DETERMINING INVENTORY BASE STOCK LEVELS OF EXPENDABLE SPARE PARTS UNDER SERVICE LEVEL AGREEMENT FOR ON-TIME DELIVERY

By

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Abstract: Availability of service parts is critical to have adequate equipment maintenance in order to avoid costs associated with unplanned shut downs, loss of production, and increase safety among others. Determining an adequate quantity of service parts to have is a challenging situation that companies have to deal with because service parts encompass intermittent demand; this type of demand is of variable size and occurring at irregular intervals. As consequence of the nature of service parts, companies have to have large quantities of parts in stock increasing their holding cost, or companies have to place expedited order to avoid late deliveries and avoid penalty fees. In this research, a model is developed in order to determine the inventory base level for all parts in order to minimize holding cost, penalty cost for late delivery and shipment cost while satisfying an agreed service level for on-time equipment delivery. Scenario based approach is utilized to provide a robust result. Given that constraints and variables increase dramatically, pre-processing techniques are utilized to reduce the model and obtain a solution for the large scale model within a reasonable time.

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CHAPTER 1

INTRODUCTION

Having adequate system maintenance is critical to avoid costs associated with unplanned shut downs and loss of production as well as to increase safety, improve equipment availability and extend useful life, as mentioned by Kutanoglu et al. [1]. Because system maintenance is of concern to both small and large companies, companies take a variety of measures to reduce the amount of system downtime. These include: system redundancy, appropriate preventive maintenance before systems fail and effective corrective maintenance after failure, as stated by Kutanoglu et al. [1].

According to Nikolopoulos et al. [2], maintenance can be classified as: Emergency (or breakdown) Maintenance where work must be done immediately; Routine Maintenance where work must be done in the finite, foreseeable future; and Preventive Maintenance where work must be carried out on a planned schedule. It is not always feasible to have back up equipment in place to be used in the event of breakdowns. This issue is specially critical in capital intensive industries such as the aviation industry.

Several factors are necessary to have effective equipment maintenance. These include: adequate maintenance policies, technicians with required training, and availability of spare parts among others. These elements are critical for any enterprise, however, this dissertation will concentrate and discuss the availability of spare parts only.

1.1 Motivation

Determining an adequate quantity of spare parts is challenging because spare parts encompass intermittent demand (Willemain et al. [3]). This type of demand is of variable size and occurs at irregular intervals (Shale et al. [4]). Predicting which materials will be required for the next time period is challenging, and, in most cases, it is prohibitively expensive to have large quantities of all the different types of spare parts used. One of the most critical effects of demand uncertainty according to Kalchschmidt et al. [5] is the simultaneous increase in inventories and decrease of customer service. For instance, Ghobbar and Friend [6] stated that when Eastern Airlines went into the bankruptcy which eventually grounded its fleet, it had spare parts inventory in excess of $700 million. That is $700 million of assets that generated no revenue nor produced capital according to the authors. They also mentioned that Pan Am had in excess of $200 million in spare parts when it collapsed. The aviation industry is a capital intensive business, with daily operations characterized by high fixed cost components and excessive inventory that affect the quality of service and the effectiveness of its maintenance and repair (Ghobbar and Friend [6]). For example, according to Canaday [7], a late flight departure could cost $10,000 per hour, a flight cancelation anywhere from $25,000 to $150,000 and an engine shutdown $500,000 per incident.

Another factor that increases the complexity of forecasting intermittent demand is lack of data. Scarf [8] mentioned that too little attention is paid to data collection, and there is not enough consideration of the usefulness of models for solving real problems through model fitting and validation.

In general, the service parts industry is a $1.5 trillion business worldwide (Muckstadt [9] and Kranenburg and Houtum [10]). This creates an incentive to manage the supply chain of these parts in a very efficient way. The aviation industry invests large amounts of money in spare parts as reflected by the market volume of the Maintenance, Repair and Overhaul (MRO) industry that was $34B in 2004, $38.8B in 2005 and which is expected to have increased to $62B in 2014, according to Cohen and Wille [11], and Flint [12]. North America accounts for 37% of the MRO market as shown in Figure 1.1. In some companies, the after sales-service and parts business accounts for more than 25% of total business, while in other companies it can account for 50% or more of total revenue generated (Kranenburg and Houtum [10]).

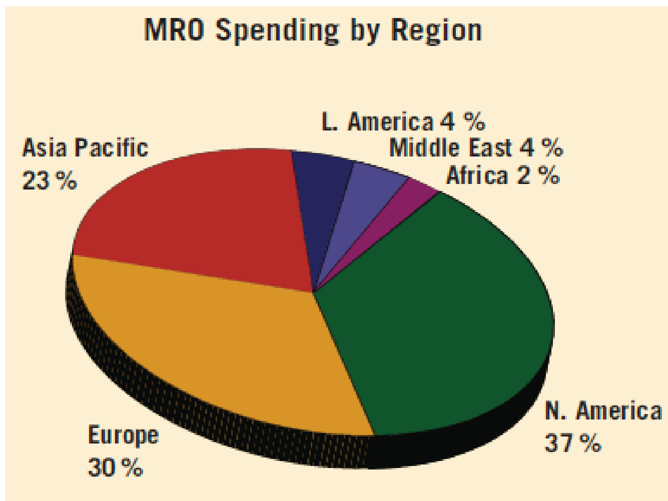


Figure 1.1: MRO Spending by Region, Flint[12]

Another important point mentioned by Cohen and Wille [11] is that the worldwide active air transport fleet was projected to expand by a growth rate of 5%, from 16,500 in 2006 to 24,000 by 2012. All of these airplanes require maintenance, which is one of the largest operational cost categories after fuel. So, maintenance becomes a great opportunity for cost reduction efforts.

The above-mentioned statements support interest in the current dissertation topic of determining inventory base stock levels of expendable spare parts under service level agreement for on-time delivery. More research is still needed in this area in order to benefit companies, customers, and society in general.

1.2 Background of Maintenance, Repair, and Overhaul in the Aviation Industry

An MRO could provide two types of services (Srinivasan et al. [13]): The first is Program Depot Maintenance (PDM), which is a heavy repair and overhaul of an aircraft following the recommendation of the aircraft manufacturer, the policies of the aircraft operators and/or the regulations of aviation authorities. The second type of service is Unscheduled Depot Level Maintenance (UDLM) which is maintenance that is required immediately for the aircraft to become serviceable again or to avoid potential problems in the near future.

This dissertation addresses expendable spare parts used during heavy maintenance services or PDM. A macro level view of the different phases followed during a PDM is depicted by Srinivasan et al. [13] and is adapted in Figure 1.2. The PDM phases are:

- Strip phase: Workers remove arms and fuel from the aircraft and remove and inspect major components.
- Order Parts/Route Components: Workers order parts and route major components to the back shops for repair.
- Inspection and Repair Phase: Inspection and repair activities on the aircraft are performed to the extent possible while awaiting parts and components.
- Buildup Phase: As parts become available workers continue to reassemble the aircraft.
- Rig Phase: Systems are reconnected, and manually operated and checked.
- Paint Phase: Workers scuff and paint aircraft, and perform check and balance procedures.

- Operational Phase: Workers power the aircraft and perform operational tests.
- Quality Assurance (QA) Audit: Aircraft is subject to quality assurance
- Functional Test Phase: Pilots perform flight tests, and mechanics prepare the aircraft and deliver it to the customer

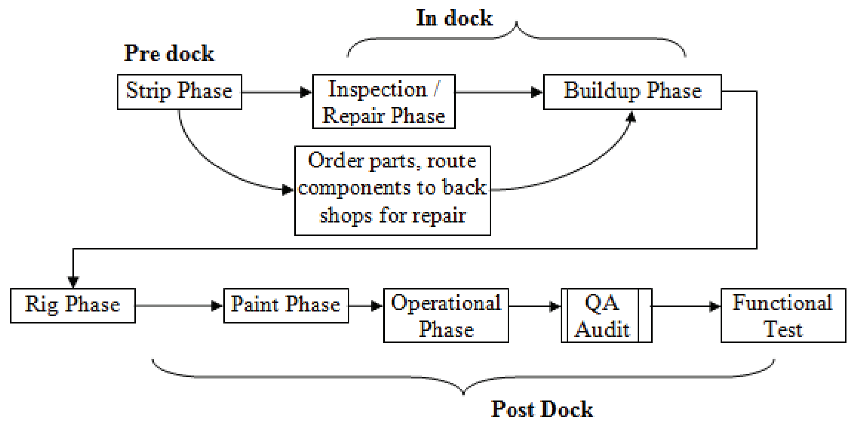


Figure 1.2: A Drill down into the different phases followed during a PDM. Adapted from Srinivasan et al. [13]

Each phase contains a different number of tasks to be performed on the aircraft as can be seen in Figure 1.3. Some organizations call each task "Work Control Document" (WCD) and others call it "Task Cards" (TC). This dissertation will refer to tasks as WCD. Some of the WCD have predecessors; some have antecessors; and some WCD have both predecessors and antecessors. The sequence in which each WCD is scheduled depends on the availability of materials, pre-work needed, tools, previous WCD (Predecessors), technicians of different skills, and the judgment and experience of supervisors, among other factors. Each WCD contains a description of the steps to perform the job, a list of tools to be used, a list of possible consumable and repairable materials, the type of technical skill required, etc. shown in Figure 1.4. Depending upon the type of findings when performing a WCD, additional WCDs can be generated based on specific needs. These new WCD's are not planned but

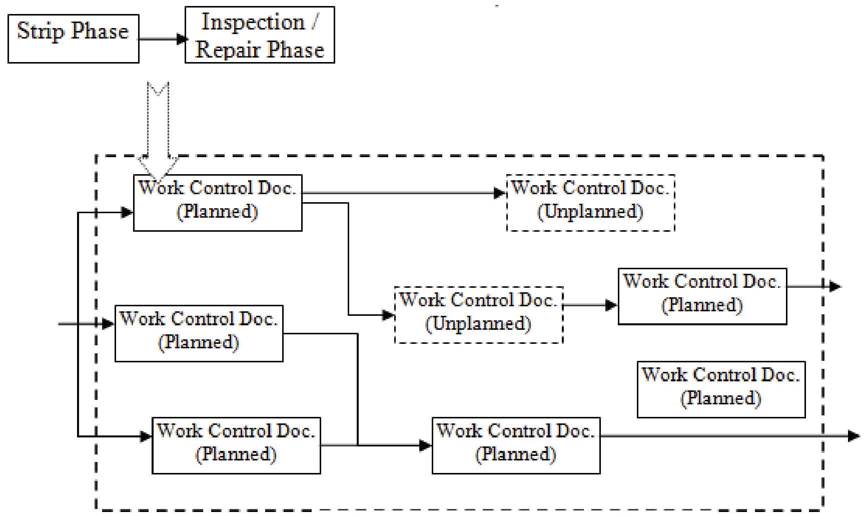


Figure 1.3: A Drill down into a phase

they might have to be completed during the maintenance check. These new WCDs could also require additional resources such as tools, materials and specific manpower. There are different types of WCDs. Some are required to clean places of the aircraft,

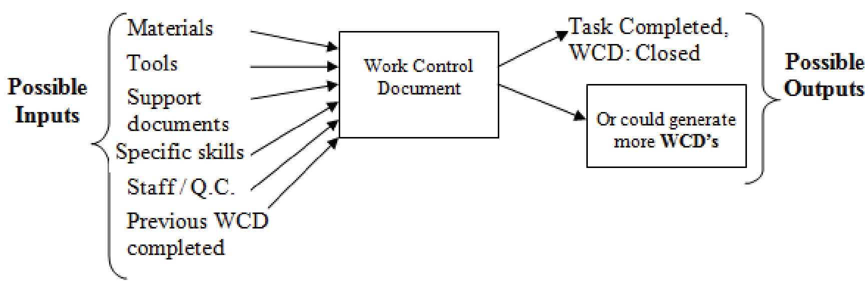


Figure 1.4: A Drill down into a Work Control Document

others specify the lubrication of certain parts, and still others specifically require the replacement of parts. However, most WCDs are related to the inspection of specific places of the aircraft, and part replacement is dependent upon inspection results. According to Cohen and Wille [14], 94% of the WCDs performed in Airbus A320 are inspections and, after they are completed, the technician will recommend replacing parts or not. If there is no available stock on hand, this creates the need to expedite orders (which is more costly than placing normal orders) in order to avoid late aircraft

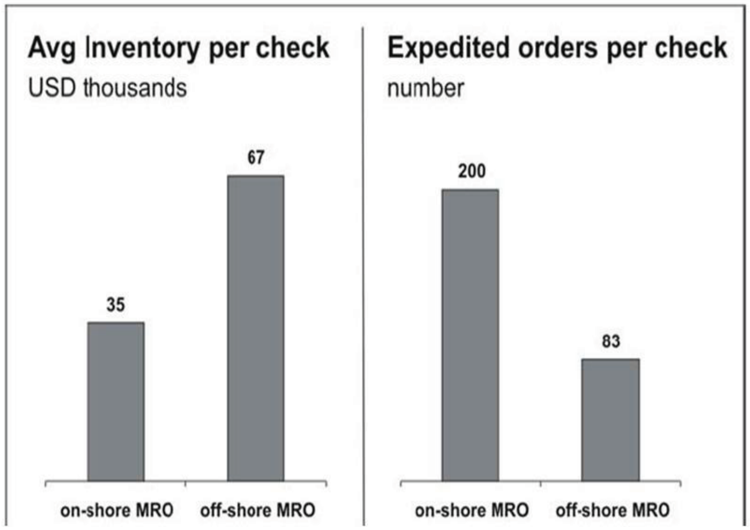


Figure 1.5: Average Inventory per Check vs Expedited Orders per Check, Cohen and Wille [14]

delivery. This issue increases uncertainty related to the parts that will be replaced in every maintenance check. This is supported by the fact that 40%-60% of spare parts used in an aircraft maintenance check are determined after the maintenance has started (Cohen and Wille [14]).

Cohen and Wille [14] present two different strategies followed by two different airline companies to deal with the nature of demand of expendable spare parts. The first company is called 'on-shore MRO' because of its geographical location and has about half of the inventory levels as the second company, the 'off-shore MRO'. However, during maintenance checks, the 'on-shore MRO' places 2.5 times more expedited orders than the 'off-shore MRO', as shown in Figure 1.5. It is also mentioned that the company that places more expedited orders has a higher number of late deliveries compared to the second company. There is no mention of which company has the overall lowest costs.

1.3 Expendable Spare Parts

According to KLM Automation Training [15], non-repairable parts or consumables/expendables are materials that are considered to be consumed when issued and are characterized by any of the following:

- Can be used only once or cannot be repaired
- Can be re-used without rework
- Have a limited lifetime according to technical information
- From Federal Aviation Authorities (FAA) or vendor direction the material cannot be repaired
- Calculated cost of repair (including costs of organization, administration, freight, etc.) should not exceed the new price plus purchase costs (Uneconomical to repair)

Sleptchenko et al. [16] mentioned that 27.2% of the parts in stock at a company are repairable and the rest are expendable/consumable. However, repairable parts account for the biggest proportion of the total investment. In other words, expendable items account for the highest volume of items in stock and the lowest investment, but the impact of not having them on hand at the moment requested could be as critical as an expensive repairable if the essentiality code is 1 even though the expendable might be cheap. The challenge is managing large quantities of different part numbers that have intermittent demand.

Nearly 800 hundred different maintenance checks of A320 aircrafts and their material consumption have been provided by Airbus within the context of a non-disclosure agreement. Figure 1.6 shows that 36% of the part numbers have been used in only one maintenance check out of the 800 possible checks. Further, it is observed that nearly 80% of the part numbers have been used in 10 different maintenance checks or fewer out of the 800 possible checks. Because of this, it can be concluded that predicting part replacement is a very challenging issue with which companies have to deal.

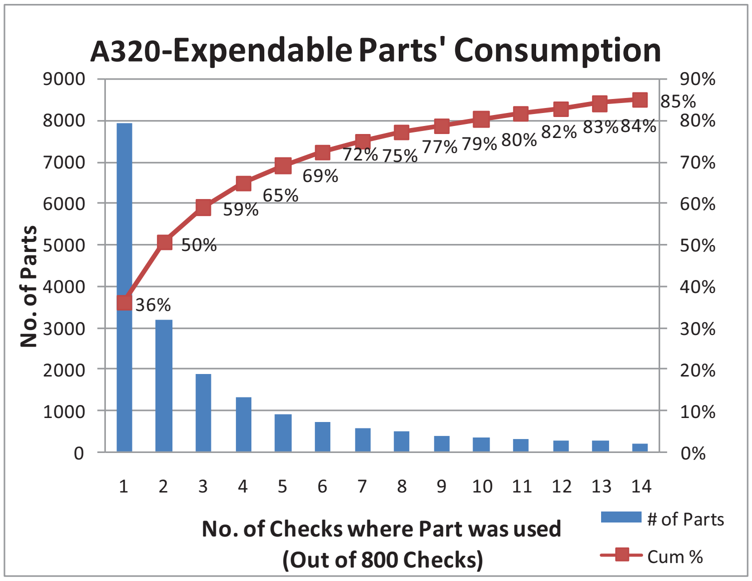


Figure 1.6: A320 Expendable Parts Usage for 800 Maintenance Checks

As we have seen, spare parts have sporadic demand; on Chapter 2, we will review some relevant literature addressing forecasting methods, classification approaches and optimization methods used to manage spare parts. Chapter 3 discusses the gaps identified in the current literature, it also describes the problem this dissertation is addressing, the research goals and expected contributions. Chapter 4 presents the data analysis of a large data set of 795 different maintenance checks provided by Airbus, this data will be used to feed the optimization model to be developed and discussed on Chapter 5. Chapter 5 also discussed the pre-processing techniques developed in order to solve large optimization models. Chapter 6 presents several cases where the optimization model is used, some of those cases are small ones and have been developed just to prove the validity of the model; it is also discussed the solution of the data set presented on chapter 4. Finally, we present the contributions of the model on Chapter 7.

CHAPTER 2

LITERATURE REVIEW

The objectives of this chapter are to identify current methodologies available to manage expendable spare parts, recognize opportunities for improvement and address spare part needs faced by the industry at this moment.

A survey of airline operators and maintenance organizations regarding their maintenance and inventory procedures was performed by Ghobbar and Friend [17]. One hundred and fifty-two (152) out of 175 respondents were using the reorder point system while the remaining 23 companies were using the material requirements planning (MRP) system. It is important to remember that the MRP objective is to provide "the right part at the right time" to meet the schedules as stated by Vollmann et al. [18]; so, it is not surprising that very few companies are using MRP because demand predictability is very challenging for spare parts.

Some of the criteria or control characteristics that need to be taken into consideration when managing spare parts are criticality, specificity, demand pattern and the value of the parts (Huiskonen [19]). The description of each of these characteristics is as follows:

- Criticality relates to the consequences caused by the failure of a part on the process in the event a replacement is not readily available.
- Specificity refers to whether the part is standard (i.e., used by many users) or if the part is tailored and only used by a particular user.
- Demand Pattern includes the aspects of volume and predictability. Predictabil-

ity means the failure process of a part and the possibilities to estimate failure patterns and rates by statistical means.

- Value of Parts refers to the price of the spare parts.

Not all methodologies used to manage spare parts take into account all of the above characteristics. Some of them concentrate on demand pattern while others take into account characteristics like criticality and value of parts. In the following sections, a brief description of the methodologies used to manage spare parts is described.

2.1 Forecasting Methods

In Ghobbar and Friend [20], the experimental results of 13 forecasting methods including those used by aviation companies, are examined using historical data from components of an airline operator. A brief description of the methods provided by Ghobbar and Friend [20] is presented below:

- Additive Winters: Assumes that seasonal effects are of constant size.
- Multiplicative Winters: Assumes that seasonal effects are proportional to the local de-seasonalized mean level.
- Seasonal Regression Model: Used in time series for modeling data with seasonal effects.
- Component Service Life: Estimates the service life characteristics of the part (Mean Time Between Removal, MTBR) derived from historical data (Flying hours or numbers of landings).
- Weighted Calculation of Demand Rates: The total demand for a given part during an experience period divided by the total activity of the aircraft during the same period, providing an average forecast rate.

- Weighted Regression Demand Forecasters: Considers forecasts based on moving regressions in terms of flying hours.
- Croston: Forecasting in circumstances of low and intermittent demand.
- Single Exponential Smoothing: Forecasting in circumstances of low and intermittent demand.
- Exponentially Weighted Moving Average: An effective forecasting tool for time series data that exhibit a linear trend.
- Trend Adjusted Exponential Smoothing: Forecasting time series data that have a linear trend.
- Weighted Moving Averages: A simple variation on the moving average technique allowing for weighting to be assigned to the data being averaged.
- Double Exponential Smoothing: Forecasting time series data that have a linear trend.
- Adaptive-response-rate Single Exponential Smoothing: Allows the smoothing parameter to be changed in a controlled manner as changes in the pattern of data occur.

According to the authors, the methods that showed superiority in the study are the weighted moving average, the exponential weighted moving average, and Croston.

A classification of the patterns of demand for forecasting purposes is described in Cavalieri et al. [21], where the two measures used are:

- The average time between two consecutive orders of the same part calculated by dividing the number of periods with no demand by the total number of periods (ADI).

- The variation of the demand size evaluated through the square of the coefficient of variation (CV). Periods with no demand are excluded from the calculation of CV since the presence or not of demand is captured by ADI.

Depending on the results of ADI and CV, the demand pattern can be classified in four categories as depicted in Figure 2.1. Smooth demand is the demand that occurs randomly, with few time periods with no demand and modest variation in demand size. Intermittent demand appears randomly with several time periods not having demand. Erratic demand is shown by part numbers with highly variable demand size, and Lumpy demand appears randomly and is highly variable with many time periods having no demand (Cavalieri et al. [21]). The authors also mention that time-series-based forecasting methods (Exponential smoothing and derivatives, ARMA models) are suitable for the smooth demand and the erratic demand quadrants. For lumpy demand and intermittent demand quadrants, the authors recommend Croston methods and derivatives.

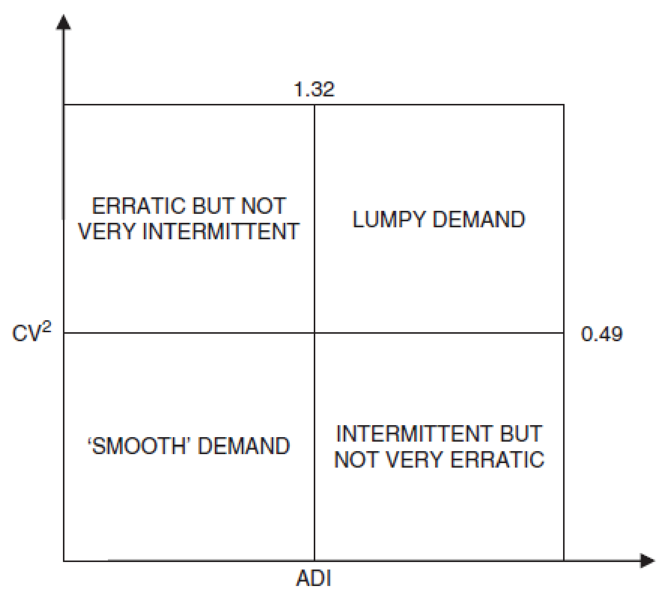


Figure 2.1: Classification of Demand Patterns, Cavalieri et al. [21]

As mentioned previously, the Croston method is used to forecast in circumstances of low and intermittent demand. This method treats the size of orders and the intervals between them as two separate series, and combines their averages to achieve a forecast of the demand per period (Shale et al. [4]). There are different derivations of the Croston method. For example, Shenstone and Hyndman [22] describe the usage of log transformations of both demand and interarrival times to restrict the sample space of the underlying model to be positive. Another variation is when interarrival times are assumed to have an independent and identically distributed (i.i.d.) geometric distribution.

The Bayesian approach is also used to manage spare parts. This approach assumes an a priori distribution of demand rate. As the demand process goes on, corrections of parameters of the a priori distribution are made according to the accumulated knowledge of past demand (Popovic [23]). Another application is done by Aronis et al. [24] where the Bayesian approach is used to specify the initial a priori distributions of failure rates. Then based on the priors, the distribution of demands for spare parts are determined and calculated for the required stock level. In Azoury and Miller [25], a comparison of the optimal ordering levels of Bayesian and non-Bayesian inventory models is performed, showing that the quantity ordered under the non-Bayesian policy would be greater than or equal to that under Bayesian policy.

There are some other approaches used to forecast spare parts. For example, Hua et al. [26] developed an approach for forecasting the intermittent demand of spare parts using a mechanism that integrates the demand autocorrelation process and the relationship between explanatory variables and nonzero demand. Another approach presented by Foote [27] discusses the philosophy, mathematical principles, and system design features of a forecasting system implemented at the Aviation Supply Office (ASO). The author concluded that the usage of statistical control techniques is a very important tool in forecasting for replenishment demand.

As a result of this literature review, we can conclude that there was no forecasting algorithm that considered all of the control characteristics mentioned (criticality, specificity, demand pattern, and value of parts). Most methods concentrate on the demand pattern, but not on the other control characteristics.

2.2 Classification Approaches

According to Cohen and Ernst [28], the number of stock-keeping units (SKU) is so large that it is not computationally feasible to set stock and service control guidelines for each individual item. For this reason, items are grouped together and generic control policies are set for each group. Service level, safety stock, etc., are applied to each item in a group under those policies.

The ABC classification scheme is the most frequently used method for item aggregation as mentioned by Cohen and Ernst [28]. It consists of separating the inventory items into three groupings according to their annual cost volume usage (unit cost x annual usage). These groups are: A, items having a high dollar usage; B, items having intermediate dollar usage; and C, items having a low dollar usage (Vollmann et al. [18]). ABC helps to identify the items that will make the largest impact on the firm's overall inventory cost performance when improved inventory control procedures are implemented.

Even though the ABC methodology is easy to implement, this process alone does not take into account other managerially significant variables such as lead time, obsolescence, availability, criticality and substitutability, among others as stated by Vollmann et al. [18]. This is why multi-criteria ABC management policies are used as well. Al Kattan and Bin Adi [29] apply ABC and 123-analysis to classify materials based on unit price in order to get better classification results. This methodology is performed to identify if high total annual cost of an item is coming from a high unit price or from a high volume of demand. Also, Cohen and Ernst [28] introduced a blend of statistical clustering procedures and operational constraints which allow the use of any collection of operational relevant attribute items. Some of the operational attributes used are price, volume of part, lead time, demand pattern per time, and criticality index.

Braglia et al. [30] present an inventory policy matrix that links the different classes of spare parts with the possible inventory management policies to identify the best control strategy for spare stocks. The basic idea of their procedure is to define a decision diagram which guides the analyst toward the best criticality classification for each type of spare part.

2.3 Optimization Models

There are different optimization models addressing spare parts management from different perspectives. For example, some literature describes models with one or multiple items, one or multiple echelons, and one or multiple locations at each echelon. Some other factors considered are lateral transshipment, emergency shipments, and different demand classes, among others. Below is a brief summary of the most relevant literature.

A model of an (s,S) inventory system in which there are two priority classes of customers is presented in Cohen et al. [31] which treat excess demand as lost sales. The model minimizes expected costs subject to a service level constraint. The single product and single location of this model is embedded into a multi-echelon, multi-product framework. When faced with insufficient stock to meet normal replenishment orders and emergency shipment orders, priority is given to demand associated with emergency shipments and direct customer requirements. Another approach is presented in Kocaga and Sen [32], where an inventory system that consists of two demand classes is studied. The orders in the first class need to be satisfied immediately, whereas the orders in the second class are to be filled in a given lead time. The model assumes a single location, using a one-for-one policy, and Poisson demand arrivals for both type of classes. The service level for the critical class is an approximation. However, the other class is exact.

According to Kranenburg and Houtum [10], one of the features required by the market is to provide differentiated service levels to different groups of customers. Critical level policies are used to exploit the differences in target service levels by inventory rationing. The model is a multi-item, single location model that minimizes the spare parts provisioning costs under the condition that aggregate mean waiting time constraints for all customer groups are met. A multi-echelon, multi-item inventory system is implemented in Cohen et al. [33] where prioritized demand classes are considered. The objective of the model is to determine stock control policies for each location and part that would minimize expected costs (replenishment costs, emergency cost and inventory holding costs) for the whole system while satisfying the service constraints for products. In another paper, a stocking policy where some of the stock is reserved for critical demand is proposed by Dekker et al. [34]; in this model, the demand is assumed to be a Poisson process and a lot by lot stocking policy with deterministic replenishment lead time assumed. The model produces an approximation for the service level for both classes of demand.

A two-echelon multi-item spare parts inventory system in which supply flexibility through both lateral transshipment and direct deliveries is considered by Wong et al. [35]. A multi-item, multi-echelon model is developed to minimize total system cost subject to a target level for the average waiting time across the items at each local warehouse. The authors conclude that the presence of lateral transshipment improves the performance of the single-echelon system considerably. In different literature, a single-echelon, N-locations, continuous review inventory system in which complete pooling of stock is permitted among the locations is studied in Kukreja et al. [36]. In this study, proactive transshipment is used as an element of inventory control policy which can significantly reduce the total inventory needed through the entire collection of stocking points.

Si et al. [37] study an optimization model and simulation algorithm for a two-echelon spare parts inventory system which includes one central warehouse and one distributive inventory system involving N sub-warehouses. The distributive inventory system optimizing model, which considers the random horizontal replenishment of spare parts among sub-warehouses, can give an optimal set of inventory policies (s,S) for each sub-warehouse while satisfying an agreed service level. On the other hand, an analysis of a multi-item, continuous review model of a two-location inventory system for repairable spare parts subject to high availability is studied in Wong et al. [38]. Lateral and emergency shipments occur in response to stock outs with the objective of minimizing the total costs of inventory holding, lateral transshipment and emergency shipments subject to a target level for the average waiting time per demanded part at each of the two locations. The authors provide a summary of the literature on multi-location inventory systems. Another study is presented by Mehrotra et al. [39], where consolidation of spare parts is modeled to reduce the overall inventory by storing parts of several locations together, taking advantage of risk pooling. The objective of the model is to minimize total cost of the spares as well as the cost of opening cluster sites. The constraints satisfied are that each location is assigned to a cluster if it is open and each location is assigned to exactly one cluster only.

Another approach to manage spare parts is presented by Yoon and Sohn [40], where the inventory level of concurrent spare parts (CSP) is determined. The model uses a two stage approach. In the first stage, a random effects model is used to predict the expected demand in a multi-echelon system consisting of depot and bases based on CSP's varying characteristics of time. In the second stage, the optimal inventory level of CSP is found while satisfying budget constraints. Similarly, Kranenburg and Houtum [41] studied the benefits of exploiting commonality for a number of groups of machines where some of the used parts are similar. A multi-item, single site spare parts inventory model is formulated with the objective of minimizing holding and transportation costs while satisfying service level constraints for each group.

In different research, Kutanoglu and Lohiya [1] presented an optimization-based model for a single-echelon, multi-facility service parts logistics with time-based service level constraints. The goal was to minimize inventory and transportation costs. The model has different transportation options and service responsiveness that can be achieved using alternate modes (slow, medium and fast). Caggiano et al. [42], describe and validate a practical method for computing channel fill rates in a multi-item, multi-echelon service parts distribution system. The goal is to determine base stock level for all items at all locations so that the service level requirements are met with minimum investment. The authors stated that the model does not consider the possibility of multiple part failures at once. Caglar et al. [43] developed a continuous review, base stock policy for a two-echelon, multi-item spare parts inventory system that minimizes system-wide inventory cost subject to a response time constraint at each field depot with no lateral transshipment allowed.

Another study is presented by Graves and Willems [44] where the supply chain places strategic safety stocks to provide a high level of service to the final customer with minimum cost. The model for stationary demand is extended to the case of non-stationary demand for products with short life cycle. The model considers a constant service time policy for which the safety stock locations are stationary but the actual safety stock changes as demand changes. Related to the behavior of the demand, Axsater and Zhang [45] present a recursive evaluation of order-up-to-S policies for a two echelon inventory system with compound Poisson demand. It is assumed that unfilled demand is backordered and the shortage costs are a linear function of the time until delivery.

According to Lau et al. [46], repairable inventory models assume that the demand for items is independent of the number of working systems, but this assumption can introduce a serious underestimation of availability when the number of working systems is small. For this reason, the authors study a multi-echelon, single-indenture repairable item inventory system under the phenomenon of passivation (system failure rate is equal to zero during repair) to compute time-varying availability. Liu and Lee [47] propose an evaluation approach to multi-item base-stock inventory policies where unidirectional substitutions are allowed (For example, a transformer with higher capacity can be used instead of one needed with a lower capacity but not vice-versa). This is considered a continuous review inventory system using base-stock policy which is frequently applied in spare parts provisioning where most items are slow-moving.

Some other models developed to manage spare parts deal specifically with repairable items. One is the Multi-Echelon Technique for Recoverable Item Control (METRIC) and its derivatives. According to Sherbrooke [48], the METRIC theory calculates for every item in a system the optimal stock level at each of several bases with the objective of minimizing the backorders across all bases. Minimizing backorders is equivalent to maximizing product availability when there is no cannibalization. Later, other authors improved the METRIC model. For example, the VARI-METRIC has the advantage of being easier to implement and reduces the METRIC's 11% gap as the optimal solution to 1%.

Sleptchenko et al. [49] states that the VARI-METRIC aims to determine initial stock levels assuming that all failed items are either repaired or replaced by new items if repair is impossible. In other words, the VARI-METRIC assumes that the original number of items remain circulating throughout the network. One deficit of the VARI-METRIC is the assumption that repair shop capacities are infinite. In Sleptchenko et al. [49], the VARI-METRIC is extended and the authors model repair shops by multi-class, multi-server priority queues which may lead to a significant reduction in the inventory investment required to attain target system availability (usually 10%-20%). Similarly, Sleptchenko et al. [16] modified the VARI-METRIC method to allocate service part stocks in the network where the repair shops are modeled by (single or multi-class) multi-server queueing systems. The authors state that under finite capacity, item throughput times can be influenced using an appropriate priority setting. For example, expensive items can be given high priority, to shorten throughput times; hence the stock level required for those items remains low. In the same way, Díaz et al [50] introduce approximations that deal with limited repair facilities under the scenarios of single-class exponentially distributed repair distributions, single-class general repair distribution, and multi-class general repair distributions. According to the authors, the assumption of ample repair capacity introduces a serious underestimation of spare part requirements in systems with high repair facility utilization.

Also, a very interesting case is presented by Smith et al. [51], where a model is formulated for optimizing multi-item inventories for repair of field equipment based on holding costs and the probability of job completion without stockout. The model determines the appropriate collection of parts to be carried by crews when they are sent to different locations to repair equipment. Overstocking increases inventory holding costs, while understocking decreases service efficiency and increases costs because equipment remains down due to unavailable parts. According to the authors, using 'job-fill' rate (fraction of jobs without stockout) is a more appropriate measure in many applications. Some of the assumptions of the model are that restock is possible between jobs, so the stocking decision is a one period inventory problem that the penalty for shortage is essentially independent of the number of unavailable parts; and that, at most, one part of each type is used on a given job.

The problem formulation of the previous model can be stated as follow: Suppose there are n possible parts that a serviceman might carry and that the fraction of jobs that require each of the parts is p_i, i = 1,2,...,n. It is assumed that part failures of different part types are independent and at most, one part of each type is used on a given job. Therefore, for any subset S of the n parts, P { parts S and no other parts are required for a given job } = $\Pi_{i \in S}\, p_i\, \Pi_{i \notin S}\,(1 - p_i)$. The serviceman performs N jobs per year, and whenever some parts are unavailable, a penalty cost L is incurred, which corresponds to the machine downtime, lost repairman time and other costs. For each part i that is carried, there is an inventory cost H_i, i = 1,2,...,n per serviceman per year; M corresponds to the stocked items. The expected cost per year per serviceman with policy M is:

$$C(M) = \sum_{i \in M} H_i + NL[1 - \Pi_{i \notin S}\,(1 - p_i)] \tag{2.1}$$

The optimal policy M^* is therefore defined by C(M^*)= Min_M C(M), where $M \subseteq \{1, 2, ..., n\}$

Similarly, a multiple-item inventory model with a job completion criterion is presented in Graves [52]. The model determines the optimal mix of components to be carried by a service representative in order to achieve the desired job completion rate. The same assumptions are made as in Smith et al. [51]: that service representatives can restock between repair visits; components fail independently; and, at most, one unit of each component type may be needed for a repair. However, in Graves [52], no penalty cost is assigned to the failure to complete a repair on the first visit by the service representative. Rather, the objective is to know the stocking policy that would guarantee a specified job completion rate with the minimum inventory holding cost. The author states that this model doesn't dominate the one presented by Smith et al. [51], but it provides additional insight into the problem and structure solution.

The problem formulation given by Graves [52] is stated as follows: It is assumed there are n components with p_i, i = 1,2,...,n, being the probability that component i has failed and needs to be replaced. It is defined h_i, i = 1,2,...,n, as the annual holding cost for a unit of component i and α is the desired completion rate ($0 \leq \alpha \leq 1$). Let x_i, i = 1,2,...,n, being a zero-one variable which denotes the stockage of component i.

The model is as follow:

$$Min \sum_{i=1}^{n} h_i \, x_i \tag{2.2}$$

$$subject \quad to \quad \Pi_{i=1}^{n}(1-p_i)^{1-x_i} \geq \alpha \tag{2.3}$$

$$x_i = 0, \, 1 \qquad\qquad i = 1, 2, ..., n \tag{2.4}$$

The objective of the model is to minimize inventory holding cost subject to a constraint on the job completion rate. The model is transformed into a binary knapsack problem in order to be solved to optimality.

Similarly, Cohen et al. [53] consider a periodic review or order up-to model that determines base stock policies for each part to minimize expected inventory costs across all parts while satisfying some service constraints on total completed customer repair services. The basic problem structure is similar to the tool-kit problem where it is considered that the recommended stock levels at the facility is complete at the beginning of the next time period. This problem is a generalization of the tool-kit problem studied in Smith et al. [51] where the repairer's kit is the equivalent of the facility for the current model.

Some of the assumptions of the model developed in Cohen et al. [53] are: 1) The repair network is single-echelon; 2) Stocking policy used by the facility is a periodic review base stock or order-up-to policy; 3) It is possible to restock to the base at the end of each period; 4) A homogeneous customer class is assumed but the model is extended to include low and high priority customer classes; 5) Primary analysis is on products that use mutually exclusive groups of parts; 6) The primary model is formulated for multiple, dependent failures across parts; 7) Service is at the product level not at the part level. The model developed is shown below:

$$MinG(\underset{\sim}{S}) = \sum_{i \in N} G_i(S_i) \tag{2.5}$$

where $\underset{\sim}{S} = (S_1, ..., S_n)$ and $G_i(S_i)$ are the expected costs per period associated with part $i \in N$, with ordering cost, holding cost, transportation cost, and shortage cost respectively as shown below:

$$G_i(S_i) = E\{K_i\,\delta(D_i) + \frac{C_ih}{2}\,[S_i{+}(S_i{-}D_i)^+] + C_{it}\,min[S_i,\ D_i] + C_{is}(D_i{-}S_i)^+\} \quad (2.6)$$

where K_i is the fixed ordering cost, D_i is the demand, S_i is the stock at the beginning of each period, C_{ih} is the holding cost per unit, C_{it} is the per unit transportation cost, and C_{is} is the cost per unit short. The model is subject to chance constraints meaning that, in the long run, excess demand should be greater than zero, for at most, a predefined fraction of the periods for which demand is nonzero; and a part availability constraint which is the required part availability level for parts in the product as a whole.

Another study addressing the problem of minimizing total inventory investment subject to constraints on the delay of the equipment due to part outage is presented in Hopp et al. [54]. The constraints ensure that the average total delay falls below a specified level. The authors describe the model as:

$$Minimize\,Annual\,inventory\,investment \quad (2.7)$$

Subject to:

$$Average\,order\,frequency\,per\,year\,per\,item\,at\,the\,Distribution\,Center \leq F \quad (2.8)$$

$$Average\,total\,delay\,at\,facility\,m\,per\,year \leq T_m, \qquad m = 1, ..., M \quad (2.9)$$

Where F is the target order frequency at the Distribution Center, T_m is the total delay per year allowed at facility m, and M is the number of facilities. Some of the assumptions of the model are:

- Demand is Poisson and constant lead time.
- Demand that cannot be fulfilled immediately is backordered.
- Each part replacement represents a separate incident. This means that if several different parts cause delays it is assumed that the total delay is given by the sum of the individual delays

- The distribution center makes use of a continuous review policy (Q, r) while the facilities use base stock policies (i. e. Q=1)
- Lateral transshipment is not allowed

2.4 Item Approach vs. System Approach

According to Canaday [7], there are two main approaches to manage spare parts: Item approach and System approach. Item approach is the conventional inventory practice that focuses on individual items that seek to keep the probability of stock-out below some specified value. According to the author, it is easy to implement; however, when there is a stock-out of a needed item, it doesn't matter how much the item cost, companies get it because it is still cheaper than having the stock-out and system down for additional time. Priority shipment, cannibalization, alternative spare, etc. could be used to solve this situation. One of the major shortcomings of the item approach is that system availability is an uncontrolled outcome of the item decisions, according to the author.

Alternatively, the system approach asks the question: how can we ensure that, x% of the time, the plant/equipment will not be shut down/delayed for lack of spare parts? The author states that, at the end of the day, the performance of spare parts inventory is measured by its success in minimizing the loss of benefits that result from system operation. It is necessary to take into account that some parts affect the system performance more than others, some cost more than others, some fail more often, some have longer lead times than others, etc. A system approach ensures that a demand-weighted average fill rate is achieved at a low inventory investment by assigning low fill rates to parts with high costs and high fill rates to parts with low costs, as stated by Thonemann et al. [55]. An item approach does not vary fill rates by parts but assigns identical fill rates to all parts.

For instance, according to Kim et al. [56], performance-based contracting is reshaping service support supply chains in capital-intensive industries such as aerospace and defense. Performance-based contracting is also known as "power by the hour" in the private sector and as "performance-based logistics" (PBL) in defense contracting. This approach aims to replace traditionally used fixed-price and cost-plus contracts to improve product availability and reduce cost of ownership by tying supplier compensation to the output value of the product generated by the customer. As is stated in Kim et al. [56] and taken from the U.S. Department of Defense (DoD) guidelines, "The essence of PBL is buying performance outcomes, not the individual parts and repair actions. Instead of buying a set level of spares, repairs, tools, and data, the new focus is on buying a predetermined level of availability to meet the customers objective."

Some of the assumptions of the model developed in Kim et al. [56] are: Failure of the subsystem 'i' is assumed to occur at a Poisson rate, and it is independent from failures of other components; each supplier maintains an inventory of spares and a repair facility; a one-for-one base stock policy is employed for spares inventory control; a failed unit is immediately replaced by a working unit and if a replacement is unavailable, a backorder occurs, and the affected system becomes inoperable. According to the authors, a common assumption in the literature is that the probability of two or more systems being down within the same system at any point in time is negligible.

According to Kutanoglu and Lohiya [1], service parts are often supplied via a multi-echelon distribution network in order to have a quick response time and the need for stock centralization to reduce holding costs. However, as mentioned by the authors, there is a trend to reduce the number of echelons for stock centralization as well as the number of locations per echelon in order to reduce fixed location costs and service parts obsolescence costs. This seeks to result in an efficient network by stocking essential parts close to customers and using fast transportation modes which vary in time and cost. The author considers that inventory stocking decisions should be integrated into the transportation mode choice decisions in order to achieve the required time-based service level.

In Kutanoglu and Lohiya [1], a model that minimizes total system cost is developed. The costs and constraints considered in the model are:

- Holding cost, which is the cost of stocking the service part at all facilities
- Transportation costs, which is the cost of transporting the parts from facility to customers
- Emergency shipment cost, which is the cost of fulfilling the demand from the central warehouse through direct emergency shipments, needed when the main facility responsible for customer's demand is out of stock at the time of the demand
- The constraints try to meet and fulfill customer demand by one mode and satisfy target time-based service levels.

One example presented by the authors is summarized in Figure 2.2 where we can see that as the total holding cost increases (more inventory in stock), emergency total cost decreases. Similarly, as discussed in Chapter 1, Cohen and Wille [14] present two different strategies followed by two different companies. The company with the highest inventory level places the lowest orders per aircraft during maintenance check (lowest emergency orders) and has the lowest late deliveries (See Figure 1.5).

It is important to consider not only purchasing and inventory costs, but also hidden costs which arise from part unavailability of MRO material (Cavalieri et al. [21]). As seen in Figure 2.3, as inventory stock level increases (Inventory holding costs increases), unavailability costs decrease. So, there is a tradeoff between them.

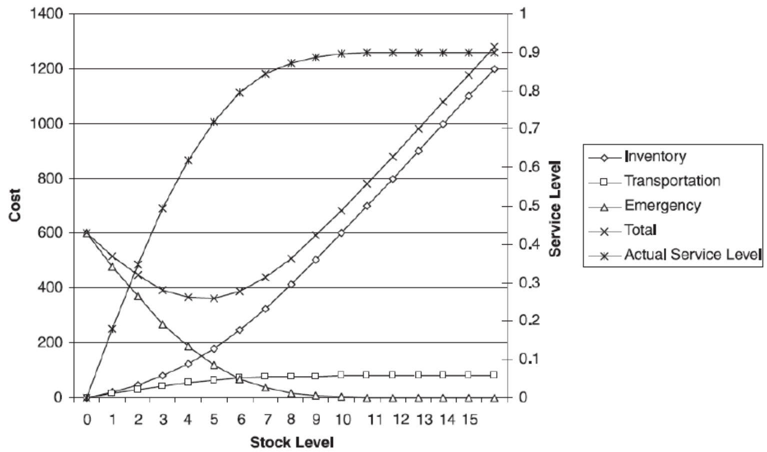


Figure 2.2: Different Costs and Service Level, taken from Kutanoglu and Lohiya [1]

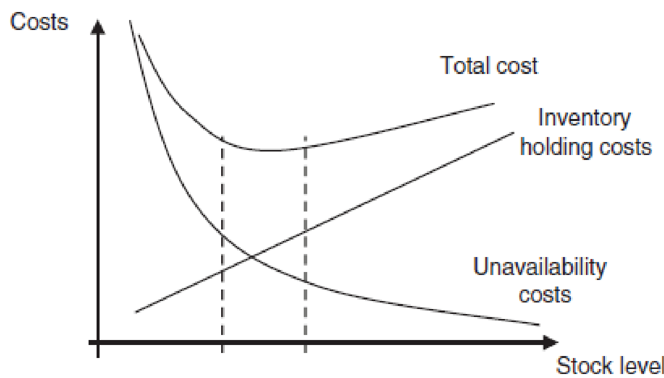


Figure 2.3: Inventory Holding Cost vs Unavailability Cost Cavaliere et al. [21]

According to the authors, when managing spare parts, it is necessary to take into account the following characteristics of the different parts: demand, criticality, value and specificity of the parts; these will help assess the most suitable stock management policy.

According to Denton [57], airframe manufacturers have developed classification systems to manage the criticality of a defective component called Essentiality Code. The detailed description of the Essentiality Codes that could be assigned to a part is presented as follow:

- Essentiality 1: No Go - Aircraft is grounded if this unit is unserviceable

- Essentiality 2: Conditional Go - aircraft is only grounded under certain conditions if this unit is unserviceable, e.g., freezing weather.
- Essentiality 3: Passenger convenience - aircraft is not grounded but this unserviceable unit causes great inconvenience to customers, e.g., toilets or coffee makers
- Essentiality 4: Minimum Equipment List (MEL) A - aircraft can fly at least one additional flight leg with the unit unserviceable
- Essentiality 5: MEL B - aircraft may fly up to three days with the unit unserviceable
- Essentiality 6: MEL C - aircraft may fly up to 10 days with the unit unserviceable
- Essentiality 7: This unit does not have to be serviceable for the aircraft to be flight worthy, or 120 day MEL dispensation

In summary, we have discussed different methodologies used to manage spare parts, some of those are forecasting methods, classification approaches, and optimization methods. The area of this dissertation is in the area of the Job Completion criteria using optimization methods. In the next chapter, we will discuss the problem we want to solve, the current gaps identified based on literature available, and the expected contributions of this dissertation.

CHAPTER 3

PROBLEM DEFINITION

The previous chapter provided a brief description of some of the methodologies used to manage spare parts. It also mentioned some relevant characteristics of spare parts that are useful to take into account when managing inventory levels. The following section presents the gap between current available literature and the problem that is going to be addressed in this dissertation.

3.1 Problem Identification and Gap

After careful consideration of the available literature in spare parts management, the author of this dissertation believes there is no available research in the MRO industry that solves the problem of determining base stock level of spare parts under service level agreement for on-time delivery during a preventive maintenance check. Some of the characteristics which together make this problem different than the previous ones studied are:

- Problem has multiple types of part failures. After the technician inspects specific areas of the equipment, he/she is going to decide if parts need to be replaced.

- Quantity of pieces of a part to be replaced could be more than one. The same type of part could be located in different places of the aircraft and it will be decided how many are going to be replaced after findings are made.

- Even though a part is needed and it is not available, it might not delay the system availability if it comes before scheduled delivery. It may be necessary to place an expedited order to avoid delay, or it could happen that normal lead time is short and the part will arrive before scheduled system delivery.

- Penalty cost for late delivery is a linear function of the number of times units the equipment is delayed. There is a penalty fee greater than or equal to zero for late delivery.

- Several items could overlap and cause late delivery, but penalty fee effect is not additive. It is calculated based on the item that causes the longest delay in equipment delivery.

- Every type of part has its own replenishment lead time (either Normal or Expedited), and instantaneous replenishment is not assumed.

- The schedule of the preventive maintenance to be performed is known in advance, but not all spare parts to be used in each case are known.

- Service level for on-time delivery is defined at the equipment level, not the item or part level. We are interested in measuring performance at the system level, not at the item level.

- The model is a multi-item, single echelon model.

On the previous chapter we discussed the most relevant literature addressing job completion criterion can be found on Smith et al. [51], Graves [52], Cohen et al. [53] and Hopp et al. [54]. In Figure 3.1 we present a summary of the main characteristics and hence current gaps this dissertation is trying to close and below is presented a discussion of it:

- Due to the nature of the characteristics of the problem, this dissertation uses a multi-period optimization model. All the four other authors are able to use a single time period given the assumptions they have made and the characteristics of the problem they are addressing.

- Three of the other authors assume that replenishment can happen between jobs, this dissertation and Hopp et al. [54] assume replenishment is not instantaneous and it depends on the lead time.

- Smith et al. [51] and Graves [52] assume that at most, only 1 part can be used. Cohen et al. [53], Hopp et al. [54] and this dissertation assume demand is stochastic.

- From all the other authors, only Smith et al. [51] assumes that the penalty cost is independent of the number of parts which is a valid characteristics of the problem we are addressing. Cohen et al. [53] and Hopp et al. [54] assumes penalty cost is additive.

- Emergency shipment is an important strategy used in the aerospace industry. Only Cohen et al. [53] and this dissertation considers emergency shipment as an alternative.

- All the four authors as well as this dissertation measure service level at the product level, not part level.

- All the four authors rely in some heuristics to solve the problem they are addressing. On this dissertation, to solve the mathematical problem we uses Xpress-MP, at the beginning, this software uses a heuristics, but later, it is able to solve the problem using branch and bound.

As mentioned in Cohen and Wille [14], 40% to 60% of the parts needed during preventive maintenance are determined after the maintenance has started because more than 90% of the tasks to be performed are inspections. After those tasks are

	Smith et al	Graves	Cohen et al	Hopp et al	Dissertation
Time Periods	1	1	1	1	Multi
Replen	Between jobs	Between jobs	Between periods	By LT	By LT
Items	Multiple	Multiple	Multiple	Multiple	Multiple
Demand Qty	1	1	Stochastic	Stochastic	Stochastic
Penalty Cost	Independent parts	n/a	Additive	Additive	Independent parts
Emergency Ship?	NO	NO	YES	NO	YES
Service Level	At Product Level	At Product Level	At Product Level	At Product Level	At Product Level
Solution	Heuristics	Heuristics	Heuristics	Heuristics	B&B
Obj Function	Min Holding, Penalty	Min Holding	Min Ordering, Holding, Transp, Shortage	Holding Cost	Min Penalty, Holding, Extra Shipment

Figure 3.1: Job Completion Criterion Literature and Dissertation Comparison

completed, the technician will recommend if parts need to be replaced or not. If there is no stock available and the part is needed, an order needs to be placed most likely an expedited order as the normal lead time of the order may well make it arrive after the expected delivery date of the aircraft. The penalty fee for late delivery is calculated based on the item that arrives latest. For example, in Figure 3.2, the penalty cost is calculated based on the arrival time of 'Part 4' minus the original expected delivery date of the equipment.

3.2 Problem Definition

Even though the schedule for preventive maintenance is known in advance, the majority of the parts and respective quantities to be used in each aircraft is not known until the maintenance has started. This causes great uncertainty and companies need to either have large quantities of stock, which increases the holding costs, or they have to place expedited orders while the aircraft is in maintenance, which increase costs, or companies have to pay penalty fees for late aircraft deliveries. All these could cause customer dissatisfaction and decreased loyalty, low employee morale and/or companies could become unprofitable. Based on the foregoing problem identification, the topic of this dissertation can be stated as follow:

"Determine inventory base level for all the parts in set I, in order to minimize total

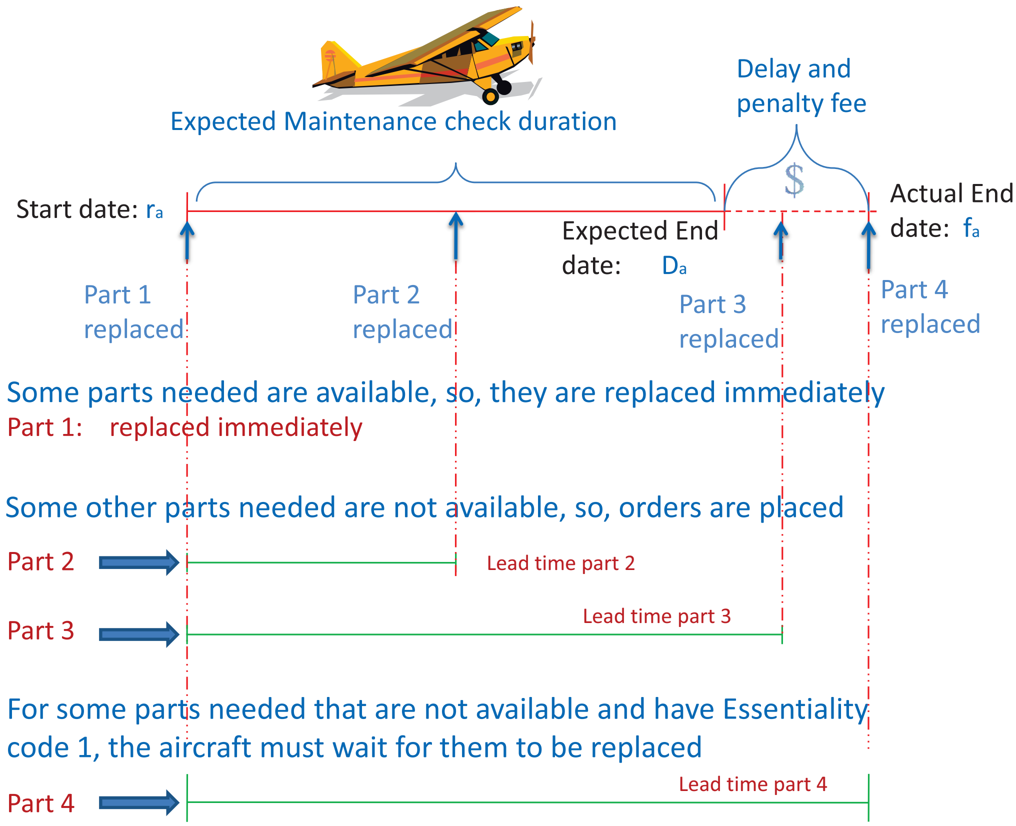


Figure 3.2: Multiple Parts Failures during Maintenance

costs (penalty cost for late delivery, holding cost and shipment cost) while satisfying an agreed service level for on-time equipment delivery."

According to Ghobbar and Friend [20], demand for air transport varies with time. So, in a competitive market, operators are trying to meet peak demand insofar as is reasonably possible. Therefore, aircraft availability has to be maximized during those peaks and maintenance must be fitted into tied slots when the planes are not required, as be seen in Figure 3.3.

In addition to incorporating several characteristics together in one model as mentioned in the previous section, the current model will be able to recalculate stock levels once the company has identified that the number of preventive maintenance checks will change significantly, or it has acquired more MRO contracts or it the

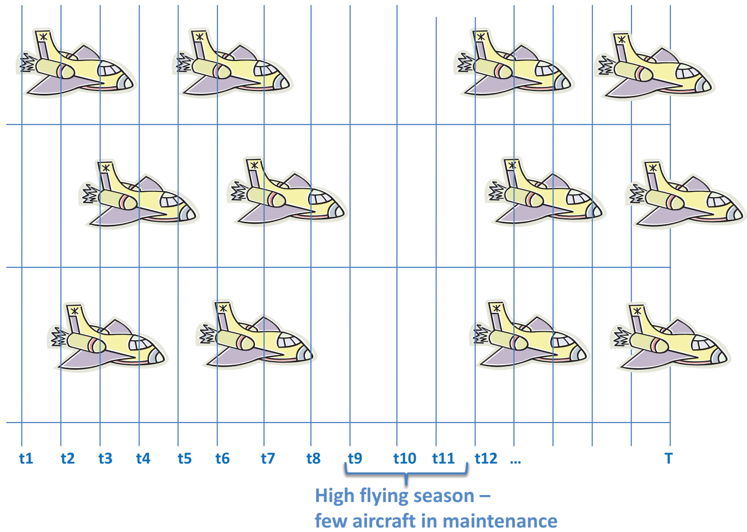


Figure 3.3: Expected Preventive Maintenance Schedule for a Time Period T (i.e. 1 year)

aircrafts are flying more frequently.

3.3 Research Goals and Objectives

In this section, the research goals and objectives are presented.

Goals:

- Develop a more realistic model addressing the MRO preventive maintenance problem while considering all the characteristics of it.
- In spite of the sporadic nature of spare parts, develop a robust model that is able to solve a large scale MRO problem within reasonable time
- Able to solve a large scale MRO problem within a reasonable time.

Objectives:

- Develop a mathematical model that minimizes costs (Penalty cost, holding cost and shipment cost) while satisfying an agreed service level for on-time equipment delivery by recommending a base stock level for each part.

- The solution of the model should be feasible over multiple scenarios with random generation or assignment of demand.
- Develop some pre-processing techniques that reduce the size of the mathematical model which will decrease the time to solve it.

3.4 Expected Contribution

We expect to develop a more realistic model to help manage spare parts in a more efficient way while minimizing total costs and satisfying service levels at the equipment level. We expect this research will be useful to airline companies and MRO operators by minimizing costs and retaining customers.

We also believe this research will have applications in other areas. In health care, for example, once a surgery has been scheduled, the health care facility could identify the need for critical devices or critical substances. Those additional items could be requested under the highest priority, if not available at the health care facility. Resources are limited, so it is not always possible to have all type of devices, or rarely used and expensive medicines, in all echelons and all locations.

On this dissertation, a case study is done using data from the commercial aviation industry. This data was provided by Airbus Industries, and in the next chapter, the main variables driving the maintenance of aircraft, as well as the characteristics of the data are being discussed.

CHAPTER 4

DATA METHODOLOGY

This chapter discusses the data collection methodology used in this research. This data relies mainly on a project implemented by Airbus. We also received additional information from an airline company in order to fill out a few data gaps. Second, the data provided is analyzed and segmented, and its groups of behavior are discussed. Third, it addresses how extra shipment cost is determined, a parameter that is critical in the model.

4.1 Data Collection

As discussed previously, a large percentage of spare parts have sporadic demand, which creates a major forecasting challenge for companies to have all the necessary parts available before a maintenance check. The majority of the parts and respective quantities to be used in each check is not known until the maintenance has started. In fact, Airbus estimates that no more than 30% of the parts are known in advance. In an effort to help the MRO and airline companies keep inventories low, limit critical orders and meet on-time completion of checks, Airbus implemented a program called Consumption Data Analysis Services (CDA). This service is free of charge, but it requires that participants share maintenance consumption data for a type of check called C-checks.

The CDA process is as follows:

- Customer requests Airbus to provide a consumption analysis list for its next maintenance check.

- Airbus, based on the latest data, performs an analysis and determines the expected materials that could be used.

- Customer receives the list and prepares for maintenance.

- Customer reports actual consumption back to Airbus.

- Airbus does some data cleaning and consolidation.

- Airbus compares actual consumption against data provided to customer.

An example of the recommended list provided by Airbus is presented below. The table contains the following data: part number, part description, material group, average consumption, usage rate, standard deviation, and the quantity recommended by Airbus. Average consumption is calculated by dividing the total consolidated demand for the part across all maintenance checks by the number of checks that have had demand. The usage rate is calculated by dividing the number of checks where demand is greater than zero by the total number of checks available in the consolidated data. The part numbers presented on Table 4.1 have been changed due to confidentiality agreements.

Part Number	Description	Mat Group	Avg Qty	Usage Rate	Std Dev	Rec Qty
ABCDE	Seal	STD	16	75.21 %	3.73	19
FGHIJ	Seal	STD	16	75.21 %	4.76	19
KLMNO	Packing	STD	3	64.46 %	1.89	4
PQRST	Washer	STD	27	59.50 %	13.97	31

Table 4.1: Example of Recommended List Provided to Customers

Upon signing a non-disclosure agreement, Airbus provided nearly 800 different maintenance check reports for the A320 aircraft that have been compiled in this program. The data comes from more than 25 different countries participating in the study. In some cases, we found more than one airline and/or MRO per country. At the time of data collection, Airbus stated they had around 4,000 A320s in operation worldwide.

The data provided contains more than 26,000 different part numbers among expendable, repairable and rotable parts, with some or all of the following attributes: part number, part description, material type, flight hours, flight cycles, lead time, price, demand consumption per maintenance check, date of maintenance, type of check, and essentiality code. However, we are only interested in the 21,000 parts that are expendable. It is also important to mention that close to 9,200 parts do not have unit price associated with them. Furthermore, 1,054 of the remaining parts, do not have supplier lead times. As such, our part population is reduced to 11,724 different parts.

In order to complete some of the missing information such as lead time and price, additional data was provided by an airline company. The data contains several thousand parts with all or some of the following parameters: part number, unit price, supplier name and address, and lead time. This data is needed by the methodology selected to solve the problem. As mentioned previously, the current dissertation considers a "system view" approach rather than an "item view" approach. In this sense, the model developed in this work is trying to minimize the total cost which includes holding cost, transportation cost and penalty cost while satisfying an agreed service level for on-time delivery. Based on this consideration and a given maintenance schedule, the model determines the recommended base stock level that the company needs to have.

After consolidating both databases, we were able to identify an additional 2,523 parts with unit price and lead time. As such, our part population was increased to 14,247 parts.

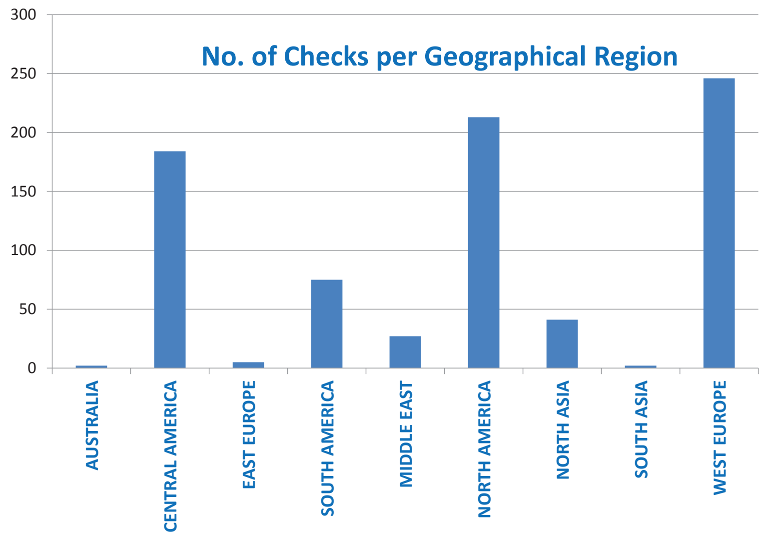


Figure 4.1: Number of Checks per Geographical Region

4.2 Data Analysis

At the time the data was provided (August, 2007), the data set contained 795 different maintenance checks from across the globe. The number of checks in each geographic region is shown in Figure 4.1. The main contributors to the data are airlines and MROs from America and Europe, with more limited participation from the other continents.

Aircraft age, flight hours and flight cycles are some of the main variables used to schedule maintenance checks for aircrafts. Based on the Maintenance Planning Document (MPD) from Airbus, the required maintenance check should be done twenty months after the previous maintenance, after 6,000 flight hours, or after 4,500 flight cycles (Table 4.2).

In order to understand the characteristics of the data set population, histograms showing flight hours, flight cycles and age of the aircraft are presented as follows: Figure 4.2 shows that around 96% of the aircraft were between 18 months to 167 months old. In other words, aircraft with demand between 18 months and 167 months

Criteria	Range
Flight Hours	6,000
Flight Cycles	4,500
Months	20

Table 4.2: Maintenance Check Interval Criteria

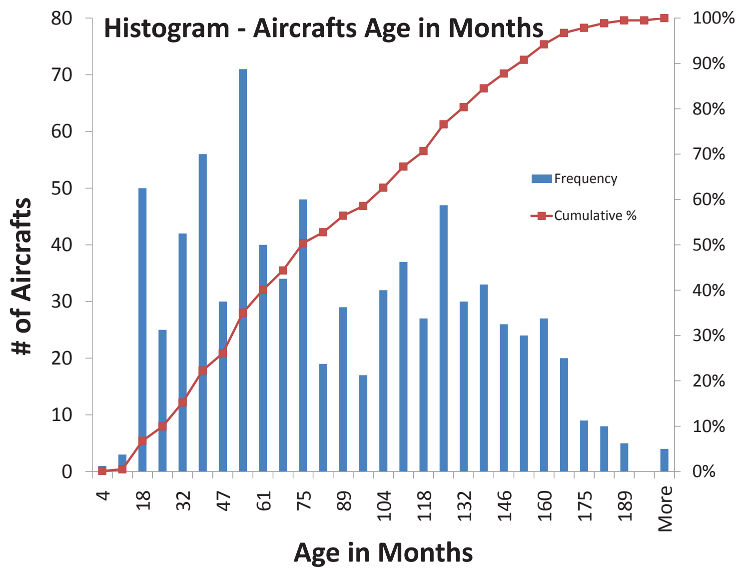


Figure 4.2: Aircraft Age in Months

is well represented on the data set provided.

Another important variable is flight hours. Figure 4.3 depicts the flight hours the aircraft had at the time of the maintenance checks; at the time of the maintenance checks, 85% of the aircraft had flown between 5,000 and 40,000 hours.

The last variable is flight cycles. Close to 89% of the data set ranges 2,300 and 23,350 flight cycles, as represented in Figure 4.4. As can be seen in all these figures, the range within each of the main variables is quite wide. The data provided can be used to represent maintenance checks for aircraft where main variables are in the ranges shown in Table 4.3.

The aircraft population is divided into three different groups in order to under-

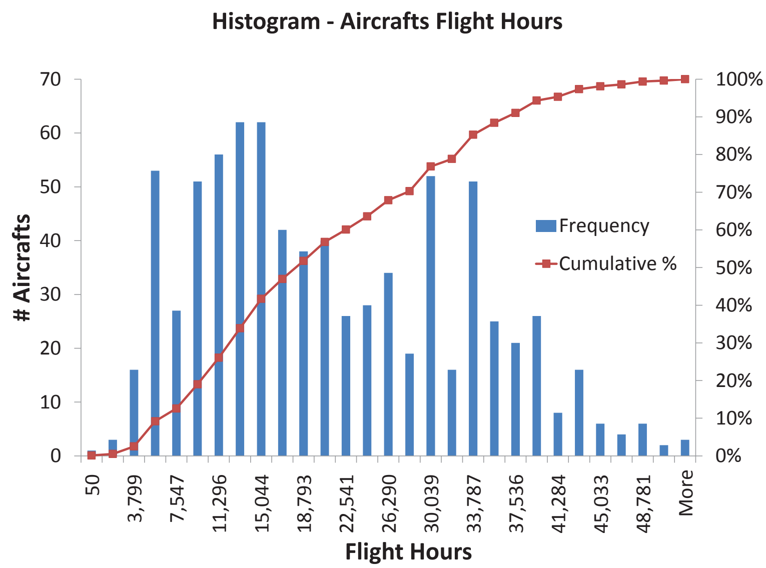


Figure 4.3: Aircrafts Flight Hours

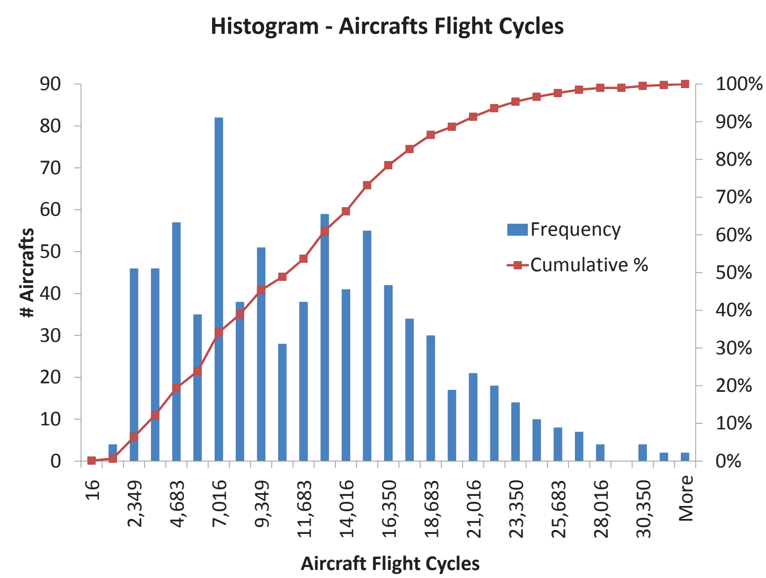


Figure 4.4: Aircrafts Flight Cycles

Criteria	Lower Bound	Upper Bound
Flight Hours	5,000	40,000
Flight Cycles	2,300	23,350
Months	18	167

Table 4.3: Aircraft Maintenance Variables

stand if there are differences in demand consumption among them. The first variable used to identify the groups is the age of the aircraft at the time of the check; and the groups are divided based on the percentile of aircraft in this category. The first group contains all aircraft from percentile 0 through the 33rd percentile; the second group contains the aircraft from percentile 33rd percentile through percentile 66th percentile; and the last group contains the remaining aircraft.

The groups are well represented with 278 aircraft in the first group, 257 aircraft in the second group and 260 aircraft in the third group. As seen in table 4.4, as the percentile increases, the average number of parts used on the check increases. Additionally, the standard deviation increases. In fact, the mean and standard deviations almost double, double or more than double in the second and third groups compared to the first group.

Criteria	Lower Percentile	Upper Percentile	Avg No Parts	Std Dev Parts
First Group	0%	33%	165	121
Second Group	33%	66%	330	208
Third Group	66%	100%	447	274

Table 4.4: Groups based on Age - Average and Standard Deviation of Parts used During Maintenance

A similar analysis is done using flight hours as the main criteria and the results are similar when compared to the prior analysis. Population is well represented among the three groups, with Group 1 having 269 aircraft, Group 2 having 271 aircraft and Group 3 having 255 aircraft. In Group 1, the average number of parts as well as the standard deviation are a little bit higher compared to the prior analysis; Group 2 has a slightly lower average; and Group 3 has similar values as seen in table **??**. It can also be stated that as the percentile increases, the mean and standard deviation of the number of parts used in maintenance increases.

Criteria	Lower Percentile	Upper Percentile	Avg No Parts	Std Dev Parts
First Group	0%	33%	183	158
Second Group	33%	66%	304	205
Third Group	66%	100%	452	266

Table 4.5: Groups based on Flight Hours - Average and Standard Deviation of Parts used During Maintenance

As discussed before, there are differences in the average number of parts used in each maintenance check based on the groups described above. On the large problem we will solve in this dissertation, we will assume that the aircraft coming to maintenance are equally distributed among the different groups presented before.

Something important to mention is that due to the sporadic nature of the demand, it would be challenging to split the data into two or more groups to generate scenarios only for aircraft which are in the same category. For instance, 44% of the parts have been used in more than one of the groups mentioned as can be seen in Figure 4.5; and 56% of the parts have been used in only one of the groups, but could have been consumed multiple times within it. For the parts used only in one of the groups, close to 42% of those parts fall into Group 2 (age criteria) and 45% fall into Group 3 (age criteria). More over, there is only one aircraft that contains maintenance records where all the parts were used in the same group. The majority of the aircraft, 772 out of the 795 aircraft, have parts that had been used by aircraft falling into any of

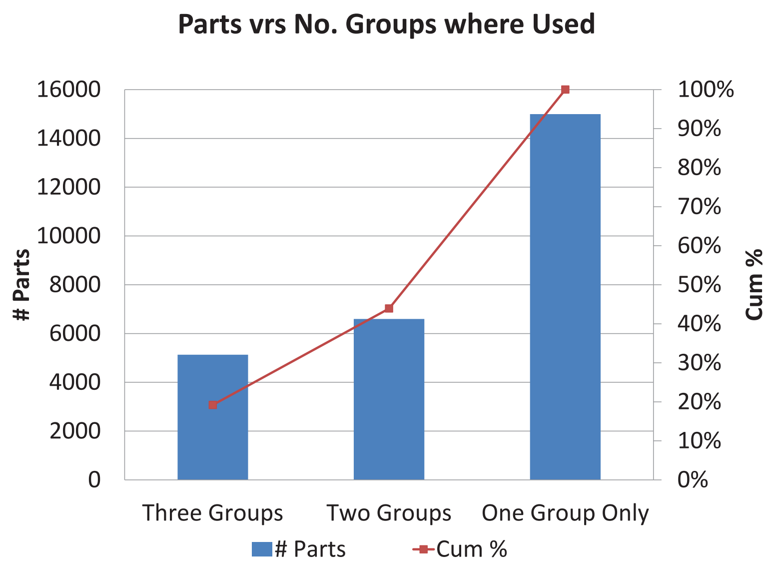


Figure 4.5: Parts and groups where they have been used

the three groups. In other words, 99.9% of the aircraft have parts that have been used by aircraft falling in two or three groups as shown in Figure 4.6. Given this information, it is better to utilize all the data together rather than splitting it into different groups.

Other important data gathered for this project includes shipment cost. In order to get estimated shipment costs, data from FEDEX was downloaded from its website (www.Fedex.com) on June 16, 2010. It is assumed that all parts are sent from the suppliers to an airline's logistics center in Miami, FL. Also, the airline provided the list of the suppliers and the list of parts supplied by them. For the cases where a part has multiple suppliers, the supplier with the greatest number of purchase orders was selected. There are some parts on the Airbus file that do not match the parts provided by the airline and, hence, do not get a supplier assigned. For these cases, a supplier is randomly assigned based on percentages of purchase orders placed to each supplier. The importance of the supplier assignment to each part is due to of its location, as a way to estimate the shipment cost from the supplier to the company's logistics center.

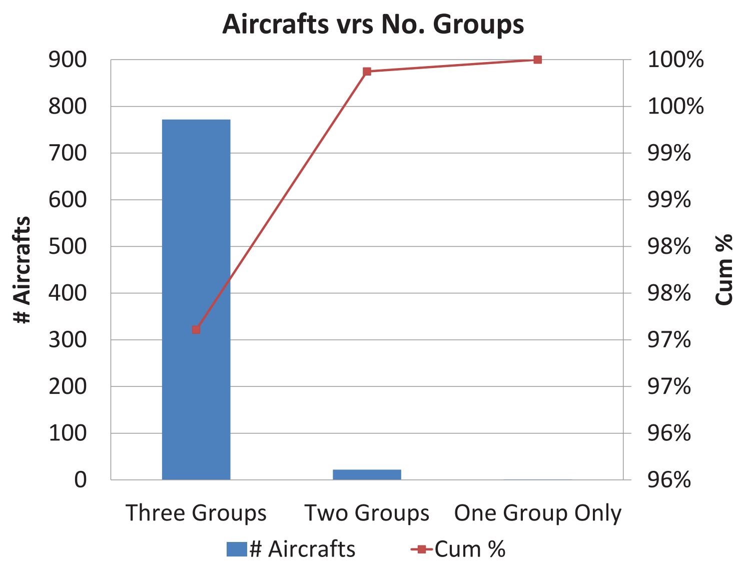


Figure 4.6: Aircrafts and groups where their parts have been used to

Table 4.6 shows the quantity and percentage of purchase orders placed to the top 10 suppliers by the company, including data used to randomly assign suppliers to parts that do not match between both files. The addresses and zip codes of the suppliers were found using the Internet. In this way, we were able to identify the table from the FEDEX site that provided the cost from supplier to the logistics center.

Another piece of data needed to calculate the shipment cost is the weight of each package. Since the parts we are dealing with are expendable items, we assume the weight for each part is between 5 to 15 pounds; and we randomly generate it for each part. Now we have all the necessary information and are able to identify the cost of shipping a part with a specific weight from one zip code (supplier) to another (company's logistics center). This research assumes both, a normal delivery method and an expedited delivery method and as experience might suggest, the expedited method is more expensive but faster.

Supplier	No. POs	Ind %	Cum %
Supplier 1	46503	23.52 %	23.52 %
Supplier 2	17864	9.03 %	32.55 %
Supplier 3	12109	6.12 %	38.67 %
Supplier 4	10194	5.15 %	43.83 %
Supplier 5	10181	5.15 %	48.98 %
Supplier 6	8156	4.12 %	53.1 %
Supplier 7	6886	3.48 %	56.58 %
Supplier 8	5474	2.77 %	59.35 %
Supplier 9	3452	1.75 %	61.1 %
Supplier 10	3113	1.57 %	62.67 %

Table 4.6: Purchase Orders Placed to the Top 10 Suppliers

4.3 Collaboration

Collaboration is one of the key components of this research. As discussed in section 4.2, based on the Maintenance Planning Document (MPD) from Airbus, the required maintenance check should be done once the aircraft completes twenty months after the previous maintenance, or 6,000 flight hours, or 4,500 flight cycles, as described in Table 4.2. Let's assume that the majority of aircraft reach the next maintenance based on the 20 months criteria. In a 5 year period, a company would be able to gather only 3 maintenance checks, or 6 maintenance checks every 10 years. So, for a company like JetBlue that has 130 aircraft of this type in use, according to wikipedia.com [58], it will take 10 years to get close to the total amount of data compiled by Airbus, $130 * 6 = 780$.

Moreover, not all the companies participating in Airbus's worldwide data collection program are as large as Jet Blue. Some regional companies are small, and, in all likelihood, would be unable to compile data such as Airbus is providing or take advantage of applications such as the one proposed in this research. Their participation grants them access to valuable information.

In the next chapter, we will discuss the methodologies to be used to solve the problem addressed in this dissertation. One key aspect of this problem is the sporadic and uncertain demand of the spare parts. We address this problem using a scenario optimization technique. One aspect of scenario optimization is that the problem size increases dramatically as the number of scenarios increase. In order to have a more tractable optimization model, some pr-processing techniques to remove unnecessary constraints are developed and discussed.

CHAPTER 5

MODELING METHODOLOGY

In this chapter, first, the scenario-based approach is introduced as the method to be used in this research due to the nature of the problem it is able to handle. Second, the mathematical model of this dissertation is developed. The model determines the inventory base stock level for all parts in set I in order to minimize total cost (penalty cost for late delivery, holding cost and extra shipment cost) while satisfying a service level for on-time equipment delivery. Also, all assumptions behind the model are explained, and descriptions of the parameters, variables and the meaning of the constraints are provided. Lastly, some pre-processing techniques are developed in order to be able to solve the model.

5.1 Scenario-Based Approach

Throughout this work, it has been shown that spare part demand is sporadic in nature, representing a big challenge for companies. One methodology that address uncertainty is the scenario-based approach. This methodology is based on two stages: The first consists of identifying the scenarios to be considered, and the second solves the optimization model based on those scenarios.

According to Sitompul and Aghezzaf [59], uncertainty is present at all levels in a production system, and until recently, sensitivity analysis has been used in post-optimality studies to discover the impact of data variability on the model's recommendation. However, the authors state that this approach doesn't solve the issue because it is a passive approach. As such, something more proactive is needed in order to produce solutions that are less sensitive to data variability.

Scenario-based optimization is an approach where the exact values of some parameters of the optimization problem are not known with absolute certainty, but may vary to a larger or lesser extent depending on the nature of the factors they represent, as mentioned by Better and Glover [60]. One advantage of this approach is that it is effective in finding a solution that is feasible for all the scenarios considered and, at the same time, minimizes the deviation of the overall solution for each scenario. One disadvantage of this methodology (Better and Glover [60]) is that this approach considers a very small subset of possible scenarios, and the size and complexity of models it can handle is quite limited. In the same paper, the authors show an approach used by Dembo [61] for solving stochastic programs based on a method for solving deterministic scenario subproblems and combining the optimal scenario solutions into a single feasible decision.

As stated by Dembo [61], the 'scenario optimization' approach to stochastic programming can be described as:

- *Stage 1*: Compute a solution to the (deterministic) problem under all scenarios.
- *Stage 2*: Solve a coordinating or tracking model to find a single, feasible policy.

This author mentioned that Stage 1 may be viewed as a sampling of the solution space of the underlying stochastic model; and Stage 2 attempts to find a single "feasible" policy that best "fits" the behavior of the system under uncertainty.

In other words, as described by Sitompul and Aghezzaf [59], the problem can be formulated as a deterministic mathematical problem for a single scenario s (the scenario sub-problem, SP) as follow:

SP:

$$Z^s = minimize \sum_{j=1}^{n} c_j^s * x_j \tag{5.1}$$

Subject to:

$$\sum_{j=1}^{n} a_{ij}^{s} * x_j \;=\; b_i^{s} \qquad\qquad \forall i \;=\; 1, 2, ..., m \tag{5.2}$$

$$x_j \;\geq\; 0 \qquad\qquad \forall j = 1, 2, ..., n \tag{5.3}$$

where: c_j is the cost of producing item j, a_{ij} is the amount of resource i needed to manufacture item j, and b_i is the amount of resource i available.

So, the model SP needs to be solved for each scenario s, and then it is necessary to solve a tracking model to find a single, feasible decision for all scenarios. A tracking model could be formulated as follows:

$$Minimize \sum_{s} p^{s}(\sum_{j} c_j^{s} * x_j \;-\; Z^{s})^2 \;+\; \sum_{s} p^{s}(\sum_{s} a_{ij}^{s} * x_j \;-\; b_i^{s})^2 \tag{5.4}$$

$$x_j \;\geq\; 0 \qquad\qquad j = 1, 2, ..., n \tag{5.5}$$

As it is stated by Better and Glover [60], the purpose of this tracking model is to find a solution that is feasible under all scenarios, and which penalizes solutions that differ greatly from the optimal solution under each scenario. The authors also mention that the objective functions are squared to avoid non-negativity and also that there are more sophisticated tracking models.

In our case, the parameter that is unknown is the demand given its sporadic nature; however, based on historical data we know what values it has taken. Given the nature of problems that the scenario-based approach is able to address, and together with the nature of the problem we are addressing in this dissertation, the scenario-based approach has been selected as part of the method to determine the base stock level for on-time equipment delivery while minimizing cost. In the next sections of this chapter, we develop the optimization model, discuss the assumptions and the pre-processing techniques developed.

5.2 Model Assumptions

The characteristics of the problem addressed by this research were listed on section 3.1. Even though the majority of the assumptions behind the mathematical model developed in this dissertation are similar to the ones presented by the tool kit problem [52], some of the characteristics of the nature of the problem are different. Below are listed the assumptions of the mathematical model:

- It is assumed that the company knows the maintenance schedule of its equipment for a given time frame T (e.g., two months). The start and delivery due dates of the equipment from maintenance are known.
- Demand scenarios are assigned based on historical data for each part number for each equipment.
- The model assumes that late delivery is due to parts only; manpower and tools are considered available with unlimited capacity.
- The stocking policy used is a continuous review order-up-to level policy. Every time there is demand, an order is placed for the same quantity either to satisfy the demand or to replenish inventory.
- It is assumed that lead times are reliable and there are only two different types of them, normal and expedited. Expedited lead time is shorter than normal lead time but has a higher cost.
- Each part number has its own lead times and the parts lead times are constant across different scenarios.
- It is assumed one type of equipment is used (i.e., A320 aircraft family).
- It is assumed that replenishment orders are received at the end of the day and material is consumed at the beginning of the day.

5.3 Model Parameters and Variables Definition

In this section, the parameters and variables used in the mathematical model are described:

- N: Total equipment to be scheduled in the selected time frame T.
- a: Set of N equipment to be scheduled in maintenance during time frame T, a={1,2,...,N}.
- R: Total number of different types of parts or SKUs to be included.
- i: Set of part numbers that potentially could be used in maintenance. i = {1,2,..., R}.
- P: Penalty cost for late delivery per time unit.
- D_a: Delivery due time of aircraft a from maintenance.
- h_i: Holding cost for part number i.
- C: Total number of different scenarios that are modeled.
- z: Set of scenarios that are modeled. z = {1,2,..., C}.
- t: Current time.
- $\lambda_{i,z}^{a,t}$: Demand of part number i for equipment a at time t for scenario z.
- $\omega_{i,z}$: Total expected demand of part number i at scenario z; mathematically, it is represented by $\sum_{a=1}^{N} \sum_{t=1}^{T} \lambda_{i,z}^{a,t}$
- $\chi_{i,z}^{t}$: Demand of part number i at scenario z that is expected to happen from time t to the end of the time horizon; mathematically, it is represented by $\sum_{a=1}^{N} \sum_{t}^{T} \lambda_{i,z}^{a,t}$

- $\upsilon_{i,z}^{t}$: Contains the demand for part number i, in scenario z that has already happened from time zero to time t'; mathematically it is represented by $\sum_{a=1}^{N} \sum_{t=0}^{t'} \lambda_{i,z}^{a,t}$
- r_a : Scheduled maintenance start time for equipment a.
- τ_i^N : Normal lead time for part number i.
- τ_i^E : Expedited lead time for part number i.
- M: A big number.
- G_i: Incremental price of placing an expedited shipment order for item i instead of a normal order.
- $Prob_z$: Probability that scenario z will occur.

Even though the main objective of the problem is to identify the recommended base stock level for each part, several other auxiliary variables are also used. The variables used in the model are described as follows:

- S_i: Base stock level for part number i - main variable.
- $Q_{i,z}^{t}$: On hand inventory of part number i available in stock at the local MRO at time t for scenario z
- $\alpha_{i,z}^{a,t}$: Gets the value of 1 if on hand quantity of part number i at time t in scenario z is assigned to equipment a; otherwise gets the value of 0.
- $E_{i,z}^{a,t}$: Gets the value of 1 if an emergency shipment is placed for part number i, equipment a at time t for scenario z; otherwise gets the value of 0.
- $f_{a,z}$: Actual delivery time for equipment a from maintenance in scenario z.
- $Qasg_{i,z}^{t,a}$: Inventory quantity of part number i assigned to aircraft a at time t in scenario z.

- $\phi^t_{i,z}$: Total quantity on order placed as expedited shipment for part number i, at time t in scenario z.

- $\rho^t_{i,z}$: Total quantity on order placed as normal shipment for part number i, at time t in scenario z.

- $\gamma^t_{i,z}$: Total received quantity of part number i at time t in scenario z; includes expedited and normal replenishment.

- $\theta^t_{i,z}$: Received quantity of expedited orders of part number i at time t in scenario z.

- $\eta^t_{i,z}$: Received quantity of normal orders of part number i at time t in scenario z.

- $IT^t_{i,z}$: In transit inventory of part number i at time t in scenario z.

- $L_{a,z}$:Number of time units that equipment a is delivered tardy in scenario z.

- $\beta_{a,z}$: Gets the value of 1 if equipment a is delivered tardy in scenario z; otherwise gets the value of 0.

5.4 Model Development

In this section, the mathematical model that this dissertation deals with is developed. This section also explains the meaning of the constraints that the model needs to satisfy.

In words, the mathematical model can be stated as follows:

Objective:

Minimize {Penalty Cost for Tardy Delivery + Inventory Holding Cost + Shipment Cost}

Subject to:

% On-time Equipment delivery ≥ Service Level

Even though the model can be described in two lines, the mathematical model requires several constraints to address inventory balancing, replenishment orders, auxiliary variables, etc.

The mathematical model is presented below. In the objective function, the first term calculates the total cost for tardy equipment delivery; the second term provides the total cost for holding inventory, and the third term provides the additional shipment costs incurred when expedited orders are placed.

Objective:

$$Min \sum_{a=1}^{N} \sum_{z=1}^{C} P * Max(0,\ f_{a,z} - D_a) * Prob_z + \sum_{i=a}^{R} h_i * S_i + \sum_{a=1}^{N} \sum_{i=1}^{R} \sum_{t=1}^{T} \sum_{z=1}^{C} G_i * E_{i,z}^{a,t} * Prob_z \qquad \forall\, z = 1, 2, ..., C \tag{5.6}$$

In equation 5.6, the first term is used to calculate tardiness. In order to avoid the 'Max' that appears on it, the term is changed as follows:

$$Min \sum_{a=1}^{N} \sum_{z=1}^{C} P * L_{a,z} * Prob_z + \sum_{i=a}^{R} h_i * S_i + \sum_{a=1}^{N} \sum_{i=1}^{R} \sum_{t=1}^{T} \sum_{z=1}^{C} G_i * E_{i,z}^{a,t} * Prob_z \forall z = 1, 2, ..., C \tag{5.7}$$

The new objective function shown in Equation 5.7 has replaced the term $Max(0,\ f_{a,z} - D_a)$ by the term $L_{a,z}$ which provides the time units that equipment 'a' is delivered tardy. It is necessary to add new constraints in order to ensure that the penalty cost is applied only if equipment is delivered tardy. This is done with equations 5.8 and 5.9. Constraint 5.8 is used to modify the objective function and avoid the 'Max' in the first term. It states that the tardiness of equipment a ($L_{a,z}$) is greater than or equal to the difference between the actual delivery date ($f_{a,z}$) and the expected delivery date (D_a). Constraint 5.9 states that equipment tardiness ($L_{a,z}$) is equal to or greater than zero.

$$L_{a,z} \geq f_{a,z} - D_a \qquad \forall\, a = 1, 2, ..., N;\ z = 1, 2, ..., C \tag{5.8}$$

$$L_{a,z} \geq 0 \qquad \forall\, a = 1, 2, ..., N;\; z : 1, 2, ..., C \tag{5.9}$$

At this moment, it is a good place to introduce some binary variables and its constraints. As mentioned previously, E_i^a is a binary variable that gets the value of 1 if an expedited order for part number i is placed, this order will satisfy demand for equipment a for scenario z at time t. Constraint 5.10 is used to indicate that E_i^a is a binary variable.

$$E_{i,z}^{a,t}\, is\, Binary \qquad \forall\; \exists\; \lambda_{i,z}^{a,t}$$
$$\forall\, a = 1, 2, ..., N;\; i = 1, 2, .., R;\; t = 0, 1, 2, ..., T\; z = 1, 2, ..., C \tag{5.10}$$

$\alpha_i^{a,t}$ is a binary variable that gets the value of 1 if on hand quantity of part number i at time t in scenario z is assigned to equipment a; otherwise gets the value of 0. Constraint 5.11 is used to indicate that $\alpha_i^{a,t}$ is a binary variable.

$$\alpha_{i,z}^{a,t}\, is\, Binary \qquad \forall\; \exists\; \lambda_{i,z}^{a,t}$$
$$\forall\, a = 1, 2, ..., N;\; i = 1, 2, .., R;\; t = 0, 1, 2, ..., T\; z = 1, 2, ..., C \tag{5.11}$$

Constraint 5.12 is used to calculate the tardiness ($L_{a,z}$) if demand for equipment a at scenario z is satisfied by placing a normal replenishment, in other words, it does not rely on any on hand inventory nor expedited order.

$$L_{a,z} + D_a \;\geq\; t \,+\, \tau_i^N - M(E_{i,z}^{a,t} \,+\, \alpha_{i,z}^{a,t}) \qquad \forall\; \exists\; \lambda_{i,z}^{a,t}$$
$$\forall\, a = 1, 2, ..., N;\; i = 1, 2, .., R;\; t = 0, 1, 2, ..., T;\; z = 1, 2..., C \tag{5.12}$$

Constraint 5.13 states that part number i for equipment a at time t during scenario z, can trigger only one type of shipment, either emergency or normal but not both at the same time.

$$E_{i,z}^{a,t} \,+\, \alpha_{i,z}^{a,t} \;\leq\; 1 \qquad \forall\; \exists\; \lambda_{i,z}^{a,t}$$
$$\forall\, a = 1, 2, ..., N;\; i = 1, 2, .., R;\; t = 0, 1, 2, ..., T;\; z = 1, 2, ..., C \tag{5.13}$$

The following two constraints deal with actual delivery date. Constraint 5.14 states that the actual delivery date ($f_{a,z}$) for equipment a at scenario z is equal to or greater than the delivery due date (D_a). Constraint 5.15 is used to determine the actual delivery date of equipment a ($f_{a,z}$) at scenario z assuming an expedited lead time is placed to satisfy demand. It is read as as follows: If an expedited shipment order was placed ($E^{a,t}_{i,z} = 1$), the arrival time of equipment a plus the expedited lead time for part number i is equal or less than the actual delivery date of the equipment.

$$f_{a,z} \geq D_a \qquad \forall\, a = 1, 2, ..., N;\ z = 1, 2..., C \tag{5.14}$$

$$f_{a,z} \geq r_a + \tau^E_i - M(1 - E^{a,t}_{i,z}) \qquad \forall\ \exists\ \lambda^{a,t}_{i,z}$$
$$\forall\, a = 1, 2, ..., N;\ i = 1, 2, .., R;\ t = 0, 1, 2, ..., T;\ z = 1, 2, ..., C \tag{5.15}$$

The following three constraints are used when on-hand inventory is assigned to an equipment. Constraint 5.16 assigns the value of 1 to $\alpha^{a,t}_{i,z}$ if there is enough inventory of part number i at the local warehouse during scenario z and it is allocated in quantity $Qasg^{a,t}_{i,z}$ to the equipment a at time t; otherwise $\alpha^{a,t}_{i,z}$ gets the value of 0. Constraint 5.17 is used to indicate that the quantity ($Qasg^{a,t}_{i,z}$) of part number i assigned to equipment a at time t during scenario z could only get one of two values: the value of the demand ($\lambda^{a,t}_{i,z}$) or zero. Constraint 5.18 is used to indicate that the quantity of part number i assigned to equipment a at time t during scenario z ($Qasg^{a,t}_{i,z}$) is equal to or greater than zero.

$$Qasg^{a,t}_{i,z} \geq \lambda^{a,t}_{i,z} - M * (1 - \alpha^{a,t}_{i,z}) \qquad \forall\ \omega_{i,z} > 0$$
$$\forall\, a = 1, 2, ..., N;\ i = 1, 2, .., R;\ t = 0, 1, 2, ..., T;\ z = 1, 2, ..., C \tag{5.16}$$

$$Qasg^{a,t}_{i,z} \leq \lambda^{a,t}_{i,z} * \alpha^{a,t}_{i,z} \qquad \forall\ \omega_{i,z} > 0$$
$$\forall\, a = 1, 2, ..., N;\ i = 1, 2, .., R;\ t = 0, 1, 2, ..., T;\ z = 1, 2, ..., C \tag{5.17}$$

$$Qasg_{i,z}^{a,t} \geq 0 \qquad \forall\ \omega_{i,z} > 0$$

$$\forall\, a = 1, 2, ..., N;\ i = 1, 2, .., R;\ t = 0, 1, 2, ..., T;\ z = 1, 2, ..., C \qquad (5.18)$$

Constraints 5.19 and 5.20 are used to indicate that the quantity of part number i assigned to all equipment at time t during scenario z is equal to or less than the on hand quantity available at the local warehouse at the beginning of the day.

$$Q_{i,z}^{t-1} \geq \sum_{a=1}^{N} Qasg_{i,z}^{a,t} \qquad \forall\ \omega_{i,z} > 0$$

$$\forall\, a = 1, 2, ..., N;\ i = 1, 2, .., R;\ t > 0\ \ and\ \ t\ \leq\ \tau_i^N;\ z = 1, 2, ..., C \quad (5.19)$$

$$Q_{i,z}^{t-1} \geq \sum_{a=1}^{N} Qasg_{i,z}^{a,t} \qquad \forall\ \omega_{i,z} > 0; \chi_{i,z}^{t-\tau_i^N} > 0$$

$$\forall\, a = 1, 2, ..., N;\ i = 1, 2, .., R;\ t > \tau_i^N;\ z = 1, 2, ..., C \qquad (5.20)$$

The following two constraints deal with expedited orders. Constraint 5.21 is used to determine the total quantity of part number i placed as an expedited order at time t during scenario z. Constraint 5.22 is used to set the expedited order term ($\phi_{i,z}^t$) as zero at time t for part number i during scenario z when at time t demand has not yet started ($\upsilon_{i,z}^t$=0).

$$\phi_{i,z}^t = \sum_{a=1}^{N} (\lambda_{i,z}^{a,t} * E_{i,z}^{a,t}) \qquad \forall\ \omega_{i,z} > 0; \chi_{i,z}^t > 0; \upsilon_{i,z}^t > 0$$

$$\forall\, a = 1, 2, ..., N;\ i = 1, 2, .., R;\ t = 0, 1, 2, ..., T;\ z = 1, 2, ..., C \qquad (5.21)$$

$$\phi_{i,z}^t = 0 \qquad \forall\ \omega_{i,z} > 0; \chi_{i,z}^t > 0; \upsilon_{i,z}^t = 0$$

$$\forall\, i = 1, 2, .., R;\ t = 0, 1, 2, ..., T;\ z = 1, 2, ..., C \qquad (5.22)$$

The next two constraints are related to normal orders. Constraint 5.23 determines the total quantity of part number i placed as a normal order at time t during scenario z. Constraint 5.24 is used to set the normal order term ($\rho_{i,z}^t$) as zero at time t for part number i during scenario z when at time t demand has not yet started ($\upsilon_{i,z}^t$=0).

$$\rho_{i,z}^t = \sum_{a=1}^{N} [\lambda_{i,z}^{a,t} * \alpha_{i,z}^{a,t}] \qquad \forall\ \omega_{i,z} > 0; \chi_{i,z}^t > 0; \upsilon_{i,z}^t > 0$$

$$\forall\, a = 1, 2, ..., N;\ i = 1, 2, .., R;\ t = 0, 1, 2, ..., T;\ z = 1, 2, ..., C \qquad (5.23)$$

$$\rho^t_{i,z} = 0 \qquad \forall\ \omega_{i,z} > 0; \chi^t_{i,z} > 0; \upsilon^t_{i,z} = 0$$

$$\forall\, i = 1, 2, .., R;\ t = 0, 1, 2, ..., T;\ z = 1, 2, ..., C \tag{5.24}$$

The next two constraints calculate the quantities received from expedited orders. Constraint 5.25 determines the received quantities coming on expedited orders for part i, received at time t during scenario z. These orders received at time t were placed t-τ_i^E time units earlier. Constraint 5.26 is used to set the received quantities from expedited orders as zero at time t for part number i during scenario z because elapsed time t is shorter than expedited lead time (τ_i^E). Hence, we do not expect any receipts at this time.

$$\theta^t_{i,z} = \phi^{t-\tau_i^E}_{i,z} \qquad \forall\ \omega_{i,z} > 0; \chi^{t-\tau_i^E}_{i,z} > 0$$

$$\forall\, i = 1, 2, .., R;\ t > \tau_i^E;\ z = 1, 2, ..., C \tag{5.25}$$

$$\theta^t_{i,z} = 0 \qquad \forall\ \omega_{i,z} > 0$$

$$\forall\, i = 1, 2, .., R;\ t \leq \tau_i^E;\ z = 1, 2, ..., C \tag{5.26}$$

The next two constraints calculate the quantities received from normal orders. Constraint 5.27 determines the received quantity coming on normal orders for part i, received at time t during scenario z. These orders received at time t were placed t-τ_i^N times units earlier. Constraint 5.28 is used to set the received quantities from normal orders at zero at time t for part number i during scenario z because elapsed time t is shorter than normal lead time (τ_i^N). Hence, we do not expect any receipts at this time.

$$\eta^t_{i,z} = \rho^{t-\tau_i^N}_{i,z} \qquad \forall\ \omega_{i,z} > 0; \chi^{t-\tau_i^N}_{i,z} > 0$$

$$\forall\, i = 1, 2, .., R;\ t > \tau_i^N;\ z = 1, 2, ..., C \tag{5.27}$$

$$\eta^t_{i,z} = 0 \qquad \forall\ \omega_{i,z} > 0$$

$$\forall\, i = 1, 2, .., R;\ t \leq \tau_i^N;\ z = 1, 2, ..., C \tag{5.28}$$

Constraints 5.29 and 5.30 calculate the total received quantity of part number i at time t in scenario z, including expedited ($\theta^t_{i,z}$) and normal ($\eta^t_{i,z}$) replenishment. The only difference between these constraints is the time frame covered. Constraint 5.29 includes $t > 0$ *and* $t \leq \tau^N_i$ while constraint 5.30 includes $t > \tau^N_i$. Also, constraint 5.30 is only added if more expected demand happens at time $t-\tau^N_i$ or beyond ($\chi^{t-\tau^N_i}_{i,z} > 0$).

$$\gamma^t_{i,z} = \theta^t_{i,z} + \eta^t_{i,z} \qquad \forall\ \omega_{i,z} > 0$$
$$\forall\, i = 1,2,..,R;\ t > 0\ \ and\ \ t \leq \tau^N_i;\ z = 1,2,...,C \tag{5.29}$$

$$\gamma^t_{i,z} = \theta^t_{i,z} + \eta^t_{i,z} \qquad \forall\ \omega_{i,z} > 0; \chi^{t-\tau^N_i}_{i,z} > 0$$
$$\forall\, i = 1,2,..,R;\ t > \tau^N_i;\ z = 1,2,...,C \tag{5.30}$$

Constraints 5.31 and 5.32 state that on hand inventory ($Q^t_{i,z}$) of part number i at time t in scenario z is equal to on hand inventory at *t-1* minus total assigned quantity ($Qasg^{a,t}_{i,z}$) at time t for all equipment, plus total received quantities minus orders placed as expedited because parts will get consumed immediately and wont be part of any inventory. Also, constraint 5.32 is only added if more expected demand happens at time $t-\tau^N_i$ or beyond ($\chi^{t-\tau^N_i}_{i,z} > 0$). The only difference between these constraints is the time frame they cover. Constraint 5.31 covers $t > 0$ *and* $t \leq \tau^N_i$, and constraint 5.32 covers $t > \tau^N_i$.

$$Q^t_{i,z} = Q^{t-1}_{i,z} - \sum_{a=1}^{N} Qasg^{a,t}_{i,z} + \gamma^t_{i,z} - \sum_{a=1}^{N} \lambda^{a,t}_{i,z} * E^{a,t}_{i,z} \qquad \forall\ \omega_{i,z} > 0$$
$$\forall\, a = 1,2,...,N;\ i = 1,2,..,R;\ t > 0\ \ and\ \ t \leq \tau^N_i;\ z = 1,2,...,C \tag{5.31}$$

$$Q^t_{i,z} = Q^{t-1}_{i,z} - \sum_{a=1}^{N} Qasg^{a,t}_{i,z} + \gamma^t_{i,z} - \sum_{a=1}^{N} \lambda^{a,t}_{i,z} * E^{a,t}_{i,z}$$
$$\forall\, \omega_{i,z} > 0; \chi^{t-\tau^N_i}_{i,z} > 0;\ a = 1,2,...,N;\ i = 1,2,..,R;\ t > \tau^N_i;\ z = 1,2,...,C \tag{5.32}$$

The following three constraints calculate in-transit inventory. Constraint 5.33 states that in-transit inventory ($IT^t_{i,z}$) of part number i at times t= 0,1 is equal to expedited and normal purchase orders ($\phi_{i,z}^t$ *and* $\rho_{i,z}^t$) placed at time t. Constraints 5.34 and 5.35 state that in-transit inventory ($IT_{i,z}^t$) of part number i at time t during scenario z is equal to the in-transit inventory at t-1, plus expedited and normal purchase orders ($\phi_{i,z}^t$ *and* $\rho_{i,z}^t$) placed at time t, minus received quantity ($\gamma_{i,z}^t$) at time t. Constraint 5.35 is only added if more demand is expected to happen at time $t-\tau_i^N$ or beyond ($\chi_{i,z}^{t-\tau_i^N} > 0$).

$$IT_{i,z}^t = \phi_{i,z}^t + \rho_{i,z}^t \qquad \forall \omega_{i,z} > 0;\ i = 1,2,..,R;\ t \le 1;\ z = 1,2,...,C \tag{5.33}$$

$$\begin{aligned} IT_{i,z}^t &= IT_{i,z}^{t-1} + \phi_{i,z}^t + \rho_{i,z}^t - \gamma_{i,z}^t \qquad \forall\ \omega_{i,z} > 0 \\ &\forall\, i = 1,2,..,R;\ t > 1\ \textit{and}\ \ t \le \tau_i^N;\ z = 1,2,...,C \end{aligned} \tag{5.34}$$

$$\begin{aligned} IT_{i,z}^t = IT_{i,z}^{t-1} + \phi_{i,z}^t + \rho_{i,z}^t - \gamma_{i,z}^t \qquad &\forall\ \omega_{i,z} > 0; \chi_{i,z}^{t-\tau_i^N} > 0 \\ &\forall\, i = 1,2,..,R;\ t > \tau_i^N;\ z = 1,2,...,C \end{aligned} \tag{5.35}$$

Thee next three constraints calculates the base stock level. Constraint 5.36 states that base stock level (S_i) of part number i is equal to or greater than zero. Constraint 5.37 states that base stock level (S_i) of part number i is equal to or greater than on-hand inventory ($Q_{i,z}^t$) at any time t for any scenario z. Constraint 5.38 states that base stock level (S_i) is equal to the on-hand inventory at time t= 0 at any scenario z for part i.

$$S_i \ge 0 \qquad \forall\, i = 1,2,..,R \tag{5.36}$$

$$S_i \ge Q_{i,z}^t \qquad \forall\, \omega_{i,z} > 0;\ i = 1,2,..,R;\ t = 1,2,...,T;\ z = 1,2,...,C \tag{5.37}$$

$$S_i = Q_{i,z}^t \qquad \forall\, \omega_{i,z} > 0;\ i = 1,2,..,R;\ t = 0;\ z = 1,2,...,C \tag{5.38}$$

The following four constraints calculate the service level for on-time equipment delivery. Constraint 5.39 and 5.40 are used to assign the value of 1 to the auxiliary variable $\beta_{a,z}$ if equipment has been delivered tardy; otherwise the auxiliary variable is valued at 0. Constraint 5.41 states that the service level for on-time equipment delivery should be equal to or greater than agreed service level (SL). Constraint 5.42 indicates that $\beta_{a,z}$ is a binary variable.

$$\beta_{a,z} \leq L_{a,z} \qquad \forall\, a = 1, 2, ..., N;\; z = 1, 2, ..., C \tag{5.39}$$

$$L_{a,z} \leq M * \beta_{a,z} \qquad \forall\, a = 1, 2, ..., N;\; z = 1, 2, ..., C \tag{5.40}$$

$$\frac{N - \sum_{a=1}^{N} \beta_{a,z}}{N} \geq SL \qquad z = 1, 2, ..., C \tag{5.41}$$

$$\beta_{a,z}\, is\, binary \qquad \forall\, a = 1, 2, ..., N;\; z = 1, 2, ..., C \tag{5.42}$$

5.5 Model Discussion

As presented, the objective of the optimization model is to minimize total cost by determining the base stock level for each part while satisfying an agreed upon on-time equipment delivery from maintenance. In order to accomplish this objective, several inventory balancing constraints and artificial variables are added. All these constraints and variables, together with the multiple scenarios, increase the columns and rows generated by the problem.

The model has three binary variables E_i^a, $\alpha_i^{a,t}$ and $\beta_{a,z}$, but the rest of the constraints have no restrictions regarding integrality. The base stock level should be a positive integer, however, we rely on the demand data to be positive and integral to achieve this objective. The rest of the variables are not required to be integers, so, the problem we have can be categorized as Mixed Integer Linear Program, or MILP. Some algorithms used to solve these problems are cutting plane algorithms, branch and bound, and branch and cut. These types of problems are well studied and known to be NP-Hard (Non-Deterministic Polynomial-time Hard).

Given that columns and rows increase when multiple scenarios are created, we need to introduce pre-processing techniques in order to reduce the size of the optimization model. According to Wolsey [62], pre-processing detects and eliminates redundant constraints and variables and tighten bounds where possible, and the resulting linear/integer program is smaller/tighter, and it will typically be solved quickly. The author mentions that pre-processing is very important in the case of branch and bound because of the large quantity of linear programs that may need to be solved.

Based on the literature, one of the pre-processing techniques used is to remove rows when all the coefficients are zero (A). By removing the row, it has no impact on the solution of the problem because most likely the right hand side ($b = 0$) is zero, or if not, the problem is infeasible $Ax = b$. In our case, each row is a different part number, but given that our problem is multi-period and multi-scenario, we utilize this technique at each scenario across all time periods. The parameter checking this criteria is $\omega_{i,z}$.

Other novel ideas on pre-processing techniques are developed on this dissertation. Given that our model is multi-period, and the demand is known, one of the novel pre-processing technique validates if there is more demand to happen from time t to the end of the horizon. If so, the constraint is added, otherwise, the constraint is avoided. Similar to that approach, the other novel technique validates if the demand has already started. In other words, it validates if the demand from time zero to time t is greater than zero. If so, the constraint is added, otherwise, it is avoided. All these concepts are discussed in more details below.

$\omega_{i,z}$: As mentioned in the previous section, $\omega_{i,z}$ represents the total demand across all time periods and all equipment of part number i at scenario z. This parameter is used to avoid adding unnecessary constraints into the model. Basically, given that spare parts have sporadic demand, many parts don't have demand at all in a complete scenario. For this reason, there might be no need to add some constraints into the model. In other words, if $\omega_{i,z}$ is greater than zero, we allow constraints to be added into the model. This parameter is used widely and, in fact, is applied from constraint 5.16 through constraint 5.38.

As an example, one of the constraints that uses $\omega_{i,z}$ is the constraint that tracks orders needed to either satisfy demand or to return the base stock level to the recommended planned quantity. But if there is no demand in the complete scenario for a given part, there is no need to add the constraint. The same can be said for the constraints that track inventory in transit and many others use this parameter to restrict the generation of more constraints.

$\chi_{i,z}^{t}$: This parameter represents the demand of part number i at scenario z that is expected to happen after time t. It is worth mentioning again that we assume that demand is known in advance. Thus, we are capable of identifying if there will be more demand for part number i at scenario z after time t. In other words, if $\chi_{i,z}^{t}$ is greater than zero, we allow some constraints to be added into the model, otherwise, we avoid creating them.

The main difference in the usage of parameters $\chi_{i,z}^{t}$ and $\omega_{i,z}$ is that with the latter, we only check if there is demand or not; if there is demand, we add all the constraints for each time t. However, it could be that demand only happened at the beginning, and if we add constraints beyond that time t, there may be no benefit because they are loose, and we are simply increasing the size of the problem which will then take more time to solve. On the other hand, when we use $\chi_{i,z}^{t} > 0$ as part of the criteria to decide whether we add a constraint or not at time t, we are potentially reducing unnecessary constraints and the size of the optimization model to be solved. As an example, some of the constraints that use $\chi_{i,z}^{t}$ are some that track replenishment orders to be placed, either expedited or normal (i.e., constraints 5.21 and 5.23, respectively). For instance, if demand only happens in time 1, the model will stop adding constraints 5.21 and 5.23 from time 2 and beyond without causing any issues in the final result.

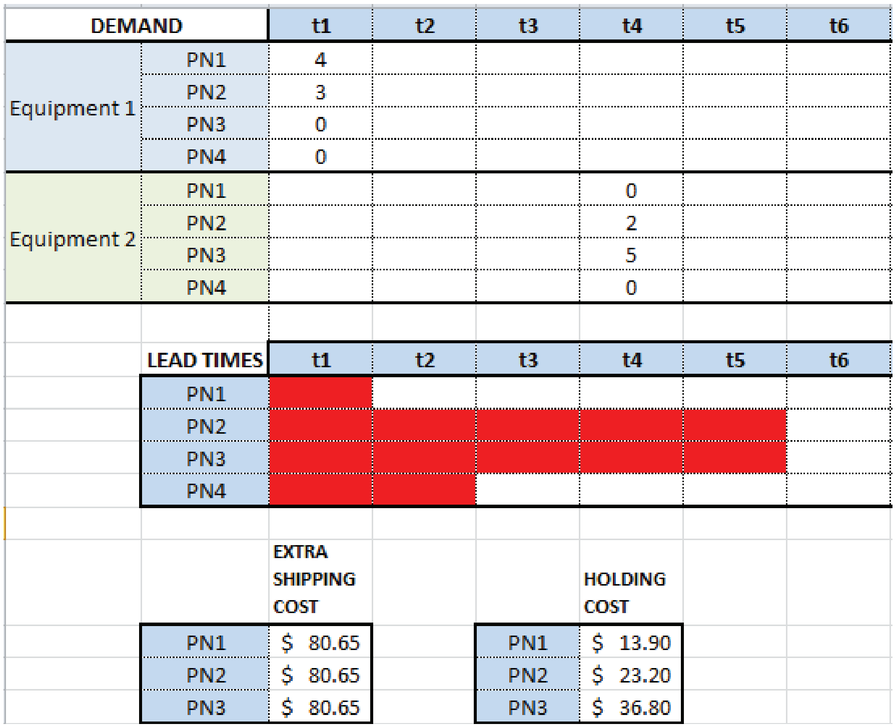
$\upsilon_{i,z}^{t}$: Following the same logic to reduce unnecessary constraints, another parameter is added, $\upsilon_{i,z}^{t}$, that contains all demand for part number i, in scenario z from time *t=0* to current time t. When using $\upsilon_{i,z}^{t}$, constraints are added only if $\upsilon_{i,z}^{t} > 0$ (only if demand has started). Similar to the previous cases, two of the constraints that use the parameter $\upsilon_{i,z}^{t}$ to limit constraints added into the model are constraints 5.21 and 5.23.

$\lambda_{i,z}^{a,t}$: Demand is also used to determine if a constraint is added into the model. Basically, if $\lambda_{i,z}^{a,t} > 0$, then a constraint might be added into the model if it satisfies any other criteria that the constraint might be subject to. As an example, constraint 5.16 helps to assign the quantity of pieces for part number i at scenario z for equipment a during time t. However, we only need to add that constraint when we have demand or $\lambda_{i,z}^{a,t} > 0$.

To illustrate the application of the parameters discussed above ($\lambda_{i,z}^{a,t}$, $\omega_{i,z}$, $\chi_{i,z}^{t}$ and $\upsilon_{i,z}^{t}$), a simple example is presented with one scenario, two equipments to be scheduled for maintenance and four different part numbers. As seen in Figure 5.1, equipment 1 will start maintenance during time 1, hence, demand will be reflected in this time period; equipment 2 will start maintenance during time 4, hence, demand is reflected in that time period. In this figure, lead times, holding and extra shipment costs can also be seen.

Based on the demand presented on Figure 5.1, we can confirm that all part numbers except the last one (PN4) have demand in either one or both of the equipment.

The first restriction we will discuss is $\lambda_{i,z}^{a,t} > 0$. Constraints 5.10- 5.13 and 5.15 are conditioned to be added into the model only if demand exist for that specific part number i, for equipment a, at scenario z at time t. As we know, demand happens only during the arrival time of the equipment, so, for the example we are considering, the model could create a maximum of four constraints for each of the 6 equations mentioned above, for a total of 24 constraints. The indices for which constraints will be created are (*a=1, t=1, i=1, z=1*); (*a=1, t=1, i=2, z=1*); (*a=2, t=4, i=2, z=1*); (*a=2, t=4, i=3, z=1*). As seen in Figure 5.1, equipment 1 has demand only for two parts, PN1 and PN2. Similarly, equipment 2 has demand for 2 parts, PN2



DEMAND		t1	t2	t3	t4	t5	t6
Equipment 1	PN1	4					
	PN2	3					
	PN3	0					
	PN4	0					
Equipment 2	PN1				0		
	PN2				2		
	PN3				5		
	PN4				0		

LEAD TIMES	t1	t2	t3	t4	t5	t6
PN1						
PN2						
PN3						
PN4						

	EXTRA SHIPPING COST
PN1	$ 80.65
PN2	$ 80.65
PN3	$ 80.65

	HOLDING COST
PN1	$ 13.90
PN2	$ 23.20
PN3	$ 36.80

Figure 5.1: Example to Illustrate Parameters used to Limit Constraints

and PN3. Those will be the constraints added into the optimization model. If we did not add the restrictions $\lambda_{i,z}^{a,t} > 0$ to generate constraints, the model would have created 72 constraints for each of the constraints 5.10- 5.13 and 5.15, for a total of 360 constraints. As added detail, the four equations generated from the constraint 5.11 are presented below:

$$\alpha_{1,1}^{1,1}\, is\ Binary \tag{5.43}$$

$$\alpha_{2,1}^{1,1}\, is\ Binary \tag{5.44}$$

$$\alpha_{2,1}^{2,4}\, is\ Binary \tag{5.45}$$

$$\alpha_{3,1}^{2,4}\, is\ Binary \tag{5.46}$$

Following the optimization model, the second parameter helping constraint reduction is $\omega_{4,1} > 0$, as all constraints but one, from 5.16 to 5.38, use $\omega_{i,z} > 0$ for validation before generating constraints. A simple example is demonstrated by constraint 5.17, $Qasg_{i,z}^{a,t} \leq \lambda_{i,z}^{a,t}.\alpha_{i,z}^{a,t}$, since we know that $\omega_{4,1} = 0$, PN4 at scenario 1 has no demand in any of the equipments. As a consequence, a total of 16 constraints will be avoided: 8 constraints avoided related to equipment 1 - $Qasg_{4,1}^{1,1}, Qasg_{4,1}^{1,2}, ...Qasg_{4,1}^{1,8}$; and 8 constraints avoided related to equipment 2 - $Qasg_{4,1}^{2,1}, Qasg_{4,1}^{2,2}, ...Qasg_{4,1}^{2,8}$. We will be only generating constraints for PN1, PN2 and PN3 but not for PN4 because the last one doesn't have demand for any equipment across all the scenarios. If we consider the other constraints from the optimization model where we have this restriction, we are reducing the total number of constraints for this small example by eight for each constraint. Given that 13 constraints were used (some constraints from 5.16 through 5.38 are mutually exclusive), we avoid $13x8 = 104$ constraints.

The impact of using these parameters to restrict the generation of constraints is larger as the data set increases. As we discussed in Tables 4.4 and 4.5, the number of parts used for each maintenance check is low compared to the total part population and data used in this research, and applied to the larger case. For instance, in both tables, the third group is the one that has the highest number of parts used per check, 447 and 452, with standard deviations of 274 and 266, respectively. As such, we will be avoiding a large number of constraint because the majority of the 4,000 parts have no demand.

The third parameter used to reduce the numbers of constraints is $\chi_{i,z}^{t}$. Again, this parameter determines if there is still demand pending from current time to the future. Since our optimization model is a multi-period model, we could potentially add many more constraints that might be loose and not needed because there are other constraints that, at that time, better describe the model. For the example under discussion, as seen in Figure 5.2, at time 1, all the parts except PN4 have demand that is going to happen during time 1 or beyond. For this reason, in the constraints where the parameter $\chi_{i,z}^{t} > 0$ is used, PN4 won't be able to generate constraints because

DEMAND	t1	t2	t3	t4	t5	t6	t7
PN1	4	0	0	0	0	0	
PN2	5	2	2	2	0	0	
PN3	5	5	5	5	0	0	
PN4	0	0	0	0	0	0	

Figure 5.2: Example to Illustrate Parameters $\chi^t_{i,z}$

it doesn't satisfy this criteria. This restriction is used on constraints 5.20 through 5.25 as well as in constraints 5.27, 5.30 and 5.35. So, for this small example, we won't generate approximately 9 constraints per time period for PN4. And, if we use six time periods, that will avoid several more constraints being added into the model. However, for the specific part number PN4, the results from using $\chi^t_{i,z} > 0$ would be the same as if we used $\omega_{i,z} > 0$. The main benefit of this restriction $\chi^t_{i,z} > 0$ can be seen in the next time period for PN1 to PN3, and are discussed next.

For example, PN1 is not going to generate constraints from time period two through six because the restriction is not satisfied as there is no more expected demand beyond time period 1. In other words, $\chi^2_{1,1} = 0$, $\chi^3_{1,1} = 0$, $\chi^4_{1,1} = 0$, $\chi^5_{1,1} = 0$ and $\chi^6_{1,1} = 0$ as can be seen in Figure 5.2. This way, the restriction is going to impact 9 constraints per time period. More constraints are avoided on PN2 and PN3, since the restriction is not satisfied for time periods five and six for each. As a result, we avoid the generation of an additional constraints. Again, on the large data set used for our industry case study, the impact of these restrictions is of great benefit.

The last parameter used to restrict the generation of constraints is $\upsilon^t_{i,z} > 0$. As discussed previously, this parameter indicates if demand has started at any given time by measuring demand from time zero to current time, and it helps avoid generating constraints for cases where demand has not yet started. For this small example, as depicted in Figure 5.3, PN3 and PN4 have cases where $\upsilon^t_{i,z} = 0$; hence, constraints won't be generated for those cases. This parameter is used on constraints 5.21 through 5.24, so, for the case of PN4, it won't generate 4 constraints per time

DEMAND	t1	t2	t3	t4	t5	t6
PN1	4	4	4	4	4	4
PN2	3	3	3	5	5	5
PN3	0	0	0	5	5	5
PN4	0	0	0	0	0	0

Figure 5.3: Example to Illustrate Parameters $\upsilon_{i,z}^{t}$

period. As we have seen before, this behavior is similar to that produced by $\omega_{i,z} > 0$. The main contribution can be seen on PN3, as the model won't generate constraints from time 1 to time 3 because $\upsilon_{1,1}^{1} = 0$, $\upsilon_{1,1}^{2} = 0$ and $\upsilon_{1,1}^{3} = 0$. As a result, the model is avoiding the generation of 12 constraints in each of the three time periods, one through three.

Another important aspect of the model that is worth discussing is related to the tracking model presented by Dembo [61]. The approach and model presented by Dembo [61] assumes that the scenarios probabilities evolve over time making them difficult to predict or model using stochastic process, for this reason, the model is solved periodically (model solved for one period only) to readjust the policy over time. The tracking model helps to select the policy for the immediate future scenarios and their associated probabilities. In our case, we assume that we know the probability for each scenario in advance, and it does not change over time.

5.5.1 Non Essentiality Constraints

As discussed before, we are looking for ways to reduce the size of the problem, hence, it can be solved quicker. After inspecting the optimization model, there are some constraints that have been identified as non-essentials and can be removed without affecting the solution of the model.

- The first constraint that can be removed is constraint 5.9. Given that constraint 5.14 indicates that $f_{a,z} \geq D_a$, the later one will dominate constraint 5.9, so, it can be dropped.

- Also, by default, Xpress-MP assumes non-negativity in the decision variables, for this reason, the later constraint and the following ones can also be dropped from the model, as well as constraints 5.18 and 5.36.

- Last, the in-transit constraints are not needed to calculate the base stock level, they can be used only if there is a need to track inventory. For this reason, constraints 5.33, 5.34, and 5.35 can be dropped.

In summary, we have discussed the optimization model, parameters and variables, and we presented examples of how to limit the generation of unnecessary constraints. In the following chapter, we will continue discussing the optimization model from a more numerical perspective by presenting small cases to prove its accuracy and a large case to prove its application in actual industry cases. The large case uses the data discussed in Chapter 4, and it is also intended to prove that the pre-processing techniques are able to help generating and solving large scale spare parts optimization models.

CHAPTER 6

CASE STUDIES

In this chapter, we are going to discuss two types of case studies, each with a different purpose. The first types of case studies are small, obvious examples to show that the model behaves as expected. The second type is based on a large data set where an industry case is solved. The software used to run the optimization model is Xpress-IVE Version 1.24.02 64 bit.

6.1 Small and Obvious Examples

In this section, small data sets are used to show the behavior of the model. Some of the data sets we are going to analyze are: examples with different replenishment lead times, examples with and without penalty cost for late delivery, examples with and without on-time service level agreement, examples with differences in shipment costs, examples with different holding costs, and small examples with multiple scenarios.

It is important to mention that the changes that will be done in the following cases are to illustrate how the model behaves. We also want to clarify that the model does not support cases where one scenario could have penalty cost 'x' and the other has penalty cost 'y'; or, one scenario has one replenishment lead time for a specific part and the other scenario has a different replenishment lead time for the same part. The penalty cost, lead times and holding costs are the same across all scenarios, however, they could differ by part. In the following examples, we change the parameters only to show that the model is behaving as expected.

In order to illustrate this case and many that follow, our basis is the example presented in the previous chapter and depicted in Figure 5.1. The model is trying to identify the recommended stock level for the 4 parts in order to minimize cost and satisfy 95% on-time delivery of the equipment. If equipment is delayed, there is a penalty cost of $1,000 per time unit. Demand, lead time and shipping cost are also depicted by Figure 5.1. This small example assumes only one scenario.

6.1.1 Case 1: "Replenishment Lead Times"

In this case, we illustrate that if replenishment arrives before the next expected demand, the model will take that into account when it recommends the base stock level to carry at the warehouse.

Following the example depicted on Figure 5.1, two equipment are expected to have maintenance, one at time 1 and the other at time 4. Each needs three different parts but in different quantities. The part PN2 is used by both equipment, 3 pieces by equipment 1 and 2 pieces by equipment 2. As can be seen, the replenishment lead time for PN2 is 5 time units. Given that demand happens in time 1 and 4, any replenishment placed in time 1 won't be able to satisfy any demand at time 4. Additional actions need to be taken. Based on current parameters, in order to satisfy on-time delivery and minimize cost, the model results can be seen in Table 6.1 and are described below:

- For PN1, 5 pieces are needed at time 1, so the model recommends having no base stock level because normal replenishment lead time is one unit. That is, parts can come before the equipment leaves maintenance. Thus, we don't invest in any holding cost or expedited shipments.

- For PN2, the model recommends having 5 units in stock, 3 pieces to be used at time 1, and 2 at time 2, as it is cheaper to pay a holding cost of $116 for the 5 pieces rather than paying two expedited costs of $80.65 each.

- For PN3, 5 pieces are needed at time 4. Given that the holding cost is high at \$36.80/piece/year, the model recommends placing an expedited shipment at a cost of \$80.65 for the whole order, rather than having any on hand because the holding cost would be \$184. Thus, the base stock level recommended is 0.

- For PN4, given that there is no expected demand for this part, the model does not recommend having it in stock. As a result, the base stock level recommended is 0.

- Given the current results, the model is able to satisfy on-time delivery greater than 95%. In the case, both equipment are delivered on time with a total cost of \$196.65 (\$116 is the holding cost of PN2 and \$80.65 is for expediting PN3).

Part Number	Base Stock
PN1	0
PN2	5
PN3	0
PN4	0

Table 6.1: Small Example - Case 1: Baseline Results

Now that we have seen results with the current parameters, let's do a small change on the replenishment lead time for part PN2 to show model behavior with replenishment. PN2's lead time will be changed from 5 units to 2 units, with post-model results presented in Table 6.2. As expected, the only change is a reduction of the base stock level of PN2. After PN2 is consumed at time 1, a replenishment order is placed, and given a lead time of 2 time units, it will be available for equipment 2 consumption when it arrives for maintenance at time 4. Just as in the baseline case, both equipment are delivered on time satisfying the 95% on-time delivery, but in this case, the total cost decreased to \$150.25 because PN2's holding cost decreased from 5 pieces to only 3.

Part Number	Base Stock
PN1	0
PN2	3
PN3	0
PN4	0

Table 6.2: Small Example - Case 1: Lead Time Changed - Results

6.1.2 Case 2: "Changes to the Extra Shipment Cost"

In this case, we illustrate the model behavior when extra shipment cost or holding cost is modified.

Continuing with the base example used in the previous case, we change the extra shipping cost for the three parts that have demand in order to validate the model. The new values can be seen in Table 6.3. Basically, the extra shipment costs for PN1 and PN2 are reduced while PN3 in increased.

Part Number	Extra Shipping Cost (Before)	Extra Shipping Cost (After)
PN1	\$80.65	\$5
PN2	\$80.65	\$10
PN3	\$80.65	\$200
PN4	\$80.65	\$80.65

Table 6.3: Small Example - Case 2: Extra Shipment Cost updates

After running the model, and in order to minimize cost and satisfy the on-time delivery of at least 95%, the results are presented in Table 6.4 and discussed as follows:

- For PN1, the model continues recommending base stock 0 because the part can arrive before maintenance is finished because of the short normal lead time.
- For PN2, the model recommends a base stock level of 0 because it is cheaper to place two expedited orders at \$10 each compared to a holding cost of \$23.20 a piece.
- For PN3, the recommendation for base stock level is 5 pieces because the holding cost is \$184 which is cheaper than an expedited order at \$200 per order.
- For PN4, there is no change.
- Given the current results, the model is able to satisfy the on-time delivery greater than 95%. In this case, both equipment are delivered on time, and the total cost is \$204. The cost breakdown is \$20 for expediting part PN2 and \$184 for the holding cost of PN3.

Part Number	Base Stock
PN1	0
PN2	0
PN3	5
PN4	0

Table 6.4: Small Example - Case 2: Extra Shipping Cost Changed - Results

6.1.3 Case 3: "Changes to the Holding Cost"

In this case, we illustrate the model behavior when holding cost is changed.

We continue with the same baseline example used in our previous cases, only changing the holding cost for PN2 from \$23.20 to \$30. As a reminder, this part has an expected demand at time 1 for 3 pieces and at time 4 for 2 pieces. After running the model and comparing results from the baseline model, the only part that changes is PN2. Results are presented in Table 6.5 and discussed as follows:

- For PN1, as expected, base stock level is the same, 0.
- For PN2, the model is suggesting a base stock level of 2 pieces of PN2 (previously it was recommending 5 pieces). In order to minimize cost and satisfy on-time delivery, the model recommends placing an expedited order for equipment 1 and using the 2 pieces in stock to satisfy the demand of equipment 2.
- For PN3, the base stock level is the same as the baseline example (0) and the model recommends an expedited order.
- For PN4, as expected, base stock level stays the same at 0.
- Given the model recommendations, total cost is $221.30, part of which is for the holding cost of 2 pieces of PN2 ($60) plus one expedited order to satisfy demand of equipment 1 ($80.65); the remaining amount is the same as the baseline case where PN3 is expedited ($80.65).

As we can see, the model is able to detect the best strategy to satisfy on-time delivery while minimizing cost, and similar to the PN2 case, it relies on a combination of base stock levels and the best use of the different types of replenishment.

Part Number	Base Stock
PN1	0
PN2	2
PN3	0
PN4	0

Table 6.5: Small Example - Case 3: Holding Cost Changed - Results

6.1.4 Case 4: "Penalty Cost for Late Delivery and Service Level Agreement for On-time Delivery"

In this case, we show how the penalty cost for late delivery and service level agreement affects the results of the model.

We continue using the same baseline example with current parameter values, and only decreasing the penalty cost from $1,000 to $0. After running the model, the results do not change: the only part with recommended base stock levels is PN2 with 5 pieces; and total cost remains the same. The main reason why the results didn't change is because the model still needs to satisfy the 95% of on-time delivery, and given that this example only has two equipment, we need to deliver both of them on-time.

When keeping the penalty cost at $0, and modifying the service level agreement from 95% to 45%, the model satisfies only 1 equipment. The results are presented in table 6.6 and discussed below.

- For PN1, base stock level continues to be 0 given the short lead time of the part.
- For PN2, the model recommends keeping only 3 pieces, which will be used to satisfy demand from equipment 1. For equipment 2, the model places a normal order.
- For PN3, no base stock level is recommended.
- For PN4, as expected, there is no demand, and therefore, and no base stock level is recommended.
- Based on current requirements that need to be satisfied, the results allow an on-time delivery of only one equipment, and the total cost is $69.60.

Part Number	Base Stock
PN1	0
PN2	3
PN3	0
PN4	0

Table 6.6: Small Example - Case 4: Penalty Cost and Service Level Changed - Results

The strategy followed by the model to minimize cost and satisfy on-time delivery is presented in Figure 6.1. As can be seen, equipment 1 is the only equipment being delivered on time; since the specified service level is 45%, this requirement is being satisfied. In order to guarantee the on-time delivery of equipment 1, the model needs to have 3 pieces of PN2 in stock at a cost of $23.20/each. And, PN1 is delivered at no extra cost due to its shorter lead time and arrival by the time the equipment is expected to depart.

The demand requirements for equipment 2 still need to be satisfied, but not its on-time delivery. The requirements for PN2 and PN3 are satisfied by placing normal orders. Given lead time of 5 time units, those orders will arrive at time 9; since the expected delivery date was at time 5, equipment 2 is delivered late by 4 time units, however, no additional cost is incurred.

One of the questions that might arise is whether equipment 1 was the cheapest equipment to deliver on time, and the answer is yes. To understand why this is the case, let's assume we want to deliver equipment 2 on time. Some options would be:

- Option 1 would have a base stock level for both parts used by equipment 2. The holding cost for PN2 is $23.20. Given that 2 pieces are needed, the total holding cost is $46.40. The holding cost for PN3 is $36.80, and given that 5 pieces are needed, the total holding cost for PN3 is $184. Thus, the total cost for this option would be $207.20, which is greater than the cost provided by the

		t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	How was Demand Fulfilled?	Cost	On-Time ?
Equipment 1	PN1	4										Placed Normal Order	$ -	
	PN2	3										From Base Sotck Level	$69.60	
	PN3	0												
	PN4	0												
Expected Maintenance														
Actual Maintenance														YES
Equipment 2	PN1				0									
	PN2				2							Placed Normal Order	$ -	
	PN3				5							Placed Normal Order	$ -	
	PN4				0									
Expected Maintenance														
Actual Maintenance														NO
													$69.60	

Figure 6.1: Small Example - Case 4: Penalty Cost and Service Level Changed

model.

- Option 2 would expedite both parts. Given an incremental cost for expediting an order of $80.65 each, the total cost for this option would be $161.30 which is greater than the cost provided by the model.

- Option 3 would be to combine of the alternatives above. PN2 is satisfied by having a base stock level of 2 pieces, for a total holding cost of $46.40; PN3 is satisfied by placing an expedited order at an incremental cost of $80.65. The total cost of this option is $127.05. As have shown, delivering equipment 1 on time is the right option from the model.

6.1.5 Case 5: "Multiple Scenarios - Same Demand"

In this case, we validate model results using multiple scenarios. Just as in the previous cases, the same baseline example will be used.

The first validation that we are going to do is very simple – we are going to create multiple scenarios using the same demand as the baseline scenario, as shown in Figure 6.2. Given that most parameters are the same, with the exception of the

DEMAND			t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
Scenario 1	Equipment 1	PN1	4									
		PN2	3									
		PN3	0									
		PN4	0									
	Equipment 2	PN1				0						
		PN2				2						
		PN3				5						
		PN4				0						
Scenario 2	Equipment 1	PN1	4									
		PN2	3									
		PN3	0									
		PN4	0									
	Equipment 2	PN1				0						
		PN2				2						
		PN3				5						
		PN4				0						
Scenario 3	Equipment 1	PN1	4									
		PN2	3									
		PN3	0									
		PN4	0									
	Equipment 2	PN1				0						
		PN2				2						
		PN3				5						
		PN4				0						

Figure 6.2: Small Example - Case 5: Example using Multiple Scenarios

number of scenarios and probabilities, we expect to have the same results compared to the baseline case. However, the model will generate more rows and columns. The probability will have the same value across the multiple scenarios, adding to 1.

First, we run a model with two scenarios. As seen in Figure 6.3, the optimal solution is exactly the same as our baseline. However, the number of rows increased from 259 to 518 and the number of columns from 272 to 540, an increase of 259 and 268, respectively. Results are as expected: with the same recommended base stock level as the baseline, the model is able to satisfy on-time delivery for at least 95% of the equipment by scenario while minimizing cost.

Next, we run the model with three scenarios, each using the same baseline demand. The results are as expected and presented in Figure 6.4. The optimal solution is the same as the baseline case and the two scenarios, however, the number of rows increases by 259 and columns increase by 268 by going from two scenarios to three scenarios.

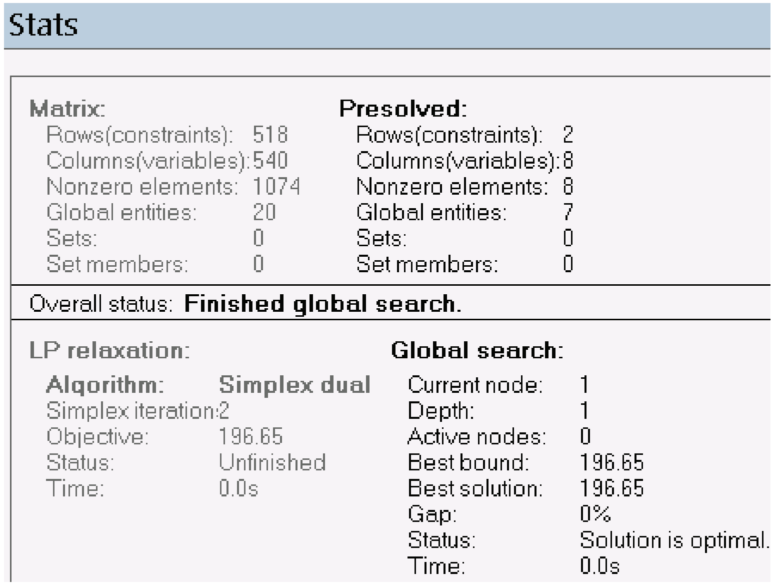


Figure 6.3: Small Example - Case 5: Two Scenarios - Demand as Baseline

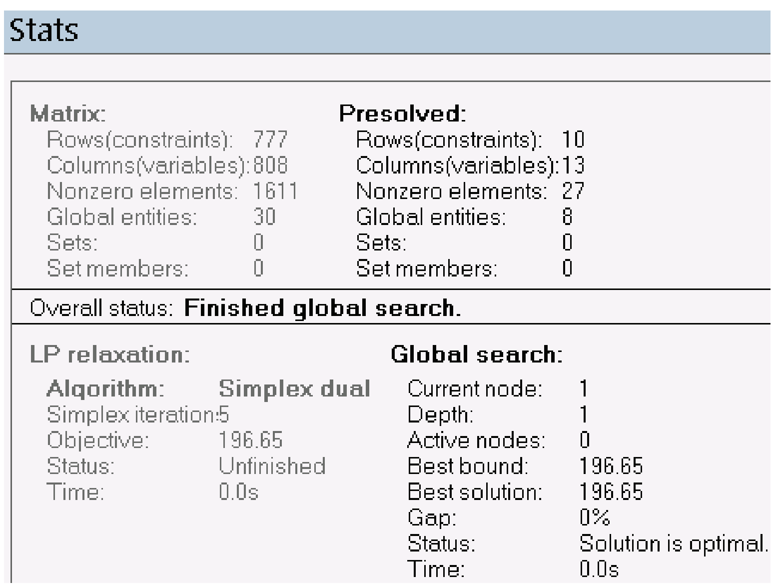


Figure 6.4: Small Example - Case 5: Three Scenarios - Demand as Baseline

6.1.6 Case 6: "Multiple Scenarios - Demand, Cost and Service Level Changes"

We continue using multiple scenarios, making several changes to the small example in order to show that the machines being delivered on-time could be different from one scenario to another; it all depends of the parameters.

In order to present the model behavior with multiple scenarios, we use the example presented in Figure 6.2 as our baseline. Several minor changes are made to the parameters:

- First, service level is changed from 95% to 45%.
- Second, penalty cost is decreased to zero.
- Third, the demand of PN2 for equipment 1 is changed from 3 to 33 in scenario 3.
- Last, the extra shipment cost for PN2 is updated from \$80.65 to \$800.65.

Since the service level has changed, we only need to satisfy 1 equipment per scenario, however, as mentioned before, the example being used has two equipment per scenario. At the same time, we need to update the penalty cost to zero or the model would try to satisfy all equipment deliveries to avoid any high penalty costs. The demand for PN2 by equipment 1 is increased to 33 in scenario 3 in order to reduce any motivation for having any in stock due to the large total holding cost. Similarly, the extra shipment cost for PN2 is increased to reduce any motivation for placing expedited orders.

Post-model results are presented in Figure 6.5. As can be seen on the right hand side, the optimal solution that satisfies at least 45% on-time delivery while reducing cost is \$96.49. On the left hand side, we see a table representing the values of variable $\beta_{a,z}$, which takes the value of 1 if the equipment is delivered late; otherwise, it takes

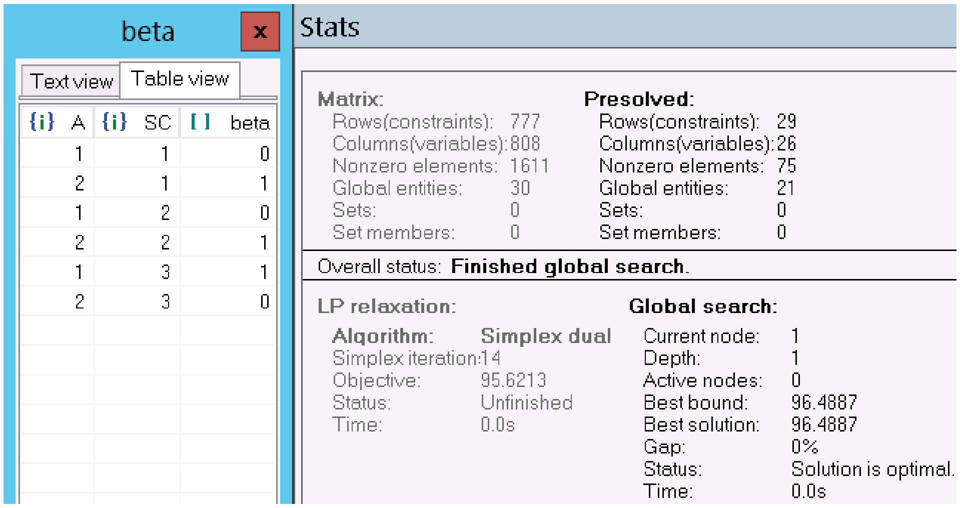


Figure 6.5: Small Example - Case 6: Three Scenarios - Demand, Service Level and Cost Changes

the value of 0. For this example, we see that in scenario 1 and scenario 2, equipment 2 is being delivered late ($\beta_{2,1} = 1$ and $\beta_{2,2} = 1$); however, in scenario 3, equipment 1 is being delivered late ($\beta_{1,3} = 1$).

As we have seen, the equipment being delivered on-time could be different among the scenarios as the only criteria that the model needs to satisfy is a specific service level while minimizing cost. The results are similar to the ones presented in Table 6.6 which are from Case 4. However, the total cost differs. In Case 4, the total cost was \$69.60, and in the current case total cost is \$96.49. Next, we are going to analyze the strategy recommended by the solution in this case to validate the accuracy of the model.

- For scenario 1 and scenario 2, the model is able to deliver equipment 1 on time by placing a normal replenishment of PN1 at no additional cost. As a reminder, PN1 has a normal lead time capable of delivering the part before the equipment departs. For PN2, the model recommends a base stock level of 3 pieces.

- For scenario 1 and scenario 2, given that equipment 1 is being delivered on time, and since there is no penalty cost for late deliveries, the model recommends placing normal orders for equipment 2 at no additional cost.

- For scenario 3, the model recommends delivering equipment 2 on time. PN2 is satisfied by the base stock level of 3 pieces being carried by the model. Since, equipment 1 is requiring 33 pieces, we are not going to use any of the base stock level of PN2; instead, an exclusive order will be place for equipment 1. PN3 for equipment 2 will be satisfied by placing an expedited order. This way the model achieves the optimal strategy.

- For scenario 3, given that equipment 2 is being delivered on time, the model recommends to satisfy demand for equipment 1 by placing normal orders at no additional cost, hence, equipment 1 is delivered late.

- For scenario 3, should equipment 1 be delivered on time, it would be more expensive because we only had two options: 1) Have a PN2 base stock level of 33 pieces, for a total holding cost of \$765.60; or, 2) place an expedited order at a cost of \$800.65. Given that the probabilities for this to happen are 1/3 (each scenario has the same probability), the total cost contribution to the total cost of the model would be an increase of \$266.88, meaning neither of these options is optimal.

- By following this strategy, the model warrants that the minimum cost is achieved while satisfying 45% on-time delivery. The total cost of \$96.49 is comprised of holding cost and extra shipment cost: PN2 has a base stock level of 3 pieces with a holding cost of \$23.20 per piece. This means total holding cost is \$69.60 (all scenarios share the same cost); the other piece is the extra shipment cost for PN3 in scenario 3. This impact is (1/3)(\$80.65) for a total cost of approximately \$96.49.

6.2 Industry Case Example

In this section, we are going to test the model using a large data set which was discussed in sections 4.1 and 4.2. We will use almost all of the historical data provided and discuss the variables, constraints generated and results of this case.

6.2.1 Industry Example Description

First, let's describe the example we are going to use. We have a company that is able to perform preventive maintenance on a specific type of commercial airplane. This company has three production lines, and thus able to schedule three airplanes for maintenance at the same time. The company is able to schedule the following 8 weeks of maintenance with a high degree of certainty. There are periods with low maintenance, especially during holidays because aircraft are being used to move passengers; but during periods of low passenger demand, the company is busy with three full production lines. It is assumed that the aircraft maintenance variables at the time of arrival are contained in the ranges presented in Table 4.3 and, further, that each has the same probability to be within any of the three groups discussed in Tables 4.4 and 4.5.

The company wants to satisfy on-time equipment delivery from maintenance of at least 95%, but at the same time, it wants to minimize cost. If the company delivers an equipment late, it must pay a penalty fee of $10,000 per week. The type of maintenance that will be scheduled typically takes 1 week to complete, so, an equipment will be considered late if it is delivered more than 1 week after its arrival.

Part lead times and shipment costs are provided in the data set discussed in sections 4.1 and 4.2. The lead times provided by Airbus and the airline are the ones we treat as normal replenishment. Given that the company has its logistics center in a very strategic area, and due to several flights per day to its maintenance location, we consider that any expedited lead time arrives during the time of maintenance. As mentioned previously, the optimization model is going to utilize the incremental cost between normal and expedited lead times. Another important piece of information needed in the model is holding cost. For our analysis, we assume a holding cost of 20% of the unit price of the part.

As discussed earlier, the original data set provided by Airbus contains about 21,000 different expendable parts. The current optimization model uses a lot by lot policy, which is typically applied to parts that are not low cost. Our initial intention with this model was to apply it to the most expensive parts, for example, the 'A' class from a typical Pareto analysis. This would typically account for 20% of the parts (4,200 parts in this case), and about 80% of the total demand value.

Given that we had missing data (discussed in section 4.2), an ABC analysis is performed based only on the 14,247 parts that have a unit price. The ABC is based on the total demand the parts have had across the 795 maintenance checks, and this value is multiplied by the unit price. Categories A and B account for 95% of the total demand value, and the total number of parts in these two categories is 3,208. It is noted that some normal lead times for fewer than a hundred parts are less than the duration of the maintenance. For this reason, we are not going to include them in the analysis, because the model would recommend a normal order anyway, and this way, we can save some computational time.

In order to increase the sample size and get closer to our initial targeted estimate of 20%, we continue looking for the data. It is noted that some parts with essentiality code 1, with valid price and lead time are being left out mainly because they are category C. As discussed in section 2.4, if a part is categorized with code 1, it must be replaced before the aircraft can fly; hence, this part is critical and its absence could carry a high penalty cost for late delivery. Some parts with essentiality code 1 are left out due to having a normal lead time that is shorter than the total duration of aircraft in maintenance. After the C parts with essentiality code 1 are added into our sample part population, we are ready to test the model with a total of 4,149 parts.

One of the leading spare parts planning software companies was willing to share the total quantity of different parts for which its customers plan for the same brand of airplane, and it is between 2,780 and 7,535, including all categories A, B and C. Based on some of the airline's data, we were able to detect that the database of the 795 maintenance checks has more than 1,400 alternative parts, probably with the same or different operators getting parts from different suppliers. Thus, there is difference in the part number even though the functionality is the same. Cases like this one might help with the decreased number of parts reported by the software company. In fact, in my current company, it is common to decrease the number of parts being planned by rolling up demand for alternate parts. This way the forecast accuracy increases and the planning workload decreases. It is not surprising to have some parts from specific commodities have as many as 40 parts rolling up demand together.

In order to reduce computational time, and given that maintenance checks are performed in a week's time, our time unit will be in weeks; hence, all part lead times are converted into this time unit. The planning horizon will be 8 weeks, and we are going to assume the busiest schedule where 3 aircraft are maintained weekly, for a total of 24 aircraft in the planning horizon, as seen in Figure 6.6.

Since we are using scenario-based methodology, the maintenance schedule is repeated several times as seen in Figure 6.7. The difference among the different scenarios is demand; though each scenario is assigned the same probability of occurrence.

6.2.2 Scenario Based on Random Generation of Part Demand

In this approach, given the historical data of maintenance checks provided by Airbus, the discrete distribution of each part was calculated. Based on the discrete distribution, 25 random values per aircraft are generated, in other words, 25 different scenarios per aircraft are generated as it can be seen on Figure 6.8. As discussed

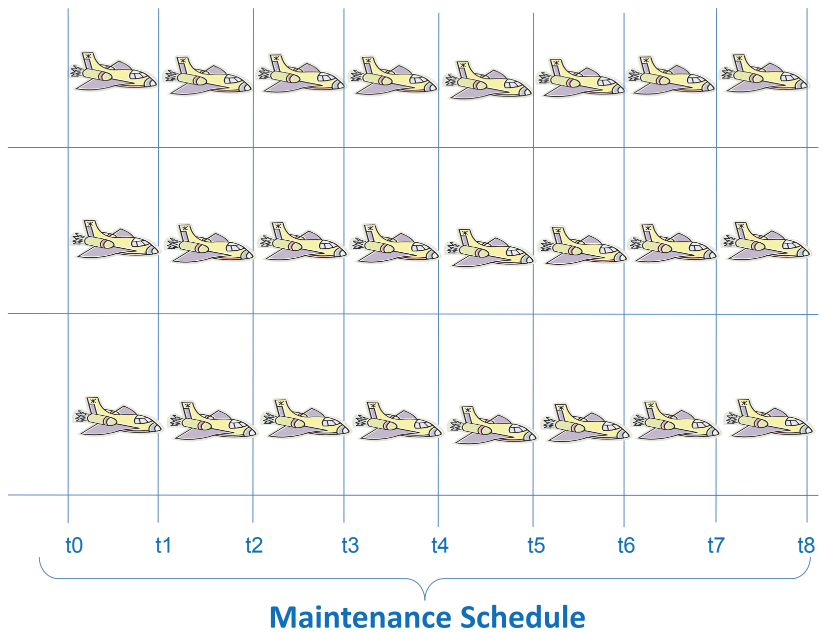


Figure 6.6: Large Example - Maintenance Schedule

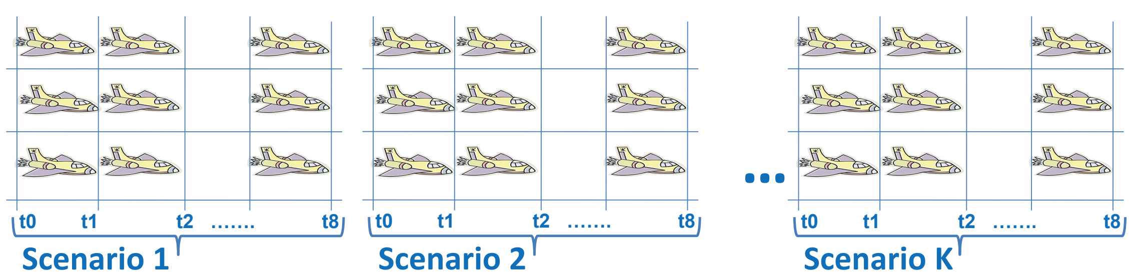


Figure 6.7: Large Example - Multiple Scenarios

	Aircraft 1						Aircraft 2					
	Scenario 1	Scenario 2	Scenario 3	....	Scenario 24	Scenario 25	Scenario 1	Scenario 2	Scenario 3	....	Scenario 24	Scenario 25
Part 1	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()
Part 2	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()
Part 3	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()
Part 4	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()
Part 5	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()
Part 6	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()
⋮	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()
Part 4149	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()	Rand ()	Rand ()	Rand ()	...	Rand ()	Rand ()

Figure 6.8: Large Example - Random Generation of Part Demand

before, a total of 24 aircrafts are coming into maintenance, so, the same process is performed for all of them.

The first observation we have in this approach is that over five hundred parts are not generating demand given the low probabilities of them. Nonetheless, we are going to discuss the most relevant results of this approach, and we are going to compare it against another approach to generate the scenarios.

The model is able to find the first feasible solution in 90 seconds, with a gap between primal and dual of 11.39%. The model quickly is able to reduce the gap to less than 1% in around 30 minutes, being the best solution \$91,011.58 and the best bound \$90,144.08. After running for a total of 6 hours, the gap gets reduce to 0.86% and keeps in that range even after the model is let run for long period of time.

Given that there were a significant number of parts that did not generate demand, we decided to solve the model again, but this time using a different approach. Instead of generating random demand of each part, we randomly choose an aircraft to repair. Accordingly, we assign the whole maintenance check randomly for each aircraft. This is explained in more details on the next section. Later on, we will also discuss how the results from both approaches compare.

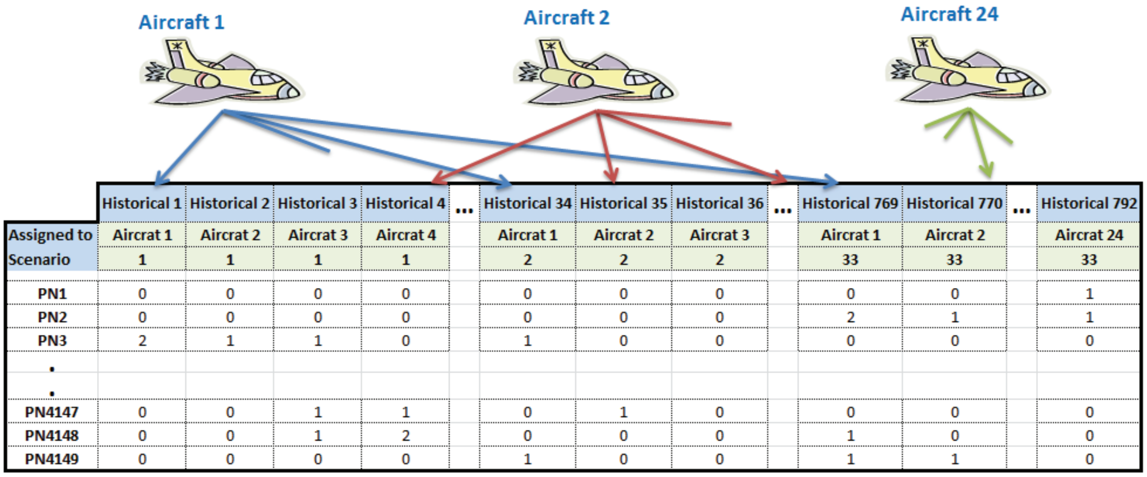
6.2.3 Scenarios Based on Random Assignment of Historical Demand Checks

In this approach, we are going to utilize almost all of the historical demand data available. As a reminder, the demand historical data contains 795 different maintenance checks. Given that the original schedule shown in Figure 6.6 has 24 aircraft, we have the ability to populate 33 scenarios (24*33=792).

When the historical data was sorted by time, it was noticed that many maintenance checks landing close to each other were performed and reported by the same company. This may well have been related to a "catch up" period, where companies were trying to report as much data as they had available when joining the program. For our data modeling, in order to avoid assigning similar data to the same aircraft, we randomly assign the maintenance checks to the aircraft to be scheduled as seen in Figure 6.9. This random assignment is without replacement, each check can only be used once. The main difference between the first approach and this one is that the first approach randomly generated demand values based on historical demand of the parts. In the approach of this section, we are randomly assigning the whole historical data to the aircraft that will be scheduled. As discussed, we are utilizing 792 maintenance checks out of the 795 available, keeping three for later use as more data becomes available.

Variables and Constraints Generated

In order to validate the robustness of the model, we run 10 different trials. In each of the trials, the maintenance checks are randomly assigned to one of the aircraft to be scheduled. Before discussing results, let's discuss the variables and constraints generated by the model. Figure 6.10 shows that the model generates an average of 17.69M constraints and 46.58M variables with a confidence interval of 95% of the values being [17.28M, 18.098M] and [46.19M, 46.97M] respectively. Even though the constraints are in the millions, the maximum number of possible constraints



	Historical 1	Historical 2	Historical 3	Historical 4	...	Historical 34	Historical 35	Historical 36	...	Historical 769	Historical 770	...	Historical 792
Assigned to	Aircrat 1	Aircrat 2	Aircrat 3	Aircrat 4		Aircrat 1	Aircrat 2	Aircrat 3		Aircrat 1	Aircrat 2		Aircrat 24
Scenario	1	1	1	1		2	2	2		33	33		33
PN1	0	0	0	0		0	0	0		0	0		1
PN2	0	0	0	0		0	0	0		2	1		1
PN3	2	1	1	0		1	0	0		0	0		0
.													
.													
PN4147	0	0	1	1		0	1	0		0	0		0
PN4148	0	0	1	2		0	0	0		1	0		0
PN4149	0	0	0	0		1	0	0		1	1		0

Figure 6.9: Large Example - Historical Data Assignment

generated could be around 246M, however, given the pre-processing techniques we discussed previously, we are only generating around 7% of them. The impact of avoiding around 93% of the constraints is substantial because we are able to solve the model efficiently as we will discuss in the next section.

The optimization model was presented in section 5.4 (page 57), and below we discuss the average number of constraints generated by the 10 trials:

- Constraints 5.8 is a function of the 33 scenarios and the 24 aircraft, with a maximum possible number of constraints of 792. In fact, that is the number of constraints generated by the model for that equations. Constraint 5.9 is non-essential, thus, it is not used in the model.

- Constraints 5.10 and 5.11 are a function of 24 aircraft, 4,149 different parts, 9 time units (from 0 to 8) and 33 scenarios. The maximum possible number of constraints that each could each generate is 29.6M. Based on the restrictions in place, each is generating only 58,413 different constraints.

- Constraints 5.12 and 5.13 are a function of the 24 aircraft, 4,149 different parts, 9 time units (from 0 to 8), and 33 scenarios. So, the maximum possible number

TRIAL No (Approach 1)	# Constraints	Variables	% Constr. Generated
1	13,250,259	35,135,605	5.37%

TRIAL No (Approach 2)	# Constraints	Variables	% Constr. Generated
1	17,838,517	46,719,218	7.23%
2	17,713,405	46,591,454	7.18%
3	17,592,589	46,483,129	7.13%
4	17,853,033	46,725,020	7.24%
5	17,581,173	46,466,034	7.13%
6	17,581,309	46,463,977	7.13%
7	17,639,417	46,526,836	7.15%
8	17,744,001	46,624,954	7.20%
9	17,503,486	46,389,638	7.10%
10	17,892,503	46,767,052	7.26%
Average	17,693,943	46,575,731	7.18%
Stdev	134,761	130,072	0.05%
LCL: -3* Std Dev	17,289,660	46,185,515	7.01%
UCL: +3* Std Dev	18,098,227	46,965,947	7.34%

Figure 6.10: Large Example - Constraints and Variables

of constraints that they could generate is 29.6M. As discussed in section 5.5, we only allow constraints to be generated under certain conditions. For this reason, these equations are only generating 58,413 constraints.

- Constraint 5.14 also is also a function of 33 scenarios and 24 aircraft, hence the maximum possible number of constraints that could be generated is 792; and that is the number of constraints that the model is actually generating.

- Constraint 5.15 is a function of 24 aircraft, 4,149 different parts, and 33 scenarios. However, it is only a function of 8 time periods (from 1 to 8). As a result, it is able to generate a maximum of 26.2M constraints, but given the restriction, it only generated 58,413 constraints.

- Constraints 5.16 is function of 24 aircraft, 4,149 different parts, 33 scenarios and 9 time periods. The maximum possible number of constraints that each could each generate is 29.6M. Based on the restrictions in place, each is generating only 7,165,267 different constraints.

- Constraint 5.17 also is function of 24 aircraft, 4,149 different parts, 9 time periods and 33 scenarios. Given that the restriction in place, the model is generating 8,060,926. Constraint 5.18 is non-essential, thus, it is removed from the model.

- Constraint 5.19 and 5.20 are a function of 8 time periods (from 1 to 8), 33 scenarios and 4,149 parts. Together these can generate a maximum of 1,095,336 constraints. Based on current restrictions, each is generating 256,404 and 36,329 constraints, respectively.

- Constraint 5.21 is a function of 9 time periods, 33 scenarios and 4,149 parts, with a maximum number of constraints of 1,232,253. However, this constraint is only generating 82,547 constraints because of the restrictions placed.

- Constraint 5.22 is similar to the prior constraint. In theory, it could generate 1,232,253 different constraints, but given the restrictions and intent for this constraint, it is generating only 107,770.

- Constraint 5.23 is a function of 9 time periods, 33 scenarios and 4,149 parts. So, it could generate 1,232,253 different constraints. However, based on the restrictions assigned, it is generating only 82,547 constraints.

- Constraint 5.24, in theory, could generate 1,232,253 different constraints. Given the restrictions and intent for this constraint, it is generating only 107,770 constraints.

- Constraints 5.25 and 5.26 together could generate a maximum of 1,232,253 constraints. Given current restrictions, they are generating 190,317 and 37,319 constraints respectively.

- Constraints 5.27 and 5.28 present similar dependency as the previous two constraints. Together they could generate a maximum of 1,232,253 constraints. Based on the restrictions placed, 5.27 generates 36,329 and 5.28 generates 293,723 constraints. However, this latter constraint simply assigns values to variables given its form $\eta_{i,z}^{t} = 0$.

- Constraints 5.29 and 5.30, depend on 8 time periods (from 1 to 8), 33 scenarios, and 4,149 parts; together could these generate a maximum of 1,095,336 constraints. Based on the restrictions given, 5.29 generates 256,404 constraints and 5.30 generates 36,329 different constraints.

- Constraints 5.31 and 5.32, similar to the prior constraints, could generate a maximum of 1,095,336 constraints. However, given the restrictions in place, these constraints generate 256,404 and 36,329 different constraints, respectively.

- Constraints 5.33, 5.34 and 5.35 are a function of 9 time periods, 33 scenarios and 4,149 parts. All together, these could generate a maximum of 1,232,253 constraints, however, since they are non-essential they are removed from the model.

- Constraint 5.36 only depends on the number of parts. Thus, the maximum number of constraints it could generate is 4,149. Given that this constraint is non-essential, we removed it from the model.

- Constraint 5.37 depends on 9 time periods, 33 scenarios and 4,149 parts. Thus, it could generate a maximum of 1,232,253 constraints; for this case, it is generating 335,872 constraints.

- Constraint 5.38 is a function of 1 time period (time 0), 33 scenarios and 4,149 different parts. The maximum number of constraints it could generate is 136,917, and in fact, it is generating all these constraints.

- Constraints 5.39 and 5.40 depend on 33 scenarios and 24 aircraft, so the maximum number of constraints that each could generate is 792. For this example, each is generating 792 different constraints.

- Constraint 5.41 depends on each scenario, so the maximum number of constraints it could generate is 33, which is the number being generated.

- Constraint 5.42 depends on 33 scenarios and 24 aircraft, for a maximum of 792 different constraints. This is the maximum being generated.

Table 6.7: Large Example - Possible No. of Constraints
vs. Avg. Constraints Generated

Constr. No.	**Possible No Constr.**	**Avg. Generated**	**Std. Dev.**
5.8	792	792	0
5.9	792	Non-Esse.	Non-Esse.
5.10	29,574,072	58,413	0
5.11	29,574,072	58,413	0
5.12	29,574,072	58,413	0
5.13	29,574,072	58,413	0
5.14	792	792	0
5.15	26,288,064	58,413	0
5.16	29,574,072	7,165,267	54,809
5.17	29,574,072	8,060,926	61,660
5.18	29,574,072	Non-Esse.	Non-Esse.
5.19 and 5.20	1,095,336	292,733	2,242
5.21	1,232,253	82,547	1,238
5.22	1,232,253	107,770	4,582
5.23	1,232,253	82,547	1,238
5.24	1,232,253	107,770	4,582
5.25 and 5.26	1,232,253	227,636	4,152
5.27 and 5.28	1,232,253	330,052	2,526
5.29 and 5.30	1,095,336	292,733	2,242
5.31 and 5.32	1,095,336	292,733	2,242
5.33, 5.34, 5.35	1,232,253	Non-Esse.	Non-Esse

Continued on next page

Table 6.7 – continued from previous page

Constr. No.	Possible No Constr.	Avg. Generated	Std. Dev.
5.36	4,149	Non-Esse.	Non-Esse.
5.37	1,232,253	335,872	2,569
5.38	136,917	136,917	0
5.39	792	792	0
5.40	792	792	0
5.41	33	33	0
5.42	792	792	0
Total	246,596,451	17,811,561	134,761

As we have seen, the possible number of constraints that could be generated by the model is more than 246M. However, thanks to the addition of the restrictions discussed in section 5.5, the model is able to avoid the creation of about 93% of constraints, or close to 228M constraints.

If we had not been able to eliminate all those constraints, the computational time to create the model and solve it would be intractable. For this large example, in order to read the data and create the model, the maximum memory Xpress-MP was utilizing from the server was close to 60 GB, as seen in Figure 6.11. It took close to 3 hours and 30 minutes for the entire process of reading and creating the model; had we not been able to avoid the creation of constraints, the process would have required more memory and more time.

In order to further reduce the number of constraints and variables, we also rely on the presolve functions of the optimization software used. As we know, presolve is able to eliminate redundant constraints, eliminate fixed variables and substitute them in constraints, enable coefficient tightening, etc. After the software presolved the model,

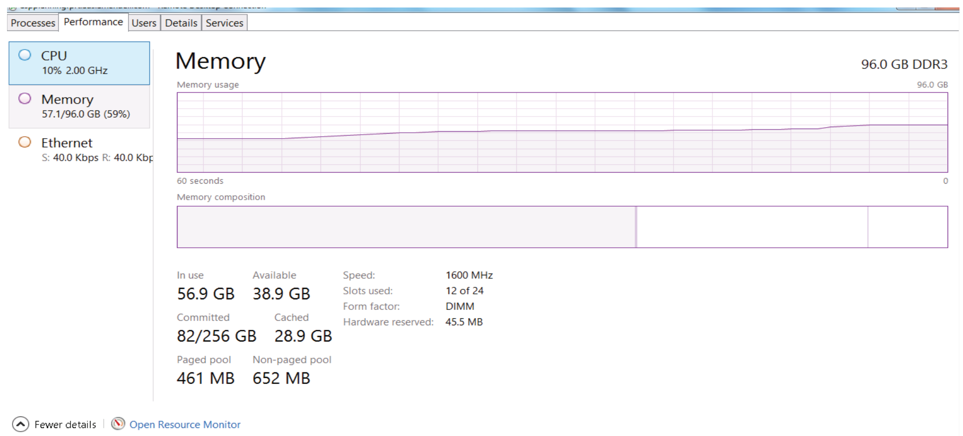


Figure 6.11: Large Example - Server Utilization when Reading and Creating the Model

the number of constraints is reduced to 170,900 and the number of variables is reduced to 129,990.

As an example of the presolving process, let's consider the following constraints:

$$c_1 + ax \leq b_1 \tag{6.1}$$

$$ax = b_2 \tag{6.2}$$

The software will keep constraint 6.2 as is, but will transform equation 6.1 as shown below, this is done in order to reduce coefficients:

$$c_1 + b_2 \leq b_1 \tag{6.3}$$

As an example of presolving, similarly, in our optimization model we have constraints 5.28 and 5.29 both needing to satisfy the following criteria: $\forall\ \omega_{i,z} > 0$; $i = 1, 2, .., R$; $t \leq \tau_i^N$; $z = 1, 2, ..., C$. The first constraint, 5.28, is used to set the received quantities from normal orders at zero at time t because elapsed time is shorter than normal lead time; and the following constraint, 5.29, calculates the total received quantity including expedited and normal orders.

$\eta^t_{i,z} = 0 \qquad\qquad Constraint \qquad 5.28$

$\gamma^t_{i,z} = \theta^t_{i,z} + \eta^t_{i,z} \qquad\qquad Constraint \qquad 5.29$

For this specific case, presolving will identify an opportunity for reducing coefficients by substituting zero instead of the term $\eta^t_{i,z}$ in constraint 5.29. Obviously, for this specific constraint, we could still modify the mathematical model developed. The new constraint is shown below:

$$\gamma^t_{i,z} = \theta^t_{i,z} + 0 \tag{6.4}$$

Similar to the previous case, there are other cases where the model was simplified, however, not all of those cases are as obvious as this one. As discussed in section 5.5.1, we were able to identify other constraints that are non-essential and can reduce the model size.

6.2.4 Model Performance and Results

In this section, we analyze the results for the large example we have been discussing.

The model performs an LP relaxation. Basically, it ignores all integer requirements, and concurrently starts solving the Dual, Primal and Barrier methods. As an example, we present the results from trial 1, however, the rest of trial present similar behavior. Based on the results presented in Table 6.8, it can be seen that the Dual method is the first method able to find a solution. The results of the Dual are shown in Figure 6.12 with a value of $87,065.54. This solution is found in 93,381 iterations and around 4.3 seconds. As we know, the value of the objective function with ignored integer requirements will be a lower bound on the optimal integer program objective value in a minimization problem.

Table 6.8: Large Example - Results from Dual in Trial 1

Dual Obj Dual Inf	**Primal Objective Primal Inf**	**Barrier: p.obj. d.obj**
dual crash		
dual crash		factorizing
D .0000000 .0000000		factorizing
D .0000000 .0000000	p 730332.83 21191.494	factorizing
D .0000000 .0000000	p 730332.83 21191.494	B -4.694E+08 .0000000
D .0000000 .0000000	p 730332.83 21191.494	B -1.181E+09 38271935.
D .0000000 .0000000	p 730332.83 21191.494	B -1.407E+09 61764211.
D .0000000 .0000000	p 730332.83 21191.494	B -1.498E+09 61074373.
D 40797.859 .0000000	p 730332.83 21191.494	B -1.498E+09 61074373.
D 40797.859 .0000000	p 730332.83 21191.494	B -1.137E+09 52547491.
D 40797.859 .0000000	p 932865.31 3136.7645	B -1.137E+09 52547491.
D 40797.859 .0000000	p 932865.31 3136.7645	B -1.043E+09 51429846.
D 40797.859 .0000000	p 932865.31 3136.7645	B -8.867E+08 48971764.
D 40797.859 .0000000	p 932865.31 3136.7645	B -6.829E+08 43129436.
D 40797.859 .0000000	p 932865.31 3136.7645	B -5.088E+08 29225551.
D 85453.182 .0000000	p 932865.31 3136.7645	B -5.088E+08 29225551.
D 85453.182 .0000000	p 932865.31 3136.7645	B -3.216E+08 16249239.
D 85453.182 .0000000	p 286083.13 .0000000	B -3.216E+08 16249239.
D 85453.182 .0000000	p 286083.13 .0000000	B -1.544E+08 7994421.6
D 85453.182 .0000000	p 286083.13 .0000000	B -60888065. 3969346.5
D 85453.182 .0000000	p 286083.13 .0000000	B -16313894. 1764326.0
——- optimal ———	—– interrupted ——–	—– interrupted ——–

```
Concurrent statistics:
      Dual: 93381 simplex iterations, 4.305628s
    Primal: 36296 simplex iterations, 2.106014s
   Barrier: 13 barrier and 0 simplex iterations, 6.302440s
            Barrier used 1 thread, L1 cache = 32K, L2 cache = 15360K
Dual solved problem
```

Figure 6.12: Large Example - LP Relaxation Dual Results from Trial 1

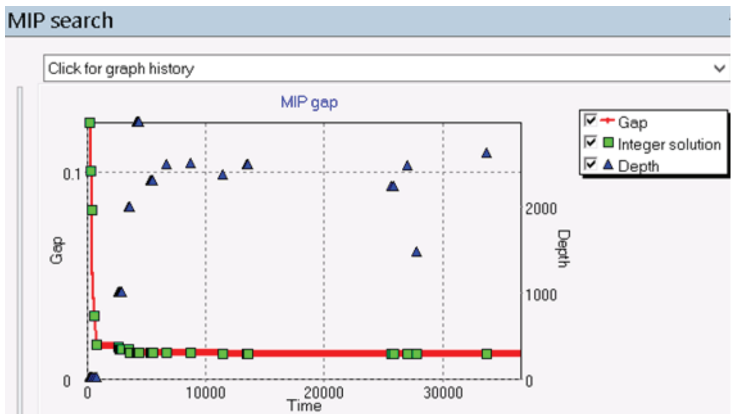


Figure 6.13: Large Example - Gap between Primal and Dual over Time

Given the value of the dual, the software is able to identify an integer solution with a value of $99,418.56. The gap between Dual and Primal is 12.43% as seen in Table A.1 in Appendix A. This same table shows that by using root cutting and heuristics, the software is able to find 5 feasible integer solutions, with a minimum gap between Dual and Primal of 1.64%, this is done in about 13 minutes.

In order to understand if the gap can further decrease, we let the model run for more additional time. At this point, the process started doing branch and bound, with the initial gap between Primal and Dual of 1.64%. Figure 6.13 shows the gap between Primal and Dual over time; and Figure 6.14 shows the objective function over time.

As we can see in Figure 6.13, we let the model run for over 10 hours in total. For 9hr and 50 minutes the model utilized branch and bound, finding 22 additional integer feasible solutions, for a total of 27 feasible solutions. During the branch and bound procedure, the gap reduction is very minimum, it goes from 1.64% to 1.19%.

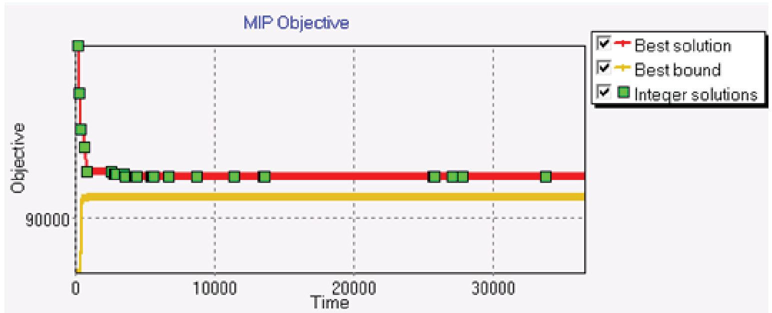


Figure 6.14: Large Example - Objective Value vs Time

After this point, the model is not able to make much progress on gap reduction. In fact, in the last 7 hours running, it is only able to go from a 1.22% gap to a 1.19% gap. At this point, the running process is manually terminated and the final results are presented in Figure 6.15; the data supporting these graphs is included in Appendix B in Table B.1.

We need to emphasize again that the model was able to reach less than 2% gap between Dual and Primal in around 12 minutes, and close to 1% (1.25%) in an 70 minutes, confirming that the model formulation is very strong. From an industry perspective, and as a practitioner, the gap is within a reasonable tolerance, and without a doubt, could help companies to model cases where the overall system needs to be considered. These results are repeated across all the different trials. Furthermore, the following results also show the robustness of the model because standard deviation is very small.

In Figure 6.16 it is shown the results from the 10 different trials. As it can be seen, all the values are very close, for this reason the standard deviation is small. Another thing to notice is that penalty cost for late delivery is zero, probably it is because the high penalty fee and the high service level it needs to satisfy. Another important thing we need to mention is that the total cost is close to the value resulted on the first approach, however, it is outside the confidence interval; the main difference between both approaches is on the shipment cost being larger on the second approach.

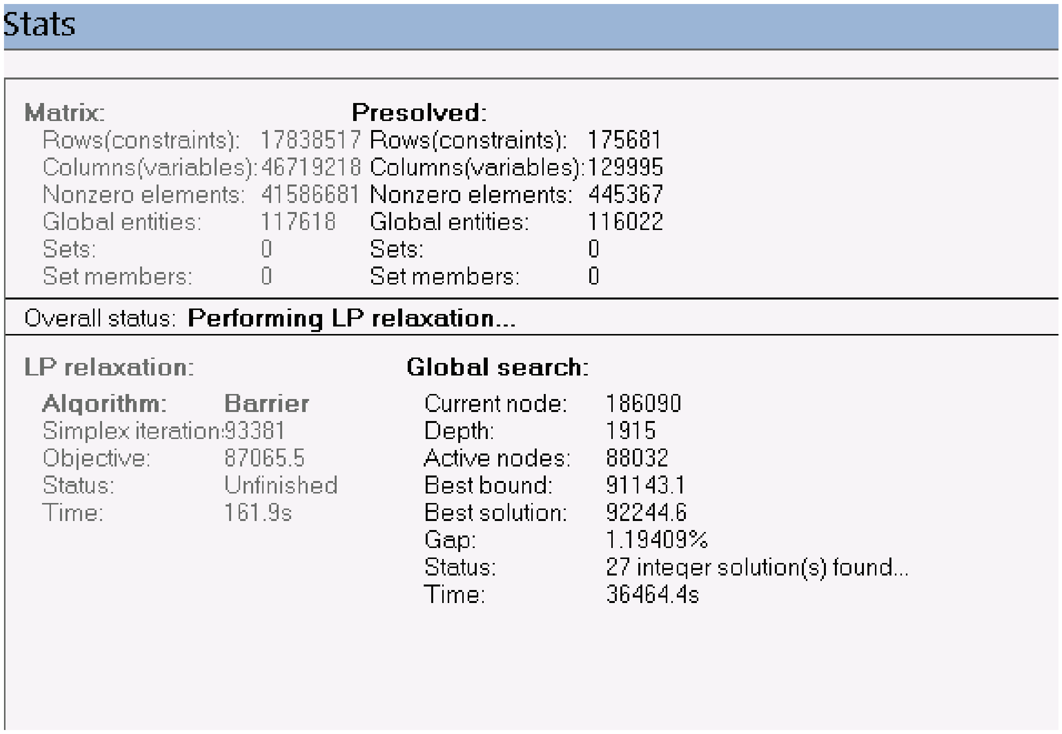


Figure 6.15: Large Example - Final Results Trial 1

The model is recommending to have base stock level only for about 924 parts, the number of parts varies depending on the trial but goes from 915 to 932. The average number of pieces recommended to stock is 11,397. Even though the total dollar amount in base stock level is very close from one trial to the other one, only 252 parts have the exact same quantity across all trials.

Let us now discuss the results for all the scenarios from one of the trials. Table C.1, available in Appendix C.1, shows the results from each scenario. As seen in Table C.1, the values of emergency delivery are widely spread ranging from $36,473 to $145,562. It is necessary to drill down into each scenario to understand the individual results. Intuitively, we know that this is related to the different part numbers required per check, hence, we organize the data from small cost to higher cost for each check modeled. Next, we add the number of different part numbers used in each check. The results are shown in Figure 6.17: as the number of parts increases, the incremental

TRIAL No (Approach 1)	Holding Cost	Shipment Cost	Penalty Cost	Total Cost	Best Bound	GAP %
1	$ 20,568.30	$ 70,393.50	$ -	$ 90,961.80	$ 90,174.33	0.866%

TRIAL No (Approach 2)	Holding Cost	Shipment Cost	Penalty Cost	Total Cost	Best Bound	GAP %
1	$ 19,862.6	$ 72,381.96	$ -	$ 92,244.60	91,143.10	1.194%
2	$ 19,921.7	$ 72,591.28	$ -	$ 92,513.00	91,425.00	1.176%
3	$ 19,993.90	$ 72,606.10	$ -	$ 92,600.00	91,485.34	1.204%
4	$ 19,643.20	$ 72,641.10	$ -	$ 92,284.40	91,088.62	1.296%
5	$ 19,320.3	$ 73,049.74	$ -	$ 92,370.00	91,224.30	1.240%
6	$ 20,089.40	$ 72,468.80	$ -	$ 92,558.20	91,471.87	1.174%
7	$ 19,904.30	$ 72,403.70	$ -	$ 92,308.06	91,238.02	1.159%
8	$ 19,664.30	$ 72,625.90	$ -	$ 92,290.30	91,123.56	1.264%
9	$ 19,627.20	$ 72,943.90	$ -	$ 92,571.00	91,496.93	1.160%
10	$ 19,888.30	$ 72,450.10	$ -	$ 92,338.40	91,212.36	1.219%
Average	$ 19,792	$ 72,616	$ -	$ 92,408	91,290.91	1.21%
Stdev	$ 226	$ 223	$ -	$ 137	162	0.05%
LCL: -3* Std Dev	$ 19,112.51	$ 71,948.33	$ -	$ 91,996.55	$ 90,805.98	1.07%
UCL: +3* Std Dev	$ 20,470.53	$ 73,284.18	$ -	$ 92,819.04	$ 91,775.84	1.35%

Figure 6.16: Large Example - Summary Results All Trials

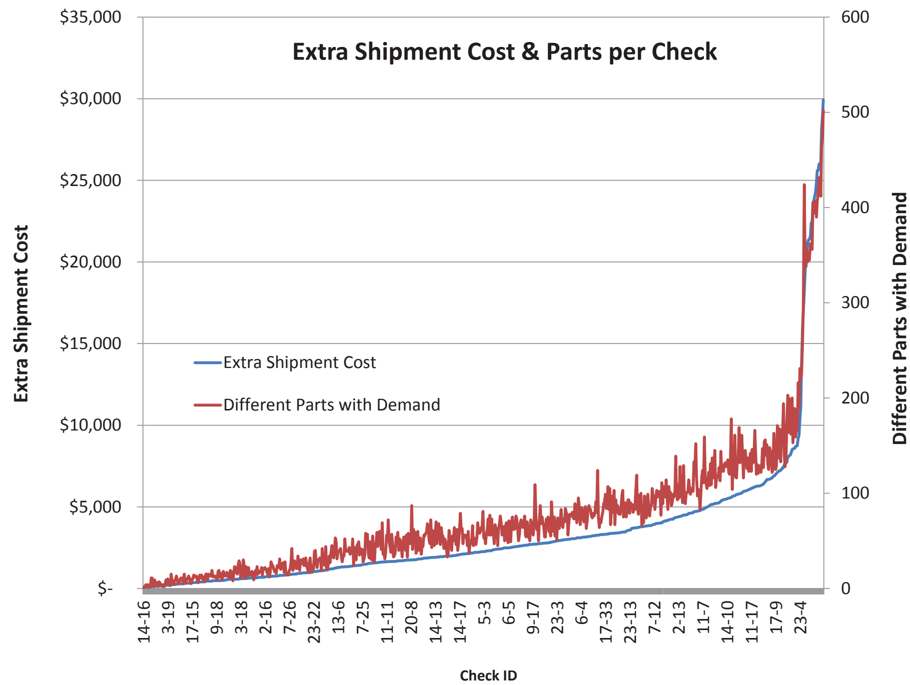


Figure 6.17: Large Example - Extra Shipment Cost and Parts per Check

cost for expediting increases.

Furthermore, we perform a linear regression analysis to understand the correlation between the number of parts used and the total expedited cost. As can be seen in Figure 6.18, the majority of the data points are around a straight line. Also, Figure 6.19 shows an RSquare of 95%, meaning that 95% of the variance can be explained. In other words, it is a very strong indicator of the correlation (97%) between both variables.

Figure 6.19 also shows the Analysis of Variance for both variables. The null hypothesis (Ho) is set as: the number of parts and the expedited cost are not linearly related, and H1 is set as the two variables are linearly related; and we use an alpha value of 0.05. The results show the F Ratio 16,731.74 and the F Critical F(0.95,1,782)=3.84. Thus, F Ratio > F Critical implying with at least 95% confidence that the variables are related.

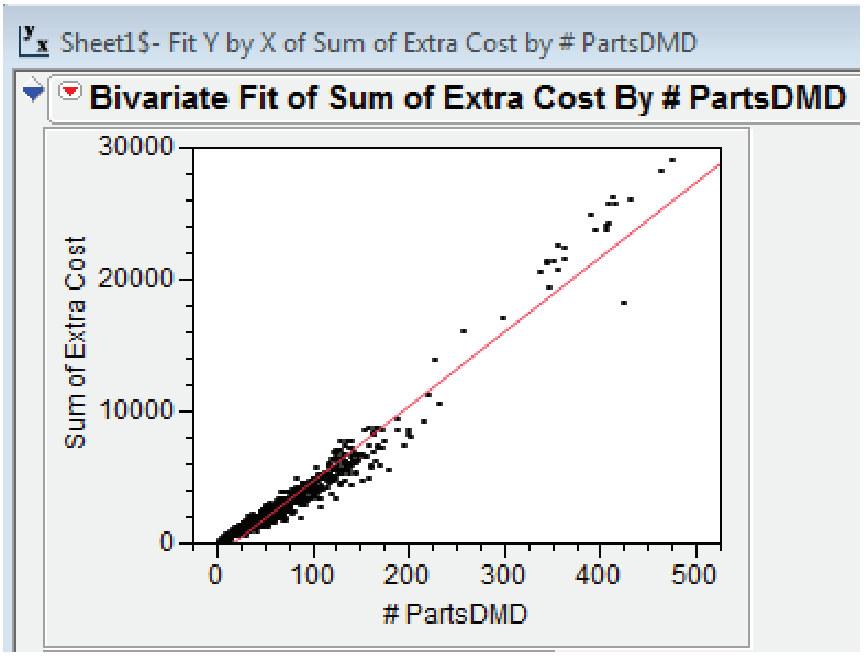


Figure 6.18: Large Example - Linear Regression Analysis

Linear Fit

Sum of Extra Cost = -899.7515 + 56.487858*# PartsDMD

Summary of Fit

RSquare	0.955349
RSquare Adj	0.955292
Root Mean Square Error	885.5359
Mean of Response	3307.729
Observations (or Sum Wgts)	784

Lack Of Fit

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	1.3121e+10	1.312e+10	16731.74
Error	782	613223950	784173.85	**Prob > F**
C. Total	783	1.3734e+10		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-899.7515	45.3681	-19.83	<.0001*
# PartsDMD	56.487858	0.436701	129.35	0.0000*

Figure 6.19: Large Example - Linear Regression Analysis

Also, the linear fit is provided and presented by equation 6.5, where Extra Cost is the total extra shipment cost incurred by each equipment and Parts is the total number of parts with demand by each equipment. In the parameter estimates section, an analysis is presented for the members of this linear equation. Given that the probability is lower than 0.05, we conclude that the parameter is significant (different than zero) and we can keep the terms in the equation. In order words, as documented in JMP help, the probability of getting by chance a t-ratio greater, in absolute value, than the computed value is less than 0.0001.

$$ExtraCost = -899.7515 + (56.487858)x(Parts) \tag{6.5}$$

We see that the model is able to predict well at the beginning of the range, however, it gets distant at the end of it, as shown in Figure 6.18. We perform one more analysis to validate if a linear model is the best fit for our data set. First, we estimate the predicted value by using the equation 6.5; second, we determine the residuals (e) by substracting the Extra Cost result predicted value as shown in equation 6.6; third, we standardize the residuals (e^*) by dividing them by their standard deviation (σ_e) as seen in equation6.7. Last, we plot the results against the different number of parts with demand.

$$e = ExtraCostResult - ExtracCostPredicted \tag{6.6}$$

$$e^* = (e - 0)/\sigma_e \tag{6.7}$$

As can be seen in Figure 6.20, the residuals are not horizontally and randomly disperse across the mean (zero). In fact, it is seen that after the value of 200, all the residuals are on the positive side of y. This behavior indicates that linear regression might not be the best model to describe the relationship between the two variables. As a result, we tried a polynomial fit, and for this specific case we use a quadratic fit.

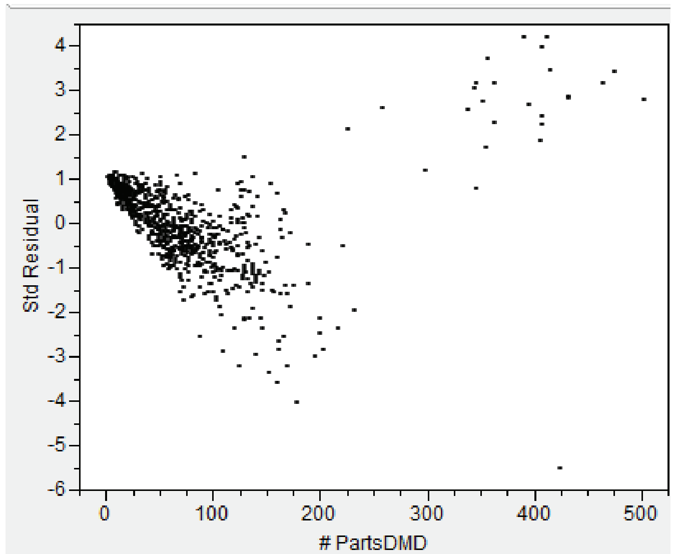


Figure 6.20: Large Example - Linear Regression Residuals plot

The results from the quadratic fit can be seen in Figure 6.21. It is noticed that this fit better describes the data points across almost the entire range whereas the linear fit was strong at the beginning, but less so at the end of the range.

The visual improvement seen on the quadratic fit is confirmed by the increase in the variation explained by the model. R square has increased to 97.7%. ANOVA is testing the same null hypothesis as before, and for this case, we also conclude that the variables are related because F ratio is greater than F critical (same value as before), $FRatio = 16,575.91 > FCrit(0.95, 1, 782) = 3.84$.

The equation fitting the data set is shown by 6.8. Extra cost is the total extra shipment cost incurred when utilizing emergency shipment, and Parts is the total number of parts with demand by each equipment in maintenance. As before, the parameter estimates section shows that all the parameters on the equation are significant, and we keep them in the equation.

$$ExtraCost = -351.72 + (45.24)(Parts) + 0.055(Parts - 74.48)^2 \tag{6.8}$$

Also, we perform a residual analysis test, and this time, we get the expected result:

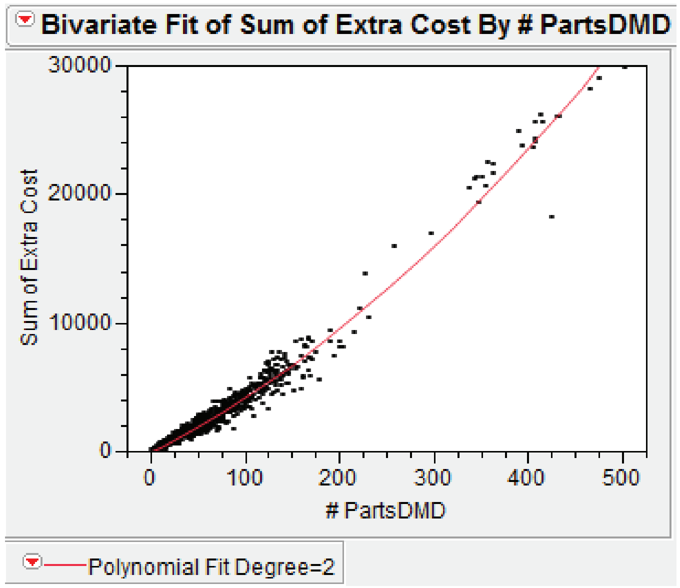


Figure 6.21: Large Example - Polynomial Fit Degree = 2

Polynomial Fit Degree=2

Sum of Extra Cost = -351.716 + 45.245548*# PartsDMD + 0.0551681*(# PartsDMD-74.4847)^2

Summary of Fit

RSquare	0.976984
RSquare Adj	0.976925
Root Mean Square Error	636.1883
Mean of Response	3307.729
Observations (or Sum Wgts)	784

Lack Of Fit

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	1.3418e+10	6.7089e+9	16575.91
Error	781	316098499	404735.59	**Prob > F**
C. Total	783	1.3734e+10		0.0000*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-351.716	38.35949	-9.17	<.0001*
# PartsDMD	45.245548	0.520187	86.98	0.0000*
(# PartsDMD-74.4847)^2	0.0551681	0.002036	27.09	<.0001*

Figure 6.22: Large Example - Polynomial Fit Analysis

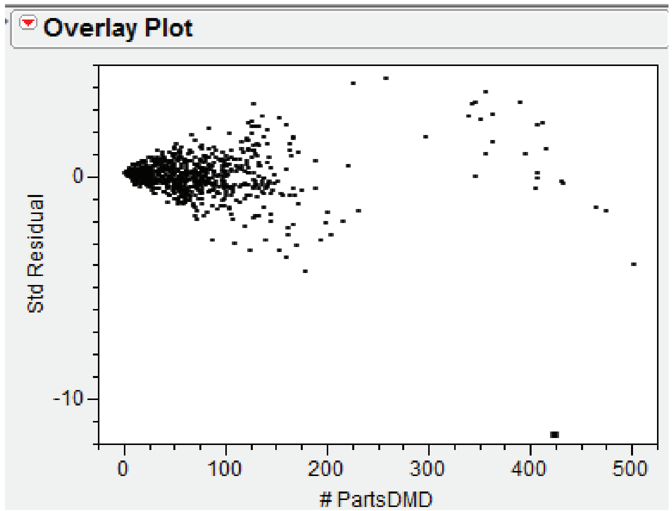


Figure 6.23: Large Example - Polynomial Fit Residuals

data points at both sides of the mean (zero) across the range as seen in Figure 6.23. As before, there is one outlier that falls outside the group, but other than that the residual is good, so, we conclude that equation 6.8 describes the relationship between the two variables.

How can we use the results of the equation discussed above? From an budgeting or quoting perspective, the previous equation can be used to estimate the total expedited cost that the company will incur in order to provide maintenance to the equipment. As we know, the three main variables used to determine the maintenance check for an aircraft are flight hours, flight cycles and age of the aircraft as seen in Table 4.2. We need to determine a model that is able to predict the numbers of part that need replacement depending on either one of the variables mentioned before.

In order to determine a predictive model that identify the number of parts to be used, we follow the same approach as before. Given that each company could be triggering maintenance relying in one variable more than another one, and, in order to reduce as much noise as possible, we develop a model using only the maintenance checks for the airline we are using as example. On the 795 checks provided, we are

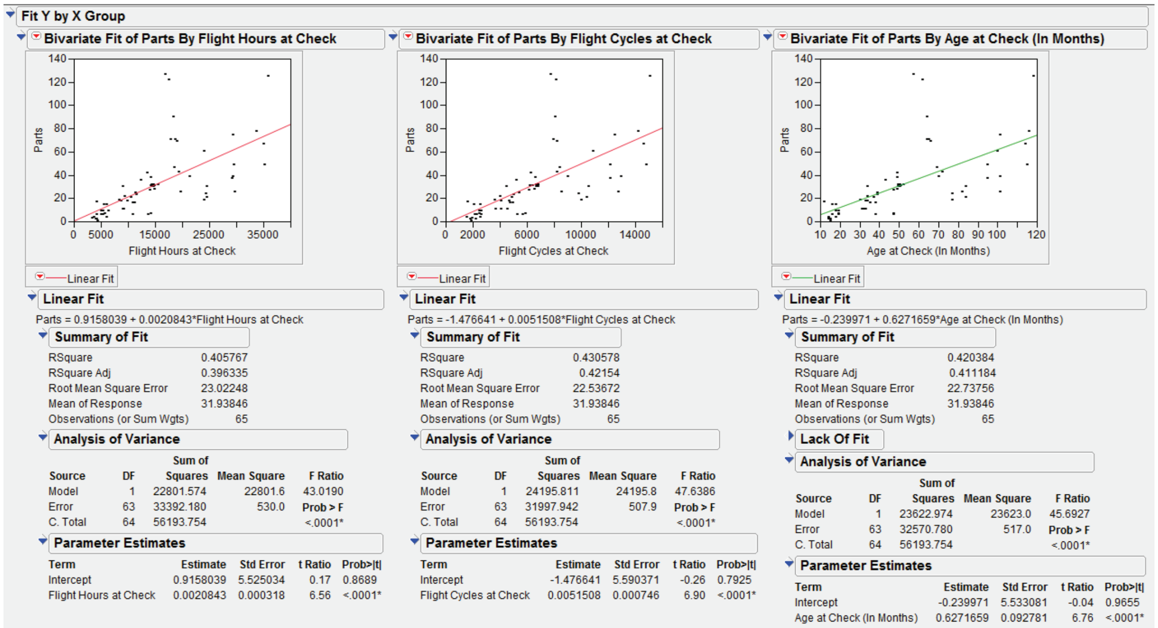


Figure 6.24: Large Example - Predicting No of Parts

able to identify 65 checks performed by the airline used in the case study.

As seen on Figure 6.24, we perform an analysis using the three variables. The correlation between flight hours and parts gives a result of 63.7%, between flight cycles and parts results in 65.6% and the correlation between age of the aircraft and parts is 64.8%. The best results are obtained with flight cycles and parts, and the relationship between both of them is presented in equation 6.9. Same as in the previous cases, we use an alpha value of 0.05. The results show that F Ratio is above 43 for the three cases, and the F Critical F(0.95,1,65)=3.988, hence, F Ratio > F Critical which implies with at least 95% confidence that the variables are related.

$$Parts = -1.476641 + 0.0051508 * FlightCycles \tag{6.9}$$

We also noticed there are three outliers data points; the company has more access to additional data to understand the reason of those points, however, we are not able to validate if they are special cases or not. If we remove those outliers, we are able to improve the predictive model as shown in Figure 6.25. The correlation between flight

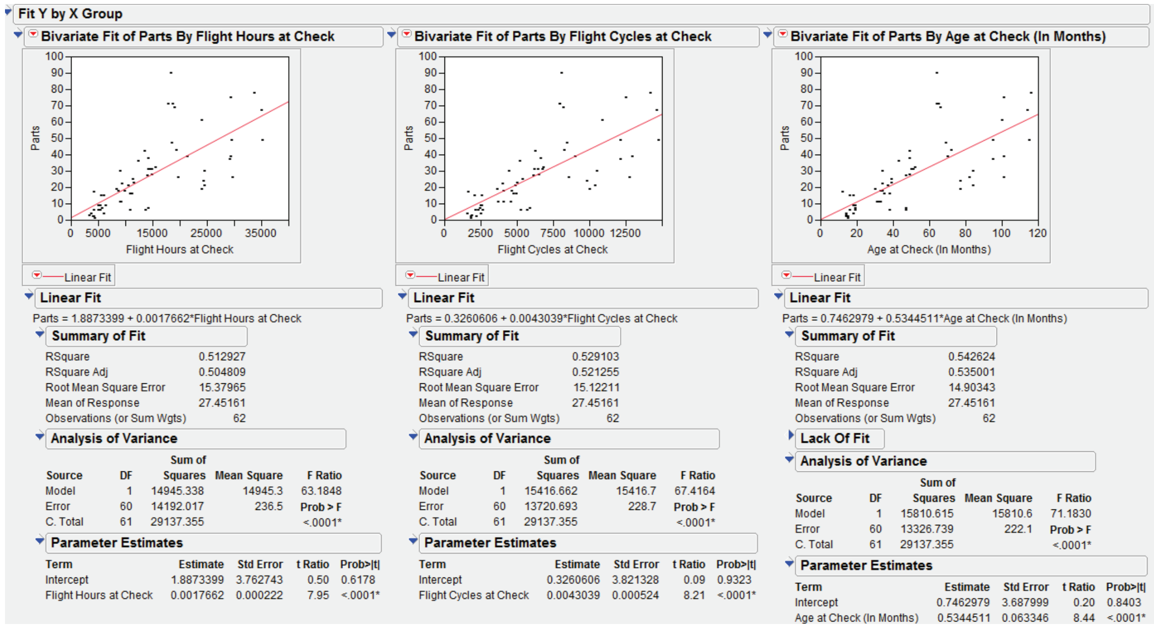


Figure 6.25: Large Example - Predicting No of Parts, Outliers Removed

hours and parts is 71.6%, between flight cycles and parts is 72.7%, and between age of the aircraft and parts is 73.7%. Based on this analysis, the best variable to predict parts is Age of the aircraft, and it is presented in equation 6.10. The results show that F Ratio is above 63 for the three cases, and the F Critical F(0.95,1,62)=3.995, hence, F Ratio > F Critical which implies with at least 95% confidence that the variables are related.

$$Parts \;= 0.7462979 + 0.5344511 * Age \tag{6.10}$$

As shown before, we have been able to developed a method (equation 6.10) to identify the potential numbers of parts to be replaced based on the flight hours of the aircraft. Since the part prediction can be done before the aircraft is in maintenance, it can be used to estimate the expected shipment cost provided by equation 6.8 and use for budgeting or quoting perspective.

In summary, we have been able to validate the mathematical model proposed in this research. We discussed some obvious examples to show it is valid. We also tested it in a real case scenario and were able to reach close to a 1% gap between dual and primal within a reasonable amount of time. We also were able to identify some predictive models that can estimate the total expedited cost based on the number of parts that will be changed replaced. Also, we were able to identify a predictive model that can estimate parts to be replaced based on the age of the aircraft; this result, can be used to improve the budgeting of the company.

In the next chapter, we present the conclusions of this research as well as some future work that could be done.

CHAPTER 7

CONCLUSIONS

In this section, conclusions about the present research are presented as well as some ideas about future work and expansion of this research.

7.1 Sporadic Demand and Optimization Model

The majority of service parts have sporadic demand, a characteristic that makes it challenging to predict demand and have adequate inventory levels. The problem is even more complex when the quantity of parts to be used in each check varies, depending on the technician's skill determining whether to replace the part(s) and on the environmental conditions in which the equipment has been operating. In order to consider the different possible quantities of each part used, a scenario based approach is utilized. One of the objectives of the dissertation was to develop a robust model, we are achieving it by using scenario-based approach. As discussed before, scenario-based approach is able to find a feasible solution for all the scenarios considered while assessing uncertainty, this address one of the objectives of the research. However, some limitations are related to the very small subset of possible scenarios it could handle due to the size and complexity of models.

As demonstrated in previous chapters, by taking advantage of the sporadic demand characteristic of spare parts, we can limit the number of constraints and variables being created: for the large case example, we avoided the creation of 93% of possible constraints. We were also able to develop a model by using only three binary variables. The rest of the variables do not have any type of integer restrictions, however, we rely on the integrity of the demand being fed into the optimization model in order for the base stock level to be an integer as well.

7.2 Results

The MILP optimization model developed is very strong as it was able to find several solutions in a short time. In fact, it was able to close the gap between dual and primal to less than 2% in around 12 minutes; and in about 70 minutes, it was able to reduce the gap to around 1.25%.

Given the current data from the large example, the model is recommending base stock for only an average of 923 out of the 4,149 different parts, or around 22% of total parts. Basically, it recommends relying on expedited shipments for the rest of the parts whenever the normal lead time is greater than the maintenance period. These results remind us of the case discussed in section 1.2 where Cohen and Wille [14] presented two different strategies followed by two different companies. The first company ('on-shore MRO'), has on average $35,000 inventory per check, which is nearly half of what the other company ('off-shore MRO') has per inventory check ($67,000). On the other hand, during maintenance checks 'on-shore MRO' places 2.5 times more expedited orders (200 orders) than the 'off-shore MRO' (83 orders), as shown in Figure 1.5. Again, the company that places more expedited orders has a higher number of late deliveries compared to the other company.

In our large case example, the value of the base stock level recommended to be held at the warehouse is around $99,000. This inventory, together with the additional orders, will help us satisfy 3 maintenance checks per week. It is challenging to compare the stock amount we got from our case with the inventory levels presented by Cohen and Wille [14] because, in our case, we did not include the C items; we left out many other parts because we didn't have all the necessary information to include them as this was a sample to test the optimization model; and also, the data used is related to only maintenance check type 'C' and not the rest of checks.

The large case example results in an average number of expedited orders of 47, with a standard deviation of 58 orders per check. However, 90% of the checks place 88 orders or less. There is no indication of how spread out the case presented in section 1.2 is. Also, it is not possible to validate if the delays are caused by lead time reliability or by not having the appropriate mix of base stock levels. Something we can conclude though, is that relying on expedited shipment is a valid strategy followed by companies, especially in the case of spare parts which have sporadic demand. This research proposes a model that is able to minimize the expedited cost, penalty cost and holding cost while satisfying an agreed service level for on-time equipment delivery.

7.3 Future Work

The present research can be further expanded by removing some assumptions made on it. For instance, we currently assume that if a plane gets delayed, it has no impact on incoming planes behind it. However, there could be cases where an aircraft operator cannot release a new aircraft into maintenance if there are none being returned from maintenance, otherwise, it could disrupt flight schedules, causing cancelations and other type of expenses. Setting up some precedent constraints might be part of expanding the model to represent a more real scenario.

Another assumption made in the current work is that when a company places an expedited order, we assume the unit price of the part is the same as the price paid when the part is acquired using a normal order. In many instances this assumption might not be true, as when the needed part is located at a supplier other than the original manufacturer, the price could be significantly higher though with almost immediate availability. By modeling a higher unit price when placing an expedited order, the strategy recommended by the model might change: for some parts, it might recommend having or increasing base stock levels or reducing expedited orders.

7.4 Summary

In summary, this research proposes a more realistic model that is able to minimize holding cost, expedited shipment cost and penalty cost for late deliveries while satisfying an agreed service level agreement for on-time delivery. Some of the characteristics which together make this problem different than previous ones studied are:

- Problem has multiple types of part failures. After the technician inspects specific areas of the equipment, he/she is going to decide if parts need to be replaced.

- Quantity of pieces of a part to be replaced could be more than one. The same type of part could be located in different places of the aircraft and it will be decided how many are going to be replaced after findings are made.

- Even though a part is needed and it is not available, it might not delay the system availability if it comes before scheduled delivery. It may be necessary to place an expedited order to avoid delay, or it could happen that normal lead time is short and the part will arrive before scheduled system delivery.

- Penalty cost for late delivery is a linear function of the number of times units the equipment is delayed. There is a penalty fee greater than or equal to zero for late delivery.

- Several items could overlap and cause late delivery, but penalty fee effect is not additive. It is calculated based on the item that causes the longest delay in equipment delivery.

- Every type of part has its own replenishment lead time (either Normal or Expedited), and instantaneous replenishment is not assumed.

- The schedule of the preventive maintenance to be performed is known in advance, but not all spare parts to be used in each case are known.
- Service level for on-time delivery is defined at the equipment level, not the item or part level. We are interested in measuring performance at the system level, not at the item level.
- The model is a multi-item, single echelon model.

We have used real case data to validate the model and run it through a sample equal to $\tilde{2}0\%$ of the original part population. Results are strong because the model is able to find 27 different and feasible solutions, and provide an answer with around 1% gap between dual and primal.

In summary, we were able to achieve the goals of the dissertation, to be able to solve a large scale MRO problem within reasonable time. By developing some preprocessing techniques, we were able to reduce the size of the mathematical model which translated in a reduction of the solution time. The mathematical model is addressing the needs of the MRO problem, and it is able to provide a robust solution which is feasible among all the scenarios.

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APPENDIX A

LARGE EXAMPLE - RESULTS FROM ROOT CUTTING & HEURISTICS

Appendix A shows the results in Table A.1 from the Large Example where the root cutting and heuristics process completed; this is discussed in section 6.2.

Table A.1: Large Example - Results from Root Cutting & Heuristics

Its	BestSoln	BestBound	Sols	Gap	Ginf	Time
+	99418.55717	87065.54072	1	12.43%	0	166
+	96814.84553	87065.54072	2	10.07%	0	260
+	94827.62145	87065.54072	3	8.19%	0	350
1	94827.62145	88039.06921	3	7.16%	14740	452
2	94827.62145	88752.06773	3	6.41%	13340	458
3	94827.62145	89259.00575	3	5.87%	12992	463
4	94827.62145	89634.44079	3	5.48%	12651	470
5	94827.62145	89923.37478	3	5.17%	12511	477
6	94827.62145	90115.04813	3	4.97%	12363	484
7	94827.62145	90260.55177	3	4.82%	12149	489
8	94827.62145	90379.10211	3	4.69%	12151	495
9	94827.62145	90465.33736	3	4.6%	12143	500

Continued on next page

Table A.1 – continued from previous page

Its	BestSln	BestBound	Sols	Gap	Ginf	Time
10	94827.62145	90558.45581	3	4.5%	11991	506
11	94827.62145	90638.01496	3	4.42%	11993	512
12	94827.62145	90699.64464	3	4.35%	11689	521
13	94827.62145	90740.23621	3	4.31%	11775	529
14	94827.62145	90803.15932	3	4.24%	11681	534
15	94827.62145	90837.50513	3	4.21%	11641	540
16	94827.62145	90892.55	3	4.15%	11531	547
17	94827.62145	90925.61213	3	4.11%	11139	552
18	94827.62145	90950.82656	3	4.09%	11193	559
19	94827.62145	90970.72343	3	4.07%	11183	567
20	94827.62145	90984.98784	3	4.05%	11289	572
21	94827.62145	91022.48875	3	4.01%	11111	575
22	94827.62145	91046.9776	3	3.99%	10701	577
+	93890.30795	91046.9776	4	3.03%	0	584
	search	started				
+	92563.79202	91046.9776	5	1.64%	0	739
	search	stopped				

APPENDIX B

LARGE EXAMPLE - RESULTS FROM BRANCH AND BOUND

Appendix B shows the branch and bound results in table B.1 from the large example data set.

Table B.1: Large Example - Results from Branch and Bound

Node	BestSoln	BestBound	Sols	Active	Depth	Gap	Time
1	92563.79202	91046.9776	5	2	1	1.64%	931
2	92563.79202	91048.19957	5	1	2	1.64%	938
3	92563.79202	91066.50611	5	2	2	1.62%	943
4	92563.79202	91076.65218	5	3	3	1.61%	950
5	92563.79202	91094.95872	5	4	3	1.59%	954
6	92563.79202	91094.98856	5	5	3	1.59%	960
7	92563.79202	91100.94512	5	6	4	1.58%	965
8	92563.79202	91111.40516	5	7	5	1.57%	968
9	92563.79202	91112.62713	5	8	6	1.57%	971
10	92563.79202	91113.2951	5	9	3	1.57%	976
20	92563.79202	91115.03395	5	11	16	1.57%	1000
30	92563.79202	91115.03395	5	11	26	1.57%	1020
40	92563.79202	91115.03395	5	11	36	1.57%	1043
50	92563.79202	91115.03395	5	11	46	1.57%	1068

Continued on next page

Table B.1 – continued from prev. page

Node	**BestSln**	**BestBound**	**Sols**	**Active**	**Depth**	**Gap**	**Time**
60	92563.79202	91115.03395	5	11	56	1.57%	1091
70	92563.79202	91115.03395	5	11	66	1.57%	1119
80	92563.79202	91115.03395	5	11	76	1.57%	1141
90	92563.79202	91115.03395	5	11	86	1.57%	1169
100	92563.79202	91115.03395	5	11	96	1.57%	1196
B-B	tree	size:	32Mb	total			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
200	92563.79202	91115.03395	5	11	196	1.57%	1435
300	92563.79202	91115.03395	5	11	296	1.57%	1668
400	92563.79202	91115.03395	5	11	396	1.57%	1875
500	92563.79202	91115.03395	5	11	496	1.57%	2056
600	92563.79202	91115.03395	5	11	596	1.57%	2221
700	92563.79202	91115.03395	5	11	695	1.57%	2357
800	92563.79202	91115.03395	5	11	795	1.57%	2478
900	92563.79202	91115.03395	5	11	891	1.57%	2519
999	92558.90415	91115.03395	6	11	991	1.56%	2563
999	92423.81395	91115.03395	7	11	991	1.42%	2690
999	92421.04245	91115.03395	8	11	991	1.41%	2814
999	92417.66396	91115.03395	9	11	991	1.41%	2900
1000	92417.66396	91115.03395	9	11	991	1.41%	3006
1100	92417.66396	91115.03395	9	11	1091	1.41%	3068
1200	92417.66396	91115.03395	9	11	1191	1.41%	3114

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Table B.1 – continued from prev. page

Node	BestSln	BestBound	Sols	Active	Depth	Gap	Time
1300	92417.66396	91115.03395	9	11	1291	1.41%	3161
1400	92417.66396	91115.03395	9	11	1391	1.41%	3204
1500	92417.66396	91115.03395	9	11	1491	1.41%	3251
1600	92417.66396	91115.03395	9	11	1590	1.41%	3286
1700	92417.66396	91115.03395	9	11	1690	1.41%	3331
B-B	tree	size:	165Mb	total			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
1800	92417.66396	91115.03395	9	11	1790	1.41%	3369
1900	92417.66396	91115.03395	9	11	1890	1.41%	3413
1999	92405.53395	91115.03395	10	11	1990	1.4%	3458
1999	92285.4792	91115.03395	11	11	1990	1.27%	3554
2000	92285.4792	91115.03395	11	11	1990	1.27%	3657
2100	92285.4792	91115.03395	11	11	2088	1.27%	3713
2200	92285.4792	91115.03395	11	11	2187	1.27%	3768
2300	92285.4792	91115.03395	11	11	2287	1.27%	3806
2400	92285.4792	91115.03395	11	11	2387	1.27%	3843
2500	92285.4792	91115.03395	11	11	2487	1.27%	3886
2600	92285.4792	91115.03395	11	11	2586	1.27%	3930
2700	92285.4792	91115.03395	11	11	2686	1.27%	3984
2800	92285.4792	91115.03395	11	11	2786	1.27%	4035
2900	92285.4792	91115.03395	11	11	2885	1.27%	4086
2999	92283.74738	91115.03395	12	11	2985	1.27%	4133

Continued on next page

Table B.1 – continued from prev. page

Node	**BestSln**	**BestBound**	**Sols**	**Active**	**Depth**	**Gap**	**Time**
2999	92279.61369	91115.03395	13	11	2985	1.26%	4222
2999	92269.75916	91115.03395	14	11	2985	1.25%	4316
3000	92269.75916	91115.03395	14	11	2985	1.25%	4405
3100	92269.75916	91115.03395	14	11	3080	1.25%	4461
3200	92269.75916	91115.17944	14	3122	13	1.25%	4516
B-B	tree	size:	304Mb	total			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
3300	92269.75916	91115.17944	14	3122	113	1.25%	4564
3400	92269.75916	91115.17944	14	3122	213	1.25%	4587
3500	92269.75916	91115.17944	14	3122	313	1.25%	4616
3600	92269.75916	91115.17944	14	3122	413	1.25%	4635
3700	92269.75916	91115.17944	14	3122	513	1.25%	4655
3800	92269.75916	91115.17944	14	3122	613	1.25%	4675
3900	92269.75916	91115.17944	14	3122	713	1.25%	4698
4000	92269.75916	91115.17944	14	3122	807	1.25%	4724
4100	92269.75916	91115.17944	14	3122	906	1.25%	4747
4200	92269.75916	91115.17944	14	3122	1004	1.25%	4776
4300	92269.75916	91115.17944	14	3122	1103	1.25%	4816
4400	92269.75916	91115.17944	14	3122	1203	1.25%	4852
4500	92269.75916	91115.17944	14	3122	1302	1.25%	4918
4600	92269.75916	91115.17944	14	3122	1400	1.25%	4968
4700	92269.75916	91115.17944	14	3122	1499	1.25%	5007

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Table B.1 – continued from prev. page

Node	**BestSln**	**BestBound**	**Sols**	**Active**	**Depth**	**Gap**	**Time**
4800	92269.75916	91115.17944	14	3122	1596	1.25%	5047
4900	92269.75916	91115.17944	14	3122	1695	1.25%	5081
5000	92269.75916	91115.17944	14	3122	1794	1.25%	5123
5100	92269.75916	91115.17944	14	3122	1894	1.25%	5162
5200	92269.75916	91115.17944	14	3122	1994	1.25%	5202
B-B	tree	size:	473Mb	total			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
5300	92269.75916	91115.17944	14	3122	2093	1.25%	5242
5400	92269.75916	91115.17944	14	3122	2193	1.25%	5274
5499	92264.82127	91115.17944	15	3122	2292	1.25%	5318
5499	92263.00093	91115.17944	16	3122	2292	1.24%	5417
5499	92262.53244	91115.17944	17	3122	2292	1.24%	5508
5500	92262.53244	91115.17944	17	3122	2292	1.24%	5607
5600	92262.53244	91115.17944	17	3122	2391	1.24%	5647
5700	92262.53244	91115.17944	17	3122	2490	1.24%	5693
5800	92262.53244	91115.17944	17	3122	2588	1.24%	5736
5900	92262.53244	91115.17944	17	3122	2686	1.24%	5775
6000	92262.53244	91115.17944	17	3122	2785	1.24%	5810
6100	92262.53244	91115.17944	17	3122	2881	1.24%	5844
6200	92262.53244	91115.17944	17	3122	2977	1.24%	5887
6300	92262.53244	91115.17944	17	6049	3146	1.24%	5909
6400	92262.53244	91116.52186	17	6035	93	1.24%	5939

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Table B.1 – continued from prev. page

Node	BestSln	BestBound	Sols	Active	Depth	Gap	Time
6500	92262.53244	91116.52186	17	6035	193	1.24%	5956
6600	92262.53244	91116.52186	17	6035	293	1.24%	5975
6700	92262.53244	91116.52186	17	6035	393	1.24%	5994
6800	92262.53244	91116.52186	17	6035	493	1.24%	6010
6900	92262.53244	91116.52186	17	6035	593	1.24%	6028
B-B	tree	size:	0.6Gb	total			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
7000	92262.53244	91116.52186	17	6035	693	1.24%	6047
7100	92262.53244	91116.52186	17	6035	790	1.24%	6066
7200	92262.53244	91116.52186	17	6035	888	1.24%	6090
7300	92262.53244	91116.52186	17	6035	985	1.24%	6117
7400	92262.53244	91116.52186	17	6035	1083	1.24%	6149
7500	92262.53244	91116.52186	17	6035	1182	1.24%	6176
7600	92262.53244	91116.52186	17	6035	1280	1.24%	6207
7700	92262.53244	91116.52186	17	6035	1379	1.24%	6239
7800	92262.53244	91116.52186	17	6035	1474	1.24%	6273
7900	92262.53244	91116.52186	17	6035	1574	1.24%	6330
8000	92262.53244	91116.52186	17	6035	1674	1.24%	6364
8100	92262.53244	91116.52186	17	6035	1774	1.24%	6402
8200	92262.53244	91116.52186	17	6035	1874	1.24%	6439
8300	92262.53244	91116.52186	17	6035	1973	1.24%	6473
8400	92262.53244	91116.52186	17	6035	2071	1.24%	6507

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Table B.1 – continued from prev. page

Node	**BestSln**	**BestBound**	**Sols**	**Active**	**Depth**	**Gap**	**Time**
8500	92262.53244	91116.52186	17	6035	2170	1.24%	6538
8600	92262.53244	91116.52186	17	6035	2270	1.24%	6572
8700	92262.53244	91116.52186	17	6035	2369	1.24%	6613
8800	92262.53244	91116.52186	17	6035	2468	1.24%	6653
8813	92255.14154	91116.52186	18	6035	2482	1.23%	6661
B-B	tree	size:	0.7Gb	total			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
8900	92255.14154	91116.52186	18	6035	2568	1.23%	6795
9000	92255.14154	91116.52186	18	6035	2665	1.23%	6839
9100	92255.14154	91116.52186	18	6035	2763	1.23%	6881
9200	92255.14154	91116.52186	18	6035	2859	1.23%	6925
9300	92255.14154	91119.24988	18	8766	13	1.23%	6947
9400	92255.14154	91119.24988	18	8766	113	1.23%	6977
9500	92255.14154	91119.24988	18	8766	213	1.23%	6999
9600	92255.14154	91119.24988	18	8766	313	1.23%	7020
9700	92255.14154	91119.24988	18	8766	413	1.23%	7039
9800	92255.14154	91119.24988	18	8766	513	1.23%	7058
9900	92255.14154	91119.24988	18	8766	613	1.23%	7080
10000	92255.14154	91119.24988	18	8766	713	1.23%	7099
11000	92255.14154	91119.24988	18	8766	1693	1.23%	7415
12000	92255.14154	91119.24988	18	8766	2677	1.23%	7773
13000	92255.14154	91119.49059	18	11637	732	1.23%	8056

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Table B.1 – continued from prev. page

Node	**BestSln**	**BestBound**	**Sols**	**Active**	**Depth**	**Gap**	**Time**
14000	92255.14154	91119.49059	18	11637	1705	1.23%	8380
14813	92254.78942	91119.49059	19	11637	2497	1.23%	8680
15000	92254.78942	91120.50346	19	14113	130	1.23%	8844
16000	92254.78942	91120.50346	19	14113	1120	1.23%	9083
17000	92254.78942	91120.50346	19	14113	2106	1.23%	9439
B-B	tree	size:	1.5Gb	total			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
18000	92254.78942	91120.74417	19	16939	226	1.23%	9795
19000	92254.78942	91120.74417	19	16939	1214	1.23%	10038
20000	92254.78942	91120.74417	19	16939	2190	1.23%	10399
21000	92254.78942	91123.26555	19	19479	577	1.23%	10700
22000	92254.78942	91123.26555	19	19479	1554	1.23%	11026
22813	92252.32578	91123.26555	20	19479	2363	1.22%	11332
23000	92252.32578	91123.26555	20	19479	2546	1.22%	11488
24000	92252.32578	91124.29585	20	22236	535	1.22%	11744
25000	92252.32578	91124.29585	20	22236	1517	1.22%	12046
26000	92252.32578	91124.29585	20	22236	2508	1.22%	12375
27000	92252.32578	91125.32538	20	25212	464	1.22%	12718
28000	92252.32578	91125.32538	20	25212	1449	1.22%	12992
29000	92252.32578	91125.32538	20	25212	2441	1.22%	13368
29046	92251.9115	91125.32538	21	25212	2488	1.22%	13389
29046	92250.89089	91125.32538	22	25212	2488	1.22%	13475

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Table B.1 – continued from prev. page

Node	**BestSln**	**BestBound**	**Sols**	**Active**	**Depth**	**Gap**	**Time**
30000	92250.89089	91128.51652	22	28259	127	1.22%	13933
31000	92250.89089	91128.51652	22	28259	1114	1.22%	14167
32000	92250.89089	91128.51652	22	28259	2096	1.22%	14541
33000	92250.89089	91129.09755	22	30686	571	1.22%	14798
34000	92250.89089	91129.09755	22	30686	1552	1.22%	15104
B-B	tree	size:	2.9Gb	total			
	1.0Mb	in	support	structures			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
35000	92250.89089	91129.09755	22	30686	2532	1.22%	15427
36000	92250.89089	91129.41629	22	33274	896	1.22%	15647
37000	92250.89089	91129.41629	22	33274	1875	1.22%	15985
38000	92250.89089	91129.41629	22	33274	2860	1.22%	16353
39000	92250.89089	91131.24775	22	36323	749	1.21%	16652
40000	92250.89089	91131.24775	22	36323	1724	1.21%	17001
41000	92250.89089	91131.42452	22	38587	366	1.21%	17326
42000	92250.89089	91131.42452	22	38587	1349	1.21%	17592
43000	92250.89089	91131.42452	22	38587	2331	1.21%	17969
44000	92250.89089	91132.49653	22	41334	496	1.21%	18287
45000	92250.89089	91132.49653	22	41334	1476	1.21%	18585
46000	92250.89089	91134.00197	22	43552	167	1.21%	18958
47000	92250.89089	91134.00197	22	43552	1153	1.21%	19216
48000	92250.89089	91134.00197	22	43552	2120	1.21%	19590

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Table B.1 – continued from prev. page

Node	**BestSln**	**BestBound**	**Sols**	**Active**	**Depth**	**Gap**	**Time**
49000	92250.89089	91135.25074	22	45638	972	1.21%	19834
50000	92250.89089	91135.25074	22	45638	1941	1.21%	20215
51000	92250.89089	91135.6504	22	47713	783	1.21%	20490
52000	92250.89089	91135.6504	22	47713	1753	1.21%	20860
53000	92250.89089	91137.14532	22	49762	642	1.21%	21170
54000	92250.89089	91137.14532	22	49762	1624	1.21%	21539
B-B	tree	size:	4.7Gb	total			
	1.2Mb	in	support	structures			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
55000	92250.89089	91137.14532	22	49762	2595	1.21%	21913
56000	92250.89089	91138.80823	22	52496	779	1.21%	22202
57000	92250.89089	91138.80823	22	52496	1762	1.21%	22543
58000	92250.89089	91138.80823	22	52496	2737	1.21%	22935
59000	92250.89089	91139.04894	22	55274	898	1.21%	23229
60000	92250.89089	91139.04894	22	55274	1872	1.21%	23613
61000	92250.89089	91139.06685	22	57762	303	1.21%	23967
62000	92250.89089	91139.06685	22	57762	1289	1.21%	24221
63000	92250.89089	91139.06685	22	57762	2273	1.21%	24605
64000	92250.89089	91139.52789	22	60578	394	1.2%	24924
65000	92250.89089	91139.52789	22	60578	1373	1.2%	25264
65874	92250.67153	91139.52789	23	60578	2231	1.2%	25609
65874	92249.76488	91139.52789	24	60578	2231	1.2%	25718

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Table B.1 – continued from prev. page

Node	BestSln	BestBound	Sols	Active	Depth	Gap	Time
66000	92249.76488	91139.52789	24	60578	2353	1.2%	25871
67000	92249.76488	91139.62953	24	63085	515	1.2%	26177
68000	92249.76488	91139.62953	24	63085	1495	1.2%	26544
68989	92247.39519	91139.62953	25	63085	2469	1.2%	26928
69000	92247.39519	91139.62953	25	63085	2479	1.2%	27028
70000	92247.39519	91140.32912	25	65485	336	1.2%	27265
71000	92247.39519	91140.32912	25	65485	1305	1.2%	27572
B-B	tree	size:	6.0Gb	total			
	2.6Mb	in	support	structures			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
71171	92245.02549	91140.32912	26	65485	1471	1.2%	27646
72000	92245.02549	91140.32912	26	67207	2628	1.2%	28013
73000	92245.02549	91140.38776	26	67032	828	1.2%	28229
74000	92245.02549	91140.38776	26	67032	1808	1.2%	28624
75000	92245.02549	91140.38776	26	67032	2778	1.2%	28989
76000	92245.02549	91140.62684	26	69811	944	1.2%	29236
77000	92245.02549	91140.62684	26	69811	1928	1.2%	29594
78000	92245.02549	91140.84229	26	72547	104	1.2%	29961
79000	92245.02549	91140.84229	26	72547	1095	1.2%	30214
80000	92245.02549	91140.84229	26	72547	2076	1.2%	30576
81000	92245.02549	91141.91501	26	75149	401	1.2%	30912
82000	92245.02549	91141.91501	26	75149	1385	1.2%	31215

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Table B.1 – continued from prev. page

Node	**BestSln**	**BestBound**	**Sols**	**Active**	**Depth**	**Gap**	**Time**
83000	92245.02549	91141.91501	26	75149	2367	1.2%	31662
84000	92245.02549	91142.40719	26	77889	528	1.2%	31981
85000	92245.02549	91142.40719	26	77889	1507	1.2%	32299
86000	92245.02549	91142.40719	26	77889	2474	1.2%	32665
87000	92245.02549	91142.934	26	80329	973	1.19%	32933
88000	92245.02549	91142.934	26	80329	1954	1.19%	33329
88674	92244.60095	91142.934	27	80329	2619	1.19%	33617
89000	92244.60095	91142.934	27	83127	2529	1.19%	33849
B-B	tree	size:	7.5Gb	total			
	2.7Mb	in	support	structures			
Node	BestSoln	BestBound	Sols	Active	Depth	Gap%	Time
90000	92244.60095	91142.93441	27	83085	977	1.19%	34128
91000	92244.60095	91142.93441	27	83085	1954	1.19%	34566
92000	92244.60095	91143.0546	27	85912	34	1.19%	35002
93000	92244.60095	91143.0546	27	85912	1026	1.19%	35238
94000	92244.60095	91143.0546	27	85912	1996	1.19%	35622
95000	92244.60095	91143.11323	27	88032	758	1.19%	35941
96000	92244.60095	91143.11323	27	88032	1735	1.19%	36380

APPENDIX C

LARGE EXAMPLE - SCENARIOS RESULTS

Appendix C shows the results in Table C.1 of all the different cost associated with each scenario. These results are discussed in section 6.2.4.

Table C.1: Large Example - Scenario Reults

Scenario	ExtraShipment
1	47949
2	70227
3	67769
4	110888
5	72372
6	88980
7	115071
8	137149
9	134159
10	142674
11	90839
12	107824
13	120707
14	145562

Continued on next page

Table C.1 – continued from prev. page

Scenario	ExtraShipment
15	110858
16	52122
17	92910
18	56536
19	76159
20	85404
21	55125
22	43863
23	45847
24	48511
25	56718
26	39676
27	85249
28	61629
29	57017
30	53420
31	40695
32	36473
33	42878

VITA

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