A DEVELOPMENT OF ELASTIC THEORY CONCERNING THE ACTION OF CERTAIN RIVETED BEAM COLUMN CONNECTIONS

A DEVELOPMENT OF ELASTIC THEORY CONCERNING THE ACTION OF CERTAIN RIVETED BEAM COLUMN CONNECTIONS

by

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Equations will be derived for the elastic behavior of three types of riveted beam column connections and the results will be checked against experiments run by Professor Rathbun.[1]

In the past, designers of continuous frames have generally adopted one of the two following assumptions: that the connection between column and beam is incapable of transmitting bending moment, or that the connection between column and beam is sufficiently rigid to prevent change in the angle between column and beam. The latter is one of the fundamental assumptions of the more common of the so-called indeterminate methods of analysis. The accuracy of the assumption is the most significant measure of the worth of the method used. Therefore, the magnitude of the elastic deformation of the connection is of considerable importance.

The experiments, to which reference is made above, were undertaken in order to secure a measure of the differential rotation between beam and column and the ratio between moment imposed and rotation obtained. Inasmuch as only a relatively few connections were tested, it is desirable that an analytical means of calculating the rotation of connection angles be obtained. Such a rational analysis has been made by Professor Lothers of the Engineering Division of Oklahoma Agricultural and Mechanical College. Professor Lothers has applied his theory to the so-called standard connections, i. e., beam web to flange of column connections. This discussion develops a relationship by

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[1] Professor J. Charles Rathbun, "Elastic Properties of Riveted Connections." Transactions, American Society of Civil Engineers, Volume 101 (1936), 524-563.

a different approach and applies the results to two other types as well as to the standard connection. The types tested by this theory will be the standard, the flange of beam to flange of column, and a combination of the two. A fourth type tested by Professor Rathbun, the tee or split I-beam connection, will not be considered in this discussion.

There has been much discussion as to whether rivets possess dependable initial tension. One of the more common methods of analyzing brackets is based on the assumption that there is no initial tension in the rivet. Wilson, Bruckner, and McCracken[2] report that if the rivets are driven hot, there exists initial tension equal approximately to 75% of the yield strength of the rivet steel. An inspection of the photographs of tested specimens in Professor Rathbun's paper shows that the initial tension was so great that the connections failed in bending in the angles and that the rivets had not elongated whatsoever, which is proof that the initial tension in the rivets due to contraction after driving was never exceeded. In this discussion, therefore, it will be assumed that the rivets possess initial tension.

The presence of initial tension affects considerably the stiffness of the joint. If there is no initial tension, the smallest moment load on the beam will cause extension of the rivets and differential rotation of the end of the beam with respect to the column. If there is initial tension, it will be reacted, before the beam is loaded, by more or less local areas of compression between the backs of the connection angles and the flanges of the columns. As the moment is gradually applied to the beam, the pull on the tension side of the beam will begin to relieve the pressure between back of angle and column flange. Not until this pressure is completely relieved will the initial tension in the rivet be exceeded. If the rivets are properly designed, therefore,

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[2] Wilbur M. Wilson, Walter Bruckner, and Thomas H. McCracken, "Tests of Riveted and Welded Joints in Low Alloy Structural Steel." University of Illinois, Engineering Experiment Station Bulletin. Series No. 337.

there will be no extension of the rivets as the beam is loaded. Consequently, there will be a point of contraflexure in the outstanding leg of the connection angle.

FLEXURE IN CONNECTION ANGLE

Consider action of angle under the loads shown (Fig. 1 & 2). The solution will be by the Method of Virtual Work. The actual moment diagrams are drawn by parts. The angle is loaded with a virtual moment loading of 1 inch pound at A and with another virtual load of 1 pound vertical at A. (Fig. 3) Note that both the rotation at A and the vertical deflection of A equal zero.

The basic virtual work equations used are:

1) $1 \cdot \Delta = \int \frac{mMds}{EI}$ for virtual loading with 1 pound vertical.

2) $1 \cdot \alpha = \int \frac{mMds}{EI}$ for virtual loading with 1 inch pound moment.

m = moment due to virtual load.

M = moment due to actual load.

Δ = deflection, vertically, in direction of 1 pound force.

α = rotation in direction of 1 inch pound moment.

ds = differential length measured along axis of member.

E = Young's modulus of elasticity.

I = moment of inertia of 1 inch strip of angle.

Actual virtual work equations:

Using virtual load A (Fig. 3).

$$1 \cdot \alpha = \int_A^B \frac{(-1)\,(-Py)\,dy}{EI} + \int_A^B \frac{(-1)\,(M_A)\,dy}{EI} + \int_B^C \frac{(-1)\,(-pg)\,dx}{EI}$$

$$+ \int_B^C \frac{(-1)\,(M_A)\,dx}{EI} + \int_B^C \frac{(-1)\,(Fx)\,dx}{EI}$$

3) $$\frac{Pg^2}{2EI} - \frac{M_A g}{EI} + \frac{Pgg_1}{EI} - \frac{M_A g_1}{EI} - \frac{Fg_1^2}{2EI} = 0$$

Using virtual load B (Fig. 3).

$$1\cdot\Delta_B = \int_B^C \frac{(x)\ (-Pg)\ dx}{EI} + \int_B^C \frac{(x)\ (M\ )\ dx}{EI} + \int_B^C \frac{(x)\ (Fx)\ dx}{EI}$$

4) $$-\frac{Pgg_1^2}{2EI} + \frac{M_A g_1^2}{2EI} + \frac{Fg_1^3}{3EI} = 0$$

In addition to above equations, one more equation is necessary to evaluate F and M_A in terms of P. Sum moments about B (Fig. 2).

5) $$M_C - M_A - Fg_1 + Pg = 0$$

By simultaneous solution of above equations:

6) $$M_A = P\,\frac{g^2 + gg_1}{2g + g_1}$$

7) $$F = P\,\frac{g^2}{g_1(2g+g_1)}$$

8) $$M_C = 0$$

Calculate the moment at B (Fig. 2).

$$M_B = -P\,\frac{g^2 + gg_1}{2g + g_1} + Pg$$

9) $$M_B = \frac{Pg^2}{2g + g_1}$$

See Figure 4 for complete moment diagram.

Write virtual work equation for Δ_B, deflection of heel of angle parallel to axis of beam.

Using virtual load C (Fig. 3).

$$1\cdot\Delta_B = \int_A^B \frac{(-Py)\ (-y)dy}{EI} + \int_A^B \frac{(M_A)\ (-y)dy}{EI} + \int_B^C \frac{(-Pg)\ (-g)dx}{EI}$$

$$+\int_B^C \frac{(M_A)\ (-g)dx}{EI} + \int_B^C \frac{(Fx)\ (-g)dx}{EI}$$

$$= P\,\frac{g^3 + 3\,g^2 g_1}{3EI} - M_A\,\frac{g + 2gg_1}{2EI} - \frac{Fgg_1^2}{2EI}$$

Substitute values of M_A and F from Equations 6) and 7)

10) $$\Delta_B = \frac{Pg^3}{6EI}\,\frac{g + 2g_1}{2g + g_1}$$

Write virtual work equation for rotation at B.

Using virtual load D (Fig. 3).

$$1\cdot\alpha_B = \int_B^C \frac{(-Pg)\ (-1)dx}{EI} + \int_B^C \frac{(M_A)\ (-1)dx}{EI} + \int_B^C \frac{(Fx)\ (-1)dx}{EI}$$

$$= P\frac{gg_1}{EI} - M_A\frac{g_1}{EI} - F\frac{g_1^2}{2EI}$$

Substitute values of M and F from Equations 6) and 7)

$$11)\ \alpha_B = \frac{P}{EI}\ \frac{g^2 g_1}{2(2g + g_1)}$$

Write virtual work equation for deflection at B perpendicular to BC, i.e., perpendicular to axis of beam.

Using virtual load E (Fig. 3).

$$1\cdot\Delta = \int_B^C \frac{(Pg)\ (x)\ dx}{EI} + \int_B^C \frac{(M_A)\ (x)\ dx}{EI} + \int_B^C \frac{(FX)\ (x)\ dx)}{EI}$$

$$= -\frac{Pgg_1^2}{2EI} + \frac{M_A g_1^2}{2EI} + \frac{Fg_1^3}{3EI}$$

Substitute values of M and F from Equations 6) and 7)

$$12)\ \Delta = -\frac{Pg^2 g_1^2}{6EI(2g + g_1)}$$

See Figure 5 for diagram showing deflections and rotations.

PRESSURE DISTRIBUTIONS ON BACKS OF ANGLES
Above Neutral Axis

Initial Condition (Fig. 6)

Due to contraction of the hot rivet, there exists initial tension in the rivet. This tension is equilibrated by local areas of compression between back of angle and column flange. The initial tension and its equilibrant are equal, opposite, and colinear, in equilibrium both as to resultant and also as to resultant moment about any axis in the plane. The shape of areas of compression may be as shown or they may be more extensive, depending on the stiffness of the angles.

Final Condition (Fig. 7)

As the moment is gradually applied to the beams, the top of the connection angle is pulled from the column flange and the bottom of the connection is forced against the column flange. There is no change in the tensile stress in the rivets, but the pressure on the back is lessened and redistributed as shown in the sketch. The resultant tension in the sketch summed up above the horizontal line of zero pressure - hereafter referred to as the "neutral axis" - composes the tension component of the couple on the back of the angle.

Below Neutral Axis

Initial Condition(Fig. 6).

Same as above neutral axis.

Final Condition (Fig. 8).

The gradual application of moment to the beam causes pressure to be applied to the outstanding leg of the angle. The equilibrant to this pressure comes from an increase of pressure on the back of the angle. In this case, there may be some change in the initial tension of the rivet if there be shortening through compression of either the angle or of column flange or of both. If there be such decrease in initial tension, it will be small and without significance. In any case, there will be resultant unbalanced compression below the neutral axis. This unbalanced compression, summed up throughout the area under the neutral axis, is equal to the summed up resultant tension above the neutral axis and composes the compression component of the couple on the back of the angle.

The resultant pressure distribution allows exactly the same method of analysis for location of the neutral axis as is used for this type of bracket when no initial tension exists. Therefore, the section for which the location of the neutral axis will be determined is composed of the area of compression on one side of the neutral axis and the area of effective tension rivets on the other side.

There will be no difficulty in locating the neutral axis for all connections possessing two rows of rivets; it is necessary to realize that the row furthest removed from the beam is not effective in resisting moment about the neutral axis of the connection. If the tension in the row of rivets closest to the beam is not exceeded, there will be only exceedingly slight deflection of the connection angle past the first row of rivets. In the absence of deflection in the angle, the rivets can exert no effective force about the neutral axis. The failures in these angles were, in most cases, failures of the angles in bending with no perceptible elongation of rivets. Since the relationships derived herein are to be applied within the elastic range of the connections, we may, therefore safely reject the second row of rivets.

In location of the neutral axis of the web of beam to column flange connections, two approximations will be used. In the first place, the area of the rivets above the neutral axis will be replaced by an equivalent area extending from the neutral axis to the top of the connection and of width equal to the nominal area of a rivet divided by the spacing of the rivets. In the second place, no deduction will be made for the area of the rivets below, or on the compression side of the neutral axis. For the connections from flange of beam to flange of column the second assumption mentioned above will be employed.

If it is desired, the exact solution for the location of the neutral axis may be used, but the improvement in accuracy will not warrant the additional computations.

<u>Calculation of the Rotation $\frac{1}{Z}$ of the Connection</u>

14) $I = \frac{t^3}{12}$ (for a 1 inch strip of angle)

15) $\phi_M = \frac{\Delta_B}{y}$

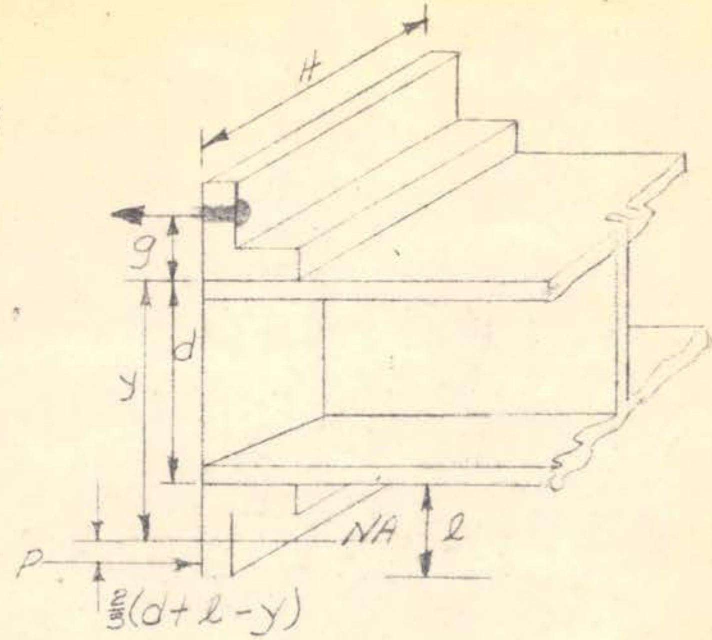
16) $M = \frac{2Phy}{3}$ (for two angles)

But,

17) $\frac{1}{Z} = \frac{M}{\phi_M}$

Therefore,

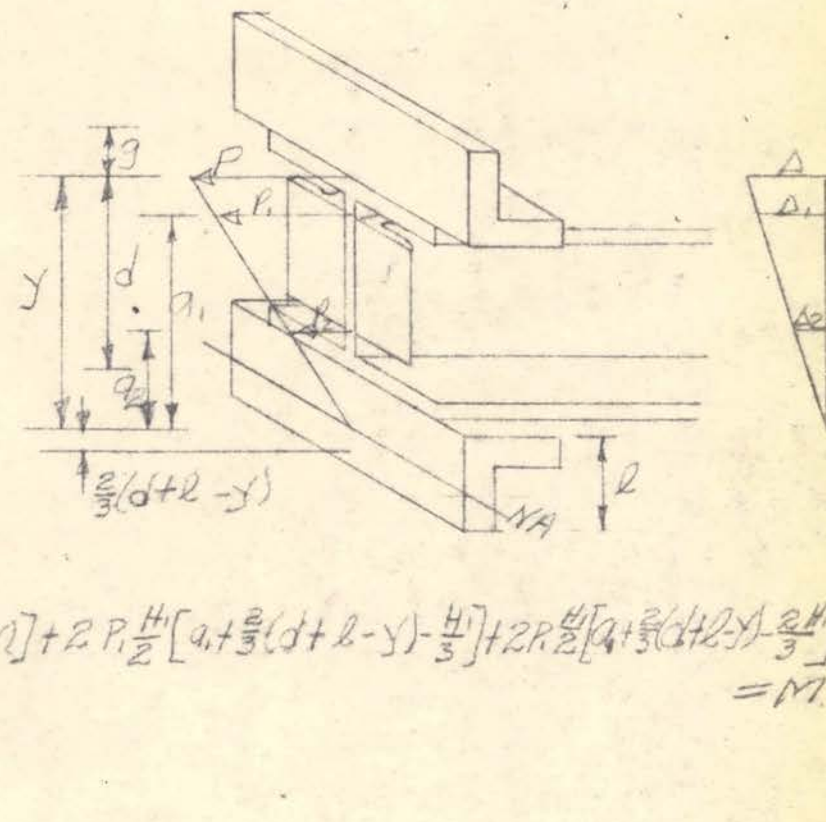
18) $\frac{1}{Z} = \frac{Et^3}{3g^3} \frac{2g + g_1}{g + 2g_1} \cdot hy^2$



19) $M = PH[g+y+\frac{2}{3}(d+\ell-g)]$

20) $\phi_M = \frac{\Delta_B}{y}$

21) $\frac{1}{Z} = \frac{M}{\phi_M}$



22) $\frac{\Delta_1}{\Delta} = \frac{a_1}{y}$

23) $\frac{P_1}{P} = \frac{\Delta_1}{\Delta} = \frac{a_1}{y}$

24) $\frac{\Delta_2}{\Delta} = \frac{a_2}{y}$

25) $\frac{P_2}{P} = \frac{\Delta_2}{\Delta} = \frac{a_2}{y}$

26) $HP[g+y+\frac{2}{3}(d+\ell-y)] + 2P_1\frac{H_1}{2}[a_1+\frac{2}{3}(d+\ell-y)-\frac{H_1}{3}] + 2P_2\frac{H_1}{2}[a_2+\frac{2}{3}(d+\ell-y)-\frac{2H_1}{3}] = M$

27) $\phi_M = \frac{\Delta_B}{y}$

28) $\frac{1}{Z} = \frac{M}{\phi_M}$

Illustrating Method of Computing Rotation

Table I

Standard Connections

Specimen	l	t	g	g_1	h	y	$\frac{1}{Z}$	$\frac{1}{Z}$ (Rathbun)
4	4	3/8	2.56	2.25	9	7.35	15,800,000	17,400,000
5	6	3/8	2.50	2.25	9	7.61	18,200,000	20,000,000
6	4	3/8	2.50	2.25	15	12.25	78,600,000	87,500,000
7	6	3/8	2.50	2.25	15	12.67	84,600,000	86,000,000

Explanation of Table I

l = length of outstanding angle leg.

t = thickness of angle.

g = distance from heel of angle to first rivet, outstanding leg.

g_1 = distance from heel of angle to first rivet, instanding leg.

h = height of angle.

y = distance from top of beam to neutral axis.

Table II

Flange to Flange Connection

Specimen	l	t	g	g	d	No Rivets Effective	h	y	$\frac{1}{Z}$	$\frac{1}{Z}$ (Rathbun)
8	6	3/8	2.25	2.25	12.00	2	6	13.48	98,000,000	100,000,000
9	6	3/8	2.25	2.25	12.00	3	8	13.22	128,000,000	145,000,000
10	6	3/8	2.25	2.25	12.00	4	14	13.65	232,000,000	132,000,000

Explanation of Table II

l = length of outstanding angle leg.

t = thickness of angle.

g = distance from heel of angle to first rivet, outstanding leg.

g_1 = distance from heel of angle to first rivet, instanding leg.

d = depth of beam.

h = length of angle.

y = distance from top of beam to neutral axis.

No Rivets Effective on tension side of connection.

Table III

Combined Connections

Specimen	Depth Beam	l	t	g	g_1	No. Rivets Effective	y	$\frac{1}{Z}$	$\frac{1}{Z}$ (Rathbun)
11	12	4	3/8	2.25	2.25	2	12.80	171,000,000	209,000,000
12	12	6	3/8	2.25	2.25	4	14.73	392,000,000	175,000,000

Explanation of Table III

l = length of outstanding angle leg on compression side.

t = thickness of angle.

g = distance from heel of angle to first rivet, outstanding leg.

g_1 = distance from heel of angle to first rivet, instanding leg.

y = distance from top of beam to neutral axis.

No Rivets Effective on tension side of connection.

CONCLUSIONS

It is apparent from a study of the tables that these results are accurate within ten to twenty percent, with the exception of specimens 10 and 12. These two connections are much stiffer than the remainder. It will be noted that in the experiments performed by Professor Rathbun, the test was run until a rotation was obtained which would produce a deflection at the center of the beam of of one - three hundred and sixtieth of the span. Since the two connections under discussion were very stiff in comparison with their fellows it was necessary to test them with a much greater moment load. Since, in the case of all connections the slope decreases as the moment increases, the stiffer specimens were tested into a much flatter range than the others. Therefore, the average slope, against which the results of this derivation are tested, is much less and the apparent check is poor. It is pointed out, however, that these connections will be rarely used together with such excessive deflection and that the results obtained on Tables II and III are a fair representation of the action of the connections in the probable range of their use.

It is again pointed out that the second row of rivets on the stiffer connections have little effect on the elastic action of the connections. Were it not for their aid in resisting the shear loads in the beam, these rivets might be omitted altogether.

The most feasible method of stiffening a given connection is to increase the thickness of the angle. Professor Batho (1) sets forth this conclusion and it is borne out by the relationships derived herein.

There is much literature in technical publications outlining methods of applying connection rotations, once obtained. Authorities in structural engineering are beginning to desire the benefits, in the form of reduced weight, accruing from the consideration of connection rotations.

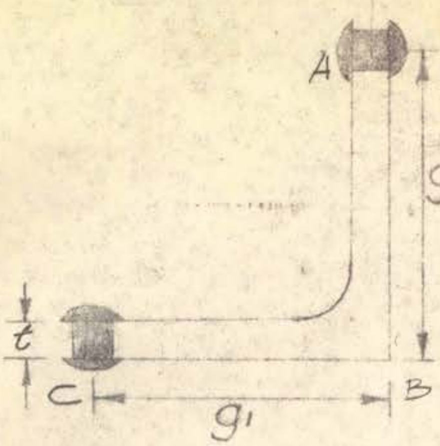


Fig 1

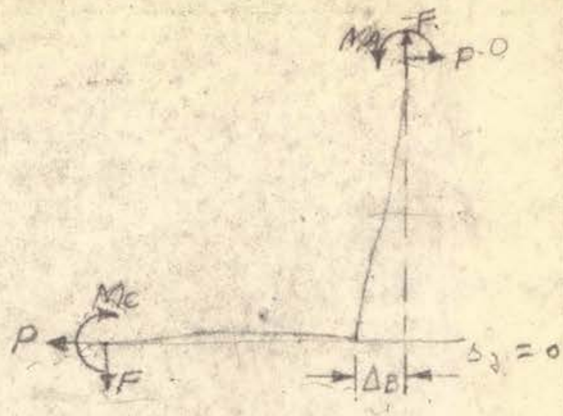
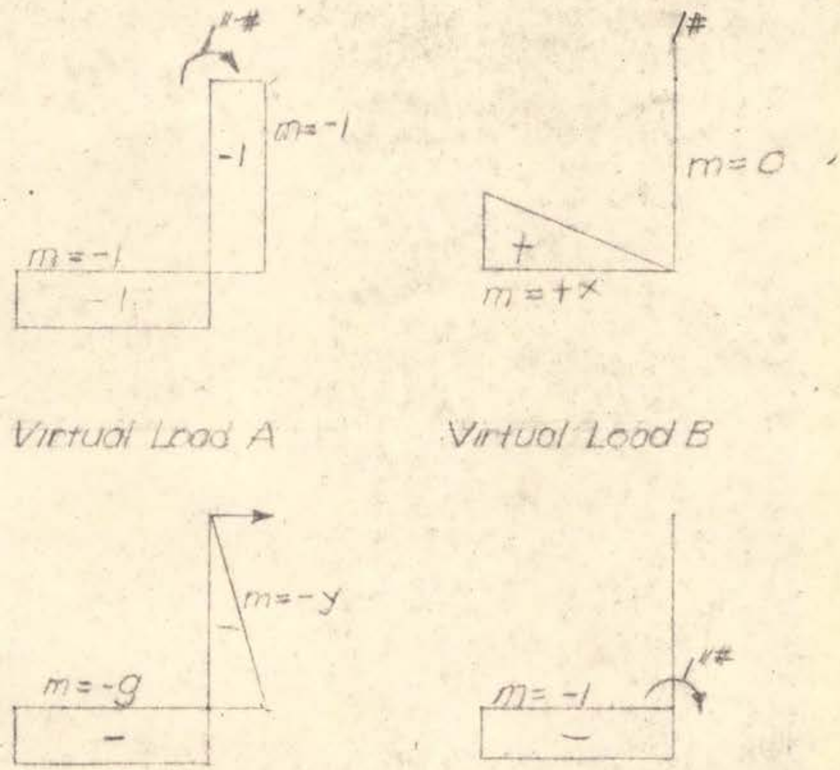


Fig 2



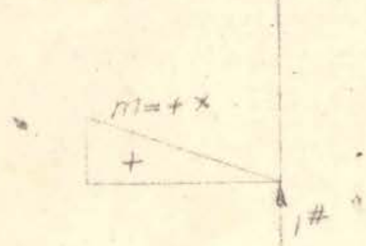
Virtual Load A

Virtual Load B

Virtual Load C

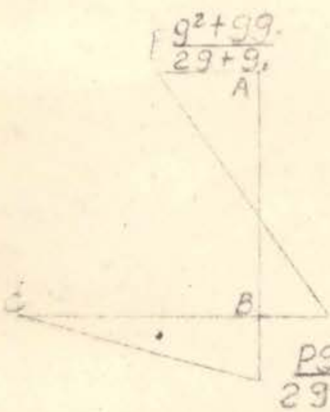
Virtual Load D

Fig 3



Virtual Load E

Fig 3 (Cont'd)

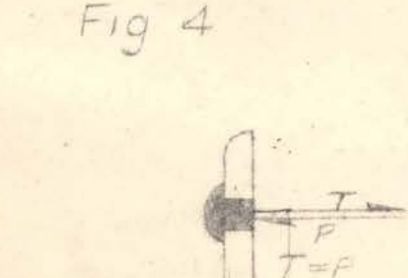


Final Moment Diagram

Fig 4

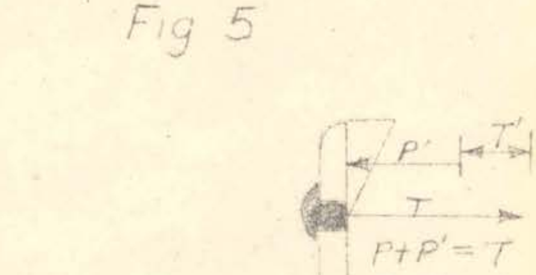
Deflection Diagram

Fig 5



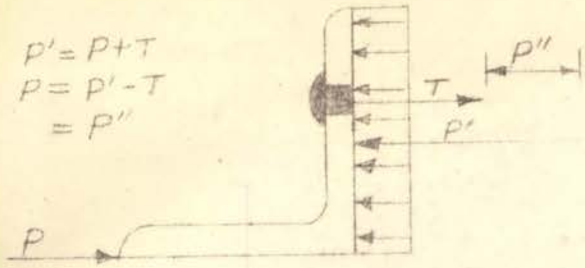
Initial Condition above NA

Fig 6



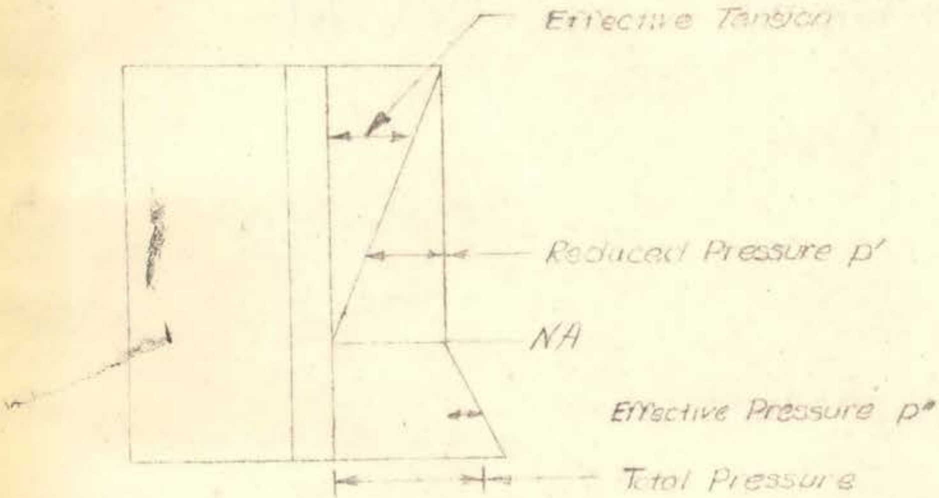
Final Condition above NA

Fig 7



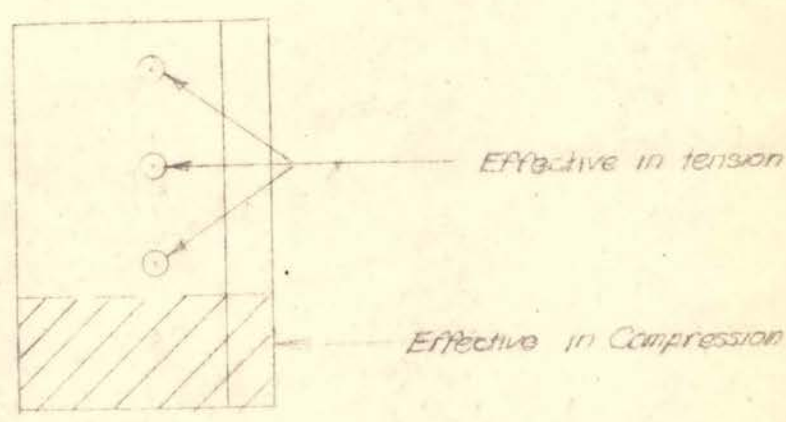
Final Condition below NA

Fig 8



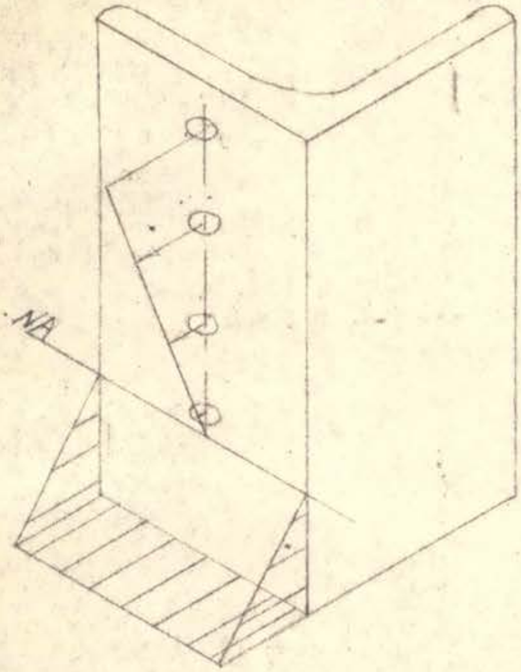
Total and Effective Forces

Fig 9



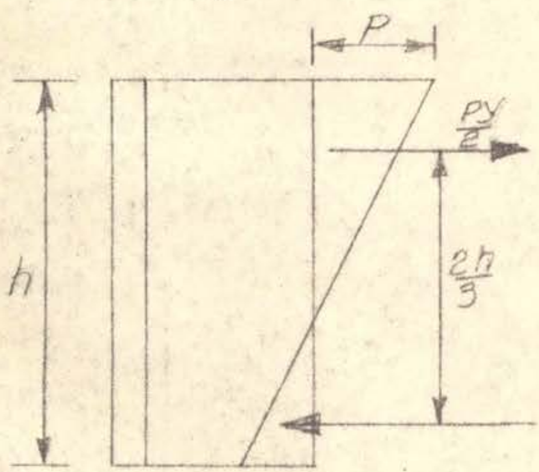
Effective Areas

Fig 10



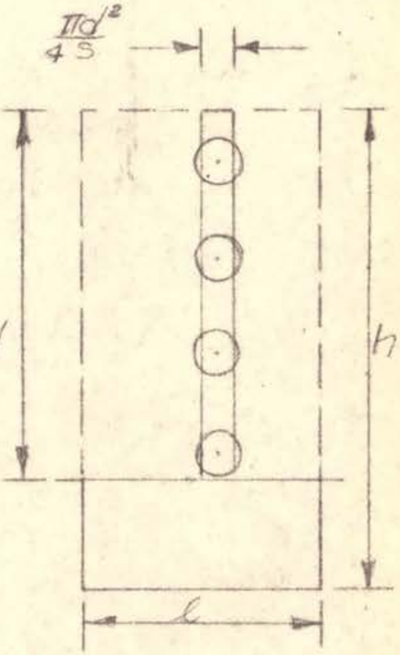
Resultant Forces

Fig. 11



Determination of Moment

Fig 13



Location of Neutral Axis

Fig 12

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Carol Ealy - typist