

IMPACT OF A NON-TRADITIONAL GEOMETRY  
COURSE ON PROSPECTIVE ELEMENTARY  
TEACHERS' ATTITUDES AND  
TEACHING EFFICACY

By

JULIANA GAIL UTLEY

Bachelor of Science in Mathematics  
Oklahoma State University  
Stillwater, OK  
1982

Master of Mathematics Education  
Oklahoma State University  
Stillwater, OK  
2000

Submitted to the Faculty of the  
Graduate College of the  
Oklahoma State University  
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By

Juliana Gail Utley

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Dissertation Approved:

*Stacy Reeder*

\_\_\_\_\_  
Dissertation Advisor

*Pat Lumpkin Jordan*

\_\_\_\_\_  
*Margaret McScott*

\_\_\_\_\_  
*John Wolfe*

\_\_\_\_\_  
*Sefer Sarlozzi*

Dean of the Graduate College

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## Chapter I

### *Introduction*

Mathematics educators are experiencing a time of change in mathematics education. These changes are being affected by current research on how children learn and recommendations by the National Council of Teachers of Mathematics (NCTM). Central themes in the *Curriculum and Evaluation Standards* (NCTM, 1989) focus on developing students who have mathematical power; changing content, "we do not assert that informational knowledge has no value, only that its value lies in the extent to which it is useful in the course of some purposeful activity" (p. 7); and teaching methods, pointing to research that indicates that learning occurs when students are actively engaged.

Mathematics teachers have typically practiced what Fiere (1970) refers to as the banking concept of education, "knowledge is a gift bestowed by those who consider themselves knowledgeable upon those whom they consider to know nothing" (p. 53). Fiere referred to teachers, practicing traditional approaches to teaching, as narrators that lead students to memorize endless facts and view students as vessels to be filled. NCTM's *Professional Standards for Teaching Mathematics* (1991) points out that teaching practice must change from a traditional lecture-mode of instruction to a style of teaching where in students are

actively engaged in the learning process through discovery and inquiry. Learning experiences based on discovery learning and inquiry methods support active engagement of students in the learning process, foster a student's natural curiosity, build upon prior knowledge of students, cause students to use higher order thinking skills, help students form positive and excited attitudes toward learning, and help students not only learn specifics but rather learn how to learn (Bruner, 1960; Conference Board of the Mathematical Sciences, 2001). Many mathematics educators believe that constructivism encompasses the essence of the proposed changes in teaching. As von Glasersfeld (as cited in Anthony, 1996) expounded upon a basic tenet of constructivism, he stated "learning is not a passive receiving of ready-made knowledge but a process of construction in which the students themselves have to be the primary actors" (p. 349).

Prospective teachers are not familiar with this type of teaching and typically have not had any experiences with learning through discovery. In their future classrooms, they typically teach how they were taught. So, the question becomes, how do we stop this cycle? A recommendation from the Conference Board of the Mathematical Sciences (CBMS) suggests that not only should mathematics content courses be concerned with building the mathematical content knowledge of teachers, but with demonstrating "flexible, interactive styles of teaching" (2001, p. 8).

The *Principles and Standards for School Mathematics* (NCTM, 2000) states "effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn

well....students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, 2000, p. 11).” Schofield (1981) points out that for elementary teachers to be effective they must have a positive attitude towards mathematics; however, Rech, Hartzell, and Stephens (1993) found that elementary education majors possess a more negative attitude towards mathematics than do the general college population. Additionally, Schofield (1981) found positive correlations between teachers’ attitudes toward mathematics and both pupil achievement in mathematics and their attitudes toward mathematics.

Teachers’ affective factors have become increasingly more important in a time when reform efforts are encouraging higher-order thinking. The *Curriculum and Evaluation Standards* (1989) include as a goal that students value mathematics and develop mathematical confidence, both of which are considered attitudes toward mathematics. If students are going to be encouraged to work non-routine problems and use higher-order thinking skills, their attitude and self-efficacy are going to be important factors for their success in mathematics. In order to foster these ideals in their own students, teachers must have both a positive attitude toward mathematics and a high mathematics teaching efficacy.

### *Foundation of the Problem*

Reform in mathematics education in the United States can be noted as far back as the early nineteenth century. In the 1820s Warren Colburn was

questioning teaching for memorization and in his book *First Lesson in Arithmetic on the Plan of Pestalozzi With Some Improvements* he advocated for a more discovery approach to student learning (Sztajn, 1995). However, according to Sztajn (1995), in the mid 1800s the "pendulum swung back to drill....instruction returned to rote memorization, and today drill is the term mainly used to characterize nineteenth century mathematics education in America" (p. 380). The swinging pendulum, used by Sztajn as a metaphor to characterize shifts in the philosophy of teaching mathematics, illustrates the ongoing debate among mathematics educators between teaching for understanding in contrast to teaching for skills acquisition or what is commonly referred to as teaching for conceptual versus procedural knowledge.

In response to a criticism of the study of mathematics in schools and a desire to reform mathematics education, the National Council of Teachers of Mathematics (NCTM) was founded in 1920. Although membership began to grow and NCTM began to make recommendations for reform the effect was minimal. In the 1950s efforts to reform mathematics curriculum were already under way when the Sputnik was launched in 1957 but this reform gained increased attention from the public, policy makers, and mathematicians and gained monetary support from the government for the improvement of mathematics education. These "new math" reform efforts continued into the 1960s; however, the 1970s brought with it a back-to basics movement (Kilpatrick & Stanic, 1995). In 1980, NCTM published its *Agenda for Action* in which it

rejected a return to basics and urge that the emphasis in mathematics teaching be put on problem solving.

Current reform in education began in 1983 with the National Commission on Excellence in Education's report, *A Nation at Risk*. Over the last two decades there have been a variety of documents published calling for reform in mathematics education beginning with *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (Mathematical Sciences Education Board, 1989) and *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). These publications were followed by *Professional Standards for Teaching Mathematics* (NCTM, 1991), *Before It's Too Late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21<sup>st</sup> Century* (U. S. Department of Education, 2000), *The Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences (CBMS), 2000), *Shaping the Future: New Expectation for Undergraduate Education in Science, Mathematics, Engineering, and Technology* (Education and Human Resources Advisory Committee, 1996), and others. All of these documents called for a new vision in the mathematical education of students and most made recommendations for the preparation of prospective teachers. This new vision calls for preparing teachers to become mathematical thinkers and problem solvers. Mathematics courses designed for the mathematical preparation of elementary teachers should initially approach the mathematics from a concrete and experientially-based direction. These courses should help

prospective elementary “learn how to learn mathematics” (CBMS, 2001, p. 8) and demonstrate an interactive and flexible teaching style.

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) promoted change in mathematics education and emphasized the mathematical empowerment of students. These standards were “viewed as facilitators of reform” (p.2). These standards dealt only with changes in curriculum and evaluation; therefore, the *Professional Standards for Teaching Mathematics* (NCTM, 1991) followed and addressed ways in which the goals of the 1989 curriculum standards could be met. Five major shifts for teaching of mathematics to ensure the mathematical empowerment of students were outlined. NCTM (1991) pointed out the need to shift:

- toward classrooms as mathematical communities - away from classrooms as simply a collection of individuals;
- toward logic and mathematical evidence as verification - away from the teacher as the sole authority for right answers;
- toward mathematical reasoning - away from merely memorizing procedures;
- toward conjecturing, inventing, and problem solving - away from an emphasis on mechanistic answer-finding;
- toward connecting mathematics, its ideas, and its applications - away from treating mathematics as a body of isolated concepts and procedures. (p. 3)



These standards for teaching pointed out that there are no recipes or prescriptions for teaching mathematics. The writers indicated that good teaching involved teachers having knowledge in several areas - mathematics, diversity of learners, school and community, how students learn mathematics, classroom setting, and the ability to engage students in mathematics. Knowledge in these areas helps the teacher make informed decisions about pedagogical practices.

Most recently NCTM published the *Principles and Standards for School Mathematics* (2000) that continue to promote reform in mathematics education and set forth a vision for school mathematics. These standards address six principles for school mathematics including Equity, Curriculum, Teaching, Learning, Assessment, and Technology. The teaching principle emphasizes that “to be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks” (NCTM, 2000, p. 17). This type of understanding was described by Ma (1999) as a “profound understanding of fundamental mathematics”. In addition to these six principles, the *Principles and Standards for School Mathematics* established ten standards overarching all grades K-12. These ten standards are comprised of five content and five process standards. The five content standards; Number & Operation, Algebra, Geometry, Measurement, and Data Analysis & Probability; lay out what students should learn. The five process standards; Problem Solving, Reasoning & Proof, Communication, Connections, and Representation; highlight ways for students to understand and use content knowledge.

### *Statement of the Problem*

Despite these efforts toward reform in mathematics education little seems to be changing in mathematics classrooms across the country. In their book, *The Teaching Gap: Best Ideas From the World's Teachers for Improving Education in the Classroom*, Stigler and Hiebert (1999) reported that "the typical U.S. lesson is consistent with the belief that school mathematics is a set of procedures" (p. 89) and that although 70% of teachers video taped as a part the Third International Mathematics and Science Study (TIMSS) believed they implemented reforms of the NCTM standards there was little evidence of reform. These results indicate an existing gap between the standards and their implementation in mathematics classrooms.

Although NCTM's standards documents and reports such as *The Mathematical Education of Teachers* (CBMS, 2001) have discussed the need for reform in the mathematical preparation of prospective teachers, most mathematics faculty still adhere to the traditional lecture format of instruction (Alsup, 2003). Prospective elementary teachers are a product of the mathematics instruction they are being asked to reform and have rarely seen or experienced reform for themselves (Ball, 1996). Additionally, Ball points out that prospective elementary teachers know that their students need to understand the mathematics they are to teach and not to just tell them how, but were not taught themselves in this fashion. Therefore, it is important for prospective elementary teachers' notions of what it means to learn be challenged and extended.

Prospective elementary teachers need to experience mathematics through active explorations and sense making of mathematical concepts.

While in the past some educators have doubted the benefits of children engaging in discovery learning activities (Friedlander, 1965), there has been little research on using guided discovery learning activities with prospective teachers. While some research has been conducted regarding the comparison of inquiry teaching to the traditional lecture methods of teaching no definitive results have been found. Generally, this research has shown only small positive results in favor of inquiry (Anderson, 2002). Researchers have argued that teachers' attitudes and self-efficacy play a significant role in the actions taken in their classroom and in the types of instructional strategies utilized (Gibson & Dembo, 1984). Teacher efficacy and attitudes that prospective elementary teachers develop about mathematics and its teaching follow them into the classroom and are related to the achievements (Schofield, 1981) and attitudes (Aiken, 1972) of their students.

Although there is a plethora of research on prospective elementary teachers in college mathematics methods courses, there is little research on prospective elementary teachers in college mathematics content courses. Recently, research studies on teaching and learning with understanding and ways children construct meaning in mathematics have emerged in the literature (Carpenter & Lehrer, 1999). However, there has been little research that describes the characteristics of non-traditional college mathematics content courses or on the impact of such courses on affective factors that influence the

perceptions of prospective elementary teachers about mathematics and their ability to teach mathematics.

### *Purpose of the Study*

The purpose of this study was to describe the characteristics of a non-traditional geometry content course designed for prospective elementary teachers and to focus on prospective elementary teachers' perceptions about these characteristics as affecting their attitude toward geometry and their mathematics teaching efficacy. This study also examined the impact the non-traditional geometry content course had on prospective elementary teachers' attitudes toward mathematics and mathematics teaching efficacy. The following research questions were addressed:

1. What are the characteristics of a non-traditional geometry content course for prospective elementary teachers?
2. What are the perceptions of prospective elementary teachers about the characteristics of this non-traditional geometry content course?
3. Are prospective elementary teachers' attitudes toward geometry influenced by this non-traditional geometry content course?
4. Are prospective elementary teachers' mathematics teaching efficacies influenced by this non-traditional geometry content course?

Results of this research contribute to literature on the mathematical preparation of prospective elementary teachers, more specifically to the literature on prospective elementary teachers learning mathematics through experiences

based in a constructivist learning theory. Additionally, it adds to the current literature on prospective teachers' attitudes and beliefs about mathematics.

#### *Assumptions*

1. It was assumed that each subject responded honestly and thoughtfully to all surveys and questionnaires.
2. It was assumed that the instructor of the observed course had a positive attitude toward mathematics, specifically geometry, and had a high mathematics teaching self-efficacy.

#### *Limitations*

1. The participants of this study were prospective elementary teachers enrolled in a content course designed for elementary majors in a Midwestern university town. Therefore, it was a sample of convenience and the findings may not be generalizable to the general population of all prospective elementary teachers.
2. The participants of this study were primarily Caucasian and female.
3. This study was carried out during a two-month summer course; therefore, findings may not be reflective of findings from a typical four-month fall or spring semester when students would have a longer time to think and reflect upon their experiences.
4. The researcher of this study had prior experiences with the course; therefore, she brought with her some preconceived notions about the

characteristics of the course and its impact on students enrolled in the course.

### *Definition of Terms*

*Attitudes Toward Geometry* – set of beliefs focusing on geometry that predisposes a person to respond in a certain way.

*Confidence to Learn Geometry* - how sure a person feels about their ability to learn and perform geometry tasks.

*Enjoyment of Studying Geometry* – how much pleasure or enjoyment a person feels while performing a geometry task.

*Prospective Elementary Teachers* - undergraduates who have declared a major in either elementary education or early childhood education.

*Mathematics Teaching Efficacy* – a person's opinion of their ability to teach mathematics and whether their teaching results in student success. It consists of two components , namely teaching outcome expectancy and personal teaching efficacy.

*Personal Teaching Efficacy* – “a belief in one's ability to teach effectively” (Enochs, Smith, & Huinker, 2000, p. 194).

*Teaching Outcome Expectancy* – “belief that effective teaching will have a positive effect on student learning” (Enochs, Smith, & Huinker, 2000, p. 194).

*Usefulness of Geometry* - how useful a person views geometry to be currently and in their future.

### *Organization of the Study*

This study is presented through a five chapter organizational format. The first chapter provides a general overview, the foundation and statement of the problem, the purpose of the study, assumptions and limitations, and definitions of terms that will be used throughout the study. A review of relevant literature that will provide the framework for the study is presented in chapter II. In chapter III, the methodology of the study will be discussed. Specifically, information relating to the participants, the research design, data collection procedures and instruments and the procedures for analysis of the data will be described. Chapter IV will present the analysis of the data and Chapter V will present the findings of the study as well as the conclusions, implications of this study, and call for additional research.

## Chapter II

### *Review of the Literature*

The purpose of this chapter is to review the research that is relevant to the examination of potential changes in prospective elementary teachers' attitudes toward geometry and mathematics teaching efficacy due to experiences during a non-traditional geometry content course. Several areas of research are relevant to the current study including:

1. constructivism as a theory of learning
2. pedagogical practices that support student knowledge construction
3. attitudes towards mathematics
4. self-efficacy.

### *Constructivism as a Theory of Learning*

Two views on the teaching of mathematics permeate mathematics education today. In one view the teacher is a transmitter of knowledge; this is known as transmissionist teaching or direct instruction (Selley, 1999). In traditional mathematics instruction, the way most of us were taught, students are passive receivers of knowledge, vessels to be filled. There is a static body of rules and algorithms invented by others that the teacher is expected to transmit to his/her students. The second view of teaching rests with the teacher serving as a facilitator of knowledge; this is known as constructivist teaching. In this view



of learning concepts are not simply transferred from teacher to student, but are constructed by the learner (von Glasersfeld, 1995). Constructivism presents a sharp contrast to the traditional mode of teaching.

Constructivism is not a set of instructional methods, but a philosophy of learning (Harris & Alexander, 1998) and a theory based on the writings of Jean Piaget during the last 10 to 15 years of his life (Fosnot, 1996). Constructivist learning theory asserts that “all mental activity is constructive” (Noddings, 1990, p. 14). According to Confrey (1990), constructivism is “a belief that all knowledge is necessarily a product of our own cognitive acts” (p. 108). Piaget (1973) believed that the basis of learning is discovery: “To understand is to discover, or reconstruct by rediscovery and such conditions must be complied with if in the future individuals are to be formed who are capable of production and creativity and not simply repetition” (p. 20). In addition, Vygotsky (1978) believes that construction of knowledge is a social act and that children internalize talk that occurs in group discussions to make meaning of their own experiences. From a Piagetian framework, Kamii (1991) asserts that knowledge is constructed “from the inside, in interaction with the environment, rather than by internalizing it directly from the outside” (p. 17).

In constructivism, learning is a process, not a product. Constructivism is based on the premise that by reflecting upon our experiences we construct meaning of the world around us. In a conversational interview, Piaget described his notion of constructivism as “knowledge is neither a copy of the object nor taking consciousness of a priori forms predetermined in the subject; it’s a

perpetual construction made by exchanges between the organism and the environment, from the biological point of view, and between thought and its object, from the cognitive point of view” (Bringuier, 1977, p. 110). Students enter a learning situation with previous knowledge and experiences they have arranged into an existing cognitive structure. As new information is encountered they attempt to understand this information based upon this cognitive structure. The construction of new knowledge occurs when this new situation challenges their existing cognitive structure. When this happens learners perceive that their present “cognitive structures do not adequately resolve, explain, predict, or allow for navigation of the problem situation” (Schifter & Simon, 1992, p. 188). A state of disequilibrium occurs when the students experience cognitive discomfort. Students arrive at new knowledge through an active process of assimilation, adding new information into existing knowledge structures, and through accommodation, modifying existing knowledge structures to be consistent with new information (Gadanidis, 1994).

Although constructivists may have different opinions about a variety of theoretical issues, they all tend to agree on the following tenets (Simpson, 2001; Gadanidis, 1994; Shifter & Simon, 1992; Clements & Battista, 1990; Noddings, 1990):

1. Knowledge is actively constructed.
2. Learners construct their own understandings through interaction with their physical and social environments.

3. A learner's cognitive structure is activated during the process of knowledge construction.
4. Learner bring to the learning situation previous experiences and knowledge they have organized into cognitive structures.
5. A learner's cognitive structure is not static but is continually being revised.
6. Learning situations should be relevant and meaningful for the learner.

Those who consider themselves to be radical constructivists add an additional controversial position:

7. For the learner, knowledge is not preexisting outside of their mind.

Piaget (1973) believed that a student's mastery of logic and reason are hindered by traditional instruction and that ineffective passive methods of teaching mathematics result in only a fraction of students absorbing mathematics knowledge. Additionally, Piaget suggests that the goal of education should be for learners to master concepts not simply to be able to repeat what they have learned or to memorize meaningless algorithms. Today in many traditional classrooms, children learn by hands-on activities, using a variety of manipulatives. However, these activities are carried out according to instructions of the teacher with little or no exploration or conjecturing encouraged.

When mathematics educators embrace constructivism as a theory of learning, their classrooms will look very different from the traditional classroom described above. In a learning environment that adheres to the constructivist learning theory, the student and teacher work together and share the

responsibility of constructing meaning. Mathematics educators must be cautious so they do not fall into the trap of believing that constructivism means if they leave their students alone they will naturally construct mathematical knowledge and understanding. Instead of utilizing algorithms that cause students to focus on the steps and not the mathematical reasoning of the problem, students should be encouraged to invent their own procedures (Kamii, Lewis, & Jones, 1991). The mathematics educator in a constructivist learning environment plays an important role to guide and support students' knowledge construction of mathematical ideas.

#### *Pedagogical Practices That Support Student Knowledge Construction*

The creation of a learning environment that supports student knowledge construction requires a paradigm shift for teachers and “the willing abandonment of familiar perspectives and practices and the adoption of new ones” (Brooks & Brooks, 1993, p. 25). Brooks and Brooks (1993) set forth a set of twelve characteristics that present the teacher as a mediator of knowledge construction rather than a giver of knowledge. The following is a summary of these characteristics:

1. Teachers encourage and accept student autonomy and initiative.
2. Teachers use raw data and primary sources along with manipulative, interactive, and physical materials.
3. When framing tasks, teachers use cognitive terminology such as “classify,” “analyze,” “predict,” and “create.”

4. Teachers allow student responses to drive lessons, shift instructional strategies, and alter content.
5. Teachers inquire about students' understandings of concepts before sharing their own understanding of those concepts.
6. Teachers encourage students to engage in dialogue, both with the teacher and with one another.
7. Teachers encourage student inquiry by asking thoughtful, open-ended questions and encouraging students to ask questions of each other.
8. Teachers seek elaboration of students' initial responses.
9. Teachers engage students in experiences that might engender contradictions to their initial hypotheses and then encourage discussion.
10. Teachers allow wait time after posing questions.
11. Teachers provide time for students to construct relationships and create metaphors.
12. Teachers nurture students' natural curiosity through frequent use of the learning cycle model.

There are a variety of pedagogical practices that are utilized in a learning environment that supports student knowledge construction including questioning, group work, worthwhile mathematical tasks, writing, and multiple solution strategies (Selley, 1999). In addition, the learning environment should foster mathematical discourse that enhances the learning of students (NCTM, 1991; Knuth & Peressini, 2001).

Traditional classroom teachers ask many questions, but these questions are often closed, convergent questions (What is the sum of 2 and 2?). These questions are usually centered on a fact or procedure that students should have memorized. However, the constructivist teacher encourages questions from students and then must decide whether to answer the question, have another student answer, or have students explore the question to find the answer themselves.

Questioning is crucial to student success in a learning environment where students are expected to construct their own mathematical meanings. These questions can not be scripted, but should help students express their thinking and aid the teacher in understanding the thought processes of their students. Teachers should be cautious not to ask questions in such a way that the questions solicit a desired response. Questions should encourage students to explain their thinking and to validate their thought processes (Ward, 2001) as well as challenge student thinking (NCTM, 1991).

According to Windshitl (1999), small groups of students talking and sharing ideas is a typical occurrence in a constructivist learning environment. Whether the students are working on a well-defined task or on an open exploration, students need to be able to discuss ideas, try out methods, and explore ideas in a collaborative manner with their peers. Talking and working with peers help students to make sense out of the problem they are working and students are more apt to be less inhibited and more likely to express their ideas (Selley, 1999). Small group interactions can create a support system and help

to counteract fear that many students may have in mathematics class. During small group time, the teacher's role is that of mediator and a facilitator in terms of asking questions that will provoke thoughtful thinking. Additionally, the teacher may encourage individual students to explain their thinking to his/her group members.

NCTM's *Professional Standards for Teaching Mathematics* (1991) points out that teachers must pose worthwhile mathematical tasks that engage their students' intellect and promote mathematical problem solving and reasoning. The use of mathematical tasks can help students see that there are multiple ways of solving a problem. According to Selley (1999), when students are finished with their investigations, they should share with each other, compare their methods, and if their methods differ they should attempt to make sense of these alternative methods. Mathematical investigations can be very structured and almost funnel the students into developing an idea, procedure or algorithm or they can be exploratory where each group may take a different direction and make discoveries or conjectures based upon their own line of thinking.

The act of writing "encourages students to examine their ideas and reflect on what they have learned. It helps them deepen and extend their understanding" (Burns, 1995, p. 13). NCTM (1989, 2000) points out the need for students to be able to communicate their mathematical thinking to others as well as being able to formalize their thoughts. Standera (1994) suggests that writing opens a line of communication between the teacher and the students, giving the teacher valuable information about their students' thinking. According to Ehrich

(1994) writing promotes student thinking; creates cognitive dissonance; clarifies, affirms and strengthens student understanding; develops language and communication skills; and aids the teacher in assessing student understanding.

An idea central to reform in mathematics education is for teachers to develop a learning environment that supports students doing and talking about mathematics (NCTM, 1991, 2000). Three of the six standards for teaching mathematics set forth in the *Professional Standards for Teaching Mathematics* (NCTM, 1991), deals with discourse in the mathematics classroom outlining the role of the teacher, the role of students, and the tools necessary for enhancing mathematical discourse. Wertsch and Toma (1995) describe two types of discourse: univocal and dialogic. Univocal refers to a passive reception of information while dialogic refers to a dynamic give-and-take communication between participants. As a part of classroom discourse, it is important for students to justify their thinking not only to explain their reasoning, but also to think about how the listener is making sense of what they are saying (Kamii, 2000b). Wertsch and Toma (1995) claim that 80% of the discourse in classrooms across America can be characterized as univocal. Knuth and Peressini (2001) point out that both univocal and dialogic discourse are important in the classroom, but that more emphasis needs to be placed on dialogic discourse in order for students to gain a deeper understanding of the mathematics they are studying.

While solving mathematical tasks students may take a variety of approaches and view them from a variety of perspectives. Student individuality



should be encouraged. In addition to multiple ways of solving a problem, mathematical tasks with multiple solutions should be given to students. This enables students to develop a real-world perspective on problem solving. In this context, most problems have multiple solutions and it is the problem solvers responsibility to decide upon the best solution for the current situation (Selley, 1999).

These pedagogical practices should not be viewed as discrete practices that are simply inserted into the current traditional mathematics classrooms. Teachers must question their vision of what it means for students to learn and develop a culture that “affects the way learners can interact with peers, relate to the teacher, and experience the subject matter” (Windschitl, 1999, p. 752). The decision is not whether to use specific pedagogical practices, but “how to use these techniques to complement rather than dominate student thinking” (Windschitl, 1999, p. 753).

Confrey (1990) summarizes constructivist teaching in mathematics as follows:

As a constructivist, when I teach mathematics I am not teaching students about the mathematical structures which underlie objects in the world; I am teaching them how to develop their cognition, how to see the world through a set of quantitative lenses which I believe provide a powerful way of making sense of the world, how to reflect on those lenses to create more and more powerful lenses and how to appreciate the role these

lenses play in the development of their culture. I am trying to teach them to use one tool of the intellect, mathematics (pp. 110-111).

A learning environment that supports student construction of knowledge effects not only students' cognitive structures, but also effects the affective domains. The affective domain includes the beliefs and attitudes of students. Attitudes such as confidence to learn mathematics play important roles in getting students to share their thinking (Ward, 2001).

### *Attitudes Toward Mathematics*

As early as 1935 a definition for attitude was purported when Allport defined attitude as a "mental and neural state of readiness, organized through experiences, exerting a directive or dynamic influence upon the individual's response to all objects and situations with which it is related" (p. 810). Rokeach (1972) defined attitude as an "organization of several beliefs focused on a specific object or situation predisposing one to respond in some preferential manner" (p. 159). Reyes (1980) defined attitudes toward mathematics as "feelings about mathematics and feelings about oneself as a learner of mathematics" (p. 164). Definitions that researchers use in mathematics education for attitudes toward mathematics varies, but each researcher should clearly explain as best they can the attitudes that they are attempting to measure (Kulm, 1980).

### *Development of Attitudes*

As soon as a child is exposed to mathematics their attitude toward mathematics begins developing. During the middle and junior high years, the development of attitudes toward mathematics appears to have the most impact. It is during these years that negative attitudes toward mathematics begin to be the most noticeable especially among girls. It is not known whether the increase in the level of abstractness of the mathematics, social preoccupations, or some other factor can be attributed to the increase in negative attitudes during these years (Aiken, 1985). Wilkins and Ma (2003) followed a group of 3,116 students from 7<sup>th</sup> grade through 12<sup>th</sup> grade measuring their attitudes toward mathematics each year. They found that the attitudes consistently became more negative each year. In 7<sup>th</sup> grade the students had a mean score of 3.78 on a five-point Likert-type scale while in 12<sup>th</sup> grade their mean score had dropped to 3.42 on a five-point Likert-type scale.

### *Effects of Attitude*

Affective variables such as attitudes toward mathematics are related to the learning of mathematics and to the learning environment in a classroom (Reyes, 1984). Some students are prevented from learning mathematics to their full potential due to their negative attitudes toward mathematics (Reyes, 1980). Therefore, to improve the learning of mathematics it is important to study students' attitudes toward mathematics (Fennema & Sherman, 1976; Reyes, 1984).

The relationship between attitudes toward mathematics (ATM) and achievement in mathematics (AIM) has been the focus of a plethora of research studies. Crosswhite (1972) found a small positive correlation between ATM and AIM of secondary students. In a review of literature, Aiken (1976) cited various studies that also showed a small positive correlation between ATM and AIM at the elementary, secondary, undergraduate, and postgraduate levels. Ma and Kishor (1997) conducted a meta-analysis of 113 studies that investigated the relationship between ATM and AIM. They found a statistically significant relationship between ATM and AIM indicating that the relationship was “positive and reliable, but not strong” (p. 35). McLeod (1992) suggests that “neither attitude nor achievement is dependent on the other; rather, they interact with each other in complex and unpredictable ways” (p. 582).

While overall measures of ATM have small positive correlation with AIM, other studies have revealed a relatively strong positive correlation between confidence and AIM (e.g. Dowling, 1978). In a study with middle school and high school age students, Fennema & Sherman (1978) found that, when there was no difference in mathematics achievement, females had a lower confidence in mathematics than males. Additionally, it has been shown that students with higher confidence in mathematics tend to have more frequent interactions with the teacher, both teacher and student initiated, than did students with a lower confidence in mathematics (Reyes & Fennema, 1980 as cited in Reyes, 1980). A student’s attitudes can affect how the teacher treats them. “Teachers seem to pay more attention to students who are sure of themselves in mathematics than

they do to students who are less sure of themselves, even when both sets of students perform equally well in mathematics” (Reyes, 1980, p. 163). Therefore, students can affect the amount of interaction they have with the teacher.

Another area of interest in attitude research is mathematics anxiety. Research has shown a consistently negative relationship between mathematics anxiety and AIM (Aiken, 1970; Crosswhite, 1972). In other words, a high level of AIM corresponds to a low level of mathematics anxiety. Betz (1978) examined the mathematics anxiety of college students. She found that mathematics anxiety was a problem for college students including those pursuing a career requiring an extensive background in mathematics and that for students in low level mathematics courses, women had lower levels of mathematics anxiety. Additionally, Betz’s study confirmed other studies’ claims that a high level of AIM corresponds to a low level of mathematics anxiety.

Reyes (1980) discovered that a student’s attitudes toward mathematics could affect their decision to enroll in mathematics courses and the amount of effort they put into learning the mathematics once they do enroll. Additionally, Aiken (1972) suggested that student’s attitudes play an important role in the mathematics course they choose to take, in their engagement of mathematical activities and in the perseverance in their efforts once they are engaged.

#### *Attitudes of Prospective Elementary Teachers*

Kulm (1980) pointed out that numerous studies have investigated prospective teachers’ attitudes toward mathematics. He then indicated two

reasons this was the potential of teachers to influence their students' attitudes toward mathematics and that prospective teachers are a readily available population with which mathematics educators can study attitudes toward mathematics. Prospective elementary teachers that prefer to teach in the primary grades tend to have a less positive attitude toward mathematics than those who prefer to teach the upper-elementary grades (Early, 1970 as cited in Kulm, 1980; Raines, 1971 as cited in Kulm, 1980). This has very important implications for the development of student attitudes since the teachers with the least favorable attitude toward mathematics are working with young children during their most formative stages.

Using a revised version of seven of the Fennema-Sherman Attitude Scales, Becker (1986) found that prospective elementary teachers' attitudes were slightly positive overall. On six of the seven subscales, they found prospective elementary teachers to be slightly anxious when it came to mathematics. In contrast to this study, Rech, Hartzell, and Stephens (1993) found that prospective elementary teachers possessed a slightly negative attitude toward mathematics. Additionally, Rech, Hartzell, and Stephens (1993) compared prospective elementary teachers' attitudes with the attitudes of the general college population. They found that while the prospective elementary teachers' attitudes were shown to be slightly negative the general college population's attitudes were shown to be slightly positive.

McDevitt, Heikkinen, Alcorn, Ambrosio, and Gardner (1993) found that when prospective teachers participated in integrated learning where they could

utilize prior knowledge that their attitudes toward mathematics improved. Additionally, Philippou and Christou (1998) found that prospective elementary teachers had significant positive changes in their attitude toward mathematics from the beginning of their first mathematics content course until they had completed all of their mathematics course work. The researchers also found that individual characteristics such as gender, grade-point averages from high school, or socioeconomic status do not predispose them to change their attitude toward mathematics; therefore, preservice teachers' experiences in their college mathematics content courses can significantly affect their attitudes.

### *Influence of Teachers' Attitudes*

A teacher's attitudes toward mathematics are influential on the development of their own students' attitudes toward mathematics (Aiken, 1972). McDevitt, Heikkinen, Alcorn, Ambrosio, and Gardner (1993) stated that a teacher's attitudes toward mathematics influences the amount of time spent on teaching mathematics and the methods they employ in its teaching. Research has shown that many teachers that are mathematically anxious tend to plan less instructional time for mathematics (Trice & Ogden, 1987). Schofeld (1981) found that teachers tend to transmit these negative attitudes to their students resulting in a decline in AIM by their students.

Meyer (1980) surveyed 120 prospective elementary teachers concerning their feelings about mathematics and what they felt accounted for their feelings. She found that teachers were the most significant factor affecting their attitudes,

regardless of whether the prospective elementary teachers' attitudes were positive or negative.

Ma (1999) found that 87% of the United States elementary teachers in her study either accepted students' claims or did not investigate their claim mathematically and she attributed this to their less favorable attitudes toward mathematics. She suggested that the two attitude factors that came into play were whether the teacher was interested in the student's claim and the confidence level of the teacher. Ma found that when the teachers were not confident in their ability to pursue the mathematics involved, they did not investigate the student's claim.

### *Measuring Mathematics Attitudes*

The measurement of mathematics attitudes can be obtained in a variety of ways. For example, attitudes toward mathematics can be determined through direct observations, interviews, questionnaires, student drawings and writings, and attitude scales. Aiken (1985) pointed out that attitude scales have proven to be the most popular method of measuring attitudes because of its "greater efficiency and apparent objectivity" (p. 3234).

The earliest instruments used to measure attitudes toward mathematics simply measured a student's like or dislike of a subject. These instruments were not of much value to teachers because they were not well defined in what they measured (Reyes, 1980). During the mid to late 1970's, there was a move in the development of attitude scales from a composite measure to a trend of



multiscore measures. For example, the Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976) consist of nine well-defined subscales including confidence, usefulness, and effectance motivation consisting of 12 questions each, 6 positively and 6 negatively worded. In addition, there are a variety of instruments that measure a specific attitude such as the Mathematics Anxiety Rating Scale (MARS) and Dowling's Mathematics Confidence Scale (1978). Aiken's (1985) reported that "the reliabilities of well-constructed scales of attitude toward mathematics are usually in the 0.80s and 0.90s" (p. 3234).

The Attitude Toward Geometry Scales (ATGS) was developed by Utley (2004). The ATGS was developed after reviewing a variety of existing instruments used to measure attitudes toward mathematics (e.g. Akin, 1974; Fennema & Sherman, 1976; Dowling, 1978; Tapia, 1996). The ATGS was created to study preservice teachers' confidence to learn geometry, perceived usefulness of geometry, and enjoyment of geometry. The Likert-type instrument has been found to be a valid and reliable instrument to use with the general college student population ( $n = 264$ ; Cronbach alpha = 0.96) and with preservice teachers ( $n = 100$ ; Cronbach alpha = 0.97) (Utley, 2004).

### *Self-Efficacy*

In addition to attitude, self-efficacy is an affective variable that is important when individuals are doing and teaching mathematics. According to Bandura (1986), how a person thinks, believes, and feels will affect how they behave.

Thus, the effects of their actions partially affect their thoughts and reactions to a task.

Self-efficacy is defined as a person's opinion of himself or herself based on prior experiences. Bandura (1977) pointed out that self-efficacy is a situation-specific belief. Perceived self-efficacy is defined by social learning theorists as a person's sense of confidence regarding his or her performance of specific tasks. Efficacy is concerned not with the skills one has but with the judgments of what one can do with whatever skills one possesses. A person with a positive self-efficacy tends to persist until they succeed while a person with a negative self-efficacy tends to stay away from or quit difficult tasks.

Bandura (1977, 1986, 1997) has set up a theoretical framework for the study of teacher efficacy. He argues that individuals' beliefs are influenced by two classes of expectations - outcome expectation and efficacy expectation. Outcome expectation refers to a person's estimate that a given behavior will lead to a certain outcome. For example, when teachers think that good instruction can offset the influence of a poor home environment they are said to have high outcome expectancy. Efficacy expectation refers to a person's conviction that he or she can successfully execute the behavior required to produce a desired outcome. For example, when teachers are confident that they are personally capable of good instruction and can offset the influence of a poor home environment they are said to have high personal teaching efficacy or efficacy expectation. Gibson and Demo (1984) state that:

Outcome and efficacy expectations are differentiated because individuals can believe that certain behaviors will produce certain outcomes, but if they do not believe that they can perform the necessary activities, they will not initiate the relevant behaviors, or if they do, they will not persist (p. 570).

Bandura (1977) suggests that self-efficacy is most malleable in the early stages of learning. Research has shown that pre-service teachers reevaluate their teaching efficacy beliefs as they engage in new tasks or when they feel a task is important for their future. In contrast, in-service teachers' teaching efficacy beliefs are resistance to change once these beliefs are firmly established (Gist & Mitchell, 1992).

### *Sources of Self-efficacy Beliefs*

The interaction between an individual's efficacy beliefs with environmental and behavior events impacts the individual's efficacy beliefs. Therefore, this relationship provides an opportunity for the individuals efficacy beliefs to be changed (Dellinger, 2002). According to Bandura (1986, 1997) self-efficacy beliefs can be enhanced or raised through four main types of influence: mastery experiences, vicarious experiences, social persuasion, and changes in physiological and emotional states. Mastery experiences are the most influential way to help individuals create a strong sense of efficacy. Individuals gauge the results of their performance on a task and their interpretation of this performance influences their self-efficacy beliefs (Pajares, 1997). Through the successful

performance of tasks, individuals can enhance their self-efficacy; however, Bandura (1997) cautions that if a person only experiences success that he or she will be easily discouraged by failures. Individuals need to develop a resilient sense of self-efficacy in order to overcome obstacles and failures.

A second influential way to enhance self-efficacy beliefs is through vicarious experiences where others model behavior. The more closely that an individual identifies with the model, the stronger the influence of that model. When an individual can identify with the model and the model performs well, the individual's belief that he or she can succeed has been raised. On the other hand, when the model does not perform well the individual's belief that he or she can succeed is undermined (Bandura, 1997; Tschannen-Moran, Hoy, & Hoy, 1998).

The third way of strengthening an individual's self-efficacy beliefs is through social persuasion. Through verbal and nonverbal communication from others, an individual can be persuaded or dissuaded of his or her capability to master a given activity. Social persuasion may involve a pep talk or feedback on a performance task (Tschannen-Moran, Hoy, & Hoy, 1998). Social persuasion has a weaker influence on self-efficacy beliefs than does mastery or vicarious experiences (Pajares, 1997). The effect of persuasion depends on the credibility, trustworthiness, and expertise of the persuader (Bandura, 1986).

A fourth way an individual's efficacy belief may be altered is through his or her physiological and emotional states. Pajares (1997) noted that when an individual has a strong emotional response to a task, cues are provided about the

anticipated reaction of the individual. How the individual perceives his or her response determines how his or her efficacy belief will be affected. When individuals experience a positive emotional response, they tend to view the experience as an energizer and their self efficacy beliefs are enhanced; however, when they experience aversive emotional responses, their perceptions of their capabilities are lowered thus lowering their self-efficacy beliefs (Bandura, 1997).

### *Teacher Efficacy*

Teacher efficacy is a form of self-efficacy relevant to education (Gibson & Dembo, 1984; Henson, 2001; Tschannen-Moran, Hoy, & Hoy, 1998) and has gained increased attention among researchers (Pajares, 1992). A teacher's efficacy belief has been defined in a variety of ways, such as "the extent to which the teacher believes he or she has the capacity to affect student performance" (Berman, McLaughlin, Bass, Pauly, & Sellman, 1977, p. 137) or as teachers' "confidence in their ability to promote students' learning" (Hoy, 2000, p. 2). Similar to self-efficacy, research has shown that teacher efficacy consists of two related constructs: personal teaching efficacy and teaching outcome expectancy (Gibson & Dembo, 1984). Personal teaching efficacy is the belief in one's competence in teaching (Tschannen-Moran & Hoy, 2001). Teaching outcome expectancy is the belief that effective teaching results in a positive effect on student learning (Enochs, Smith & Huinker, 2000).

The study of teacher efficacy originated with RAND researchers' examination of whether teachers believed they could control the reinforcement of

their actions (Armor et al., 1976). As a construct, teacher efficacy is about a quarter of a century old; its meaning and measure are still a subject of debate (Henson, 2001; Tschannen-Moran, Hoy, & Hoy, 1998). Enochs, Smith, and Huinker (2000) pointed out that Bandura asserts that self-efficacy beliefs are situational specific; therefore measures should be used to measure a specific task such as teaching. Teacher efficacy has been the focus of study by several researchers (e.g. Enochs & Riggs, 1990; Gibson & Dembo, 1984; Guskey, 1988; Pajares, 1997; Woolfolk & Hoy, 1990). Philippou and Christou (2002) point out that with respect to the learning and teaching of mathematics, teacher efficacy is an area in need of further research.

### *Effects of Teacher Efficacy*

The construct of efficacy has been acknowledged as having important implications for the field of education. The belief that one can effectively teach is critical to quality instruction as well as to adherence to reform issues of best practice (Allinder, 1994; Coladarci, 1992; Guskey, 1988; Stein & Wang, 1988) and to the types of instruction utilized (Gibson & Dembo, 1984). Coladarci (1992) found that personal teaching efficacy and teaching outcome expectancy are strong predictors of a teacher's commitment to teaching with teaching outcome expectancy as the stronger of the two as a predictor. Teacher efficacy has been shown to be related to both student outcomes and to teacher behavior.

*Teacher efficacy and student outcome.* Teacher efficacy has been found to be correlated to student achievement in reading (Armor, et al, 1976; Ashton &

Webb, 1986) and mathematics (Allinder, 1995; Ashton & Webb, 1986; Tracz & Gibson, 1986). Researchers have found that students of efficacious teachers tend to outperform students in other classes on achievement tests such as the Iowa Test of Basic Skills (Moore & Esselman, 1992) and the Ontario Assessment Instrument Pool (Ross, 1992). Additionally, teacher efficacy has been found to be related to students' sense of self-efficacy (Anderson, Greene, & Loewen, 1988) and student motivation (Midgley, Feldlaufer, & Eccles, 1989).

*Teacher efficacy and teacher behavior.* Teacher efficacy has been found to be related to teacher behaviors. Gibson and Dembo (1984) found that the more efficacious the teacher the more they tended to persist with struggling students, the less they criticized incorrect student responses and the more flexible they were if the classroom routine was interrupted. Teachers with high teaching efficacy tend to be more open to try new instructional methods and search for improved teaching strategies (Allinder, 1994; Czernaik & Schriver, 1994; Guskey, 1988). Highly efficacious teachers tend to use inquiry, discovery and student-centered teaching strategies (Allinder, 1994; Czernaik, 1990; Enochs, Smith, & Huinker, 2000). Allinder (1995) found that teachers with a high sense of teacher efficacy set higher goals for their students. Teachers exhibiting a high degree of teacher efficacy tend to be more likely to use learning centers, observation activities, simulations, and small group discussions (Czernaik & Schriver, 1994).

Bandura (1977) suggests that teachers with a low teaching efficacy tend to believe that student motivation and performance rests with the home

environment rather than the teacher. Low efficacious teachers tend to be more concerned with student behavior rather than student learning and tend to rely on traditional teacher-directed strategies such as lectures (Czerniak & Schriver, 1994). Wenner (2001) examined the science and mathematics efficacy beliefs of both in-service (N=101) and prospective teachers (N=187). Based upon participant responses to the Science Teaching Efficacy Belief Instrument (Enochs & Riggs, 1990) and a variation of this instrument developed by substituting mathematics for science, his study suggests that low teacher efficacy is a significant contributing factor to teachers' reluctance to teach mathematics and science. Similarly, Ashton & Webb (1986) found that teachers whose self-efficacy is low with reference to a particular subject would reduce teaching time for that subject or avoid teaching that subject altogether.

#### *Efficacy Beliefs of Prospective Teachers*

Bandura (1977) suggested that teaching efficacy is malleable early in the learning process then tends to be resistant to change. Therefore, researchers have studied how teaching efficacy develops among prospective teachers. Woolfolk and Hoy (1990) found that teaching efficacy of prospective teachers is linked to their attitudes toward children and issues of control. Researchers have found that student teachers' personal teaching efficacy was positively, although weakly, related to lesson presentation, questioning, and management of student behavior when rated by their supervising teacher (Saklofske, Michaylu, & Randhawa, 1988). Teaching efficacy of prospective teachers has been found to



increase during methods courses and then fall during student teaching (Hoy & Woolfolk, 1990; Spector, 1990). Huinker and Madison (1997) found that mathematics methods courses have a positive effect on prospective teachers' mathematics teaching efficacy.

### *Measuring Teacher-Efficacy*

Teacher efficacy as a construct was first conceived in 1976 by Rand researchers in their study of teacher characteristics and student learning. This first measure of teacher efficacy was grounded in Rotter's social learning theory (Tschannen-Moran & Hoy, 2001). The Rand researchers placed two statements on their questionnaire and asked teachers to indicate their level of agreement with the statements. The sum of the two statements was called teacher efficacy (TE). These two statements were:

1. When it comes right down to it, a teacher really can't do much because most of a student's motivation and performance depends on his or her home environment.
2. If I really try hard, I can get through to even the most difficult or unmotivated students.

When a teacher agrees with the first statement, he or she is indicating that environmental factors are a stronger influence on student learning than the teacher's ability to effectively teach. Statements that deal with environmental factors were labeled general teaching efficacy (GTE); Bandura (1977) referred to this construct as teaching outcome expectancy (TOE). When a teacher agrees

with the second statement, he or she is indicating his or her confidence in his or her ability to effectively help students learn. Statements of this type have been referred to as personal teaching efficacy (PTE).

In the early 1980s, Gibson and Dembo (1984) sought to construct and validate a reliable way to measure teaching efficacy. They used the two items from the RAND study that corresponded to Bandura's (1977) self-efficacy and outcome expectancy of social cognitive theory to develop their 30-item Teacher Efficacy Scale. Factor analysis revealed a two construct instrument that directly related to Bandura's two-factor theoretical model of self-efficacy.

There have been a variety of subject specific modifications of the Gibson and Dembo instrument. In 1990, Enochs and Riggs published a Science Teaching Efficacy Belief Instrument (STEBI) for prospective elementary teachers to measure efficacy of teaching science. Consistent with the Gibson and Dembo TES, they found two uncorrelated factors that they called personal science teaching efficacy and science teaching outcome expectancy. Using the STEBI, several researchers developed other subject-specific teacher efficacy instruments. Sia (1992) developed the Environmental Education Efficacy Belief Instrument (EEEEBI) for preservice teachers. Additionally, Rubeck and Enochs (1991) established an instrument to measure chemistry teaching efficacy. Huinker and Enochs (1995) developed the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) for prospective teachers by modifying the STEBI. Similar to the STEBI, the MTEBI is a 5-point Likert-scaled instrument consisting of two scales - personal mathematics teaching efficacy (PMTE) and mathematics

teaching outcome expectancy (MTOE). The Reading Teachers' Self-Efficacy Instrument (RTSEI) for prospective teachers was developed by Szabo, Mokhtari, and Walker (in review) following the pattern of the STEBI-B and the MTEBI.

### *Summary*

Researchers have argued that attitudes and self-efficacy play a significant role in the actions taken by teachers in the classroom. Additionally, NCTM's *Principles and Standards for School Mathematics* (2000) suggest that in order for students to be successful in mathematics they must believe they can do mathematics and that they must see the usefulness of the mathematics they are studying. Exploring the impact of a non-traditional geometry content course on prospective elementary teachers' attitudes and mathematics self-efficacy will enable mathematicians and mathematics educators to better plan and implement mathematics content courses in the future. In addition, changing the attitudes and self-efficacy of prospective elementary teachers should be an important part of teacher education programs.

## Chapter III

### *Methodology*

The purpose of this study was to describe the characteristics of non-traditional geometry content course designed for prospective elementary teachers and examine prospective elementary teachers' perceptions about these characteristics as affecting their attitude toward geometry and their mathematics teaching efficacy. Both qualitative and quantitative data were collected and analyzed to determine the characteristics of the non-traditional geometry content course and to gain information concerning the attitudes toward geometry and mathematics teaching efficacy of each prospective elementary teacher. The appropriateness of combining qualitative and quantitative methodology is addressed in this chapter along with data collection and data analysis procedures unique to each research paradigm.

The research questions guiding this study were:

1. What are the characteristics of a non-traditional geometry content course for prospective elementary teachers?
2. What are the perceptions of prospective elementary teachers about the characteristics of this non-traditional geometry content course?

3. Are prospective elementary teachers' attitudes toward geometry influenced by this non-traditional geometry content course?
4. Are prospective elementary teachers' mathematics teaching efficacies influenced by this non-traditional geometry content course?

### *Combining Qualitative and Quantitative Designs*

This study employed both qualitative and quantitative data. Creswell (1994) and Tashakkori and Teddlie (1998, 2003) refer to this as combining or mixing of approaches. Debate continues today about the appropriateness of combining the two approaches within a single study, with some researchers arguing that only quantitative methods are truly scientific and others arguing the reverse. Researchers adhering to either of these arguments are considered purists. Purists argue that the two approaches should not be mixed because their assumptions are in opposition to each other. Purists also tend to adhere to the methodology that fits their particular worldview. According to Creswell (1994), the situationalists believe that the approach should be appropriate to the situation. In other words, the research questions drive whether the researcher should follow a quantitative or qualitative methodology rather than the researcher's worldview. Pragmatists have begun to argue the value of combining qualitative and quantitative methods within a single study and argue against the existence of a dichotomy between the two approaches. Additionally, some social scientists are leaning toward acceptance of a mixing of methods.

Researchers first suggested combining methods for purposes of triangulation of data (Patton, 2002). They believed that triangulation strengthened the validity of the results. After reviewing some fifty-seven articles using mixed methods, Greene, Caracelli & Graham (1989) posited five major purposes for the combination of qualitative and quantitative methods. The current study was guided by three of these purposes. First, the combination of methods helps in the classical sense of triangulation of data. In other words, it helps in testing for the convergence and consistency of findings from various data sources. Second, the methods are complementary to each other. They tend to clarify and illustrate findings from one method with findings from the other method. The data overlap and different aspects of the phenomena are revealed. Patton (2002) also stated that they are used in a complementary fashion in order to “answer different questions that do not easily come together to provide a single, well-integrated picture of the situation” (p. 557). Third, combining of methods can expand the study by providing richness and details, therefore adding scope and breadth to the study.

Creswell (1994) and Tashakkori and Teddlie (1998, 2003) describe several designs for combining methods. These researchers describe what they independently refer to as a mixed-model design. In this design qualitative and quantitative approaches are mixed at a variety of steps in the research process. For example, the approaches might be mixed in the introduction, in how the literature review is put together, in the statement of the purpose and research questions, in the methodology section and/or in the reporting of the results.

Creswell suggests that this design uses the advantages of both qualitative and quantitative methods; however, the researcher must be well versed in both methods to be effective.

### *Research Design*

According to Creswell (1994), a phenomenological study is conducted when the researcher plans to examine and describe human experiences – a telling of their life-world experiences. Mertens (1998) states that phenomenology “seeks the individual’s perceptions and meaning of a phenomenon or experience” (p. 169). Therefore in this study, a form of phenomenology was conducted in order to examine and describe the perceptions of preservice elementary teachers about the characteristics of the non-traditional geometry content course as affecting their attitude toward geometry and their mathematics teaching efficacy.

For triangulation of data, videotaping occurred during approximately half of the class periods, field notes were taken by the researcher, course documents were examined, interviews with four students and the instructor were conducted, and self-reported surveys were solicited. According to Mertens (1998), in qualitative research the researcher is the instrument since he or she decides what to observe and what to write down. In this study the researcher will take on the observation role that Adler and Adler (1994 as cited in Mertens, 1998) have described as peripheral-member-researcher. In this role the researcher

observes and interacts “closely enough with members to establish an insider’s perspective, without participating in the activities of the core group” (p. 318).

### *Participants and Instructional Setting*

The participants for this study were twenty-one (19 female and 2 male) prospective elementary teachers who were enrolled in a non-traditional geometry content course at a land grant university in the Midwestern United States. While the participants were not selected at random, all students who were enrolled in the non-traditional geometry content course were invited to participate and only those who agreed to participate were included in this study. Demographic information was collected on each participant using the form in Appendix A.

The average age of the participants was 21.7 years. Twenty (95%) of the participants were of traditional age ( $\leq 25$  years old), with the remaining participant’s age being 27. Eighteen (86%) Caucasians were the largest group of the participants, with non-Caucasians (Asian American and Native American) making up the remaining three (14%) participants.

A non-traditional geometry content course designed for prospective elementary teachers was the instructional setting for this study and the course is specifically designed to help prospective elementary teachers gain conceptual understanding of geometric concepts. Student participation in this study was a part of the normal course work with the exception of the pre and post data collected.



### *Data Collection*

In this study, data were collected over an extended period of time primarily in a classroom situation. There were multiple data sources including both qualitative and quantitative data. These multiple data sources helped the researcher gain a more complete and accurate picture thus helping to confirm the interpretations made by the researcher.

### *Qualitative Measures*

*Observations.* Field observations were an important part of this study in order to make sense of the classroom interactions and happenings that would have been difficult to capture and record at a later time from reflection. Field observations occurred throughout the course and each of these sessions was videotaped. Careful attention was made not to video any student who wished not to participate in this study. Field notes were taken by the researcher during each of these class periods including teaching strategies that were used, reminder notes to be sure and review certain scenarios on the videotape, and general notes on the happenings in the classroom.

*Written Responses.* Prospective teachers were asked to provide information about their attitudes and perceptions of themselves as an elementary teacher focusing particularly on those aspects that relate to their attitude toward geometry and to their own mathematics teaching efficacy. These were in the form of a pre- and a post questionnaire (Appendix B). Additionally, the post survey asked participants to tell which aspect (CD problems, class discussions,

projects, group activities, writing or seeing multiple solution strategies) of the course impacted them the most as a future teacher and why.

Journal prompts were a normal part of the course and solicited students' thoughts and feelings about their experiences in the course including various instructional strategies. At the end of the course the instructor asked students to respond to journal prompts about how their experiences in the course had effected their confidence to learn geometry, their confidence in their ability to teach geometric concepts, their belief in the usefulness of learning geometry, and their enjoyment of learning geometry. See Appendix C for samples of the journal prompts.

*Interviews.* Audio taped, semi-structured interviews were conducted with prospective elementary teachers and with the course instructor. From those prospective elementary teachers willing to be interviewed six were selected based upon their responses to the pre/post attitude survey. Three students interviewed had an increase in their overall attitude score and three students had a decrease in their overall attitude score. The interview protocol (Appendix D) consisted of questions in three areas: (a) past and present experiences in geometry content courses, (b) attitude, and (c) teaching efficacy. These interviews allowed the researcher to gain a better picture of the perceptions of prospective teachers about the characteristics of the non-traditional geometry content course and how these characteristics affected the prospective teachers' attitude toward geometry and their mathematics teaching efficacy. Audio recordings of the interviews were transcribed and analyzed. The researcher used

what Maxwell (1996) refers to as a categorizing strategy that utilizes coding and thematic analysis. According to Rubin & Rubin (1995) “coding is the process of grouping interviewees’ responses into categories that bring together the similar ideas, concepts, or themes you have discovered, or steps or stages in a process” (p. 238). Additionally, Schuman (as cited in Glesne & Peshkin, 1992) states that data gained from the interview can be a form of validity check for the responses of the participants to the various surveys and instruments.

*Course Documents.* Course documents were evaluated to give the researcher some insight into the characteristics of the course. Included in this analysis was the student workbook style text, the course web site, and student work samples. Glesne and Peshkin (1992) suggest that “documents corroborate your observations and interviews and thus make your findings more trustworthy... They also provide you with historical, demographic, and sometimes personal information that is unavailable from other sources” (p. 52). According to Patton (2002), these documents can provide the researcher with information that can not be observed.

### *Quantitative Measures*

*Attitude Toward Geometry Scales (ATGS).* The Attitude Toward Geometry Scales (Appendix E) was designed to specifically measure the attitudes of prospective elementary teachers toward geometry. The instrument contains three subscales: 1) confidence to learn geometry, 2) usefulness of studying geometry, and 3) enjoyment of studying geometry. The instrument is a

5-point Likert-scaled survey consisting of thirty-two statements, seventeen positively and fifteen negatively worded statements. Negatively worded items were recoded prior to analysis. Scores on the instrument can range from 32 to 160, with higher scores indicative of an overall higher attitude toward geometry. Illustrative items are “I believe that I will need geometry for my future” and “I feel sure of myself when doing geometry problems”. Utley (2004) reported the instrument to have content, criterion, and construct validity. Additionally, she reported the Cronbach alpha coefficients of 0.96 for the instrument as a whole and subscale Cronbach alpha coefficients of 0.95 for the confidence subscale, 0.93 for the usefulness subscale, and 0.92 for the enjoyment subscale.

*Mathematics Teaching Efficacy Beliefs Instrument.* Huinker and Enochs (1995) developed the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) for prospective teachers. The MTEBI is a 5-point Likert-scaled instrument consisting of two scales - personal mathematics teaching efficacy (PMTE) and mathematics teaching outcome expectancy (MTOE). The PMTE scale consists of thirteen statements and the MTOE scale consists of eight statements. The instrument contains eight negatively worded items that were recoded prior to analysis. Scores on the PMTE can range from 13 to 65 with higher scores indicating greater teaching efficacy. Scores on the MTOE can range from 8 to 40, with higher scores indicating a greater belief in their ability to impact student learning. Enochs, Smith, and Huinker (2000) performed a confirmatory analysis on the instrument and found that the two scales are independent of each other. Additionally, they reported Cronbach alpha coefficients of 0.88 on the PMTE

scale and 0.77 for the MTOE scale. Illustrative items are “I know how to teach mathematics concepts effectively” from the PMTE scale and “When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach” from the MTOE scale. A copy of this instrument is included in Appendix F.

### *Procedure*

This study was conducted in four phases. After initial IRB approval (Appendix H), the first phase involved outlining the study to students in the non-traditional geometry content course and asking students to participate in the study. Those agreeing to participate signed an informed consent outlining the study, completed a demographic survey, completed both the MTEBI and the ATGI instruments, and completed a pre-questionnaire.

The second phase of the study involved the researcher conducting observations of twenty class sessions. Each of these observations involved the researcher taking field notes and videotaping. The field notes included comments by the instructor and students, general observations of the workings of the class and to review an episode from the classroom on the video. Additionally, during this phase course documents and journal prompts were collected.

The third phase of the study involved participants again completing the MTEBI and the ATGI. Additionally, a post-questionnaire and a journal prompt concerning how their experiences in the course had effected their confidence to

learn geometry, feelings about their ability to teaching geometric concepts, belief in the usefulness of learning geometry, and enjoyment of geometry.

The fourth phase of the study involved conducting interviews with six of the prospective elementary teachers that were willing to be interviewed. Interviewees were selected based on their responses to the questionnaires and their responses to the self-reporting MTEBI and ATGI instruments. Each interview was audio taped and lasted approximately forty-five minutes. Interviews were conducted during the fall academic semester, 2003. Additionally, the instructor was interviewed to gain information about the course and his perceptions of the course as affecting attitudes and teaching efficacy of the students. Table 1 shows an overview of data collected for the study.

Table 1  
Overview of Data Collected

Phase 1	Phase 2	Phase 3	Phase 4
<ul style="list-style-type: none"> <li>• Demographic Data</li> </ul>	<ul style="list-style-type: none"> <li>• Course Documents</li> </ul>	<ul style="list-style-type: none"> <li>• MTEBI</li> </ul>	<ul style="list-style-type: none"> <li>• Interviews</li> </ul>
<ul style="list-style-type: none"> <li>• MTEBI</li> </ul>	<ul style="list-style-type: none"> <li>• Video of Class Sessions</li> </ul>	<ul style="list-style-type: none"> <li>• ATGS</li> </ul>	
<ul style="list-style-type: none"> <li>• ATGS</li> </ul>	<ul style="list-style-type: none"> <li>• Observation of Class Sessions</li> </ul>	<ul style="list-style-type: none"> <li>• Post-Questionnaire</li> </ul>	
<ul style="list-style-type: none"> <li>• Pre-Questionnaire</li> </ul>	<ul style="list-style-type: none"> <li>• Journal Prompts</li> </ul>		

### *Data Analysis*

Quantitative data was analyzed in order to determine to what extent the non-traditional geometry content course had an impact on prospective elementary teachers' attitudes toward geometry and to their mathematics teaching efficacies. Item responses from each instrument was coded and entered into SPSS 10.0 (SPSS, 1999) exactly as each appeared on the instrument. All negatively worded items were recoded in order to allow for consistency in reporting of the scoring, the higher the score the more positive the respondents' attitude or teaching efficacy. Descriptive statistics (means, standard deviations, ranges, and confidence intervals) were reported on each administration of all instruments as well as for the subscales of all instruments. Additionally, Cronbach's alphas are reported on each administration of all instruments as well as for the subscales of all instruments. Paired t-tests were conducted to check for a significant difference between the pre- and posttest scores on the ATGI and the MTEBI.

A constant comparative method (Strauss & Corbin, 1998) for analysis was used for the qualitative data. To understand the changing attitudes or teaching efficacy of prospective teachers requires openness to emergent themes and areas for exploration that can only be revealed through continual analysis of the data as it is collected. Thus, this study employed the constant comparative method for analysis of data, creating a process of identifying categories, themes, and patterns in the data. These categories, themes, and patterns, grounded in the data, were used to describe the characteristics of the course and the

perceptions of prospective teachers about the characteristics of the non-traditional geometry content course and how the characteristics affected the students' attitude toward geometry and their mathematics teaching efficacy. While data were analyzed for emergent themes the themes of whole class discussion, group activities, geometric constructions, projects, and reflective writing were predetermined. Cresswell (1998) recommends that a general review of data be conducted to help the researcher get an overall picture and to help the researcher begin the process of examining the data. Next, he advocates creating charts or displays of the data in order for the researcher to begin to develop codes or categories. At this point he suggests the researcher develop five or six categories to sort data into with the idea that this coding scheme can be expanded to include more categories as necessary.

### *Ethical Considerations*

All participants' responses were coded to protect their anonymity. The use of pseudonyms for all participants was used to help ensure the privacy and confidentiality of all participants. An assurance of privacy and confidentiality was presented in writing to all participants.



## *Summary*

A summary of each research question and the related measure and statistic follows:

1. What are the characteristics of a non-traditional geometry content course for prospective elementary teachers? Videotapes, field notes, course documents, and interview transcripts were examined and a coding scheme established in order to see emergent themes and patterns.
2. What are the perceptions of prospective elementary teachers about the characteristics of this non-traditional geometry content course? Videotapes, field notes, course documents, and interview transcripts were examined and a coding scheme established in order to see emergent themes and patterns.
3. Are prospective elementary teachers' attitudes toward geometry influenced by this non-traditional geometry content course? Descriptive statistical analysis (means, standard deviations, and confidence intervals) and a paired t-test between the pre- and post ATGS were conducted. In addition, journal prompts, open-ended questionnaires, and interview transcripts were analyzed for emerging themes and patterns.
4. Are prospective elementary teachers' mathematics teaching efficacies influenced by this non-traditional geometry content course? Descriptive statistical analysis (means, standard deviations, and

confidence intervals) and a paired t-test between the pre and post MTEBI were conducted. In addition, journal prompts, open-ended questionnaires, and interview transcripts were analyzed for emerging themes and patterns.

The results of the data analysis are presented in Chapter IV with a discussion of the findings presented in Chapter V.

## Chapter IV

### *Results*

This study combined quantitative and qualitative data gathered from prospective elementary teachers to characterize a non-traditional geometry content course designed for prospective elementary teachers and to investigate prospective elementary teachers' perceptions about these characteristics as affecting their attitude toward geometry and their mathematics teaching efficacy. In this chapter research data will be presented that was gleaned from observations, written responses, interviews, course documents, and pre/post attitude and mathematics teaching efficacy surveys. The research questions guiding this study were:

1. What are the characteristics of a non-traditional geometry content course for prospective elementary teachers?
2. What are the perceptions of prospective elementary teachers about the characteristics of a non-traditional geometry content course for prospective elementary teachers?
3. Are prospective elementary teachers' attitudes toward geometry influenced by this non-traditional geometry content course?
4. Are prospective elementary teachers' mathematics teaching efficacies influenced by this non-traditional geometry content course?

In this chapter, four major sections will be presented. First, the characteristics of the non-traditional geometry course will be described. Second, qualitative data will be examined to determine the perceptions of the prospective elementary teachers about these characteristics. In the third section, quantitative and qualitative data will be examined to determine the influence of the characteristics of this non-traditional geometry content course on the attitude toward geometry of prospective elementary teachers. The fourth section includes an examination of both quantitative and qualitative data to determine the influence of the characteristics of this non-traditional geometry content course on the mathematics teaching efficacy of prospective elementary teachers.

#### *Characteristics of a Non-Traditional Geometry Content Course*

The course examined in this study is a non-traditional geometry content course that was designed for prospective elementary teachers and referred to as Geometric Structures. The goal of the course was “to provide a discovery-based and creative experience with geometry...[and to] support each student’s growth toward being a confident, independent learner empowered to make sense of the geometric world” (Aichele & Wolfe, in press, p. iii). The topics covered in the course were aligned with topics that prospective elementary teachers encounter on competency and certification examinations. The course was organized around the interweaving of four types of manipulatives paper folding, geoboards, straightedge and compass, and miras to help students make sense of and represent basic geometric ideas.

During the first two weeks of classes, the instructor insured through discussion with the students that his view of teaching mathematics and student responsibility was understood. He shared his belief that most mathematics courses designed for prospective elementary teachers are “too authoritarian on the part of the teacher and too passive on the part of the student” (Fieldnotes, 6-17-03). In line with this, the course text stated that “practices for this course place substantially more of the burden for initiating and assessing learning on the shoulders of the student” (Aichele & Wolfe, in press, p. iv). Analysis of course documents, video tapes, and fieldnotes suggest that the instructor encourages and seeks to develop autonomous thinking among the prospective elementary teachers.

One of the hallmarks of this non-traditional geometry course is whole class discussion fueled by a variety of activities including daily activity sheets, group activities, and geometric constructions. Additionally, students are given projects to work on outside of class. While projects are not a specific focus of whole class discussion, they are inextricably connected to students’ sense making as it relates to their understanding of geometric concepts that are central in the whole class discussions.

### *Creating An Environment Through Whole Class Discussions*

Implicit in the above stated goal for this course is the creation and development of a dynamic environment wherein students can discuss and grapple with significant mathematics. Contrary to the traditional lecture-oriented

structure of most college mathematics courses where students depend on the teacher to impart explanations and answers, this course provided students with the opportunity to listen and converse with each other about concepts they were struggling to understand and about concepts that made sense to them. Whole class discussions provided students with an avenue to continuously learn and/or reinforce their understanding of geometric ideas and terminology. Additionally, the opportunity to verbalize their thinking allowed students to think about and construct meaning about the various geometric concepts including the vocabulary studied throughout the course. The whole class discussions in this non-traditional geometry course can best be characterized as dialogic discourse (Wertsch & Toma, 1995). In other words, whole class discussions could be seen as give-and-take or two-way communication among students whose function was to generate meaning. Thus, the whole class discussions aided the creation of a classroom community wherein meaningful geometry concepts could be wrestled with.

Examination of course documents, video tapes, and fieldnotes revealed that whole class discussions served a variety of purposes. These purposes included providing opportunities for students to raise questions they had, a source of ideas to help students make sense of the material, an opportunity for students to confirm their understanding of the material, an opportunity for students to share their ideas and strategies with the rest of the class, and an opportunity to encourage thoughtful reflection by the students. Additionally, whole class discussions allowed students to vocalize their struggles and

frustrations as they gained support and encouragement from others. The instructor indicated that it is important to honor and accept this frustration because it is an essential part of the learning process.

Whole class discussions were a major part of the class accounting for at least three fourths of a typical class period. Class discussions usually began with questions and/or issues that arose from the assigned daily activity sheets, geometric constructions, or group activities. Two students described a typical class discussion as “we went over the homework from the night before and various people went over how they have worked the problems” and “everybody would just tell how they got the answer and the different ways they came to that answer ...so you see a lot of different points of views.”

The instructor and developer of this course indicated that the pedagogical practices he used in teaching this course had gradually evolved over the past ten years through critical reflection of his own teaching and how students learn. His continual asking himself how can he ask better questions and how can he allow the students to do more of the thinking fueled much of his own personal evolution as a teacher and the design of this course. His current pedagogical practices in this course support students' construction of their own knowledge and his role could best be described as a facilitator.

Observation of the class revealed that the instructor asked questions to facilitate and keep the discussion going and made decisions about how much time was spent on each activity. Questions were asked by the instructor to solicit student input on how they made sense of a particular problem such as “how did

someone do that problem,” “did anyone do this problem in a different way,” “someone else want to read their conclusions,” or “did anyone describe this relationship differently?” Additionally, questions were asked to confirm student understanding. For example, the instructor would ask “do you all understand what Lori said”, “how many follow what Becky said” or “does that make sense to you?” The instructor’s continual probing, paraphrasing of student questions and answers, and efforts to encourage participants’ vocalization of their mathematical understandings is characteristic of what Davis (1997) referred to as interpretive listening.

The instructor and students worked together throughout the course to help students make sense of the material being studied. The instructor, together with the students, was responsible for creating a learning environment wherein meaningful geometry was grappled with and discussed while student construction of knowledge was supported. This dynamic and interactive environment, created by the ongoing whole class discussions, was fueled by the instructor’s questioning and probing of student thinking and ideas.

### *Daily Activity Sheets*

For this course, students used a 195-page packet that included general course information, group activities, and geometric constructions along with daily activity sheets. Daily activity sheets occasionally included pages for students to read introducing a new concept such as congruence conditions prior to the student working the activity sheets on that new concept. These activity pages



were identified with a tree in the upper right hand corner. At the end of each class period, the instructor would place page numbers corresponding to various pages in the student packet on the chalkboard for assigned activity sheets followed occasionally by a brief comment. These brief comments by the instructor included instructions that students were required to read a certain page, that a certain page indicated a new concept being added for discussion, or that certain pages were cutouts to be used with the previous page. For samples of the daily activity sheets see Appendix G.

The daily activity sheets were used on a *try first* basis and “provide[d] an experiential basis for understanding the geometric concepts and relationships presented” (Aichele & Wolfe, in press, p. ii). The idea behind *try first* was that students were to attempt to complete and make sense of each of the activity sheets independently prior to discussion in class. The comment “do the assignment in advance, so we can talk about it” (fieldnotes 6-9-03) made by the instructor illustrates the idea behind *try first* and emphasizes to the students that it was okay if they did not fully understand an activity because they could bring their questions to class for discussion, thus setting the stage for the next whole class discussion. Typically, the instructor assigned four to six daily activity sheets at the end of each class period. The students came to the next class period with a myriad of questions. From a student’s question, the instructor could glean whether they had no understanding, some understanding, or only slight holes in their understanding. Their questions also revealed when students thought they were on the right track and had realized they needed to reevaluate

their own understanding based on the discussion going on around them. Student questions fueled the whole class discussions along with the students various solution strategies. The instructor was always careful to read directions on each activity sheet prior to discussion of that sheet. This seemed to allow students time to focus on and think about their own reasoning on the activity as well as determining what their questions were on that particular activity.

Each daily activity sheet consisted of a series of questions, activities, and/or writing prompts that assisted student sense making of various geometric ideas. The activity sheets allowed students to investigate, explore, make conjectures, ask questions, and communicate their thinking both in writing and orally during whole class discussions. For example, while studying the area of solid tile shapes, participants were asked, as part of a daily activity sheet, to describe any relationship they saw between the number of edge pegs and the area of a tile shape. Students' written responses consisted of descriptions in sentence form and/or a kind of algebraic-type equation. See Appendix G for samples of students' written responses.

### *Group Activities*

In this course, the instructor referred to cooperative learning opportunities as group activities. As part of these group activities, students worked on and discussed a variety of group activities. While working in groups, students explored, observed, and made conjectures while having the support of their peers. Observations of group activities revealed that students who were shy or

hesitant to talk during whole class discussions were more open and willing to talk and discuss. During group activities, students were encouraged to listen to each other's ideas, ask questions, and explain their reasoning to aid in their understanding of the mathematical ideas involved in the activities. Group activities were used on average about every three or four class periods with the first group activity of the semester occurring during the third class period.

Group activities in this course were used in two ways. First, group activities were used as a *first try* giving students an initial look at a geometric concept with the assistance of their group members. *First try* group activities tended to be centered on concepts that were being introduced for the first time in the course or to help students get started thinking about particularly difficult material. For many students, this *first try* was important to help them make sense of and understand the group activity. For example, students were asked to work with their group and figure out how to construct the perpendicular bisector of a line segment they had drawn on paper using paper folding. As a part of this group activity, students had to discuss and determine the meaning of the term perpendicular bisector. This sense making of mathematical vocabulary was typical of most of the group activities. This group activity allowed students to attempt and begin thinking about their first CD problem of the semester with the aid and support of their group members. *First try* group activities occurred during the last five to ten minutes of a class period. The expectation of the *first try* group activities was for the activity to be started in class, finished individually, and followed up the following class period with a whole class discussion.

For one student an understanding of the term “convex” was important for her understanding and ability to complete a *first try* group activity. The group activity involved students deciding which statements would make a good definition and which statements would make a bad definition for a kite. Katie and Becky discussed while the other two members of their group listened. Becky was trying to make sense of the term “convex”.

Becky: Do any of you guys know what a convex quadrilateral is?

Katie: Convex goes out and concave goes in.

Becky: Yes.

Katie: And that is about all I know.

Becky: Okay. [laughs] So convex would be one that a quadrilateral goes on.

Katie: Yes, all I know is that you know cave is like, as in, cave in. So,

Becky: [interrupts] So, no cave ins.

Katie: Yes.

Becky: Okay, so that means that all of these [meaning figures on the activity sheet] would be convex.

Katie: I think so, yea.

Becky: So if it doesn't go back into its self it should be convex.

After the group seemed to have made sense of the term “convex”, they went on to discuss the “What is a Kite” group activity (see Appendix G). For these two students, making sense of the term convex was important for them to understand the statements that represented the possible definitions of a kite; while for the

remaining two members of their group and for many other groups this term did not seem to be an issue.

Second, group activities were used as a *second try*. *Second try* group activities gave students a second look at a geometric idea and allowed them to validate their understanding of the idea or to clarify where their confusion or grappling with a concept lay. These *second try* group interactions occurred during the first five to ten minutes of the class period and allowed students to ask others in their group about things that had confused them or did not make sense as well as check their solutions with others in their group. One example of a *second try* group activity was when students were assigned two informal proofs, called four-step problems in the course (see Appendix G). Although the instructor had given a rare mini-lecture the previous day on solving these problems, few students came to class with these problems completed. According to the instructor, this is a common occurrence in the course, thus, he used the idea of a *second try* group activity for students to help each other make sense of these types of problems. In his estimation, this allowed the whole-class discussion to be more rich in that more students had participated in completing and making sense of the two four-step problems.

While students worked on their assigned task in their groups, the instructor walked around observing and listening to the various groups' sense-making of the task. The instructor's facilitation of group activities came in the form of his posing questions to help students initiate dialogue among their group

members. Additionally, the instructor acted as an encourager in order to get all students engaged in the group activity.

### *Geometric Constructions*

Geometric constructions were an integral part of this course and were referred to as CD problems. The initials CD stand for construct and describe. On each geometric construction students were required to perform the construction and then describe their process in their own words. The instructor emphasized to students that describing their process was an important ability to help them make sense of the vocabulary and that the written and oral descriptions would give students practice communicating their descriptions. Throughout the semester CD problems were performed using three techniques: paper folding, straight edge and compass, and mira. Each of these techniques were used for about one-third of the course. An example of a student's response to a CD problem can be found in Appendix G.

On the second day of class, CD problems were introduced using paper folding. The following excerpt illustrates how CD problems were typically incorporated into the daily whole class discussions:

Instructor: On your sheet of paper draw a line segment. [Instructor illustrates this on his own sheet of paper.] By folding this paper, I would like you to find a line which is perpendicular to this line [segment] and passing through the middle of this line [segment].

[Instructor walks around and encourages students to talk and work together.]

Instructor: Can I have a student describe what they did?

Amy: You put the two dots together on top of each other.

Instructor: [Repeats and illustrates Amy's instructions] Put the two dots together.

Amy: And fold it.

Instructor: Put the two dots together and fold it. What do you think Mandy? Does that look perpendicular?

Mandy: Yes.

Throughout the semester, students would orally verbalize their method of constructing a CD problem in class. During this verbalization, other students could be seen performing the construction and possibly writing out what they were doing. The instructor either had the students demonstrate the CD construction process themselves or he would illustrate the construction according to a student's description. When the instructor illustrated the construction, he was very careful to do exactly what the student said. This helped the students to see when their wording was missing a step or if they needed to alter their wording to fit their meaning.

### *Projects*

Projects were also a part of the complex myriad of tasks used throughout the semester to allow students to demonstrate their understanding of various geometric concepts. Analysis of course documents, video tapes, and fieldnotes

revealed that projects were used to provide students a creative way to illustrate their understanding. The instructor encouraged the students to be creative and imaginative as well as to connect new concepts with previously discussed concepts. Projects provided an avenue for students to deepen their understanding of the geometric concepts. The instructor shared with students that he was interested in how they thought about and communicated their understanding of geometric ideas.

Projects during this course included creating three-dimensional origami models, examining relationships between two geometric concepts, and demonstrating an understanding of geometric concepts. The prospective elementary teachers created origami models of cubes, stellated octahedrons, and stellated icosahedrons. They were shown how to fold a basic parallelogram unit and the end product, then participants were required to assemble their stellated polyhedron without assistance from instructor. Some students relied heavily on help from other students to get their model together.

*Relationship projects* required participants to examine two related geometric concepts in detail. For example, during this course the prospective elementary teachers were asked to think about two terms - definition and property. They were then asked to describe in their own words what was meant by each term and demonstrate through illustrations the relationship between the two terms. See Appendix G for an example of this project.

In a third type of project, prospective elementary teachers were asked to demonstrate their understanding of a geometric concept. For example, during



this course students studied the idea of symmetry. As a real world application of symmetry students examined the seven types of border patterns. The prospective elementary teachers were shown examples of previous students' border projects and asked to create their own set of all seven-border patterns. Students used a variety of creative techniques to create their border patterns including computer graphics, hand-drawn graphics, stickers, and shapes cut from a die-cut machine. A couple of students border pattern sets were centered around a theme such as a picnic or sports.

#### *Perceptions of Prospective Elementary Teachers About These Characteristics*

In order to determine the perceptions of prospective elementary teachers about the characteristics of this non-traditional geometry course, data were examined from post-questionnaires, journal prompts, and interviews. From this data, prospective elementary teachers' perceptions of whole class discussions, group activities, geometric constructions, and projects were examined along with their feelings about the course as a whole. Additionally, a theme that emerged from student interviews was the perceptions of prospective elementary teachers about the learning environment of this non-traditional geometry course.

#### *Perceptions About The Course*

Six prospective elementary teachers were asked to describe what they would tell someone else about their experiences during this non-traditional geometry course. Additionally, on the post-questionnaire, twenty prospective

elementary teachers described their feelings about geometry based on their experiences in this course. Analysis of interviews and post-questionnaires revealed that participants used words like frustrating, overwhelming, fun, worthwhile, and hands-on to describe their experiences. Participants tended to be perturbed by the class, but felt it was a worthwhile course to take. Below are excerpts from participant interviews about their experiences during this course.

Nancy: "I hated, I liked the class....I got frustrated, because I just felt there were lots of times that you can learn from other people, but you never knew if you were right or wrong....I did think it was a fun class and it was a worthwhile class."

Jean: "I found so much use and I get so excited about this class....It was a hands-on learning situation. I am a very visual person and I learn better that way."

Amy: "It wasn't necessarily hard, but there was a lot of outside work. There was a lot of things that I hadn't done before like the CD problems, using miras, and origami. I liked doing those things, it was fun once I figured it out."

Ruth: "Probably frustrating because I was so used to being told whether this is the right way, this is the wrong way to do it, that I always had the backup that you were correct. And when you get thrown into this environment all of a sudden, what do you think, how did you get that, and not being reassured that you were correct, it was

frustrating...It was not difficult, it's was just an adjustment to get used to after 12 years or so of being told you were correct."

As Ruth's and Nancy's responses reveal, students were frustrated that they did not have that reassurance of a correct answer. Additionally, the excerpts below from participant responses on the post-questionnaire about their experiences during this course indicated students were frustrated that they did not have reassurance that their answers were correct.

Ashley: "I dislike math because it is not my best subject and frustrates me. I feel dumb when in math class. This course was okay, but I wanted more direct answers than what I got."

Kristina: "I feel that I could have learned a lot more if I was given the chance....I think I would have done a lot better if I was taught."

During observations these same comments of frustration about whether participants were right or wrong were overheard frequently at the beginning of the course, but much less by the end of the course. Kristina's comment also typifies most of the students' notions of what it means to teach at least this was the impression from student comments heard at the beginning of the course. In other words, they expect to be told what to do and how to do it by the teacher. This sense of "not being taught" captures the essence of the non-traditional nature of this course.

Prospective elementary teachers responses' to the post-questionnaire revealed similar responses to Nancy's, Jean's, and Amy's interview responses indicating that the class was worthwhile, fun, and hands-on. For Anna,

“Geometry was more interesting and could be fun.” Amanda felt that she “learned a lot more geometry than in the past” while Becky liked “all the hands-on stuff.” For Beth, it was an enjoyable experience and “proved that we can basically figure out a lot of this material on our own.”

### *Whole Class Discussion*

Participants were asked to respond to a journal prompt about whole class discussions. The prompt read as follows:

Most of the class time in this course is taken up by whole class discussion of the day’s assignment. This replaces the more traditional lecture. My thoughts and feelings about this are...

Two mathematics professors and two graduate students coded each response according to whether they felt the student’s response indicated a positive, negative, or neutral feeling about whole class discussion. Nine (42.9%) students responded with positive thoughts and feelings about whole class discussion. Three examples of students’ positive thoughts follow:

- I like it better because it is more interactive. The students get to contribute their ideas instead of just listening to the teacher’s ideas.
- I like it. I often do things a ‘hard way’, hearing others ways of doing things helps me to be sure I understand the reasons behind an idea and not just the process. I feel that many students didn’t like this method because they have to think, but I feel it builds confidence in

thinking and as teachers we need confidence in our thinking or our students won't feel comfortable in theirs.

- I learn better by doing things myself rather than listening how to do things. I believe it improves my learning.

Six (28.6%) students expressed negative feelings about whole class discussions.

Three example statements of negative feelings follow:

- In all honesty, I learn better when there is a set lecture and I am able to ask the professor questions and get an answer from him. When the discussion is left to the class I tend not to ask questions and leave the class wondering/doubting myself.
- I do not feel if the class as a whole is struggling for answers, that expecting them to work through it without the aid of guidance is the answer. Call me a traditionalist, but I believe a teacher does just that, teaches. It is okay to guide students to an answer, but I have no confidence I will remember what we covered because the points a) were not reiterated and b) I'm not sure we had the correct answer.
- Well, I feel that I would be doing a lot better if I was being taught by a teacher. Since we do not get straight forward answers I am never sure if what I am doing is correct. Therefore, I am left with doubt of my ability in this class. . . .

While some students tended to have either a positive or negative feeling about whole class discussions, six (28.6%) students expressed thoughts and feelings that seemed to be neutral. Three examples of neutral responses follow:

- Sometimes it is helpful to hear other student's ideas, but when there is confusion among everyone sometimes it is best for the teacher to step in with the right answer on how to do it.
- I like that we don't have to sit through lecture everyday, but sometimes I get frustrated by the lack of explanation.
- It works great some days because I find that there are different ways to work problems and not just one teacher way. Some days it is very frustrating when no one knows how to do the problem.

Interviews revealed that for some students these discussions were equated with increased understanding and confidence building. It made them "feel more confident" and that "if there is something you don't know you can ...get it right then" without having to go "until the end of the year and still not understand." Active listening during the whole class discussions helped many students "to understand and it also helps you to teach it." Students also felt that talking about it helped them understand because "by you saying something out loud...[it] makes even more sense when I hear myself say it, instead of just writing it down on paper." For Ruth, whole class discussions were both helpful and a waste of time. For example, Ruth stated "I felt like whole class discussions were a waste of time, because you are telling people to get up there and they would do it incorrectly and then other times it was helpful because I didn't know how to do a problem and someone else did and it was helpful."

On the post-questionnaire, participants were asked to respond to a 5-point Likert-type scale consisting of a series of five questions about whole class

discussion. Approximately two-thirds of the prospective elementary teachers indicated that whole class discussions were an enjoyable part of the course and that it helped them to make sense of the geometric ideas. Seventeen (81%) did not believe whole class discussions were a waste of time thus indicating they were worthwhile. Additionally, slightly more than one-half of the participants believed that whole class discussions helped them gain confidence to learn and teach geometric concepts.

### *Group Activities*

Observations of group work revealed that students who were more often quiet and reserved during whole class discussions seemed to be more open to participate in the small group discussions. Some students found the group activities to be very helpful, while others did not find them to be helpful at all. Jean felt that the group work “helped a lot because even if you didn’t get it or if your like I got it right and they are like no I got this you know you can still work the problem over.” For Anna, “group activities gave [her] a chance to get feedback from others and interact with classmates.” Additionally, Anna commented “I would say that what helped me the most would be group work, just because I was able to get into a small group, and when I’m in smaller groups I’m more likely to answer a question, to throw out my answer versus the entire classroom.” On the other hand, Jamie felt group activities were a “waste of time.” She also indicated that she might feel differently if she had had another group to work in or if they had had “more time in groups.” Amy stated “I’m not a huge fan

of group work;" however, she also thought "it was nice to have help and work it out together, and that always helped."

On the post-questionnaire, participants were asked to respond to a 5-point Likert-type scale consisting of a series of five questions about group activities. Approximately three-fourths of the prospective elementary teachers felt that hearing other student's ideas during group activities were an enjoyable part of the class and that it helped them make sense of the geometric concepts. The majority (85.7%) of the participants indicated that hearing another student's ideas during group activities was not a waste of their time, thus suggesting that it was a worthwhile part of the course. Slightly more than one-half of the participants believe that hearing other students' idea's during group activities helped them gain confidence to learn and to teach geometry.

### *Geometric Constructions*

On the post-questionnaire, prospective elementary teachers were asked what part of the course had impacted them the most as future teachers. More than half of all participant responses indicated that CD problems had impacted them the most. Examination of responses revealed that participants believed that CD problems improved their mathematical thinking and their ability to communicate that thinking. For example, Joseph felt they helped him "think about math," while Beth suggested that "CD problems, though not my favorite, showed me that I have the ability to both figure out a solution and explain my thinking," and Jean believed that she had "gained the necessary skills to be able



to not only work these problems, but to verbalize my process as well.”

Furthermore, for Jane they provided an “interesting way to look at things.”

Additionally on the post-questionnaire, participants were asked to respond to a 5-point Likert-type scale consisting of a series of five questions about CD problems. One-third of the prospective elementary teachers felt they were an enjoyable part of the class while another one-third did not find them an enjoyable part of the class. Two-thirds of the participants did not find CD problems a waste of time; thus, indicating they believed they were a worthwhile part of the class. Slightly less than half of the prospective elementary teachers believed that CD problems helped them make sense of geometric concepts and helped them gain confidence to teach geometric concepts effectively. In response to whether CD problems helped them gain confidence to learn geometric concepts, eight (38.1%) participants felt CD problems did help while eight (38.1%) participants indicated CD problems did not help them gain confidence to learn geometric concepts.

Analysis of interview data revealed that participants found the CD problems to be frustrating, valuable, interesting, and increased understanding. For Jamie, CD problems were “frustrating at first, but they were valuable and definitely a keeper... at the end I got better at figuring them out on my own.” Although she tried the CD problems at home “it was frustrating for me because I wasn’t getting it. I didn’t understand what I was doing.” Similarly, Amy felt “they could be very frustrating, because I don’t really like the trial and error stuff, I like to go, this is how you do it...When you did figure it out you definitely

remembered it and you definitely knew it.” Jean liked the idea that there were different ways to do the constructions and “it kept your interest.” Jamie thought it was “really cool to see that there was so many different ways” to do a CD problem. Additionally, Ruth “found them kind of interesting. I liked messing around with the folding.” Participants felt the CD problems helped them to understand the geometric concept better. For example, Jamie indicated they helped her “understand and visualize the problems better” and Jean stated that “you learned by others doing them in different ways.”

### *Projects*

For some prospective elementary teachers, projects helped develop their understanding of a geometric idea and provided them with self-assurance that they truly did understand. For example, Lucy stated “once I did the projects, I understood that portion or section better” and Jamie commented “they helped you understand better and it helped you remember the material because you totally understood.” For Anna, projects allowed her to express her understanding. She wrote, “projects gave me a chance to put into words/art what/how I understood a subject and I was able to break it down in my own words.”

On the post-questionnaire, participants were asked to respond to a 5-point Likert-type scale consisting of a series of five questions about projects. Slightly more than one-half of the participants found completing projects to be enjoyable and helped them to make sense of geometric ideas while approximately one

quarter of the participants did not find completing projects enjoyable and did not feel they helped them make sense out of geometric ideas. Three-fourths of the participants felt that completing projects was an important use of their time indicating they thought the projects were a worthwhile part of the class. About one-half of the prospective elementary teachers indicated that completing projects helped them gain confidence in learning and teaching geometry.

### *Classroom Community/Environment*

Although no explicit questions were asked about the classroom environment, the classroom community/environment emerged, primarily from interviews, as a major characteristic of the course and seemed to be highly important to students. Students expressed that a positive learning environment was created in this course. During class discussions, some students "felt comfortable enough to speak in the class... [and] to say no this is what I got and this is how I derived this answer." Students appeared to connect this comfort to their ability to learn as one student pointed out "[if] you feel comfortable speaking...then you feel comfortable to learn" and that as they spent more time discussing their grades improved and "they felt more comfortable to learn." Additionally, students equated this comfort within the environment with their confidence to learn with statements such as "I think when you feel more comfortable you feel more confident in the class." One student even commented that if the professor "makes the classroom feel comfortable and you don't feel like you're going to get in trouble or embarrassed when you don't have a right answer

it [referring to the non traditional class structure] would work." In addition to discussing the importance of their comfort level, students pointed out that "at first, until you get used to it [referring to the non traditional class structure], it is overwhelming" and "at first it [referring to the non traditional class structure] is very overwhelming...[but] it gets better with the discussion and as you get used to trying to figure out things rather than just using a formula."

### *Attitude Toward Geometry*

A total of twenty-one prospective elementary teachers completed the 32-item Attitude Toward Geometry Scales (ATGS)(Utley, 2004) at the beginning of the first class period of the semester and again during the last week of coursework to determine if their attitude toward geometry changed during the course of the semester. With a possible range of 32 to 160 on the ATGS, prospective elementary teachers' scores on the first administration of the ATGS ranged from 55 to 143. On the second administration the scores ranged from 48 to 157. A higher score on the scale is indicative of an overall higher attitude toward geometry. Individual participant scores are reported in Table 2.

Utley (2004) reported the ATGS to have good reliability with Cronbach alpha coefficients of 0.96 for the instrument and subscale Cronbach alpha coefficients of 0.95 for the confidence subscale, 0.93 for the usefulness subscale, and 0.92 for the enjoyment subscale. Using Cronbach alpha, the reliability of the data on the ATGS for all twenty-one participants was 0.97 for the pre-test and 0.98 for the post-test. Cronbach alpha coefficients for the three sub scales revealed a reliability of 0.93 for the pre-test and 0.97 for the post-test for the

confidence subscale, 0.95 for the pre-test and 0.96 for the post-test for the usefulness subscale, and 0.95 for the pre-test and 0.96 for the post-test for the enjoyment subscale.

An examination of the individual differences (see Table 2) on the ATGS from the pre-test to the post-test revealed that two-thirds of the students had an increase in their overall attitude toward geometry score ranging from 4 to 59 points. The remaining one-third of the students had a decrease in their overall attitude toward geometry score ranging from 1 to 11 points. Descriptive analysis (see Table 3) of the students responses to the surveys showed an increase in the groups mean score from the pre-test ( $M = 94.10$ ,  $SD = 22.82$ ) to the post-test ( $M = 105.00$ ,  $SD = 25.80$ ). The mean difference was 10.90 with a 95% confidence interval for the difference in the means of 3.05 to 18.75. A paired t-test showed a significant difference ( $t_{20} = 2.90$ ,  $p = .009$ ) between the pre-test and post-test scores of the participants indicating that the prospective elementary teachers had a higher overall attitude toward geometry upon completion of the non-traditional geometry content course.

Table 2

## Individual Results for Attitude Toward Geometry Scales

	Sex	Pre ATGS	Post ATGS	Pre Conf	Post Conf	Pre Joy	Post Joy	Pre Use	Post Use
1	F	116	105	39	33	36	34	41	38
2	M	107	132	35	49	36	40	36	43
3	F	101	121	41	48	30	36	30	37
4	F	55	48	18	17	16	17	21	14
5	F	55	84	18	28	13	25	24	31
6	F	105	109	35	39	29	30	41	40
7	M	68	98	21	29	19	36	28	33
8	F	93	110	27	42	33	37	33	31
9	F	114	143	34	53	38	44	42	46
10	F	77	136	38	58	18	36	21	42
11	F	87	91	29	30	22	21	36	40
12	F	123	116	51	47	34	32	38	37
13	F	78	91	31	37	24	27	23	27
14	F	106	105	37	36	32	29	37	40
15	F	72	67	25	24	22	20	25	23
16	F	101	114	34	40	30	34	37	40
17	F	97	88	41	46	23	15	33	27
18	F	143	157	52	58	43	49	48	50
19	F	86	82	33	32	26	25	27	25
20	F	117	121	42	47	36	36	39	38
21	F	75	87	41	43	17	20	17	24

Note: Conf refers to the confidence to learn geometry subscale; Joy refers to the enjoyment of studying subscale; Use refers to the usefulness to study geometry subscale; ATGS refers to the overall instrument

TABLE 3

Means, Standard Deviations, and Results of the Paired Samples T-Test for the Attitude Toward Geometry Scale (N=21)

Administration	Mean	Standard Deviation	
Pre-Test	94.10	22.82	
Post-Test	105.00	25.80	

Mean Difference	Standard Error of Mean Difference	95% Confidence Interval of the Difference		t(20)	p
		Lower	Upper		
10.90	3.76	3.05	18.75	2.90	.009**

\*\*p<.01

#### *Confidence to Learn Geometry Subscale*

*Quantitative results.* The twelve *confidence to learn geometry* subscale items were analyzed to determine if the prospective elementary teachers confidence levels significantly changed. An examination of individual differences revealed that fifteen (71%) of the students' scores increased from the pre-test to the post-test ranging from 1 to 10 points and six (29%) of the students' scores decreased from 1 to 6 points from the pre-test to the post-test. Descriptive statistics reporting the mean and standard deviation of the pre- and post-tests are reported in Table 4.

TABLE 4

Means, Standard Deviations, and Results of the Paired Samples T-Test for the Confidence Subscale of the Attitude Toward Geometry Scale (N=21)

Administration	Mean	Standard Deviation
Pre-Test	34.38	9.27
Post-Test	39.81	10.95

Mean Difference	Standard Error of Mean Difference	95% Confidence Interval of the Difference		t(20)	p
		Lower	Upper		
5.43	1.55	2.19	8.67	3.50	.002**

\*\*p<.01

Against a potential range in scores of 12 to 60 on the *confidence to learn geometry* subscale, the prospective elementary teachers scores ranged from 18 to 52 on the pre-test and 17 to 58 on the post-test. Examination of data presented in Table 4 revealed that prospective elementary teachers achieved a higher mean score on the post-test (M = 39.81, SD = 10.95) than on the pre-test (M = 34.38, SD = 9.27). After noting these descriptive differences, a paired t-test was used to determine if the mean difference was statistically significant. A 95% confidence interval was calculated for the mean difference. Results of the paired t-test are shown in Table 4. The mean difference was statistically significant ( $t_{20} = 3.50$ ,  $p = .002$ ), indicating that the prospective elementary teachers felt more confident to study geometry. Thus, quantitative results



indicate the characteristics of this non-traditional geometry content course had a positive effect on prospective elementary teachers' confidence to learn geometry.

*Qualitative results.* A more in-depth examination of the confidence of participants was provided through qualitative data collection and analysis. Data analysis was conducted on twenty journal prompt responses, twenty-one open-ended questionnaires, and six semi-structured interviews. In order to ensure confidentiality of participants, pseudonyms were assigned to participants.

Twenty of the twenty-one participants responded to the journal prompt:

"Have the experiences in this course had an effect on your confidence to learn geometry? Please comment."

Three mathematics education professors, one mathematics professor, and one doctoral student examined participant responses to the journal prompt and coded each response according to whether the response indicated a positive, negative, or no effect on the participants confidence to learn geometry. When at least four out of five of the raters agreed upon a code for a response, the code was considered to be reliable. Two responses were eliminated due to unreliable coding. All percentages are reported based on the number of responses with reliable codes.

Results of this journal prompt revealed that fourteen (78%) of the prospective elementary teachers' felt their experiences in this non-traditional geometry course had a positive effect on their confidence to learn. In other words, they felt their confidence to learn geometry had increased. In this response, Mandy indicated, "Yes, this class forced me to think on my own.

Before in math classes I would just do what I needed to get by.” Similarly, Julia commented, “I understand some of the concepts better; therefore, I have more confidence in doing geometry problems.”

While more than three-fourths of the participants felt their experiences had a positive effect on their confidence to learn, three (17%) felt their experiences had a negative effect. Nancy indicated that her experiences had a negative impact, stating, “I felt that it was really hard to learn throughout this course. I really need a teacher who will lecture or at least someone whom you feel you can talk to and ask questions.” Additionally, one participant’s comment suggested that her experiences in the course had not affected her confidence. She stated “I do feel as if I have more knowledge of geometry but my confidence level in learning hasn’t really increased.”

Examination of data from the journal prompt responses, open-ended questionnaires, and semi-structured interviews revealed that prospective elementary teachers tended to equate changes in their confidence to learn geometry to their level of understanding and to the characteristics of the course. Participants felt that as their level of understanding increased so did their confidence to learn geometry. Jean indicated, “I think you are more confident when you understand the material better. It has to improve your confidence. If you don’t know what you are talking about, you are obviously not going to be confident talking about it.” Lynn commented, “I feel that I have learned a lot more geometry than in the past. It has made me more confident in the fact that when I apply myself my mind really does do math.” Similarly, Mandy stated, “I feel that I

have learned a lot more geometry than in the past. It has made me more confident in the fact that when I apply myself my mind really does do math.”

Several prospective elementary teachers attributed the characteristics of the class as effecting their confidence to learn geometry. For some prospective elementary teachers, the structure of the course positively effected their confidence. For example, Beth stated “I feel the structure of the class allowed me to confidently learn the material” while Mandy commented “yes, this class forced me to think on my own. Before, in math classes, I would just do what I needed to get by.” Additionally, Jamie felt her confidence had increased but suggested that “you really learn a lot more when you have to figure out things on your own, as opposed to lecture.” Jamie also attributed the whole class discussions as allowing her to see a variety of ways to solve a problem; therefore, she indicated, “I think your confidence level is increased by having so many different ways to explain it and to understand it.” Only a few prospective elementary teachers indicated that the structure negatively effected their confidence. Nancy indicated, “I felt it was really hard to learn throughout this course. I really need a teacher who will lecture.” Kristina stated “I feel that I could have learned a lot more if I was given the chance. What I mean is that I left this classroom maybe five times feeling good (confident) about my ability in geometry. However, I think I would have done a lot better if I was taught.”

In summary, approximately three-fourths of the prospective elementary teachers believed that their experiences in this non-traditional geometry course had a positive effect on their confidence to learn geometry. Additionally, the

prospective elementary teachers tended to attribute this positive effect to the characteristics of the course and/or to their increased understanding of geometric concepts.

### *Enjoyment Subscale*

*Quantitative results.* The ten enjoyment subscale items were analyzed to determine if the prospective elementary teachers' enjoyment levels as related to geometry significantly changed. The construct ranges from a lack of involvement in studying geometry to active enjoyment of studying geometry. An examination of individual differences revealed that thirteen (62%) students' scores increased from the pre-test to the post-test ranging from 1 to 18 points, seven (33%) students' scores decreased from 1 to 8 points from the pre-test to the post-test, and one student's score remain unchanged. Descriptive statistics reporting the mean and standard deviation of the pre- and post-tests are reported in Table 5.

TABLE 5

Means, Standard Deviations, and Results of the Paired Samples T-Test for the Enjoyment Subscale of the Attitude Toward Geometry Scale (N=21)

Administration	Mean	Standard Deviation	
Pre-Test	27.48	8.32	
Post-Test	30.62	8.98	

Mean Difference	Standard Error of Mean Difference	95% Confidence Interval of the Difference		t(20)	p
		Lower	Upper		
3.14	1.39	0.24	6.04	2.26	.035*

\*p<.05

Against a potential range in scores of 10 to 50 on the enjoyment subscale, the prospective elementary teachers' scores ranged from 13 to 43 on the pre-test and 17 to 49 on the post-test. Examination of data shown in Table 5 revealed that prospective elementary teachers achieved a higher mean score on the post-test (M = 30.62, SD = 8.98) than on the pre-test (M = 27.48, SD = 8.32). After noting these descriptive differences, a paired t-test was used to determine if the mean difference was statistically significant. A 95% confidence interval was calculated for the mean difference. Results of the paired t-test are shown in Table 5. The mean difference was statistically significant ( $t_{20} = 2.26$ ,  $p = .035$ ), indicating that the prospective elementary teachers were more involved and felt more enjoyment to study geometry. Thus, quantitative results indicate the

characteristics of this non-traditional geometry content course did have a positive effect on prospective elementary teachers' enjoyment to study geometry.

*Qualitative results.* A more in-depth examination of the enjoyment and motivation of participants was provided through qualitative data collection and analysis. Data analysis was conducted on twenty journal prompt responses, twenty-one open-ended questionnaires, and six semi-structured interviews. In order to ensure confidentiality of participants, pseudonyms were assigned to participants.

Twenty of the twenty-one participants responded to the journal prompt: "Has your enjoyment of studying geometry been affected by your experiences in this course? Please Comment."

Three mathematics education professors, one mathematics professor, and one doctoral student examined participant responses to the journal prompt and coded each response according to whether the response indicated a positive, negative, or no effect on the participants motivation and enjoyment to study geometry. When at least four out of five of the raters agreed upon a code for a response, the code was considered to be reliable. Two responses were eliminated due to unreliable coding. All percentages are reported based on the number of responses with reliable codes.

Analysis of the eighteen responses to this journal prompt revealed that twelve (67%) of the prospective elementary teachers' felt their experiences in this non-traditional geometry course had a positive effect on their level of motivation and enjoyment to study geometry. In other words, they believed that their level

of motivation and enjoyment had increased. For instance, Beth stated, "Prior to this class, I did not enjoy geometry. My attitude has changed because I have been successful in this class, but my enjoyment came with understanding the material." Cheri commented that she "enjoyed this class a lot" and Amy stated "Some of the things that I enjoyed doing I did not realize were even part of geometry." Additionally, Carol commented "I used to dislike geometry because I really did not know how to work most of the problems, but now I know I can handle some of it."

While two-thirds of the participants felt their experiences had a positive effect on their level of enjoyment to study geometry, two (11%) felt their experiences had a negative effect. Nancy commented that she had "never really enjoyed geometry...I hate it now and can't wait for it to be over." Similarly, Ruth indicated that she "dislike[d] geometry even more, probably because you do not like what you don't understand."

The remaining four (22%) students did not perceive their enjoyment of studying geometry changed as a result of their experiences in the non-traditional geometry course. Katie commented "I have never really enjoyed it. So, I believe that whether good or bad the experiences wouldn't affect me." Lucy stated "Math is math. But of the [various] types, geometry is the best but nothing to get me excited."

An examination of data from the journal prompt responses, open-ended questionnaires, and semi-structured interviews revealed that prospective elementary teachers attributed changes in their level of motivation or enjoyment

to characteristics of the course and to their level understanding. Several participants felt their level of motivation and enjoyment changed with their level of understanding. Beth commented, “my enjoyment came with understanding the material.” Ashley stated, “I enjoyed working with my fellow students because they added to my understanding of the concepts.” Similarly, Carol indicated, “I used to dislike geometry because I really did not know how to work most of the problems, but now I know I can handle some of it.” Ruth stated, “I dislike geometry even more, probably because you do not like what you don’t understand.”

Several prospective elementary teachers cited their experiences in this non-traditional geometry course as instrumental in changing their level of motivation and/or enjoyment. For example, Jamie stated “I really enjoyed several of the projects and learning neat things to make.” Lynn believed that specific topics and projects helped her increase her confidence. She commented “I have enjoyed the mandalas, borders, tessellations, and hands-on projects.” For Jamie, seeing multiple ways of doing problems made studying geometry “more enjoyable” and motivated her to keep trying “If I start one way and don’t understand I start over.” Additionally, Jamie indicated that whole class discussions “helped me try harder.” For another student the characteristics this course motivated her to get her assignments done. Jean commented, “Even when I was with my family I was still thinking about geometry.... And even though it is going to be explained in class you still wanted to try to get it right before class started.”



In conclusion, approximately three-fourths of the prospective elementary teachers perceived that their experiences in this non-traditional geometry course positively effected their level of motivation and enjoyment in studying geometry. In addition, the prospective elementary teachers attributed this positive effect to the characteristics of the course and/or to their increased understanding of the geometric concepts.

#### *Usefulness to Study Geometry Subscale*

*Quantitative results.* The ten *usefulness to study geometry* subscale items were analyzed to ascertain whether the prospective elementary teachers' beliefs about the usefulness to study geometry significantly changed. An examination of individual differences revealed that twelve (57%) of the students had their scores increase from the pre-test to the post-test ranging from 2 to 21 points and nine (43%) of the students had their scores decrease from 1 to 7 points from the pre-test to the post-test. Descriptive statistics reporting the mean and standard deviation of the pre- and post-tests are reported in Table 6.

Against a potential range in scores of 10 to 50 on *the usefulness to study geometry* subscale, the prospective elementary teachers' scores ranged from 17 to 48 on the pre-test and 14 to 50 on the post-test. Examination of data shown in Table 6 revealed that prospective elementary teachers achieved a higher mean score on the post-test ( $M = 34.57$ ,  $SD = 8.81$ ) than on the pre-test ( $M = 32.24$ ,  $SD = 8.34$ ). After noting these descriptive differences, a paired t-test was used to determine if the mean difference was statistically significant. A 95% confidence

interval was calculated for the mean difference. Results of the paired t-test are shown in Table 6. The mean difference was not statistically significant ( $t_{20} = 1.77, p = .091$ ), indicating that the prospective elementary teachers' beliefs about the usefulness to study geometry did not significantly change. Thus, the characteristics of this non-traditional geometry content course did not have an effect on prospective elementary teachers' beliefs about the usefulness to study geometry.

TABLE 6

Means, Standard Deviations, and Results of the Paired Samples T-Test for the Usefulness Subscale of the Attitude Toward Geometry Scale (N=21)

Administration	Mean	Standard Deviation
Pre-Test	32.24	8.34
Post-Test	34.57	8.81

Mean Difference	Standard Error of Mean Difference	95% Confidence Interval of the Difference		t(20)	p
		Lower	Upper		
2.33	1.32	-0.41	5.08	1.77	.091

*Qualitative results.* A more in-depth examination of participants' belief in the *usefulness to study geometry* was provided through qualitative data collection and analysis. Data analysis was conducted on twenty journal prompt responses

and six semi-structured interviews. In order to ensure confidentiality of participants, pseudonyms were assigned to participants.

Twenty of the twenty-one participants responded to the journal prompt “Has your belief in the usefulness of geometry been affected by your experiences in this course? Please comment.”

Three mathematics education professors, one mathematics professor, and one doctoral student examined participant responses to the journal prompt and coded each response according to whether the response indicated a positive, negative, or no effect on the participants notion of the usefulness of geometry. When at least four out of five of the raters agreed upon a code for a response, the code was considered to be reliable. No responses were eliminated due to unreliable coding. All percentages are reported based on the number of responses with reliable codes.

Analysis of the twenty responses to the journal prompt revealed that eleven (55%) of the prospective elementary teachers' felt their experiences in this non-traditional geometry course had a positive effect on their view of the usefulness of geometry. In other words, they felt their perception of how useful geometry is increased. For example, Carol stated, “I hadn't realized all the places that geometry was present, so learning about all the concepts that go along with it has been helpful to me.” Lynn believed that her experiences in the class had helped her to see geometry as more useful and commented that she believed that “geometry concepts are useful and needed in many real-life experiences and careers.”

While slightly more than half of the participants believed their experiences helped them to see the usefulness of geometry, five (25%) believed that their experiences had an adverse effect on their belief. Ruth stated, "I believe this class was a waste of my time. I don't believe I learned anything compared to the time spent in this classroom." Nina felt that some topics were useful, but that she "really did not see why [she] need[ed] to know most of the information."

The remaining four (20%) participants did not feel that their experiences in the non-traditional geometry course had altered their perceptions of the usefulness of geometry. Mandy stated, "I have always believed that geometry is useful." Anna commented, "In all reality, no they haven't changed. I do feel that having basic knowledge of geometry is good ..."

Analysis of data from both the journal prompts and the semi-structured interviews revealed that prospective elementary teachers attributed their beliefs about the usefulness of geometry to their perception of its uses in the world around them and/or to its usefulness to them in their future careers as elementary teachers. Several students believed their perception of the usefulness of geometry could be equated with their newly acquired perception of geometry in the world around them. Carol commented that she "hadn't realized all the places that geometry was present. As Lynn put it "geometry concepts are useful and needed in many real-life experiences and careers." Additionally, Jean felt that studying geometry had helped her mind think more mathematically and had helped her while working at her summer job.

A few students attributed their belief in the usefulness of geometry to their future careers as elementary teachers. Anna felt that “having a basic knowledge for geometry is good for [her] and the students [she] plan[s] to teach.” Similarly, Mandy felt that her experiences helped her to see that it was “very useful for [her] to learn how to teach geometric ideas” and Kristina believed that she could utilize the study of “simpler problems with first, second and third graders.”

Several students equated their belief in the usefulness of geometry to both their perception of its uses in the world around them and to its usefulness to them in their future careers as elementary teachers. Ashley felt that it was useful to her career as an elementary teacher because “you are teaching little kids and it is something they need to know” and to her everyday life because she felt there “were lots of things that you could relate it to.” Additionally, Ashley believed that geometry was a good tool for many people’s occupations. Amy felt that the geometric concepts being studied in this course could be “used in everyday life experiences;” however, she questioned the value of some of the topics because she felt they were too “advanced for the field [she] was entering.”

In summary, slightly more than one-half of the prospective elementary teachers felt their experiences in this non-traditional geometry course had positively effected their belief in the usefulness of geometry. Additionally, they attributed these changes to their belief in the usefulness of geometry to their perception of its uses in the world around them and/or to its usefulness to them in their future career as elementary teachers.

### *Perceptions of Prospective Elementary Teachers About Teaching Geometry*

In order to determine prospective elementary teachers' perceptions of how their experiences in a non-traditional geometry course influenced their belief in their ability to teach geometric concepts a variety of qualitative and quantitative data was examined. At the beginning and end of the course participants were asked to complete the 21-item Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) (Huinker & Enochs, 1995) and an open-ended questionnaire with two questions pertaining to teacher efficacy. At the end of the course students were asked to respond to a journal prompt and five prospective elementary teachers were interviewed.

#### *Quantitative Results*

A total of twenty-one prospective elementary teachers completed the 21-item MTEBI at the beginning of the first class period of the semester and again during the last week of coursework to determine whether their mathematics teaching efficacy had changed during the course of the semester. The MTEBI is a 5-point Likert-scaled instrument that contains two scales: 1) personal mathematics teaching efficacy (PMTE) and 2) mathematics teaching outcome expectancy (MTOE). These two scales are viewed as separate constructs and not pieces of a larger construct; therefore, each scale will be examined separately. Individual participant scores on the scales are reported in Table 7.

Table 7

## Individual Results for Mathematics Teaching Efficacy Beliefs Instrument

	Sex	Pre PMTE	Post PMTE	Pre MTOE	Post MTOE
1	F	54	53	29	31
2	M	36	46	30	32
3	F	46	51	27	29
4	F	54	51	33	38
5	F	47	46	34	33
6	F	47	47	25	25
7	M	42	43	30	24
8	F	44	44	37	20
9	F	57	64	33	37
10	F	52	59	31	36
11	F	51	52	26	24
12	F	49	51	32	32
13	F	37	34	29	28
14	F	41	40	32	32
15	F	51	47	21	20
16	F	49	52	33	32
17	F	50	51	31	27
18	F	47	47	32	28
19	F	41	40	31	24
20	F	36	34	31	35
21	F	54	54	26	32

Enochs, Smith, and Huinker (2000) reported the MTEBI to have good reliability with Cronbach alpha coefficients of 0.88 on the PMTE scale and 0.77 on the MTOE scale. Using Cronbach alpha, reliability of the data in this study for

all 21 participants was 0.81 for the pre-test and 0.87 for the post-test for the PMTE scale. Additionally, reliability analyses for the MTOE scale revealed Cronbach alpha coefficients of 0.77 for the pre-test and 0.87 for the post-test.

*Personal Mathematics Teaching Efficacy.* With a possible range of 13 to 65 on the PMTE scale, prospective elementary teachers' PMTE scores on the first administration of the MTEBI ranged from 36 to 57 and on the second administration scores ranged from 34 to 64. A higher score is more indicative of a higher personal mathematics teaching efficacy. In other words, the higher their score the greater their belief in their ability to teach mathematics concepts effectively. Examination of the individual differences on the PMTE scale from the first administration to the second administration revealed that slightly less than half of the students had a decrease in their personal mathematics teaching efficacy ranging from 1 to 4 points and a slightly less than half of the students had an increase in their personal mathematics teaching efficacy ranging from 1 to 10 points. The remaining four students showed no change in their PMTE.

Descriptive analysis (see Table 8) of participant scores showed an increase in the groups mean score from the pre-test ( $M = 46.90$ ,  $SD = 6.20$ ) to the post-test ( $M = 47.90$ ,  $SD = 7.34$ ). However, a paired t-test revealed no significant difference ( $t_{20} = 1.27$ ,  $p = .218$ ) between the pre-test and post-test scores of the participants indicating that the prospective elementary teachers personal mathematics teaching efficacy was not effected by their experiences in this non-traditional geometry course.



TABLE 8

Means, Standard Deviations, and Results of the Paired Samples T-Test for the Personal Mathematics Teaching Efficacy Subscale (N=21)

Administration	Mean	Standard Deviation
Pre-Test	46.90	6.20
Post-Test	47.90	7.34

Mean Difference	Standard Error of Mean Difference	95% Confidence Interval of the Difference		t(20)	p
		Lower	Upper		
1.00	0.79	-0.64	2.64	1.27	.218

*Mathematics Teaching Outcome Expectancy.* With a possible range of 8 to 40 on the MTOE scale, prospective elementary teachers' MTOE scores on the first administration of the MTEBI ranged from 21 to 37 and on the second administration scores ranged from 20 to 38. A higher score is more indicative of a higher mathematics teaching outcome expectancy. In other words, the higher their score the greater their belief that effective mathematics teaching will have a positive effect on student learning. Examination of the individual differences on the MTOE scale from the first administration to the second administration revealed that slightly less than half of the students had a decrease in their mathematics teaching outcome expectancy belief ranging from 1 to 17 points and slightly less than half of the students had an increase in their personal

mathematics teaching efficacy ranging from 2 to 6 points. The remaining three students showed no change in their MTOE.

Descriptive analysis (see Table 9) of participant scores showed a decrease in the group's mean score from the pre-test ( $M = 30.14$ ,  $SD = 3.58$ ) to the post-test ( $M = 29.48$ ,  $SD = 5.24$ ). However, a paired t-test revealed no significant difference ( $t_{20} = 0.59$ ,  $p = .561$ ) between the pre-test and post-test scores of the participants indicating that the prospective elementary teachers' mathematics teaching outcome expectancy was not effected by their experiences in this non-traditional geometry course.

TABLE 9

Means, Standard Deviations, and Results of the Paired Samples T-Test for the Mathematics Teaching Outcome Expectancy Subscale (N=21)

Administration	Mean	Standard Deviation	
Pre-Test	30.14	3.58	
Post-Test	29.48	5.24	

Mean Difference	Standard Error of Mean Difference	95% Confidence Interval of the Difference		t(20)	p
		Lower	Upper		
-0.67	1.13	-3.02	1.69	0.59	.561

### *Qualitative Results*

A more in-depth examination of the perceptions of participants about their ability to teach mathematics was provided through qualitative data collection and analysis. Data analysis was conducted on twenty journal prompt responses, twenty-one open-ended questionnaires, and six semi-structured interviews. In order to ensure confidentiality of participants, pseudonyms were assigned to participants.

Twenty of the twenty-one prospective elementary teachers participating in this study responded to the journal prompt “Has your feelings about your ability to teach geometric concepts been effected by your experiences in this course? Please comment.” Three mathematics education professors, one mathematics professor, and one doctoral student examined participant responses to the journal prompt and coded each response according to whether the response indicated a positive, negative, or no effect on the participants ability to teach geometric concepts. When at least four out of five of the raters agreed upon a code for a response, the code was considered to be reliable. Five responses were eliminated due to unreliable coding. All percentages are based on the number of responses with reliable codes.

Analysis of responses to this journal prompt revealed that twelve (80%) of the prospective elementary teachers felt that their experiences in the non-traditional geometry course had a positive effect on their belief in their ability to teach mathematics concepts. In other words, they had a more positive belief in their ability to teach mathematics effectively. One student felt her experiences

had a positive effect due to her increased knowledge about geometric concepts. She stated, "I believe I have more ability to teach geometric concepts because I have learned more than I can imagine." Some other students equated the positive effect to their belief they would be more effective in the classroom teaching geometric concepts. For example, Jamie commented "I think I will be clearer and more effective since taking this class" and Sally suggested that she knew she could "teach [geometric] concepts and others would understand." Becky pointed out that she was more confident to learn "new ways to teach math and also to make it more interesting." Some students believed that at the conclusion of the course they felt their ability to reason mathematically and explain concepts to others had improved. Lynn believed she could "express [her] reasoning more and in doing so [she would] be able to teach the concepts better."

Of the remaining three reliably coded responses, two prospective elementary teachers' felt their experiences had a negative effect and one prospective elementary teacher felt her experiences had no effect on her ability to teach geometric concepts. One of the two prospective elementary teachers who experienced a negative effect felt that she could not explain geometric concepts to students even though she herself knew how to answer questions related to those concepts. The other commented she felt more confident about geometric concepts, but that "there is always many different ways to get an answer that it could be a problem in my teaching. I don't understand even all my college peers' ways at arriving at the answer, so how could I ever teach that

way.” She seemed to be equating the structure of the class with what would be expected of her as a future teacher and seemed unsure of her ability to teach in that manner. The participant’s response that was coded as no effect simply stated “No” indicating that she did not feel that her feeling had been altered due to her experiences.

On the pre-post open-ended questionnaire (see Appendix B), participants were asked to respond to the following three questions that relate to their personal teaching efficacy:

1. Describe how you now perceive yourself as a teacher of mathematics for young children.
2. Describe how comfortable you now feel about answering your student’s questions about geometric concepts.
3. Describe whether you now feel you have the necessary skills to teach geometric ideas to young children.

Twenty of the twenty-one participants responded to these questions. Each response from the pre to post questionnaire was evaluated as to whether each participant’s perception had changed positively, negatively, or no change had occurred.

Five participants did not respond to the actual first question posed. Rather, they responded with how they would teach, citing such examples as using manipulatives. Additionally, two participants did not respond to this question on the post questionnaire. Analysis of the remaining fourteen participants’ responses revealed that nine felt better prepared, had an increased

understanding of mathematics, and felt more confident to teach young children. Typical post-questionnaire responses were “I feel better prepared more confident in my ability to figure [out] and explain problems” and “I think I’ll be able to teach them with confidence now that I understand all the basic components.” One student’s response indicated a negative effect. On the pre-questionnaire she responded that she felt she “could be good at anything,” but on the post-questionnaire she did not feel she could teach geometry to young children. The three prospective elementary teachers with responses that revealed no effect on their perceptions indicated that they would be an “effective teacher” on both the pre- and post-questionnaires.

On the second of these three questions, participants were asked to respond to how comfortable they felt about answering their students’ questions about geometric concepts. On one student’s responses from the pre- to post-questionnaire, the researcher was unable to code a possible effect due to inconsistency in the responses. One-half of the participants felt more comfortable and confident to answer their students questions. Typical pre-post questionnaire responses are as follows:

Example #1:

Pre: “Somewhat comfortable.”

Post: “I think that I will be able to answer just about any geometric question that may be asked of me in the future.”

Example #2:

Pre: “At this moment not very confident.”

Post: "I definitely feel more comfortable about answering my students' questions and what I don't know I'm sure I can always find out."

Analysis of participant responses with no change in perceived ability to answer student questions revealed that they already felt comfortable answering student questions, with the majority feeling very comfortable. Examination of the two participants whose responses showed a negative effect on their perceived ability to answer their students' questions revealed that while at the beginning of the course they felt comfortable, by the end they did not feel comfortable. One participant's post response equated this to the fact that she did not feel that she had "learned very much or enough from this course."

The third question prompted the prospective elementary teachers to respond to whether they felt they had the necessary skills to teach geometric ideas to young children. On this question one participant did not respond; therefore, a total of twenty pre-post responses were analyzed. Fifteen (75%) of these responses reflected a positive effect on their personal teaching efficacy while the remaining five (25%) responses reflected no effect on their personal teaching efficacy. Typical pre-post questionnaire responses that reflected this positive effect were:

Example #1:

Pre: "Probably not. I could teach the basics as of now."

Post: "I feel, because of this class, that I can teach geometric ideas because of all the discussion put forth in this class."

Example #2:

Pre: "I feel I do not have the necessary skills to teach geometry. I do not understand completely and feel I would not be able to successfully explain the subject."

Post: "Not fully."

Example #3:

Pre: "Not yet."

Post: "I feel that I may be closer to being able to teach."

Although three-fourths of the prospective elementary teachers' responses reflected a positive effect, the effect appeared to be provisional for three (15%) of these participants. For example, Nina responded that she did not have the necessary skills at the beginning of the course and at the end of the course she commented, "I do for the most part but it just scares me that there are so many different ways to do problems and I don't think I understand all the ways. How could I teach it?"

During the interviews, five of the six prospective elementary teachers' responses were consistent with the pattern of responses from participants' responses on the open-ended questionnaire. They felt better prepared, had an increased understanding of mathematics, and felt more confident and comfortable to teach young children. For example, Jean felt "comfortable that [she] could teach anything in that book [referring to the course textbook] and teach it to any grade in school." Jamie believed that her experiences in the course would help her "explain the material better" because she learned so many



different ways to understand a concept. Amy suggested that her experiences in the course helped her feel better prepared, that she learned “different ways of teaching” and that she had an increased understanding of the material. Amy stated, “learning that doing things yourself is better than someone telling you that this is how you are going to do it because even if you get the right answer, you may not understand how you got it, how you came to that solution. So, being able to see myself doing the problems myself, messing round with it until I had the right answer showed me that that works better than just someone telling you how to do it.”

From the interviews, the remaining prospective elementary teacher’s experiences had a negative effect on her perceptions about her ability to teach mathematics. Nancy felt that due to her negative attitude about “the way it [referring to the course] was structured and organized and the discussions” that she did not “learn a lot of geometry.” Additionally, she stated, “I know I didn’t come out of it learning a lot of geometry. I don’t feel I could teach it real well.”

In summary, more than one-half of the prospective elementary teachers felt their experiences in this non-traditional geometry course had positively influenced their belief in their ability to teach young children mathematics, specifically geometry. Additionally, they attributed these positive changes to their feelings of preparedness, understanding, confidence, and comfort with the material.

### *Conclusion*

This study investigated the characteristics of a non-traditional geometry course and prospective elementary teachers' perceptions about these characteristics. This study also investigated the attitudes toward geometry and mathematics teaching efficacy beliefs of twenty-one prospective elementary teachers for the purpose of noting differences in these factors over the course of their participation in this non-traditional geometry course.

Regarding the investigation of the characteristics of this non-traditional geometry course, qualitative data analysis showed that whole class discussion was the prominent characteristic of the course fueled by daily activity sheets, group activities, geometric constructions and projects. The instructor played a key role in this course as a facilitator of these discussions and other activities.

In order to explore the perceptions of prospective elementary teachers about the characteristics of this non-traditional geometry course, qualitative and quantitative data were examined. Analysis of data collected through questionnaires, journal prompts, and semi-structured interviews indicated that participants found the course to be frustrating and overwhelming at first, but on the whole a fun and worthwhile experience. Slightly more than three-fourths of participants felt the whole class discussions were worthwhile and almost half of them responded with positive thoughts and feelings about whole class discussions. Group activities were found to be helpful by the majority of participants; however, some participants found them to be a waste of time. Construct and describe problems are the portion of the course that most

participants felt had impacted them the most as future teachers because they felt they helped them improve their ability to communicate their thinking to others. The prospective elementary teachers believed that projects helped to deepen their understanding and/or provided reassurance they understood a geometric concept. Participants indicated that a positive learning environment existed where they felt comfortable sharing and asking questions.

The results of this study indicate that participation in this non-traditional geometry course had a positive influence on participants' attitude toward geometry. Results have also shown that participants had a significantly higher overall attitude toward geometry. Both quantitative and qualitative analyses revealed that participants experience an increase in their confidence to learn geometry and had increased levels of motivation and enjoyment to study geometry. While quantitative data did not reveal a significant effect on participants' belief in the usefulness to study geometry, qualitative data indicated that slightly more than one-half of the prospective elementary teachers felt that their experiences had positively effected their believe in the usefulness of studying geometry.

Regarding the investigation of the mathematics teaching efficacy of the prospective elementary teachers both quantitative and qualitative data were analyzed. Examination of quantitative data revealed no statistically significant difference in prospective elementary teachers' personal mathematics teaching efficacy (PMTE) or mathematics teaching outcome expectancy (MTOE). Analysis of qualitative data indicated that prospective elementary teachers felt

that their experiences in this non-traditional geometry course had a positive impact on their belief in their ability to teach mathematics effectively.

In the next chapter, a summary of the findings and conclusions will be presented. Chapter V also discusses the implications of the study's findings for teacher education along with recommendations for future research.

## Chapter V

### *Summary, Conclusions, and Recommendations*

In universities across the United States, undergraduate mathematics courses play a critical role in the mathematical preparation of future teachers. Although efforts have been made toward reform in mathematics classrooms, the teaching practices in few undergraduate mathematics courses have changed to adhere to recommendations made by such documents as the *Professional Standards for Teaching Mathematics* (NCTM, 1991) and *The Mathematical Education of Teachers* (CBMS, 2001). Ball (1996) points out that prospective elementary teachers tend to teach the way they have been taught and have rarely experienced reform teaching. Therefore, if the experiences that prospective teachers have while learning mathematics impacts the way they teach mathematics to their students, efforts need to be made in the mathematical education of future elementary teachers to utilize pedagogical strategies that encourage knowledge construction and that model effective mathematics teaching.

This research study was designed to describe the characteristics of a non-traditional geometry course and the perceptions of prospective elementary teachers about these characteristics. Additionally, this study examined the impact of a non-traditional geometry course on the attitude toward geometry and

mathematics teaching efficacy of prospective elementary teachers. The research questions guiding this study were:

1. What are the characteristics of a non-traditional geometry content course for prospective elementary teachers?
2. What are the perceptions of prospective elementary teachers about the characteristics of a non-traditional geometry content course for prospective elementary teachers?
3. Are prospective elementary teachers' attitudes toward geometry influenced by this non-traditional geometry content course?
4. Are prospective elementary teachers' mathematics teaching efficacies influenced by this non-traditional geometry content course?

The participants in this study were twenty-one prospective elementary teachers enrolled in a non-traditional geometry course. Participants were primarily Caucasian females. The study employed both quantitative and qualitative data collection and analysis. Participants completed a background questionnaire, pre/post questionnaires, pre/post ATGS, and pre/post MTEBI. In addition, as a part of the normal course students completed journal prompts. Course documents were collected and analyzed. Semi-structured interviews were conducted with six of the prospective elementary teachers and observations of the course were conducted. Results of quantitative and qualitative data were used to determine the characteristics of the course, perceptions of participants about these characteristics, and the influence of these

characteristics on attitude toward geometry and mathematics teaching efficacy of participants.

### *Characteristics of a Non-Traditional Geometry Course*

The first two research questions explored the characteristics of a non-traditional geometry course designed for prospective elementary teachers and their perceptions of these characteristics. This course was non-traditional in that it did not adhere to the typical lecture – homework structure of mathematics classrooms. Simply using the course text could not duplicate the essence of this course. At the heart of this course is the creation of a dynamic environment by the instructor wherein the instructor and students continuously worked together to aid in the students' sense making of the geometry being studied. The instructor's role in this course was that of a facilitator; choosing appropriate tasks, encouraging dialogue, asking questions, and supporting risk-taking on the part of the participants. The instructor did not play the role of evaluator, but rather encouraged students to become their own authority on various problems. Thus, the instructor was encouraging and accepting of autonomous thinking on the part of the students (Brooks & Brooks, 1993). The instructor supported students' construction of knowledge along with helping them make meaningful connections to prior knowledge. The instructor's role in this course is consistent with NCTM's (2000) vision of teaching mathematics.

The aim of this course was to create and develop a dynamic environment wherein students could discuss and wrestle with significant mathematics. One hallmark of this course was whole class discussions that were dynamic and

interactive in nature and involved all of the participants. Whole class discussions provided students with an opportunity to participate in mathematical discourse that helped students generate meaning about a geometric concept. This type of give-and-take communication was characterized by Lotman (as cited in Knuth & Peressini, 2001) as discourse that helps students generate meaning and is referred to as dialogic discourse (Wertsch & Toma, 1995).

Whole class discussions could not happen without the careful selection of tasks, thus these whole class discussions were fueled by participants' completion of daily activity sheets, group activities, geometric constructions, and projects. Completion of daily activity sheets, group projects, and geometric constructions assisted the students in grappling with making sense of the geometric concepts being studied and discussed. Each of these activities allowed students to explore their own understanding and infused the whole class discussions. In addition to these characteristics, students perceived a positive learning environment; one in which they were encouraged to think for themselves, make conjectures, and express their mathematical thinking and frustrations. The combined characteristics of this non-traditional geometry course created an environment wherein students were allowed and encouraged to reflect upon the mathematics being studied, to communicate their understanding in a variety of ways, and to feel they were a part of a community of mathematical learners.

The characteristics of this non-traditional geometry course are consistent with the first standard for the professional development of teachers of mathematics (NCTM, 1991). This standard outlines what mathematics



instructors of prospective teachers should model in their teaching. The standard states that mathematics instructors should model good mathematics teaching by:

- posing worthwhile mathematical tasks;
- engaging [prospective] teachers in mathematical discourse;
- enhancing mathematical discourse through the use of a variety of tools, including calculators, computers, and physical and pictorial models;
- creating learning environments that support and encourage mathematical reasoning and [prospective] teachers' dispositions and abilities to do mathematics;
- expecting and encouraging [prospective] teachers to take intellectual risks in doing mathematics and to work independently and collaboratively;
- representing mathematics as an ongoing human activity;
- affirming and supporting full participation and continued study of mathematics by all students (NCTM, 1991, p. 127).

The perceptions of prospective elementary teachers about the characteristics of this non-traditional geometry course were explored. The results of the data analysis provided a description of how the prospective elementary teachers felt about these characteristics including whole class discussion, group activities, geometric constructions, projects, and the classroom environment. The majority of the prospective elementary teachers felt that whole class discussions, group activities, and projects were enjoyable, helped them

make sense of the geometric concepts studied, were a worthwhile part of the class, and helped them to gain confidence to learn and teach geometry. While less than half of the prospective elementary teachers may not have found the CD problems to be enjoyable or to positively influence their confidence to learn geometry, they did feel such problems were worthwhile and helped students to understand the geometric ideas along with gaining confidence to teach geometry.

The students felt that a positive learning environment was fostered during the course. One student commented, “[if] you feel comfortable speaking...then you feel comfortable to learn.” Additionally, students felt that the environment had a positive effect on their confidence to learn geometry. The learning environment developed as part of this non-traditional geometry course, as described by participants, is analogous with the supportive learning environment described by the NCTM (1991) in its *Professional Standards for Teaching Mathematics*.

### *Attitudes Toward Geometry*

The third research question examined the influence of a non-traditional geometry course on the attitudes toward geometry of prospective elementary teachers. Both quantitative and qualitative data were collected and analyzed. Descriptive statistical analysis (means, standard deviations, and confidence intervals) and a paired t-test between the pre- and post-ATGS were conducted to determine if the attitude toward geometry of prospective elementary teachers changed over the duration of their participation in a non-traditional geometry

course. Each of the three subscales of the ATGS was similarly analyzed. In addition, journal prompts, open-ended questionnaires, and interview transcripts were analyzed to examine the perceptions of prospective elementary teachers about whether their experiences in a non-traditional geometry course had an effect on their attitude toward geometry.

This study shows that experiences in a non-traditional geometry course do have an impact on prospective elementary teachers' overall attitude toward geometry, as there was a statistically significant difference ( $t_{20} = 2.90$ ;  $p = .009$ ) between responses on the pre- and post-ATGS. Therefore, these prospective elementary teachers had a more positive attitude toward geometry upon completion of the non-traditional geometry course. As teachers they should have a positive influence on the attitudes of their own students, be more likely to investigate the mathematical conjectures of their students, and be more likely to spend an adequate time on teaching mathematics.

A statistically significant difference was found on the confidence to learn geometry subscale ( $t_{20} = 3.50$ ;  $p = .002$ ) and on the enjoyment to study geometry subscale ( $t_{20} = 2.26$ ;  $p = .035$ ) of the ATGS. These results indicate that the characteristics of this non-traditional geometry course had a positive influence on prospective elementary teachers' confidence to learn geometry and enjoyment or motivation to study geometry. Qualitative data analysis suggests that approximately three-fourths of the prospective elementary teachers felt their experiences in this non-traditional geometry course had a positive effect on their confidence and enjoyment to study geometry. More specifically, they equated

these positive effects to the characteristics of the course and/or to their increased understanding of geometric concepts. This increase in confidence being attributed to increased understanding or achievement is consistent with findings reported by Reyes (1984) and Dowling (1978). However, the findings of this study concerning the enjoyment to study geometry can add to what McLeod (1992) considers to be a small body of literature related to the emotional reactions of students to mathematics.

While there was not a significant difference ( $t_{20} = 1.77$ ;  $p=.091$ ) regarding beliefs of prospective elementary teachers about the usefulness to study geometry on the usefulness subscale of the ATGS, analysis of qualitative data indicated that slightly more than one-half of these prospective elementary teachers believed their perceptions of the usefulness to study geometry had increased. The prospective elementary teachers who felt their belief in the usefulness of studying geometry had been positively influenced attributed this change in their beliefs to an increase in their perception of the uses of geometry in the world around them and/or to its usefulness in their future career as elementary teachers.

### *Mathematics Teaching Efficacy*

The final research question explored how the experiences of prospective elementary teachers in a non-traditional geometry course influenced their mathematics teaching efficacy. In other words, did their belief in their ability to teach geometry change over the course of these experiences? In order to examine the perceptions of these prospective elementary teachers both

quantitative and qualitative data was collected and analyzed. Descriptive statistical analysis (means, standard deviations, and confidence intervals) and a paired t-test between the two administrations of the MTEBI were conducted to determine if the personal mathematics teaching efficacy (PMTE) and the mathematics teaching outcome expectancy (MTOE) of prospective elementary teachers changed over the duration of their participation in a non-traditional geometry course. In addition, journal prompts, open-ended questionnaires, and interview transcripts were analyzed to examine the perceptions of prospective elementary teachers about whether their experiences in a non-traditional geometry course had an effect on their personal mathematics teaching efficacy.

The prospective elementary teachers had a mean score of 46.90 on the pre-test and a slightly higher mean score of 47.90 on the post-test for the PMTE scale of the MTEBI. The results of the paired t-test used to determine if the mean difference was statistically significant indicated no significant difference between the pre- and post-tests. These results can be seen in Table 8 of Chapter IV. On the MTOE scale of the MTEBI the prospective elementary teachers' mean score decreased slightly from 30.14 to 29.48. Once again the results of the paired t-test indicated no significant difference between the pre- and post-test results on the MTOE scale. These quantitative results suggest that participants' experiences in a non-traditional geometry course had no effect on their mathematics teaching efficacy.

In contrast to quantitative results, analysis of qualitative responses suggests that approximately three-fourths of the prospective elementary teachers

felt their experiences in this non-traditional geometry course had a positive effect on their belief in their ability to teach geometric concepts to their future students. These prospective elementary teachers indicated that they felt better prepared to teach, more comfortable to answer student questions, and had the necessary skills to teach geometric concepts.

One possible reason for the inconsistency between the qualitative and quantitative results could be the specificity of the measure of teacher efficacy of the instrument used. The MTEBI measures the mathematics teaching efficacy beliefs of prospective teachers and the qualitative data collected in this study explored the geometry teaching efficacy beliefs of prospective elementary teachers. Tschannen-Moran and Hoy (2001) point out problems with measurement of teaching efficacy and question the appropriate level of specificity in measuring of teacher efficacy.

### *Implications for Teacher Programs*

The results of this study signify important implications for teacher education programs. This study reveals first, that the characteristics of this non-traditional course were consistent with recommendations supported by such organizations as the National Council of Teachers of Mathematics (e.g. NCTM, 1991) and the Mathematical Association of America (CBMS, 2001). Thus, it is possible to design and implement a mathematics content course for undergraduates that adheres to the vision of the reform efforts outlined by the NCTM (1989, 1991, 2000). As pointed out in the standards for the professional

development of teachers of mathematics (NCTM, 1991), teachers need to “experience good mathematics teaching” (p. 127) because through these experiences they develop ideas about how mathematics should be taught. Frykholm (1999) reported that students felt they had heard a lot about the theories behind reform efforts in mathematics, but they had not experienced these reform efforts. Therefore, if mathematics content courses continue to adhere to the traditional lecture format prospective elementary teachers’ view of teaching mathematics will remain unchanged and reform efforts will be hindered. Thus, the ideal situation would be for the teaching of all mathematics content courses taken by prospective elementary teachers to model pedagogical strategies that support student knowledge construction and adhere to the vision set forth by NCTM in its *Principles and Standards for School Mathematics* (2000).

Second, this course could not be duplicated by simply incorporating whole class discussions, group activities, CD problems, projects, and the use of daily activity sheets. The heart of this course is the creation of a classroom community by the instructor wherein students’ knowledge construction was nurtured and students felt comfortable to autonomously grapple with the material. The orchestration of whole class discussion by the instructor could not have happened without the careful choice of mathematical tasks that engage students to think and question. As pointed out in the NCTM *Principles and Standards for School Mathematics* (2000), “worthwhile mathematical tasks alone are not

sufficient for effective teaching” (p. 19), but must be carefully orchestrated to facilitate student learning.

Third, the perceptions of the prospective elementary teachers about these characteristics suggest that the role of the mathematics course can be more than just to enhance the content knowledge of the students. The perceptions of the prospective elementary teachers about these characteristics revealed that when students perceive themselves as part of learning environment that fosters mathematical thinking and genuine respect between students and between the instructor and students their confidence to learn and teach is increased. Additionally, since one of the goals of education is to develop autonomous learners (Kamii, 2000a; Kamii 2000b), the type of environment fostered in this course is imperative. This study indicated that while students were at first perturbed by not being reassured of “correct” answers, this frustration eventually gave way to students becoming more confident of their explanations. Therefore, this study has implications for future research into the effect of the characteristics of this non-traditional course on student autonomy in relation to studying geometry or mathematics.

Fourth, this study reveals that prospective elementary teachers’ attitudes toward geometry and mathematics teaching efficacy can be enhanced by their experiences in the non-traditional practices described in this study. It has been established that teachers attitudes and teacher efficacy effects the attitude and efficacy of their students (e.g. Aiken, 1972; Anderson, Greene, & Loewen, 1988) and their behavior in the classroom and the methods they incorporate into their



mathematics teaching (e.g. Gibson & Dembo, 1984; McDevitt, et. al., 1993). This realization along with findings of this study provides a basis to consider attitudes and teacher efficacy in teacher education programs.

### *Recommendations for Future Research*

Further research on affect in mathematics education is needed. The more that teacher educators understand about attitudes toward geometry/mathematics and mathematics teacher efficacy and how they are influenced, the more effectively teacher education programs can address the needs of prospective teachers. Recommendations for further research based on findings from this study leads to the following possible explorations:

- Reliability and validity are always an issue; therefore, this study, using the newly develop ATGS, should be replicated with other groups of participants and at other universities.
- Since this study was conducted during a two-month summer session, the study should also be conducted during a typical four-month semester and the results compared.
- Additional studies should be conducted that involve a comparison of the change in attitudes of prospective elementary teachers in a traditional lecture-oriented geometry course verses a non-traditional geometry course in which the students are active participants.
- Longitudinal studies should be conducted to determine whether this increase in attitude toward geometry carries over into the methods

courses, student teaching, and first year of teaching of prospective elementary teachers participating in non-traditional geometry courses.

- Other affective variables such as mathematics anxiety, self-concept, and learned helplessness should be investigated to determine the influence of a non-traditional mathematics course on these affective factors. McLeod (1992) points out that the role of affect is prominent in the current reform efforts within mathematics education.
- Results of this study suggest that future research should be conducted to explore the influence of a non-traditional mathematics course on student achievement and on conceptual understanding of the mathematics being taught.
- The construction and validation of an instrument that focuses on the geometry teaching efficacy of prospective elementary teachers needs to be developed.
- This study revealed some inconsistencies between quantitative and qualitative results; therefore, additional studies should be conducted to examine the issue of techniques for measuring both attitudes and teaching efficacy. Results of this study reinforced a suggestion by Kulm (1980) that the construction of self-reported scales should continue to be developed and improved but more attention should be paid to observation of student behavior and use of responses to open-ended questionnaires to construct items on these scales.

### *Concluding Comments*

Often times prospective elementary teachers have a background and general knowledge of the mathematics they will teach, but do not have a good conceptual understanding. The characteristics of this non-traditional geometry course lie in stark contrast to those of typical traditional lecture style mathematics content courses. In this non-traditional geometry course students were allowed and encouraged to think about, mull over ideas, and gain a firm foundation of the concepts whereas, in most traditional mathematics content courses students tend to mimic the procedure outlined by the teacher over and over resulting in little or no understanding.

The role of research into the affective domain of mathematics education can be important to reform efforts for mathematics teaching and learning. Although the sample in this study was small, the results of the study provide evidence that certain classroom experiences, such as those highlighted as the characteristics of the non-traditional geometry course described here, have the potential to effect the mathematical attitudes and teaching efficacy beliefs of prospective elementary teachers. Therefore, the implementation of a nontraditional geometry course for prospective elementary teachers holds promise for positively modifying the attitudes toward studying geometry and the mathematics teaching efficacy of these prospective elementary teachers. As pointed out in the *Professional Standards for Teaching Mathematics* (NCTM, 1992), it is the teacher's responsibility to foster a disposition for doing mathematics in their students. If teachers do not exhibit a disposition to do mathematics themselves, how can they foster it in their students? Thus,

developing positive attitudes and efficacy beliefs about mathematics is an important goal of any teacher education program, which includes those mathematics courses taken by prospective elementary teachers.

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## APPENDICES



APPENDIX A  
BACKGROUND QUESTIONNAIRE

ID# \_\_\_\_\_

Date \_\_\_\_\_

### Demographic Information

Gender: M or F (circle one)

Age: \_\_\_\_\_ years

Major: \_\_\_\_\_

Ethnicity: \_\_\_ Native-American \_\_\_ Latino \_\_\_ African-American

\_\_\_ Caucasian \_\_\_ Asian

\_\_\_ Other (please specify: \_\_\_\_\_)

Did you take a geometry course in high school? Yes or No (circle one)

Have you taken this course previously? Yes or No (circle one)

I consider myself:

\_\_\_ Poor in Math  
Math

\_\_\_ Average in Math

\_\_\_ Excellent in

Place an X beside each Mathematics course listed below that you took in High School.

\_\_\_ Algebra I

\_\_\_ Pre-Calculus or Math Analysis

\_\_\_ Algebra II

\_\_\_ Calculus

\_\_\_ Geometry

\_\_\_ Statistics

\_\_\_ Algebra III

\_\_\_ Other (specify): \_\_\_\_\_

\_\_\_ Trigonometry

Place an X beside each Mathematics course listed below that you have taken in college.

\_\_\_ College Algebra

\_\_\_ Mathematical Structures

\_\_\_ Functions

\_\_\_ Statistics

\_\_\_ Applications of Modern Math

\_\_\_ Other (specify): \_\_\_\_\_

APPENDIX B  
PRE/POST QUESTIONNAIRES





Post-Questionnaire - Part II

ID# \_\_\_\_\_

Date: \_\_\_\_\_

Please indicate the degree to which you agree or disagree with each statement below by circling the appropriate number to the right of the statement.

	1	2	3	4	5
	Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
1. Doing CD problems was enjoyable.	1	2	3	4	5
2. Doing CD problems was a waste of my time.	1	2	3	4	5
3. Doing CD problems helped me make sense of geometric ideas.	1	2	3	4	5
4. Doing CD problems helped me gain confidence to learn geometry.	1	2	3	4	5
5. Doing CD problems helped me gain confidence in my ability to teach geometric concepts effectively.	1	2	3	4	5
6. Completing projects was enjoyable.	1	2	3	4	5
7. Completing projects was a waste of my time.	1	2	3	4	5
8. Completing projects helped me make sense of geometric ideas.	1	2	3	4	5
9. Completing projects helped me gain confidence to learn geometry.	1	2	3	4	5
10. Completing projects helped me gain confidence in my ability to teach geometric concepts effectively.	1	2	3	4	5
11. Hearing other students idea's during group activities was enjoyable.	1	2	3	4	5
12. Hearing other students idea's during group activities was a waste of my time.	1	2	3	4	5
13. Hearing other students idea's during group activities helped me make sense of geometric ideas.	1	2	3	4	5
14. Hearing other students idea's during group activities helped me gain confidence to learn geometry.	1	2	3	4	5
15. Hearing other students idea's during group activities helped me gain confidence in my ability to teach geometric concepts effectively.	1	2	3	4	5
16. Class discussions were an enjoyable part of this course.	1	2	3	4	5
17. Class discussions were a waste of my time.	1	2	3	4	5
18. Class discussions helped me make sense of geometric ideas.	1	2	3	4	5

- |   |   |   |   |   |   |
|---|---|---|---|---|---|
| 19. Class discussions helped me gain confidence to learn geometry.  | 1 | 2 | 3 | 4 | 5 |
| 20. Class discussions helped me gain confidence in my ability to teach geometric concepts effectively.                              | 1 | 2 | 3 | 4 | 5 |
| 21. The opportunity to write about geometric ideas was enjoyable.   | 1 | 2 | 3 | 4 | 5 |
| 22. The opportunity to write about geometric ideas was a waste of my time.  | 1 | 2 | 3 | 4 | 5 |
| 23. The opportunity to write about geometric ideas helped me make sense of geometric concepts.                                      | 1 | 2 | 3 | 4 | 5 |
| 24. The opportunity to write about geometric ideas helped me gain confidence to learn geometry.                                     | 1 | 2 | 3 | 4 | 5 |
| 25. The opportunity to write about geometric ideas helped me gain confidence in my ability to teach geometric concepts effectively. | 1 | 2 | 3 | 4 | 5 |
| 26. Seeing multiple solution strategies was enjoyable.  | 1 | 2 | 3 | 4 | 5 |
| 27. Seeing multiple solution strategies was a waste of my time.   | 1 | 2 | 3 | 4 | 5 |
| 28. Seeing multiple solution strategies helped me make sense of geometric concepts.   | 1 | 2 | 3 | 4 | 5 |
| 29. Seeing multiple solution strategies helped me gain confidence to learn geometry.  | 1 | 2 | 3 | 4 | 5 |
| 30. Seeing multiple solution strategies helped me gain confidence in my ability to teach geometric concepts effectively.            | 1 | 2 | 3 | 4 | 5 |

APPENDIX C  
JOURNAL PROMPTS



Journal Prompt

ID# \_\_\_\_\_

Date: \_\_\_\_\_

*Please take a few minutes to jot down your thoughts on the following questions.*

- **Have the experiences in this course had an affect on your confidence to learn geometry? Please comment.**

- **Has your feelings about your ability to teach geometric concepts been affected by your experiences in this course? Please comment.**

**(OVER)**

- **Has your belief in the usefulness of geometry been affected by your experiences in this course? Please comment.**

- **Has your enjoyment of geometry been affected by your experiences in this course? Please comment.**

*Please take a few minutes to jot down your thoughts on these ideas. As before, we will type up all responses anonymously so that you can see how other members of this class respond. (Dr. Wolfe will not see your names until after the class is over.)*

- Most of the class time in this course is taken up by whole class discussion of the day's assignment. This replaces the more traditional lecture. My thoughts and feelings about this are...**
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
- Many course activities (e.g. CD problems, describing relationships, expressing your thinking, etc.) provide opportunities for you to write about your ideas or insights about a geometric situation. My thoughts and feelings about this are...**

APPENDIX D  
INTERVIEW PROTOCOL

## Interview Protocol

Good morning \_\_\_\_\_ I am so glad you agreed to talk with me. As you know I am working on a study that deals with students' perceptions and feelings about the geometric structures class you took this past summer. I wanted to talk with you because I thought you would be a good person to give me some information about geometric structures and your feelings about the course.

- Tell be about your experiences studying geometry prior to taking the geometric structures class.
  - What kinds of things did you find valuable?
  - Describe your feelings about how confident you felt.
  - Can you describe a few of the effects of your participation in class?
- If you were to tell someone else about your experience in geometric structures, what would you tell them?
  - Suppose someone came to observe in your class this summer for a typical class period; in what ways would they have seen you participating?
    - How do you think your participation in geometric structures affected your attitude toward studying geometry?
  - Suppose I told you that you were going to teach a geometry lesson to 5<sup>th</sup> graders tomorrow, how would this make you feel?
  - Others have commented that this course had an effect on their belief in their ability to teach geometry and indicated this in their journal prompts, what about you?
    - Do you now feel that you can teach geometric concepts to elementary children effectively?
    - Do you now feel that you are responsible for the achievements of your students while studying geometry topics?
    - If you are given a choice, would you now invite the principal in to evaluate you during a geometry lesson?
  - Could you tell me what aspect(s) of geometric structures that you feel had an affect on your attitude toward the study of geometry?
    - In what way?
    - I've heard that some students' confidence in studying geometry changes during the semester they take geometric structures; do you feel that geometric structures had an affect on your confidence?
      - (if they respond in a single word) Can you tell why you feel that way?
  - How valuable do you feel that studying geometry is?
    - Do you feel that this changed as a result of taking GS?
  - What, if anything, do you now feel about studying geometry that you didn't feel before taking geometric structures?
- Now lets focus on various aspects of the class?
- Whole class discussions seemed to be a major part of this class. What is the first thing that comes to your mind when you think about these whole class discussions?
- What was a typical class discussion like?
  - How did these discussions affect your confidence to learn geometry?
  - How did these discussions affect your enjoyment of studying geometry?
  - How did these discussions affect your belief in the usefulness to study geometry?
  - How did these discussions affect your notion of your ability to teach geometry?(Repeat with projects, CD problems, writing experiences, seeing of multiple strategies, group work)
- Now that you have an idea about what my study is about, is there anything that I should have asked you that I didn't think to ask?

APPENDIX E  
ATTITUDES TOWARD GEOMETRY SCALES (ATGS)

**Attitudes Toward Geometry Scales  
(ATGS)**

ID# \_\_\_\_\_

Date: \_\_\_\_\_

For the following statements, circle your level of agreement with each of the following statements.

SD - if you strongly disagree

D - if you disagree

N - if your feeling is neutral

A - if you agree

SA - if you strongly agree

- |  |    |   |   |   |    |
|--|----|---|---|---|----|
| 1. I am sure that I can learn geometry concepts.   | SD | D | N | A | SA |
| 2. I believe that I will need geometry for my future.  | SD | D | N | A | SA |
| 3. Geometry problems are boring.   | SD | D | N | A | SA |
| 4. When I leave class with a geometry question unanswered, I continue to think about it.             | SD | D | N | A | SA |
| 5. I often have trouble solving geometry problems.   | SD | D | N | A | SA |
| 6. When I start solving a geometry problem, I find it hard to stop working on it.                    | SD | D | N | A | SA |
| 7. Time drags during geometry class.   | SD | D | N | A | SA |
| 8. I am confident I can get good grades in geometry.   | SD | D | N | A | SA |
| 9. When I can't figure out a geometry problem, I feel as though I am lost and can't find my way out. | SD | D | N | A | SA |
| 10. Geometry has no relevance in my life.  | SD | D | N | A | SA |
| 11. I lack confidence in my ability to solve geometry problems.                                      | SD | D | N | A | SA |
| 12. Geometry is not a practical subject to study.  | SD | D | N | A | SA |
| 13. I feel sure of myself when doing geometry problems.  | SD | D | N | A | SA |
| 14. Geometry is fun.   | SD | D | N | A | SA |
| 15. I just try to get my homework done for geometry class in order to get a grade.                   | SD | D | N | A | SA |
| 16. Geometry is an interesting subject to study.   | SD | D | N | A | SA |
| 17. I can see ways of using geometry concepts to solve everyday problems.                            | SD | D | N | A | SA |
| 18. For some reason even though I study, geometry seems unusually hard for me.                       | SD | D | N | A | SA |
| 19. Geometry is not worthwhile to study.   | SD | D | N | A | SA |

20. I often see geometry in everyday things.	SD	D	N	A	SA
21. Geometry problems often scare me	SD	D	N	A	SA
22. I am confident that if I work long enough on a geometry problem, I will be able to solve it.	SD	D	N	A	SA
23. Solving geometry problems is enjoyable.	SD	D	N	A	SA
24. I will need a firm understanding of geometry in my future work.	SD	D	N	A	SA
25. Working out geometry problems does not appeal to me.	SD	D	N	A	SA
26. I do not expect to use geometry when I get out of school.	SD	D	N	A	SA
27. Geometry tests usually seem difficult.	SD	D	N	A	SA
28. I will not need geometry for my future.	SD	D	N	A	SA
29. I can usually make sense of geometry concepts.	SD	D	N	A	SA
30. Geometry has many interesting topics to study.	SD	D	N	A	SA
31. Geometry is a practical subject to study.	SD	D	N	A	SA
32. I have a lot of confidence when it comes to studying geometry.	SD	D	N	A	SA



APPENDIX F  
MATHEMATICS TEACHING EFFICACY BELIEF INSTRUMENT  
(MTEBI)

**MATHEMATICS TEACHING EFFICACY BELIEFS INSTRUMENT  
(MTEBI - Preservice)**

Date: \_\_\_\_\_

ID#: \_\_\_\_\_

Please indicate the degree to which you agree or disagree with each statement below by circling the appropriate number to the right of the statement.

1	2	3	4	5	
Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree	
			Strongly Disagree	Strongly Agree	
1. When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.	1	2	3	4	5
2. I will continually find better ways to teach mathematics.	1	2	3	4	5
3. Even if I try very hard, I will not teach mathematics as well as I will most subjects.	1	2	3	4	5
4. When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.	1	2	3	4	5
5. I know how to teach mathematics concepts effectively.	1	2	3	4	5
6. I will not be very effective in monitoring mathematics activities.	1	2	3	4	5
7. If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.	1	2	3	4	5
8. I will generally teach mathematics ineffectively.	1	2	3	4	5
9. The inadequacy of a student's mathematics background can be overcome by good teaching.	1	2	3	4	5
10. When a low-achieving child progresses in mathematics, it is usually due to extra attention by the teacher.	1	2	3	4	5
11. I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	1	2	3	4	5
12. The teacher is generally responsible for the achievement of students in mathematics.	1	2	3	4	5
13. Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.	1	2	3	4	5
14. If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.	1	2	3	4	5

15. I will find it difficult to use manipulatives to explain to students why mathematics works. 1 2 3 4 5
16. I will typically be able to answer students' questions. 1 2 3 4 5
17. I wonder if I will have the necessary skills to teach mathematics. 1 2 3 4 5
18. Given a choice, I will not invite the principal to evaluate my mathematics teaching. 1 2 3 4 5
19. When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better. 1 2 3 4 5
20. When teaching mathematics, I will usually welcome student questions. 1 2 3 4 5
21. I do not know what to do to turn students on to mathematics. 1 2 3 4 5

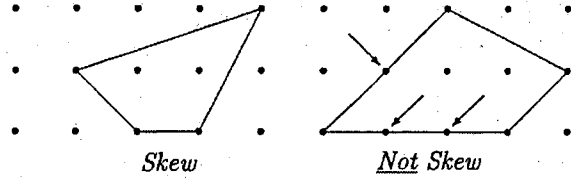
*APPENDIX G*  
COURSE DOCUMENT SAMPLES



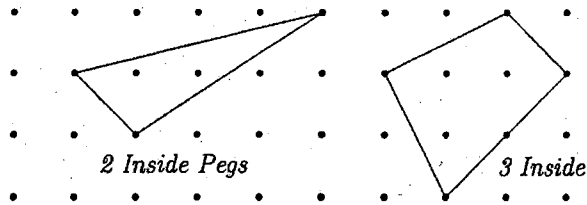
**6.1** Areas of Skew Quadrilaterals

Name: \_\_\_\_\_

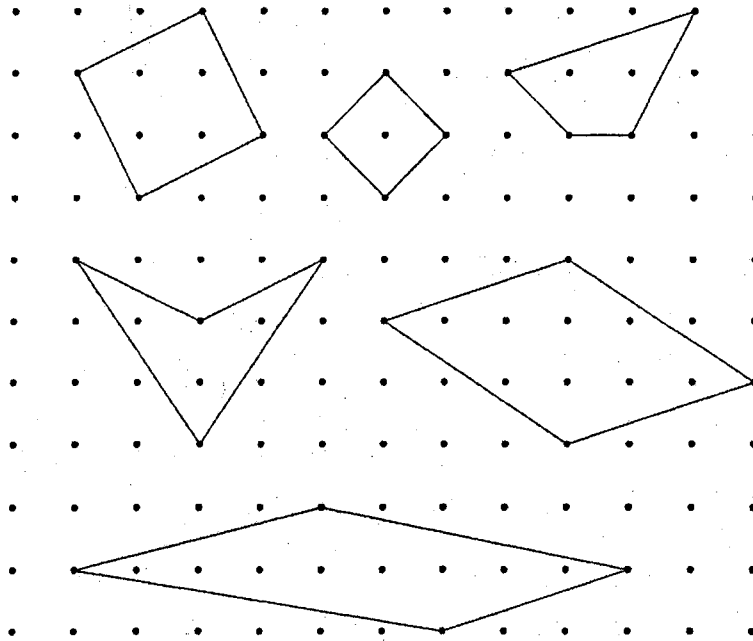
On a geoboard, a figure is a *skew* figure if each edge touches exactly 2 pegs (one at each end).



*Inside Pegs* are the pegs entirely inside the figure.

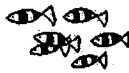


Using any method you like, find the area of the skew quadrilaterals pictured and write the value of the area inside the figure. Check your answers with others.



Do you see any relationship between the area and the number of internal dots? Describe the relationship that you see.

Do you think this relationship is always true? Under what conditions is it true? Describe briefly.

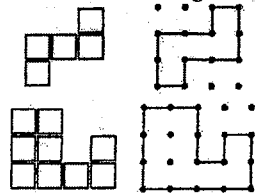


**6.2 Solid Tile Shapes**

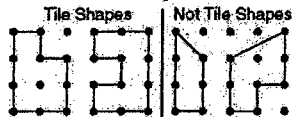
Name: \_\_\_\_\_

**Solid Tile Shapes**

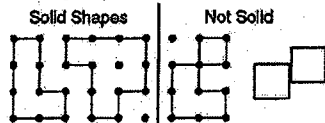
A *tile shape* is a shape which can be made by putting together square tiles. We can make tile shapes either by putting together square pieces or by using a geoboard. When representing a tile shape we can show the square tiles or we can make the outline with a rubber band on a geoboard (or draw it on dotpaper). Here are two examples shown both as tiles and geoboard figures:



Here are some more examples.



We will be interested in *solid* tile shapes. A tile shape is *solid* if (a) when two tiles touch they touch either at a corner or along an entire side and (b) each tile is attached to the whole shape by at least one side.

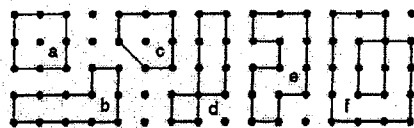


Some tile shapes are *skinny*. When drawn on dot paper skinny tile shapes have no pegs inside of them.



**Problems**

1. Draw a circle around the figures below which are solid tile shapes. Also, if the shape is not circled, write "Not Solid" or "Not Tile Shape" to indicate why it is not a solid tile shape.



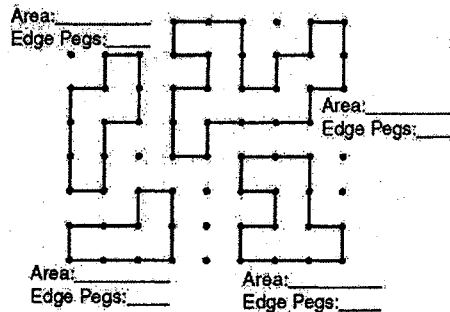
Also write "Skinny" inside the skinny shapes.

2. Figure the area of these three solid tile shapes.



The area of tile shapes is really easy to figure out. What do you think is a good way to get the area of a tile shape? Describe:

3. Several skinny tile shapes are given below. Under each shape write the area and also write down the number of *edge pegs*. An edge peg is a peg that is touched by the rubber band surrounding the figure.



- (a) Do you see a relationship between the number of edge pegs and the area? Describe the relationship that you see. Your description:

- (b) If the area were 9 units, how many edge pegs would be around the skinny figure?
- (c) If there were 16 edge pegs, what would the area of the skinny tile shape be?
- (d) If you know the number of edge pegs, how can you figure out the area? Describe your way.



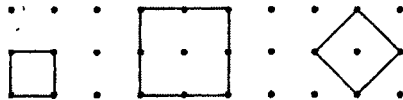
# 8.5 Diagonal Lengths on a Geoboard

Name: \_\_\_\_\_

We can find diagonal lengths by two different methods.

### Method I

- Write the area and side length of each of these squares as in the first example.



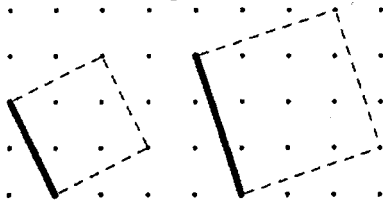
Area =	Area =	Area =
Side length =	Side length =	Side length =

If you know the area of a square, how can you figure the length of a side? Describe:

- If the area is 100, how long is a side? \_\_\_\_\_  
If the area is 5, how long is a side? \_\_\_\_\_

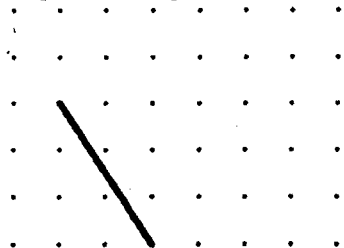
*Note:* Square roots often occur as the lengths of diagonal (or slanted) lines.

- Figure these diagonal lengths by figuring the areas of the square.



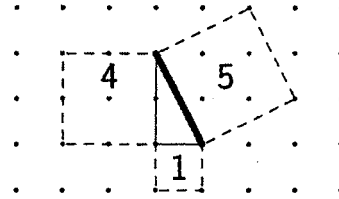
Area =	Area =
Side (as square root) =	Side (as square root) =
Side (as decimal) =	Side (as decimal) =

- Figure the following slanted length by making a square and figuring its area.



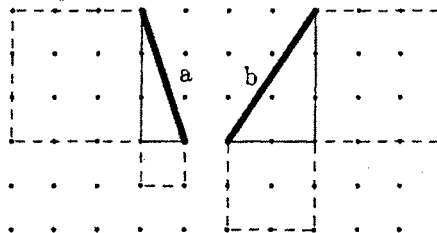
### Method II

The principle of right triangles of squares (Pythagorean Theorem) gives a fast way to find diagonal lengths.

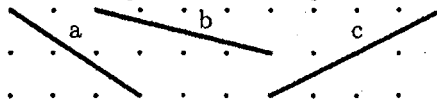


The diagonal is a side of a square whose area can be found by adding the areas of the squares on the legs (smaller squares).

- Find the lengths of these two diagonals by this method. Express your answer as both a square root and as a decimal.



- Find the lengths of these diagonals.



### Summary

- Describe, in your own words, the best way you see to figure the length of a slanted line on the geoboard.



## 11.4 Congruence Conditions for Triangles and CPCT

There are several ways for an architect to exactly describe a triangular shape.

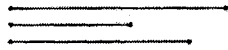
### Triangle Congruence Conditions

We have had 4 worksheets titled "Making Triangles". In a way these worksheets were about congruence conditions for triangles. We saw that, in the first three cases (SSS, SAS and ASA), there was only one possible triangle that could be constructed satisfying the given conditions.

These experiences are summarized here in terms of the idea of a congruence condition.

For triangles, SSS is a congruence condition.

Imagine that you and others in class were each given three straws, say with lengths as pictured here.

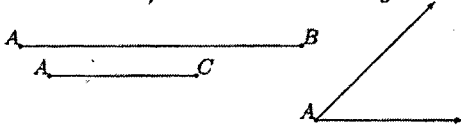


If you were asked to put the straws together to form a triangle, do you think that each student would get the same (congruent) triangle?

In fact, they would. When we did the worksheet "Making Triangles I: Side-Side-Side" everybody came up with the same triangle. This means that SSS is a congruence condition.

For triangles, SAS is a congruence condition.

Again, imagine that you are given two straws and are required to form a specified angle between the two straws (abbreviated SAS). This information is diagramed here.

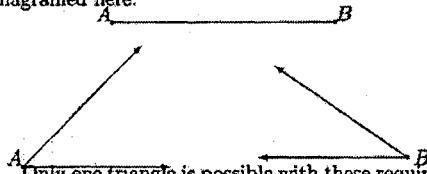


Notice that the straws and angle are labeled so that the angle is between the two given straws.

There is only one triangle which can be made consistent with this information. That is, SAS is a congruence condition.

For triangles, ASA is a congruence condition.

Imagine that you are given one straw and are required to form certain angles at both ends (ASA for short) as diagramed here.

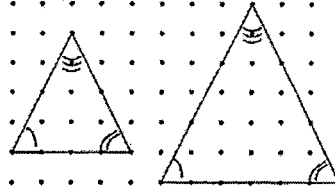


Only one triangle is possible with these requirements. Again, ASA is a congruence condition.

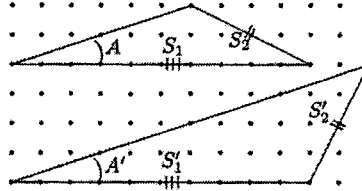
### Not Triangle Congruence Conditions

If there is more than one triangle which matches a condition then it is not a congruence condition. Both AAA (meaning three angles are given) and SSA (meaning two sides and an angle not between the two sides are given) are not congruence conditions. Pairs of non-congruent triangles are given below to illustrate this.

AAA is not a congruence condition:



SSA is not a congruence condition:



This example above is similar to the construction we did in the worksheet *Making Triangles IV: The Ambiguous Case*.

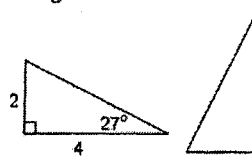
### CPCT Principle

When two triangles are congruent this just means that one could be stacked directly on top of the other and they would line up exactly. Thus each side of one lines up with a side of the other and each angle of one lines up with an angle of the other. These pairs of sides or angles, one from each triangle, which line up are called *corresponding parts*. Corresponding parts can be either *corresponding sides* or *corresponding angles*.

This fact that congruent triangles always line up is called the CPCT Principle (CPCT stands for *Corresponding Parts of Congruent Triangles*).

CPCT Principle: Corresponding parts of congruent triangles are congruent.

For example, the two triangles below are congruent. Using the CPCT principle, see if you can figure out all of the sides and angles of the second triangle.





## Sample Student Responses From "Solid Tile Shape" Daily Activity Sheet

Do you see a relationship between the number of edge pegs and the area? Describe the relationship that you see. **Your description:**

$$\text{Area} \times (2) + 2 = \text{Edge Pegs}$$

Do you see a relationship between the number of edge pegs and the area? Describe the relationship that you see. **Your description:**

You double your area + add 2 to get your edge pegs.

Do you see a relationship between the number of edge pegs and the area? Describe the relationship that you see. **Your description:**

$$A = EP \div 2 - 1$$

Do you see a relationship between the number of edge pegs and the area? Describe the relationship that you see. **Your description:**

45

$(\text{area} \times 2) + 2 = \# \text{ of edge pegs}$

$\frac{\text{edge pegs}}{2} - 1 = \text{area}$

Do you see a relationship between the number of edge pegs and the area? Describe the relationship that you see. **Your description:**

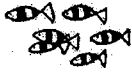
The Area goes by one's and the edge pegs goes by two's.

4	10
5	12
6	14
...	...
11	24

Do you see a relationship between the number of edge pegs and the area? Describe the relationship that you see. **Your description:**

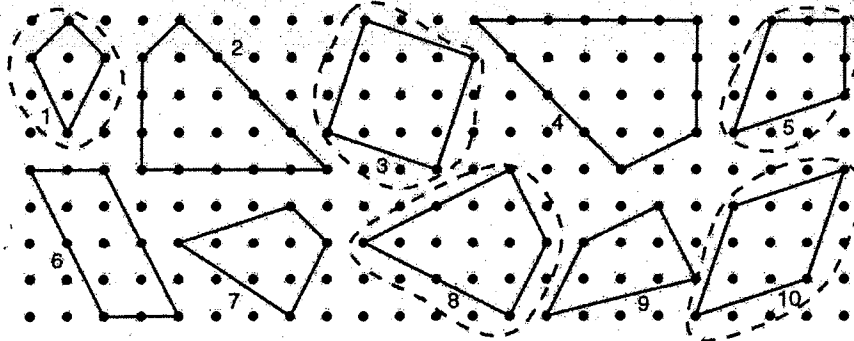
You can multiply the area by 2 and then add 2 and that is the number of edge pegs.

$$2a + 2 = p$$



**2.14** What is a Kite? – Equivalent Definitions

Name: \_\_\_\_\_ Group members: \_\_\_\_\_



Among the examples of quadrilaterals drawn above, the *kites* are circled with a dashed line.

*Note:* Included among the kites are some special cases: number 3 is a square, and number 10 is a rhombus, both special cases of kites. This means we are using an *inclusive* definition of a kite.

**Your Problem**

Often there are many different ways to define a concept, that is, different ways to describe the same thing. We will call these *equivalent definitions*. In this activity page we will work with this idea of equivalent definitions.

Several possible definitions for a kite are given below. For each of these possible definitions carefully work through the examples drawn above to determine which ones satisfy the definition.

After *Example Numbers* write down all of the numbers of the quadrilaterals which satisfy the definition. Check to make sure you are in agreement with others in your group on these.

After *Is it an equivalent definition (yes or counterexample)?* write *yes* if it seems to be a good definition for a kite or, if it does not work, give an example number from above which satisfies the possible definition but is not a real kite.

**Possible Definition A**

*A kite is a convex quadrilateral which has at least one pair of congruent opposite angles.*

Example Numbers:

Is it an equivalent definition (yes or counterexample)?

**Possible Definition B**

*A kite is a convex quadrilateral that has perpendicular diagonals.*

Example Numbers:

Is it an equivalent definition (yes or counterexample)?

**Possible Definition C**

*A kite is a convex quadrilateral in which at least one diagonal is a line of symmetry.*

Example Numbers:

Is it an equivalent definition (yes or counterexample)?

**Possible Definition D**

*A kite is a convex quadrilateral that has two pairs of adjacent congruent sides.*

Example Numbers:

Is it an equivalent definition (yes or counterexample)?

**Possible Definition E**

*A kite is a convex quadrilateral in which at least one diagonal is a perpendicular bisector of the other.*

Example Numbers:

Is it an equivalent definition (yes or counterexample)?

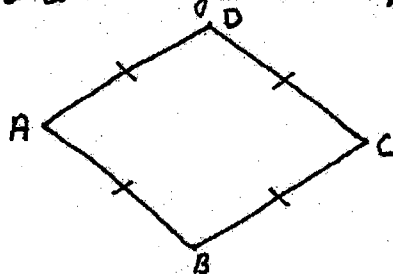
**Q&A 11.7 Example: Model Problem A**

**Official Definition:** An official rhombus is a quadrilateral where all 4 sides are equal.

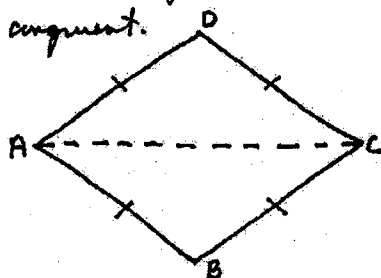
**Property:** For official rhombuses, opposite angles are congruent (by SSS).

Note that we are given the hint to use the congruence condition SSS for this problem.

**Step 1** The four sides of this rhombus are marked as equal since this is given in the official definition.



**Step 2** Notice that the diagonal divides the rhombus into 2 triangles ( $\triangle ACD$  and  $\triangle ACB$ ) which look congruent.



Note: We could also have used the other diagonal.

**Step 3** We apply the congruence condition SSS to the two triangles. First  $AD \cong AB$  and  $CD \cong CB$  from the official definition and the diagonal  $AC \cong AC$ . So  $\triangle ACD \cong \triangle ACB$  by SSS.

**Step 4** Now  $\angle B$  corresponds to  $\angle D$  and so, by CPCT,  $\angle B \cong \angle D$ . This shows that these opposite angles are congruent.

**Four Step Model**

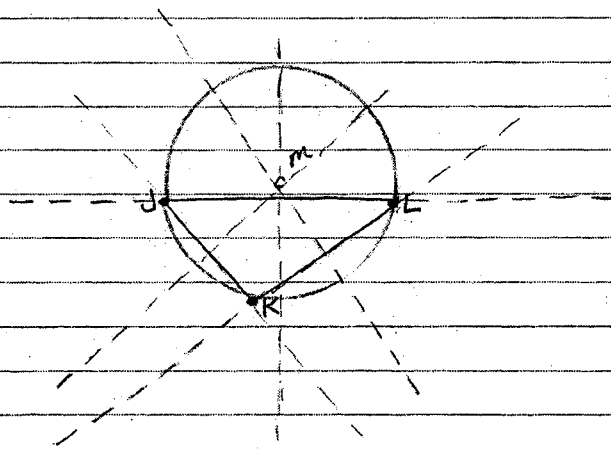
- Step 1: Mark given information on figure: official definition, constructions, related definitions, earlier results.
- Step 2: Draw and identify apparently congruent triangles.
- Step 3: Cite and fully apply CC to triangles.
- Step 4: Apply CPCT for results needed for the property.

## Sample Student Responses to a CD Problem

CD Problem asked students to "Locate the center of the circle passing through points J, K, and L; then use a compass to draw the circle."

-Locate the center of the circle passing thru J, K, L  
-use a compass to draw the circle.

- 1 Trace the 3 dots label J, K, L
- 2 make folds along JK, KL, JL. Connecting the vertices, trace over with a pencil to make the  $\Delta$ .
- 3 Fold J on top of K;  $\perp$  bisector
- 4 Fold J on top of L;  $\perp$  bisector
- 5 Fold K on top of L;  $\perp$  bisector



- 6 where all 3  $\perp$  bisectors meet you have the circumcenter
- 7 put the point of the compass on the circumcenter (m) & draw a circle that passes through pts. J, K, L
- 8 you have a circumscribed circle

## Instructions for Sample Project

### **Basic Ideas Project: Definitions and Properties**

Making sense out of the idea of definitions and properties can be difficult for college students and even more so for kids. As we discuss worksheets about quadrilaterals you are encouraged to think about what these two words mean. As a project you will be asked to develop a display to illustrate these ideas in as basic and simple a way as you can.

Your project will take several pages in a notebook. These pages need to be presented in such a way that they would be readable and attractive if displayed on the wall of a geometry classroom. Be creative and visual – we want to support the idea that geometry can be both fascinating and beautiful.

**Condition 1d:** Express as clearly and simply as you can what it means for a statement to be a definition.

**Condition 2d:** Express as clearly and simply as you can how you can tell if a statement is a “good” definition or a “bad” definition.

**Condition 3d:** Give at least two examples of “good” definitions, one from “real life” and one from geometry. Be sure to indicate why your examples are “good.”

**Condition 4d:** Give at least two examples of “bad” definitions, one from “real life” and one from geometry. Be sure to indicate why your examples are “bad.”

**Condition 5p:** Express as clearly and simply as you can what it means for a statement to be a property.

**Condition 6p:** All definitions are properties; however, some properties are not definitions. Express as clearly and simply as you can how you can tell that a property is not a definition.

**Condition 7p:** Give at least two examples of properties that are not definitions, one from “real life” and one from geometry.

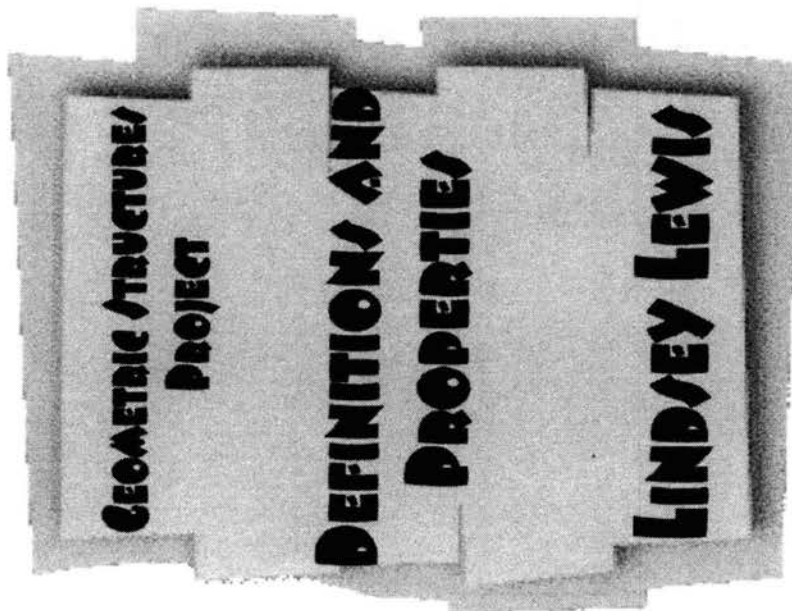
There are many examples of student projects on the websites for this course:

<http://www.math.okstate.edu/~rpssc>  
<http://www.math.okstate.edu/~geoset>

Eventually all web materials for this course will be on the “GeoSET” site (the second of the two listed above). During the transition you will find resources on both sites.

Perhaps the examples most related to this project will be found in the “RPSC” site under the link Geometry: Main Ideas. In the “GeoSET” site these student examples will be under the “Student” tab through the link Student Projects: Basic Ideas.

As part of the GeoSET project we will use some of the best projects as models to go either in the project website or in the materials for teachers who use these curricular materials. If your project is selected for inclusion you will get bonus project points.



## Good Definition or Bad Definition

A statement is a **good definition** when you can't think of another example that might fit in the definition.

A statement is a **bad definition** when you can think of just one other thing or many other things that could also fit in the definition besides what you are trying to define.

## Good & Bad Definitions

Here is an example of a good definition.

A pencil is a tool that uses lead to write and usually has an eraser.

This is a good definition because no other writing tool fits in this definition.

Here is an example of a bad definition.

A pencil is a tool used for writing.

This is a bad definition because there are many tools for writing such as a pen or a marker.

## Good & Bad Geometry Definitions

Here is an example of a good definition from geometry.

A parallelogram is a four-sided figure that has two pair of parallel sides and their opposite angles are equal.

This is a good definition because no other four-sided figure fits this definition that is not a parallelogram.

Here is an example of a bad definition.

A parallegram is a four-sided figure.

This is a bad example because any figure that has four sides fits this definition.

# PROPERTY

A statement is a **PROPERTY** when it describes only one trait of something.



A definition is a combination of properties. So, definitions are always properties, but not all properties are definitions.

Here is an easy way to tell the difference between a definition and a property. If you can think of something else that also fits the statement then the statement is a property and not a definition.

Here are some examples of properties that are not definitions.

A frame is an object that you put pictures in. This is a property of a frame, because you can also put pictures in an album.

A rectangle is a four-sided figure. This must also be a property because we know that a kite has four sides, but it is not a rectangle.

APPENDIX H  
IRB APPROVAL FORM

Oklahoma State University  
Institutional Review Board

Protocol Expires: 5/1/2004

Date: Friday, May 02, 2003

IRB Application No ED03113

Proposal Title: THE IMPACT OF A REFORM BASED GEOMETRY CONTENT COURSE ON  
PRESERVICE ELEMENTARY TEACHERS' ATTITUDES AND TEACHING EFFICACY

Principal  
Investigator(s):

Juliana Utley  
249 Willard  
Stillwater, OK 74078

Stacy Reeder  
238 Willard  
Stillwater, OK 74078

Reviewed and  
Processed as: Exempt

Approval Status Recommended by Reviewer(s): Approved

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Dear PI :

Your IRB application referenced above has been approved for one calendar year. Please make note of the expiration date indicated above. It is the judgment of the reviewers that the rights and welfare of individuals who may be asked to participate in this study will be respected, and that the research will be conducted in a manner consistent with the IRB requirements as outlined in section 45 CFR 46.

As Principal Investigator, it is your responsibility to do the following:

1. Conduct this study exactly as it has been approved. Any modifications to the research protocol must be submitted with the appropriate signatures for IRB approval.
2. Submit a request for continuation if the study extends beyond the approval period of one calendar year. This continuation must receive IRB review and approval before the research can continue.
3. Report any adverse events to the IRB Chair promptly. Adverse events are those which are unanticipated and impact the subjects during the course of this research; and
4. Notify the IRB office in writing when your research project is complete.

Please note that approved projects are subject to monitoring by the IRB. If you have questions about the IRB procedures or need any assistance from the Board, please contact Sharon Bacher, the Executive Secretary to the IRB, in 415 Whitehurst (phone: 405-744-5700, sbacher@okstate.edu).

Sincerely,



Carol Olson, Chair  
Institutional Review Board

VITA

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Juliana Gail Utley

Candidate for the Degree of  
Doctor of Philosophy

Dissertation: IMPACT OF A NON-TRADITIONAL GEOEMTRY COURSE  
ON PROSPECTIVE ELEMENTARY TEACHERS' ATTITUDES  
AND TEACHING EFFICACY

Major Field: Professional Education Studies with emphasis in Mathematics

Biographical:

Personal Data: Born in Tulsa, Oklahoma, the daughter of Sarah Mosteller and Tommy Clemmer. Married to Gary Utley and mother of Jason and Kristina.

Education: Graduated Valedictorian from Milburn High School, Milburn, Oklahoma in May 1977; attended Murray State College, Tishomingo, Oklahoma receiving an associates degree in May 1979; transferred to Oklahoma State University, Stillwater, Oklahoma and received a Bachelor of Science degree in 1982 with a Major in Mathematics and a minor in Mathematics Education. Earned a Master of Science Degree in Education with an emphasis in Mathematics from Oklahoma State University, Stillwater, Oklahoma in August, 2000. Completed the requirements for a Doctor of Philosophy degree with a focus on Mathematics Education at Oklahoma State University in July, 2004.

Experience: Middle school mathematics classroom teacher in Stillwater, Oklahoma 1982-1983; High school mathematics, physics, and computer science teacher in Claremore, Oklahoma 1983-1988; High school mathematics teacher and mathematics department head in Tuttle, Oklahoma 1988-1997; Junior high/high school mathematics teacher in Yale, Oklahoma 1997-1999. Mathematics Adjunct Faculty Oklahoma State University 1999-2003; Graduate teaching and research assistant at Oklahoma State University 1999-present.