UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

DECISION ANALYSIS TOOLS FOR MULTIOBJECTIVE TRADEOFFS IN PROJECT RISK MANAGEMENT

A THESIS

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

Degree of

MASTER OF SCIENCE

By

MATTHEW L. STEWART
Norman, Oklahoma
2016

DECISION ANALYSIS TOOLS FOR MULTIOBJECTIVE TRADEOFFS IN PROJECT RISK MANAGEMENT

A THESIS APPROVED FOR THE
SCHOOL OF INDUSTRIAL AND SYSTEMS ENGINEERING

BY

____
Dr. Kash Barker, Chair

____
Dr. Hank Grant

____
Dr. Charles Nicholson

Acknowledgements

I would like to thank Dr. Barker for giving me the opportunity to work on this project. I must thank Matt McCarter for his assistance in coding the NSGA-II algorithm. Another thank you must go to my parents for always supporting me. Finally, I want to thank my brother and fellow graduate student, Andrew Stewart, for his consistent help and constant encouragement.

Table of Contents

List of Tables

List of Figures

Abstract

The need to manage project risk, through the use of decision analysis tools and other approaches will only increase in the future. Several statistical methods are applied to simulate, in a project risk environment, a simple multiobjective optimization problem. Through the use of the NSGA-II genetic algorithm, coded in Python, and the employment of PROMETHEE to rank the alternatives generated, a method for determining where along a Pareto frontier to focus effort upon to produce the maximum amount risk mitigation will be introduced. The combined use of these decision analysis tools and the presentation, in context to project risk within project networks, is innovative in the area of Multi-Criteria Decision Making and Project Risk Management.

Key Terms: Project Risk, Multi-Criteria Decision Making, NSGA-II, PROMETHEE

Chapter 1: Introduction

Risk is the probability of loss or injury (Merriam-Webster, 2016). Project risk is an uncertain event or condition that, if it occurs has an effect on at least one project objective (Project Management Institute, 2008). Risk management focuses on identifying and assessing the risks to the project and managing those risks to minimize the impact on the project. There are no risk-free projects because there are an infinite number of events that can have a negative effect on any given project. Risk management is not about eliminating risk but about identifying, assessing, and managing as much risk as possible (Wiley, et al., 2016). Specifically, risk management can be considered an iterative process and should be implemented throughout the life cycle of a project to ensure that the minimum risk possible has been achieved upon completion of the project.

Raz, Shenhar, and Dvir (2002) have studied risk management practices on over one hundred projects in a variety of industries. The results of this study suggested the following about risk management practices (Raz et al., 2002):

- Risk management is not widely used.
- The projects that were most likely to have a risk management plan were those that were perceived to be high risk.
- When risk management practices were applied to projects, they appeared to be positively related to the success of the project.
- The risk management approach influenced project schedules and cost goals but exerted less influence on project product quality.
- Good risk management increases the likelihood of a successful project.

Holland & Holland (2010) define the following process to implement to manage risk: set the objective, analyze the risk, take an action, and monitor and control throughout the life-cycle process. By following this simple process, project risk can be effectively minimized (Holland & Holland, 2016).

There are many applications of risk management from construction management to the creation of a software life-cycle process to analyzing project networks. Many of these examples allow for us to use statistical tools to examine project risk management in a Multi-Criteria Decision Making (MCDM) environment. An MCDM setting is, primarily, done with an operations research approach that allows for some optimization and the ranking of alternatives. Through the optimization of MCDM methods, we can find alternatives, rank them using statistical tools, and determine the best course of action based on those rankings.

The Multi-Criteria Decision Aid, also referred to as MCDA, has been one of the fastest growing areas of Operations Research (Behzadian, 2010). The MCDA often deals with the ranking of alternatives from best to worst based on a predetermined criterion. The MCDA method that is implemented in this thesis is called the Preference Ranking Organization Method for Enrichment Evaluation, or PROMETHEE, which was developed by Brans in 1982 and further expanded upon by Vincke and Brans in 1985 (Behzadian, 2010). Brans et al. (1985) explain, PROMETHEE is an outranking method for a finite set of actions to be ranked and selected among criteria, which often conflict. It is a simple ranking method when compared to alternate methods of multi-criteria analysis but has become a widely adopted MCDA method (Brans et al., 1985).

1.1 Past Work

Most of the work done in the area of project risk management is done in application to an industry problem but this doesn't necessarily mean that the application of statistical tools has been performed. The application, included in this work, includes the implementation of a project network problem, the minimization of a multi-objective formulation to create a Pareto Optimal Frontier, a ranking of the alternatives using a statistical tool, PROMETHEE, applying randomly generated weights, and generation of a heat graph to visually confirm our rankings in a project risk environment. This application will allow for future work to be done in the field of MCDM and project risk management by implementing a more complex multi-objective formulation in relation to project networks.

1.2 Contribution

It is important to note that this specific application of statistical tools differs from past work in several ways: (i) implementing a genetic algorithm, the NSGA-II algorithm, to populate a Pareto Optimal Frontier, (ii) employing a Multi-Criteria Decision Making tool, PROMETHEE, to rank our alternatives utilizing randomly generated weights, (iii) generating a Heat Graph to illustrate the ranking of alternatives, and (iv) encompassing these methods within a project risk management environment to assist decision-makers in maximizing the amount of project risk mitigated by focusing effort expended upon a certain region along a Pareto optimal curve. These tools have been chosen because of the dual relevance in academia as well as in industry.

1.3 Thesis Structure

Chapter 2 will introduce applicable concepts, and provide context for the rest of the thesis. Chapter 3 will consist of the methodology implemented in this thesis. Chapter

4 will consist of results. In Chapter 5, conclusions will be made and future work discussed. Appendix A will include various tables, relevant code, and datasets.

Chapter 2: Supporting Literature

2.1 Project Risk Management

As defined earlier, risk is the probability of loss or injury (Merriam-Webster, 2016). Project risk is an uncertain event or condition that, if it occurs has an effect on at least one project objective (Project Management Institute, 2008). In other words, a project manager may question what problems might be encountered during a project and how to avoid them. This is project risk management.

Risk management is not always taken seriously, though. When project managers assess risk, many often just add a "random margin of risk" (Cervone, 2006)." This is often just a wild guess but the likelihood that this guess will be significantly underestimated is equal to the likelihood that it will be a valid overestimation (Cervone, 2006). This seems like, and is, a crude method of assessing risk but is often implemented because project managers know that there are a lot of risks that, ultimately, are out of their influence. Cervone (2006) provides an example:

> "A construction company could be using an outside vendor to supply plywood. Project managers know that unforeseen delays occur with suppliers but without hard data would rather "throw out a guess" to assess this risk than to ignore the possibility of risk completely. There are many unknown factors that contribute to project risk management. These are called "risk factors."

By minimizing this risk, there is a higher likelihood of success and that the project will likely complete on time. Risk cannot be avoided completely, especially when there are multiple steps prior to completion of a project or when there are multiple criteria defining levels of success. There are several applications of multi-objective optimization problems in risk management, such as: optimal portfolio selection, pricing tolling agreements, project network problems, and other project management scenarios (Steponavičė, 2016).

Not only can the risk be minimized but also by applying this combination of techniques, mitigation efforts can take place to prevent additional risk. These efforts can save businesses and industries money, time and effort, just by employing statistical tools to understand project risk management.

2.2 Project Networks and Risk Management

Increasingly, organizations execute projects by employing project networks. These networks consist of relationships between organizations or individual parts within a process to achieve a predetermined objective. Located within project networks are inherent risks that, feasibly, could derail the completion, or successful achievement, of the project as a whole. Project risk management attempts to minimize these risks by providing suggestive actions to manage uncertainty. Risk, in the network context, can be defined as an uncertain event or condition that results from the network form of work and has an impact that contradicts the expectations or desired outcome (Pekkinen and Aaltonen, 2015).

A random project network generation can be completed by implementing the R code from Floyd (2015). This code randomly generates project network from defined input variables: (i) number of runs and (ii) number of nodes. By changing these user inputs, one can randomly generate any size project network. Figure 1, shown below, is an example project network generated from the code in Floyd (2015), using number of runs equal to 1 and number of nodes equal to 30.

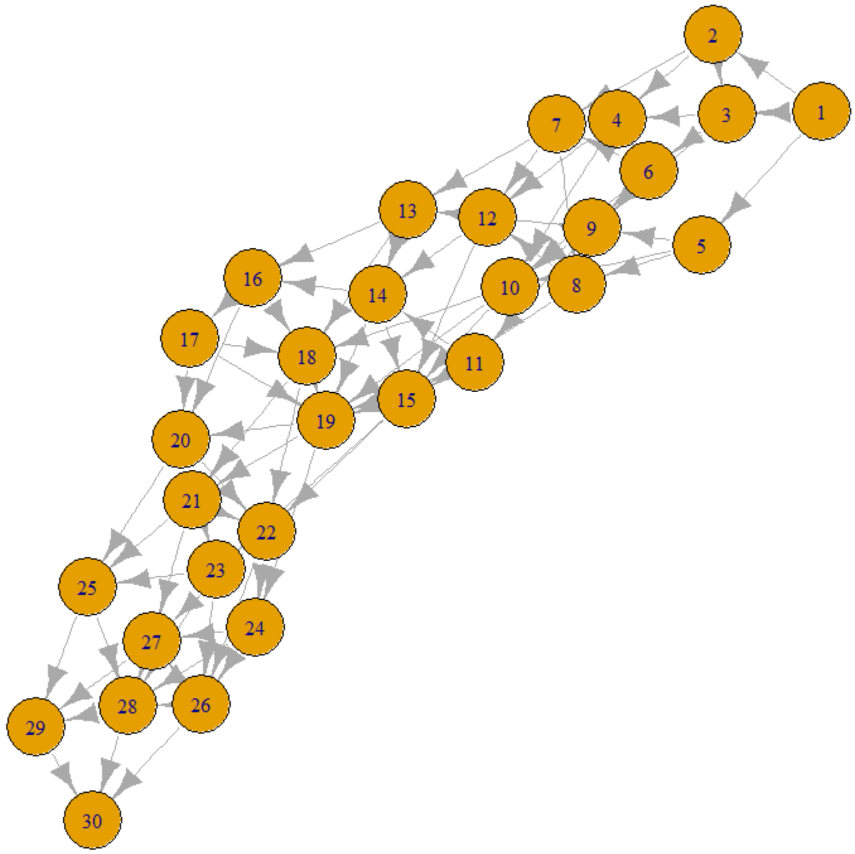


Figure 1 - Example Project Network Generated From Floyd (2015)

It is important to note that when project networks increase the number of nodes and activities that the complexity also increases. When applying statistical tools and other applications of techniques or methodologies, complexity needs to be considered. Figure 2, shown below, illustrates what increasing the complexity of a project network visually. For the application to this paper, 30 nodes were selected to include a moderate amount of complexity while still allowing for some straightforwardness to remain, inherently.

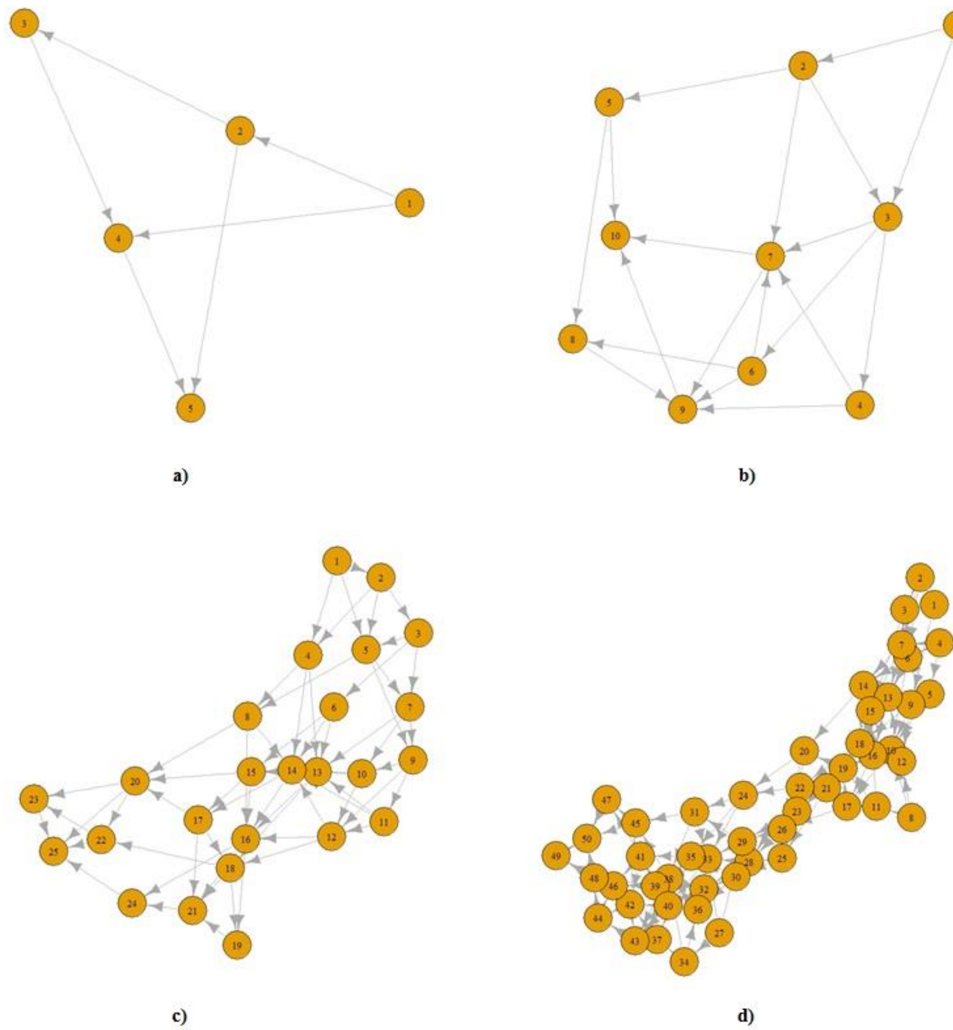


Figure 2 - Generated Project Network with Different Number of Nodes a) 5 b) 10 c) 25 and d) 50

Figure 2 illustrates what the R code from Floyd (2015) generates when asked to create a random project network with increasing complexity. This is a useful tool that can be implemented in a variety of ways, including problems regarding project networks, project risk and many other areas of interest, including Multi-Criteria Decision Making in academia or industry.

2.3 Multi-Criteria Decision Making

Decision-making permeates everything human. From choosing which hairbrush to buy or deciding what route to drive to get to a destination, making decisions is inherent. The way we study alternatives and evaluate them based on criteria is a natural process (Sánchez-Lozano, 2013). This, which at first sight seems to be simple, forms part of the whole discipline that is called Multi-Criteria Decision Making, or MCDM (Sánchez-Lozano, 2013).

A review of the literature suggests that to a large extent, Multi-Criteria Decision Making (MCDM) tools have been developed in Operations Research and numerous other fields. These methods, which can handle both quantitative and qualitative criteria, share the common characteristics of incommensurable units, and hard in design/selection of alternatives (Pohekar and Ramachandran, 2004). MCDM has three main concepts: choosing the best alternative, ranking the alternatives from best to worst, and sorting all alternatives into different pre-ordered groups (Marle, 2012). Beyond the concept of MCDM, consideration for the overall optimality of these alternatives must be given. Optimality is an important idea in MCDM, primarily the concept of Pareto Optimality.

2.3 Pareto Optimality

In engineering, computing, and many other fields, there are a large number of optimization problems, most of them being complex and multifaceted with numerous objective functions or dynamic parameters. In Pareto optimization, the central concept is named the *non-dominated solution* (Tomoiaga et al., 2013). This solution must satisfy the following two conditions: (i) there is no other solution that is superior at least in one objective function; (ii) it is equal or superior with respect to other objective function values (Tomoiaga et al., 2013). For example, Moitra and O'Donnell (2012) explain:

> "In choosing a driving route between two points one might want to minimize distance, tolls, the number of turns, and expected traffic; in choosing a vacation hotel one might want to minimize price and distance to the beach, while maximizing quality. In such cases there is rarely a single solution which is best on all criteria simultaneously. The most popular way to handle the trade-off is to determine the set of all Pareto optimal solutions, meaning those solutions which are not dominated in all measures of quality by some other solution. This idea, originating in microeconomics, has been very extensively studied in computer science, especially in operations."

Since there is more than one objective function in the scenario to be outlined, linear programming cannot be utilized. This means that there are two focal considerations: (i) formulate a problem with multiple objectives and (ii) find the optimal solution before applying other statistical methods (Tomoiaga et al., 2013). By completing the former, utilizing the NSGA-II genetic algorithm, a ranking of alternatives can be achieved from the application of PROMETHEE. Finally, a heat graph will visually show the alternative rankings and probabilities associated with the risks involved.

2.4 Genetic Algorithms and the NSGA-II

A Genetic Algorithm (GA) is an adaptive heuristic search algorithm based on the ideas of natural selection and genetics ("Introduction to Genetic Algorithms", 2016). They represent the idea of implementing random searches used to solve optimization problems. GAs work with a population of potential solutions, within a search space, and iteratively move this population toward the optimum (Tsai, 2014).

A typical genetic algorithm requires two things: (i) a genetic representation of the solution domain and (ii) a fitness function to evaluate the solution domain. Most standard representations of a candidate solution are arrays of bits, although other types and structures can also be applied in the same way (Tsai, 2014). One excellent property of these genetic algorithms is that they are convenient representations because they are aligned due to their fixed size. This allows for easy, simple crossover operations. There are further applications to genetic algorithms other than simply optimization, like genetic programming, gene expression programming, and or evolutionary programming.

The NSGA is a popular non-domination-based genetic algorithm for multi-objective optimization. It differs from other genetic algorithms in only how the selection operator works (Sarkar & Mordak, 2005). Sarkar and Mordak (2005) explain, the efficiency of the NSGA algorithm is in the way that multiple objectives are reduced to a single fitness measure by the creation of a number of fronts that are sorted according to non-domination. An example is shown below in Figure 3. Rank 1 is preferred to Rank 2 which is preferred to Rank 3 in Figure 3 because Rank 1 is closer to the Pareto optimal solution. It is difficult to know the Pareto optimal solutions before applying GAs to find them.

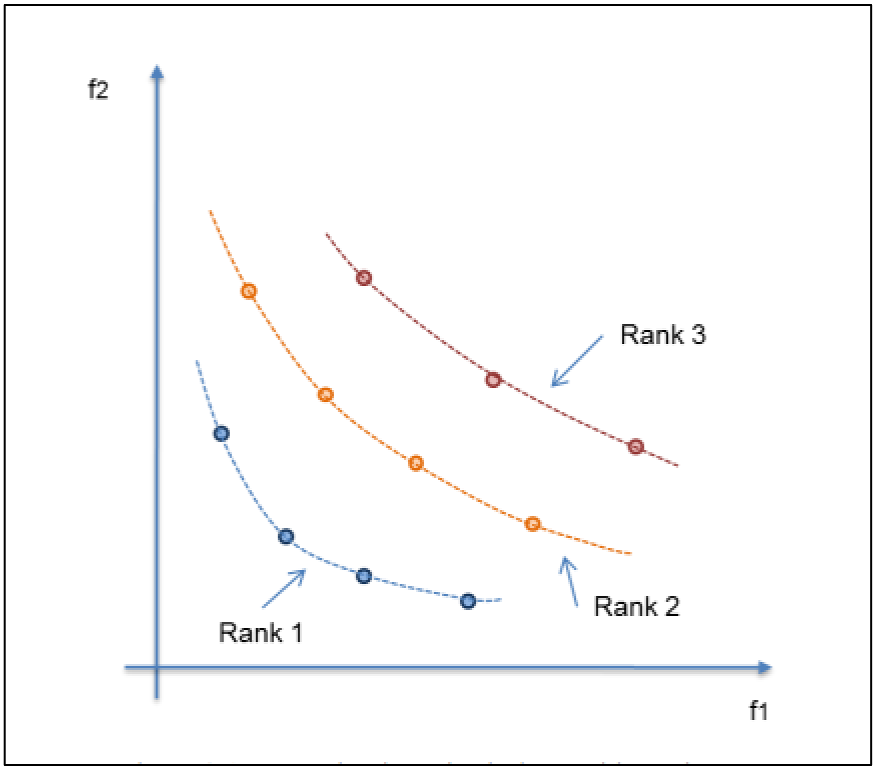


Figure 3 - Example of Non-Dominated Solution Ranking (Seshadri, 2016)

There have been criticisms of the NSGA, though, leading to the improved NSGA-II, which introduces a fast non-dominated sorting procedure, parameter-less niching operator for diversity preservation, and the elitist preservation approach (Sarkar & Mordak, 2005). The NSGA-II is implemented in this thesis.

Seshadri (2006) explains, the population (Pt) is randomly initialized, and once initialized, it is sorted based on non-dominance into each front with the first being a completely non-dominant set and the second front (Qt) being dominated by the individuals in the first front only and so on. Each individual, in each front, is assigned a fitness, or rank value, with the individuals in the first front being given a rank value of 1 and individuals in the second being given fitness of 2 and so on (Seshadri, 2006).

Chromosomes, which are a set of parameters defining a proposed solution to the problem that the GA is trying to solve, are then sorted and put into Pareto non-dominated sets, where the chromosomes are then ranked, based on the crowding distance, which is the measure of how close an individual chromosome is to its neighbor (Deb, Pratap, Agarwal, and Meyarivan, 2002). Solutions that are further, in distance, from the other solutions are given a higher ranking, in order to make a diverse solution set and to avoid a crowded solution set (Deb et al., 2002). The best chromosomes are selected from the current population and put into a mating pool, where a binary tournament selection, based on the rank and crowd distance, occurs (Deb et al., 2002). The "winner" of each binary tournament (with the best fitness) is selected for crossover, where the algorithm takes more than one "parent" solution to create a child solution.

After the binary tournament selection transpires, the mating pool and current population are combined, resulting in a sorted set where the best N chromosomes make it into the new population, where N is the population size (Deb et al., 2002). The selection is based on rank and on the crowding distance on the last front (Deb et al., 2002). This process is repeated unless a maximum number of generations have been achieved (Deb et al., 2002). The process flow example of this procedure is shown below in Figure 4. Utilizing the NSGA-II is critical to the overall application of this thesis as it allows for any number of generations and populations to be generated for further data analysis.

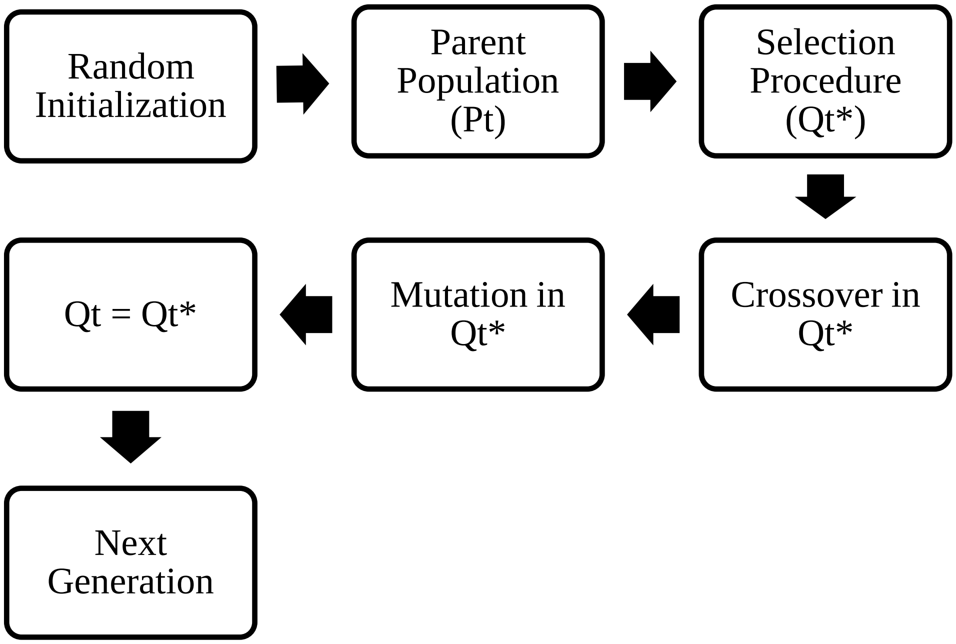


Figure 4 - NSGA-II Example Process Flow

2.5 PROMETHEE

The Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) is a family of outranking methods (Brans & Vincke, 1985). PROMETHEE wants to identify the pros and cons of the alternatives and in turn rank them, accordingly. The key question is whether there is enough information to state that one alternative is at least as good as another (Ferretti, 2013). This ranking relationship obtained from PROMETHEE does not determine if there is a relationship between an alternative *a* or an alternative *b*, just that "the alternative *a* is at least as good as alternative *b*" (Brans & Mareschal, 1984). Multiple criteria are considered when ranking these alternatives and can be weighted separately.

Ferretti (2013) explains that the PROMETHEE process has two major phases: (i) assigning a preference function and (ii) estimating the outranking degree of options.

To begin phase one, the starting point is the evaluation matrix. This matrix presents the performance of each alternative in relation to a criterion. The alternatives are compared pairwise to each criterion (example of a criterion in Figure 5 and Table 2), resulting in preference functions, calculated for each pair of options, which can range from 0 to 1. Zero meaning that there is no difference between the pair of options and one indicating that there is a large difference (Ferretti, 2013).

Phase two begins by multiplying the preferences with the criterion's weight, adding the values, and creating a matrix of global preferences. The sums of the rows show the strength of an alternative, or dominance. The sums of the columns show how much an alternative is dominated by the other alternatives (subdominance). By subtracting the sums of the columns from the sums of the rows, a linear ranking is obtained (Ferretti, 2013).

PROMETHEE assumes that the decision maker is able to weigh the criteria appropriately and does not provide guidelines for determining weights to criteria (Macharis, Springael, De Brucker, & Verbeke, 2004). The *usual criterion* was employed for this project because it requires no parameters be asked of the decision maker. Figure 5, below, presents the various criterions that could have been applied and also the pairwise comparisons associated with them.

- Usual

$$P_j(d_j) = \begin{cases} 0 & \text{if } d_j \le 0 \\ 1 & \text{if } d_j > 0 \end{cases}$$

- U-shape

$$P_j(d_j) = \begin{cases} 0 & \text{if } |d_j| \le q_j \\ 1 & \text{if } |d_j| > q_j \end{cases}$$

- V-shape

$$P_j(d_j) = \begin{cases} \frac{|d_j|}{p_j} & \text{if } |d_j| \le p_j \\ 1 & \text{if } |d_j| > p_j \end{cases}$$

- Level

$$P_j(d_j) = \begin{cases} 0 & \text{if } |d_j| \le q_j \\ \frac{1}{2} & \text{if } q_j < |d_j| \le p_j \\ 1 & \text{if } |d_j| > p_j \end{cases}$$

- Linear

$$P_j(d_j) = \begin{cases} 0 & \text{if } |d_j| \le q_j \\ \frac{|d_j| - q_j}{p_j - q_j} & \text{if } q_j < |d_j| \le p_j \\ 1 & \text{if } |d_j| > p_j \end{cases}$$

- Gaussian

$$P_j(d_j) = 1 - e^{-\frac{d_j^2}{2s_j^2}}$$

Figure 5 - PROMETHEE Criterions

2.5 Randomly Generated Weights for Use in PROMETHEE

Table 3 and Table 4 provide the results of 10,000 generated weights, simulated from a uniform distribution between 0 and 1 (UNI (0, 1)) in Microsoft Excel. Table 3 represents the frequency of these generated weights based on the equally likely set of bins. Alternatively, since there are 30 alternatives (A1-A30), the number of bins selected in Table 3 was also 30. This is presented in similar fashion in Table 4, so similarly. Table 4 illustrates the randomly generated weights using the approach described in Tervonen and Lahdelma (2007). The frequency of the 10,000 generated weights is represented in

Table 3 and Table 4. Figure 6, below, presents the frequency of each bin$\left(\frac{n}{30}\right)$, while Figure 7 represents the frequency of each bin with random bin size, for comparison. This is a simple illustration of how randomly generated weights can affect the outcome on a system or strategy, even when applying a statistical method like PROMETHEE.

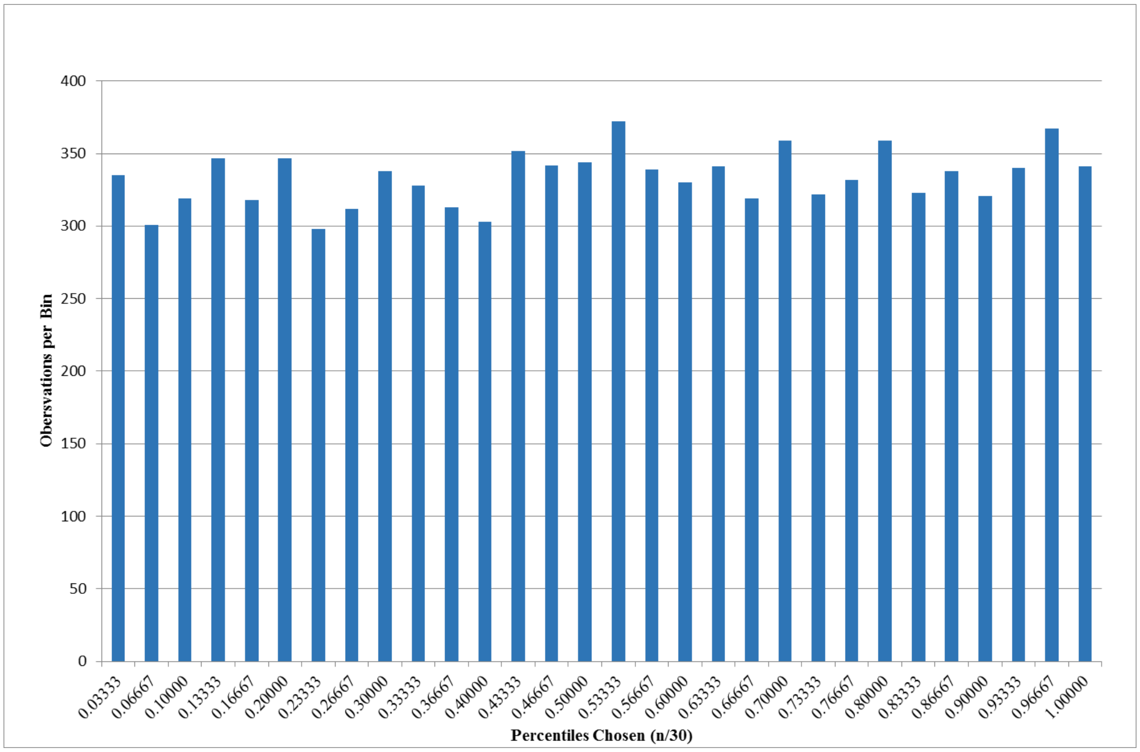


Figure 6 - Frequency of Equally Likely Weights with Equal Bin

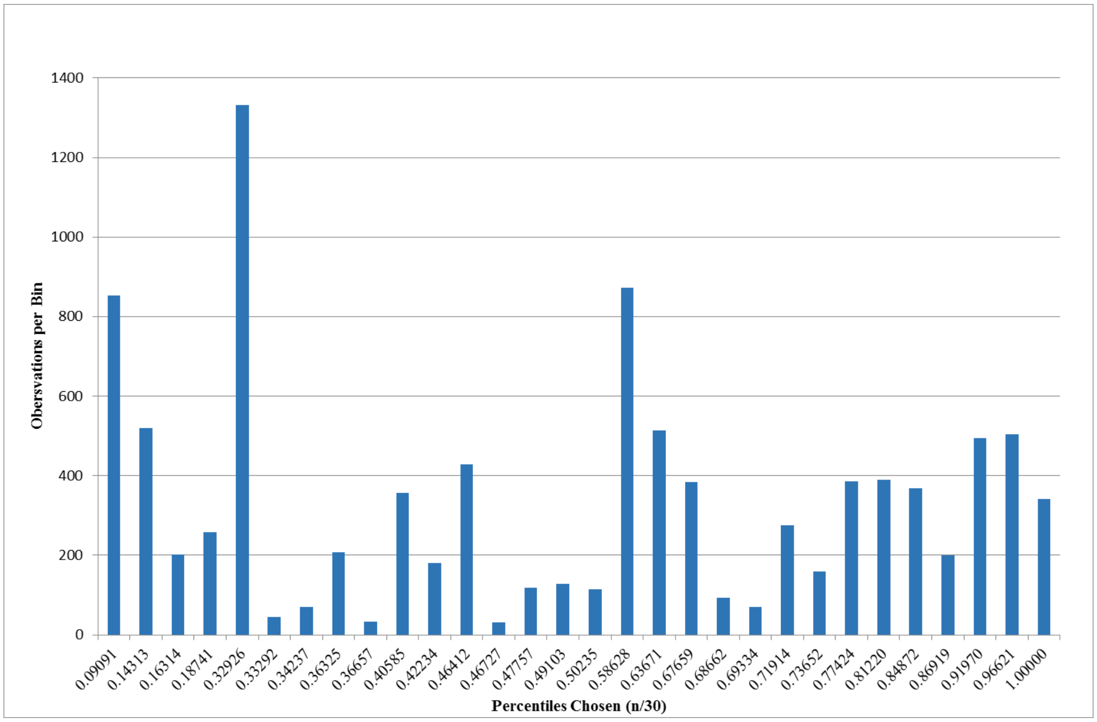


Figure 7 - Randomly Generated Bin Sizes for Comparison

PROMETHEE is appropriate to use in many applications. Prime examples include selecting a single alternative from a given set of alternatives where multiple decision criteria are concerned, prioritization, resource allocation, ranking, and conflict resolution. In this case, PROMETHEE will be implemented to rank the alternatives generated from the application of the NSGA-II to a multiobjective optimization problem. Since a complete ranking is desired, anytime PROMETHEE is referenced in this thesis, it will be in reference to PROMETHEE II.

2.6 Scenario Creation

In general, multiobjective optimization problems can be described by the following:

Find a vector x = (x_1, x_2, …, x_n) for

$$\boldsymbol{Min_{x\in\Omega}} \qquad \boldsymbol{F(x) = (f_1(x), f_2(x) \dots, f_n(x))^{\tau}} \qquad \textbf{(1)}$$

s.t.

$$\boldsymbol{q_i(x) \leq 0 \ (i = 1, 2, \dots, k),} \qquad \textbf{(2)}$$
$$\boldsymbol{h_j(x) = 0 \ (j = 1, 2, \dots, l),} \qquad \textbf{(3)}$$

Where Ω is the set of the decision vector, *m* is the number of objectives, $q_i(x)$ (i = 1, 2,…, k) are k additional inequality constraints and $h_j(x)$ = (j =1,2,…, l) are l additional equality constraints. In a word, it aims to find vectors subject to some constraints, which make all the objective values as small as possible or minimized.

In an attempt to find the optimal set of alternatives for the above general formulation, one would solve for these vectors using the NSGA-II genetic algorithm, which would provide a set of alternatives but not necessarily the globally optimal set of alternatives. In general, however, multiobjective optimization problems have no feasible solution, minimizing all the objective functions simultaneously. For this reason, the topics of Pareto optimal solutions and Pareto dominance are mentioned. A Pareto optimal solution is a feasible solution, though; it cannot be improved without degrading at least one set of objectives. Therefore, no feasible solution can dominate it. These alternatives, or Pareto optimal solutions, can be referred to as the Pareto front. The previously mentioned alternatives would form a Pareto optimal front which can be represented by the following:

$$PF = \{f(x) = (f_1(x), f_2(x), \dots, f_m(x))^T | x \in P\}$$

where PF includes the values of all objective functions corresponding to the solutions in P.

2.6.1 The ZDT1 Test Function

Five functions belong to the Zitzler-Deb-Thiele (ZDT) set of test functions (Zitzler and Thiele, 1999). They are generally considered the most reliable benchmark for evaluating multiobjective genetic algorithms. The ZDT1 test function was selected because of the low complexity and ease of illustration. The ZDT1 test function consists of the following multiobjective optimization problem:

$\min\{f_1, f_2\}$

where the objective functions are

$$\boldsymbol{f_1(x) = x_1} \quad \textbf{(4)}$$

$$f_2(x) = g(x)\left[1 - \sqrt{\frac{x_1}{g(x)}}\right] \quad \textbf{(5)}$$

and *g(x)* is defined as:

$$g(x) = 1 + \left(\frac{9}{n-1}\right)\left(\sum_{t=2}^{n} x_i\right) \quad \textbf{(6)}$$

s.t.

$$0 \leq x_i \leq 1, \quad \textbf{(7)}$$

$$1 \leq n \leq 30 \quad \textbf{(8)}$$

The application of the ZDT1 function is to prove that the code used in Python was implemented correctly. Figure 8 illustrates the Pareto front that the ZDT1 is expected to look like. Figure 9 demonstrates the generated front that results from the administration of the code in Appendix A.

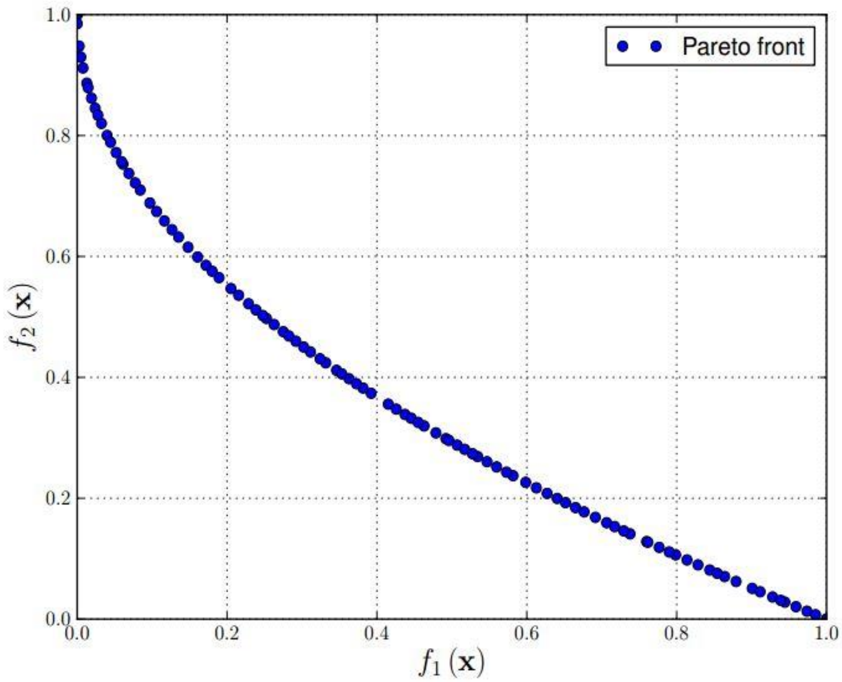


Figure 8 - Expected ZDT1 Function Pareto Front

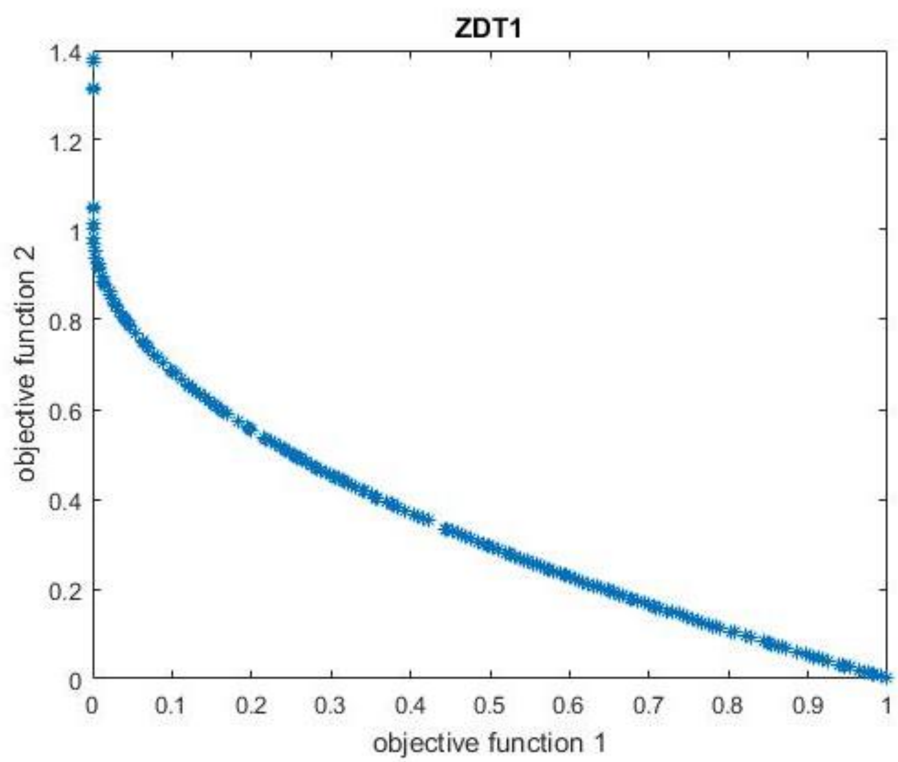


Figure 9 - Generated ZDT1 from NSGA-II

2.6.2 Scenario Implemented

Using the approach from (Better, Glover, Kochenberger, & Wang, 2008) to create the scenario, a basic project network risk management problem is employed as a multiobjective problem with three objective functions. The problem discussed is from Cebi and Otay (2013). A contractor wishes to handle a project network problem by mitigating risks and maximizing associated bonuses or incentives. Several conventional factors will be introduced and used to illustrate the general approach of this thesis to a project risk environment.

The model introduced by Cebi and Otay (2013) seeks to minimize three objective functions. Objective Function 1 (f_1) aims to minimize completion time of the project, while Objective Function 2 (f_2) minimizes the total project cost, and finally Objective Function 3 (f_3) deals with minimization of the earliest time of an event that the management may give special attention due to various reasons (Cebi and Otay, 2013). The nomenclature and model come from work previously done by Cebi and Otay (2013). Shown below are the objective functions that were coded into Python.

$\min\{f_1, f_2, f_3\}$

$$f_1(x) = E_p - E_1 \qquad \textbf{(9)}$$

$$f_2(x) = \sum(c_{ij} * y_{ij}) + \left(I * E_p\right) - (B * y_s^-) + \sum Z_f * C(b) \qquad \textbf{(10)}$$

$$f_3(x) = E_s - E_1 \qquad \textbf{(11)}$$

s.t.

$$E_j - E_i - y_{ij} \geq D_{ij} \quad \text{for } \forall i, j \qquad \textbf{(12)}$$

$$y_{ij} \leq D_{ij} - d_{ij} \qquad \text{for } \forall i, j \qquad \textbf{(13)}$$

$$E_p + s^+ - s^- = T \qquad \textbf{(14)}$$

$$s^- \geq e y_{s^-} \qquad \textbf{(15)}$$

$$s^+ \leq t y_{s^+} \qquad \textbf{(16)}$$

$$y_{s^-} + y_{s^+} \leq 1 \qquad \textbf{(17)}$$

The nomenclature is taken from Cebi and Otay (2013) and is presented below in Figure 10. With the Equations 9-11 explained, above, further explanation is necessary for the constraints. Equation 12 refers to the time between events *i* and *j*. Equation 13 pertains to the crashing time of an activity (*i*, *j*). Equation 14 is about planned project completion time (*T*). Equation 15 either activates a bonus if the project is completed earlier than *T* or removes the potential bonus. Equation 16 activates the term of the bonus if the project is completed at least *e* weeks before *T*. Equation 17 implies that only one of the variables take the value of “1” if the project cannot be completed on the planned completion time *T*.

With these objective functions coded into Python, the next step is to apply the NSGA-II genetic algorithm to generate a Pareto optimal front to rank using

PROMETHEE. In the next section, the methodology will be discussed and the application of statistical tools will be implemented to generate data for further analysis.

Nomenclature.

Indices	
(i,j)	Index for activities between event i and event j
b	Index for break points of the penalty cost
k	Index for goals
Decision variables	
E_i	Earliest event time for node i $i = \{1,2,3,\ldots,s,\ldots,p\}$
E_j	Earliest event time for node j $j = \{2,3,\ldots,s,\ldots,p\}$
y_{ij}	Crash time for activity (i,j) (Week) $y_{ij} = D_{ij} - d_{ij}$
s^-	Total earliness (Week)
s^+	Total tardiness (Week)
$y_{s^-} = \begin{cases} 1, & \text{if the project is completed at least e weeks earlier than } T \\ 0, & \text{otherwise} \end{cases}$	
$y_{s^+} = \begin{cases} 1, & \text{if the project is completed later than } T \\ 0, & \text{otherwise} \end{cases}$	
Parameters	
T	Planned project completion time (week)
D_{ij}	Normal time of an activity (i,j) (week)
d_{ij}	Crash time of an activity (i,j) (week)
$C_{D_{ij}}$	Normal cost of an activity (i,j) (€)
$C_{d_{ij}}$	Crash cost of an activity (i,j) (€)
c_{ij}	Unit crashing cost (€/week) $\left[c_{ij} = (C_{d_{ij}} - C_{D_{ij}})/(D_{ij} - d_{ij})\right]$
I	Indirect cost (€/week)
B	Bonus (€)
$C(b)$	Total penalty cost at break points $b = \{0,3,6,9,12\}$ (€)
t	Maximum tardiness (week)
e	Minimum earliness for gaining bonus (week)

Figure 10 - Nomenclature from Cebi and Otay (2013)

Chapter 3 Methodology

3.1 Software Utilized

To perform the outlined scenario, Python inside the Wing IDE 101, RStudio, and Microsoft Excel were used to generate the data and execute the analysis. All code and tables that are relevant are located in Appendix A, below. No other tools were applied to this scenario.

3.2 Python Implementation

Python was selected because of previous knowledge and availability of resources to complete the coding on time. Python is highly recommended because of the flexibility, ease of learning, and free resources out there to code effectively. The NSGA-II can be applied in other mediums but was not in this thesis.

With the presented objective functions entered into the code, the implementation of the NSGA-II algorithm to find alternatives via Python can be utilized. Table 1, in Appendix A, shows the example output from the initial iterations of the NSGA-II algorithm. Shown below, Figure 11 illustrates the NSGA-II being applied to multiple known test functions, as a frame of reference, to prove the algorithm can be applied in different context and to provide perspective. The NSGA-II was executed a number of different times and example data for a single iteration is shown below in Appendix A in Table 1.

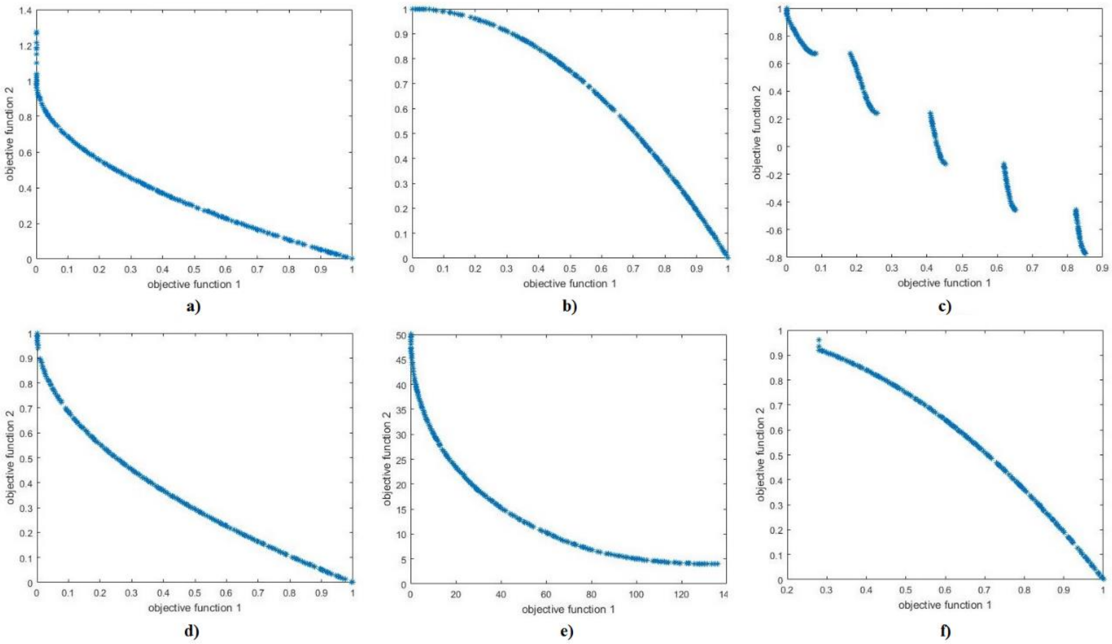


Figure 11 - NSGA-II Applied to Known Test functions:
a) ZDT1, b) ZDT2, c) ZDT3, d) ZDT4, e) ZDT6, and f) Binh & Korn

Figure 11 shows each test function run 1 time, while figure 12 shows the NSGA-II run over 1000 times to illustrate what increasing the number of runs from 1 to 1000 visually represents. While the overall curvature does not change, it is important to note that the number of points generated on the curve increased exponentially and that the data from a run of this scale would be substantial.

It can helpful to plot data using different bin sizes to visualize and understand the results or to use heuristics and visually inspect results. Figure 11 shows the implementation of the NSGA-II algorithm to 6 different test functions. This technique has been implemented in previous research to provide context and allows for descriptive analysis to be done visually.

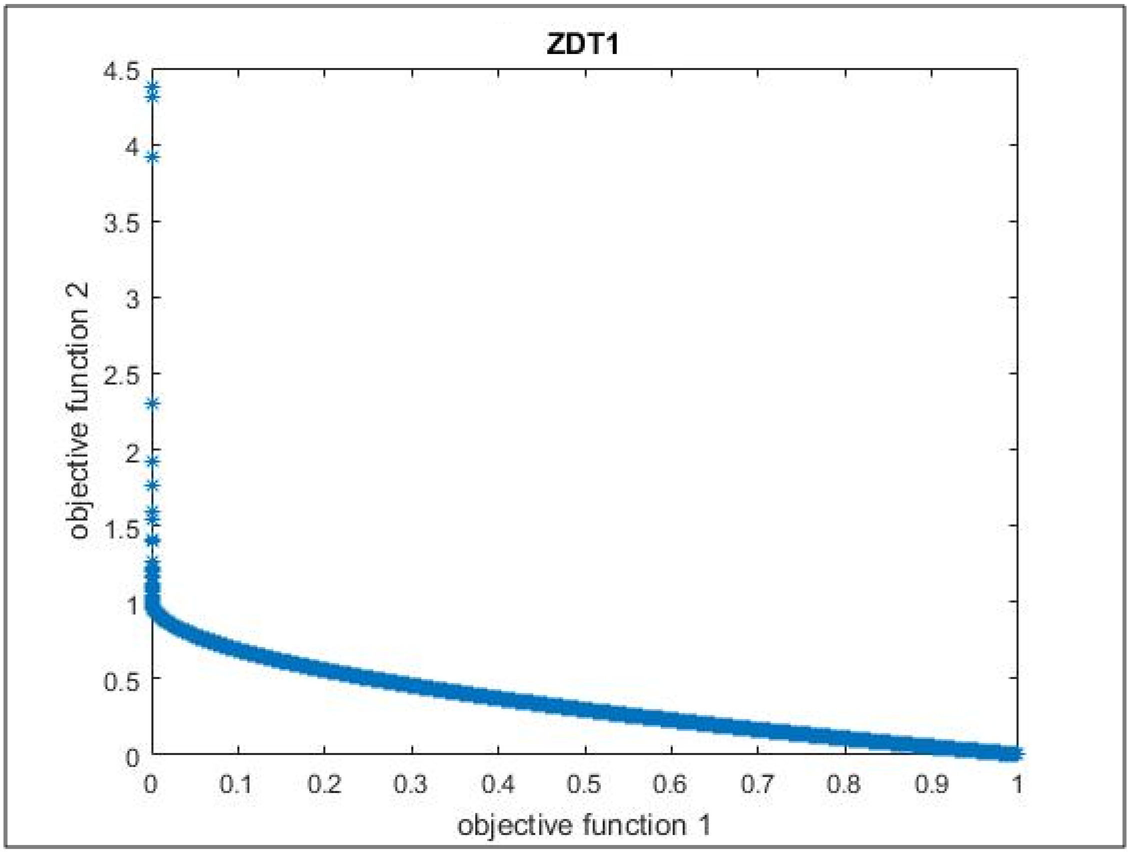


Figure 12- Graphical Representation of 1000 Runs of the ZDT1

Table 6, shown below in Appendix A, presents descriptive statistics about the data collected from our simulations in respect to the generated points above. These statistics, provided through the Data Analysis tool in Excel, are an effective way to gather important information on significant amounts of data, quickly, before moving forward on to further analysis, such as PROMETHEE or TOPSIS.

When applying PROMETHEE, with randomly generated weights, and employing the *usual criterion,* illustrated in Table 2 in Appendix A, the results of the ranking from Table 1 alternatives, from Figure 12, are observed. To effectively show how the rankings and graphs represent the data generated, a selection of 30 random points, from the generated 100 data points, will be selected for ranking and be known as "alternatives" moving forward. This method of randomly selecting 30 out of 100 points will allow for conclusions to be drawn. The alternatives presented are A1 though A30, obtained from our simulation of the NSGA-II. This notation will be used throughout the remainder of this thesis. This illustrates a few key concepts of PROMETHEE, primarily, the ranking of alternatives A1 through A30 and the Net Phi, which will be explained presently.

During the application of PROMETHEE, the use of a positive Phi (ϕ+) and negative Phi (ϕ-) are used to calculate the rank of the different alternatives, using a pairwise comparison. The difference between the positive Phi and the negative Phi result in the net Phi or net flow. The net Phi is then used to rank the alternatives (Ferretti, 2013). This technique is known as Multicriteria Preference Flows, where the positive preference flow (ϕ+) quantifies how a give action, a_i, is globally preferred *to all* other actions, while the negative preference flow (ϕ-) quantifies how a given action, a_i, is being globally preferred *by all* the other actions ("Promethee | Multiple criteria optimization", 2016). Figure 13, shown below, demonstrates the process flow of PROMETHEE, visually.

Table 3 presents the frequency of the equally likely weights. Table 4 displays the frequency of the randomly generated weights. It is important to note that there will be an adjustment in the ranking of alternatives. Some of this can be attributed to the implementation of the usual criterion, which employs a pairwise comparison during PROMETHEE.

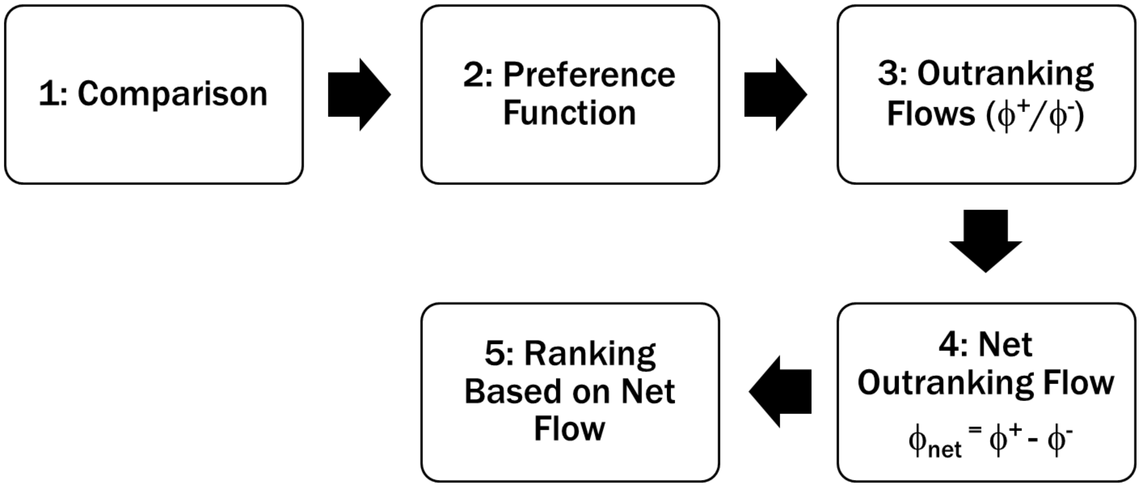


Figure 13 – Sample of a PROMETHEE Process Flow

Chapter 4: Results

The implementation of the combination of statistical techniques was successful: a genetic algorithm (NSGA-II) to generate a Pareto optimal front and the ranking of the alternatives from our simulation using PROMETHEE. Shown below is a Pareto frontier from the Python code, in Appendix A section A.2. This code employs several assumptions that are stated in the code, for simplicity and ease of application. The generated Pareto frontier, from the NSGA-II, shows a concave frontier, which is to be expected because of the nature of the problem and application of numerous, strict, constraints. Figure 14, shown below, represents a Pareto frontier with user inputs of: (i) population size = 100, (ii) number of generations = 500, and (iii) the inclusion of all Objective Functions. The 3d graph can be difficult to interpret, thus the 2 dimension figures are included below in Figure 15. The overall curvature represents the associate Pareto front without a superimposed arc.

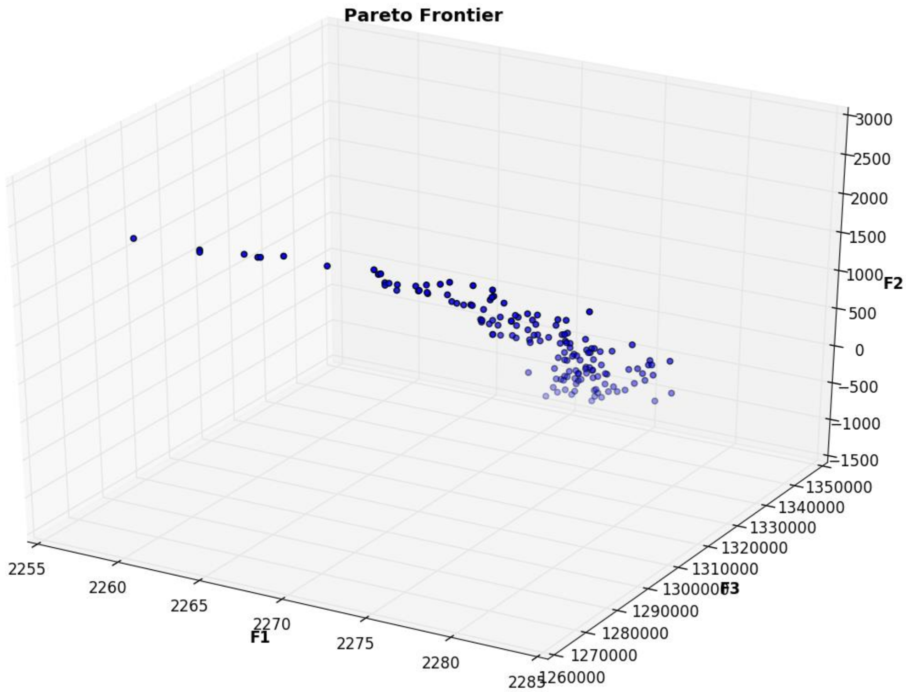


Figure 14 – Objective Function 1 vs Objective Function 3

Figure 15, shown below, illustrates the associated Pareto frontiers of the different Objective Functions and their associated plots from Cebi and Otay (2013). The differences in the curvatures and the overall scatter of the data-points are interesting and should be interpreted. The code was run for several instances and the resulting plots were all of similar construction. By presenting different Pareto fronts, it can be noted that the associated curves are different and that the with different Objective Functions included that the overall curves should be different, in nature.

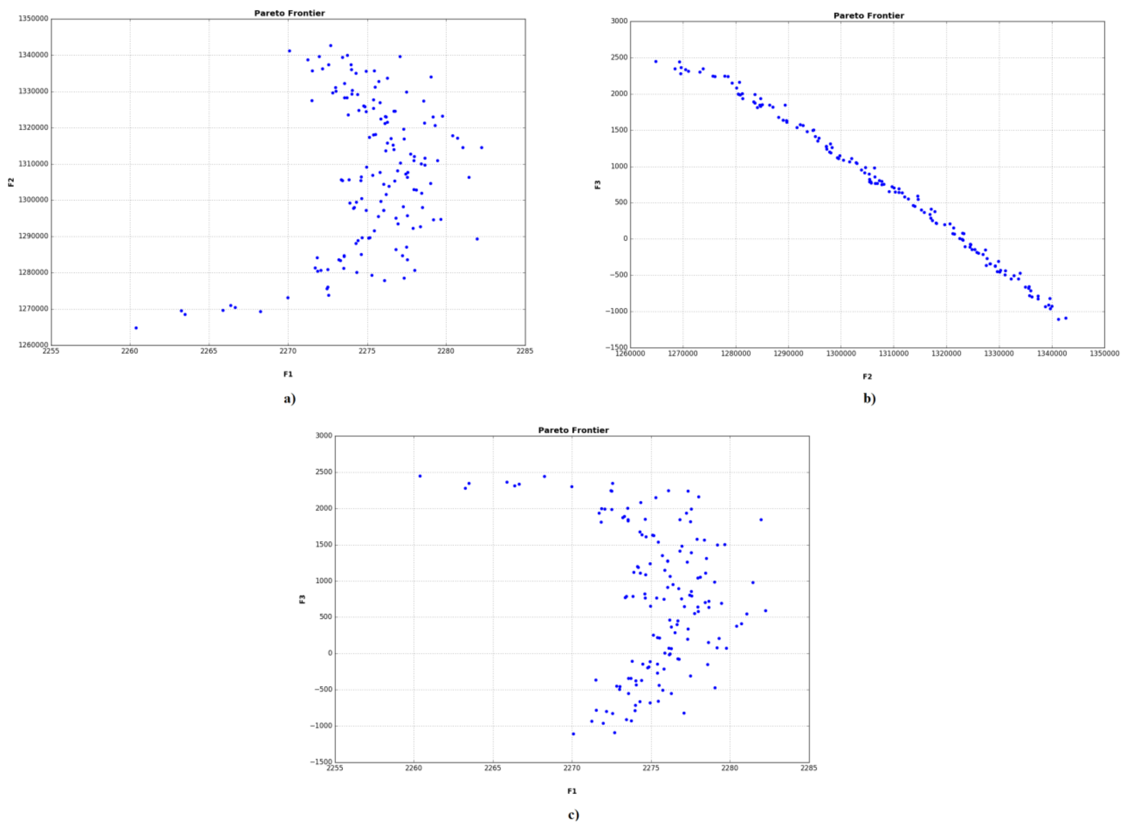


Figure 15 – Run of NSGA-II in Python – a) F1 vs F2, b) F2 vs F3, and c) F1 vs F3

For the above figures, the number of generations is equal to 500 and the number of populations is also equal to 100. Table 5, in Appendix A, represents rankings generated from the NSGA-II data associated with Figure 14 after the application of PROMETHEE, simulated 1000 times. This ranking will be important moving forward. Table 7 represents the order of alternatives by ranking. Table 8 shows the probabilities of the rankings shown in Table 7. Table 9 illustrates the average ranking per alternative, for context. Table 10 demonstrates the percent that each alternative was ranked at each ranking. Table 11 presents the data from the Microsoft Excel application to count the number of time that an alternative, A1-A30, was ranked at each ranking. Table 13 illustrates how inserting random weights, slightly, changes the ranking of alternatives. Table 15 – 18 show how changing the weights to known values changes the rankings of the alternatives. This may be important for the contractor to note because it could shift the attention from areas that might need more attention than others.

It is important to understand what the data represents once PROMETHEE has been employed to rank alternatives. Simply generating the ranks does not solve the risk minimization problem. The alternatives ranked lower (at 1) represent the lowest risk possible to the system proposed in Cebi and Otay (2013). Conversely, the highest ranked alternative presents the highest risk to the project network, or system. Figures 16-18 illustrate where the 30 alternatives selected for ranking fall along the Pareto frontier. Each figure represents two objective functions and their graphically shown values. This is important and will allow for some conclusions to be drawn about the rankings from PROMETHEE and the area on the Pareto frontier that the alternatives lie.

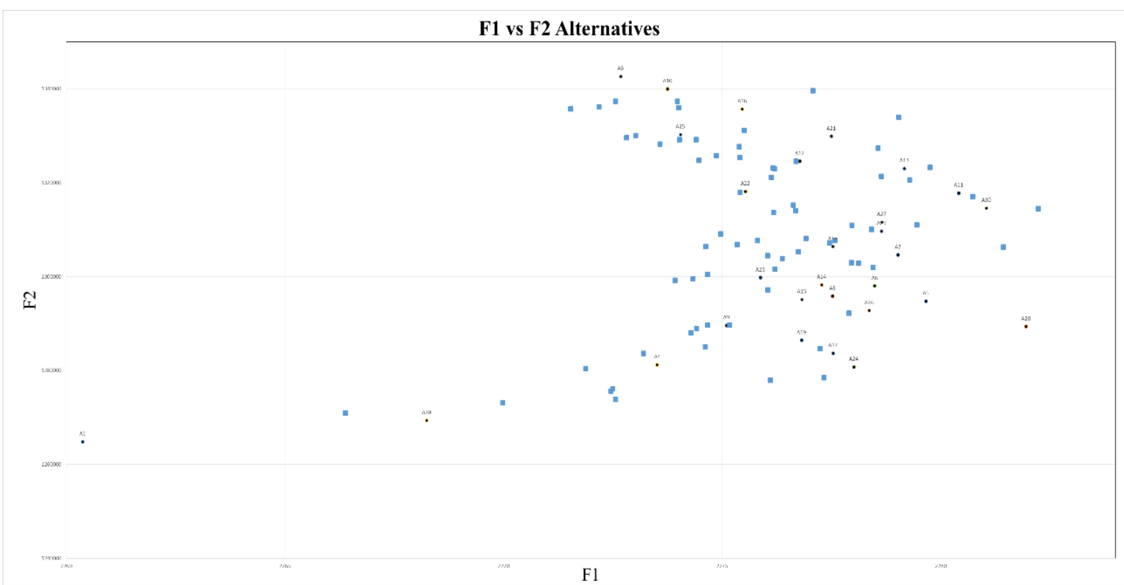


Figure 16 – F1 and F2 Values of 30 Alternatives Used in Ranking Superimposed Over 100 Other Data Points

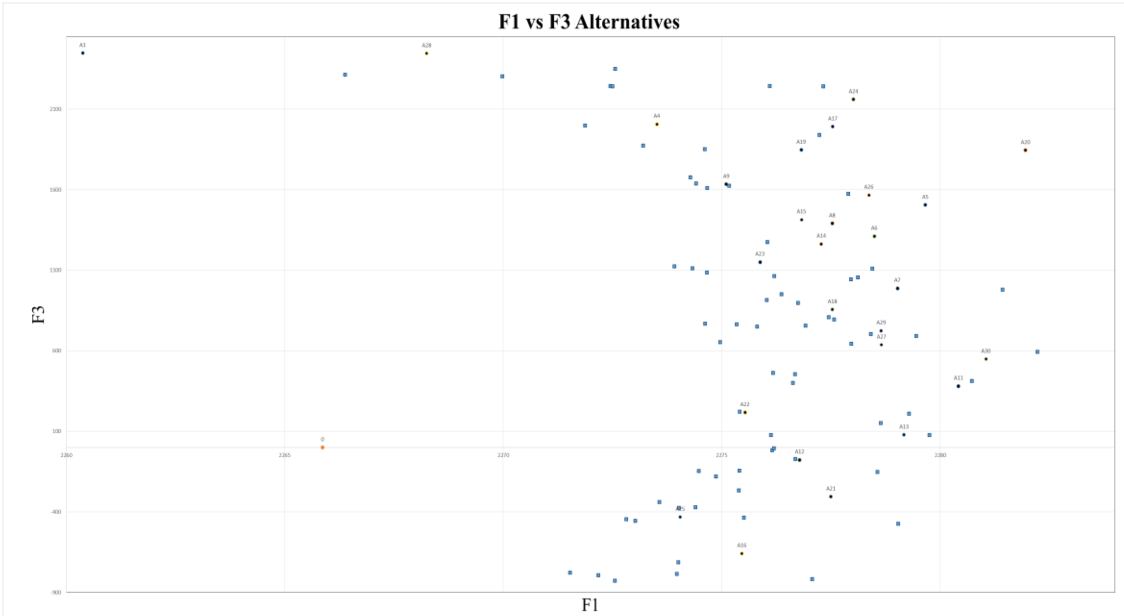


Figure 17 - F1 and F3 Values of 30 Alternatives Used in Ranking Superimposed Over 100 Other Data Points

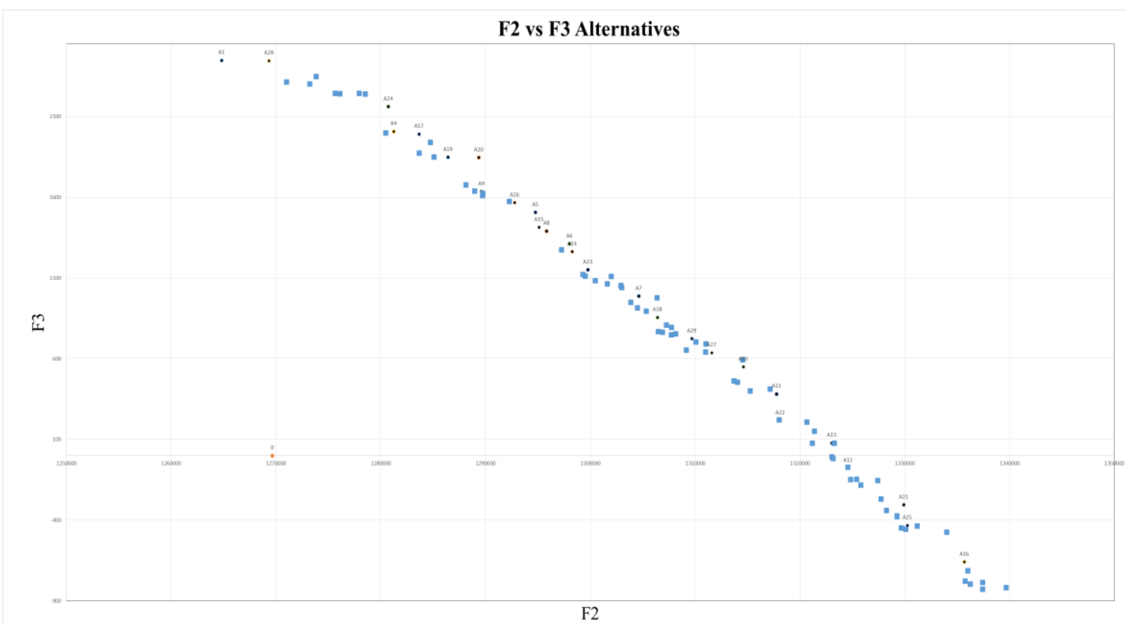


Figure 18 - F2 and F3 Values of 30 Alternatives Used in Ranking Superimposed Over 100 Other Data Points

Heat graphs are heuristics that can easily show the relationship between two factors. Generally, some examples of heat graphs include: segmentation of a target market, geographical heat maps, and many other instances where two factors can be compared. In this case, the factors are: (i) Ranking and (ii) Alternative.

Figure 19 is a heat graph that visually demonstrates the ranking of alternatives from PROMETHEE and their associated probability of the ranking per that alternative. The darker the color associated with an alternative and the respective ranking, the higher the probability that this ranking should be applied to that alternative. The key in the upper left hand of Figure 19 offers a quick and easy legend to illustrate some probabilities and the accompanying color. Shown in the upper left portion of the heat graph are the alternatives that offer the least risk to the system. Inversely, shown in the bottom right of the heat graph are the alternatives that offer the most risk to the overall system, per our objective functions and constraints from Cebi and Otay (2013).

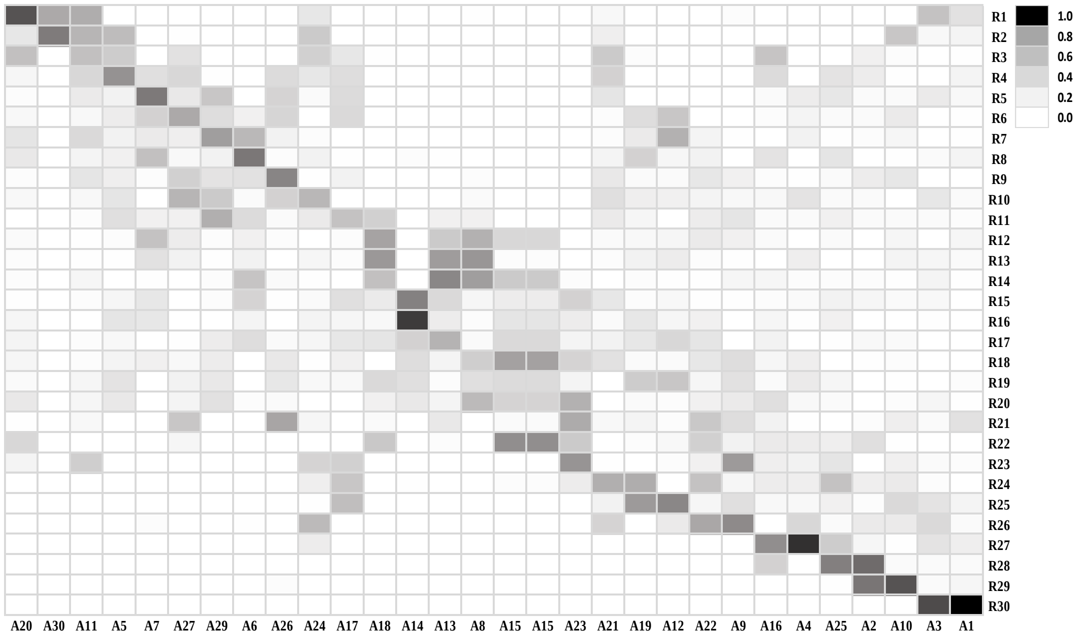


Figure 19 - Heat Map Showing the Probability of Each Ranking (Vertical Axis) of Each Alternative (Horizontal Axis) Under Randomly Assigned Weights for Each Percentile

Since the problem calls for a minimization of the objective functions, it should be noted that A20 was ranked 1st, offering the least amount of risk to the overall system and that it is ranked 1st for most iterations. Conversely, it can also be noted that alternative A1 is ranked 30th and offers the most amount of risk to the overall system. The nature of the heat graph can be explained by a few things: (i) the best ranked alternatives are associated with R1 and offer the least amount of risk while the highest amount of risk of those alternatives ranked the worst are ranked R30, (ii) with only 3 Objective Functions, it is expected for the Heat Graph to be variable like shown, and (iii) PROMETHEE ranks the alternatives from rank 1 to rank 30. There should be a linear relationship relative to the alternatives and risk coupled to them.

Chapter 5: Conclusions

Applying these tools in a project management risk setting is helpful because the team can immediately understand deficiencies and inadequacies of the proposed endeavor. Understanding project risk is important to addressing inadequacies in existing processes and mitigating any further reduction in efficiency. Additionally, companies, employees, and contractors deal with multiobjective problems frequently. There needs to be some way to elicit from a decision-maker, a solution, based on these multiple objectives and the desired outcomes associated with them. The application motivation is that multiobjective problems exist in project management and this general application can also be applied to project risk.

The proposed approach to manage risk and to rank alternatives to achieve the minimum risk possible is not new to the field of risk management but allows several questions to be answered. This approach assists the project manager of the proposed system to understand risk and the inherent dangers. The proposed problem, being primarily related to project duration and project cost allow for analysis to be done on the final rankings. Objective 1 (f_1) aimed to minimize completion time of the project, while Objective 2 (f_2) expected to minimize the total project cost. Objective 3 (f_3) minimized the detection time of an event needing special attention.

Once the rankings were gathered. Figures 16, 17 and 18 were generated to determine where on the associated graphs the ranked alternatives fell. There are a few conclusions that can be drawn from this application of the PROMETHEE rankings in comparison to the Pareto frontier: (i) higher values of f_1 produced better ranked alternatives, (ii) lower values of f_1 displayed lower ranked alternatives, (iii) moderate f_2

and f_3 values are preferred to extreme values, illustrated in the rankings where the most moderate f_2 and f_3 values were the best ranked alternatives, and (iv) the best ranked alternatives, A20, A30, and A11 were all located in the "middle" of the curvature of the Pareto front with the highest values of f_1.

How does this approach help with understanding project management? It combines project risk management concepts with the theories of Pareto optimality. Many different studies have been done to address how to achieve a Pareto optimal curve or how to use the curve in certain instances. This approach provides a way to determine *where on the Pareto optimal curve does the project manager want to focus to minimize the most risk.* This relationship between the plotted alternatives and the rankings could significantly alter decisions to mitigate project risk. With the application of PROMETHEE to rank alternatives and by graphing the generated alternatives from the NSGA-II to provide additional data to the project manager or contractor, the overall understanding of risk mitigation has increased. These conclusions empower the decision-maker by providing a ranking of strategies to mitigate the most risk and offer the decision-maker a method to approach additional general risk management scenarios. Without knowing what alternatives have the most impact upon a system, how can the system be effectively managed or better yet, improved?

Minimizing risk isn't a new concept but addressing *where* along the curve is *best* to achieve risk minimization is a newer topic of discussion. By empowering the project manager to see where the most and least risks lie (from the ranking and the heat graph), better, and possibly more initially beneficial, decisions can be made. This is beneficial to the contractor involved in the applied problem from Cebi and Otay (2013) and can also benefit the company who is employing said contactor because they now have: (i) a more informed employee, (ii) a possibly more efficient contractor, (iii) information for future improvements, and (iv) risk mitigation of their process.

Furthermore, the use of randomly generated weights, in the application of PROMETHEE, addresses another concern. When choosing a weighting schema, what weights would the project manager when deciding between subsets of alternatives? The idea that adding additional randomization to PROMETHEE helps address this topic because: (i) with added randomization incorporated, risk mitigation inherently improves (ii) by taking the decision from the contractor to choose a weighting schema, the company removes the possibility of any bias's skewing results, and (iii) the inherent harshness of the usual criterion can be mitigated to a minor degree.

5.1 Future Work

The application of statistical methods to the area of project risk management will always be in demand. Limiting risk and minimizing cost, while maximizing profitability will only increase in the future. The applied methodology aimed to minimize project completion time, project cost, and detection time of an event needing special attention. A limitation of this specific application is primarily due to the simple nature of the employed multiobjective optimization problem.

Future work would include: selecting a more complex multiobjective optimization problem, applying the NSGA-II algorithm (or any number of GAs) to generate a Pareto optimal front, performing more analysis on how random weights affect alternative rankings in comparison to equal weighting or specifying weights, ranking the alternatives using a statistical tool (PROMETHEE or another preferred method) to additional project risk scenarios, and performing more analysis in the area of comparing generated rankings to their alternatives and how that location along a Pareto front is important to decision-makers.

References

Alireza, V., Mohammadreza, Y., Zin, R. M., Yahaya, N., & Noor, N. M. (2013). An enhanced multi-objective optimization approach for risk allocation in public–private partnership projects: a case study of Malaysia. *Canadian Journal of Civil Engineering, 41*(2), 164-177. doi:10.1139/cjce-2013-0342

Azaron, A., Perkgoz, C., & Sakawa, M. (2005). A genetic algorithm approach for the time-cost trade-off in PERT networks. *Applied Mathematics and Computation, 168*(2), 1317-1339. doi:http://dx.doi.org/10.1016/j.amc.2004.10.021

Balali, V. , Zahraie, B. , & Roozbahani, A. (2014). A Comparison of AHP and PROMETHEE Family Decision Making Methods for Selection of Building Structural System. American Journal of Civil Engineering and Architecture, 2(5), 149-159.

Bandyopadhyay, S., & Bhattacharya, R. (2014). Solving a tri-objective supply chain problem with modified NSGA-II algorithm. *Journal of Manufacturing Systems, 33*(1), 41-50. doi:http://dx.doi.org/10.1016/j.jmsy.2013.12.001

Behzadian, M., Kazemzadeh, R. B., Albadvi, A., & Aghdasi, M. (2010). PROMETHEE: A comprehensive literature review on methodologies and applications. *European Journal of Operational Research, 200*(1), 198-215. doi:http://dx.doi.org/10.1016/j.ejor.2009.01.021

BETTER, M., GLOVER, F., KOCHENBERGER, G., & WANG, H. (2008). SIMULATION OPTIMIZATION: APPLICATIONS IN RISK MANAGEMENT. *International Journal of Information Technology & Decision Making, 07*(04), 571-587. doi:doi:10.1142/S0219622008003137

Bonnal, P. (2012). Project Management. *Construction Management & Economics, 30*(3), 248-250. doi:10.1080/01446193.2012.655253

Brans, J., & Mareschal, B. (1992). PROMETHEE V: MCDM problems with segmentation constraints. INFOR, 30(2), 85. Retrieved from http://search.proquest.com/docview/228469041?accountid=12964

Brans, J. P., Mareschal, B., and Vincke, P. 1984. PROMETHEE: A New Family of Outranking Methods in Multi-criteria Analysis. Operational Research, J.P Brans (ed.). North Holland: Amsterdam.

Brans, J., & Vincke, P. (1985). A Preference Ranking Organisation Method (The PROMETHEE Method for Multiple Criteria Decision-Making). *Management Science, 31*(6), 647.

Burnett, R. (2007). Project management. *ITNOW, 49*(1), 34. doi:10.1093/combul/bwl107

Cervone, H. F. (2006). Project risk management. *OCLC Systems & Services: International digital library perspectives, 22*(4), 256-262. doi:doi:10.1108/10650750610706970

Chen, M., Guo, Y., Liu, H., & Wang, C. (2015). The Evolutionary Algorithm to Find Robust Pareto-Optimal Solutions over Time. *Mathematical Problems in Engineering, 2015,* 18. doi:10.1155/2015/814210

Corrente, S., Figueira, J. R., & Greco, S. (2014). The SMAA-PROMETHEE method. *European Journal of Operational Research, 239*(2), 514-522. doi:http://dx.doi.org/10.1016/j.ejor.2014.05.026

Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *Trans. Evol. Comp, 6*(2), 182-197. doi:10.1109/4235.996017

Ehrgott, M. *Multicriteria Optimization*, Springer, New York, 2005. (accessed January 6, 2016)

Ferhan Çebi, İrem Otay, A fuzzy multi-objective model for solving project network problem with bonus and incremental penalty cost, Computers & Industrial Engineering, Volume 82, April 2015, Pages 143-150, ISSN 0360-8352, http://dx.doi.org/10.1016/j.cie.2015.01.007.

Ferretti, J. (2013). *Outranking Methods - PROMETHEE | LIAISE KIT*. *Liaise-kit.eu*. Retrieved 14 January 2016, from http://www.liaise-kit.eu/content/outranking-methods-promethee

Fettaka, S., Thibault, J., & Gupta, Y. (2014). A new algorithm using front prediction and NSGA-II for solving two and three-objective optimization problems. *Optimization and Engineering, 16*(4), 713-736. doi:10.1007/s11081-014-9271-9

Floyd, M. (2015). Multicriteria Decision Analysis Technique for Stochastic Ranking: Application to Project Network Risk. Unpublished Master's Thesis, the University of Oklahoma, Norman, Oklahoma.

Franca, R. B., Jones, E. C., Richards, C. N., & Carlson, J. P. (2010). Multi-objective stochastic supply chain modeling to evaluate tradeoffs between profit and quality. *International Journal of Production Economics, 127*(2), 292-299. doi:http://dx.doi.org/10.1016/j.ijpe.2009.09.005

Gabriel, S. A., Kumar, S., Ordóñez, J., & Nasserian, A. (2006). A multiobjective optimization model for project selection with probabilistic considerations. *Socio-Economic Planning Sciences, 40*(4), 297-313. doi:http://dx.doi.org/10.1016/j.seps.2005.02.002

Greenberg, R. R., & Nunamaker, T. R. (1994). Integrating the analytic hierarchy process (AHP) into the multiobjective budgeting models of public sector organizations. *Socio-Economic Planning Sciences, 28*(3), 197-206. doi:http://dx.doi.org/10.1016/0038-0121(94)90005-1

Han, Y.-Y., Gong, D.-w., Sun, X.-Y., & Pan, Q.-K. (2014). An improved NSGA-II algorithm for multi-objective lot-streaming flow shop scheduling problem. *International Journal of Production Research, 52*(8), 2211-2231. doi:10.1080/00207543.2013.848492

Hartono, B., Sulistyo, S. R., Praftiwi, P. P., & Hasmoro, D. (2014). Project risk: Theoretical concepts and stakeholders' perspectives. *International Journal of Project Management, 32*(3), 400-411. doi:http://dx.doi.org/10.1016/j.ijproman.2013.05.011

Holland & Holland Enterprises, Ltd. (2016). *Project Risk Management*. *Successful-project-management.com*. Retrieved 2 January 2016, from http://www.successful-project-management.com/project-risk-management.html

Hu, Y.-C. (2013). Rough sets for pattern classification using pairwise-comparison-based tables. *Applied Mathematical Modelling, 37*(12–13), 7330-7337. doi:http://dx.doi.org/10.1016/j.apm.2013.03.007

Huchzermeier, A., & Loch, C. H. (2001). Project Management Under Risk: Using the Real Options Approach to Evaluate Flexibility in R…D. *Management Science, 47*(1), 85-101. doi:doi:10.1287/mnsc.47.1.85.10661

Introduction to Genetic Algorithms. (2016). *Doc.ic.ac.uk*. Retrieved 7 January 2016, from http://www.doc.ic.ac.uk/~nd/surprise_96/journal/vol1/hmw/article1.html

Kapur, K. C. (2010). Risk Modeling, Assessment, and Management, 3rd edition. *Journal of Quality Technology, 42*(2), 227-228. Retrieved from http://search.proquest.com/docview/214491816?accountid=12964

Kou, G., Lu, Y., Peng, Y. I., & Shi, Y. (2012). EVALUATION OF CLASSIFICATION ALGORITHMS USING MCDM AND RANK CORRELATION. *International Journal of Information Technology & Decision Making, 11*(1), 197-225. Retrieved from http://libraries.ou.edu/access.aspx?url=http://search.ebscohost.com/login.aspx?direct=true&db=buh&AN=71669839&site=ehost-live

Lakshminarasimman, N., Baskar, S., Alphones, A., & Willjuice Iruthayarajan, M. (2010). Evolutionary multiobjective optimization of cellular base station locations using modified NSGA-II. *Wireless Networks, 17*(3), 597-609. doi:10.1007/s11276-010-0299-2

Li, J., Sun, X., Wang, F., & Wu, D. (2014). Risk integration and optimization of oil-importing maritime system: a multi-objective programming approach. *Annals of*

Operations Research, 234(1), 57-76. doi:10.1007/s10479-014-1550-5

Li, K., & Zhang, X. (2012). Using NSGA-II and TOPSIS Methods for Interior Ballistic Optimization Based on One-Dimensional Two-Phase Flow Model. *Propellants, Explosives, Pyrotechnics, 37*(4), 468-475. doi:10.1002/prep.201000113

Macharis, C., Springael J., De Brucker, K., Verbeke, A., 2004: PROMETHEE and AHP: The design of operational synergies in multicriteria analysis. Strengthening PROMETHEE with ideas of AHP. European Journal of Operational Research 153: 307−317.

Marle, F., & Gidel, T. (2012). A multi-criteria decision-making process for project risk management method selection. *International Journal of Multicriteria Decision Making, 2*(2), 189-223. doi:doi:10.1504/IJMCDM.2012.046948

Marques, G., Gourc, D., & Lauras, M. (2011). Multi-criteria performance analysis for decision making in project management. *International Journal of Project Management, 29*(8), 1057-1069. doi:http://dx.doi.org/10.1016/j.ijproman.2010.10.002

Massam, B. H. (1988). Multi-Criteria Decision Making (MCDM) techniques in planning. *Progress in Planning, 30, Part 1*, 1-84. doi:http://dx.doi.org/10.1016/0305-9006(88)90012-8

Merriam-Webster Online, s.v. “risk,” http://www.merriam-webster.com/dictionary/Risk (accessed January 2, 2016).

Mee, J. F. (1963). Project management. *Business Horizons, 6*(3), 53-54. doi:http://dx.doi.org/10.1016/0007-6813(63)90049-1

Miner, G. (2004). Risk Modeling, Assessment, and Management. *American Water Works Association. Journal, 96*(9), 120-121. Retrieved from http://search.proquest.com/docview/221570001?accountid=12964

Moitra, A., & O'Donnell, R. (2012). Pareto Optimal Solutions for Smoothed Analysts. *SIAM Journal on Computing, 41*(5), 1266-1284. doi:doi:10.1137/110851833

Murugan, P., Kannan, S., & Baskar, S. (2009). NSGA-II algorithm for multi-objective generation expansion planning problem. *Electric Power Systems Research, 79*(4), 622-628. doi:http://dx.doi.org/10.1016/j.epsr.2008.09.011

Pekkinen, L. and Aaltonen, K. (2015) Risk Management in Project Networks: An Information Processing View. Technology and Investment, 6, 52-62. doi: 10.4236/ti.2015.61005.

Peng, Y., Kou, G., Wang, G., & Shi, Y. (2011). FAMCDM: A fusion approach of MCDM methods to rank multiclass classification algorithms. *Omega, 39*(6), 677-689.

doi:http://dx.doi.org/10.1016/j.omega.2011.01.009

Peng, Y., Zhang, Y., Kou, G., & Shi, Y. (2012). A Multicriteria Decision Making Approach for Estimating the Number of Clusters in a Data Set. *PLoS ONE, 7*(7), e41713. doi:10.1371/journal.pone.0041713

Pohekar, S. D., & Ramachandran, M. (2004). Application of multi-criteria decision making to sustainable energy planning—A review. *Renewable and Sustainable Energy Reviews, 8*(4), 365-381. doi:http://dx.doi.org/10.1016/j.rser.2003.12.007

Pourjavad, E., & Shirouyehzad, H. (2011). A MCDM Approach for Prioritizing Production Lines: A Case Study. *International Journal of Business and Management, 6*(10), 221-229. Retrieved from http://search.proquest.com/docview/899223047?accountid=12964

Project Management Institute, Inc., A Guide to the Project Management Body of Knowledge (PMBOK Guide), 4th ed. (Newtown Square, PA: Project Management Institute, Inc., 2008), 273.

Promethee | Multiple criteria optimization. (2016). Brasil.cel.agh.edu.pl. Retrieved 15 February 2016, from http://brasil.cel.agh.edu.pl/~13sustrojny/en/promethee/

Raz, T., Shenhar, A. J., & Dvir, D. (2002). Risk management, project success, and technological uncertainty. *R & D management, 32*(2), 101-110.

Rolstadås, A., Pinto, J. K., Falster, P., & Venkataraman, R. (2015). Project Decision Chain. *Project Management Journal, 46*(4), 6-19. doi:10.1002/pmj.21517

San Cristobal, J. (2012). Critical Path Definition Using Multicriteria Decision Making: PROMETHEE Method. *Journal of Management in Engineering, 29*(2), 158-163. doi:10.1061/(ASCE)ME.1943-5479.0000135

Sánchez-Lozano, J. M., Teruel-Solano, J., Soto-Elvira, P. L., & Socorro García-Cascales, M. (2013). Geographical Information Systems (GIS) and Multi-Criteria Decision Making (MCDM) methods for the evaluation of solar farms locations: Case study in south-eastern Spain. *Renewable and Sustainable Energy Reviews, 24*, 544-556. doi:http://dx.doi.org/10.1016/j.rser.2013.03.019

Sarkar, D., & Modak, J. M. (2005). Pareto-optimal solutions for multi-objective optimization of fed-batch bioreactors using nondominated sorting genetic algorithm. *Chemical Engineering Science, 60*(2), 481-492. doi:http://dx.doi.org/10.1016/j.ces.2004.07.130

Seshadri, A. (2016). *A Fast Elitist Multiobjective Genetic Algorithm: NSGA-II* (1st ed.). New Jersey: New Jersey Institute of Technology. Retrieved from https://web.njit.edu/~horacio/Math451H/download/Seshadri_NSGA-II.pdf

Steponavičė, I. (2016). *Algorithms for Uniform Distribution of Solutions Over Pareto Set, and Their Applications in Risk Management* (Phd). Vytauta Magnus University.

Tamošaitienė, J., Zavadskas, E. K., & Turskis, Z. (2013). Multi-criteria Risk Assessment of a Construction Project. *Procedia Computer Science, 17*, 129-133. doi:http://dx.doi.org/10.1016/j.procs.2013.05.018

Tervonen, T and Lahdelma, R.. (2007). Implementing stochastic multicriteria acceptability analysis. European Journal of Operational Research, 178(2).

Tomoiagă, B., Chindriş, M., Sumper, A., Sudria-Andreu, A., & Villafafila-Robles, R. (2013). Pareto Optimal Reconfiguration of Power Distribution Systems Using a Genetic Algorithm Based on NSGA-II. *Energies*, *6*(3), 1439. Retrieved from http://www.mdpi.com/1996-1073/6/3/1439

Tsai, S. C., & Fu, S. Y. (2014). Genetic-algorithm-based simulation optimization considering a single stochastic constraint. *European Journal of Operational Research, 236*(1), 113-125. doi:http://dx.doi.org/10.1016/j.ejor.2013.11.034

Wang, L., Wang, T.-g., & Luo, Y. (2011). Improved non-dominated sorting genetic algorithm (NSGA)-II in multi-objective optimization studies of wind turbine blades. *Applied Mathematics and Mechanics, 32*(6), 739-748. doi:10.1007/s10483-011-1453-x

Wiley, et al. (2016). *11.1 Defining Risk | Project Management for Instructional Designers | Page 2*. *Pm4id.org*. Retrieved 02 Janurary 2016, from http://pm4id.org/chapter/11-1-defining-risk/2/

Xu, Z., & Lu, S. (2011). Multi-objective optimization of sensor array using genetic algorithm. *Sensors and Actuators B: Chemical, 160*(1), 278-286. doi:http://dx.doi.org/10.1016/j.snb.2011.07.048

Yu, X., Xu, Z., & Ma, Y. (2013). Prioritized Multi-Criteria Decision Making based on the Idea of PROMETHEE. *Procedia Computer Science, 17*, 449-456. doi:http://dx.doi.org/10.1016/j.procs.2013.05.058

Zhang, C., & Ma, X. (2015). NSGA-II Algorithm with a Local Search Strategy for Multiobjective Optimal Design of Dry-Type Air-Core Reactor. *Mathematical Problems in Engineering, 2015*, 9. doi:10.1155/2015/839035

Zhang, Y., & Fan, Z.-p. (2014). An optimization method for selecting project risk response strategies. *International journal of project management : the journal of The International Project Management Association, 32*(3, (4)), 412-422.

Zitzler, E. and Thiele, L. (1999). Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach. IEEE Transactions on

Evolutionary Computation, 3(4):257–271

Appendix A:

A.1 Tables

Table 1 - Example Output from NSGA (Truncated)

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
1	-39.38390	75.36695	66.55935	73.63086	68.97343	80.31033	72.99608	65.48189	83.98210	76.02914
2	-37.34166	76.81083	67.16131	72.52930	68.50953	79.03486	72.50131	66.09669	84.51501	76.47786
3	-37.97220	76.07027	67.96264	72.50527	68.47686	78.90010	73.48723	65.99798	84.24377	77.08752
4	-35.95655	79.41026	67.59699	74.09652	68.31789	80.22324	74.36884	65.22704	85.97398	77.26453
5	-36.37936	77.11882	66.83138	74.92473	68.22865	78.71249	73.37642	66.14040	84.12275	77.13961
6	-37.66485	77.29906	67.22875	73.92100	68.39910	79.20109	72.46494	67.53392	83.87198	78.20558
7	-37.38085	76.20384	67.71282	73.37270	68.30055	80.78342	72.97963	65.97350	84.27271	76.33077
8	-38.06335	76.70934	66.72946	73.59654	68.66645	80.06933	73.22054	65.59345	83.65767	76.70234
9	-37.89167	77.32704	68.27283	75.77351	68.59234	80.33553	71.93045	65.95271	84.75434	75.68809
10	-37.20195	76.40568	67.20872	72.91075	68.40379	81.41322	72.85853	65.92578	84.61155	76.20777
11	-38.33204	75.74311	66.61150	73.54148	67.26801	78.97310	72.69179	65.84302	84.29641	76.19261
12	-38.01885	76.48477	68.28264	73.30211	68.06150	82.25530	72.84693	65.83643	84.52074	73.30259
13	-36.11927	76.37079	66.96191	73.96169	70.36997	80.47458	73.17987	66.27680	83.97346	78.32214
14	-38.02873	76.22452	67.12898	74.04327	69.94386	80.07908	72.97381	65.80104	84.83444	73.82761
15	-36.08777	76.40884	67.35851	75.95517	69.09606	82.09092	73.72683	66.45516	84.71303	76.07983
16	-37.75826	77.36542	67.08578	73.89645	68.02414	78.55059	73.29990	66.23879	83.96436	76.75693
17	-37.36550	75.91920	66.94683	75.10702	69.62368	79.18013	74.06980	66.75017	83.71739	76.78316
18	-37.22009	74.86493	67.66396	72.76298	68.21548	78.40911	74.40604	66.08754	85.01922	76.56561
19	-37.17076	75.48176	67.55103	72.76493	68.43930	78.93263	73.73863	65.92120	84.67568	76.49885
20	-35.70108	75.64568	67.79090	74.32144	69.94051	80.40547	73.18966	66.33790	84.40412	76.87619
21	-35.83056	78.73178	67.88078	74.39920	69.18304	79.97639	74.25404	65.25787	84.85980	77.21422
22	-36.74847	75.78142	66.37475	72.66108	69.11107	82.13766	73.49575	66.27066	84.43248	76.85383
23	-37.57701	76.87487	67.28931	73.93181	68.94934	79.12051	73.31881	66.23500	83.95754	76.90617
24	-36.39118	75.36272	66.86214	73.40514	70.54034	80.23190	73.17224	64.63238	83.96291	77.93392
25	-36.12237	76.10275	67.29761	74.85954	69.64593	81.32650	73.76471	66.43534	85.62740	75.74325
26	-36.31053	76.72261	66.92974	74.27455	68.69907	79.01375	76.22999	66.81336	83.84438	77.10223
27	-36.61979	75.86317	66.81578	74.23614	68.50539	80.01965	73.71889	66.20496	84.63415	77.09857
28	-37.17836	76.07003	67.35713	75.06924	68.39858	79.45404	73.13542	66.32139	84.15105	75.77026
29	-36.98673	75.84850	66.83507	73.62057	70.15749	79.86411	72.40448	65.39359	83.66317	76.25599
30	-36.27806	75.62027	67.11561	74.47477	69.15870	79.39865	74.01164	66.75329	85.08941	77.18476
31	-36.96977	76.68819	67.08589	73.96867	68.87115	79.16493	72.60445	66.94478	85.09539	77.09974
32	-36.09244	75.78888	65.85835	74.44118	69.93955	79.79829	73.12951	66.65592	83.95745	78.11308
33	-37.37324	76.25546	67.95406	73.86160	68.28653	82.30733	72.85721	66.23072	84.24999	75.37503
34	-37.75144	76.17078	68.23078	74.52308	68.31326	80.85423	72.86084	65.64172	84.17536	75.48597
35	-37.04893	75.50431	66.83748	73.53828	70.42436	79.95190	72.81421	65.39024	83.59989	77.05987
36	-37.92737	75.19066	67.12920	73.44998	68.31371	80.58467	72.58904	65.21068	84.95704	77.27406
37	-37.26812	77.43142	67.10159	74.00401	68.66747	78.89545	72.97205	67.35651	87.03488	76.39040
38	-37.13868	75.95006	67.12849	74.32651	70.14787	79.38164	72.94044	66.73418	84.19486	77.09469
39	-36.28174	75.88099	67.08118	74.28847	69.43581	79.92597	73.95909	65.25731	84.05986	77.63508
40	-37.68741	75.84449	66.75253	74.42043	70.39911	79.26178	73.02873	65.75069	84.43176	75.75145
41	-36.27734	76.58960	66.23323	73.27326	70.70647	79.29488	73.50314	65.68424	85.12621	77.19881
42	-37.06076	75.23572	66.52843	73.60043	68.56651	78.20607	72.92472	66.84429	84.06240	77.05007
43	-35.52914	75.78492	67.79923	74.15987	69.29405	80.88183	73.16358	66.54000	84.12374	76.60558
44	-36.37058	76.98518	67.79400	73.32098	68.44776	79.60518	72.83754	67.25899	85.30784	77.16209
45	-36.09490	75.84953	66.49488	73.73182	70.53091	80.48263	72.85748	68.39561	83.86880	77.38101
46	-35.30097	76.14416	67.75704	73.33574	69.29695	80.98678	72.77013	65.57692	84.66358	77.40458
47	-37.70806	76.34651	68.03805	73.76262	68.06296	81.74672	72.81464	66.40697	84.53838	74.32925
48	-36.22567	76.66108	66.86527	74.25049	68.89589	80.21006	72.46706	66.46083	84.60752	77.11227
49	-35.16853	76.69586	67.00266	74.03733	69.19013	80.21345	72.45759	67.21202	84.69891	77.81932
50	-37.39027	75.34483	66.50350	72.93628	70.68140	78.20446	72.49641	65.75798	85.44825	77.13467

Table 2- Examples of Preference Function

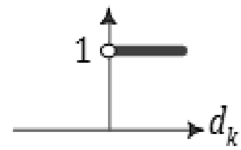
Table 2 – Examples of Preference Function, adapted from (Brans et al. 1984).		
Difference comparison	Preference function	Functional shape
Usual criterion	$f_k(a,b) = \begin{cases} 0 & \text{if} d_k = 0 \\ 1 & \text{if } d_k > 0 \end{cases}$	

Table 3 - Frequency of Weights in 10,000 Samples

Equally Likely Weights		
Number	**Percentiles**	**Frequency**
1	0.03333	333
2	0.06667	300
3	0.10000	317
4	0.13333	344
5	0.16667	318
6	0.20000	344
7	0.23333	296
8	0.26667	308
9	0.30000	336
10	0.33333	327
11	0.36667	313
12	0.40000	303
13	0.43333	351
14	0.46667	341
15	0.50000	344
16	0.53333	371
17	0.56667	337
18	0.60000	329
19	0.63333	341
20	0.66667	319
21	0.70000	356
22	0.73333	320
23	0.76667	332
24	0.80000	358
25	0.83333	323
26	0.86667	337
27	0.90000	320
28	0.93333	340
29	0.96667	367
30	1.00000	340

Table 4 - Frequency of Random Weights in 10,000 Samples

	Random Weights	
Number	Percentiles	Frequency
1	0.09091	851
2	0.14313	519
3	0.16314	199
4	0.18741	257
5	0.32926	1330
6	0.33292	44
7	0.34237	70
8	0.36325	207
9	0.36657	34
10	0.40585	354
11	0.42234	181
12	0.46412	427
13	0.46727	32
14	0.47757	118
15	0.49103	129
16	0.50235	114
17	0.58628	867
18	0.63671	512
19	0.67659	382
20	0.68662	93
21	0.69334	69
22	0.71914	275
23	0.73652	158
24	0.77424	385
25	0.81220	389
26	0.84872	367
27	0.86919	198
28	0.91970	492
29	0.96621	503
30	1.00000	341

Table 5 – Ranking Shown for Each Alternative

Ranking From PROMETHEE	
Alternative	**Ranking**
A1	30
A2	26
A3	29
A4	24
A5	4
A6	8
A7	5
A8	15
A9	22
A10	28
A11	3
A12	20
A13	14
A14	13
A15	16
A16	23
A17	11
A18	12
A19	19
A20	1
A21	18
A22	21
A23	17
A24	10
A25	25
A26	9
A27	6
A28	27
A29	7
A30	2

Table 6 - Descriptive Statistics for ZDT1 Test Function

Descriptive Statistics About S1 - S32

S1		S2		S3		S4		S5		S6		S7		S8	
Mean	0.474129365	Mean	0.000717108	Mean	0.001577249	Mean	0.001131481	Mean	0.001350882	Mean	0.001571252	Mean	0.001272751	Mean	0.00112231
Standard Error	0.009700929	Standard Error	0.000102632	Standard Error	0.000730112	Standard Error	0.000305101	Standard Error	0.000466436	Standard Error	0.00044544	Standard Error	0.00043246	Standard Error	0.000546221
Median	0.4809	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003
Mode	0	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001
Standard Deviation	0.326677974	Standard Deviation	0.003456119	Standard Deviation	0.024586454	Standard Deviation	0.010274254	Standard Deviation	0.015707177	Standard Deviation	0.015000144	Standard Deviation	0.014563063	Standard Deviation	0.018393955
Sample Variance	0.106718499	Sample Variance	1.19448E-05	Sample Variance	0.000604494	Sample Variance	0.00010556	Sample Variance	0.000246715	Sample Variance	0.000225004	Sample Variance	0.000212083	Sample Variance	0.000338338
Kurtosis	-1.39320114	Kurtosis	474.8241484	Kurtosis	856.3684314	Kurtosis	565.6778433	Kurtosis	376.207022	Kurtosis	278.6697041	Kurtosis	818.2325912	Kurtosis	1116.556595
Skewness	-0.001234629	Skewness	21.06390929	Skewness	28.24352959	Skewness	22.20352242	Skewness	19.21252228	Skewness	16.3412949	Skewness	27.59650917	Skewness	33.29820935
Range	1	Range	0.0803	Range	0.7702	Range	0.2871	Range	0.3299	Range	0.2816	Range	0.4503	Range	0.6176
Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0
Maximum	1	Maximum	0.0803	Maximum	0.7702	Maximum	0.2871	Maximum	0.3299	Maximum	0.2816	Maximum	0.4503	Maximum	0.6176
S9		**S10**		**S11**		**S12**		**S13**		**S14**		**S15**		**S16**	
Mean	0.001468607	Mean	0.001438272	Mean	0.001011287	Mean	0.000807584	Mean	0.001133824	Mean	0.001716981	Mean	0.000960239	Mean	0.001195858
Standard Error	0.000625775	Standard Error	0.000617112	Standard Error	0.000238005	Standard Error	0.000159583	Standard Error	0.000248659	Standard Error	0.000539054	Standard Error	0.000171932	Standard Error	0.000397878
Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003
Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001
Standard Deviation	0.021072926	Standard Deviation	0.020781186	Standard Deviation	0.008014813	Standard Deviation	0.005373931	Standard Deviation	0.011591371	Standard Deviation	0.02512823	Standard Deviation	0.00801467	Standard Deviation	0.018547268
Sample Variance	0.000444068	Sample Variance	0.000431858	Sample Variance	6.42372E-05	Sample Variance	2.88791E-05	Sample Variance	0.00013436	Sample Variance	0.000631428	Sample Variance	6.42349E-05	Sample Variance	0.000344001
Kurtosis	540.2226438	Kurtosis	820.7209285	Kurtosis	330.6343401	Kurtosis	397.6571587	Kurtosis	939.4715218	Kurtosis	679.7257867	Kurtosis	530.2930417	Kurtosis	1413.810916
Skewness	23.07194073	Skewness	27.54728491	Skewness	17.63719952	Skewness	19.01588862	Skewness	28.1397794	Skewness	25.19535455	Skewness	21.64369518	Skewness	35.88048579
Range	0.5108	Range	0.6434	Range	0.1676	Range	0.1311	Range	0.4325	Range	0.7384	Range	0.2285	Range	0.7718
Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0
Maximum	0.5108	Maximum	0.6434	Maximum	0.1676	Maximum	0.1311	Maximum	0.4325	Maximum	0.7384	Maximum	0.2285	Maximum	0.7718
S17		**S18**		**S19**		**S20**		**S21**		**S22**		**S23**		**S24**	
Mean	0.001357662	Mean	0.001441878	Mean	0.001370548	Mean	0.001230097	Mean	0.001405292	Mean	0.001343948	Mean	0.001152554	Mean	0.001730143
Standard Error	0.000370022	Standard Error	0.000376718	Standard Error	0.000355481	Standard Error	0.00045841	Standard Error	0.000330144	Standard Error	0.000389382	Standard Error	0.000268881	Standard Error	0.000546656
Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.0003
Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001
Standard Deviation	0.01724874	Standard Deviation	0.017560897	Standard Deviation	0.016570923	Standard Deviation	0.021369001	Standard Deviation	0.015389832	Standard Deviation	0.018151212	Standard Deviation	0.012534025	Standard Deviation	0.025482594
Sample Variance	0.000297519	Sample Variance	0.000308385	Sample Variance	0.000274595	Sample Variance	0.000456634	Sample Variance	0.000236847	Sample Variance	0.000329466	Sample Variance	0.000157102	Sample Variance	0.000649363
Kurtosis	831.158886	Kurtosis	662.4266063	Kurtosis	1108.781354	Kurtosis	1935.978117	Kurtosis	414.3312565	Kurtosis	800.4019567	Kurtosis	1254.497638	Kurtosis	666.4510822
Skewness	27.75650838	Skewness	24.91816651	Skewness	30.82360665	Skewness	43.08561719	Skewness	19.90288614	Skewness	27.56409967	Skewness	33.09932107	Skewness	25.41705557
Range	0.5378	Range	0.5368	Range	0.647	Range	0.9682	Range	0.3521	Range	0.5811	Range	0.5061	Range	0.7143
Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0
Maximum	0.5378	Maximum	0.5368	Maximum	0.647	Maximum	0.9682	Maximum	0.3521	Maximum	0.5811	Maximum	0.5061	Maximum	0.7143
S25		**S26**		**S27**		**S28**		**S29**		**S30**		**S31**		**S32**	
Mean	0.000996272	Mean	0.001563046	Mean	0.001951358	Mean	0.001323746	Mean	0.001056006	Mean	0.001366682	Mean	0.445844547	Mean	0.409919512
Standard Error	0.000228106	Standard Error	0.000390226	Standard Error	0.000669684	Standard Error	0.00026148	Standard Error	0.000210413	Standard Error	0.000351225	Standard Error	0.006876551	Standard Error	0.007146455
Median	0.0003	Median	0.0003	Median	0.0002	Median	0.0003	Median	0.0003	Median	0.0003	Median	0.427	Median	0.3498
Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0.0001	Mode	0	Mode	0.6747
Standard Deviation	0.010633265	Standard Deviation	0.018190558	Standard Deviation	0.031217636	Standard Deviation	0.012188997	Standard Deviation	0.009808496	Standard Deviation	0.016372496	Standard Deviation	0.32055352	Standard Deviation	0.333135197
Sample Variance	0.000113066	Sample Variance	0.000330896	Sample Variance	0.000974541	Sample Variance	0.000148572	Sample Variance	9.62066E-05	Sample Variance	0.000268059	Sample Variance	0.102754559	Sample Variance	0.110979059
Kurtosis	1230.895595	Kurtosis	770.1323486	Kurtosis	808.1958883	Kurtosis	353.2223926	Kurtosis	659.4231678	Kurtosis	423.7349269	Kurtosis	-1.354957	Kurtosis	22.8914575
Skewness	33.03382905	Skewness	26.37830915	Skewness	27.75891033	Skewness	17.92380864	Skewness	24.1267447	Skewness	20.48569637	Skewness	0.142595573	Skewness	2.556223142
Range	0.425	Range	0.5696	Range	0.951	Range	0.2992	Range	0.2993	Range	0.3527	Range	1	Range	4.3721
Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0	Minimum	0.0014
Maximum	0.425	Maximum	0.5696	Maximum	0.951	Maximum	0.2992	Maximum	0.2993	Maximum	0.3527	Maximum	1	Maximum	4.3735

Table 7 – Ordered Alternatives By Ranking

Ranking From PROMETHEE	
Alternative	**Ranking**
A20	1
A30	2
A11	3
A5	4
A7	5
A27	6
A29	7
A6	8
A26	9
A24	10
A17	11
A18	12
A14	13
A13	14
A8	15
A15	16
A23	17
A21	18
A19	19
A12	20
A22	21
A9	22
A16	23
A4	24
A25	25
A2	26
A28	27
A10	28
A3	29
A1	30

Table 8 - Probabilities of Each Ranking to the Associated Alternative

Ordered By Ranking		
Alternative	**Rank**	**Probability of Ranking**
A20	1	0.376
A30	2	0.276
A11	3	0.172
A5	4	0.228
A7	5	0.280
A27	6	0.180
A29	7	0.204
A6	8	0.284
A26	9	0.256
A24	10	0.152
A17	11	0.136
A18	12	0.216
A14	13	0.440
A13	14	0.252
A8	15	0.220
A15	16	0.236
A23	17	0.224
A21	18	0.168
A19	19	0.212
A12	20	0.252
A22	22	0.184
A9	23	0.244
A16	24	0.236
A4	25	0.468
A25	26	0.268
A2	27	0.312
A28	28	0.292
A10	28	0.376
A3	29	0.396
A1	30	0.592

Table 9 – Average Ranking of Each Alternative, Simulated 1000 times

Average Ranking	
Alternative	**Rank**
A1	23.5
A2	23.3
A3	20.9
A4	21.6
A5	8.1
A6	10.8
A7	8.9
A8	14.9
A9	20.8
A10	19.9
A11	7.3
A12	16.1
A13	14.8
A14	16.3
A15	18.0
A16	17.7
A17	14.9
A18	14.9
A19	17.3
A20	7.3
A21	13.4
A22	19.4
A23	20.3
A24	13.1
A25	20.3
A26	11.7
A27	10.2
A28	22.0
A29	10.2
A30	6.3

Table 10 - Alternatives and Percentile at Each Rank per Simulation of 1000 Times

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A1	0.064	0.024	0.004	0.024	0.016	0.004	0.004	0.024	0	0.012	0.008	0.02	0.012	0.004	0	0	0	0	0.008	0.004	0.064	0	0.008	0	0.024	0.012	0.036	0.012	0.02	0.592
A2	0	0	0.032	0.04	0.012	0.012	0.008	0	0.04	0.012	0.012	0	0	0.008	0.02	0	0.016	0.004	0	0	0.008	0.068	0	0.036	0.008	0.044	0.02	0.312	0.288	0
A3	0.128	0.016	0	0	0.044	0	0	0.012	0	0.052	0.012	0	0.02	0.024	0.016	0.004	0	0	0	0.02	0.012	0	0.012	0.008	0.06	0.08	0.06	0.012	0.012	0.396
A4	0	0	0	0	0.04	0.024	0.028	0	0.004	0.06	0.012	0	0.036	0.008	0.012	0.004	0	0.036	0.044	0.016	0.004	0.04	0.028	0.036	0.016	0.084	0.468	0	0	0
A5	0	0.14	0.108	0.228	0.028	0.04	0.028	0.032	0.036	0.056	0.068	0.008	0.004	0	0.02	0.056	0.016	0.024	0.06	0.048	0	0	0	0	0	0	0	0	0	0
A6	0	0	0	0	0	0.032	0.148	0.284	0.064	0.012	0.076	0.032	0.028	0.124	0.092	0.024	0.072	0	0	0.008	0	0	0	0	0	0.004	0	0	0	0
A7	0	0	0	0.068	0.28	0.096	0.044	0.132	0.008	0	0.032	0.128	0.064	0.004	0.052	0.052	0	0.032	0	0	0	0	0	0	0	0.008	0	0	0	0
A8	0.004	0	0	0	0	0	0	0	0.008	0.012	0.032	0.164	0.22	0.204	0.02	0.008	0.012	0.104	0.068	0.144	0	0	0	0	0	0	0	0	0	0
A9	0.008	0	0	0	0	0	0.004	0	0.032	0.016	0.056	0.032	0.004	0.02	0.004	0	0	0.072	0.064	0.044	0.072	0.028	0.212	0.02	0.068	0.244	0	0	0	0
A10	0	0.12	0.008	0.004	0.012	0.044	0.02	0.004	0.052	0	0.012	0.02	0.024	0.004	0.012	0	0.012	0.008	0.004	0.008	0.036	0.004	0.032	0.044	0.08	0.044	0.004	0.012	0.376	0
A11	0.172	0.156	0.132	0.084	0.044	0.016	0.08	0.024	0.056	0.016	0.008	0.004	0.004	0.02	0.008	0	0.008	0.008	0.02	0.02	0.016	0	0.104	0	0	0	0	0	0	0
A12	0	0	0	0	0	0.12	0.164	0.02	0.012	0.032	0	0.02	0.04	0.004	0.016	0.02	0.084	0.012	0.12	0.008	0.008	0.016	0.008	0	0.252	0.044	0	0	0	0
A13	0	0	0	0	0	0	0	0.004	0	0	0.032	0.112	0.208	0.252	0.08	0.04	0.16	0.024	0.008	0.024	0.048	0.008	0	0	0	0	0	0	0	0
A14	0	0	0	0	0	0	0	0.008	0	0	0	0	0	0	0.264	0.44	0.096	0.072	0.068	0.048	0	0	0	0.004	0	0	0	0	0	0
A15	0	0	0	0	0	0	0	0	0	0	0	0.084	0.008	0.112	0.04	0.056	0.08	0.196	0.076	0.092	0.012	0.236	0	0.008	0	0	0	0	0	0
A16	0	0	0.124	0.076	0.012	0.008	0	0.06	0.008	0.016	0.012	0.012	0	0.02	0.008	0.02	0.028	0.02	0.012	0.068	0.028	0.044	0.036	0.04	0.016	0	0.236	0.096	0	0
A17	0	0	0.052	0.072	0.076	0.08	0	0.004	0.028	0	0.128	0.008	0.008	0	0.068	0.02	0.052	0.032	0.016	0	0	0	0.1	0.12	0.136	0	0	0	0	0
A18	0	0	0.004	0	0	0	0	0	0	0	0.1	0.192	0.216	0.132	0.044	0.016	0.056	0	0.08	0.032	0.012	0.116	0	0	0	0	0	0	0	0
A19	0	0	0.008	0	0	0.072	0.044	0.096	0.012	0.036	0.02	0.016	0.028	0.008	0.008	0.052	0.052	0.012	0.108	0.012	0.024	0.008	0	0.172	0.212	0	0	0	0	0
A20	0.376	0.052	0.132	0.02	0.008	0.016	0.056	0.048	0.008	0.016	0	0.012	0.008	0.008	0.004	0.02	0.024	0.02	0.012	0.048	0.004	0.084	0.024	0	0	0	0	0	0	0
A21	0.016	0.036	0.112	0.096	0.056	0.008	0.008	0.024	0.048	0.068	0.044	0.012	0.008	0	0.052	0.012	0.028	0.064	0	0	0.016	0	0.004	0.168	0.028	0.092	0	0	0	0
A22	0	0	0	0	0	0	0.024	0.028	0.052	0.024	0.04	0.044	0.008	0.004	0.004	0.032	0.056	0.052	0.02	0.032	0.116	0.1	0.032	0.128	0.02	0.184	0	0	0	0
A23	0	0	0	0	0	0	0	0	0	0	0	0	0	0.008	0.096	0.04	0.024	0.092	0.024	0.164	0.176	0.112	0.224	0.04	0	0	0	0	0	0
A24	0.052	0.112	0.1	0.052	0.012	0	0.008	0.028	0.004	0.152	0.044	0	0.02	0.004	0.012	0.024	0.02	0.004	0.016	0	0.02	0	0.092	0.036	0.004	0.144	0.04	0	0	0
A25	0	0	0	0.06	0.052	0.012	0.004	0.056	0.012	0.012	0.032	0.012	0	0.008	0.028	0.032	0.004	0	0.02	0.012	0	0.036	0.056	0.128	0.032	0.012	0.108	0.268	0.004	0
A26	0	0	0	0.076	0.092	0.088	0.028	0.008	0.256	0.096	0.012	0.008	0	0.012	0.004	0	0.012	0.048	0.052	0.012	0.188	0	0	0	0	0	0	0	0.008	0
A27	0	0.004	0.064	0.084	0.048	0.18	0.036	0.016	0.1	0.156	0.032	0.04	0.028	0	0	0	0.004	0.016	0.028	0.024	0.12	0.02	0	0	0	0	0	0	0	0
A28	0	0.064	0.028	0.004	0.02	0.02	0.032	0.004	0.012	0.024	0.004	0.012	0	0	0.008	0	0.02	0	0.008	0.048	0	0.008	0.028	0.012	0.044	0.004	0.028	0.276	0.292	0
A29	0	0	0	0.004	0.12	0.072	0.204	0.028	0.06	0.112	0.168	0.008	0.004	0.008	0.008	0	0.04	0.04	0.048	0.064	0	0	0	0	0	0	0	0.012	0	0
A30	0.18	0.276	0.092	0.008	0.028	0.056	0.028	0.056	0.088	0.008	0.004	0	0	0	0	0.028	0.024	0.008	0.016	0	0.016	0.072	0	0	0	0	0	0	0	0.012

Table 11 - Count of Each Alternative at Each Rank

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A1	64	24	4	24	16	4	4	24	0	12	8	20	12	4	0	0	0	0	8	4	64	0	8	0	24	12	36	12	20	592
A2	0	0	32	40	12	12	8	0	40	12	12	0	0	8	20	0	16	4	0	0	8	68	0	36	8	44	20	312	288	0
A3	128	16	0	0	44	0	0	12	0	52	12	0	20	24	16	4	0	0	0	20	12	0	12	8	60	80	60	12	12	396
A4	0	0	0	0	40	24	28	0	4	60	12	0	36	8	12	4	0	36	44	16	4	40	28	36	16	84	468	0	0	0
A5	0	140	108	228	28	40	28	32	36	56	68	8	4	0	20	56	16	24	60	48	0	0	0	0	0	0	0	0	0	0
A6	0	0	0	0	0	32	148	284	64	12	76	32	28	124	92	24	72	0	0	8	0	0	0	0	0	4	0	0	0	0
A7	0	0	0	68	280	96	44	132	8	0	32	128	64	4	52	52	0	32	0	0	0	0	0	0	0	8	0	0	0	0
A8	4	0	0	0	0	0	0	0	8	12	32	164	220	204	20	8	12	104	68	144	0	0	0	0	0	0	0	0	0	0
A9	8	0	0	0	0	0	4	0	32	16	56	32	4	20	4	0	0	72	64	44	72	28	212	20	68	244	0	0	0	0
A10	0	120	8	4	12	44	20	4	52	0	12	20	24	4	12	0	12	8	4	8	36	4	32	44	80	44	4	12	376	0
A11	172	156	132	84	44	16	80	24	56	16	8	4	4	20	8	0	8	8	20	20	16	0	104	0	0	0	0	0	0	0
A12	0	0	0	0	0	120	164	20	12	32	0	20	40	4	16	20	84	12	120	8	8	16	8	0	252	44	0	0	0	0
A13	0	0	0	0	0	0	0	4	0	0	32	112	208	252	80	40	160	24	8	24	48	8	0	0	0	0	0	0	0	0
A14	0	0	0	0	0	0	0	8	0	0	0	0	0	0	264	440	96	72	68	48	0	0	0	4	0	0	0	0	0	0
A15	0	0	0	0	0	0	0	0	0	0	0	84	8	112	40	56	80	196	76	92	12	236	0	8	0	0	0	0	0	0
A16	0	0	124	76	12	8	0	60	8	16	12	12	0	20	8	20	28	20	12	68	28	44	36	40	16	0	236	96	0	0
A17	0	0	52	72	76	80	0	4	28	0	128	8	8	0	68	20	52	32	16	0	0	0	100	120	136	0	0	0	0	0
A18	0	0	4	0	0	0	0	0	0	0	100	192	216	132	44	16	56	0	80	32	12	116	0	0	0	0	0	0	0	0
A19	0	0	8	0	0	72	44	96	12	36	20	16	28	8	8	52	52	12	108	12	24	8	0	172	212	0	0	0	0	0
A20	376	52	132	20	8	16	56	48	8	16	0	12	8	8	4	20	24	20	12	48	4	84	24	0	0	0	0	0	0	0
A21	16	36	112	96	56	8	8	24	48	68	44	12	8	0	52	12	28	64	0	0	16	0	4	168	28	92	0	0	0	0
A22	0	0	0	0	0	0	24	28	52	24	40	44	8	4	4	32	56	52	20	32	116	100	32	128	20	184	0	0	0	0
A23	0	0	0	0	0	0	0	0	0	0	0	0	0	8	96	40	24	92	24	164	176	112	224	40	0	0	0	0	0	0
A24	52	112	100	52	12	0	8	28	4	152	44	0	20	4	12	24	20	4	16	0	20	0	92	36	4	144	40	0	0	0
A25	0	0	0	60	52	12	4	56	12	12	32	12	0	8	28	32	4	0	20	12	0	36	56	128	32	12	108	268	4	0
A26	0	0	0	76	92	88	28	8	256	96	12	8	0	12	4	0	12	48	52	12	188	0	0	0	0	0	0	0	8	0
A27	0	4	64	84	48	180	36	16	100	156	32	40	28	0	0	0	4	16	28	24	120	20	0	0	0	0	0	0	0	0
A28	0	64	28	4	20	20	32	4	12	24	4	12	0	0	8	0	20	0	8	48	0	8	28	12	44	4	28	276	292	0
A29	0	0	0	4	120	72	204	28	60	112	168	8	4	8	8	0	40	40	48	64	0	0	0	0	0	0	0	12	0	0
A30	180	276	92	8	28	56	28	56	88	8	4	0	0	0	0	28	24	8	16	0	16	72	0	0	0	0	0	0	0	12

Table 12 - Example Random Weights for PROMETHEE

Example of Random Weights Employed			
	W1	**W2**	**W3**
1	0.03496575	0.8736536	0.091380657
2	0.37082023	0.16569261	0.463487157
3	0.81783168	0.15356615	0.028602174
4	0.08955004	0.62501743	0.285432529
5	0.34333941	0.48543661	0.171223974
6	0.5999421	0.00102498	0.39903292
7	0.33513423	0.20092566	0.463940108
8	0.06622044	0.26963794	0.664141621
9	0.60440079	0.22069746	0.174901748
10	0.42616799	0.2371074	0.336724605
11	0.20180276	0.13356579	0.664631451
12	0.14780046	0.25356412	0.598635424
13	0.18014688	0.64287117	0.176981951
14	0.11755096	0.43514239	0.447306656
15	0.08494507	0.44762588	0.467429056
16	0.58538423	0.09551308	0.319102686
17	0.45919418	0.52055121	0.020254617
18	0.41679844	0.53827968	0.044921882
19	0.63707842	0.05119987	0.311721705
20	0.51918096	0.27430792	0.206511123

Table 13 - Equal Weights vs Random Weights

	Ranking From PROMETHEE	
Alternative	**Equal Weights**	**Random Weights**
A1	30	30
A2	29	26
A3	27	29
A4	26	24
A5	4	4
A6	8	8
A7	5	5
A8	14	15
A9	23	22
A10	25	28
A11	3	3
A12	19	20
A13	11	14
A14	16	13
A15	17	16
A16	22	23
A17	12	11
A18	13	12
A19	18	19
A20	1	1
A21	15	18
A22	21	21
A23	20	17
A24	10	10
A25	24	25
A26	9	9
A27	6	6
A28	28	27
A29	7	7
A30	2	2

Table 14 - All Rankings For PROMETHEE

			Equal Weights vs Specified Weights		
Alternative	Equal Weights	Random Weights	(0.50, 0.25, 0.25)	(0.25, 0.50, 0.25)	(0.25, 0.25, 0.50)
A1	30	30	30	30	19
A2	29	26	29	28	22
A3	27	29	27	14	30
A4	26	24	26	27	18
A5	4	4	4	9	3
A6	8	8	8	11	6
A7	5	5	5	6	8
A8	14	15	14	20	12
A9	23	22	23	26	20
A10	25	28	25	13	29
A11	3	3	3	1	10
A12	19	20	19	7	25
A13	11	14	11	15	17
A14	16	13	16	19	16
A15	17	16	17	22	14
A16	22	23	22	10	27
A17	12	11	12	24	5
A18	13	12	13	12	21
A19	18	19	18	25	9
A20	1	1	1	8	1
A21	15	18	15	4	24
A22	21	21	21	17	26
A23	20	17	20	21	23
A24	10	10	10	23	2
A25	24	25	24	16	28
A26	9	9	9	18	4
A27	6	6	6	3	13
A28	28	27	28	28	15
A29	7	7	7	5	11
A30	2	2	2	2	7

A.2 NSGA-II Python Code

```
from random import Random, randint
from time   import time
from ecspy  import emo
from ecspy  import variators
from ecspy  import terminators
from ecspy  import benchmarks
from math   import exp
from ecspy import ec
from random import uniform, randint, sample, shuffle, Random, seed
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
import pylab

show_graph = 1
show_graph2 = 1
show_graph3 = 1
show_graph3d = 1

prng = None

M = 1000000000

unitcrashcost = {}
duration = {}
y = {}
E = {}
d = {}
D = {}
f_set = []

pairs = [(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,8),(8,9),(9,10),(10,11),(11,12),(12,13),
(13,14),(14,15),(15,16),(16,17),(17,18),(18,19),(19,20),(20,21),(21,22),(22,23),(23,2
4),(24,25),(25,26),(26,27),(27,28),(28,29),(29,30)]

for f in f_set:
        Zf = randint(0,1)
        Cb = randint(0,10)
```

```
for i,j in pairs:
        unitcrashcost[i,j] = randint(0,10)
        duration[i,j] = randint(0,10)
        y[i,j] = 0
        E[i] = 0
        E[j] = 1
        D[i,j] = 0
        d[i,j] = 0

E[29,30] = 20

def generatorx(random,args):
        size = args.get('num_inputs',30)
        return [random.uniform(0,100) for i in xrange(size)]

def evaluatorx(candidates,args):

        fitness = []

for cs in candidates:

                Ej = 1
                Ei = 0
                #yij = 1
                Dij = 1
                dij = 0
                yij = Dij - dij
                Ep = sum(cs)
                B = 10
                I = 20
                cdij = randint(10,15)
                cDij = randint(1,10)
                cij = cdij - cDij
                T = 92
                t = 0
                e = 1
                ys_m = randint(0,1)
                ys_p = randint(0,1)
                s_m = 0
                s_p = 0
```

```
constraints_violated = []

        if (Ej - Ei - yij >= Dij):
            constraints_violated.append(0)
        else:
            constraints_violated.append(1)

        if (yij <= Dij - dij):
            constraints_violated.append(0)
        else:
            constraints_violated.append(1)

        if (Ep + s_m + s_p >= T):
            constraints_violated.append(0)
        else:
            constraints_violated.append(1)

        if (e*ys_m - s_m >= T):
            constraints_violated.append(0)
        else:
            constraints_violated.append(1)

        if (t*ys_p >= s_p):
            constraints_violated.append(0)
        else:
            constraints_violated.append(1)

        if (ys_m + ys_p <= 1):
            constraints_violated.append(0)
        else:
            constraints_violated.append(1)

        if sum(constraints_violated) >= 0:
            f1 = M
            f2 = M
            f3 = M
```

```
result1 = 0
            result1 = sum(cs) - cs[0]
            print result1
            f1 =  result1

            result2 = 0
            result4 = 0
            y_minus_s = 1

            for i,j in pairs:
                  result2 += cij + (I*sum(cs)) - (B*y_minus_s)
            for f in f_set:
                  result4 += Zf*Cb
            f2 = result2 + result4
            print f2
            result3 = 0
            for i,j in pairs:
                  i >= 1
                  j >= 2
                  result3 += duration[i,j] - cs[0]
            f3 = result3
            print f3
            fitness.append(emo.Pareto([f1,f2,f3]))

      return fitness



if prng is None:
      prng = Random()
      prng.seed((0))


ea = emo.NSGA2(prng)
ea.variator = [variators.blend_crossover, variators.gaussian_mutation]
ea.terminator = terminators.generation_termination
final_pop = ea.evolve(generator=generatorx,
              evaluator=evaluatorx,
              num_inputs = 30,
              pop_size=150,
              maximize=[0,0,0],
              max_generations=100)
print final_pop
```

```
for f in final_pop:
    print f.fitness

if show_graph:
    import pylab
    figure2 = pylab.figure(figsize=(15,10), facecolor='w')
    x = []
    y = []
    pylab.title('\n Pareto Frontier', fontweight = 'bold', linespacing =2.0)
    pylab.grid(True)
    pylab.xlabel('\n F1 ', fontsize = 12, fontweight ='bold', linespacing = 2.0)
    pylab.ylabel('\n F3 ', fontsize = 12, fontweight ='bold', linespacing = 2.5)

    for f in final_pop:
        x.append(f.fitness[0])
        y.append(f.fitness[2])

    pylab.scatter(x, y, color='b')

    pylab.show()

    for f in final_pop:
        f
if show_graph2:
    import pylab
    figure = pylab.figure(figsize=(15,10), facecolor='w')
    x = []
    y = []
    pylab.title('\n Pareto Frontier', fontweight = 'bold', linespacing =2.0)
    pylab.grid(True)
    pylab.xlabel('\n F1 ', fontsize = 12, fontweight ='bold', linespacing = 2.0)
    pylab.ylabel('\n F2 ', fontsize = 12, fontweight ='bold', linespacing = 2.5)

    for f in final_pop:
        x.append(f.fitness[0])
        y.append(f.fitness[1])

    pylab.scatter(x, y, color='b')

    pylab.show()

    for f in final_pop:
        f
```

```
if show_graph3:
    import pylab
    figure = pylab.figure(figsize=(15,10), facecolor='w')
    x = []
    y = []
    pylab.title('\n Pareto Frontier', fontweight = 'bold', linespacing =2.0)
    pylab.grid(True)
    pylab.xlabel('\n F2 ', fontsize = 12, fontweight ='bold', linespacing = 2.0)
    pylab.ylabel('\n F3 ', fontsize = 12, fontweight ='bold', linespacing = 2.5)

    for f in final_pop:
        x.append(f.fitness[1])
        y.append(f.fitness[2])

    pylab.scatter(x, y, color='b')

    pylab.show()

    for f in final_pop:
        f

if show_graph3d:
    import pylab
    figure =plt.figure(figsize=(15,10), facecolor='w')
    ax = figure.add_subplot(111, projection='3d')

    x = []
    y = []
    z = []

    for f in final_pop:
            x.append(f.fitness[0])
            y.append(f.fitness[1])
            z.append(f.fitness[2])
    ax.scatter(x, y, z)
    ax.set_xlabel('F1', fontsize = 12, fontweight ='bold', linespacing = 2.0)
    ax.set_ylabel('F3', fontsize = 12, fontweight ='bold', linespacing = 2.0)
    ax.set_zlabel('F2', fontsize = 12, fontweight ='bold', linespacing = 2.0)
    ax.set_title('\n Pareto Frontier', fontweight = 'bold', linespacing = 5.0)
    ax.grid(True)

    plt.show()

    for f in final_pop:
        f
```

```
import csv
newlist = []
with open('nsga2.csv', 'wb') as csvfile:
	writer = csv.writer(csvfile)
	for f in final_pop:
		newlist = list(f.candidate)
		writer.writerow(newlist)
```

A.4 NSGA-II Data

1	2	3	4	5	6	7	8	9	10
-79.641	79.94871	71.65126	83.56262	77.65908	83.71137	91.11468	63.65197	71.09334	83.65502
-76.7187	79.54282	73.14427	83.98176	78.58052	83.05592	92.51002	62.67601	71.40432	82.99804
42.36616	81.54131	72.21041	83.70124	79.12009	82.90252	93.02178	63.36292	71.7782	85.23089
-64.444	80.29606	72.73659	85.64106	78.19639	82.99068	93.31124	63.65284	71.41464	83.4204
-47.1614	80.59497	72.65048	85.57671	77.82645	84.87869	92.57637	63.66386	71.76243	84.35263
-40.445	80.83181	72.63333	84.79227	77.72266	84.96822	94.08333	63.99215	71.78053	85.45494
-29.3034	80.51576	71.55311	84.13975	77.69773	84.93968	94.10948	63.88135	71.77579	85.51415
-43.1937	80.97158	72.64185	84.71713	77.82494	83.57438	94.15265	63.51942	72.42813	84.65889
-51.5946	80.01844	73.16458	83.87864	77.76217	83.1349	93.16747	64.07818	72.01745	84.68482
36.63563	80.62941	73.33205	83.08001	78.04767	83.68502	93.01218	63.37671	71.67989	84.86059
-8.38105	81.1698	72.31018	85.30081	79.49891	84.12362	92.9391	63.2752	73.1093	84.09136
7.430866	80.80695	72.09771	84.8684	77.92177	82.94353	94.33455	62.65624	72.34553	84.99059
2.056045	81.04094	72.47985	83.54235	78.28553	82.84737	94.09276	65.45922	72.80728	84.21026
-38.7928	80.75671	72.70985	84.9646	77.71028	85.05184	93.52858	63.1905	70.88997	85.56909
-43.9916	81.54971	72.57486	83.62288	77.75425	85.21293	93.19475	64.06392	71.09332	83.77913
27.45258	81.03837	72.67193	84.07839	81.38747	83.62895	93.39103	63.06429	72.53625	84.96431
-63.9141	80.67167	72.61464	85.25771	79.45951	83.67059	93.25132	64.0973	73.16965	83.72567
-24.7615	80.13704	72.15698	83.6831	78.97609	83.66467	93.18232	63.65347	72.08548	85.27668
-58.9391	80.37194	71.85157	84.42675	78.17533	83.62649	93.38151	64.08897	71.79147	86.34571
-58.8667	80.53197	72.60622	84.41483	79.26355	84.72295	94.13013	64.28249	72.57151	84.68123
15.28058	82.20566	72.34362	83.21395	78.64878	82.92913	93.30465	64.41679	71.94095	83.70162
-2.71005	81.52931	72.16292	84.50844	78.81278	82.83779	93.30185	63.39957	71.93323	83.29261
-34.8987	81.19941	72.91403	84.22957	78.57929	86.41897	93.52234	63.68398	69.91641	84.78151
-69.751	82.77669	72.94608	83.79796	78.15267	85.10186	93.29699	66.26166	72.532	83.83564
19.65554	80.99841	72.66836	83.69123	79.88085	83.29815	93.37382	63.842	71.87424	83.26403
-49.257	79.57051	72.97123	84.75446	78.66959	82.27365	93.95568	62.37985	73.66021	87.27037
-17.1962	80.62996	73.86512	84.30639	78.80474	83.91638	93.45504	64.02431	71.80606	83.85007
-79.5784	83.92581	74.71089	84.79451	77.92968	82.926	93.10483	63.00378	71.57874	83.40281
-20.2114	80.55359	73.75727	84.3559	79.14468	85.14575	93.2632	63.79508	71.82724	86.19663
-14.1628	81.38804	73.1559	85.37223	79.35054	83.34907	92.95691	63.4008	71.84064	84.91061
-72.6249	82.09296	72.67889	84.70776	78.19379	86.64172	93.23436	65.56172	72.39755	84.60676
-72.5042	82.49726	72.79427	84.33018	78.23544	84.63286	93.10759	64.57023	72.40282	84.98819
-15.7177	81.48305	72.50979	84.83747	77.71457	85.0715	93.48465	64.01044	73.39635	84.33108
-62.1248	80.76701	72.5792	84.06586	78.19472	83.13525	92.72214	65.13819	71.6312	84.10726
21.08135	80.90098	72.5406	83.70457	79.38191	83.3366	94.53263	64.25688	71.40805	83.82791
-9.46816	80.40201	72.38832	85.20261	78.57523	83.62092	93.28767	64.04355	74.96795	84.66374
-2.43507	80.13214	72.42641	84.00773	78.65948	83.42746	93.2008	63.14	74.3392	83.94563
31.84981	81.89618	72.24616	84.88091	79.89599	83.50709	93.49099	64.1024	71.47101	83.0708
7.249906	80.88663	72.93679	84.96313	79.48783	83.69925	93.16631	64.21568	71.54848	84.4145
2.102977	81.2531	72.5488	83.58179	78.61994	83.37126	94.03603	64.61727	71.81283	84.24059
-0.46712	81.4046	73.97167	83.55556	78.54494	83.94616	94.04588	65.51531	72.65855	84.30848
-51.2798	80.57441	73.05416	84.68704	77.86669	84.9336	92.54183	65.69856	72.03327	83.99523
9.755562	80.97658	72.44196	83.56474	79.7216	83.49774	93.3266	63.87276	71.34699	83.77211
-19.5213	80.66325	72.84351	84.19387	80.48584	84.23848	93.01203	63.58702	72.90264	84.26894
-59.0509	81.9114	72.51396	83.90913	78.15669	84.52233	93.29945	64.5564	72.07354	85.37782
-31.2026	80.37844	73.63014	85.25874	78.30527	82.58026	93.13243	62.57447	74.27067	85.38922
-33.5005	80.54229	73.47193	83.93154	78.27665	83.38642	93.88089	62.68171	71.5854	86.69449
-31.61	81.48636	73.48528	83.24462	78.03139	83.7086	92.6641	63.79192	71.96905	84.3061
-22.6397	80.1764	72.74993	83.75581	78.7672	83.61069	93.17237	65.3623	72.01149	85.09956
-23.1268	80.28218	72.87471	83.91213	78.83072	83.3988	93.25817	63.90409	72.1002	85.34473

11	12	13	14	15	16	17	18	19	20
85.60386	76.01939	77.56116	67.56141	83.535	90.03728	65.93322	85.34131	76.1847	69.95721
84.22519	76.32639	77.1034	68.15759	83.58736	89.46256	65.77574	84.96017	76.16883	70.49782
84.20194	77.85489	77.7545	69.35727	82.71074	89.51125	66.38664	84.86924	76.07571	69.46123
84.71357	78.02972	77.67218	68.21809	83.38103	89.88137	66.01979	84.89813	75.90041	69.56606
83.81402	78.16927	77.21769	68.19891	83.17502	89.57388	65.98055	85.01362	75.78381	71.57942
85.04615	77.02689	77.85902	68.28398	84.33923	92.03133	66.1607	85.38582	76.46915	69.63759
84.76781	77.53934	78.02253	68.23216	85.08375	91.70329	66.09315	85.53148	76.37052	69.49753
84.73751	76.95738	77.90694	68.04011	82.35949	90.56499	66.2617	85.25371	76.46342	69.86984
84.37985	77.49381	78.176	69.25685	84.77166	89.49156	65.76521	84.95353	76.49738	70.72327
83.96609	77.8776	77.75013	68.3978	82.68598	90.46402	66.36707	85.36886	76.04488	69.92135
84.15408	77.11737	76.91267	69.08006	83.70159	90.24313	66.41563	84.80696	76.19377	70.08363
83.52133	78.50384	78.38893	69.58388	83.77309	90.75562	66.60011	85.39203	76.27276	69.75322
84.92307	76.54119	77.29756	68.148	83.55726	89.73467	66.05115	84.96999	77.62009	69.92663
85.36046	77.38706	77.80608	68.29479	85.38281	91.73441	66.12973	85.37594	76.16377	69.79003
84.08193	77.15223	77.95206	68.88894	84.68218	89.48821	66.25482	84.99515	76.4168	70.47625
82.21284	77.35871	77.62853	68.58645	83.58008	90.07745	66.5264	85.21101	75.04468	69.82759
83.94508	76.7042	77.23238	68.98046	82.62571	89.38266	67.51799	85.01175	76.09759	69.75961
84.17873	77.05273	77.86592	69.01773	84.7906	89.79937	66.84321	84.74223	76.17716	70.35605
85.47293	77.05367	77.3855	69.86633	84.63043	91.13068	66.0848	85.53162	76.38094	69.52369
85.59639	77.35887	77.81808	69.77667	84.49671	91.66494	66.01905	85.00945	77.05633	69.66807
85.14326	77.02822	78.7413	68.70808	83.68403	91.73162	65.72885	84.77079	76.88563	71.87027
87.14788	77.17189	77.05843	69.3571	83.42285	92.33972	66.34965	84.77882	76.31789	69.74018
86.16805	76.36386	78.29716	68.23128	84.05311	90.2328	66.10498	85.10219	76.37503	70.38326
83.72386	76.83623	78.05527	67.92605	83.63173	91.0116	66.72665	85.16601	76.34855	70.4688
86.75765	76.89563	77.53836	68.18737	84.11057	91.5197	66.76517	85.02651	75.22592	70.28561
84.70095	76.3466	77.64877	70.40748	84.39421	91.0987	66.12705	83.62827	76.32056	69.75316
85.86991	77.71241	77.7977	71.08319	83.30254	90.2469	66.20628	85.74048	76.44935	70.27249
84.59029	78.91807	76.87354	68.4305	82.68739	89.76455	66.02552	83.84947	74.26626	70.32938
84.78511	76.25201	78.58339	68.10772	83.54326	89.82379	67.16785	85.02176	75.95724	70.96032
84.09826	77.6399	76.7619	69.08694	84.38734	90.14998	67.34738	85.06483	76.24956	69.93297
84.08657	76.92306	78.17349	67.9615	83.4546	90.72193	65.86156	85.15119	76.38805	70.5405
83.75836	77.40384	78.16252	67.84358	84.45513	90.80718	66.17513	85.3267	76.36892	71.372
85.15867	77.42925	77.81329	69.23848	85.39307	92.86192	66.12506	83.94744	76.31204	69.67157
85.61302	76.51727	78.13479	68.33469	83.33351	89.79428	66.86181	84.97511	76.6194	69.86989
84.37116	77.0371	80.46268	69.13635	83.35568	90.74316	66.15281	84.42935	76.08721	70.95293
86.41918	76.93387	78.14869	70.1811	83.54358	89.89872	66.25961	84.64413	76.63958	70.75537
86.72929	76.86491	78.05258	68.34698	84.30253	89.8618	66.09617	84.92036	76.57767	71.68935
83.48921	78.12763	77.66342	68.10772	83.40084	89.64127	66.41491	84.78548	76.22985	69.7074
86.40027	75.55873	77.68664	68.5151	83.88535	91.58644	65.13883	85.04144	76.42797	70.14044
84.83268	76.66549	80.84053	68.25311	83.45975	90.82806	66.01004	84.99453	76.96563	70.36443
84.21639	76.56211	79.19108	68.22203	83.45925	90.60157	66.04353	84.97939	74.86155	70.36921
83.81566	77.14468	77.87207	68.18321	83.21507	89.59508	65.37155	85.01507	76.46873	70.52121
85.53416	76.4997	78.66773	67.82159	84.03198	91.09088	66.09357	85.49163	77.28888	70.32872
84.14104	76.31127	78.07546	68.13278	84.60185	89.99908	66.40279	84.96466	76.15292	70.66758
84.48095	77.00854	77.64201	68.67404	83.6728	91.1177	66.2733	85.47884	76.34561	70.41043
84.31748	76.36241	77.53948	68.60001	84.05321	91.10877	66.6827	84.26123	76.50418	69.72126
84.3852	76.36166	77.65752	69.46257	84.02418	90.57382	66.7133	83.9326	76.73084	69.44656
84.89508	76.45184	77.67936	69.37288	84.188	89.73521	66.38336	86.42988	75.07957	69.5134
84.04588	77.01919	77.88894	68.55066	84.30868	91.37956	66.64935	84.67607	76.50994	70.26466
83.97667	77.01163	80.24631	68.08439	84.36651	90.09016	66.79576	84.71084	76.21485	70.28918

21	22	23	24	25	26	27	28	29	30
80.42199	76.13613	76.3862	80.29256	72.43117	76.94819	80.86033	93.21331	71.78406	68.11853
80.44366	76.7185	76.75756	82.95267	72.76025	76.47167	82.31511	90.7743	72.46226	70.0526
79.79326	76.5433	76.7753	81.00329	75.05696	76.76385	83.43247	90.74873	72.8213	68.68958
79.49802	77.26388	76.93855	81.15921	74.09103	76.0047	82.83404	90.67596	73.27777	71.83299
81.77408	77.46312	77.34801	80.51121	73.02294	77.94183	82.27381	93.49225	75.03988	68.40466
80.03089	78.11755	76.60355	79.80844	73.90669	76.48957	82.22346	91.79609	72.94783	68.06894
79.98744	78.07956	76.64862	81.26544	73.6148	76.48789	82.25955	91.25693	72.49016	69.96404
80.02278	77.75488	76.55413	80.48541	73.88643	76.72825	81.59135	91.79539	75.2849	70.52161
81.88704	77.9894	77.58828	80.34994	71.96304	78.45087	81.73326	90.66322	71.83299	69.22667
80.53931	77.65645	76.73309	81.10616	73.27104	76.57585	82.84201	90.84429	74.84732	68.78911
81.4431	77.6611	76.54498	82.16714	74.19988	77.75001	83.55896	91.29981	73.35374	67.90817
79.38453	76.95825	76.93546	79.47948	73.38735	77.07666	81.2912	91.22378	74.66936	70.86425
81.02131	78.98333	76.7082	81.16536	73.84476	76.20852	81.85746	90.82122	75.7726	69.25099
80.63003	78.0859	76.8903	80.12761	71.92286	76.55867	81.42724	91.84066	73.0447	68.95261
81.31336	77.98634	77.067	80.97567	72.73966	76.90719	81.86402	90.66631	72.21629	71.85717
78.97168	77.46903	77.47631	80.18064	72.40851	76.77189	81.23782	91.16584	76.08588	70.87939
82.88109	78.88255	76.96129	80.0711	74.032	77.8059	82.1164	91.13905	72.82756	67.64493
82.13864	77.71207	77.06532	80.711	72.15537	76.49298	82.36803	90.75751	73.6127	70.87856
79.08311	78.03789	76.44736	81.29194	73.1865	76.72365	81.49568	90.93341	72.58182	69.91792
79.82647	78.01483	76.46074	80.23303	72.92893	76.56171	82.30213	90.93293	73.23296	69.78821
80.58817	77.77409	76.88324	80.25424	73.4576	77.45876	81.56163	91.3891	71.96307	69.17236
79.8043	76.75817	76.76372	81.01127	74.6322	76.77834	81.77125	90.73576	72.60507	69.20962
79.7722	76.10565	76.23093	80.36685	73.20558	77.8791	81.89909	90.6533	73.73582	69.47054
79.72879	76.43306	77.41791	81.24844	72.63789	77.36653	81.55717	90.82984	72.1742	70.02581
79.97767	76.31628	77.05741	79.88458	73.22579	77.46305	81.42316	90.70621	73.50418	69.28901
80.18087	77.91617	76.29221	81.12989	73.91239	76.47472	83.8848	91.84652	72.64659	68.15238
79.57568	76.50638	78.04554	80.10932	71.57992	77.11317	81.70798	91.6699	73.28165	69.72565
78.89369	76.23908	78.02454	80.77024	72.27314	76.26655	82.38781	89.68058	71.98739	70.6093
79.2076	77.42183	76.56022	80.51783	73.1595	76.9041	81.41397	92.91946	73.84463	68.45618
80.88108	77.61797	76.54136	82.15522	74.52054	76.91955	82.73913	91.24376	73.87532	68.10897
80.3967	76.42527	76.79962	80.84017	72.63163	76.93169	80.75986	91.11211	72.61805	68.20742
80.92026	76.42373	78.1805	81.30238	72.65222	76.71545	81.46817	91.07314	70.62287	68.73491
80.50101	78.03707	76.77246	80.91399	73.1467	76.45923	81.93946	91.45864	73.24092	68.9684
78.70853	76.2646	78.27709	81.56091	73.67916	79.42271	82.49159	92.12726	72.32995	69.98177
81.03681	76.93617	76.57807	80.39964	73.30159	77.45178	82.21102	91.21971	72.2662	71.01686
80.80723	76.94489	75.65251	80.17055	72.73978	77.53545	81.47858	91.25098	73.09854	70.47199
81.06519	76.82639	77.15583	81.03842	72.71661	77.16083	81.40717	90.86734	75.16573	69.16168
80.794	77.71848	77.89797	79.9978	73.2333	77.3735	81.51343	91.05199	71.51843	70.7474
79.22475	76.13049	76.62553	80.37999	72.9997	77.6453	82.00423	93.15774	73.41702	69.40246
80.42072	78.09674	76.28816	80.4818	74.81216	76.09268	81.88951	90.77277	74.1421	69.4608
80.78318	78.94171	76.71797	79.94036	73.50426	76.97887	81.92461	90.53857	73.27214	69.57832
80.71885	77.40551	77.32767	81.59407	72.7452	77.72273	82.1223	90.80437	73.66866	68.47144
79.11929	77.89194	76.32628	80.35082	73.99746	77.18254	81.17306	90.61898	72.27289	71.10157
80.24496	76.41681	76.62898	81.34799	73.37559	77.38162	80.9054	91.62605	73.79074	71.05032
79.74774	76.62421	76.76015	81.27448	71.17535	77.3313	81.5258	90.87119	71.95856	69.9196
80.07347	77.76866	76.17612	81.12866	73.9862	77.94227	83.31818	91.90341	72.41242	68.57607
80.17568	77.64262	77.95795	81.12415	73.93908	77.22068	83.45168	91.95006	72.66307	68.58213
79.6823	81.08664	76.87891	80.63954	71.98954	77.77413	82.68246	91.47443	72.73786	70.75291
79.75116	76.63954	76.92181	81.14361	72.72285	77.22993	81.99247	91.23941	73.08914	70.84381
81.87333	76.04402	76.88584	80.29848	72.224	77.30809	81.77526	91.27951	73.22748	70.84093

-19.1024	81.06349	72.90416	84.4918	78.7563	83.85918	93.91526	62.39277	73.66822	84.70345
-28.9831	81.28761	74.4737	83.8511	78.2433	85.014	93.12876	67.57596	70.28215	84.8454
-21.2964	81.71912	72.75491	84.00333	78.38836	83.91003	93.27165	65.75119	71.87652	83.78554
-49.5434	80.09845	72.72137	84.21857	78.15984	83.57653	92.73523	63.60174	73.17219	84.58026
10.02405	80.64933	72.76998	84.12293	76.50073	83.39578	93.93409	63.14789	71.62282	84.99846
-2.90251	80.26821	73.48596	85.81119	78.87817	83.38982	92.58433	63.46609	71.4887	82.22921
19.74916	80.0672	73.3004	86.5794	78.61902	83.75521	93.34419	63.39501	71.65142	83.05932
-76.2456	81.23731	73.02526	83.7908	77.95951	84.44741	92.25994	63.95123	71.08656	83.6548
32.91767	80.41712	73.38592	83.97374	78.43044	82.48769	93.25347	64.58661	73.16609	83.73649
13.98521	80.8851	72.43653	84.18741	78.05557	83.83645	93.8445	63.96766	71.7041	84.98333
29.35628	80.65092	72.96765	83.48603	78.59755	83.72415	93.17498	63.61156	71.71325	84.85406
-21.0916	81.40479	72.40807	83.632	78.4098	83.7243	92.97028	64.5642	71.74935	83.42274
-21.7304	80.40956	72.92379	84.26053	78.74169	82.65028	93.32142	65.68641	71.843	83.57498
-17.4112	81.42969	72.77224	83.80884	78.67783	85.17348	93.26021	65.95339	71.84438	83.67119
-10.9132	82.09489	72.59257	83.71403	78.71892	83.72465	93.30138	64.65367	71.15681	83.74178
5.35785	80.71613	72.8014	84.10342	78.14287	82.89146	94.67558	62.00668	71.54959	85.10038
-9.03767	80.24689	72.33258	84.76709	77.72229	85.25134	93.46655	63.99311	73.68451	84.31383
-75.0273	80.03068	72.32949	83.57046	77.54529	83.4874	93.34858	63.92936	71.07777	83.65907
32.14428	80.60458	72.84137	83.66685	78.42278	83.92455	93.1397	64.52823	71.57352	83.0961
-32.6504	80.51268	73.6102	83.50771	78.08778	83.36714	93.05144	64.082	71.11236	83.98151
9.822734	81.33606	72.99162	83.61919	78.72914	83.199	93.35922	63.88356	71.33366	84.12192
-72.6236	80.27459	73.15077	84.2983	78.09568	84.0152	92.95423	66.97078	71.24938	83.67577
33.27761	81.37515	73.80489	83.71661	78.40303	82.5153	93.28747	64.59034	72.0972	83.72071
-74.6569	80.65533	73.06679	84.08606	77.79127	83.41179	93.04027	62.67026	71.39333	82.75814
-11.2053	81.06353	72.36875	84.09791	78.06861	82.86485	93.4327	63.53829	72.48207	85.39781
-50.7857	81.09418	72.29487	83.78133	78.64681	83.91978	93.25665	65.37033	71.13515	83.52749
-21.6134	79.96284	74.23805	83.75028	79.84321	83.69574	93.17866	63.80748	71.84463	82.24247
-72.5378	80.61735	72.98638	85.03093	78.73153	83.88325	93.24009	63.86792	71.55298	83.97852
-31.9077	81.29897	72.91361	84.78678	78.69645	83.34138	93.35599	63.27432	72.33441	84.4624
-26.1396	82.15647	73.50001	84.19959	77.56921	83.8528	93.22465	64.0533	71.82445	83.51757
11.01184	80.77563	73.02268	83.68387	78.72905	83.74373	93.37316	63.92299	71.28608	83.98493
31.57047	80.27457	72.74048	86.26466	79.55859	83.77117	93.48416	64.27613	71.30649	82.99515
-53.063	79.89312	73.30408	83.30722	77.75273	83.17635	93.25407	64.05233	72.03636	84.3205
-17.8036	80.26942	72.64279	85.0814	78.00723	84.63684	92.87764	63.64526	71.00019	83.90501
-33.9602	80.3084	72.66504	84.25714	78.65137	83.13694	93.13032	63.03338	72.84914	84.26481
16.45933	80.35548	73.32666	83.9703	78.07748	82.47586	92.80832	63.01487	72.00672	83.86749
20.46444	80.06008	72.68022	83.66858	77.67398	83.26169	93.18684	63.09925	72.99125	83.9535
20.16184	80.6594	72.5382	83.967	78.55357	83.94466	93.33677	63.68461	71.9821	83.58936
17.58354	81.41919	72.59473	84.24044	81.66947	83.78757	93.44678	63.71188	72.83528	83.65566
-33.576	81.1573	72.74156	84.24103	78.18231	82.86291	93.28033	65.51984	71.87901	83.79429
-59.8541	80.34327	72.60333	84.44062	79.24907	83.32929	93.38141	64.63236	71.0452	84.13325
-28.0034	81.31317	72.23771	84.14946	78.21212	83.85628	92.41972	63.78669	71.71147	86.09279
-64.167	82.17826	73.56818	83.39288	77.11696	82.98969	93.26163	64.02904	71.04267	83.3577
-26.8073	80.57023	72.1947	84.63596	78.81141	83.3386	92.66724	63.43927	71.44272	84.7362
-39.1855	80.03587	73.22151	84.27739	79.04635	84.68959	93.67821	64.18288	71.42802	83.89067
-51.7657	79.90474	73.49946	83.46193	78.18099	83.9648	93.54466	63.668	71.58536	84.76246
4.972879	80.54239	73.30047	84.47097	77.66803	83.53149	92.98755	62.59349	71.22929	83.12967
17.72997	81.06906	72.4262	83.54123	78.03099	83.18207	92.84935	62.93199	72.80375	84.18338
2.090886	81.21373	72.80307	84.43305	78.76434	83.81553	93.19207	63.33458	71.29161	83.79761
-38.0051	81.06654	72.04485	84.11394	78.60745	83.71424	93.33261	64.34363	72.47977	83.26139

85.14638	76.35832	77.78184	70.66719	84.09374	90.96141	66.18444	85.33666	76.43104	69.77591
83.88963	76.97674	78.07599	67.81386	84.89598	91.47026	66.57117	85.2085	76.59336	71.18782
83.832	77.00419	78.02064	68.42383	83.60741	91.52356	66.5646	84.86044	76.58187	71.21527
84.63015	76.74731	77.48381	67.5028	84.16142	89.5302	66.99145	84.96611	76.37957	72.54603
85.01224	77.44342	77.36353	68.25309	83.5636	91.96169	66.90042	85.37023	77.99229	69.75199
84.1343	77.99041	77.94678	68.39837	84.9099	89.42456	66.01865	85.04615	77.64137	70.87776
84.59852	77.01428	77.23506	68.9529	84.51219	89.37454	66.01714	85.14918	77.82293	70.82067
84.75053	78.96687	77.64588	68.52711	83.5958	90.57493	66.02669	85.28867	76.16472	70.25148
87.31911	77.77329	77.53881	69.95433	83.81056	89.15917	66.56592	84.51424	76.69349	71.49212
83.79635	76.87294	78.44265	69.2753	83.54806	90.4988	65.84397	85.16109	76.63533	69.85508
84.23133	77.77041	77.64872	68.48465	82.91541	91.98907	66.20048	84.87907	76.22305	70.01376
85.73236	76.90464	77.43743	68.26426	84.58144	90.59276	66.27709	84.70872	76.79208	70.21742
83.82278	77.01532	77.94263	67.71798	83.73965	92.05412	66.49755	84.67775	76.59241	70.19542
83.81732	76.90582	78.10042	68.59351	83.76956	91.51992	66.51088	84.89579	76.60215	71.55553
85.33565	76.70313	80.70157	69.10869	83.5839	91.73585	65.83171	84.42934	76.08192	69.92649
84.15266	77.94455	77.24289	68.09571	83.44866	92.28691	66.66472	85.55816	76.64891	69.92506
85.1386	77.45884	77.95916	68.25753	85.34753	90.69175	66.12545	83.52309	76.37023	68.75626
84.77285	76.48919	77.55624	68.01342	83.50383	89.74012	66.87188	84.52715	76.68136	68.87632
86.32691	77.51591	77.38829	68.24632	83.75869	90.02932	66.89494	84.68442	76.16263	69.65573
84.2528	76.44485	77.52791	68.29806	83.59147	89.69186	66.67127	84.3988	76.71425	69.44933
86.8763	76.43579	77.60287	67.86236	84.10117	92.1183	66.10483	85.15208	76.50985	70.41793
84.28252	77.41679	76.89386	68.16231	83.86486	90.29482	65.86884	84.2438	76.13434	70.65404
87.29364	77.76289	77.30614	68.89065	83.51804	89.32245	66.62567	84.52174	76.70479	70.60962
85.43122	77.24361	77.20854	68.27258	83.71629	91.09833	66.19401	84.89941	76.16833	70.32202
83.70441	79.42531	77.91448	69.01506	82.68111	90.17855	67.23847	84.63936	76.5558	71.0736
83.74681	78.11148	78.1292	68.54215	83.78645	92.12125	65.87894	84.95426	76.61917	70.40145
84.21825	77.01338	77.90097	68.97763	84.86203	89.48377	66.23562	85.28728	77.88171	70.41511
83.81481	76.42446	77.27943	68.00435	82.74085	89.72397	67.03989	84.90672	75.43828	69.99508
86.19961	76.41896	78.31444	68.33058	83.84017	89.38072	66.1149	86.11028	76.26722	70.22209
84.67484	76.41054	77.62099	69.40181	83.47544	89.73917	66.10254	86.42294	75.1341	69.59521
86.7499	77.00063	77.34308	69.24092	84.07147	90.15274	67.02657	85.68956	76.17783	70.02094
85.36101	77.66226	77.40494	67.98523	83.37831	90.03944	66.0147	84.92214	76.33109	69.78594
85.29902	77.46102	78.52667	68.59182	84.78184	89.29224	66.037	85.06134	76.33247	70.66121
84.29636	77.1906	78.19854	69.61928	83.90177	90.37036	66.39632	84.80575	76.21892	68.8547
83.97977	76.59469	77.8615	68.06301	84.72204	89.90771	67.1551	84.9786	76.17407	70.74754
85.91729	77.89452	77.65213	70.4245	84.73762	89.0893	66.55275	84.51342	76.17849	71.5681
83.93401	76.74357	78.75987	68.74581	84.52962	89.86263	66.61589	84.96191	76.33198	70.26843
86.48628	77.7571	77.65123	68.47839	83.96458	89.96733	66.99907	84.89536	76.4536	70.23827
83.64737	76.86048	78.80877	69.12725	83.55246	89.34222	66.26137	85.20673	76.67438	69.49748
83.90773	77.01587	77.92623	67.87166	83.55904	91.42281	66.60546	84.81163	76.58235	70.99776
86.02134	76.95064	77.28979	70.88905	83.56535	89.8821	66.01065	84.85438	76.40467	69.66372
83.83888	76.87255	77.9933	69.38085	84.23962	91.66916	65.76904	84.65217	76.52578	70.81288
84.67308	76.42807	77.5258	69.4592	83.4811	89.71596	66.7046	84.61661	74.56858	69.87664
84.74788	77.43658	78.04567	69.72121	84.0002	91.78648	65.48644	84.8442	76.55158	70.84241
85.36056	76.1282	77.92061	69.20923	83.52742	89.65521	66.33415	85.07139	76.12663	69.57839
83.75416	77.24064	77.69804	68.2038	84.64147	90.56447	67.34732	85.24152	76.3241	69.75362
84.65501	77.65979	77.26541	68.99016	83.79125	90.88119	66.59236	85.74939	76.56097	69.75617
84.69107	76.48616	77.14886	68.04537	83.44776	89.94731	66.94873	84.92641	75.72241	70.46646
83.89325	76.80953	77.21818	69.04809	83.7581	90.54556	66.24429	85.52559	77.28551	70.07501
85.86957	76.85555	77.79512	68.6322	82.72314	90.20724	66.62825	84.83519	76.69738	70.80324

79.40458	77.97489	76.40201	81.37422	72.44737	76.47465	82.88061	91.30613	73.12871	69.56859
79.31323	76.43031	76.70975	81.66999	72.97855	77.37887	81.80788	90.60929	72.81072	70.33489
79.29891	76.41765	76.6856	81.73984	72.97957	77.32788	81.56762	90.69351	72.79216	70.31971
79.54941	77.18294	76.02532	80.50005	72.79774	78.53857	80.36521	93.31247	76.96055	68.86144
79.96182	77.58276	76.55025	79.78941	72.92751	77.27417	81.59896	91.70202	73.94132	72.47952
79.63964	77.62467	77.52712	80.80599	72.42042	77.51258	81.43332	92.55694	71.83816	70.06175
79.98782	77.01863	77.23886	80.9753	72.51955	78.21606	81.69299	90.81821	71.76363	70.0101
79.76184	75.89204	76.43642	80.10613	72.57018	76.71577	83.16325	92.28866	72.58392	69.83582
80.62134	77.01076	76.833	80.87873	73.76791	76.30497	81.31404	91.56342	72.16894	68.35048
78.96006	76.95356	77.7676	80.21024	73.02476	79.03905	81.28435	90.62186	72.7351	70.96331
80.10257	76.92162	76.99849	80.75132	72.56072	76.91023	82.20696	91.71813	73.67029	69.02787
81.50049	78.133	76.30583	80.3182	74.64401	77.07426	81.38505	90.66117	72.59875	69.39714
79.05964	76.04262	76.72272	81.95237	73.03734	77.28293	81.68566	91.24085	73.08914	70.83976
79.40252	76.42238	76.74901	81.62131	72.99021	77.41128	81.23945	90.82402	72.17156	70.27112
80.27069	76.89257	76.8756	80.19533	73.64415	77.4607	81.33187	91.34989	72.07126	69.45042
79.52121	77.56338	76.93159	79.53761	73.36952	77.27441	81.26174	91.47459	74.91504	70.35158
79.62078	78.02776	76.64476	81.00348	73.47876	76.3405	82.25802	91.2408	73.47469	69.13258
80.39661	76.0757	76.64793	81.72707	72.9447	77.48025	81.57879	91.55617	70.54279	71.41841
81.15609	78.07072	76.38498	80.1434	73.19912	77.13808	81.40239	91.03381	72.35506	68.83473
79.98263	76.81194	78.26862	81.12318	73.9808	77.89211	81.89251	91.98872	72.41265	71.95472
79.89628	76.14203	76.29708	80.22662	73.88795	77.66577	80.98978	90.60783	73.52701	69.47689
80.49063	76.37256	76.16516	81.41206	72.87664	76.48426	82.17731	91.11525	71.93026	70.92731
79.82953	77.0089	76.74009	80.67639	72.2703	77.0615	81.18763	91.64977	71.91659	68.14823
79.32362	77.87018	76.82928	81.65845	72.71536	76.57055	81.31757	90.93811	73.23859	70.09207
79.91187	77.70169	76.65686	81.59991	72.7595	77.10466	81.42522	91.25243	72.09939	69.92257
78.67744	76.34534	76.55573	80.25008	73.93798	79.31425	81.26023	90.41314	72.60452	69.99348
79.23093	78.04911	77.52744	80.74895	72.36019	78.10999	81.78161	90.67925	71.91046	70.10394
81.23165	77.34055	76.65068	82.09692	72.7281	77.78515	83.45289	91.2354	72.80835	67.91298
79.86635	76.12606	77.07268	80.39627	73.29573	77.68851	82.34131	90.6883	73.59095	69.47582
79.93151	77.23515	77.01825	80.15994	74.20807	77.80369	82.86515	91.49304	72.95337	70.6002
79.90223	76.73099	76.7543	80.16655	73.31483	77.3336	81.4747	90.68444	73.5397	68.96719
80.09013	77.37547	76.40912	80.11832	73.19178	77.45184	81.43958	91.11263	71.75252	69.03346
82.52785	77.99216	76.47099	81.15966	72.06457	76.50137	81.90105	90.74311	72.90017	68.88106
82.43211	76.22693	77.68091	80.83083	72.3928	78.23604	83.43058	91.25981	73.50215	67.05545
79.79426	76.29513	76.6935	81.77532	72.70695	77.03026	81.39645	90.3966	74.40626	70.94082
80.97808	77.01505	76.8688	80.9725	73.63837	76.338	81.36676	90.91092	72.4921	68.56169
81.59423	76.4931	76.62496	81.1197	72.71644	77.07804	81.48616	90.36803	74.35932	69.85257
79.56007	76.48767	77.20335	79.838	72.52992	77.11561	81.48051	91.65616	72.75366	69.0421
78.95093	77.2341	76.49935	79.95206	72.41647	76.56201	81.25606	90.62607	72.87708	71.68369
79.26425	76.21156	76.54047	81.74709	72.9768	77.17249	81.75783	90.58194	73.352	70.36522
79.59169	77.89377	76.35563	80.3095	72.77788	76.72099	81.48397	90.38629	73.20335	69.78685
79.07589	76.95321	78.28526	80.27304	73.01971	79.23233	81.44953	90.57928	72.65641	69.31082
79.87414	76.35086	77.91693	80.37151	74.02114	78.17468	81.91097	90.63625	73.21091	71.42154
79.21925	76.94842	77.38269	80.58264	74.12557	78.41631	81.32656	90.53374	72.89562	69.30375
79.52086	77.49908	76.64763	80.37409	73.56835	76.66221	82.50806	92.8466	73.16489	69.89122
82.71004	77.55669	76.31162	80.48258	73.12343	76.61531	82.21052	90.591	72.87004	68.59857
79.55212	78.09511	76.10939	80.44555	74.61354	76.82545	81.62147	91.1051	74.26231	72.20903
81.19416	78.5558	76.60172	81.34199	73.28643	76.57545	81.87412	91.51276	74.57289	69.65894
81.26168	77.8251	77.34335	82.0264	73.6206	77.22688	81.7602	90.95847	72.67955	68.3761

-36.6843	80.76164	72.62314	84.90329	76.04577	84.1395	93.51819	63.14632	71.73194	82.92866	85.4881
16.60716	80.67033	72.59837	83.54464	79.6776	83.29476	93.44738	62.93478	72.02802	83.15882	83.82902
-5.26568	80.74316	72.91264	84.6874	78.80795	83.94401	93.05356	63.29347	71.98646	84.20946	85.38347
-58.275	80.5749	73.02513	83.62541	78.41418	84.10834	93.26916	65.72799	72.11734	83.77603	83.78772
-63.86	80.43257	72.45613	85.12387	78.13618	83.23903	93.37687	64.30775	71.39745	82.29237	84.2627
-22.4365	80.46594	72.95411	84.86113	78.59591	84.99013	93.58803	63.92238	71.83183	84.46646	85.03585
-23.6921	80.37994	72.92569	85.18992	78.43377	84.24654	93.12689	64.57898	71.83453	84.59915	84.95173
-62.087	80.65385	73.66357	83.16616	76.63361	82.6412	93.27302	62.8489	72.37928	83.55954	86.25915
27.60093	80.25623	71.94502	84.03392	78.6657	83.78411	93.57808	64.30401	71.43056	82.99268	85.8155
-6.91722	80.41621	72.40508	85.513	78.5636	83.14435	93.4038	63.98443	72.90116	84.52987	84.85314
-21.9763	80.54331	71.87587	84.47294	79.42558	85.11178	93.18879	65.38852	71.78783	83.75456	84.87575
-15.242	81.38812	73.98395	84.13384	79.98078	83.8355	92.95505	63.32265	71.87451	84.08865	84.1019
21.81088	81.81199	72.71697	84.14471	78.98995	83.38634	92.63847	63.62904	71.8322	84.01382	84.10087
-48.3277	81.07028	72.19822	83.96233	78.45471	83.88012	93.13864	63.78931	72.86277	83.38947	84.32353
-41.954	80.58819	72.6018	84.56283	77.79509	83.60307	94.21113	64.11302	72.20283	85.33463	84.84912
-46.853	80.53514	72.60043	84.63222	77.89563	84.17456	94.05204	64.04767	71.9669	84.95948	85.14999
8.402355	81.0586	73.27286	84.49353	77.57096	83.59507	93.12648	64.13825	71.24278	83.33265	84.16887
-7.85453	81.33558	72.35306	84.96943	77.7195	85.16477	93.02022	63.73602	71.42115	84.52297	85.26499
-60.4296	80.68455	72.89795	84.13475	77.76694	82.96729	93.35301	63.37918	72.1092	83.18577	85.23283
-14.3417	81.40587	72.62063	83.74916	78.64296	83.67215	93.28374	65.47079	69.63759	84.14181	84.83366
-39.3797	80.67535	73.64854	83.25182	77.14787	83.02251	93.03362	62.84852	71.88047	84.64789	85.67599
2.366628	82.40119	71.7217	84.1222	78.28626	83.01516	94.16371	61.95949	71.94484	84.79394	84.04538
12.01761	80.03864	73.29034	85.14996	78.80271	83.5361	93.71849	64.49095	71.10532	82.37412	86.38426
-36.1784	81.23838	72.68821	83.94892	77.53559	84.09337	93.24583	63.16376	71.73623	82.60621	84.76849
23.64854	80.41726	72.78944	83.63777	78.35072	84.86842	93.12096	63.70834	71.3933	83.48272	84.71338
-2.05495	80.58236	72.55145	82.94914	77.95595	84.74208	94.14539	63.1353	71.40809	85.78285	84.64081
8.494939	80.81545	72.43425	83.93597	79.16009	83.73497	93.06684	64.38033	71.1781	83.88403	85.33414
22.10673	80.41164	72.7385	83.66598	78.41016	83.8052	93.00151	64.56724	71.74855	83.35848	86.28454
-22.4627	81.14185	72.6823	83.82764	78.52429	82.88139	94.69048	63.1808	72.48122	85.57126	83.80452
-69.4075	80.61146	72.71858	84.25288	78.37536	84.37881	93.72629	63.44159	71.06891	85.04461	85.84598
-57.9879	80.74095	72.81961	84.15551	78.15174	83.95458	93.37717	65.08987	72.2723	85.36451	85.52781
28.17272	81.01709	73.52136	84.83306	80.52415	83.84735	93.23766	63.86975	72.93111	83.94941	83.4173
-17.7013	80.55671	74.26543	84.03084	79.01005	84.82716	93.14459	63.70633	71.12773	86.84023	84.20161
36.1397	80.40133	72.88162	83.67992	78.74477	82.96851	93.04226	63.32601	73.32829	83.22747	86.65817
-75.826	80.15411	73.97206	84.23418	78.14172	83.21603	93.11717	63.1333	71.39478	83.96388	84.25831
-76.0964	79.99122	73.71794	83.5322	77.80069	83.72538	91.62555	63.4241	71.1585	83.7934	85.05057
11.57443	80.65506	72.15616	84.55521	78.06461	83.63113	93.36821	63.31513	71.53892	85.06032	84.23265
-67.0694	80.29047	73.44706	85.31074	78.26219	82.96353	93.27364	63.44244	71.58662	83.67553	83.90197
4.476558	80.64394	73.20788	84.30989	78.86963	83.39608	93.3768	63.29209	71.5782	84.79759	84.36193
-64.0269	81.20004	72.59934	84.25511	78.45237	83.87616	93.27543	66.38343	71.20553	83.6945	84.18049
42.91273	81.57213	72.73608	82.79884	78.37667	83.37042	93.33825	62.45266	70.95841	84.15808	85.57275
-58.9448	79.68675	71.87514	84.40469	78.72399	83.51707	93.41213	63.96639	71.35914	86.44897	83.68866
-4.03795	81.36333	72.39556	84.18322	81.71526	83.83075	92.34413	63.73326	72.93834	84.36431	83.72574
37.78567	80.6586	72.92558	83.9845	78.56974	83.7919	92.99822	63.43487	71.94612	84.30386	84.12777
-73.9087	80.81885	72.48513	83.45954	78.67368	82.7102	91.66441	63.01814	71.93756	83.70277	84.45527
36.97996	81.81377	72.19292	81.71736	79.17554	82.78784	92.95344	63.05675	71.71347	83.87773	85.55822
-46.3348	81.27096	72.95536	84.16598	78.40254	83.20542	93.35532	63.64117	71.38557	83.92666	86.36183
17.3335	81.6207	72.57119	84.02407	77.99457	83.55007	93.25548	63.96084	70.4802	83.60841	83.65351
-57.7008	80.59733	73.31413	83.27473	76.80256	83.39905	93.54764	64.66492	71.10401	84.33682	84.03877
23.65684	80.61787	73.38764	83.81283	78.25447	82.50702	92.82018	64.42694	71.99812	83.7338	87.07885

77.48679	77.53342	68.22184	84.08154	91.23331	66.23415	85.71631	76.29584	69.64643	79.98976
77.31556	78.80259	68.62278	83.54194	90.08129	66.67591	85.21169	76.26125	69.7782	78.95569
76.73901	77.2995	69.52154	84.57993	90.06531	66.31459	85.00693	76.48554	69.94014	81.81428
76.64825	78.0554	68.16011	83.29552	90.43783	67.39727	85.01641	76.4096	70.56999	80.13172
77.43636	77.6415	68.18545	83.50092	89.54605	66.53996	84.78622	76.10596	69.66763	79.87868
76.24887	78.44886	68.34746	83.36821	89.61394	65.97248	85.1053	76.35139	70.12664	79.58081
76.45503	77.74969	68.52222	83.42741	89.62201	65.14221	85.02324	76.35563	70.16254	79.65002
78.01959	77.42423	67.73818	83.37497	89.22981	67.98299	85.34395	75.55549	68.904	81.19854
77.60745	77.98627	68.12944	83.35589	89.87395	68.01342	84.84567	75.65409	69.71137	80.26485
76.9437	78.36036	70.3717	83.81011	89.91349	66.83188	84.64414	76.77974	69.87594	79.828
74.90269	77.29695	69.49867	83.54318	89.86015	65.11536	84.95791	76.40708	70.34974	79.22208
77.29138	76.98874	68.90669	83.34379	90.20488	66.35409	85.41728	76.21037	70.08743	81.40197
76.42611	78.32387	68.62116	84.5939	90.00329	65.84157	84.80373	76.37182	70.47031	80.44954
77.92427	77.9729	68.57366	83.78031	91.10508	65.583	85.6982	76.22312	70.20838	79.55035
77.197	77.45148	67.99327	83.58214	91.1016	66.08201	85.2545	76.4577	69.74875	79.48441
77.03971	77.43036	67.63313	84.52522	90.8609	66.21326	84.51435	76.37366	69.87393	80.86002
76.72698	77.50143	68.19123	83.78901	90.89127	66.00504	85.21095	76.60201	70.06686	79.12197
77.45925	77.82015	68.29701	85.34481	91.60807	66.1256	84.78579	76.19081	68.78922	80.10092
76.91789	77.24936	68.14415	81.45312	89.74894	66.20399	84.91384	77.64961	69.91955	81.41142
76.69267	77.83666	67.73295	84.19675	92.42687	66.39966	84.70367	77.91867	70.53996	81.68449
77.90627	77.42914	68.48523	83.83197	89.44109	67.89261	84.97113	75.27581	68.89433	83.08731
77.57817	76.48123	69.61377	84.32246	91.02623	66.91262	85.16417	77.87578	70.03292	79.60296
77.46491	78.08003	69.39699	83.8219	89.63855	65.94299	84.3092	76.3302	69.0217	80.00669
77.34701	77.78201	68.9155	84.24937	89.82542	66.23426	85.30651	76.40963	70.27407	79.60275
77.60064	76.6206	68.87842	84.25533	90.09365	66.91692	84.89541	76.37941	69.75234	81.35655
76.73059	78.61587	68.26605	86.14841	90.90539	66.90906	85.38994	76.24821	70.88953	79.32897
76.78446	80.04738	68.75849	83.51274	89.91797	66.09665	84.42982	76.07446	71.02197	79.67047
77.47672	77.43408	68.2557	84.39086	90.33145	66.25295	84.70761	76.65337	70.06966	81.27076
78.22359	77.18635	68.69973	82.85977	89.7823	66.2961	85.14037	76.38935	70.31994	79.09972
77.00801	78.3503	68.23704	83.85555	90.07662	66.05846	85.05866	76.4173	71.23076	81.38338
77.0311	77.38125	69.19625	83.65993	91.12519	66.34422	85.46604	76.3612	70.12597	79.17564
76.56292	77.34209	67.99285	82.77238	89.74075	66.22883	84.92462	76.35705	69.98296	79.27125
77.35102	78.07169	67.80027	83.54125	90.5788	66.40326	85.26793	75.93089	71.21777	79.39938
77.68555	77.6228	68.40876	83.58552	90.74665	66.46578	85.17419	75.88477	69.71862	80.10199
76.72161	77.72123	68.25662	83.413	89.81232	66.53946	85.72062	75.93333	70.29719	80.03239
76.75567	77.07886	67.77566	83.59731	90.32757	66.03721	85.69014	76.32456	70.20609	80.4399
77.05063	77.56923	68.58272	83.68185	89.72307	67.5953	84.77723	76.60273	69.95585	80.09237
77.71844	77.42785	68.5273	83.16756	89.81242	66.24457	85.23808	76.18871	69.79583	80.27717
77.772	77.57774	68.92363	83.3888	90.76375	66.60748	85.72107	76.43824	70.25853	79.53842
77.0612	76.92403	68.41234	83.60631	90.24411	65.85669	84.44458	76.11532	70.13029	80.4607
77.57084	77.88403	68.26578	83.59137	89.22675	66.05911	84.89778	76.28637	69.9425	80.61977
77.09934	77.68973	69.46722	84.4198	90.35661	67.05522	85.54722	75.66064	68.62455	80.23294
76.87161	78.87655	69.25044	83.72597	89.15955	65.06234	85.20279	76.63893	70.26797	78.96102
77.79735	77.72892	68.40376	83.7561	90.06047	66.36407	84.56547	76.42844	70.21048	80.01116
76.48391	78.41515	68.37175	83.51566	88.81497	66.14199	84.84924	76.54957	69.54208	80.81905
77.67029	78.15179	67.82845	83.41123	89.35359	66.05395	84.79949	76.39428	71.29385	80.88285
79.23233	77.61049	67.88996	82.40697	91.98839	66.02001	84.89926	76.10818	69.49399	79.6719
78.58376	77.12494	68.54184	83.70895	90.17969	65.84833	85.48568	76.18209	71.53635	79.72932
76.22204	77.58218	68.15781	83.6036	89.34904	66.73263	84.76582	76.38041	70.06533	80.1437
77.81507	77.36615	68.81817	83.63612	89.10255	66.6103	84.52136	76.5271	71.61226	80.07338

77.95014	76.93433	80.32824	73.25079	76.56956	81.94899	91.80824	73.25291	70.36337
77.76777	76.65618	80.19049	72.82742	76.73991	81.25464	91.31732	74.75688	71.77798
76.55523	76.78018	80.34856	73.24432	76.97493	81.88177	91.67858	72.7527	69.49819
76.32721	76.97785	80.61387	72.73321	77.70723	81.60819	90.86996	72.1737	69.97898
76.88321	77.43236	82.07224	73.26072	76.45846	82.00932	90.82897	73.23799	72.01674
76.40853	76.4687	80.39011	73.06664	77.41518	82.31688	91.37214	73.23524	69.29417
76.33558	76.69047	80.44942	73.0592	77.96958	82.2622	93.00887	73.08755	69.35287
78.6376	77.04969	79.93079	72.03965	77.7594	82.1486	93.62064	72.15812	68.51438
77.88945	77.19919	80.13032	73.26651	77.22343	81.60255	92.40713	72.46463	69.85547
76.71456	76.49345	79.5017	72.38864	76.98712	82.50071	91.22811	73.71407	70.72239
76.9604	76.56544	80.76176	72.67088	77.09782	81.51577	93.2372	73.86433	69.10958
76.83953	76.5453	81.2393	74.50351	76.74949	83.54844	90.8063	73.11522	68.76408
76.42383	76.60577	80.61544	73.41091	77.10637	81.09476	91.38559	74.31094	68.87756
77.9755	76.59276	79.58102	73.17236	77.74158	82.68545	91.70245	72.61696	69.70272
77.91943	76.507	81.15978	73.87073	76.72815	81.57178	91.07833	72.56818	70.07889
77.92546	76.49944	80.80895	73.6114	76.72584	82.21995	91.30881	74.19226	70.56872
77.75847	76.98746	81.16051	73.53102	77.22703	81.53745	90.5718	73.1408	71.78481
78.07347	76.77677	80.88471	72.6998	76.52683	81.72849	91.33082	73.25836	68.9701
78.45078	76.6974	81.23809	73.90592	76.20945	81.63889	90.88954	74.8018	70.14203
77.94607	76.5951	80.28765	73.85267	77.7228	81.08941	90.59839	72.3708	69.70125
78.66229	77.01407	79.86962	72.8916	77.52309	82.02046	92.38323	74.274	68.34909
77.40529	76.31624	79.72725	73.34877	77.27334	82.04153	91.48846	72.53868	71.0794
77.49787	76.51256	80.34707	74.44337	77.09348	82.36664	91.21268	72.72873	70.70647
78.00764	79.12193	80.36435	73.08456	77.08028	81.82925	90.95281	72.57339	70.1961
77.57537	76.51775	80.58557	74.01888	76.97008	81.94413	91.22068	72.25458	71.96529
77.00247	76.63198	80.02174	73.39271	76.91121	81.37421	91.73337	74.26766	68.65348
77.03885	76.66017	80.89669	72.7294	77.46025	81.63215	91.17741	72.23655	70.86028
78.09587	76.37936	80.15791	74.75947	77.08267	81.39872	91.55623	72.3838	69.093
80.0373	76.46263	79.96339	72.71999	77.3724	81.40045	91.51649	72.26721	68.91097
76.2736	76.15604	80.53746	73.23697	77.28648	81.90738	91.16139	72.84697	68.74413
77.59806	76.62223	81.26559	72.82344	77.0369	82.79943	89.79375	72.19742	70.0338
77.88373	76.82869	80.85183	72.69798	77.52164	82.20232	90.79082	72.86346	70.97202
77.30559	76.59431	80.65172	72.89013	76.71514	82.05148	90.95067	73.85185	68.8236
77.39355	76.24465	81.8577	73.2789	77.19789	81.48292	90.81897	73.14319	68.37091
76.56695	76.21507	81.19784	72.7299	75.44681	82.3983	90.82667	72.28957	68.9411
76.31477	76.29668	80.6102	72.76342	77.18382	81.03379	90.41127	71.84855	68.97146
76.82921	76.10228	78.91937	73.42031	77.39874	81.90938	91.12	74.02306	72.85377
78.35592	77.21731	81.14255	72.74599	75.91536	82.91683	90.97679	73.25579	71.26765
77.4188	77.78693	79.75967	73.12803	77.18262	80.24616	91.48073	74.13902	69.90551
77.21035	76.66644	80.23834	73.52609	76.852	81.48047	91.37308	72.41586	69.93555
77.49625	77.0522	80.12768	72.79047	76.52243	83.12837	91.24788	73.09775	68.94038
77.49401	76.67765	81.23239	72.60203	76.84386	81.4982	90.91998	72.62653	70.39994
77.24696	76.379	79.9759	72.7035	79.41798	81.24837	90.59721	72.82781	70.13816
77.42941	76.7575	80.60484	72.48642	76.69616	81.37761	91.56205	73.32679	69.65455
76.94125	76.91839	80.05508	72.71372	76.55979	82.443	90.72949	72.18108	68.28472
76.78321	77.31973	80.19352	73.70984	77.18041	82.58581	91.22314	72.38396	69.18402
77.1437	76.46962	80.85962	74.01812	77.17674	81.81143	91.78353	73.34877	70.34768
75.83003	77.43497	80.15841	73.11258	77.10224	81.94736	90.96532	73.24032	70.06619
75.87123	77.39092	81.96778	72.37647	78.60156	81.93581	91.50887	72.46845	71.64112
77.01505	76.85082	80.9659	73.66709	76.59395	81.33258	91.48905	72.34166	68.58319

A.5 Possible Alternatives

	F1	F2	F3
1	2276.100473	1277928.834	2244.122031
2	2277.325828	1278506.516	2240.623244
3	2282.227585	1314517.713	593.8143332
4	2277.238176	1284707.755	1939.619353
5	2279.036414	1333981.303	-473.359115
6	2280.726353	1317126.755	412.5765039
7	2279.285713	1320631.374	208.6169597
8	2273.975559	1337407.713	-785.6444067
9	2276.687	1324567.406	-72.24727752
10	2279.753305	1323273.643	77.01367702
11	2278.637256	1321367.677	151.5465652
12	2275.167909	1289710.085	1625.115105
13	2275.40476	1325392.987	-144.9112922
14	2278.413479	1310070.491	704.1163318
15	2274.613323	1285055.192	1850.476754
16	2277.955863	1302971.886	1042.875705
17	2278.446688	1301981.769	1109.515512
18	2278.114718	1302885.746	1054.689526
19	2277.57242	1307715.954	794.552494
20	2277.448936	1307245.835	808.677394
21	2279.453313	1311003.555	691.968334
22	2281.428752	1306360.489	978.5093509
23	2276.916924	1308143.927	755.5944303
24	2277.896727	1292241.913	1574.759422
25	2278.562259	1327409.057	-152.6973223
26	2275.410519	1317996.648	222.1726491
27	2275.50972	1331163.153	-434.7257446
28	2272.559535	1273833.09	2349.122003
29	2277.072233	1339649.146	-816.6125533
30	2275.390103	1327721.684	-267.5712138
31	2274.004335	1336007.158	-713.3321766
32	2275.811651	1307708.629	749.6564306
33	2274.620312	1306473.158	768.1811097
34	2277.965031	1310976.214	642.9252159
35	2276.679442	1314028.414	454.48309
36	2276.156408	1323133.269	-17.3776399
37	2276.628753	1315231.828	400.0924028
38	2266.378864	1271012.883	2313.792902
39	2272.179229	1336246.635	-794.1841145
40	2274.661306	1300453.353	1084.860212
41	2274.472095	1324804.001	-146.8592986
42	2272.452308	1275639.651	2244.084374
43	2272.555269	1337412.072	-827.05081
44	2269.981366	1273201.178	2303.050727
45	2276.174783	1313682.308	462.9532925
46	2274.669954	1289707.842	1610.786534
47	2275.340977	1306871.988	764.7889223
48	2272.49946	1276093.745	2241.597078
49	2276.205279	1301605.576	1063.324297
50	2276.744022	1305321.577	896.0477775

	F1	F2	F3
51	2274.864273	1325779.147	-181.3434238
52	2271.531308	1335741.032	-777.5436652
53	2274.283401	1288104.833	1676.826982
54	2274.965981	1309125.163	654.3052768
55	2273.91611	1299261.434	1122.8455
56	2273.573568	1328248.079	-339.320476
57	2273.021665	1330076.942	-455.4688062
58	2272.813958	1329635.966	-446.6934971
59	2274.397336	1329232.907	-371.9226202
60	2274.328782	1299491.637	1111.702822
61	2273.199424	1283653.268	1873.769883
62	2276.369146	1303849.105	950.0999734
63	2271.875585	1280471.005	1998.841726
64	2276.033534	1304493.221	915.4114558
65	2276.045277	1297233.693	1274.378411
66	2274.411334	1288960.455	1639.205919
67	2276.19413	1323018.866	-6.213496766
68	2274.022803	1329245.609	-376.1691509
69	2276.12702	1321163.386	77.36431119
70	2275.40476	1325392.987	-145.64123
71	2260.375061	1264825.756	2447.588977
72	2265.867345	1269648.237	2362.841138
73	2272.680815	1342640.243	-1090.618524
74	2273.516377	1281232.982	2006.875804
75	2279.660614	1294762.517	1505.68198
76	2278.49211	1298009.311	1310.905613
77	2279.022785	1304634.239	987.7988178
78	2277.529208	1295827.596	1390.617269
79	2275.100456	1289604.38	1634.244223
80	2273.751946	1339995.794	-924.4332787
81	2280.414024	1317779.122	381.0505901
82	2276.780421	1324581.547	-77.49510606
83	2279.168919	1323023.479	78.37468875
84	2277.277057	1298262.849	1262.992222
85	2276.827348	1295102.712	1413.75749
86	2275.461717	1335690.292	-658.1247929
87	2277.53737	1283669.482	1991.509659
88	2277.531751	1306403.719	856.0848163
89	2276.819636	1286428.739	1847.232516
90	2281.951371	1289360.102	1845.134683
91	2277.499456	1329899.424	-305.1369494
92	2275.532587	1318121.07	216.5914916
93	2275.876305	1299766.988	1150.06344
94	2278.015918	1280735.635	2160.77989
95	2274.050914	1330262.746	-432.0107793
96	2278.367819	1292768.302	1566.451629
97	2278.654505	1311587.802	636.6905565
98	2268.244362	1269339.246	2445.77419
99	2278.647137	1309689.709	724.1315262
100	2281.046696	1314589.632	548.7225945

A.6 Selected Alternatives for PROMETHEE Ranking

	F1	**F2**	**F3**
1	2260.375061	1264825.756	2447.588977
2	2265.867345	1269648.237	2362.841138
3	2272.680815	1342640.243	-1090.618524
4	2273.516377	1281232.982	2006.875804
5	2279.660614	1294762.517	1505.68198
6	2278.49211	1298009.311	1310.905613
7	2279.022785	1304634.239	987.7988178
8	2277.529208	1295827.596	1390.617269
9	2275.100456	1289604.38	1634.244223
10	2273.751946	1339995.794	-924.4332787
11	2280.414024	1317779.122	381.0505901
12	2276.780421	1324581.547	-77.49510606
13	2279.168919	1323023.479	78.37468875
14	2277.277057	1298262.849	1262.992222
15	2276.827348	1295102.712	1413.75749
16	2275.461717	1335690.292	-658.1247929
17	2277.53737	1283669.482	1991.509659
18	2277.531751	1306403.719	856.0848163
19	2276.819636	1286428.739	1847.232516
20	2281.951371	1289360.102	1845.134683
21	2277.499456	1329899.424	-305.1369494
22	2275.532587	1318121.07	216.5914916
23	2275.876305	1299766.988	1150.06344
24	2278.015918	1280735.635	2160.77989
25	2274.050914	1330262.746	-432.0107793
26	2278.367819	1292768.302	1566.451629
27	2278.654505	1311587.802	636.6905565
28	2268.244362	1269339.246	2445.77419
29	2278.647137	1309689.709	724.1315262
30	2281.046696	1314589.632	548.7225945