MATHEMATICAL ANALYSIS OF AN ARTIFICIAL PULSE LINE

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By

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TABLE OF CONTENTS

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Introduction. In ultra-high frequency radar equipment a special circuit is used to produce a pulse of energy closely approximating a square pulse. This pulse is used to trigger the transmitter. The leading edge of the pulse must be steep in order to get a more accurate ranging on the target while the trailing edge must be steep in order to receive echoes as soon as possible after the transmitter is triggered thus shortening the effective minimum range. Between the leading and trailing edge of the pulse the amplitude must be maintained at full value in order to have a strong signal to produce echoes.

Since the period of the pulse is of the order of one micro-second a special circuit is needed to produce it. There are many variations of this special circuit, but since all have their origin in a type that is supposed to approximate a transmission line in its action, these circuits are called "artificial transmission lines" or "artificial pulse lines".

The Circuit and its Differential Equations. The circuit considered here is a basic circuit and similar in action to the others. It consists of a series of pi section filters terminated by a resistance equal to the characteristic impedance of the network. A four-section pi network and a three-section pi network will be considered. These are shown in figure one.

In each network all inductances are of equal value. The

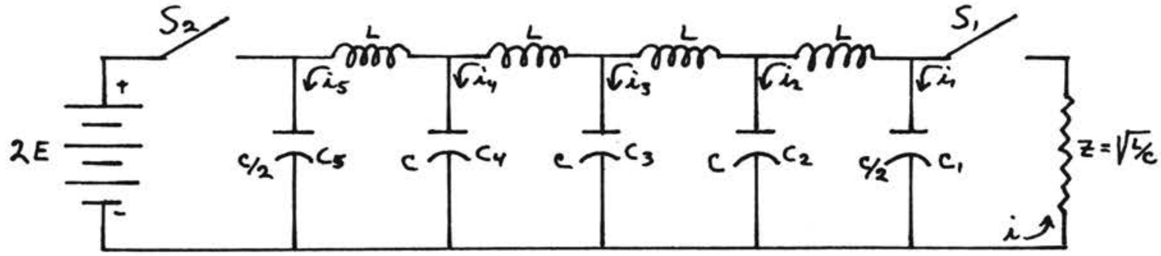

Figure 1(a) Four-section Pi Network

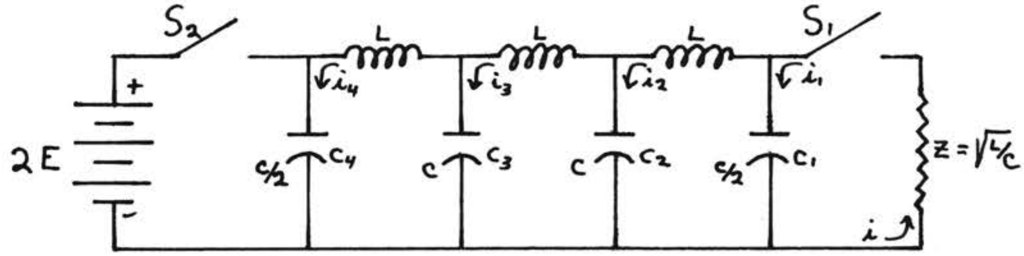

Figure 1(b) Three-section Pi Network

capacitors are equal in value, with the exception of the ones on each end which are one half the value of the others. The characteristic impedance is a resistance equal in value to the square root of the quotient of one inductance divided by the value of an interior capacitor.

The switch S_1 will be left open while S_2 is closed to allow the capacitors to charge up to full battery voltage, 2E, and then opened. Then S_1 will be closed. In general the action is similar to this: capacitor C_1 will immediately discharge through the characteristic impedance Z to approximately half its value, then C_2 will discharge to half its value, the reaction preventing C_1 from discharging completely. Then C_3 will discharge to half its value, the process continuing until the last capacitor is reached which discharges completely and allows the one next to it to discharge completely, this process continuing to the other end of the line until capacitor C_1 is completely discharged. The voltage across the output impedance Z is the same as that across C_1, so that as S_1 is closed the voltage across Z jumps from zero to full battery voltage but rapidly falls off to one half battery voltage, remains there for a period of time depending on the network, and then drops to zero. Communications engineers[1] explain this action by comparing this to a transmission line, and saying that a square wave travels down the line and back.

The previous discussion gives a fairly vivid but highly inaccurate picture of what actually happens. The purpose of this dissertation is to make a mathematical analysis, according to known laws, of what occurs in this circuit.

In this dissertation, whenever current is discussed, electron flow is meant instead of the conventional current flow from positive to negative. The equations are set up on this basis.

Applying Kirchoff's voltage law to each of the loops in the four-section pi network, the following equations are obtained:

$$L \frac{d^2q_5}{dt^2} + \frac{2q_5}{C} - \frac{q_4}{C} = 0$$

$$L \frac{d^2q_5}{dt^2} + L \frac{d^2q_4}{dt^2} + \frac{q_4}{C} - \frac{q_3}{C} = 0$$

$$L \frac{d^2q_5}{dt^2} + L \frac{d^2q_4}{dt^2} + L \frac{d^2q_3}{dt^2} + \frac{q_3}{C} - \frac{q_2}{C} = 0$$

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[1] M.I.T. Staff, <u>Principles of Radar</u>, pp. 6-2 to 6-14.

$$L \frac{d^2q_5}{dt^2} + L \frac{d^2q_4}{dt^2} + L \frac{d^2q_3}{dt^2} + L \frac{d^2q_2}{dt^2} + \frac{q_2}{C} - \frac{2q_1}{C} = 0$$

$$\frac{2q_1}{C} + Z \frac{dq}{dt} = 0$$

These second order differential equations may be reduced to first order differential equations by substituting the relationship $dq/dt = -i$. If we make the reduction and add the equation resulting from the application of Kirchoff's current law the following equations are obtained:

$$\frac{di_5}{dt} = \frac{1}{LC} (2q_5 - q_4)$$

$$\frac{di_4}{dt} = \frac{1}{LC} (2q_4 - q_3 - 2q_5)$$

$$\frac{di_3}{dt} = \frac{1}{LC} (2q_3 - q_2 - q_4)$$

$$\frac{di_2}{dt} = \frac{1}{LC} (2q_2 - 2q_1 - q_3)$$

$$\frac{dq_5}{dt} = -i_5$$

$$\frac{dq_4}{dt} = -i_4$$

$$\frac{dq_3}{dt} = -i_3$$

$$\frac{dq_2}{dt} = -i_2$$

$$\frac{dq_1}{dt} = -i_1$$

$$\frac{-dq}{dt} = i = \frac{2q_1}{\sqrt{LC}}$$

$$i = i_1 + i_2 + i_3 + i_4 + i_5$$

The initial conditions for this set of equations are that when $t = 0$: $q_1 = q_5 = CE$; $q_2 = q_3 = q_4 = 2CE$; $i_2 = i_3 = i_4 = i_5 = 0$; and $i = i_1 = 2CE/\sqrt{LC}$.

The differential equations for the three-section network are obtained by a similar procedure. Applying Kirchoff's voltage law to the loops we obtain:

$$L\frac{d^2q_4}{dt^2} + \frac{2q_4}{C} - \frac{q_3}{C} = 0$$

$$L\frac{d^2q_4}{dt^2} + L\frac{d^2q_3}{dt^2} + \frac{q_3}{C} - \frac{q_2}{C} = 0$$

$$L\frac{d^2q_4}{dt^2} + L\frac{d^2q_3}{dt^2} + L\frac{d^2q_2}{dt^2} + \frac{q_2}{C} - \frac{2q_1}{C} = 0$$

$$\frac{2q_1}{C} + Z\frac{dq}{dt} = 0$$

After the substitution we obtain:

$$\frac{di_4}{dt} = \frac{1}{LC}(2q_4 - q_3)$$

$$\frac{di_3}{dt} = \frac{1}{LC}(2q_3 - q_2 - 2q_4)$$

$$\frac{di_2}{dt} = \frac{1}{LC}(2q_2 - 2q_1 - q_3)$$

$$\frac{dq_4}{dt} = -i_4$$

$$\frac{dq_3}{dt} = -i_3$$

$$\frac{dq_2}{dt} = -i_2$$

$$\frac{dq_1}{dt} = -i_1$$

$$\frac{-dq}{dt} = i = \frac{2q_1}{\sqrt{LC}}$$

$$i = i_1 + i_2 + i_3 + i_4$$

The initial conditions are when $t = 0$: $q_1 = q_4 = CE$; $q_2 = q_3 = 2CE$; $i_2 = i_3 = i_4 = 0$; and $i_1 = i = 2CE/\sqrt{LC}$.

Method of Solution. The method used to solve the differential equations was applied by F. R. Moulton[2] to exterior ballistics problems during World War I, and has proven to be a very effective method of numerical integration of simultaneous differential equations.

This method consists essentially of using third degree parabolas as approximations to the right hand members of the expressions:

$$\frac{dx_1}{dt} = f_1(x_1, x_2, \ldots, x_n; t)$$

$$\frac{dx_2}{dt} = f_2(x_1, x_2, \ldots, x_n; t)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\frac{dx_n}{dt} = f_n(x_1, x_2, \ldots, x_n; t)$$

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[2] F. R. Moulton, New Methods in Exterior Ballistics.

The integral of the third degree parabola is obtained as a finite difference equation. Values of x_1, x_2, ..., x_n are guessed for a particular value of t. These values are then substituted into f_1, f_2, ..., f_n, and the results used in the finite difference equation to obtain a better value of x_1, x_2, ..., x_n. These better values are then substituted into f_1, f_2, ..., f_n and the process is repeated until the desired degree of accuracy of x_1, x_2, ..., x_n is obtained.

To obtain the finite difference equation of the integral of the third degree parabola, values of f are assumed to be known for $t = t_{n-2}$, t_{n-1}, t_n, and t_{n+1}. A third degree parabola is passed through these points with the origin at t_n. It is of the form

$$y = a_o + a_1(t-t_n) + a_2(t-t_n)^2 + a_3(t-t_n)^3$$

The integral from t_n to t_{n+1} is

$$\int_{t_n}^{t_{n+1}} f(t)\,dt = \int_{t_n}^{t_{n+1}} \left[a_o + a_1(t-t_n) + a_2(t-t_n)^2 + a_3(t-t_n)^3\right] dt$$

$$= h\left[a_o + a_1h/2 + a_2h^2/3 + a_3h^3/4\right]$$

where $h = t_i - t_{i-1}$ for $i = n-1,\ n,\ n+1$.

We define difference relationships by these equations

$$\Delta_1 f_{n+1} = f_{n+1} - f_n$$

$$\Delta_2 f_{n+1} = \Delta_1 f_{n+1} - \Delta_1 f_n$$

$$\Delta_3 f_{n+1} = \Delta_2 f_{n+1} - \Delta_2 f_n$$

The values of y at t_{n-2}, t_{n-1}, t_n, and t_{n+1} are

$$f_{n-2} = a_o - 2a_1h + 4a_2h^2 - 8a_3h^3$$

$$f_{n-1} = a_o - a_1h + a_2h^2 - a_3h^3$$

$$f_n = a_o$$

$$f_{n+1} = a_o + a_1h + a_2h^2 + a_3h^3$$

and

$$\Delta_1 f_{n+1} = a_1h + a_2h^2 + a_3h^3$$
$$\Delta_2 f_{n+1} = 2a_2h^2$$
$$\Delta_3 f_{n+1} = 6a_3h^3$$

From these are obtained

$$a_o = f_{n+1} - \Delta_1 f_{n+1}$$
$$a_1h = \Delta_1 f_{n+1} - \Delta_2 f_{n+1}/2 - \Delta_3 f_{n+1}/6$$
$$a_2h^2 = \Delta_2 f_{n+1}/2$$
$$a_3h^3 = \Delta_3 f_{n+1}/6$$

Substituting these back into the value for the integral it becomes:

$$\int_{t_n}^{t_{n+1}} f(t)dt = h\left[f_{n+1} - \Delta_1 f_{n+1}/2 - \Delta_2 f_{n+1}/12 - \Delta_3 f_{n+1}/24\right]$$

Since the values of $f(t)$ for four successive values of t are needed, this finite difference equation is insufficient for starting the procedure. Therefore if the initial conditions are given the following equations are used for beginning the solution:

$$x_1 = x_o + hf_o$$
$$x_1 = x_o + h(f_1 - \Delta_1 f_1/2)$$
$$x_2 = x_1 + h(f_2 - \Delta_1 f_2/2)$$
$$x_3 = x_2 + h(f_3 - \Delta_1 f_3/2 - \Delta_2 f_3/12)$$

and the following equations are used for correcting these values by iteration:

$$x_1 = x_o + h(f_3 - 5\Delta_1 f_3/2 + 23\Delta_2 f_3/12 - 3\Delta_3 f_3/8)$$
$$x_2 = x_1 + h(f_3 - 3\Delta_1 f_3/2 + 5\Delta_2 f_3/12 + \Delta_3 f_3/24)$$
$$x_3 = x_2 + h(f_3 - \Delta_1 f_3/2 - \Delta_2 f_3/12 - \Delta_3 f_3/24)$$

The Solution. The values of q (charge in coulombs) for each capacitor of the four-section network are given in table I. The corresponding values of E (voltage) are given in table II and are also shown graphically in figure 2. The values of i (current) for the same network are given in table III and plotted in figure 3.

The values of q for the three-section network are given in table IV. The corresponding values of E are given in table V and plotted in figure 4. The values of i for the same network are given in table VI and shown graphically in figure 5.

The voltage output of the four-section network, and the voltage output of the three-section network are plotted against each other for the same time length of pulse in figure 6. Also indicated is the ideal square pulse that is being approximated.

TABLE I

t	q_1	q_2	q_3	q_4	q_5
0.0	1.0000	2.0000	2.0000	2.0000	1.0000
0.1	.8193	1.9994	2.0000	2.0000	1.0000
0.2	.6747	1.9952	2.0000	2.0000	1.0000
0.3	.5621	1.9847	1.9999	2.0000	1.0000
0.4	.4777	1.9658	1.9997	2.0000	1.0000
0.5	.4177	1.9372	1.9992	2.0000	1.0000
0.6	.3786	1.8982	1.9980	2.0000	1.0000
0.7	.3568	1.8489	1.9958	2.0000	1.0000
0.8	.3493	1.7897	1.9922	1.9999	1.0000
0.9	.3529	1.7217	1.9867	1.9997	1.0000
1.0	.3651	1.6462	1.9786	1.9994	1.0000
1.1	.3834	1.5648	1.9674	1.9989	1.0000
1.2	.4055	1.4796	1.9525	1.9981	1.0000
1.3	.4297	1.3923	1.9333	1.9969	.9999
1.4	.4543	1.3052	1.9094	1.9950	.9999
1.5	.4780	1.2201	1.8803	1.9924	.9998
1.6	.4998	1.1390	1.8457	1.9887	.9996
1.7	.5189	1.0635	1.8055	1.9836	.9993
1.8	.5348	.9951	1.7597	1.9770	.9988
1.9	.5471	.9352	1.7085	1.9683	.9982
2.0	.5557	.8845	1.6521	1.9573	.9973
2.1	.5607	.8437	1.5911	1.9437	.9959
2.2	.5623	.8132	1.5261	1.9270	.9942
2.3	.5608	.7929	1.4581	1.9069	.9918
2.4	.5566	.7825	1.3879	1.8831	.9886
2.5	.5501	.7814	1.3166	1.8553	.9845
2.6	.5420	.7889	1.2453	1.8233	.9792
2.7	.5326	.8039	1.1753	1.7868	.9726
2.8	.5226	.8252	1.1076	1.7458	.9644
2.9	.5125	.8515	1.0435	1.7003	.9544
3.0	.5026	.8815	.9841	1.6503	.9423
3.1	.4934	.9138	.9302	1.5959	.9278
3.2	.4852	.9469	.8829	1.5375	.9108
3.3	.4782	.9796	.8427	1.4755	.8909
3.4	.4728	1.0108	.8102	1.4101	.8680
3.5	.4689	1.0393	.7857	1.3421	.8418
3.6	.4667	1.0643	.7693	1.2718	.8122
3.7	.4660	1.0850	.7608	1.2001	.7790
3.8	.4669	1.1010	.7599	1.1276	.7423
3.9	.4691	1.1119	.7662	1.0550	.7020
4.0	.4725	1.1176	.7787	.9829	.6583
4.1	.4768	1.1182	.7967	.9122	.6112
4.2	.4818	1.1140	.8190	.8434	.5610
4.3	.4873	1.1054	.8445	.7771	.5081
4.4	.4929	1.0928	.8720	.7139	.4527
4.5	.4985	1.0769	.9001	.6542	.3955
4.6	.5038	1.0585	.9274	.5983	.3368
4.7	.5086	1.0383	.9529	.5464	.2775

TABLE I (cont'd)

t	q_1	q_2	q_3	q_4	q_5
4.8	.5127	1.0170	.9751	.4987	.2180
4.9	.5160	.9953	.9929	.4551	.1592
5.0	.5184	.9740	1.0055	.4155	.1018
5.1	.5198	.9537	1.0118	.3797	.0464
5.2	.5201	.9347	1.0112	.3473	- .0060
5.3	.5194	.9176	1.0033	.3180	- .0549
5.4	.5177	.9025	.9877	.2912	- .0995
5.5	.5152	.8896	.9642	.2665	- .1392
5.6	.5118	.8789	.9331	.2433	- .1735
5.7	.5077	.8701	.8945	.2211	- .2019
5.8	.5031	.8630	.8490	.1994	- .2240
5.9	.4980	.8572	.7971	.1778	- .2397
6.0	.4926	.8522	.7396	.1557	- .2488
6.1	.4869	.8474	.6775	.1330	- .2514
6.2	.4811	.8421	.6116	.1094	- .2476
6.3	.4753	.8357	.5429	.0848	- .2378
6.4	.4695	.8276	.4727	.0591	- .2224
6.5	.4638	.8170	.4018	.0326	- .2020
6.6	.4580	.8034	.3314	.0053	- .1772
6.7	.4523	.7862	.2624	- .0222	- .1489
6.8	.4466	.7649	.1959	- .0496	- .1177
6.9	.4407	.7393	.1325	- .0765	- .0848
7.0	.4346	.7090	.0731	- .1022	- .0509
7.1	.4282	.6740	.0184	- .1261	- .0170
7.2	.4212	.6342	- .0313	- .1476	.0160
7.3	.4137	.5899	- .0755	- .1662	.0472
7.4	.4053	.5413	- .1140	- .1813	.0758
7.5	.3960	.4889	- .1466	- .1924	.1011
7.6	.3856	.4332	- .1733	- .1991	.1224
7.7	.3741	.3747	- .1942	- .2011	.1393
7.8	.3612	.3144	- .2095	- .1983	.1514
7.9	.3470	.2528	- .2194	- .1906	.1586
8.0	.3313	.1910	- .2244	- .1781	.1606
8.1	.3143	.1297	- .2247	- .1610	.1577
8.2	.2957	.0699	- .2208	- .1399	.1500
8.3	.2759	.0124	- .2133	- .1152	.1379
8.4	.2547	- .0420	- .2025	- .0875	.1220
8.5	.2324	- .0925	- .1889	- .0577	.1027
8.6	.2091	- .1384	- .1731	- .0266	.0808
8.7	.1849	- .1791	- .1554	.0049	.0570
8.8	.1602	- .2140	- .1364	.0359	.0322
8.9	.1351	- .2429	- .1164	.0655	.0070
9.0	.1099	- .2653	- .0959	.0928	- .0176
9.1	.0849	- .2813	- .0752	.1169	- .0410
9.2	.0604	- .2906	- .0546	.1370	- .0624
9.3	.0366	- .2935	- .0345	.1527	- .0812
9.4	.0138	- .2902	- .0151	.1634	- .0968
9.5	- .0078	- .2809	.0034	.1687	- .1089

TABLE II

t	E_1	E_2	E_3	E_4	E_5
0.0	2.0000	2.0000	2.0000	2.0000	2.0000
0.1	1.6387	1.9994	2.0000	2.0000	2.0000
0.2	1.3493	1.9952	2.0000	2.0000	2.0000
0.3	1.1242	1.9847	1.9999	2.0000	2.0000
0.4	.9553	1.9658	1.9997	2.0000	2.0000
0.5	.8354	1.9372	1.9992	2.0000	2.0000
0.6	.7571	1.8982	1.9980	2.0000	2.0000
0.7	.7136	1.8489	1.9958	2.0000	2.0000
0.8	.6985	1.7897	1.9922	1.9999	2.0000
0.9	.7059	1.7217	1.9867	1.9997	2.0000
1.0	.7303	1.6462	1.9786	1.9994	2.0000
1.1	.7668	1.5648	1.9674	1.9989	2.0000
1.2	.8110	1.4796	1.9525	1.9981	2.0000
1.3	.8594	1.3923	1.9333	1.9969	1.9998
1.4	.9086	1.3052	1.9094	1.9950	1.9998
1.5	.9560	1.2201	1.8803	1.9924	1.9996
1.6	.9996	1.1390	1.8457	1.9887	1.9992
1.7	1.0378	1.0635	1.8055	1.9836	1.9986
1.8	1.0695	.9951	1.7597	1.9770	1.9976
1.9	1.0941	.9352	1.7085	1.9683	1.9964
2.0	1.1114	.8845	1.6521	1.9573	1.9946
2.1	1.1214	.8437	1.5911	1.9437	1.9918
2.2	1.1246	.8132	1.5261	1.9270	1.9884
2.3	1.1216	.7929	1.4581	1.9069	1.9836
2.4	1.1132	.7825	1.3879	1.8831	1.9772
2.5	1.1003	.7814	1.3166	1.8553	1.9690
2.6	1.0840	.7889	1.2453	1.8233	1.9584
2.7	1.0653	.8039	1.1753	1.7868	1.9452
2.8	1.0453	.8252	1.1076	1.7458	1.9288
2.9	1.0249	.8515	1.0435	1.7003	1.9088
3.0	1.0051	.8815	.9841	1.6503	1.8846
3.1	.9867	.9138	.9302	1.5959	1.8556
3.2	.9703	.9469	.8829	1.5375	1.8216
3.3	.9565	.9796	.8427	1.4755	1.7818
3.4	.9456	1.0108	.8102	1.4101	1.7360
3.5	.9379	1.0393	.7857	1.3421	1.6836
3.6	.9334	1.0643	.7693	1.2718	1.6244
3.7	.9321	1.0850	.7608	1.2001	1.5580
3.8	.9338	1.1010	.7599	1.1276	1.4846
3.9	.9382	1.1119	.7662	1.0550	1.4040
4.0	.9449	1.1176	.7787	.9829	1.3166
4.1	.9535	1.1182	.7967	.9122	1.2224
4.2	.9636	1.1140	.8190	.8434	1.1220
4.3	.9745	1.1054	.8445	.7771	1.0162
4.4	.9859	1.0928	.8720	.7139	.9054
4.5	.9971	1.0769	.9001	.6542	.7910
4.6	1.0077	1.0585	.9274	.5983	.6736
4.7	1.0173	1.0383	.9529	.5464	.5550
4.8	1.0255	1.0170	.9751	.4987	.4360

TABLE II (cont'd)

t	E_1	E_2	E_3	E_4	E_5
4.9	1.0321	.9953	.9929	.4551	.3184
5.0	1.0368	.9740	1.0055	.4155	.2036
5.1	1.0395	.9537	1.0118	.3797	.0928
5.2	1.0402	.9347	1.0112	.3473	- .0120
5.3	1.0388	.9176	1.0033	.3180	- .1098
5.4	1.0355	.9025	.9877	.2912	- .1990
5.5	1.0304	.8896	.9642	.2665	- .2784
5.6	1.0236	.8789	.9331	.2433	- .3470
5.7	1.0155	.8701	.8945	.2211	- .4038
5.8	1.0062	.8630	.8490	.1994	- .4480
5.9	.9960	.8572	.7971	.1778	- .4794
6.0	.9851	.8522	.7396	.1557	- .4976
6.1	.9738	.8474	.6775	.1330	- .5028
6.2	.9623	.8421	.6116	.1094	- .4952
6.3	.9506	.8357	.5429	.0848	- .4756
6.4	.9390	.8276	.4727	.0591	- .4448
6.5	.9275	.8170	.4018	.0326	- .4040
6.6	.9161	.8034	.3314	.0053	- .3544
6.7	.9047	.7862	.2624	- .0222	- .2978
6.8	.8932	.7649	.1959	- .0496	- .2354
6.9	.8814	.7393	.1325	- .0765	- .1696
7.0	.8692	.7090	.0731	- .1022	- .1018
7.1	.8563	.6740	.0184	- .1261	- .0340
7.2	.8424	.6342	- .0313	- .1476	.0320
7.3	.8273	.5899	- .0755	- .1662	.0944
7.4	.8106	.5413	- .1140	- .1813	.1516
7.5	.7920	.4889	- .1466	- .1924	.2022
7.6	.7712	.4332	- .1733	- .1991	.2448
7.7	.7481	.3747	- .1942	- .2011	.2786
7.8	.7224	.3144	- .2095	- .1983	.3028
7.9	.6940	.2528	- .2194	- .1906	.3172
8.0	.6627	.1910	- .2244	- .1781	.3212
8.1	.6285	.1297	- .2247	- .1610	.3154
8.2	.5915	.0699	- .2208	- .1399	.3000
8.3	.5517	.0124	- .2133	- .1152	.2758
8.4	.5094	- .0420	- .2025	- .0875	.2440
8.5	.4647	- .0925	- .1889	- .0577	.2054
8.6	.4181	- .1384	- .1731	- .0266	.1616
8.7	.3698	- .1791	- .1554	.0049	.1140
8.8	.3204	- .2140	- .1364	.0359	.0644
8.9	.2702	- .2429	- .1164	.0655	.0140
9.0	.2199	- .2653	- .0959	.0928	- .0352
9.1	.1699	- .2813	- .0752	.1169	- .0820
9.2	.1208	- .2906	- .0546	.1370	- .1248
9.3	.0731	- .2935	- .0345	.1527	- .1624
9.4	.0275	- .2902	- .0151	.1634	- .1936
9.5	- .0156	- .2809	.0034	.1687	- .2178

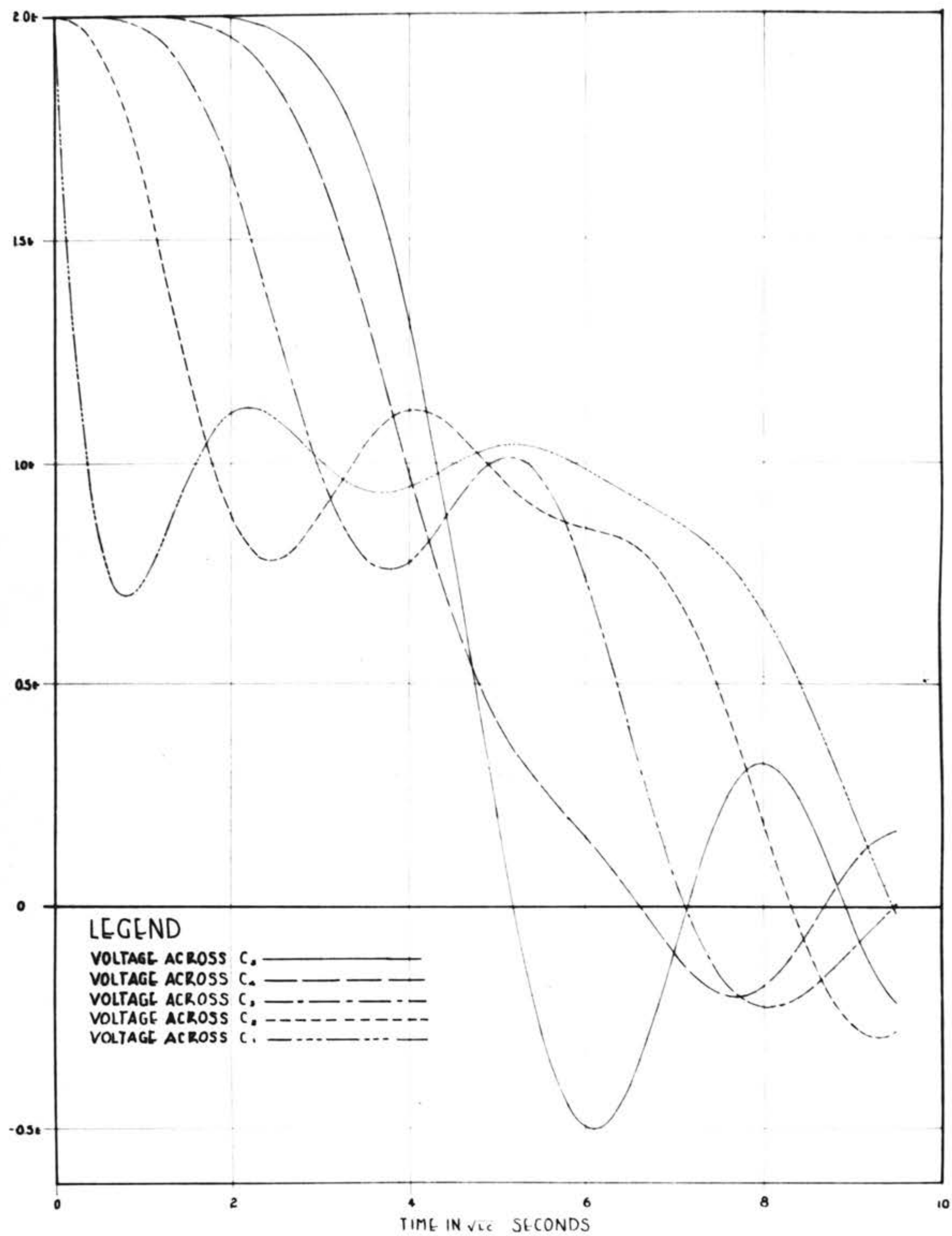

2.0E
1.5E
1.0E
0.5E
0
-0.5E
0
2
4
6
8
10
TIME IN √LC SECONDS
LEGEND
VOLTAGE ACROSS C
VOLTAGE ACROSS C
VOLTAGE ACROSS C
VOLTAGE ACROSS C
VOLTAGE ACROSS C

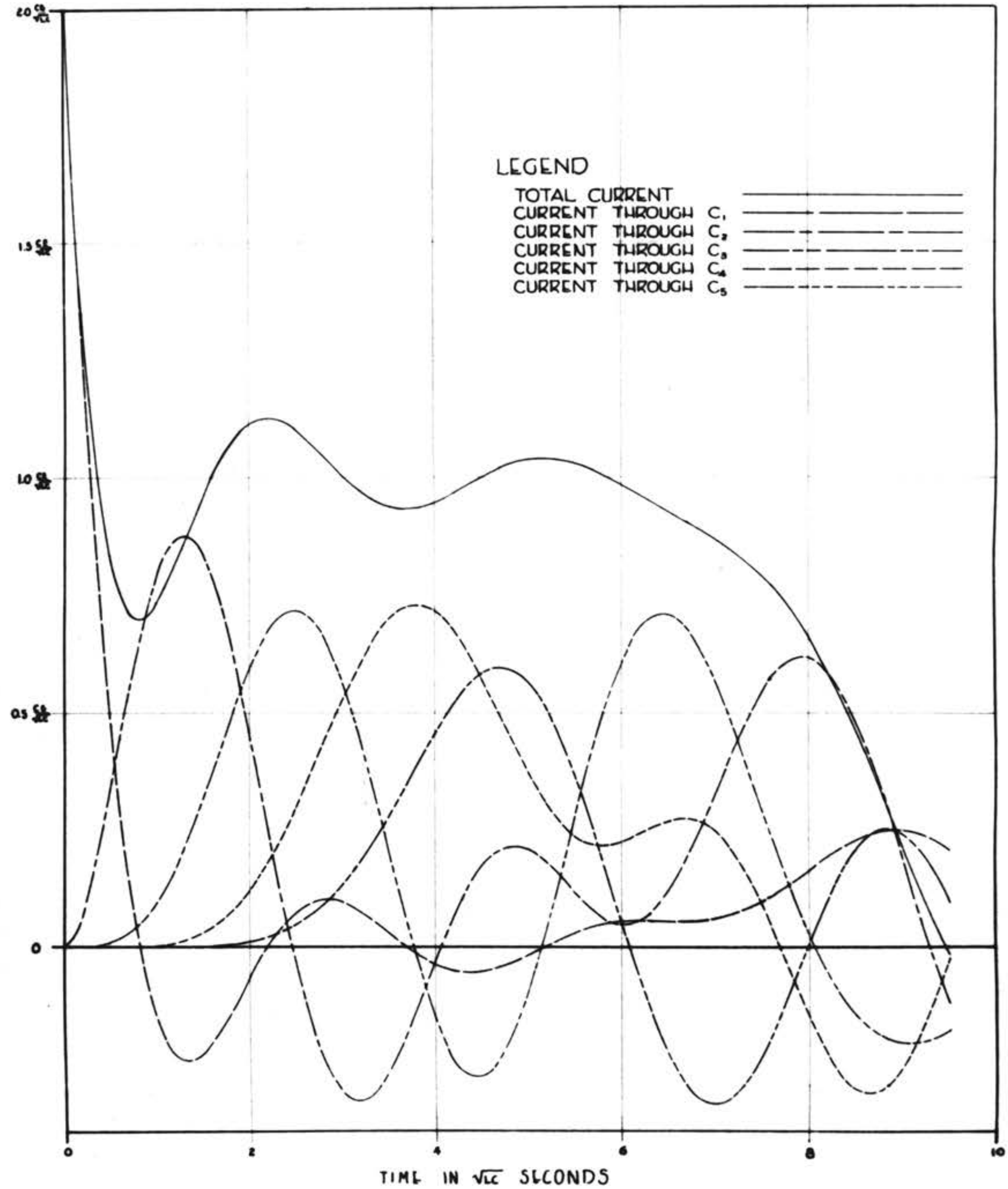
FIGURE 2

TABLE III

t	i	i_1	i_2	i_3	i_4	i_5
0.0	2.0000	2.0000	0.0000	0.0000	0.0000	0.0000
0.1	1.6387	1.6200	.0187	.0000	.0000	.0000
0.2	1.3493	1.2797	.0694	.0002	.0000	.0000
0.3	1.1242	.9787	.1443	.0012	.0000	.0000
0.4	.9553	.7158	.2360	.0035	.0000	.0000
0.5	.8354	.4898	.3374	.0082	.0001	.0000
0.6	.7571	.2989	.4419	.0161	.0002	.0000
0.7	.7136	.1413	.5438	.0280	.0005	.0000
0.8	.6985	.0146	.6380	.0449	.0011	.0000
0.9	.7059	- .0836	.7202	.0672	.0021	.0000
1.0	.7303	- .1560	.7872	.0953	.0037	.0001
1.1	.7668	- .2053	.8363	.1295	.0062	.0002
1.2	.8110	- .2345	.8659	.1694	.0099	.0003
1.3	.8594	- .2462	.8753	.2147	.0151	.0005
1.4	.9086	- .2435	.8644	.2646	.0222	.0009
1.5	.9560	- .2291	.8341	.3180	.0314	.0015
1.6	.9996	- .2056	.7858	.3738	.0433	.0024
1.7	1.0378	- .1755	.7214	.4303	.0580	.0036
1.8	1.0695	- .1411	.6434	.4859	.0760	.0054
1.9	1.0941	- .1047	.5546	.5390	.0974	.0078
2.0	1.1114	- .0680	.4580	.5880	.1224	.0110
2.1	1.1214	- .0326	.3567	.6310	.1510	.0153
2.2	1.1246	.0002	.2540	.6664	.1833	.0207
2.3	1.1216	.0293	.1528	.6930	.2189	.0276
2.4	1.1132	.0540	.0561	.7094	.2576	.0361
2.5	1.1003	.0739	- .0338	.7148	.2990	.0465
2.6	1.0840	.0884	- .1143	.7085	.3424	.0589
2.7	1.0653	.0976	- .1836	.6904	.3873	.0736
2.8	1.0453	.1017	- .2404	.6606	.4327	.0906
2.9	1.0249	.1010	- .2838	.6195	.4780	.1102
3.0	1.0051	.0961	- .3134	.5680	.5221	.1324
3.1	.9867	.0875	- .3291	.5072	.5641	.1571
3.2	.9703	.0759	- .3315	.4387	.6030	.1843
3.3	.9565	.0621	- .3214	.3641	.6379	.2138
3.4	.9456	.0467	- .2999	.2854	.6680	.2454
3.5	.9379	.0306	- .2687	.2047	.6925	.2788
3.6	.9334	.0144	- .2295	.1241	.7108	.3136
3.7	.9321	- .0012	- .1842	.0459	.7224	.3492
3.8	.9338	- .0155	- .1348	- .0279	.7270	.3850
3.9	.9382	- .0282	- .0832	- .0952	.7245	.4203
4.0	.9449	- .0389	- .0315	- .1542	.7150	.4545
4.1	.9535	- .0472	.0185	- .2034	.6988	.4868
4.2	.9636	- .0529	.0653	- .2413	.6763	.5163
4.3	.9745	- .0561	.1072	- .2670	.6482	.5423
4.4	.9859	- .0567	.1432	- .2798	.6153	.5638
4.5	.9971	- .0549	.1725	- .2795	.5786	.5803
4.6	1.0077	- .0508	.1945	- .2661	.5391	.5910
4.7	1.0173	- .0448	.2089	- .2401	.4981	.5952
4.8	1.0255	- .0372	.2158	- .2022	.4565	.5925
4.9	1.0321	- .0284	.2157	- .1536	.4157	.5826

TABLE III (cont'd)

t	i	i_1	i_2	i_3	i_4	i_5
5.0	1.0368	-.0187	.2092	-.0956	.3767	.5652
5.1	1.0395	-.0085	.1972	-.0299	.3404	.5402
5.2	1.0402	.0018	.1808	.0418	.3079	.5079
5.3	1.0388	.0118	.1613	.1176	.2797	.4685
5.4	1.0355	.0212	.1399	.1954	.2565	.4225
5.5	1.0304	.0298	.1181	.2732	.2385	.3707
5.6	1.0236	.0374	.0973	.3492	.2259	.3138
5.7	1.0155	.0438	.0787	.4214	.2185	.2530
5.8	1.0062	.0490	.0637	.4882	.2160	.1893
5.9	.9960	.0529	.0532	.5481	.2179	.1239
6.0	.9851	.0556	.0482	.5997	.2233	.0583
6.1	.9738	.0573	.0493	.6420	.2314	-.0063
6.2	.9623	.0580	.0570	.6744	.2412	-.0684
6.3	.9506	.0581	.0714	.6963	.2516	-.1267
6.4	.9390	.0578	.0925	.7075	.2613	-.1800
6.5	.9275	.0573	.1200	.7081	.2693	-.2272
6.6	.9161	.0570	.1533	.6984	.2744	-.2670
6.7	.9047	.0572	.1916	.6791	.2757	-.2989
6.8	.8932	.0580	.2340	.6509	.2723	-.3220
6.9	.8814	.0597	.2794	.6147	.2636	-.3359
7.0	.8692	.0626	.3265	.5717	.2490	-.3406
7.1	.8563	.0668	.3740	.5230	.2284	-.3359
7.2	.8424	.0724	.4207	.4699	.2017	-.3223
7.3	.8273	.0795	.4651	.4136	.1693	-.3002
7.4	.8106	.0881	.5059	.3554	.1317	-.2704
7.5	.7920	.0981	.5419	.2964	.0896	-.2340
7.6	.7712	.1094	.5720	.2378	.0440	-.1919
7.7	.7481	.1219	.5952	.1806	-.0040	-.1456
7.8	.7224	.1353	.6108	.1257	-.0529	-.0965
7.9	.6940	.1493	.6182	.0738	-.1015	-.0459
8.0	.6627	.1637	.6170	.0257	-.1482	.0046
8.1	.6285	.1781	.6069	-.0184	-.1916	.0535
8.2	.5915	.1921	.5882	-.0579	-.2303	.0994
8.3	.5517	.2054	.5609	-.0927	-.2630	.1411
8.4	.5094	.2177	.5256	-.1226	-.2886	.1773
8.5	.4647	.2285	.4830	-.1478	-.3060	.2071
8.6	.4181	.2376	.4337	-.1682	-.3148	.2297
8.7	.3698	.2447	.3789	-.1841	-.3143	.2446
8.8	.3204	.2495	.3196	-.1956	-.3045	.2515
8.9	.2702	.2517	.2569	-.2031	-.2856	.2503
9.0	.2199	.2513	.1921	-.2068	-.2581	.2413
9.1	.1699	.2482	.1264	-.2070	-.2226	.2249
9.2	.1208	.2423	.0611	-.2041	-.1803	.2018
9.3	.0731	.2336	-.0027	-.1983	-.1323	.1728
9.4	.0275	.2222	-.0637	-.1898	-.0803	.1391
9.5	-.0156	.2083	-.1209	-.1790	-.0258	.1018



LEGEND
TOTAL CURRENT
CURRENT THROUGH C1
CURRENT THROUGH C2
CURRENT THROUGH C3
CURRENT THROUGH C4
CURRENT THROUGH C5
2.0
1.5
1.0
0.5
0
2
4
6
8
10
TIME IN √LC SECONDS

FIGURE 3

TABLE IV

t	q_1	q_2	q_3	q_4
0.0	1.0000	2.0000	2.0000	1.0000
0.1	.8193	1.9994	2.0000	1.0000
0.2	.6747	1.9952	2.0000	1.0000
0.3	.5621	1.9847	1.9999	1.0000
0.4	.4777	1.9658	1.9997	1.0000
0.5	.4177	1.9372	1.9992	1.0000
0.6	.3786	1.8982	1.9980	1.0000
0.7	.3568	1.8489	1.9958	1.0000
0.8	.3493	1.7897	1.9922	.9999
0.9	.3530	1.7217	1.9866	.9997
1.0	.3651	1.6461	1.9786	.9994
1.1	.3834	1.5648	1.9674	.9990
1.2	.4055	1.4795	1.9524	.9982
1.3	.4297	1.3923	1.9332	.9969
1.4	.4543	1.3052	1.9092	.9951
1.5	.4780	1.2201	1.8800	.9924
1.6	.4998	1.1389	1.8452	.9887
1.7	.5189	1.0634	1.8047	.9837
1.8	.5348	.9951	1.7585	.9770
1.9	.5471	.9351	1.7065	.9684
2.0	.5557	.8844	1.6492	.9575
2.1	.5607	.8435	1.5869	.9439
2.2	.5623	.8129	1.5202	.9273
2.3	.5608	.7924	1.4497	.9074
2.4	.5566	.7818	1.3764	.8838
2.5	.5502	.7804	1.3010	.8563
2.6	.5420	.7874	1.2245	.8247
2.7	.5326	.8017	1.1479	.7889
2.8	.5226	.8221	1.0721	.7488
2.9	.5124	.8472	.9981	.7044
3.0	.5024	.8757	.9267	.6559
3.1	.4931	.9059	.8586	.6036
3.2	.4847	.9364	.7944	.5478
3.3	.4776	.9659	.7347	.4890
3.4	.4718	.9929	.6797	.4278
3.5	.4676	1.0164	.6296	.3648
3.6	.4648	1.0351	.5843	.3009
3.7	.4635	1.0483	.5437	.2367
3.8	.4635	1.0553	.5074	.1733
3.9	.4647	1.0556	.4750	.1115
4.0	.4668	1.0487	.4459	.0522
4.1	.4696	1.0348	.4194	- .0037
4.2	.4727	1.0137	.3948	- .0553
4.3	.4759	.9857	.3713	- .1019
4.4	.4787	.9513	.3482	- .1428
4.5	.4810	.9110	.3248	- .1773
4.6	.4824	.8652	.3004	- .2050
4.7	.4827	.8149	.2747	- .2256
4.8	.4815	.7606	.2471	- .2390

TABLE IV (cont'd)

t	q_1	q_2	q_3	q_4
4.9	.4787	.7032	.2173	- .2452
5.0	.4741	.6436	.1854	- .2443
5.1	.4675	.5823	.1514	- .2367
5.2	.4590	.5203	.1154	- .2228
5.3	.4484	.4582	.0778	- .2033
5.4	.4357	.3967	.0393	- .1790
5.5	.4210	.3364	.0003	- .1507
5.6	.4044	.2777	- .0383	- .1195
5.7	.3860	.2212	- .0758	- .0862
5.8	.3660	.1672	- .1112	- .0519
5.9	.3444	.1161	- .1438	- .0177
6.0	.3216	.0681	- .1727	.0154
6.1	.2977	.0235	- .1972	.0464
6.2	.2730	- .0177	- .2166	.0746
6.3	.2476	- .0552	- .2303	.0991
6.4	.2219	- .0889	- .2381	.1194
6.5	.1960	- .1188	- .2396	.1349
6.6	.1702	- .1448	- .2347	.1453
6.7	.1446	- .1669	- .2238	.1505
6.8	.1196	- .1850	- .2071	.1505
6.9	.0953	- .1990	- .1850	.1453
7.0	.0718	- .2090	- .1584	.1355
7.1	.0494	- .2150	- .1280	.1213
7.2	.0283	- .2170	- .0947	.1035
7.3	.0085	- .2150	- .0597	.0826
7.4	- .0099	- .2092	- .0240	.0595

TABLE V

t	E_1	E_2	E_3	E_4
0.0	2.0000	2.0000	2.0000	2.0000
0.1	1.6387	1.9994	2.0000	2.0000
0.2	1.3493	1.9952	2.0000	2.0000
0.3	1.1242	1.9847	1.9999	2.0000
0.4	.9553	1.9658	1.9997	2.0000
0.5	.8354	1.9372	1.9992	2.0000
0.6	.7571	1.8982	1.9980	2.0000
0.7	.7136	1.8489	1.9958	2.0000
0.8	.6985	1.7897	1.9922	1.9998
0.9	.7059	1.7217	1.9866	1.9994
1.0	.7303	1.6461	1.9786	1.9988
1.1	.7668	1.5648	1.9674	1.9980
1.2	.8111	1.4795	1.9524	1.9964
1.3	.8594	1.3923	1.9332	1.9938
1.4	.9086	1.3052	1.9092	1.9902
1.5	.9560	1.2201	1.8800	1.9848
1.6	.9996	1.1389	1.8452	1.9774
1.7	1.0378	1.0634	1.8047	1.9674
1.8	1.0695	.9951	1.7585	1.9540
1.9	1.0941	.9351	1.7065	1.9368
2.0	1.1114	.8844	1.6492	1.9150
2.1	1.1214	.8435	1.5869	1.8878
2.2	1.1246	.8129	1.5202	1.8546
2.3	1.1216	.7924	1.4497	1.8148
2.4	1.1132	.7818	1.3764	1.7676
2.5	1.1003	.7804	1.3010	1.7126
2.6	1.0840	.7874	1.2245	1.6494
2.7	1.0653	.8017	1.1479	1.5778
2.8	1.0452	.8221	1.0721	1.4976
2.9	1.0247	.8472	.9981	1.4088
3.0	1.0048	.8757	.9267	1.3118
3.1	.9861	.9059	.8586	1.2072
3.2	.9694	.9364	.7944	1.0956
3.3	.9551	.9659	.7347	.9780
3.4	.9436	.9929	.6797	.8556
3.5	.9351	1.0164	.6296	.7296
3.6	.9296	1.0351	.5843	.6018
3.7	.9270	1.0483	.5437	.4734
3.8	.9271	1.0553	.5074	.3466
3.9	.9294	1.0556	.4750	.2230
4.0	.9336	1.0487	.4459	.1044
4.1	.9392	1.0348	.4194	- .0074
4.2	.9454	1.0137	.3948	- .1106
4.3	.9517	.9857	.3713	- .2038
4.4	.9575	.9513	.3482	- .2856
4.5	.9620	.9110	.3248	- .3546
4.6	.9649	.8652	.3004	- .4100
4.7	.9653	.8149	.2747	- .4512
4.8	.9630	.7606	.2471	- .4780

TABLE V (cont'd)

t	E_1	E_2	E_3	E_4
4.9	.9574	.7032	.2173	- .4904
5.0	.9482	.6436	.1854	- .4886
5.1	.9351	.5823	.1514	- .4734
5.2	.9180	.5203	.1154	- .4456
5.3	.8967	.4582	.0778	- .4066
5.4	.8714	.3967	.0393	- .3580
5.5	.8421	.3364	.0003	- .3014
5.6	.8089	.2777	- .0383	- .2390
5.7	.7721	.2212	- .0758	- .1724
5.8	.7319	.1672	- .1112	- .1038
5.9	.6889	.1161	- .1438	- .0354
6.0	.6432	.0681	- .1727	.0308
6.1	.5955	.0235	- .1972	.0928
6.2	.5460	- .0177	- .2166	.1492
6.3	.4953	- .0552	- .2303	.1982
6.4	.4438	- .0889	- .2381	.2388
6.5	.3920	- .1188	- .2396	.2698
6.6	.3403	- .1448	- .2347	.2906
6.7	.2893	- .1669	- .2238	.3010
6.8	.2392	- .1850	- .2071	.3010
6.9	.1905	- .1990	- .1850	.2906
7.0	.1436	- .2090	- .1584	.2710
7.1	.0988	- .2150	- .1280	.2426
7.2	.0565	- .2170	- .0947	.2070
7.3	.0169	- .2150	- .0597	.1652
7.4	- .0198	- .2092	- .0240	.1190

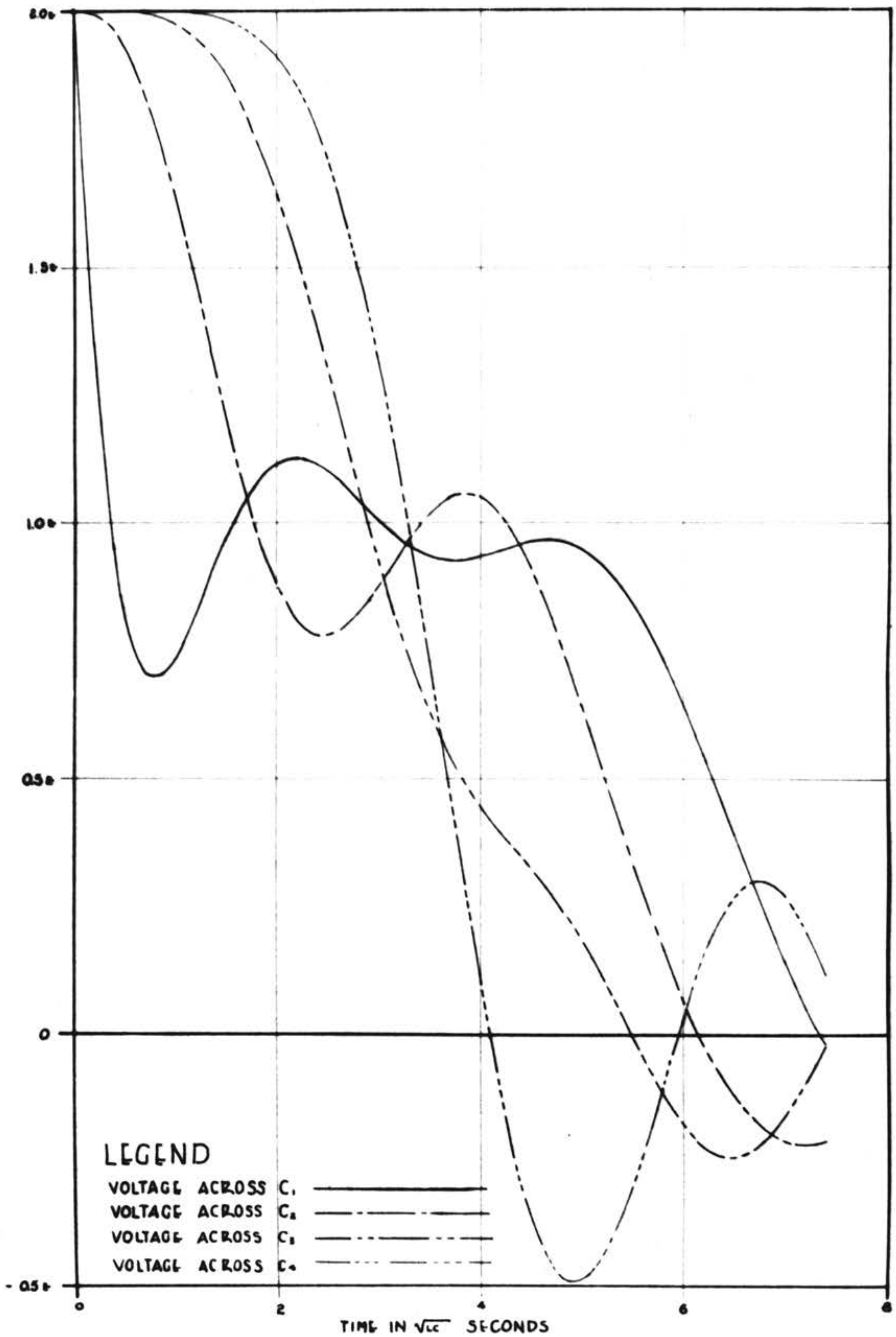

0
0
2
4
6
8
TIME IN √LC SECONDS
LEGEND
VOLTAGE ACROSS C1
VOLTAGE ACROSS C2
VOLTAGE ACROSS C3
VOLTAGE ACROSS C4

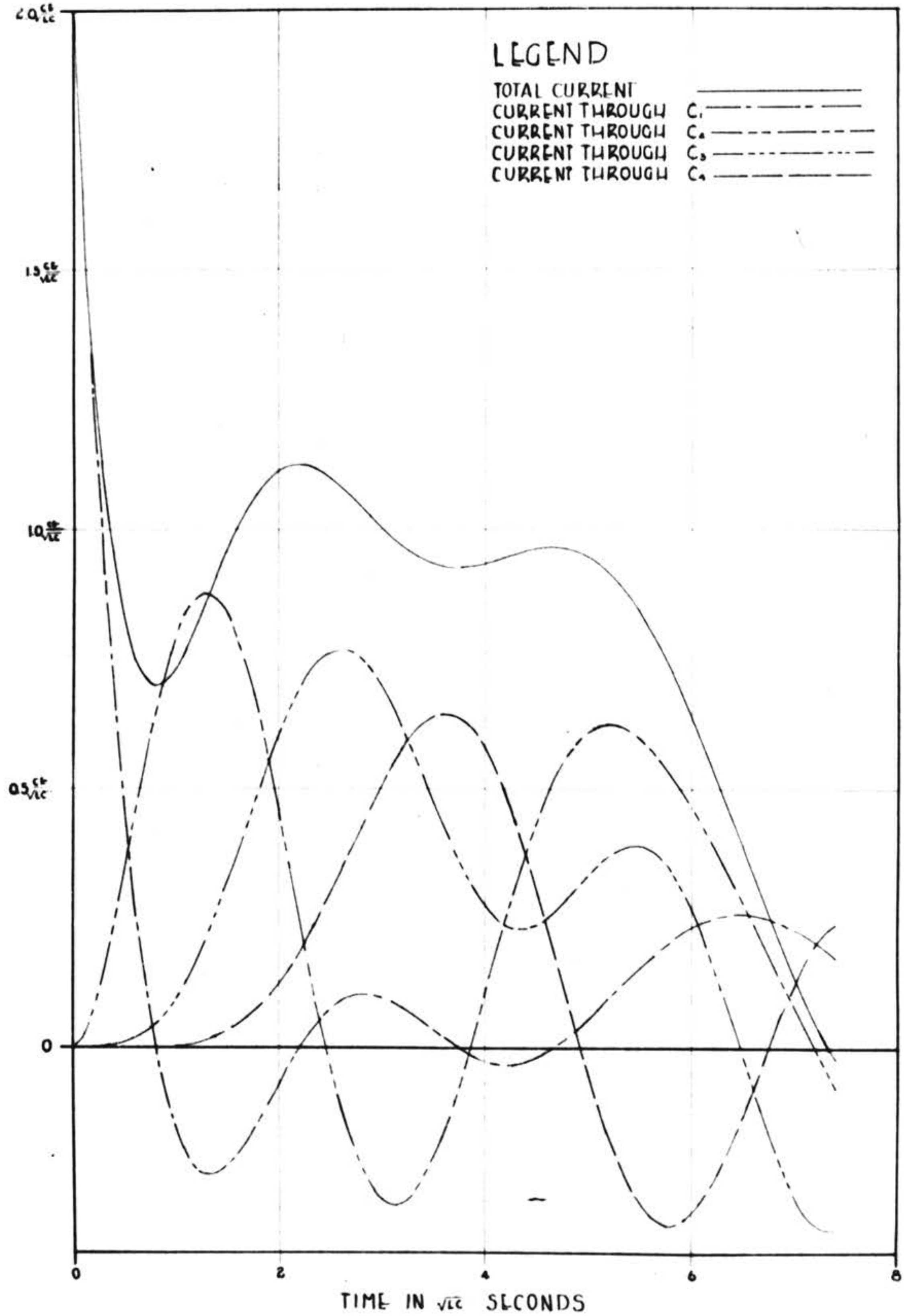
FIGURE 4

TABLE VI

t	i	i_1	i_2	i_3	i_4
0.0	2.0000	2.0000	0.0000	0.0000	0.0000
0.1	1.6387	1.6200	.0187	.0000	.0000
0.2	1.3493	1.2797	.0694	.0002	.0000
0.3	1.1242	.9787	.1443	.0012	.0000
0.4	.9553	.7158	.2360	.0035	.0000
0.5	.8354	.4898	.3374	.0082	.0001
0.6	.7571	.2989	.4419	.0161	.0002
0.7	.7136	.1413	.5438	.0280	.0005
0.8	.6985	.0146	.6380	.0449	.0011
0.9	.7059	- .0836	.7203	.0672	.0021
1.0	.7303	- .1560	.7872	.0954	.0037
1.1	.7668	- .2053	.8363	.1296	.0062
1.2	.8111	- .2344	.8659	.1697	.0099
1.3	.8594	- .2462	.8753	.2152	.0151
1.4	.9086	- .2435	.8644	.2655	.0222
1.5	.9560	- .2291	.8341	.3195	.0314
1.6	.9996	- .2056	.7858	.3761	.0432
1.7	1.0378	- .1755	.7215	.4339	.0580
1.8	1.0695	- .1412	.6435	.4913	.0758
1.9	1.0941	- .1048	.5549	.5469	.0971
2.0	1.1114	- .0681	.4585	.5991	.1219
2.1	1.1214	- .0326	.3576	.6463	.1503
2.2	1.1246	.0002	.2553	.6871	.1820
2.3	1.1216	.0293	.1548	.7205	.2170
2.4	1.1132	.0541	.0589	.7453	.2549
2.5	1.1003	.0739	- .0296	.7610	.2951
2.6	1.0840	.0885	- .1085	.7670	.3370
2.7	1.0653	.0978	- .1758	.7634	.3798
2.8	1.0452	.1021	- .2300	.7504	.4226
2.9	1.0247	.1017	- .2701	.7286	.4645
3.0	1.0048	.0971	- .2955	.6988	.5044
3.1	.9861	.0889	- .3062	.6622	.5412
3.2	.9694	.0778	- .3023	.6201	.5738
3.3	.9551	.0646	- .2847	.5741	.6011
3.4	.9436	.0501	- .2543	.5257	.6221
3.5	.9351	.0350	- .2127	.4768	.6360
3.6	.9296	.0201	- .1613	.4289	.6419
3.7	.9270	.0061	- .1020	.3836	.6393
3.8	.9271	- .0064	- .0367	.3425	.6278
3.9	.9294	- .0168	.0326	.3066	.6071
4.0	.9336	- .0248	.1040	.2770	.5774
4.1	.9392	- .0299	.1755	.2545	.5390
4.2	.9454	- .0319	.2456	.2394	.4923
4.3	.9517	- .0307	.3125	.2318	.4382
4.4	.9575	- .0264	.3749	.2314	.3776
4.5	.9620	- .0190	.4315	.2377	.3119
4.6	.9649	- .0087	.4816	.2497	.2423
4.7	.9653	.0043	.5244	.2664	.1703
4.8	.9630	.0196	.5594	.2864	.0976

TABLE VI (cont'd)

t	i	i_1	i_2	i_3	i_4
4.9	.9574	.0368	.5866	.3082	.0258
5.0	.9482	.0555	.6058	.3302	- .0434
5.1	.9351	.0754	.6174	.3508	- .1085
5.2	.9180	.0958	.6216	.3684	- .1679
5.3	.8967	.1164	.6190	.3815	- .2202
5.4	.8714	.1368	.6102	.3888	- .2644
5.5	.8421	.1565	.5958	.3892	- .2994
5.6	.8089	.1752	.5765	.3817	- .3245
5.7	.7721	.1925	.5530	.3659	- .3394
5.8	.7319	.2082	.5260	.3416	- .3439
5.9	.6889	.2221	.4960	.3089	- .3380
6.0	.6432	.2339	.4636	.2682	- .3224
6.1	.5955	.2435	.4292	.2203	- .2976
6.2	.5460	.2509	.3934	.1664	- .2647
6.3	.4953	.2559	.3564	.1079	- .2249
6.4	.4438	.2586	.3184	.0462	- .1795
6.5	.3920	.2590	.2797	- .0167	- .1300
6.6	.3403	.2572	.2405	- .0792	- .0782
6.7	.2893	.2532	.2007	- .1392	- .0255
6.8	.2392	.2472	.1607	- .1949	.0263
6.9	.1905	.2392	.1204	- .2446	.0756
7.0	.1436	.2294	.0800	- .2868	.1209
7.1	.0988	.2180	.0398	- .3200	.1610
7.2	.0565	.2051	- .0001	- .3432	.1947
7.3	.0169	.1907	- .0393	- .3557	.2211
7.4	- .0198	.1752	- .0774	- .3571	.2396



LEGEND
TOTAL CURRENT
CURRENT THROUGH C_1
CURRENT THROUGH C_2
CURRENT THROUGH C_3
CURRENT THROUGH C_4
$2.0\frac{CE}{\sqrt{LC}}$
$1.5\frac{CE}{\sqrt{LC}}$
$1.0\frac{CE}{\sqrt{LC}}$
$0.5\frac{CE}{\sqrt{LC}}$
0
2
4
6
8
TIME IN $\sqrt{LC}$ SECONDS

FIGURE 5

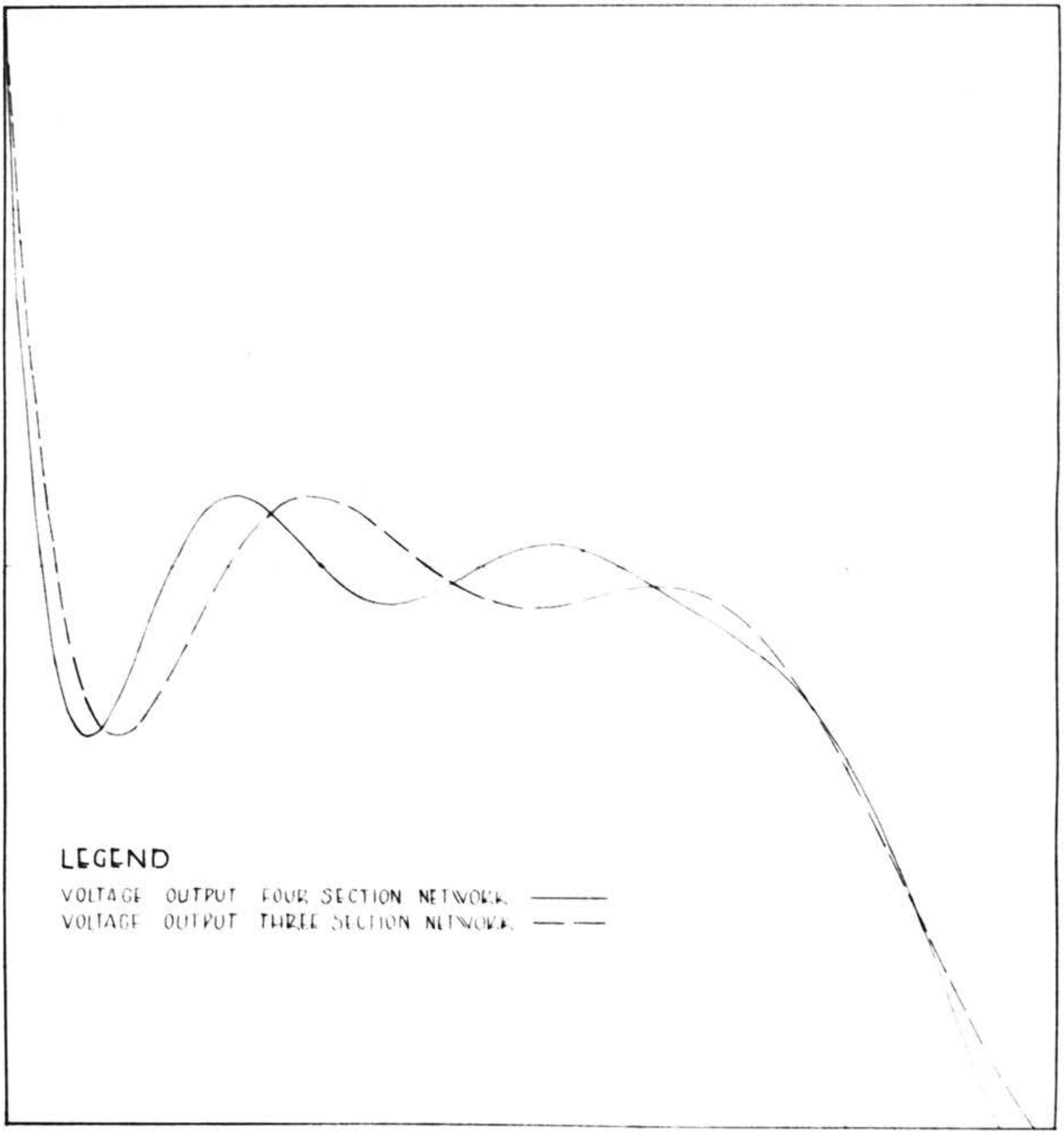

LEGEND
VOLTAGE OUTPUT FOUR SECTION NETWORK
VOLTAGE OUTPUT THREE SECTION NETWORK

FIGURE 6

<u>Conclusion</u>. The voltage output of the pulse line does not form a square wave as can be readily seen from the graphs. The leading edge as shown in the graph is very sharp, but in an actual circuit, capacitance across the load would cause the voltage to rise more slowly.

The top of the wave is not absolutely flat, but has a certain amount of oscillations. The more sections a particular network has for the same period of pulse, the higher the frequency of oscillation. If enough sections are used, this oscillation will be high enough that its effect will be nullified in the transmitter. The number of sections does not have any effect on the amplitude of the first cycle, but with a larger number of sections the amplitude of the last cycle will be smaller.

The average value of voltage across the top of the wave drops off towards the end of the pulse. In actual practice this is sometimes corrected by changing the relative value of certain of the capacitors.

The trailing edge of the pulse does not have a sharp descent, but the voltage falls off gradually. The voltage of the four-section network drops more rapidly than that of the three-section network. The more sections the network contains the sharper would be the descent.

In general this network does not produce the ideal square pulse desired for pulsing the radar transmitter, but does produce a pulse that is a good enough approximation for practical purposes. The more sections used the closer the approximation would be to the ideal square pulse. In practice the tendency has been to use other circuits to produce the square wave, but the other circuits, while having certain advantages, such as fewer components, still operate on the same general principle that this circuit operates on.

BIBLIOGRAPHY

Members of the Staff of the Radar School of the Massachussett's Institute of Technology. Principles of Radar. Second Edition. New York: McGraw-Hill Book Co., Inc., 1946.

Moulton, F. R. New Methods in Exterior Ballistics. Chicago: The University of Chicago Press, 1926.

Moulton, F. R. Differential Equations. New York: The Macmillan Co., 1930.

Typist: Mrs. Esther K. Miller