SETTLEMENT ANALYSIS OF FOOTINGS

SETTLEMENT ANALYSIS OF FOOTINGS

BY

CLAUDE R. ZIMMERMAN, Jr.

Bachelor of Science

Oklahoma Agricultural and Mechanical College

Stillwater, Oklahoma

1948

Submitted to the Department of Civil Engineering

Oklahoma Agricultural and Mechanical College

In Partial Fulfillment of the Requirements

for the Degree of

MASTER OF SCIENCE

1948

APPROVED BY:

Chairman, Thesis Committee

Member of the Thesis Committee

Head of the Department

Dean of the Graduate School

CONTENTS

Settlement Analysis of Footings

Predicted settlement of structures on compressible soils followed by observations after construction has been undertaken only in recent years. William S. Housel of the University of Michigan published an article in the Engineering News Record in 1933 called "Bearing Power of Clay is Determinable". In this article Mr. Housel developed equations by which theoretical settlements could be determined. He also gave examples of a large water storage tank and a highway grade separation in which the actual settlements agreed very closely with the predicted settlements.

In the early 1940's R. E. Means of the Department of Architecture, Oklahoma A & M College was studying at Harvard University. During the period he wrote "Building Foundations on Compressible Soils". This paper has not been published, but is being prepared for publication in the A.S.C.E. Proceedings in the near future. Mr. Means computed settlements due to the external loads, allowing no deformation in the structural frame. He also computed the change in load, ΔP, on the various footings due to a deformation, $\Delta Y = 1"$. These ΔP's produced unknown settlements which had to be determined by a series of simultaneous equations, there being one equation for every footing. The number of unknowns in each equation is equal to the number of footings and the constant term is the settlement produced allowing no deformation of the frame. By solving the equations for the unknowns, the settlements of the various footings were determined. After the settlements had been determined, Mr. Means discussed the effect of these settlements upon the moments and stresses in the beams and columns of the structural frame.

George Geoffrey Meyerhof, an English engineer, made a study of this same subject and published his paper, "The Settlement Analysis of Building Frames", in the September 1947 issue of The Structural Engineer. This is an English publication. Mr. Meyerhof made the same analysis that was carried out by Mr. Means. About the only difference was in the method of analysis. Mr. Meyerhof used the slope deflection method, and Mr. Means used moment distribution.

The discussion that follows will be based upon Mr. Means' paper[1].

In this discussion, the term foundation will include all compressible soil between the bottom of the footings and an incompressible layer such as rock or dense sand. The extent and properties of this soil must be determined in order to make an analysis. The fact that these properties cannot be determined accurately does not detract much from the analysis, because many assumptions must be made in order to design a reinforced concrete frame. The degree of accuracy is probably about as good in the foundation analysis as in the structural analysis.

At the present time foundations on compressible soils and the structural frame are usually designed as separate units, the frame being assumed to rest on a foundation which settles uniformly, producing no settlement deformation of the frame. The foundation is usually designed assuming equal settlements at all points by one of the following methods:

1. Enough overburden is excavated below the building and above the footings to compensate for the building load to be distributed over the loaded area so as not to change the state of stress in the soil beneath the footings.
2. The building load may be distributed over the site in such a manner that the corners and exterior portions are loaded heavier than the interior in the correct proportions to produce equal settlements over the entire area.

---

[1] R. E. Means, "Building Foundations on Compressible Soils", Unpublished

3. The foundation system may be designed stiff enough to redistribute the loads to footings so that the differential settlements will be samll enough that the structural frame will withstand the deformations produced without failure.

4. Equal settlements are assumed to be produced by varying the individual footing sizes according to the loads carried.

In all of these methods the building frames and foundations are designed independently and usually with little consideration given to the effect of the action of one upon the other.

Theoretically, if the extent and properties of the compressible soil under the footings are known, equal settlements of all footings could be made to occur at any given time; but the settlements would not be equal before nor after that time, except for ultimate settlements in which case there would be no change after complete consolidation of all material.

As an illustration consider three loads P_1 , P_2 , and P_3 on independent footings spaced b distance apart on a thick bed of compressible clay drained at the top only. Just beneath the footing the stress in the clay is dependent almost entirely upon the size of the footing but at depths a little greater than the size of the footing the stress is almost independent of the size of the footing and is dependent upon the loads on other footings as well as its own load.

If the stress distribution curves were plotted at depth b in a vertical plane for loads P_1 and P_3 it would be found that the two curves overlap and the effect of the two are added, which makes the stress at any point for the two loads greater than for either load acting alone. Now if P_2 is applied half way between P_1 and P_3 it will produce settlement of P_1 and P_3 and it will settle more than either of the outside loads.

In order to compensate for the greater settlement under the middle footing the sizes of the footings may be adjusted so that the sole pressures of 1 and 3 are greater than 2 which will produce greater compression of the soil immediately beneath the outside footings.

Since the soil is drained at the top only consolidation will take place first at the top layer and progress downward with time. This means that a given degree of consolidation of the soil just below the footings will occur earlier than the same degree at depth, and footings 1 and 3 will settle faster than 2 in the period immediately following the application of the loads. As consolidation nears completion in the upper part of the layer, the compression of the soil at depth will begin to contribute to the settlement and No. 2 will gain on 1 and 3, and at some time the settlements of all three will become equal. After this time the middle footing will continue to settle faster than the outside footings and differential settlements will occur in the opposite direction from those produced earlier.

Deformations in building frames may be caused by unequal settlement or unequal uplift. Settlements may be caused by consolidation of saturated soils below or slightly above the ground water level due to the addition of building loads. Settlements may also be produced in soils above the zone of perpetual saturation by drying out of the soil from the surface by hot winds and sun during dry seasons. During wet seasons this same soil swells as it becomes saturated.

The method of analysis used by Mr. Means is an adaptation of the method developed by Dr. Karl Terazghi based on his theory of consolidation.

The analysis covered in this discussion applies only to those soils, such as clay, in which settlement is produced by the application of load only. Sand under the pressure of an overburden has a very high shearing resistance and is practically incompressible under static pressure. Settlements in sand are produced by vibration of the sand in the loose state, or by removing the overburden which removes the confining lateral pressure.

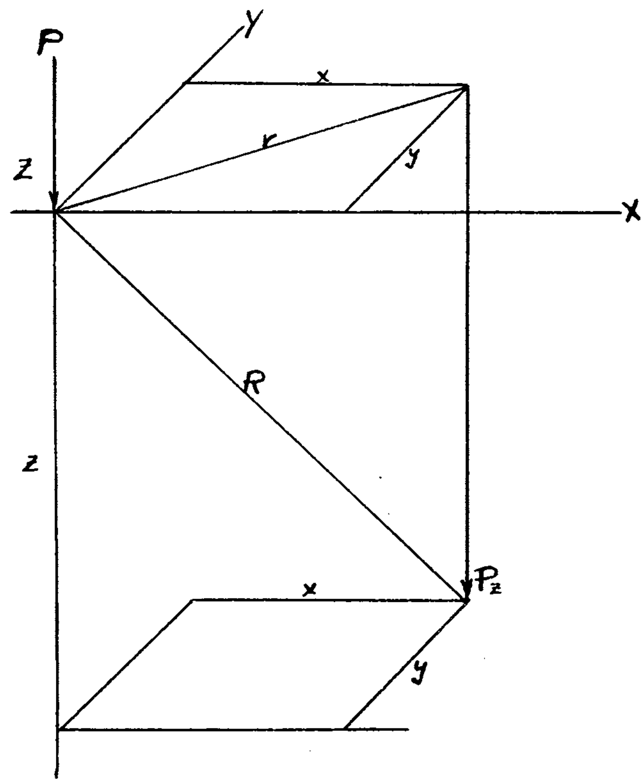
The following assumptions were made by Mr. Means in his development[2]. For a specific case a comparison of the assumed and actual conditions should be made in order to intelligently interpret the results.

1. Pressures are determined by the Boussinesq solution. This assumes a semi-infinite elastic soil mass, loaded on the surface.
2. Pressure is assumed to be applied uniformly to the soil at the bottom of the footings in determining the soil pressure at depths under the footings. This is true only of a flexible footing loaded uniformly or of a footing of such stiffness as to deform the same as the unequal settlement produced by a load on the surface. If the footing is rigid, the pressure distribution in clay is less than average at the middle and high at the edges.
3. Pressure under a footing in a group due to loads on other footings of the group are computed as the summation of pressures due to the other footings applied as point loads. This means that the influence of one footing upon another is assumed to be that produced by the total footing load applied at a point.
4. The settlement under a footing is assumed to be due to consolidation produced by the vertical pressure under the middle of the footing. This is the maximum pressure and is applied to only a very small area and probably for most cases indicates settlements slightly in excess of the actual settlement of the footing.

---

[2] _Ibid._, pp. 32-34

Boussinesq determined the vertical stress at a point due to a concentrated or point load at the surface as $p_z = \frac{3P}{2\pi} \frac{z^3}{R^5}$



This may be expressed in terms of r and z as

$$p_z = \frac{3P}{2\pi} \frac{z^3}{(x^2 + y^2 + z^2)^{2.5}}$$

$$p_z = \frac{3P}{2\pi} \frac{z^3}{(r^2 + z^2)^{2.5}}$$

$$p_z = \frac{3P}{2\pi} \frac{1}{z^2 \left[(\frac{r}{z})^2 + 1 \right]^{2.5}}$$

This may also be expressed as

$$p_z = \frac{P}{z^2} \; \frac{3}{2\pi} \left[\frac{1}{\left(\frac{r}{z}\right)^2 + 1} \right]^{2.5}$$

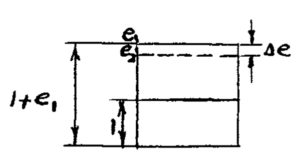
$$p_z = \frac{P}{z^2} P_o$$

where P_o is a function of the dimensionless ratio $\frac{r}{z}$. Values of P_o for $\frac{r}{z}$ have been computed and published by Glennon Gilboy[3].

For this analysis, curves have been drawn for different values of r showing the unit pressure, p'_r, at depth z due to a load of 1 kip at the origin. These curves are drawn for r = 10', 20', 30' and 50' and are shown in Diagram No. II.

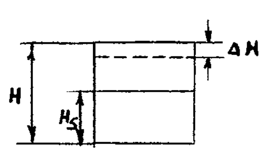
Since this analysis deals only with square footings and only the maximum pressure under the center of the load is required, a diagram is presented showing the variation of vertical pressure with depth in terms of the half width of the footing for a uniform load of w per unit area. See Diagram No. I.

Settlement of clay is due to a volume change or decrease in void ratio. If the reduction in void ratio, e, is known, the unit change in height may be determined from the following relationship $\delta e = \frac{\Delta e}{1 + e_1}$



Δe = total decrease in height of $1 + e_1$

δe = deformation per unit height



The change in height of a layer of H thickness equals the unit change times H or $\Delta H = H \frac{\Delta e}{1 + e_1}$

---

[3] Glennon Gilboy, "Earths and Foundations", Progress Report of Special Committee, Proceedings A.S.C.E. (May, 1933).

DIAGRAM SHOWING VERTICAL UNIT PRESSURE UNDER CENTER OF SQUARE AREA UNIFORMLY LOADED WITH w/Sq. Ft.

Boussinesq Solution

$p_z = w \times w_0$

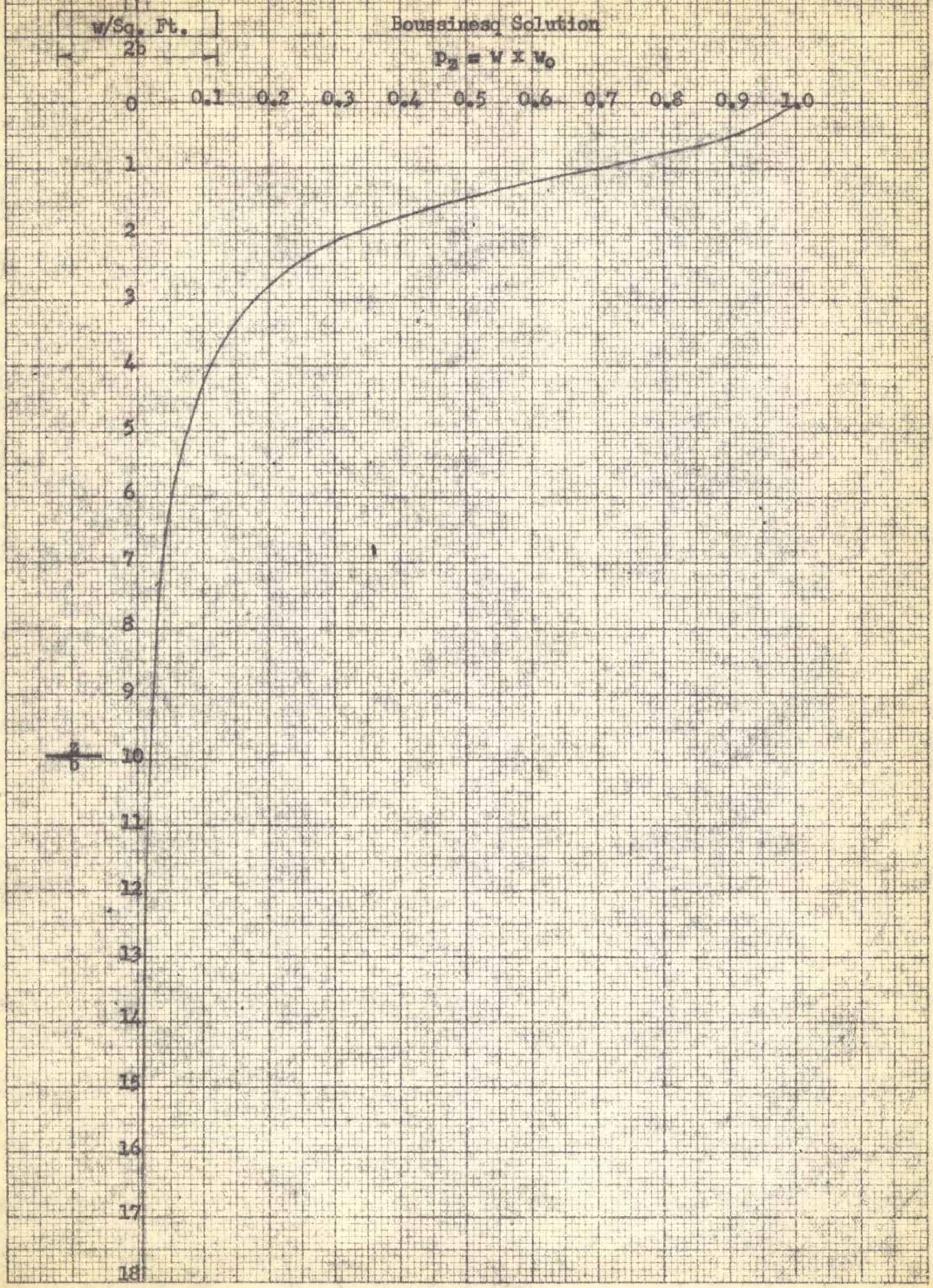


DIAGRAM No. I

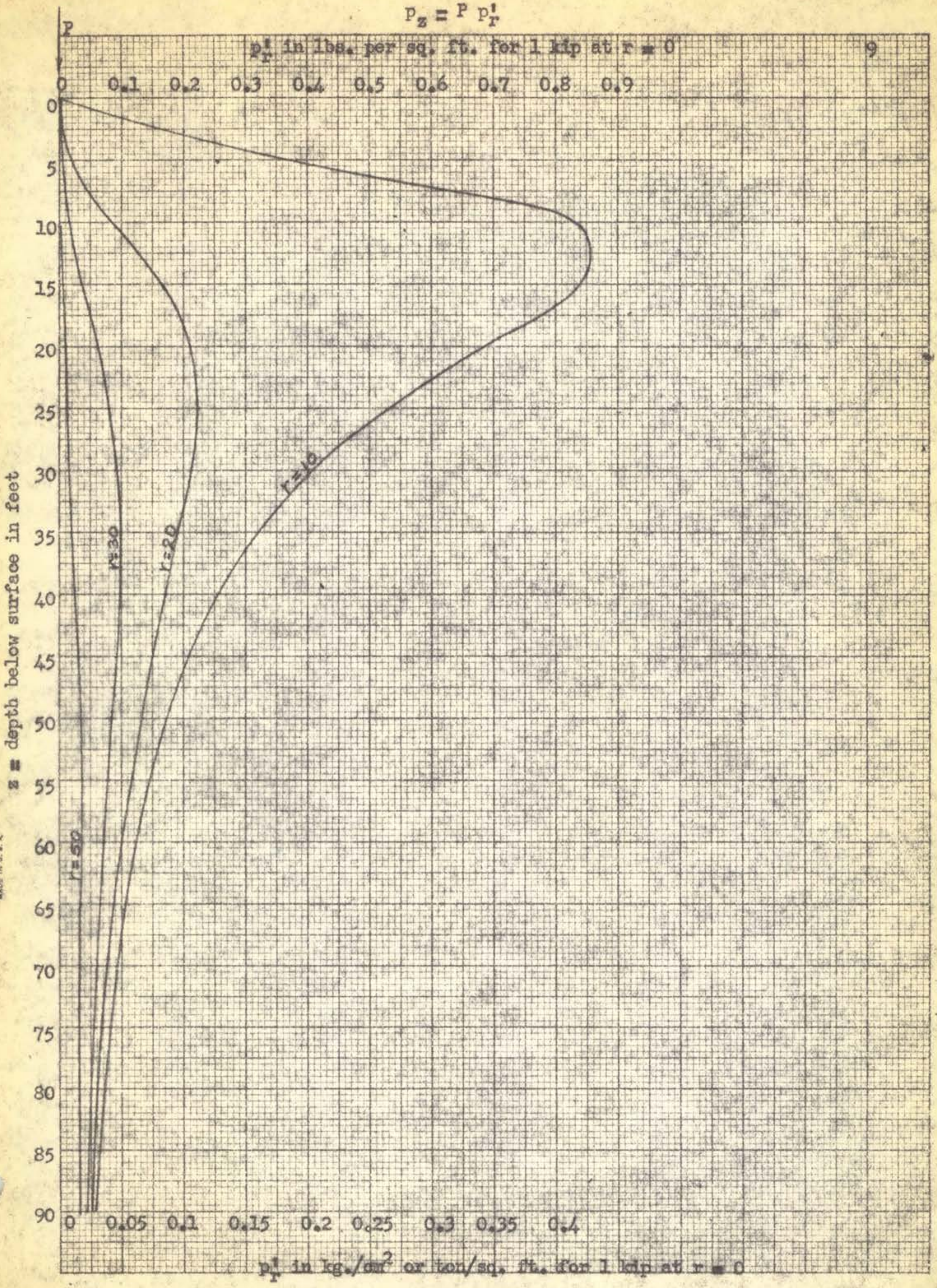
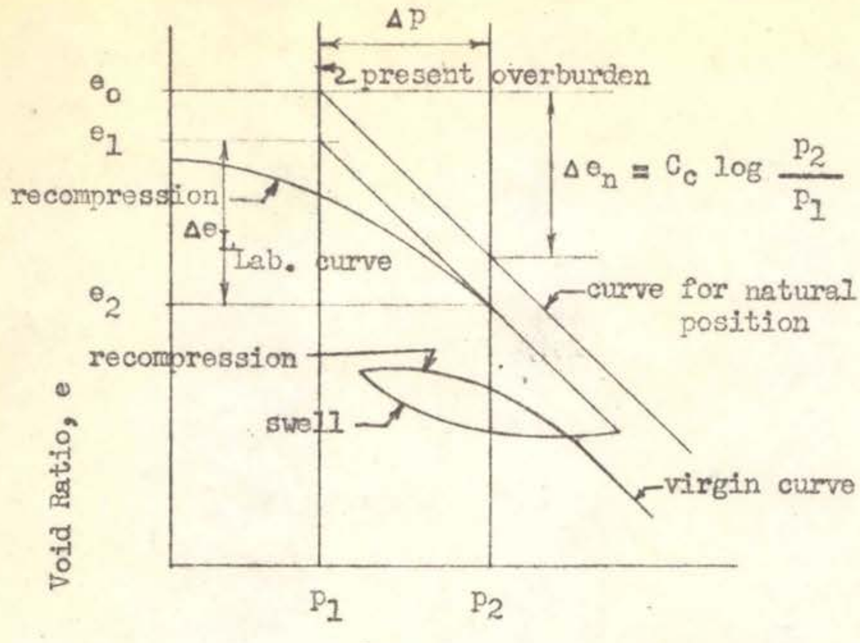

$p_z = P\, p'_r$
P
p'_r in lbs. per sq. ft. for 1 kip at r = 0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
9
z = depth below surface in feet
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
r=10
r=20
r=30
r=50
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4
p'_r in kg./cm^2 or ton/sq. ft. for 1 kip at r = 0

DIAGRAM II

If a consolidation test is run on an undisturbed sample of clay taken from a test pit, allowing complete consolidation between loads, and the void ratio computed for each load, a pressure-void ratio curve may be drawn for that soil. When e is plotted on an arithmetic scale, and P on a log scale, the line is curved up to the void ratio produced by previous pressure and straight over that portion of the curve for pressures applied to the soil for the first time. The straight portion of the curve is called the virgin curve.

If the clay is subjected to a pressure which is removed and applied again and continued to greater pressures than before, a second recompression and virgin curve is obtained which is parallel to the first curve but not an exact continuation of it. The virgin curve may be expressed $e_1 - e_2 = C_c \log \frac{p_2}{p_1}$ in which C_c is the slope of the virgin curve on a semi-log plot and is called the compression index.

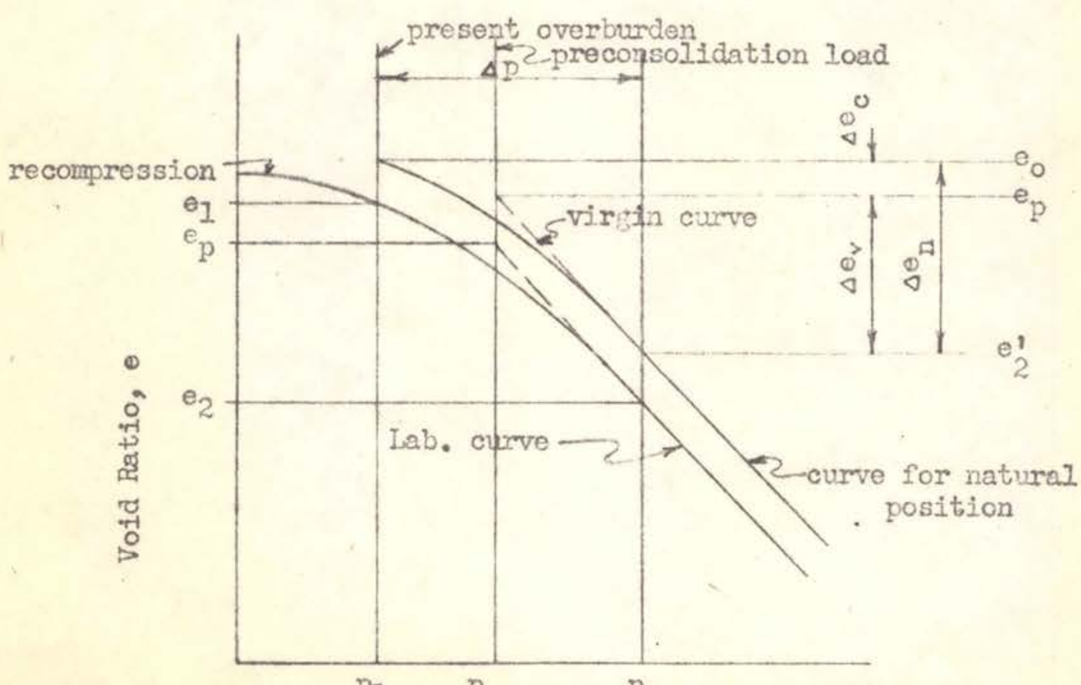
A clay which has not been subjected to pressure greater than that due to the present overburden is called a normally consolidated clay. When this clay is subjected to additional pressure in its natural state without removing any of the overburden it will be compressed along a curve as indicated by the virgin portion of the laboratory curve but which will be parallel and slightly above the virgin portion of the laboratory curve. Let p_1 be the present overburden pressure and p_2 the total pressure of $p_1 + \Delta p$. As shown by Figure 1 the actual Δe_n produced by the addition of ΔP equals $e_o - e$ determined by the intersection of p_2 with the virgin curve for the natural position, and equals $C_c \log \frac{p_2}{p_1}$. The Δe_L determined by extending the virgin laboratory curve to an intersection with p_1 is the same as Δe_n because the two lines are parallel. Thus in using a laboratory determined curve for normally consolidated clay, the settlement should be determined entirely along the virgin curve or $\Delta e = C_c \log \frac{p_2}{p_1}$.



$\Delta e_n = \Delta e_L$

e - p curve for normally consolidated clay

Figure 1



$\Delta e_n = \Delta e_L$

e - p curve for overconsolidated clay

Figure 2

A clay which at some earlier period has been subjected to a preconsolidation pressure, p_p , greater than its present overburden pressure, p_1 , is an overconsolidated clay. This clay will be recompressed in its natural position from p_1 to p_p , and for pressures, p_2 , greater than p_p the curve will be a straight virgin curve. See Figure 2. The total settlement, Δe, in this case is equal to $\Delta e_c + \Delta e_v$.

$$\Delta e_v = \frac{C_c}{1 + e_1} \log \frac{p_2}{p_p}$$

Often the settlement for loads less than the preconsolidation load is very small and may be neglected in the computation.

Here again the laboratory curve is parallel to the curve for natural position and $\Delta e_L = \Delta e_n$. For compression along the virgin curve the settlement may be determined directly from the pressures and slope of the virgin curve.

$$\Delta H = H \frac{C_c}{1 + e_1} \log \frac{p_2}{p_p}$$

For normally consolidated clay the preconsolidation pressure, p_p , and the overburden pressure, p_1 , are equal and the ΔH thus determined is the total settlement.

The first major step in the settlement analysis is the determination of settlements due to column loads, allowing no deformation of the frame. In this analysis the settlement due to the column load and the settlement due to pressure from loads on other columns are determined independently and added to determine the total settlement of each footing.

It is assumed that the footing sizes are proportional to the loads, that is, that the unit sole pressure under all footings is approximately equal.

The following procedure was followed by Mr. Means[4] to determine the settlements, allowing no frame deformation:

A. Settlement Due to Own Footing Load Only.

a. Determine depth, thickness and description of compressible soil layers below footings, taking undisturbed samples for testing. Determine the elevation of the water table.

b. Run consolidation tests on undisturbed samples of compressible soil and prepare pressure-void ratio curves, and determine preconsolidation loads.

c. Divide beds into layers of convenient thicknesses and determine present overburden pressure at average depth of layers chosen.

d. From e - p curve determine values of e_1 at average depth of layer. Determine value of slope of virgin curve C_c, and compute $\frac{C_c}{1 + e_1}$ for each layer.

e. From Diagram No. I determine the pressure at average depth for each layer due to unit pressure under footing for footings of four different sizes within the range of sizes used, using nominal footing sizes; say 4, 6, 8, and 10 feet square.

Determine settlement under each of these nominal size footings in each layer chosen, which is for that layer, $\Delta H = H \frac{C_c}{1 + e_1} \log \frac{p_2}{p_1}$.

f. Divide the total settlement under each footing by the total load to determine the amount of settlement per unit of load on the column and plot a curve showing the relationship between the size of footing and the settlement per unit of column load. See Figure 3.

---

[4] Means, <u>op</u>. <u>cit</u>., pp. 40-42.

g. To determine the settlement under a footing of given size loaded to the load per sq. ft. for which the curve is prepared, determine from the curve the settlement per unit of column load on the footing of the size required under the column load, and multiply by the total column load carried by the footing.

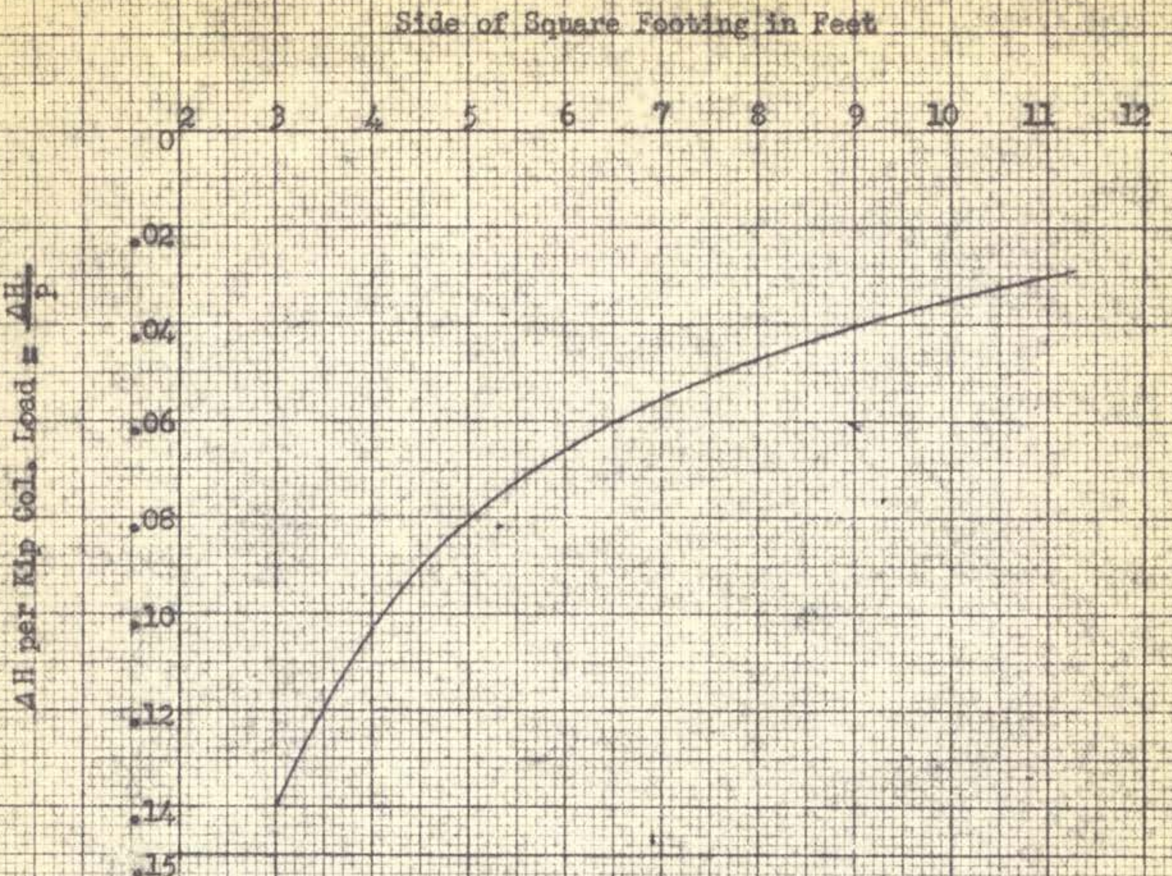
B. Settlement of One Footing Due to Loads on Other Footings.

a., b., c., d., same as under A.

e. The stress in the soil under a footing produced by another column load is essentially the same as that produced by a concentrated load at the other column location.

Therefore, determine from Diagram No. II the stress at the average depths of all layers due to 1 kip load at arbitrary distances, say $r = 10'$, $r = 20'$, $r = 30'$, and $r = 50'$.

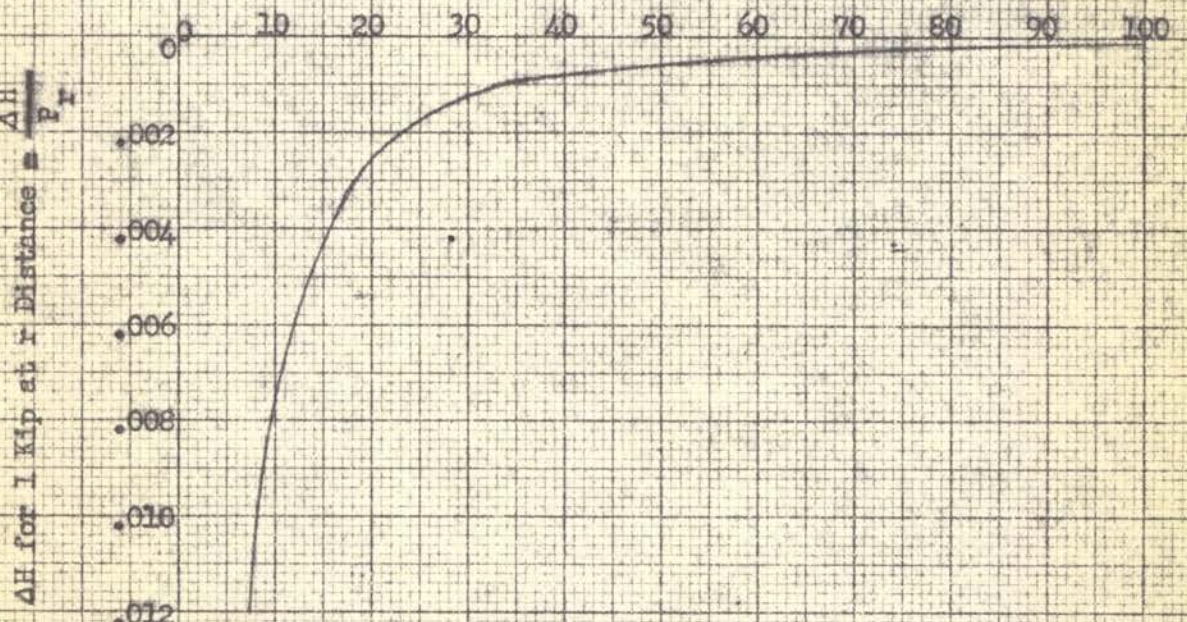
It can be seen from Diagram I that for layers at depths greater than about twice the footing width, the pressure is almost independent of the footing size; and the ratio of the additional stress to the overburden, $\frac{\Delta p}{p_1}$, is small and there is little change in deformation per unit stress under different size footings. From Diagram II it can be seen that loads at some distance produce very little stress near the elevation of their application. Since a distant load produces little stress at the bottom of a footing where $\frac{\Delta H}{\Delta p}$ is greater for different footing sizes, and since $\frac{\Delta H}{\Delta p}$ is nearly constant for any size footing at depths where appreciable stresses are produced by distant loads, it may be assumed that the additional settlement produced by distant loads is dependent only upon the distance and the column load on the distant footing and independent of the size of the footing under which settlement is produced.



Relation Between Footing Size and

Settlement per Kip Column Load

Figure 3



Settlement Produced by 1 Kip Load at r Distance

Figure 4

Following this assumption, the additional settlement is computed under an average size footing by multiplying the $\frac{\Delta H}{\Delta p}$ of each layer by the pressure at average depth of the layer due to unit load at r distance. The summation of these products for all layers under the footing is the settlement due to a unit load at r distance.

Determine settlements as outlined above for the four arbitrary distances chosen.

f. Plot curve showing relationship between r and settlement for unit load at r distance. See Figure 4.

g. Multiply settlement for unit load at r_1 distance by column load at r_1 to obtain settlement produced by load at r_1 distance.

Determine settlements due to each load at distance from footing in a similar manner, and add to obtain the total settlement of the footing due to loads on footings at a distance. The settlement of the footing is equal to ΔH due to own footing load plus ΔH due to loads at r distances.

The next step is to allow the structural frame to deform and compute the resulting settlements which will be added to those determined above to get the actual settlements of all footings.

The following symbols will be used:

ΔP_1 = change in load on column No. 1

$\Delta P_2^{\delta_1 = 1''}$ = change in load on column No. 2 due to settlement of 1" in footing No. 1.

Imagine an elastic continuous frame resting on independent footings on an elastic soil mass. When the frame is loaded it will produce a deformation of the soil. Now assume that there are jacks in the columns just above the footings and that as the footings settle the jacks are extended the same amount so as to allow no deformation of the frame. Under these conditions the column loads and settlements (jack extensions) can be determined as previously outlined. Let ρ^{PD} represent this settlement due to the footing load for no deformation of the frame.

Now release one of the jacks and the change in length of the column will be partly taken up by a deformation of the frame and partly by expansion of the elastic soil, the two amounts depending upon the stiffness of the frame and the soil.

The deformation of the frame produces a redistribution of loads, changing the load on a column by an amount ΔP. The change from ρ_1^{PD} will be the settlement produced by $\Delta P_1^{\rho_1}$ on the footing itself plus the settlement produced by $\Delta P_2^{\rho_1}$, plus the settlement produced by $\Delta P_3^{\rho_1}$ etc., and

$$\rho_1 = \rho_1^{PD} + \rho_1^{\Delta P_1^{\rho_1}} + \rho_1^{\Delta P_2^{\rho_1}} + \rho_1^{\Delta P_3^{\rho_1}} + \cdots$$

$$\text{or} \quad \rho_1 = \rho_1^{PD} + \sum \rho_1^{\Delta P^{\rho_1}}$$

Now if the jack in column 2 is released ρ_1^{PD} will be changed due to the change in column load which is produced by the accompanying deformation. The settlement of footing No. 1 for the settlement of both footing No. 1 and No. 2 will be

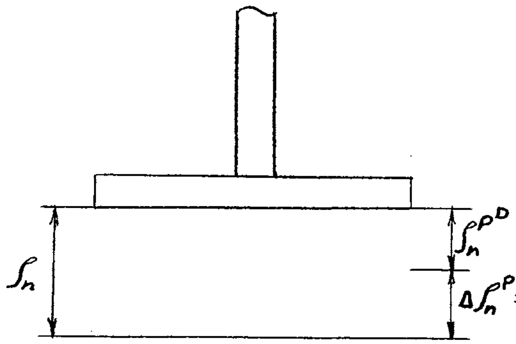
$$\rho_1 = \rho_1^{PD} + \sum \rho_1^{\Delta P^{\rho_1}} + \sum \rho_1^{\Delta P^{\rho_2}}$$

If all of the jacks are released

$$\rho_1 = \rho_1^{PD} + \sum \rho_1^{\Delta P^{\rho_1}} + \sum \rho_1^{\Delta P^{\rho_2}} + \sum \rho_1^{\Delta P^{\rho_3}} + \cdots$$

As stated above ρ^{PD} can be determined by methods discussed previously but ρ_1, ρ_2, etc. are not known. We can, however, determine the changes in column loads due to the deformation of the frame for a settlement of 1 inch in one column and from these changes in loads determine the change in settlement due to a frame deformation of 1 inch in one column only. The change in settlement of footing No. 1 due to settlement of footing No. 2, then, is the settlement of footing No. 1 produced by a settlement of 1 inch in footing No. 2 times the actual settlement of No. 2 or $\rho_1^{\Delta P^{\rho_2 = 1''}} \rho_2$ and the effect of ρ_3 on No. 1 is $\rho_1^{\Delta P^{\rho_3 = 1''}} \rho_3$ etc.

By following this procedure an equation can be written for the settlement in any footing.



$$\Delta \rho_n^{P^D} = \rho_n^{\Delta P^{\rho_n = 1''}} \rho_n + \rho_n^{\Delta P^{\rho_1 = 1''}} \rho_1 + \rho_n^{\Delta P^{\rho_2 = 1''}} \rho_2 + \cdots\cdots$$

Since the effect of the deformation due to settlement of the footing on itself is to decrease the load on that footing the value of $\rho_n^{\Delta P^{\rho_n = 1''}}$ is always negative, so in setting up the equation the coefficient of ρ_n , $\rho_n^{\Delta P^{\rho_n = 1''}}$ is always increased by unity.

The details of the analysis can be carried out by first determining the column loads for no settlement deformation of the frame. Next, produce a deformation ΔY of 1 inch in each bay independently and determine the ΔP produced in each column for a settlement of 1 inch in one column only, thenthe ΔP in each column for 1 inch settlement of one other column and so on until the ΔP in each column has been determined for a settlement of 1 inch in each column independently. This can readily be handled by moment distribution where $FEM = \frac{6EI\Delta}{L^2}$. After the moments are distributed the ΔP's can be computed by dividing the difference in moment by the span length.

Now, choose one footing only, say No. 1 and determine the settlement due to the loaded frame without settlement deformation, ρ_1^{PD}. Then determine the change in settlement due to ΔP for 1 inch settlement of column No. 1, then the change in settlement of footing No. 1 due to ΔP from 1 inch settlement of column No. 2, and so on until the effect upon footing No. 1 of 1 inch settlement of each column independently has been determined.

The change in settlement of footing No. 1 due to the actual settlement, ρ_1, of column No. 1 is the change in settlement due to 1 inch settlement times ρ_1; and due to the actual settlement, ρ_2, of column No. 2, is the change in settlement due to 1 inch settlement of No. 2 times ρ_2, etc. Using the settlement of footing No. 1 due to a settlement of 1 inch in each of the columns of the frame as coefficients of the actual settlement of each footing, an equation can be written for the settlement of column No. 1 in terms of the unknown settlements of all the footings.

$$-\rho^{PD} = \rho_1\left(\rho_1^{\Delta P \rho_1 = 1''} - 1\right) + \rho_2 \rho_1^{\Delta P \rho_2 = 1''} + \rho_3 \rho_1^{\Delta P \rho_3 = 1''} + \cdots$$

This equation contains an unknown for each footing. A similar equation can be written for each footing. This provides as many equations and as many unknowns as there are footings. These equations are solved simultaneously to determine the settlement of each footing.

After the settlements are determined the moments produced by differential settlements may readily be found because the moments due to a differential settlement of 1" in each bay will have been determined earlier in the analysis. The actual moments due to differential settlements are merely the product of the differential settlements and the moments for 1" differential settlement. The total moments existing in the frame are found by adding the moments due to differential settlement to those due to loads.

When this analysis is carried through for a specific structure it will be found that the total moment will be opposite in sign to the moment due to loads for some of the members. This is especially bad in reinforced concrete because the design for external loads may not provide enough anchorage for the steel in the event that the concrete should crack due to the moment changing signs.

As building materials of higher working stresses come into use the consideration of differential settlement will be even more important, because these materials will not redistribute the stresses as readily as will the more ductile materials.

BIBLIOGRAPHY

Gilboy, Glennon. "Earths and Foundations". Progress Report of Special Committee, Proceedings A.S.C.E. (May 1933).

Housel, William S. "Bearing Power of Clay is Determinable". Engineering News Record, (February 1933).

Means, R. E. "Building Foundations on Compressible Soils", Unpublished.

Meyerhof, George Geoffrey. "Settlement Analysis of Building Frames". The Structural Engineer, (September 1947)

Fern B. Hall