THE CURRENT LOCUS OF THE HYSTERESIS MOTOR

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PREFACE

The hysteresis motor has sometimes been referred to as an "unloaded synchronous motor." This type of machine has not been used as a producer of appreciable amounts of power because of its low torque per ampere and low efficiency. As a result the hysteresis motor has been used mainly in electric clocks and in timing devices.

A search of the literature reveals that no method of prediction has been published for the hysteresis motor. The search also reveals that a satisfactory qualitative analysis of the hysteresis motor is lacking. An attempt is made to present a more satisfactory analysis of this type of machine other than by the use of the magnetic field theory used heretofore.

Different methods of prediction are discussed to illustrate the reason why perhaps no method has been applied to the hysteresis motor. An analysis is then made to determine the feasibility of prediction in the future. This thesis developes an equation to represent the current locus of a particular machine from the manufacturer's curve. A breakdown of the losses is then presented to aid in the discussion of the operation of the hysteresis motor.

ACKNOWLEDGEMENT

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CHAPTER I

THE THEORY OF OPERATION OF THE HYSTERESIS MOTOR

In 1908, C. P. Steinmetz discussed the production of torque by magnetic hysteresis. The following discussion is a condensation of the explanation by Dr. Steinmetz.

If a circular iron disk or cylinder of uniform magnetic reluctance is placed in a revolving magnetic field, the disk is caused to rotate in the direction of the magnetic field even though laminated to reduce eddy currents. This rotation is the result of the hysteresis effect of the disk, thus this machine may be termed a hysteresis motor.

This disk will be exposed to a revolving field. The axis of the magnetization of the disk does not coincide with the axis of the revolving field, but lags behind it, thus producing a couple. This means that the component of magnetism in the direction of the rotating disk, as a result of the lag of magnetism in the hysteresis loop, lags behind that of the revolving field, and thus the axis of magnetism in the disk does not coincide with the axis of the revolving field, but is shifted backward by an angle, α, which is called the hysteretic lead.

The induced magnetism gives the following couple with the revolving field

$$C = k F R \sin \alpha \qquad (1)$$

where

k = a constant

F = revolving field

R = induced magnetism

α = angle of hysteretic lead

The apparent input of the motor is

$$P = k\ F\ R \tag{2}$$

Thus the apparent torque efficiency is

$$P/Q = \sin \alpha \tag{3}$$

where

Q = volt-ampere input

and the power of the motor is

$$P = (1-s)\ D = (1-s)\ k\ F\ R \sin \alpha \tag{4}$$

where

s = slip as a fraction of synchronous speed

The apparent efficiency is

$$P/Q = (1-s) \sin \alpha \tag{5}$$

Since a motor must contain an air gap, the angle, α, is small, usually a few degrees only, the apparent efficiency is necessarily low, and, consequently, unsuitable for the production of large amounts of power.

From the torque equation it is evident that for a constant applied voltage, or current, F is constant, the torque is constant and independent of the speed.

For

$$s < 0 \qquad \alpha < 0 \tag{6}$$

and the apparatus becomes a hysteresis generator.

This same result can be reached from a different point of view. In a magnetic circuit comprising a rotating iron disk of uniform magnetic reluctance in a revolving field, the magnetic reluctance, and the distribution of magnetism, is obviously independent of the speed, and thus the current and the energy expenditure of the impressed field is independent of the speed also. If, now:

V = volume of iron of the disk

$\mathcal{B}$ = magnetic density

η = coefficient of hysteresis

the energy expended by hysteresis in the disk per cycle is

$$W_o = V \eta \mathcal{B}^{1.6} \tag{7}$$

If

f = frequency of the supply source

the power supplied to the disk by the revolving field through the hysteresis loop is

$$P_o = f V \eta \mathcal{B}^{1.6} \tag{8}$$

At slip frequency, sf or speed $(1-s)f$, the power expended by hysteresis in the disk is

$$P_1 = s f V \eta B^{1.6} \tag{9}$$

In the transfer from the stationary member to the revolving member, the magnetic power

$$P = P_0 - P_1 = (1-s) f V \eta B^{1.6} \tag{10}$$

has disappeared, and reappears a mechanical work, and the torque is

$$D = P / (1-s) f = V \eta B^{1.6} \tag{11}$$

which is independent of the speed.

Since $\sin \alpha$[1] is the ratio of the energy of the hysteresis loop to the total apparent energy of the magnetic cycle, the apparent efficiency can never exceed the value

$$(1-s) \sin \alpha \tag{12}$$

or a fraction of the primary hysteretic energy.

The primary hysteretic energy of an induction motor being a

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[1] Steinmetz, C. P. Theory and Calculation of Alternating Current Phenomena, Chapter XII.

part of the losses, and thus only a small part of its output, the output of a hysteresis motor is only a small portion of the output which the same magnetic structure could give with the secondary short-circuited, as in the regular induction motor.

As the rotary effort of the magnetic structure as a hysteresis motor appears in all induction motors, though it is small, the hysteresis torque of the hysteresis motor decreases at a lesser rate than the induction torque as the size of the motor decreases. The torque as a hysteresis motor is comparable to the induction motor for extremely small motors.

The following discussion and analysis is taken from Mr. H. C. Rotor's article published in the March 1948 issue of the ELECTRICAL ENGINEERING.

The hysteresis motor is known as a scientific curiosity and as an unloaded synchronous motor. It is unknown as a practical type of power motor. Usual output is from five (5) to ten (10) milliwatts and the input is from two (2) to three (3) watts. Recent models have an output of one-quarter horsepower and is eighty per cent efficient. This high output and efficiency is a result of elimination of spurious hysteresis loss in the rotor and reduction of the exciting current. This new feature has resulted in the absence of transient oscillations of the rotor when drastic load changes occur. Damping by hysteresis loss is associated with these oscillations.

An ideal magnetic field distribution is one that causes no undulation of the rotor flux density, when slipping behind the revolving field. The flux density at any point in the rotor must follow the major loop continuously with no momentary recessions to cause minor loops. When at synchronous speed, in steady state, the flux density at any point in the rotor must remain constant. This condition exists if the vector distribution of the stator magnetomotive force is absolutely invariant, revolving as a whole at constant angular velocity, and the rotor turns at a constant angular velocity in synchronism with the field.

The actual power converted by means of the hysteresis loop is

$$P = 4.44 f \beta (H_s \sin \alpha)/\sqrt{2} \qquad (13)$$

where

f = frequency in cycles per second

β = magnetic density of the rotor

H_s = intensity of the stator field

α = angle of hysteretic lead

Disregarding eddy current effects in the rotor, this is the maximum torque which just can be brought into synchronism. When the rotor slips, the axis of the rotor field remains at a constant position with respect to the stator field, lags by the angle, α, but will be slipping with respect to the rotor. When synchronism just has been reached the rotor axis becomes stationary with respect to the rotor but still lags the stator axis by the angle, α.

When the load torque is removed the angle, α, will close and the rotor will advance in phase until the rotor axis coincides with the stator axis and the mechanical torque is zero. If the rotor is advanced more by driving torque then generator action results. It is obvious that the component of the voltage induced in the stator will shift its phase angle from a power absorbing component to a generating component.

The magnetization of the rotor changes when the motor is operated at full load and then deprived of its load. Thus the pull-out torque is larger than the pull-in torque. The rotor axis is oriented fully to the stator when at no load and more magnetized than when slipping. Thus accurate quantitative results must be predicted on a known state of magnetization. Such a state is more easily obtained when slipping.

Another method of analysis is based on an energy concept of the hysteresis loop. The flux density-magnetic intensity relationship in each particle of the rotor steel is described by a magnetic hysteresis loop, the various hysteresis loops for the particles being identical except that they are displaced in phase depending on the electrical angular position of the particles. Each particle goes through a complete loop for each pair of poles it slips by, and heat energy equal to the area of the loop for the particle released. The energy stored in the rotor is transmitted from the stator through the magnetic field as a torque times the speed of the field. The hysteresis power developed in the rotor is

$$P = f V \int_{B=0}^{B=0} H \, dB \qquad (14)$$

and the torque developed is:

$$T = (f V \int H dB) / 2\pi n \tag{15}$$

$$= (22.6 f V W_h) / n \tag{16}$$

where

W_h = area of the loop in joules per cu. in. per cycle

V = volume of the rotor in cu. in.

n = speed of stator field in revolutions per second

f = frequency in cycles per second

The largest loss is excitation copper loss. It can be decreased greatly by overexcitation momentarily after synchronism is reached. This can be accomplished by raising the input voltage. The benefits obtained depend on using the rotor in the same manner as a permanent magnet is used, that is, the rotor must supply a magnetomotive force to the external magnetic circuit. Any condition of operation tending to weaken the retained magnetism gradually will change the rotor to the conditions it had before it had been overexcited. Oscillation of the rotor caused by heavy load changes causes demagnetization of the rotor. For best operation, the motor should be overexcited and the load inertia should be low and not operated too close to the pull-out point.

CHAPTER II

FIELD ANALYSIS OF THE TORQUE PRODUCTION IN THE HYSTERESIS MOTOR

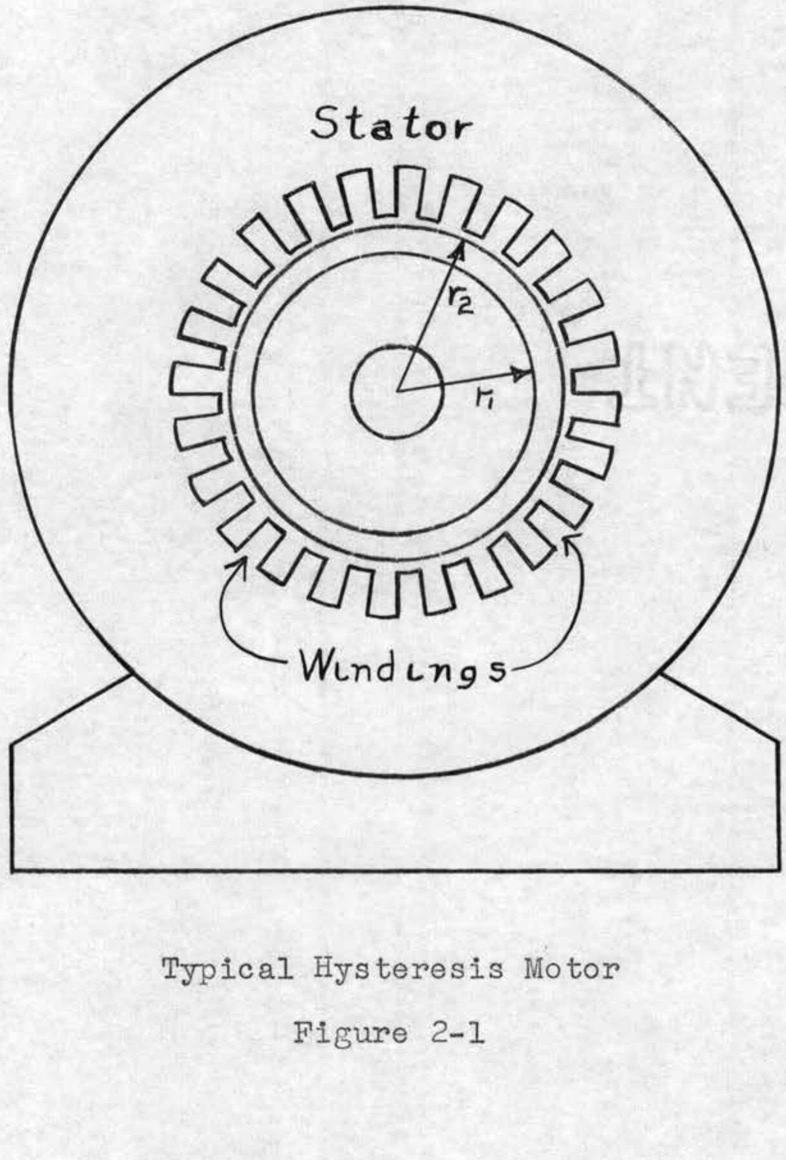
A general expression of the instantaneous torque in terms of the field within the rotor is obtained from the energy exchange caused by a virtual displacement of the rotor. This method is discussed in many textbooks.

The stator field and the induced rotor magnetization at a given instant will be treated as a stationary configuration. The impelling torque, will be balanced by an opposing external force. Each unit of volume of the rotor is situated in a magnetic field of strength, $\mathcal{B}$, which is the sum of, $\mathcal{B}'$, the strength of the stator field and, $\mathcal{B}''$, the strength of the rotor magnetization. The impelling torque may be considered as the result of the interaction of these two fields, $\mathcal{B}'$ and $\mathcal{B}''$. The stator will be represented by permanent magnets of appropriate strength and distribution to give an equivalent field, $\mathcal{B}'$, within the rotor.

If the opposing torque is decreased slightly the rotor will be given a forward angular displacement, $\delta\phi$, and the intensity of the magnetization of the rotor, $\mathcal{M}$, is held constant at each point, the amount of mechanical work done at the expense of the magnetic energy of the system will be

$$T\,\delta\phi = -\,\delta\,W \qquad (1)$$

In this and the following equations, the centimeter-gram-second



Typical Hysteresis Motor

Figure 2-1

electromagnetic units are employed. Since, M, is constant, the only portion of the energy which changes is that associated with the interaction of the stator field, B', and the rotor magnetization, B''. The energy of any unit volume, dV, is given in terms of components by the following equations on the basis of Ampere's theory of magnetism[1]

$$dW = -(M_r B_r' + M_\theta B_\theta' + M_z B_z')\, dV \qquad (2)$$

where, r, θ, and z, are the cylindrical co-ordinates fixed with respect to the stator. The displacement moves the unit volume to a position where the field components are

$$B_r' + (\partial B_r'/\partial\theta)\,\delta\phi \qquad (3)$$

$$B_\theta' + (\partial B_\theta'/\partial\theta)\,\delta\phi \qquad (4)$$

$$B_z' + (\partial B_z'/\partial\theta)\,\delta\phi \qquad (5)$$

thus the energy decreases by the amount

$$-\delta(dW) = [M_r(\partial B_r'/\partial\theta) + M_\theta(\partial B_\theta'/\partial\theta) + M_z(\partial B_z'/\partial\theta)]\,\delta\phi\, dV \qquad (6)$$

Integration over the volume, V, of the rotor gives

$$-\delta W = \delta\phi \iiint_V [M_r(\partial B_r'/\partial\theta) + M_\theta(\partial B_\theta'/\partial\theta) + M_z(\partial B_z'/\partial\theta)]\, dV \qquad (7)$$

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[1] Page, L. Introduction of Theoretical Physics, 430-9.

This expression is substituted into equation 1. Then both sides are divided by $\delta\phi$, and the volume, V, is expressed in terms of the co-ordinates. The resulting equation is

$$T=\int_0^L\int_{r_1}^{r_2}\int_0^{2\pi}[M_r(\partial B_r'/\partial\theta)+M_\theta(\partial B_\theta'/\partial\theta)+M_z(\partial B_z'/\partial\theta)]r\,d\theta\,dr\,dz \qquad (8)$$

where

L = length of the rotor

An expression for the torque involving the total field, B, instead of B', lends itself more convenient for application. The induced rotor magnetization, B'', produces only pairs of equal and opposite torques. Therefore, the torque resulting from impressing an external field identical with B'' on the rotor is zero, or

$$0=\int_0^L\int_{r_1}^{r_2}\int_0^{2\pi}[M_r(\partial B_r''/\partial\theta)+M_\theta(\partial B_\theta''/\partial\theta)+M_z(\partial B_z''/\partial\theta)]r\,d\theta\,dr\,dz \qquad (9)$$

If this equation is added to equation 8, the result is

$$T=\int_0^L\int_{r_1}^{r_2}\int_0^{2\pi}[M_r(\partial B_r/\partial\theta)+M_\theta(\partial B_\theta/\partial\theta)+M_z(\partial B_z/\partial\theta)]r\,dr\,d\theta\,dz \qquad (10)$$

The component of B in each term may be used as the variable of integration in place of θ, and since B and M go through a cycle of values in twice the pole pitch of the stator winding, the integral around the rotor can be expressed in terms of the cyclic integral, and thus

$$T=P/2\int_0^L\int_{r_1}^{r_2}\int_0^{2\pi}\oint(M_r\,dB_r+M_\theta\,dB_\theta+M_z\,dB_z)\,r\,dr\,dz \qquad (11)$$

where

P = number of poles

Each integration of MdB must be performed in the direction of positive θ. The torque may be expressed in terms of H instead of M by using

$$M = (B - H)/4\pi \tag{12}$$

For the r term of equation 11

$$\oint M_r dB_r = 1/4\pi \oint B_r dB_r - 1/4\pi \oint H_r dB_r$$

$$= -1/4\pi \oint H_r dB_r \tag{13}$$

since $\oint B_r dB_r$ disappears when integrated over a cycle.

When the last member of equation 13 is integrated by parts, there results

$$\oint M_r dB_r = -1/4\pi H_r B_r + 1/4\pi \oint B_r dH_r$$

$$= 1/4\pi \oint B_r dH_r \tag{14}$$

since $H_r B_r$ has the same value at the beginning and end of one cycle. Equation 11, after similar treatment of the other terms, becomes

$$T = P/8\pi \int_0^L \int_{r_1}^{r_2} \int_0^{2\pi} \oint (B_r dH_r + B_\theta dH_\theta + B_z dH_z) r\, dr\, dz \tag{15}$$

This equation gives the torque in terms of a triple integral of all three field components, and requires a knowledge of both B and H at all points within the rotor in order to apply it to a machine. Each component of B and H varies cyclically along a circular arc of the rotor corresponding to a fixed r and z. The relation of B_θ to H_θ is represented by a hysteresis loop, the area of which, $\oint B_\theta \, dH_\theta$, appears in the torque equation along with the loop areas of the other field components.

Equation 10 can be simplified by neglecting the axial component of the field. This results in the last term of the integrand being dropped, making the other components independent of z, and

$$T = PL/8\pi \int_{r_1}^{r_2} \oint r(B_r \, dH_r + B_\theta \, dH_\theta) \, dr \qquad (16)$$

From the expression of the magnetic field, the radial component of the field decreases, and the tangential field becomes independent of r as r_1 approaches r_2. If the rings are sufficiently thin, the radial component may be dropped. After multiplication and division by 2π, the equation becomes

$$T = PL/16\pi^2 \int_{r_1}^{r_2} 2\pi r \, dr \oint B_\theta \, dH_\theta \qquad (17)$$

If, the volume is

$$V = L \int_{r_1}^{r_2} 2\pi r \, dr \qquad (18)$$

then

$$T = PV/16\pi^2 \oint \beta_\theta \, d H_\theta$$

$$= K \oint \beta_\theta \, d H_\theta \tag{19}$$

where

V = volume of the rotor magnetic material

K = constant for a particular machine

If r_1 , and r_2 are of such values that the tangential component varies but little with r , and the radial component is large enough to contribute up to ten (10) per cent of the total torque, and T_r need not be determined accurately so that r may be replaced by $(r_1 + r_2)\ 1/2$, then

$$T_r = (PL/16\pi^2[2\pi(r_1+r_2)/2] \oint \beta_r \, d H_r \tag{20}$$

The integral is the average value of $\oint \beta_r d H_r$ with respect to r , multiplied by $(r_2 - r_1)$ and

$$T_r = (PL/16\pi^2)\pi(r_1+r_2)(r_2-r_1) \oint \overline{\beta_r d H_r}$$

$$= K \oint \overline{\beta_r d H_r} \tag{21}$$

where the bar denotes the average of the whole integral. The tangential component is determined by replacing β_θ by its average $\overline{\beta_\theta}$, with respect to r , and the total torque of the machine becomes

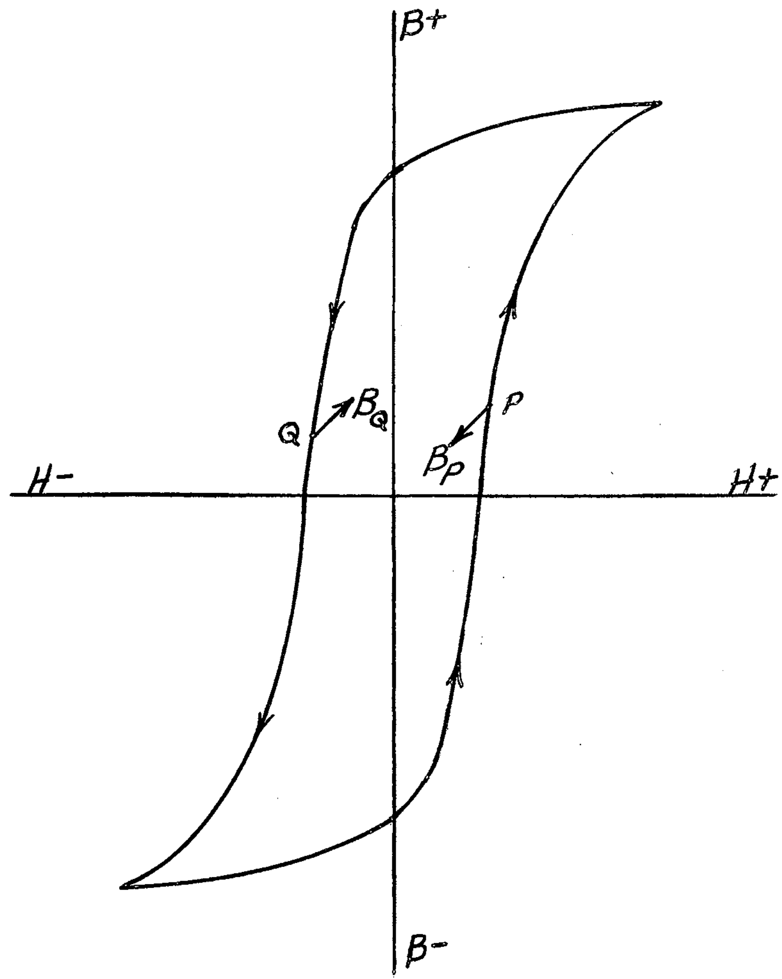
$$T = K\left(\oint \bar{B}_\theta \, d\bar{H}_\theta + \oint \overline{B_r \, d\, H_r} \right) \qquad (22)$$

If a machine having dimensions such that equation 19 may be used, that is sufficiently thin rings so that the radial component may be neglected, and the revolving stator field is sinusoidal, and the reluctance of the air gap is large compared to that of the rotor; the rotor flux density will be tangential, varying sinusoidal with θ, and independent of r.

When the rotor turns at less than synchronous speed, any element of volume moves with respect to the field and experiences a sinusoidal variation of B in time, and eventually reaches a cyclic state in which corresponding values of B and H are given by an alternating hysteresis loop, Figure 2-2. Other elements of volume experience exactly similar magnetic cycles displaced in time phase; thus the loop gives either time variation of the magnetic state at a fixed point on the rotor, or the space variation around the rotor at any instant of time. The integral $\oint B_\theta \, d\, H_\theta$ of equation 19 represents the area, A, of the loop, and

$$T = K A \qquad (23)$$

From this equation it can be seen that the torque is the same from zero to synchronous speed since the torque depends only on the area of the loop and not the speed. This fact was



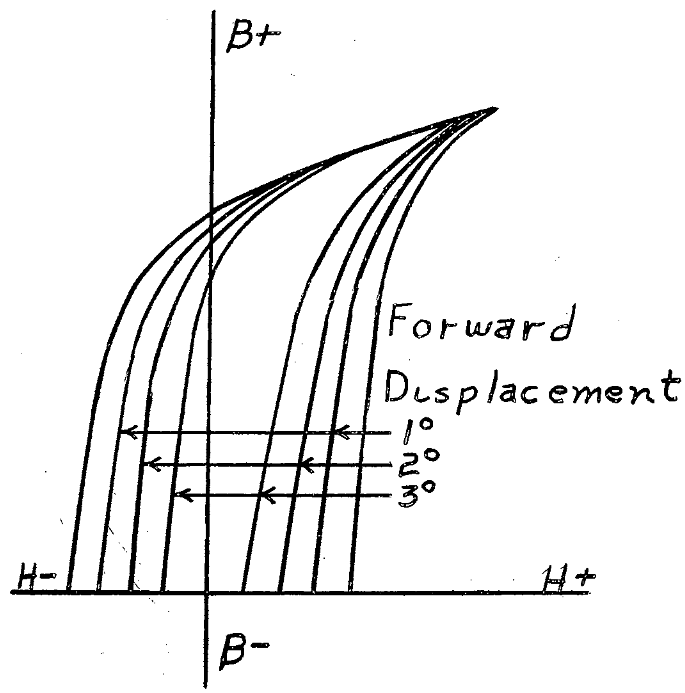
Hysteresis Loop

Figure 2-2

also noted by Dr. Steinmetz.

During the acceleration to synchronous speed, the rotor slips backward with respect to the rotating field at a diminishing rate. The field at a given point, P, goes through a magnetic cycle, Figure 2-2, slower and slower until the flux density reaches the value B_P and stops changing momentarily. The last change of B may have been either an increase or decrease depending upon the location of P with respect to the flux density wave. If the last change in B was an increase, then B_P lies on the ascending portion of the hysteresis loop. At another point, Q, where the last change was a decrease, then B_Q lies on the descending portion of the hysteresis loop. The relation of B and H, around the rotor, is still given by Figure 2 at the instant of synchronous speed, and the impelling torque is the same as that at subsynchronous speeds. If the load torque is just equal to the impelling torque at this instant, the motor speed will not change further, and the machine will operate at synchronous speed.

If the load torque is less than the impelling torque, there will be an increase in speed and a forward displacement of the rotor with respect to the field. This relative displacement is in a direction opposite to the last displacement that occurred. Thus the field at P, which was reached by an increase, now decreases to a point, B_P; this change occurring on a minor loop. Thus, after a forward displacement, the B-H relation from point to point around the rotor is given by a closed curve similar to the original hysteresis loop and derived from it by displacement of all its points inward along their respective minor loops. The new loop is thus smaller and the torque is less than before the change. Such B-H loops are illustrated in Figure 2-3. These loops were obtained graphically, with the minor loops being taken as straight lines. On this basis the torque decreases as the displacement angle increases, and at some point is in equilibrium with the load. However, at this point, the speed of the rotor is larger than synchronous and the displacement continues to increase, and the torque becomes less than that of the load and causes the net torque to be a retarding torque. This causes the speed to decrease and the rotor will oscillate about the point of equilibrium, with decreasing amplitude because of the losses. When the oscillations cease, the rotor turns at synchronous speed and has a position with respect to the stator field that makes the impelling torque just equal to the load.



Several Hysteresis Loops

Figure 2-3

CHAPTER III

THE PERMANENT MAGNET

Permanent magnets are used in different types of instruments and devices, such as compasses, electrical measuring instruments, relays, telephone receivers, microphones, magnetos, phonograph pick-ups, coin separators and numerous other devices. Some applications do not require great constancy, but in other applications, such as electrical measuring instruments, the highest degree of constancy possible is of prime importance.

The usefulness of permanent magnets has greatly expanded in recent years because of the development of superior materials. The advantages gained by their use can not be noted unless magnets are designed taking into consideration the magnetic properties of the materials used in making them.

The characteristics of permanent magnets and the properties of the materials used in their manufacture are expressed in terms of magnetic quantities as follows:

MAGNETOMOTIVE FORCE is the measure of the magnetizing force caused by current flowing in coils surrounding a magnetic circuit composed of the material to be magnetized. It is defined by the following equation

$$F = 0.4 \pi N I \qquad (1)$$

where

F = magnetomotive force in gilberts

N = number of turns in the coil

I = current in amperes

MAGNETIC FLUX is the measure of the magnetized condition of a magnetic circuit caused by the magnetomotive force. It is characterized by the fact that a change in the magnitude gives rise to an electromotive force in any electrical circuit linked with it. The electromotive force induced is directly proportional, at any instant, to the change of flux with respect to time. It is defined by the following equation

$$e = -N(d\phi/dt)10^{-8} \qquad (2)$$

where

e = induced electromotive force in volts

N = number of turns linked with the flux

ϕ = magnetic flux in maxwells

t = time in seconds

MAGNETIC RELUCTANCE is the expression of the ability of the circuit to oppose the establishment of the magnetic flux. It is defined by the following equation

$$\phi = F/\mathcal{R} \qquad (3)$$

where

ϕ = magnetic flux in maxwells

F = magnetomotive force in gilberts

$\mathcal{R}$ = magnetic reluctant in rels

In a magnetic circuit of uniform cross-sectional area and uniform permeability (to be defined later)

$$\mathcal{R} = l / \mu A \qquad (4)$$

where

$\mathcal{R}$ = magnetic reluctance in rels

l = length in centimeters

μ = magnetic permeability

MAGNETIC PERMEANCE is the reciprocal of the magnetic reluctance, thus

$$\phi = F \mathcal{P} \qquad (5)$$

where

ϕ = magnetic flux in maxwells

F = magnetomotive force in gilberts

$\mathcal{P}$ = magnetic permeance in cgs units

In a circuit of uniform cross-sectional area and uniform permeability

$$\mathcal{P} = \mu A / l \qquad (6)$$

MAGNETIZING FORCE is the magnetomotive force per unit length along a circuit in a manner dependent upon the distribution of the magnetizing windings and the reluctance of the circuit. It is also called MAGNETIC INTENSITY and is defined by the following equation

$$H = F/l \qquad (7)$$

where

H = magnetizing force in oersteds

F = magnetomotive force in gilberts

l = length in centimeters

At the center of a very long, uniformily wound solenoid having N turns per centimeter in which a current of I amperes flows, the magnetizing force in oersteds is

$$H = 0.4\pi N I \qquad (8)$$

MAGNETIC INDUCTION is the magnetic flux per unit area of a section normal to the direction of the flux. It is also called the MAGNETIC FLUX DENSITY and is defined by the following equation

$$B = \phi / A \qquad (9)$$

where

B = magnetic induction in gausses

ϕ = magnetic flux in maxwells

A = area in square centimeters

INTRINSIC INDUCTION is that part of the magnetic induction which is in excess of that induction which would exist in a vacuum under the influence of a given magnetizing force. It is defined by the following equation

$$B_i = B - H \tag{10}$$

It is known generally that the only source of magnetic effects is accomplished as a result of the flow of electric current. Ferromagnetic effects are caused by groups of electrons within a ferromegnetic material which will be called "domains" and consist of electrons spinning on their own axes.[1] The magnetic axes of the electrons within a single domain are held parallel by mutual forces, which will be called "exchange forces", thus, each domain behaves as a unit. These domains are effectively current-turns and thus account for the magnetomotive forces inherent in ferromagnetic materials. The total of the magnetomotive forces of the domains is the "permanent" quantity in a permanent magnet.

In the unmagnetized condition, these domains are oriented in such a manner with respect to each other that the net effect is zero in any direction. Under the influence of an external magnetic field, the magnetic axes of the domains tend to orient themselves in the direction of the external field, and thus, their effect is added to that of the external field. The magnetic induction results from the combined effects of the external field

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[1] R. M. Bozorth, "Present Status of Ferromagnetic Theory," Electrical Engineering, 1935, 1251.

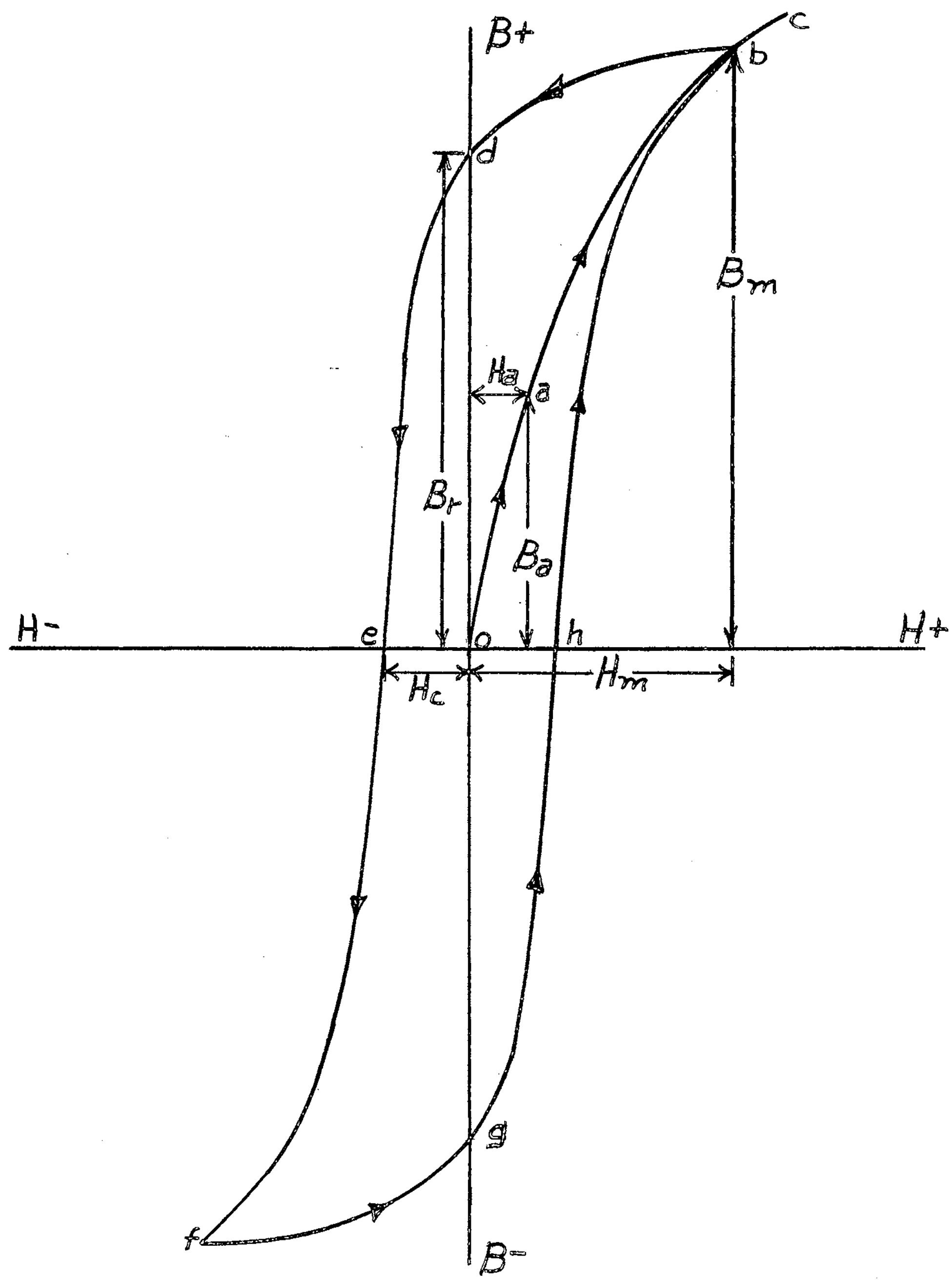
and the domains. The intrinsic induction results from the effect of the domains only.

When the external field is removed the orientations of the domains tend to persist depending on their distribution in space, which depends on the internal structure of the material. It is necessary to apply a demagnetizing force to reduce the induction further. The relationships between induction and magnetizing force applied are shown in Figure 3-1, with the magnetizing force as the abscissa and the magnetic induction as the ordinate. The curve, *oabc* is the normal induction curve, and represents the variation of the induction as the magnetizing force is increased when applied to material that is initially demagnetized. At any point on this curve the ratio of the induction to the magnetizing force required to produce this induction is called MAGNETIC PERMEABILITY

$$\mu = B_a / H_a \qquad (11)$$

Since the line *oabc* is not straight, the permeability is not constant, but varies with the degree of magnetization. This variation is one of the distinguishing characteristics of ferro-magnetic materials.

After the magnetizing force has reached a certain point, (point *b* on the curve), it is then decreased, the induction does not follow line *bao*, but lags behind (curve *bd*). Thus, when the magnetizing force has been reduced to zero, the induction still has an appreciable value and is called the RESIDUAL



Hysteresis Loop

Figure 3-1

INDUCTION, B_r.

In order to reduce the induction to zero, a magnetizing force must increase in the opposite direction. The value of this force is called COERCIVE FORCE, H_c.

HYSTERESIS is the lagging of the induction behind the magnetizing force, and the curve $bdefghb$ is called the hysteresis loop. Hysteresis is another characteristic of ferromagnetic materials and is the property which makes permanent magnets a possibility. The size of the loop, and also the magnitudes of B_r and H_c, depends upon the magnitudes of B_m and H_m at the tip of the loop. The size of the loop approaches a maximum as H_m is increased. The magnitudes of B_r and H_c corresponding to the maximum loop for any given material is called the RETENTIVITY and the COERCIVITY, respectively.

The part of the hysteresis loop from B_r to H_c, or from d to e on the curve is called the DEMAGNETIZING CURVE. Most of the important characteristics of a permanent magnet material can be determined from this curve or the area within this curve and the axes.

CHAPTER IV

OPERATING CHARACTERISTICS OF THE HYSTERESIS MOTOR

SUMMARY OF EXISTING METHODS OF PREDICTING MOTOR PERFORMANCE

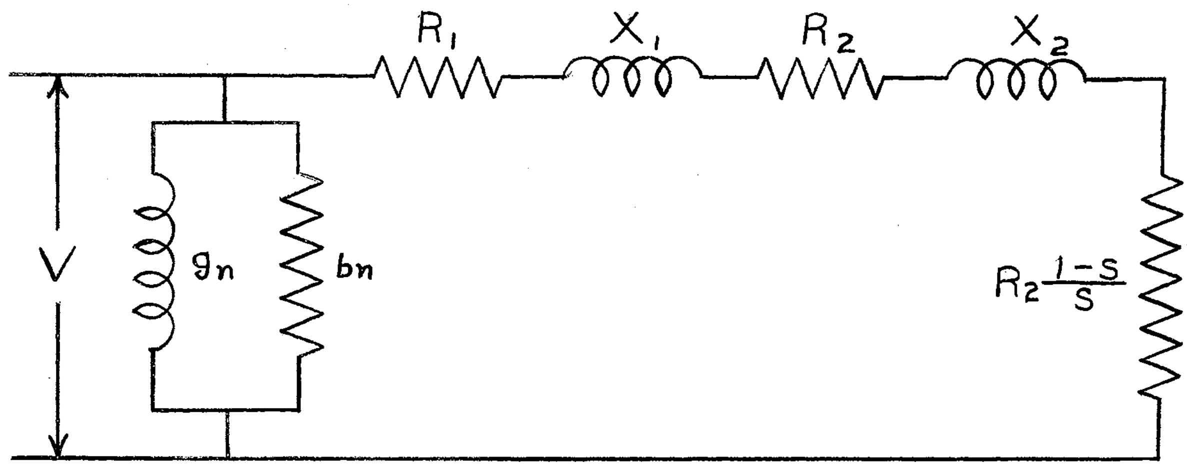
It is generally known that the most widely used method of prediction of induction motor performance is called the "Circle Diagram Method." In order to apply this method to a particular motor, several assumptions should be made. This method is based on the approximate equivalent network shown in Figure 1. This network neglects the effect of the exciting current causing a drop in the stator. It also neglects the effect of the stator resistance drop on the internal voltage.

If a constant voltage V is applied to this circuit, the load current flowing through the stator and rotor winding will be

$$I_2 = V/\sqrt{(R_1+R_2+R)^2+(X_1+X_2)^2} \quad (1)$$

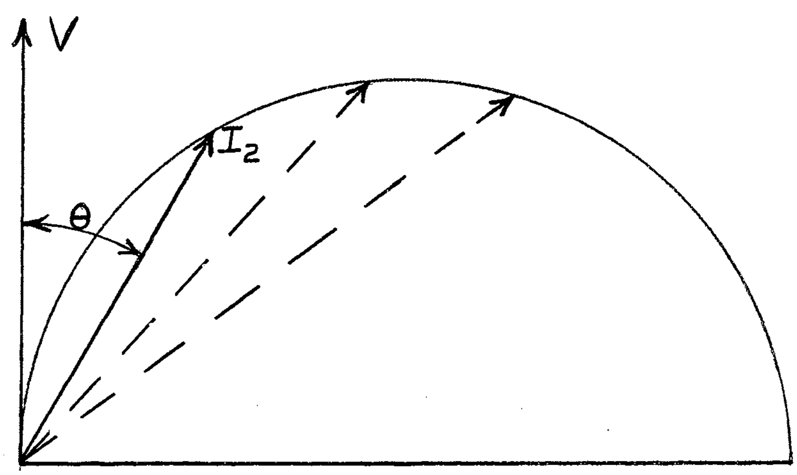
This circuit requires all of the rotor constants to be expressed in terms of the stator. When the rotor constants have been so expressed then I_2 is the same as the load component of the total stator current. This current is out of phase with the applied voltage by an angle whose sine is

$$\sin\theta = (X_1+X_2)/\sqrt{(R_1+R_2+R)^2+(X_1+X_2)^2} \quad (2)$$



The Approximate Equivalent Network

Figure 4-1



Locus of I_2

Figure 4-2

By substituting equation 2 in equation 1 there results

$$I_2 = (V/X_1 + X_2) \sin \theta \qquad (3)$$

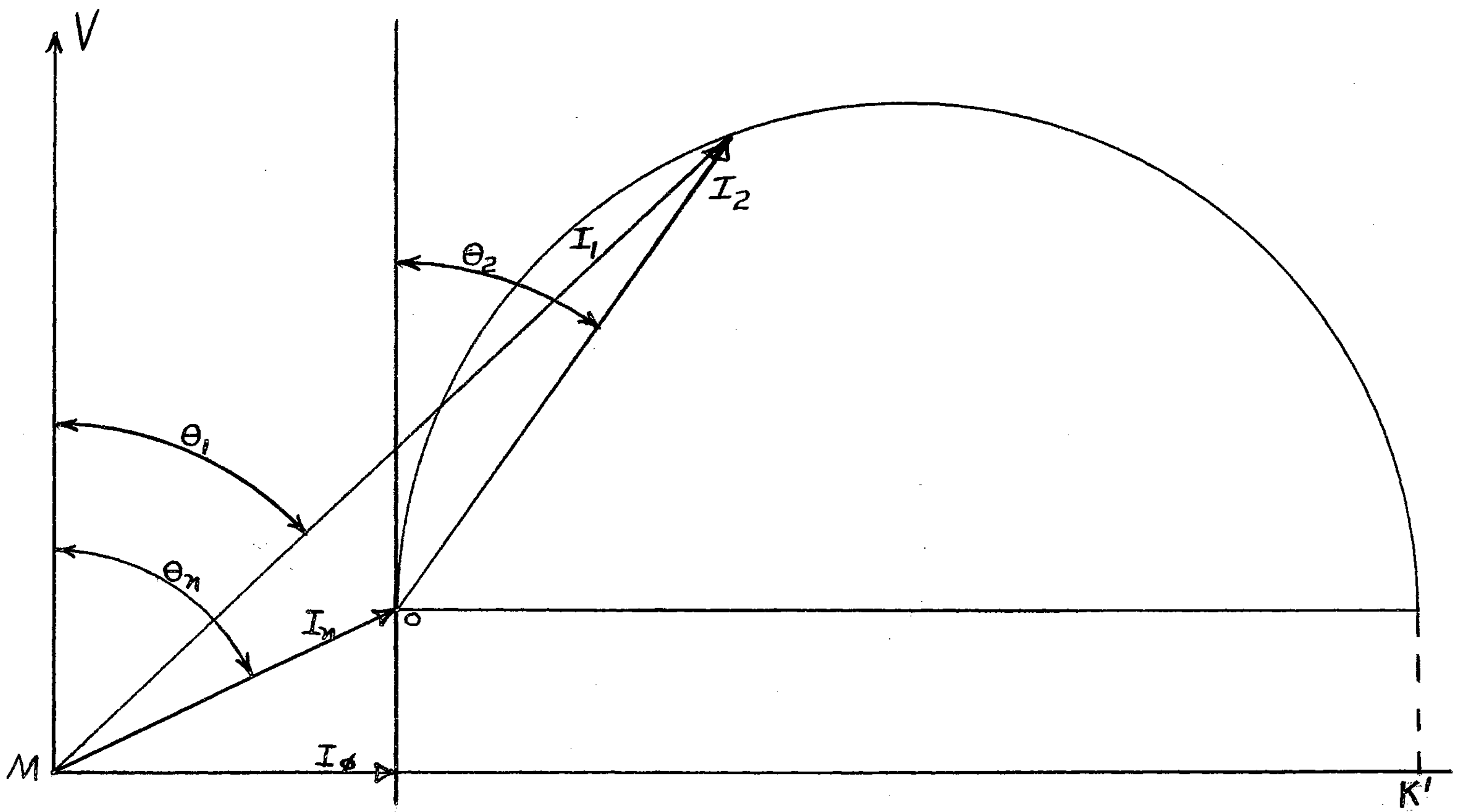
The usual application of the circle diagram is to assume that the leakage reactance remains constant. If the applied voltage is constant then equation 3 is the polar equation of a circle whose diameter is $V/(X_1 + X_2)$. If R (the load) is changed, the sine of θ will change and I_2 will vary in magnitude and direction with respect to the voltage V . Such a locus is shown in Figure 4-2. In order to have a plot of the total stator current, the exciting current I_n must be added. This is shown in Figure 4-3. In order for this diagram to truly represent the circuit of Figure 4-1, the applied voltage and the induced voltage must be assumed to be equal in magnitude and in phase.

To use the circle diagram to predict motor performance, two tests are necessary. They are the no-load test and the blocked-rotor test.

The applied voltage is drawn as a vertical line. It is to be the reference line. The no-load current is drawn to a length determined by arbitrary current scale behind the voltage by its power factor angle.

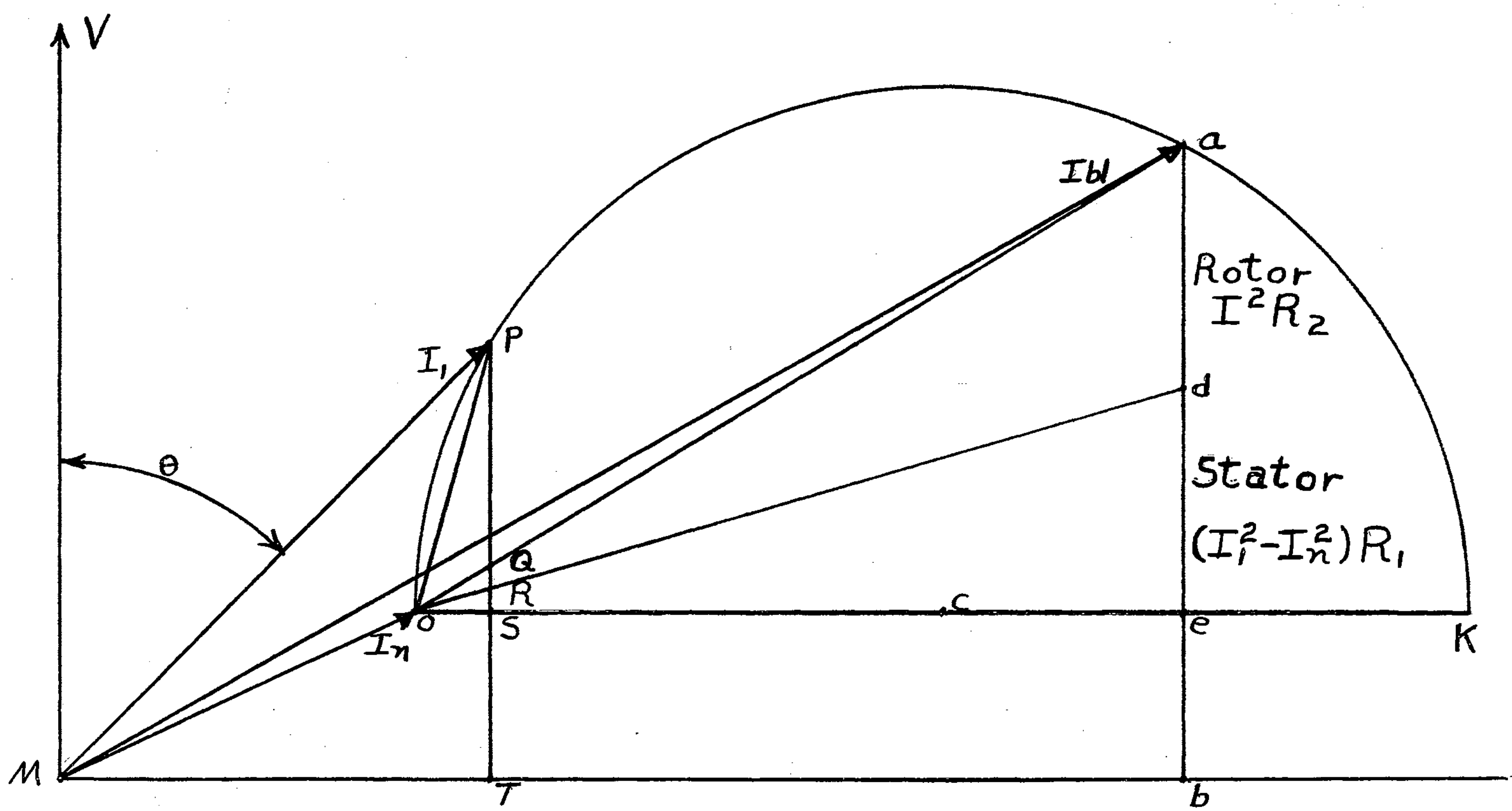
The blocked-rotor current is drawn behind the voltage at its power factor angle so that ab on Figure 4-4 represents the inphase component at rated voltage.

The points o and a must lie on the locus circle. The center C of the locus circle can be found by constructional



Plot of Total Stator Current

Figure 4-3



Circle Diagram of Induction Motor

Figure 4-4

means. With c as the center, draw the semi-circle oak.

The point d represents the division between the stator and rotor resistances. In other words; ad is equal to the rotor resistance expressed in terms of the stator and de is equal to the stator resistance. The length I_{ed} can be found also by the following equation:

$$I_{ed} = (I_{bl}^2 - I_n^2) R_1 / V \tag{4}$$

Draw I_{ed} to the current scale to determine the position of point d. Draw od and oa and ok.

Draw the line MP equal to the stator current. Draw the vertical line PT. Using the correct scales the following values may now be determined from Figure 4-4

MO--No-load current
MP--Stator current
OP--Rotor current
TP--Power input to stator
TS--Constant losses
SR--Stator added copper loss
RQ--Rotor copper loss
RP--Power transferred across air gap
QP--Useful output
PT/MP--Power factor
RQ/RP--Slip
QP/TP--Efficiency

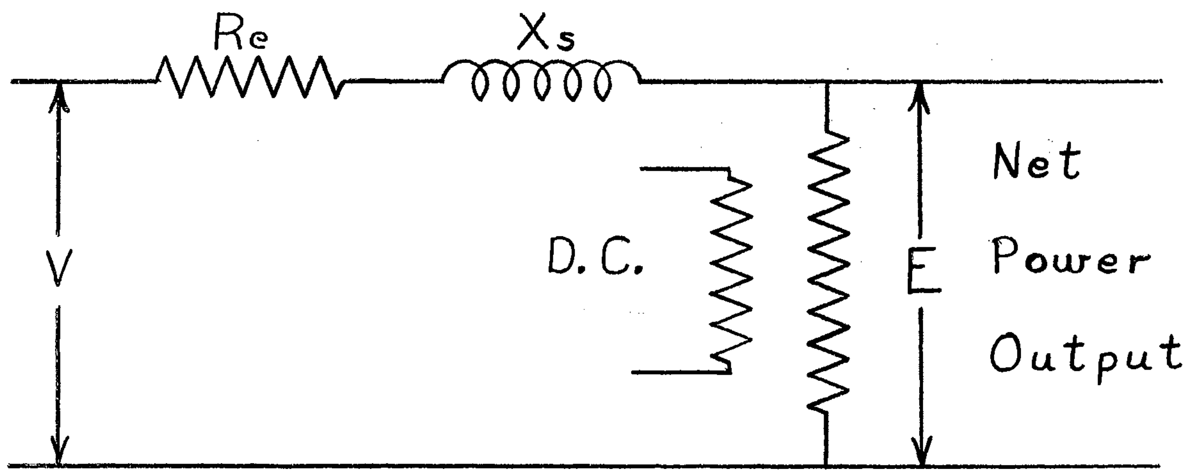
The watt scales are determined by multiplying the current by the applied voltage. The torque in pound-feet may be determined by multiplying the watt scale by the quantity

$$33{,}000/(746 \times 2\pi \times \text{Synchronous Speed}) \tag{5}$$

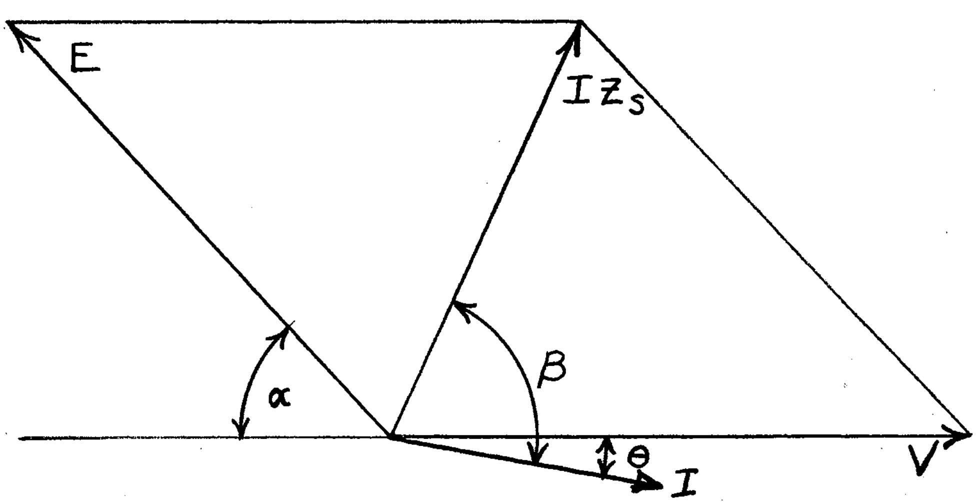
By assuming different values of I_1, the performance of the motor may be calculated. All of the values in the above discussion are per-phase values.

Since the circle diagram proved relatively satisfactory for the induction motor, more research was completed on the application of the circle diagram method of prediction for the synchronous motor performance. As with any analysis of motor action, the circle diagram method must assume a constant resistance and synchronous reactance.

Figure 4-5 shows the approximate equivalent circuit of the synchronous motor along with the vector diagram of the circuit for any one load and any one value of field excitation. The vector diagram of Figure 4-5 may be redrawn for convenience as shown in Figure 4-6a. As the torque angle, α, varies, point M will describe a circle about point O. The angle, β, is the impedance angle of the stator winding. Since R_a and Z_s is constant, the angle, β, will be constant. Hence, as the angle, α, varies, the point, I, will also describe another circle. If I is moved to the end of the vector V, Figure 4-6b the circle described by I will be more clearly seen. If M is on OG (indicating no-load on the machine) I will be on GH. If M is at O (indicating zero excitation, since the length of E is determined by the amount of direct current applied to the field) I will be at GC. As α increases (indicating increasing load) OM describes an arc and I moves along the semicircle HB. The diameter of the semicircle HB is proportional to the excitation E. This can be verified by letting OM equal

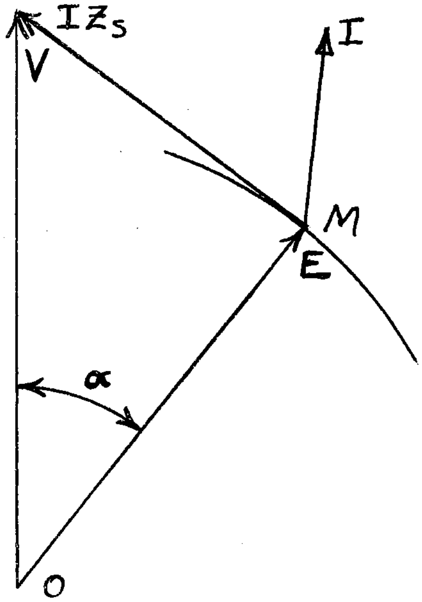


a. Approximate Equivalent Circuit

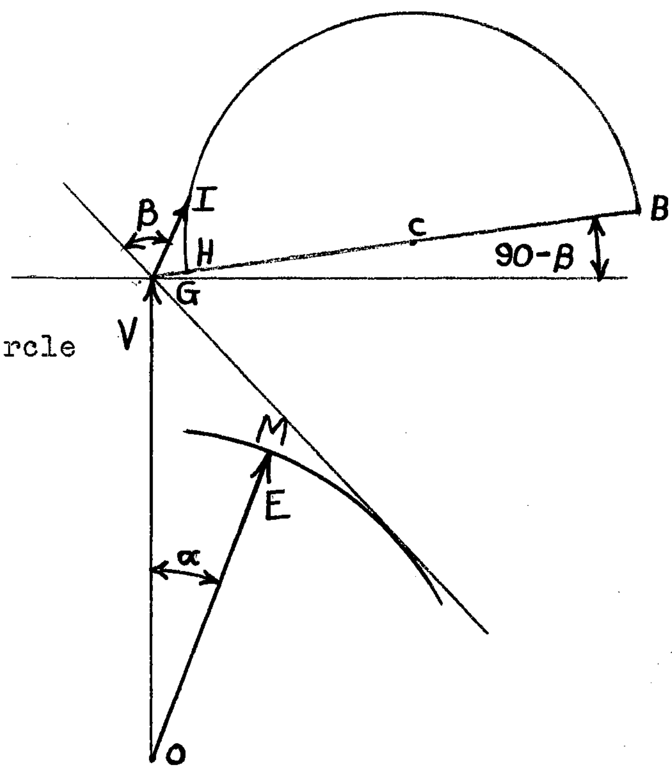


b. Vector Diagram

Figure 4-5



a. Rearranged Vector Diagram



b. Construction of Circle Diagram

Figure 4-6

OG. Under this condition the difference between V and E is zero and H will be at G. When M is at O, E is zero and H will be at C. GH can be determined when M lies on OG, thus

$$GH = (V - E)/Z_s \tag{6}$$

When M is at O (zero excitation)

$$GC = V/Z_s \tag{7}$$

Since CH is proportional to E, a family of circles can be drawn using C as a center and representing various ratios of E to V as radii. The voltage triangle GOM may be dropped as it is no longer necessary and the vector V drawn vertically from point G. This is done in Figure 4-7.

It can be clearly seen from Figure 4-7 that for a current I that

GT--in-phase component of current GI
TI--quadrature component of current GI
GT/GI--power factor
GTxGV--input watts

If the input watts remain constant, the current will follow a horizontal line for various values of excitation, as shown by ab in Figure 4-7.

To determine the current characteristics at a constant developed power, a power circle may be drawn about the point S which represents the locus of current for constant developed

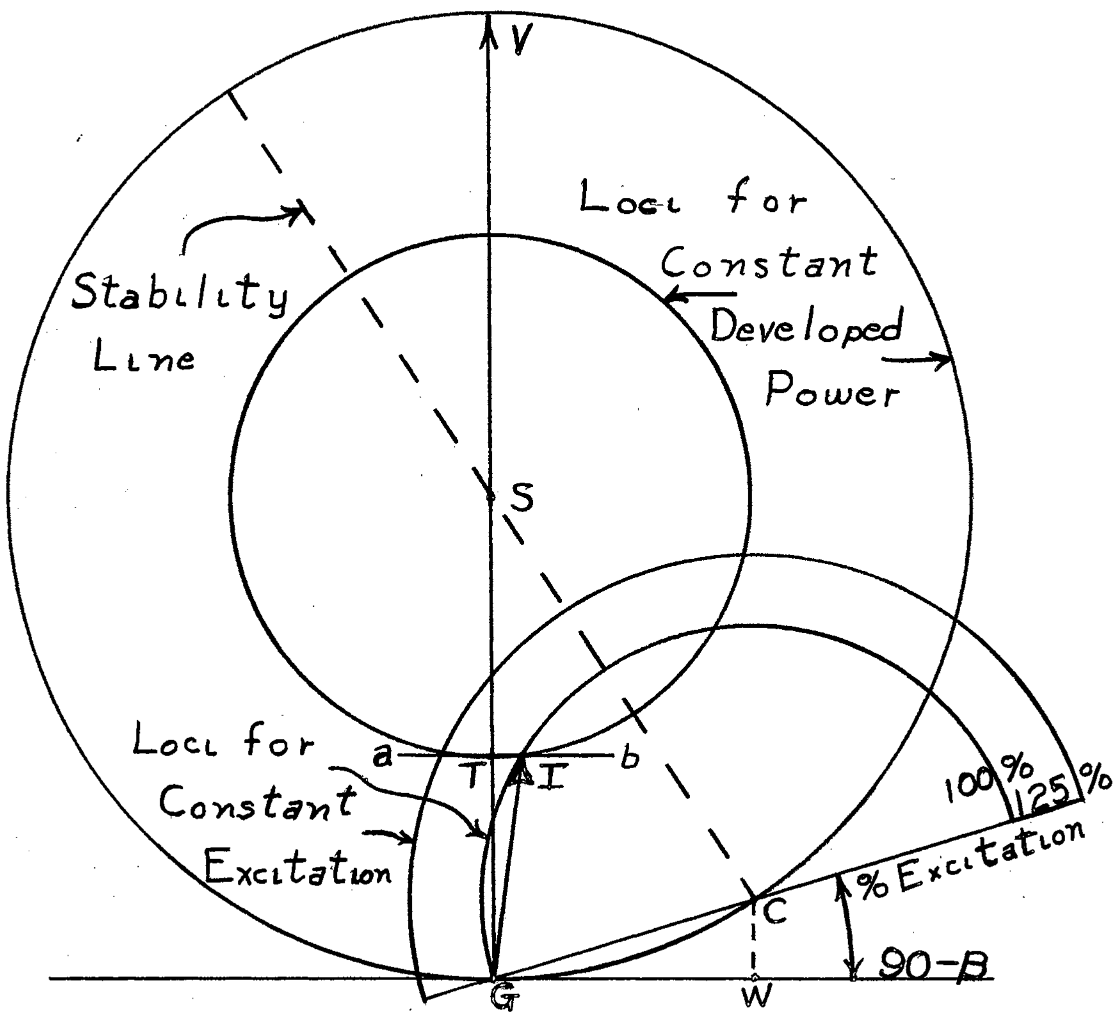


Figure 4-7

Circle Diagram for Synchronous Motor

power. The following procedure may be used to determine this locus:

The current taken by the motor at zero excitation is GC. CW represents the power component of the current GC. When the excitation is zero the input to the machine is exactly equal to the copper loss in the stator, thus

$$V \times CW = I^2 R_a \tag{8}$$

By similar triangles

$$CW/GC = GC/2GS \tag{9}$$

from which

$$GS = (GC)^2/2CW \tag{10}$$

when

$$GC = V/Z_s \tag{11}$$

and

$$CW = VR_a/Z_s^2 \tag{12}$$

then equation 10 becomes

$$GS = V/2R_a \tag{13}$$

With S as the center the constant developed power circle may be drawn passing through points G and C .

To obtain the constant developed power circle for any other load, the law of cosines may be used as follows

$$(SI)^2 = (GS)^2 + (GI)^2 - 2(GS)(GI)\cos\theta \qquad (14)$$

transposing

$$(GS)^2 - (SI)^2 = 2(GS)(GI)\cos\theta - (GI)^2 \qquad (15)$$

since

$$2(GS) = V/R_a \qquad (16)$$

and

$$(GI)\cos\theta = \text{Power Component} \qquad (17)$$

then

$$V^2/4R_a^2 - (SI)^2 = [\text{Power Input} - R_a(GI)^2]/R_a \qquad (18)$$

Since

$$\text{Power Input} - R_a(GI)^2 = \text{Developed Power} \qquad (19)$$

then

$$SI = \sqrt{(V^2/4R_a^2) - (\text{Developed Power}/R_a)} \tag{20}$$

and

$$SI = (1/R_a)\sqrt{(V^2/4) - (R_a)\,\text{Developed Power}} \tag{21}$$

As the constant developed power circles decrease in radii, the power developed becomes greater.

For any one value of constant developed power, the end of the current vector will follow one of these circles. For a given value of excitation, the current will follow along the excitation circle.

Since the hysteresis motor operates at synchronous speed the following summary will deal only with the synchronous type of machine.

Any type of motor which has only one input circuit and one output circuit may be represented by a network called the "Four-Terminal Network."

The general equation used in the solution of any four-terminal network are as follows

$$\begin{aligned} \bar{V}_s &= \bar{A}\bar{V}_r + \bar{B}\bar{I}_r \\ \bar{I}_s &= \bar{C}\bar{V}_r + \bar{D}\bar{I}_r \end{aligned} \tag{22}$$

where

$\bar{V}_s$ = Sending-end voltage

$\bar{I}_s$ = Sending-end current

$\bar{V}_r$ = Receiver-end voltage

I_r = Receiver-end current

$\bar{A}$-$\bar{B}$-$\bar{C}$-$\bar{D}$= Circuit parameters or constants

In equations 22, $\bar{A}$ and $\bar{D}$ are simple numerals while $\bar{B}$ has the characteristic of an impedance and $\bar{C}$ has the characteristic of an admittance.

If equations 22 were solved simultaneously for $\bar{V}_r$ and $\bar{I}_r$, there results

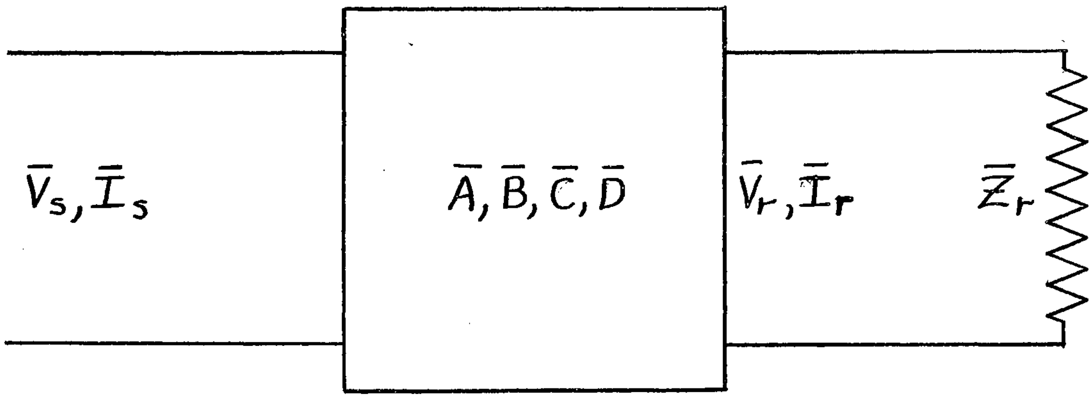
$$\bar{V}_r = (\bar{D}\bar{V}_s - \bar{B}\bar{I}_s)/(\bar{A}\bar{D} - \bar{B}\bar{C})$$
$$\bar{I}_r = (-\bar{C}\bar{V}_s + \bar{A}\bar{I}_s)/(\bar{A}\bar{D} - \bar{B}\bar{C}) \qquad (23)$$

When a four-terminal network as that one shown in Figure 4-8 is loaded with an impedance, the transfer impedance from the sending end to the receiver end is

$$\bar{Z}_{ts} = \bar{V}_s / \bar{I}_r \qquad (24)$$

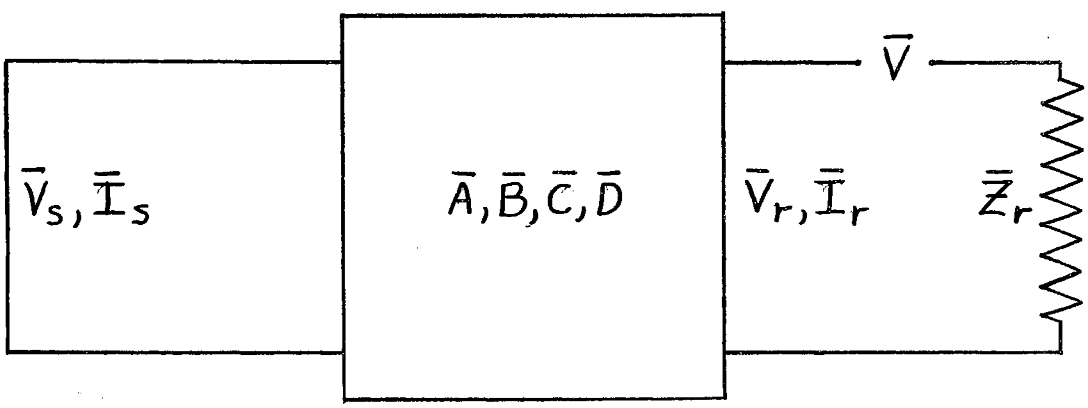
This can be derived from equations 22 by dividing by I_r and obtaining

$$\bar{Z}_{ts} = \bar{A}\,\bar{Z}_r + \bar{B} \qquad (25)$$



Four-Terminal Network With Impedance Load

Figure 4-8



Four-Terminal Network with Sending End Terminals Shorted

Figure 4-9

If the supply voltage $\bar{V}$ is introduced at the receiver end and the sending end is short circuited, there results Figure 4-9, from which

$$\bar{V}_r = (\bar{D}\bar{V}_s + \bar{B}\bar{I}_s)/(\bar{A}\bar{D} - \bar{B}\bar{C})$$

$$\bar{I}_r = (\bar{C}\bar{V}_s + \bar{A}\bar{I}_s)/(\bar{A}\bar{D} - \bar{B}\bar{C}) \qquad (26)$$

Since the sending end is short circuited, $\bar{V}_S = 0$ and the voltage $\bar{V}_r$ applied to the receiver terminals is

$$\bar{V}_r = \bar{V} - \bar{I}_r\bar{Z}_r \qquad (27)$$

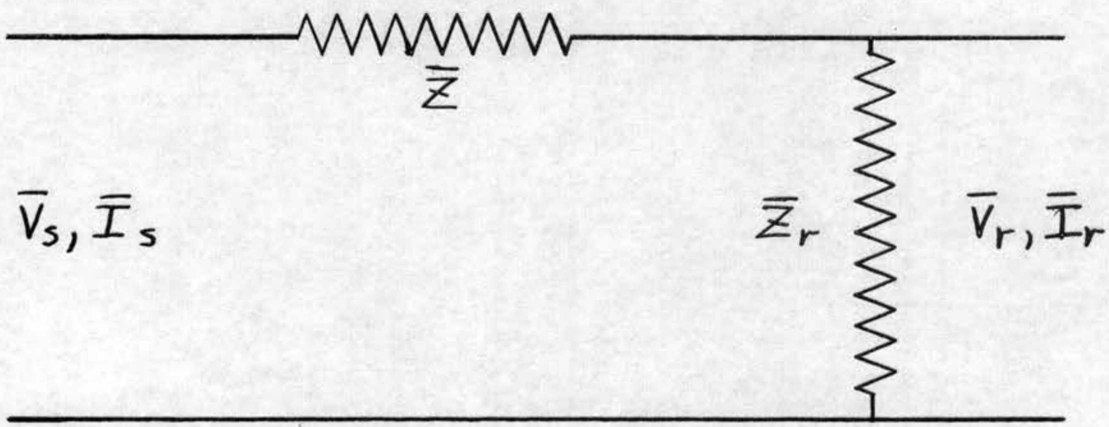
This value of $\bar{V}_r$ may be substituted into equations 26 and by dividing by $\bar{I}_r$ there results

$$\bar{\bar{Z}}_{tr} = (\bar{A}\bar{\bar{Z}}_r + \bar{B})/(\bar{A}\bar{D} - \bar{B}\bar{C}) \qquad (28)$$

In accordance with the reciprocity theorem, a circuit is fully reciprocal when $\bar{\bar{Z}}_{ts}$ is equal to $\bar{\bar{Z}}_{tr}$. Thus, if equation 25 were set equal to equation 28, there results

$$\bar{A}\bar{D} - \bar{B}\bar{C} = 1 \qquad (29)$$

The former presentation may now be used to investigate the cantilever circuit which most nearly represents the hysteresis motor. From Figure 4-10 the following equations are obtained



Cantilever Circuit

Figure 4-10

$$\bar{V}_s = (1 + \bar{Z}/\bar{Z}_r)\bar{V}_r + \bar{Z}\,\bar{I}_r$$
$$\bar{I}_s = \bar{V}_r/\bar{Z}_r + \bar{I}_r \qquad (30)$$

then

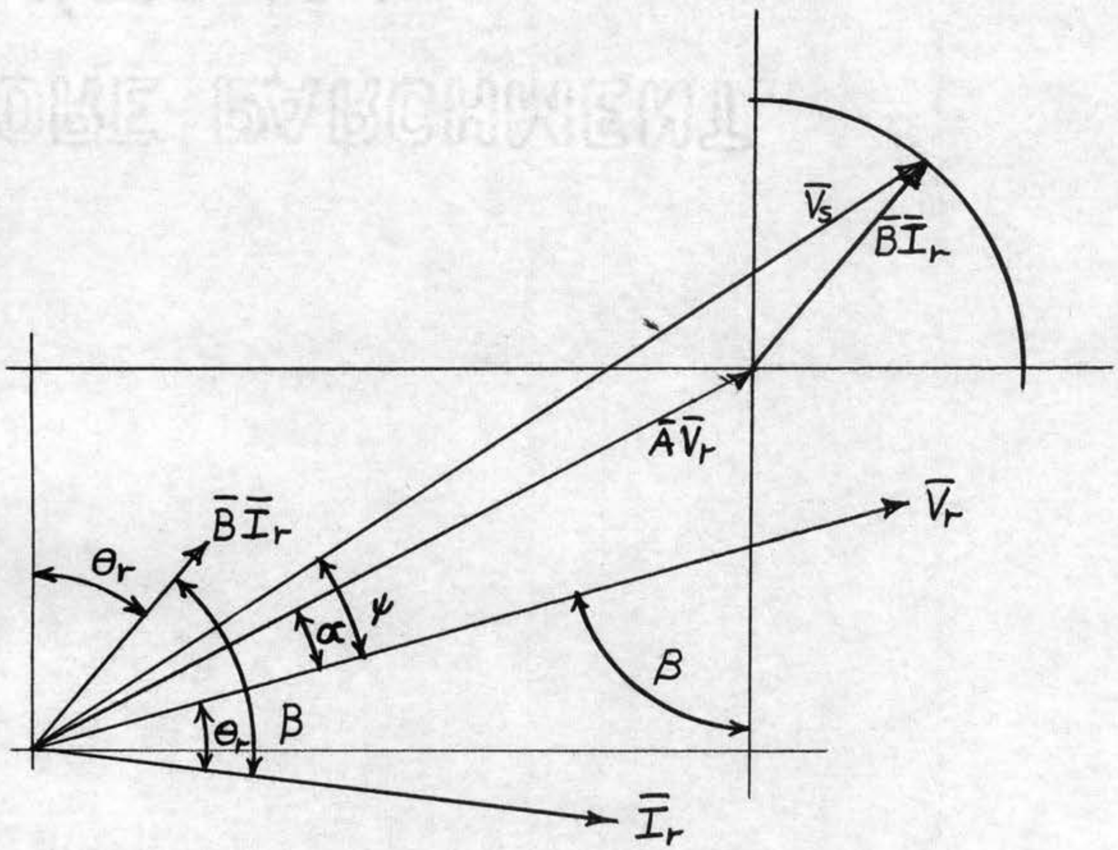
$$\bar{A} = 1 + \bar{Z}/\bar{Z}_r$$
$$\bar{B} = \bar{Z}$$
$$\bar{C} = 1/\bar{Z}_r \qquad (31)$$
$$\bar{D} = 1$$

and

$$\bar{V}_r = \bar{V}_s - \bar{Z}\,\bar{I}_s$$
$$\bar{I}_r = -\bar{V}_s/\bar{Z}_r + (1 + \bar{Z}/\bar{Z}_r)\bar{I}_s \qquad (32)$$

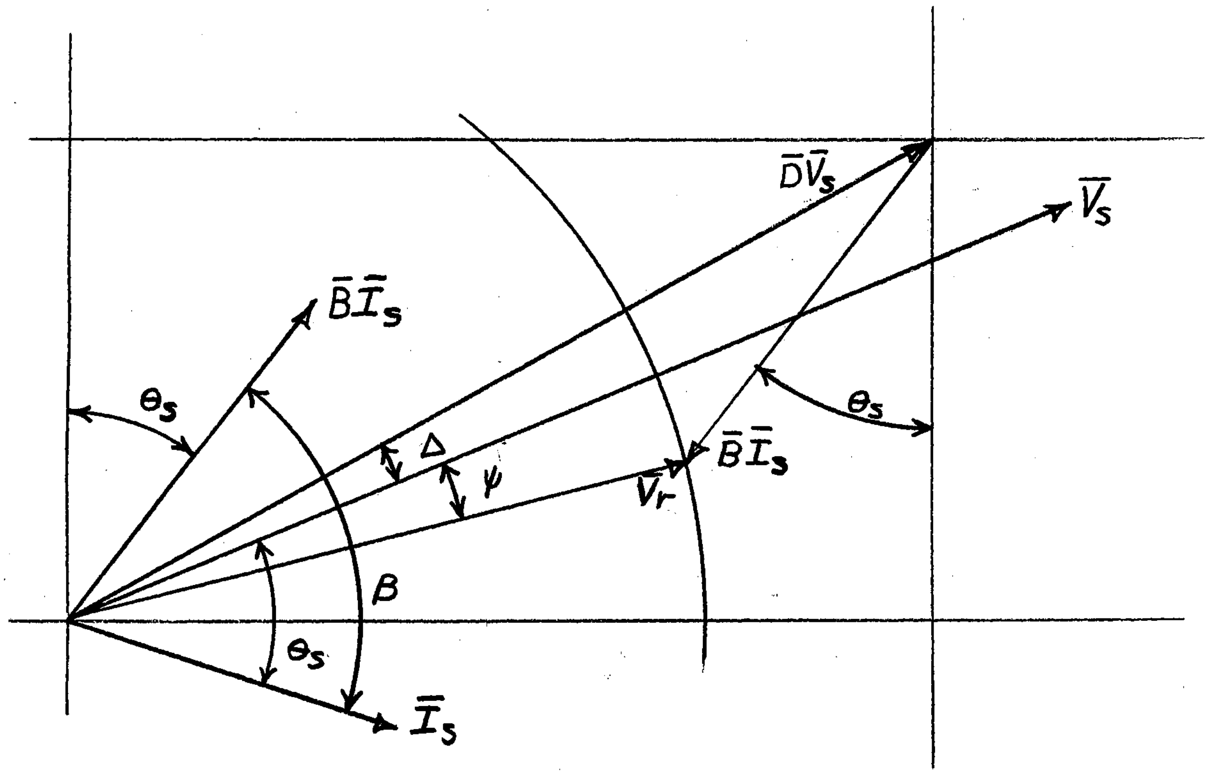
Since the synchronous motor operates at constant terminal voltage, a type-B circle diagram will be used.

Using equation 30, the vector diagram of Figure 4-11 may be drawn, assuming that $\bar{V}_s$ and $\bar{V}_r$ are maintained constant in magnitude, to represent the receiver circle diagram. Using equation 32, the vector diagram of Figure 4-12 may be drawn using the same assumptions to represent the sending circle diagram. These two diagrams may be combined for convenience to determine different operating characteristics. From Figures 4-11 and 4-12



Receiver Circle Diagram

Figure 4-11



Sending Circle Diagram

Figure 4-12

it can be seen that when $\bar{V}_s$ and $\bar{V}_r$ are assumed constant in magnitude, so must the constants $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ be constant.

In the preceding discussions, several assumptions were made to allow these methods to be used for the performance prediction of electrical machinery. With the application of the assumptions, the solution for a particular machine of the induction or synchronous type has been relatively accurate in comparison with the normal dynamometer tests of these machines.

The assumptions that the preceding methods must apply are: (1) the parameters of the machine must remain constant, (2) the stator impedance does not affect the internal voltage, (3) the exciting current does not cause a drop in the stator windings. If these assumptions were not made then a circle could not be used to predict the performance accurately.

It will be shown later that the parameters of the hysteresis motor may not be assumed constant with any degree of accuracy since the resistance of the machine is so high that its drop should be considered. As a result, the hysteresis motor may not, with any accuracy, be analyzed using the preceding methods of performance prediction. A search of the existing material on prediction methods does not reveal any general method for the hysteresis motor.

DEVELOPMENT OF AN EQUATION FOR THE CURRENT LOCUS OF A HYSTERESIS MOTOR

An analysis of the hysteresis motor will reveal that the general application of the circle diagram method may not be used because the current locus, as the load increases, does not generate a circle.

In this work, a hysteresis motor, with the following nameplate data, was used:

Robbins and Myers Inc.
Frame SKH 85
Type Hysteresis
1/150 Horsepower
115 Volts
0.6 Amperes
Single Phase
60 Cycles
1800 RPM

Table 4-1 is the data taken from the manufacturer's curves illustrated in Figure 4-13.

Figure 4-14 is a plot of the current vectors of the hysteresis motor used in this work. In the determination of the type of curve that would fit this curve, several different types of curves were investigated. By a process of trial and error it is found that a rotated parabola most nearly represents this current locus.

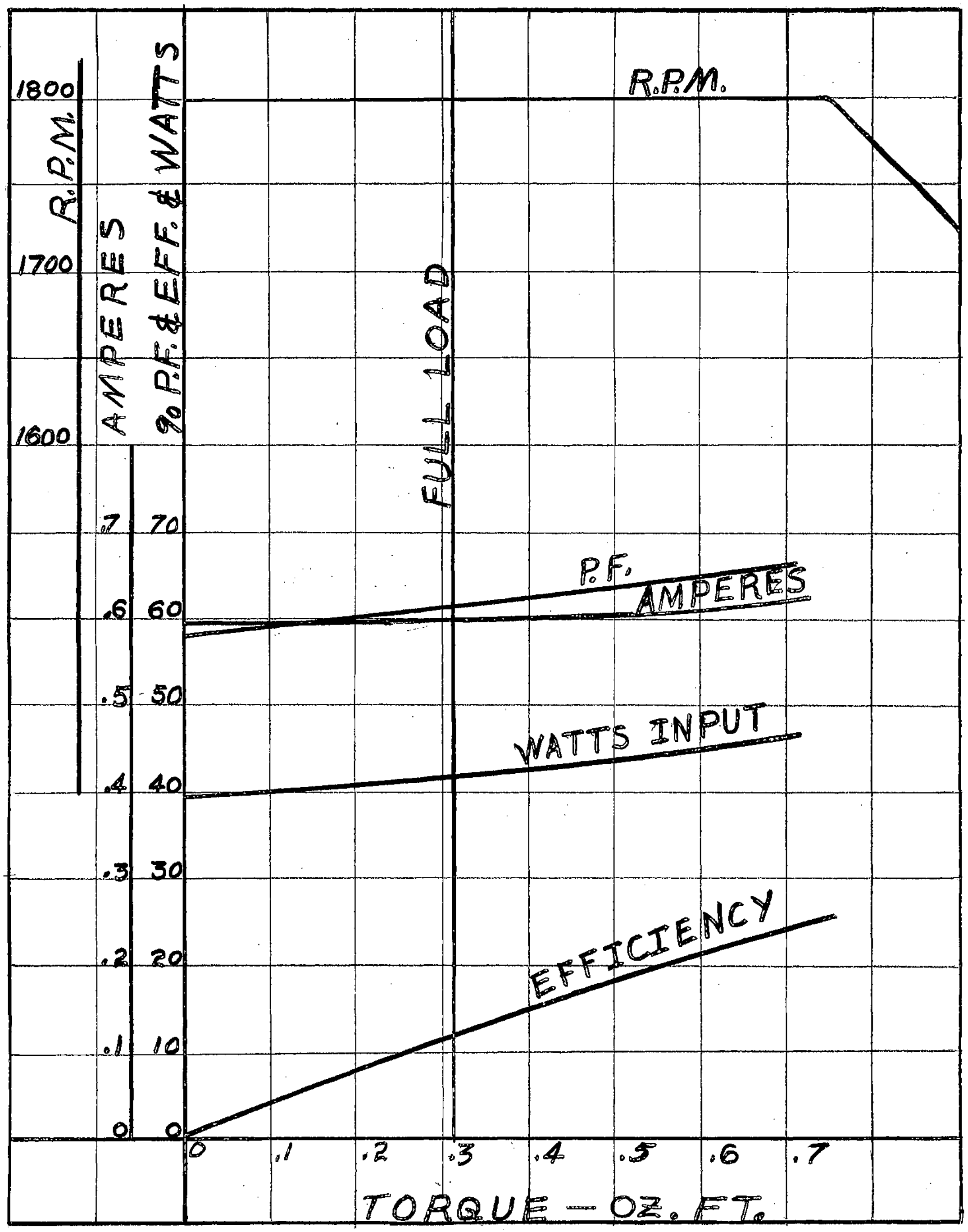
The general parabolic formula was solved using different values of p. This curve was imposed on the current curve and rotated until the correct value of p was found so that the current curve and the parabola coincided.

It is found that the origin of the parabola must be translated to the origin of the current vectors, and that the parabola must also be rotated. The general equation for a parabola is

$$y^2 = 2px \tag{33}$$

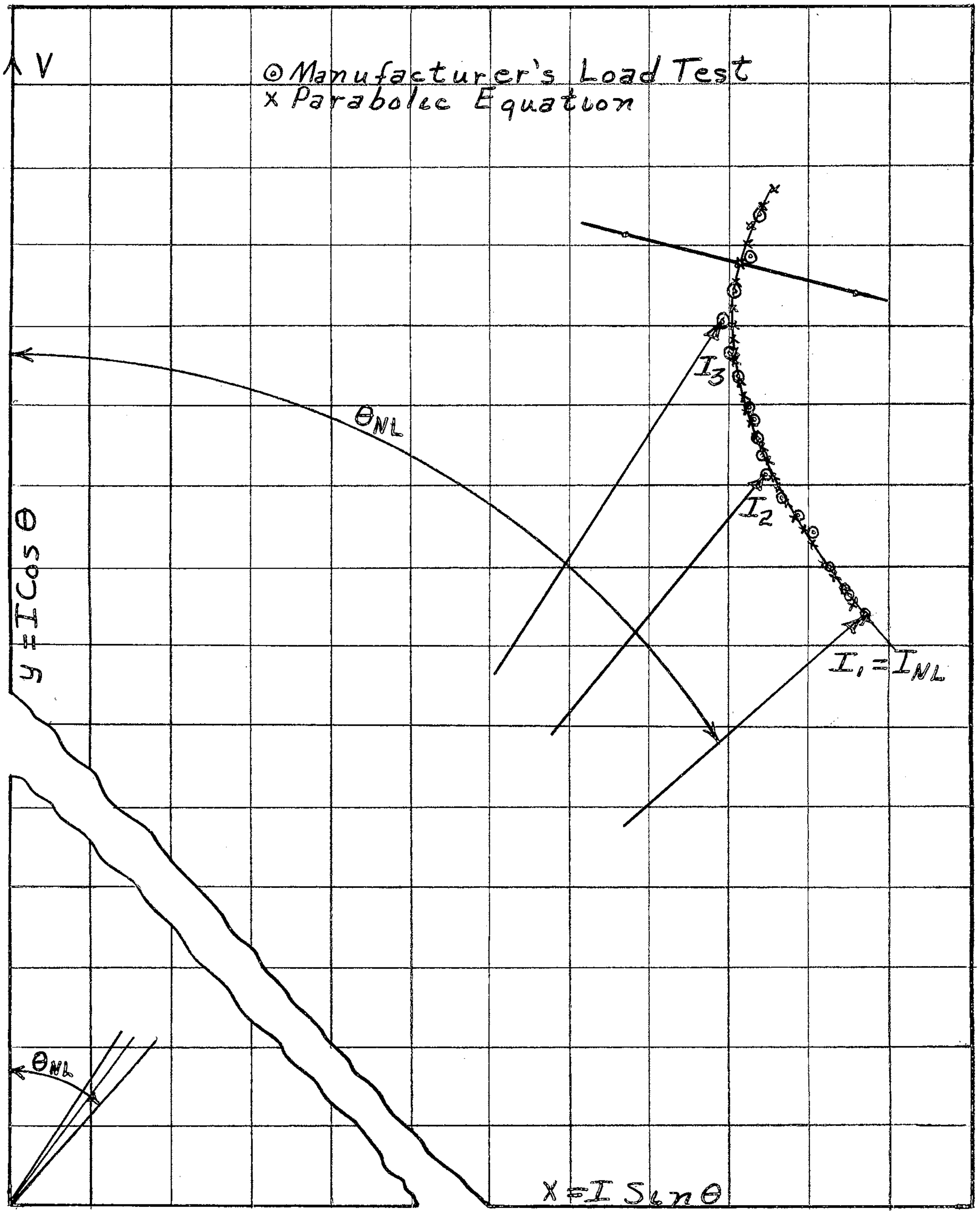
TABLE 4-1

Voltage Volts	Torque oz-ft	Current Amperes	Input Watt	Power Factor	Efficiency
115	0.00	0.5930	39.4	0.5780	0.000
115	0.04	0.5930	39.7	0.5820	0.014
115	0.06	0.5930	39.9	0.5850	0.031
115	0.12	0.5930	40.2	0.5900	0.047
115	0.16	0.5930	40.6	0.5960	0.061
115	0.20	0.5935	40.8	0.5985	0.073
115	0.24	0.5935	41.1	0.6030	0.089
115	0.28	0.5935	41.5	0.6085	0.105
115	0.311	0.5940	41.9	0.6140	0.115
115	0.36	0.5945	42.2	0.6175	0.133
115	0.40	0.5955	42.6	0.6210	0.148
115	0.44	0.5970	42.9	0.6250	0.162
115	0.48	0.5990	43.2	0.6270	0.175
115	0.52	0.6000	43.7	0.6335	0.189
115	0.56	0.6010	44.2	0.6380	0.203
115	0.60	0.6040	44.7	0.6440	0.214
115	0.64	0.6080	45.2	0.6470	0.227
115	0.68	0.6130	45.9	0.6510	0.237



Manufacturer's Curves

Figure 4-13



Parabolic Equation and Current Locus of a Hysteresis Motor

Figure 4-14

The equation used to rotate the parabola are

$$x = x'\cos\gamma - y'\sin\gamma \tag{34}$$

$$y = x'\sin\gamma + y'\cos\gamma \tag{35}$$

When equations 34 and 35 are substituted into equation 33, there results

$$(x'\sin\gamma)^2 + 2(x'y'\sin\gamma\cos\gamma) + (y'\cos\gamma)^2 =$$

$$= 2p(x'\cos\gamma - y'\sin\gamma) \tag{36}$$

The equations for translation are

$$x' = x - h \tag{37}$$

$$y' = y - k \tag{38}$$

When equations 37 and 38 are substituted into equation 36 and terms are collected, there results

$$x^2\sin^2\gamma + 2xy\sin\gamma\cos\gamma + y^2\cos^2\gamma + 2xh\sin^2\gamma +$$

$$-2xk\sin\gamma\cos\gamma - 2xp\cos\gamma - 2yh\sin\gamma\cos\gamma +$$

$$-2yk\cos\gamma + 2yp\sin\gamma + h^2\sin^2\gamma + 2hk\sin\gamma\cos\gamma +$$

$$+k^2\cos^2\gamma + 2ph\cos\gamma - 2pk\sin\gamma = 0 \tag{39}$$

From Figure 4-14 the following quantities were determined:

$2p = 0.075$

$h = 0.4643$

$k = 0.3973$

$\gamma = 14° \ 51.5'$

$\cos\gamma = 0.9709$

$\sin\gamma = 0.2395$

Substitution into equation 39 will yield

$$0.05736x^2 + 0.46506xy + 0.94265y^2 +$$

$$-0.31085x - 0.947y + 0.27362 = 0 \qquad 40)$$

Equation 40 may be reduced still further by substituting

$$x = I \sin\theta \qquad (41)$$

$$y = I \cos\theta \qquad (42)$$

This curve, represented by equation 40 is not a circular function. Thus, it is evident that the parameters do not remain constant. One particular parameter, usually assumed constant, is the internal or generated voltage. Ordinarily the internal voltage will stay relatively constant as it is a large value. However, in the hysteresis motor the winding is constructed of a large number of turns of small size wire, causing the resistance to be large. Under these circumstances the reactance of the winding is also going to be large. Thus, the internal voltage is going to be small with respect to the applied voltage. Any change however slight, that occurs in the current, will cause a sufficient change in the stator impedance drop that the internal voltage will vary considerably. With this point in mind, it may be said that the general performance prediction methods may not be used with sufficient accuracy with the hysteresis motor. Furthermore a search of the literature has failed to reveal a satisfactory prediction method for the hysteresis motor.

THE SEPARATION OF LOSSES IN A HYSTERESIS MOTOR

The manner of separating the losses in the hysteresis motor has not been discussed in any previous literature. Following is the manner in which the author has separated these losses from the manufacturer's data.

The output in watts was subtracted from the input watts to obtain the total losses of the machine. The direct current resistance of the stator winding was measured with a Wheatstone Bridge and found to be 34.35 ohms. This value was multiplied by the factor 1.1 to obtain the alternating current resistance of the winding, the value of which was 37.8 ohms. The current was squared and multiplied by this resistance to determine the stator copper loss. This loss was then subtracted from the total losses to obtain the windage and friction plus eddy current plus hysteresis losses in the machine.

The windage and friction has been assumed to have a constant value of 2 watts. This value is so assumed because the machine is small and the small air gap necessitates accurate construction. This value is constant because the machine operates at synchronous speed. This value is then subtracted to obtain the sum of the eddy current and hysteresis losses. The following equation is used to determine the eddy current loss

$$P_c = k_e B^2 f^2 \tag{43}$$

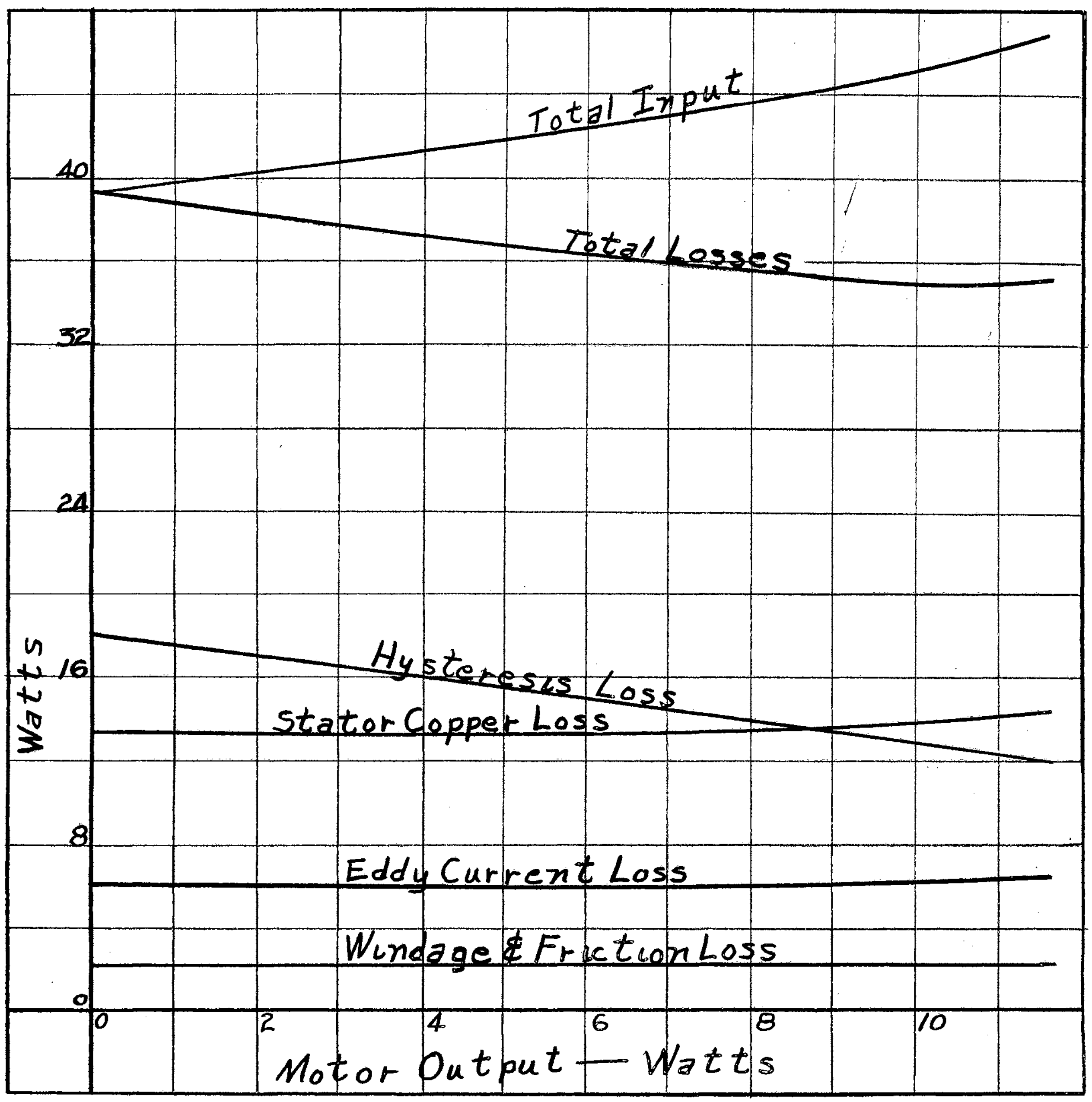
The assumption was made that the flux density, B, would vary directly with the current. This assumption was made on the basis that the machine was not in a saturated or overexcited condition. As the machine was operating at a constant frequency, then f would be constant also. This loss was then subtracted to obtain the hysteresis loss in the machine. Table 4-2 is a compilation of the losses of this machine. Figure 4-15 is the curves taken from Table 4-2.

The eddy current loss has been assumed to be equal to 6 watts at no-load. Design considerations dictate low flux densities. Since the eddy current loss serves no useful purpose in the stator, they are kept at a low value relative to hysteresis input through the use of thinly laminated high quality steel in the stator. The eddy current loss component in the rotor is non-existant during normal operation at synchronous speed.

The circle diagram procedures for the induction motor and synchronous motor were taken from "Alternating-Current Machines" by Puchstein and Lloyd. The four-terminal network discussion was taken from "Alternating-Current Machinery" by Tarboux.

TABLE 4-2

Input	Output	Losses	I^2R	W F and Eddy Current and Hysteresis	Eddy Current and Hysteresis	Eddy Current	Hysteresis
39.4	—	39.40	13.292	26.108	24.108	6	18.108
39.7	.64	39.06	13.292	25.768	23.768	6	17.768
39.9	.96	38.94	13.292	25.648	23.648	6	17.648
40.2	1.92	38.28	13.292	24.988	22.988	6	16.988
40.6	2.56	38.04	13.292	24.748	22.748	6	16.748
40.8	3.20	37.60	13.315	24.285	22.285	6.010	16.275
41.1	3.84	37.26	13.315	23.945	21.945	6.010	15.935
41.5	4.48	37.02	13.315	23.705	21.705	6.010	15.695
41.9	4.98	36.92	13.337	23.583	21.583	6.020	15.563
42.2	5.76	36.54	13.360	23.180	21.180	6.030	15.150
42.6	6.40	36.20	13.405	22.795	20.795	6.051	14.744
42.9	7.04	35.86	13.472	22.388	20.388	6.081	14.307
43.2	7.68	35.52	13.563	21.957	19.957	6.122	13.835
43.7	8.32	35.38	13.608	21.772	19.772	6.142	13.630
44.2	8.96	35.24	13.653	21.587	19.587	6.163	13.424
44.7	9.60	35.10	13.790	21.310	19.310	6.224	13.086
45.2	10.24	34.96	13.973	20.987	18.987	6.307	12.680
45.9	10.88	35.02	14.204	20.816	18.816	6.411	12.405
46.7	11.52	35.18	14.484	20.696	18.696	6.537	12.159



Breakdown of the Losses of a Hysteresis Motor

Figure 4-15

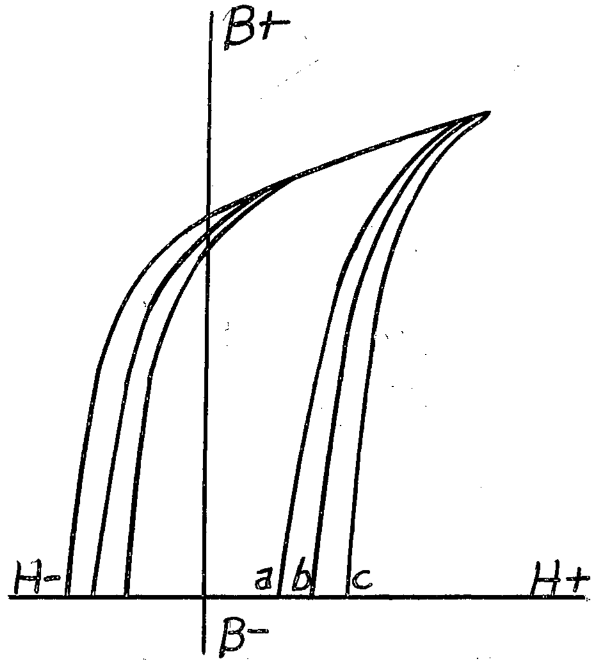
CHAPTER V

CONCLUSIONS

The rotor of the hysteresis motor has a hard steel shell and for this reason is of high resistance. As power is supplied to the stator there will be currents induced within this rotor shell. Since the rotor does not contain conductors, these currents must then be eddy currents. The interaction of these currents, though they are small, is sufficient to cause a torque to be produced. If this torque is sufficiently large the rotor will begin to rotate. As rotation continues at an increasing rate, the induced field in the rotor shell slips less and less with respect to the stator field. During this increase in speed a point on the rotor has been experiencing a hysteresis loop. As the speed of the machine approaches the synchronous value this point on the rotor slows down its movement along the hysteresis loop. It may also be considered that the movement of this point along its hysteresis loop is retarded by the permanent magnetic properties of the material of the rotor. It may be said that this retarding effect will cause the machine to pull into synchronism at a faster rate as the rotor approaches synchronous speed. At synchronous speed the rotor may possibly go past this point in speed. If this happens then generator action exists. This action will cause the machine to slow down. The rotor may possibly oscillate below and above synchronous speed until the losses cause the oscillations to die out. At synchronous speed any point on the rotor may be represented by a point on a hysteresis loop. As long as the load and speed remain constant this point will remain stationary on the loop.

The induced magnetism in the rotor will be stationary with respect to the field of the stator at synchronous speed. As a result of the hysteretic lag of the rotor material, the induced field of the rotor lags behind the field of the stator. This angle between the two fields permits them to act as a couple thus producing a torque. At no-load the angle is just large enough so that the torque produced by the couple just overcomes the frictional countertorque of the rotor structure. Since the rotor and the induced rotor magnetism rotate at synchronous speed, there are no currents within the rotor structure. Thus the only origin of torque must come from the couple produced by the angle between the stator field and the rotor induced magnetism. From this analysis it may be said that the output torque of this machine is supplied by the advent of hysteresis. Thus as the load increases and the machine continues to operate at synchronous speed, the load torque is supplied by hysteresis. This phenomena has been shown in Chapter IV in the fact that the hysteresis loss of this machine decreases as the load increases.

With the preceding discussion as a basis, the hysteresis loop can be used to represent the operating characteristics of this type of machine. Figure 5-1 shows several hysteresis loops of the same material. These loops may be said to represent the total hysteresis input to this machine. The horizontal axis represents input current to the machine, and the vertical axis represents the field strength of the stator field. The smallest



Several Hysteresis Loops

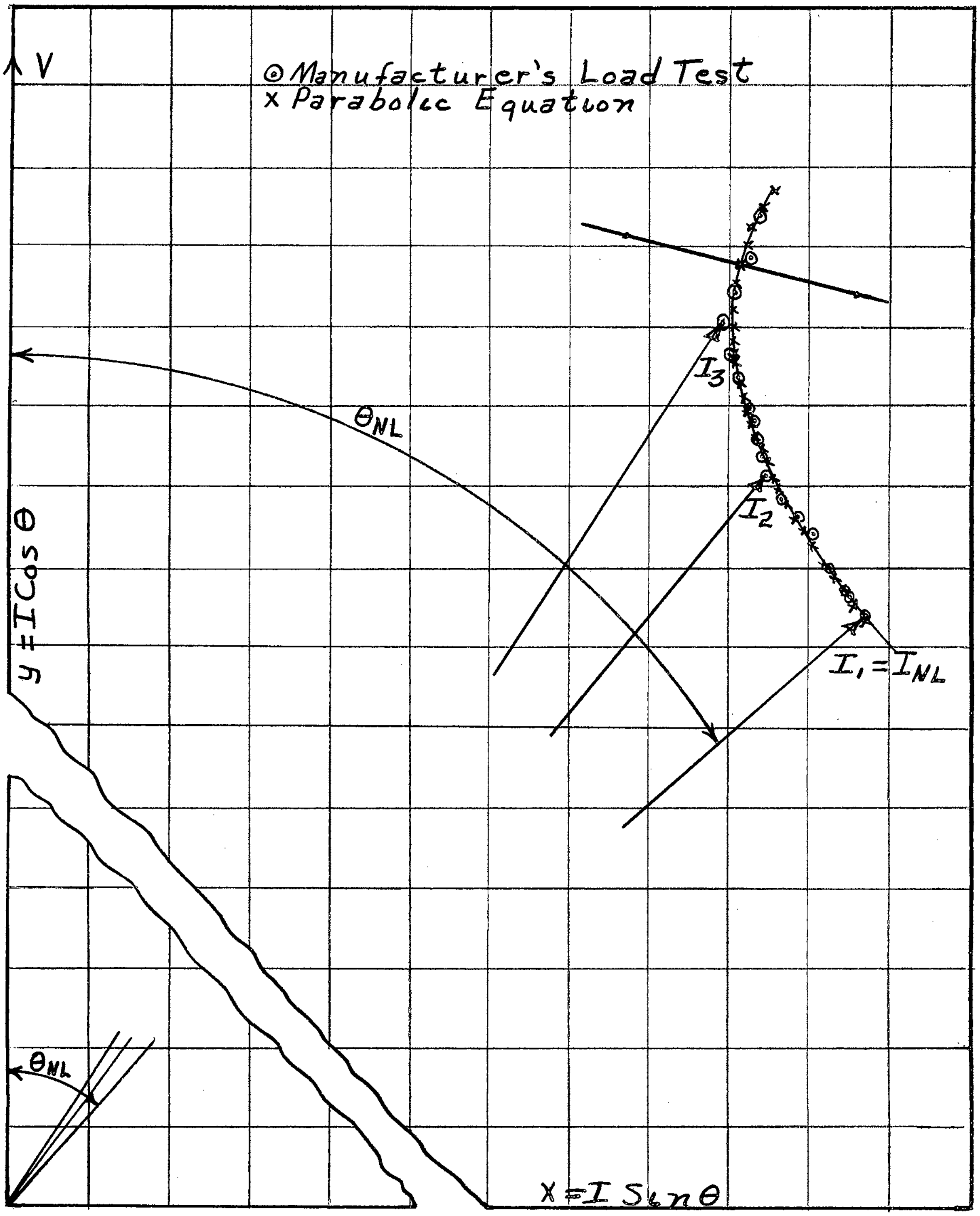
Figure 5-1

curve represents the no-load curve. The curves have a larger area as the load increases. Since the load has been determined to come from hysteresis, the area of these curves represents the load plus the hysteresis loss of the machine. When the saturation point is reached, the area of the curve does not get any larger with increase in current. Curve "a" may be called the no-load curve, and as such, it represents wholly hysteresis loss. Curve "b" represents some operating point where the area represents the load and the hysteresis loss. Curve "c" represents the largest area that may be obtained for a particular machine. It may be said that it is at this point that the machine breaks down. In other words any attempt to cause the machine to take more load will result in loss of synchronism as there is no more hysteretic power available. The hysteresis loss could not possibly go to zero because of the material making up the machine. Thus the saturation point of the magnetic material could be spoken of as the breakdown point.

Since the rotor is composed of a magnetic material and is the coupling between the stator and load, it too may be the locus of a hysteresis loop. As the machine operates at synchronous speed, there will be no hysteresis loss in the rotor itself. Thus the only hysteresis loop pertaining to the rotor is that corresponding to the load as the torque depends upon the hysteretic coupling for its existance. It has been determined that the saturation point of the stator is the breakdown point and thus the same thing may be said of the rotor because it is the intermediary between the stator and the load.

In Chapter IV the breakdown of the losses clearly shows that the hysteresis loss decreases until very nearly the breakdown point of operation. This phenomena occurs because the load is derived from hysteretic coupling and thus some of the input hysteresis actually appears as the load and the hysteresis loss. At the point where the hysteresis loss increases the area of the input loop is increasing only slightly with relatively large increase of current. The hysteretic coupling is also becoming weak and thus less hysteretic transfer will occur and the machine begins to become unstable and finally breaks down. When breakdown occurs the speed of the rotor is different from the speed of the induced rotor field and thus eddy currents flow in the rotor causing a large increase in losses. Also the rotor now has a frequency which causes it to have a hysteresis loss which will increase as the speed goes down. Thus the speed of the machine will decrease rapidly until the machine stops.

The decrease in hysteresis loss may also be shown from an inspection of Figure 5-2. This Figure is a reproduction of Figure 4-14. The horizontal component of this plot represents the out-of-phase component drawn by the machine. It will be noticed that as the load increases the out-of-phase component also decreases. This means that the hysteresis and eddy current effects must have decreased. Since the eddy current loss increased as shown in Chapter IV, the hysteresis effect must be the one that decreases. Thus the preceding analogy should be well founded. The out-of-phase component decreases until almost the breakdown point before increasing again. This occurrence



Parabolic Equation and Current Locus of a Hysteresis Motor

Figure 5-2

coupled with the hysteresis loss variation should validate the use of the saturation curve to approximate the extent to which a machine may be loaded.

Several interesting facts may be found concerning the plot of the current and the losses of this machine. A close survey of Figures 4-14 and 4-15 will reveal that the vertex of the parabola falls at the same operating point when there is a minimum of losses in the machine. Whether or not this phenomena, the angle of rotation, and the value of "p" has any significance as to the size of machine is beyond the scope of this work. These points and others will be left for further research when several sizes of machines are available.

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