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THE EFFECT OF HARMONICS ON THE
PREDICTION OF SINGLE-PHASE MOTOR PERFORMANCE

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THE EFFECT OF HARMONICS ON THE
PREDICTION OF SINGLE-PHASE MOTOR PERFORMANCE

by

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PREFACE

The induction motor is one of the most important types of electric machinery in use today; it might be said that it is second only to the transformer in importance. It is to be expected then, that a great amount of research would be done to determine the operating characteristics of the induction motor.

There are two methods available for determining the performance of induction motors and they are (1) the method of loading and (2) the equivalent circuit method. It is generally conceded that the load test is the most accurate for all types of motors; however, the equivalent circuit method requires less equipment and gives comparable results. The equivalent circuit method, in its usual form, requires the following tests of the machine: (1) the no-load test, (2) the blocked rotor test and (3) the measurement of stator resistance. The performance of single- and polyphase machines from the equivalent circuit has been covered fairly well in present day electrical literature, but much work is left to be done. The method of the four-terminal network is given in some detail in this paper and will be found to be one of the more accurate and expedient methods available, but the use of this method, as well as all others, results in inaccuracies because of:

- (1) unjustified treatment of exciting current,

- (2) the assumption that circuit parameters are constant,
- (3) the failure to consider stray load loss,
- (4) the assumption of sinusoidal currents and voltages throughout the motor.

Some work may be found in the literature on the first three items listed above; however, very little work has been done on the fourth, especially as applied to single-phase motors. The usual procedure, when considering the performance characteristics of induction motors, is to neglect the effect of harmonics and in most instances the results obtained are sufficiently accurate. With harmonics present, however, the motor can be expected to show some departure from normal operation.

It has been shown¹ that two types of harmonics exist in induction motors;

- (a) time harmonics, introduced by the impressed emf,
- (b) space harmonics, introduced by the counter emf of the motor.

In analyzing a motor considering harmonics, the motor could be thought of as several motors connected on the same shaft, each identical to the motor under consideration, and drawing power; one from the fundamental, one from the third, one from the fifth, and one from the seventh harmonic emf. The reactance of the harmonic motors is n times that of the fundamental, where n is the order of the harmonic. The resistance is also somewhat greater

¹ Doggett, L. A. and Queer, E. R., "Induction Motor Operation With Non-sinusoidal Impressed Voltages", Electrical Engineering, XLVIII, (October, 1929) p. 759.

for the harmonic motors, but not n times as great. It was further shown that to all but the fundamental frequency, the running induction motor acts practically as though blocked; i.e., the slip is in the neighborhood of one. The conclusions were that, with harmonics of 10% or less, the effect of harmonics are negligible for all types of induction motors and for all conditions except the no-load condition. It was found that, at no-load, an increase of I^2X occurred, but it was not considered serious and the lightly loaded induction motor could be expected to smooth out the impressed emf wave.

In the split-phase single-phase motor it is possible to have all the harmonics present, but modern winding practice does lessen their magnitudes. The problem is complicated by the fact that there is no way to obtain the wave forms of rotor current and voltage. In this thesis the problem is approached from the input current and voltage and the waveforms of these quantities were obtained through the use of the magnetic oscillograph and then analyzed for harmonics. The question is how much the effects of these harmonics are and how much is being neglected.

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CHAPTER I

PREDICTING MOTOR PERFORMANCE

INTRODUCTION. The alternating current induction motor has been classified in many different ways, but may simply be classified as single-phase or polyphase. In each class will be found many types of motors, each built for a particular service and each requiring special operating characteristics. For example: one load requirement may make desirable a constant speed at all loads, while another load may require variable characteristics; i.e., a change in slip with a change in load. To classify all induction motors, it may be said that they belong to that group of electrical apparatus which is known as asynchronous; i.e., they operate at a speed somewhat less than the synchronous speed of their rotating fluxes.

The induction motor was invented by Nikola Tesla in 1888, and much improvement has been made in its operating characteristics since its invention. The common use of this motor may be largely attributed to its simple, rugged and inexpensive construction.

The theory of operation and construction of the polyphase motor is simpler than that of the single-phase motor and although the single-phase motor lacks the symmetry inherent in the polyphase motor it is very commonly used. This may be largely attributed to the growth of the single-phase system of trans-

mission and distribution in this country. Since the use of the induction motor is so common it follows that much research would be done on the operating characteristics of the motor and a search of electrical literature will reveal a wealth of information on both the single-phase and the polyphase motor.

PERFORMANCE OF INDUCTION MOTORS. In general, the performance of induction motors may be obtained by direct or indirect methods. The direct method necessitates the use of the load test which may be performed in two different ways depending upon whether the output of the machine is measured directly or calculated from the input and losses. The performance of the motor is usually obtained by the use of the dynamometer, prony brake, or a calibrated generator. One manufacturer¹ states that most induction motors, rated from 1 to 300 horsepower, are tested by use of the electric dynamometer, as it offers the most satisfactory method of obtaining complete performance and speed-torque characteristics; that fractional horsepower motors are tested by use of the prony brake, although this is not suitable for large motors; that the equivalent circuit method is used for machines too large to be loaded conveniently. Loading devices are quite often unavailable, not to mention that they are bulky and costly for even medium sized machines, therefore, they are applied only to the smaller machines. Load tests will not be covered further, as sufficient information may be found in any good text on alternating current machinery.

¹ Westinghouse Electric Manufacturing Co., Factory Testing of Electric Apparatus, East Pittsburgh, Pa., 1943.

The indirect method, which may be referred to as the equivalent circuit method, makes use of tests that may be applied to a motor without special equipment. The indirect method always begins with the equivalent circuit and the accuracy obtainable is dependent upon the choice of an equivalent circuit which will closely represent the motor under all load conditions.

The induction motor is essentially a transformer, the magnetic path of which is separated by the air gap of the motor into two relatively moving parts, one part carrying the primary and the other the secondary windings. The secondary is usually of the form of a squirrel cage winding; i.e., it consists of identical copper or aluminum bars cast solidly to conducting end rings. Thus, the secondary is short-circuited, or as in the case of the wound rotor machine, may be closed through an external resistance. The distinguishing feature of the induction machine is the fact that current is "induced" into the secondary, that is, no emf is directly applied to the rotor. Motion is created by electromagnetic forces corresponding to the power that is transferred across the air gap by induction. Thus, with the machine being considered as a transformer, it may be represented by an equivalent circuit, and the performance predicted fairly accurately if the circuit constants are carefully determined in advance. The tests for the determination of the circuit constants are essentially the same for single- or polyphase machines. The American Institute of Electrical Engineers test code² for in-

² AIEE, Test Code For Polyphase Induction Machines, No. 500, (August, 1937).

duction machines gives authoritative methods for all the usual tests on polyphase machines and will not be discussed further.

THE SINGLE-PHASE MOTOR. The performance of the single-phase motor is inferior to the polyphase, but many applications require the single-phase motor and, because of the extent of the single-phase distribution system, they are very common. There are two popular theories for explaining the operation of the single-phase induction motor: (1) the double-revolving field theory³ and (2) the cross-field theory.^{4,5} Both theories lead to the same conclusions and the results of each are identical.

The end result is to pre-determine the performance of the single-phase motor and to do so necessitates the calculation of the current. Many expressions for the current have been derived, but for this investigation the method of the four-terminal network theory was used. The use of this theory for the prediction of motor performance from the equivalent circuit has been presented in a text⁶ and is an excellent treatment of the four-terminal network method as applied to induction motors in general. The four-terminal network theory was recently extended⁷ and improvement was made

³ Puchstein, A. F. and Lloyd, T. C., Alternating-Current Machines, p. 333.

⁴ Ibid., p. 343.

⁵ Robin Beach, "A Physical Conception of Single-Phase Motor Operation", Electrical Engineering, LXIII, (July, 1944) pp. 254-263.

⁶ Tarboux, J. G., Alternating-Current Machinery.

⁷ Harmon Reeder, Jr., "The Four Terminal Network Method of Predicting Single-Phase Induction Motor Performance Characteristics From Test Data", Oklahoma A. & M. College (Thesis), 1948.

in it for predicting the performance of single-phase motors. The necessary developments and equations are presented here briefly.

PREDICTING PERFORMANCE BY FOUR-TERMINAL NETWORK METHOD. Figure 1 shows an exact equivalent circuit for the split-phase single-phase motor and it has been shown that the rotor exciting branch of this circuit is the complicating parameter. The constants of this circuit cannot be found from the usual tests of a single-phase motor, however the circuit of Figure 2, the approximate equivalent circuit, may be solved and the constants determined by test.

The circuit of Figure 2 is a four-terminal network with the input terminals at a and b and the output terminals at c and d. It has been shown that the voltage and current relations in a four-terminal network may be represented as

$$V_1 = AV_2 + BI_2 \quad (1)$$

$$I_1 = CV_2 + DI_2 \quad (2)$$

where the subscript 1 denotes sending end and subscript 2 denotes receiving end.

Referring to Figure 2 and by applying Kirchhoff's Law

$$V_1 = I_1 Z_1 + I_2 Z_2 + V_2 \quad (3)$$

$$I_1 = I_0 + I_2 \quad (4)$$

$$V_0 = I_2 Z_2 + V_2 \quad (5)$$

but

$$I_0 = V_0 Y_0 = Y_0 (I_2 Z_2 + V_2) \quad (6)$$

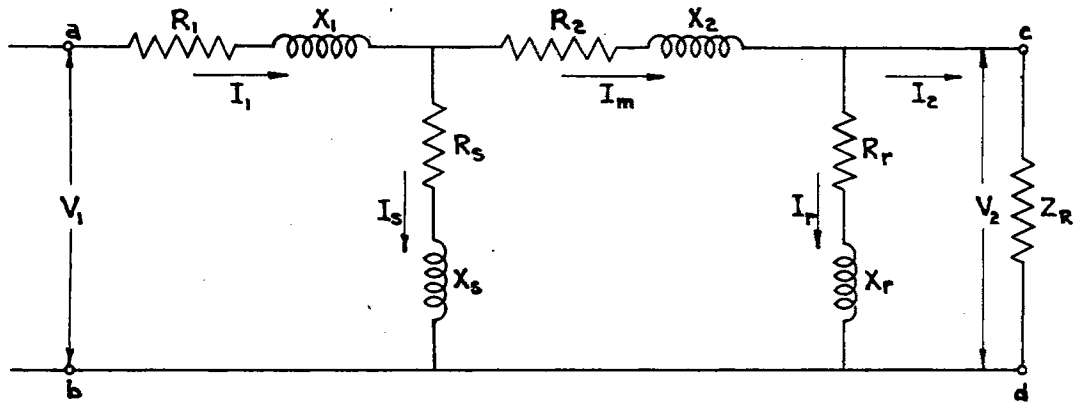


Figure 1. The exact equivalent circuit of the single-phase induction motor.

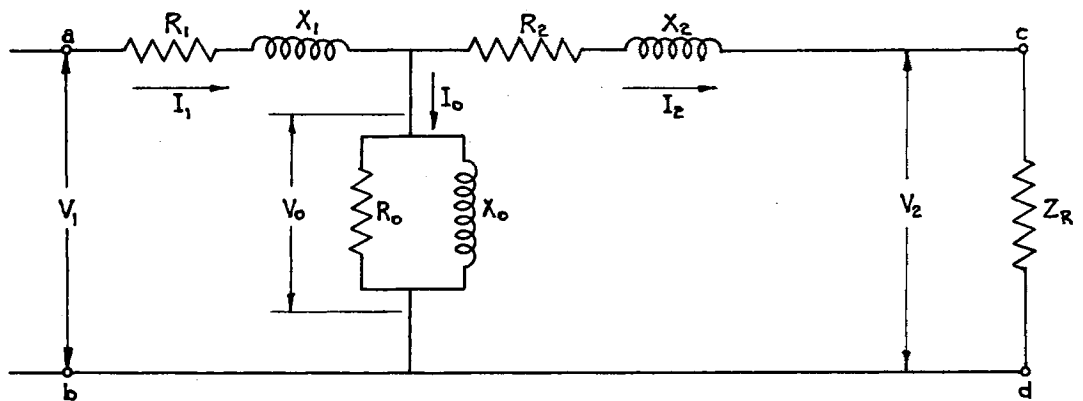


Figure 2. The approximate equivalent circuit of the single-phase induction motor.

Substituting the value of I_o of equation (6) into equation (4),

$$I_1 = Y_o(I_2 Z_2 + V_2) + I_2 \quad (7)$$

and substituting equation (7) into equation (3),

$$\begin{aligned} V_1 &= Z_1[I_2 + Y_o(I_2 Z_2 + V_2)] + I_2 Z_2 + V_2 \\ V_1 &= (Z_1 Y_o + 1)V_2 + (Z_1 + Z_1 Z_2 Y_o + Z_2)I_2 \end{aligned} \quad (8)$$

Comparing equation (8) with equation (1) shows that

$$A = Z_1 Y_o + 1 \quad (9)$$

$$B = Z_1 + Z_1 Z_2 Y_o + Z_2 \quad (10)$$

Rearranging equation (7),

$$I_1 = Y_o V_2 + (Z_2 Y_o + 1)I_2 \quad (11)$$

and comparing equation (11) with equation (2) shows that

$$C = Y_o \quad (12)$$

$$D = Z_2 Y_o + 1 \quad (13)$$

Returning to the two general equations of the equivalent circuit, equations (1) and (2), from equation (1),

$$V_2 = \frac{V_1 - BI_2}{A} \quad (14)$$

but from Figure 2,

$$V_2 = I_2 Z_R \quad (15)$$

and

$$I_2 = \frac{V_1}{AZ_R + B} \quad (16)$$

Substituting this value of current into equation (15),

$$V_2 = \frac{V_1 Z_R}{AZ_R + B} \quad (17)$$

and using equations (6) and (7) in equation (2),

$$I_1 = \frac{V_1 (CZ_R + D)}{AZ_R + B} \quad (18)$$

Multiplying both the numerator and denominator of the right hand side of equation (18) by A, equation (18) becomes,

$$I_1 = \frac{V_1 (ACZ_R + AD)}{A(AZ_R + B)} \quad (19)$$

Adding and subtracting, simultaneously, BCV_1 in the numerator of the right hand side of equation (19) and rearranging gives

$$I_1 = V_1 \frac{C}{A} + \frac{\frac{V_1}{A^2}}{Z_R + \frac{B}{A}} \quad (20)$$

Equation (20) gives the input current in terms of the four-terminal network constants and it may be seen that the following relations hold.

$$\left. \begin{aligned} K' &= C/A && \text{(an admittance) (a)} \\ K' &= 1/A^2 && \text{(a complex ratio) (b)} \\ K &= K_1 + jK_2 = B/A && \text{(an impedance) (c)} \end{aligned} \right\} \quad (21)$$

Using the above constants in equation (20) and grouping Z_R with the resistive component of K (Z_R is considered a variable load resistance whose value depends upon the rotor speed), equation (20) takes the form,

$$I_1 = K'V_1 + \frac{K''V_1}{(Z_R + K_1) + jK_2} \quad (22)$$

Equation (22) represents the input current and consists of a constant component and a variable component. It has been shown that $K'V_1 = I_n$, the no-load current.⁸ This, however, cannot be true unless the machine is driven at synchronous speed, but this error is considered slight compared to the errors in obtaining the test data.

The proper substitutions in equations (21) gives the values of K and $K''V_1$ as

$$K = \frac{B}{A} = Z_o - K'Z_1 \quad (23)$$

$$K''V_1 = V_o - I_n Z_1 (1 - K'Z_1) \quad (24)$$

where $K' = I_n/V_n$ (the no-load voltage is the rated voltage V_1). It should be noted here that, unless otherwise specifically stated, all quantities are vector quantities and must be handled as such.

CALCULATION SHEET. The calculation sheet is a means to an end. It is used for the sole purpose of saving time in the calculation of the performance. In any such procedure it is necessary that the sheet be as simple and straight-forward as possible

⁸ Ibid., p. 24.

and for that reason certain simplifying substitutions are made. In this case the procedure involves actually two sheets; one a preliminary calculation sheet and the other the final or calculation sheet.

The network constants are obtained from the usual no-load and blocked rotor tests. These tests yield the following:

- (a) Blocked rotor test, main winding, I_b , V_b , W_b .
- (b) No-load test, main winding, I_n , V_n , W_n .
- (c) Resistance of the stator winding R_1 ; which may be measured by the "IR drop" method or by the use of a bridge.

From the blocked rotor test the following equivalent values are obtained.

$$Z_e = \frac{V_b}{I_b}$$

$$R_e = \frac{W_b}{I_b^2}$$

The total resistance R_e is also equal to

$$R_e = R_1 + \frac{R_2 R_o}{R_2 + R_o}$$

but since the resistance to the shunt branch is so much greater than R_2 , the resistance of the secondary, the resistance R_e may be assumed to be equal to

$$R_e = R_1 + R_2$$

from which the value of R_2 may be obtained.

Since $Z_e = R_e + jX_e$, it follows that the value of the equiv-

alent reactance may be obtained from the relation

$$X_e = \sqrt{Z_e^2 - R_e^2}$$

and making the assumption that the stator and rotor reactances are equal, then, $X_1 = X_2 = 0.5 X_e$. From this the stator and rotor impedances are readily obtained.

$$\begin{aligned} Z_1 &= R_1 + jX_1 = Z_1/\theta_1 \\ Z_2 &= R_2 + jX_2 = Z_2/\theta_2 \end{aligned}$$

The no-load current is completely determined with the phase angle between it and the applied voltage given. The phase angle θ_n may be found from the relation,

$$\theta_n = \cos^{-1} \frac{W_n}{V_n I_n}$$

and using the applied voltage, $V/90^\circ$, as a reference, the no-load current will be $I_n/90^\circ - \theta_n$.

Applying Kirchhoff's Law to the circuit at no-load, the voltage drop across the shunt or exciting branch is obtained as

$$\begin{aligned} V_o &= V_n - I_n Z_1 \\ &= V_n/90^\circ - (I_n/90^\circ - \theta_n)(Z_1/\theta_1) \end{aligned}$$

and

$$Y_o = \frac{I_n}{V_o}$$

From equivalent circuit theory, the variable load impedance is

$$Z_R = \frac{S^2 R_2}{1-S^2}$$

If it is further assumed that the rotor speed is practically a linear function of the power output, then values of S may be assumed a starting points for the calculation of performance. The equation may be written

$$S = \frac{N_s - k(N_s - N_{fl})}{N_s}$$

where k is the per cent of rated load at which the performance is desired, N_s is the synchronous speed and N_{fl} is the full load speed.

From equation (22) the stator current could be re-written as

$$I_1 = (m + jn) + \frac{r + js}{x + jy} \quad (25)$$

where,

$$m = V_1(C/A) \cos(90^\circ + \beta)$$

$$n = V_1(C/A) \sin(90^\circ + \beta)$$

$$r = (V_1/A^2) \cos(90^\circ + \phi)$$

$$s = (V_1/A^2) \sin(90^\circ + \phi)$$

$$x = Z_R + (B/A) \cos \gamma$$

$$y = (B/A) \sin \gamma$$

$$u = x^2 + y^2$$

After making the proper substitutions and rationalizing equation (25) the expression for the current becomes

$$I_1 = \left[m + \left(\frac{rx + sy}{u} \right) \right] + j \left[n + \left(\frac{xs - ry}{u} \right) \right] \quad (25a)$$

and it will be seen that the phase angle between I_1 and V_1 is

$$\theta = \tan^{-1} \frac{m + \left(\frac{rx + sy}{u}\right)}{n + \left(\frac{xs - ry}{u}\right)}$$

from which the power factor may be found by taking the cosine of the angle θ .

Now, by making a comparison of equations (25) and (25a) with reference to the circle diagram given in Figure 3, it will be seen that since I_1 is made up of the components OP and Pa ,

$$Pa = \left(\frac{rx + sy}{u}\right) + j\left(\frac{xs - ry}{u}\right)$$

and

$$Pe' = \frac{rx + sy}{u}$$

$$ae' = \frac{xs - ry}{u}$$

$$\theta_R = \tan^{-1} \frac{Pe'}{ae'} = \tan^{-1} \left(\frac{rx + sy}{xs - ry}\right)$$

Referring to the triangle Pad , Figure 3, by trigonometry

$$ad = Pa \cos(\theta_R + \phi)$$

$$Pd = Pa \sin(\theta_R + \phi)$$

Referring to the triangle Pfh' , Figure 3, by trigonometry

$$fh' = I_b \cos \theta_b - I_n \cos \theta_n$$

$$Ph' = I_b \sin \theta_b - I_n \sin \theta_n$$

$$Pf = \sqrt{(Ph')^2 + (fh')^2}$$

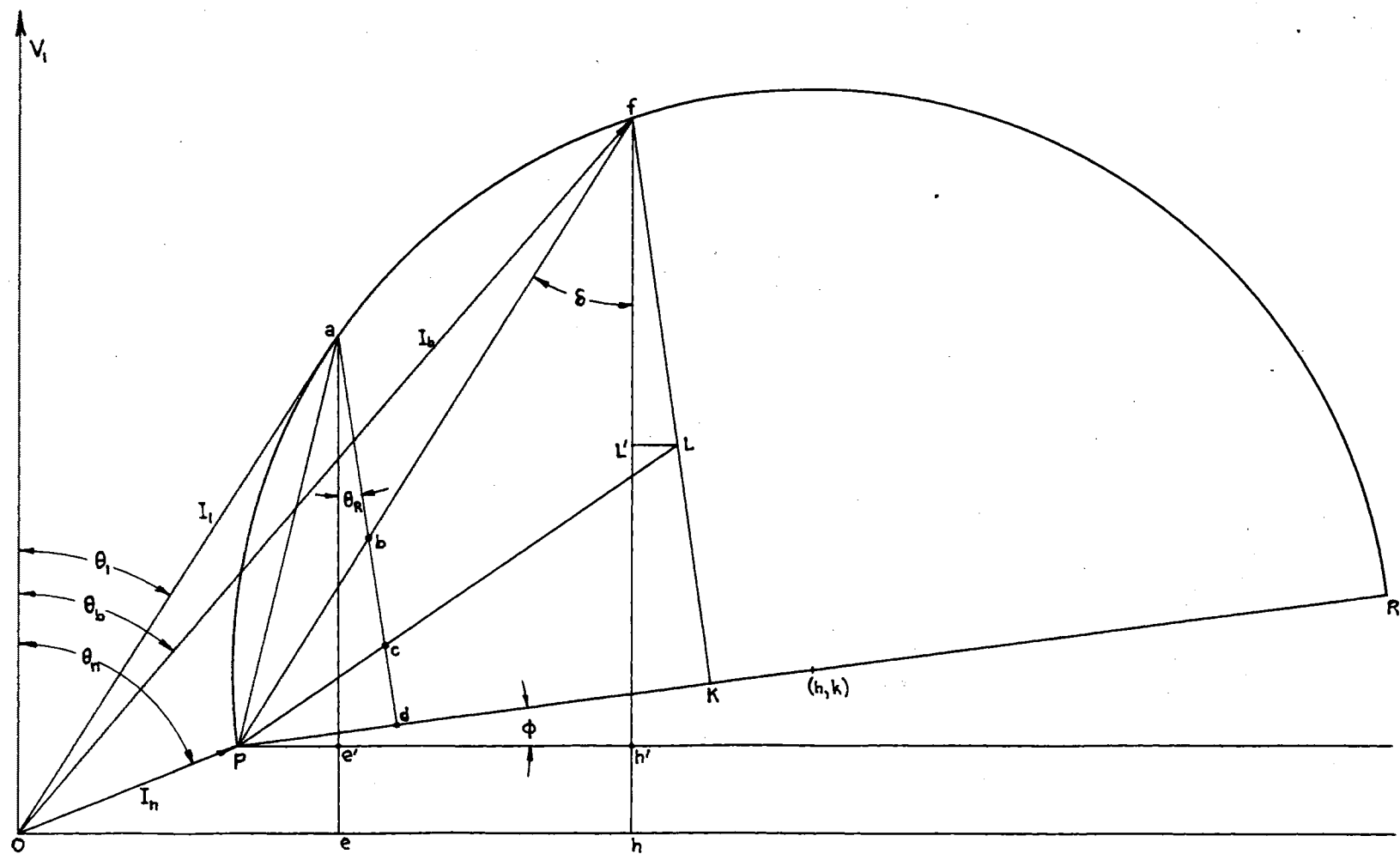


Figure 3. Circle diagram of a single-phase induction motor.

$$\delta = \tan^{-1} \frac{Ph'}{fh'}$$

and

$$fK = Pf \cos(\delta + \phi)$$

It is now necessary that the rotor and stator copper losses be separated. The length $h'L'$ represents the stator copper loss and may be calculated by

$$h'L' = \frac{(I_b^2 - I_n^2)R_1}{V_1} \quad (26)$$

From the circle diagram it will be seen that

$$fL' = fh' - h'L'$$

$$fL = \frac{fL'}{\cos \phi}$$

and the point L is located.

Referring again to the circle diagram, Figure 3, from the triangles Pdb and Pfk, the following relations may be written

$$bd = Pd \cot(\delta + \phi)$$

$$bc = bd \left(\frac{fL}{fK} \right)$$

It follows then, from the above calculations, that the power output, P_o , may be obtained from

$$P_o = V_1 \times ab = V_1(ad - bd) \quad (27)$$

and the torque, in synchronous watts, may be found from

$$T = V_1 \times ac = V_1(ad - cd) \quad (28)$$

The torque in oz-ft may now be obtained from the equation

$$T = \frac{112.8(ad-cd)V_1}{N_s} \quad (29)$$

The power input, watts, is given by the equation

$$P_i = ae \times V_1 \quad (30)$$

and the efficiency, the ratio of the power output to the power input, is obtained from

$$\text{Eff.} = \frac{ab}{ae} \quad (31)$$

The calculation sheets⁹ are given in Figures 4 and 5 and the complete performance characteristics may be obtained and plotted. Considering the assumptions made in the foregoing development, the accuracy is fairly good. All the values obtained from the blocked rotor and no-load tests and given on the calculation sheet of Figure 4 are considered to remain constant for a particular motor.

CIRCLE DIAGRAM. The circle diagram is another popular method of determining the performance of induction machines. The circle diagram method, however, is not as accurate as the method using the calculation sheets. This method is based upon the fact that the equation for the current I_1 , equation (22), is made up of a constant component and a variable component; the variable component being the locus of a circle. The four-terminal network theory may be used to advantage in finding the necessary parameters for

⁹ Ibid., pp. 35-45.

$$\begin{aligned}
V_b &= \underline{\hspace{2cm}} & I_b &= \underline{\hspace{2cm}} & W_b &= \underline{\hspace{2cm}} & R_1 &= \underline{\hspace{2cm}} \\
V_n &= \underline{\hspace{2cm}} & I_n &= \underline{\hspace{2cm}} & W_n &= \underline{\hspace{2cm}} \\
\cos \theta_b &= W_b/V_b I_b = \underline{\hspace{2cm}} & \cos \theta_n &= W_n/V_n I_n = \underline{\hspace{2cm}} \\
\theta_b &= \underline{\hspace{2cm}} & \theta_n &= \underline{\hspace{2cm}} & 90^\circ - \theta_n &= \underline{\hspace{2cm}} \\
Z_e &= V_b/I_b = \underline{\hspace{2cm}} & R_e &= W_b/I_b^2 = \underline{\hspace{2cm}} & R_2 &= R_e - R_1 = \underline{\hspace{2cm}} \\
X_e &= \sqrt{Z_e^2 - R_e^2} = \underline{\hspace{2cm}} & X_1 &= X_2 = 0.5 X_e = \underline{\hspace{2cm}} \\
Z_1/\theta_1 &= R_1 + jX_1 = \underline{\hspace{2cm}} & Z_2/\theta_2 &= R_2 + jX_2 = \underline{\hspace{2cm}} \\
Z_e/\theta_e &= R_e + jX_e = \underline{\hspace{2cm}} & V_o &= V_n - I_n Z_1 = \underline{\hspace{2cm}} \\
Y_o &= I_n/V_o = \underline{\hspace{2cm}} & V_1(C/A)/90^\circ + \beta &= I_n/90^\circ - \theta_n = \underline{\hspace{2cm}} \\
m &= I_n \cos(90^\circ - \theta_n) = \underline{\hspace{2cm}} & n &= I_n \sin(90^\circ - \theta_n) = \underline{\hspace{2cm}} \\
K' &= (I_n/V_n)/-\theta_n = \underline{\hspace{2cm}} & K'Z_1^2 &= K'Z_1^2/2\theta_1 - \theta_n = \underline{\hspace{2cm}} \\
K/\gamma &= Z_e - K'Z_1^2 = \underline{\hspace{2cm}} & y &= K \sin \gamma = \underline{\hspace{2cm}} \\
1 + K'Z_1 &= 1 + K'Z_1/\theta_1 - \theta_n = \underline{\hspace{2cm}} & I_n Z_1 &= I_n Z_1/90 + (\theta_1 - \theta_n) = \underline{\hspace{2cm}} \\
I_n Z_1(1 + K'Z_1) &= \underline{\hspace{2cm}} & K''V_1/90 + \phi &= V_o - I_n Z_1(1 + K'Z_1) = \underline{\hspace{2cm}} \\
r &= K''V_1 \cos(90 + \phi) = \underline{\hspace{2cm}} & s &= K''V_1 \sin(90 + \phi) = \underline{\hspace{2cm}} \\
fh' &= I_b \cos \theta_b - I_n \cos \theta_n = \underline{\hspace{2cm}} & Ph' &= I_b \sin \theta_b - I_n \sin \theta_n = \underline{\hspace{2cm}} \\
Pf &= \sqrt{(fh')^2 + (Ph')^2} = \underline{\hspace{2cm}} & \delta &= \tan^{-1}(Ph')/(fh') = \underline{\hspace{2cm}} \\
\delta + \phi &= \underline{\hspace{2cm}} & \cot(\delta + \phi) &= \underline{\hspace{2cm}} & ry &= \underline{\hspace{2cm}} & sy &= \underline{\hspace{2cm}} \\
fK &= Pf \cos(\delta + \phi) = \underline{\hspace{2cm}} & h'L' &= R_1(I_b^2 - I_n^2)/V_1 = \underline{\hspace{2cm}} \\
A/\phi_e &= Z_1 Y_o + 1 = \underline{\hspace{2cm}} & fL &= (fh' - h'L')/\cos \phi = \underline{\hspace{2cm}}
\end{aligned}$$

Figure 4. Preliminary calculation sheet.

CALCULATION SHEET					
①	Slip				
②	$Z_R = (R_2 S^2) / (1 - S^2)$				
③	$K \cos \gamma + ②$				
④	$y^2 + ③^2$				
⑤	$r \times ③$				
⑥	$⑤ + sy$				
⑦	$⑥ / ④$				
⑧	$m + ⑦$				
⑨	$s \times ③$				
⑩	$⑨ - ry$				
⑪	$⑩ / ④$				
⑫	$n + ⑪$				
⑬	$P_1 = V_1 \times ⑫$				
⑭	$I = [⑧^2 + ⑫^2]^{1/2}$				
⑮	$(90 - \theta) = \tan^{-1} ⑫ / ⑧$				
⑯	$P.F. = \sin (90 - \theta)$				
⑰	$P_a = [⑦^2 + ⑪^2]^{1/2}$				
⑱	$\lambda = \tan^{-1} ⑪ / ⑦$				
⑲	$ad = ⑰ \times \sin (\lambda - \phi)$				
⑳	$Pd = ⑰ \times \cos (\lambda - \phi)$				
㉑	$bd = ⑳ \times \cot (\delta + \phi)$				
㉒	$cd = ㉑ \times (fK - fL) / (fK)$				
㉓	$P_o = V_1 \times (⑲ - ㉑)$				
㉔	$T = 112.8 / N_s (⑲ - ㉒)$				
㉕	$E_{ff.} = ㉓ / ⑬$				
㉖	$H_p. = ㉓ / 746$				

Figure 5. Calculation sheet for a single-phase induction motor.

the construction of the circle diagram.

It has been shown that the diameter of the circle diagram¹⁰ is

$$\text{Diameter} = \frac{K''V_1}{jK_2} \quad (32)$$

and if the voltage reference is taken as jV_1 this expression for the diameter becomes

$$\text{Diameter} = \frac{V_1 K''}{K_2}$$

It was further shown that the coordinates of the circle are

$$h = \frac{V_1}{2K_2 A^2} \cos \phi$$

$$k = \frac{V_1}{2K_2 A^2} \sin \phi$$

also it was shown that the diameter of the circle should be tilted by the angle ϕ , which is easily determined from the network constant A .

As previously mentioned the quantity $K''V_1$ is the no-load current and displaces the center of the circle by the magnitude of the no-load current and in the direction of its angle from the original coordinates, (h,k) . The circle diagram is shown in Figure 3, page 14.

The separation of the rotor and stator copper losses is as follows: it was stated previously that the length

¹⁰ Ibid., pp. 13-16.

$$h'L' = \frac{(I_b^2 - I_n^2)R_1}{V_1} \quad (26)$$

but when the circle is tilted there is no length on the conventional circle diagram which represents the stator copper loss. The point L may be located approximately by the use of the above formula as follows: locate the point L' first, then the horizontal projection to the line representing the in-phase component of the blocked rotor current, such as line Kf on Figure 3, will locate the point L. Some simplification can be made on the above method by making more assumptions, but the simplified formulas hold only for motors rated at less than $\frac{1}{8}$ horsepower.

The procedure for using the circle diagram to predict the motor performance is as follows. The usual blocked rotor and no-load tests are made on the motor and from these tests and the formulas on pages 10 and 11 of this chapter, I_b , I_n , θ_b , θ_n and the network constants Z_e , R_e , R_1 , R_2 , X_e , X_1 , X_2 and Y_o are first obtained. Next the network constants are calculated from equations (9), (10), (12), (13) and equations (21). The diameter of the circle may then be found from equation (32). To find the current assume a value of rotor speed as a fraction of synchronous speed and determine the load impedance.

$$Z_R = \frac{S^2 R_2}{1 - S^2} \quad (33)$$

Then the current I_1 may be found from equation (22). The next step is to determine the stator copper loss as previously indicated and locate the point L from equation (26). The

performance may now be found from the circle diagram as follows:
 for any value of input current, I_1 (the distance Oa on the circle diagram), the performance characteristics are represented by the following quantities from the circle diagram (Figure 3):

$(ae)V_1$ - Power input (watts)	(a)	}-(34)
$(ac)V_1$ - Torque (synchronous watts)	(b)	
$(ab)V_1$ - Power Output (watts)	(c)	
ae/Oa - Power factor (decimal)	(d)	
ab/ae - Efficiency (decimal)	(e)	

CHAPTER II

THE HARMONIC ANALYSIS

The importance of the equivalent circuit has already been shown, however, certain assumptions are necessary in the usual solution of the circuit and certain inaccuracies result from these assumptions. One of the assumptions usually made is that the impressed voltage is sinusoidal. Although great care is taken to minimize the harmonic content of the output of modern alternators, it is practically impossible to remove all harmonics from the alternator output. It has been shown that only the higher odd harmonics are present in a three-phase alternator; the third and multiples of the third harmonic are cancelled, or very nearly so, in the machine.

Figures 6, 7, 8, 9 and 10 show the almost negligible harmonic content of an ordinary commercial supply voltage. These figures also show the harmonic content of the input current when applied to a single-phase motor and it will be seen that the harmonic content is somewhat increased.

Even when the applied voltage is a pure sinusoidal wave, harmonics are produced in a motor. In this case the harmonics are the result of a non-sinusoidal distribution of flux in space which is caused by winding distribution and the limitation of the number of slots per pole. The result is that the counter emf and the primary current are non-sinusoidal waves. Harmonics

produced in this manner are given the name "space" harmonics.

The flux in a motor is the result of current flowing in the stator and rotor and just as the fundamental produces flux so do the harmonics. Thus, it will be seen that the flux in a motor must adapt itself to the time variation of the input voltage and when the impressed voltage contains harmonics these same harmonics appear in the motor. These harmonics are given the name "time" harmonics.

To complete the distortion in the motor, there exists the effects of magnetic saturation which causes the magnetizing current to be non-sinusoidal also.

It is logical, then, that the harmonic content of the input current may contain a larger amount of each harmonic than the impressed voltage. In the analysis that follows no attempt is made to differentiate between the types of harmonics or to determine their effects separately. The purpose is, however, to analyze the effect of the total harmonic content of the input current and impressed voltage as waveforms of these quantities may be obtained easily.

It has been shown¹ that it is theoretically possible to have all the possible harmonics present in a single-phase winding. This is readily seen from the equation

$$n = k - 1 \quad (35)$$

where k is a positive integer exclusive of zero and n is the

¹ Liwschitz-Garick, M. and Whipple, C. C., Electric Machinery Vol. II, p. 496.

order of the harmonic. Equation (35) is independent of the number of phases and when applied to a single-phase motor it yields, for integral slot windings, all digits from 0 to infinity indicating that it is possible to have an infinite number of harmonics in a single phase winding. With proper winding distribution, it is possible to eliminate many, and reduce the remainder, of the possible harmonics.

WAVEFORMS AND DATA. The waveforms presented here were obtained by use of a magnetic oscillograph; the traces being obtained on photographic film. Three tests were used and they include (1) the synchronous speed test, (2) the no-load test and (3) the blocked rotor test. In addition oscillograms were obtained at two speeds slightly less than synchronous.

The synchronous speed test gave the waveforms of voltage and current shown in Figure 6 with rated voltage applied. The waveforms of voltage and current at less than synchronous speed are shown in Figures 7 and 8 and were also obtained with rated voltage applied. Figure 9 is the waveforms of voltage applied to, and the current in the motor at no-load. No appreciable change will be noted in these first four oscillograms with respect to harmonic content. However, the blocked rotor condition, shown in Figure 10, indicated an appreciable increase in the harmonic content of the input current and no increase in the harmonic content of the impressed voltage. Since the blocked rotor waveforms were also obtained at rated voltage it may be assumed that the effects of saturation caused the increase in harmonic content.

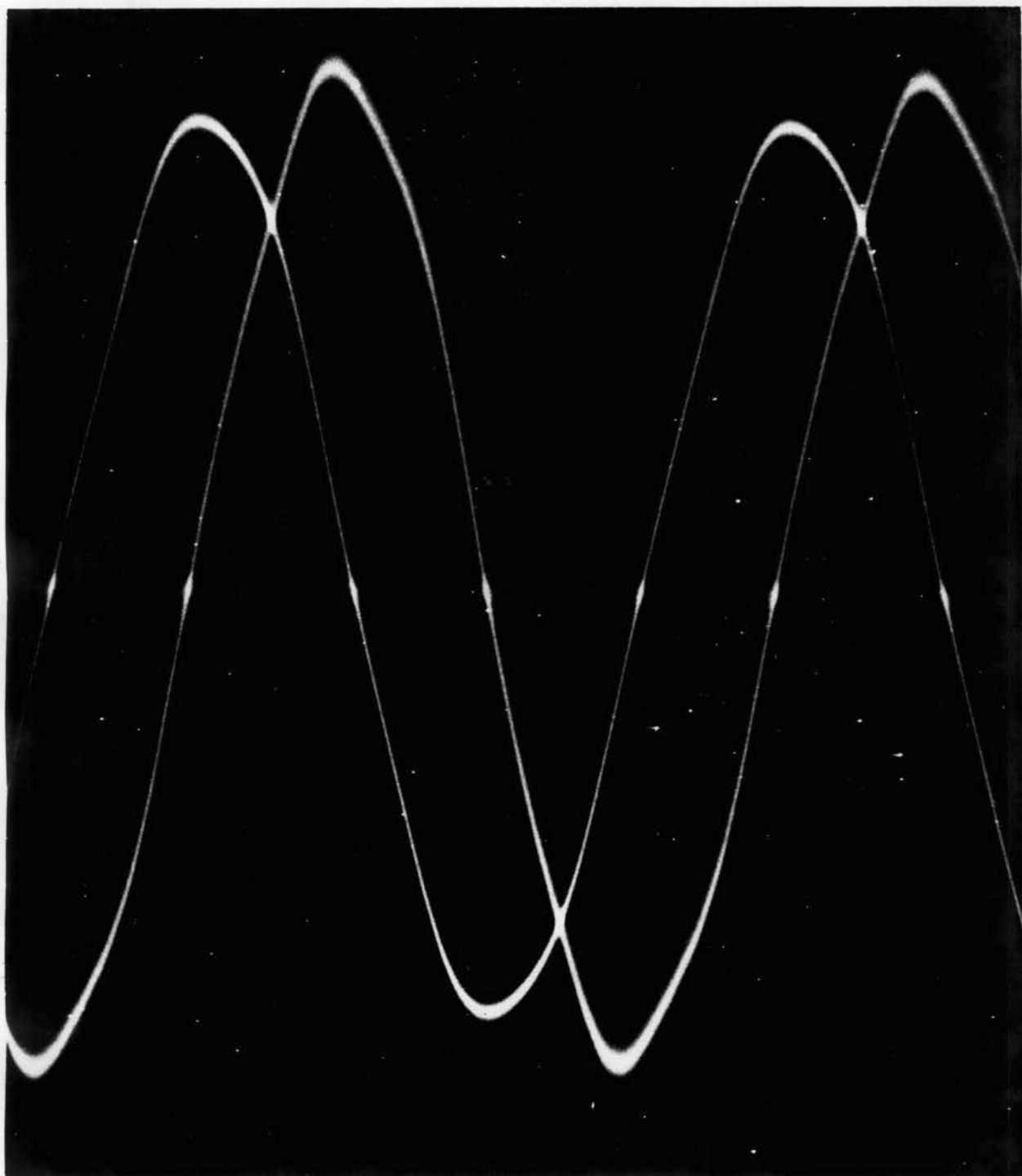


Figure 6. Waveforms of input current and applied voltage in a Century single-phase motor at synchronous speed. The larger waveform represents the input current.

$V = 118.5$ volts, $I = 2.92$ amps, $W = 49$ watts, $N_s = 1800$ rpm

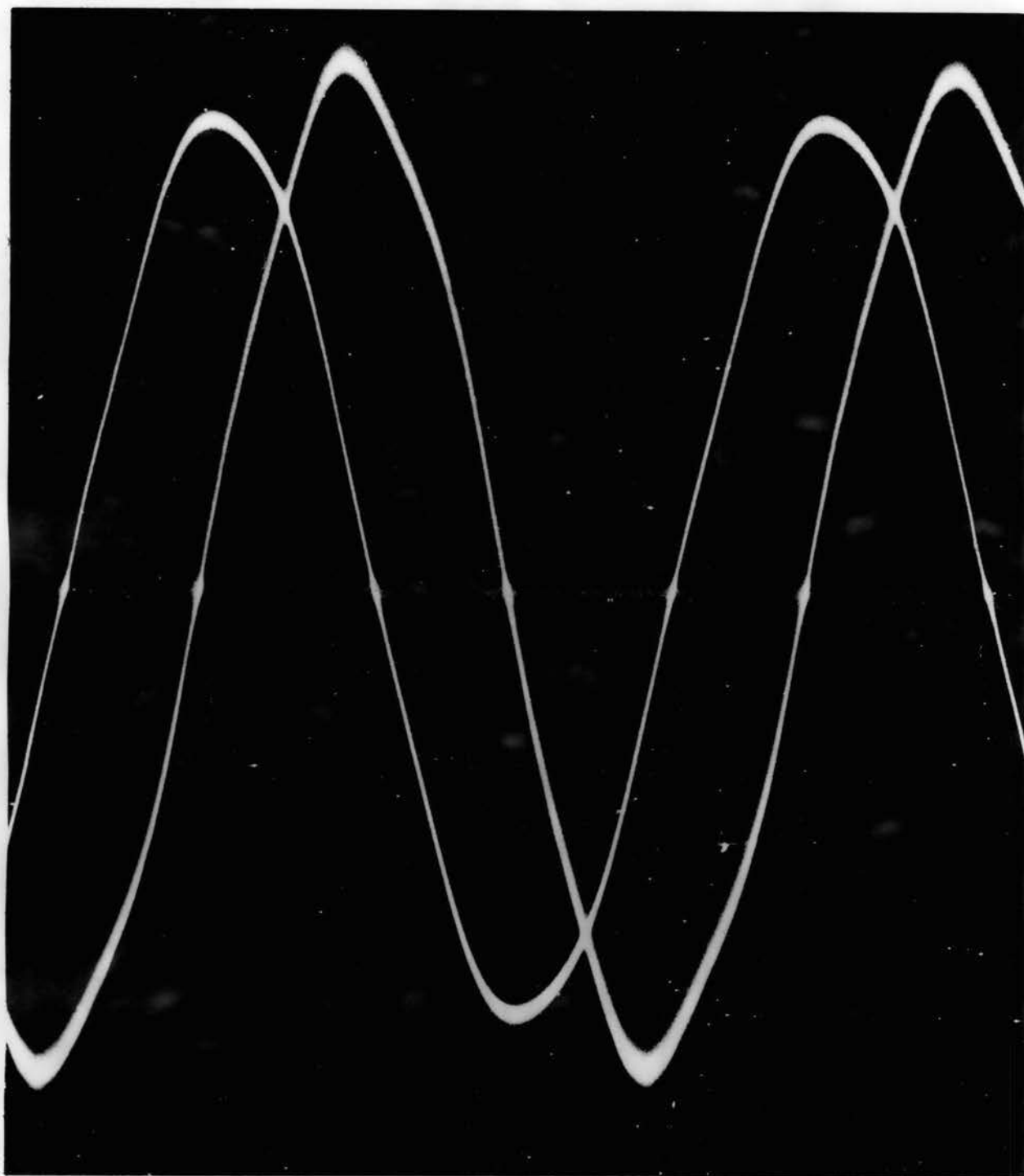


Figure 7. Waveforms of input current and applied voltage in a Century single-phase motor at slightly less than synchronous speed. The larger waveform represents the input current.
 $V = 118.5$ volts, $I = 2.92$ amps, $W = 76$ watts, $N = 1794$ rpm

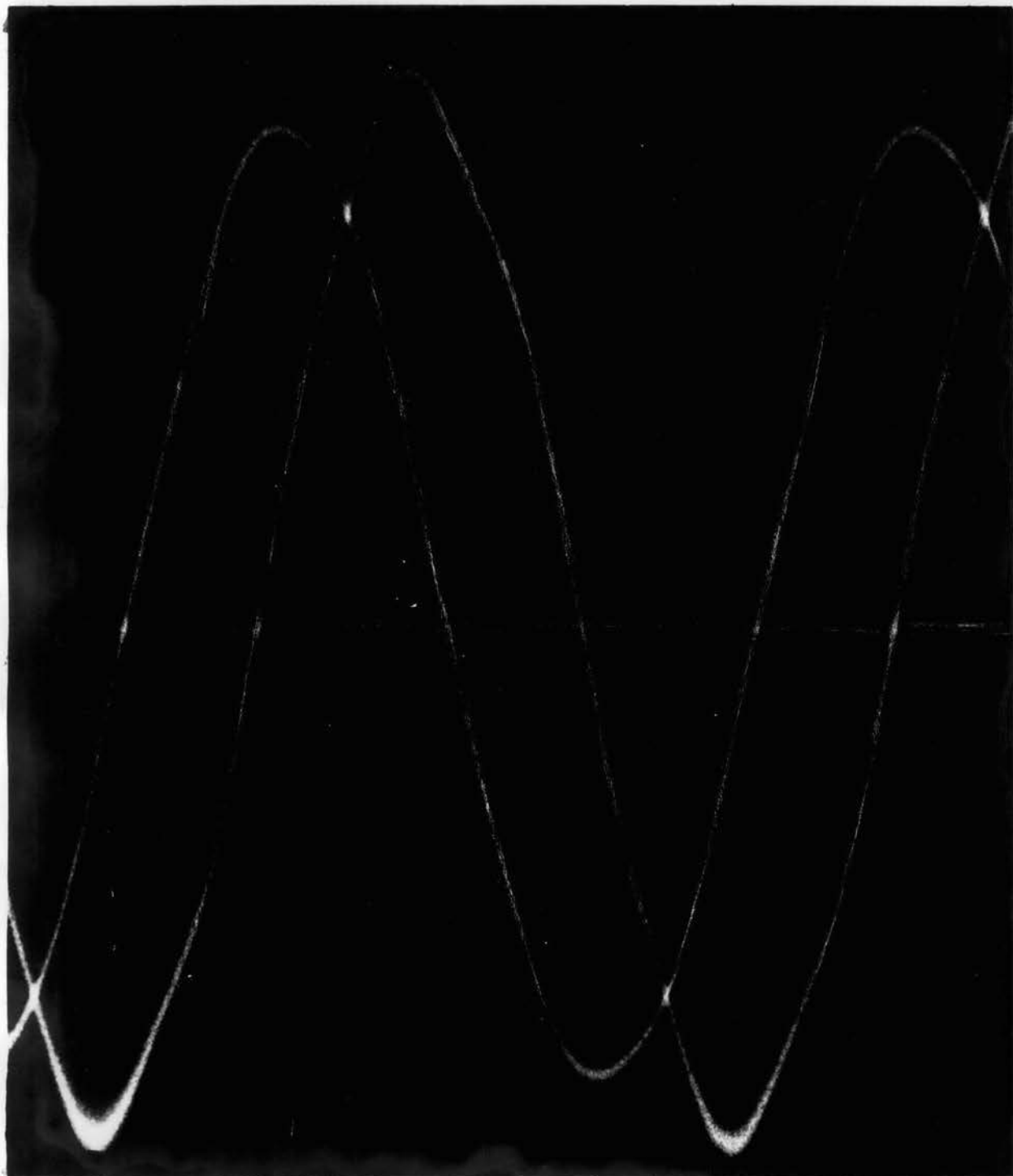


Figure 8. Waveforms of input current and applied voltage in a Century single-phase motor at slightly less than synchronous speed. The larger waveform represents the input current.
 $V = 118$ volts, $I = 2.92$ amps, $W = 91$ watts, $N = 1789$ rpm

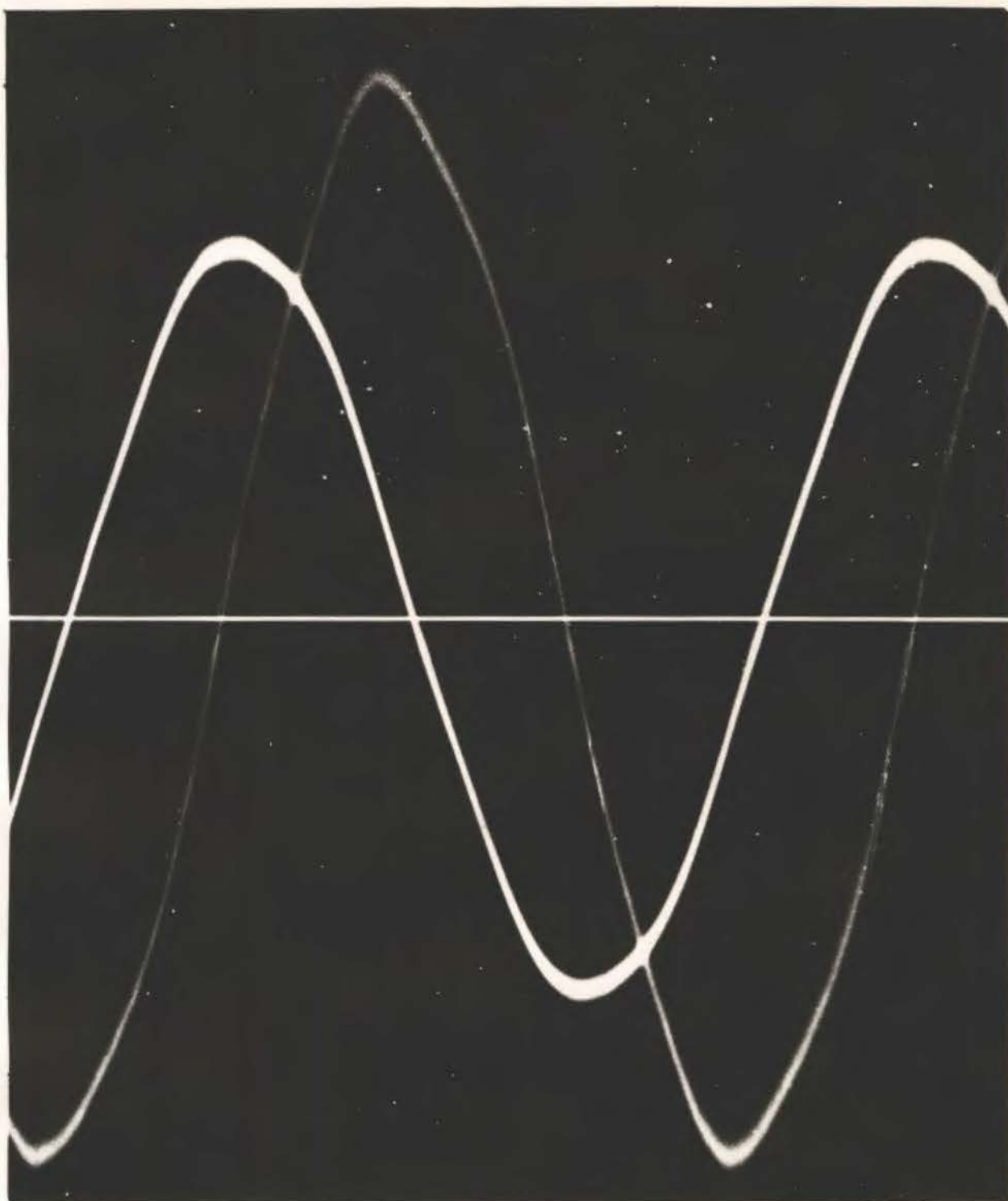


Figure 9. Waveforms of input current and applied voltage in a Century single-phase motor at no-load. The larger waveform represents the input current.

$V = 116$ volts, $I = 2.83$ amps, $W = 66$ watts, $N = 1783$ rpm

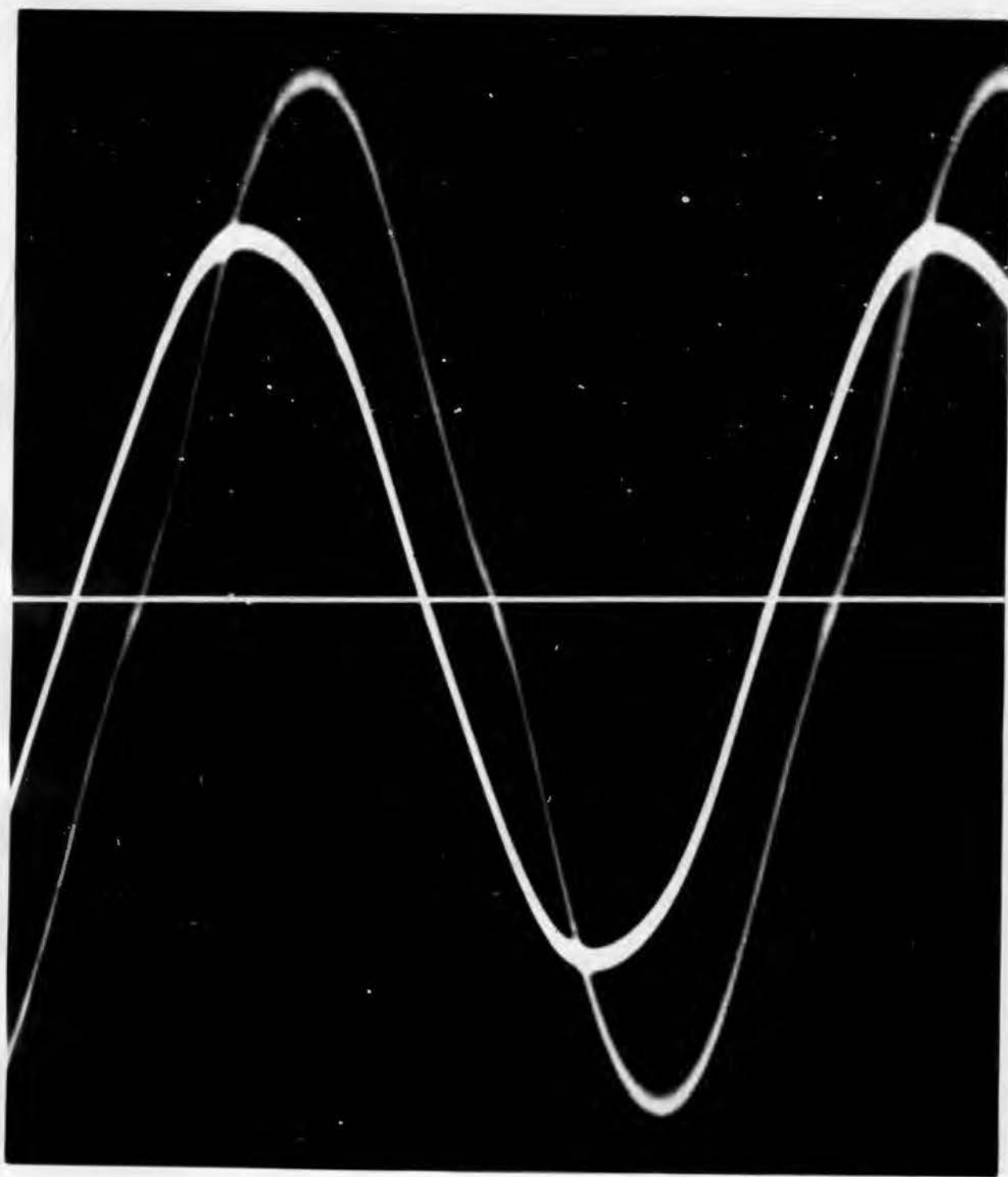


Figure 10. Waveforms of input current and applied voltage in a Century single-phase motor under blocked rotor conditions. Rated voltage was applied. The larger waveform represents the current.

$V = 114$ volts, $I = 22.6$ amps, $W = 2.2$ kw, $N = 0$

ANALYSIS OF FINDINGS ON HARMONICS. A graphical method² was used to determine the harmonics present in the voltage and current waves shown by the oscillograms. Analyzing the waveforms of voltage applied to the motor revealed a very small harmonic content as was expected. In per cent of the fundamental it was found that there was

- (a) 0.26% third harmonic,
- (b) 1.53% fifth harmonic,
- (c) 0.13% seventh harmonic.

These values bear out the theory of negligible third and multiples of the third harmonic present in ordinary commercial supplies and it is seen that the higher odd harmonics are also very small.

At synchronous speed with rated voltage applied the single-phase motor acts as an energy absorption device and the power applied to the motor represents the friction and windage, core and a small stator copper loss. This may be shown as follows: Since, at synchronous speed, $S = 1$, Z_R approaches infinity and I_2 is essentially zero³. In the conventional circle diagram I_n is not equal to I_s (synchronous speed current) as is assumed to be true of an ideal motor, therefore, I_n of Figure 3 could be more accurately represented by the input current at synchronous speed, I_s . Some error might be attributed to the difference in power

² See Appendix for graphical method of harmonic analysis.

³ S is the ratio of rotor speed to the synchronous speed of the motor flux.

Z_R is equal to $(S^2 R_2)/(1-S^2)$, the load which can be represented as a variable resistance.

factor of I_n and I_s . It was observed that there was a change in power factor as the speed was varied from synchronous speed to no-load speed. The data and findings are given in the table below.

rpm	Syn. Speed			No-load 1783
	1800	1794	1789	
Voltage	118.5	118.5	118	116
Current	2.92	2.92	2.92	2.83
Watts	49	76	91	66
Power Factor	0.142	0.220	0.264	0.201

As the speed is varied from synchronous, Z_R is no longer infinity, although it is still much larger than the impedance of the exciting branch, and some current flows in the rotor circuit of the motor supplying some of the rotor core and copper losses. There is also some increase in losses because of the addition of stray load loss caused by the harmonics of the current.

At synchronous speed it was found that the input current contained the following per cent harmonics

- (a) 1.15 % third harmonic,
- (b) 2.16 % fifth harmonic,
- (c) 0.54 % seventh harmonic.

At no-load, the power input represents the rotor and stator copper loss, core loss, friction and windage and stray load loss. Some increase in the harmonic content of the input current was, therefore, expected and found. At no-load there was

- (a) 1.55 % third harmonic,
- (b) 2.35 % fifth harmonic,
- (c) 0.32 % seventh harmonic.

Under blocked rotor conditions, the motor is essentially a transformer with a short-circuited secondary. The losses are the stator and rotor copper losses and core losses. However, there is an increase in the harmonic content of the input current. Under blocked rotor conditions there was found to be

- (a) 6.53% third harmonic,
- (b) 1.82% fifth harmonic,
- (c) 0.60% seventh harmonic.

Since the reactances X_1 and X_2 are found from the blocked rotor data, there will be a definite error because of the greatly increased third harmonic.

FOURIER EQUATIONS OF CURRENT AND VOLTAGE. From the graphical analysis, the Fourier Series equations of the current and voltage may be obtained. At synchronous speed,

$$i = 4.04 \cos(\alpha - 89.4^\circ) + 0.0467 \cos(3\alpha - 169.7^\circ) + \\ 0.0872 \cos(5\alpha - 26.1^\circ) + 0.0218 \cos(7\alpha + 162.95^\circ)$$

At no-load,

$$i = 3.9 \cos(\alpha - 90^\circ) + 0.0605 \cos(3\alpha - 170.35^\circ) + \\ 0.0917 \cos(5\alpha - 43.5^\circ) + 0.0126 \cos(7\alpha + 128.5^\circ)$$

Under blocked rotor conditions,

$$i = 30.35 \cos(\alpha - 87.9^\circ) + 1.98 \cos(3\alpha + 113.3^\circ) + \\ 0.551 \cos(5\alpha + 129.2^\circ) + 0.182 \cos(7\alpha + 83.45^\circ)$$

The voltage under blocked rotor conditions is

$$v = 161.5 \cos(\alpha - 90.4^\circ) + 0.42 \cos(3\alpha + 135^\circ) + \\ 2.46 \cos(5\alpha + 33.6^\circ) + 0.21 \cos(7\alpha + 87.5^\circ)$$

DIAGRAM OF CONNECTIONS. The connection diagram used to obtain the data for this investigation are shown in Figure 12. The direct current motor provided the driving force for obtaining the synchronous speed data and the speed was determined by the use of a stroboscope and target mounted on the coupled shafts of the two machines. Synchronous speed was then easily obtained by adjusting the speed of the direct current motor until the target image remained stationary.

The oscillograph was connected directly to the input of the induction motor for all tests with one exception. Under blocked rotor conditions, because of the high short-circuit current, it was necessary to use a shunt for the current element of the oscillograph.

The testing was greatly facilitated by use of the induction motor test table upon which it was possible to place all motors and instruments. The complete set-up of test equipment is shown in the photograph of Figure 11.

CIRCUIT CONSTANTS CONSIDERING HARMONICS. In the foregoing analysis it was found that under blocked conditions the harmonic content of the current was fairly large; i.e., in per cent of the fundamental, the third harmonic was 6.53%, the fifth harmonic was 1.82% and the seventh harmonic was 0.6%. It follows, then, that since the circuit constants are almost entirely determined from the blocked rotor data, these constants will be in error.



Figure 11. Photograph showing testing table and complete test set-up.

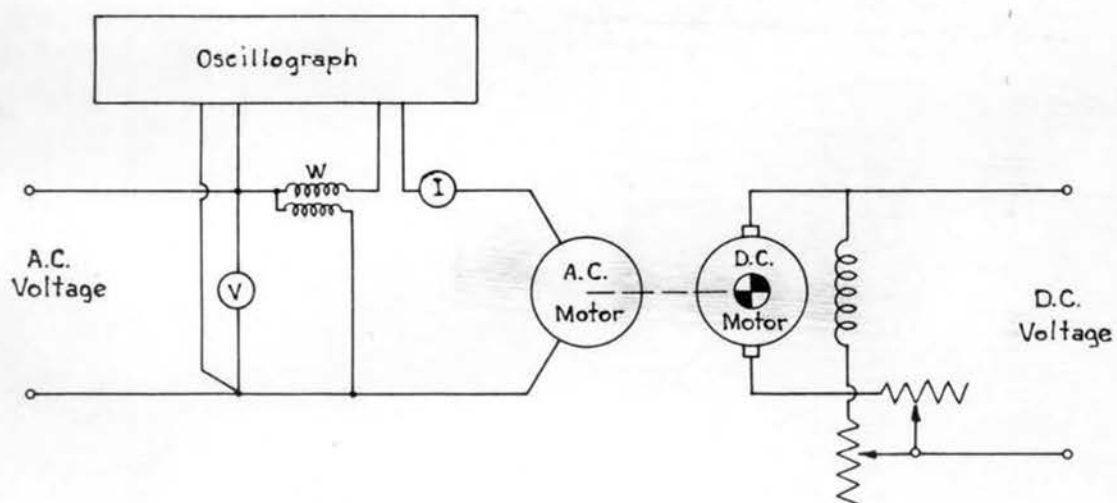


Figure 12. Diagram of connections for obtaining data for harmonic analysis.

It will be remembered that the equivalent impedance is given by

$$Z_e = \frac{V_b}{I_b}$$

and the equivalent resistance is given by

$$R_e = \frac{W_b}{I_b^2}$$

where $R_2 = R_e - R_1$. The equivalent reactance may be found from the equation

$$X_e = \sqrt{Z_e^2 - R_e^2}$$

where $X_1 = X_2 = 0.5 X_e$. The voltage across the exciting branch is given by

$$V_o = V_n - I_n Z_1$$

where V_n is the no-load voltage, I_n is the no-load current and Z_1 is the primary impedance. The admittance of the exciting branch may then be found from the relation

$$Y_o = \frac{I_n}{V_o}$$

The above equations may be corrected for the error caused by harmonics through the use of the harmonic analysis and the following symbols will be used. The subscript "1" will be added to denote the fundamental, the subscript "3" will be added to denote

the third harmonic, etc., and the added subscript "n" will mean the "nth" harmonic.

The equivalent impedance of the motor may be found from

$$Z_{e1} = \frac{V_{b1}}{I_{b1}}$$

The equivalent resistance will be given by the equation below if it is assumed that the input power of the harmonics is negligible. Thus,

$$R_{e1} = \frac{W_b}{I_{b1}^2}$$

where $R_2 = R_{e1} - R_1$. The equivalent reactance may be found from

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$

where $X_{11} = X_{21} = 0.5 X_{e1}$.

Since the reactance of an alternating current circuit is proportional to the frequency, the reactance to each of the harmonics may be found from

$$X_{en} = n X_{e1}$$

where n is the order of the harmonic. The resistance to each harmonic will be slightly greater but not n times as great and the increase in resistance will be considered negligible for this analysis.

It was pointed out that the no-load speed of the single-phase motor is not synchronous for an ideal motor and it was suggested

that the synchronous speed test be used instead. Then,

$$V_{o1} = V_{s1} - I_{s1}Z_{11}$$

where V_{s1} is the applied fundamental voltage and I_{s1} is the fundamental input current, both being obtained from the test at synchronous speed. The admittance of the exciting branch may be found from

$$Y_{o1} = \frac{I_{s1}}{V_{o1}}$$

To determine Y_{o1} , the equivalent circuit at synchronous speed must be analyzed. As previously stated the motor acts as an energy absorption device at synchronous speed and with rated voltage applied the losses represented are (1) the stator copper loss and (2) the core loss. Figure 13a shows the equivalent circuit of the motor at synchronous speed with the usual parallel representation of the exciting branch and the circuit of Figure 13b represents the parallel exciting branch with a series branch needed to calculate R_{o1} and X_{o1} .

The impedance, Z_{oe} , is related to Y_{o1} and

$$Z_{oe} = \frac{1}{Y_{o1}}$$

and

$$Z_{oe} = R_{oe} + j X_{oe}$$

The equivalent series resistance may be found by first finding the equivalent resistance of the circuit of Figure 13b.

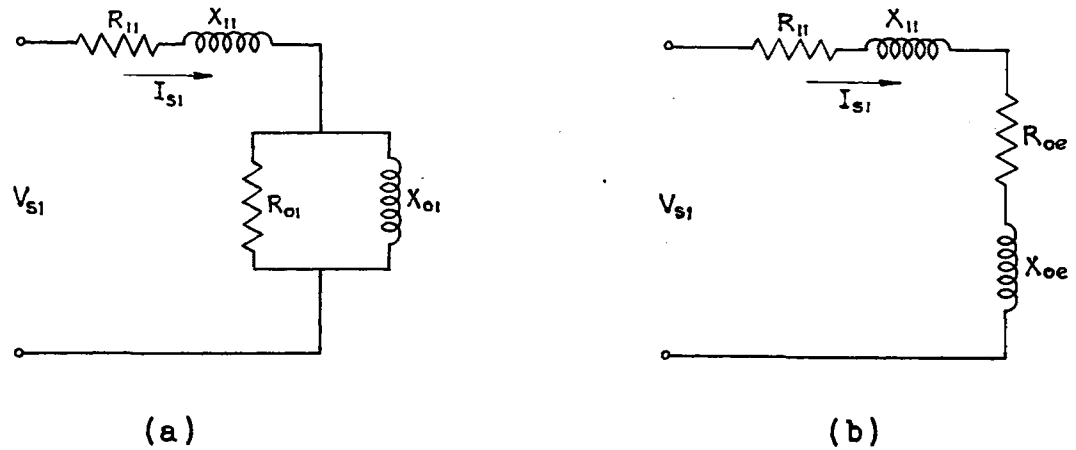


Figure 13. Equivalent circuits of a single-phase split-phase motor when operated at synchronous speed. (a) Parallel representation of the exciting branch. (b) Exciting branch represented by an equivalent series resistance and reactance.

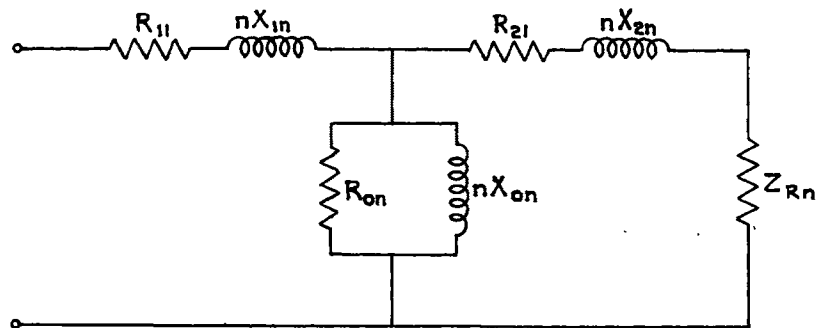


Figure 14. General equivalent circuit which may be used for the fundamental or any of the harmonics, where n is the order of the harmonic.

$$R_{es} = \frac{W_{s1}}{I_{s1}^2}$$

and $R_{oe} = R_{es} - R_1$, then, the equivalent series reactance is given by

$$X_{oe} = \sqrt{Z_{oe}^2 - R_{oe}^2}$$

The parallel reactance X_{ol} and the parallel resistance R_{ol} may be found from the following relations:

$$g = \frac{1}{R_{ol}} = \frac{R_{oe}}{R_{oe}^2 + X_{oe}^2}$$

$$b = \frac{1}{X_{ol}} = \frac{X_{oe}}{R_{oe}^2 + X_{oe}^2}$$

and

$$R_{ol} = \frac{R_{oe}^2 + X_{oe}^2}{R_{oe}}$$

$$X_{ol} = \frac{R_{oe}^2 + X_{oe}^2}{X_{oe}}$$

The reactance to the "nth" harmonic may then be found from

$$X_{on} = n X_{ol}$$

and

$$Y_{on} = \frac{1}{R_{on}} - j \frac{1}{X_{on}}$$

Thus, all of the constants of the equivalent circuit have been determined considering harmonics. The equivalent circuit of Figure 15 may be applied to the fundamental or any of the harmonics present.

SLIP AND TORQUE AT HARMONIC FREQUENCY. In any harmonic analysis, applying to induction motors, it is necessary to determine whether the harmonics aid or oppose the fundamental in producing torque. This relationship may be found by obtaining the angular displacement, along the time axis, of the harmonics with respect to the fundamental throughout the entire load range. This relationship is not constant and varies over the load range which means, for example, that although the third harmonic may aid the fundamental at no-load it may oppose the fundamental at half load and aid it again near full load. There is no method at hand by which this variation may be predicted and any attempt to apply performance calculations to the higher harmonics results in pure conjecture.

It is possible to determine the slip at the various harmonic frequencies. For example: if the third harmonic is considered for a four pole single-phase motor operating on 60 cycles (no-load speed 1782 rpm), then, if the third harmonic is assumed to oppose the fundamental.

$$\text{Synchronous speed} = 3 \times 1800 = 5400 \text{ rpm}$$

$$s = 5400 + 1782 = 7182 \text{ rpm}$$

$$S = \frac{7182}{5400} = 1.33$$

If it is assumed that the third harmonic aids the fundamental

$$s = 5400 - 1782 = 3618 \text{ rpm}$$

$$S = \frac{3618}{5400} = 0.67$$

If some method were available for determining the effect of harmonics in producing torque, the performance of the machine could be determined by considering the fundamental and each harmonic separately in their proper magnitudes, phase relation and frequency and combining the results. However, the overall effect of harmonics may be found without the aid of this analysis as will be shown in the following chapter.

CHAPTER III

RESULTS

The performance was calculated for a $\frac{1}{8}$ horsepower Century split-phase single-phase motor which was chosen because complete performance characteristics were available for comparison of results.

Test data on this machine was obtained as follows:

At Synchronous speed (1800 rpm)

$$V = 118.5 \text{ volts, } I = 2.29 \text{ amps, } W = 49 \text{ watts.}$$

At no-load (1782 rpm)

$$V = 116 \text{ volts, } I = 2.83 \text{ amps, } W = 66 \text{ watts.}$$

Under blocked rotor conditions

$$V = 114 \text{ volts, } I = 22.6 \text{ amps, } W = 2200 \text{ watts.}$$

In addition, the waveforms of voltage and current were obtained as shown in Figures 6, 9 and 10 of Chapter II and analyzed for harmonics. The resistance of the stator winding, R_1 , was measured and found to be 2.42 ohms.

CALCULATION AND COMPARISON OF EQUIVALENT CIRCUIT CONSTANTS.

The sample calculation shown below is the calculation of circuit constants using the fundamental components of current and voltage under blocked rotor conditions. This removes the error caused by higher harmonics in determining these constants.

From the harmonic analysis of voltage and current under

blocked rotor conditions

$$V_{bl} = 114 \text{ volts}, \quad I_{bl} = 21.45 \text{ amps}, \quad W_b = 2200 \text{ watts.}$$

From the harmonic analysis of voltage and current at synchronous speed

$$V_{sl} = 118.5 \text{ volts}, \quad I_{sl} = 2.86 \text{ amps}, \quad W_s = 49 \text{ watts.}$$

Detailed calculations of constants are as follows:

$$\theta_{bl} = \cos^{-1} W_b / V_{bl} I_{bl} = 2200 / (114 \times 21.45) = 26.2^\circ$$

$$\theta_{sl} = \cos^{-1} W_s / V_{sl} I_{sl} = 49 / (118.5 \times 2.86) = 81.7^\circ$$

$$90^\circ - \theta_{sl} = 90^\circ - 81.7^\circ = 8.3^\circ$$

$$Z_{el} = V_{bl} / I_{bl} = 114 / 21.45 = 5.3 \text{ ohms}$$

$$R_{el} = W_b / I_{bl}^2 = 2200 / 21.45^2 = 4.76 \text{ ohms}$$

$$R_2 = R_{el} - R_1 = 2.34 \text{ ohms}$$

$$X_{el} = \sqrt{Z_{el}^2 - R_{el}^2} = \sqrt{5.3^2 - 4.76^2} = 2.34 \text{ ohms}$$

$$X_{11} = X_{21} = 0.5 X_{el} = 0.5 \times 2.34 = 1.17 \text{ ohms}$$

$$Z_{11} = R_1 + j X_{11} = 2.42 + j 1.17 = 2.69 / 25.8^\circ \text{ ohms}$$

$$Z_{21} = R_2 + j X_{21} = 2.34 + j 1.17 = 2.62 / 26.6^\circ \text{ ohms}$$

$$Z_{el} = R_{el} + j X_{el} = 4.76 + j 2.34 = 5.3 / 26.2^\circ \text{ ohms}$$

$$\begin{aligned} V_{ol} &= V_{sl} - I_{sl} Z_{11} = 114 / 90^\circ - (2.86 / 8.3^\circ \times 2.69 / 25.8^\circ) \\ &= -6.37 + j 109.69 = 109.9 / 93.3^\circ \text{ volts} \end{aligned}$$

$$Y_{ol} = I_{sl} / V_{ol} = 2.86 / 8.3^\circ / 109.9 / 93.3^\circ = 0.026 / -85^\circ \text{ mhos}$$

$$Z_{ol} = 1 / Y_{ol} = 1 / 0.026 = 38.5 \text{ ohms}$$

$$R_{es} = W_s / I_s^2 = 49 / 2.86^2 = 5.99 \text{ ohms}$$

$$R_{oe} = 5.99 - 2.42 = 3.57 \text{ ohms}$$

$$X_{oe} = \sqrt{Z_{ol}^2 - R_{oe}^2} = \sqrt{38.5^2 - 3.57^2} = 38.6 \text{ ohms}$$

$$R_{ol} = \frac{R_{oe}^2 + X_{oe}^2}{R_{oe}} = \frac{3.57^2 + 38.6^2}{3.57} = 414 \text{ ohms}$$

$$X_{ol} = \frac{R_{oe}^2 + X_{oe}^2}{X_{oe}} = \frac{3.57^2 + 38.6^2}{38.6} = 38.4 \text{ ohms}$$

Table I gives a comparison of equivalent circuit constants as determined from the usual four-terminal network theory with the equivalent circuit constants as determined by the harmonic analysis using the fundamental components of current and voltage. The per cent error was computed using the values obtained by the method of the harmonic analysis as being the correct values for the equivalent circuit constants.

TABLE I

	Usual Four-Terminal Network Method	Harmonic Analysis	Per Cent* Error
R_1	2.42	2.42	
R_2	1.89	2.34	19.4
R_e	4.31	4.76	10.4
X_1	1.32	1.17	12.8
X_2	1.32	1.17	12.8
X_e	2.64	2.34	12.8
Z_1	$2.76/28.6^\circ$	$2.69/25.8^\circ$	2.6
Z_2	$2.31/34.9^\circ$	$2.62/26.6^\circ$	13.4
Z_e	$5.05/31.4^\circ$	$5.30/26.2^\circ$	4.7
Y_o	$0.0255/-81.5^\circ$	$0.026/-85.0^\circ$	1.9

* Error in the magnitude only.

Table II shows the variation of circuit constants for the various harmonics through the seventh harmonic. This table shows that the reactances increase with the order of the harmonic; i.e., the reactance for a particular harmonic is equal to n times the

reactance of the fundamental. The assumption is made that although the resistances in the equivalent circuit will increase they will not be n times the resistances of the fundamental and the increase will be considered negligible.

TABLE II

	Fundamental	3rd	5th	7th
R_1	2.42	2.42	2.42	2.42
R_2	2.34	2.34	2.34	2.34
R_e	4.76	4.76	4.76	4.76
X_1	1.17	3.51	5.85	8.19
X_2	1.17	3.51	5.85	8.19
X_e	2.34	7.02	11.70	16.38
Z_1	$2.69/25.8^\circ$	$4.26/55.4^\circ$	$6.33/67.5^\circ$	$8.54/73.5^\circ$
Z_2	$2.62/26.6^\circ$	$4.22/56.3^\circ$	$6.30/68.2^\circ$	$8.52/74.05^\circ$
Z_e	$5.30/26.2^\circ$	$8.48/55.85^\circ$	$12.65/67.9^\circ$	$17.06/73.8^\circ$
Y_o	$0.026/-85.0^\circ$	$0.00901/-74.5^\circ$	$0.00574/-65.2^\circ$	$0.00443/-57.1^\circ$

Comparing the values of Table I above shows that the error in calculating the circuit constants is quite high, being over 10% in most cases. Some error is also apparent in the angles computed for the various parameters. These errors will affect the calculated performance of the motor and lead to false conclusions.

COMPARISON OF CALCULATED PERFORMANCE CHARACTERISTICS. To compare the calculated performance using the two sets of constants

previously calculated in this chapter the method of the four-terminal network theory was used throughout.

The performance characteristics were first calculated and plotted using the usual form of the four-terminal network theory. The next set of characteristic curves were calculated and plotted using the circuit constants obtained from the harmonic analysis in conjunction with the test values of voltage, current and power. The third set of curves were obtained by using constants obtained from the harmonic analysis and the fundamental components of voltage and current. Detailed calculations will be shown for the second case only.

Using the values of the circuit constants as calculated on page 42 and listed in Table I, column 3.

$$I_n/90^\circ - \theta_n = 2.83/11.6^\circ$$

$$m = 2.83 \cos 11.6^\circ = 2.77$$

$$n = 2.83 \sin 11.6^\circ = 0.569$$

$$K' = C/A = Y_o/(Z_1 Y_o + 1) = (0.026/-85^\circ)/(1.037/-3.3^\circ) \\ = 0.0251/81.7^\circ$$

$$K'Z_1^2 = 0.0251/-81.7^\circ (2.69/25.8^\circ)^2 = 0.1817/-30.1^\circ$$

$$K/\gamma = Z_o - K'Z_1^2 = 4.76 + j2.34 - 0.1572 + j0.091 \\ = 4.6 + j2.43 = 5.21/27.8^\circ$$

$$y = K \sin \gamma = 2.43$$

$$K \cos \gamma = 4.6$$

$$A/\phi_2 = Z_1 Y_o + 1 = 1.037/-3.3^\circ \quad \phi = -6.6^\circ$$

$$K'' = 1/A^2 = 1/(1.037/-3.3^\circ)^2 = 0.93/6.6^\circ$$

$$K''V_1/90^\circ + \phi = 0.93/6.6^\circ \times 115/90^\circ = 107/96.6^\circ$$

$$r = K''V_1 \cos (90^\circ + \phi) = 107 \cos 96.6^\circ = -12.3$$

$$s = K''V_1 \sin (90^\circ + \phi) = 107 \sin 96.6^\circ = 106.3$$

$$ry = -12.3 \times 2.43 = -29.9$$

$$sy = 106.3 \times 2.43 = 259$$

$$\begin{aligned} fh' &= I_b \cos \theta_b - I_n \cos \theta_n \\ &= 22.6 \cos 31.4^\circ - 2.83 \cos 78.4^\circ = 18.73 \end{aligned}$$

$$\begin{aligned} Ph' &= I_b \sin \theta_b - I_n \sin \theta_n \\ &= 22.6 \sin 31.4^\circ - 2.83 \sin 78.4^\circ = 9.01 \end{aligned}$$

$$Pf = \sqrt{(fh')^2 + (Ph')^2} = \sqrt{18.73^2 + 9.01^2} = 20.8$$

$$\delta = \tan^{-1} (Ph')/(fh') = \tan^{-1} 9.01/18.73 = 25.7^\circ$$

$$\delta + \phi = 25.7^\circ + 6.6^\circ = 32.3^\circ$$

$$\cot (\delta + \phi) = \cot 32.3^\circ = 1.584$$

$$fK = Pf \cos (\delta + \phi) = 20.8 \cos 32.3^\circ = 17.58$$

$$\begin{aligned} h'L' &= R_1(I_b^2 - I_n^2)/V_1 = 2.42(22.6^2 - 2.83^2)/115 \\ &= 10.57 \end{aligned}$$

$$\begin{aligned} fL &= (fh' - h'L')/\cos \phi = (18.73 - 10.57)/\cos 6.6^\circ \\ &= 8.21 \end{aligned}$$

$$(fK - fL)/fK = (17.58 - 8.21)/17.58 = 0.533$$

The above calculations are the calculations outlined in Chapter I, Figure 4, on the preliminary calculation sheet. The remainder of the calculations are made using the calculation sheet of Figure 5, Chapter I, and are shown in Figure 15. The performance curves, plotted from the data of Figure 15, are shown in Figure 16 as solid lines and the dashed curves are the performance characteristics supplied by the manufacturer. It will be seen that the calculated curves do not follow the manufacturer's data perfectly. However,

CALCULATION SHEET					
(1) Slip	0.991	0.986	0.980	0.975	0.969
(2) $Z_R = (R_2 S^2) / (1 - S^2)$	127.6	78.3	56.2	44.4	36.0
(3) $K \cos \gamma + (2)$	132.2	82.9	60.8	49.1	40.6
(4) $y^2 + (3)^2$	17450	6875	3700	2410	1650
(5) $r \times (3)$	-1626	-1020	-748	-604	-499
(6) (5) + sy	-1367	-761	-489	-345	-240
(7) (6) / (4)	-0.078	-0.111	-0.132	-0.143	-0.146
(8) $m + (7)$	2.69	2.66	2.64	2.63	2.62
(9) $s \times (3)$	14050	8830	6460	5220	4315
(10) (9) - ry	14080	8850	6510	5250	4345
(11) (10) / (4)	0.806	1.289	1.76	2.18	2.635
(12) $n + (11)$	1.375	1.858	2.329	2.749	3.204
(13) $P_1 = V_1 \times (12)$	158.1	213.5	268.0	316.1	368.5
(14) $I = [(8)^2 + (12)^2]^{1/2}$	3.02	3.24	3.52	3.80	4.14
(15) $(90 - \theta) = \tan^{-1} (12) / (8)$	27.1°	34.9°	41.4°	46.25°	50.7°
(16) P.F. = $\sin (90 - \theta)$	0.456	0.573	0.662	0.722	0.774
(17) $P_a = [(7)^2 + (11)^2]^{1/2}$	0.810	1.293	1.762	2.185	2.636
(18) $\lambda = \tan^{-1} (11) / (7)$	95.6°	94.9°	94.3°	93.8°	93.2°
(19) $ad = (17) \times \sin (\lambda - \phi)$	0.810	1.292	1.761	2.183	2.63
(20) $Pd = (17) \times \cos (\lambda - \phi)$	0.014	0.038	0.071	0.107	0.156
(21) $bd = (20) \times \cot (\delta + \phi)$	0.022	0.061	0.112	0.169	0.248
(22) $cd = (21) \times (fK - fL) / (fK)$	0.012	0.032	0.060	0.090	0.132
(23) $P_o = V_1 \times (19 - 21)$	90.7	141.6	189.7	231.5	274.0
(24) $T = 112.8 / N_s (19 - 22)$	0.05	0.079	0.107	0.131	0.156
(25) Eff. = (23) / (13)	0.573	0.663	0.708	0.733	0.744
(26) $H_p = (23) / 746$	0.122	0.19	0.255	0.311	0.368

Figure 15. Performance calculations using constants from the harmonic analysis and test values.

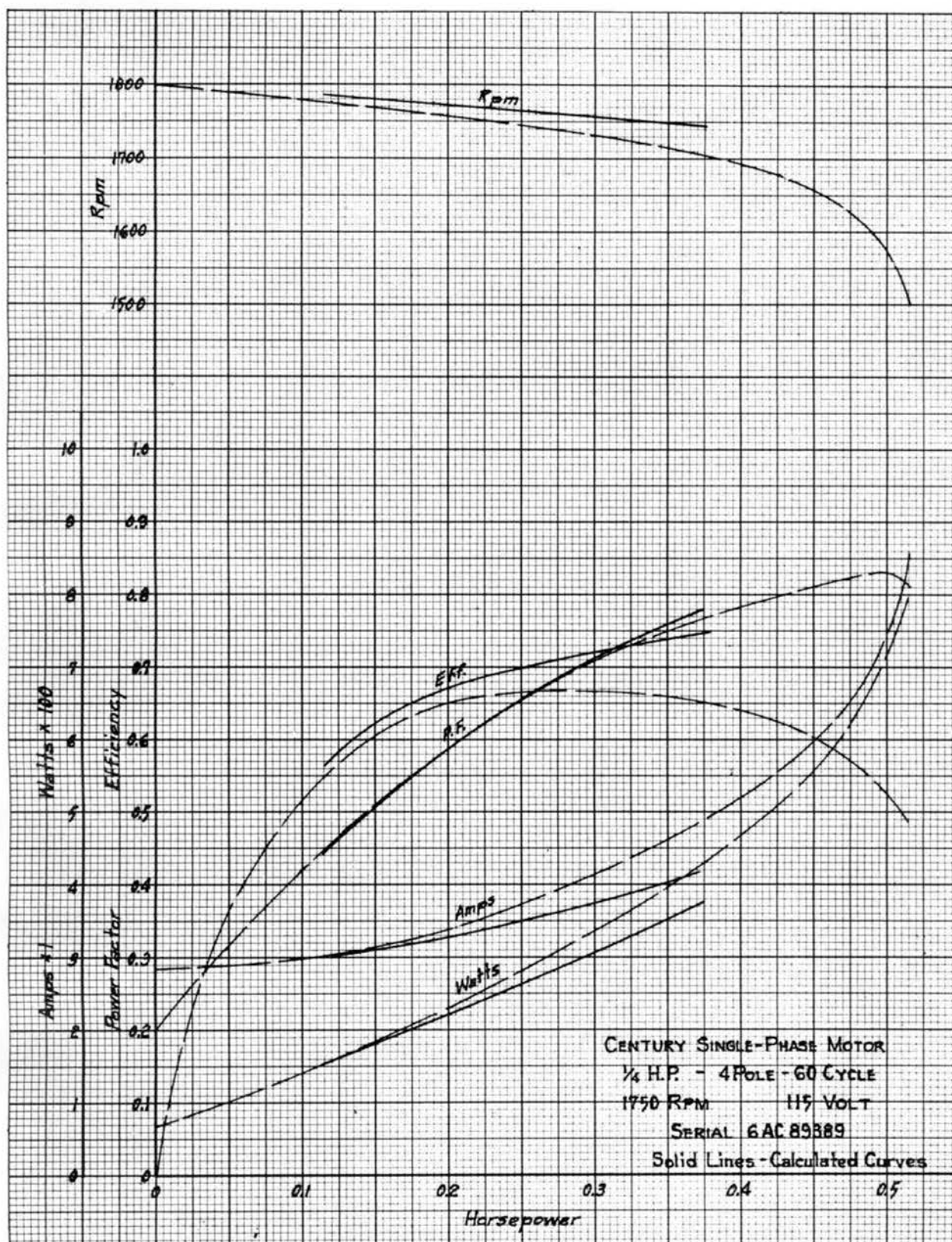


Figure 16. Comparison of calculated performance curves with the manufacturers curves of performance. The calculated curves were obtained by use of the four-terminal network method using the constants obtained from the harmonic analysis with the test values of current and voltage.

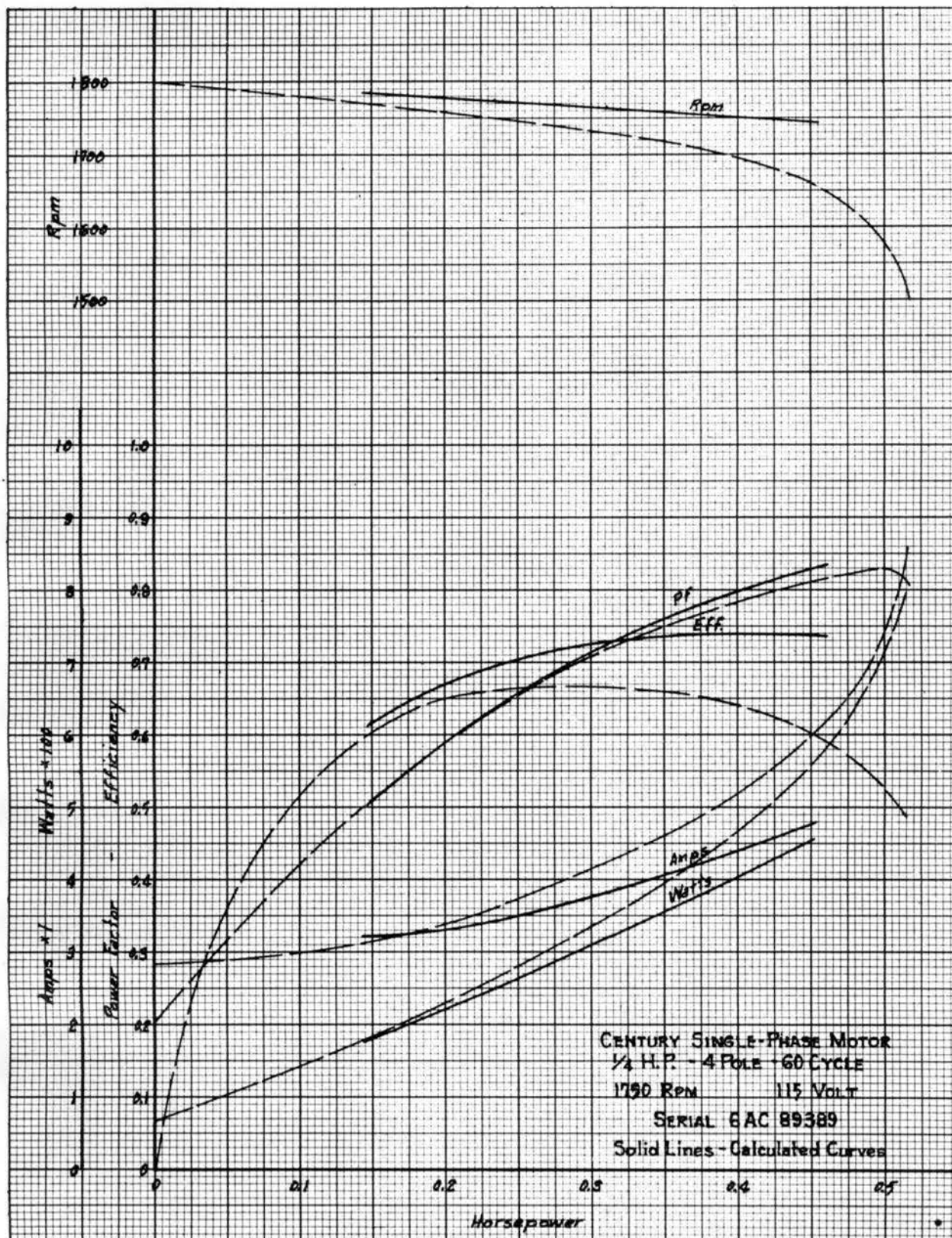


Figure 17. Comparison of calculated performance curves with the manufacturers curves of performance. The calculated curves were obtained from the usual four-terminal method.

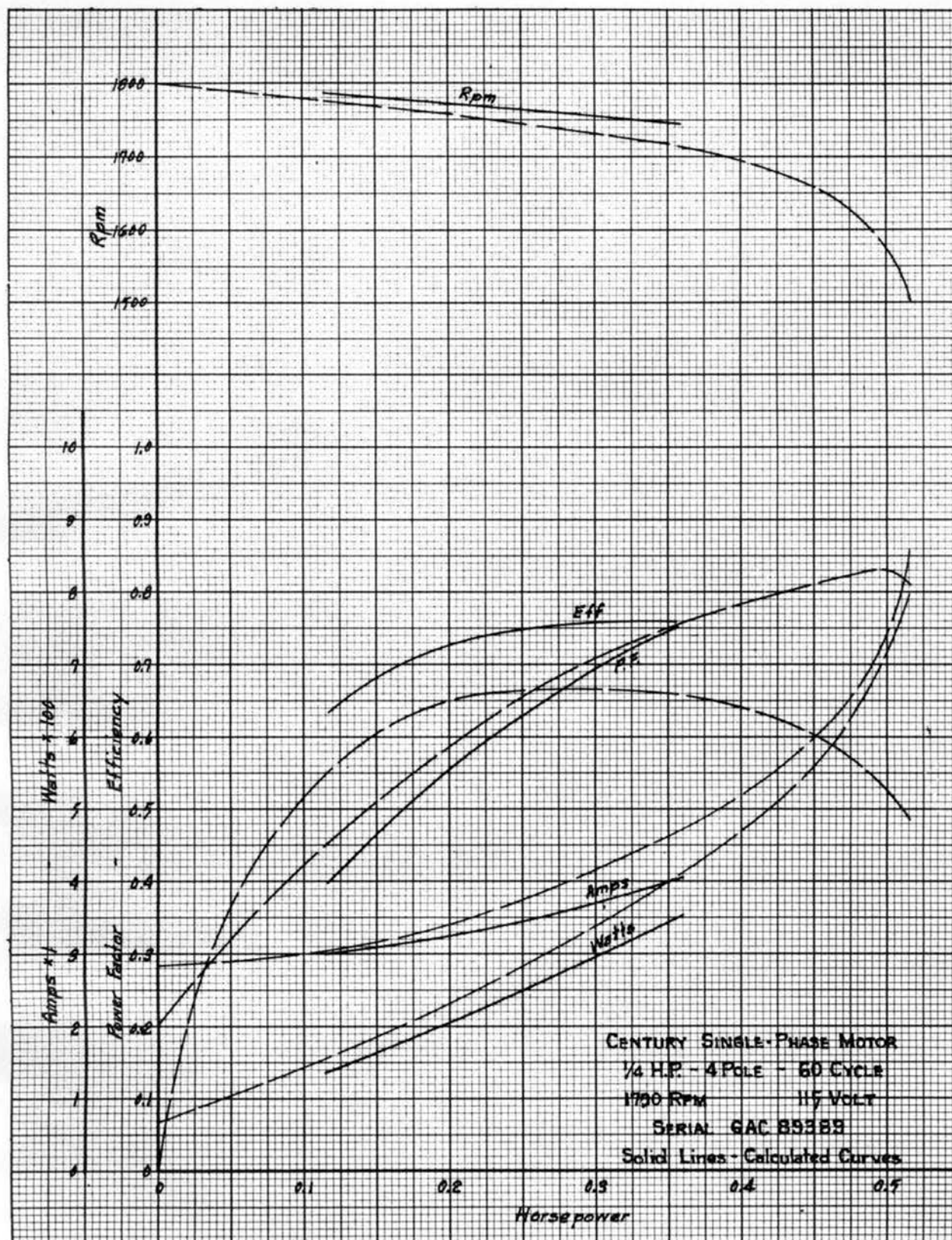


Figure 18. Comparison of calculated performance curves with the manufacturers curves of performance. The calculated curves were obtained by use of the four-terminal network method using the constants obtained from the harmonic analysis with the fundamental components of current and voltage.

the large errors are considerably above full load.

Figure 17 gives the calculated performance by the usual four-terminal network theory (solid lines) as compared to the manufacturer's performance data. The calculated curves do not follow the manufacturer's data and the larger error is above full load.

Figure 18 shows the calculated performance using the fundamental components of voltage and current with the circuit constants determined from the harmonic analysis. Here again the calculated curves do not fit the manufacturer's performance curves and further, do not fit as well as in Figures 16 and 17. The curves of Figure 18 are given here to show that the harmonics do affect the performance of the motor.

A comparison, at full load, of the per cent error in performance with reference to the manufacturer's performance data is shown below. This comparison is made between the data of Figures 16 and 17.

Per Cent Error

	Usual Four-Terminal Network Theory	Harmonic Analysis
Watts	6.4%	6.4%
Current	6.2%	5.9%
Power Factor	0.46%	0 %
Efficiency	6.0%	4.5%

CONCLUSIONS. Single-phase motor theory is a difficult theory to master. Similarly, the performance of single-phase motors is difficult to predict and in many cases the results

obtained from usual methods are unsatisfactory. This is readily seen when the many assumptions used to calculate the performance characteristics are pointed out. However, some method is necessary for predicting single-phase motor performance and the four-terminal network method is presented in this investigation as being no worse and in many cases a little better than most methods. It is evident, then, that the work is not complete. Much work is left to be done. Perhaps, with further investigation, other improvements can be made by use of new tests and new calculation procedures. With the improvement in prediction methods must come improvement of single-phase motor theory. This does not mean that existing methods of analyzing single-phase motors are not good. These methods are very good in so far as illustrating what happens inside the motor.

The results of this investigation show that considerable error is introduced into the calculation procedure by the usual method of determining the equivalent circuit constants. A method for correcting for this error is presented in this thesis. Since the circuit parameters are determined from the blocked rotor test and since the effect of harmonics is greatest for this test, it follows that considerable error in the circuit parameters can be expected using usual methods of calculation. To correct for this error it is necessary that the voltage and current waves be analyzed for harmonics. The corrected values of circuit constants may then be calculated from the fundamental components of voltage and current. Table I of this chapter shows the magnitude of this error to be of the order of 10 per cent. The error is not con-

finer to the magnitudes of the circuit parameters. A study of Table I will show that some error exists in the computed angles of these parameters. The conclusion must not be drawn that these corrections are always necessary. The necessity of such corrections will depend upon the accuracy desired, the instruments available and the harmonic content of the source.

It is also necessary to note that the improvement, gained in the harmonic analysis to determine circuit parameters, was lost in the method of calculation to determine the performance. At this point, to illustrate the overall improvement made, the table of page 51 will be repeated.

	Per Cent Error	
	Usual Four-Terminal Network Theory	Harmonic Analysis
Watts	6.4%	6.4%
Current	6.2%	5.9%
Power Factor	0.46%	0 %
Efficiency	6.0%	4.5%

The overall improvement will be seen to be extremely small in most cases, but the errors remaining to be corrected for are small also.

This investigation was conducted to determine the magnitude of the errors introduced in making the usual performance calculations and not in an effort to improve upon existing methods of predicting motor performance. It is probable that other schemes can be devised to take into account the affects of harmonics. Perhaps, the four-terminal network method can be improved. On the

other hand, the search for an entirely new scheme for predicting motor performance may be necessary.

The investigation of the effect of harmonics on single-phase motor performance is important to the general knowledge of the entire subject of single-phase motors. The harmonic method of calculating the circuit parameters is presented as an improvement over conventional methods. To logically neglect the affects of harmonics, it is necessary to know how much is being neglected and what errors are being introduced.

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APPENDIX

WAVEFORM ANALYSIS

The waveform to be analyzed is that of Figure 10, page 29, and was obtained by means of an oscillograph. The diagram of connections is shown in Figure 12, page 34. A study of the oscillograms of Chapter II revealed that the wave of stator input current contains a definite harmonic and that this wave is symmetrical with the time axis, thus, it is safe to assume that only odd harmonics are present and, therefore, only one-half cycle need be analyzed. It will be remembered that if even order harmonics are present in a wave, the wave will not be symmetrical with the x or time axis.

The general equation of the Fourier Series is

$$y = A_0 + A_1 \cos \alpha + A_3 \cos 3\alpha + A_5 \cos 5\alpha + A_7 \cos 7\alpha + \dots \\ + B_1 \sin \alpha + B_3 \sin 3\alpha + B_5 \sin 5\alpha + B_7 \sin 7\alpha + \dots$$

however, for any alternating current wave that is symmetrical with respect to the horizontal time axis $A_0 = 0$, hence, A_0 can be omitted from the expression given above having only odd harmonics.

As shown in Figure 19, the enlarged wave was traced and the time axis (from 0° to 180°) is divided into 36 parts, divisions of 5° , and the magnitude of the ordinates at the midpoint of each

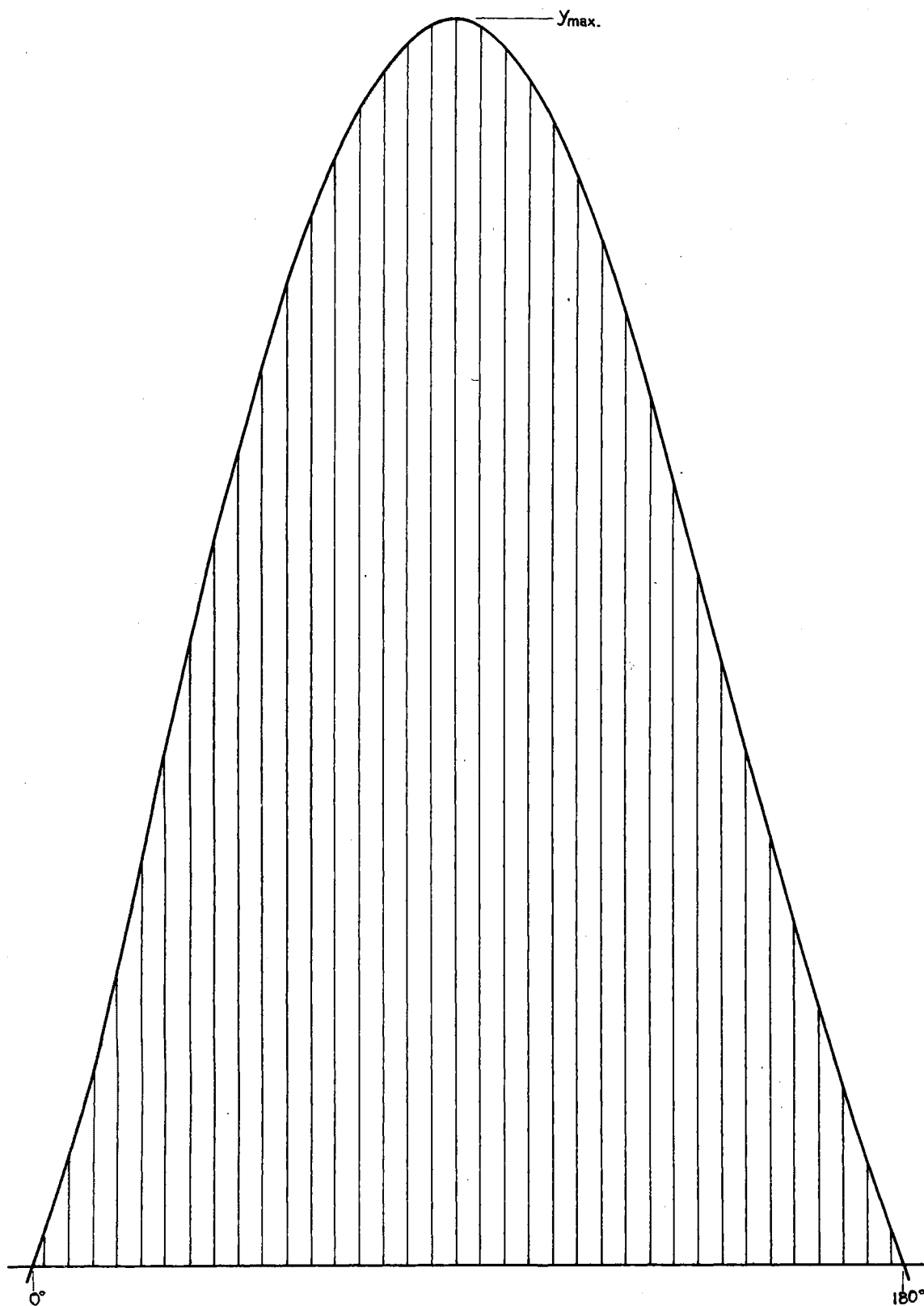


Figure 19. Trace of enlarged wave of blocked rotor current to be analyzed for harmonics. Ordinates (y) were erected and measured at each 5 degree interval starting with 2.5 degrees. The maximum value of y is 48.7 units.

section is recorded (measured ordinate y of Tables). Values of " α ", " $\sin \alpha$ ", " $\cos \alpha$ ", " y ", " $y \sin \alpha$ " and " $y \cos \alpha$ " are tabulated in parallel columns.

The resultant summation of products of " $y \sin \alpha$ " and " $y \cos \alpha$ " are determined separately. The value of B_1 may then be found by taking the algebraic sum of the positive and negative parts of " $y \sin \alpha$ " and dividing by 18. Similarly, the value of A_1 may be found by taking the algebraic sum of the positive and negative parts of " $y \cos \alpha$ " and dividing by 18.

$$B_1 = \frac{y \sin \alpha}{36} \times 2 = \frac{y \sin \alpha}{18}$$

$$A_1 = \frac{y \cos \alpha}{36} \times 2 = \frac{y \cos \alpha}{18}$$

The values of B_3 , A_3 , B_5 , A_5 , B_7 and A_7 are found in the similar manner using " $y \sin 3 \alpha$ ", " $y \cos 3 \alpha$ ", " $y \sin 5 \alpha$ ", " $y \cos 5 \alpha$ ", " $y \sin 7 \alpha$ " and " $y \cos 7 \alpha$ " respectively.

The series can be further simplified as follows, let

$$A_n = C_n \cos \theta_n \quad (1)$$

$$B_n = -C_n \sin \theta_n \quad (2)$$

Substituting these values for A_n and B_n in the series given above,

$$y = C_1 \cos \alpha \cos \theta_1 + C_3 \cos 3 \alpha \cos \theta_3 + \dots \dots \dots$$

$$-C_1 \sin \alpha \sin \theta_1 - C_3 \sin 3 \alpha \sin \theta_3 - \dots \dots \dots$$

and collecting terms gives the general equation of the harmonics

$$y = C_n (\cos n\alpha \cos \theta_n - \sin n\alpha \sin \theta_n)$$

which is equal to

$$y = C_n [\cos (n\alpha + \theta_n)] \quad (3)$$

The value of C_n is obtained as follows: square equation (1) and (2) and add, then,

$$A_n^2 + B_n^2 = C_n^2 (\sin^2 \theta_n + \cos^2 \theta_n) = C_n^2$$

from which

$$C_n = \sqrt{A_n^2 + B_n^2}$$

The value of θ_n may be obtained as follows: dividing equation (2) by equation (1),

$$\tan \theta_n = \frac{-B_n}{A_n}$$

There will, of course, be two values of θ_n determined by the above equation, but an inspection of equations (1) and (2) will indicate which is to be used.

The Fourier Series equation will then be of the form

$$y = C_1 \cos(\alpha + \theta_1) + C_3 \cos(3\alpha + \theta_3) + C_5 \cos(5\alpha + \theta_5) \\ + C_7 \cos(7\alpha + \theta_7) + \dots \dots \dots (4)$$

In this analysis,

y is proportional to i (the instantaneous current)

A_0 is zero (as previously shown)

C will be replaced by I (the amplitude of the harmonic).

TABLE I
FUNDAMENTAL

sin x	Products (y sin x)		Angle x	Meas. Ord. y	cos x	Products (y cos x)	
	Pos.	Neg.				Pos.	Neg.
.04362	0.06		2.5	1.30	.99905	1.30	
.13053	0.56		7.5	4.30	.99144	4.26	
.21644	1.70		12.5	7.85	.97630	7.66	
.30071	3.48		17.5	11.60	.95372	11.05	
.38268	6.07		22.5	15.85	.92388	14.65	
.46175	9.19		27.5	19.90	.88701	17.65	
.53730	13.10		32.5	24.40	.84339	20.55	
.60876	17.25		37.5	28.30	.79335	22.45	
.67559	21.55		42.5	31.90	.73728	23.50	
.73728	25.90		47.5	35.20	.67559	23.80	
.79335	30.50		52.5	38.45	.60876	23.40	
.84339	34.60		57.5	41.00	.53730	22.00	
.88701	38.30		62.5	43.20	.46175	19.90	
.92388	41.70		67.5	45.10	.38268	17.27	
.95372	44.40		72.5	46.60	.30071	13.98	
.97630	46.50		77.5	47.70	.21644	10.30	
.99144	48.00		82.5	48.40	.13053	6.32	
.99905	48.70		87.5	48.70	.04362	2.12	
.99905	48.40		92.5	48.40	-.04362		2.11
.99144	47.10		97.5	47.60	-.13053		6.21
.97630	45.20		102.5	46.35	-.21644		10.01
.95372	42.60		107.5	44.70	-.30071		13.40
.92388	39.25		112.5	42.50	-.38268		16.28
.88701	35.50		117.5	40.05	-.46175		18.50
.84339	31.35		122.5	37.20	-.53730		20.00
.79335	26.95		127.5	34.00	-.60876		20.70
.73728	22.55		132.5	30.60	-.67559		20.70
.67559	18.25		137.5	27.00	-.73728		19.90
.60876	14.30		142.5	23.50	-.79335		18.65
.53730	10.85		147.5	20.20	-.84339		17.05
.46175	7.65		152.5	16.60	-.88701		14.73
.38268	5.09		157.5	13.30	-.92388		12.30
.30071	2.97		162.5	9.90	-.95372		9.44
.21644	1.49		167.5	6.90	-.97630		6.73
.13053	0.52		172.5	4.00	-.99144		3.96
.04362	0.06		177.5	1.35	-.99905		1.35
Sum of Products	831.64					262.16	232.02
	831.64					30.14	

TABLE II
THIRD HARMONIC

sin 3x	Products (y sin 3x)		Angle x	Meas. Ord. y	cos 3x	Products (y cos 3x)	
	Pos.	Neg.				Pos.	Neg.
.13053	0.17		2.5	1.30	.99144	1.29	
.38268	1.65		7.5	4.30	.92388	3.97	
.60876	4.78		12.5	7.85	.79335	6.23	
.79335	9.20		17.5	11.60	.60876	7.06	
.92388	14.65		22.5	15.85	.38268	6.06	
.99144	19.72		27.5	19.90	.13053	2.60	
.99144	24.20		32.5	24.40	-.13053		3.18
.92388	26.15		37.5	28.30	-.38268		10.85
.79335	25.30		42.5	31.90	-.60876		19.40
.60876	21.40		47.5	35.20	-.79335		27.90
.38268	14.70		52.5	38.45	-.92388		35.50
.13053	5.35		57.5	41.00	-.99144		40.60
-.13053		5.63	62.5	43.20	-.99144		42.80
-.38268		17.26	67.5	45.10	-.92388		41.70
-.60876		28.35	72.5	46.60	-.79335		36.90
-.79335		37.80	77.5	47.70	-.60876		29.00
-.92388		44.70	82.5	48.40	-.38268		18.55
-.99144		48.25	87.5	48.70	-.13053		6.36
-.99144		48.00	92.5	48.40	.13053	6.32	
-.92388		43.90	97.5	47.60	.38268	18.21	
-.79335		36.75	102.5	46.35	.60876	28.20	
-.60876		27.20	107.5	44.70	.79335	35.45	
-.38268		16.28	112.5	42.50	.92388	39.30	
-.13053		5.23	117.5	40.05	.99144	39.70	
.13053	4.86		122.5	37.20	.99144	36.90	
.38268	13.02		127.5	34.00	.92388	31.40	
.60876	18.64		132.5	30.60	.79335	24.30	
.79335	21.40		137.5	27.00	.60876	16.45	
.92388	21.70		142.5	23.50	.38268	9.00	
.99144	20.00		147.5	20.20	.13053	2.63	
.99144	16.45		152.5	16.60	-.13053		2.17
.92388	12.28		157.5	13.30	-.38268		5.09
.79335	7.85		162.5	9.90	-.60876		6.03
.60876	4.20		167.5	6.90	-.79335		5.47
.38268	1.53		172.5	4.00	-.92388		3.70
.13053	0.18		177.5	1.35	-.99144		1.34
Sum of Products	309.38	359.35				315.07	336.54
	-49.97					-21.47	

TABLE III
FIFTH HARMONIC

sin 5x	Products (y sin 5x)		Angle x	Meas. Ord. y	cos 5x	Products (y cos 5x)	
	Pos.	Neg.				Pos.	Neg.
.21644	0.28		2.5	1.30	.97630	1.27	
.60876	2.62		7.5	4.30	.79335	3.37	
.88701	6.96		12.5	7.85	.46175	3.62	
.99905	11.60		17.5	11.60	.04362	0.51	
.92388	14.65		22.5	15.85	-.38268		6.07
.67559	13.45		27.5	19.90	-.73728		14.66
.30071	7.32		32.5	24.40	-.95372		23.30
-.13053		3.69	37.5	28.30	-.99144		28.10
-.53730		17.12	42.5	31.90	-.84339		26.90
-.84339		29.70	47.5	35.20	-.53730		18.90
-.99144		38.10	52.5	38.45	-.13053		5.02
-.95372		39.10	57.5	41.00	.30071	12.30	
-.73728		31.80	62.5	43.20	.67559	29.20	
-.38268		17.25	67.5	45.10	.92388	41.70	
.04362	2.03		72.5	46.60	.99905	46.60	
.46175	22.00		77.5	47.70	.88701	42.25	
.79335	38.40		82.5	48.40	.60876	29.50	
.97630	47.50		87.5	48.70	.21644	10.53	
.97630	47.25		92.5	48.40	-.21644		10.48
.79335	37.70		97.5	47.60	-.60876		28.95
.46175	21.40		102.5	46.35	-.88701		41.10
.04362	1.95		107.5	44.70	-.99905		44.70
-.38268		16.30	112.5	42.50	-.92388		39.30
-.73728		29.50	117.5	40.05	-.67559		27.10
-.95372		35.50	122.5	37.20	-.30071		11.16
-.99144		33.70	127.5	34.00	.13053	4.44	
-.84339		25.80	132.5	30.60	.53730	16.42	
-.53730		14.50	137.5	27.00	.84339	22.70	
-.13053		3.07	142.5	23.50	.99144	23.30	
.30071	6.06		147.5	20.20	.95372	19.26	
.67559	11.22		152.5	16.60	.73728	12.24	
.92388	12.29		157.5	13.30	.38268	5.09	
.99905	9.90		162.5	9.90	-.04362		0.43
.88701	6.12		167.5	6.90	-.46175		3.18
.60876	2.44		172.5	4.00	-.79335		3.17
.21644	0.29		177.5	1.35	-.97630		1.32
Sum of Products	323.43	335.13				324.30	333.84
	-11.7					-9.54	

TABLE IV
SEVENTH HARMONIC

sin 7x	Products (y sin 7x)		Angle x	Meas. Ord. y	cos 7x	Products (y cos 7x)	
	Pos.	Neg.				Pos.	Neg.
.30071	0.39		2.5	1.30	.95372	1.24	
.79335	3.41		7.5	4.30	.60876	2.62	
.99905	7.85		12.5	7.85	.04362	0.34	
.84339	9.78		17.5	11.60	-.53730		6.23
.38268	6.07		22.5	15.85	-.92388		14.65
-.21644		4.31	27.5	19.90	-.97630		19.43
-.73728		17.98	32.5	24.40	-.67559		16.50
-.99144		28.05	37.5	28.30	-.13053		3.69
-.88701		28.30	42.5	31.90	.46175	14.70	
-.46175		16.24	47.5	35.20	.88701	31.20	
.13053	5.02		52.5	38.45	.99144	38.10	
.67559	27.70		57.5	41.00	.73728	30.20	
.97630	42.15		62.5	43.20	.21644	9.34	
.92388	41.60		67.5	45.10	-.38268		17.25
.53730	25.00		72.5	46.60	-.84339		38.30
-.04362		2.08	77.5	47.70	-.99905		47.70
-.60876		29.45	82.5	48.40	-.79335		38.40
-.95372		46.40	87.5	48.70	-.30071		14.60
-.95372		46.20	92.5	48.40	.30071	14.52	
-.60876		28.95	97.5	47.60	.79335	37.70	
-.04362		2.02	102.5	46.35	.99905	46.30	
.53730	24.00		107.5	44.70	.84339	37.60	
.92388	39.25		112.5	42.50	.38268	16.27	
.97630	39.10		117.5	40.05	-.21644		8.66
.67559	25.15		122.5	37.20	-.73728		27.40
.13053	4.44		127.5	34.00	-.99144		33.70
-.46175		14.10	132.5	30.60	-.88701		27.10
-.88701		23.95	137.5	27.00	-.46175		12.45
-.99144		23.30	142.5	23.50	.13053	3.07	
-.73728		14.89	147.5	20.20	.67559	13.65	
-.21644		3.59	152.5	16.60	.97630	16.20	
.38268	5.11		157.5	13.30	.92388	12.29	
.84339	8.35		162.5	9.90	.53730	5.32	
.99905	6.90		167.7	6.90	-.04362		0.30
.79335	3.17		172.5	4.00	-.60876		2.44
.30071	0.40		177.5	1.35	-.95372		1.29
Sum of Products	324.84	329.81				330.66	330.09
	-4.97					0.57	

The remainder of the analysis is devoted to the evaluation of the fundamental, third, fifth and seventh harmonics of the input current under blocked rotor conditions. Tables I, II, III and IV show the preliminary calculations necessary for obtaining the constants of the Fourier Series equations.

From Table I the constants A, B and C are determined as follows:

$$B_1 = \frac{y \sin \alpha}{18} = \frac{831.4}{18} = 46.2$$

$$A_1 = \frac{y \cos \alpha}{18} = \frac{30.14}{18} = 1.675$$

$$C_1 = \sqrt{A_1^2 + B_1^2} = \sqrt{1.675^2 + 46.2^2} = 46.25$$

$$\theta_1 = \tan^{-1} \frac{-B_1}{A_1} = \tan^{-1} \frac{-46.2}{1.675} = -87.9^\circ \text{ or } 92.1^\circ$$

The correct angle to use may be found from the relation of equation (2)

$$B_n = -C \sin \theta_n$$

$$46.2 = -46.25 \sin \theta_1$$

and to make the equality true, θ_1 must be negative. Therefore, θ_1 is equal to -87.9° and the equation may be written as

$$y_1 = 46.25 \cos (\alpha - 87.9^\circ)$$

The above method may be used in conjunction with Tables II, III and IV and the results are as follows:

$$y_3 = 3.02 \cos (3\alpha + 113.3^\circ)$$

$$y_5 = 0.839 \cos (5\alpha + 129.2^\circ)$$

$$y_7 = 0.278 \cos (7\alpha + 83.45^\circ)$$

The per cent harmonic may be determined from the above equations and

$$\% C_3 = \frac{C_3}{C_1} \times 100 = \frac{3.02}{46.25} \times 100 = 6.53\%$$

$$\% C_5 = \frac{C_5}{C_1} \times 100 = \frac{0.839}{46.25} \times 100 = 1.82\%$$

$$\% C_7 = \frac{C_7}{C_1} \times 100 = \frac{0.278}{46.25} \times 100 = 0.60\%$$

It is next necessary to determine the instantaneous current equations. Since the equations are now in terms of arbitrary units, they must be converted to current. This may simply be done as follows: the complete expression, arbitrary units, is

$$y = 46.25 \cos(\alpha - 87.9^\circ) + 3.02 \cos(3\alpha + 113.3^\circ) + 0.839 \cos(5\alpha + 129.2^\circ) + 0.278 \cos(7\alpha + 83.45^\circ)$$

From the blocked rotor test, $I = 22.6$ amps and

$$i = \sqrt{2} \times 22.6 = 31.95 \text{ amps.}$$

From Figure 19, $y_m = 48.7$ units and

$$i_{m1} = \frac{46.25}{48.70} \times 31.95 = 30.35 \text{ amps}$$

$$i_{m3} = \frac{3.02}{48.70} \times 31.95 = 1.98 \text{ amps}$$

$$i_{m5} = \frac{0.839}{48.70} \times 31.95 = 0.551 \text{ amps}$$

$$i_{m7} = \frac{0.278}{48.70} \times 31.95 = 0.182 \text{ amps.}$$

The equation for the instantaneous current may now be written as

$$i = 30.35 \cos(\alpha - 87.9^\circ) + 1.98 \cos(3\alpha + 113.3^\circ) + \\ 0.551 \cos(5\alpha + 129.2^\circ) + 0.182 \cos(7\alpha + 83.45^\circ)$$

Thus, the waveform analysis has given the amount of harmonic in the input current of the motor under blocked rotor conditions.

- Margaret Hitt -