QUANTITATIVE MANAGEMENT

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INTRODUCTION

Revoutionary advancement has been made in recent years in the economic, social, and physical aspects of life. In fact, advancement seems to be the very essence of human existence. It is felt that an itemizing or listing of all advancement in the various fields of knowledge would prove interesting. Although a detailed study would be required for definite conclusions, it is believed that a more rapid advancement has occurred in three fields of knowledge: (1) chemical science, (2) engineering, and (3) medicine. In addition to leading in relative advancement, it is felt that the three fields now surpass all others in available facts to be applied to known problems.

There should be some reason, common to these three fields, that results in this rapid advancement which seems to have left so many other fields of endeavor behind. It should be noted that the basis of teaching and learning within the three fields listed is <u>fact</u> and that such <u>fact</u> is substantiated by very concrete and quantitative methods. The whole theory of the teaching within these fields is that a fact must be proven conclusively before it becomes a part of the formal knowledge of the field.

So many other areas of teaching and practice find their background and current development in the opinions of those who proclaim themselves, or have been proclaimed by others, as "authorities" within the field. The student of these subjects is prone to argue with such authorities because proof is usually lacking and is very seldom complete and final, if attempted. The result of this type of teaching is that effort is continually expended in argument on opinions, whereas, in the field of engineering for example, such argument is seldom encountered since a single positive mathematical proof leaves only one alternative and that is acceptance. Once present knowledge is mastered in such a field, efforts can be concentrated on future developments.

In the fields usually termed "less exact sciences" or "the arts" very little or none of the proclaimed knowledge is supported by unquestionable and concrete proof. This means that the practice in these fields is a matter of the exercising of personal opinion and judgement. It is granted that the opinions and judgments may be conditioned by previous opinions and judgments. However, the possibility of erroneous and controversial opinions on various phases of the activity of the field would tend to promote erratic and possibly very harmful decisions on the part of those attempting to apply such knowledge.

It is believed that the only way that advancement will be fostered and that the learning obtained by students in the field will be consistent is to express all knowledge numerically. Further, the opinions must be capable of conclusive numerical proof to be accepted as fact and until such proof is made, it will be rightfully considered as theory. It is granted that opinions advanced as theory may be of value yet they are much more valuable when proclaimed and accepted as theory which may be proven later than to be proclaimed as fact when the future may disprove them.

The purpose of this discussion is to advance the theory that all knowledge is utilized by application through the medium of decisions made by people and that such decisions will prove more consistently correct when made through utilization of quantitative means. This implies that all information affecting the decision must be expressed numerically. Such a practice will perhaps be difficult for many areas of knowledge until those working in those areas become accustomed to such expression.

In an effort to present the theory outlined above, Chapter I presents the various aspects of making decisions with illustrations of the utilization of quantitative evaluation of knowledge in various fields; Chapter II presents a few available quantitative tools and, although the number is limited and the coverage is brief, it is felt that sufficient basis is established to illustrate and advance the idea that the utilization of proper quantitative tools will achieve desired results; Chapter III is an attempt to establish methods of utilization of, not only the tools discussed in Chapter II, but all numerical analyses in general.

CHAPTER I

ORGANIZATIONAL MANAGEMENT AND ITS FUNCTION

The Industrial Revolution of the late 19th Century is usually credited with almost universal conversion to mass production methods and mass utilization of labor. This condition has been tremendously intensified during recent years by the revolutionary scientific and technological advances. The fact that more advanced equipment and techniques have been discovered and developed has required more available advanced education and training. This in turn has resulted in a greater demand for excellence of knowledge and skill which has required more specialization by the individual to attain this excellence. The net effect of all this specialization is that there are few activities today that do not have such wide scope that their successful accomplishment does not require the efforts of more than one person.

Building a house, which was accomplished by many of our grandfathers single-handedly, now requires the cooperative efforts of a carpenter, a bricklayer, a plasterer, a plumber, an electrician, a paper hanger, and perhaps an architect, a cabinet maker, and other specialized craftsmen. However, an undertaking does not have to involve even the scope of building a house. The housewife who wishes to bake bread must have cooperation from the grocer, the milkman, the gas or electric company, and even her neighbors who might choose to call her on the telephone just as the bread should be taken from the oven.

Fifty years ago, the housewife would perhaps have ground her flour from grain she or her husband raised, used milk procured by her from her cow, gathered wood and built her fire, and in all probability would not have been interrupted by her neighbors.

When two or more persons contribute their efforts to accomplish a common objective and organization is formed. The organization may be in existence for only a few minutes as in the case of two strangers who stop and assist a motorist, whose automobile is stalled in a snow drift, by pushing the automobile until it is no longer in the drift. On the other hand, an organization may exist for many years as in the case of many of the churches or some of our large industrial concerns. A very strict interpretation would be that a new organization is formed when any of the persons contributing to the organization's activities leave or new persons are added. Such an interpretation is of little value in this discussion since it is the functioning of any organization that is of interest and not a specific organization.

Organization Objectives

An organization usually begins its existence with a single and sometimes very broad objective. This objective may be of the nature "to manufacture automobiles" or "to improve city cleanliness" or "to impart religious information." As the organization becomes active in the achievement of this objective, many subsidiary objectives may be adopted and the primary objective may be modified and even radically changed. However, the organization will continue to exist only so long as there is an objective.

The objective or objectives of the organization are accomplished through the utilization and combination of one or more of the following: (1) Men, (2) Materials, and (3) Methods.

Men, or human energies of people, is one element found in all activities of an organization. These energies may be physical or mental and may also include contributions which are described as moral, spiritual, or inspirational.

Materials include natural resources, tools, and equipment, as well as materials that have resulted from previous processing of natural resources.

Methods deal with the means of utilizing or combining the men and materials to accomplish the objective of the organization. One of the principal considerations under this factor is the element of time. This includes both <u>when</u> particular actions of utilization or combination are made and also the <u>length of time</u> required to accomplish the utilization or combination.

Management of the Organization

In any organization there is a necessity for planning and directing the activities to facilitate the furnishing, utilization, and combination of the men, materials, and methods to attain the objective of the organization; and to achieve the objective efficiently. This function or contribution is <u>management</u> of the organization.

In an organization of two members the management function may be performed through utilization of only a portion of one person's efforts. In large organizations the management of the organization will require the efforts of many persons. The function and not the individual is "management," and the term will be used in this meaning regardless of the number of persons contributing all or a portion of their efforts to this function.

The importance of management in a society where practically all accomplishment is through group activity cannot be over - emphasized. Management, as it has been defined here, is responsible to a great extent for the success or failure of the activity of an organization through the planning and directing of the three basic essentials to organization activity. For this reason, a better understanding and more efficient application of management will result in greater organizational efficiency.

Management and Men -- Since men are essential to activity by any organization, means must be established for furnishing these men. The problem is greater than the mere furnishing of a given number of men. First, the mental and physical qualifications must be such that the men selected are <u>capable</u> of performing the activities of the organization so as to attain the objective. Second, the objective of the organization must not be in conflict with the personal characteristics of the men selected and must be accepted by the men so that they <u>will</u> perform the tasks for which they are selected. Third, there must be means devised to so condition the men that they will continue to perform the tasks necessary.

The knowledge that will prove helpful in the three steps in furnishing men to perform activity of the organization is quite extensive, covering recruiting, selection, placement, training, and industrial relations. The object of this discussion is not to present such knowledge. It is suffice at this point that these duties are present and their accomplishment is a function to be performed by persons engaged in management.

Management and Materials -- The furnishing of materials required to perform the activities of an organization varies considerably with the size and the objective of the organization. Regardless of the type or amount of these materials, the function of management is the same. The proper materials must be obtained and dispensed in the proper quantities, of the proper quality, at the proper place, at the proper time, and in the proper manner.

Management and Methods -- This aspect furnishes perhaps the most complex of the basic functions of management. If both men and materials are available in the most desirable manner, nothing is accomplished without methods for their combination. Also, the method of combination is the most important factor in determining the efficiency of the organization. Methods includes not only functions usually described as "executive functions" such as communication procedures and designation of activities to be performed by the organization in order to attain objectives, but also functions of technical and supervisory personnel who design and put into practice the specific means of accomplishing the designated activities.

The Basic Function of Management

From the discussion thus far the function of the management of an organization is to furnish the proper men, material, and methods to accomplish the organization objectives. When some of our large industrial organizations are considered, the scope of the duties and obligations of management can be seen to cover numerous fields of specialized knowledge requiring a variety of individual qualifications and capabilities. If an attempt is made to analyze the management function for purposes of study and improvement from the standpoint of the furnishing of men, materials, and methods for specific organizations, it will appear that the function differs radically with different organizations. Yet, since the function is basically the same for all organizations, this basis for analysis must not be basic enough for a general application.

Let us consider the means utilized by management to accomplish its function. In considering the problem of furnishing men for the performance of activities of an organization, the usual approach is to think in terms of recruiting programs, methods of selection and placement, and so on through the series of personnel programs which will satisfactorily solve the problem. It is only natural that somewhat different solutions would be obtained when the objective in one case is harvesting a wheat crop and in another case is the development of practical application of atomic energy to the generation of electricity. However, such an analysis is not of the function itself, but rather the results of the function.

The one process which is common to all management efforts is the process of making decisions. This element is then the basic element upon which concentration must be centered if the function of management and, hence, the efficiency of an organization is to be improved. Any action by personnel engaged in work of management of an organization is the result of a decision or decisions.

Let us consider some widely varied aspects of management to illustrate this point. The personnel functions which were discussed previously will serve as a good example. The person delegated to recruit men to harvest a wheat crop would probably decide that the means to be utilized would be newspaper advertisement. The person delegated to hire a group of engineers to join a corporation as trainees and later be utilized in various aspects of the organization's engineering work would probably decide to visit colleges to interview graduating engineers or possibly write to colleges inviting graduating engineers of certain qualifications to visit the organization for an interview. The chairman of a civic club who needs certain organizational work accomplished would probably decide to do his recruiting through personal contact or by telephone.

Any directive issued by an executive is the result of many decisions. The decision is made that the directive should be issued. The wording of the directive is the result of one or more decisions. The time of issuance, to whom issued, and the manner in which it is issued are all the result of decisions.

A football team furnishes an example of the effect of decision making. The coach decides after each play which players he thinks should be in the game and makes any necessary substitutions. The quarterback decides which of several plays shall be used and each player must decide, under certain conditions, what he will do.

A physical expansion of a plant is the result of a decision by an executive or board of directors after many related decisions concerning the plans, estimates of cost of installation and operation, and estimates of production and market potentials. The curriculum of an educational institution is the result of a decision to include or exclude each of many courses considered. This decision results from supporting decisions concerning student ability, value in practical application, and employer's evaluation for each course. The speech by the politician, the educator, or the industrial executive, and its success or failure, is a result of decisions by the speaker as to the general content of the speech, even individual sentence construction, and the manner in which each word is uttered.

The number of decisions involved in making a speech or any other action is numerically large. Many of these decisions are made more by the sub-conscious and involve such an infinitesimal amount of time that they would seldom be recognized as decisions without detailed analysis. Chester I. Barnard gives an example to illustrate this.

> "Thus if the president of a telephone company for good reasons orders two poles carrying a cable removed from the north side of First Street between A and B streets to the opposite side of First Street, it can,

> I think, be approximately demonstrated that carrying out that order involves perhaps 10,000 decisions of 100 men located at 15 points, ----. If inquiry be made of those responsible, probably not more than a half-a-dozen decisions will be recalled or deemed worthy of mention --."[1]

It should also be stated that inaction is also the result of decision. This may be as important as action. Often action is recommended, requested, or even demanded of persons engaged in management activities. These items are often "shelved" or "filed" by the person. This "shelving" is a result of the decision to either delay or refuse to perform the action.

It should be borne in mind throughout this discussion that the activities of those persons in a position to make decisions are analyzed from the standpoint that even the smallest action is the result of a decision and, as such, could have occurred in a much different manner. It is a composite of the actions resulting from decisions that make up the activities of an organization; therefore, if the functions of the management group are to be analyzed and improved, the means and methods of making decisions must be analyzed.

The Nature of Decisions

Decision in the organizational sense shall be defined as the setting forth of a course of action to be followed in so far as it is possible to control the men, materials, and methods which may be necessary to attain the objectives desired. With

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1 Chester I. Barnard, The Functions of the Executive (Cambridge, Massachusetts: Harvard University Press, 1948) P. 198.

this definition in mind, it is apparent that all decisions are made as a basis for or as a guide to future actions. Past actions have become fact and no longer require decision, even though some decisions may be based on information furnished by these facts. It is always the proposed action for the next minute, the next hour, or the next day that is the object of a decision.

Actions are the result of decisions which in turn are the result of a mental process or series of mental processes. Actions, then, never "just occur" but are caused by people. It, therefore, becomes necessary to determine the steps in a mental process that accomplishes decision and strive to minimize the human error of such processes.

A decision results from a combination of three factors:

(1) A knowledge of the subject or subjects which are involved in the decision.

(2) The ability to analyze and evaluate available information.

(3) The formation of conclusions based on evaluated information.

Any successful decision involves a certain amount of specialized knowledge. The variety and importance of the decisions to be made by any one person dictate how varied and extensive that person's knowledge must be. The decision of a construction gang-leader to have his men use a certain type of shovel to dig a hole of given dimensions and in a known type of soil requires a certain amount of specialized knowledge since it is doubtful that a metropolitan housewife would possess the knowledge to select the proper shovel. However, it should not be assumed that the construction gang-leader could make correct decisions concerning the design or construction of a 60-story apartment building or a suspension bridge. It should also be apparent that for a person to make decisions concerning personnel and industrial relations policies should have a thorough knowledge of such subjects as psychology, sociology, labor law, and perhaps several related fields. A knowledge of these same subjects would be of minimum value to an accountant who must make decisions to establish and maintain the accounts of an organization.

Sources where knowledge may be obtained are available in any field of endeavor. The content of such information is both extensive and, in some cases, quite technical. No further effort will be made in this discussion to present such specialized knowledge but concentration will be placed on application of the knowledge.

While specialized knowledge is essential to consistently correct decisions, such knowledge is valueless unless it is properly applied in the analysis and evaluation of the alternatives available. The ability to analyze and evaluate alternatives for decision is a science within itself and is similar in all decisions. The primary precept of decision making is to express quantitatively the specialized knowledge as it applies to the problem requiring the decision, to weigh this information quantitatively, and arrive at a quantitative result. The great failure of those making decisions is that they improperly apply knowledge they possess because they have failed to realize they must have a common medium of expression of all knowledge to arrive at a correct result when various fields of knowledge have a bearing on that result. The only common medium of expression of values is numbers; hence, those making decisions must learn to reduce their knowledge from various fields to numbers in order to properly combine them and arrive at the correct result.

Lord Kelvin writing in 1883 observed, "when you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind." This fact is just as true today as in the days of Lord Kelvin and is of even greater importance, especially as related to management decisions. Keener competition requires that organizational activities be more efficient which, in turn, requires that management decisions be correct. Correctness can result more consistently through quantitative analysis and decisions based on such analysis.

The formation of conclusions is closely allied to the analysis and evaluation of the factors affecting the decision. However, there must be a mental conclusion and mental confirmation of the quantitative analysis. Since this step is a summation process, it should follow if a complete quantitative analysis is made.

The presumption is made in this discussion that the person expected to make decision has the required knowledge or has the opportunity to utilize the knowledge of other persons to assist him. The problem then is one of proper analysis and the purpose of the remainder of this discussion is that more consistent and successful decisions will result from quantitative analysis. Further, that quantitative analysis can be applied to all decisions.

Proof of the fact that quantitative analysis can be applied, and applied to achieve more desirable results, in any situation is possible only by illustrating quantitative analysis for every different type of situation. Since this is impractical, examples will be presented from various fields of activity in an effort to provide verification of the theory.

Can Quantitative Analysis Be Applied

Types of analysis and methods of application will be discussed in more detail later; however, before this is done, a few illustrations of how quantitative analysis can be applied to several different types of situations may promote a more receptive attitude of the techniques and methods.

Problems of Educational Institutions -- The problems of a large educational institution are quite varied. Two actual problems and the basis of their ultimate solution will be presented.

A committee of college instructors and administrators was appointed to plan the arrangements for graduation ceremonies to occur in the month of January. Preliminary estimates were that a crowd of 10,000 persons, in addition to those graduating, was expected. The committee had decided upon much of the program when one of the committee members suggested that means be provided to "check" the coats of those attending the ceremonies. The idea was thought to be a good one by many of the committee members and would probably have been accepted as a committee decision except for the following analysis which was offered by one member.

The facilities available would permit four check rooms to be utilized. The arrangement of the rooms were such that two persons could work in each as checkers. This would mean that each checker would be required to check 1,250 coats assuming all persons utilized the service. Since college students would be utilized as checkers and they would be inexperienced in the work, an estimate of one-half minute to check each coat was made. This would mean that $(1250 \div 60)\frac{1}{2}$ or between 10 and 11 hours would be required for persons to check their coats and at least a like period for them to regain their coats. Allowing 3 hours for the graduating ceremonies, it would require about 24 hours from the time the first person must arrive until the last person could leave. The above analysis, although apparently very simple, prevented a decision that would have resulted in confusion and, perhaps, disrupted the entire ceremonies.

Another problem that often arises in colleges is discipline. Without legal status a college must develop methods of discipline that will achieve the desired results. Several colleges handle this situation by stating a rule which is usually to the effect that, "Any student who conducts himself so as to bring discredit upon the school is subject to expulsion."

This particular discipline problem concerned a student who had failed to pay his debts on several occasions. Merchants had notified the college of this and had intimated that the college should take some action. Attempts had been made by several college officials and instructors to get the student to settle his debts, but these attempts were not successful. The student, when questioned by a committee of college instructors and college students, offered no defense except to contend that the debts were his private affairs and should be of no concern of the college. The decision of the committee was to recommend expulsion on the basis of the following analysis presented by one of the committee members.

The objective of a college is to provide the means for a formal education for those who attend. This can be done economically only so long as enrollment is maintained at a high level, since the income of the college is in approximate proportion to the number of students enrolled. The problem is merely what decision will result in the largest enrollment. It is estimated that expulsion will result in the loss of one student immediately and two students each year for the next five years. These are students and prospective students who are friends of the expelled student and will be adversely influenced to the extent of selecting a different college. The expulsion of the student, it is estimated, will increase enrollment an average of three students for the next 10 years above what it would be if the decision were "not to expel the student." This is estimated to be the result of favorable rather than unfavorable influence on those merchants and individuals whom the student owes. The decision is merely whether the college had rather lose 11 students or 30 students from their potential enrollment for the next 10 years.

Problems of Entertainment Groups -- In considering varied types of organizations, let us consider an orchestra and the decisions that might be required. The popularity and hence the success of an orchestra, both from the standpoint of reknown and financial benefit, is determined by the number of persons making an effort to listen to the particular orchestra. It is conceivable that a musical group could be formed for an objective that was not affected by the number of listeners but since this is the objective of most orchestras it will be assumed the objective for this illustration. Suppose that a vacancy exists in the orchestra. The vacancy can be filled by any one of three musicians. Musician No. 1 is one who has previously established a reputation and has a large personal following of fans. It is estimated that this musician would increase the number of listeners 5%. Musician No. 2 is a good musician and it is estimated the present number of listeners will be retained without loss or gain. Musician No. 3 is of lower caliber than the musician whose loss resulted in the vacancy and will consequently result in a lessening of the number of listeners by 3%.

The following is an example of the analysis that could be made of such a situation.

Musician No. 1 --

Annual value of 5% increase in audience	$17,500
Annual cost of services in excess of previous musician	7,500
Net Gain	$10,000

Musician No. 2 --

Annual value of audience change	$ 0
Annual change in cost of services	0
Net Gain	$ 0

Musician No. 3 --

Annual value of 3% decrease in audience	$ 7,000
Annual cost of services less than for previous musician	2,000
Net Loss	$ 5,000

The choice of Musician No. 1 would result in a net gain of $10,000 to the orchestra as compared to no change for Musician No. 2 and a loss of $5,000 for Musician No. 3. The choice becomes obvious after such an analysis.

A situation similar to the above example exists in most professional sports, the movies, radio, and television. However, it must not be assumed that the items evaluated in the above example are the ones that apply in all situations. Personal traits such as temperament, cooperativeness, and personal habits when not working may have either a positive or negative value in a particular situation. Ability may have value other than as a means of drawing an audience. For example, the professional baseball player, who, primarily through demonstrative antics while playing, is credited with increasing the customers by 1,000 on each of 60 days but is also credited, through lack of ability, in causing the team to lose the opportunity of playing in the World Series, which is equivalent to having 100,000 customers on each of 3 days, is of questionable value to his team.

Problem of a Recreational Group -- An organization consisting of 100 business and professional men of a town of 10,000 population considered and approved plans for the construction of a lake and lodge to be used by members and their families for boating, fishing, and camping. The cost of the project was $75,000.

After construction of the project was completed most of the members were very displeased. They made their quantitative analysis after the decision and the action, which was too late. As one member stated the analysis: "the annual investment cost is $8,200. (Method of obtaining this presented in Chapter II). Annual cost of operation and upkeep is $7,000, making a total cost of $15,200 each year. Since other facilities are available, although not as desirable, the utilization benefit is valued at an average of $100 each year for each member, or $10,000. The organization is then losing $5,200 a year which means $52 each year for each member."

Had a complete analysis been made previous to the decision, the ill feeling of the members and the financial loss could have been prevented.

Problems of an Industrial Organization -- Situations requiring decision in connection with the utilization of personnel, engineering and investment problems, and research analysis are discussed in Chapter II. Many of the functions of an industrial organization are thought to be incapable of numerical analysis. Two such situations are selected for discussion.

The manager of the personnel and industrial relations department of an industrial plant was asked to submit his recommendation to the plant manager as to whether or not a particular individual should be discharged. The individual had been absent from work on two occasions without notifying his supervisor. The employee was assigned to a unit that operated 24 hours a day and his absence meant that a fellow worker must work 16 hours without relief or that the supervisor must find some other employee work the required shift after it was known that the man was absent. The employee, replacing the absent employee, is paid at one and one-half times his regular rate. For this reason, failure of the man to notify his supervisor prior to the absence was considered a serious offense and previous practice gave the plant manager the prerogative of discharging the man if he felt the circumstances justified.

The manager of personnel, in discussing this problem informally with one of the members of the Engineering Department, was bemoaning the fact that his task was so difficult since he could not add 2 and 2 and get four as an engineer could always do. Through a series of questions and answers, the engineer obtained the following facts, using the personnel manager's knowledge of the situation. The cause of the absences, known to be true because the man in question had confirmed them, was an out-of-town visit by the employee on one occasion and engagement by the employee in a sporting event on the other. Both absences could be considered as being for pleasure and it was possible in both cases for the employee to notify his supervisor, but he did not do so because "he was in a rush." Because of the independent and somewhat arrogant attitude of the employee, some of his fellow employees feel he should be discharged. The personnel manager estimated that there will be a decrease in efficiency of an average of 7% for 15 working days of 8 hours each on the part of 10 employees if the employee in question is discharged. He also estimates that there will be a decrease in efficiency of 5% for 10 working days on the part of 30 employees if the employee in question is retained. Employees whose efficiency is affected in either case receive an average rate of $1.60 per hour. It is estimated that the recruiting, employing, and training of a replacement will cost $200. Future unexcused absences by other workers, occurring as a direct result of a decision to retain the employee, is estimated at 4 with an estimated average rate of $1.60 for such employees. The possibility of future absences by the same employee should be considered. The personnel manager estimates that the probability of such an occurrence within a period of one year is 1/100. In the event of this occurrence, it is assumed the employee will be discharged with no effect on efficiency of fellow employees.

The following quantitative analysis is made:

If the employee is discharged --

Cost of lost efficiency 10 x 15 x 8 x \$1.60 x .07		\$135
Cost of training replacement		200
	Total Cost	\$335

If the employee is retained --

Cost of lost efficiency 30 x 10 x 8 x \$1.50 x .05		\$192
Cost of probable future replacement 1/100 x \$200		2
Cost of absences by all employees 4 x 1½ x \$1.60 x 8		77
	Total Cost	\$271

The analysis indicates that discharge of the employee would not be economical. This result in this particular case may be due to the value placed on the effect of retaining an employee on the future conduct of his fellow employees. This analysis was made using the personnel manager's evaluation which are based on knowledge he should possess and were not and should not be questioned merely because of personal feelings on the part of the person making the analysis.

Another example of a situation often thought to be incapable of quantitative analysis is the utilization of the time of an executive. Suppose the president of an organization is requested to speak at a luncheon of a local women's Civic Betterment Club. The objective of the organization, of which he is president, is the design, making, and sale of women's clothing. This situation can be analyzed by weighing the value of estimated increase in sales against the estimated value of the president in some other activity, which is possible, during the time required to prepare and deliver the speech to the group of women.

Conclusions on the Application of Quantitative Analysis

The illustrations just discussed were not extensive in number nor were they complicated in nature. The illustrations were given to establish the fact that quantitative analyses can be and are made in actual situations in varied fields of activity. In Chapter II and Chapter III, the means and methods of performing quantitative analysis will be discussed more fully.

A very important conclusion that may be drawn from the illustrations given is that the ability and the inclination to utilize quantitative analysis is often not possessed by those required to make decisions where such ability and its use would be helpful.

Usually, one of two attitudes is taken when the suggestion is made that quantitative analysis be applied to problems requiring decision. One of the attitudes is that quantitative measurement cannot be made within a particular field. One of the principal contentions of this discussion is that this claim or supposition is a result of either a lack of sufficient knowledge of the subject or a failure to comprehend numerical expression adequately. Sufficiency within a particular field, to enable a person to be a part of the management of an organization where decisions will be required, should include both knowledge of the subject and the ability to express this knowledge, as applied to a particular problem, numerically.

The other attitude often encountered from persons failing to utilize quantitative analysis as a basis of decision is that "any fool knows that." This implies that when the subject of quantitative analysis is broached the individual has the knowledge of the subject and the ability to evaluate the knowledge numerically but does not do so. There may be several reasons for this. The person may fail to recognize that certain actions or statements made by him are decisions. This is often the case in those repetitive situations where it is assumed that experience has taught the correct answer and individual analysis is not made, even mentally. It may also be especially true in fields such as psychology, industrial relations, or public relations where few people express themselves numerically. Another reason for failure to analyze a situation quantitatively is that the stress upon the urgency of the decision causes one unpracticed in numerical expression and evaluation to resort to less exact and possibly more rapid methods.

CHAPTER II

TOOLS AVAILABLE FOR QUANTITATIVE ANALYSIS

The complexities of situations requiring decision may vary greatly. It may be simple a matter of deciding whether to furnish a clerk with wooden pencils or a mechanical pencil; or, it may be a decision as to whether or not a new plant is to be constructed where the plant includes numerous technical and non-technical processes requiring skilled and unskilled labor, considerable processing equipment, facilities for furnishing utilities, and the many other varied factors that must be included in a large manufacturing plant. The complexity of the situation determines the number of tools that must be available and the extent to which they must be applied to arrive at a quantitative result.

There are as many tools available as there are subdivisions in the field of applied numbers.[1] Some of these tools will have application in the formation of practically all decisions while others will be limited in their application. Several of the tools which have wide application and also aid in the development of quantitative approach to many problems are discussed in this chapter.

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1 The term "applied numbers" is used here because of lack of a term to cover all of those areas where numerical representations and combinations are the basis of the area of knowledge. Ordinarily one would think "mathematics" would serve as well, but in common usage mathematics does not cover such fields as accounting where mathematical principles are used, but the methods of application are an area of study not included under mathematics.

Mathematics

The majority of decisions that must be made in the day to day operation of the organization are of a nature that they must be made rapidly either because of the urgency of the situation requiring decision, or because the associates of the person making the decision expect the decision to be made without the necessity of prolonged calculations or discussion. In the illustrations cited, to the effect that 10,000 decisions were involved in the transfer of two telephone poles from one side of a street to the other, it must be realized that many of these decisions are of an instantaneous or nearly instantaneous nature.

If quantitative analysis is to be utilized in the making of these decisions, then means of making rapid calculations must be available and the person making the decisions must have the ability to utilize these means. In an effort to accomplish this, the four fundamental mathematical processes, addition, subtraction, multiplication, and division, will be discussed. Since these processes were studied and supposedly mastered in grade school, it is not the mere essential of being able to set down a group of numbers and add them that is important, but to make rapid mental calculations which are required in so many decisions.

During World War II, the president of an organization that produced small metal parts for the automotive industry was interested in acquiring a contract from the Federal Government to produce certain small metal items for the Army. The president placed a telephone call to a government official in Washington to discuss the contract when he suddenly realized that he had little idea what an adequate building to house the operation would cost and that such information was desirable prior to the discussion. He hurriedly summoned an engineer and asked him the cost of the building. From previous discussions, the type of construction was known and the space needed would require a building 100 feet long by 50 feet wide. The engineer stated that he would compute the cost and have it available in a few hours, whereupon the president very excitedly exclaimed that he had the telephone call placed and needed to know immediately. The engineer then, mentally in less than a minute, estimated the cost to be $30,000. This may not seem unusual since the president or any one else could have selected such a figure at random; however, the engineer's estimate was not a random figure but was determined quantitatively, using the best information available considering the time permitted to make the decision.

The engineer knew that the company had recently completed a building of the same type of construction which was 200 feet by 75 feet. This building had cost $86,000. The engineer then reasoned that 15,000 square feet had cost $86,000, which was $86,000 ÷ 15,000, or approximately $5.75 per square foot. However, he reasoned the unit cost would be slightly higher on the smaller building, so he estimated a cost of $6.00 per square foot. A building of 5,000 square feet would then cost 5,000 x $6.00, or approximately $30,000. This may sound like a very unique example, but when it is considered that the building was actually constructed a short time later for less than five per cent in excess of the engineer's impromptu estimate, the means utilized to arrive at such a decision must have some merit.

This procedure is not unique to the field of engineering. The office manager who is given a job to be accomplished on a given day which will require the services of several of his clerks must decide the number of clerks to assign to the job. In doing so the quantitative analysis upon which his decision will be based should be somewhat as follows: if five men are assigned to the job, it will take them five hours each or twenty-five man hours to accomplish the task. If six men are assigned to the job, they will require four hours each or twenty-four man hours; whereas, if seven men are assigned to the job, three and one-half hours each or a total of twenty-four and one-half man hours. Since the job is one in which a total of eight hours per man is allowed, then the manager would assign six men to the task to achieve the most economical utilization of his clerks.

Can Everyone Make Such Decisions

The procedure outlined in the examples above may be very commonplace for some few people who deal with numerical situations continually; however, it is not a common practice of a majority of those persons making decisions. On the assumption that decisions based on quantitative thinking are desirable, the question arises "can those who are not now using such analysis develop the ability such that decisions that must be rendered in a very short time will have a quantitative basis?"

A member of the purchasing department of an industrial organization was interested in deciding which of four brands of paint would be purchased by his organization for future use. In order to determine this, it was decided that each of the four brands would be tested under actual conditions of use. The respective paint companies agreed to furnish the paint if the company would furnish the labor necessary to apply the paint. The purchasing department representative picked up the telephone and made inquiry of the chief engineer, "do you have four tanks of the same size that would be subjected to similar conditions so that we could run a test of four different brands of paint?" The engineer replied that he did and the second inquiry was, "how much will it cost us to paint them if the paint is furnished?" In less than a minute, the chief engineer gave the estimate of $200.

In discussing this later with the chief engineer, it was found that his mental process was as follows: the tanks are 20 feet in diameter and 30 feet in height and have a cone roof which is 10 feet at its apex. The square footage to be painted is, therefore, π x 20 x 30, or approximately 1,900 feet for the side of each tank and x 10^2, or approximately 315 square feet for each roof if the roof were flat. Making an allowance of 40 per cent for the cone, the area of the roof would be approximately 435 square feet for the roof, or a total of 1900 + 435 or 2335 square feet for each tank. Knowing that the cost of painting tanks was two cents per square foot would make 2335 x $.02, or approximately $46.70 per tank. Four tanks would then cost 4 x $46.70, or approximately $187.00. The engineer then allowed for contingencies to make his estimated $200.00

Since the mental calculations made by the chief engineer were somewhat difficult, the above problem was stated to each of five engineers, some engaged in engineering design and construction, while others were engaged in a supervisory capacity for processing departments, and each was asked to make a mental estimate of the cost of painting the tanks if it were known that labor cost would be $.02 per square foot. All of the replies were between $180 and $200; however, the time to make the estimate was considerably longer than that required by the chief engineer.

The question of the ability to make decisions was then discussed with each of the five engineers and each expressed confidence in his ability and the ability of any other engineer to make such decisions, although each of them also stated that he seldom used this method in practice. This same idea was discussed with persons engaged in accounting, city management, and civic organizations. The reply received from all of them indicated that such analysis could be made in most of their decisions, but that such analysis was seldom used except in those instances where they made a detailed written calculation on some specific problem.

On the basis of these findings, it is believed that those who are in a position to make decisions are capable of mental quantitative analysis of any of the situations arising within the scope of their specific jobs. However, it is apparent that few have developed or tried to develop the ability to utilize this method of analysis.

Development for Mental Quantitative Analysis

It is doubtful that one will find the facilities for formal education to develop the ability for mental mathematical analysis. Rather, it must be developed through individual initiative which will require above all else a constant mental quickness toward quantitative thinking and continued mental practice in the four basic mathematical processes. Some few educational institutions offer instruction in mathematical short cuts with the aim of developing rapid use of the four basic mathematical processes.

Listed below are a number of mathematical short cuts. This list is not intended to be exhaustive, but is given more to indicate the trend of these short cuts and how they might assist in mental calculations.[2]

1. To square a number ending in 5 or to multiply any two numbers where the sum of the last digit of the two numbers is equal to 10 and the first digit of the two numbers is the same, multiply the last digits of the two numbers and then multiply the portion of one number preceding the last digit by one greater than the portion of the other number preceding the last digit.

   Example: 65 x 65: 5 x 5 = 25 and 6 x 7 = 42
   therefore, the answer is 4225.

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2 Recommended for a complete treatment of mathematical shortcuts: Lester Meyers, High Speed Mathematics (New York: D. Van Nostrand Company, Inc., 1947)

Example: 124 x 126: 6 x 4 = 24 and 13 x 12 = 156 therefore, the answer is 15,624.

2. To multiply any number by 10 or a multiple of 10, move the decimal point as many places to the right as there are zeros in the multiplier.

   Example: 24 x 1000 = 24,000

3. To divide a number by 10 or a multiple of 10, move the decimal point as many places to the left as there are zeros in the divisor.

   Example: 24 ÷ 1000 = 0.024

4. To divide by a number that itself divides easily into 100 or greater multiples of 10, divide by the multiple of 10, using Rule No. 3 above and multiply by the number of times the original divisor was contained in that multiple of 10.

   Example: 24000 ÷ 25 would be (24000 ÷ 100)4 or 960

5. To multiply by a number that is easily divisible into 100 or greater multiple of 10, multiply the number by 100 or the greater multiple of 10 and divide by the number of times the original multiplier is contained in the chosen multiple.

   Example: 24 x 25 would be (24 x 100) ÷ 4 or 600

6. Utilizing some of the principles outlined above, if it were desired to multiply 38 x 98 the short cut method would be to multiply 38 x 100 and subtract 2 x 38 or (38 x 100) - (2 x 38) or 3800 - 76, which is 3724.

7. Similar to No. 6, if it were desired to multiply 38 x 102, the short cut method would be (38 x 100) + (38 x 2) or 3800 + 76, or 3876.

The development of the ability to make mental calculations as an aid in quantitative decisions so that these calculations can be made with speed and reasonable assurance of accuracy will require mastery of many short cuts similar to those illustrated above and will require considerable practice in the uses of these short cuts as well as the mental performance of the four basic mathematical processes without aid of short cuts.

The importance of this type of aid in the making of decisions cannot be stressed too highly because of two facts. One is that the majority of decisions must be made in a minimum of time, and the other is that a great majority of decisions can be made quantitatively without the aid of mathematical knowledge other than the four basic processes. With the exception of a very few fields, of which engineering and research are notable examples, it is estimated that seventy-five per cent of all decisions can be made based on quantitative analysis using only the four basic processes and that approximately ninety per cent of these decisions could be made much more rapidly and just as efficiently using mental calculations as using written calculations.

The Value of Mental Calculations:

Although it is imperative that the abil ty to perform mental calculations be developed in order to make good decisions consistently, there is another definite advantage to the utilization of this tool. It is estimated that the consistent use of this practice enhances the value to the organization of the person making decisions by two or three times. In the example cited above of the chief engineer making the estimate of the cost of painting petroleum storage tanks, it is doubtful that the same result could have been obtained by precise written calculations in less than three minutes. It then follows that the facility increases that chief engineer's capacity of making decisions by three times. It should be apparent that much more efficient utilization of supervisory, administrative, or executive employees will result through development of the quantitative approach to situations requiring decision, followed by the ability to mentally analyze and arrive at a quantitative result in those situations.

Engineering Economy

Engineering Economy is an area of study in the field of engineering that has developed importance in recent years. It deals primarily with the application of quantitative analysis to engineering proposals for the purpose of determining their economic justification, hence, furnishing a basis for decision concerning these proposals.

It is felt that a discussion of this subject is important for three reasons: (1) Specific application of quantitative analysis to the field of engineering has become a fact through utilization of the principles included in engineering economy. (2) A thorough understanding of how quantitative analysis has been applied in one area will stimulate its adoption in other areas and do much toward development of a mental attitude on the part of those making decisions which will propagate quantitative analysis of all problems. (3) The concepts and methods utilized in engineering economy will be applicable to other fields. Once numerical values are assigned to the factors affecting a situation, the methods of combining and evaluating these factors are basically the same in most situations whether it involves engineering or not.

In the present industrial world, with continual building, expanding, and modernizing of industrial plants and equipment, a vast number of decisions bear a relationship to engineering. This relationship may include the entire situation on which a decision is required or may involve only portions of the situation. Because of this fact, it is not only essential that engineers be capable of making quantitative analysis of their problems, but those administrative and executive officers who must work with, or evaluate the results of, engineers should also be familiar with the means by which quantitative results are obtained.

The basic principles included in the field of engineering economy are discussed in the following pages. This discussion is merely a synopsis for the purpose of familiarization and general information. A complete treatment of the subject should be secured through use of a good textbook.[3]

Interest

A large portion of today's business is transacted with borrowed money. One of the expenses of transacting this business is the rental, or interest, on the money used. It is reasonable then to consider interest on borrowed money as much an expense of the operation of an organization as labor or materials.

Now considering business which is transacted on funds belonging to the organization, suppose that a sum of money is to be invested in a building for use by the organization. If this money is not invested, it could be loaned to some other organization and interest would be received by the organization loaning the money. Therefore, if the money is invested in the building, the potential interest income is sacrificed. It is a basic economic fact that failure to receive an income is equivalent to an expense; therefore, an amount equivalent to interest that could be received is chargeable as an expense for that business transacted on organization funds.

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3 Recommended: H. G. Thuesen, Engineering Economy (New York: Prentice-Hall, Inc., 1950)

There are two types of interest usually explained in any discussion of interest: (1) Simple interest and (2) Compound interest.

In simple interest the amount of interest is in direct proportion to the length of time the money is borrowed. On a loan of $100 at a rate of 6% per year, the total amount due at the end of 6 months would be $103; the amount due at the end of 1 year would be $106; and the amount due at the end of 2 years would be $112.

In compound interest the interest due at the end of a compounding period, if not paid, becomes part of the loan and draws interest during succeeding compounding periods. On a loan of $100 at 6% per year compounded semi-annually, the total amount due at the end of 6 months would be $103; the amount due at the end of 1 year would be $106.09; and the amount due at the end of 2 years would be $112.55.

It should be noted that in simple interest it is necessary to express the rate of interest and the period for which the interest is chargeable. In compound interest, it is necessary to express the interest rate, the period for which the interest is chargeable, and the period for compounding. Under normal circumstances, the period for which the interest is chargeable is one year unless otherwise specified. In keeping with this, this period will not be expressed in giving compound interest rates.

Through business custom and practice it has become practically universal to use compound interest. Practically the only usage of simple interest is for determination of interest for short-term loans, usually less than one year. For this reason the remainder of this discussion will deal with compound interest only.

Equivalence

The term equivalence is used to signify two sums or groups of sums of money being equal considering interest. It is universally accepted that $100 today is more desirable than the promise of $100 five years from today. Observation tells one that the two sums are equal; yet, common sense tells one that the two sums are not equivalent. One hundred dollars today is equivalent to $133.80 five years from today at a rate of 6% compounded semi-annually.

Although a simple comparison involving only two sums may be apparent from observation, comparison of two or more groups of sums where each group covers a number of years requires a means of comparison. Suppose that two proposals are offered to accomplish a certain production process. For simplicity, suppose that the income would be the same for the two proposals and that the expenditures are as shown in the following table.

Table 1. Comparison of Annual Expenditures of Two Alternate Plans

End of Year No.	Expenditures for Proposal No. 1	Expenditures for Proposal No. 2
0	$10,000	$13,000
1	3,000	2,000
2	3,000	3,000
3	4,000	3,000
4	5,000	4,000
5	5,000	5,000

It is difficult to evaluate two such series of expenditures from observation alone. However, when it can be stated either that Proposal 1 is equivalent to a present expense of $34,392 while Proposal 2 is equivalent to a present expense of $33,870, or that Proposal 1 is equivalent to an annual expense of $4,031.87 for the next 5 years while Proposal 2 is equivalent to an annual expense of $3,970.58 for the next 5 years, it is apparent that Proposal 2 is more desirable economically.

Depreciation

Today's industry is predominately comprised of production, manufacturing, and distribution organizations that utilize large and complicated machinery and equipment. The cost of this machinery and equipment must be passed on to the goods and services produced through its use. Since the majority of the machinery and equipment lasts for a number of years, it is essential that means be used to quantitatively proportion the costs of the machinery and equipment to the units of goods and services. When it is contemplated to purchase machinery and equipment, it is necessary to have means of distributing the cost over the years of life of the machinery and equipment so that proper comparison can be made among different types of equipment that will accomplish the objective. This is the evaluation of the annual depreciation of the machinery and equipment.

There are three more commonly used methods of determining the depreciation to be charged in connection with a given asset.

They are:

1. Straight Line Depreciation
2. Sinking Fund Depreciation
3. Fixed Percentage of Diminishing Balance Depreciation

The three factors that are combined to obtain the desired depreciation by any of the methods above are the life of the asset, the cost of the asset, and the sale or salvage value at the end of its life.

The life of the asset will depend upon the circumstances of its usage. The asset may be a pump in a remote location where maintenance is difficult. Under such circumstances, it is logical that the usable life might be much less than the same pump located in a pump house with almost constant inspection and maintenance. It is the usable life of an asset that is used in depreciation calculations.

The cost and salvage value are important in depreciation calculations since it is the difference between these two values that must be recovered through charges against the products of the equipment. Many organizations adopt the policy of disregarding salvage value in determining depreciation. This policy is detrimental to determining what investments will result in the most economical course of action. In many cases, salvage values are great enough to cause a complete reversal of decision. It is conceivable that, during an inflationary period, a machine used for 2 years could be sold for more than the original cost in which case salvage value is of more importance than cost.

In any situation where depreciation is an element of cost, it should be understood that the depreciation obtained by any method is an estimate. At the time the purchase of an asset is being considered the annual depreciation must be estimated since it is essential to have this and other costs to weigh against contemplaced incomes as an aid in the decision of whether or not to make the purchase. At this time the usable life and salvage value cannot be known with certainty and, since these factors are estimates, the depreciation determined from them will be an estimate.

Straight Line Depreciation -- In straight line depreciation the amount of depreciation is uniform for each year of the usable life. If P represents the cost of an asset, L represents the salvage value at the end of the usable life, and n represents the number of years of usable life, the formula for the annual depreciation becomes $\frac{P-L}{n}$. If it is known that a certain machine can be purchased for $800 and the estimated usable life is 7 years at which time it is estimated the machine can be sold for $240, the annual depreciation would be $\frac{\$800-\$240}{7}$ or $80.

Sinking Fund Depreciation -- In sinking fund depreciation it is assumed that a uniform amount will be deposited into a sinking fund each year. The amount of depreciation for any year is then the amount of the deposit plus interest due for that year on deposits for previous years. Using the example where

P = \$800, L = \$240, and n = 7 years, a table of annual depreciation is compiled below assuming 4% interest compounded annually on the sinking fund.

Table 2. Illustration of Sinking Fund Depreciation

Year No.	Deposit	Add Interest Due	Annual Depreciation	Balance
0	\$ 0.00	0.00	0.00	\$800.00
1	70.90	0.00	70.90	729.10
2	70.90	2.84	73.74	655.36
3	70.90	5.79	76.69	578.67
4	70.90	8.85	79.75	498.92
5	70.90	12.05	82.95	415.97
6	70.90	15.36	86.26	329.71
7	70.90	18.81	89.71	240.00

The amount of the annual sinking fund deposit can be determined through use of any good sinking fund interest tables realizing that the amount of the fund must equal (P-L) at the end of n years.

Fixed Percentage of Diminishing Balance Depreciation-- This method of depreciation is based on the theory that each year an asset depreciates a certain percentage of its value at the beginning of the year. This then means that the depreciation is greater during the first year of the life of an asset and becomes less as the asset becomes older and has less value. The percentage, F, is obtained by the formula $F = 1 - \sqrt[n]{\frac{L}{P}}$. A table of depreciation values for the situation where P = \$800, L = \$240, and n = 7 follows.

Table 3. Illustration of Fixed Percentage of Diminishing Balance Depreciation

Year No.	Value At Beginning of Year	Depreciation Percentage	Amount of Percentage	Balance
0				$800.00
1	$800.00	.158	$126.40	673.60
2	673.60	.158	106.43	567.17
3	567.17	.158	89.61	477.56
4	477.56	.158	75.45	402.11
5	402.11	.158	63.53	338.58
6	238.58	.158	53.50	285.03
7	285.03	.158	45.03	240.00

Comparison of Depreciation Methods -- A consideration of the factors affecting depreciation should serve as a guide as to which method of depreciation is most applicable. Conditions of use, amount of expected use, chances of the asset becoming inadequate or obsolete, and the expected standards of maintenance and repair will all be important in determining whether straight line, sinking fund, or fixed percentage of diminishing balance should be used as the method of depreciation.

In addition to the explanation of each of the three methods above, a graphic comparison of the methods may prove helpful in selecting the proper method. In Fig. 1, the curves of each of the three methods are given for the example P = $800, L = $240, and n = 7 years.

An analysis of Fig. 1 is that straight line depreciation is uniform for each year of the usable life, sinking fund depreciation is less during the first years of life and greater in the later years, while the fixed percentage of diminishing balance depreciation is greater the first years of the life and less during the later years.

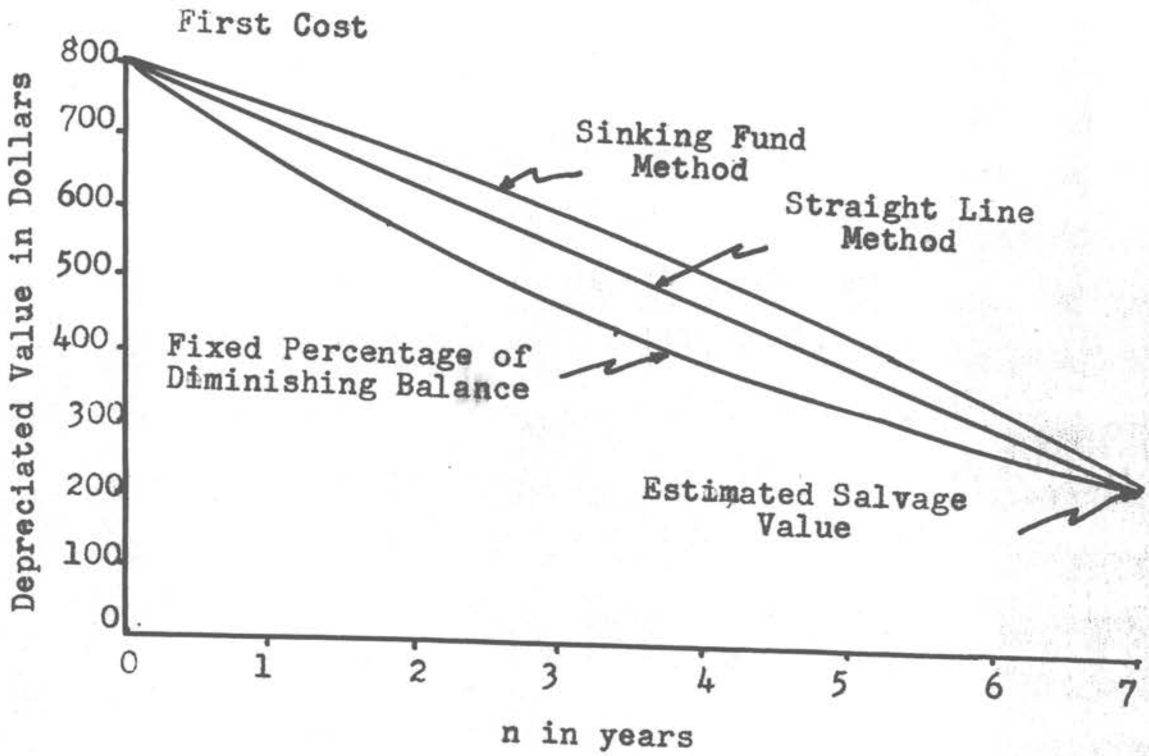

Fig. 1 -- Comparison of Methods of Depreciation

The difference in annual depreciation as determined by any of the three methods is not great and, in view of the fact that the depreciation is an estimate, the important factor is not the method utilized but the fact of recognizing depreciation as an expense of operation, and estimating it by a reasonable method.

Comparison in Short-Term Situations

This aspect of engineering economy is for the purpose of evaluating alternate methods of accomplishing an objective where the time required for the accomplishment is such that interest on money invested is not an item for consideration. Since interest is based ordinarily upon one year's use, the principles outlined under this heading will be limited to the following situations:

1. Any utilization of labor or materials for the accomplishment of organization objectives where a change of investment is not involved.
2. Any situation involving utilization of labor or materials for the purposes of accomplishing organization objectives where the life of any investment to assist in the activities is less than one year.

In the day to day activities of any organization there are situations which arise that will require only a day or perhaps a week to complete. Decisions in these situations based on quantitative analysis is of prime importance. The combined effect of all such situations within an organization will amount to a large percent of the total activity of the organization and can determine the success or failure of that organization.

Situations Involving Utilization of Men -- Efficient utilization of human energy is always an important consideration in any activity. Even though this is supposedly "the age of the machine", most organizations find that labor cost is usually about as important as the cost of the materials. Three examples are cited to illustrate methods of quantitative analysis of short-term situations involving men.

The first to be discussed is the situation of deciding which of several optional work assignments to perform where some of the assignments must be deferred because of a shortage of personnel. Most supervisors and foremen find themselves at frequent intervals with too few men to perform all the available work and the decision as to which work to perform may be an important one.

An organization sold a group of 720 used lubricating oil drums. The purchaser offered $.25 per drum if the organization would load the drums in a box car. A supervisor was asked if he could provide two men to perform the work. His reply was that he could not afford to have his men do that because he needed them too badly and that it would be much better to contract the loading task to some commercial trucking organization. The supervisor was then asked how long it would take two men to load the drums and the reply was 8 hours. The next question asked of the supervisor was whether he had any men performing work that would be worth $80 to $90 for an 8-hour day. "No, those guys are worth $20 to $30 to me though," was the reply. The supervisor was then very surprised when the following simple mathematical analysis was explained to him and he realized he had refused to furnish men for work that would be 3 to 4 times more profitable:

$$720 \times \$.25 \div 2 = \$90 \text{ per man per 8-hour day.}$$

This situation is not unusual. Those making decisions often become so agitated with apparent urgency of getting "caught up" with work needing to be done that they "cannot see the woods for the trees."

Another situation that arises often in efficient utilization of human energy is the decision of how many men to assign to a particular task. This determination of crew-size may be for a one-hour task or for crews that will work at the same task each day.

For tasks of short duration the "preparation time" and "clean-up time" are important factors. The cleaning out of a tank that has contained acid will require 5 hours when performed by 1 man but can be accomplished in 1 hour by a crew of 4 men. It requires 15 minutes for each man to prepare for the task since rubber clothing and a mask must be worn, and upon completion of the task, it will require 15 minutes for each man to "clean up." The analysis would be as follows:

For one man:

Time for preparation		$\frac{1}{4}$ hour
Time to perform work		5 hours
Time to clean up		$\frac{1}{4}$ hour
	Total	$5\frac{1}{2}$ hours

For four men:

Time for preparation ($4 \times \frac{1}{4}$)		1 hour
Time to perform work (4×1)		4 hours
Time to clean up ($4 \times \frac{1}{4}$)		1 hour
	Total	6 hours

Although the actual work time is less for the larger crew, the total time required when preparation and clean-up time is considered is greater.

In many activities cooperative effort increases the results obtained for each man hour devoted to the activity. This is usually true only so long as the forced idleness of part of the group does not become excessive. Such a problem is present in most all maintenance and repair work and in much clerical work.

Consider the case of determining the size of crew to assign to a given job. A maintenance supervisor is requested to repair the concrete floor of a storeroom. The floor is pitted in several places and is to be patched with a prepared patching compound. The supervisor assigns a crew of 3 men to the job on the basis of the analysis shown in the following table:

Table 4. Number of Men to Assign to a Job

No. of Men Assigned	Hours Required to Complete	Total Man-hours for Job
1	15	15
2	7	14
3	4	12
4	3½	14
5	3	15

This analysis is on the basis that all men receive the same hourly wage and that no other operations are affected by a delay in the repair job. It should always be borne in mind that there may be many ramifications to such a problem and they must be considered to arrive at a correct result.

A problem that faces many executives is the decision as to what extent supervisory and executive training is an economical venture. As has been pointed out previously, the supervisor or executive is a non-productive individual and his efficiency is determined by the efficiency of productive workers under his supervision. This implies that there is a multiplying effect as one progresses up the supervisory line. A first line supervisor may supervise only 7 men and, hence, the extent of training for this supervisor will be determined by the extent to which the training will result in an increase in the efficiency of the 7 men. On the other hand a plant manager may supervise 5 subordinates who each supervise 5 subordinates and so on through about four or five supervisor levels until the productive worker is reached. At the productive level there will then be 300 to 1,000 men whose increased efficiency will determine the extent of training for the higher executive. However, it must not be assumed that the amount of training that can be participated in economically is in direct proportion to the number of productive workers under the particular executive or supervisor. An increase in worker efficiency as a result of supervisory training is definitely a function of the distance between the supervisor and the worker on the organization chart as well as being a function of the number of workers.

It is desired to determine the extent to which first line supervisors should participate in supervisory training. Each supervisor is paid an average of $2.50 per hour and supervises 7 workers who receive an average of $1.65 per hour. The training will cost $1.00 per hour per supervisor for training materials and instructions.

Table 5. Efficiency Increase Required to Justify Supervisory Training

Hours of Training Per Week	Cost of Training Per Week	Weekly Pay of Supervised Workers	Increase in Efficiency Required
¼	$ .875	$462.00	0.19%
½	1.75	462.00	0.38%
1	3.50	462.00	0.75%
2	7.00	462.00	1.51%
3	10.50	462.00	2.27%
4	14.00	462.00	3.03%

An analysis, such as that illustrated above, for each level of supervision would furnish a sound basis for the decision as to the extent of training that could be practiced economically.

Situations Involving Utilization of Material -- There are many situations that arise in day to day operations where a decision must be rendered as to the coice of materials to be used for parts of an item being produced or the tools and equipment to be utilized to perform a given operation.

Consider the manufacture of items for sale where the investment in specific materials to be used is usually of short duration. Two types of laminated plastic are being considered for the manufacture of table tops. Both materials have equal sales value. The following analysis is made of the two materials.:

Type 1: (for each table top)

Cost of material	\$3.87
Cost of cleaning anf finishing	4.76
Average freight cost paid by manufacturer	.42
Total	\$9.05

Type 2: (for each table top)

Cost of material	\$4.12
Cost of cleaning and finishing	2.17
Average freight cost paid by manufacturer	.53
Total	\$6.82

The selection of Type 2 plastic will then result in a savings of \$2.23 on each table top manufactured.

The problem often arises as to what tools and equipment shall be utilized when the job can be accomplished through use of two or more types of equipment.

A contractor has agreed to install 6,000 feet of sewer lines for a city. An analysis reveals that of the 6,000 feet of line, 1,000 feet must be 6.0 feet deep, 2,000 must be 7.0 feet deep, 1,000 feet must be 8.0 feet deep and 2,000 feet must be 9.0 deep. All of the ditch must be 16 inches wide. The contractor has a ditching machine that will dig to a maximum of 7.0 feet deep. The cost of this machine and its operator will be $8.00 per hour. The machine is capable of opening 15 feet of ditch per hour considering the type of soil. The remainder of the dirt must be removed by hand at the rate of $7.00 per cubic yard. The contractor can also perform the excavation by securing a larger ditching machine on a rental basis. This machine is capable of a maximum depth of 10.0 feet and the rental fee is $200, plus $.60 per linear foot within the depth range of 6.0 to 9.0 feet, providing it is used for the entire job.

Cost using small ditching machine:

For machine - $\frac{6000}{15}$ x \$8.00	$3200
For hand excavation -	
$\left[(1 \times 1000) + (2 \times 2000\right] \times \frac{16}{12} \times \$7.00$	1728
Total Cost	$4928

Cost using large ditching machine:

Fixed cost for moving	$ 200
For use of machine 6000 x $.75	4500
Total Cost	$4700

From the above analysis, selection of the larger equipment will save $228 on the entire ditching job.

Situations Involving Utilization of Methods -- Methods, as has been discussed previously, is concerned primarily with the time element in the efficient combination of men and materials to produce goods or services. As such, the economical choice of method, in many cases, will overlap the economical choice of utilization of men and materials. In many cases, however, the method may be altered where no change in the number of men or the type or amount of material is made. This aspect of economy in organizational activity is the prime consideration in the fields of Time Study, as originated by Frederick W. Taylor about 1881, and Motion Study, developed to a great extent through the efforts of Frank B. and Lillian M. Gilbreath.

Consider the example of the female worker whose job is to place nuts on bolts. At the present time the worker holds the bolt in one hand and the nut is held in the other hand and screwed onto the bolt. The worker is paid an hourly rate of $1.30. By installing two small holders, two bolts may be placed in the holders and a nut screwed onto each bolt simultaneously, one with each hand. The holders will cost $6 and will increase the worker's efficiency by 40%. Assuming the operation will last 90 working days for 8 hours each day, the following comparison might be made:

Return due to change of method - .40 x $1.30 x 90 x 8		$374.40
Cost of change of method		6.00
	Net Gain	$368.40

Although all changes may not result in returns this great, there is always a large potential savings for any organization in the area of methods improvement. These method improvements may vary from a better arrangement on an automobile assembly line to a better method to be used by an executive in signing his name. There is seldom any method of performance of activity that cannot be improved and, with proper analysis of cost and savings, those which are economically justified may result in large savings for the organization.

Comparison in Long-Term Situations

When two or more alternatives are to be compared where investments in assets having a life greater than one year are contemplated or incomes and disbursements occur in connection with the alternatives over a period of years, the means by which the alternatives are compared become more complicated. It now becomes essential to consider interest as a factor of cost which implies that methods must be available for transposing sums of money from one time to an equivalent sum expressed as of some earlier or later time. This transposition was briefly discussed earlier under the heading, "Equivalence."

To make the transpositions it is necessary to utilize compound interest at the applicable rate. This rate is usually that rate which must be paid for money if business is performed on borrowed funds or the rate that could be received for money if it were loaned for a venture of similar risk where business is performed with organization funds. The transposition can be made one compound interest period at a time or through formulas; however, utilization of interest tables which have been developed for this purpose makes the transpositions much easier and more rapid.

Interest tables for this purpose are usually compiled in six parts or, in other words, the tables list a value for various interest rates and numbers of periods for each of six factors. These factors will be defined but no effort will be made to show their formula derivations or to express values for them in table form since tables may be found in any textbook on the subject or any good mathematical handbook.

The six factors are:

1. Single Payment Compound Amount Factor --
   This factor when multiplied by a known present sum of money gives the sum's equivalent value at the rate of interest and at the end of the number of compounding periods for which the factor was chosen.
2. Single Payment Present Worth Factor --
   This factor is utilized when it is desired to express some known future income or expense as its equivalent present amount. The factor is selected for the desired interest rate and the number of compounding periods intervening between the present and the time of the income or expense. The known sum times the factor gives the equivalent present value.

3. Uniform Series Compound Amount Factor --
   This factor is utilized to evaluate a series of equal amounts occurring at the end of each compounding period as of the end of the last compounding period in which one of the amounts occur. For example, to express the equivalent of an income of $2000 each year for 5 years as of the end of the 5th year. The uniform amount is multiplied by the appropriate Uniform Series Compound Amount Factor.
4. Uniform Series Sinking Fund Factor --
   This factor is the reciprocal of the Uniform Series Compound Amount Factor and is utilized to determine the uniform series that is equivalent to or that will accumulate to a desired amount a given number of compounding periods in the future.
5. Uniform Series Present Worth Factor --
   This factor when multiplied by a known uniform series of incomes or expenses determines the equivalent of the series expressed as of the present.
6. Uniform Series Capital Recovery Factor --
   This factor is the reciprocal of the Uniform Series Present Worth Factor. When this factor is multiplied by a given present amount, the equivalent series of end-of-period amounts is determined. This factor is sometimes referred to as the "Annuity Factor."

There are four basic methods of making comparisons in long-term situations using the above factors:

1. Comparison by Annual Cost
2. Comparison by Present Worth
3. Comparison by Rate of Return
4. Comparison by Service Life

Comparison by Annual Cost -- In making an economic comparison of alternatives it is necessary to consider three and only three factors: (1) Recovery of capital invested, (2) Annual interest on the unrecovered portion of capital, and (3) Incomes and disbursements.

Recovery of capital invested is for all practical purposes synonymous with depreciation. In this aspect of annual cost comparisons we are determining the amount of the investment that should be recovered each year of the life of the investment such that the total amount will be recovered at the end of the usable life of the asset.

Interest on the unrecovered portion of the capital as has been explained previously is a cost of operating and, as such, should be used in the comparison of alternatives since the capital investment for two alternate methods of accomplishing an objective may vary greatly and, hence, the interest chargeable to the alternatives would not be the same.

Combinations of these first two items of annual cost might be expressed as "Capital Recovery with a Return." It is interesting to note that capital recovery with a return is equivalent for all methods of depreciation. This is illustrated in tabular form in Tables 6, 7, and 8, using the example in the section on "Depreciation." The interest rate used in all three tables is 4% compounded annually.

Table 6. Equivalent Capital Recovery with a Return Using Straight Line Depreciation

Year No.	Capital Recovered	Interest on Unrecovered Capital	Total	Single Payment Present Worth Factor	Equivalent Present Worth Value
1	$80.00	$32.00	$112.00	.9615	$107.69
2	80.00	28.80	108.80	.9246	100.60
3	80.00	25.60	105.60	.8890	93.88
4	80.00	22.40	102.40	.8548	87.53
5	80.00	19.20	99.20	.8219	81.53
6	80.00	16.00	96.00	.7903	75.87
7	80.00	12.80	92.80	.7599	70.52
				Total Present Worth	$617.62

Table 7. Equivalent Capital Recovery with a Return Using Sinking Fund Depreciation

Year No.	Capital Recovered	Interest on Unrecovered Capital	Total	Single Payment Present Worth Factor	Equivalent Present Worth Value
1	$70.90	$32.00	$102.90	.9615	$ 98.94
2	73.74	29.16	102.90	.9246	95.14
3	76.69	26.21	102.90	.8890	91.48
4	79.75	23.15	102.90	.8548	86.96
5	82.95	19.95	102.90	.8219	84.57
6	86.26	16.64	102.90	.7903	81.32
7	89.71	13.19	102.90	.7599	78.19
				Total Present Worth	$617.60

Table 8. Equivalent Capital Recovery with a Return Using Fixed Percentage of Diminishing Balance Depreciation

Year No.	Capital Recovered	Interest on Unrecovered Capital	Total	Single Payment Present Worth Factor	Equivalent Present Worth Value
1	$126.40	$32.00	$158.40	.9615	$152.30
2	106.43	26.94	133.37	.9246	123.31
3	89.61	22.69	112.30	.8890	99.83
4	75.45	19.10	94.55	.8548	80.82
5	63.53	16.08	79.61	.8219	65.43
6	53.50	13.54	67.04	.7903	52.98
7	45.03	11.40	56.43	.7599	43.88
				Total Present Worth	$617.55

The preceeding comparison should not necessarily be interpreted to mean that the method of depreciation is unimportant. It may be relatively unimportant in determining the equivalent cost of an investment but it may be of major importance in determining unit costs as a basis for sales prices, in estimating time of replacement of equipment, or in estimating taxes to be paid.

An approximate method of computing capital recovery with a return is based on straight line depreciation plus average interest. This method can be utilized without interest tables and the error will not exceed 10% for a life less than 15 years and an interest rate less than 10%. Above these limits the error increases rapidly. The formula for this method using symbols previously designated is: $(P-L)\left[1/n + \left(\frac{n+1}{n}\right) i/2\right] + Li$.

The more practical and theoretically correct method is based on Sinking Fund Depreciation and interest on the unrecovered Capital. The equal annual cost is: (P-L) x Sinking Fund

Factor + P x Interest. This is equivalent to (P-L) x Capital Recovery Factor + L x Interest. Any of the three methods give annual values which are equal for each year of the assets life and whose combined value are equivalent to the total cost of using the asset.

The third item to be considered in comparing alternatives by the Annual Cost Method is the incomes and expenses occurring each year. The possible variation in these items and the fact that they do occur each year instead of at one time are what make the evaluation of such alternatives difficult. To illustrate the method of evaluation and comparison by Annual Cost, the following example will be considered.

Two plans are offered to furnish the same service. Plan 1 requires an investment of \$9,500 in automatic equipment which will have an estimated life of 5 years and can be sole for \$1500 at the end of the fifth year. Estimated labor and operating expenses will be \$8,300, \$9,400, \$9,200, \$10,300, and \$10,200 for the five years. Plan II will require an investment of only \$2,500 in equipment which will have an estimated life of 5 years and can be salvaged for \$300 at the end of the fifth year. Estimated labor and operating expenses will be \$9,700, \$10,600, \$10,800, \$11,700, and \$12,000. Assume an interest rate of 6%, compounded annually. All other items will be the same for the two plans. There are a number of methods for the solution of this problem, one of which is given below.

Annual Cost of Plan I:

(\$9500-\$1500) x Uniform Series Capital Recovery Factor (.23740) + \$1500 (.05) =	\$ 1,989.20

\$8,300 x Single Payment Present Worth Factor (.9434) x Uniform Series Capital Recovery Factor (.23740) =	\$ 1,858.89
\$9,400 x Single Payment Present Worth Factor (.8900) x Uniform Series Capital Recovery Factor (.23740) =	1,986.09
\$9,200 x Single Payment Present Worth Factor (.8396) x Uniform Series Capital Recovery Factor (.23740) =	1,833.75
\$10,300 x Single Payment Present Worth Factor (.7921) x Uniform Series Capital Recovery Factor (.23740) =	1,939.86
\$10,200 x Single Payment Present Worth Factor (.7473) x Uniform Series Capital Recovery Factor (.23740) =	1,809.57
Equivalent Annual Cost for 5 years	\$11,414.36

Annual Cost of Plan II:

(\$2500-\$300) x Uniform Series Capital Recovery Factor (.23740 + \$300 (.06) =	\$ 540.28
\$10,000 x Single Payment Present Worth Factor (.9434) x Uniform Series Capital Recovery Factor (.23740) =	2,239.63
\$10,900 x Single Payment Present Worth Factor (.8900) x Uniform Series Capital Recovery Factor (.23740) =	2,303.02
\$11,100 x Single Payment Present Worth Factor (.8396) x Uniform Series Capital Recovery Factor (.23740) =	2,212.46
\$12,000 x Single Payment Present Worth Factor (.7921) x Uniform Series Capital Recovery Factor (.23740) =	2,256.53
\$12,300 x Single Payment Present Worth Factor (.7473) x Uniform Series Capital Recovery Factor (.23740) =	2,182.13
Equivalent Annual Cost for 5 years	\$11,734.05

Plan I is equivalent to spending $11,414.36 each year for the 5 years while Plan II is equivalent to spending $11,734.05 each year for the 5 years; or, Plan I will result in an equivalent savings of $319.69 each year for five years. It must be remembered that the equivalent annual cost figures of $11,414.36 and $11,734.05 include recovery of the respective investments with 6% interest.

Comparison by Present Worth -- In present worth comparison of alternatives, all expenses and incomes that are estimated at times in the future are transposed to their equivalent values at the present. The values can then be combined to obtain a single equivalent present value for each alternative and comparisons are then merely the comparison of one composite value against another composite value.

Suppose it is desired to compare Plan I and Plan II outlined under "Comparison by Annual Cost" by the Present Worth Method.

Plan I

Year End No.	Amount of Expense	Single Payment Present Worth Factor	Equivalent Present Value
0	$ 9,500	1.0000	$ 9,500.00
1	8,300	.9434	7,830.22
2	9,400	.8900	8,366.00
3	9,200	.8396	7,724.32
4	10,300	.7921	8,158.63
5	10200-1500 = 8,700	.7473	6,501.51
	Total Equivalent Present Expense		$48,080.68

Plan II

Year End No.		Amount of Expense	Single Payment Present Worth Factor	Equivalent Present Value
0		$ 2,500	1.000	$ 2,500.00
1		10,000	.9434	9,434.00
2		10,900	.8900	9,701.00
3		11,100	.8396	9,319.56
4		12,000	.7921	9,505.20
5	12,300-300 =	12,000	.7473	8,967.60
	Total Equivalent Present Expense			$49,427.36

It can now be stated that Plan I is equivalent to making a present investment of $48,080.68, while Plan II is equivalent to making a present investment of $49,427.36. It is readily seen that Plan I is more economical.

The situation often arises in industry where a certain investment will save an annual expense. For example, automatic office equipment may be installed and operated by one operator to do the work formerly performed by three clerks. The equipment will cost $25,000 and the company selling the equipment will maintain it for $2,500 per year for the next 15 years, which is the estimated usable life of the equipment. The annual savings in the salaries of the two clerks will be $6,000. Therefore, a present investment of $25,000 will result in a net annual saving of $6,000-$2,500 or $3,500 each year for the next 15 years. Three thousand and five hundred dollars times the Uniform Series Preset Worth Factor (10.380) is $36,330, assuming 5% interest compounded annually. A $25,000 investment results in an equivalent present savings of $36,330; therefore, the decision to purchase the equipment would result in a net savings of \$36,330-\$25,000, or \$11,330 evaluated at the present. Such a single decision each year will, in many cases, pay the salary of the person making the decision.

A type of present worth comparison, often considered as a separate method of comparison, is "Capitalized Cost." Capitalized Cost is the present worth of expenditures necessary to perform a given operation for an infinite period of time. The above usage of capitalized cost is not universal. It is often used synonymously with Present Worth in the sense of, "What is the Capitalized Cost of the expenditures for a given operation that will last only 10 years?" For mutual understanding in economic discussions, the equivalent present worth of an operation for a limited time should be expressed as the "present worth of X years of operation." The term Capitalized Cost should be reserved for the present worth evaluation for an infinite period of time.

Capitalized cost is computed by determining the annual cost for a selected period of time and dividing this annual cost by the interest rate. This means then that the capitalized cost is an amount such that, if invested at the interest rate used, will produce sufficient annual interest to meet annual cost payments leaving the principal constant such that this principal will produce this amount of interest for an infinite number of years.

The assumption, when capitalized cost is utilized, is that investment costs, operation expenses, and incomes occurring during a selected period of years will be repeated during succeeding periods. The basis for such an assumption is not too sound and comparison by Capitalized Cost should be limited to situations where the life of investments is long and annual income and expense are comparatively stable.

Comparison by Rate of Return -- Although this method of comparison is utilized extensively at the present time, the true significance of its use is seldom realized. The method of comparison can be utilized to compare only two alternatives at a time. In most cases, one of these alternatives is the present method where the present method is understood to include inactivity within the area being studied. A rate of return comparison is merely determining the rate of interest at which an investment will "break even" or "pay for itself" in annual savings or income.

Let us consider a situation where it is desired to manufacture a certain article for sale. At the present time there is no activity in this area by the organization contemplating the venture. A detailed study of the situation reveals the following summary figures. An investment of $300,000 is required in equipment. For simplicity we will assume that all equipment has an estimated usable life of 8 years with a salvage value at that time of $50,000. Annual expenses will be $150,000 and contemplated annual income will be $200,000.

A common solution to this problem utilizes the straight line method of depreciation but neglects proper consideration of interest. This incorrect solution is performed as follows:

$200,000-$150,000 or $50,000 is net annual savings

$300,000-$50,000 ÷ 8 or $31,250 is annual depreciation

then $50,000-$31,250 or $18,750 is interest

$18,750 ÷ $300,000 or .0625 or 6.25% is the rate of interest the investment pays.

A correct solution can be performed only through trial and error utilizing the compound interest tables although a close approximation can be obtained through proper use of straight line depreciation plus average interest. These two methods are illustrated below:

A. Straight Line Depreciation plus Average Interest Method --

$$(P-L)\left[\frac{1}{n}+\left(\frac{n+1}{n}\right)\frac{i}{2}\right] + Li = \text{Annual Cost}$$

$$(\$300{,}000-\$50{,}000)\left[\frac{1}{8}+\left(\frac{8+1}{8}\right)\frac{i}{2}\right] + \$50{,}000i =$$

$200,000-$150,000   i = .0984 or 9.84%

B. Method using Interest Tables --

at 8% compounded annually:

($300,000-$50,000) x Uniform Series Capital Recovery Factor (.17401) + $50,000 (.04) = $47,502

at 10% compounded annually:

($300,000-$50,000) x Uniform Series Capital Recovery Factor (.18744) + 50,000 (.05) = $51,860

at the unknown interest rate, i, it is known that the annual return is $200,000-$150,000 or $50,000;

therefore, by interpolation,

$$i = 8\% - \frac{\$50,000-\$47,502}{\$51,860-\$47,502} \times 100 \quad \times 2 = 9.15\%$$

In addition to using incorrect methods of arriving at the rate of return, incomplete and often inaccurate interpretations are concluded from such a comparison. Although the statement that "9.15% is the rate of return on the $300,000 investment" is true in the above example, the interest rate secured when two alternatives, both requiring investments, are compared may not warrant this interpretation.

Consider Plan I and Plan II for the accomplishment of a manufacturing operation. Plan I requires an investment of $200,000 with annual income of $190,000 and annual expenses of $160,000. The investment will have a usable life of 7 years with a salvage value of $40,000 at that time. Plan II will require an investment of $400,000 with annual income of $190,000 and annual expense of $140,000. The investment will have a usable life of 15 years and a salvage value of $60,000 at that time.

Two such plans may be compared by the approximate method using straight line depreciation plus average interest or by the exact method using factors from interest tables and trial and error methods. Solution by the approximate method follows:

$$(\$200,000-\$40,000)\left[\frac{1}{7} + \left(\frac{8}{7}\right)\left(\frac{i}{2}\right)\right] + \$40,000\ i + \$160,000-$$

$$\$190,000 = (\$400,000-\$60,000)\left[\frac{1}{15} + \left(\frac{16}{15}\right)\left(\frac{i}{2}\right)\right]$$

$$+ \$60,000\ i + \$140,000-\$190,000$$

$$i = 18.3\%$$

This then means that 18.3% is the interest rate at which Plan I and Plan II are equivalent. Further, if a higher interest rate is desired for the life assumed the plan requiring the lesser investment, Plan I, is more economical; whereas, at a rate lower than 18.3% the plan requiring the higher investment is more economical. Yet, it is impossible to state in this case what actual rate would be paid by either of the two plans without a solution of each one separately against an alternative of no investment. Such a comparison reveals that Plan I would pay a rate of return at 5.43%, while Plan II would pay a rate of return at 6.47%. It should be apparent that 18.3% is not the rate of return for either Plan I or Plan II and that such an interpretation of the results of the above comparison could lead to very erroneous decisions.

Comparison by Service Life -- This method of comparison, like that of comparison by interest rate, is a solution for a life of an investment at which it will "break even" or "pay for itself."

Considering the situation used under interest rate comparisons where the investment required was $300,000, annual income is $200,000 and annual expense is $150,000. With interest at 8% compounded annually, it is desired to find the number of years required for the investment to pay for itself. An approximate solution can be accomplished through use of the formula for straight line depreciation plus average interest or a correct solution through use of trial and error methods using interest factors. No salvage value is considered in this case.

A. Straight Line Depreciation plus Average Interest Method --

$$\$300{,}000 \left[\frac{1}{n} + \left(\frac{n+1}{n}\right) \frac{.08}{2}\right] = \$50{,}000$$

$n = 8.21$ years

B. Method using Interest Tables --

at $n = 8$:

$300,000 x Uniform Series Capital Recovery Factor (.17401) = $52,203

at $n = 9$:

$300,000 x Uniform Series Capital Recovery Factor (.16008) = $48,024

It is known that at the unknown number of years the annual amount is $200,000-$150,000 or $50,000; therefore, by interpolation,

$$n = 8 - \frac{\$52{,}203-\$50{,}000}{\$52{,}203-\$48{,}024} \text{ or } 8.53 \text{ years}$$

It can then be stated that in 8.53 years the investment will pay for itself with 8% interest. Again in the case of a comparison of two alternatives where each requires an investment, the interpretation of results is slightly confusing. Consider again a Plan I requiring an investment of $200,000 with annual income of $190,000 and annual expense of $160,000 and a Plan II requiring an investment of $400,000 with annual income of $190,000 and annual expense of $140,000. With interest at 8% compounded annually, the two plans may be equated using the straight line depreciation plus average interest method and solving for n. The solution reveals n equal to 17.67 years. The life of the investments at which they are equivalent then is 17.67 years. Then, if the lives of the investments are expected to be greater than 17.67 years, the plan requiring the larger investment, Plan II, is more economical; and also, if the lives of the investments are expected to be less than 17.67 years, the plan requiring the smaller investment, Plan I, is the more economical.

However, if the lives are expected to be less than 17.67 years there is little indication as to whether either investment would be economically sound and further comparison is required. Also, if the expected life of the investment for Plan I is different from the life of the investment for Plan II, the use of this method of comparison is not desirable especially if the expected life of either or both of the investments is near the break-even figure.

An incomplete and somewhat erroneous understanding of calculations of payout periods often causes organizations to adopt a policy that "an investment must pay for itself in 3 years" or it is not approved. Such a policy has only one merit and that is that practically all organization funds are invested in short-lived assets and there is reasonable assurance that any invested capital will be recovered within a maximum of 3 years. However, an understanding of the true significance of such a policy reveals that it may be disastrous in competitive industry. Such a policy places an automobile with an expected usable life of 3 years on the same basis as a large turbine with an expected usable life of 25 years. Under the policy stated above, the turbine could pay a rate of return 4 or even 5 times greater than the automobile and yet not be purchased because it will not pay for itself in 3 years. A graphic illustration of this concept is presented in Fig. 2.

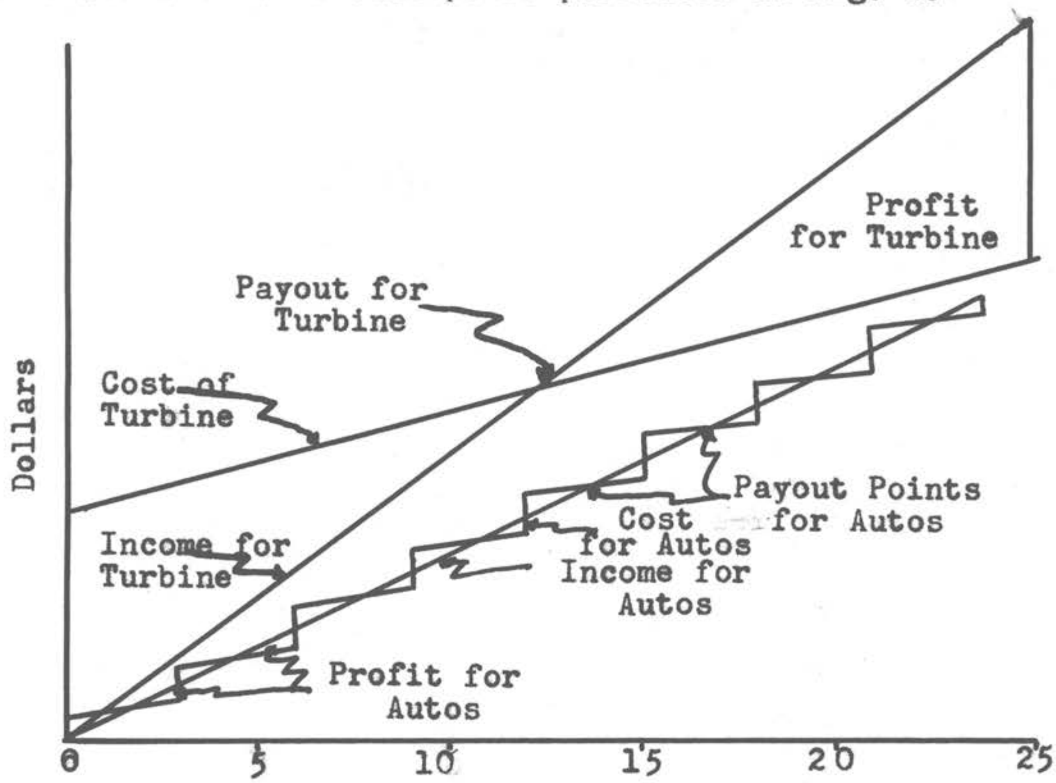

Fig. 2. Contrast of Potential Profit and Payout Periods

From Fig. 2, it can be seen that the requirement of a short payout period for all investments regardless of expected life may result in the sacrifice of a potentially large profit in an asset with a long life even though the payout period for the asset with the long life may be 3 or 4 times the payout period for the asset with the short life.

Summary of Methods of Comparison -- Economic comparison of alternate methods of accomplishing desired objectives may be made correctly in a number of methods. However, many of these methods have limited application and are often erroneously interpreted by those utilizing them. It therefore seems sound that an organization should select one method of comparison that is universally applicable and utilize this method to the exclusion of all others. Such a policy would lead to a better understanding of economic comparisons by all those within the organization and would do much to eliminate confusion in joint discussions of such comparisons. In many such discussions it is not unusual for one individual to think entirely in terms of "what rate of return will the investment pay" while another thinks only of "how soon will it pay for itself" and still a third asks "well, how much does that mean we have to spend each year." All these attitudes are related in that they mean to ask "is this a paying venture" and then in their own minds they ask "will this then pay better than some alternate venture."

Individuality of thought may be highly desirable but, when such thinking does not reach a mutual result with the other individuals, the result can be confusion and delay of action, or possibly inaction where prompt action is desirable.

The method of comparison by Annual Cost is applicable to any situation requiring economic comparison of alternate plans. At the same time, this method offers less opportunity for confusion in the interpretation of results. Most executives are already trained to think in terms of annual costs since most organizations prepare their Profit and Loss Statement, Balance Sheet, Tax Reports, and other financial reports on an annual basis.

The organization should determine what rate of interest offers them a fair return on investments. Comparisons made by the Annual Cost Method would indicate that a certain investment could be recovered with a fair return and, in addition, will add a certain sum to the Profit and Loss Statement. Such an interpretation should prove valuable in making sound decisions concerning proposed investments.

Sunk Cost

An investment is made with the intent that the total amount will be recovered through sale of goods or services resulting from use of the asset which is purchased by the investment or through sale of the asset itself. A certain return above recovery of the investment is usually anticipated. All situations do not result in recovery of the investment and at this point a full understanding of sunk cost is essential. For simplicity, let us assume that the following situation exists. An 80-mile pipeline has been laid between a source of crude oil supply and a refinery at a cost of \$700,000. One week after the pipeline is completed the question arises as to whether or not it is economical to use the pipeline. The pipeline cannot be sold in place and any value that might be received for the pipe would be less than the cost of removing it from the ground. The \$700,000 investment at this point is a "sunk cost." That is, it cannot be recovered and, hence, so long as operating incomes exceed operating expenses the pipeline can be economically operated even though the $700,000 will not be recovered. It is often difficult to understand why investment costs in such a situation should not be considered as much an expense as they would be at the time the installation is contemplated. The following comparisons are made using 6% interest compounded annually with a life of 50 years estimated for the pipeline with annual cost computed by straight line depreciation plus average interest. Annual income is estimated at $200,000 and annual operating expenses at $195,000.

A. Considering Investment Costs:

Investment Costs		
$\$700{,}000 \; \frac{1}{20} - \left(\frac{21}{20}\right)\left(\frac{.06}{2}\right)$		$ 57,050
Operating Costs		195,000
	Total Cost	$252.050
Less Income		200,000
	Net Loss	$ 52,050

B. Not Considering Investment Costs:

Income		$200,000
Less Operating Expense		195,000
	Net Gain	$ 5,000

From Comparison A, a $52,050 loss annually would possibly indicate the proper action; and yet, since there is no value to be received from the pipeline if it is not used, it is apparent from Comparison B that the amount of $5,000 annually would be sacrificed if operation of the pipeline were abandoned.

The failure to recognize sunk cost is a failure to admit that an erroneous decision was made previously. In the situation of the pipeline, the $700,000 must be accepted as a loss and a result of the decision to install the pipeline which was erroneous because of errors in evaluation or unforeseen changes in conditions, and should not continue to be charged against future operations.

Another aspect of sunk cost is the utilization of "book value" for present equipment when contemplating replacement. Consider that Machine A was purchased 2 years ago for $3,000 and depreciation at that time was estimated at $300 per year. Today, Machine B can be purchased for $2,400 and will result in a savings of $200 annually in operating costs. The first inclination is to simply state that the value of Machine A is $3,000-(2 x $300) or $2,400 and, since this is exactly the cost of Machine B, the installation of Machine B will result in a savings of $300 annually. However, on closer examination, it is found that the maximum that can be received for Machine A is $400. The difference between the "book value" of $2,400 and the actual sale value of $400 is $2,000 and this $2,000 is a sunk cost not chargeable against future operations. The comparison is then between a $400 machine and a $2,400 machine which will save $300 annually. Using a life of 4 years for Machine A and 6 years for Machine B with interest at 6% compounded annually, the following comparison would result.

Salvage values are assumed to be \$0.

Comparative Annual Costs:

Annual Cost Machine A --

$$\$400 \left[\frac{1}{4} + \left(\frac{5}{4}\right) \left(\frac{.06}{2}\right)\right] = \qquad \$115$$

Annual Cost Machine B --

$$\$2,400 \left[\frac{1}{6} + \left(\frac{7}{6}\right) \left(\frac{.06}{2}\right)\right] - \$300 = \quad \$184$$

When sunk cost is considered the advantage is now with the old machine. The sunk cost can only be credited to the past decision which was to depreciate Machine A at the rate of \$300 a year when it should have been \$1,300 a year.

Increment Costs

Increment cost is defined as the additional cost of making additional units of product above the basic production. Increment cost is of considerable economic importance in activities of producing items for sale. The competitive nature of industry demands the products be sold at or below the price charged by competitors and yet it is desired to obtain as large a margin of profit as possible.

Let us consider a situation where the economic importance of increment cost is apparent. An organization is producing children's tricycles. They produce an average of 400 tricycles a month which is 80% of their capacity. It is anticipated that the sale of 400 tricycles each month will continue at the present selling price of \$20 each, although no increase in sales is contemplated. Total monthly costs for the 400 tricycles are as follows.

Material	$ 800.00
Labor	2,600.00
Costs of equipment and space usage	2,000.00
Administrative and sales	1,200.00
Total	$6,600.00

A salesman reports that he has the possibility of a large order of 100 tricycles to be delivered in 30 days and wishes to know how cheaply he can sell them and still maintain a fair profit for his organization The costs inherent in producing an additional 100 tricycles during the next month are as follows:

Material	$ 200.00
Labor	500.00
Equipment and space usage	200.00
Administrative and sales	100.00
Total	$1,000.00

The following analysis is presented:

Present production --

Average cost per unit	$6,600 ÷ 400	$16.50
Average profit per unit	$20-$16.50	3.50

Proposed production --

Average Cost per unit	($6,600 + $1000) ÷ 500	$15.20
Increment unit cost	$1,000 ÷ 100	10.00

On the assumption that the present profit of $3.50 per unit is a fair profit, the sales price would be $15.20 + $3.50 or $18.70 is the concept of increment cost is ignored. However, since the actual unit cost is only $10, the sales price could be $10 + $3.50, or $13.50. Any sales price in excess of $13.50

would result in a greater unit profit than now being received and should, therefore, be of financial advantage to the organization.

It is necessary that the idea of increment costs be used with care. For example, had an additional order for 100 tricycles been possible, the increment unit cost of $10 could not be assumed to apply to it because this order would increase the demand for product above present production capacity and might entail investments that could be utilized for only one month. Pricing on the basis of increment costs should be reserved to those situations where analysis indicates that such a policy will result in a net gain to the organization.

Break-Even and Minimum Cost Points

The break-even point is defined as that value of a variable factor at which two situations are equal. This concept is important in determining the length of time, the interest rate, the extent of use, or value of other factors that will make two alternate methods of accomplishing an objective equal. It is also important in determining the value of such variables at which income is equal to expense for a given operation.

The method of determining the break-even value of the number of years or interest rate in comparing two long-term alternatives has been discussed previously. Examples will be given to illustrate other usages of this concept.

A part used in a product manufactured by an organization is made entirely in a machine operation where the operator is paid \$2.00 per hour. The material costs per unit produced is \$.30. Machine costs, supervisory, and other costs comprising the total cost of the unit total \$4.00 per hour. The part can be purchased for \$1.80. How many parts must be produced per hour by the operator to break-even with the alternate of purchasing the parts?

$(\$2 + \$4) \div n + \$.30 = \$1.80 \quad n = 4$

The indication would be that so long as the operator produces an average of 4 or more units per hour the organization will continue to produce the part itself. If the production is less than 4 per hour, the parts should be purchased.

An example of utilization of the point at which expenses break-even with incomes might be as follows. A certain operation in petroleum refining costs \$400 per hour regardless of the amount of product processed, assuming that the unit is not shut down. This cost represents labor, investment costs, and all other costs inherent in operating the unit which are not variable with the amount of product processed. In addition, there is a variable cost of \$.07 per barrel of product processed. The value of the product before this operation is \$3.12 per barrel and after the operation the value is \$4.70 per barrel. How many barrels must be processed on the average each hour for the unit to break even?

$\$400 + \$.07\ n = (\$4.70 - \$3.12)\ n$

$n = 265$ barrels per hour

Then, when an average of less than 265 barrels per hour are processed, the unit is operating at a loss. Above this rate of operation, a profit is realized. A graphic solution of break-even problems is often ehlpful. The graphic solution not only gives the break-even point but indicates the relationship between the two alternate methods for values of the variable above and below the break-even value.

A contractor can purchase a dump truck for \$3,000. He estimates that the truck can be used for 4 years and will have a sale value of \$600 at that time. It will cost him \$12 per day for a driver and operating expenses on those days the truck is operated. He can rent a dump truck any time he needs it for \$20 per day where the driver and all expenses are paid by the party from whom the truck is rented. If interest is assumed at 6% compounded annually, how many days must the services of a dump truck be required to justify the purchase of a truck? The break-even point is

(\$3,000-\$600) x Uniform Series Capital Recovery Factor

(.28859) + \$600 (.06) + \$12 n = \$18 n   n = 122 days

A graphic solution is obtained by plotting the annual cost for various values of n for both the purchase of a truck and the rental of a truck. The graphic solution gives the break-even point of 122 days and, in addition, shows the relationship of the two alternates on both sides of this point.

Minimum cost point is defined as that value of a variable factor that will result in a minimum cost for an operation. This concept can also be utilized in another sense; that is, the value of the variable that will result in maximum profit.

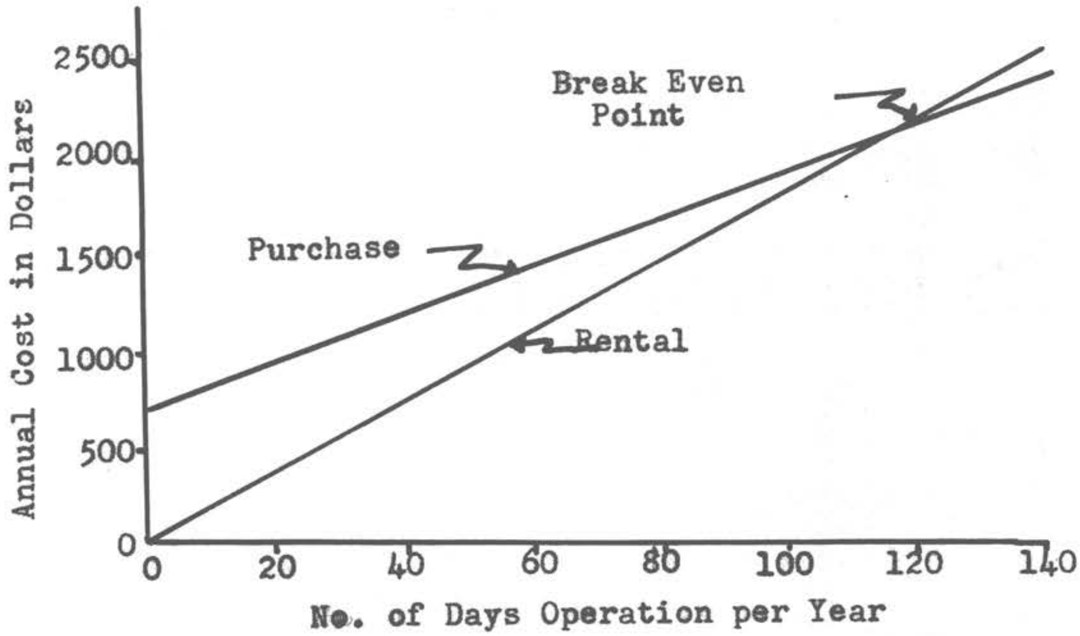

Fig. 3. Graphic Solution of Break-Even Point

Minimum cost determination is often utilized in determining the cost of maintaining inventories of materials for use in an activity. Formulas have been developed for this purpose whereby values are merely substituted into the formula and the number of lots of material to order a year for a known annual consumption is obtained. Derivation of the formula is somewhat long and will be merely stated here for illustrative purposes. The formula is

$$X = \sqrt{\frac{NCI - 2NS}{2L}}$$

where

X = the number of lots per year
N = the total units of material used per year
C = purchase price per unit
I = Investment costs in per cent per year
S = Storage cost per unit per year
L = Cost per lot of placing and receiving an order

With the above definitions, assume that N is 7,000 pounds of a phenolic plastic, C is \$.13, I is 12%, S is \$.07, and L is \$5, then the number of lots to order is

$$X = \sqrt{\frac{(7000)\ (\$.13)\ (.12) - 2(7000)\ (\$.07)}{2(\$5)}} = 104\text{- or } 104 \text{ lots}$$

and of course the number of pounds per lot would be

$$\frac{N}{X} = \frac{7000}{104} = 67.3 \text{ pounds}$$

The minimum cost can also be found by solving for the annual cost for various values of X and seeing which value results in the minimum cost. Letting Y be the annual cost, the solution would be obtained by substituting values for X in the formula

$$Y = NC - XL - \frac{NCI}{ZX} - \frac{NS}{X}$$

As X is increased, values obtained for Y will become progressively less until the minimum point is reached and will then increase.

This concept is used to obtain a minimum cost in situations where income from the activity is constant. When income varies with the degree of an activity the concept is utilized to obtain a maximum profit. This idea is applicable in determining the extent to which it will prove most economical to indulge in an activity.

An organization contemplating the building of an office building would desire such information with relation to the number of stories high to construct the building. Also, an organization contemplating a processing plant would desire information with regard to the capacity in units of product.

Accounting

In this discussion, accounting is interpreted to include not only the formal accounts of the organization, but also such information as salary records, performance and rating records of employees, production records that might be maintained by subdivisions of the organization, and cost and time records for operation and maintenance. Also included would be sales records recorded by both quantities and price. In general, it could be stated that all numerical informational data recorded and retained by the organization is considered as accounting for purposes of this discussion.

Although there are legal and other reasons for various accounting procedures, there are two utilizations of these records and procedures for purposes of decision making.

1. Accounting records contain numerical information of past performance which can be utilized when properly interpreted as a basis for decisions.
2. Accounting records serve as a recording device through which the success or failure of past decisions is recorded.

It is essential that the nature of accounting records be fully understood so that proper utilization of these records can be made in decision making.

It should first be pointed out that accounting records are numerical evidence of <u>past</u> actions. Decisions, on the other hand, are a statement of intended <u>future</u> actions. Because of this seemingly small but very important difference, a proper understanding and interpretation of accounting records must be made if they are to be of any value in making decisions.

Personnel concerned primarily with production or operational activities of an organization are often wholly ignorant of the type of records maintained by the accounting and clerical staff of the organization. As a result, many foremen and supervisors either make an attempt to keep their own records which are usually inadequate and incomplete, or they base decisions on their memory of past performances.

Many times the numerical data maintained can serve as a beginning such that, when re-evaluation of certain variables is made, numerical basis for an important decision is obtained. In addition, utilization of such records by personnel required to make decisions will soon result in that data being compiled which will be valuable in forming decisions concerning future activities. Quite often clerical staffs continue compiling and filing data that has lost its utility and fail to compile data that would be of immense value to supervisory and managerial personnel. This is usually due to either a distrust of the accounting staff or procedures, or an ignorance of the function of the accounting and clerical staff. Either reason is inexcusable when it is considered that success in organizational activities can be obtained much better through mutual faith and cooperation.

Utilization of Production and Operational Records

These records shall be defined to include all numerical data recorded and retained which are not a part of the formal accounts of the organization. These records, then, supplement the formal accounts and should be compiled and retained only so long as their utilization value exceeds the cost of compiling and retaining them.

In one instance, a production process utilized 7 similar machines, all producing the same product. Records had been compiled and retained for the process showing daily production, labor cost, material cost, maintenance and repair costs, and miscellaneous costs allotable to the process. Five years previously the clerk retaining these records was instructed to keep the same information by individual machines as well as for the entire process. This required additional time, not only for the clerk but for the foreman supervising the process and the maintenance and repair foreman. Upon questioning the clerk who had maintained these records for the entire five years, he stated that no one, to his knowledge, had ever requested any of the information contained in the records showing the machine break-down, although the production and cost data for the process as a whole had been utilized on several occasions.

The decision to compile the records for individual machines may have been sound even though their actual utilization would indicate otherwise. These records could serve as a partial basis for decisions in the following situations:

1. The more productive machines in the event only a portion of the 7 machines were to be operated.
2. The most productive operators, providing it were known which operator worked on a given machine, to be used as a basis for layoff or transfer.

3. Type of machine to purchase for a replacement, since the 7 machines were not the same type.
4. Rating of operators.
5. Determination of efficiency of maintenance and repair work

The records might also be valuable in evaluating decisions made during the past five years with respect to operational changes. Although such information can in no way alter the past decision, it may be helpful in an instructional sense in the making of better decisions.

Information contained in records of past salaries and ratings for employees might serve as a basis for a quantitative approach to decisions concerning promotions, discharges, wage increases, the position of the organization concerning labor grievances or any other possible decisions concerning the employee and his relation to the organization.

A special effort should be made to have those records that will prove valuable in future decisions. This requires that persons required to make decisions become familiar with records now kept that are related to their area of activity and further, that they make every effort to cooperate with the accounting staff in discontinuing old records and originating new records as operating conditions require them.

Formal Accounts

The formal accounts of an organization are usually established to accomplish a dual purpose: (1) To furnish information as a basis for payment of taxes and for a numerical picture of the organization's activities to insure compliance with legal and governmental requirements, and (2) To furnish information to the management of the organization concerning cost with respect to the organization as a whole and also with respect to designated subdivisions. In this sense, all accounting could be considered under the heading of "cost accounting" although cost accounting is usually understood to be a specialized form of accounting. The only difference between general accounting and cost accounting in their usual usage is that in cost accounting costs are recorded for various subdivisions rather than the organization as a whole in an effort to furnish separate cost data for various products.

Since cost information is, in most situations, a major consideration, an understanding of the cost information utilized in cost accounting is necessary if the information is to be utilized satisfactorily.

Costs are usually classified under the following headings:

1. Direct Labor
2. Direct Material
3. Indirect Labor
4. Indirect Material
5. Administrative Expense
6. Sales Expense
7. Miscellaneous Expense

Direct labor is that labor which can be directly and readily identified with a particular job or process. As such, it can be accurately charged to the job or process account desired.

Direct material can also be readily and directly identified with a particular job or process. This is easily done if adequate order, receipt, and storeroom records are maintained.

Indirect labor is that labor performed by service or maintenance and repair departments. Some organizations require that such departments show a break-down of each employee's time with reference to jobs or processes such that it can be handled in much the same way as direct labor. However, if this is not done it becomes necessary to take the total cost of maintenance, for example, and proportion it to the various jobs or processes. If this situation exists, the amount of indirect labor charged to each job or process will not necesarily be correct. Often such costs are distributed on the basis of direct labor costs which would probably overcharge a process accomplished mostly with hand labor and very little machinery, while it would undercharge the process using automatic equipment and little labor. Although the basis for the distribution of indirect costs is usually equitable, it is a matter that should be questioned if the cost information is to be utilized as a basis for decisions.

Indirect material is material utilized by service, repair, or maintenance groups. It is also charged to particular jobs of processes as it is used in some organizations, while in others it is considered as a total and proportioned to the jobs or processes in proportion to direct material or other basis.

Administrative expense is the expense of all administrative employees that cannot be charged to a particular job or operation. For example, the salary of the president of an organization is a cost of getting the goods or services to the consumer and, as such, must be charged to them to obtain correct cost data. Again this item must be proportioned to the various jobs and processes on whatever basis seems most equitable for the particular activity.

Sales expense is all advertising, freight, salaries, and any other expense inherent in getting the product into consumers' hands after production is completed. This expense is very similar to administrative expense from the standpoint of its being a cost item and the method it must be charged to various jobs or processes.

Miscellaneous expense usually includes such expenses as rental on building, telephone expense, insurance, utilities, depreciation, and any other expenses incidental to the production of the goods or services. In some instances, expenses such as electric power, water, or telephone may be capable of being charged directly to a job or process. In such cases, these expenses would be handled as direct expenses much the same as direct labor. In many cases, however, the expense is chargeable to the organization as a whole and must be proportioned to various jobs or processes for cost purposes.

Special attention should be directed to depreciation expense. It is one item that is usually charged to jobs and processes before its true value is known. Depreciation expense is only an estimate until the asset is disposed of or abandoned. At that time it is usually too late to charge true depreciation as a cost of production. This is another factor that should always be considered when accounting records are utilized as a basis for decision.

It seems indicative that even though it is realized that some of the miscellaneous costs would be difficult or perhaps impossible to allot exactly to the proper job or process, accounting records are never maintained in vague terms utilized in some fields of knowledge as discussed in Chapter I. It is never expressed that "Job A used more water that Job B." The water charges for both Job A and Job B are determined as nearly as it is possible to do and are then recorded and utilized as numerical values.

Statistics

The general conception of statistics ordinarily includes the collection of masses of data and presentation of such data in tables or charts. It may also include the calculation of totals, averages, and certain percentages. This conception of statistics is at least 30 years out of date, because such minor and routine calculations and presentations are a very incidental part of the theory and application of statistics.

Statistics as a specialized area in the field of mathematics has been developed mostly in recent years to furnish a quantitative method of analyzing research and experimental data and also includes analysis of data obtained through specified sampling procedures. The statistical analysis furnishes positive methods of obtaining and expressing the amount of uncertainty that is connected with conclusions drawn from experimental data.

The scope of experimentation is quite broad. An experiment may simply be the evaluation of a constant such as the exact weight of a molecule of some element, or it may be the evaluation of a complex relationship of variables. In organizational activities, experimentation is usually for one of the following purposes:

1. The testing of an hypothesis that two factors are unrelated. This situation exists in practically all research where the objective is to determine the effect of certain treatment upon the characteristics of some unit. It might be to test to see if certain types of advertising affect public opinion or sales; to see if a certain company policy affects employee morale adversely; to determine whether a certain change in a process increases the tensile strength of metal; or to determine whether the addition of certain operating practices affect employee behavior or whether they affect product quality.
2. Determination of whether or not a given process, where there is an amount of variation, is remaining within specified operating limits. This application of statistics can be utilized in many situations where repetitive methods of production exist. Examples of this application would be to determine whether the paint on automobiles has the desired characteristics; whether the diameter of a part

being produced on an automatic lathe can be expected to be within specified tolerances; or whether a process of blending two constituents to produce a product is producing the blend consistently within a desired range of proportion of constituents.

The Uncertainty of Experimental Data

It is apparent that statistics, as a means of analysis, is applied to those situations where the value or effect of some factor cannot be determined without there being a possibility of error--there is a degree of uncertainty surrounding the results of any experiment. It stands to reason that, if each shaft coming off the production line is to be measured, statistical methods of determining the percentage of shafts expected to be within certain limits would be of very little value. However, statistics might well be applied to the problem of determining the relationship between the frequency of sharpening the cutting tool and the number of shafts rejected because of not meeting specifications.

In an attempt to clarify the area in which statistics may be applied, let us state that when it is not desired, for physical, financial, or other reason, to test each item for a given characteristic or reaction, and it is desired to know the characteristic or reaction, a sample of items may be selected and the characteristic of each item in the sample is determined. This group of characteristics is then analyzed statistically. The statement of conclusions will then be stated somewhat as follows: "Any item selected can be expected to have a certain characteristic unless the sample used was one which would occur only 3 times in 100," or "the treatment being tested does affect the outcome unless the sample was one which would occur only 5 times in 100.

When only a portion of a large group of items are measured for a certain characteristic, there is always a possibility that the portion selected for measurement is not truly representative of the entire group. It is very important to know the probability that a non-representative sample was selected because this is, in a sense, the probability that conclusions drawn from the sample will be in error.

Phases of Statistics to be Discussed

The statistical techniques and methods of application are quite extensive in number. The application of statistical analysis is a specialized field and should be performed by a specialist. However, there are certain basic concepts in statistics which should be understood by any member of the management group of an organization whose duties require him to plan or supervise research, experimental, or any other type of repetitive operation.

Three areas of statistical theory and application will be discussed in an attempt to achieve a basic understanding of statistics and its application. However, no attempt will be made to discuss actual methods and techniques of application.

The entire basis for the development of statistics as a means of quantitative analysis originated with the theory of probability. The result of a statistical analysis is the statement of the probability that conclusions drawn from the experiment will be correct. For these reasons the theory of probability will be discussed in an effort to gain familiarity with probability to promote an understanding of the basis of statistics and the meaning of statements of the results of statistical analysis.

A second phase of statistics to be discussed is distribution functions and their characteristics and application. This area of the theory of statistics should prove helpful in forming a general understanding of statistical applications and significance of statistical analysis.

The third area of statistics to be discussed is its application to the control of quality in repetitive production processes. As a means of furnishing means of exercising control readily to a production process, it is felt that statistical quality control will find increasing applications in the near future.

The Theory of Probability

Probability is a measure of the likelihood of the occurrence of a chance event. Words such as "probable," "probably", "likely", and "chance" are used in discussions and there seldom seems to be any difficulty of understanding their meaning. And yet, this may be due to the fact that they have little or no meaning of value; that the understanding is of what is said rather than what is meant. Probability on the other hand is a quantitative statement having no vague range of meaning that will vary with individuals.

Consider the statement that "a 15/16 inch cable is not likely to break under the anticipated load of 400 pounds." This statement has very little real meaning since the individual making the statement may mean by "not likely" that there is only one chance in four, or that there is only one chance in ten thousand, that the cable will break. On the other hand, the statement that "the probability that the cable will break under a maximum load of 400 pounds is .995" is very exact and definite. Going one step further, if it is known that the breaking of the cable will result in a cost of $10,000, the probable cost of using the 15/16 inch cable under a load of 400 pounds can be computed.

Definition of Probability -- The following statement is known as the classical definition of probability:

> If an event can happen in "a" ways and fail to happen in "b" ways, and all these ways are mutually exclusive and equally likely to happen, the probability of the event happening is given by $a/(a + b)$ where $a + b$ are all possible happenings.

Consider as an illustrative example that a single card is drawn from an ordinary deck of playing cards. The probability that this is any specified card, say the four of clubs, is 1/52. Likewise, the probability that the card is of a specified suit, say spades, is 13/52. The probability that the card is between four and seven inclusive is 16/52.

Careful attention should be given the terms "mutually exclusive" and "equally likely" in the definition of probability. Suppose that, in the above example, one desired to determine whether the card were a three or a spade. One might reason that there are four threes and 13 spades and conclude the probability was 17/52. However, occurrence of a three does not necessarily preclude the occurrence of a spade simultaneously. It now becomes apparent that there are only 16 mutually exclusive successful occurrences and the probability is 16.52.

Equal likelihood of occurrence is also essential. Suppose one wished to compute the probability of getting two heads if a coin were tossed twice. One might reason that there are three possible occurrences: two heads, two tails, or one head and one tail. One of these occurrences gives the desired result and, hence, the probability would be 1/3. This reasoning is faulty in that the three occurrences are not equally likely. There are two ways in which one tail and one head can occur and, hence, there are four possible occurrences: HH, HT, TT, TH. The correct probability is therefore 1/4.

Theorems of the Theory of Probability -- There are three important theorems that must be understood for the application of the theory of probability to practical situations:

1. The theorem of total probability
2. The theorem of compound probability
3. The theorem of conditional probability

The theorem of total probability is stated as follows:

> The probability of the occurrence of either one or another of any number of mutually exclusive events is the sum of the probabilities for the separate events.

This may be illustrated by letting A correspond to drawing a spade from an ordinary deck of playing cards and B correspond to drawing a heart. Then, if a single card is drawn, the probability that it is either a spade or a heart is

P(A) + P(B) or 13/52 + 13/52 or 26/52.

The theorem of compound probability is stated as follows:

> If a compound event is made up of a number of separate and independent subevents, and the occurrence of the compound event be the result of <u>each</u> of the subevents happening, the probability of occurrence of the compound event is the product of the probabilities that each of the subevents will happen.

This may be illustrated by letting A correspond to drawing a spade from an ordinary deck of playing cards and B correspond to drawing a heart. Then the probability that of two cards drawn from the deck, one will be a spade and one a heart will be P(A) x P(B) or 13/52 x 13/52 or 1/16. Notice that no order is specified.

The theorem of conditional probability is stated as follows:

> The probability that both of two dependent events will occur is the probability of the first multiplied by the probability that if the first has occurred, the second will also happen.

This is illustrated by letting A correspond to drawing a spade from an ordinary deck of playing cards and B correspond to drawing a heart. Then the probability of drawing two cards from the deck such that the first is a spade and the second a heart would be P(A) x P(B/A) or 13/52 x 13/51 or 13/204. The symbol P(B/A) is read "the probability of B knowing A has occurred."

Permutations and Combinations -- Evaluation of probability can be somewhat difficult when the number of possible occurrences is not obtained by simply knowing the number of items in a group. It is often required to know the number of orders in which n things can be combined. For example, how many ways can three items, designated by a, b, and c, be arranged. Since there are to be three items in each arrangement, it can be stated that there are 3 ways to choose the first item. However, after the first item is selected, there are only two ways to select a second item and then, after two are selected, only one item remains for the third position. The three items can then be arranged in $3 \times 2 \times 1 = 6$ different arrangements. This can be seen by writing all the possible arrangements:

abc, acb, bac, bca, cab, cba

The solution is of the form $n \times (n-1) \times (n-2) \times \text{----} \times n-(n-1)$ and is written as n! or "n factorial." The different arrangements are known as permutations and symbolically would be written

$$P(n,n) = n!$$

which is the permutation of n things taken n at a time equals n factorial. For example,

$$P(6,6) = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

It is apparent that determination of the arrangements possible in a large group of items would be difficult to evaluate. For this reason, most books of mathematical tables contain tables of the value of factorial numbers.

In continuing the idea of permutations it is often desired to know the arrangements possible with n items selecting r at a time. The number of such permutations is:

$$P(n,r) = \frac{n!}{(n-r)!}$$

For example, if there are 10 items in a particular group, the different arrangements possible in a sample of 3 items would be:

$$P(10,3) = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 720$$

It should be noted that, in the discussion of permutations, order within arrangements were considered as separate events. That is, an arrangement abc was considered as being different from bac because, even though the items are the same, the order is different. In many cases such an occurrence would not be considered as two separate happenings; the occurrence of the three items together being important but the order in which they occur being of no significance. Since, as explained previously, the number of different arrangements or permutations of r things is r!, then the number of arrangements of n items taken r at a time where at least one of the r items is different each time would be the number of permutations divided by r! or

$$\frac{n!}{(n-r)!\,r!}$$

This is known as the <u>combination</u> of n items taken r at a time and is written $C(n,r)$.

Application of Combinatorial Formulas to Probability Problems -- Consider a group of 50 items of which 4 are defective. The sampling acceptance procedure is to inspect 5 items of each lot of 50. It is desired to determine the probability that a sample of 5 items will have exactly one defective item. In review, the probability will be the ratio of the possibilities of this occurring to the total possibilities of drawing a sample of 5 items from a group of 50 items. Since the one defective item will be from a group of 4 defectives, it can be selected in $C(4,1)$ ways. Likewise, the 4 non-defective items are selected from a group of 46 items and can be selected in $C(46,4)$ ways. Thus the numerator of the fraction will be $C(4,1)$ times $C(46,4)$. From the combinatorial formulas the total number of ways different samples of 5 can be selected from a group of 50 items is C(50,5) and this becomes the denominator of the fraction. (Notice that order is not of importance in this example. Had the probability of drawing one defective item and then 4 non-defective items in that order, P(50,5) would have been the denominator). The probability of exactly one defective in a sample of 5 items is:

$$P(1) = \frac{C(4,1)\ C(46,4)}{C(50,5)} = .30808$$

Likewise

$$P(0) = \frac{C(4,0)\ C(46,5)}{C(50,5)} = .64696$$

$$P(2) = \frac{C(4,2)\ C(46,3)}{C(50,5)} = .04299$$

$$P(3) = \frac{C(4,3)\ C(46,2)}{C(50,5)} = .00195$$

$$P(4) = \frac{C(4,4)\ C(46,1)}{C(50,5)} = .00002$$

Total of all probabilities is:

$$P(0) - P(1) - P(3) - P(4) = 1$$

$$.64696 - .30808 - .04299 - .00195 - .00002 = 1.00000$$

The Binomial Expansion and Its Relation to Probability -- Let p be the probability that an event will occur at a single trial, and let $q = 1-p$ denote the probability that it will fail to occur. If the event occurs at a given trial, it will be called a success, otherwise a failure. Let n independent trials be made, and let x denote the number of successes occurring in the n trials. Then consider the problem of determining the probability of obtaining precisely x successes in n trials.

First, determine the probability of obtaining x successes followed by n-x failures. The events are independent, therefore the theorem of compound probability can be applied. The probability would be p x p x p . . . for x terms, times q x q x q . . . for n-x terms or $p^x q^{n-x}$. This is the probability for any one specific arrangements of successes and failures but there are many such arrangements. Referring to the discussion on combinatorial formulas, it will be recalled that n things combined x at a time will furnish $\frac{n!}{x!(n-x)!}$ combinations. It now follows that the total probability of x successes and n-x failures is the total of the probabilities for all arrangements or

$$P(x) = \frac{n!}{x!(n-x)!} \quad p^x q^{n-x}$$

This function is referred to as the binomial or Bernoulli distribution function. The name binomial comes from the relationship to the binomial expression $(a + b)^n$. In the expansion, it will be found that a series of terms containing a and b are obtained as follows: $a^n + n\, a^{n-1}\, b + \frac{n(n-1)}{2}\, a^{n-2}\, b^2$, etc. The general term of this expansion is given by

$$\frac{n!}{x!\,(n-x)!} \quad b^x a^{n-x}$$

where x has values from 0 to n. Notice that the function obtained to express the probability of x successes in n trials is identical in form to the general term of the binomial expansion. It follows then that, knowing p and q, the probability of any designated number of successes in a given number of trials can be obtained by substituting the values for p, q, x, and n into the formula. Also, the sum of the probabilities for several values of x may be obtained by adding the values obtained when the respective values of x are substituted in the formula.

Suppose that for a given process, it is known that 5% of the articles are defective. If samples of 10 are drawn, what is the probability there will be 3 defectives in a given sample? In this example p = .05, q = .95, n = 10, and x = 3, therefore

$$\frac{n!}{x!(n-x)!} p^x q^{n-x} = \frac{10!}{3!\,(10-3)!} (.05)^3 (.95)^7 = .009673$$

Now suppose it is desired to obtain the probability that there will be one or more defectives in a sample. This means that the above procedure must be followed for x = 1, x = 2, x = 3, etc., to x = 10 and the values added to obtain the probability of 0.4013. This probability can also be obtained by 1-P(0) since the total of all probabilities must equal 1.

Frequency Distributions

In order to obtain statistical methods that will be sufficiently versatile to handle a wide variety of practical problems, it is necessary to work mathematically with theoretical frequency distributions which represent actual distributions satisfactorily.

A frequency distribution may be thought of as the representation, either graphically or symbolically, of the values of the variable being studied and the number of times each value occurs. To illustrate a frequency distribution, assume that the diameters of 100 shafts were determined as they came off of a production line. The following table represents the summary of the diameters found.

Table No. 9. Diameters of 100 Shafts

Diameter, in inches	Number of shafts having the diameter
.405	1
.410	3
.415	5
.420	10
.425	18
.430	27
.435	17
.440	11
.445	5
.450	2
.455	1

The graphical representation of the above frequency distribution is most conveniently illustrated as a histogram which is one of several methods.

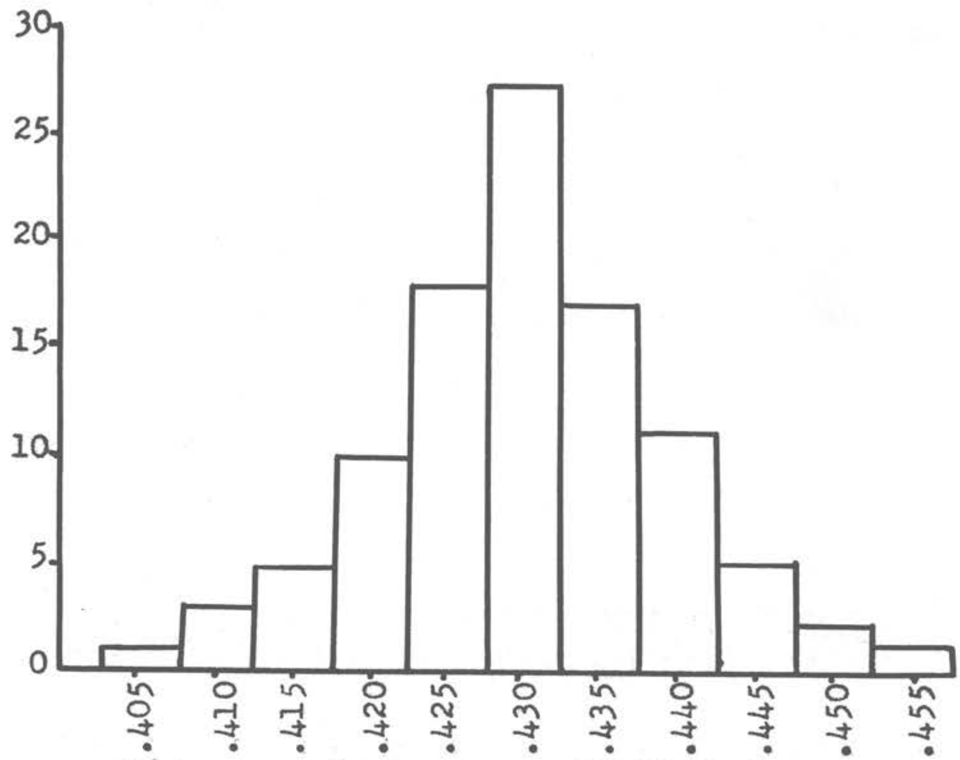

Fig. 4. Histogram of Diameters of 100 Shafts

Frequency distributions are of two types, discrete or continuous, depending upon the nature of the variable being studied. A discrete variable is one which can take on only isolated values. Illustrations of discrete variables are: the number of units of product per unit of time resulting from a production process, the number of units meeting specifications from a sample of units, or coded indications of the color of automobiles passing a given observation point where 1 would represent black, 2 represent green, etc. The value of each of these variables must be integers and cannot have any values between two successive integers.

A continuous variable is one that is not restricted to isolated values; it may have any value within a certain interval. Hence, the graph of the distribution of a continuous variable is a smooth curve, while for a discrete variable it has a step appearance as in the case of the shaft diameters. A continuous variable is considered because it lends itself to mathematical manipulation more readily than a discrete variable and the theoretical distribution of such a variable approximates many actual situations.

Distribution Function -- For greater ease in analysis and applications, the theoretical frequency distributions are expressed as distribution functions. The distribution function is such that the area included under the graph of the function is equal to 1. Hence, the area under the graph between any two values of the variable is the probability that the actual value will be within that interval.

Any distribution function, discrete or continuous, has characteristics known as moments which completely define and describe the function. The moments of a function are, therefore, of major importance in the analysis of any distributions. Although there are an infinite number of moments for any function, the first four moments are the ones of primary importance in statistical analysis. These four moments represent respectively: the central tendency, the variation, the skewness, and the peakedness. A verbal description of these characteristics will be given, but, since the mathematical derivations and designations require an understanding of fairly advanced mathematics, the mathematics will not be presented.

The first moment about the origin is called the mean (designated by m) or sometimes the weighted average. This value is such that, if the graph of the function could be considered as made of metal and a pivot point were placed at the mean, the metallic graph would be perfectly balanced. The mean is the value about which the data tend to concentrate and can be thought of as the value expected for any single item.

The concept of variation of dispersion which is described by the second moment is of paramount importance in statistics. It is assumed that variation means variation of the data about the mean. The importance of this measure can be easily illustrated. Suppose that a consumer of wire cable desired a mean tensile strength of 500 pounds. Also the use of the cable is such that a strength of less than 300 pounds cannot be tolerated. There are two sources of supply and a graphical representation of the frequency distribution of the strength of the cables from the two sources follows.

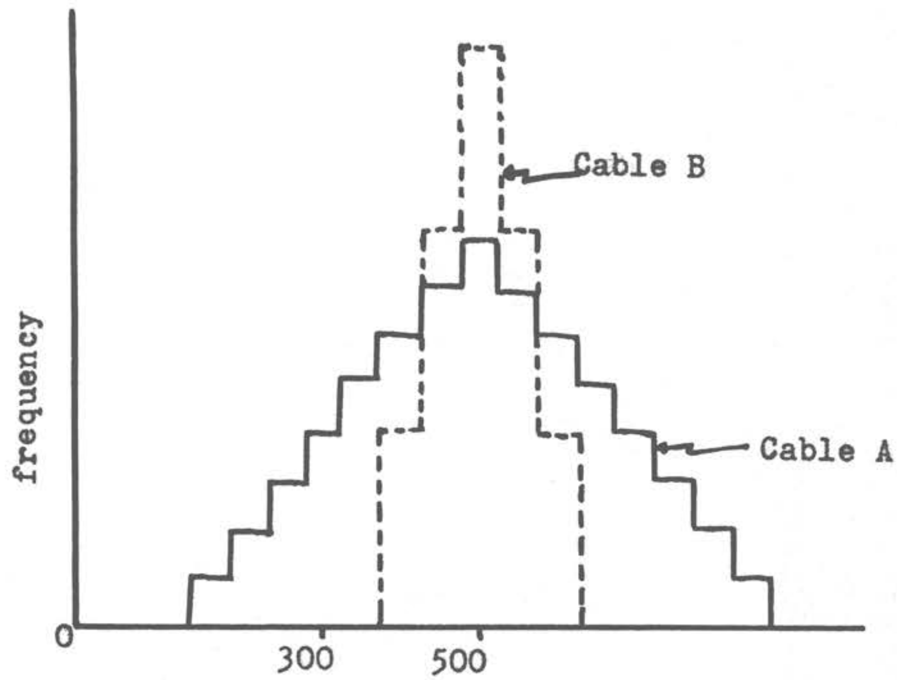

Fig. 5. Distribution of Strength of Cables from Two Sources

Since the mean strength is the same for both cables, a further analysis would be required to furnish a basis of choice. It is apparent from observation that the variation of Cable A is such that there is a possibility of obtaining a cable having less than the required strength, while Cable B should satisfy the required strength. The second moment about the mean gives a measure of variation and is called the <u>variance</u>, designated by s^2. Since the second moment includes the second power of the variable, the square root of the variance of <u>standard deviation</u> designated by s is sometimes valuable since it is expressed in the same units as the mean.

The third moment about the mean describes the symmetry of the distribution about the mean. The following graphical representation illustrates the concept of skewness. Curve 1 is symmetrical about the mean and will have zero for a third moment. Curve 2 is said to be skewed to the right and will have a positive third moment. Curve 3 is said to be skewed to the left and will have a negative third moment.

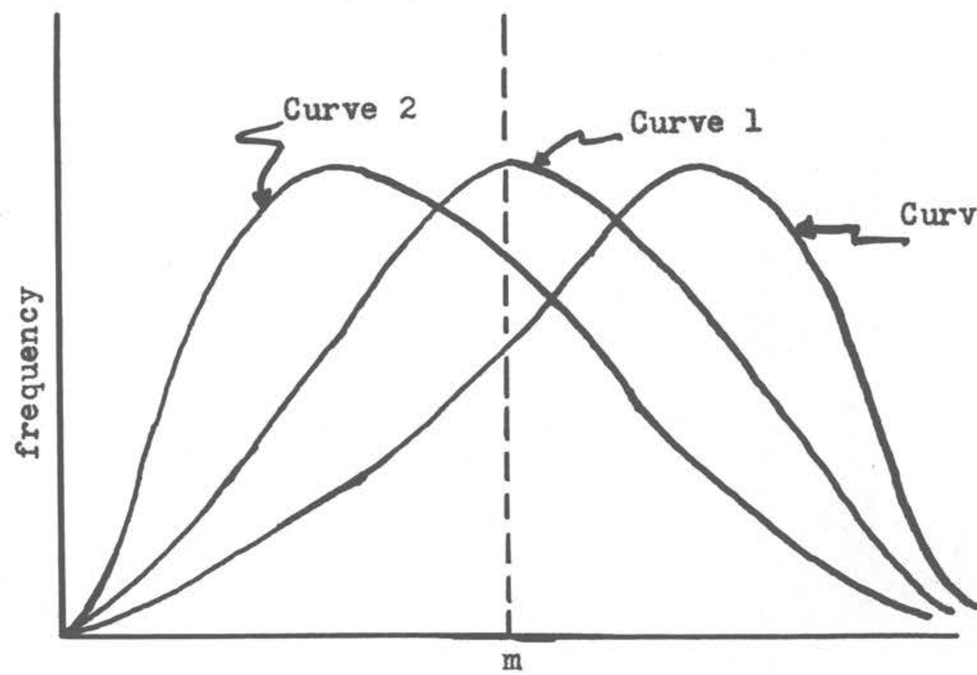

Fig. 6. Illustration of Skewness

The fourth moment about the mean is a measure of the peakedness of the distribution. If two distributions should be identical with respect to the first three moments but one distribution has a larger concentration of its data about the mean and consequently longer "tails" (this is necessary if the variance of the two distributions is the same) the fourth moment will represent the situation mathematically. It can generally be stated that the more peaked of two distributions has the larger fourth moment.

Discrete Distribution Functions -- There are several discrete distribution functions which adapt themselves to the analysis of particular types of situations. The most universally applicable discrete function is the binomial distribution function discussed previously. This distribution function is associated with repeated trials of the same event. The equation of this distribution function, as given previously, is

$$\frac{n!}{x!\,(n-x)!} \quad p^x q^{n-x}$$

This is a discrete distribution of one random variable, x. The other values, n, p, and q, are called parameters since they will have definite values for a given distribution while x may vary from 0 to n. The mean of the theoretical binomial distribution is

$$m = np$$

and the variance is

$$s^2 = npq$$

The distribution is simlar to the example in Fig. 4, with the theoretical distribution being exactly symmetrical about the mean.

The Multinomial Distribution -- The multinomial distribution function is associated with repeated trials of an event which may have more than two outcomes, whereas the binomial distribution was limited to two outcomes. If the possible outcomes are denoted by x_1, x_2, x_3 - - - x_k, then there are probabilities for the occurrence of each of the outcomes represented by p_1, p_2, p_3 - - - p_k and the sum of all these probabilities must be equal to 1. The equation of the multinomial distribution function is

$$f(x_1,x_2,x_3 - - x_k) = \frac{n!}{(x_1!)(x_2!)(x_3!) - - (x_k!)} p_1^{x_1} p_2^{x_2} p_3^{x_3} - - p_k^{x_k}$$

The probability of obtaining a designated set of outcomes can easily be determined by substitution of values in this formula. Suppose that there are three possible outcomes which are dimensions of a shaft ending in 0, ending in 5, and ending in numbers other than these two. Suppose that it is known that the probability of a measurement ending in 0 is 1/10, the probability of ending in a 5 is 1/10 and the probability of ending in other than 0 or 5 is 8/10. For a sample of 10 items, what is the probability there will be 2 ending in 0, 3 ending in 5, and 5 ending in some other number. $x_1 = 2$, $x_2 = 5$, and $x_3 = 5$; $p_1 = 1/10$, $p_2 = 1/10$, and $p_3 = 8/10$; $n = 10$, then

$$p = \frac{10!}{2!\,3!\,5!} \left(\frac{1}{10}\right)^2 \left(\frac{1}{10}\right)^3 \left(\frac{8}{10}\right)^5 = 0.0083$$

Other Discrete Distributions -- Other discrete distributions, which are less frequently applicable to actual situations but are at times very essential, will be described briefly.

The hypergeometric distribution function is defined by

$$f(x) = \frac{C(m,n)\ C(n,r-x)}{C(m + n,r)}$$

This distribution function is applicable to situations where the probability is not constant on succeeding trials.

m represents the possible successes; n represents the total number of possibilities; r represents the number of trials; and x represents the designated successes.

The uniform distribution function is defined by

$$f(x) = \text{a constant}$$

This distribution applies to the situation where each of several possible outcomes have the same probability of occurrence. Then, regardless of the outcome specified, the density for that value of the variable will be the same as for any other value.

The negative binomial distribution function is defined by

$$f(x) = p^r \frac{(x + r-1)!}{(r-1)!\,(x)!} q^x$$

where x + r represents the number of trials, r represents the number of successes, p represents the probability of a success, and q represents the probability of a failure. This distribution applies to the situation where the last trial is predetermined to be a success. f(x) then is the probability that r successes will be achieved in x + r trials. For example, what is the probability that exactly 10 trials or bolts must be selected to secure 8 within tolerance limits. To satisfy this condition, the tenth bolt must be within tolerances and, in addition, 2 of the 9 preceeding bolts must fail to meet tolerances.

Continuous Distribution Functions

Just as there are several discrete distribution functions, each having its area of application, there are several continuous distribution functions, each having a specific utilization in statistical analysis. A few of these will be described to indicate the scope of their application.

Uniform Distribution -- This is the simplest distribution for a continuous variate and is given by

$$f(x) = \frac{1}{b-a}$$

where a must be less than the variate, x, and b must be greater than x. The graphic representation of this distribution is given in Fig. 7. The primary importance of this distribution lies in the fact that it is easy to deal with mathematically. Since any other continuous distribution function may be converted to the uniform distribution, the ease of mathematical manipulation may be helpful in analysis of other types of distributions.

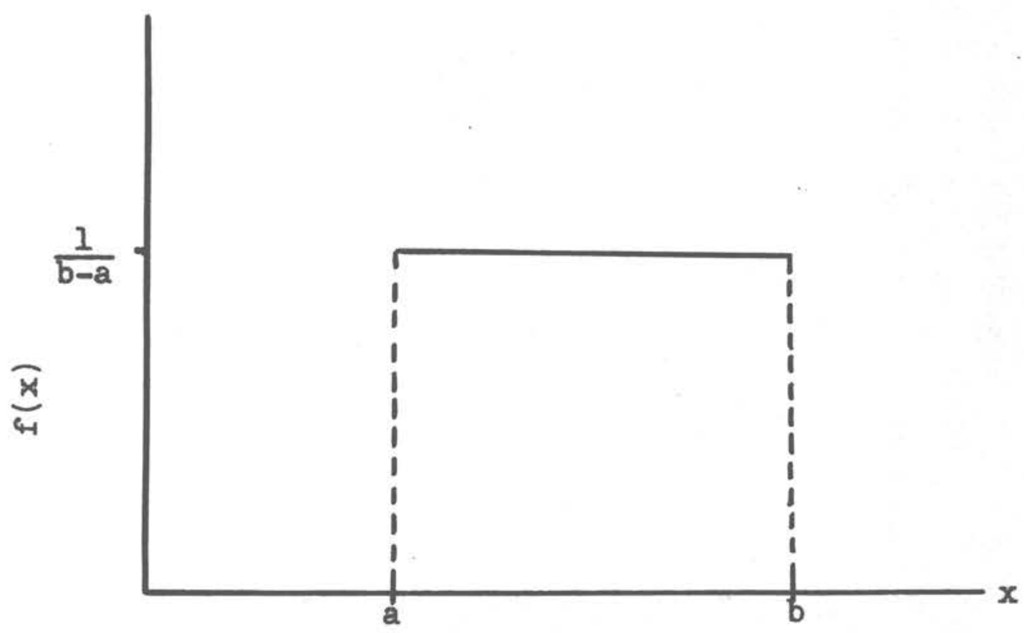

Fig. 7. Illustration of Uniform Distribution

The Gamma Distribution -- The gamma distribution function is given by

$$f(x) = \frac{1}{a!\, b^{a+1}} \quad x^a \, e^{-x/b}$$

where x must be greater than zero. e represents the base of Naperian logarithms and is approximately equal to 2.71828. b must be greater than zero and a must be greater than minus one. The function is plotted in Fig. 8 for various values of a and b.

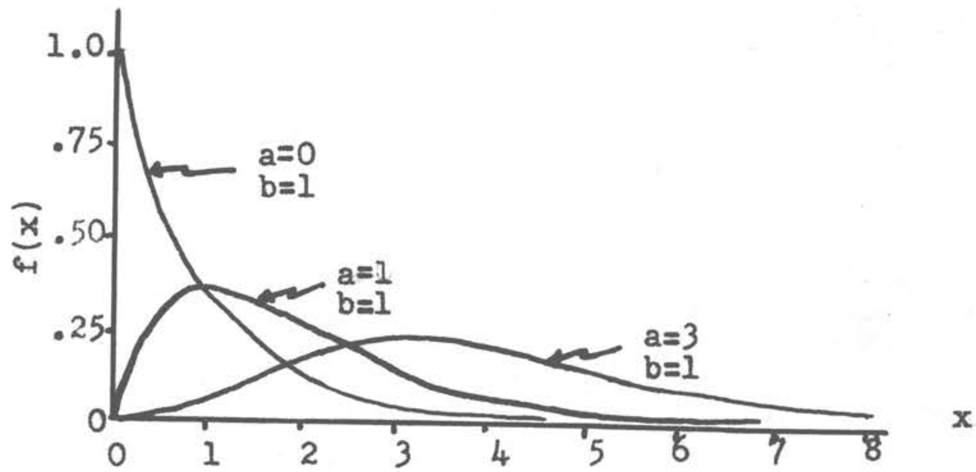

Fig. 8. Illustration of Gamma Distribution

Changing the value of b merely changes the scale used on the axes. The effect of changing the value of a is illustrated by Fig. 8, since the three curves plotted are all for $b = 1$. The mean of the gamma distribution function is

$$m = b\,(a + 1)$$

and the variance is

$$s^2 = b^2\,(a + 1)$$

The Beta Distribution -- The beta distribution function is given by

$$f(x) = \frac{(a + b + 1)!}{a!\, b!} \quad x^a \, (1-x)^b$$

where x must be between 0 and 1. The function is plotted in Fig. 9 for various values of a and b.

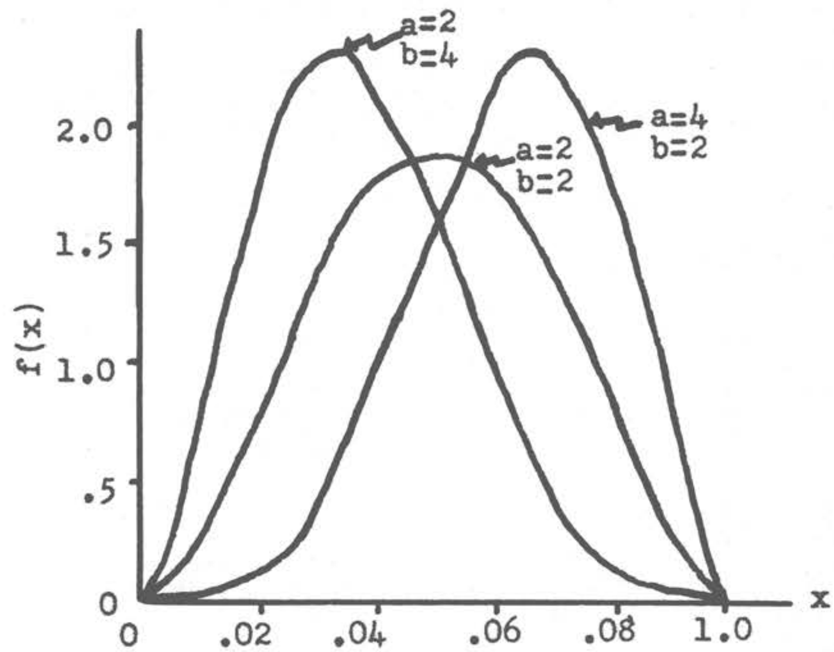

Fig. 9. Illustration of Beta Distribution

For any value of a and b where a = b, the function is symmetrical about the point x = .05. This means that m = .05. As a and b increase, the variance approaches zero. As a increases with relation to b, the distribution becomes skewed to the left with the mean approaching one and the variance decreasing, approaching zero. As b increases with relation to a, the function becomes skewed to the right with the mean approaching zero and the variance decreasing, approaching zero.

The Normal Distribution -- A great many of the techniques used in applied statistics are based on the normal distribution because so many populations encountered in the course of experimentation in many fields seem to approximate the normal distribution closely. For this reason it was decided to discuss the normal distribution last. The normal distribution function is given by

$$f(x) = \frac{1}{s\sqrt{2\pi}} \quad e^{-\frac{1}{2}\left(\frac{x-m}{s}\right)^2}$$

where m is the mean and s^2 the variance of the distribution. Some representative distributions are plotted in Fig. 10.

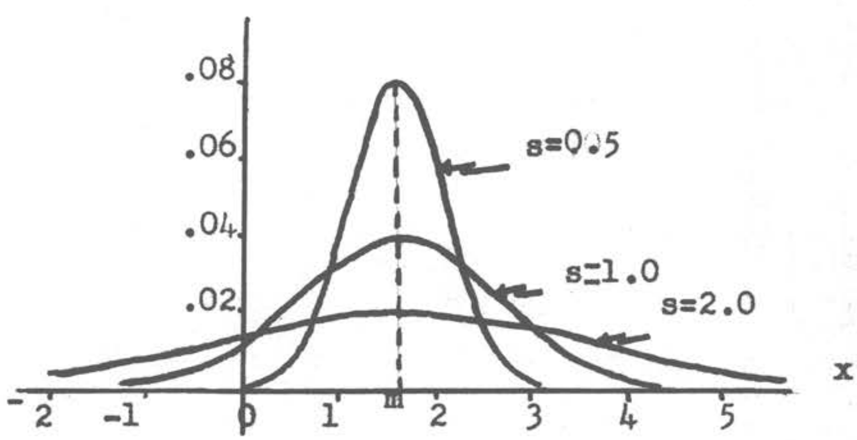

Fig. 10. Illustration of Normal Distribution

A change in m will shift the function along the x axis while an increase in s will tend to spread the function and lower its apex. Because of the great amount of application of the normal distribution, methods have been devised to transform all normal distributions into the same form. This is accomplished by setting $t = \frac{x-m}{s}$ and the formula for the distribution function becomes

$$f(x) = \frac{1}{\sqrt{2\pi}} \quad e^{-\frac{1}{2}t^2}$$

This is called the standard or normalized form of the distribution function and has a mean of zero and a variance of one. The probability for any range of x can easily be determined through use of tables of the standard values by making the substitution $t = \frac{x-m}{s}$ and looking up the area under the standard normal curve between the two values of t obtained.

The Nature of Statistical Hypothesis -- The statistical hypothesis is usually an assumption about a parameter of the distribution. In the normal distribution the parameters are m, the mean, and s^2, the variance. The hypothesis is not made as to the type of distribution function. This is determined from other information relative to the population to be tested and the known characteristics of the distribution functions. A typical statistical hypothesis might be made in the following situation. The present process for producing incandescent light bulbs results in bulbs with an average life of 1,700 hours burning time. A new process is installed and it is desired to know whether there is significant change in the average life of bulbs produced. The statistical hypothesis would be that the mean life for the old process is the same as the mean life for the new process.

Tests of Statistical Hypothesis -- A test of a statistical hypothesis is a procedure for deciding whether to accept or reject the hypothesis. This procedure is based on the assumption that any sample taken may be such that it does not represent the population. Therefore, it cannot be stated with certainty that values based on the sample are true of the population. For this reason the hypothesis is proven within a certain confidence interval, which is usually based on the value of s. The most common interval used is plus or minus 2s. This range includes 95% of the normal curve and leaves 2½% on the plus end and 2½% on the minus end that is not included. Thus, a statement that the hypothesis is accepted within a .95 confidence level means, in effect, that the hypothesis is true unless a 5 in 100 chance occurred in taking the sample. Assume that analysis of samples of bulbs produced by the new process gives a mean life of 1,900 hours. The first inclination is to state that the new process is better; however, this is not necessarily true since the dispersion resulting from the new process may greatly affect the interpretation. Suppose that s for the means obtained for the new process is 140. Assuming a confidence interval of .95, we say that the mean of 1,900 could easily have come from a population having a mean of from 1620 hours to 2180 hours. Since the mean obtained by the old process, 1700 hours, is within this range and the hypothesis would be accepted meaning that there is not a significant difference in the mean life resulting from the two processes. However, suppose that s for the means obtained from the new process is 15, the sample mean of 1900 hours would be interpreted to have come from a population whose mean was between 1870 hours and 1930 hours. Since the mean life obtained with the old process does not fall within this range, it can be stated that within a confidence interval of .95 the hypothesis is rejected and the new processes are better than the old process.

Summary of Distribution Functions -- A concise description of the most common discrete and continuous distribution functions has been presented in the hope of accomplishing two purposes:

1. An understanding of the scope of situations that may be quantitatively evaluated through statistical methods and, hence, furnish the necessary quantitative basis for decision.
2. A generalized knowledge of various repetitive situations and the form of their distribution such that the person required to make decisions can understand the language and to some degree appraise the implication of results furnished him by the statistician.

A full understanding of statistics and the applicable techniques and methods can only be obtained through fairly extensive study in the field. This does not seem entirely feasible for every person engaged in activities of organizational management. However, it is essential if improved decisions and, consequently, improved organizational efficiency is expected, that the capabilities of this tool, which was developed especially for evaluation of the very prevalent situation of repetitive activity, be understood by management personnel.

Statistical Quality Control

The quality of any manufactured product is always subject to a certain amount of variation as a result of chance. The sources of this variation lies in one or both of two types of causes: (1) Assignable, and (2) Unassignable. Assignable causes are those causes which may be eliminated to improve the quality of the product. Hence, it is a cause of variation of output that can be removed. Unassignable causes are those causes that occur strictly within some stable system of causes which occur wholly by chance and cannot be eliminated by any known method.

The function of statistical quality control is to separate out the assignable causes of variation in product quality. This makes possible the diagnosis and correction of many production troubles which will result in substantial improvements in product quality and reduction of spoilage and rework. In addition, by identifying certain quality variations as occurring wholly from chance, quality control methods tell when to leave a process alone and, thus, prevent unnecessarily frequent adjustments and work stoppages.

Another important function of statistical quality control is the revealing of "natural" tolerances which will promote a wiser choice of engineering tolerances and provide a basis for comparison of alternate designs and production methods.

There are three basic tools that are ordinarily classified under the field of statistical quality control although all statistical theory and techniques could be considered applicable to quality control. The three tools which have been specifically developed to maintain very nearly an instantaneous indication of the variations of product quality are:

1. The Shewart control charts for measurable variables indicating quality characteristics. There are two charts to a "set." In one case, a chart for the mean, m, and one for the <u>range</u>, R, is used. The range is the spread between the smallest and largest value within a sample. The other set of charts combines a chart for the mean, m, and the <u>standard deviation</u>, s. The standard deviation being the square root of the variance which has been discussed previously.
2. The Shewart control chart for the fraction defective, ordinarily called a p chart.
3. The Shewart control chart for the number of defects per unti, ordinarily called a c chart.

Control Charts for Variables -- The simplicity and yet the sound statistical basis of the Shewart control charts for variables make them readily applicable to a vast majority of production situations. Use of the charts is undertaken for one of the following reasons:

1. To analyze a process to secure information to be used in establishing or changing specifications, production procedures, or inspection and acceptance procedures.

2. To provide a basis for current decisions as to when to attempt to locate causes of variations and take action to correct them.
3. To provide a basis for current decisions on acceptance or rejection of manufactured or purchased product.

The charts are established by taking a group of samples and measuring the variable which it is desired to study. For each sample, the mean and range is determined. The next step is to decide the range to be permitted between the control limits for both m and R. The usual procedure is to use 3s or 3 standard deviations above and below the average value for the variable. The selection of the control limits is based on the standard normal distribution. Tabulated values for this distribution will indicate the percentage of the population that will normally be expected to fall outside the control limits selected, whether those limits be 3s, 2.5s, or 2s. Once the mean value for m and R and the control limits are determined, the control chart can be constructed. Fig. 11 shows sample charts for the mean and range.

After the chart is established, samples continue to be taken and the values for m and R plotted on the chart. At intervals, the most recent samples may be included and new values for the average of the variable and the control limits determined. The control chart then becomes a continuous indication of product quality.

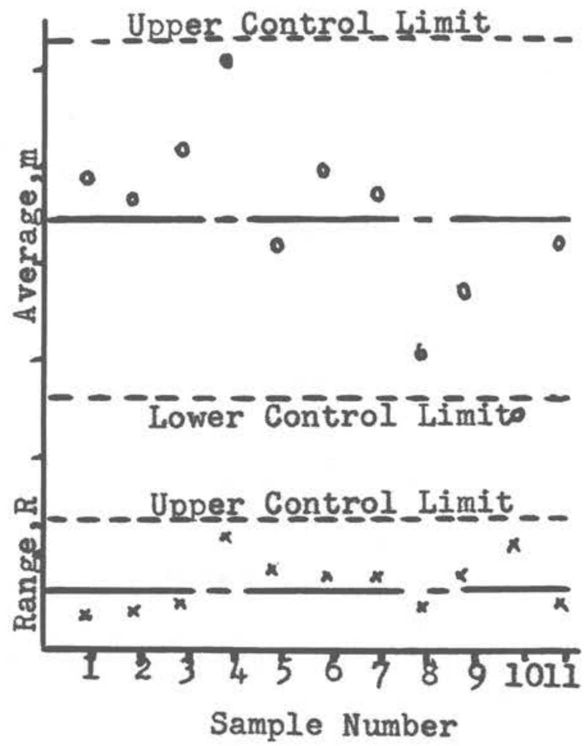

Fig. 11. Mean and Range Control Chart

The charts for the mean and standard deviation are established by a similar process. It should be noted that, if the m,R charts are used, the m chart gives an indication of the consistency of the central tendency of average values while the R chart gives an indication of the dispersion of individual measurements about the mean. In m,s charts, the m chart is the same as before and the s chart indicates the dispersion of individual measurements about the mean. The only known guide as to which set of charts to select is that for samples of size 15 or smaller the m,R charts are recommended, while for samples of size greater than 15, the m,s charts are recommended. Fig. 12 illustrates control charts for m and s.

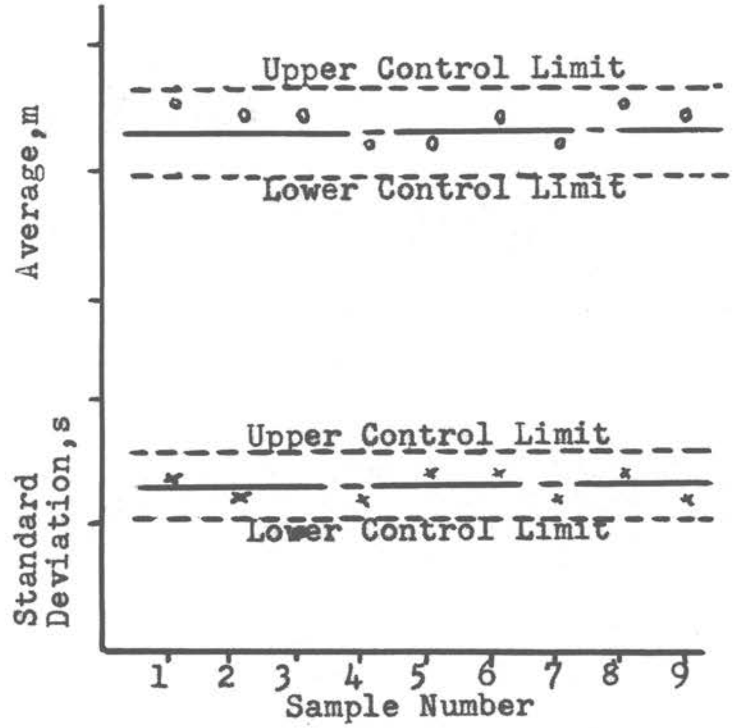

Fig. 12. Mean and Variance Control Chart

There are a group of criteria to indicate when a process is "out of control." The following are suggested:

1. When a larger percentage of values fall outside the control limits than are to be expected considering the spread of the control limits.
2. Whenever, in 7 successive points on the control chart, all are on the same side of the established average value of the variable.
3. Whenever, in 11 successive points on the control chart, at least 10 are on the same side of the established average value of the variable.
4. Whenever, in 14 successive points on the control chart, at least 12 are on the same side of the established average value of the variable.

5. Whenever, in 17 successive points on the control chart, at least 14 are on the same side of the established average value of the variable.
6. Whenever, in 20 successive points on the control chart, at least 16 are on the same side of the established average value of the variable.

Whenever, because of any one of the six criteria listed above, a process is "out of control", this means that there are assignable causes of variation present.

Control Chart for Fraction Defective -- The control chart for fraction defective, or p chart, was developed as a quality control measure for those situations where an article is either acceptable or not acceptable, but involves no measurement of individual items. The steps in establishing and maintaining the p chart are as follows:

1. Record the data for a number of samples. This data includes the number of items in the sample and the number of defectives or non-acceptable items in the sample.
2. Calculate p for each sample where
$$p = \frac{\text{number of defectives in sample}}{\text{number of items in sample}}$$
3. Compute the average fraction defective $\bar{p}$, where
$$\bar{p} = \frac{\text{total number of defectives in all samples}}{\text{total number of items in all samples}}$$

4. Compute the control limits for each sample where

$$\text{Upper control limit} = \bar{p} + \frac{3\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$$

$$\text{Lower control limit} = \bar{p} - \frac{3\sqrt{\bar{p}(1-\bar{p})}}{\sqrt{n}}$$

5. Plot the values for p obtained for the original group of samples and continue to plot p for succeeding samples.

6. Revise values for $\bar{p}$ and control limits at intervals using the recent sample values.

Fig. 13 illustrates the p chart.

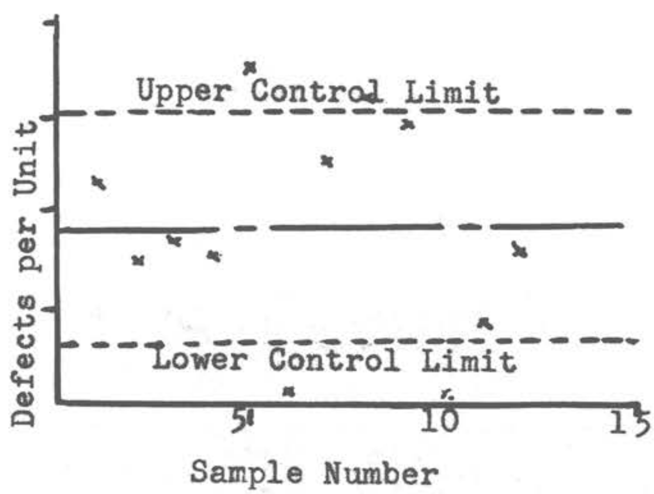

Fig. 13. Fraction Defective Control Chart

Control Chart for Defects Per Unit -- The control chart for the number of defects per unit or c chart applies to situations similar to the following:

1. c is the number of defective rivets in an aircraft wing.
2. c is the number of surface defects in a galvanized sheet, or an enameled surface.
3. c is the number of imperfections in a bolt of cloth

The procedure for setting up and maintaining the c chart is very similar to that for the p chart and is given in the following steps:

1. Record the data--the number of defects per unit--for several units of product.
2. Compute the average value of c -- that is $\bar{c}$.
3. Compute the control limits where

   Upper control limit = $\bar{c} + 3\sqrt{\bar{c}}$

   Lower control limit = $\bar{c} - 3\sqrt{\bar{c}}$
4. Plot the values of c for the units used in Step No. 1 and succeeding values as determined.
5. Revise the values of $\bar{c}$ and the control limits at intervals using recent sample values.

Fig. 14 illustrates the c chart.

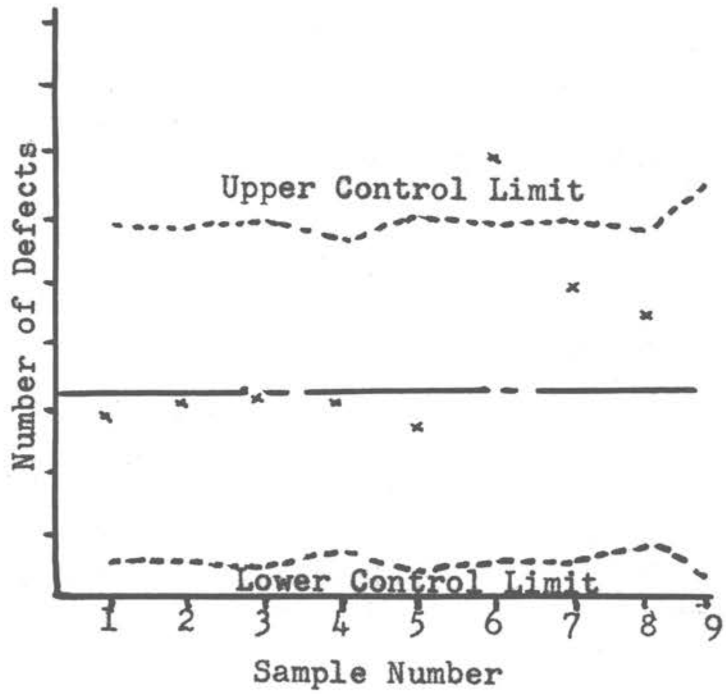

Fig. 14. Number of Defects Control Chart

CHAPTER III

APPLICATION OF TOOLS TO QUANTITATIVE ANALYSIS

The four methods discussed in Chapter II do not constitute a complete list of available tools for quantitative analysis. They were discussed on the basis that they are believed to be the four most important from the standpoint of the number of decisions to which they may be applied. It is unquestionable that the ideal situation would be for all persons belonging to the management group to know all phases of the field of mathematics as well as any fields, such as accounting, which are based on numerical representation, and, at the same time, to possess complete knowledge in any area which might be pertinent to the formation of decisions the particular person might be required to make.

Since it is usually impossible to obtain the services of such personnel, a desirable combination of abilities should be sought. The mathematician who has little or no knowledge of processing procedures, personnel relations, the factors effecting group dynamics, or finance would be of questionable value in making decisions where knowledge of these subjects is required. However, a condition just as faulty has been a practice to a great extent in many industrial for the last several years. That is, the employment of personnel who will be required to make decisions who have sufficient knowledge of a particular field, such as industrial relations, but have little or no ability to make quantitative analysis of problems which arise. The result has been that these persons resort to vague and oftentimes inaccurate methods of application of their knowledge, usually described as "judgment" or "opinion."

There have been numerous occasions where persons in responsible positions who are known to possess the knowledge relative to their position have failed in its administration. The comment of wonderment expressed so often as to why "Joe could not succeed in his job when he knows it from A to Z" should indicate that knowledge in itself is not enough. Success of an individual in any position requiring the making of decisions is believed to be a combination of knowledge of the subject matter and the practice of quantitative analysis and these two factors are believed to affect success equally.

The four methods discussed in Chapter II can be utilized in the following situations: (1) the gathering and utilization of historical numerical data and its application to a decision to be made for future action, (2) the combination of numerical facts in the day to day situations where long-term investment costs or analysis requiring higher mathematics is not needed, (3) means of converting all expenses and incomes in connection with any situation to an equivalent basis for a comparison, (4) the analysis of research and experimental problems and situations where repetitive processes are utilized. A full understanding of the four methods will furnish the basis necessary for quantitative analysis of the great majority of situations requiring decision.

When is Knowledge Sufficient

The contention that the knowledge possessed by many persons who consider themselves authorities within a particular field is insufficient for consistently correct application would probably cause much heated comment. However, the contention is, in keeping with the quotation of Lord Kelvin quoted earlier in this discussion, that no knowledge is of appreciable value until that knowledge can be expressed in numbers. The reason for such a contention is basic enough when it is considered that few, if any, fields of knowledge can stand alone, but must be combined or added to knowledge from other fields to be of value in application to any particular situation. This implies that there must be a common means for the combination of knowledge from various fields so that application to actual problems is possible. It is not felt that there is any other adequate means of accomplishing this except through the utilization of numbers for the evaluation of the knowledge and subsequent combination with other knowledge.

On the basis of the preceding discussion, there are many areas of proclaimed knowledge which are of little practical value since their application thus far is dependent solely on judgment and opinion. Therefore, it becomes necessary to devise means of numerical expression of such knowledge as well as the instilling of these means in the minds of those who propose to apply such knowledge.

The contention of many writers with respect to the act of making decisions is that a problem can be analyzed to a certain point and, after this point is reached, there are certain intangible factors which cannot be expressed numerically which must be considered. This means that the quantitative analysis is carried through a portion of the analysis after which the person must mentally jump to a conclusion. This mental leap is merely a judgment consideration of those factors where ignorance of the subject prevents quantitative expression, although, in many cases, it may be more desirable to make this sort of analysis rather than delay decision until the knowledge can be obtained or persons possessing the knowledge be consulted. The "leap" in itself should be the result of a quantitative analysis as to whether numerical data will support such action. Realizing that such a procedure introduces the element of human error to a great extent, is such action justified if sufficient time is available for quantitative analysis?

Procedure for Quantitative Analysis

It is believed that best results will be obtained in the making of quantitative analysis of problems requiring decision when a definite procedure which is applicable to all situations is followed. A procedure can be established which involves four steps. These can be followed rapidly for those situations requiring decisions in a minimum amount of time, or in those situations which involve only one or two arithematical combinations of knowledge.

Step 1. Enumerate or list all facts pertinent to the situation.

Step 2. Evaluate each of these facts numerically.

Step 3. Combine these numerical values and arrive at a numerical result.

Step 4. Form the decision based on the analysis.

Step 1 is the verbal statement of pertinent facts and should never be eliminated in any analysis. The attempted elimination or abbreviation of this step will do much to hinder satisfactory results in the analysis of a problem through failure to include pertinent facts or the inclusion of facts that have no bearing on the problem being analyzed. This, then, should be considered a precautionary step to facilitate a proper beginning in any analysis which is essential to obtaining a correct result.

Step 2 has been discussed throughout this report and is of primary concern in that it is not practiced at the present time in so many areas of knowledge. The statement has been made numerous times that knowledge is insufficient when it cannot be evaluated numerically, but very little has been said to this point as to how such evaluation can be accomplished. This problem necessarily involves considerable research by those persons engaged in particular fields, but the primary factor that has prevented such evaluation so long is believed to be the mental fear of numbers possessed by many people. It becomes necessary that any person making a decision should think in terms of numbers and the accomplishment of this will require a complete mental reversal in many instances. On the assumption that the theory of quantitative analysis and its application to the making of decisions is correct, a knowledge of numbers and their application becomes a basic element in all knowledge and, as such, should be mastered to a fair degree by all persons. This knowledge of numbers is then added to by specialization within a particular field.

Before numerical expression of knowledge, as it applies to making decisions, can be expected to become a fact two things must be accomplished: (1) Evaluation of the knowledge must be accomplished through research and study, and (2) persons expecting to practice within these areas of knowledge must have sufficient background in the methods of numerical expression and combination. This second point can be accomplished only through the requirement of sufficient mathematical study from elementary school through college or university.

Step 3 can be accomplished in a majority of cases through the four methods of application discussed in Chapter II. However, there are numerous occasions where the combination of factual data involves specialized mathematical knowledge and can only be performed by specialists within particular branches of mathematics. Since the four methods outlined in Chapter II apply to such a large percentage of decisions, training of members of the organizational management group in these four methods should do much to promote more consistent decisions and, consequently, increase the efficiency of the organization.

Step 4, in the ideal case, will involve only the consideration of two numerical quantities to see which is greater and selecting the most desirable quantity. Even though it would be highly desirable to have this condition exist in all situations requiring decision, it is not believed that such could prove practical. Therefore, it should be mentioned again that Step 4 may include a separate decision which is to make the primary decision on the basis of incomplete numerical data. There could be many reasons causing such a situation to exist. However, these reasons are all to the effect that a delay of decision or failure to decide may be far more disastrous than the making of a decision based on incomplete data, even though such a decision may be slightly in error.

CONCLUSION

A concise statement of the major points advanced and discussed within this report is made in an effort to summarize and emphasize these factors.

1. When two or more persons combine their efforts to accomplish an objective, an organization is formed.

2. The objective of an organization is accomplished through the combination and utilization of men, materials, and methods.

3. The function of planning and directing the activities of an organization to facilitate the furnishing, utilization, and combination of men, materials, and methods to attain the objective of the organization is management.

4. Decision in the organizational sense is defined as the setting forth of a course of action to be followed in so far as it is possible to control the men, materials, and methods which may be necessary to attain the objective.

5. The primary duty of a person engaged in the management of an organization is to make decisions.

6. Decisions will be more nearly correct when they are based on quantitative analysis of the problem.

7. Quantitative analysis involves the application of numbers to facts and the subsequent combination of these numbers to obtain a numerical result.

8. Knowledge within an area is sufficient only when it can be expressed in numbers when applied to a particular problem or situation.

9. Four areas--arithmetic, accounting, engineering economy, and statistics--will furnish the basis for quantitative analysis in a vast majority of situations requiring decision.

10. Quantitative analysis should be accomplished through establishing and following a definite procedure of analysis.

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