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A HEURISTIC PROGRAMMING APPROACH.

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DETERMINING A HOSPITAL'S OPTIMAL PATIENT MIX: A HEURISTIC PROGRAMMING APPROACH

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DETERMINING A HOSPITAL'S OPTIMAL PATIENT MIX:

A HEURISTIC PROGRAMMING APPROACH

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PREFACE

I would like to thank the members of my dissertation committee for their aid throughout the various stages of this work: Professors Robert Lusch, Robert Ford, James Hibdon, Bill Schumacher, and Benjamin Taylor. Professor Lusch, acting as chairman, gave thoughtful and speedy consideration during the latter stages of this project. Also, I am grateful to Howard Harris of the Comanche County Memorial Hospital for giving invaluable insights into the problems facing the modern hospital, and to John Sherrill, Charlotte Southerland, and John Uzzo for their help in gathering the necessary information.

Most especially, I wish to thank Francis and Mildred Cella for their steady encouragement and warm support through the lean years of this effort. They make it seem worthwhile.

LIST OF EXHIBITS

CHAPTER I

INTRODUCTION

General Background

America's medical industry has grown significantly since the Second World War, reflecting an increasing demand for health service. Many reasons exist for the growing interest Americans have in medicine, but two reasons seem to be of particular importance. First is the increasing real incomes and real wealth of the population. As real disposable incomes rise, there is a tendency to spend more on superior goods, of which health care is an example.

The second reason for increased demand is the growing acceptance of public and private insurance. Many private employers now offer their employees life, accident, and health care group insurance policies which are carried by most major insurance firms. Even more important, during the last decade the federal government has begun to provide hospitalization insurance for both the aged and the indigent. Therefore, much of the growth in insurance-related demand can be attributed to the Medicare and Medicaid programs of the federal government.

Doubtless there are many other reasons for rising medical care demand. In any case, as can be seen in Exhibit 1-1, the rise in medical spending has been both substantial and sustained. Beginning with $12 billion in

Exhibit 1-1

U.S. HEALTH CARE EXPENDITURES 1950-1975

	National Health Expenditures		Hospital Expenditures	
Year	Total in billions	Per Capita	Total in billions	Per Capita
1950	$ 12.027	$ 78.98	$ 3.845	$ 25.25
1955	17.330	104.44	5.929	35.73
1960	25.856	143.66	9.044	50.06
1965	38.892	200.97	13.520	69.61
1970	69.201	339.54	27.528	134.36
1971	77.944	376.45	30.850	148.99
1972	86.687	416.30	34.215	163.83
1973	95.383	454.54	37.808	179.70
1974	104.031	492.15	43.500	205.79
1975	118.499	556.02	46.600	218.66

Source: Statistical Abstract of the U.S.: 1976, pp. 72-84; U.S. Bureau of the Census.

1950, the nation's health and medical care expenditures have grown to over $118 billion in 1975. This amounts to an increase in expenditures from $79 to $556 per capita.

Hospital Background

Hospitals are the chief users of funds in the over $100 billion health care industry and are continuing to grow in importance. Hospitals now consume approximately $46 billion of the total health care expenditures in the United States, compared to $3.8 billion in 1950. Further, current hospital consumption represents 39 percent of all health care payments as opposed to 32 percent in 1950. Restated on a per capita basis, consumption in 1950 was $25.25 and in 1975 was $212.66. Importantly, this amounts to an eight-fold increase in just 26 years.

One major cause of the rapid increase in hospital expenditures is the growth in the number of patients. As seen in Exhibit 1-2, the number of patients has grown from fewer than 17 million in 1950 to almost 33 million in 1974. This 94 percent jump forced the expansion in the number of hospitals, beds, and employees. Over 940 hospitals and almost 430 thousand new beds were added to handle the additional 16 million patients. Exhibit 1-3 shows that hospital bed utilization increased impressively during the period. In 1950, the hospital industry used 73.22 percent of its bed-day capacity, whereas in 1974 it used 75.30 percent.

While the industry was increasing its bed utilization, it was unfortunately decreasing its employee utilization. Tripling the number of employees from 1950 to 1974 caused the ratio of personnel per thousand patients to jump from about 40 to almost 68. Thus, the industry required

Exhibit 1-2

PATIENTS, HOSPITALS, BEDS, AND PERSONNEL
IN THE U.S., 1950-1975

Year	Patients (000)	Hospitals	Beds (000)	Personnel (000)
1950	16,663	5,031	505	662
1955	19,100	5,237	568	826
1960	22,970	5,407	639	1,080
1965	26,463	5,736	741	1,386
1970	29,300	5,839	848	1,929
1971	30,100	5,865	867	1,999
1972	30,800	5,843	884	2,056
1973	31,761	5,891	903	2,149
1974	32,900	5,977	931	2,240

Source: Guide to the Health Field, 1976, Table 1, American Hospital Association. Statistical Abstract of the U.S.: 1976, U.S. Bureau of the Census, pp. 72-84.

Exhibit 1-3

SELECTED CHARACTERISTICS OF U.S. HOSPITALS
1950-1974

Year	Average Beds Per Hospital	Bed Utilization	Length of Patient Stay	Personnel Per Thousand Patients
1950	100	73.22%	8.1 days	39.73
1955	108	71.86	7.8	43.25
1960	118	74.85	7.6	47.02
1965	129	76.32	7.8	52.38
1970	145	77.62	8.2	65.84
1971	148	76.09	8.0	66.41
1972	151	75.41	7.9	66.75
1973	153	75.16	7.8	67.66
1974	156	75.30	7.8	68.09

Source: Guide to the Health Field, 1976, American Hospital Association, Table 1. Statistical Abstract of the U.S.: 1976, U. S. Bureau of the Census, pp. 72-84.

70 percent more workers to treat each thousand patients in 1974 than it did in 1950.

A second reason for the increase in hospital expenditures is inflation. For whatever reason, the cost of buying hospital care has risen to the point where it is causing concern. Greenfield points out that because hospital prices have risen far faster than the prices for all goods and services, hospitals have priced themselves into the public eye[2]. Exhibit 1-4 presents the reasons for the increased public awareness. Since 1950, the prices of the items included in the Consumer Price Index increased approximately 134 percent. However, the price of medical care rose 240 percent, and the price of hospital care rose about 810 percent. The daily hospital charge has increased an incredible six times faster than all other prices.

Another way of looking at price increases is shown in Exhibit 1-5. Compared to the percentage change in the Consumer Price Index, hospital charges increased 3.82 times faster in 1950-1955; 5.55 times faster in 1960-1965; and 2.30 times faster in 1976 alone.

The startling impact of these figures is more fully realized when actual dollar amounts are considered. For example, in 1950 a patient would expect to pay $21.67 a day for hospital care. If he stayed the customary 8.1 days, he would be billed $176. But, if a patient stayed the normal 7.8 days today, he would be billed $1,539, excluding physician fees. In large metropolitan areas, the total bill could reach as high as $3,000.

Many explanations have been suggested for this rapid increase. They include over-building and under-utilization of facilities, high initial

Exhibit 1-4

CONSUMER PRICE, MEDICAL CARE PRICE, AND HOSPITAL ROOM PRICE INDEXES FOR 1950-1976 (1967=100)

Year	Consumer Price Index	Medical Care Price Index	Hospital Room Price Index
1950	72.1	53.7	28.9
1955	80.3	64.8	41.5
1960	88.7	79.1	56.3
1965	94.5	89.5	76.6
1970	116.3	120.6	143.9
1971	121.3	128.4	163.1
1972	125.3	132.5	173.9
1973	133.1	137.7	182.2
1974	147.7	150.5	201.5
1975	161.2	168.6	236.1
1976	169.2	182.6	263.2

Source: Statistical Abstract of the U.S.: 1976, U.S. Bureau of the Census, pp. 439-441.

Exhibit 1-5

PERCENTAGE INCREASES IN CONSUMER PRICE, MEDICAL CARE PRICE, AND HOSPITAL ROOM PRICE INDEXES, 1950-1976

Year	Consumer Price	Medical Care Price	Hospital Room Price
1950-1955	11.4%	20.7%	43.6%
1955-1960	10.5	22.1	35.7
1960-1965	6.5	13.1	36.1
1965-1970	23.1	34.7	87.9
1970-1971	4.3	6.5	13.3
1971-1972	3.3	3.2	6.6
1972-1973	6.2	3.9	4.8
1973-1974	11.0	9.3	10.6
1974-1975	9.1	12.0	17.2
1975-1976	5.0	8.3	11.5

Source: Exhibit 1-4.

construction cost, urban versus rural location, differing factor prices, poor regional planning, and lack of competition[3]. Much of the difficulty with these variables is industry oriented and can only be corrected by substantial redesign of the health care delivery system. The disheartening fact is that the individual hospital has little control over these areas. However, part of the problem of rising costs does fall within the sphere that hospitals can control, and importantly it is this sphere that can readily produce a measure of cost control.

For example, one of the factors that a hospital can control is its patient mix, which is its combination of the various types of patients receiving treatment. Feldstein has shown that the hospital industry can effect significant increases in economic welfare by manipulating patient mix[1]. By employing standard linear programming techniques, Feldstein has demonstrated that economic welfare can be maximized by varying patient mix. He defines the optimal patient mix as that mix associated with maximized welfare. In the British National Health Service, maximizing welfare or "value" as Feldstein calls it, can act as the objective for the medical system. But in a less controlled economy, the extremely difficult if not impossible measure of value can be replaced by a cost objective. Controlling patient mix may offer a way to substantially reduce the costs of providing health care with no apparent sacrifice in quality. Unfortunately, no attention has been given to this problem thusfar.

The standard method of locating an optimum is via mathematical programming techniques. Feldstein, for example, employed linear programming to estimate the optimal patient mix. However, as discussed in the next chapter, many of the relationships needed to solve for the optimum may not be linear. If that is true, the identified mix may be incorrect. In order to circumvent that possibility, heuristic programming can be used. In heuristic programming the number of iterations needed to evaluate an optimization model is successively reduced. This technique is especially attractive when more sophisticated non-linear formulations are used, and when no adequate generalized non-linear scheme is available.

Purpose of the Study

The purpose of this study is to demonstrate that if a hospital is subject to cost-effective patient mixes, and if these mixes combine toward an optimum that succeeds in minimizing the average daily cost of treating a patient, then that optimum mix can be estimated by using heuristic programming. Knowing that an optimum exists that is capable of being estimated would be valuable information for hospital administrators trying to control patient costs.

Outline of the Study

In order to research this problem, the investigation will be organized along the following lines. Chapter II, which is divided into three parts, presents more detailed work relating to the purpose of the study. The first part gives a general discussion on hospital inefficiency, its possible causes, and ways suggested to reduce it. The second part offers a discussion of economic principles which govern efficiency and optimality. Variations from these principles as they apply to hospitals are also presented. The theoretical model of patient-mix optimization is presented in generalized form, along with a discussion of data requirements. Finally, the third part describes heuristic programming, the method which will be used to investigate the optimal patient mix.

Chapter III divides conveniently into two parts. Part I presents a discussion of the sampling methodology used to gather the necessary data, coupled with a brief examination of the data. In part II a description of data adequacy is given.

The fourth chapter presents the optimization model, which involves both linear and non-linear equations. It is demonstrated that the model is statistically significant, both in the individual equations and the output. In addition, the simulated results from the model are presented. Finally, each sampled hospital is examined by heuristic programming to ascertain if there is an optimal patient mix.

Conclusions are drawn in the fifth chapter as to the potential uses and possible modifications of the model. In addition, several research implications are discussed.

References

1. Martin Feldstein, Economic Analysis of Health Service Efficiency (Amsterdam: North-Holland Publishing Co., 1967), chapter 6.

2. Harry I. Greenfield, Hospital Efficiency and Public Policy(New York: Praeger Publishers, 1973), p. 3.

3. Judith R. Lave and Lester B. Lave, "Hospital Cost Functions," American Economic Review, Vol. 60, No. 3(September, 1970), pp. 379-395.

CHAPTER II

DISCUSSION OF HOSPITAL RESEARCH, THE GENERALIZED MODEL, AND HEURISTIC PROGRAMMING

This chapter consists of four parts. The initial section discusses hospital inefficiency, its likely causes, and ways suggested to reduce it. This discussion is based on a survey of the relevant published literature. The second section discusses the economic principles that govern efficiency and their impact on hospital operations. In addition, the generalized model of patient-mix optimization is presented. The third section discusses heuristic programming and data needs, while the fourth section summarizes the chapter.

Survey of the Literature

In the popular literature, there are numerous references to the inefficiencies that can be found at most levels of hospital operation. The levels range from the seemingly endless paperwork associated with being admitted, to the disorganization involved in receiving and paying the bill. Fortune suggests that hospitals should be forced to be at least as efficient as private industry, and decries the fact that they are not[11]. In discussing the hospital industry, Thurlow states that "to most consumers, service has often come to mean non-service: inefficiency, ineptitude, and indifference--at all levels and at frequently distressing prices[31]. Hill, writing in Hospital, suggests that now some of the more powerful third-party insurers, principally Blue Cross, are growing impatient with continued hospital inefficiencies[15]. Finally, commenting editorially in the Wall Street Journal on the proposed national health insurance program, Melloan discusses the "escalation in hospital inefficiencies" and the damage to the industry that national health programs would add to an already damaged industry[25].

A considerable body of research suggests that these opinions are more often than not correct. The Laves, who are active in researching matters that relate to medical economics, discuss the inefficiencies that hospitals are encountering[19]. Long believes that in the absence of the usual forces which work to effect acceptable resource allocation, production of hospital services will be accomplished only at high cost and great inefficiency[22]. Weisbrod agrees with Long and adds that the insulating from the market pressures allows hospitals the opportunity to be inefficient[33]. Finally, Greenfield probably summarizes the views of most when he flatly states that hospitals are nothing if not inefficient organizations[12].

However, the terms "efficiency" and "inefficiency" oftentimes are used loosely. Martin Feldstein makes the clear distinction between technical and economic efficiencies, and declares that technical efficiency has to do with the relationships of the quantity of inputs to the quantity of outputs[10, p.3]. Technical efficiency is an indication of productivity.

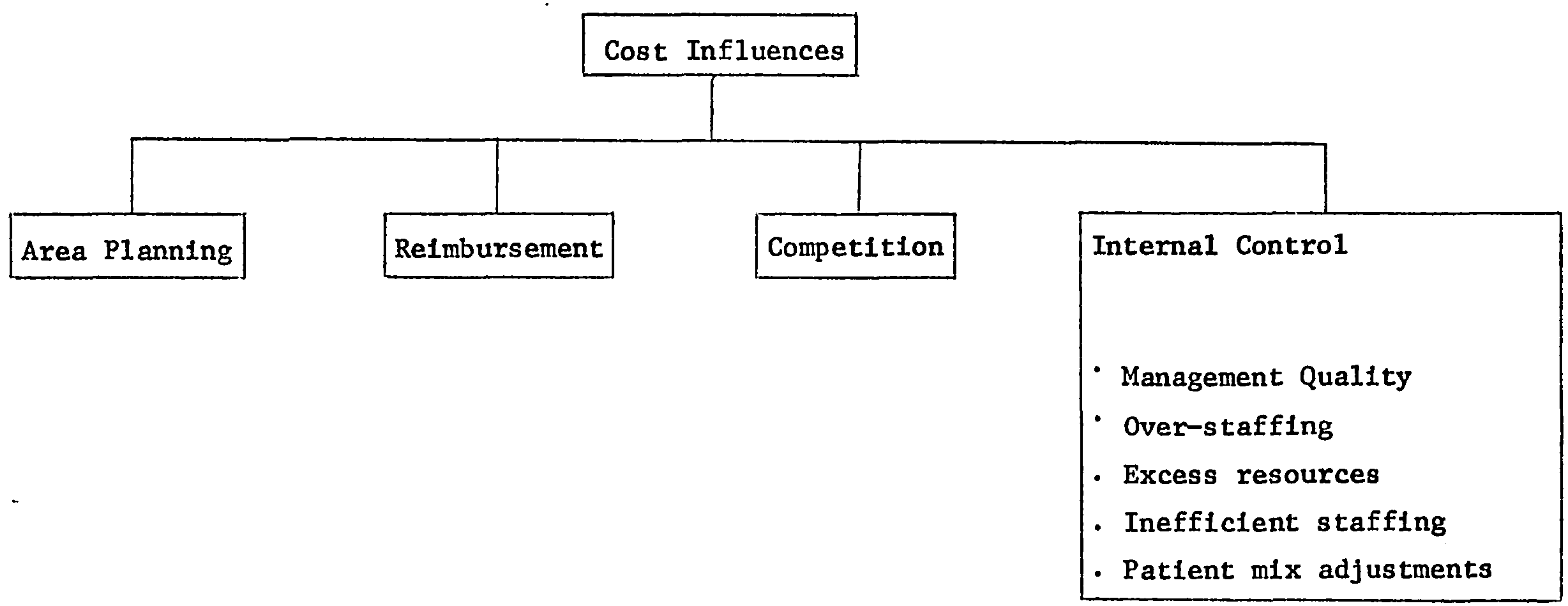
It considers inputs and outputs only in an engineering and physical sense, and ignores both quality and cost. Economic efficiency, on the other hand, refers to the production of a certain output at minimum cost; it introduces money into the definition of efficiency. Naturally, the two are highly interrelated, but sometimes produce different results. For example, a church-connected hospital may be staffed with a considerable number of voluntary workers. These workers may receive little or no compensation for their efforts. The hospital would consequently have a smaller wage and salary expense, which presumably would indicate a more attractive level of economic efficiency. But that same hospital may be technically inefficient because it makes extensive use of voluntary, unskilled labor.

For today's average hospital patient, technical efficiency may be of little concern. What is perhaps more important is the cost of being hospitalized. Traditional economic theory would suggest that a patient will have increased utility as the cost of care declines, all else held constant. As long as quality is maintained, the patient will probably care little if the hospital is or is not technically efficient. Predictably the patient's interest is primarily in economic efficiency. It is the definition of economic efficiency that will be used here.

In any event, various reasons have been offered to explain the cost-inefficiency of many hospitals. Four very broad reasons have been presented and are displayed in Exhibit 2-1. They are area planning, reimbursement methods, competition, and internal control. Each is discussed below, along with possible ways to improve them.

Exhibit 2-1

Selected Hospital Cost Influences



Area Planning

Feldstein suggests that governmental planning may be the most important source of help for reducing rising hospital costs[10, p. 79]. He offers several ideas that may help which center around increased government control through area planning agencies. These agencies would have the power to withhold or grant financial support for area hospitals. Primarily this support is limited to long term capital improvements. Today in the United States, the Comprehensive Health Planning Agencies have the power to deny reimbursements for federal patient care programs if their plans are not accepted by local hospitals.

However, this power is seldom exercised. Two reasons explain why. First, most agencies are under very vague constraints. Chiefly, they are charged with using "good judgment" in their decision-making and that sort of judgment is difficult to define. Second, they must determine which projects are in the public's best interest, which involves the almost impossible task of defining the "public's best interest". These two factors are important limitations to the success of the agencies. In fact, there is some question as to whether or not the planning groups will succeed. The Laves flatly predict that their planning efforts will fail[19, pp. 58-61]. And as a result, there will be increased pressure for direct government involvement in hospital affairs.

Many people, the Laves included, fear more government involvement. They believe simply that the hospital administrators are the best judges of hospital policies and operations. But even the best administrators may have goals imposed upon them that are not in the best interest of the community. The current system of hospital costing forces the administrator to give his attention to satisfying the demands of the medical staff and governing board for more elaborate facilities. The Laves soundly condemn this system as inefficient; Blue Cross has condemned it; and even the American Hospital Assocation has implied its disapproval.

Importantly, not all planning is negative. In fact, some area planning must be done in order to avoid useless waste of resources. U.S. News and World Report provides information on the ways some area hospitals are lowering patient cost[27]. Of the many ways listed, area-wide planning is the first. However, this planning is done on a voluntary basis, with no penalty imposed for non-participation. In this case, the market forces involvement by local hospitals.

Reimbursement

Some writers believe that the American system of refunding a hospital for its costs is improper. Typically, the private consumer, his insurers, or the government simply reimburse the hospital for whatever costs the hospital incurs. However, this method allows substantial opportunities for waste. The Laves suggest that reimbursement be separated from costing[19, p.53]. They offer a scheme for accomplishing this, and it involves a formulation that ties together the relevant factors that influence costs, such as teaching credits, types of services, and patient mix[21]. Their incentive plan would enable the hospital to finance its short-term operations and provide all the capital needed for long-term improvements.

Also, Michael Bromber, the director of the Federation of American Hospitals, suggests that cost-reimbursement be phased out because it penalizes the efficient hospital and rewards the inefficient[4]. Instead, Bromberg would substitute negotiated rates, which would provide incentives for management.

Competition

Another way to lower hospital costs is to promote hospital competition. In their writings, the Laves and Bromberg have recommended this very thing. Today, several groups around the country are experimenting with mini-hospitals. These so-called "day surgery" facilities have generated a great deal of enthusiasm. Generally, these little hospitals have concentrated on more localized surgery, such as the removal of tonsils and skin tumors[13]. In addition, they have provided general medical care. Their operational results have been impressive: quality of care has been improved, duplication of services has been reduced, and patient costs have been lowered. The Armstrongs point out that patients recuperating in these mini-hospitals, before going home, find their daily costs about 45 percent less than a full service hospital[2].

Kernaghan very correctly notes, however, that the American Hospital Association has recognized one negative aspect of this atomizing movement. That is the longer-term and thus more expensive types of patients must go to the full service hospital[13]. By forcing only longer-term patients into the normal hospital, the cost per patient day must necessarily increase, because these costs are not balanced off by the costs of the shorter-term and hence less expensive patients.

Internal Control

The first three reasons which identify problem areas are all macro in nature and are concerned with the delivery of health care throughout the entire nation. As a result, they are largely beyond the control of the individual hospital. However, the fourth method is not beyond the local hospital's control. By controlling internal operations more effectively, a hospital can strive for improved efficiency and lower costs. Apparently there are many areas that need controlling. Mecklin suggests that the chief problem is inferior management[24]. Very little management, he says, is done by trained professionals. He argues that, with few exceptions, physicians tend to dominate hospital policy-making, and are largely indifferent to economic considerations. He also believes that some of the usual characteristics of poorly managed organizations occur along with this "dictatorship of the doctors": personnel favoritism, empire-building, and reluctance to reveal internal information.

According to the Laves, intentional overstaffing may contribute substantially to inefficiency[20]. Many hospitals consistently employ more workers than are needed during normal times, simply as a protection against shortages during emergencies. Also, numerous hospitals over-purchase expensive equipment. Nader reports that of the fifteen hospitals in the Philadelphia area that are equipped to handle open-heart surgery, four do over 80 percent of the work[26].

In addition to over-consumption of resources, Nader has also identified four other reasons for hospital waste:

1. Hospital overbuilding. He estimates that over 300 thousand beds are not needed.

2. Unnecessary tests. Nader believes that over-testing adds up to 15 percent to a patient's bill.

3. Service duplication. Because of prestige, many hospitals offer more services than are economically feasible.

4. Unnecessary hospitalization. The American Medical Association has for some years been calling for a reduction of the number of surgical residencies.

In addition to these, Greenfield suggests that there is a functional misallocation of resources and that the opportunity costs are substantial of having highly-trained personnel performing tasks which less well-trained personnel could perform[12, p. 14]. He estimates that for 1968 almost $1 billion was lost because of poor matching of workers with their jobs.

The last internal factor given in Exhibit 2-1 is patient mix, which is the balance of the various types of patients that a hospital treats. For example, one hospital may treat many pediatric patients, and very few obstetric patients, while another may treat many obstetric patients and few pediatric patients. The combination of the number of each type is the patient mix.

Including patient mix in the list of internal control factors has been supported by Feldstein[9, p.39]. He shows that with a mix of nine patient types, 27.5 percent of the inter-hospital cost variation is explained by case mix. When 28 types of patients are considered, he shows that 32 percent of the cost variation is explained by case mix. The work of Ingbar and Taylor also supports the conclusion that mix influences costs[16]. Their results show that the types of patients substantially contribute to an explanation of average cost variation. While on a smaller scale, their results reinforce Feldstein's.

Interestingly, Feldstein reveals that the make up of the patient mix is largely irrelevant to the shape of the average cost curve[9, p132]. He reasons that if the coefficients for the independent variables other than mix variables remain the same when the mix is enlarged from 9 types to 28 types, the composition of the patient mix is not important to determining the shape of the average cost curve. He then demonstrates that the coefficients do remain much the same.

Regardless of the patient-mix composition, failure to consider a source of cost variation as large as this would be unfortunate. If cost control is important to the hospital, then certainly patient mix should be entered into any analysis of hospital efficiency.

However, controlling costs through patient mix requires knowledge of cost and resource-usage functions for each type of patient served by the hospital. Estimating hospital cost and production functions has occupied the energies of many researchers since 1964. One of the problems that arise from their work relates to the general shape of the estimated cost curves. Traditionally, economists have believed that business firms operate under a U-shaped cost curve in the short run. But, some writers argue that empirical studies show that the short-run cost curves may not be U-shaped, but rather L-shaped. Johnston, summarizing the results of 31 studies on the cost functions of various industries, writes that more often than not the marginal cost of a firm is constant[17]. He says that this finding directly supports the theory that the cost curve is L-shaped and not U-shaped.

This empirical work has not escaped criticism and importantly contains two specific objections. First, most of the studies were of oligopolistic or highly regulated industries. As a consequence, unwanted and unmeasured bias may have caused distortion of the data. Second, other writers believe that there is a bias toward linearity built into the cost functions themselves. Ruggles, for example, argues that pronounced curvature in marginal and average cost curves will give very little curvature in the total cost function[28]. His concern is with analyses that try to establish the nature of a cost-output relationship solely by graphical considerations. Thus, statistical evaluation of a cost curve depends a great deal on the definition of cost--marginal, average, or total.

If these shortcomings are valid, there is no real indication of whether or not a typical firm is subject to the more traditional theory. Likewise, there is no indication of whether or not the hospital is subject to the traditional theory, either.

A great deal of empirical work has been done in an effort to determine if the hospital has U-shaped short-run cost curves. The conclusions are conflicting. Three studies present results that seem consistent with the L-shaped curve theory. These studies are by Martin Feldstein[9], Judith and Lester Lave[20], and Mary Ingbar and Lester Taylor[16]. John Carr and Paul Feldstein[5], Harold Cohen[6], and Judith Mann and Donald Yett[23] have found health care costs consistent with traditional theory. In any case, if costs are to be reduced, there must be knowledge of hospital cost curves. Because each of the six studies makes an important contribution to this end, the conclusions of each will be briefly reviewed.

Feldstein

In his comprehensive Economic Analysis of Health Service Efficiency, Feldstein concludes that the cost curves for hospitals are L-shaped. This finding, however, appears to be judgmental. Specifically, his research indicates that the average cost curve is U-shaped, with beds related to cost per case. However, he points out that the minimum cost is reached at over 1,000 beds, which is near the upper limit of the hospital size found in his sample. He thus concludes that the significant second-order coefficient in the quadratic function is simply an artifact of the function itself. Because of this problem, Feldstein estimated logarithmic functions which decreased monotonically, the degree of fit equal to that of the quadratic function. In light of the shape of the cost curve, he concludes that if there are increasing returns, they are not important to the operations of the hospital.

Lave

The Laves studied hospitals in western Pennsylvania with data comprised of fourteen semi-annual observations for each hospital. They employed multiple regression techniques to functionally relate cost per patient day to occupancy rate and size(number of beds). Patient mix was ignored because of its stability over time. They found that a quadratic specification for occupancy rate was almost always statistically significant, whereas the quadratic specification for size was not. From these figures, they conclude that the hypothesized L-shaped curve is appropriate.

Ingbar and Taylor

Mary Ingbar and Lester Taylor studied 72 Massachusetts hospitals, with data being pooled for the years 1958 and 1959. Using multiple regression, they specified three different cost models. The first functionally related operating expense per bed day to eleven independent variables, with no otherwise believed important variables testing out to be significant. Their second model used expense per patient day as the dependent variable along with the same eleven previous independent factors. Five independent variables tested out to be significant, the most important being beds and beds-squared. However, all relationships produced cost curves with an inverted U-shape, which apparently so baffled Ingbar and Taylor that they abandoned their specifications and concluded that all the variables' impacts were constant. Thus, averages could be used. With their third model they test cost per patient day against various occupancy rates, with the negative coefficient of utilization showing great significance. In this regard, they essentially agree with the Laves that increased utilization is associated with lowered patient costs.

Carr and Feldstein

In an effort to determine the optimal hospital size, Carr and Paul Feldstein used partial regression analysis on over 3,000 hospitals. Relating several independent variables to adjusted costs, they found the partial coefficients of both the first and second-degree elements of the size variable to be significant. This suggests a U-shape average cost curve, with the minimum cost found with approximately 190 patients in residence.

Cohen

Harold Cohen has studied over 80 northeastern hospitals in order to explore the effects of size on adjusted patient cost. He used an index for cost that sought to eliminate the expense differences brought about by the wage and salary variation in urban and non-urban areas. When relative costs were regressed on either total patient days or number of beds, U-shaped average cost curves resulted. Cohen shows that cost per patient reaches a minimum at 160-170 beds or 80-85,000 patient days.

Mann and Yett

Taking Martin Feldstein's data from 177 British hospitals, Mann and Yett respecify his model and conclude that even though hospitals have failed to take advantage of available economies, increasing returns nevertheless exist that are important. They believe that Feldstein errs when he ignores the second-degree coefficients and substitutes a continually declining function for them.

There is no uniform conclusion from these studies. Each of them produces different results. Because of this uncertainty of whether or not a hospital's cost curves are flat, it is essential that the optimization model be specified initially in only the broadest terms. The next section presents the economic principles governing optimization and the generalized optimization model for hospitals.

Generalized Model

Much of the economic theory of the firm concerns maximizing well-being. For the individual firm, this sometimes means maximizing profit or minimizing losses. In the short-term, maximized profit can be achieved by producing that amount of goods which equates the marginal revenue and marginal cost of production, regardless of the degree of competition. Departures from this equilibrium point will force reductions in profit and therefore, the firm will not be operating as efficiently as possible. In short, its performance will be sub-optimal.

A firm that produces several different products can determine an optimal product mix. According to traditional theory, a firm that seeks to maximize profit, given costs, will necessarily seek to operate on its highest iso-revenue curve[14]. Using a simple two-product case, the revenues to the firm will be maximized if it produces that combination of the two products which will result in the firm's iso-revenue curve being tangent to the production possibilities curve. This output is the most efficient from the producer's standpoint.

When the interests of the consumer are introduced in the form of his indifference map, the combination of products that maximizes consumer satisfaction can be determined. The consumer's interests are best served at a point of production where his indifference curve is tangent to his budget line.

Since the slope of the firm's iso-revenue curve is equal to the consumer's budget line, the level of output that best suits both the buyer and seller is where the production possibilities curve is tangent to the buyer's indifference curve. At that point, the firm's marginal rate of substitution is equal to the consumer's marginal rate of substitution. When that combination of products is provided, both the producer and buyer have maximized their well-being. Consequently, this combination maximizes efficiency and is optimal.

This result is based upon the premise that the firm seeks to maximize profit. But, several writers question if that is the most important goal of the firm. Most recently, Stonehill and others found that twenty financial executives of various large American corporations ranked profit maximization eighth in importance out of eleven possibilities, and this finding indicates that in practical terms, profit maximization is possibly not the most important goal to business[30].

This conclusion has significant meaning for the individual hospital. Essentially, it means that parts of traditional economic theory are not appropriate to the hospital industry. There are two reasons for this statement. First, profit maximization is not universally recognized as the chief goal of the organization, implying that the application of portions of economic theory is suspect. Second, most hospitals are not profit oriented and in fact, many try <u>not</u> to earn profits.

All this suggests that other goals have been substituted. In the situation where there is inadequate revenue information or revenue does not exist, Bilas offers an alternative objective[3]. Ignoring long-run considerations, he believes that the level of output in a single-product situation can be optimized when the firm produces at the low point on the average cost curve. When more products are added, their optimal output levels can also be identified as the lowest point on their average cost curves. However, producing at the individual optimums may not be feasible because of input constraints. If that is true, then another goal must be substituted. One possibility is to produce that output which minimizes the average cost for the entire firm. Doing this requires treating the various products as homogeneous. However, this in no way compromises the logic of minimizing average cost. Rather, it simply recognizes that products which appear on the surface to be generally uniform and possess equal selling prices, do not necessarily have the same production costs. Thus, treating the cost curves of each product independently while totalling the number of the products produced allows the individual outputs to be determined which results in minimizing the total average cost.

In the hospital setting, each patient type has a separate cost curve with a respective minimum point. However, operating at the minimum point may violate the input constraints. Nevertheless, operating at the individual sub-optimal positions may still allow the hospital to minimize its overall average costs. For example, consider a three patient-type operation. If all three levels of output are sub-optimal because of input considerations, the respective average daily costs of each type can be combined with the number of patients treated by type in order to determine total cost. Then, by dividing by the total number of patients(treated homogeneously), the average cost per patient day can be estimated. Chiefly,

$$Z = (a_1X_1 + a_2X_2 + a_3X_3)/(X_1 + X_2 + X_3)$$

where Z = average cost per patient day

a_i = average daily cost for the i^{th} patient type

X_i = number of patients for the i^{th} patient type

$i = 1, 2, 3$

The hospital then needs to be able to identify the objective function Z, which is to be minimized, and the pertinent input constraints. Estimating the objective function requires knowledge of the cost curve for each type of patient. In this illustration, the a_i represent the average daily cost of treating patients(by type), so that the functional relationship between the number of patients and average daily cost can be generalized as $f(a_i)$, where i is the number of different types of patients. Again, these functions should be specified in only the broadest form so that the empirically determined equations truly represent the real cost relationships.

Input constraints must also be considered in this generalized statement. Most hospitals do not have unlimited resources, and thus must carefully marshall their inputs in such a way as to achieve their goals. This implies that resources must be allocated to the production of different types of patients. If this is the case, then there must be some knowledge of the production functions, or resource-consumption functions of each patient type in order to allocate resources intelligently. In general terms, the function $f(r_{ij})$ identifies the production function, where j is the j^{th} resource to be used by the i^{th} patient type. Thus, there must be a function for each resource that reveals how much of each resource is consumed by each type of patient.

However, there is a limit to available resources. These limitations take the form of absolute constraints. They can be identified by c_j, which means there is only c_j amount of the j^{th} resource available for allocation to the X_i patients.

In general terms, the optimization model discussed here can be summarized as:

$$\text{Minimize } Z = \sum_{i=1}^{n} f(a_i)X_i / \sum_{i=1}^{n} X_i$$

$$\text{subject to } f(r_{ij}) \leq c_j$$

In order to quantify this model, a sampling of hospitals must provide three bits of data. First, the average daily cost of treating each type of patient must be estimated for each hospital. These cost figures can be functionally related to the number of each patient type in order to estimate the coefficients of $f(a_i)$. Second, the average amount of each resource used by each hospital for treating each patient type must be known. These average consumption figures can then be related to patient numbers in order to estimate the coefficients of $f(r_{ij})$. Finally, the maximum amount of those resources, c_j, need to be identified for each hospital.

Once the model has been quantified, it will be necessary to vary patient mix in order to isolate that level of patient mix which minimizes average daily cost. The method employed in this study is heuristic programming, and its general nature is discussed in the following section.

Heuristic Programming

A solution for a model which is stated in a series of inequalities typically cannot be easily determined mathematically. Instead, it must be found by alternative optimization methods, which usually are known collectively as mathematical programming. The most common form is linear programming, and it requires all the model's relationships to be couched in linear terms. But, in order to determine the optimum patient mix that will minimize average daily cost, the specification of the stated model must be general enough to allow for non-linear relationships.

Optimization problems involving non-linear properties are almost always more difficult to solve than those with linear ones because as yet no generalized theory has been developed for their solutions. Many computational procedures have been introduced, but these tend to relate only to exactly specified models, and thus lack the generality that is needed for an all-purpose model.

The non-linear optimization problem will usually take one of three typical forms. First, it can be a statement of non-linear functions re-cast into linear form. This is the standard method of handling simple non-linear problems, and is normally accomplished by either logarithmic transformation or respecification of the function. In either case, the relationship could then be processed by the usual linear programming techniques.

Second, the problem can involve a non-transformable non-linear function in conjunction with linear constraints. In this case quadratic programming may be used. There are many published quadratic routines available for computer processing.

Finally, the third type of problem involves non-transformable non-linear objective functions and constraints. These problems are far more difficult to solve analytically than either of the other two general types. The classical optimization method, which is based on calculus and Lagrangian multipliers, can theoretically be used to solve this type of problem. For practical purposes, however, the use of this method is restricted to simpler types of problems. Therefore, as a generalized technique it fails to provide the necessary flexibility that repeated modifications and iterations would require.

The most practical way to overcome this limitation is to employ simulation algorithms which can discover solutions by experimentation rather than by expansive mathematical formulation. Many operations research analysts gloomily view computer simulation as a method of last resort, to be used only when all else fails. The chief reason for this viewpoint centers on the nature of the problems themselves. Specifically, if a system is so complicated that it cannot be solved by the normal techniques, such as linear and non-linear programming, then the required model-building effort and subsequent analysis should also prove to be difficult, if not impossible. Wagner points out that the simulated world is really just about as unfathomable as the real world[32, p.887-892]. However, since the real world oftentimes cannot be reduced to simple mathematical statements that easily lend themselves to standard-technique solutions, life-like problems must be approached and solved with simulation. Thus, one of the key advantages of simulation is that it can consider more complex situations than can other tools. In that regard, a simulation model can provide exact quantitative results without distorting the theory underlying the model.

One of the attractive offshoots of this ability to consider complex problems is that a simulation may provide greater insights into the dynamics of the theory. This is an important contribution, especially since analysis tends to limit rather than broaden those insights. Because realistic problems probably are complex, it is valuable to know how alterations in the model's specifications would change the output. A simulation is very easy to modify. Thus, the range of a model's dynamics operates in conjunction with its ease of operation. That is to say, the simulation model can be very complex as befitting its real world counterpart, and yet be so accessible and changeable in contrast to the more analytical techniques.

Another positive feature of simulation is that it can be handled quite easily by non-mathematicians. A great deal of mathematical knowledge is needed to set up and solve analytically any complex problem. This necessarily restricts the number of persons able to formulate and solve their own problems. Simulation, however, since it so simple conceptually, can be employed advantageously by people who are largely unfamiliar with mathematics.

Finally, simulation is particuarly well-suited to evaluate micro-level models. Individual operations can be described in objective form, and the resulting model can then be tailored toward a simulation-type analysis. This is advantageous, since it is difficult to adapt a generalized technique for small, simple analyses, due to the level of technical know how required and the related expense.

However, this introduces two large disadvantages of simulation. First, a simulation model does not have universal applicability. Normally, it is limited to solving only the problem at hand. Second, most simulation models must be built from the ground up. There is usually no middle area from which to start. In most cases this means that all work must begin with data which can be used to develop the relationships in the model. Then the computer work must be geared specifically for the one-time processing of the model. Naturally, all of this work is expensive and time consuming. But, when contrasted with the advantages of simulation and the shortcomings of other methods, it seems that simulation can be useful in many situations.

There is a great variety of simulation techniques available. They range from the simple to the very complex, with the degree of difficulty being directly related to the model's level of sophistication. Most applications using simulation models encompass random phenomena, such as queuing problems, inventory control, and research and development models. Frequently, such simulations require millions or even hundreds of millions of randomly generated values to be used in the evaluation of the model. These values can be produced by a computer formulation based on pre-established probability distributions. In some cases, though, probabilistic formulations can be replaced with deterministic ones. This latter method avoids considering probabilities, which often are difficult to estimate.

Sivazlian has identified two broad, multiple variable categories of deterministric search routines which can be used to resolve simulation problems [29, p.346]. First is simultaneous search, in which no simulated outcomes of observations are used in locating subsequent decision values. Second is sequential search, or heuristic programming, in which individual outcomes are used to determine which observation values will be processed next.

The most comprehensive simultaneous search method is exhaustive or universal processing. Parenthetically, while the general simulation methods are relatively few, the number of different names for them is legion. So, the identifying labels used here are chosen because they appear more often in the literature than do any of the others. In any event, an interval δ normally is selected which indicates the distance from one point of evaluation to the next. For continuous data, the smaller the interval, the more precise will be the results, but also the more expensive the results will be to obtain.

The following simple numerical example demonstrates how the technique works. Suppose that the objective function to be minimized is:

$$Z = 10A - 0.5A^2 + 6B + 0.2B^2$$

$$\text{subject to:} \quad 100A - 0.5A^2 + 50B - 0.1B^2 \leq 5{,}400$$

$$60A + 3A^2 + 25B - 2B^2 \leq 10{,}980$$

$$96 \geq A \leq 100$$

$$196 \geq B \leq 200$$

When $\delta = 1$, there are twenty five different combinations of A and

B that can occur, and each one has been processed through the objective function with the results listed in Exhibit 2-2. Of the possible combinations, the one in which A = 98 and B = 196 results in the least value of the objective function, a value of 37,240. However, this combination violates the second constraint of 10,980 minimum units and is therefore unobtainable. As a matter of fact, only seven of the twenty five groupings are feasible, with the smallest value of Z being generated when A = 100 and B = 199.

This example is small in scale and does not show how involved problems can become using this form of simulation. If there were four different variables specified in the model and if each could take on a hundred possible values, there would be a hundred million different combinations produced from the model. Each of these would require testing. While the magnitude of the testing job may be breath-taking, this form of simulation does result in one clear advantage: the global optimum is known with certainty. Most other simulation methods can provide approximations of the optimum, but can give no guarantee that it has actually been found--universal processing can.

To circumvent the high cost of such processing, three standard heuristic programming search routines have been developed. All are reasonably sophisticated. First is the gradient method, which uses a set of partial derivatives which can indicate the direction of fastest increase in the objective function in the vicinity of some trial point [32, p. 534]. The second is a modified gradient, the optimal steepest ascents method, which allows one isolated factor to fluctuate so that the

Exhibit 2-2

EXAMPLE OF UNIVERSAL PROCESSING

A	B	Objective Function	Constraint B	Constraint II
96	196	38,524	5211	10,950
	197	39,285	5295	10,961
	198	40,050	5380	10,971
	199	40,819	5466	10,981
	200	41,592	5552	10,992
97	196	37,885	5124	10,953
	197	38,646	5209	10,964
	198	39,411	5294	10,975
	199	40,180	5379	10,985
	200	40,953	5465	10,995
98	196	37,240	5037	10,956
	197	38,001	5121	10,967
	198	38,766	5206	10,977
	199	39,535	5292	10,987
	200	40,308	5378	10,998
99	196	36,589	4948	10,957
	197	37,350	5033	10,968
	198	38,115	5118	10,979
	199	38,884	5203	10,989
	200	39,657	5289	10,999
100	196	35,932	4859	10,958
	197	36,643	4943	10,969
	198	37,458	5028	10,979
	199	38,227	5114	10,989
	200	39,000	5200	11,000
Constraints			5400 Maximum	10,980 Minimum

objective function is optimized locally [29, pp.391-394]. After that, another variable is allowed to fluctuate until an even better local optimum is found. This process continues until no further improvements can be made. Finally, the various parallel tangent(PARTAN) methods involve optimizing on planes that are successively parallel to one another[3, p. 221]. This method is the most efficient, given its formulation requirements.

Each of these methods is extremely valuable, but unfortunately has questionable practical use. Aoki criticizes the group as a whole and suggests that these heuristic search routines cannot be used if the objective function has discontinous first derivatives, if the objective function is given as a set of equations written for various subsystems of the whole system, and if the objective function has various local extremes which are likely to trap the points generated by one of these methods[1, pp. 152-153].

It may be disheartening to some that these objections are valid for the hospital model formulated previously. As a result, none of these methods can be used successfully. However, the heuristic programming umbrella covers three additional, howbeit less sophisticated, methods for simulating the model. Emshoff has identified the first as single factor search[3, p.219]. This technique allows one component of the model to change while all others remain fixed. It is similar to the optimal steepest ascent, but it does not require the sometimes unavailable partial derivatives. One variable, say pediatric patients, is allowed to vary toward a point where the objective function is optimized. Then the next variable is freed, and so on. In essence, this relaxation method forces substantial juggling of numbers, but can move in the direction of the optimum.

Aoki offers the pattern search, which is a modification of the single factor search[1, p.153]. Once the direction of the optimization has been found, he would reduce the interval δ for successive iterations. Fundamentally, this technique tries to locate the ravines in the objective function, and allows smaller and smaller increments to test for them. So long as smaller step sizes continue to be successful, the pattern continues. However, when a step fails to move closer toward the optimum, the pattern is broken and a new directional search is begun.

The third offering is a random search. Cooper believes that random forays have just about as much chance of quick success as do the other methods, and as a consequence holds that this method is as acceptable as any[7]. It does appear, however, that the feasible surface must be reasonably flat, without important nooks and crannies requiring investigation. This may be an important drawback, because knowledge of the surfact of a multi-dimensional sphere probably will be lacking.

Of the latter three heuristic techniques, the pattern search seems to offer the greatest promise. It has several important advantages. First, it is intuitively appealing, which is especially valuable for the non-mathematician. Second, no explicit knowledge of the objective function is required. Third, it is easy to deal with constraints on individual variables as well as complex constraints that have irregular boundaries and isolated excluded regions. Finally, it can handle sets of interacting equations which may comprise the whole system.

The major disadvantage of pattern search is that it may get stuck, and may not be able to make further improvements toward a local or global optimum. This may be especially true if the objective function has sharp turns or very curved ridges. Also, there is no guarantee that the global optimum has been found.

Thus, there are many heuristic methods that can be used to study the hospital problem. One seems to stand out, though, and it is pattern searching because of its obvious advantages to the analyst. In addition, in order to avoid the problem of not finding the global optimum, universal processing, which is a non-heuristic approach, can be used as a back-up to make certain that the optimum generated by the pattern search routine is the true one. This is the combination of techniques that will be used for this study.

Summary

In essence, this chapter has shown that hospitals have an entire array of measures from which to choose in order to control costs. Several of these are possible only on an area or regional scale, and are beyond the power of most administrators to implement. However, all administrators can effectively try to control costs by better supervision of the internal machinery of the hospital. Several alternatives were discussed. One of the most promising appeared to be controlling patient mix. By a judicious manipulation of its mix, a hospital may be able to lower its patient-related costs.

Naturally, it would be comforting to know that costs, as associated with variations in mix, could be minimized. Estimating that optimal mix requires knowledge of the cost and production functions for the hospital. In the past, most published studies relating to hospital optimization have used linear programming techniques. However, one important shortcoming of those methods is their inability to consider non-linear functions. At present, though, no generalized non-linear programming methods have been used. But, heuristic programming, which is a form of simulation, can be used to estimate the optimal patient mix, if one exists.

Thus, information can be generated which will enable hospital administrators to plan their long-range admission policies more effectively. In the following chapter, the sampling methodology, data acquisition for the optimization model, and data adequacy are discussed.

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CHAPTER III

DATA

This chapter is divided into two parts. First is the discussion of the sampling methodology used to gather the necessary data, along with a brief examination of the data themselves. Second is the very important discussion of the adequacy of the data obtained.

Sampling Methodology

As summarized earlier, three bits of data are needed in order to process the proposed model. First is the average daily cost of treating each type of patient; second is the average amount of each resource used by each hospital to treat each patient type; and third is the maximum amount of each resource available for each hospital.

Hospital administrators have identified five major types of patients for which information must be gathered. They are: nursery, pediatric, obstetric, medical-surgery, and intensive care patients. The billing procedure for most hospitals forces a patient into one of these purely definitional categories. This classification will be used for pragmatic reasons, because if optimum patient mix information is to be useful, it should appear in a form similar to what hospitals actually use.

However, other alternative classifications have been suggested. Easily the most important alternative is classifying by specialty. Feldstein tested a specialty mix that involved 28 different types of patients[2]. Knotts used a definition schedule which considered 54 types[7]. The trouble with such classifications is that they cannot be exhausted--as long as physicians recognize many different specialties that require individualized attention, the list of specialty types can become unreasonably long.

Another shortcoming with specialty lists is the obvious limited applicability. The longer the list becomes, the fewer the number of hospitals that would be able to use a model based on that list. A short classification, while perhaps sacrificing some theoretical purity, probably is managerially better.

One individual has suggested combining the standard patient mix classification with a more extensive specialty mix[6]. The five patient types would be retained, but each would be broken down into groupings based upon acuity of need. This would allow a broader spectrum of variety to be evaluated, but still be grounded on the highly usable patient-type classification.

The simple five patient-type classification is used here, chiefly because there is a lack of information on how to subdivide the types into smaller groupings. Processing data through this classification will allow hospital administrators to evaluate their present operations using the pragmatic patient-type definition.

Hospital administrators have also identified three general classes of resources available to the hospital. These are: number of bed-days, nurse-hours, and laboratory procedures. These data are necessary in order to estimate the production equations for the model, and as a result must be separated into resources used by each type of patient. This separation will provide the total consumption of a particular resource by a specific type of patient.

The third piece of data needed is the maximum amount of each resource. Obviously there is only a fixed amount of nurse-hours that a hospital can use in the short run, as well as fixed amounts of bed-days and procedures. Certainly those amounts can be enlarged over time, but for practical purposes they should be treated as constants during short periods. As such, these amounts act as constraints on the equations of the hospital model.

When selecting a sample of hospitals, two problems must be considered so that they can be avoided. They are differing factor prices and differing services. It is important that cost figures for all sampled hospitals be equivalent in purchasing power, so that direct comparisons can be made between hospitals without having to adjust for differences in cost of living and relative wages. For example, comparisons between New York City and Boston hospitals with those in Memphis and Birmingham are probably meaningless, because of the real differences in prices and costs.

The impact of differing prices can be reduced by selecting hospitals that operate within a restricted geographical region, say, the state of Oklahoma. Unfortunately, no published data have been found that support this selection, but it is appealing strictly on intuitive grounds. One hospital administrator has suggested that there are at least three factors that would lead to such a conclusion[4]. First, most Oklahoma hospitals are supplied by firms operating either in Oklahoma City or Tulsa. Furthermore, the prices that the hospitals pay are uniform and do not vary according to the location of the hospital. In all fairness, though, it is obvious that transportation costs will vary, but the importance of this feature is not known. Second, the average salary of newly registered nurses is approximately equal throughout the state. For inexperienced nurses, the factor prices do not vary substantially. Third, most Oklahoma hospitals pay their unskilled workers the minimum wage. Thus, unit material prices and employee wages, two major hospital expenses, are approximately equal for most Oklahoma hospitals. While no proof exists, these three items do tend to support the belief that material and labor prices for Oklahoma hospitals are equal or almost so.

The second problem with designing a sample is the differing services that hospitals may provide. It would be unwise to compare the cost and production figures of hospitals with extensive services with those having more modest service offerings. For instance, a teaching and research hospital, like the University of Oklahoma Medical Center in Oklahoma City, should not be compared to a typical hospital, such as Norman Municipal, because of the great differences in types of services offered. Selecting hospitals that have similar service offerings is tentative at best, but at least one other hospital administrator believes that service equivalency can be approximated by sampling hospitals of the same general size[5].

For Oklahoma, hospitals can be broken down into three categories: small hospitals with less than 140 beds, medium size hospitals with 140-215 beds, and large hospitals with more than 215 beds. All large hospitals are located in the Oklahoma City and Tulsa Standard Metropolitan Statistical Areas, and most of the small ones are located in rural cities. Middlesized facilities are scattered in both the major urban centers and in the rural areas. This study used the medium size group because it offers a balance of urban and rural institutions that may cause the resulting model to have wider acceptance.

In 1976, there were fifteen medium-size hospitals in Oklahoma and they are listed in Exhibit 3-1. Three of them are probably not usable because they are either comprised of geographically separate facilities or they have recently moved into new quarters. A preliminary random sampling of four of the remaining twelve was conducted as a means of gathering information which would be used to determine the sample size for the entire study. Average cost per patient day figures were collected from the four hospitals. The cost data obtained were as follows:

Hospital	Average Cost
A	\$114
B	110
C	95
D	135

The formula for determining the sample size is

$$n = \left[\frac{S}{D/Z} \right]^{\frac{1}{2}}$$

where n = the sample size
S = the standard deviation of the preliminary sample
D = the desired error
Z = the confidence coefficient

For this study, the desired error(D) of \$1.00 about the mean at the 95 percent confidence level($Z = 1.96$) was selected. The mean cost from the

Exhibit 3-1

Medium Size Hospitals in Oklahoma

Hospital	Location
Valley View	Ada
Memorial	Ardmore
Grady Memorial	Chickasha
Bass Memorial	Enid
Commanche	Lawton
McAlester Gen.[1]	McAlester
Midwest City	Midwest City
Norman Mun.	Norman
Deaconess	Oklahoma City
Mercy[2]	Oklahoma City
Presbyterian[2]	Oklahoma City
South Community	Oklahoma City
Ponca City	Ponca City
Bartlett	Sapulpa
Doctors'	Tulsa

Source: Hospital Guide, 1976

[1] Composed of two geographically separate facilities.

[2] Recently moved to entirely new facilities.

preliminary study is $113.50 with a standard deviation of $16.50. Processing these figures through the formula gives the sample size of 5.689, which should be rounded up to 6[8]. Two additional hospitals were chosen randomly to combine with the four selected for the preliminary work. All hospitals will remain anonymous because of commitments made to safeguard hospital privacy.

In selecting any size sample, two factors surface which are vastly more important than any others and must be considered[3]. First, the precision of the sample, or the error that is tolerable, is critical. If the consequences of failure are not serious, undertaking risk posses no threat even if the odds for failure are great. On the other hand, if failure could cause severe complications, the level of precision can be beefed up in order to compensate for risk. The selection of the precision level, here $1.00 at 95 percent, will thus depend on what the results of catastrophic failure would be.

The second factor is cost. If cost is excessive, some balancing of precision against cost must be considered. For example, if the usefulness of the results would not be compromised by lowering precision, the study can be conducted at reduced cost. But, if precision cannot be sacrificed, additional funds should be obtained, or the scope of the survey altered. Cost considerations were minor since this study used only six hospitals.

Several key officers of the hospitals indicated that data acquired for a two-week period are generally representative of an entire year. That is to say, the cost and production averages are approximately the same for a two-week period as for a full year. Consequently, data were gathered from each hospital during a two-week span in the Spring, 1976.

No holidays appeared during the periods. The average cost per patient day, nurse-hour usage, laboratory procedure consumption, and bed-days were calculated by patient type for each of the six hospitals.

Questions can be raised, though, about the feasibility of using a single hospital's data in order to construct an optimization scheme, such as Knott[7] and Dowling[1] did. This would certainly be attractive as far as sampling costs are concerned. However, the single hospital sample is fraught with conceptual difficulties. First, aside from variations caused by major holidays and emergencies, administrators feel that a hospital's patient load per week and its patient mix are fairly stable throughout an entire year. As a result, cost and resource-usage figures would have little variation during most short-term periods. Second, designing an optimization scheme around only one operation, even with multiple observations over time, may tend to hide any shortcomings a hospital might have. A multiple hospital sample can reveal that an individual unit, while locally successful, may be under-achieving and over-costing when compared to other units. Finally, the model obtained would not be valid for any other hospital, because using it to evaluate other facilities would presume that the single unit is representative of the group, which of course is unlikely. Thus, the limitations of a one unit sample seem to outweigh its major advantage of reduced cost.

The procedure for gathering the necessary data from the six hospitals is discussed below.

Average Daily Cost

The actual cost to the hospital of caring for a patient is the most difficult figure of all to determine because most hospitals do not have and perhaps do not wish to have adequate cost accounting systems. As a consequence, most hospitals' financial officers have not measured and hence do not know what their true costs are. However, most controllers can estimate those costs by examining some of the other data, such as average stay, average number of laboratory procedures, and the average number of nurse hours. Using that kind of information, one controller has estimated that the total two-week cost of treating the hospital's obstetric patients was $21,250, or about $139 a patient day.

Bed-days

Admission and discharge dates are on each patient's billing sheet, and can be used to determine the total number of days a patient spent in the hospital. For example, during the two weeks, one hospital treated 30 intensive care patients for a total of 410 days, or an average of 13.67 days each.

Nurse-hours

Most hospitals do not keep records indicating the amount of time a nurse spends with each patient. Consequently, determining the amount of time nurses spend with patients is largely a matter of averaging aggregated data provided by the personnel office. Specifically, most patient-types and nurses are assigned to a certain area of the hospital. The man-hour reports reveal how much total nurse time is spent in any given section, say, the pediatric section. The number of pediatric bed-days for the same period can be determined as mentioned previously. When combined, the two aggregates will give an indication of how much time, on average, nurses spend caring either directly or indirectly for individual patients. For instance, one hospital recorded 1,177 nurse hours in the pediatric section for two weeks, while during that same time 42 pediatric patients were hospitalized a total of 167 days. The average number of hours of nurse care for each pediatric patient for each day was 1,177/167 = 7.04 nurse hours per day.

Laboratory Procedures

All hospitals keep records of the laboratory tests that physicians order for their patients. Many of the tests are routine to all patients while others are not. However, the data among hospitals may not be consistent because many of the definitions of the number of procedures or steps comprising each test are not fixed. Hospitals can process and record a test using any definition they wish. In order to avoid the possibility of data inconsistencies, a classification should be imposed. The definition schedule used for this study is listed in Exhibit 3-2.

The ease of gathering these laboratory data depend upon automation. For the computerized facilities, the name of each test is listed on the billing sheets, thus enabling the simple comparison of the name and arbitrary definition of the number of procedures associated with that test. With non-automated hospitals, the job is more difficult. For these hospitals, records are kept listing in random order the name of the patient and the kind of test ordered by the physician. For the patient discharged during the sample period, a search must be made through laboratory files to glean the information required. Using another test hospital, 42 obstetric patients were served during the two weeks, and the women's physicians ordered 427 procedures for an average of 10.17 each.

Exhibit 3-2

LABORATORY PROCEDURES PER TEST

Test	Procedures
CBC	4
Cultures (all)	2
Electrolite Series	
Chloride	1
CO_2	1
Potassium	1
Sodium	1
SMA 12	1
Cross-Match	4
Serium-iron	2
Cordosone blood	3
Urinalysis	1
Protymes	1
Sensitivity	1
VDRL	1
Glucose	1
Urea Nitrogen (BUN)	1
CPK	1
CDH	1
Transaminase (OT)	1
Transaminase (PT)	1
Cedrate	1
T/3 (thyroid)	1
T/4	1
Bilirubin	1

Resource Maximums

Each of the three resources has an absolute maximum available in the short run. These figures are easy to estimate. For example, the maximum number of bed-days is found by multiplying the number of beds times the number of days during the sampled period. One of the six hospitals has 200 beds, which means it has 2,800 bed days for any two week period. The maximum number of nurse-hours must be estimated from man-hour reports. Figures for direct and administrative hours are typically kept for payroll purposes and are thus readily available. For the same period, the hospital mentioned above has 24,050 hours of nurse care available. In addition, that hospital also has the capacity to produce 13,400 laboratory procedures. This estimate is based upon the beliefs of the laboratory supervisor.

All of the data were reduced to averages and are shown in Exhibit 3-3. These are the data that are used to construct the optimization model.

Data Quality

Significant questions can be raised about the quality of the data obtained. The gathering process was consistent for all hospitals and the periods for data collection were approximately the same. In fact, in most areas the quality of the data is not suspect. But in that regard, there appear to be five areas that may cause some uneasy feelings.

First, the definition of size is based upon casual observation of all hospitals in Oklahoma. The desire was to restrict the sample to hospitals of similar size so that possible service differences could be minimized. Importantly, as the definition becomes narrower, the number of hospitals available for sampling becomes fewer. However, this shortcoming must be

Exhibit 3-3

INPUT DATA BY PATIENT-TYPE

Hospital	Patients	Bed-Days	Procedures	Nurse-hours	Cost Per Day
			Nursery Patients		
I	37	5.27	10.86	4.71	$ 26.00
II	30	4.05	4.25	5.15	21.90
III	8	3.25	1.00	8.00	41.82
IV	15	4.33	6.73	7.22	23.00
V	76	3.30	9.87	5.22	19.50
VI	21	4.00	4.84	8.00	30.05
			Pediatrics Patients		
I	42	3.98	9.29	7.04	$120.00
II	42	1.00	11.00	5.89	116.61
III	46	4.22	12.48	5.30	115.92
IV	129	7.22	11.67	8.15	83.81
V	74	7.39	11.02	8.24	95.00
VI	23	4.89	11.34	5.30	135.00
			Obstetrics Patients		
I	42	3.64	10.17	13.96	$139.00
II	32	3.89	9.00	15.10	69.00
III	8	4.25	7.63	4.50	135.18
IV	26	3.88	11.19	11.61	45.00
V	100	3.90	9.58	8.35	90.00
VI	21	4.10	10.31	10.40	114.85
			Medical-Surgery Patients		
I	250	6.70	13.78	6.06	$110.00
II	290	10.00	10.60	5.89	58.19
III	157	6.36	19.78	6.70	107.44
IV	131	4.97	16.82	8.15	125.44
V	215	5.69	11.41	6.99	140.00
VI	210	7.76	12.43	7.30	130.84
			ICU Patients		
I	30	13.67	50.40	23.54	$161.00
II	30	11.60	55.60	16.42	172.50
III	8	3.63	27.00	20.00	203.30
IV	28	7.21	24.29	18.74	93.84
V	46	7.84	16.50	14.90	135.00
VI	31	5.90	43.90	22.75	228.69

balanced against the opposite requirements of geographical proximity and service similarity. Thus, the potential loss from a restricted definition does not seem to outweigh the gains.

Second, the limited number of observations may be a problem. Statistically, the sample size is significant, but no comments can be made about its representativeness[9]. No published data are available to check if the six hospitals are representative of the twelve medium size Oklahoma hospitals, and gathering the necessary data would amount to a census of the population, in which data scarcity is a known problem. However, a sampling of half of the hospitals tends to mollify the negative impact of being unable to make such tests.

Third, factor price differences may not have been eliminated by using geographically-close hospitals. As stated earlier, there is no proof of price similarity, but the arguments offered support of the belief that prices are approximately equal. Unfortunately, the controllers of the various hospitals were not authorized by their boards to release purchase invoices and payroll records that would have been helpful in determining price equality. Consequently, data gathering and processing proceeded as though the data were pure of any substantial input price difference.

Fourth, questions may arise about the accuracy of the average cost figures. There does not seem to be any way to check on the accuracy, other than a truly comprehensive examination of the hospitals' accounting data. Unfortunately once again, no controller was authorized to provide those data. As a result, the controllers' estimates must be used simply because nothing else is available. From a practical point of view, however, this necessity offers no difficulties at all, because these controllers work with cost figures every day and are acutely aware of their organizations' costs. To suggest that they are ill-equipped to make such cost estimates is unwise.

Finally, there is the problem of the independence of patient mix and average cost per patient day by type. Of cource, each hospital's mix has an influence on costs, but how much influence is not known. One way to estimate the level of interaction would be by way of a correlation matrix, which would show how the variables interrelate. Exhibit 3-4 presents such a matrix for the averages from Exhibit 3-3. Only pediatric cost and intensive care cost appear related; all others fail to show any important relationships. The pediatric-ICU relation probably has no theoretical or even practical foundation, and most likely results from quirks in the data. This conclusion is supported by the fact that no other ICU relationship has a correlation coefficient greater than 0.50. Thus, this exhibit fuels the presumption that there are no practical interrelationships which need evaluation.

Exhibit 3-4

CORRELATION MATRIX FOR AVERAGE PATIENT
COST BY PATIENT-TYPE

	Nursery	Pediatric	Obstetric	Medical	ICU
Nursery	1.000	.202	.415	.000	.401
Pediatric		1.000	.463	.065	.867
Obstetric			1.000	.017	.460
Medical				1.000	.025
ICU					1.000

Source: Exhibit 3-3

References

1. William Dowling, "A Linear Programming Approach to the Analysis of Hospital Production"(Ph.D. dissertation, University of Michigan, 1971).

2. Martin S. Feldstein, Economic Analysis of Health Service Efficiency, (Amsterdam: North-Holland Publishing Co., 1967), p. 39.

3. Morris H. Hansen, William N. Hurwitz, and William G. Madow, Sample Survey Methods and Theory, Vol. 1(New York: John Wiley and Sons, Inc., 1953), pp. 31-34.

4. Interview with John Coffey, President, Oklahoma Hospital Association, Lawton, Oklahoma, October 16, 1975.

5. Interview with Richard Luttrell, Past-President, Oklahoma Hospital Association, Norman, Oklahoma, June 4, 1974.

6. Interview with E. Paul Smith, California State College, Phoenix, Arizona, March 18, 1977.

7. Ulysses S. Knotts, Jr., "Linear Programming Analysis of a Community Hospital to Determine Optimum Response Utilization'(Ph.D. dissertation, University of Nebraska, 1971).

8. Donald H. Sanders, A. Franklin Murph, and Robert J. Eng, Statistics, (New York: McGraw-Hill Book Co., 1976), p. 186.

9. W. Allen Wallis and Harry V. Roberts, Statistics,(Glencoe, Illinois: The Free Press, 1958), pp. 338-339.

CHAPTER IV

FINDINGS

This chapter is separated into two parts. Part I presents the quantified form of the generalized model discussed in the second chapter. Part II gives the results of the simulation for each hospital's patient mix.

Model

All of the data discussed in previous chapters form the foundation of this economic model of the hospital. The cost and resource functions, which are the main components of the model, were specified in non-linear form, chiefly $Y = a + bX + cX^2$, and then estimated with the actual data. If the resulting coefficients did not test out to be significant, the non-linear specification was abandoned and substituted by a linear function. If the linear relationship, such as $Y = a + bX$, tested out to be insignificant, the average cost or resource-usage figure was used. In all cases the Conversational Statistical Package[1], a series of canned programs provided by the IBM Corp., was used to test the statistical significance of the relationships. All of the equations fit into the generalized model, which is reproduced below:

$$Z = \sum_{i=1}^{n} f(a_i)X_i \Big/ \sum_{i=1}^{n} X_i$$

$$\text{subject to } f(r_{ij}) \leq c_j$$

where $f(a_i)$ and $f(r_{ij})$ are the cost and production equations to be estimated.

For business applications, a confidence coefficient of 95 or 99 percent is typically used when testing a hypothesis for statistical significance. This coefficient is a convenient way of expressing the sampling error by giving an interval that is likely to include the true population parameter. The larger the confidence coefficient the more likely that the population parameter will be included within the confidence interval, and the less likely that an error will be committed in accepting the hypothesis. However, by decreasing the chance for this Type I error, the chance of accepting a false hypothesis(Type II error) is increased. Unfortunately, these two types of errors work against one another, so that it is impossible to reduce one without increasing the other, given the sample size.

The classical approach to statistical inference would leave the balancing of the risks of the two errors to the judgment of the analyst. Usually, the coefficient can be chosen in a way that trades the value of a precise estimate against the cost of missing the true value. For this study, a larger chance for committing the Type I error, as denoted by the critical probability α, was selected in order to shrink the chance for a Type II error. The critical probability α was set equal to 20 percent, and the t-value at that level with three degrees of freedom is 1.638.

Estimates of the cost and production functions with non-linear specifications are given in Exhibit 4-1. The numbers in parentheses under the functions are the t-values for the coefficients. Also, the correlation coefficient for each function is given. Of the five cost functions, only Pediatric and Medical-surgery are non-linearly significant. Nursery and Obstetric functions are the only non-linearly significant bed-day and nurse-hour relationships, while the ICU function is the only significant laboratory relationship. All the remaining test out to be insignificant. Thus, seven of the twenty functions display non-linear characteristics.

The thirteen functions that tested insignificant were re-specified to a linear form. For linearity, the t-value with four degrees of freedom is 1.533, which means that seven of the functions listed in Exhibit 4-2 are linearly significant. They are: (1) nursery cost, (2) pediatric beds, (3) medical-surgery bed days, (4) pediatric nurse hours, (5) medical-surgery nurse hours, (6) nursery laboratory, and (7) medical-surgery laboratory.

The six remaining relationships must be stated as averages, since apparently for the hospitals at hand there are no important connections between the respective dependent and independent variables.

Exhibit 4-3 lists all twenty relationships that appear to be important. These are the functions that describe the connection between patient numbers and average cost and resource-usage. Taken by themselves, each function adequately describes a single relationship. But a check on the entire model is needed. Such a check is summarized in Exhibit 4-4.

Exhibit 4-1

Non-Linear Hospital Cost and Product Functions

Patient Type	Average Daily Cost Function	Correlation Coefficient
Nursery	$\$42.27 - 0.8042N + 0.00673N^2$ (1.46227) (1.0914)	.7700
Pediatrics	$\$162.65 - 1.279P + 0.00517P^2$ (11.16625) (7.248)	.99730
Obstetrics	$\$125.70 - 1.29751B + 0.00974B^2$ (.42121) (.37197)	.25758
Medical-Surgery	$\$-111.70 + 2.62834M - 0.007M^2$ (2.574) (2.882)	.90928
ICU	$\$214.26 - 1.982I + 0.009071I^2$ (.25645) (.06382)	.38157

Exhibit 4-1 (Cont.)

Non-Linear Hospital Cost and Production Functions

Patient Type	Bed-day Function	Correlation Coefficient
Nursery	$2.582 + 0.10593N - 0.00126N^2$ (2.4207) (2.579)	.83344
Pediatric	$2.107 + 0.0497P - 0.00006P^2$ (.327) (.061)	.65017
Obstetric	$4.508 - 0.02832B + 0.00022B^2$ (5.232) (4.8213)	.95246
Medical-Surgery	$8.467 - 0.0437M + 0.00016M^2$ (.540) (.846)	.84101
ICU	$-1.024 + 0.659I - 0.01014I^2$ (1.3487) (1.1274)	.65069

Exhibit 4-1 (Cont.)

Non-Linear Hospital Cost and Production Functions

Patient Type	Nurse-hour Function	Correlation Coefficient
Nursery	$9.873 - 0.1923\ N + 0.00171\ N^2$ $(2.500) \quad (1.9865)$	.88019
Pediatric	$3.122 + 0.0914\ P - 0.0004\ P^2$ $(1.521) \quad (1.0711)$	.85051
Obstetric	$0.893 + 0.5403 - 0.0047\ B^2$ $(6.3201) \quad (6.4164)$	.96548
Medical-Surgery	$9.1314 - 0.0099\ M - 0.00001\ M^2$ $(.2538) \quad (.0492)$	.82983
ICU	$17.583 + 0.357\ I - 0.0888\ I^2$ $(.7658) \quad (1.02491)$	.60949

Exhibit 4-1 (Cont.)

Non-Linear Hospital Cost and Production Functions

Patient Type	Procedures Function	Correlation Coefficient
Nursery	$-0.646 + 0.354N - 0.0028N^2$ $(1.515) \quad (1.078)$	.80387
Pediatric	$11.349 - 0.0136P + 0.00013P^2$ $(1.5814) \quad (.2341)$	.25823
Obstetric	$7.441 + 0.1202B - 0.0011B^2$ $(1.484) \quad (1.5892)$	.68373
Medical-Surgery	$31.285 - 0.1198M + 0.00017M^2$ $(.6691) \quad (.3990)$	.80556
ICU	$4.854 + 3.1574I - 0.0625I^2$ $(1.650) \quad (1.7753)$	.71816

where N = number of Nursery patients

P = number of Pediatric patients

B = number of Obstetric patients

M = number of Medical-Surgery patients

I = number of ICU patients

Exhibit 4-2

Linear Hospital Functions

Patient Type	Average Cost Function	Correlation Coefficient
Nursery	\$33.88 - 0.2192N (1.742)	.65670
Obstetrics	\$105.60 - 0.17734B (.30881)	.15260
ICU	\$209.24 - 1.508I (.8218)	.38005
	Bed-day Function	
Pediatric	2.3733 + 0.04062P (1.709)	.64963
Medical-Surgery	1.83458 + 0.02433M (2.65214)	.79842
ICU	4.5823 + 0.12923I (.93425)	.42323

Exhibit 4-2 (Cont.)

Linear Hospital Functions

Patient Type	Nurse-hour Function	Correlation Coefficient
Pediatric	4.987 + 0.0281 P (2.542)	.78588
Medical-Surgery	9.317 - 0.0118 M (2.972)	.82968
ICU	22.495 - 0.107 I (.8306)	.38352
	Procedure Function	
Nursery	2.877 + 0.1085 N (2.037)	.71351
Pediatric	10.765 + 0.00621 P (.457)	.22278
Obstetric	9.694 - 0.0056 B (.2797)	.13851
Medical-Surgery	24.37 - 0.0488 M (2.6114)	.79391

Exhibit 4-3

Chosen Cost and Production Functions

Patient Type	Function
Nursery Cost	$33.88 - 0.2192 N
Pediatric	$162.65 - 1.279 P + 0.00517 P^2
Obstetric	$94.81
Medical-Surgery*	$-111.70 + 2.6283 M - 0.007 M^2
ICU	$159.35
Nursery bed-days	2.582 + 0.106 N - 0.00126 N^2
Pediatric	2.3733 + 0.0406 P
Obstetric	4.51 - 0.283 B + 0.00022 B^2
Medical-Surgery	1.835 + 0.02433 M
ICU	8.86
Nursery nurse-hours	9.873 - 0.1923 N + 0.00171 N^2
Pediatric	4.987 + 0.0281 P
Obstetric	0.893 + 0.5403 B - 0.0047 B^2
Medical-Surgery	9.317 - 0.0118 M
ICU	18.93
Nursery procedures	2.877 + 0.1085 N
Pediatric	11.26
Obstetric	9.79
Medical-Surgery	24.37 - 0.0488 M
ICU	4.854 + 3.574 I - 0.0625 I^2

* This function takes on an inverted U shape similar to the ones found by Ingbar and Taylor. Average cost rises as patient number increases toward 188, and begins to fall beyond that. It is important that none of the individual coefficients, $-111.70 for instance, be evaluated in isolation. The entire function is pertinent here, not its parts.

Exhibit 4-4

Actual and Estimated Costs and Resources

Hospital	Cost		Bed-Days		Procedures		Nurse-hours	
	Actual	Est.	Actual	Est.	Actual	Est.	Actual	Est.
I	114.09	103.85	2,600	2,749	6,176	5,481	24,034	21,973
II	69.27	74.34	3,536	3,277	5,620	5,233	25,548	23,708
III	110.30	122.15	1,282	1,215	3,965	3,458	8,661	9,581
IV	94.71	100.05	1,950	2,048	4,860	5,369	18,320	19,894
V	112.55	104.03	2,774	2,968	5,736	6,462	23,012	25,198
VI	134.91	125.16	2,095	1,983	4,550	4,861	18,221	17,046
Correlation Coefficient		.90394		.97421		.79721		.95452

There are four sets of actual and estimated data, one for each of the resources and one for cost. From these data a vector of correlation coefficients was calculated, the results of which are also given in the same exhibit. With the single exception of laboratory procedures, the coefficients are all greater than 0.90. This indicates that a satisfactory relationship exists between actual and estimated values, and that the model can be used with confidence.

In addition to the cost and resource coefficients, the hospital model must also include constraints. Hospitals, like businesses, find themselves constrained in their operations. The constraints will normally take one or both of two forms. First, the hospital can provide health care to some maximum number of patients which is determined by the available resources--number of bed-days, nurse-hours, and laboratory procedures.

Second, there will also likely be a minimum number of patients that a hospital will want to have. This policy will usually be set by the interacting considerations of the hospital's board, administration, and medical staff, and the public. For example, the hospital will probably need a certain minimum number of a certain type of patient before it installs an expensive treatment center. This minimum number is a part of the hospital's admission policy.

These minimum and maximum figures will vary according to each hospital. Unlike the cost and production curves which describe an association relative to patient numbers, these constraints are absolutes and are not functionally related to patient mix or patient load. As a result, each hospital will have its own maximum number of patients it can serve and a minimum number that it wishes to serve. It is entirely plausible to presume that average costs can be minimized somewhere between these two extremes.

What is needed now is an application of the model in such a way as to aid hospital management in reducing average daily cost. This can be accomplished by using the simulation technique described earlier.

Simulation

A summary of the model as found in Exhibit 4-3 is provided in Exhibit 4-5 and is stated in standard mathematical programming form. Values for the maximum available resources are denoted by a subscripted "C". These are variables because of the relationship of those maximums to the individual hospital. Each hospital can produce only so much health care; the limit is set by the amount of resources it has available. All hospitals will have different amounts, and it would be unwise to insert a maximum value for the three resources when that value may only apply to a particular hospital. Rather, what should be done is to use the maximums for each hospital and process them through the model in order to determine the level of efficient operation for each institution. Values will be inserted later in order to show how the system works.

In addition to the available resource maximums, there will usually be a minimum number of each patient type that a hospital will want to serve. These minimums are listed under the fourth constraint. The five figures selected represent the smallest number of patients cared for during the two-week period by any of the six hospitals. As a matter of information,

Exhibit 4-5

Hospital Model

$$\text{Minimize } Z = [(33.88 - 0.2192N((N) + (162.65 - 1.279\,P + 0.00517\,P^2)\,(P) + + (94.81)\,(B) + (-111.70 + 2.6283M - 0.007\,M^2)\,(M) + + (159.35)\,(I)]/[N + P + B + M + I]$$

Subject to:

1. $$\text{Bed-days} = [BN + BP + BB + BM + BI] \leq C_1$$

   where
   $BN = (2.582 + 0.106N - 0.00126N^2)\,(N)$
   $BP = (2.3733 + 0.0406P)\,(P)$
   $BB = (4.51 - 0.0283B + 0.00022B^2)\,(B)$
   $BM = (1.835 + 0.02433M)\,(M)$
   $BI = (8.86)\,(I)$
   C_1 = Maximum available bed-days

2. $$\text{Nurse-hours} = [(9.873 - 0.1923N + 0.0017N^2)\,(BN) + (4.987 + .0281P)\,(BP) + (0.893 + 0.5403B - 0.0047B^2)\,(BB) + + (9.317 - 0.0118M)\,(BM) + (18.93)\,(BI)] \leq C_2$$

   where C_2 = Maximum available Nurse-hours

3. $$\text{Laboratory procedures} = [(2.877 + 0.1085N)\,(N) + (11.26)\,(P) + +(9.79)\,(B) + (24.36 - 0.0488M)\,(M) + (4.854 + 3.1574I - - 0.0625I^2)\,(I)] \leq C_3$$

   where C_3 = Maximum available Laboratory procedures

4. $8 \leq N \leq 76$ where N = nursery patients

   $23 \leq P \leq 129$ P = pediatric

   $8 \leq B \leq 100$ B = obstetric

   $130 \leq M \leq 290$ M = medical-surgery

   $8 \leq I \leq 46$ I = ICU

three of the hospitals operated above the minimum number of each patient-type. As a result, these minimums are not appropriate for any single hospital, but are appropriate only as a starting point of the simulation.

Also listed under the fourth constraint are maximum patient numbers. As with the minimums, these figures represent the largest values for all six hospitals. They are listed for purely informational reasons, since they may be rendered useless by the resource maximums.

Finally, general constraint 2 has been altered somewhat and perhaps should be explained. The numbers within each pair of parentheses determine the average daily consumption of nurse-hours by patient-type. Usage of the other two resources is stated in terms of the patient's length of stay. For example, the value for the nursery laboratory equation is a statement of how many laboratory resources each nursery patient will use for his entire stay in the hospital. The same is true for the bed-day resource. Thus, in order to be compatible with them, nurse-hour usage must be reformulated so that the ending value reflects how much of that resource is consumed by each patient type over the full two weeks. This can be accomplished by simply multiplying the average daily nurse-hour figure by the total number of bed days used by each type of patient. As an illustration, consider nurse-hour usage by the ICU patients. If there are 20 ICU patients they will use up a total of 177.2 bed days. Further, each of them will require 18.9 hours of daily nurse care, which yields a total nurse-hour consumption by the 20 ICU patients of 3,354 hours.

As it now stands, the model is ready for processing with only the values for the upper limits of the resources needed. All the hospitals'

Exhibit 4-6

Resources Available by Hospital

Hospital	Bed-days	Nurse-hours	Laboratory Procedures
I	2,800	24,050	13,400
II	3,010	25,500	12,200
III	2,142	8,700	7,400
IV	2,044	18,400	10,200
V	2,758	23,000	12,400
VI	1,988	18,300	9,800

resource maximums are shown in Exhibit 4-6. For the first run of the model, the resource limits of Hospital I were employed. In this case,

$$C_1 = 2{,}800 \text{ bed days}$$

$$C_2 = 24{,}050 \text{ nurse hours}$$

$$C_3 = 13{,}400 \text{ procedures}$$

The first iteration through this system using the smallest allowable patient numbers produced the following results:

Cost	=	\$112.64
Bed days	=	858
Procedures	=	2,920
Nurse hours	=	7,222

These 177 patients used up 858 bed days of the available 2,800, resulting in 1,942 unused bed days. Of the total 200 beds available for use throughout the full two weeks, 138 would be idle. Naturally, this represents an enormous under-consumption of health care facilities. Likewise, both the laboratory and nurses were under-consumed, leaving about 10,500 procedures and 16,800 nurse hours left idle.

Consumption of each of the three major resources increases as the number of patients increases. This means that by increasing the number of patients from 177, the total consumption of the resources will also increase, but perhaps at a decreasing rate. For example, the use of beds by Pediatric, Medical-surgery, and ICU patients will force an accelerating usage rate, which, in all likelihood, will be slowed by the declining averages of Nursery and Obstetric patients. This argument can be applied in similar fashion for nurse hour and laboratory procedure usage.

In any case, there are ample resources left unused. Because of that availability, patient numbers can be expanded to take advantage of the economies that are produced by Nursery, Pediatric, Obstetric, and Medical-surgery cost functions. The ICU function results only in increases in average daily cost up to $159.35; beyond that, however, increasing ICU numbers will reduce average daily cost.

Subsequent iterations of this system produce lower and lower average daily cost figures, all the while being constrained by the resources available. If the entire range of patient numbers found in each patient grouping were to be simulated, there would be 429 million iterations. That's a lot. The vast majority of these combinations violates one or all of the resource constraints. For instance, if all the largest patient numbers were used, then

Cost	= $85.87
Bed-days	= 6,365
Procedures	= 7,907
Nurse-hours	= -999,571

even though average daily cost is attractively low, it is unobtainable because two of the constraints are violated. Predictably therefore, larger costs must be balanced off with non-violated constraints.

To facilitate the search, many of the unnecessary iterations can be avoided by allowing only those combinations that will reduce average daily cost to be evaluated. As discussed previously, increasingly larger numbers of ICU patients result only in increasing average daily costs. On the other hand, greater numbers of Nursery and Obstetric patients cause costs to fall. Further, certain numbers of both Pediatric and Medical-surgery patients also force declining costs. Average Pediatric costs will fall below, say $90 a day, when bi-weekly admissions range from 89 to 159 patients. Similarly, Medical-surgery daily costs fall below $90 when less than 108 or more than 268 patients are admitted. But, 108 patients fall below the minimum acceptable number, forcing the iterations to begin where Medical-surgery admissions exceed 268.

However, average costs when all five patient types are considered simultaneously may not tend toward minimum where the individual categories indicate. But, knowing these separate values does help to speed up the search process. To repeat, pattern search is being used to estimate the optimums.

All this considerably reduces the number of iterations required. Now, only about six million are necessary, presuming, of course, that average cost can be contained to under $90. Many of those possible combinations are infeasible, because they violate one or more of the constraints. Of those that are feasible, the combinations which produce the ten lowest average costs are listed in Exhibit 4-7.

With the sole exception of laboratory procedures, available resources are moving toward depletion. Only five bed days are left of the total 2,800 and 4,593 nurse hours remain of the 24,050. Both represent opportunity costs, the values of which are unknown. But, of the two, the

Exhibit 4-7

Ten Lowest-Cost Combinations

	Nursery	Ped.	OB	MS	ICU	Cost	Bed-days	Nurse-hours	Procedures
1.	76	51	40	258	8	$87.64	2,795	19,457	5,059
2.	76	51	41	258	8	87.66	2,798	19,531	5,069
3.	76	50	40	258	8	87.68	2,789	19,410	5,048
4.	76	50	41	258	8	87.69	2,792	19,483	5,058
5.	76	50	42	258	8	87.71	2,795	19,557	5,068
6.	76	50	43	258	8	87.73	2,798	19,630	5,077
7.	75	51	40	258	8	87.85	2,798	19,456	5,040
8.	75	50	40	258	8	87.88	2,792	19,408	5,029
9.	75	50	41	258	8	87.90	2,795	19,482	5,039
10.	75	50	42	258	8	87.91	2,798	19,555	5,048

excess nurse hours can be easily reduced to an acceptable level by simply exercising normal management control. Such a reduction cannot be easily accomplished with the laboratory, for the obvious reason that much of the testing and measuring is done on and with machinery. Even though there is substantial laboratory excess capacity, it is unlikely that hospital administrators will be able to narrow the gap other than by restricting the number of extra personnel within that department.

For all intents and purposes, the ten listed combinations are equal. Costs will be held to under $88 if the hospital allows 75 or 76 Nursery patients, 50 or 51 Pediatric patients, 40 to 43 Obstetric patients, 258 Medical-surgery, and 8 ICU patients. With its present combination of patients, Hospital I's average daily cost is $114.09, which results from weighting the actual costs by type by the actual number of patients by type. This figure represents an opportunity loss of some $26.45 for each of the 2,600 bed days produced. This loss amounts to $68,770 in unnecessary patient billings for the two weeks, or $1.79 million for the entire year. This loss is due completely to an economically improper mix of patient types, and can be reduced by altering the present mix toward the "optimal" listed in Exhibit 4-7.

The mixes described in Exhibit 4-7 are not without their faults. Of particular interest is the balance between Nursery and Obstetric patients. Using the lowest cost combination, Hospital I ought to admit an average of 40 Obstetric and 76 Nursery patients every two weeks. Over the year, 1,040 maternity patients would be cared for and would presumably give birth to the optimal 1,976 babies. But, even after considering false labors, it is unlikely that each expectant mother would deliver 1.9 babies--twins! Consequently, there appears to be a misspecification of the Nursery and Obstetric relationships. Rather than being treated independently, the two types should be combined. While this idea does conflict with the manner in which hospitals account for their patients, considerably more realism would be added to the problem and its solution if the two were joined.

Combining Nursery and Obstetric data in order to determine new functions can be handled in at least two ways. Two plausible methods are:

1. Calculate new cost and resource-usage rates based upon an average weighted by numbers.
2. Calculate new totals by adding Nursery cost and resource data to Obstetric data, and determine new averages based solely on the number of Obstetric patients.

The data for both techniques are given in Exhibit 4-8 and Exhibit 4-9. As before, the information was processed first as non-linear; those results are shown in Exhibit 4-10, which reflect the data used from Exhibit 4-8; and in Exhibit 4-11, which used data from Exhibit 4-9. From the first data set, both bed-days and nurse-hour relationships are highly significant second-order polynomials, whereas the cost and laboratory procedures are not. Using the second data set, laboratory and nurse-hour functions are significant, while cost and bed days are not. When the cost function is specified to be linear, neither data set yields significant coefficients. In that cose, an average must be used for cost.

Each method gives both strong and weak relationships, and each

Exhibit 4-8

Maternity Averages Based on Total
Maternity Patients*

Hospital	Patients	Bed-days	Procedures	Nurse-hours	Cost
I	79	4.40	10.49	9.63	$86.08
II	62	3.97	6.70	10.29	46.21
III	16	3.75	4.32	6.25	88.50
IV	41	4.04	9.56	10.00	36.95
V	176	3.64	9.70	7.00	59.56
VI	42	4.05	7.58	9.20	72.45

*Maternity patients equal total of Nursery and Obstetric patients.

Source: Exhibit 3-3.

Exhibit 4-9

Maternity Averages Based on Number
of Obstetric Patients

Hospital	Patients	Bed-days	Procedures	Nurse-hours	Cost
I	42	8.28	19.73	18.11	$161.91
II	32	7.69	12.98	19.94	89.53
III	8	7.50	8.64	12.50	177.00
IV	26	6.37	15.08	15.77	58.27
V	100	6.41	17.07	12.32	104.83
VI	21	8.10	15.16	18.40	144.90

Source: Exhibit 3-3.

Exhibit 4-10

Maternity Functions Using Data
from Exhibit 4-8

Factor	Function	Correlation Coefficient
Cost	$\$78.86 - 0.363\ NB + 0.0015\ NB^2$ (0.3701) (0.3200)	0.2349
Bed-days	$3.487 + 0.0167\ NB - 0.00009\ NB^2$ (2.9900) (3.3310)	0.8989
Procedures	$3.207 + 0.1222\ NB - 0.0005\ NB^2$ (1.7577) (1.4492)	0.7801
Nurse-hours	$5.274 + 0.1136\ NB - 0.0006\ NB^2$ (3.3090) (3.5820)	0.9053

Note: NB is the sum of Nursery and Obstetric patients.

Exhibit 4-11

Maternity Functions Using Data from Exhibit 4-9

Factor	Function	Correlation Coefficient
Cost	$171.87 - 2.313B + 0.0168 B^2 (0.6468) (0.5530)	0.3967
Bed-days	7.09 + 0.0302B - 0.00037 B^2 (0.5337) (0.7604)	0.5863
Procedures	5.79 + 0.4268 B - 0.00314 B^2 (2.7760) (2.3982)	0.8765
Nurse-hours	10.72 + 0.3427 B - 0.00328 B^2 (2.7140) (3.0490)	0.8856

Note: B equals the number of Obstetric patients is adequate. But, since no theory exists showing which method is better, the choice between the two may be judgmental. Moreover, no theoretical concepts are violated if only the strong portions of each set are used and the weak parts discarded. Combining functions from both methods results in a more acceptable formulation for estimating optimal mixes, because the resulting equations can be used with a higher degree of reliability. Even so, some uneasy feelings may result from this maneuver, which may seem like bunching apples and oranges together. But, so long as the model is used to evaluate only the sampled hospitals, the combination of methods will not cause difficulty. Problems may arise only when the model formulations are applied to additional facilities. In the absence of an underlying theory opposing such a move, the optimization scheme may be best served by allowing empiricism to dictate which functions to choose.

Thus, for the highly interrelated Nursery and Obstetric patient-types, the four cost and production functions that should be used in the optimization model are found in Exhibit 4-12. Bed-day and nurse-hour relationships are from the first data set, the latter being more acceptable because of its higher correlation coefficient. The laboratory procedures function is significant only from the second data group. Finally, the cost function is a simple average based upon figures in the first data grouping. This choice was made because the t-value, F-value, and correlation coefficient were all larger when the initial data were used.

The revised model is presented in Exhibit 4-13. Processing the

Exhibit 4-12

Maternity Functions Chosen
for Revised Model

Factor	Function
Cost	\$62.79
Bed-days	$3.487 + 0.0167\ NB - 0.00009\ NB^2$
Procedures	$5.790 + 0.4268\ B - 0.00314\ B^2$
Nurse-hours	$5.274 + 0.1136\ NB - 0.00059\ NB^2$

Exhibit 4-13

Revised Hospital Model

$$\text{Minimize } Z = [(\$62.79)\ (NB) + (\$162.65 - 1.279\ P + 0.00517\ P^2)\ (P) + (\$-111.70 + 2.6283\ M - 0.007\ M^2)\ (M) + (\$159.35)\ (I)]/[NB + P + M + I]$$

Subject to:

1. $\text{Bed-days} = [BP + BNB + BM + BI] \leq C_1$

   where $BP = (2.3733 + 0.406\ P)\ (P)$

   $BNB = (3.487 + 0.0167\ NB - 0.00009\ NB^2)\ (NB)$

   $BM = (1.835 + 0.0243\ M)\ (M)$

   $BI = 8.86\ I$

   C_1 = maximum available bed days

2. $\text{Nurse-hours} = [(4.987 + 0.0281\ P)\ (BP) + (5.274 + 0.1136\ NB - 0.00059\ NB^2)\ (BNB) + (9.317 - 0.0118\ M)\ (BM) + (18.93)\ (BI)] \leq C_2$

   where C_2 = maximum available nurse-hours

3. $\text{Procedures} = [(11.26)\ (P) + (5.79 + 0.4268\ B - 0.00314\ B^2)\ (B) + (24.37 - 0.0488\ M)\ (M) + (4.854 + 3.1574\ I - 0.0625\ I^2)\ (I)] \leq C_3$

   where C_3 = maximum available procedures

4. $16 \leq NB \leq 176$

   $23 \leq P \leq 129$

   $8 \leq B \leq 100$

   $131 \leq M \leq 290$

   $8 \leq I \leq 46$

Definitions:

NB = Nursery and Obstetric patients

P = Pediatric

M = Medical-surgery

I = ICU

B = Obstetric

model is easier than before because there is one less variable to consider.

Exhibit 4-14 gives the five combinations of patient types that result in lowest costs for Hospital I. As before, most if not all of the available bed days are completely used up. In fact, for the lowest cost combination, all 2,800 bed-days are consumed. But, the other two resources have ample in reserve. There are 9,666 procedures and 5,764 nurse hours still available. Once again, much of the excess laboratory capacity probably results from the fixed nature of the equipment, whereas the excess nurse hours seem to indicate true waste.

In general terms, the five low-cost mixes are equal. Costs can be held below $77 a day if the hospital admits 42 to 45 maternity patients, 23 Pediatric patients, 283 or 284 Medical-surgery patients, and 8 ICU patients. Hospital I's minimal cost combination saves $38.56 a day for each bed-day provided, when compared to actual cost. This opportunity loss sums to $108,000 for the two weeks, and to $2.8 million for the entire year. Improper patient mix is the sole cause of this loss. The optimal allocations for all six hospitals are listed in Exhibit 4-15.

Earlier, realistic considerations forced modification in the formulation of Nursery and Obstetric cost and production functions. For the same reason the model must be modified once again. This time there are two fundamental changes that must be made. First, the data in Exhibit 4-16, which contrasts the actual and optimal mixes for the six hospitals, reveal a great divergence between the two. For example, Hospital VI's optimal Pediatric load is 73, whereas actual is 23. Thus, the model produces an optimal that would require the hospital to treat

Exhibit 4-14

Five Lowest-Cost Combinations for
Hospital I

Cost	NB	P	M	I	Bed-days	Nurse-hours	Procedures
$75.53	42	23	284	8	2,800	18,286	3,734
76.49	45	23	283	8	2,797	18,418	3,767
76.53	44	23	283	8	2,793	18,352	3,757
76.57	43	23	283	8	2,788	18,286	3,747
76.61	42	23	283	8	2,784	19,222	3,737

Exhibit 4-15

Lowest Cost Patient-Mixes
for Sampled Hospitals

Hospital	Cost	NB	Ped.	M	ICU
I	$75.53	42	23	284	8
II	68.57	55	23	290	8
III	106.36	41	31	130	8
IV	114.55	69	23	222	8
V	78.69	43	23	281	8
VI	108.64	92	73	180	8

Exhibit 4-16

Actual and Optimal Patient Mixes

Hospital	Maternity Act.	Maternity Opt.	Pediatric Act.	Pediatric Opt.	Medical Act.	Medical Opt.	ICU Act.	ICU Opt.
I	79	42	42	23	250	284	30	8
II	62	55	42	23	290	290	30	8
III	16	41	46	31	157	130	8	8
IV	41	69	129	23	131	222	28	8
V	176	43	74	23	215	281	46	8
VI	42	92	23	73	210	180	31	8

Source: Exhibits 3-3 and 4-15.

50 more Pediatric patients than it is currently doing. Instead of forcing the hospital to go out and beat the bushes for 50 more Pediatric patients, a much more rewarding avenue would be to further constrain the model in order to consider this incongruity.

One way this can be done is to allow the search for optimums to be restricted by the proportion of individual patient-types to the total[2]. For example, the average proportion of maternity patients to the total is about 19.0 percent. All of the hospitals could be limited to a range about that average, say ± 10 percent, or from 17 to 21 percent of total admissions. The averages for all the patient types along with 10 percent ranges are listed in Exhibit 4-17.

The second change results from the wishes of most hospital administrators to use up as much of the available resources as possible. Bed days and nurse hours probably are the resources most likely to be used up first, leaving the laboratory with excess procedure capacity. One way to handle this is to impose a restriction that each hospital must use up either, say 95 percent of available bed days or 95 percent of nurse hours. If that were done, Hospital I would be able to locate its optimal mix as long as more than 2,660 bed days or 22,848 nurse hours were consumed. All resource minimums, using a 95 percent figure, are given in Exhibit 4-18, including figures for laboratory procedures.

Both modifications were designed into the model program and the optimal mixes for all six hospitals were re-estimated.[1] These figures are provided in Exhibit 4-19 along with cost and resource consumption

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[1] See Appendix for program

Exhibit 4-17

Average Percentage Admissions with

Ten Percent Ranges

Patient Type	Average Percentage	Range
Maternity	19.0%	17.0 - 21.0%
Pediatric	16.0	14.4 - 17.6
Medical-surgery	56.5	50.9 - 62.2
ICU	8.5	7.7 - 9.4

Exhibit 4-18

Resource Minimums

Hospital	Bed-days	Nurse-hours	Procedures
I	2,660	22,848	12,730
II	2,860	24,225	11,590
III	2,035	8,265	7,030
IV	1,942	17,480	9,690
V	2,620	21,850	11,780
VI	1,889	17,385	9,310

Source: Exhibit 4-6.

Exhibit 4-19

Optimal Cost, Resource Use, and Patient Mixes

Hospital	Cost	Beds	Nurse-hours	Labs	N+B	Ped.	MS	ICU
I	$105.34	2,798	23,544	5,637	70	59	247	32
II	99.30	3,005	25,003	5,751	73	65	257	33
III	91.85	842	8,363	3,191	39	32	100	16
IV	115.75	1,793	17,602	5,194	67	56	167	30
V	107.30	2,722	22,965	5,595	69	58	243	31
VI	115.75	1,793	17,602	5,194	67	56	167	30

data. As the numbers show, Hospital I does in fact meet the minimum requirements for both major resources, but quite obviously falls far short of using up the minimum number of procedures. Other hospitals do not fare so well. The most glaring example is Hospital III, which manages to satisfy its minimum nurse-hour requirement, but fails completely to make adequate use of its available bed days. In fact, it uses only 39 percent of its 2,142 bed days.

Hospital III points out the practical need to consider the minimum for only one of the resources, because in some cases, if both major minimums were required to be met, some hospitals could not be evaluated, since imposing a minimum restriction on one resource would in all likelihood violate a maximum restriction on the other.

Exhibit 4-20 contrasts the actual patient mixes with the revised optimums. Many of these data are interesting. For instance, Hospital V would find the costs per patient day falling if it could reduce its maternity admissions by over a hundred during the two-week period. Similarly, seventy Pediatric admissions less would benefit Hospital IV.

More interesting still are the figures shown in Exhibit 4-21, which gives the excess resources. Immediately obvious, of course, is the fact that hospitals have vastly too much laboratory capacity. No single unit is able to come even close to using all of its capability. In fact, the most any hospital can do is muster a 60 percent utilization level. Arguments have been made, however, that this excess results from having to use very sophisticated equipment, which oftentimes is designed to handle a larger workload than most hospitals can provide. While the

Exhibit 4-20

Actual and Revised Optimal Patient Mixes

Hospital	Maternity Act.	Maternity Opt.	Pediatric Act.	Pediatric Opt.	Medical Act.	Medical Opt.	ICU Act.	ICU Opt.
I	79	70	42	59	250	247	30	32
II	62	73	42	65	290	257	30	33
III	16	39	46	32	157	100	8	16
IV	41	67	129	56	131	167	28	30
V	176	69	74	58	215	243	46	31
VI	42	67	23	56	210	167	31	30

Source: Exhibits 3-3 and 4-19.

Exhibit 4-21

Excess Resources

Hospital	Bed-days	Nurse-hours	Procedures
I	2	506	7,763
II	5	497	6,449
III	1,300	337	4,209
IV	251	798	5,006
V	36	35	6,805
VI	193	698	4,606

Source: Exhibits 4-6 and 4-19.

statement may be true, the argument is empty. Some hospitals have joined together to operate a single laboratory which makes far better use of very expensive equipment and personnel. When more than one hospital operates in an area, this excess capacity could vanish with co-operation.

Another fact is that Hospital III has too many beds--it has almost three times as much as it optimally needs. Hospitals IV and VI have too many beds also, but clearly not so much excess as III. Hospitals I and II have optimals essentially at bed-day capacity, but do have excess nurse hours. If these two hospitals were to operate at their optimum levels, significant reductions in nursing personnel would result. If laboratory procedures are ignored, Hospital V could easily be the most efficient user of resources. For all intents and purposes, its optimal would have it operating at full bed and nurse capacity. None of the other five units can boast of such effectiveness.

Finally, the data from Exhibit 4-19 can be used to determine the opportunity losses associated with the hospitals operating sub-optimally. These figures are given in Exhibit 4-22. The opportunity cost per patient day is found by subtracting the optimal cost from the actual cost. For Hospital I, this amounts to \$8.75 a day. For the two weeks when the optimal bed days equals 2,798, the opportunity loss is \$24,500, and \$636,500 for the year. Hospital VI has the largest yearly opportunity loss--almost \$900 thousand.

Instead of subtracting the optimal cost from actual to determine losses, another way is to subtract optimal cost from the "actual" costs that result from processing the actual patient mixes through the model.

Exhibit 4-22

Opportunity Losses for Sub-Optimal Allocations

Hospital	Actual Cost	Optimal Cost	Opportunity Loss Per Patient Day	Opportunity Loss For Two Weeks	Opportunity Loss For Year
I	$114.09	$105.34	$ 8.75	$24,500	$ 636,500
II	69.27	99.30	(30.03)	(90,200)	(2,346,000)
III	110.30	91.85	18.45	15,500	403,900
IV	94.71	115.75	(21.04)	(37,700)	(980,800)
V	112.55	107.30	5.25	14,300	371,600
VI	134.91	115.75	19.16	34,400	893,200

Source: Exhibits 4-4 and 4-19.

The figures in Exhibit 4-23 reflect this second method. Hospital III has the largest opportunity loss using this method, losing just over $600,000.

Both sets of opportunity figures are correct, but obviously result in different numbers. The first set compares the model calculations with actual performance, while the second compares the model output with figures also generated by the model. This latter places double reliance on model results and avoids consideration of the actual data altogether. Whereas those hypothetical values may have passing interest for the analyst, they may also lack relevance for a hospital administrator who perhaps wishes to contrast what he should be doing with what he has really done, and not with what a model says he has done. Consequently, the first data set seems to be preferable.

In any case, the figures in Exhibit 4-22 introduce a problem. Hospitals II and IV have optimal costs that are larger than actual costs. For instance, Hospital II would be worse off if it adjusted its patient mix to the optimal and would lose over $2 million if it did. Hospital IV would also enjoy substantial losses if any patient mix adjustments were made.

There are at least two possible explanations for this case. First, in some instances the actual allocations violate actual resource constraints, thus enabling the hospital to treat more patients than the model allows. The best illustration of this is Hospital II, which has 2,800 bed days available for use. However, a quick check of the figures in Exhibit 3-3 shows that the actual number of bed days used is over 3,500, which is 700 more than the hospital is supposed to have. What has

Exhibit 4-23

Opportunity Losses for Sub-Optimal
Allocations Using Hypothetical Actual Costs

Hospital	Actual Cost	Optimal Cost	Patient-day Opportunity Loss	Bi-weekly Opportunity Loss	Yearly Opportunity Loss
I	$103.91	$105.34	($ 1.43)	($ 4,000)	($ 104,000)
II	74.42	99.30	(24.88)	(74,800)	(1,944,800)
III	120.10	91.85	28.25	23,800	618,400
IV	110.93	115.75	(4.82)	(8,600)	(224,700)
V	104.53	107.30	(2.77)	(7,500)	(196,000)
VI	125.66	115.75	9.91	17,800	462,000

Source: Exhibits 3-3, 4-13, and 4-19.

happened is multiple counting. According to hospital records, a patient's bill records the number of different days that a patient is hospitalized, even if he stays for only a fraction of a day. He may check out in the morning and still be billed for the entire day. Once his room is freed, it becomes available for a new occupant, who may check in that afternoon. The second patient will be billed for a full first day, even though he arrived late and did not have complete use of the hospital. Unfortunately, the double counting is built into the hospital industry's accounting system, and it would take a substantial re-designing of the account reporting system to revise it. Needless to say, it results in some curious figures.

The second possible explanation relates to the quality of the estimated relationships. Some of the variation could be caused by the specification of non-linear equations, in this case second-order polynomials. Some other form of non-linear function may have been better, say e^{-ax}. A second source may result from the selection of the critical probability. If the figure has been different from the 0.20 level, certainly the values generated by the model would also have been different. Third, some of the problem may be caused by a model that is constructed from only six observations. As stated earlier, while the sample size is statistically significant, it may not be representative.

These three explanations should produce caution when it comes to interpreting these figures. While the results may be appropriate for the hospitals included in the sample, they should not be used to instruct other hospitals on how to operate. Importantly, this model and the methods used to generate it do show that a procedure is available to demonstrate how hospitals can adjust their patient mixes in order to effect substantial savings for the hospital and certainly for the patients. What is needed is a more comprehensive study to provide a more representative model that can be used to generate optimal mixes for non-sampled hospitals.

References

1. Conversational Statistical Package. White Plains, New York: International Business Machines, Corp, 1968.

2. The averages come from the following sources:

   a. In a personal interview, Dr. James Goldring, Secretary of the American Pediatrical Society, estimated that roughly 16 percent of all admissions are pediatric.

   b. In 1973-1974, 9.5 percent of all admissions were nursery patients. Doubling this figure would give an approximate maternity value.

   c. In personal interviews, hospital administrators state that ICU patients account for between 8 and 9 percent of total admissions. The midpoint was used.

   d. Medical-surgery patients account for the remaining 56.5 percent of total admissions.

Chapter V

SUMMARY AND CONCLUSIONS

This final chapter presents a brief summary of the study and provides conclusions based upon it.

The chief goal of this research was to present a methodology that hospital administrators can use in order to take advantage of the cost-effectiveness of patient mix. This study has shown that efficiencies may be obtained by adjusting patient mix. This basically can be achieved by allowing realistic substitution of more costly patient types for less costly. Also it results from the fact that several of the functional relationships display non-linear movements with patient numbers. These relations contradict some other hospital research findings.

The model's sixteen functions, specifying a casual relationship between patient numbers and average daily cost and resource-usage, were combined to form a model which attempts to describe the interactions among the variables. However, solving for an optimal solution by normal mathematical programming procedures proved unacceptable. Because non-linear optimization problems are considerably more complex than linear ones, the techniques used in searching for solutions are usually restrictive and oftentimes impractical. Rather than limit the usefulness of the model because of the inability of traditional approaches to effect solutions, the traditional mathematical programming methods were discarded. Computer-simulation models do not have to be restricted since no attempt is made to solve problems analytically. Indeed, the great power of simulation techniques lies in their ability to handle unrestricted, complex, non-linear relationships. Because of this, heuristic programming techniques were employed in the search for optimums.

The optimum balances of patient type were found by combining Nursery and Obstetric functions together and by restricting the search-technique to a feasible solution-space. Optimal values for patient-mix were obtained by pattern search, a form of heuristic programming, and were confirmed by universal processing. These values reveal the combination of Maternity, Pediatric, Medical-surgery, and ICU patients which will result in minimized costs, given certain policy limitations imposed on the hospital by internal and external pressures.

Hospital administrators should find these results helpful, since by using the simplest admission control processes, they could effectively balance the number of patients so that costs could be reduced. Even though a small cost reduction, say 6 percent, may seem insignificant, it amounts to at least $2.7 billion for the nation's hospitals and patients. Thus, one of the chief results of this study is to demonstrate how the industry can save vast sums of the patients' money.

Another result is the support this study can provide for increased control of the proportion of patient types admitted for treatment. In that regard, it can specifically help administrators economically support their positions in limiting or at least controlling those physicians who tend to over-admit. This problem is recognized even by the American Medical Association and the American College of Surgeons, which repeatedly cite evidence indicating than an alarmingly large percentage of admissions are unnecessary. By showing the implicit and explicit costs of malproportioned admissions, administrators should be more equipped to handle the problem.

Area wide planning organizations would also benefit from these findings. Admission policies for all area hospitals could be meshed together in a way as to optimize the patient mixes for all hospitals. For example, some institutions tend to treat a larger proportion of a certain type of patient, say Pediatric, because of the facilities themselves and because of the interests of the medical staff. If one facility is particularly adept, both professionally and economically, then the other area hospitals can simply adjust the acceptable proportions of desired Pediatric admissions, thus forcing more Pediatric patients to seek admission at the specializing institution.

In addition, these findings may provide skeletal information regarding the size of the hospitals themselves. This would naturally be of great interest to area planning groups, but much more work would be required before any helpful conclusions could be reached.

Another important result of this paper is the demonstration of the acceptability of using computer simulation models. Computer models can be made as complex and realistic as the theories permit. This is possible since no matter how complicated the formulation of the model is, simulation techniques allow for tracing out the consequences of it. Thus, real world descriptions may be considered without restriction or limitation.

Moreover, computer models provide opportunities for working with realistic and answer-providing models without requiring mathematical sophistication. This considerably expands the usefulness of a hospital model based upon the procedure developed here, for it can allow any hospital administrator to employ a model in his own computer in order to discover the ways to minimize his patients' cost.

One final advantage of computer simulation modeling is the ease with which the models can be modified to reflect changes occurring in the economy. Some hospital administrators may believe that certain of the specifications are inappropriate for their operations. If so, they can respecify the model to allow for adjustments required by different situations.

Another result of this study is its ability to grant administrators information needed to properly evaluate hospital expansion. By simply altering the number of bed-days and nurse-hours in the model, new optimal combinations and costs can be determined. These new average costs can be weighed against the old costs in order to learn whether or not the proposed hospital expansion is economically feasible.

A final result of this research is in pointing out, indirectly to be sure, the poor state of the hospital industry's accounting systems. A fair and honest appraisal of the industry would find the industry critically unaware of what the costs of treatment truly are. With the continuing transformations to electronic data processing, most hospitals will be capable of providing the input needed for analysis with little extra effort. However, for the information system alone, the materials required to generate useful patient-mix and cost data should be available anyway. It is truly remarkable that none of the officers of the six hospitals know what their treatment costs really were. If a system like the one described here were widely employed, administrators would at the very least be required to have more detailed knowledge of their operations.

Despite all the positive results and obvious advantages of such a study, it is not without shortcomings. Specifically, the model itself has its limitations. Of the many possible shortcomings, two seem most important. First is the data themselves, both quantity and quality. Only six observations were obtained to provide all the information for the model. To be more widely acceptable, many more hospitals would be needed. This would help to avoid regional bias, but at the same time, introduce varying factor prices, both of which are undesirable. Clearly, much more study is needed in this area of regional pricing differences before expanded sampling can be done. While more data would be attractive, the quality of them must be given serious consideration. Because of the absence of accepted hospital accounting procedures and almost total absence of hospital information systems, the quality of much additional data would be suspect. Here, work leading toward implementation of more sophisticated accounting information systems would probably be well rewarded.

The second major shortcoming of the model concerns the quality of the established relationships. An 80 percent confidence coefficient was used to determine whether or not each of the estimated functions was statistically significant. This level may be unattractively low. Furthermore, five of the relationships were not significant either non-linearly or linearly. As a result, average values were used for them. This also may not be acceptable. However, many of the shortcomings with the relationships could possibly be eliminated with improved data.

While these shortcomings may appear to compromise these findings, in fact they do not. For this entire research has established that the hospital industry can very easily use the procedures outlined here in an effort to improve cost efficiency. Certainly with an enlarged data set, the coefficients for many if not most of the relationships would change. Most importantly, that change is expected and desirable, for the model would then be more acceptable for general use.

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APPENDIX

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PAGE 1      HEWLETT-PACKARD 32201A.4.03 EDIT/3000 TUE, AUG 10, 1976,  2:20 PM

      $CONTROL USLINIT
    1 FORMAT(5X,3HN+B,5X,4HPEDS,5X,2HMS,5X,3HICU,5X,4HCOST,
     15X,4HLABS,5X,5HNURSE,5X,4HBEDS)
      WRITE(6,1)
      IM=0
      IX=0
      IP=0
      IC=0
      ZB=2800
      ZL=13400
      ZN=24050
      ZB1=ZB*.95
      ZL1=ZL*.95
      ZN1=ZN*.95
      DO 100 IX=65,72
      DO 100 IP=55,62
      DO 100 IM=240,250
      DO 100 IC=30,35
      XIP=IP
      XIX=IX
      XIM=IM
      XIC=IC
      SUMP=XIP+XIX+XIM+XIC
      PP=XIP/SUMP
      PX=XIX/SUMP
      PM=XIM/SUMP
      PI=XIC/SUMP
      IF(PX-.170)100,5,5
    5 IF(PX-.210)6,6,100
    6 IF(PP-.144)100,7,7
    7 IF(PP-.176)8,8,100
    8 IF(PM-.509)100,9,9
    9 IF(PM-.622)10,10,100
   10 IF(PI-.077)100,11,11
   11 IF(PI-.094)18,18,100
   18 CNB=62.79*XIX
      CP=(162.65-1.279*XIP+.00517*XIP**2)*XIP
      CM=(-111.7+2.62834*XIM-.007*XIM**2)*XIM
      CI=159.35*XIC
      COST= (CNB+CP+CM+CI)/(XIP+XIX+XIM+XIC)
      IF(COST-106.00)20,20,100
   20   BP=(2.3733+.0406*XIP)*XIP
      BM=(1.835+.02433*XIM)*XIM
      BNB=(3.487+.0167*XIX-.00009*XIX**2)*XIX
      BI=8.86*XIC
      BEDS=BP+BM+BI+BNB
      IF(BEDS-ZB  )15,15,100
   15 XLP=11.26*XIP
      XLNB=(5.79+.4268*XIX*.5-.00314*(XIX*.5)**2)*XIX*.5
      XLM=(24.37-.0488*XIM)*XIM
      XLI=(4.854+3.157*XIC-.0625*XIC**2)*XIC
      XLAB=XLP+XLM+XLI+XLNB
      IF(XLAB- ZL  )12,12,100
   12 XNP=(4.987+.0281*XIP)*BP
      XNM=(9.317-.0118*XIM)*BM
```

PAGE 2   HEWLETT-PACKARD 32201A.4.03 EDIT/3000 TUE, AUG 10, 1976, 2:20 PM

```
      XNNB=(5.274+.1136*XIX)*BNB
      XNI=18.93*BI
      XNUR=XNP+XNM+XNI+XNNB
      IF(XNUR-ZN   )40,40,100
   40 IF(XNUR-ZN1) 41,52,52
   41 IF(BEDS-781)100,52,52
   52 WRITE(6,51)IX,IP,IM,IC,COST,XLAB,XNUR,BEDS
   51 FORMAT(4X,I3,5X,I3,7X,I3,5X,I2,5X,F6.2,5X,F5.0,4X,F6.0,3X,F5.0)
  100 CONTINUE
      STOP
      END
```