ELLIPTICAL DIAGRAMS FOR INDUCTION MACHINERY

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Preface

In many electrical engineering problems it is of great help in the understanding of the physical nature of the phenomena that are being studied if the actual apparatus can be represented by an equivalent circuit which has the same properties. An example of such an equivalent circuit is that for the induction machine. There is, however, a disadvantage with such equivalent circuits. When an equivalent circuit has been drawn, there is a tendency for the student to think solely in terms of the equivalent circuit, thus forgetting what takes place in the actual apparatus.

As a result of the equivalent circuit for the induction machine a circle diagram has been developed. This circle diagram is familiar to all students of the induction machine. Such a circle diagram, however, is possible only if the parameters of the equivalent circuit are constant; and only if this is the case, may the performance characteristics of the induction machine as obtained from the circle diagram be expected to be right.

Because of the fact that the parameters of the equivalent circuit do change, the true performance of the induction machine does not form a circle. The question then arises: Would it be possible to obtain some other kind of geometric pattern that would give the true performance of the induction machine? If some general geometric figure could be found that would fit the true performance better than the general circle diagram, then a research along this line would be justified.

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Chapter 1

Introduction

Since the induction machine was invented by Nickola Tesla in 1888, different schemes have been employed in determining the characteristics of this machine. At an early stage in the history of the induction machine, Steinmetz developed a set of equations by which the characteristics of this machine can be calculated.[1] From 1894 to 1896 Heyland constructed a circle diagram from which the characteristics of the induction machine can be determined geometrically.[2] Later different methods have been used to obtain similar circle diagrams, some of which were based on an exact equivalent circuit and some on approximate equivalent circuits. Fig. 1 below shows the exact equivalent circuit, and Fig. 2 shows a common approximate equivalent circuit.

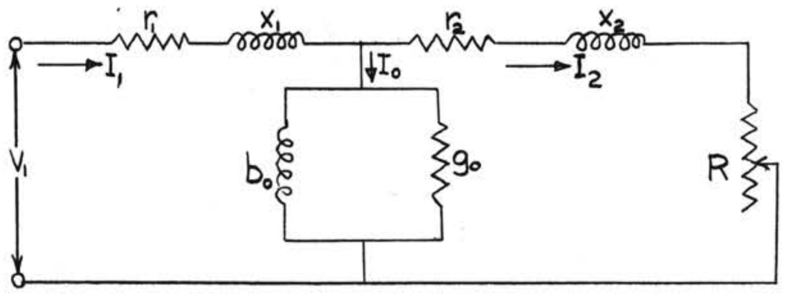

Figure 1. The theoretical exact equivalent circuit representing one phase of an induction machine. All secondary quantities are referred to the primary.

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[1]C. P. Steinmetz, "The Alternating Current Induction Motor," A.I.E.E. Transaction, XIV (1897), pp. 185-217.

[2]Thomälen, Electrical Engineering, p. 398.

In the circuit shown in Fig. 1.

V_1 represents the applied voltage.

I_1 represents the current in the stator or primary windings.

I_2 represents the current in the rotor referred to the primary.

I_o represents the vectorial sum of the magnetizing current and the eddy and hysteresis current.

r_1 represents the resistance of the stator.

r_2 represents the resistance of the rotor referred to the primary.

x_1 represents the leakage reactance of the stator.

x_2 represents the leakage reactance of the rotor referred to the primary.

g_o represents a conductance of such magnitude as to permit a current to flow through it equal to the eddy current and hysteresis loss current.

b_o represents a susceptance of such magnitude as to permit a current to flow through it equal to the magnetizing current.

R represents the load on the machine.

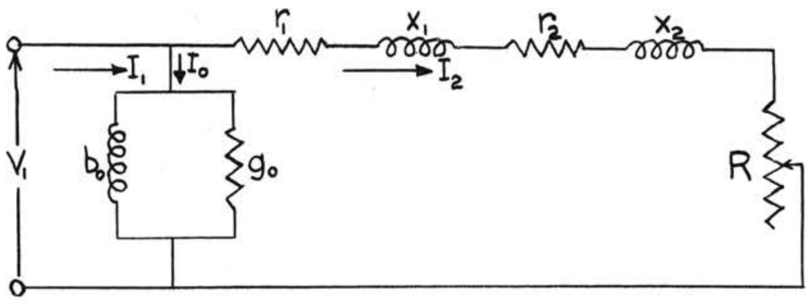

Figure 2. The approximate equivalent circuit representing one phase of an induction machine.

The circle diagram based on the approximate equivalent circuit was soon found to be of little value as the result obtained when using it is far off in most cases. Common for most of the approximate circle diagrams is that the magnetizing current is to be assumed constant, independent of load, and that the voltage drop caused by it is disregarded.

Different authors have succeeded in developing circle diagrams based on the exact equivalent circuit, hoping that the result obtained from an exact circle diagram would be much more accurate than that obtained from the circle diagram based on the approximate equivalent circuit. One of the exact circle diagrams is the Ho's Circle Diagram which is introduced on Page 6. The exact circle diagram, however, did not prove to be of much more value than the approximate one due to poor accuracy and has not come into general usage.

Manufacturers of induction machinery, not being able to use the circle diagrams as a means of obtaining the performance characteristics, have developed their own methods of changing them so as to fit their particular machines.[3]

Common for all circle diagrams so far developed is the authors in their development have assumed no saturation. Most of them have pointed out that there is some saturation, and that the more saturation there is present, the greater the error will be in the result. This is where most authors leave the question of saturation. Any general attempt of introducing the saturation effects in the calculation of the performances of the induction machine is not found in any textbook.

Rudolf Richter goes a little bit further in his discussion of saturation than do most authors of induction machinery.[4] In his discussion of saturation, he points out that the error

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[3]Allis Chalmer Bulletin #05R6393, "The Circle Diagram and the Induction Motor."

[4]Rudolf Richter, Elektrische Maschinen, IV, p. 53.

introduced by assuming b_o constant in Fig. 1 is negligible in most cases. This circumstance can easily be understood from the fact that in the equation for the secondary current most authors do not even include b_o.

The error by assuming x_1 and x_2 constant, however, thus neglecting the saturation effects in the leakage reactances, may cause the result to be in great error, especially at higher currents. Mr. Richter further points out that if the effect of saturation on x_1 and x_2 was taken into account, the current locus would follow a curve which resembles an ellipse. Mr. Richter, however, does not attempt to find any method by which such an elliptical locus can be predicted.

The first attempt of actually using an elliptical diagram instead of a circle diagram in predicting the characteristics of the induction machine can be found in "Induction Motor Characteristics," Bulletin #65, by C. F. Cameron, H. Webking, and J. Grantham. Two methods by which the elliptical diagram can be determined are shown in this bulletin. The first one, which has some discrepancies, assumes the d/D ratio, the ratio of the minor axis of the ellipse to the major axis of the ellipse, to be equal to saturation ratio at rated load. The second method, which probably is more accurate than any method developed up to now, requires one extra test run besides the no-load running test and the blocked rotor test. This extra test run is to be made at maximum power output.

The elliptical diagram from Method II in Bulletin #65 will generally give results with much better accuracy than the approximate or exact circle diagram, but has the disadvantage of requiring an extra test run, which in some cases may be difficult to perform. This leads to the question: Would it be possible to determine a reliable elliptical diagram by running an extra test at full load? This test would in most cases be easier to perform than one at maximum power output. It would be even better if a dependable elliptical diagram could be determined from only the no-load running test and the blocked rotor test.

Chapter II

H. Ho's Circle Diagram

Analytical and graphical methods have been developed in proving that the exact equivalent circuit leads to a circle. One of these methods was derived by a Japanese engineer, Mr. H. Ho.[1] This method will be introduced here.

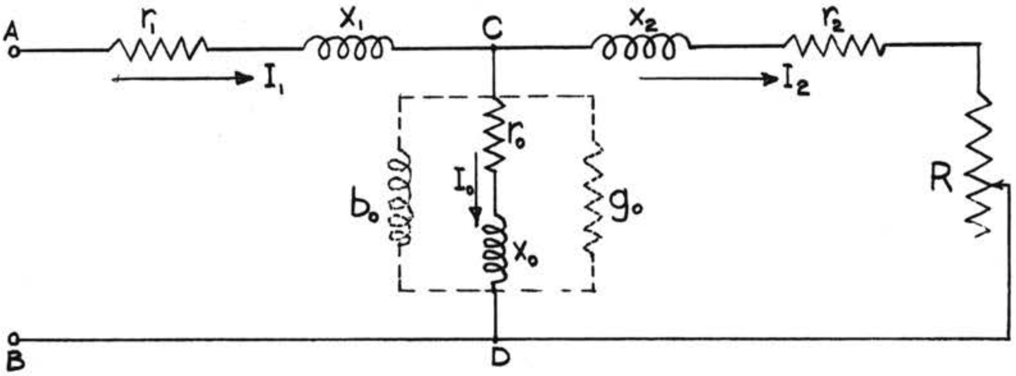

Figure 3. The theoretical exact equivalent circuit representing one phase of the induction machine. All secondary quantities are referred to the primary.

Fig. 3 above shows the exact equivalent circuit where the parallel exciting impedance, $(g_0 - jb_0)$, is replaced by an equivalent series impedance, $(r_0 + jx_0)$. The other components of the circuit are as follows:

r_1 = primary resistance

x_1 = primary leakage reactance

r_2 = secondary resistance referred to the primary

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[1] Karapetoff and Dennison, Experimental Electrical Engineering, II, p. 407.

x_2 = secondary leakage reactance referred to the primary

R = resistance representing the load on the machine

All values are given per phase.

Each actual current in the exact equivalent diagram will be obtained by a superposition of the currents in two fictitious circuits, Fig. 4 and Fig. 5.

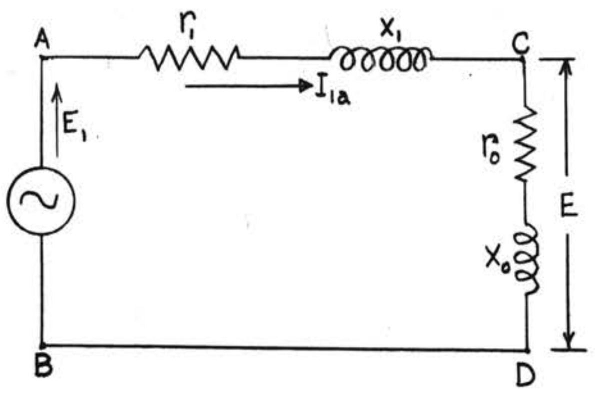

Figure 4

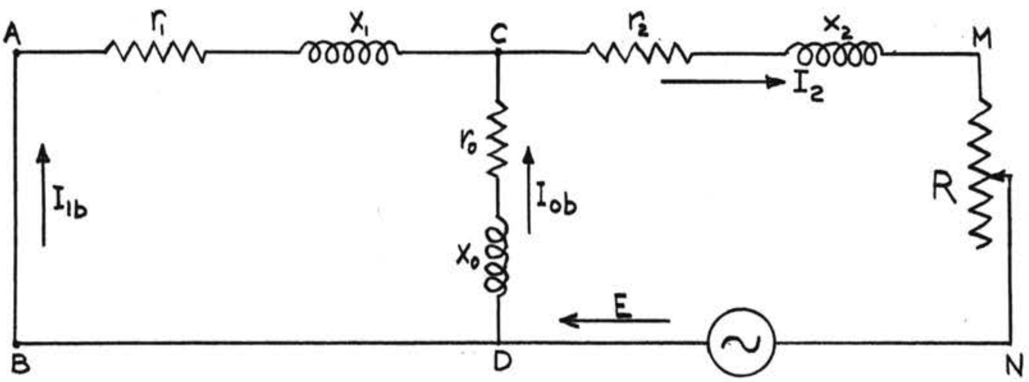

Figure 5

Figures 4 and 5. Ho's resolution of the exact equivalent circuit into two fictitious circuits.

From Fig. 4 the following relations may be obtained:

$$E = \frac{r_o + jx_o}{(r_o + r_1) + j(x_o + x_1)} E_1 \tag{1}$$

$$I_{1a} = \frac{E_1}{(r_o + r_1) + j(x_o + x_1)} \tag{2}$$

I_{1a} lags E_1 by:

$$\theta_{1a} = \tan^{-1} (x_o + x_1)/(r_o + r_1) \tag{3}$$

The fictitious circuit shown in Fig. 5 may be simplified to the one shown in Fig. 6 by replacing the parallel combination $(r_1 + jx_1)$ and $(r_o + jx_o)$ by an equivalent series impedance $(r_h + jx_h)$.

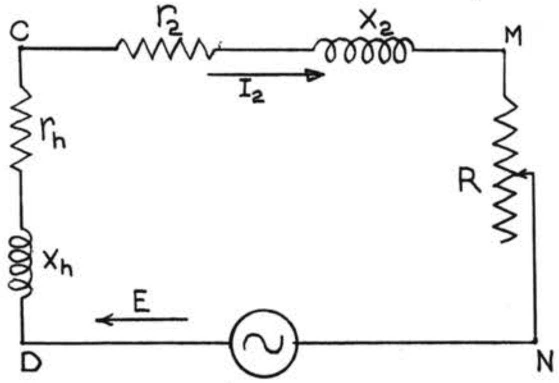

Figure 6. A simplified equivalent circuit of Fig. 5.

From Fig. 6 the following relationships may be derived:

$$r_h + jx_h = \frac{(r_o + jx_o)\,(r_1 + jx_1)}{(r_o + r_1) + j(x_o + x_1)} \tag{4}$$

$$I_2 = \frac{E}{(r_h + r_2 + R) + j(x_h + x_1)} \tag{5}$$

From Fig. 5: $$I_{1b}(r_1 + jx_1) = I_{ob}(r_o + jx_o) \tag{6}$$

Also: $$I_{ob} + I_{1b} = I_2 \tag{7}$$

Solving for I_{ob} and I_{1b} gives:

$$I_{ob} = \frac{r_1 + jx_1}{(r_o + r_1) + j(x_o + x_1)} I_2 \tag{8}$$

$$I_{1b} = \frac{r_o + jx_o}{(r_o + r_1) + j(x_o + x_1)} I_2 \tag{9}$$

The three currents in Fig. 3 can now be found from the following equations:

$$I_o = I_{1a} - I_{ob} \tag{10}$$

$$I_1 = I_{1a} + I_{1b} \tag{11}$$

$$I_2 = I_2 + 0 \tag{12}$$

The circuit shown in Fig. 6 represents a series combination of constant reactance and variable resistance. It can be found in textbooks that the current in such a circuit will form a circular locus.[2] Thus I_2 in circuit Fig. 6 will form a circle. The diameter of this circle will, according to equation 5, be equal to $E/(x_h + x_2)$, and be lagging 90 degrees behind E.

From equations 1 and 9 the numerical ratios E_1/E and I_2/I_{1b} may be found. Let this ratio be called m:

$$m = E_1/E = I_2/I_{1b} = \sqrt{(r_o + r_1)^2 + (x_o + x_1)^2} \Big/ \sqrt{r_o^2 + x_o^2} \tag{13}$$

E leads E_1, and I_{1b} leads I_2 by an angle:

$$\alpha = \tan^{-1} x_o/r_o - \tan^{-1}(x_o + x_1)/(r_o + r_1)$$

Or: $$\alpha = \tan^{-1}(r_o + r_1)/(x_o + x_1) - \tan^{-1} r_o/x_o \tag{14}$$

From these relationships it can be concluded that also I_{1b} will form a circle. Its diameter will be 1/m times the diameter for I_2; thus it will be $\frac{E}{m(x_h + x_2)}$. But since $E = E_1/m$, its diameter can also be written as $\frac{E_1}{m^2(x_h + x_2)}$. Then since I_{1b} leads I_2 by α degrees, the diameter of the circle formed by I_{1b} will lag $(90 - \alpha)$ degrees behind E. But as E leads E_1 by α degrees, the

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[2] Puchstein and Lloyd, <u>Alternating Current Machines</u>, p. 266.

diameter of the circular locus described by I_{1b} will lag $(90 - 2\alpha)$ degrees behind E_1.

The exact circle diagram can now be drawn as shown in Fig. 7.

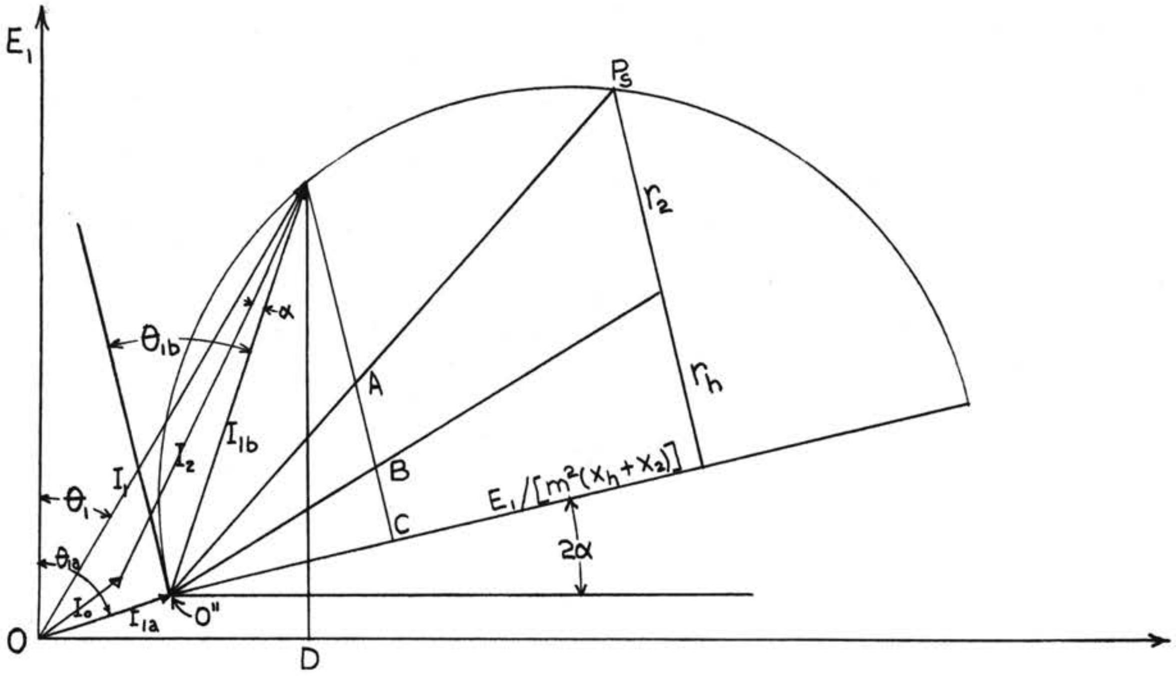

Figure 7. The exact circle diagram for the polyphase induction motor.

From the circle diagram shown in Fig. 7 the following equation can be written:

$$I_{1b} = \frac{E_1}{m^2(x_h + x_2)} \sin \theta_{1b} \qquad (15)$$

From Fig. 7 the following characteristics may be predicted:

Input. The component PD of the current multiplied by the phase voltage E_1 gives the power input per phase.

Power factor. Power factor may be obtained by dividing the in phase current PD by the total current OP, or by measuring the angle θ_1 and find the cosine of this angle.

Primary copper loss. There is no distance shown on the circle diagram which represents the primary copper loss; this loss is to be found from the formula.

$$\text{Primary copper loss per phase} = I_1^2\, r_1 = (OP)^2 r_1 \quad (16)$$

Secondary copper loss. The semi-circle with origin at O" correctly represents the secondary circuit of the actual motor, except that the current I_{1b} must be multiplied by m to obtain I_2. Therefore, the secondary copper loss is equal to m(AB)E, but E_1 is equal to mE so that:

$$\text{Secondary copper loss per phase} = (AB)\, E_1 \quad (17)$$

Output. PA represents the power delivered to the load resistance R, or in other words, PA represents the power output plus friction and windage.

$$\text{Output per phase} = (PA)\, E_1 - F\&W \quad (18)$$

Torque. Torque developed in the motor is equal to the input into the secondary circuit and is therefore represented by PB. In order to obtain shaft torque, torque due to friction and windage must be subtracted.

Thus:

$$\text{Torque per phase} = (PB \times E_1 - F\&W) \times \frac{7.04}{\text{syn. rpm}} \quad (19)$$

Slip. Slip may be found by dividing PB into AB:

$$\text{Slip} = (AB)/(PB) \quad (20)$$

Possible Reasons why the Current Locus does not follow Ho's Exact Circle Diagram

It has already been pointed out that the current locus in most cases does not follow the exact circle diagram derived by Ho. The question therefore arises: What are the possible reasons why the current locus does not follow the exact circle diagram? In order to find an answer to this question, Ho's circle diagram and the main equations governing it will be summarized below.

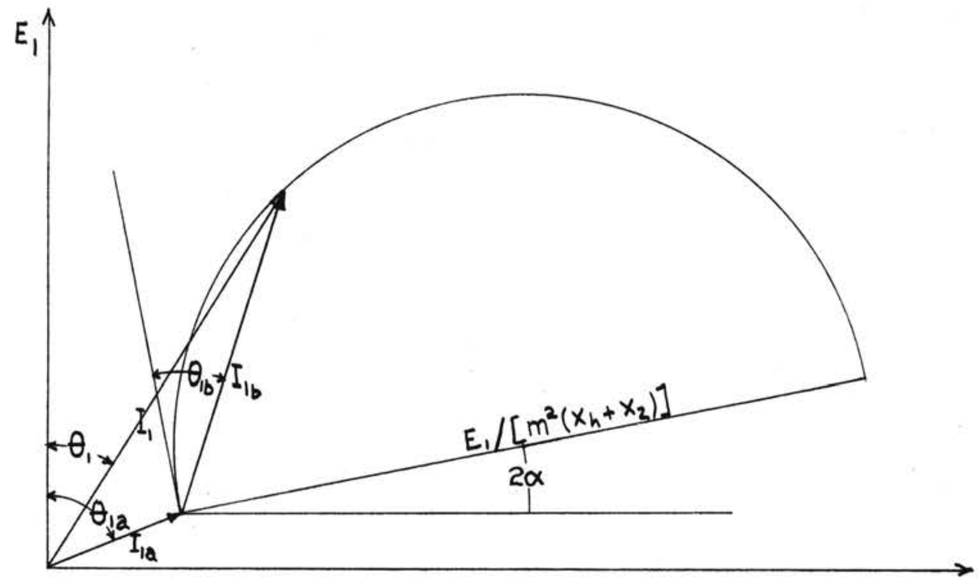

Figure 8. Ho's Exact Circle Diagram of the polyphase induction motor.

Referring to Fig. 8:

$$I_1 = I_{1a} + I_{1b} \quad (21)$$

$$I_{1a} = \frac{E_1}{(r_o + r_1) + j(x_o + x_1)} \quad (22)$$

$$\theta_{1a} = \tan^{-1} (x_o + x_1)/(r_o + r_1) \quad (23)$$

$$I_{1b} = \frac{E_1}{m^2(x_h + x_2)} \sin \theta_{1b} \quad (24)$$

$$\theta_{1b} = \tan^{-1} (x_h - x_1)/(r_h - r_2 - R) \quad (25)$$

$$\alpha = \tan^{-1}(r_o + r_1)/(x_o + x_1) - \tan^{-1} r_o/x_o \qquad (26)$$

$$m = \sqrt{(r_o + r_1)^2 + (x_o + x_1)^2} \;/\; \sqrt{r_o^2 + x_o^2} \qquad (27)$$

In order for the current locus to follow a circle as shown in Fig. 8, I_{1a}, α , m, and $(x_h + x_2)$ must be constant quantities.

From tests on different kinds of induction machines it can be shown that some machines have current loci which follow fairly closely the exact circle diagram derived by Mr. Ho, while other machines have current loci which deviate largely from the exact circle diagram. Therefore, one or more of the four quantities mentioned above must change and cannot be assumed constant.

First only I_{1a}, α , and m shall be considered. It will be shown that in most cases these three quantities may be assumed constant; in other words, the changes in these quantities are so small that they cannot account for any appreciable deviation from the circle diagram. By observing equations 22, 26, and 27 it is seen that in order for I_{1a}, α , and m to remain constant, r_o, r_1, x_o, and x_1 must stay constant. All four of these parameters, however, change to some degree due to different causes:

The value of r_o will change due to change in core losses, which in turn change with the load.

The value of r_1 will change with the resistivity of the primary windings, and resistivity will change with the temperature which usually changes with the load.

The value of x_o will change because of saturation in iron due to the main flux. In most cases, however, this saturation effect is very small.[3]

The value of x_1 will change due to saturation in the teeth of the stator.[3]

$$I_{1a} = \frac{E_1}{(r_o + r_1) + j(x_o + x_1)} \tag{28}$$

As pointed out already, saturation due to the main flux is negligibly small so that x_o may be considered a constant. Usually, x_o is much greater than either of the other three parameters. For this reason the magnitude of I_{1a} will change but very little. Due to change in r_o and r_1, the direction of I_{1a} will change some, but for most practical purposes, I_{1a} may be considered constant.

$$\alpha = \tan^{-1}(r_o + r_1)/(x_o + x_1) - \tan^{-1} r_o/x_o \tag{29}$$

The value of α will also change very little. A change in x_o or r_o, for example, will change $(r_o + r_1)/(x_o + x_1)$ and r_o/x_o almost in the same proportion. A change in x_1 or r_1 may change the value of α, but not enough to account for any appreciable error in the circle diagram. Usually, the value of α is very small in the first place, from one-half to two degrees.

$$m = \sqrt{(r_o + r_1)^2 + (x_o + x_1)^2} \;/\; \sqrt{r_o^2 + x_o^2} \tag{30}$$

Regarding the value of m, a change in x_o or r_o will change its numerator and denominator almost proportionally, so the net change in the fraction will be negligible. As was said regarding α, a change in x_1 or r_1 will change m too, but not enough to account for any appreciable error in the circle diagram.

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[3] Richter, loc. cit.

As the three quantities discussed on page 14 very unlikely can account for the relatively large deviation from the circle diagram sometimes encountered, a change in the quantity $(x_h + x_2)$, which appears in the equation for I_{1b}, must be the main reason why the current locus forms an ellipse rather than a circle. The quantity $(x_h + x_2)$ is approximately equal to and directly dependent upon the quantity $(x_1 + x_2)$. Therefore, the primary and secondary leakage reactances must change. It is found in textbooks that x_2 may decrease due to skin effect. In other words, because of skin effect, x_2 is less at standstill, when the rotor frequency is equal to that of the machine input, than under normal operating condition when the rotor frequency is only a few per cent of machine input frequency. Within the range of normal operating condition, however, the change in x_2 due to skin effect is so little that it can be neglected.

But $(x_1 + x_2)$ also changes due to saturation. At high value of slips the currents are large in stator as well as in rotor, and they produce high leakage flux which may result in high saturation in the teeth of stator and rotor. For a fixed current high saturation means that the leakage flux (the leakage reactance) is smaller than if there were no saturation. Thus when the load on the machine is small, $(x_1 + x_2)$ is greater than when the load is large. For this reason, for a certain angle, θ_{1b}, the I_{1b} below standstill will actually be smaller than Ho's circle diagram will show. This fact explains why the current locus will follow some sort of an ellipse rather than a circle, as shown in Fig. 9.

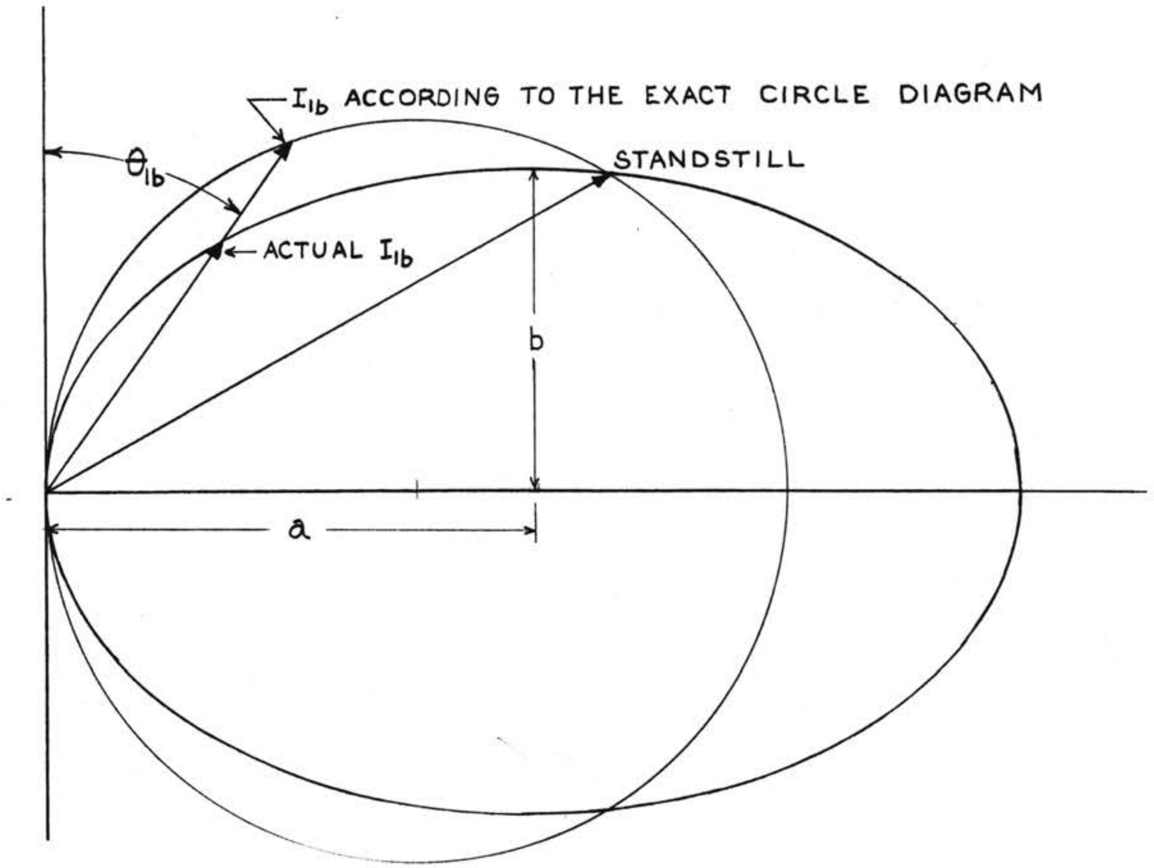

Figure 9. An illustration showing a comparison between Ho's exact circular current locus and the proposed elliptical current locus.

Chapter III

Elliptical Diagram - The Two Tests Method

A method of determining the current locus from only two tests will be introduced in this chapter. The two tests are the no-load running test and the blocked rotor test. By example problems it will be shown that the elliptical diagrams obtained by the Two Tests Method will follow very closely the current loci as obtained from actual tests.

Derivation of the Polar Equation of an Ellipse

The ellipse shown in Fig. 9 is drawn below:

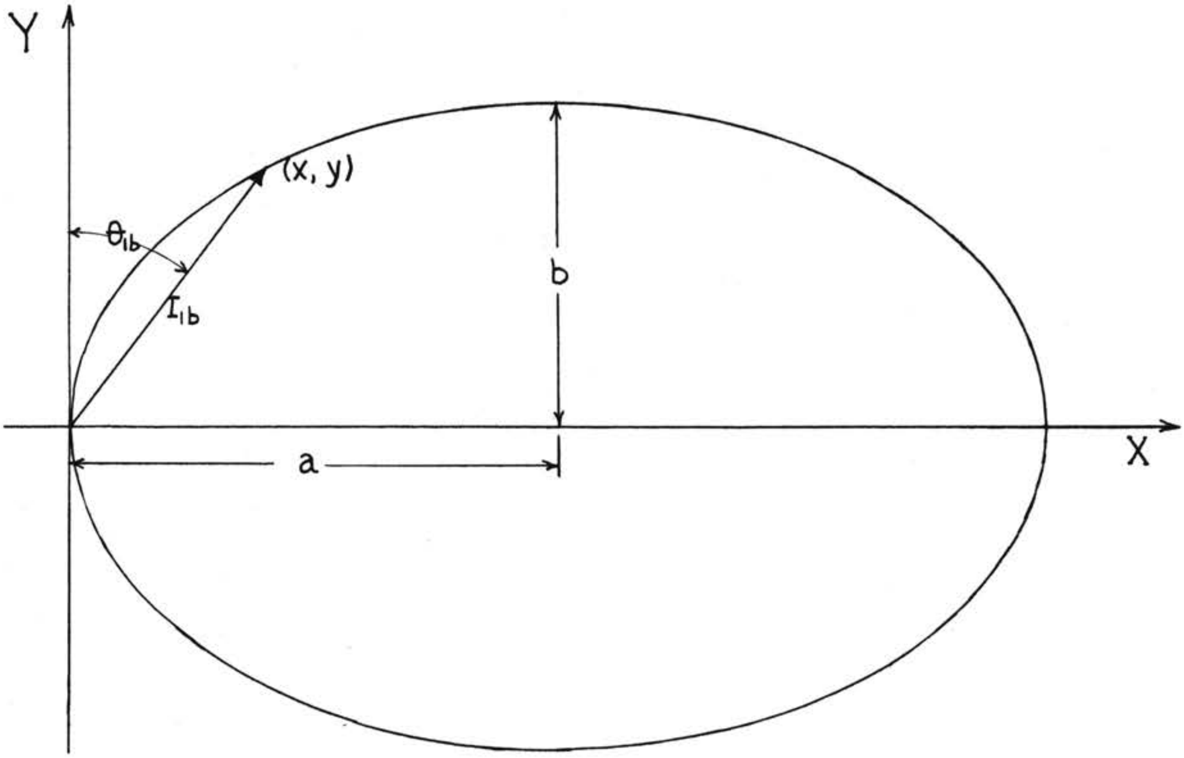

Figure 10. An elliptical current locus of the polyphase induction machine.

The equation of the ellipse above may be written as follows:[1]

$$\frac{(x - a)^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{31}$$

From Fig. 10 above

$$x = I_{1b} \sin \theta_{1b} \tag{32}$$

$$y = I_{1b} \cos \theta_{1b} \tag{33}$$

By substituting the values of x and y as given by equations 32 and 33 into equation 31 above, the following relation is obtained:

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[1]Burington, Handbook of Mathematical Tables and Formulas, 2 Ed. p. 28.

$$\frac{(I_{1b} \sin \theta_{1b} - a)^2}{a^2} + \frac{I_{1b}^2 \cos^2 \theta_{1b}}{b^2} = 1 \qquad (34)$$

Multiplying each side of equation 34 by a^2 gives:

$$(I_{1b} \sin \theta_{1b} - a)^2 + \frac{a^2}{b^2} I_{1b}^2 \cos^2 \theta_{1b} = a^2 \qquad (35)$$

Multiplying out the square $(I_{1b} \sin \theta_{1b} - a)^2$, gives:

$$I_{1b}^2 \sin^2 \theta_{1b} - 2 I_{1b} a \sin \theta_{1b} + a^2 + \frac{a^2}{b^2} I_{1b}^2 \cos^2 \theta_{1b} = a^2 \qquad (36)$$

Subtracting a^2 on each side, gives:

$$I_{1b}^2 \sin^2 \theta_{1b} - 2 I_{1b} a \sin \theta_{1b} + \frac{a^2}{b^2} I_{1b}^2 \cos^2 \theta_{1b} = 0 \qquad (37)$$

Dividing through by I_{1b}, gives:

$$I_{1b} \sin^2 \theta_{1b} - 2 a \sin \theta_{1b} + \frac{a^2}{b^2} I_{1b} \cos^2 \theta_{1b} = 0 \qquad (38)$$

Solving for I_{1b}, gives:

$$I_{1b} = \frac{2 a \sin \theta_{1b}}{\sin^2 \theta_{1b} + \frac{a^2}{b^2} \cos^2 \theta_{1b}} \qquad (39)$$

Substituting $(1 - \sin^2 \theta_{1b})$ for $\cos^2 \theta_{1b}$, gives:

$$I_{1b} = \frac{2 a \sin \theta_{1b}}{\sin^2 \theta_{1b} + \frac{a^2}{b^2} (1 - \sin^2 \theta_{1b})} \qquad (40)$$

Or:

$$I_{1b} = \frac{2 a}{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \sin^2 \theta_{1b}} \sin \theta_{1b} \qquad (41)$$

Determination of the Ratio of the Major Axis to the Minor Axis of the Ellipse.

The equations for the circle and the ellipse shown in Fig. 9 are according to equations 15 and 41 as follows:

For the exact circle,

$$I_{1b} = \frac{E_1}{m^2 \ (x_h + x_2)} \ \sin \theta_{1b} \tag{42}$$

For the ellipse,

$$I_{1b} = \frac{2a}{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \sin^2 \theta_{1b}} \ \sin \theta_{1b} \tag{43}$$

By comparing the two equations which are given above, it is seen that in order for I_{1b} to follow an ellipse rather than a circle, the following relationship must hold true:

$$(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \sin^2 \theta_{1b} \ = \ k \ (x_h + x_2) \tag{44}$$

where k is a proportionality factor

From equation 42, $$\sin^2 \theta_{1b} = I_{1b}^2 \ m^4 \ (x_h + x_2)^2 / E_1^{\ 2} \tag{45}$$

The substituting of equation 45 into equation 44 gives:

$$k = \frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \frac{I_{1b}^2 \ m^4 \ (x_h + x_2)^2}{E_1^{\ 2}}}{(x_h + x_2)} \tag{46}$$

Equation 46 contains two unknowns: the proportionality factor k, and the desired ratio a/b. This ratio, namely the ratio between the major and minor axis of the ellipse, can be determined if it would be possible to find two different values of $(x_h + x_2)$ and their corresponding values of I_{1b}.

Under blocked rotor condition a certain voltage E'_{1s} will cause a certain current I'_{1s} to flow and a certain power input W'_s. From equation 11 the following relation may be derived:

$$I_{1b} = I_1 - I_{1a} \tag{47}$$

It has been pointed out on page 14 that I_{1a} may be considered constant and be determined by the equation:

$$I_{1a} = \frac{E_1 \text{ (rated)}}{(r_o + r_1) + j(x_o + x_1)} \tag{48}$$

where r_1, r_o, x_1, and x_o are determined according to procedure given below, except for certain cases where a more accurate method of calculation should be followed.[2]

The value of r_1 may be determined by means of a Wheatstone bridge, or by the voltmeter-ammeter method.

The value of r_o may be determined from the no load test according to the equation:

$$r_o = \frac{W_n - I_{1n}^2 r_1 - F\&W}{I_{1n}^2} \tag{49}$$

where, W_n = no load input watts per phase

I_{1n} = no load phase current in amperes

$F\&W$ = friction and windage losses per phase

The value of x_1 may be determined from the blocked rotor test, preferably at or close to rated current, according to the equation:

$$x_1 = \tfrac{1}{2} \sqrt{(E_{1s}/I_{1s})^2 - (W_s/I_{1s}^2)^2} \tag{50}$$

where, E_{1s} = blocked rotor phase voltage used

I_{1s} = blocked rotor phase current

W_s = blocked rotor input watts per phase

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[2] Liwschitz-Garik and Whipple, Electric Machinery, II, p. 187.

The value of x_o may be determined from no load test according to the equation:

$$x_o = E_1/I_{1n} - x_1 \tag{51}$$

where:

E_1 = rated voltage per phase

I_{1n} = no load phase current

Having determined I_{1a}, I'_{1b} may be found from the equation:

$$I'_{1b} = I'_{1s} - I_{1a} \tag{52}$$

The quantity $(x_h + x_2)'$ may be approximately calculated from the equation:

$$(x_h + x_2)' = \sqrt{(E'_{1s}/I'_{1s})^2 - (W'_s/I'^{2}_{1s})^2} \tag{53}$$

where:

E'_{1s} = blocked rotor phase voltage

I'_{1s} = blocked rotor phase current

W'_s = blocked rotor input per phase

By applying a different voltage, E''_{1s}, under blocked rotor condition, another set of I_{1b} and $(x_h + x_2)$, I''_{1b} and $(x_h + x_2)''$ can be found. The two sets of $(x_h + x_2)$ and I_{1b} should both satisfy equation 46. Therefore, the following relation must hold true:

$$\frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \dfrac{I'^2_{1b}\, m^4 (x_h + x_2)'^2}{E_1^2 \text{ (rated/ph)}}}{(x_h + x_2)'} = \frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \dfrac{I''^2_{1b}\, m^4 (x_h + x_2)''^2}{E_1^2 \text{ (rated /ph)}}}{(x_h + x_2)''} \tag{54}$$

By solving equation 54, the desired ratio a/b is determined.

From calculation it can be shown that the value of a/b determined by two sets of I_{1b} and $(x_h + x_2)$ at currents close to rated current is somewhat different from the a/b ratio determined from two sets of I_{1b} and $(x_h + x_2)$ at currents close to rated voltage, blocked rotor current. But since it is of most interest to obtain correct performance characteristics near rated value, a/b should be determined from two sets of I_{1b} and $(x_h + x_2)$ near rated current preferably.

Determination of each Axis of the Ellipse separately

It has already been shown how to determine the ratio a/b. The next step will be to find a method whereby a and b can be determined separately.

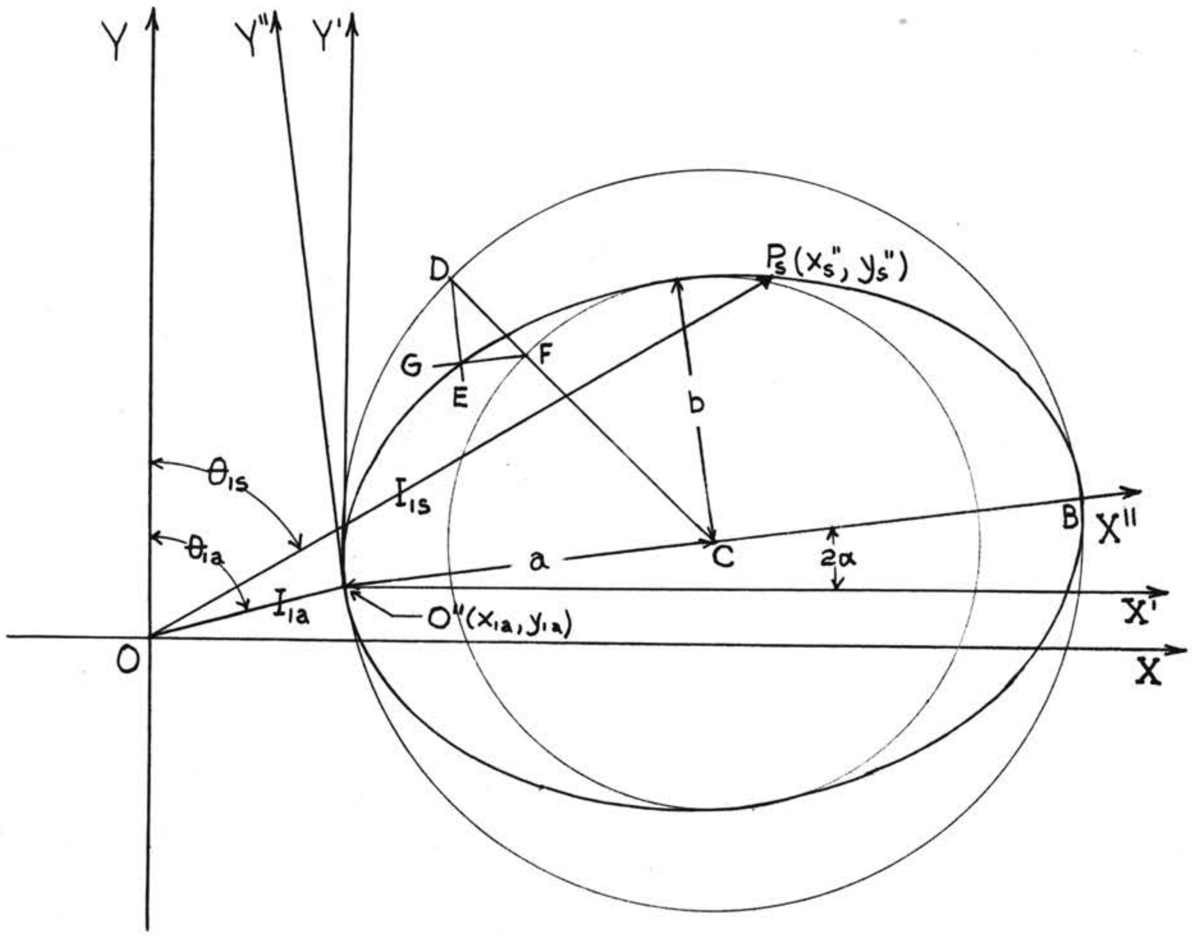

Figure 11. Construction procedure for the elliptical diagram.

I_{1s} refers to the blocked rotor, rated voltage current and θ_{1s} its corresponding power factor angle. For I_{1a}, θ_{1a}, and α see pages 12 and 13, equations 21, 22, and 26 respectively.

The equation for the ellipse shown in Fig. 11 using X" and Y" for coordinate axes is as follows:

$$\frac{(x_s'' - a)^2}{a^2} + \frac{y_s''^2}{b^2} = 1 \tag{55}$$

Substituting $a^2/(1/\frac{a}{b})^2$ for b^2 in equation 55 gives:

$$\frac{(x_s'' - a)^2}{a^2} + \frac{y_s''^2}{a^2/(1/\frac{a}{b})^2} = 1 \tag{56}$$

Solving this equation for a gives:

$$a = \frac{(x_s'')^2 + (\frac{a}{b}\, y_s'')^2}{2x_s''} \tag{57}$$

After a has been determined, b may be determined from the equation:

$$b = a/(a/b) \tag{58}$$

In order to find x_s'' and y_s'' in equation 57, first find the abcissa x_s and the ordinate y_s for point P_s using the coordinate system X, Y.

$$x_s = I_{1s} \sin|\theta_{1s}| \tag{59}$$

$$y_s = I_{1s} \cos|\theta_{1s}| \tag{60}$$

where:

$$\theta_{1s} = \cos^{-1} \frac{W_s}{I_{1s} \cdot E_1} \tag{61}$$

For equation 59:

W_s = watts input per phase at blocked rotor rated voltage condition.

I_{1s} = phase current at blocked rotor rated voltage condition.

E_1 = rated voltage.

Next using the coordinate system X', Y' find the abcissa x_s' and the ordinate y_s' for point P_s as follows:

$$x_s' = x_s - x_{1a} \tag{62}$$

$$y_s' = y_s - y_{1a} \qquad (63)$$

where:

$$x_{1a} = I_{1a} \sin|\theta_{1a}| \qquad (64)$$

$$y_{1a} = I_{1a} \cos|\theta_{1a}| \qquad (65)$$

Finally, determine the abcissa x_s'' and the ordinate y_s'' for point P_s using the coordinate system X", Y".

$$x_s'' = x_s' \cos 2\alpha + y_s' \sin 2\alpha \qquad (66)$$

$$y_s'' = y_s' \cos 2\alpha - x_s' \sin 2\alpha \qquad (67)$$

When I_{1a}, θ_{1a}, 2α , a, and b have been found, everything is known in order to draw the elliptical current locus. Referring to Fig. 11, first draw I_{1a} at an angle θ_{1a} with the Y-axis. Next draw the line O"B = 2a at an angle 2α with the X-axis. On the middle of this line, point C, draw two circles, one with radius a and one with radius b. From C draw any line CD; where this line crosses the outer circle, draw a line DE perpendicular to the X"-axis; and where the line CD crosses the inner circle, draw a line FG parallel with the X"-axis. Where the line DE and FG intersect, there is a point on the ellipse. By successful repetitions of this procedure, enough points can be obtained so that the ellipse can be drawn by free hand or with a French curve.

Example Problem I

Machine tested: Westinghouse Lifeline Induction Motor
5-hp, 1750-rpm, 220/440-volt, 3-phase,
60-cycle, Nema Design A.

All values below refer to phase values if not otherwise designated.

No load test gave the following result:

$$W_n = 100 \text{ watts}$$

$$I_{1n} = 6.00 \text{ amps}$$

$$E_1 = 127 \text{ volts}$$

$$F\&W = 20 \text{ watts}$$

Blocked rotor test gave the following result:

W_s watts	I_{1s} amps	E_{1s} volts	θ_{1s} degrees
1040	34.6	60	- 59.8
2110	48.2	80	- 56.7
6450	85.7	127	- 53.6

Primary resistance measurement gave the following result:

$$r_1 = .45 \text{ ohm}$$

From equation 49:

$$r_o = (W_n - I_{1n}^2 r_1 - F\&W)/I_{1n}^2$$

$$r_o = (100 - 6^2 \times .45 - 20)/6^2 = 1.79 \text{ ohm}$$

From equation 50:

$$x_1 = \frac{1}{2}\sqrt{(E_{1s}/I_{1s})^2 - (W_s/I_{1s}^2)^2}$$

$$x_1 = \frac{1}{2}\sqrt{(60/34.6)^2 - (1040/34.6^2)^2} = .75 \text{ ohm}$$

From equation 51:

$$x_o = E_1/I_{1n} - x_1$$

$$x_o = 127/6 - .75 = 20.4 \text{ ohm}$$

From equation 48: $$I_{1a} = \frac{E_1 \text{ (rated)}}{(r_o + r_1) + j(x_o + x_1)}$$

$$I_{1a} = \frac{127}{(1.79 + .45) + j(20.4 + .75)}$$

$$I_{1a} = 5.98 \; \underline{/-83.95^\circ} \text{ amps}$$

From equation 13: $$m^2 = \frac{(r_o + r_1)^2 + (x_o + x_1)^2}{r_o^2 + x_o^2}$$

$$m^2 = \frac{(1.79 + .45)^2 + (20.4 + .75)^2}{1.79^2 + 20.4^2} = 1.064$$

From equation 14: $$\alpha = \tan^{-1} (r_o + r_1)/(x_o + x_1) - \tan^{-1} r_o/x_o$$

$$\alpha = \tan^{-1} \frac{1.79 + .45}{20.4 + .75} - \tan^{-1} \frac{1.79}{20.4} = 1.01^\circ$$

$$\text{Or } 2\alpha = 2.02^\circ$$

Let:

$E'_{1s} = 60$ volts

$I'_{1s} = 34.6$ amps

$W'_s = 1040$ watts

$\theta'_{1s} = -59.8^\circ$

And let:

$E''_{1s} = 80$ volts

$I''_{1s} = 48.2$ amps

$W''_s = 2110$ watts

$\theta''_{1s} = -56.7^\circ$

From equation 53: $$(x_h + x_2)' = \sqrt{(E'_{1s}/I'_{1s})^2 - (W'_s/I'^2_{1s})^2}$$

$$(x_h + x_2)' = \sqrt{(60/34.6)^2 - (1040/34.6^2)^2}$$

$$(x_h + x_2)' = 1.50 \text{ ohms}$$

From equation 52:

$$I'_{1b} = I'_{1s} - I_{1a}$$

$$I'_{1b} = 34.6 \underline{/-59.8} - 5.98 \underline{/-83.95} = 29.2 \underline{/-55.0} \text{ amps}$$

Similarly:

$$(x_h + x_2)'' = (80/48.2)^2 - (2110/48.2^2)^2 = 1.39 \text{ ohms}$$

$$I''_{1b} = 48.2 \underline{/-56.7} - 5.98 \underline{/-83.95} = 43.0 \underline{/-53.0} \text{ amps}$$

Using equation 54:

$$\frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \dfrac{I'^2_{1b}\, m^4 (x_h + x_2)'^2}{E_1^2 \text{ (rated)}}}{(x_h + x_2)'} =$$

$$\frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \dfrac{I''^2_{1b}\, m^4 (x_h + x_2)''^2}{E_1^2 \text{ (rated)}}}{(x_h + x_2)''}$$

$$\frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \dfrac{29.2^2 \times 1.064^2 \times 1.50^2}{127^2}}{1.50^2} =$$

$$\frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \dfrac{43^2 \times 1.064^2 \times 1.39^2}{127^2}}{1.39^2}$$

Solution of this equation gives:

$$\frac{a}{b} = 1.54$$

From equations 59 and 60:

$$x_s = I_{1s} \sin|\theta_{1s}| = 85.7 \sin 53.6 = 69$$

$$y_s = I_{1s} \cos|\theta_{1s}| = 85.7 \cos 53.6 = 50.8$$

From equations 64 and 65:

$$x_{1a} = I_{1a} \sin|\theta_{1a}| = 5.98 \sin 83.95 = 5.94$$

$$y_{1a} = I_{1a} \cos|\theta_{1a}| = 5.98 \cos 83.95 = .54$$

From equations 62 and 63:

$$x_s' = x_s - x_{1a} = 69 - 5.94 = 63.06$$

$$y_s' = y_s - y_{1a} = 50.8 - .54 = 50.26$$

From equations 66 and 67:

$$x_s'' = x_s' \cos 2\alpha + y_s' \sin 2\alpha = 63.6 \cos 2.02 + 50.26 \sin 2.02 = 64.8$$

$$y_s'' = y_s' \cos 2\alpha - x_s' \sin 2\alpha = 50.26 \cos 2.02 - 63.6 \sin 2.02 = 48.1$$

From equations 57 and 58:

$$a = \frac{(x_s'')^2 + (\frac{a}{b}\, y_s'')^2}{2x_s''} = \frac{64.8^2 + (1.54 \times 48.1)^2}{2 \times 64.8} = 74.6$$

$$b = a/(a/b) = 74.6/1.54 = 48.5$$

The elliptical diagram resulting from these calculations is shown in Fig. 12. Also on the same curve sheet is plotted the actual current locus as obtained from test data. Test data points are designated by the symbol, ⊙ . It is seen that the ellipse follows closely the current locus as obtained from test data. Ho's exact circle diagram, which also is drawn in on Fig. 12, is seen to deviate considerably from actual test performance.

The fact that the elliptical diagram follows closely the actual current locus as obtained from test means that for any

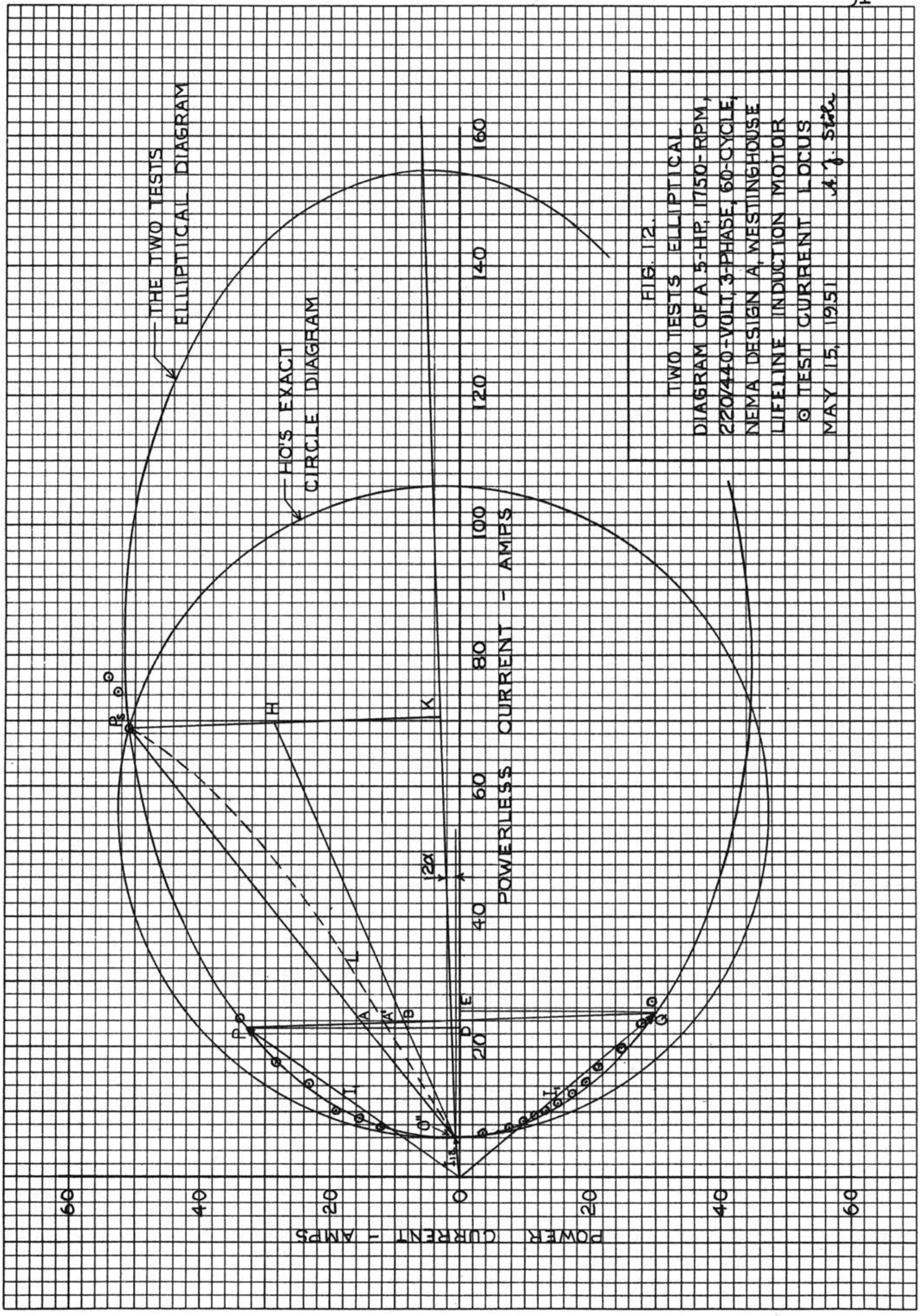

FIG. 12.
TWO TESTS ELLIPTICAL
DIAGRAM OF A 5-HP, 1750-RPM,
220/440-VOLT, 3-PHASE, 60-CYCLE,
NEMA DESIGN A, WESTINGHOUSE
LIFELINE INDUCTION MOTOR
⊙ TEST CURRENT LOCUS
MAY 15, 1951

current the power factor as well as the power input to the motor and the power output of the machine, if used as a generator, will be correct, if determined from the elliptical diagram.

In order to obtain and check torque, power output of the motor (or power input for generator action), slip, and efficiency, the line O"H has to be determined. This may be done by finding the distance KH which in ampere units is equal to $(r_h I_{1s}^2)/127$. The resistance r_h may be found from equation 4 as follows:

$$r_h + jx_h = \frac{(r_o + jx_o)\ (r_1 + jx_1)}{(r_o + r_1) + j(x_o + x_1)}$$

$$r_h + jx_h = \frac{(1.79 + j20.4)\ (.45 + j.75)}{(1.79 + .45) + j(20.4 + .75)} = (.415 - j.73) \text{ ohm}$$

Or:

$$r_h = .415 \text{ ohm}$$

When calculating the value for r_h above, r_1 was taken as the direct current primary resistance. Actually, however, the 60-cycle value of r_1 should have been used. Usually, the 60-cycle value of the primary resistance is at least 1.1 times its direct current value. Thus,

$$r_1 \text{ (60-cycle)} = 1.1 \times .45 = .495 \text{ ohm}$$

Calculating r_h using $r_1 = .495$ ohm gives:

$$r_h + jx_h = \frac{(1.79 + j20.4)\ (.495 + j.75)}{(1.79 + .495) + j(20.4 + .75)} = (.46 + j.73) \text{ ohm}$$

Or:

$$r_h = .46 \text{ ohm}$$

It is seen from the calculation above that r_h is approximately equal to the direct current resistance of the primary circuit.

Therefore, to simplify the calculation, r_h may be assumed equal to the $r_{1(d.c.)}$, which in this example was .45 ohm. Consequently:

$$KH = \frac{.45 \times 85.7^2}{127} = 26$$

If a reliable measurement can be taken of torque, T_s, at blocked rotor, rated voltage condition, then P_sH can be calculated as follows:

$$P_sH = \frac{1}{3} \times \frac{T_s}{E_1} \times \frac{\text{syn. rpm}}{7.04} \qquad (68)$$

where, T_s = total torque in lb x ft

The value of P_sH may then be used as a check for the value of KH.

Table I

PERFORMANCE CHARACTERISTICS FOR A 5-HP, 1750-RPM, 220/440-VOLT, 3-PHASE, 60-CYCLE, WESTINGHOUSE LIFELINE INDUCTION MACHINE, NEMA DESIGN A. AS OBTAINED FROM THE ELLIPTICAL DIAGRAM - THE TWO TESTS METHOD.

Current at 127 Volt/ph AMP	Elect. power input KW	Torque LBxFT	Mech. power output HP	Effi-ciency	Power factor	Slip
Motor Action:						
8	1.975	6.33	2.10	.794	.645	.0273
10	2.915	9.61	3.16	.807	.751	.0394
12	3.735	12.44	4.03	.807	.815	.0506
14	4.490	15.00	4.80	.797	.843	.0637
16	5.220	17.35	5.49	.784	.858	.0746
18	5.940	19.60	6.13	.769	.868	.0864
20	6.630	21.62	6.66	.750	.871	.0990
22	7.290	23.47	7.13	.730	.871	.1120
24	7.950	25.27	7.56	.710	.870	.1245
Generator Action:						
	Output KW		Input HP			
8	1.734	7.99	2.81	.828	.626	.0269
10	2.655	11.86	4.21	.846	.700	.0372
12	3.470	15.60	5.60	.830	.760	.0485
14	4.215	18.87	6.82	.828	.792	.0560
16	4.920	22.13	8.10	.814	.808	.0680
18	5.600	25.43	9.35	.803	.817	.0728
20	6.250	28.73	10.62	.788	.823	.0817
22	6.860	31.83	11.88	.774	.824	.0887
24	7.470	34.83	13.10	.765	.821	.0970

Table II

PERFORMANCE CHARACTERISTICS FOR A 5-HP, 1750-RPM, 220/440-VOLT, 3-PHASE, 60-CYCLE, WESTINGHOUSE LIFELINE INDUCTION MACHINE, NEMA DESIGN.

DYNAMOMETER TEST CHARACTERISTICS

Current at 127 Volt/ph AMPS	Elect. power input KW	Torque LBxFT	Mech. power output HP	Efficiency	Power factor	Slip
Motor Action:						
6.00	.300	0.00	0.00	.000	.131	
6.43	1.025	2.15	0.75	.545	.418	.0042
8.45	2.220	6.50	2.12	.713	.659	.0105
11.03	3.450	10.50	3.52	.760	.821	.0170
14.08	4.525	14.70	4.91	.808	.843	.027
17.66	5.825	18.90	6.23	.797	.861	.039
21.50	7.225	23.10	7.56	.780	.881	.043
27.00	8.725	27.30	8.79	.751	.847	.061
33.00	10.700	31.50	9.91	.691	.851	.083
41.50	12.800	35.70	10.72	.625	.810	.122
85.70	19.350		0.00	.000	.594	1.000
Brake Action:	Input KW		Input HP			
91.00	20.400	42.0	20.60		.59	1.43
93.80	21.200	43.6	25.90		.59	1.73
96.00	21.600	45.2	32.40		.59	2.09
Generator Action:	Output KW		Input HP			
7.48	1.400	6.20	2.135	.881	.492	.00732
10.50	2.920	12.6	4.40	.889	.730	.01495
13.17	3.780	16.3	5.72	.885	.753	.0195
14.87	4.400	18.90	6.66	.887	.777	.0234
16.50	5.000	21.50	7.61	.881	.796	.0268
18.70	5.705	25.20	8.93	.857	.801	.0315
21.36	6.520	29.30	10.45	.836	.803	.0389
24.20	7.375	33.60	12.08	.820	.800	.0472
27.33	8.200	36.80	13.23	.830	.783	.0500
31.33	9.325	45.15	16.40	.765	.782	.0583
36.50	10.680	52.50	19.40	.742	.768	.0722
40.00	11.230	57.80	21.50	.701	.737	.0833

Sample Calculation for Table I

The results shown in Table I were found by drawing an elliptical diagram similar to that shown in Fig. 12, but of larger scale, so that more accurate results could be obtained.

Motor Action:

For the last reading, $I_1 = 24$ amps. With center at the origin, point O, a circular arc with a radius representing 24 amps. was drawn. Where this arc intersected the motor action side of the elliptical diagram is marked as point P. A line was drawn from origin to this point. The angle between the vertical axis and this line was measured and found to be 29.6 degrees. From point P, two lines were drawn, one perpendicular to the horizontal axis and the other perpendicular to the major axis of the ellipse. The following distances were measured:

$$PD = 20.85$$
$$PB = 17.10$$
$$PA = 14.97$$
$$AB = 2.13$$

Input Power = $3 \times E_1/ph \times PD = 3 \times 127 \times 20.85 = 7950$ watts = 7.95 KW

Torque $= (3 \times E_1/ph \times PB - 3 \times F\&W/ph) \dfrac{7.04}{\text{syn. R.P.M.}}$

$= (3 \times 127 \times 17.10 - 3 \times 20) \dfrac{7.04}{1800} = 25.27$ lb x ft

Output power $= 3 \times E_1/ph \times PA - 3 \times F\&W/ph$

$= 3 \times 127 \times 14.97 - 3 \times 20 = 5640$ watts

Or output power = 5640/746 = 7.56 HP

Efficiency = Output power/Input power = 5640/7950 = .710

Power factor = cos 29.6 = .870

Slip = AB / PB = .1245

Generator Action:

For the last reading, I_1 = 24 amps. With center at the origin, point O, a circular arc with a radius representing 24 amps was drawn. Where this arc intersected the generator action side of the elliptical diagram is designated as point Q. A line was drawn from origin to this point. The angle between the vertical axis and this line was measured and found to be 34.80 degrees. From Q two lines were drawn, one perpendicular to the horizontal axis and the other perpendicular to the major axis of the ellipse. The following distances were measured:

QE = 19.60

QB = 23.20

QA = 25.45

AB = 2.25

Output power = $3 \times E_1/ph \times QE = 3 \times 127 \times 19.60 = 7470$ watts = 7.47 KW

Torque = $(3 \times E_1/ph \times QB + 3 \times F\&W/ph) \dfrac{7.04}{\text{syn. R.P.M.}}$

= $(3 \times 127 \times 23.2 + 3 \times 20) \dfrac{7.04}{1800} = 34.83$ lb x ft

Input power = $3 \times E_1/ph \times QA + 3 \times F\&W/ph$

= $3 \times 127 \times 25.45 + 3 \times 20 = 9760$ watts

Or input power = 9760/746 = 13.10 HP

Efficiency = Output power / Input power = 7470/9760 = .765

Power factor = cos 34.8 = .821

Slip = AB/QB = 2.25/23.20 = .0970

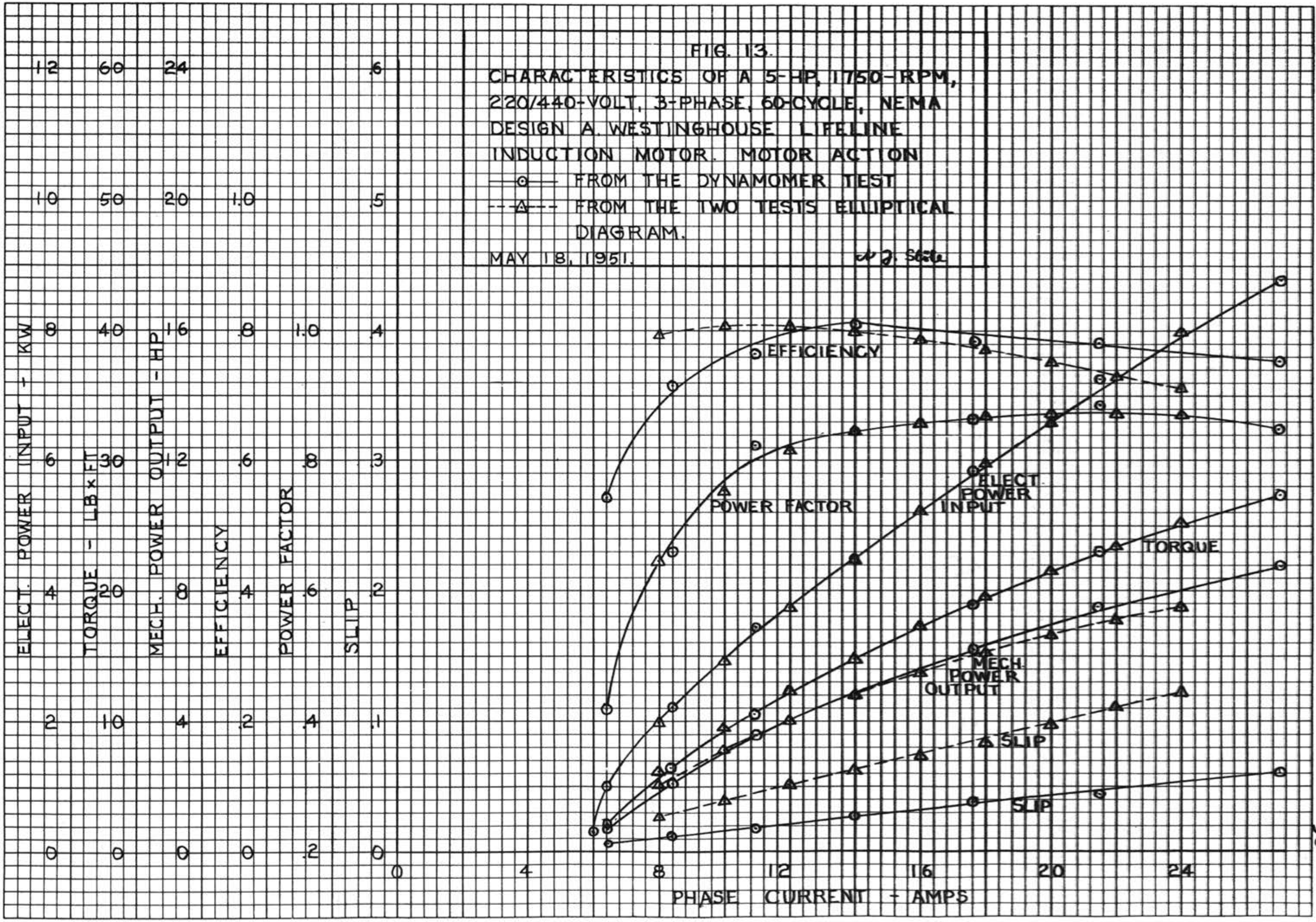

FIG. 13.
CHARACTERISTICS OF A 5-HP, 1750-RPM, 220/440-VOLT, 3-PHASE, 60-CYCLE, NEMA DESIGN A, WESTINGHOUSE LIFELINE INDUCTION MOTOR. MOTOR ACTION

—⊙— FROM THE DYNAMOMER TEST

--△-- FROM THE TWO TESTS ELLIPTICAL DIAGRAM.

MAY 18, 1951.

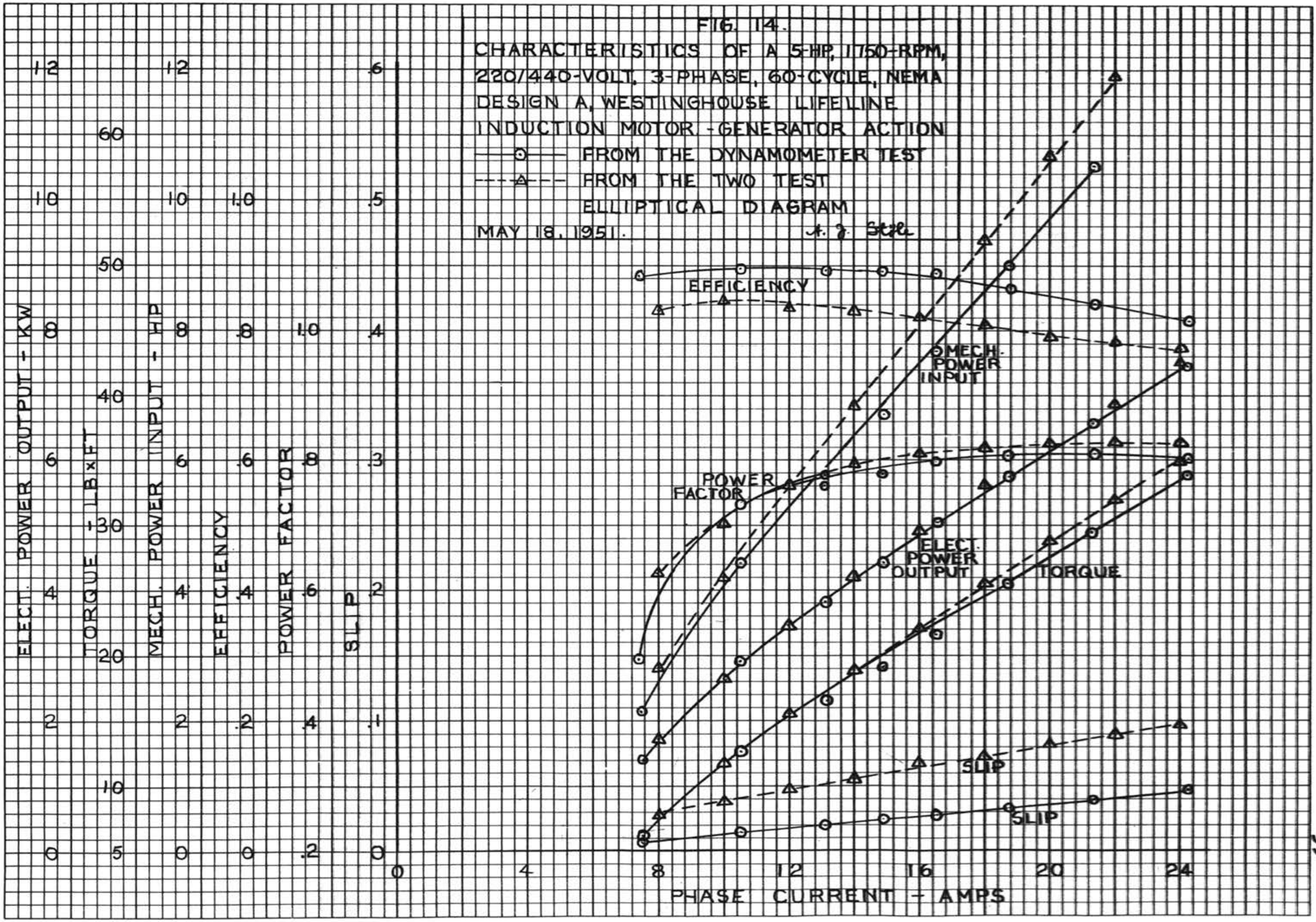
FIG. 14.
CHARACTERISTICS OF A 5-HP, 1750-RPM, 220/440-VOLT, 3-PHASE, 60-CYCLE, NEMA DESIGN A, WESTINGHOUSE LIFELINE INDUCTION MOTOR - GENERATOR ACTION
—⊙— FROM THE DYNAMOMETER TEST
---△--- FROM THE TWO TEST ELLIPTICAL DIAGRAM
MAY 18, 1951.
EFFICIENCY
MECH. POWER INPUT
POWER FACTOR
ELECT. POWER OUTPUT
TORQUE
SLIP
ELECT. POWER OUTPUT - KW
TORQUE - LB×FT
MECH. POWER INPUT - HP
EFFICIENCY
POWER FACTOR
SLIP
PHASE CURRENT - AMPS
0
4
8
12
16
20
24

Derivation of An Analytical Calculating Chart to be used with The Two Tests Elliptical Diagram

In example problem I, the induction machine characteristics were obtained by measuring distances on the elliptical diagram. A rather large diagram had to be drawn for this purpose in order to measure the distances precisely. For this reason it will be in many cases inconvenient to obtain the performance characteristics of the induction machine by measuring distances on the diagram. Therefore, an analytical method of obtaining the performance characteristics will be introduced, followed by an example problem.

Referring to Fig. 15:

The values of x_{1a}, y_{1a}, x_s'', and y_s'' are determined from equations 59-65.

$$KH = \frac{r_h\, I_{1s}^2}{E_1(\text{rated})/ph} \qquad (69)$$

where: r_h can be taken as $r_{1(d.c.)}$

$$c = \sqrt{a^2 - b^2} \qquad (70)$$

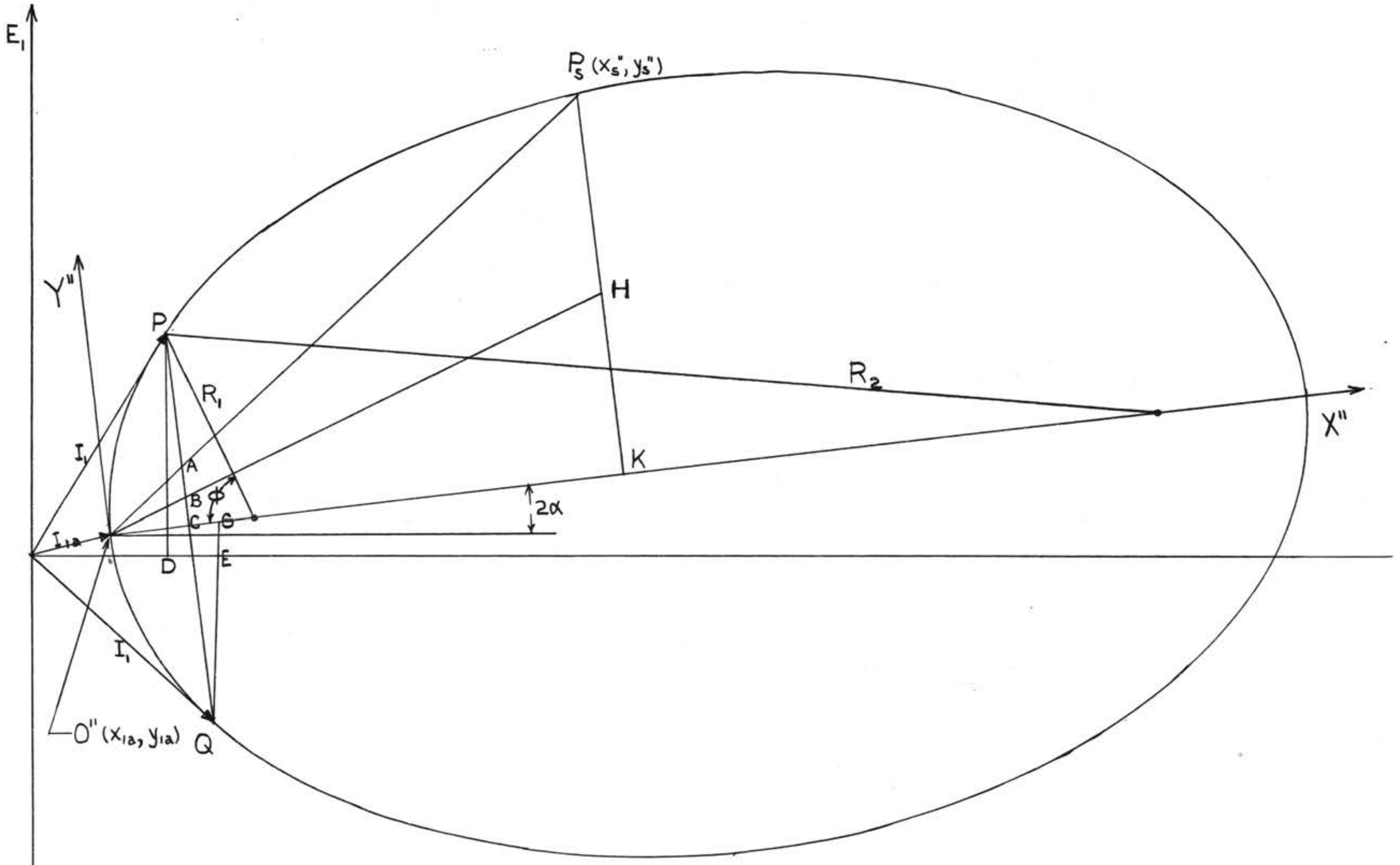

FIG. 15. ELLIPTICAL DIAGRAM OF A POLYPHASE INDUCTION MACHINE.

Table III

Analytical Calculating Chart for a Three Phase Induction Machine

1. Choose R_1, R_1 being greater than (a - c)
2. $R_2 = 2a - R_1$
3. $\phi = \cos^{-1} \dfrac{R_2^2 - R_1^2 - 4c^2}{4cR_1}$
4. $PC = R_1 \sin \phi$
5. $O''C = (a - c) - R_1 \cos \phi$
6. $BC = O''C \dfrac{KH}{x_s''}$
7. $AC = O''C \, (y_s''/x_s'')$
8. $PD = y_{1a} + O''C \sin 2\alpha + PC \cos 2\alpha$

Motor Action:

9. Output watts $= 3 \times E_1/\text{ph} \times (PC - AC) - 3 \times F\&W/\text{ph}$;

$$HP = \frac{\text{Output watts}}{746}$$

10. Input watts $= 3 \times E_1/\text{ph} \times PD$
11. Power factor $= \cos \left(\tan^{-1} \dfrac{x_{1a} + (O''C - PC \tan 2\alpha) \cos 2\alpha}{PD}\right)$
12. Phase current = PD/(power factor)
13. Torque $= \left[3 \times E_1/\text{ph} \times (PC - BC) - 3 \times F\&W/\text{ph}\right] \dfrac{7.04}{\text{syn. R.P.M.}}$
14. Slip $= (AC - BC)/(PC - BC)$
15. Efficiency = Output watts / Input watts

Generator Action:

16. $O''G = O''C + PC \tan 2\alpha$
17. $QE = PC/\cos 2\alpha - (O''G \sin 2\alpha + y_{1a})$
18. Output watts $= 3 \times E_1/\text{ph} \times QE$

19. Input watts $= 3 \times E_1/ph \times (PC + AC) + 3 \times F\&W/ph;$

$$HP = \frac{\text{Input watts}}{746}$$

20. Power factor $= \cos\left(\tan^{-1} \frac{x_{1a} + O''G \cos 2\alpha}{QE}\right)$

21. Phase current = QE/(power factor)

22. Torque $= \left[3 \times E_1/ph \times (PC + BC) + 3 \times F\&W/ph\right] \frac{7.04}{\text{syn. R.P.M.}}$

23. Slip = (AC - BC)/(PC + BC)

24. Efficiency = Output watts / Input watts

Example Problem II

Machine tested: Reliance Type A-A, 5-hp, 1155 rpm, 220-volt, 3-phase, 60-cycle, induction motor.

All values refer to phase value if not otherwise designated.

No-load test gave the following result:

$$W_n = 58.33 \text{ watts}$$

$$I_{1n} = 5.25 \text{ amps}$$

$$E_1 = 127 \text{ volts}$$

$$F\&W = 6 \text{ watts}$$

Blocked rotor test gave the following result:

W_s watts	I_{1s} amps	E_{1s} volts	θ_{1s} degrees
638	25.0	57.8	- 63.8
1062	33.5	72.7	- 64.15
4450	71.5	127.0	- 60.65

Primary resistance measurement gave the following result:

$$r_1 = .355 \text{ ohm}$$

From equation 49:

$$r_o = (W_n - I_{1n}^2 r_1 - F\&W)/I_{1n}^2$$

$$r_o = (58.33 - 5.25^2 \times .355 - 6)/5.25^2 = 1.54 \text{ ohm}$$

From equation 50:

$$x_1 = \frac{1}{2}\sqrt{(E_{1s}/I_{1s})^2 - (W_s/I_{1s}^2)^2}$$

$$x_1 = \frac{1}{2}\sqrt{(57.8/25)^2 - (638/25^2)^2} = 1.036 \text{ ohm}$$

From equation 51:

$$x_o = E_1/I_{1n} - x_1$$

$$x_o = 127/5.25 - 1.036 = 23.2 \text{ ohms}$$

From equation 48:

$$I_{1a} = \frac{E_1(\text{rated})}{(r_o + r_1) + j(x_o + x_1)}$$

$$I_{1a} = \frac{127}{(1.54 + .355) + j(23.2 + 1.036)} = 5.23 \underline{/-85.52^o} \text{ amps}$$

From equation 13:

$$m^2 = \frac{(r_o + r_1)^2 + (x_o + x_1)^2}{r_o^2 + x_o^2}$$

$$m^2 = \frac{(1.54 + .355)^2 + (23.2 + 1.036)^2}{1.54^2 + 23.2^2} = 1.1$$

From equation 14:

$$\alpha = \tan^{-1} (r_o + r_1)/(x_o + x_1) - \tan^{-1} r_o/x_o$$

$$\alpha = \tan^{-1} \frac{1.54 + .355}{23.2 + 1.036} - \tan^{-1} \frac{1.54}{23.2} = .68 \text{ degrees}$$

Or $2\alpha = 1.36$ degrees

Let:

$E'_{1s} = 57.8$ volts

$I'_{1s} = 25$ amps

$W'_s = 638$ watts

$\theta'_{1s} = -63.8$ degrees

And let:

$E''_{1s} = 72.7$ volts

$I''_{1s} = 33.5$ amps

$W''_s = 1062$ watts

$\theta''_{1s} = -64.15$ degrees

From equation 53:

$$(x_h + x_2) = \sqrt{(E'_{1s}/I'_{1s})^2 - (W'_s/I'^2_{1s})^2}$$

$$(x_h + x_2)' = \sqrt{(57.8/25)^2 - (638/25^2)^2} = 2.07 \text{ ohms}$$

From equation 52:

$$I'_{1b} = I'_{1s} - I_{1a}$$

$$I'_{1b} = 25 \underline{/-63.8} - 5.23 \underline{/-85.52} = 20.2 \underline{/-58.3} \text{ amps}$$

Similarly:

$$(x_h + x_2)'' = \sqrt{(72.7/33.5)^2 - (1062/33.5^2)^2} = 1.955 \text{ ohm}$$

$$I''_{1b} = 33.5 \underline{/-64.15} - 5.23 \underline{/-85.52} = 28.05 \underline{/-60.4} \text{ amps}$$

From equation 54:

$$\frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \frac{I'^2_{1b}\ m^4 (x_h + x_2)'^2}{E^2_{1(rated)}}}{(x_h + x_2)'} =$$

$$\frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \frac{I''^2_{1b}\ m^4 (x_h + x_2)''^2}{E^2_{1\ (rated)}}}{(x_h + x_2)''}$$

$$\frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \frac{20.2^2 \times 1.1^2 \times 2.07^2}{127^2}}{2.07} =$$

$$\frac{(\frac{a}{b})^2 - \left[(\frac{a}{b})^2 - 1\right] \frac{28.05^2 \times 1.1^2 \times 1.955^2}{127^2}}{1.955}$$

The solution of this equation gives:

$$\frac{a}{b} = 1.51$$

From equations 59 and 60:

$$x_s = I_{1s} \sin |\theta_{1s}| = 71.5 \sin 60.65 = 62.3$$

$$y_s = I_{1s} \cos |\theta_{1s}| = 71.5 \cos 60.65 = 35.1$$

From equations 64 and 65:

$$x_{1a} = I_{1a} \sin |\theta_{1a}| = 5.23 \sin 85.52 = 5.22$$

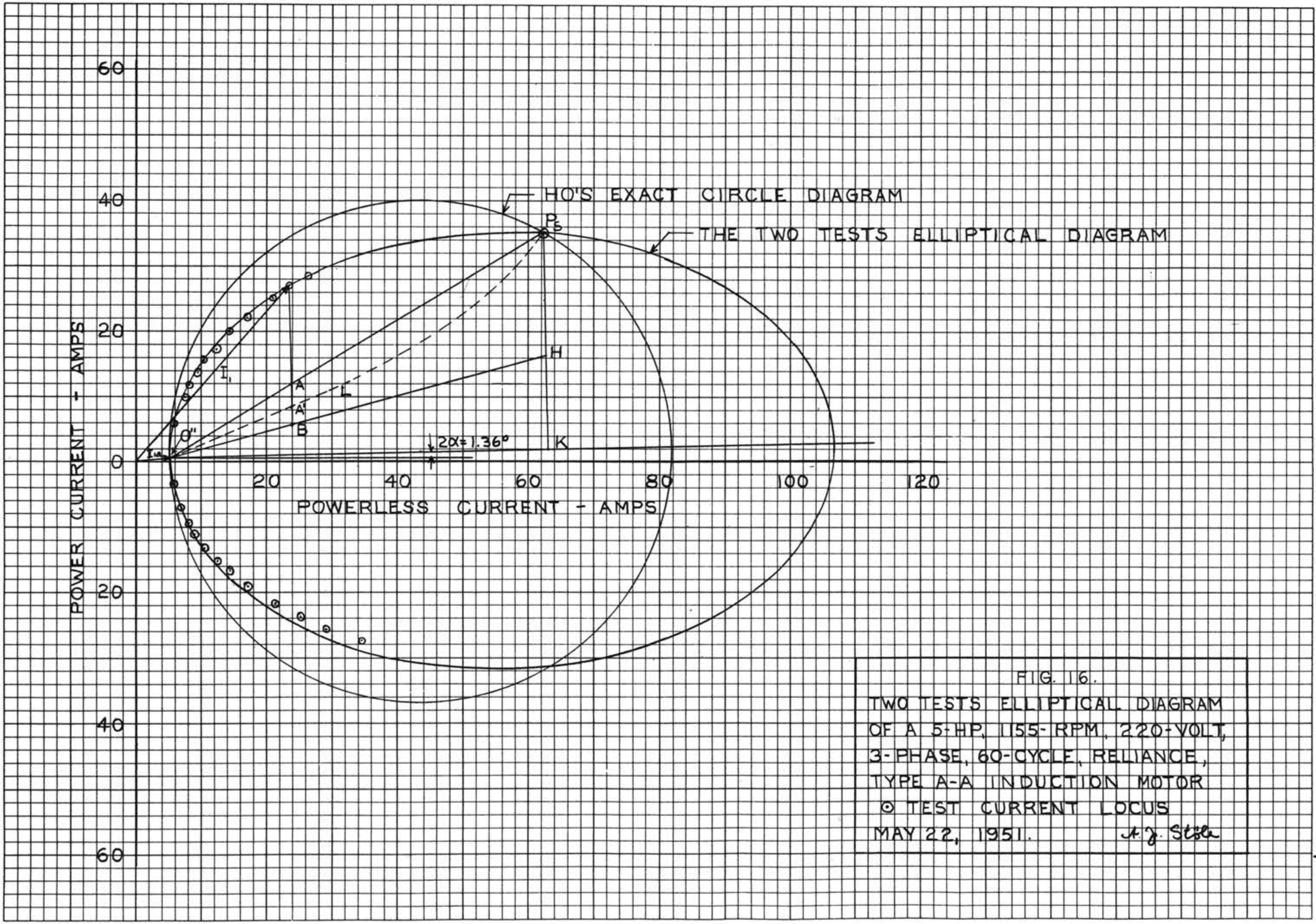

FIG. 16.

TWO TESTS ELLIPTICAL DIAGRAM OF A 5-HP, 1155-RPM, 220-VOLT, 3-PHASE, 60-CYCLE, RELIANCE, TYPE A-A INDUCTION MOTOR

⊙ TEST CURRENT LOCUS

MAY 22, 1951.

$$y_{1a} = I_{1a} \cos|\theta_{1a}| = 5.23 \cos 85.52 = .41$$

From equations 62 and 63:

$$x_s' = x_s - x_{1a} = 62.3 - 5.22 = 57.18$$

$$y_s' = y_s - y_{1a} = 35.1 - .41 = 34.70$$

From equations 66 and 67:

$$x_s'' = x_s' \cos 2\alpha + y_s' \sin 2\alpha = 57.18 \cos 1.36 + 34.7 \sin 1.36 = 58.0$$

$$y_s'' = y_s' \cos 2\alpha - x_s' \sin 2\alpha = 34.7 \cos 1.36 - 57.18 \sin 1.36 = 33.3$$

From equations 57 and 58:

$$a = \frac{(x_s'')^2 + (\frac{a}{b} y_s'')^2}{2x_s''} = \frac{58^2 + (1.51 \times 33.3)^2}{2 \times 58} = 50.8$$

$$b = a/(a/b) = 50.8/1.51 = 33.6$$

The elliptical diagram resulting from the calculation shown on pages 44-48 is drawn on Fig. 16. Also on the same diagram sheet is plotted the actual current locus as obtained from test data. Test data points are designated by the symbol, ⊙ . It is seen that the elliptical diagram follows very closely the current locus as obtained from test data, while Ho's exact circle diagram, which also is drawn in on Fig. 16, deviates considerably from the actual current locus. Accordingly, it is to be expected that more accurate performance characteristics may be predicted from the elliptical diagram than from Ho's circle diagram.

In order to derive the performance characteristics from the elliptical diagram shown in Fig. 16, the analytical calculating chart shown in Table III will be used. First, however, the distance KH, and the distance, c, from the center of the ellipse to either focus, will be calculated according to equations 69 and 70 respectively:

$$KH = \frac{r_h I_{1s}^2}{E_1 \text{ (rated)/ph}} = \frac{.355 \times 71.5^2}{127} = 14.3$$

$$c = \sqrt{a^2 - b^2} = \sqrt{50.8^2 - 33.6^2} = 38$$

The results obtained from the analytical calculating chart are shown in Table IV, and rearranged in Table V. Table VI shows actual test performance characteristics.

Table IV

PERFORMANCE CHARACTERISTICS OF 5-HP, 1155-RPM, 220-VOLT 3-PHASE, 60-CYCLE, RELIANCE TYPE A-A INDUCTION MOTOR AS PREDICTED FROM THE TWO TESTS ELLIPTICAL DIAGRAM.

Following the Analytical Calculating Chart, Table III:

1.	13	14	15	16	18	20
2.	88.6	87.6	86.6	85.6	83.6	81.6
3.	15.7	37.2	48.8	57.9	71.03	80.54
4.	3.52	8.46	11.29	13.54	17.02	19.72
5.	.3	1.65	2.93	4.30	6.95	9.51
6.	.0738	.406	.721	1.058	1.71	2.34
7.	.1722	.947	1.683	2.47	3.99	5.45
8.	3.93	8.91	11.77	14.05	17.59	20.35
9.	1260	2842	3642	4202	4940	5422
10.	1499	3400	4480	5350	6700	7750
11.	.585	.800	.831	.837	.832	.819
12.	6.72	11.13	14.16	16.78	21.10	24.80
13.	7.61	17.90	23.50	27.75	34.10	38.70
14.	.0287	.0672	.091	.113	.149	.179
15.	.840	.836	.813	.785	.738	.699
16.	.3836	1.85	3.20	4.62	7.35	9.98
17.	3.10	8.01	10.80	13.03	16.44	19.08
18.	1180	3050	4120	4965	6265	7270
19.	1425	3600	4968	6118	8020	9618
20.	.484	.750	.789	.798	.795	.782
21.	6.4	10.68	13.69	16.33	20.7	24.4
22.	8.13	19.90	26.95	32.70	41.90	49.40
23.	.0274	.061	.0801	.0966	.1218	.141
24.	.828	.847	.829	.812	.782	.756

Table V

PERFORMANCE CHARACTERISTICS OF A 56-HP, 1155-RPM, 220-VOLT, 3-PHASE, 60-CYCLE, RELIANCE TYPE A-A INDUCTION MOTOR AS PREDICTED FROM THE TWO TESTS ELLIPTICAL DIAGRAM

Rearrangement of Table IV

Current at 127 volts/ph	Elect. power input	Torque	Mech. power output	Effi-ciency	Power factor	Slip
AMPS	KW	lb x ft	HP			
Motor Action:						
6.72	1.499	7.61	1.69	.840	.585	.0287
11.13	3.400	17.90	3.81	.836	.800	.0672
14.16	4.480	23.50	4.88	.813	.831	.091
16.78	5.350	27.75	5.63	.785	.837	.113
21.10	6.700	34.10	6.62	.738	.832	.149
24.80	7.750	38.70	7.27	.699	.819	.179
Generator Action:						
	KW output		HP input			
6.4	1.180	8.13	1.91	.828	.484	.0274
10.68	3.050	19.90	4.82	.847	.750	.0610
13.69	4.120	26.95	6.66	.829	.789	.0801
16.33	4.965	32.70	8.21	.812	.798	.0966
20.70	6.265	41.90	10.76	.782	.795	.1218
24.40	7.270	49.40	12.90	.756	.782	.1410

Table VI

PERFORMANCE CHARACTERISTICS OF A 5-HP, 1155-RPM, 220-VOLT, 3-PHASE, 60-CYCLE, RELIANCE TYPE A-A INDUCTION MOTOR.

Dynamometer Test Characteristics

Current at 127 volts/ph	Elect. power input	Torque	Mech. power output	Effi- ciency	Power factor	Slip
AMPS	KW	lb x ft	HP			
Motor Action:						
5.25	.175	0	0	0	.0875	.0017
5.43	.495	.84	.191	.288	.239	.0042
6.43	1.272	5.20	1.175	.689	.519	.0117
7.27	1.715	7.88	1.78	.775	.618	.0141
8.37	2.247	10.50	2.37	.786	.705	.0150
9.68	2.788	13.68	3.07	.820	.756	.0171
10.77	3.180	15.75	3.53	.828	.776	.0208
12.33	3.740	18.90	4.21	.840	.803	.0267
14.35	4.450	22.10	4.89	.820	.815	.0333
16.40	5.105	25.20	5.54	.810	.817	.0358
18.53	5.795	28.35	6.22	.801	.821	.0417
21.13	6.475	31.50	6.84	.788	.805	.0508
24.63	7.465	35.70	7.62	.762	.796	.0666
27.80	8.290	38.90	8.17	.736	.782	.0816
32.57	9.385	42.00	8.56	.681	.757	.108
35.76	10.160	44.10	8.81	.647	.747	.125
39.13	10.850	45.20	8.61	.593	.727	.166
71.50	13.350		0	0	.490	1.000
Generator Action						
	KW output		HP input			
5.73	.637	4.20	.965	.885	.292	.0058
6.85	1.344	8.50	1.97	.914	.515	.0125
7.80	1.805	10.82	2.51	.964	.608	.0142
9.22	2.374	14.18	3.30	.963	.676	.0192
10.18	2.756	17.02	3.97	.931	.711	.0208
12.33	3.487	21.35	5.00	.934	.743	.0233
14.20	4.170	26.60	6.25	.893	.771	.0283
15.83	4.635	29.40	6.97	.890	.770	.0366
17.20	5.040	32.40	7.70	.877	.770	.0408
19.83	5.725	37.30	8.91	.862	.762	.0450
22.00	6.285	41.20	9.93	.848	.750	.0525
25.63	7.225	47.60	11.61	.834	.739	.0667
30.50	8.235	57.00	14.05	.785	.708	.0791
34.80	9.025	64.80	16.17	.749	.680	.0917
39.17	9.625	71.90	18.21	.708	.646	.1108
44.13	10.145	79.50	20.50	.663	.601	.1291

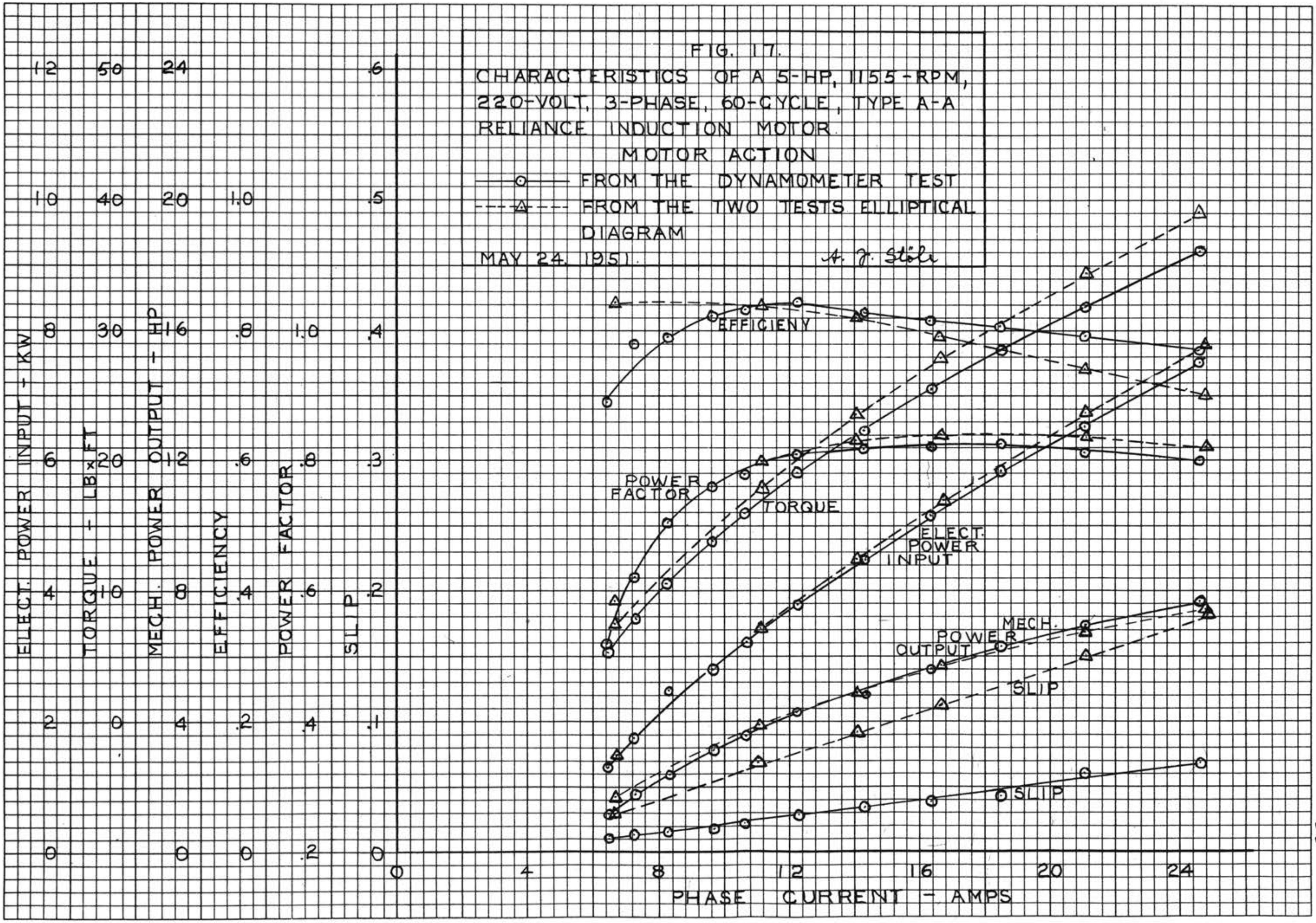
FIG. 17.
CHARACTERISTICS OF A 5-HP, 1155-RPM, 220-VOLT, 3-PHASE, 60-CYCLE, TYPE A-A RELIANCE INDUCTION MOTOR
MOTOR ACTION
FROM THE DYNAMOMETER TEST
FROM THE TWO TESTS ELLIPTICAL DIAGRAM
MAY 24, 1951
A. J. Stöli
EFFICIENY
POWER FACTOR
TORQUE
ELECT. POWER INPUT
MECH. POWER OUTPUT
SLIP
SLIP
ELECT. POWER INPUT - KW
TORQUE - LB×FT
MECH. POWER OUTPUT - HP
EFFICIENCY
POWER FACTOR
SLIP
PHASE CURRENT - AMPS
0
4
8
12
16
20
24

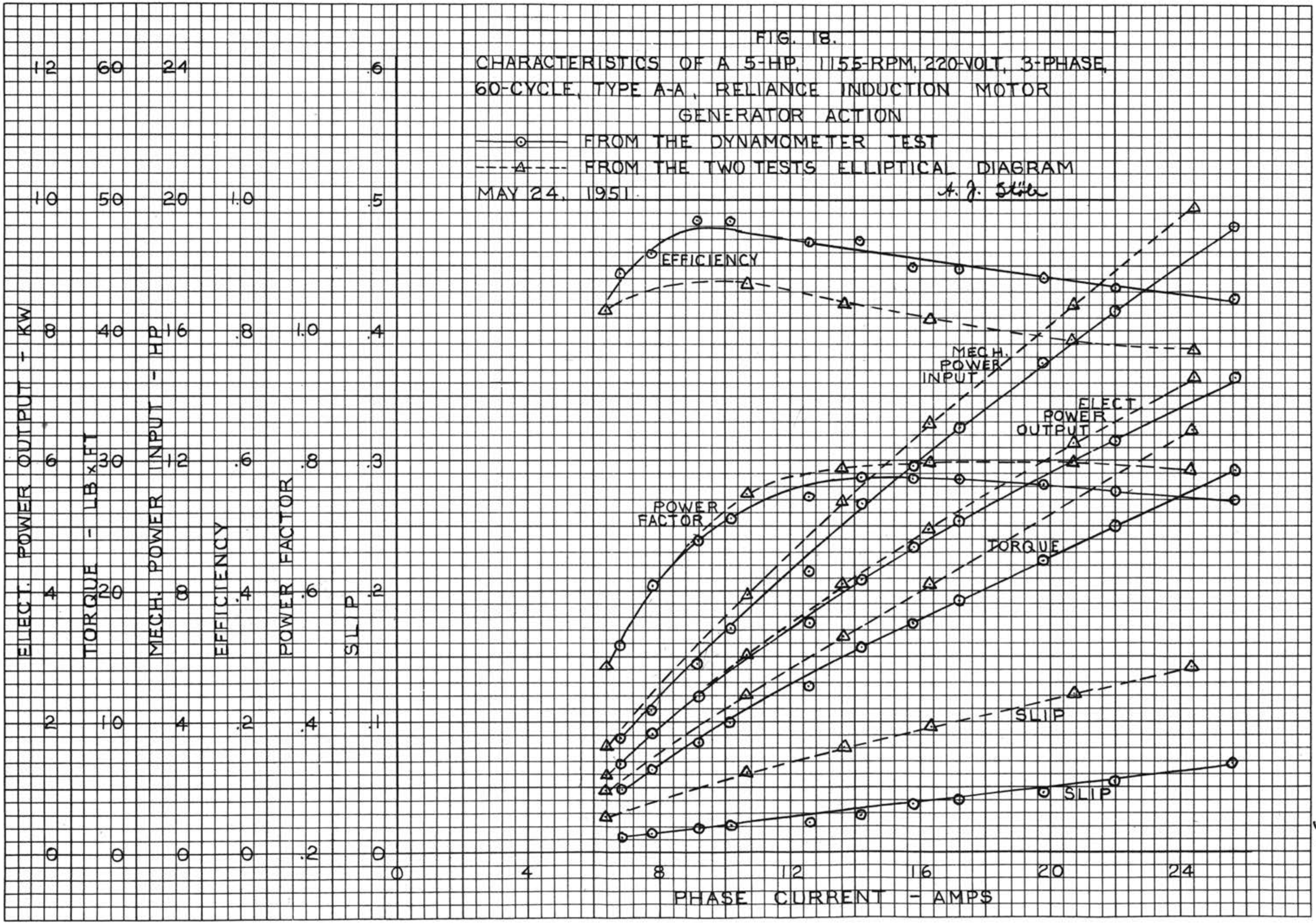

FIG. 18.
CHARACTERISTICS OF A 5-HP, 1155-RPM, 220-VOLT, 3-PHASE, 60-CYCLE, TYPE A-A, RELIANCE INDUCTION MOTOR
GENERATOR ACTION
—⊙— FROM THE DYNAMOMETER TEST
- -△- - FROM THE TWO TESTS ELLIPTICAL DIAGRAM
MAY 24, 1951 A. J. Stöhr

Discussion of the Results in Example I and II

The performance characteristics as obtained from the two tests elliptical diagrams of the two machines were plotted together with the performance characteristics as obtained from the dynamometer tests. This was done in order that the results derived from the elliptical diagrams could be checked. Thus Fig. 13 and Fig. 14 show a comparison between the dynamometer test performance characteristics of the 5-hp Westinghouse Lifeline induction machine, motor and generator action respectively, and performance characteristics as obtained from the elliptical diagram of the same machine. In a similar way, Fig. 17 and Fig. 18 show a comparison between the dynamometer test performance characteristics of the 5-hp Reliance induction machine, motor and generator action respectively, and performance characteristics as obtained from the elliptical diagram for the same machine.

By observing these curves, it is seen that the dynamometer test performance characteristics fall close together with the performance characteristics as obtained from the two tests elliptical diagrams regarding torque, motor input, generator output, and power factor. Motor output as obtained from the elliptical diagrams is a little smaller than the dynamometer tests show, while generator input as obtained from the elliptical diagrams is a little smaller than the dynamometer tests show. For this reason the efficiency as obtained from the elliptical diagrams is a little smaller than the dynamometer tests show. Slip as obtained from the elliptical diagrams is much greater than the test values. Generally speaking, all performance characteristics as predicted from the two tests elliptical diagrams, except slip, are fairly close to actual test performances at and near normal operating conditions.

Chapter IV

The Three Tests Elliptical Diagram

As pointed out in Chapter III, fairly reliable results can be predicted from the two tests elliptical diagram except for slip. In order to use the two tests elliptical diagram, only two kinds of tests were necessary to run: namely, the no-load running test and the blocked rotor test. In many cases it is inconvenient to run more than these two kinds of tests. If, however, it would be possible to run a test at full load, for example, the elliptical diagram would in the first place be more easily found, and secondly, all performance characteristics including the slip could be determined accurately.

Using the test data as obtained for the 5-hp Westinghouse Lifeline induction machine and the 5-hp Reliance induction machine, the distances PB, PA', and A'B and the distances QB, QA', and A'B were calculated following the opposite procedure of that shown on pages 36 and 37. The results of these calculations are shown in Table VII and Table VIII. These distances were plotted on diagrams Fig. 12 and Fig. 16. The result shows that the line O"H falls together with the one previously calculated. The line O"A'P_s, however, does not fall together with the line O"AP_s. This fact explains the errors encountered on Figs. 13, 14, 17, and 18. The following question arose: Why does not the dotted line O"A'P_s follow the straight line O"AP_s? Any distance, A'B, between line O"H and line O"A'P_s is a function of the secondary resistance, r_2.

Table VII

TRANSFORMATION OF PERFORMANCE CHARACTERISTICS SHOWN IN TABLE II INTO EQUIVALENT DISTANCES ON THE ELLIPTICAL DIAGRAM, FIG. 12

Motor Action:

I_1	PB	PA'	A'B
8.45	4.51	4.46	.048
11.03	7.19	7.07	.122
14.08	10.01	9.74	.270
17.66	12.84	12.34	.501
21.50	15.75	15.07	.677
27.00	18.47	17.34	1.128
33.00	21.30	19.53	1.768
41.50	24.10	21.16	2.940

Generator Action:

I_1	QB	QA'	A'B
7.48	4.01	4.04	.029
10.50	8.30	8.32	.124
13.17	10.78	10.99	.210
14.87	12.53	12.82	.293
16.50	14.29	14.67	.383
18.70	16.75	17.28	.528
21.36	19.50	20.26	.758
24.20	22.40	23.46	1.058
27.33	24.55	26.78	1.230
31.33	30.15	31.91	1.760
36.50	35.05	37.58	2.530
40.00	38.65	41.87	3.220

Table VIII

TRANSFORMATION OF PERFORMANCE CHARACTERISTICS SHOWN IN TABLE VI INTO EQUIVALENT DISTANCES ON THE ELLIPTICAL DIAGRAM, FIG. 16.

Motor Action:

I_1	PB	PA'	A'B
5.43	.419	.418	.0017
6.43	2.37	2.34	.0277
7.27	3.57	3.52	.0503
8.37	4.74	4.67	.0711
9.68	6.16	6.06	.105
10.77	7.08	6.93	.147
12.33	8.50	8.27	.227
14.35	9.94	9.61	.331
16.40	11.33	10.93	.406
18.53	12.72	12.19	.532
21.13	14.15	13.43	.719
24.63	16.02	14.96	1.065
27.80	17.45	16.02	1.430
32.57	18.73	16.71	2.020
35.76	19.78	17.31	2.470
39.13	21.25	16.89	3.360

Generator Action:

I_1	QB	QA'	A'B
5.73	1.88	1.89	.0109
6.85	3.80	3.85	.0476
7.80	4.85	4.92	.0688
9.22	6.35	6.47	.122
10.18	7.61	7.77	.158
12.33	9.56	9.78	.223
14.20	11.90	12.24	.337
15.83	13.15	13.63	.480
17.20	14.50	15.09	.592
19.83	16.70	16.45	.752
22.00	18.40	19.36	.966
25.63	21.30	22.70	1.42
30.50	25.45	27.46	2.01
34.80	28.90	31.55	2.65
39.17	32.10	35.66	3.56
44.13	35.50	40.38	4.58

Since the line $O''A'P_s$ is curved and falls below the straight line $O''AP_s$, the secondary resistance must change, being less at no-load running condition than at standstill. Several factors can explain these circumstances. As the phase current increases from no-load current to blocked rotor current, the frequency of the rotor increases from close to zero cycle per second up to input source frequency. Past standstill, when the machine is running in opposite direction to that of the rotating magnetic field, the frequency of the rotor quickly increases to values several times that of the source frequency.

It is a known fact that the hysteresis losses increase proportionally with the frequency and that the eddy current losses increase with the square of the frequency.[1] This fact will make the value of the secondary resistance at rated load be less than its value at standstill. Also due to skin effect the secondary resistance will increase with an increase in frequency. According to Elektrische Maschinen by Rudolf Richter, skin effect alone in some cases may cause the resistance of the rotor at 50 cycle per second to be 1.75 times its d.c. value.[2] However, it would be a hard job, if not impossible, to find the line $O''A'P_s$ by trying to determine the influence of hysteresis losses, eddy current losses and skin effect.

As seen from Fig. 12 and Fig. 16, the curved line $O''A'P_s$ is fairly straight from O" up to at least 3/2 rated load. This can be explained by the fact that from no-load and up to 3/2 rated

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[1] Puchstein and Lloyd, op. cit., pp. 160-163.

[2] Richter, op. cit., IV, p. 221.

load the frequency of the rotor is very small compared to the 60-cycle source frequency. Thus, in effect, the value of the rotor resistance from no-load up to 3/2 full load may be considered constant and equal to the d.c. value of the rotor resistance.

If, besides the no-load and the blocked rotor tests, it would be possible to run a test at, for example, full load, the straight portion, O"A'L, of the line O"A'P_s may be drawn. In this extra test it would not be necessary to measure the torque, but a dependable measurement should be taken of the slip.[3]

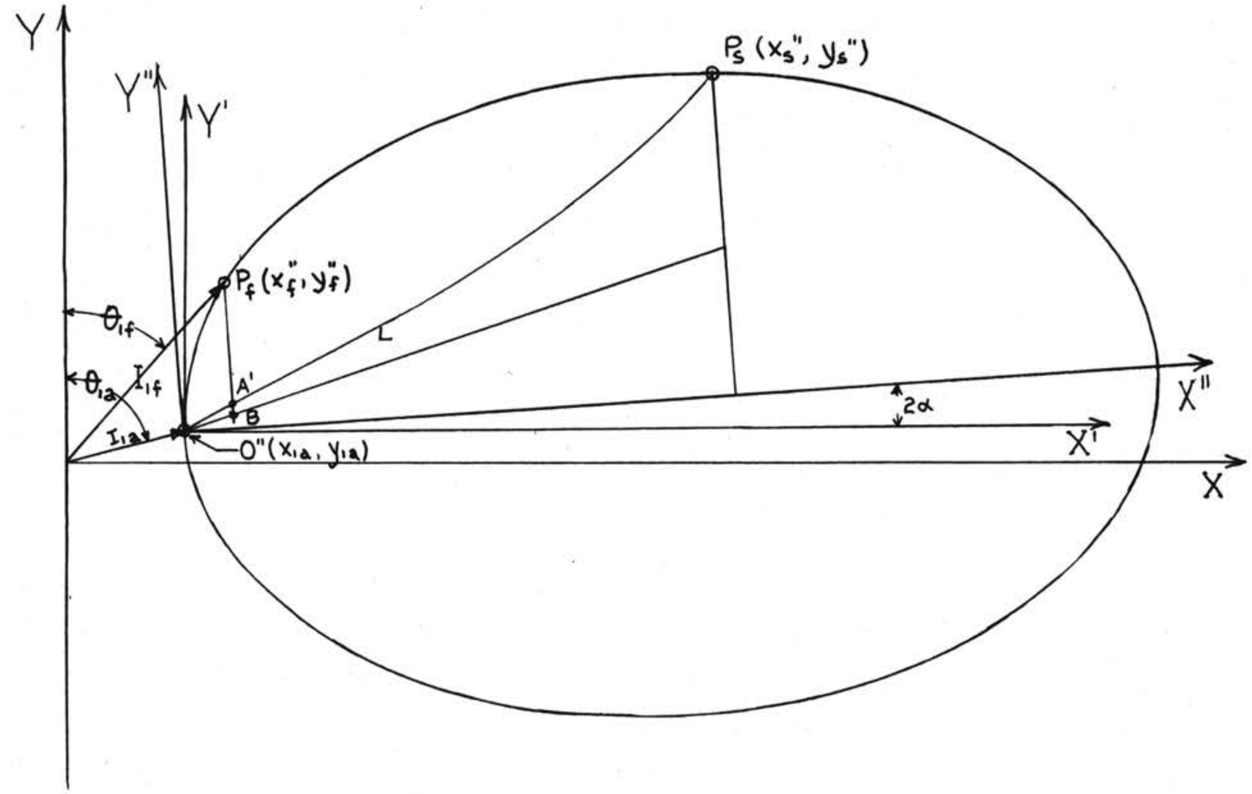

Figure 19. Elliptical diagram for the polyphase induction machine.

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[3]Karapetoff and Dennison, Electrical Laboratory Experiments, p. 391.

From the test at full load a point, P_f, is located on the current locus, see Fig. 19. In the coordinate system, X", Y", point P_f has the abcissa x''_f and the ordinate y''_f.

Since the point, P_f, on the current locus is known, the axes of the ellipse may be determined by a much easier method than was used in Chapter III. By substituting the coordinates x''_s, y''_s and x''_f, y''_f in turn for x and y in the equation below, two equations with a and b as unknowns are obtained.

$$\frac{(x'' - a)^2}{a^2} + \frac{y''^2}{b^2} = 1 \qquad (71)$$

By solving the resulting equations, a and b may be obtained. An example will clarify the procedure to follow.

Example Problem III

From Table II, page 35, which gives the dynamometer test performance characteristics of the 5-hp, Westinghouse Lifeline induction motor, the following data are obtained:

Current at 127 volts/ph	Watts input	Power factor	Slip
14.08	4525	.843	.027

From page 28:

$$2 \quad = 2.02 \text{ degrees}$$

From page 30: $x_{1a} = 5.94$

$y_{1a} = .54$

From page 30: $x_s'' = 64.8$

$y_s'' = 48.1$

A power factor of .843 for motor action corresponds to an angle of -32.6 degrees.

Following a procedure similar to the one shown on page 26, x_f'' and y_f'' may be found:

$$x_f' = 14.08 \sin 32.6 - x_{1a} = 7.58 - 5.94 = 1.64$$

$$y_f' = 14.08 \cos 32.6 - y_{1a} = 11.89 - .54 = 11.35$$

$$x_f'' = 1.64 \cos 2.02 + 11.35 \sin 2.02 = 2.04$$

$$y_f'' = 11.35 \cos 2.02 - 1.64 \sin 2.02 = 11.29$$

By substituting the coordinates x_s'', y_s'' and x_f'', y_f'' into equation 71, the following two equations are obtained:

$$\frac{(64.8 - a)^2}{a^2} + \frac{48.1^2}{b^2} = 1$$

$$\frac{(2.04 - a)^2}{a^2} + \frac{11.29^2}{b^2} = 1$$

The solution of these two equations gives:

$$a = 74.2$$

$$b = 48.5$$

These values check satisfactorily with the values obtained on page 30 by the two tests method. The elliptical diagram can now be drawn in the same way as explained on page 26. The result is shown on Fig. 20.

The distance KH, which is given on page 33, was set off by a compass. Next the line O"H was drawn. Point P_f was found by drawing an arc with radius equal to 14.08. From P_f a line was drawn perpendicular to the major axis of the ellipse. Where this line crosses the line O"H, is the point B. The distance P_fB can now be measured and the distance A'B calculated from the equation A'B = slip x P_fB. P_fB was found to be 10.2; therefore, A'B = .027 x 10.2 = .276. By setting this distance off from B along the line BP_f, point A' is located and the auxilliary line O"A'L may be drawn. The performance characteristics can now be derived from the elliptical diagram in the same way as shown in example I, Chapter III, except that A' is used instead of A.

Performance characteristics could also be determined from the elliptical diagram by first geometrically determining the distance MK, and then substituting this distance for y_s'' in the analytical calculating chart shown on page 42. In this example, however, the performance characteristics were obtained by measuring the distances on the elliptical diagram. The result is given in Table IX.

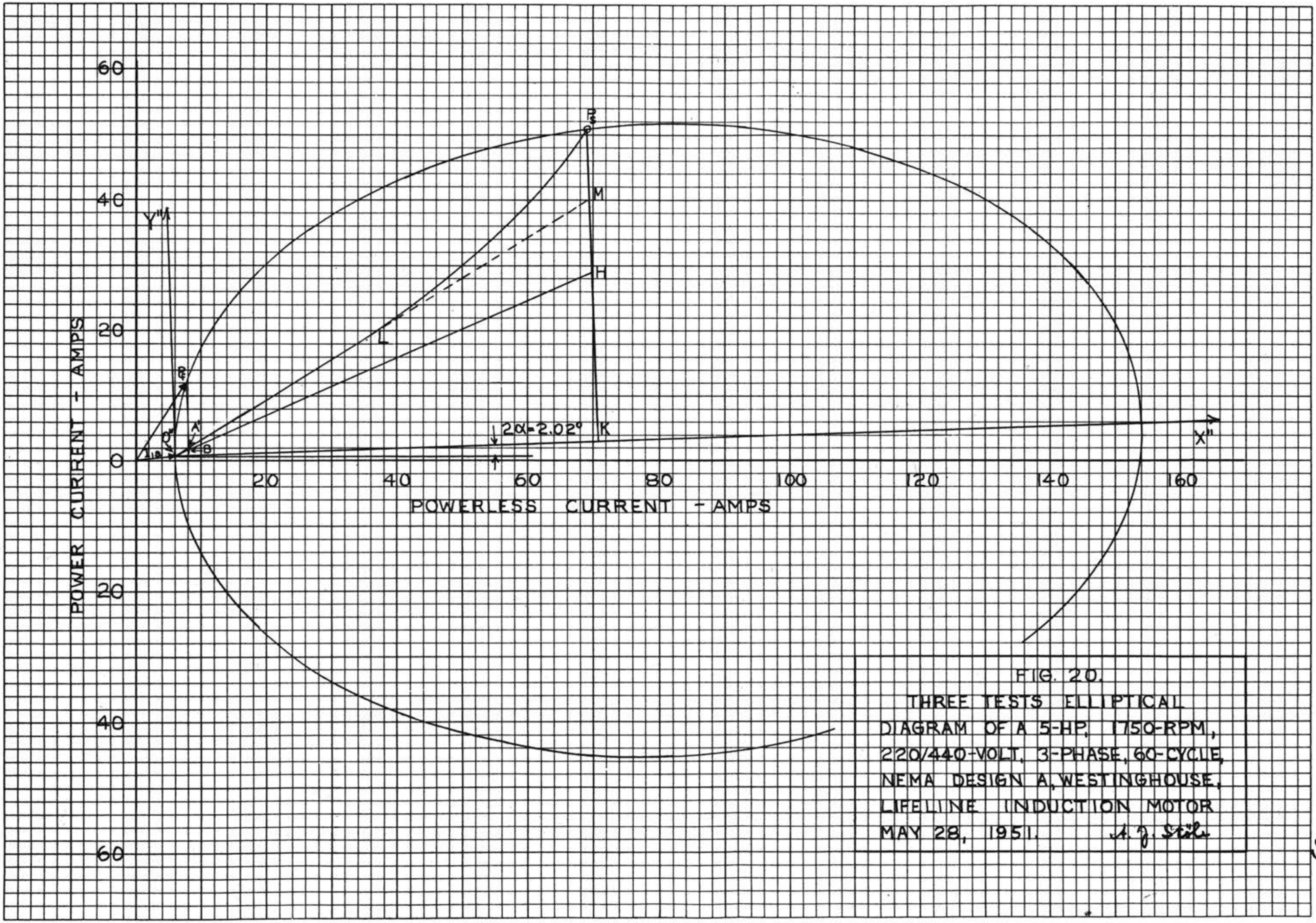

FIG. 20.
THREE TESTS ELLIPTICAL DIAGRAM OF A 5-HP, 1750-RPM, 220/440-VOLT, 3-PHASE, 60-CYCLE, NEMA DESIGN A, WESTINGHOUSE, LIFELINE INDUCTION MOTOR
MAY 28, 1951. A. J. Stiel

Table IX

PERFORMANCE CHARACTERISTICS OF A 5-HP, 1750-RPM, 220/440-VOLT, 3-PHASE, 60-CYCLE, WESTINGHOUSE LIFELINE INDUCTION MOTOR, NEMA DESIGN A.

Data as obtained from the three tests elliptical diagram.

Current at 127 volts/ph AMPS	Elect. power input KW	Torque lb x ft	Mech. power output HP	Effi-ciency	Power factor	Slip
Motor Action:						
8	1.975	6.33	2.14	.807	.645	.0114
10	2.915	9.61	3.24	.828	.751	.0167
12	3.735	12.44	4.17	.832	.815	.0212
14	4.490	14.99	4.98	.828	.843	.0265
16	5.220	17.35	5.84	.824	.858	.0305
18	5.940	19.60	6.47	.813	.868	.0353
20	6.630	21.62	7.09	.798	.871	.0410
22	7.290	23.47	7.66	.784	.871	.0459
24	7.950	25.27	8.21	.770	.870	.0514
Generator Action:						
	KW output		HP input			
8	1.734	7.99	2.76	.842	.626	.0112
10	2.655	11.86	4.13	.862	.700	.0154
12	3.470	15.60	5.45	.852	.760	.0204
14	4.215	18.87	6.61	.853	.792	.0232
16	4.920	22.13	7.80	.845	.808	.0272
18	5.600	25.43	8.98	.836	.817	.0302
20	6.250	28.73	10.18	.823	.823	.0355
22	6.860	31.83	11.33	.812	.824	.0368
24	7.470	34.83	12.42	.807	.821	.0401

Table X

PERFORMANCE CHARACTERISTICS OF A 5-HP, 1750-RPM, 220/440-VOLT, 3-PHASE, 60-CYCLE, WESTINGHOUSE LIFELINE INDUCTION MOTOR, NEMA DESIGN A.

Data as obtained from Ho's exact Circle Diagram, Fig. 12

Current at 127 volts/ph AMPS	Elect. power input KW	Torque lb x ft	Mech. power output HP	Effi-ciency	Power factor	Slip
Motor Action:						
8	2.040	6.78	2.27	.831	.668	.0191
10	2.990	10.21	3.40	.847	.785	.0271
12	3.850	13.34	4.41	.854	.840	.0341
14	4.685	16.10	5.27	.839	.871	.0438
16	5.400	18.57	6.03	.833	.887	.0516
18	6.135	21.07	6.79	.826	.896	.0594
20	6.880	23.22	7.45	.807	.903	.0677
22	7.585	25.57	8.07	.795	.906	.0774
24	8.290	27.77	8.72	.783	.908	.0846
Generator Action:						
	KW output		HP input			
8	1.792	8.27	2.89	.832	.597	.0185
10	2.750	12.30	4.33	.851	.726	.0259
12	3.600	15.88	5.63	.857	.786	.0333
14	4.380	19.23	6.85	.858	.821	.0392
16	5.130	22.58	8.11	.849	.842	.0473
18	5.850	25.93	9.36	.838	.855	.0527
20	6.580	29.13	10.60	.833	.862	.0593
22	7.240	32.33	11.82	.822	.868	.0639
24	7.910	35.63	13.10	.811	.870	.0708

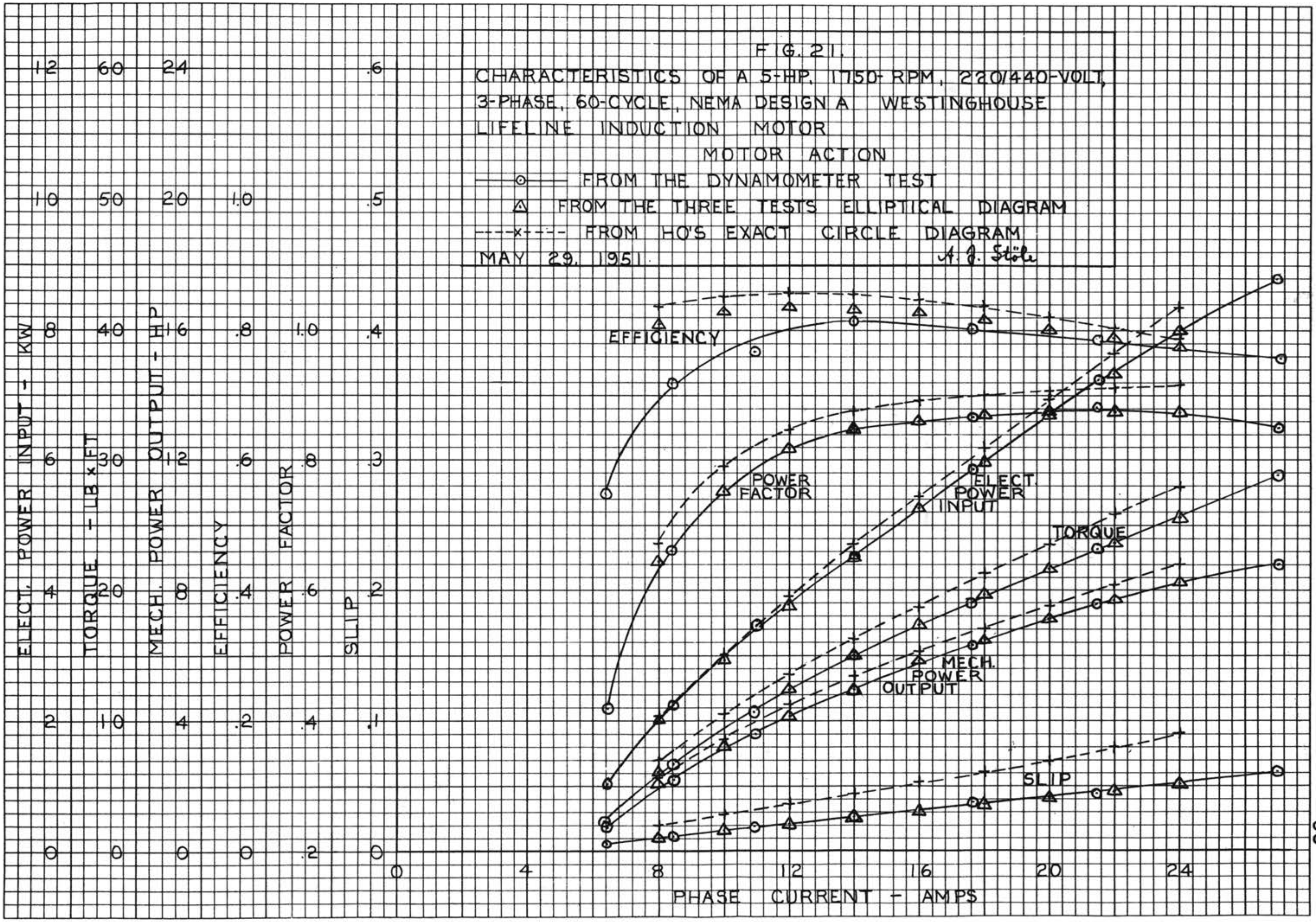
FIG. 21.
CHARACTERISTICS OF A 5-HP, 1750-RPM, 220/440-VOLT,
3-PHASE, 60-CYCLE, NEMA DESIGN A WESTINGHOUSE
LIFELINE INDUCTION MOTOR
MOTOR ACTION
FROM THE DYNAMOMETER TEST
FROM THE THREE TESTS ELLIPTICAL DIAGRAM
FROM HO'S EXACT CIRCLE DIAGRAM
MAY 29, 1951
A. J. Stöle
ELECT. POWER INPUT - KW
TORQUE - LB × FT
MECH. POWER OUTPUT - HP
EFFICIENCY
POWER FACTOR
SLIP
EFFICIENCY
POWER FACTOR
ELECT. POWER INPUT
TORQUE
MECH. POWER OUTPUT
SLIP
PHASE CURRENT - AMPS

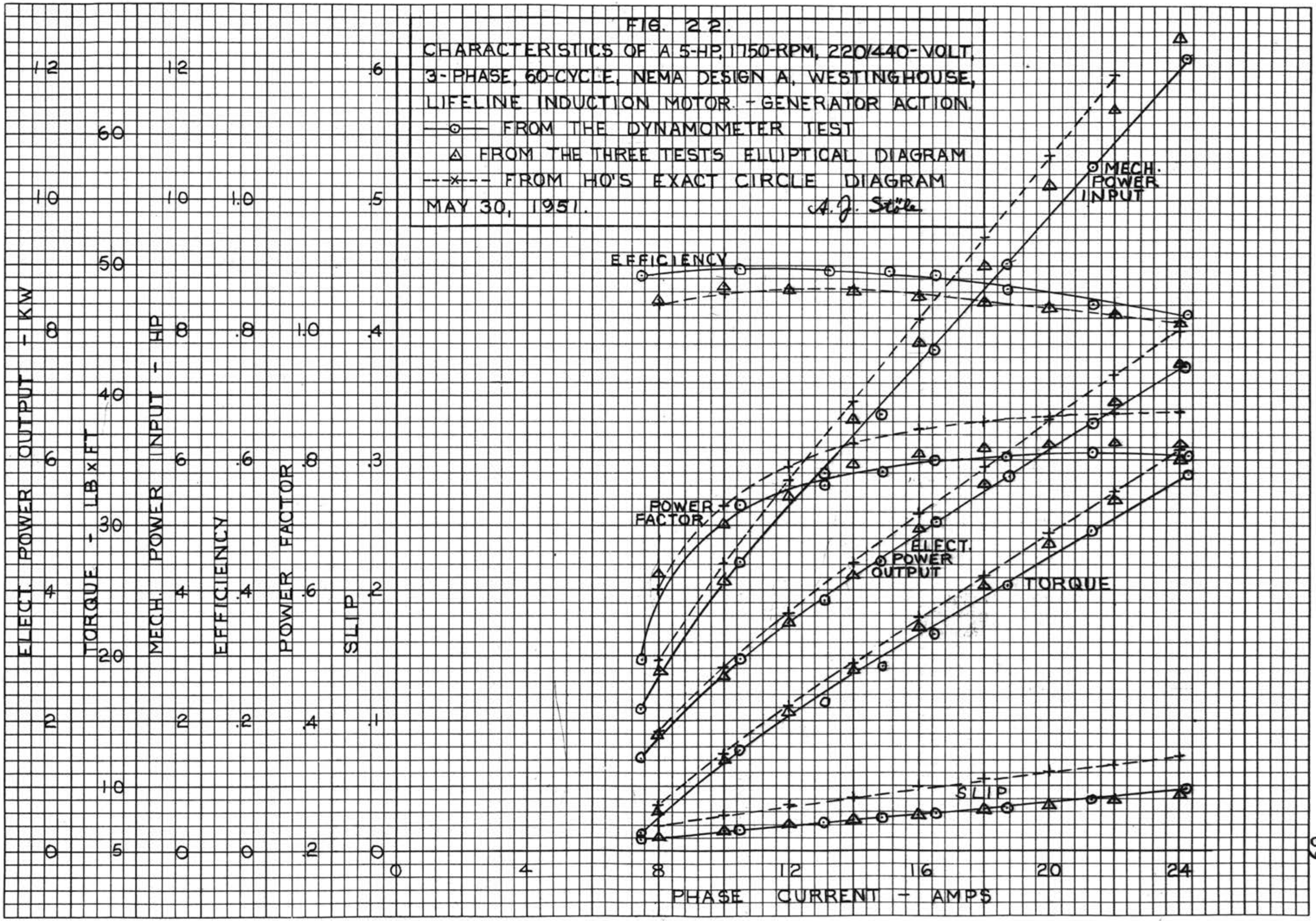

FIG. 22.

CHARACTERISTICS OF A 5-HP, 1750-RPM, 220/440-VOLT, 3-PHASE, 60-CYCLE, NEMA DESIGN A, WESTINGHOUSE, LIFELINE INDUCTION MOTOR - GENERATOR ACTION.

—⊙— FROM THE DYNAMOMETER TEST

△ FROM THE THREE TESTS ELLIPTICAL DIAGRAM

---×--- FROM HO'S EXACT CIRCLE DIAGRAM

MAY 30, 1951. A. J. Stile

Fig. 21 and Fig. 22 show graphically the result obtained from the three tests elliptical diagram, together with the performance characteristics as obtained from the dynamometer test. Also performance characteristics as obtained by Ho's exact circle diagram are shown on Fig. 21 and Fig. 22. By observing these two figures, it is seen that the performance characteristics as obtained from the three tests elliptical diagram fall so to speak together with the performance characteristics as obtained from the dynamometer test. Performance characteristics as obtained from Ho's exact circle diagram, on the other hand, are seen to deviate considerably from the dynamometer test performance characteristics.

Chapter V

Conclusions

The polyphase induction machine has been subject to considerable amount of study since it first was invented. Different approaches have been made toward obtaining circle diagrams from which the performance characteristics could accurately be predicted. In most cases, however, these circle diagrams fail to be sufficiently accurate.

Considering all the discrepancies of the exact equivalent circuit, it is obvious that the circular locus as developed from the ideal exact equivalent circuit is bound to deviate from the true current locus.

It is seen from Fig. 12 and Fig. 16 that the difference between the actual current locus and the one formed by the exact circle diagram is relatively large. Instead of following the circular locus, the current locus seems to describe part of an ellipse. These circumstances and also the fact that a circle is just a special form of an ellipse were the fundamental reasons why an ellipse rather than a circle was chosen to represent the current locus. It was determined that the current locus tends to follow what seems to be an ellipse because of the fact that the leakage reactances change due to saturation.

In Chapter III a method was developed whereby the effects of saturation directly was used to determine the ellipse. Using only the no-load running test and the blocked rotor test, an elliptical diagram was obtained. From Fig. 12 and Fig. 16 it is seen that this elliptical diagram almost falls together with the current locus as obtained from actual test. Except for slip the performance characteristics obtained from the "Two Tests Elliptical Diagram" follow closely the performance characteristics as obtained from the dynamometer test at and near rated load.

The fact that the slip as obtained from the two tests elliptical diagram was in great error led to an investigation originally not planned. By converting test data into equivalent distances on the elliptical diagram, it was shown that the value of the rotor resistance is subject to great changes from blocked rotor condition down to normal operating condition. It was found that the rotor resistance at blocked rotor condition, due to hysteresis and eddy current losses and due to skin effect, was approximately twice the rotor resistance at normal operating condition for both machines that were tested. There is, however, no reason that this should be taken as a rule regarding any induction machine.

The rotor resistance at normal operating condition may be assumed equal to the direct current resistance of the rotor, since under this condition hysteresis, eddy current, and skin effect are negligibly small. Unfortunately, the direct current rotor resistance of a squirrel cage rotor cannot be directly measured. For this reason a three tests method was introduced. This method, as the name implies, has the disadvantage of requiring an extra test run, preferably one at or around rated load condition. On the other hand, this method has the advantage of a more satisfactory result including slip.

The two tests as well as the three tests elliptical diagram requires more calculation than the circle diagram method. So even though the result obtained by either of the elliptical diagram methods is better than the result from the circle diagram, it is not expected that any of them will be taken into immediate use. It is hoped, however, that this paper will be of help to those who are interested in trying to find a simple and, at the same time, accurate method of predicting the characteristics of the polyphase induction machine.

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