derivative. So, the additional forces exerted due to a rate of change in momentum are

$$F_{cxm} = -\bar{V}_{cx} \frac{d}{dt}(\triangle W_L) , \qquad \text{EQ. (B.106)}$$

$$F_{cym} = -\bar{V}_{hy} \frac{d}{dt}(\triangle W_L) , \qquad \text{EQ. (B.107)}$$

$$F_{czm} = -\bar{V}_{cz} \frac{d}{dt}(\triangle W_L) , \qquad \text{EQ. (B.108)}$$

$$\frac{d}{dt}(\triangle W_L) = \frac{d}{dt}[\eta_y(x_c, b, z_c) - Y - \triangle y_h]$$

$$= - \frac{H}{2} \sin B_1(t)$$

$$- \frac{\pi H^2}{4\lambda} \frac{\cosh kh}{\sinh^3 kh}(\cosh 2kh + 2)\sin B_{21}(t)$$

$$- Y + \cos\theta\theta z_c + (\cos\psi\cos\theta\psi - \sin\psi \sin\theta\theta)x_c$$

$$- (\sin\psi \cos\theta\psi + \cos\psi\sin\theta\theta)y_h \quad . \qquad \text{EQ. (B.109)}$$

The perturbation of the hydrostatic bouyancy force due to simultaneous change in bou ancy with a change in the column wetted length is

$$F_{ch} = \rho g \frac{\pi d_c^2}{4} \Delta W_L \, . \qquad \text{EQ. (B.110)}$$

Preliminary Summary, Equations of Motion

The theory and derivation of forces exerted thus far have been concerned only with the forces that arise due to the vessels motion when subjected to a uniform wave train. Wind forces and the restraints of a mooring system are yet to be derived. At this time, the force terms (just derived) are to be grouped and substituted into the equations of motion, (B.23 thru B.28).

Surge Equation

$$M\ddot{X} = \sum (F_{cxj} + F_{hxj}) \qquad \text{EQ. (B.111)}$$

Dropping the "j" subscript for brevity the hull and column forces derived are superimposed to give a total force perturbation which is summarized as

$$\begin{aligned} F_{cx} = \; & -S(t)[\Delta M_{cx}V_{cx} + D_{cx}\bar{V}_{cx}|\bar{V}_{cx}| - D9\sin B1(t) \\ & -D29\sin B21(t) + D39\sin B41(t) + D49\cos B1(t)\sin B1(t) \\ & +D59\sin B1(t)] \, \delta_{jm} \quad , \qquad \text{EQ. (B.112)} \end{aligned}$$

$$\begin{aligned} F_{hx} = - \; & \Delta M_{hx}V_{hx} + D_{hx}\bar{V}_{hx}|\bar{V}_{hx}| + D7(\cos B4(t) - \cos B5(t)) \\ & +D27(\cos B24(t) - \cos B25(t)) - D37(\cos B44(t) - \cos B45(t)) \end{aligned}$$

$$-D47(\cos^3 B4(t)-\cos^3 B5(t)) - D57(\cos B4(t)-\cos B5(t))\Big\}\delta_{jn} ,$$

EQ. (B.113)

where the new parameters (D — —'s) are equated to the constants involved in the integrated pressure gradients. These terms are defined as:

$$D9 = \frac{\pi d_c^2}{2}\cos\alpha(A_1+A_5)[\sinh kh-\sinh k(h+y_h+\tfrac{1}{2}d_h-b)] ,$$

EQ. (B.114)

$$D29 = \frac{\pi d_c^2}{2}\cos\alpha A_2[\sinh 2kh-\sinh 2k(h+y_h+\tfrac{1}{2}d_h-b)]$$

$$-\pi d_c^2\cos\alpha A_3 k(b-y_h-\tfrac{1}{2}d_h) ,$$ EQ. (B.115)

$$D39 = 2\pi d_c^2\cos\alpha A_4 k(b-y_h-\tfrac{1}{2}d_h) ,$$ EQ. (B.116)

$$D49 = 3\pi d_c^2\cos\alpha A_5[\sinh kh-\sinh k(h+y_h+\tfrac{1}{2}d_h-b)] ,$$ EQ. (B.117)

$$D59 = \pi d_c^2\cos\alpha A_5[\sinh^3 kh-\sinh^3 k(h+y_h+\tfrac{1}{2}d_h-b)] ,$$ EQ. (B.118)

$$D7 = \frac{\pi d_h^2}{2}\cot\alpha(A_1+A_5)\cosh k(h+y_h-b) ,$$ EQ. (B.119)

$$D27 = \pi d_h^2\cot\alpha[A_2\cosh 2k(h+y_h-b) - A_3] ,$$ EQ. (B.120)

$$D37 = \pi d_h^2 A_4\cot\alpha ,$$ EQ. (B.121)

$$D47 = \frac{\pi d_h^2}{2} A_5\cot\alpha\cosh k(h+y_h-b) ,$$ EQ. (B.122)

$$D57 = \frac{\pi d_h^2}{2} A_5\cot\alpha\sinh^2 k(h+y_h-b)\cosh k(h+y_h-b) .$$ EQ. (B.123)

<u>Heave Equation</u>

$$M\ddot{Y} = \Sigma F_{yj} ,$$ EQ. (B.124)

where:

$$F_y = -\Delta M_{hy}\dot{\bar{V}}_{hy} - D_{hy}\bar{V}_{hy}|\bar{V}_{hy}| - [D1(\sin B2(t) - \sin B3(t))$$

$$+D21(\sin B22(t) - \sin B23(t)) - D31$$

$$+D41(\sin^3 B2(t) - \sin^3 B3(t))$$

$$-D51(\sin B2(t) - \sin B3(t))$$

$$+D2\cos B1(t) + D22\cos B22(t) - D32$$

$$+D42\sin^2 B1(t)\cos B1 - D52\cos B1(t)]\,\delta_{jm}$$

$$-[D3(\sin B4(t) - \sin B5(t) + D23(\sin B24(t) - \sin B25(t))$$

$$-D33 + D43(\sin^3 B4(t) - \sin^3 B5(t)$$

$$-D53(\sin B4(t) - \sin B5(t))]\,\delta_{jn}$$

$$-\rho\frac{\pi d_e^2}{4}\bar{V}_{hy}\frac{d}{dt}\Delta W_L$$

$$+\frac{\pi\rho d_h^2}{4}\begin{cases} -\bar{V}_{hy}\bar{V}_{cx}, & j = 1,2,3,6 \\ 0, & j \neq 1,2,3,6,11,14,15,16, \\ +\bar{V}_{hy}\bar{V}_{cx}, & j = 11,14,15,16 \end{cases} \quad \text{EQ. (B.125)}$$

$$D1 = \frac{\pi d_h^2}{2}(A_1 - A_5)\sinh k(h+y_h-b)/\cos\alpha\,, \quad \text{EQ. (B.126)}$$

$$D21 = \frac{\pi d_h^2}{2}A_2\sinh 2k(h+y_h-b)/\cos\alpha\,, \quad \text{EQ. (B.127)}$$

$$D31 = \pi d_h^2 A_3 L_h \sinh 2k(h+y_h-b)/\cos\alpha$$

$$+2\pi d_h^2 A_4 L_h \sinh 4k(h+y_h-b)/\cos\alpha\,, \quad \text{EQ. (B.128)}$$

$$D41 = \frac{1}{3}\pi d_h^2 A_5 \sinh k(h+y_h-b)/\cos\alpha\,, \quad \text{EQ. (B.129)}$$

$$D51 = 2\pi d_h^2 A_5 \cosh^2 k(h+y_h-b)\sinh k(h+y_h-b)/\cos\alpha\,, \quad \text{EQ. (B.130)}$$

$$D2 = \frac{\pi d_c^2}{2}(A_1 - A_5)[\cosh kh - \cosh k(h+y_h+\tfrac{1}{2}d_h-b)]\,, \quad \text{EQ. (B.131)}$$

$$D22 = \frac{\pi d_c^2}{4} A_2[\cosh 2kh - \cosh 2k(h+y_h+\tfrac{1}{2}d_h-b)] , \qquad \text{EQ. (B.132)}$$

$$D32 = \frac{\pi d_c^2}{4} A_3[\cosh 2kh - \cosh 2k(h+y_h+\tfrac{1}{2}d_h-b)]$$

$$+ \frac{\pi d_c^2}{4} A_4[\cosh 4kh - \cosh 4k(h+y_h+\tfrac{1}{2}d_h-b)] , \qquad \text{EQ. (B.133)}$$

$$D42 = \frac{\pi d_c^2}{2} A_5[\cosh kh - \cosh k(h+y_h+\tfrac{1}{2}d_h-b)] , \qquad \text{EQ. (B.134)}$$

$$D52 = \frac{\pi d_c^2}{2} A_5[\cosh^3 kh - \cosh^3 k(h+y_h+\tfrac{1}{2}d_h-b)] ; \qquad \text{EQ. (B.135)}$$

$$D3 = D1\cot\alpha , \qquad \text{EQ. (B.136)}$$

$$D23 = D21\cot\alpha ; \qquad \text{EQ. (B.137)}$$

$$D33 = D31\cot\alpha ; \qquad \text{EQ. (B.138)}$$

$$D43 = D41\cot\alpha , \qquad \text{EQ. (B.139)}$$

$$D53 = D51\cot\alpha . \qquad \text{EQ. (B.140)}$$

Sway Equation

$$M\ddot{Z} = \Sigma (F_{ezj} + F_{hzj}) , \qquad \text{EQ. (B.141)}$$

where:

$$F_{cz} = \Big\{ -S(t)[\Delta M_{cz}\dot{\bar{V}}_{cz} + D_{cz}\bar{V}_{cz}|\bar{V}_{cz}| - D4\sin B1(t)$$

$$-D4\sin B21(t) + D34\sin B41(t) + D44\cos^2 B1(t)\sin B1(t)$$

$$+D54\sin B1(t)\Big] + \rho\frac{\pi d_c^2}{4}\bar{V}_{cz}\frac{d}{dt}\Delta W_L\Big\}\,\delta_{jm} , \qquad \text{EQ. (B.142)}$$

$$F_{hz} = -\Big\{\Delta M_{hz}\dot{\bar{V}}_{hz} + D_{hz}\dot{\bar{V}}_{hz}|\dot{\bar{V}}_{hz}| - D5(\cos B2(t) - \cos B3(t))$$

$$-D25(\cos B22(t) - \cos B23(t)) + D35(\cos B42(t) - \cos B43(t))$$

$$+D45(\cos^3 B2(t) - \cos^3 B3(t)) + D55(\cos B2(t) - \cos B3(t))\Big\}\,\delta_{jm}$$

$$+\frac{\pi\rho d_h^2}{4}\begin{cases}-\bar{V}_{hz}\bar{V}_{cx} & , j = 1,2,3,6 \\ 0 & , j \neq 1,2,3,6,11,14,15,16 \; , \\ +\bar{V}_{hz}\bar{V}_{cx} & , j = 11,14,15,16\end{cases}$$

EQ. (B.143)

$D4 = D9\tan\alpha$, EQ. (B.144)

$D24 = D29\tan\alpha$, EQ. (B.145)

$D34 = D39\tan\alpha$, EQ. (B.146)

$D44 = D49\tan\alpha$, EQ. (B.147)

$D54 = D59\tan\alpha$; EQ. (B.148)

$D5 = D7\tan^2\alpha$; EQ. (B.149)

$D25 = D27\tan^2\alpha$; EQ. (B.150)

$D35 = D37\tan^2\alpha$; EQ. (B.151)

$D45 = D47\tan^2\alpha$, EQ. (B.152)

$D55 = D57\tan^2\alpha$; EQ. (B.153)

Roll Moment

Consider the hull-column segment with a positive roll angle θ as shown in Figure B.9. The moment around the C.G. in roll due to the applied forces is readily seen to be

$$M(\theta) = -F_{yj}(y_{hj}\sin\theta + z_{cj}\cos\theta) + F_{hzy}(y_{hj}\cos\theta - z_{cj}\sin\theta)$$

$$+ F_{czj}(\lambda_j\cos\theta - z_{cj}\sin\theta) \; , \qquad \text{EQ. (B.154)}$$

where λ_j designates the distance from the z-axis to the mid-way point of the column's wetted length. This term may be represented by

$$\lambda_j = \tfrac{1}{2}[y_h + \tfrac{1}{2}d_h + b + \eta_y(x_c, b, z_c) - Y + z_c\sin\theta - x_c\sin\psi\cos\theta$$
$$+ y_h(1 - \cos\theta\cos\psi)] \quad . \qquad \text{EQ. (B.155)}$$

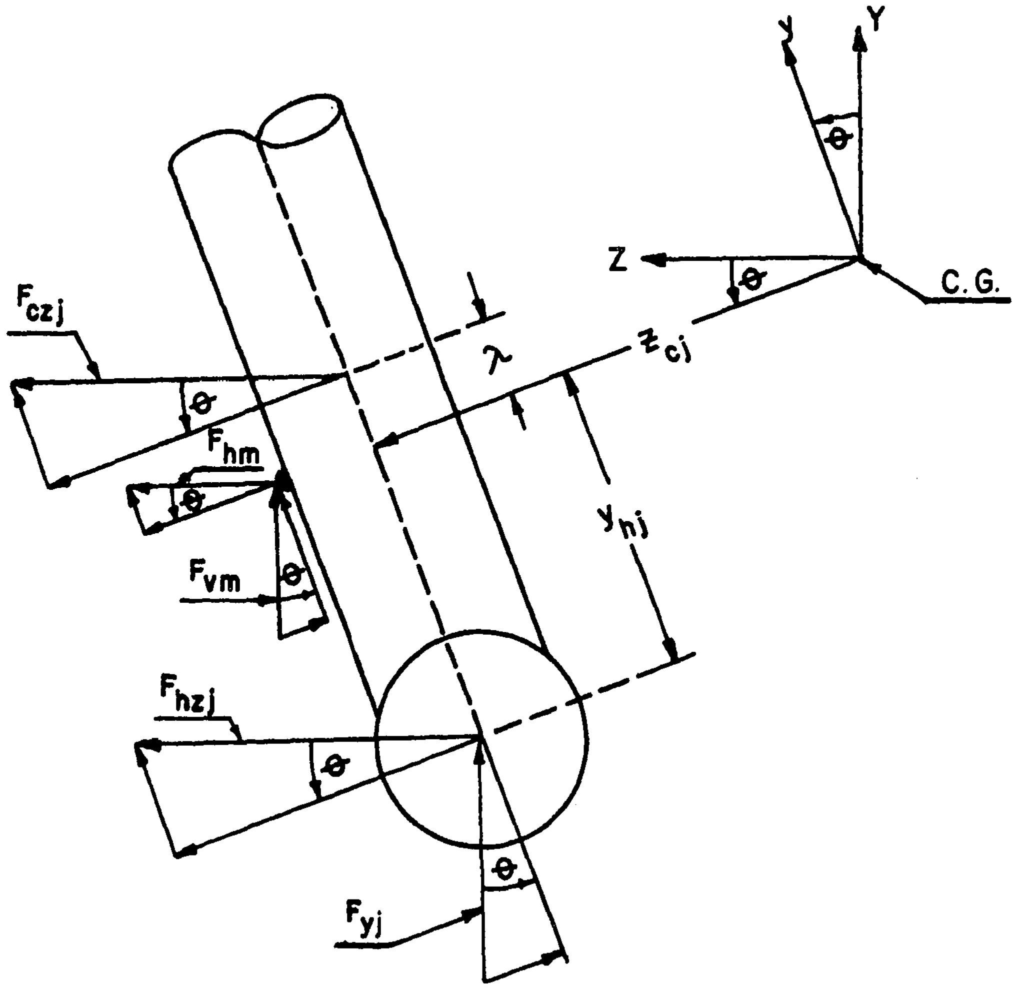


Figure B.9¥ Hull-column in large roll.

The total moment given by Equation (B.125) is to be corrected. The total buoyant force is equal and opposite in direction of the weight of the vessel acting through the center of gravity. In roll and pitch the total buoyant moves to a new line of action where it exerts a restoring moment on the vessel for positive stability as shown in Figure B.10.

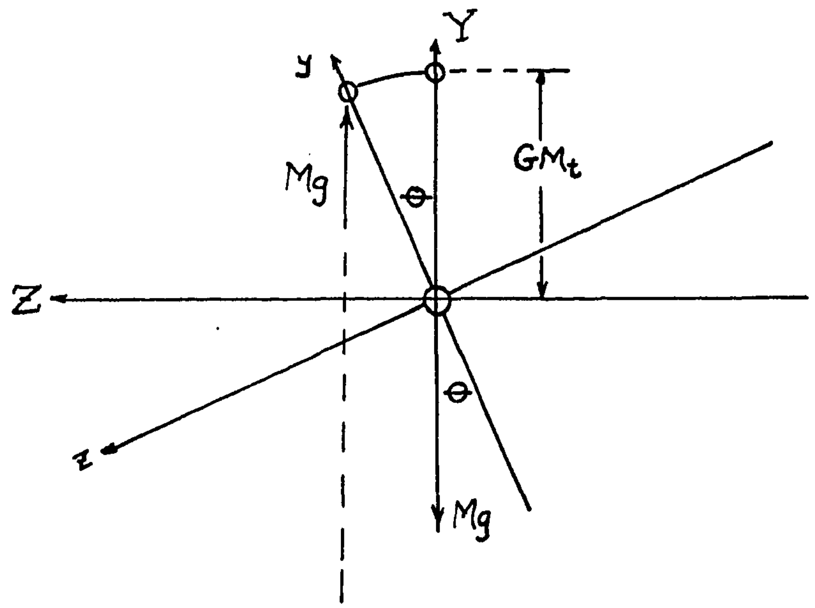


Figure B.10: Restoring moment and metacentric height.

The couple formed by the buoyant force in roll is

$$M_t(\theta) = -MgGM_t\sin\theta \;, \qquad \text{EQ. (B.156)}$$

where GM_t is the transverse metacentric height. For small angles GM_t is independent of θ. This numerical distance will be treated as a constant even though the remainder of the analysis includes the affects of large angle rotation. Thus far, the total moment on the vessel around the x-axis is found to be

$$M(\theta) = \sum_{j=1}^{24} \Big[-F_{yj}(y_{hj}\sin\theta + z_{cj}\cos\theta) + F_{hzj}(y_{hj}\cos\theta - z_{cj}\sin\theta)$$

$$+ F_{czj}(\lambda_j\cos\theta - z_{cj}\sin\theta)] - MgGM_t\sin\theta \quad .$$

EQ. (B.157)

Pitch Moment

A typical hull-column segment in large pitch is shown in Figure B.11. The moment due to the applied forces in pitch is easily shown to be

$$M(\psi) = F_{yj}(x_{cj}\cos\psi - y_{hj}\sin\psi) - F_{hxj}(x_{cj}\sin\psi + y_{hj}\cos\psi)$$
$$+ F_{cxj}(\lambda_j\cos\psi - x_{cj}\sin\psi) \quad . \qquad \text{EQ. (B.158)}$$

The concepts of a couple being formed due to a new line of action for the total buoyant force are also valid for pitching motion. If the angle in Figure B.10 is changed to and GM_t to GM_L (longitudinal metacentric height) the corresponding restoring moment in pitch is found to be

$$M_L(\psi) = - MgGM_L\sin\psi \qquad \text{EQ. (B.159)}$$

With this addition, the total moment developed thus far is determined as

$$M(\psi) = \sum_{j=1}^{24}[F_{yj}(x_{cj}\cos\psi - y_{hj}\sin\psi) - F_{hxj}(x_{cj}\sin\psi + y_{hj}\cos\psi)$$
$$- F_{cxj}(\lambda_j\cos\psi + x_{cj}\sin\psi)] - MgGM_L\sin\psi.$$

EQ. (B.160)

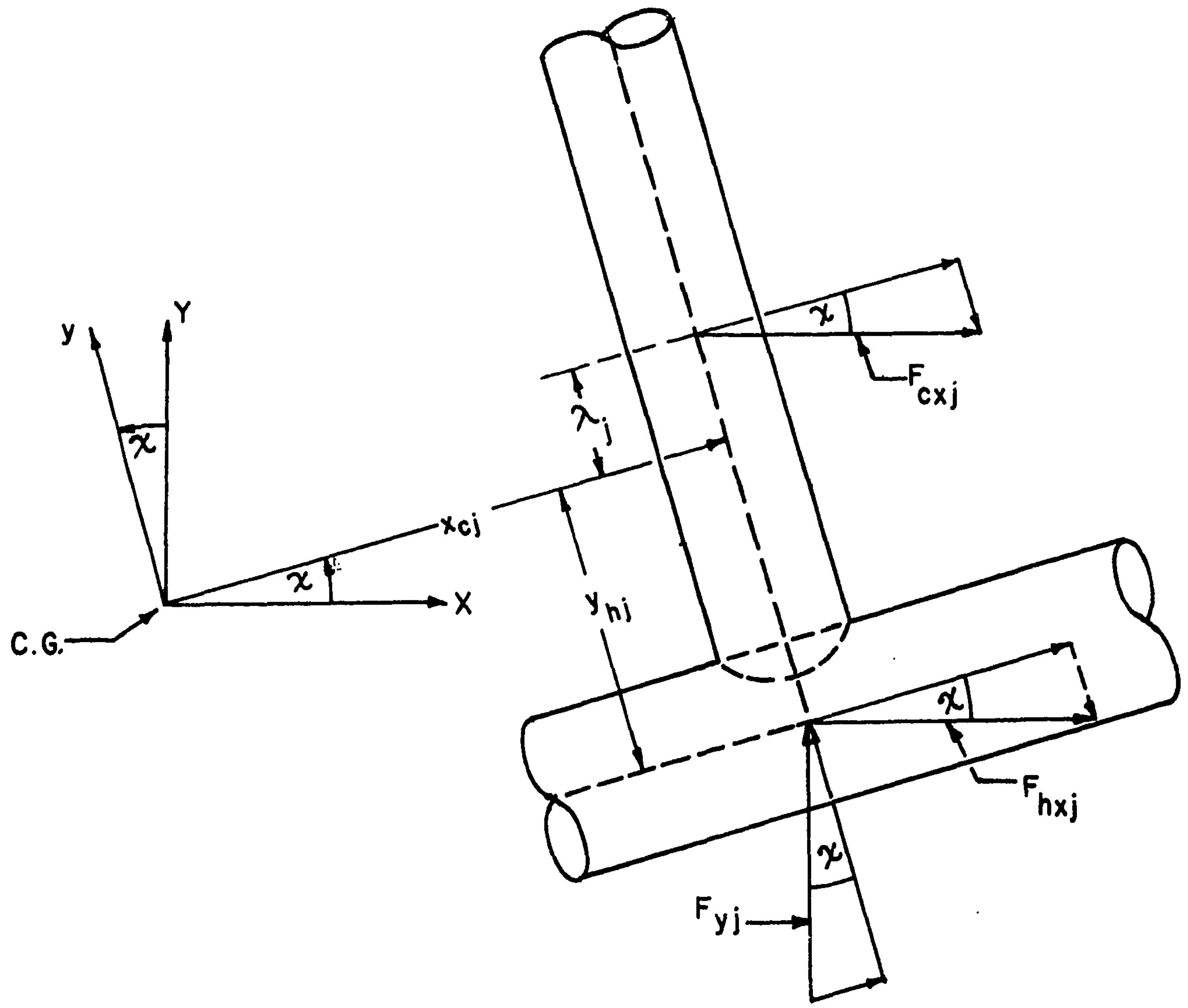


Figure B. 11: Typical hull - column with large pitch angle.

Yaw Moment

In determining the moment in yaw, terms for the total force (sum of separate forces on hull and column) in the x and z directions may be used rather than hull and column forces. Consider Figure B.12 which depicts the forces exerted on a typical hull-column with a large yaw angle.

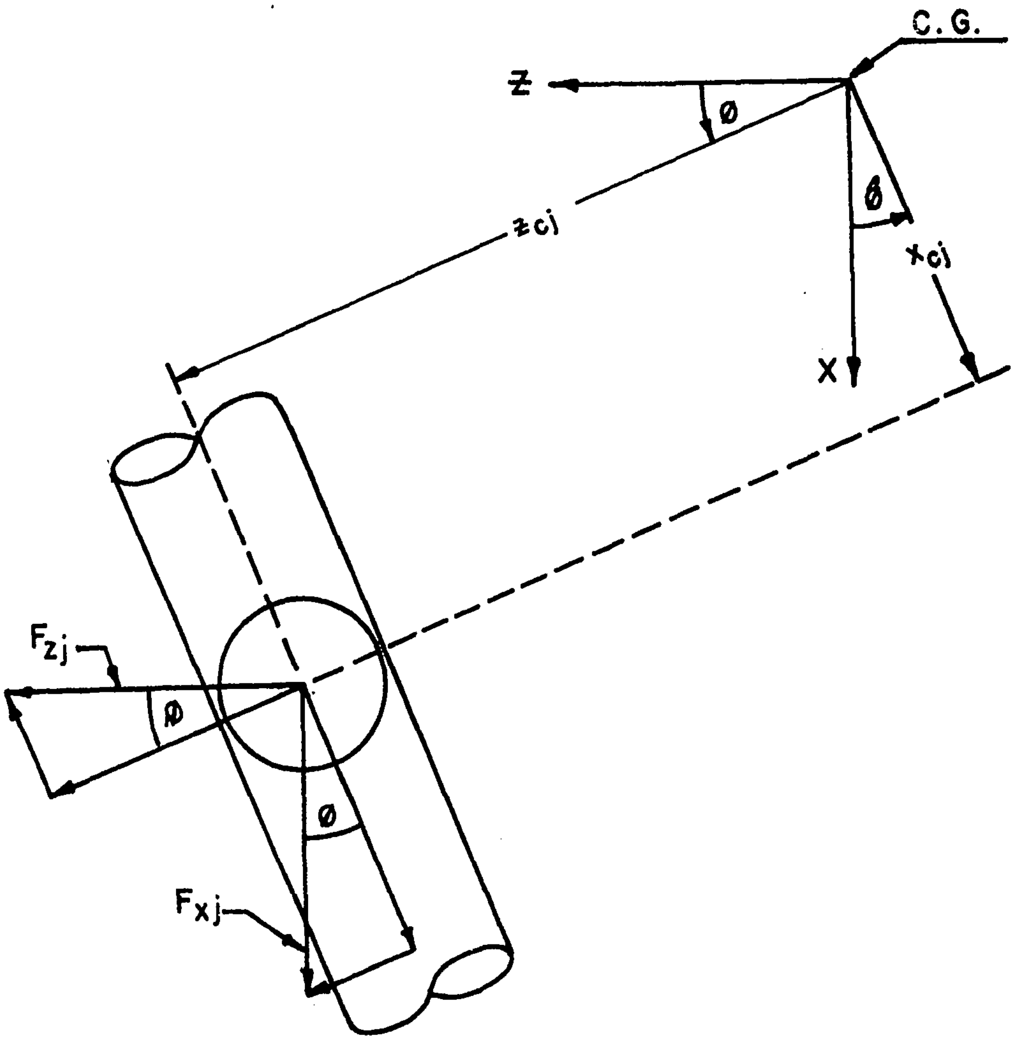


Figure B.12: Typical hull-column in large yaw.

The moment around the vertical axis is written as

$$M(\phi) = F_{xj}(z_{cj}\cos\phi - x_{cj}\sin\phi) - F_{zj}(x_{cj}\cos\phi + z_{cj}\sin\phi) \;.$$

EQ. (B.161)

The moments defined by Equations (B.129), (B.132), and (B.133) are substituted into Equations (B.29), (B.30), and (B.31) to define the equations of motion in roll, pitch and yaw. These equations are as follows:

$$I_{xx}\ddot{\sigma}_x - I_{xy}\ddot{\sigma}_y - I_{xz}\ddot{\sigma}_z - \dot{\sigma}_z(I_{yy}\dot{\sigma}_y - I_{xy}\dot{\sigma}_x - I_{yz}\dot{\sigma}_z)$$

$$+ \dot{\sigma}_y(I_{zz}\dot{\sigma}_z - I_{xz}\dot{\sigma}_x - I_{yz}\dot{\sigma}_y)$$

$$= \sum_{j=1}^{24}[-F_{yj}(y_{hj}\sin\theta + z_{cj}\cos\theta) + F_{hzj}(z_{cj}\sin\theta - y_{hj}\cos\theta)$$

$$+ F_{czj}(\lambda_j\cos\theta - z_{cj}\sin\theta)] - MgGM_t\sin\theta \ , \qquad \text{EQ. (B.162)}$$

$$I_{yy}\ddot{\sigma}_y - I_{xy}\ddot{\sigma}_x - I_{yz}\ddot{\sigma}_z - \dot{\sigma}_x(I_{zz}\dot{\sigma}_z - I_{xz}\dot{\sigma}_x - I_{yz}\dot{\sigma}_y)$$

$$+ \dot{\sigma}_z(I_{xx}\dot{\sigma}_x - I_{xy}\dot{\sigma}_y - I_{xz}\dot{\sigma}_z),$$

$$= \sum_{j=1}^{24}[F_{x_j}(z_{c_j}\cos\phi - x_{c_j}\sin\phi) - F_{z_j}(x_{c_j}\cos\phi + z_{c_j}\sin\phi)] \ , \qquad \text{EQ. (B.163)}$$

$$I_{zz}\ddot{\sigma}_z - I_{xz}\ddot{\sigma}_x - I_{yz}\ddot{\sigma}_y - \dot{\sigma}_y(I_{xx}\dot{\sigma}_x - I_{xy}\dot{\sigma}_y - I_{xz}\dot{\sigma}_z)$$

$$+ \dot{\sigma}_x(I_{yy}\dot{\sigma}_y - I_{xy}\dot{\sigma}_x - I_{yz}\dot{\sigma}_z)$$

$$= \sum_{j=1}^{24}[F_{yj}(x_{cj}\cos\psi - y_{hj}\sin\psi) - F_{hxj}(x_{cj}\sin\psi + y_{hj}\cos\psi)$$

$$- F_{cx}(\lambda_j\cos\psi + x_{cj}\sin\psi)] - MgGM_L\sin\psi \ , \qquad \text{EQ. (B.164)}$$

where

$$F_{xj} = F_{cxj} + F_{hxj} \ , \qquad \text{EQ. (B.165)}$$

$$F_{zj} = F_{czj} + F_{hzj} \ . \qquad \text{EQ. (B.166)}$$

The equations of motion for the six degree of freedom rigid body system accounts for the interaction of the vessel and sea with the excitation of second-order waves. These equations do not yet include the restraining forces of a mooring system or applied wind loads. These effects are to be derived in Appendices C and D.

APPENDIX C

MOORING SYSTEM

Vessels of this type are moored by a spread mooring system generally containing at least eight mooring lines and anchors. The mooring lines in this analysis are made of very heavy chain which may be idealized as a uniform heavy string whose shape may be described by the catenary equations.

Derivation of Catenary Equations

Consider the curve formed by an ideal perfectly flexible uniform string hanging freely between two points as shown in Figure C.1.

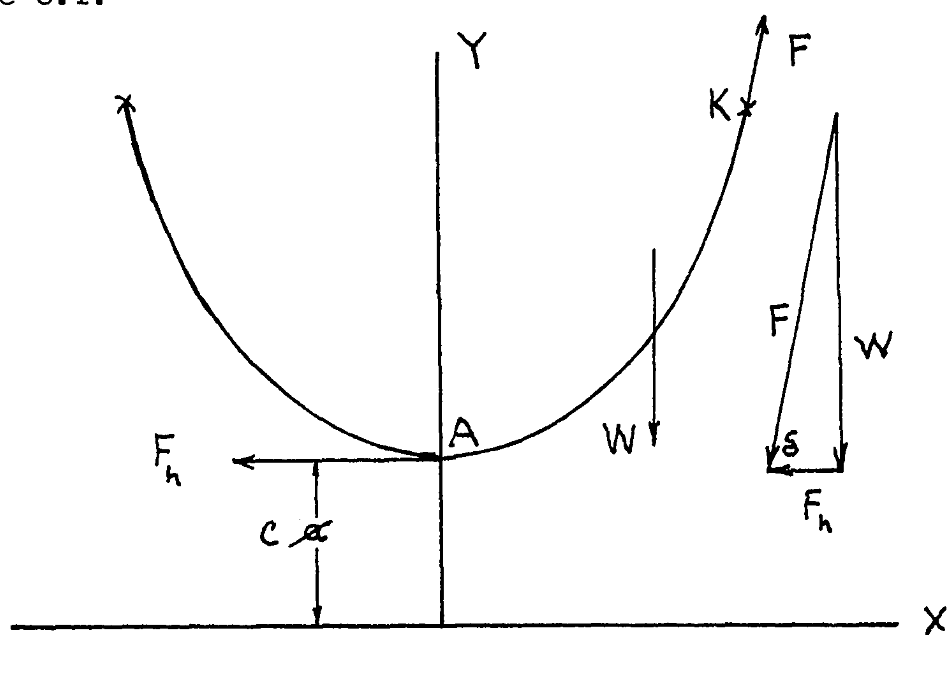


Figure C.1: A ideal string hanging freely between two points.

The objective at this point is to derive the differential equation which describes the system shown in Figure C.1, solve and obtain a relation for the tension F in the mooring line.

The portion AK is in equilibrium under a horizontal tension F at A, the tension F directed along the tangent at K, and the weight W of the arc AK. It is easy to see that

$$W = W_c S, \qquad \text{EQ. (C.1)}$$

$$dW = W_c ds , \qquad \text{EQ. (C.2)}$$

and by dividing by dx

$$\frac{dW}{dx} = W_c \frac{ds}{dx} . \qquad \text{EQ. (C.3)}$$

In general, dx/dx is defined by the arc length equation as

$$\frac{ds}{dx} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} , \qquad \text{EQ. (C.4)}$$

which may be substituted into Equation (C.3) to give

$$\frac{dW}{dx} = W_c\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} . \qquad \text{EQ. (C.5)}$$

From the force triangle in Figure C.1, one should note that

$$\tan\delta = \frac{dy}{dx} = \frac{W}{F_h} ,$$

which upon differentiation leads to

$$\frac{d^2y}{dx^2} = \frac{1}{F_h}\frac{dW}{dx} . \qquad \text{EQ. (C.6)}$$

Substituting Equation (C.5) into (C.6) gives the differential equation of the uniform hanging string, otherwise known as the catenary. This result is

$$\frac{d^2y}{dx^2} = \frac{W_c}{F_h}\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} , \qquad \text{EQ. (C.7)}$$

with the boundary conditions

$$y = c \quad \text{at} \quad x = 0 , \qquad \text{EQ. (C.8)}$$

$$\frac{dy}{dx} = 0 \quad \text{at} \quad x = 0 . \qquad \text{EQ. (C.9)}$$

This differential equation is non-linear; but fortunately, it is quite easily solved by reducing the order when letting

$$P = \frac{dy}{dx} , \qquad \text{EQ. (C.10)}$$

and separating variables. The result is

$$\int \frac{dP}{(1+P^2)^{\frac{1}{2}}} = \frac{W_c}{F_h}\int dx . \qquad \text{EQ. (C.11)}$$

Integration and some manipulation leads to

$$P + (1 + P^2)^{\frac{1}{2}} = C \exp\left(\frac{W_c}{F_h}\right) , \qquad \text{EQ. (C.12)}$$

where C is a constant of integration. Applying Equation (C.9) which dictates that the slope (P) is zero at $x = 0$ gives $C = 1$, which leaves

$$P + (1 + P^2)^{\frac{1}{2}} = \exp\left(\frac{W_c}{F_h}\right) , \qquad \text{EQ. (C.13)}$$

a first order non-linear differential equation. Observing that

$$(1 + P^2)^2 - P^2 = 1 ,$$

and factoring

$$(1 + P^2 + P)(1 + P^2 - P) = 1 ,$$

Equation (C13) ma-- be changed to

$$1 + P^2 - P = \exp(-\frac{W_c}{F_h}) \quad . \qquad \text{EQ. (C.14)}$$

Subtracting Equation (C.14) from (C.13) eliminates the radical and leaves the result in a form conducive to straight forward integration

$$\frac{dy}{dx} = \tfrac{1}{2} \exp\left(\frac{W_c}{F_h}\right) - \exp\left(\frac{W_c}{F_h}\right) \quad . \qquad \text{EQ. (C.15)}$$

Integration gives

$$y = \tfrac{1}{2} \frac{F_h}{W_c} \exp\left(\frac{W_c}{F_h}\right) + \exp\left(-\frac{W_c}{F_h}\right) + C_2 \; . \qquad \text{EQ. (C.16)}$$

Applying Equation (C.8), the remaining boundary condition is

$$C_2 = c - \frac{F_h}{W_c} \quad . \qquad \text{EQ. (C.17)}$$

Now by suitably shifting the x-axis C_2 can be set to zero to give

$$c = \frac{F_h}{W_c} \quad , \qquad \text{EQ. (C.18)}$$

which is called the parameter of the catenary.

The final solution is

$$y = \frac{c}{2}\left(\exp\left(\frac{x}{c}\right) + \exp\left(-\frac{x}{c}\right)\right) = c\cosh\left(\frac{x}{c}\right) \; , \qquad \text{EQ. (C.19)}$$

which is the familar catenary equation.

Integrating Equation (C.4) leads to the companion equation for arc length

$$s = c\sinh\left(\frac{x}{c}\right) \; . \qquad \text{EQ. (C.20)}$$

The tension F from the force triangle in Figure C.1 is easily observed to be

$$F = F_h \sec \delta \quad ,$$

but

$F_h = Wc$,

so

$F = W_c c \sec \delta$. EQ. (C.21)

It was shown previously that

$\tan \delta = \frac{W_c s}{F_h} = \frac{s}{c}$,

so the arc length may be written in the form

$s = c\tan \delta$. EQ. (C.22)

Squaring and subtracting Equations (C.19) and (C.20) leads to

$y^2 = c^2 + s^2$, EQ. (C.23)

a result which is needed now and will be quite useful later. Substituting Equation (C.22) into (C.23) and using a trigonometric identity gives

$y = c \sec \delta$, EQ. (C.24)

and making use of this relation in Equation (C.21) indicates that the tension in the catenary is given by

$F = cy$. EQ. (C.25)

These equations are transcendental in nature are very difficult to use in obtaining an explicit equation for tension at the vessel. For example consider a typical mooring line with part laying flat on the seabed and the remainder forming the catenary just described in theory. The water depth and total length of chain are known.

This system may be solved for F at the vessel (which is of supreme interest) if the horizontal distance b or the arc length are known. But if the vessel is displaced in either positive or negative directions a solution for F must be found graphically or by trial and error. The difficulty lies in the varying arc length. In essence, a very heavy chain as shown is a non-linear spring that can be classified as either "hard or soft" depending entirely on how far the catenary is stretched out.

x = vessel displacement

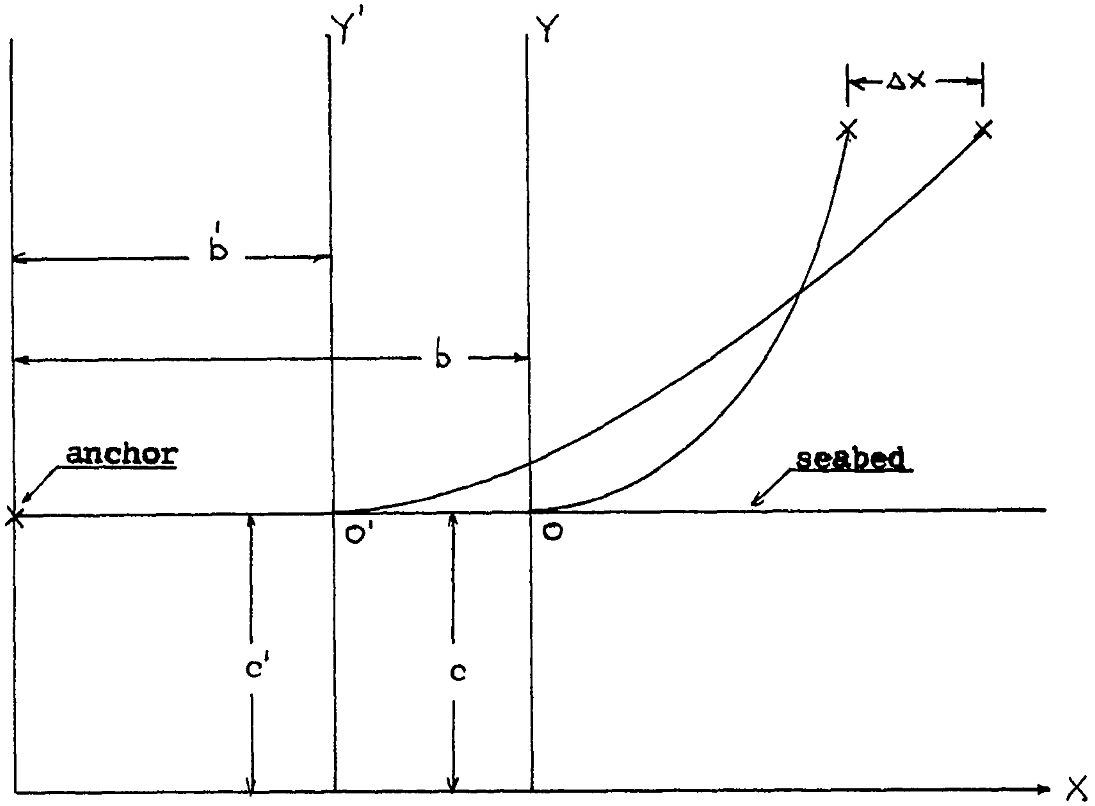


Figure C.2: Typical mooring line in static equilibrium.

The forces that arise in the mooring system due to the vessel's motion must be included in the equations of motion derived in Appendix B. It is very desirable that this force be an explicit term that can be evaluated easily rather than in trancendental form. This can be done if two simplifications are made:

1) Assume that vertical motion of the vessel produces a relative small change in the tension F and may be neglected.
2) The inertia effects in addition to fluid drag of the chain are negligible.

The general approach to solving this problem will be to assume values for arc length and calculating the corresponding tension at the vessel as data points on a curve F vs X_D the distance from the anchor point which is stationary. These data points will be fitted by "least squares" to a finite series of Chebyshev polynominals, which will result in an explicit equation for the tension F as a function of vessel displacement in the form

$$F(x) = \sum_{n=1}^{m} B_n F_{n-1}(x) \text{ ,} \qquad \text{EQ. (C.26)}$$

where the polynominal is calculated in the form of its Chebyshev expansion

$$\sum_{n=1}^{m} B_n P_{n-1}(x) = B_1 P_0(x) + B_2 P_1(x) + \ldots + B_m P_{m-1}(x) \text{ .} \qquad \text{EQ. (C.27)}$$

Calculating Data Points

The total length of chain has been defined as S_T. This would be the upper limit of s if the vessel was displaced to a point where the total length of chain would form the catenary; i.e., no chain would be laying flat on the seabed. The lower limit of s may be said to correspond to the arc length when the chain is completely slack. This length would equal to the vertical distance from the fairlead to the seabed, W_D.

At the fairlead, one should note that

$$y = c + W_D . \qquad \text{EQ. (C.28)}$$

Substituting this relation into Equation (C.23) and simplifying leads to

$$c = \frac{1}{2W_D}(W_D^2 + s^2) , \qquad \text{EQ. (C.29)}$$

which may be used to calculate the value of the parameter a for assumed values of s. Setting Equation (C.28) equal to the catenary, Equation (C.19)

$$c + W_D = c\cosh(\tfrac{x}{c}) ,$$

or

$$\frac{c + W_D}{c} = \cosh(\tfrac{x}{c}) , \qquad \text{EQ. (C.30)}$$

the variable x is solved for by taking the inverse

$$x = c\cosh^{-1}(\tfrac{c+W_D}{c}) , \qquad \text{EQ. (C.31)}$$

where

$$\cosh^{-1}\xi = \ln 2\xi - \frac{1}{2\cdot 2\xi^2} - \frac{1\cdot 3}{2\cdot 4\cdot 4\xi^4} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6\xi^6} - \cdots \qquad \text{EQ. (C.32)}$$

The x in the above relations should not be confused with the x in the equations of motion; x as shown in Figure C.2 is the horizontal distance from the point where $dy/dx = 0$ to the fairlead.

The amount of chain laying on the seabed may now be determined by

$$b_a = S_T - c\sinh\left(\frac{x}{c}\right). \qquad \text{EQ. (C.33)}$$

Similarly the horizontal distance from the anchor point to the fairlead is

$$X_D = x + b_a , \qquad \text{EQ. (C.34)}$$

and the tension as defined by Equation (C.25). This is the calculation required for one data point on the curve $F vs X_D$. Any number of data points may be calculated between the limits of arc length; the more the better insofar as s is reasonably equidistant. In this study 417 data points were calculated for a chain whose total length is set at 5000 feet.

Least Squares Fit of Chebyshev Polynominals

The problem in any "least squares" technique is to determine the coefficients of the series expansion such that

$$\sum_{j=1}^{n} [f(x_i) - p(x_i)]^2 = \text{minimum.}$$

This problem leads to a system of linear equations

$$[A]\{B\} = \{R\} \qquad \text{EQ. (C.35)}$$

where B is the vector of unknown coefficients.

Let X_L and X_D denote the lower and upper limits of X_D.

By means of the linear transformation

$$t(X_D) = \frac{2X_D - (X_L + X_u)}{X_u - X_L} , \qquad \text{EQ. (C.36)}$$

the argument range $X_L \leq X_D \leq X_u$ is reduced to $-1 \leq X_D \leq 1$. The polynominal is then calculated by Equation (C.27), the Chebyshev expansion.

The vector B of the unknown coefficients is a solution of the matrix Equation (C.35), where A is an m by m positive definite matrix whose elements may be determined by

$$A_{jk} = \sum_{i=1}^{n} T_{j-1}[t(x_i)]\ T_{k-1}[t(x_i)] . \qquad \text{EQ. (C.37)}$$

The elements of the vector R are calculated by

$$r_j = \sum_{i=1}^{n} T_{j-1}[t(x_i)]\ f\ (x_i) . \qquad \text{EQ. (C.38)}$$

The Chebyshev polynominals of degree k are determined by the recurrance relation

$$T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x) , \ k \geq 2 \qquad \text{EQ. (C.39)}$$

where

$$T_0(x) = 1 \qquad \text{EQ. (C.40)}$$

and

$$T_1(x) = x . \qquad \text{EQ. (C.41)}$$

Numerical Results

Using a mooring system of chain whose unit weight is 91.4 lbs/ft. in seawater, total length of 5000 ft. and W_D = 1000 ft., a computer program was written to calculate the data and matrix equation for "least squares" fit for the Chebyshev expansion of tension F at the fairlead. The matrix equation is then solved by "Gauss" elimination. The results which are given below are used directly in the equations of motion which are solved by a separate computer program. The coefficients of the Chebyshev expansion are listed in Table 5a.

It was determined that eighteen terms would give a relative error of less than 0.01 % by the Chebyshev approximation as opposed to the calculated data points in the range 100,000 lbs. < F < 1,000,000 lbs.. This error is calculated by the relation

$$\text{error} = \frac{F_D - F_C}{F_D} \times 100 \quad . \qquad \text{EQ. (C.42)}$$

Additional terms will improve the accurracy significently if needed.

It must be remembered that the tension F at the fairlead formed by Equation (C.26) is directed along the tangent. What is needed for the equations of motion are the horizontal and vertical components. Therefore, a similar curve-fitting procedure is used to fit a Chebyshev expansion to the curve $\tan \delta$ vs X_D where $\tan \delta$ is the tangent of the angle relative to the horizontal at the fairlead. Much of the initial calculations may be used since

$$\tan \delta = \sinh \frac{x}{c} \quad . \qquad \text{EQ. (C.43)}$$

TABLE 5a

Coefficients for Mooring

Line Tensions

B(1) = 3.3094528	B(10) = 0.0353964
B(2) = 3.9090561	B(11) = 0.01815837
B(3) = 2.5616797	B(12) = 0.0090104
B(4) = 1.5184452	B(13) = 0.0046683
B(5) = 0.8621944	B(14) = 0.0021917
B(6) = 0.4723058	B(15) = 0.0011888
B(7) = 0.2532643	B(16) = 0.0005230
B(8) = 0.1331087	B(17) = 0.0002541
B(9) = 0.0692389	B(18) = 0.0001899

TABLE 5b

Coefficients for Mooring Line

Tangent Angles

C(1) = 0.2861405	C(10) = 0.0884618
C(2) =-0.4993399	C(11) = -0.0841822
C(3) = 0.3677220	C(12) = 0.0730596
C(4) = -0.2397937	C(13) = -0.0583060
C(5) = 0.1294944	C(14) = 0.0428919
C(6) = -0.0427505	C(15) = -0.0286052
C(7) = -0.0198204	C(16) = 0.0169394
C(8) = 0.0603033	C(17) = -0.0081828
C(9) = -0.0818071	C(18) = 0.0026316

An identical procedure of a "least squares" fit yields the coefficient vector of the tan δ at the fairlead as a function of distance from the anchor. These coefficients are listed in Table 5b. The relative error for the tangent angle is slightly less for the same range of tension used previously. Once the coefficients are known, the tangent angle is found by the inverse as

$$\delta = \arctan\left\{\sum C_n P_{n-1}(X_D)\right\} . \qquad \text{EQ. (C.44)}$$

Multiplying Equation (C.26) by $\cos\delta$ and $\sin\delta$ the respective horizontal and vertical components of force are obtained

$$F_h(X_D) = \cos\delta\left\{\sum B_n P_{n-1}(X_D)\right\} , \qquad \text{EQ. (C.45)}$$

and

$$F_v(X_D) = \sin\delta\left\{\sum C_n P_{n-1}(X_D)\right\} . \qquad \text{EQ. (C.46)}$$

This is the form that will be used and added to the equations of motion derived in Appendix B.

<u>Computer Program for Least-Squares Fit of Chebyshev Expansion</u>

The following Fortran IV computer program simultaneously calculates and solves for the two coefficient vectors B_n and and C_n just given.

COMPUTER PROGRAM

CHEBYSHEV POLYNOMINAL

APPROXIMATION FOR THE

CATENARY

```
      FUNCTION T(J,X)
      IMPLICIT REAL*8(A-H,C-Z),INTEGER*4(I-N)
      IF(J.GT.0) GO TO 1
      T=1.
      RETURN
1     IF(J.GT.1)GO TO 2
      T=X
      RETURN
2     IF(J.GT.2)GO TO 3
      T=2*X**2-1.
      RETURN
3     IF(J.GT.3)GO TO 4
      T=4*X**3-3*X
      RETURN
4     IF(J.GT.4)GO TO 5
      T=8*X**4-8*X**2+1.
      RETURN
5     IF(J.GT.5)GO TO 6
      T=16*X**5-20*X**3+5*X
```

```
      RETURN
6     IF(J.GT.6)GO TO 7
      T=32*X**6-48*X**4+18*X**2-1.
      RETURN
7     IF(J.GT.7)GO TO 8
      T=64*X**7-112*X**5+56*X**3-7*X
      RETURN
8     IF(J.GT.8)GO TO 9
      T=128*X**8-256*X**6+160*X**4-32*X**2+1.
      RETURN
9     IF(J.GT.9)GO TO 10
      T=256*X**9-576*X**7+432*X**5-12C*X**3+9*X
      RETURN
10    IF(J.GT.10)GO TO 11
      T=512*X**10-1280*X**8+1120*X**6-400*X**4+82*X**2-1.
      RETURN
11    IF(J.GT.11)GO TO 12
      T=1024*X**11-2816*X**9+2816*X**7-1232*X**5+284*X**3-11*X
      RETURN
```

```
12    IF(J.GT.12)GO TO 13
      T=2048*X**12-6144*X**10+6912*X**8-2576*X**6+968*X**4-104*X**2+1.
      RETURN
13    T=4096*X**13-13312*X**11+16640*X**9-7968*X**7+3168*X**5-492*X**3
     1+13*X
      RETURN
      END

      SUBROUTINE SIMQX(A,N,B,KS)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER*4(I-N)
      DIMENSION A(1),B(1)
      TOL=0.
      KS=1
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0.
      IT=JJ-J
```

```
      DO 30 I=J,N
      IJ=IT+I
      IF(DABS(BIGA)-DABS(A(IJ)))20,30,30
20    BIGA=A(IJ)
      IMAX=I
30    CONTINUE
      IF(DABS(BIGA)-TOL)35,35,40
35    KS=0
      RETURN
40    I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(12)
      A(I2)=SAVE
50    A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
```

```
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
      IF(J-N)55,70,55
55    IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
60    A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
65    B(IX)=B(IX)-(B(J)*A(IXJ))
70    NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
```

```
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
80    IC=IC-1
      RETURN
      END

      SUBROUTINE CSHIV(AA,YY,XX)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER*4(I-N)
      DIMENSION A(500)
      IF(AA.GT.0.)GO TO 1
      XX=0.
      RETURN
1     Z=YY/AA
      A(1)=DLOG(2*Z)
      A(2)=.25/Z/Z
      DO Z K=3,500
      KK=K-1
      KL=2*K-4
      KM=2*K-3
```

```
      KN=2*K-2
      A(K)=A(KK)*KL*KM/(KN*KN*Z*Z)
      KSAV=K
      IF(A(K).LE.1.E-70)GO TO 3
2     CONTINUE
3     AX=0.
4     AX=AX+A(KSAV)
      KSAV=KSAV-1
      IF(KSAV-2),5,1,4
5     XX=(A(1)-AX)*AA
      RETURN
      END

      IMPLICIT REAL*8(A-H,O-Z),INTEGER*4(I-N)
      DIMENSION A(417),S(417),Y(417),X(417),F(417),DIST(417)
      DIMENSION TX(417),R(20),Q(20,20),DYDX(417),RD(20),QD(20,20)
      NDIM=20
      NPT=417
      DELS=6.25
```

```
      IW=6
      WD=400.
      WC=60.
      ST=3000.
      S(1)=WD
      DYDX(1)=100.
      DDD=DYDX(1)
      DO 1 I=2,NPT
      II=I-1
1     S(I)=S(II)+DELS
      DO 2 J=1,NPT
      A(J)=.5*(S(J)*S(J)-WD*WD)/WD
      Y(J)=DSQRT(A(J)*A(J)+S(J)*S(J))
      AA=A(J)
      YY=Y(J)
      CALL CSHIV(AA,YY,XX)
      X(J)=XX
      F(J)=WC*Y(J)
      DIST(J)=ST+X(J)-S(J)
```

```
2     CONTINUE
      DO 3 J=2,NPT
      H=DSQRT(F(J)*F(J)-(WC*S(J))**2)
3     DYDX(J)=WC*S(J)/H
      FFC=F(1)
      DDC=DIST(1)
      DO 140 J=1,NPT
      DYDX(J)=DYDX(J)/DDD
      F(J)=F(J)/FFC
      DIST(J)=DIST(J)/DDC
140   CONTINUE
      DO 138 J=1,NPT
138   TX(J)=(2*DIST(J)-DIST(1)-DIST(NPT))/(DIST(NPT)-DIST(1))
      DO 139 J=1,NDIM
      RD(J)=0.
      R(J)=0.
      DO 139 K=1,NDIM
      QD(J,K)=0.
139   Q(J,K)=0.
```

```
      DO 141 J=1,NDIM
      DO 141 K=1,NDIM
      DO 141 I=1,NPT
      Q(J,K)=Q(J,K)+T(J-1,TX(I))*T(K-1,TX(I))
  141 QD(J,K)=Q(J,K)
      DO 142 J=1,NDIM
      DO 142 I=1,NPT
      RD(J)=RD(J)+T(J-1,TX(I))*DYDX(I)
  142 R(J)=R(J)+T(J-1,TX(I))*F(I)
      CALL SIMQX(Q,NDIM,R,KS)
      CALL SIMQX(QD,NDIM,RD,KS)
      DO 144 J=1,NPT
      TEST=R(1)*T(0,TX(J))+R(2)*T(1,TX(J))+R(3)*T(2,TX(J))+R(4)*T(3,TX(
     1J))+R(5)*T(4,TX(J))+R(6)*T(5,TX(J))+R(7)*T(6,TX(J))
     2+R(8)*T(7,TX(J))+R(9)*T(8,TX(J))+R(10)*T(9,TX(J))+R(11)*T(10,TX
     3(J))+R(12)*T(11,TX(J))+R(13)*T(12,TX(J))+R(14)*T(13,TX(J)
      TEST=TEST*FFC
      TDYDX=RD(1)*T(0,TX(J))+RD(2)*T(1,TX(J))+RD(3)*T(2,TX(J))+RD(4)*
     1T(3,TX(J))+RD(5)*T(4,TX(J))+RD(6)*T(5,TX(J))+RD(7)*T(6,TX(J))
```

```
     2+RD(8)*T(7,TX(J))+RD(9)*T(8,TX(J))+RD(10)*T(9,TX(J))+RD(11)*
     3T(10,TX(J))+RD(12)*T(11,TX(J))+RD(13)*T(12,TX(J))+RD(14)*T(13,T
     4X(J))
      TDYDX=TDYDX*DDD
      DYDX(J)=DYDX(J)*DDD
      F(J)=F(J)*FFC
      DIFF=(F(J)-TEST)/F(J)*100.
      WRITE(IW,145)J,F(J),TEST,DIFF,DYDX(J),TDYDX
  145 FORMAT(I5,5E25.15)
  144 CONTINUE
      DO 147 I=1,NDIM
      WRITE(IW,146)I,R(I),RD(I)
  147 CONTINUE
  146 FORMAT(1HO,I5,5X,'R=',E22.15,5X,'RD=',E22.15)
      CALL EXIT
      END
```

APPENDIX D

PHYSICAL DATA

WIND FORCES AND MOMENTS

The forces and moments acting on the vessel for a given wind velocity in the general direction of wave propagation are to be determined. Basic data and calculations of areas and moment arms furnished by the vessel owner are included at the end of this appendix.

The data determines the transverse and longitudinal wind moments for wind velocity and wind components in the X and Z directions. For wind velocity V_w at an angle of incidence α the components in the X and Z directions are $V_w \cos\alpha$ and $V_w \sin\alpha$ respectively. It is easily shown that the normal components of force due to wind are

$$F_{wx} = -\ 69.3056(V_w \cos\alpha \cos\ )^2 \qquad \text{EQ. (D.1)}$$

and

$$F_{wz} = -\ 73.4496(V_w \sin\alpha \cos\theta)^2 \qquad \text{EQ. (D.2)}$$

The transverse moment (around the z-axis) has been determined to be

$$M_w(\ ) = +\ 6967.07(V_w \cos\alpha \cos\ )^2 \qquad \text{EQ. (D.3)}$$

and the longitudinal wind moment (around the x-axis) as

$$M_w(\theta) = -\ 6411.55(V_w \sin\alpha \cos\theta)^2\ . \qquad \text{EQ. (D.4)}$$

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and the longitudinal wind moment (around the x-axis) as

$$M_w(\theta) = - 6411.55(V_w\sin\alpha\cos\theta)^2\ . \qquad \text{EQ. (D.4)}$$

The yaw moment (around the vertical axis) due to wind is omitted due to insufficient data.

The above terms for forces and moments are to be included in the appropriate equations of motion derived in Appendix B.

MASS AND MASS PROPERTIES*

Mass = 55,321,734 lbs.^{2}-kips

I_{xx} = 430,724,040 ft.^{2}-kips

I_{yy} = 632,448,210 ft.^{2}-kips

I_{zz} = 426,417,240 ft.^{2}-kips

I_{xy} = 633,484 ft.^{2}-kips

I_{xz} = 1,709,465 ft.^{2}-kips

I_{yz} = 11,837,974 ft.^{2}-kips

CENTERS OF GRAVITY

Vertical C.G. Height is 49.45 ft.

Longitudinal C.G. is .02 ft. starboard of centerline.

Transverse C.G. is 1.57 ft. aft of centerline.

METACENTRIC HEIGHTS

GM_L(Longitudinal) = 24.40.

GM_T(Transverse) = 23.50 ft.

* Note: Above properties are for a zero topside load only.

PHYSICAL DIMENSIONS AND COORDINATES

Member	Col. O.D. ft.	Hull O.D. ft.	Hull Length ft.	x_c	y_h	z_c
1	22.50	25.50	61.00	124.57	-36.70	44.98
2	22.50	25.50	61.00	124.57	-36.70	-45.02
3	22.50	25.50	64.00	61.57	-36.70	119.28
4	12.00	25.50	64.00	61.57	-36.70	44.98
5	12.00	25.50	64.00	61.57	-36.70	-45.02
6	22.50	25.50	64.00	61.57	-36.70	-119.32
7	12.00	25.50	57.00	1.57	-36.70	119.28
8	12.00	25.50	57.00	1.57	-36.70	44.98
9	12.00	25.50	57.00	1.57	-36.70	-45.02
10	12.00	25.50	57.00	1.57	-36.70	-119.32
11	22.50	25.50	64.00	-58.43	-36.70	119.28
12	12.00	25.50	64.00	-58.43	-36.70	44.98
13	12.00	25.50	64.00	-58.43	-36.70	-45.02
14	22.50	25.50	64.00	-58.43	-36.70	-119.32
15	22.50	25.50	61.00	-121.43	-36.70	44.98
16	22.50	25.50	61.00	-121.43	-36.70	-45.02

Member	Col. O.D. ft.	Hull O.D. ft.	Hull Length ft.	x_c	y_h	z_c
17	0.0	12.00	67.00	124.57	- 9.45	- 0.02
18	0.0	18.00	51.00	61.57	- 9.45	81.73
19	0.0	18.00	51.00	61.57	- 9.45	-81.77
20	0.0	18.00	51.00	61.57	- 9.45	-81.77
21	0.0	18.00	51.00	- 58.43	- 9.45	81.73
22	0.0	18.00	67.00	- 58.43	- 9.45	- 0.02
23	0.0	18.00	51.00	- 58.43	- 9.45	-81.77
24	0.0	12.00	67.00	-121.43	- 9.45	- 0.02

APPENDIX E

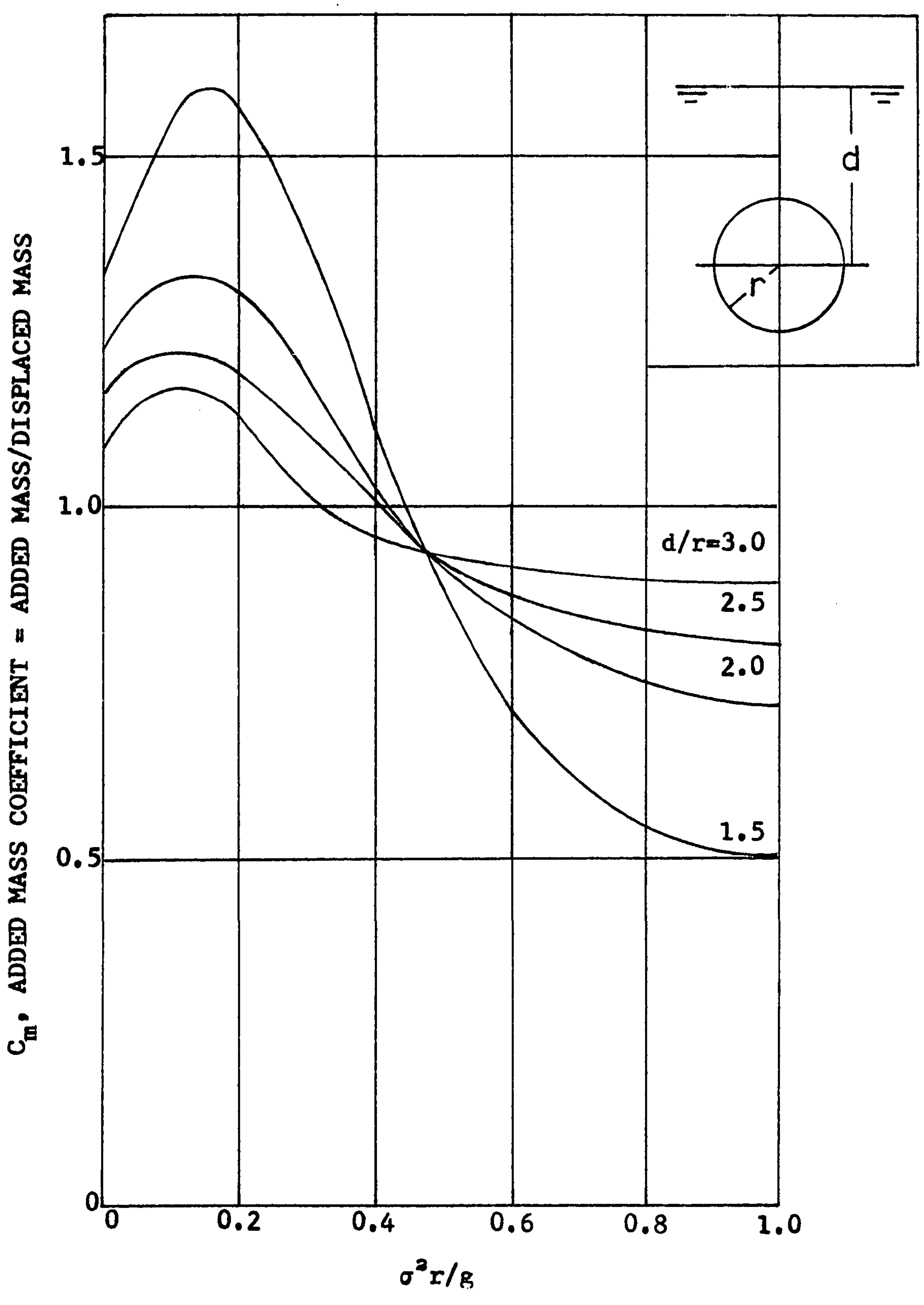

Figure E.3: Added Mass Coefficient for an Oscillating Cylinder

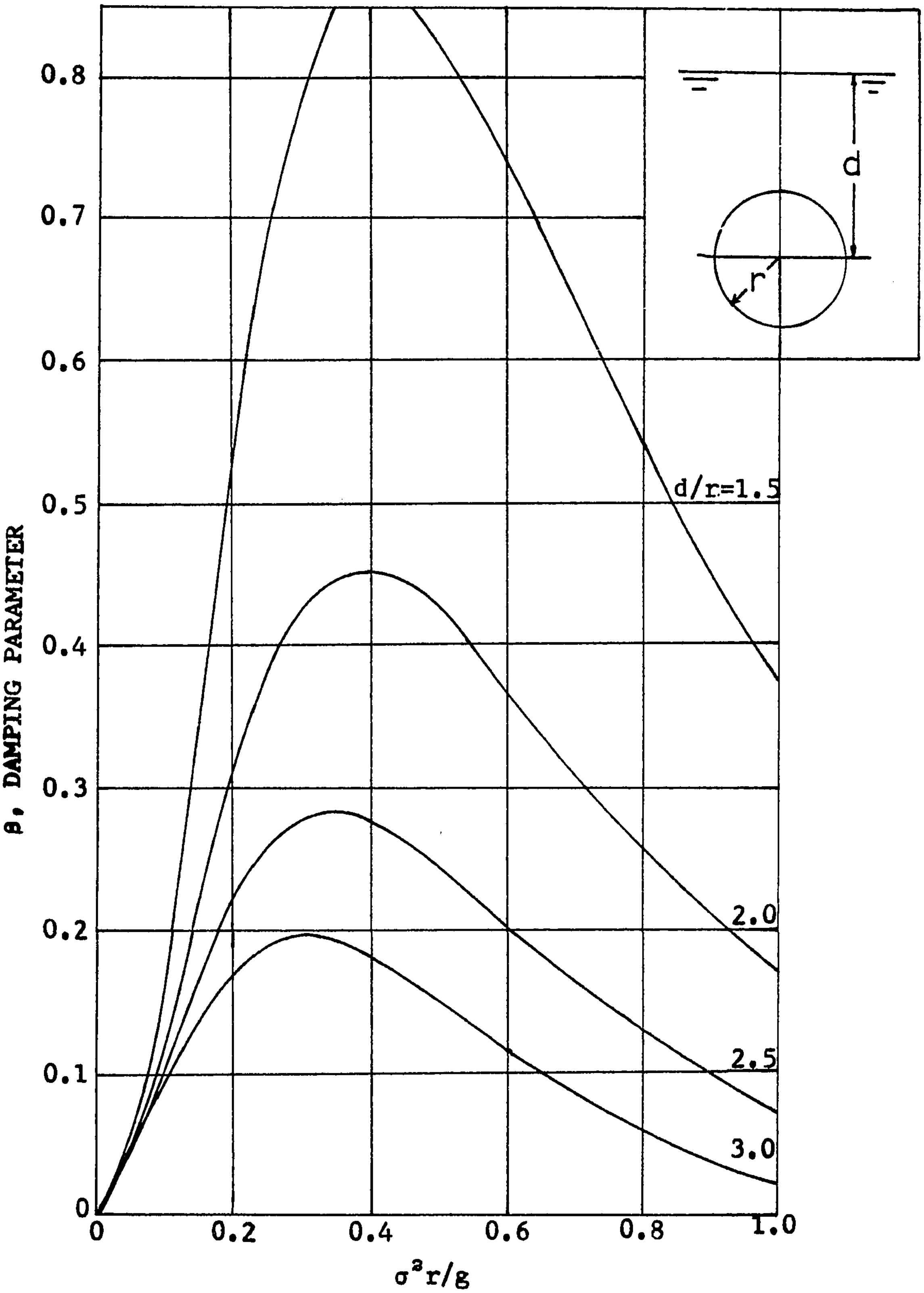

Figure E.4: Damping Parameter for an Oscillating Cylinder

APPENDIX F

"PROSPCTR"

COMPUTER PROGRAM

THRUST REQUIREMENT

DETERMINATION

```
C       MAIN PROGRAM
        DIMENSION Y(12),DERY(12),PRMT( 5),DMCX(24),DMHY(25),DMHZ(24),
       1    DMCZ(24),ALFA(24),CAPK(24),PDH(24),YH(24),DC(24),HL(24),
       2    SJ(24),YBAR(24),AF(8),DCX(24),DHY(24),DCZ(24),DHZ(24),
       3    AUX(8,12),Q(24),ANU(24),FCX(24),FHX(24),FCZ(24),FHZ(24),
       4    DHX(24),DMHX(24),XI(24),YI(24),ZI(24)
        DIMENSION A1(24),A2(24),A3(24),A4(24),A5(24),A6(24),A7(24),A8(24),
       1    A9(24),A10(24),A11(24),A13(24),E1(24),E2(24),E3(24),E4(24),
       2    E5(24),E6(24),E7(24),E8(24),E9(24),E10(24),E11(24),E13(24),
       3    A21(24),A22(24),A23(24),A24(24),A25(24),A26(24),A27(24),
       4    A28(24),A29(24),A210(24),A211(24),A213(24),E21(24),E22(24),
       5    E23(24),E24(24),E25(24),E26(24),E27(24),E28(24)
        DIMENSION E29(24),E210(24),E211(24),E213(24),C1(24),C2(24),C3(24),
       1    C4(24),C5(24),C7(24),C9(24),D1(24),D2(24),D3(24),D4(24),D5(24),
       2    D7(24),D9(24),C21(24),C22(24),C23(24),C24(24),C25(24),C27(24),
       3    C29(24),D21(24)D22(24),D23(24),D24(24),D25(24),BYD(24),D27(24),
       4    D29(24),B1(24),B2(24),B3(24),B4(24),B5(24),DSJ(24),A35(24),
       5    A36(24),E35(24),E36(24),C31(24)
```

```
      DIMENSION C32(24),C33(24),D31(24),D32(24),D33(24),D69(24),C41(24),
     1   C51(24),C61(24),C43(24),C53(24),C63(24),C34(24),C44(24),
     2   C54(24),C35(24),C45(24),C55(24),C65(24),C37(24),C47(24),
     3   C57(24),C67(24),C39(24),C49(24),C59(24),C69(24),D41(24),
     4   D51(24),D61(24),D42(24),D52(24),D62(24),D43(24),D53(24),
     5   D63(24),D34(24),D44(24),D54(24),D64(24),D35(24),D45(24),
     6   D55(24),D65(24),D37(24),D47(24),D57(24),D67(24),D39(24)
      DIMENSION D49(24),D59(24)
      DIMENSION DATA(20)
      COMMON A35,A36,E35,E36,C31,C32,C33,D31,D32,D33,D59,D69,C41,C51,
     1   C61,C43,C53,C63,C34,C44,C54,C35,C45,C55,C65,C37,C47,C57,C67,
     2   C39,C49,C59,C69,D41,D51,D61,D42,D52,D62,D43,D53,D63,D34,D44,
     3   D54,D64,D35,D45,D55,D65,D37,D47,D57,D67,D39,D49
      COMMON A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A13,E1,E2,E3,E4,E5,E6,
     1   E7,E8,E9,E10,E11,E13,A21,A22,A23,A24,A25,A26,A27,A29,A210,A211,
     2   A213,E21,E22,E23,E24,E25,E26,E27,E28,E29,E210,E211,E213,C1,C2,
     3   C3,C4,C5,C9,C7,D1,D2,D3,D4,D5,D7,D9,C21,C22,C23,C24,C25,C27,C29
     4   ,D21,D22,D23,D24,D25,D27,D29,B1,B2,B3,B4,DSJ,BYD
```

```
      COMMON GML,GMT,SINA,COSA,CAPH,ANU,SJ,DMCX,DMHY,DMHZ,DMCZ,ALFA,
     1    DHX,DMHX,XI,YI,ZI,EIXY,EIXZ,EIYZ,DCX,DHY,DCZ,DHZ,OMEGA,PI,
     2    WAMP,G,PIR04,AK,WAVEL,DEG,PI2,EMASS,EIX,EIY,EIZ,B,HH,T1,CAPK,
     3    CSHKH,CSH2KH,SNHKH,SNH2KH,A28,B5,PDH,HL,DC,YBAR,RO,CM,VCMAX,
     4    SNHKH3
      COMMON TMOM,THRUST,WIND,ALPHA,WNVX2,WNVZ2,WX,WZ,WMX,WMZ,AF,DIR,
     1    FCX,FHX,FCZ,FHZ,Q,DOMX8,DOMX10,DOMX12,DOMY8,DOMY10,DOMY12,
     2    DOMZ8,DOMZ10,DOMZ12,PER
      COMMON SNHKH2,SNHKH4,SNHKH5,CSHKH3,CSH4KH,SNH4KH,OMEGAC,PIHL,
     1    PIHL2,CMC,AC1,AC2,AC3,AC4,AC5,C,H
      COMMON SINY7,COSY7,SINY9,COSY9,COSY11,SINY11,Y7,Y8,Y9,Y10,Y11,
     1    Y12,C9S11,C7C11,C9S7C1,S9S7S1,S9S11,C9C11,C7S11,S9S7C1,C9C7,
     2    S9C7,S9C11,C9S7S1,S9C7S1
      COMMON XD(8),ZD(8),YD(8),DH(8)HD(8),DIAG(8),RT(14),RD(14),AX(4),
     1    AY(4),AZ(4),ISIN(24),JSIN(24),KSIN(24),NOEL,NOADM
      WRITE (6,2)
      PI=3.141592653897932
```

```
C       ******* THE FOLLOWING COEFFICIENTS ARE DERIVED BY SEPARATE PROGRAM
        RT(1)=.585486207567359E01
        RT(2)=-.655207363767616E02
        RT(3)=.700250076790167E01
        RT(4)=-.390504932656121E02
        RT(5)=.347744093045743E01
        RT(6)=-.120584678480321E02
        RT(7)=.110999717336903E01
        RT(8)=-.119988117142609E01
        RT(9)=.312869779676840E00
        RT(10)=.258947534989535E00
        RT(11)=.689791595390375E-01
        RT(12)=.123090560725601E00
        RT(13)=-.474291002936958E-02
        RT(14)=.575121457054698E-01
        RD(1)=.315570220253098E1
        RD(2)=-.379663428077203E1
        RD(3)=.448416616629841E1
```

```
      RD(4)=-.204884702675601E1
      RD(5)=.159798259161104E1
      RD(6)=-.574389211513435E0
      RD(7)=.222001561998078E0
      RD(8)=-.440981032013393E-01
      RD(9)=-.340374970989804E-01
      RD(10)=.307661923105107E-01
      RD(11)=-.232107931651822E-01
      RD(12)=.147666199923824E-01
      RD(13)=-.738303718839151E-02
      RD(14)=.231931035865417E-02
      DEG=180./PI
   80 READ (5,23,END=99) DATA
   23 FORMAT (20A4)
C     ********MASS AND PRINCIPAL INERTIA TERMS*********
      READ(5,4)EMASS,EIX,EIZ,EIY
C     ********CROSS-PRODUCTS OF INERTIA AND UPPER ERROR BOUND*********
      READ(5,4)EIXY,EIXZ,EIYZ,EPSI
```

```
C       *********B=+VALUE OF CG BELOW CALM WATER LINE************
C       *********H=WATER DEPTH*************
C       *********GML=LONGITUDINAL METACENTRIC HEIGHT**************
C       *********GMT=TRANSVERSE METACENTRIC HEIGHT**********
        READ(5,32)B,H,GML,GMT,NOEL,NOADM
     32 FORMAT (4F8.3,2I8)
C       *********CONSTANTS FOR WIND FORCES AND MOMENTS**********
C       *********VCMAX=CURRENT VELOCITY IN KNOTS**********
        READ(5,3)WX,WZ,WMX,WMZ,VCMAX
C       *********ALPHA=ANGLE OF INCIDENCE,POSITIVE CLOCKWISE FROM POS
C       X-AXIS
C       *********NOTE THE ANGLE OF INCIDENCE ALPHA SHOULD NOT BE A
C       MULTIPLE OF PI/2.********************
C       *********WIND=WIND VELOCITY IN KNOTS***********
        READ(5,1)ALPHA,WIND,CMC,CM,CDF
        ALPH=ALPHA
        ALPHA=ALPHA/DEG
C       *********WAVEL=WAVE LENGTH IN FT.********
```

```
C       ********WAMP=WAVE=WAVE AMP OR ONE-HALF OF WAVE HEIGHT******
        READ(5,1)WAVEL,WAMP,ADM,CD,C,CASE,TMIN,CYC
        DO 30 I=1,NOEL
        READ (5,37) DC(I),PDH(I),HL(I),XI(I),YI(I),ZI(I),DSJ(I),C69(I),
       1   ALFA(I),ISIN(I),JSIN(I),KSIN(I)
     37 FORMAT (2X,9F8.2,3I2)
        IF(DSJ(I).LE.0.0) DSJ(I) = ADM
        IF (C69(I).LE.0.0) C69(I) = CM
     30 CONTINUE
        IF (NOADM.LE.0) GO TO 49
        DO 35 I=1,NOADM
     35 READ (5,36) DH(I),HD(I),XD(I),YD(I),ZD(I)
     36 FORMAT (10X,5F8.2)
     49 WRITE (6,24) DATA
     24 FORMAT (25X,20A4/)
        DO 39 I=1,4
     39 READ (5,38) AX(I),AY(I),AZ(I)
     38 FORMAT (26X,3F8.2)
```

```
      WRITE(6,4)EMASS,EIX,EIY,EIZ
      WRITE(6,4)EIXY,EIXZ,EIYZ,EPSI
      WRITE(6,1)B,H,GML,GMT
      WRITE(6,3)WX,WZ,WMX,WMZ,VCMAX
      WRITE(6,15)ALPHP,WIND,CMC,CM,CDF
      WRITE(6,15)WAVEL,WAMP,ADM,CD,D,CASE,TMIN,CYC
      WRITE (6,55)
   55 FORMAT (6X, "ELEM  COLUMN     HULL      HULL",  8X,"X",11X,
     1"Y",11X,"Z", 8X, "DAMPING  COEFF.     HULL    ISIN  JSIN  KSIN"/
     2 7X,"NO  DIAMETER  DIAMETER  LENGTH     COORD.      COORD.",6X, "
     3COORD.       COEFF.    MASS     ANGLE"/
     4 13X, 4("(FT.)",5X),2X, "(FT.)",7X, "(FT.)",26X, "(DEG.)"//)
      DO 60 I=1,NOEL
      WRITE (6,56) I, DC(I), PDH(I), HL(I), XI(I), YI(I), ZI(I),
     1 DSJ(I), C69(I), ALFA(I), ISIN(I), JSIN(I), KSIN(I)
   56 FORMAT (5X,I4,1X, 3(F8.2,2X), 3(F9.2,3X), F8.2, 2X, F7.2, 3X,
     1 F8.2, 2X, 3(I4,2X))
   60 CONTINUE
```

```
      IF(NOADM.LE.0) GO TO 65
      WRITE (6,27)
   27 FORMAT (1H0,40X, "*** ADDED DAMPING ***"/)
      DO 64 I=1,NOADM
   64 WRITE (6,28) I,DH(I), HD(I),XD(I),YD(I),ZD(I)
   28 FORMAT (5X,I4,11X,2(F8.2,2X), 3(F9.2,3X))
   65 WRITE (6,40)
   40 FORMAT (1H0,40X, "***  ANCHOR LOCATIONS  ***"/)
      WRITE (6,41)  (I,AX(I),AY(I),AZ(I),I=1,4)
   41 FORMAT (5X,I4,5X, "FWD PORT", 14X,3(F9.2,3X)/ 5X,I4,5X,
     1  "FWD STBD" , 14X,3(F9.2,3X)/ 5X,I4, 5X, "AFT PORT", 14X,
     2  3(F9.2,3X)/ 5X,I4,  5X, "AFT STBD",14X,3(F9.2,3X))
      WRITE (6,5) CASE
    1 FORMAT (9F8.3,2X,3I2)
2     FORMAT(1H1)
3     FORMAT(5F14.4)
4     FORMAT(4E14.8)
5     FORMAT(1H1,58X,"CASE ",F6.3)
```

```
6     FORMAT(8F15.4)
14    FORMAT(4F14.7)
15    FORMAT (2X,8F8.4)
16    FORMAT(1H0,1X,"WAVE HEIGHT =",F5.0,"FT.",5X,"WAVE LENGTH =",F6.0,"
     1FT.",5X,"WAVE PERIOD =",F5.1," SEC")
17    FORMAT(1H0,1X,"WAVE IS APPROACHING BOW AT",F5.1," DEGREES TO STB")
18    FORMAT(1H0,1X,"WATER DEPTH =",F6.1,"FT.")
19    FORMAT(1X,"WIND SPEED =",F5.0," KNOTS")
20    FORMAT(1H0,1X,"CURRENT =",F5.0,"KNOTS")
      G=32.17
      EMASS=EMASS/G
      EIX=EIX/G
      EIY=EIY/G
      EIZ=EIZ/G
      EIXY=EIXY/G
      EIXZ=EIXZ/G
      EIYZ=EIYZ/G
      CD=1.05
```

```
D=1.05
RO=64./G
AK=2.*PI/WAVEL
WAVH=2*WAMP
NDIM=12
PIRO4=PI*RO/4.
PI2=PI/2.
HH=2*WAMP
XAKH= EXP(AK*H)
XMAKH= EXP(-AK*H)
TNHKH=(XAKH-XMAKH)/(XAKH+XMAKH)
SNHKH=.5*(XAKH-XMAKH)
SNHKH2=SNHKH*SNHKH
SNHKH3=SNHKH**3
SNHKH4=SNHKH**4
SNHKH5=SNHKH**5
CSHKH=.5*(XAKH+XMAKH)
CSHKH3=CSHKH**3
```

```
X2AKH= EXP(2*AK*H)
XM2AKH= EXP(-2*AK*H)
X4AKH= EXP(4*AK*H)
XM4AKH= EXP(-4*AK*H)
SNH2KH=.5*(X2AKH-XM2AKH)
CSH2KH=.5*(X2AKH+XM2AKH)
CSH4KH=.50*(X4AKH+XM4AKH)
SNH4KH=.5*(X4AKH-XM4AKH)
PIHL=PI*HH/WAVEL
PIHL2+PIHL*PIHL
C= SQRT(G*TNHKH/AK)
OMEGA=2*PI*C/WAVEL
OMEGA2=OMEGA*OMEGA
PER=WAVEL/C
TGAP=TMIN*PI/(180*OMEGA)
TLAST=PER*CYC
OMEGAC=OMEGA*C
CAPH=PI*HH*HH*CSHKH*(CSH2KH+2.)/(8*WAVEL*SNHKH**3)
```

```
      WRITE(6,16)WAVH,WAVEL,PER
      WRITE (6,17) ALPHP
      WRITE(6,18)H
      WRITE(6,19)WIND
      WRITE(6,20)VCMAX
      VCMAX=VCMAX*1.689
      AOMEG2=WAMP*WAMP*OMEGA*OMEGA
      AC1=.5*RO*HH*C*OMEGA/SNHKH
      AC2=.375*RO*PI*HH*HH*C*OMEGA/(WAVEL*SNHKH4)
      AC3=.25*RO*(.5*HH*C*AK/SNHKH)**2
      AC4=9*RO/256.*(PI*HH*HH*C*AK/(WAVEL*SNHKH4))**2
      AC5=3*RO/16.*(PI*HH*HH*HH*C*C*AK*AK)/(WAVEL*SNHKH5)
      WRITE(6,903) AC1,AC2,AC3,AC4,AC5
  903 FORMAT (1X,"  AC  ",5E15.5)
      IF (NOADM.LE.0) GO TO 101
      DO 105 I=1,NOADM
  105 DIAG(I) = ADM * OMEGA * PIRO4 * DH(I)**2 * HD(I)
  101 DO 100 I=1,NOEL
```

```
PIROC=.25*PI*   DC(I)*DC(I)
PIROH=.25*PI*   PDH(I)*PDH(I)
ADM=DSJ(I)
CM=C69(I)
YH(I)=YI(I)
BYDB   =AK*(H+YH(I)-B)
B2YDB = 2.0 * BYDB
B4YDB=2*B2YDB
XBY   = EXP(BYDB)
XMBY   = EXP(-BYDB)
X2BY   = EXP(B2YDB)
XM2BY   = EXP(-B2YDB)
X4BY= EXP(B4YDB)
XM4BY= EXP(-B4YDB)
CBSH1=.5*(XBY+XMBY)
CBSH2=.5*(X2BY+XM2BY)
SBNH1=.5*(XBY-XMBY)
SBNH2=.5*(X2BY-XM2BY)
```

```
SBNH4=.5*(X4BY-XM4BY)
CBSH4=.5*(X4BY+XM4BY)
CBSH12=CBSH1*CBSH1
SBNH12=SBNH1*SBNH1
SBNH13=SBNH1*SBNH12
A2(I)=PIHL*OMEGAC*CBSH1/(AK*HL(I)*SNHKH)
A22(I)=.75*PIHL2*OMEGAC*CBSH2/(AK*HL(I)*SNHKH4)
A4(I)=A2(I)
A24(I)=A22(I)
A5(I)=PIHL*OMEGAC*SBNH1/(AK*HL(I)*SNHKH)
A25(I)=.75*PIHL2*OMEGAC*SBNH2/(AK*HL(I)*SNHKH4)
A6(I)=A5(I)
A26(I)=A25(I)
A8(I)=PIHL*C*CBSH1/(AK*HL(I)*SNHKH)
A28(I)=.75*PIHL2*C*CBSH2/(2*AK*HL(I)*SNHKH4)
A10(I)=PIHL*C*SBNH1/(AK*HL(I)*SNHKH)
A210(I)=.75*PIHL2*C*SBNH2/(2*AK*HL(I)*SNHKH4)
A11(I)=A210(I)
```

```
A211(I)=A210(I)
A13(I)=A8(I)
A213(I)=A28(I)
C1(I)=  (AC1-AC5)*SBNH1*PIROH
C21(I)=  AC2*SBNH2*PIROH
C31(I)=2*AC3*AK*HL(I)*SBNH2*PIROH+4* AC4*AK*HL(I)*SBNH4*PIROH
C31(I)=C31(I)
C41(I)=2*AC5*SBNH1/3.*PIROH
C51(I)=(4*AC5*SBNH1*CBSH12*PIROH+2*AC5*SBNH13*PIROH)
C3(I)=C1(I)
C23(I)=C21(I)
C33(I)=C31(I)
C43(I)=C41(I)
C53(I)=C51(I)
C5(I)=  PIROH*(AC1+AC5)*CBSH1
C25(I)=  PIROH*(AC2*CBSH2-AC3)
C35(I)=PIROH*AC4
C45(I)=2*PIROH*AC5*CBSH1
```

```
C55(I)=2*PIROH*AC5*SBNH12*CBSH1
C7(I)=C5(I)
C27(I)=C25(I)
C37(I)=C35(I)
C47(I)=C45(I)
C57(I)=C55(I)
D61(I)=PIRO4*PDH(I)*PDH(I)*G*WAMP*SBNH1/SNHKH
D62(I)=PIRO4*PDH(I)*PDH(I)*G*3*HH*HH*SBNH2/(16*WAVEL*SNHKH4)*PI
BYD(I)=1./(B-YH(I)-.5*PDH(I))
A2YDB   =2*AK*(H+YH(I)+.5*PDH(I)-B)
AYDB   =AK*(H+YH(I)+.5*PDH(I)-B)
A4YDB=2*A2YDB
XA2    = EXP(A2YDB)
XMA2    = EXP(-A2YDB)
XA1    = EXP(AYDB)
XMA1    = EXP(-AYDB)
XA4= EXP(A4YDB)
XMA4= EXP(-A4YDB)
```

```
CSH1=.5*(XA1+XMA1)
CSH2=.5*(XA2+XMA2)
SNH1=.5*(XA1-XMA1)
SNH2=.5*(XA2-XMA2)
SNH4=.5*(XA4-XMA4)
CSH4=.5*(XA4+XMA4)
CSH13=CSH1**3
SNH13=SNH1**3
A1(I)=BYD(I)*PIHL*OMEGAC*(SNH1-SNHKH)/(AK*SNHKH)
A21(I)=.75*BYD(I)*PIHL2*OMEGAC*(SNH2-SNH2KH)/(AK*SNHKH4)*KSIN(I)
A3(I)=A1(I)
A23(I)=A21(I)
A7(I)=BYD(I)*PIHL*C*(SNH1-SNHKH)/(AK*SNHKH) *KSIN(I)
A27(I)=.75*BYD(I)*PIHL2*C*(SNH2-SNH2KH)/(2*AK*SNHKH4)*KSIN(I)
A9(I)=A7(I)
A29(I)=A27(I)
D2(I)=  PIROC*(CSHKH-CSH1)*(AC1-AC5)
D22(I)=  PIROC*AC2*(CSH2KH-CSH2)
```

```
 D32(I)= PIROC*AC3*(CSH2KH-CSH2)+PIROC*AC4*SNHKH4**2*(CSH4KH/(SNHK
1H4**2)-CSH4/(SNHKH4**2))
 D42(I)=  PIROC*2*AC5*(CSHKH-CSH1)
 D52(I)=4*PIROC*AC5*SNHKH5*SNHKH5*(CSHKH3/SNHKH5-CSH13/SNHKH5)/3.0
1+2*AC5*((CSHKH3/3.-CSHKH)-(CSH13/3.-CSH1)) * PIROC
 C4(I)=  (AC1+AC5)*(SNHKH-SNH1)*PIROC
 C24(I)+ (PIROC*AC2*(SNH2KH-SNH2)-2*AC3*AK/BYD(I)*PIROC)
 C34(I)=PIROC*4*AC4*AK/BYD(I)
 C44(I)= 6*PIROC*AC5*(SNHKH-SNH1)
 C54(I)=2*PIROC*AC5*SNHKH5*(SNHKH3/SNHKH5- SNH13/SNHKH5)/3.0
 C9(I)=C4(I)
 C29(I)=C24(I)
 C39(I)=C34(I)
 C49(I)=C44(I)
 C59(I)=C54(I)
 DHY(I)=.5*D*RO*HL(I)*PDH(I)
 CAPK(I)=PIRO4*G*DC(I)*DC(I)
 C67(I)=CDF*ADM*OMEGA*PIROC*RO/BYD(I)
```

```
      D67(I)=ADM*PIROH*RO*HL(I)*OMEGA
      DCX(I)=CD*.5*RO*DC(I)/BYD(I)
      DCZ(I)=DCX(I)
      DMCX(I)=CMC*PIRO4*DC(I)*DC(I)/BYD(I)
      DMCZ(I)=DMCX(I)
      DHZ(I)=.5*D*RO*HL(I)*PDH(I)
      DHX(I)=DHZ(I)
      DMHY(I) = CM*PIRO4*PDH(I)*PDH(I)*HL(I)
      DMHX(I)=DMHY(I)*SIN(ABS(ALFA(I))/DEG)/CM
      DMHZ(I)=DMHY(I)*COS(ABS(ALFA(I))/DEG)/CM
      ALFA(I)=ALPHA-ALFA(I)/DEG
100   CONTINUE
      PRMT(1)=0.
      PRMT(2)=TLAST
      PRMT(3)+TGAP
      PRMT(4)=EPSI
      DO 1000 I=1,12
1000  Y(I)=0.
```

```
C       ********DERY IS INITIALLY A VECTOR OF WEIGHTS, LATER THE VECTOR OF
C                 DERIVATIVES ***********
        DO 2000 I=1,5
2000    DERY(I)=.01D0
        DERY(6)=.02D0
        DO 3000 I=7,11
3000    DERY(I)=.15D0
        DERY(12)=.18D0
C       ********PRKGS IS INTEGRATION SUBROUTINE REFER IBM PACKAGE********
        CALL PRKGS(PRMT,Y,DERY,NDIM,IHLF,AUX)
        GO TO 80
     99 CALL EXIT
        END
```

```
      SUBROUTINE OUTP(T,Y,DERY,IHLF,NDIM,PRMT)
      DIMENSION Y(12),DERY(12),PRMT( 5),DMCX(24),DMHY(24),DMHZ(24),
     1    DMCZ(24),ALFA(24),CAPK(24),PDH(24),DC(24),
     2    HL(24),SJ(24),YBAR(24),AF(8),DCX(24),DHY(24)DCZ(24),DHZ(24),
     3    AUX(8,12),Q(24),ANU(24),FCX(24),FHX(24),FCZ(24),FHZ(24),
     4    DHX(24),DMHX(24),XI(24),YI(24),ZI(24)
      DIMENSION A1(24),A2(24),A3(24),A4(24),A5(24),A6(24),A7(24),A8(24),
     1    A9(24,A10(24),A11(24),A13(24),E1(24),E2(24),E3(24),E4(24),
     2    E5(24),E6(24),E7(24),E8(24),E9(24),E10(24),E11(24),E13(24),
     3    A21(24),A22(24),A23(24),A24(24),A25(24),A26(24),A27(24),
     4    A28(24),A29(24),A210(24),A211(24),A213(24),E21(24),E22(24),
     5    E23(24),E24(24),E25(24),E26(24),E27(24),E28(24)
      DIMENSION E29(24),E210(24),E211(24),E213(24),C1(24),C2(24),C3(24),
     1    C4(24,C5(24),C7(24),C9(24),D1(24),D2(24),D3(24),D4(24),D5(24),
     2    D7(24,D9(24),C21(24),C22(24),C23(24),C24(24),C25(24),C27(24),
     3    C29(24),D21(24),D22(24),D23(24),D24(24),D25(24),BYD(24),D27(24)
     4    ,D29(24),B1(24),B2(24),B3(24),B4(24),B5(24),DSJ(24),A35(24),
     5    A36(24,E35(24),E36(24),C31(24)
```

```
      DIMENSION C32(24),C33(24,D31(24),D32(24),D33(24),D69(24),
     1   C41(24),C51(24),C61(24),C43(24),C53(24),C63(24),C34(24),C44(24)
     2   ,C54(24),C35(24),C45(24),C55(24),C65(24),C37(24),C47(24),
     3   C57(24),C67(24),C39(24),C49(24),C59(24),D41(24),D51(24),
     4   D61(24),D42(24),D52(24),D62(24),D43(24),D53(24),D63(24),
     5   D34(24),D44(24),D54(24),D64(24),D35(24),D45(24),D55(24),D65(24)
     6   ,D37(24),D47(24),D57(24),D67(24),D39(24),D49(24),D59(24)
      COMMON A35,A36,E35,E36,C31,C32,C33,D31,D32,D33,D59,D69,C41,C51,
     1   C61,C43,C53,C63,C34,C44,C54,C35,C45,C55,C65,C37,C47,C57,C57,
     2   C67,C39,C49,C59,C69,D41,D51,D61,D42,D52,D62,D43,D53,D63,D34,
     3   D44,D54,D64,D35,D45,D55,D65,D37,D47,D57,D67,D39,D49
      COMMON A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A13,E1,E2,E3,E4,E5,
     1   E6,E7,E8,E9,E10,E11,E13,A21,A22,A23,A24,A25,A26,A27,A29,A210,
     2   A211,A213,E21,E22,E23,E24,E25,E26,E27,E28,E29,E210,E211,E213,
     3   C1,C2,C3,C4,C5,C9,C7,D1,D2,D3,D4,D5,D7,D9,C21,C22,C23,C24,C25,
     4   C27,C29,D21,D22,D23,D24,D25,D27,D29,B1,B2,B3,B4,DSJ,BYD
      COMMON GML,GMT,SINA,COSA,CAPH,ANU,SJ,DMCX,DMHY,DMHZ,DMCZ,ALFA,
     1   DHX,DMHX,XI,YI,ZI,EIXY,EIXZ,EIYZ,DCX,DHY,DCZ,DHZ,OMEGA,PI,
```

```
     2    WAMP,G,PIRO4,AK,WAVEL,DEG,P12,EMASS,EIX,EIY,EIZ,B,HH,T1,CAPK,
     3    CSHKH,CSH2KH,SNHKH,SNH2KH,A28,B5,PDH,HL,DC,YBAR,RO,CM,VCMAX,
     4    SNHKH3
      COMMONTMOM,THRUST,WIND,ALPHA,WNVX2,WNVZ2,WX,WZ,WMX,WMZ,AF,DIR,
     1    FCX,FHX,FCZ,FHZ,Q,DOMX8,DOMX10,DOMX12,DOMY8,DOMY10,DOMY12,
     2    DOMZ8,DOMZ10,DOMZ12,PER
      COMMON SNHKH2,SNHKH4,SNHKH5,CSHKH3,CSH4KH,SNH4KH,OMEGAC,PIHL,
     1    PIHL2,CMC,AC1,AC2,AC3,AC4,AC5,C,H
      COMMON SINY7,COSY7,SINY9,COSY9,COSY11,SINY11,Y7,Y8,Y9,Y10,Y11,
     1    Y12,C9S11,C7C11,C9S7C1,S9S7S1,S9S11,C9C11,C7S11,S9S7C1,C9C7,
     2    S9C7,S9C11,C9S7S1,S9C7S1
      COMMON XD(8),ZD(8),YD(8),DH(8),HD(8),DIAG(8),RT(14),RD(14),
     1    AX(4),AY(4),AZ(4),ISIN(24),JSIN(24),KSIN(24),NOEL,NOADM
      IF(T.GT.0.)GO TO 4
      WRITE(6,5)
5     FORMAT(1HO,3X,"T",4X,"WAVE",8X,"SURGE",10X,"HEAVE",8X,"SWAY",
     1    10X,"ROLL",12X,"YAW",10X,"PITCH",4X,"THRUST/DEG",4X,"MOMENT"//)
4     WAVE=WAMP* COS(OMEGA*T)+.125*HH*HH*CSHKH*(CSH2KH+2.)* COS(2*
```

```
     1   OMEGA*T)*PI/(WAVEL*SNHKH3)
      TX=Y(7)*DEG
      F=Y(9)*DEG
      U=Y(11)*DEG
      WRITE(6,1) T,WAVE,Y(1),Y(3),Y(5),TX,F,U,THRUST,DIR,TMOM,IHLF
1     FORMAT(1X,F6.2,3X,F6.2,6(3X,F11.4),3X,F8.0,"/",F5.1,F10.0,13)
      WRITE(6,2) (AF(I),I=1,8)
2     FORMAT(10X,"ANCHOR TENSIONS NO. 1 THRU 8 ",8(3X,F8.0)/)
      TIMER = 0.8 * PRMT(2)
      IF(T.LT.TIMER)GO TO 8
      OMD2TX=DOMX8*DERY(8)+DOMX10*DERY(10)+DOMX12*DERY(12)
      OMD2TY=DOMY8*DERY(8)+DOMY10*DERY(10)+DOMY12*DERY(12)
      OMD2TZ=DOMZ8*DERY(8)+DOMZ10*DERY(10)+DOMZ12*DERY(12)
      DO 7 I=1,24
      FCXT=-DMCX(I)*SJ(I)*(DERY(2)+ZI(I)*OMD2TY-.5*(B+YI(I)+.5*PDH(I)
     1   *OMD2TZ)+FCX(I)
      FHXT=-DMHX(I)*(DERY(2)+ZI(I)*OMD2TY-YI(I)*OMD2TZ)+FHX(I)
      FCZT=-DMCZ(I)*SJ(I)*(DERY(6)+.5*(B+YI(I)+.5*PDH(I))*OMD2TX-XI(I)*
```

```
     1   OMD2TY)+FCZ(I)
      FHZT=-DMHZ(I)*(DERY(6)+YI(I)*OMD2TX-XI(I)*OMD2TY)+FHZ(I)
      FHYT=-DMHY(I)*(DERY(4)+XI(I)*OMD2TZ-ZI(I)*OMD2TX)+Q(I)
      WAV=WAMP*COS(B1(I))+CAPH*COS(2*B1(I))
      AGAP=50.-WAV+YBAR(I)
      WRITE(6,6)I,FCXT,FHXT,FCZT,FHZT,FHYT,ANU(I),AGAP
6     FORMAT(1X,I5,5E15.4,2F15.4)
7     CONTINUE
8     RETURN
      END
```

```
      SUBROUTINE FCT(T,Y,DERY)
      DIMENSION Y(12),DERY(12),DMCX(24),DMHY(24),DMHZ(24),DMCZ(24),
     1    ALFA(24),CAPK(24),PDH(24),YH(24),ZC,(24),XC(24),DC(24),
     2    HL(24),SJ(24),YBAR(24),AF(8),RHS(6),A(6,6),DCX(24),DHY(24),
     3    DCZ(24),DHZ(24),Q(24),ANU(24),FCX(24),FHX(24),FCZ(24),FHZ(24),
     4    DHX(24),DMHX(24),XI(24),YI(24),ZI(24)
      DIMENSION A1(24),A2(24),A3(24),A4(24),A5(24),A6(24),A7(24),A8(24),
     1    A9(24),A10(24),A11(24),A13(24),E1(24),E2(24),E3(24),E4(24),
     2    E5(24),E6(24),E7(24),E8(24),E9(24),E10(24),E11(24),E13(24),
     3    A21,(24),A22(24),A23(24),A24(24),A25(24),A26(24),A27(24),A28(24
     4    ),A29(24),A210(24),A211(24),A213(24),E21(24),E22(24),E23(24),
     5    E24(24),E25(24),E26(24),E27(24),E28(24)
      DIMENSION E29(24),E210(24),E211(24),E213(24),C1(24),C2(24),C3(24),
     1    C4(24),C5(24),C7(24),C9(24),D1(24),D2(24),D3(24),D4(24),D5(24),
     2    D7(24),D9(24),C21(24),C22(24),C23(24),C24(24),C25(24),C27(24),
     3    C29(24),D21(24),D22(24),D23(24),D24(24),D25(24),BYD(24),
     4    D27(24),D29(24),B1(24),B2(24),B3(24),B4(24),B5(24),DSJ(24),
     5    A35(24),A36(24),E35(24),E36(24),C31(24)
```

```
      DIMENSION C32(24),C33(24),D31(24),D32(24),D33(24),D69(24),C41(24),
     1   C51(24),C61(24),C43(24),C53(24),C63(24),C34(24),C44(24),C54(24)
     2   ,C35(24),C45(24),C55(24),C65(24),C37(24),C47(24),C57(24),C67
     3   (24),C39(24),C49(24),C59(24),C69(24),D41(24),D51(24),D61(24),
     4   D42(24),D52(24),D62(24),D43(24),D53(24),D63(24),D34(24),D44
     5   (24),D54(24),D64(24),D35(24),D45(24),D55(24),D65(24),D37(24),
     6   D47(24),D57(24),D67(24),D39(24),D49(24),D59(24)
      COMMON A35,A36,E35,E36,C31,C32,C33,D31,D32,D33,D59,D69,C41,
     1   C51,C61,C43,C53,C63,C34,C44,C54,C35,C45,C55,C65,C37,C47,C57,
     2   C67,C39,C49,C59,C69,D41,D51,D61,D42,D62,D43,D53,D63,D34,D44,
     3   D54,D64,D35,D45,D55,D65,D37,D47,D57,D67,D39,D49
      COMMON A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A13,E1,E2,E3,E4,E5,
     1   E6,E7,E8,E9,E10,E11,E13,A21,A22,A23,A24,A25,A26,A27,A29,A210,
     2   A211,A213,E21,E22,E23,E24,E25,E26,E27,E28,E29,E210,E211,E213,
     3   C1,C2,C3,C4,C5,C9,C7,D1,D2,D3,D4,D5,D7,D9,C21,C22,C23,C24,
     4   C25,C27,C29,D21,D22,D23,D24,D25,D27,D29,B1,B2,B3,B4,DSJ,BYD
      COMMON GML,GMT,SINA,COSA,CAPH,ANU,SJ,DMCX,DMHY,DMHZ,DMCZ,ALFA,
     1   DHX,DMHX,XI,YI,ZI,EIXY,EIXZ,EIYZ,DCX,DHY,DCZ,DHZ,OMEGA,PI,
```

```
2    WAMP,G,PIRO4,AK,WAVEL,DEG,P12,EMASS,EIX,EIY,EIZ,B,HH,T1,CAPK,
3    CSHKH,CSH2KH,SNHKH,SNH2KH,A28,B5,PDH,HL,DC,YBAR,RO,CM,VCMAX,
4    SNHKH3
 COMMONTMOM,THRUST,WIND,ALPHA,WNVX2,WNVZ2,WX,WZ,WMX,WMZ,AF,DIR,
1    FCX,FHX,FCZ,FHZ,Q,DOMX8,DOMX10,DOMX12,DOMY8,DOMY10,DOMY12,
2    DOMZ8,DOMZ10,DOMZ12,PER
 COMMON SNHKH2,SNHKH4,SNHKH5,CSHKH3,CSH4KH,SNH4KH,OMEGAC,PIHL,
1    PIHL2,CMC,AC1,AC2,AC3,AC4,AC5,C,H
 COMMON SINY7,COSY7,SINY9,COSY11,SINY11,Y7,Y8,Y9,Y10,Y11,Y12,
1    C9S11,C7C11,C9S7C1,S9S7S1,S9S11,C9C11,C7S11,S9S7C1,C9C7,S9C7,
2    S9C11,C9S7S1,S9C7S1
 COMMON XD(8),ZD(8),YD(8),DH(8),HD(8),DIAG(8),RT(14),RD(14),AX(4),
1    AY(4),AZ(4),ISIN(24),JSIN(24),KSIN(24),NOEL,NOADM
 NDIM=6
 T2=T
 IF(T.LE.0.)T1=0.
 SINY7= SIN(Y(7))
 COSY7= COS(Y(7))
```

```
SINY9= SIN(Y(9))
COSY11= COS(Y(11))
SINY11= SIN(Y(11))
Y7=Y(7)
Y8=Y(8)
Y9=Y(9)
Y10=Y(10)
Y11=Y(11)
Y12=Y(12)
C9S11=COSY9*SINY11
C7C11=COSY7*COSY11
C9S7C1=COSY9*SINY7*COSY11
S9S7S1=SINY9*SINY7*SINY11
S9S11=SINY9*SINY11
C9C11=COSY9*COSY11
C7S11=COSY7*SINY11
S9S7C1=SINY9*SINY7*COSY11
C9C7=COSY9*COSY7
```

```
S9C7=SINY9*COSY7
S9C11=SINY9*COSY11
C9S7S1=COSY9*SINY7*SINY11
S9C7S1=SINY9*COSY7*SINY11
OMEGAT=OMEGA*T
DELT=T2-T1
WNVX2=(WIND* COS(ALPHA)*COSY11)**2
WNVZ2=(WIND* SIN(ALPHA)*COSY7)**2
OMGY=-SINY7*Y12+C7S11*Y8+C7C11*Y10
DO 1 I=1,24
ALFR=ALFA(I)+OMGY*DELT
ALFA(I)=ALFR
SINA= SIN(ALFR)
COSA= COS(ALFR)
TANA=SINA/COSA
ZC(I)=C9C7*ZI(I)+(C9S7S1-S9C11)*XI(I)+(C9S7C1+S9S11)*YI(I)
XC(I)=S9C7*ZI(I)+(S9S7S1+C9C11)*XI(I)+(S9S7C1-C9S11)*YI(I)
DMYI=   (YI(I)+.5*PDH(I)+B)
```

```
B1(I)=AK*((Y(1)+XC(I))*COSA+(Y(5)+ZC(I))*SINA)+OMEGAT
B2(I)=AK*((Y(1)+XC(I)+.5*HL(I))*COSA+(Y(5)+ZC(I))*SINA)+OMEGAT
B3(I)=AK*((Y(1)+XC(I)-.5*HL(I))*COSA+(Y(5)+ZC(I))*SINA)+OMEGAT
B4(I)=AK*((Y(1)+XC(I))*COSA+(Y(5)+ZC(I)+.5*HL(I))*SINA)+OMEGAT
B5(I)=AK*((Y(1)+XC(I))*COSA+(Y(5)+ZC(I)-.5*HL(I))*SINA)+OMEGAT
COSB1=COS(B1(I))
COSB21=COS(2*B1(I))
WAVE=WAMP*COSB1+CAPH*COSB21
E1(I)=A1(I)*SINA
E21(I)=A21(I)*SINA
E2(I)=A2(I)*TANA
E22(I)=A22(I)*TANA
E3(I)=A3(I)*COSA
E23(I)=A23(I)*COSA
E4(I)=A4(I)/TANA
E24(I)=A24(I)/TANA
E5(I)=A5(I)/COSA
E25(I)=A25(I)/COSA
```

```
E6(I)=A6(I)/SINA
E26(I)=A26(I)/SINA
E7(I)=A7(I)*COSA
E27(I)=A27(I)*COSA
E8(I)=A8(I)/TANA
E28(I)=A28(I)/TANA
E9(I)=A9(I)*SINA
E29(I)=A29(I)*SINA
E10(I)=A10(I)/SINA
E210(I)=A210(I)/SINA
E11(I)=A11(I)/COSA
E211(I)=A211(I)/COSA
E11(I)=A11(I)/COSA
E211(I)=A211(I)/COSA
E13(I)=A13(I)*TANA
E213(I)=A213(I)*TANA
D1(I)=C1(I)/COSA
D21(I)=C21(I)/COSA
```

```
D31(I)=C31(I)
D41(I)=C41(I)/COSA
D51(I)=C51(I)/COSA
D3(I)=C3(I)/SINA
D23(I)=C23(I)/SINA
D33(I)=C33(I)
D43(I)=C43(I)/SINA
D53(I)=C53(I)/SINA
D4(I)=C4(I)*SINA
D24(I)=C24(I)*SINA
D34(I)=C34(I)*SINA
D44(I)=C44(I)*SINA
D54(I)=C54(I)*SINA
D5(I)=C5(I)*TANA
D25(I)=C25(I)*TANA
D35(I)=C35(I)*TANA
D45(I)=C45(I)*TANA
D55(I)=C55(I)*TANA
```

```
      D7(I)=C7(I)/TANA
      D27(I)=C27(I)/TANA
      D37(I)=C37(I)/TANA
      D47(I)=C47(I)/TANA
      D57(I)=C57(I)/TANA
      D9(I)=C9(I)*COSA
      D29(I)=C29(I)*COSA
      D39(I)=C39(I)*COSA
      D49(I)=C49(I)*COSA
      D59(I)=C59(I)*COSA
      SINB1=SIN(B1(I))
      SINB21=SIN(2*B1(I))
      YBAR(I)=Y(3)-ZI(I)*SINY7+XI(I)*SINY11*COSY7-YI(I)*(1.-COSY7*COSY11
     1)
      SJ(I)=1.-BYD(I)*(YBAR(I)-WAMP*COSB1-CAPH*COSB21)
      DUMY=SJ(I)
      IF(DUMY.LT.0.)DUMY=0.
      IF(DUMY.GT.2.)DUMY=2.
```

```
      SJ(I)=DUMY
      ANU(I)=.5*(DMYI+WAMP*COSB1+CAPH*COSB21-YBAR(I))
1     CONTINUE
      T1=T2
C     ********RHSV CALCULATES THE RHS VECTOR***************
      CALL RHSV(T,Y,RHS)
C     ********AMAT CALCULATE THE COEFFICIENT MATRIX**************
      CALL AMAT(A,Y)
C     ********SIMQX SOLVES THE MATRIX EQ FOR THE DERIVATIVE VECTOR,REFER
               TO IBM PACKAGE SIMQ**************
      CALL SIMQX(A,RHS,NDIM,KS)
      IF(KS.EQ.1)GO TO 1000
      DO 3 I=1,6
      IODD=2*I-1
      IEVEN=2*I
3     DERY(IODD)=Y(IEVEN)
      RETURN
1000  WRITE(6,1100)
```

```
1100  FORMAT(1X,"SINGULAR MATRIX")
      RETURN
      END
```

```
      SUBROUTINE SIMQX(A,B,N,KS)
      DIMENSION A(36),B(6)
C     FORWARD SOLUTION
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0
      IT=JJ-J
      DO 30 I=J,N
C     SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ)))20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
```

```
C       TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)
        IF(ABS(BIGA)-TOL)35,35,40
     35 KS=1
        RETURN
C       INTERCHANGE ROWS IF NECESSARY
     40 I1=J+N*(J-2)
        IT=IMAX-J
        DO 50 K=J,N
        I1=I1+N
        I2=I1+IT
        SAVE=A(I1)
        A(I1)=A(I2)
        A(I2)=SAVE
C       DIVIDE EQUATION BY LEADING COEFFICIENT
     50 A(I1)=A(I1)/BIGA
        SAVE=B(IMAX)
        B(IMAX=B(J)
        B(J)=SAVE/BIGA
```

```
C     ELIMINATE NEXT VARIABLE
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(AIXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
C     BACK SOLUTION
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
```

```
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

```
      SUBROUTINE AMAT(A,Y)
      DIMENSION Y(12),DMCX(24),DMHY(24),DMHZ(24),DMCZ(24),ALFA(24),
     1    CAPK(24),PDH(24),YH(24),ZC(24),XC(24),DC(24),HL(24),SJ(24),
     2    YBAR(24),AF(8),A(6,6),DCX(24),DHY(24),DCZ(24),DHZ(24),Q(24),
     3    ANU(24),FCX(24),FHX(24),FCZ(24),FHZ(24),DHX(24),DMHX(24),
     4    XI(24),YI(24),ZI(24)
      DIMENSION A1(24),A2(24),A3(24),A4(24),A5(24),A6(24),A7(24),A8(24)
     1    ,A9(24),A10(24),A11(24),A13(24),E1(24),E2(24),E3(24),E4(24),
     2    E5(24),E6(24),E7(24),E8(24),E9(24),E10(24),E11(24),E13(24),
     3    A21(24),A22(24),A23(24),A24(24),A25(24),A26(24),A27(24),
     4    A28(24),A29(24),A210(24),A211(24),A213(24),E21(24),E22(24),
     5    E23(24),E24(24),E25(24),E26(24),E27(24),E28(24)
      DIMENSION E29(24),E210(24),E211(24),E213(24),C1(24),C2(24),
     1    C3(24),C4(24),C5(24),C7(24),C9(24),D1(24),D2(24),D3(24),
     2    D4(24),D5(24),D7(24),D9(24),C21(24),C22(24),C23(24),C24(24),
     3    C25(24),C27(24),C29(24),D21(24),D22(24),D23(24),D24(24),
     4    D25(24),BYD(24),D27(24),D29(24),B1(24),B2(24),B3(24),B4(24),
     5    B5(24),DSJ(24),A35(24),A36(24),E35(24),E36(24),C31(24)
```

```
      DIMENSION C32(24),C33(24),D31(24),D32(24),D33(24),D69(24),C41(24)
     1   ,C51(24),C61(24),C43(24),C53(24),C63(24),C34(24),C44(24),C54
     2   (24),C35(24),C45(24),C55(24),C65(24),C37(24),C47(24),C57(24),
     3   C67(24),C39(24),C49(24),C59(24),C69(24),D41(24),D51(24),
     4   D61(24),D42(24),D52(24),D62(24),D43(24),D53(24),D63(24),
     5   D34(24),D44(24),D54(24),D64(24),D35(24),D45(24),D55(24),D65
     6   (24),D37(24),D47(24),D57(24),D67(24),D39(24),D49(24),D59(24)
      COMMON A35,A36,E35,E36,C31,C32,C33,D31,D32,D33,D59,D69,C41,C51,
     1   C61,C43,C53,C63,C34,C44,C54,C35,C45,C55,C65,C37,C47,C57,C67,
     2   C39,C49,C59,C69,D41,D51,D61,D42,D52,D62,D43,D53,D63,D34,D44,
     3   D54,D64,D35,D45,D65,D37,D47,D57,D67,D39,D49
      COMMON A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A13,E1,E2,E3,E4,E5,
     1   E6,E7,E8,E9,E10,E11,E13,A21,A22,A23,A24,A25,A26,A27,A29,
     2   A210,A211,A213,E21,E22,E23,E24,E25,E26,E27,E28,E29,E210,E211,
     3   E213,C1,C2,C3,C4,C5,C9,C7,D1,D2,D3,D4,D5,D7,D9,C21,C22,C23,
     4   C24,C25,C27,C29,D21,D22,D23,D24,D25,D27,D29,B1,B2,B3,B4,
     5   DSJ,BYD
      COMMON GML,GMT,SINA,COSA,CAPH,ANU,SJ,DMCX,DMHY,DMHZ,DMCZ,ALFA,
```

```
     1    DHX,DMHX,XI,YI,ZI,EIXY,EIXZ,EIYZ,DCX,DHY,DCZ,DHZ,OMEGA,PI,WAMP
     2    ,G,PIRO4,AK,WAVEL,DEG,PI2,EMASS,EIX,EIY,EIZ,B,HH,T1,CAPK,CSHKH
     3    ,CSH2KH,SNHKH,SNH2KH,A28,B5,PDH,HL,DC,YBAR,RO,CM,VCMAX,SNHKH3
      COMMONTMON,THRUST,WIND,ALPHA,WNVX2,WNVZ2,WX,WZ,WMX,WMZ,AF,DIR
     1    FCX,FHX,FCZ,FHZ,Q,DOMX8,DOMX10,DOMX12,DOMY8,DOMY10,DOMY12,
     2    DOMZ8,DOMZ10,DOMZ12,PER
      COMMON SNHKH2,SNHKH4,SNHKH5,CSHKH3,CSH4KH,SNH4KH,OMEGAC,PIHL,
     1    PIHL2,CMC,AC1,AC2,AC3,AC4,AC5,C,H
      COMMON SINY7,COSY7,SINY9,COSY9,COSY11,SINY11,Y7, 8,Y9,Y10,Y11,
     1    Y12,C9S11,C7C11,C9S7C1,S9S7S1,S9S11,C9C11,C7S11,S9S7C1,C9C7,
     2    S9C7,S9C11,C9S7S1,S9C7S1
      COMMON XD(8),ZD(8),YD(8),DH(8),HD(8),DIAG(8),RT(14),RD(14),AX(4),
     1    AY(4),AZ(4),ISIN(24),JSIN(24),KSIN(24),NOEL,NOADM
C     ********** AMAT CALCULATES THE COEFFICIENT MATRIX **********
C        THE COEFFICIENT MATRIX IS SET UP AS FOLLOWS"
C          THE FIRST ROW IS SURGE ACCELERATION
C          THE SECOND ROW IS HEAVE ACCELERATION
C          THE THIRD ROW IS SWAY ACCELERATION
```

```
C             THE FOURTH ROW ISROLL MOMEMTUM
C             THE FIFTH ROW IS YAW MOMENTUM
C             THE SIXTH ROW IS PITCH MOMENTUM
        DO 1 I=1,6
        DO 1 J=1,6
1       A(I,J)=0
        S9C7C1=SINY9*COSY7*COSY11
        S7C11=SINY7*COSY11
        C9C7C1=COSY9*COSY7*COSY11
        C9C7S1=COSY9*COSY7*SINY11
        S9S7=SINY9*SINY7
        C9S7=COSY9*SINY7
        C9S11=COSY9*SINY11
        S7S11=SINY7*SINY11
        C9S7C1=COSY9*SINY7*COSY11
        DOMX8=S9S7S1+C9C11
        DOMY8=C7S11
        DOMZ8=C9S7S1-S9C11
```

```
DOMX10=S9S7C1-C9S11
DOMY10=C7C11
DOMZ10=C9S7C1+S9S11
DOMX12=S9C7
DOMZ12=C9C7
A2C2=EMASS
A2C8=0
A2C10=0
A2C12=0
A4C4=EMASS
A4C8=0
A4C10=0
A4C12=0
A6C6=EMASS
A6C8=0
A6C10=0
A6C12=0
A8C4=0
```

```
A8C6=0
A8C8=EIX*DOMX8-EIXY*DOMY8-EIXZ*DOMZ8
A8C10=EIX*DOMX10-EIXY*DOMY10-EIXZ*DOMZ10
A8C12=EIX*DOMX12-EIXY*DOMY12-EIXZ*DOMZ12
A10C2=0
A10C6=0
A10C8=EIY*DOMY8-EIXY*DOMX8-EIXZ*DOMZ8
A10C10=EIY*DOMY10-EIXY*DOMX10-EIXZ*DOMZ10
A10C12=EIY*DOMY12-EIXY*DOMX12-EIXZ*DOMZ12
A12C2=0
A12C4=0
A12C8=EIX*DOMZ8-EIXZ*DOMX8-EIYZ*DOMY8
A12C10=EIZ*DOMZ10-EIXZ*DOMX10-EIYZ*DOMY10
A12C12=EIZ*DOMZ12-EIXZ*DOMX12-EIYZ*DOMY12
DO 2 I=1,24
XC(I)=XI(I)
YH(I)=YI(I)
ZC(I)=ZI(I)
```

```
C        ********ARM"S ARE THE MOMENT ARMS*************
         ARMX1=ABS(YH(I))*SINY7-ZC(I)*COSY7
         ARMX2=ABS(YH(I))*COSY7+ZC(I)*SINY7
         ARMX3=-ANU(I)*COSY7+ZC(I)*SINY7
         ARMY1=ZC(I)*COSY9-XC(I)*SINY9
         ARMY2=XC(I)*COSY9+ZC(I)*SINY9
         ARMZ1=ABS(YH(I))*SINY11+XC(I)*COSY11
         ARMZ2=ABS(YH(I))*COSY11-XC(I)*SINY11
         ARMZ3=-ANU(I)*COSY11-XC(I)*SINY11
         DMYI=.5*(YH(I)+.5*PDH(I)+B)
         FHX8=DMHX(I)*(ZC(I)*DOMY8-YH(I)*DOMZ8)
         FCX8=DMCX(I)*SJ(I)*(ZC(I)*DOMY8-DMYI*DOMZ8)
         FHX10=DMHX(I)*(ZC(I)*DOMY10-YH(I)*DOMZ10)
         FCX10=DMCX(I)*SJ(I)*(ZC(I)*DOMY10-DMYI*DOMZ10)
         FHX12=DMHX(I)*(ZC(I)*DOMY12-YH(I)*DOMZ12)
         FCX12=DMCX(I)*SJ(I)*(ZC(I)*DOMY12-DMYI*DOMZ12)
         FHY8=DMHY(I)*(XC(I)*DOMZ8-ZC(I)*DOMX8)
         FHY10=DMHY(I)*(XC(I)*DOMZ10-ZC(I)*DOMX10)
```

```
FHY12=DMHY(I)*(XC(I)*DOMZ12-ZC(I)*DOMX12)
FHZ8=DMHZ(I)*(YH(I)*DOMX8-XC(I)*DOMY8)
FCZ8=DMCZ(I)*SJ(I)*(DMYI*DOMX8-XC(I)*DOMY8)
FHZ10=DMHZ(I)*(YH(I)*DOMX10-XC(I)*DOMY10)
FCZ10=DMCZ(I)*SJ(I)*(DMYI*DOMX10-XC(I)*DOMY10)
FHZ12=DMHZ(I)*(YH(I)*DOMX12-XC(I)*DOMY12)
FCZ12=DMCZ(I)*SJ(I)*(DMYI*DOMX12-XC(I)*DOMY12)
A2C2=A2C2+DMHX(I)+DMCX(I)*SJ(I)
A2C8=A2C8+FHX8+FCX8
A2C10=A2C10+FHX10+FCX10
A2C12=A2C12+FHX12+FCX12
A4C4=A4C4+DMHY(I)
A4C8=A4C8+FHY8
A4C10=A4C10+FHY10
A4C12=A4C12+FHY12
A6C6=A6C6+DMHZ(I)+DMCZ(I)*SJ(I)
A6C8=A6C8+FHZ8+FCZ8
A6C10=A6C10+FHZ10+FCZ10
```

```
      A6C12=A6C12+FHZ12+FCZ12
      A8C4=A8C4+DMHY(I)*ARMX1
      A8C6=A8C6-DMHZ(I)*ARMX2-SJ(I)*DMCZ(I)*ARMX3
      A8C10=A8C10+FHY10*ARMX1-FHZ10*ARMX2-FCZ10*ARMX3
      A8C12=A8C12+FHY12*ARMX1-FHZ12*ARMX2-FCZ12*ARMX3
      A10C2=A10C2+DMHX(I)*ARMY1+DMCX(I)*SJ(I)*ARMY1
      A10C6=A10C6-DMHZ(I)*ARMY2-DMCZ(I)*SJ(I)*ARMY2
      A10C8=A10C8+FHX8*ARMY1+FCX8*ARMY1-FHZ8*ARMY2-FCZ8*ARMY2
      A10C10=A10C10+FHX10*ARMY1+FCX10*ARMY1-FHZ10*ARMY2-FCZ10*ARMY2
      A10C12=A10C12+FHX12*ARMY1+FCX12*ARMY1-FHZ12*ARMY2-FCZ12*ARMY2
      A12C2=A12C2+DMHX(I)*ARMZ2+DMCX(I)*SJ(I)*ARMZ3
      A12C4=A12C4+DMHY(I)*ARMZ1
      A12C8=A12C8+FHY8*ARMZ1+FHX8*ARMZ2+FCX8*ARMZ3
      A12C10=A12C10+FHY10*ARMZ1+FHX10*ARMZ2+FCX10*ARMZ3
2     A12C12=A12C12+FHY12*ARMZ1+FHX12*ARMZ2+FCX12*ARMZ3
      A(1,1)=A2C2
      A(1,4)=A2C8
      A(1,5)=A2C10
```

```
A(1,6)=A2C12
A(2,2)=A4C4
A(2,4)=A4C8
A(2,5)=A4C10
A(2,6)=A4C12
A(3,3)=A6C6
A(3,4)=A6C8
A(3,5)=A6C10
A(3,6)=A6C12
A(4,2)=A8C4
A(4,3)=A8C6
A(4,4)=A8C8
A(4,5)=A8C10
A(4,6)=A8C12
A(5,1)=A10C2
A(5,3)=A10C6
A(5,4)=A10C8
A(5,5)=A10C10
```

```
A(5,6)=A10C12
A(6,1)=A12C2
A(6,2)=A12C4
A(6,4)=A12C8
A(6,5)=A12C10
A(6,6)=A12C12
RETURN
END
```

```
      SUBROUTINE PRKGS(PRMT,Y,DERY,NDIM,IHLF,AUX)
      DIMENSION Y(1),DERY(1),AUX(8,1),A(4),B(4),C(4),PRMT(1)
      DO 1 I=1,NDIM
    1 AUX(8,I)=.066666666666666667D0*DERY(I)
      X=PRMT(1)
      XEND=PRMT(2)
      H=PRMT(3)
      PRMT(5)=0.
      CALL FCT(X,Y,DERY)
C     ERROR TEST
      IF(H*(XEND-X))38,37,2
C     PREPARATIONS FOR RUNGE-KUTTA METHOD
    2 A(1)=.5
      A(2)=.29289321881345248
      A(3)=1.7071067811865475
      A(4)=.16666666666666667
      B(1)=2.
      B(2)=1.
```

```
      B(3)=1.
      B(4)=2.
      C(1)=.5
      C(2)=.29289321881345248
      C(3)=1.7071067811865475
      C(4)=.5
C     PREPARATIONS OF FIRST RUNGE-KUTTA STEP
      DO 3 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=0.
    3 AUX(6,I)=0.
      IREC=0
      H=H+H
      IHLF=-1
      ISTEP=0
      IEND=0
C     START OF A RUNGE-KUTTA STEP
```

```
    4 IF((X+H-XEND)*H)7,6,5
    5 H=XEND-X
    6 IEND=1
C       RECORDING OF INITIAL VALUES OF THIS STEP
    7 CALL OUTP(X,Y,DERY,IREC,NDIM,PRMT)
      IF(PRMT(5))40,8,40
    8 ITEST=0
    9 ISTEP=ISTEP+1
C       START OF INNERMOST RUNGE-KUTTA LOOP
      J=1
   10 AJ=A(J)
      BJ=B(J)
      CJ=C(J)
      DO 11 I=1,NDIM
      R1=H*DERY(I)
      R2=AJ*(R1-BJ*AUX(6,I))
      Y(I)=Y(I)+R2
      R2=R2+R2+R2
```

```
   11 AUX(6,I)=AUX(6,I)+R2-CJ*R1
   12 J=J+1
      IF(J-3)13,14,13
   13 X=X+.5E0*H
   14 CALL FCT(X,Y,DERY)
      GO TO 10
C     END OF INNERMOST RUNGE-KUTTA LOOP
C     TEST OF ACCURACY
   15 IF(ITEST)16,16,20
C     IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY
   16 DO 17 I=1,NDIM
   17 AUX(4,I)=Y(I)
      ITEST=1
      ISTEP=ISTEP+ISTEP-2
   18 IHLF=IHLF+1
      X=X-H
      H=.5E0*H
      DO 19 I=1,NDIM
```

```
      Y(I)=AUX(1,I)
      DERY(I)=AUX(2,I)
   19 AUX(6,I)=AUX(3,I)
      GO TO 9
C     IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE
   20 IMOD=ISTEP/2
      IF(ISTEP-IMOD-IMOD)21,23,21
   21 CALL FCT(X,Y,DERY)
      DO 22 I=1,NDIM
      AUX(5,I)=Y(I)
   22 AUX(7,I)=DERY(I)
      GO TO 9
C     COMPUTATION OF TEST VALUE DELT
   23 DELT=0.
      DO 24 I=1,NDIM
   24 DELT=DELT+AUX(8,I)*ABS(AUX(4,I)-Y(I))
      IF(DELT-PRMT(4))28,28,25
C     ERROR IS TOO GREAT
```

```
   25 IF(IHLF-10)26,36,36
   26 DO 27 I=1,NDIM
   27 AUX(4,I)=AUX(5,I)
      ISTEP=ISTEP+ISTEP-4
      X=X-H
      IEND=0
      GO TO 18
C      RESULT VALUES ARE GOOD
   28 CALL FCT(X,Y,DERY)
      DO 29 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=AUX(6,I)
      Y(I)=AUX(5,I)
   29 DERY(I)=AUX(7,I)
      CALL OUTP(X-H,Y,DERY,IHLF,NDIM,PRMT)
      IF(PRMT(5))40,30,40
   30 DO 31 I=1,NDIM
```

```
      Y(I)=AUX(1,I)
   31 DERY(I)=AUX(2,I)
      IREC=IHLF
      IF(IEND)32,32,39
C     INCREMENT GETS DOUBLED
   32 IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      IF(IHLF)4,33,33
   33 IMOD=ISTEP/2
      IF(ISTEP-IMOD-IMOD)4,34,4
   34 IF(DELT-.02E0*PRMT(4))35,35,4
   35 IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      GO TO 4
C     RETURNS TO CALLING PROGRAM
   36 IHLF=11
```

```
      CALL FCT(X,Y,DERY)
      GO TO 39
   37 IHLF=12
      GO TO 39
   38 IHLF=13
   39 CALL OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
   40 RETURN
      END
```

```
      SUBROUTINE RHSV(T,Y,RHS)
      DIMENSION Y(12),DMCX(24),DMHY(24),DMHZ(24),DMCZ(24),ALFA(24),CAPK
     1 (24),PDH(24),YH(24),ZC(24),XC(24),DC(24),HL(24),SJ(24),YBAR
     2 (24),AF(8),RHS(6),DCX(24),DHY(24),DCZ(24),DHZ(24),P(24),Q(24),
     3 R(24),TH(24),FF(24),FR(24),ANU(24),FCX(24),FHX(24),FCZ(24),
     4 FHZ(24),DHX(24),DMHX(24),XI(24)YI(24),ZI(24)
      DIMENSION A1(24),A2(24),A3(24),A4(24),A5(24),A6(24),A7(24),A8(24)
     1 ,A9(24),A10(24),A11(24),A13(24),E1(24),E2(24),E3(24),E4(24),
     2 E5(24),E6(24),E7(24),E8(24),E9(24),E10(24),E11(24),E13(24),
     3 A21(24),A22(24),A23(24),A24(24),A25(24),A26(24),A27(24),
     4 A28(24),A29(24),A210(24),A211(24),A213(24),E21(24),E22(24),
     5 E23(24),E24(24),E25(24),E26(24),E27(24),E28(24)
      DIMENSION E29(24),E210(24),E211(24),E213(24),C1(24),C2(24),
     1 C3(24),C4(24),C5(24),C7(24),C9(24),D1(24),D2(24),D3(24),D4(24)
     2 ,D5(24),D7(24),D9(24),C21(24),C22(24),C23(24),C24(24),C25(24),
     3 C27(24),C29(24),D21(24),D22(24),D23(24),D24(24),D25(24),BYD(24)
     4 ,D27(24),D29(24),B1(24),B2(24),B3(24),B3(24),B5(24),DSJ(24),
     5 A35(24),A36(24),E35(24),E36(24),C31(24),
```

```
 DIMENSION C32(24),C33(24),D31(24),D32(24),D33(24),D69(24),C41
1    (24),C51(24),C61(24),C43(24),C53(24),C63(24),C34(24),C44(24),
2    C54(24),C35(24),C45(24),C55(24),C65(24),C37(24),C47(24),C57
3    (24),C67(24),C39(24),C49(24),C59(24),C69(24),D41(24),D51(24),
4    D61(24),D42(24),D52(24),D62(24),D43(24),D53(24),D63(24),D34
5    (24),D44(24),D54(24),D64(24),D35(24),D45(24),D55(24),D65(24),
6    D37(24),D47(24),D57(24),D67(24),D39(24),D49(24),D59(24)
 DIMENSION DELX(4),DELZ(4),ARM(12),FX(4),FZ(4),TENSX(4),TENSZ(4),
1    FY(4)
 COMMON A35,A36,E35,E36,C31,C32,C33,D31,D32,D33,D59,D69,C41,C51,
1    C61,C43,C53,C63,C34,C44,C54,C35,C45,C55,C65,C37,C47,C57,C67,
2    C39,C49,C59,C69,D41,D51,D61,D42,D52,D62,D43,D53,D63,D34,D44,
3    D54,D64,D35,D45,D55,D65,D37,D47,D57,D67,D39,D49
 COMMON A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A13,E1,E2,E3,E4,E5,E6,
1    E7,E8,E9,E10,E11,E13,A21,A22,A23,A24,A25,A26,A27,A29,A210,
2    A211,A213,E21,E22,E23,E24,E25,E26,E27,E28,E29,E210,E211,E213,
3    C1,C2,C3,C4,C5,C9,C7,D1,D2,D3,D4,D5,D7,D9,C21,C22,C23,C24,
4    C25,C27,C29,D21,D22,D23,D24,D25,D27,D29,B1,B2,B3,B4,DSJ,BYD
```

```
      COMMON GML,GMT,SINA,COSA,CAPH,ANU,SJ,DMCX,DMHY,DMHZ,DMCZ,ALFA,
     1   DHX,DMHX,XI,YI,ZI,EIXY,EIXZ,EIYZ,DCX,DHY,DCZ,DHZ,OMEGA,PI,
     2   WAMP,G,PIR04,AK,WAVEL,DEG,P12,EMASS,EIX,EIY,EIZ,B,HH,T1,CAPK,
     3   CSHKH,CSH2KH,SNHKH,SNH2KH,A28,B5,PDH,HL,DC,YBAR,RO,CM,VCMAX,
     4   SNHKH3
      COMMONTMOM,THRUST,WIND,ALPHA,WNVX2,WNVZ2,WX,WZ,WMX,WMZ,AF,DIR,
     1   FCX,FHX,FCZ,FHZ,Q,DOMX8,DOMX10,DOMX12,DOMY8,DOMY10,DOMY12,
     2   DOMZ8,DOMZ10,DOMZ12,PER
      COMMON SNHKH2,SNHKH4,SNHKH5,CSHKH3,CSH4KH,SNH4KH,OMEGAC,PIHL,
     1   PIHL2,CMC,AC1,AC2,AC3,AC4,AC5,C,H
      COMMON SINY7,COSY7,SINY9,COSY9,COSY11,SINY11,Y7,Y8,Y9,Y10,Y11,
     1   Y12,C9S11,C7C11,C9S7C1,S9S7S1,S9S11,C9C11,C7S11,S9S7C1,C9C7,
     2   S9C7,S9C11,C9S7S1,S9C7S1
      COMMON XD(8),ZD(8),YD(8),DH(8),HD(8),DIAG(8),RT(14),RD(14),AX(4),
     1   AY(4),AZ(4),ISIN(24),JSIN(24),KSIN(24),NOEL,NOADM
C     ********* RHSV CALCULATES THE RHS VECTOR *********
C     RHSV IS SET UP AS FOLLOWS
C        1) RHSV SUMS ALL THE FORCES AND MOMENTS FOR SIX DEGREES OF
```

```
C          FREEDOM
C          2) IN ADDITION, THE VECTOR CONTAINS THE FIRST DERIVATIVE
C          PORTIONS OF ACCELERATION AND ANGULAR MOMENTUM WITH A CHANGE
C          IN SIGN
C          3) RHS(1)=SURGE FORCE
C          4) RHS(2)=HEAVE FORCE
C          5) RHS(3)=SWAY FORCE
C          6) RHS(4)=ROLL MOMENT
C          7) RHS(5)=YAW MOMENT
C          8) RHS(6)=PITCH MOMENT
      DO 6 I=1,24
      XC(I)=XI(I)
      YH(I)=YI(I)
    6 ZC(I)=ZI(I)
C     ********WIND FORCES IN X AND Z DIRECTIONS*********
      FWX=WX*WNVX2
      FWZ=WZ*WNVZ2
C     ********WIND MOMENTS ABOUT X AND Z***************
```

```
      WMY7=WMZ*WNVZ2
      WMY11=WMX*WNVX2
C     ********DISPL CALCULATES THE DISPLACEMENT OF THE FAIRLEADS*******
      CALL DISPL(Y,DELX,DELZ,ARM,AX,AZ,AY)
C     FOXYZ CALCULATES THE MOORING LINE TENSIONS**********
      CALL FOXYZ(DELX,DELZ,FX,FY,FZ,TENSX,TENSZ,DIRC,THR,RT,RD)
      DIR=DIRC
      THRUST=THR
      AF(1)=TENSZ(1)
      AF(2)=TENSX(1)
      AF(3)=TENSX(2)
      AF(4)=TENSZ(2)
      AF(5)=TENSZ(4)
      AF(6)=TENSX(4)
      AF(7)=TENSX(3)
      AF(8)=TENSZ(3)
      Y8Y8=Y8*Y8
      Y10Y10=Y10*Y10
```

```
      Y12Y12=Y12*Y12
      Y8Y10=Y8*Y10
      Y8Y12=Y8*Y12
      Y10Y12=Y10*Y12
      S9C7C1=SINY9*COSY7*COSY11
      S7C11=SINY7*COSY11
      C9C7C1=COSY9*COSY7*COSY11
      C9C7S1=COSY9*COSY7*SINY11
      S9S7=SINY9*SINY7
      C9S7=COSY9*SINY7
      C9S11=COSY9*SINY11
      S7S11=SINY7*SINY11
      C7C11=COSY7*COSY11
C     ********ANGULAR VELOCITIES**************
      OMEGX=S9C7*Y12+(S9S7S1+C9C11)*Y8+(S9S7C1-C9S11)*Y10
      OMEGY=-SINY7*Y12+C7S11*Y8+C7C11*Y10
      OMEGZ=C9C7*Y12+(C9S7S1-S9C11)*Y8+(C9S7C1+S9S11)*Y10
C     ********FIRST DERIVATIVE PART OF ANGULAR ACCELLERATION********
```

```
      DOMGX=-S9S7*Y8Y12+C9C7*Y10Y12+C9S7S1*Y8Y10+S9C7S1*Y8Y8+S9S7C1*
     1 Y8Y12-S9C11*Y8Y10-C9S11*Y8Y12+C9S7C1*Y10Y10+S9C7C1*Y8Y10
     2 -S9S7S1*Y10Y12+S9S11*Y10Y10-C9C11*Y10Y12
      DOMGZ=-C9S7*Y8Y12-S9C7*Y10Y12-S9S7S1*Y8Y10+C9C7S1*Y8Y8+C9S7C1*
     1      Y8Y12-C9C11*Y8Y10+S9S11*Y8Y12-S9S7C1*Y10Y10+C9C7C1*Y8Y10-C9S7S
     2      1*Y10Y12+C9S11*Y10Y10+S9C11*Y10Y12
      DOMGY=-S7S11*Y8Y8-C7S11*Y10Y12-S7C11*Y8Y10-COSY7*Y8Y12+C7C11*Y8Y1
     2      2
      HPX=EIX*OMEGX-EIXY*OMEGY-EIXZ*OMEGZ
      HPY=-EIXY*OMEGX+EIY*OMEGY-EIYZ*OMEGZ
      HPZ=-EIXZ*OMEGX-EIYZ*OMEGY+EIZ*OMEGZ
      HDOTX=EIX*DOMGX-EIXY*DOMGY-EIXZ*DOMGZ
      HDOTY=-EIXY*DOMGX+EIY*DOMGY-EIYZ*DOMGZ
      HDOTY=-EIXY*DOMGX+EIY*DOMGY-EIYZ*DOMGZ
      HDOTZ=-EIXZ*DOMGX-EIYZ*DOMGY+EIZ*DOMGZ
C     ********FIRST DERIVATIVE PART OF ANGULAR MOMENTUM**********
      RHS4=-(HDOTX-OMEGZ*HPY+OMEGY*HPZ)
      RHS5=-(HDOTY-OMEGX*HPZ+OMEGZ*HPX)
```

```
      RHS6=-(HDOTZ-OMEGY*HPX+OMEGX*HPY)
C     ********THRUST MOMENT************
      TMOM=FX(1)*((AZ(2)+.5*DC(7))*COSY9-AX(2)*SINY9)-FX(2)*((-AZ(1)+
     1 .5*DC(3))*COSY9+AX(1)*SINY9)-FX(3)*((AZ(4 )+.5*DC(24))*COSY9
     2 -AX(4)*SINY9)+FX(4)*((-AZ(3)+.5*DC(23))*COSY9+AX(3)*SINY9)-
     3 FZ(1)*(AX(2)*COSY9+(AZ(2)+.5*DC(7))*SINY9)+FZ(2)*(AX(1)*COSY
     4 9-(-AZ(1)+.5*DC(3))*SINY9)+FZ(3)*(-AX(4)*COSY9-(AZ(4)+.5*DC
     5 (24))*SINY9)-FZ(4)*(-AX(3)*COSY9+(-AZ(3)+.5*DC(23))*SINY9)
      OMEGXX=OMEGX*OMEGX
      OMEGYY=OMEGY*OMEGY
      OMEGZZ=OMEGZ*OMEGZ
      OMEGXY=OMEGX*OMEGY
      OMEGXZ=OMEGX*OMEGZ
      OMEGYZ=OMEGY*OMEGZ
      VCSINA=VCMAX*SINA
      VCCOSA=VCMAX*COSA
      DO 2 I=1,24
      COSB1=COS(B1(I))
```

```
COSB21=COS(2*B1(I))
SINB1= SIN(B1(I))
SINB21= SIN(2*B1(I))
COSB2= COS(B2(I))
COSB3= COS(B3(I))
COSB22= COS(2*B2(I))
COSB23= COS(2*B3(I))
COSB4= COS(B4(I))
COSB5= COS(B5(I))
COSB24= COS(2*B4(I))
COSB25= COS(2*B5(I))
SINB2= SIN(B2(I))
SINB3= SIN(B3(I))
SINB22= SIN(2*B2(I))
SINB23= SIN(2*B3(i))
SINB4= SIN(B4(I))
SINB5= SIN(B5(I))
SINB41= SIN(4*B1(I))
```

```
      SINB24= SIN(2*B4(I))
      SINB25= SIN(2*B5(I))
      COSB42= COS(4*B2(I))
      COSB43= COS(4*B3(I))
      COSB44= COS(4*B4(I))
      COSB45= COS(4*B5(I))
      SIN3B2=SINB2**3
      SIN3B3=SINB3**3
      COS3B3=COSB3**3
      COS3B2=COSB2**3
      SIN3B4=SINB4**3
      SIN3B5=SINB5**3
      COS3B4=COSB4**3
      COS3B5=COSB5**3
      DMYI=.5*(YH(I)+.5*PDH(I)+B)
C     ********RELATIVE VELOCITIES****************
      VBHX=JSIN(I)*(Y(2)+ZC(I)*OMEGY-YH(I)*OMEGZ+E8(I)*(SINB4-SINB5)
     1   +E28(I)*(SINB24-SINB25)+VCCOSA)
```

```
      VBHY=Y(4)+XC(I)*OMEGZ-ZC(I)*OMEGX-JSIN(I)*E10(I)*(COSB4-COSB5)-
     1    JSIN(I)*E210(I)*(COSB24-COSB25)-KSIN(I)*E11(I)*(COSB2-COSB3)-
     2    KSIN(I)*E211(I)*(COSB22-COSB23)
      VBHZ=KSIN(I)*Y(6)+YH(I)*OMEGX-XC(I)*OMEGY+E13(I)*(SINB2-SINB3)
     1    +E213(I)*(SINB22-SINB23)+VCSINA)
      VBCZ=(Y(6)-XC(I)*OMEGY+DMYI*OMEGX-E9(I)*COSB1-E29(I)*COSB21+
     1    VCSINA)*KSIN(I)
      VBCX=(Y(2)+ZC(I)*OMEGY-DMYI*OMEGZ-E7(I)*COSB1-E27(I)*COSB21+
     1    VCCOSA)*KSIN(I)
C     ********WAVE PART OF RELATIVE ACCELERATION**********
      TACZW=SJ(I)*DMCZ(I)*(E1(I)*SINB1+E21(I)*SINB21)*KSIN(I)
      TAHZW=-DMHZ(I)*KSIN(I)*(E2(I)*(COSB2-COSB3)+E22(I)*(COSB22-
     1    COSB23))
      TACXW=-SJ(I)*DMCX(I)*(E3(I)*SINB1+E23(I)*SINB21)*KSIN(I)
      TAHXW=-DMHX(I)*JSIN(I)*(E4(I)*(COSB4-COSB5)+E24(I)*(COSB24-
     1    COSB25))
      TAHYW=-DMHY(I)*KSIN(I)*(E5(I)*(SINB2-SINB3)+E25(I)*(SINB22-
     1    SINB23))-DMHY(I)*JSIN(I)*(E6(I)*(SINB4-SINB5)+E26(I)*
```

```
2    SINB24-SINB25))
C      ********WAVE PRESSURE FORCES*************
       WAVFY1=-(D1(I)*(SINB2-SINB3)+D21(I)*(SINB22-SINB23)-D31(I)+D41(I)
1    *(SIN3B2-SIN3B3)-D51(I)*(SINB2-SINB3))*KSIN(I)
       WAVFY2=-(D2(I)*COSB1+D22(I)*COSB21-D32(I)+D42(I)*SINB1*SINB1*
1    COSB1-D52(I)*COSB1)*KSIN(I)
       WAVFY3=-(D3(I)*(SINB4-SINB5)+D23(I)*(SINB24-SINB25)-D33(I)+D43(I)
1    *(SIN3B4-SIN3B5)-D53(I)*(SINB4-(SINB5))*JSIN(I)
       WAVCZ=-(-D4(I)*SINB1-D24(I)*SINB21+D34(I)*SINB41+D44(I)*COSB1*
1    COSB1*SINB1+D54(I)*SINB1)*KSIN(I)
       WVFHZ1=-(D5(I)*(COSB2-COSB3)+D25(I)*(COSB22-COSB23)-D35(I)*(COSB4
1    2-COSB43)-D45(I)*(COS3B2-COS3B3)-D55(I)*(COSB2-COSB3))*KSIN(I)
       WVFHX2=-(D7(I)*(COSB4-COSB5)+D27(I)*(COSB24-COSB25)-D37(I)*(COSB
1    44-COSB45)-D47(I)*(COS3B4-COS3B5)-D57(I)*(COSB4-COSB5))*JSIN(I
2    )
       WVFCX=-(-D9(I)*SINB1-D29(I)*SINB21+D39(I)*SINB41+D49(I)*COSB1*
1    COSB1*SINB1+D59(I)*SINB1)*KSIN(I)
       MSIGN=ISIN(I)
```

```
C        ********MOMENTS ARMS***********
         ARMX1= ABS(YH(I))*SINY7-ZC(I)*COSY7
         ARMX2= ABS(YH(I))*COSY7ZC(I)*SINY7
         ARMX3=-ANU(I)*COSY7+ZC(I)*SINY7
         ARMY1=ZC(I)*COSY9-XC(I)*SINY9
         ARMY2=XC(I)*COSY9+ZC(I)*SINY9
         ARMZ1= ABS(YH(I))*SINY11+XC(I)*COSY11
         ARMZ2= ABS(YH(I))*COSY11-XC(I)*SINY11
         ARMZ3=-ANU(I)*COSY11-XC(I)*SINY11
C        ********NORMAL ACELERATIONS************
         ANCZ=XC(I)*OMEGXZ-ZC(I)*OMEGXX-ZC(I)*OMEGYY+DMYI*OMEGYZ
         ANHZ=XC(I)*OMEGXZ-ZC(I)*OMEGXX-ZC(I)*OMEGYY+YH(I)*OMEGYZ
         ANHZ=KSIN(I)*ANHZ
         ANCX=DMYI*OMEGXY-XC(I)*OMEGYY-OMEGZZ*XC(I)*OMEGXZ
         ANHX=YH(I)*OMEGXZ-XC(I)*OMEGYY-XC(I)*OMEGZZ+ZC(I)*OMEGXZ
         ANHX=JSIN(I)*ANHX
         ANHY=ZC(I)*OMEGYZ-YH(I)*OMEGZZ-YH(I)*OMEGXX+XC(I)*OMEGXY
         ANCX=KSIN(I)*ANCX
```

```
      ANCZ=KSIN(I)*ANCZ
      ANCX=-SJ(I)*DMCX(I)*ANCX
      ANHX=-DMHX(I)*ANHX
      ANCZ=-SJ(I)*DMCZ(I)*ANCZ
      ANHZ=-DMHZ(I)*ANHZ
      ANHY=-DMHY(I)*ANHY
C     ********FIRST DERIVATIVE PORTION OF TANGENTIAL ACCELERATION*****
      TACX=-DMCX(I)*SJ(I)*(ZC(I)*DOMGY-DMYI*DOMGZ)*KSIN(I)
      TAHY=-DMHY(I)*(XI(I)*DOMGZ-ZI(I)*DOMGX)
      TACZ=-DMCZ(I)*SJ(I)*(DMYI*DOMGX-XC(I)*DOMGY)*KSIN(I)
      TAHZ=-DMHZ(I)*(YH(I)*DOMGX-XC(I)*DOMGY)*KSIN(I)
      TAHX=-DMHX(I)*(ZI(I)*DOMGY-YI(I)*DOMGZ)*JSIN(I)
      XCB=S9C7*ZI(I)+(S9S7S1+C9C11)*XI(I)+(S9S7C1-C9S11)*YI(I)+Y(1)
      ZCB=C9C7*ZI(I)+(C9C7S1-S9C11)*XI(I)+(C9S7C1+S9S11)*YI(I)+Y(5)
      BETA=AK*(XCB-MSIGN*.5*HL(I))*COSA+AK*ZCB*SINA+OMEGA*T
      BETA2=2*BETA
C     ********Q IS VERTICAL FORCE***********
      Q(I)=-DHY(I)*VBHY*ABS(VBHY)+KSIN(I)*WAVFY1+WAVFY2+JSIN(I)
```

```
     1    WAVFY3+PIRO4*PDH(I)*PDH(I)*MSIGN*VBHY*VBCX+CAPK(I)*(WAMP*
     2    COSB1+CAPH*COSB21-YBAR(I))-D67(I)*VBHY-C67(I)*VBHY*SJ(I)
      Q(I)=Q(I)+ANHY+TAHY+TAHYW
C     ********FCZ IS FORCE ON COL IN Z DIRECTION****
      FCZ(I)=-SJ(I)*(DCZ(I)*VBCZ*ABS(VBCZ)-WAVCZ)
      FCZ(I)=FCZ(I)+ANCZ+TACZ+TACZW-C67(I)*VBCZ*SJ(I)
      FCZ(I)=KSIN(I)*FCZ(I)
C     ********FHZ IS FORCE ON HULL IN Z DIRECTION********
      FHZ(I)=-D67(I)*VBHZ-DHZ(I)*VBHZ*ABS(VBHZ)+KSIN(I)*WVFHZ1+
     1    PIRO4*PDH(I)*PDH(I)*MSIGN*VBHZ*VBCX
      FHZ(I)=FHZ(I)+ANHZ+TAHZ+TAHZW
      FHZ(I)=KSIN(I)*FHZ(I)
C     ********FHX IS FORCE ON HULL IN X DIRECTION*************
      FHX(I)=-D67(I)*VBHX-DHX(I)*VBHX*ABS(VBHX)+JSIN(I)*WVFHX2
      FHX(I)=FHX(I)+ANHX+TAHX+TAHXW
C     ********FHYX IS FORCE ON HULL DUE TO END PRESSURE*******
      FHYX=D61(I)*MSIGN*COS(BETA)+MSIGN*D62(I)*COS(BETA2)
      FHX(I)=JSIN(I)*FHX(I)+FHYX
```

```
C       ********SUM UP MEMBER FORCES AND MOMENTS***********
        DO 5 I=1,NOEL
        RHS(1)=RHS(1)+P(I)
        RHS(2)=RHS(2)+Q(I)
        RHS(3)=RHS(3)+R(I)
        RHS(4)=RHS(4)+TH(I)
        RHS(5)=RHS(5)+FF(I)
5       RHS(6)=RHS(6)+FR(I)
        IF(NOADM.LE.0) GO TO 470
        DO 469 I+1,NOADM
        ARMDX=ABS(YD(I))*SINY7-ZD(I)*COSY7
        ARMDZ=ABS(YD(I))*SINY11+XD(I)*COSY11
        FYD=-DIAG(I)*(Y(4)+XD(I)*OMEGZ-ZD(I)*OMEGX)
        FDMX=DYD*ARMDX
        FDMZ=FYD*ARMDZ
        RHS(2)=RHS(2)+FYD
        RHS(4)=RHS(4)+FDMX
469     RHS(6)=RHS(6)+FDMZ
```

```
  470 CONTINUE
C       ********ADD MOORING FORCES AND MOMENTS*********
        RHS(1)=RHS(1)+FX(1)+FX(2)-FX(3)-FX(4)-FWX
        RHS(2)=RHS(2)-FY(1)-FY(2)-FY(3)-FY(4)
        RHS(3)=RHS(3)+FZ(1)-FZ(2)+FZ(3)-FZ(4)-FWZ
        RHS(4)=RHS(4)+FY(1)*(AZ(2)+.5*DC(7))*COSY7-FY(2)*(-AZ(1)+.5*DC(3)
     1  )*COSY7+FY(3)*(AZ(4)+.5*DC(24))*COSY7-FY(4)*(-AZ(3)+.5*DC(23))
     2  *COSY7-FZ(1)*(AZ(2)+.5*DC(7))*SINY7-FZ(2)*(-AZ(1)+.5*DC(3))*
     3  SINY7-FZ(3)*(AZ(4)+.5*DC(24))*SINY7-FZ(4)*(-AZ(3)+.5*DC(23))
     4  *SINY7-WMY7
        RHS(5)=RHS(5)+TMOM
        RHS(6)=RHS(6)-FX(1)*AX(2)*SINY11-FX(2)*AX(1)*SINY11+FX(3)*AX(4)
     2  *SINY11+FX(4)*AX(3)*SINY11-FY(1)*AX(2)*COSY11-FY(2)*AX(1)
     3  *COSY11-FY(3)*AX(4)*COSY11-FY(4)*AX(3)*COSY11+WMY11
        RETURN
        END
```

```
      SUBROUTINE DISPL(Y,DELX,DELZ,ARM,X,Z,YH)
      DIMENSION X(4),Z(4),YH(4),XX(4),ZZ(4),YY(4),DELX(4),DELZ(4),
     1   ARM(12),Y(12)
      C1=COS(Y(7))
      C2=COS(Y(9))
      C3=COS(Y(11))
      S1=SIN(Y(7))
      S2=SIN(Y(9))
      S3=SIN(Y(11))
      E1=C1*C2
      F1=S2*C1
      G1=-S1
      E2=S3*S1*C2-S2*C3
      F2=C2*C3+S2*S1*S3
      G2=C1*S3
      E3=S2*S3+C2*S1*C3
      F3=S2*S1*C3-C2*S3
      G3=C1*C3
```

```
      DO 2 I=1,4
      ZZ(I)=E1*Z(I)+E2*X(I)+E3*YH(I)
      XX(I)=F1*Z(I)+F2X(I)+F3*YH(I)
      YY(I)=G1*Z(I)+G2*X(I)+G3*YH(I)
      DELX(I)=Y(I)+XX(I)-X(I)
2     DELZ(I)=Y(5)+ZZ(I)-Z(I)
      DO 3 I=1,4
      ARM(I)=XX(I)
      ARM(I+4)=YY(I)
3     ARM(I+8)=ZZ(I)
      RETURN
      END
```

```
      SUBROUTINE FOXYZ(DELX,DELZ,FX,FY,FZ,TENSX,TENSZ,DIRC,THR,RT,RD)
      DIMENSION RD(14),RT(14),DELX(4),DELZ(4),FX(4),FY(4),FZ(4),TX(4),
     1   TZ(4),DISTX(4),DISTZ(4),TENSX(4),TENSZ(4)
      NDIM=14
C     ********DISTL IS THE HORIZ DISTANCE BETWEEN FAIRLEAD AND ANCHOR
C             WHEN CHAIN IS FULLY STRETCHED OUT, THIS VALUE TAKEN
C             FROM SEPARATE PROGRAM**********
      DISTL=2964.32
C     ********DISTI IS THE DISTANCE FROM THE ANCHOR TO GIVE THE DESIRED
C             INITIAL CHAIN TENSION. THIS VALUE IS TAKEN FROM SEPARATE
C             PROGRAM**********
      DISTI=2889.66
      DD=100.
C     ********FFC IS THE CHAIN TENSION WHEN THE CHAIN IS COMPLETELY
C             SLACK ITS VALUE IS PRE-DETERMINED BY MULTIPLYING THE
C             EFFECTIVE UNIT WEIGHT OF CHAIN BY WATER DEPTH**********
      FFC=24000.
C     ********XO IS THE LENGTH OF CHAIN LAYING FLAT WHEN CHAIN IS
```

```
C              COMPLETELY SLACK**********
        XO=2600.
        XL=1.
        XR=DISTL/XO
        DISTX(1)=(DISTI-DELX(1))/XO
        DISTX(2)=(DISTI-DELX(2))/XO
        DISTX(3)=(DISTI+DELX(3))/XO
        DISTX(4)=(DISTI+DELX(4))/XO
        DISTZ(1)=(DISTI-DELZ(1))/XO
        DISTZ(2)=(DISTI+DELZ(2))/XO
        DISTZ(3)=(DISTI-DELZ(3))/XO
        DISTZ(4)=(DISTI+DELZ(4))/XO
        DO 1 I=1,4
        DELX(I)=0.0
        DELZ(I)=0.0
        TX(I)=(2*DISTX(I)-(XL+XR))/(XR-XL)
1       TZ(I)=(2*DISTZ(I)-(XL+XR))/(XR-XL)
        DO 3 I=1,4
```

```
      FXI=0
      FZI=0
      DYDX=0
      DYDZ=0
      TXI=TX(I)
      TZI=TZ(I)
      DO 2 J=1,NDIM
      CHEBX=P(J-1,TXI)
      CHEBZ=P(J-1,TZI)
      FXI=FXI+RT(J)*CHEBX*FFC
      FZI=FZI+RT(J)*CHEBZ*FFC
      DYDX=DYDX+RD(J)*CHEBX*DD
2     DYDZ=DYDZ+RD(J)*CHEBZ*DD
      ANGX=ATAN(DYDX)
      FX(I)=FXI*COS(ANGX)
      FZ(I)=FZI*COS(ANGZ)
      TENSX(I)=SQRT(FX(I)*FX(I)+(FXI*SIN(ANGX))**2)
      TENSZ(I)=SQRT(FZ(I)*FZ(I)+(FZI*SIN(ANGZ))**2)
```

```
3       FY(I)=FXI*SIN(ANGX)+FZI*SIN(ANGZ)
        FZT=FZ(1)-FZ(2)+FZ(3)-FZ(4)
        FXT=FX(1)+FX(2)-FX(3)-FX(4)+.01
        DIRC=ATAN(FZT/FXT)*180./3.14159
        THR=SQRT(FXT*FXT+FZT*FZT)
        RETURN
        END
```

```
      FUNCTION P(J,X)
      IF(J.GT.0) GO TO 1
      P=1.
      RETURN
1     IF(J.GT.1)GO TO 2
      P=X
      RETURN
2     IF(J.GT.2)GO TO 3
      P=2*X**2-1.
      RETURN
3     IF(J.GT.3)GO TO 4
      P=4*X**3-3*X
      RETURN
4     IF(J.GT.4)GO TO 5
      P=8*X**4-8*X**2+1.
      RETURN
5     IF(J.GT.5)GO TO 6
      P=16*X**5-20*X**3+5*X
```

```
      RETURN
6     IF(J.GT.6)GO TO 7
      P=32*X**6-48*X**4+18*X**2-1.
      RETURN
7     IF(J.GT.7)GO TO 8
      P=64*X**7-112*X**5+56*X**3-7*X
      RETURN
8     IF(J.GT.8)GO TO 9
      P=128*X**8-256*X**6+160*X**4-32*X**2+1.
      RETURN
9     IF(J.GT.9)GO TO 10
      P=256*X**9-576*X**7+432*X**5-120*X**3+9*x
      RETURN
10    IF(J.GT.10)GO TO 11
      P=512*X**10-1280*X**8+1120*X**6-400*X**4+82*X**2-1.
      RETURN
11    IF(J.GT.11)GO TO 12
      P=1024*X**11-2816*X**9+2816*X**7-1232*X**5+284*X**3-11*X
```

```
      RETURN
12    IF(J.GT.12)GO TO 13
      P=2048*X**12-6144*X**10+6912*X**8-2576*X**6+968*X**4-104*X**2+1.
      RETURN
13    P=4096*X**13-13312*X**11+16640*X**9-7968*X**7+3168*X**5-492*X**3
     1  +13*X
      RETURN
      END
```