

STRATEGIC AND MYOPIC BEHAVIOR UNDER
AVERAGE-REVENUE-LAGGED
AND LASPEYRES PRICE
CAP REGULATION

By

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1. INTRODUCTION

1.1. *Motivation*

This is a report of research conducted for the purpose of evaluating the effectiveness of two well-known price cap schemes in meeting common regulatory objectives. Traditionally in the United States, regulatory methods have been of the rate-of-return type with an aim to guarding against the undesirable natural monopoly behavior of profit-maximizing pricing, while still allowing for profits in degree sufficient to attract necessary capital investment. Given that marginal cost pricing can force negative profits in the presence of sufficiently large economies of scale, rate-of-return, as a way of attaining second-best outcomes and guaranteeing firm viability, made a compelling case for its position as *the* method of choice for regulatory authorities in the United States until economists of the late twentieth century began to publicize its shortcomings. Of course, much of the criticism stems from the fact that rate-of-return regulation generally requires significant auditing responsibilities of the regulator to ensure that the firm accurately reports all costs of production. Asymmetric familiarities with the production process often place the regulator in an inferior position to confidently verify or falsify the firm's reports. Indeed, a firm under rate-of-return may succeed in disguising excessive spending on nonessential goods and services (e.g., extravagant office furnishings, expensive dinners with clients, or frivolous research and development projects) as legitimate costs, thereby deceiving the regulator into believing that the firm is earning

only a “fair” return on capital investment. Even outright fraudulent marginal cost reporting can escape regulatory scrutiny. Moreover, rate-of-return regulation creates perverse incentives for even the honest and frugally minded firm to utilize its resources in inefficient ways. Averch and Johnson (1962) enumerate several possible scenarios. Consequently, much study has been devoted to alternative price-incentive arrangements requiring less onerous investigative duties of the regulator. Some of these methods utilize a price cap designed to force a coincidence of interests between the regulator and the firm, i.e., as the firm maximizes profit over a set of prices restricted by the cap, the firm also moves in the direction of increased consumer surplus.

We see then that the primary impetus behind the historical development of price caps is that since rate-of-return regulation is necessarily cost based, i.e., rates move with reported costs in a more or less one-to-one manner, any incentives the regulated firm may have for trimming costs is effectively nullified. The firm faces a very real moral hazard problem because cost reductions are essentially confiscated by the regulatory authority. Price caps, in contrast with rate-of-return, sever the link between costs and prices (at least during the time span between successive reviews of the price cap) because the regulator commits to a guaranteed price for the firm. As a result, the firm is more inclined to implement cost-saving production technologies, reduce nonessential and extravagant expenditures, and seek for more efficient organizational strategies. Periodic reviews of the cap can serve only the public interest by permitting at least some portion of the cost savings achieved by increased production efficiencies to be ultimately transferred to the consumer (presumably, production efficiency always increases over time).

It is precisely these advantages of price-incentive regulation over traditional rate-

of-return that makes our analysis here so important. However, before this report can be appreciated fully by the reader, some background information about the development of regulatory thought in the U.S. and in Great Britain must be in place. The following two sections of this chapter are not intended to be an exhaustive review of the extensive literature on the subject, but rather the author offers them as an overview of the most significant contributions to the present evolved understanding of incentive regulation's advantages over rate-of-return.

1.2. An Overview of the Literature Related to Non-Price-Cap Regulatory

Approaches

The most common rate-of-return formula employed is rate-of-return on capital invested (return-on-output and return-on-sales are other methods used). If we assume only one non-capital input, L , purchased by the firm at rate w , the rate-of-return on capital K is $(PQ - wL)/K$ where output Q is sold at price P . The requirement that this be less than some fair rate f is stated mathematically as $(PQ - wL)/K \leq f$. Averch and Johnson (1962) demonstrate that rate-of-return formulas induce the firm to substitute between production factors in an uneconomical fashion that is difficult for the regulatory agency to detect. Train (1991) succinctly verifies Averch and Johnson's claim by altering the above equation to $PQ - wL \leq fK$ and subtracting the total cost of capital (rK where r is the price of capital) from both sides of this inequality to obtain

$\pi = PQ - wL - rK \leq (f - r)K$. Allowed profit for the firm is therefore a linear function of capital and is maximized at a capital-labor ratio higher than that corresponding to the feasible profit maximum. Hence social cost is not minimized at the output it

selects. Additional negative aspects of rate-of-return as listed by Train include:

1. The potential for higher prices and lower outputs compared to the unregulated outcome
2. The inability to induce the firm to expand output into the inelastic portion of demand.

Moreover, Averch and Johnson's research determined that firms operating under rate-of-return often tend to expand into other markets, even if they operate at a long run loss in those markets. A firm might build up its rate base by selling competitive outputs at a price below marginal cost. Such behavior can result when a fully-associated cost basis is employed for the rate-of-return calculation. As a result, these peripheral markets can actually become less competitive.

Cost incentive mechanisms are designed to induce the firm to move toward efficient prices and outputs over time. A condition of quasi-optimal prices (a second-best solution forced by the imposition of a revenue requirement) first appears in Ramsey's "A Contribution to the Theory of Taxation" (1927) where the author examines the issue of determining the most efficient set of prices that will allow for the financing of fixed production costs which are not covered by marginal cost pricing. Ramsey determined that unit prices for different goods should exceed marginal costs the most when demand is inelastic (provided cross-elasticities of demand are zero), and the least when demand is elastic. Such "inverse elasticity" pricing requires that consumers with the highest willingness-to-pay contribute the most toward the fixed costs. Baumol and Bradford (1970) go on to derive equivalent necessary conditions of "quasi-optimal" pricing by abstracting from a hypothetical economy where all industry is nationalized and where a

central planning agency is dedicated to the maximization of social welfare. The term “quasi-optimal” is employed to highlight the requirement that government revenues must suffice to cover all deficits of the individual firms that constitute the economy.

A well-known cost incentive mechanism was proposed by Vogelsang and Finsinger (1979). The basic assumptions of VF regulation are a convex welfare function and decreasing ray-average costs. The process repeatedly drives the firm in the direction of efficient prices of ever-increasing consumer surplus even though the regulator knows virtually nothing about production costs. Hence the information asymmetry problem is effectively dealt with. The mechanism essentially takes the prices, outputs, and costs incurred by the firm in a given period of time to constrain the firm in its selection of prices for the next period. Specifically, the firm’s prices in period t when multiplied by the quantities corresponding to these prices from the previous period cannot exceed the previous period’s total costs. Mathematically stated, the set of feasible prices for the firm in period t , $t \in N$, is given by $R_t = \{p \mid q_{t-1} \cdot p \leq C(q_{t-1})\}$. Consequently, the firm is encouraged in any period to exploit the potential for cost decreases so as to increase that period’s profit. The profit is then turned over to the consumer in the next period. Note that the regulator does not need to know the demand and cost curve parameters, only the demands and costs actually occurring in each period. Market prices and quantities along with explicitly reported costs substantiated by invoices, wage payments and capital accounting is all that is required by the regulator.

A severe criticism of VF regulation was presented in a paper by Sappington (1980) in which he explains how the single-product monopoly with constant costs might be tempted to subvert the process by wasting inputs in any given period prior to reaching

the equilibrium so that higher prices will be allowed in the next period. Any costs reported but not actually incurred can be retained by the firm as profits. A non-VF regulated firm will almost certainly find it advantageous to waste if it is expecting VF regulation to be imposed in the very near future. Additionally, overstatement of costs under VF can slow the inevitable convergence to equilibrium. Further, equilibrium can occur at a price above the true average costs enabling the firm to earn positive profits indefinitely. Nevertheless, Sappington shows that a monopoly firm subject to VF regulation will not engage in pure waste if the cost of capital differs from the allowed rate of return.

1.3. An Overview of the Literature Related to Price-Cap Regulation

Stephen Littlechild of Great Britain is largely to be credited with making the first politically compelling case for regulation by price caps when during the time of his report on the subject in 1983 to the British Department of Industry the British government was considering the privatization of much of its public services. Littlechild contended that prices rather than profits should be the principal concern of regulatory authorities. He pointed to systemic cost inefficiencies in the rate-of-return approaches that typified U.S. regulatory arrangements. As a result of his arguments, the British government began to apply price caps to British Telecom and other privatized monopolies. Since then, price caps have increasingly displaced rate-of-return approaches as the method of choice by regulators in many countries including the United States for setting rates for public utility and telecommunication services. A price cap for a one-product / multi-market firm is simply a price ceiling on the weighted average price of the firm's product over all

markets supplied. Additionally, a typical price cap incorporates the provision that the price ceiling increase from one period to the next by no more than $CPI-X$ percent¹ where CPI is the Consumer Price Index and X is a predetermined number, referred to as the “X-factor”, included to encourage production efficiency. With prices no longer directly tied to production costs, the firm obtains an incentive for cost reduction with the benefits of engaging in high-risk ventures and inefficient operating practices greatly diminished. Moreover, pricing flexibility is greatly enhanced. Hence, price caps have incentive properties that more closely reflect those of a competitive marketplace than does rate-of-return. Typically, a price cap is set for only a fixed number of years at the end of which it is reset so as to transfer to consumers a surplus from any increased production efficiencies. The obvious problem of determining an appropriate X-factor attends the $CPI-X$ formula however. This is an issue explored in several recent papers which, nevertheless, is separate from the main concern of this particular study - to examine the technical merits between certain well-known, structurally different price cap approaches. For this reason, the price cap formulas discussed in the chapters following do not include the $CPI-X$ adjustment.

Acton and Vogelsang (1989) list the following characteristics of price cap regulation:

1. The regulator sets the price cap and the regulated firm must price at or below this cap. Any profits earned may be retained by the firm.
2. The price cap may be adjusted over time. Input prices, demands, and profits will probably be used to adjust the cap over time.
3. For multi-output monopolies, the regulator may specify a price cap that is

¹ The retail price index (RPI) minus 3 percent was initially used in the regulation of British Telecom.

essentially a price index of the outputs. The firm can adjust prices of the goods up or down so long as a weighted average of these prices remains below the cap.

Lehman and Weisman (2004) describe certain advantages to price caps including:

1. Price-cap regulation provides the incumbent firm with increased pricing flexibility necessary to compete more effectively with new market entrants.
2. Price caps offer rate stability that in many cases improves upon historical trends.

In contrast, Lehman and Weisman add the insightful comment that imposition of price caps in place of rate-of-return regulation involves the exchange of one moral hazard problem for another. Under traditional rate-of-return regulation, the moral hazard problem entails the firm failing to provide service at least-cost because it is fully compensated by the regulator for the costs that it incurs. On the other hand, a price-cap regime that permits no earnings sharing involves a moral hazard of temptation for the regulator to induce excessive competitive entry. Lehman and Weisman provide empirical evidence that regulators in price cap states of the U.S. indeed adopt more liberal competitive entry policies in comparison with regulators in rate-of-return states (or regulators in states requiring earnings sharings with price caps).

Liston (1993) observes similarities and differences between rate-of-return and price cap regulation, and notes that information requirements for both are often essentially the same. Also, because of incentives to minimize costs, a decrease in product quality and a retardation of quality innovations can accompany price caps. Further, when

there is considerable uncertainty about cost fluctuations, the regulator may need to set prices so high that consumer surplus is greatly diminished and excessive deadweight losses are created. Generally, to ensure viability the regulator must set the cap above perceived costs. As a result, potential cross-subsidization across markets by the multiproduct firm may require the regulator to guard against predatory pricing of competitive services. Of course, this requires that firm products be separable into regulated and non-regulated groups. Alternatively, Liston notes that where partial entry in a regulated industry is possible, price caps may not really be advantageous to rate-of-return because the caps are likely to be binding upon increased competition, and thus ineffective once the firm successfully petitions the regulatory board.

The Federal Communication Commission implemented one of the first price cap schemes in the U.S. in its regulation of AT&T in 1989. While the cap required that annual price increases could not exceed a rate of 3 percent less than the inflation rate as measured by the Gross National Product Price Index, it nevertheless placed no limit upon the rate-of-return AT&T could earn during any year under the cap. Since that time, where the possibility of windfall profits for a regulated firm exists because of an excessively high level for the cap, the imposition of a sliding scale is often called for by the regulator. Braeutigam and Panzar (1993) note that a common sliding scale mechanism employed by many individual states in their regulation of the local exchange carriers (LEC's) resulting from the breakup of AT&T required that the LEC's refund a portion of earnings that result from a rate-of-return in excess of 13 percent. Other multi-tiered approaches, where the earnings portion to be rebated increases with the degree to which the rate-of-return exceeds 13 or some other target percentage, have been utilized. While sliding scales

reintroduce some of the unattractive accounting and monitoring features of rate-of-return, Braeutigam and Panzar are nevertheless sanguine about the advantages of sliding-scale price caps. They mention that in addition to improving the adversarial nature of the regulatory process, sliding scales expand the scope of bargaining between the firm and the regulator by making the productivity offsets as subjects of explicit bargaining rather than treating them as knowable data as is erroneously done under rate-of-return.

Currier (2005) notes that three types of pure price-cap regulation have received the bulk of attention in recent regulatory literature. These are:

1. Average Revenue (AR)
2. Laspeyres (L)
3. Average Revenue Lagged (ARL).

Under AR the firm selects prices in period t satisfying $\frac{p^t \cdot q(p^t)}{Q(p^t)} \leq \bar{p}^0$ where

$p^0 = (\bar{p}^0, \bar{p}^0)$ is the price cap vector. Cowan (1997) demonstrates that, in general, steady-state prices under AR are inefficient, and the steady-state welfare can be less than that which would prevail in the absence of regulation. Under L the firm satisfies

$\frac{p^t \cdot q(p^{t-1})}{p^{t-1} \cdot q(p^{t-1})} \leq 1$. As L is predicated on logic similar to the VF regulation discussed

earlier (period $t - 1$ revenue is used in place of the period $t - 1$ costs), the myopic firm converges to efficient prices. Under ARL the firm selects prices in period t satisfying

$\frac{p^t \cdot q(p^{t-1})}{p^0 \cdot q(p^{t-1})} \leq 1$. Hence ARL is a combination of AR and L. ARL utilizes a cap $(\bar{p}^0,$

where $p^0 = (\bar{p}^0, \bar{p}^0)$) on the weighted average prices of the firm's product during each

period of regulation where the previous period's demand quantities are used as the weights. An obvious advantage of ARL over AR regulation is that while AR regulation requires the regulator to forecast demand quantities in advance and have in place a mechanism for either the firm or consumers to be compensated in case the forecasts are in error, ARL has no such requirement because it utilizes lagged quantities. Another advantage of ARL regulation over AR is that consistently myopic behavior by the firm leads to steady-state price vectors that are efficient. However, enthusiasm for ARL based on this fact must be tempered by the knowledge that a firm can instead undermine the imposed price cap by forgoing profit in an early period in order to achieve the global profit-maximum in some later period. Such strategic behavior results in the firm selecting prices for which consumer surplus is reduced and firm revenue is increased from that which probably existed under the pre-ARL regime. Alternatively, any strategic behavior performed under L regulation must lead only to increased consumer surplus, a fact the research reported in chapter four of this paper has utilized.

Bradley and Price (1988) compare efficiency properties of L against AR for industries in the United Kingdom. The AR approach as imposed upon British Gas and the British Airports Authority is shown to be clearly inefficient. In contrast, the L approach as applied to British Telecom is shown to be clearly superior as leading to efficient prices. They demonstrate that in a two-market setting with identical associated marginal costs of production, AR results in a higher price in the more inelastic demand market compared to Ramsey pricing at the same profit level. Additionally, for identical product demands across both markets, price will be higher than the corresponding Ramsey price at the same profit level in the market with higher associated marginal costs, and lower in

the less costly market.

Sappington and Sibley (1992) demonstrate how intertemporal linkages resulting from calculating average revenue in each period by using the firm's sales in the previous period can provide incentives for the firm to engage in strategic nonlinear pricing to undermine an ARL price cap. The example of the Federal Communications Commission's price cap regulation of AT&T is employed to show how current sales, expanded by setting low current usage prices, can in turn permit larger entry fees in a subsequent period. The larger entry fees outweigh the consumer surplus derived from the increased sales. As a consequence, aggregate welfare may very well decline.

Currier (2005) demonstrates the possibility of ARL price cap manipulation by a firm whose product demand functions satisfy the following reasonable desirability property:

(D) If $\{p^n\}$ is a sequence of price vectors converging to \hat{p} (i.e., $p^n \rightarrow \hat{p}$) where only $\hat{p}_i = 0$, then $q_i(p^n) \rightarrow +\infty$.

The firm may still achieve the global profit-maximum provided the price cap is set above the average revenue that corresponds to this maximum and the firm is willing to sacrifice some profit in the first period of regulation. A purpose of this research has been to determine whether any like strategic behavior can occur under ARL given the presence of linear demand functions that necessarily preclude the desirability assumption. As demand parameters commonly vary little if at all across the set of price vectors assumed by a regulated firm, this seems to be a particularly important question and one whose resolution may yet provide insights into the nonlinear situation. Any demonstration of

how strategic behavior can occur under linear demand functions is an additional indictment against the wisdom of employing ARL regulation toward the goal of protecting consumer surplus. However, as it is unlikely that ARL will be completely relegated to the dustbin of history any time soon, the matters of concealment and detection of strategic behavior will certainly continue to be topics of interest for the foreseeable future. Accordingly, an additional aim of this study is to delimit observable signs of ARL price cap manipulation for which the regulator may be on the alert.

The possibility for manipulation of L regulation in particular and tariff basket approaches in general to the benefit of the firm but detriment to the consumer is demonstrated by Law (1997). The firm via adjusting revenue shares (weight manipulation) in the period immediately preceding L regulation can position itself in the first period under L to reap a higher profit than that intended by the regulator. Elsewhere in this paper we discuss Law's results as they apply to the one-product / two-market firm. In a related paper, Foreman (1996) provides specific conditions under which weight manipulation within a two-period / L-regulation window occurs and proposes a particular weighting scheme that eliminates the incentives for strategic behavior. Foreman illustrates how deadweight losses resulting from manipulation increase with demand elasticity and how profits are affected. He offers the following insightful rationale for two-period strategic behavior under revenue-share weighting-scheme price caps: "The key intuition is that by pricing one of the services at zero, the firm can drive one of the period-two weights to zero and thereby create infinite "leverage" with which to charge higher period-two prices."

Cowan (1997) compares allocative efficiency properties of the AR, ARL and L

approaches and proves the following:

1. The firm prefers ARL to Laspeyres regulation.
2. With ARL regulation consumer surplus is higher in period 1 than with equal prices, but lower in the steady state.
3. Steady-state prices when the firm is myopic satisfy the necessary condition for efficiency. Profits are higher, consumer surplus is lower, and welfare is higher than with equal prices.
4. Under binding ARL regulation and with a positive discount factor, steady-state prices that are not equal are inefficient.
5. With marginal costs that vary across markets, steady-state welfare under ARL can be below that obtained with no regulation if the price cap is below at least one cost level.

One aspect of this paper is to focus on Cowan's assertions as they pertain to ARL and L regulation of a one-product / multi-market monopoly firm operating with linear demands and constant marginal costs of production. Some refinements of his conclusions in this particular context will be obtained.

1.4. Focus of Research

Because of Average Revenue regulation's well-known weakness of producing allocatively inefficient steady-state outcomes, we set AR aside and devote the remaining chapters of this study to a comparative analysis of Average-Revenue-Lagged and Laspeyres price incentive approaches with respect their susceptibilities to being manipulated by a strategic firm. Further, myopic behavior within ARL and L under

certain realistic demand and cost conditions is explored to give a more thorough contrast between the two.

2. STRATEGIC BEHAVIOR UNDER AVERAGE-REVENUE-LAGGED REGULATION

2.1. *Strategic Behavior Under Average-Revenue-Lagged (ARL) Regulation (Linear Product Demands and Constant Marginal Costs)*

The objective of this section is to determine if deficiencies exist for Average-Revenue-Lagged regulation as applied to single-product / multimarket monopolies when product demands are linear and marginal costs are constant. Specifically, is the price cap set under ARL with linear demands and constant marginal costs susceptible to being undermined by the monopoly firm? The simplifying assumptions on demands and marginal costs are in fact realistic for any situation in which prices can feasibly vary little, and they do allow us to begin our examination of price cap deficiencies within a somewhat tractable context.

Our question here is important because manipulation of the ARL price cap comes only at the expense of total consumer surplus. For a product sold in only one market, the consumer surplus V corresponding to the price p_1 where demand is given by

$q_1 = ap_1 + b$ is $V(p_1) = -\frac{1}{2} \left(ap_1^2 + 2bp_1 + \frac{b^2}{a} \right)$. This expression is obtained by

multiplying one-half by the base $ap_1 + b$ and the height $-\frac{b}{a} - p_1$ of the shaded triangle in figure 1.

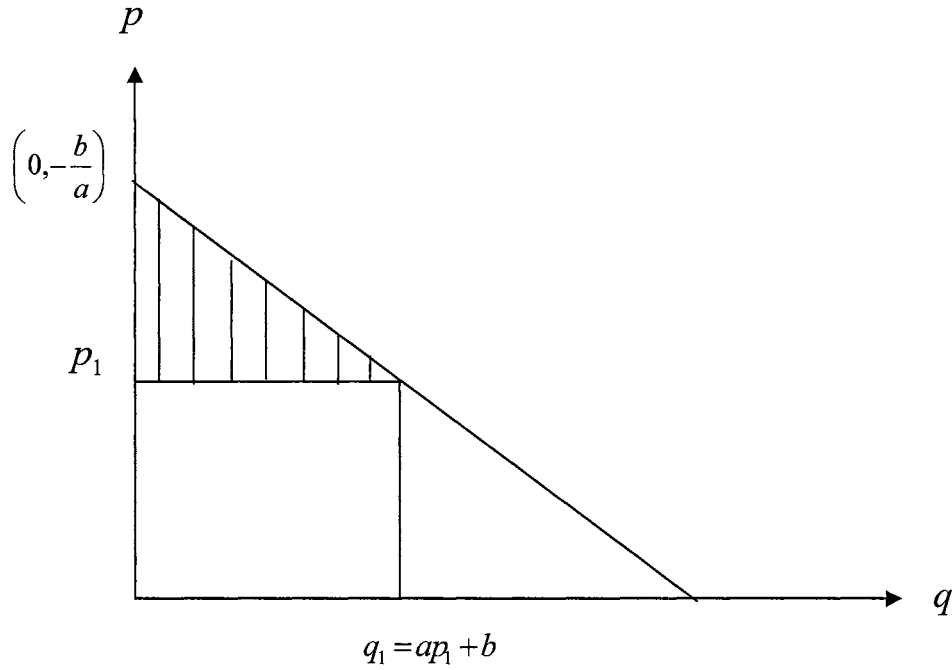


Figure 1. Consumer surplus triangle given a linear demand function.

Consumer surplus is therefore a quadratic function of price. It follows that the consumer surplus corresponding to the n -dimensional price vector $p = (p_1, p_2, \dots, p_n)$ for demand functions $q_i = a_i p_i + b_i$ where $i = 1, 2, \dots, n$ is given by:

$$V(p) = \sum_{i=1}^n -\frac{1}{2} \left(a_i p_i^2 + 2b_i p_i + \frac{b_i^2}{a_i} \right) = \sum_{i=1}^n -\frac{a_i}{2} \left(p_i + \frac{b_i}{a_i} \right)^2 \quad (1)$$

which is C^2 and convex since the Hessian matrix of $V(p)$, which is

$$H(p) = \begin{bmatrix} -a_1 & 0 & \cdots & 0 \\ 0 & -a_2 & \cdots & 0 \\ 0 & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & -a_n \end{bmatrix}, \text{ is positive definite. We are assuming zero cross-price}$$

elasticities of demands between markets. For $n = 2$ the consumer surplus function is a paraboloid having domain $\{(p_1, p_2) \mid 0 \leq p_i \leq -b_i / a_i, i = 1, 2\}$ and possessing a family of concentric elliptically-shaped iso-surplus curves of common eccentricity and

orientation² centered at $\left(-\frac{b_1}{a_1}, -\frac{b_2}{a_2}\right)$ (figure 2). An explanation of the term *eccentricity*

as used here is presented in Appendix A. In general, the consumer surplus function is monotone decreasing in any price p_i over its domain. Additionally, total differentiation of (1) gives the slope of any iso-surplus in the p_1, p_2 -plane as:

$$\frac{dp_2}{dp_1} = -\frac{a_1 p_1 + b_1}{a_2 p_2 + b_2} = -\frac{q_1(p_1)}{q_2(p_2)}. \quad (2)$$

Equivalently, the gradient of V associated with p is:

$$\nabla V(p) = (-a_1 p_1 - b_1, -a_2 p_2 - b_2, \dots, -a_n p_n - b_n) = (-q_1, -q_2, \dots, -q_n). \quad (3)$$

This, of course, is simply a re-statement of Roy's identity with the assumption of independence between consumer surplus and income.

² Alternatively, we could apply the term "homothetic" to this set of elliptically-shaped curves. Two curves are homothetic if they are related by a geometric expansion or contraction. A similarity transformation preserving orientation relates any two curves in a homothetic family.

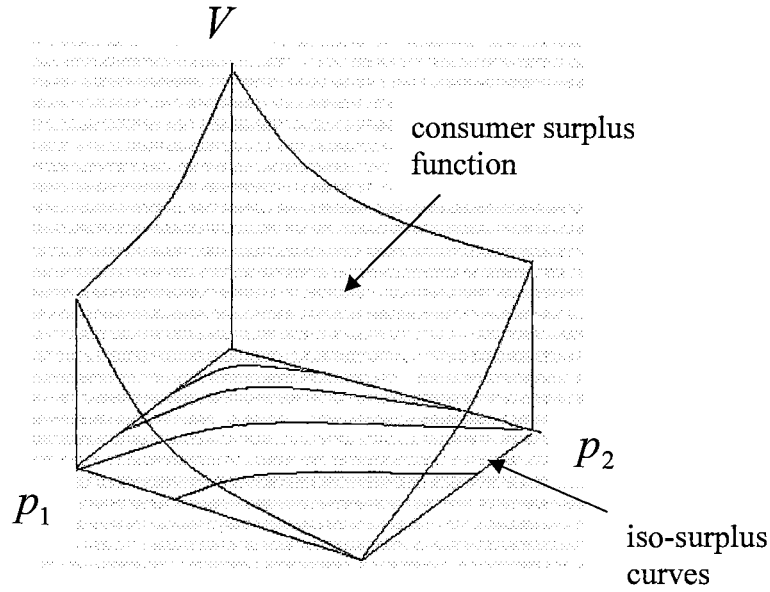


Figure 2. Parabolic consumer surplus function over the price plane.

The firm's profit function is $\pi(p) = p \cdot q(p) - C[q(p)]$. For n prices it takes the form:

$$\pi(p) = -F + \sum_{i=1}^n (p_i - c_i) q_i(p_i) \quad (4)$$

or:

$$\pi(p) = -F + \sum_{i=1}^n (p_i - c_i) (a_i p_i + b_i) \quad (5)$$

giving for $n = 2$:

$$\pi(p_1, p_2) = a_1 p_1^2 + (b_1 - a_1 c_1) p_1 - b_1 c_1 + a_2 p_2^2 + (b_2 - a_2 c_2) p_2 - b_2 c_2 - F, \quad (6)$$

where we have assumed constant marginal costs c_1, c_2 of production and fixed cost F .

This function is C^2 , concave, and maximized at some unique price vector p^* . Moreover,

the slope of any iso-profit in the p_1, p_2 -plane is:

$$\frac{dp_2}{dp_1} = -\frac{2a_1p_1 + b_1 - a_1c_1}{2a_2p_2 + b_2 - a_2c_2}. \quad (7)$$

Expanding and completing the square in p_1 and p_2 allows us to rewrite the iso-profit

locus equation for $\pi = k$ in the standard form:

$$\begin{aligned} k + F &= a_1(p_1)^2 + (b_1 - a_1c_1)p_1 - b_1c_1 + a_2(p_2)^2 + (b_2 - a_2c_2)p_2 - b_2c_2 \\ &= a_1 \left[(p_1)^2 + \frac{(b_1 - a_1c_1)}{a_1} p_1 \right] - b_1c_1 + a_2 \left[(p_2)^2 + \frac{(b_2 - a_2c_2)}{a_2} p_2 \right] - b_2c_2 \\ &= a_1 \left[p_1 + \frac{(b_1 - a_1c_1)}{2a_1} \right]^2 - \frac{(b_1 - a_1c_1)^2}{4a_1} \\ &\quad + a_2 \left[p_2 + \frac{(b_2 - a_2c_2)}{2a_2} \right]^2 - \frac{(b_2 - a_2c_2)^2}{4a_2} \end{aligned} \quad (8)$$

So for any profit level $\pi = k$ there is a corresponding iso-profit ellipse centered at

$\left(-\frac{(b_1 - a_1c_1)}{2a_1}, -\frac{(b_2 - a_2c_2)}{2a_2} \right)$. This derivation, which has given us one approach to finding

the global profit-maximum price vector:

$$p^* = \left(-\frac{(b_1 - a_1c_1)}{2a_1}, -\frac{(b_2 - a_2c_2)}{2a_2} \right), \quad (9)$$

implies an important result; the family of iso-profit curves is also elliptically-shaped, concentric and of common eccentricity and orientation. Of course, the global profit maximum p^* may be obtained also by differentiating the profit function (6) with respect to p_1 and p_2 . Substitution of (9) into (5) gives the maximum profit attainable for the firm as:

$$\pi(p^*) = -\frac{(a_1c_1 + b_1)^2}{4a_1} - \frac{(a_2c_2 + b_2)^2}{4a_2} - F. \quad (10)$$

Additionally, substitution of (9) into (1) yields the consumer surplus corresponding to the global profit maximum:

$$V(p^*) = -\frac{(a_1c_1 + b_1)^2}{8a_1} - \frac{(a_2c_2 + b_2)^2}{8a_2}. \quad (11)$$

Equating the expression for the iso-profit slope with that for the iso-surplus slope gives the following linear equation for the set of efficient price vectors in the p_1, p_2 -plane:

$$p_2 = \frac{a_1(a_2c_2 + b_2)p_1 + a_2b_1c_2 - a_1b_2c_1}{a_2(a_1c_1 + b_1)}. \quad (12)$$

This equation also represents the line connecting the center of the iso-profit family with the center of the iso-surplus family. For purposes of the discussion which follows, we define E to be the set of efficient price vectors in the interior of the domains for $V(p)$ and $\pi(p)$, i.e.,

$$E = \left\{ (p_1, p_2) \mid 0 < p_i < -\frac{b_i}{a_i}, (i = 1, 2), \text{ and } p_2 = \frac{a_1(a_2c_2 + b_2)p_1 + a_2b_1c_2 - a_1b_2c_1}{a_2(a_1c_1 + b_1)} \right\}.$$

2.2. *The Mechanics of Average-Revenue-Lagged Regulation*

Denoting prices immediately before imposition of ARL regulation by $p^r = (p_1^r, \dots, p_n^r)$ and defining social welfare as the sum of consumer surplus and profit (i.e., $W(p) = V(p) + \pi(p)$) provides a starting point and rationale for the first step in the ARL process. Here we assume that p^r is a zero-profit price vector to which the firm has been driven under some inefficient rate-of-return regulatory scheme.

Imposition of ARL regulation requires that the firm immediately transition from p^r to an equal-price vector $p^0 = (\bar{p}^0, \dots, \bar{p}^0)$ where \bar{p}^0 (called the price cap) is at most the weighted average of the pre-ARL prices p_1^r, \dots, p_n^r using the demand quantities q_1^r, \dots, q_n^r as the weights, i.e.,

$$\bar{p}^0 \leq AR^r = \frac{p^r \cdot q(p^r)}{\sum_{i=1}^n q_i(p^r)}, \quad (13)$$

and at least as large as the length of the equal-price vector contained in the iso-profit curve that passes through p^r . Letting $\pi^+ = \{p \mid \pi(p) > \pi(p^r)\}$, $V^+ = \{p \mid V(p) > V(p^r)\}$ and $\Delta^+ = \{p \in R_+^n \mid p_1 = p_2 = \dots = p_n\}$, we see that \bar{p}^0 is selected by the regulator so as to allow for an immediate increase in welfare over that existing at p^r because

$p \in \Delta^+ \cap \pi^+ \cap V^+$ (See figure 3 for $n = 2$, and observe that for $n = 2$ the set Δ^+ is simply

the 45 degree line in quadrant I). Most significantly, the consumer and the firm are immediately better off.

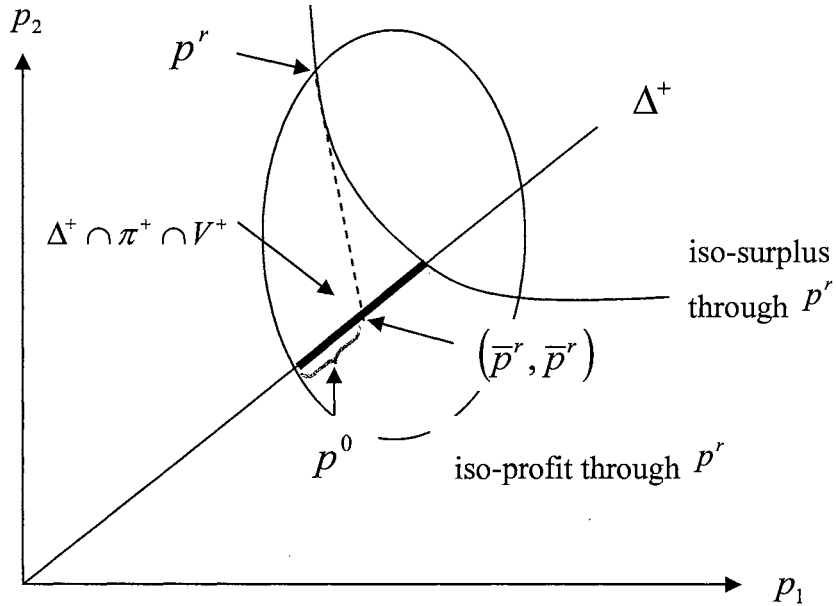


Figure 3. Region for the initial level of the price cap.

Now during the first period of ARL regulation, the firm is allowed to adjust its prices to any p^1 where the average revenue for the firm at p^1 (each price in the average being weighted by its corresponding element in the demand vector q^0) is at most equal to the price cap \bar{p}^0 . We are of course assuming that the regulator can observe these demands. Mathematically we may write:

$$AR^1 = \frac{p^1 \cdot q(p^0)}{\sum_{i=1}^n q_i(\bar{p}^0)} \leq \frac{p^0 \cdot q(p^0)}{\sum_{i=1}^n q_i(\bar{p}^0)} = \bar{p}^0 \quad (14)$$

(AR^1 denotes the weighted average revenue for period 1) giving:

$$(p^1 - p^0) \cdot q(p^0) = -(p^1 - p^0) \cdot \nabla V(p^0) \leq 0 \quad (15)$$

and thus implying that p^1 must lie on or beneath the plane containing the point p^0 that is just tangent to the iso-surplus surface passing through p^0 . For the binding case (p^* is above the tangent plane), p^1 will be on the tangent plane itself. At p^1 the regulator observes the demand vector q^1 and restricts the selection of a period 2 price vector p^2 by requiring that p^2 lie on or beneath the plane that contains the point p^0 and is perpendicular to the demand vector q^1 . Roy's identity guarantees that this plane is parallel to the iso-surplus through p^1 at p^1 . This step demonstrates the recursive aspect of the ARL process. In any period t , the myopic firm is expected to search for a p^t that solves the following problem P:

$$(P) \quad \underset{p^t}{\text{Maximize}} \pi(p^t) \text{ subject to } (p^t - p^0) \cdot q(p^{t-1}) = -(p^t - p^0) \cdot \nabla V(p^{t-1}) \leq 0.$$

Essentially, the average revenue at p^t will be less than or equal to the price cap \bar{p}^0 where the prices in the average-revenue quotient are weighted by the demands noted in the previous period, hence the label – Average-Revenue-*Lagged* regulation.

The Lagrangian for this problem is:

$$L(p^t, \lambda^t) = \pi(p^t) + \lambda^t [p^0 \cdot q(p^{t-1}) - p^t \cdot q(p^{t-1})] \quad (16)$$

and Kuhn-Tucker necessary conditions for a solution are:

$$\nabla \pi(p^t) = \lambda^t q(p^{t-1}) \quad (17)$$

$$\lambda^t [(p^0 - p^t) \cdot q(p^{t-1})] = 0 \quad (18)$$

$$\lambda^t \geq 0. \quad (19)$$

The desired price vector p^t is represented by a unique point of tangency between the constraint plane and the highest iso-profit attainable because $\pi(p)$ is strictly concave. The firm will attain the global profit maximum p^* in period t when $\lambda^t = 0$. Moreover, binding ARL results for $\lambda^t > 0$ giving the period t constraint $(p^t - p^0) \cdot q(p^{t-1}) = 0$ as illustrated in figure 4.

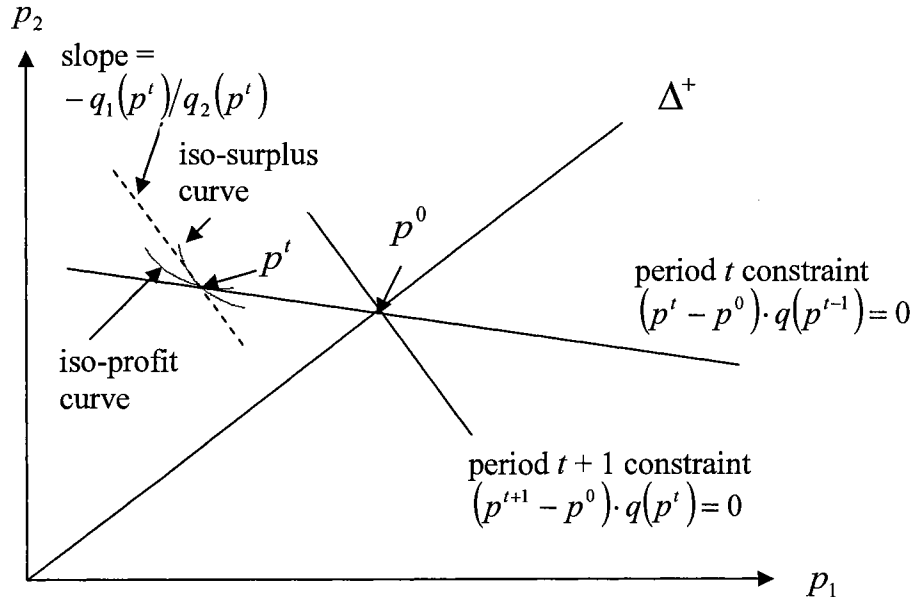


Figure 4. Average-Revenue-Lagged regulation.

Note that steady-state variables p^s , λ^s , and $q^s \equiv q(p^s)$ imply:

$$\nabla \pi(p^s) = \lambda^s q^s \quad (17')$$

$$\lambda^s [(p^0 - p^s) \cdot q^s] = 0 \quad (18')$$

$$\lambda^s \geq 0 \quad (19')$$

It follows that a global profit-maximizing price vector p^* is attainable as a steady state by the firm operating under binding ARL regulation with price cap p^0 , given that all demand functions are linear, only if it satisfies:

$$(p^0 - p^*) \cdot \nabla V(p^*) = \sum_{i=1}^n (\bar{p}^0 - p_i^*) (-a_i p_i^* - b_i) = 0. \quad (20)$$

Geometrically, since $(p - p^*) \cdot \nabla V(p^*) = 0$ is the equation of the plane tangent to an iso-suplus at p^* , the angle between the vectors $\nabla V(p^*)$ and $p^* p^0$ must be exactly 90 degrees. While for any p^* to be attainable under ARL the price cap must satisfy $\bar{p}^0 \geq \bar{p}^* = AR^*$, we can now say that if p^* is a steady state and the ARL constraint is binding, then $\bar{p}^0 = \bar{p}^* = AR^*$ (Appendix B demonstrates how \bar{p}^* can be obtained from the parameters a_1 , a_2 , b_1 , b_2 , c_1 , and c_2). This is surely a point of primary concern for a firm allowed to influence the setting of the cap. See figure 5.

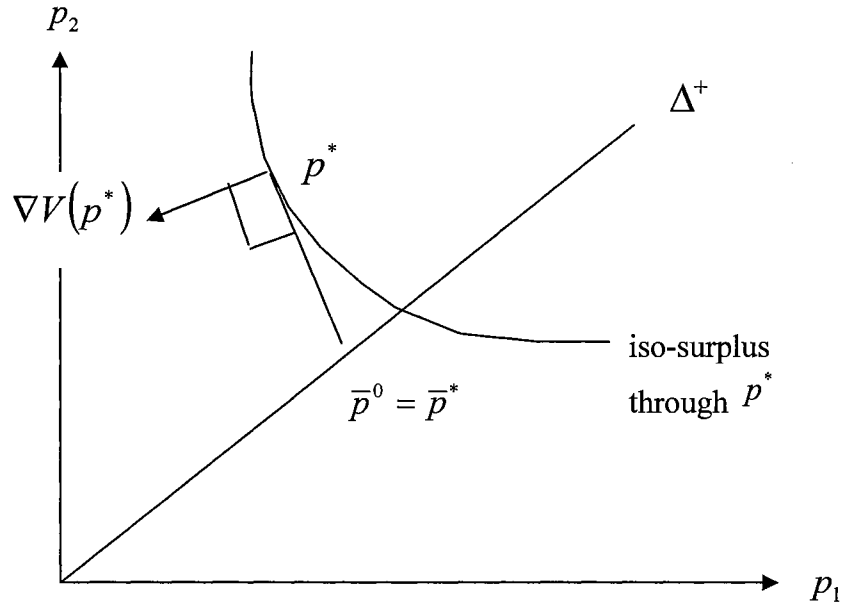


Figure 5. Geometric restriction on the ARL price cap vector in order for the global profit maximum to be a steady state.

**2.3. Strategic Behavior Under ARL Attaining p^* as a Steady State in Period 2
(Linear Demands and Constant Marginal Costs)**

Consider now a firm whose price cap is set at p^0 where p^* is above Δ^+ . The slope of the line containing p^0 and tangent to the iso-surplus curve through p^0 is

$$m_1 = -\frac{q_1^0}{q_2^0} = -\frac{a_1 \bar{p}^0 + b_1}{a_2 \bar{p}^0 + b_2}, \text{ giving the equation for this tangent line as}$$

$$p_2^1 = m_1 p_1^1 + (1 - m_1) \bar{p}^0, \text{ or better:}$$

$$(a_2 \bar{p}^0 + b_2) p_2^1 + (a_1 \bar{p}^0 + b_1) p_1^1 = [(a_1 + a_2) \bar{p}^0 + b_1 + b_2] \bar{p}^0. \quad (21)$$

This is the period 1 ARL constraint boundary. Now Currier (2005) has demonstrated that

under desirability assumptions (D) the ARL regulated firm may attain p^* as a steady state in the second period of ARL regulation and thus essentially undermine the price cap p^0 provided it satisfies $\bar{p}^* \leq \bar{p}^0$ (figure 6). Although in theory the regulator could ensure against this circumstance by setting $\bar{p}^0 < \min(p_1^*, p_2^*)$, it is extremely unlikely for the regulator to know the components of p_1^* . The success of strategic behavior under a price cap p^0 satisfying $\bar{p}^* \leq \bar{p}^0$ when demands are linear and marginal costs are constant is guaranteed by Proposition 1 below. Observe firstly that to achieve p^* in period 2 the firm can utilize the fact that the slope of the line connecting p^0 with p^* is

$$\frac{a_1(2a_2\bar{p}^0 + b_2 - a_2c_2)}{a_2(2a_1\bar{p}^0 + b_1 - a_1c_1)}, \text{ and therefore start at a period 1 price vector } p^1 = (p_1^1, p_2^1)$$

satisfying:

$$-\frac{a_1p_1^1 + b_1}{a_2p_2^1 + b_2} = \frac{a_1(2a_2\bar{p}^0 + b_2 - a_2c_2)}{a_2(2a_1\bar{p}^0 + b_1 - a_1c_1)}. \quad (22)$$

Accordingly, it will first solve:

$$-\frac{a_1p_1^1 + b_1}{a_2 \left[-\frac{a_1\bar{p}^0 + b_1}{a_2\bar{p}^0 + b_2} p_1^1 + \frac{(a_1 + a_2)\bar{p}^0 + b_1 + b_2}{a_2\bar{p}^0 + b_2} \bar{p}^0 \right] + b_2} = \frac{a_1(2a_2\bar{p}^0 + b_2 - a_2c_2)}{a_2(2a_1\bar{p}^0 + b_1 - a_1c_1)}. \quad (23)$$

for p_1^1 . Doing so yields:

$$p_1^1 = \frac{2a_1a_2^2(\bar{p}^0)^3(a_1 + a_2) - a_1a_2(\bar{p}^0)^2(a_1(a_2c_2 - b_2) + a_2(a_2c_2 - 4b_1 - 5b_2))}{a_1a_2(\bar{p}^0(a_1(a_2(c_1 - c_2) - b_2) + a_2b_1) + a_1b_2c_1 - a_2b_1c_2)}$$

$$\begin{aligned}
& \frac{+ a_2 \bar{p}^0 \left(a_2 b_1^2 - a_1 (a_2 (b_1 (c_1 + c_2) + 2b_2 c_2) - b_2 (3b_1 + 4b_2)) \right)}{\cdot 1} \sim \\
& \frac{- b_2 \left(a_1 (a_2 (b_1 c_1 + b_2 c_2) - b_2^2) - a_2 b_1^2 \right)}{\cdot 1} \cdot 3
\end{aligned} \tag{24}$$

Substituting this expression for p_1^1 in (21) and solving produces p_2^1 .

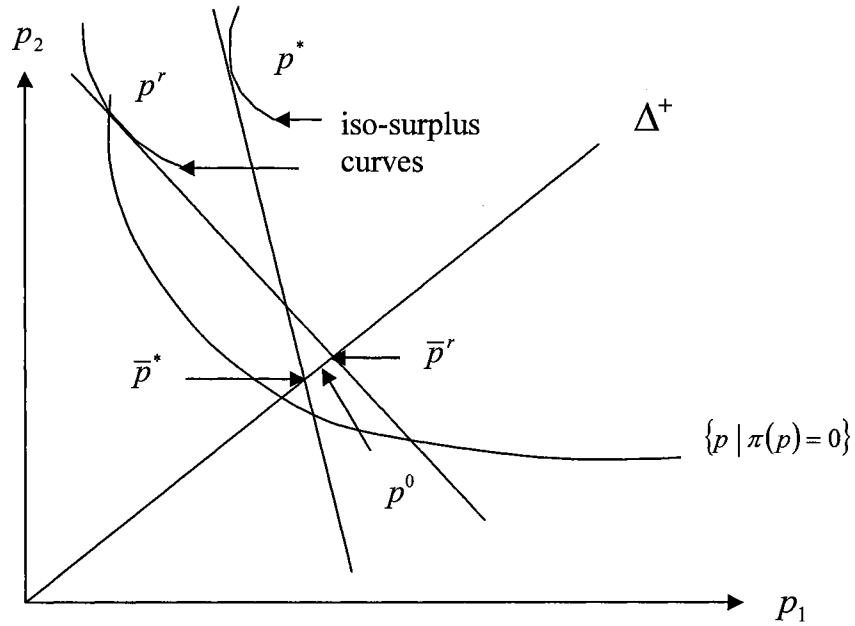


Figure 6. The possibility of $\bar{p}^* < \bar{p}^0 < \bar{p}^r$. A requirement for undermining the price cap.

2.3.1. Proposition 1

Assume linear demands and binding ARL regulation with $\bar{p}^* \leq \bar{p}^0$. If the firm is allowed to price initially at the vector (\bar{p}^*, \bar{p}^*) and select any p^1 satisfying the period 1

³ Obtained by use of the computer algebra system *Derive*.

constraint, it can achieve p^* as a steady state in period 2 of ARL regulation.

Proof of Proposition 1:

This proposition follows from the fact that the family of iso-surplus curves is concentric and of common eccentricity and orientation (homothetic). Therefore any ray emanating from the family center and intersecting two curves in the family will form congruent corresponding angles with lines tangent to these curves at the given intersection points. A proof of this fact is presented in Appendix C. In particular, the line passing through $\left(-\frac{b_1}{a_1}, -\frac{b_2}{a_2}\right)$ and p^* intersects any nonparallel line containing (\bar{p}^*, \bar{p}^*) at a point p' where the slope of the iso-surplus through p' equals the slope of the line segment connecting p^* and (\bar{p}^*, \bar{p}^*) . See figure 7.

Now if the firm accepts its price cap at $\bar{p}^0 = \bar{p}^*$, and it lets p^1 be the point at which the period 1 constraint intersects the line through $\left(-\frac{b_1}{a_1}, -\frac{b_2}{a_2}\right)$ and p^* (i.e., set E), then the period 2 constraint line will pass through the price vector p^* and be tangent to the iso-surplus curve through p^* because p^* is an efficient price vector. Substitution of \bar{p}^* for \bar{p}^0 in (24) will give p_1^1 . Substitution of the resulting expression for p_1^1 in (21) will produce p_2^1 .

Finally, p^* is a steady state price vector because (17'), (18'), and (19') are satisfied for $p^s = p^*$, $\lambda^s = 0$, and $\bar{p}^0 = \bar{p}^*$.

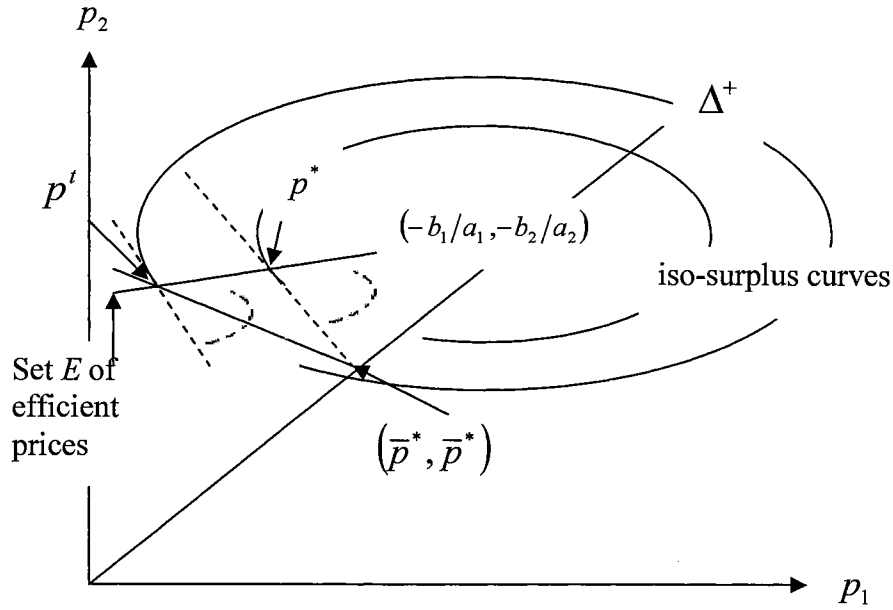


Figure 7. Congruent angle property allows for the global profit maximum to be achievable in period 2 of ARL regulation.

While price vectors positioned on the period 1 constraint further from Δ^+ than the price vector p^1 given by (24) and (21) may produce a period 2 constraint rotated beyond p^* thus also making p^* assumable in period 2, the profit associated with any of these must be less than that for this particular p^1 . Hence, the firm desirous of achieving p^* in period 2, while sacrificing as little profit as possible in period 1, will follow the course of action specified above.

2.3.2. An Example of Strategic Behavior Under ARL Attaining p^* as a Steady State in Period 2 (Linear Demands and Constant Marginal Costs)

Consider a monopolist who sells its product in two different markets with

independent linear demands: $q_1 = -5p_1 + 20$ in market 1, and $q_2 = -6p_2 + 40$ in market

2. Also, the cost of production for the firm is given by $C = q_1 + 3q_2 + 16.1369$. Equation

(9) gives the global profit maximum for this firm as $p^* = (p_1^*, p_2^*) = (2.5000, 4.833\bar{3})$

producing the corresponding demand vector $q^* = (q_1^*, q_2^*) = (7.5000, 11.0000)$.

Now suppose that previous to imposition of ARL regulation the firm has been constrained to operate at the price vector $p^r = (p_1^r, p_2^r) = (3.91, 3.89)$ by some rate-of-return regulatory scheme. It can be shown that since the demand functions produce the demand vector $q^r = (q_1^r, q_2^r) = (0.45, 16.66)$ then p^r is a zero-profit price vector.

Additionally, the average revenue at p^r is calculated as $AR^r = \frac{p_1^r q_1^r + p_2^r q_2^r}{q_1^r + q_2^r} = 3.8905$.

If ARL regulation is suddenly imposed on the firm, it must accept a price cap

$p^0 = (\bar{p}^0, \bar{p}^0)$ on its product where $\bar{p}^0 \leq AR^r$. Suppose, for the sake of illustration,

$\bar{p}^0 = 3.8874$ which is also the value for \bar{p}^* (figure 8). This is a realistic value for a price

cap because it still allows the firm to earn a non-negative profit at its initial ARL

regulated price vector p^0 ($\pi(p^0) = 0.2867 > 0$). We will now proceed to show that the

firm will be able to achieve p^* as a steady state in period 2 and thus undermine the price

cap even though p^* is actually unattainable in period 1.

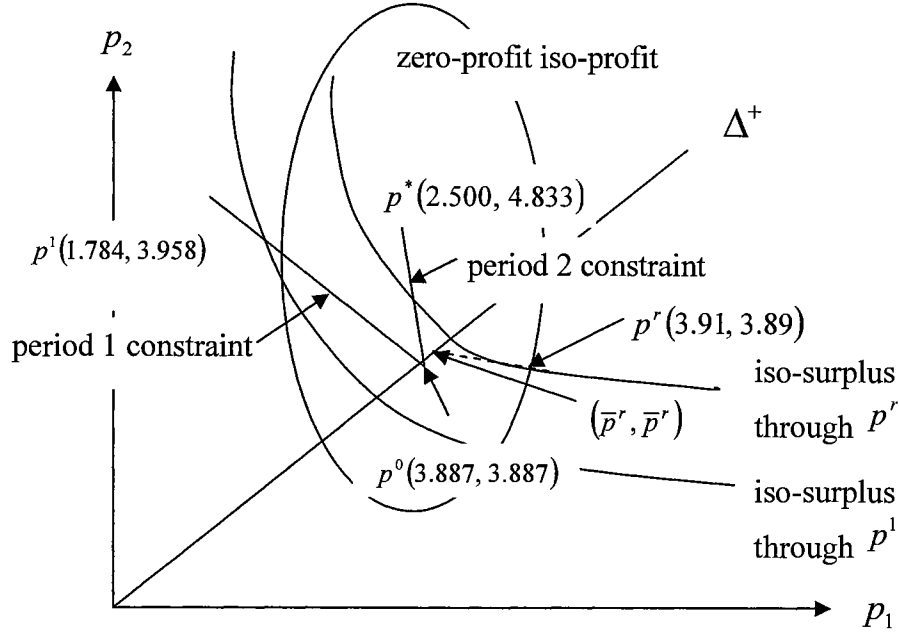


Figure 8. Example of strategic behavior attaining p^* in period 2 of binding ARL regulation.

To demonstrate, we firstly recognize that the period 1 constraint line equation is given by $0.0338p_1^1 + p_2^1 = 4.0186$. Accordingly, the firm may move to any new price vector p^1 whose coordinates satisfy this equation. Suppose the firm selects

$p^1 = (p_1^1, p_2^1) = (1.784, 3.958)$ thus producing $q^1 = (q_1^1, q_2^1) = (11.079, 16.250)$. Then since

$\nabla V(p^1) = (-q_1^1, -q_2^1)$ we can obtain the slope of the period 2 constraint as

$m_{period\ 2} = -\frac{11.079}{16.250}$ and consequently produce the period 2 constraint equation

$0.6818p_1^2 + p_2^2 = 6.5379$. Observe that the coordinates of the global profit maximum p^* satisfy this new constraint even though p^* lies beyond the period 1 constraint. Moreover,

p^* is a steady-state price vector because $\nabla \pi(p^*) = (2a_1p_1^* + b_1 - a_1c_1, 2a_2p_2^* + b_2 - a_2c_2)$

$$= (2(-5)(2.500) + 20 - (-5)(1), 2(-6)(4.83\bar{3}) + 40 - (-6)(3)) = (0, 0).$$

Cowan (1997) asserts, “With ARL regulation consumer surplus is higher in period 1 than with equal prices, but lower in steady state.” The period 1 increase in consumer surplus from $V(p^0) = 23.205$ to $V(p^1) = 34.287$ results from the fact that the iso-surplus passing through p^0 , whose slope at p^0 equals the slope of the period 1 constraint, is convex. Additionally, $V(p^*) = 15.712 < V(p^0) = 23.205$ comports with the other half of Cowan’s assertion. That imposition of ARL regulation can ultimately prove inimical to the regulator’s goal of improving consumer welfare is also demonstrated here because $V(p^*) = 15.708 < V(p^r) = 23.150$. Finally, this example illustrates that it is entirely possible to have both $V(p^r) > V(p^*)$ and $AR^r > AR^*$ simultaneously true.

2.4. Existence and Uniqueness of a Steady-State Price Vector for any Price Cap Under ARL (Linear Demands and Constant Marginal Costs)

Now we see that for linear demands and constant marginal costs each price-cap vector will have exactly one steady-state price vector associated with it. A one-to-one correspondence between vectors in Δ^+ and elements in the set E is guaranteed by the linearity of each set and the congruency property of corresponding angles referred to in the proof of Proposition 1 (figure 9). If a price-cap vector p^0 had more than one steady-state price vector associated with it, the line segments connecting each steady-state price vector to the price-cap vector could not be parallel. However, this would imply that the

angles formed by the set of efficient price vectors and each of these line segments could not be congruent thus violating our congruency property.

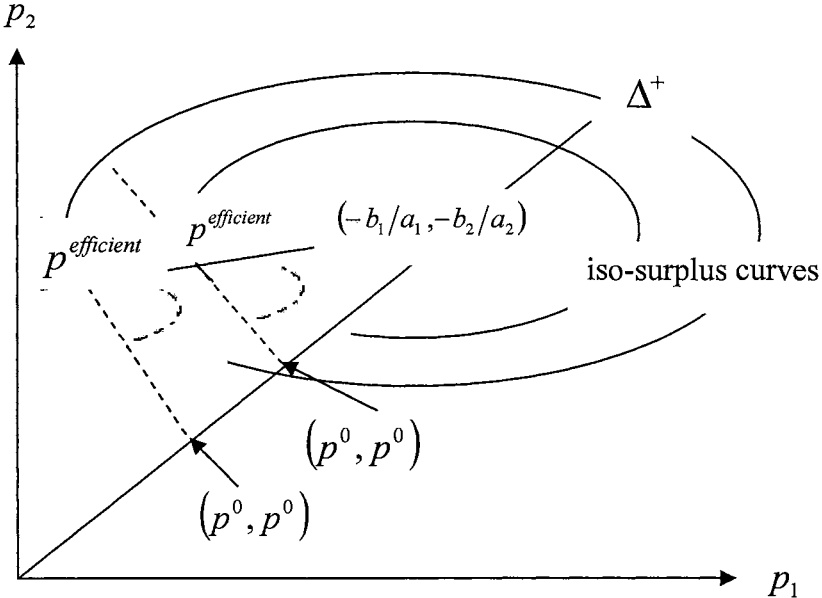


Figure 9. One-to-one relationship between price cap vectors and efficient price vectors.

**2.5. *Concealment and Detection of Strategic Behavior Under ARL Regulation
(Linear Demands and Constant Marginal Costs)***

Although we shall see that profit maximization behavior over all periods of regulation (consistently myopic behavior) may enable the firm operating under linear demands and constant marginal costs to converge over time to the global profit-maximum price vector, normally a firm must behave strategically by forgoing some profit in at least one period in order to attain p^* within a *finite* total number of periods. Generally speaking, for the firm to apply Proposition 1 in this quest it must deceive the regulator into thinking that the required p^1 maximizes period 1 profit. The most common approach is to misrepresent marginal costs in period 1. For the regulator to have any hope of verifying or falsifying the reported marginal costs it must have another independent relationship involving marginal costs with which to work. If the firm anticipated ARL regulation while operating at p^r and was able to misreport either profit or fixed costs, then such devising would preclude the regulator from evaluating the profit function at p^r to obtain any independent information here. Likewise, if the firm can misreport profit or fixed cost at p^0 , it successfully foils any like attempt by the regulator to employ the profit function at this price vector. In fact, if the firm can misreport values for all of four parameters – namely, marginal cost of producing the good for market #1, marginal cost of producing the good for market #2, profit, and fixed cost – then there is an indeterminacy in any systematic approach to verifying any reported values for these parameters by applying the profit function to only the three vectors p^r , p^0 , and p^1 . The problem is made worse for the regulator if it is not a given that marginal costs are

constant over all assumed prices. Hence, the firm expecting imminent ARL regulation while operating at p^r may be successful in evading detection of any strategic behavior through at least the first full period of regulation. The firm can position itself to achieve p^* in period 2 simply by first accepting $\bar{p}^0 = AR^*$ and then moving to the period 1 efficient price vector $p^{1, \text{Efficient}}$ (note: $p^{1, \text{Efficient}}$, p^* and $\left(-\frac{b_1}{a_1}, -\frac{b_2}{a_2}\right)$ are all collinear in the price plane, i.e., $p^{1, \text{Efficient}} \in E$). This it can do without detection by misrepresenting $p^{1, \text{Efficient}}$ as a period 1 profit-maximizing price vector. The resulting period 2 constraint will contain p^* and so the firm will undermine the price cap. Specifically, if the firm reports marginal costs \tilde{c}_1 and \tilde{c}_2 satisfying:

$$-\frac{2a_1 p_1^{1, \text{Efficient}} + b_1 - a_1 \tilde{c}_1}{2a_2 p_2^{1, \text{Efficient}} + b_2 - a_2 \tilde{c}_2} = \frac{p_2^{1, \text{Efficient}} - \bar{p}^*}{p_1^{1, \text{Efficient}} - \bar{p}^*}, \quad (25)$$

the price vector $p^{1, \text{Efficient}}$ will mimic a period 1 profit-maximizing price vector and the regulator may be oblivious to the presence of strategic behavior.

When a firm reports marginal costs \tilde{c}_1 and \tilde{c}_2 differing from the actual marginal costs c_1 and c_2 , a fictitious profit-maximizing price vector $\tilde{p}^* = (\tilde{p}_1^*, \tilde{p}_2^*)$ is implied whose components will satisfy the equation:

$$\begin{aligned} \nabla \pi(\tilde{p}_1^*, \tilde{p}_2^*) &= \tilde{p}_1^* q_1(\tilde{p}_1^*) + \tilde{p}_2^* q_2(\tilde{p}_2^*) - \tilde{c}_1 [q_1(\tilde{p}_1^*)] - \tilde{c}_2 [q_2(\tilde{p}_2^*)] - F \\ &= \left(\tilde{p}_1^* \frac{\partial \tilde{q}_1}{\partial \tilde{p}_1^*} + \tilde{q}_1^* - \tilde{c}_1 \frac{\partial \tilde{q}_1^*}{\partial \tilde{p}_1^*}, \tilde{p}_2^* \frac{\partial \tilde{q}_2}{\partial \tilde{p}_2^*} + \tilde{q}_2^* - \tilde{c}_2 \frac{\partial \tilde{q}_2^*}{\partial \tilde{p}_2^*} \right) \end{aligned}$$

$$\begin{aligned}
&= \left(\tilde{q}_1^* \frac{(\tilde{p}_1^* - \tilde{c}_1)}{\tilde{p}_1^*} \frac{\partial \tilde{q}_1^*}{\partial \tilde{p}_1^*} \frac{\tilde{p}_1^*}{\tilde{q}_1^*} + \tilde{q}_1^* \tilde{q}_2^* \frac{(\tilde{p}_2^* - \tilde{c}_2)}{\tilde{p}_2^*} \frac{\partial \tilde{q}_2^*}{\partial \tilde{p}_2^*} \frac{\tilde{p}_2^*}{\tilde{q}_2^*} + \tilde{q}_2^* \right) \\
&= \left(\tilde{q}_1^* \cdot \left[\frac{\tilde{p}_1^* - \tilde{c}_1}{\tilde{p}_1^*} \tilde{\varepsilon}_{11} + 1 \right], \tilde{q}_2^* \cdot \left[\frac{\tilde{p}_2^* - \tilde{c}_2}{\tilde{p}_2^*} \tilde{\varepsilon}_{22} + 1 \right] \right) = (0,0) \tag{26}
\end{aligned}$$

where $\tilde{\varepsilon}_{11} = \frac{\tilde{p}_1}{\tilde{q}_1} \frac{\partial \tilde{q}_1}{\partial \tilde{p}_1}$ and $\tilde{\varepsilon}_{22} = \frac{\tilde{p}_2}{\tilde{q}_2} \frac{\partial \tilde{q}_2}{\partial \tilde{p}_2}$, for else the regulator will not be convinced that

the firm is not sacrificing some period t profit for the purpose of pursuing an ulterior goal. Often information regarding elasticities, instead of demand function parameters per se, is all that is available to the regulator which is why we present the elasticity form of the equation. A reasonable assumption is that there is sufficient information, if only from the demand vectors at p^r and p^0 , for the regulator to independently calculate the price elasticities ε_{11} and ε_{22} . If this is so, then the firm must ensure that $\tilde{\varepsilon}_{11} = \varepsilon_{11}$ and

$\tilde{\varepsilon}_{22} = \varepsilon_{22}$. Additionally, note that linear demands here imply $\frac{\partial q_i}{\partial p_i} \equiv a_i$ allowing us to

write the familiar form:

$$2a_i \tilde{p}_i^* + b_i - a_i \tilde{c}_i = 0, \quad i = 1, 2. \tag{27}$$

The regulator is now persuaded of an “ideal” iso-profit ellipse - in contrast to the “real” iso-profit ellipse - for $\pi = k$ of:

$$k + F = a_1 p_1^2 + (b_1 - a_1 \tilde{c}_1) p_1 - b_1 \tilde{c}_1 + a_2 p_2^2 + (b_2 - a_2 \tilde{c}_2) p_2 - b_2 \tilde{c}_2 \tag{28}$$

or

$$\begin{aligned}
& k + F \\
& = \\
& a_1 \left[p_1 + \frac{(b_1 - a_1 \tilde{c}_1)}{2a_1} \right]^2 - \frac{(b_1 - a_1 \tilde{c}_1)^2}{4a_1} + a_2 \left[p_2 + \frac{(b_2 - a_2 \tilde{c}_2)}{2a_2} \right]^2 - \frac{(b_2 - a_2 \tilde{c}_2)^2}{4a_2} - (b_1 \tilde{c}_1 + b_2 \tilde{c}_2).
\end{aligned} \tag{29}$$

Figure 10 illustrates for a firm reporting \tilde{c}_1 , and \tilde{c}_2 that is also consistent with $\pi(p^r) = 0$.

The ideal iso-profit ellipse is centered at:

$$\tilde{p}^* = \left(-\frac{(b_1 - a_1 \tilde{c}_1)}{2a_1}, -\frac{(b_2 - a_2 \tilde{c}_2)}{2a_2} \right) \tag{30}$$

and will be tangent to the period t constraint at the strategic period t price vector provided the firm reports marginal costs \tilde{c}_1 and \tilde{c}_2 satisfying (27).

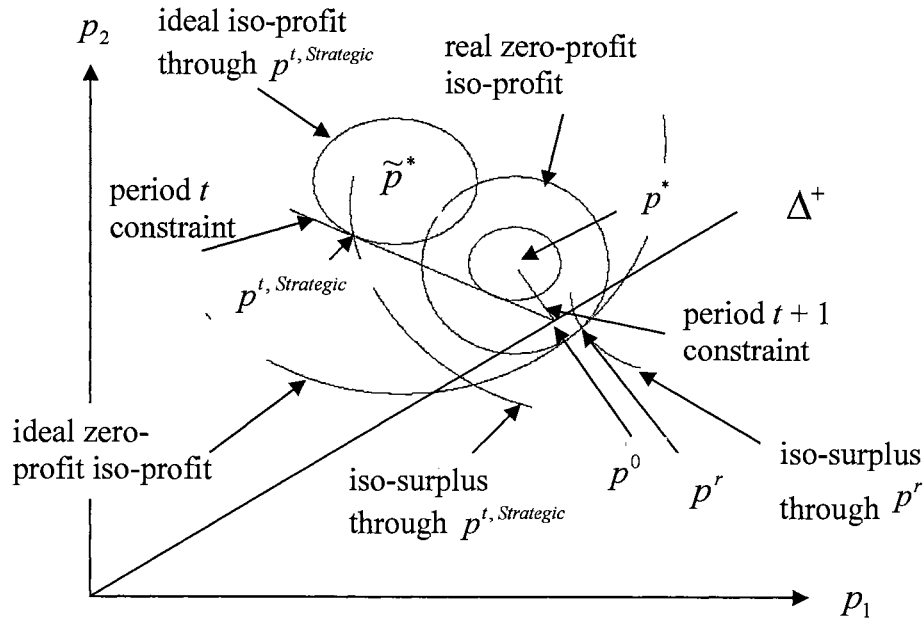


Figure 10. Ideal and real iso-profit curves corresponding to strategic behavior in period t .

We see then that the regulator may recognize certain instances of when $p^{1, \text{Efficient}}$ is misrepresented as a period 1 profit-maximizing vector, if so doing forces the firm to report an unrealistic value for a fixed or marginal cost. Consider the following situations that assume the ideal situation of the regulator knowing all demand parameters:

1. The ideal zero-profit iso-profit curve does not intersect the set of equal price vectors Δ^+ . In this case the firm could not have earned non-negative profit at the price cap p^0 and should have protested. The ideal zero-profit iso-profit curve ($k = 0$) should intersect Δ^+ at point(s) $p' = (\bar{p}', \bar{p}')$ such that:

$$\bar{p}' = \frac{-(b_1 + b_2 - a_1\tilde{c}_1 - a_2\tilde{c}_2) \pm \sqrt{(b_1 + b_2 - a_1\tilde{c}_1 - a_2\tilde{c}_2)^2 + 4(a_1 + a_2)(b_1\tilde{c}_1 + b_2\tilde{c}_2 + F)}}{2(a_1 + a_2)} \quad (31)$$

Hence, if the firm reports marginal costs satisfying (26), but nevertheless

\tilde{F} is so large that:

$$(b_1 + b_2 - a_1\tilde{c}_1 - a_2\tilde{c}_2)^2 + 4(a_1 + a_2)(b_1\tilde{c}_1 + b_2\tilde{c}_2 + \tilde{F}) < 0, \quad (32)$$

then the regulator may surmise that the firm willfully sacrificed profit at

p^0 .

2. The price vector p^r is outside of the zero-profit iso-profit ellipse

($\pi(p^r) < 0$). Such a configuration would be impossible if the firm were complying with a zero-profit regulatory scheme previous to the imposition of ARL. Hence, instead of Equation (28) the firm should attempt to satisfy:

$$k + \tilde{F} = a_1 p_1^2 + (b_1 - a_1\tilde{c}_1)p_1 - b_1\tilde{c}_1 + a_2 p_2^2 + (b_2 - a_2\tilde{c}_2)p_2 - b_2\tilde{c}_2 \quad (28')$$

where \tilde{F} is the reported fixed cost that must be revised downward (upward) from F in order to offset any increase (decrease) in the reported marginal costs, for else the regulator will not accept that p^r was a pre-ARL zero-profit price vector. Since the regulator will believe only $\tilde{F} > 0$,

it follows that there is a limit to how much the marginal costs can be over reported.

3. The average revenue at p^r is too small even though p^r is located on the ideal zero-profit iso-profit (figure 11). Here the regulator knows that if the firm were truthfully reporting marginal and fixed costs in period 1 leading to the zero-profit iso-profit as pictured, it could not have earned non-negative profit at the price cap because the cap is necessarily too low. The firm's assent to the price cap at p^0 is at least tacit acknowledgement that $\pi(p^0) \geq 0$, but if $\bar{p}^0 < AR^r$ in this type of configuration, then $\pi(p^0) < 0$. The regulator is alerted to this occurrence because the firm reports marginal costs satisfying (26), but nevertheless has forced average revenue at p^r to satisfy:

$$\begin{aligned}
 AR^r &= \frac{q_1^r p_1^r + q_2^r p_2^r}{q_1^r + q_2^r} = \frac{p_1^r (a_1 p_1^r + b_1) + p_2^r (a_2 p_2^r + b_2)}{a_1 p_1^r + b_1 + a_2 p_2^r + b_2} \\
 &< p^r = \\
 &\frac{-(b_1 + b_2 - a_1 \tilde{c}_1 - a_2 \tilde{c}_2) + \sqrt{(b_1 + b_2 - a_1 \tilde{c}_1 - a_2 \tilde{c}_2)^2 + 4(a_1 + a_2)(b_1 \tilde{c}_1 + b_2 \tilde{c}_2 + F)}}{2(a_1 + a_2)}.
 \end{aligned} \tag{33}$$

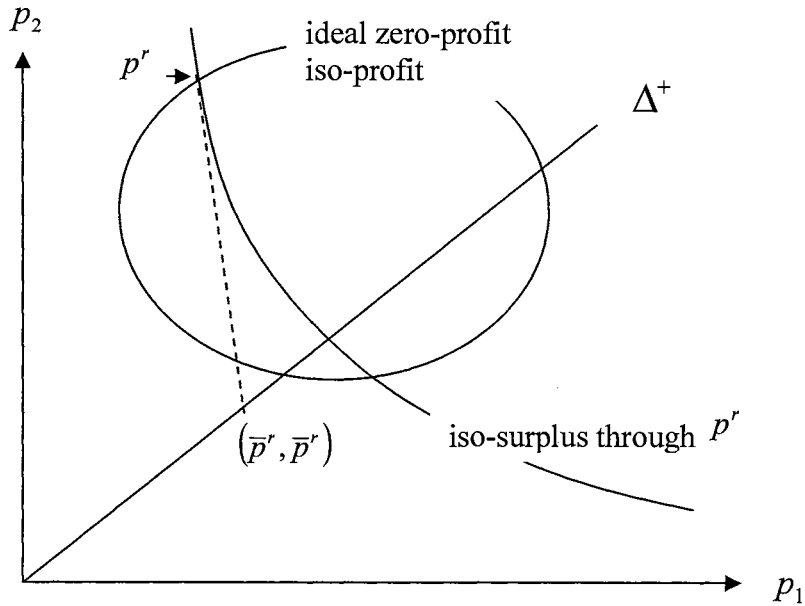


Figure 11. Reported marginal costs force average revenue of pre-ARL regulation price vector and consequently initial price cap to be outside corresponding zero-profit iso-profit.

It must be strongly emphasized, however, that absent the certainty that marginal costs are constant over all price vectors assumed by the firm, the regulator cannot appeal to any of the points listed above as indicators of strategic behavior.

2.6. Strategic Behavior Under ARL Performed Over Multiple Periods (Linear Demands and Constant Marginal Costs)

In contrast to the 2-period strategic behavior described previously, a firm may attempt a “go slow approach” by manipulating the ARL scheme over multiple periods, thus delaying its arrival at p^* . Reasons for doing so might include an aversion by the firm to absorbing the necessary single-period sacrifice of forgone profits entailed in the shorter approach, or the fear that the required differential between reported and actual

marginal costs in the shorter approach would be so great as to raise the suspicions of the regulator. It should be obvious, however, that it becomes more difficult for the firm to misreport any parameter values in later and over multiple periods because the regulator gleans additional information about the firm as the process progresses. Consequently, the firm will do any strategic activity as early as possible.

To demonstrate how the regulator could be alerted to strategic behavior performed over multiple periods, recall that a purported period t profit-maximizing price vector implies an ideal iso-profit center. Since under linear demands and constant marginal costs a family of iso-profits, whether real or ideal, must be concentric and of a common eccentricity and orientation, the myopic firm moves from one period to the next in such a way that the profit gradient is continually rotating toward that iso-profit center established in period 1. The path taken by the “consistently” myopic firm under linear demands and constant marginal costs will be discussed in Proposition 2 of Chapter 3.

2.7. *Strategic Behavior Under ARL Attaining p^* as a Steady State in Period 2 (Homogeneous Consumer Welfare Function)*

We explore now strategic behavior of the firm under ARL given a strictly-convex homogeneous consumer welfare function. To illustrate, consider the homogeneous consumer surplus function $V = kp_1^{-\alpha} p_2^{-\beta}$. Such a consumer surplus function would exist under a Cobb-Douglas utility function. The gradient of this function is $\nabla V = k(-\alpha p_1^{-\alpha-1} p_2^{-\beta}, -\beta p_1^{-\alpha} p_2^{-\beta-1})$. Now the slope of any line tangent to an iso-surplus curve at the point (p_1, p_2) is given by $m = -\frac{\alpha p_2}{\beta p_1}$. Therefore, we can solve

$\frac{p_2^* - \bar{p}^*}{p_1^* - \bar{p}^*} = -\frac{\alpha p_2^*}{\beta p_1^*}$ for \bar{p}^* to obtain the average revenue for the global profit-maximum

price vector:

$$\bar{p}^* = \frac{(\alpha + \beta)p_1^* p_2^*}{\beta p_1^* + \alpha p_2^*}. \quad (34)$$

Now certainly, there is an interdependence between the consumer surplus function and the profit function which, depending on the parameter values in each, will constrain where p^* may be located. Therefore, we are specifying only how the firm can achieve p^* wherever it may be located. Necessarily then, the strategic price vector whose expression we now obtain must be stated in terms of p_1^* and p_2^* .

If the firm accepts the price cap vector p^0 to be (\bar{p}^*, \bar{p}^*) , then it may achieve p^* as a steady-state in period 2 of ARL regulation provided it can select any non-profit-maximizing price vector in period 1. To see this, first note that the period 1 constraint is:

$$p_2^1 = \left(1 + \frac{\alpha}{\beta}\right) \bar{p}^0 - \frac{\alpha}{\beta} p_1^1. \quad (35)$$

Using equation (34) we may write:

$$p_2^1 = \left(1 + \frac{\alpha}{\beta}\right) \left(\frac{(\alpha + \beta)p_1^* p_2^*}{\beta p_1^* + \alpha p_2^*}\right) - \frac{\alpha}{\beta} p_1^1. \quad (36)$$

Now the strategic firm will seek a period 1 price vector $p^{1, Strategic}$ satisfying:

$$-\frac{\alpha p_2^{1,Strategic}}{\beta p_1^{1,Strategic}} = -\frac{\alpha p_2^*}{\beta p_1^*} \quad (37)$$

in order to achieve p^* in period 2. Replacing p_1^1 with $p_1^{1,Strategic}$ in equation (36) and then substituting the resulting p_2^1 for $p_2^{1,Strategic}$ in equation (37) produces the following equation involving $p_1^{1,Strategic}$:

$$\frac{(\alpha + \beta)\bar{p}^0 - \alpha p_1^{1,Strategic}}{\beta p_1^{1,Strategic}} = \frac{p_2^*}{p_1^*}. \quad (38)$$

Solving equation (38) for $p_1^{1,Strategic}$ gives:

$$p_1^{1,Strategic} = \frac{(\alpha + \beta)p_1^* \bar{p}^0}{\alpha p_1^* + \beta p_2^*}. \quad (39)$$

Substitution of the right side of (34) into (39) yields:

$$p_1^{1,Strategic} = \frac{(\alpha + \beta)p_1^* \left(\frac{(\alpha + \beta)p_1^* p_2^*}{\beta p_1^* + \alpha p_2^*} \right)}{\alpha p_1^* + \beta p_2^*} = \frac{(\alpha + \beta)^2 (p_1^*)^2 p_2^*}{(\alpha p_1^* + \beta p_2^*)(\beta p_1^* + \alpha p_2^*)}. \quad (40)$$

Therefore we can obtain the period 1 strategic price vector from p_1^* and p_2^* simply by using equations (40) and (36). If the firm is permitted to price at this $p_1^{1,Strategic}$ in period 1, it will successfully attain p^* in period 2. Additionally, p^* is a steady state since

$$\bar{p}^0 = \bar{p}^*.$$

2.7.1. An Example of Strategic Behavior Under ARL Attaining p^* as a Steady

State in Period 2 (Homogeneous Consumer Welfare Function)

Suppose a firm under ARL regulation has a global profit maximum at $p^* = (4.00, 5.50)$. Also suppose consumer surplus is given by $V = k(p_1)^{-1}(p_2)^{-2}$. Though indeed the consumer welfare and profit functions are dependant here, for appropriate marginal cost values this p^* is certainly plausible. Average revenue at the global profit maximum is therefore by equation (34) $\bar{p}^* = \frac{(1+2)(4.00)(5.50)^2}{2(4.00)+(1)(5.50)} = 4.888\bar{8}$. The period 1 constraint given by (35) is $p_2^1 = \left(1 + \frac{\alpha}{\beta}\right)\bar{p}^0 - \frac{\alpha}{\beta}p_1^1 = 7.333\bar{3} - 0.5p_1^1$. The point $p^{1,Strategic}$ which lies on this line has coordinates given by (40) and (36):

$$\begin{aligned} p_1^{1,Strategic} &= \frac{(\alpha + \beta)^2 (p_1^*)^2 p_2^*}{(\alpha p_1^* + \beta p_2^*)(\beta p_1^* + \alpha p_2^*)} \\ &= \frac{(1+2)^2 (4.00)^2 (5.50)}{(1(4.00) + 2(5.50))(2(4.00) + 1(5.50))} = 3.911\bar{1} \end{aligned}$$

and

$$\begin{aligned} p_2^{1,Strategic} &= \left(1 + \frac{\alpha}{\beta}\right) \left(\frac{(\alpha + \beta) p_1^* p_2^*}{\beta p_1^* + \alpha p_2^*} \right) - \frac{\alpha}{\beta} p_1^{1,Strategic} \\ &= \left(1 + \frac{1}{2}\right) \left(\frac{(1+2)(4.00)(5.50)}{2(4.00) + 1(5.50)} \right) - \frac{1}{2} (3.911\bar{1}) = 5.377\bar{7}. \end{aligned}$$

Observe that the slope of the iso-surplus through $p^{1,Strategic}$ at $p^{1,Strategic}$ is

$$m = -\frac{\alpha p_2^{1,Strategic}}{\beta p_1^{1,Strategic}} = -\frac{(1)(5.377\bar{7})}{(2)(3.911\bar{1})} = -0.6875. \text{ Therefore, the period 2 constraint is}$$

$0.6875 p_1^2 + p_2^2 = 8.25$. Finally, observe that $p^* = (4.00, 5.50)$ satisfies this equation.

Hence, we have demonstrated via an example how - for a homogeneous consumer surplus function - a price cap \bar{p}^0 satisfying $\bar{p}^0 \leq \bar{p}^*$ can be undermined by the strategic firm.

2.8. *Summary of Chapter 2*

To summarize the main points of this chapter, a price cap set under ARL regulation may be subject to being undermined by the strategic firm if the level of the cap is greater than the average revenue of the global profit maximum. Currier (2005) has demonstrated this possibility for a consumer surplus function possessing a very reasonable desirability property. The content of this chapter demonstrates the possibility of like strategic behavior for the situation of linear product demands and constant marginal costs, and for the situation of a homogeneous consumer surplus function. While the regulator may be able to detect such behavior by using demand and cost function parameter values, in practice the likelihood of such information being accessible by the regulator is remote. Average-revenue-lagged regulation, its property of inducing firms to efficient prices notwithstanding, manifests deficiencies as a regulatory scheme designed for protecting consumer surplus.

3. MYOPIC BEHAVIOR UNDER AVERAGE-REVENUE-LAGGED REGULATION

3.1. *Convergence of the Myopic Firm Under Binding ARL to the Global Profit Maximum*

The objective of this chapter is to determine the conditions under which consistently *myopic* behavior under ARL given linear demands and constant marginal costs will inevitably lead to the profit-maximum price vector.

Depending on the level of the price cap, it is possible that consistently myopic behavior can eventually lead the regulated firm to the global profit-maximum price vector. The following propositions address this possibility for linear demands and constant marginal costs.

3.1.1. *Proposition 2*

Suppose a firm with linear demands and constant marginal costs operates under ARL regulation with a price cap \bar{p}^0 satisfying $\bar{p}^* \leq \bar{p}^0$. If the firm prices initially at (\bar{p}^*, \bar{p}^*) and is myopic over periods $t = 1, 2, \dots$, then the sequence of price vectors $\{p^t\}$ assumed by the firm lies along an ellipse centered at the price vector

$\left(-\frac{b_1 - a_1 c_1 - 2a_1 \bar{p}^*}{4a_1}, -\frac{b_2 - a_2 c_2 - 2a_2 \bar{p}^*}{4a_2} \right)$ that contains p^* , (\bar{p}^*, \bar{p}^*) , and the point

$\left(\frac{a_1 c_1 - b_1 + a_2 c_2 - b_2}{2(a_1 + a_2)}, \frac{a_1 c_1 - b_1 + a_2 c_2 - b_2}{2(a_1 + a_2)} \right)$. See figure 12.

Proof of Proposition 2:

We assume that the myopic firm moves to a price vector p^t in period t that is the point of tangency between the period t constraint line and the highest iso-profit possible. The constraint line is therefore binding.

Equating the slope of the iso-profit through p^t from (7) with the slope of the line segment through (\bar{p}^*, \bar{p}^*) and p^t yields:

$$\frac{dp_2}{dp_1} = -\frac{2a_1 p_1^t + b_1 - a_1 c_1}{2a_2 p_2^t + b_2 - a_2 c_2} = \frac{\bar{p}^* - p_2^t}{\bar{p}^* - p_1^t}. \quad (41)$$

Expanding about p_1^t and p_2^t gives:

$$\begin{aligned} & 2a_2 (p_2^t)^2 + (b_2 - a_2 c_2 - 2a_2 \bar{p}^0) p_2^t - (b_2 - a_2 c_2) \bar{p}^* \\ &= -2a_1 (p_1^t)^2 - (b_1 - a_1 c_1 - 2a_1 \bar{p}^0) p_1^t + (b_2 - a_2 c_2) \bar{p}^* \end{aligned} \quad (42)$$

yielding:

$$\begin{aligned} & 2a_2 \left[(p_2^t)^2 + \frac{b_2 - a_2 c_2 - 2a_2 \bar{p}^*}{2a_2} p_2^t \right] - (b_2 - a_2 c_2) \bar{p}^* \\ &= -2a_1 \left[(p_1^t)^2 + \frac{b_1 - a_1 c_1 - 2a_1 \bar{p}^*}{2a_1} p_1^t \right] + (b_1 - a_1 c_1) \bar{p}^* \end{aligned} \quad (43)$$

which, upon completing the square in p'_1 , and p'_2 , leads to the equation:

$$\begin{aligned}
& 2a_2 \left[(p'_2)^2 + \frac{b_2 - a_2 c_2 - 2a_2 \bar{p}^*}{2a_2} p'_2 + \left(\frac{b_2 - a_2 c_2 - 2a_2 \bar{p}^*}{4a_2} \right)^2 \right] \\
& - \frac{(b_2 - a_2 c_2 - 2a_2 \bar{p}^*)^2}{8a_2} - (b_2 - a_2 c_2) \bar{p}^* \\
& = -2a_1 \left[(p'_1)^2 + \frac{b_1 - a_1 c_1 - 2a_1 \bar{p}^*}{2a_1} p'_1 + \left(\frac{b_1 - a_1 c_1 - 2a_1 \bar{p}^*}{4a_1} \right)^2 \right] \\
& + \frac{(b_1 - a_1 c_1 - 2a_1 \bar{p}^*)^2}{8a_1} + (b_1 - a_1 c_1) \bar{p}^*. \tag{44}
\end{aligned}$$

This produces the standard form for an ellipse:

$$\begin{aligned}
& 2a_1 \left[p'_1 + \frac{b_1 - a_1 c_1 - 2a_1 \bar{p}^*}{4a_1} \right]^2 + 2a_2 \left[p'_2 + \frac{b_2 - a_2 c_2 - 2a_2 \bar{p}^*}{4a_2} \right]^2 \\
& = \frac{(b_1 - a_1 c_1 - 2a_1 \bar{p}^*)^2}{8a_1} + (b_1 - a_1 c_1) \bar{p}^* + \frac{(b_2 - a_2 c_2 - 2a_2 \bar{p}^*)^2}{8a_2} + (b_2 - a_2 c_2) \bar{p}^*. \tag{45}
\end{aligned}$$

Note that the center of this ellipse is $\left(-\frac{b_1 - a_1 c_1 - 2a_1 \bar{p}^*}{4a_1}, -\frac{b_2 - a_2 c_2 - 2a_2 \bar{p}^*}{4a_2} \right)$. That

$$p^* = \left(-\frac{(b_1 - a_1 c_1)}{2a_1}, -\frac{(b_2 - a_2 c_2)}{2a_2} \right), (\bar{p}^*, \bar{p}^*), \text{ and}$$

$\left(\frac{a_1 c_1 - b_1 + a_2 c_2 - b_2}{2(a_1 + a_2)}, \frac{a_1 c_1 - b_1 + a_2 c_2 - b_2}{2(a_1 + a_2)} \right)$ lie on the ellipse can be shown by

substitution of the coordinates for each of these price vectors into (41) (See Appendix B for how the coordinates of (\bar{p}^*, \bar{p}^*) are defined in terms of the demand and cost function parameters).

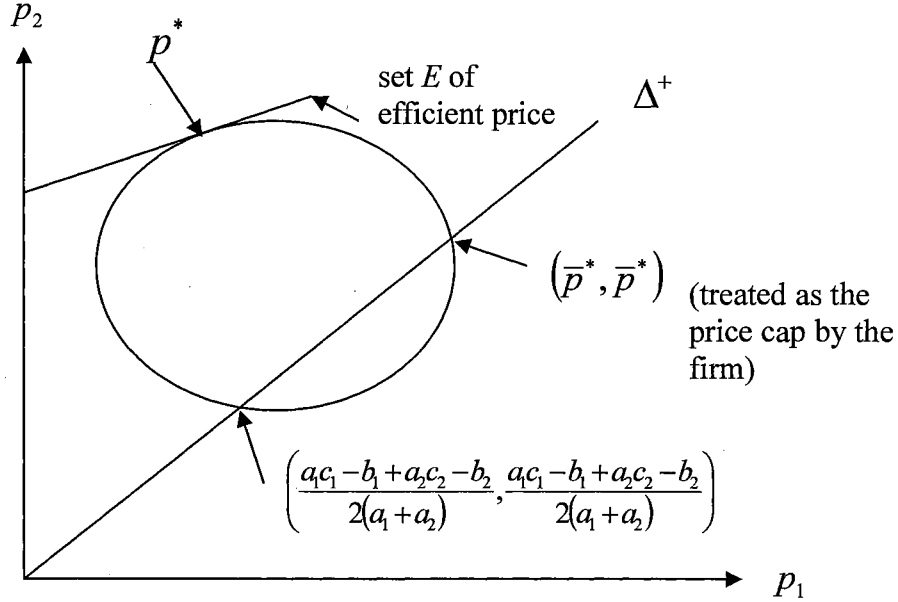


Figure 12. Elliptical path of the consistently myopic firm where the price cap is set at the average revenue of the global profit maximum.

3.1.2. Proposition 3

The sequence of price vectors $\{p^{t, Myopic}\}$ (profit-maximizing price vectors) for periods $t = 1, 2, \dots$, not only exists along the elliptical path of Proposition 2, but for a price cap set at $\bar{p}^0 = \bar{p}^*$ will converge over time to the global profit-maximum price vector p^* . Moreover, p^* is a steady state.

Proof of Proposition 3:

Without loss of generality assume p^* is above the line Δ^+ and $\bar{p}^0 = \bar{p}^*$. Also, assume p^* is not attainable in period t under ARL regulation (figure 13). Assume linear demands and constant marginal costs.

Let $p^{t,Myopic}$ be the point on the period t constraint line at which period t profit is a maximum. Then at $p^{t,Myopic}$ the slope of the iso-profit must be greater than the slope of the iso-surplus curve. This follows because at the period t efficient price vector $p^{t,Efficient}$

$$\text{we have } -\frac{\frac{\partial \pi}{\partial p_1}}{\frac{\partial \pi}{\partial p_2}} \bigg|_{p^{t,Efficient}} = -\frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}} \bigg|_{p^{t,Efficient}} \quad \text{and to traverse the period } t \text{ constraint from}$$

$p^{t,Efficient}$ to $p^{t,Myopic}$ we must increase p_1 and decrease p_2 . This implies decreasing $\frac{\partial \pi}{\partial p_1}$

with increasing $\frac{\partial \pi}{\partial p_2}$ because π is C^2 and strictly concave. Also, we have increasing

$\frac{\partial V}{\partial p_1}$ with decreasing $\frac{\partial V}{\partial p_2}$ because V is C^2 and strictly convex. This leads to the

$$\text{inequality } -\frac{\frac{\partial \pi}{\partial p_1}}{\frac{\partial \pi}{\partial p_2}} \bigg|_{p^{t,Myopic}} > -\frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}} \bigg|_{p^{t,Myopic}} \quad . \text{ Now the slope of the period } t \text{ constraint is}$$

$$m_t = -\frac{\frac{\partial \pi}{\partial p_1}}{\frac{\partial \pi}{\partial p_2}} \bigg|_{p^{t,Myopic}} \quad \text{because the iso-profit through } p^{t,Myopic} \text{ is tangent to the period } t$$

constraint. Also, the slope of the period $t + 1$ constraint is $m_{t+1} = - \left. \frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}} \right|_{p^{t, Myopic}}$ by

definition of the ARL process (provided $p^{t, Myopic}$ is assumed by the firm in period t).

Hence $m_{t+1} < m_t$ and there is clockwise rotation about p^0 from the period t constraint to the period $t + 1$ constraint. The sequence of price vectors $\{p^{t, Myopic}\}$ moves rightward and upward from $p^{1, Myopic}$ along the elliptical path of Proposition 2. Moreover, for any period

$t + 1$ we must have $\frac{p_2^* - \bar{p}^*}{p_1^* - \bar{p}^*} < m_{t+1}$ because $m_{t+1} = - \left. \frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}} \right|_{p^{t, Myopic}} > - \left. \frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}} \right|_{p^{t, Efficient}} =$

$- \left. \frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}} \right|_p = \frac{p_2^* - \bar{p}^*}{p_1^* - \bar{p}^*}$. Hence, $\{m_t\}$ is a monotone decreasing sequence bounded from

below.

Now suppose there exists \hat{m} such that $m_1 > \hat{m} > \frac{p_2^* - \bar{p}^*}{p_1^* - \bar{p}^*}$ where \hat{m} is a lower

bound for the sequence $\{m_t\}$. Then there is a price vector $\hat{p} \neq p^*$ satisfying

$- \left. \frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}} \right|_{\hat{p}} \geq - \left. \frac{\frac{\partial \pi}{\partial p_1}}{\frac{\partial \pi}{\partial p_2}} \right|_{\hat{p}} = \hat{m}$, for otherwise \hat{m} could not be a lower bound. But this

contradicts the implication of Proposition 2 that there is only one price vector on the

elliptical path of the consistently myopic firm for the case $\bar{p}^0 = \bar{p}^*$, namely p^* , where

$$-\frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}} \geq -\frac{\frac{\partial \pi}{\partial p_1}}{\frac{\partial \pi}{\partial p_2}} \geq \frac{p_2^* - \bar{p}^*}{p_1^* - \bar{p}^*}. \text{ Therefore, } \frac{p_2^* - \bar{p}^*}{p_1^* - \bar{p}^*} \text{ is the greatest lower bound and the}$$

sequence $\{m_t\}$ converges to $\frac{p_2^* - \bar{p}^*}{p_1^* - \bar{p}^*}$.

So if the price cap \bar{p}^0 is set at \bar{p}^* , then the consistently myopic firm converges over time to the global profit-maximum price vector p^* (figure 13). Observe

$$-\frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}} \bigg|_{p^*} = \frac{p_2^* - \bar{p}^*}{p_1^* - \bar{p}^*} \text{ guarantees that } p^* \text{ will be a steady state.}$$

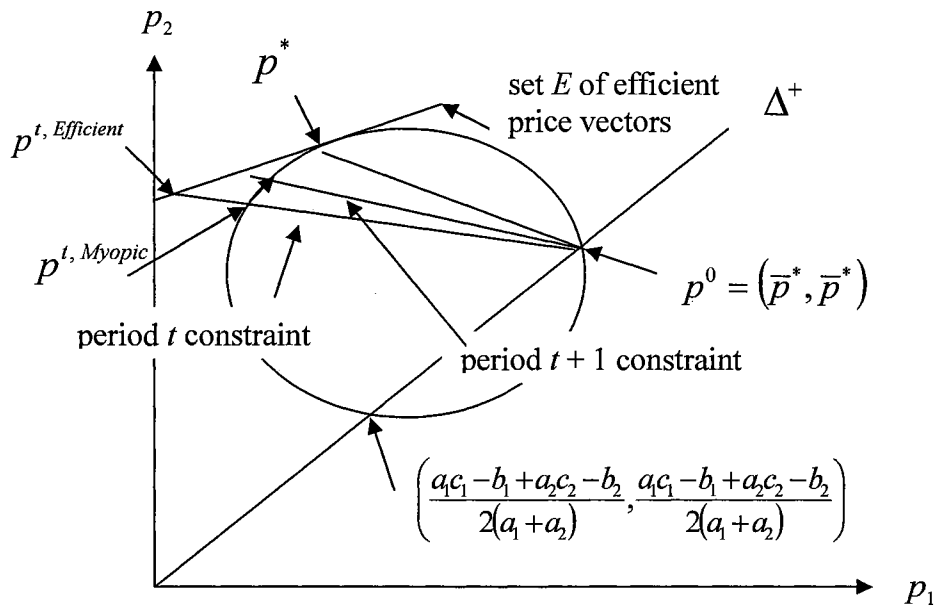


Figure 13. Consistently myopic firm moves along elliptical path towards the global profit maximum.

3.1.3. An Example of Consistently Myopic Behavior Under ARL Regulation

Approaching p^ as a Steady State (Linear Demands and Constant Marginal Costs)*

To illustrate a case of inevitable convergence of the consistently myopic firm under ARL regulation to p^* as a steady state (linear demands and constant marginal costs), consider the monopolist examined earlier. Recall that the product demand functions were $q_1 = -5p_1 + 20$ and $q_2 = -6p_2 + 40$, while the cost function was $C = q_1 + 3q_2 + 16.1369$. The global profit-maximum price vector of $p^* = (2.5000, 4.833\bar{3})$ can be attained as a steady state via strategic behavior within just two periods of regulation given that the firm can accept the price cap to be $\bar{p}^* = 3.887387$ (a little more precision here for purposes of our illustration). Now note that the consistently myopic firm will in each period t attempt to solve equations (17) – (19). Doing so produces the following expressions for the period t myopic price vector given the linear demand functions $q_i = a_i p_i + b_i$ where $i = 1, 2, \dots, n$ and cost function

$$C(p_1, p_2) = c_1 [q_1(p_1)] + c_2 [q_2(p_2)] + F:$$

$$p_1^t = \frac{1}{\frac{q_1^{t-1}}{q_2^{t-1}} + \frac{a_1 q_2^{t-1}}{a_2 q_1^{t-1}}} \left\{ \frac{q_1^{t-1} \bar{p}^0}{q_2^{t-1}} - \frac{1}{2a_2} \left[\frac{q_2^{t-1}}{q_1^{t-1}} (b_1 - a_1 c_1) - b_2 + a_2 c_2 \right] + \bar{p}^0 \right\}$$

$$p_2^t = \frac{1}{2a_2} \left[\frac{q_2^{t-1}}{q_1^{t-1}} (2a_1 p_1^t + b_1 - a_1 c_1) - b_2 + a_2 c_2 \right], \quad t = 1, 2, \dots^4 \quad (46)$$

⁴ Obtained by use of the computer algebra system *Derive*.

Appendix D gives alternative recursion formulas for $p^{t, Myopic}$ in terms of $p^{t-1, Myopic}$.

Of course we have $p_i^0 = \bar{p}^0$ and $q_i^0 = a_i p_i^0 + b_i$. Several iterations of this procedure were performed for identical demand and cost function parameter values as before to produce Table 1. Each row of the table represents one period of the ARL process. Perhaps surprisingly, this firm also converges to $p^* = (2.5000, 4.833\bar{3})$ as a steady state, although the convergence is not as rapid as that corresponding to the strategic behavior described earlier.

Consistently myopic behavior (demands $q_1 = -5p_1 + 20$ and $q_2 = -6p_2 + 40$, cost function $C = q_1 + 3q_2 + 16.1369$, and price cap $\bar{p}^0 = 3.887387 = \bar{p}^*$)				
Period	Prices	Quantities Demanded	Consumer Surplus and Firm Profit	Sequence of Projections of Proposition 4
Initial Price Vector	$p_1^0 = 3.887387$ $p_2^0 = 3.887387$	$q_1^0 = 0.563063$ $q_2^0 = 16.67568$	$V(p^0) = 23.204891$ $\pi(p^0) = 0.286666$	
Period 1	$p_1^1 = 2.463619$ $p_2^1 = 3.935462$	$q_1^1 = 7.681903$ $q_2^1 = 16.38723$	$V(p^1) = 28.27961$ $\pi(p^1) = 10.43611$	$u_1 = -3.753987$
Period 2	$p_1^2 = 2.468426$ $p_2^2 = 4.599436$	$q_1^2 = 8.157868$ $q_2^2 = 12.40338$	$V(p^2) = 19.4754$ $\pi(p^2) = 14.86496$	$u_2 = -1.86983$
Period 3	$p_1^3 = 2.482625$ $p_2^3 = 4.811318$	$q_1^3 = 7.586877$ $q_2^3 = 11.13209$	$V(p^3) = 16.08302$ $\pi(p^3) = 15.27535$	$u_3 = -1.1117$
Period 4	$p_1^4 = 2.499792$ $p_2^4 = 4.833079$	$q_1^4 = 7.501042$ $q_2^4 = 11.00153$	$V(p^4) = 15.7127$ $\pi(p^4) = 15.27977$	$u_4 = -1.023012$
Period 5	$p_1^5 = 2.500000$ $p_2^5 = 4.833333$	$q_1^5 = 7.500000$ $q_2^5 = 11.00000$	$V(p^5) = 15.70833$ $\pi(p^5) = 15.27977$	$u_5 = -1.021957$
Period 6	$p_1^6 = 2.500000$ $p_2^6 = 4.833333$	$q_1^6 = 7.500000$ $q_2^6 = 11.00000$	$V(p^6) = 15.70833$ $\pi(p^6) = 15.27977$	$u_6 = -1.021957$

Table 1

Hence, even the consistently myopic firm can undermine any price cap $\bar{p}^0 \geq \bar{p}^*$ given linear demands and constant marginal costs provided it may effectively treat \bar{p}^* as its price cap. This would seem to present a severe indictment against binding ARL. Observe that Cowan's (1997) assertion regarding changes in consumer surplus is demonstrated once again since taking the steady-state price vector p^s to be p^6 gives

$V(p^s) = 15.70833 < V(p^0) = 23.204891 < V(p^1) = 28.27961$. Cowan also states,
 “Steady-state prices when the firm is myopic satisfy the necessary condition for efficiency. Profits are higher, consumer surplus is lower, and welfare is higher than with equal prices.” This proposition is born out by the results in Table 1. The steady-state price vector $p^s = (2.50000, 4.83333)$ belongs to the set of efficient prices. Also,
 $\pi(p^s) = 15.27977 > \pi(p^0) = 0.286666$, $V(p^s) = 15.70833 < V(p^0) = 23.204891$, and
 $W(p^s) = 30.9881 > 23.491557 = W(p^0)$.

Nevertheless, note there is one redeeming note regarding the ability of the regulator to detect when the strategic firm is seeking to quickly arrive at the global profit-maximum price vector. Any period t price vector p^t which would be required for the firm to attain in order to ultimately achieve p^* in a finite number of periods later will force the firm to move outside of the path traced by the consistently myopic firm. Marginal cost values implied after the reporting of profit by the firm at the price vector p^1 , even if selected under bogus reporting, set the firm on a particular course from which it cannot stray. The regulator needs only substitute these marginal cost values into (42). The firm is expected to converge to the ideal iso-profit center

$$\tilde{p}^* = \left(-\frac{(b_1 - a_1 \tilde{c}_1)}{2a_1}, -\frac{(b_2 - a_2 \tilde{c}_2)}{2a_2} \right) \text{ established in period 1 instead of the global profit}$$

maximum p^* it surely desires. Any deviation from this path is easily detectable.

The inability of the firm to perform strategic behavior over multiple periods notwithstanding, ARL regulation is a very limited approach for the regulator to use in the effort of securing a satisfactory level of consumer surplus. With no constraints on

marginal or fixed costs beyond the standard ones attending application of the profit function to the price vectors p^r and p^0 , the firm under ARL regulation can succeed in undermining the price cap and achieving the global profit maximum within the first two periods of regulation.

3.2. *Social Welfare Movement Under ARL and Policy Recommendations*

Summing the right-hand sides of equations (1) and (5) gives the following for the social welfare function:

$$W(p) = -F + \sum_{i=1}^n -\frac{a_i}{2} \left(p_i + \frac{b_i}{a_i} \right)^2 + (p_i - c_i)(a_i p_i + b_i). \quad (47)$$

Iso-welfare curves are therefore of the form:

$$W = -F + \sum_{i=1}^n \left[\frac{a_i}{2} p_i^2 - a_i c_i p_i - \frac{2a_i b_i c_i + b_i^2}{2a_i} \right] \quad (48)$$

or

$$W = -F + \sum_{i=1}^n \frac{a_i}{2} (p_i - c_i)^2 - \frac{(b_i + a_i c_i)^2}{2a_i}. \quad (49)$$

For $n = 2$, (49) becomes:

$$W = -F + \frac{a_1}{2} (p_1 - c_1)^2 - \frac{(b_1 + a_1 c_1)^2}{2a_1} + \frac{a_2}{2} (p_2 - c_2)^2 - \frac{(b_2 + a_2 c_2)^2}{2a_2} \quad (50)$$

which is an ellipse in the price plane centered at the social welfare optimum price vector

(c_1, c_2) (marginal cost pricing). Substituting $\pi(p') + V(p')$ for W in (50) allows us to determine what, if any, price vectors are Pareto superior to p' . For example, in the case examined previously where product demand functions are $q_1 = -5p_1 + 20$ and $q_2 = -6p_2 + 40$, while the cost function is $C = q_1 + 3q_2 + 16.1369$, we obtain the following locus of price vectors that are Pareto equivalent to

$$p^{1,Myopic} = (2.463619, 3.935462):$$

$$\begin{aligned} & V(p^{1,Myopic}) + \pi(p^{1,Myopic}) + F \\ &= \frac{a_1}{2}(p_1 - c_1)^2 - \frac{(b_1 + a_1c_1)^2}{2a_1} + \frac{a_2}{2}(p_2 - c_2)^2 - \frac{(b_2 + a_2c_2)^2}{2a_2} \end{aligned} \quad (51)$$

which yields $54.85262 = -\frac{5}{2}p_1^2 + 5p_1 - 3p_2^2 + 18p_2 + \frac{100}{3}$. A plot of this ellipse along

with the period 1 and 2 constraint lines and the vector $p^{1,Myopic}$ are pictured in figure 14.

Price vectors in the interior of this ellipse are Pareto superior to $p^{1,Myopic}$.

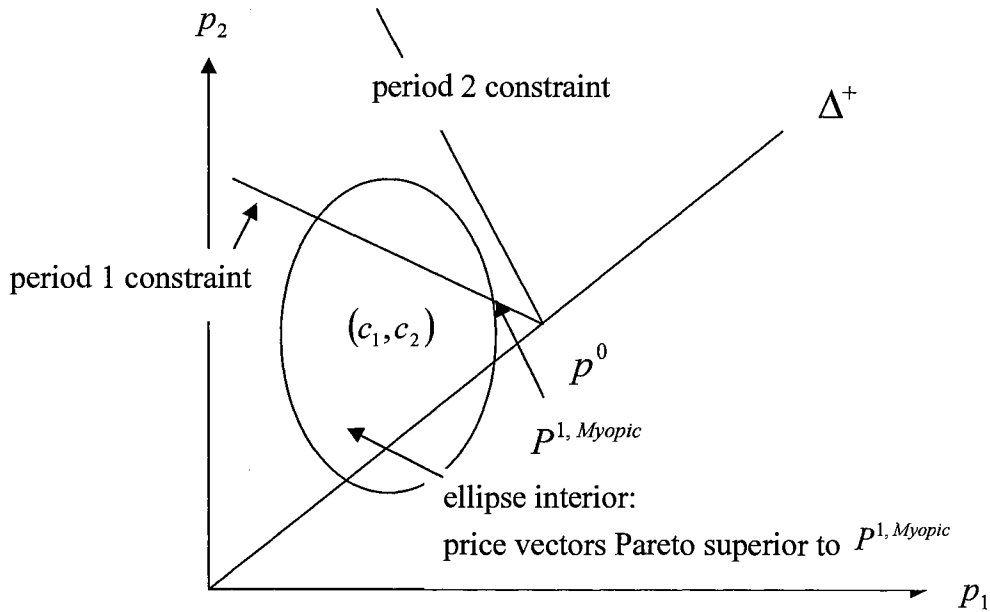


Figure 14. Period 2 price vectors in example are Pareto inferior to $p^{1, Myopic}$.

In this example we see that there is no price vector on the period 2 constraint that is Pareto superior or equivalent to $p^{1, Myopic}$. Moreover, an examination of the social welfare values in Table 1 indicates that social welfare accompanying myopic behavior may decrease continually over time beginning in period 1. This is not necessarily always the case and Cowan (1997) notes only that consumer surplus is higher in period 1 than at the price cap vector, but is lower in the steady state. Nevertheless, as this example illustrates, welfare can move in the wrong direction, which possibility justifies some precaution against it. Accordingly, note that if upon moving to any price vector on the period 2 constraint the firm were required to fully compensate the consumer for the consumer surplus lost in moving from $p^{1, Myopic}$, the firm would prefer to remain at $p^{1, Myopic}$. The author therefore suggests a modification to the ARL process that would inure to the benefit of society generally and to the consumer particularly.

3.2.1. Policy Recommendation 1 (Applicable to Binding ARL Regulation of Two-Product-Market Firms Where Product Demand Functions Are Linear)

The regulator should observe product demand vectors $q^0 = (q_1^0, q_2^0)$ and $q^1 = (q_1^1, q_2^1)$ corresponding to the price vectors $p^0 = (\bar{p}^0, \bar{p}^0)$ and $p^1 = (p_1^1, p_2^1)$ assumed by the firm initially and in period 1, respectively. Estimates of the demand parameters a_1 , a_2 , b_1 , and b_2 can be determined by the regulator from solving the linear systems

$$\begin{cases} a_1 \bar{p}^0 + b_1 = q_1^0 \\ a_1 p_1^1 + b_1 = q_1^1 \end{cases} \text{ and } \begin{cases} a_2 \bar{p}^0 + b_2 = q_2^0 \\ a_2 p_2^1 + b_2 = q_2^1 \end{cases}. \text{ From these parameter estimates, the regulator}$$

should estimate consumer surplus at p^1 according to the formula

$$V(p^1) = \sum_{i=1}^2 -\frac{a_i}{2} \left(p_i^1 + \frac{b_i}{a_i} \right)^2. \text{ The firm should then be permitted to select any period 2}$$

price vector p^2 on the period 2 constraint line only if it agrees to compensate the

$$\text{regulatory board by the amount } V(p^1) - V(p^2) \text{ where } V(p^t) = \sum_{i=1}^2 -\frac{a_i}{2} \left(p_i^t + \frac{b_i}{a_i} \right)^2.$$

Otherwise, the firm is permitted only to select a period 2 price vector that lies on the period 1 constraint line. In all likelihood, any necessary compensation would take the form of a customer rebate program. Regulators often do require firms to do this.

Implementation of this policy recommendation will not only ensure that welfare is non-decreasing from period 1 to period 2, but can offset the decrease in consumer surplus accompanying ARL regulation if compensations from the firm to the regulatory board are forwarded as rebates to the consumer. Further, this requirement can induce the firm away

from any non-profit-maximizing strategic price vector as outlined in Proposition 1 during period 1 because such non-optimal pricing will only increase the differential

$V(p^1) - V(p^2)$. Observe from Table 1 that the consistently myopic firm positioned at

$p^{1, Myopic} = (2.463619, 3.935462)$ will not elect to proceed to $p^{2, Myopic} = (2.368426,$

$4.599436)$ since $\frac{\pi(p^{2, Myopic}) - \pi(p^{1, Myopic})}{V(p^{1, Myopic}) - V(p^{2, Myopic})} = \frac{14.86496 - 10.43611}{28.27961 - 19.4754} = 0.50304 < 1.00$.

3.2.2. Policy Recommendation 2 (Applicable to Binding ARL Regulation of Two Product-Market Firms Where Product Demand Functions Are Linear)

An alternative recommendation offered here draws upon a procedure constructed by Tam (1981) which has been demonstrated to lead a myopic firm in a Barone-Lange socialist economy to efficient prices. Tam's scheme, as modified by Finsinger and Vogelsang (1985) to disallow certain possible strategic behavior, induces a firm to efficient prices by rewarding the firm according to price improvements as it maximizes profit in each time period. In the case of a firm desirous of raising its prices, a tax of $(p^t - p^{t-1}) \cdot q^{t-1}$ in each period is assessed against the firm. The firm therefore will not raise its prices from period $t - 1$ to period t unless the increased profit exceeds this measure of reduction in price performance. Essentially this scheme encourages profit-maximizing behavior by the firm through penalizing it for setting high prices. Observe from Table 1 that under such a system our consistently myopic firm positioned at $p^{1, Myopic} = (2.463619, 3.935462)$ will not elect to proceed to $p^{2, Myopic} = (2.368426, 4.599436)$ because the tax $(p^t - p^{t-1}) \cdot q^{t-1} = (2.368426 - 2.463619)(7.681903) + (4.599436 - 3.935462)(16.38723) = 10.14943$ is greater than the additional profit to the

firm of $14.86496 - 10.43611 = 4.42885$. However, there may be a period t price vector beneath the period t constraint the firm would desire to move to if the additional profit for doing so exceeds the corresponding tax.

The Lagrangian for this problem is:

$$L(p^t, \lambda^t) = \pi(p^t) - \pi(p^{t-1}) - (p^t - p^{t-1}) \cdot q(p^{t-1}) + \lambda^t [p^0 \cdot q(p^{t-1}) - p^t \cdot q(p^{t-1})] \quad (52)$$

and Kuhn-Tucker necessary conditions for a solution are:

$$\nabla \pi(p^t) - q(p^{t-1}) = \lambda^t q(p^{t-1}) \quad (53)$$

$$\lambda^t [(p^0 - p^t) \cdot q(p^{t-1})] = 0 \quad (54)$$

$$\lambda^t \geq 0. \quad (55)$$

For $\lambda^t > 0$, we have binding ARL and the firm will move to the period t myopic price vector $p^{t, Myopic}$ guaranteed by Proposition 2. For $\lambda^t = 0$, ARL is nonbinding and the firm may select a period t price vector beneath the period t constraint line. In fact, the firm converges to the global social-welfare maximum (c_1, c_2) provided the tax applied to the firm that chooses to lower its profit and increase social welfare from period $t - 1$ to t can be negative, i.e., a subsidy is paid to the firm. In this case, Tam's scheme essentially overrides ARL regulation. To demonstrate that this convergence to (c_1, c_2) will occur, note that for $\lambda^t = 0$ equation (53) becomes:

$$\nabla \pi(p^t) - q(p^{t-1}) = 0 \quad (56)$$

which, for the profit function given by (5) corresponding to the linear case, yields:

$$2a_i p_i^t + b_i - a_i c_i = a_i p_i^{t-1} + b_i \text{ for } i = 1, 2, \dots, n. \quad (57)$$

Consequently, we obtain the recursive formula:

$$p_i^t = \frac{p_i^{t-1} + c_i}{2}. \quad (58)$$

Hence, the firm in period t will move to the vector of prices p^t , where each p_i^t is the simple average of the preceding period's price p_i^{t-1} and the marginal cost of production c_i for market i . The subsidy (negative tax) provided to the firm must increase in each period t by:

$$\begin{aligned} \Delta s^t &= -(p^t - p^{t-1}) \cdot q^{t-1} = \sum_{i=1}^n (p_i^{t-1} - p_i^t) q_i^{t-1} \\ &= \sum_{i=1}^n \left(p_i^{t-1} - \frac{p_i^{t-1} + c_i}{2} \right) q_i^{t-1} = \frac{1}{2} \sum_{i=1}^n (p_i^{t-1} - c_i) q_i^{t-1} \\ &= \frac{1}{2} \sum_{i=1}^n (p_i^{t-1} - c_i) (a_i p_i^{t-1} + b_i) \end{aligned} \quad (59)$$

where $\Delta s^t = s^t - s^{t-1}$ and $s^1 = 0$.

3.3. *Behavior of the Myopic Price Vector Sequence Within ARL About Steady States and Multiple Steady States*

Let p^t be the price vector assumed by the firm under binding ARL in period t .

Also let $p^{t,y \min}$ be the price vector at which consumer surplus is minimized over the

period t constraint, i.e., $\nabla V(p^{t,V \min}) \cdot (p^{t,V \min} - p^0) = 0$. Now the strict convexity of the consumer surplus function tells us that the slope of the iso-surplus through p^t exceeds the slope of the period t constraint when both p^t and $p^{t,V \min}$ are below the set Δ^+ if and only if p^t is a greater distance from p^0 than is $p^{t,V \min}$. Alternatively, the slope of the iso-surplus through p^t is less than the slope of the period t constraint when both p^t and $p^{t,V \min}$ are above the set Δ^+ if and only if p^t is a greater distance from p^0 than is $p^{t,V \min}$. Since ARL stipulates that the period $t + 1$ constraint be parallel to the iso-surplus through p^t at p^t , the ARL constraint line rotates from period t to period $t + 1$ in a predictable way depending on the positions of p^t and p^0 relative to $p^{t,V \min}$. Considering p^t and $p^{t,V \min}$ above the set Δ^+ , the constraint line will rotate clockwise about p^0 from period t to period $t + 1$ if $\sqrt{(p_1^t - \bar{p}^0)^2 + (p_2^t - \bar{p}^0)^2} > \sqrt{(p_1^{t,V \min} - \bar{p}^0)^2 + (p_2^{t,V \min} - \bar{p}^0)^2}$, and will rotate counterclockwise if $\sqrt{(p_1^t - \bar{p}^0)^2 + (p_2^t - \bar{p}^0)^2} < \sqrt{(p_1^{t,V \min} - \bar{p}^0)^2 + (p_2^{t,V \min} - \bar{p}^0)^2}$. Similarly, for p^t and $p^{t,V \min}$ below the set Δ^+ , the constraint line will rotate clockwise about p^0 from period t to period $t + 1$ if $\sqrt{(p_1^t - \bar{p}^0)^2 + (p_2^t - \bar{p}^0)^2} < \sqrt{(p_1^{t,V \min} - \bar{p}^0)^2 + (p_2^{t,V \min} - \bar{p}^0)^2}$, and will rotate counterclockwise if $\sqrt{(p_1^t - \bar{p}^0)^2 + (p_2^t - \bar{p}^0)^2} > \sqrt{(p_1^{t,V \min} - \bar{p}^0)^2 + (p_2^{t,V \min} - \bar{p}^0)^2}$.

In general, we may say that for any price vector p^{t+1} satisfying the period $t + 1$

constraint, we have $\frac{(p^t - p^0) \cdot p^0}{\sqrt{(p_1^t - \bar{p}^0)^2 + (p_2^t - \bar{p}^0)^2}} < \frac{(p^{t+1} - p^0) \cdot p^0}{\sqrt{(p_1^{t+1} - \bar{p}^0)^2 + (p_2^{t+1} - \bar{p}^0)^2}}$ if and only

if $\sqrt{(p_1^t - \bar{p}^0)^2 + (p_2^t - \bar{p}^0)^2} > \sqrt{(p_1^{t,Y \min} - \bar{p}^0)^2 + (p_2^{t,Y \min} - \bar{p}^0)^2}$. We are using the fact that the angle θ between two vectors $u = p^0$ and $v = p^t - p^0$ is obtainable by the well-known formula $u \cdot v = |u||v|\cos\theta$ where $|u|$ and $|v|$ are the Euclidean lengths of u and v , respectively. Hence, the expression $\frac{(p^t - p^0) \cdot p^0}{\sqrt{(p_1^t - \bar{p}^0)^2 + (p_2^t - \bar{p}^0)^2}}$ is in absolute value equal

to the length of the projection of p^0 onto the period t constraint vector $p^t - p^0$. The following proposition guarantees the existence of a steady-state efficient price vector within a certain set of prices given binding ARL regulation with a C^2 strictly convex consumer surplus function and a C^2 strictly concave profit function. Neither linear demands nor constant marginal costs need be assumed.

3.3.1. Proposition 4

Define the sequence

$$u_t = \frac{(p^{t,Myopic} - p^0) \cdot p^0}{\sqrt{(p_1^{t,Myopic} - \bar{p}^0)^2 + (p_2^{t,Myopic} - \bar{p}^0)^2}} \quad (60)$$

where $p^{t,Myopic}$ is the price vector assumed by the myopic firm in period t under binding ARL with C^2 convex consumer surplus function and C^2 concave profit function. If for some t we have $u_t < u_{t+1}$ and $u_{t+2} < u_{t+1}$, then there exists a steady-state efficient price

vector p^s satisfying $u_t < \frac{(p^s - p^0) \cdot p^0}{\sqrt{(p_1^s - \bar{p}^0)^2 + (p_2^s - \bar{p}^0)^2}} < u_{t+1}$. The scalar u_t is the

magnitude of the projection of the price-cap vector p^0 onto $p^t - p^0$ ($u_t < 0$ simply

indicates that the angle between p^0 and the vector $p^t - p^0$ is greater than 90°). See figure 15.

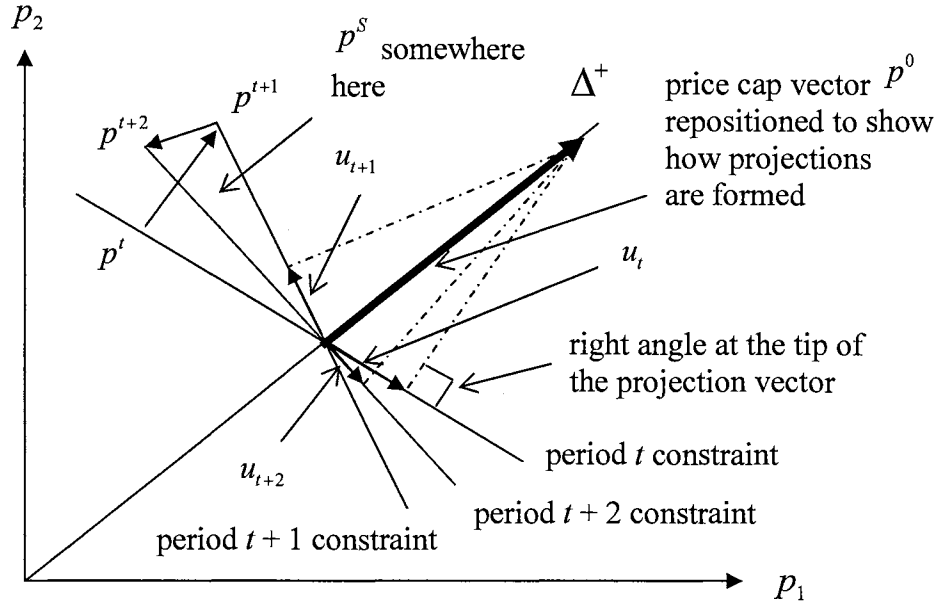


Figure 15. When $u_t < u_{t+1}$ and $u_{t+2} < u_{t+1}$ a steady-state price vector p^s must exist somewhere between the period t and period $t + 1$ constraints. (For the figure here we have $u_t < 0$ and $u_{t+2} < 0$, but $u_{t+1} > 0$).

Proof of Proposition 4:

For every $u^{Myopic} \in [u_t, u_{t+1}]$ there exists a unique price vector p^{Myopic} such that

$$u^{Myopic} = \frac{(p^{Myopic} - p^0) \cdot p^0}{\sqrt{(p_1^{Myopic} - \bar{p}^0)^2 + (p_2^{Myopic} - \bar{p}^0)^2}} \text{ and } \nabla \pi(p^{Myopic}) \cdot (p^{Myopic} - p^0) = 0. \text{ This}$$

follows because π is C^2 and strictly concave. Also, for every $u^{V \min} \in [u_t, u_{t+1}]$ there

$$\text{exists a unique price vector } p^{V \min} \text{ such that } u^{V \min} = \frac{(p^{V \min} - p^0) \cdot p^0}{\sqrt{(p_1^{V \min} - \bar{p}^0)^2 + (p_2^{V \min} - \bar{p}^0)^2}} \text{ and}$$

$\nabla V(p^{V \min}) \cdot (p^{V \min} - p^0) = 0$. This follows because V is C^2 and strictly convex. Define

the functions $f(u^{Myopic}) = \sqrt{(p_1^{Myopic} - \bar{p}^0)^2 + (p_2^{Myopic} - \bar{p}^0)^2}$ and

$g(u^{V \min}) = \sqrt{(p_1^{V \min} - \bar{p}^0)^2 + (p_2^{V \min} - \bar{p}^0)^2}$. Now the ARL process implies that since

$u_t < u_{t+1}$ we have $f(u_t) > g(u_t)$, and since $u_{t+2} < u_{t+1}$ we have $f(u_{t+1}) < g(u_{t+1})$. So since

π and V are both continuous, then by the Intermediate Value Theorem there exists

$u_s \in (u_t, u_{t+1})$ such that $f(u_s) = g(u_s)$. But this implies there is a price vector p^s such

that $u_t < \frac{(p^s - p^0) \cdot p^0}{\sqrt{(p_1^s - \bar{p}^0)^2 + (p_2^s - \bar{p}^0)^2}} < u_{t+1}$ and $\nabla \pi(p^s) \cdot (p^s - p^0) = 0 =$

$\nabla V(p^s) \cdot (p^s - p^0)$.

An important use of Proposition 4 is to alert the regulator when the consistently myopic firm is converging to a steady-state price vector. In Table 1 we see that the sequence of projections $\{u_t\}$ is monotone because the price cap is set at the average revenue of the profit maximum. In a vacuous sense, the sequence obeys Proposition 4 as its movement never reverses direction.

Now we must acknowledge the possibility that in the general case multiple steady states can correspond to the price cap. However, if the sequence $\{u_t\}$ reverses direction, the steady state guaranteed by Proposition 4 cannot be p^* because the constraint never needs to bind. Oscillation in the sequence $\{u_t\}$ signifies convergence to a steady state other than p^* .

For the instance of where the period t constraint line is positioned between two steady states corresponding to the price cap p^0 , the direction of rotation to the period

$t + 1$ constraint is determined by the positions of p^t and p^0 relative to $p^{t,V \min}$. If two steady-state price vectors p^a and p^b are related to the period t constraint as in figure 16, then the constraint line rotates towards p^b in period $t + 1$. When the path of $p^{t, Myopic}$ is outside the path of $p^{t,V \min}$ (i.e., $p^{t, Myopic}$ is a greater distance from Δ^+ than is $p^{t,V \min}$), the constraint line rotates away from the origin and the sequence $\{u_t\}$ is increasing. Alternatively, when the path of $p^{t, Myopic}$ is inside the path of $p^{t,V \min}$, the constraint line rotates toward the origin and the sequence $\{u_t\}$ is decreasing.

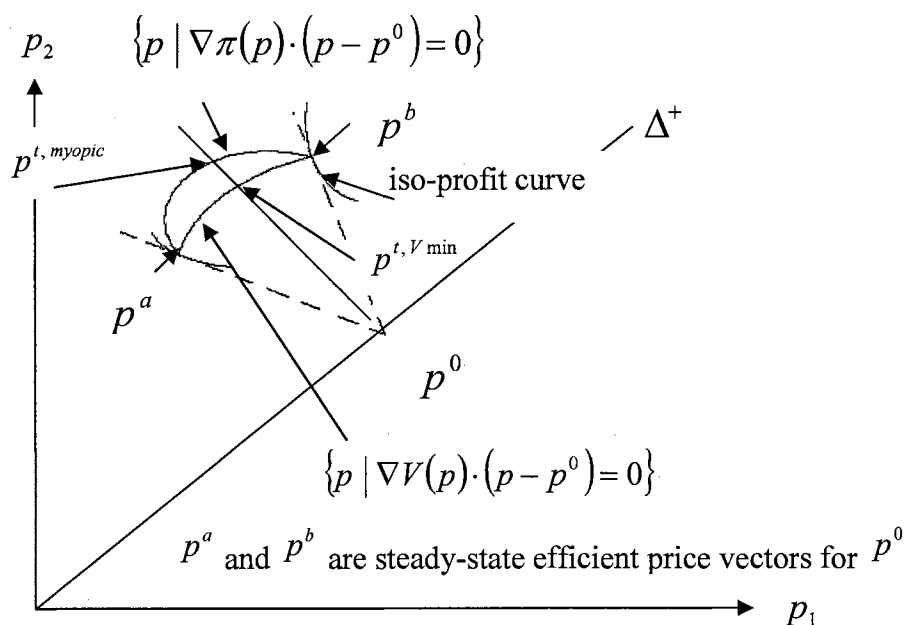


Figure 16. Period t constraint line between two steady states.

3.4. Firm Behavior Under ARL in the Face of a Positive Discount Factor

Cowan (1997) states that for a strictly positive discount factor steady-state prices will be inefficient. A positive discount factor is due to the fact that because of uncertainty

a firm values the prospect of obtaining a specific sum in the future less than it values possessing the identical sum in the present. The relevant infinite-horizon Lagrangian is:

$$\begin{aligned}
L(p^1, p^2, \dots, \lambda^1, \lambda^2, \dots; \beta) &= \pi(p^1) + \lambda^1(p^0 - p^1) \cdot q(p^0) \\
&\quad + \beta[\pi(p^2) + \lambda^2(p^0 - p^2) \cdot q(p^1)] \\
&\quad + (\beta)^2[\pi(p^3) + \lambda^3(p^0 - p^3) \cdot q(p^2)] + \dots \\
&= \sum_{t=1}^{\infty} (\beta)^{t-1} [\pi(p^t) + \lambda^t(p^0 - p^t) \cdot q(p^{t-1})] \tag{61}
\end{aligned}$$

where β is the discount factor and λ^t is the Lagrange multiplier in period t . The first-order conditions for prices yield:

$$\nabla \pi(p^t) = \lambda^t q^{t-1} + \beta \lambda^{t+1} H(p^t) (p^0 - p^{t+1}). \tag{62}$$

The extra term on the right hand side of (62) is due to the influence of the slope of the iso-surplus through p^t at p^t on the slope of the period $t + 1$ constraint. Steady-state prices satisfy:

$$\nabla \pi(p^s) = \lambda q(p^s) + \beta \lambda H(p^s) (p^0 - p^s). \tag{63}$$

Cowan notes that because of the term $\beta \lambda H(p^s) (p^0 - p^s)$ on the right-hand side of (63) the necessary condition of efficient prices for a steady-state price vector no longer applies. For demand functions $q_i = a_i p_i + b_i$ the profit gradient is:

$$\nabla \pi(p) = (2a_1 p_1 + b_1 - a_1 c_1, 2a_2 p_2 + b_2 - a_2 c_2, \dots, 2a_n p_n + b_n - a_n c_n). \tag{64}$$

Therefore at the steady-state price vector for p^0 satisfying $\bar{p}^0 \leq \bar{p}^*$ we have:

$$2a_i p_i^S + b_i - a_i c_i = \lambda(a_i p_i^S + b_i) - \beta \lambda a_i (\bar{p}^0 - p_i^S) \quad (65)$$

for all i . Solving for λ yields:

$$\lambda = \frac{2a_i p_i^S + b_i - a_i c_i}{a_i(1 + \beta)p_i^S + b_i - \beta a_i \bar{p}^0}. \quad (66)$$

For $n = 2$, steady-state price vectors corresponding to the positive discount factor β will necessarily satisfy:

$$\frac{2a_1 p_1^S + b_1 - a_1 c_1}{a_1(1 + \beta)p_1^S + b_1 - \beta a_1 \bar{p}^0} = \frac{2a_2 p_2^S + b_2 - a_2 c_2}{a_2(1 + \beta)p_2^S + b_2 - \beta a_2 \bar{p}^0} \quad (67)$$

which defines p_2^S as a linear function of p_1^S . Note that

$$p^* = \left(-\frac{(b_1 - a_1 c_1)}{2a_1}, -\frac{(b_2 - a_2 c_2)}{2a_2} \right) \text{ always satisfies this equation. A steady state may}$$

therefore be obtained by solving the system comprised of this equation and:

$$(p^S - p^0) \cdot \nabla V(p^S) = 0. \quad (68)$$

To illustrate, consider the previously considered demand and cost functions

$$q_1 = -5p_1 + 20, \quad q_2 = -6p_2 + 40, \quad \text{and} \quad C = q_1 + 3q_2 + 16.1369. \text{ Also let the price cap be}$$

$$\bar{p}^0 = \bar{p}^* = 3.8874. \text{ The locus for (67) becomes}$$

$$\frac{-10p_1^S + 25}{-5(1 + \beta)p_1^S + 20 + 19.437\beta} = \frac{-12p_2^S + 58}{-6(1 + \beta)p_2^S + 40 + 23.3244\beta}. \text{ Suppose } \beta = 0.95. \text{ The}$$

locus for (67) now becomes $56.0818p_1^S - 169.0818p_2^S = -677.0242$. The price vector $(p_1, p_2) = (5.362666, 5.782835)$ is on this line and satisfies (68). However, this vector is not actually in the domain of the functions $V(p)$ and $\pi(p)$ (Recall that the domain of both functions is $\{(p_1, p_2) \mid 0 \leq p_i \leq -b_i/a_i, i = 1, 2\}$ which here becomes $\{(p_1, p_2) \mid 0 \leq p_1 \leq 4, 0 \leq p_2 \leq 20/3\}$). Hence, p^* is really the only meaningful steady-state price vector here. Cowan's assertion actually does not apply in this context. Proposition 5 gives conditions under which p^* will be the steady state in the face of a positive discount factor given linear demands and constant marginal costs.

3.4.1. Proposition 5

For positive discount factor β , linear demand functions, constant marginal costs c_1 and c_2 , and a price cap p^0 whose level satisfies $\bar{p}^0 = \bar{p}^*$, $\bar{p}^0 > c_1$ and $\bar{p}^0 > c_2$, there are no steady states in the domains of $V(p)$ and $\pi(p)$ other than at p^* .

Proof: of Proposition 5:

Solve (67) for p_2^β to obtain:

$$p_2^\beta = \frac{a_1 p_1 (2\beta a_2 \bar{p}^0 + \beta(b_2 - a_2 c_2) - a_2 c_2 - b_2) + \beta \bar{p}^0 (a_2 b_1 - a_1(a_2(c_1 - c_2) + b_2))}{a_2 (2\beta a_1 \bar{p}^0 + \beta(b_1 - a_1 c_1) - a_1 c_1 - b_1)}$$

$$\sim \frac{+ a_1 b_2 c_1 - a_2 b_1 c_2}{.1} .5 \quad (69)$$

⁵ Obtained by use of the computer algebra system *Derive*.

Letting $\beta = 0$ in (69) produces:

$$p_2^\beta = \frac{a_1 p_1 (a_2 c_2 + b_2) - a_1 b_2 c_1 + a_2 b_1 c_2}{a_2 (a_1 c_1 + b_1)}. \quad (70)$$

which, as we should expect, is equation (12) again defining the set of efficient prices.

Also, letting $\beta = 1.0$ in (69) yields:

$$p_2^\beta = \frac{2a_1 a_2 p_1 (\bar{p}^0 - c_2) + \bar{p}^0 (a_2 b_1 - a_1 (a_2 (c_1 - c_2) + b_2)) + a_1 b_2 c_1 - a_2 b_1 c_2}{2a_1 a_2 (\bar{p}^0 - c_1)}. \quad (71)$$

A plot of (68) with (70) and (71) in the price plane is provided in figure 17. Observe that

for all values of β satisfying $0 < \beta \leq 1.00$ the intersection of (68) and (69) is properly

outside the domain set $\{(p_1, p_2) \mid 0 \leq p_i \leq -b_i / a_i, i = 1, 2\}$ except for the price vector

p^* . Confirmation of this result is obtained by substituting $p_1 = -\frac{b_1}{a_1}$ in the right-hand

side of (70) thus yielding $p_2^\beta = -\frac{b_2}{a_2}$, and by noting that since $\bar{p}^0 > c_1$ and $\bar{p}^0 > c_2$, the

slope of the line represented by (71) which is:

$$m = \frac{\bar{p}^0 - c_2}{\bar{p}^0 - c_1} \quad (72)$$

must be positive, and hence cannot intersect the ellipse of (68) in the aforementioned

domain.

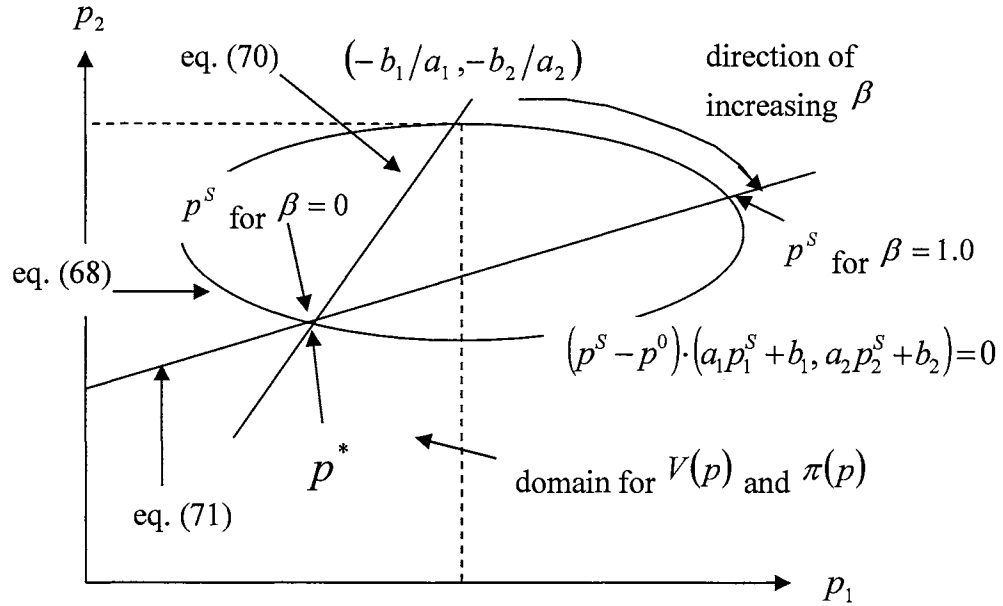


Figure 17. Locus of possible steady-state price vectors corresponding to a positive discount factor β ($\bar{p}^0 = \bar{p}^*$, $\bar{p}^0 > c_i$ for $i = 1, 2$).

We have here a particular story not presented by Cowan regarding steady states attained by a firm when discounting future profits. Specifically, for linear demands, constant marginal costs, and price cap \bar{p}^0 satisfying $\bar{p}^0 = \bar{p}^*$, $\bar{p}^0 > c_i$ for both $i = 1, 2$, where c_i is the marginal cost of production for market i , if the firm seeks to maximize the present value of a future stream of profits, the resulting steady state is the price vector p^* which indeed is efficient.

However, Cowan does demonstrate by example that if \bar{p}^0 is less than c_i for some i , there can be a feasible inefficient steady state that will manifest a perversity of producing a welfare level below that obtained with no regulation, i.e., $W(p^s) < W(p^*)$. The question of what precise circumstances (given linear demands and constant marginal

costs) lead to this outcome for welfare is addressed by the statement of Proposition 6.

3.4.2. Proposition 6

Given linear demand functions $q_i = a_i p_i + b_i$, constant marginal costs c_1 and c_2 , and a price cap \bar{p}^0 whose level satisfies $\bar{p}^0 < c_i$ for exactly one i , if

$$3a_1(c_1)^2 + 6b_1c_1 + 3a_2(c_2)^2 - 2(4a_2\bar{p}^0 + b_2)c_2 + \frac{a_1(4(a_2)^2(\bar{p}^0)^2 - (b_2)^2) + 3a_2(b_1)^2}{a_1a_2} < 0,$$

then there exists a critical value β^C for the discount factor satisfying $0 < \beta^C < 1.0$ such that for any β^S where $\beta^C < \beta^S < 1.0$ there exists a non-efficient steady-state price vector p^S in the domains of $V(p)$ and $\pi(p)$ satisfying $W(p^S) < W(p^*)$.

Proof of Proposition 6:

Without loss of generality, assume $c_2 > c_1$. Consider the system of equations formed by:

$$\begin{aligned} W(p^*) &= V(p^*) + \pi(p^*) = -\frac{3(a_1c_1 + b_1)^2}{8a_1} - \frac{3(a_2c_2 + b_2)^2}{8a_2} - F \\ &= -F + \frac{a_1}{2}(p_1 - c_1)^2 - \frac{(b_1 + a_1c_1)^2}{2a_1} + \frac{a_2}{2}(p_2 - c_2)^2 - \frac{(b_2 + a_2c_2)^2}{2a_2} \end{aligned} \quad (73)$$

$$(p_1 - \bar{p}^0)(a_1p_1 + b_1) + (p_2 - \bar{p}^0)(a_2p_2 + b_2) = 0 \quad (74)$$

$$\frac{2a_1p_1 + b_1 - a_1c_1}{a_1(1 + \beta)p_1 + b_1 - \beta a_1\bar{p}^0} = \frac{2a_2p_2 + b_2 - a_2c_2}{a_2(1 + \beta)p_2 + b_2 - \beta a_2\bar{p}^0} \quad (75)$$

in variables p_1 , p_2 , and β . Equation (73) is the iso-welfare curve through p^* (cp. equation 50) obtained by employing (10) and (11). Equation (74) is the set of steady-state price vectors for p^0 under ARL regulation, i.e., $(p - p^0) \cdot \nabla V(p) = 0$. Equation (75) follows from the first-order conditions on the Lagrangian (cp. equation 67). Now it can be shown by direct substitution in (74) that $\left(\bar{p}^0, -\frac{b_2}{a_2}\right)$, $\left(-\frac{b_1}{a_1}, \bar{p}^0\right)$, and $\left(-\frac{b_1}{a_1}, -\frac{b_2}{a_2}\right)$

are on the ellipse of steady-state price vectors associated with p^0 . Additionally, for $\beta = 0$ we have the first-order condition line (75) giving the equation of the set E of efficient prices which is the positively sloped line passing through (c_1, c_2) , p^* , and

$\left(-\frac{b_1}{a_1}, -\frac{b_2}{a_2}\right)$. For $\beta = 1.0$ we have (75) producing the first-order condition line (71). If

$\bar{p}^0 < c_i$ for exactly one i , then by (72) $m < 0$ and the line of (75) is sloped downward.

Note: if $\bar{p}^0 < c_i$ for both $i = 1, 2$, then $\pi(p^*) < 0$ results thus putting the firm out of business. Hence we do not consider this possibility. A graph of this system for

$c_2 < \bar{p}^0 < c_1$ and $-\frac{b_1}{a_1} > -\frac{b_2}{a_2}$ is represented in figure 18.

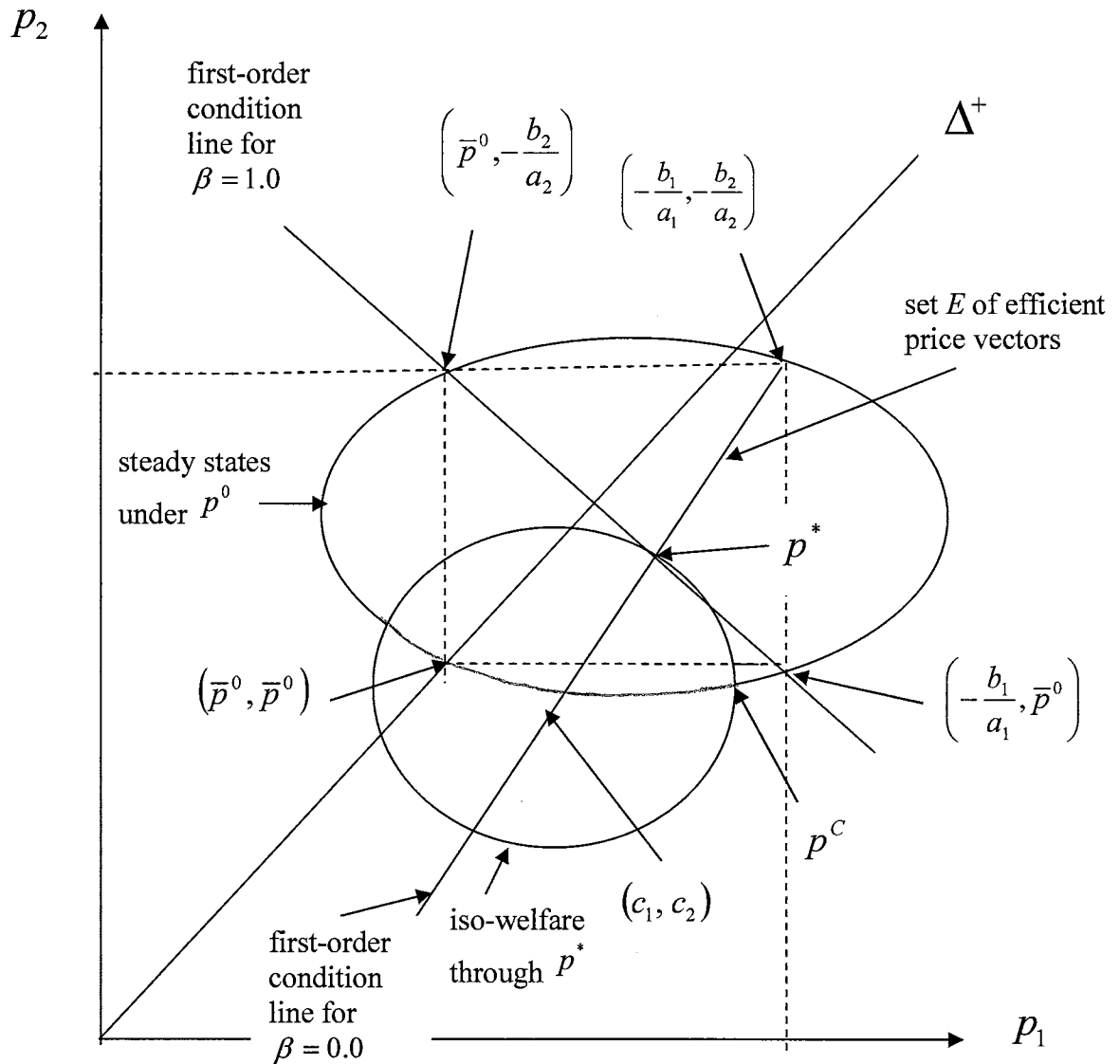


Figure 18. Iso-welfare through p^* , steady states for p^0 , first-order condition lines, and p^c for discount factor β .

Observe the price vector $p^c = (p_1^c, p_2^c)$ at the intersection of the iso-welfare curve (73) and the set of steady states (74). For particular c_1 and c_2 satisfying $c_2 > c_1$,

this vector could exist to the right of $\left(-\frac{b_1}{a_1}, \bar{p}^0\right)$ and thus outside the domain of $\pi(p)$

and $V(p)$. To obtain the situation of figure 19 we simply require that c_1 and c_2 force

$W(p^*) > W\left(-\frac{b_1}{a_1}, \bar{p}^0\right)$. The price vector $\left(-\frac{b_1}{a_1}, \bar{p}^0\right)$ is then outside the iso-welfare curve

passing through p^* . Hence, we use (51) to produce the additional necessary condition

that c_1 and c_2 satisfy:

$$\begin{aligned} & -\frac{3(a_1c_1 + b_1)^2}{8a_1} - \frac{3(a_2c_2 + b_2)^2}{8a_2} \\ & > \frac{a_1}{2} \left(-\frac{b_1}{a_1} - c_1\right)^2 - \frac{(b_1 + a_1c_1)^2}{2a_1} + \frac{a_2}{2} (\bar{p}^0 - c_2)^2 - \frac{(b_2 + a_2c_2)^2}{2a_2} \end{aligned} \quad (76)$$

to establish the existence of such a β^C . Expanding (76) about c_1 and c_2 yields:

$$3a_1(c_1)^2 + 6b_1c_1 + 3a_2(c_2)^2 - 2(4a_2\bar{p}^0 + b_2)c_2 + \frac{a_1(4(a_2)^2(\bar{p}^0)^2 - (b_2)^2) + 3a_2(b_1)^2}{a_1a_2} < 0 \quad (77)$$

for the feasible set of marginal costs in the c_1, c_2 dual space. Ordered pairs (c_1, c_2) for which β^C , as pictured in figure 19, exist are points in the interior of the elliptical-curve graph in c_1, c_2 space of the equation corresponding to (77).

Now for any β^S where $\beta^C < \beta^S < 1.0$ we see there is a non-efficient steady-state price vector p^S in the domains of $V(p)$ and $\pi(p)$ satisfying $W(p^S) < W(p^*)$ because p^S is outside the iso-welfare ellipse through p^* . See figure 19.

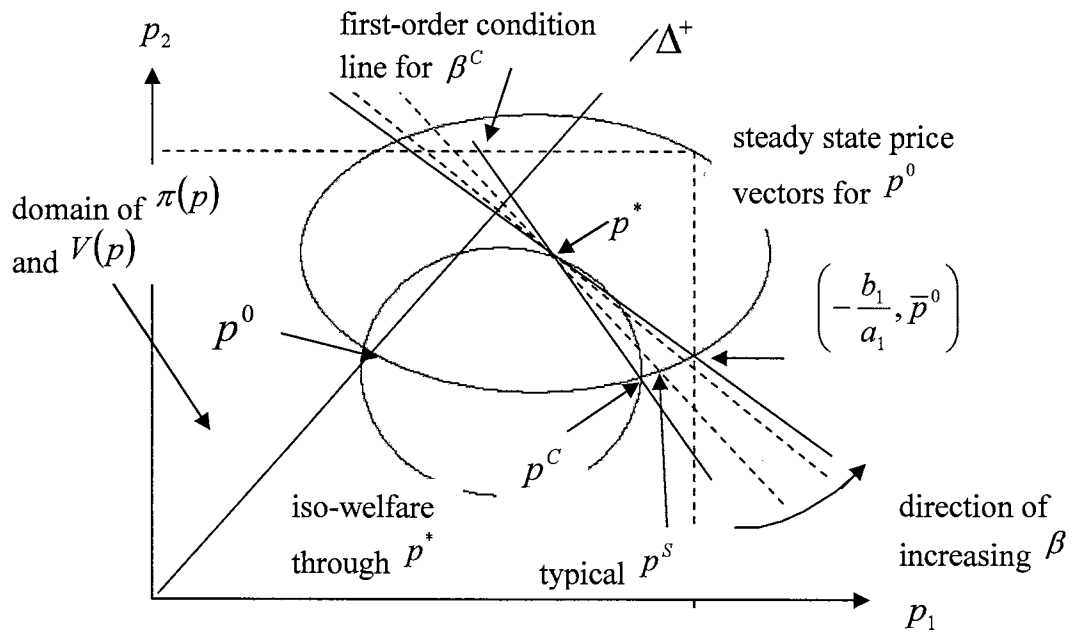


Figure 19. Rotation of first-order condition line with increasing β .

As a final caveat, however, note that even under nonlinear demands and non-constant marginal costs a firm cannot be successful in convincing the regulator that a steady-state price vector corresponding to a positive discount factor β , or for that matter any price vector other than p^* , is an efficient price vector indefinitely. Though a firm may reasonably be expected to misjudge the precise location of $p^{t, Myopic}$ in any period t , the firm will not be able to pass off the steady state p^β for p^* indefinitely because the observed product demand vector $q(p^\beta)$ will not be orthogonal to $p^0 - p^\beta$.

3.5. *Summary of Chapter 3*

In summary, this chapter has investigated myopic behavior of the ARL regulated firm. If product demands are linear and marginal costs are constant, even the consistently myopic firm will inevitably arrive at the global profit maximum. Moreover, this convergence may be fairly rapid as demonstrated with our example. This deficiency of ARL exists again when the level of the price cap exceeds the average revenue of the profit maximum. Policy recommendations are herein offered to offset this failing, yet each has its limitations and is not so automatic in implementation. Additionally, as multiple steady states associated with the cap is a real possibility, this chapter suggests a method by which the regulator should be able to detect whether or not the steady state approached by the firm is in fact the profit maximum. Also, we have complemented Cowan's finding that under binding ARL with a positive discount factor steady-state prices that are not equal are inefficient, by showing that when the price cap level is set above all marginal costs of production the firm will nevertheless approach the global profit-maximum which is indeed an efficient price vector. Finally, to expand on Cowan's example demonstrating that steady-state welfare under ARL can be below that associated with no regulation, we have provided more specific conditions under which this situation can occur.

4. STRATEGIC AND MYOPIC BEHAVIOR UNDER LASPEYRES PRICE CAP REGULATION

4.1. *Movement of Profit, Consumer Surplus, and Social Welfare Under Laspeyres Price Cap Regulation*

The objective of this chapter is to determine necessary and sufficient conditions under which Laspeyres price cap regulation will not be susceptible to manipulation by the regulated firm.

There are some general reasons for preferring Laspeyres price cap regulation to ARL. The firm can change prices in each period to maximize profit, but only in a manner that increases consumer surplus. Figure 20 illustrates for the general situation of nonlinear demands. Suppose prices at the beginning of period t are at p^{t-1} . The

Laspeyres constraint for period t , $\frac{p^t \cdot q(p^{t-1})}{p^{t-1} \cdot q(p^{t-1})} \leq 1$, dictates that the firm selects for

period t only those price vectors lying along, or beneath, the line containing p^{t-1} that is tangent to the isosurplus curve through p^{t-1} . Because of the convexity of this iso-surplus curve, any such price vector p^t selected by the firm will increase consumer surplus, i.e., $V(p^t) \geq V(p^{t-1})$. Hence, successive iterations of Laspeyres price cap regulation as applied to the consistently myopic firm can result only in a monotone increasing sequence of welfare levels because $W(p^t) = V(p^t) + \pi(p^t)$.

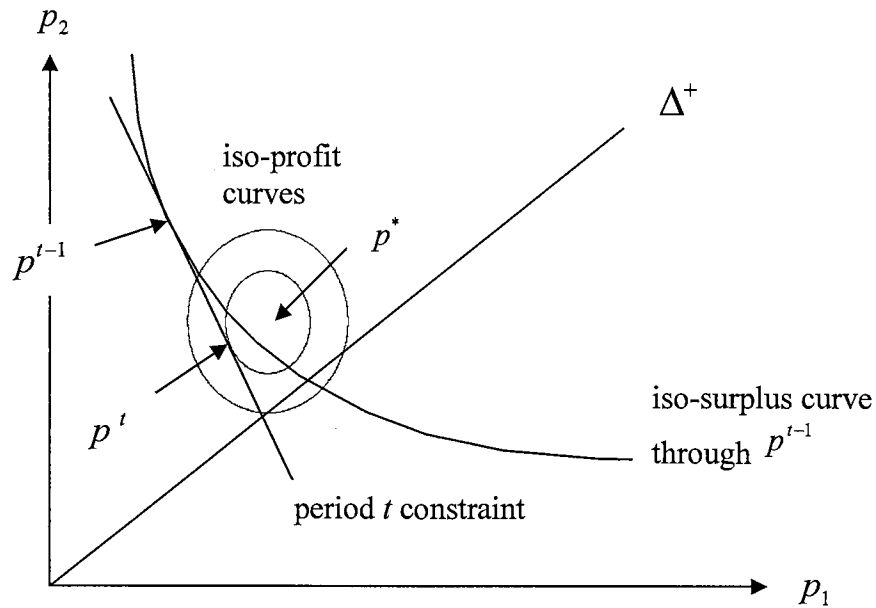


Figure 20. Laspeyres price cap regulation.

4.2. Implications of Consumer Surplus for Strategic Behavior Under Laspeyres Price Cap Regulation

We consider herein the question of whether it is possible for Laspeyres price cap regulation to be subject to the same type of strategic manipulation that we have shown to be associated with ARL regulation. We firstly assume that the pre-price cap regulation regime required that the firm operate at a price vector lying on the undominated portion of the zero-profit iso-profit curve. The undominated portion U of an iso-profit corresponds to the set of price vectors from which it is impossible to move to another price vector on the same iso-profit without increasing at least one of the market prices. In figure 21, the undominated portion of the illustrated iso-profit is the downward sloping segment extending from p^A to p^B . Under rate-of-return regulation it is reasonable to

assume that the regulator requires the zero-profit firm to operate in this region, for otherwise the firm is allowed to operate at a socially inefficient position – all prices could be cut without reducing the rate of return. Hence, U is the only portion of the zero-profit iso-profit of interest to the regulator.

Vectors in the plane of ordered price pairs that allow for immediate attainment of p^* under Laspeyres regulation belong to the set $A = \{p \mid p^* \cdot q(p) \leq p \cdot q(p)\}$. This set, whose boundary contains p^* , is convex and lies above the isosurplus passing through p^* . Therefore, it is possible for the firm to move directly to p^* upon institution of Laspeyres regulation if $A \cap U$ is nonempty (assuming the pre-price cap price vector is in this intersection). While this possibility certainly exists in nonlinear cases (see figure 21), the following proposition claims that such possibility cannot exist with linear demands and constant marginal costs.

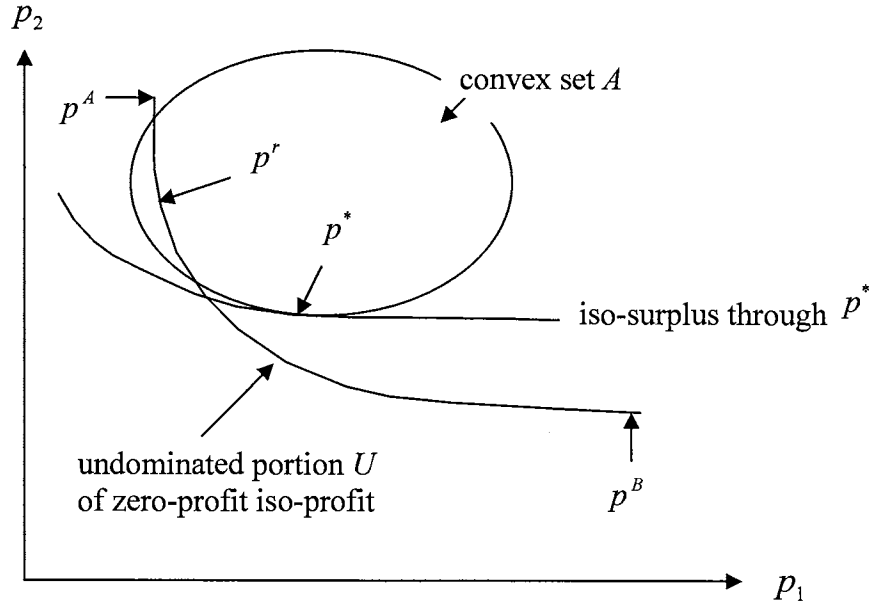


Figure 21. Nonlinear case allowing the possibility of immediate attainment of p^* under Laspeyres price cap regulation.

4.2.1. Proposition 7

If the price vector assumed by a firm with linear demands and constant marginal costs before imposition of Laspeyres price cap regulation is in the undominated portion of the zero-profit iso-profit curve, the firm will not be able to assume the global profit maximum p^* under Laspeyres price cap regulation.

Proof of Proposition 7:

Suppose a firm has demand vectors $q_i(p_i) = a_i p_i + b_i$, marginal costs c_i for $i = 1, 2$, and fixed cost F . By (8) the iso-profit locus for $\pi = 0$ is curve C represented by:

$$C: \quad F = a_1(p_1)^2 + (b_1 - a_1 c_1)p_1 - b_1 c_1 + a_2(p_2)^2 + (b_2 - a_2 c_2)p_2 - b_2 c_2. \quad (78)$$

Letting U represent the undominated portion of C implies:

$$\frac{dp_2}{dp_1} = \frac{2a_1p_1 + b_1 - a_1c_1}{2a_2p_2 + b_2 - a_2c_2} < 0 \quad (79)$$

for p in the interior of U . This is satisfied by the downward sloping portion of C

extending between p^A and p^B where $p_2^A = -\frac{(b_2 - a_2c_2)}{2a_2}$ and $p_1^B = -\frac{(b_1 - a_1c_1)}{2a_1}$. Now the

global profit maximum is by (9) $p^* = \left(-\frac{(b_1 - a_1c_1)}{2a_1}, -\frac{(b_2 - a_2c_2)}{2a_2} \right)$, so the firm cannot

move from a price vector $p \in U$ to p^* while decreasing the price in a given market

(figure 22). It follows that $V(p^*) \leq V(p)$ for all $p \in U$, and since consumer surplus under

Laspeyres is monotone increasing over time for convex V , the firm cannot move from U

to p^* .

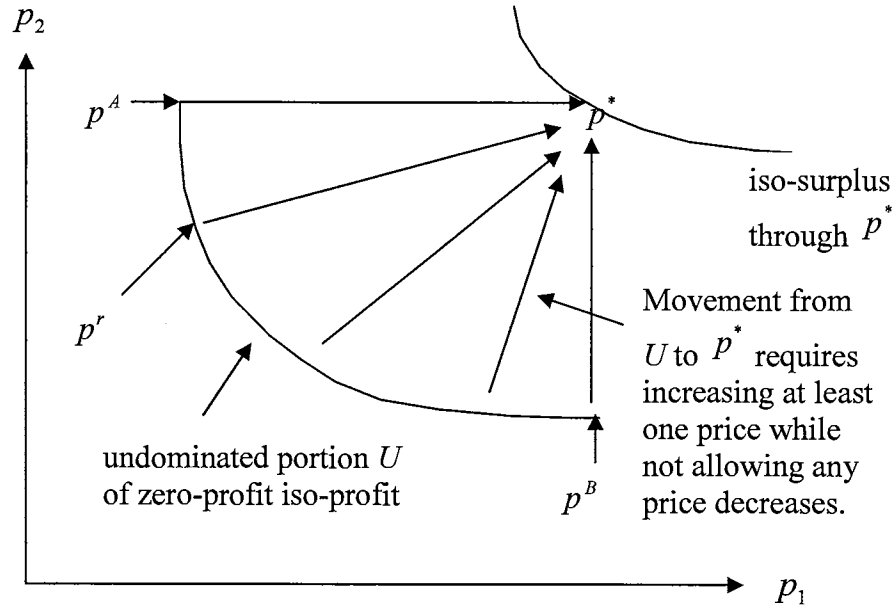


Figure 22. Linear case disallowing the possibility of attainment of p^* under Laspeyres price cap regulation.

The above argument can be extended to the case of n markets where for each $i = 1, \dots, n$ we have demand vectors $q_i(p_i) = a_i p_i + b_i$ and marginal costs c_i . More generally, we need only stipulate that $\frac{\partial^2 \pi}{\partial p_j \partial p_i} = 0$ for $i \neq j$ for the same result to obtain.

The zero-profit iso-profit surface for linear demands and constant marginal costs will be radially symmetric about each axis ξ_i through p^* and the zero-profit vector solving

$$\frac{\partial \pi}{\partial p_i} = 0.$$

4.3. Strategic Behavior Under Laspeyres Price Cap Regulation Attaining p^*

Now because consumer surplus is monotone increasing under Laspeyres for

strictly convex V , it follows that a firm positioned at a price vector p contained in the set $V(p^*)^+ = \{p \mid V(p) > V(p^*)\}$ cannot attain the global profit maximum. However, the question of whether the firm can converge to p^* from any price vector in $V(p^*)^- = \{p \mid V(p) < V(p^*)\}$ but outside the convex set A (note $A \subset V(p^*)^-$) is perhaps not so amenable to our intuition. The firm may find itself in $V(p^*)^-$ before imposition of Laspeyres regulation for reasons that here deserve brief consideration. Baumol, Fischer and ten Raa (1979) stress that fully distributed cost pricing may be called for by competitors of the regulated firm. As a result, the regulatory board may actually mandate an increase in the firm's prices to a dominated point on the zero-profit iso-profit locus. Additionally, Currier (2005) mentions that information asymmetries inherent in the regulatory procedure itself may induce the firm to fabricate cost data with the result that the regulatory board is deceived into believing that the undominated portion exists at artificially higher prices (i.e., above and to the right of its actual location). In such an event, the pre-Laspeyres regulated firm may be permitted to operate in $V(p^*)^-$ without the regulatory board's cognizance. For whatever reason the firm is permitted to price in $V(p^*)^-$, the firm may eventually move to p^* under Laspeyres regulation. To substantiate this possibility for the case of linear demands and constant marginal costs, we here utilize the justification for a familiar numerical method applied to the problem of approximating the solution to simple first-order initial value problems in ordinary differential equations. The method will be used to demonstrate the feasibility of convergence under Laspeyres from any price vector in $V(p^*)^-$ to p^* .

4.3.1. Proposition 8

Assume a strictly convex consumer surplus function $V(p_1, p_2)$ that is C^2 in some open region R containing $p^0 = (p_1^0, p_2^0)$ and for which $\frac{\partial V}{\partial p_2} \neq 0$ and

$$\left| \frac{\frac{\partial V}{\partial p_2} \frac{\partial^2 V}{\partial (p_1)^2} - \frac{\partial V}{\partial p_1} \frac{\partial^2 V}{\partial p_1 \partial p_2}}{\left(\frac{\partial V}{\partial p_2} \right)^2} \right| \text{ is bounded for } p = (p_1, p_2) \in R. \text{ Then there is a } C_1 > 0,$$

$C_2 > 0$ for which p^0, p^1, \dots, p^t generated by the iteration:

$$\begin{aligned} p^1 = (p_1^1, p_2^1) &= \begin{pmatrix} p_1^0 + h, p_2^0 - h \frac{\frac{\partial V}{\partial p_1} \Big|_{(p_1^0, p_2^0)}}{\frac{\partial V}{\partial p_2} \Big|_{(p_1^0, p_2^0)}} \\ \frac{\partial V}{\partial p_2} \Big|_{(p_1^0, p_2^0)} \end{pmatrix} \\ p^2 = (p_1^2, p_2^2) &= \begin{pmatrix} p_1^1 + h, p_2^1 - h \frac{\frac{\partial V}{\partial p_1} \Big|_{(p_1^1, p_2^1)}}{\frac{\partial V}{\partial p_2} \Big|_{(p_1^1, p_2^1)}} \\ \frac{\partial V}{\partial p_2} \Big|_{(p_1^1, p_2^1)} \end{pmatrix} \\ &\vdots \\ p^t = (p_1^t, p_2^t) &= \begin{pmatrix} p_1^{t-1} + h, p_2^{t-1} - h \frac{\frac{\partial V}{\partial p_1} \Big|_{(p_1^{t-1}, p_2^{t-1})}}{\frac{\partial V}{\partial p_2} \Big|_{(p_1^{t-1}, p_2^{t-1})}} \\ \frac{\partial V}{\partial p_2} \Big|_{(p_1^{t-1}, p_2^{t-1})} \end{pmatrix} \end{aligned} \tag{80}$$

satisfy:

$$\left| \tilde{p}_2^j - p_2^j \right| \leq hC_1 \left(e^{C_2jh} - 1 \right), \quad (81)$$

where \tilde{p}^j is the price vector on the iso-surplus passing through p^0 satisfying

$$\tilde{p}_1^j = p_1^0 + jh, \text{ provided the iterates remain in } R.$$

Proof of Proposition 8:

We appeal here to a theorem we will call Euler's tangent line theorem (Leader 2004). Appendix E presents a statement of this theorem and how it is adapted to the discussion that follows. The "solution" curve is the iso-surplus that passes through the price vector p^0 . Along this curve in the region R we may consider p_2 to be a variable

dependent on p_1 because V is strictly convex. The function $f(p_1, p_2) = -\frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_2}}$, which

is continuously differentiable with respect to p_1 and p_2 in R , gives us the slopes of nearby iso-surplus curves used to construct the polygonal approximation of the solution

curve. Also, $\left| \frac{\partial f}{\partial p_1} \right|$ is bounded for $p = (p_1, p_2) \in R$. The absolute error associated with

Euler's approximation at j steps forward from p^0 is bounded by $hC_1 \left(e^{C_2jh} - 1 \right)$ where h is the step size.

4.3.2. An Implication of Proposition 8

Recall that for any given j , $\tilde{p}_1^j = p_1^0 + jh$. Consequently (81) yields:

$$\left| \tilde{p}_2^j - p_2^j \right| \leq hC_1 \left(e^{C_2(\tilde{p}_1^j - p_1^0)} - 1 \right) \quad (82)$$

as a bound for the absolute error. We now write (82) as:

$$\left| \tilde{p}_2^j - p_2^j \right| \leq hC_1 M \quad (83)$$

where $M = e^{C_2(\tilde{p}_1^j - p_1^0)} - 1$. Now let \tilde{p}^j represent a fixed p^A on the iso-surplus through p^0 . Reducing the step size h does not affect M . However, the value of j that identifies \tilde{p}_1^j with p_1^A is increased. Reducing the step size requires more steps to get from p_1^0 to p_1^A . At any rate, (83) tells us that the error associated with approximating the solution curve at p^A by the polygon decreases as $O(h)$ (i.e., the error bound is proportional to h). Hence we can make it as small as desired. This, of course, is a well known property of Euler's method.

4.3.3. Corollary to Proposition 8

Assume a consumer surplus function $V(p)$ satisfying the hypothesis of Proposition 8. Also define $V(p^*)^- = \{p \mid V(p) < V(p^*)\}$ and $A = \{p \mid p^* \cdot q(p) \leq p \cdot q(p)\}$. The firm under Laspeyres price cap regulation may attain p^* starting from any price vector $p^0 \in V(p^*)^- - A$ within a finite number of time intervals.

Proof of Corollary to Proposition 8:

Let $p^0 \in V(p^*)^- - A$. Also let $p^A = (p_1^A, p_2^A)$ be the price vector satisfying $p_1^A = p_1^*$ and $V(p^A) = V(p^0)$. Therefore, p^A is the price vector on the iso-surplus curve through p^0 that is directly above p^* ($V(p^A) = V(p^0) < V(p^*)$). Observe that $p^A \in A$.

Beginning at p^0 , generate the sequence of price vectors p^1, p^2, \dots, p^t by use of (80). Each p^1, p^2, \dots, p^t may be considered to have been produced by the Laspeyres

price cap scheme starting at the initial price vector p^0 . To see this, observe that

$(p^t - p^{t-1}) \cdot \nabla V(p^{t-1}) = 0$ for all t . An iteration of Euler's method is a Laspeyres

movement. Let $p(p^A; h)$ represent the price vector immediately below p^A on the

polygonal approximation resulting from a step size of h . The vertical distance between

this point and p^A is the truncation error $E(p^A; h)$ (see figure 23). Now since Euler's

method is $O(h)$, there exists an $h > 0$ sufficiently small making $E(p^A; h) < |p_2^A - p_2^*|$.

Selecting such a value for h forces $p(p^A; h) \in A$ when p^A is in A because A is convex.

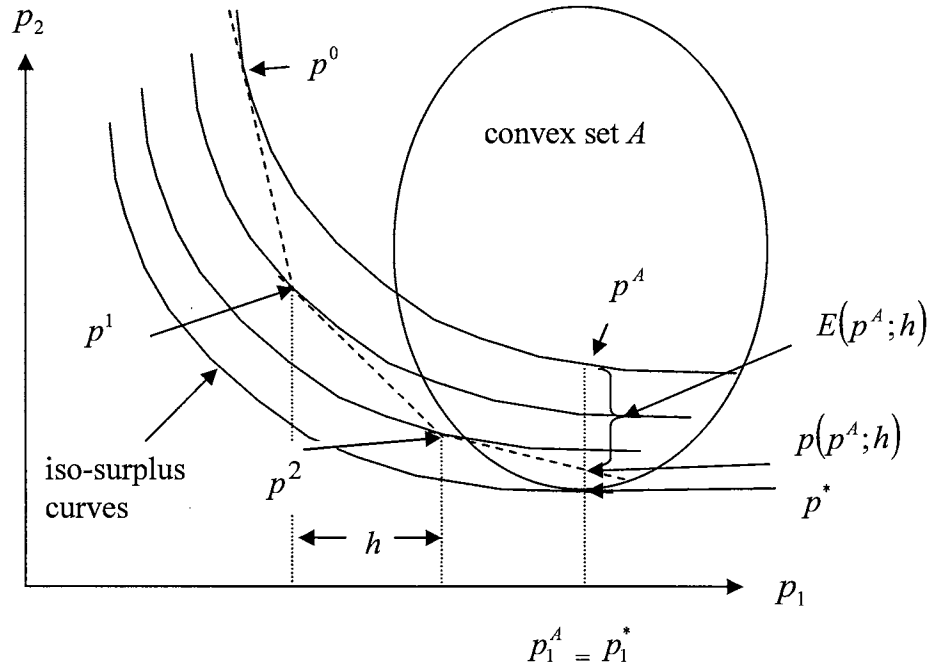


Figure 23. For sufficiently small h , Euler's polygonal approximation of the iso-surplus curve through p^0 and p^A passes between p^* and p^A .

4.3.4. An Example of Strategic Behavior Under Laspeyres Attaining p^* as a Steady State Within a Finite Number of Periods

When given linear demands $q_i(p_i) = a_i p_i + b_i$ and constant marginal costs c_i for $i = 1, 2$, the hypothesis of Proposition 8 are satisfied by the associated consumer surplus function, and the iteration formula (80) takes the form:

$$p^t = (p_1^t, p_2^t) = \left(p_1^{t-1} + h, p_2^{t-1} - h \frac{a_1 p_1^{t-1} + b_1}{a_2 p_2^{t-1} + b_2} \right). \quad (84)$$

To illustrate Euler's method here, consider again the linear demand functions $q_1 = -5p_1 + 20$ and $q_2 = -6p_2 + 40$, but now under Laspeyres price cap regulation. For

cost function $C = q_1 + 3q_2 + 16.1369$ we have a global profit maximum at

$p^* = (2.5000, 4.833\bar{3})$. Now the price vector $p^0 = (1.742, 5.875)$ belongs to the set

$V(p^*) - A$. We demonstrate that the firm may be able to move from p^0 to p^* in a finite number of periods.

Let p^A be the price vector directly above p^* that is on the same iso-surplus curve as p^0 . So $p^A = (2.5000, 4.9345)$. Since demand functions are linear in prices, and marginal costs are constant, (84) should give a sequence of feasible price vectors that enters the convex set A provided a small enough value for h is chosen. It turns out that $h = 0.758$ is the largest such value that will allow for a price vector p^t in the sequence satisfying both $p_1^t = p_1^* = p_1^A$ and $p_2^* \leq p_2^t < p_2^A$. With this value for h , five iterations are necessary to produce the desired p^t . A computer generated graph of several iso-surplus curves (including the one through p^* and the one through p^0 and p^A), iso-profit curves, and the approximating polygon appear in figure 24. The polygon intersects the boundary of A at the point $(2.2828, 5.0223)$. Once inside the set A , the firm can assume p^* in the very next period.

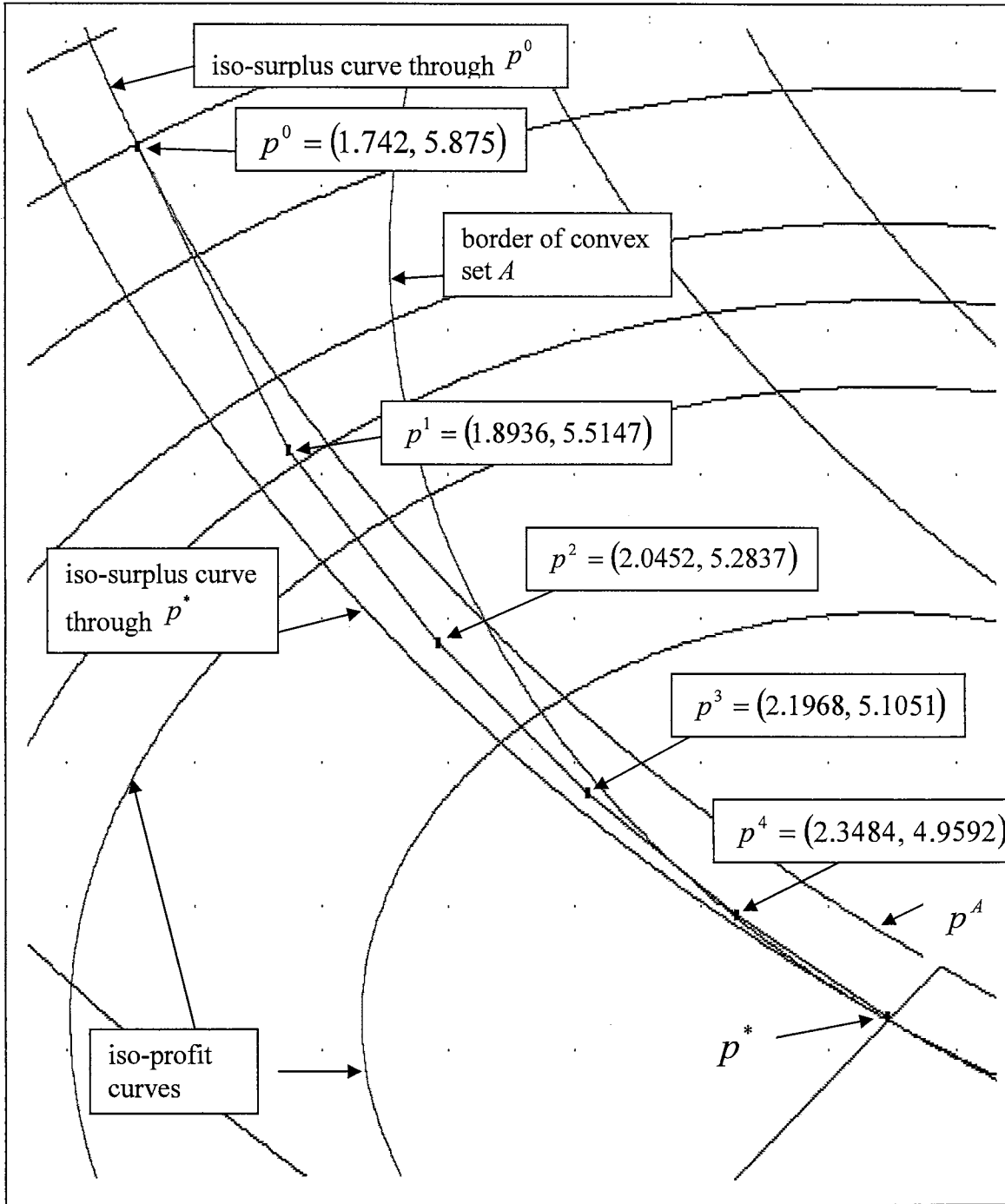


Figure 24. Sequence of Laspeyres price cap vectors generated by Euler's tangent line method for demands $q_1 = -5p_1 + 20$, $q_2 = -6p_2 + 40$, and cost $C = q_1 + 3q_2 + 16.1369$.⁶

⁶ Graph generated by use of the computer algebra system *Derive*.

4.3.5. Strategic Behavior Under Laspeyres Given n Markets

We can extend Euler's method to multiple dimensions to consider the possibility of a firm moving from $p^0 \in V(p^*) - A$ to p^* under Laspeyres in a finite number of steps when given n markets. Assume a consumer surplus function V resulting from independent linear product demand functions of the form $q_i(p_i) = a_i p_i + b_i$. It follows that V is C^2 and strictly convex in R^n . As a result, sufficiently small h_1, h_2, \dots, h_{n-1} exist whereby the iteration formula:

$$p^t = (p_1^t, p_2^t, \dots, p_n^t) = \left(p_1^{t-1} + h_1, p_2^{t-1} + h_2, \dots, p_{n-1}^{t-1} + h_{n-1}, p_n^{t-1} - \left(h_1 \frac{\frac{\partial V}{\partial p_1}}{\frac{\partial V}{\partial p_n}} + h_2 \frac{\frac{\partial V}{\partial p_2}}{\frac{\partial V}{\partial p_n}} + \dots + h_{n-1} \frac{\frac{\partial V}{\partial p_{n-1}}}{\frac{\partial V}{\partial p_n}} \right) \right)_{p^{t-1}}$$

$t = 1, 2, \dots \quad (85)$

provides a sequence of price vectors that eventually enters the set A . To see this, let p^A be the unique price vector on the iso-surplus surface through p^0 satisfying $p_i^A = p_i^*$ for all $i = 1, 2, \dots, n-1$. Consider now the curve φ connecting p^0 with p^A that is geodesic⁷ in the iso-surplus surface containing p^0 and p^A (see figure 25). Along the projection of this curve in the p_i, p_n -plane, which we label φ_i , we have p_n as a convex function of

⁷ A curve is geodesic between p^0 and p^A if it is the shortest of all possible curves connecting these points and is contained within the surface.

p_i . We can approximate φ_i with a planar polygon beginning at $(p_i^0, \varphi_i(p_i^0))$ so that the truncation error associated with estimating $p_n^A = \varphi_i(p_i^A)$ is $O(h)$. The slope field required for Euler's method here is obtained by projecting the n -dimensional slope field for V into the p_i, p_n -plane. Independence between the product demand functions ensures that the projected field is well-defined and gives the slope of $\varphi_i(p_i)$.⁸

Accordingly, the slope field in the p_i, p_n -plane will satisfy

$$\left. \frac{dp_n}{dp_i} \right|_{(p_i, p_n)} = \frac{\frac{\partial V}{\partial p_i}}{\frac{\partial V}{\partial p_n}} \bigg|_{(0, \dots, p_i, 0, \dots, p_n)} \quad . \text{ If in every } p_i, p_n\text{-plane } \left| \frac{d^2 p_n}{dp_i^2} \right| \text{ is bounded, then}$$

the tangent line method may be applied. For each market i we can select a step size \tilde{h}_i

$$\text{for use in } p^t = (p_i^t, p_n^t) = \left(p_i^{t-1} + \tilde{h}_i, p_n^{t-1} - \tilde{h}_i \frac{\frac{\partial V}{\partial p_i} \big|_{(p_i^{t-1}, p_n^{t-1})}}{\frac{\partial V}{\partial p_n} \big|_{(p_i^{t-1}, p_n^{t-1})}} \right) \text{ so that the truncation}$$

error $E_i((p_i^A, p_n^A); \tilde{h}_i)$ of the polygon in the p_i, p_n -plane is less than $|p_n^A - p_n^*|$. Finally,

$$\text{letting } M = \text{Max} \left\{ \frac{|p_1^A - p_1^0|}{\tilde{h}_1}, \frac{|p_2^A - p_2^0|}{\tilde{h}_2}, \dots, \frac{|p_{n-1}^A - p_{n-1}^0|}{\tilde{h}_{n-1}} \right\} \text{ and subsequently defining}$$

$$h_i = \frac{|p_i^A - p_i^0|}{M} \text{ for all } i = 1, 2, \dots, n-1 \text{ will estimate the geodesic curve by a polygon}$$

⁸ Independence is a reasonable assumption in many cases. The example offered by Brennan (1989) of long-distance telephone illustrates. The demand for calling city A from city B is not likely to be too sensitive to the demand for calling city C. Brennan claims that where demands are interdependent, the elasticities can be replaced with "superelasticities" (Brock 1983) that embody zero cross-elasticities.

whose projection in any p_j, p_i -plane is the line connecting the projection of p^0 with the projection of p^A . Moreover, the resulting p^t generated by (85) after M iterations will have a truncation error at p^A less than $|p_n^A - p_n^*|$.

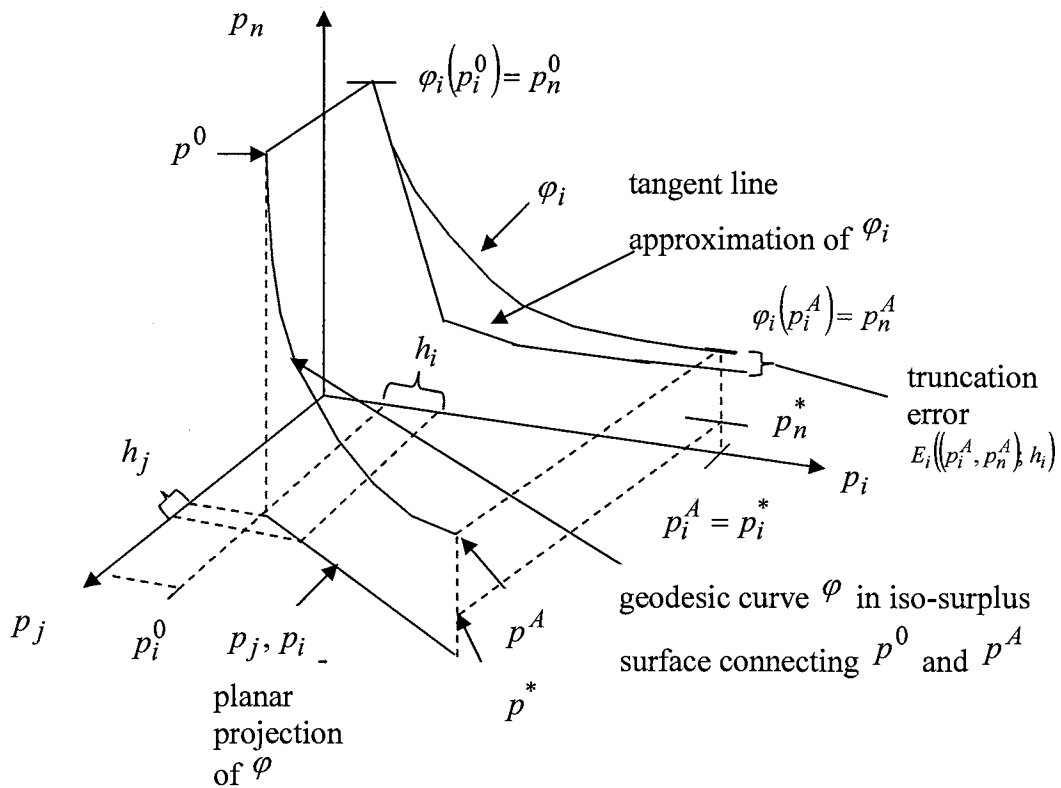


Figure 25. Tangent line approximation of p_i, p_n -planar projection (φ_i) of geodesic curve.

We can verify that (85) does in fact give a Laspeyres movement in n -space by

noting that Roy's Identity implies $\frac{\partial V / \partial p_i}{\partial V / \partial p_n} = \frac{q_i}{q_n}$ during any regulatory period. From

(85) then we obtain:

$$p_n^t = p_n^{t-1} - \left(h_1 \frac{q_1^{t-1}}{q_n^{t-1}} + h_2 \frac{q_2^{t-1}}{q_n^{t-1}} + \dots + h_{n-1} \frac{q_{n-1}^{t-1}}{q_n^{t-1}} \right) \quad (86)$$

thus giving:

$$\left(h_1 \frac{q_1^{t-1}}{q_n^{t-1}} + h_2 \frac{q_2^{t-1}}{q_n^{t-1}} + \dots + h_{n-1} \frac{q_{n-1}^{t-1}}{q_n^{t-1}} \right) + p_n^t - p_n^{t-1} = 0 \quad (87)$$

which implies:

$$\left(h_1, h_2, \dots, h_{n-1}, p_n^t - p_n^{t-1} \right) \cdot \left(\frac{q_1^{t-1}}{q_n^{t-1}}, \frac{q_2^{t-1}}{q_n^{t-1}}, \dots, \frac{q_{n-1}^{t-1}}{q_n^{t-1}}, 1 \right) = 0 \quad (88)$$

and thus:

$$\left(p_1^t - p_1^{t-1}, p_2^t - p_2^{t-1}, \dots, p_n^t - p_n^{t-1} \right) \cdot \left(\frac{q_1^{t-1}}{q_n^{t-1}}, \frac{q_2^{t-1}}{q_n^{t-1}}, \dots, \frac{q_{n-1}^{t-1}}{q_n^{t-1}}, 1 \right) = 0. \quad (89)$$

So that finally:

$$\begin{aligned} & \left(p_1^t - p_1^{t-1}, p_2^t - p_2^{t-1}, \dots, p_n^t - p_n^{t-1} \right) \cdot \left(q_1^{t-1}, q_2^{t-1}, \dots, q_{n-1}^{t-1}, q_n^t \right) \\ &= \left(p^t - p^{t-1} \right) \cdot q^{t-1} = 0. \end{aligned} \quad (90)$$

It is this author's conviction that an approximation scheme similar to that of (85) applies even when independence between demands cannot be assumed. The geometric

problem of estimating a geodesic curve within a convex iso-surplus surface by a polygon remains.

The firm may actually favor the incremental approach for modifying its prices as detailed above because of the potential for increasing the present value of its profit stream over the anticipated periods of the regulatory regime. In a similar vein, Sappington (1980) observes that for a one-product firm expecting imposition of V-F regulation, a natural strategy of forgoing short-term profits over multiple regulatory periods by expending resources on unproductive inputs (i.e., waste) with an aim to relaxing the regulatory constraint in subsequent periods will likewise produce such behavior. More precisely, he demonstrates that with higher rates of discounting, the V-F regulated firm should choose to waste over a greater number of periods. In the extreme case when the discount factor β equals 1.00, the firm will actually elect to waste over all periods of the regime. As V-F regulation is quite similar in form to Laspeyres in that the period t constraint line is simply the line tangent to the iso-surplus curve passing through the previous period's price vector, it seems reasonable that Sappington's logic should apply to the Laspeyres regulated firm. It is only because the firm values future profits that it will sacrifice present profits in the manner prescribed by (85). Conceivably, a Laspeyres regulated firm beginning at a non-global profit-maximizing price vector in $V(p^*)^- - A$ would decide to depart any polygonal path leading to p^* in the face of a sufficiently small value for β . Alternatively, if the firm begins in $V(p^*)^+$, β is very close to 1.00, and many regulatory periods are anticipated, then while p^* is unattainable the firm may yet move on very short linear segments that together comprise a nearly smooth path

which more closely approximates the iso-surplus curve through p^0 . Not surprisingly, the discount factor β has direct implications on Laspeyres regulation performance for profit, consumer surplus, and social welfare.

A potential lesson for regulators exists here. Regulators should beware of small Laspeyres movements. Although to protect consumers regulators often place upper limits on the size of allowed price changes under a price cap to protect consumers from dramatic price swings,⁹ it nevertheless follows from the discussion here that there may be good rationale for forbidding too many small price changes as well. A firm that moves from period to period in such a way that it remains close to the iso-surplus curve passing through the initial price vector p^0 essentially over time appropriates almost all of the welfare increase to itself as profits. If the firm may select the step size as small as it wishes, then it can effectively place an arbitrarily low upper bound on the increase in consumer surplus.

4.4. Myopic Behavior Under Laspeyres Price Cap Regulation

The profit-maximizing firm in period t of Laspeyres regulation will move from price vector p^{t-1} to price vector p^t satisfying:

$$\nabla V(p^{t-1}) = \gamma^t \nabla \pi(p^t) \quad (91)$$

$$\nabla V(p^{t-1}) \cdot (p^t - p^{t-1}) = 0. \quad (92)$$

⁹ Typically, pricing bands restrict prices in each market to deviate in any period by no more than a certain percentage from their levels at the beginning of the period. See Sappington's article in Cave, Majumdar, and Vogelsang (2002) for a discussion on pricing bands.

For demand vectors $q_i(p_i) = a_i p_i + b_i$, and marginal costs c_i for $i = 1, 2$, (91) and (92)

imply:

$$\frac{a_1 p_1^{t-1} + b_1}{2a_1 p_1^t + b_1 - a_1 c_1} = \frac{a_2 p_2^{t-1} + b_2}{2a_2 p_2^t + b_2 - a_2 c_2} \quad (93)$$

$$(a_1 p_1^{t-1} + b_1)(p_1^t - p_1^{t-1}) + (a_2 p_2^{t-1} + b_2)(p_2^t - p_2^{t-1}) = 0 \quad (94)$$

Now under linear demands and constant marginal costs, if the firm initially prices at $p^0 \in V(p^*)^- - A$ and behaves myopically in the first period of regulation, it will not be able to attain p^* in any subsequent period of regulation. In other words, to attain p^* from $p^0 \in V(p^*)^- - A$ the firm must behave strategically beginning at the very first opportunity to do so. This follows from the convexity of the iso-surplus curves. The solution $p^t = (p_1^t, p_2^t)$ to (91) and (92) where $p^{t-1} \in V(p^*)^- - A$ must be in the set $V(p^*)^+$ because $\nabla V(p^{t-1}) \cdot (p^* - p^{t-1}) < 0$. For linear demands and constant marginal costs, strategic behavior following myopic behavior may lead the firm to a price vector corresponding to a higher profit than that resulting from consistently myopic behavior, but this price vector cannot be p^* . If p^* is unattainable in period 1, then myopic behavior during any subsequent period precludes eventual attainment of p^* . This presents an important difference between Average-Revenue-Lagged and Laspeyres regulation, for with ARL we have seen that even consistently myopic behavior performed over multiple periods can inevitably lead to p^* (see Proposition 3).

4.5. Firm Behavior In Anticipation of Laspeyres Price Cap Regulation in the Face of a Positive Discount Factor

Law (1997) demonstrates that the convex set A described previously is actually a limiting case for a family of convex sets applicable to a monopolist's pricing behavior in a simple two-period model in which the firm correctly anticipates that it will be subject to Laspeyres price-cap regulation in the second period. Strategic pricing occurs in the period before the cap is imposed when the firm is unregulated. For the two-market case with linear, independent, time-invariant demands and constant marginal costs, the profit-maximizing firm will select period 1 prices satisfying:

$$(b_1 + a_1 p_1^1)(p_1^* - \delta p_1^1) + (b_2 + a_2 p_2^1)(p_2^* - \delta p_2^1) = 0 \quad (95)$$

where $\delta = \frac{(1 + L^2 \beta)}{(1 + L\beta)}$, given discount factor β , Laspeyres index reduction cap L (i.e., L

satisfies $\frac{p^t \cdot q(p^{t-1})}{p^{t-1} \cdot q(p^{t-1})} \leq L$) and that $0 < L < \delta < 1$.¹⁰ The set A actually corresponds to

$L = \delta = 1$ in (95), but for smaller values of δ and still smaller corresponding values of L , the strategic period 1 price vector set is represented graphically by elliptically-shaped

closed curves passing through the price vectors $(p_1^* / \delta, p_2^* / \delta)$, $\left(-\frac{b_1}{a_1}, -\frac{b_2}{a_2}\right)$ and

centered at the midpoint between these two price vectors (figure 26). Law verifies that since the period 1 ellipse corresponding to δ is tangent to the iso-surplus curve passing through $(p_1^* / \delta, p_2^* / \delta)$ at this price vector, then certain welfare consequences under

¹⁰ Law obtains this result by maximizing with respect to prices a Lagrangian expression involving discounted profits (objective function) subject to the Laspeyres constraint.

Laspeyres necessarily follow from the firm's ability to adjust price relatives and period 1 revenue shares for the markets. In contrast to optimal pricing under Uniform Regulation (UR) which requires $p_i^2 \leq Lp_i^1$ and therefore provides a natural benchmark of comparison with no opportunities for strategic manipulation, strategic manipulation of market weights under Laspeyres with identical index reduction cap L lowers consumer surplus (at least in period 1) and raises producer welfare. The potential for consumer surplus in period 2 to be lower under Laspeyres than under UR derives from the fact that the necessary condition for optimum period 2 prices:

$$a_1(p_1^* - p_1^2)(p_1^2 - Lp_1^*/\delta) + a_2(p_2^* - p_2^2)(p_2^2 - Lp_2^*/\delta) = 0 \quad (96)$$

and the iso-surplus curve through the period 2 UR prices have unequal slopes at the period 2 UR prices where they intersect. Finally, social welfare may be lower under Laspeyres than under UR.

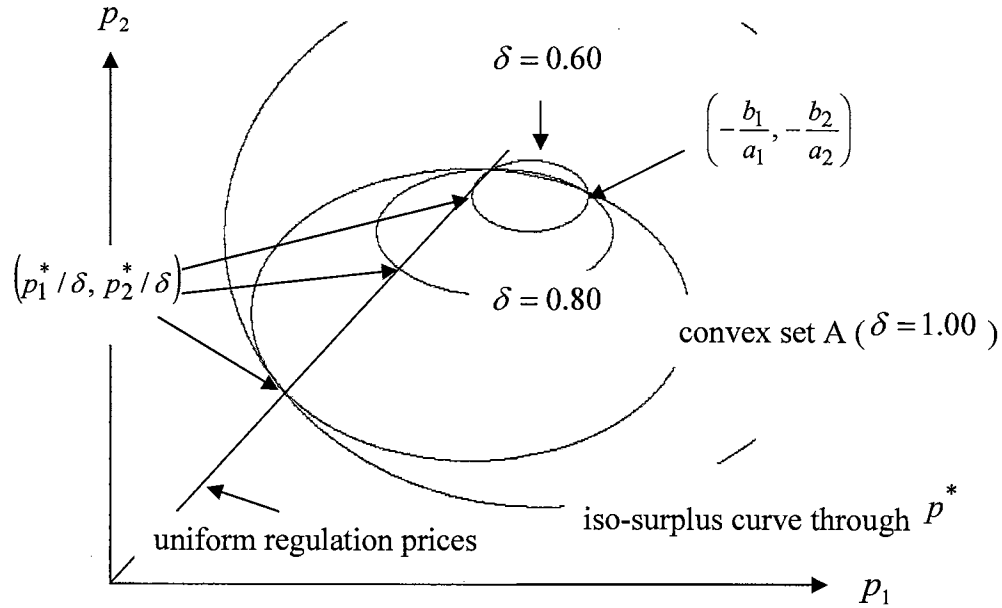


Figure 26. Optimum period 1 price vector sets when Laspeyres price cap regulation is correctly anticipated for period 2.

4.6. Comparison of Average-Revenue Lagged to Laspeyres Price Cap Regulation

We have discussed the possibility of strategic behavior leading to the global profit maximum for both ARL and L regulation when product demands are linear and marginal costs are constant (provided $\bar{p}^* \leq \bar{p}^0$ for ARL, and the initial price vector belongs to the set $V(p^*) - A$ for L). However, while strategic behavior under ARL can be commenced during later periods of regulation, strategic behavior under Laspeyres, to be effective in achieving p^* , must be initiated during the very first period of regulation. Further, strategic behavior under Laspeyres may necessarily involve several regulatory periods, whereas strategic behavior under ARL need involve only two periods of regulation. If the firm can perform strategically only under the guise that it is acting myopically, then of course Laspeyres is a superior regulatory scheme because the regulator has a greater

opportunity to scrutinize the behavior of the firm. Under both regimes, marginal cost values reported by the firm in any period constrain the firm in what price vectors it can select during subsequent periods. If several periods are required before the firm can achieve an ulterior objective, then the firm may not be able to pull off its strategy simply because the regulator becomes wise to the firm's intentions.

Additionally, we have observed how consistently myopic behavior under ARL can inevitably lead to the global profit maximum. Consistently myopic behavior under Laspeyres, however, must lead to a price vector of lower profit and higher consumer surplus than those at p^* , if begun at any price vector outside of the convex set A . Moreover, while consumer surplus under ARL will increase once before decreasing steadily to a value less than that associated with the initial price vector; consumer surplus, firm profit, and consequently social welfare are monotone increasing throughout under myopic behavior within Laspeyres. As we have considered it necessary to suggest corrective mechanisms to interpose within ARL for the purpose of ensuring against erosion of welfare (see Sections 3.2.1. and 3.2.2.), no such mechanisms are needed in Laspeyres regulation. Table 2 presents a summary of these results. For these reasons and others adduced in Chapters 2 and 3, it appears that Laspeyres is superior to ARL as a regulatory scheme designed for furthering the typical objectives of public oversight authorities.

A comparison of some properties of Average-Revenue-Lagged and Laspeyres regulation for linear product demands and constant marginal costs.		
	Average-Revenue-Lagged	Laspeyres
Strategic Behavior	Requires only two periods to achieve the global profit maximum p^* if $\bar{p}^* \leq \bar{p}^0$. Can be commenced in any period.	May require many periods. Must be commenced in period 1 to achieve the global profit maximum p^* .
Myopic Behavior	Can still lead to the global profit maximum if $\bar{p}^* \leq \bar{p}^0$.	Cannot lead to the global profit maximum if begun outside of the convex set A.
Consumer Surplus	Increases in the first period then decreases below that for p^0 under strategic and consistently myopic behavior	Increases monotonically under strategic and consistently myopic behavior.
Welfare	Can decrease over time. Policy recommendations necessary.	Increases monotonically under strategic and consistently myopic behavior.

Table 2

BIBLIOGRAPHY

- Acton, J. P., and I. Vogelsang. 1989. "Introduction," to Price Caps Symposium, *Rand Journal of Economics* 20: 369-372.
- Averch, H., and L. L. Johnson. 1962. "Behavior of the Firm Under Regulatory Constraint." *American Economic Review* 52: 1053-1069.
- Baumol, W. J., and D. Bradford. 1970. "Optimal Departures From Marginal Cost Pricing." *American Economic Review* 60: 265-283.
- Baumol, W. J., D. Fischer, and T. ten Raa. 1979. "The Price-Iso Return Locus and Rational Rate Regulation." *Bell Journal of Economics* 10: 644-660.
- Beesley, M. E., and S.C. Littlechild. 1989. "The Regulation of Privatized Monopolies in the United Kingdom." *Rand Journal of Economics* 20: 454-472.
- Bradley, I., and C. Price. 1988. "The Economic Regulation of Private Industries by Price Constraints." *Journal of Industrial Economics* 37: 99-106.
- Braeutigam, R., and J. C. Panzar. 1993. "Effects of the Change from Rate-of-Return to Price Cap Regulation." *American Economic Review Papers and Proceedings* May: 191-198.
- Brennan, T. J. 1989. "Regulating by Capping Prices." *Journal of Regulatory Economics* 1: 133-148.
- Cabral, L., and M. Riordan. 1989. "Incentives for Cost Reduction Under Price Cap Regulation." *Journal of Regulatory Economics* 1: 93-102.
- Cowan, S. 1997. "Price Cap Regulation and Inefficiency in Relative Pricing." *Journal of Regulatory Economics* 12: 53-70.
- Currier, K. M. 2005. "Strategic Firm Behavior Under Average-Revenue-Lagged Regulation." *Journal of Regulatory Economics* 27: 67-79.
- Currier, K. M. 2005. "Existence and Uniqueness of Equilibrium Under Average-Revenue-Lagged Regulation." (Oklahoma State University working paper).
- Faulhaber, G., and S. Levinson. 1981. "Subsidy-Free Prices and Anonymous Equity." *The American Economic Review* 71: 1083-1091.
- Finsinger, J., and I. Vogelsang. 1985. "Strategic Management Under Reward Structures in a Planned Economy." *The Quarterly Journal of Economics* 100: 263-269.

- Foreman, R. D. 1995. "Pricing Incentives Under Price Cap Regulation." *Information Economics and Policy* 7: 331-351.
- Fraser, R. 1995. "The Relationship Between the Costs and Prices of a Multi-Product Monopoly: The Role of Price-Cap Reg." *Journal of Regulatory Economics* 8: 23-31.
- Gravelle, H. S. E. 1985. "Reward Structures in a Planned Economy: Some Difficulties." *The Quarterly Journal of Economics* 100: 271-278.
- Law, P. 1997. "Welfare Effects of Pricing in Anticipation of Laspeyres Price-Cap Regulation: An Example." *Bulletin of Economic Research* 491: 17-27.
- Leader, J. 2004. *Numerical Analysis and Scientific Computation*. Boston, MA: Pearson-Addison-Wesley.
- Lehman, D. E., and D. L. Weisman. 2000. "The Political Economy of Price Cap Regulation." *Review of Industrial Organization* 16: 343-356.
- Lewis, T., and D. Sappington. 1989. "Regulatory Options and Price-Cap Regulation." *Rand Journal of Economics* 20: 405-416.
- Liston, C. 1993. "Price-Cap Versus Rate-of-Return Regulation." *Journal of Regulatory Economics* 5: 24-48.
- Mathios, A., and R. Rogers. 1989. "The Impact of Alternative Forms of State Regulation of AT&T on Direct-Dial, Long-Distance Telephone Rates." *Rand Journal of Economics* 20: 437-453.
- Ramsey, F. P. 1927. "A Contribution to the Theory of Taxation." *Economic Journal* 37: 47-61.
- Sappington, D. 1980. "Strategic Firm Behavior Under a Dynamic Regulatory Adjustment Process, *Bell Journal of Economics* 11: 360-372.
- Sappington, D. E. M. 2002. "Price Regulation." In M. Cave, S. Majumdar, and I. Vogelsang (Eds.), *Handbook of Telecommunications Economics*. Amsterdam: Elsevier Publishers.
- Sappington, D. E. M., and D. S. Sibley. 1992. "Strategic Nonlinear Pricing Under Price Cap Regulation." *RAND Journal of Economics* 23: 1-19.
- Schmalensee, R. 1989. "Good Regulatory Regimes." *Rand Journal of Economics* 20: 392-404.

- Sibley, D. 1989. "Asymmetric Information, Incentives and Price-Cap Regulation." *Rand Journal of Economics* 20: 392-404.
- Tam, M. S. 1981. "Reward Structures in a Planned Economy: The Problem of Incentives and Efficient Allocation of Resources." *The Quarterly Journal of Economics* 96: 111-128.
- Train, K. E. 1991. *Optimal Regulation: The Optimal Theory of Natural Monopoly*. Cambridge, MA: MIT Press.
- Vogelsang, I. 2002. "Incentive Regulation and Competition in Public Utility Markets: A 20-Year Perspective." *Journal of Regulatory Economics* 22: 1, 5-27.
- Vogelsang, I., and J. Finsinger. 1979. "A Regulatory Adjustment Process for Optimal Pricing by Multiproduct Monopoly Firms" *Bell Journal of Economics* 10: 157-171.

APPENDIXES

APPENDIX A

The standard form equation for an ellipse centered at the point (h, k) is

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ where a is half the length of the horizontal axis and b is half the

length of the vertical axis. The eccentricity of an ellipse is given by the quotient $e = c/a$

where c (the distance from the center to either focus) satisfies $c = \sqrt{|a^2 - b^2|}$. Two

ellipses whose equations are identical except for the constant term are therefore of the same eccentricity.

APPENDIX B

The components of the price vector (\bar{p}^*, \bar{p}^*) can be expressed in terms of the parameters of the demand and cost functions (i.e., a_1, a_2, b_1, b_2, c_1 , and c_2). The average revenue for p^* is given by:

$$\bar{p}^* = \frac{p_1^* q_1^* + p_2^* q_2^*}{q_1^* + q_2^*}. \quad (97)$$

So since $q_i = a_i p_i + b_i, i = 1, 2$, we have:

$$\bar{p}^* = \frac{p_1^* (a_1 p_1^* + b_1) + p_2^* (a_2 p_2^* + b_2)}{(a_1 p_1^* + b_1) + (a_2 p_2^* + b_2)} \quad (98)$$

which from (9) yields:

$$\begin{aligned} \bar{p}^* &= \frac{\left(-\frac{b_1 - a_1 c_1}{2a_1} \right) \left[a_1 \left(-\frac{b_1 - a_1 c_1}{2a_1} \right) + b_1 \right] + \left(-\frac{b_2 - a_2 c_2}{2a_2} \right) \left[a_2 \left(-\frac{b_2 - a_2 c_2}{2a_2} \right) + b_2 \right]}{\left[a_1 \left(-\frac{b_1 - a_1 c_1}{2a_1} \right) + b_1 \right] + \left[a_2 \left(-\frac{b_2 - a_2 c_2}{2a_2} \right) + b_2 \right]} \\ &= \frac{\frac{a_1^2 c_1^2 - b_1^2}{2a_1} + \frac{a_2^2 c_2^2 - b_2^2}{2a_2}}{a_1 c_1 + b_1 + a_2 c_2 + b_2}. \end{aligned} \quad (99)$$

APPENDIX C

This property can be easily verified by noting that an ellipse is concentric and of identical eccentricity and orientation with another ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ only if

its equation can be written in the form $\frac{(x-h)^2}{(ta)^2} + \frac{(y-k)^2}{(tb)^2} = 1$. Translating both so that

their centers are $(h, k) = (0, 0)$ gives the equations $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{(ta)^2} + \frac{y^2}{(tb)^2} = 1$. So

if the ray $y = mx$ intersects $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) then it intersects

$\frac{x^2}{(ta)^2} + \frac{y^2}{(tb)^2} = 1$ at the point (tx_1, ty_1) . Now the slope of a line tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at

(x_1, y_1) is $\left. \frac{dy}{dx} \right|_{(x_1, y_1)} = -\frac{b^2 x_1}{a^2 y_1}$ and the slope of a line tangent to $\frac{x^2}{(ta)^2} + \frac{y^2}{(tb)^2} = 1$ at

(tx_1, tx_2) is $\left. \frac{dy}{dx} \right|_{(tx_1, tx_2)} = -\frac{b^2 tx_1}{a^2 ty_1} = -\frac{b^2 x_1}{a^2 y_1}$.

APPENDIX D

Since under ARL the period t constraint must be perpendicular to $\nabla V(p^{t-1})$, the firm operating with the linear demands $q_i(p_i) = a_i p_i + b_i$ and constant marginal costs c_i may obtain the period t profit maximizing price vector p^t by solving:

$$\begin{cases} 2a_1 p_1^t + b_1 - a_1 c_1 = \lambda^t (a_1 p_1^{t-1} + b_1) \\ 2a_2 p_2^t + b_2 - a_2 c_2 = \lambda^t (a_2 p_2^{t-1} + b_2) \\ (\bar{p}^0 - p_1^t)(a_1 p_1^{t-1} + b_1) + (\bar{p}^0 - p_2^t)(a_2 p_2^{t-1} + b_2) = 0 \end{cases} \quad (100)$$

and thus obtaining:

$$\begin{aligned} p_1^t = & \frac{2(a_1)^2 a_2 (p_1^{t-1})^2 \bar{p}^0 + a_1 p_1^{t-1} (a_2 p_2^{t-1} (2a_2 \bar{p}^0 - c_2 a_2 + b_2) + 2a_2 \bar{p}^0 (2b_1 + b_2) - b_2 (c_2 a_2 - b_2)) + (a_2)^2 (p_2^{t-1})^2 (c_1 a_1 - b_1) + a_2 p_2^{t-1} (2a_2 b_1 \bar{p}^0 + 2c_1 a_1 b_2 - b_1 (c_2 a_2 + b_2)) + 2a_2 (a_1 a_2 b_1 p_1^{t-1} + a_1 (a_2)^2 (p_2^{t-1})^2 + 2a_1 a_2 b_2 p_2^{t-1} + a_1 (b_2)^2 + a_2 (b_1)^2)}{2((a_1)^2 a_2 (p_1^{t-1})^2 + b_1 \bar{p}^0 (b_1 + b_2) + b_2 (c_1 a_1 b_2 - c_2 a_2 b_1))} \\ & \cdot 1 \end{aligned} \quad (101)$$

and

$$p_2^t = \frac{(a_1)^2 (p_1^{t-1})^2 (c_2 a_2 - b_2) + a_1 p_1^{t-1} (a_2 p_2^{t-1} (2a_1 \bar{p}^0 - c_1 a_1 + b_1) + 2a_1 b_2 \bar{p}^0 - c_1 a_1 b_2 + b_1 (c_2 a_2 - b_2)) + 2a_1 (a_2)^2 (p_2^{t-1})^2 (c_1 a_1 - b_1) + a_2 p_2^{t-1} (2a_1 b_1 \bar{p}^0 + 2c_1 a_1 b_2 - b_1 (c_2 a_2 + b_2)) + 2a_1 (a_1 a_2 b_1 p_1^{t-1} + a_1 (a_2)^2 (p_2^{t-1})^2 + 2a_1 a_2 b_2 p_2^{t-1} + a_1 (b_2)^2 + a_2 (b_1)^2)}{2((a_1)^2 a_2 (p_1^{t-1})^2 + b_1 \bar{p}^0 (b_1 + b_2) + b_2 (c_1 a_1 b_2 - c_2 a_2 b_1))}$$

$$\frac{2c_2a_2 - b_2)) + 2a_1(a_2)^2(p_2^{t-1})^2\bar{p}^0 + a_2p_2^{t-1}(2a_1\bar{p}^0(b_1 + 2b_2) - b_1(c_1a_1 - b_1)) + 2a_1}{2a_1a_2b_1p_1^{t-1} + a_1(a_2)^2(p_2^{t-1})^2 + 2a_1a_2b_2p_2^{t-1} + a_1(b_2)^2 + a_2(b_1)^2} \sim$$

$$\frac{b_2\bar{p}^0(b_1 + b_2) - b_1(c_1a_1b_2 - c_2a_2b_1)}{.1} \quad (102)$$

APPENDIX E

Euler's Method:

For the initial value problem:

$$y' = f(x, y)$$

$$y(x_0) = y_0$$

the iterates:

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_2 = y_1 + hf(x_1, y_1)$$

⋮

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$$

is known as **Euler's Method**. Correspondingly, we have the following theorem as presented by Leader (2004). The proof is contained in his text and in any standard numerical analysis text:

Let $y(x)$ be the solution of $y' = f(x, y)$, $y(x_0) = y_0$. If $f(x, y)$ is continuously differentiable with respect to both its independent variables in some open region R containing (x_0, y_0) , and $|y''(x)|$ is bounded for x within R , then there is an $A > 0$, $B > 0$ for which the iterates y_0, y_1, \dots, y_n generated by Euler's method satisfy

$|y(x_i) - y_i| \leq hA(e^{Bih} - 1)$, provided the iterates and the solution $y(x)$ remain in R .

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