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WITH QUANTITATIVE CHOICE.

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ABSTRACT

AGGREGATE BEHAVIORAL TRAVEL DEMAND MODELING

WITH QUANTITATIVE CHOICE

by

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The present study of aggregate behavioral travel demand modeling is based on the translog model with a quantitative choice. This is the comparative approach with the disaggregate behavioral demand models. In the model, the demand for travel is measured in terms of passenger miles of travel which is a continuously divisible unit.

The income-compensated elasticities which exclude the income effects from the market demand elasticities by holding the utility level constant are a correct measure for the substitution effects among the trip modes. The sensitivity analyses based on income-compensated elasticities give no counter-intuitive results; they give very reasonable results.

This study further indicates that such a correct understanding of the passenger travel demand has a very important policy implication.

As for the parameter estimation process, the non-linear maximum likelihood estimation method is used.

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CONTENTS

AGGREGATE BEHAVIORAL TRAVEL DEMAND MODELING WITH QUANTITATIVE CHOICE

CHAPTER I

INTRODUCTION AND SUMMARY

A. Purpose of the Study

The purpose of this study is: first, to formulate aggregate behavioral travel demand models in contrast to disaggregate behavioral travel demand models; second, to find a better way to measure quantitative assessments of the alternative transportation policies and plans.

The present study of aggregate behavioral demand modeling is based on the translog model with a quantitative choice approach, which is based on consumer theory; whereas, most of the disaggregate behavioral demand models are based on the multilogit model with a qualitative choice approach.

In the model, the demand for travel is measured in terms of passenger miles of travel instead of number of trips. The use of passenger miles as a dependent variable in the demand equation has a great appeal because many recent and traditional policy issues, such as energy consumption, air and noise pollution, accidents, revenue information, transit vehicle productivity and the efficiency of alternative transit management are more directly related to passenger and vehicle miles of travel than to number of trips. Therefore, it is more reasonable to consider travel choices as being not only qualitative but also quantitative. In this study, many comparisons are done between qualitative choice and quantitative choice in connection with travel demand modeling.

In the quantitative assessments of alternative transportation policies, the use of Hicksian income-compensated elasticities gives the correct measure of substitution effect.
The market demand cross elasticities which have long been used in the aggregate travel demand models are not a good measure to see the substitution effect among the trip modes, unless signs of the parameters are constrained according to the prior knowledge.

The sensitivity analysis based on Hicksian income-compensated elasticities gives very reasonable results too.

B. Statement of the Problem and its Importance

(Reference: Chapter II)

The traditional Urban Transportation Model System (UTMS) has been useful for the transportation forecasting purpose in many metropolitan areas. But its shortcomings have long been demonstrated on both theoretical and empirical aspects. Without considering any theory of trip makers behavior, it only correlates the existing data. Hence it is not satisfactory for the forecasting purposes.

The aggregate joint models (or econometric models) developed mainly for intercity travel demands in connection with the North East Corridor Transportation Project (NECTP) get over several of the shortcomings of the UTMS by the adoption of joint structure instead of sequential structure and the partial implementation of economic consumer theory in the model. But only limited improvements are made.

Recent developments on disaggregate behavioral models (referring to multinomial logit models) are based on probabilistic choice (qualitative choice) with the adoption of joint structure of trip making decisions. Disaggregate models are demonstrated to be superior than the previous models.

But many of the already existing data are not in the form useful for disaggregate models, therefore a huge amount of data collection cost is needed to apply disaggregate models. In many cases, money and time are not readily available to collect disaggregate data sets.

Although the transferability of the disaggregate models is demonstrated to be possible, it is often necessary to collect a subsampling of the data for the analysis. Only the transferability of mode-choice in the entire travel decision processes is considered and more study is needed.

In order to use the disaggregate models in travel demand forecasting, aggregation of the disaggregate models is necessary. But still, there is an aggregation problem with disaggregate models. More study is needed on how far aggregation can be carried such that errors are within an acceptable range.

Further developments and more applications are necessary with disaggregate behavioral models but at the same time, re-developments of aggregate models are needed because no one model will handle all cases of travel demands. Each type of model has limited scope of applicability.

From these considerations, the approach to aggregate travel demand models with consistent trip makers behavior and with quantitative choice context is sensible. In aggregate behavioral travel demand modeling, no data collection is required. The use of existing data will be sufficient and no aggregation problem is necessary. Since it is based on solid theory, once estimated it is readily transferable to any area and is also useful for forecasting purposes.

As for the estimation method, the disaggregate behavioral models (logit models) employ the use of the non-linear maximum likelihood estimation method which is the most advanced method existing today.

The aggregate travel demand models are estimated by either linear regression, log-linear regression, constrained linear regression or one equation non-linear regression. To go comparatively with the disaggregate behavioral models, the present aggregate behavioral models with quantitative choice employ the use of the non-linear maximum likelihood estimation method.

Thus the way in which the two types of models (qualitative and quantitative choice approach) complement each other, gives rise to significant reasons for further developments on both approaches.

C. Outline of the Report

The present study is mainly divided into four parts. Chapter II discusses current travel demand models and theories on which they are based. Current results and shortcomings are briefly discussed with regard to aggregate and disaggregate models.

Chapter III goes over some review on qualitative choice theory based on strict and random utility models. Then some review on conventional consumption theory is done and comparisons are made between qualitative and quantitative choice theory. In the last part of Chapter III, transportation demand modeling is presented with translog models. After demand modeling is presented, elasticities of both Marshallian demand and Hicksian income-compensated demand and elasticities of substitution of the present model are derived and their importances are discussed.

Chapter IV describes the data and model specification. The estimation methods in both quantitative and qualitative choice cases are comparatively discussed. In both cases, the non-linear maximum likelihood estimation method is applied.

Chapter V goes over the estimation results under different model specifications. The parameter estimates are evaluated with respect to their signs, t-statistics and χ^2-test statistics for the group of parameter estimates and for the restrictions. Marshallian elasticities, Hicksian income-compensated elasticities and elasticities of substitution are calculated and evaluated. Sensitivity analyses are further examined to test models' performances.

Finally, Chapter VI discusses the application of the model and the conclusive results. Several research directions are proposed for further study.

CHAPTER II

PRESENT STATE OF KNOWLEDGE

A. Current Theories and Models

Travel demand models may be divided into choice models and volume models. In a choice model, the dependent variable is a share (e.g. proportion of people taking mode m); in a volume model, the dependent variable is a volume (e.g. number of trips). Travel demand models may also be divided according to their structure, either joint or sequential. They are either disaggregate or aggregate depending upon their use of data.

1. Aggregate Sequential Models

The Urban Transportation Model System (UTMS) that has long been used in urban transportation planning studies is composed of a variety of aggregate sequential (indirect) travel demand models. The prediction of travel flows is divided into four sequential steps: trip generation, trip distribution, mode split and route assignment models. The shortcomings of the UTMS model system have been well discussed elsewhere (Talvitie 1971, CRA 1972, Manheim 1973, Ben-Akiva 1973, Talvitie 1975). But it is useful to go over their shortcomings in this study.

interzonal trips, mode split, vehicle miles of travel and statistics derived thereof, are incorrectly predicted. Further, the level of service attribute variable '$\omega_{k\ell}$' should include travel cost and other variables in addition to travel time.

Third, the mode-split models behave as if the level of service variables affecting modal choice have no effect on trip generations and trip distributions.

Modal split: $V_{k\ell m} = g_3(V_{k\ell}, L_{k\ell m}, S_k, A_\ell)$

where $V_{k\ell m}$ = interzonal trips by mode m.

Fourth, neither all level of service attributes nor the same parameters ("weights" on the attributes) in the modal split prediction are used in the network assignment model.

Route choice: $V_{k\ell mr} = g_4(V_{k\ell m}, L_{k\ell mr})$

where $V_{k\ell mr}$ = volume of flow from k to ℓ of mode m by path r, and

$L_{k\ell mr}$ = level of service from k to ℓ of mode m by path r.

The last shortcoming is its inaccuracy and its time consuming requirement to get predicted flows. The UTMS has many variants and their application ranges from small urban areas to large metropolitan areas.

2. Aggregate Joint Models (or Econometric Models)

The aggregate joint models are based on the concept that transportation is a derived demand, and it is natural to develop an intercity travel demand model based on the theory of consumer behavior.

Lancaster (1966) defined the utility functions over the attributes of goods. Hence the demand for transportation as a derived demand may be expressed in terms of level of service variables, socio-economic variables and attraction variables.

The structural specifications of the direct travel demand models are expressed in terms of direct and cross elasticities which are the result of the application of economic consumer theory.

The direct demand models include trip generation, trip distribution and mode split in a single equation. They are expressed as follows:

$$V_{k\ell m} = g(S_k, A_\ell, L_{k\ell m}, L_{k\ell m'}, \forall m' \neq m)$$

where S_k = characteristics describing origin k,

A_ℓ = characteristics describing destination ℓ,

$L_{k\ell m}$ = level of service variables by mode m, and

$L_{k\ell m'}$ = level of service variables by other mode m'.

This type of model was first developed for intercity travel between two cities in connection with the North East Corridor Transportation Project by Kraft (1963), Quandt and Baumol (1966), Blackburn (1966), Kraft and Wohl (1967), Quandt and Young (1969), McLynn and Woronka (1969) and others. Later, the direct demand models were applied to urban travel demand by CRA (1967), Domencich et al (1968) and Talvitie (1971). The common characteristic of these models is that they do not consider competition among alternative destinations in order to avoid the estimation of excessive parameters. The exclusion of competition among alternative destinations is not serious for work trips but for other kinds of trips, such as shopping and recreational trips, it is significant. Expressing per capita trips with route choice,

$$\frac{V_{k\ell m}}{P_k} = f\left(S_k,\ (A_\ell,\ A_{\ell'}\ \forall\ell'\neq\ell),\ (L_{k\ell m},\ L_{k\ell' m'}\ \forall\ell'\neq\ell,\ \forall m'\neq m)\right)$$

f is not only a function of its own attributes but is also a function of attributes of the substitute destinations and of the substitute modes. Hence the models can be extended to add route choice, provided the number of observations exceeds the number of parameters to be estimated.

The direct demand models result in smaller errors by escaping the drawbacks of the UTMS but these errors are still substantial. There are basically two kinds of direct demand models developed for the NECTP. One is mode-specific, which does not allow new modes to be added to the market. The other is mode independent which allows the introduction of a new mode to the market. The Kraft-Sarc model is an example of the mode-specific model, and the Baumol-Quandt model (abstract mode model) is an example of the mode independent model.

Kraft-Sarc model (product form):

$$V_{k\ell m} = f_m(S_k,\ A_\ell,\ L_{k\ell m},\ L_{k\ell m'}\ \forall m'\neq m)$$

$$V_{k\ell m} = (P_k P_\ell)\cdot(Y_k Y_\ell)^{\phi_m} \cdot \prod_q (t_{k\ell q}^{\theta_{mq}} \cdot c_{k\ell q}^{\psi_{mq}})$$

where $c_{k\ell q}$, $t_{k\ell q}$ = travel cost and time by mode q.

Kraft estimated the demand equations for four modes separately.

Baumol-Quandt abstract mode model:

$$V_{k\ell m} = f(L_{k\ell m},\ L_{k\ell b},\ S_k,\ S_\ell)$$

$$V_{k\ell m} = a_0 \cdot P_k^{a_1} \cdot P_k^{a_2} \cdot \left(\frac{P_k Y_k + P_\ell Y_\ell}{P_k + P_\ell}\right)^{a_3} \cdot c_{k\ell b}^{a_4} \cdot t_{k\ell b}^{a_5} \cdot f_{k\ell b}^{a_6} \left(\frac{c_{k\ell m}}{c_{k\ell b}}\right)^{a_7} \cdot \left(\frac{t_{k\ell m}}{t_{k\ell b}}\right)^{a_8} \cdot \left(\frac{f_{k\ell m}}{f_{k\ell b}}\right)^{a_9}$$

where $f_{k\ell m}$ = frequency of service from k to ℓ by mode m, and

$c_{k\ell b}', t_{k\ell b}', f_{k\ell b}$ = cost, time and frequency of service from k to ℓ by "best" mode (the cheapest cost, the fastest time and the most frequent service)

Baumol-Quandt estimated each demand equation by mode separately. The model is based on the gravity model approach with elasticity considerations. The primary motivation for the abstract mode formulation is that of data saving because it includes a smaller number of parameters, and it reduces the multicollinearity problem which the mode-specific models have difficulty in. The multicollinearity problem in the direct model were handled by constrained regression techniques (CRA 1967, Talvitie 1973).

Critiques of the Kraft-Sarc and Quandt-Baumol models:

The inclusion of all possible cross-elasticities in the Kraft-Sarc is an advantage over the Baumol-Quandt model.

But it is impossible to introduce a new mode in the Kraft-Sarc model while the Baumol-Quandt model can predict the demand for new modes without changing the functional form of the model or its parameters.

The specification of the Baumol-Quandt model is based on the assumption that cross-elasticity (competition) between modes exists only with respect to variables that qualify as "bests". Hence any changes in level of service variables, if they do not qualify that mode as the best, have no effect on demand for other modes in the system. It will affect use of the given mode only.

McLynn Model:

The travel demand model developed by McLynn and Woronka (1969) for the Northeast Corridor Transportation Project (NECTP) can be classified either aggregate simultaneous or partially sequential. The model can be written as follows:

$$V_{k\ell m} = g_2(S_k, A_\ell, L_{k\ell q}) \frac{g_{1m}(L_{k\ell m})}{\sum_q g_{1q}(L_{k\ell q})}$$

The model includes two functions, one is g_2 to predict total trips from k to ℓ and a second g_{1m} to predict the share of $V_{k\ell}$ which will use mode m. In the McLynn model,

$$V_{k\ell} = g_2(S_k, A_\ell, L_{k\ell q}) = b_o \cdot P_k^{b_1} \cdot P_\ell^{b_2} \cdot Y_k^{b_3} \cdot Y_\ell^{b_4} \cdot \left[\sum_q g_{1q}\right]^{b_5}$$

$$\frac{V_{k\ell m}}{V_{k\ell}} = g_{1m}(L_{k\ell m}) = a_{om} \cdot c_{k\ell m}^{a_{1m}} \cdot t_{k\ell m}^{a_{2m}} \cdot [1 - \exp(a_{3m} f_{k\ell m})]^{a_{4m}}$$

where $c_{k\ell m}, t_{k\ell m}, f_{k\ell m}$ = cost, time and frequency of service from city k to city ℓ by mode m.

The two functions are estimated sequentially: First the g_{1q} functions are estimated, and then their sum is obtained as a variable to be used in g_2. The calibration of this model was done for each of the four modes separately. McLynn's model is also based on gravity model formulation along with the elasticity consideration based on consumer theory. McLynn's model performs better than the Kraft-Sarc and the Baumol-Quandt models and it was adopted as a mode-split model. However problems were encountered in the McLynn model too. Considering binary mode-split case in the McLynn model,

$$V_{k\ell} = V_{k\ell m} + V_{k\ell m'}$$

then

$$E_{t_{k\ell m}}^{V_{k\ell}} = E_{t_{k\ell m}}^{V_{k\ell m}} \cdot \frac{V_{k\ell m}}{V_{k\ell}} + E_{t_{k\ell m}}^{V_{k\ell m'}} \cdot \frac{V_{k\ell m'}}{V_{k\ell}}$$

where $E_{t_{k\ell m}}^{V_{k\ell}}$ = elasticity of trip volume $V_{k\ell}$ with respect to travel time from k to ℓ by mode m,

$E_{t_{k\ell m}}^{V_{k\ell m}}$ = direct elasticity of $V_{k\ell m}$ with respect to $t_{k\ell m}$

hence $E_{t_{k\ell m}}^{V_{k\ell m}} < 0$, and

$E_{t_{k\ell m}}^{V_{k\ell m'}}$ = cross elasticity of $V_{k\ell m'}$ with respect to $t_{k\ell m}$

and hence $E_{t_{k\ell m}}^{V_{k\ell m'}} \geq 0$.

If the share of m, $(\frac{V_{k\ell m}}{V_{k\ell}})$, is small and the first direct elasticity term is smaller than the second cross elasticity term, then $E_{t_{k\ell m}}^{V_{k\ell}}$ will be positive. This is not a valid result. In actuality, as travel time of a mode improves (i.e. $t_{k\ell m}$ decreases), trip volume from k to ℓ increases. Hence in the McLynn model, there is no guarantee that $E_{t_{k\ell m}}^{V_{k\ell}}$ will be negative.

Talvitie Model:

The Talvitie model is based on the Kraft-Sarc model but it was applied to a three-mode case for downtown worktrips. The functional form of the model is

$$V_{k\ell m} = f\{S_k, A_\ell, L_{k\ell m}\} .$$

For the detailed model specification, see (P. R. Stopher and A. H. Meyburg, 1975). In the demand model formulation, Talvitie considered different kinds of mathematical forms and chose the mixed-form (i.e. $\ell ny = a+bx+c\ell nx$) on the grounds that this form provides both absolute and relative effects of a change in an explanatory variable, and also that the demand for travel is not implied to be sensitive to travel volumes. After predictive and structural accuracy tests were done in relation to other traditional transit models, he shows that the model is superior in its predictive accuracy and that it has both lower mean error and lower variation of the error than the traditional models.

Critiques of Aggregate Joint Model:

All four models presented above try to use economic consumer theory in the form of elasticity and the sign of elasticity but none of the models are well grounded in consumer theory.

All three models except the Talvitie model have constant elasticity with respect to each of the variables. This is a strong restriction.

The calibrations of all the models are done with linear regression on the log transformed demand equation according to each mode separately.

None of the four models consider the relation between short-run travel choices and long-run mobility choices.

3. Disaggregate Models

The difficulties with aggregate travel demand models have encouraged new modeling approaches. The disaggregate models have superior predictive power over the conventional aggregate travel demand models. The disaggregate models are free from the drawbacks of the aggregate models. When we consider individual's discrete choice behavior, then the travel choice is qualitative. Usually disaggregate travel demand models are formulated as probabilistic models while the aggregate models have been formulated to be deterministic although a share form may be considered as probabilistic in form.

McFadden (1968) constructively derived the multinomial logit model from the theoretical foundation and formulated the estimation of the parameters of the multilogit model (more detail in Chapter IV).

The probabilistic models express the functional relationship between the transportation system and the socioeconomic attributes of a random individual and the probability that the individual will choose to make a certain trip. Statistical inference on this functional dependence is made possible if travel surveys are interpreted as drawings from a statistical distribution with these probabilities (Domencich and McFadden 1974).

Disaggregate Sequential Models:

Categorizing the disaggregate models, the choice structure of individuals over the set of travel alternatives--trip frequency, destination, time of day, mode and route (f,d,h,m,r) may be either sequential or simultaneous.

sequential: $P_i(f) \cdot P_i(d|f) \cdot P_i(h|d,f) \cdot P_i(m|f,d,h) \cdot P_i(r|f,d,h,m)$

joint: $P_i(f,d,h,m,r)$

The sequential choice model is the result of assuming that travel choices are made sequentially. Expressing mathematically:

$$P_i(f,d,h,m,r) = P_i(f) \cdot P_i(d|f) \cdot P_i(h|d,f) \cdot P_i(m|f,d,h) \cdot P_i(r|f,d,h,m)$$

That is, both simultaneous and sequential models can be expressed as a joint probability or as a sequence of marginal and conditional probabilities. However joint probability derived from the simultaneous model is different from that derived from the sequential model. The same is true with marginal probability. The differences are due to degrees of freedom. Sequential logit is the same as joint logit if the coefficient of the inclusive price is 1. The sequential choice model is as important as the simultaneous choice model in the study of travel choice behavior. Disaggregate sequential models have been studied by many (Stopher, 1969; McGillivray, 1970; Stopher and Lisco, 1970; Talvitie, 1972; CRA, 1972; and McFadden, 1974).

Disaggregate Simultaneous Models:

The disaggregate probabilistic simultaneous smodels were developed by Ben-Akina (1973). The assumption is that the decision process of an individual traveller is simultaneous (joint) in nature. Hence a complete trip is based on one simultaneous decision. Under a simultaneous choice situation, the problems of a large number of possible combinations of choices and the number of explanatory variables and the interactions among variables are handled by the explicit choice hierarchy. Travel decisions are divided into two sets of choices: the long-run mobility decisions of location, housing, automobile ownership and mode to work and the short-run travel choices of frequency, mode, destination, route and time of day. The set of models is termed block-conditional, where the blocks of mobility and travel choices as single units have a conditional structure, while each block by itself has a joint structure.

In recent years many studies have been made using disaggregate simultaneous choice models (Ben-Akiva, 1974; Liou and Talvitie, 1974; Lerman, 1975; Lerman and Ben-Akiva, 1976; Ben-Akiva and Richards, 1976; and others).

Issues on Disaggregate Models:

Disaggregate probabilistic models have been demonstrated to be best fit primarily for urban area travel demand forecasting.

(i) Data Set:

The existing travel information obtained by the origin-destination survey is not in a format that is compatible with the calibration of disaggregate models. A great amount of time and cost are required to collect data from individuals. Furthermore the individual data is collected in such a way such that individual's travel choice is discrete in nature.

In the disaggregate model, the level of service variables are not obtained from individual data because individuals do not observe the service attributes "correctly" and hence engineering estimates are used predominantly. This has been argued by many because it does not comply with qualitative choice theory.

(ii) Use of Extrapolated Figures:

For the purpose of travel forecasting, projection of future independent variables such as population or employment are required. Such projected figures are obtained from the application of extrapolated factors which have potential errors and the higher the level of disaggregation, the less reliable the extrapolated figures become.

(iii) Transferability and Updating:

Brand (1973) emphasized the possible transferability of the behavioral travel demand models. The transferability and updating of disaggregate travel demand models were carried out by Atherton and Ben-Akiva (1975). Their empirical results assure that a well-specified model is transferable. But no model is perfectly specified and no model is perfectly transferable. Therefore, the updating procedures for the model coefficients were required.

The use of aggregate data in adjusting constant terms results in biases.

The use of maximum likelihood estimation technique with a small disaggregate sample gives very unstable coefficient estimates. (The resulting biases and standard deviations are large.)

They found that the Bayesian updating procedure using a small disaggregate sample is the most effective procedure for well-specified models.

Their empirical results were limited to the conditional probability of mode choice which is only one component of the entire travel decision processes. Therefore, further study on transferability and updating is needed in areas of more complex choice situations.

(iv) Aggregation:

Aggregation is always needed in travel demand forecasting processes. Aggregation of disaggregate choice probabilities is the way in which aggregate share or volumes can be obtained from disaggregate models.

There have been five methods developed for the aggregation of the disaggregate travel demand models. Let utility function be denoted as:

$$U(x_i) = V(x_i) + e(x_i) \text{ where } V(x_i) = (x_i'\beta)$$

$$x_i = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_n \end{pmatrix}$$

1. Naive Procedure. This is a procedure of using the zonal means of the independent variables in the disaggregate models. It is computationally simple but this procedure ignores the within zone variance. The expectation of people choosing an alternative is:

$$E(p_i) = \bar{p}_i = f_i(\bar{x}'\beta)$$

where $\bar{p}$ is the probability evaluated at the zonal means of the independent variables by logit or probit function depending on which assumptions have been made on the independently distributed error term $e(x_i)$.

2. Numerical Integration Procedure. It is a method of numerical integration to compute the expectation of the people choosing an alternative. McFadden and Reid (1973) used the assumption that independent variables have multivariate normal distributions in the case of binary choice probit models of the form:

$$P_i(k:A) = \Phi(x_i'\beta) \quad \text{where } \Phi(\omega) = \int_{-\infty}^{\omega} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-y^2}{2}\right) dy,$$

i.e. probability of an individual i choosing an alternative k from the alternative set A is expressed with the assumption that $(x_i'\beta)$ is normally distributed with mean $(\bar{x}_i'\beta)$ and covariance $\sigma^2 = \beta' A \beta$. By integration, they obtained an aggregate share probit model:

$$E(P) = \Phi\left(\frac{\bar{x}_i'\beta}{\sqrt{1+\sigma^2}}\right)$$

Westin (1974) used the same assumption with binary logit models.

3. Statistical Differential Procedure. This method is done by Talvitie (1973). The expectation of aggregate share is computed analytically by expanding the logit model using Taylor series expansion and truncating after second order terms to get a manageable expression for E(P). The expected aggregate share in the binary choice case is:

$$E(P_k) = \bar{P}_k\left(1 + \sigma^2(\bar{P}_k - 1)(\bar{P}_k - \tfrac{1}{2})\right)$$

where σ^2 = variance of net utility distribution in the prediction group, and

$\bar{P}_k = P_k$ evaluated at the mean of net utility function.

For multiple choice situation, (assuming that each utility function is stochastically independent)

$$E(P_k) = \bar{P}_k\left[1 + \sum_j \operatorname{var}(g(x_j))(\bar{P}_j - \theta)(\bar{P}_j - \tfrac{1}{2})\right],$$

$\theta=1$ when $k=j$

$\theta=0$ when $k \neq j$

$g(x_j)$ = utility function of alternative j.

4. Enumeration Procedure. The expected number of people choosing alternative k is

$$V_k = \sum_{i=1}^{T} P_{ik}$$

where P_{ik} is the probability of an individual i to choose alternative k.

Share of people choosing alternative k is

$$P_k = \frac{V_k}{T}$$

where T is the number of individuals in the prediction group. The above formula are shown by Talvitie (1973).

Although this enumeration procedure is the most theoretically consistent way, it requires complete knowledge of all individuals in the group and the attributes of available alternatives, which are not easily available.

Koppelman (1975) revised the above formulation as the random sample enumeration method.

$$V_k = \sum_{i=1}^{T_S} P_{ik} \cdot \frac{T}{T_S}$$

$$\bar{P}_k = \frac{V_k}{T} = \sum_{i=1}^{T_S} P_{ik} \cdot \frac{1}{T_S}$$

where T_S = number of individuals in the prediction sample, and

P_{ik} = their corresponding probabilities to choose alternative k.

In other words, a sample of individuals is used for the entire forecasting process. And aggregate forecasts are made by simply applying the appropriate sampling factors to the probabilities. Here, the within-zone variance still remains.

5. Classification Procedure (Koppelman, 1975). It is based on

--assigning the aggregate group into two or more homogeneous classes.

--predicting aggregate choice chares of each class using the naive method.

--computing the overall aggregate share as the weighted average of the class shares.

$$E(P) = \sum_G \bar{P}_G \quad \frac{T_G}{T} = \sum_G f(\bar{X}_G)\frac{T_G}{T}$$

where T_G = number of individuals in subgroup G,

T = number of individuals in the prediction group. and

$\bar{X}_G$ = vector of average variable values for individuals in subgroup G. Note that f is logit or probit function.

This method is used to reduce the variance of the net utility distributions by selecting variables for classification.

Among the five aggregation procedures, only the complete enumeration procedure is consistent with relevant theories of travel behavior. The second and third methods are applicable in the binary choice situations, but in the multiple choice situations numerical integration would be quite cumbersome and the Taylor series approximation is also quite unstable.

Koppelman conducted an analysis of the different procedures under different conditions (different levels of variances of net utility distribution, symmetric and skew distribution, and mean net utility values). Under symmetric distribution, the integration procedure has lowest biases followed by classification with two classes. Under skewed distribution, classification with three classes has the lower biases than integration. Bias is the greatest in naive procedure. But when distributions are skewed, the naive procedure has the least bias within a range of mean net utility values. The statistical differential procedure gives a high error with the prediction for groups of large variances. Simulation analysis indicates that numerical integration and classification perform better than naive and statistical differential procedures.

The result indicates that it is feasible to predict aggregate travel demand using an aggregation of the disaggregate choice model. But the result doesn't say how far aggregation can actually be carried such that the errors are within an acceptable limit.

The two procedures that are proposed as the most applicable in the multidimensional model are the classification (classification of subgroups according to differences in choice set availability, and according to variables which contribute most to the variance in the distribution of the net utility in order to increase the homogeniety) and random sample enumeration. Here the problem of within zone variance is reduced but still remains.

Furthermore, there must be more research to compare predictions based on aggregated disaggregate models with disaggregate data and predictions based on aggregate models (well-structured on behavioral foundation) with aggregate data.

B. Current Results to Intercity Passenger Demand Models

1. Aggregate Models

There have been studies in multimodal intercity passenger demand models with number of trips (or persons) as a dependent variable.

The descendants of the classic gravity distribution models (indirect demand model) which represent traffic between zones as a function of trip generation, attraction characteristics and some measure of impedance term (supply characteristics such as distance, price or travel time) performed poorly in predicting intercity passenger demand on a wide range of distances and city sizes.

Blackburn (1966) formulated a model which includes differences in tastes and income among individuals based on consumer's choice theory. But the difficulty involves solving nonlinear functions over many definite integrals and the unavailability of data which are quite different from the existing data.

The Kraft-Sarc, Baumol-Quandt and McLynn models which are based on gravity model are the representatives for intercity passenger demand models.

Billheimer (1972) used the intercity passenger demand model that was initially developed by McLynn (1969) to predict travel by mode within and around the state of Michigan. He carried out a complicated task requiring calibration process to obtain each of the model's parameters using constrained log-linear regression. The idea of segmentation on different city sizes is a noticeable point.

Bennett et al (1974) compared the seven intercity modal-split models, which are all calibrations of cross elasticity models that were initially developed by McLynn (1967, 1969), with the adoption of abstract-mode and stratification. The model parameters are estimated by linear regressions.

Recently, the direct demand models (aggregate simultaneous) were again applied to intercity application by Peers et al (1976). They considered both mode-specific and mode-abstract models. The use of non-linear regression significantly improved the standard estimation procedures over the linear regression technique which normally requires linear transformations and constraining variables. They have found that the aggregate direct-demand models successfully satisfied such objectives as policy sensitivity and demand response to alternative transportation systems. This brings enough attention that aggregate as well as disaggregate demand models should be developed further.

Watson (1974) demonstrated the superior predictability of disaggregate models over the aggregate models. But the question of different estimation methods (e.g. linear methods to aggregate models versus non-linear methods to disaggregate models) also contributes to errors in the prediction of the aggregate models. Poor predictability of aggregate model is due partly to sampling variances, partly to model specification errors and partly to aggregation errors (Talvitie, 1973) along with errors associated with estimation methods. Hence the poor predictability of aggregate models is not wholly due to the data aggregation (within zone variance) only. Talvitie (1973) further stated that the within zone variance problem still persists when aggregation of disaggregate model is performed (see previous aggregation section).

As have been done with disaggregate probabilistic models, there should be a well-specified structural or behavioral representation of the decision process in the aggregate models to apply to the intercity passenger travel demand models. That is, instead of just correlating existing travel behavioral pattern with socio-economic, activity system and level of service variables, the model specification must represent the casual relationships between these variables to obtain aggregate models with aggregate data.

2. Disaggregate Models

Watson's (1974) study was the mode-choice case on a relatively short distance city pair. He compared the predictions of mode-choice behavior by applying linear regression to the aggregate models and logit analysis to the disaggregate models in which both models are based on the same data set.

Stopher and Prashker (1976) applied the multinomial logit model to intercity passenger travel forecasting. Their analysis was only partially disaggregate because the models were developed by creating a record for each trip made in each corridor. That is, their data is 'quasi-disaggregate' data. Although they concluded that the data set they used was not suitable for the analysis, the models show a greater sensitivity to cost than to time and frequency. This again emphasizes that intercity trips over relatively long distances are more price-sensitive than intra-urban trips.

This brings enough attention that consumers tend to consider budget constraint when trip distance gets longer partly because of transportation cost and partly because of other accompanying costs.

CHAPTER III

QUALITATIVE VERSUS QUANTITATIVE CHOICE THEORY

A. Introduction

This chapter deals with the comparisons between qualitative and quantitative choice theory which are applicable to transportation demand function formulations. When the choice is discrete, it is more appropriate to apply probabilistic choice theory. When the choice is continuous, it is more appropriate to apply the classical consumption theory.

This chapter falls into four parts. The first part is the description of the probabilistic choice theory based on strict and random utility models. The second part is the review of the conventional consumption theory based on various utility functions of consumer preferences. The third part is the comparisons between qualitative and quantitative choice theory. In the fourth part , which is the most important, the transportation demand formulation based on the transcendental logarithmic utility function is introduced.

Before going into the qualitative and quantitative choice theory, some basic differences need to be considered.

In the qualitative choice, the choice is discrete (a trip versus no trip). We assume a probabilistic behavior which is the result of the probabilistic choice behavior based on strict utility or random utility models. The probabilistic choice theory is suitable to the qualitative choice.

In the quantitative choice, there is a continuous divisible commodity which may be demanded in any quantities. We assume an economic consumer behavior which is the result of the assumption that the consumer maximizes utility subject to budget constraint. The economic consumption theory is suitable to the quantitative choice.

In both cases, the utility is used to express the consumer preferences. However, the utility in the probabilistic choice theory is defined as a function of the attributes of a single alternative and hence each utility function is defined corresponding to each alternative. Whereas the utility in the consumption theory is defined over a space of n-commodities and attributes and hence only one utility function is defined.

McFadden (1974) had clarified the suitability of economic consumption theory to the continuous choice cases as well as the suitability of probabilistic choice theory to the discrete choice cases. It is noteworthy to review his discussion briefly regarding the qualitative choice and the quantitative choice comparisons.

In economic consumer behavior, the individual has a utility function $u=u(x,s,e)$ where x is the attribute variables such as prices, s is the socioeconomic characteristics and e is the unobserved disturbance term which is assumed to account for the taste variations among individuals and unmeasured attributes. The utility function is maximized subject to budget constraint and the system of demand functions derived are denoted as x=f(B;s,e). This demand analysis can be done with either cross-sectional data or time-series data.

Most empirical demand studies based on economic consumer behavior is (1) to ignore the possibility of taste variations in the sample, and (2) to make the assumption that the consumers have observed demands which are distributed randomly about the exact values for some common tastes. In the conventional demand study, where quantities vary continuously, it is reasonable to expect errors in the measurement of the chosen alternative to be significant, and perhaps dominate the effect of taste variations (McFadden, 1974). Hence in this case, demand modeling with a quantitative choice approach is suitable.

But in the case where the individual's discrete choice is made from the set of finite number of alternatives, each utility should be a function of each alternative. The discreteness of the travel alternatives implies that those alternatives are perfect substitutes for each other. Here, the probabilistic choice theory that considers a finite set of mutually exclusive alternatives from which only one alternative will be chosen can be easily applied (Ben-Akiva, 1973).

B. Qualitative Choice Theory

McFadden (1973) adequately discusses the qualitative choice theory. The following presentations are heavily drawn from McFadden (1973). Luce and Suppes (1965) approach two different ways to relate the choice probabilities and the utility functions. One way is called "constant utility (or strict utility) models" and the other is called "random utility models".

1. Strict Utility Models

The utility function is a fixed function over the set of alternatives and the choice probabilities are determined as a specific assumed function of the utility measures. Suppressing the socio-economic characteristics in the utility function in order to simplify the notation, we get:

$$V(x) = \sum_i z^i(x)\beta_i = z(x)'\beta$$

where $z^i(x)$ are empirical functions with no unknown parameters,

x is the vector of attributes of alternative k,

$z' = (z^1, z^2, \ldots, z^k)$ a row vector of the empirical function, and

$\beta = (\beta_1, \ldots, \beta_k)'$ a column vector of unknown parameters.

V is a linear function of the parameter vector .

z^i may be complex transformations of row data.

Let the probability of the individual choosing alternative i from the set A be P(i:A). Here, the axiom of usual probability theory should hold.

(axiom 1): $P(i:A) > 0$, $\sum P(i:A) = 1$ (positivity and summability)
Since there is no distinction between empirically zero probability and extremely small probability, one may assume without loss of generality that $P(i:A) > 0$.

(axiom 2): (Independence of irrelevant alternatives) This axiom is introduced by D. Luce (1959). The relative odds of one alternative being chosen over a second should be independent of the presence or absence of unchosen third alternatives. The probabilistic analog is that the ratio of the probabilities of choice of two alternatives is fixed and does not vary by changing the set of alternatives, i.e.

$\frac{P(i:A)}{P(j:A)} = \frac{P(i:B)}{P(j:B)}$ where $i,j \varepsilon A$, $A c B$

(axiom 3): (Irrelevance of alternative set effect) The utility function $V(x,y)$ has the additively separable form, $V(x,y) = V(x) - V(y)$.

Using the axioms and defining the logarithmic relationship between utility function (indirect) V and the ratio of choice probabilities, the strict utility model is constructively derived as

$$P(i:A) = \frac{e^{v_i}}{\sum_{j \varepsilon A} e^{v_j}} .$$

2. Random Utility Models

In the random utility models, the utility function is not a fixed function but a random variable. The utility function is expressed as $U(x) = V(x) + \varepsilon(x)$

where V is non-stochastic function and reflects "representative's tastes" of the consumer, and

ε is the stochastic and reflects the idiosyncracies of this individual.

Assuming an individual chooses an alternative which maximizes his utility, he will choose the alternative that gives him the most satisfaction. Then the probability that an individual drawn randomly from the population will choose alternative i from the set A equals:

$$\begin{aligned} P(i:A) &= P(U_i > U_j,\ j \neq i) \\ &= P(V_i + \varepsilon_i > V_j + \varepsilon_j,\ \forall j \neq i) \\ &= P(\varepsilon_j < \varepsilon_i + V_i - V_j,\ \forall j \neq i) \quad \ldots\ldots\ldots\ldots\ldots \text{(1)} \end{aligned}$$

where $V_i = V(x_i)$, $\varepsilon_i = \varepsilon(x_i)$, $U_i = V_i + \varepsilon_i$.

Let $F(\varepsilon_1, \ldots, \varepsilon_J)$ denote the cumulative joint distribution function of $(\varepsilon_1, \ldots, \varepsilon_J)$ and let F_i denote the partial derivative of F with respect to its i^{th} argument. Then,

$$P_i = \int_{-\infty}^{\infty} F_i(\varepsilon_i+V_i-V_1,\ \varepsilon_i+V_i-V_2,\ \ldots,\ \varepsilon_i+V_i-V_J)\, d\varepsilon_i$$

To simplify the notation, we drop the subscript i on ε and $d\varepsilon$, and we get

$$P_i = \int_{-\infty}^{\infty} F_i(\varepsilon+V_i-V_1,\ \varepsilon+V_i-V_2,\ \ldots,\ \varepsilon+V_i-V_j)\, d\varepsilon \quad \ldots\ldots \text{(2)}$$

For the binary choice case as well as the multiple choice case, different probability models result from different assumptions on the distribution of the error terms. For the binary choice case, rewriting the above equation,

$$P_1 = F\big(V(x_1) - V(x_2)\big)$$

where F is the cumulative distribution function of the difference of the random variables $\varepsilon(x_2) - \varepsilon(x_1)$.

If $\varepsilon(x_j)$ are distributed jointly and normally, then the binary probit model will result.

$P_1 = \Phi[V(x_1) - V(x_2)]$

where Φ is the cumulative standard normal curve.

This probit model holds when ε_1 and ε_2 are independent with identical means and variances. But the probit model also holds with the inclusion of covariance $(\varepsilon_1, \varepsilon_2)$ when ε_1 and ε_2 are dependent. Other random utility models will result with different assumptions on the distribution of the stochastic error terms (Domencich and McFadden, 1975).

For the multiple choice case, multinomial probit model results if the error terms are assumed to be distributed independently normal distributions. But the numerical integration technique to obtain the choice probabilities is considered too cumbersome and costly for practical usage (for more discussion see Chapter IV, C. Estimation Methods).

For binary and multiple choice cases, the multi-logit model results if the error terms are independently identically Weibull distributed. Reviewing the derivation of the multi-logit model, error terms are Weibull distributed, i.e.

$$P(\varepsilon_j \le \varepsilon) = e^{-e^{-\varepsilon}}$$

Then,

$$F(\varepsilon+V_i-V_i,\ \varepsilon+V_i-V_2,\ \ldots,\ \varepsilon+V_i-V_J) = \prod_{j=1}^{J} e^{-e^{-(\varepsilon+V_i-V_j)}}$$

$$= \left(e^{-e^{-\varepsilon}}\right)^{\sum_{j=1}^{J} e^{(V_j-V_i)}} \quad \ldots (3)$$

$$F_i(\varepsilon+V_i-V_1,\ \varepsilon+V_iV_2,\ \ldots,\ \varepsilon+V_i-V_J) = \frac{\partial}{\partial\varepsilon}\left(e^{-e^{-\varepsilon}}\right)^{\sum_{j=1}^{J} e^{(V_j-V_i)}}$$

$$= e^{-\varepsilon} \prod_{j=1}^{J} e^{-e^{-(\varepsilon+V_i-V_j)}} \quad \ldots (4)$$

from Eqs. (2), (4),

$$P_i = \int_{-\infty}^{\infty} e^{-\varepsilon}\left(e^{-e^{-\varepsilon}}\right)^{\sum_{j=1}^{J} e^{(V_j - V_i)}} d\varepsilon = \frac{e^{V_i}}{\sum_{j=1}^{J} e^{V_j}}$$

$$P_i = \frac{e^{V_i}}{\sum_{j=1}^{J} e^{V_j}}$$

This is the multi-logit model which has equivalent expression as the strict utility model. The strict utility model is an independent random utility model. The multilogit model is the model most predominantly used in the qualitative choice analysis of transportation.

Critique on the Independence of Irrelevant Alternatives Axiom:

The advantage of assuming this axiom is above all, it simplifies the analysis. This axiom allows the introduction of new alternatives without the re-estimation of the model. Addition of a new alternative involves the introduction of the corresponding utility function and its choice probability with the introduction of one more term in the denominator term in all the choice probabilities of the alternatives.

On the other hand, this axiom is at the same time a disadvantage because all the alternatives are assumed to be completely distinct and independent of each other. In actuality, some of the alternatives are related. Hence this axiom precludes the differential substitutability and complementarity between alternatives.

This axiom allows the separability of choice probability and also the separability of utility function. Here the separability refers to weak separability (see section on utility function). The separability of utility function implies that the marginal rate of substitution among variables in the alternative set (e.g. mode choice set) is independent of other variables in other alternative set (e.g. destination choice set). The separability of choice probability implies that the conditional probability (e.g. probability of mode m given destination d) for a given choice depends only on the alternatives for the given set (e.g. destination). It is independent of the alternatives (modes) to all other sets (destinations); but it is dependent in reality.

The separability of choice probability enables the use of the recursive (sequential) methods to factor a series of separate choice models. Again it is a strong assumption (Ben-Akiva, 1973).

C. Quantitative Choice Theory

The economic consumption theory is applied to the quantitative choice theory. The consumption theory is concerned with a "representative" individual. It assumes that this individual maximizes utility subject to budget constraint.

In order to see the advantage and significance of the translog utility model, knowledge on consumption theory is necessary.

1. Restrictions on Economic Demand Functions

Properties any demand system should have if it is derived from any utility function are as follows (Theil, 1975; Brown and Deaton, 1972):

(a) Equilibrium Condition (Summability). The reallocations of the budget due to income and price changes respectively must separability of utility function implies that the marginal rate of substitution among variables in the alternative set (e.g. mode choice set) is independent of other variables in other alternative set (e.g. destination choice set). The separability of choice probability implies that the conditional probability (e.g. probability of mode m given destination d) for a given choice depends only on the alternatives for the given set (e.g. destination). It is independent of the alternatives (modes) to all other sets (destinations); but it is dependent in reality.

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Properties any demand system should have if it is derived from any utility function are as follows (Theil, 1975; Brown and Deaton, 1972):

(a) Equilibrium Condition (Summability). The reallocations of the budget due to income and price changes respectively must continue to exhaust total expenditure.

$$\sum_{i=1}^{n} \frac{p_i x_i}{M} = 1$$

i.e. budget constraint has to be satisfied before and after income change.

(b) Homogeneity Restrictions. The demand equations are homogeneous of degree zero in income and prices. (Proportional changes in all prices and income leave the choice of commodities unchanged.)

$$x(kM,kp) = x(M,p) \text{ for any } k>0$$

(c) Symmetry Restrictions of Substitution (often called Slutsky condition). When income and prices change, the substitution terms are symmetric.

$$\frac{\partial x_i}{\partial P_j} + x_j \frac{\partial x_i}{\partial M} = \frac{\partial x_j}{\partial P_i} + x_i \frac{\partial x_j}{\partial M}$$

i.e. the effect of a change in the i^{th} price on the j^{th} compensated demand is the same as the effect of a change in the j^{th} price on the i^{th} compensated demand (confer pp. 66 and 67).

(d) Negativity. If the demand equation $x=x(M,p)$ represents a maximum of the utility function, small changes in x in the neighborhood of the optimum must lead to a decrease in utility. The diagonal terms of the Hessian of the utility function are all negative. This is the famous "law of demand" that own-price compensated elasticities of demand are negative.

(e) Integrability of the difference equation is necessary if that equation is to be derivable from the demand equation at all, i.e. if fundamental equations

$$\begin{pmatrix} U & P \\ P' & 0 \end{pmatrix} \begin{pmatrix} dx \\ -d\lambda \end{pmatrix} = \begin{pmatrix} \lambda dP \\ dM - x'dP \end{pmatrix}$$

where U is the utility function, and

λ is the Lagrangian multiplier (also marginal utility of income),

are to be derivable from $x=x(M,p)$. The economic meaning of the integrability is consistency of choice. Consequently if demand functions exist satisfying the four constraints or if differential demand functions exist satisfying all five, then for all practical purposes we may regard the utility theory as valid and we are guaranteed that there is no conflict between that theory and the evidence before us.

Aggregation:

For the empirical analysis of the consumer demand model system, the use of aggregate data is inevitable. If the postulates of the theory turn out to be rejected by aggregate data, we may not be wise to reject the basic model but rather we should reconsider the appropriateness of the implicit method of aggregation. In the economic demand theory we assume that the "representative consumer" acts like an ideal consumer and use the micro-theory in order to derive demand equations. In the application of the economic demand theory, data almost inevitably relate to groups of consumers. Hence aggregation error always exists but it may not significantly add to errors of measurement and omission of variables which are inevitably present (see Chapter III, Introduction).

Gorman (1959), Green (1964), and many others (Pearce, 1964; Theil, 1965; Barten and Turnovsky, 1966; etc.) have investigated the conditions under which aggregation of consumer demand equations can be made. Some of the results turn out to be very stringent and have some limitations for practical usage.

Hence some aggregation is always necessary but at the present stage, we may consider that aggregation is less important when we are dealing with consumer theory.

2. Study of Utility Functions

(a) Direct and Indirect Utility Functions. The basic notion of classical consumption theory is that of a preference ordering, a relation P('preferred to') which applies to points x in the 'commodity space'. However, a preference ordering does not determine a utility function uniquely, but only up to a monotonic transformation. (Thus, if we replace U(x) by a monotonically increasing function $e^{U(x)}$, then the resulting utility function corresponds to exactly the same preference ordering.)

The direct utility function $U(x_1, x_2, \ldots, x_n)$ and the indirect utility function $V(p_1, p_2, \ldots, p_n, M)$ are the two ways of describing a given preference ordering. The direct utility function is a function of all elements of the commodity bundles x, which is constant on any indifference surface for which $U(x_1,x_2,\ldots,x_n)>U(x_1^o,x_2^o,\ldots,x_n^o)$ iff xPx^o.

To see the relationship between direct and indirect utility functions, consider the following:

Maximize $U = U(x_1, x_2, \ldots, x_n)$ subject to $\sum_{i=1}^{n} p_i x_i = M$.

Then the primal function is the Lagrangian function

$$L = U(x_1, \ldots, x_n) - \lambda(\sum_{i=1}^{n} p_i x_i - M)$$

The necessary conditions for an optimum are

$$\frac{\partial L}{\partial x_i} = \frac{\partial U}{\partial x_i} - \lambda p_i = 0 \qquad \text{for } i = 1, 2, \ldots, n$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} p_i x_i - M = 0$$

then

$$\frac{\partial U}{\partial x_i} = \lambda p_i \quad (\text{for } i = 1, \ldots, n) \quad \ldots \quad (1)$$

$$\sum_{i=1}^{n} p_i x_i = M \quad \ldots \quad (2)$$

where p_i's and M are known. Solving the equations (1) and (2), we get the demand equations:

$$x_i = x_i(p_1, \ldots, p_n, M)$$

When substituting demand equations to the utility function,

$$U = U[x_1(p_1,\ldots,p_n,M),\ldots,x_n(p_1,\ldots,p_n,M)] = V(p_1,\ldots,p_n,M)$$

This utility function V expressed in terms of prices and expenditure is called the indirect utility function.

Duality exists between the direct and indirect utility function (Lau 1970).

	Primal	Dual
Function	Direct utility function	Indirect utility function
Variables	Quantities $(x_1,\ldots,x_n)$	Prices and expenditure $(p_1,\ldots,p_n,M)$
Lagrangian multiplier	λ	λ

(b) Separability of Utility Function (Goldman and Uzawa, 1964). Let the set of all finite number of n commodities (n variables) be denoted by N. Then the partition of the set N into mutually exclusive and exhaustive subsets is denoted as

$$N = N_1 \cup \ldots \cup N_S, \ N_s \cap N_t = \phi \quad \text{for } s \neq t .$$

Let $\{N_1, \ldots, N_s\}$ be a partition of the set N (the set of variables in the utility function) and U(x) be a utility function for a preference relation.

The utility function U(x) is called strongly separable with respect to the partition $\{N_1, \ldots, N_s\}$ if the marginal rate of substitution between two variables x_i and x_j (where $x_i \varepsilon N_s$, $x_j \varepsilon N_t$) is independent of the variables outside of N_s and N_t; i.e.

$$\frac{\partial MRS_{ij}}{\partial x_k} = 0 ,$$

where

$$MRS_{ij} = \frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial x_j}} \neq 0 \text{ and } x_k \ \varepsilon \ N_k .$$

The utility function U(x) is called weakly separable with respect to the partition $\{N_1, \ldots, N_s\}$ if marginal rate of substitution between two variables x_i and x_j from N_s is independent of the rest of the variables outside of N_s.

$$\frac{\partial MRS_{ij}}{\partial x_k} = 0 \quad \text{where } x_k \ \varepsilon \ N_k .$$

Separability by Strotz (1957) is equal to weak separability.

(c) Quadratic Utility Function. The quadratic utility function is expressed as:

$U(x) = a_0 + a'x + x'H\,x$

where a' is a row vector consisting of n coefficients,

x is a column vector of n commodity quantities, and

H is a symmetric Hessian matrix (n by n).

The utility function has continuous derivatives up to the 3rd order and the 1st order derivatives are all positive, which insures that a larger quantity of any commodity leads to higher utility.

Since H (Hessian matrix $\frac{\partial^2 U}{\partial x_i \partial x_j}$) is negative definite, H^{-1} is also negative definite. In particular, elements of H^{-1} along the principal diagonal are negative ($\frac{\partial^2 U}{\partial x_i^2} < 0$). This property implies that the marginal utility of each commodity is a decreasing function of its own quantity.

The symmetry of the Hessian ($\frac{\partial^2 U}{\partial x_i \partial x_j} = \frac{\partial^2 U}{\partial x_j \partial x_i}$) is due to the fact that the effect of a change in the i^{th} commodity price on the j^{th} commodity demand is the same as the effect of a change in the j^{th} commodity price on the i^{th} commodity demand. By assuming that the Hessian matrix is not only symmetric but negative definite, we can ensure that the demand equation corresponds to a constrained maximum rather than a minimum or a saddle point (see Theil, 1975).

(d) Homotheticity, Additivity and Elasticity of Substitution.

(Dfn. 1): A utility function is homothetic if it can be written in the form $U = F[f(x_1,\ldots,x_n)]$ where F is a positive, finite, continuous and strictly monotonically increasing function of one variable with $F(0) = 0$ and f is a homogeneous function of degree 1 of n variables. Intuitively, if a utility function is homothetic, the shape of the indifference curves is same, hence Engel curves are straight lines.

(Dfn. 2): A function is homogeneous of r^{th} degree if

$f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) = \lambda^r f(x_1, \ldots, x_n)$.

If r=1, then f is called linear homogeneous function.

(Remark): Every utility function of homogeneous of degree r is homothetic (because any homogeneous of r^{th} degree is the transformation of monotonically increasing function of homogeneous of degree 1.)

(Property 1): If a utility function is homothetic, expenditure proportions are independent of total expenditure, to show:

$$\frac{\partial}{\partial M}\left(\frac{x_i p_i}{M}\right) = 0 \; .$$

If a utility function is homothetic, its income elasticities of the demand equation is equal to 1. (Every homogeneous utility function has its income elasticities equal to one.)

I.e.,

$$\frac{\partial x_i}{\partial M} \cdot \frac{M}{x_i} = 1$$

but homotheticity implies

$$\frac{\partial x_i}{\partial M} = C \text{ (constant)}$$

hence

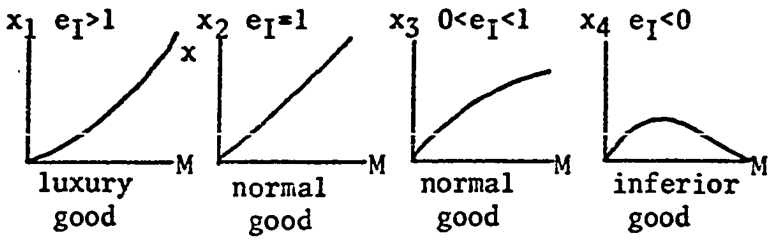
$$\frac{M}{x_i} = \frac{1}{C} \rightarrow \frac{x_i}{M} = C$$

$$\frac{p_i x_i}{M} = p_i C$$

$$\frac{\partial}{\partial M}\left(\frac{p_i x_i}{M}\right) = \frac{\partial}{\partial M}(p_i C) = 0 \; .$$

Therefore, expenditure proportions are independent of total expenditure.

(Remark 2): The relationship between the quantity of commodity demanded and income (prices fixed) is called Engel curve.



If the utility function is non-homothetic, the Engel curve need not be a straight line. And in actuality, as income increases, the budget share changes.

(Dfn.3): Additive utility function (or preference independence).

A utility function is additive if it can be written as:

$$U(x) = \sum_{i=1}^{n} u_i(x_i): \text{ direct additivity}$$

$$V\left(\frac{P}{M}\right) = \sum_{i=1}^{n} v_i\left(\frac{P_i}{M}\right): \text{ indirect additivity}$$

The marginal utility of the i^{th} commodity du_i/dx_i depends only on the i^{th} quantity. This implies that all 2nd order cross derivatives of the utility function are identically zero, so that the Hessian of the utility function H and H^{-1} are diagonal matrices. Hence each entry $u_{ij}=0$ whenever $i \neq j$. No commodity is a specific substitute or complement of any other commodity. I.e. additivity assumption implies the neglect of "related goods" in which additivity assumptions appear too strong.

(Property 2): If a utility function is additive and homothetic, elasticities of substitution among all pairs of commodities are constant and equal (Jorgenson, Christensen and Lau, 1975).

E.g. CES (constant elasticity of substitution) function

$$u^{\sigma} = \sum_{i=1}^{n} a_i x_i^{\sigma}$$

Elasticity of Substitution. Let the utility function be represented by $U(x_1, x_2, \ldots, x_n)$. Then the marginal rate of substitution of the commodity x_j for the commodity x_i (the tangent gradient along the utility curve):

$$MRS = -\frac{dx_j}{dx_i} = \frac{Ux_i}{Ux_j} = \frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial x_j}} = \frac{P_i}{P_j}$$

MRS represents the additional amount of x_j, necessary to maintain the utility unchanged when a small unit reduction is made in x_i. The convexity of the utility function implies the increasing marginal rate of substitution and it becomes increasingly more difficult to substitute x_j for x_i as the substitution proceeds. In order to determine how fast MRS increases, the definition of elasticity of substitution is necessary.

(Dfn.4): The elasticity of substitution between x_i and x_j is defined as taking values between zero and infinity according to the ease with which x_i and x_j can be substituted in consumption to maintain a given level of indifference.

$$\delta_{ij} = \frac{d\ \ell n(x_j/x_i)}{d\ \ell n(MRS_{x_i,x_j})}$$ but $MRS_{x_i,x_j} = P_i/P_j$ when utility maximization subject to budget is done.

Therefore, $\delta_{ij} = \dfrac{d\ \ell n(x_j/x_i)}{d\ \ell n(P_i/P_j)}$.

In the case of the CES function, the elasticity of subsitution is given δ_{ij} = constant (Arrow, Chenery, Minhas and Solow, 1961). $\delta_{ij} = 1$ if the utility function is of the Cobb-Douglas type.

3. Systems of Economic Demand Functions

Note that in the discussion of consumer theory, utility functions are expressed only in terms of quantities (x) or prices (p) and income (M). No attribute variables are considered in the utility functions and also in the demand functions. But attribute variables can be included in the utility functions and hence in the demand functions of comsumer theory (see section on translog utility model).

(a) System of Double Logarithmic Functions.

$$U = x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n} \text{ (Cobb-Douglas utility function)}$$

$$\ln U = \alpha_1 \ln x_1 + \alpha_2 \ln x_2 + \ldots + \alpha_n \ln x_n$$

where x_i stands for the quantity of i^{th} commodity.

The demand function is:

$$\ln x_i = \alpha_i + \alpha_{iM} \ln M + \sum_{j=1}^{n} \alpha_{ij} \ln p_j$$

where $i=1,2,\ldots,n$

Note that all intercity travel demand models (McLynn, Kraft-Sarc, and Baumol-Quandt) have double logarithmic demand functional forms.

(b) Linear Expenditure System. This system of demand function is proposed by Stone (1954). Given the Klein-Rubin utility function (1947-48),

$$U(x) = \sum_i \alpha_i \ln(x_i - r_i)$$

where α_i, r_i are parameters.

The demand equation is:

$$p_i x_i = p_i r_i + \alpha_i (M - \sum_{i=1}^{n} p_i r_i)$$

$$x_i = r_i + \frac{\alpha_i}{p_i} (M - \sum_{i=1}^{n} p_i r_i)$$

(c) Rotterdam Demand System. The following system of equations is proposed by Theil (1965). The Rotterdam model is not based for one type of utility function in particular. It provides an approximation to a demand equation system based on an arbitrary utility function which satisfies the assumptions:

--All variables (p,x,M) can be varied continuously.

--When utility is maximized subject to budget constraint for any price-income point in the region (p,M>0), the solution x° is unique and has strictly positive component.

--The utility has continuous derivatives up to 3rd order and 1st order derivatives are all positive. By assuming that the Hessian of the utility is not only symmetric but also negative definite, the solution x corresponds to a constrained maximum rather than a minimum or a saddle point.

The Rotterdam system of demand functions is consistent with utility maximization only if the utility function is linear logarithmic.

Suppose that the Rotterdam model has strictly constant price coefficients in its infinitesmal demand equations. The Rotterdam model can be expressed as:

$$\omega_i d(\ell n\ x_i) = B_i[d(\ell n\ M) - \sum_{k=1}^{n} \omega_k d(\ell n\ P_k)] + \sum_{j=1}^{n} C_{ij}[d(\ell n\ p_j) - \sum_{k=1}^{n} B_k d(\ell n p_k)]$$

where d stands for differential, $(i=1,2,\ldots,n)$

$$\omega_i = \frac{p_i x_i}{M} \text{ and } B_i \text{ and } C_{ij} \text{ are constants}$$

satisfying $C_{ij} = C_{ji}$

$\sum_i B_i = 1$

$\sum_j C_{ij} = \psi B_i$ and $\psi = \frac{\partial \ \ln M}{\partial \ \ln \lambda}$

λ is lagrangian multiplier.

Using $\sum_{j=1}^{n} C_{ij} = \psi B_i$, the system reduces to partial differential equations (McFadden) and the resulting demand system is of the double logarithmic function which is the result of the maximization of a utility function of Cobb-Douglas type ($U = \sum_{i=1}^{n} B_i \ \ln x_i$).

(d) Indirect Addilog Model. The addilog model is proposed by Houthakker (1960). The indirect utility is:

$$V = a_1\left(\frac{p_1}{M}\right)^{\alpha_1} + a_2\left(\frac{p_2}{M}\right)^{\alpha_2} + \dots + a_n\left(\frac{p_n}{M}\right)^{\alpha_n}$$

The demand equation is:

$$x_i = \frac{a_i\alpha_i\left(\frac{p_i}{M}\right)^{\alpha_i - 1}}{\sum_{k=1}^{n} a_k\alpha_k\left(\frac{p_k}{M}\right)^{\alpha_k}}$$

(Note: The unknown parameters a_k, α_k are in the product form.) The following double-log expression is preferable for empirical calculation:

$$\ln x_i - \ln x_j = \ln(a_i\alpha_i) - \ln(a_j\alpha_j) + (\alpha_i - 1)\ln\left(\frac{p_i}{M}\right) - (\alpha_j - 1)\ln\left(\frac{p_j}{M}\right)$$

Note that the share form is similar to the form of gravity formula.

The trip volume share in the transportation gravity formula is:

$$\frac{V_{k\ell}}{V_k} = \frac{V_\ell \cdot \omega_{k\ell}}{\sum_{\ell'} V'_\ell \cdot \omega_{k\ell}}$$

whereas the indirect addilog share is:

$$\frac{p_i x_i}{M} = \frac{a_i \alpha_i \left(\frac{p_i}{M}\right)^{\alpha_i}}{\sum_{k=1}^{n} a_k \alpha_k \left(\frac{p_k}{M}\right)^{\alpha_k}}$$

4. Comparisons of Demand Functions

(Christensen, Jorgenson and Lau, 1975).

(a) Double Logarithmic Demand Functions.

--Linear logarithmic utility function is both additive and homothetic (Cobb-Douglas utility function).

--All expenditure proportions are constant.

--Income elasticity is unity.

--Elasticities of substitution among all pairs of commodities are constant and equal to unity.

(b) Linear Expenditure System. The utility function is linear in the logarithms of quantity consumed less a constant for each commodity. The constants are interpreted as initial commitments.

--If constants = 0, the utility is linear logarithmic in form.

--If constants $\neq$ 0, then expenditure proportions vary with total expenditure. The incremental expenditure proportions derived from quantities consumed in excess of the initial commitments are constant for all variations in total expenditure and in prices.

--Income elasticity $\neq$ 1.

(c) Rotterdam System of Demand Functions. The Rotterdam system of demand functions is consistent with utility maximization only if the utility function is linear logarithmic. Then all the properties are the same as the double logarithmic demand function explained above.

(d) Indirect Addilog System. Utility function is additive in functions that are homogeneous in quantity consumed for each commodity.

--The degree of homogeneity may differ from commodity to commodity, permitting expenditure proportions to vary with total expenditure.

--If the degree of homogeneity is the same for all commodities, the addilog utility function is additive and homothetic (CES function), elasticities of substitution among all pairs of commodities are constant and equal.

--Income elasticity = 1.

(Note): Since the well-known McLynn, Kraft-Sarc and Baumol-Quandt models have double logarithmic functional form while the Talvitie's model has mixed form (logarithmic terms and non-logarithmic terms), it is useful to go over some critiques on double logarithmic demand.

The double logarithmic functional form is widely used in empirical analysis because of its functional simplicity (log-linear). The elasticities are constants at all values of the exogeneous variables. It is a convenient methodology but we should not expect it to be true all the time. Past studies have shown that consumer's trip making decreases as residential density rises, increasing population increases residential density, the number of trips increases at a decreasing rate as income increases, and also the number of trips increases as the level of service of the modes improves.

Thus even if the model fits the data well when estimated, we know that if it is used to project forward it will eventually lead to unsatisfactory results. Obviously we need a model with changing elasticities and we need some theory describing how we might expect the elasticities to change.

(e) Translog Utility Model. The translog utility function which will be introduced later is quadratic in the logarithm of the quantities consumed. This utility function is neither additive nor homothetic. They allow expenditure proportions to vary with the level of total expenditure and permit a greater variety of substitution patterns among commodities than functions based on constant and equal elasticities of substitution among all pairs of commodities. The income elasticity is not unitary in general. Hence the use of the translog utility function is more realistic than other utility functions.

D. Comparative Study of Qualitative and Quantitative Choice Theory

Talvitie (1975) investigated the limitations of behavioral probabilistic qualitative choice theory and some comparisons between qualitative and quantitative choice theory.

In this section, comparison between qualitative and quantitative choice theory is carried out by means of the comparisons between multi-logit models and the translog models.

1.

In the qualitative choice case, the analysis is based on disaggregate data, i.e. individual data are used in the analysis.

In the quantitative choice case, the analysis is based on aggregate data. The 'per capita' terms are used to neutralize the variation in the population. The demand equation derived is a 'per capita' demand equation of the 'representative' consumer.

2.

The utility function in the qualitative choice case is linear and additively separable. The linearity of the utility function is a strong assumption in the demand theory. Additivity implies neglect of related goods or related variables. But both additivity and linearity simplifies the analysis. The utility function is homogeneous and also homothetic. By assuming that the income spent on a particular group of commodity (transportation) is very small, the utility function is maximized without budget constraint.

The utility function in the quantitative choice case (translog utility) is quadratic and hence non-linear which is a more realistic start. The utility function is maximized subject to budget constraint.

The utility function can be varied under the restrictions implied by the demand theory (equality and symmetry) and under the restrictions on the form of the utility function. Under equality and symmetry restriction, income elasticity is not unitary and the expenditure pattern of a consumer changes with rising income, i.e. Engel curves are not straight lines.

3.

The 'independence of irrelevant alternatives' property in the qualitative choice theory precludes the differential substitutability between alternatives. In actuality, there are cases in which the alternatives are dependent upon each other and the random error terms of the random utility function are correlated with each other (Talvitie, 1975).

The demand functions based on translog utility model have the property that the elasticity of substitution is not constant and hence permit a greater variety of substitution patterns among alternatives. This is a more realistic behavioral observation.

4.

In the qualitative choice case, the aggregation of the disaggregate travel demand predictions is needed for transportation forecasting purposes. (See aggregation section in Chapter II.)

In quantitative choice case, microeconomic theory is applied to the 'per capita' terms of the aggregate data. When we are dealing with consumer theory, we may ignore the aggregation.

5.

In the qualitative choice case, the independence of irrelevant alternative property gives the separability of choice probability as well as separability of utility functions. The separability of choice probability enables the recursive methods to factor simultaneous travel decisions into the set of travel alternatives--trip frequency (f), destination (d), time of day (h), mode (m) and route (r).

In the quantitative choice case, especially with translog model, we can assume the separability of choice decisions by assuming separability of utility functions and the resulting restrictions on the parameters among the independent variables. Christensen, Jorgenson and Lau (1975) developed tests of a series of possible restrictions on the underlying structure of consumer preferences. They have considered groupwise separability, overall homotheticity, groupwise homotheticity restrictions on preferences and groupwise linear logarithmic utility as a possible restriction on preferences. More study is needed on the separability of the utility function and on its application to the travel choice situations in either simultaneous choice or recursive choice cases.

6.

As for the cost-benefit analysis, the conventional consumer surplus arguments is useful for transportation project evaluation. The consumer surplus consideration is based on the maximization of a 'representative' consumer's utility subject to constraint and is typically translated to the maximization of the value of consumer benefits.

However, the area under the market demand curve does not properly represent the consumer surplus. To measure the consumer surplus correctly, it is assumed that marginal utility of money (income) is constant and utility is additively separable into money (Diamond and McFadden, 1974). Under these assumptions, the market demand curve (Marshallian) becomes a Hicksian income compensated demand curve on which the level of utility is the same along the demand curve.

The translog utility model which is based on the conventional consumption theory is directly applicable to consumer surplus calculations with proper restrictions on the utility function which will generate zero income effect.

In the qualitative choice case where the choice is discrete and the individual tastes vary among population, the conventional consumer surplus arguments are not directly applicable. McFadden and Domencich (1975) have considered the applicability of the consumer surplus approach in the qualitative choice situation with proper assumptions on the structure of utility function and the definition of social welfare. They define the measure of social welfare by assuming that the utility is additively separable in money and in other attributes of the alternatives and that this money is transferable across individuals. In the analysis, both benefits and costs are expressed in per capita terms. Also the demand which is estimated by the choice probability is expressed in per capita terms. In effect, it is the translation of the qualitative choice situation to the quantitative choice case such that the conventional consumer surplus arguments may be applied.

7.

The comparisons of the estimation methods between logit model and translog model are shown in Chapter IV. C. Estimation Methods.

As a final remark, the development of quantitative choice in the transportation demand modeling is as important as that of qualitative choice because of its complementarity and the direct interrelationship between the two types of the theory.

E. Translog Utility Models

1. Derivation of Translog Models

There have been many studies in travel demand modeling based on consumer theory but with qualitative choice approach.

McGillivrary (1970) applied consumer theory in the construction of demand and choice models of binary modal split. The utility function is assumed to be linear by assuming the covariance matrices of the two populations of auto and transit users being equal. In his analysis, the choice of mode is discrete, that is the quantity of trip is 1 if auto is chosen and 0 if transit is chosen for each individual.

Golob and Beckman (1971) approached the problem of predicting individual's travel behavior in terms of economic utility theory and derived different travel demand models from different utility functions. Their work also indicated the potential applicability of economic consumption theory to travel demand modeling. But their work was not specific enough to carry out an actual travel demand modeling.

Before going into the translog utility model, we admit the fact that all transportation demand functions are formulated with the assumption that there exists a demand function to allocate the expenditure on each group (food, clothing, housing, transportation, etc.) on the first stage. Here the allocation of income to the transportation group is a fixed proportion of the total income. And all transportation demand functions are derived with the assumption of separability on their utility functions (the separability refers to weak separability(Ben-Akiva, 1973)). In other words, marginal rate of substitution between two variables of the transportation group is independent of the rest of the commodities outside the transportation sector.

Based on the theoretical work of Christensen, Jorgenson and Lau (1975), the passenger miles of travel demand model is derived. The model is based on a consistent system of consumer demand theory. But the model includes other transportation attributes besides prices and income. Jorgenson (1974) considered time trend variable in addition to prices and income in the translog demand modeling. Going over briefly the derivation of translog model, the direct transcendental logarithmic utility function (instead of the plain utility function $U = U(x_1, \ldots, x_n, A_1, \ldots, A_m)$) can be represented as:

$$\ell n\, U = \ell n\, U(x_1, \ldots, x_n, A_1, \ldots, A_m)$$

where x_i is the quantity consumed of the i^{th} trip commodity, and

A_j is the j^{th} transportation attribute of a 'representative individual'.

The consumer maximizes utility subject to the budget constraint.

$$\sum_{i=1}^{n} p_i x_i = M$$

For the present demand equation formulation, the use of indirect translog utility function is desirable instead of direct translog utility function because the budget share equation derived from the direct translog utility function has dependent variable appearing not only on the left hand side but also on the right hand side of the equation. That is, the budget share equation using direct utility function is as follows:

Maximize $U = U(x_1, \ldots, x_n, A_1, \ldots, A_m)$ subject to $\sum_{i=1}^{n} p_i x_i = M$

$$\frac{\partial U}{\partial x_i} = \lambda p_i \quad \ldots \quad (1)$$

The transcendental logarithmic utility function is $\ell n\ U = \ell n\ U(x_1, x_2, \ldots, x_n, A_1, \ldots, A_m)$, then

$$\frac{\partial\ \ell n\ U}{\partial\ \ell n\ x_i} = \frac{x_i}{U} \cdot \frac{\partial U}{\partial x_i} \ \underline{\underline{\text{by (1)}}}\ \frac{x_i}{U} \cdot \lambda p_i = \frac{\lambda}{U} \cdot p_i x_i \quad \ldots \quad (2)$$

$$\sum_i \frac{\partial\ \ell n\ U}{\partial\ \ell n\ x_i} = \frac{\lambda}{U} \sum_i p_i x_i = \frac{\lambda}{U} \cdot M$$

$$\frac{\lambda}{U} = \frac{1}{M} \sum_{i=1}^{n} \frac{\partial\ \ell n\ U}{\partial\ \ell n\ x_i} \quad \ldots \quad (3)$$

Substituting (3) to (2), we get

$$\frac{p_i x_i}{M} = \frac{\left(\frac{\partial\ \ell n\ U}{\partial\ \ell n\ x_i}\right)}{\sum_{i=1}^{n} \left(\frac{\partial\ \ell n\ U}{\partial\ \ell n\ x_i}\right)} \quad \ldots \quad (4)$$

Expressing direct translog utility function by functions that are quadratic in the logarithm of x_i's and A_j's,

$$\ell nU = a_0 + \sum_{i=1}^{n} a_i \ell nx_i + \sum_{j=1}^{m} a_{n+j} \ell nA_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{n} b_{ij} \ell nx_i \cdot \ell nx_j$$

$$+ \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij} \ell nx_i \cdot \ell nA_j + \frac{1}{2} \sum_{k=1}^{m} \sum_{j=1}^{m} d_{jk} \ell nA_j \cdot \ell nA_k \ .$$

Using equation (4),

$$\frac{p_i x_i}{M} = \frac{a_i + \sum_{j=1}^{n} b_{ij} \ell nx_j + \sum_{j=1}^{m} c_{ij} \ell nA_j}{a_M + \sum_{j=1}^{n} b_{Mj} \ell nx_j + \sum_{j=1}^{m} c_{Mj} \ell nA_j}$$

where

$$a_M = \sum_{i=1}^{n} a_i ,$$

$$b_{Mj} = \sum_{i=1}^{n} b_{ij} , \text{ and}$$

$$c_{Mj} = \sum_{i=1}^{n} c_{ij} .$$

The budget share has implicit expression in terms of dependent variable x_i's. Here, we get 'indirect' demand functions.

The indirect utility function is useful in characterizing the system of direct demand functions ('direct' means the endogeneous variable (x_i) is expressed by only exogeneous variables (prices, income and other attributes)), giving total passenger miles of travel by each mode as functions of ratios of price to total expenditure and as functions of its transportation attributes. We can express the indirect translog utility function by functions that are quadratic in the logarithms of the ratios of prices to total expenditure and in its transportation attributes.

The resulting indirect utility function provides a local second-order approximation to any indirect utility function. These indirect utility functions are not required to be additive or homothetic; therefore, the expenditure proportion is dependent on expenditure, while additive and homothetic utility function implies that the expenditure proportion is invariant. In reality, expenditure proportion depends on income as Engel shows that the proportion of food items decline as income increases (as expenditure increases).

For simplicity, the indirect utility function involving three trip commodities with two attributes, time trend and speed factor is considered:

$$\ell n\, V = a_0 + a'Z + \frac{1}{2} Z'BZ$$

where a_0 = constant,

$$a' = (a_1, a_2. a_3, a_t, a_s)$$

$$Z' = (\ell n \frac{P1}{M}, \ell n \frac{P2}{M}, \ell n \frac{P3}{M}, \ell nA_1, \ell nA_2)$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{1t} & b_{1s} \\ b_{21} & b_{22} & b_{23} & b_{2t} & b_{2s} \\ b_{31} & b_{32} & b_{33} & b_{3t} & b_{3s} \\ b_{t1} & b_{t2} & b_{t3} & b_{tt} & b_{ts} \\ b_{s1} & b_{s2} & b_{s3} & b_{st} & b_{ss} \end{bmatrix} \text{ ; symmetric matrix}$$

This utility function allows expenditure shares to vary with the level of total expenditure and permit a greater variety of substitution patterns among trip modes than functions based on constant and equal elasticities of substitution among all pairs of trip modes.

Rewriting,

$$\ell nV = a_0 + \sum_{i=1}^{3} a_i \ell n(\frac{P_i}{M}) + a_t \ell nA_1 + a_s \ell nA_2$$

$$+ \frac{1}{2} \sum_{j=1}^{3} \sum_{i=1}^{3} b_{ij} \ell n(\frac{P_i}{M}) \ell n(\frac{P_j}{M}) + \sum_{i=1}^{3} b_{it} \ell n(\frac{P_i}{M}) \ell nA_1$$

$$+ \sum_{i=1}^{3} b_{is} \ell n(\frac{P_i}{M}) \cdot \ell nA_2 + b_{ts} \ell nA_1 \cdot \ell nA_2 + \frac{1}{2} b_{tt} (\ell nA_1)^2$$

$$+ \frac{1}{2} b_{ss} (\ell nA_2)^2 .$$

In order to determine the budget share for the i^{th} commodity (trip mode), Roy's identity $x_i = - \frac{\partial V/\partial P_i}{\partial V/\partial M}$ is used.

(Note): R. Roy's Identity.

Maximize $U = U(x_1, \ldots, x_n, A_1, \ldots, A_m)$ subject to $\sum_{i=1}^{n} p_i x_i = M$.

Then $\partial U/\partial x_i = \lambda p_i$. Consider indirect utility function $V = U[x_1(p_1, \ldots, p_n, M, A_1, \ldots, A_m), \ldots, x_n(p_1, \ldots, p_n, M, A_1, \ldots, A_m), A_1, \ldots, A_m]$

$$\partial V/\partial p_j = \sum_i \partial U/\partial x_i \cdot \partial x_i/\partial p_j = \lambda \sum_i p_i \cdot \partial x_i/\partial p_j \quad \ldots\ldots\ldots \quad (1)$$

$$\partial V/\partial M = \sum_i \partial U/\partial x_i \cdot \partial x_i/\partial M = \lambda \sum_i p_i \cdot \partial x_i/\partial M \quad \ldots\ldots\ldots \quad (2)$$

Differentiating budget constraint $\sum_i p_i x_i = M$ with respect to p_j and M (assuming all prices and attributes are constant except p_j), we get,

$$\begin{cases} x_j + \Sigma\, p_i \cdot \dfrac{\partial x_i}{\partial p_j} = 0 \rightarrow \Sigma\, p_i \cdot \dfrac{\partial x_i}{\partial p_j} = -x_j \quad \ldots\ldots\ldots \quad (3) \\[2ex] \Sigma\, p_i \cdot \dfrac{\partial x_i}{\partial M} = 1 \quad \ldots\ldots\ldots \quad (4) \end{cases}$$

$$\left.\begin{array}{l} \text{by (3), (1), } \partial V/\partial p_j = -\lambda x_j \\ \text{by (4), (2), } \partial V/\partial M = \lambda \end{array}\right\} \rightarrow x_j = -\frac{\partial V/\partial p_j}{\partial V/\partial M}$$

Using Roy's identity, the budget share of the i^{th} trip mode is:

$$\frac{p_i x_i}{M} = -\frac{\partial V/\partial P_i}{\partial V/\partial M} \cdot \frac{P_i}{M} = -\frac{\partial V/\partial P_i}{\partial V/\partial M} \cdot \frac{P_i/V}{M/V} = -\frac{\partial \ell n V/\partial \ell n p_i}{\partial \ell n V/\partial \ell n M}$$

therefore,

$$\frac{p_i x_i}{M} = -\frac{\partial \ell n V/\partial \ell n p_i}{\partial \ell n V/\partial \ell n M}$$

where x_i is the quantity (passenger miles) of the trip mode i of a consumer,

p_i is the price of the i^{th} trip mode per passenger mile,

M is the expenditure of trips during a time period, and

p_i and M are exogeneously determined.

Then the budget shares are the simultaneous equations as:

$$\frac{p_1x_1}{M} = \frac{a_1 + b_{11}\ell n(p_1/M) + b_{12}\ell n(p_2/M) + b_{13}\ell n(p_3/M) + b_{1t}\ell nA_1 + b_{1s}\ell nA_2}{a_M + b_{M1}\ell n(p_1/M) + b_{M2}\ell n(p_2/M) + b_{M3}\ell n(p_3/M) + b_{Mt}\ell nA_1 + b_{Ms}\ell nA_2}$$

$$\frac{p_2x_2}{M} = \frac{a_2 + b_{21}\ell n(p_1/M) + b_{22}\ell n(p_2/M) + b_{23}\ell n(p_3/M) + b_{2t}\ell nA_1 + b_{2s}\ell nA_2}{a_M + b_{M1}\ell n(p_1/M) + b_{M2}\ell n(p_2/M) + b_{M3}\ell n(p_3/M) + b_{Mt}\ell nA_1 + b_{Ms}\ell nA_2}$$

$$\frac{p_3x_3}{M} = \frac{a_3 + b_{31}\ell n(p_1/M) + b_{32}\ell n(p_2/M) + b_{33}\ell n(p_3/M) + b_{3t}\ell nA_1 + b_{3s}\ell nA_2}{a_M + b_{M1}\ell n(p_1/M) + b_{M2}\ell n(p_2/M) + b_{M3}\ell n(p_3/M) + b_{Mt}\ell nA_1 + b_{Ms}\ell nA_2}$$

where $a_M = a_1+a_2+a_3$,

$b_{M1} = b_{11}+b_{21}+b_{31}$,

$b_{M2} = b_{12}+b_{22}+b_{32}$,

$b_{M3} = b_{13}+b_{23}+b_{33}$,

$b_{Mt} = b_{1t}+b_{2t}+b_{3t}$, and

$b_{Ms} = b_{1s}+b_{2s}+b_{3s}$.

The restrictions on the parameters of the budget share equations are:

(1) Equality. The budget constraint implies that

$$\sum_i \frac{p_ix_i}{M} = 1 .$$

So, the parameters of the last equation can be obtained from the definitions of a_M and b_{Mj} ($j = 1, 2, \ldots, n$), b_{Mt}, b_{Ms} and n-1 equations (in the three mode choice case above, only two equations) are required for a complete econometric model of demand.

(2) Normality and Negativity. The budget share equations are

homogeneous of degree zero in the parameters; normalization of these parameters is required for estimation. Since utility is non-increasing in the prices, the logarithm of utility is non-increasing in the logarithms of the prices. Therefore, all a_i's must be negative and normalization is done by the following relationship:

$$a_M = a_1 + a_2 + a_3 = -1$$

(3) Symmetry. Symmetry of the Hessian of the indirect utility function gives rise to symmetry restrictions:

$$b_{ij} = b_{ji} \quad (i \neq j, \quad i,j = 1,2,3)$$

Since the three budget share equations sum to unity, the sum of the disturbances across the three equations is zero at each observation. This implies that the disturbance variance-covariance matrix is singular. Since the disturbance variance-covariance matrix of the three equations is singular, we could arbitrarily drop one equation and estimate the remaining two equations.

2. Elasticities

Elasticities are useful to measure the quantitative assessment of alternative transportation policies.

(a) Marshallian Elasticities.

Expressing the budget share form of the least restricted translog model, equality and symmetry case, where the explanatory variables are prices, income and other attribute variables;

$$\left.\frac{p_j x_j}{M}\right|_{\substack{M \\ p_i,\ i\neq j \\ A_1, A_2 \\ \text{all} \\ \text{fixed}}} = \frac{a_j + \sum_i b_{ji}\ln(p_i/M) + b_{jt}\ln A_1 + b_{js}\ln A_2}{-1 + \sum_i b_{Mi}\ln(p_i/M) + b_{Mt}\ln A_1 + b_{Ms}\ln A_2} = \frac{b_{jj}\ln p_j + A}{b_{Mj}\ln p_j + B} = \frac{L}{K}$$

where $A = a_j + \sum_{\substack{i \\ i\neq j}} b_{ji}\ln(p_i/M) + b_{jt}\ln A_1 + b_{js}\ln A_2 - b_{jj}\ln M$,

$$B = -1 + \sum_{i \neq j} b_{Mi}\ln(p_i/M) + b_{Mt}\ln A_1 + b_{Ms}\ln A_2 - b_{Mj}\ln M \ ,$$

$$L = b_{jj}\ln p_j + A \ ,$$

$$K = b_{Mj}\ln p_j + B \ , \text{ and}$$

$$S_j = \frac{p_j x_j}{M} \ .$$

Direct elasticity.

Partial differentiation of x_j and p_j gives:

$$\frac{1}{M}(p_j \partial x_j + x_j \partial p_j) = \frac{b_{jj}\partial(\ln p_j)\cdot K - b_{Mj}\partial(\ln p_j)\cdot L}{(b_{Mj}\ln p_j + B)^2}$$

$$p_j \partial x_j + x_j \partial p_j = \frac{(b_{jj}/p_j \cdot \partial p_j \cdot K - b_{Mj}/p_j \cdot \partial p_j \cdot L)M}{K^2}$$

$$\frac{p_j}{x_j} \cdot \frac{\partial x_j}{\partial p_j} = \frac{(b_{jj}/p_j x_j \cdot M \cdot K - b_{Mj}/p_j x_j \cdot M \cdot L)}{K^2} - 1$$

$$E_{jj} = \frac{p_j}{x_j} \cdot \frac{\partial x_j}{\partial p_j} = \frac{M}{p_j x_j}\left(\frac{b_{jj}}{K} - b_{Mj}\cdot\frac{L}{K^2}\right) - 1$$

Similarly with other attribute variables, (i.e. partial differentiation of x_j and an attribute variable, say, A_1)

$$E_{jA_1} = \frac{\partial x_j}{\partial A_1} \cdot \frac{A_1}{x_j} = -\frac{b_{jA_1}}{S_j}$$

if $A_1 = e^T$ (time trend variable is usually represented as exponential form),

$$E_{jt} = -\frac{b_{jt}}{S_j} \cdot T$$

if $A_1 = Sp$,

$$E_{jSp} = -\frac{b_{js}}{S_j}$$

Cross elasticity.

$$\left.\frac{p_j x_j}{M}\right|_{\substack{M \\ p_i,\, i\neq j\neq k \\ A_1, A_2}} = \frac{b_{jk}\ln p_k + A_k}{b_{Mk}\ln p_k + B_k} = \frac{L}{K}$$

where

$$A_k = a_j + \sum_{\substack{i \\ i\neq k}} b_{ji}\ln\left(\frac{p_i}{M}\right) + b_{jt}\ln A_1 + b_{js}\ln A_2 - b_{kk}\ln M$$

$$B_k = -1 + \sum_{\substack{i \\ i\neq k}} b_{Mi}\ln\left(\frac{p_i}{M}\right) + b_{Mt}\ln A_1 + b_{js}\ln A_2 - b_{Mk}\ln M.$$

Partial differentiation of x_j and p_k:

$$\left(\frac{p_j}{M}\right)\partial x_j = \left(\frac{K\cdot(b_{jk}\cdot 1/p_k) - L(b_{Mk}\cdot 1/p_k)}{K^2}\right)\partial p_k$$

$$E_{jk} = \frac{p_k}{x_j}\cdot\frac{\partial x_j}{\partial p_k} = \frac{p_k}{x_j}\left(\frac{M}{p_j}\,\frac{(b_{jk} - L b_{Mk})}{p_k\cdot K}\right)$$

$$= \frac{(M/p_j x_j)b_{jk} - b_{Mk}}{K} \qquad \text{Since } \frac{L}{K} = \frac{p_j x_j}{M} = S_j$$

Hence

$$E_{jk} = \frac{b_{jk}/S_j - b_{Mk}}{K}$$

Income elasticity.

$$\left.\frac{p_j x_j}{M}\right|_{\substack{\text{all prices} \\ \text{and attributes} \\ \text{variables are} \\ \text{constant.}}} = \frac{-\sum_i b_{ji}\ln M + A_M}{-\sum_i b_{Mi}\ln M + B_K} = \frac{L}{K}$$

where $A_M = a_j + \sum_i b_{ji}\ln p_i + b_{jt}\ln A_1 + b_{js}\ln A_2$, and

$$B_K = -1 + \sum_i b_{Mi}\ln p_i + b_{Mt}\ln A_1 + b_{Ms}\ln A_2 .$$

Partial differentiation of x_j and M,

$$\frac{M(p_j\partial x_j) - (p_j x_j)\partial M}{M^2} = \frac{K(-\sum_i b_{ji})\ 1/M\cdot\partial M - L(-\sum_i b_{Mi})\ 1/M\cdot\partial M}{K^2}$$

$$\frac{p_j}{M}(M\partial x_j) = \frac{1}{M}\, p_j x_j \cdot \partial M - \frac{1}{K}\sum_i b_{ji} \cdot \partial M + \frac{1}{K}\left(\frac{p_j x_j}{M}\right)\sum_i b_{Mi} \cdot \partial M$$

$$\frac{p_j x_j}{M}\left(\frac{\partial x_j}{\partial M} \cdot \frac{M}{x_j}\right) = S_j - \frac{1}{K} \cdot \sum_i b_{ji} + \frac{1}{K} \cdot S_j \cdot \sum_i b_{Mi}$$

Hence

$$E_{Mj} = 1 - \frac{\sum_i b_{ji}}{S_j \cdot K} + \frac{\sum_i b_{Mi}}{K} .$$

Marshallian elasticities are:

(i) direct elasticity: $E_{jj} = \frac{\partial x_j}{\partial p_j} \cdot \frac{p_j}{x_j} = \frac{b_{jj}/S_j - b_{Mj}}{K} - 1$

$$E_{jt} = \frac{\partial x_j}{\partial T} \cdot \frac{T}{x_j} = \frac{-b_{jt}}{S_j} \cdot T$$

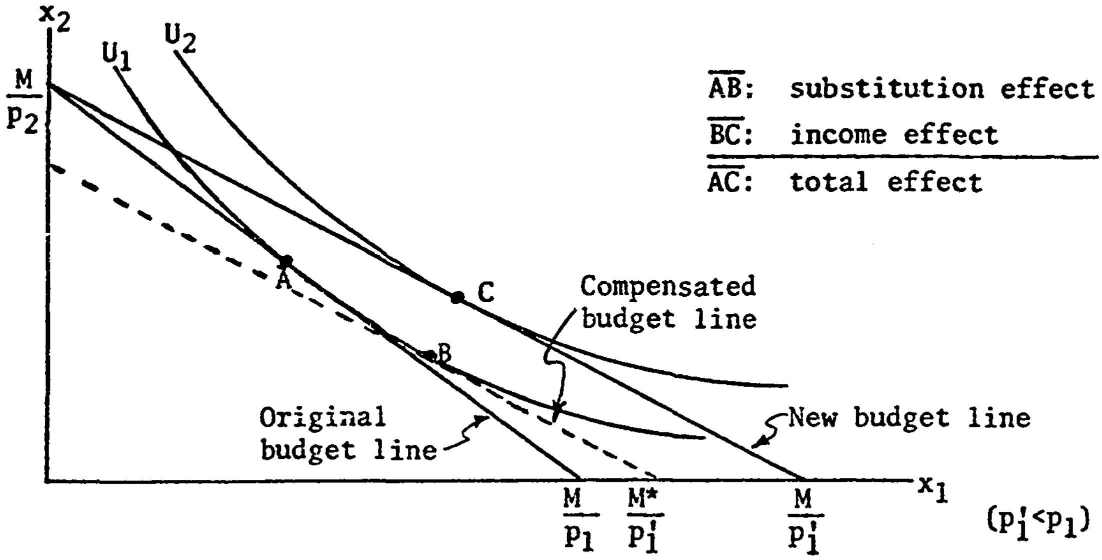
$$E_{jSp} = \frac{\partial x_j}{\partial Sp} \cdot \frac{Sp}{x_j} = \frac{-b_{js}}{S_j}$$

(ii) cross elasticity: $E_{jk} = \frac{\partial x_j}{\partial p_k} \cdot \frac{p_k}{x_j} = \frac{b_{jk}/S_j - b_{Mk}}{K}$

(iii) income elasticity: $E_{Mj} = \frac{\partial x_j}{\partial M} \cdot \frac{M}{x_j} = 1 - \frac{\sum_i b_{ji}}{S_j \cdot K} + \frac{\sum_i b_{Mi}}{K}$

So far, the demand equations referred imply market demand equations which may be called as "Marshallian demand equations". Both the own-price and cross-price elasticities based on Marshallian demand equations are not correct measures of substitutability. The price elasticities of Marshallian demand equations are calculated as the change in quantity demanded that results from a change in price. But as the price varies in the Marshallian demand (i.e. the price p_1 falls to p_1' with all other prices being held constant), two effects occur (reference: <u>Theory of Resource Allocation and Prices</u> by McFadden and

Winter, Microeconomic Theory by Henderson and Quandt).



First, money income is adjusted so that the utility level is held constant. The shift from A to B is called the substitution effect, with real income (utility level) held constant.

Second, money income is increased from the compensated level M* to the actual level M, holding all other prices fixed, and a shift occurs from B to C which is called the income effect.

The total effect is the algebraic sum of the income effect and the substitution effect. The substitution effect by compensating income to hold the consumer's indifference curve constant is due to J. R. Hicks (1946, Value and Capital). An alternative analysis is done by Slutsky and Slutsky's substitution effect is equivalent to Hicksian substitution effect for infinitesimal changes. Hence, the use of Slutsky equation is employed in the computation of Hicksian income-compensated elasticities which deal only with substitution effect. Thus, both the own-substitution effect and cross-substitution effect are correctly measured by Hicksian income-compensated own elasticities and cross elasticities respectively. And two commodities i and j are:

$$\begin{cases} \text{substitutes if } E_{ij}\Big|_{U_o} \text{ (Hicksian income- compensated cross elasticity)} > 0 \\ \text{complements if } E_{ij}\Big|_{U_o} \text{ (Hicksian income-compensated cross elasticity)} < 0 \end{cases}$$

(b) Hicksian Income-Compensated Elasticities.

From Slutsky equation,

$$\left(\frac{\partial x_i}{\partial p_j}\right)_{U_o} = \frac{\partial x_i}{\partial p_j} + x_j\left(\frac{\partial x_i}{\partial M}\right)_{p_o}$$

Multiplying each side of the equation by $\frac{p_j}{x_i}$,

$$E_{ij}\Big|_{U_o} = E_{ij} + \frac{p_j x_j}{M} \cdot E_{Mi}\Big|_{p_o}$$

$$\underbrace{E_{ij}\Big|_{U_o}}_{\substack{\text{Hicksian} \\ \text{income-compensated} \\ \text{elasticity}}} = \underbrace{E_{ij}}_{\substack{\text{Marshallian} \\ \text{elasticity}}} + S_j \cdot \underbrace{E_{Mi}\Big|_{p_o}}_{\substack{\text{Marshallian} \\ \text{income-elasticity}}}$$

(i) direct elasticity: $E_{jj}\Big|_{U_o} = E_{jj} + S_j \cdot E_{Mj}$

(See Appendix for derivation)

(ii) cross elasticity: $E_{jk}\Big|_{U_o} = E_{jk} + S_k \cdot E_{Mj}$

(iii) elasticity for attribute variables:

$$E_{jt}\Big|_{U_o} = E_{jt} - \sum_i b_{it}\ell n(p_i)$$

(See Appendix for derivation)

$$E_{jSp}\Big|_{U_o} = E_{jSp} - \sum_i b_{is}(\ell n p_i)$$

*Appendix

Hicksian income-compensated elasticities in relation to Marshallian elasticities. (Slutsky equations are derived for the own-price and cross elasticities and for the attribute variable elasticities.) (Reference: Henderson, J. M., and R. E. Quandt, 1958).

1. Own-price and cross-price elasticities

Maximize $U = U(x_1, x_2, x_3; e^T, Sp)$ subject to $p_1x_1+p_2x_2+p_3x_3 = M$.

The Lagrangian function is:

$$\pounds = U(x_1,x_2,x_3,e^T,Sp) - \lambda(p_1x_1+p_2x_2+p_3x_3-M)$$

From the Lagrangian 1st order condition,

$$\begin{aligned} \frac{\partial \pounds}{\partial x_1} &= U_1 - \lambda p_1 = 0 \\ \frac{\partial \pounds}{\partial x_2} &= U_2 - \lambda p_2 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)} \\ \frac{\partial \pounds}{\partial x_3} &= U_3 - \lambda p_3 = 0 \end{aligned}$$

where U_i is the partial derivative of U with respect to x_i's, e^T, Sp.

Total differentiation of equation (1) gives:

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} & -p_1 \\ U_{21} & U_{22} & U_{23} & -p_2 \\ U_{31} & U_{32} & U_{33} & -p_3 \\ -p_1 & -p_2 & -p_3 & 0 \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda dp_1 - U_{14}d(e^T) - U_{15}dSp \\ \lambda dp_2 - U_{24}d(e^T) - U_{25}dSp \\ \lambda dp_3 - U_{34}d(e^T) - U_{35}dSp \\ -dM + x_1dp_1 + x_2dp_2 + x_3dp_3 \end{bmatrix} \ldots \text{(2)}$$

where $U_{ij} = \dfrac{\partial^2 U}{\partial x_i \partial x_j}$.

Then

$$\begin{aligned} dx_1 = \frac{1}{D}\big(&D_{11}[\lambda dp_1-U_{14}d(e^T)-U_{15}dSp]-D_{21}[\lambda dp_2-U_{24}d(e^T)-U_{25}dSp] \\ &+D_{31}[\lambda dp_3-U_{34}d(e^T)-U_{35}dSp]-D_{41}[-dM+x_1dp_1+x_2dp_2+x_3dp_3]\big) \quad \ldots \text{(3)} \end{aligned}$$

where D_{ij} = cofactor of the element in the i^{th} row and j^{th} column of D, and

D = the bordered Hessian determinant $\begin{bmatrix} U_{11} & U_{12} & U_{13} & -p_1 \\ U_{21} & U_{22} & U_{23} & -p_2 \\ U_{31} & U_{32} & U_{33} & -p_3 \\ -p_1 & -p_2 & -p_3 & 0 \end{bmatrix}$

letting $dp_1 = dp_3 = dM = d(e^T) = dSp = 0$,

$$dx_1 = \frac{-D_{21}\lambda dp_2 - D_{41}x_2dp_2}{D}$$

then

$$\frac{\partial x_1}{\partial p_2} = \frac{-D_{21}\lambda}{D} - \frac{D_{41}x_2}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (a)$$

This is Marshallian rate of change in demand of commodity 1 with respect to change in the price of commodity 2.

Now, considering constant utility level,

$$\begin{aligned} dU &= U_1dx_1 + U_2dx_2 + U_3dx_3 + U_4d(e^T) + U_5dSp = 0 \\ &= \lambda p_1dx_1 + \lambda p_2dx_2 + \lambda p_3dx_3 + U_4d(e^T) + U_5dSp \\ &= \lambda(p_1dx_1 + p_2dx_2 + p_3dx_3) + U_4d(e^T) + U_5dSp \\ &= -\lambda(-dM + x_1dp_1 + x_2dp_2 + x_3dp_3) + U_4d(e^T) + U_5dSp \end{aligned}$$

$$\lambda(-dM + x_1dp_1 + x_2dp_2 + x_3dp_3) = U_4d(e^T) + U_5dSp$$

$$(-dM + x_1dp_1 + x_2dp_2 + x_3dp_3) = 1/\lambda \ (U_4d(e^T) + U_5dSp) \quad \ldots\ldots\ldots \quad (4)$$

Substituting equation (4) to equation (3),

$$dx_1\Big|_{U=constant} = 1/D \ \big(D_{11}[\lambda dp_1-U_{14}d(e^T)-U_{15}dSp]-D_{21}[\lambda dp_2-U_{24}d(e^T)-U_{25}dSp] + D_{31}[\lambda dp_3-U_{34}d(e^T)-U_{35}dSp]-D_{41}[1/\lambda \ (U_4(de^T)+U_5dSp)]\big)$$

letting

$$dp_1 = dp_3 = dM = d(e^T) = dSp = 0 \ ,$$

$$\left.\frac{\partial x_1}{\partial p_2}\right|_{U=\text{constant}} = \frac{-D_{21}\lambda}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(b)}$$

by (a) and (b),

$$\left.\frac{\partial x_1}{\partial p_2}\right|_{U=\text{constant}} = \frac{\partial x_1}{\partial p_2} + \frac{D_{41}\cdot x_2}{D} \quad \text{(Slutsky equation)}$$

Multiplying p_2/x_1 both sides,

$$\left(\frac{\partial x_1}{\partial p_2}\cdot\frac{p_2}{x_1}\right)_{U=\text{constant}} = \frac{\partial x_1}{\partial p_2}\cdot\frac{p_2}{x_1} + \frac{D_{41}\cdot x_2}{D}\cdot\frac{p_2}{x_1} \quad \ldots\ldots\ldots \quad (5)$$

Also cetris paribus ($dp_1 = dp_2 = dp_3 = dT = dSp = 0$) except dM,

then from eq. (3),

$$\frac{\partial x_1^*}{\partial M} = \frac{D_{41}}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Substituting (6) to (5),

$$\left(\frac{\partial x_1}{\partial p_2}\cdot\frac{p_2}{x_1}\right)_{U=\text{constant}} = \left(\frac{\partial x_1}{\partial p_2}\cdot\frac{p_2}{x_1}\right) + \left(\frac{\partial x_1}{\partial M}\cdot\frac{M}{x_1}\right)\cdot\left(\frac{x_2 p_2}{M}\right)$$

$$\left.E_{12}\right|_{U=\text{constant}} = E_{12\,\text{Marshallian}} + E_{M1} \cdot S_2$$

Hence, denoting with indirect utility function, in general,

$$\begin{cases} \left.E_{jj}\right|_{V=\text{constant}} = E_{jj} + E_{Mj}\cdot S_j \\ \left.E_{jk}\right|_{V=\text{constant}} = E_{jk} + E_{Mj}\cdot S_k \end{cases}$$

2. <u>Elasticities of other attributes</u> (time trend, speed ratio)

$$\left.dx_1\right|_{U=\text{constant}} = \frac{1}{D}\Big(D_{11}[\lambda dp_1 - U_{14}d(e^T) - U_{15}dSp] - D_{21}[\lambda dp_2 - U_{24}d(e^T) - U_{25}dSp]$$
$$+ D_{31}[\lambda dp_3 - U_{34}(de^T) - U_{35}dSp] - D_{41}[1/\lambda\ (U_4(de^T) + U_5 dS)]\Big)$$

$dp_1 = dp_2 = dp_3 = dM = dS = 0$

$$\left.\partial x_1\right|_{U=\text{constant}} = -\frac{D_{11}}{D}U_{14}\partial(e^T) + \frac{D_{21}}{D}U_{24}\partial(e^T) - \frac{D_{31}}{D}U_{34}\partial(e^T) - \frac{D_{41}}{D}\frac{U_4}{\lambda}\partial(e^T)$$

$$\left(\frac{\partial x_1}{\partial T}\right)_{U=\text{constant}} = \underbrace{\left(-\frac{D_{11}}{D} U_{14} + \frac{D_{21}}{D} U_{24} - \frac{D_{31}}{D} U_{34}\right)}_{\left(\frac{\partial x_1}{\partial e^T}\right) \text{ Marshallian}} - \left(\frac{D_{41}}{D} \cdot \frac{U_4}{\lambda}\right)$$

$$\left(\frac{\partial x_1}{\partial e^T}\right)_{U=\text{constant}} = \left(\frac{\partial x_1}{\partial e^T}\right) - \left(\frac{D_{41}}{D} \cdot \frac{U_4}{\lambda}\right)$$

$$\left.\frac{\partial x_1}{\partial e^T} \cdot \frac{e^T}{x_1}\right|_{U=\text{constant}} = \frac{\partial x_1}{\partial e^T} \cdot \frac{e^T}{x_1} - \frac{\partial x_1}{\partial M} \cdot \frac{M}{x_1} \cdot \frac{\partial U/\partial e^T}{\partial U/\partial M} \cdot \frac{e^T/U}{M/U}$$

$$E_{1t}\Big|_{U_o} = E_{1t_{\text{Marshallian}}} - E_{M1} \cdot \frac{\partial U/\partial e^T \cdot e^T/U}{\partial U/\partial M \cdot M/U}$$

If the translog indirect utility function ($\partial \ell nV/\partial \ell nM = +1$) is homogeneous and non-additive, then $E_{M1}=1$ and

$$E_{1t}\Big|_{V_o} = E_{1t_{\text{Marshallian}}} - (1) \cdot \frac{\partial \ell nV}{\partial \ell n(e^T)}$$

in general,

$$E_{jt}\Big|_{V_o} = E_{jt_{\text{Marshallian}}} - \frac{\partial \ \ell nV}{\partial \ell n(e^T)}, \ i=1,2,3$$

Similarly, the speed ratio attribute variable results in,

$$\left.\frac{\partial x_1}{\partial Sp} \cdot \frac{Sp}{x_1}\right|_{U=\text{constant}} = \left(\frac{\partial x_1}{\partial Sp}\right)\left(\frac{Sp}{x_1}\right) - \frac{\partial x_1}{\partial M} \cdot \frac{M}{x_1} \cdot \frac{\partial U/\partial Sp}{\partial U/\partial M} \frac{Sp/U}{M/U}.$$

$$E_{1Sp}\Big|_{U_o} = \underset{\text{Marshallian}}{E_{1Sp}} - E_{M1} \cdot \frac{\partial \ell nV/\partial \ell nSp}{(+1)}$$

If the translog utility function is homogeneous and non-additive, then

$$E_{1Sp}\Big|_{V_o} = \underset{\text{Marshallian}}{E_{1Sp}} + \frac{\partial \ell nV}{\partial \ell nSp}$$

in general

$$E_{jSp}\Big|_{V_o} = E_{jSp_{Marshallian}} - \frac{\partial \ell nV}{\partial \ell nSp}\ ,\ i = 1,\ 2,\ 3$$

(c) Allen Uzawa Elasticities of Substitution

Elasticity of substitution between two commodiites x_i and x_j is defined as:

$$\delta_{ij} = \frac{d\ \ell n(x_j/x_i)}{d\ \ell n(p_i/p_j)} \quad \text{(See Chapter III.C.3 (dfn. 4).)}$$

There is a relationship between Hicksian income-compensated elasticity and elasticity of substitution (Liew, 1977). Uzawa showed elasticity of substitution in terms of the expenditure function and its derivatives as (1) below (1962).

$$\delta_{ij} \overset{(1)}{=} \frac{M(V;p)M_{ij}(V;p)}{M_i(V;p)M_j(V;p)} = \frac{M\cdot\partial/\partial p_j\ (x_i^*)}{x_i^*\quad x_j^*} \quad \frac{\partial x_i^*/\partial p_j \cdot p_j/x_i^*}{p_j x_j^*/M}$$

$$= \frac{E_{ij}\big|_{V_o}}{S_j}$$

where M(V;p) is the expenditure function, and

x_i^*, x_j^* are the Hicksian income-compensated i^{th} and j^{th} demand equations.

By the use of Shepherd Lemma,

$\underline{M_K(V'p) = x_k^*}$

Maximize U(x) subject to p'x=M.

then $\partial U/\partial x_i = \lambda p_i \qquad (i=1,2,\ldots,n)$

$\Sigma p_i x_i = M$

and $\partial M/\partial p_j = \sum_i p_i \cdot \partial x_i/\partial p_j + x_j = \sum_i \frac{1}{\lambda}\frac{\partial U}{\partial x_i}\frac{\partial x_i}{\partial p_j} + x_j$

but under the Hicksian income-compensated demand equations,

$$\frac{\partial U}{\partial p_j} = \sum_i \frac{\partial U}{\partial x_i}\cdot\frac{\partial x_i}{\partial p_j} = 0$$

hence $\frac{\partial M}{\partial p_j} = x_j$

i.e. $M_j(V;p) = x_j^*$

CHAPTER IV

METHOD AND DATA

A. Descriptions of Data

Most of the data used for the empirical analysis of this study are obtained from "Transportation Facts and Trends" (12th edition, 1975). It contains annual intercity travel data for the period from 1947 to 1974:

(1) total intercity passenger mileage (in billions) of airline, bus and rail: XA, XB, XR;

(2) prices per passenger mile (in cents) by each mode: PA, PB, PR;

(3) number of passengers (in millions) carried by airline, bus and rail: PNA, PNB, PNR;

(4) consumer price index with base year 1967.

The average speed (in miles per hour) of the domestic airline passenger carriers is obtained from the Handbook of Airline Statistics. The average speeds of passenger bus and rail modes are obtained from FHWA and AMTRAK respectively. The speed data of the rail mode are computed differently from those of airline and bus modes. The speed data of the rail mode include not only intercity trains but also suburban trains with waiting time included, whereas the speed data of airline and bus modes are the average maximum speed of the trip with waiting time not included. Considering this factor in the rail speed data, there is not much difference in speed between the bus and rail modes. Hence only the speed of the airline mode versus speed of the bus-rail mode is considered. And the speed factor is derived as the logarithm of the ratio of the speed of the airline mode versus the speed of the bus mode (representing both bus and rail speed).

For the present study, the intercity travel by auto is excluded, due primarily to a lack of data.

From the available data,

(1) Average revenue per passenger mile is divided by the consumer price index (consumer price index of 1967 = 100) to obtain deflated average revenue per passenger-mile, which is the price that individual passenger pays per passenger mile. Here, the use of average revenue per passenger-mile assumes the linear relationship between the prices and the passenger miles.

RPA = PA/CP , RPB = PB/CP , RPR = PR/CP (cents/passenger-mile).

(2) The quantities of travel (passenger-miles per person) by each mode are obtained by dividing total passenger mileage of travel by the number of passengers carried by each mode.

QXA = XA/PNA , QXB = XB/PNB , QXR = XR/PNR

(billions of passenger miles/millions of passengers).

(3) Price per passenger mile (revenue per passenger mile) is multiplied by the quantities (passenger miles per person) of each trip mode to get expenditure of a 'representative individual' of a specific year.

XM = (RPA·QXA) + (RPB·QXB) + (RPR·QXR) (where XM is in thousands of cents/passenger).

(4) Time trend variable T ranges from 1 to 28.

(5) Speed factor, SPF = ℓn(SPA/SPBR): let Sp = SPA/SPBR.

where SPA is the speed of airline mode in miles per hour, and

SPBR is the speed of bus, rail modes in miles per hour.

(6) Since the intercity passengers carried by bus (PNB) are quite different before and after the 1959 data, linear regression is applied in PNA and PNR with respect to PNB to obtain uniformly increasing PNB values. Therefore, 13 regressed values of PNB's (from 1947 to 1959) are used with the rest of the data in the analysis.

B. Model Specifications

The model specifications are based on the validity of demand theory. Normality and negativity conditions are also imposed (i.e. Sum of the constant terms should be equal to minus one).

1. Equality and Symmetry Restriction Case

The independent variables considered are the prices per passenger mile of the airline, bus and rail modes (RPA, RPB, RPR) and the expenditure M.

Equality: the parameters b_{Mj} (j=1,2,3) occur in both of the share equations.

Symmetry: Hessian matrix is a symmetric matrix.

The entries of the Hessian Matrix satisfy

$$\beta_{ij} = \beta_{ji} \quad (i,j = 1,2,3)$$

The share equations based on the variable notations of the data are:

$$SA = \frac{RPA \cdot QXA}{XM} = \frac{AA+BAA \cdot \ell n(RPA/XM)+BAB \cdot \ell n(RPB/XM)+BAR \cdot \ell n(RPR/XM)}{-1+BMA \cdot \ell n(RPA/XM)+BMB \cdot \ell n(RPB/XM)+BMR \cdot \ell n(RPR/XM)}$$

$$SR = \frac{RPR \cdot QXR}{XM} = \frac{AR+BAR \cdot \ell n(RPA/XM)+BRB \cdot \ell n(RPB/XM)+BRR \cdot \ell n(RPR/XM)}{-1+BMA \cdot \ell n(RPA/XM)+BMB \cdot \ell n(RPB/XM)+BMR \cdot \ell n(RPR/XM)}$$

$$SB = 1 - SA - SR$$

For simplicity, the share equation is denoted as:

$$S_j = \frac{P_j X_j}{M} = \frac{a_j + \sum_i b_{ji} \ \ell n(\frac{P_i}{M})}{-1 + \sum_i b_{Mi} \ \ell n(\frac{P_i}{M})} \quad \text{for } i,j=1,2,3$$

and the demand equation (Marshallian) is:

$$x_j = (\frac{M}{P_j})\frac{a_j + \sum_i b_{ji} \ \ell n(P_i/M)}{-1 + \sum_i b_{Mi} \ \ell n(P_i/M)} \quad \text{for } i,j=1,2,3$$

Elasticities (refer to Chapter III).

Marshallian elasticity:

(i) Direct elasticity

$$E_{jj} = \frac{\partial x_j}{\partial P_j} \cdot \frac{p_j}{x_j} = \frac{b_{jj}/S_j - b_{Mj}}{K} - 1 \quad (\text{where } K = -1 + \sum_i b_{Mi} \ell n(\frac{P_i}{M}))$$

(ii) Cross elasticity

$$E_{jk} = \frac{\partial x_j}{\partial P_k} \cdot \frac{P_k}{x_j} = \frac{b_{jk}/S_j - b_{Mk}}{K}$$

(iii) Income elasticity

$$E_{Mj} = \frac{\partial x_j}{\partial M} \cdot \frac{M}{x_j} = 1 - \frac{\sum_i b_{ji}}{S_j \cdot K} + \frac{\sum_i b_{Mi}}{K}$$

Hicksian Income-Compensated elasticity:

$$E_{ij}\Big|_{U_o} = E_{ij} + S_j \cdot E_{M_i}(P_o) \quad \text{for } i,j=1,2,3$$

Allen-Uzawa Elasticity of Substitution:

$$\sigma_{ij} = \frac{E_{ij}\Big|_{U_o}}{S_j} = \frac{E_{ij}}{S_j} + E_{M_i}(P_o)$$

2. Homogeniety and Non-additivity Restriction Case

The independent variables considered are the prices per passenger mile of the airline, bus and rail modes (RPA, RPB, RPR), the expenditure M, the time trend T and the speed ratio Sp = SPA/SPBR. Homogeniety restriction is also well discussed in Christenson, Jorgenson and Lau (1975). But it is useful to relate their discussion to the present model specification which has attribute variables in addition to prices and income variables.

Homogeniety is a special case of homotheticity. Hence restrictions for homotheticity condition should be included in the homogeniety condition.

If the direct utility function is homothetic, we can write

$$\ln U = F[\ln H(x_1,x_2,x_3,t,Sp)] = F[\ln H(x_1,x_2,x_3,x_4,x_5)]$$

where $x_4 = t$, $x_5 = Sp$,

H is a homogeneous function of degree one, and

F is a continuous and strictly monotonic function.

Partial differentiation of the translog direct utility function gives:

$$\frac{\partial \ln U}{\partial \ln x_i} = a_i \quad (i = 1, 2, \ldots, 4, 5)$$

$$\frac{\partial^2 \ell nU}{\partial \ell nX_i \partial \ell nX_j} = \frac{\partial F}{\partial \ell nH} \cdot \frac{\partial^2 \ell nH}{\partial \ell nX_i \partial \ell nX_j} + \frac{\partial^2 F}{\partial \ell nH^2} \cdot \frac{\partial \ell nH}{\partial \ell nX_i} \cdot \frac{\partial \ell nH}{\partial \ell nX_j}$$

$$= b_{ij} \quad (i,j=1,\ 2,\ \ldots,\ 4,\ 5)$$

Homogeniety of degree 1 of the function H implies that (R.G.D. Allen, 1956):

$$\sum_i \frac{\partial \ell nH}{\partial \ell nX_i} = 1$$

$$\sum_i \frac{\partial^2 \ell nH}{\partial \ell nX_i \partial \ell nX_j} = 0$$

Summing over i and using homogeniety of degree 1 of the function H,

$$b_{Mj} = \sum_i \frac{\partial^2 \ell nU}{\partial \ell nX_i \partial \ell nX_j} = \sigma a_j$$

$$\text{where } \sigma = \frac{\partial^2 F/\partial \ell nH}{\partial F/\partial \ell nH}$$

$\underline{b_{Mj} = \sigma a_j}$ (for j=1, 2, ..., 4, 5)..homotheticity restriction (1)

Note that translog approximation to a homothetic utility function is not necessarily homothetic. The translog utility function is homothetic iff it is homogeneous. i.e. $\underline{\sigma=0}$ . . . . . . .homogeniety restriction (2)

Now, the translog approximation to a homogeneous function is homogeneous. Samuelson (1965) showed that the direct utility function is homothetic if and only if the indirect utility function is homothetic. Hence the share equations derived from the indirect translog utility function with homotheticity and homogeniety restrictions along with normality restrictions ($a_1+a_2+a_3 = -1$) are:

$$S_A = \frac{RPA \cdot QXA}{XM}$$

$$= \frac{AA+BAA \cdot \ell N(RPA/XM)+BAB \cdot \ell n(RPB/XM)+BAR \cdot \ell n(RPR/XM)+BAT \cdot T+BAS \cdot \ell n(Sp)}{(-1)}$$

$$S_R = \frac{RPR \cdot QXR}{XM}$$

$$= \frac{AR+BAR \cdot \ell n(RPA/XM)+BRB \cdot \ell n(RPB/XM)+BRR \cdot \ell n(RPR/XM)+BRT \cdot T+BRS \cdot \ell n(Sp)}{(-1)}$$

$$S_B = 1 - S_A - S_R$$

For simplicity, the share equation is denoted as:

$$S_j = \frac{P_j x_j}{M} = \frac{a_j + \sum_i b_{ji}\ \ell n(P_i/M) + b_{jt} \cdot T + b_{js}\ \ell n(Sp)}{(-1)}$$

for i,j = 1, 2, 3.

Note that the share equation is in log-linear form but S_j (the dependent variable) is not in log form which is different from conventional aggregate travel demand models of log-linear form on both sides of the equations.

The demand equation (Marshallian) is:

$$x_j = (\frac{M}{P_j})\frac{a_j + \sum_i b_{ji}\ \ell n(P_i/M) + b_{jt} \cdot T + b_{js}\ \ell n(Sp)}{(-1)} \quad \text{for } i,j = 1, 2, 3.$$

Elasticities (refer to Chapter III.E.2)

Marshallian elasticity:

(i) Direct elasticity:

$$E_{jj} = \frac{\partial x_j}{\partial P_j} \cdot \frac{P_j}{x_j} = - \frac{b_{jj}}{S_j} - 1 \quad \text{for } j=1,2,3$$

$$E_{jt} = \frac{\partial x_j}{\partial T} \cdot \frac{T}{x_j} = - \frac{b_{jt} \cdot T}{S_j} \quad \text{for } j=1,2,3$$

$$E_{jSp} = \frac{\partial x_j}{\partial Sp} \cdot \frac{Sp}{x_j} = - \frac{b_{js}}{S_j} \quad \text{for } j=1,2,3$$

(ii) Cross elasticity:

$$E_{jk} = \frac{\partial X_j}{\partial P_k} \cdot \frac{P_k}{X_j} = - \frac{b_{jk}}{S_j} \quad \text{for } j=1,2,3$$

(iii) Income elasticity:

$$E_{Mj} = \frac{\partial X_j}{\partial M} \cdot \frac{M}{X_j} = 1 \quad \text{for } j=1,2,3$$

Hicksian income-compensated elasticity (see Chapter III.E.2):

(i) $E_{ij}\Big|_{U_o} = E_{ij} + S_j$ for $i,j=1,2,3$

(ii) Elasticities with attribute variables, time trend and speed ratio (see Chapter III.E.2. Appendix).

$$E_{jt}\Big|_{U_o} = E_{jt} - \frac{\partial \ell nU}{\partial \ell ne^T}$$

since $\ell nU = F[\ell nH(x_1,x_2,x_3,e^T,Sp)]$ and

$\ell nV = V[\ell nH(P_1,P_2,P_3,M,e^T,Sp)]$,

$$E_{jt}\Big|_{V_o} = E_{jt} - \frac{\partial \ell nV}{\partial \ell ne^T} .$$

The following is the indirect translog utility function with time and speed varying preferences:

$$\ell nV^* = a_o + \sum_i a_i \ell n(P_i/M) + a_t \ell n(e^T) + a_s \ell n(Sp) + \frac{1}{2} \sum_j \sum_i b_{ij} \ell n(P_i/M) \ell n(P_j/M)$$

$$+ \sum_i b_{it} \ell n(P_i/M) \ell n(e^T) + \sum_i b_{is} \ell n(P_i/M) \ell n(Sp) + b_{ts} \ell n(e^T) \ell n(Sp)$$

$$+ \frac{1}{2} b_{tt} [\ell n(e^T)]^2 + \frac{1}{2} b_{ss} [\ell n(Sp)]^2$$

Since a_o, a_t, a_s, b_{ts}, b_{tt}, b_{ss} are not estimated in the demand equations (see Chapter III, Translog utility models), define time and speed adjusted indirect utility function as:

$$\ell nV = \ell nV^* - [a_o + a_t \ell n(e^T) + a_s \ell n(Sp) + b_{ts}(\ell ne^T)\ell n(Sp) + \frac{1}{2} b_{tt}(\ell ne^T)^2 + \frac{1}{2} b_{ss}(\ell n(Sp))^2]$$

With homogeniety restriction ($\sum_i b_{it} = 0$, $\sum_i b_{is} = 0$),

$$\frac{\partial \ell nV}{\partial \ell n(e^T)} = \sum_i b_{it} \; \ell n(P_i)$$

hence

$$E_{jt}\Big|_{V_o} = E_{jt} - \sum_i b_{it} \; \ell n(P_i) \; , \; j=1,2,3$$

Similarly with speed ratio variable,

$$E_{jSp}\Big|_{V_o} = E_{jSp} - \frac{\partial \ell nV}{\partial \ell nSp} \; .$$

$$E_{jSp}\Big|_{V_o} = E_{jSp} - \sum_i b_{is}(\ell nP_i) \quad j=1,2,3$$

<u>Allen-Uzawa elasticity of substitution:</u>

$$\sigma_{ij} = \frac{E_{ij}\Big|_{V_o}}{S_j} \quad i,j=1,2,3$$

C. Estimation Methods

In this section, both quantitative choice and qualitative choice estimation methods are discussed.

1. Quantitative Choice Estimation Methods

In the calibration of aggregate travel demand models, parameters of each demand equation are estimated by means of linear regression, constrained linear regression, and non-linear regression.

For the present study, parameters of non-linear share equations are simultaneously determined by using non-linear maximum likelihood estimation method. No work has ever been made to employ the maximum likelihood method in aggregate travel demand model estimation. The well-known multilogit models or probit models have employed the use of maximum likelihood method in disaggregate travel demand model estimation.

Consider the p non-linear system of stochastic simultaneous equations in Y given X (reference: Malinvaud, 1970; Bard, 1974). The standard reduced model is of the form

(1) $Y = F(X,\theta) + \varepsilon$ (p equation with n observations of Y and X and k unknown parameters)

where the errors ε are distributed normally $N(0,\Sigma\otimes I)$ with zero mean and non-singular covariance matrix

$$\Sigma\otimes I = \begin{bmatrix} \sigma_{11}I & \sigma_{12}I & \cdots & \sigma_{1p}I \\ \vdots & & & \vdots \\ \sigma_{p1}I & \cdots & \cdots & \sigma_{pp}I \end{bmatrix}$$ and I is a nxn identity matrix,

then the maximum likelihood function is

(2) $$L = (2\pi)^{\frac{1}{2}nP} \left|\Sigma\otimes I\right|^{-\frac{1}{2}} e^{-\frac{1}{2}\varepsilon'(\Sigma\otimes I)^{-1}\varepsilon} .$$

Since the covariance matrix $\Sigma\otimes I$ is generally unknown, the concentrated maximum likelihood function (p.66, Bard) is considered.

(3) $$\tilde{\mathcal{L}}(\theta) = (\frac{n\cdot P}{2})[\ell n(\frac{n}{2\pi}) - 1] - (\frac{n}{2})\ell n|M(\theta)|$$

where M(θ) is a moment matrix defined as,

$$\begin{cases} M(\theta) = \sum_{i=1}^{n} e(\theta)e'(\theta) \\ \quad = \begin{bmatrix} \sum_{i=1}^{n} e_{1i}^2 & \sum_i e_{1i}e_{2i} & \cdots & \sum_i e_{1i}e_{Pi} \\ \vdots & \Sigma e_{2i}^2 & \ddots & \vdots \\ \Sigma e_{P_i} e_{1i} & \cdots & \cdots & \sum_i e_{Pi}^2 \end{bmatrix}_{PxP} \\ |M(\theta)| = \det M(\theta) \end{cases} \qquad \text{where } e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_P \end{bmatrix}, \quad e_1 = \begin{bmatrix} e_{1i} \\ e_{2i} \\ \vdots \\ e_{ni} \end{bmatrix},$$

(4) Maximizing $\mathcal{L}$ is equivalent to minimizing $\Phi(\theta) = \frac{n}{2} \ln|M|$

(5) By Taylor series expansion at the initial point θ_i,

$$\Phi(\theta) \cong \Phi_i + g_i^T(\theta-\theta_i) + \frac{1}{2}(\theta-\theta_i)^T H_i(\theta-\theta_i)$$

where g^T = gradient vector of Φ, $g_\alpha(\theta) = \frac{\partial\phi}{\partial\theta_\alpha}$

H = Hessian matrix of Φ, $H_{\alpha\beta}(\theta) = \frac{\partial^2\Phi}{\partial\theta_\alpha\partial\theta_\beta}$

Minimizing Φ given y and x,

$$\frac{\partial\Phi}{\partial\theta} \cong g_i + H_i(\theta-\theta_i) = 0$$

$$\theta = \theta_i - H_i^{-1} \cdot g_i \quad \text{letting } \theta = \theta_{i+1}, \; \theta_{i+1} = \theta_i - H_i^{-1} \cdot g_i$$

(6) Iterative scheme: Starting with a given point as the initial guess, generate a sequence of points θ_1, θ_2, ... which will converge to the point θ^*, at which $\Phi(\theta)$ is minimum.

$\theta_{i+1} = \theta_i + \rho_i V_i$ where ρ_i is step size, and V is the directional vector (step direction).

Newton-Rapson (or Newton) iteration method:

Let $\rho_i=1$, given an arbitrary small number $\varepsilon>0$,

$$\theta_{i+1} = \theta_i - H_i^{-1} g_i \quad : \quad i^{th} \text{ iteration}$$

till $\left|\dfrac{\theta_{i+1} - \theta_i}{\theta_i}\right| \leq \varepsilon$ if yes, stop; θ_{i+1} is optimal.

if no, iterate back.

McFadden used the Newton-Rapson (or Newton) iteration method in the multilogit model parameter estimation.

(7) In the case of logit model, the likelihood function is easily computed, and the gradient and Hessian of the log-likelihood function is easily obtained. But in general, the computation of gradient and Hessian is tedious especially when there are many number of simultaneous non-linear equations. The Newton-Gauss method is an approximation to the Newton method by eliminating the need for computing the second derivatives. For simplicity, single equation non-linear least square is illustrated.

$$\text{Minimize } \Phi(\theta) = \sum_{i=1}^{n} [Y_i - f(X_i,\theta)]^2 = \sum_{i=1}^{n} [Y_i - f_i]^2 = \sum_{i=1}^{n} e_i^2$$

$$\text{then gradient } g_\alpha = \frac{\partial\Phi}{\partial\theta_\alpha} = 2 \sum_{i=1}^{n} e_i \cdot \frac{\partial e_i}{\partial\theta_\alpha} = -2 \sum_{i=1}^{n} e_i \frac{\partial f_i}{\partial\theta_\alpha}$$

$$\text{Hessian } H_{\alpha\beta} = \frac{\partial\Phi}{\partial\theta_\alpha \partial\theta_\beta} = -2 \sum_{i=1}^{n} e_i \frac{\partial^2 f_i}{\partial\theta_\alpha\partial\theta_\beta} - 2 \sum_{i=1}^{n} \frac{\partial f_i}{\partial\theta_\alpha} \cdot \frac{\partial e_i}{\partial\theta_\beta}$$

$$= -2 \sum_{i=1}^{n} e_i \frac{\partial^2 f_i}{\partial\theta_\alpha\partial\theta_\beta} + 2 \sum_{i=1}^{n} \frac{\partial f_i}{\partial\theta_\alpha} \cdot \frac{\partial f_i}{\partial\theta_\beta}$$

Since in general, residuals are small and neglecting the 1st term,

$$H_{\alpha\beta} \cong N_{\alpha\beta} = 2 \sum_{i=1}^{n} \left(\frac{\partial f_i}{\partial \theta_\alpha}\right)\left(\frac{\partial f_i}{\partial \theta_\beta}\right)$$

and

$$\theta = \theta_o - N_\theta^{-1} \cdot g_\theta$$: This is called Newton-Gauss (or Gauss) method.

where θ_o is initial guess, and

$N_\theta^{-1} \cdot g_\theta$ is the directional vector.

For practical applications, gradient is approximated to reduce computational burden.

$$g_\theta = -2\Sigma e_i \left(\frac{\partial e_i}{\partial \theta}\right) \cong -2\Sigma e_i \left(\frac{\Delta e_i}{\Delta \theta}\right)$$

(8) Notes on convergence and initial guess:

The essence of Gauss method is to use the directional vector $\nu_i = N_i^{-1} \cdot g_i$ to determine the direction of ρ(step-size) not the length of ρ, where ν_i is the solution of the set of simultaneous linear equations $N_i \nu_i = -g_i$. Convergence achieved by Gauss method gives local minimum. But the local minimum by Gauss method is a global minimum if measurement errors are not significant, the model fits the data well and specification errors are not significant. Even after one obtains a convergence, it is better to restart the estimation procedure with different sets of initial guess to ensure the convergence gives a global minimum. But when excessive measurement errors or specifications errors exist, or the model doesn't fit the data, then either divergence occurs or convergence to local minimum occurs or certain parameters increase beyond bound. The state of the art of non-linear optimization is such that one cannot obtain correct parameter estimates in a single computer run. The choice of a good initial guess is also an important factor in order to reach convergence to the solution. In choosing an initial guess, one must rely heavily on intuition and prior knowledge of parameters.

In the case of the translog model, an alternative way of finding an initial guess is to change the original model into a simpler model by placing restrictions on the utility functions, and hence by placing restrictions on the parameters.

Recently Smale (1976) showed theoretically the convergence of Newton method (and hence Gauss method) to the global minimum under an arbitrarily given initial guess. The practical application of this theoretical development has not yet been demonstrated.

(9) The statistical properties of the parameter estimates are well discussed in Malinvaud (1970). The parameter estimates are shown to be asymtotically efficient, consistent and normal.

2. Qualitative Choice Estimation Methods

In the qualitative choice estimation methods where the choice is discrete, logit model or probit model is used depending upon the distributional assumption of the random error term of the utility function. In both cases, the method of maximum likelihood is used for the estimation process. In addition to logit and probit models, discriminant analysis is used in qualitative choice estimation methods. But in discriminant analysis, no dependent variable exists. The discriminant analysis is a procedure to find the utility functions of the two groups such that their joint distribution has almost no overlap.

The discrimination rule classifies a new observation of an unknown mode to which mode it is most likely to be chosen. Usually the populations of the two groups are assumed to be normally distributed in the set of variables specified.

Most often, the multilogit model has been used in the qualitative choice travel demand models.

The multinomial probit model has not been used previously because of the computational problems involving cumbersome numerical integration technique in the estimation procedure. However, recent work (Lerman and Manski, 1976) demonstrates the feasibility of the use of the multinomial probit model. The probit model has an advantage over logit model in the sense that probit model allows the cases where the random error terms of the utility function are dependent. More developments are expected in the multinomial probit model.

McFadden (1968) used the Newton-Rapson iteration method in the multilogit model parameter estimation. In the logit model,

$$P(i;A_t) = \frac{e^{X_{it}'\theta}}{\sum_{j\varepsilon A_t} e^{X_{jt}'\theta}} .$$

Then the likelihood function

$$\text{(i)}\quad L = \prod_{t=1}^{T} \prod_{i\varepsilon A_t} P(i;A_t)^{f_t}$$

where T is the number of observations, and

$$f_{it} = \begin{cases} 1 \text{ if alternative i is chosen in observation t,} \\ 0 \text{ otherwise.} \end{cases}$$

(ii) The log likelihood function is:

$$L^* = \sum_{t=1}^{T} \sum_{i \varepsilon A_t} f_{it} \cdot \ell n P(i;A_t)$$

(iii) Gradient:

$$\frac{\partial L^*}{\partial \theta} = \sum_{t=1}^{T} \left[\sum_{i \varepsilon A_t} \left(f_{it} - P(i;A_t)\right) x_{it} \right] = 0$$

where $x_{it} = (x_{it1}, x_{it2}, \ldots, x_{itk})$

(iv) Hessian:

$$\frac{\partial^2 L}{\partial \theta \partial \theta'} = - \sum_{t=1}^{T} \left\{ \sum_{j \varepsilon A_t} (x_{it} - \bar{x}_t) P(i;A_t)(x_{it} - \bar{x}_t)' \right\}$$

where $\bar{x}_t = \sum_{i \varepsilon A_t} x_{it'} P(i;A_t)$

As is seen above, the calculation of Hessian is not tedious and hence the approximation to Hessian is not necessary. McFadden (1973) showed that the estimator L* has optimal asymtatic properties and that there is a unique MLE whenever a maximum exists.

Maximization of L is equivalent to solving the systems of equations (iii) by applying Newton-Rapson Method.

Research has progressed in the alternative logit estimation methods to be useful for disaggregate transportation demand model parameter estimation.

Manski's (1974) maximum score estimation methods do not require the independently Weibull distribution of the random error terms. Only the disturbances are required to be order preserving (i.e. alternative with the highest utility has the greatest probability of being selected). While maximum score estimators are consistent under a broader range of conditions, they are neither asymtotically efficient nor normal. Hence the asymtotic statistical tests of the significance of the coefficients are impossible. But more study in this area is necessary.

Lerman and Manski (1976) categorized three sampling techniques (random, stratified and choice-based sampling) for the calibration of disaggregate travel demand models. Existing logit estimation method yields consistent parameter estimates for random and stratified sampling techniques. But the weighted maximum likelihood estimation method whose estimators are both consistent and asymtotically normal is applicable to choice-based sampling techniques. This estimation method to choice-based sampling can be applied by using existing logit estimation programs with minor changes.

McFadden and Manski (1976) developed further in the area of parameter estimation for discrete choice cases. They showed the consistent and asymtotically efficient estimation methods under alternative sampling processes. They developed and analyzed the maximum likelihood and the pseudo maximum likelihood estimators (or weighted maximum likelihood estimators) into fully constrained MLE, partially constrained MLE and unconstrained MLE depending upon whether B and Q are both known, only one of the two is known or neither of them are known respectively. (P is the marginal distribution of attributes z in the population, $P(z) = \sum_{i \varepsilon C} f(i,z)$ and Q is the marginal distribution of an alternative $i \varepsilon C$, $Q(i) = \sum_{z \varepsilon Z} f(i,z)$ (or aggregate choice shares in the population).

It is well known that MLE's are efficient under a correct model specification and no data measurement errors. But practically these conditions don't hold. Furthermore, the MLE of the logit model perform poorly in the rare choice case. Hence, they developed new estimators (the MM-class estimators) which are relatively inefficient but relatively robust when specification and measurement errors are present.

Further research in this area will contribute not only the estimation methods of the discrete choice case but also the general non-linear simultaneous parameter estimation cases.

3. Test Statistics Used

Since both of the quantitative choice and qualitative choice cases involve non-linear simultaneous estimation methods, the measure of fit used in regression analysis (R^2) is not a good indicator. Instead, the significance of a group of coefficients can be tested using the likelihood ratio. In the qualitative choice estimation case (logit-model case), an index similar to R^2 is used. This index ρ^2 is obtained by transforming likelihood function.

$\rho^2 = 1 - \frac{L^*(\theta)}{L^*(0)}$ and $\bar{\rho}^2$ is ρ^2 adjusted for degrees of freedom

where $L^*(\theta)$ is the log-likelihood function evaluated for the vector of estimated coefficients, and

$L^*(0)$ is the log-likelihood function evaluated for $\theta=0$, where setting $\theta=0$ assumes the equally likelihood alternatives.

In the quantitative choice estimation case (translog-model case), the significance of the group of coefficients or the validity of restrictions is tested by using the likelihood ratio (S.S. Wilks, 1962; H. Theil, 1971).

(i) Test for the significance of the group of coefficients. The likelihood ratio λ is defined as the ratio of the maximum value of the likelihood function with the added group of coefficients $L^*(\theta)$ to the maximum value of the likelihood function without the added group of coefficients $L^*(\theta_o)$, where $\theta \supset \theta_o$.

$$\lambda = \frac{L^*(\theta_o)}{L^*(\theta)}$$

For normally distributed disturbances the likelihood ratio may be written as

$$\lambda = \left(\frac{|\hat{\Sigma}_{\theta_o}|}{|\hat{\Sigma}_{\theta}|}\right)^{-n/2}$$

where $|\hat{\Sigma}_{\theta}|$ = determinant of the θ-included estimator of the var-cov matrix of the disturbances,

$|\hat{\Sigma}_{\theta_o}|$ = determinant of the θ_o-included estimator of the var-cov matrix of the disturbances, and

n = degrees of freedom = number of newly added parameters.

Hence

$$-2\ \ln\lambda = n(\ln|\hat{\Sigma}_{\theta_o}| - \ln|\hat{\Sigma}_{\theta}|)\ .$$

(ii) Test for the Validity of restrictions. The likelihood ratio λ is defined as the ratio of the maximum value of the likelihood function with restriction $L^*_{restr.}$ to the maximum value of the likelihood function without restriction $L^*_{w/o\ restr.}$

$$\lambda = \frac{L^*_{restr.}}{L^*_{w/o\ restr.}}$$

For normally distributed disturbances the likelihood ratio may be written as

$$\lambda = \left(\frac{|\hat{\Sigma}_{restr.}|}{|\hat{\Sigma}_{w/o\ restr.}|}\right)^{-n/2}$$

where $|\hat{\Sigma}_{restr.}|$ = the determinant of the restricted estimator of the var-cov. matrix of the disturbances,

$|\hat{\Sigma}_{w/o\ restr.}|$ = the determinant of the without restricted estimator of the var-cov. matrix of the disturbances, and

n = degrees of freedom = no. of restrictions.

Hence

$$-2\ell n\lambda = n(\ell n|\hat{\Sigma}_{restr.}| - \ell n|\hat{\Sigma}_{w/o\ restr.}|).$$

(iii) Under the null hypothesis, this test statistic '$-2\ell n\lambda$' is distributed asymptotically, as chi-square with the number of degrees of freedom equal to the number of added parameters or number of restrictions to be tested. That is, if H_o is true, then

$$\lim_{n\to\infty} P(-2\ell n\lambda_{H_o} < \chi^2) = \frac{1}{2^{r/2}\Gamma(r/2)} \cdot \int_o^{\chi^2} u^{\frac{1}{2}(r-1)} e^{-\frac{1}{2}u}\, du$$

which means that $-2\ell n\lambda_{H_o}$ converges to chi-square distribution. For the actual use of this test statistic in the present study, see Chapter V.A.3.

CHAPTER V

ESTIMATION RESULTS AND INTERPRETATION

(See Chapter IV.B. Model specifications for reference.)

A. Parameter Estimations

The parameters of the share equations are estimated by the non-linear maximum likelihood method. The TSP (Time Series Processor) program developed by R. E. Hall and B. H. Hall (1974) is used in the parameter estimation process.

Although R^2, the standard error of regression and sum of squared residuals are computable, the χ^2-statistic is a more powerful tool to test the significance of the group of coefficients or the restrictions. Hence we will mainly focus on the log of likelihood function values instead of R^2.

Parameter estimations are done under the following:

- homogeniety and non-additivity case
- equality and symmetry case
- equality case (share equations are free from symmetry restrictions)
- addition of new parameters starting from share equations with constant term only.

1. Equality and Symmetry Restriction Case

This is the most general case (minimum restriction case) of the translog models. The model with prices and income as independent variables is considered under the equality and symmetry restriction case. Its share equations are expressed as,

$$S_j = \frac{P_j x_j}{M} = \frac{a_j + \sum_i b_{ji} \ \ln(P_i/M)}{-1 + \sum_i b_{M_i} \ \ln(P_i/M)} \quad \text{for } i,j=1,2,3$$

Explicitly expressing the share equations in terms of the data variable names,

$$SA = \frac{RPA \cdot QXA}{XM} = \frac{AA+BAA \cdot \ln(RPA/XM)+BAB \cdot \ln(RPB/XM)+BAR \cdot \ln(RPR/XM)}{-1+BMA \cdot \ln(RPA/XM)+BMB \cdot \ln(RPB/XM)+BMR \cdot \ln(RPR/XM)}$$

$$SB = \frac{RPR \cdot QXB}{XM} = \frac{AR+BAB \cdot \ln(RPA/XM)+BBB \cdot \ln(RPB/XM)+BRB \cdot \ln(RPR/XM)}{-1+BMA \cdot \ln(RPA/XM)+BMB \cdot \ln(RPB/XM)+BMR \cdot \ln(RPR/XM)}$$

$$SR = \frac{RPB \cdot QXR}{XM} = \frac{AB+BAR \cdot \ln(RPA/XM)+BRB \cdot \ln(RPB/XM)+BRR \cdot \ln(RPR/XM)}{-1+BMA \cdot \ln(RPA/XM)+BMB \cdot \ln(RPB/XM)+BMR \cdot \ln(RPR/XM)}$$

(Table I-1) shows the parameter estimates and their t-statistics of the above share equations. (Table I-2) shows that the parameter estimates become stationary under different sets of initial guesses. This guarantees that the system converges to the global optimum. (Table I-3) shows the parameter estimates and their t-statistics under (1) homogeneous and non-additivity case, (2) equality and symmetry case, and (3) equality case where the prices and income are independent variables.

<u>Analysis</u> of the parameter estimates

(i) The signs of the parameter estimates all came out as expected. That is, all the constant terms of the share equations AA, AB, AR are negative whose sum equals -1, which satisfies the normality condition. The diagonal elements of the Hessian matrix BAA, BBB, BRR have negative signs as expected.

$$BAA < 0, \quad \det \begin{bmatrix} BAA & BAB \\ BAB & BBB \end{bmatrix} > 0, \quad \text{and } \det \begin{bmatrix} BAA & BAB & BAR \\ BAB & BBB & BRB \\ BAR & BRB & BRR \end{bmatrix} < 0$$

ensure the negative definiteness of the Hessian matrix of the utility function. Among the parameters, the symmetry of the Hessian matrix is satisfied as well as the equality restriction.

(ii) Number of observation $n=28$

number of parameters to be estimated = 8

(AA, BAA, BAB, BAR, AR, BRB, BRR, BMB)

number of share equations to be estimated = 2

hence, degrees of freedom = $28 - \frac{8}{2} = 24$.

The t-values with 24 degrees of freedom with significance level (2-tail t-test) is:

significance level	.40	.20	.10
t-values at d.f. = 24	.857	1.318	1.711

At 20% significance level, parameters AA, BRB, AR, AB are significant.

2. Homogeniety and Non-Additivity Case

This is more restrictive case than equality and symmetry case but this case can accommodate more variables than the equality and symmetry case. The share equations with prices, income, time trend and speed ratio may be expressed as:

$$S_j = \frac{P_j x_j}{M} = \frac{a_j + \sum_{i=1}^{3} b_{ji} \, \ell n(P_i/M) + \sum_{j=1}^{3} b_{jt} \cdot T + \sum b_{js} \, \ell n(Sp)}{(-1)}$$

for $j = 1, 2, 3$.

Several combinations of attribute variables (time trend and speed ratio) are tried and are compared. Model 4 with prices, income, time trend and speed ratio as explanatary variables is chosen to be the representative model of homogeniety and non-additivity restriction case. (Table II-1) has parameter estimates for model 4. All the models considered and compared are shown in (Table II-2). (Table II-3) shows restrictions under the same independent variables (PA,PB,PR,M,T, Sp). The restrictions are (1) homogeniety and non-additivity, (2) equality and symmetry.

In Table II-2, the independent variables considered in each of the models are:

model 1 . . . PA,PB,PR,M (prices, income)

model 2 . . . PA,PB,PR,M,T (prices, income, time trend)

model 3 . . . PA,PB,PR,M,Sp where Sp = SPA/SPBR (prices, income, speed ratio)

model 4 . . . PA,PB,PR,M,T,Sp where Sp = SPA/SPBR (prices, income, time trend, speed ratio)

model 5 . . . PA,PB,PR,M,T,SPA,SPBR (prices, income, time trend, speed of air, speed of bus-rail)

Note that in the homogeniety and non-additivity restriction case, share equation is independent of M since $\sum_i b_{ji} \ell n M = 0$.

Explicitly expressing the share equations in terms of the data variable names,

$$SA = \frac{RPA \cdot QXA}{XM} = \frac{AA + BAA \cdot \ell n(RPA/XM) + BAB \cdot \ell n(RPB/XM) + BAR \cdot \ell n(RPR/XM) + BAT \cdot T + BAS \cdot \ell n(Sp)}{(-1)}$$

$$SB = \frac{RPB \cdot QXB}{XM} = \frac{AB + BAB \cdot \ell n(RPA/XM) + BBB \cdot \ell n(RPB/XM) + BRB \cdot \ell n(RPR/XM) + BBT \cdot T + BBS \cdot \ell n(Sp)}{(-1)}$$

$$SR = \frac{RPR \cdot QXR}{XM} = \frac{AR + BAR \cdot \ell n(RPA/XM) + BRB \cdot \ell n(RPB/XM) + BRR \cdot \ell n(RPR/XM) + BRT \cdot T + BRS \cdot \ell n(Sp)}{(-1)}$$

In the evaluation of the signs of the parameter estimates, the negativity and normality conditions of the constant terms are checked. The negativity of the diagonal terms of the Hessian is checked along with symmetry condition of the Hessian. As for the evaluation of the signs of the attribute variables (e.g. time trend, speed ratio), all the elements of the Hessian are not defined on the share equations. Therefore, only the elements of the Hessian that are defined in the share equations are judged by the prior common knowledge.

The negative value of R^2 in model 1 implies that model 1 is no better than the prediction of shares by using mean values of observed shares.

Model 1 (prices, income) and model 2 (prices, income, time trend) have reasonable signs and reasonable t-statistics of the parameter estimates except that BRR (rail price coefficient of rail share) in model 1 and BBB (bus price coefficient of bus share) in model 2 have positive signs. Both BRR and BBB are diagonal elements of the Hessian matrix and hence they should be negative.

Model 3 (prices, income, speed ratio) has reasonable signs and reasonable t-statistics of its parameter estimates, except that BBB has positive sign and BBS (speed ratio coefficient of bus share) has negative sign.

Model 4 (prices, income, time trend, speed ratio) has reasonable signs and reasonable t-statistics of its parameter estimates, except that BBS has negative sign.

In model 5 (prices, income, time trend, speed of air, speed of bus-rail), AB (constant term of bus share) and BBB have positive signs in its parameter estimates but AB and BBB should both be negative.

From model 1 through model 5, as more variables are added, the log of likelihood functions increase, R^2s increase, standard errors of regression decrease and sums of residual squares decrease for the corresponding airline and rail share equations.

Comparing model 4 and model 5, better results are obtained if one speed variable in the speed ratio term (Sp = SPA/SPBR) is added as in model 4, instead of two separate speed variables SPA and SPBR as in model 5, because model 5 produces incorrect signs of both AB and BBB.

Comparing model 3 and model 4, the inclusion of the time trend variable changes the incorrect positive sign of BBB in model 3 into a reasonable negative sign of BBB in model 4. In both models 3 and 4, BBS is negative.

Comparing model 4 and model 2, the parameters of the time trend variable in model 2 and those in model 4 have exactly opposite signs. In model 2, the parameter estimates of the time trend variable of the three share equations are:

	t-stat.		
BAT = -.000194	(-.430)	:	time coefficient of air share
BRT = .000657	(1.52)	:	time coefficient of rail share
BBT = -.000463	(-2.33)	:	time coefficient of bus share

and for illustration the air share equation is,

$$SA = \frac{RPA \cdot QXA}{XM} = \frac{AA + BAA \cdot \ln(RPA/XM) + BAB \cdot \ln(RPB/XM) + BAR \cdot \ln(RPR/XM) + BAT \cdot T}{(-1)}$$

Therefore, the air share and bus share, SA and SB, increase with time trend but SR decreases with time trend. Note that BAT has a low t-value.

Actual time series data shows that budget share of air SA decreases, while budget share of bus SB increases very slightly as the time trend approaches the present period. Budget share of rail fluctuates somewhat over the years.

In model 4, the parameter estimates of the time trend variable and speed ratio variables are:

BAT = .00335 (4.97) BAS = -.136 (-6.20): time & speed coeff. of air share

BRT = -.00354 (-6.17) BRS = .152 (7.96): time & speed coeff. of rail share

BBT = .000195 (.833) BBS = -.0157 (-2.79): time & speed coeff. of bus share

and the share equation of the air mode is,

$$SA = \frac{RPA \cdot QXA}{XM} = \frac{AA+BAA \cdot \ln(RPA/XM)+BAB \cdot \ln(RPB/XM)+BAR \cdot \ln(RPR/XM)+BAT \cdot T+BAS \cdot \ln(Sp)}{(-1)}$$

Therefore air and bus shares decrease with time, and rail share increases with time. But BBT (time coefficient of bus share) has a smaller value than BAT (time coefficient of air share) or BRT (time coefficient of rail share) and it also has low t-value. Hence one may consider parameter estimates of the time trend variable fit the actual data better than those in model 2.

In model 4, the parameter estimates of the speed ratio variable have the same corresponding signs as those of the time trend variable of model 2. In model 4, the speed ratio variable (Sp = SPA/SPBR) has a positive effect on air and bus shares while it has a negative effect on rail share. Since model 2 does not have the speed ratio variable, the time trend variable explains vaguely people's tastes preference over the time period. Thus, in model 4, the introduction of the speed ratio variable explicitly explains why people's preference changes over time. For example, people's taste preference to the air mode has increased over time because of the technological changes of the air mode over other modes such as speed as well as other service attributes. This assures that people prefer a specific transport mode not because of the attractiveness of the name of that mode but because of the characteristics of the level of service and other attributes that the mode offers.

In model 4, all the parameters have reasonable signs and reasonable t-values, except that BBS (speed ratio coefficient of bus share) is negative. This is because of the homogeniety restriction of the model (BAS+BBS+BRS = 0) and positive BRS (speed ratio coefficient of rail share) is greater than negative BAS (speed ratio coefficient of air share), resulting BBS (speed ratio coefficient of bus share) to be negative. At 10% significance level, all the parameters are significant except BBT (time trend coefficient of bus share) and BBB (bus price coefficient of bus share).

Hence, model 4 is chosen as the representative model for homogeniety and non-additivity case.

Table I-1. Parameter Estimates (equality and Symmetry)

PA, PB, PR, M are independent variables.

n = 28, Log of likelihood function = 227.837

Initial guess	Parameter name	Parameter estimates	t-statistic
AA=-.70	AA	-.815	-65.6
AR=-.15	BAA	-.685	-.409
BMB=-.03	BAB	-.00986	-.125
BAA=-.50	BAR	-.155	-.573
BAB=.05	BMB	-.0575	-.465
BAR=.10	BRB	-.0232	-1.36
BRB=-.06	BRR	-.0887	-1.12
BRR=-.10	AR	-.124	-17.0
	BMA	-.850	-.420
	BMR	-.267	-.742
	BBB	-.0245	-.808
	AB	-.0617	-5.69

	SA	SR
R^2	.214	.207
Standard error of regression	.00901	.00907
$\sum_i e_i^2$	.00227	.00231

Table I-2. Parameter Estimates (Equality and Symmetry).

n=28; PA, PB, PR, M are independent variables.

log of likelihood fn. = 227.837				log of likelihood fn. = 227.837			
Initial guess	Parameter name	Parameter estimates	t-statistic	Initial guess	Parameter name	Parameter estimates	t-statistic
AA=-.70	AA	-.815	(-65.6)	AA=-.83	AA	-.815	(-65.6)
AR=-.15	BAA	-.685	(-.409)	AR=-.12	BAA	-.687	(-.409)
BMB=-.03	BAB	-.00986	(-.125)	BMB=.02	BAB	-.00993	(-.126)
BAA=-.50	BAR	-.155	(-.573)	BAA=.40	BAR	-.155	(-.573)
BAB=.05	BMB	-.0575	(-.465)	BAB=.02	BMB	-.0576	(-.465)
BAR=.10	BRB	-.0232	(-1.36)	BAR=-.06	BRB	-.0232	(-1.36)
BRB=-.06	BRR	-.0887	(-1.12)	BRB=-.06	BRR	-.0887	(-1.12)
BRR=-.10	AR	-.124	(-17.0)	BRR=-.10	AR	-.124	(-16.9)
	BMA	-.850	(-.420)		BMA	-.852	(-.420)
	BMR	-.267	(-.742)		BMR	-.267	(-.742)
	BBB	-.0245	(-.808)		BBB	-.0245	(-.807)
	AB	-.0617	(-5.69)		AB	-.0617	(-5.68)

	SA	SR	SA	SR
R^2	.214	.207	.214	.207
Standard error of regression	.00901	.00907	.00901	.00907
$\sum_i e_i^2$	.00227	.00231	.00227	.00231

Table I-3. Parameter Estimates under Various Restrictions.

PA, PB, PR, M are independent variables (n=28 obs.)

Log of likelihood fn.	Homogeniety and non-additivity case 223.860		Equality and symmetry case 227.837		Equality case 231.689	
Parameter name	Parameter estimates	t-statistic	Parameter estimates	t-values	Parameter estimates	t-statistic
AA	-.823	(-116.)	-.815	(-65.6)	-.817	(-136.0)
BAA	-.0147	(-1.29)	-.685	(-.409)	.151	(.163)
BAB	.0233	(10.8)	-.00986	(-.125)	-.872	(-.911)
BAR	-.00860	(-.746)	-.155	(-.573)	-.0947	(-.180)
BMA			-.850	(-.420)	.144	(.130)
BMB			-.0575	(-.465)	-1.11	(-.970)
BMR			-.267	(-.742)	-.152	(-.244)
AR	-.120	(-16.8)	-.124	(-17.0)	-.137	(-26.4)
BRA					-.0304	(-.231)
BRB	-.0108	(-2.41)	-.0232	(-1.36)	-.197	(-1.45)
BRR	.0194	(1.51)	-.0887	(-1.12)	-.0443	(-.597)
AB	-.0571	(-39.4)	-.0617	(-5.69)	-.0459	(-21.6)
BBA					.0228	(.455)
BBB	-.0125	(-3.45)	-.0245	(-.808)	-.0378	(-.751)
BBR					-.0132	(-.510)
	SA	SR	SA	SR	SA	SR
R^2	-.0094	-.0354	.214	.207	.546	.427
Standard error of regression	.0102	.0104	.00901	.00907	.00685	.00772
$\sum_i e_i^2$	.00292	.00301	.00227	.00231	.00131	.00167

Table II-1. Parameter Estimates (Homogeniety and Non-Additivity) [Model 4] of Table II-2.

PA, PB, PR, M, T, Sp are independent variables.

n=28 obs. Log of likelihood function = 248.259.

Initial guess	Parameter name	Parameter estimates	t-statistic
AA=-.70	AA	-.642	-20.2
AR=-.15	BAA	-.0457	-2.87
BAA=-.50	BAR	.0330	2.53
BAR= .10	BAT	.00335	4.97
BRR=-.10	BAS	-.136	-6.20
BAT= .1	AR	-.330	-12.2
BRT=-.1	BRR	-.0258	-2.26
BAS= .1	BRT	-.00354	-6.17
BRS=-.1	BRS	.152	7.96
	BAB	.0127	1.94
	BRB	-.00719	-1.93
	BBT	.000195	.833
	BBS	-.0157	-2.79
	AB	-.0275	-2.71
	BBB	-.00554	-.786

	SA	SB
R^2	.607	.706
Standard error of regression	.00637	.00553
Σe_i^2	.00114	.000856

Table II-2. Parameter Estimates (Homogeniety and Non-Additivity Case).
n = 28 obs.

	Model 1 PA,PB,PR,M		Model 2 PA,PB,PR,M,T		Model 3 PA,PB,PR,M,Sp		Model 4 PA,PB,PR,M,T,Sp		Model 5 PA,PB, PR,M,T,SPA,SPBR		Constants only	
Log of ll fn.	223.860		227.098		235.020		248.259		256.484		199.207	
Param name	Param estim	(t-stat)	Param estim	(t-stat)	Param estim	(t-stat)	Param estim	(t-stat)	Param estim	(t-stat)	Param estim	(t-stat)
AA	-.823	(-116.)	-.818	(42.9)	-.753	(-22.1)	-.642	(-20.2)	-.356	(-1.55)	-.836	(-436.)
BAA	-.0147	(-1.29)	-.0239	(-1.15)	-.0549	(-2.72)	-.0457	(-2.87)	-.0358	(-2.13)		
BAR	-.00860	(-.746)	.0172	(.848)	.0489	(2.64)	.0330	(2.53)	.0259	(1.86)		
BAT			-.000194	(-.430)			.00335	(4.97)	.00436	(4.32)		
BAS					-.0316	(-2.20)	-.136	(-6.20)				
BAX									-.143	(-6.51)		
BAY									.0692	(1.21)		
AR	-.120	(-16.8)	-.146	(-8.15)	-.230	(-7.41)	-.330	(-12.2)	-.762	(-3.89)	-.123	(-63.9)
BRR	.0194	(1.51)	-.00450	(-.215)	-.0371	(-2.11)	-.0258	(-2.26)	-.0120	(-1.01)		
BRT			.000657	(1.52)			-.00354	(-6.17)	-.00480	(-5.62)		
BRS					.0477	(3.63)	.152	(7.96)				
BRX									.161	(8.64)		
BRY									-.0520	(-1.07)		
BAB	.0233	(10.8)	.00667	(.888)	.00596	(1.21)	.0127	(1.94)	.00994	(1.81)		
BRB	-.0108	(-2.41)	-.0127	(-3.02)	-.0118	(-3.24)	-.00719	(-1.93)	-.0138	(-3.62)		
BBT			-.000463	(-2.33)			.000195	(.833)	.000437	(1.80)		
BBS					-.0161	(-3.96)	-.0157	(-2.79)				
BBX									-.0182	(-3.57)		
BBY									-.0172	(-1.20)		
AB	-.0571	(-39.4)	-.0365	(-4.08)	-.0177	(-1.76)	-.0275	(-2.71)	.118	(1.98)	-.0405	(-44.5)
BBB	-.0125	(-3.45)	.00607	(.711)	.00587	(1.17)	-.00554	(-.786)	.00391	(.641)		
	SA	SR	SA	SR	SA	SR	SA	SR	SA	SR	SA	SR
R^2	-.00940	-.0354	.0713	.0290	.215	.309	.607	.706	.630	.735	.001	.000
S.e.r.	.0102	.0104	.00979	.0100	.00901	.00847	.00637	.00553	.00618	.00524	.0101	.0102
$\sum_i e_i^2$	.00292	.00301	.00269	.00282	.00227	.00201	.00114	.000856	.00107	.000770	.00288	.00291

Table III-3. Parameter Estimates with PA, PB, PR, M, T, Sp (n = 28 obs.)

Log of likelihood fn.	Homogeniety & non-additivity 248.259		Equality & Symmetry 261.737		Equality 263.715	
Parameter name	Parameter estimates	(t-statistics)	Parameter estimates	(t-statistics)	Parameter estimates	(t-statistics)
AA	-.642	(20.2)	-.792	(-15.7)	-.810	(-37.7)
BAA	-.0457	(-2.87)	1.11	(1.61)	.999	(2.15)
BAB	.0127	(1.94)	.0482	(1.57)	-.275	(-.402)
BAR	.0330	(2.53)	.134	(1.80)	-.0129	(-.0465)
BAT	.00335	(4.97)	-.00576	(-.498)	.00713	(.460)
BAS	-.136	(-6.20)	.0281	(.0609)	-.0621	(-.133)
BMA			1.29	(1.63)	1.17	(2.14)
BMB			.0363	(1.41)	-.319	(-.393)
BMR			.117	(1.84)	.0500	(-.149)
BMT			-.00857	(-.607)	.00681	(.385)
BMS			.0794	(.156)	-.0340	(-.0638)
AR	-.330	(-12.2)	-.191	(-2.72)	-.165	(-5.49)
BRA					.123	(2.05)
BRB	-.00719	(-1.93)	-.0103	(-.698)	-.0310	(-.325)
BRR	-.0258	(-2.26)	-.00662	(-.320)	-.0245	(-.538)
BRT	-.00354	(-6.17)	-.00272	(-1.00)	-.000729	(-.429)
BRS	.152	(7.96)	.0678	(2.34)	.0421	(.973)
AB	-.0275	(-2.71)	-.0174	(-.809)	-.0246	(-2.43)
BBA					.0443	(1.96)
BBB	-.00554	(-.786)	-.00161	(-.236)	-.0134	(-.394)
BBR					-.0126	(-.773)
BBT	.000195	(.833)	-.0000895	(-.176)	.000410	(.551)
BBS	-.0157	(-2.79)	-.0164	(-.426)	-.0140	(-.490)
	SA	SR	SA	SR	SA	SR
R^2	.607	.706	.858	.891	.866	.890
Standard error of regression	.00637	.00553	.00383	.00337	.00372	.00337
$\sum_i e_i^2$	.00114	.000856	.000411	.000318	.000388	.000319

3. Use of χ^2 Test Statistics (refer to Chapter IV.C.3)

(i) Test the significance of the group of coefficients under homogeniety and non-additivity case.

Critical values of χ^2								
Degrees of freedom	1	2	3	4	5	6	7	8
5% sig. level	3.84	5.99	7.81	9.49	11.1	12.6	14.1	15.5
2.5% sig. level	5.02	7.38	9.35	11.1	12.8	14.4	16.0	17.5

		2.5% sig.level	5% sig.level

Constant only → PA,PB,PR,M
# param's=2 # param's=5 	
l.l.f.=199.207 l.l.f.=223.860
	$-2\ln\lambda=49.3$
	d.f.=3
		3 parameters significant	3 newly added parameters significant

PA,PB,PR,M → PA,PB,PR,M,T
# param's=5 # param's=7 	
l.l.f.=223.860 l.l.f.=227.098
	$-2\ln\lambda=6.48$
	d.f.=2
		2 parameters insignificant	2 parameters significant

PA,PB,PR,M → PA,PB,PR,M,Sp
# param's=5 # param's=7 	
l.l.f.=223.860 l.l.f.=235.020
	$-2\ln\lambda=22.3$
	d.f.=2
		2 parameters significant	2 parameters significant

PA,PB,PR,M,T → PA,PB,PR,M,T,Sp
# param's=7 # param's=9 	
l.l.f.=227.098 l.l.f.=248.259
	$-2\ln\lambda=42.3$
	d.f.=2
		2 parameters significant	2 parameters significant

The above says that the addition of new variables and therefore new groups of parameters are all significant. The addition of time trend variable is insignificant at 2.5% but significant at 5% level.

(ii) Test the validity of restrictions under the same set of independent variables.

(a) PA,PB,PR,M

		2.5% sig. level	5% sig. level
Equality and symmetry ← ← ← ← ← ← ← ← ← ← Equality 3 restrictions Ll.f.=227.837 Ll.f.=231.689	$-2\ell n\lambda=7.70$ d.f.=3	Insig. (accept restrictions)	Insig.
Homogeneous and non-additivity ← ← ← ← ← ← ← ← ← ← Equality and symmetry 3 restrictions Ll.f.=223.860 Ll.f.=227.837	$-2\ell n\lambda=7.95$ d.f.=3	Insig. (accept restrictions)	Sig. (reject restrictions)

The insignificance of the symmetry restriction implies that symmetry restriction on the utility function is consistent with the evidence. Equality and symmetry case of PA, PB, PR, M is chosen as representative instead of homogeniety and non-additivity case of PA,PB,PR,M because of the more reasonable signs of the parameter estimates.

(b) PA,PB,PR,M,T,Sp

		2.5% sig. level	5% sig. level
Equality and symmetry ← ← ← ← ← ← ← ← ← ← Equality 3 restrictions Ll.f.=261.737 Ll.f.=263.715	$-2\ell n\lambda=3.96$ d.f.=3	Insig. (accept restrictions)	Insig.
Homogeneous and non-additivity ← ← ← ← ← ← ← ← ← ← Equality and symmetry 5 restrictions Ll.f.=248.259 Ll.f.=261.737	$-2\ell n\lambda=27.0$ d.f.=5	Sig. (reject restrictions)	Sig.

Again, the insignificance of the symmetry restriction implies that symmetry restriction on the utility function is consistent with the evidence. The homogeneous and non-additivity restrictions of the equality and symmetry case is significant. But homogeneous and non-additivity case with PA, PB, PR, M, T, Sp is chosen to be representative because of the more reasonable signs of the parameter estimates.

B. Elasticities

[Refer to: Chapter III.E.1. Translog Utility Models
Chapter IV.B. Model Specification]

For the present study, the Marshallian elasticities, Hicksian income-compensated elasticities and Allen-Uzawa elasticities of substitution are all computed and analyzed under equality and symmetry restricted model and also under homogeniety and non-additivity restricted model.

1. Equality and Symmetry Restriction Model (Table III-1)

(a) In the Marshallian elasticities, own-price elasticities, cross-price elasticities and income elasticities are calculated using yearly data and average data of independent variables.

The own-price elasticities of all three modes are negative, which implies that all three modes are normal goods. The own-price elasticities of airline demand are fairly stable over the years and they are elastic. The own-price elasticities of the bus mode increase somewhat in magnitude over the years and they are inelastic. The own-price elasticities of the rail decreased somewhat in magnitude over the years and they are inelastic.

Income elasticities of the three modes are all positive which implies that all three modes are superior goods and the income elasticity of the air mode is the highest among the three modes (1.13). Income elasticities of the air mode are stable but in the increasing direction over the years. Income elasticities of the bus mode increase over the years, while income elasticities of the rail mode decrease over the years.

Both the own-price and income elasticities of the market demand (Marshallian demand) explain the characteristics of all three modes. But the Marshallian own-price elasticities are not a correct measure to see the pure own substitution effect. The cross-price elasticities of the market demand are partly negative and partly positive. But in transportation, the three available competing modes are normally considered as substitutes for each other. Therefore the cross elasticities should be positive. Constraining the signs of parameters were frequently used in the aggregate travel demand model calibration handled the undesired signs of elasticities.

(b) The Hicksian income-compensated own and cross price elasticities are the correct measure to see the substitution effect (Henderson and Quandt, 1958; Intriligator, 1971).

The Slutsky equation is used (reference: Chapter III.E.2) to get Hicksian income-compensated elasticities to measure the pure substitution effects.

Note again that in the market demand, as price changes both income and substitution effects are counted and money income M is fixed with the utility level being varied. But in the income-compensated demand, as price changes only substitution effect is counted and the utility level is held constant with money income being varied.

Looking at the Hicksian income-compensated own-price elasticities, the air mode has very small own-price elasticities ($HE_{AA} = -.0828$). Bus and rail modes have similar magnitude in their own price Hicksian elasticities ($HE_{BB} = -.525$, $HE_{RR} = -.614$). That is, air mode has the smallest own substitution effect. Bus and rail mode have some own substitution effect with rail mode having larger own substitution effect than bus mode. All of the own-price elasticities are negative in sign, which is reasonable. Furthermore, it is interesting to notice that the own air price Marshallian elasticity ($E_{AA} = -1.02$) and the own air price Hicksian elasticity ($HE_{AA} = -.0828$) differ quite much, while those of rail and bus modes do not differ much. From Slutsky equations, one can observe that the Marshallian elasticity differs from that of the Hicksian by the product of the budget share to the income elasticity. This difference is the income effect that the Marshallian elasticity includes in addition to the usbstitution effect. Therefore, air mode has greater income effect than the bus and rail modes. Bus and rail modes have very little income effect. This means that the demand of air mode is not mainly determined by airline fare changes but is mainly determined by the income change of the consumers. And the demand of bus and rail mode is mainly determined by bus and rail fair changes. This further indicates that there is a market segmentation between air passengers and bus, rail passengers. Air passengers are the ones whose income group is high while bus, rail passengers are the ones whose income group is low.

The Hicksian income-compensated cross elasticities have all positive signs, which ensure that the three trip modes are substitutes for each other. The cross elasticity of rail demand with respect to air price change (HE_{RA} = .501) is larger than the cross elasticity of bus demand with respect to air price (HE_{BA} = .177). There is also some cross substitution effect between bus and rail modes, giving cross elasticity of bus demand with respect to rail price as HE_{BR} = .348 and cross elasticity of rail demand with respect to bus price as HE_{RB} = .112. There are very weak cross substitution effects between air and rail and also between air and bus, giving cross elasticity of air demand with respect to rail price as HE_{AR} = .0743 and cross elasticity of air demand with respect to bus price as HE_{AB} = .00847. One may notice that cross substitution effect between air and rail is slightly greater than the cross substitution effect between air and bus. One may also notice that more cross substitution effect exists between bus and rail than between air and bus or between air and rail.

Over the years, HE_{AB}, HE_{BA} and HE_{RB} increase while HE_{AR}, HE_{BR} and HE_{RA} decrease. Actual data shows that bus is used more for the short intercity travel. The use of air mode for the short intercity travel nowadays, may influence cross substitution effect between bus and air modes. The popular use of air mode for the long trips as consumer's income increases, may reduce the cross substitution effect between rail and air modes over the years.

(c) Elasticities of substitution (reference: Chapter III.C.3 and E.2) measure the extent to which the quantity ratio of commodities in response to changes in the price ratio of the corresponding commodities (or other corresponding attribute variable ratio) by holding utility level constant. Note that $\sigma_{ij} = \sigma_{ji}$.

$$\text{From Allen-Uzawa, } \sigma_{ij} = \frac{M \cdot M_{ij}}{M_i \cdot M_j} \quad \text{and} \quad \sigma_{ji} = \frac{M \cdot M_{ji}}{M_i \cdot M_j}$$

while $M_{ij} = \frac{\partial}{\partial P_j}(x_i^*)$ and $M_{ji} = \frac{\partial}{\partial P_i}(x_j^*)$, $M_i = x_i^*$, $M_j = x_j^*$

but $\frac{\partial}{\partial P_j}(x_i^*) = \frac{\partial}{\partial P_i}(x_j^*)$ (p. 158, M. D. Intriligator)

hence $\sigma_{ij} = \sigma_{ji}$.

$AE_{BR} = AE_{RB} = 2.81$, $AE_{AR} = AE_{RA} = .600$ and $AE_{AB} = AE_{BA} = .212$, under the average independent variables. That is, greater substitution is between bus and rail than between air and rail or between air and bus. Slightly more substitution exists between air and rail than between air and bus. This indicates that bus and rail are close substitutes. Again, one can deduce that there is a market segmentation between air passengers and bus, rail passengers. Elasticity of substitution is slightly greater between air and rail than between air and bus. This is reasonable because rail is more used than bus for the long distance travel and air and rail are competing more than air and bus for the long distance travels. But bus is used much for the short distance travels. Over the years, elasticity of substitution increased slightly between bus and air while it decreased slightly between air and rail and it fluctuated somewhat between bus and rail. The usage of air for the short distance intercity travel may result in a slight increase in the elasticity of substitution between bus and air. The increase in in the usage of air for the longer intercity travel may result in the slight decrease in the elasticity of substitution between air and rail.

The results of elasticities of substitution is consistent with those of Hicksian cross elasticities. Elasticities of substitution further ensures the interrelated reasonings between Marshallian and Hicksian elasticities.

2. Homogeniety and Non-Additivity Restriction Model

(Table III-2)

(a) In the Marshallian demand, own-price elasticities are all negative, which is reasonable. There is not much gap among own-price elasticities of three modes. (E_{AA} = -.945, E_{BB} = -.863, E_{RR} = -.790 evaluated under average independent variables.) Note also that the own-price elasticity of bus is greater than that of rail in the homogeniety and non-additivity case, while the own price elasticity of rail is greater than that of bus in the equality and symmetry case. This is because of the log-linearity of the homogeniety and non-additivity model and the inclusion of other attribute variables in the model. Over the years, the own-price elasticities of the air mode are stable, those of the bus mode increased somewhat but those of rail mode decreased somewhat in magnitude (same as in equality and symmetry case).

The time trend elasticities are: E_{AT} = -.0580, E_{BT} = -.0699 and E_{RT} = .418 (where $E_{jt} = -b_{jt} \cdot T/S_j$)

Over the years, time trend elasticities of all three modes increased in magnitude because time trend variable is put in the model to increase yearly by 1 starting from 1 and ending to 28.

The speed elasticities are: $E_{ASp} = .163$, $E_{BSp} = .390$ and $E_{RSp} = -1.24$ (where $E_{jSp} = -b_{js}/S_j$).

Over the years, the speed elasticities of the air mode increased, those of the bus mode decreased and those of the rail mode fluctuated in magnitude mainly because share of the rail mode fluctuated over the years.

Under homogeniety and non-additivity restriction, the model has unitary income elasticities (Chapter IV.B.2). This is rather a restricted situation but many demand systems have unitary income elasticities such as double logarithmic demand system and Rotterdam-demand system.

As for the cross-price elasticities of the Marshallian demand, some of them are positive and some of them are negative. Hence the Hicksian income-compensated elasticities are needed.

(b) Looking at the Hicksian income-compensated elasticities, the air mode has the smallest own-price elasticities next the rail mode and then the bus mode. All the own-price elasticities are negative as expected. Again the air mode has the smallest own-substitution effect. Hence, air mode has the greater income effect than the bus, rail modes. Similar interpretations may be drawn as in equality and symmetry model (see page 111). The Hicksian income-compensated cross elasticities have all positive signs which again ensure that the three trip modes are substitutes for each other. ($HE_{RA} = .568$, $HE_{BA} = .522$, $HE_{BR} = .301$, $HE_{RB} = .0989$, $HE_{AR} = .0835$ and $HE_{AB} = .0252$.) Over the years HE_{AB}, HE_{BA} and HE_{RB} are increased while HE_{AR}, HE_{BR} and HE_{RA}

are decreased. Again see equality and symmetry case (page 113) for the similar interpretations of the Hicksian cross elasticities.

The Hicksian income-compensated time trend and speed ratio elasticities are $HE_{AT} = -.0600$, $HE_{BT} = -.0719$, $HE_{RT} = .416$ and $HE_{ASp} = .242$, $HE_{BSp} = .469$, $HE_{RSp} = -1.16$. Comparing these with Marshallian time trend and speed ratio elasticities, they have correspondingly same signs and corresponding values are similar.

(c) Elasticities of substitution are calculated with only price variables. $AE_{BR} = AE_{RB} = 2.45$, $AE_{AR} = AE_{RA} = .679$ and $AE_{AB} = AE_{BA} = .624$ computed under average independent variables. Here, they show that greater substitution exists between bus and rail than any other pairs. But there is not much gap between elasticity of substitution of air, rail and elasticity of substitution of air, bus. See equality and symmetry case (page 114) for the interpretations of the elasticities of substitution.

In summary, both the Hicksian own and cross elasticities are the correct measures to see the own and cross substitution effects. By comparing Marshallian and Hicksian elasticities, one can find out how much income effect is in each mode.

For the present analysis, both restriction case models indicate that income effect is big and own substitution effect (price effect) is small in the air mode, while income effect is small and own substitution effect (price effect) is big in both bus and rail modes. The big income effect and very small price effect of the air mode may partly be due to the inclusion of business trips in the data. But the underlying income and price effects will still hold even if business trips are excluded.

For policy implication, the above results are useful in determining whether deregulation of prices is necessary or not. As for the elasticities of other attribute variables, both Marshallian and Hicksian approaches give similar results.

Allen-Uzawa elasticities of substitution is also a good measure to see the differential substitutability between pairs of trip commodities. Greater substitution exists between bus and rail than between other modes. This indicates that market segmentation exists between air users and bus, rail users. Actual data indicates that bus is used for the short intercity trips, hence air and rail compete more than air and bus for the long distance trips, as is seen from the elasticity of substitution in equality and symmetry case. This reveals that stratification by distances is also necessary for further developments.

Table III-1. Equality and Symmetry Case with PA,PB,PR,M as independent variables.

Marshallian Elasticities

	Direct Elast.			Cross Elast.						Income Elast.		
Year	E_{AA}	E_{BB}	E_{RR}	E_{AB}	E_{AR}	E_{BA}	E_{BR}	E_{RB}	E_{RA}	E_{MA}	E_{MB}	E_{MR}
1950	-1.02	-.545	-.693	-.0334	-.0593	-.420	.276	.0891	.256	1.11	.689	.348
1954	-1.02	-.553	-.675	-.0345	-.0614	-.445	.263	.0943	.279	1.12	.735	.302
1958	-1.02	-.561	-.656	-.0358	-.0636	-.471	.250	.0997	.302	1.12	.782	.254
1962	-1.03	-.515	-.598	-.0378	-.0687	-.487	.285	.115	.386	1.14	.717	.0962
1966	-1.03	-.552	-.597	-.0386	-.0693	-.517	.246	.116	.381	1.14	.823	.101
1970	-1.03	-.565	-.530	-.0427	-.0765	-.597	.214	.134	.463	1.15	.947	.0674
1974	-1.02	-.581	-.568	-.0420	-.0741	-.592	.202	.124	.402	1.14	.971	.0417
	Marshallian Elasticities Under Average Independent Variables											
	-1.02	-.557	-.641	-.0366	-.0653	-.484	.250	.104	.321	1.13	.790	.216

Average Independent Variables and Shares are:

$\overline{RPA} = .0645$, $\overline{RPB} = .0285$, $\overline{RPR} = .0350$, $\overline{M} = .0436$,

$\overline{QXA} = .576$, $\overline{QXB} = .0619$, $\overline{QXR} = .154$,

$\overline{SA} = .836$, $\overline{SB} = .0405$, $\overline{SR} = .123$.

Table III-1 (continued)

Hicksian Income-Compensated Elasticities

	Direct Elast.			Cross Elast.					
Year	HE_{AA}	HE_{BB}	HE_{RR}	HE_{AB}	HE_{AR}	HE_{BA}	HE_{BR}	HE_{RB}	HE_{RA}
1950	-.0912	-.520	-.648	.00668	.0845	.155	.365	.102	.547
1954	-.0885	-.525	-.637	.00763	.0808	.169	.357	.106	.531
1958	-.0859	-.530	-.624	.00865	.0772	.182	.348	.110	.514
1962	-.0704	-.488	-.587	.00532	.0651	.119	.369	.119	.468
1966	-.0746	-.518	-.585	.00861	.0660	.174	.344	.120	.465
1970	-.0668	-.520	-.538	.0110	.0559	.197	.323	.131	.407
1974	-.0756	-.534	-.563	.0124	.0632	.216	.319	.126	.437
Hicksian Income-Comp. Elast. under Avg. Indep. variables									
	-.0828	-.525	-.614	.00847	.0743	.177	.348	.112	.501

Allen-Uzawa Elasticity of Substitution

Year	$\delta_{AB}=\delta_{BA}$	$\delta_{AR}=\delta_{RA}$	$\delta_{BR}=\delta_{RB}$
1950	.186	.655	2.83
1954	.202	.636	2.80
1958	.219	.615	2.77
1962	.141	.554	3.14
1966	.207	.554	2.89
1970	.235	.485	2.81
1974	.259	.525	2.65
Elast. of Subs. under Avg. Ind. Var.			
	.212	.600	2.81

Table III-2. Homogeniety and Non-Additivity Case with PA,PB,PR,M,T,Sp as Independent Variables.

	Marshallian Elasticities														
	Direct Elasticities									Cross Elasticities					
Year	E_{AA}	E_{BB}	E_{RR}	E_{AT}	E_{ASp}	E_{BT}	E_{BSp}	E_{RT}	E_{RSp}	E_{AB}	E_{AR}	E_{BA}	E_{BR}	E_{RB}	E_{RA}
1950	-.946	-.846	-.791	-.0159	.162	-.0217	.439	.115	-1.23	-.0151	-.0393	-.354	.200	.0581	-.267
1954	-.945	-.852	-.796	-.0320	.163	-.0417	.422	.224	-1.20	-.0152	-.0394	-.341	.193	.0569	-.261
1958	-.945	-.855	-.812	-.0487	.165	-.0613	.413	.310	-1.11	-.0154	-.0400	-.334	.189	.0525	-.241
1962	-.946	-.860	-.787	-.0638	.162	-.0792	.400	.468	-1.25	-.0152	-.0393	-.323	.183	.0594	-.273
1966	-.946	-.870	-.781	-.0797	.162	-.0917	.371	.600	-1.29	-.0152	-.0393	-.300	.169	.0609	-.280
1970	-.946	-.882	-.771	-.0956	.162	-.0993	.334	.754	-1.35	-.0151	-.0393	-.270	.153	.0637	-.293
1974	-.946	-.888	-.762	-.111	.162	-.110	.318	.913	-1.40	-.0151	-.0392	-.257	.145	.0662	-.304
	Marshallian Elasticities under Average Independent Variables														
	-.945	-.863	-.790	-.0580	.163	-.0699	.390	.418	-1.24	-.0152	-.0394	-.315	.178	.0585	-.268

Table III-2 (continued)

Hicksian Income-Compensated Elasticities

Year	Direct Elasticities									Cross Elasticities					
	HE_{AA}	HE_{BB}	HE_{RR}	HE_{AT}	HE_{ASp}	HE_{BT}	HE_{BSp}	HE_{RT}	HE_{RSp}	HE_{AB}	HE_{AR}	HE_{BA}	HE_{BR}	HE_{RB}	HE_{RA}
1950	-.105	-.810	-.668	-.0182	.252	-.0240	.529	.113	-1.14	.0208	.0844	.486	.324	.0940	.574
1954	-.109	-.814	-.669	-.0342	.251	-.0439	.511	.222	-1.11	.0221	.0868	.496	.319	.0942	.575
1958	-.120	-.817	-.675	-.0508	.248	-.0634	.496	.308	-1.03	.0227	.0969	.491	.326	.0906	.584
1962	-.106	-.820	-.666	-.0663	.260	-.0817	.498	.466	-1.16	.0242	.0818	.516	.304	.0987	.567
1966	-.106	-.827	-.663	-.0818	.245	-.0938	.454	.598	-1.20	.0273	.0787	.540	.287	.103	.560
1970	-.105	-.835	-.658	-.0969	.215	-.101	.387	.753	-1.29	.0319	.0735	.570	.265	.111	.548
1974	-.104	-.839	-.654	-.112	.191	-.111	.348	.912	-1.37	.0344	.0694	.585	.254	.116	.538
	Hicksian Income-Compensated Elasticities under Avg. Indep. Variables														
	-.109	-.823	-.667	-.0600	.242	-.0719	.469	.416	-1.16	.0252	.0835	.522	.301	.0989	.568

Table III-2 (continued)

Allen-Uzawa Elasticity of Substitution

Year	$\delta_{AB}=\delta_{BA}$	$\delta_{AR}=\delta_{RA}$	$\delta_{BR}=\delta_{RB}$
1950	.578	.683	2.62
1954	.592	.688	2.53
1958	.595	.708	2.38
1962	.615	.676	2.51
1966	.643	.667	2.44
1970	.678	.652	2.35
1974	.695	.639	2.34
Elast. of Sub. under Avg. Ind. Var.			
	.624	.679	2.45

Average Independent Variables and Shares are:

$\overline{RPA} = .0645$, $\overline{RPB} = .0285$, $\overline{RPR} = .0350$,

$\overline{M} = .0436$, $\overline{QXA} = .576$, $\overline{QXB} = .0619$,

$\overline{QXR} = .154$, $\overline{T} = 14.5$, $\overline{Sp} = 1.564$,

$\overline{SA} = .836$, $\overline{SB} = .0405$, $\overline{SR} = .123$.

C. Sensitivity Analyses

Sensitivity analyses concretely give us more information about the estimated models' performances in relation to elasticities.

In the sensitivity analyses, both the aggregate intercity models developed for NECTP and the disaggregate models applied for intercity passenger forecasting (P. Stopher and J. Prashker) have been found to have some counter-intuitive results. That is, as the price of a mode increases, not only the use of that mode is decreased but also the use of the other modes is decreased. They explained that the counter-intuitive result is due to the use of the simple average of the independent variable in the model and in the cases where the market share is small and where its variable value is relatively large compared with the other modes, the counter-intuitive result may occur.

For the present study, the use of Hicksian income-compensated simulation gives us <u>no</u> counter-intuitive results. It gives us very reasonable results.

The Hicksian income-compensated simulation is based on the Hicksian income-compensated demand equations, which have the same form as Marshallian demand equations except that M in the Hicksian income-compensated demand equations is the expenditure function instead of a fixed value. The expenditure function M is a function of prices, other attribute variables and the utility index. The translog expenditure function can be expressed in terms of the parameters of the Marshallian demand equations which are already estimated from the prices, income and other attribute variable data. Note that double logarithmic demand system is a special case of translog models. Other type of demand system such as addilog system do not have such property (expenditure function cannot be expressed in terms of the parameters of the Marshallian demand equations).

In the Hicksian simulation, M is varied and utility level is fixed while in the Marshallian simulation, utility is varied and M is fixed. Both simulations are done under equality and symmetry model and homogeneity and non-additivity model.

1. Equality and Symmetry Model

Marshallian share equations and demand equations are:

$$S_j = \frac{P_j x_j}{M} = \frac{a_j + \sum_i b_{ji} \ \ell n(P_i/M)}{-1 + \sum_i b_{Mi} \ \ell n(P_i/M)} \quad \text{for } j = 1,2,3 \quad \text{where M is fixed as } P_i \text{ varies.}$$

$$x_j = \left(\frac{M}{P_j}\right) \frac{a_j + \sum_i b_{ji} \ \ell n(P_i/M)}{-1 + \sum_i b_{Mi} \ \ell n(P_i/M)} \quad \text{for } j = 1,\ 2,\ 3$$

where M is fixed and ℓn V varies as P_i varies.

$$\ell n\ V = \sum_i a_i \ell n(P_i/M) + \frac{1}{2} \Sigma\Sigma\ b_{ij}\ \ell n(P_i/M)\ \ell n(P_j/M)$$

$$= \sum_i a_i \ell n(P_i) + \ell nM + \frac{1}{2} \sum_{ij}\sum b_{ij} \ell n(P_i)\ \ell n(P_j)$$

$$- \ell nM \sum_{ij}\sum b_{ij}(\ell n P_i) + \frac{1}{2}(\ell nM)^2 \sum_i \sum_j b_{ij} .$$

Hicksian income-compensated share equations and demand equations are:

$$S_j = \frac{P_j x_j}{M} = \frac{a_j + \sum_i b_{ji} \ \ell n(P_i/M)}{-1 + \sum_i b_{Mi} \ \ell n(P_i/M)} \quad \text{for } j = 1,\ 2,\ 3$$

where the expenditure function M is varied as P_i varies.

$$x_j = \left(\frac{M}{P_j}\right) \frac{a_j + \Sigma\ b_{ji} \ \ell n(P_i/M)}{-1 + \sum_i b_{Mi} \ \ell n(P_i/M)} \quad \text{for } j = 1,\ 2,\ 3$$

where the expenditure function M is varied and ℓn V is fixed as P_i varies.

$$\ell n\ V = A_o + A_1\ \ell nM + A_2(\ell n\ M)^2$$

where $A_o = \sum_i a_i \ln(P_i) + \frac{1}{2} \sum_i \sum_i b_{ij} \ln(P_i) \ln(P_i)$

$$A_1 = 1 - \sum_i \sum_j b_{ij} \ln P_i$$

$$A_2 = \frac{1}{2} \sum_i \sum_j b_{ij}$$

Solving for $\ln M$, log of the expenditure function is

$$\ln M = \frac{-A_1 \pm \sqrt{A_1^2 - 4A_2(A_o - \ln V)}}{2A_2}$$

hence

$$M_1 = \mathrm{Exp}\left(\frac{-A_1 + \sqrt{A_1^2 - 4(A_o - \ln V)}}{2A_2}\right)$$

$$M_2 = \mathrm{Exp}\left(\frac{-A_1 - \sqrt{A_1^2 - 4(A_o - \ln V)}}{2A_2}\right)$$

Since utility level is fixed, the smaller positive value of the two possible M's is chosen. (Table IV-1) and (Table IV-2) give the results of both Marshallian and Hicksian simulation under equality and symmetry model.

In the Marshallian simulation, only the effects on its own demand work reasonably as its own-price changes. However, some of the effects on the demand of the competing modes are counter-intuitive.

But in the Hicksian income-compensated simulation, the model predicts very reasonably. That is, as the price of a mode increases, its direct effect (effect on the demand of that mode) is the decrease in the demand of that mode, but its cross effect (effect on the demand of competing modes) is the increase in the demands of competing modes.

In the Marshallian simulation, fixed M = .0436 and the varying utility levels are calculated for each price change and are shown in Table IV-1. As the price of a trip mode increases, utility level is decreased. Oppositely as the price of a trip mode decreases, utility level is increased. That is, utility function is decreasing with increasing prices and increasing with decreasing prices. This is consistent with the utility theory.

In the Hicksian simulation, fixed V = .735 and the varying money incomes are calculated for each price change and are shown in Table (IV-2). Money income increases with increasing prices while it decreases with decreasing prices. Considering one example:

In the Marshallian simulation, as air price increases by 10%, 53 air passenger miles are decreased, 2.6 bus passenger miles are decreased while 5 rail passenger miles are increased. Decrease in the bus passenger miles when air price increases is unreasonable.

In the Hicksian simulation, as air price increases by 10%, 4 air passenger miles are decreased while .9 bus passenger miles and 8 rail passenger miles are increased. These are never counter-intuitive.

As air price increases by 10%, 4 air passenger miles decrease in the Hicksian simulation is due to substitution effect (price effect) only but 53 air passenger miles decrease in the Marshallian simulation is due to both substitution and income effect. Cross effects may be analyzed in a similar manner.

2. Homogeniety and Non-Additivity Model

Marshallian share equations and demand equations are:

$$S_j = \frac{P_j x_j}{M} = \frac{a_j + \sum_i b_{ji} \ln P_i + b_{jt} T + b_{js} \ln(Sp)}{(-1)} \quad \text{for } j = 1, 2, 3$$

$$x_j = \left(\frac{M}{P_j}\right)\left(\frac{a_j + \sum_i b_{ji} \ \ln P_i + b_{jt} \cdot T + b_{js} \ \ln(Sp)}{(-1)}\right) \quad \text{for } j = 1, 2, 3$$

where M is fixed and $\ln V$ varies as P_i varies.

The time and speed adjusted utility function is:

$$\ln V = a_i \ \ln(P_i/M) + 1/2 \sum_i \sum_j b_{ij} \ \ln(P_i/M) \ \ln(P_j/M)$$

$$+ \sum_i b_{it} \ \ln(P_i/M) \cdot T + \sum_i b_{is} \ \ln(P_i/M) \ \ln(Sp)$$

$\ln V$ with homogeniety restriction $= \sum_i a_i \ \ln P_i + \ln M + 1/2 \sum_i \sum_j b_{ij} (\ln P_i)(\ln P_j)$

$$+ \sum_i b_{it} (\ln P_i) T + \sum_i b_{is} (\ln P_i)(\ln Sp)$$

Hicksian income-compensated share equations and demand equations are:

$$S_j = \frac{P_j x_j}{M} = \frac{a_j + \sum_i b_{ji} \ \ln P_i + b_{jt} \cdot T + b_{js} \ \ln(Sp)}{(-1)} \quad \text{for } j=1,2,3$$

(Note that share equation in this case doesn't carry M in the right hand side, hence varying M as P_i varies will not affect share equations.)

$$x_j = \left(\frac{M}{P_j}\right)\left(\frac{a_j + \Sigma b_{ji} \ \ln P_i + b_{jt} \cdot T + b_{js} \ \ln(Sp)}{(-1)}\right) \quad \text{for } j=1,2,3$$

where the expenditure function M is varied and $\ln V$ is fixed as P_i varies.

$$\ln V = \sum_i a_i \ \ln(P_i) + \ln M + 1/2 \sum_i \sum_j b_{ij} (\ln P_i)(\ln P_j) + \sum_i b_{it} (\ln P_i) \cdot T$$

$$+ \sum_i b_{is} (\ln P_i)(\ln Sp)$$

and solving for $\ln M$, log of the expenditure function is:

$\ln M = \ln V - \sum_i a_i \ln(P_i) - \frac{1}{2} \sum_i \sum_j b_{ij} (\ln P_i)(\ln P_j) - \sum_i b_{it} (\ln P_i) \cdot T - \sum_i b_{is} (\ln P_i)(\ln Sp)$, and

$M = \exp(\ln V - \sum_i a_i \ln(P_i) - \frac{1}{2} \sum_i \sum_j b_{ij} (\ln P_i)(\ln P_j) - \sum_i b_{it} (\ln P_i) \cdot T - \sum_i b_{is} (\ln P_i)(\ln Sp))$.

(Table V-1) and (V-2) give the results of both Marshallian and Hicksian simulation under homogeniety and non-additivity model.

In the Marshallian simulation, only the effects on the direct demand are reasonable as its own-price changes and attribute variable (time trend and speed ratio) changes. But again, as in equality and symmetry model, some of the effects on the demand of the competing modes are counter-intuitive.

However, in the Hicksian income-compensated simulation, the model predicts very reasonably. The direct and cross effects as the price changes are not at all counter-intuitive. The effects as the attribute variable (time trend and speed ratio) changes are also reasonable.

Fixed M = .0436 and the varying utility levels of the Marshallian simulations are found in (Table V-1).

Fixed V = .762 and the varying money incomes of the Hicksian simulations are found in (Table V-2).

In the Marshallian simulation, utility level decreases with increasing prices, increases with increasing time trend, decreases with increasing speed ratio and vice versa. The change of utility level with respect to price change is consistent with utility theory. Also the change of utility level is realistic with time trend and speed ratio.

In the Hicksian simulation, money income increases with increasing prices, decreases with increasing time trend, increases with increasing speed ratio and vice versa. Since expenditure function has inverse relationship with utility function, the change of money income with price, time trend and speed ratio changes must be opposite to that of utility level.

In effect, both elasticity and sensitivity considerations are very important for transportation policy implementation. The approach with both Hicksian and Marshallian considerations brings far better idea about the demand system.

Table IV-1. Marshallian Simulation (Equality and Symmetry)

PA,PB,PR,M are independent variables.	X_A	X_R	X_B	Utility level (V)
Observed quantity	.576	.154	.0619	
Predicted quantity	.566	.154	.0613	.735
M = .0436 (fixed)				
increase PA by 10%	.513	.159	.0587	.663
increase PA by 25%	.451	.164	.0556	.572
decrease PA by 10%	.631	.149	.0647	.817
decrease PA by 25%	.769	.137	.0719	.964
increase PR by 10%	.563	.145	.0627	.724
increase PR by 25%	.558	.133	.0646	.708
decrease PR by 10%	.570	.165	.0597	.746
decrease PR by 25%	.577	.183	.0566	.765
increase PB by 10%	.564	.156	.0581	.731
increase PB by 25%	.561	.158	.0538	.726
decrease PB by 10%	.568	.153	.0649	.738
decrease PB by 25%	.572	.150	.0712	.744

Sensitivity analysis (simulation) is done with respect to average independent variables on both Marshallian and Hicksian simulations.

$\overline{RPA} = .0645$, $\overline{RPB} = .0285$, $\overline{RPR} = .0350$, $\overline{M} = .0436$,

$\overline{QXA} = .576$, $\overline{QXB} = .0619$, $\overline{QXR} = .154$, $\overline{SA} = .836$

$\overline{SB} = .0405$, $\overline{SR} = .123$

Note: For simple notation, PA, PB, PR stands for RPA, RPB, RPR respectively, and X_A, X_B, X_R stands for QXA, QXB, QXR respectively. X_A, X_B, X_R are in 'thousands of passenger miles' that a representative individual travels during one year period.

Table IV-2.

Hicksian Income-Compensated Simulation (Equality and Symmetry).

PA,PB,PR,M are independent variables

	X_A	X_R	X_B	Money income M
Observed quantity	.576	.154	.0619	.0436
Predicted quantity	.566	.154	.0613	.0436
Utility V = .735 (fixed)				
increase PA by 10%	.562	.162	.0622	.0473
increase PA by 25%	.557	.172	.0628	.0527
decrease PA by 10%	.571	.146	.0600	.0400
decrease PA by 25%	.582	.133	.0572	.0344
increase PR by 10%	.570	.145	.0634	.0442
increase PR by 25%	.576	.134	.0662	.0447
decrease PR by 10%	.562	.165	.0591	.0431
decrease PR by 25%	.555	.183	.0553	.0422
increase PB by 10%	.567	.156	.0583	.0438
increase PB by 25%	.567	.158	.0543	.0440
decrease PB by 10%	.566	.153	.0647	.0435
decrease PB by 25%	.565	.149	.0707	.0432

Table V-1.

Marshallian Simulation (Homogeniety and Non-Additivity)

PA,PB,PR,M,T,Sp are independent variables

	X_A	X_R	X_B	Utility level (V)
Observed quantity	.576	.154	.0619	
Predicted quantity	.566	.153	.0619	.762
M = .0436 (fixed)				
increase PA by 10%	.518	.149	.0600	.703
increase PA by 25%	.459	.144	.0575	.631
decrease PA by 10%	.626	.157	.0639	.832
decrease PA by 25%	.743	.165	.0675	.967
increase PR by 10%	.564	.142	.0629	.753
increase PR by 25%	.561	.128	.0643	.741
decrease PR by 10%	.569	.166	.0607	.772
decrease PR by 25%	.573	.192	.0587	.788
increase PB by 10%	.566	.154	.0570	.759
increase PB by 25%	.564	.155	.0510	.755
decrease PB by 10%	.567	.152	.0674	.765
decrease PB by 25%	.569	.150	.0792	.770
increase T by 5 years	.555	.175	.0604	.769
increase T by 10 years	.544	.197	.0589	.771
decrease T by 5 years	.578	.131	.0634	.754
decrease T by 10 years	.589	.109	.0648	.747
increase Sp by 10%	.575	.135	.0642	.756
increase Sp by 50%	.604	.0764	.0716	.738
decrease Sp by 10%	.557	.173	.0593	.768
decrease Sp by 50%	.503	.284	.0452	.805

Sensitivity analysis (simulation) is done with respect to average independent variables on both Marshallian and Hicksian simulations.

$\overline{RPA} = .0645$, $\overline{RPB} = .0285$, $\overline{RPR} = .0350$, $\overline{M} = .0436$, $\overline{T} = 14.5$,

$\overline{SPF} = 1.56$, $\overline{QXA} = .576$, $\overline{QXB} = .0619$, $\overline{QXR} = .154$, $\overline{SA} = .836$,

$\overline{SB} = .0405$, $\overline{SR} = .123$.

Note: For simple notation, PA, PB, PR stands for RPA, RPB, RPR respectively, and X_A, X_B, X_R stands for QXA, QXB, QXR respectively. Also SpF = $\ln(Sp)$ = $\ln(SPA/SPBR)$. X_A, X_B, X_R are in 'thousands of passenger miles' that a representative individual travels during one year period.

Table V-2.

Hicksian Simulation (Homogeniety and Non-Additivity)

PA, PB, PR, M, T, Sp are independent variables

	X_A	X_R	X_B	Money income M
Observed quantity	.576	.154	.0619	.0436
Predicted quantity	.566	.153	.0619	.0436
Utility V = .762 (fixed)				
increase PA by 10%	.561	.162	.0650	.0473
increase PA by 25%	.553	.174	.0694	.0527
decrease PA by 10%	.573	.144	.0585	.0400
decrease PA by 25%	.585	.130	.0531	.0344
increase PR by 10%	.571	.144	.0637	.0442
increase PR by 25%	.577	.132	.0662	.0449
decrease PR by 10%	.562	.164	.0599	.0431
decrease PR by 25%	.554	.185	.0567	.0422
increase PB by 10%	.568	.155	.0572	.0438
increase PB by 25%	.570	.156	.0515	.0440
decrease PB by 10%	.565	.151	.0675	.0435
decrease PB by 25%	.562	.149	.0783	.0431
increase T by 5 years	.550	.173	.0598	.0432
increase T by 10 years	.533	.193	.0577	.0428
decrease T by 5 years	.584	.132	.0640	.0441
decrease T by 10 years	.601	.111	.0662	.0445
increase SP by 10%	.580	.136	.0647	.0440
increase SP by 50%	.624	.0789	.0740	.0451
decrease Sp by 10%	.552	.172	.0588	.0433
decrease Sp by 50%	.476	.269	.0427	.0413

CHAPTER VI

CONCLUSIONS

A. Applications of the Model

1.

In the model formulation, the prices and other attribute values are exogeneously determined and we assume that the supply is readily available. In fact, many demand models are formulated assuming that the supply is readily available. But the transportation planner has to predict the equilibrium in the transportation system or the pattern of flows in the transportation network. Hence complete policy assessments require the analysis of both demand and supply simultaneously. From the short run equilibrium, the resource consumption such as energy consumption, air and noise pollution, vehicle productivity, etc. can be forecast and from the long run equilibrium, both resource consumption and activity shifts (socio-economic changes such as population, production costs, etc.) are to be forecast, while activity shifts are at the same time the influencing variables to the demand model. Hence the effective utilization of the model requires well structured supply models which express the supply of the transport system.

2.

The model is very useful in measuring the substitutability among different trip decisions (e.g. in the case of mode choice decisions, the elasticity of substitution between different trip modes measures the different substitutability among trip modes). The elasticities and simulation results are very useful in transportation policy implications. The results indicate whether the deregulation of public carriers prices is necessary, how much prices or other attribute variables should be increased or decreased and what the impacts are to the competing modes as well as its own mode.

3.

The consumer surplus argument is easily attacked since the present model is based on consistent consumer theory. Consumer surplus can be estimated for each demand equations and we can observe how the consumer surplus changes when alternative transportation policies are implemented.

Any travel demand models which use the quantitative choice approach may adopt the translog models. In air travel demand modeling, FAA used an econometric model to forecast RPM (revenue passenger miles) in its 1974 aviation forecast report. RPM per capita is equivalent to 'passenger miles of travel' in the present study. The application of the translog model to air travel demand modeling will have some significant results.

In freight demand modeling, the dependent variable of the aggregate models is defined as 'tons of shipments' which is a continuous, divisible commodity. Here again, quantitative choice approach with aggregate behavioral modeling will be quite suitable. Both aggregate and disaggregate models are used in freight demand modeling. But other aggregate models performed poorly and disaggregate models have been applied to only a few situations because most of the data are in the aggregate form.

B. Conclusions and Research Directions

This study is the first attempt to apply translog models into travel demand modeling with quantitative choice approach. Although a highly aggregated data set is used, the model still performs quite satisfactorily. The use of stratified data by socio-economic groups or by trip purposes should give better performing model. Yet present study reveals that aggregate behavioral models based on translog models will be useful for transportation forecasting.

The use of Hicksian income-compensated demand elasticity and its sensitivity analysis give us very satisfactory results in measuring the substitution effect among trip modes. The parallel developments of both disaggregate and aggregate behavioral demand modeling are necessary.

Several areas are proposed for research directions.

1.

For the present study, auto travel is excluded because of the unavailability of the data. Although the intercity travel by auto differs from that of public carriers (airline, bus and rail), the inclusion of auto travel in the model after proper calculation of prices and other attribute variable values will be possible. Both time series analysis to see the long run impact and cross sectional analysis to see the short run impact will give interesting results with the stratified data according to distances and trip purposes.

2.

For the further extension of the present study, sequential structure as well as simultaneous structure of travel choice decisions need to be developed under proper imposition of the separability of the translog utility functions.

3.

The incorporation of both quantitative and qualitative choices in the travel demand model formulation is necessary because some of the travel choices are qualitative while others are quantitative.

4.

As for the estimation methods, presently no inequality constraints can be placed into the parameter estimation process. It will be quite useful if the plausibility of the inequality constraints applied to the non-linear maximum likelihood estimation method, is handled both theoretically and empirically as has been done with linear regression case (C. K. Liew, 1976). Finally, aggregate behavioral travel demand models need to be developed further as well as disaggregate models for the application to transportation demand modeling.

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