WIND STRESS ANALYSIS OF ONE STORY BENTS
BY NEW DISTRIBUTION FACTOR

By

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THESIS AND ABSTRACT APPROVED:

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Jan Joseph Tuma

Table of Contents

Part I

General Theory

Table of Contents

Part II

Special Cases

Table of Contents

Part III

Illustrative Examples

P A R T I

General Theory

1. Introduction

The purpose of this thesis is to present a new type of Analysis of Wind Stresses for one-story bents. The work is based on Papers No. 2, 3, 4/50 worked out by the same author at the Oklahoma Institute of Technology in 1950.

The present method is dependent upon the system of balancing fixed-end moments.[1]

Each wind-moment is derived as an infinite, geometrical series. It is shown that each step forms one external and one internal circle, which in summation gives a very simple algebraic expression. The author calls this summation of the circles,

New Distribution Factor.

The total working moment multiplied by this factor gives the direct wind moment at the corresponding part of the loaded bent.

A problem given by Prof. J. E. Lothers in Graduate course 534 at OIT as a one hour test

was solved by this method in
1 minute and 48 seconds

by a student who had not read this thesis and had only been instructed to use the last four formulas.

The solution obtained by this
method is as precise as the
slope-deflection method and
varies from 10 to 42 times
shorter.

For checking his own numberical results, the author used the same two legged bent as that of Mr. Morris' of 20 years ago. The author's results were the same as Mr. Morris' Slope-deflection results with only one exception in the last moment - in column B at the bottom there was a very small difference: 0.0105%. Encouraged by some of the professors at the school, the author checked the old computation of Mr. Morris and found a small error in his numerical solution of elastic equations...an error equal to the difference of 0.0105%. The author's method is exact.

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[1] This method was developed in connection with the calculation of secondary stresses in trusses and is described in the book by O. Mohr "Abhandlugen aus dem Gebiete der Technischen Mechanik,"p. 429, 1906. In the USA, the method was first used by S. Hardesty and is fully explained in the book by J. A. L. Waddell, "Bridge Engineering," 1916. The extension of the method to the analysis of highly statically indeterminate frame structures is due to K. A. Calisev, who used it in analysis of building frames with and without lateral constraints. See "Technicne Listy," 1923, No. 17-21, Zagreb. A German translation of his paper appeared in Pub. Inter. Assoc. of Bridge Structural Eng., vol. 4, pp. 199-215, 1936. The final form of the method of successive approximations was obtained in the paper by H. Cross, Trans. ASCE, vol. 96, 1932 and in the same paper-discussion by C. T. Morris.

By means of this method:

(1) No advanced mathematics are used in practical computations.

(2) There are no simultaneous equations to solve.

By means of this method in illustrative example No. 1:

only 36 moves on the slide-rule were used and exact results reached.

By means of the Morris-Cross Method:

in 4 step approximation, 88 moves on the slide-rule were used and the results differed 1%.

These points show as well as any explanation the big advantage of the new method.

2. Wind Analysis - general notes

Internal stresses involved by lateral pressure of the wind are being computed by three types of methods:

I. Very Approximate (max. dif.: 10-30%)

II. Approximate (max. dif.: 6-10%)

III. Exact (classic) (no dif.)

I. Very approximate methods working with max. error,[1] 10-30% are:

(a). A. Smith Method.[2]

(b). Fleming's Method I-III.[3]

II. Approximate methods working with max. error, 6-10%, are:

(a). Morris-Cross Method.[4]

(b). Morris-Cross Method Simplified.[5]

(c). Grinters Correction moments Method.[6]

(d). Morris-Ross Method.[7]

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1 Maximum error is the maximum difference between the results worked out by the very approximate method and the classic method.

2 Theory of Frameworks with Rectangular Panels, Transactions ASCE 1915, vol. 55, p. 418.

3 Engineering News, March 13, 1913.

4 Transactions ASCE 1932, p. 66.

5 Theory of Modern Steel Structures, by L. Grinter, New York 1949, p. 123.

6 Ibid, p. 122.

7 The design of Tall Building Frames to Resist Wind, by C. Morris and A. Ross, Jr. The Engineering Experiment Station Bulletin No. 48, 1929; State University of Ohio.

III. Exact (classic) methods working without any error (theor.) are the following:

(a). Slope deflection method.[8]

(b). Work methods.[9]

The methods of the first group are for rapid calculation and are so inaccurate that their use is limited to only some types of structures and usually for preliminary analysis only.

The methods of the second group are more exact but the maximum deviation of the classic solution is significant enough if only a few steps are taken. They are nearly correct when many steps of distribution are applied. In such cases, when many steps are required to obtain the desired accuracy, the use of classic methods is to be recommended.[10]

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[8] Bulletin No. 80, University of Illinois, 1915 by Wilson and Maney - For symetrical, three-span bent, 12 stories high. These calculations are summarized in 12 tables covering 31 pages. Calculations involve 60 simultaneous equations with 60 unknowns. The method is perfectly feasible, but for practical use unworkable.

[9] Stress in Tall Buildings, Bulletin No. 8, College of Engineering, Ohio State University, by Cyrus Melick, 1918. The method was developed for a 4 story building (6 weeks work); for more, it is unworkable.

[10] Wind Stress Analysis simplified by L. E. Grinter, Transactions ASCE, vol. 99, 1934, a discussion by Raymond C. Reese, p. 649; "The method of starting with no wind moments in the girders and gradually transferring from the columns into the girders will bring convergence fairly rapidly in simple, symmetrical bents in which the relative stiffnesses of the different members do not vary too much. When the members vary in their relative stiffness, the method of starting with no moments in the girders is too slow and tedious a process. In this particular case, twenty-three cycles of operations failed to come very close to the desired results."

3. Base for NDF

The author's idea was to find an easy, short method of wind stress analysis which could produce results close to those of the exact methods.

The author hoped to find the easiest method in the same way as presented in his paper,

"New Elastic Theory."

This New Elastic Theory solves the beam-, girder-, and column-moments by direct multiplication of Fixed End Moment and a special factor (Stupen vetknuti .. Einspannung Grad .. which could be translated as the "grade of fixing").

This idea expressed mathematically should be:

When

M = Total working moment
= Total shear times story-height then:

M times Factor of fixed end is equal to the resisting moment at the corresponding end of the member in question.

This special factor was really found and is called

Final Distribution Factor.

A ratio of

New Distribution Factor: of all column-NDF of the computed story.

The whole paper is an algebraic derivation of these three types of factors:

(1). New Distribution Factor.

(2). Summation of all Stories NDF.

and (3). Final Distribution Factor.

The base for derivations of these three types of factors is the idea of geometric deformation by an infinite distribution in every step of successive correction.

Partial realization of this idea can be found in the Cross-Morris Method of Wind Analysis. It was shown by Mr. Morris that the idea of geometric deformation applied in Wind Analysis must be combined with the method of successive correction by additional moments. (The n-step distribution method by Cross gives lower resting moments as it is the value of the total working moment). The Morris expression for this case is: Loss of moment involved by distribution.

Mr. Morris tries to solve this loss of moment by additional moments added to the first working moment in n-external circles (n-correction's steps). By this new approximation, the errors increase doubly:

(1). Cross n-step distribution (not finished appr.)

(2). Morris n-step circles (not finished appr.)

and by unprecise distribution.

These errors of the Morris-Cross Method try to correct Mr. Grinter by his Corrected Method based on the New Working Moment. The working moment corrected is: Total shear times stories height times correction factor (usually - 50%).[1]

This method is a practical short-cut of the Morris-Cross Method and gives results much closer (Max. error: 4%). But the error is not defined and the method with his 3rd grade of approximation cannot be called a scientific method.[2]

The author chose another way to eliminate all these possible errors and to get results very close to those of slope deflection. He works the whole problem with general algebraic symbols, looking for general relations between moments and stiffness factors of members and transforms these relations in the final formulas, which are multiplication factors of the total working moment.

The author was very surprised when, at the end of his long and sometimes very difficult research, the investigations gave very simple formulas which can be used in any practical problem and solved by anyone who knows the basic operations of arithmetic.

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[1] This method is the most popular method in the USA at the present time.

[2] The author shows, in his Engineering Paper No. Tu 4/1950, that this Correction factor can be found exactly and is equal to the ratio of

$$\frac{1}{\sum NDF}$$

4. A summation for NDF

(1). Calculate the moments in the columns due to the lateral forces, considering the joints fixed against rotation, but free to deflect laterally.

(2). The sum of the moments at the top and bottom of all the columns of a story is equal to the shear in the story, multiplied by the story height, and, as the deflections of the columns in the story due to the lateral forces are equal, the column moments and shears are proportional to the values

$$\frac{I}{L^2}$$ of the columns.[1]

(3). Distribute the moments at the joints, considering them free to rotate but not changing their location.

(4). Carry over the distributed moments using a carry-over factor.

(5). Balance the column moments in each story by making their sum equal to the shear in the story times the story height (total working moment for the story).

(6). The difference between the sum of all the first circle moments and the working moment is called the first moment difference.

(7). This completes the first external circle and the next external circle for distribution of the first moment difference can be repeated.

(8). Internal circles (distribution of each working moment) and external circles (first step dif. moment, second step dif. moments, etc.) form infinite series.

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[1] When all columns of a story are of equal height, the value of

$$\frac{I}{L^2} \text{ can be replaced by } \frac{I}{L} .$$

5. Nomenclature

a = Stiffness Factor of member I.-II.
b = Stiffness Factor of member III.-IV.
c = Stiffness Factor of member V.-VI.
d = Stiffness Factor of member VII.-VIII.
$d_{(1,2)}$ = Distance of conc. load from the column's ends.
$d_{(I,II)}$ = Distribution Constant.
k = Stiffness Factor of any member.
n = Any number.
p = Internal Factor of NDF.
q = Internal Factor of NDF.
r = Ratio of two successive members of geom. series.
u = Column ratio ($L_1:L_3$).
w = Fix End Moment ratio ($FEM_I:FEM_{II}$).
x = a + b.
y = b + c.
z = a + c.

A,B,C,D,E,F,G,H = Members of geometric series.
N = Member of any geometric series.
S = Sum of any geometric series.
I = Moment of inertia.
K = Total New Factor of Distribution (NDF).
L = Length of member.
M = Total working moment.
V = Total working shear.

$K_{(I,II,III,IV,V,VI,VII,VIII)}$ = New Distribution Factor (NDF) at point I, or II, or III, ...
$\sum K$ = Sum of all column-NDF.
M_{D_n} = n-circle moment difference.
U_{I_1} = First circle moment at I.
U_{I_n} = n-th circle moment at I.
$\frac{K_I}{\sum K}$ = Final NDF at I.
M_1 = Starting moment at column I.-II.
M_2 = Starting moment at column V.-VI.
$M_{(I,II,III,IV,V,VI,VII,VIII)}$ = Final column or girder moments at point I, or II, or III, ...

6. Investigated bent

For theoretical investigation a simple two-legged bent with the bottom of the columns fixed will be used. The load and dimensions are shown on Fig. 1. The stiffness factors are a, b, c.

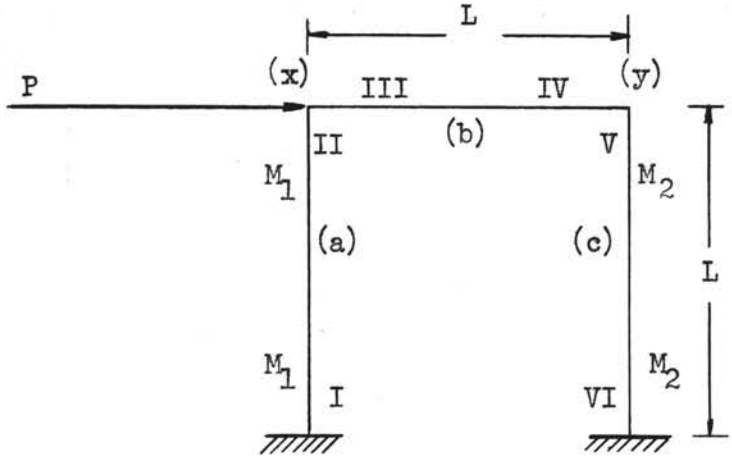

Fig. 1

The wind pressure is working from the left towards the right side and involves total working moment[1]

$$M = PL \tag{1}$$

According to the shear equation the total working moment is equal to the sum of all column moments

$$M = M_I + M_{II} + M_V + M_{VI} \tag{2}$$

The total working moment will be distributed to 4 starting moments (Fig. 1)

$$M_1 = \frac{Ma}{2z} \tag{3}$$

$$M_2 = \frac{Mc}{2z} \tag{4}$$

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[1] Expression "total working moment" is a special term for this work and is based on the shear equation. In reality this moment does not exist.

7. First external and internal circle

Determination of M by eq. (1) will be the base for the first external and internal circles. First internal circle is an infinite distribution of M_1 and M_2 by the Cross Method and is forming an infinite series of members. Sum of one series will be called the first circle moment.

$$U_{I_1} \quad ; \quad U_{II_1} \quad ; \quad U_{V_1} \quad ; \quad U_{VI_1}$$

Sum of column-first circle moments will be called first external circle. Difference between total working moment and first external circle is

$$M_{D_1} = M - U_{I_1} - U_{II_1} - U_{V_1} - U_{VI_1} \tag{5a}$$

and will be called first moment difference.

Figs. 2 and 3 show graphically the second and nth circle.

First moment difference replaces the function of the base in the second circle for external and internal circle then the whole procedure of the 1st circle will be repeated for the 2nd circle. The second moment difference then replaces the function of the base and the procedure can be repeated for the third time, etc.

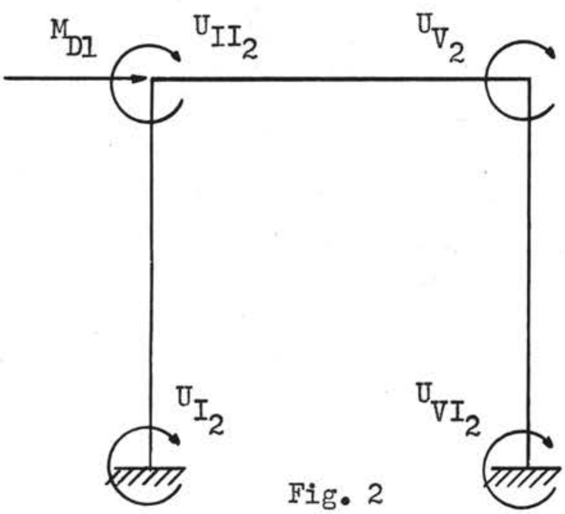

Fig. 2

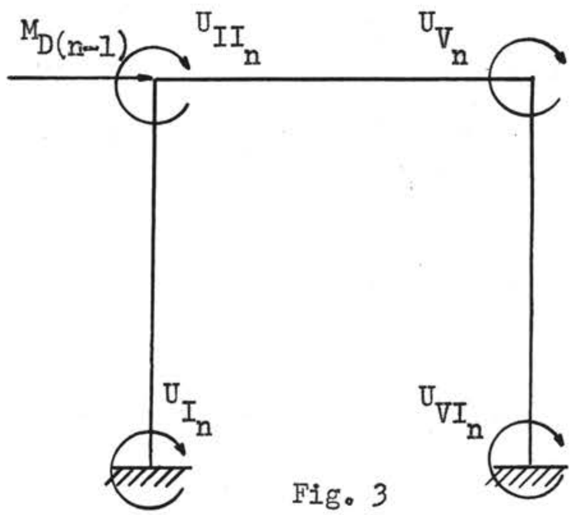

Fig. 3

$$M_{D2} = M_{D_1} - U_{I_2} - U_{II_2} - U_{V_2} - U_{VI_2} \tag{5b}$$

$$M_{Dn} = M_{D(n-1)} - U_{I_n} - U_{II_n} - U_{V_n} - U_{VI_n} \tag{5}$$

8. First Internal Circle Distribution

U_{I_1}	U_{II_1}	U_{III_1}	U_{IV_1}	U_{V_1}	U_{VI_1}
$-M_1$	$-M_1$			$-M_2$	$-M_2$
$\frac{M_1a}{2x}$	$\frac{M_1a}{x}$	$\frac{M_1b}{x}$	$\frac{M_2b}{y}$	$\frac{M_2c}{y}$	$\frac{M_2c}{2y}$
		$\frac{M_2b}{2y}$	$\frac{M_1b}{2x}$		
$-\frac{M_2ab}{4xy}$	$-\frac{M_2ab}{2xy}$	$-\frac{M_2b}{2xy}$	$-\frac{M_1b}{2xy}$	$-\frac{M_1bc}{2xy}$	$-\frac{M_1bc}{4xy}$
		$-\frac{M_1b^2}{4xy}$	$-\frac{M_2b^2}{4xy}$		
$\frac{M_1ab^2}{8x^2y}$	$\frac{M_1ab^2}{4x^2y}$	$\frac{M_1b^3}{4x^2y}$	$\frac{M_2b^3}{4xy^2}$	$\frac{M_2b^2c}{4xy^2}$	$\frac{M_2b^2c}{8xy^2}$
		$\frac{M_2b^3}{8x^2y}$	$\frac{M_1b^3}{8xy^2}$		
$-\frac{M_2ab^3}{16x^2y^2}$	$-\frac{M_2ab^3}{8x^2y^2}$	$-\frac{M_2b^4}{8x^2y^2}$	$-\frac{M_1b^4}{8x^2y^2}$	$-\frac{M_1b^3c}{8x^2y^2}$	$-\frac{M_1b^3c}{16x^2y^2}$
		$-\frac{M_1b^4}{16x^2y^2}$	$-\frac{M_2b^4}{16x^2y^2}$		
$\frac{M_1ab^4}{32x^3y^2}$	$\frac{M_1ab^4}{16x^3y^2}$	$\frac{M_1b^5}{16x^3y^2}$	$\frac{M_2b^5}{16x^2y^3}$	$\frac{M_2b^4c}{16x^2y^3}$	$\frac{M_2b^4c}{32x^2y^3}$
		0	0		
0	0	0	0	0	0

Fig. 4

Table of Series A and B

Series A			Series B		
No. of Member	Symbol of Member	Algebraic Value	No. of Member	Symbol of Member	Algebraic Value
1	A_1	$\frac{M_1 ab^o}{2\ x\ y^o}$	1	B_1	$-\frac{M_2 ab}{4\ x\ y}$
2	A_2	$\frac{M_1 ab^2}{2^3 x^2 y}$	2	B_2	$-\frac{M_2 ab^3}{4^2 x^2 y^2}$
3	A_3	$\frac{M_1 ab^4}{2^5 x^3 y^2}$	3	B_3	$-\frac{M_2 ab^5}{4^3 x^3 y^3}$
4	A_4	$\frac{M_1 ab^6}{2^7 x^4 y^3}$	4	B_4	$-\frac{M_2 ab^7}{4^4 x^4 y^4}$
5	A_5	$\frac{M_1 ab^8}{2^9 x^5 y^4}$	5	B_5	$-\frac{M_2 ab^9}{4^5 x^5 y^5}$
⋮	⋮	⋮	⋮	⋮	⋮
n-1	A_{n-1}	$\frac{M_1 ab^{2(n-2)}}{2^{2n-3} x^{n-1} y^{n-2}}$	n-1	B_{n-1}	$-\frac{M_2 ab^{2n-3}}{4^{n-1} x^{n-1} y^{n-1}}$
n	A_n	$\frac{M_1 ab^{2(n-1)}}{2^{2n-1} x^n y^{n-1}}$	n	B_n	$-\frac{M_2 ab^{2n-1}}{4^n x^n y^n}$
⋮	⋮	⋮	⋮	⋮	⋮
∞	A_∞	0	∞	B_∞	0

Fig. 5

9. Investigation of moment U_{I_1}

Exact investigation of all members forming column-first circle moment U_{I_1}
shows that it is a function of three algebraic groups

$$U_{I_1} = - M_1 + \sum_{0}^{\infty} A + \sum_{0}^{\infty} B \qquad (6a)$$

The starting moment (M_1) was determined from eq. (3). The summations of the (A) members and (B) members (Fig. 5) are to be investigated here.

The derivations of the (A) and (B) series taken from Fig. 4 and distributed and presented by Fig. 5 show that both series are:

(a). infinite series - number of members is infinite.

(b). convergent series - condition of convergency:

$$A_{n-1} > A_n \quad ; \quad B_{n-1} > B_n$$

(c). geometric series - ratio of two successive members is constant:

$$r_A = \frac{A_n}{A_{n-1}} = \frac{b^2}{4xy} \qquad (6b)$$

$$r_B = \frac{B_n}{B_{n-1}} = \frac{b^2}{4xy} \qquad (6c)$$

Using the formula for the sum of members of the infinite convergent geometric series

$$\sum_{0}^{\infty} N = S_N = \frac{N}{1 - r_N}$$

For our series the sums are:

$$\sum_{0}^{\infty} A = S_A = \frac{\dfrac{M_1 ab^o}{2xy^o}}{1 - \dfrac{b^2}{4xy}} \qquad (6d)$$

and

$$\sum_{0}^{\infty} B = S_B = - \frac{\dfrac{M_2 ab}{4xy}}{1 - \dfrac{b^2}{4xy}} \qquad (6e)$$

By substitution of eqs. (3), (4), (6d) and (6e) into eq. (6a) the relation between the column-first circle moment and stiffness of the members of the bent can be determined.

$$U_{I_1} = - M_1 + \left[\frac{\dfrac{M_1 ab^o}{2xy^o}}{1 - \dfrac{b^2}{4xy}} \right] - \left[\frac{\dfrac{M_2 ab}{4xy}}{1 - \dfrac{b^2}{4xy}} \right]$$

$$= - M_1 + \left[\frac{2M_1 ay - M_2 ab}{4xy - b^2} \right] \qquad (6f)$$

$$= - \frac{Ma}{2z} + \left[\frac{2\dfrac{Ma}{2z}ay - \dfrac{Mc}{2z}ab}{4xy - b^2} \right]$$

$$= M\frac{a}{2z} \left(\frac{2ay - bc}{4xy - b^2} - 1 \right) \qquad (6g)$$

The expression

$$\frac{2ay - bc}{4xy - b^2} = p \qquad (6i)$$

is the internal factor (p) and

$$\frac{a}{2z} = d_{I-II} \qquad (6j)$$

is the distribution constant.

A function of the internal factor and distribution constant is

$$d_{I-II} (p - 1) = K_I \qquad (6h)$$

and will be called

<u>NEW DISTRIBUTION FACTOR FOR I = NDF_I</u>

Total working moment (M) multiplied by NDF_I gives the direct column-first circle moment.

$$\underline{U_{I_1} = MK_I} \qquad (6)$$

Table of Series C and D

Series C			Series D		
No. of Member	Symbol of Member	Algebraic Value	No. of Member	Symbol of Member	Algebraic Value
1	C_1	$\frac{M_1 ab^o}{2^o xy^o}$	1	D_1	$-\frac{M_2 ab}{2\ x\ y}$
2	C_2	$\frac{M_1 ab^2}{2^2 x^2 y}$	2	D_2	$-\frac{M_2 ab^3}{2^3 x^2 y^2}$
3	C_3	$\frac{M_1 ab^4}{2^4 x^3 y^2}$	3	D_3	$-\frac{M_2 ab^5}{2^5 x^3 y^3}$
4	C_4	$\frac{M_1 ab^6}{2^6 x^4 y^3}$	4	D_4	$-\frac{M_2 ab^7}{2^7 x^4 y^4}$
5	C_5	$\frac{M_1 ab^8}{2^8 x^5 y^4}$	5	D_5	$-\frac{M_2 ab^9}{2^9 x^5 y^5}$
⋮	⋮	⋮	⋮	⋮	⋮
n-1		$\frac{M_1 ab^{2(n-2)}}{2^{2(n-2)} x^{n-1} y^{n-2}}$	n-1	D_{n-1}	$-\frac{M_2 ab^{2n-3}}{2^{2n-3} x^{n-1} y^{n-1}}$
n		$\frac{M_1 ab^{2(n-1)}}{2^{2(n-1)} x^n y^{n-1}}$	n	D_n	$-\frac{M_2 ab^{2n-1}}{2^{2n-1} x^n y^n}$
⋮	⋮	⋮	⋮	⋮	⋮
∞	C_∞	0	∞	D_∞	0

Fig. 6

10. Investigation of moment U_{II_1}

Exact investigation of all members forming column-first circle moment U_{I_1}
shows that it is a function of three algebraic groups

$$U_{I_1} = -M_1 + \sum_{0}^{\infty} C + \sum_{0}^{\infty} D \qquad (7a)$$

The starting moment (M_1) was determined from eq. (3). The summations of the (C) members and (D) members (Fig. 6) are to be investigated here.

The derivations of the (C) and (D) series taken from Fig. 4 and distributed and presented by Fig. 6 show that both series are:

(a). infinite series - number of members is infinite.

(b). convergent series - condition of convergency:

$$C_{n-1} > C_n \quad ; \quad D_{n-1} > D_n$$

(c). geometric series - ratio of two successive members is constant:

$$r_C = \frac{C_n}{C_{n-1}} = \frac{b^2}{4xy} \qquad (7b)$$

$$r_D = \frac{D_n}{D_{n-1}} = \frac{b^2}{4xy} \qquad (7c)$$

Using the formula for the sum of members of the infinite convergent geometric series

$$\sum_{0}^{\infty} N = S_N = \frac{N}{1 - r_N}$$

For our series the sums are:

$$\sum_{0}^{\infty} C = S_C = \frac{\frac{M_1 ab^0}{2^0 xy^0}}{1 - \frac{b^2}{4xy}} \tag{7d}$$

and

$$\sum_{0}^{\infty} D = S_D = - \frac{\frac{M_2 ab}{2xy}}{1 - \frac{b^2}{4xy}} \tag{7e}$$

By substitution of eqs. (3), (4), (7d) and (7e) into eq. (7a) the relation between the column-first circle moment and stiffness of the members of the bent can be determined.

$$U_{I_1} = - M_1 + \left[\frac{\frac{M_1 ab^0}{2^0 xy^0}}{1 - \frac{b^2}{4xy}} \right] - \left[\frac{\frac{M_2 ab}{2xy}}{1 - \frac{b^2}{4xy}} \right]$$

$$= - M_1 + \left[\frac{2M_1 ay}{4xy} \;\; \frac{-}{-} \;\; \frac{M_2 ab}{b^2} \, 2 \right] \tag{7f}$$

$$= - \frac{Ma}{2z} + \left[\frac{2\frac{Ma}{2z}ay}{4xy} \;\; \frac{-}{-} \;\; \frac{\frac{Mc}{2z}ab}{b^2} \, 2 \right]$$

$$= M\frac{a}{2z} \left(2 \, \frac{2ay - bc}{4xy - b^2} - 1 \right) \tag{7g}$$

The expression

$$\frac{2ay - bc}{4xy - b^2} = p \qquad (6i) = (7i)$$

is the internal factor (p) and

$$\frac{a}{2z} = d_{I-II} \qquad (6j) = (7j)$$

is the distribution constant.

The function of the internal factor and distribution constant is

$$d_{I-II}\ (2p - 1) = K_{II} \qquad (7h)$$

and will be called

<u>NEW DISTRIBUTION FACTOR FOR II = NDF_{II}</u>

Total working moment (M) multiplied by NDF_{II} gives the direct column-first circle moment

$$\underline{U_{II_1} = MK_{II}} \qquad (7)$$

11. Investigation of moments U_{III_1} and U_{IV_1}

Beam-first circle moments according to conditions of static equilibrium must be equal to their corresponding column-first circle moments (equilibrium in every joint) and we can assume that

$$U_{III_1} = -U_{II_1} \quad (8a)$$

and

$$U_{IV_1} = -U_{V_1} \quad (9a)$$

Substituting from eqs. (7) and (10) into eqs. (8a) and (9a) we derive directly the formulas for beam-first circle moments

$$\underline{U_{III_1} = -MK_{II}} \quad (8)$$

and

$$\underline{U_{IV_1} = -MK_{V}} \quad (9)$$

Other investigations at points III and IV are not necessary.

Table of Series E and F

	Series E			Series F	
No. of Member	Symbol of Member	Algebraic Value	No. of Member	Symbol of Member	Algebraic Value
1	E_1	$\frac{M_2 b^o c}{2^o x^o y}$	1	F_1	$-\frac{M_1 b\ c}{2\ x\ y}$
2	E_2	$\frac{M_2 b^2 c}{2^2 x\ y^2}$	2	F_2	$-\frac{M_1 b^3 c}{2^3 x^2 y^2}$
3	E_3	$\frac{M_2 b^4 c}{2^4 x^2 y^3}$	3	F_3	$-\frac{M_1 b^5 c}{2^5 x^3 y^3}$
4	E_4	$\frac{M_2 b^6 c}{2^6 x^3 y^4}$	4	F_4	$-\frac{M_1 b^7 c}{2^7 x^4 y^4}$
5	E_5	$\frac{M_2 b^8 c}{2^8 x^4 y^5}$	5	F_5	$-\frac{M_1 b^9 c}{2^9 x^5 y^5}$
⋮	⋮	⋮	⋮	⋮	⋮
n-1	E_{n-1}	$\frac{M_2 b^{2(n-2)} c}{2^{2(n-2)} x^{n-2} y^{n-1}}$	n-1	F_{n-1}	$-\frac{M_1 b^{2n-3} c}{2^{2n-2} x^{n-1} y^{n-1}}$
n	E_n	$\frac{M_2 b^{2(n-1)} c}{2^{2(n-1)} x^{n-1} y^n}$	n	F	$-\frac{M_1 b^{2n-1} c}{2^{2n-1} x^n y^n}$
⋮	⋮	⋮	⋮	⋮	⋮
∞	E_∞	0	∞	F_∞	0

Fig. 7

12. Investigation of moment U_{V_1}

Exact investigation of all members forming column-first circle moment U_{V_1}

shows that it is a function of three algebraic groups

$$U_{I_1} = -M_2 + \sum_0^\infty E + \sum_0^\infty F \tag{10a}$$

The starting moment (M_2) was determined from eq. (4). The summations of the (E) members and (F) members (Fig. 7) are to be investigated here.

The derivations of the (E) and (F) series taken from Fig. 4 and distributed and presented by Fig. 7 show that both series are:

(a). infinite series - number of members is infinite.

(b). convergent series - condition of convergency:

$$E_{n-1} > E_n \; ; \; F_{n-1} > F_n$$

(c). geometric series - ratio of two successive members is constant:

$$r_E = \frac{E_n}{E_{n-1}} = \frac{b^2}{4xy} \tag{10b}$$

$$r_F = \frac{F_n}{F_{n-1}} = \frac{b^2}{4xy} \tag{10c}$$

Using the formula for the sum of members of the infinite convergent geometric series

$$\sum_0^\infty N = S_N = \frac{N}{1 - r_N}$$

For our series the sums are:

$$\sum_0^\infty E = S_E = \frac{\dfrac{M_2 b^0 c}{2^0 x^0 y}}{1 - \dfrac{b^2}{4xy}} \tag{10d}$$

and

$$\sum_0^\infty F = S_F = -\frac{\dfrac{M_1 bc}{2xy}}{1 - \dfrac{b^2}{4xy}} \tag{10e}$$

By substitution of eqs. (3), (4), (10d) and (10e) into eq. (10a) the relation between the column-first circle moment and stiffness of the members of the bent can be determined.

$$U_{V_1} = -M_2 + \left[\frac{\dfrac{M_2 b^0 c}{2^0 x^0 y}}{1 - \dfrac{b^2}{4xy}}\right] - \left[\frac{\dfrac{M_1 bc}{2xy}}{1 - \dfrac{b^2}{4xy}}\right]$$

$$= -M_2 + \left[\frac{2M_2 cx - M_1 bc}{4xy - b^2} 2\right] \tag{10f}$$

$$= -\frac{Mc}{2z} + \left[\frac{2\dfrac{Mc}{2z}cx - \dfrac{Ma}{2x}bc}{4xy - b^2} 2\right]$$

$$= M\frac{c}{2z} \left(2\frac{2xc - ab}{4xy - b^2} - 1 \right) \tag{10g}$$

The expression

$$\frac{2xc - ab}{4xy - b^2} = q \qquad (10i)$$

is the internal factor (q) and

$$\frac{c}{2z} = d_{V-VI} \qquad (10j)$$

is the distribution constant.

The function of the internal factor and distribution constant is

$$d_{V-VI} \; (2q - 1) = K_{II} \qquad (10h)$$

and will be called

<u>NEW DISTRIBUTION FACTOR FOR V = NDF_V</u>

Total working moment (M) multiplied by NDF_V gives the direct column-first circle moment

$$\underline{U_{V_1} = MK_V} \qquad (10)$$

Table of Series G and H

Series G			Series H		
No. of Member	Symbol of Member	Algebraic Value	No. of Member	Symbol of Member	Algebraic Value
1	G_1	$\frac{M_2 b^o c}{2\, x^o y}$	1	H_1	$-\frac{M_1 b\, c}{4\, x\, y}$
2	G_2	$\frac{M_2 b^2 c}{2^3 x\, y^2}$	2	H_2	$-\frac{M_1 b^3 c}{4^2 x^2 y^2}$
3	G_3	$\frac{M_2 b^4 c}{2^5 x^2 y^3}$	3	H_3	$-\frac{M_1 b^5 c}{4^3 x^3 y^3}$
4	G_4	$\frac{M_2 b^6 c}{2^7 x^3 y^4}$	4	H_4	$-\frac{M_1 b^7 c}{4^4 x^4 y^4}$
5	G_5	$\frac{M_2 b^8 c}{2^9 x^4 y^5}$	5	H_5	$-\frac{M_1 b^9 c}{5^5 x^5 y^5}$
⋮	⋮	⋮	⋮	⋮	⋮
n-1	G_{n-1}	$\frac{M_2 b^{2(n-2)} c}{2^{2n-3} x^{n-2} y^{n-1}}$	n-1	H_{n-1}	$-\frac{M_1 b^{2n-3} c}{4^{n-1} x^{n-1} y^{n-1}}$
n	G_n	$\frac{M_2 b^{2(n-1)} c}{2^{2n-1} x^{n-1} y^{n}}$	n	H_n	$-\frac{M_1 b^{2n-1} c}{4^{n} x^{n} y^{n}}$
⋮	⋮	⋮	⋮	⋮	⋮
∞	G_∞	0	∞	H_∞	0

Fig. 8

13. Investigation of moment U_{VI_1}

Exact investigation of all members forming column-first circle moment U_{VI_1}
shows that it is a function of three algebraic groups

$$U_{VI_1} = - M_2 + \sum_0^\infty G + \sum_0^\infty H \qquad (11a)$$

The starting moment (M_2) was determined from eq. (4). The summations of the (G) members and (H) members (Fig. 8) are to be investigated here.

The derivations of the (G) and (H) series taken from Fig. 4 and distributed and presented by Fig. 8 show that both series are:

(a). infinite series - number of members is infinite.

(b). convergent series - condition of convergency:

$$G_{n-1} > G_n \; ; \; H_{n-1} > H_n$$

(c). geometric series - ratio of two successive members is constant:

$$r_G = \frac{G_n}{G_{n-1}} = \frac{b^2}{4xy} \qquad (11b)$$

$$r_H = \frac{H_n}{H_{n-1}} = \frac{b^2}{4xy} \qquad (11c)$$

Using the formula for the sum of members of the infinite convergent geometric series

$$\sum_0^\infty N \doteq S_N = \frac{N}{1 - r_N}$$

For our series the sums are:

$$\sum_{0}^{\infty} G = S_G = \frac{\dfrac{M_2 b^o c}{2x^o y}}{1 - \dfrac{b^2}{4xy}} \tag{11d}$$

and

$$\sum_{0}^{\infty} H = S_H = -\frac{\dfrac{M_1 bc}{4xy}}{1 - \dfrac{b^2}{4xy}} \tag{11e}$$

By substitution of eqs. (3), (4), (11d) and (11e) into eq. (11a) the relation between the column-first circle moment and stiffness of the members of the bent can be determined.

$$U_{VI_1} = -M_2 + \left[\frac{\dfrac{M_2 b^o c}{2x^o y}}{1 - \dfrac{b^2}{4xy}} \right] - \left[\frac{\dfrac{M_1 bc}{4xy}}{1 - \dfrac{b^2}{4xy}} \right]$$

$$= -M_2 + \left[\frac{2M_2 cx - M_1 bc}{4xy - b^2} \right] \tag{11f}$$

$$= -\frac{Mc}{2z} + \left[\frac{2\dfrac{Mc}{2z}cx - \dfrac{Ma}{2z}bc}{4xy - b^2} \right]$$

$$= M\frac{c}{2z} \left(\frac{2cx - ab}{4xy - b^2} - 1 \right) \tag{11g}$$

The expression

$$\frac{2cx - ab}{4xy - b^2} = q \qquad (10i) = (11i)$$

is the internal factor (q) and

$$\frac{c}{2z} = d_{V-VI} \qquad (10j) = (11j)$$

is the distribution constant.

The function of the internal factor and distribution constant is

$$d_{V-VI} (q - 1) = K_{VI} \qquad (11h)$$

and will be called

NEW DISTRIBUTION FACTOR FOR VI $= NDF_{VI}$

Total working moment (M) multiplied by NDF_{VI} gives the direct column-first circle moment

$$U_{VI_1} = MK_{VI} \qquad (11)$$

14. Conclusions of the first circle

Investigations of the first circle moments in points I, II, V, VI result in the following conclusions:

(1). Any column-first circle moment is equal to the total working moment times the corresponding New Distribution Factor

$$^{U}(I,II,V,VI)_1 = {}^{MK}(I,II,V,VI) \qquad (12)$$

(2). The New Distribution Factor is a function of the distribution constant and internal factor

$$K_I = \frac{a}{2z}(p-1) \qquad (6h)$$

$$K_{II} = \frac{a}{2z}(2p-1) \qquad (7h)$$

$$K_V = \frac{c}{2z}(2q-1) \qquad (10h)$$

$$K_{VI} = \frac{c}{2z}(q-1) \qquad (11h)$$

(3). First moment difference according to eqs. (5a), (7), (8), (10) and (11) is

$$M_{D_1} = M - MK_I - MK_{II} - MK_V - MK_{VI}$$

$$= M(1 - K_I - K_{II} - K_V - K_{VI})$$

The expression

$$(1 - K_I - K_{II} - K_V - K_{VI}) = K \qquad (13)$$

and the first moment difference

$$\underline{M_{D_1} = MK} \qquad (14)$$

15. Investigation of second circle

According to Chapter 7, in the second circle the first moment difference (M_{D_1}) replaces the base for the external and internal circles and the whole procedure of the first circle can be repeated with this new base.

Using Figs. 4, 5, 6, 7, 8 transformed for the second circle, in the condition of transformation,

$$(M) \text{ will be replaced by } M_{D_1} = MK \tag{14}$$

and using the conclusions of the first circle from Chapter 14 every column-second circle moment can be determined directly.

The column-second circle moments are:

$$U_{I_2} = M_{D_1} K_I = MKK_I \tag{15}$$

$$U_{II_2} = M_{D_1} K_{II} = MKK_{II} \tag{16}$$

$$U_{V_2} = M_{D_1} K_V = MKK_V \tag{17}$$

$$U_{VI_2} = M_{D_1} K_{VI} = MKK_{VI} \tag{18}$$

The second moment difference according to eqs. (5b),(15), (16), (17), (18) and (13) is

$$M_{D_2} = M_{D_1} - U_{I_2} - U_{II_2} - U_{V_2} - U_{VI_2} \tag{5b}$$

$$= MK - MKK_I - MKK_{II} - MKK_V - MKK_{VI} \tag{19a}$$

$$= MK\ (\ 1 - K_I - K_{II} - K_V - K_{VI}\ ) \tag{19b}$$

$$M_{D_2} = MK^2 \tag{19}$$

16. Investigation of nth circle

For our investigation it is important to derive general formulas for any column-circle moment and for any moment difference. The procedures of Chapters 9 - 14 can be repeated.[1]

Thus column-third circle moments are:

$$U_{I_3} = MK^2K_I \tag{20}$$

$$U_{II_3} = MK^2K_{II} \tag{21}$$

$$U_{V_3} = MK^2K_V \tag{22}$$

$$U_{VI_3} = MK^2K_{VI} \tag{23}$$

and the third moment difference is again

$$M_{D_3} = MK^3 \tag{24}$$

Column-fourth circle moments are:

$$U_{I_4} = MK^3K_I \tag{25}$$

$$U_{II_4} = MK^3K_{II} \tag{26}$$

$$U_{V_4} = MK^3K_V \tag{27}$$

$$U_{VI_4} = MK^3K_{VI} \tag{28}$$

and fourth moment difference is again

$$M_{D_4} = MK^4 \tag{29}$$

---

[1] Complete derivations of the third and fourth circle can be found in the author's Engineering Paper Tu - No.2/50, Oklahoma Institute of Technology, Stillwater 1950.

Column-nth circle moments are:

$$U_{I_n} = MK^{(n-1)}K_I \qquad (30)$$

$$U_{II_n} = MK^{(n-1)}K_{II} \qquad (31)$$

$$U_{V_n} = MK^{(n-1)}K_V \qquad (32)$$

$$U_{VI_n} = MK^{(n-1)}K_{VI} \qquad (33)$$

and nth moment difference is

$$M_{D_n} = MK^n \qquad (34)$$

17. Conclusions concerning circle moments

Closing the investigation of column-circle moments we can derive the following final principles:

(1). column-nth circle moment is equal to the total working moment (M)

times

(n-1) power of total New Distribution Factor (K)

times

New Distribution Factor for (I, II, V, VI)

$$U^{1}_{(I,\ II,\ V,\ VI)_n} = MK^{(n-1)}K_{(I,\ II,\ V,\ VI)} \qquad (35)$$

(2). nth moment difference is equal to the total working moment (M)

times

(n) power of total New Distribution Factor (K)

$$M_{D_n} = MK^n \qquad (36)$$

(3). column-circle moments form 4 series, which are

(a). infinite series - number of members is infinite

(b). convergent series - condition of convergency

$$U_{(I,\ II,\ V,\ VI)_{(n-1)}} > U_{(I,\ II,\ V,\ VI)_{(n)}}$$

(c). geometric series - ratio of two successive members is constant

$$\frac{U_{(I,\ II,\ V,\ VI)_{(n)}}}{U_{(I,\ II,\ V,\ VI)_{(n-1)}}} = \frac{MK^{(n-1)}K_{(I,\ II,\ V,\ VI)}}{MK^{(n-2)}K_{(I,\ II,\ V,\ VI)}} = K \qquad (37)$$

---

[1] Index (I, II, V, VI) is to be read: "at point I, or II, or V, or VI".

(4). moment differences form one series which is

(a). infinite series - number of members is infinite

(b). convergent series - condition of convergency

$$M_{D_{n-1}} > M_{D_n}$$

(c). geometric series - ratio of two successive members is constant

$$\frac{M_{D_n}}{M_{D_{n-1}}} = \frac{MK^n}{MK^{n-1}} = K \qquad (38)$$

18. Summations of column-circle moments

Using the formula for summation of members of infinite convergent geometric series

$$\sum_{0}^{\infty} N = S_N = \frac{N}{1 - r_N}$$

the summation of all column-circle moments at I is

$$\sum_{0}^{\infty} U_{(I,\ II,\ V,\ VI)} = S_{U_{(I,\ II,\ V,\ VI)}} = \frac{MK_{(I,\ II,\ V,\ VI)}}{1 - K} \qquad (39)$$

(K), according to eq. (13), $(1 - K_I - K_{II} - K_V - K_{VI})$ substituted in eq. (39)

$$1 - (1 - K_I - K_{II} - K_V - K_{VI}) =$$

$$= K_I + K_{II} + K_V + K_{VI} = \sum K \qquad (40)$$

the expression $\sum K$ will be called

SUM OF ALL COLUMN NEW DISTRIBUTION FACTORS

the ratio of any NDF at point (I, II, V, VI) and the sum of NDF

$$\frac{K_{(I,\ II,\ V,\ VI)}}{\sum K} \qquad (41)$$

will be called

FINAL NEW DISTRIBUTION FACTOR

This Final New Distribution Factor will hereafter be called simply FINAL DISTRIBUTION FACTOR

19. General conclusions

To the conclusions of the first circle, Chapter 14, and the conclusions of all circle moments, these general conclusions can be added :

(1). final column moments $M_{(I,\ II,\ V,\ VI)}$ are equal to the total working moment times the Final Distribution Factor for (I, II, V, VI)

$$M_{I} = M\frac{K_{I}}{\Sigma K} \qquad (42)$$

$$M_{II} = M\frac{K_{II}}{\Sigma K} \qquad (43)$$

$$M_{V} = M\frac{K_{VI}}{\Sigma K} \qquad (44)$$

$$M_{VI} = M\frac{K_{VI}}{\Sigma K} \qquad (45)$$

(2). summation of all New Distribution Factors

$$\Sigma K = \frac{3(ap + cq) - 2z}{2z} \qquad (46)$$

and

(3). ratio of $K_{(I,\ II,\ V,\ VI)}$ and ΣK

$$\frac{K_{I}}{\Sigma K} = \frac{a(p - 1)}{3(ap + cq) - 2z} \qquad (47)$$

$$\frac{K_{II}}{\Sigma K} = \frac{a(2p - 1)}{3(ap + cq) - 2z} \qquad (48)$$

$$\frac{K_{V}}{\Sigma K} = \frac{c(2q - 1)}{3(ap + cq) - 2z} \qquad (49)$$

$$\frac{K_{VI}}{\Sigma K} = \frac{c(q - 1)}{3(ap + cq) - 2z} \qquad (50)$$

20. Review of internal and external circles

Circle: Ext.	Circle: Inter.	Base:	Column Circle Moments:				Circle Moment Difference:	
	1.	MK^o	MK^oK_I	MK^oK_{II}	MK^oK_V	MK^oK_{VI}		
1.							$MK^o(1-K_I-K_{II}-K_V-K_{VI})$	$\rightarrow MK^1$
	2.	MK^1	MK^1K_I	MK^1K_{II}	MK^1K_V	MK^1K_{VI}		
2.							$MK^1(1-K_I-K_{II}-K_V-K_{VI})$	$\rightarrow MK^2$
	3.	MK^2	MK^2K_I	MK^2K_{II}	MK^2K_V	MK^2K_{VI}		
3.							$MK^2(1-K_I-K_{II}-K_V-K_{VI})$	$\rightarrow MK^3$
	4.	MK^3	MK^3K_I	MK^3K_{II}	MK^3K_V	MK^3K_{VI}		
4.							$MK^3(1-K_I-K_{II}-K_V-K_{VI})$	$\rightarrow MK^4$
	n+1	MK^n	MK^nK_I	MK^nK_{II}	MK^nK_V	MK^nK_{VI}		
n+1							$MK^n(1-K_I-K_{II}-K_V-K_{VI})$	$\rightarrow MK^{n-1}$
	∞	0	0	0	0	0		
∞							0	0
$\sum_0^\infty$ Inter. Cir.			$\frac{MK_I}{\Sigma K}$	$\frac{MK_{II}}{\Sigma K}$	$\frac{MK_V}{\Sigma K}$	$\frac{MK_{VI}}{\Sigma K}$		

$$\sum_0^\infty \text{Ext. Circles} \quad \frac{M}{1 - K} = \frac{M}{K_I+K_{II}+K_V+K_{VI}} = \frac{M}{\Sigma K}$$

Fig. 9

P A R T II

Special Cases

21. Case No. 1 - Simple two-legged bent with the bottom fixed - both columns are the same length

For calculation of wind stresses in case No. 1 (Fig. 10) equations (47), (48), (49), and (50) can be directly applied and simplified in the following way.[1]

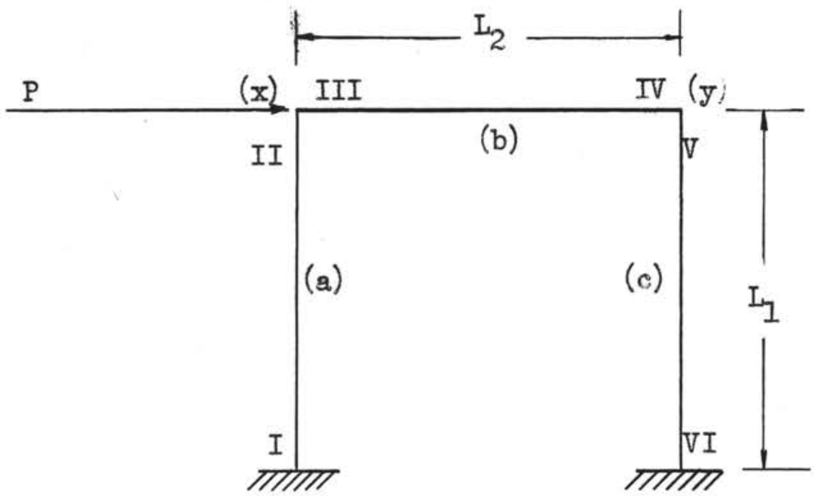

Fig. 10

$$K_I = a(2ay - bc - 4xy + b^2) \quad (51)$$

$$K_{II} = a\left[2(2ay - bc) - 4xy + b^2\right] \quad (52)$$

$$K_V = c\left[2(2cx - ab) - 4xy + b^2\right] \quad (53)$$

$$K_{VI} = c(2cx - ab - 4xy + b^2) \quad (54)$$

---

[1] By substituting for (p) and (q) the right values according to eqs. (6i) and (10i) the term $(4xy - b^2)$ can be eliminated and eqs. (47 - 50) can be written

$$\frac{K_I}{K_I + K_{II} + K_V + K_{VI}} = \frac{a(2ay - bc - 4xy + b^2)}{\Sigma K} \quad (51a)$$

from which $K_I = a(2ay - bc - 4xy + b^2)$ (51)

and analogically for K_{II}, K_V, K_{VI}.

then final column moments according to eqs. (42 - 45) are:

$$M_I = \frac{a(2ay - bc - 4xy + b^2)}{\Sigma K} M \qquad (55)$$

$$M_{II} = \frac{a\left[2(2ay - bc) - 4xy + b^2\right]}{\Sigma K} M \qquad (56)$$

$$M_V = \frac{c\left[2(2cx - ab) - 4xy + b^2\right]}{\Sigma K} M \qquad (57)$$

$$M_{VI} = \frac{c(2cx - ab - 4xy + b^2)}{\Sigma K} M \qquad (58)$$

Numerical value of ΣK is the sum of $K_I + K_{II} + K_V + K_{VI}$ according to eqs. (51 - 54) and eq. (40).

For practical purposes the so-called "NDF - Chess board" will be used.[2]

NDF	Algebraic NDF	Numerical NDF	$M\frac{NDF}{\Sigma K}$	Final Moment
K_I	$a(2ay - bc - 4xy + b^2)$	X_I	$M\frac{X_I}{\Sigma X}$	M_I
K_{II}	$a\left[2(2ay - bc) - 4xy + b^2\right]$	X_{II}	$M\frac{X_{II}}{\Sigma X}$	M_{II}
K_V	$c\left[2(2xc - ab) - 4xy + b^2\right]$	X_V	$M\frac{X_V}{\Sigma X}$	M_V
K_{VI}	$c(2xc - ab - 4xy + b^2)$	X_{VI}	$M\frac{X_{VI}}{\Sigma X}$	M_{VI}
		$\Sigma X = X_I + X_{II} + X_V + X_{VI}$	M =	ΣM

2 See illustrative examples No. 1, 2, and 3.

22. Case No. 2 - Simple two-legged bent with the bottom hinged - both columns are the same length

For calculation of wind stresses in case No. 2 (Fig. 11), eqs. (47 - 50) can be directly applied and simplified into eqs. (51 - 54),[1] then transformed for the condition

$$M_I = M_{VI} = 0 \qquad (59)$$

and the condition

$$a = a' = 3/4\ a \qquad (60)^{2}$$

$$c = c' = 3/4\ c \qquad (61)$$

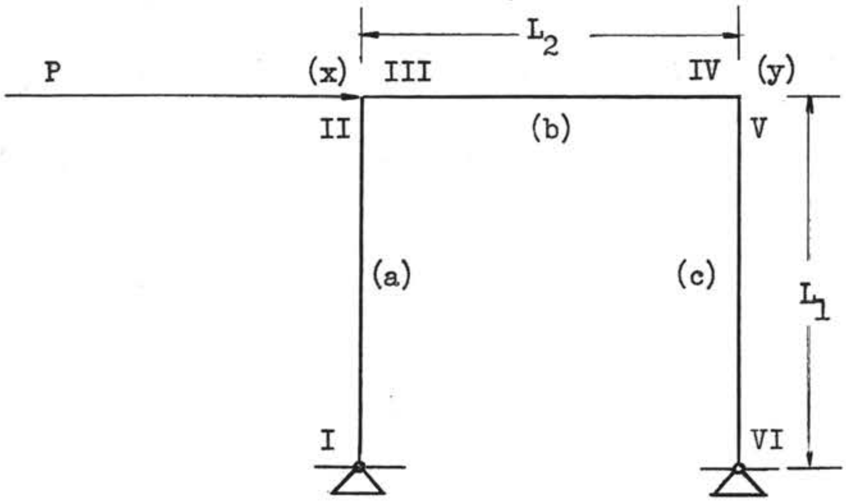

Fig. 11

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[1] The simplification of eqs. (47 - 50) is made for convenience of faster solution. If this simplification and transformation should be involved in practice, eqs. (47 - 50) could be applied.

[2] This substitution, valid for hinged ends only, was applied according to the slope-deflection relations for a structural member with one hinged end. See Elementary Structural Analysis by J. B. Wilbur and C. H. Norris, New York, 1948, page 421.

$$M_I = 4Ea\theta_I + 2Ea\theta_{II} = 0$$

from which

$$\theta_I = -\frac{\theta_{II}}{2} \quad \text{and} \quad a' = 3/4\ a$$

By substitution of eqs. (59 - 61) in the eqs. (47 - 50) we can write directly the final moment equations:

$$M_{II} = \frac{M}{1 + \frac{c(b + 2a)}{a(b + 2c)}} \quad (62)$$

[3]

$$M_{V} = \frac{M}{1 + \frac{a(b + 2c)}{c(b + 2a)}} \quad (63)$$

[4]

---

[3] Algebraic transformation presented in Chapter 22 can be checked with the complete derivations in Engineering Paper Tu - No. 3/50 worked by the author at O.I.T., Stillwater, 1950.

[4] In numerical computation the moment $M_V = M - M_I$.

23. Case No. 3 - Simple two-legged bent with the bottom fixed - the columns are different lengths

For calculation of wind stresses in case No. 3 (Fig. 12), eqs. (47 - 50) can be again applied when corrected for the new condition.

The starting moments in this case will be :

$$M_1 = \frac{M\frac{a}{L_1}}{2\left(\frac{a}{L_1} + \frac{c}{L_3}\right)} = \frac{Ma}{2z'L_1} \qquad (64)$$

$$M_2 = \frac{M\frac{c}{L_3}}{2\left(\frac{a}{L_1} + \frac{c}{L_3}\right)} = \frac{Mc}{2z'L_3} \qquad (65)$$

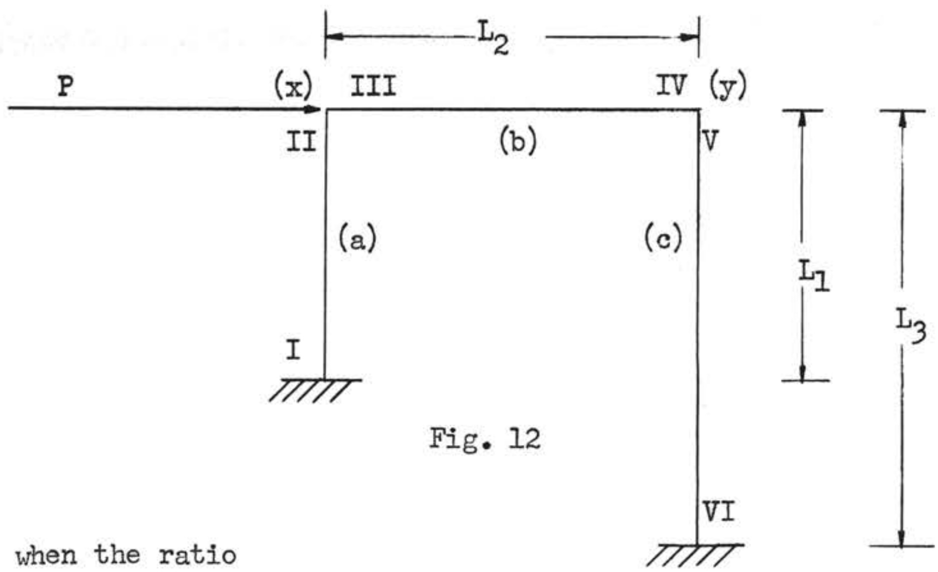


Fig. 12

when the ratio

$$\frac{L_1}{L_3} = u \qquad (66)$$

eq.. (40) has to be corrected

$$\sum K = K_I + K_{II} + uK_V + uK_{VI} \qquad (67)$$

Repeating the same procedure as presented in Chapter (6 - 20) with new starting moments given by eqs. (64 - 65) the following final eqs. may be derived:

$$K_I = a(2ay - ubc - 4xy + b^2) \qquad (51a)$$

$$K_{II} = a\left[2(2ay - ubc) - 4xy + b^2\right] \qquad (52a)$$

$$K_V = uc\left[2(2cx - \frac{ab}{u}) - 4xy + b^2\right] \qquad (53a)$$

$$K_{VI} = uc(2cx - \frac{ab}{u} - 4xy + b^2) \qquad (54a)$$

from which we can define eqs. (51 - 54) as transformations of eqs. (51a - 54a) for the special condition u = 1. Eqs. (51a - 54a) are fundamental and all others, (eqs. (51 - 54, 62 - 63, 62a - 63a) are algebraic transformations for special conditions. <u>Every two-legged bent can be computed by eqs. (51a - 54a)</u>.

24. Case No. 4 - Simple two-legged bent with the bottom hinged - the colums are different lengths

For calculation of wind stresses in case No. 4 (Fig. 13), eqs. (62 - 63) can be used when the new conditions are to be considered .

These new conditions are:

$$a = a' \quad = \quad 0,75a \qquad (60)$$

$$c = c' \quad = \quad 0,75c \qquad (61)$$

$$M_I = M_{VI} = 0 \qquad (59)$$

and

$$\frac{L_1}{L_3} = u \qquad (66)$$

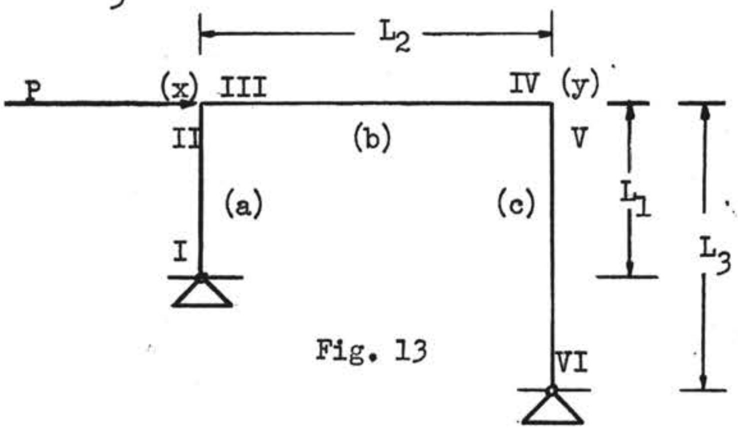


Fig. 13

Repeating the transformations by which eqs. (62 - 63) were derived, we may find the following final eqs.[1]

$$M_{II} = \frac{M}{1 + \dfrac{1 + 2u - \dfrac{3b^2u}{2a}}{1 + 2u - \dfrac{3b^2}{2c}}} \qquad (62a)$$

$$M_V = \frac{M}{1 + \dfrac{1 + 2u - \dfrac{3b^2}{2c}}{1 + 2u - \dfrac{3b^2u}{2a}}} \frac{1}{u} \qquad (63a)$$

---

1 Eqs. (52a - 53a) can be applied directly.

25. Case No. 5 - Three-legged bent with the bottom fixed or hinged - the columns are different lengths.

For calculation of wind stresses in case No. 5, eqs. (47 - 50) can be used and corrected for condition represented by Fig. 14 and derived in a new series in Fig. 15.[1]

Final moment equations are:

$$M_{I} = \frac{a(ay - cb - 2xy + b^2)}{\sum K} M \qquad (68)$$

$$M_{II} = \frac{a\left[2(ay - cb) - 2xy + b^2\right]}{\sum K} M \qquad (69)$$

$$M_{V} = \frac{c\left[2(cx - ab) - 2xy + b^2\right]}{\sum K} M \qquad (70)$$

$$M_{VI} = \frac{c(cx - ab - 2xy + b^2)}{\sum K} M \qquad (71)$$

The sum of all $K_{(I,\ II,\ V,\ VI)}$ is equal to

$$\sum K = 2K_{I} + 2K_{II} + K_{V} + K_{VI} \qquad (72)$$

When the columns are of different lengths, the proportional constants have to be used (case No. 3)[2]

---

[1] Complete derivation of the infinite series for bents with three or more legs is presented in Engineering Paper No. 4/50.

[2] Applications of the proportional constant in case No. 5 is shown in illustrative example No. 5.

Three-legged bent with bottom fixed ---- the columns are the same length

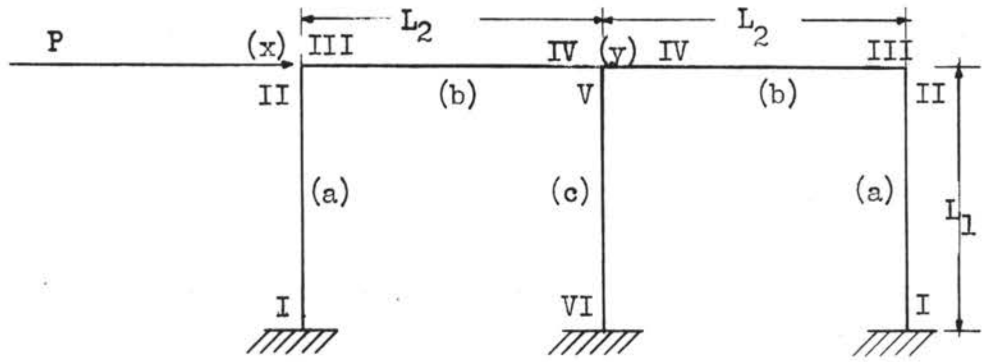


Fig. No. 14

First Internal Circle Distribution for Case No. 5

I	II	III	IV	V	VI	IV	III	II	I
M_1	M_1			M_2	M_2			M_1	M_1
	$\frac{-M_1 a}{x}$	$\frac{-M_1 a}{x}$	$\frac{-M_2 b}{y}$	$\frac{-M_2 c}{y}$		$\frac{-M_2 b}{y}$	$\frac{-M_1 b}{x}$	$\frac{-M_1 a}{x}$	
$\frac{-M_1 a}{2x}$		$\frac{-M_2 b}{2y}$	$\frac{-M_1 b}{2x}$		$\frac{-M_2 c}{2y}$	$\frac{-M_1 b}{2x}$	$\frac{-M_2 b}{2y}$		$\frac{-M_1 a}{2x}$
	$\frac{M_2 ab}{2xy}$	$\frac{M_2 b^2}{2xy}$	$\frac{M_1 b^2}{xy}$	$\frac{M_1 bc}{xy}$		$\frac{M_1 b^2}{xy}$	$\frac{M_2 b^2}{2xy}$	$\frac{M_2 ab}{2xy}$	
$\frac{M_2 ab}{4xy}$		$\frac{M_1 b}{2xy}$	$\frac{M_2 b}{4xy}$		$\frac{M_1 bc}{2xy}$	$\frac{M_2 b}{4xy}$	$\frac{M_1 b}{2xy}$		$\frac{M_2 ab}{4xy}$
	$\frac{-M_1 ab^2}{2x^2 y}$	$\frac{-M_1 b^3}{2x^2 y}$	$\frac{-M_2 b^3}{2xy^2}$	$\frac{-M_2 b^2 c}{2xy^2}$		$\frac{-M_2 b^3}{2xy^2}$	$\frac{-M_1 b^3}{2x^2 y}$	$\frac{-M_1 ab^2}{2x^2 y}$	
$\frac{-M_1 ab^2}{4x^2 y}$					$\frac{-M_2 bc}{4xy^2}$				$\frac{-M_1 ab^2}{4x^2 y}$
0	0	0	0	0	0	0	0	0	0

Fig. No. 15

26. Case No. 6 - Four-legged bent with the bottom fixed or hinged - the columns are different lengths.

Calculation of wind stresses in case No. 6 (Fig. 16), by NDF requires an exact investigation of additional[1] convergent series, in which new members of the bent (four-legged bent is a complexity of three portals), are involved.

Direct derivation of basic and additional series is represented by Fig. 17 and Fig. 18. For convenience of investigation the derivation is resolved into two figures. The first represents derivation of basic series without influence of a third portal (the joint (3) is locked). When the basic series are determined and NDF found, the joint (3) will be relocked and additional series added to the basic ones. This method which is called by the author "separation of primary series of secondary, tertiary and so forth series" is based upon the principles of superposition and will be shown to be one of the most important tools of investigation. Without this method, complicated cases, like No. 7 and No. 8 could not be solved.

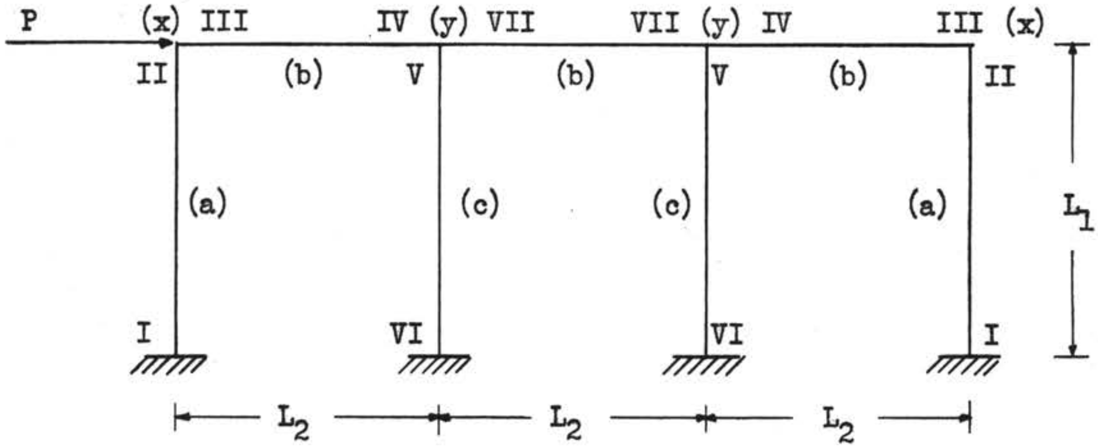


Fig. 16

Basic series need not be investigated and their summation are expressed by eqs. (47 - 50). Summations of additional series can be determined directly from Fig. 18 and added to the first ones.[2]

---

[1] Complex trusses and bents can be solved by NDF in two ways. Exact - when the additional series will be taken into account. Approximately - when additional series will be neglected and NDF will be computed from basic eq. (47 - 50) only.

[2] It is shown in Engr. Paper No. 5 that influence of secondary series is very small (1-3%) and the error involved by neglecting additional series is usually not important in practical use.

First Internal Circle Distribution for Case No. 6, Joint No. 3 - Locked

II	III.	IV.	V.	IV.
$-M_1$			$-M_2$	
$+\frac{M_1 a}{x}$	$+\frac{M_1 b}{x}$	$+\frac{M_2 b}{y}$	$+\frac{M_2 c}{y}$	$+\frac{M_2 b}{y}$
	$+\frac{M_2 b}{2y}$	$+\frac{M_1 b}{2x}$		
$-\frac{M_2 ab}{2xy}$	$-\frac{M_2 b^2}{2xy}$	$-\frac{M_1 b^2}{2xy}$	$-\frac{M_1 b^c}{2xy}$	$-\frac{M_1 b^2}{2xy}$
	$-\frac{M_1 b^2}{4xy}$	$-\frac{M_2 b^2}{4xy}$		
$+\frac{M_1 ab^2}{4x^2y}$	$+\frac{M_1 b^3}{4x^2y}$	$+\frac{M_2 b^3}{4xy^2}$	$+\frac{M_2 b^2 c}{4xy^2}$	$+\frac{M_2 b^3}{4xy^2}$

Fig. 17

<u>Joint No. 3 - Relocked</u>

II	III	IV	V	IV
				$+\frac{M_2b}{2y}$
		$-\frac{M_2b^2}{2y^2}$	$-\frac{M_2bc}{2y^2}$	$-\frac{M_2b^2}{2y^2}$
	$-\frac{M_2b^2}{4y^2}$			$-\frac{M_1b^2}{4xy}$
$+\frac{M_2ab^2}{4xy^2}$	$+\frac{M_2b^3}{4xy^2}$	$+\frac{M_1b^3}{4xy^2}$	$+\frac{M_1b^2c}{4xy^2}$	$+\frac{M_1b^3}{4xy^2}$
	$+\frac{M_1b^3}{8xy^2}$	$+\frac{M_2b^3}{8xy^2}$		$+\frac{M_1b^3}{8xy^2}$
$-\frac{M_1ab^3}{8x^2y^2}$	$-\frac{M_1b^4}{8x^2y^2}$	$-\frac{M_2b^4}{8xy^3}$	$-\frac{M_2b^3}{8xy^3}$	$-\frac{M_2b^4}{8xy^3}$

Fig. 18

Final moment equations for case No. 6 are:

$$M_I = a\left[\frac{2ay - bc}{4xy - b^2} + \frac{b^2(2cx - ab)}{8xy(2xy-b^2)} - 1\right]M \qquad (73)$$

$$M_{II} = a\left[\frac{2(2ay - bc)}{4xy - b^2} + \frac{b^2(2cx - ab)}{4xy(2xy - b^2)} - 1\right]M \qquad (74)$$

$$M_V = c\left[\frac{2(2cx - ab)}{4xy - b^2} - \frac{b(2cx - ab)}{2y(2xy - b^2)} - 1\right]M \qquad (75)$$

$$M_{VI} = c\left[\frac{2cx - ab}{4xy - b^2} - \frac{b(2cx - ab)}{4y(2xy - b^2)} - 1\right]M \qquad (76)$$

When the columns are not of the same length the proportional constant (u) eq. (68) has to be applied. For our case

$$u_1 = \frac{L_1}{L_2} \quad \text{for 2nd column} \qquad (66a)$$

$$u_2 = \frac{L_1}{L_3} \quad \text{for 3rd column} \qquad (66b)$$

and

$$u_3 = \frac{L_1}{L_4} \quad \text{for 4th column} \qquad (66c)$$

The shear eq. (40) may be found in the following way:

$$M = MK_I + MK_{II} + MK_V u_1 + MK_{VI} u_1 +$$

$$+ MK_V u_2 + MK_{VI} u_2 + MK_I u_3 + MK_{II} u_3 \qquad (77)$$

from which the sum of all $K_{(I,\ II,\ V,\ VI)}$ may be determined

$$\Sigma K = (K_I + K_{II})(1 + u_3) + (K_V + K_{VI})(u_1 + u_2) \qquad (78)$$

27. Case No. 7 - Unsymmetrical rectangular frame

One of the most complicated cases of wind analysis by NDF is case No. 7 (Fig. 19). The bottom of the columns are connected with horizontal beams through which new additional series can circulate and be carried-over into the upper part of the frame.

Without the help of the method of separation this case could not be solved. Probably the easiest procedure for investigation of this type of structure is the following:

(1). resolve the investigated frame into two, free-body sketches and solve each separately. (Figs. 20 and 21)

(2). carry-over the bottom series from one sketch to the other and then work as an additional series.

For convenience of investigation we may work with M_1 and M_2 separately.

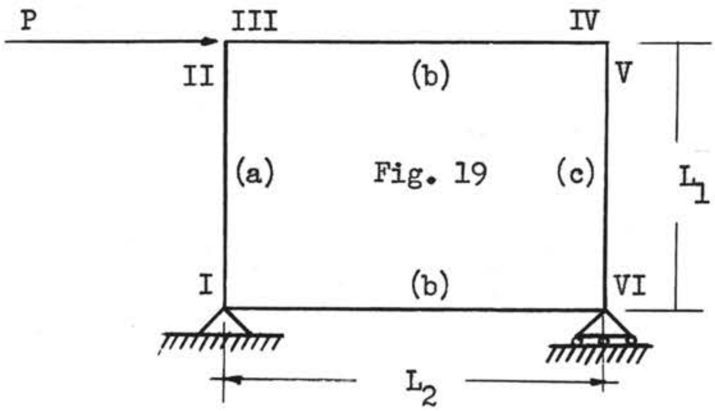


Fig. 19

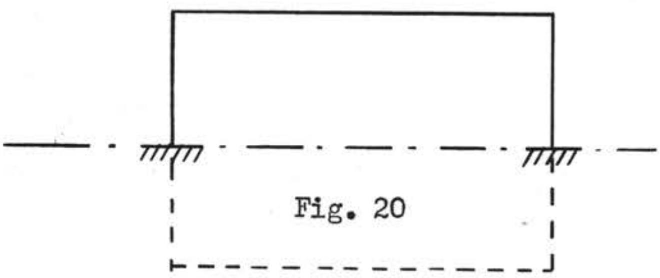


Fig. 20

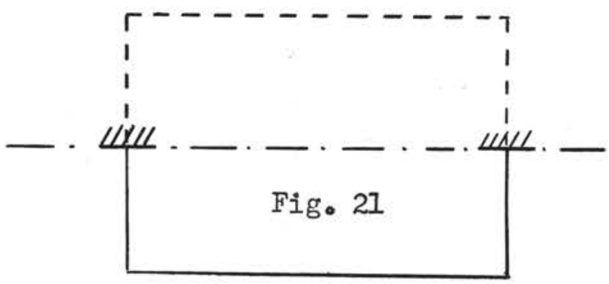


Fig. 21

<u>Series involved by M_1 at I:</u>

Series involving M_1 at (I) were derived in Fig. 6. "series C" and the summation is

$$\sum_{0}^{\infty} C = - \frac{\frac{M_1 a}{x}}{1 - \frac{b^2}{4xy}} \qquad (7d)$$

If joint (II) can rotate freely, the summation of all carry-over moments from (II) to (I) for base M_1 is exactly one-half of "series C". This summation of all carry-over moments forms a new base for distribution and replaces in eq. (7d) the expression M_1. The new summation of all carry-over moments is again one-half of the preceding series. Thus this procedure can be repeated indefinitely.

1st member $\quad \frac{4M_1 ay}{4xy - b^2} + \frac{2M_1 ay}{4xy - b^2} = \frac{6M_1 ay}{4xy - b^2}$

2nd member $\quad - \frac{6(2M_1)(ay)^2}{(4xy - b^2)^2}$

3rd member $\quad + \frac{6(2^2)M_1(ay)^3}{(4xy - b^2)^3}$

nth member $\quad \pm \frac{6(2^{n-1})M_1(ay)^n}{(4xy - b^2)^n}$

∞ member $\quad 0$

All members of this series form an infinite convergent geometric series.

Summation[1] of all members

$$\sum_{0}^{\infty} N_1 = \frac{6Ma^2y}{4xy - b^2 + 2ay} \qquad (79)$$

Similarly the same procedure can be repeated for base M_2 and the summation of all members

$$\sum_{0}^{\infty} N_2 = \frac{3Mabc}{4xy - b^2 + ab} \qquad (80)$$

from which, according to eqs. (6a and 7a), the column-first circle moment at I and II is

$$U_{(I,\ II)} = M\frac{K_{(I,\ II)}}{\sum K} \qquad (42\text{-}43)$$

and then

$$K_{(I,\ II)} = a\left[\frac{(4xy - b^2) - 4ay}{(4xy - b^2) + 2ay} + \frac{3bc}{(4xy - b^2) + ab}\right] \qquad (81)$$

Similarly

$$K_{(V,\ VI)} = c\left[\frac{(4xy - b^2) - 4cx}{(4xy - b^2) + 2cx} + \frac{3ab}{(4xy - b^2) + bc}\right] \qquad (82)$$

Since the $M_I = M_{II}$ and $M_V = M_{VI}$

$$\sum K = 2(K_I + K_{VI}) \qquad (83)$$

---

[1] Complete investigation and derivations of box-frame-series can be found in Engineering Paper Tu - 5/50. Graphically these three types of series form a broken plane of 4th degree in space.

28. Case No. 8 - Series of two rectangular box-frames

Derivation of NDF for a series of two rectangular box-frames (Fig. 22) may be determined by the same way as in case No. 7. The frame will be resolved into two horizontal parts. Both will be solved separately and later carry-over series added.

Final equations for NDF are:

$$K_{(I,\ II)} = a\left[\frac{(2xy - b^2) - 2ay}{(2xy - b^2) + ay} + \frac{3bc}{2(2xy - b^2) + ab}\right] \tag{84}$$

$$K_{(V,\ VI)} = c\left[\frac{(2xy - b^2) - 2cx}{(2xy - b^2) + cx} + \frac{3ab}{2(2xy - b^2) + cb}\right] \tag{85}$$

Summation of (K) is

$$\sum K = 4K_I + 2K_{VI} \tag{87}$$

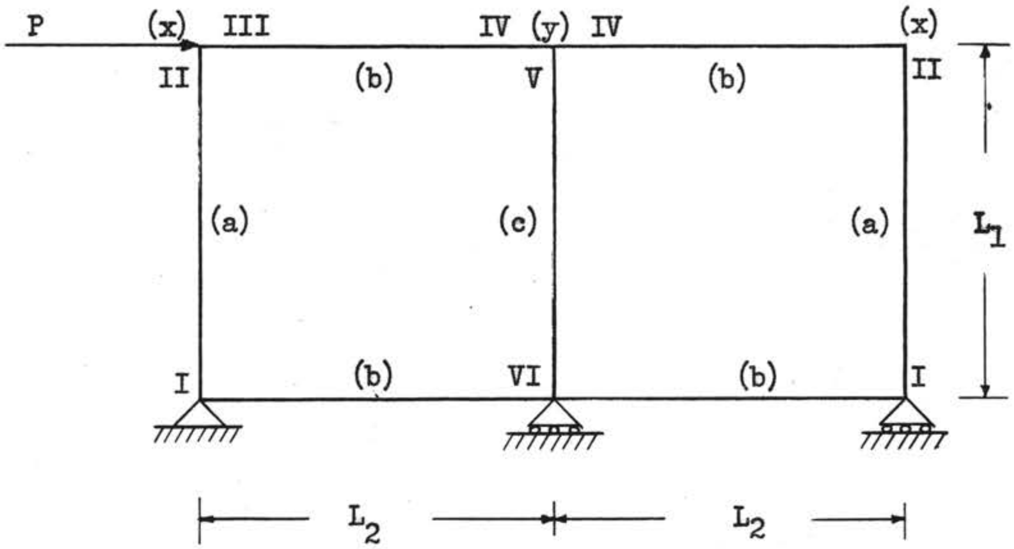


Fig. 22

29. Case No. 9 - Simple two-legged bent, of which, joints can not translate

Derivation of NDF for structures in case 9 (Fig. 23, 24, and 25) may be determined by the same way as cases No. 1 - 4, when the new condition of joints, which can not translate, is considered.

The base for the series is not a simple distributed total working moment, but an FEM at the member where pressure is affected. A second new condition in which external circles do not exist, essentially simplifies the computation.

The new procedure will be shown first on the simplest structure (Fig. 23a). Assuming that the final eq. should serve for computation of any lateral pressure, the load (P) is placed unsymmetrically between point I and point II.

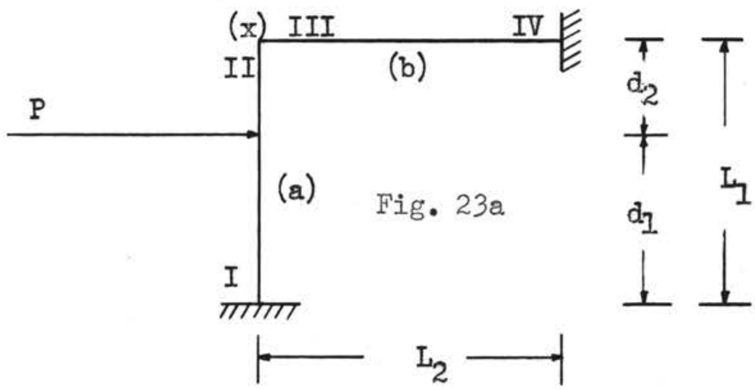


Fig. 23a

The FEM at I and II are FEM_I and FEM_{II}. With this, base derivation of series can start.

I	II	III	IV
FEM_I	FEM_{II}		
	$-\frac{FEM_{II}a}{x}$	$-\frac{FEM_{II}b}{x}$	
$-\frac{FEM_{II}a}{2x}$			$-\frac{FEM_{II}b}{2x}$

Fig. 23b

When the ratio

$$\frac{FEM_I}{FEM_{II}} = w \tag{88}$$

the final formulas for (M) are:

$$M_I = FEM_{II}\frac{2wx - a}{2x} \tag{89}$$

$$M_{II} = FEM_{II}\frac{b}{x} = -M_{III} \tag{90-91}$$

$$M_{IV} = -FEM_{II}\frac{b}{2x} \tag{92}$$

Derivation of NDF in case No. 9a is a very simple transaction and serves to show the new procedure of investigation. It is of interest to show the same case for columns with hinged bottoms and to prove that the substitution used in eqs. (60-61) is exactly correct for one story bents.

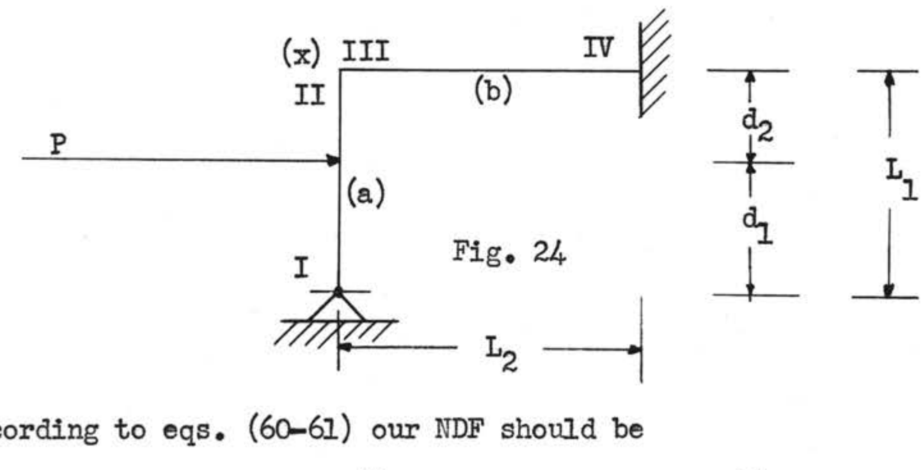


Fig. 24

According to eqs. (60-61) our NDF should be

$$K_{II} = \left[1 - \frac{a'}{x} - \frac{w}{2} - \frac{wa'}{2x} \right]$$

$$= \frac{b(1 - \frac{w}{2})}{\frac{3}{4}a - b} = - K_{III} \tag{93-94}$$

The series in the structure of (Fig. 24) can be developed by normal procedure of the NDF and checked with eqs. (93-94).

Fig. 25 presents the series:

I	II	III	IV
FEM_I	FEM_{II}		
$-FEM_I$	$\frac{-FEM_{II}a}{x}$	$\frac{-FEM_{II}b}{x}$	
$\frac{-FEM_{II}a}{2x}$	$\frac{-FEM_I}{2}$		$\frac{-FEM_{II}b}{2x}$
$\frac{FEM_{II}a}{2x}$	$\frac{FEM_I a}{2x}$	$\frac{FEM_I b}{2x}$	
$\frac{FEM_I a}{4x}$	$\frac{FEM_{II}a}{4x}$		$\frac{FEM_I b}{4x}$
$\frac{-FEM_I a}{4x}$	$\frac{-FEM_{II}a^2}{4x^2}$	$\frac{-FEM_{II}ab}{4x^2}$	
0	0	0	0

Fig. 25

It is evident that series I is equal to zero and, therefore, NDF_I is equal to zero also. Summation of all members of series II must be the summation of two geometric series.

$$\sum_0^{\infty} N_{FEM_{II}} = \frac{\frac{FEM_{II}b}{x}}{1 - \frac{a}{4x}} = \frac{4FEM_{II}b}{4x - a} \qquad (95a)$$

and summation of series with FEM_I at II.

$$\sum_0^\infty N_{FEM_I} = \frac{-\frac{FEM_I b}{2x}}{1 - \frac{a}{4x}} = -\frac{2FEM_I b}{4x - a} \qquad (95b)$$

The final moment is

$$U_{II} = \sum_0^\infty N_{FEM_{II}} - \sum_0^\infty N_{FEM_I} \qquad (95c)$$

$$U_{II} = \frac{4FEM_{II} b(1 - \frac{W}{2})}{4x - a} \qquad (95d)$$

from which $4x - a = 4a + 4b - a = 4b + 3a$

Substituting this expression in eq. (95d)

$$U_{II} = \frac{FEM_{II} b(1 - \frac{W}{2})}{b + \frac{3a}{4}} \qquad (95e)$$

By visual inspection it is evident that NDF_{II}

$$K_{II} = \frac{b(1 - \frac{W}{2})}{b + \frac{3a}{4}} = -K_{III} \qquad (95)$$

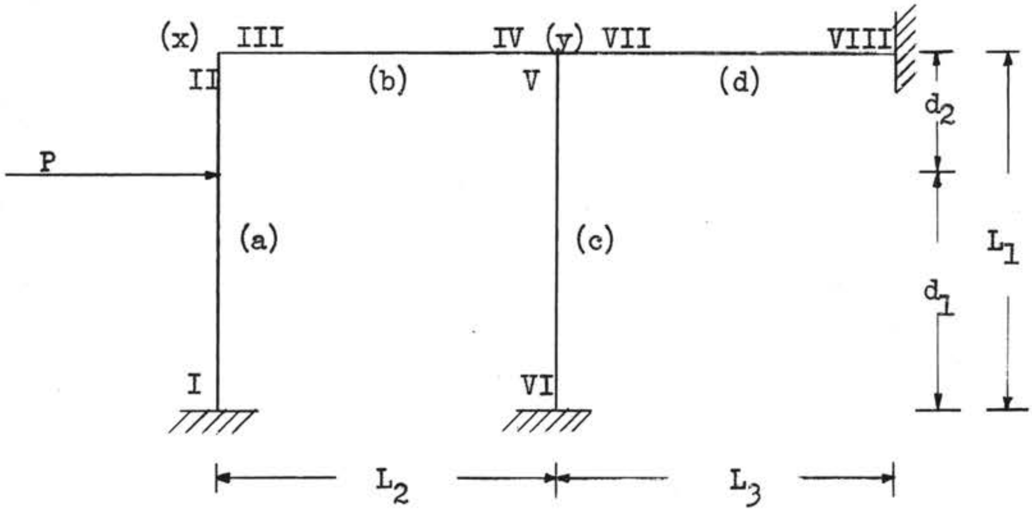
and is exactly the same one as derived by eqs. (60-61)[1] and expressed by eqs. (93-94).

---

[1] This proof is probably the first one made by exact mathematical check for this recommended short-cut.

30. Case No. 10 - Three-legged bent which joints can not translate

Derivation of NDF for structures in case No. 10 (Fig. 26) can be done by the procedure of case No. 9. Series forming the first circle moments are derived in Fig. 27.



NDF at corresponding points of structure are:

$$K_I = w - \frac{a}{2(x - \frac{b^2}{4y})} \qquad (96\text{-}97)$$

$$K_{II} = 1 - \frac{a}{(x - \frac{b^2}{4y})} \qquad (98)$$

$$K_V = \frac{2bc}{4xy - b^2} \qquad (99)$$

$$K_{VI} = \frac{bc}{4xy - b^2} \qquad (100)$$

$$K_{VII} = \frac{2bd}{4xy - b^2} \qquad (101)$$

$$K_{VIII} = \frac{bd}{4xy - b^2} \qquad (102)$$

First Internal Circle Distribution for Case No. 10

I.	II.	III.	IV.	
FEM_I	FEM_{II}			1
	$-\frac{FEM_{II}a}{x}$ ↙	$-\frac{FEM_{II}b}{x}$ ↘		2
$-\frac{FEM_{II}a}{2x}$			$-\frac{FEM_{II}b}{2x}$	3
		$+\frac{FEM_{II}b^2}{4xy}$ ←	$\frac{FEM_{II}b^2}{2xy}$	4
$-\frac{FEM_{II}ab^2}{8x^2y}$ ←	$-\frac{FEM_{II}ab^2}{4x^2y}$	$-\frac{FEM_{II}b^3}{4x^2y}$ →	$-\frac{FEM_{II}b^3}{8x^2y}$	5
0	0	0	0	∞

V.	VI.	VII.	VIII.	
$\frac{FEM_{II}bc}{2xy}$ →	$\frac{FEM_{II}bc}{4xy}$	$\frac{FEM_{II}bd}{2xy}$ →	$\frac{FEM_{II}bd}{4xy}$	4
$\frac{FEM_{II}b^3c}{8x^2y^2}$ →	$\frac{FEM_{II}b^3c}{16x^2y^2}$	$\frac{FEM_{II}b^3d}{8x^2y^2}$ →	$\frac{FEM_{II}b^3d}{16x^2y^2}$	6
0	0	0	0	∞

Fig. 27

P A R T III

Illustrative Examples

31. Illustrative example No. 1[1]

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 1, (Fig. 28).

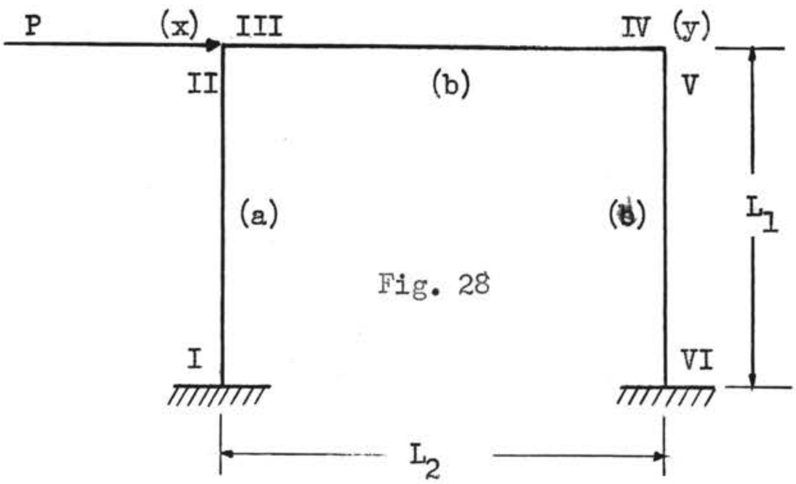


Fig. 28

Load:

P = 12 lb

Dimension:

L_1 = 12 ft

L_2 = 20 ft

Stiffness factors:

a = 1 in^3

b = 3 in^3

c = 2 in^3

a + b = x = 4 in^3

b + c = y = 5 in^3

NDF:	Alg. NDF:	Num. NDF:		$M\frac{NDF}{\sum K}$:	Moments:		
K_I	$a(2ay-bc-4xy+b^2)$	1(4-71)	-67	$144\frac{67}{336}$	28.714	M_I	
K_{II}	$a\left[2(2ay-bc)-4xy+b^2\right]$	$1\left[2(4)-71\right]$	-63	$144\frac{63}{336}$	27.000	M_{II}	$-M_{III}$
K_V	$c\left[2(2cx-ab)-4xy+b^2\right]$	$2\left[2(13)-71\right]$	-90	$144\frac{90}{336}$	38.572	M_V	$-M_{IV}$
K_{VI}	$c(2cx-ab-4xy+b^2)$	2(13-71)	-116	$144\frac{116}{336}$	49.714	M_{VI}	
		$\sum K$ =	-336		144.000 = M		
All moments in lb-ft							

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[1] Transactions ASCE, Vol. 96, 1932, page 66.

32. Illustrative example No. 2[1]

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 2, (Fig. 29).

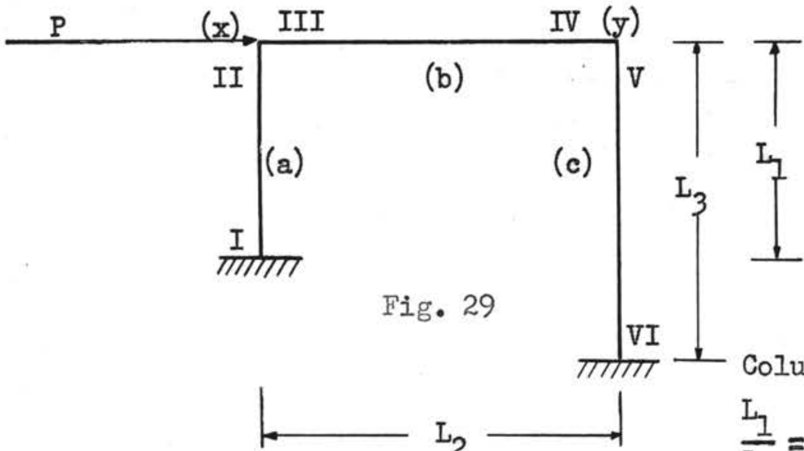

Fig. 29

Column ratio:

$\frac{L_1}{L_3} = \frac{1}{2} = u$

Load:

$P = 167$ lb

Dimension:

$L_1 = 15$ ft

$L_2 = 20$ ft

$L_3 = 30$ ft

Stiffness factors:

$a = 1\ in^3$

$b = 2\ in^3$

$c = 4\ in^3$

$x = 3\ in^3$

$y = 6\ in^3$

NDF	Alg. NDF	Num. NDF	$M\frac{NDF}{\Sigma K}$	Moments
K_I	$a\left[2ay-ubc-4xy+b^2\right]$	$1(8-68) = -60$	$2504.37\frac{60}{188}$	799.37 lb ft
K_{II}	$a\left[2(2ay-ubc)-4xy+b^2\right]$	$1\left[2(8)-68\right] = -52$	$2504.37\frac{52}{188}$	702.30 lb ft
K_V	$cu\left[2(2cx-\frac{ab}{u}-4xy+b^2\right]$	$\frac{1}{2}(4)\left[2(20)-68\right] = -56$	$2504.37\frac{56}{188}$	727.50 lb ft
K_{VI}	$cu\left[(2cx-\frac{ab}{u})-4xy+b^2\right]$	$\frac{1}{2}(4)(20-68) = -96$	$2504.37\frac{96}{188}$	1279.10 lb ft
$\Sigma K = K_I + K_{II} + u(K_V + K_{VI}) = -188$				

[1] Theory of Modern Steel Structures, by L. E. Grinter, Vol. II, page 124, Problem 100.

33. Illustrative example No. 3

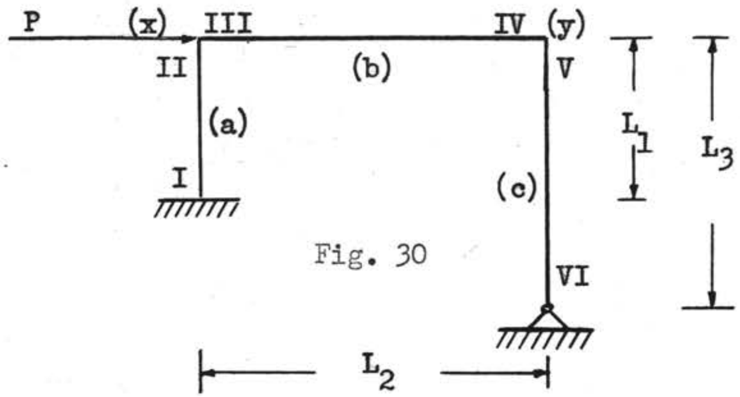

Fig. 30

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 3 and No. 4 - (Fig. 30).

a	1 in^3	P	.	100 lb
b	2 in^3	M	.	1000 lb ft
c	4 in^3	c'	0.75c	3 in^3
L_1	10 ft	d	0.50c	2 in^3
L_2	15 ft	x	a + b	3 in^3
L_3	20 ft	y	b + c'	5 in^3

NDF:	Alg. NDF:	Num. NDF:	NDF:	$\frac{NDF}{\sum K}M$	Moments:	
K_I	$a(2ay-ubd-4xy+b^2)$	$1(8-56)$	-48	$\frac{48}{104}M$	461 lb ft	M_I
K_{II}	$a\left[2(2ay-ubd)-4xy+b^2\right]$	$1(16-56)$	-40	$\frac{40}{104}M$	385 lb ft	M_{II}
K_V	$du\left[3(2dx-abu^{-1})-4xy+b^2\right]$	$2(\frac{1}{2})(24-56)$	-32	$\frac{32}{104}M$	308 lb ft	M_V
K_{VI}	0	0	0	0	0	M_{VI}
$\sum K = K_I + K_{II} + uK_V$			-104	$u = L_1 : L_3 = 0.5$		

34. Illustrative example No. 4

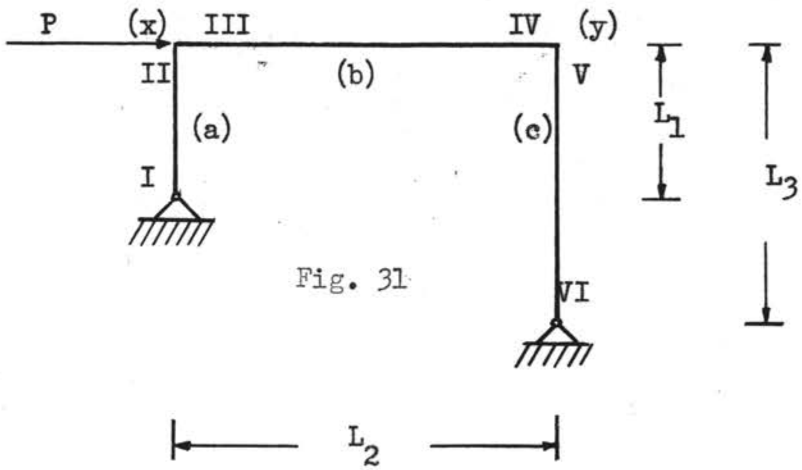

Fig. 31

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 4, (Fig. 31).

P		100.00 lb	a	1.33 in^3
M	$P(L_1)$	1000.00 lb-ft	b	2.00 in^3
a'	0.75a	1.00 in^3	c	2.66 in^3
c'	0.75c	2.00 in^3	L_1	10.00 ft
u	$L_1 : L_3$	0.33	L_2	15.00 ft
			L_3	30.00 ft

NDF:	Alg. NDF:	Num. NDF	$\frac{NDF.M}{\Sigma K}$	Moment:	
$\frac{1}{K_{II}}$	$\frac{1}{1+\frac{1+2u-\frac{3b^2u}{2a}}{1+2u-\frac{3b^2}{2c}}}$	$\frac{1}{1+\frac{1.66-2}{1.66-3}}$	$\frac{1000}{1.254}$	798.00 lb-ft	M_{II}
$\frac{1}{K_V}$	$\frac{1}{1+\frac{1+2u-\frac{3b^2}{2c}}{1+2u-\frac{3b^2u}{2a}}}\left(\frac{1}{u}\right)$	$\frac{1(3)}{1+\frac{1.66-3}{1.66-2}}$	$\frac{1000(3)}{4.930}$	606.00 lb-ft	M_I

35. Illustrative example No. 5

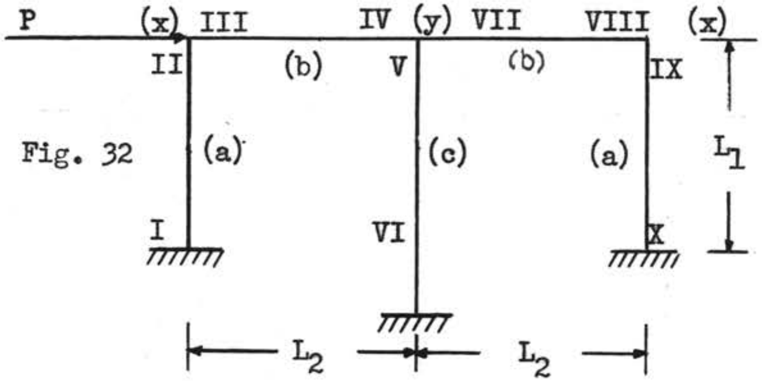

Fig. 32

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 5, (Fig. 32).				
P		100 lb	a	1 in^3
M	P_1L_1	1000 ft lb	b	2 in^3
x	a + b	3 in^3	c	4 in^3
y	2b + c	8 in^3	L_1	10 ft
u	$L_1:L_2$	0.25	L_2	20 ft
			L_3	40 ft

NDF:	Alg. NDF:	Num. NDF:	NDF:	$\frac{NDF}{\sum K}M$	Moments:	
$K_{(I, X)}$	$a\left[(ay-ubc)-2xy+b^2\right]$	1(6-44)	-38	$\frac{38}{159}M$	239	$M_{(I, X)}$
$K_{(II, IX)}$	$a\left[2(ay-ubc)-2xy+b^2\right]$	1(12-44)	-32	$\frac{32}{159}M$	201	$M_{(II, IX)}$
K_V	$cu\left[2(cx-\frac{ab}{u})-2xy+b^2\right]$	$4(\frac{1}{4})(8-44)$	-36	$\frac{36}{159}M$	227	M_V
K_{VI}	$cu\left[(cx-\frac{ab}{u})-2xy+b^2\right]$	$4(\frac{1}{4})(4-44)$	-40	$\frac{40}{159}M$	253	M_{VI}
$\sum K$	$= 2(K_I + K_{II}) + u(K_V + K_{VI})$		- 159	All moments in lb ft		

35. Illustrative example No. 6

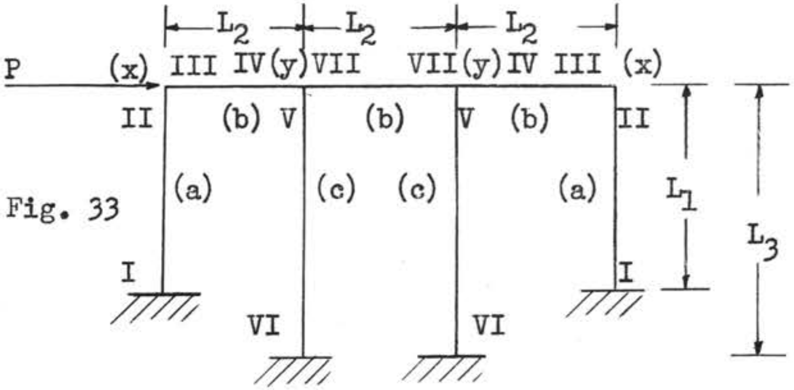

Fig. 33

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 6, (Fig. 33).				
P		41.60 lb	a	1.00 in^3
M	P(L_1)	1000.00 lb ft	b	3.00 in^3
x	a + b	4.00 in^3	c	2.00 in^3
y	2b + c	8.00 in^3	L_1	24.00 ft
u	L_1:L_3	0.666	L_2	12.00 ft
			L_3	36.00 ft

NDF:	Alg. NDF:	Num. NDF:	NDF:	$\frac{NDF}{\Sigma K}M$:	Moment:	
K_I	$a\left[\frac{2ay-bcu}{4xy-b^2}+\frac{b^2(2cxu-ab)}{8xy(2xy-b^2)}-1\right]$	$1\left[\frac{12}{119}+\frac{9(7.66)}{256(55)}-1\right]$	-0.893	$\frac{0.893}{6.515}M$	137	M_I
K_{II}	$a\left[\frac{2(2ay-bcu)}{4xy-b^2}+\frac{b^2(2cxu-ab)}{4xy(2xy-b^2)}-1\right]$	$1\left[\frac{2(12)}{119}+\frac{9(7.66)}{128(55)}-1\right]$	-0.785	$\frac{0.785}{6.515}M$	121	M_{II}
K_V	$cu\left[\frac{2(2cx-abu^{-1})}{4xy-b^2}-\frac{b(2cx-abu^{-1})}{2y(2xy-b^2)}-1\right]$	$2(\frac{2}{3})\left[\frac{2(11.5)}{119}-\frac{3(11.5)}{16(55)}-1\right]$	-1.130	$\frac{1.130}{6.515}M$	173	M_V
K_{VI}	$cu\left[\frac{2cx-abu^{-1}}{4xy-b^2}-\frac{b(2cx-abu^{-1})}{4y(2xy-b^2)}-1\right]$	$2(\frac{2}{3})\left[\frac{11.5}{119}-\frac{3(11.5)}{32(55)}-1\right]$	-1.235	$\frac{1.235}{6.515}M$	189	M_{VI}
$\sum K$	$-\ 2\left[K_I + K_{II}\right] + 2u\left[K_V + K_{VI}\right]$	=	-6.515	All moments in lb ft		

37. Illustrative example No. 7

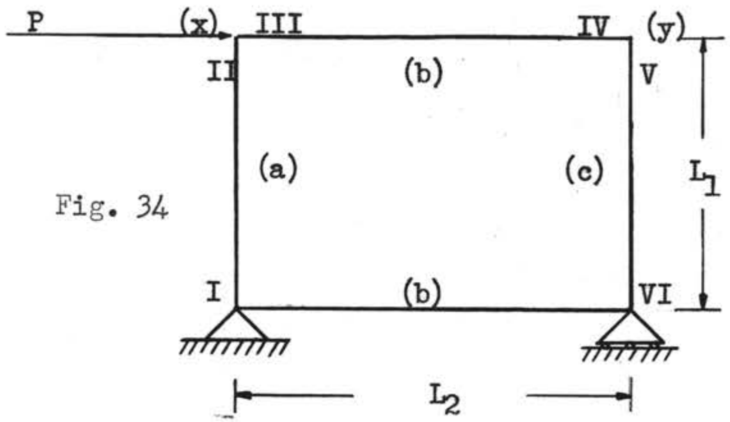

Fig. 34

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 7, (Fig. 34).

a	1.00 in^3	P		100 lb
b	3.00 in^3	M	$P(L_1)$	1500 lb ft
c	2.00 in^3	x	a + b	4 in^3
L_1	15.00 ft	y	b + c	5 in^3
L_2	20.00 ft			

NDF:	Alg. NDF:	Num. NDF:	NDF:	$\frac{NDF}{\Sigma K}M$:	Moment:	
$K_{(I,\ II)}$	$a\left[\frac{(4xy-b^2)-4ay}{(4xy-b^2)+2ay}+\frac{3bc}{(4xy-b^2)+ab}\right]$	$1\left[\frac{71-20}{71+10}+\frac{18}{71+3}\right]$	0.87	$\frac{0.87}{4.00}M$	326.25 lb ft	$M_{(I,\ II)}$
$K_{(V,\ VI)}$	$c\left[\frac{(4xy-b^2)-4cx}{(4xy-b^2)+2cx}+\frac{3ab}{(4xy-b^2)+bc}\right]$	$2\left[\frac{71-32}{71+16}+\frac{9}{71+6}\right]$	1.13	$\frac{1.13}{4.00}M$	423.75 lb ft	$M_{(V,\ VI)}$
ΣK	$= 2(K_I + K_V) =$	$2\left[0.87 + 1.13\right] =$	4.00			

38. Illustrative example No. 8

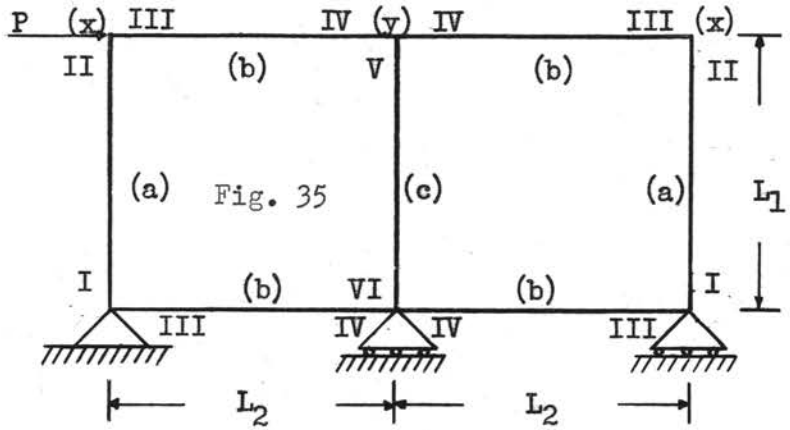

Fig. 35

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 8, (Fig. 35).				
a	2.00 in^3	P		300 lb
b	3.00 in^3	M	$M(L_1)$	3600 lb ft
c	5.00 in^3	x	a + b	5 in^3
L_1	12.00 ft	y	2b + c	11 in^3
L_2	15.00 ft			

NDF:	Alg. NDF:	Num. NDF:	NDF:	$\frac{NDF}{\sum K}M$	Moment:	
$K_{(I,\ II)}$	$a\left[\frac{(2xy-b^2)-2ay}{(2xy-b^2)+ay}+\frac{3bc}{2(2xy-b^2)+ab}\right]$	$2\left[\frac{110-44}{110+22}+\frac{45}{220+6}\right]$	1.420	$\frac{1.420}{10.670}M$	480.00 lb ft	$M_{(I,\ II)}$
$K_{(V,\ VI)}$	$c\left[\frac{(2xy-b^2)-2cx}{(2xy-b^2)+cx}+\frac{3ab}{2(2xy-b^2)+bc}\right]$	$5\left[\frac{110-50}{110+25}+\frac{18}{220+15}\right]$	2.500	$\frac{2.500}{10.670}M$	840.00 lb ft	$M_{(V,\ VI)}$
$\sum K$ = $4K_{(I,\ II)} + 2K_{(V,\ VI)}$ = $4[1.420] + 2[2.500] = 10.670$						

39. Illustrative example No. 9[1]

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 9, (Fig. 36).

a	4.00 in^3	P	100.00 lb
b	2.00 in^3	FEM_I	-125.00 lb ft
a'	3.00 in^3	FEM_{II}	125.00 lb ft
L_1	10.00 ft	d(1,2)	5.00 ft
L_2	10.00 ft	$x = a' + b$	5.00 in^3

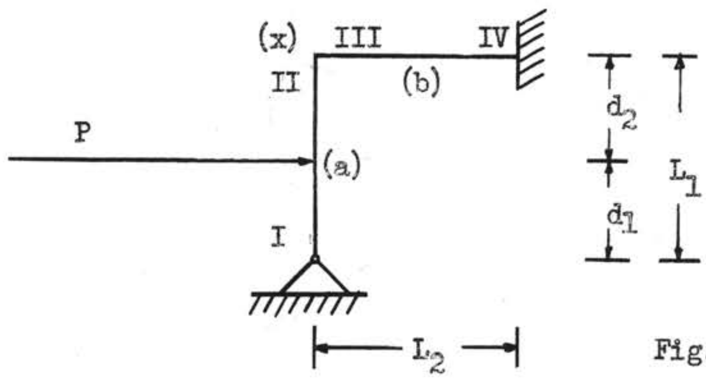

Fig. 36

M_I			0.00 lb ft
M_{II}	$FEM_{II}\frac{b}{x}$	$125\frac{3}{5}$	75.00 lb ft
M_{III}	$-FEM_{II}\frac{b}{x}$	$-125\frac{3}{5}$	-75.00 lb ft
M_{IV}	$-FEM_{II}\frac{b}{2x}$	$-125\frac{3}{10}$	-37.50 lb ft

[1] Example No. 9 was given as a one hour test in Course Civ. En. 423 at Oklahoma Institute of Technology in 1950. By NDF it can be solved in a few minutes.

40. Illustrative example No. 10

To illustrate the procedure of computation the calculations will be presented for the wind stresses in a bent - Case No. 10, (Fig. 37).

a	2.00 in^3	P		100.00 lb
b	4.00 in^3	FEM_I		-111.11 lb ft
c	8.00 in^3	FEM_{II}		222.22 lb ft
d	4.00 in^3	d_1		10.00 ft
L_1	15.00 ft	d_2		5.00 ft
L_2	25.00 ft	c'	0.75c	6.00 in^3
L_3	15.00 ft	d'	0.75d	3.00 in^3
		x	a + b	6.00 in^3
		y	b + c' + d'	13.00 in^3

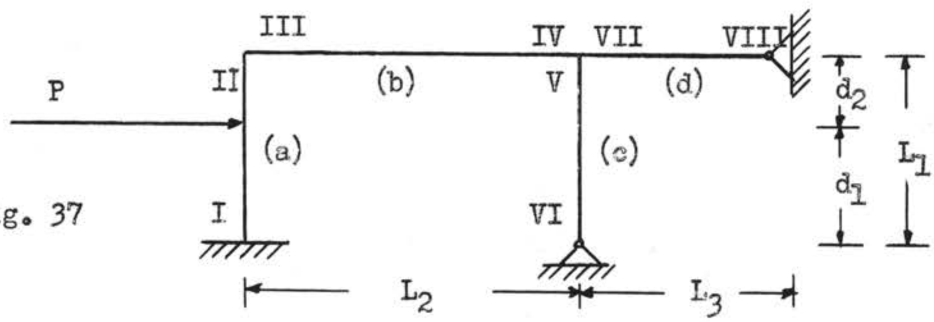

Fig. 37

M_I	$FEM_{II}\left[w - \frac{a}{2(x - \frac{b^2}{4y})}\right]$	$222.22(-\frac{1}{2} - \frac{52}{296})$	-149.00 lb ft
M_{II}	$FEM_{II}\left[1 - \frac{a}{(x - \frac{b^2}{4y})}\right]$	$222.22(1 - \frac{104}{296})$	142.50 lb ft
M_V	$FEM_{II}\left[\frac{2bc}{4xy - b^2}\right]$	$222.22\frac{48}{296}$	36.10 lb ft
M_{VII}	$FEM_{II}\left[\frac{2bd}{4xy - b^2}\right]$	$222.22\frac{24}{296}$	18.05 lb ft
M_{VI} = 0.00 lb ft		M_{VIII} = 0.00 lb ft	

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