SOME ASPECTS OF SINGLE PHASE INDUCTION MOTOR THEORY

By

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PREFACE

For more than 50 years the two distinct theories of single-phase motor operation, the cross-field theory and the double-revolving field theory, have been used to explain the characteristics of these motors. Neither is entirely satisfactory either in its calculation of characteristics or in the physical conception of the motor's operation. The complexity and actual mystery involved have defied through this half century efforts to simplify and correlate completely the theories now extant. This paper is designed, not to perform this task, nor to develop a new concept, but rather to bring together a few ideas which have been helpful in the work which was carried on in the belief that there can be a more satisfactory explanation of single-phase motor operation.

ACKNOWLEDGMENT

The writer wishes to express his sincere appreciation to Professor C. F. Cameron for his encouragement, his reading of this material and his many helpful suggestions concerning it.

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CHAPTER I

The analysis of single-phase motor operation based on the idea that a single pulsating force can be represented by two forces rotating in opposite directions dates back to Ferraris. For more than fifty years the analogy has been used in explaining and computing the performance of single-phase motors.

THE DOUBLE REVOLVING FIELD

The concept is simple to explain but can become quite complex when applied to the motor. Figure 1 shows two vectors equal in length rotating in opposite directions at the same speed with a common center of rotation. The sum of these two vectors at every instant is a vector lying along the vertical axis. When the vectors cross at their uppermost position their resultant is a vector in the same plane with length double that of the rotating vectors. As they continue to rotate vector addition will result in nullification of their horizontal components and addition of their vertical components. As they reach

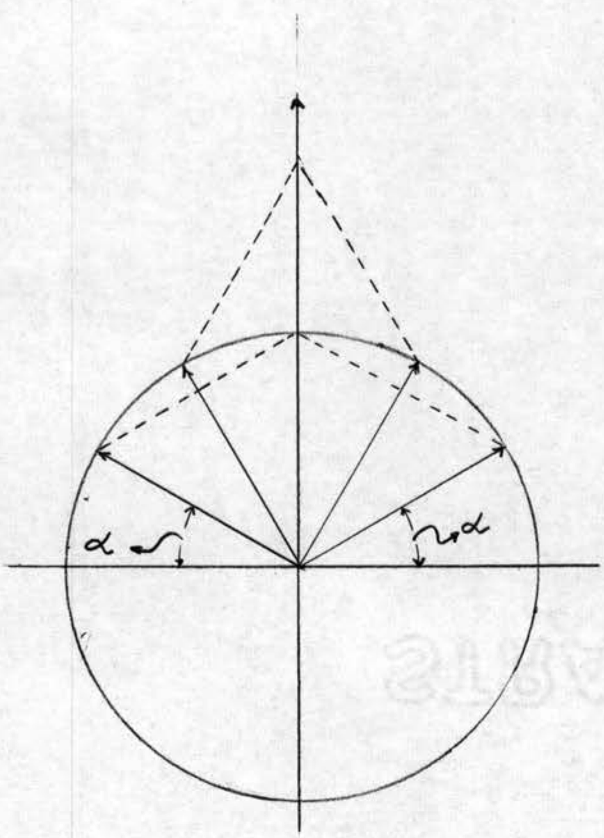

Figure 1

the horizontal position their vector sum is zero; and at their nadir, again they combine to make a vector of double their length.

One of these vectors, as it rotates, provides a component (half of the sum) equal to its length times the sine of the angle, α, it makes with the horizontal. Then at any instant the vector resultant equals two times length of one vector times sine α. When α is 90° and the sine is one the resultant is two times the length of one vector. When α is 180° the resultant is zero. Thus there can be developed by this simple device, an equivalence between these two revolving vectors and a single vector lying along a line and varying sinusoidally in length from a maximum of positive twice length to a minimum of negative twice length.

It was this analogy that caused the early theorists[1] to apply the double-revolving field concept to the single-phase motor. The smoothly gliding flux field of the polyphase motor was well understood. Its effect could be measured and its characteristics predicted. On the other hand, the flux field of the single-phase motor was more difficult. It did not glide around the stator but pulsed along the axis of the stator field, varying sinusoidally with time. So it is easy to see that an opportunity to analyze this machine on the basis of two smoothly rotating fields, each very much like the field of the polyphase

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[1] Galileo Ferraris, in 1893, published his theory of single-phase motors based on the double-revolving field.

machine, was a welcome one.

However, the theory has not been without its critics. The lack of rigor in the analysis was one very evident weakness. Some mathematical support for the idea came from Euler's relation:

Expansion by Maclaurin's Series can be expressed by the formula,

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^n(0)}{n!}x^n + \ldots \text{ etc.} \tag{1}$$

Where $f(x)$ equals function of x

$f(0)$ equals function of x evaluated at x equals 0

$f'(0)$ equals first derivative of $f(x)$ evaluated at x equals 0

$f''(0)$ equals second derivative of $f(x)$ evaluated at x equals 0

By use of this series cos x can be expanded:

$$\text{Cos } x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots \text{ etc.} \tag{2}$$

By the same method e^{jx} and e^{-jx} can be expanded,

$$e^{jx} = 1 + jx - \frac{x^2}{2!} - j\frac{x^3}{3!} + \frac{x^4}{4!} + j\frac{x^5}{5!} \ldots \text{ etc.} \tag{3}$$

$$e^{-jx} = 1 - jx - \frac{x^2}{2!} + j\frac{x^3}{3!} + \frac{x^4}{4!} - j\frac{x^5}{5!} \ldots \text{ etc.} \tag{4}$$

Adding (3) and (4)

$$e^{jx} + e^{-jx} = 1 + 1 - \frac{x^2}{2!} - \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^4}{4!} \ldots \tag{5}$$

$$e^{jx} + e^{-jx} = 2\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} \ldots\right) \tag{6}$$

The sum in parentheses is equal to the cos x as expanded in (2) above, So

$$\cos x = \frac{e^{jx}}{2} + \frac{e^{-jx}}{2} \tag{7}$$

The main field flux can be expressed,

$$\phi = \phi_m \cos \omega t \tag{8}$$

Where ϕ equals instantaneous value of flux

ϕ_m equals maximum value of flux

t equals time

ω equals 2π frequency

Using the development in (7) above,

$$\phi = \phi_m\left(\frac{e^{jx}}{2} + \frac{e^{-jx}}{2}\right) \tag{9}$$

Since x is an angle the operator e^{jx} indicates rotation of a unit vector in the clockwise direction and, conversely, e^{-jx} means rotation of a unit vector in a counterclockwise direction. So,

$$\phi = \frac{\phi_m}{2} e^{jx} + \frac{\phi_m}{2} e^{-jx} \tag{10}$$

This could be interpreted to represent two oppositely rotating vectors of length $\frac{\phi_m}{2}$ combining at every instant to produce ϕ, the sinusoidally varying uniaxial main flux of the one-phase motor. This, then, offered some degree of mathematical basis for the illustration which opened this discussion.

It did not answer all criticism, however. The operators e^{jx} and e^{-jx} could represent a rotation in time or in space but could not represent both. And argue some[2], the rotating fluxes must have both time and space relationships made clear.

If the flux is sinusoidally distributed in space as well as varying sinusoidally in time then an additional factor must be added to the expression $\phi = \phi_m \cos \omega t$ to account for the space distribution. If the wavelength of the space wave is 2T then $\sin \frac{x}{T}\pi$ is the new factor and,

$$\phi = \phi_m \cos \omega t \sin \frac{x}{T}\pi \tag{11}$$

By trigonometry,

$$\phi = \frac{\phi_m}{2} \left[\sin\left(\frac{x}{T}\pi - \omega t\right) + \sin\left(\frac{x}{T}\pi + \omega t\right) \right] \tag{12}$$

To examine the expression $\sin\left(\frac{\pi}{T}x - \omega t\right)$, one must first determine the form of

$$f(x) = \sin(x - \omega t) \tag{13}$$

This function, of course, can be plotted for various values of t. At times when t equals 0, and $t = \frac{\pi}{2\omega}$, it can be shown thus:

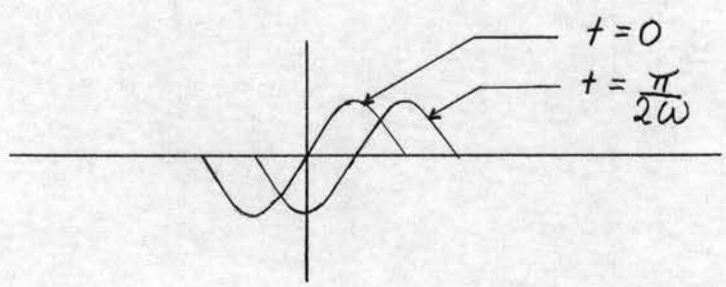

Figure 2

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[2] L. V. Bewley, Alternating Current Machinery, p. 194.

Thus, the function proves to be a traveling sine wave moving in the positive direction. $f(x) = \sin(x + \omega t)$ is a sine wave in space traveling in the negative direction. The expression,

$$\phi = \frac{\phi_m}{2}\left[\sin\left(\frac{\pi}{T}x - \omega t\right) + \sin\left(\frac{\pi}{T}x + \omega t\right)\right] \tag{14}$$

represents a forward rotating wave plus a backward rotating wave. Both space and time distribution have been taken into consideration in this analysis.[3]

ARMATURE REACTION

Another way to begin the analysis of single-phase motor operation is with a review of the well known principles that explain armature reaction in the single-phase alternator.

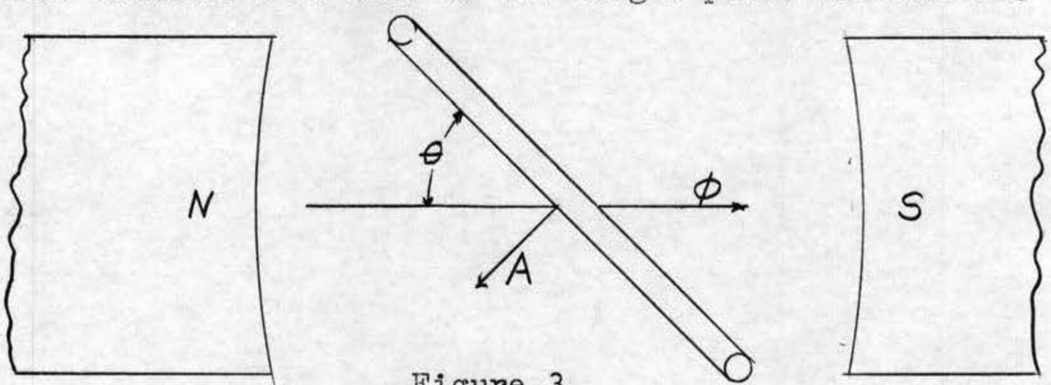

Figure 3

In Figure 3, as the coil rotates it cuts the main field flux, ϕ. The voltage generated causes a current to flow through the coil and load reaching its maximum at the instant shown. This means that a lagging power factor load of angle θ exists. The flux resulting from this flow of current would be along A, perpendicular to the plane of the coil. This, of course, is

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[3] Ibid., p. 195.

armature reaction described in the textbooks[4], which with lagging power-factor, reduces the mmf of the main field.

The successive positions of this single coil as it rotates can be shown in this way, again assuming a low lagging power factor load:

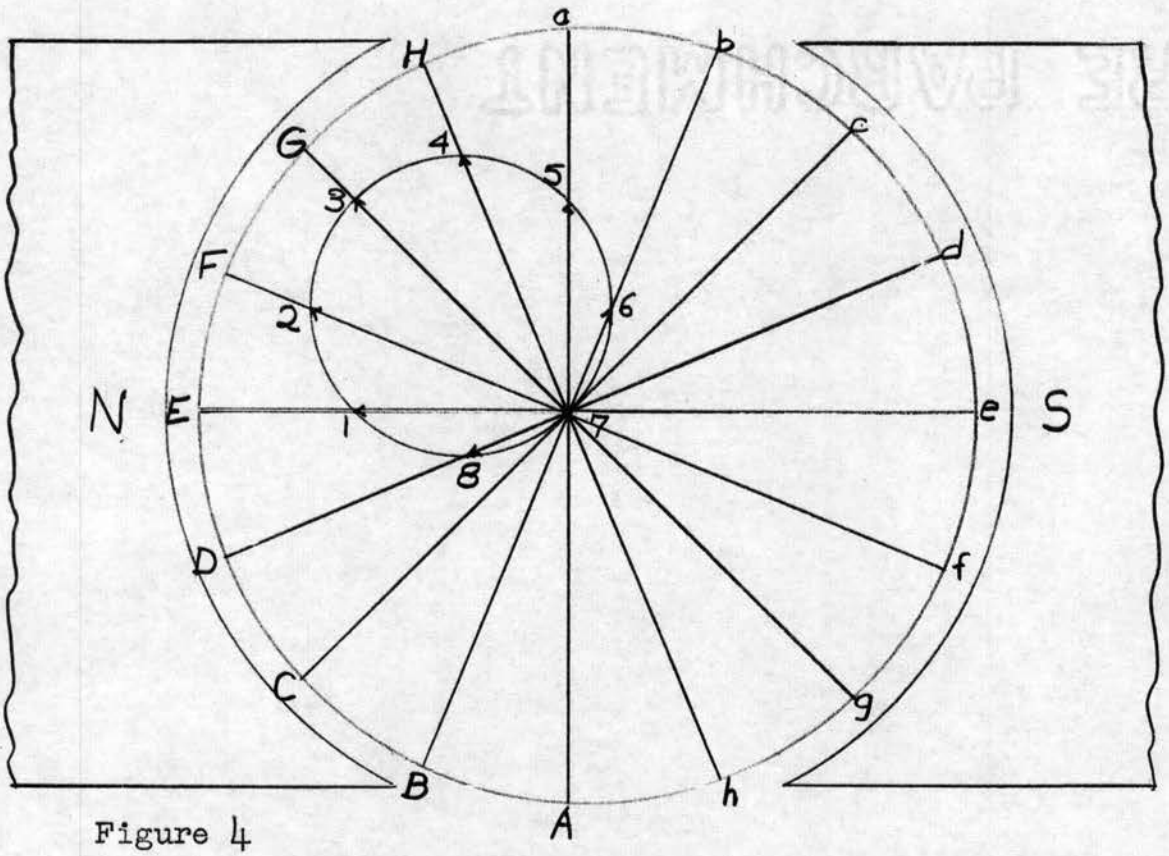

Figure 4

Referring to Figure 4, since position Aa produced flux vector 1, Bb vector 2, Cc vector 3------and Hh vector 8 the coils have rotated half a revolution. To complete the revolution of the coils would simply duplicate the eight vectors and their resultant circle in the same position. The armature flux, then, is double frequency de-magnetizing and cross-magnetizing

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4 Frederick W. Hehre and George T. Harness, Electrical Circuits and Machinery, Vol. II Alternating Currents, p. 315.

with respect to the main field flux.

Relative to the rotating coil (the armature), however, the armature reaction is a vector, constant in position varying sinusoidally with time. In this respect the condition is identical to that existing in a single-phase stator of a motor.

Some texts[5] treat alternator armature reaction by use of the double revolving theory. The reaction vector is replaced by two oppositely rotating vectors of half the length of the reaction vector. One is constant in position with respect to the poles and changes their strength. The other rotates at twice synchronous speed relative to the poles and in the opposite direction, generating fundamental and third harmonic voltages in the alternator.

The obvious difference between these two machines is that the poles of the alternator are not the same as the rotor of the motor. The closed coils of the rotor, providing circuits for flow of the currents due to the voltages generated in the coils by the two oppositely rotating fluxes offer a more complex problem than that of the armature reaction in a single-phase alternator. For example, if the short circuited secondary (stationary here as were the poles for the alternator illustration) be excited by the same coil shown as alternator armature except now the a.c. supply is provided by an outside source, this means the same double frequency flux pattern will be established. The flux threading a turn formed by any pair of

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[5] Ralph R. Lawrence, Principles of Alternating-Current Machinery, p. 58.

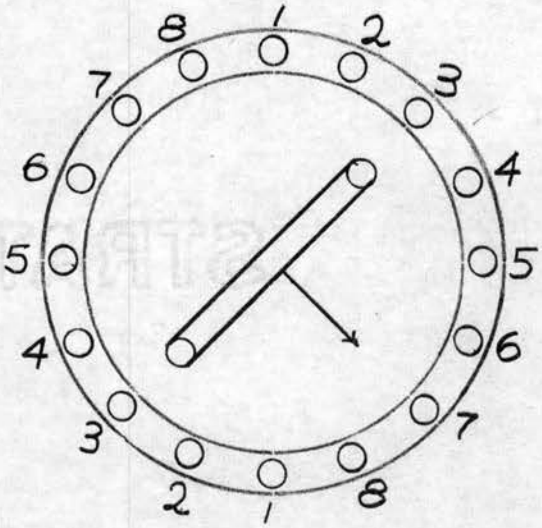

Figure 5

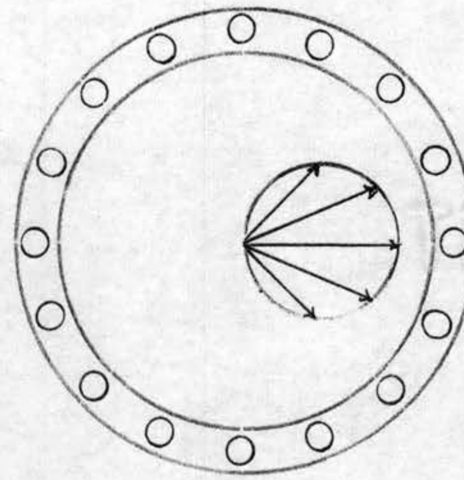

Figure 6

conductors (1-1, 2-2, or 3-3, etc.) would be at any instant[6],

$$\phi = \phi_m \sin \omega t \cos \theta \tag{15}$$

Where ϕ_m = maximum value of the coil flux.

$\omega = 2\pi f$

θ = angle measured between the turn and the flux so that when $\theta = 0$, the maximum flux threads.

But since θ is also a function of time, the equation can be written.

$$\phi = \phi_m \sin \omega t \cos \left[1 - s\, \omega t + a \right] \tag{16}$$

Where s = slip in percent

a = initial angle between coil and turn when flux is 0.

By trigonometry,

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6 C. T. Button, "Single-Phase Motor Theory - A Correlation of the Cross-Field and Revolving Field Concepts," A.I.E.E. Transactions, Vol 60 (1941), p. 507.

$$\phi = \phi_m\left[\frac{1}{2}\sin\left(\omega t - \left\{(1 - s)\omega t + a\right\}\right) + \frac{1}{2}\left(\omega t + \left\{(1 - s)\omega t + a\right\}\right)\right] \tag{17}$$

$$\phi = \frac{\phi_m}{2}\left[\sin\left(s\omega t - a\right) + \sin\left(\left\{2 - s\right\}\omega t + a\right)\right] \tag{18}$$

The flux of conductors Aa, Gg, and Ee can be calculated by substitution in equation 18. For turn Aa, $a = 0^\circ$ and let slip = 0.

$$\phi_{Aa} = \frac{\phi_m}{2}\sin 2\omega t \tag{19}$$

For Gg, $a = 45^\circ$, $s = 0$

$$\phi_{Gg} = \frac{\phi_m}{2}\left[\sin(-45^\circ) + \sin(2\omega t + 45)\right] \tag{20}$$

For coil Ee, $a = 90^\circ$, $s = 0$

$$\phi_{Ee} = \frac{\phi_m}{2}\left[\sin(-90^\circ) + \sin(2\omega t + 90)\right] \tag{21}$$

The motor can be shown this way:

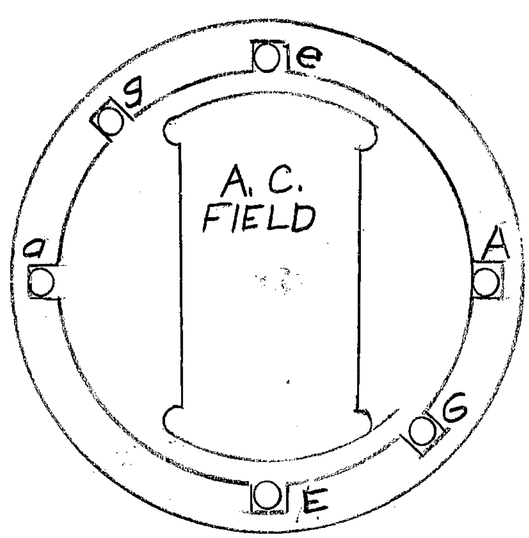
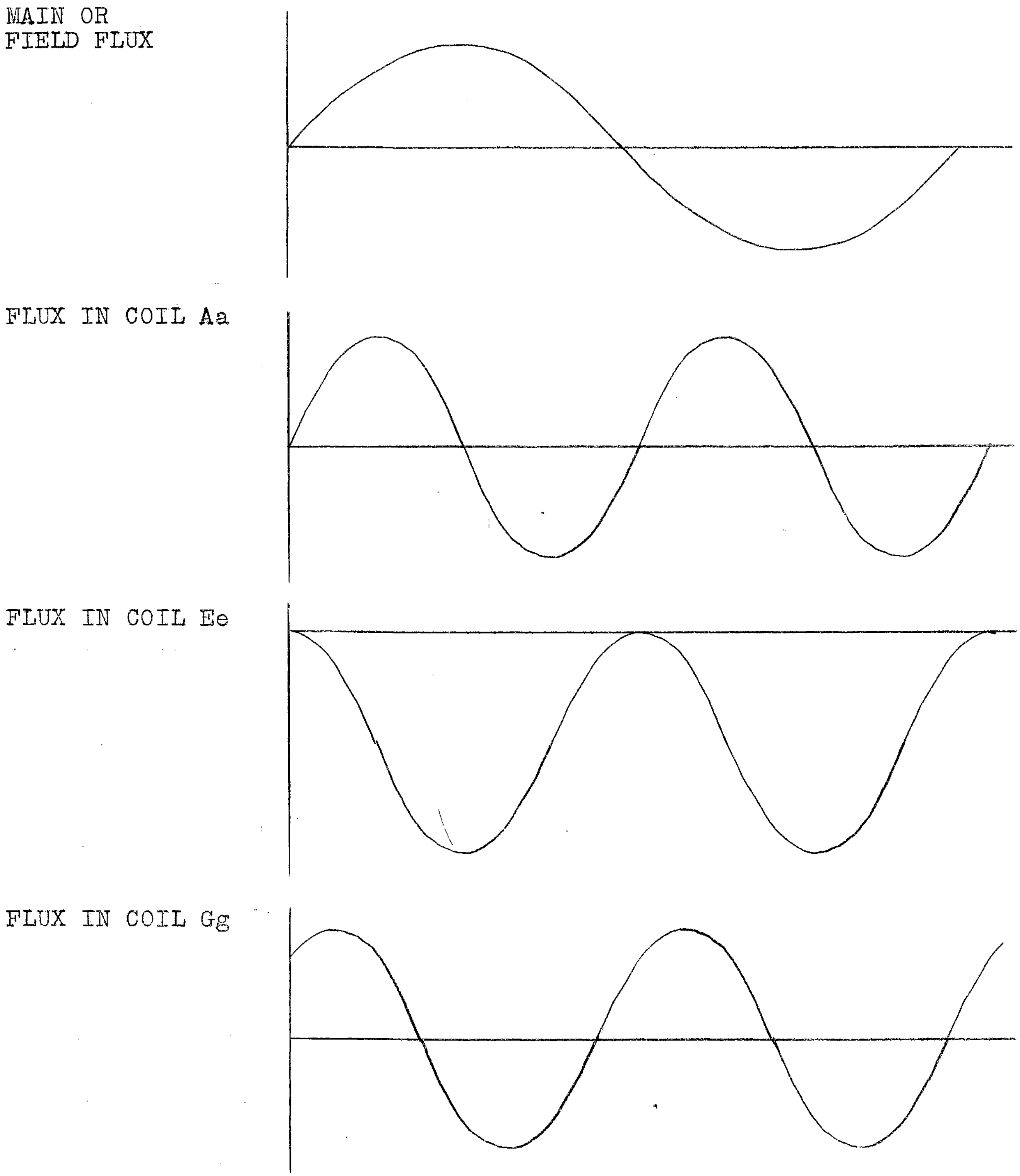


Figure 7

These fluxes can be plotted this way:



All fluxes in rotor coils are of double frequency.
Figure 8

In spite of these mathematical developments which point to the presence of the two fields, there is still some criticism of the fundamental postulate of the theory.

Lawrence says, "Although the method of Ferraris, just outlined, serves to explain the general action of the single-phase induction motor a vigorous analysis must include the reaction of the rotor. This factor is neglected in the method of Ferraris."[8]

Bretch also is critical of the theory because of this fundamental postulate:

> In the theoretical consideration of the single-phase squirrel-cage motor it is rather difficult to reconcile some of the assumptions with practical conditions. In the two-field theory it is assumed that two rotary fields of half the amplitude of the single-phase field, rotating at equal speeds in opposite directions is the equivalent of a single-phase field. In a single-phase field the flux goes through zero twice during each cycle. As the flux is confined to one path only, all components must go through zero when the total flux does as all flux disappears at the moment of reversal. Thus all components of a single-phase flux must also go through zero and since all components go through zero there could be no rotary components as a rotary field does not go through zero but shifts its position. It is generally admitted that a polyphase system cannot be transformed directly into a single-phase, or the reverse, since the polyphase represents a continuous flow while the single-phase is intermittent. It would seem that the same reasoning would apply to magnetic fluxes and it is difficult to see how a single-phase field can have any rotary components. Experiment also indicates that the single-phase field cannot be resolved into oppositely rotating components.[9]

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8 Ralph R. Lawrence, Principles of Alternating-Current Machinery, First Edition, p. 515.

9 Edward Bretch, "Letter," A.I.E.E. Transactions, Vol. 60 (1941), p. 668.

The experiment suggested by Mr. Bretch has been carried out and a discussion of it appears in the next section.

TWO OPPOSITELY ROTATING FIELDS IN A SINGLE STATOR

Since the theory depends so completely on the equivalence of the single uniaxial alternating flux to the two oppositely rotating fields some laboratory work has been done to investigate this.

The stator which was used can be described as follows: A Westinghouse Motor type CS frame 225 with 48 slots and with a connecting board providing terminal posts for each end of the 48 coils in the machine. These 96 coil terminals are shown in Figure 9. There are 16 turns per coil of size #16 wire. The 48 slots in the stator permitted a double-layer winding. The coil span is 1-8 or 7 slots. With this equipment it is possible to secure two or three-phase windings of 2, 4, 6, 8, or 10 poles on the stator by the use of different connecting schemes.

The first tests were made on three-phase windings. The stator was so connected that a three-phase two-pole winding used only half of the coils. These coils were chosen so that adjacent coils were not used; alternate coils made up this first winding. The second winding then was made up of the remaining coils, none of which were adjacent coils. The result was two three-phase, two-pole stators in one frame with the least possible (7 1/2°) displacement one from another. They were Y-connected with their phases in series. See Figure 10. The connecting sheet shows details of the winding scheme. Connections may be tabulated

Three-Phase Input at Terminals 1, 17, and 40								
Phases	Motor 1	Phases	Motor 2			Phases		
A	————————		—+——+—			A	B	C
B	———— – —		——— + ——		Motor 1	—	|	+
C	– – – – – –		– + – – +		Motor 2	=	||	‡

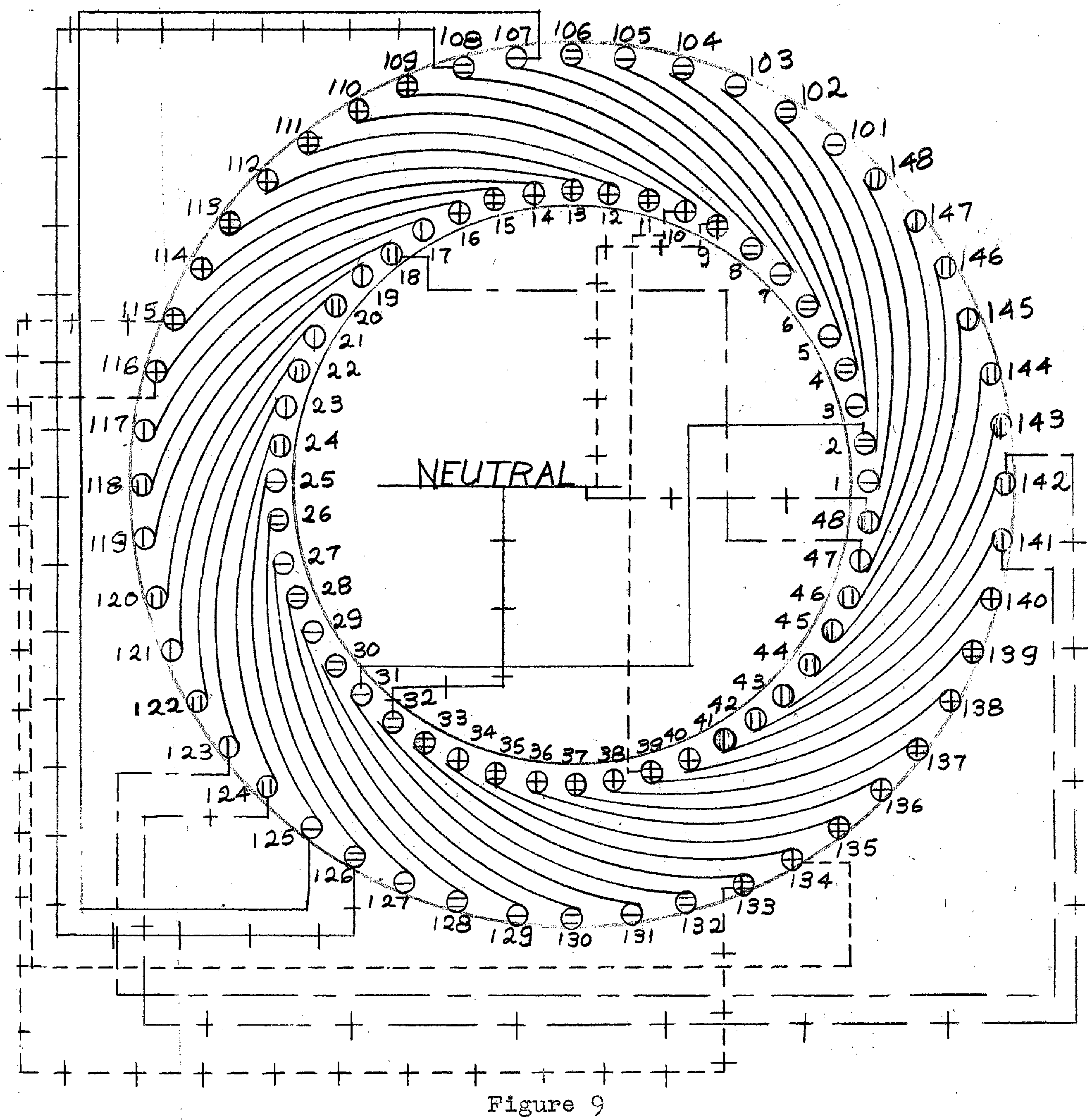
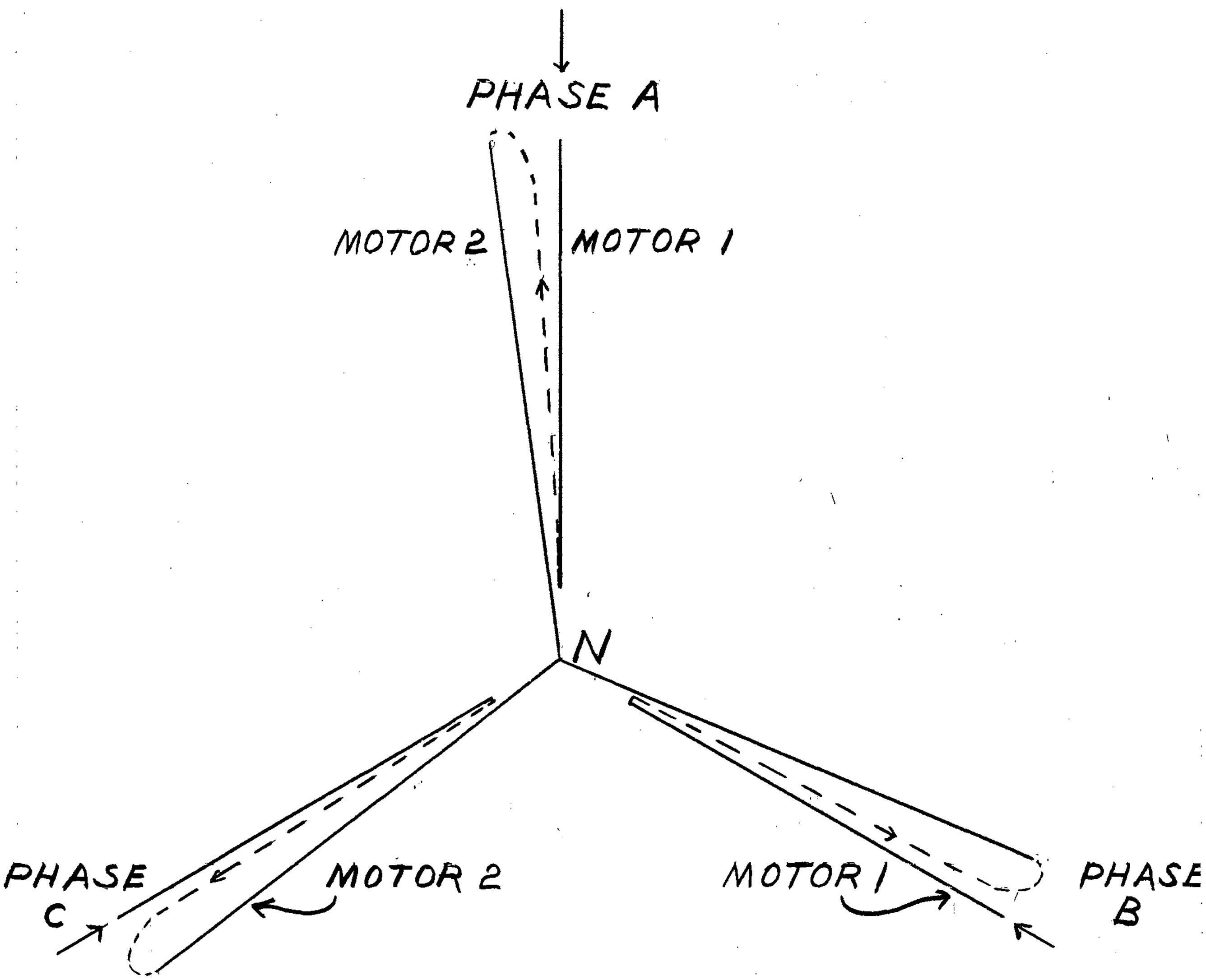


Figure 9



Two Three-Phase Windings 7 1/2° Apart Connected in Series

Figure 10

thus:

MOTOR #1	MOTOR #2
Phase A	Phase A
Begins at 1	Begins at 2
101 to 3	102 to 4
103 to 5	104 to 6
105 to 7	106 to 8
107 to 125 (next pole)	108 to 126 (next pole)
125 begins	126 begins
25 to 127	26 to 128
27 to 129	28 to 130
29 to 131	30 to 132
31 to 2 (beginning of Motor 2 Phase A which is connected in series with Motor 1 Phase A.)	32 to NEUTRAL
Phase B	Phase B
Begins at 17	Begins at 18
117 to 19	118 to 20
119 tc 21	120 to 22
121 to 23	122 to 24
123 to 141 (next pole)	124 to 142 (next pole)
141 begins	142 begins
41 to 143	42 to 144
43 to 145	44 to 146
45 to 147	46 to 148
47 to 18 (beginning ot Motor 2	48 to NEUTRAL

Motor #1 Phase B (Cont)	Motor #2 Phase B (Cont)
Phase B which is connected in series with Motor 1 Phase B)	
Phase C	**Phase C**
Begins at 40	Begins at 39
140 to 38	139 to 37
138 to 36	137 to 35
136 to 34	135 to 33
134 to 116 (next pole)	133 to 115 (next pole)
116 begins	115 begins
16 to 114	15 to 113
14 to 112	13 to 111
12 to 110	11 to 109
10 to 39 (beginning of Motor 2 Phase C with is connected in series with Motor 1 Phase C)	9 to NEUTRAL

This arrangement performed well when both stators were connected in series with same phase rotation. In order to secure opposite rotation in the second motor the connections to 18 and 2 were interchanged. Instead of operating as a single-phase motor the set came immediately to a stop and excessive currents flowed.

Many other schemes were tried without success. Changing the number of poles had no apparent affect. One arrangement which did not work was this:

The two stators were connected to separate variable three-phase voltage supplies. Each stator had its phases Y-connected in two independent neutrals. With the machine running with one stator at about 100 volts the voltage on the other was varied from a lower voltage up to 100 volts. With the same phase-rotation no difficulty was encountered except a circulating current through the lower voltage source. But when opposite rotation was used excessively high currents flowed at all voltages. The reason, of course, is readily apparent.

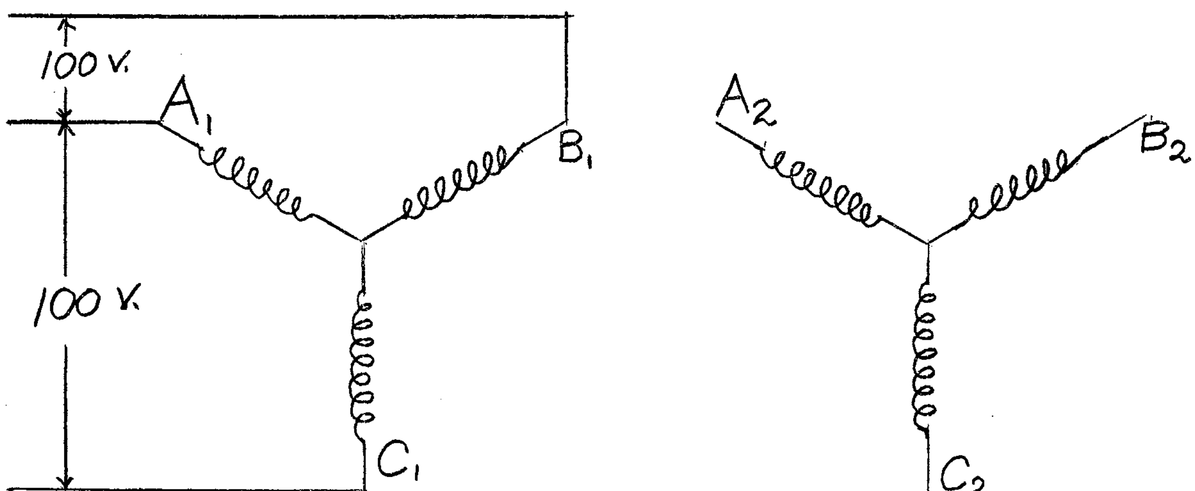


Figure 11

With no connection to the second stator a voltage about 90% of the value of the voltage applied to stator terminals A, B, and C appeared at terminals A_2, B_2, C_2; and it was of the same phase rotation as the applied voltage. It is readily apparent that connection of a voltage source of opposite phase rotation to $A_2B_2C_2$ would cause excessive currents to flow. This eliminates the possibility of duplicating single-phase motor action by use of two electrically independent stators with variable voltage sources. Because of the failure in the three-phase system it was abandoned in favor of a two-phase set.

Several two-phase arrangements were made, using a Scott transformer connection as well as separately generated two-phase voltages for the supply.

This four-pole two-phase motor, supplied with the two shaft connected generators was the only connection tried which gave single-phase operation:

MOTOR #1	MOTOR #2
Phase A (Line 1)	Phase A
Begins on 1	Begins on 2
101 to 3	102 to 4
103 to 5	104 to 6
105 to 114 (next pole)	106 to 113 (next pole)
114 begins	113 begins
14 to 116	13 to 115
16 to 118	15 to 117
18 to 25 (next pole)	17 to 26 (next pole)
25 begins	26 begins
125 to 27	126 to 28
127 to 29	128 to 30
129 to 138 (next pole)	130 to 137 (next pole)
138 begins	137 begins
38 to 140	37 to 139
40 to 142	39 to 141
42 to 2 (beginning of Motor 2 Phase A which is connected in series with Motor 1, Phase A)	41 to Line 3

MOTOR #1	MOTOR #2
Phase B (Line 2)	Phase B
Begins on 7	Begins on 8
107 to 9	108 to 10
109 to 11	110 to 12
111 to 120 (next pole)	112 to 119 (next pole)
120 begins	119 begins
20 to 122	19 to 121
22 to 124	21 to 123
24 to 31 (next pole)	23 to 32 (next pole)
31 begins	32 begins
131 to 33	132 to 34
133 to 35	134 to 36
135 to 144 (next pole)	136 to 143 (next pole)
144 begins	143 begins
44 to 146	43 to 145
46 to 148	45 to 147
48 to 8 (beginning to Motor 2 Phase B, which is connected in series with Motor 1, Phase B)	47 to Line 4

When connected in this way the rotor would turn at near-synchronous speed with either motor stator connected. Provision to reverse Motor #2 Phase B was made by interchanging the leads of terminals 8 and 47. See Figure 12.

This reversal resulted in higher current flowing in Phase B of both motors, but only a slight reduction of speed. The motor

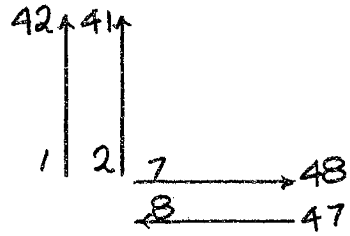


Figure 12

would not start under this condition. The mmf of Motor 1, Phase B was so reduced by the opposing mmf of Motor 2, Phase B that only Phase A of the two windings can be effective. Single-phase operation was the result.

During the test the system was supplied by two three-phase generators connected in Y, each supplying one phase of the two required. There was a 30° mechanical shift between the two machines to secure the 90° angle and four wires were used.

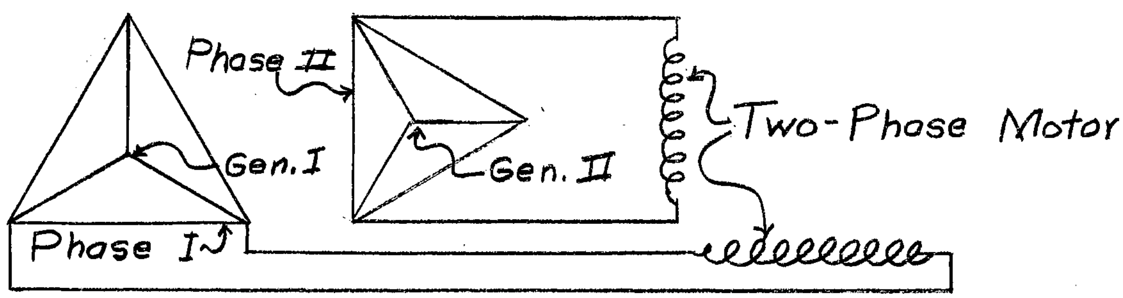


Figure 13

These conditions resulted in high harmonic content for the second phase when reversed rotation for one stator was used.

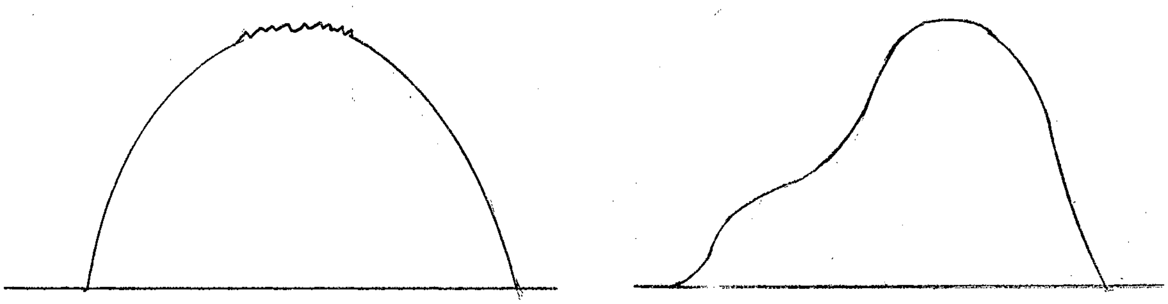


Figure 14

When an attempt was made to use the Scott-connected transformers the motor would not perform as a single-phase machine but came immediately to a stop and drew high current. On the other hand, the internal impedance of the alternator and its regulation had the effect of retarding this current flow and permitted successful performance of the experiment.

Data from this test is tabulated below:

Resistance of stator groups,

Terminal 8 to Terminal 47	1.594 ohm
Terminal 7 to Terminal 48	1.616 ohm
Terminal 1 to Terminal 42	1.608 ohm
Terminal 2 to Terminal 41	1.656 ohm

Resistance between lines:

Line 1 to line 3	3.32 ohm
Line 2 to line 4	3.22 ohm

When both were rotating counterclockwise:

$V_1 = 116$	$V_2 = 116$
$I_1 = 1.24$	$I_2 = 1.3$
$W_1 = 66$	$W_2 = 66$

With lines 2 and 4 open and one stator operating:

$V_1 = 109$	$V_2 = 127$
$I_1 = 2.23$	$I_2 = 0$
$W_1 = 131.5$	$W_2 = 0$

With the 2-4 phase of the second motor reversed:

$V_1 = 101$	$V_2 = 46$
$I_1 = 3.85$	$I_2 = 9.9$
$W_1 = 307.5$	$W_2 = 315.5$

When the polarity of 1-3 was reversed the motor rotated clockwise with both stators aiding:

$V_1 = 117$ $V_2 = 117$

$I_1 = 1.3$ $I_2 = 1.3$

$W_1 = 64$ $W_2 = 64$

With the 2-4 lines open:

$V_1 = 110$ $V_2 = 127$

$I_1 = 2.2$ $I_2 = 0$

$W_1 = 130$ $W_2 = 0$

With the 2-4 phase of the second motor reversed:

$V_1 = 101$ $V_2 = 46$

$I_1 = 3.8$ $I_2 = 9.8$

$W_1 = 305$ $W_2 = 310$

With rotor blocked; that is, with the 2-4 second motor phase reversed for starting counterclockwise,

$V_1 = 65$ $V_2 = 47$

$I_1 = 7.2$ $I_2 = 9.6$

$W_1 = 270$ $W_2 = 312.5$

Under similar conditions except for clockwise rotation,

$V_1 = 64$ $V_2 = 48.5$

$I_1 = 7.25$ $I_2 = 9.7$

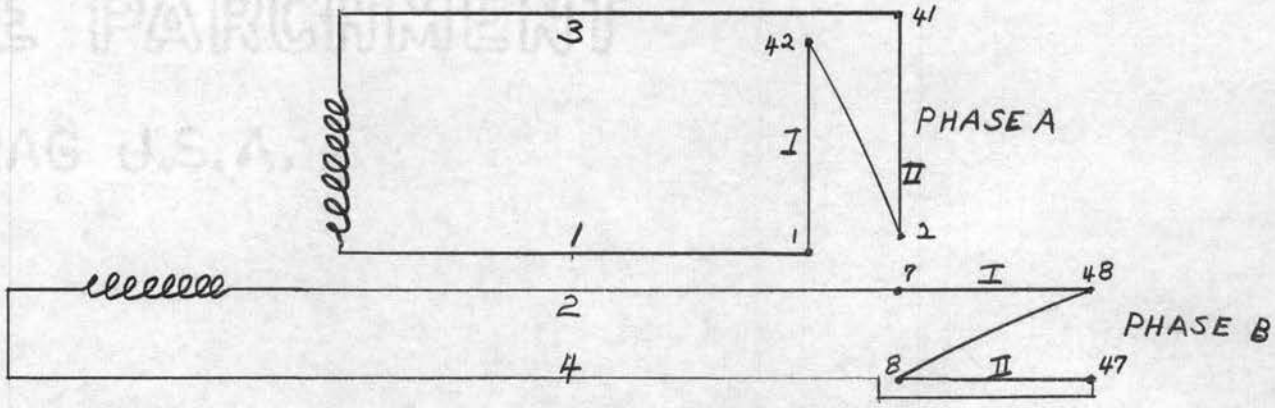
$W_1 = 263$ $W_2 = 330$

With the field switch open on the 1-3 alternator 2.85 amperes flowed through it with 32 volts measured across its terminals. The 2-4 machine showed 27 volts and 2.7 amps under the same conditions.

During normal operation the voltage drop across 8-47 was

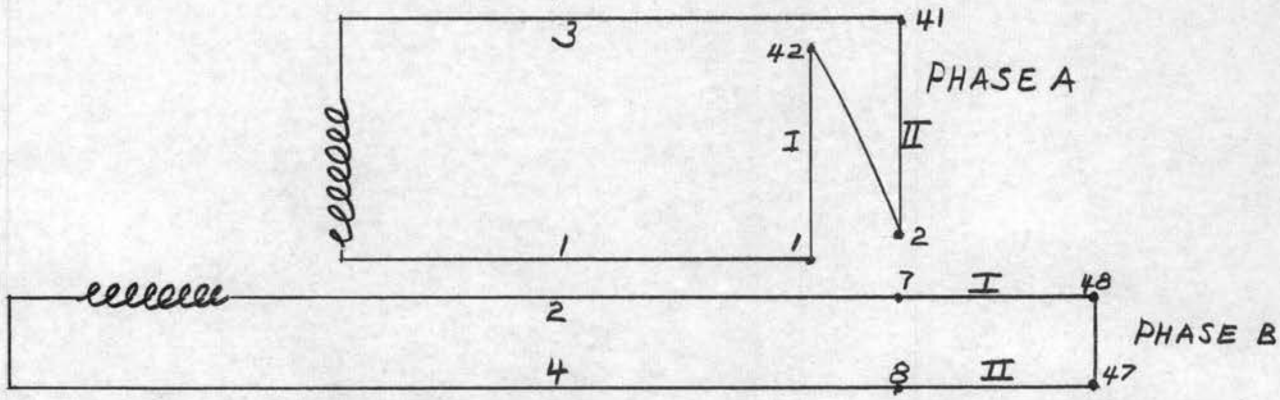
Figure 14A

(a)



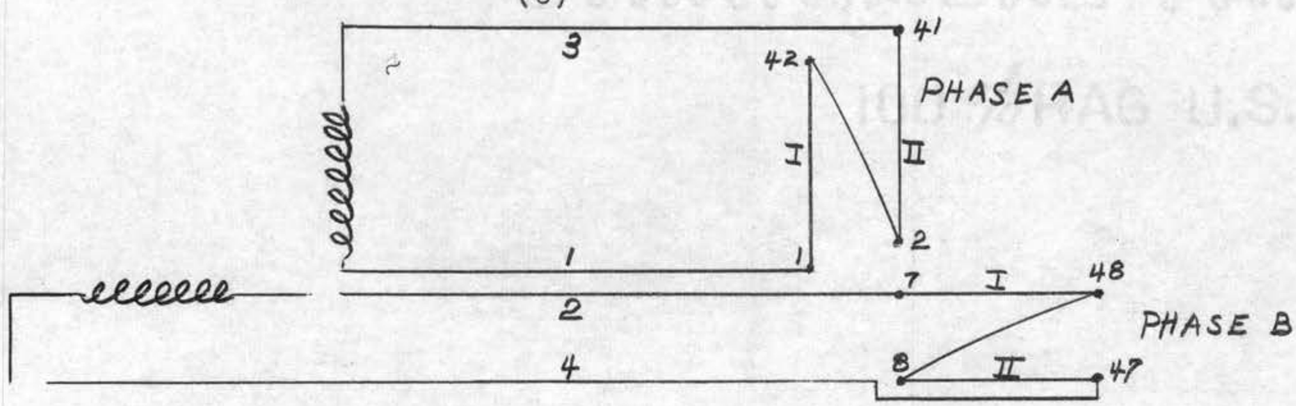
Both stators connected for same direction of rotation

(b)



Phase B of Motor II reversed

(c)



Motor operating on Phase A with lines 2 and 4 open

60 volts. While this group was reversed during operation of the motor the voltage was 19.3 volts. The voltage across 7-48 remained at 60 volts under both conditions.

TWO OPPOSITELY ROTATING FIELDS IN TWO STATORS

Another method of duplicating the double-revolving field involves the use of two similar polyphase machines with stator fluxes rotating in opposite directions. The system used in this case consisted of two Westinghouse 5 KW laboratory alternators with their shafts coupled. Their stators were connected in series Y:

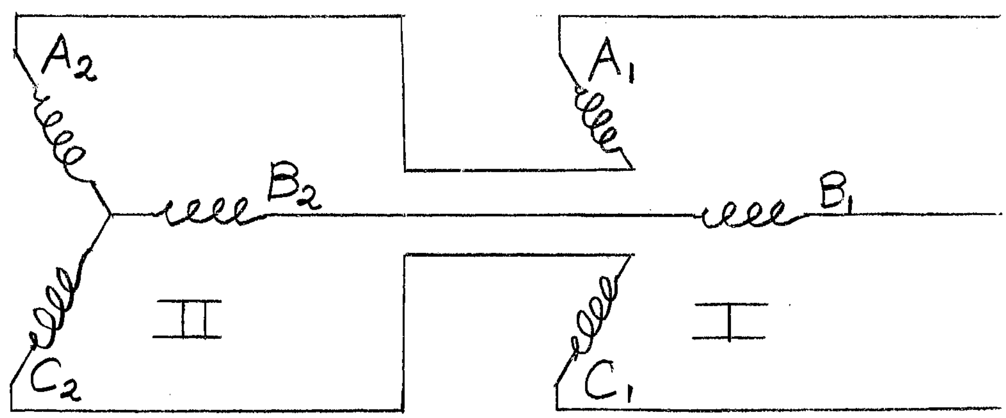


Figure 15

Provision was made to reverse the direction of flux rotation in the second machine. The machines were operated as induction motors (field open) and the test run as follows:

With both machines having the same direction of rotation for the stator flux the power, currents, and line voltages were read. Also the phase voltages of all six of the phases involved were read.

MOTOR	LINE	VOLTS	CURRENT		PHASE	VOLTS
	1-2	108.5	1	4.24A	Line 1 to N	32.7
I	2-3	108.5	2	4.33	2 to N	32.7
	1-3	108.0	3	4.33	3 to N	32.9
	1-2	53.0	1	4.24A	Line 1 to N	31.3
II	2-3	53.5	2	4.33	2 to N	31.2
	3-1	53.0	3	4.33	3 to N	31.5

Total power input 250 watts, speed 1100 RPM.

With the neutral in the second machine opened, and the first machine connected Y,

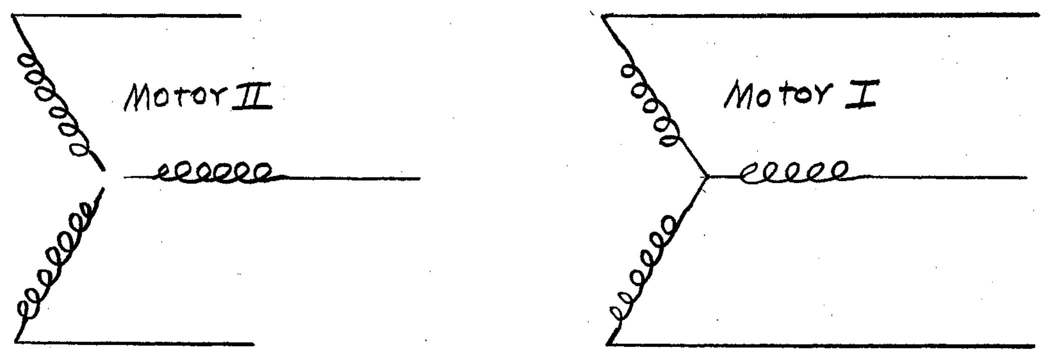


Figure 16

these conditions existed:

MOTOR	PHASE	VOLTS	AMPS
	1-N	65	8.60
I	2-N	64.6	8.66
	3-N	65	8.74

Total power input 330 watts, speed 1140 RPM CCW

No voltage was generated in Machine II, of course.

With the second machine alone running and driving the first:

MOTOR	PHASE	VOLTS	AMPS
	1-N	63.5	8.9
II	2-N	64.0	8.7
	3-N	64.2	8.75

Total power input 330 watts, apeed 1140 RPM CCW

With the first running, the second was energized with opposite phase rotation:

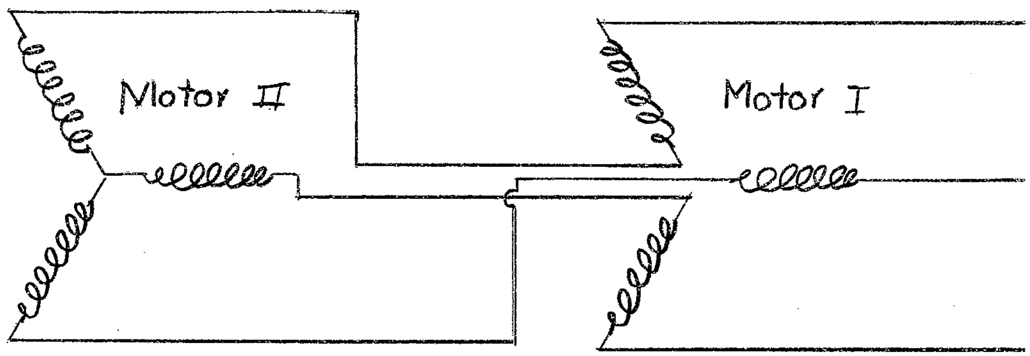


Figure 17

The results were:

MOTOR	LINE	VOLTS	CURRENT	PHASE	VOLTS
	1-2	121	8.4	1-N	54
I	2-3	121	8.4	2-N	54
	3-1	121	8.4	3-N	54
			8.4	1-N	20
II			8.4	2-N	20
			8.4	3-N	20

Total power input 800 watts, speed 910 RPM

In all the tests to this point the two coupled machines were running free.

Blocked rotor tests gave these results:

	VOLTS APPLIED	CURRENT AMPS	TOTAL POWER WATTS
BOTH STATORS AIDING	108	10.6 10.2 8.6	1300
FIRST MOTOR ALONE	105	17.5 19.0 16.0	2220
SECOND MOTOR ALONE	105	24.5 18.4 17.4	2720
TWO MOTORS BUCKING	108	9.7 9.0 8.5	1240

This arrangement gave single-phase motor action.

The equivalent single-phase motor is a very poor one since 800 watts is the running-light power input as compared with 1240 watts blocked input. Also actual load to a very slight degree stalled the system. But it does indicate that actual induction machines could be used to better advantage in this type of test carried to greater lengths. The loading tests described by Lamme[10] show high correlation between the actual

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10 B. G. Lamme, "A Physical Conception of the Operation of the Single-Phase Induction Motor," AIEE Transactions, 37 (April, 1918), 627.

single-phase motor characteristics and the conditions with two polyphase motors operated in the manner described here.

MATHEMATICAL DEVELOPMENT BY EQUIVALENT CIRCUIT

Although the physical application of two revolving fields to a single stator is unsatisfactory, the development of the theory on this suspect foundation is quite convincing.

One approach has been through use of an equivalent circuit which is based on the equivalence of the single-phase motor to two polyphase motors.[11]

The two stators are taken to be connected in series, therefore the same current will pass through both.

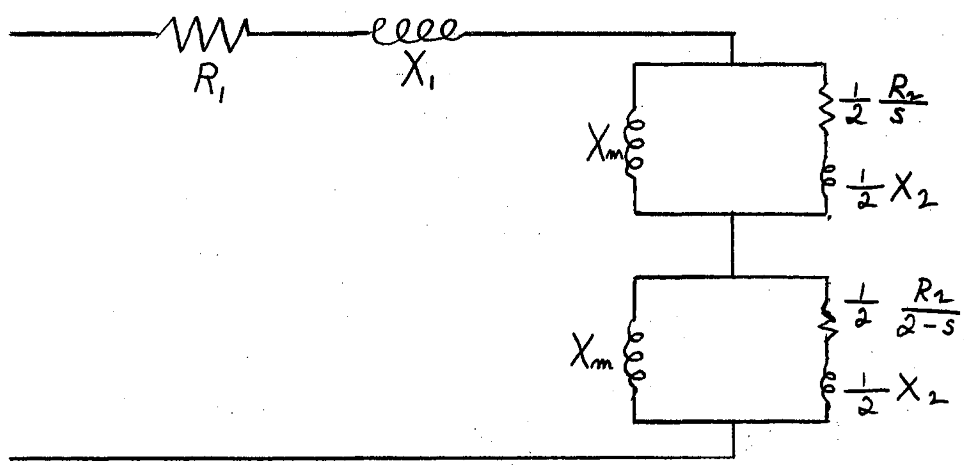

Figure 18

Neglecting the iron loss, and assigning one-half the rotor resistance and reactances to each rotor and one-half the

11 A. F. Puchstein and T. C. Lloyd, Alternating Current Machines, Second Edition, pp. 335-337.

magnetizing reactance to each rotor, Figure 18 represents an equivalent circuit based on the double-revolving field.

The impedance of the forward rotor is the equivalent impedance of the two branches:

$$Z_f = \frac{jx_m \left(\frac{r_2}{s} + j\,x_2\right)}{\frac{r_2}{s} + j\left(x_2 + x_m\right)} \tag{22}$$

The impedance of the backward rotor:

$$Z_b = \frac{jx_m \left(\frac{r_2}{2 - s} + j\,x_2\right)}{\frac{r_2}{2 - s} + j\left(x_2 + x_m\right)} \tag{23}$$

The stator impedance is,

$$Z_s = R_1 + jX_1 \tag{24}$$

The total impedance,

$$Z_T = Z_s + Z_f + Z_b \tag{25}$$

Then the stator current for applied voltage V will be by Ohm's law,

$$I = \frac{V}{R_1 + jX_1 + \frac{jx_m\left(\frac{r_2}{s} + jx_2\right)}{\frac{r_2}{2} + j(x_2 + x_m)} + \frac{jx_m\left(\frac{r_2}{2 - s} + jx_2\right)}{\frac{r_2}{2 - s} + j(x_2 + x_m)}} \tag{26}$$

For any assumed value of slip and with the stator and rotor resistances, and stator and rotor reactances known, the characteristics of the motor may be determined:

$$\text{Power factor} = \frac{R_T}{Z_T} \tag{27}$$

Rotor voltages:

Forward is $I_{2f} = e_f$ (28)

Backward is $I_{2b} = e_b$ (29)

The current through the branches containing resistances can be determined. For the resistive branch of the forward rotor,

$$I_f = \frac{e_f}{\sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2}} \tag{30}$$

For the backward rotor,

$$I_b = \frac{e_b}{\sqrt{\left(\frac{r_2}{2 - s}\right)^2 + x_2^2}} \tag{31}$$

The torque in synchronous watts equals the rotor current (I_f or I_b) squared times the rotor resistance:

Torque of forward rotor = $T_f = I_f^2 \frac{r_2}{s}$ synchronous watts (32)

and for the backward rotor,

$$\text{Torque} = T_b = I_b^2 \frac{r_2}{2 - s} \text{ synchronous watts} \tag{33}$$

Since the two torques are opposing the resultant torque becomes

$$T = T_f - T_b \tag{34}$$

Output in watts,

$$W = T(1 - s) \tag{35}$$

And,

$$\text{efficiency} = \frac{W}{VT \cos \theta} \tag{36}$$

ANOTHER APPROACH TO PERFORMANCE EQUATIONS[12]

In a single-phase stator with a sine-wave distribution of conductors the conductor density at any point is:

$$D_c = D_{cm} \sin \frac{\pi}{\lambda} x \tag{37}$$

Where λ = pole pitch in inches

x = distance in inches from reference point measured in air-gap

D_{cm} = maximum density of conductors expressed as conductors per inch

Integrating over one pole, $x = 0$ to $x = \lambda$

Conductors per pole, $$C_p = \frac{\lambda}{\pi} D_{cm} \int_0^{\lambda} \sin\left(\frac{\pi}{\lambda} x\right) \frac{\pi}{\lambda} \tag{38}$$

$$C_p = \frac{\lambda}{\pi} D_{cm} \Big[\cos \frac{\pi}{\lambda} x \Big]_0^{\lambda} \tag{39}$$

And,

$$D_{cm} = C_p \frac{\pi}{2\lambda} \tag{40}$$

Substituting (40) into (37)

$$D_c = \frac{\pi}{2\lambda} C_p \sin \frac{\pi}{\lambda} x \tag{41}$$

The current in the stator at any time, t, is,

$$i = i_{max} \cos \omega t \tag{42}$$

The current density at any point, x, and any time, t, is the product of current and conductor density.

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12 Wayne J. Morrill, "The Revolving Field Theory of the Capacitor Motor," AIEE Transactions, 49 (April, 1929), 614.

Multiplying (41) by (42)

$$D_i = D_c i = (i_{max} \cos \omega t) \left[\frac{\pi}{2\lambda} C_p \sin\left(\frac{\pi}{\ } x\right)\right] \tag{43}$$

$$D_i = i_{max} \frac{\pi}{2\lambda} C_p \cos \omega t \sin\left(\frac{\pi}{\lambda} x\right) \tag{44}$$

By trigonometry,

$$D_i = i_{max} \frac{\pi}{2\lambda} C_p \left[\frac{1}{2} \sin\left(\frac{\pi}{\lambda} x - \omega t\right) + \frac{1}{2} \sin\left(\frac{\pi}{\lambda} x + \omega t\right)\right] \tag{45}$$

$$D_i = i_{max} \frac{\pi}{4\lambda} C_p \left\{\left[\sin\left(\frac{\pi}{\lambda} x - \omega t\right)\right] + \left[\sin\left(\frac{\pi}{\lambda} x + \omega t\right)\right]\right\} \tag{46}$$

Where $i_{max} \frac{\pi}{4\lambda} C_p \left[\sin\left(\frac{\pi}{\lambda} x - \omega t\right)\right]$ is the forward wave and $i_{max} \frac{\pi}{4\lambda} C_p \left[\sin\left(\frac{\pi}{\lambda} x + \omega t\right)\right]$ is the backward wave.

The MMF at any point and time is the integral of the current density. For the forward wave:

$$MMF_f = \int i_{max} \frac{\pi}{4\lambda} C_p \left[\sin\left(\frac{\pi}{\lambda} x - \omega t\right)\right] \tag{47}$$

$$MMF_f = i_{max} \frac{\pi}{4\lambda} C_p \frac{\lambda}{\pi} \int \sin\left(\frac{\pi}{\lambda} x - \omega t\right) \frac{\pi}{\lambda} \tag{48}$$

$$MMF_f = i_{max} \frac{C_p}{4} \cos\left(\frac{\pi}{\lambda} x - \omega t\right) \tag{49}$$

The MMF due to the backward wave:

$$MMF_b = i_{max} \frac{C_p}{4} \cos\left(\frac{\pi}{\lambda} x + \omega t\right) \tag{50}$$

The MMF multiplied by the permeance per square inch of air gap section gives flux density. Flux density times the length of the stator gives flux per inch of periphery of air gap.

Forward Flux per inch of periphery:

$$\phi_f = MMF_f \, \ell \, P = \ell \, Pi_{max} \frac{C_p}{4} \cos\left(\frac{\pi}{\lambda}x - \omega t\right) \tag{51}$$

Backward Flux per inch of periphery:

$$\phi_b = MMF_b \, \ell \, P = \ell \, Pi_{max} \frac{C_p}{4} \cos\left(\frac{\pi}{\lambda}x + \omega t\right) \tag{52}$$

The voltage generated, per inch of periphery at a point X is equal to the product of flux, conductor density and velocity of the flux relative to the conductors. A factor must be used to convert to volts.

The velocity of the forward flux with respect to the stator conductors is $2\pi f$ radians per second which becomes $2\lambda f$ inches per second.

The voltage per inch of periphery generated by the forward flux is

$$e_f = 2\lambda f \ell \, Pi_{max} \, 10^{-8} \frac{C_p}{4} \cos\left(\frac{\pi}{\lambda}x - \omega t\right) \frac{\pi}{2\lambda} C_p \sin\left(\frac{\pi}{\lambda}x\right) \tag{53}$$

$$e_f = \pi f \ell \, Pi_{max} \, 10^{-8} \frac{C_p^2}{4} \left[\frac{1}{2} \sin\left(\frac{\pi}{\lambda}x - \omega t + \frac{\pi}{\lambda}x\right) + \frac{1}{2} \sin\left(\frac{\pi}{\lambda}x - \omega t - \frac{\pi}{\lambda}x\right)\right] \tag{54}$$

$$e_f = \pi f \ell \, Pi_{max} \, 10^{-8} \frac{C_p^2}{8} \left[\sin\left(2\frac{\pi}{\lambda}x - \omega t\right) + \sin(-\omega t)\right] \tag{55}$$

Integrating over one pole as x goes from 0 to λ, the voltage per pole is,

$$E_f = \frac{1}{8}\pi f Pi_{max} \, C_p^2 \, 10^{-8} \lambda \sin \omega t \tag{56}$$

Calculations similar to this give,

$$E_b = \frac{1}{8}\pi f P i_{max} C_p^2 10^{-8} \lambda \sin \omega t \tag{57}$$

The stator voltages generated by the forward and backward rotor fluxes can be computed as Morrill has done.[13] The torque is

$$T = i^2 \left[(R_f - R_b) + (R_f - R_b) \cos 2\omega t - (X_f - X_b) \sin 2\omega t \right] \tag{58}$$

Where,

R_f = Apparent resistance to forward field

R_b = Apparent resistance to backward field

X_f = Apparent reactance to forward field

X_b = Apparent reactance to backward field.[14]

$$\text{Output} = W = T(1-S) \tag{57}$$

13 Ibid., p. 614.

14 Ibid., p. 614.

CHAPTER II

THE CROSS-FIELD THEORY

In 1894, Potier[1] applied the cross-field theory to single-phase induction motor action. Others who have led in its development and use are McAllister, Steinmetz, and H. R. West. The theory is more complex than the two revolving field approach; but, like it, has some advantages over other methods.

Stated in the most elementary terms, the cross-field theory describes the action of a single-phase motor on the basis of two fixed fluxes in time and space quadrature caused by the currents flowing in the stator and rotor windings. These fluxes are of approximately equal magnitude at synchronous speed, producing a circular flux pattern which becomes elliptical at intermediate speeds and a straight line at standstill because of the reduction of the rotor flux with a reduction of speed.

Steinmetz describes it this way:

> The cause of this cross magnetization in the single-phase induction motor near synchronism is that the secondary armature currents lag 90° behind the magnetism[2] and are carried by synchronous rotation 90° in space before reaching their maximum thus giving the same magnetic effect as a quarter-phase emf impressed upon the primary system in quadrature position with the main coil................ Below synchronism the secondary currents are carried less than 90° and thus the cross magnetization due to them is

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[1] Bulletin de la Societe Internationale des Electriciens, Paris, May, 1894.

[2] Steinmetz, "Single-Phase Induction Motor," A.I.E.E. Transactions, (February, 1898). In this paper he calls this "induced magnetism."

correspondingly reduced and becomes zero at standstill.[3]

McAllister was critical of this description of the theory saying: "It is greatly doubted if these statements convey any physical idea whatever to a mind not already thoroughly familiar with the facts.[4]

In a later book Steinmetz put it this way:

> The magnetic flux at standstill is a single-phase alternating flux of constant direction and the line of polarization of the armature or secondary currents, that is, the resultant mmf of the armature currents coincides with the axis of magnetic flux impressed by the primary circuit. When revolving, however, even at low speeds, torque appears in the single-phase induction motor, due to the axis of armature polarization being shifted against the axis of primary impressed magnetic flux, by the rotation. That is, the armature currents, lagging behind the magnetic flux which induces them, reach their maximum later than the magnetic flux, thus at a time when their conductors have already moved a distance or an angle away from coincidence with the inducing magnetic flux. That is, if the armature currents lag $\pi/2 = 90^\circ$ beyond the primary main flux and reach their maximum 90° in time behind the magnetic flux, at slip s and thus speed (1-s) they reach their maximum in the position $(1-s)\ \pi/2 = 90(1-s)$ electrical degrees behind the direction of the main magnetic flux.[5]

Lamme expressed in different words the same concept:

> In some methods of considering the single-phase induction motor problem, the single-phase primary winding is assumed to generate a magnetomotive force in the secondary which, by rotation of the core, is carried around until it generates a second magnetic field or flux at right angles to the original primary flux, thus giving the equivalent of a polyphase magnetic field.[6]

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[3] C. P. Steinmetz, Theoretical Elements of Electrical Engineering, p. 374.

[4] A. S. McAllister, Alternating Current Motors, p. 139.

[5] C. P. Steinmetz, Theory and Calculation of Electrical Apparatus, p. 93.

[6] B. G. Lamme, op. cit., 37, p. 632.

THE SQUIRREL-CAGE ROTOR

Before a description of the cross-field theory can be made a discussion of the squirrel-cage rotor should be undertaken. This idea that a rotor current is carried by rotor rotation around the stator when the motor is running is not a part of the modern theory. Button points this out in listing the factors that complicate the evaluation of the cross flux:

> Angular velocity of rotor: the rotor is a moving object; and the angular position of a given conductor will be considerably different at the instant when one voltage component is a maximum and the instant when the current due to that voltage is a maximum. This fact is completely forgotten in the usual simple textbook "explanation" of the single-phase induction motor.[7]

Bretch points to the same problem:

> In the cross-flux theory it is assumed that the rotor currents generated by the speed action of the rotor conductors cutting across the stationary alternating main or transformer flux lag approximately 90 time degrees. The electromotive force developed in the rotor conductors by speed action is in position to set up a flux at right angles to this assumed transformer flux. However, at synchronism with the rotor current lagging 90 time degrees, the rotor will move 90 space degrees during the time it takes the 90 degree current to build up, so that when the rotor magnetomotive force appears it will have moved 90 space degrees from the quadrature position into a position of opposition to the main flux. This shifting action of the magnetomotive force is observed in the a c generator and confirms the 90 degree space shift of the rotor magnetomotive force with the 90 degree lagging rotor currents. Thus it is physically impossible for uncommutated rotor currents generated by speed action to develop a cross-flux that is in both time and space quadrature with the main flux as assumed in the cross-flux theory.[8]

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[7] Button, _op. cit._, 60, 508.

[8] Edward Bretch, "Discussion," _A.I.E.E. Transactions_, 64 (1945), p. 943.

There can hardly be doubt that the currents in this axis are lagging the voltages. McFarland says the lag is about 70°.[9] Tarboux says: "The current in the XX conductors will lag the voltage by a considerable angle because of the high reactance of this winding."[10]

Lawrence says, speaking of the cross-field current path: "Its reactance for the axis XX is high and the current producing the quadrature field must lag nearly 90° behind the speed voltage."[11]

These arguments by Button and Bretch have been answered in two ways. The first may be found to be obscure:

> I have found considerable confusion existing in the minds of students and engineers concerning the fundamental analytical reasoning employed in the cross-field analysis of the single-phase induction motor. We are accustomed to consider a conductor or circuit. We determine the voltage generated in this conductor, the resistance and reactance of its circuit, and we say that the current lags the voltage by an amount determined by the ratio of resistance to reactance, or we may consider the reactance as a voltage, and determine the net voltage acting in the circuit. The current flow will then be in phase with this net voltage, which is the I R drop.
>
> In analysis of polyphase motors, we sometimes use these methods of thought. We consider the voltage induced in conductor d. We say that the current will flow later, because of the reactance of the circuit, and that d will then be in some other position, where e or f were at first. This same attack may be applied to the single-phase motor. But it leads to unnecessary complexity.
>
> So in single-phase-motor analysis another method of attack

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9 T. C. McFarland, <u>Alternating Current Machines</u>, p. 466.

10 J. G. Tarboux, <u>Alternating Current Machinery</u>, p. 324.

11 Lawrence, <u>op. cit.</u>, Third Edition, 585.

is used, as in this paper, but this new method has never to my knowledge, been clearly and definitely described. It has been rather assumed without distinct description, resulting in much confusion of thought on the part of many readers.

The new idea, which needs to be clearly stated, is that E_3 is not the voltage in any one conductor. If we plotted the value of the voltage in each successive conductor as it passed through the position d, we should obtain a sine wave, for ϕ_2 varies thus with time. A vector, E_3, may obviously be used to represent this sine wave, but it must be constantly kept in mind that E_3 does not represent the voltage in any one conductor, but represents the successive values of the voltage in each conductor as it passes through the position d. Our usual ideas of reactance and resistance will no longer apply, since we have no circuit, but a succession of circuits. Hence Mr. Perkins abandons the idea of reactance and uses the method of determining the net voltage and considering that the instantaneous current flow in any of the successive circuits is directly proportional to the instantaneous value of this net voltage, which is the I R drop at the successive instants of passing through position d.

Thus, in Figure 20, E_3 represents the successive values of voltage in the rotor conductors due to cutting ϕ_2 at the instant that each passes through position d. E_6 represents the successive values of transformer voltage due to ϕ_c at the instant that each conductor passes through position d. I_3 is not the current in a coil d or in any coil, but represents the successive values of the current in each conductor as it passes through position d. With this distinction clearly in mind some erroneous conclusions may be avoided, and some hazy conceptions made clearer.[12]

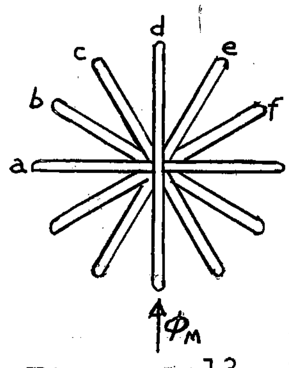

Figure 19[13]

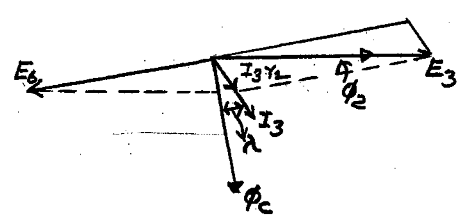

Figure 20[13]

12 W. B. Hall, "Discussion," A.I.E.E. Journal, XLIV (May, 1925), 522.

13 L. M. Perkins, "Single-Phase Induction Motor," A.I.E.E. Journal, XLIV (May, 1925), 499.

A second answer to the difficulty can be found in assumptions regarding the rotor as a circuit. A simple, but unconvincing explanation is found in several sources. That given by Robin Beach is an example of this, discussing first the transformer action:

> When the rotor is in rotation, the successive inductors, as can be reasoned from the geometry of the squirrel-cage winding and shown in Figure 21 momentarily form new pairs as they pass through the vertical plane, the trace of which is the Y-axis. On the basis of this hypothesis of how the rotor inductors, when in rotation, may be considered to form themselves continually into pairs distributed about the Y-axis, the resulting magnetic axis for these pairs is maintained coincident at all times with the magnetic Y-axis of the primary winding.[14]

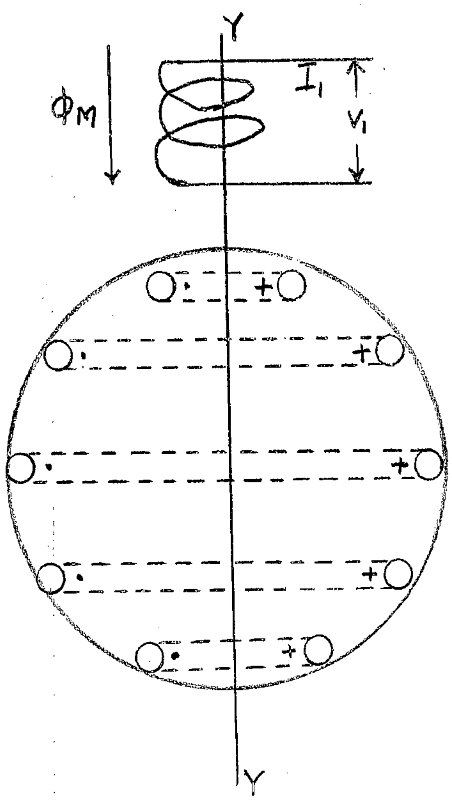

Figure 21[14]

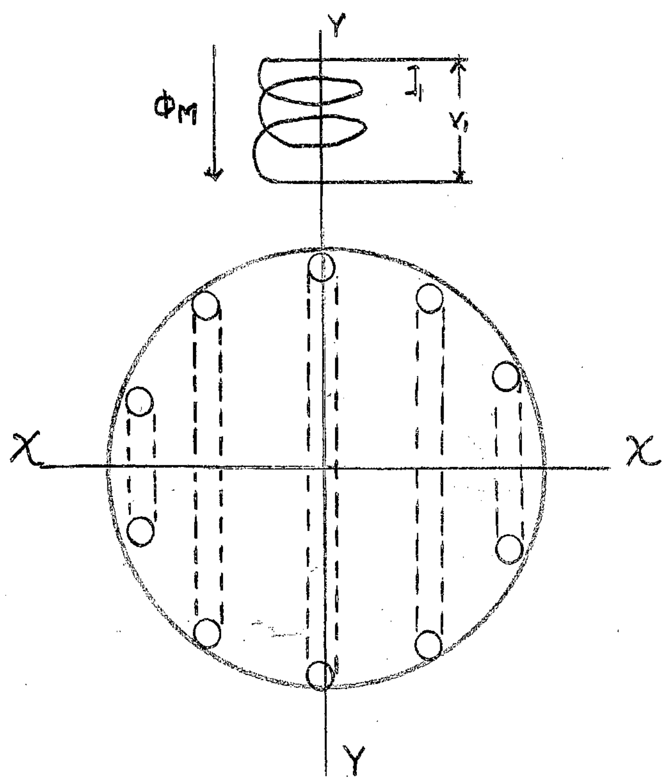

Figure 22[14]

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14 Robin Beach, "The Physical Conception of Single-Phase Motor Operation," Electrical Engineering, 63 (July, 1944), p.

Then, concerning the speed voltages Mr. Beach continues:

When the rotor inductors rotate through the main field ϕ_m, speed voltages are generated in one direction in all those inductors which are, at the instant, above the horizontal plane and voltages in the opposite direction are produced in the inductors occupying positions below the horizontal plane. These inductors, in which voltages are generated by their speed action through the main field, may also be regarded as paired, except that, in so doing, one above and a corresponding one below the X axis are considered to lie in a vertical plane, each pair with its magnetic axis individually coincident with the X axis as shown in Figure 22.[15]

The difficulty appears to be here that no matter what is said, it seems that certain conductors will have heavier current than others at any instant.

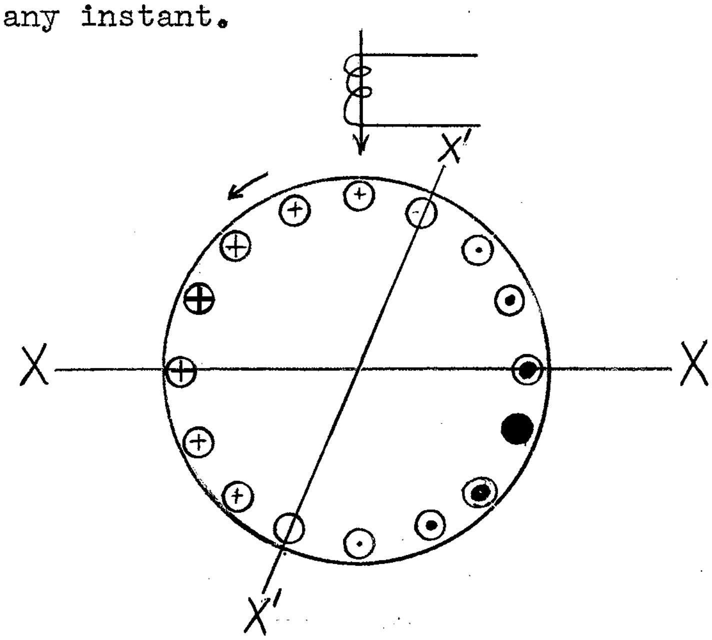

Figure 23

The same reasoning as applied to alternator armature reaction would have the speed-generated currents lagging the voltage in the arrangement as Figure 23 giving an axis rotated from XX.

A more satisfactory explanation is based on an assumption

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[15] Beach, loc. cit.

mentioned only rarely. McFarland states it this way: "The squirrel-cage is considered equivalent to a commutated winding with brushes bearing on the commutator short circuited on themselves in both the transformer and field axes."[16]

West in his fundamental paper stated it also: "A squirrel-cage is considered as equivalent to a commutated winding with brushes bearing on the commutator short circuited on themselves in the transformer and field axes."[17]

A detailed consideration of this equivalence is given by Langsdorf.[18] An approach based on this proof is much more satisfying.

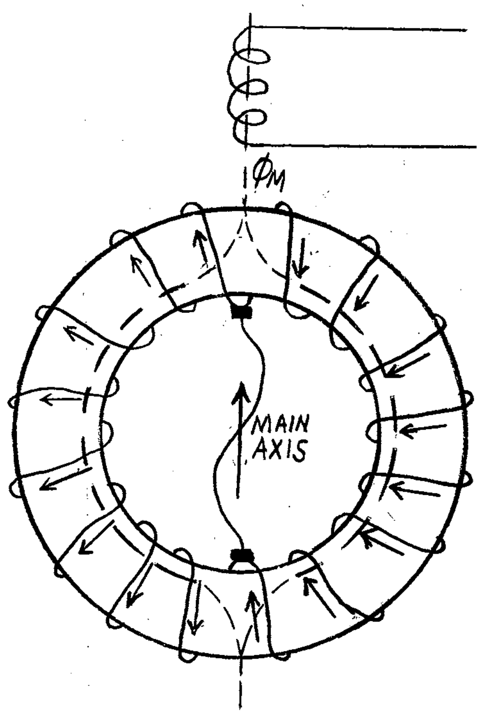

Figure 24

At standstill there is no cross field and the rotor may be represented by a Gramme ring with one pair of shorted brushes. All the current flowing in the armature generates a flux field in the main axis. A pair of brushes set 90° from these and short-circuited would be the pair required for the cross-field currents when the motor is

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16 McFarland, op. cit., 465.

17 H. R. West, "The Cross-Field Theory of Alternating-Current Machinery," Journal of the A.I.E.E., 45 (February, 1926), 160.

18 A. S. Langsdorf, Theory of Alternating Current Machinery, p. 525.

running. Then, in any rotor conductor a current composed of two components would flow: One flowing through the main axis brushes, the other through the cross axis brushes. The first generates a flux in the main axis; the second generates the cross axis flux.

Now a more detailed explanation of the cross field theory may be given. Upon assuming the presence of two fields in the air gap there are four emfs to be dealt with:

1. Transformation voltage (caused by sinusoidal time variation of field strength) in the rotor due to main winding flux.
2. Speed voltage (caused by cutting of lines by the rotating conductors) in the rotor due to the main winding flux.
3. Transformation voltage in rotor caused by the cross-field.
4. Speed voltage in the rotor caused by the cross-field.

See Figure 26 for vector representation.

THE FOUR ROTOR VOLTAGES

At standstill the single-phase motor is like a transformer with its secondary short-circuited. The main winding sends the mutual flux ϕ_m threading through the rotor conductors. Since these bars are interconnected current will flow in them because of the transformation voltage generated by the mutual flux. There will be no resultant torque induced in the motor because of the flux and current time phase, and because one-half of the rotor currents give clockwise torque and the other half gives counterclockwise torque.

Without rotation the only rotor voltage is the transformer voltage caused by the main flux. Rotation does not affect the generation of this transformer voltage. The other three voltages appear only when the rotor is rotating.

As the rotor bars cut the main flux a voltage will be generated in them. This is called the speed voltage caused by the main flux. The resultant current flow will generate a cross flux in space quadrature with the main flux. The direction of the voltage and current resulting from the action can be shown by referring to Figure 25.

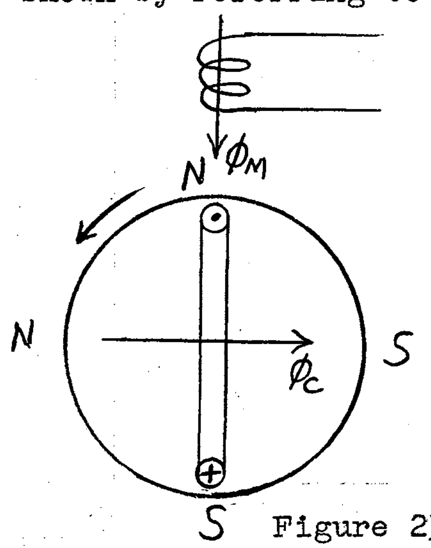
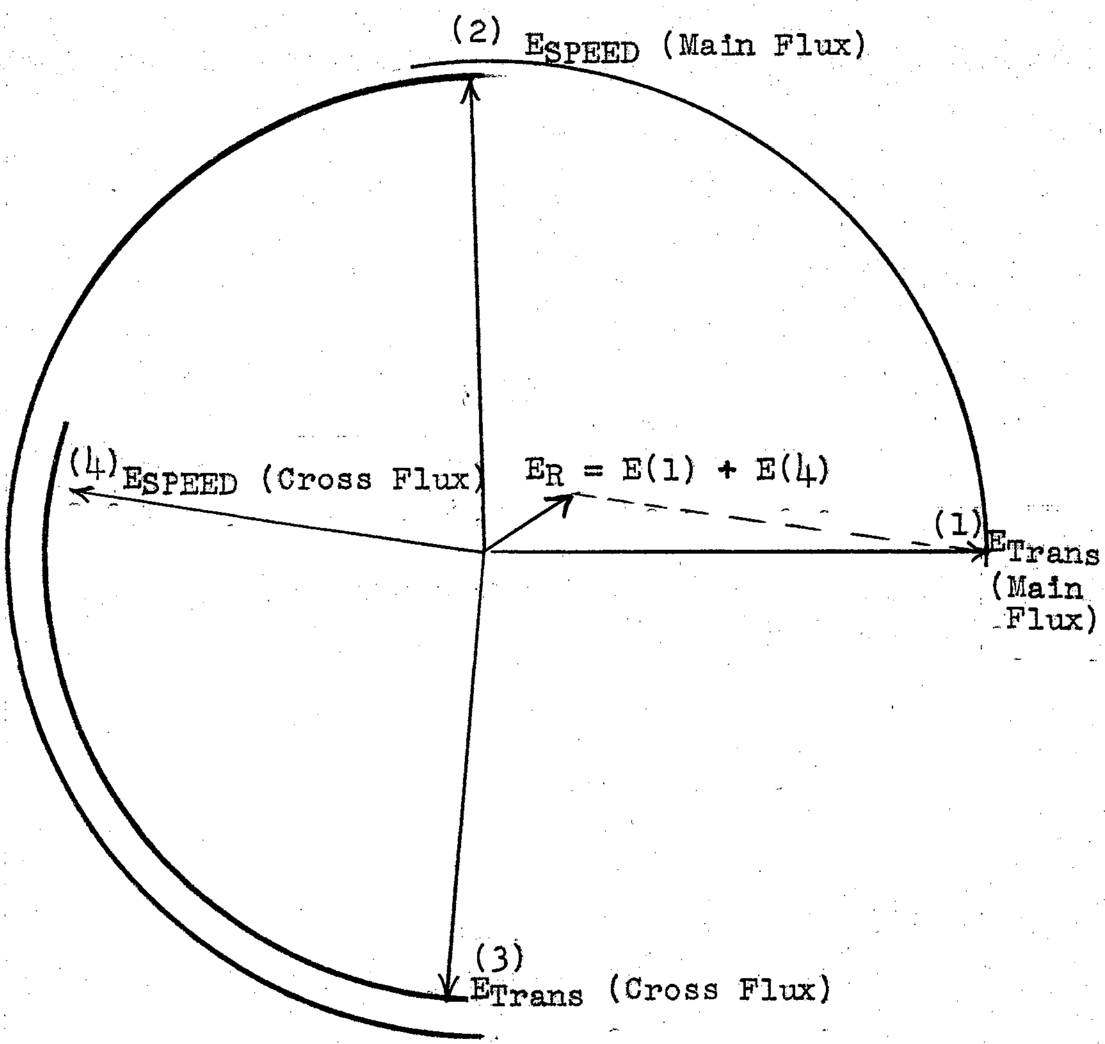


Figure 25

For the direction of rotation and main flux polarity chosen the rotor currents and cross-field flux will flow as indicated. The combination of the two fluxes will produce a magnetic field revolving counterclockwise, the same direction as the assumed rotor rotation.

Since these turns on the rotor do not react against the primary they are like a choke coil and their currents lag the voltages by a large angle. Because of the nature of the rotor circuit just described, however, their magnetic axis is in space quadrature with the main flux axis. The magnitude of the voltage is directly proportional to the speed of the armature, becoming almost equal to the main transformer voltage at synchronous speed.



Approximate Position and Relative Magnitude of the Four Voltages Showing One Resultant, E_R, in the Main Axis.

Figure 26

The time variation of the cross-field leads to an induced voltage called the transformation voltage caused by the cross-field. This voltage in the rotor bars causes a current to flow which will, by Lenz's law, oppose the rise and decay of the cross-field flux. Since the speed voltage (main field) is causing this variation in the cross-field, it can be seen that this third voltage is directed oppositely to the main field speed voltage.

The action of the rotor bars cutting the cross-field flux leads to the generation of the fourth voltage, speed voltage caused by the cross-field. The direction of this voltage is opposed to that of the main field transformer voltage. The magnitude of this voltage is governed by the strength of the cross-field (proportional to speed) and also by the rate of cutting flux (also proportional to speed); therefore this speed voltage caused by the cross-field is proportional to the square of the rotor speed. This makes it the least of the four voltages generated. Since this emf and the first, transformer emf caused by main flux, are both in the main axis their resultant E_R (See Figure 26) is the emf that drives the main axis current in the rotor. Similarly the other two voltages combine to produce the cross axis current in the rotor.

TORQUE

From the foregoing it is evident that the main-axis conductors have current flowing in them while they are cutting the quadrature flux and the cross axis conductors are carrying current while they cut the main flux. Because of the periodic reversals of fluxes and currents which occur at different times during the cycle there are alternately motor torques in the direction of rotation and generator torques in the opposite direction produced by both combinations of current and flux. The cross-field and main axis current produces most of the torque, and it is predominantly motor torque, while the main field and cross axis currents produce most of the generator torque, which is, of course, less than the motor torque for running conditions. Both are of double frequency time variation. This alternation of motor and generator action in both flux fields results in the higher frequency torque pulsations characteristic of single-phase motors.[19]

MATHEMATICAL ANALYSIS BASED ON THE CROSS-FIELD THEORY[20]

The cross-field theory divides the fluxes in the motor into four:

1. Transformer flux, ϕ_m. This is the flux mutual to stator and rotor.
2. Cross-field flux ϕ_c produced by rotor currents in cross axis.
3. Leakage flux ϕ_1 of the stator.
4. Leakage flux ϕ_2, rotor in main-field axis.

The equations of these fluxes can be written:

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19 Beach, loc. cit.

20 West, loc. cit.

$$\phi_m = \frac{X_m(I_1 - I_t)}{2\pi fN} \tag{1}$$

Where X_m = mutual inductive reactance of rotor and stator.

I_1 = line current.

I_t = rotor current in the main axis.

N = effective turns in circuit.

f = frequency applied.

$$\phi_c = \frac{(X_m - x_2)I_c}{2\pi fN} \tag{2}$$

Where X_2 = leakage reactance of rotor circuit.

I_c = rotor current in cross axis.

$$\phi_1 = \frac{x_1 I_1}{2\pi fN} \tag{3}$$

Where X_1 = stator leakage reactance

$$\phi_2 = \frac{x_2 I_t}{2\pi fN} \tag{4}$$

The applied voltage, E, must equal the sum of the drops:

$$E = I_1 r_1 + jx_1 I_1 + jX_m(I_1 - I_t) \tag{5}$$

Where r_1 = stator resistance.

In the rotor the sum of the voltages in each axis must equal zero:

For counterclockwise rotation,

$$0 = -jX_m(I_1 - I_t) - S(X_m + x_2)I_c + r_2 I_t + jx_2 I_t \tag{6}$$

Where $S = (1 - \text{slip})$

and,

$$0 = j(X_m + x_2)I_c - SX_m(I_1 - I_c) + Sx_2I_t + r_2I_c \tag{7}$$

These three equations, (5), (6), and (7) can be solved for the three currents:

$$I_1 = E\frac{\left[-r_2^2 + (1 - S^2)(X_m + x_2)^2 - j2r_2(X_m + x_2)\right]}{U_1 + jW_1} \tag{8}$$

$$I_t = EX_m\frac{(1 - S^2)(X_m + x_2) - jr_2}{U_1 + jW_1} \tag{9}$$

$$I_c = -EX_m\frac{Sr_2}{U_1 + jW_1} \tag{10}$$

Where,

$$U_1 = -r_1r_2^2 + 2r_2x_1(X_m + x_2) + r_2X_m(X_m + 2x_2) + (1 - S^2)r_1(X_m + x_2)^2 \tag{11}$$

$$W_1 = -r_2^2x_1 - 2r_1r_2(X_m + x_2) - r_2^2X_m + (1 - S^2)\left[x_1(X_m+x_2)^2 + x_2X_m(X_m + x_2)\right] \tag{12}$$

If the expressions for the currents found in (8), (9), and (10) be substituted in (1), (2), and (4) the expressions for the fluxes in terms of voltage applied and motor constants will be:

$$\phi_m - \phi_2 = \frac{-EX_m\left[r_2^2 + jr_2(X_m + x_2)\right]}{2\pi fN(U_1 + jW_1)} \tag{13}$$

$$\phi_c = \frac{SEX_m(X_m + x_2)r_2}{2\pi fN(U_1 + jW_1)} \tag{14}$$

The torque is then the product of the flux and the in-phase component of the current.

From (10) and (13) the torque due to the main axis flux and the cross axis rotor current is,

$$T_2 = \frac{-E^2X_m{}^2r_2{}^3S}{U_1{}^2 + W_1{}^2} \tag{15}$$

and the torque due to the main axis rotor current and the cross flux is:

$$T_1 = \frac{E^2X_m{}^2(X_m + x_2)^2r_2S(1 - S^2)}{U_1{}^2 + W_1{}^2} \tag{16}$$

The total torque is the sum of these two,

$$T = T_1 + T_2 = \frac{E^2X_m{}^2r_2S\left[(1 - S^2)(X_m + x_2)^2 - r_2{}^2\right]}{U_1{}^2 + W_1{}^2} \tag{17}$$

From this equation and the expression for the line current, I, (8), the performance of the motor of given design constants can be calculated.

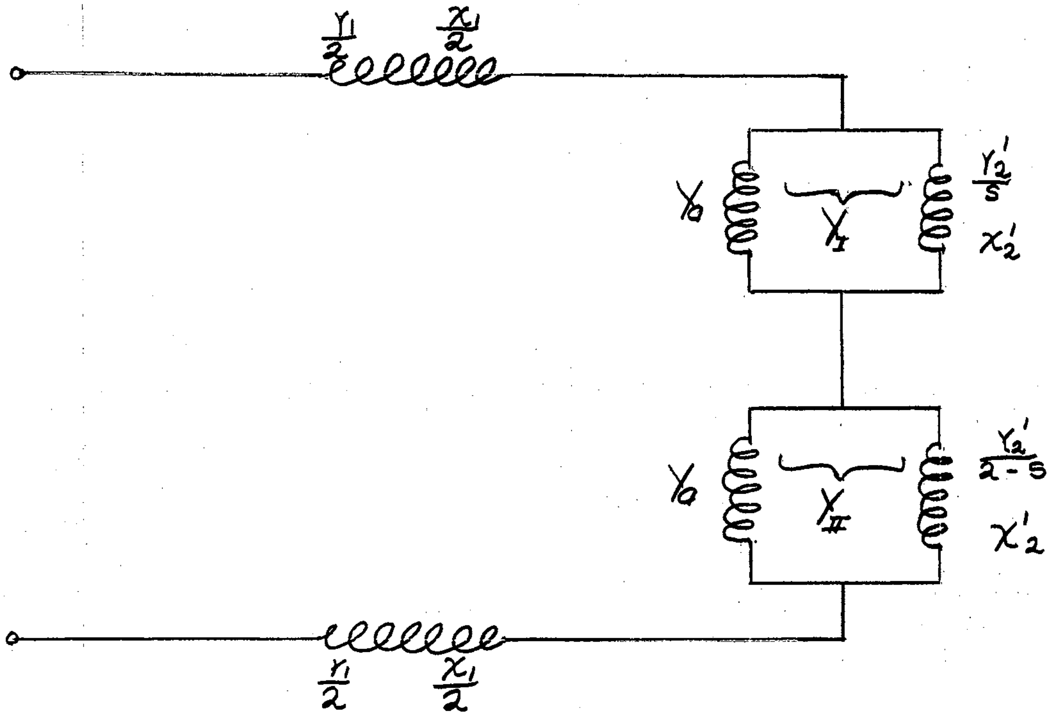
CHAPTER III

The fact that the two theories, double-revolving field and cross-field, are equivalent has been known for many years. Three ways of showing this equivalence will be demonstrated here.

THE CIRCLE LOCUS OF THE PRIMARY CURRENT

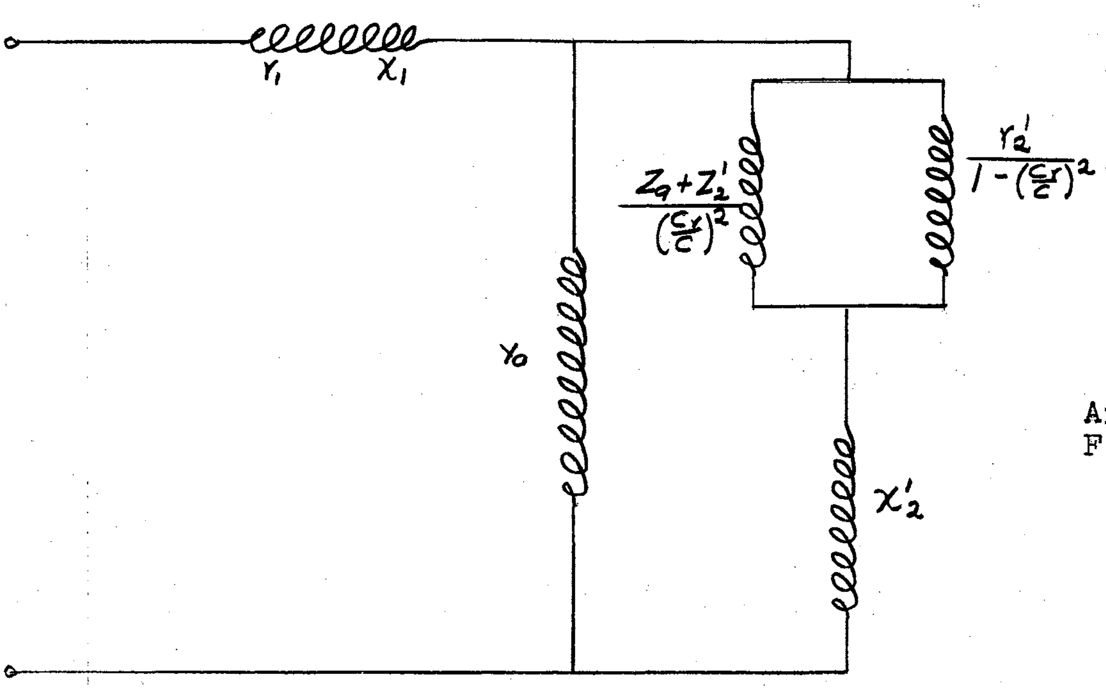
Arnold in 1909 showed a comparison of the results of the cross-field theory and the rotating-field theory:

> We can determine if the circle so obtained is identical with the circle obtained by the rotating-field theory in Chapter VII.
>
> This can be done by comparing the two equivalent circuits of Figure 79 page 126 (Figure 27) and Figure 89 (Figure 28).
>
> But the general expressions become complex. Since the circles are determined by three points, it suffices to compare the three points, standstill, synchronism, and infinite speed, where the equations are simplified. By this comparison it must be considered that the admittance, Y_a, for the rotating-field theory pertains to a polyphase winding of 1/2 the number of turns of a single-phase winding, i.e., by this theory Y_a is twice as great as for the cross-field theory. Likewise the impedance, $Z'_2 = (r'_2 - jx'_2)$, is half as large for the rotating-field theory as for the cross-field theory.
>
> For standstill we obtain from the equivalent circuit for the rotating-field theory (Figure 79) the impedance:
>
> $$Z_1 + \frac{2}{\frac{1}{Z'_2} + Y_a}$$
>
> Substituting $\frac{Z'_2}{2}$ for Z'_2 and 2 Y_a for Y_a we have,
>
> $$Z_1 + \frac{2}{\frac{2}{Z'_2} + 2Y_a} = Z_1 + \frac{1}{\frac{1}{Z'_2} + Y_a}$$
>
> and this is the expression which Figure 89 gives also for



Arnold's
Figure 79

Figure 27



Arnold's
Figure 89

Figure 28

$\frac{c_r}{c} = 0$, therefore these points coincide.

For infinite speed one obtains similar expressions as for standstill only in the place of the impedance, Z'_2, we have the reactance $-jx_2$. Hence these points also coincide.

For synchronism we obtain from Figure 79, neglecting the stator impedance Z_1, the admittance,

$$\frac{1}{\frac{1}{Y_a} + \frac{1}{Y_a + \frac{1}{\frac{r_2'}{2} - jx'_2}}} = \frac{Y_a\left[1 + Y_a\left(\frac{r_2'}{2} - jx'_2\right)\right]}{1 + 2Y_a\left(\frac{r_2'}{2} - jx'_2\right)}$$

If we substitute again $2Y_a$ for Y_a and $\frac{r'_2}{4} - \frac{jx'_2}{2}$ for $\frac{r'_2}{2} - jx'_2$ we obtain,

$$\frac{2Y_a\left[1 + 2Y_a\left(\frac{r'_2}{4} - \frac{jx'_2}{2}\right)\right]}{1 + 4Y_a\left(\frac{r'_2}{4} - \frac{jx'_2}{2}\right)} = \frac{Y_a\left[2 + Y_a\left(r'_2 - j2x'_2\right)\right]}{1 + Y_a\left(r'_2 - 2jx'_2\right)}$$

From Figure 89 we obtain for $\frac{c_r}{c} = 1$, neglecting the stator impedance,

$$Y_a + \frac{1}{Z_a + Z'_2 - jx'_2} = Y_a + \frac{1}{\frac{1}{Y_a} + (r_2' - jx'_2)}$$

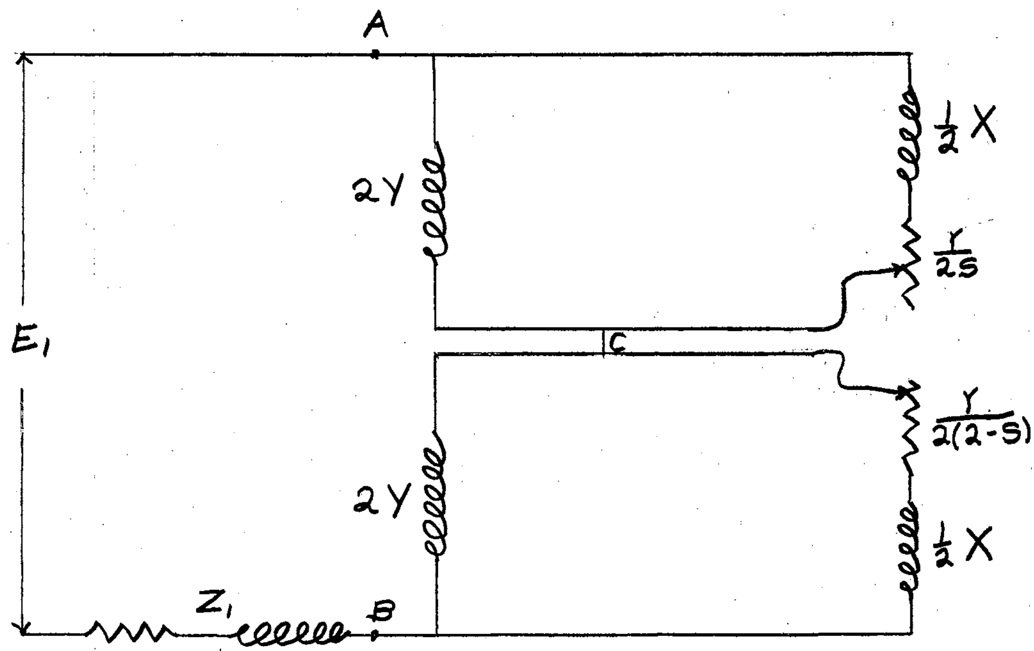
$$= \frac{Y_a\left[2 + Y_a(r'_2 - 2jx'_2)\right]}{1 + Y_a(r'_2 - 2jx'_2)}$$

Thus again the same expression; the three points therefore coincide and thereby the circuits also; both theories give the same results.[1]

THE EQUIVALENT CIRCUITS

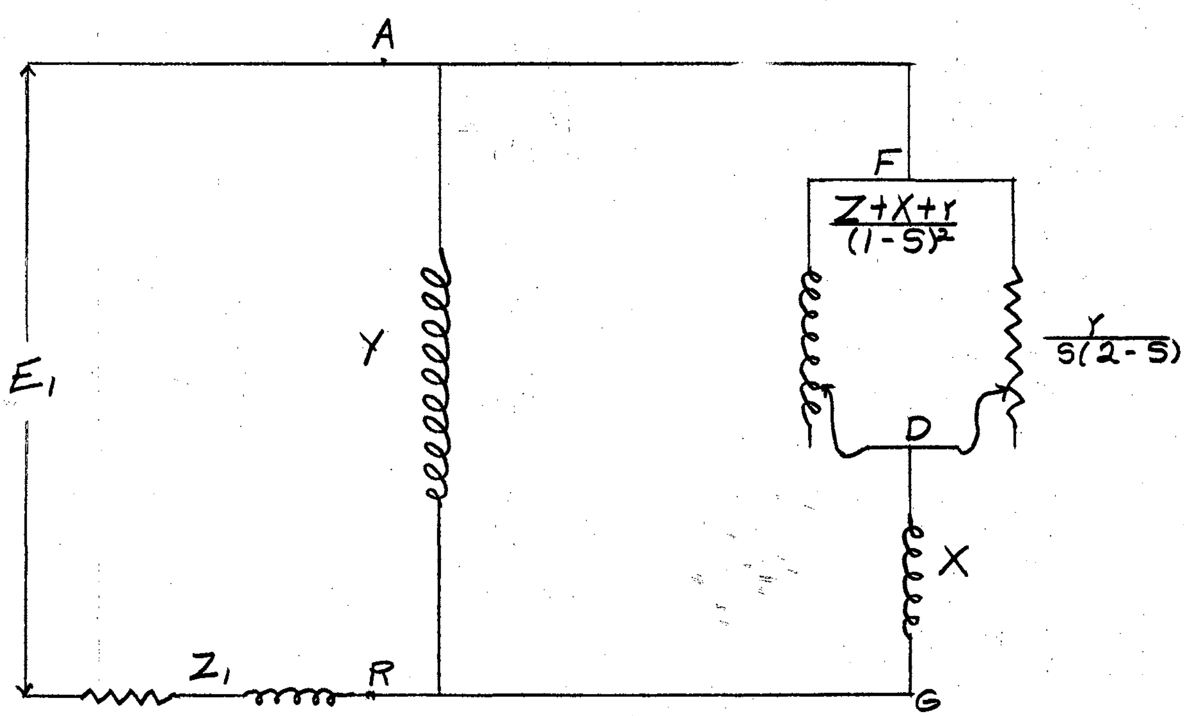
Twelve years after Arnold's book, V. Karapetoff published a

[1] E. Arnold, Die Wechselstromtechnik, Vol. 5, Part I, p. 152.



Rotating-Field Equivalent Circuit

Figure 29



Cross-Field Equivalent Circuit

Figure 30

paper which, demonstrated the equivalence of the two theories by a method based on the equivalent circuits.

The diagram for the two-motor theory shows each motor with half the resistance and reactance of the single-phase motor and each has double the exciting admittance of the single-phase motor, making it 2Y for each polyphase motor. Z_1 is the primary impedance and E_1 is the applied voltage. The following further notation is used,

$$Z = \frac{1}{Y} = \text{impedance of main exciting current in the cross-field diagram.} \tag{1}$$

$$Z' = \frac{1}{Y'} = \frac{1}{2}X + \frac{1}{2}\frac{r}{s} \tag{2}$$

$$Z'' = \frac{1}{Y''} = \frac{1}{2}X + \frac{1}{2}\frac{r}{2 - s} \tag{3}$$

In Figure 29 the impedance between A and B is

$$Z_{AB} = \frac{1}{2Y + Y'} + \frac{1}{2Y + Y''} \tag{4}$$

$$Z_{AB} = \frac{4Y + Y' + Y''}{(2Y + Y')(2Y + Y'')} \tag{5}$$

If the diagrams shown here are equivalent then Equation 5 above must also be the impedance between A and B in Figure 30.

Now, beginning with the other figure, (30), based on the cross-field theory, the admittance between the two points, F and D is,

$$Y_{FD} = \frac{s(2 - s)}{r} + \frac{(1 - s)^2}{Z + X + r} \tag{6}$$

Rearranging for a common denominator,

$$Y_{FD} = \frac{(Z + X + r)s(2 - s) + r(1 - s)^2}{r(Z + X + r)} \tag{7}$$

Combining terms,

$$Y_{FD} = \frac{1}{r}\,\frac{(Z + X)s(2 - s) + r}{Z + X + r} \tag{8}$$

$$Z_{FD} = r\,\frac{Z + X + r}{(Z + X)s(2 - s) + r} \tag{9}$$

$$Z_{FD} = \frac{rZ + \frac{r}{2}(2x + 2r + Xs - Xs)}{(Z + X)s(2 - s) + r} \tag{10}$$

$$Z_{FD} = \frac{\frac{rZ}{s(2 - s)} + r\left(\frac{X(2 - s) + r}{2s(2 - s)} + \frac{Xs + r}{2s(2 - s)}\right)}{Z + X + \frac{r}{s(2 - s)}} \tag{11}$$

$$Z_{FD} = \frac{\frac{rZ}{s(2 - s)} + r\left(\frac{\frac{1}{2}X + \frac{1}{2}\,\frac{r}{2 - s}}{s}\right) + r\left(\frac{\frac{1}{2}X + \frac{1}{2}\,\frac{r}{s}}{2 - s}\right)}{Z + X + \frac{r}{s(2 - s)}} \tag{12}$$

Substituting Z' and Z" for their equivalent values, Equation 2 and Equation 3,

$$Z_{FD} = \frac{\frac{Z''r}{s} + \frac{Z'r}{2 - s} + \frac{Zr}{s(2 - s)}}{Z + X + \frac{r}{s(2 - s)}} \tag{13}$$

By equations (2) and (3),

$$\frac{r}{s} = 2Z' - X \tag{14}$$

$$\frac{r}{2 - s} = 2Z'' - X \tag{15}$$

and

$$\frac{r}{s(2 - s)} = Z' + Z'' - X \tag{16}$$

Using (14), (15) and (16) in (13),

$$Z_{FD} = \frac{(2Z' - X)Z'' + Z'(2Z'' - X) + Z(Z' + Z'' - X)}{Z + X + Z' + Z'' - X} \tag{17}$$

$$Z_{FG} = Z_{FD} + X = \frac{4Z'Z'' + ZZ' + ZZ''}{Z + Z' + Z''} \tag{18}$$

Dividing numerator and denominator by $ZZ'Z''$,

$$Z_{FG} = \frac{\frac{4}{Z} + \frac{1}{Z''} + \frac{1}{Z'}}{\frac{1}{Z'Z''} + \frac{1}{Z''Z} + \frac{1}{ZZ'}} \tag{19}$$

$$Z_{FG} = \frac{4Y + Y'' + Y'}{Y'Y'' + YY'' + YY'} \tag{20}$$

$$Y_{FG} = \frac{YY' + YY'' + Y'Y''}{4Y + Y' + Y''} \tag{21}$$

$$Y_{AB} = Y_{FG} + Y = \frac{Y'Y'' + 2YY'' + 2YY' + 4\,Y^2}{4Y + Y' + Y''} \tag{22}$$

$$Y_{AB}^{-1} = Z_{AB} = \frac{4Y + Y' + Y''}{(2Y + Y')(2Y + Y'')} \tag{23}$$

But this value of Z_{AB} derived from the cross-field diagram is identical to the value of Equation 5 derived from the revolving field diagram. Therefore, the two equivalent circuits based on the two theories are equivalent each to the other.[2]

THE ROTOR FLUX LOCUS[3]

In both the proofs of equivalence just shown it is impossible to get a physical conception of what is happening in

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[2] V. Karapetoff, "On the Equivalence of the Two Theories of the Single-Phase Induction Motor," Journal of the A.I.E.E., XL (August, 1921), 640.

[3] Button, loc. cit.

the motor. This last demonstration has a decided advantage over the other two in this regard.

A set of reference axes is fixed in relation to the rotor. Then if synchronous clockwise rotation is assumed for the motor an observer stationed on the rotor would see the stator winding rotating at synchronous speed in a counterclockwise direction. If it be assumed for the present that rotor currents will not flow the flux from the main field will trace a double-frequency circular locus as shown in Figure 31. This flux path never goes below the horizontal rotor axis. This pattern has been discussed in the section on armature reaction.

This flux, changing as it is in magnitude and direction, will induce currents to flow in the short-circuited rotor bars which by Lenz's law will oppose every change of this flux.

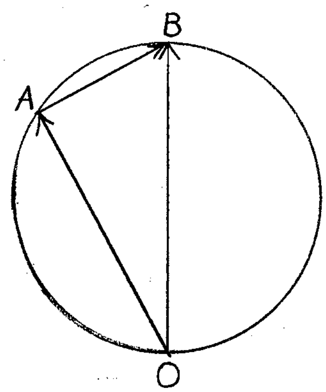
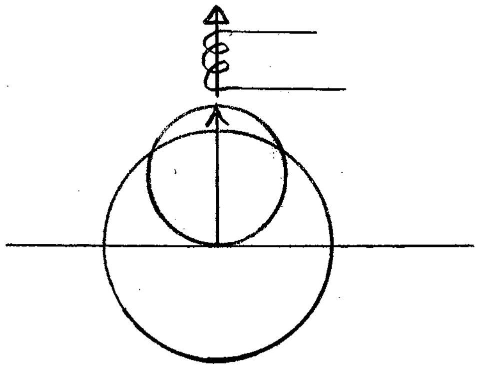


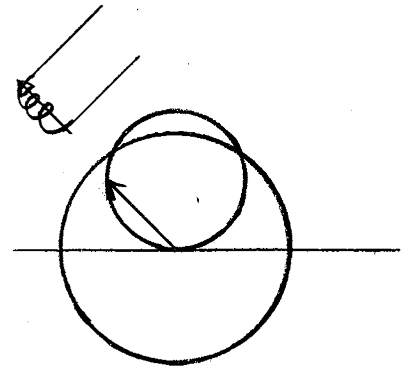
Figure 32

Beginning at maximum flux OB the stator field will rotate and the main flux will decrease to some other value, OA. Assuming that the rotor currents are 100% effective in their maintenance of the status quo - OB - then the component flux AB will be established by the rotor currents which adds to OA to give OB. At any instant rotor currents would provide the exact flux component required to combine with the main flux and maintain the maximum vertical flux, OB:

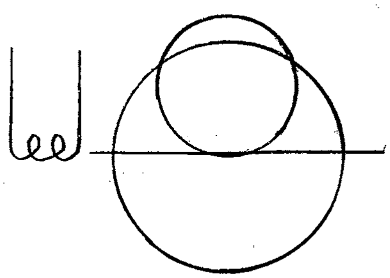
Figure 31



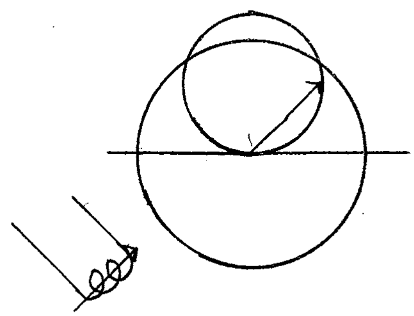
(a)
Assuming Maximum Flux At
This Initial Position



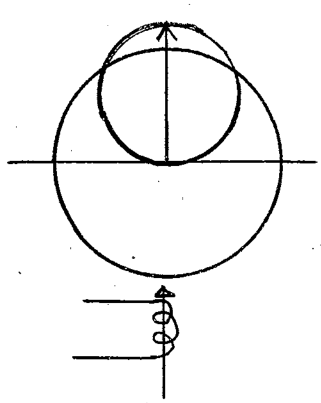
(b)
The Field Has Rotated And
The Flux Magnitude Is Less



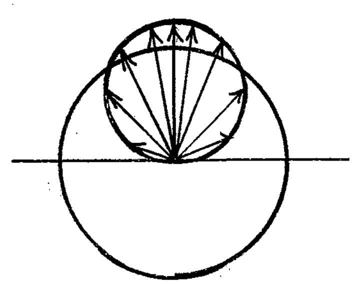
(c)
At 90° The Flux Goes
Through Zero



(d)
Now Reversed, The Flux
Continues To Rotate As it
Increases

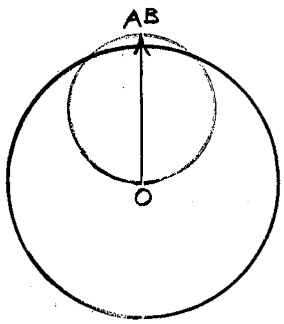


(e)
At 180° The Flux Is At
Another Peak Although The
Main Current Is Opposite
That of The First Figure



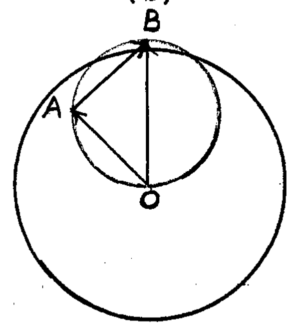
(f)
This Is The Resultant Double-
Frequency Flux Locus Set Up
By The Main Field In The
Rotor

(a)

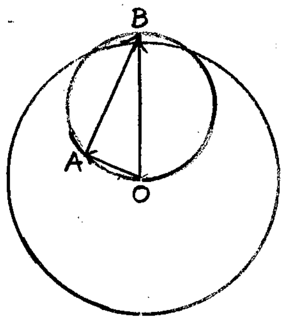


AB = 0, OA = MAX

(b)

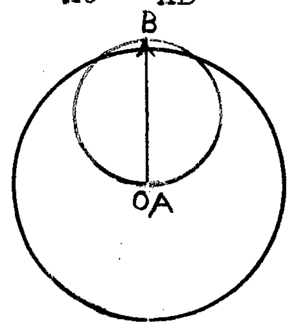


AO = AB



(c)

OA decreases AB increases



(d)

AB = MAX = OB, OA = 0

Figure 33

The rotor flux, AB, is the cross-flux of the cross-field theory always acting in time and space quadrature with the main flux. At synchronous speed, by the cross-field theory, it is equal to the main-field flux as shown here.

Now since the inductive effect of the rotor does not totally succeed in the maintenance of the vertical flux vector the resultant flux will trace a small circular locus at its tip (See Figure 34). Starting at OB the main flux moves to OA', the cross flux produced by rotor currents is A'B', and the resultant flux (not shown) is OB'. Moving around the circle makes the resultant flux become OB'', OB''', until finally when the main flux is zero the cross flux alone maintains the vertical flux OB^{iv}.

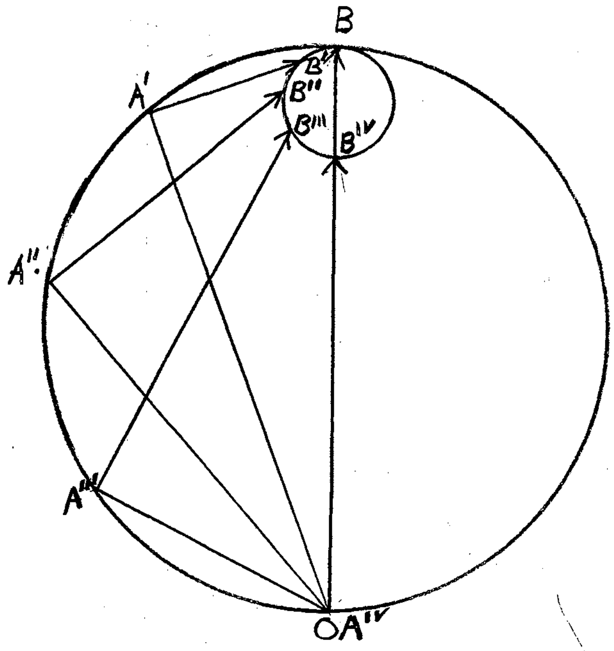


Figure 34

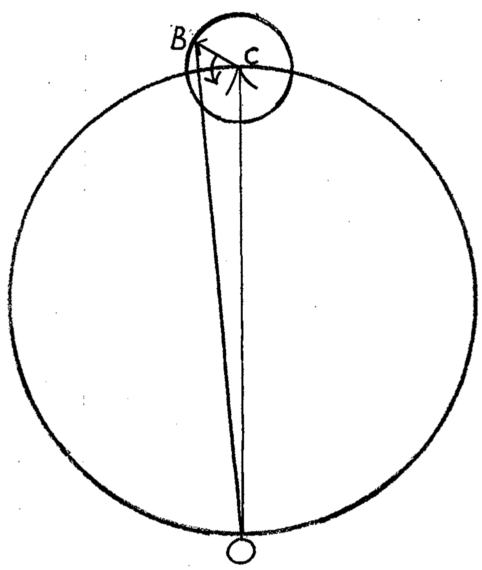


Figure 35

Another way to picture this same action is shown in Figure 35. The resultant OB is the sum at any instant of OC and CB. OC is a stationary vector while BC is rotating at twice synchronous speed in a counterclockwise direction. It will be seen at once that this is motor action expressed in the factors of the double-revolving field - a forward rotating flux (stationary with respect to a synchronously rotating rotor) and a backward rotating flux (of double-frequency at synchronous speed).

The two theories when approached in this way are seen to be equivalent. The rotation of the rotor at speeds less than synchronous results in the rotation of the vector OB clockwise around the rotor, since this flux vector must travel at synchronous speed with respect to the stator. The variations of this vector in speed

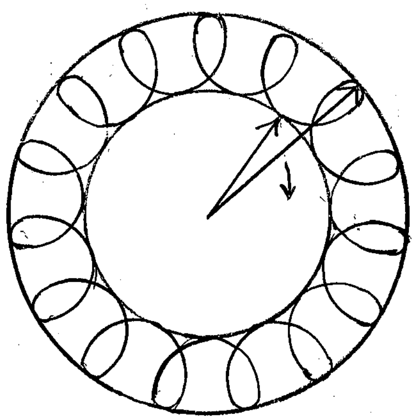


Figure 36

and magnitude give the torque pulsations characteristic of the single-phase motor.

CHAPTER IV

Following the discussions of the two theories and their equivalence, consideration of their advantages and disadvantages is in order. Since the inception of the single-phase motor those dealing with it have usually preferred one theory above the other. Steinmetz was a proponent of the cross-field theory and counted the double-revolving field passe in 1898:

> I held the same opinion some time ago, but in attempting to get results agreeing with experience from this theory of two magnetic fields of half intensity revolving in opposite directions, I have found that the theory does not represent the facts, and had to be given up, for several reasons.
>
> 1st--At standstill the magnetic field of the single-phase induction motor is undoubtedly alternating, and can be resolved into two equal and oppositely revolving fields. At speed, however, and especially at synchronism, one component has disappeared altogether and the other component is of full intensity, that is the field is a uniformly revolving field as shown by the fact that in a turn at right angles to the primary coil the same E.M.F. is induced, as in a turn parallel to the primary coil, but the E.M.F. is displaced in phase by 90°.
>
> 2nd--The current consumed by the single-phase induction motor when running light, contradicts the theory of the two oppositely revolving fields of half intensity. According to this theory the current running light should be equal to the sum of the exciting current of a polyphase motor of half impressed E.M.F. plus the current taken by a polyphase motor of half impressed E.M.F. when driven backward at full speed. This latter current, however, is frequently many times larger than the current actually observed in a single-phase induction motor at synchronism.
>
> 3rd--The torque curve of the polyphase induction motor with low resistance secondary is as shown in t in Figure 37. The torque curve of the same motor with oppositely revolving field is as shown in Figure 37 by t^1, and thus the torque curve of the single-phase induction motor should be the difference between t and t^1, or T in Figure 37. It is in reality only approximately of similar shape, in a motor with low resistance armature, but it entirely disagrees in a motor with high resistance armature. With

BACKWARD AND FORWARD TORQUE CURVES OF POLYPHASE MOTORS WITH NORMAL RESISTANCE IN ROTOR AND WITH HIGH RESISTANCE IN THE ROTOR

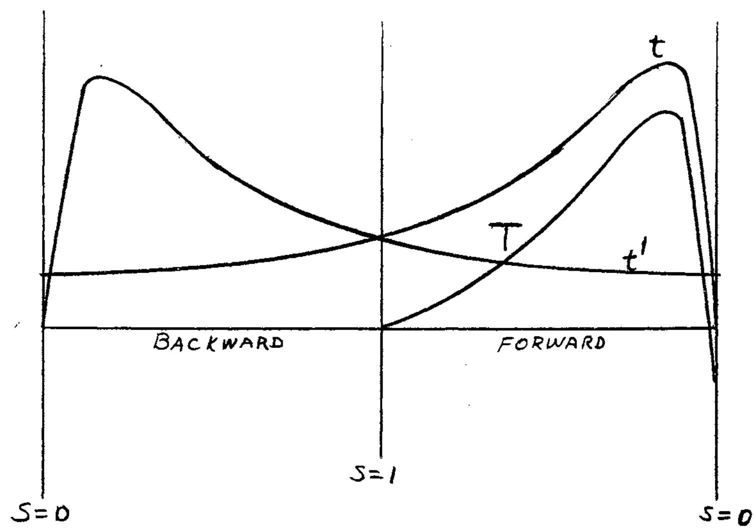
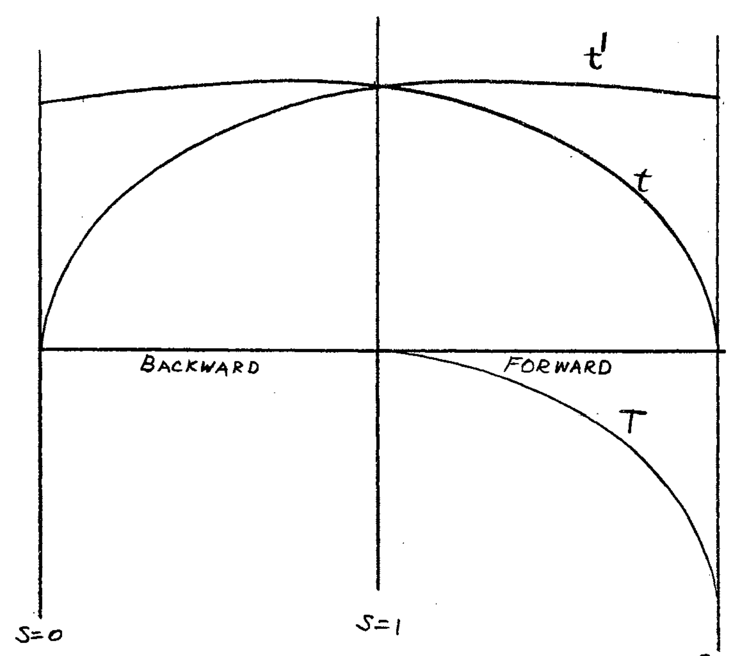


Figure 37

Figure 38



> very high resistance in the armature or secondary, the torque curve of the polyphase motor is as shown by t in Figure 38, with the maximum beyond standstill. The same motor with oppositely revolving field gives a torque curve t^1, and a single-phase motor should thus have as torque curve the difference $T = t - t^1$ as shown in Figure 38, that is, should have negative torque over almost the entire range. This does not agree with experience since we know that no matter how high the secondary resistance of the motor is, the torque still remains positive.
>
> For these reasons, the theory of two equal and oppositely revolving fields must be given up, although it is still occasionally used in publications.[1]

In the discussion that followed it was pointed out that objections numbered one and two could be answered if the rotor currents were taken into account; also in the two-motor theory the voltage on the machines is not divided equally but on a basis as unequal as 90% - 10%. The third point was not answered.

McAllister, in advancing the cross-field theory, said that by the method of Ferraris "the actual significance of the results obtained, as viewed by the average reader, are greatly obscured by the difficulty in distinguishing the imaginary from the real when the two are so closely interwoven."[2]

The two-revolving field method suffered from mistakes and misunderstanding. Behrend says:

> All manner of errors and mistakes have been made, even by leading authors in the interpretation of the theory by the two motor method. Twenty-six years ago (1895?) in a famous and otherwise brilliant book, an author assumed that the two poly-phase motors were connected in parallel and this same curious mistake has recently been reproduced in a

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[1] Steinmetz, Charles P., Discussion, A.I.E.E. Transactions, (February 23, 1898), pp. 107-109.

[2] McAllister, op. cit.,

noted textbook[3] and also in handbook of wide circulation.[4]

On the other hand protagonists of the double-revolving field have not been lacking. Their arguments seemed to begin to outweigh the ones for the other theory in middle years of this half-century. Professor M. I. Pupin is one example of this group:

> The presence of these two magnetic fields rotating in opposite directions is not a theory; it is a fact. We do not have to make an assumption of that kind at all. It is a fact from which you cannot escape. You have in each secondary winding two alternating currents. Take an oscillograph and photograph the emf in the secondary winding and you will find that it consists of two components of different frequencies; one which I call the additive frequency and another which I call the differential frequency...............By photographing suitably you will find that these two frequencies are distributed in such a way that the additive frequency..........produces a rotary magnetic field contrary to the motion of the rotor and the differential frequency produces a rotation in the direction of the rotor..............
>
> It is not necessary to assume two rotary magnetic fields produced by the stator current at all; in fact they have no physical existence; but the presence of two rotary magnetic fields produced by the rotor currents is a fact.[5]

The lines of argument have become fairly well stabilized in the last few years. This exchange between P. L. Alger and L. M. Perkins is an example:

> (ALGER) The cross-field theory is, according to all the books, precisely equivalent to the revolving-field theory, so that any result may be obtained by either method.. This equivalence of the two theories was again brought out by Mr. Kimball and myself in our recent paper on "Torque

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3 Lawrence's first edition of <u>Principles of Alternating Current Machinery</u> (1915) has this mistake. See footnote 8 Chapter I.

4 Behrend, B. A., <u>The Induction Motor</u>, p. 211.

5 Pupin, M. I., Discussion, <u>A.I.E.E. Transactions</u>, (April 10, 1918), p. 685.

Pulsations of Single-Phase Motors", A.I.E.E. Journal, December, 1924, page 1142, when we derived the same formulas for the double-frequency torque by the two methods independently. However, there is one phenomenon, that of eddy currents in the squirrel-cage winding, that can be better taken into account by the revolving-field theory. If a double squirrel-cage winding, or its equivalent is employed, the secondary resistance is much higher at double, line frequency than at slip frequency. Thus, different values of secondary resistance must be used for the forward and backward field currents in the revolving-field theory. But in the cross-field theory, the secondary current in either axis is considered as a whole, instead of being divided into its slip frequency and double-frequency components. Thus the resistances used in the secondary circuits of the cross-field theory must be intermediate between the high and the low value, but just what values they should have is a problem that has not been solved. In short no method has been published of properly taking into account eddy currents in the secondary by the cross-field theory, and so in this respect the revolving-field theory is superior.[6]

(L. M. PERKINS) A good illustration of the trouble that you get into when you consider the rotating-field method is apparent when you wish to determine the effect of increasing the reactance of the secondary winding. According to the rotating-field method, there are two separate currents in the secondary winding, one of slip frequency and one of (2-s) frequency. But what happens if you increase the reactance as far as it affects the slip frequency? There is no large change, but according to the normal way of looking at it the current of the (2-s) frequency would be cut out almost entirely. It is rather hard to see how the performance is really affected, unless you follow this method, which shows that all currents in the rotor are really of line frequency and that, therefore, an increase in reactance of the secondary will cut down or will affect all rotor currents; the slip frequency current, as well as the (2-s) frequency current due to the oppositely rotating field.

The original purpose of this paper was to bring out the theory of the single-phase induction motor in such a way that the average engineer could follow it, although of course it is still very complicated. There are no assumptions as are necessary when you consider the oppositely-

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[6] Alger, P. L., Discussion, A.I.E.E. Transactions, 44 (May, 1925), p. 523.

rotating-field theory.[7]

SOME WEAKNESSES OF THE CROSS-FIELD THEORY

The cross-field theory has disadvantages that limit its usefulness. Those mentioned often are:

1. Its complexity. In theory the motor action is not easy to visualize and follow. In practice its calculation becomes tedious and involved.
2. Its foundation on a mathematical abstraction.
3. Its failure to take saturation into account.
4. Its neglect of harmonics.
5. Its blindness to two frequencies in the rotor.

It is of interest to note in connection with its complexity that one author in the integration of an expression used 24 sheets of letter size paper to perform the integration!

Many who use the theory are some of the ones who point out difficulties. Puchstein and Lloyd say,

> It is not the intention of the present writers to advance the cross-field theory as being wholly superior. Each method has certain advantages and defects. For instance, the cross-field theory is blind to the existence of two separate skin effects in rotor conductors and to a pulsating torque superposed on a steady torque. (This latter difficulty could be avoided by the use of instantaneous instead of vector quantities.) But difference in saturation and core losses in the two axes are much easier to visualize with the cross-field theory.[8]

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[7] Perkins, L. M., Discussion, A.I.E.E. Transactions, 44 (May, 1925), p. 523.

[8] Puchstein, A. F. and Lloyd, T. C., "The Cross-Field Theory of the Capacitor Motor," A.I.E.E. Transactions, 60 (February, 1941), p. 58.

Veinott is critical of this statement in his discussion of their paper: "The authors' statements that the cross-field theory is blind to the existence of the pulsating torque is a little surprising since Kimball and Alger developed formulas for the pulsating torque from the cross-field theory."[9]

E. J. Schaefer also criticizes the statement of Puchstein and Lloyd, saying,

> There are certain advantages stated by the authors to be inherent in the cross-field theory, however, which are based on assumptions that are not rigid and therefore do not actually provide the advantages mentioned.
>
> I believe, for instance, that there is too much of a tendency to regard the two axes into which the machine is broken down as actual physical fact rather than analytical artifice. I believe that this tendency is evidenced by the statement that it is easier to visualize differences in saturation and core losses in the two axes by the cross-field theory.
>
> It must not be forgotten that the fluxes in the two axes in fact do <u>not</u> exist separately and that in the presence of unequal saturation the fluxes in two axes cannot be assumed to remain independent of each other which is a basic assumption on which this method of analysis depends. To take into account different saturations in the two axes in making calculations is an attempt to take out of the formulas more than was included in the original assumptions and therefore cannot give correct results.
>
> In visualizing these component fields as separate and physical entities there is also the danger of considering that one or the other represents the maximum flux axis. This, of course, is not the case, for, at some angle between the main and start axis there will be found, by properly adding the two components an axis of maximum flux and 90° displacement from this an axis of minimum flux.[10]

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9 Veinnot, C. G., Discussion, A.I.E.E. Transactions, 60 (June, 1941), p. 668.

10 Schaefer, E. J., Discussion, A.I.E.E. Transactions, 60 (1941), p. 667.

In a recent textbook Liwschitz-Garik and Whipple do not include the cross-field theory, using only the two-motor method for two reasons:

> First, it is based on the same fundamental conceptions on which the polyphase motor is based..........Secondly, it makes it possible to treat the harmonics in the same way as in the polyphase machine.[11]

SOME WEAKNESSES OF THE DOUBLE REVOLVING FIELD THEORY

The first four of the items listed for the cross-field theory are also failings of the double-motor theory:

1. Complexity.
2. Foundation on a mathematical abstraction.
3. Saturation neglected.
4. Harmonics neglected.

In addition other points have been mentioned in the quotations from Steinmetz and Perkins. L. W. Buchanan offers this:

> The apportioning of iron loss and the in phase magnetizing current has caused quite a bit of trouble in the calculation of capacitor motors by the double revolving-field method.[12]

The double-revolving field is more complex than the cross-field in the case of the A C commutator motor.

In spite of these shortcomings the double-revolving field theory has gained ground because of several things. The development of the use of symmetrical components since 1918 and its

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[11] Liwschitz-Garik, Michael and Whipple, Clyde C., Electric Machinery, Volume II, p. 271.

[12] Buchanan, L. W., Discussion, A.I.E.E. Transactions, 60 (1941), p. 663.

natural adaptation to the double revolving-field theory has led to greater gains for this theory in popularity. Further explanation of these gains is made by Morrill:

> It might be questioned why two approaches should be used to attain the same end and it might further be questioned as to whether one of these theories does not have important advantages over the other. In view of these natural questions I should like to consider what a good theory should be expected to do.
>
> In my opinion some of the advantages which we should expect of a good motor theory are:
>
> 1. A good theory should present an accurate picture by means of which mathematical expressions for the current torque, etc. may be obtained.
>
> 2. It should furnish the simplest possible picture concerning the important phenomena consistent with the accuracy desired.
>
> 3. The theory should furnish means for readily considering the effects of minor phenomena when such consideration becomes necessary or desirable.
>
> 4. It should, if possible, present a picture readily susceptible to high-handed simplification for the determination of limiting conditions such as no load, standstill, balanced operation, etc.
>
> 5. The theory should preferably not change form or require a new mathematical analysis when slight normal modifications of structure are made.
>
> 6. Finally, if with no extra complication, the theory can show the relation between the motor in hand and motors of other types this is desirable, particularly in an operating design department where motors of many types and connections must be handled simultaneously.[13]

Morrill then proceeds to show that on every one of the six points the double-revolving-field theory has some advanatage over the cross-field theory.

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13 Morrill, Wayne J., Discussion, A.I.E.E. Transactions, 60 (1941), pp. 669-670.

OTHER THEORIES

Because of the serious criticisms and faults of the two classical concepts two beginnings have been made in recent years which offer a new approach.

The most consistent critic of the old way has been for many years Edward Bretch. His letters to the electrical journals have appeared frequently to point out and discuss the weaknesses of the corss-field and double revolving-field theories. From two of these letters criticisms concerning the flux addition of the two motor concept and the current lag in the speed axis of the cross-field theory have been quoted (page 12 and 38). Another objection was raised by him:

> ......an attempt to apply the formula developed by the cross-flux theory stating that the quadrature flux varies directly with the speed, results in contradiction. Tests show that not only does the quadrature flux not increase with the speed above synchronism, but that it really decreases with overspeed. Thus the straight-line relation between rotor speed and quadrature flux assumed in the cross-flux theory does not exist.[14]

In addition to calling attention to the limitations of the classical methods he has advanced a theory based on the constant synchronous flux existing in the air-gap of the single-phase motor. It was published under the title, "Amortisseur Action of the Squirrel Cage."[15]

Another, earlier, paper showing the action of the single-

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[14] Bretch, Edward, Discussion, A.I.E.E. Transactions, 64 (1945), p. 943.

[15] Bretch, Edward, "Amortisseur Action of the Squirrel Cage," Electrical Engineering, 65 (1946), p. 255.

phase motor in terms of a constant air-gap flux was Button's first paper on the rotor flux locus concept described in Chapter III, page 58.[16]

A comparison of the two views shows some remarkable similarities in them. The conclusions given by Bretch are listed below in the left column. Remarks by the present author concerning Button's work are on the right:

The synchronous or near synchronous squirrel-cage in the single-phase motor operates as an amortisseur opposing and suppressing sudden flux changes either as to strength or position with relation to the rotor conductors	In Chapter III diagrams, Figure 33 and Figure 34, illustrating the effect of the rotor currents in maintaining a constant flux were shown. This agrees exactly with Bretch's idea.
The mmf impressed on the synchronous rotor is unidirectional and pulsating at double frequency between zero and a maximum.	If by "unidirectional" Bretch means that the flux does not reverse, then the exact situation he describes is illustrated by Figure 33, Figure 34, and Figure 35 showing Button's theory.
The flux produced in the synchronous rotor by this pulsating unidirectional primary mmf is leveled off and maintained substantially constant throughout the cycle by the amortisseur action of the squirrel cage.	Same as Button.
The induced amortisseur rotor currents, leveling and sustaining the flux constant throughout the cycle are due to the transformer action, are of double-frequency and	Button's work agrees with this except that he deals exclusively with components of rotor current rather than total current. The last sentence from Bretch concerning

[16] Button, C. T., op. cit., and "A Suggested Rotor Flux Locus Concept of Single-Phase Induction Motor Operation," Electrical Engineering, 56 (March, 1937), pp. 331-332.

are equal to one half the primary exciting current. The synchronous mechanical rotation of this amortisseur sustained rotor flux produces the rotary field.

mechanical rotation does not have a counterpart given explicitly by Button, but the concept is there.

The speed action opposing relative motion between the flux and the rotor conductors, tending to pull the rotor into synchronism with the primary exciting impulses, produces a torque which pulsates at double frequency. This synchronously rotating and substantially constant rotating flux in its quadrature position is the so-called cross-flux.

This relative motion is shown in Figure 36 as given by Button. The in-phase component of the currents so generated in the rotor provide the torque in reaction with the air-gap flux.

The speed action producing the torque and the transformer action sustaining the flux constant take place in space quadrature in the rotor and, as they do not react on each other can operate either singly or simultaneously.[17]

Button's work substantially agrees with this.

These two methods, although different in their development, are very much alike in their conclusions. Their advantages are summed up by Button:

> Present methods of calculating the performance of single-phase induction motors are quite tedious, and probably do not convey to the mind of the practical engineer a logical concept of the operation of the motor such as is the case with generally used methods of polyphase motor design calculation. It seems desirable to have a unified and co-ordinated theory for single phase induction motors including formulas which are simplified as much as possible (with justifiable approximations) and which convey unabstruse concepts. The formulas developed may be used as the basis of a routine calculation procedure.
>
> An advantage of formulas of this type is that they deal

[17] Bretch, *loc. cit.*

with real values-actual rotor current in a rotor bar in amperes, and so on. The value of rotor impedance does not change in using the same rotor with two different stator windings for example, as it does when equivalent primary values are used. Flux densities in the cross field axis and other similar values are explicitly evident.

In applying the equations developed, it is necessary first to determine motor constants by established methods (primary impedance, magnetizing current and flux value with open-circuited rotor, and effective rotor resistance and reactance for a turn consisting of two rotor bars 180° apart with connecting end rings common to all turns).[18]

Others concur that this constant flux theory is helpful:

Mr. Button's approach is interesting and helpful......As the analysis unfolds the reader discovers that neither concept is used exclusively, but he does get a helpful picture of just what is happening in the rotor bars. The picture is undoubtedly clearer than that obtained from the usual presentation of the cross-field theory in which consideration is given not to the actual currents and voltages in the individual rotor bars but to those same currents and voltages referred to the stator, wherein the only frequency is line frequency.[19]

However Puchstein holds little hope for the immediate adoption of a new theory:

During the past two years a number of discussions on the theories used to explain and calculate the behavior of the single-phase induction motor have appeared in AIEE publications. The general tenor of these is to the effect that, though the two principal methods in use give accurate numerical results for performance, they are in error or inadequate in other respects. As some of the statements printed are misleading and erroneous, an attempt to correct them should be made.

The methods in question are the double-revolving field (in either classical or symmetrical co-ordinate dress) and the cross field theories. Each of these has its sphere of usefulness, and both admittedly have certain limitations. Thus, neither is simple, and neither takes into account

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18 Button, loc. cit., 60, 511.

19 Veinott, C. G., Discussion, A.I.E.E. Transactions, 60 (1941), p. 664.

> the disturbing effects of saturation or harmonics. The first theory easily can be made to include skin effects in the rotor, while the second in its simple vector form is blind to the existence of two frequencies in the rotor, and so forth. Now, as no simpler methods are on the horizon, we shall have to live with them for some time yet. Most of the questions at issue have been settled long ago........
>
> In conclusion, there is no indication yet that the "amortisseur theory" will be more illuminating, more exact, easier to apply, or more comprehensive as a tool than are the older methods. In its present form, the amortisseur theory requires the interposition of graphical work, which makes it less rapid, less convenient, and in other respects not so incisive and up to date as those now in use. The latter are suitable for the slide rule, avoid the use of drawing instruments, and are (theoretically) precise in application.[20]

CONCLUSIONS

Any study of single-phase motor theory leads immediately to the fact that there is wide divergence of opinion on the subject among those who work with motors. It began in the last years of the 18th Century when the motor was introduced. At that time Fresnel's theorem developed into the double-revolving field explanation and the cross-field theory came immediately into competition with it. Ever since, the proponents of concepts have freely contributed arguments on the matter.

Both theories are based on sound mathematical abstractions, which although they give theoretically precise and equivalent results, lead immediately to difficulties in actual practise. For example, the double-revolving field theory holds the single-phase motor to be equivalent to two polyphase fluxes, oppositely

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20 Puchstein, A. F., Letter, Electrical Engineering, 66 (February, 1947), p. 208.

rotating, combining to form a single uniaxial pulsating flux. The present work showed this to be sound, mathematically. But when attempts were made in the laboratory to secure this condition in a single stator some difficulty was encountered. The three-phase system was abandoned in favor of the two-phase system which gave successful results only under carefully controlled conditions of supply; that is, separate voltage sources for the two phases, maintenance of equal voltages from these sources and high impedance in the sources to reduce the high current flow.

On the other hand, results from two three-phase fluxes in two separate stators with rotor shafts coupled together gave results highly consistent with the theory. Lamme's work on this test is very interesting. It is evident that the difference in saturation in the two machines and their equivalent single-phase motor will lead to differences in the results. Also the one-phase machine has line voltage continually applied, whereas the voltages on the two machines vary considerably, with speed, but never are as high as line voltage.

Difficulties in securing other practical results consistent with the theories and misunderstandings of the assumptions involved have led to other arguments. For example it is necessary that some assumption be made regarding the lag of the speed current behind its voltage in the cross-flux theory. If this is not done in the manner suggested by Hall or by use of the equivalence of the rotor to a commutated winding with brushes shorted, then the currents will shift the magnetic axis of the cross-field against the main field. As pointed out in Chapter II very few authors mention this assumption.

Both theories have their uses, advantages, and disadvantages. Both will continue to be used in spite of their limitations. It appears that the cross-field theory, at first most prevalent in the literature has been becoming less popular in recent years. The most promising hope for a new attack is in the theories advanced by Button and Bretch. These have both been used in solution of actual problems of performance. Development of these may lead to a much simpler theory for the single-phase induction motor.

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