AN IMPROVED METHOD

FOR PREDICTING

INDUCTION MOTOR CHARACTERISTICS

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By

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PREFACE

The induction motor is one of the most useful electric machines in industry today. Since its invention in 1888 by Nikola Telsa, it has constantly been replacing other types of machines, both electric and mechanical, as a means of supplying mechanical power.

Made in its diversified forms, an induction motor can be made to fit almost any torque-speed requirement. In its most common form, the normal-starting-current, normal-starting-torque, squirrel-cage induction motor, it offers such advantages as high efficiency, practically constant speed, extremely simple operation, and small electrical maintenance requirements.

Because of the induction motor's wide use, it is advantageous to both the manufacturer and the consumer to be able to ascertain, as accurately and simply as possible, how a given motor will operate under varying conditions of load. Although an actual load test will supply the desired information, it requires considerable time, effort and equipment to perform such a test.

Instead of making a load test, a device known as the circle diagram is commonly used to predict induction motor characteristics. This circle diagram is essentially a graphical representation of the various currents and voltages in the induction motor equivalent circuit. Its use makes it possible to predict the complete operating characteristics of a given induction motor from the no-load and blocked-rotor tests or from the design data.

Seemingly, this circle diagram for induction motors would be especially advantageous to the motor manufacturers, who must predict the performance characteristics of a multitude of motors. This is proven not to be the case however, by the fact that many manufacturers do not even use the circle diagram. Instead, they perform brake tests upon a given motor to determine its characteristics.

There are two main reasons for this seemingly needless expenditure of time and money. First, the two common methods of making predictions from the circle diagram or the equivalent circuit upon which it is based are not entirely satisfactory and second, even though the circle diagram be accurately solved, the accuracy of the predictions thus made may vary over quite a large range, depending upon the particular motor.

The first method for predicting induction motor characteristics is a mathematical solution of the equivalent circuit. Although a solution obtained by this method is very accurate, the number of complex equations which must be solved for each value of load renders this method impractical for ordinary use.

The second method, which is by far the more common, is a graphical solution of the circle diagram. The larger use of this method may be attributed to its comparative simplicity and speed. There are however, serious disadvantages to this method. First of all, the need for drawing instruments and a large-scale drawing is a disadvantage in itself. In addition, unless the diagram is made excessively large, the measurement of small quantities such as are present in the circle diagram is not only tedious but also likely to introduce considerable error. Added to this are the inherent errors of graphical construction arising from such things as the width of pencil lines and enlarged compass center-holes.

Another objection to both of these methods lies in the fact that several attempts are usually necessary in order to determine the characteristics for a particular value of load. That is, with the common methods of solution, it is not possible to select some value of output and then solve for the characteristics at that output. Usually it is necessary to assume some value of another parameter, say primary current, and then solve for the output and other characteristics at that value of current.

Consequently, there is a definite need for a method of predicting induction motor characteristics which will offer simplicity and speed without sacrificing the desired accuracy. In 1930, an article by Mr. W.I. Branson appeared in the Transactions of the American Institute of Electrical Engineers, which attempted to alleviate this problem somewhat.[1] The method presented, however, was still somewhat complex and required a large number of calculations in addition to the use of about 100 curve sheets. As far as is known, these curve sheets have never been published.

It is the intent of this thesis to offer a method of making predictions from the circle diagram for induction

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1 W.I. Branson, "Induction Motors," Transactions of American Institute of Electrical Engineers, V49 (January 1930), 319-32.

motors that requires far less time and effort than any of the present methods and yet gives results which are easily within the accuracy required by commercial standards. It is the further purpose of this thesis to show why the accuracy of the circle diagram itself varies and to offer at least one means for improving that accuracy.

This thesis is presented with the hope that the material will be of some value especially to those engaged in the manufacture of induction motors but also to anyone who has reason to deal with or study induction motors.

ACKNOWLEDGEMENT

The writer wishes to express his sincere appreciation to Professor C.F. Cameron for his many helpful suggestions and careful scrutiny of this material and to Professor A. Naeter for his detailed reading of the subject matter.

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CHAPTER I

The following is a method for predicting the performance characteristics of an induction motor which embodies most of the advantages of the graphical and mathematical methods but eliminates many of their disadvantages. It is a method based upon an analytical solution of the circle diagram[2] for which most of the equations have been solved in advance and their results plotted in the form of curves. Thus the accuracy of the mathematical solution is approached without the inherent laborious calculations.

ANALYTICAL SOLUTION OF THE CIRCLE DIAGRAM

Since this method for the solution of the circle diagram is based upon a new analytical solution, that solution together with its derivation will be presented first.

In the diagram of Figure 1, V represents the applied voltage per phase and is taken as a reference. OA represents the no-load current which lags the applied voltage by an angle θ_0. OB represents the blocked rotor current lagging the applied voltage by an angle θ_B. These values may either be obtained by tests or from the motor design data.

The equation of the line AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \qquad (1)$$

or

$$y - y_1 = m(x - x_1)$$

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[2] Paper by Professor C.F. Cameron - Oklahoma Agricultural and Mechanical College.

where m equals the slope of the line.

Then $y = mx - mx_1 - y_1$.

Since the coordinates of H are

$$x_3 = \frac{x_1 + x_2}{2}$$

and

$$y_3 = \frac{y_1 + y_2}{2}$$

the equation for HZ is

$$y - y_3 = -\frac{1}{m}(x - x_3)$$

or

$$y = -\frac{x}{m} + \frac{x_3}{m} + y_3. \qquad (2)$$

When $y = y_4$, equation (2) may be solved for x or x_4.

Since $y_1 = y_4$,

$$y_1 = -\frac{x_4}{m} + \frac{x_3}{m} + y_3$$

and

$$x_4 = x_3 + m\,(y_3 - y_1); \qquad (3)$$

but

$$x_3 = \frac{x_1 + x_2}{2}$$

and

$$y_3 = \frac{y_1 + y_2}{2}$$

so

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (4)$$

The coordinates of A and B may be found from the no-load and blocked rotor tests. These values are:

$$x_1 = I_o \cos \theta_o,$$

$$y_1 = I_o \sin \theta_o,$$

$$x_2 = I_B \cos \theta_B,$$

and

$$y_2 = I_B \sin \theta_B.$$

An equation which gives x_4 in terms of the coordinates of A and B may be determined by substituting values of x_3

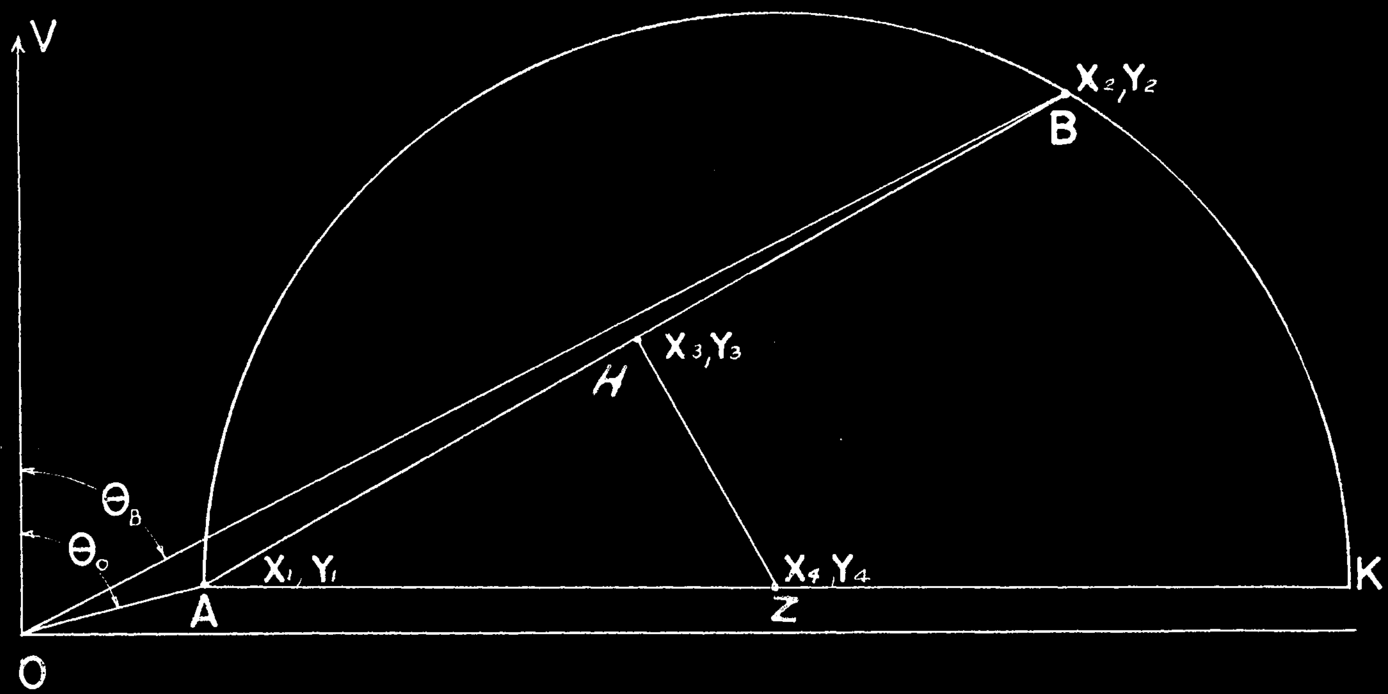

Figure 1. Primary Current locus of an induction motor.

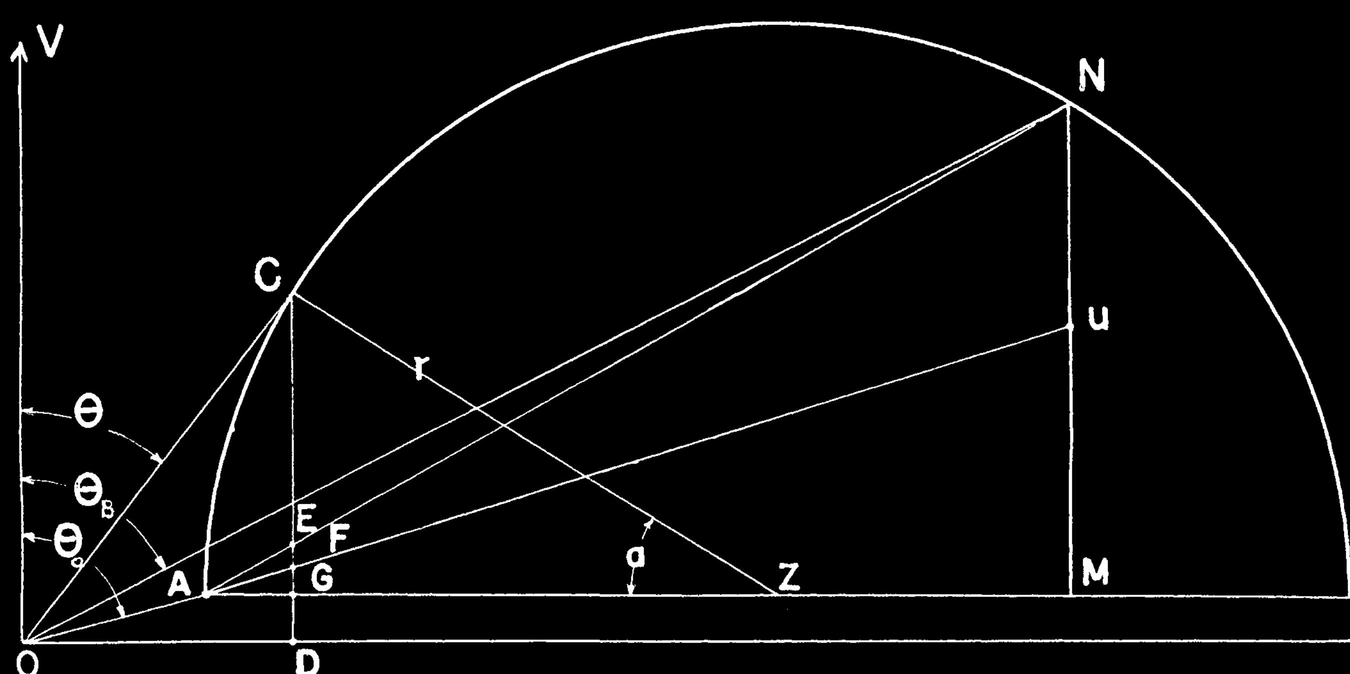

Figure 2. Approximate circle diagram for the prediction of induction motor characteristics.

and y_3 in equation (3). Then

$$x_4 = \frac{x_1 + x_2}{2} + m \left(\frac{y_1 + y_2}{2} - y_1\right). \tag{5}$$

The coordinates of Z are x_4 and y_4 but $y_4 = y_1$ and y_1 is known which locates Z.

From the diagram of Figure 1,

$$r = x_4 - x_1$$

so from equation (5),

$$r = \left(\frac{x_1 + x_2}{2} - x_1\right) + m \left(\frac{y_1 + y_2}{2} - y_1\right)$$

or

$$r = \frac{x_2 - x_1}{2} + m \frac{y_2 - y_1}{2} . \tag{6}$$

Referring to Figure 2, the formulas for the analytical solution of the circle diagram are derived as follows:

$$CG = r \sin a \tag{7}$$

$$AZ = r \tag{8}$$

$$GZ = r \cos a \tag{9}$$

$$AG = AZ - GZ = r - r \cos a$$

$$AG = r (1 - \cos a) \tag{10}$$

$$CD = CG + GD$$

$$CG = r \sin a = (7)$$

$$GD = y_1$$

$$CD = (7) + y_1 \tag{11}$$

$$\frac{MN}{EG} = \frac{AM}{AG}$$

$$EG = AG \frac{MN}{AM} = r (1 - \cos a) \frac{y_2 - y_1}{x_2 - x_1}$$

$$EG = mr (1 - \cos a) \tag{12}$$

$$UM = \frac{(I_B^2 - I_N^2) R_1}{V_P} \tag{13}$$

$$UM = I_{um}$$

$$\frac{FG}{AG} = \frac{UM}{AM} = \frac{I_{um}}{x_2 - x_1} = K_m$$

$$FG = r\ (1 - \cos a)\frac{I_{um}}{x_2 - x_1} = r\ (1 - \cos a)\ K_m \qquad (14)$$

$$EF = EG - FG \qquad (15)$$

$$(15) = (12) - (14)$$

$$CE = CG - EG \qquad (16)$$

$$(16) = (7) - (12)$$

$$FC = EF + CE \qquad (17)$$

$$(17) = (15) + (16)$$

$$OC = \text{Current/phase} \qquad (18)$$

$$= \sqrt{\left[OA \cos\ (90 - \theta_n) + AG\right]^2 + CD^2}$$

$$OA \cos\ (90 - \theta_n) = x_1$$

$$AG = (4)$$

$$CD = (5)$$

$$I_p = \sqrt{\left[x_1 + (10)\right]^2 + (11)^2}\ .$$

Power Input to stator/phase = (19)

CD x volts/phase

$$P_s = (12) \times V_p.$$

Torque, synchronous watts/phase = (20)

FC x volts/phase

$$= (17) \times V_p\ .$$

Slip, percent = (21)

$$\frac{EF}{FC} = \frac{(15)}{(17)}\ .$$

Efficiency, percent = (22)

$$\frac{CE}{CD} = \frac{(16)}{(11)}$$

Power factor, percent = (23)

$$\frac{CD}{OC} = \frac{(11)}{(18)}$$

3 Phase output = 3 x CE x volts/phase (24)

$$= 3 \times (16) \times V/ph.$$

Maximum power output will occur when CE is a maximum

$$CE = CG - EG = (7) - (12)$$

$$= r \sin a - m\, r\, (1 - \cos a).$$

For maximum power,

$$\frac{dP}{da} = 0 = r \cos a - m\, r \sin a$$

and, $r \cos a = m\, r \sin a$

or $\tan a = \frac{1}{m}$.

Maximum torque will occur when FC is a maximum.

$$FC = EF + CE$$

$$= (12) - (14) + (7) - (12) = (7) - (14)$$

$$= r \sin a - r\,(1 - \cos a)\,\frac{I_{um}}{x_2 - x_1}$$

$$= r \sin a - r\,(1 - \cos a)\,K_m$$

$$\text{where } K_m = \frac{I_{um}}{x_2 - x_1}.$$

For maximum torque,

$$\frac{dT}{da} = 0 = r \cos a - K_m \times r \times \sin a$$

or $\tan a = \frac{1}{K_m}$.

DETERMINATION OF DATA FOR CURVES

In order to plot the results of the preceding equations, it is necessary to express the equations in terms of the variables whose values may be determined from the no-load and blocked rotor tests on any motor and the independent variable, (a).

Starting with the equation for power output, the three-phase power output is given by equation (24) as

$$W_{3\phi} = 3 \times (16) \times V/ph.$$

or the watts output per phase

$$= W = (16) \times V/ph.$$

Substituting, $W = r \sin a - m\, r\, (1 - \cos a)$

or $$\frac{W}{Vr} = \sin a - m\,(1 - \cos a)$$

In order to plot the results of this equation, typical values are assumed for m and for each value of m, a complete set of values for (a), 0° to 60°, are substituted in the equation. If the equation is then solved for $\frac{W}{Vr}$ and the results plotted against the corresponding values of (a), a set of curves such as in Figure 3 will result.

Equation (18) gives the input current per phase as

$$I_p = \sqrt{\left[x_1 + (11)\right]^2 + (12)^2}$$

Substituting;[3] $$I_p = r\sqrt{\left[K_r + (1 - \cos a)\right]^2 + \left[\sin a + K_e\right]^2}$$

where $$K_r = \frac{x_1}{r} \text{ and } K_e = \frac{y_1}{r} \qquad (1)$$

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[3] Complete substitutions given in Appendix I.

Then $$\frac{I_p}{r} = \sqrt{\left[K_r + (1 - \cos a)\right]^2 + \left[\sin a + K_e\right]^2}.$$

By assuming typical values for K_r and K_e and then substituting a complete range of values for (a) in the equation, a set of multiple-curve graphs like the one shown in Figure 4 may be derived. It may be seen that Figure 4 gives a set of curves for only one value of K_e. If more accuracy is desired such a set of curves may be plotted for each value of K_e. However, since the variations of K_e have such little effect upon the final value of I and because the value of K_e is fairly constant for most motors, K_e has been assumed to have a constant value of 0.01. The assumption seems justifiable because it has been used even in the solution of the exact circle diagram.[4]

The efficiency as given by equation (16) is

$$\text{Eff} = \frac{(16)}{(11)}.$$

Substituting, $$\text{Eff} = \frac{\sin a - m \cos a}{\sin a + K_e}.$$

Since K_e is assumed to be constant, typical values of m and a complete set of values for (a) for each value of m may be substituted in the equation and the results plotted as in Figure 5.

From equation (23) the power factor is

$$\text{P.F.} = \frac{(11)}{(18)}.$$

Substituting, $$\text{P.F.} = \frac{\sin a - K_e}{\sqrt{\left[K_r + (1 - \cos a)\right]^2 + \left[\sin a - K_e\right]^2}}$$

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[4] Branson, op. cit. 1, p. 324.

In this equation, K_r and (a) are the only variables since K_e has already been assumed to be constant. Consequently, typical values may be substituted for K_r, and for each value, the equation may be solved using various values of (a). The results, when plotted, will give a set of curves such as is shown in Figure 6.

The torque, in synchronous watts per phase, according to equation (20), is

$$T = (17) \times \text{volts/phase}.$$

Substituting, $T = V \ r \sin a - r (1 - \cos a) K_m$

and $$\frac{T}{Vr} = \sin a - K_m (1 - \cos a).$$

Here again, a complete set of values for (a) may be substituted in the equation for each value of K_m used. The results of the solutions of this equation may then be used to plot a set of curves giving a relation between $\frac{T}{Vr}$ and (a).

Equation (21) gives the slip in per cent as

$$S = \frac{(15)}{(17)}.$$

Substituting, $$S = \frac{m (1 - \cos a) - K_m (1 - \cos a)}{\sin a - K_m (1 - \cos a)}$$

In this equation there are three variables; m, K_m and (a). The slip then requires a separate set of curves for each value of K_m used. The procedure for determining the values to use in plotting the curves is as follows. Assume one value of K_m. Then substitute typical values for m and for each value of m used, solve the above equation for each of a complete range of values for (a). The process must then be repeated for each value of K_m that is used. The results of these solutions will produce several sets of curves. The number of values to be substituted for K_m will, of course, depend upon the accuracy required. For ordinary calculations, about four values of K_m (0.05, 0.15, 0.25, 0.35) will give sufficient accuracy.

At first glance, the determination of enough points to accurately plot a complete set of curves appears to be a very laborious and complex process. True, a multitude of calculations are required but the use of mimeographed forms can greatly reduce the complexity and number of the calculations involved. Three such sets of forms were used to calculate the points for the accompanying curves. A sample of each of the forms is given in Appendix II. A close scrutiny of the foregoing equations will reveal that the required calculations are not so numerous as they might at first appear. The reason being that many of the combinations, such as (1 - cos a) and (r sin a), are used over and over again.

The calculations for the accompanying curves were made with the use of a calculating machine thus making possible four-place accuracy. While these curves are plotted for values of (a) from zero to 60 degrees, curves for ordinary calculations, say up to 150% of full load, need only be drawn for values of (a) up to about 30 degrees.

The range of values of K_m, K_r and m, for which these curves were plotted is believed to be sufficient for most calculations. The values used were determined from the test data upon a range of motors from 1½ to 100 H.P.

Use of the Curves

In order to make use of the curves, it is first necessary to determine the values of the constants y_1, m, r, K_m, and K_r from the no-load and blocked-rotor tests on the motor or from the motor design data. In solving for these constants, the following procedure is used.

From the no-load test, determine I_o θ_o

From the blocked-rotor test, determine I_B θ_B

Then $I_o \sin \theta_o = x_1$;

$I_o \cos \theta_o = y_1$,

$I_B \sin \theta_B = x_2$,

and $I_B \cos \theta_B = y_2$.

By equation (4),

$$m = \frac{y_2 - y_1}{x_2 - x_1} .$$

From equation (6),

$$r = \frac{x_2 - x_1}{2} + m \frac{y_2 - y_1}{2} .$$

Equation (13) gives

$$K_m = \frac{(I_B^2 - I_o^2) \times R_1}{\text{Volts/phase} \times (x_2 - x_1)}$$

where R_1 = stator resistance per phase.

Then

$$K_r = \frac{x_1}{r} \text{ by definition.}$$

For the particular output at which the characteristics are desired,

$$\frac{W}{Vr} = \frac{\text{horsepower output x 746}}{\sqrt{3} \text{ x rated voltage x r}}$$

It should be noticed that the foregoing equations need only be solved once for any particular motor as all of the values except $\frac{W}{Vr}$ are constant. $\frac{W}{Vr}$ varies directly as the horsepower output. Thus if its value is first determined for rated load, the values will then change in accordance with the percentage of full load for which the characteristics are to be found.

Once these constants have been solved for, the determination of the motor characteristics is a relatively simple matter. First, from the curves of $\frac{W}{Vr}$ versus (a), the value of (a), corresponding to the chosen output is found. Using this value of (a), and the subsequent curve sheets, values may be determined for efficiency, power factor, $\frac{I}{r}$ (I equals amperes per phase), $\frac{T}{Vr}$ (T equals synchronous watts per phase), and slip. The quantity $(\frac{I}{r})$ must be multiplied by r in order to find the current per phase in amperes. Likewise, $\frac{T}{Vr}$ must be multiplied by the phase voltage and (r) to ascertain the torque output in synchronous watts per phase. Efficiency, power factor, and slip are read directly from the curves.

Example

The data for the following example was taken from a commercial test sheet on a 100 H.P. motor.

H.P. = 100

V = 440 volts

R_s = 0.1024 ohms per phase

P_o = 2400 watts (no-load power)

$$P_B = 180{,}000 \text{ watts}$$

$$I_o = 46.4 \text{ amps}$$

$$I_B = 581.6 \text{ amps}$$

Then

$$\cos \theta_o = \frac{2400}{\sqrt{3} \times 440 \times 46.4} = 0.068$$

and $$\theta_o = 86.1^\circ.$$

Hence, $$x_1 = 46.4 \sin 86.1^\circ = 46.2$$

and $$y_1 = 46.4 \cos 86.1^\circ = 3.16.$$

Also

$$\cos \theta_B = \frac{180{,}000}{\sqrt{3} \times 440 \times 581.6} = 0.416$$

so $$\theta_B = 66^\circ.$$

Then $$x_2 = 581.6 \sin 66^\circ = 531$$

and $$y_2 = 581.6 \cos 66^\circ = 236$$

Next, $$m = \frac{236 - 3.16}{531 - 46.2} = 0.481$$

and $$r = \frac{485}{2} - 0.481 \times \frac{233}{2}$$

$$= 298.5 \text{ by equation (6).}$$

From (13),

$$K_m = \frac{\left[581.6^2 - 46.4^2\right] \times 0.1024}{440/\sqrt{3} \times (531 - 46.2)} = 0.28$$

By definition,

$$K_r = \frac{46}{298.5} = 0.154.$$

Then for full load

$$\frac{W}{Vr} = \frac{100 \times 746}{\sqrt{3} \times 440 \times 298.5}$$

or $$\frac{W}{Vr} = 0.328$$

Using $\frac{W}{Vr} = 0.328$ and $m = 0.481$, from Figure 3, a_1 is found to be 21.2°.

From Figure 5, the efficiency at full load equals 88.5%.

Figure 6 gives the full-load power factor as 0.86.

Using Figure 4, $\frac{I}{r}$ at full load equals 0.430 so the full load current is

$$0.430 \times 298.5 = 128.1 \text{ amps.}$$

According to Figure 7,

$$\frac{T}{Vr} = 0.335$$

so, $T = 0.335 \times \frac{440}{\sqrt{3}} \times 298.5 = 25{,}400$ synchronous watts per phase at full load.

Since $K_m = 0.28$, it is necessary to interpolate between the curves of Figures 10 and 11 for the full-load slip. From Figure 10, the slip equals 4.6% and from Figure 11, the slip equals 2.7%. Then the full-load slip is equal to 4.0%.

It may be noted that there is a possibility of some small error in determining the slip due to the straight-line method of interpolation used. Compared to the errors in determining slip imposed by the circle diagram itself however, this small error is negligible.

Figure 12 gives a comparison of values determined by the use of the curves sheets and the values determined from a brake test on the motor. The brake-test values were supplied by the manufacturer and apply to the same 100 H.P. motor used in the preceding example.

						Taken from
H.P.	122.7	99.3	74.7	49.9	24.6	
I/ph.	158.7	127.2	98.6	74.0	54.6	Curves
	156.6	126.8	99.2	74.8	54.6	Brake test
Efficiency %	86.9	88.5	90.2	90.5	87.2	Curves
	88.8	89.7	90.9	89.2	85.5	Brake test
P.F.	0.872	0.859	0.825	0.735	0.505	Curves
	0.867	0.854	0.811	0.734	0.515	Brake test
Slip %	5.2	3.9	2.8	1.8	0.81	Curves
	4.6	3.6	2.7	1.7	0.82	Brake Test

Figure 12. Comparison of induction motor characteristics as determined from curves with the characteristics determined by brake tests.

Since the method described here will give an accurate solution of the approximate circle diagram, it seemed expedient to make a study of the accuracy of the approximate circle diagram and to determine, if possible, any practical means for improving that accuracy. The results of this study, together with a means for improving the accuracy of the approximate circle diagram, is given in the following chapter.

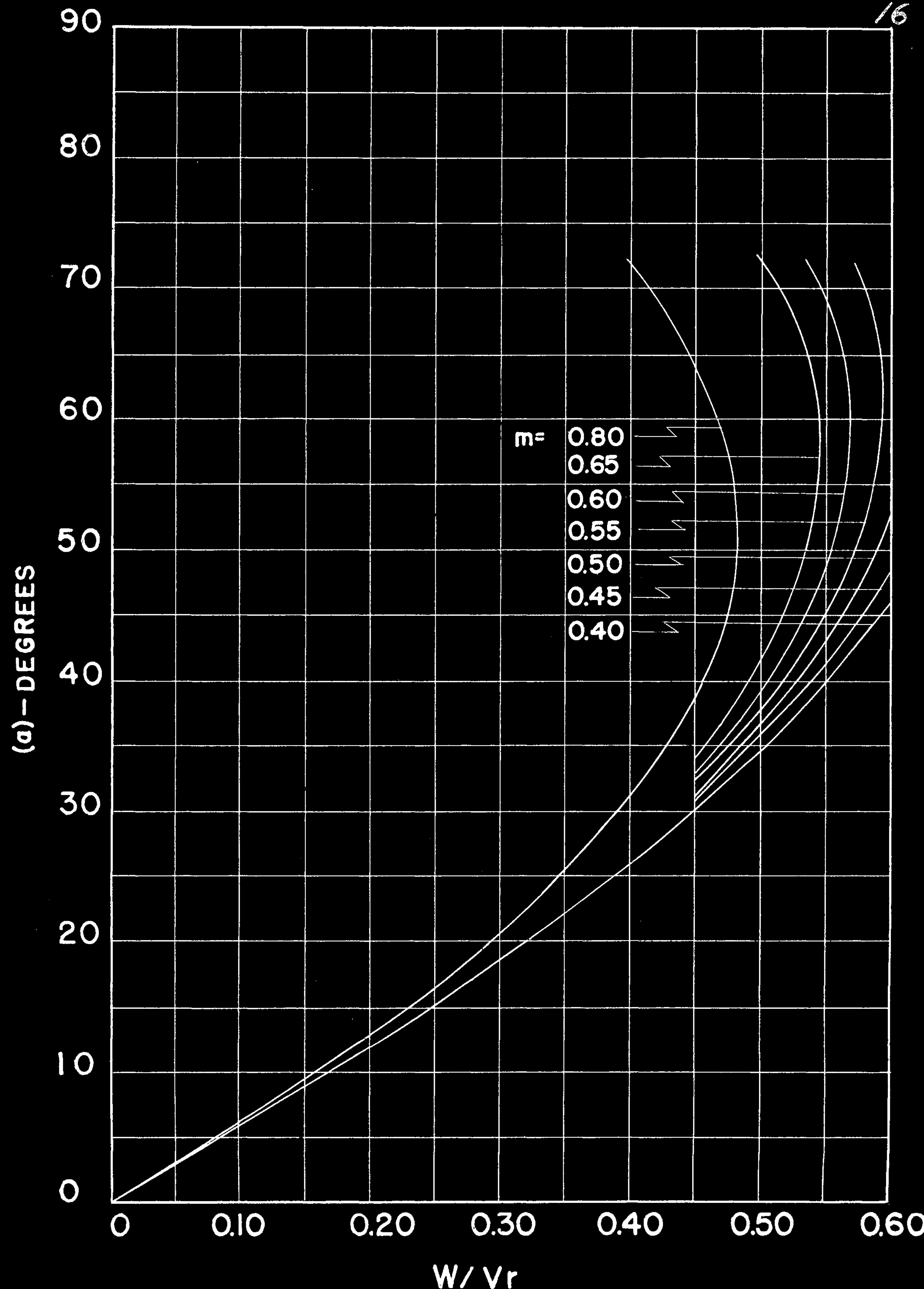

Figure 3. Variations of the angle (a) with power output.

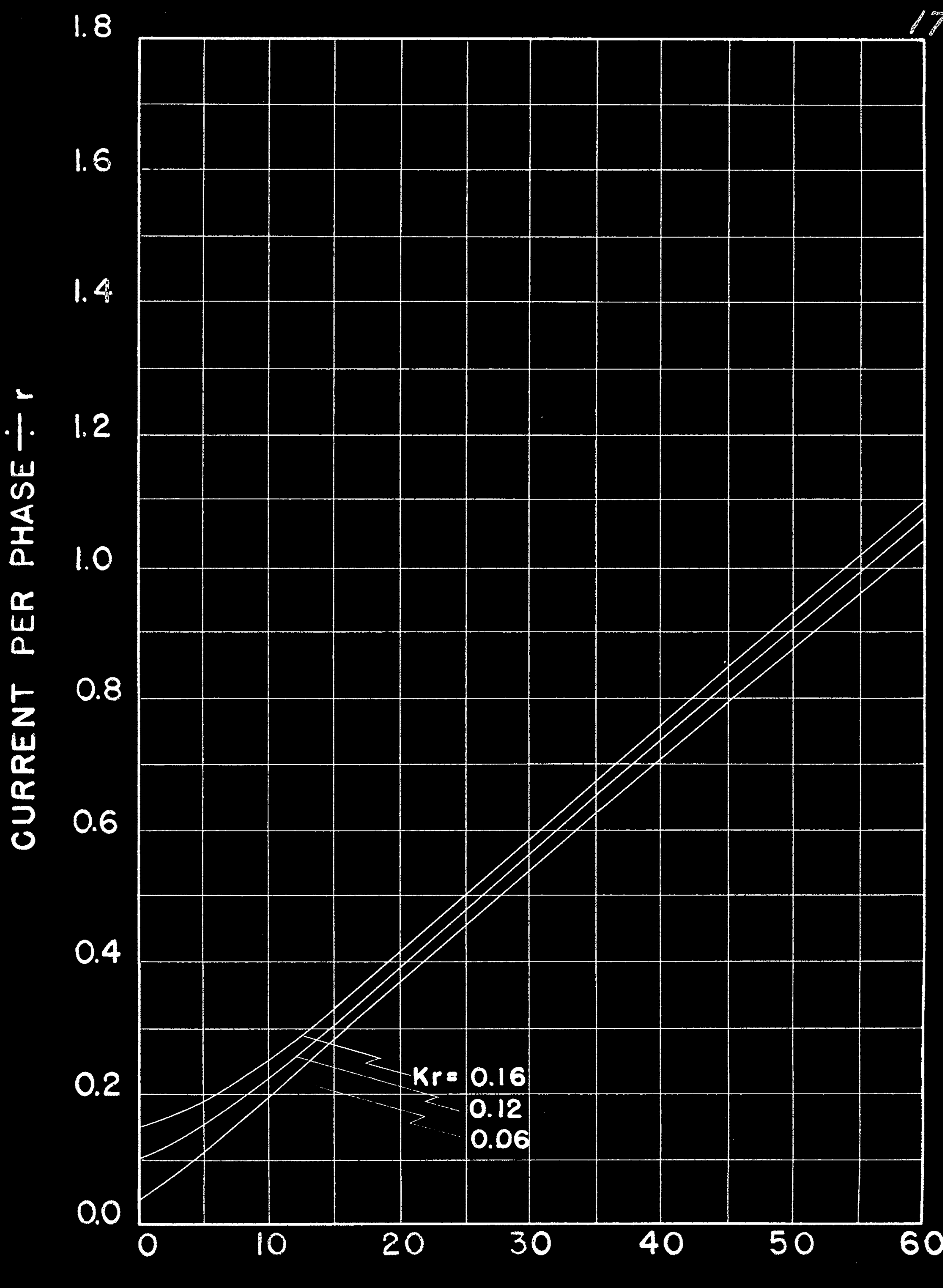

Figure 4. Induction motor primary current curves.

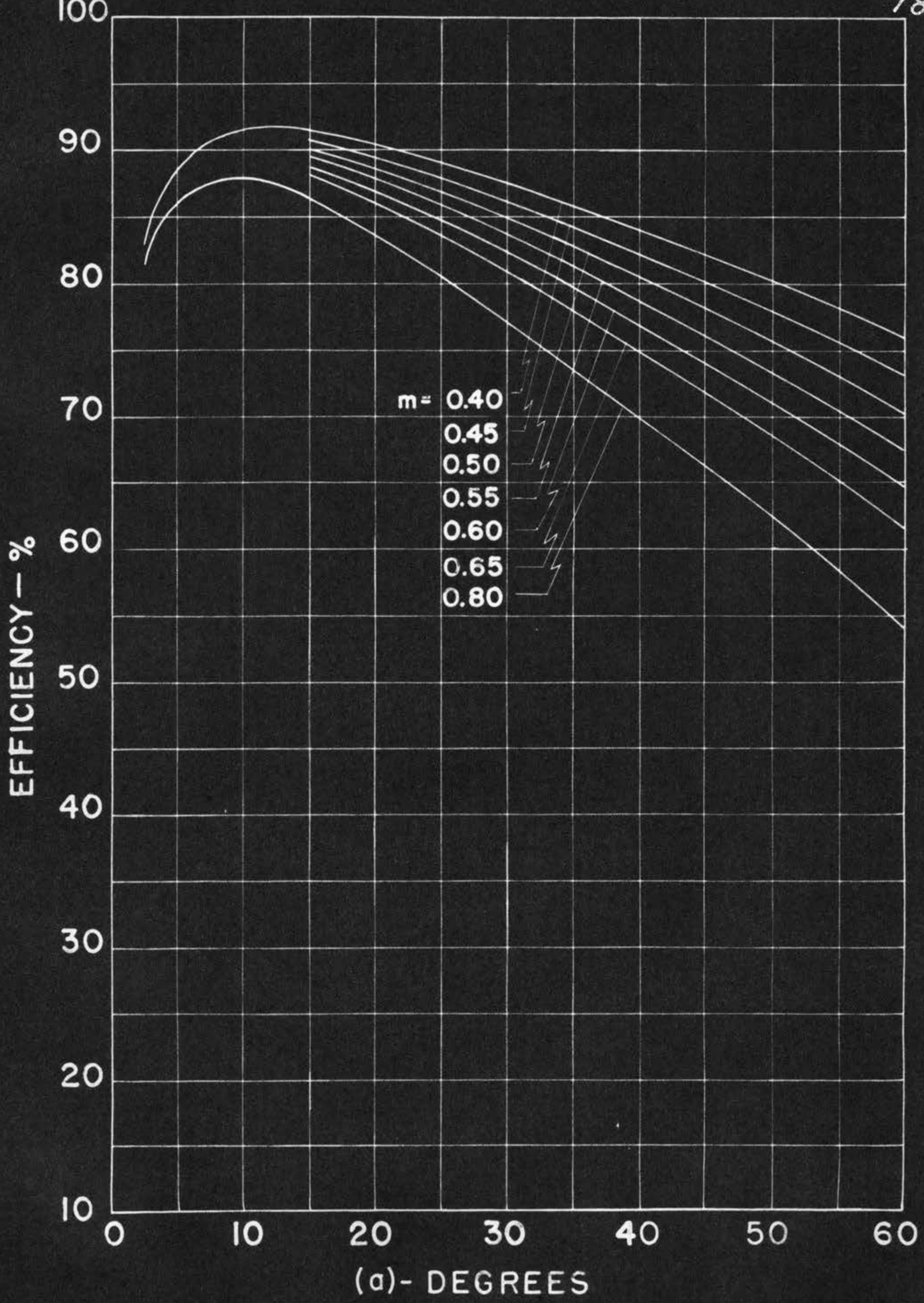

Figure 5. Induction motor efficiency curves.

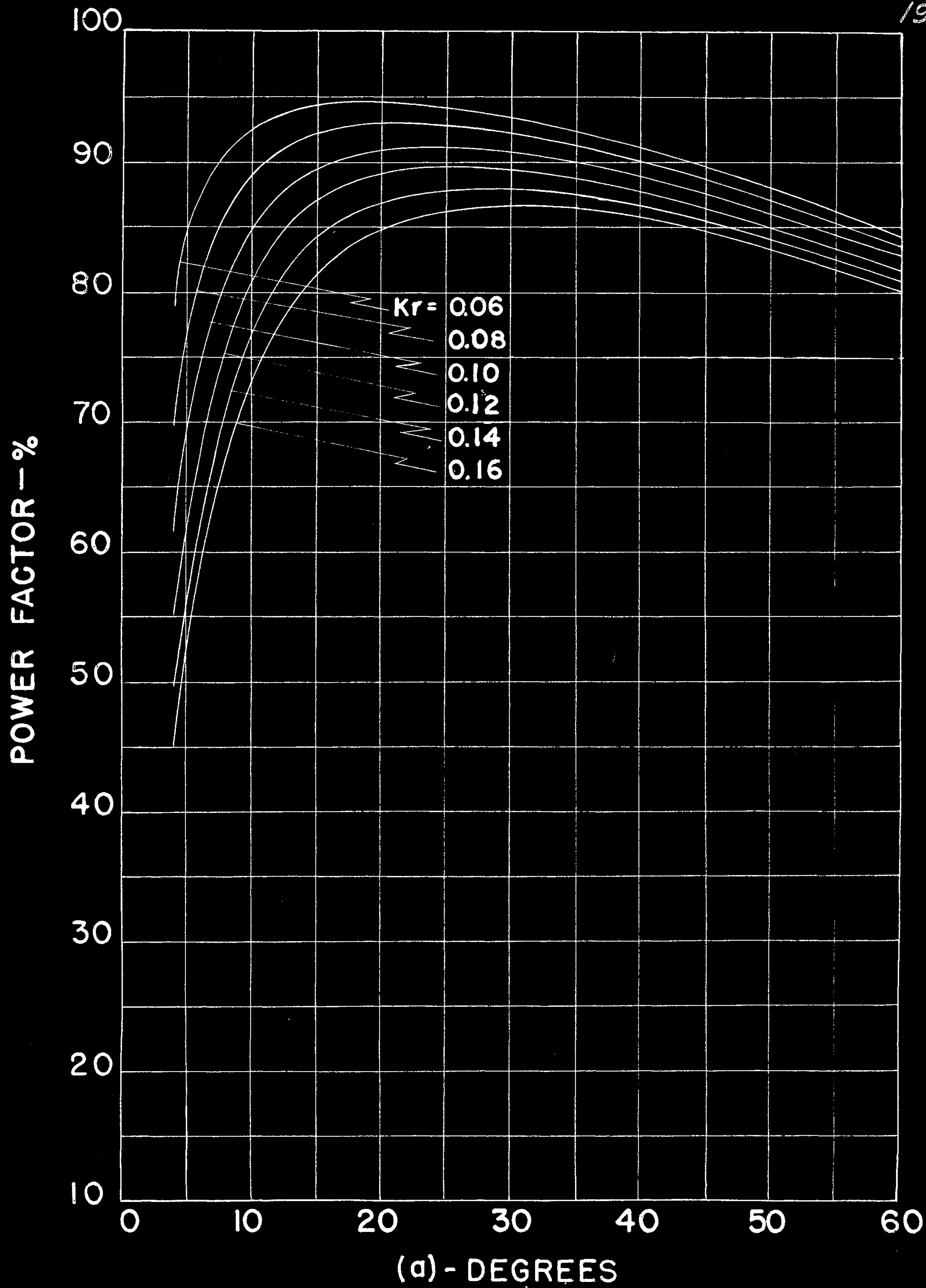

Figure 6. Power factor curves for induction motors.

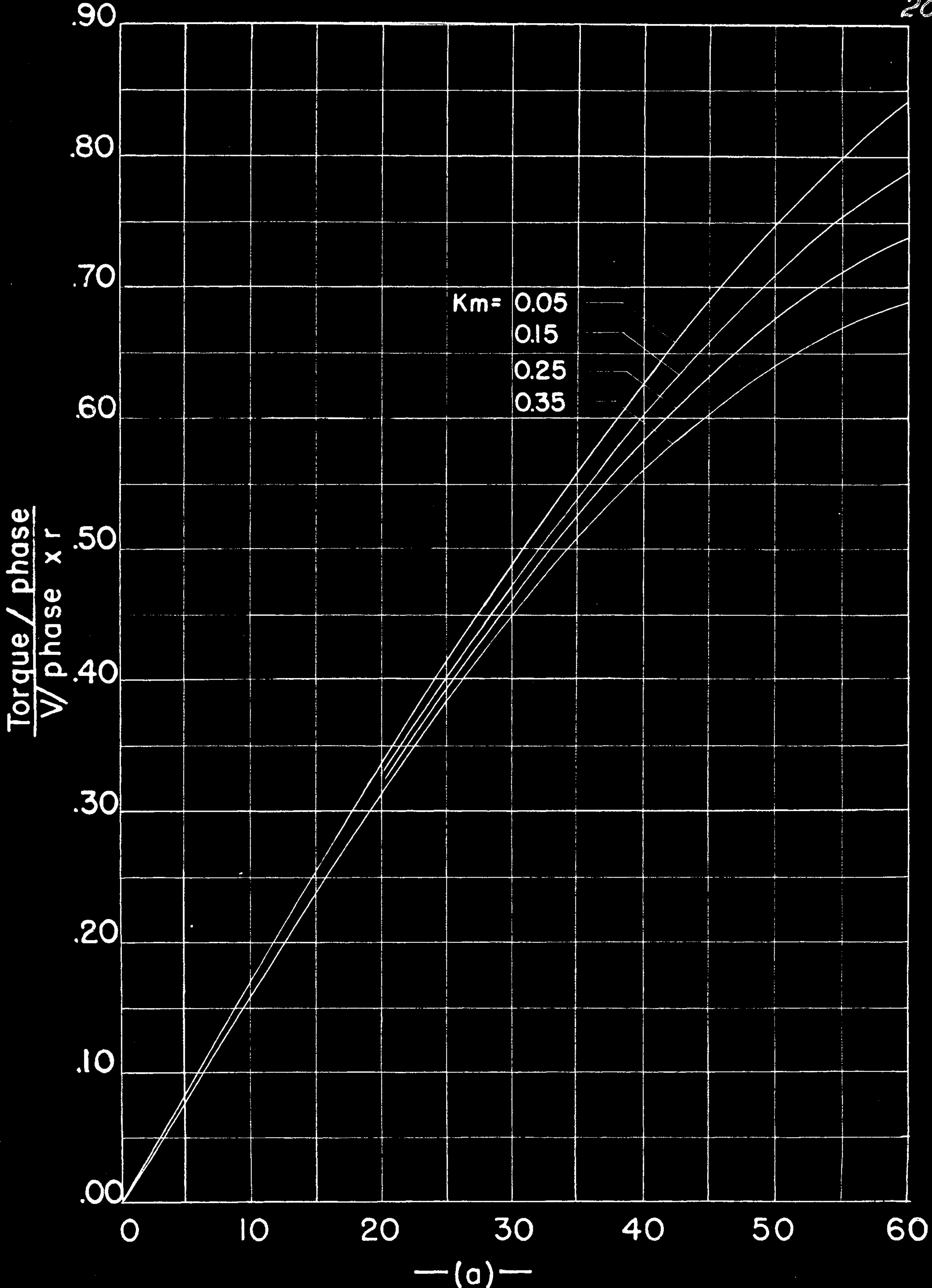

Figure 7. Induction motor torque curves.

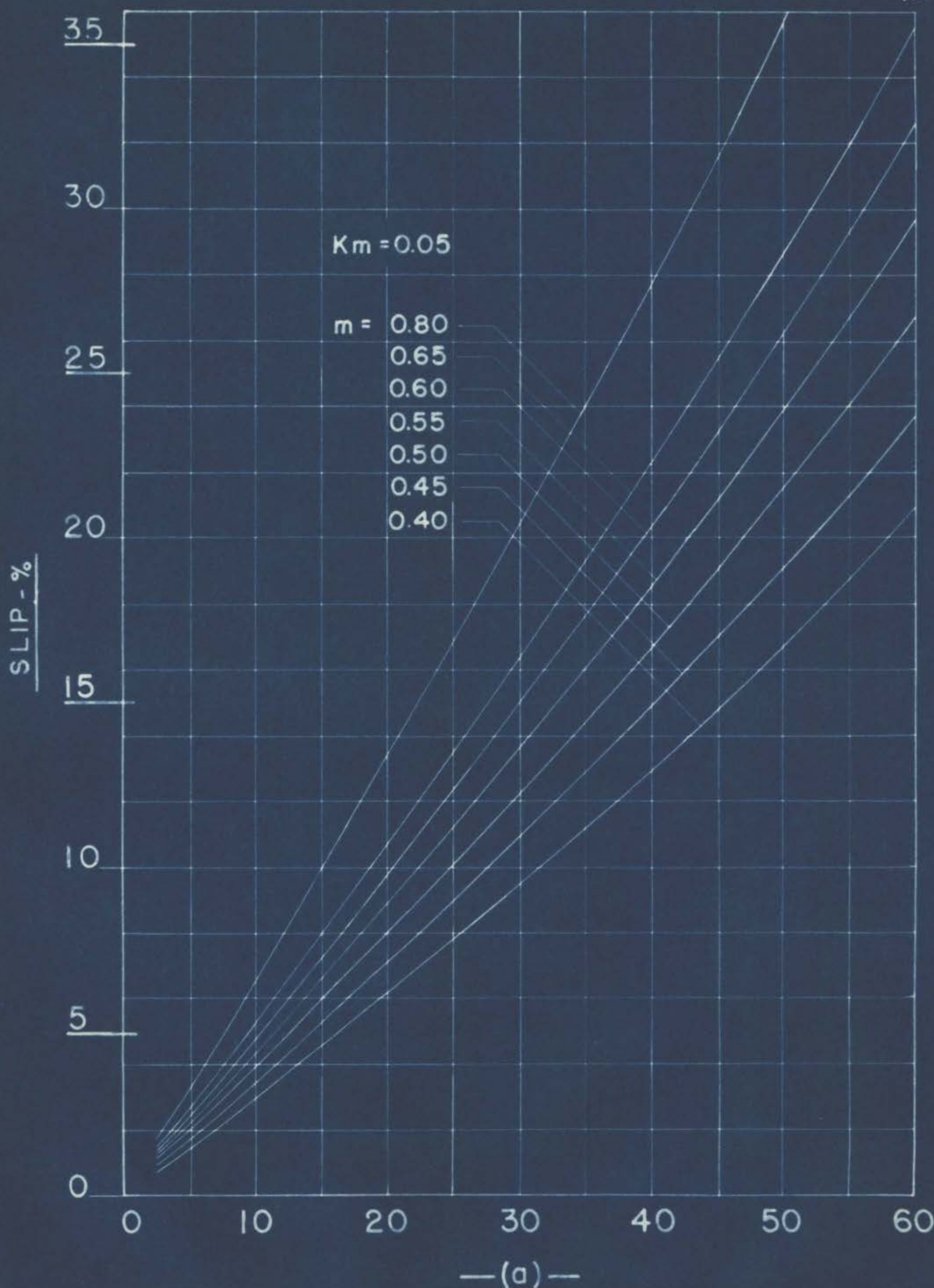

Figure 8. Induction motor slip curves.

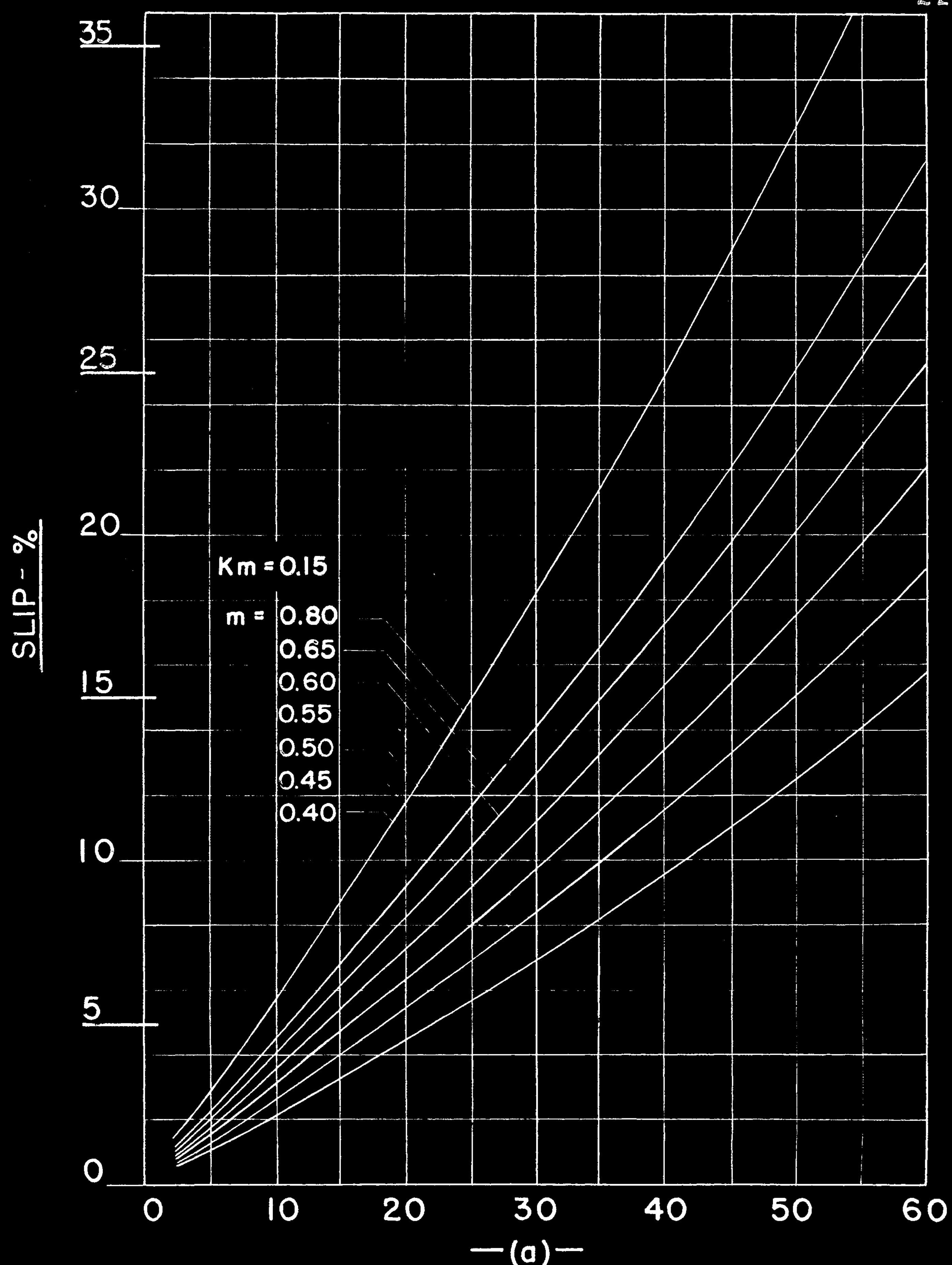

Figure 9. Induction motor slip curves.

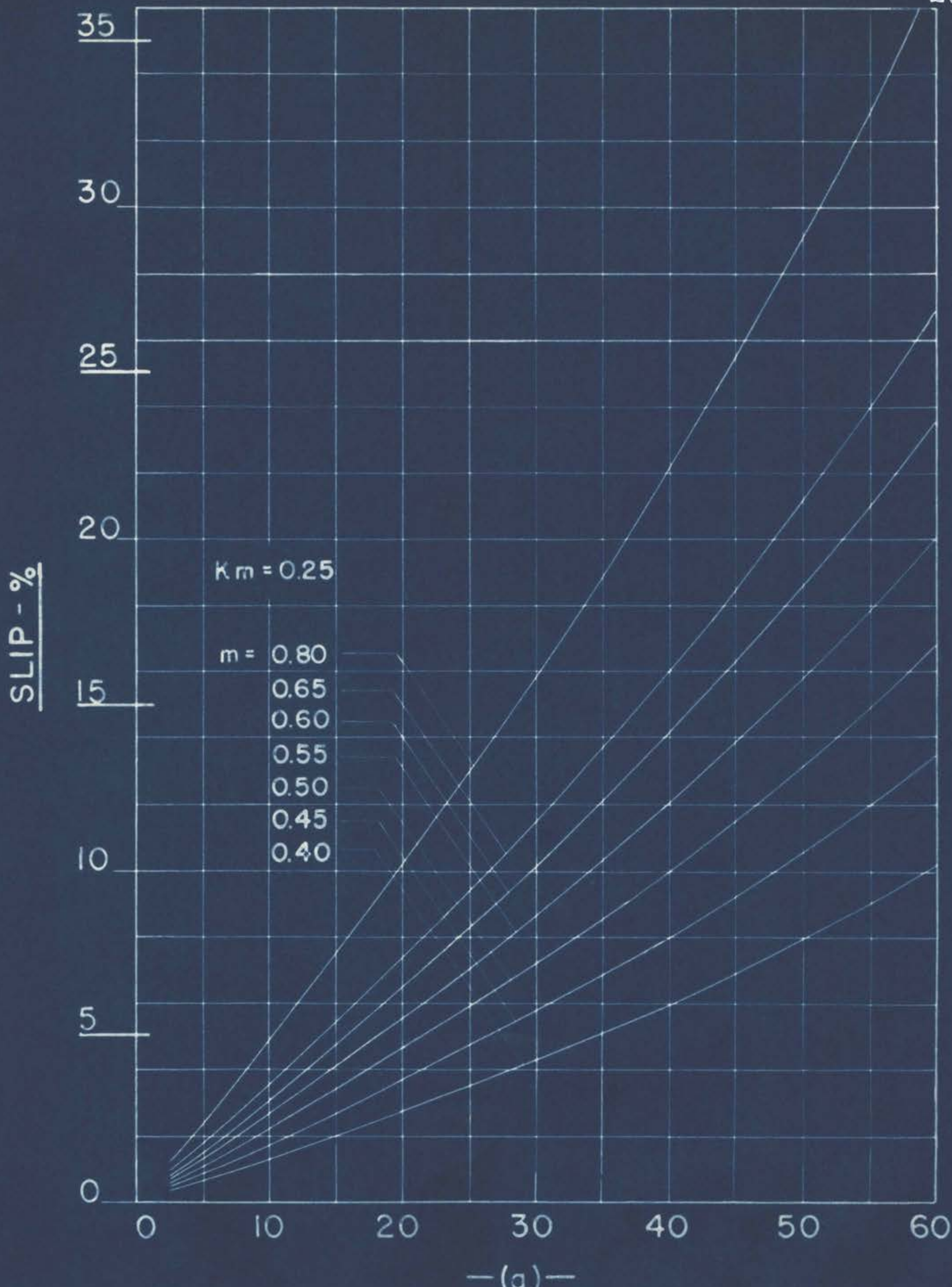

Figure 10. Induction motor slip curves.

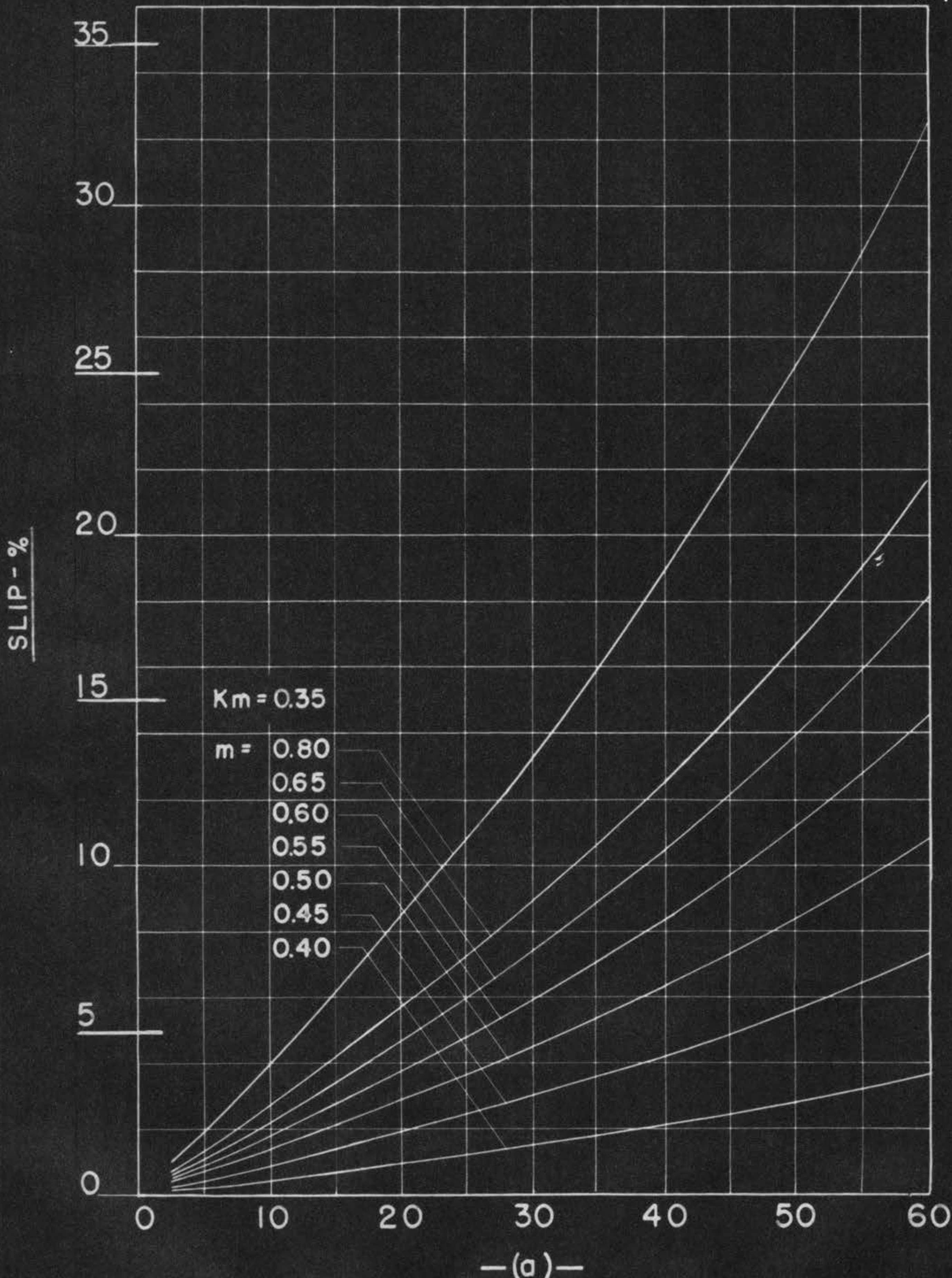

Figure 11. Induction motor slip curves.

CHAPTER II

ERRORS IN THE APPROXIMATE CIRCLE DIAGRAM

One of the most obvious causes for error in the circle diagram is the use of the approximate, rather than the exact, equivalent circuit. The induction motor may be represented by the exact equivalent circuit of Figure 13. However, in order that the current locus of the equivalent circuit may be represented by the approximate circle diagram, a slight change is made in the circuit. Namely, the exciting admittance, Y_o, is moved from point B to point A as shown in Figure 14. This is then the approximate equivalent circuit.[1]

This network neglects the effect of the exciting current in causing a drop in the stator, and the effects of the stator resistance drop on the voltage.

> "It will be observed that the errors involved relate merely to the quadrature exciting current circuit and the core loss current circuit; the voltage across these circuits is not constant, but it varies with the load current. Since that portion of the voltage drop across the primary impedance which is due solely to the core loss current and the exciting current, is quite negligible in comparison to that due to the load current, it is permissible to assume that the voltage at point B of Figure 13 depends entirely upon the load current. This assumption is equivalent to neglecting terms of higher order. These may be taken into consideration graphically without difficulty but the gain by doing so is not sufficient to justify the added complications."[2]

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[1] A.F. Puchstein and T.C. Lloyd, Alternating Current Machinery, p. 223-225.

[2] A.S. McAllister, Alternating Current Motors, p. 105.

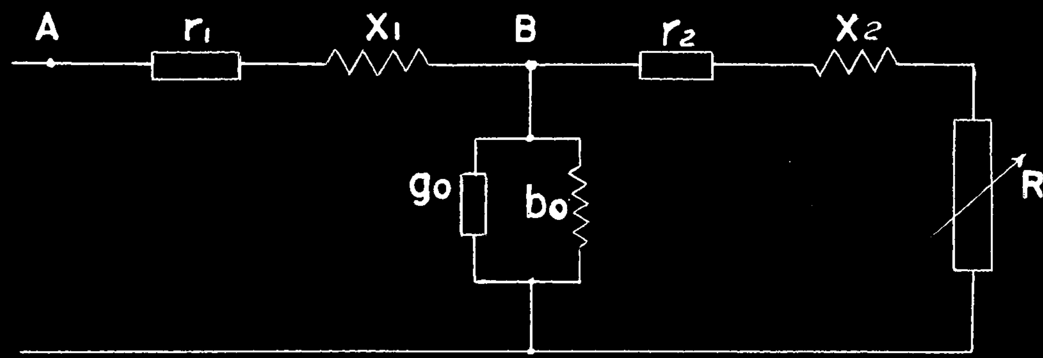

Figure 13. Induction motor exact equivalent circuit.

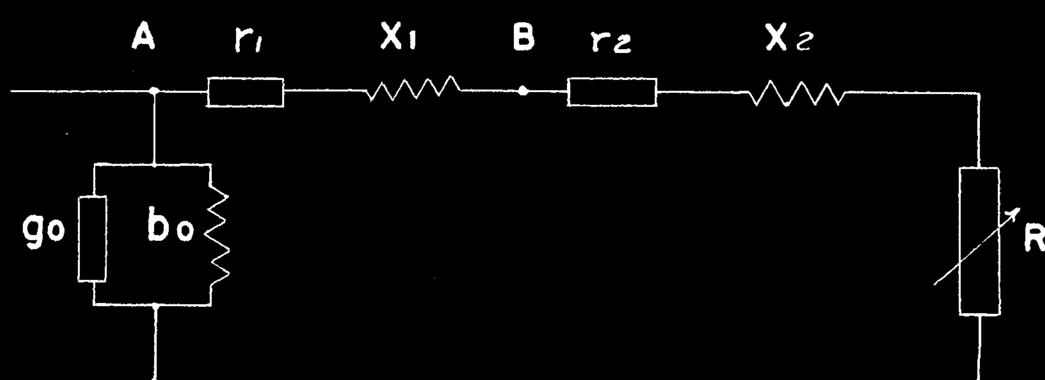

Figure 14. Induction motor approximate equivalent circuit.

Thus it is seen that the use of the approximate equivalent circuit obviously introduces an error but that the error is negligible for most ordinary calculations.

Still another cause for error in the circle diagram arises from the assumption that the windage and core losses of the motor are constant, regardless of the load and speed.[3]

Such is obviously not the case since the windage losses vary approximately as the square of the speed and the core losses vary somewhat with load. However, since the speed of a squirrel-cage induction motor varies only slightly over the working range (approximately 0 to 6%), the errors involved in assuming constant windage losses are very slight. The motor core loss is caused by both main and leakage fluxes but as the load increases, the loss due to the mutual flux decreases and that due to the leakage fluxes increases.

> "Because of this partially neutralizing effect, it is sufficiently accurate to regard the core loss as constant at all loads and speeds within the working range of the motor, under constant voltage conditions."[4] "Due to the drop in speed as the load increases, the windage and friction losses decrease and due to the increase in frequency of the rotor flux, the iron loss in the rotor core increases, so that it is reasonable to assume that the sum of these two losses is constant at all speeds."[5]

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[3] M.G. Say and E.N. Pink, The Performance and Design of A.C. Motors, p. 242.

[4] Puchstein and Lloyd, op. cit., p. 241.

[5] A. Gray, Electrical Machine Design, p. 337.

Evidently then, the errors introduced into the circle diagram by assuming that the windage, friction, and core loss remain constant are also of little consequence because the errors tend to cancel each other.

There are various other minor errors introduced by assumptions made in the use of the circle diagram but the literature treats these as being of very little consequence.

The only other causes for such large errors in the circle diagram would seem to be the effects of changes in parameters with changes in load. On this point, the references available were somewhat vague. Many of the authorities whose books were consulted omitted these effects entirely. Others mentioned that such things as changing parameters, caused by saturation and changes in leakage reactance, would probably have an effect upon the accuracy of the circle diagram but failed to reveal what those effects would be or how they could be taken into account. For instance, one author states that in general, r_1, x_1, r_2, and x_2 (Figure 13) will be less for large currents than for small currents and that the reactances will probably decrease faster than the resistances.[6]

EFFECTS OF SATURATION

Because of the scarcity of accurate and direct information dealing with the effects of saturation, an

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[6] McAllister, <u>op. cit.</u>, p. 242

investigation of the subject was made. The data used in the investigation is test data for a number of commercial motors, ranging in size from 1½ to 100 H.P.

If it be assumed that no saturation exists in the motor, curves of input current and power plotted as functions of the applied voltage should be straight lines. However, this is definitely not the case of the present day normal-starting-torque, normal-starting-current, squirrel-cage motor. Instead, curves of current and power as functions of the applied voltage, under blocked-rotor conditions, are of the general shape shown in Figure 15. The fact that these are not linear relations indicates very clearly that the values of the circuit parameters are not constant but vary with the circuit current.

A large percentage of the books which discuss or explain the circle diagram make no reference to this non-linear relation but rather give directions for determining the blocked-rotor point such as,

> "With the rotor held firmly, apply rated voltage to the motor and read current, power, and voltage. If the current with rated voltage is too high, apply a reduced voltage, read current, power and voltage and then multiply the current value by the ratio of rated to applied voltage and the power by the square of this ratio."

EFFECTS ON CIRCLE DIAGRAM OF VARYING BLOCKED ROTOR VOLTAGE

The test data used for these calculations included four different values of blocked-rotor input current and power, each for a different value of applied voltage. If there were no saturation in the motor, the circle diagrams

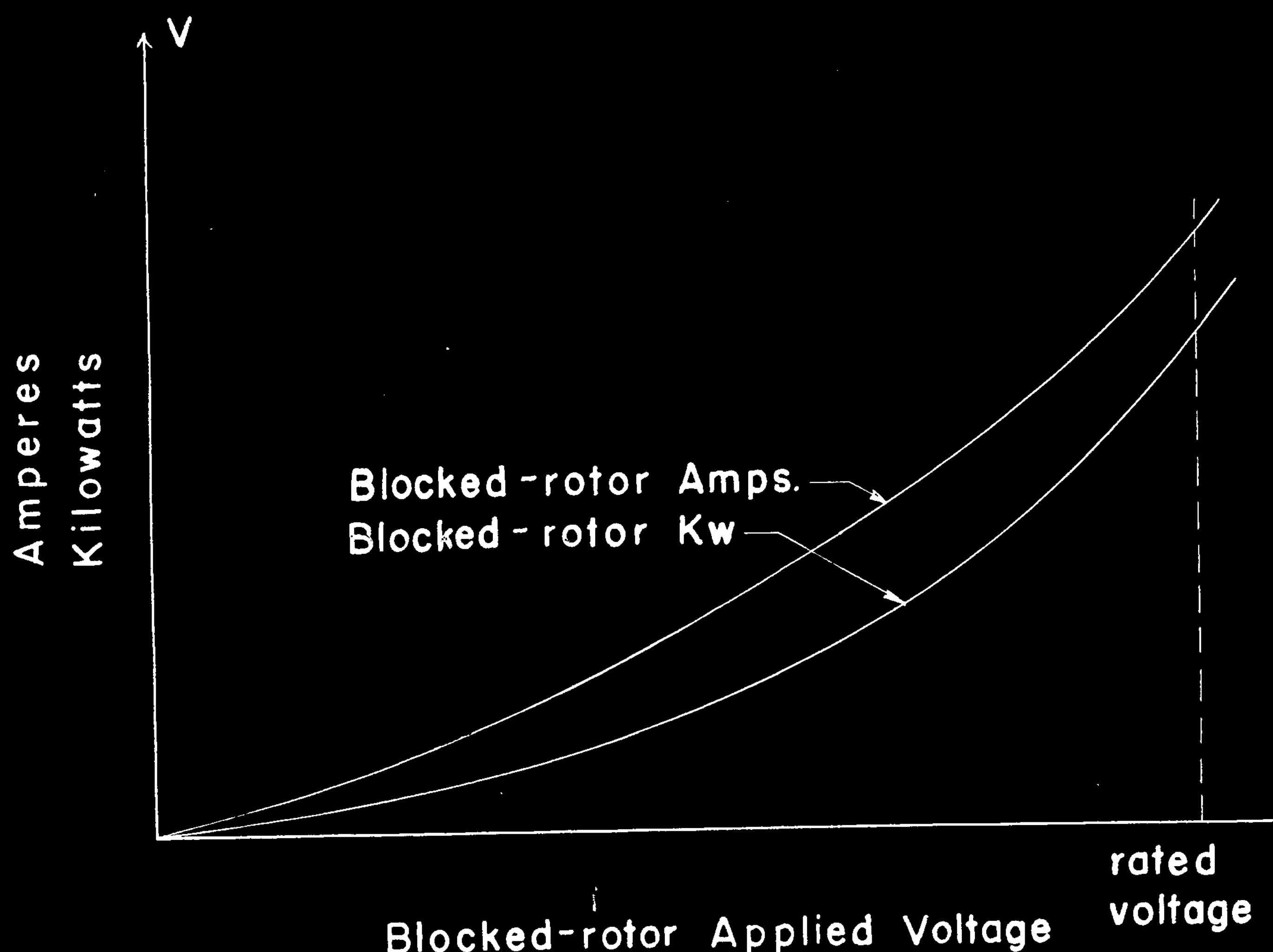
V
Amperes
Kilowatts
Blocked-rotor Amps.
Blocked-rotor Kw
rated voltage
Blocked-rotor Applied Voltage

Figure 15

drawn for each set of blocked-rotor current and power values, according to the preceding instructions, should coincide. Actually however, there is a very large difference in the size of these circles.

Figure 16 shows, to scale, the circle diagrams drawn from values of blocked-rotor current and power corresponding to four different values of applied voltage. The numbers on each circle indicate the magnitude of applied voltage used to determine the blocked rotor data. It may be seen that the diameter of the circle increases as the voltage applied under blocked-rotor conditions is increased and consequently as the degree of saturation increases.

The question next arises as to what effect the variations in the diameter of the circle will have upon the accuracy of the circle diagram within the common range of load values. That is, up to about 150% load.

Figure 17 shows, to scale, an enlarged portion of Figure 16. In order to show the true current locus, points are shown for the primary current locus corresponding to 1/4, 1/2, 3/4, 4/4, and 5/4 of full load. These points are actual operating values and were determined from brake tests on the motor. A study of this diagram reveals that the diameter of the circle used will determine in a very large degree, the accuracy of the predictions derived from the circle diagram. It is certainly not logical to assume that accurate predictions may be made

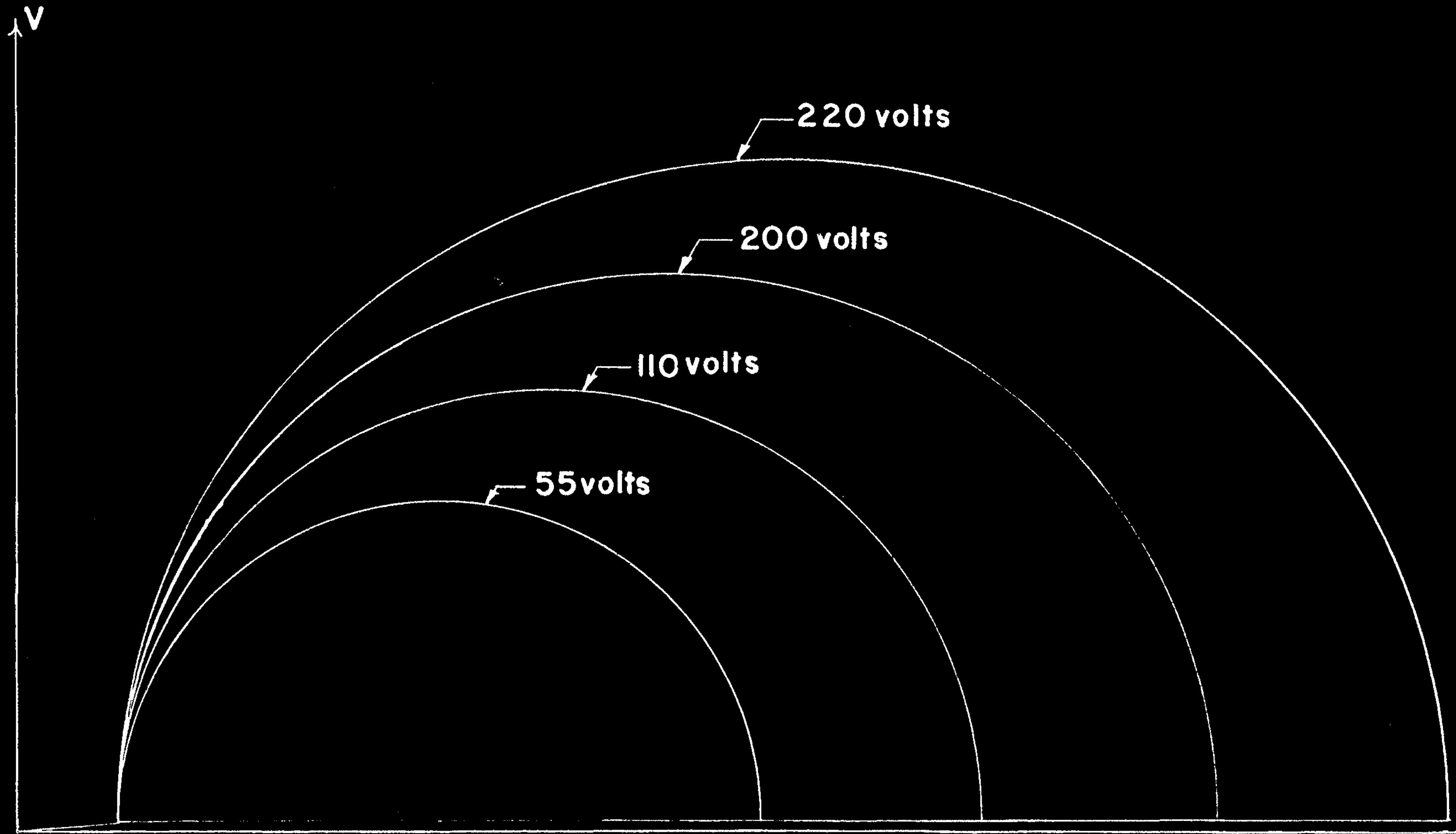

Figure 16

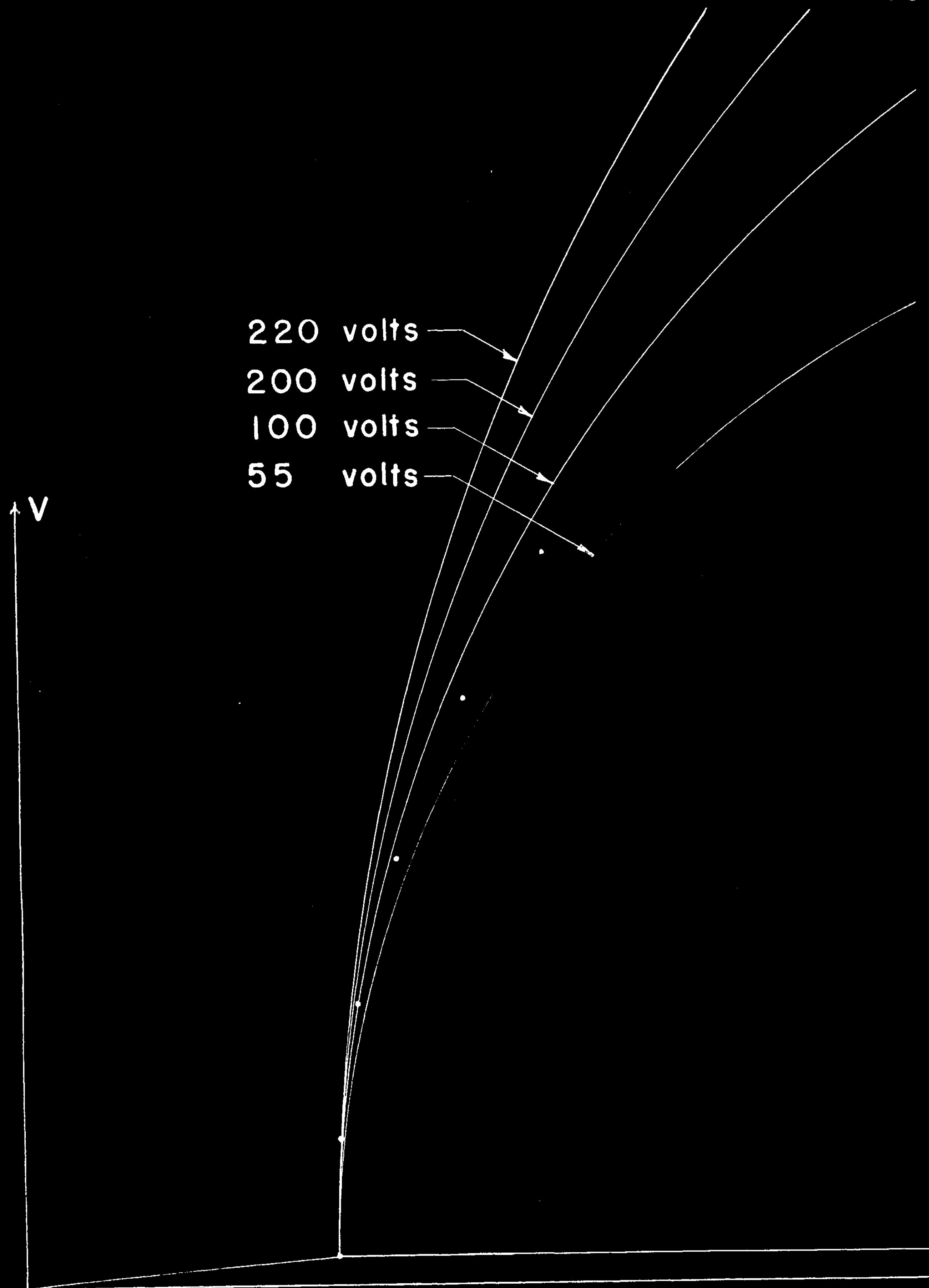

Figure 17

from a circle diagram whose circumference does not give a true indication of the primary current locus since, "The circle diagram is primarily intended to give the locus of the primary current as the load is varied."[7] This is indeed a basic requirement which, if not met, renders all calculations based upon it unreliable as far as accuracy is concerned.

There have been numerous schemes developed to improve the accuracy of the approximate equivalent circuit. Curiously enough, however, none of them have been developed to take into account saturation. Instead, they are concerned with making corrections for the error imposed by making use of the approximate equivalent circuit rather than the exact equivalent circuit. The magnitude of this error has been previously discussed. Most of these schemes are acknowledged to be mathematically exact solutions. Nowhere has it been found, where one of these schemes is derived or presented, that a comparison is given between the characteristics as predicted by the particular scheme and the actual characteristics as found by brake tests on the motor. Instead, if any comparison is given, it is usually a comparison between the results obtained by the Stienmetz equations and the results obtained by the scheme in question.

Two of the more common methods for predicting the

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[7] V. Karapetoff and B.C. Dennison, Experimental Electrical Engineering, VII, p. 411.

induction motor characteristics from the exact equivalent circuit are; (1) an analytical solution of the exact equivalent circuit by C.P. Stienmetz[8] and (2) a graphical solution by Mr. H. Ho.[9]

Since the results of Mr. Ho's Theorem can easily be shown graphically, its effect upon the accuracy of the circle diagram will be taken up. It should be kept in mind however, that whatever is true for one of the above methods should automatically be true for the other since both methods are commonly acknowledged to be mathematically correct solutions of the exact equivalent circuit.

The solid lines of Figure 18 show a typical circle diagram as derived from the approximate equivalent circuit. Ho's Theorem states, in substance, that in order to make the circumference of the circle represent the locus of the input current to the exact equivalent circuit, the semi-circle should be tilted as shown by the dotted lines. The angle by which the circle is tilted is a function of r_1, x_1, and r_2, and for normal-starting-torque, normal-starting-current, squirrel-cage motors varies from about 0 to 3 degrees.

The accuracy of this method and of Stienmetz equations cannot be questioned. They will, without doubt, obliterate

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[8] C.P. Stienmetz, Theory and Calculation of Electrical Apparatus.

[9] Karapetoff and Dennison, op. cit., p. 407.

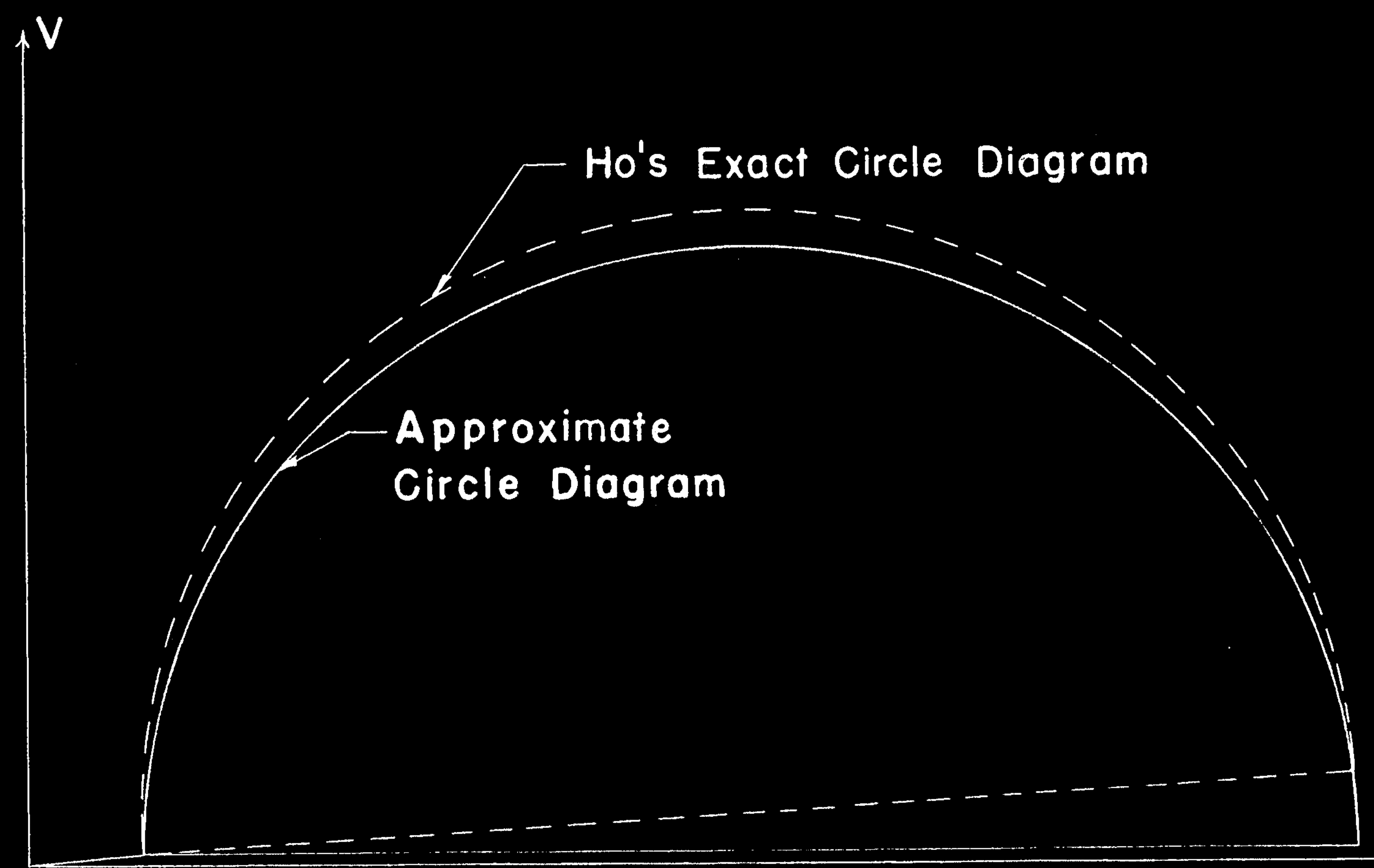

Figure 18

those errors in the approximate circle diagram which are caused by the use of the approximate rather than the exact equivalent circuit. It would seem then, and one would logically deduce such from the available literature, that by using a method such as Ho's Theorem, the performance characteristics could be predicted with a very high degree of accuracy. Such would indubitably be the case if, and only if, saturation effects were not present or were so minute as to be negligible. The diagram of Figure 19 and the explanation following will substantiate this statement.

The solid lines of Figure 19 are a reproduction of Figure 16. For the sake of clarity, only a portion of one circle is shown. The blocked-rotor point for this circle was determined from data obtained with rated voltage applied to the motor under blocked-rotor conditions. Here again, the points indicate the actual primary current locus as determined by brake tests on the motor. It may be easily seen from the diagram that if rated voltage is applied to the motor in order to determine the blocked-rotor point and then the resultant semi-circle is tilted according to Ho's Theorem that the primary current locus, as described by the circle diagram, will be even further removed from the true current locus than it was originally. Accordingly then, instead of tilting the semi-circle in a counter-clockwise direction, more accurate results would accrue from tilting the circle in the opposite, clockwise, direction.

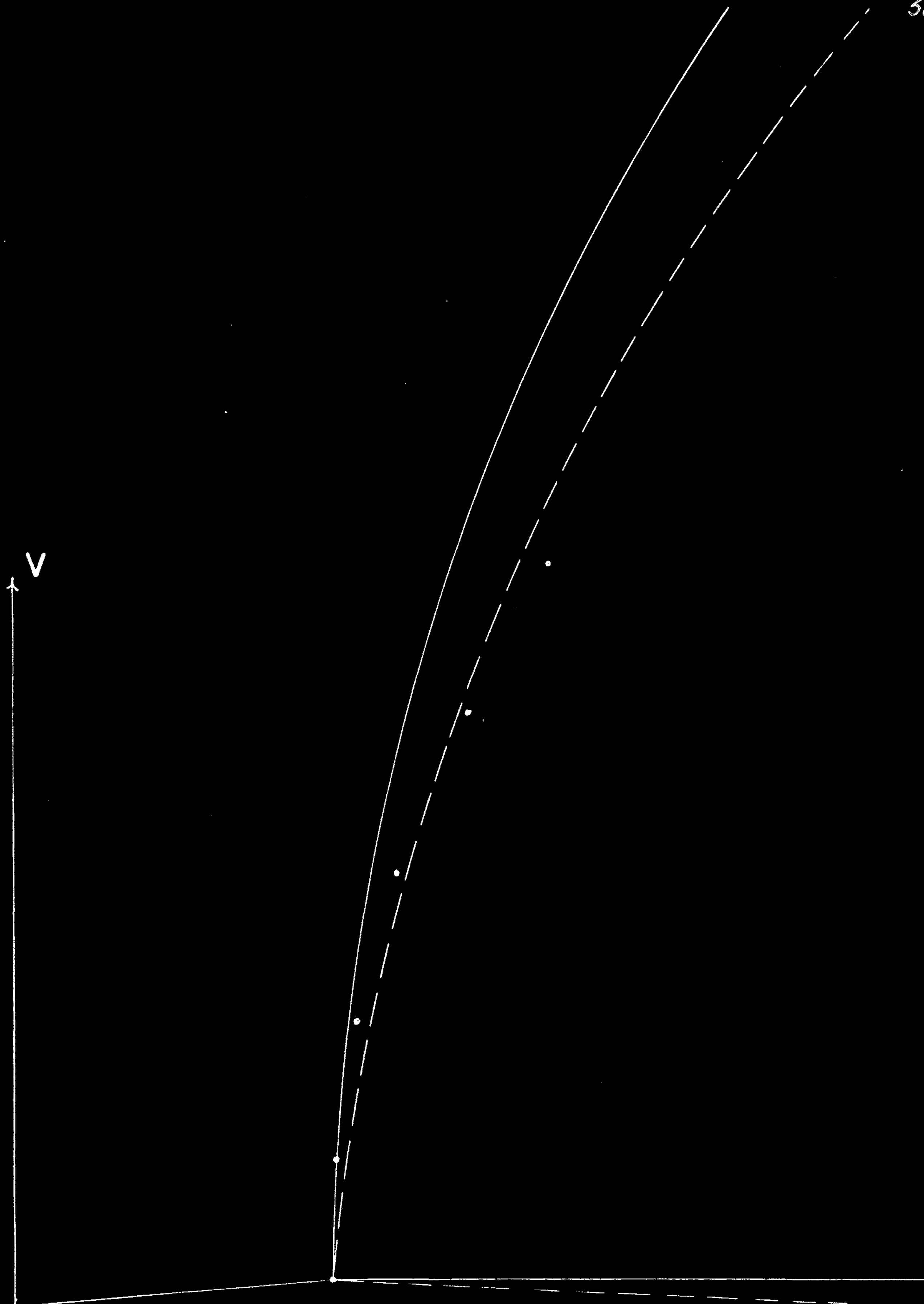

Figure 19

The circle of Figure 19, drawn with broken lines, shows the original circle tilted in a clockwise direction. Obviously, the predictions made using this circle would be more accurate than those made using the circle in its original position or in the position prescribed by Ho's Theorem. Even so, the predictions made from this circle can hardly be of any great accuracy since the circle, because of its size, cannot be made to fit all of the test points regardless of the angle through which it is tilted.

Figure 20 gives, with solid lines, a reproduction of another part of Figure 16. The blocked-rotor point for this circle was determined from data obtained with one-fourth rated voltage applied to the motor under blocked-rotor conditions. As before, the points indicate the actual values of the primary current locus. It may be seen that the approximate circle diagram, under these conditions, gives a poor indication of the primary current locus. If the circle is tilted in a counter-clockwise direction, as prescribed by Ho's Theorem, (circle shown by dotted lines) the accuracy will be somewhat improved. Here again, however, no great degree of accuracy can be expected because the circle is not of the proper size to fit all of the test points and because the angle through which the circle is to be tilted does not depend upon the degree of saturation.

It has been shown that the diameter of the circle used

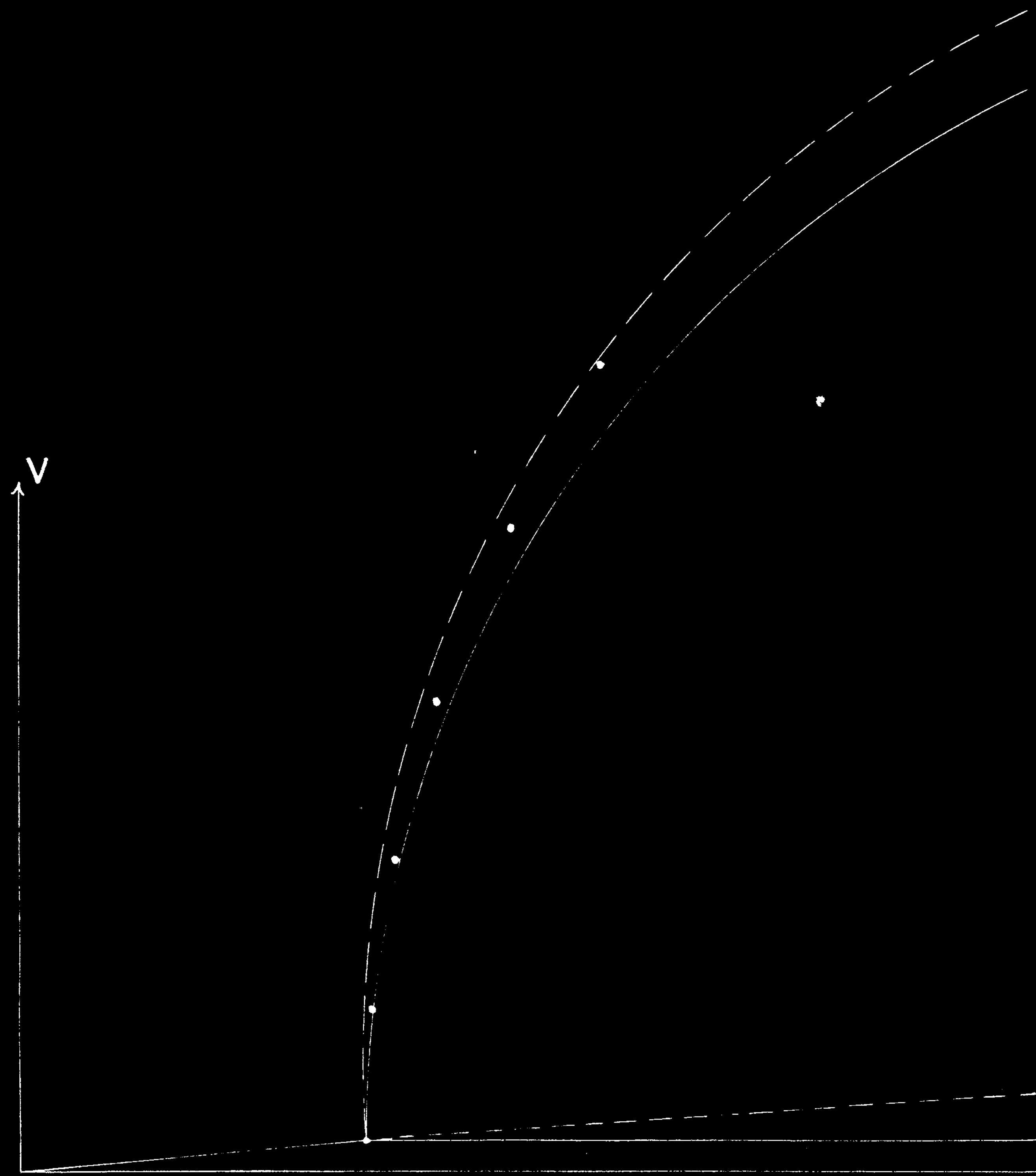

Figure 20

in the circle diagram is dependent upon the value of voltage used in determining the blocked-rotor point for the construction of the circle diagram. It has further been shown that when rated voltage is used, the resultant circle is too large and when a low value of voltage is used, the circle is too small to give a true indication of the primary current locus. The obvious question then is, "What value of voltage should be applied to the motor under blocked-rotor conditions in order that the circle diagram constructed from the resultant data will give the most accurate indication of the true primary current locus?"

OPTIMUM VALUE OF BLOCKED-ROTOR VOLTAGE

Since no better method for finding a solution to this problem presented itself, a method of more or less trial and error was adopted. This method is explained in the following paragraph.

First, a circle diagram was drawn from the no-load and blocked-rotor data for a particular motor. Then by trial and error process, the diameter of the circle was varied until the circle was found which would most nearly fit all the test points determined from brake tests on the motor. Next, circles were drawn from the no-load and blocked-rotor test data, for various values of blocked-rotor voltage. This process was continued until a circle was found which coincided with the circle drawn to most nearly fit the actual test points. The blocked-rotor data from which this circle was drawn was then recorded.

This same process was repeated for a large number of motors, ranging in size from $1\frac{1}{2}$ to 100 horsepower. From the resultant accumulated data it was concluded that the proper voltage to apply to the motor under blocked-rotor conditions is that value of voltage which will cause approximately 1.1 times the rated primary current to flow in the motor. If this value of voltage is used for the determination of the blocked-rotor point for the approximate circle diagram, the predictions, for the normal load range of the motor, will as a general rule be more accurate than those made from any other circle, regardless of any way it may be tilted.

It should be noticed, however, that very minute accuracy cannot be expected in any case because no one circle will fit all the test points even for the normal load range. This is due to the fact that the degree of saturation varies with the primary current. Therefore, for more accuracy, a different circle should be used for each value of load at which the characteristics are desired. The blocked-rotor data for each of these circles would in turn have to be obtained with a different value of applied voltage.

CONCLUSIONS

The method presented here for the prediction of induction motor characteristics is essentially a simple, yet exact solution of the approximate circle diagram. It has the advantages over the graphical solution of being more accurate and not requiring the use of drawing instruments. It compares very favorably with the mathematical solution as to accuracy and yet requires only a very small percentage of the time and work inherent in the mathematical or graphical solutions. An added advantage is the fact that the characteristics may be determined directly for any particular value of load. That is, it is not necessary to assume some value for another variable such as input current and then solve for the load which will require that current. Such a process usually requires several attempts in order to determine the characteristics at some particular value of load.

The accuracy mentioned in the preceding paragraph pertains to the accuracy of solution of the approximate circle diagram or the approximate equivalent circuit. In other words, the method presented herein, for the prediction of induction motor characteristics, will give accurate results when the approximate circle diagram itself is accurate.

It has been shown that the accuracy of both the exact and approximate circle diagrams is dependent upon the value of voltage applied when determining the blocked-rotor test data and that the best results are obtained when a value of blocked-rotor voltage is used which will cause 1.1 times the rated primary current to flow in the motor. Furthermore, the discrepancies in the predictions made from the approximate circle diagram and those made from the exact circle diagram are so far overshadowed by the errors introduced in neglecting saturation effects that they become insignificant. In no case can it be claimed that any exact equivalent circuit or exact circle diagram, in which effects of saturation are neglected, will give truly accurate predictions of the motor characteristics. In fact in some cases, where a high value of voltage is used to determine the blocked-rotor data, the exact circle diagram will give less accurate results than the approximate circle diagram.

If the circle diagram is to be drawn from design data instead of from the no-load and blocked-rotor data, the same rule should be observed. That is, the effective values of r_1, r_2, x_1, and x_2 (taking into account the proper leakage reactances) should be calculated for a value of primary current equal to about 1.1 times the rated value.

Therefore it may be concluded that the performance characteristics for the normal load range of a normal-starting-torque, normal-starting-current, squirrel-cage induction motor may best be predicted from an approximate circle diagram - the blocked-rotor data for which is determined with an applied voltage which causes 1.1 times rated primary current to flow in the motor. Consequently, the method presented here for the prediction of induction motor characteristics, since it is based upon the approximate circle diagram, will, if the previously mentioned value of voltage is used in determining the blocked-rotor data, give practical results as accurate as and generally more accurate than those obtainable by any other method which neglects the effects of saturation.

Compared to the method presented by Mr. Branson,[10] the method presented herein requires about one-tenth as many curve sheets and an almost insignificant number of calculations. Also, many of the characteristics may be read directly from the curves while the curve sheets presented by Mr. Branson merely give constants to be substituted in mathematical equations. The assumptions made in both cases are the same except that this method is based upon the approximate equivalent circuit while Mr. Branson's is based upon the exact. Since it has been shown that the practical results obtainable from the approximate equivalent circuit are as accurate and sometimes more accurate than those obtainable from the exact equivalent circuit, this assumption is thoroughly justified.

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[10] Branson, op. cit., 1.

APPENDIX I

SUBSTITUTIONS MADE TO GET EQUATIONS IN TERMS OF FUNDAMENTAL VARIABLES

Equation for power output:

$$W = (16) \times V/ph.$$

$$= \left[(7) - (13)\right] V/ph.$$

$$\frac{W}{V} = r \sin a - m\, r\,(1 - \cos a)$$

and $$\frac{W}{Vr} = \sin a - m(1 - \cos a).$$

Current per phase:

$$I_p = \sqrt{\left[x1 + (10)\right]^2 + (12)^2}$$

$$= \sqrt{\left[x1 + r\,(1 - \cos a)\right]^2 + \left[r \sin a + y_1\right]^2}$$

$$= r\sqrt{\left[r + (1 - \cos a)\right]^2 + \left[\sin a + K_e\right]^2}$$

Efficiency:

$$Eff = \frac{(16)}{(11)}$$

$$= \frac{(7) - (12)}{(7) + y_1}$$

$$= \frac{r \sin a - m\, r \cos a}{r \sin a + y_1}$$

Dividing by r,

$$Eff = \frac{\sin a - m \cos a}{\sin a + K_e}$$

Power Factor:

$$P.F. = \frac{(11)}{(18)}$$

$$= \frac{(7) + y_1}{I_p}$$

$$= \frac{r \sin a + y_1}{r\sqrt{\left[K_r + (1 - \cos a)\right]^2 + \left[\sin a + K_e\right]^2}}$$

Dividing by r,

$$P.F. = \frac{\sin a + K_e}{\sqrt{\left[K_r + (1 - \cos a)\right]^2 + \left[\sin a + K_e\right]^2}}$$

Torque:

$$T = (17) \times V/ph$$

$$= \left[(15) + (16)\right] \times V$$

$$= \left[(12) - (14) + (7) - (12)\right] \times V$$

$$= \left[(7) - (14)\right] \times V$$

$$= \left[r \sin a - r (1 - \cos a) K_m\right] \times V$$

Slip:

$$S = \frac{(15)}{(17)}$$

$$= \frac{(6) - (8)}{(9) + (10)} \quad \frac{(12) - (24)}{(15) - (16)}$$

$$= \frac{m\, r\, (1 - \cos a) - r\, K_m\, (1 - \cos a)}{(12) - (14) + (7) - (12)}$$

$$= \frac{m\, r\, (1 - \cos a) - r\, K_m\, (1 - \cos a)}{(7) - (14)}$$

$$= \frac{m\, r\, (1 - \cos a) - r\, K_m\, (1 - \cos a)}{r \sin a - r\, K_m\, (1 - \cos a)}$$

$$= \frac{m(1 - \cos a) - K_m\, (1 - \cos a)}{\sin a - K_m\, (1 - \cos a)}$$

APPENDIX II

1 M										
2	4	8	12	16	20	24	30	40	50	60
3 sin	.06976	.1392	.2079	.2756	.3420	.4067	.5000	.6428	.7660	.8660
4 (1 - cos)	.00244	.00973	.02185	.03874	.06031	.08645	.1340	.2340	.3572	.5000
5 1 x 4										
6 3 - 5										
7 3 - K_e										
8 6 - 7										

APPENDIX II

1 M										
2	4	8	12	16	20	24	30	40	50	60
3 sin	.06976	.1302	.2079	.2756	.3420	.4067	.5000	.6428	.7660	.8660
4 (1-cos)	.00244	.00973	.02185	.03874	.06031	.08645	.1340	.2340	.3572	.5000
5 K_m										
6 1 - 5										
7 6 x 4										
8 4 x 5										
9 3 - 8										
10 7 9										

APPENDIX II

1 M										
2	4	8	12	16	20	24	30	40	50	60
3 sin	.06976	.1392	.2079	.2756	.3420	.4067	.5000	.6428	.7660	.8660
4 (1 - cos)	.00244	.00973	.02185	.03874	.06031	.08645	.1340	.2340	.3572	.5000
5 K_r										
6 5 - 4										
7 $(6)^2$										
8 K_e										
9 3 - 8										
10 $(9)^2$										
11 7 - 10										
12 11										
13 9 - 12										

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- Eddie Presgrove -