

THE CONTRIBUTION OF COLUMN FLANGES TO  
THE ROTATION OF PARTIALLY RESTRAINED  
RIVETED CONNECTIONS IN BUILDING FRAMES

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By

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## PREFACE

The question of the rigidity of riveted connections first arose in the author's mind while privately engaged in the design of a riveted structural frame. At that time information on the subject was extremely limited and that which had been published was difficult to find. The problem next was encountered during the study of the design of steel buildings under Professor J. E. Lothers of the Oklahoma A & M College Structural Department. Although intensive reference work was undertaken to seek some answer, most of the available works avoided the question of the amount contributed by the column flanges to the rotation of the joint. This investigation, then, was undertaken in an effort to supply information about a phase of the problem on which little or no information had been published.

The semi-rigid connection may be defined as a locally weakened section between the beam and column face. The fact that it is welded does not necessarily make the joint fixed. A welded joint can be considered fully fixed only when there is no yield of the column flanges nor of the connecting angles. This condition can be attained by stiffening the column flanges and at the same time welding the upper and lower flanges of the beam to the column.

The writer wished to acknowledge his indebtedness to the Civil Engineering Department of Oklahoma A & M College, Ren G. Saxton, Head, for the provision of the laboratory facilities and to the Engineering Experiment Station, Dr. C. A. Dum, Vice-Director, for their aid in obtaining the specimens and in supplying valuable information.

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## INTRODUCTION

Most structural steel designers and those concerned with the analysis of steel building frames have in the past made the erroneous assumption that riveted beam-column connections are simply supported and no end restraint can be afforded from this type of joint. On the other hand, a welded joint is assumed to offer full fixity at the connection. Tests have proved that the riveted connection can, and does withstand a certain amount of moment. If this moment, or end restraint, is taken into account it is estimated that the average savings in weight of steel used would amount to 20% as compared to the same beams designed as simply supported members.

Extensive tests related to this subject have been conducted both here and abroad. The first article covering this problem was published in May 1934 in England<sup>1</sup>. Then, in 1936, Professor J. Charles Rathbun<sup>2</sup> published his article. Other articles related to this problem have been published since then, and are listed in the bibliography at the end of this report. In each of these articles mention is made of the wind bracing connection (use of standard tee beams) with which this report deals but, the amount of rotation contributed by the column flanges is not recorded.

The angle change at any joint in a semi-rigid building frame is caused by the elastic action of both the connection angles, or tees, and the column flanges. Since column flanges vary in thickness, some contribute more to this angle change than others, and thus some analysis must be made of the column

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<sup>1</sup> Professor Cyril Batho, First, Second, and Final Report, Steel Structures Research Committee, Department of Scientific and Industrial Research, H. M. Stationery Office, London, 1931-1936.

<sup>2</sup> J. Charles Rathbun, Transactions of the American Society of Civil Engineers, Vol. 101, pp. 525-596.

flanges as well as the connectors. This, of course, brings up the problem concerning the amount of the column flange affected (i.e. effective length of flange). This effective length of the flange was found by the tests conducted for the preparation of this report and a more thorough discussion will be made later.

Therefore, the specimens tested were designed to first give the amount of rotation due directly to the connectors, and then the amount due to the column flange and the connectors. The difference being the amount that the column flange contributes to this rotation. The method by which this was accomplished will be described under "Description of Specimens".

#### THE SPECIMENS USED IN TESTING

In each case the specimens used for testing were made under normal shop fabrication methods. No particular care was given them in the fabrication and no inspection for acceptance or rejection was made. As a result, some parts of the specimens showed signs of poor workmanship, including burned rivets, over sized holes, webs of tees "beat" into shape by the rivet gun, etc., while other parts were quite acceptable. The physical properties of the materials used were not required by the writer, since the analysis would have been rather costly.

Specimen three contained the poorest workmanship, the webs of the beams being out of alignment with the web of the column, although they were parallel to each other. This made testing rather difficult, since one side of the one-inch bearing plates had to be shimmed so as to bring the center line of the column to the center line of the moving head of the testing machine.

## DESCRIPTION OF SPECIMENS

Three different specimens (see Fig. 1 and 2) were tested. In each case the connections were the same, the difference being the parts to which they were attached. Specimen 1 contained a one-inch thick plate between the two beams, while specimens 2 and 3 contained columns 5'-6 inches in height. Specimen 2 had the lightest 10 inch column obtainable and specimen 3 had one of the heavier sizes. The reason for the plate and the two sizes of columns was to ascertain the amount the connection riveted to the plate would rotate and then any additional rotation measured at the beam to column connection in specimens 2 and 3 would be due to the deflection of the flanges. The variations in the column sizes aided in determining the effective length of the column flange.

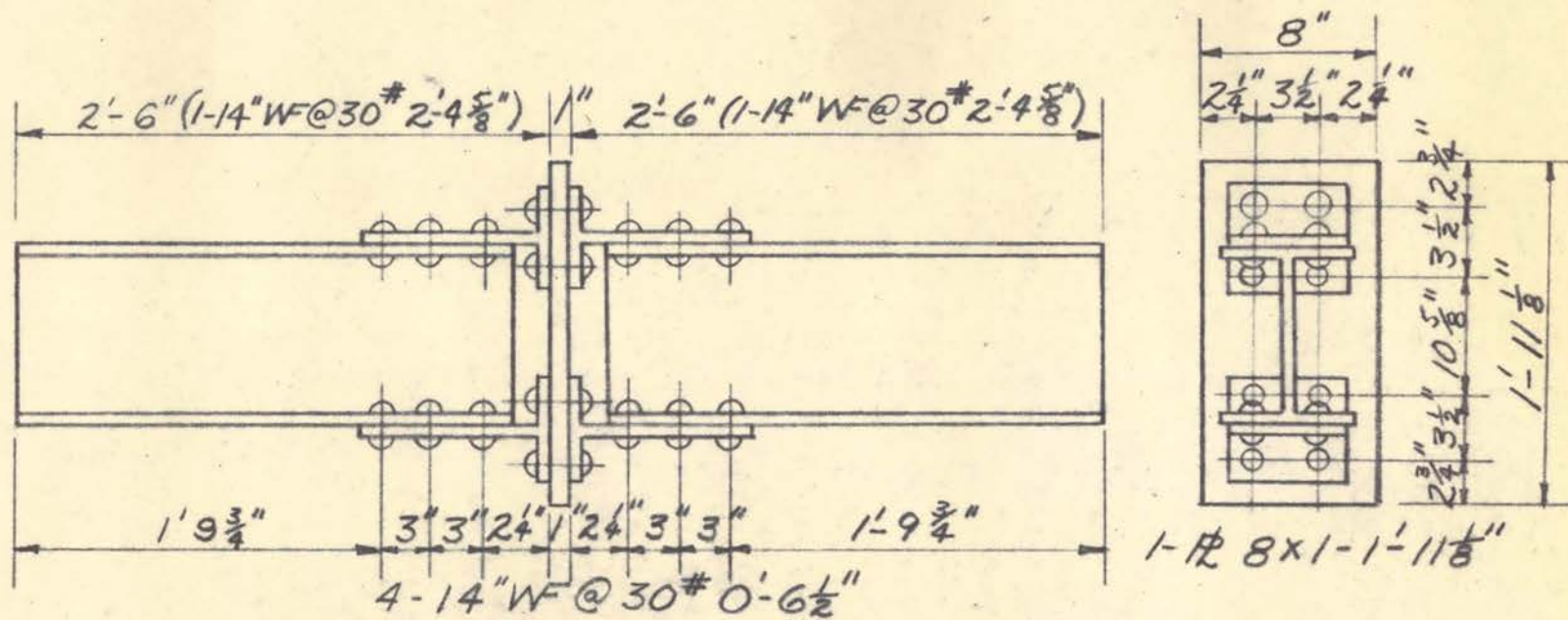
The fabricators of these specimens did not mill the surfaces on which the load was applied, and some difficulty was encountered in getting good bearing. This condition was finally remedied by the use of shims.

In the preliminary design of these members other sections were selected but, due to the shortages of steel during fabrication, only certain sections could be purchased. As a result, substitution had to be made for every size of beam and column originally selected.

Ordinarily it would be assumed that this joint would fail in tension across the net section of the web of the tee at the pair of rivets adjacent to the column. Since none of the test specimens were loaded to their ultimate strength, the actual method of failure is not known. It is the intent of the writer to do additional testing on these specimens with SR-4 strain gauge equipment, and thus the maximum applied load was limited to that which would not exceed the yield point of the material.

The notch, which was cut in the flanges of the column, (see pg. 12), affected the results slightly and any additional specimens which might be made





$\frac{5}{16}'' \phi$  Holes  
 $\frac{7}{8}'' \phi$  Rivets

FIG. 1 DETAILS FOR TEST SPECIMEN 1.

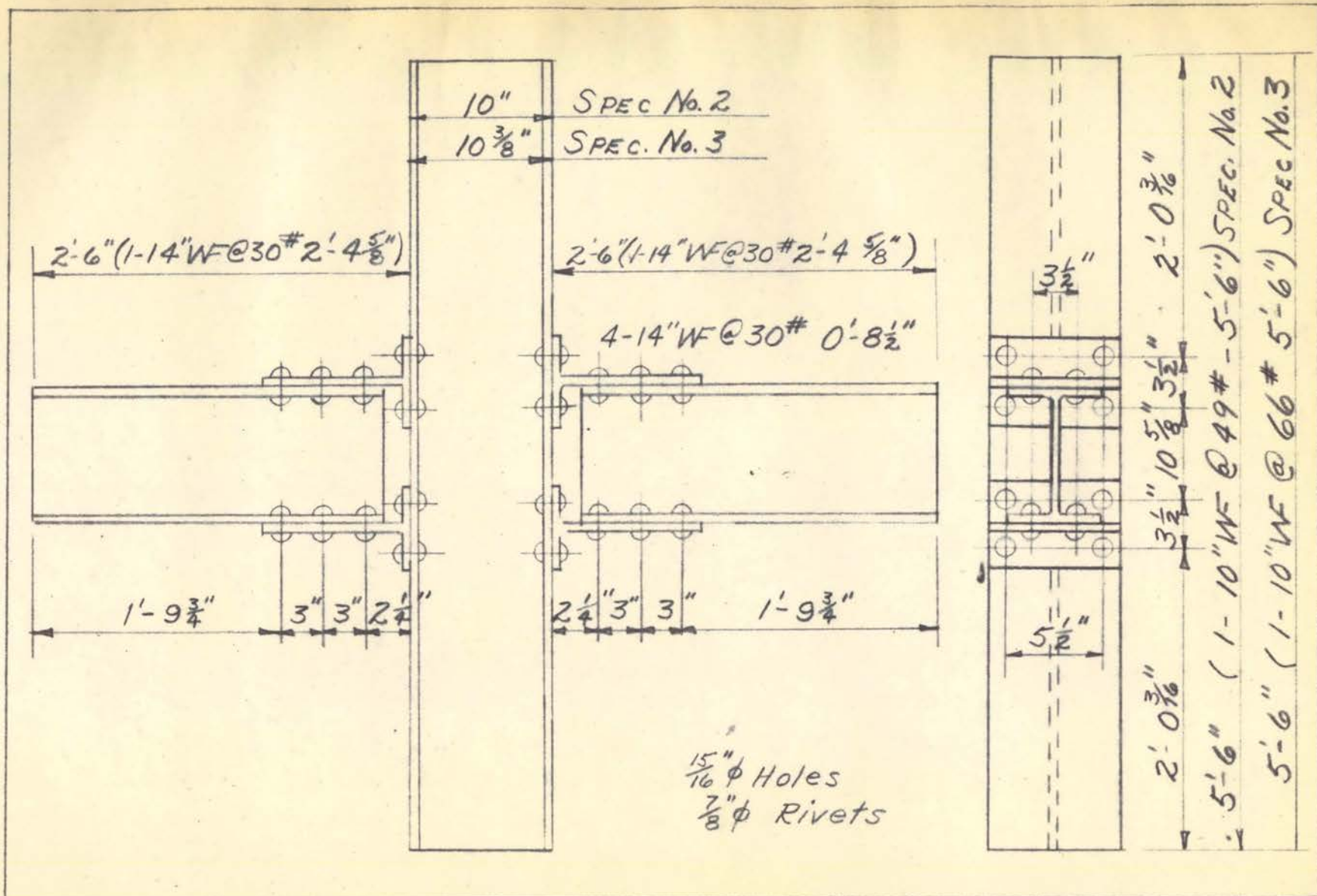


FIG. 2. DETAILS FOR TEST SPECIMENS 2 & 3.

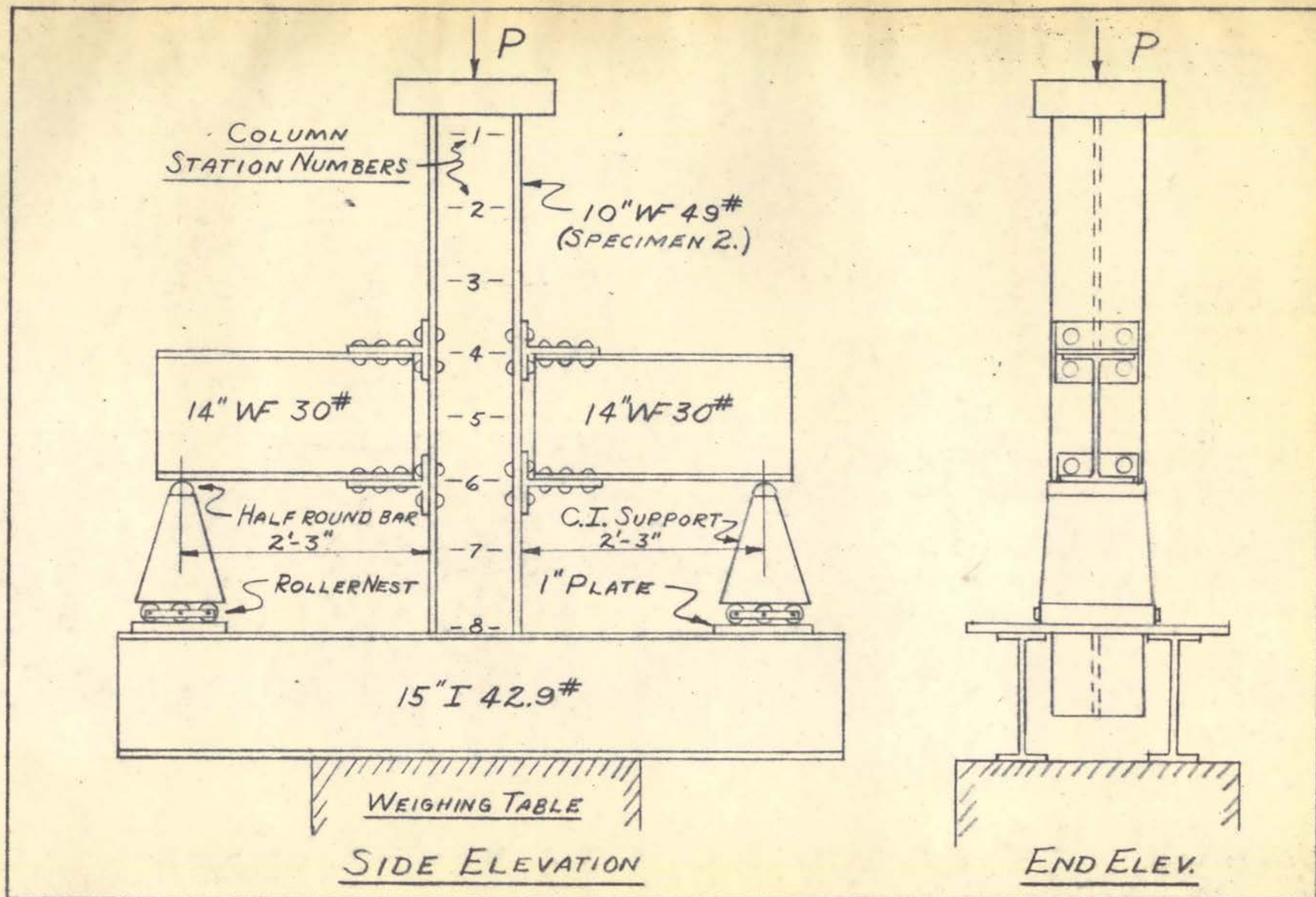


FIG. 3. GENERAL TESTING SETUP.

for testing later should have the flanges of the tees flush with the edge of the column to which they connect.

#### METHODS OF TESTING AND APPARATUS USED

The tests were conducted in the Civil Engineering Testing Laboratories of Oklahoma Agricultural and Mechanical College during the spring and summer of 1948. Professors Robert P. Witt and James V. Parcher aided the writer in designing apparatus and testing these specimens and their aid was of inestimable value.

The setup for the three different test specimens was practically identical. A 200,000 pound screw type Tinius Olsen universal testing machine was used throughout the tests. The load was applied at a rate of 0.05" per minute.

In order that the load could be transmitted to the weighing table, two 15"-42.9# I-beams were placed on the table and spaced 10½" apart as shown in Fig. 3. Then, one inch thick plates having a machine finish on the top surface spanned these I-beams at each end. A specially designed roller nest consisting of three two inch diameter rollers, twelve inches in length, connected together by a steel spacer bar on each end, was placed on the one-inch plate. The fact that the rollers were connected together aided in placing and handling them. Cast iron supports, machine finished on both top and bottom, were placed on the roller nests. Half-round steel bars which acted as fulcrums were located on top of the cast iron supports. The test specimens were located on these supports in such a manner that the distance from each face of either the plate or column to the center line of each support was 2'-3". Then, for any given load which the testing machine applied, the moment on the plane of the connecting rivets (in tension) was identical and the elastic curve of the beams the same.

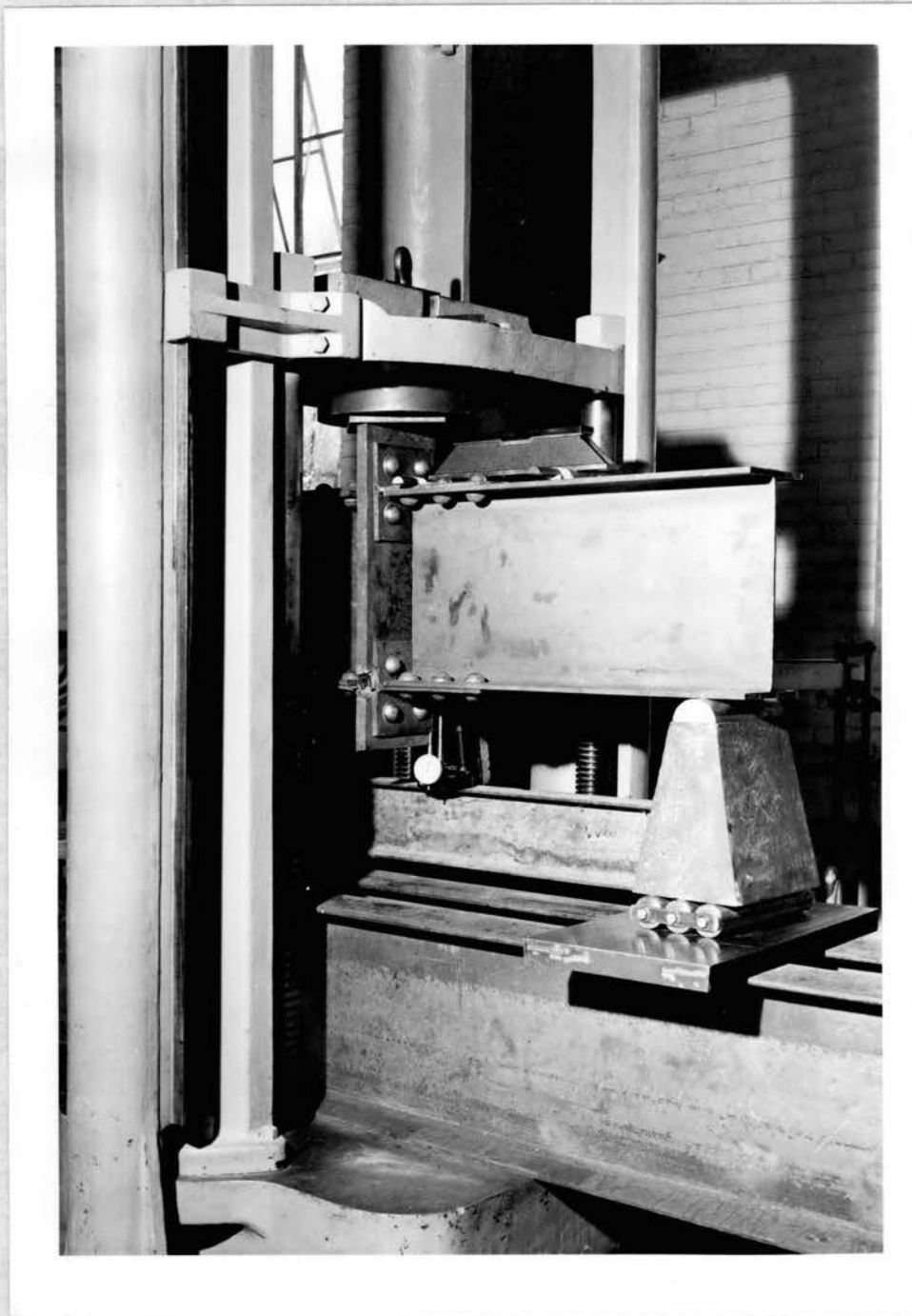


Fig. 4. View of Specimen 1 in testing machine.  
Note roller nest and cast iron support.

The apparatus used to obtain the data included a master precision Starrett Level (No. 199). When the level bubble of this instrument moved one division it represented a change in elevation of  $0.0005''$  per foot or  $0.000042$  radians. Since the observer could estimate one-tenth of a division very accurate results were obtained as is evidenced by the plotted points on the graph Fig. 13.

The precision level was located on the top of one of the  $1\frac{1}{4}''$  WF 30# beams being supported at each end by one-inch half-round bars taped in place as shown in Fig. 5. As additional load was applied to the test specimen  $0.003''$  shim stock was required to relevel the level the maximum allowable movement of the bubble being ten divisions. Any additional movement of the bubble beyond the ten divisions would have necessitated recalibration of the bubble tube, which was not deemed advisable from the standpoint of time. An observer watched the bubble at all times during the application of the load in order that the bubble would not go beyond it's limits.

An Ames dial ( $1/1000''$  divisions) was attached to a rigid steel beam, which in turn was attached at each end to the one inch thick machine finished plates. This dial was placed so that it read the deflection of the center line of the column or the center of the plate (Specimen 1). This necessitated special clips on both the columns and the plate. The data obtained and the curves plotted from these dial readings are not included in this report since a number of different sources contributed to this deflection. These sources included normal deflection of the beam (elastic curve), deflection due to shear, deflection from elastic action of the flanges of the tee sections and shearing deformation due to the web of the tee beam being deflected. The latter is covered more thoroughly on page 24.

During the testing of Specimen 1, after the maximum load had been applied,

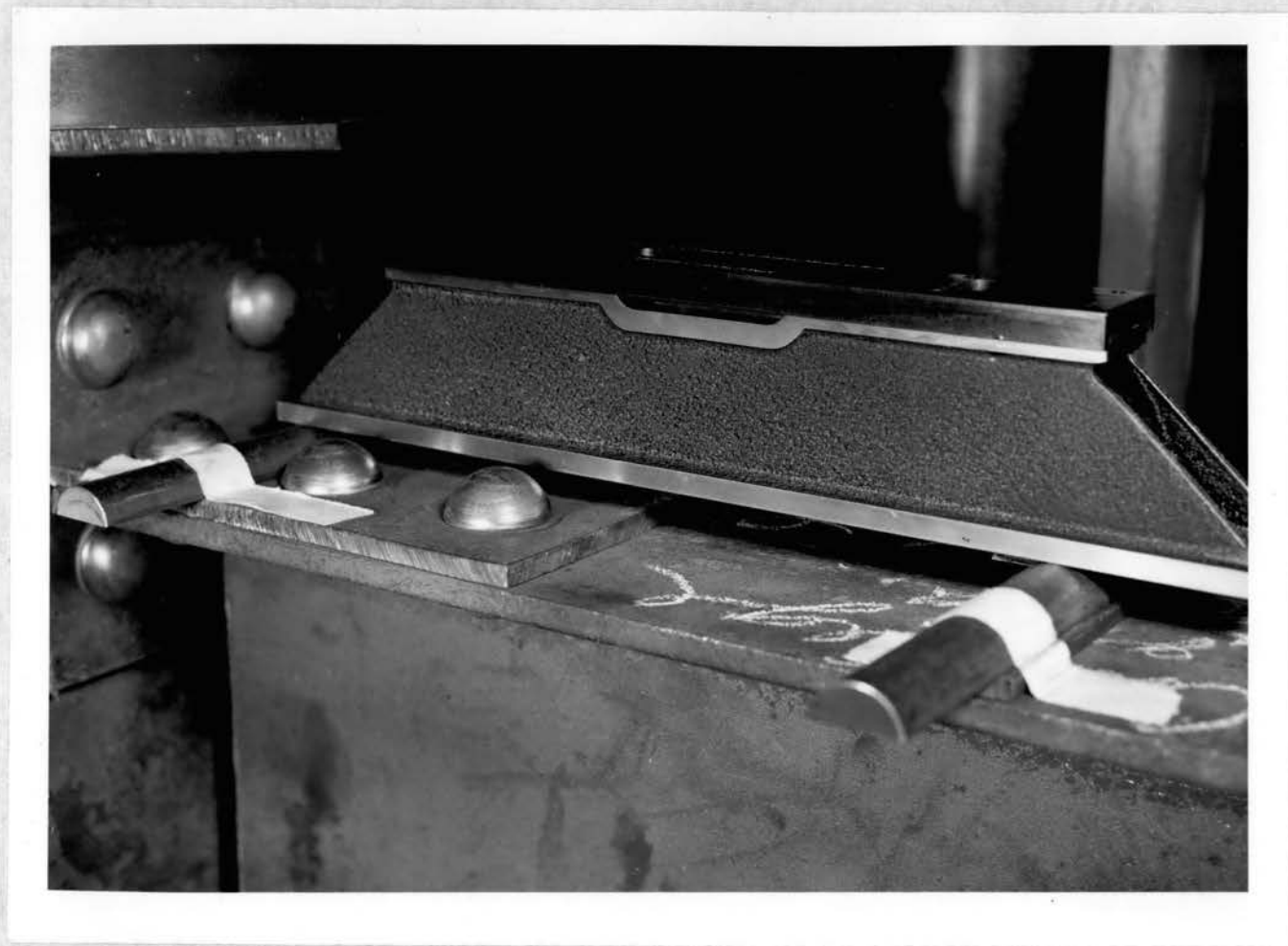


Fig. 5. Starrett precision level located in position on Specimen 1.

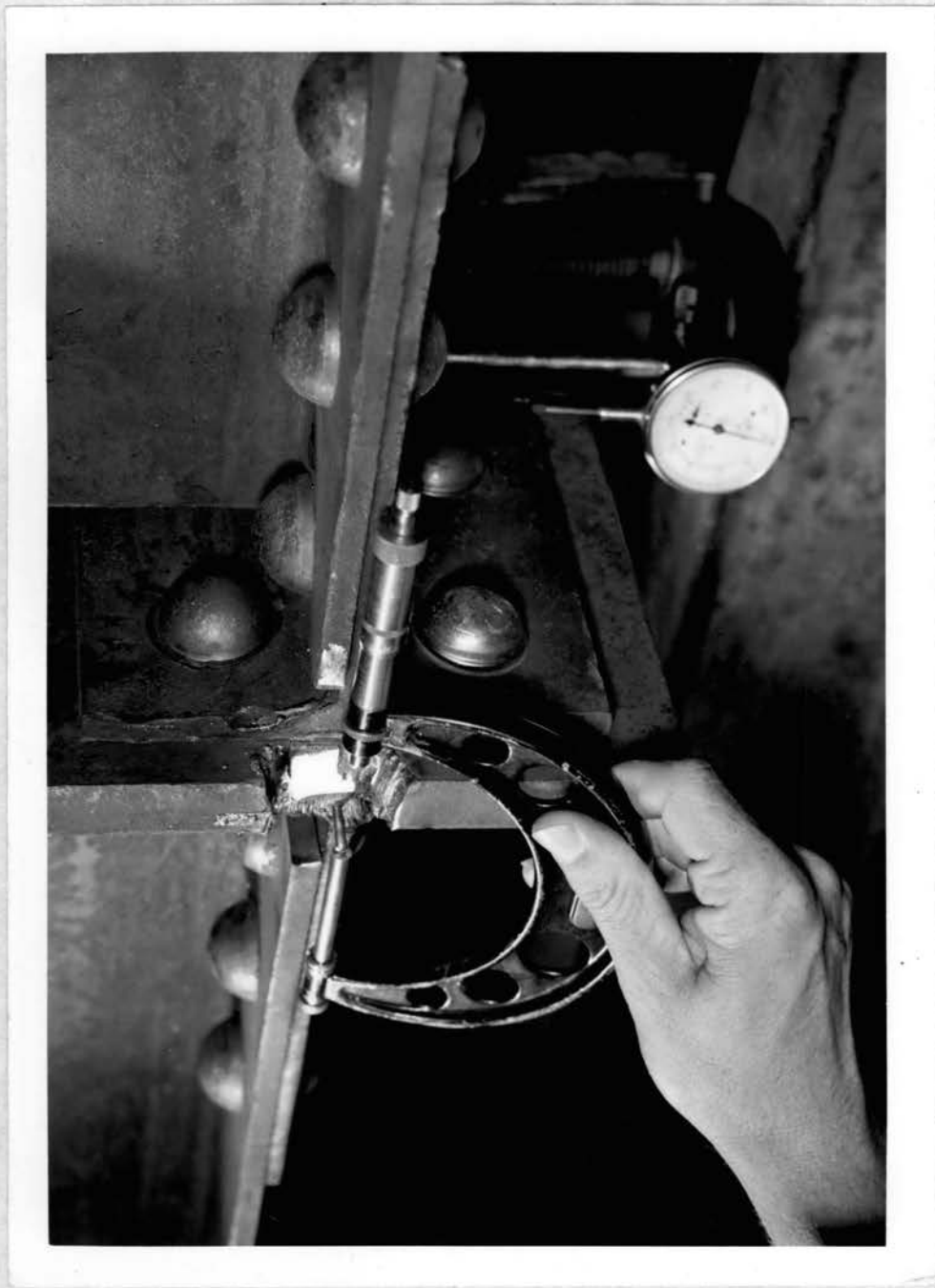


Fig. 6. Method of measuring Tee flange separation, Specimen 1.



the dial was removed and securely placed against the rollers. Then the load was released slowly and the movement of the rollers was recorded on the dial. It was assumed that the rollers on the other end moved the same amount, since the solid plate at the center of the beam was in contact with the head of the testing machine and friction alone would hold it fixed against motion.

In order that the movement of the flanges of the tee beams could be measured, notches were cut by an acetylene torch. The notch was cut so that the tee flange was flush with the depth of the notch.

For the first test on Specimen 1, gauge points were placed in the junction of the flange and web of the two tee's on the tension side of the beam. A "Last Word" strain gauge was then placed on these gauge points and readings were taken. As this gauge had limited travel, approximately one-third the amount required, it was abandoned. For the remaining tests, one-eighth inch holes were drilled in the junction of the flange and web of each tee. These holes were reamed with a tapered reamer and tapered pins installed. To find the movement of these pins, an outside micrometer was used on Specimen 1 and an inside micrometer on the second and third specimens. The outside micrometer and taper pins are shown in Fig. 6.

On Specimens 2 and 3, the column flanges were marked at approximately eight inch intervals. These marks were numbered in the manner shown in Fig. 3. Stations 4 and 6 were in the same horizontal plane as the upper and lower flanges, respectively, of the wide flange beams. Station 5 was at the midpoint. All other stations were located at eight inch intervals measured from stations 4 and 6. The distance between flanges at each station was measured with an inside micrometer after each additional load was applied. A higher degree of accuracy could have been accomplished if dial gauges had been used. None were available at the time of testing and as a result, the inside micrometer had to be employed.



Fig. 7. General setup for Specimens 2 and 3.

Before any of the tests were performed, a straight edge was used to align the roller nests relative to each other and in correct relationship to the beams. Also, all other parts were checked with a steel square to make certain that each part was square with the axes of the column and beams.

#### DESCRIPTION OF TESTS

Each specimen was loaded at the center, so that a moment of one-half the load times the distance from the support to the connection occurred at the vertical plane of the connection, and a shear of one-half the total load resulted at each connection.

As previously stated, none of the specimens were loaded to failure so that no comment can be made relative to this subject.

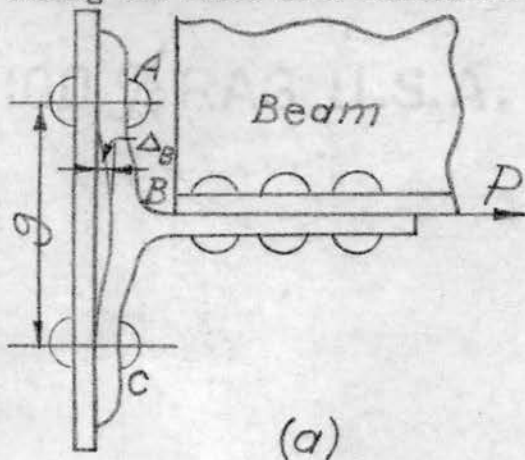
Among the other measurements which could have been made, is the measurement on the relative slip of the riveted connections. Tests have been conducted previously on this problem and, in general, show that the friction between the two surfaces due to the initial tension in all rivets is sufficiently great that the joint does not slip until after the design load for these rivets has been attained. As pointed out, this is a general statement and may not hold for every connection. Poor riveting could very easily disprove this statement.

The properties of columns compared to the rigidity of riveted joints is another factor which has been omitted entirely from this discussion. Those who have worked on problems similar to this, mention it quite frequently. No attempt has been made to cover this problem to date, since it involves so many unknowns and since there are so many combinations of columns, beams, and connections. Briefly, the problem deals with the relative stiffness of the intersecting members. It is evident that in the case of a long, slender column having an infinitely rigid beam-column connection made to it, that, when a

moment is applied to the beam the column would rotate at the connection through some angle change while this connection merely transmitted the moment and would not contribute to this angle change by virtue of its own elastic action. The tests, as conducted by the writer, did not include the stiffness of the column but rather the contribution of the column flange to the rotation of the attaching beam. In effect, the column carried only an axial load with no moment induced into it due to the beam connections.

## ROTATION COMPUTATIONS

The rotation or angle change of a riveted connection of the type tested is due partially to the elastic and inelastic deformation of the flange of the beam. The equation derived below checks the actual deflection as measured during the tests to a reasonable degree of accuracy.



It is assumed that the load  $P$  acts along the center-line of the tee web and that the rivets at  $A$  and  $C$  when driven hot have sufficient initial tension<sup>3</sup> to hold the flanges fixed.

If points  $A$  and  $C$  are equidistant from  $B$ , the rotation of the web of the tee is zero, and point  $B$  moves in a line at right angles to the plane  $AC$ , then,  $M_a = M_b = M_c$ . Then, using the second theorem of moment areas<sup>4</sup>, the deflection of point  $B$  is equal to the following:

$$\Delta_B = \frac{M_a}{EI} \times \frac{g}{4 \times 2} \times \frac{5g}{12} - \frac{M_a}{EI} \times \frac{g}{4 \times 2} \times \frac{g}{12}$$

$$\Delta_B = \frac{M_a g^2}{24EI} \quad (1)$$

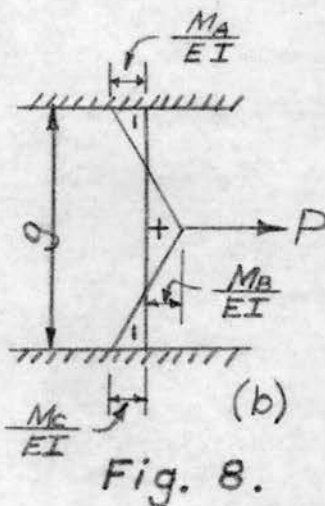


Fig. 8.

From the sketch Fig. 8b, it may be seen that the moment at  $A$  is,

$$M_a = \frac{P}{2} \times \frac{g}{4} = \frac{Pg}{8} \quad (2)$$

<sup>3</sup> C. R. Young and W. B. Dunbar, "Permissible Stresses Rivets in Tension", Bulletin 8, Sect. No. 16, School of Engineering Research, University of Toronto, 1928.

<sup>4</sup> F. B. Seely, Resistance of Materials, pp. 150-155.

When this is included in equation (1), the deflection is then,

$$\Delta_B = \frac{P_g}{8} \times \frac{g^2}{24EI} = \frac{P_g^3}{192EI} \quad (3)$$

and the rotation of the beam due to this deflection is

$$\Theta = \frac{P_g^3}{192EIh} \quad (4)$$

where  $h$  is the total depth of the beam. This rotation is measured in radians and can be converted to degrees by use of the proper conversion factor.

Now, applying equation (3) to the data received from the tests conducted, for a total load of 20,000# on Specimen 1, the load  $P$  on the tee section is

$$P = \frac{20000}{2} \times \frac{27}{14} = 19,300\#$$

the gauge length is 3.5", the modulus of elasticity is 30,000,000 and the moment of inertia is  $I = \frac{1}{12} (6.5)(.383)^3 = .03045 \text{ in.}^4$  then, the deflection

$$\text{is } \Delta_B = \frac{P_g^3}{192EI} = \frac{19300(3.5)^3}{192(3 \times 10^7)(.03045)} = .00473''$$

When the actual deflection was plotted accurately, the value of the flange deflection found by actual tests on Specimen 1 for this given load was .0045" which is in close agreement with the value just calculated. This causes an angle change of 0.000338 radians for the beam with reference to the center-line of the column.

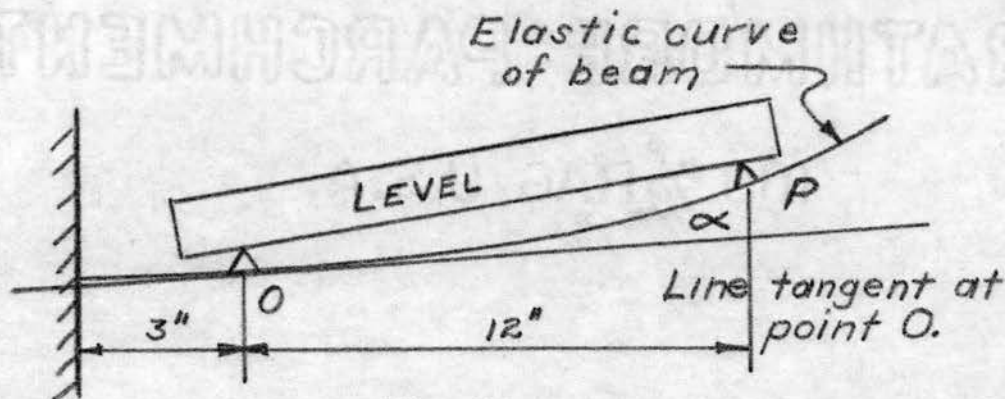


Fig. 9

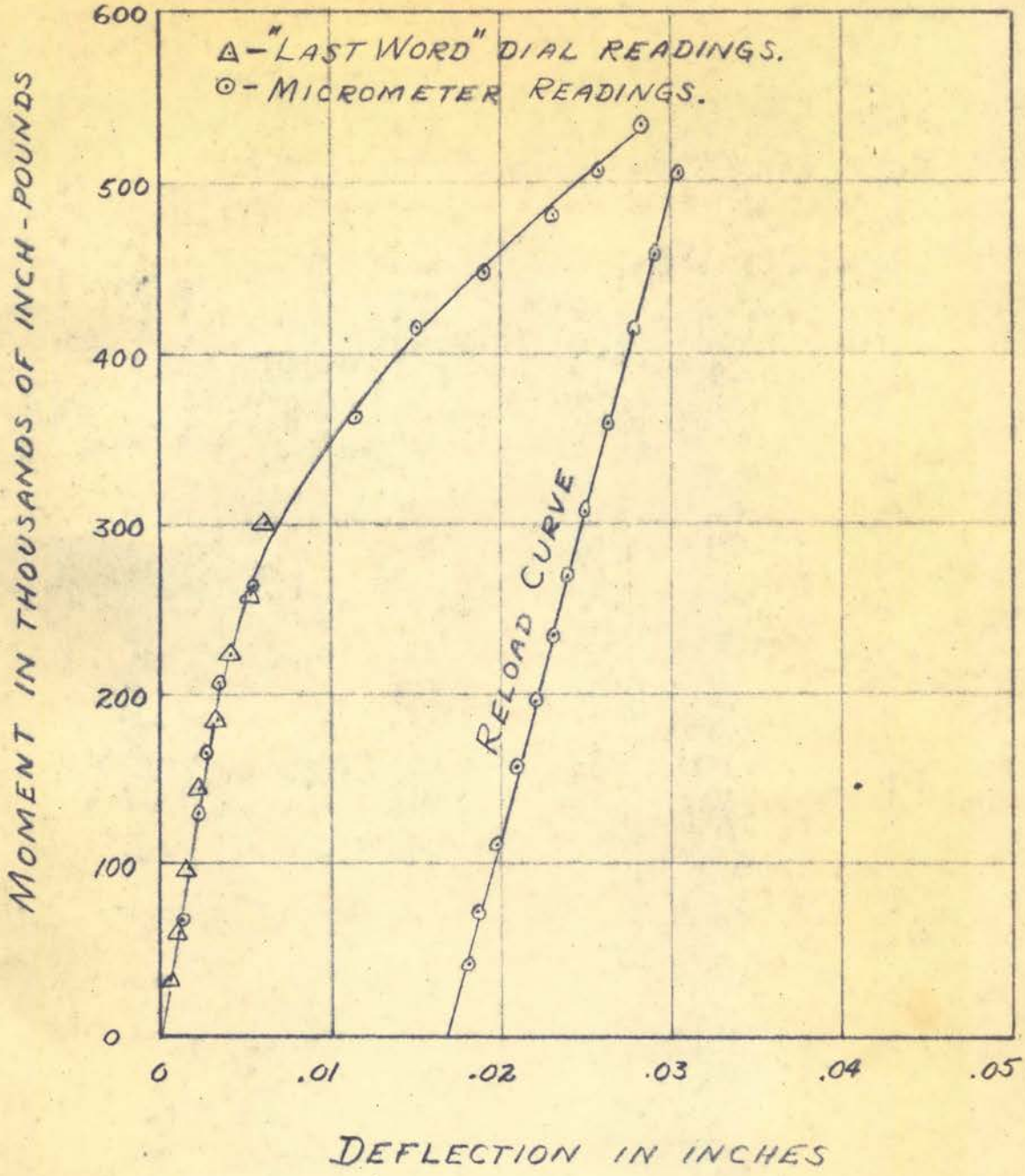


FIG. 10 DEFLECTION OF TEE FLANGE -  
SPECIMEN 1.



Fig. 11. Close up view of Tee flange for Specimen 1 showing slight crack between the plate and flange due to permanent set from a previous load.



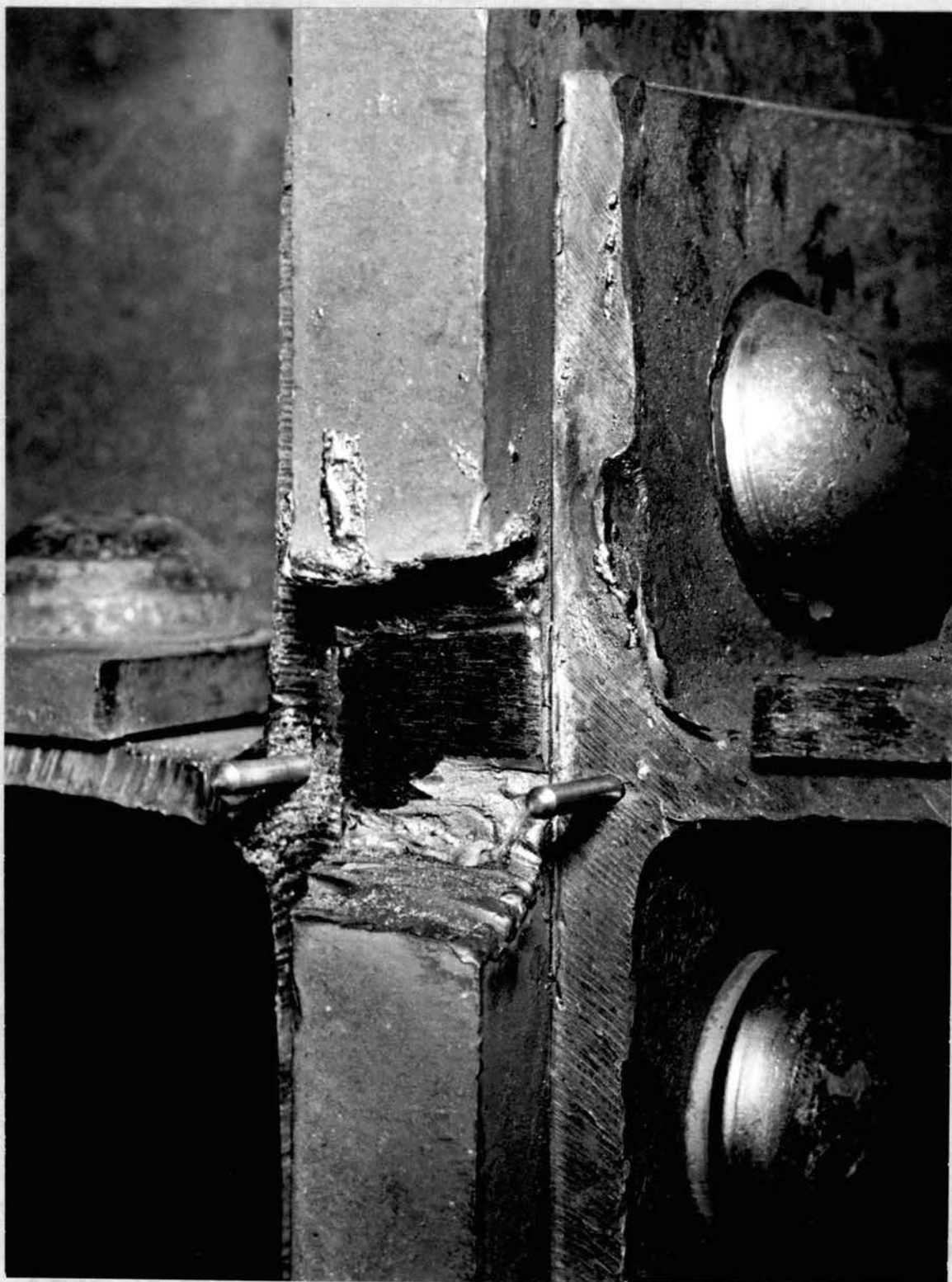


Fig. 12. Close up view of Tee flange for Specimen 1 showing crack in Fig. 11 enlarged due to 40,000# total load.

In plotting the values of the angle change against the moment in Fig. 13, the values of this angle change were reduced by the amount the elastic curve of the beam diverges from a line drawn tangent to this curve at the support adjacent to the connection, see Fig. 9.

Since the level which measured this angle change was located on supports 12" apart, the angle measured in radians is equal to that divergence divided by 12. The left and right contact points were located 3" and 15" respectively from the face of the plate or column flange as shown in Fig. 12a.

To determine the deflection of point P with respect to a tangent drawn at point O, using the moment area method<sup>4</sup>, this deflection is

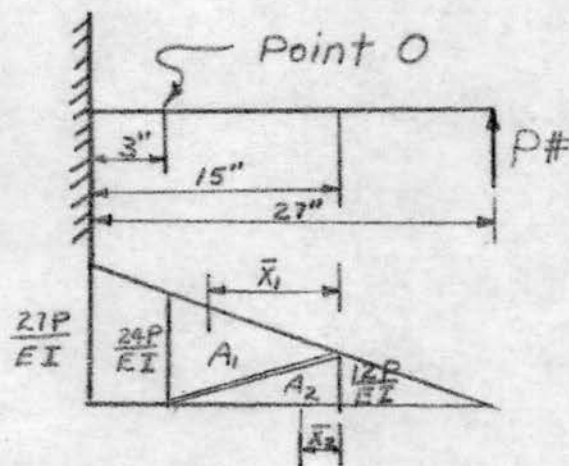


Fig. 12 a.

$$\Delta = A_1 \bar{x}_1 + A_2 \bar{x}_2$$

$$\Delta = \frac{24P}{EI} \times \frac{12}{2} \times 8 + \frac{12P}{EI} \times \frac{12}{2} \times 4$$

$$\Delta = \frac{1152P}{EI} + \frac{288P}{EI}$$

$$\Delta = \frac{1440P}{EI}$$

where,  $E = 30,000,000$  p.s.i.

$$I = 290 \text{ in.}^4$$

With the constant values of  $E$  and  $I$

given above, the deflection and rotation

can be found for any given load  $P$ . These values of rotation found by this method were subtracted from the actual values in plotting the curves on the graph Fig. 13.

The tees which attached the beams to the solid plate were  $6\frac{1}{2}$ " in width and those attaching the beams to the columns were  $8\frac{1}{2}$ " in width. So, to get a better comparison of the relative angle change, the values of the angle change used in plotting the curve for Specimen 1 on Fig. 13 were multiplied by  $6.5/8.5$  since the resistance to rotation varies inversely as the ratio of the widths of the tee. Perhaps it should be mentioned that the original curve for

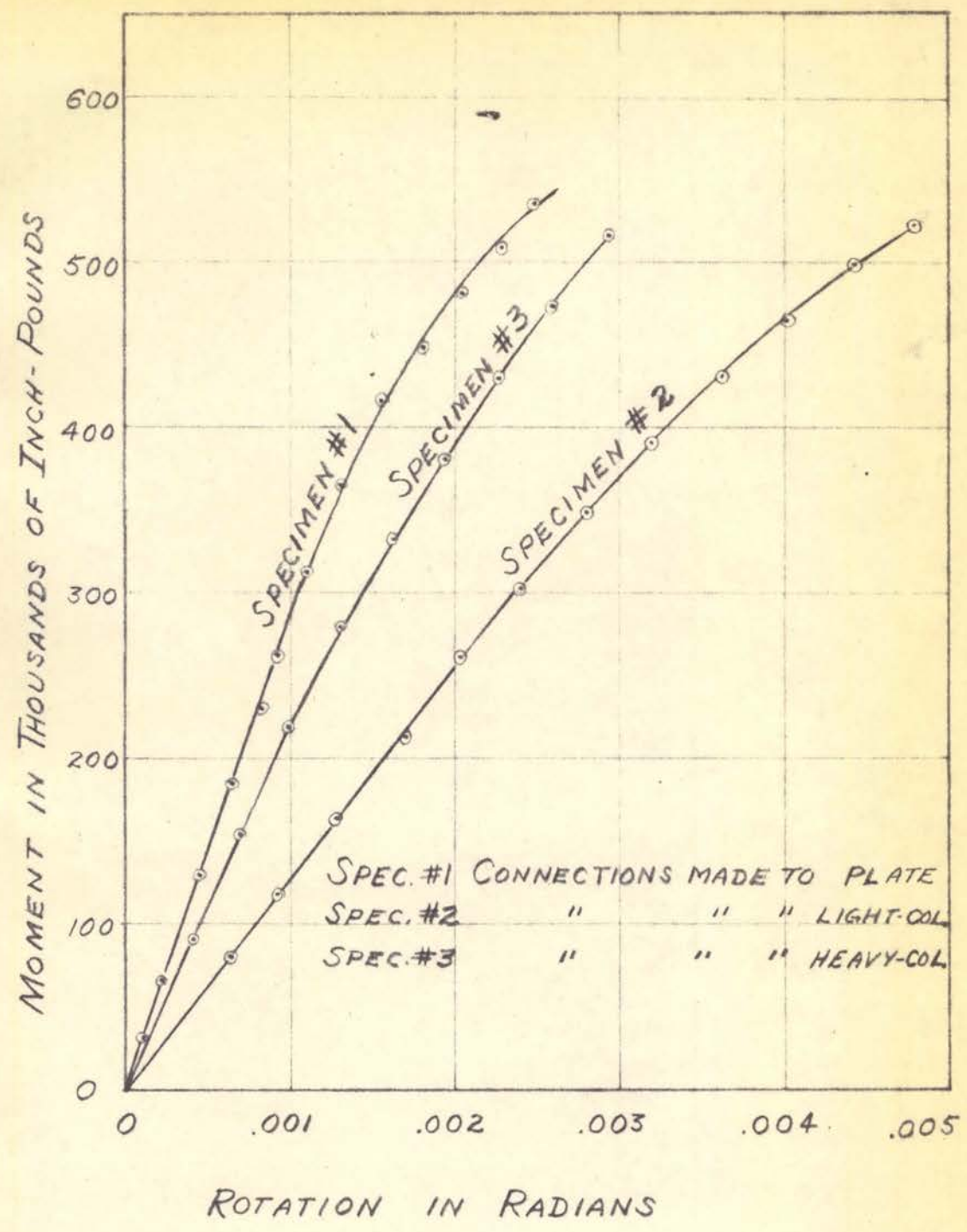


FIG. 13. COMPARATIVE ELASTICITY OF CONNECTIONS

Specimen 1 (width of tee =  $6\frac{1}{2}$ " ) fell directly upon the curve for Specimen 3 in Fig. 13.

The values obtained from any tests performed where plates are used between the connections would be of value in design for connections of beams to column webs and beams to beam webs. An enormous number of these connections are made and, in practically every case, the connection is assumed to transmit no moment. This is true for all types of connections and is not limited to the particular connection tested in the preparation of this report.

The actual rotation is made up of a combination of variables and is not the result of any particular or single deformation. Among the traceable values are: deformation of the tee flange as mentioned previously, the deformation due to shear, the deformation (due to tension or compression) of the webs of the tees, both upper and lower and the deflection of the elastic curve of the beam previously mentioned. All of these values may be calculated. Other sources which contribute to this rotation, but are not easily obtained include slip of the joint, deformation due to shearing action on the rivets, high stress concentration at the rivet holes, deformation due to shear on the tee stems etc.

The rotation for one specific load on Specimen 1 due to those traceable values was found to be only 60% of the actual rotation measured during the test (for the same load). An electric strain gauge (SR-4 type) would aid in obtaining this actual rotation, since the actual deformations and stresses could be found for any point required by the use of this equipment.

The deflection of the tee stems due to shear can be found by an approximate method. Several assumptions have to be made to get this deflection and as there is some doubt concerning the validity of these assumptions, the calculated deflection could be quite different from the actual case.

To find the deflection due to this cause, it must be assumed that both ends are fully fixed; then, the moment area method states that the area under the  $M/EI$  diagram is equal to the angle change between these two points.

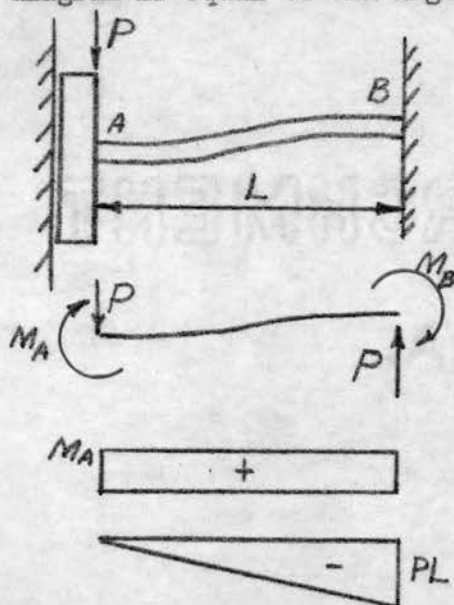


Fig. 14

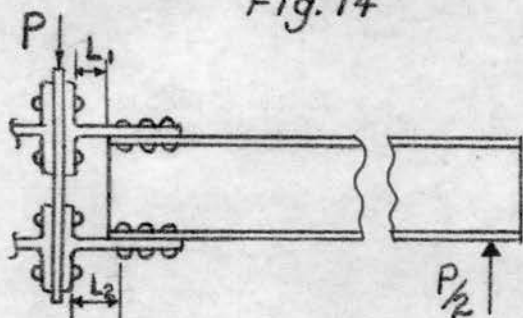


Fig. 15

There is some doubt as to the end restraint at point B, but with special strain gauges this could be determined. Since the length  $L_1$  is less than  $L_2$ , each will contribute a different amount of rotation as the result of their deformation.

Mention was made previously of the movement of the roller nests at each support. Although the value of this movement was not considered in any of the calculations, it is gratifying to know that the roller nests did move since they involved a great amount of time and expense in their production. The measured amount of movement for one roller was 0.0037". This movement took place during the removal of a 40,000# total load on Specimen 1.

Then,

$$\theta_b - \theta_a = 0 = \Sigma A$$

$$M_a L = PL \frac{L}{2}$$

$$M_a = \frac{PL}{2}$$

In like manner, the deflection is the first moment of this area.

$$\Delta = \frac{M_a L}{EI} \frac{L}{2} - \frac{PL^2}{2EI} \frac{2L}{3}$$

$$\Delta = \frac{M_a L^2}{2EI} - \frac{PL^3}{3EI} \quad \text{and from}$$

above  $M_a = \frac{P}{2}$  hence,

$$\Delta = \frac{PL^3}{4EI} - \frac{PL^3}{3EI} = -\frac{PL^3}{12EI}$$

The value of the  $L$  is different for the upper tee than for the lower tee, as shown in Fig. 15.

## INTERPRETATION OF GRAPHS

The graph on page 22 shows the relative restraints for the various specimens tested. The vertical axis in this graph represents a fully fixed beam, while the horizontal axis is equivalent to a simply supported beam. Any diagonal line between these two principal axes, passing through zero, represents a beam which is partially restrained at its support or connection. The degree of this fixity or restraint is covered in "Application".

It is noted from this graph that Specimen 1 is the nearest condition to full fixity while Specimen 3 is intermediate between Specimens 1 and 2. The reason that the beam in Specimen 2 rotated a greater amount than Specimen 3 was due to the fact that its flanges were much thinner and thus could not resist as great a moment as Specimen 3. The variation in these lines show the effect which the column flange has on the connection, since each case was identical except for the section to which the beams were fastened.

Fig. 10 is a graph showing the flange separation for Specimen 1. The tee's begin to yield at a point when the moment at the face of the plate is about 300,000"#. This specimen was reloaded after some permanent set had taken place in the tee flange. The straight line curve shown on the graph represents this reloading. The slope of the reloading curve is not parallel with the initial curve for moment values up to 300,000"#. This merely substantiates previous results of other tests performed on similar specimens.

The graphs in Fig. 16 and Fig. 17 show the relative movement of the column flanges and the deflection of the flanges and tee's combined. Since the deflection of the column flanges is shown, the deflection of the tee flanges may be found by subtracting the value of the deflection of the former from the latter. This graph indicates that the column as well as the tee's, contribute to the total deformation.

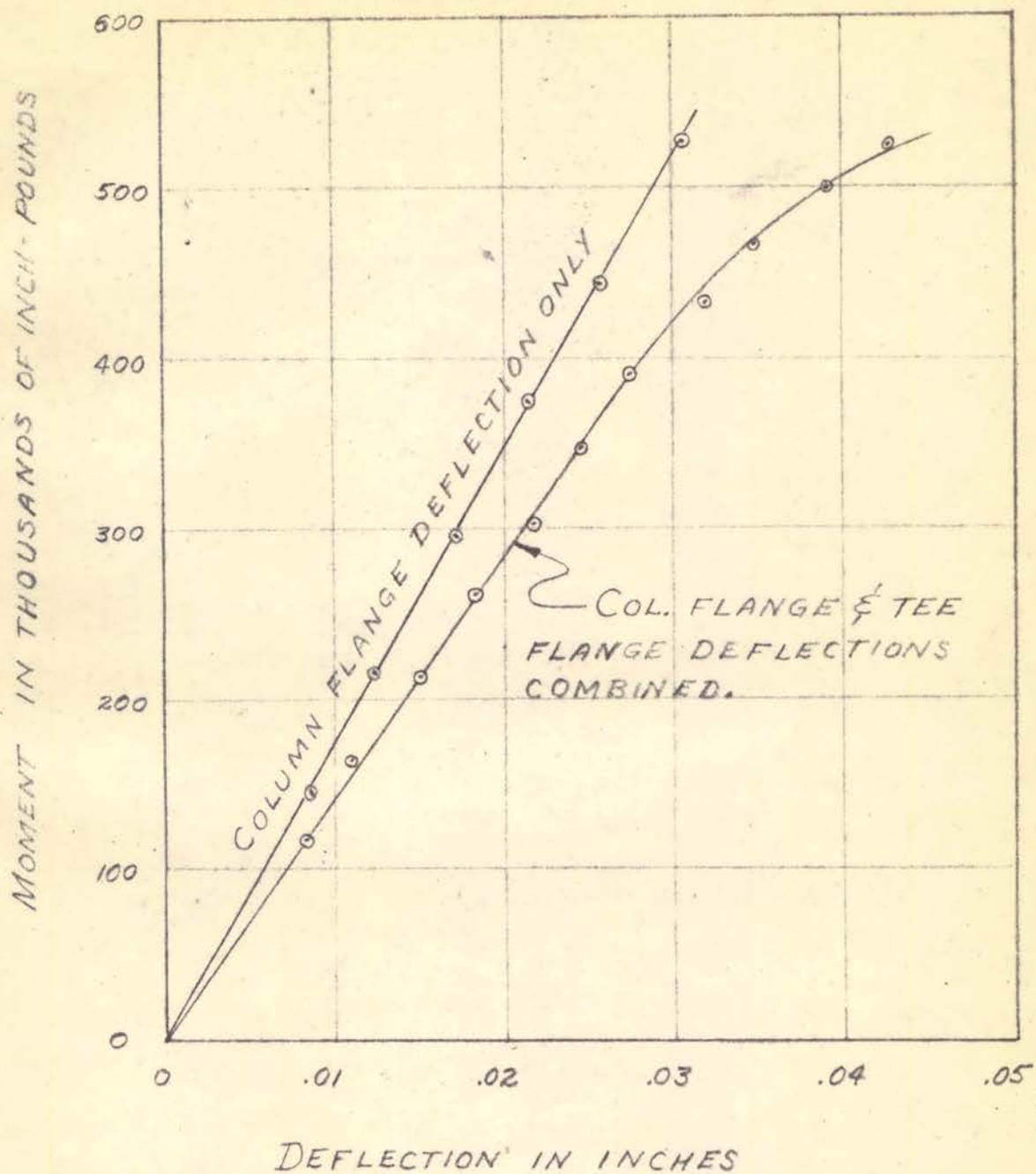


FIG. 16 RELATIVE DEFLECTIONS OF COLUMN  
AND TEE FLANGES - SPECIMEN 2.

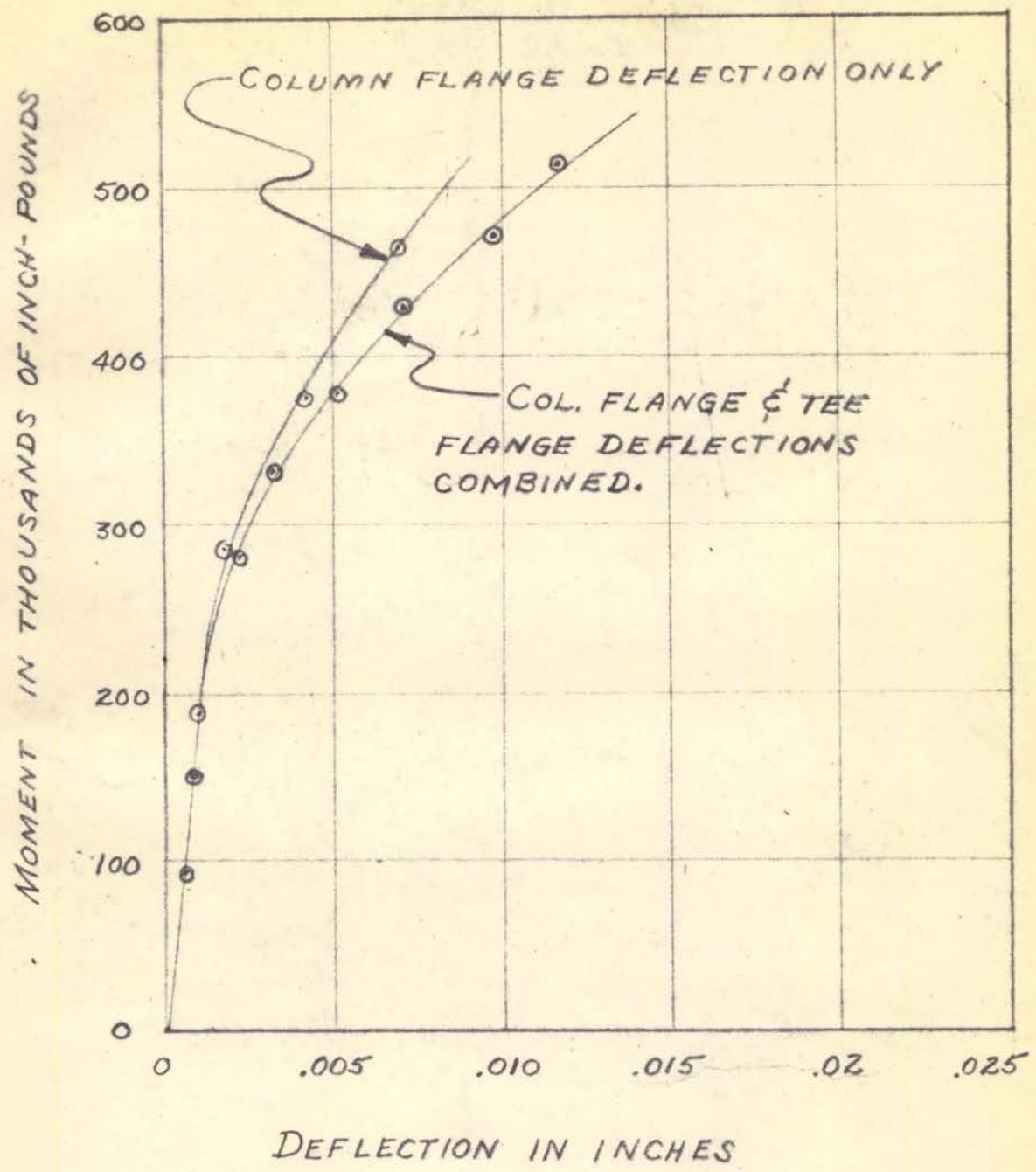


FIG. 17 RELATIVE DEFLECTIONS OF COLUMN  
AND TEE FLANGES - SPECIMEN 3.



The two curves of Specimen 3 (Fig. 17) are concurrent to the point where the moment is equal to about 220,000<sup>#</sup>, and at this point they begin to diverge. This would indicate that the tee did not deflect. Actually, this deflection was so slight that the small deviation could not be picked up by the micrometer used in taking these measurements.

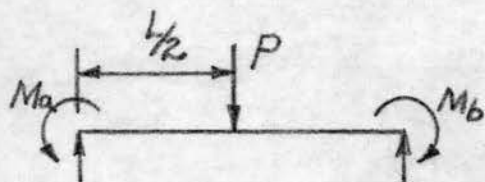
The graph which deals with the effective length of the column flange will be discussed later.

#### APPLICATION

The information gained from the tests run, regarding the amount of fixity, can be used in actual analysis of semi-rigid building frames. The slope of the lines from Fig. 13 must first be calculated. Probably the easiest method of computing this slope is to find the value of the moment when the rotation in radian measure is .001. These values for Specimens 1, 2, and 3 are  $2.85 \times 10^8$ ,  $1.25 \times 10^8$  and  $2.2 \times 10^8$  respectively.

The moment coefficient for any particular loading can be calculated in the manner which follows. A concentrated load at the center of the span will be used as the example. Fig. 18 (a) shows the example to be used. The ends are not fully fixed, but only partially restrained and it is assumed that the column is infinitely rigid so that any angle change is due entirely to the strain in the connection. The value of  $M_g$ , moment on a simple beam, would be  $PL/4$ , and the value of  $M_g$  would be  $kPl$ , where  $k$  is the moment coefficient.

By moment area, the change in slope between any two points is equal to the area under the  $M/EI$  diagram. For the example above, the slope at point A would be the area between the center-line of the beam and point A.



(a)

$$\Theta_a = \frac{1}{2} \times \frac{PL}{4EI} \times \frac{L}{2} - \frac{kPL}{EI} \times \frac{L}{2} \quad (1)$$

$$\Theta_a = \frac{PL^2}{16EI} (1-8k) \quad (2)$$

$$\text{but, } M_a = kPL \text{ or } PL = \frac{M_a}{k} \quad (3)$$

$$\Theta_a = \frac{M_a L}{16kEI} (1-8k) \quad (4)$$

$$\frac{M_a}{\Theta_a} = \frac{16kEI}{L(1-8k)} \quad (5)$$

Johnston and Mount<sup>5</sup> use the value of  $1/Z$

for  $M_a/\phi$  which is the same as  $M_a/\Theta_a$  above.

Making this substitution in the equation (5)

$$\text{we obtain, } \frac{1}{Z} = \frac{16kEI}{L(1-8k)} \quad (6)$$

$$\text{This equation may be reduced to } k = \frac{L}{8L + 16EI Z} = \frac{1}{8 + 16EKZ} \quad (7)$$

where  $Z$  is the reciprocal of the slope previously determined, and  $K$  is the stiffness ratio of the beam.

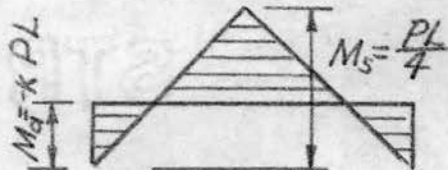
The same method could be applied to a uniformly loaded beam or any symmetrically loaded beam. Beams having unsymmetrical loadings would each be a special case, and thus will not be considered here.

The cantilever beams used in the tests simulate beams 10 feet in length partially fixed at each end. For a fixed end beam with a concentrated load at the center the moment coefficient is 0.125. Applying equation (7) the moment coefficients for Specimens 1, 2 and 3 are 0.1212, 0.1165 and 0.120 respectively.

The limiting value of the moment for any particular connection can be found by applying the method used by Prof. Batho<sup>6</sup> in his report (p.282) published in 1936.

<sup>5</sup> Bruce Johnston and Edward H. Mount, Transactions, American Society of Civil Engineers, Vol. 107 (1942), pp. 524-530.

<sup>6</sup> Batho, op. cit., p. 282.



(b)

Fig. 18

## EFFECTIVE LENGTH OF COLUMN FLANGES

The deflection of the outer edge of the column flanges is shown on the graph in Fig. 19. The lighter column flange deflected a greater amount than did the other column as could be expected. Actually the lighter column (Specimen 3) deflected inward in the vicinity of Stations 3 and 4 but is not shown on the graph since values were almost negligible.

Quite peculiar results were obtained in the region of Stations 3 and 4. The graph shows deflections of the column flange for the heavy (thick flanged) column in this area while the deflection of the light column flanges was too small to be shown on the graph. It is evident that the web between the flanges of the columns must transmit a large load and thus a small amount of deflection should be found in all cases. This would indicate that the column flanges of Specimen 2 (heavy column) were concave in shape and the stiff tee flanges pressing on the column faces deflect or straighten these flanges. In the case of the light column the flanges were parallel or convex and thus only the web deflected. This may not be the actual condition but it would seem logical to make this assumption.

The effective length of the flanges for the light column is approximately 35 inches while it is 22 inches for the heavy column. This "effective length" means that the flanges of the column contribute to the resistance of the load and thus deflection throughout this length.

It was found that the area between the vertical axis and the curve (Fig. 19) for any load divided by the maximum deflection for that load would be a value of 13.5 to 14.5 inches with the average being 14 inches. In other words, this means that if a section of the column flange 14 inches in length supports a concentrated load uniformly distributed along a line coincident with the rivet line and equivalent to the total load applied to that flange the

COLUMN STATION NUMBER

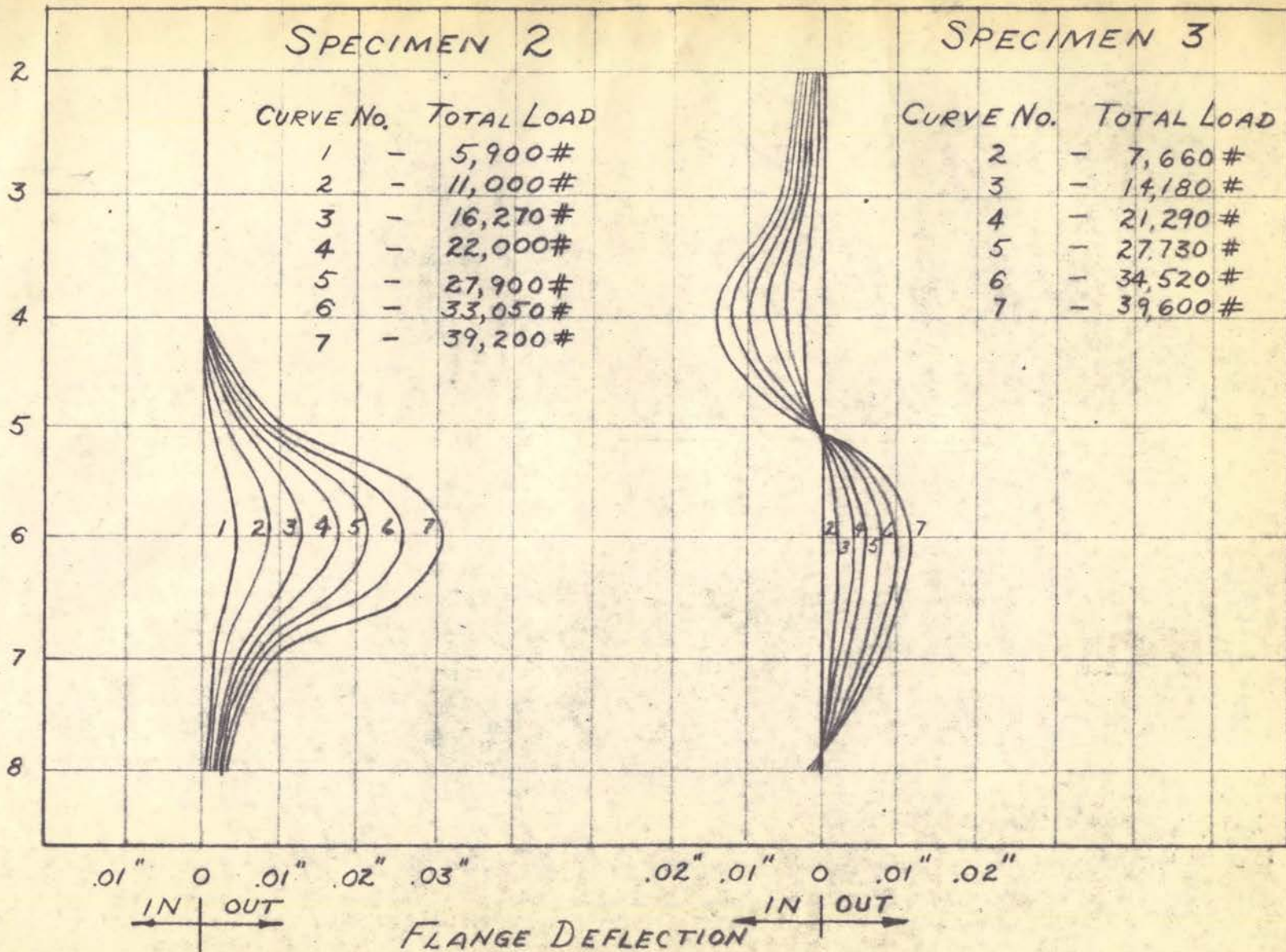


FIG. 19. FLANGE DEFLECTION FOR VARIABLE LOADS

deflection all along the 14 inch length would be the same as the maximum value of the deflection recorded on the graph for that particular load. This was found true for both the light and heavy columns.

An attempt was made to derive the equations so as to predict the deflection of the flanges. The values obtained from the equations derived were found to be in error with the test results. At this time, only an empirical equation could be employed with little or no basis for its use on any other column. As a result, this has been omitted from the report.

#### GENERAL DISCUSSION

The specimens tested were actually inverted with their normal position in the building. The tests were performed in this manner so as to simplify the setup.

It has been pointed out in the discussion of other problems similiar in nature that when the connections are encased in concrete and the partitions and walls add to the stiffness and rigidity of the building frame, the problem is no longer the same. This is quite true, but the problem is of an academic nature and should be viewed in that manner.

The connection in this test is not designed as a shear attachment for the reason that only the rotation due to the yielding of the tee flanges and their component parts was to be measured. Any connection placed on the beam to carry the shear would have increased the rigidity of the joint and a true value for the proportion of the rotation contributed by the tee could not have been found. The connection was found to transmit the shearing load suprisingly well. Others conducting similiar tests have also found this to be true.

Some thought should be given to the values for moment coefficients found. These were based on the original curve and not the reload curve. The beam

once loaded will from then on act on the reload curve values. Perhaps the values taken from the reload curve would be more indicative of the actual conditions than the original values.

No attempt was made to invert the beams which in effect would reverse the moment. This condition would arise in a building frame which first has wind forces in one direction and then in the opposite direction.

#### CONCLUSIONS

The conclusions drawn from the tests performed are based on three specimens and, it should be pointed out that other columns, other tee connections and other rivet sizes could each affect the results in a slightly different manner. As a result, the conclusions made for this particular set of specimens may not apply to any other case.

The method of finding the change in slope was not too well designed, and the values obtained are considered to be only relative. Additional equipment in the form of strain gauges, both electric and mechanical type, would be of great value. Additional readings should have been made on the opposite side of the beam and column, and an average value recorded, but due to insufficient equipment this was not done.

If the rotation of the connection riveted to the solid plate is represented by  $\Theta_1$  then the rotation of Specimen 2 is  $2.25\Theta_1$  and that of Specimen 3 is  $1.25\Theta_1$ . Although the value of  $2.25\Theta_1$  seems rather large, it is based upon a quite rigid connection. When comparing the moment coefficients for the various cases tested, they were found to be 93% to 97% of the value for a fully fixed beam. These are the values on which a design is based, and thus, even the 93% effective joint will give economy in design as compared to the freely supported ends which are usually assumed.

The inadequacy of the proper equipment made it impossible to predict the deflection of a column flange. It was found that the effective length of the column flange varies between two and three feet. This indicates that column flanges in buildings are affected only in local areas of connections, and not affected throughout the full story height. Although the deflection of the column flanges forms a sine curve at the edge of the flange, it converges quite rapidly and the effective length is rather small.

It seems regrettable that a country such as ours whose supply of steel is diminishing at a rapid rate should annually continue to design structural steel worth millions of dollars, with little regard to conservation, when only a small portion of the savings would promote sufficient research to repay the industry many, many times.

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