A COMPARISON OF THE RATINGS OF SINGLE-PHASE AND THREE-PHASE INDUCTION MOTORS

By

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PREFACE

Since its practical inception by Nikoli Tesla in 1888, no one part of our electrical industry has enjoyed so great an industrial and domestic expansion and application as has the principle of the induction motor. Yet with this tremendous expansion and development, much is to be learned and recorded with respect to its ultimate design and the predetermination of its operating characteristics.

That this is true is most vividly brought out in a paper by Messrs. T. C. Lloyd, P. H. Trickey, W. R. Hough, and C. P. Potter entitled, "Is there a Doctor in the House?" (Unsolved motor design problems suggested for Postgraduate Theses) published in the September, 1949, issue of Electrical Engineering. Quoting in part from this paper, the electrical machinery field as a whole is reviewed as follows:

> One sometimes hears the statement that the electric machinery field is so well established that relatively few opportunities exist for further investigations and developments. Nothing could be further from the truth. In the electric motor business alone, many problems appear which are not solved rigorously but are by-passed by estimates based on experienced judgment; or they are solved by empirical data in use in design offices, but not generally available in the literature. In either case, further study is justified.

Specifically, with respect to comparisons of single-phase and polyphase motors, the authors further state:

> Comparisons. The following comparisons are for single and polyphase motors.

1. A single-phase motor is built in exactly the same stator and rotor laminations and with the same stack and rotor cage as a polyphase motor. Suppose each is designed with the same flux per pole. What are the relative values of maximum torque obtained from each? (Some assumptions must be made concerning relative conductor length and winding factors.) On the basis of these maximum torques, what relative horsepower ratings should be assigned to each?

Because of the increased magnetizing currents of single-phase motors, the circular mils per ampere of the two will differ. On the basis of equal copper loading in both, what full-load horsepower ratings should be assigned to each?

2. Suppose the foregoing single-phase motors were built as capacitor-start induction-run types. Does good design result from the use of the same rotor resistances in both cases? If not, what should be the relative resistances of squirrel cages for single-phase and polyphase motors built in the same laminations and stacks?

3. It seems to be accepted as more or less axiomatic that double-layer lap windings are best for polyphase and concentric type are best for single-phase motors. For single-phase motors what are the relative merits of concentric and lap windings? This could be investigated on the bases of relative winding parameters, copper weights, and resulting performance.

It is with respect to the first comparison that this paper is concerned, namely the relative horsepower ratings of single-phase and polyphase induction motors designed with the same stator and rotor and with the same flux per pole.

It is obvious at the outset that any conclusions that may be drawn by the writer must of necessity be very general in as much as each particular design may have innumerable possibilities.

ACKNOWLEDGEMENT

The writer wishes to express his sincere appreciation to Professor Charles F. Cameron for his expert guidance and suggestions in the preparation of this thesis.

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CHAPTER I

INTERPRETATION OF THE PROBLEM AND GENERAL DESIGN

In attempting to analyze a problem of this type it would be well to first state the problem and to then give an interpretation. The problem[1] is as follows:

> A single-phase motor is built in exactly the same stator and rotor laminations and with the same stack and rotor cage as a polyphase motor. Suppose each is designed with the same flux per pole. What are the relative values of maximum torque obtained from each? (Some assumptions must be made concerning relative conductor length and winding factors.) On the basis of these maximum torques, what relative horsepower ratings should be assigned to each?
>
> Because of the increased magnetizing currents of single-phase motors, the circular mils per ampere of the two will differ. On the basis of equal copper loading in both, what full-load horsepower ratings should be assigned to each?

By the same stator and rotor laminations and the same stack and rotor cage is meant as the use of the same physical stator in both the single-phase and polyphase design. The stator may or may not be of the open type but in any case the effects as to heat dissipation would not exactly be the same whether the stator is wound with a single or a polyphase winding because of different coil distribution. Since the exact physical stator is used in each, there will be a definite slot dimension and shape which as far as the slot itself is concerned will present the same per conductor reactance in each case except for the end

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[1] T. C. Lloyd, P. H. Trickey, W. R. Hough, C. P. Potter, "Is There a Doctor in the Ouse," Electrical Engineering, 68, Part II (September, 1949), 759.

turns.

It can also be assumed that with the same lamination thickness and outside dimensions of the stator, the core loss for the same flux per pole, in both the single-phase and polyphase design, will be the same. The core loss made up of the hysteresis and eddy current losses, as is later shown is proportional to a power of the maximum flux density. For the same flux per pole and with a sinusoidally distributed flux in space in the design in each case, it is reasonable to make the above assumptions.

The stator will be wound in the one case with a polyphase (three-phase) winding and in the other with a commonly used type of single-phase winding. In the former, a distributed, double layer, diamond shape coil, lap winding is used. Versatility as to arrangement and connection and saving in cost to manufacture are the main advantages of this type winding for three-phase machines. The full diamond shape coil being preformed is a considerable saving in labor. Any three-phase machine may be connected either delta or wye, however, in most cases the wye connection has the advantage of eliminating multiples of the third harmonic with the neutral ungrounded, as well as generally giving a better wave form.

A typical three-phase distributed, double layer, lap winding is given as an example. Consider the stator as having 36 slots and is to be wound as a four pole, 1800 rpm synchronous speed induction motor. Thus 36/4 gives 9 slots per pole and 9/3 produces 3 slots per pole per phase. A full pitch winding will be shown. For some cases a fractional slot winding may have been used. Using diamond shaped coils the stator winding diagram will appear as in Figure 1. The beginning and ending of each coil as can be seen is easily accessible for connection. In the case of the single-phase machine the most generally used winding is the concentric type. The single-phase concentric winding has the advantage like the diamond shape lap winding of being flexible in its use.

Using the same 36 slot stator and winding it for a single-phase operation with a concentric winding will produce a layout as shown in Figure 2. As can be seen from the diagram ample slot space is left for placement of a starting winding. This center space can not be economically used for part of the main winding because the effective turns of the coils are reduced when the pitch of the coil is less than full pitch and the winding distributed. Considerable saving in copper and labor may be had with only a slight reduction in voltage. This will be discussed in detail in Chapter 4.

To determine to a certain degree the type of flux curve the previous windings will produce, a flux plot for each winding will be calculated and shown using the method[2] outlined in <u>Connecting Induction Motors</u> by A. M. Dudley. In the former case the distributed lap winding will be investigated for a 36 slot stator as before.

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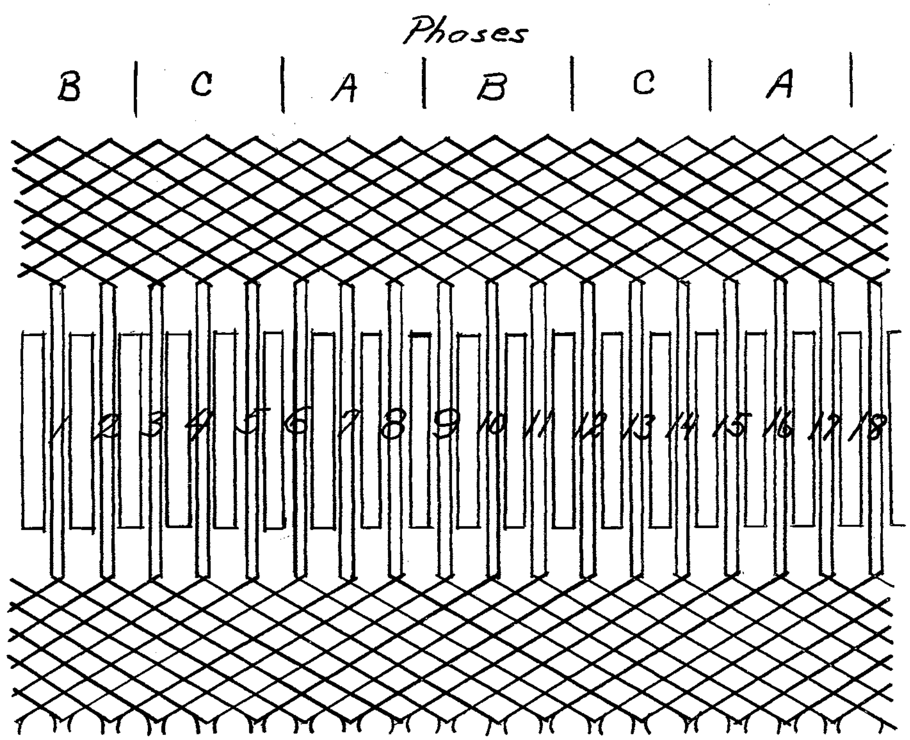
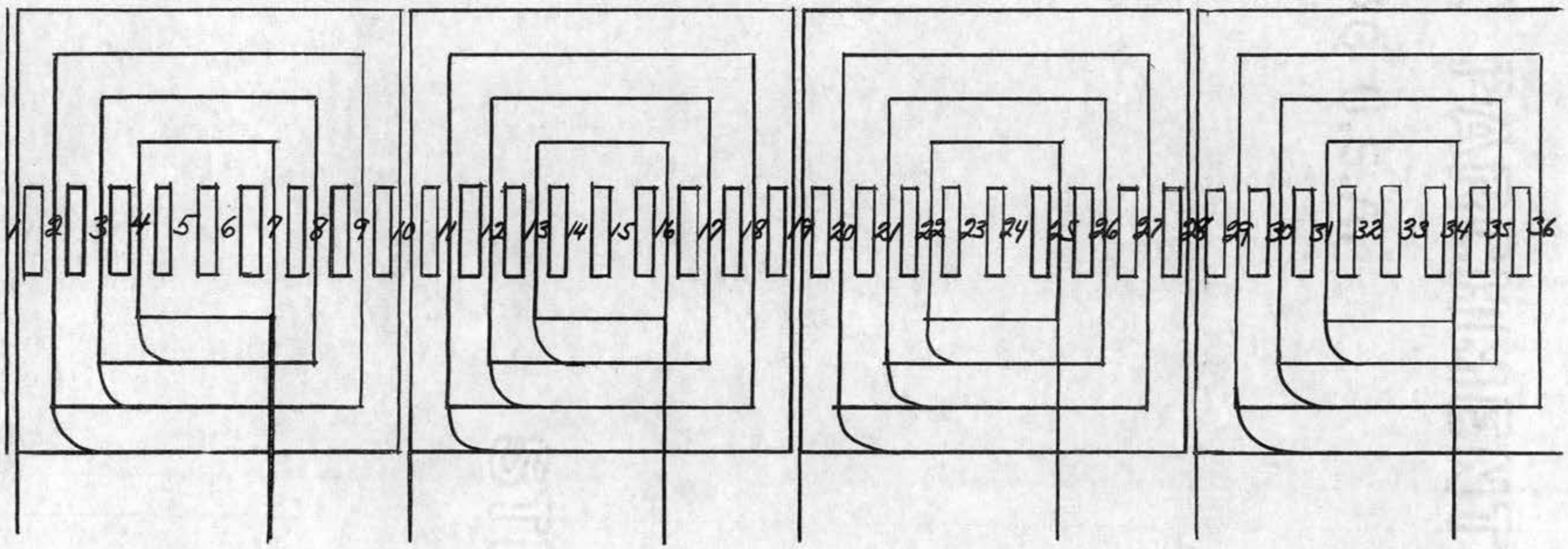
[2] A. M. Dudley, <u>Connecting Induction Motors</u>, p. 115.



3Ø distributed lap winding, double layer
36 slots, 4 pole, 3 slots/pole/phase
(only 18 slots shown)

Figure 1

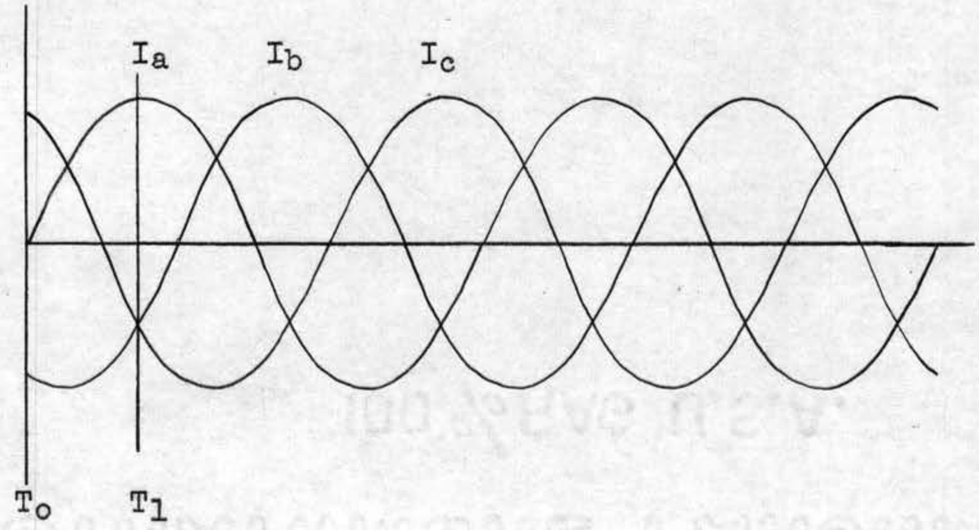


Single-Phase Concentric Winding

36 slot, 4 poles, 4 coils/pole

Figure 2

In order to plot the flux, one particular instant of time must be chosen and the relative values of the currents in the three-phase determined. It also must be noted that in laying out the winding of the motor, one phase must be reversed so as to produce the rotating field. Assume the three symmetrical currents I_a, I_b, I_c as shown in Figure 3, are flowing in the three respective phases of the motor as steady state values. If the time t_1 is chosen for the flux plot, the instantaneous value of the phase currents will be I_a equal plus I maximum, I_b equal minus one-half I maximum and I_c equal minus one-half I maximum. These values are obvious since the algebraic sum of the currents of a balanced three-phase system will always be zero.



Balanced Three-Phase Currents

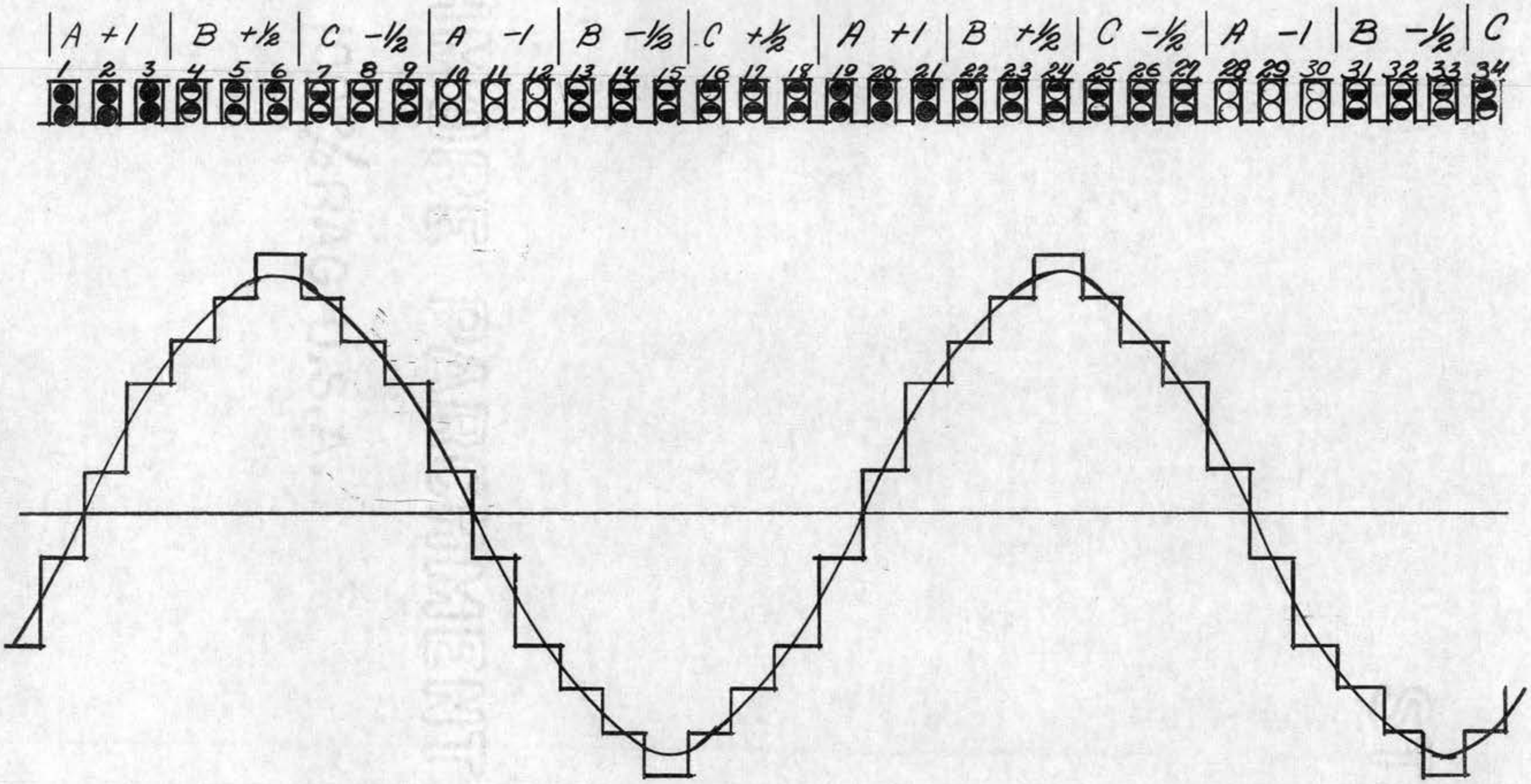
Figure 3

As shown in Figure 4, the winding layout is a full pitch, three-phase winding with phase b reversed. The currents in the respective phases are as at time, t_1, in Figure 3.

By using the full pitch winding there is no overlapping of the phases, that is, slots 1, 2, and 3 are used only by phase a, slots 4, 5, and 6 by phase b, etc. The fully shaded conductors indicate maximum positive current at the particular instant of time whereas the shaded top half indicates one-half maximum negative in line with the currents at time t_1 above. The blank conductors indicate maximum negative current. In all conductors positive and negative representing only the relative directions of the phase currents.

To investigate the field plot, start at any slot and proceed in one direction around the stator. Each time a slot is crossed the magnetic potential is raised or lowered, depending upon the direction and in proportion to the number of conductors in the slot and the current flowing. For simplicity in Figure 4 each phase is assumed to have only one conductor per turn. The procedure will produce the stair step flux wave shape, but the rotor currents and flux will smooth the curve as is shown. By chording the winding the top of the flux wave curve would be flattened slightly, thus further approximating the sine wave curve. It should also be noted that a concentrated winding would produce a greater area under the flux curve, but the wave shape would be almost square and thus contain many harmonics.

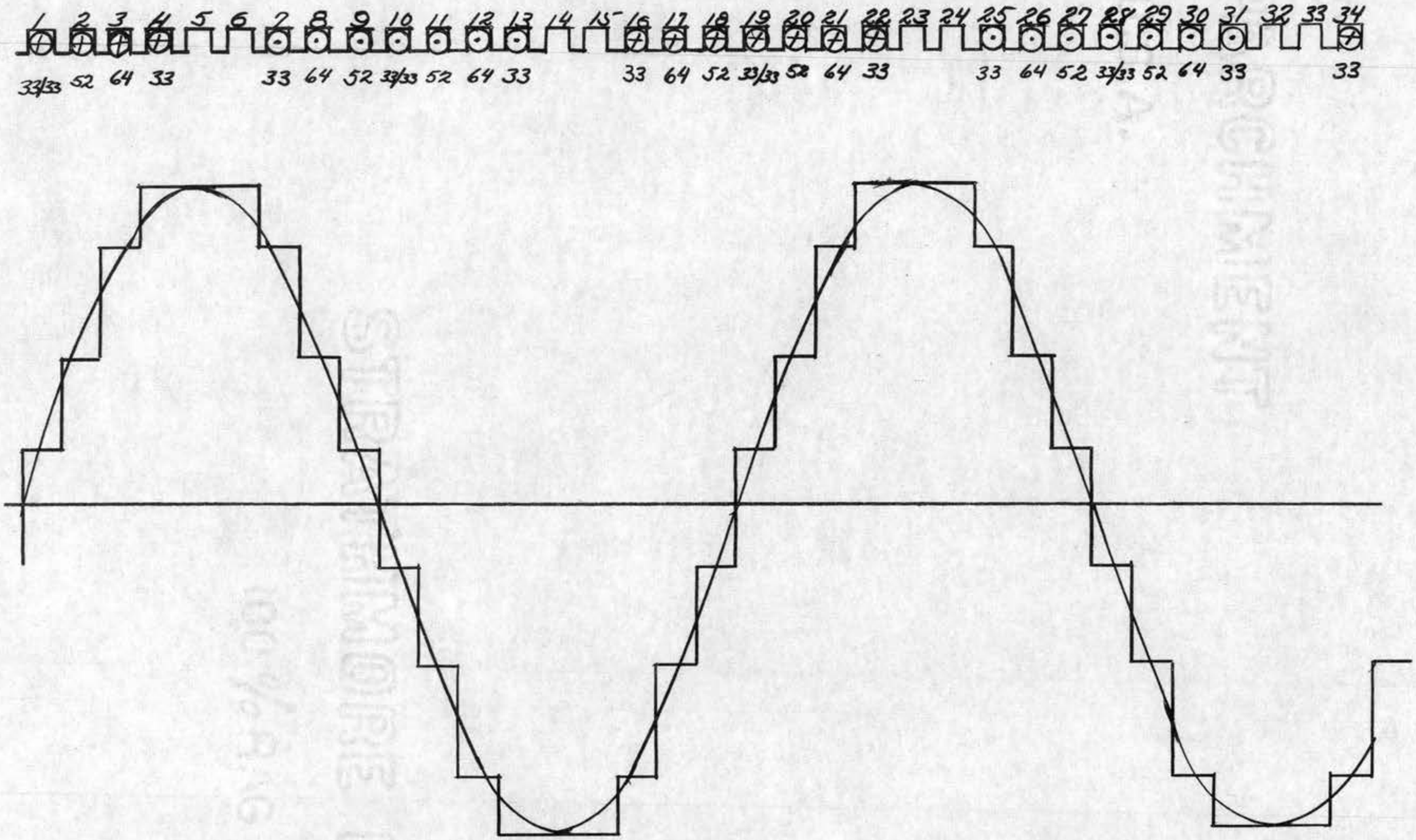
By the same procedure the magnetic field plot for a single-phase machine with a concentric winding is given in Figure 5.



Magnetic Field Plot

36 slot, 3Ø, 4 pole, lap winding

Figure 4



Magnetic Field Plot
36 slot, 4 pole, single-phase concentric winding
4 coils/pole 33, 52, 64, 33, turns per coil

Figure 5

The concentric winding in this figure is an actual winding taken from a Robbins and Meyers 1/4 HP, 4 pole, 36 slot, single-phase, capacitor start, induction run, induction motor with 33, 52, 64, and 33 turns per coil. While the turns are not exactly sinusoidally distributed in space the resulting flux wave is a very good approximation of the sine wave. As in the polyphase machine winding the stair step flux plot would smooth to approximately the curve of the sine wave as shown.

Of general significance at this point is the fact that the windings of the polyphase winding as shown in Figure 1 are excited by a three-phase voltage source with balanced voltages 120 electrical degrees out of phase. This coupled with the fact that the phase belts of the winding have 120 degrees space separation in the stator will produce a very nearly constant flux value rotating at synchronous speed around the periphery of the stator. In contrast, however, the single-phase windings of Figure 2 are placed 180 degrees apart in space and excited by a single-phase voltage source. It is obvious, therefore, that the stator winding alone, when excited, will produce only a pulsating flux, stationary with respect to the stator. Such is not the case with the rotor in place and rotating at synchronous speed. By means of the cross-field theory, which will not be discussed here, it is shown that a quadrature flux will be set up by the induced currents in the rotor. This flux will be in both space and time quadrature with respect to the main winding flux and will thus produce a synchronously rotating flux. At synchronous speed of the rotor, the main field flux and the quadrature flux will have very nearly the same magnitude. When this resultant flux wave is equal to the resultant flux of the three-phase machine, it is assumed that the two machines will have the same flux per pole.

In connection with the problem as a whole it is assumed the rotor is of the squirrel-cage type.

In both the three-phase and single-phase windings as just discussed, the effective value of the voltage[3] that would be induced per phase is

$$E = 4\, f\, k_b k_w k_c w \phi\, 10^{-8} \quad \text{volts} \tag{1}$$

where

e = the effective value of voltage
f = frequency in cycles per second
k_b = form factor
k_w = winding distribution factor
k_c = chord factor
w = number of turns in series per phase
ϕ = flux per pole

With the same flux per pole in each motor the hypothetical total flux[4] of the machines will be;

$$\phi_t \text{ equals } \frac{\phi\, p}{k_d} \text{ lines} \tag{2}$$

where

ϕ_t = total flux of motors
ϕ = flux per pole of motors
p = number of poles
k_d = air gap flux distribution factor

Solving for ϕ gives $\phi = \dfrac{\phi_t\, k_d}{p}$

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[3] John H. Kuhlmann, Design of Electrical Apparatus, p. 167.

[4] Ibid., p. 167.

Substituting this value of ϕ in Equation 1 and solving for ϕ_t[5] gives

$$\phi_t = \frac{E_p \, 10^{-8}}{4 \, f \, k_b \, k_c \, k_w \, k_d \, w}$$

The frequency[6] of the voltage generated is $f = \frac{n \, p}{120}$

where n is the synchronous rpm.

w equals $\frac{N}{2}$ where N equals the total number of conductors per phase.

Substituting their values in Equation (3) and solving for E[7] gives

$$E = \frac{\phi_t \, n \, N \, k_c \, k_b \, k_w \, k_d}{60 \times 10^8} \text{ volts} \tag{4}$$

The horsepower[8] output and thus the rating of any polyphase induction motor is;

$$HP = \frac{m \, E \, I \, PF \, eff}{746} \tag{5}$$

where

HP = horsepower output of the motor
m = number of phases
E = phase voltage
I = line current
PF = power factor of the motor
eff = efficiency

This above equation will hold for single-phase motors;

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5 Ibid., p. 167.

6 Ibid., p. 168.

7 Ibid., p. 168.

8 Ibid., p. 168.

where

$m = 1$
E = line voltage
I = line current
PF = power factor
eff = efficiency

Substituting the value of E in Equation (4) in Equation (5) gives:

$$HP = \frac{\phi_t\ n\ m\ N\ k_c\ k_b\ k_w\ k_d\ I\ PF\ eff}{60 \times 10^8 \times 746} \qquad (6)$$

or

$$HP = \frac{\phi_t\ n\ m\ N\ k_c\ k_b\ k_w\ I\ PF\ eff}{4.476 \times 10^{12}} \qquad (7)$$

In terms of the air gap flux density[9] and stator dimensions

$$\phi_t = \pi D\ l_g\ B_g$$

where

D = inside diameter of stator
l_g = length of stator air gap section
B_g = air gap flux density

The total number of ampere conductors per inch of stator gap circumference[10] Q is equal to

$$\frac{n\ I\ N\ k_c}{\pi D}$$

Substituting the values of ϕ_t in Equation (7) gives;

9 Ibid., p. 168.

10 Ibid., p. 168.

$$HP = \frac{\pi D\ l_g\ B_g\ m\ n\ N\ k_c\ k_b\ k_w\ k_d\ I\ PF\ eff}{4.476 \times 10^{12}} \tag{8}$$

Equation (8) will hold for single-phase induction motors where

$m = 1$
I = line current
N = single-phase conductors

Substituting Q in the above equation and solving the equation in terms of D and l_g will yield;

$$D^2\ l_g = \frac{HP \times 4.476 \times 10^{12}}{\pi^2\ Q\ B_g\ n\ k_b\ k_w\ k_d\ PF\ eff} \tag{9}$$

Thus equations (8) and (9) give a basis upon which a comparison of the single-phase and polyphase induction motor may be made.

CHAPTER II

EXPLANATION AND DERIVATION OF DESIGN FACTORS

In considering the horsepower output Equation (8) in Chapter I, it immediately becomes apparent that the winding factors of the stator winding; distribution factor, pitch factor, flux distribution factor, and form factor have a direct effect on the output of the motor. Not evident from Equation (8), however, is the winding connection factor of the stator and the skew factor of the rotor, both of which slightly effect the motor output. A brief discussion of these factors follows in order to be able to determine their relative contribution to induction motor performance.

Distribution Factor

In the polyphase winding the distribution or breadth factor is commonly defined[1] as the ratio of the resultant voltage of the coils per phase per pole to the arithmetic sum of the effective values of the individual coil voltages. (Each of the latter being displaced electrically, depending upon the number of slots per pole.) It may be further defined as the ratio of the actual voltage to the voltage that would have resulted from the total turns, had they been located in one pair of slots. For example[2], in Figure 6, $E_3 = E_1 + E_2$.

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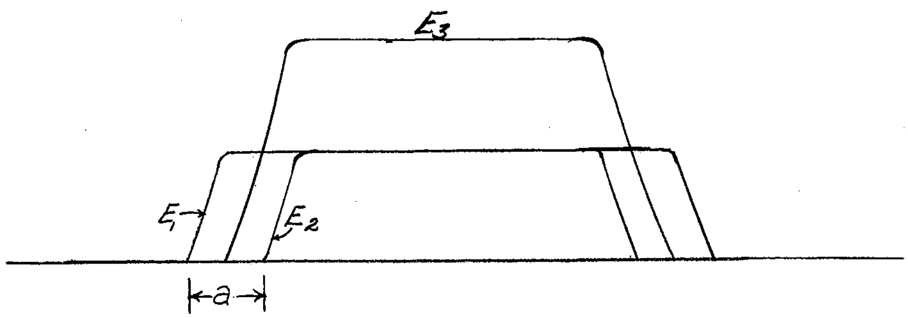
[1] A. F. Puchstein and T. C. Lloyd, Alternating Current Machine, p. 18.

[2] Ibid., p. 18.

Where a = the electrical phase difference between slots
E_1 = voltage generated in first slot
E_2 = voltage generated in adjacent slot



Effect of Distributed Winding on Wave Shape

Figure 6

from definition

$$K_W = \text{distribution factor} = \frac{E_3}{E_1 + E_2}$$

and the electrical phase difference (a) $= \dfrac{180 \text{ degrees}}{\text{slots per pole}}$

Using vectors representing generated voltages as shown in Figure 7 (a) $K_W = \dfrac{\text{length of long chord}}{\text{sum of the lengths of the short chords}}$

$$= \frac{\sqrt{(E + E \cos a)^2 + (E \sin a)^2}}{2\ E}$$

$$= \frac{\sqrt{E^2 + 2\ E^2 \cos a + E^2 \operatorname{Cos}^2 a + E^2 \sin^2 a}}{2\ E}$$

$$= \frac{\sqrt{E^2 + 2\ E^2 \cos a + E^2 (\sin^2 a + \cos^2 a)}}{2\ E}$$

$$= \frac{\sqrt{2\ E^2 + E^2 \cos a}}{2\ E}$$

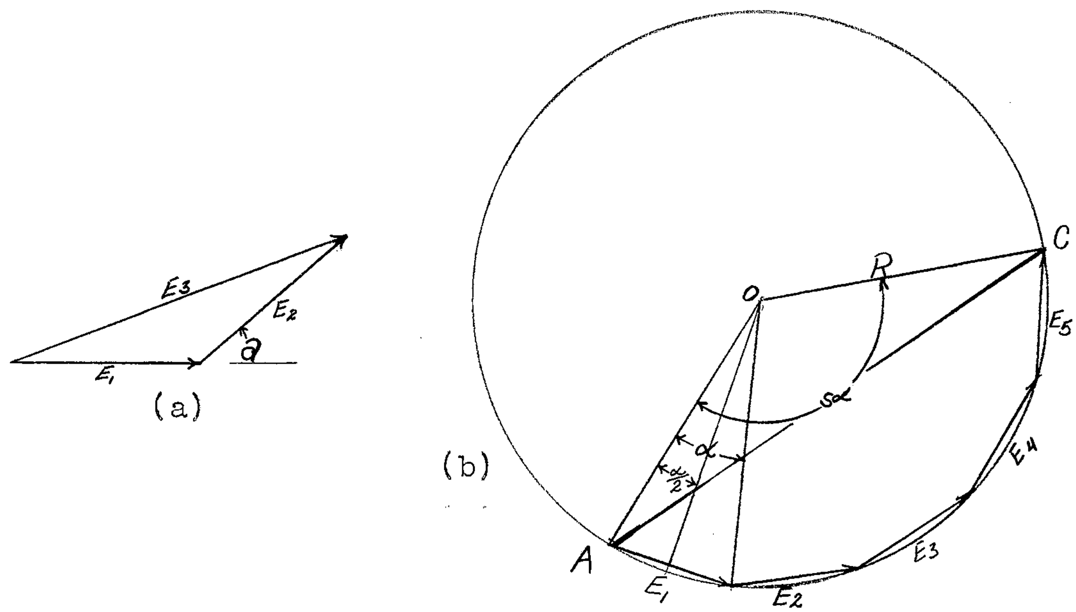
$$= \frac{\sqrt{2}\ E \sqrt{1 + \cos a}}{2\ E}, \text{ substituting } \sqrt{1 + \cos a} = a \cos \frac{a}{2}$$

$$= \frac{2\ E \cos \frac{a}{2}}{2\ E} = \cos \frac{a}{2} \qquad (10)$$

In the general case of a large number of coils per phase where the number of slots per pole per phase is an integer.

let N = number of slots
P = number of poles
m = number of phases

then the slots per pole per phase will be N/mP.



Vector Diagram of Voltages of Distributed Winding

Figure 7

Referring to Figure[3] 7 (b), the number of vectors to be added is $S = \frac{N}{mP}$ from which $k_w = \frac{AC}{SE}$

$$= \frac{2 R \sin \frac{S\alpha}{2}}{S\, 2 R \sin \frac{\alpha}{2}} = \frac{\sin S \frac{\alpha}{2}}{S \sin \frac{\alpha}{2}} \tag{11}$$

3 Reink Andriessen, "A Common Error in the Distribution Factor of Electric Machines," Electrical Engineering, 62 (February, 1943), p. 69.

This equation[4] is true for any number of phases when S is an integer, i.e., there is a whole number of slots per pole, the coils for all phases and poles being identical. The winding may be single layer, but all phases are symmetrical with respect to each other and with respect to the poles. It can be seen from Equation (11) that the limits of the distribution factor can be determined by substituting a minimum of two coils per phase per pole to a theoretical maximum of an infinite number of coils per phase per pole. (For one coil per phase per pole the distribution factor is unity.) In so doing it is found that for three-phase windings the distribution factor for integral slot windings has as its limiting minimum value 0.955. In the case of an infinite number of slots per pole per phase, the polygon will approach a circle and the distribution factor will be equal to a chord divided by an arc. The net effect of distributing the winding in the phases of an induction motor is to produce a sinusoidal resultant flux wave, thus reducing the relative effect of harmonics. Chording the winding reduces leakage reactance and gives better heat distribution in the coils. It will further reduce the coil axial length thus helping to eliminate end brackets. The above, plus a reduction in overall resistance and reactance produce better torque, power factor and efficiency in the polyphase machine.

Using the above method of determining the distribution

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[4] Ibid., p. 69.

factor, (other methods yield the same results), Table I[5] is produced for comparison. (Values as shown for single-phase machines apply only when a full pitch winding is used and all slots contain an equal number of coil sides.)

Distribution Factor K_W

Slots per pole per phase	Number of phases 1	2	3
1	1.000	1.000	1.000
2	0.707	0.924	0.966
3	0.667	0.911	0.960
4	0.653	0.906	0.058
5	0.647	0.904	0.957
6	0.644	0.903	0.956
7	0.642	0.902	-----
8	0.641	0.902	-----
9	0.640	-----	-----
10	0.639	-----	-----
infinity	0.637	0.900	0.955

Table I

As was stated previously, the above calculation and table suffice for stator with an integral number of slots per phase per pole. In general this will be the case; however, if the number of slots per pole per phase is a fraction, as in some modern polyphase induction motors, there will exist two different size coil

[5] *Ibid.*, p. 70

groups, the larger group containing one or more coils than the smaller. Such a winding adds vectorially a large number of coil voltages which are slightly out of phase. The higher harmonics in the coil voltages will have a much greater phase difference resulting in closer approximation to the sine wave of voltage than would exist when the number of slots per phase per pole is an integer. This, of course, is the ideal case, such ideal conditions being assumed for later calculations. In many polyphase induction motors, the number of slots per pole is not an integer in which case it is called a fractional slot winding. Such a fractional slot winding, in order to give a balanced polyphase voltage must be such as to be divisible into as many identical belts as there are phases, these belts being displaced in the case of a three-phase winding, 120 electrical degrees apart.

The fractional slot winding[6] improves wave form and locked torque as well as having the advantage of behaving like a winding with many slots per pole per phase, thus reducing the distribution factor of the harmonics.

To determine if a fractional slot machine is capable of producing a balanced three-phase winding, the ratio of the number of slots to the number of poles is reduced to its lowest terms. The numerator will give the number of belts per pole in a repeatable section and it must be divisible by the number of phases in order to give the number of belts per phase per pole, other-

[6] R. R. Lawrence, Principles of Alternating Current Machinery, p. 47.

wise the machine is incapable of producing a balanced three-phase winding. The denominator reduced to its lowest term gives the number of poles after which the belt is repeated.

As in the case of the integral slot windings the voltage in the different belts are not in phase, consequently, the ratio of the vectorial sum of the voltages to the algebraic sum, or the distribution factor, must be determined. Since all the belts are not identical, a weighted average distribution factor is determined, taking into account the different number of coils in some belts. The phase voltage of a fractional slot winding can also be determined by an equivalent winding. The coil sides of such a winding will be separated by small angles equal to the angle between belts in a fractional slot winding. A vector diagram representing these voltages will form the spokes of a fan from which the phase voltage may be determined.

The following example[7] will be used to clarify the above discussion: A three-phase machine has 20 poles and a total of 84 slots. As 84 is not exactly divisible by the number of phases, it is readily seen that an integral slot winding is not possible. The ratio of the total slots to the number of poles reduced to its lowest terms is 21/5. Thus 21 is divisible by 3, and a balanced fractional slot winding is possible with 7 slots per phase per pole. The winding will be repeated every fifth pole. For these five poles there must be five belts with a total of seven slots per pole. This will be accomplished with two belts

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[7] Ibid., p. 47.

of two slots and three belts of one slot each, taking the necessary 7 slots per pole. The best arrangement of these belts is one which will produce the best distribution factor, such being a symmetrical arrangement. The arrangement of 1 slot, 2 slots, 1 slot, 2 slots, 1 slot will be used for the phases A, B, C. One repeatable section is shown below.

Slots

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	21	21
a	-c	-c	b	-a	-a	c	-b	a	a	-c	b	b	-a	c	-b	-b	a	-c	-c	b

As shown, the belts, when passing around the armature are of the order a, -c, b, -a, c, -b, etc. in order to provide the correct phase displacement. This order occurs whether in an integral or fractional slot winding.

In this example the angle between adjacent slots will be $\frac{20 \times 180}{84}$ or 42 6/7 electrical degrees.

Thus the phase angle between the conductor voltage of slot 1 and slot 8 of the first two coil groups will be

$$\frac{20 \times 180 \times 7}{84} = 300 \text{ electrical degrees.}$$

This is equivalent to a displacement of 300-360 or -60 degrees. However since -b is the conductor in slot 8 the actual phase displacement due to the reversal of phase will be 180 -60 or 120 degrees from phase A in slot 1.

By the same means it will be found that phase A in slot 15 will lag phase A in slot 1 by 240 degrees. Thus the correct phase displacement between the three phases is achieved and as the groups in each phase are equal, the machine will have balanced voltages. The machine considered has a two layer winding; however, only the first layer is considered above. This second layer will be identical but will be displaced a number of slots determined by the pitch of the coils. Thus if the pitch is 3 slots or $\frac{20 \times 180 \times 3}{84} = 128\ 4/7$ electrical degrees. The second layer will start in slot 4 instead of slot 1.

As the group factor K_g is the ratio of the vector sum of the voltages in the belts for any phase to the algebraic sum of these voltages for any phase, the weighted breadth or distribution factor K_W and the group factor may be found as follows. Angle between slots is 42.86 electrical degrees. For phase A the belts will be noted 1, 2, 3, 4, and 5. Belts 1, 4, and 5 as shown will have a breadth factor on one since there is only one coil per belt. Belts 2 and 3 occupying two slots per belt will have a breadth or distribution factor in each case as follows:

$$K_W = \frac{\text{Sin } 2\ \frac{42.86}{2}}{2 \sin \frac{42.86}{2}} \quad \text{or } 0.931 \text{ for each belt alone.}$$

The average weighted breadth factor for the whole winding will be

$$K_W = \frac{1 + 2(0.931) + (2)\ (0.931) + 1 + 1}{7}$$

$$= \frac{6.724}{7} = 0.96.$$

The group factor will be found by adding all of the individual coil side voltages vectorially, thus for phase A

Belt	No. of slots per belt	Voltage per belt	Angle between middle of belt and first belt
+1	1	1 x 1 or 1.000	$\theta_1 = 0.0$ degrees
-2	2	2 x .931 or 1.862	$\theta_2 = 4\ 1/2 \times 42.86 = 192.9^\circ$
+3	2	2 x .93 or 1.862	$\theta_3 = 8\ 1/4 \times 42.86 = 364.3^\circ$
-4	1	1 x 1 or 1.000	$\theta_4 = 13 \times 42.86 = 557.2^\circ$
+5	1	1 x 1 or 1.000	$\theta_5 = 17 \times 42.86 = 728.6^\circ$

θ_1 may be written 0.00°
θ_2 may be written $+12.90^\circ$
θ_3 may be written $+4.30^\circ$
θ_4 may be written $+17.20^\circ$
θ_5 may be written $+8.60^\circ$

The equivalent spoke diagram of the above voltage will appear as in Figure 8.

Voltage Equivalent

Spoke Diagram

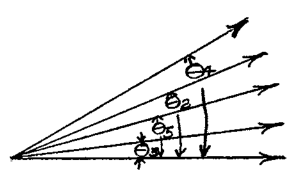

Figure 8

The vector sum of the above voltages can be determined in the following manner to give the group factor K_g.

Belt	V cos θ	V sin θ
+1	1 x 1 or 1000	1 x 0.000 or 0.000
-2	1.863 x .9748 or 1.8150	1.862 x .2232 or .4155
+3	1.862 x .9972 or 1.8568	1.862 x .0750 or .1397
-4	1 x .9953 or .9553	1 x .2957 or .2957
+5	1 x .9887 or .9887	1 x .1498 or .1498
	Summation V cos θ = 6.6158	Summation V sin θ = 1.0007

$$\text{The group factor } K_g = \frac{((6.616^2) + (1.0007^2))^{1/2}}{1 + 1.862 + 1.863 + 1 + 1}$$

$$= \frac{6.691}{6.724} = 0.995$$

The product of the breadth factor and the group factor will determine the voltage per phase of a fractional slot winding.

Pitch Factor

Next of importance of the factors to be taken into consideration in the design of polyphase and single-phase machines is pitch factor. It is of importance because it will, in effect reduce the copper weight required for windings, reduce winding resistance and by shortening the end turns will effectively reduce the leakage reactance and will reduce if not completely eliminate various harmonics in the space distribution of the flux wave. In so doing, however, the effective voltage per turn is reduced but not to the extent of the advantages mentioned above. The pitch factor is the ratio of the voltage generated in a fractional pitch winding to that voltage which would be generated if the winding were full pitch and had an equal number of windings.

Thus in a winding of full pitch the voltages generated in the two coil sides are in phase and will add directly. In the fractional pitch machine the voltages generated in the two coil sides will not be in phase and so will have to be added vectorially. In Figure[8] 9 it is readily seen that the two voltages in the full pitch coil are 180 electrical degrees out of phase, but are in series and will add directly. By the same token it is seen that the voltage in the two coil sides of the fractional pitch coil will have to be added vectorially.

Full and Fractional Pitch Coils

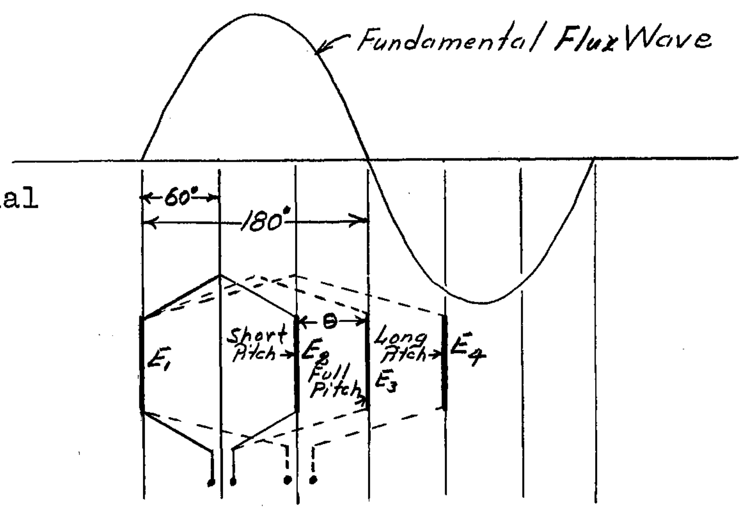

Figure 9

Therefore $E_1 + E_2 = 2\ E \cos\frac{\theta}{2}$ (12)

Letting[9] v = the ratio of the coil pitch to the pole pitch.
θ = the angular difference between pole and coil pitch as shown above.

It is seen from Figure 9 that the coil span is $v\pi$ where v is

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[8] L. V. Bewley, Alternating Current Machinery, p. 120.

[9] A. F. Puchstein and T. C. Lloyd, Alternating Current Machines, p. 17.

less than unity. Then $\Theta = \pi(1 - v)$, from which the pitch or chord factor may be conveniently expressed as:

$$K_c = \cos\frac{\Theta}{2} = \sin\frac{v\pi}{2} \tag{13}$$

The above derivation is for the fundamental voltage alone. If a coil pitch is shortened by the ratio of 1/n of the pole pitch, the n^{th} harmonic will be eliminated. The other effects of harmonics will be discussed briefly later in the thesis.

Distribution Factor and Pitch Factor in Single-Phase Machines

The discussion of distribution and pitch factors up to this point pertained only to polyphase windings. Most single-phase windings are of the concentric or of the spiral concentric type. Determination[10] of the distribution factor for concentric windings is a weighted average pitch factor and is found by multiplying the chord or pitch factor of each coil per pole group by the number of turns. As an example[11] consider the winding as shown in Figure 10 with four concentric coils per pole. As each coil does not contain the same number of conductors, the distribution and pitch factors must be weighted in proportion.

Thus; Distribution Factor =

$$\frac{N_1 \sin\Theta_1 + N_2 \sin\Theta_2 + N_3 \sin\Theta_3 + N_4 \sin\Theta_4}{N_1 + N_2 + N_3 + N_4}$$

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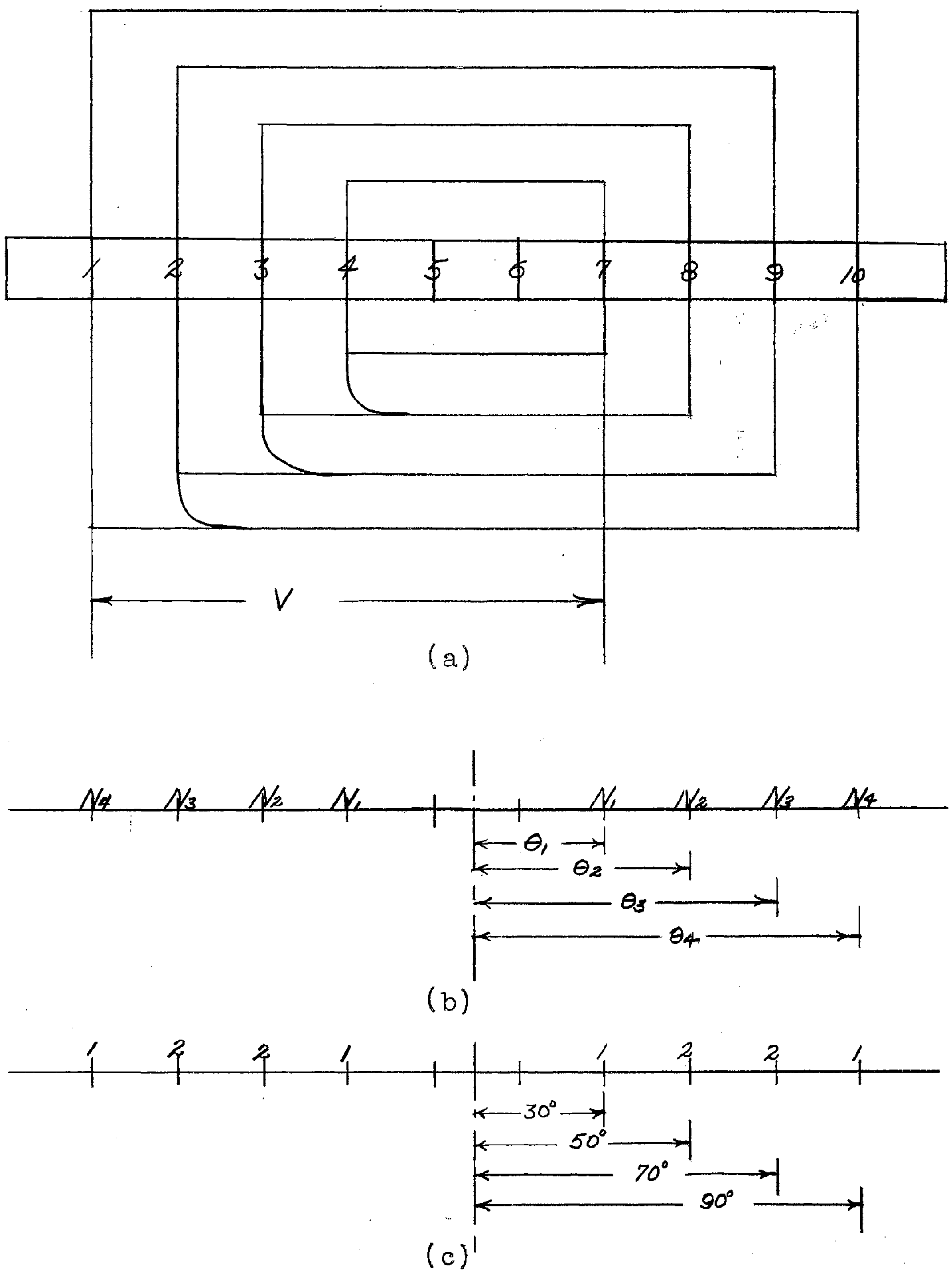
10 Veinott, Fractional Horsepower Electric Motors, p. 401.

11 Ibid., p. 401.



Single-Phase Concentric Winding

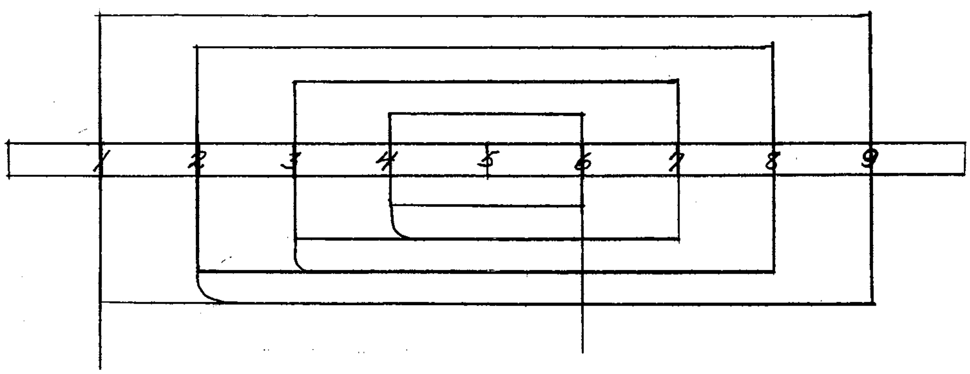
Figure 10

Referring to Figure 10 c, the numerical solution for a conductor ratio of 1 - 2 - 2 - 1, per pole is as follows;

$$\text{Distribution Factor} = \frac{1 \sin 30 + 2 \sin 50 + 2 \sin 70 + 1 \sin 90}{1 + 2 + 2 + 1}$$

$$= 0.8185$$

While non-sinusoidal space distribution of conductors in a single-phase stator will give good results. Sinusoidal distribution[12] of the winding will tend to produce sinusoidal distribution of flux and thus reduce the harmonics in the air gap flux. Consider the winding in Figure 11 of nine slots per pole.



Single-Phase Concentric Winding
(9 slots/pole)
Figure 11

Sinusoidal distribution of conductors can be determined as follows:

Coil 4&6 - sin 1/2 coil span - sin (2/9x90) - 0.342

Coil 3&7 - sin 1/2 coil span - sin (4/9x90) - 0.643

Coil 2&8 - sin 1/2 coil span - sin (6/9x90) - 0.866

Coil 1&9 - sin 1/2 coil span - sin (8/9x90) - <u>0.985</u>

2.836

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[12] John H. Kuhlmann, <u>Design of Electrical Apparatus</u>, p. 345.

Percent turns per pole in coil 4-6 - 0.342/2.836 x 100 - 12.10

Percent turns per pole in coil 3-7 - 0.643/2.836 x 100 - 22.70

Percent turns per pole in coil 2-8 - 0.866/2.836 x 100 - 30.60

Percent turns per pole in coil 1-9 - 0.985/2.836 x 100 - 34.60

With this winding the mmf produced would be a close approximation to the sine wave and thus contain only negligible harmonics.

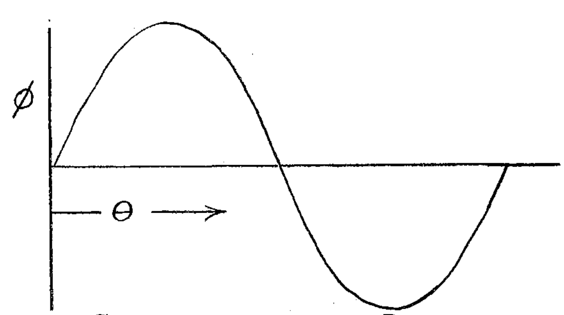
Flux Distribution Factor and Form Factor

The flux distribution factor k_d and the form factor k_b of the air gap flux are of importance in determining the shape of the wave. The flux distribution factor[13] is defined as the ratio of the area under the flux wave to the area of a rectangle with the same base and maximum ordinate of the flux wave. This ratio may be determined by means of a planimeter or by analyzing the flux wave by the methods of Fourier series. Either will suffice for any air gap shape of wave. In case of a sine wave of flux distribution, however, the wave and rectangle have the same base, consequently, for this case the flux distribution factor may be easily determined as the ratio of the average ordinate to the maximum ordinate. For the wave $\phi = \phi_m \sin \theta$ in Figure 12

$$k_d = \frac{1/\pi \int_0^{\pi} \phi_m \sin \theta \, d\theta}{\theta_m}$$

Integrating;

$$= \frac{1/\pi \, \phi_m(-\cos \theta)_0^{\pi}}{\phi_m}$$



Sine Wave Flux Distribution

Figure 12

13 Ibid., p. 26

Upon substitution of limits;

$$k_d = \frac{\phi_m}{\pi\phi_m} \left[- (-1-1) \right]$$

$$k_d = \frac{2}{\pi} = 0.637$$

On the other hand the form factor k_b is defined as the ratio of effective or the root mean square ordinate of the air gap flux wave to average ordinate and for the sine wave distribution of flux may be determined for $\phi_\theta = \phi_m \sin\theta$ as follows;

$$k_b = \frac{\sqrt{1/\pi \int_0^\pi \phi_m^2 \sin^2\theta \, d\theta}}{1/\pi \int_0^\pi \phi_m \sin\theta \, d\theta}$$ Substituting $(1/2 - 1/2 \cos 2\theta) = \sin^2\theta$

$$= \frac{\sqrt{\frac{\phi^2{}_m}{\pi} \int_0^\pi (1/2 - 1/2 \cos 2\theta) \, d\theta}}{\frac{\phi_m}{\pi} \int_0^\pi \sin\theta \, d\theta}$$

$$= \frac{\sqrt{\frac{\phi_m^2}{2\pi} \int_0^\pi d\theta - 1/2 \int_0^\pi \cos 2\theta \, d\theta}}{\frac{\phi_m}{\pi} \int_0^\pi \sin\theta \, d\theta}$$

Upon integrating and substituting of limits;

$$k_b = \frac{\sqrt{\frac{\phi_m^2}{2}}}{\frac{2\phi_m}{\pi}} = \frac{\frac{\phi_m}{\sqrt{2}}}{\frac{2\phi_m}{\pi}} = \frac{\pi}{2\sqrt{2}} = 1.11$$

While the form factor is not a definite indication of the flux wave shape, waves with large form factor imply peaked flux waves, whereas, waves of low form factor indicate flat top waves of flux distribution. The minimum form factor is unity for the square wave.

The product of the flux distribution factor and form factor for a sine wave of flux is;

$$\frac{1/\pi\ \phi_m \int_0^\pi \sin\theta\, d\theta}{\phi_m} \cdot \frac{\sqrt{\frac{\phi_m^2}{\pi}\int_0^\pi \sin^2\theta\, d\theta}}{\frac{\phi_m}{\pi}\int_0^\pi \sin\theta\, d\theta}$$

$$= \frac{\sqrt{\frac{\phi_m^2}{\pi}\int_0^\pi \sin^2\theta\, d\theta}}{\phi_m}$$

Upon integration, substitution of limits and simplifying becomes

$$= \frac{1}{\sqrt{2}} = .707.$$

Which is the effective value or root mean square of the sine wave flux distribution.

The shape of flux wave is also to be considered in the hysteresis and eddy current losses in the iron portion of the magnetic circuit. A highly peaked flux wave will effect these losses as follows.

By Steinmetz empirical equation.[15]

$$P_h = K_n\ f\ B_m^x\ V$$

P_h - Hysteresis loss in watts

K_n - Constant depending upon the chemical analysis, heat and mechanical treatment of the metal

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[15] R. P. Ward, Introduction to Electrical Engineering, p. 160.

f - Frequency in cycles per second

B_m - Maximum flux density in kilolines per square inch

x - Exponent, depends upon material (1.6 being average for most materials)

Flux densities

V - Volume of material in cubic inches

Eddy current losses[16] may be determined by:

$$P = K_e f^2 c^2 B_m^2 V$$

P - Eddy current loss in watts

K_e - Constant, depending upon the resistivity of the material

f - Frequency in cycles per second

c - Lamination thickness in inches

B_m - Maximum flux density in kilolines per square inch

V - Volume of material in cubic inches

In each case the losses are proportional to a power of the maximum flux density. Peaked flux waves therefore, will increase the hysteresis and eddy current losses and not necessarily increase the effective value of the generated voltage.

As the writer is assuming a sine wave distribution of flux in this thesis, both the flux distribution factor and the form factor will appear as a constant in both polyphase and single-phase calculations.

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16 Ibid., p. 166.

The Skewing of Rotor Bars

This factor, of slight importance, in the design of induction machines must be taken into account. The magnitude of the skewing of the rotor bars will affect the maximum torque as well as the starting torque. Other effects of skewing the rotor slots are as follows, and each may or may not be to the advantage of the designer, depending upon the specific design.

1. Eliminates noise and reduces motor frame vibration.
2. Results in smooth torque curve for different positions of the rotor (reduces cogging).
3. Increased rotor resistance due to increased length of bars.
4. Increase in the effective ratio of transformation between stator and rotor.
5. Increased machine impedance at a given slip.
6. Increased slip for a given torque.
7. Indiscriminate skew, especially in machines of many poles leads to a reduction in short circuit current, starting torque, and torque at high slips.
8. Increase in skew angle has an effect similar in respect to decrease in voltage, and this the output is reduced by the square of the skew factor.
9. Reduces space harmonics in air gap flux.

The increase in rotor resistance[17], due to skewing of rotor

[17] A. F. Puchstein and T. C. Lloyd, <u>Alternating Current Machines</u>, p. 288.

bars, can be determined by

$$R'_b = R_b \sqrt{1 + \frac{\tau\alpha}{L_c \times 180}}$$

where

τ = pole pitch in inches

L_c = core length in inches

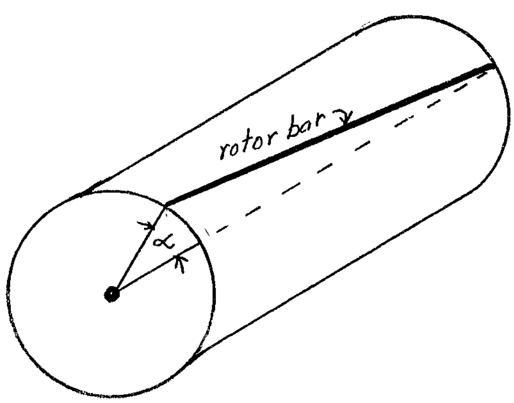
R_b = resistance of bar without skewing

R_r = end ring resistance

R_2 = secondary resistance or $R'_b + R_r$

α = angle of skew in degrees

$$\alpha = \frac{\text{slot pitches of skew}}{\text{total rotor slots}} \times \frac{\text{poles}}{2} \times 360 \text{ degrees}$$



Skew Angle of a
Two Pole Motor

Figure 13

The skew factor K, as shown in Figure 13, is $\frac{\sin \alpha/2}{\alpha/2}$ (α in radians). The effect of the skewing of rotor slots in the operation of induction machines is readily seen from Table II of comparisons as given by Puchstein and Lloyd in Alternating Current Machines, page 291.

	Zero Skew $\alpha = 0$	Skew Angle $= \alpha$ Radians
Slip	s	s
Secondary induced emf	sE_2	ksE_2
Secondary leakage reactance	X_2	$X'_2 > X_2$
Secondary resistance	R_2	$R'_2 > R_2$
Secondary current	$I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$	$\frac{ksE_2}{\sqrt{R'^2_2 + (sX'_2)^2}}$
Torque produced	$K_2 I_2 \phi_m \cos(\phi, I_2)$ $= K_2 \phi_m \frac{sE_2R_2}{R_2^2 + (sX_2)^2}$	$K_2 \phi_m \frac{k^2 sE_2 R'_2}{R'^2_2 + (sX'_2)^2}$
Primary current required to balance the rotor mmf	I_b	kI_b
Ratio of transformation	$a_1 = \frac{m_1 N_1 k_{w1}}{m_2 N_2 k_{w2}}$	$\frac{a_1}{k}$

Table II

The rotor being common to both the single and polyphase machines in this comparison will have equal effect in each case.

Harmonics

In the present day design of induction machines the effects of applied harmonic voltages and of the induced harmonic currents must be considered as they will indeed affect the operational characteristics of the machine. Even though commerical power line voltages are a fairly close approximation to sine wave voltages, it is almost impossible to have a perfect sine wave varying voltage for a source of power.

As actual tests, as well as theory, have shown that a sine wave input of voltage will give smoother operating characteristics, designers, by means of certain harmonic reduction factors are able to reduce and in some cases completely eliminate certain harmonic effects in induction machines.

Harmonics[18] as found in the air gap flux of induction motors are of two types: 1. Time harmonics occuring in the flux due to the harmonics in the applied voltage and current varying with respect to time. It is easily seen that time harmonic components will have the same number of poles as does the fundamental in the induction motor. 2. Space harmonics of flux which may be caused by the stator winding, slots of the rotor and stator, non-uniform air gap or saturation of the magnetic circuit are likely to be of more trouble since the space harmonics of flux will have a greater number of poles in induction motors. The n^{th} space harmonic will have k times as many poles as the fundamental and

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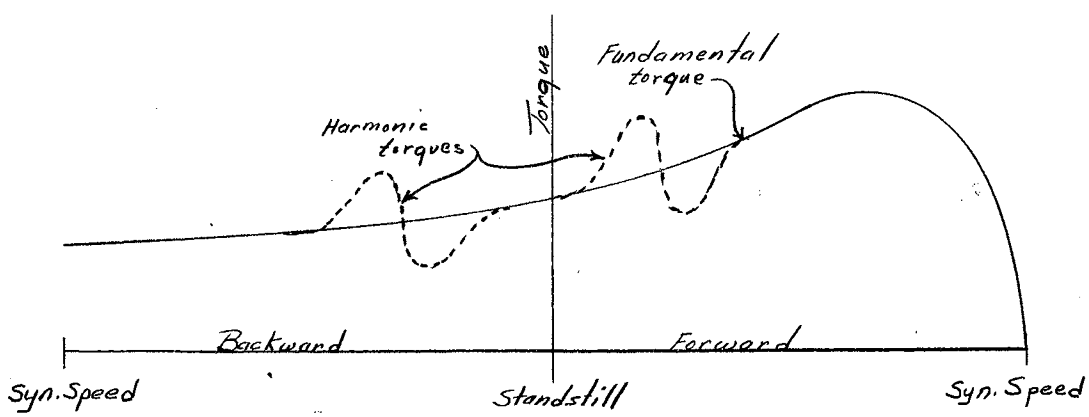
18 <u>Ibid</u>., p. 278.

its torque-speed curve will pass through zero at 1/k of synchonous speed. Both types of harmonic components of flux will increase iron losses and the latter will affect the torque of the motor due to the larger number of poles, some of which may rotate in a direction opposite to that of the fundamental.

If the air gap flux is non-sinusoidally distributed in space, as is the case in all motors because of the limit on stator and rotor slots, all odd space harmonics may be present. Because of the symmetry of the machine the even harmonics are absent or negligible. Because of the large number of poles of space harmonics, the speed[19] at which they rotate is;

$$\text{Rpm(harmonic)} = \frac{\text{synchronous Rpm (fundamental)}}{2\ n\ m \pm 1}$$

n = Any assumed integer equal to or greater than 1
m = Number of phases
h = (2 nm $\pm$ 1) Order of the space harmonic



Torque-Speed Curve With Harmonic Torques

Figure 14

19 Ibid., p. 280.

It is seen, therefore, that if the harmonic or parasitic torques is of significant magnitude it may cause the torque-speed curve[20] of an induction motor to appear as shown in Figure 14.

The induction motor designer has at his disposal four harmonic reduction factors which are capable of suppressing certain harmonics and may eliminate or reverse the sequence of specific harmonics. In each case the harmonic reduction factors, skew, pitch, distribution, and connection suppress harmonics in essentially the same manner. That is, by arranging the windings such that induced harmonic voltages in the same conductors as well as in series conductors are in partial or complete opposition and therefore partially or completely cancel each other. Figure[21] 15 is an example of each reduction factor in the case of suppression of the third harmonic.

As in Figure 15, it the rotor bars are skewed such that one half the bar is cutting the positive loop of flux and the other half is cutting the negative loop of flux, the induced voltages will be in direct opposition and will cancel. It is obvious that only the higher frequencies can be eliminated in this manner. It is only practical to eliminate the eleventh harmonic and above. It is not practical for reducing the lower harmonics because of circuit complication and the cost of material and manufacturing.

By either short or long pitch certain harmonics may be suppressed. Both sides of the coil are cutting a positive loop

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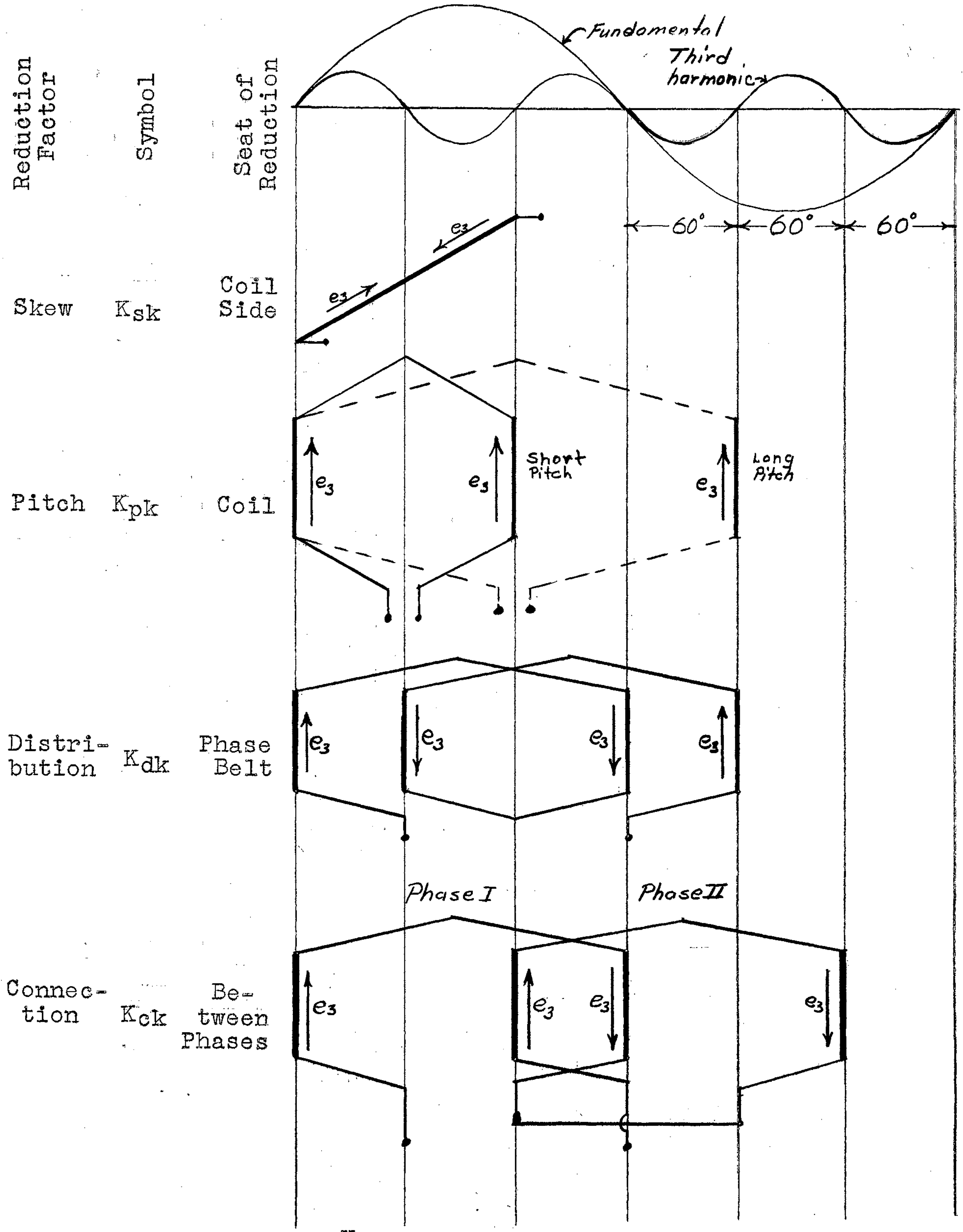
20 Ibid., p. 281.

21 L. B. Bewley, Alternating Current Machinery, p. 120.



Harmonic Reduction
Figure 15

of flux, but are in opposition as shown in Figure 15.

The distributed winding reduces certain harmonics by placing the coil sides in a belt such that the harmonic voltages induced will be in opposition.

By connecting the phases of polyphase machines in series, all harmonics which are odd multiples of the number of machine phases may be suppressed. For this reason most commercial three-phase systems have negligible 3rd harmonics and odd multiples of the 3rd harmonics. The even harmonics being absent because of symmetry.

The actual determination of the reduction factors in each case is given by Bewley is given in Table III.

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22 *Ibid.*, p. 121.

	Reduction factor	Seat of the reduction
Skew factor	$K_{sk} = \frac{\sin(K\lambda/2)}{k\lambda/2}$	In the coil side
Pitch factor	$K_{pk} = \sin\frac{kp}{2}$	Between coil sides of the coil
Distribution factor of belt	$K_{dk} = \frac{\sin\ (k\pi q\sigma/2r)}{q\ \sin\ (K\pi\sigma/2r)}$	Between coils of the phase belt
Connection factor	$K_{ck} = \frac{\sqrt{(\sum \sin k\theta_r)^2 \quad (\sum \cos k\theta_r)^2}}{c}$	Between phases

K = Order of the space harmonic
$\lambda =)\pi\tan\alpha/\tau$ = skew coefficient
P = Pitch
q = Slots per phase belt
σ = Slot pitch in inches
τ = Pole pitch in inches
θ_r = Phase angle
c = Phases connected in series

Table III

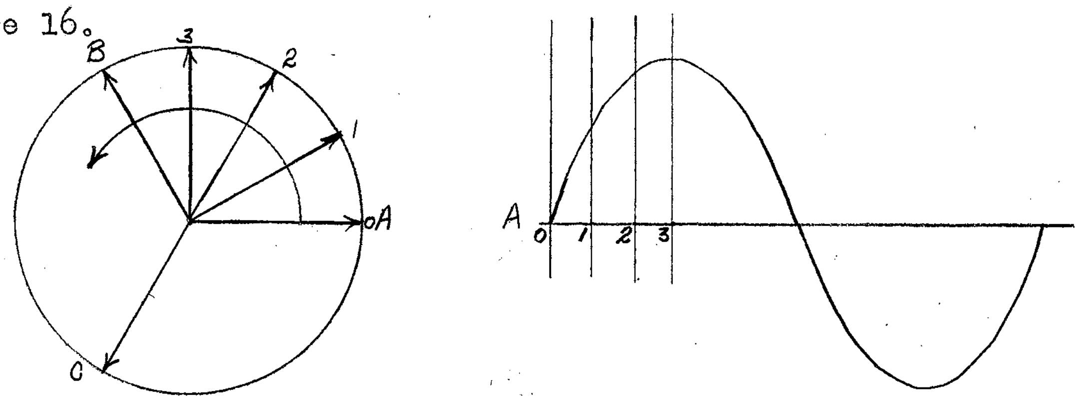
CHAPTER III

EXCITING CURRENT OF SINGLE-PHASE AND THREE-PHASE INDUCTION MOTORS

Mentioned briefly in Chapter I, yet of importance in the comparison of the three-phase and single-phase induction motors, is the relative magnetizing currents in each. It is of consequence, because it has a decided effect upon the power factor and efficiency of the two machines.

It is the purpose of this chapter to show how the magnetizing current in both the single-phase and the three-phase motor sets up the rotating flux field and to compare the over all performance of the two motors.

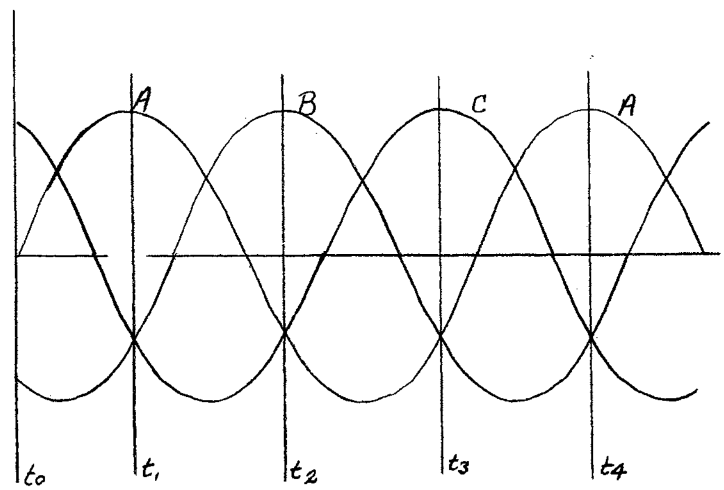
It is commonly known that a sine wave may be represented by a vector rotating in a counterclockwise direction and that the sine wave itself may be constructed by rotating a vector in a counterclockwise direction, taking vertical projections at each angle as the vector rotates. Using the circumference of the circle formed by the rotating vector as a reference base, the vertical projection points may be plotted measuring vertically from the base so as to construct the sine wave as shown in Figure 16.



Vector Representation of a Sine Wave
Figure 16

The sine wave will also represent in magnitude and space a flux wave similar to that shown in Figure 5 of Chapter I, the base being the space distribution around the stator itself and the magnitude of the wave representing the magnitude and polarity of the flux at any point on the stator circumference.

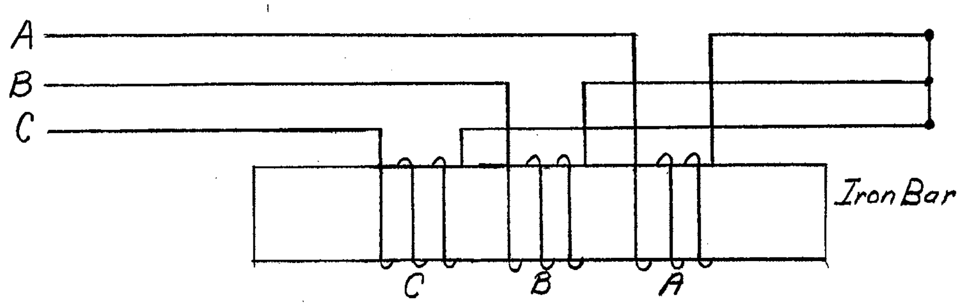
If in Figure 16 the curves of the three vectors A, B, and C are plotted simultaneously, the three sine waves would appear as in Figure 17, and would represent a balanced set of three-phase currents. It could also represent the flux waves of such a set of currents, as for example the three-phase winding of Figure 4 in Chapter I.



Balanced Three-Phase Flux Waves

Figure 17

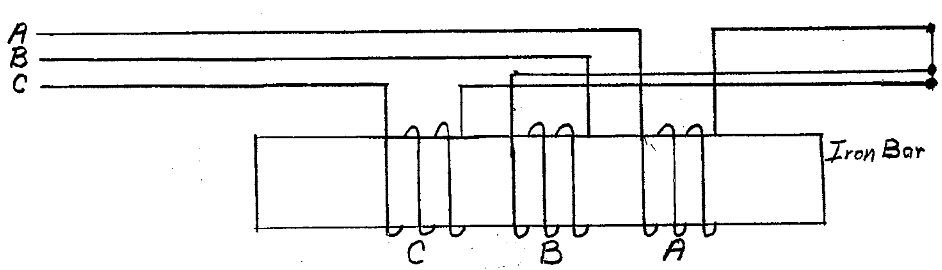
Suppose the polyphase currents of Figure 17 are applied to the three coils A, B, and C wound identically and with the same number of turns on the bar magnet as shown in Figure 18A.



Three-Phase Currents Acting on Iron Bar

Figure 18A

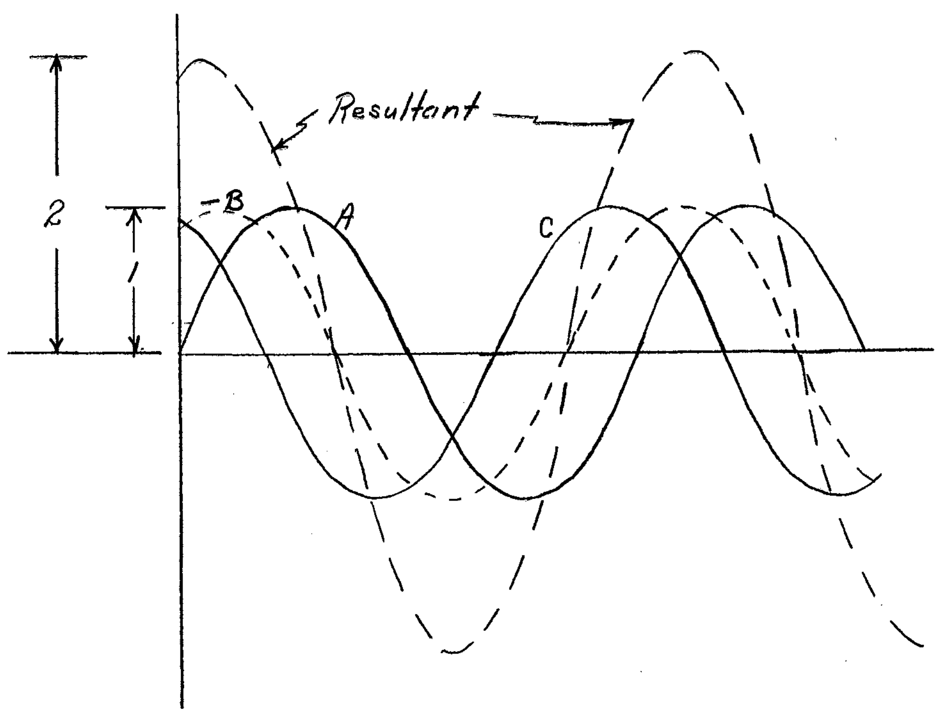
The mmf of the bar magnet will be proportional to the number of turns and the magnitude of currents in each coil. The number of turns of each coil is the same, and as can be seen by referring to Figure 17, the sum of the currents in the coils at anytime t_1, t_2, t_3, etc. will be zero. Therefore, no flux will be set up in the bar magnet. However, if the bar magnet coils are wound as shown in Figure 18B, the middle coil reversed, the mmf of this reversed coil will be in a relative direction opposite to the direction of the other coils with a positive current flowing in each. In this case the resultant mmf will appear as shown in Figure 19 and will be represented by the sum total of the mmf waves as shown. The resultant mmf wave will have a maximum value of two times the maximum value of any one of the phase values.



Three-Phase Currents Acting on Iron Bar (Middle Coil Reversed)
Figure 18B

It is for the above reason that the middle phase of a three-phase motor winding is always reversed, and in this case a strong resultant magnetism is set up in the bar magnet varying alternately north and south.

The coils of the winding of a three-phase motor, unlike the bar magnet, have a space displacement of 120 degrees which in conjunction with the three-phase currents set up the rotating magnetic field. However, the winding of the single-phase machine sets up a pulsating field, and only in conjunction with the field set up by the rotating rotor does a rotating field actually exist in the single-phase motor. That the rotating field does exist is discussed as follows by B. F. Bailey in his book <u>The Induction Motor</u>, page 106.

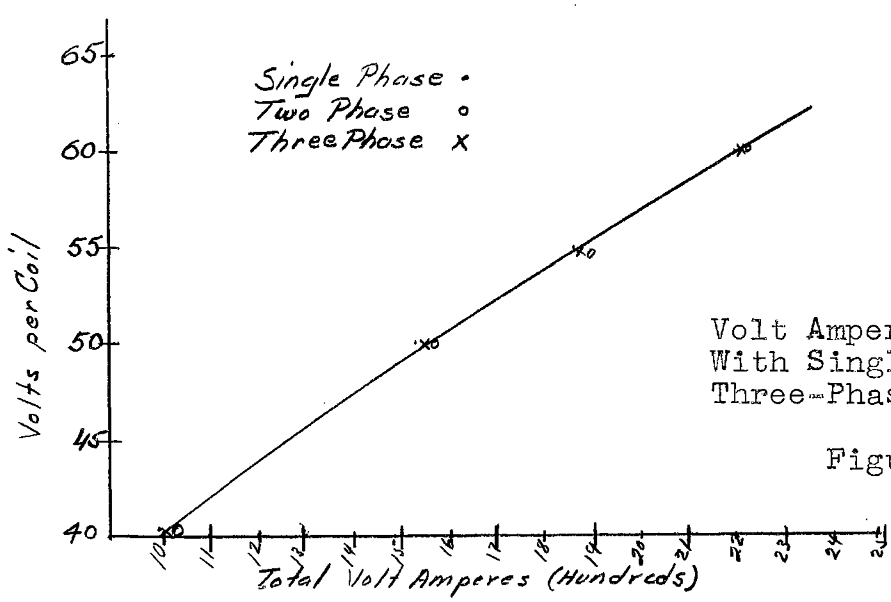


Resultant Flux Wave (Coil B Reversed)

Figure 19

To test the above theory, use was made of a small squirrel-cage induction motor. This machine was wound with 72 coils in the same number of slots. The coils were connected in six sets of 12 coils each. The circuits were located 30 electrical degrees apart on the core. By various groupings of the coils, the machine could be operated as a single, two- three- or six-phase machine. To insure the same value of the flux in all of the cases, the applied voltage was varied so as to give 40, 50, 55 and 60 volts over one section of the winding. Thus assuming that the flux was harmonic or at least did not change with the change in connections, the readings were taken for the same value of the flux in each case.

Readings were made as above indicated with the machine connected for one, two, three and six phases. Having adjusted the applied voltage until a certain voltage was indicated across one of the coils, readings were taken of the current in the various phases and of the voltage across the phases. The average current per phase was then multiplied by the average voltage per phase and by the number of phases. The result is the total no-load volt-amperes. To get the true magnetizing current of the motor, it would have been necessary to subtract vectorially from the no-load current the power component of the no-load current. This was not done in this case, as it would have changed the results but little. The resultant volt-amperes with the different connections are plotted in Figure 20. The ordinates are the volts across one coil, and are therefore proportional to the flux density. Only the points for the single, two and three-phase connection are plotted. The points for the six-phase fall very accurately on the line as drawn, and are omitted to avoid confusion.



Volt Amperes Excitation
With Single, Two- and
Three-Phase Connections

Figure 20

It will be seen that the total volt-amperes are practically the same in all of the cases. It is true that the volt-amperes seem a little more in the case of the two-phase winding, and a little less in the case of the single phase. The difference between the two and three phase is possibly on account of wave shape, although the wave in both cases approximated a sine shape. The connections in the case of the two-phase and the single-phase readings was the same, one of the two circuits being merely opened. Operating single-phase, as will appear later, the flux ceases to be of exactly constant value in the different positions, being somewhat weaker when at right angles to the position of the stator winding. Hence on the whole a slightly lower volt-ampere excitation will be required.

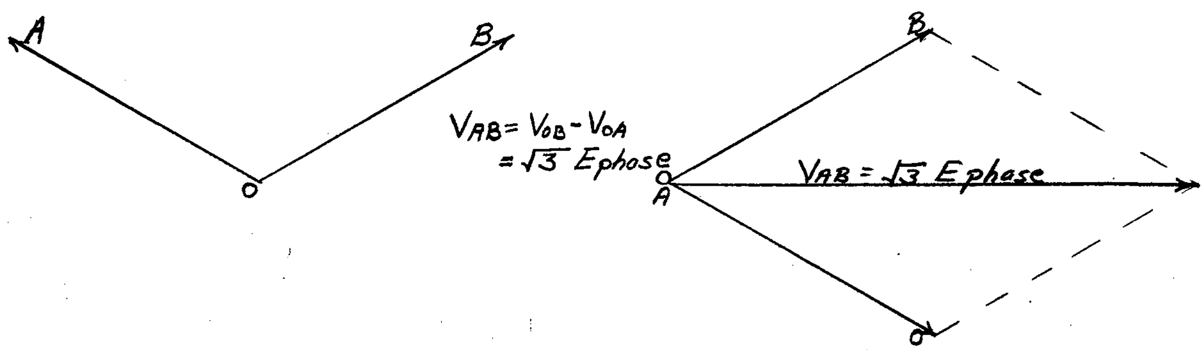
From Mr. Bailey's discussion, it is seen that a field rotating at synchronous speed does actually exist. From the facts as given in the above quotation and further information to follow, the single-phase operation of the three-phase induction motor can be analyzed.

Suppose for instance that a three-phase wye connected induction motor operated at 220 volts line voltage, and that the magnetizing current for no-load operation is 5 amperes (neglecting losses of the machine). Since the flux set up per phase is directly proportional to the current, it might be said that each phase will set up a flux whose maximum value is 5 (on a relative basis). From induction motor theory, a constant rotating magnetic field of one and one half maximum phase value or 7.5 will rotate at synchronous speed around the circumference of the stator. The line voltage being 220, the phase voltage is $200/\sqrt{3} = 127$, and the total voltamperes applied to the motor is $200/\sqrt{3} \times 5 \times 3$ or 1905.

Now if one phase of the motor is disconnected the motor will run as a single-phase machine. In compliance with the experiment conducted by Mr. Bailey, the single-phase magnetizing current will be;

$$\frac{1905}{220} = 8.66 \text{ amperes}$$

Since there is a rotating magnetic field, the back voltage generated in each of the two excited phase of the three-phase machine operating single-phase will be 120 degrees out of phase. Thus to oppose the applied voltage of 220 volts, the back voltage must be at a rate of 127 volts per phase winding. This can be seen in Figure 21.



Vector Addition of Phase Voltages

Figure 21

Thus $\sqrt{3} \times 127$ equals 220 or the line voltage. It is seen that the magnitude of the rotating flux of the three-phase machine operating as such and operating as a single-phase motor is in each case the same value. 7.5

Discussing the method of establishing a rotating flux in the single-phase induction motor, Mr. Bailey further states:

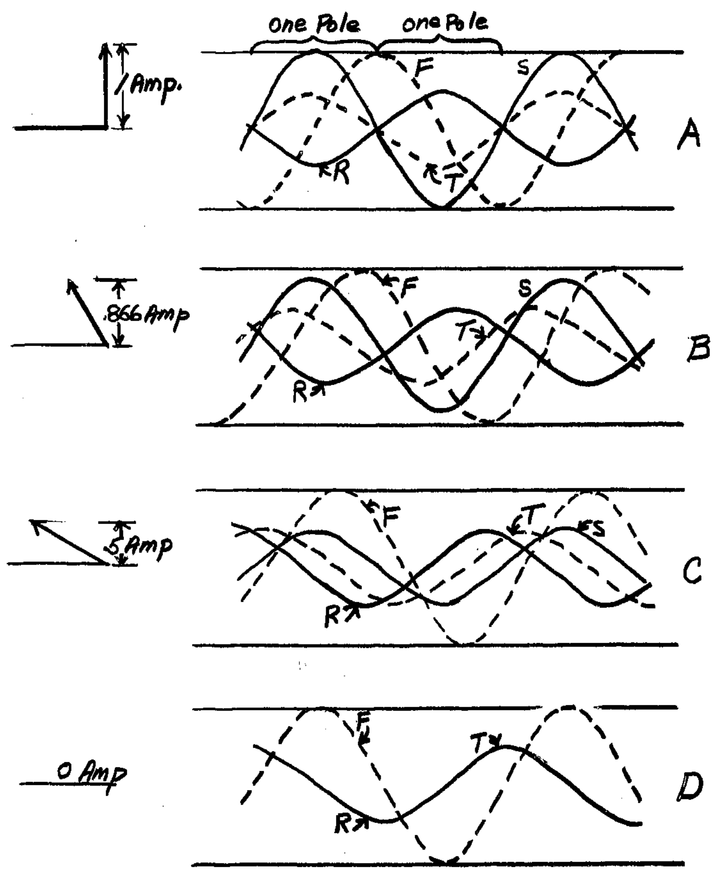
The single-phase induction motor may be considered as a special case of the polyphase motor. If a two-phase motor is operating without load and one of the phases is opened, the motor continues to run at almost exactly the same speed as before. The only apparent change is that the nature of the hum emitted by the motor changes slightly in character. If an ammeter is used to measure the current taken by the machine (in this case principally magnetizing current) it will be found that the current in the phase still connected has approximately doubled, as has likewise the power taken by this phase. The total current and the total power are nearly unchanged.

If when operating in this manner, a voltmeter be applied to the idle phase it will be found that nearly the full line voltage is present there, and a further investigation will show that this e.m.f. differs 90 degrees in phase from the line voltage. The application of test coils at various angles with the active phase would show the presence of nearly the same e.m.f. regardless of the position of the coil, and an angular difference of phase corresponding to the angle of the coil with the stator winding. This experiment proves the existence of a rotating magnetic field, and a fuller investigation would show that this field is harmonic in its space distribution and rotates with uniform angular velocity.

In the light of what has previously been said, it is almost self-evident that this will be the case. The flux tends to assume such a distribution and value that the minimum cutting of the rotor conductors and consequently the minimum expenditure of power will take place. Each change of flux sets up a rotor current in such a position and phase as to tend to prevent the change of flux. With a rotor operating at synchronism, a uniform flux rotating at uniform velocity would not cut the rotor bars at all. Hence the flux tends to assume this distribution and velocity.

However, to produce the rotating magnetic field, in a single-phase induction motor it is evident that there must exist a component of m.m.f. at right angles to the axis of the stator winding. Since the stator can not carry a current in the proper position to produce this it must exist in the rotor. To produce this rotor current requires a change in the stator flux. Hence the field can not be of absolutely constant value at all times. The change however is slight.

The nature of the current required in the rotor to produce with the stator current a rotating magnetic field in the motor, will be apparent from Figure 22. The four diagrams are drawn for successive values of the current taken at intervals of 30 degrees. The flux is shown displaced 30 degrees to the left for each change of 30 degrees in the



F = Flux
S = Stator Current
R = Rotor Current
T = Resultant Current

Current Distribution in Stator and Rotor of A Single-Phase Inudction Motor. Sinusoidal Distribution of Stator Coils, Rotor Squirrel-Cage.

Figure 22

current.

The solid line marked stator current represents the distribution of the stator current over the stator surface. The conductors are assumed to be arranged on the stator core in such a manner that the number of conductors at any given point is proportional to the sine of the angle corresponding to the point. The arc between two poles is of course taken as 180 electrical space degrees. The current is the same in all of the conductors and hence the sinusoidal curve represents the distribution of the current over the stator core. The curve of distribution is stationary in space, but variable in magnitude, changing from a positive to a negative maximum, in accordance with the change in the current.

In order that an harmonic, uniformly rotating magnetic flux be maintained, it is necessary that a <u>resultant</u> harmonic band of current rotate uniformly around the stator. This resultant is due to currents in both the stator and the rotor. It is shown as a dotted line in each of the figures and is drawn 90 degrees ahead of the flux. The resultant band of current is due to the algebraic sum of the current sheets in both the stator and rotor at any given point. The rotor current is then the difference between the resultant current and the stator current. It is shown by the curve marked rotor current. In order that the required rotor current may circulate, it is necessary that the rotor be of the squirrel-cage variety with many bars. With a phase-wound rotor, the current could not assume the exact values required at all points and the resulting rotating flux would not have exactly harmonic distribution.

A study of the construction of the diagrams will reveal the following facts:

A. The flux has harmonic distribution, is constant in magnitude and rotates uniformly in the direction of rotation of the rotor.

B. The stator current sheet is stationary in space distribution, and has harmonic variation in magnitude.

C. The rotor current sheet is harmonic in space distribution, of constant maximum value, and rotates backward, i.e., opposite to the direction of rotation of the rotor at synchronous speed. Its maximum value is half that of the stator current sheet.

It is apparent that at the time shown in Figure D, the rotor current sheet must be sufficient to force the total flux across the gap. Hence its value is the same as would be required in a second phase of the stator if one were present.

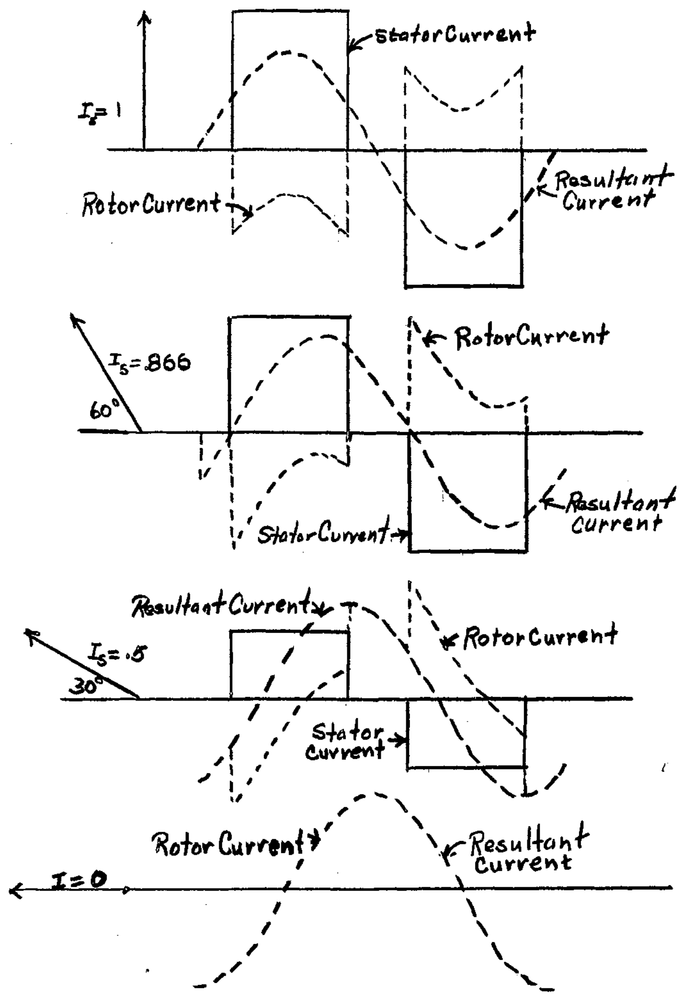
Its space location is 90 degrees from the stator winding. The conclusion readily follows, that a single-phase motor requires twice the magnitizing current that would be taken by one phase of the same motor-wound two-phase with the same number of turns per phase. Similar reasoning would apply to a three-phase motor compared with a single-phase, or in general we may say that the volt-amperes required are the same whatever the number of phases. The same principle was explained in developing the formula for the magnetizing current of a polyphase motor. The foregoing may be considered as a proof that the same formula applies to the single-phase motor.

The curves of Figure 22 are constructed on the supposition that the conductors on the stator core are distributed in such a manner that the number of conductors at any given point is proportional to the sine of the angle at that point, counting from some fixed point of reference on the stator core. It is hardly necessary to point out that such a distribution is not feasable in practice. In many motors, some attempt is made to approximate this condition by winding some coils with less turns than others. To do this requires coils of the concentric type, as shown in Figure 2. For this reason, and for the sake of symmetry, such coils are frequently employed in single-phase motors.

The curves shown in Figure 23 may be considered as an example of the extreme opposite condition. This represents the currents in the stator and rotor of a three-phase motor, wound with full pitch coils, and operated on a single-phase circuit. The curve of distribution of the stator current is of course rectangular as shown. The resultant band of current is sinusoidal, and the rotor current is of the proper value to give, in combination with the stator current, the resultant harmonic band of current. As before, it will be seen that the rotor current sheet moves in the opposite direction from the resultant current sheet, but that it is now very much distorted from the sine shape.

These curves, like those of Figure 22, are for the no-load condition. With the motor under load, the value of the rectangular stator current would be increased in proportion to the current. There would be added in the rotor a corresponding rectangular current distribution, almost exactly equal and opposite to the added stator current. In the case of Figure 22 a corresponding sine distribution of current would be added. It is this added component, in connection with the component of the flux at right angles to the direction of the stator winding, that produces the rotor torque.

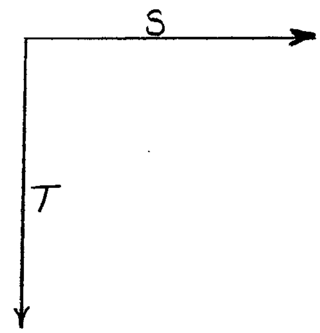
Returning to the ideal case of sinusoidal distribution, as shown in Figure 22, instead of considering the rotor



Current Distribution in Stator and Rotor of Single-Phase Induction Motor. Stator Coils in 120 Degree Bands. Rotor Squirrel-cage.

Figure 23

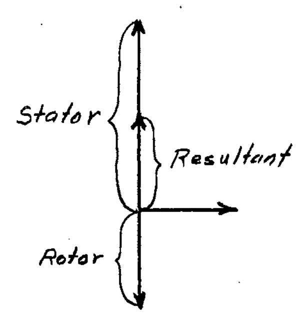
current sheet as a band of current rotating backward in space, we may perhaps gain a better idea of the phenomena involved if we separate the band of rotor current into two component current sheets, each stationary in space but varying harmonically in magnitude. The bands differ 90 degrees in time phase and are displaced 90 degrees in space. As we have already shown the combination of two such stationary bands is equivalent to one rotating band. Each band of current may be represented by a vector as shown in Figure 24. The resultant current sheet will be of constant value, and will rotate as shown.



Vector Diagram of Current Bands in Rotor.

Figure 24

Under the condition of no-load and synchronous speed, we have just seen that we have in the stator a current sheet of double the value of the rotor current sheet. This sheet can be represented by a vector of double the value of one of the vectors representing the rotor current sheets and at an angle of 180 degrees with one of them. The relations are then as shown in Figure 25, and the direction of rotation of the resultant of the three current sheets will be in the opposite direction to that of the rotor current sheet. The net result then is that we may consider that we have three stationary current sheets, one in the stator and two in the rotor. One of the rotor sheets is directly opposed to and offsets half of the stator current sheet. This resultant then combines with the remaining rotor sheet to form a rotating current sheet of constant value. This rotating current sheet sets up a corresponding rotating flux sheet which is likewise constant in value and rotates in synchronism with the current sheet. The above applies of course to the no-load condition only.



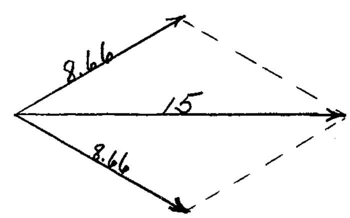
Vector Diagram of Current Bands in Rotor and Stator.

Figure 25

From the above discussion it is seen that the single-phase current of 8.66 amperes and thus the flux per phase coil will set up the same rotating field as did the three-phase machine. Since the two phases of the three-phase machine operated single-phase will always be 60 degrees out of space phase, the resultant flux will be 15 as shown in Figure 26.

Vector Addition of Flux.

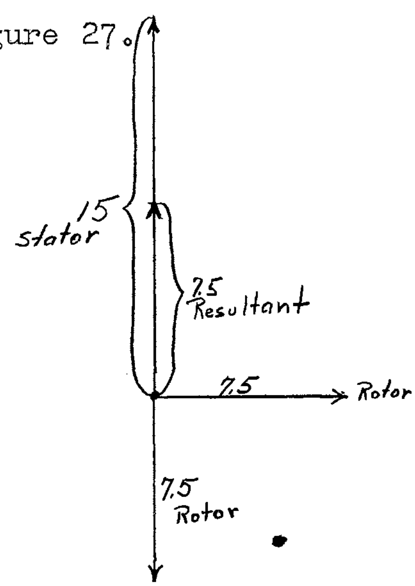
Figure 26



Then from Figure 25 in Mr. Bailey's discussion it is apparent that a rotating field of value 7.5 will exist in the single-phase operation of the motor as was in the three-phase operation of the same motor. The two fluxes are in both space and time quadrature as shown in Figure 27.

Vector Diagram of Rotating Flux in Single-Phase Motor.

Figure 27



It is therefore apparent that the magnetizing current in the single-phase induction motor is nearly twice that of the same size three-phase induction motor, effectively reducing the power factor and efficiency of the single-phase machine.

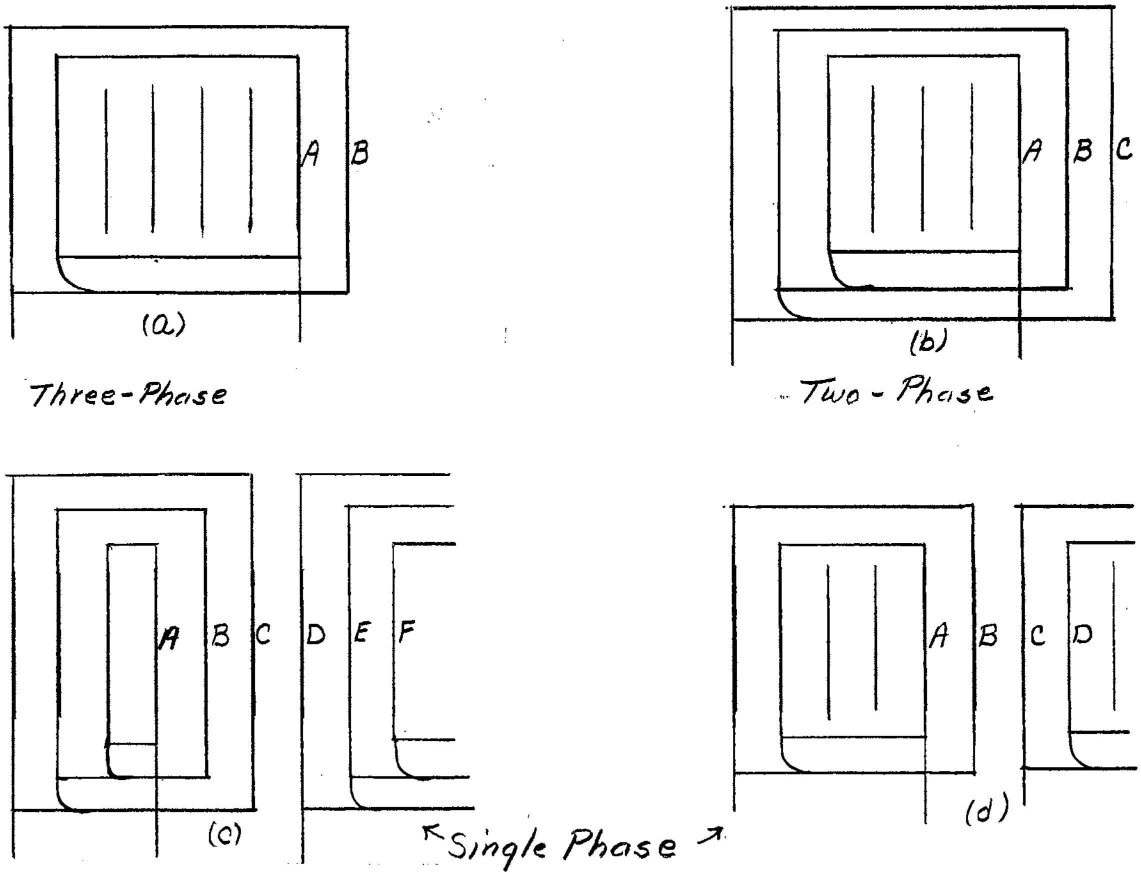
CHAPTER IV

COMPARISON OF HORSEPOWER RATING OF SINGLE-PHASE AND POLYPHASE INDUCTION MOTORS ON THE BASIS OF EQUAL COPPER LOADING

In general, the back voltage at no load of single-phase and polyphase induction motor will be of the order of 95% of the terminal voltage. This is not exactly true however, because of the higher magnetizing current necessary for single-phase machine in comparable ratings to polyphase machines. The higher magnetizing current in the single-phase machine is due to the retarding effect and the cross-axis field losses of the single-phase machine. The result being that the single-phase induction motor does not run at, or very close to, synchronous speed at no load as does the polyphase induction motor and consequently the single-phase motor has greater power losses. It might be assumed without much error, however, that the back voltage of both machines will be 95% of terminal voltage, and that the variation due to load will be approximately the same in each motor up to full load. Making the above assumption an example[1] of a stator of six slots per pole will be utilized to show the effect of economical and efficient design for equal copper loading on the three-phase, two-phase, and single-phase induction motors.

Shown in Figure 28 are the winding diagrams for three-phase, two-phase, and single-phase induction motors, all having six slots per pole. The same stator is assumed to be used in each case. With six slots per pole the back voltage generated in the

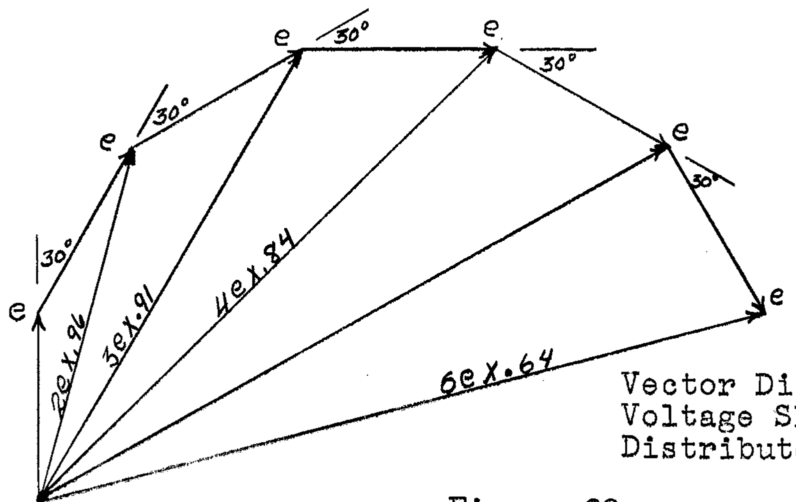
[1] Alexander Gray, Electrical Machine Design, p. 187.



Effect of the Number of Slots on Induced Voltage
Figure 28

conductors in each adjacent slot of the stator will be 180/6 or 30 electrical degrees out of phase.

Referring to one phase of the three-phase winding of Figure 28 it is apparent that the back voltage generated in slots A and B will not be in phase, and since the conductors are in series, their resultant voltage will not be twice the voltage or one of them but somewhat less than this value. In Figure 29 the two voltages are plotted in their relative phase relations and letting e be the conductor voltage, the resultant of the two conductors will be 2e x .96. Since the windings as shown for the three-phase motor in Figure 28 a are full pitch, the reduction factor in voltage due to the distributed winding is .96.



Vector Diagram of Induced Voltage Showing Effect of Distributed Winding

Figure 29

For the two-phase winding, Figure 28 b, the three conductors of one phase A, B, and C will have a back voltage generated of equal magnitude in each conductor. The voltages in conductor B will be 30° out of phase with that of conductor A, and the back voltage of conductor C will be 60 electrical degrees out of phase with conductor A. Thus as shown in the vector diagram of Figure 29 the resultant voltage will be 3e x .91, .91 being the distribution factor of the two-phase winding.

In the single-phase winding, Figure 28 c, it is seen that all six slots are used in the winding and consequently the back voltage of the conductor will be 150° out of phase with the voltage of conductor A, or almost in phase opposition. The resultant back voltage of the six conductors of the single-phase winding will be 6e x .64. In comparison, if only four of the slots are used for the single-phase winding as in Figure 28 d. The resultant back voltage is 4e x .84 or only about 10% less than when all six slots of the stator are used for the winding. Therefore, by a 50% increase in the amount of copper, there is a 10% increase in voltage. For reasons of economy as well as the fact that all single-phase induction motors of the capacitor start, induction run and the split phase start, induction run types will require about 1/3 of the winding slots for the starting winding. About 2/3 of the slots are generally used for the main winding. Even for commutator start single-phase induction motors, the middle slots are usually not used so as to take advantage of the saving of copper. Using the concentric type winding makes it expedient to leave out these windings.

In the four windings of Figure 28, if the conductors in each slot or the group of conductors in each slot, as the case may be, were to have the same current per circular mil of copper cross section, a comparison on the basis of equal copper loading could be made. It is true that the single-phase coils would not all have the same number of turns, more being concentrated in some slots than in others. This would effectively reduce the back voltage, but the equal copper loading could remain the same. This slight decrease in voltage would occur because arranging the coils so as to approach sine wave space distribution will decrease the distribution and pitch factor of the single-phase machine windings. Unequal grouping of the conductors will cause the hot spot temperature to further limit the current.

The three induction motors with windings of diagrams as shown in Figure 28 a, b, d, having the same number of conductors and equal loading, will have generated back voltages[2] per phase as shown:

Single-phase (2/3 slots used)	K x 4e x .84
Two-phase	K x 3e x .91
Three-phase	K x 2e x .96

Under the conditions of equal copper loading and the same number of conductors, the volt ampere rating of the different machines will be determined by the temperature rise and the hot spot temperature. This rating[3] would be the product of the terminal voltage per phase, the phase current, and the number of phases, or as shown for the different machines.

Single-phase (2/3 slots used)	
	K x 4e x .84 x I x 1 = .56 K
Two-phase	K x 3e x .91 x I x 2 = .91 K
Three-phase	K x 2e x .96 x I x 3 = .96 K

I is the stator conductor current which is the same in each motor. The flux per pole being the same in each case, e is the conductor voltage.

From the above and under the conditions assumed, the rating of the single-phase motor will be only about 60% of the rating of the three-phase machine. As pointed out in Chapter IV, the single-phase motor inherently has a lower power factor and

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2 _Ibid._, p. 188.

3 _Ibid._, p. 188.

efficiency, this plus a reduction in distribution and pitch factor will thus tend to reduce the rating of the single-phase induction to nearer 50% of the three-phase rating.

CHAPTER V

COMPARISON OF HORSE POWER RATINGS ON THE BASIS OF MAXIMUM TORQUE

The minimum breakdown[1] or maximum torque for polyphase induction motors, set by NEMA, is to be at least 200% of the rated full load torque. Recent commercial publications by different manufacturers, however, indicate that the breakdown torque of most general purpose polyphase induction motors will be in the neighborhood of 250% of rated torque.

For single-phase induction motors, values of breakdown torque vary widely with the different types of motors. Tables[2] IV and V give a comparison of the general characteristics and application of single-phase and polyphase induction motors. The average breakdown torque required of the general purpose single-phase induction motor will, like the polyphase motor, be near 200% of full load torque.

As is seen, the table for single-phase motors shows breakdown torque only for fractional horsepower motors. These fractional horsepower motors represent by far the major portion of single-phase motor production. Although single-phase induction motors are built in standard sizes up to and including 10 horsepower, only a few are used in special cases. The necessity of special starting devices, poor power factor and low efficiency make the polyphase machine advantageous, and, in some

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[1] Puchstein and Lloyd, Alternating Current Machines, p. 293.

[2] Liwschita-Garik and Whipple, Electric Machinery, Volume II, pp. 438-439 and 453-454.

TABLE IV

POLYPHASE INDUCTION MOTOR CHARACTERISTICS

Type Classification	Hp Range	Starting Torque (%)*	Pull-Out Torque (%)*	Starting Current (%)*	Slip (%)	Power Factor (%)	Efficiency (%)	Typical Application
General-purpose, normal torque and starting current NEMA Class A	0.5 to 200	Poles - Torque 2 - 150 4 - 150 6 - 135 8 - 125 10 - 120 12 - 115 14 - 110 16 - 105	Up to 250 but not less than 200	500-1000	Low, 3-5	High, 87-89	High, 87-89	Constant-speed loads where excessive starting torque is not needed and where high starting current is tolerated. Fans, blowers, centrifugal pumps, most machine tools, woodworking tools, line shafting. Lowest in cost. May require reduced voltage starter. Not to be subjected to sustained overloads, because of heating. Has high pull-out torque.

* Figures are given in percent of rated full-load values.

TABLE IV (CONTINUED)

Type Classification	Hp Range	Starting Torque (%)*	Pull-Out Torque (%)*	Starting Current (%)*	Slip (%)	Power Factor (%)	Efficiency (%)	Typical Application
General-purpose, normal torque low starting current, NEMA Class B	0.5 to 200	Same as above or larger.	About the same as Class A but may be less	About 500-550, less than average of Class A	3-5	A little lower than Class A	87-89	Same as Class A - advantage over Class A is lower starting current, but power factor slightly less.
High torque, low starting current, NEMA Class C	1 to 200	200 to 250	Usually a little less than Class A but not less than 200	About same as Class B	3-7	Less than Class A	82-84	Constant-speed loads requiring fairly high starting torque and lower starting current. Conveyors, compressors, crushers, agitators, reciprocating pumps. Maximum torque at standstill

TABLE IV (CONTINUED)

Type Classification	Hp Range	Starting Torque (%)*	Pull-Out Torque (%)*	Starting Current (%)*	Slip (%)	Power Factor (%)	Efficiency (%)	Typical Application
High torque, medium and high slip, NEMA Class D	0.5 to 150	Medium slip 350 high slip 250-315	Usually same as stand-still torque	Medium slip 400-800 high slip 300-500	Medium 7-11, high 12-16	Low	Low	Medium slip. Highest starting torque of all squirrel-cage motors. Used for high-inertia loads such as shears, punch presses, die stamping, bulldozers, boilers. Has very high average accelerating torque. High slip used for elevators, hoists, etc., on intermittent loads.
Low starting torque, either normal starting current, NEMA Class E, or low starting current NEMA Class F	40 to 200	Low, not less than 50	Low but not less than 150	Normal 500-1000, low 350-500	1 to 3 1/2	About same as Class A or Class B	About same as Class A or Class B	Direct-connected loads of low inertia requiring low starting torque, such as fans and centrifugal pumps. Has high efficiency and low slip.

TABLE V

SINGLE-PHASE INDUCTION MOTOR CHARACTERISTICS

Type Designation	Starting Torque (% Of Normal)	Pull-Up Torque (% of Normal)	Pull-Out Torque (% of Normal)	Starting Current at 115 V	Power Factor	Efficiency (%)	hp Range	Application and General Remarks
General-purpose split-phase motor	90-200 Medium normal	200-250	185-250 Medium	23 1/4 hp	56-65	62-67	1/20 to 3/4	Fans, blowers, office appliances, food-preparation machines. Low- or medium-starting-torque, low-inertia loads. Continuous operation loads. May be reversed.
High-torque split-phase motor	200-275 High	160-250 High	Up to 350	32 High 1/4 hp	50-62	46-61	1/6 to 1/3	Washing machines, sump pumps, home workshops, oil burners. Medium- to high-starting-torque loads. May be reversed.

TABLE V (CONTINUED)

Type Designation	Starting Torque (% of Normal)	Pull-Up Torque (% of Normal)	Pull-Out Torque (% of Normal)	Starting Current at 115 V	Power Factor	Efficiency (%)	hp Range	Application and General Remarks
Permanent-split-capacitor motor	60-75 Low	60-75 Low	Up to 225	Medium	80-95	55-65	1/20 to 3/4	Direct-connected fans, blowers, centrifugal pumps. Low-starting-torque loads. Not for belt drives. May be reversed.
Permanent-split capacitor motor	Up to 200 Normal	200	260		80-95	55-65	1/6 to 3/4	Belt-driven or direct-drive fans, blowers, centrifugal pumps, oil burners. Moderate-starting-torque loads. May be reversed.
Capacitor-start general-purpose motor	Up to 435 Very High	265 High	Up to 400		80-95	55-65	1/8 to 3/4	Dual voltage. Compressors, stokers, conveyors, pumps. Belt-driven loads with high static friction. May be reversed.

TABLE V (CONTINUED)

Type Designation	Starting Torque (% of Normal)	Pull-Up Torque (% of Normal)	Pull-Out Torque (% of Normal)	Starting Current at 115 V	Power Factor	Efficiency (%)	hp Range	Application and General Remarks
Capacitor-start capacitor-run motor	380 High	260	Up to 260		80-95	55-65	1/8 to 3/4	Compressors, stokers, conveyors, pumps. High-torque loads. High power factor. Speed may be regulated.
Repulsion-start induction-run motor	350-500 Very high	225	Up to 275	350%	70-80	55-65	Up to 10	Very-high-starting-torque loads. Pumps, compressors, conveyors, machine tools. Reversed by shifting brushes.

cases, less expensive in the large sizes.

The expression for the torque of the polyphase induction motor is relatively simple and is given in most test books as alternating current machinery. As given by Dean A. S. Langsdorf, the equation[3] for torque in pound feet, derived from the equivalent diagram is

$$T = \frac{33,000}{2\pi N_1 \times 746} \times \frac{M_1 V_1 R_{2e} S}{(SR_1 + R_{2e}) + S^2(X_1 + X_{2e})} \text{ lb. ft.} \quad (14)$$

where

N_1 = synchronous speed in rpm
V_1 = impressed voltage per phase
R_{2e} = equivalent rotor resistance
S = slip in percent synchronous
M_1 = number of phases
R_1 = stator resistance per phase
X_1 = stator reactance per phase
X_{2e} = equivalent rotor reactance

It was found by Dean Langsdorf that the condition for maximum torque exists when the slip is as follows:

$$S = \frac{R_{2e}}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \quad (15)$$

Upon substituting this value in Equation (14), the expression[4] for maximum torque becomes:

$$T_{(max)} = \frac{33,000}{2\pi N_1 \times 746} \cdot \frac{M_1 V_1^2}{2\left[R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2}\right]} \text{ lb. ft.} \quad (16)$$

The above equations show that the slip at which maximum torque

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3 Langsdorf, <u>Alternating Current Machinery</u>, p. 579.

4 <u>Ibid.</u>, p. 580.

occurs is directly proportional to the rotor resistance, whereas the maximum torque itself is independent of rotor resistance. This is not true in the case of the single-phase machine. From Equations (15) and (16) it is seen that maximum torque may be made to occur at any speed by varying the rotor resistance.

In general, the stator resistance will be very small as compared to the stator leakage reactance and may be neglected for comparison purposes without excessive error. In so doing the above equation will reduce to:

$$T_{(max)} = \frac{33,000}{(2\pi n_1 \times 746)} \cdot \frac{mV^2}{2\sqrt{(X_1 + X_2)^2}} \text{ lb. ft.} \qquad (17)$$

Letting $(X_1 + X_2) = X =$ the total leakage reactance of the motor, the maximum torque is

$$T_{max} = K\frac{MV^2}{2X} \qquad (18)$$

where $K = (\frac{33,000}{2\pi n_1 \times 746})$

Thus the maximum torque of a polyphase induction motor is directly proportional to the applied voltage squared and inversely proportional to the motor leakage reactance.

Contrary to the above expression, the maximum torque equation for the single-phase induction motor is not quite so simple. The equation[5] for maximum torque in the single-phase induction motor is given as follows:

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[5] Puchstein and Lloyd, "Single-Phase Induction Motor Performance," Electrical Engineering, 56 (October, 1937), p. 1277.

$$T = \frac{112}{N}\,\frac{1}{S_{po}}\left[\frac{V^2}{X}\,tr\,\frac{Q}{Q^1} - (F + Fe^{(c)})\right] \text{ oz. ft.} \tag{19}$$

where

V = applied voltage
r_1 = stator winding resistance
r_2 = rotor resistance in stator terms
X_1 = stator winding leakage reactance
X_2 = rotor leakage reactance
X_m = mutual reactance in stator terms
$X_o = X_m + X_1$ = magnetizing reactance for air gap and primary leakage fluxes
$K_r = \frac{X_o + X}{X}$
$X = X_1 + X_2 = X_1 + \sqrt{Kr_1}\,X_2$
F = watts, friction, and windage loss
Fe = watts core loss
I_1 = primary current
S = percent synchronous speed
S_{po} = percent synchronous speed at which pull out torque occurs
$P = \frac{r_2}{x}$, $P_1 = \frac{r_1}{x}$

$$t_r = \frac{K_r}{2(2 - K_r)}\left[\sqrt{\left[1 + P^2(1 - K_r)^2\right]\left[1 - \frac{P^2\,(2 - K_r)^2}{(P^2+1)\left[1+P^2(1-K_r)^2\right]}\right]} + P(1 - K_r)\right]$$

$$Q^1 = \left[(1 + P_1 t_r)^2 + (P_1 0^1{}_r)^2\right]$$

$$Q = \frac{1}{1 + a}$$

$$0^1{}_r = \left[1 - .5\left(\frac{K_r}{2 - K_r}\right) - \frac{P}{2\sqrt{1 + P^2}}\right]$$

$$A = \frac{\text{total secondary losses}}{\text{output} + F + Fe^{(c)}} = \frac{1 - S^2}{S^2} + \frac{2\left(\frac{r^2}{x_o}\right)^2}{\left[(1-S^2) - \left(\frac{r^2}{x_o}\right)^2\right]S^2}$$

C = series conductor of stator winding.

It is thus apparent that a comparison of the maximum torque equation of single-phase and polyphase induction motors as shown in Equations (18) and (19) would require extensive study and analysis. However a partial comparison may be made from the equation for torque[6] in terms of air gap flux as follows;

$$T = \frac{2.2}{10^8} (p\ \phi\ Z\ I\ k_p\ k_d \cos\theta) \text{ lb. inches} \qquad (20)$$

Where

p = number of poles
ϕ = effective flux per pole
Z = total conductors of stator
I = stator current in amperes
k_p = pitch factor of stator winding
k_d = distribution factor of stator winding
$\cos\theta$ = angle between stator current and rotating flux

It is brought out by Messrs. Puchstein and Lloyd and by Messrs. Liwschitz-Garik and Whipple that the above equation applies for single-phase as well as polyphase machines. The equation may be applied in terms of the stator or rotor. Equation (20) is the developed torque exclusive of friction and windage and core losses.

In examining Equation (20) maximum torque will depend upon the winding factors k_p and k_d, the current density of the conductors and upon $\cos\theta$. The number of poles, and flux per pole is the same in both the single-phase and polyphase machines. It is assumed that the single-phase motor will have only approximately 2/3 the number of series conductors as has the polyphase induction machine, the remainder of the slots being used for the starting windings.

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[6] Puchstein and Lloyd, Alternating Current Machines, p. 256. Liwschitz-Garik and Whipple, Electric Machinery, Volume II, p. 179.

It can be seen from the above discussion and from Equation (18) that maximum torque will also depend upon the leakage reactance of the windings of both stator and rotor. The total leakage reactance[7] is made up of (1) slot reactance, (2) zigzag reactance, (3) belt reactance and (4) end-connection leakage reactance. The slot and end-connection leakage reactances are functions of the number of conductors, type of winding, winding factors, and physical dimensions of the machine, whereas the zigzag leakage reactance is a function of the applied voltage, magnetizing current, winding, and machine dimensions. In induction motors with squirrel-cage rotors the belt leakage reactance is zero. The total leakage reactance of the induction motor directly affects the maximum torque and should be kept as low as is possible. In the three-phase motor the maximum torque is independent of rotor resistance, but in the single-phase machines the rotor resistance is instrumental in determining pull out torque and should be kept low. If the assumption is made that the total leakage reactance and rotor resistance are approximately the same in both the single-phase and three-phase motor, then the current density and stator winding will be the determining factors.

In general the single-phase induction motor will have approximately 10 per cent greater current density[8] than does the

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[7] Kuhlmann, Design of Electrical Apparatus, p. 314-315.

[8] Ibid., p. 291-331.

polyphase machine. This is because of the relatively fewer conductors and thus more heat dissipating surface of the stator.

Under the above circumstances, the winding factors will greatly affect the maximum torque, as was the conclusion in Chapter 4. Taking into account the winding factors, current density, number of stator conductors, power factor, and efficiency, the maximum torque of a single-phase machine will be less than 2/3 that of a polyphase induction motor. On the basis of pull out torque being 200 per cent of full load torque, the horsepower rating of the single-phase induction motor will be between 1/2 and 2/3 the rating of the polyphase induction motor.

CONCLUSION

As stated in the preface, any conclusion drawn by the writer would necessarily be of a very general nature. There are many different types of motors and all are designed for a particular operation or a particular type of job. Therefore certain design features are sacrificed so as to emphasize other design features. Thus there is no definite set design procedure for either the single-phase or polyphase induction motors. Too, there are many empirical equations as well as various data gained from experience, not generally available in the literature on the subject.

In studying the different windings of the stators of both single-phase and polyphase machines, the writer finds that the winding factors, distribution factor and pitch factor, have an important effect upon the economy of manufacture, efficiency and performance of the machines. Since it is assumed in this thesis that the same rotor, same stator and same flux per pole are used in both the single-phase and polyphase machines, the winding factors of the stator of the two machines are most important.

It is true that if the single-phase motor were designed, using all the stator slots and having as high current density as possible, then the rating will be a little less than the rating of a polyphase induction motor with the same stator, rotor and flux per pole.

In the interest of good motor design (reduction of harmonics, sine wave of flux, etc.), economy of manufacture, and economy of material, it is the opinion of the writer that the rating of the single-phase induction motor will be approximately one-half that of the three-phase machine.

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