HYPERFINE SPLITTING OF THE $7s^3S_1$-$6p^3P_2$ LINE OF MERCURY FROM INTERFERENCE FRINGE MEASUREMENTS

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1948

Submitted to the Department of Physics

Oklahoma Agricultural and Mechanical College

In Partial Fulfillment of the Requirements

for the Degree of

MASTER OF SCIENCE

1950

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PREFACE

In previous work on hyperfine separation by interference fringe measurements, the reduction formulae required measurements of the radius of the fringe systems as measured from the center of the interference pattern. It is impossible to find the center accurately, so measurements were made of diameters of the circular fringe systems. To eliminate this inconvenience Dr. R. A. Fisher and his students at Northwestern University have devised a new set of reduction formulae where only differences in adjacent fringe orders are measured without recourse to radii measurements. This simplifies the measurements immensely and greatly increases the possible accuracy of measurement. The purpose of this paper is to experimentally verify the Fisher reduction formulae.

ACKNOWLEDGEMENT

The author wishes to express his deep appreciation for the many invaluable conferences held with his adviser, Dr. Alvin V. Pershing, on problems met within the preparation of this paper.

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INTRODUCTION

The well-known green line $\lambda 5460.74$ of mercury is due to the atomic transition 6s7s to 6s6p. It can be shown that atomic transition levels in Hg are due to Russell-Saunders type coupling. In general atoms with few valence electrons (almost empty outer Stoner shells) will obey Russell-Saunders coupling and atoms with few holes (almost completed outer Stoner shells) will obey (j-j) coupling scheme rules. Russell-Saunders or (LS) coupling is due to an orbital moment L precessing about a resultant J and a spin resultant S precessing about the same resultant J, or $\sum l_i = L$, $\sum s_i = S$, $L + S = J$. For (j-j) coupling each l_i and s_i add to form a resultant j_i and the summation of these j_i's gives the final J, and the precession is that of the individual j_i around the final J, or $l_i + s_i = j_i$, $\sum j_i = J$.

The possible orientations in (LS) and (j-j) coupling for the two-valence electron are shown in Fig. 1.

The possible transitions are shown in Fig. 2. The intercombination lines (singlet to triplet) are very weak in most cases, as here, although in some cases[1]

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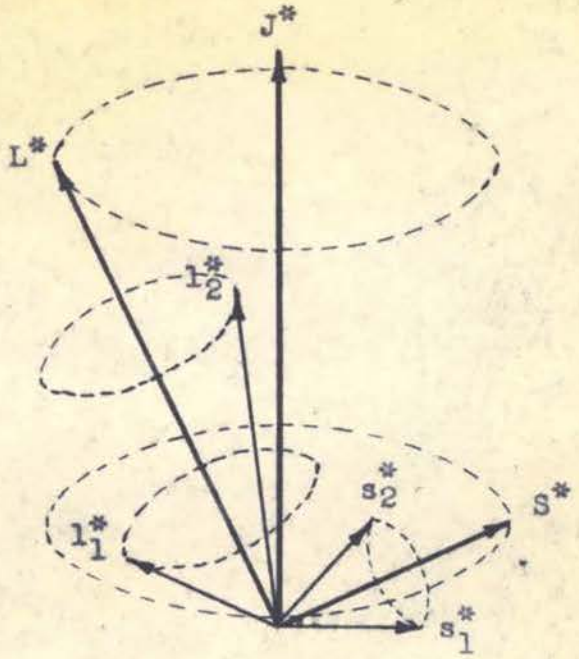
[1] Richtmyer, F. K. and E. H. Kennard, Introduction to Modern Physics, p. 359.

LS - Coupling



jj - Coupling

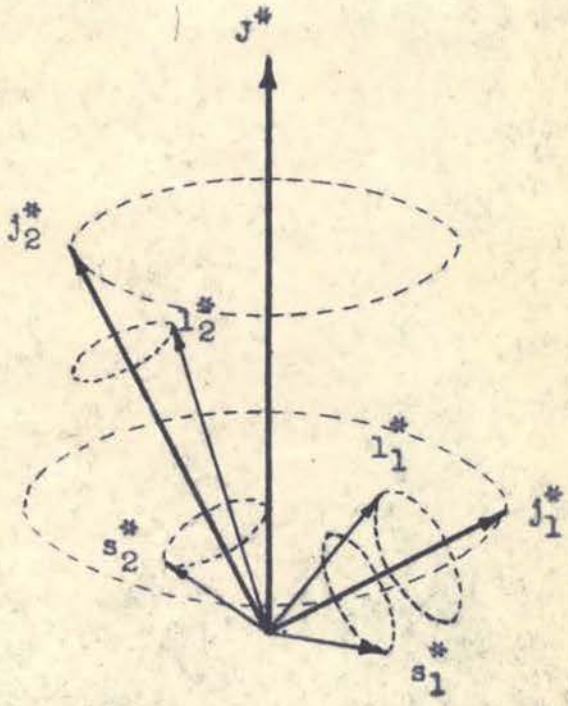
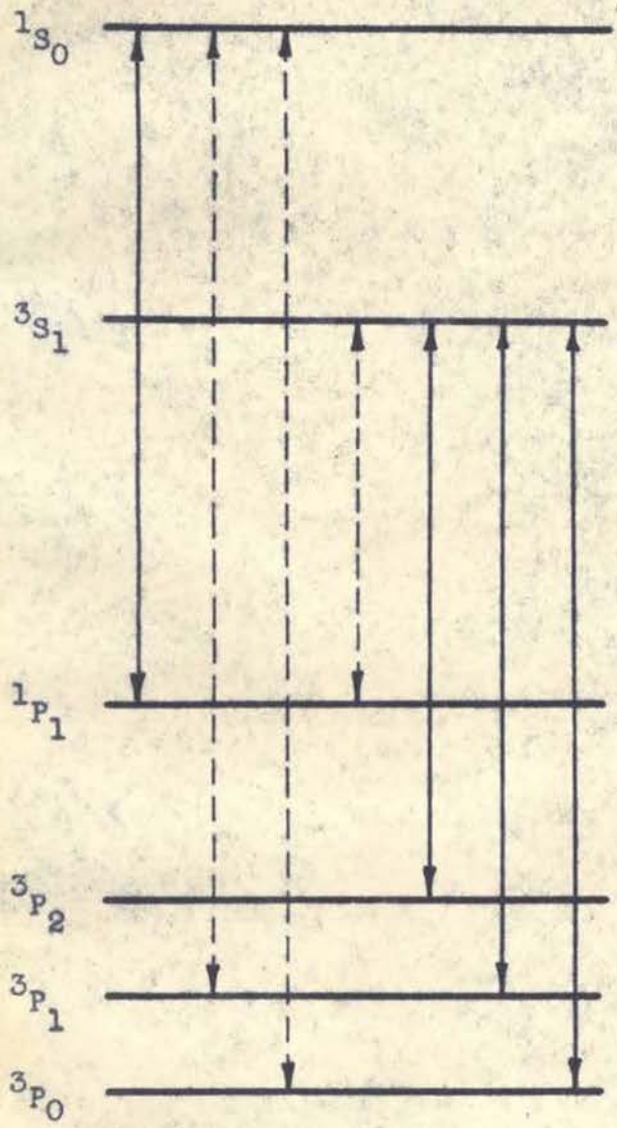


FIGURE 1

Hg 6s7s-6s6p



6s 7s

l	s	S
0	$\frac{1}{2}$	
0	$\frac{1}{2}$	0,1
0; 1,0		

6s 6p

l	s	P
0	$\frac{1}{2}$	
1	$\frac{1}{2}$	2,1,0; 1
1; 1,0		

FIGURE 2

intercombination lines are very strong due to a piling up of atoms on one level, which spill to a lower one in sufficient numbers to give intense lines. An example of this "spilling" effect is Hg $6s6p\,^3P_1$-$6s6s\,^1S_0$, λ2536.52, where the electrons pile up in the $6s6p\,^3P_1$ level. However, these cases are uncommon and the transition probabilities for intercombination lines are quite low. Intercombination lines occur only in fine structure, not in hyperfine structure. In the figure the intercombination lines are shown as dashed lines, and these transitions are of no interest here.

The intensities of lines are greatest for transitions in which (1) J and L change in the same direction, i. e., $^3P_2$-$^3S_1$ is more intense than $^3P_1$-$^3S_1$ which in turn is more intense than $^3P_0$-$^3S_1$; and (2) the transitions have the highest intensities which have the highest J values, i. e., a transition $^3D_3$-$^3P_2$ is more intense than $^3D_2$-$^3P_1$, in general.

From the first of the relations above we see that in Fig. 2, the strongest lines correspond to the transitions $^1S_0$-$^1P_1$ and $^3S_1$-$^3P_2$ where we ignore the intercombination line $^1S_1$-$^3P_2$, and from (2) above we see that $^3S_1$-$^3P_2$ is the strongest line in this group. From energy level diagrams of mercury[2,3,4] it can be seen that the

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[2] Richtmyer, F. K. and E. H. Kennard, Ibid., p. 358.

[3] Herzberg, G., Atomic Spectra and Atomic Structure, p. 202.

atomic transition $^3S_1 - {}^3P_2$ gives rise to the very intense green line, λ5460.74. From the intensity and the fact that this line lies in a part of the spectrum easily accessible for visual observation, it was decided to attempt a study of this line.

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4 White, H. E., Introduction to Atomic Spectra, p. 179.

THEORETICAL CONSIDERATIONS

The hyperfine structure of spectral lines has been suggested by Pauli[5] to arise from a nuclear magnetic moment and its associated mechanical moment. There is also an isotope effect (due to differences in nuclear structure, not mass differences), which for our discussion we will not designate as hyperfine structure. Here we will understand hyperfine structure to result from Pauli's hypothesis of a nuclear "ausgezeichnete Achse." As in the case of the spinning electron, this axis will be associated with a magnetic moment, whose interaction with extranuclear electrons would give rise to the energy terms causing hyperfine splitting of the levels. This magnetic moment will be associated with a mechanical moment of magnitude $i\hbar$, where i is a new quantum number, the nuclear spin quantum number. The magnetic moment is probably e/Mc times the mechanical moment, where M denotes the proton mass and is thus only 1/1838th of the electron moment.

The hyperfine structure of energy levels and spectral lines arising from a magnetic nucleus can be

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[5] Pauli, W. Naturwissenschaften, 12 (May, 1924), 741. See also Pauling, L. and S. Goudsmit, The Structure of Line Spectra, p. 203.

described by the use of the vector model. It is only necessary to add the nuclear moment vector to the model used to give the fine structure of energy levels and spectral lines (Fig. 1). The vector model for (LS) coupling for hyperfine structure differs from the model used for fine structure only in the fact that instead of L^*, S^*, and J^*, we use the vectors I^*, J^*, and F^* respectively, where the value of J^* is obtained from fine structure considerations and the nuclear moment I^* is added to J^* and the resultant F^* is the total mechanical moment. The vector model for hyperfine structure will not be repeated here as it is the same as the model for fine structure provided the above substitutions are made.

Returning to Fig. 2, we see that for the 6s7s state we have $l_1 = l_2 = 0$ and $s_1 = s_2 = 1/2$, thus $L = 0$ and $S = 0$ or 1 and therefore $J = 0$ or 1. For the 6s6p state we have $l_1 = 0$ and $l_2 = 1$, and $s_1 = s_2 = 1/2$, thus $L = 1$, $S = 1$, and therefore $J = 0$, 1, or 2. Thus we have the fine structure separation shown. However, as mentioned earlier, the line we wish to consider, $\lambda 5460.74$, is due to the transition $^3P_2$-$^3S_1$. Now Hg has eight isotopes. All even atomic weight Hg isotopes 196, 198, 200, 202, 204 have no nuclear spin moment I^*. The nuclear spin moment I^* of Hg 199 is 1/2 and that of Hg 201 is 3/2. The I^* value of Hg 203 is unknown due to the fact

the relative abundance of this isotope is only approximately 0.006%[6] of all Hg found in nature, and separation of this isotope has never been carried out so no work has been done on it.

The splitting of the $^3P_2$ line for I = 3/2 is shown in Figs. 3a and 3b as an example of the results. Here F* is the vector sum of I* and J*. Thus we obtain the splitting of energy levels for $^3S_1$ and $^3P_2$ which is shown in Fig. 3c. with I = 0, 1/2, and 3/2 corresponding to the isotopes which are respectively, even numbered, 199, and 201.

The rules for the multiplicity of the splitting are m = 2J + 1 for J < I or m = 2I + 1 for I < J. Thus terms with J = 0 are always single; and with I = 1/2 all other terms with J > 0 show a splitting into two components, and so forth.

The width of the hyperfine structure is determined not by the mechanical moment, but by the magnetic moment of the nucleus. The effect of the nuclear mechanical spin is to create a nuclear magnetic field, which, coupling with the magnetic field produced by electron spin and orbital moment, gives the observed hyperfine structure pattern. The proposed theories of nuclear spin attribute the nuclear magnetic properties to protons and

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6 Rasetti, F., Elements of Nuclear Physics, p. 160.

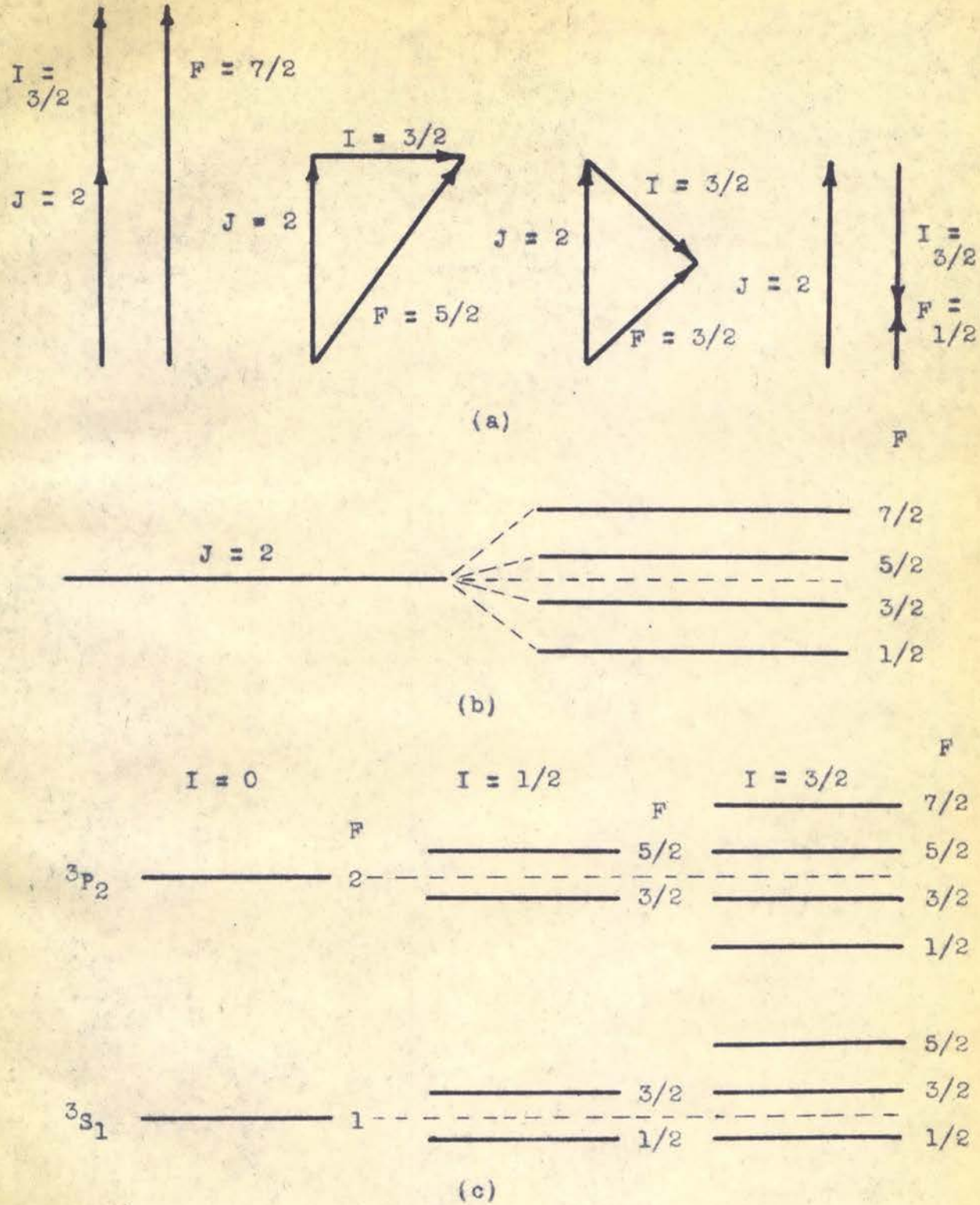
I = 3/2
F = 7/2
J = 2
I = 3/2
J = 2
F = 5/2
I = 3/2
J = 2
F = 3/2
J = 2
I = 3/2
F = 1/2
(a)
F
J = 2
7/2
5/2
3/2
1/2
(b)
I = 0
I = 1/2
I = 3/2
F
F
F
$^3P_2$
2
5/2
3/2
7/2
5/2
3/2
1/2
$^3S_1$
1
3/2
1/2
5/2
3/2
1/2
(c)

FIGURE 3

neutrons. From the small values of nuclear spin (9/2 is the largest observed nuclear spin) it is probable that only a very few protons and neutrons in the nucleus contribute to the magnetic properties.

The interaction energy of a single valence electron screened from the nucleus by a shell of electrons, is given by[7]

$$\Gamma_F = 1/2\ a' \ (F^{*2} - I^{*2} - J^{*2})$$

where

$$a' = \frac{g_I\ RK^2 Z_i Z_o^2}{1838\ n_o^3\ (1 + \frac{1}{2})\ j(j+1)}$$

Here R is the Rydberg constant for Hg (109737); K is the Sommerfeld fine structure constant (1/137); Z_i is the value of Z inner and Z_o is Z outer; n_o is the effective value of the quantum number n; and g_I is the ratio of the nuclear mechanical spin moment to that of the nuclear spin magnetic moment.

Normally, for a single valence electron atom, $Z_o = 1$, but in mercury we have two valence electrons and hence $Z_o = 2$. The value of Z_i for the s electron is the same as the atomic number, or for mercury, Z_i is 80 for the s electron. For the p electron, the rule is that $Z_i = Z - 4$[8] or Z_i is 76 for the mercury p electron.

The value of n_o^3 is gotten from the following

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7 White, H. E., Ibid., p. 361.

8 Tolansky, S., Hyperfine Structure in Line Spectra and Nuclear Spin, p. 106.

equation[9]:

$$T = R/n_o^2.$$

From the energy level diagrams[2,3] we get T equals 21700 cm^{-1}., or

$$n_o^2 = R/T = 109737/21700 = 5.057$$

whence $n_o = 2.219$ and $n_o^3 = 11.373$.

Considering the transitions for splitting in the S state, we have

$$a_s' = g_I\ (0.089034)\ cm^{-1}.$$

For the P state we have

$$a_p' = g_I\ (0.009398)\ cm^{-1}.$$

For Hg 199, I = 1/2 and $g_I = 1.1$[10], thus

$$a_s' = 0.097938\ cm^{-1}.$$

$$a_p' = 0.010338\ cm^{-1}.$$

Then, for the S level,

$$\left[{}^{1}_{2}S = -\ a_s'\right.$$

$$\left[{}^{3}_{2}S = +\ 1/2\ a_s'\right.$$

Similarly, for the P level,

$$\left[{}^{3}_{2}P = -\ 3/2\ a_p'\right.$$

$$\left[{}^{5}_{2}P = +\ a_p'\right.$$

For Hg 201, I = 3/2, $g_I = -\ 0.41$, and following the same procedure as above, we get

$$a_s' = g_I\ (0.089034) = -\ 0.036504\ cm^{-1}.$$

$$a_p' = g_I\ (0.009308) = -\ 0.003853\ cm^{-1}.$$

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9 White, H. E., Ibid., p. 89.

10 White, H. E., Ibid., p. 372.

Then for the S state,

$$\Delta_{5/2}S = 3/2\ a_s'$$
$$\Delta_{3/2}S = -\ a_s'$$
$$\Delta_{1/2}S = -\ 5/2\ a_s'$$

Similarly, for the P state,

$$\Delta_{1/2}P = -\ 9/2\ a_p'$$
$$\Delta_{3/2}P = -\ 3\ a_p'$$
$$\Delta_{5/2}P = -\ 1/2\ a_p'$$
$$\Delta_{7/2}P = 3\ a_p'$$

The resulting separations are shown in Fig. 4. From the rule for intensities, $I = 2F + 1$, it can be seen that the center of gravity is not shifted. The actual separations in terms of wave numbers are noted on the figure also. These have of course been obtained by substituting the four values for a_s' and a_p' from Hg 199 and Hg 201.

The allowable transitions for Hg 199 are

$$^3P_{5/2}2 - {}^3S_{3/2}1 = 0.038631 \text{ cm.}^{-1}$$
$$^3P_{3/2}2 - {}^3S_{3/2}1 = 0.064476 \text{ cm.}^{-1}$$
$$^3P_{3/2}2 - {}^3S_{1/2}1 = -\ 0.082431 \text{ cm.}^{-1}$$

The allowable transitions for Hg 201 are

$$^3P_{7/2}2 - {}^3S_{5/2}1 = -\ 0.043197 \text{ cm.}^{-1}$$
$$^3P_{5/2}2 - {}^3S_{5/2}1 = -\ 0.056682 \text{ cm.}^{-1}$$
$$^3P_{5/2}2 - {}^3S_{3/2}1 = 0.034578 \text{ cm.}^{-1}$$
$$^3P_{3/2}2 - {}^3S_{5/2}1 = -\ 0.066315 \text{ cm.}^{-1}$$
$$^3P_{3/2}2 - {}^3S_{3/2}1 = 0.024945 \text{ cm.}^{-1}$$

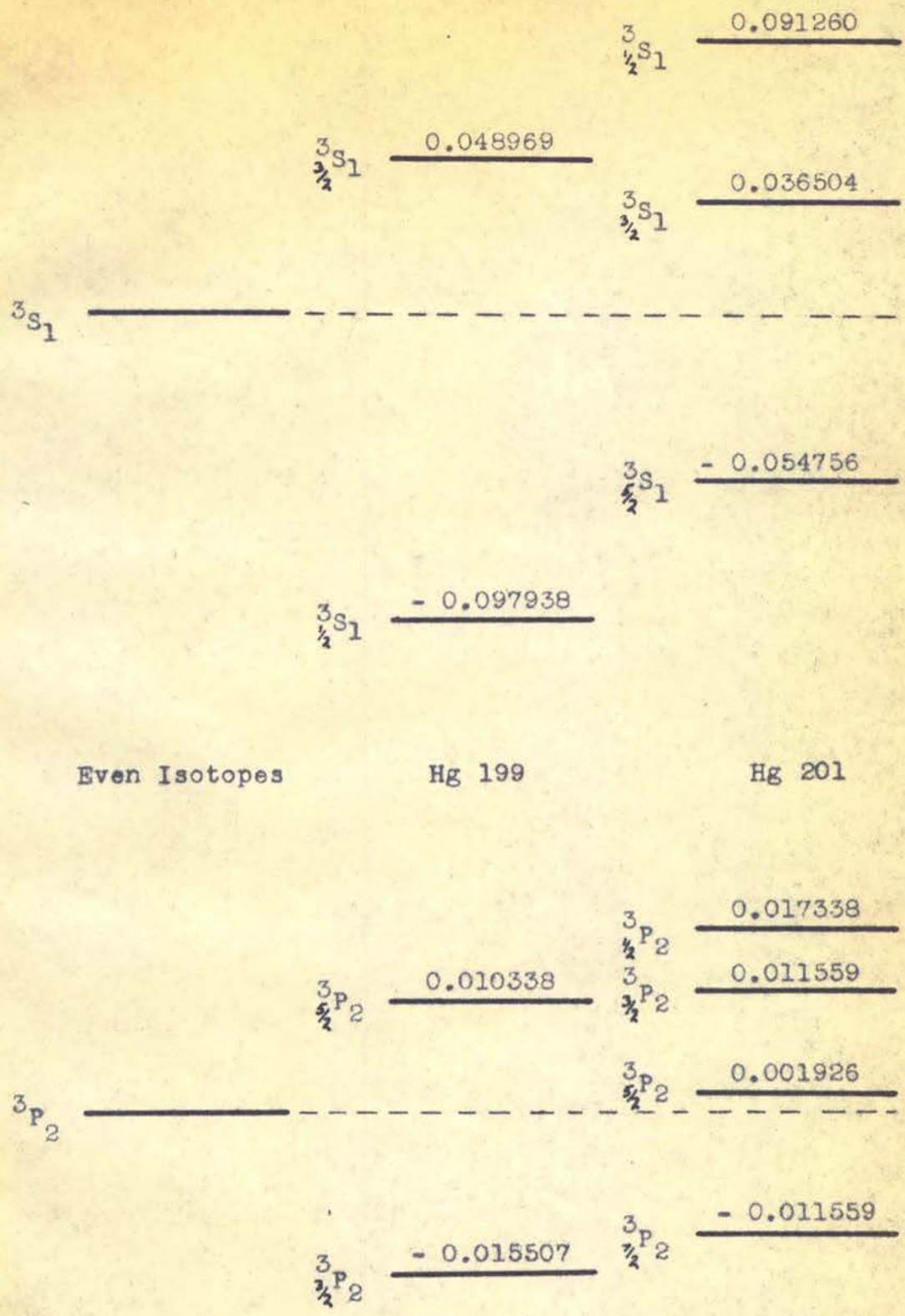
$^3S_1$
$^3S_1$ 0.091260
$^3S_1$ 0.048969
$^3S_1$ 0.036504
$^3S_1$ - 0.054756
$^3S_1$ - 0.097938
Even Isotopes
Hg 199
Hg 201
$^3P_2$
$^3P_2$ 0.017338
$^3P_2$ 0.010338
$^3P_2$ 0.011559
$^3P_2$ 0.001926
$^3P_2$ - 0.011559
$^3P_2$ - 0.015507

FIGURE 4

$$^3P_2 - {}^3S_1 = 0.079701 \text{ cm.}^{-1}$$
$$^3P_2 - {}^3S_1 = 0.019166 \text{ cm.}^{-1}$$
$$^3P_2 - {}^3S_1 = 0.073922 \text{ cm.}^{-1}$$

The wave numbers following the above transitions are expressed in terms of separations from the normal $^3P_2 - {}^3S_1$ line in the even numbered isotopes.

Actually, it will be seen later that the measurements are of differences in hyperfine lines of Hg 201.

The line differences for Hg 201 are

$$(^3P_2 - {}^3S_1) - (^3P_2 - {}^3S_1) = 0.023118 \text{ cm.}^{-1}$$
$$(^3P_2 - {}^3S_1) - (^3P_2 - {}^3S_1) = 0.013485 \text{ cm.}^{-1}$$
$$(^3P_2 - {}^3S_1) - (^3P_2 - {}^3S_1) = 0.085481 \text{ cm.}^{-1}$$
$$(^3P_2 - {}^3S_1) - (^3P_2 - {}^3S_1) = 0.091260 \text{ cm.}^{-1}$$
$$(^3P_2 - {}^3S_1) - (^3P_2 - {}^3S_1) = 0.009633 \text{ cm.}^{-1}$$

THE LIGHT SOURCE

The choice of light source is dependent upon the purpose of the investigation. The arc source is not used at present for observing hyperfine structures.

The width of a spectral line is affected by many factors. The most important causes of line broadening are:

(1) Radiation line width

(2) Doppler width

(3) Pressure Broadening

(4) Resonance broadening

(5) Stark broadening

(6) Interatomic Stark broadening

(7) Self-reversal width

(8) Spurious reversal structure.

For the purposes at hand, it is not possible to go into detail on these line broadening effects. The interested reader will find a complete treatment given by Tolansky[11].

At present arcs are only used for special cases where it is difficult to excite the required weak lines by other means. The highest resolution can never be

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[11] Tolansky, S., High Resolution Spectroscopy, Chapter I.

employed for the resulting lines are too broad, and there is a permanent danger of encountering self-reversal, or at least spurious self-absorption effects. The considerable number of contradictory measurements in the earliest published measurements of hyperfine structure are mostly due to the erratic effects of self-absorption in the arcs at first so regularly employed[12].

Without going into detail on the theory of line broadening which is covered in the reference given, a few of the more serious effects for the mercury arc will be mentioned.

The Doppler broadening can be shown[13] to be given by

$$w = 0.71 \times 10^{-6} \, (T/M)^{\frac{1}{2}} \nu \text{ cm.}^{-1}$$

In the case of the mercury green line this results in a broadening of 0.016 cm.$^{-1}$ at an arc temperature of 3600° Kelvin. The broadening is least for the heaviest elements and at low temperatures. It is now almost universal to use either liquid air cooling on Geissler tubes or atomic beams for hyperfine work.

The effect of self-reversal is that resonance lines have the strongest tendency to self-absorption. This results in a serious broadening effect. If the density of the absorbing column progressively increases a more

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12 Tolansky, S., Op. cit., p. 22.

13 Tolansky, S., Op. cit., p. 6.

serious phenomenon appears, namely the production of spurious structure by self-reversal. Owing to the much higher opacity of the central portion of the line, the center of the line is more absorbed than the wings until with a sufficient concentration complete self-reversal sets in, with the appearance of a spurious doublet. This consideration is of much more than theoretical interest. An extreme example is the earliest researches on the intercombination resonance line λ2537 of mercury in which 25 components were observed. When a source free from self-reversal is employed it can be definitely established that this line has no more than five readily observable components.

The other effects enumerated are not of great importance unless very poor vacuum conditions prevail or very high voltages are used.

In light of what has been said it can be seen that the mercury arc constitutes a very poor source for study of hyperfine structure. The only justification for its use in this case was that it was the best mercury source available.

THE FABRY-PEROT INTERFEROMETER

This interferometer[14] consists of a pair of plane parallel plates coated with a partially transparent layer of a highly reflecting metal, having a reflecting coefficient of the order of 80 per cent to 90 per cent. If light of wavelength L, coming from an extended source, falls upon such a pair of plates separated by an air gap t centimeters thick, fringes of equal inclination are formed. All light incident along the surface of a cone of semi-angle A contributes to form a single circular fringe when

$$mL = 2nt \cos A,$$

m being the order of interference and n the refractive index of the medium between the plates.

At and near the center of the fringe system, cos A equals 1, and

$$m = 2ntv$$

thus,

$$dv = dm/2nt.$$

Now dv is the change in wave number corresponding to the change in order dm. On moving from one fringe to

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[14] Tolansky, S., Op. cit., p. 97.

the next dm equals 1. Hence the wave number change between orders is

$$\Delta v = 1/2nt.$$

By varying t the range between orders can be adjusted.

For a specified dv for hyperfine separation in the line structure to be studied, it is apparent that if the different components are to be in phase with each other we must adjust t so dm is integral. If dm is not integral from one fringe ring to the next, then the components will overlap and an out-of-phase relationship will exist.

There are five readily observed components of the Hg green line, all having different dv separations relative to each other. There must be a least common multiple of these separations found such that the value of t chosen brings all the component hyperfine lines into phase with each other. Since the separations are not known in advance, t cannot be computed. It is a rather tedious task to adjust the air gap to this critical t, as the whole method is simply one of continuous adjustment combined with visual observation of the resulting fringe system until the desired phase relationship appears.

A careful inspection of Plate II will show that every sixth order fringe has six components instead of five. A final adjustment of t would be necessary to

correct this slight overlap. However, in view of the complete lack of correlation of preliminary measurements with theoretical calculations, it was deemed fruitless to pursue this line of attack further and attention was directed to the photograph of Ritter and Lau[15].

Ordinarily air is used as the medium in the gap, so n is usually considered to be one. Actually, in air n = 1.003, but as long as one is measuring line differences as in hyperfine work, this is relatively unimportant.

Ritter and Lau have developed a new system. They use two Fabry-Perot interferometers in series. For the gap they used dense flint glass with a refractive index of 1.775 for the green line of mercury. The gap in the first interferometer was 0.5000 cm. and in the second 0.2940 cm. They increase the resolving power by a factor of $(2)^{\frac{1}{2}}$ as compared to a single interferometer. This gives an equivalent plate separation of 1.1227 cm. Since the photograph due to Ritter and Lau is used in this paper their value of the equivalent plate separation will be used. Using their data, we get 2nt = 3.9856 cm.

To appreciate the fine work possible with an instrument of this type, it is interesting to note that Ritter and Lau claim a resolving power of 5,000,000.

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15 Ritter, E. and E. Lau. Zeitschrift fur Physik, 76 (May 31, 1932), 190.

EXPERIMENTAL RESULTS

The hope was that hyperfine structure might be observed with equipment available and correlated with the theory. This was found to be impossible, and as a final resort measurements were taken on a photograph due to Ritter and Lau[15]. The paper by Ritter and Lau contained photographs of the green line of mercury which were taken to demonstrate the superior resolving power of a new double Fabry-Perot interferometer developed by them. No further use was made of the photographs in their paper, so it was determined to make use of a reproduction of their plate to have something to compare with the calculated separations.

The procedure used in the experimental work attempted was to direct the light due to a mercury arc lamp into a monochromator. The monochromator was set to $\lambda 5461$ and a collimating lens was used to yield parallel light. In the final stage a converging lens was used to converge the parallel rays into the field of the Fabry-Perot interferometer. The equipment as used is shown in Plate I. The lower photograph shows a close-up view of the Fabry-Perot interferometer.

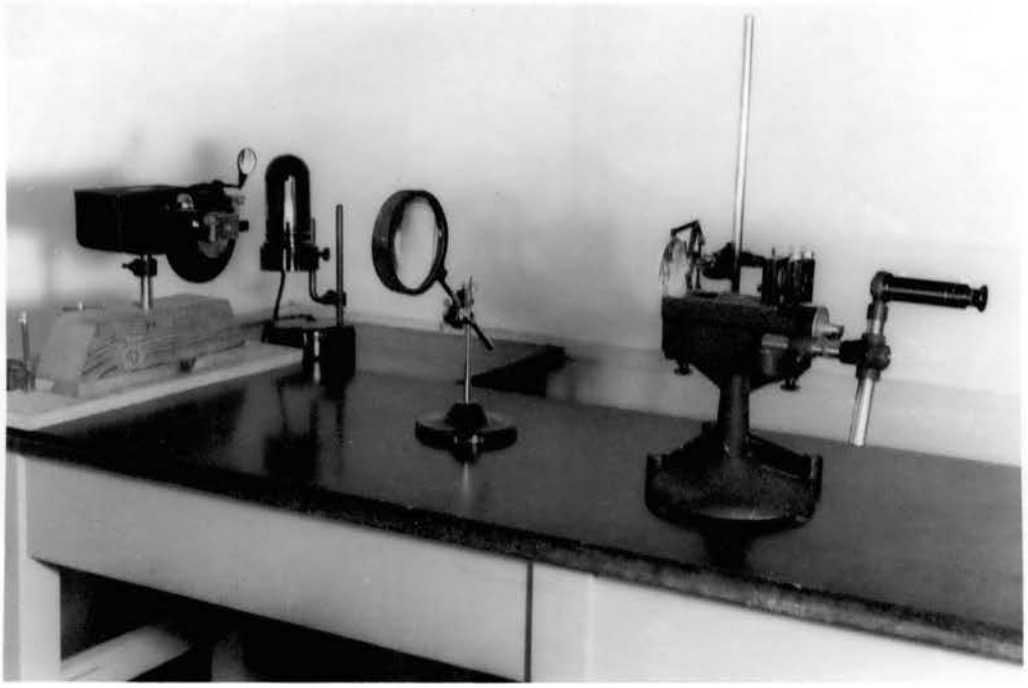

PLATE I

The photographic process used was extremely simple. The eyepiece of the interferometer was focused at infinity and the camera (also focused at infinity) was placed directly behind the eyepiece, as close as possible. It was not necessary to take any precaution to shield the lens from stray light as the exposure was made in a darkened room due to the low intensity of the interference fringe system. To obtain the desired hyperfine structure in the field of the interferometer it was necessary to narrow the slit widths of the monochromator to such an extent that the exposure time was 45 minutes at f6.3 on film with a tungsten rating of 160. The light intensity was not sufficient to form a discernible image on the ground glass screen for focusing purposes. For this reason it was felt advisable to focus everything at infinity and then expose the film. The size of the aperture at the camera was found to have no noticeable effect on the image sharpness at the edges, so the aperture was left at its maximum opening to shorten exposure time as much as possible. Actually, the diameter of the eyepiece was so much less than that of the camera lens that the eyepiece in all probability amounted to an equivalent stop of about f16 for the camera lens. It was felt best to leave the lens wide open so the alignment of the camera behind the eyepiece would not be so critical. It was found sufficiently satisfactory for mounting purposes to merely stand the camera on wooden blocks high

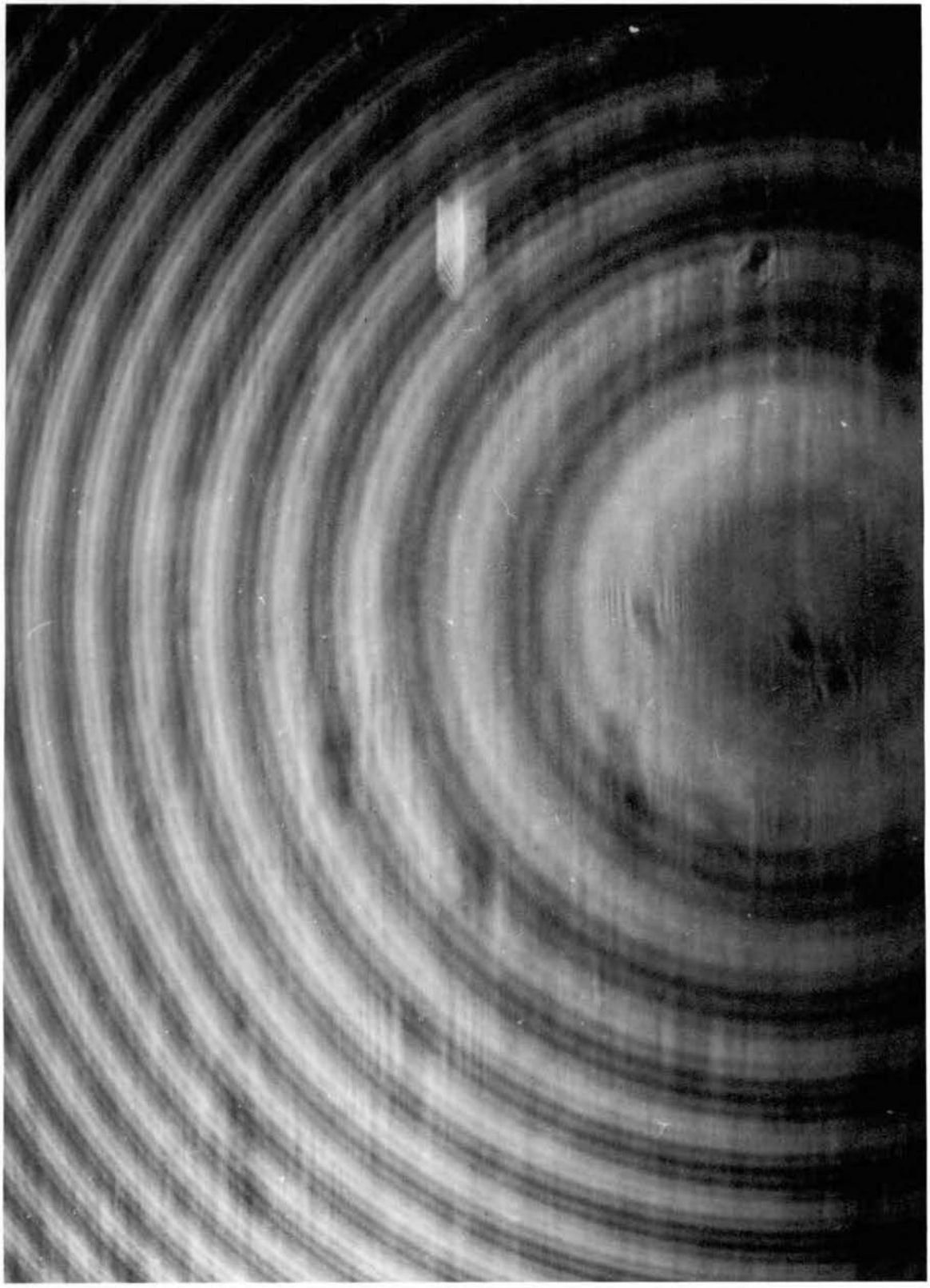

PLATE II

enough to bring it up to the level of the interferometer eyepiece. With such a long exposure, any ordinary vibrations in the building which were not of a continuous nature would have no appreciable effect on the sharpness of the negative image.

No correlation was obtained between measurements of line separations taken on the author's photograph and those calculated by the theory. This was undoubtedly due to many causes, but the two primary ones are felt to be: (1) the fact that the interferometer plates were not separated by the proper distance to have the lines in phase in successive interference orders, and (2) the light source available was unsuitable for hyperfine observation. The resolving power is very much decreased by plates with a poor reflection coefficient. This factor was very apparent as the silver coating is in rather poor condition on the interferometer used, however this is relatively unimportant as the full resolving power of the instrument could not be utilized due to the lack of a suitable light source.

In view of the lack of correlation between the calculated results and the measurements of the author's photograph, no data will be given on it, although the photograph is reproduced in Plate II.

The photograph that was used for the measurements is reproduced on Plate III. An enlarged view of the

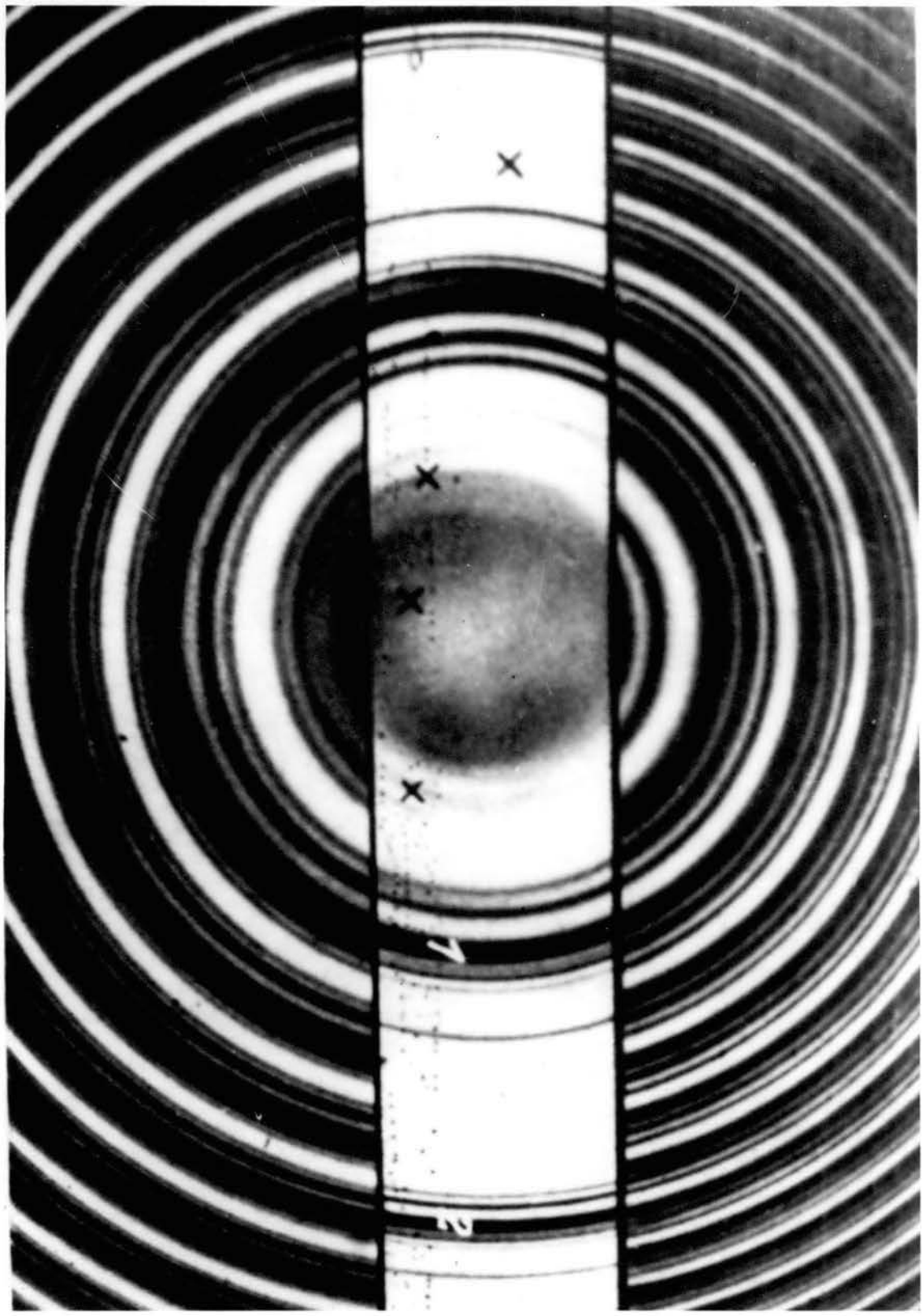

PLATE III

portion measured is reproduced on Plate IV for the purpose of showing the points clearly at which measurements were taken.

Since in the case of Ritter and Lau, the interference takes place within the glass, the ring diameters are magnified relative to air gap diameters by a factor equal to n, the refractive index of the glass. Thus the angular dispersion is effectively increased and the multiple reflections within the glass impose a severe test on the uniformity of refractive index. This effect is felt by technical opticians[16] to rule out use of the central fringe system for purposes of measurement of hyperfine separations. The accuracy increases as one goes outward from the center of the fringe pattern, so it is best to take the measurements on orders beyond the first few central ones if possible. In the photograph used, the only way it was possible to get enough measurements to use the three order reduction formulae on all components was to use the central fringe system as the field is cut off at the middle of the fourth order, counting the central order as the first order. In view of this fact, it is felt that this paper presents a very inadequate test of the validity of the three order reduction formulae, although the results may be expected

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[16] Tolansky, S. High Resolution Spectroscopy, p. 164.

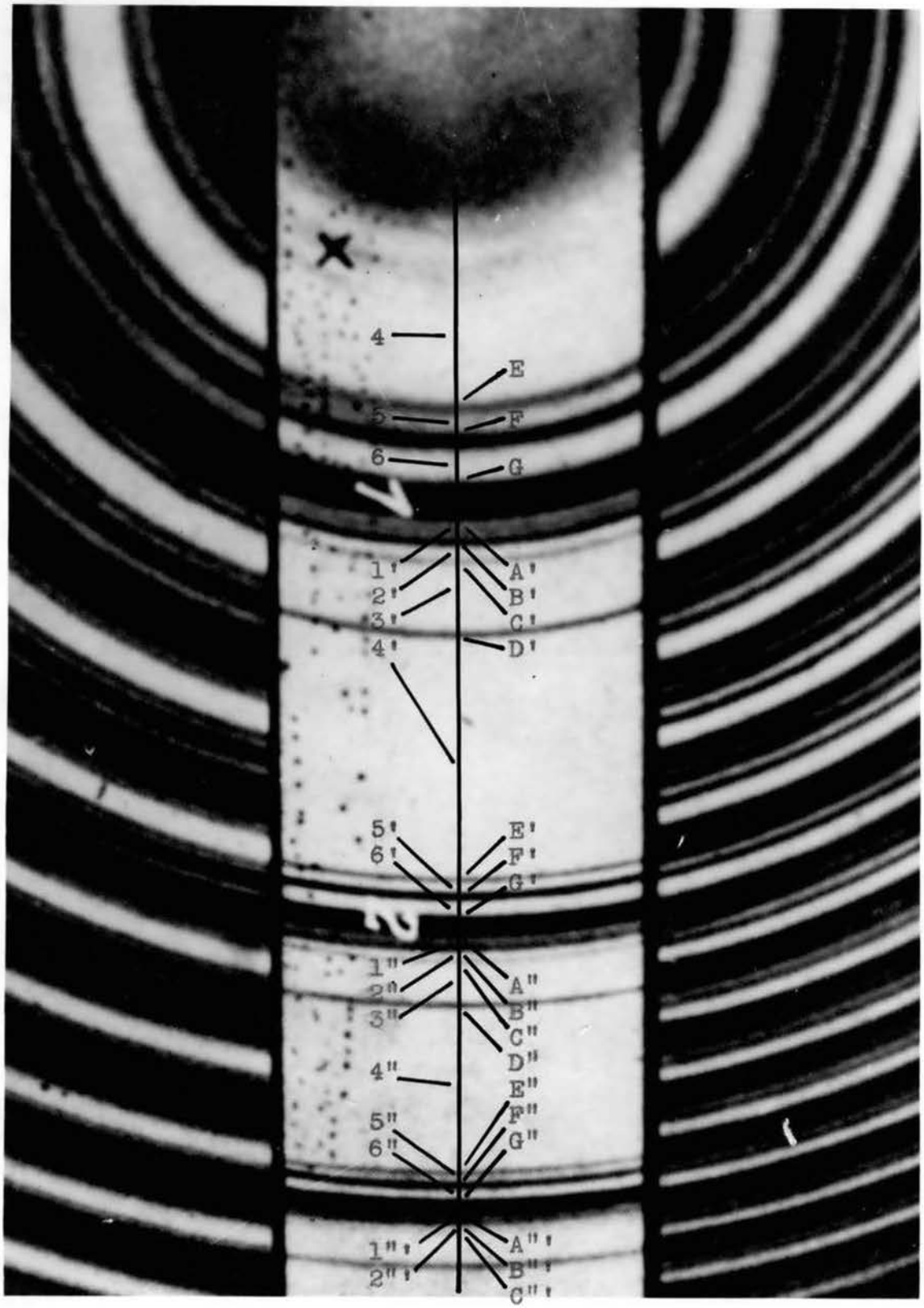

PLATE IV

to be of fair accuracy for the two order reduction formula as the main source of distortion seems to be in the central fringe order.

The Fisher reduction formulae were conveyed to the author's adviser, Dr. Alvin V. Pershing, by private communication. As previously mentioned in the preface, the purpose of this paper is to experimentally verify the Fisher reduction formulae. In view of the preceding discussion, unfortunately the best to be hoped for is an agreement between the two order formula and the theoretical separation.

The measurements to be reduced were taken by means of a cathetometer, a photograph of which appears on Plate V.

Due to the fact that the line centers could not be determined sufficiently accurately, the measurements were taken on the edges of the lines, and the edge measurements were averaged to find the centers. The averages of thirty readings on each line edge in centimeters are as follows:

C	0.89429
D	0.99952
E	1.16016
F	1.30190
G	1.54520
A'	1.94877
B'	2.15462
C'	2.33402
D'	2.99320
E'	5.23198
F'	5.37343
G'	5.55645

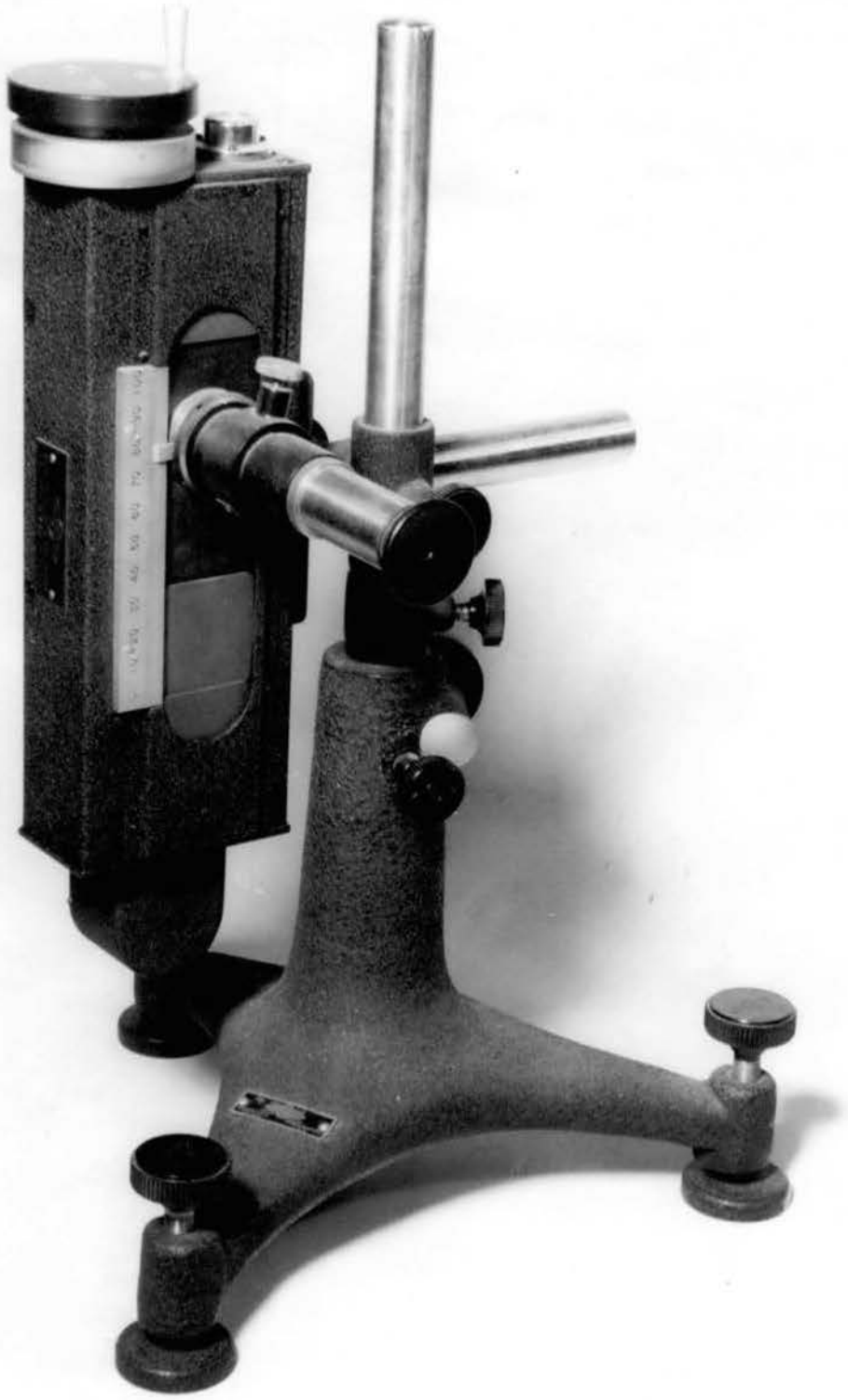

PLATE V

A"	5.74352
B"	5.84983
C"	5.96595
D"	6.36385
E"	7.90883
F"	8.03175
G"	8.14880
A"'	8.28087
B"'	8.33450
C"'	8.38753
D"'	8.76310

The letters correspond to the letters on Plate IV.

From this data the resulting line centers are:

3	0.94690
4	1.07984
5	1.23103
6	1.42355
1'	2.05170
2'	2.24432
3'	2.66361
4'	4.11259
5'	5.30170
6'	5.46494
1"	5.79668
2"	5.90789
3"	6.16490
4"	7.13634
5"	7.97029
6"	8.09028
1"'	8.30768
2"'	8.36102
3"'	8.57532

The numbers correspond to the numbers on Plate IV.

The second order reduction formula for separations in wave numbers is

$$\nu - \nu' = \frac{1}{2nt} \frac{(K'-K)}{(J'-K)-(J-K')} \left[\frac{(J'-K)}{(J-K)} - \frac{(J-K')}{(J'-K')} \right]$$

For the separation between lines designated 2 and 1,

$J' = 5.90789$, $J = 5.79668$, $K' = 2.24432$, $K = 2.05170$.

For the separation between the lines designated 3 and 2,

$J' = 6.16490$, $J = 5.79668$, $K' = 2.66361$, $K = 2.24432$.

For the separation between the lines designated 4 and 3, $J' = 7.13634$, $J = 6.16490$, $K' = 4.11259$, $K = 2.66361$.
For the separation between the lines designated 5 and 4, $J' = 7.97029$, $J = 7.13634$, $K' = 5.30170$, $K = 4.11259$.
For the separation between the lines designated 6 and 5, $J' = 8.09028$, $J = 7.97029$, $K' = 5.46494$, $K = 5.30170$.
Substituting these values in the above two order formula, we get the following separations:

(2-1) $0.038071/2nt = 0.009552$ cm.$^{-1}$

(3-2) $0.089002/2nt = 0.022331$ cm.$^{-1}$

(4-3) $0.358424/2nt = 0.089928$ cm.$^{-1}$

(5-4) $0.345792/2nt = 0.086759$ cm.$^{-1}$

(6-5) $0.054169/2nt = 0.013591$ cm.$^{-1}$

The value of 2nt is 3.9856 cm. as was mentioned in the previous discussion of the multiplex instrument used by Ritter and Lau.

The correlation between these values and those on page 10 as calculated from the quantum mechanical formulas will be mentioned later after considering the reductions of the line separations using the three order formulae.

The three order reduction formula is in three parts; the first for the outer order, the second for the middle order, and the third for the inner order. The separations due to each of the three parts are added and the total is then divided by three for the resulting separation.

The formulae are:

$$\nu - \nu' = \frac{1}{2nt}\frac{(J'-J)}{(K-L)}\left[\frac{(J'-L)}{(J-K)} - \frac{2(J'-K)}{(J-L)}\right]$$

$$\nu - \nu' = \frac{1}{2nt}\frac{(K'-K)}{(J-L)}\left[\frac{(J-K')}{(K-L)} + \frac{2(K'-L)}{(J-K)}\right]$$

$$\nu - \nu' = \frac{1}{2nt}\frac{(L'-L)}{(J-K)}\left[\frac{(J-L')}{(K-L)} - \frac{2(K-L')}{(J-L)}\right]$$

For the separation between the lines designated 2 and 1,

$J' = 8.36102 \quad K' = 5.90789 \quad L' = 2.24432$
$J = 8.30768 \quad K = 5.79668 \quad L = 2.05170$

For the separation between the lines designated 3 and 2,

$J' = 8.57532 \quad K' = 6.16490 \quad L' = 2.66361$
$J = 8.36102 \quad K = 5.90789 \quad L = 2.24432$

For the separation between the lines designated 4 and 3,

$J' = 7.13634 \quad K' = 4.11259 \quad L' = 1.07984$
$J = 6.16490 \quad K = 2.66361 \quad L = 0.94690$

For the separation between the lines designated 5 and 4,

$J' = 7.97029 \quad K' = 5.30170 \quad L' = 1.23103$
$J = 7.13634 \quad K = 4.11259 \quad L = 1.07984$

For the separation between the lines designated 6 and 5,

$J' = 8.09028 \quad K' = 5.46494 \quad L' = 1.42355$
$J = 7.97029 \quad K = 5.30170 \quad L = 1.23103$

Substituting these values in the three above formulas gives the following line separations:

(2-1)	0.010853 cm.$^{-1}$
(3-2)	0.052578 cm.$^{-1}$
(4-3)	0.079767 cm.$^{-1}$
(5-4)	0.083060 cm.$^{-1}$
(6-5)	0.008278 cm.$^{-1}$

CONCLUSIONS

The three sources of the line separations are tabulated below in $cm.^{-1}$

	Calculated	Two Order	Three Order
(2-1)	0.009633	0.009552	0.010853
(3-2)	0.023118	0.022331	
(4-3)	0.091260	0.089928	0.079767
(5-4)	0.085491	0.086759	0.083060
(6-5)	0.013485	0.013591	

The agreement between the calculated values and those due to the two order formula is very good. The largest percentage deviation from the average between these two values is 1.7%, which is for the (3-2) separation. In reality it is meaningless to list the calculated separations to more than two significant figures as the values of g_I for Hg 199 and Hg 201 are only given to two significant figures in the reference used, and the uncertainty in these cases is 10 and 3 per cent respectively. These uncertainties are minimum values. Within the uncertainty of the calculations we must conclude that the two order formula gives results well within the probable accuracy of the theoretical calculations.

The photograph used was really unsuitable for the

three order formulas. As mentioned earlier, the central fringe system is distorted. In the case of the (6-5) line, measurements cannot be made on the fourth order due to the edge of the plate cutting off the last half of the fourth order. This makes it necessary to measure the central fringe system, which makes the resulting separation for the (6-5) line somewhat doubtful because of the inherent difficulty of the central fringe distortion.

In the case of the fourth fringe system (counting the central order as the first) the lines designated 3 and 2 cannot be seen very clearly, hence this measurement was one involving some estimation. This would invalidate the measurements of the (3-2) lines and the adjacent lines (2-1) and (4-3) as well.

The lines 4 and 5 can be clearly seen in the fourth fringe system, so this measurement should be satisfactory. The (5-4) line separation does agree with the theory within 3% which is quite good, owing to the uncertainty in the nuclear g_I factors.

In view of these difficulties, no conclusive statement can be made about the validity of the three order formulas.

It can definitely be concluded that the two order formula and the theory agree very well. This would seem to indicate that the two order formula should prove to be very useful for reduction of observations.

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