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INTERACTION OF AN ADVANCE ORGANIZER AND PERCEPTUAL STYLE IN THE LEARNING AND RETENTION OF MATHEMATICS

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DONALD A. JOSEPHSON

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INTERACTION OF AN ADVANCE ORGANIZER AND PERCEPTUAL STYLE

IN THE LEARNING AND RETENTION OF MATHEMATICS

APPROVED BY

Thomas J. Hill

Larry E. Toothaker

Harold Huneke

Herbert R. Hengst

DISSERTATION COMMITTEE

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CHAPTER I

INTRODUCTION

Background and Theoretical Framework

During the past century many educators and psychologists have attempted to set forth a theory of learning. Perhaps the most ambitious attempt was that of Clark Hull, who listed a set of axioms postulating mathematical relationships among his variables (Hill, 1963). His theory was modified as scientific experimentation dictated until it became one of less precision and containing more intervening variables. Today no learning theory exists that explains all the learning we observe taking place. In fact no theory exists that explains only the learning of mathematics we observe. Such a broad theory of learning may not even exist. But if it does exist, it seems that it would best be approached via a synthesis of research carefully conducted at the molecular level rather than at the molar level.

Within our formal educational system occur many kinds of learning: Motor skills, perceptual learning, concept formation, problem-solving, rote learning, etc. And these various kinds of learning are affected by variables such as emotion, motivation, attitude, etc. Acting as a preserver and transmitter of culture only, formal education is saddled with the gargantuan task of teaching people a vast quantity of knowledge.

David Ausubel (Ausubel, 1960) asserts that meaningful verbal exposition is the most efficient way of teaching subject matter. Verbal learning is not equivalent to rote learning, and meaningful learning is not equivalent to discovery learning. But in order for verbal exposition to imply meaningful learning rather than rote learning, the material must be relatable in a substantive, non-arbitrary, and non-verbatim way to the learner's existing cognitive structure.

According to Ausubel the subsumption process is the key organizational principle of meaningful learning. This model of cognitive organization postulates the existence of a cognitive structure comprised of highly inclusive traces under which are subsumed less inclusive traces and traces of specific informational data. Meaningful learning occurs when more differentiated and less inclusive material is related to more general and highly inclusive knowledge in cognitive structure. Hence, a prerequisite to learning a new body of knowledge in a meaningful way is to have broad anchoring concepts in cognitive structure with which to associate the new body of knowledge.

One strategy of obtaining these broad anchoring concepts in cognitive structure is the use of an advance organizer. An advance organizer is material introduced prior to the learning material itself and presented at a higher level of abstraction, generality, and inclusiveness. Advance organizers provide relevant ideational scaffolding, enhance the discriminability of the new learning material from previously learned related ideas, and effect integrative reconciliation at a level of abstractness, generality, and inclusiveness which is much higher than that of the learning material itself. The use of advance organizers is an attempt to manipulate cognitive structure so that learning will be facilitated.

Cognitive style refers to a psychological dimension which represents a consistency in an individual's approach to acquiring knowledge. During the last half century educators and psychologists have produced a substantial collection of terms to describe individual differences in cognitive style. In recent years these various labels have been consolidated into the following four categories (Ausburn, 1976): (1) perceptual, (2) tempo, (3) differentiation, and (4) memory.

Lowenfeld (1957) identified two types of perceptual style, which he called visual and haptic. He defined the visual type individual as one who reacts to his environment as a spectator and whose main sensory intermediaries are his eyes. He defined the haptic type individual as one who uses his eyes as primary sensory intermediaries only when he is compelled to do so and who reacts to his environment subjectively even though he has normal vision. The haptic prefers to rely on muscular sensations, kinesthetic experiences, and tactile impressions. Among the people that Lowenfeld used in his extensive research, he found that approximately 50% showed visual tendency, 25% showed haptic tendency, and 25% showed no tendency toward either extreme. Lowenfeld listed the following four distinctions between the perception of visual and haptic individuals: (1) While the visual has the ability to see a whole, break it up, see its parts, and resynthesize the parts back into a whole, the haptic is unable to do this. (2) While the visual tends to react to stimuli as a spectator and "see" experiences, the haptic tends to react emotionally to stimuli as one who puts himself into the experience and to "feel" experiences. (3) While the visual has the tendency to visualize tactile experiences and to visually complete partial experiences, the haptic cannot do this. (4) While the visual has the ability to retain visual imagery, the haptic cannot do this.

Kagan (1966) identified two types of tempo style, which he called reflective and impulsive. The reflective-impulsive dimension of cognitive style measures the speed with which alternatives are selected and information is processed in a learning situation. The impulsive individual usually selects the first alternative which occurs to him and is usually subject to error. The reflective individual usually considers all possibilities, takes considerable time before responding, and is usually correct in his response.

Witkin (1962) contrasted two types of psychological differentiation, which he called field independent and field dependent. This dimension of cognitive style measures an individual's ability to overcome an embedding context in a stimulus field. The field dependent individual tends to experience his environment in a global fashion, passively conforming to the influence of the prevailing contextual framework. The field independent individual tends to experience his environment in an analytic fashion, perceiving objects as distinct from their backgrounds.

Klein (1958) identified two types of memory functioning, which he called leveling and sharpening. The leveling-sharpening dimension of cognitive style measures an individual's ability to remember gradual changes in sequentially experienced stimuli. Levelers tend to assimilate new experiences with memories of earlier experiences, constructing relatively undifferentiated impressions of ongoing experiences. Sharpeners tend to maintain discrete impressions and memories of sequentially presented stimuli so that elements do not lose their individuality.

Ausburn (1976) asserts that individuals tend to increase with age in absolute degree of reflectivity, field independence, and sharpening, but to remain stable in relationship to age-group peers. This developmental trend has not been demonstrated for perceptual orientation. Two similarities among these four categories of cognitive style are lack of relationship to general intelligence and strong resistance to external modification. Visuals do tend to display the cognitive style traits of field independence, reflectivity, and sharpening; whereas haptics tend to display the cognitive style traits of field dependence, impulsivity, and leveling.

Purpose of the Study

The purpose of this study is to design an advance organizer, based on the concept of the pre-image of a range element under a function, for antidifferentiating $\int f'[g(x)]\, g'(x)\, dx$ and then to investigate the interaction of this advance organizer with perceptual style of cognition, as measured by the Successive Perception Test I, in the learning and retention of such antidifferentiation. Moreover, this study will investigate the main effect of an advance organizer and the main effect of perceptual style on the learning and retention of such antidifferentiation.

Need for the Study

Important to educators is the location of instructional situations in which learning is facilitated for those of a certain cognitive composition exposed to a certain instructional method. Ausubel and Robinson (1963) assert that the most important factor influencing the meaningful learning of any new idea is the individual's existing cognitive structure at the time of learning. In some instances the state of one's cognitive structure can be advantageously prepared by employing an advance organizer, and in other instances not so. What then are the conditions under which advance organizers prepare advantageously one's cognitive structure to incorporate new learning meaningfully? At the molecular level it would be important to know whether or not exposure to an advance organizer facilitates the learning of mathematics for visually or haptically oriented individuals. No known study has investigated the interactive effects of advance organizers with student perceptual style. If relationships exist between perceptual style and beneficial effects of advance organizers, then the question of when to use advance organizers in a classroom setting is partially answered. Moreover, further research in interactive effects of advance organizers with other dimensions of cognitive style would indeed appear to be worthwhile.

Design of the Study

The sample for the proposed study will consist of students enrolled in either section six or section seven of mathematics 1744 at the University of Oklahoma during the fall semester, 1977. All students in one of the sections will receive the advance organizer and will be designated the experimental group. All students in the other section will receive additional applications of the calculus of exponential functions in lieu of receiving the advance organizer and will be designated the control group.

The advance organizer will be constructed according to the guidelines set forth by Ausubel and will be administered orally. Perceptual style will be assessed by the motion picture testing instrument entitled *Successive Perception Test I*. Learning material will be taken from section 6-1 of Saltz's *A Short Calculus*. Two written tests will be constructed-- one for measuring initial learning and one for measuring retention.

Hypotheses

Data collected from the initial learning and retention tests and from the motion picture testing instrument will be used to test the following six hypotheses:

1) Initial learning is facilitated by employing an advance organizer.

2) There is a differential relationship between initial learning and perceptual style.

3) Advance organizers better facilitate initial learning for haptic individuals than for visual individuals.

4) Retention is facilitated by employing an advance organizer.

5) There is a differential relationship between retention and perceptual style.

6) Advance organizers better facilitate retention for haptic individuals than for visual individuals.

The following two reasons are given for stating the direction of interaction in hypotheses three and six. First, Caponecchi (1973) concluded that low ability students benefited from receiving either an advance organizer or an introductory overview. Second, Ausburn (1976) asserted that haptic individuals are not quite as skilled in reading ability as are visual individuals of the same age group.

Analysis of Data

In this study the unit of statistical analysis will be the student. A 2 X 3 factorial analysis of variance will test the hypotheses, the six cells being experimental-visual, experimental-neutral, experimental-haptic, control-visual, control-neutral, and control-haptic. F-ratios for the main effects and the interaction will be reported. In addition, an individual comparison involving four cell means will be used to test the third and sixth hypotheses. The level of significance will be .05, and type I error will be controlled for at the per hypothesis level. In the event of failure to reject a null hypothesis, the statistical power to detect a difference of one standard deviation from the mean under the null hypothesis will be determined from the Pearson-Hartley charts.

CHAPTER II

REVIEW OF RELATED LITERATURE

Advance Organizers

During the past decade there has been a considerable amount of research investigating the facilitating effects of advance organizers on learning mathematics. But the results have been conflicting and confusing. Under what conditions are advance organizers beneficial remains an unanswered question. Scandura and Wells (1967) compared an organizer in game format with an historical introduction for facilitation of learning group theory and combinatorial topology in a college course for elementary education majors. They determined that the group receiving the organizer performed significantly better on posttests.

Grotelueschen and Sjogren (1968) found that advance organizers facilitated learning and transfer of number base concepts for adults, especially for those of superior intelligence when the material was partially sequenced. Gubrud (1970) conjectured that advance organizers facilitated the learning of vector addition for junior and senior high students with high abstract thinking ability.

However, Ratzlaff (1970) could not support Ausubel's theory when he applied an advance organizer to seventh grade students learning addition and multiplication in base five and base eight arithmetic. Moreover, Nixt (1972) obtained no significant differences on posttest scores between college students receiving advance organizers and a control group. All students tested were enrolled in a pre-calculus mathematics course for non-physical science majors.

Romberg and Wilson (1973) defined "cognitive set" as information given to students prior to instruction that informs them of anticipated associations they can expect to acquire in the instruction. They attempted to separate cognitive set from both advance organizer and post organizer. They then investigated the effects of no organizer, advance organizer only, and both organizers under presence and absence of cognitive set. The subjects were eleventh grade algebra students studying the mathematics of radioactive decay. They found that irrespective of cognitive set, learning was superior when either the advance organizer or post organizer was used, but learning was inferior when no organizer or both organizers were used. They also found that retention was superior in the presence of cognitive set.

Peterson, Thomas and Lovett, and Bright (1973) each independently attempted to substantiate the findings of Romberg and Wilson. Their learning material was tracing of networks. Of the three replications only Thomas and Lovett confirmed the findings of Romberg and Wilson. Their subjects were eighth grade students, as were Peterson's. Bright used students enrolled in an undergraduate class in mathematics for elementary school teachers. Peterson and Bright detected no significant effects among advance organizer, post organizer, and cognitive set.

Johnson (1973) administered advance organizers in the form of games, manipulatives, and applications to fourth grade students studying transformational geometry. His purpose was to synthesize Ausubel's theory of advance organizers with Piaget's theory of centering and egocentrism. On the basis of posttest scores, the advance organizer significantly increased the child's ability to decenter and the child's ability to view his own percepts, concepts, and viewpoints as one of many possible interpretations of reality.

Caponecchi (1973) investigated the effect of an advance organizer on the learning of matrix algebra in a pre-calculus mathematics class for non-physical science majors. The concept of a ring was used as the advance organizer, and the effect of the organizer was compared to that of an introductory overview and of an introductory historical passage. Although Ausubel's theory was not supported, the low ability students receiving either the organizer or the overview scored significantly higher on posttests than the low ability students receiving the historical passage.

Montano Midence (1974) compared the effects of an audiovisual advance organizer, a written advance organizer, and no advance organizer on the learning of permutations in a lower division college mathematics class. Ability was considered a concomitant variable. There were no significant differences.

Kennedy (1974) investigated the effects of an advance organizer, an historical introduction, and a control with respect to cognitive structure (the number of high school and college courses in mathematics and science completed) and student ability (the subject's grade point average). Subjects were enrolled in a physical science course for elementary school teachers, and the learning material was metric system concepts. Results showed that the advance organizer was significantly more effective than the historical introduction, which in turn was more effective than the control.

Hartje (1975) constructed two advance organizers for the learning of concepts from elementary group theory: (1) a discussion of axiomatic systems, and (2) a discussion of mathematical systems. Subjects were of two kinds: (1) those enrolled in an advanced mathematics course in twelfth grade, and (2) those enrolled in a mathematics course for elementary education majors with minimal secondary mathematics preparation. The college and high school samples were analyzed separately. There were three treatment levels: (1) both organizers, (2) the mathematical systems organizer only, and (3) control. No significant differences were found for the college group. For the high school group both the multiple organizer and the single organizer facilitated learning that involved algorithmic thinking. Post hoc analysis revealed that the multiple organizer was superior to the single organizer in learning identity and inverse concepts.

Swaney (1974) defined a summarizer to be information introduced in advance of relearning previously studied material. He called it a new organizer because it presented the material at a higher level of abstraction, generality, and inclusiveness than during the original instructional phase. Swaney investigated the effects of three types of summarization. In order of increasing levels of abstraction they were: A summary (recapitulation of previously learned material) with problem format, a summarizer with problem format, and a summarizer with behavioral objective format. The subjects were college students enrolled in a first semester calculus course, and the learning material was the entire course content. No significant differences were obtained.

Andreozzi (1975) defined written verbalizations as written responses to a series of questions designed to (1) relate new concepts to relevant concepts in the learner's cognitive structure, (2) recall similar concepts in cognitive structure, and (3) discriminate salient features of new concepts. He considered the effects of an advance organizer with written verbalizations on the learning of ninth grade algebra. Although no significant results were obtained, Andreozzi concluded that written verbalizations used in conjunction with advance organizers seemed more effective in increasing retention than written verbalizations used in conjunction with control introductions (historical passages and examples similar to those solved in class).

Graber (1975) investigated the effects of three types of organizers with three rates of questioning used in the lessons. Subjects were undergraduates enrolled in a precalculus mathematics course, and the material to be learned was a lesson on the ellipse. The nine treatment levels led to the following four hypotheses, only the last of which was supported: (1) One organizer will facilitate learning more than another organizer. (2) A given amount of time spent on advance organizers facilitates learning more than the same amount of time spent on concluding examples. (3) Different rates of questioning will differentially facilitate learning. (4) Different combinations of organizers and rates of questioning will differentially facilitate student learning. Students who were presented one of the organizers and four questions scored significantly higher than both those who were presented no organizer and four questions and also those who were presented the same organizer and eight questions. Though not significantly different, the students receiving organizers and four questions scored slightly higher than those receiving organizers and twelve questions. This trend was reversed for students receiving no organizer.

Lesh (1976) conducted three experiments comparing advance organizers with post organizers and in each experiment concluded that advance organizers facilitate learning significantly better than do post organizers. In the first experiment both fourth graders and seventh graders were involved in studying motion geometry. The design was a 2 X 2 X 2 X 2 factorial analysis of variance in which the independent variables were type of organizer (advance vs. post), type of examples (models vs. applications), number of examples (one vs. several), and number of children working together on the organizer (individual vs. small group). For fourth grade students the models organizers were superior to the applications organizers, and the organizers with several examples were superior to those with only one example; whereas for seventh grade students, those who worked in groups performed better than those working individually. In the second experiment college geometry students were involved in studying finite geometries. The design was a 2 X 2 analysis of covariance using the independent variables, type of organizer (advance vs. post) and type of unit (hierarchy vs. spiral), and using midterm examination scores as covariate. Posttest scores were significantly higher for students receiving the spiral unit than for those receiving the hierarchy unit. The following significant interaction was obtained: The difference between the advance organizer and post organizer posttest scores was greater for the hierarchy unit than for the spiral unit. In the third experiment modern algebra students were involved studying finite groups. The design was a 2 X 2 analysis of covariance using independent variables, type of organizer (advance vs. post) and type of models (examples vs. counterexamples), and using midterm examination scores as covariate. It was found that counterexamples in organizers were more effective than examples in organizers.

Bright (1976) conducted two studies, each using two advance organizers at different levels of abstraction. Subjects were elementary education majors, and learning material was concepts of integer addition. The two organizers for this learning material were mathematical field and mathematical system, the latter being at a higher level of abstraction. The second study differed from the first in that programmed recall of an advance organizer during instruction was used to determine differential achievement resulting from several levels of reinforcement of the organizer. The two hypotheses tested were the following: (1) The level of abstraction and inclusiveness of advance organizers does not affect learning of the concepts of integer addition; and (2) Repeated recall of an advance organizer during instruction does not affect learning of the concepts of integer addition. Bright concluded that although students of the second study receiving the field organizer scored higher on posttests than students receiving the system organizer, results were not meaningful in a practical sense, both because this trend did not appear in the first study and also because Ausubel's theory might suggest the opposite result, field axioms being less general and inclusive than mathematical systems.

Zakkour (1977) investigated the interactive effects of an organizer with personality types as measured by Myers and Briggs. Subjects were enrolled in a pre-calculus mathematics course for non-physical science majors. The learning material was matrix algebra with the concept of operation serving as organizer. There were three treatment levels (advance organizer, post organizer, and control) and four personality dimensions (extraversion vs. introversion, sensing vs. intuition, thinking vs. feeling, and judging vs. perceiving). The only significant result obtained was that the advance organizer was superior to the control in facilitating learning restricted to the judging-perceiving dimension.

Goodman (1977) conducted a study consisting of four treatments: (1) An advance organizer, (2) an example of a problem that can be solved using statistics and instructions to write summaries for each learning section (generative processing cues), (3) a combination of the first two, and (4) an historical passage. Subjects were ninth and tenth grade geometry students, and subject matter was descriptive statistics. Students were classified as lower ability or higher ability based on SAT mathematics scores. The results showed no significant effect due to treatment and no significant interactions between treatment levels and ability. However, there was a significant effect due to ability. From post hoc analysis of the data, the following three conclusions were drawn: (1) Students can learn the concepts and skills in descriptive statistics; (2) For many students it may be necessary to include relevant structuring cues throughout the instructional sequence; (3) Having students write summaries of learning material can be expected to enhance learning of mathematics, especially for higher ability students; however, the effect of writing summaries may depend on the presence of relevant advance organizers.

Cognitive Style

Only two studies have been found which attempt to investigate the interactive effects of advance organizers and cognitive styles--both in areas of learning other than mathematics. Price (1973) constructed a single paragraph advance organizer for learning about neurotic disorders and a single paragraph advance organizer for learning about psychotic disorders. Subjects were junior college students enrolled in an introductory psychology course. Ausubel's Cognitive Style Instrument (unpublished) was used to measure cognitive style along the dimension labeled "generalizing and particularizing". Generalizers are individuals who tend toward broad categorization, subsuming new material under more inclusive existing concepts than do the particularizers, who are more specific in the information they incorporate and retain. Generalizing seems to be analogous to the Kagan clan's "global", and particularizing to "analytic". Ausubel's instrument consists of seven paragraphs dealing with various customs of cultures foreign to Americans. Each subject was classified as either a generalizer or a particularizer, and each was aware from the beginning that he was involved experimentally. The experimental results did not support an affirmative answer to any of the following questions: (1) Does acquisition or retention differ for the two cognitive styles? (2) Do advance organizers facilitate acquisition or retention? (3) Is there an interaction between advance organizers and cognitive style?

Shmurak (1974) posed the following two questions: (1) Is there an interaction between categorization style of student and style of advance organizer such that a match between the two facilitates learning and retention of expository science material? (2) Are there sex differences in the interaction between categorization style of student and style of organizer? Categorization style refers to an individual's preference for criteria used to group objects, as measured by the Sigel Cognitive Style Test. Categorical-inferential style refers to a categorization style in which objects are grouped on the basis of inferred attributes or class membership ("mammels", "public servants", etc.). Relational style refers to a categorization style in which objects are grouped on the basis of functional interdependence or other relationship attributes ("boy lives in house", "woman is baby's mother", etc.). Descriptive style refers to a categorization style in which objects are grouped on the basis of similar physical attributes. This may be subdivided into two styles, descriptive-global and descriptive-part-whole, depending on whether the attribute selected characterizes the entire object ("male", "black", etc.) or a part of it ("wearing shoes", "holding something", etc.). Here again relational seems to be analogous to "global", and descriptive-part-whole to "analytic". Subjects were eighth grade students, and the learning passage was entitled "The Pancreas, Insulin, and Diabetes". Three single page advance organizers were constructed to match the three categorization styles of categorical-inferential, relational, and descriptive-part-whole. The three corresponding experimental groups were determined from results on the Sigel Cognitive Styles Test. There was also a control group receiving an interest-creating introductory passage. An affirmative answer to neither question could be given. Moreover, Ausubel's theory could not be substantiated.

Perceptual Style

The potential and importance of perceptual style to the investigation of meaningful learning is underscored by the literature supportive of Lowenfeld's classification, very impressive among which is the research in physiology. Drewes (1958) used an electroencephalograph to measure brain alpha rhythms of subjects who were manipulating geometric figures on a table top to form various combinations. Since alpha rhythm typically ceases when a visual image is seen or suggested, Drewes concluded that those individuals recording continuous change in alpha rhythm patterns were forming no visual images, while those recording no change in alpha rhythm patterns were continuously forming visual imagery. Based on his experiment, Drewes classified individuals as visualizers, non-visualizers, and responsives, the first two groups corresponding rather well to Lowenfeld's groups of visual and haptic, respectively.

Walter (1963) also used an electroencephalograph to study alpha patterns. His results indicated that individuals with continuous change in alpha patterns tended to tactile perceptions rather than visual ones. He suggested that these alpha rhythm patterns are hereditary, thus explaining their robustness to change, even when attempts are made to block them with mental effort.

Nebes (1975) hypothesized that whereas the left hemisphere of the brain is the seat of verbal literacy, the right hemisphere of the brain is the seat of visual literacy in most people. This concept of hemispheric lateralization raises the possibility that visuals have a greater degree of development or dominance of the right hemisphere of the brain than have the haptics.

Wheatley, Mitchell, Frankland, and Kraft (1978) reviewed recent studies that support hemispheric lateralization of the brain. In addition to electroencephalography, these studies have involved lesions of one hemisphere, differences in size and shape between the hemispheres, surgical disconnection of the hemispheres, dichotic listening, tachistoscopic presentation of stimuli to only one hemisphere, and anesthetization of just one hemisphere with sodium amytal. A synthesis of these studies does indicate that the left hemisphere is superior in performing logical tasks, whereas the right hemisphere is superior in performing visio-spatial tasks.

CHAPTER III

IMPLEMENTATION OF THE STUDY

Determination of the Sample

In order to test the six hypotheses listed in the first chapter, the experimenter was given sections six and seven of Mathematics 1744 as his teaching assignment at the University of Oklahoma during the fall semester of 1977. Mathematics 1744 is intuitive differential and integral calculus of the elementary functions with associated analytic geometry and applications. There were sixteen sections of this four credit hour course offered at the University of Oklahoma during the fall semester of 1977. In each section class size was not allowed to exceed forty students. A course outline is provided in Appendix C.

Enrollment for sections six and seven of Mathematics 1744 was not controlled. However, section six met from 9:30--10:20 AM, MTThF, PHSC 321; and section seven met from 10:30--11:20 AM, MTThF, PHSC 116. This proximity in time and this variance in location helped to reduce contamination in the experiment. Each Wednesday there was an optional help session for each section at the regularly scheduled time in the regularly scheduled location. One week prior to the administration of the advance organizer, section six was randomly selected as the experimental group.

With but very few exceptions the students enrolled in Mathematics 1744 are non-physical science majors, the majority of whom are business majors. Only those students enrolled in section six or section seven of Mathematics 1744 who successfully completed every phase of the study constituted the final sample. The final sample consisted of fifty-one students, thirty-five of which were males. There were eleven freshmen, eighteen sophomores, thirteen juniors, six seniors, and three graduate students.

The Advance Organizer

On November 7 the experimental group was presented the advance organizer (Appendix D), constructed according to the guidelines suggested by Ausubel. The concept of pre-image of a range element under a function served as the abstraction and highly inclusive material, under which the learning unit as well as several ideas from prior mathematical training was subsumed. The organizer was presented during the first half of the fifty minute period in an oral fashion with use of the chalk board to display equations and diagrams. During the second half of the period, the special quiz (Appendix E) was administered to obtain some indication of reception of the organizer.

There was insufficient class time for students to receive immediate feedback on the special quiz performance. Therefore, after examining the special quiz results, the instructor attempted to telephone those students whose performances were weak. Only those students who responded correctly to all parts of question #1 and at least two parts of question #4 or who revealed understanding of these parts during the course of a telephone conversation were deemed to have adequate reception of the organizer. And only these students were allowed in the experimental group.

During the week preceding November 7, both sections had studied the calculus of the exponential and logarithm functions and applications thereof. On November 7 the control group spent class time analyzing the graph of $f(x) = x^2e^x$ and working the following problem:

> A population is known to experience exponential decay. The initial population (that is, the population at time 0) is found to be 10,000 objects. At the end of 20 minutes, the population is found to be 5000 objects. When will there be 1000 objects in the population? When will there be 100 objects in the population? How fast is the size of the population changing when there are 100 objects in the population?

The Learning Material

The unit of learning under experimental consideration was the antidifferentiation, $\int f'[g(x)]\, g'(x)\, dx$, especially where f represents a power function, an exponential function, or the natural logarithm function. This technique of integration is described in Saltz's A Short Calculus section 6-1 by way of the substitutions, $u = g(x)$ and $du = g'(x)\, dx$. However, instead of being assigned this reading portion of the text, the classes were taught to observe directly that $f'[g(x)]\, g'(x)$ is the derivative of $f[g(x)]$, applying the chain rule for differentiating a composition of functions. If the integrand were different from $f'[g(x)]\ g'(x)$ by only a constant factor, the classes were instructed to multiply the integrand by that constant factor together with its reciprocal and then to apply judiciously the fact that the antiderivative of a constant times a function is the constant times the antiderivative of the function.

On November 8 both sections were instructed in this method of integration--reversing the chain rule for differentiating a composition of functions. This instruction lasted for the first half of the period. During this learning session students were reminded of the following three applications of the chain rule for differentiating a composition of functions:

1) $D_x[f^r(x)] \quad = \quad r\ f^{r-1}(x)\ f'(x)$

2) $D_x[a^{f(x)}] \quad = \quad a^{f(x)}\ f'(x)\ \ln(a)$

3) $D_x[\ln|f(x)|] \quad = \quad f'(x)/f(x)$

Each of these three differentiation formulas yields a corresponding antidifferentiation formula. The following three problems were then presented, illustrating how to facilitate integration by making the integrand look exactly like the right-hand side of one of the above three equations:

1) $\int x^2(x^3+1)^4\ dx \quad = \quad \frac{1}{15}\int 5(x^3+1)^4 3x^2\ dx$

2) $\int x2^{x^2}\ dx \quad = \quad \frac{1}{2\ \ln(2)}\int 2^{x^2}(2x)\ln(2)\ dx$

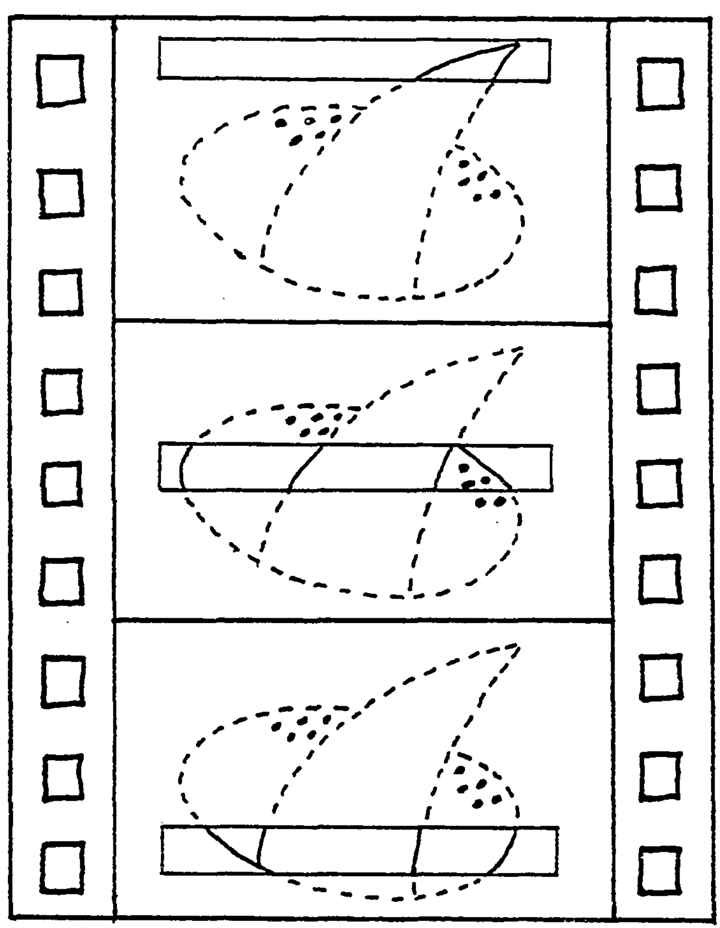
3) $\int \frac{x}{4x^2+1}\ dx \quad = \quad \frac{1}{8}\int \frac{8x}{4x^2+1}\ dx$

The initial learning quiz was administered during the second half of the period. (See Appendix E.)

Determination of Perceptual Style

On December 12 both sections were administered the Successive Perception Test I (SPT-1) (Army Air Corps, 1944). SPT-1 is a motion picture designed by J. J. Gibson in 1944 at the suggestion of Lowenfeld (Erickson, 1969). Although this instrument was developed for military use, the precedent exists for its use in educational settings (Erickson, 1969; Ausburn,

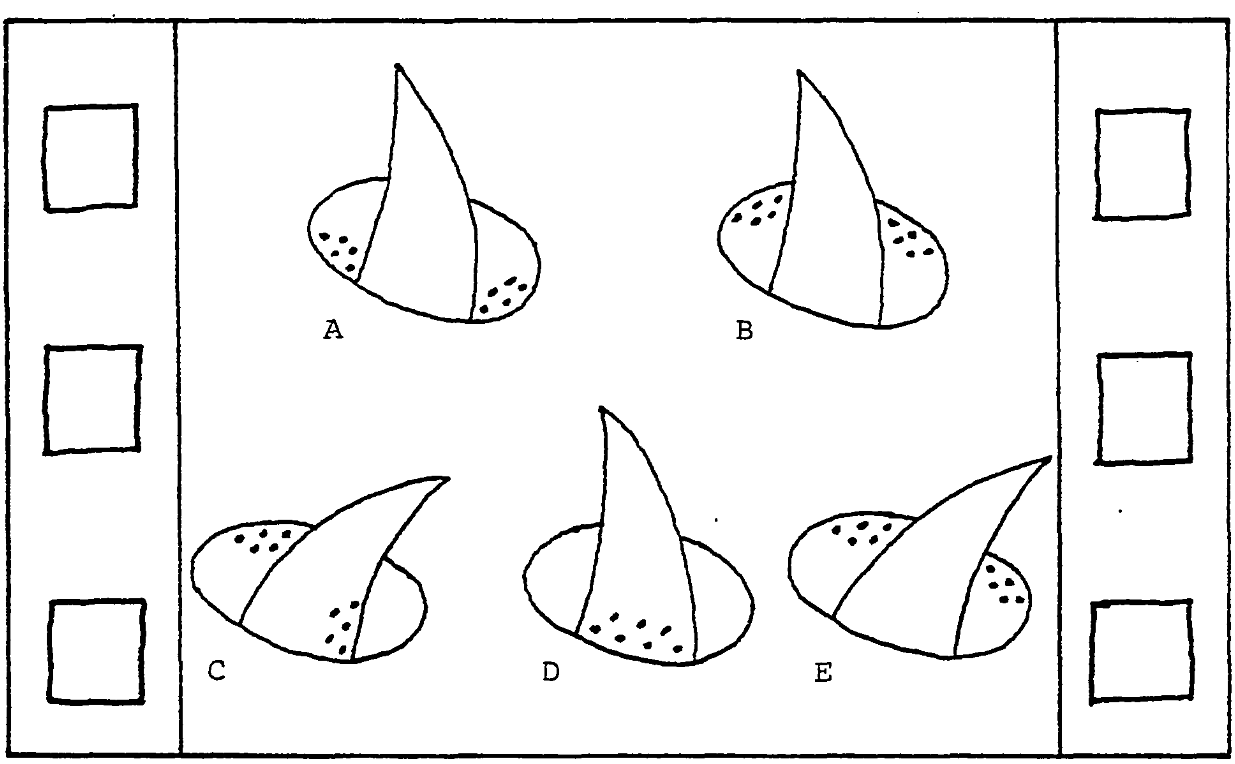
Figure 1. Stages in a sample item of SPT-1. The dotted lines are hidden lines in an actual test item.



1976). The basis on which this test classifies individuals as to perceptual type is the visual's ability to see first a whole without an awareness of details, then to analyze this whole into partial impressions, and finally to build these partial impressions into a new synthesis of the whole. The haptic individual does not possess this capability, and is satisfied with internalizing the separate segments in their partial forms.

SPT-1 consists of thirty-eight items, the first three of which serve as practice items. This instrument utilizes a horizontal slot moving from top to bottom across the screen, behind which a small portion of a figure is shown (Figure 1). The students are then shown five similar figures from which

Figure 2. The five alternative responses corresponding to the sample item shown in Figure 1.



they are required to match the correct response with the figure already shown in partial form (Figure 2).

Those students who were absent on December 12 arranged with the instructor a time to respond to SPT-1 before the conclusion of the semester. That these absentee testings were individually administered rather than group administered was considered to be immaterial in obtaining a measure of perceptual type.

The Retention Test

The retention test was built into the final examination. The experimental group took the final examination on Friday, December 16, from 8:00--10:00 AM. The control group took the final examination on Tuesday, December 20, from 8:00--10:00 AM. Although there were two forms of the final examination, the retention test was the same for both sections. The retention test consisted of the integration parts of problems 1b, 1c, and 1d, together with 3b, 3d, 3e, and the finding of g(x) in problem 4. (See Appendix E.)

The students were never told that they were a part of experimental research. During the administration of SPT-1, the students were told that the instructor wanted information obtained from their response to the film in order to help with the evaluation process in a general sense, and that their grade depended in no way upon their response. The results on SPT-1 were not evaluated until after the retention test was graded.

CHAPTER IV

ANALYSIS OF THE DATA

Scoring

The initial learning test consisted of ten antidifferentiation problems, each problem worth one point. No fractional credit was given. The retention test consisted of seven problems, each problem worth one point. Again no fractional credit was given. Scoring of these two tests was done by the instructor. The raw score distribution of these two dependent variables is presented in Appendix A.

Since the directions given in SPT-1 are for students to answer each question and since the test is multiple choice with five selections for each answer, a penalty of 20% of the number of wrong answers was exacted. After this correction factor for guessing was made, the visual student was operationally defined to be one who responded correctly to more than 57% of the items in SPT-1. Those who responded correctly to less than 43% of the items in SPT-1 were operationally defined as haptic individuals. And those who scored between 43% and 57% on SPT-1 were operationally defined as neutral. The number of items missed on SPT-1 before applying the correction factor is presented in Appendix A. Responses to SPT-1 were scored by the instructor.

The number of students classified as visual, neutral, and haptic were sixteen, sixteen, and nineteen, respectively. Therefore, it was hypothesized that for the population of students enrolled in mathematics 1744 there was an equal number of students in each of the three classifications, visual, neutral, and haptic. Since unequal cell sizes occurred for reasons unrelated to the experimental design, an unweighted means two-way analysis of variance was used.

Tests of Hypotheses for Initial Learning

The three null hypotheses corresponding to the hypotheses for initial learning listed in the first chapter are the following:

1) There is no significant difference in initial learning test mean scores between the experimental group and the control group.

2) There is no significant difference in initial learning test mean scores among the visual, neutral, and haptic subjects.

3) There is no significant positive difference between the following two numbers: (1) The initial learning test mean score of the haptic individuals in the experimental group minus the initial learning test mean score of the visual individuals in the experimental group, and (2) the initial learning test mean score of the haptic individuals in the control group minus the initial learning test mean score of the visual individuals in the control group.

The summary table for initial learning, listing cell sizes n_{ij} and cell means $\overline{X}_{ij}$, is displayed in Table 1. The analysis of variance for initial learning is displayed in Table 2. The formulas used to obtain these figures are given in Appendix B. (See Kirk, 1969, pages 200-204.) One sees from studying Table 2 that there are no F scores significant at the .05 level. Since the F-interaction score is extremely low, the t-statistic for testing the third hypothesis was not employed.

Table 1. Summary Table for Initial Learning

	Visual	Neutral	Haptic
Experimental	$n_{11} = 8$ $\overline{X}_{11} = 4.88$	$n_{12} = 7$ $\overline{X}_{12} = 2.00$	$n_{13} = 10$ $\overline{X}_{13} = 2.60$
Control	$n_{21} = 8$ $\overline{X}_{21} = 4.88$	$n_{22} = 9$ $\overline{X}_{22} = 3.56$	$n_{23} = 9$ $\overline{X}_{23} = 3.11$

Table 2. Analysis of Variance for Initial Learning

Source	MS	DF	F	P
Total	10.64	50		
Between	11.74	5		
Treatment	5.97	1	.57	.53
Perceptual Type	23.73	2	2.26	.11
Interaction	2.64	2	.25	.78
Within	10.52	45		

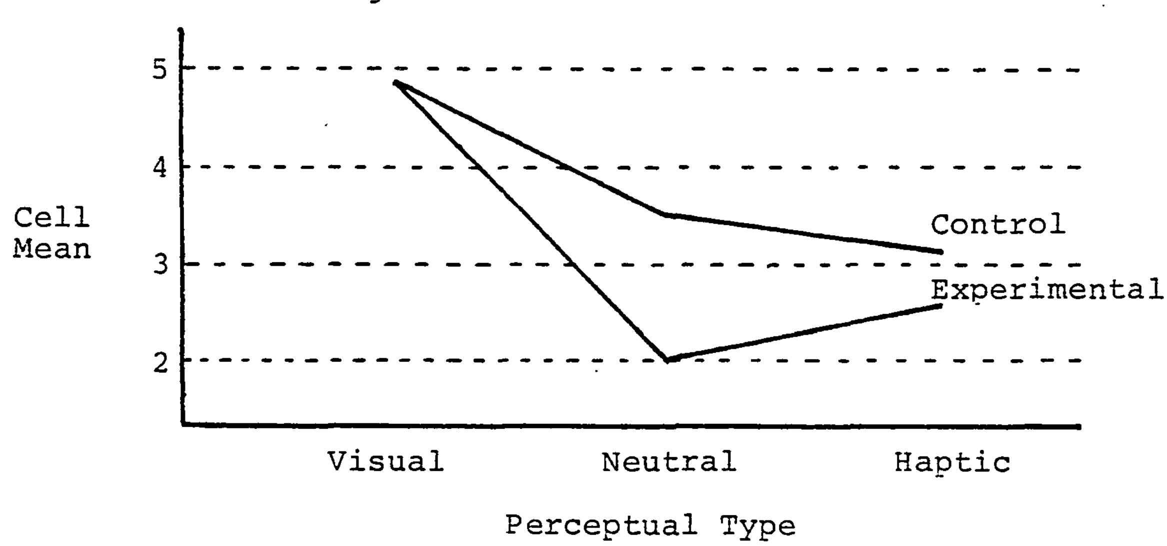
Therefore, none of the three null hypotheses concerning initial learning is rejected.

However, the following trend is observed from the relatively high F-statistic on the perceptual type. The visuals tend to perform better on initial learning than either the neutrals or haptics--irrespective of the presence of an advance organizer. This is clearly indicated in Figure 3, which displays the effect of the organizer across the three levels of perceptual style. It is also seen from Figure 3 that the control group had a higher mean score on the initial learning test than had the experimental group.

Tests of Hypotheses for Retention

The three null hypotheses corresponding to the hypotheses for retention listed in the first chapter are the following:

Figure 3. Graphic Representation of Interaction of Treatment and Perceptual Type on Initial Learning.



4) There is no significant difference in retention test mean scores between the experimental group and the control group.

5) There is no significant difference in retention test mean scores among the visual, neutral, and haptic subjects.

6) There is no significant positive difference between the following two numbers: (1) The retention test mean score of the haptic individuals in the experimental group minus the retention test mean score of the visual individuals in the experimental group, and (2) the retention test mean score of the haptic individuals in the control group minus the retention test mean score of the visual individuals in the control group.

The summary table and the analysis of variance for retention are given in Table 3 and Table 4, respectively. One sees from studying Table 4 that there are no F scores significant at the .05 level. Hence, hypotheses four and five are not rejected. However, the relatively high F-statistic on the interaction suggests that hypothesis six (the planned comparison) may be significant. This strong interaction is

Table 3. Summary Table for Retention

	Visual	Neutral	Haptic
Experimental	$n_{11} = 8$ $\overline{X}_{11} = 3.75$	$n_{12} = 7$ $\overline{X}_{12} = 3.14$	$n_{13} = 10$ $\overline{X}_{13} = 4.50$
Control	$n_{21} = 8$ $\overline{X}_{21} = 4.75$	$n_{22} = 9$ $\overline{X}_{22} = 3.67$	$n_{23} = 9$ $\overline{X}_{23} = 3.11$

displayed graphically in Figure 4.

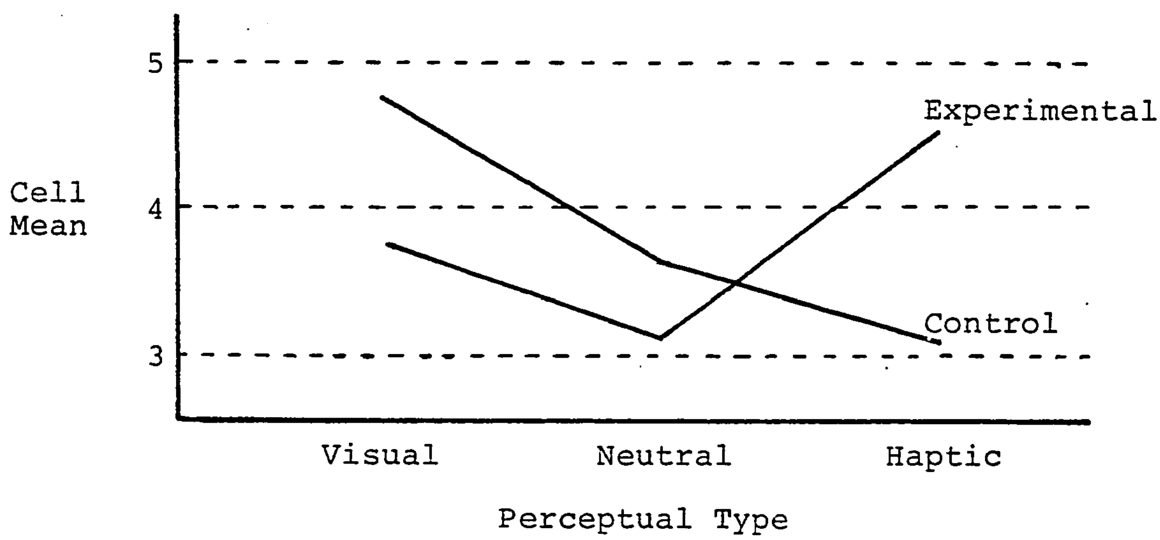
In order to test the sixth hypothesis, the following t-statistic was used:

$$t = \frac{1}{2}[(\overline{X}_{13}-\overline{X}_{11}) - (\overline{X}_{23}-\overline{X}_{21})]\tilde{n}^{.5}[MS(within)]^{-.5}$$

Table 4. Analysis of Variance for Retention

Source	MS	DF	F	P
Total	3.53	50		
Between	3.89	5		
Treatment	.03	1	.01	.98
Perceptual Type	3.00	2	.86	.57
Interaction	6.71	2	1.92	.16
Within	3.41	45		

Figure 4. Graphic Representation of Interaction of Treatment and Perceptual Type on Retention.



where $\tilde{n}$ represents the harmonic mean of the six cell sizes, and MS(within) represents the within mean square (Appendix B). The value of this t-statistic is 1.85. The value .035 is obtained when Student's t-distribution with forty-five degrees of freedom is integrated over the interval $[1.85, \infty)$. Therefore, the sixth hypothesis is rejected.

Power

Since there was failure to reject the first five null hypotheses, a word about statistical power is in order. If in fact a null hypothesis is false, then some alternative hypothesis is true. Each null hypothesis of this study can be stated in terms of a zero difference of means. In such terminology one often says that the mean under the null hypothesis is zero--meaning that the mean of the sampling distribution of differences of means is zero. If the mean of the sampling distribution of differences of sample means is non-zero--i.e., if an alternative hypothesis is true--then one can be concerned with the error of failing to reject the null hypothesis. Seen from a bit different perspective, the concern can be expressed as the following question: What is the probability of rejecting the null hypothesis, given the truth of some alternative hypothesis? This probability is called the power of the test. Of course this probability depends upon the mean under the alternative hypothesis. Often this mean is expressed in units of standard deviations of the sampling distribution under the null hypothesis. The larger the absolute value of the mean under the alternative hypothesis, the greater is the power to detect this difference, other factors being held constant.

If the null hypothesis is false, then the F-statistic obtained from the analysis of variance actually has a sampling distribution that is not an F distribution, but rather is a noncentral F distribution. In addition to depending upon the degrees of freedom in the numerator and denominator of the F quotient, the noncentral F distribution depends upon a noncentrality parameter ϕ. For the design of this study, there are three noncentrality parameters. These correspond to the treatment F, the perceptual type F, and the interaction F. These noncentrality parameters are defined as follows:

1) Treatment F: $\phi = [3\tilde{n}(a_1{}^2+a_2{}^2)]^{.5}(2s^2)^{-.5}$

2) Perceptual Type F: $\phi = [2\tilde{n}(b_1{}^2+b_2{}^2+b_3{}^2)]^{.5}(3s^2)^{-.5}$

3) Interaction F: $\phi =$
$[\tilde{n}(ab_{11}{}^2+ab_{12}{}^2+ab_{13}{}^2+ab_{21}{}^2+ab_{22}{}^2+ab_{23}{}^2)]^{.5}(3s^2)^{-.5}$

In the above formulas $\tilde{n}$ represents the harmonic mean of the cell sizes, s^2 represents the variance of the sampling distribution of the mean under the null hypothesis, a_i represents a fixed treatment effect, b_j represents a fixed perceptual type effect, and ab_{ij} represents an interaction effect.

Differences of effects (and hence differences of means) are often expressed in units of s. If the mean of the sampling distribution under the alternative hypothesis is rs, where r is a positive real number, then each of the following possibilities minimizes the appropriate noncentrality parameter:

1) $a_1 = \frac{-rs}{2}$ and $a_2 = \frac{rs}{2}$.

2) $b_1 = \frac{-rs}{2}$, $b_2 = \frac{rs}{2}$, and $b_3 = 0$.

3) $ab_{11} = ab_{22} = \frac{-rs}{2}$, $ab_{12} = ab_{21} = \frac{rs}{2}$, and $ab_{13} = ab_{23} = 0$.

With r=1 the minimum values for noncentrality parameters corresponding to sampling distributions whose means are one standard deviation from the mean under the null hypothesis can be obtained. These values are displayed in Table 5. With the level of significance set at .05 and with ø and the degrees of freedom for numerator and denominator corresponding to each F-statistic known, the power is read from the Pearson-Hartley charts. Since power is an increasing function of ø, Table 5 lists the minimum power to detect a difference of one standard deviation from the mean under the null hypothesis.

Table 5. Noncentrality Parameters and Corresponding Power to Detect Differences of one Standard Deviation.

F	ø	Power
Treatment	2.51	.93
Perceptual Type	1.67	.70
Interaction	1.67	.70

CHAPTER V

CONCLUSION

Recapitulation of the Study

According to Ausubel, meaningful learning occurs when the learner has within his cognitive structure broad anchoring concepts under which the learning material can be subsumed. This ideational scaffolding can be constructed by employing an advance organizer. Although Ausubel provides no operational definition of an advance organizer, he does set forth guidelines. An advance organizer should be material that is presented at a higher level of abstraction, generality, and inclusiveness than the material to be learned.

Many studies have been conducted during the past decade to investigate the effect of advance organizers upon learning mathematics. The results of the research have been inconclusive, and in some cases apparently contradictory. Under what conditions advance organizers facilitate learning is therefore a question of interest to mathematics educators. With Ausubel's emphasis upon cognitive structure as the key to meaningful learning, it seems reasonable to suppose that there may be a relationship between cognitive style and beneficial effects of an advance organizer. The purpose of this study was to investigate the relationships among an advance organizer, perceptual style, and the learning and retention of a rather complicated integration process in calculus. Only two known studies have heretofore attempted to relate advance organizers and cognitive styles in school learning--both studies in areas other than mathematics and along cognitive dimensions other than the perceptual dimension suggested by Lowenfeld.

In order to investigate the relationship between advance organizers and perceptual style in the learning of mathematics, the experimenter was given two sections of basic calculus for non-physical science majors as his teaching assignment at the University of Oklahoma during the fall semester, 1977. One section (the experimental group) received an advance organizer for learning an integration technique. The other section (the control group) did not receive the organizer. Both sections were administered the <u>Successive Perception Test I</u> (SPT-1) to determine perceptual style. Subjects were classified as visual, neutral, or haptic, according to their responses on SPT-1.

The instructor constructed both an initial learning test and a retention test to measure acquisition of the integration technique. The data obtained for the 2 X 3 factorial analysis of variance design was used to test the following six hypotheses:

1) Initial learning is facilitated by employing an advance organizer.

2) There is a differential relationship between initial

learning and perceptual style.

3) Advance organizers better facilitate initial learning for haptic individuals than for visual individuals.

4) Retention is facilitated by employing an advance organizer.

5) There is a differential relationship between retention and perceptual style.

6) Advance organizers better facilitate retention for haptic individuals than for visual individuals.

The type I error rate of .05 was assigned per hypothesis. Also the power to detect a difference of one standard deviation from the mean under the null hypothesis was calculated when the null hypothesis was not rejected.

Inferences

Only hypothesis six was supported by the statistical analysis of the data. It was concluded that haptically oriented individuals receiving the advance organizer scored significantly higher on the retention test when compared to the visuals receiving the advance organizer than haptically oriented individuals in the control group scored when compared to the visuals in the control group. The inference drawn is that, although the advance organizer did not facilitate retention for the experimental group when compared to the control group, the advance organizer did better facilitate retention for the haptically oriented individuals than for the visually oriented individuals. This inference needs to be considered carefully, since two of the three experimental cell means were lower than the corresponding cell means of the control group. However, the experimental-haptic cell mean was larger than both the experimental-visual cell mean and also the control-haptic cell mean. This interactive effect observed on retention was not exhibited on initial learning.

Although the F-statistic for a main effect of perceptual style on initial learning was not significant, the trend was observed for visuals to score better than haptics on the initial learning test, independent of the presence of an advance organizer. This trend was not observed on the retention test.

Evaluation of the Study

Ausubel claims that learning is meaningful when it is related in a non-arbitrary, substantive, and non-verbatim way to existing cognitive structure. The adjective "meaningful" implies other kinds of learning. This study does not measure the "meaningfulness" of learning; it only measures learning as indicated by written responses on a test. Ausubel classifies learning as either meaningful or rote. There is a temptation to say that there is more chance for rote learning to be measured on the initial learning test rather than on the retention test, especially when the retention test occurred after a much greater time interval and was concealed among much other learning material. That the visuals scored better than the haptics on initial learning might be rationalized by saying that the visuals had a better chance at a rote memorization (perhaps akin to "photographic memory") of the learning material. Evidently there are students who are accustomed to rote learning and who are quite adept at such. For these students exposure to an advance organizer could be purposeless and confusing. However, until the intervening variables, rote learning and meaningful learning, are operationally defined, distinguishing between the two types of learning is at best conjectural and at worst unwarranted.

Various studies investigating the effects of advance organizers on learning mathematics have been conducted in which many sections of a particular course were involved and in which written organizers were used. A strength of this study is that instructor variability did not occur and that the organizer was presented essentially as was other material during the semester--in a lecture-question-discussion environment.

Whenever an advance organizer is presented, the connection between it and the learning material should be indicated. It may be then that the group receiving the organizer has more exposure to the learning material than the control group. However, in this study the control group experienced rehearsal of the derivatives and antiderivatives of exponential functions, counteracting the experimental group's rehearsal of simple derivative and antiderivative formulas.

Some studies have involved written organizers that were only a paragraph or a page in length. Although the organizer used in this study was somewhat longer, an even more detailed presentation of the organizer may have better prepared cognitive structure to assimilate the learning material.

Actually the experimental group did have more exposure to the organizer by the time of the retention test than they had at the initial learning test, because on two occasions between administration of the two tests the instructor referred to the organizer.

A negative aspect that affected the study was the short time of presentation of learning material before the initial learning test. Ideally there would have been a seventy-five minute session with the first fifty minutes devoted to presentation of the learning material. But with only fifty minute sessions, the alternative was to spend one session in presenting the material and then to test initial learning the following day. This would have allowed several uncontrolled variables to appear, among which would have been outside study of the lesson, which the instructor wanted eliminated from the initial learning aspect.

Suggestions

As far as mathematics teaching is concerned, there is no guarantee that an advance organizer will facilitate learning in a classroom setting. On an individual basis, though, there do exist possibilities. This study has indicated a beneficial effect of advance organizers for haptically oriented individuals as far as retention or long term learning is concerned. This significant result of the study suggests research in the following areas.

The effect of advance organizers administered to haptic individuals both in areas of learning other than mathematics and also in levels of mathematics different from calculus should be investigated. Confirmation of the results of this study might be obtained.

The interaction of advance organizers with dimensions of cognitive style other than perceptual should be examined. With some degree of relationship between the several dimensions of cognitive style, it may be that there is a stronger facilitating effect of advance organizers for certain types of individuals along dimensions other than the perceptual. And there exist tests that measure cognitive style along these dimensions that have been extensively used in educational research.

Research should be done to arrive at a standardization of classifying individuals according to perceptual style. Perhaps a new test should be devised to replace SPT-1 for measuring perceptual style. Such a test may evolve from the continuing research in hemispheric lateralization of the brain. Be that as it may, there should be a consensus among educators for operationally defining visual and haptic individuals.

As a final thought, the effect of organizers on learning appears to be weak, amid the present confusion and conflicting results. But it is precisely this weakness that contributes intrigue and challenge to the study of organizers. Educators are well aware of strong factors which they can employ to facilitate school learning--enthusiasm, a sense of humor, respect for the student as a "human becoming", etc. Use of advance organizers in a classroom setting would add to variety, as would other innovative techniques. However, until more research is done, that organizers facilitate learning is questionable. With future research into the areas mentioned above and with the latitude to group students with more homogeneity of cognitive style, advance organizers may very well become a strong factor in school learning under a sufficiently localized environment.

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APPENDIX A

RAW DATA

Table 6. Raw Scores for the Experimental Group

Subject	Initial Learning Test	Retention Test	SPT-1
1	1	6	23
2	0	3	15
3	0	6	23
4	0	5	23
5	4	5	18
6	0	2	18
7	2	7	9
8	9	5	14
9	10	7	9
10	0	0	16
11	2	6	18
12	6	1	7
13	6	4	21
14	0	2	15
15	0	4	14
16	1	1	13
17	3	4	19
18	3	4	9
19	0	3	15
20	0	3	11
21	8	2	11
22	10	7	17
23	4	4	14
24	7	4	8
25	3	2	8

Table 7. Raw Scores for the Control Group

Subject	Initial Learning Test	Retention Test	SPT-1
26	5	3	8
27	6	6	21
28	4	6	12
29	5	5	14
30	10	7	16
31	0	3	17
32	7	4	13
33	5	5	12
34	9	6	8
35	2	2	12
36	0	4	11
37	0	4	15
38	2	3	19
39	3	2	13
40	4	3	14
41	0	4	13
42	2	3	14
43	0	1	16
44	5	4	16
45	3	1	17
46	9	6	14
47	2	2	13
48	6	6	7
49	8	6	10
50	1	1	19
51	1	2	18

APPENDIX B

STATISTICAL FORMULAS

The computational formulas used in the unweighted means two-way analysis of variance are listed below. Let X_{ijk} represent the k^{th} score in the $i \times j$ cell, which cell has n_{ij} subjects.

1) $$\overline{X}_{ij} = (n_{ij})^{-1} \sum_{k=1}^{n_{ij}} X_{ijk}$$

2) $$\tilde{n} = 6\left[\sum_{i=1}^{2}\sum_{j=1}^{3}(n_{ij})^{-1}\right]^{-1}$$

3) MS(treatment) =

$$\tilde{n}\left[\frac{1}{3}\sum_{i=1}^{2}\left(\sum_{j=1}^{3}\overline{X}_{ij}\right)^2 - \frac{1}{6}\left(\sum_{i=1}^{2}\sum_{j=1}^{3}\overline{X}_{ij}\right)^2\right]$$

4) MS(perceptual type) =

$$\frac{\tilde{n}}{2}\left[\frac{1}{2}\sum_{j=1}^{3}\left(\sum_{i=1}^{2}\overline{X}_{ij}\right)^2 - \frac{1}{6}\left(\sum_{i=1}^{2}\sum_{j=1}^{3}\overline{X}_{ij}\right)^2\right]$$

5) MS(interaction) =

$$\frac{\tilde{n}}{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\overline{X}_{ij}^{\,2} - \frac{\tilde{n}}{4}\sum_{j=1}^{3}\left(\sum_{i=1}^{2}\overline{X}_{ij}\right)^2 - \frac{1}{2}\,\text{MS(treatment)}$$

6) MS(within) =

$$\frac{1}{45}\left[\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{n_{ij}} X_{ijk}^{\,2} - \sum_{i=1}^{2}\sum_{j=1}^{3}\frac{1}{n_{ij}}\left(\sum_{k=1}^{n_{ij}} X_{ijk}\right)^2\right]$$

7) MS(total) =

$$\frac{1}{50}\left[\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{n_{ij}} x_{ijk}^{2} - \frac{1}{51}\left(\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{n_{ij}} x_{ijk}\right)^{2}\right]$$

8) MS(between) = 10 MS(total) − 9 MS(within)

9) F(treatment) = $\frac{\text{MS(treatment)}}{\text{MS(within)}}$

10) F(perceptual type) = $\frac{\text{MS(perceptual type)}}{\text{MS(within)}}$

11) F(interaction) = $\frac{\text{MS(interaction)}}{\text{MS(within)}}$

12) P(treatment) =

$$\int_{c}^{\infty} \text{F(1,45) distribution} \quad \text{, where } c = \text{F(treatment)}$$

13) P(perceptual type) =

$$\int_{c}^{\infty} \text{F(2,45) distribution} \quad \text{, where } c = \text{F(perceptual type)}$$

14) P(interaction) =

$$\int_{c}^{\infty} \text{F(2,45) distribution} \quad \text{, where } c = \text{F(interaction)}$$

APPENDIX C

COURSE OUTLINE

MATHEMATICS 1744

Syllabus

Fall Semester 1977

August 29: Overview & Introduction

August 30: The Cartesian Plane

September 1: Lines

September 2: The Function Concept

September 6: Functions Continued

September 8: New Functions from Old Ones

September 9: Introduction to the Limit Concept

September 12: The Meaning of $\lim_{x \to c} f(x) = L$

September 13: Limits & Infinity

September 15: The Meaning of $\lim_{x \to c} f(x) = f(c)$

September 16: The Derivative Concept

September 19: The Equation of a Tangent Line to a Curve at a Given Point.

September 20: Review

September 22: Test I

September 23: Test I in Retrospect

September 26: The Technique of Differentiation

September 27: Rules of Differentiation

September 29: Implicit Differentiation

September 30: How to Differentiate a Composition

October 3: Relationship of Sign of First Derivative to Direction

October 4: Candidates for Relative Extrema

October 6: Relationship of Sign of Second Derivative to Concavity

October 7: Candidates for Inflection Points

October 11: Applications

October 13: Review

October 14: Test II

October 17: Test II in Retrospect

October 18: Antiderivatives

October 20:	The Area Under a Curve
October 21:	The Fundamental Theorem of Calculus
October 24:	The Fundamental Theorem in Retrospect
October 25:	A Business Application of the Definite Integral
October 27:	A Function Having Domain N
October 28:	Summation Properties
October 31:	Exponential Functions
November 1:	The Inverse of an Exponential Function
November 3:	Algebra of Logarithms
November 4:	Natural Law of Growth (Decay)
November 7:	More About Integration
November 8:	A Technique of Integration
November 10:	Integration by Parts
November 11:	Further Techniques of Integration
November 14:	Approximate Integration
November 15:	Integration from a Different Perspective
November 17:	Improper Integrals
November 18:	Applications of the Definite Integral
November 21:	Review
November 22:	Test III
November 28:	Test III in Retrospect
November 29:	Differential Equations
December 1:	Functions of Two Independent Variables
December 2:	Partial Differentiation
December 5:	The Chain Rule for Partially Differentiating a Composition
December 6:	Candidates for Relative Extrema of Functions of Two Independent Variables
December 8:	Lagrange Multipliers
December 9:	Applications
December 12:	Semester Review
December 13:	Semester Review Continued
December 15:	Some Specifics Concerning the Final Examination

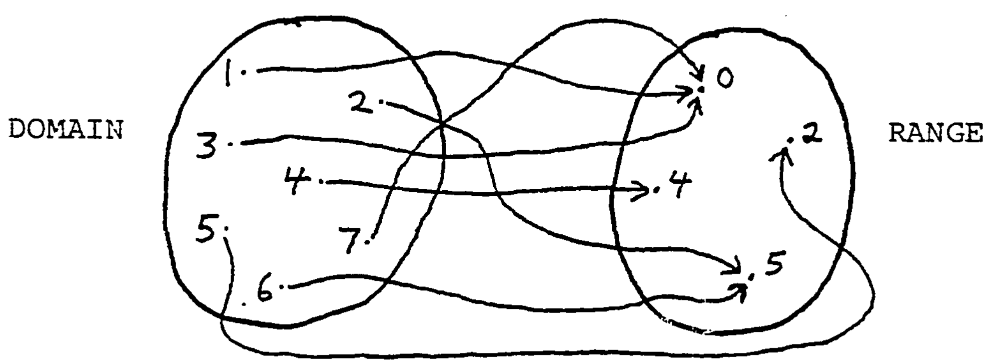
APPENDIX D

ADVANCE ORGANIZER

AN ADVANCE ORGANIZER FOR ACQUIRING INTEGRATION SKILLS

A function is a set of ordered pairs having no repetition in first coordinates. Implied by this definition is an association between the elements of a pair of sets--the set of all first coordinates of the ordered pairs (the domain) and the set of all second coordinates of the ordered pairs (the range). The fact that there are no repetitions in the first coordinates of the ordered pairs guarantees that for each element in the domain there is associated exactly one element in the range. Thus a function can be viewed as a mapping from one set (the domain) onto a second set (the range) such that to each element in the domain there corresponds exactly one element in the range.

$\{(1,0), (2,5), (3,0), (4,4), (5,2), (6,5), (7,0)\}$ is a function viewed as a set of ordered pairs. This same function is viewed as a mapping in the following way.



A word about terminology: If we let f be a symbol for the function ($f = \{(1,0), (2,5), (3,0), (4,4), (5,2), (6,5), (7,0)\}$) and if x represents a domain element (a first coordinate), then the symbol f(x) represents the unique second coordinate (element in the range) corresponding to x. In our particular example $f(1) = 0$, $f(2) = 5$, $f(3) = 0$, $f(4) = 4$, $f(5) = 2$, $f(6) = 5$, and $f(7) = 0$. $f(x)$ is called the image of x under function f.

A concept which is not always covered in an initial study of functions is that of pre-image. If y is an element in the range of function f, then the pre-image of y under function f, denoted by $f^{\leftarrow}(y)$, is defined to be $\{x : y = f(x)\}$. That is, $f^{\leftarrow}(y)$ is the set of all domain elements which have y as image under function f. In our example above $f^{\leftarrow}(0) = \{1, 3, 7\}$, $f^{\leftarrow}(2) = \{5\}$, and $f^{\leftarrow}(5) = \{2, 6\}$.

Solving the equation $x^3 - 2x^2 - 5x + 6 = 0$ has the concept of pre-image hidden behind the scene in the following way: If P is the cubic polynomial function defined by $P(x) = x^3 - 2x^2 - 5x + 6$, then $P^{\leftarrow}(0)$ is the solution set for the above equation. In this case $\overleftarrow{P}(0) = \{-2, 1, 3\}$, which you should verify.

If function f is a 1-1 function (i.e., there are no repetitions in second coordinates), then for each element y in the range of f, $f^{\leftarrow}(y)$ consists of just a single element. Recall that a 1-1 function f has an inverse function, denoted by f^{-1}. The single element in $f^{\leftarrow}(y)$ then is simply $f^{-1}(y)$. The linear function $f(x) = 2x + 3$ is a 1-1 function; and you should recall how to find $f^{-1}(x)$, which in this case is $\frac{x-3}{2}$. Observe that $f^{-1}(13) = 5$ and that $f^{\leftarrow}(13) = \{5\}$ because $f(x) = 13$ if and only if $x = 5$.

We have recently studied the exponential functions, suggestively denoted by $\exp_b$, defined by $\exp_b(x) = b^x$, where $0 < b \neq 1$. Each of these functions is 1-1. Notice that $\exp_2(3) = 2^3 = 8$. Thus $\exp_2^{\leftarrow}(8) = \{3\}$. We have written this in the form $\log_2(8) = 3$. You see $\log_b(x)$ is just another symbol for the single element in $\exp_b^{\leftarrow}(x)$.

Consider now a bit different kind of function, which we shall label D. The domain of D is not a subset of real numbers but rather is a subset of functions--those functions that are differentiable. If we agree to let f(x) be a symbol for a differentiable function, then D shall be defined by $D[f(x)] = f'(x)$. Notice that $(x^2, 2x)$, $(7, 0)$, (e^x, e^x), $[\ln(x), \frac{1}{x}]$, and $[(x^3 + 7x - 19)^8, 8(x^3 + 7x - 19)^7(3x^2 + 7)]$ are five of the infinitely many ordered pairs of function D.

Since $D(x^2) = 2x$, we know that x^2 is a member of $D^{\leftarrow}(2x)$. But x^2 is not the only element in $D^{\leftarrow}(2x)$. If C is any real number, then $x^2 + C$ is a member of $D^{\leftarrow}(2x)$. In fact $D^{\leftarrow}(2x)$ is precisely the set $\{x^2 + C : C \text{ is a real number}\}$. Isn't this last set $\int 2x\,dx$? Then the problem of antidifferentiating $\int f(x)\,dx$ can be considered as the problem of determining $D^{\leftarrow}[f(x)]$. And it is advantageous to consider it as such.

One of the reasons that integration is more difficult than differentiation is the following: We know how to differentiate any elementary function (algebraic, exponential, logarithmic, or composition thereof). But we can't antidifferentiate all such functions. We can only antidifferentiate those elementary functions that are also derivatives of elementary functions. So integration will be facilitated if we have learned well the techniques of differentiation and if we can also look at the function we are antidifferentiating as a derivative of some elementary function.

As an example, suppose we wish to determine $\int x^5\,dx$. Recall that rx^{r-1} is the derivative of x^r, where r is any real number. How does rx^{r-1} compare with x^5? $x^5 = \frac{1}{6}(6x^5) = \frac{1}{6}(rx^{r-1})$, with r=6. Hence, $\int x^5\,dx = \int \frac{1}{6}(6x^5)\,dx = \frac{1}{6}\int 6x^5\,dx = \frac{1}{6}x^6 + C = D^{\leftarrow}(x^5)$.

It is viewing $\int f(x)\,dx$ as $D^{\leftarrow}[f(x)]$ that will aid us in performing more complicated integration problems. These more complicated problems will be studied tomorrow and will be seen as instances of reversing the chain rule for differentiating a composition.

APPENDIX E

TESTS

MATHEMATICS 1744

Special Quiz

Name________________________

1. Suppose that function f is defined as follows:
   $f = \{(0,5), (1,5), (2,3), (3,5), (4,3), (5,0)\}$
   Find each of the following pre-images:

   a. $f^{\leftarrow}(5) =$ ______________

   b. $f^{\leftarrow}(3) =$ ______________

   c. $f^{\leftarrow}(0) =$ ______________

2. If $P(x) = x^2 - 9x - 10$, then $P^{\leftarrow}(0) =$ ______________

   and $P^{\leftarrow}(-10) =$ ______________

3. a. $\exp_2^{\leftarrow}(16) =$ ______________

   b. $\log_2^{\leftarrow}(5) =$ ______________

4. Let $D[f(x)] = f'(x)$. Obtain each of the following pre-images:

   a. $D^{\leftarrow}(3x^2) =$ ______________

   b. $D^{\leftarrow}(5e^x) =$ ______________

   c. $D^{\leftarrow}(1/x) =$ ______________

   d. $D^{\leftarrow}(x^5 - 4x + \sqrt{x} - 3) =$ ________________________

5. Let g be the function that assigns to each team on OU's 1977 football schedule the letter W, T, L, or F, according to whether OU won, tied, lost, or has yet to play the team (future), respectively. What are $g^{\leftarrow}(L)$ and $g^{\leftarrow}(F)$?

MATHEMATICS 1744

Initial Learning Quiz

Name________________________

Find each of the following families of antiderivatives:

1. $\int (3x + 1)^7 \, dx$ ____________________

2. $\int e^{5x} \, dx$ ____________________

3. $\int \frac{1}{3x} \, dx$ ____________________

4. $\int (x^2 + 2x + 3)^{13}(x + 1) \, dx$ ____________________

5. $\int x \, e^{x^2+3} \, dx$ ____________________

6. $\int \frac{x^2}{x^3 + 5} \, dx$ ____________________

7. $\int 2^x \, dx$ ____________________

8. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$ ____________________

9. $\int 3^{x^3+3x-4}(x^2+1) \, dx$ ____________________

10. $\int f'[g(x)] \, g'(x) \, dx$ ____________________

MATHEMATICS 1744

Final Exam

Name________________________

Directions: This test is worth 122 points, with 120 considered as a perfect score. Write all answers and show all work you wish to have evaluated on these test sheets, using backs if additional space is needed. Please see the instructor if you have any question.

1. Find both the derivative and also the family of antiderivatives of each of the following functions: (22 points)

a. $f(x) = 9x^8 - x^5 + 7e^x - \frac{1}{x} + \sqrt{3}$

$f'(x) =$ ________________________

$\int f(x)\,dx =$ ________________________

b. $f(x) = \frac{\ln(x)}{x}$ $f'(x) =$ ________________

$\int f(x)\,dx =$ ________________

c. $f(x) = (4x-3)^5$ $f'(x) =$ ________________

$\int f(x)\,dx =$ ________________

d. $f(x) = 3x^2(x^3+1)^{.5}$ $f'(x) =$ ________________

$\int f(x)\,dx =$ ________________

2. State each of the following limits in simplified form. (22 points)

a. $\lim_{x \to 64} \log_4(x)$ ______

b. $\lim_{x \to \infty} e^{-x}$ ______

c. $\lim_{x \to 6} \dfrac{x - 6}{x^2 - 8x + 12}$ ______

d. $\lim_{x \to -5} [\![x/2]\!]$ ______

e. $\lim_{x \to 0^-} \int_{-1}^{x} t^{-2/3}\, dt$ ______

f. $\lim_{(x,y) \to (2,1)} (x^2 + y^2 + x \ln(y))$ ______

g. $\lim_{x \to \infty} \dfrac{2 - x^2}{5x}$ ______

h. $\lim_{n \to \infty} \dfrac{2 - 3n}{4n + 5}$ ______

i. $\lim_{x \to \infty} \int_{8}^{x} t^{-2}\, dt$ ______

j. $\lim_{h \to 0} \dfrac{\exp(4+h) - \exp(4)}{h}$ ______

k. $\lim_{n \to \infty} \left(\sum_{i=1}^{n} (4/5)^i \right)$ ______

3. Evaluate each of the following definite integrals: (18 points)

a. $\int_{-1}^{4} (2x-3)\ dx$ ______________

b. $\int_{0}^{2} \frac{6x^2}{x^3 + 1}\ dx$ ______________

c. $\int_{-2}^{-.6} [\![x]\!]\ dx$ ______________

d. $\int_{0}^{1} 5(x^2+1)^4 2x\ dx$ ______________

e. $\int_{0}^{1} 10^{2x}\ dx$ ______________

f. $\int_{-\infty}^{\infty} \frac{1}{3e^{|x|}}\ dx$ ______________

4. Find the family of antiderivatives $\int 15x^3\sqrt{x^2+1}\ dx$ by using the integration by parts technique letting $g'(x) = 3x(x^2+1)^{1/2}$. (6 points)

5. Let function f be defined by $f(x) = (x-1)^3e^x$.
(15 points)

Observe that $f'(x) = (x-1)^2(x+2)e^x$ and
$f''(x) = (x-1)(x^2+4x+1)e^x$.

$f'(x) = 0$ if $x =$ _____ or _____ .

$f''(x) = 0$ if $x =$ _________ , _________ , or _________ .

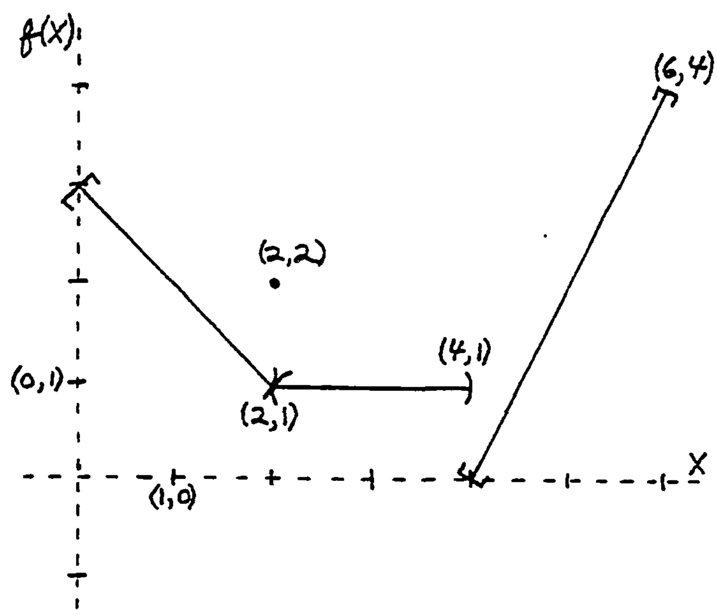
f is increasing on interval ___________ .

f is concave up on intervals ___________ and ___________ .

f attains a minimum value at $x =$ _____ .

f has inflection points at $x =$ _____ , _____ , and _____ .

6. Consider the function f defined by the graph displayed on the coordinate system to the right.
(10 points)



range(f) = __________

$\int_0^6 f(x)\,dx =$ ________

Find a real number c so that $\lim_{x \to c} f(x)$ exists but f is not continuous at c.

c = ______

On the same coordinate system (above) sketch the graph of f', the derivative function of f.

7. Suppose the position s of an object is given by the following function of time t: $s(t) = t^2 - 4t$.
(10 points)

a. Find the average velocity of the object during the time interval [5,7]. __________

b. Use the definition of the derivative to find the instantaneous velocity at time $t = 5$. (Work this part on back of previous page.)

c. Find the equation of the line tangent to the graph of s at the point (5,5). __________

8. Let z be defined implicitly as a function of independent variables x and y by the equation
$x^2 - y^2 + z^3 = 3yz$. (9 points)

a. What value is z when $x = \sqrt{21}$ and $y = 5$? __________

b. Observe that $(\sqrt{3},1,-2)$ and the origin are two points in space on the graph of this function. What is the distance between these two points? __________

c. $\frac{\partial z}{\partial x}$ = __________

d. $\frac{\partial z}{\partial y}$ = __________

9. Work two of the following four problems. (Extra credit will be given for correctly working more than two. Show work below and on back of previous page.) (10 points)

   a. Find the area of the region in the plane bounded by the two parabolas $y = (x-1)^2$ and $y = 13 - x^2$.

   b. Find two positive real numbers whose sum is 35 and such that the product of the cube of the one and the square of the other is a maximum value.

   c. Solve the differential equation $y' = 2xy - 3y$ with boundary condition $y=5$ when $x=3$.

   d. Solve the following logarithmic equation for x:

$$2 \ln(x) + \ln(x-3) - \ln(108) = 0$$