THE EFFECT OF MUTUAL IMPEDANCE UPON HARMONICALLY RELATED ANTENNA ELEMENTS

By

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THESIS AND ABSTRACT APPROVED:

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PREFACE

The advancements of the electronic art in the past decade have probably been greater than those of any other science in a similar period. In spite of this, the behavior of the simple antenna remains one of its unsolved mysteries. Many assumptions render results that are fairly accurate but the final adjustment is always one based upon empirical data and experimentation. The commercial antenna designer encounters many problems involving the mutual impedance between antennas of varied lengths and must always rely upon the laboratory to supply the necessary data. It was felt that a study of the effect of the mutual impedance would be of great value to the designer as well as add another clue to the absolute solution of the simple antenna.

ACKNOWLEDGMENT

The author wishes to express his thanks to Dr. C. H. Harrington of the Research Foundation of the Oklahoma Agricultural and Mechanical College for making certain equipment available in this study. He is greatly indebted to Professor H. T. Fristoe for his many valuable suggestions and assistance in interpretation of data.

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INTRODUCTION

Radio services are often assigned frequencies that are harmonically related in order that harmonic radiation will not interfere with other stations. Typical examples are the amateur frequencies which are multiples of 1.75 megacycles and the aerial watch frequencies of 3105 kcs. and 6210 kcs. Where the same directivity is desired, it is necessary that the antennas be parallel and their lengths must be multiples of each other. This condition causes great concern upon shipboard and in other limited space applications due to the high mutual impedances encountered between close spaced harmonic antennas. Not only does the pattern become distorted due to absorptions and reflections but the feed point impedance is altered to such a degree that serious mismatches may result. Phased antenna arrays may require redesign and construction of phasing networks.

It is economically desirable to operate amateur ten and twenty meter beam antennas from the same rotatable mount and manufacturers, in their advertising literature, often state that it is possible to operate their beams under these conditions without serious interaction. This is generally true in the case of the larger twenty meter beam where the ten meter elements are then but one-eighth wave long at the operating frequency yet it is far from the case in ten meter operation where the unused twenty meter elements are one wavelength long and located very close to the half wave elements in use. The radiation patterns as well as the feed point impedances are effected to the degree that the ten meter beams offer little advantage over the ordinary dipole.

Although various means have been devised to determine the mutual impedance between antennas, it is difficult to predict the degree of coupling. An engineering approximation that would enable the designer and user of multiple length antennas to predict the actual distortion of the radiation pattern and the change of feed point impedance would be of great value.

OBJECTIVE

It is the object of this thesis to compare the calculated, theoretical effect of mutual coupling between two close coupled harmonically related antenna elements to the actual measured values obtained with antennas of several finite diameters in order to determine some means of approximating the degree of coupling and more accurately predict the feed point impedance.

THEORY

The antenna element may be compared to a passive series circuit of resistance, capacitance and inductance for a circuit analogy. It may be shown as in Figure 1, where V_1 represents the external source of excitation such as a generator, oscillator, etc.

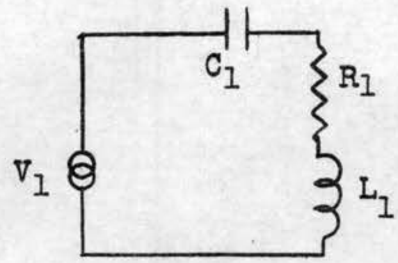

Figure 1.

The antenna located in the near range may be shown as in Figure 2.

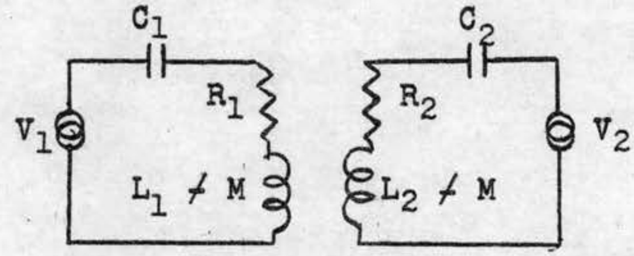

Figure 2.

The circuit may be further simplified by lumped impedances as in Figure 3.

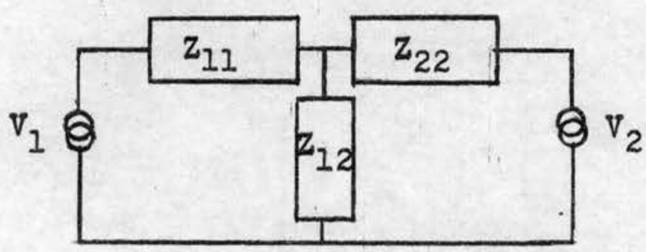

Figure 3.

The values of the lumped impedances are as follows:

$$Z_{11} = R_1 + j(X_{L1} - X_{C1})$$

$$Z_{22} = R_2 + j(X_{L2} - X_{C2})$$

$$Z_{12} = jwM$$

The network equations for solution of the above are:

$$(1)\quad V_1 = I_{01}Z_{11} + I_{02}Z_{12}$$

$$(2)\quad V_2 = I_{01}Z_{21} + I_{02}Z_{22}$$

In the case of excitation to only element number one, V_2 is equal to zero. Further simplification is obtained from the fact that

$$(3) \quad Z_{12} = Z_{21}$$

where all elements of the network are linear and of constant dielectric. Making the necessary substitutions, we have

$$(1) \quad V_1 = I_{01}Z_{11} + I_{02}Z_{12}$$

$$(4) \quad 0 = I_{01}Z_{12} + I_{02}Z_{22}$$

A solution of the equations for the impedance as seen by the generating source may be obtained by solving for V_1/I_{01} and is found to be

$$(5) \quad \frac{V_1}{I_{01}} = Z_{11} - \frac{Z_{12}{}^2}{Z_{22}}$$

Assuming the notation

$$Z_{12} = Z_{12} \underline{|\theta_{12}}$$

$$Z_{22} = Z_{22} \underline{|\theta_{22}}$$

Our solution is of the form

$$(6) \quad \frac{-(Z_{12})^2 \underline{|2\theta_{12} - \theta_{22}}}{Z_{22}}$$

The resistive component is

$$(7) \quad R = \frac{-(Z_{12})^2}{Z_{22}} \cos (2\theta_{12} - \theta_{22})$$

The reactive component is

$$(8) \quad X = \frac{-(Z_{12})^2}{Z_{22}} \sin (2\theta_{12} - \theta_{22})$$

The value of Z_{12} is equal to jwM which necessitates a solution for the value of M. A method of determining this value is outlined in an article by Cox.[1] Based on vertical antennas, values computed may be compared to the horizontal antenna of double length with multiplication by two.

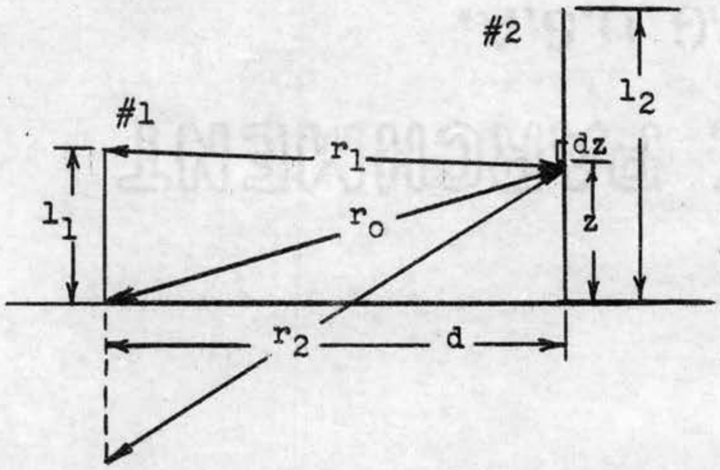

Figure 4.

l_1 = height of antenna #1

l_2 = height of antenna #2

d = horizontal spacing

$$w_o = v_o = B(\sqrt{d^2 + l_1^2} + l_1)$$

$$w_1 = B(\sqrt{d^2 + L^2} + L)$$

$$x_o = u_o = B(\sqrt{d^2 + l_1^2} - l_1)$$

$$x_1 = B(\sqrt{d^2 + L^2} - L)$$

$$y_1 = u_1 = B(\sqrt{d^2 + l_2^2} + l_2)$$

$$s_o = B(d)$$

$$s_1 = B(\sqrt{d^2 + l_2^2} - l_2)$$

$$v_1 = B(\sqrt{d^2 + \Delta^2} - \Delta)$$

$$\Delta = l_2 - l_1$$

$$L = l_2 + l_1$$

$$y_o = B(d)$$

Assuming the above configuration and an antenna base current of I_{01} for antenna #1 and current at any other

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1 Russell Cox, "Mutual Impedance Between Vertical Antennas of Unequal Heights," Proceedings of the I.R.E., XXXV (November, 1947), 1367-1370.

point I_{z1} with similar notation for antenna #2, we may determine the mutual impedance by determining the value of V_{12} (excitation voltage of antenna #2) and dividing by the value of I_{01}.

$$(9) \quad Z_{21} = \frac{-V_{21}}{I_{01}}$$

$$(10)\ (11) \text{ where } V_{21} = \int_0^{l_2} \frac{E_{z21} I_{z2}}{I_{o2}}\, dz$$

$$\text{and } I_{z2} = \frac{I_{02} \operatorname{Sin} B(l_2 - z)}{\operatorname{Sin} Bl_2}$$

assuming sinusoidal distribution of current.

The mutual impedance is determined to be

$$(12) \quad Z_{21} = -\int_0^{l_2} \frac{E_{z21} \operatorname{Sin} B\,(l_2 - z)}{I_{01} \operatorname{Sin} Bl_2}\, dz$$

or, in exponential form,

$$(13) \quad Z_{21} = -\int_0^{l_2} \frac{E_{z21}\,\left(e^{jB(l_2 - z)} - e^{-jB(l_2 - z)}\right)}{2j\, I_{01} \operatorname{Sin} Bl_2}\, dz$$

Using Brown's[2] expression for the vertical component of an electric field,

$$(14) \quad E_{z21} = \frac{-j30 I_{01} e^{jwt}}{\operatorname{Sin} B\, l_1} \left(\frac{e^{-jBr_1}}{r_1} + \frac{e^{-jBr_2}}{r_2} - \frac{2e^{-jBr_o}}{r_o} \operatorname{Cos} Bl_1 \right)$$

The mutual impedance determined by substituting E_{z21} into the expression for Z_{21} is the sum of six integrals

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2 G. H. Brown, "Directional Antennas", Proceedings of the I.R.E., XXIV (1936), 81-145.

$$(15)\quad Z_{21} = \frac{15}{\sin Bl_1 \ \sin Bl_2}\Bigg[\int_0^{l_2} \frac{e^{-jB(r_1 - l_2 + z)}}{r_1}\, dz$$

$$- \int_0^{l_2} \frac{e^{-jB(r_2 + l_2 - z)}}{r_2}\, dz$$

$$- \int_0^{l_2} \frac{e^{-jB(r_1 + l_2 - z)}}{r_1}\, dz$$

$$+ \int_0^{l_2} \frac{e^{-jB(r_2 - l_2 + z)}}{r_2}\, dz$$

$$-2 \int_0^{l_2} \frac{e^{-jB(r_o - l_2 + z)}}{r_o}\, dz \quad \cos Bl_1$$

$$+2 \int_0^{l_2} \frac{e^{-jB(r_o + l_2 - z)}}{r_o}\, dz \quad \cos Bl_1\Bigg]$$

The values of resistive and reactive components are determined to be

$$(16)\quad R_{21} = \frac{15}{\sin Bl_1 \ \sin Bl_2}\Big[\cos B\Delta \big| Ci(u_1) - Ci(u_o) + Ci(v_1)$$

$$- Ci(v_o) + 2Ci(y_o) - Ci(y_1)$$

$$- Ci(s_1)\big|$$

$$+ \sin B\Delta \big| Si(u_1) - Si(u_o) + Si(v_o)$$

$$- Si(v_1) - Si(y_1) + Si(s_1)\big|$$

$$+ \cos BL \big| Ci(w_1) - Ci(v_o) + Ci(x_1)$$

$$- Ci(u_o) + 2Ci(y_o) - Ci(y_1)$$

$$- Ci(s_1)\big|$$

$$+ \sin BL \big| Si(w_1) - Si(v_o) + Si(u_o)$$

$$- Si(x_1) - Si(y_1) + Si(s_1)\big|\Big]$$

$$
\begin{aligned}
(17)\quad X_{21} = \frac{15}{\sin Bl_1 \sin Bl_2} \Big[& \cos B\Delta \big| Si(u_o) - Si(u_1) + Si(v_o) \\
& - Si(v_1) + Si(y_1) - 2Si(y_o) + Si(s_1) \big| \\
& + \sin B\Delta \big| Ci(u_1) - Ci(u_o) + Ci(v_o) \\
& - Ci(v_1) - Ci(y_1) + Ci(s_1) \big| \\
& + \cos BL \big| Si(v_o) - Si(w_1) + Si(u_o) \\
& - Si(x_1) + Si(y_1) - 2Si(y_o) + Si(s_1) \big| \\
& + \sin BL \big| Ci(w_1) - Ci(v_o) + Ci(u_o) \\
& - Ci(x_1) - Ci(y_1) + Ci(s_1) \big| \Big]
\end{aligned}
$$

In order to determine the theoretical value of Z_{22}, the solution for the impedance at the center of an antenna as outlined by King and Blake[3] enables us to form a complete solution,

$$
\begin{aligned}
(18)\quad R_{22} = 30 \Big[& (1-\cot^2 H)\, \overline{Ci}\ 4H + 4\cot^2 H\ \overline{Ci}\ 2H \\
& + 2\cot H\ (Si\ 4H - 2Si\ 2H) \Big]
\end{aligned}
$$

$$
\begin{aligned}
(19)\quad X_{22} = 30 \Big[& (1-\cot^2 H)\ Si\ 4H + 4\cot^2 H\ Si 2H \\
& + 2\cot H (2\overline{Ci} 2H - \overline{Ci} 4H - 2\ln \frac{h}{a}) \Big]
\end{aligned}
$$

where $\overline{Ci} = .5772\ldots + \ln x - Ci\ x$

$H = \frac{2\pi h}{\lambda}$

h = antenna half length

a = radius of conductor

A solution of (16) and (17) for values of spacing from .1 to 1.0 wavelength was made using the experimental lengths

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3 R. King and G. H. Blake, "The Self-impedance of a Symmetrical Antenna," Proceedings of the I.R.E., XXX (July, 1942), 335-349.

and is summarized in Figure 15. Values for Z_{22} are included in the data for the various antennas used in the experimental work and used with Figure 15 enable the calculation of the theoretical curves of $-\frac{Z_{12}^2}{Z_{22}}$ shown in Figures 8, 9, and 10.*

EXPERIMENTAL RESULTS

Measuring Equipment.--A General Radio type 874B slotted line was made available for use in this work. This line is fifty centimeters long and has a characteristic impedance of fifty ohms. The terminals have a guaranteed standing wave ratio of less than 1.02 and accuracy of the characteristic impedance is within $\pm$ 1%. The accuracy of probe penetration is stated to be better than $\pm$ 2½%. It was found that the line had a slight taper or a variation in probe penetration which produced slightly different SWR values along the line. Inasmuch as the measurements to be made were of a difference character, this fault of the line had no effect and was checked by making several data runs on each end of the line for the same load condition.

Type RG 8/U coaxial line was used throughout for transmission from the signal generator to the slotted line, line to antenna and probe to meter.

*An example of this calculation is included in Appendix A.

The signal generator was an Army type APT/5 radar jammer transmitter. The APT/5 employs a type 3C22 lighthouse tube in a cavity resonator and has a frequency range of 300 to 1400 mcs. It was found to be extremely useful due to its versatility and great frequency coverage. The only conversion necessary for its use is the removal of the noise modulator section and the addition of a source to supply 24 volts D. C. for the blower motor and a 300 volt D. C. supply for plate voltage in addition to the 110 volts A. C. required for the filament transformer.

Method of Measure.--Wherever crystal diodes are used for detection in accurate measurements, it is necessary that calibration curves be constructed for the crystal in use. Perhaps the best method of calibrating crystals used in slotted line work consists of shorting the line and plotting the voltage distribution along the line. The resultant curve should be sinusoidal and the crystal may be calibrated against a sinusoid plotted for the same maxima. The calibration curve used in this work was Figure 16.

The actual measurement procedure consisted of first removing the antenna and shorting the load end of the coaxial cable. Notation was made of a voltage minima on the slotted line and the short was then removed. Replacing the dipole, the voltage minima was then relocated and notation was made of the displacement of the minima, the value of the minima and the value of the nearest maxima. It was necessary to recheck the frequency of the transmitter as the oscillator showed a tendency to shift slightly with a different reflected load impedance.

Calculation of the terminating impedance was made with the use of circular Smith charts as illustrated in Appendix B. By determining the value of the SWR from the corrected voltage readings and locating the shift of the minima, a normalized impedance was determined and converted to the actual terminating impedance by multiplying by the characteristic impedance of the line.

Initial readings were made with the long antennas out of the near field of the excited antenna. Successive readings were then made as the antenna was moved closer to the excited antenna. A continual check of the frequency was necessary as the spacing was decreased in order to maintain a constant frequency at the transmitter with different reflected loads.

In order that the balanced antennas could be fed with coaxial line and not be seriously detuned by the unbalanced line, it was necessary to construct an isolation section or line choke. This section in addition to transferring the balance to unbalanced line also serves to offer a high impedance to radio frequency currents that would otherwise travel on the outer conductor and seriously upset radiation and line conditions. This section constructed as in Figure 5 is one quarter wave long at the operating frequency. The complete measuring set-up is shown in Figure 6.

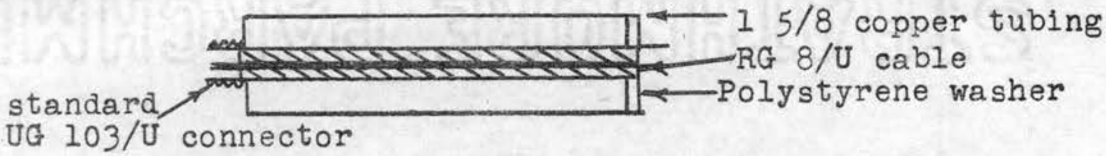


Figure 5.

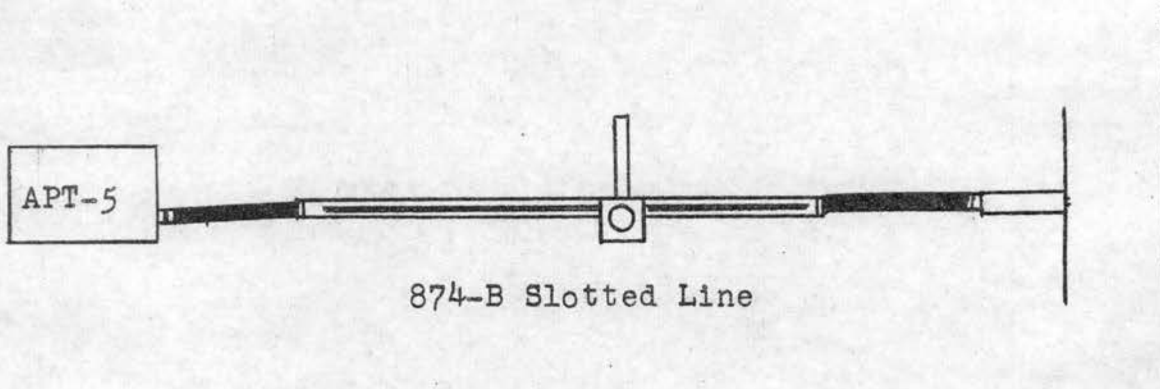


Figure 6.

Analysis of Results.--Experimental data were obtained for antennas of three different diameters. A plot of the theoretical value of $-\frac{w^2M^2}{Z_{12}}$ and the measured value reveal that the actual maxima were less than the calculated; the entire curve shifted to the right and successive peaks spread over greater distance with regard to spacing. It may be said that the curves were also shifted downward with smaller diameter antennas. In the case of the .127 cm. copper wire antenna, the resistive components did not appear positive until a spacing of .45 wavelength was reached as compared to .3 for both the .483 cm. brass and the .953 cm. aluminum tubing. However, it was felt that the accuracy of the .127 cm. wire measurements was impaired due to sag of the longer element. This effect would be more pronounced at close spacing and would be a probable explanation of the irregularity in the resistive curve occurring between .2 and .4 wavelength.

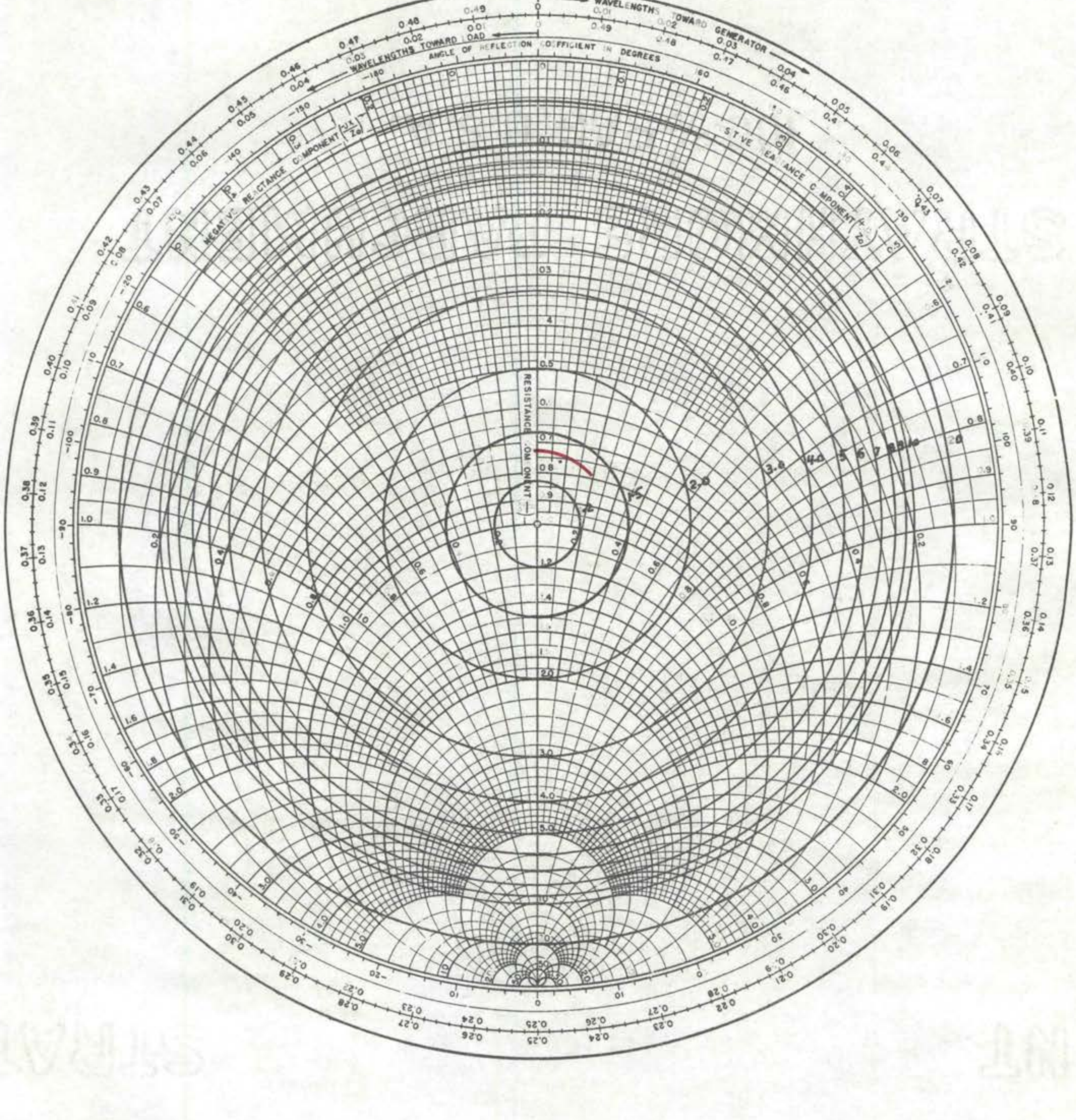
Examination of the curves plotted of the absolute impedances and their phase angles reveal that the actual mutual effect varies from twenty to fifty per cent of the calculated. It is evident that there is a complex coefficient of coupling whose magnitude varies from .20 to .50 and whose phase angle is varied from -100° to -215°. Maximum coefficient of coupling appears with spacing in the order of .3 wavelength but the phase angle continues to increase in a negative direction with decreased spacing. This would serve to indicate that the amount of coupling is greatly affected by the capacity between the two antennas as well as their mutual inductance.

CONCLUSIONS

The results of the study are rather disappointing in that a definite means of approximating the effect of the mutual impedance was not determined. It was realized that the problem is a very complex one and cannot be solved by merely making a series of measurements at one frequency. The frequency used was an arbitrary one dictated by the length of slotted line available and convenience of isolating the antenna from other objects. Lower frequencies would greatly increase ground effects as well as being very inconvenient from the standpoint of the larger antenna equipment required. Higher frequencies require increased accuracy in the cutting of antennas and isolating networks as well as more precise measuring techniques. The distributed capacity is greatly increased due to the much closer spacing involved with very short wavelengths.

In view of the close correlation between the calculated values and the actual measured values, it was felt that a step has been made in the right direction but more studies are needed before the formulas as developed by Cox will be worthwhile at the higher frequencies. Accurate measurements of capacitance between antennas as well as studies at different frequencies are needed. The arrangement of antennas in this problem is only one of a number that should be investigated. Other harmonic lengths should be studied as a means of more accurately determining the coefficient of coupling and effect of distributed capacity.

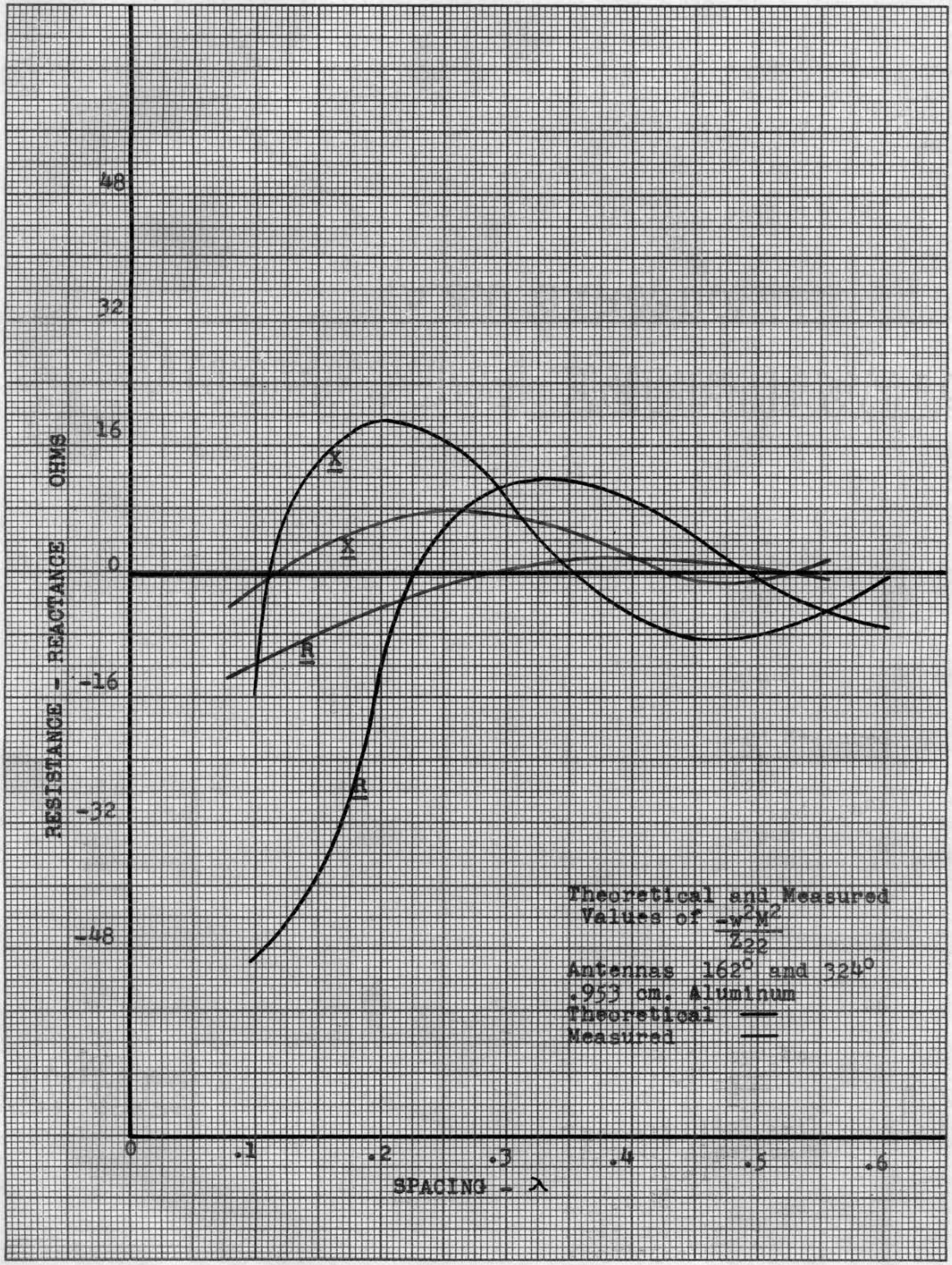
THE EMELOID CO., INC.
HILLSIDE 5, N. J.
JANUARY 1948



Ref: Phillip H. Smith "Transmission Line Calculator" Electronics Jan. 1939 and Jan. 1944

Printed in U.S.A.

Figure 7. Solution of Measured Impedance by Smith Chart



48
32
16
0
-16
-32
-48
RESISTANCE - REACTANCE OHMS
X
X
R
R
0
.1
.2
.3
.4
.5
.6
SPACING - λ
Theoretical and Measured
Values of $\frac{-w^2M^2}{Z_{22}}$
Antennas 162° and 324°
.953 cm. Aluminum
Theoretical
Measured

Figure 8.

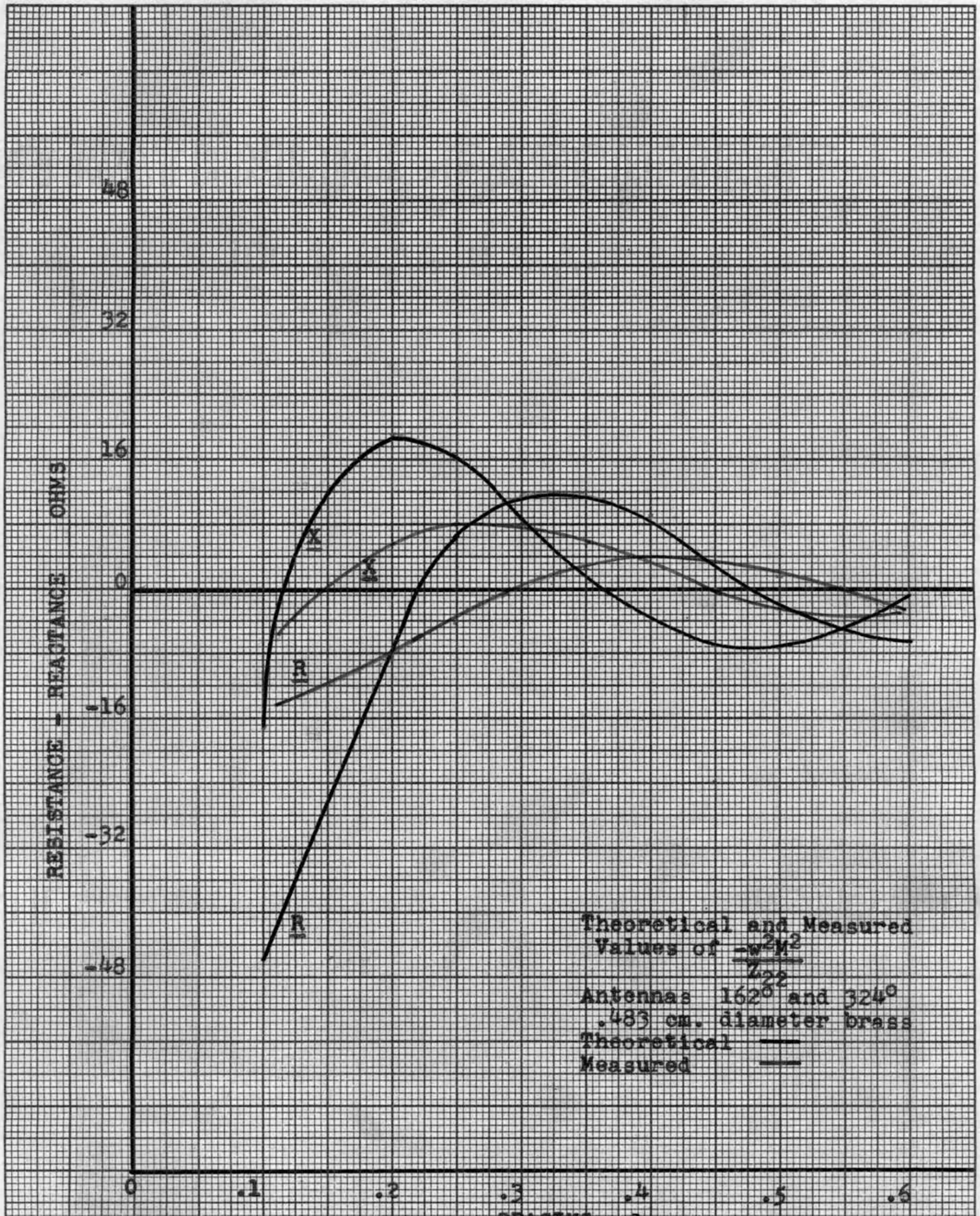
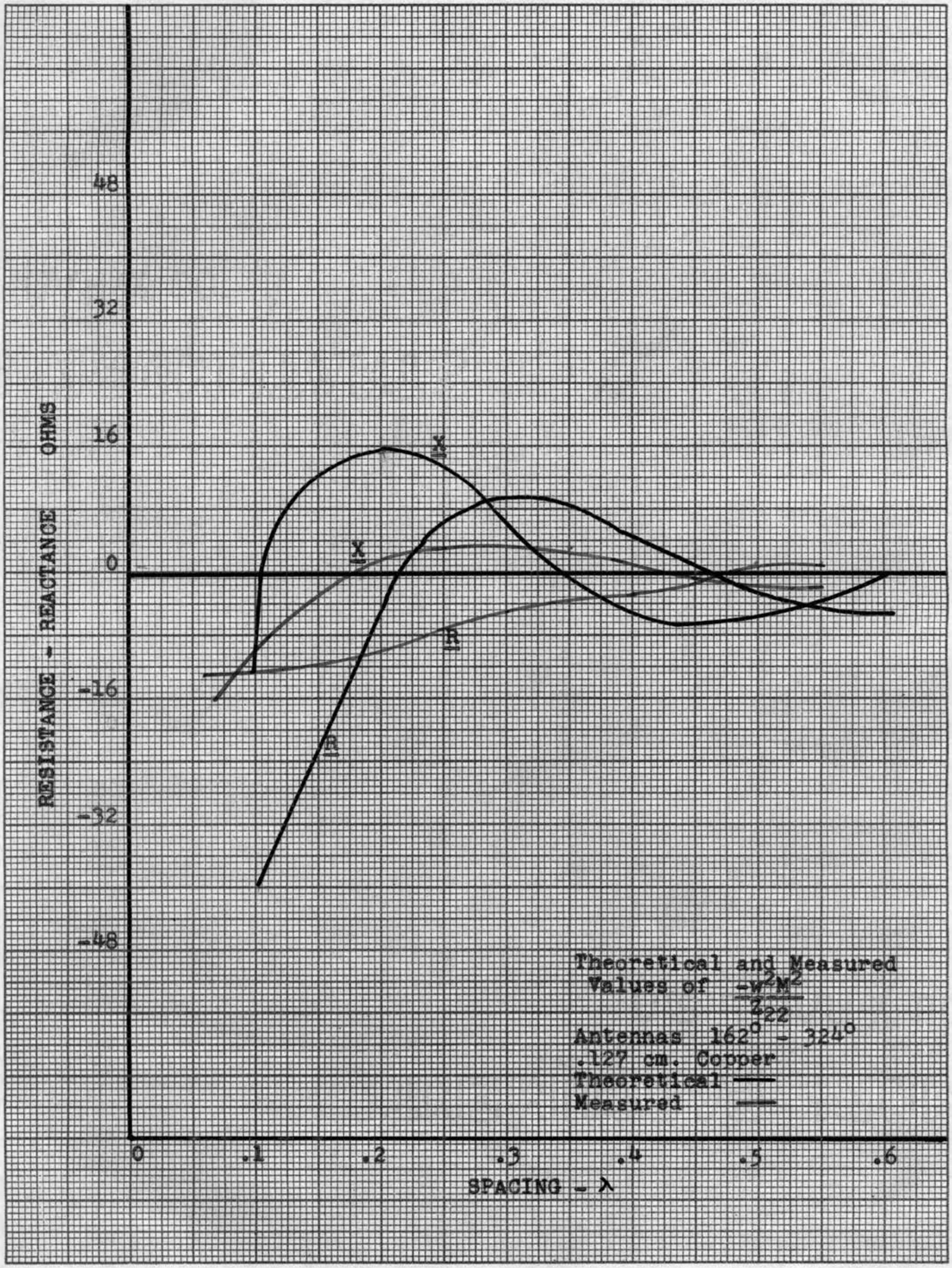

RESISTANCE - REACTANCE OHMS
48
32
16
0
-16
-32
-48
0
.1
.2
.3
.4
.5
.6
SPACING - λ
X
X
R
R
Theoretical and Measured Values of $\frac{-w^2M^2}{Z_{22}}$
Antennas 162° and 324°
.483 cm. diameter brass
Theoretical
Measured

Figure 9.



RESISTANCE - REACTANCE OHMS
48
32
16
0
-16
-32
-48
X
X
R
R
Theoretical and Measured Values of $\frac{-w^2M^2}{Z_{22}}$
Antennas $162^\circ - 324^\circ$
.127 cm. Copper
Theoretical
Measured
0
.1
.2
.3
.4
.5
.6
SPACING - λ

Figure 10.

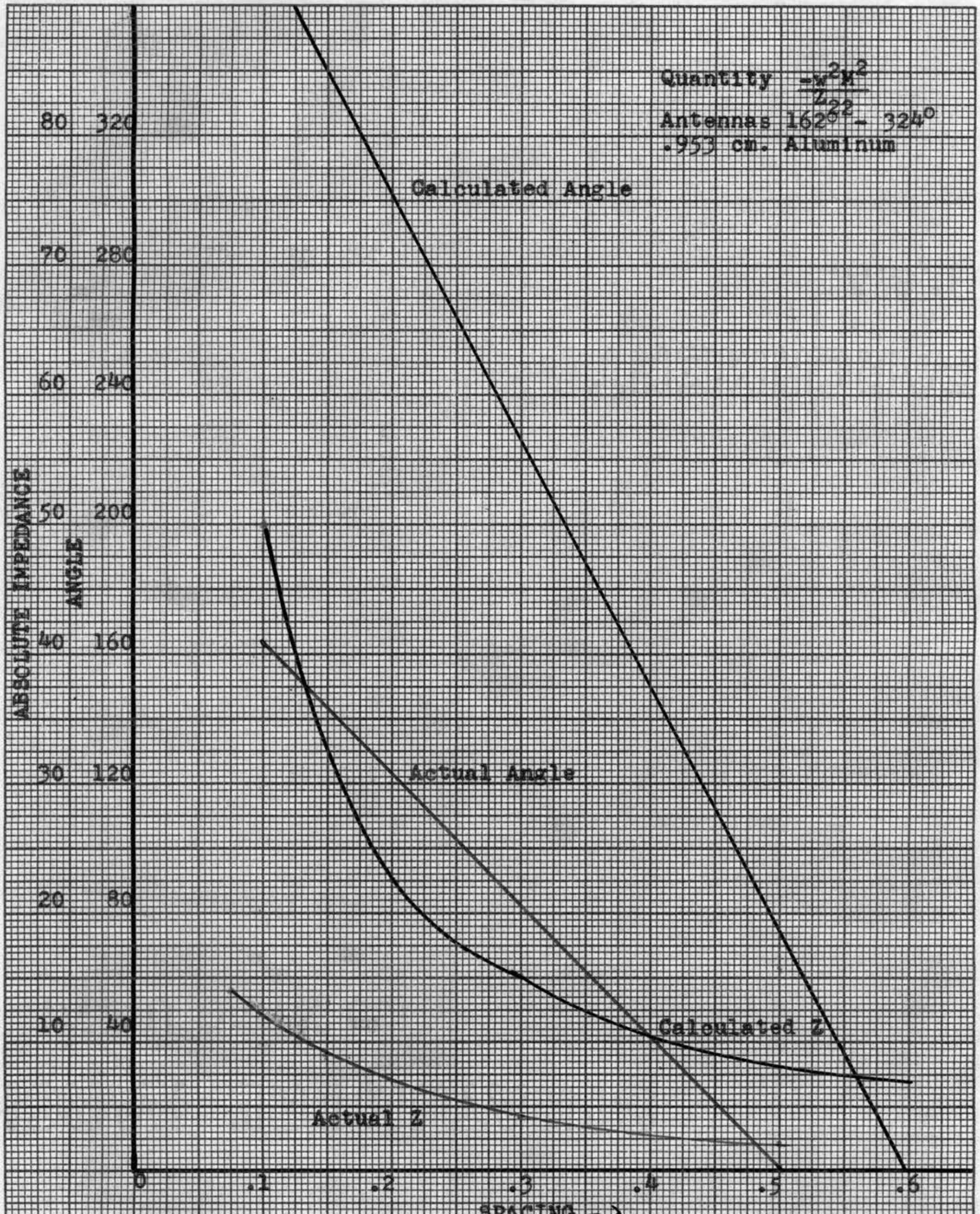
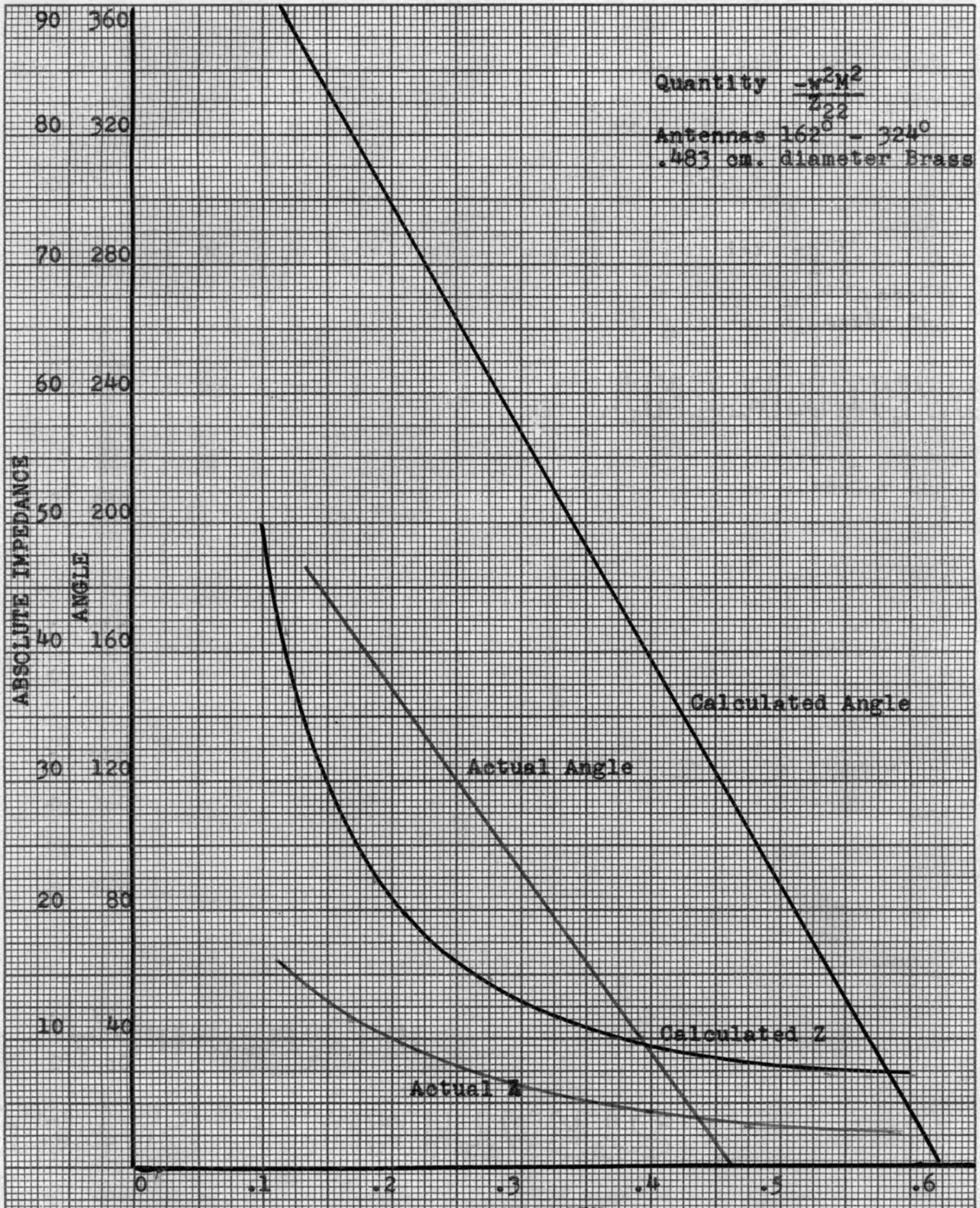
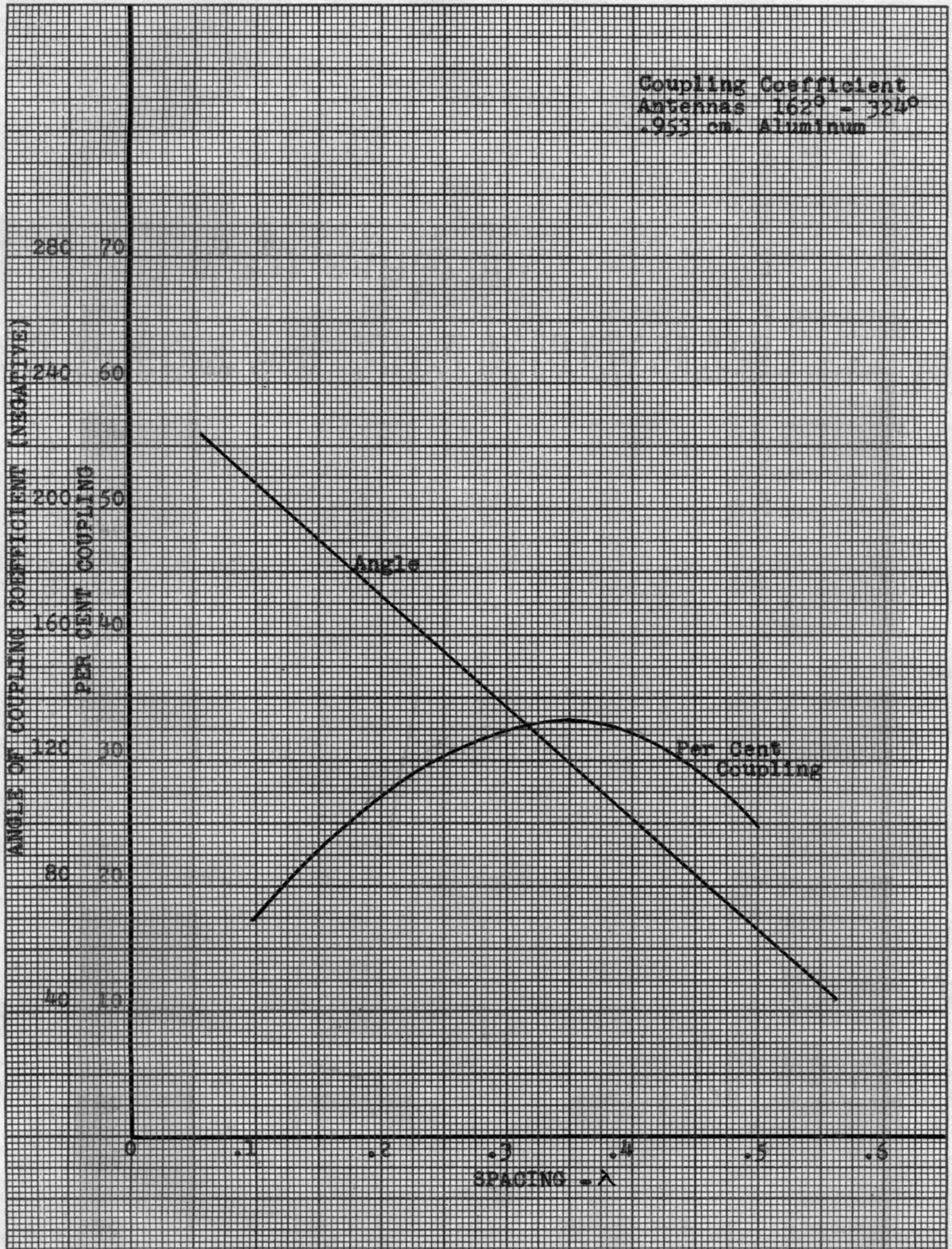

Quantity $\frac{-w^2M^2}{Z_{22}}$
Antennas 162° - 324°
.953 cm. Aluminum
Calculated Angle
Actual Angle
Calculated Z
Actual Z
ABSOLUTE IMPEDANCE
ANGLE
80 320
70 280
60 240
50 200
40 160
30 120
20 80
10 40
0
.1
.2
.3
.4
.5
.6
SPACING - λ

Figure 11.



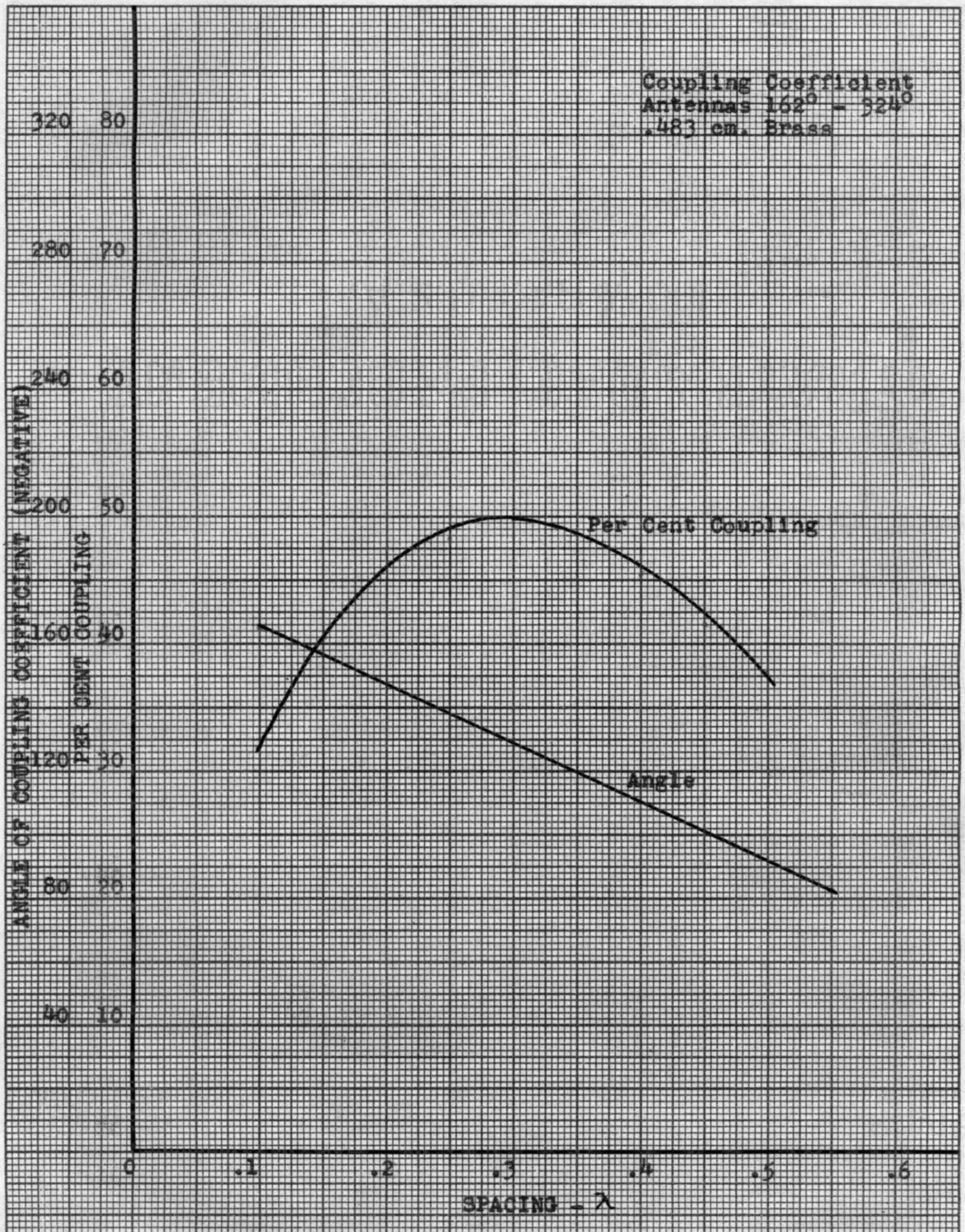
Quantity $\frac{-w^2M^2}{Z_{22}}$
Antennas $162^0 - 324^0$
.483 cm. diameter Brass
Calculated Angle
Actual Angle
Calculated Z
Actual Z
ABSOLUTE IMPEDANCE
ANGLE
90 360
80 320
70 280
60 240
50 200
40 160
30 120
20 80
10 40
0 .1 .2 .3 .4 .5 .6
WAVELENGTH

Figure 12.



Coupling Coefficient
Antennas 162° - 324°
.953 cm. Aluminum
Angle
Per Cent Coupling
ANGLE OF COUPLING COEFFICIENT (NEGATIVE)
PER CENT COUPLING
280
240
200
160
120
80
40
70
60
50
40
30
20
10
0
.1
.2
.3
.4
.5
.6
SPACING - λ

Figure 13.



Coupling Coefficient
Antennas 162° - 324°
.483 cm. Brass
Per Cent Coupling
Angle
ANGLE OF COUPLING COEFFICIENT (NEGATIVE)
PER CENT COUPLING
320 80
280 70
240 60
200 50
160 40
120 30
80 20
40 10
0 .1 .2 .3 .4 .5 .6
SPACING - λ

Figure 14.

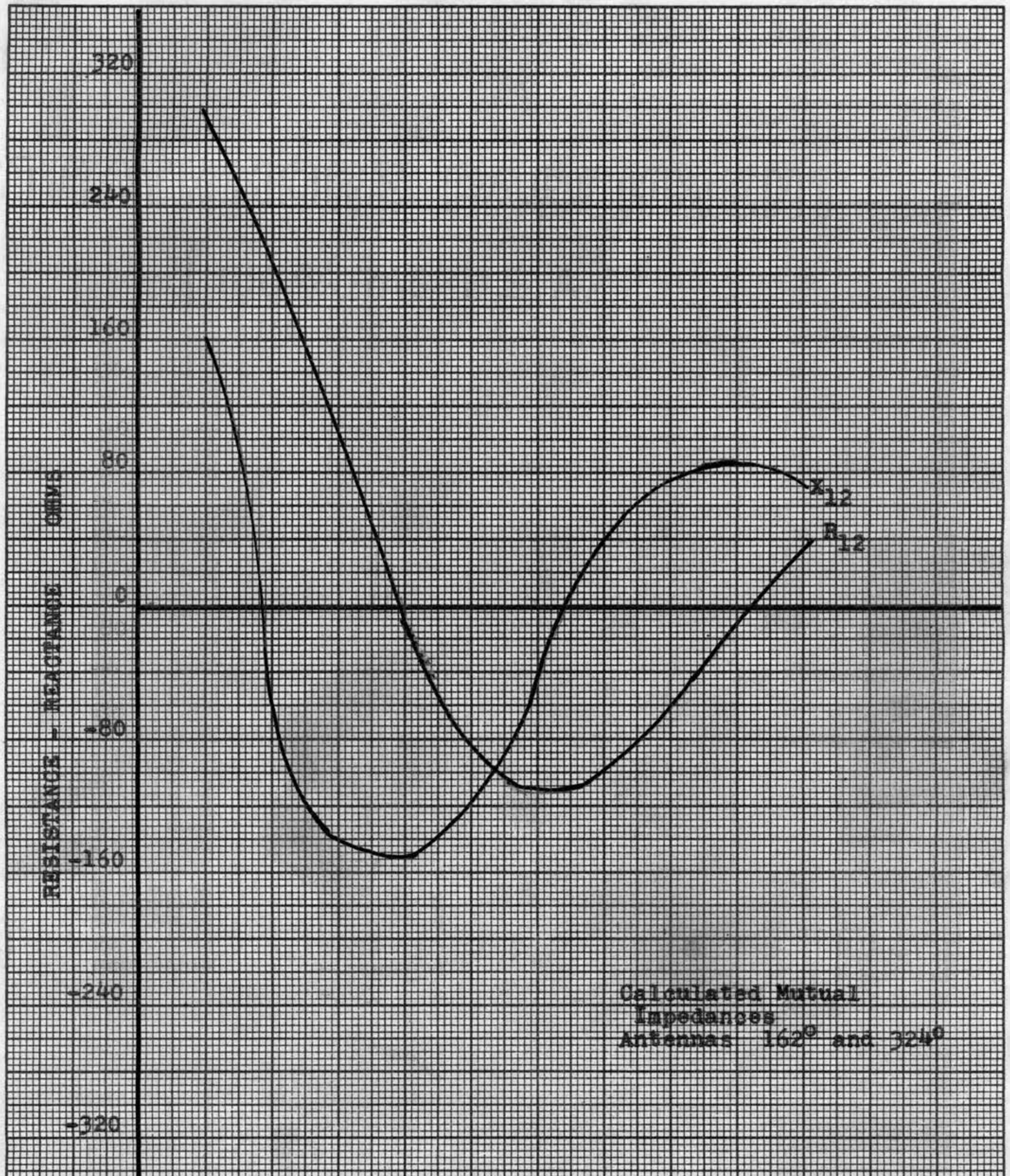

320
240
160
80
0
-80
-160
-240
-320
RESISTANCE - REACTANCE OHMS
0
.2
.4
.6
.8
1.0
SPACING - λ
X12
R12
Calculated Mutual
Impedances
Antennas 162° and 324°

Figure 15.

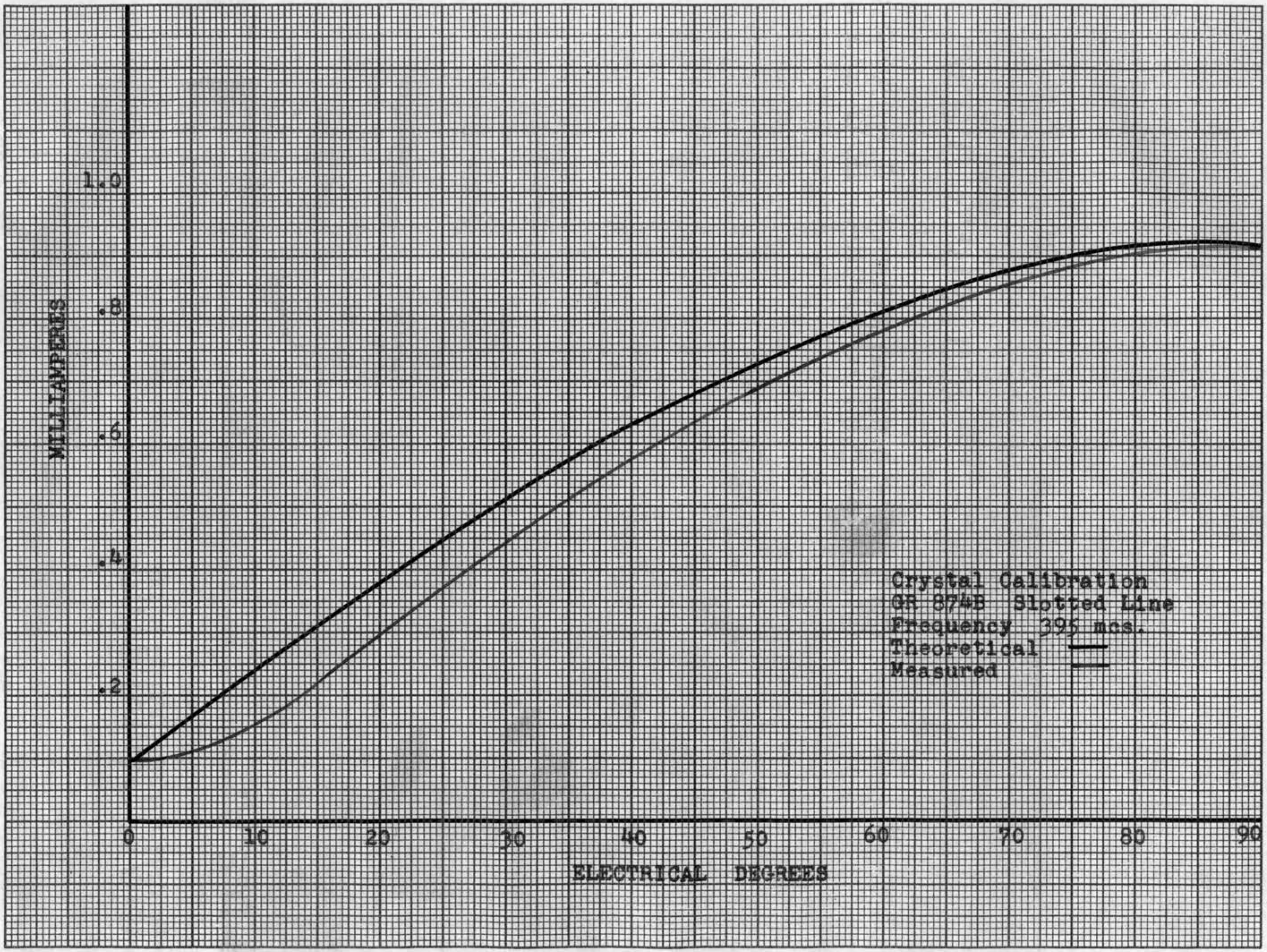

1.0
.8
.6
.4
.2
MILLIAMPERES
0
10
20
30
40
50
60
70
80
90
ELECTRICAL DEGREES
Crystal Calibration
GR 874B Slotted Line
Frequency 395 mcs.
Theoretical
Measured

Figure 16.

APPENDIX A.

Sample Calculation of M

Spacing = $d = .1\lambda = .62832$ radians

$l_1 = 81^\circ = .225\lambda$

$l_2 = 162^\circ = .45\lambda$

$\Delta = l_2 - l_1 = .45 - .225 = .225\lambda$

$L = l_2 + l_1 = .45 + .225 = .675\lambda$

$l_1^{\,2} = .050625$

$l_2^{\,2} = .2025$

$\Delta^2 = .050625$

$L^2 = .455625$

$f = 395$ mcs.

$w_o = v_o = B(\sqrt{d^2 + l_1^{\,2}} + l_1) = 2\pi(\sqrt{.01 + .050625} + .225)$
$= 2.9604$

$w_1 = B(\sqrt{d^2 + L^2} + L) = 2\pi(\sqrt{.01 + .455625} + .675) = 8.5295$

$x_o = u_o = B(\sqrt{d^2 + l_1^{\,2}} - l_1) = 2\pi(\sqrt{.01 + .050625} - .225)$
$= .13257$

$x_1 = B(\sqrt{d^2 + L^2} - L) = 2\pi(\sqrt{.01 + .455625} - .675) = .04712$

$y_o = B(d) = 2\pi \times .1 = .62832$

$y_1 = u_1 = B(\sqrt{d^2 + l_2^{\,2}} + l_2) = 2\pi(\sqrt{.01 + .2025} + .45)$
$= 5.7240$

$s_o = B(d) = 2\pi \times .1 = .62832$

$s_1 = B(\sqrt{d^2 + l_2^{\,2}} - l_2) = 2\pi(\sqrt{.01 + .2025} - .45) = .06911$

$v_1 = B(\sqrt{d^2 + \Delta^2} - \Delta) = 2\pi(\sqrt{.01 + .050625} - .225) = .13257$

Quantity	Si	Ci
wo_1	1.84651	.13277
v_o	1.84651	.13277
w_1	1.63233	.09730
x_o	.1323	-1.4477
u_o	.1323	-1.4477
x_1	.0470	-2.4786
y_o	.61440	.01501
y_1	1.44392	- .11087
u_1	1.44392	- .11087
s_o	.61440	.01501
s_1	.0690	-2.0960
v_1	.1323	-1.4477

Substituting in (16)

$$R_{21} = \frac{15}{\sin 81^\circ \sin 162^\circ} \Big(\cos 81^\circ\big[-.11087 + 1.4477 - 1.4477 - .13277 + .03002 + .11087 + 2.0960\big]$$

$$+ \sin 81^\circ\big[1.44392 - .1323 + 1.84651 - .1323 - 1.44392 + .0690\big]$$

$$+ \cos 243^\circ\big[.09730 - .13277 - 2.4786 + 1.4477 + .03002 + .11087 + 2.0960\big]$$

$$+ \sin 243^\circ\big[1.63233 - 1.84651 + .1323 - .0470 - 1.44392 + .0690\big]$$

$$= 49.145\,(.615954 + 1.63058 - .531404 + 1.339927)$$

$$= 150.1409\,\Omega$$

Substituting in (17)

$$
\begin{aligned}
X_{21} = \frac{15}{\sin 81^\circ \sin 162^\circ} \Big(& \cos 81^\circ \big[.1323 - 1.44392 + 1.84651 \\
& - .1323 + 1.44392 - 1.22880 + .0690 \big] \\
& + \sin 81^\circ \big[-.11087 + 1.4477 + 1.3277 \\
& + 1.4477 + .11087 - 2.0960 \big] \\
& + \cos 243^\circ \big[1.84651 - 1.63233 + .1323 \\
& - .0470 + 1.44392 - 1.22880 + .0690 \big] \\
& + \sin 243^\circ \big[.09730 - .13277 - 1.4477 \\
& + 2.4786 + .11087 - 2.0960 \big] \Big)
\end{aligned}
$$

$$= 49.145\ (.212207 + .92069 - .264948 + .881832)$$

$$= 85.9929\,\Omega$$

Doubling these values to connect from vertical antenna to horizontal of double length we obtain

$$R_{21} = 300.2818\,\Omega \qquad (.1\lambda \text{ spacing})$$

$$X_{21} = 171.9858\,\Omega$$

$$Z_{21} = 300.28 + j\,171.98 = 356 \underline{|28.9^\circ}$$

Using values from Blake and King's article[3] for Z_{22}*

$$-\frac{(Z_{21})^2}{Z_{22}} = \frac{(356)^2}{2250} = 56.5$$

$$R = -\frac{(Z_{21})^2}{Z_{22}} \cos(2\theta_{12} - \theta_{22}) = -49.8\,\Omega$$

$$X = -\frac{(Z_{21})^2}{Z_{22}} \sin(2\theta_{12} - \theta_{22}) = -20.2\,\Omega$$

---

* $1800 + j1300$ for .953 cm. tubing of 324° length.

APPENDIX B.

Sample Calculation of Load Impedance

Data: Minima shift 5 cm. toward load (.066 wavelength at 395 mcs.)

Maximum value .9 ma.

Minimum value .65 ma.

$$SWR = \frac{.9}{.65} = 1.385$$

On SWR circle of 1.385 of the circular Smith chart, start at the line of resistance only and in direction of R less than 1 and rotate in the $+ jx$ (clockwise) direction a distance equivalent to .066 wavelength. Read normalized impedance of $.79 + j.19$. The load impedance is found by multiplying the normalized impedance by the characteristic impedance of the line.

$$50(.79 + j.19) = 39.5 + j9.5$$

This is based on the assumption of negligible losses in the line and a constant characteristic impedance of 50 ohms. This is a fairly valid assumption although manufacturing irregularities will cause discrepancies of over one ohm in the RG type cables.

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THESIS TITLE: THE EFFECT OF MUTUAL IMPEDANCE

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