ELECTRIC HEATING ELEMENT ECONOMICS

By

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PREFACE

The history of electric resistance furnaces is one of progressive development and regression.

The resistance furnace was first introduced to industry in the period of 1920 to 1922. Very effective sales campaigns were conducted and the industries made up the proving ground for the equipment. Sales increased steadily until industry, through new gained experience, realized the resistance furnace was not the cure-all as they were lead to believe. As a result, the rapid growth in the use of resistance furnaces suffered a reversal and an equally rapid decline in sales in many industrial heating processes occurred.

In recent years the industries, under heavy expansion, began to realize the outstanding features offered by the resistor furnace. The designers and users began to compare the resistor furnaces with fuel fired furnaces from a different standpoint than their predecessors. They began to understand that the value of the resistor furnace did not lie in thinking in comparative terms of electricity and fuel costs, but rather the furnace's value was hidden in the improved quality of the product and overall cost figures, not just those of fuels alone.

In spite of the many advances being made, the designers of the electric resistance element of the resistor furnace are neglecting to consider one very important factor, namely, economy.

The purpose of this thesis is to show the design engineer and the user of electric resistance elements that economy, based on sound engineering fundamentals, can be obtained and incorporated into the resistance element.

It is the hope of the author that the material presented here may be of some value to the engineers working in the field of electric heating elements and their users, and that it may prove to be beneficial to others interested in conducting more research along the same general lines.

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Professor C. F. Cameron who aroused the author's interest in the topic of this thesis. He not only gave freely of his time and counsel, but also made invaluable suggestions while reading the material.

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CHAPTER I

In the consideration of economics of resistor heating elements, it is of prime importance to review the conditions the element must meet, the conditions under which the element must operate, and its applications in order to fully understand the factors involved.

THE RESISTOR FURNACE

The resistor furnace offers a large number of distinctive features over other kinds available. The main features are: (1) simplicity of application, (2) reproducibility of any process resulting in the improvement of the quality, (3) the heat distributed or radiated is uniform, and (4) freedom of choice of the atmosphere. In addition, several minor factors may be mentioned, such as: (1) easier metering, (2) ease with which small units can be constructed, (3) simpler construction, (4) simplified operation, (5) no combustion products to dispose of, (6) the amount of heat radiated to surroundings is relatively small, and (7) less space is required.

At the present time, electrically produced heat is available for almost any application, i.e., heating air, liquids, solids, melting metals, heat treating, drying, sterilizing, brazing, and many others. Electric heating units can be suspended in air, immersed in liquids, or embedded in solid material insuring highest efficiency in the transfer of heat for the two latter cases because all of the heat must pass into the liquid or solid being heated.

Electrically produced heat does not require a flow of air, does not emit fumes, or create soot nor is there a flame. The electric furnace will, in general, take less floor space and can more easily be located than a fuel fired furnace when such factors as fuel storage and ash disposal are taken into account.

The accuracy of control over the temperature or flow of heat applies to both uniformity in space and uniformity in time, both having their effect on the uniformity of the temperature of the charge. In regard to flow of energy or temperature uniformity in space, proper furnace design coupled with the proper placing of the charge in the furnace will provide uniformly generated heat or that the heat transferred to the charge is uniform. For example, heating elements can be properly distributed within the heating chamber and then easily grouped in separate sections, each with its own control. Used this way they can compensate for local high heat losses avoiding cool areas near doors or other openings. This uniformity is very seldom, if ever, accomplished in the fuel-fired furnace. Automatic controls have been developed for all types of furnaces, fuel-fired as well as electric, and on the surface it appears that the furnaces should be alike as far as uniformity of energy flow or temperature with time is concerned. However, no fuels are consistantly alike. The changes in composition requires different air-fuel ratios which cannot be achieved automatically. This obstacle is not present in electric furnaces resulting in a superiority of uniformity of temperature with time.

Rapid developments have been and are being made in controlled atmosphere processes which is a very important advantage of resistance furnaces. The heating chamber and the source of heat in a resistor furnace are practically independent making possible the control of the atmosphere and its function to almost the same degree of precision as is electrical energy.

Some engineers have classified atmospheres in heating chambers of resistance furnaces as (1) natural atmosphere and (2) artificial atmosphere.[1]

The natural atmosphere, as the name implies, is the result of air entering the heating chamber by intent or leakage and mixing with the gases given off by the charge. The natural atmosphere continually changes during the heating process being dependent upon the amount of gas liberated from the charge which is naturally a function of the temperature of the charge, the chemical reactions taking place in the chamber, and the rate at which air is allowed to enter. The only controllable factor is the rate at which air enters, such control can be obtained by sealing the chamber and using throttling vents. Natural atmospheres can be used to control the oxidation of the charge in processes not warranting the use of an artificial atmosphere.

Gases of known and previously determined composition introduced into a tightly sealed heating chamber constitutes an artificial atmosphere. The gases so injected may enter

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[1] N. R. Stansel, "Industrial Electric Heating," <u>General Electric Review</u>, XXXI (October, 1928), 560.

directly into the chemical reaction taking place in the heating chamber, combining readily at the high temperatures, or they may remain neutral to reactions. In controlling the flow of the artificial atmosphere one must bear in mind that gases may be liberated from the charge and the reactions taking place occur at elevated temperatures and various pressures; all this must be considered when selecting the gases and determining the rate of flow. Also grouped under artificial atmospheres are the atmospheres maintained at a pressure below atmospheric.

Below is a classification of the use of gas atmospheres, being defined on the basis of the material to be heated and the process in view.

(1) Processes in which the result, wholly or in part, is due to formation of an oxide. One example is the baking of oil-bonded sand cores. This is an oxidation process. A second example is vitreous enamelling in which the bond between the ground coat and the base metal is formed by an oxide. For processes of this kind natural atmospheres are used.

(2) Processes for which the oxygen content of a natural atmosphere is permissible provided it is restricted by a closed, but not necessarily sealed, heating chamber. An example is the annealing of machine parts preliminary to machining. A certain limited amount of scaling, i.e., surface oxidation, is acceptable, the limitation being that the degree of this action must not make machining difficult.

(3) Processes for which the atmosphere of the heating chamber must be neutral, i.e., without effect upon the material of the charge. The annealing of copper in an atmosphere of steam is an illustration of processes of this kind.

(4) Processes which require an atmosphere which will bring about the reduction of an oxide. The use of hydrogen as a flux in the brazing of steel parts is an example.

(5) Processes which require holding the charges at an elevated temperature for a period of time. Here excessive oxidation would result if oxygen is permitted access to the

charge, and the atmosphere of the heating chamber must be neutral or, as is generally used, reducing in character. An example is the heat treatment of steel to improve its magnetic properties.

(6) Processes in which a constituent of the heating chamber atmosphere combines with the charge to change some physical property of the material as in gas carburizing, nitriding, etc.

(7) Processes for the purification of metals.[2]

It would not be fair to look only into the advantages of resistance furnaces; it, like everything else, has decided disadvantages. Among these are: high first cost, fuel storage is impossible, repairs are more expensive, the furnace is more subject to abuse, and forcing of the furnace is impossible. Several of these points may be argued; for example, certainly fuel storage is impossible but perhaps this is not objectionable in certain installations with our present practice of interconnecting power systems and through the use of a loop primary feeder. Another example might be the higher first cost item. Comparing furnaces in regard to Btu per dollar for first cost makes the resistor furnace usually appear higher than the other types. This would be somewhat like looking at only part of a mathematical equation. Complete consideration should be given of the entire cost figures. This will, in many instances, justify the purchase of a resistance furnace with the decision based on this one item alone, especially when such factors are considered as the ability of an electric furnace to be located

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[2] N. R. Stansel and C. Dantsizen, "Industrial Electric Heating," General Electric Review, XXXII (March, 1929), 163.

directly in the line of material flow reducing handling costs, accurate control and reproducibility of any process minimizing rejected work loss, and comfortable working conditions can easily be maintained.

RESISTANCE FURNACE TYPES

Electric resistance furnaces are divided into the two general classes of batch and continuous furnaces. Each main classification is further divided into several types and these types are given a descriptive title according to the method used to convey the charge to and from the heating chamber, the shape, or the means of support.

The batch type furnaces, as the name implies, is designed for heating individual stationary or non-moving charges. Its use is mainly for heat treating of castings and forgings; annealing of steel castings, bar stock, and plates; relieving of stresses in welded structures; nitriding; carburizing; and other artificial atmosphere treatments.

The continuous furnace has a continuous or intermittent conveyor feed. Furnaces of this classification are used for hardening gears, bolts, ball and roller races, and other parts; annealing steel strips; heat treating glass; drawing; brazing; normalizing; vitreous enamelling; and other processes. These furnaces also are frequently used with controlled artificial atmospheres. A slow cooling or a quenching chamber may be attached to this type of furnace and be incorporated in its feed.

The selection of the type of furnace depends upon the method of handling the charge, character of material to be heated; the process; weight, size, shape, and volume of pieces; and the desired rate of production.

THE RESISTOR ELEMENT

The electrical element of the furnace, the resistor, is the most important single item in the design of resistance furnaces. The resistors are the element of highest temperature placed directly in the heating chamber and hence more subject to the danger of failure than any component.

The resistors, or sources of heat, are separated from the charge only by the atmosphere, natural or artificial, therefore, the transfer of heat is direct. This direct transfer of heat is an asset to the resistance furnace. Synonymous with it is that a small temperature gradient exists between the resistor element and the heating chamber, since there is practically no thermal storage capacity in the atmosphere, and immediate response is therefore obtained from the temperature control equipment. This, of course, leads to simplicity itself.

What are the desired properties of the resistor element material? The conditions the ideal material must meet are many and no material can fulfill each and every one. The more important properties that a material should possess are:

(1) Freedom from action with the atmosphere whether natural or artificial. The material should resist oxidation and scaling in a natural atmosphere or chemical reaction

with the artificial atmosphere. This is a primary requisite.

(2) The melting point of the material should be high and confined to a narrow range. The narrow range assures the designer that the resistance material will not soften and sag until well above the operating temperature.

(3) The cost of the material should be reasonable.

(4) The coefficient of expansion should be low eliminating the need of special supports to allow for expansion and contraction.

(5) It should be mechanically strong at high temperatures and must not undergo a permanent increase in length.

(6) The material should have a comparatively high coefficient of resistivity. Without it, heat will not be developed economically. The current in the resistor must be limited by its resistance. Without high resistivity the length of wire required to limit the current would perhaps not fit in the space available and would not produce the required high temperature.

(7) Its temperature coefficient of resistivity from room temperature to the operating temperature should be low and preferably positive. If this is not possible, it should at least be low within the working temperature range of the heating chamber.

(9) The material should be a rather poor conductor of heat, cool terminals are desired.

(10) The material should be workable, that is, it should be easy to shape into any form desired for the element.

(11) No chemical reactions should occur between the resistor and its supports or terminals.

(12) The material should be non-magnetic.

(13) The resistor element should have a long useable life.

With these desired properties in mind it is readily seen that no material can meet all of these requirements, and as operating temperatures increase the search becomes more fruitless.

The materials in general use can be grouped into four classifications:

(1) nickel-chromium alloys
(2) iron and steel alloys
(3) molybdenum
(4) non-metallic resistors such as the silicon-carbide products.

The nickel-chromium alloys are largely responsible for the present state of development of resistor furnaces. These alloys do not meet the ideal material requirements but do come sufficiently close to be the most useful material at the present time.

When a resistor made of nickel-chromium alloy is first heated an oxide film forms on its surface and this film acts as a protector preventing to a large degree the formation of additional oxide by resisting the penetration of oxygen. Fortunately, this oxide has the same coefficient of expansion as the material itself.

As the chromium content of the nickel-chromium alloy is increased the resistance to oxidation increases and the element can withstand higher temperatures before the resistance to excessive oxidation is broken down. However, as the chromium content is increased the material becomes less workable, that is, it becomes increasingly difficult to form into the desired shapes. Alloys containing 80% nickel and 20% chromium have been found to be a justifiable limit. This composition has satisfactory oxidation resistance up to 1150° C. or approximately 2100° F. 80-20 nickel-chromium alloys have a rather high resistivity, 650 ohms at 70° F. The resistance increase for this alloy from room temperature to operating temperatures over 1000° F. is approximately 7% which is not large and can be operated from constant voltage circuits without necessitating voltage regulating equipment. These alloys have excellent strength at high temperatures and only a slight amount of expansion.

As mentioned previously, the oxide coating formed on the surface of the material acts as a protector to oxidation but does not eliminate it completely. The thickness of this coating slowly increases throughout the life of the element, resulting in a smaller cross-section of alloy available for the passage of current or an increase in resistance. This increase is very slight if the material temperature is below its operating limit.

For low temperatures, usually below 1850° F., alloys of lower chromium content are used.

The main object in attempting to use iron or steel resistors is to obtain higher furnace temperatures but these elements by themselves are practically of no value. For instance, steel has a melting point of approximately 2650° F. and yet, by itself, it cannot be used at a temperature in excess of 800 to 1000° F. because it would oxidize too rapidly. Used in an artificial atmosphere the material is still not satisfactory due to its excessive expansion coefficient. The large difference between the cold and hot resistance of iron and steel makes it necessary to provide means of limiting the current on starting from a cold state. Another objection to its use is that it is a magnetic material in a cold state resulting in a low power factor. The power factor does become unity when heated to the non-magnetic state. In general, the same objections are encountered in the alloys of iron and steel, namely, excessive expansion, poor strength at high temperatures, and many of the others mentioned above. Certain alloys do overcome the low oxidation resistance and have a high specific resistance. It might be remembered that nearly all pure metals such as copper, iron, aluminum or nickel have specific resistances that are too low to develop heat economically and oxidize too easily without the formation of a protective oxide coating.

Molybdenum, as a pure metal, is sometimes used as a material for high temperature laboratory furnaces. It must be protected from oxidation having oxidation resistance only up to 700° C. By using an artificial atmosphere the material can be very useful for high temperature work having an extremely high melting point of 2625° C. or 4750° F. The melting point is higher than the softening point of refractory materials resulting in the heating chamber temperature being limited by the supports. Molybdenum has a low specific resistance but due to its high melting point it can be operated at a high value of current. When used, voltage control equipment is necessary because of the large change in resistance with temperature. The high cost of this material limits its use to laboratory units.

In the field of non-metallic resistors the most important type is made of silicon-carbide sold in the United States under the trade name of Globar. Resistors of this material are available for furnace temperatures up to 2400° F. The silicon-carbide products are not subject to oxidation. They do have serious objections and offer a field open to much improvement. The resistivity of this material is not constant; starting at room temperature the resistivity drops by about one-third reaching a minimum about 800° F. and then increases. The material has a change of resistivity with time as well as temperature following an irregular pattern with magnitudes on the order of 1:5 or more. After a period the resistance of the element becomes too high reducing the current through it and thus its temperature. The remedy is to increase the applied voltage or renew the element. The manufacturers of Globar recommend the use of transformers having as high as thirty-six voltage taps.

Other materials are in use but their use is confined to the lower temperatures of 1200° F. or less and thus have limited applications, primarily that of small low temperature furnaces and appliances.

CHAPTER II

In the study of electric heating element economics, what factors are going to determine the point of economical operation? This question goes back to the grass roots of the use of heating elements and can best be answered by another question, what is the purpose of the resistance element? Naturally, it is to produce a useful output of heat.

Heat energy can be transmitted from one place to another by three distinct methods: conduction, convection, and radiation. Conduction occurs if the transmission of heat takes place by a point-by-point process within solid bodies or between solid bodies if they are placed in intimate contact (also occurs in liquids in combination with convection). For instance, if one end of a metal rod is placed in a hot furnace, the heat will travel along the rod by the process of conduction. Convection is the transference of heat in liquids and gases due to moving masses of matter caused by the differences in densities. As the molecules move from hotter to colder parts they convey the heat contained within themselves. The transfer of heat by "heat waves" or radiant energy is called radiation. This method of transfer involves the conversion of internal energy into radiant energy. At the receiver this radiant energy is converted back to internal energy with no heating of the intermediate gas or vacuum.

For temperatures above 1200° F., resistors are located within the heating chamber so as to transmit heat directly to the charge by radiation. At the high temperatures and low temperature gradients that exist in a resistance furnace, the natural convection component is small and heat transferred in this manner can usually be neglected, leaving only heat transferred by radiation to be considered in a heating chamber.

A brief review of the laws of radiation will be necessary to fully understand the problem at hand.

LAWS OF RADIATION

A black body is defined as a body that absorbs all the radiant energy that falls on it and reflects none. When a body of this nature is heated so as to act as a source, the radiation emitted is called black body radiation.

By using a source that approaches the idealized black body and after making necessary corrections for absorption in various parts of the apparatus, curves can be obtained showing the energy distribution in the spectrum of a black body at various temperatures. These curves, which are reproduced in Figure 1, were first obtained by Lummer and Pringsheim[1] after a thorough study of a radiation. The curves show that the spectrum is continuous and has a definite characteristic shape.

The German physicist, Max Planck, found these curves could be fit by an equation which bears his name. Planck's empirical equation is

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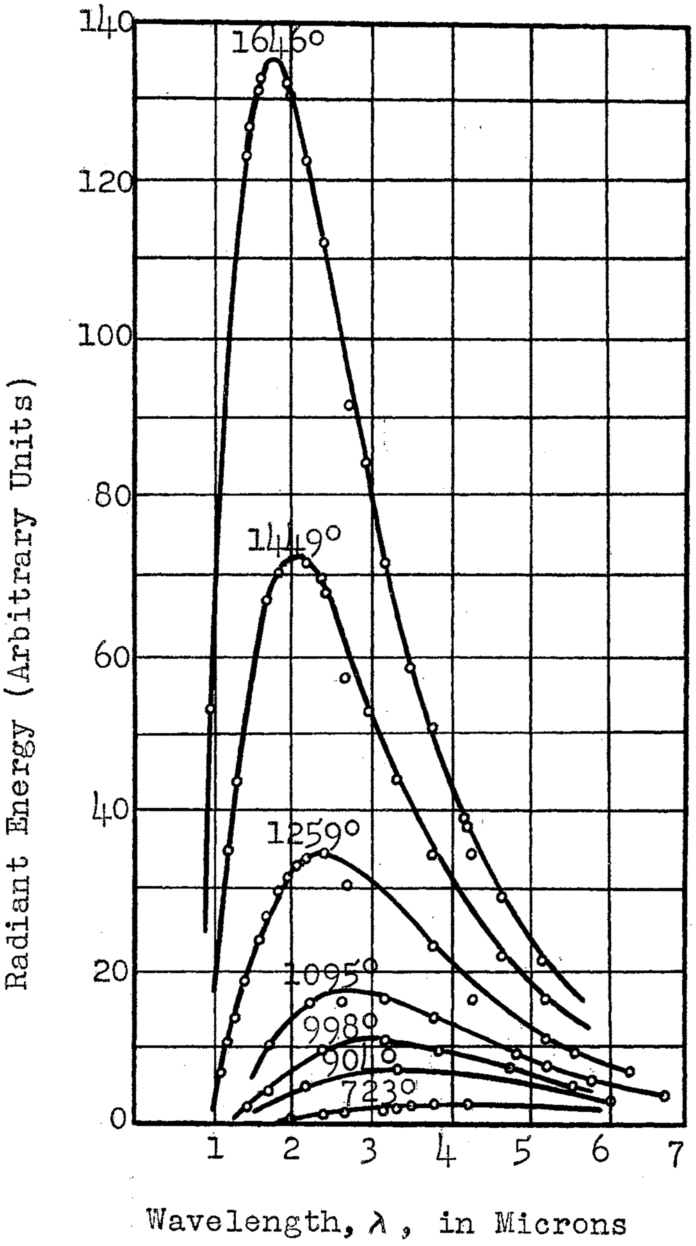
[1] Lummer and Pringsheim, "Die verteilung der Energie im Spektrum des schwarzen Korpers," Verhandlungen der Deutschen Physikalischen Gessellschaft, I (1899), pp. 23 and 215, II (1900), p. 163.



Experimental Radiation Curves of A Black Body at Various Absolute Temperatures (Kelvin)

Figure 1

$$W_\lambda = \frac{C_1}{\lambda^5} \frac{1}{\epsilon^{\frac{C_2}{\lambda T_1}} - 1} \qquad (1)$$

where ϵ = Naperian base of logarithms (2.718...)

W_λ = Radiant energy emitted per unit area per unit range of wavelength

λ = Wavelength in microns

C_1 = First radiation constant

C_2 = Second radiation constant

T_1 = Absolute temperature of radiating body.

Referring again to the energy distribution curves it may be noted that the energy of each wavelength rises rapidly with an increase in temperature and that the higher the temperature the shorter is the wavelength at which maximum energy occurs. By taking Planck's equation and differentiating it with respect to wavelength, λ, it is possible to find the position of the peak energy relative to wavelength. This relation can be expressed as

$$\lambda_{max} T = 2897 \qquad (2)$$

which is often called Wien's displacement law.

The total energy radiated from a black body can be found by intergrating Planck's equation from zero to infinity or, in other words, finding the area under the curve of W_λ shown in Figure 1. This relationship is expressed by the Stefan-Boltzmann law, which is

$$W = \sigma T_1^4 \qquad (3)$$

where

W = Radiant energy emitted per unit area

T_1 = Absolute temperature of radiating body

σ = Stefan-Boltzmann constant which depends upon the units used for expressing W and T_1.

The above equation yields the total energy radiated to cold space (absolute zero) and has very little use in dealing with a practical problem. The law is usually expressed in a modified form as

$$W = e\sigma(T_1^4 - T_2^4) \tag{4}$$

where

e = Emissivity factor (black body = 1)

T_2 = Absolute temperature of the surroundings.

and W and T_1 are as defined above.

The emissivity of a body is a physical property indicative of the amount of radiation. It is the ratio of the emissive power of an actual surface to that of a black body. Typical values for various surfaces can be found in W. H. Adams, Heat Transmission, McGraw-Hill, New York, 1942.

Some of the typical values of the Stefan-Boltzmann constants bases on Degrees Kelvin appear in Table I.

TABLE I[2]

5.672×10^{-5} erg cm^{-2} deg^{-4} sec^{-1}
5.672×10^{-8} watts $meter^{-2}$ deg^{-4}
1.356×10^{-12} cal cm^{-2} deg^{-4} sec^{-1}
3.71×10^{-11} watts in^{-2} deg^{-4}
1.825×10^{-8} Btu ft^{-2} deg^{-4} hr^{-1}
5.348×10^{-12} kw-hr ft^{-2} deg^{-4} hr^{-1}
4.602×10^{-9} kg-cal ft^{-2} deg^{-4} hr^{-1}

THE ECONOMIC PROBLEM

The temperature attained by an electric heating element is determined by the input expressed in watts per square inch of surface area and the rate at which energy is lost by conduction, convection, and radiation. The life of an element is dependent on its temperature. Since the electrical energy input varies as the square of the impressed voltage, it is readily seen that both the temperature of the element and the life of the element are dependent on the impressed voltage.

Mr. G. C. Stauffer experimentally determined the relations of these values which are graphically shown in Figure 2.[3] These relations were obtained by performing tests similar to the A.S.T.M. Standard Life Tests[4] on samples of Nichrome V,[5] an 80-20 nickel-chromium alloy, of 0.025 inch diameter at a temperature

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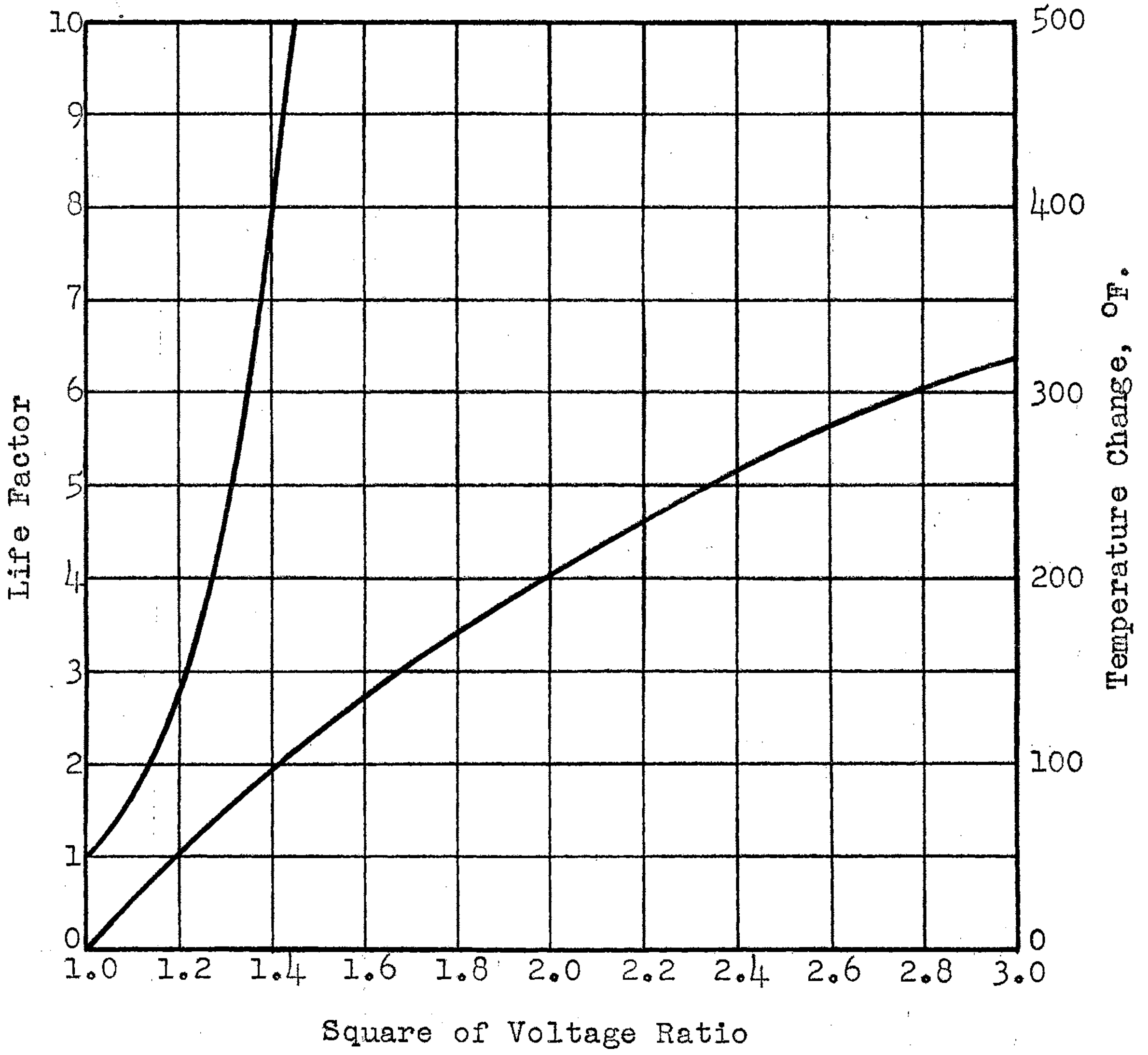
[2] A. H. Canada, "Simplified Calculation of Black-Body Radiation," General Electric Review, XLI (December, 1948), p. 52.

[3] F. E. Bash, "Life Expectancy of Electrical Heating Elements," Metal Progress, XXXVII (January, 1940), p. 41.

[4] Specification B76-36 of the American Society for Testing Materials.

[5] A product of the Driver-Harris Company, Harrison, New Jersey.



Effect of Voltage Change on Life and Temperature
for Nichrome V of 0.025" Diameter at 2150° F.
(Standard Intermittent Heating)

Figure 2

of 2150° F.

Committee B-4 of the American Society for Testing Materials which arrived at the standard life test spent two years in its development. Many cycles of heating and cooling were tried and the committee definitely established that a cycle of two minutes heating and two minutes cooling gave the shortest life for #22 wire tested under the most severe oxidizing conditions. Committee B-4 also found that the size of wire is immaterial between 18 and 24 gage. These results were later verified in Europe. The curves of Figure 2 may, therefore, be applied to wires of this range, #18 to #24 gage.

The curves are plotted in terms of ratio of voltages squared. This ratio is that of applied to rated voltage or vice versa making the ratio a number greater than one. The ordinates of the curves show temperature change in degrees Fahrenheit which is positive for applied voltages greater than rated and negative for applied voltages less than rated. Also plotted as an ordinate is the life factor which for increasing voltages is to be divided into the normal life and for decreased voltages the normal life is to be multiplied by this factor to find the new life.

The use of these curves can best be illustrated by an example. Consider a small furnace that is rated at 110 volts, having a normal life of 5 years. If the furnace were to be operated at 125 volts, its new life would be 5/4.5 or only 1.1 years. This is obtained by taking the voltage ratio 125/110 and squaring, resulting in a ratio of 1.29. Entering the curve with this value we find the life factor to be approximately 4.5 or its new life would be 5/4.5 or only 1.1 years. The curves hold true for voltages lower than rated also. For example, if we were going to operate the above furnace at 100 volts the voltage ratio squared would be $(110/100)^2$ or 1.21 and the new life would be approximately 5 x 3 (the life factor) or 15 years. When operated with voltage higher than rated the life is shortened but the temperature is increased while the reverse is true at voltages less than rated. The curves of Figure 2 apply only within the limitations of the maximum operating temperatures of the alloy.

Certainly it is evident that there must be a voltage at which it would be economical to operate. This value of voltage can best be found by determining the minimum cost per unit of heat output. In general the plan would be to arrive at an equation involving all costs in the numerator and the heat output in the denominator of an equation, such as,

$$Q = \frac{\text{energy cost} + \text{total element cost}}{\text{heat output}} \tag{5}$$

then by taking the derivative with respect to the voltage ratio a minimum can be determined. Before doing this however, many factors must first be determined.

Considering the proposed method of approach it will first be necessary to evaluate the terms in the equation, namely, the energy cost, the total element cost, and the unit heat output in terms of the voltage ratio.

ENERGY COST

The energy cost of operating an element will depend on the electrical energy input which can be expressed as

$$E_c \, H \, W \, y^2 \qquad (6)$$

where

E_c = energy cost in dollars per kw-hr

H = hours element in use

W = rated kilowatts of element

y = voltage ratio, $\frac{\text{actual voltage}}{\text{rated voltage}}$

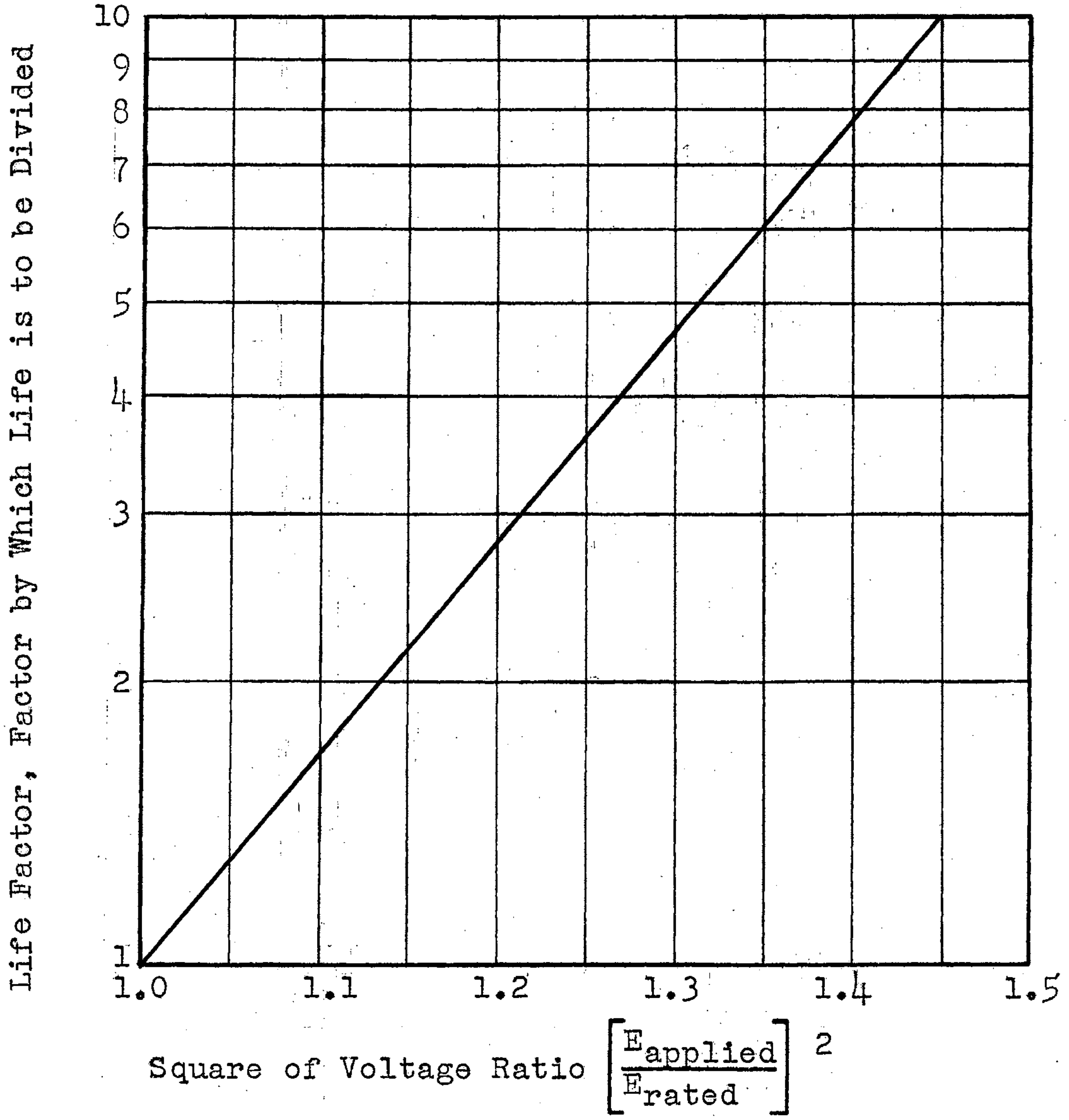
TOTAL ELEMENT COST

The total element cost expressed in terms of the voltage ratio is somewhat more difficult to determine. The total element cost must include such items as replacement cost, the first cost of the complete resistor, and maintenance costs. These costs are dependent on the voltage ratio as shown in Figure 2, that is, the higher the ratio of voltages the shorter life resulting in more replacement, maintenance, and element cost. This relation to voltage can be found from the curve of Figure 2 which involves the life factor. The curve is replotted in Figure 3 on semi-logarithmic coordinates yielding a straight line variation.

The general equation for a straight line is

$$AX + BY + C = 0$$

where X in this case is the voltage ratio y^2 and Y is log F, the logarithm of the life factor. By using any of the available



Effect of Voltage Change on Life for Nichrome V
of 0.025" Diameter at 2150° F.
(Standard Intermittent Heating)

Figure 3

methods of determining the equation of a straight line, the resulting relation is

$$Y = 2.237X - 2.237$$

since

$$Y = \log F$$

and

$$X = \text{the voltage ratio, } y^2$$

the equation after substitution gives

$$\log F = 2.237\,y^2 - 2.237$$

or

$$F = 10^{2.237y^2 - 2.237}$$

$$= (0.0058)10^{2.237y^2} \qquad (7)$$

which is the life factor expressed in terms of the voltage ratio. The total element cost can then be expressed as

$$\frac{A_c\ H\ (0.0058)\ 10^{2.237y^2}}{L} \qquad (8)$$

where

A_c = first cost of element + maintenance costs + replacement costs

H = hours used

y = voltage ratio

L = normal hours of life with standard intermittent heating. The normal hours of life with standard intermittent heating must be included to obtain the cost in dollars or the same units as the energy cost of Equation 8.

To determine the normal hours of life more information is necessary. The additional information must show the relation between the normal hours of life and the element temperature for the same size wire and under the same test conditions as previously used, that is, the normal hours of life with standard intermittent heating as a function of temperature.

Test results[6] are plotted in Figure 4 showing the relation between the total hours of life and the temperature of the element under the life tests as devised by the A.S.T.M.

The three curves indicate the advances made in recent years in the development of heating element materials. The "X-100B" was a later development then the "A.S.T.M. 1934" and "Lot 1406" represents the most recent development. The three nichrome V's are alloys having a nominal composition of 80% nickel and 20% chromium. It is assumed that X-100B is typical of the 80-20 alloys and is used as the present standard of comparison.[7]

The value of Equation (8) can now be determined with the aid of the curve of the reference standard X-100B in the study of 80-20 nickel-chromium alloys. By knowing the operating

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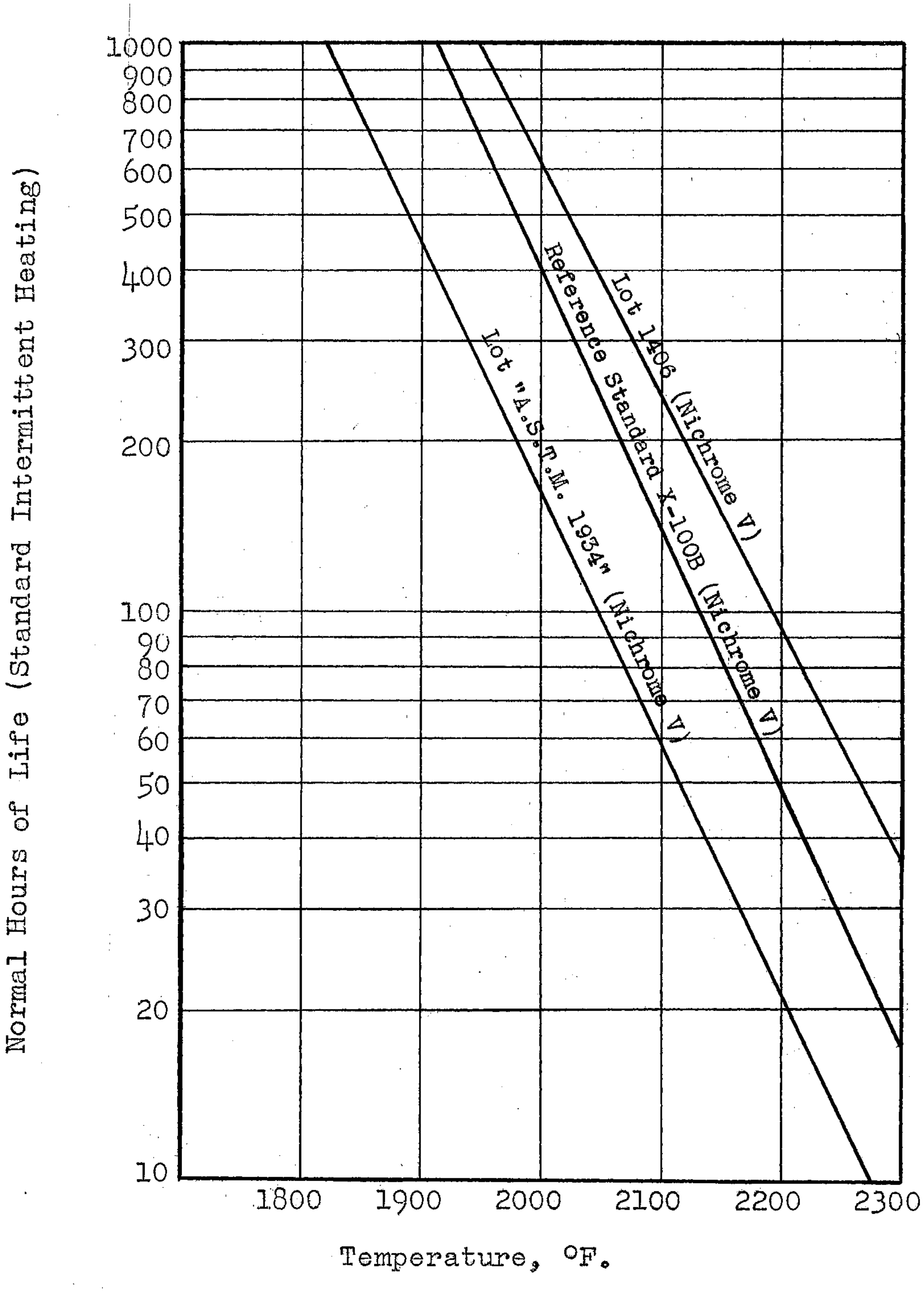
[6] F. E. Bash, "Estimating Life of Electrical Heating Elements," Metal Progress, XXXIII (February, 1938), p. 144.

[7] *Ibid.*, p. 145.



Normal Hours Life of Nichrome V with Standard Intermittent Heating in Relation to Element Temperature (Wire size, 0.025 inches diameter)

Figure 4

temperature the normal hours of life can be found directly from this curve or its equation

$$L = 10^{-0.00457t_1 + 11.73} \tag{9}$$

where

L = the normal hours of life with standard intermittent heating

and

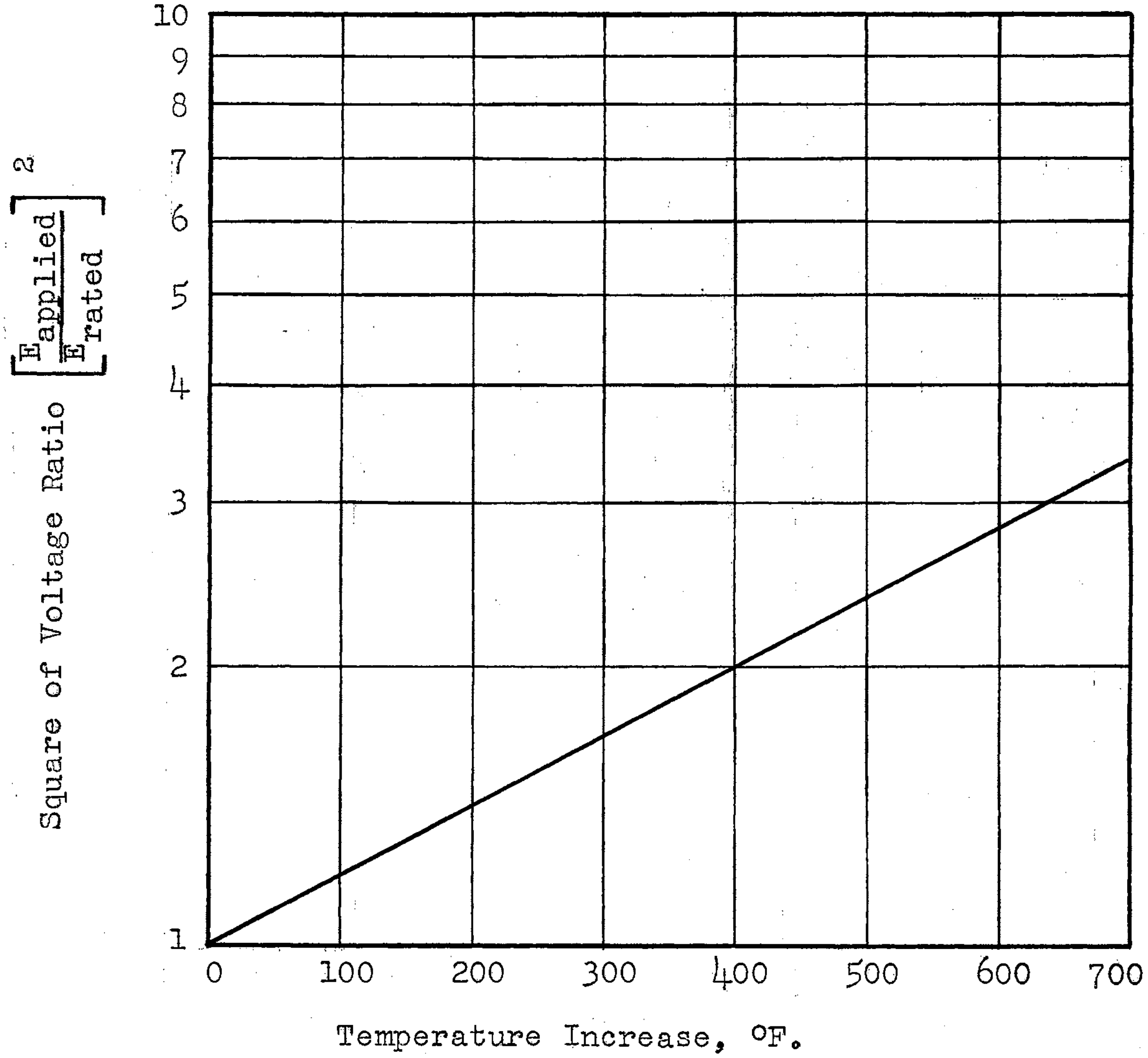
t_1 = element temperature in degrees Fahrenheit.

HEAT OUTPUT

As pointed out previously, the transmission of heat in a resistance furnace is usually accomplished by radiation. To determine the total energy radiated it is necessary to apply our laws of radiation. The Stefan-Boltzmann law, Equation (4), deals with the total energy radiated. This law expresses the relation between the total energy radiated per unit area and the temperature as

$$W = e\sigma(T_1^4 - T_2^4) \tag{4}$$

It must be kept in mind that the temperature is dependent on the impressed voltage. The curve of Figure 1 graphically shows the relation of temperature change with the voltage ratio squared. Although this curve was obtained at an initial temperature of 2150° F. it can be applied within limits to the higher furnace temperatures near this value. This curve is replotted in Figure 5 on semi-logarithmic coordinates giving a straight



Effect of Voltage Change on Temperature
for Nichrome V of 0.025" Diameter at 2150° F.
(Standard Intermittent Heating)

Figure 5

line relation. The equation of this line is

$$\Delta t = 2700 \log y \tag{10}$$

where

Δt = the change in temperature (°F)

and

y = the voltage ratio

The Stefan-Boltzmann law was originally derived with the units of temperature to be in degrees Kelvin (°K = °C + 273). Since the English system of units is commonly used in this country it is desired that the law be expressed in Rankine temperature, (°R = °F + 460), or

$$W = e\sigma (T_1^4 - T_2^4) \tag{4'}$$

where

T_1 = absolute temperature of the radiating body in degrees Rankine

T_2 = absolute temperature of the surroundings in degrees Rankine

$\sigma = 0.173 \times 10^{-8}$ Btu per hr, sq ft, $°R^4$

and W and e having the same definition as given previously.

A change in the applied voltage will change the temperature of the radiating body, the resistance element, in accordance with Equation (7), that is,

$$\Delta t = 2700 \text{ Log } y \tag{7}$$

Substituting Equation (7) in Equation (4') yields an expression for the radiant energy output in terms of the voltage ratio.

$$W = e\sigma\left[(T_1 + \Delta t)^4 - T_2^4\right]$$

or $$W = e\sigma\left[(T_1 + 2700 \log y)^4 - T_2^4\right] \qquad (11)$$

SOLUTION OF THE ECONOMIC PROBLEM

All of the quantities in the proposed equation (Equation 5) are now expressed in terms of the voltage ratio.

Equations (6), (8), and (11) may now be substituted in Equation (5).

$$Q = \frac{\text{energy cost + total element cost}}{\text{heat output}}$$

$$= \frac{E_cHWy^2 + \frac{A_cH}{L}(0.0058)10^{2.237y^2}}{e\sigma\left[(T_1 + 2700 \log y)^4 - T_2^4\right]} \qquad (12)$$

The first derivative set equal to zero to find the minimum gives

$$2\left[(T_1 + 2700 \log y)^4 - T_2^4\right]\left[E_cWy + 0.0299\frac{A_cy}{L}10^{2.237y^2}\right]$$
$$- 4690\left[T_1 + 2700 \log y\right]^3\left[E_cWy + 0.0058\frac{A_c}{Ly}10^{2.237y^2}\right]$$
$$= 0 \qquad (13)$$

This equation shows the relation that exists between the energy cost per kw-hr, the kilowatt rating of the resistance element, the normal life with standard intermittent heating, the temperature of the surroundings, the temperature of the element with rated voltage applied, and the voltage ratio. Values may be substituted in this equation to determine the voltage at which a resistance heating element should be operated for minimum cost. Unfortunately, the equation is too involved to solve for the voltage ratio, y, directly; a trial and error solution is necessary.

APPLICATION

The following design data of a resistance heating element for an open type range unit is taken from a slide rule printed by Hoskins Manufacturing Company for use with Chromel "A", an 80-20 nickel-chromium alloy.

$E = 110$ volts $\qquad W = 0.6$ kilowatts

$I = 5.45$ amperes when hot $\qquad R = 18.9$ ohms (at 75°F)

#21 gage $\quad t_1 = 1650°$ F. $\quad$ length of wire = 23.6 feet

Assume the following values apply to this element.

energy cost = $0.03 per kw-hr

total element cost = $4.00 (initial element cost plus replacement and maintenance costs)

and the temperature gradient to be 100° F. or the receiver of the radiant energy is at a temperature of 1550° F.

The normal life under standard intermittent heating conditions can be found from Equation (9)

$$L = (5.37 \times 10^{11})10^{-0.00457t_1}$$

$$= (5.37 \times 10^{11})10^{-0.00457(1650)}$$

$$= (5.3 \times 10^{11})10^{-7.54} = 15,500 \text{ hrs.}$$

Substituting the above values in Equation (13) gives

$$2\left[(T_1 + 2700 \log y)^4 - T_2^4\right]\left[E_c W y + 0.0299\frac{A_c y}{L}10^{2.237y^2}\right]$$

$$- 4690\left[T_1 + 2700 \log y\right]^3\left[E_c W y + 0.0058\frac{A_c}{Ly}10^{2.237y^2}\right] = 0$$

$$= 2\left[(2110 + 2700 \log y)^4 - 2010^4\right]\left[0.03(0.6)y + \frac{0.0299(4)}{15500}y10^{2.237y^2}\right]$$

$$- 4690\left[2110 + 2700 \log y\right]^3\left[0.018y + \frac{1.496 \times 10^{-6}}{y}10^{2.237y^2}\right]$$

$$= 0$$

$$= 2\left[(2110 + 2700 \log y)^4 - 2010^4\right]\left[0.018 + (7.71 \times 10^{-6})y10^{2.237y^2}\right]$$

$$- 4690\left[2110 + 2700 \log y\right]^3\left[0.018 + \frac{1.496 \times 10^{-6}}{y}10^{2.237y^2}\right]$$

$$= 0$$

The easiest method to follow in solving this equation is to equate its two parts,

$$2\left[(2110 + 2700 \log y)^4 - 2010^4\right]\left[0.018 + (7.71 \times 10^{-6})y10^{2.237y^2}\right]$$

$$= 4690\left[2110 + 2700 \log y\right]^3\left[0.018 + \frac{1.496 \times 10^{-6}}{y}10^{2.237y^2}\right]$$

then solve by trial and error carrying on a plot of the two sides of the equation at the same time. The intersection point of the two curves thus formed will result in its solution which in this case gives $y = 1.25$.

The heating element can then be redesigned for a voltage of 1.25 times 110 volts or its new kilowatt rating will be its original rating multiplied by the voltage ratio squared,

$$W' = Wy^2 = 0.6(1.56) = 0.936 \text{ kw.}$$

To obtain this wattage it is necessary to go back to the original design data which gives the current drawn, when hot, as 5.45 amps and original wattage of 600 watts at 110 volts. From this it can be determined that the resistance is 20.2 ohms (hot) for the 23.6 feet that compose the resistance unit. It is now desired that its resistance be changed so as to consume 0.936 kw at rated voltage or the new resistance will be $\frac{110^2}{936}$ or 12.9 ohms. The new length of wire can be found by proportion,

$$\frac{23.6}{20.2} = \frac{\text{length}}{12.9}$$

or the new length will be 15.1 feet.

This will give the most economical element.

Economy is not achieved through the shorter length of wire. In developing the conditions for economical operation the total element cost, A_c, which consists of the initial element cost plus the replacement and maintenance cost, is taken as a constant or fixed value for any given element. (See Equation 8). The change in the length of the wire would represent only a very small part of this term. Economy of operation <u>is achieved</u> through obtaining the minimum cost per unit of heat.

ASSUMPTIONS

In this development of the economical operating voltage several assumptions were made which should be brought to the reader's attention.

First, the emissivity was considered to be a constant. Strictly speaking, the emissivity varies with the temperature or, as presented here, would be dependent on the impressed voltage. This variation is slight and in no practical problems involving small temperature changes is it considered.

Second, the resistance of the heating element material was considered to be constant over the temperature range considered. The assumption would not introduce an appreciable error since the resistance change with temperature of the 80-20 nickel-chromium alloys is usually less than 1% in its working range.

Third, all the heat lost or transferred by conduction and convection is neglected. This has been discussed in detail in the text of this thesis.

Fourth, the curve of Figure 4 showing the total hours of life under standard intermittent heating in relation to temperature has been extended through the use of its equation (Equation 9). The amount of extension necessary to cover furnace temperatures would be small and the errors introduced in this manner would be slight.

Fifth, the curve showing temperature increase as a function of the square of the voltage ratio, Figure 5, was determined from an initial temperature of 2150° F. In other words, this curve shows the increase in temperature that accompanies an increase in applied voltage over and above the temperature of 2150° F. obtained when operated at an impressed voltage equal to the rated voltage. This is perhaps the largest source of error. The lower the element temperature the greater the error introduced. An extreme example is: assume an element operated with an impressed voltage of two volts at a temperature of 100° F. has its voltage increased by one volt giving a squared voltage ratio of 2.25. It is evident that the increase in temperature would not be 475° F. as indicated by Figure 5.

The relations expressed by this curve would apply to only the higher operating temperatures of the alloy.

CONCLUSIONS

The cost of energy, resistance element temperature, temperature of the surroundings, rating of the element, normal hours of life with standard intermittent heating, the voltage ratio, and the total element cost--initial cost plus replacement and maintenance costs--determine the economical operating voltage of a resistance heating element.

The basis of any furnace problem is--or should be--a heat problem: how to heat the charge to the required uniformity at lowest cost.[8] The engineers in the field of heating element design appear to be unaware of the economics involved. Designers prove this statement by the nature of their product and deceive the consumer by advertizing. At the present time, elements are designed with a certain energy density, watts per square inch of surface area, in mind. The energy density is chosen to give a very long life with no economic justification.

One of the important factors which determines the economical operating point is the total element cost consisting of the initial cost of the element, replacement cost, and maintenance costs. The initial cost and the maintenance costs are items without much possible improvement. The largest component in the total element cost in many cases is the replacement cost. To decrease the replacement cost which will extend the

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[8] V. Paschkis, Industrial Electric Furnaces and Appliances, II, p. v.

economical life, design engineers should also devote their attention to the elements physical construction. A plug-in unit would result in an enormous decrease in the replacement cost and, at the same time, an increase in the economical life.

Economy is very seldom thought of by the consumer, however, he should be informed and be aware of the factors involved. The only way this can be accomplished is for the engineers to change their designs with the full-hearted backing of their companies, keeping in mind the relations determining the minimum cost per unit of heat.

The other factors determining the point of most economical operation are not as easily varied. The resistance element temperature and the temperature of the surroundings are fixed by the charge to be heated, the furnace design, and the emissivity of the resistance element; all of which are out of the control of the heating element designer. The cost of energy is another of the determining factors out of the control of the design engineer. The reduction of the energy cost to a minimum value is the problem of the consumer.

The developments made here are based on information available which has been determined under standard intermittent heating tests as proposed by the A.S.T.M. As stated before, these tests are used since their application results in the shortest life of the material. It should be kept in mind that the heating and cooling cycle of two minutes on and two minutes off is seldom, if ever, encountered in practice. The life of an element in normal use would be several times to as high as thirty times longer than indicated here.[9]

The range of wire size discussed is rather limited, #18 to #24 gage. This size of wire would only be found in small units. To apply these principles to large units, where the application of economics would be of the most value, further experimental work is necessary to establish the relations between life factor, temperature, normal hours of life, and voltage ratio for resistors of larger cross-section and rectangular shapes.

Economy based on sound engineering principles can be adapted to the design of electric resistance heating elements. More research along the same general lines of this thesis would be desirable. The economics of heating element design is of value not only to the user of electric furnaces and heating devices but also to the manufacturer.

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[9] Bash, *op. cit.*, XXXIII, 145.

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