GENERATOR CHARACTERISTICS OF SINGLE-PHASE INDUCTION MACHINES WITH CAPACITOR EXCITATION

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Preface

There are many products of electrical engineering development which because of the degree of their technical perfection and because of the great demand for the services they render, have found wide acceptance by the general public and therefore become commonplace. One example of this is the single-phase induction motor.

The list of machines and home appliances to which these electric drives are applied continues to grow. Washing machines, refrigerators, stokers, oil burners, blowers, circulating and water pumps are but a few of the most common pieces of equipment which depend upon the single-phase induction motor for mechanical energy. Millions of these motors are in operation today, and the fact that they operate continuously for extended periods of time with few failures and very little supervision or maintenance is a tribute to the success of the design engineers and the motor manufacturers who produce them.

Present trends indicate that the single-phase induction motor will be of greater economic importance in multi-motor machines of industry and in the still-growing home appliance field. This economic importance justifies continued research in theory of design and operation of this electric motor. There are steps in design procedure that may be improved; there are methods of analysis of performance that may be explored; there are unusual conditions of operation yet to be analyzed.

An interest in one of the unusual conditions of operation of the single-phase, capacitor-start induction motor led the writer to investigate the performance characteristics of this machine as a thesis project in electrical engineering. The results of the investigation show a method for predicting the generator characteristics of single-phase induction machines which draw their excitation from static capacitors.

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GENERATOR CHARACTERISTICS OF SINGLE-PHASE INDUCTION MACHINES WITH CAPACITOR EXCITATION

Chapter I

The General Problem of Self-excitation of Induction Machines

The induction machine has for many years been of great industrial importance as a means for converting electrical energy into mechanical energy. As a constant-speed driving source, it has been useful in a wide variety of applications. Its simple, rugged construction and relatively good operating characteristics make it one of the most economical types of electric motors.

The theory of operation and the general performance characteristics of the induction machine as a motor have been thoroughly investigated and are relatively well understood by engineers who have worked with rotating electric machinery. The fact that this machine will also operate as a generator has been known since shortly after its invention by Nickola Tesla in 1888. However, the details of its generator characteristics are not so well known. There has been little incentive to explore those characteristics, since only a few practical applications for them have been found.

The fact that induction machines can operate as self-excited generators seems to have been ignored by all but a few researchers. Self-excitation is accomplished by means of static capacitors placed across the terminals of the machine. The technical literature contains a few papers which analyze specific applications of this phenomenon in polyphase machines. Only passing reference is made to the possibility of similar characteristics in single-phase machines. This thesis deals with applications of self-excitation in capacitor-start, single-phase machines.

Since the true single-phase motor has no starting torque, several methods of starting have been devised and are in common use today. The method which employs an electrolytic capacitor and a "start" winding, in addition to the main winding, is perhaps most widely used to obtain high starting torque. The start winding is assembled into the machine in space quadrature with the main winding. The starting circuit, consisting of the capacitor in series with the start winding, is connected in parallel with the main winding.

When voltage of correct magnitude and frequency is applied to the terminals of the machine, its rotor accelerates to approximately 75 per cent of normal operating speed. At this point in the starting cycle, a centrifugally operated switch opens the starting circuit. The rotor continues to accelerate to normal speed, running as a true single-phase motor. The speed at which the starting circuit is opened during acceleration is called "cut-out" speed.

When the source of electric power is removed from the terminals of the motor, the rotor begins to decelerate. During deceleration, the switch in the starting circuit closes. The speed at which this occurs is called "cut-back" speed and depends upon the design of the centrifugal mechanism. "Cut-back" speed is just below "cut-out" speed for most centrifugally operated devices.

With the supply voltage removed and the speed of the rotor less than "cut-back" speed, the main winding, the start winding, and the capacitor become a closed circuit. If the capacitance is of sufficient magnitude, the conditions for induction generation may be satisfied. Voltages and current may rise to values considerably higher than normal.

Machines which utilize the self-excited generator characteristics have found commercial application. Under certain conditions these generator characteristics are found to be objectionable and thus limit application of the machine. It is the purpose of this thesis to present a method for predicting the conditions under which self-excited induction generation may occur in single-phase machines. In the following discussion, such operation is referred to as "regeneration". A method for obtaining the approximate values of voltages, current, and torque during regeneration is also presented.

A study of this nature can be justified both by its application to machines in commercial use and by the fact that basic induction machine phenomena are more fully explored and understood.

Single-phase, capacitor-start motors are used to drive machine tools where quick reversing or deceleration is important. Regeneration is used to provide braking torque which quickly reduces the speed to approximately 25 per cent of normal running speed. Although regeneration is not maintained below this speed, approximately 90 per cent of the rotational energy has been removed from the rotor. Where quick reversing is necessary, regeneration is used during the reversing cycle.

Capacitor-start motors are used to drive hoists of limited lifting capacity. Regeneration is employed to prevent overspeeding of hoists when lowering a heavy overhauling load without electric power. Also, it is an effective safety device if the mechanical brake fails at a time when electric power is off.

In some applications the occurrence of regeneration is highly undesirable. Capacitor-start motors are used on automatic oil burners which are usually connected to a large amount of thin, sheet metal ductwork. If regeneration occurs during the deceleration of the motor, the pulsating torque causes an excessive vibration of the installation, resulting in undesirable noises. If solenoids or other electrically actuated devices are connected in parallel with the motor, the voltage during regeneration can cause false operation.

The above examples are cited to show the importance of a method for predicting regeneration characteristics. It is hoped that the following analysis can form a basis for controlling design to obtain desired regenerative operation.

Conventional Analysis of Polyphase Induction Machines

It will be helpful in the analysis of the generator action in a single-phase induction machine to review the characteristics of the polyphase machine as a motor, generator, and brake.

The polyphase machine consists essentially of the following components: (1) stator, made up of a laminated iron core containing slots into which the main, or primary, armature winding is assembled; (2) rotor, made up of a laminated iron core with a winding in slots on its periphery; (3) shaft, end-shields, bearings, and other mechanical parts.

The conventional analysis of balanced polyphase machine performance is based on the idea of a magnetomotive force (mmf) of constant magnitude revolving at constant speed around the air gap. This revolving magnetomotive force is produced by the simultaneous action of currents in two or more windings on the stator, physically displaced in space at an angle which depends upon the number of phases and poles. The windings must be supplied with currents which vary sinusoidally with time and which are displaced in time at an interval depending upon the number of phases. The direction of rotation depends on the phase sequence of the applied voltages and the polarity of the windings. The speed of the field in the air gap is directly proportional to the frequency of the currents and inversely proportional to the number of poles. If the frequency is constant and the number of poles fixed, a definite value of speed results, and this value is known as synchronous speed.

$$n_s = \frac{120\ f_1}{p_1} \tag{1}$$

where

n_s = synchronous speed in revolutions per minute (rpm)

f_1 = frequency in cycles per second (cps)

p_1 = number of poles in stator armature winding

For mathematical analysis, it has been found convenient to use the relative speed between the revolving field and the rotor instead of the actual rotor speed. The difference between synchronous speed and actual rotor speed is known as "slip". Slip can be expressed in terms of synchronous speed as follows:

$$s = \frac{n_s - n}{n_s} \tag{2}$$

where

s = slip

n = actual rotor speed in rpm

If the rotor of the machine is not turning at the same speed as the revolving field, there is a relative speed between the rotor conductors and the field. A voltage will therefore be generated in the rotor conductors, and the value of that voltage can be expressed as follows:

$$e \propto Blv \tag{3}$$

where

e = instantaneous electromotive force

B = flux density of magnetic field

l = length of rotor conductor

v = velocity of rotor conductor with respect to the field

If the rotor circuit is closed, currents will flow. The physical force which is developed between the rotor currents and the revolving field produce a torque in the same direction as field rotation at all rotor speeds between standstill and synchronous speed.

$$F \propto Bli_r \tag{4}$$

where

$F =$ force on the rotor conductor

$i_r =$ current in the rotor conductor

This force, acting at a fixed radius from the center line of the shaft, produces motor torque. The torque of a polyphase machine is constant with respect to time when the speed is constant.

At synchronous speed the rotor conductors have no relative motion with respect to the revolving field. Therefore, no currents are induced in the rotor, and no torque is produced. Normal motor operation, then, is at a speed slightly below the speed of the field.

Since motor operation is between synchronous speed and standstill, the values of slip which represent motor operation are between 0 and 1. If the rotor is driven in a direction opposite to the field rotation, the condition is represented by positive values of slip greater than 1. This is a condition of brake action, since the electromagnetic torque is in the direction of field rotation, but opposite to shaft rotation.

If the rotor is driven at speeds above synchronism, the relative motion between the rotor conductors and the magnetic field is reversed. Rotor voltages, currents, and torques are also reversed. Because the actual rotor speed is greater than synchronous speed, the slip is negative. An induction machine operating at a negative slip is known as an induction generator.

Turning now to a more detailed analysis of the voltages, currents, and torques in the rotor winding, it will be noted that the frequency of rotor voltages and currents can be expressed in terms of stator values as follows:

$$f_2 = sf_1 \tag{5}$$

where

f_2 = frequency of rotor voltage and current

Since equation (1) gives the speed of the field with respect to the stator, and

$$n_2 = \frac{120\ f_2}{p_2} \tag{6}$$

is the speed of the field with respect to the rotor, then

$$n = n_s - n_2 = \frac{120\ f_1}{p_1}\ (1-s) \tag{7}$$

is the actual rotor speed.

When the rotor circuit is open and the rotor is at standstill, $(s = 1)$, the machine is electrically equivalent to an open-circuited, static transformer. Assuming that the rotating magnetic flux in the air gap is distributed sinusoidally, the effective, or root-mean-square (rms), value of the stator induced voltage is

$$E_1 = 4.44\ f_1 N_1 \phi_m\ 10^{-8}\ K_{p_1}\ K_{d_1} \tag{8}$$

where

E_1 = rms volts per phase in stator winding

N_1 = series turns per phase on stator

ϕ_m = maximum flux linking a full-pitch turn on stator

K_{p_1} = pitch factor of stator winding

K_{d_1} = distribution factor of stator winding

and the rotor induced voltage at standstill is

$$E_2 = 4.44\ f_1 N_2 \phi_m\ 10^{-8}\ K_{p2}\ K_{d2} \tag{9}$$

where

E_2 = rms volts per phase in rotor winding at standstill

N_2 = series turns per phase on rotor

K_{p_2} = pitch factor of rotor winding

K_{d_2} = distribution factor of rotor winding

When the rotor is turning, the value of the rotor voltage is sE_2.

The usual form of the rotor winding is that of a squirrel cage, and this type of winding produces an effect equivalent to that of a wound rotor with the same number of poles and phases as the stator. When the rotor and stator have the same number of phases, the rotor voltage can be expressed in terms of the stator voltage in the manner of transformer equivalent values using

$$a = \frac{N_1 K_{p_1} K_{d_1}}{N_2 K_{p_2} K_{d_2}} \tag{10}$$

where

a = ratio of transformation between rotor and stator.

Then

$$aE_2 = E_2' = E_1 \tag{11}$$

where

E_2' = equivalent rotor voltage in terms of stator

If the rotor circuit is now closed and at standstill, $(s = 1)$, then

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{R_2+jX_2} = \frac{E_2}{\sqrt{R_2^2+X_2^2}} \Big| \tan^{-1} \frac{X_2}{R_2} \tag{12}$$

where

I_2 = rms amperes per phase rotor current

R_2 = ohms per phase effective resistance of rotor winding

X_2 = ohms per phase leakage reactance of rotor winding at frequency f_1, or

$$X_2 = 2\pi f_1 L_2 \tag{13}$$

where

L_2 = henrys per phase rotor leakage inductance.

The frequency, f_2, of the rotor voltage at any positive slip is given in equation (5), and the rotor leakage reactance is $2\pi f_2 L_2$, or

$$2\pi(sf_1) L_2 = sX_2 \tag{14}$$

When the rotor is turning at slip $= s$, the rotor voltage is sE_2 and the current is

$$I_2 = \frac{sE_2}{R_2 + jsX_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \qquad (15)$$

Dividing numerator and denominator by s ,

$$I_2 = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + X_2^2}} \qquad (16)$$

This equation implies that the rotor may be represented by the series circuit shown in Fig. 1.

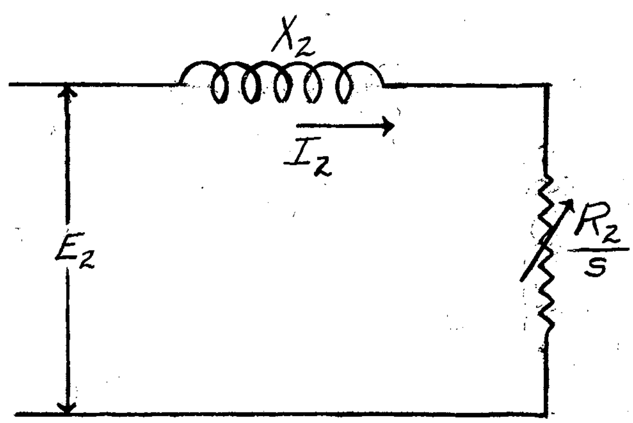


Fig. 1
Equivalent circuit of the rotor
of a polyphase induction motor

If E_2 is constant, a vector diagram may be drawn to represent the current and voltage which are varying sinusoidally with time.

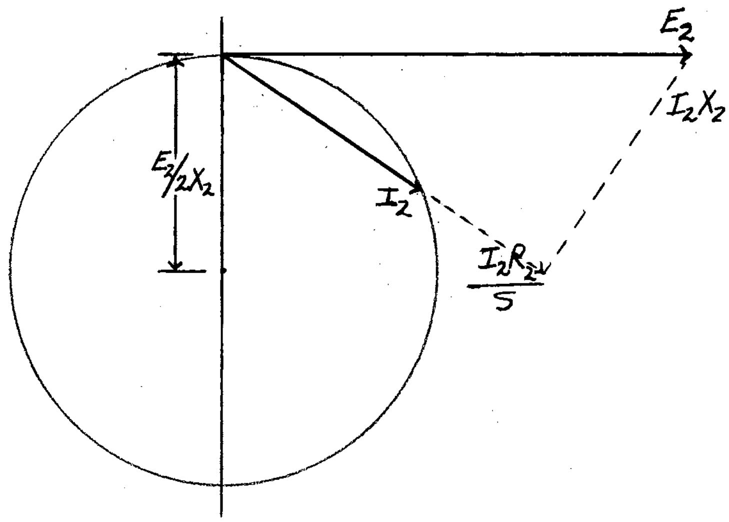


Fig. 2
Vector diagram of the rotor
of a polyphase induction motor

The heavy solid lines in Fig. 2 represent E_2 and I_2 for a constant value of slip. The light solid line shows the circular locus of I_2 when slip is allowed to vary from $-\infty$ to $+\infty$.

The total power input to the rotor circuit is $I_2^2 \left(\frac{R_2}{s}\right)$. Since the actual loss in the rotor circuit is $I_2^2 R_2$, the remainder represents the electromagnetic power converted to mechanical form.

$$P_{dev} = I_2^2 \left(\frac{R_2}{s}\right) - I_2^2 R_2$$

$$= I_2^2 R_2 \left(\frac{1}{s} - 1\right) = I_2^2 R_2 \left(\frac{1-s}{s}\right) \qquad (17)$$

The circuit of Fig. 1 can now be modified to represent power lost and power developed.

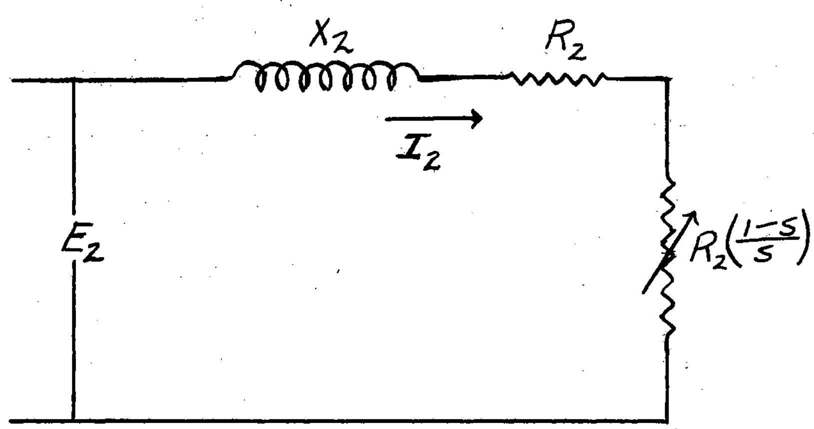


Fig. 3
Modified rotor circuit showing fictitious resistance which represents mechanical load

Substituting equation (16) into equation (17), the power developed is given in terms of rotor voltage,

$$P_{dev} = \frac{E_2^2 R_2 (1-s)}{\left[\left(\frac{R_2}{s}\right)^2 + X_2^2\right] s} . \qquad (18)$$

In rotation, power can be expressed as

$$P_{dev} = \frac{T\ n}{7.04} \qquad (19)$$

where

P_{dev} = power developed in watts

T = torque in pound-feet

$$T = 7.04 \frac{P_{dev}}{n} \qquad (20)$$

Substituting equation (7) and (18) in equation (20),

$$T = 7.04 \frac{p_1}{120\ f_1} \left(\frac{E_2^2 R_2}{s\left[\left(\frac{R_2}{s}\right)^2 + X_2^2\right]} \right) . \qquad (21)$$

Often the torque of an induction motor is expressed by the term in parentheses of equation (21). The name of the unit of torque obtained by this expression is the "synchronous watt". The actual torque in pound-feet is

$$T = \frac{7.04}{n_s} \text{ times (synchronous watts)} \qquad (22)$$

Thus far in the analysis, it has been assumed that E_2 was constant. Actually, there is a stator impedance voltage drop that causes E_2 to vary, even when the terminal voltage of the machine is held constant. This effect may be analyzed by considering the condition of stator input when the rotor is turning at synchronous speed, $(s = 0)$. From equation (15), sE_2 equals zero, and I_2 equals zero. Also by equation (16), as slip approaches zero, $\frac{R}{s}$ approaches infinity, and the rotor impedance approaches infinity. In the limit, the rotor circuit acts like an open circuit and I_2 equals zero.

The input to the stator under these conditions is equivalent to the input of an open-circuited transformer. The magnetizing current is much greater than that of a transformer because of the air gap between the rotor and stator. The power input supplies stator copper losses, $I_1^2 R_1$, and the iron losses due to the main flux. The equivalent circuit which represents this condition is shown in Fig. 4.

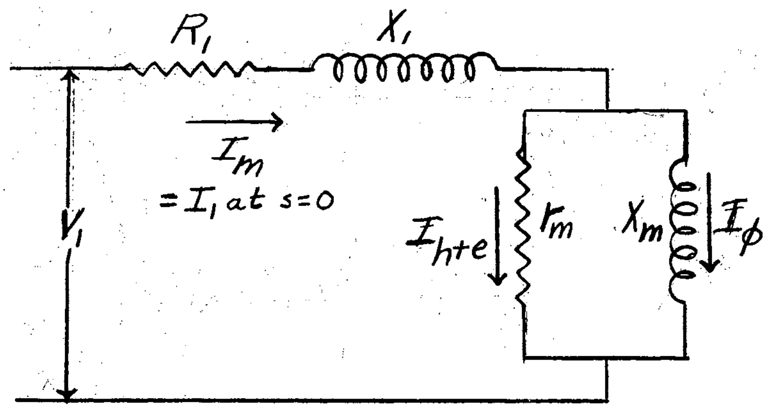


Fig. 4
Equivalent circuit of a polyphase induction motor running at synchronous speed ($s = 0$)

$$I_m = I_\phi + I_{h+e} \quad \text{(vector addition)} \tag{23}$$

where

I_1 = rms amperes per phase stator current

I_ϕ = Magnetizing component of stator current

I_{h+e} = iron-loss component of stator current

I_m = vector sum of I_ϕ and I_{h+e}

r_m = fictitious resistance through which iron-loss component of stator current, I_{h+e}, flows (Fig. 4).

If the power component of the no-load current is neglected, then

$$E_1 = V_1 - (-jI_1)(R_1+jX_1) = I_\phi X_m = E_2' \tag{24}$$

where

R_1 = ohms per phase effective resistance of stator winding

X_1 = ohms per phase leakage reactance of stator winding at frequency f_1

X_m = magnetizing reactance through which the magnetizing component of stator current, I_ϕ, flows

V_1 = volts per phase applied stator voltage.

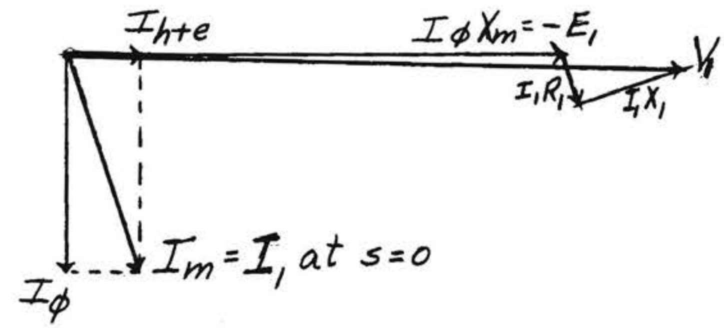


Fig. 5
The vector diagram of a polyphase induction motor at synchronous speed (s=0)

The magnetizing component of the stator current, I_ϕ, is very large compared to the iron-loss component, I_{h+e}, because of the high reluctance of the air gap. Therefore, the machine at synchronous speed has a very low, lagging power-factor. If the machine is allowed to develop power sufficient to overcome its own windage and friction losses, only a small in-phase component of current will be added to the input current. Then the induced stator voltage at no-load as a motor is

$$E_1 \approx V_1 - I_1X_1 \tag{25}$$

because the I_1R_1 drop is almost perpendicular to V_1 and has a negligible effect on the magnitude of E_1.

The rotor voltage, E_2, is proportional to the stator voltage, E_1, and therefore the magnitude of E_2 varies with the magnitude and phase angle of I_1. The stator current, I_1, varies with load and therefore the rotor voltage changes with load.

If all of the values shown on Fig. 3 are expressed in terms of the stator by means of the ratio of transformation, a, the circuit components have the values shown in Fig. 6.

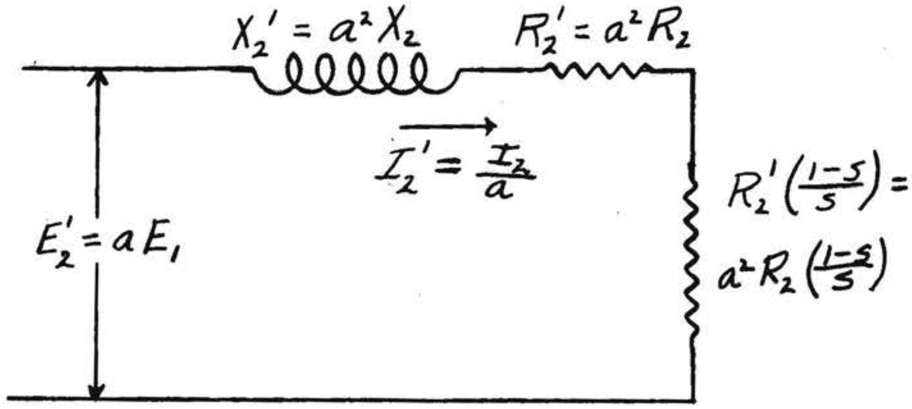


Fig. 6
Rotor circuit in terms of the stator

A combination of the circuits in Fig. 4 and Fig. 6 yields the exact equivalent circuit of the polyphase induction motor, Fig. 7.

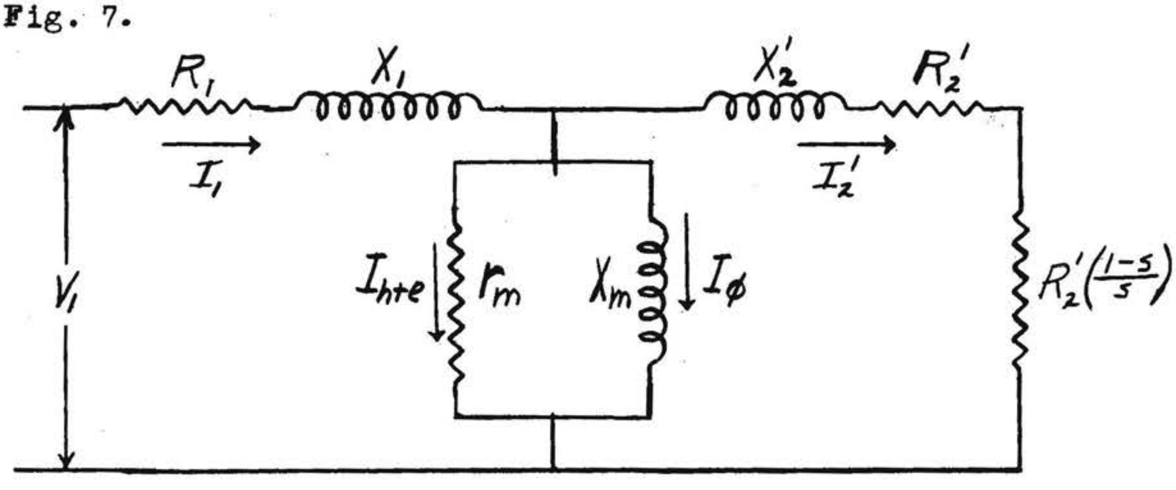


Fig. 7
Exact equivalent circuit of a polyphase induction motor

This equivalent circuit derived from the revolving-field theory shows lumped machine constants in a network similar to the equivalent circuit of a two-winding transformer with a pure resistance load. Fig. 8 shows a vector diagram representing motor operation with a normal load.

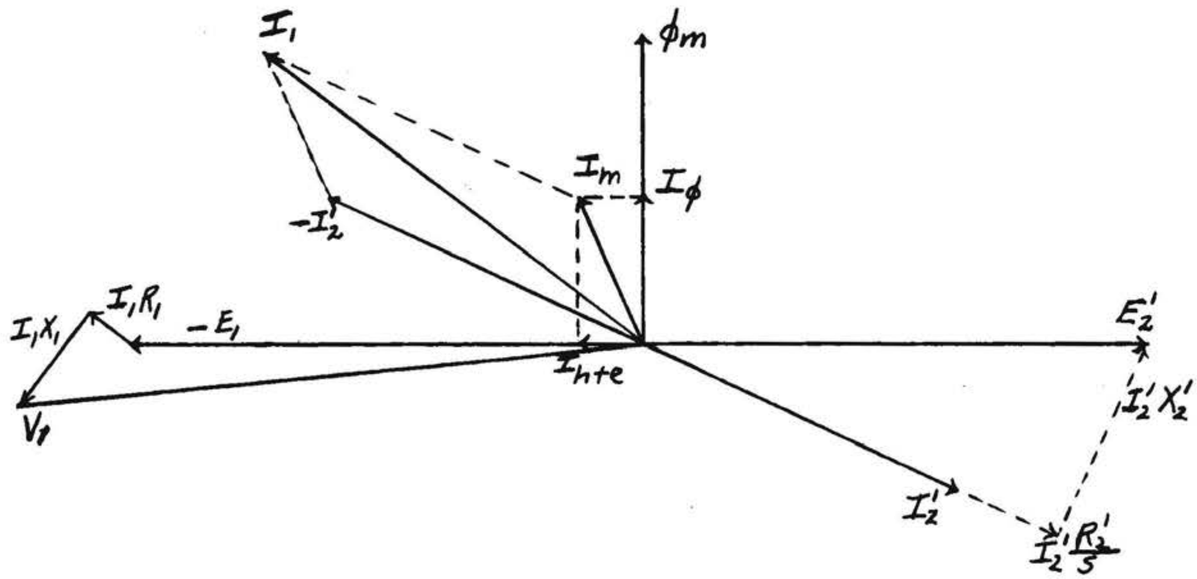


Fig. 8
Vector diagram of a polyphase induction motor with a normal load

The exact expression of developed torque in terms of applied voltage is somewhat unwieldy, but a very good approximation is as follows: [1]

$$T = \frac{7.04}{n_s} \quad \frac{V_1^2 R_2}{s(R_1 + \frac{R_2'}{s})^2 + s(X_1 + X_2')^2} \qquad (26)$$

1 A.F.Puchstein and T.C.Lloyd, Alternating-Current Machines, p. 251

The constants for a 3-hp, 220-volt, 3-phase, 60-cycle, 4-pole induction machine are as follows:

R_1 = 0.851 ohms per phase, X_1 = 1.1 ohms per phase

R_2' = 0.589 ohms per phase, X_2' = 1.1 ohms per phase

r_m = 374.0 ohms per phase, X_m = 37.3 ohms per phase

Core loss (P_{h+e}) = 42 watts per phase

Friction and windage loss(P_{f+w}) = 23 watts per phase

The above constants are used in the exact equivalent circuit of Fig. 7. Table I shows values of stator current, power factor, power input, power output, and torque for various assumed values of slip from plus two to minus two. The magnitude of the air-gap voltage ($E_1 = E_2'$) is also shown. Fig. 9 is a plot of torque, mechanical power, and air-gap voltage vs. slip.

The most interesting points shown by the curves of Fig. 9 are:

(1) Maximum torque input of the machine running as a generator is approximately 2.6 times the maximum torque output when operating as a motor.

(2) The air-gap voltage, ($E_1 = E_2'$), drops to approximately 50 per cent of the applied voltage at slip equal to one and remains practically constant at all higher values of slip.

TABLE I

Polyphase Induction Machine Characteristics
3-hp, 220-volt, 3-phase, 60-cycle, 4-pole

1	2	3	4	5	6	7	8	9
Speed (rpm)	Slip	Current	Power-factor angle	Power-factor (cos θ)	Elect. power	Mech. power	Torque lb.-ft.	Air-gap voltage
					input	input		
-1800	2.0	52.0	62°30'	.463	3050	-750	2.84	57.5
- 900	1.5	50.7	60°30'	.492	3150	-471	3.60	57.2
			Motor action		input	output		
0	1.0	48.2	57°	.545	3260	0	4.86	58.2
360	0.8	47.0	53°30'	.595	3530	321	6.18	61.0
900	0.5	42.6	47°20'	.678	3660	1015	8.16	68.3
1260	0.3	36.3	40°	.767	3540	1680	9.29	78.5
1440	0.2	29.6	33°20'	.837	3120	1890	9.13	89.0
1620	0.1	18.5	25°20'	.905	2115	1615	6.92	105.0
1710	0.05	10.7	25°	.906	1235	1055	4.25	114.0
1800	0.0	3.3	84°25'	.100	42	0	0.00	123.3
			Generator action		output	input		
1890	-0.05	11.6	150°	-.866	-1273	-1507	- 6.0	130.2
1980	-0.1	23.0	146°36'	-.836	-2440	-3190	-12.6	133.2
2160	-0.2	43.4	125°	-.574	-3160	-5410	-21.3	123.0
2340	-0.3	50.8	113°36'	-.402	-2600	-6370	-25.0	112.0
2700	-0.5	56.9	98°	-.104	-1010	-5700	-22.4	91.4
					input	input		
3240	-0.8	57.7	87°18'	.047	345	-4500	-17.7	76.2
3600	-1.0	58.0	82°	.139	1025	-3700	-14.6	70.3
4500	-1.5	56.8	77°30'	.216	1560	-2990	-11.7	64.3
5400	-2.0	56.6	75°	.259	1860	-2650	-10.4	62.4

1. Positive speed is in the direction of the field rotation.
2. Positive slip indicates rotor speed lower than synchronous speed. Negative slip indicates rotor speed greater than synchronous speed.
3. Effective (rms) current magnitude
4. Time angle between the current and the applied voltage
5. Cosine of the angle between the current and the applied voltage
6. Power in watts per phase. Positive power means that electric power is being supplied to the machine.
7. Positive power is motor action.
8. Mechanical torque is in the direction of rotation for motor action. It is opposite to shaft rotation for generator and brake action.
9. Magnitude of $E_1 = E_2'$

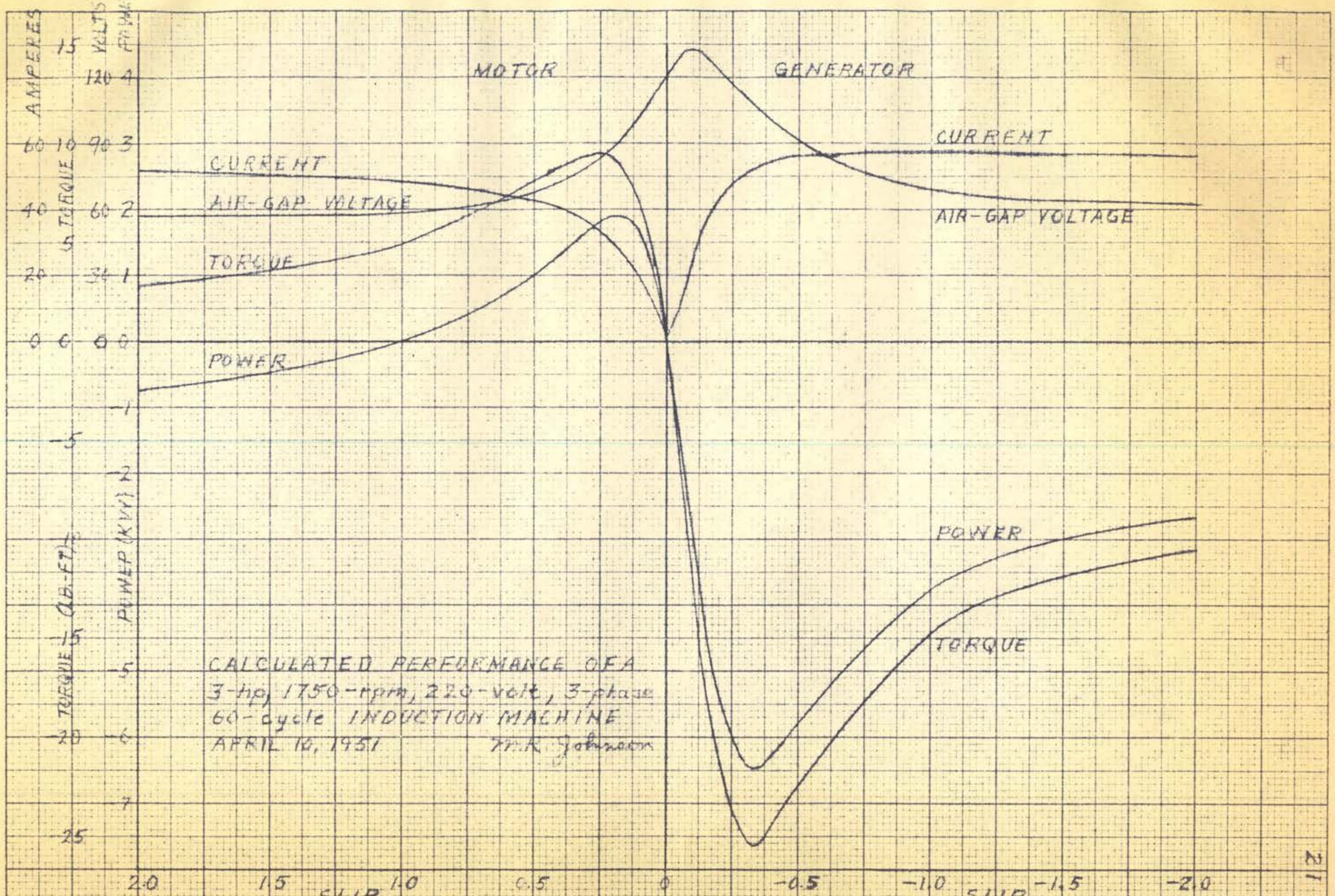

AMPERES
VOLTS
POWER
TORQUE
MOTOR
GENERATOR
CURRENT
AIR-GAP VOLTAGE
TORQUE
POWER
CURRENT
AIR-GAP VOLTAGE
POWER
TORQUE
TORQUE (LB-FT)
POWER (KW)
CALCULATED PERFORMANCE OF A
3-hp, 1750-rpm, 220-volt, 3-phase
60-cycle INDUCTION MACHINE
APRIL 10, 1951
M.K. Johnson
2.0
1.5
SLIP
1.0
0.5
0
-0.5
-1.0
SLIP
-1.5
-2.0

FIG. 9

One assumption, which adds some error but greatly simplifies the circuit calculations, allows the magnetizing branch to be placed across the terminals of the machine as shown in Fig. 10.

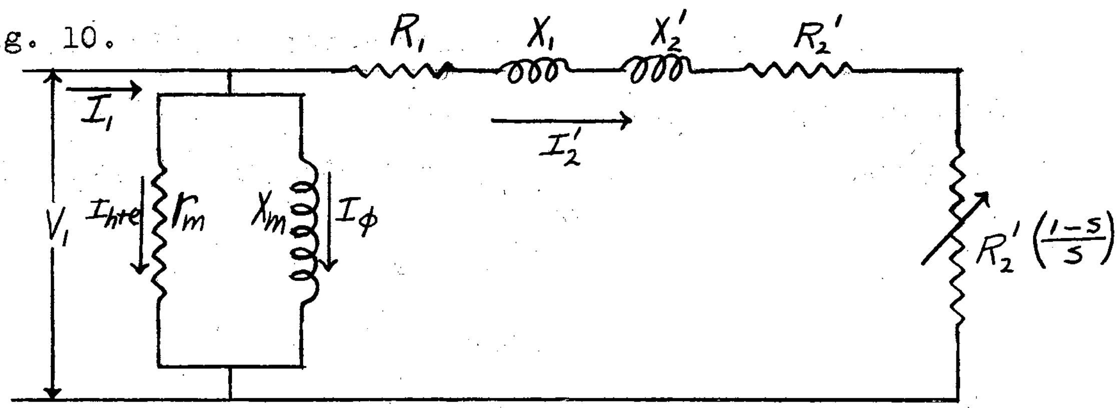


Fig. 10
Approximate equivalent circuit of a polyphase induction motor

If V_1 is constant, a vector diagram representing the sinusoidal currents and voltage can be drawn as in Fig. 11.

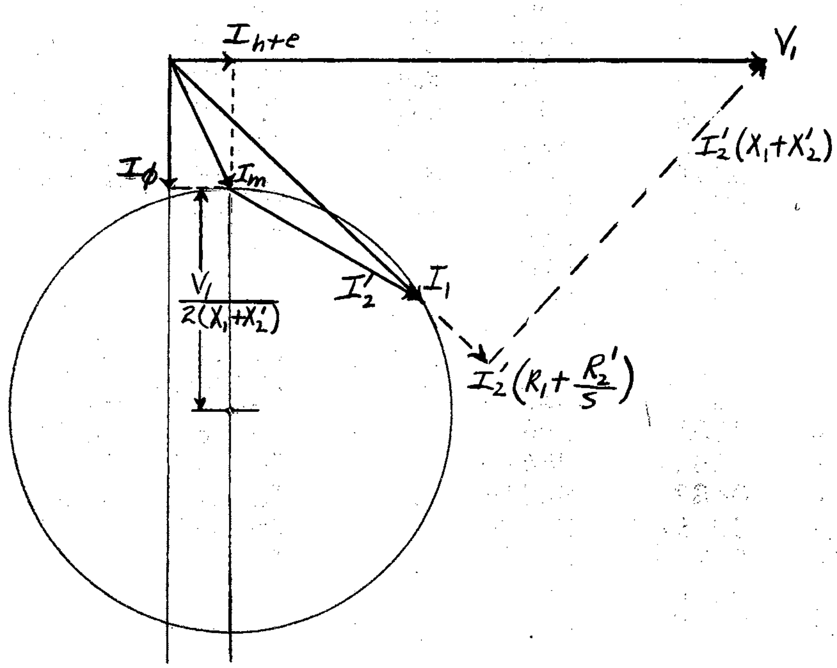


Fig. 11
Vector diagram representing the approximate equivalent circuit of the polyphase induction motor

The locus of the tip of the current vector as slip varies from $-\infty$ to $+\infty$ is again a circle. This fact is the basis for the familiar "circle diagram" which may be constructed from no-load and blocked rotor test data on a polyphase machine.

Table II shows the characteristics of a 5-hp, 1750 rpm, 220-volt, 3-phase, 60-cycle, NEMA Design A, Westinghouse Lifeline motor from dynamometer tests.

Table II

Dynamometer Tests on a 5-hp, 1750-rpm, 220/440-volt, 3-phase, 60-cycle, Westinghouse Lifeline Induction Motor, NEMA Design A

Speed	Slip	Current at 220 volts	Power-factor angle	Power-factor	Elect. power input	Mech. power output	Torque lb.-ft.
0	1.0	83.0	51.6°	.620	6533	0	9.5
1500	0.167	--	--	--	----	2820	13.2
1580	0.122	41.8	36.5°	.805	4267	2660	11.9
1650	0.083	33.2	32.4°	.846	3567	2460	10.5
1690	0.061	27.2	32.7°	.843	2907	2190	9.1
1720	0.044	21.6	28.8°	.879	2408	1880	7.7
1730	0.039	17.7	30.4°	.864	1942	1550	6.3
1750	0.028	14.1	32.8°	.842	1507	1220	4.9
1770	0.0167	11.0	37.4°	.797	1117	875	3.5
1780	0.0111	8.44	46.5°	.690	739	548	2.17
1792	0.0045	6.4	65.0°	.421	342	181	0.72
1802	0.0	5.92	90.0°	.000	0	- 63	- 0.24
					output	input	
1810	-0.0055	7.5	119.5°	-.492	- 467	- 530	- 2.06
1820	-0.0111	9.1	128.5°	-.622	- 720	- 829	- 3.22
1830	-0.0167	10.8	135.0°	-.708	- 973	-1092	- 4.2
1850	-0.028	14.9	140.8°	-.775	-1467	-1660	- 6.3
1860	-0.033	18.7	142.8°	-.797	-1902	-2230	- 8.4
1885	-0.047	24.2	143.0°	-.799	-2458	-2990	-11.2
1900	-0.055	30.0	140.4°	-.770	-2933	-3730	-13.8
1930	-0.072	37.0	139.4°	-.758	-3560	-4810	-17.5
1950	-0.083	40.5	136.9°	-.730	-3743	-5350	-19.3

Fig. 12 is a circle diagram of the same machine obtained from no-load and blocked rotor tests. Data from those tests are as follows:

	I_1 per phase	Power factor angle	Power-factor	Voltage per phase	Power per phase
No-load	6.0	82.5°	.131	127	100
Blocked rotor	83.0	51.8°	.618	127	6533

$R_1 = .46$ ohms per phase at 25 c.

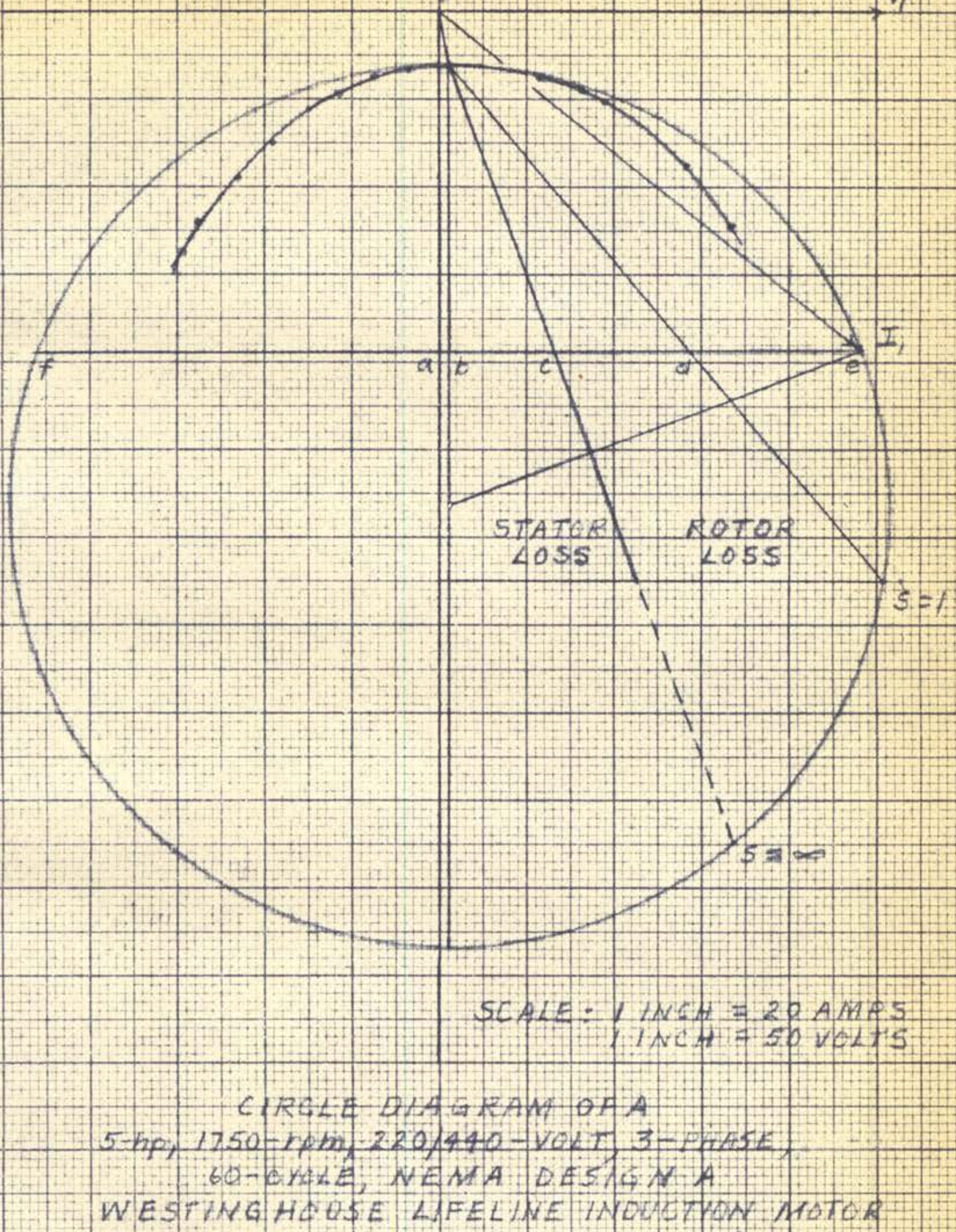
$R_2' = .4$ ohms per phase at 25 c.

$X_1 = X_2' = .7$ ohms per phase at 60 cps.

A plot of the locus of the tip of the current vector obtained from the dynamometer tests is also shown.

Table III
Operating characteristics which may be obtained from the approximate circle diagram Fig. 12

	Motor	Generator
Current	oe	of
Power factor	oa/oe	oa/of
Power input	ae	df
Power output	de	af
Efficiency	de/ae	af/df
Torque	ce	cf
Slip	cd/ce	cd/df
Stator copper loss	bc	bc
Rotor copper loss	cd	cd
Constant losses (P_{w+f} and P_{h+e})	ab	ab



SCALE: 1 INCH = 20 AMPS
1 INCH = 50 VOLTS

CIRCLE DIAGRAM OF A
5-hp, 1750-rpm, 220/440-VOLT, 3-PHASE,
60-CYCLE, NEMA DESIGN A
WESTINGHOUSE LIFELINE INDUCTION MOTOR
APRIL 10, 1951 M.R. Johnson

FIG. 12

Conventional Approach to Single-phase Induction Motor Performance

The stator of a single-phase motor, having only one winding, is capable of producing a magnetomotive force in only one space axis of the air gap. With a sinusoidal distribution of conductors in the slots of the stator, the practical machine can be made to have very nearly sinusoidal distribution of mmf and flux in its air gap. Assume the single-phase stator flux to be sinusoidally distributed in space, and varying sinusoidally in time. Then if 2τ is the wave length of the space distribution, the flux may be represented by [2]

$$\phi = \phi_{max} \sin \frac{\pi x}{\tau} \cos \omega t \qquad (27)$$

$$= \frac{\phi_{max}}{2} \left[\sin\left(\frac{\pi x}{\tau} - \omega t\right) + \sin\left(\frac{\pi x}{\tau} + \omega t\right) \right] \qquad (28)$$

where

τ = stator pole pitch

ϕ = instantaneous value of the air-gap flux

ϕ_{max} = maximum value of the air-gap flux

π = 3.1416

x = distance around the air gap from reference point (variable)

ω = angular velocity $(2\pi f)$

t = time (variable)

---

2. L.V. Bewley, Alternating Current Machinery, pp.195-196

Equation (28) says that the instantaneous value of the field along the axis of the stator winding is made up of the sum of two equal and constant quantities revolving in opposite directions at the same angular velocity. The constant magnitude of the revolving quantities is one-half of the maximum magnitude of the original alternating field. A vector diagram of this concept is shown in Fig. 13.

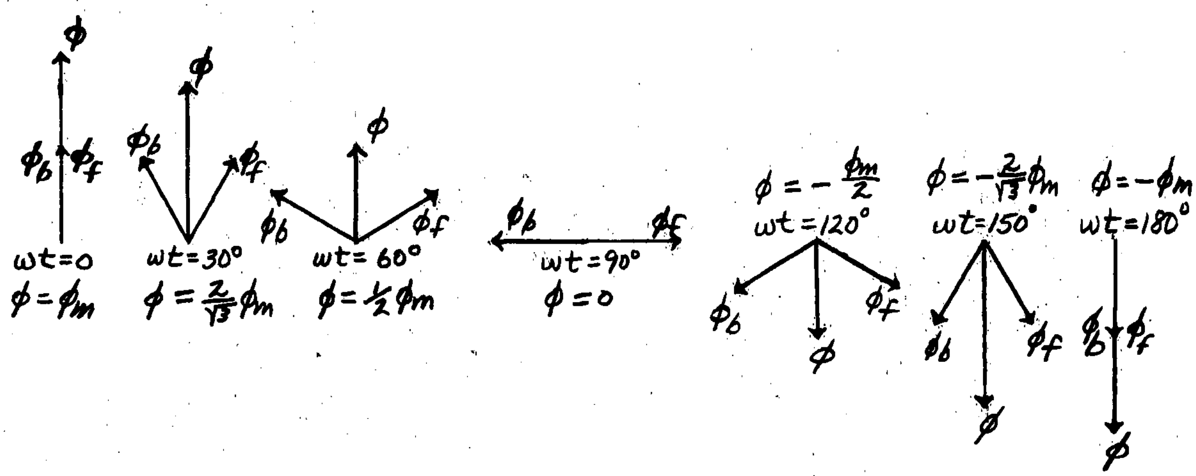


Fig. 13
Vector diagrams representing a stationary alternating field by means of two equal fields revolving in opposite directions

The effects of each of the revolving fields may be considered separately, using the same approach as was developed for the polyphase machine. This method of analysis of the single-phase machine is called the double-revolving-field method. Another prominent theory which explains single-phase induction motor operation is called the cross-field theory. Other methods of explanation are possible, but none of them have received full mathematical treatment.

When the rotor of the single-phase machine is at standstill, the voltage induced in the rotor winding is due only to single-phase transformer action. If the winding is short-circuited, rotor current will flow. The rotor mmf will be in the same space axis as the stator mmf and will oppose it. Analyzing this condition by the double-revolving-field method yields a solution of equal and opposite torques due to the two fields. No resultant torque is produced at standstill.

As the rotor speed increases, the value of torque due to the forward field increases, while that due to the backward field decreases. The average resultant torque is in the direction of rotation, tending to accelerate the machine. This condition may be represented by the action of two magnetic fields of different magnitudes revolving in opposite direction at the same angular velocity. Fig. 14 shows a series of vector diagrams representing the effect at approximately one-half synchronous speed.

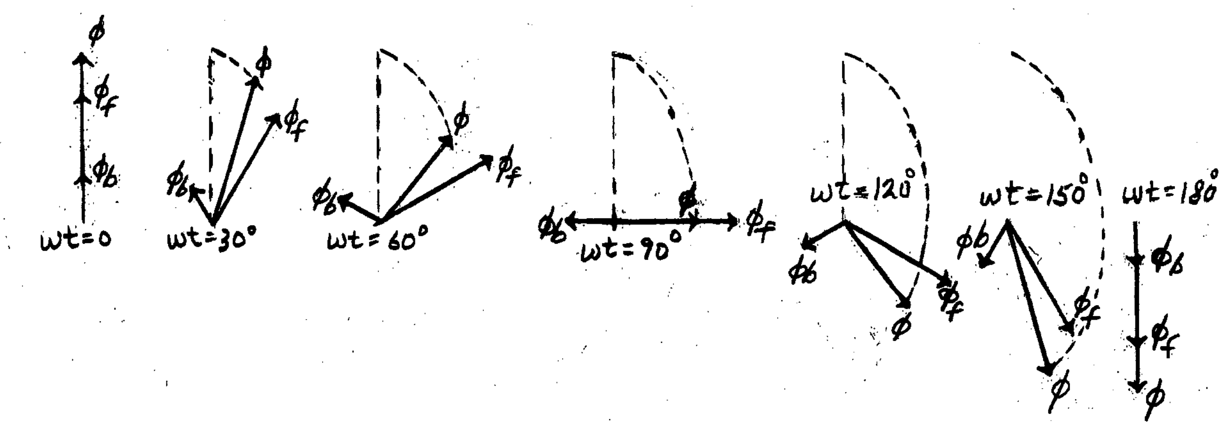


Fig. 14
Vector diagrams representing a revolving "elliptical" field by means of two unequal fields revolving in opposite directions

Note that the locus of the tip of the resultant flux vector is an ellipse. This type of rotating magnetic field is often referred to as an "elliptical" field.[3] The average speed is synchronous speed, but the instantaneous angular velocity changes. The magnitude of the field also changes at a frequency which is twice the frequency of the stator voltage. Thus the rotor voltage and current are made up of components due to both the forward and backward fields. The current due to the forward field has a frequency of sf_1 , while the current due to the backward field has a frequency of $(2-s)f_1$. Although the average resultant torque is in the direction of rotation, it pulsates at a frequency $(2-s)f_1$. Under certain operating conditions, its instantaneous value may even be in the direction opposite to rotation.

When the rotor of the single-phase induction machine is near synchronous speed, the effect of the backward revolving field upon the air-gap flux is greatly reduced. The resultant revolving field, due to the combined effect of stator and moving rotor, is almost "circular".

---

3 A stator which has two windings displaced 90° in space, supplied with two equal currents displaced 90° in time, produces a "circular"field. If any one of these conditions is not satisfied, an elliptical field results. Practically, all single-phase machines rely on the action of an elliptical field during the starting cycle.

The equivalent circuit of the single-phase motor which evolves from the double-revolving-field analysis is shown in Fig. 15.

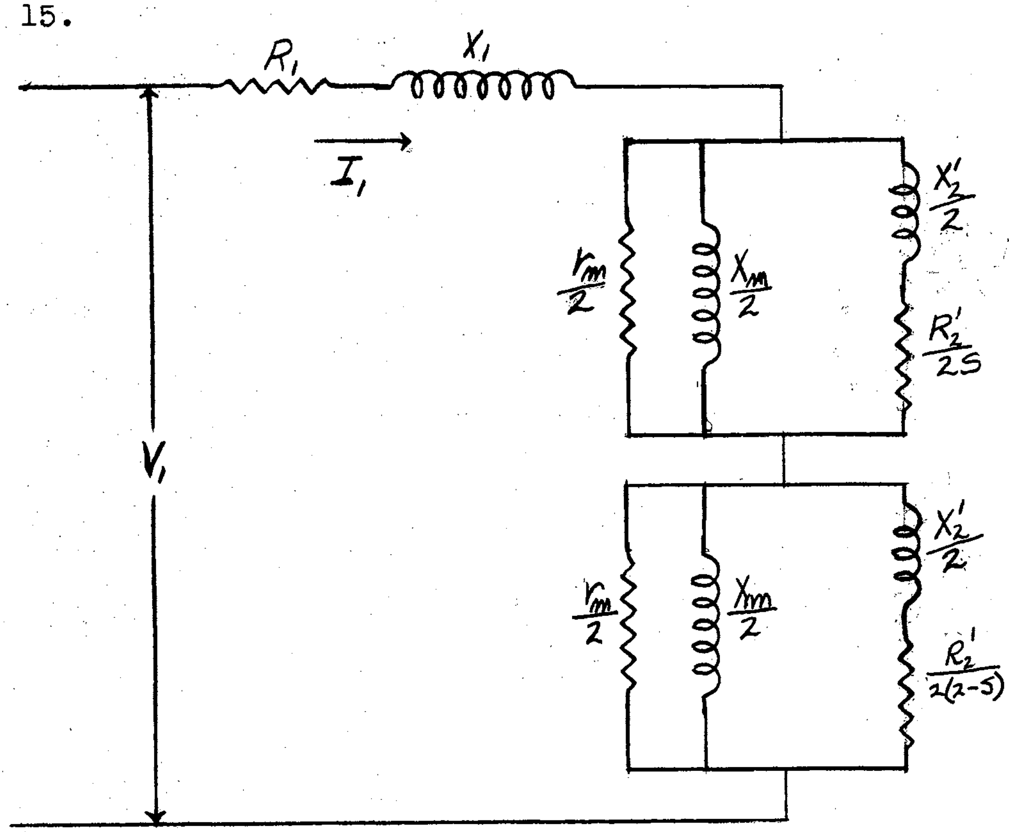


Fig. 15
Exact equivalent circuit of the single-phase induction machine

A comparison between Fig. 7 and Fig. 15 shows the terms that must be added to the polyphase circuit to represent the effects of the backward revolving field for single-phase operation.

Note that only one-half the value of rotor impedance components is assigned to each of the fictitious rotors. The no-load or magnetizing current is therfore approximately twice the value required for polyphase operation with equivalent machine constants.

The constants for a ¼-hp, 110-volt, 60-cycle, single-phase, 4-pole, induction machine are as follows:

R_1	$=$ 1.86	ohms,	X_1	$=$ 2.76	ohms
$R_2'/2$	$=$ 1.78	ohms.	$X_2'/2$	$=$ 1.28	ohms
r_m	assumed infinite,		$X_m/2$	$=$ 26.8	ohms
P_{h+e}	neglected,		P_{f+w}	13.5	watts

The above constants are used in the equivalent circuit of Fig. 15. Table IV shows values of stator current, power-factor, power input, power output, torque, "forward" rotor voltage, and "backward" rotor voltage for various assumed values of speed from standstill to three times synchronous speed. Since the forward field for one rotation becomes the backward field for opposite rotation, the curves are symmetrical about the axis of zero speed. It is sufficient to complete calculations for positive slips from zero to one, and for negative slips from zero to minus two. Fig. 16 is a plot of torque, mechanical power, and rotor voltages versus slip.

The most interesting points shown by the curves of Fig. 16 are:

(1) Maximum torque of the machine running as a generator is approximately three times the maximum torque when operating as a motor.

(2) The voltage across both the forward and backward rotor circuits drops to approximately 30 per cent of the applied voltage at $s = 1$.

(3) At synchronous speed, the voltage on the forward rotor is approximately 85 per cent of the applied voltage

while the voltage on the backward rotor is only five per cent of the applied voltage.

(4) When the slip is approximately 0.15 negative, the voltage on the forward rotor reaches its maximum value of 95 per cent of the applied voltage. As slip increases negatively, this voltage again decreases, approaching 30 per cent of the applied voltage in the limit.

(5) When the negative slip reaches 0.5, the voltage on the backward rotor increases from its minimum value to approximately 30 per cent of the applied voltage. It remains practically constant at this value as the slip becomes increasingly negative.

The complete equation of the current of a single-phase induction machine is quite cumbersome, but with algebraic manipulation it can be reduced to a form which may be recognized as a circle.[4] Fig. 17 shows the locus of the tip of the current vector for the machine mentioned above. These values were obtained by the step-by-step method, assuming various values of slip.

---

4 A. S. Langsdorf, Theory of Alternating Current Machinery, pp. 668-671, and 678-680.

Table IV

Single-phase induction machine characteristics
$\frac{1}{4}$-hp, 110-volt, 60-cycle, 4-pole, single-phase

Speed	Slip	Current	Power factor angle	Power factor (cos θ)	Elect. power	Mech. power	Torque lb.-ft.	Forward rotor voltage	Backward rotor voltage
					input	output	output		
0	1.0	15.3	45.5°	.700	1178	0	0	32.2	32.2
360	0.8	14.8	45.0°	.707	1150	25	0.49	36.2	28.1
900	0.5	13.4	41.6°	.747	1102	180	1.41	47.5	22.2
1260	0.3	11.0	37.0°	.800	968	362	2.02	62.4	17.4
1440	0.2	8.7	36.0°	.810	779	385	1.88	71.5	13.3
1620	0.1	5.8	41.0°	.755	482	316	1.37	84.0	8.7
1710	0.05	4.4	53.8°	.591	287	190	0.78	90.0	6.6
1800	0.0	3.6	87.0°	.053	24	- 24	-0.094	96.0	5.3
					output	input	input		
1890	-0.05	5.0	115.0°	-.423	- 233	- 336	-1.25	101.0	7.3
1980	-0.1	7.2	126.0°	-.588	- 465	- 730	-2.59	105.0	10.7
2160	-0.2	12.5	123.0°	-.531	- 731	-1548	-5.03	102.5	18.1
2340	-0.3	16.6	113.1°	-.392	- 716	-2180	-6.55	94.0	23.8
2700	-0.5	20.2	97.0°	-.122	- 268	-2310	-6.05	71.6	29.0
					input				
3240	-0.8	21.1	85.0°	.087	202	-2110	-4.60	51.8	29.0
3600	-1.0	21.3	81.8°	.145	340	-1990	-3.89	45.0	28.6
4500	-1.5	21.0	76.2°	.239	552	-1750	-2.73	35.0	27.5
5400	-2.0	20.9	73.8°	.280	645	-1630	-2.13	31.2	27.2

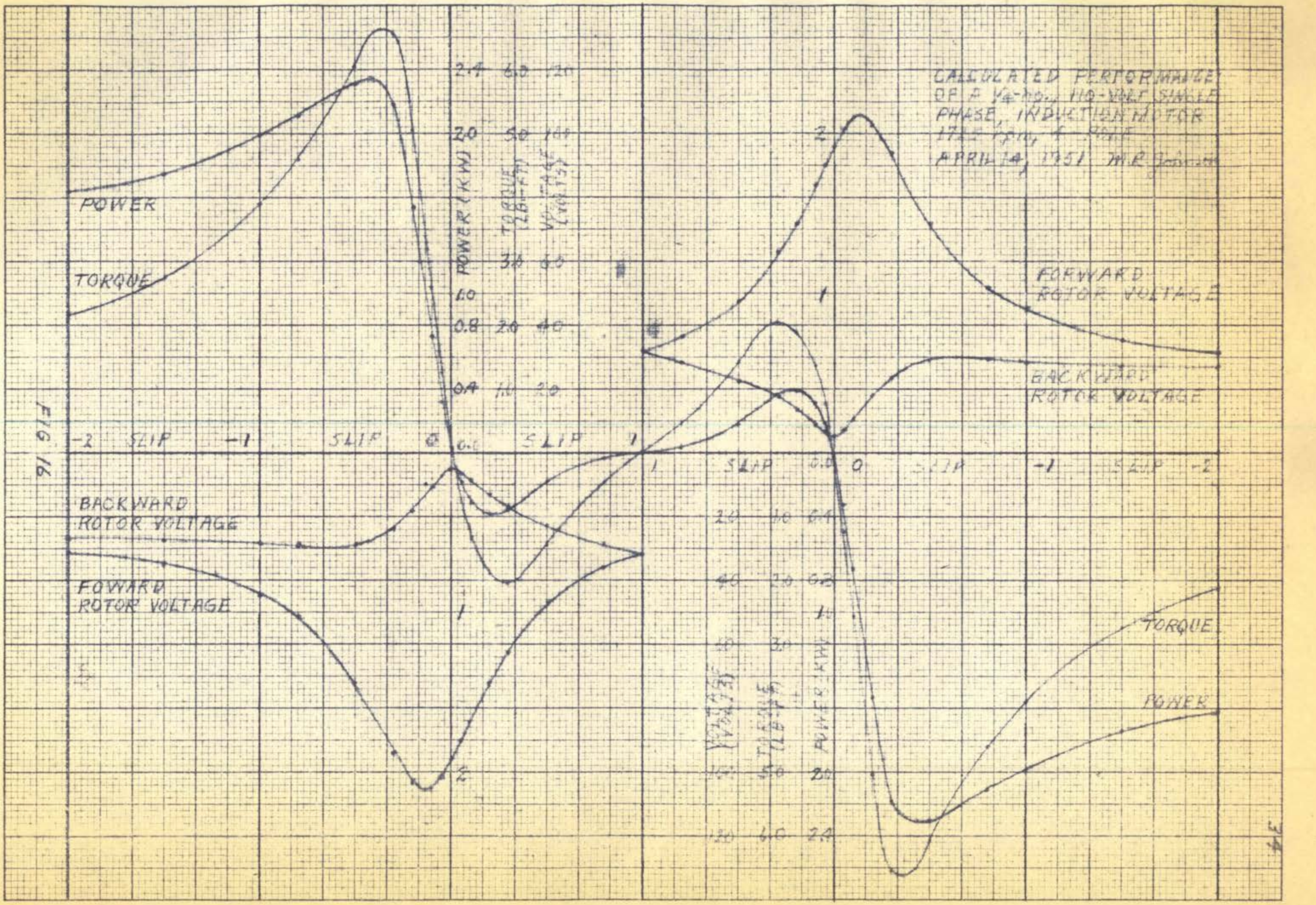

CALCULATED PERFORMANCE
OF A 1/4-hp., 110-VOLT SINGLE
PHASE, INDUCTION MOTOR
APRIL 14, 1951 M.R.
POWER
TORQUE
POWER (KW)
TORQUE (LB-FT)
VOLTAGE (VOLTS)
SLIP
FORWARD ROTOR VOLTAGE
BACKWARD ROTOR VOLTAGE
FOWARD ROTOR VOLTAGE

FIG 16

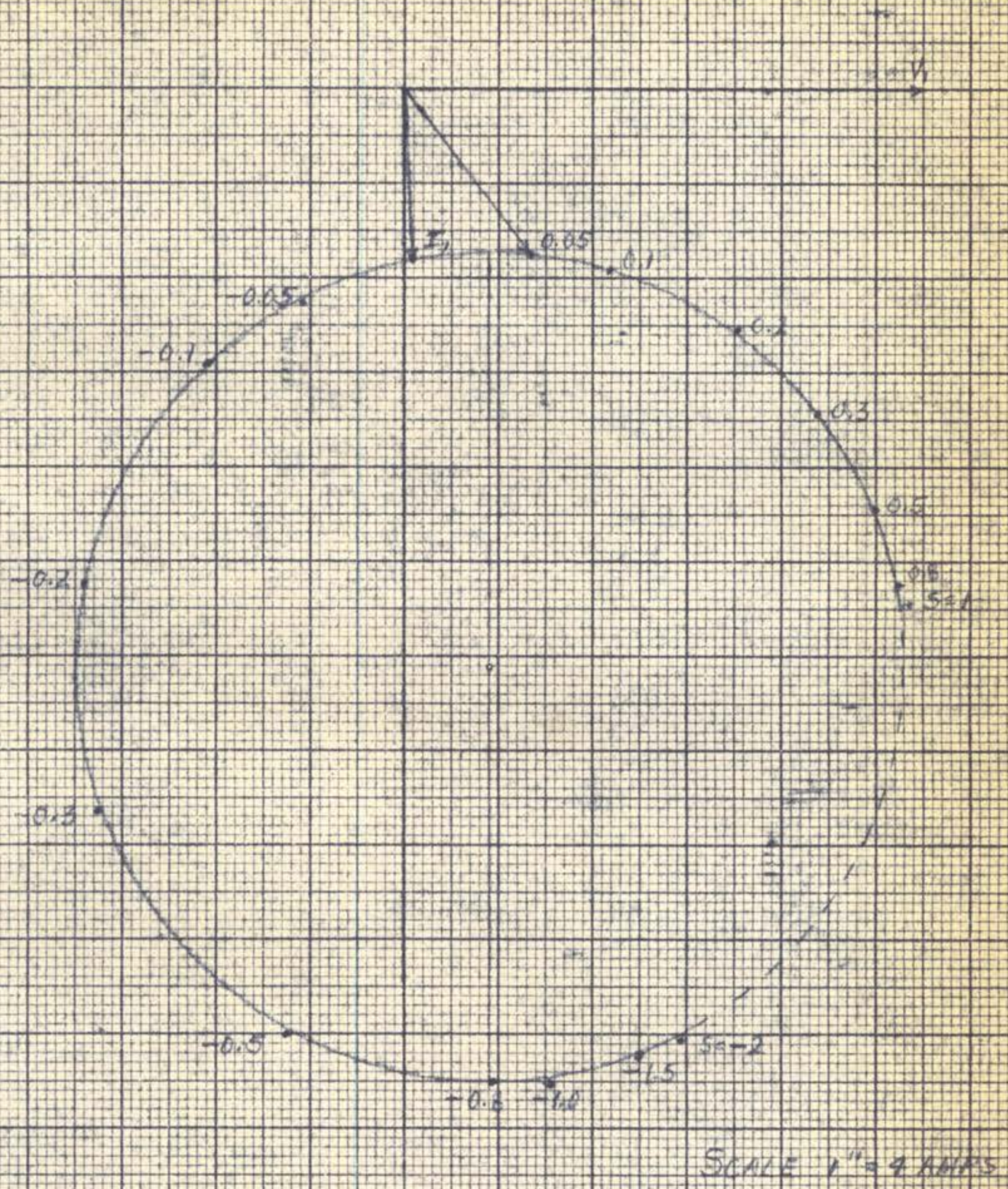


Fig 17

Locus of the current of a 1/4-hp., 1725-rpm, 110-volt, 60-cycle, single-phase induction machine (calculated), (Table IV)

Chapter II

Self-excited Polyphase Induction Machines

It has long been known that an isolated polyphase machine can operate as a self-excited generator under certain conditions. Mr. C. F. Wagner explains why there has been a lack of interest in the characteristics of such operation until fairly recently.

> "The reason for this paucity of papers and articles lies in the relative minor practical importance of the subject. However, the increased use during the past few years, of capacitors for power-factor correction, has placed a new aspect upon this problem. If the power supply to an induction motor is disconnected, the inertia of the connected rotating load tends to continue the rotation of the armature. The extent to which this occurs is dependent upon the nature of the load and in certain cases the armature may continue to rotate for seconds or minutes. In addition, applications are known in which gas or gasoline motors are connected to the same shaft with the induction motor and the utilization device, so that, in the event of the removal of the electric-power source, the armature can actually increase in speed and remain at the increased speed until manual readjustments are made. With capacitors connected across the terminals of induction machines which have been disconnected from the electrical source and in which the armature continues to rotate, the value to which the terminal voltage will rise due to self-excitation is dependent upon the speed, value of the capacitor, and load. With the regulatory function of the power source removed the terminal voltage may rise to dangerously high voltages - dangerous with regard to human life or dangerous with regard to insulation breakdown. Parallel-connected lights might also burn out with but a nominal increase in voltage. It may be seen, therefore, that this problem has been removed from one of purely academic interest to one of practical importance." [1]

---

1 C. F. Wagner, "Self Excitation of Induction Motors", A.I.E.E. Transactions, Vol. 58 (February, 1939), pp. 47-51.

The problem of dynamic braking of polyphase machines by self-excitation was analyzed by Srinivasan and Thomas in 1947.[2]

Since the theory of generation in polyphase induction machines presented in the above references can be used as a basis for analysis of generation in single-phase machines, the pertinent information recorded in those papers will be reviewed.

The conventional equivalent circuit of the polyphase induction motor as shown in Fig. 7 is used as a basis for the analysis. The copper loss in the rotor circuit is neglected. The impedance of the magnetizing branch is assumed to vary in accordance with the saturation curve of the machine.

> "The particular volt-ampere characteristic for any motor will depend upon the saturation of the iron of that machine. The wave form of the voltage is nearly sinusoidal and little error is introduced by such an assumption. The frequency is assumed to be that determined by speed alone without regard to slip. This will introduce considerable error at low voltages where the generator action is disappearing, but does permit a simple graphical solution for the operating conditions. A more exact method based on a cut and try solution is available if thought necessary. . . The very considerable tolerance in the capacitance of capacitor units makes a simple solution sufficiently accurate for the purpose of specifying the capacitors for a given application."[2]

---

2 A. Srinivasan and M. A. Thomas, "Dynamic Breaking by Self-Excitation of Squirrel-Cage Motors", A.I.E.E. Transactions, Vol. 66 (1947), pp. 145-148.

With the above assumptions, the equivalent circuit reduces to the approximate form shown in Fig. 18.

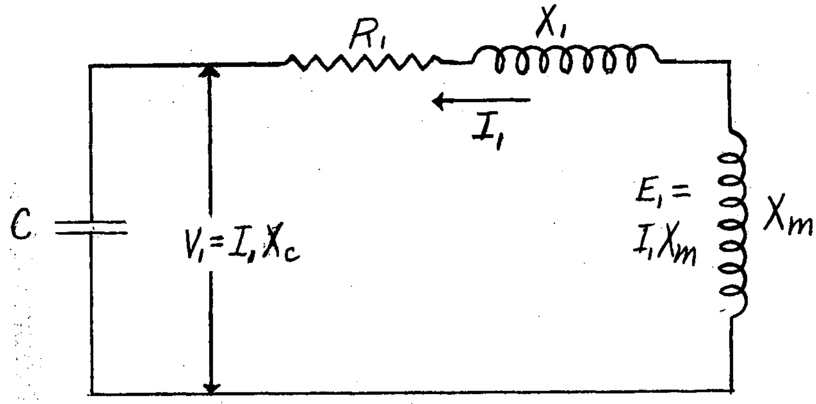


Fig. 18
Approximate equivalent circuit for the determination of operating conditions

Assuming a perfect capacitor, the current I_c will lead the voltage by 90 degrees as shown in the vector diagram of Fig. 19. Then the generated voltage E_1 may be expressed as:

$$E_1 = V_1 + jI_1 (R_1 + jX_1) \tag{29}$$

or approximately

$$E_1 \approx V_1 - I_1 X_1 \tag{30}$$

because the $I_1 R_1$ drop is perpendicular to V_1 and has a negligible effect upon the magnitude of E_1.

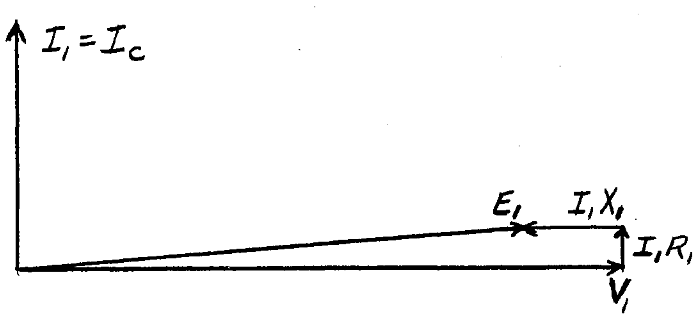


Fig. 19
Vector diagram of a capacitor-excited induction generator at no-load

In general, the induced voltage in the armature winding of a rotating machine is a function of flux and rotor speed.

$$E_1 \propto \phi_1 n \tag{31}$$

or

$$\phi \propto \frac{E_1}{n} \tag{32}$$

The flux is a function of the magnetizing current, and therefore

$$I_\phi \propto \frac{E_1}{n} \tag{33}$$

If the values of the effective resistance, R_1, and the leakage reactance, X_1, are known, the induced (air-gap) voltage, E_1, may be determined from readings of input voltage, current, and power when the machine is running at no-load as a motor. A plot of current versus induced voltage yields a saturation curve for the speed at which the test is taken.

The impedance of the capacitor is

$$X_c = \frac{1}{2\pi f_1 C} \tag{34}$$

where

X_c = ohms capacitive reactance of capacitor

C = capacitance of capacitor in farads

Then

$$I_c = V_1 (2\pi f_1 C) \tag{35}$$

where

I_c = rms amperes through capacitor

If the effect of slip upon the frequency of the generated voltage is neglected, then

$$f_1 = \frac{p_1 n}{120} \tag{36}$$

Substituting equation (36) into equation (35)

$$I_c = \frac{2\pi p_1 n C}{120} \cdot V_1 \tag{37}$$

Equation (37) can be made a function of V_1/n by multiplying numerator and denominator by n.

$$I_c = \frac{2\pi p_1 n^2 C}{120} \cdot \frac{V_1}{n} \tag{38}$$

Note that both equations (25) and (30) give the stator induced voltage and that

$$V_1 \approx E_1 + I_1 X_1 \tag{39}$$

in both no-load motor and no-load generator operation. It is therefore possible to use the plot of terminal voltage versus current obtained from no-load tests as a generator or a motor to obtain the saturation curve which is the basis for predicting generator characteristics. For greater utility the saturation curve should be plotted as a function of V_1/n.

$$I_\phi \propto \frac{V_1}{n} \tag{40}$$

If p_1, n, and C are constant, then equation (38) is a straight line when plotted on the no-load saturation curve.

The intersection of the straight line and the saturation curve yields a graphical solution to the simultaneous equations

(38) and (40). This solution is the steady-state operating condition of current and voltage for the given speed and capacitance.

Table V is the results of a no-load, running-free test on the 5-hp, 1750-rpm, 220-volt, 3-phase, 60-cycle, Westinghouse Lifeline motor to which previous reference has been made. Fig. 20 is a saturation curve for the same machine. Several straight lines, representing the solution of equation (38) for different values of capacitance are also shown. The particular values which represent rated voltage conditions at rated speed should be noted.

Also shown on Table V and Fig. 20 are results of a test on the machine when driven at 1800 rpm and excited with capacitors. The saturation curve for the generator follows the curve obtained from the motor no-load test very closely.

Equation (38) may be written as follows

$$\frac{V_1}{n} = \frac{120}{2\pi p_1} \cdot \frac{1}{n^2 C} \cdot I_c \qquad (41)$$

Differentiating equation (41) with respect to I_c yields the slope of the line.

$$\frac{d\left(\frac{V_1}{n}\right)}{dI_c} = \frac{120}{2\pi p_1} \cdot \frac{1}{n^2 C} \qquad (42)$$

The slope of the line varys inversely as the capacitance and inversely as the square of the rotor speed.

Table V

No-load saturation data on a 5-hp, 1750-rpm, 220-volt, 3-phase, 60-cycle, Westinghouse Lifeline Motor

Motor (no-load)			Generator (1800 rpm)			
V_1 per phase (volts)	I_1 per phase (amps)	V_1 per rpm (volts)	C per phase (microfarads)	V_1 per phase (volts)	I_1 per phase (amps)	V_1 per rpm (volts)
46.2	1.8	0.0256	93	71.6	2.7	0.0398
53.3	2.1	0.0296	99	102.5	4.2	0.0570
57.8	2.3	0.0321	114	123.0	5.6	0.0684
63.1	2.5	0.0351	129	134.0	6.9	0.0745
69.6	2.7	0.0387	135	139.0	7.7	0.0772
75.1	2.9	0.0418	153	145.0	8.9	0.0806
80.1	3.2	0.0445	159	147.0	9.4	0.0816
87.0	3.4	0.0484				
104.0	4.2	0.0579				
115.0	4.8	0.0639				
123.1	5.6	0.0685				
127.0	6.0	0.0707				
133.5	6.8	0.0741				
139.8	7.6	0.0776				
145.6	8.8	0.0809				

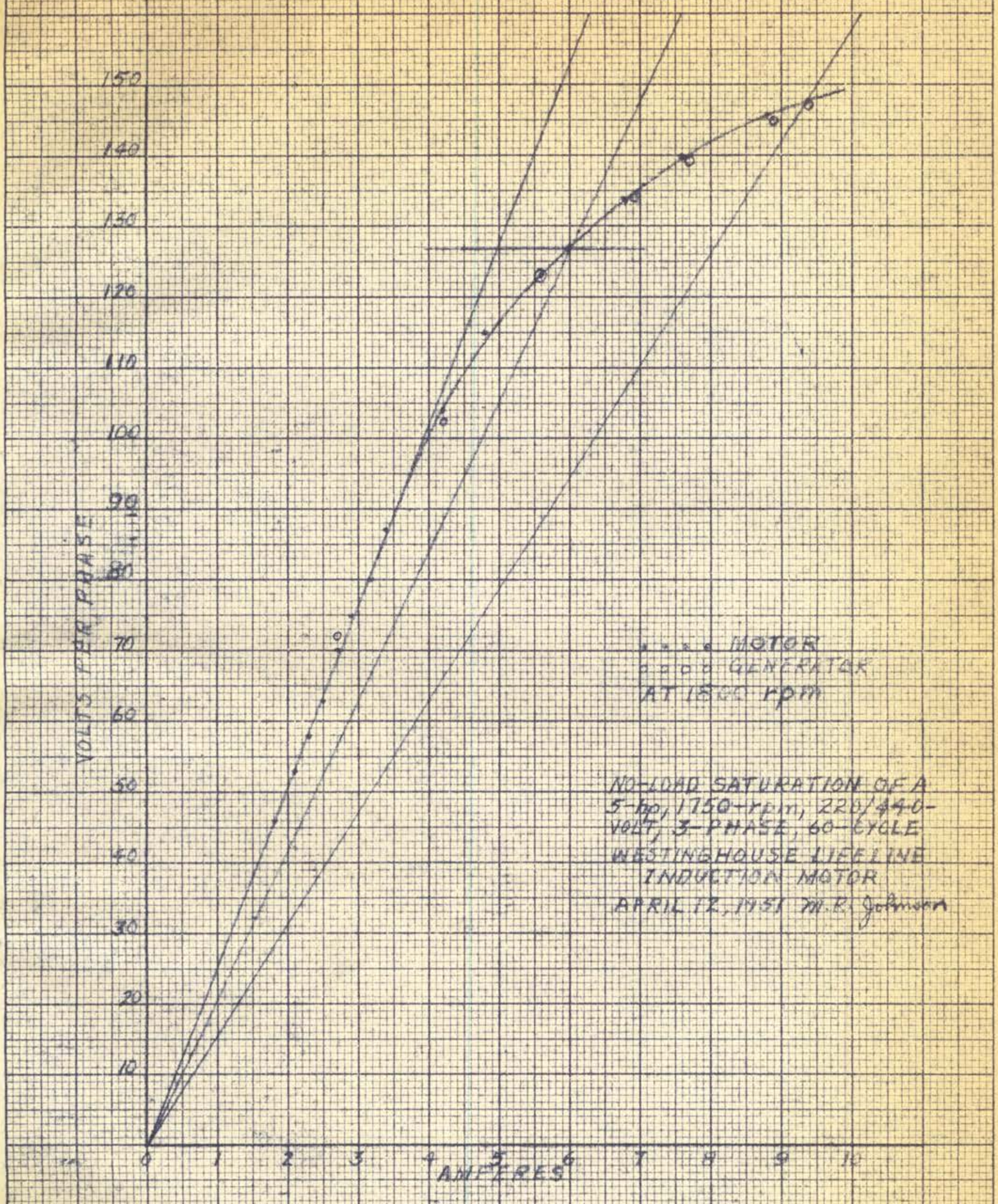

VOLTS PER PHASE
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
AMPERES
• • • • MOTOR
○ ○ ○ ○ GENERATOR
AT 1800 rpm
NO-LOAD SATURATION OF A 5-hp, 1750-rpm, 220/440-VOLT, 3-PHASE, 60-CYCLE WESTINGHOUSE LIFELINE INDUCTION MOTOR
APRIL 12, 1951 M.R. Johnson

FIG. 20

The power delivered to the machine during regeneration must be sufficient to supply friction and windage loss, iron loss, stator winding $I_1^2 R_1$ loss, and rotor winding $I_2^2 R_2$ loss. Friction and windage loss for rated speed may be obtained from the standard no-load saturation test. The equivalent retarding torque may then be calculated. To obtain the values of frictional torques at other speeds, a dynamometer test may be advisable.

Iron loss as a function of voltage at rated frequency is also obtained from the no-load saturation test. However, in applications of regeneration involving braking action, or deceleration, the frequency and flux density of the magnetic field may vary through wide ranges. Iron loss is made up of hysteresis loss and eddy current loss, each of which varies with frequency and flux density as shown in equation (43).

$$P_{h+e} = P_h + P_e$$

$$P_h = f_1 B^{1.6} \; ; \; P_e = f_1^2 B^2 \tag{43}$$

No-load saturation tests at several different frequencies may be necessary to obtain the effect of the individual components. From a family of curves, the total loss may be obtained at various frequencies and flux densities. The total iron loss may become an appreciable part of the total loss if the flux density rises to a high value. It will be shown that high voltage and flux density are not uncommon in some applications of self-excited generation.

The stator $I_1^2 R_1$ loss is readily calculated from the current obtained in the graphical solution of equations (38) and (40) for the particular speed and capacity being considered.

Rotor $I_2^2 R_2$ losses are neglected since slip is assumed negligible throughout this method of analysis.

For steady-state operation the summation of the retarding torque caused by the above losses must be overcome by some continuous driving source. If no source is present, the torque will decelerate the revolving mass. Problems involving deceleration time are solved using the rotational equations from mechanics.

$$T = \mathcal{I} \alpha \qquad (44)$$

where

$\mathcal{I}$ = moment of inertia

α = rotational acceleration

When α is in terms of speed and time, equation (44) may be written as follows:

$$T = \frac{\mathcal{I}}{K} \frac{dn}{dt} \qquad (45)$$

The above differential equation may be solved by separating the variables and integrating.

$$t = \frac{\mathcal{I}}{K} \int_{n'}^{n''} \frac{1}{T} \, dn \qquad (46)$$

A graphical integration process may be necessary when $\frac{1}{T}$ is not readily expressed in simple mathematical terms as a function of n.

Chapter III

Self-excitation of Single-phase Induction Machines

In the analysis of generator action of a single-phase induction machine with capacitor excitation, the following assumptions are made.

1. Voltages and currents vary sinusoidally with time.

   It has been shown that capacitor-excited machines develop a voltage which is very nearly sinusoidal with respect to time.[1] Unless extreme saturation is present, the current also approximates a sine wave.

2. All machine impedances, except the magnetizing reactance, are lumped, linear constants.

   Even though machine impedances actually vary with different operating conditions, any attempt to introduce the effects of such variation would so complicate the problem as to make its solution practically impossible. In most conventional analyses, this assumption is made.

3. The magnetizing reactance, X_m, varies in accordance with the saturation curve of the machine.

   It is not difficult to obtain the value of X_m as a function of voltage if the no-load saturation curve (motor or generator) is available. When it is necessary to rely upon design data for the value of X_m, not only must the

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1 E. D. Bassett and F. M. Potter, "Capacitive Excitation for Induction Generators", Electrical Engineering, Vol. 54 (May, 1935), pp. 540-545.

B-H curve for the iron be known, but also the dimensions of the magnetic circuit must be available. Since stable operation with self-excitation depends upon saturation of the iron circuit, it is imperative that the effects of saturation upon the value of magnetizing reactance be known.

4. The effect of slip upon frequency is negligible.

A study of the curves of slip versus torque near synchronous speed in Fig. 16 shows that slip varies much less for generator operation than for motor operation. In machines with relatively low rotor resistance, the error introduced by neglecting the effect of slip upon frequency is small.

5. The excitation capacitor has no power loss.

The capacitors used for power-factor correction and other continuous-duty operations are designed for extremely low loss, and no noticiable error is introduced by assuming a perfect capacitor. However, the capacitors in starting circuits of single-phase motors are electrolytic capacitors, designed for high capacity with minimum volume. They are rated for intermittant duty and have a relatively low safety factor on voltages higher than rated value. Capacitors rated at twenty-five to sixty cycles per second and 110-220 volts have a power-factor angle of approximately eighty-six degrees and a power factor of 0.07.

6. The effect of iron loss upon the magnitude of voltage and current is negligible.

At voltages near the rated voltage of the machine, little error is introduced by this assumption. At high voltages where the magnitude of the third harmonic in the exciting current due to saturation may be appreciable, additional error is possible.

The actual iron loss will be accounted for as an additional input after all other calculations are completed.

7. The effect of the backward revolving field may be represented as a constant, lumped, linear impedance.

8. The copper loss in the rotor circuit may be neglected.

This assumption introduces some error at high values of slip. A "cut-and-try" process may be used to solve a circuit which includes the "forward rotor" components, but it will not be considered here.[2]

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2 C. F. Wagner, op. cit., p. 49.

With the above assumptions, the equivalent circuit of Fig. 15 may be simplified for generator action to that shown in Fig. 21.

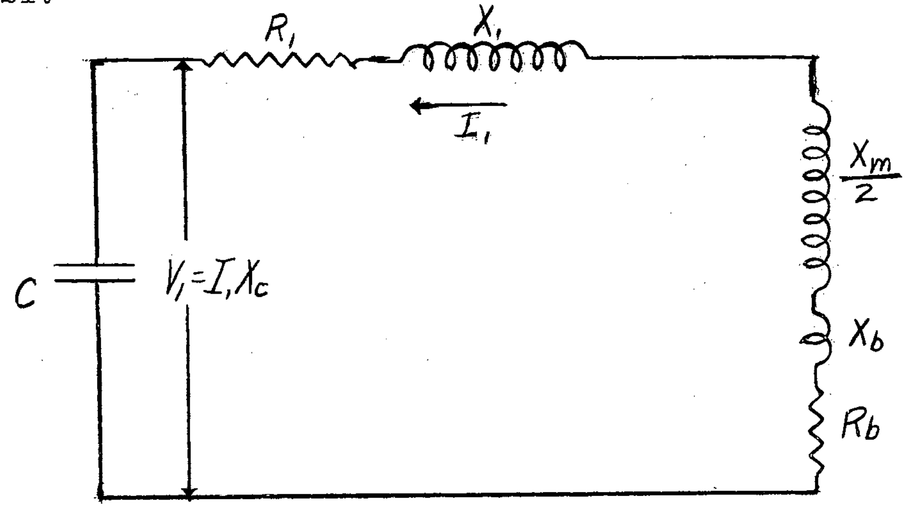

Fig. 21
Approximate equivalent circuit of a
single-phase induction generator with capacitor excitation

The impedance of the backward field is represented by the components, R_b and X_b, which may be added to the components of the stator impedance. With this addition, the circuit of Fig. 21 becomes similar to the circuit used in the development of the theory of self-excitation in polyphase machines. The magnetizing current which must flow is approximately twice the value required for polyphase operation with equivalent machine constants. It has been shown that the single-phase motor also requires approximately twice the magnetizing current of equivalent polyphase machines.

Referring again to equations (25) and (30), it is seen that either the air-gap voltage or the terminal voltage obtained from the no-load motor saturation test may be used for determining the self-excited generator voltage for steady-state operation. When the saturation curve is plotted as V_1/n versus I_1, the straight lines obtained by equation (38) may be used to obtain the operating point for a particular speed and capacitance. When the current has been determined, the power input may be found by a summation of copper, iron, and frictional losses. The retarding torque can then be calculated. The speed-torque curve may be plotted using a step-by-step method assuming different values of speed for each step. For determining time, equations (43), (44), and (45) may be used.

It has previously been pointed out that the most important applications of self-excited induction generation involve deceleration or dynamic braking. Some writers have suggested that self-excited induction generator characteristics are suitable as a source of steady-state, sine-wave voltage.[3] The above theory can be used for predicting the operating conditions for both applications. A series of tests have been completed on a 1/4-hp, single-phase, capacitor-start motor to show the correlation between calculated data and actual performance during transient as well as steady-state operation.

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3 E. D. Bassett, and F. M. Potter, Ibid, p 542.

Tests and Results

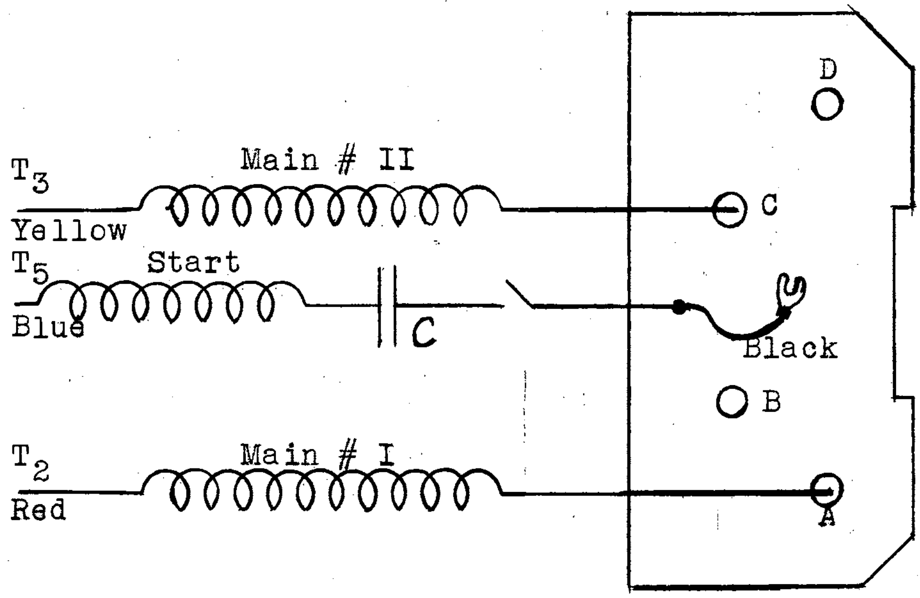
A 1/4-hp, 1725-rpm, 115/230-volt, 60-cycle, single-phase, capacitor-start motor manufactured by Robbins and Myers, Inc., Springfield, Ohio, was selected for performance tests of single-phase generator operation with capacitor excitation. Certain features of the construction of this machine affect its performance as a generator, therefore a rather complete description is deemed advisable.

The frame number is KL F56 and the serial number is MS 89287. It is of open construction and is rated for continuous duty with a normal temperature rise of 40° C.

The stator has an outside diameter of 6.000 inches and an inside diameter of 3.876 inches with an active core length of 2.500 inches. There are 36 slots, and each of the four poles of the main winding has four concentrically wound coils. The inner coil has 33 turns and spans three teeth; the second coil has 52 turn and spans five teeth; the third coil has 64 turns and spans seven teeth; the outer coil has 33 turns and spans nine teeth. The 230-volt rating is accomplished by placing all four poles in series. The total effective number of turns for the 230-volt connection is 149. For 115-volt rating, opposite poles are connected in series and the two pairs of poles are connected in parallel. The total effective number of turns for the 115-volt connection is 74.5. Hereafter this connection will be referred to as the 115-volt main winding.

Each of the four poles of the start winding has three concentrically wound coils, the inner coil having 20 turns and spanning four teeth, the middle coil having 40 turns and spanning six teeth, the outer coil having 40 turns and spanning eight teeth. The total effective number of turns is 86.8. The four poles are connected in series and this winding is connected in series with an electrolytic capacitor and a centrifugally operated switch. The capacitor has 145 microfarads capacitance and a voltage rating of 110 volts (rms) at 60 cycle, intermittent duty. The starting circuit is connected across the motor terminals for 115-volt operation. For 230-volt operation, it is connected in parallel with only one pair of the main poles. The centrifugal mechanism opens the starting circuit during acceleration for all speeds above 1200 rpm. During deceleration, the starting circuit is closed by the mechanism for all speeds below 1000 rpm. Fig. 22 shows a connection diagram for both low voltage and high voltage.

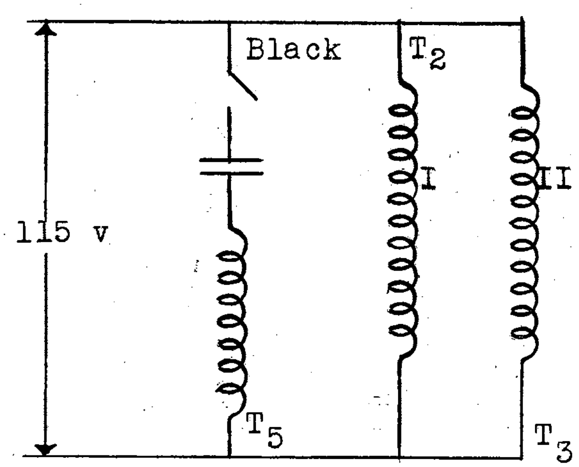
The rotor has an outside diameter of 3.845 inches which provides an air gap of 0.015 inches. There are 48 slots and the rotor winding is a cast-aluminum squirrel cage. The bars of the rotor cage are skewed slightly more than one rotor slot pitch. The moment of inertia of the rotor about the axis of normal rotation was obtained by measuring its period of oscillation when suspended by a calibrated wire and found to be .146 pound-feet^2.



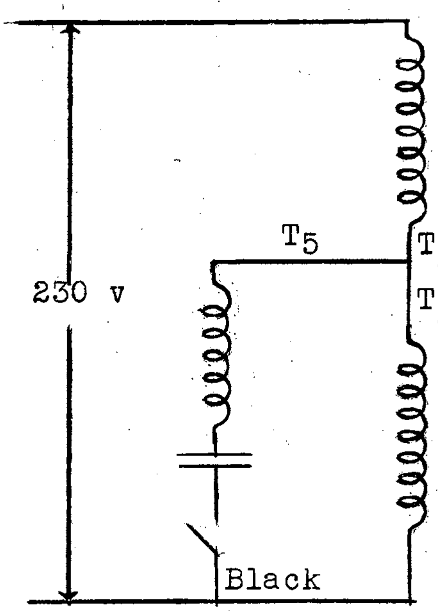
(a)

Capacitor: 145 μf., 110v
Centrifugal Mechanism
cuts out at 1200 rpm,
cuts back at 1000 rpm.

To reverse rotation
interchange T_5 and
Black leads.



(b) Low voltage



(c) High voltage

Fig. 22
Connection diagrams for a Robbins and Myers
$\frac{1}{4}$-hp, 1735-rpm, 115/230-volt, 60-cycle,
single-phase, capacitor-start induction motor

Before testing the machine for self-excited generator characteristics, the standard motor no-load and blocked-rotor tests were run. Machine constants for the equivalent circuit were calculated for various connections of the main and start windings. Table VI shows the machine constants for the connections of main and start windings that may be encountered during motor or generator operation.

Dynamometer tests were run for normal motor performance and also for generator action with the excitation supplied from a source of rated voltage and frequency. Table VII shows the data from a dynamometer test using the 115-volt connection. Fig. 23 shows the performance characteristics for both motor and generator action.

Table VIII (a) shows the data from the motor no-load test using the 115-volt connection. Fig. 24 is a plot of the saturation data of Table VIII.

Table IX (a) shows the data from a motor no-load test with the 115-volt main winding in series with the start winding. Fig. 25 is a plot of the saturation data of Table IX.

The series connection of main and start windings is important because it is the circuit which is effective for producing generator action when the motor is rotating at speeds below 1000 rpm during deceleration.

Referring to Fig. 22 (c) it can be seen that for the 230-volt connection only one main winding will be in series with the start winding for generator action during normal deceleration.

Table VI

Blocked-rotor data and impedance values of a Robbins and Myers $\frac{1}{4}$-hp, 1725-rpm, 115/230-volts, 60-cycle, single-phase, capacitor-start induction motor

Winding and Connections	Blocked rotor			Impedance components (ohms)			
	Voltage (volts)	Current (amps)	Power (watts)	R_1	R_2'	X_1	X_2'
Main windings							
No. I (red)	95.8	8.85	580	3.56	3.8	4.3	
No. II (yellow)	96.0	8.83	585	3.56	3.9	4.3	
I & II in series (230-volt conn.)	182.0	7.50	940	7.10	9.7	8.8	
I & II in parallel (115-volt conn.)	99.5	16.18	1160	1.78	2.6	2.1	
Start winding	100.0	9.50	805	5.04	3.9	2.7	
Main I & Start in series	135.5	6.33	648	8.60	7.6	7.0	
Main II & Start in series	136.0	6.34	649	8.60	7.6	7.0	
Mains (I & II in parallel) in series with Start	103.0	6.08	487	6.80	6.4	5.3	

Reactance of 145 microfarad capacitor at 60-cycles = 18.3 ohms

$$a' = \frac{\text{number of effective turns on start winding}}{\text{number of effective turns on main winding}} = 1.165$$

Table VII

Dynamometer test for motor and generator action of a $\frac{1}{4}$-hp, 1725-rpm, 115/230-volt, 60-cycle, single-phase, capacitor-start, induction machine.

115-volt Connection
(115-volts, 60-cycles applied)

Current	Elect. power	Power factor	Power factor angle	Torque lb.-ft.	Speed rpm	Mech. Power
	input					output
9.90	967	0.850	31.8°	1.85	1510	398
8.00	785	0.854	31.4°	1.75	1600	398
6.35	611	0.835	33.4°	1.50	1670	356
5.20	483	0.807	36.2°	1.25	1700	301
4.33	389	0.781	38.5°	1.00	1720	243
3.74	307	0.715	44.4°	0.75	1740	186
3.32	237	0.622	51.5°	0.50	1760	125
2.95	165	0.486	60.9°	0.25	1780	63
2.77	113	0.354	69.3°	0.075	1790	19
						input
2.71	79	0.253	75.3°	0.00	1795	0
2.70	42	0.135	82.3°	-0.15	1800	- 38
2.73	0	0.0	90.0°	-0.375	1810	- 97
	output					
2.82	- 34	-0.105	96.0°	-0.50	1815	-129
3.30	-145	-0.381	112.4°	-1.00	1830	-260
3.67	-196	-0.454	117.6°	-1.25	1840	-327
4.10	-252	-0.533	122.2°	-1.50	1855	-396
4.45	-296	-0.578	125.2°	-1.75	1870	-465
4.90	-346	-0.615	127.9°	-2.00	1880	-534
5.40	-390	-0.632	129.2°	-2.25	1890	-604
5.85	-432	-0.642	130.0°	-2.50	1895	-675
6.20	-464	-0.650	130.5°	-2.75	1900	-742
6.60	-496	-0.653	130.7°	-2.96	1910	-800
7.20	-548	-0.662	131.4°	-3.30	1920	-900
7.60	-582	-0.665	131.6°	-3.40	1935	-955

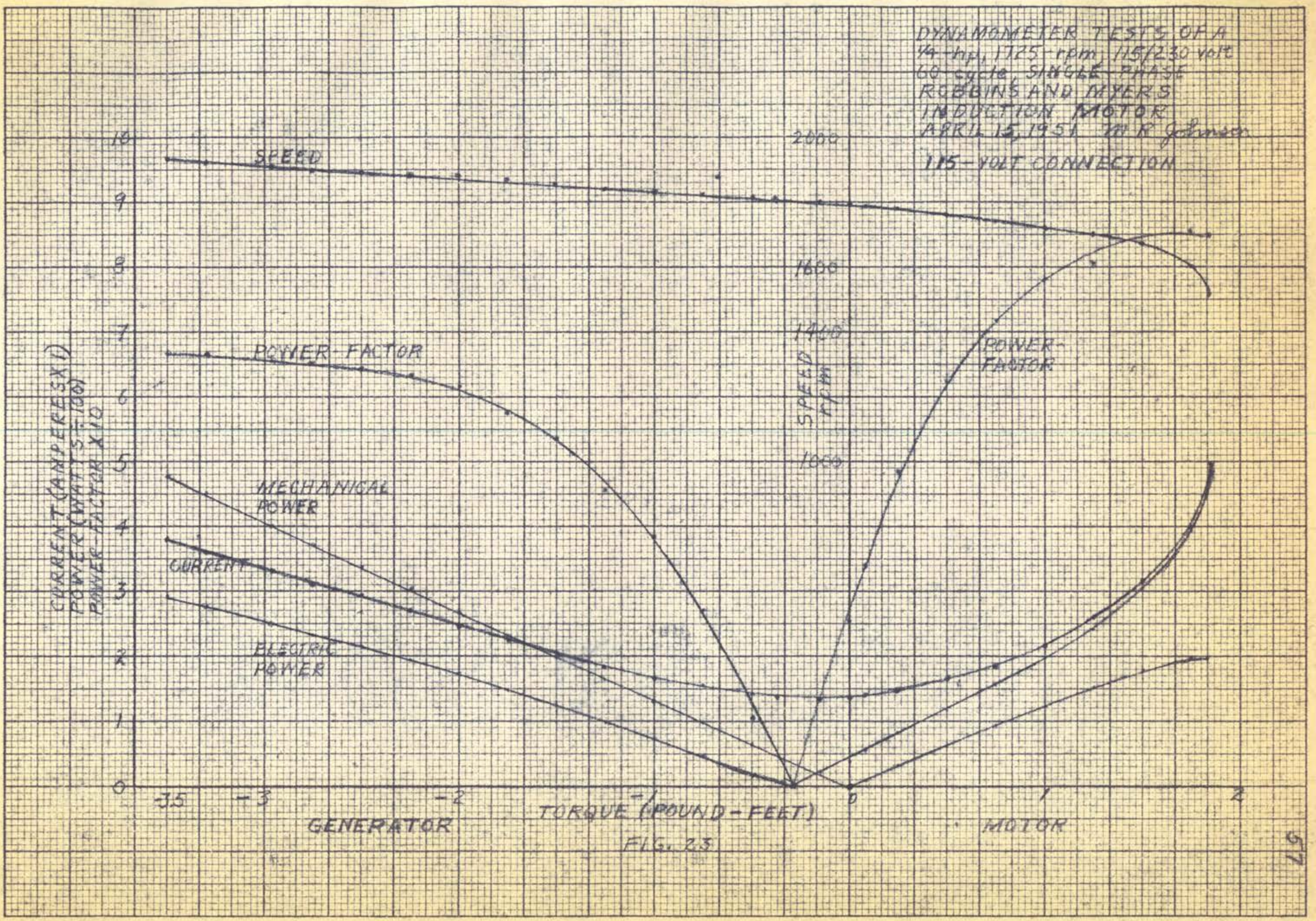
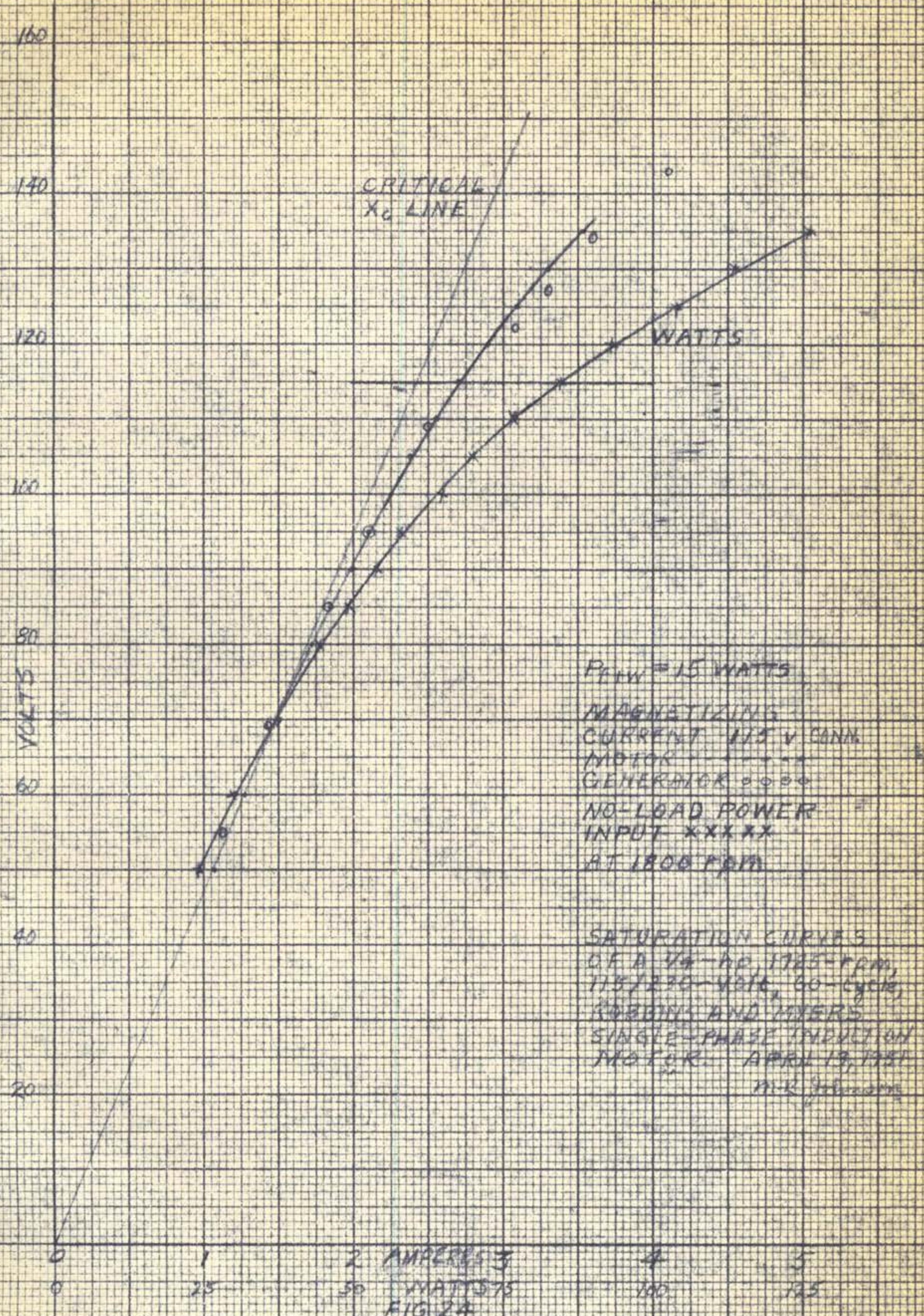

DYNAMOMETER TESTS OF A
¼-hp, 1725-rpm, 115/230 volt
60-cycle, SINGLE-PHASE
ROBBINS AND MYERS
INDUCTION MOTOR
APRIL 15, 1951 W R Johnson
115-VOLT CONNECTION
SPEED
POWER-FACTOR
MECHANICAL POWER
CURRENT
ELECTRIC POWER
POWER-FACTOR
SPEED rpm
2000
1600
1400
1000
CURRENT (AMPERES X 1)
POWER (WATTS ÷ 100)
POWER-FACTOR X 10
0
1
2
3
4
5
6
7
8
9
10
-3.5
-3
-2
-1
0
1
2
GENERATOR
TORQUE (POUND-FEET)
MOTOR
FIG. 23

Table VIII

Saturation tests on a Robbins and Myers ¼-hp, 1725-rpm, 115/230-volt, 60-cycle, single-phase, capacitor-start induction motor

115-volt connection (Main I & II in parallel)

(a) Motor (No-load)				(b) Generator (60-cycles)			
Volts	Current	V_1 per rpm	Watts	Volts	Current	V_1 per rpm	C
50	1.07	0.0280	24	55	1.13	0.0306	49
60	1.27	0.0330	30	63	1.28	0.0350	50
70	1.50	0.0390	37	69	1.44	0.0384	51
80	1.72	0.0444	44	76	1.60	0.0422	52
85	1.85	0.0472	49	85	1.83	0.0472	53
90	2.00	0.0500	54	90	1.97	0.0500	54
95	2.10	0.0528	59	95	2.11	0.0528	55
100	2.25	0.0556	65	102	2.24	0.0567	56
105	2.38	0.0584	70	105	2.45	0.0584	57
110	2.55	0.0610	77	109	2.50	0.0606	58
115	2.70	0.0640	85	113	2.70	0.0629	59
120	2.90	0.0677	93	117	2.85	0.0650	60
125	3.10	0.0695	104	120	2.98	0.0666	61
130	3.30	0.0720	114	122	3.07	0.0678	62
135	3.54	0.0750	126	126	3.19	0.0700	63
				127	3.30	0.0706	65
				134	3.62	0.0745	67
				143	4.10	0.0795	70



160
140
120
100
80
60
40
20
VOLTS
CRITICAL
X_c LINE
WATTS
0
1
2
AMPERES
3
4
5
0
25
50
WATTS 75
100
125
P_{F+W} = 15 WATTS
MAGNETIZING
CURRENT 115 V CONN.
MOTOR
GENERATOR
NO-LOAD POWER
INPUT
AT 1800 rpm
SATURATION CURVES
OF A 1/4-hp, 1725-rpm,
115/230-volt, 60-cycle,
ROBBINS AND MYERS
SINGLE-PHASE INDUCTION
MOTOR. APRIL 13, 1951

FIG 24

Table IX

Saturation tests on a Robbins and Myers $\frac{1}{4}$-hp, 1725-rpm, 115/230-volt, 60-cycle, single-phase, capacitor-start induction motor

115-volt main winding in series with the start winding

(a) Motor (No-load)				(b) Generator (60-cycles)			
Volts	Current	V_1 per rpm	Watts	Volts	Current	V_1 per rpm	C
89	.83	0.0495	34	130	1.20	0.0722	23
100	.91	0.0556	38	152	1.43	0.0845	24
120	1.09	0.0667	48	168	1.60	0.0935	25
140	1.30	0.0778	60	184	1.90	0.1022	26
160	1.52	0.0890	77	194	2.05	0.1079	27
180	1.78	0.1000	97	203	2.24	0.1130	28
193	1.97	0.1072	112	210	2.40	0.1169	29
200	2.07	0.1112	123	220	2.61	0.1222	30
220	2.46	0.1222	160	226	2.79	0.1256	31
240	2.92	0.1333	206	232	2.95	0.1290	32
				238	3.12	0.1322	33
				243	3.25	0.1350	34
				248	3.45	0.1379	35
				251	3.58	0.1397	36
				254	3.75	0.1412	37
				258	3.90	0.1424	38

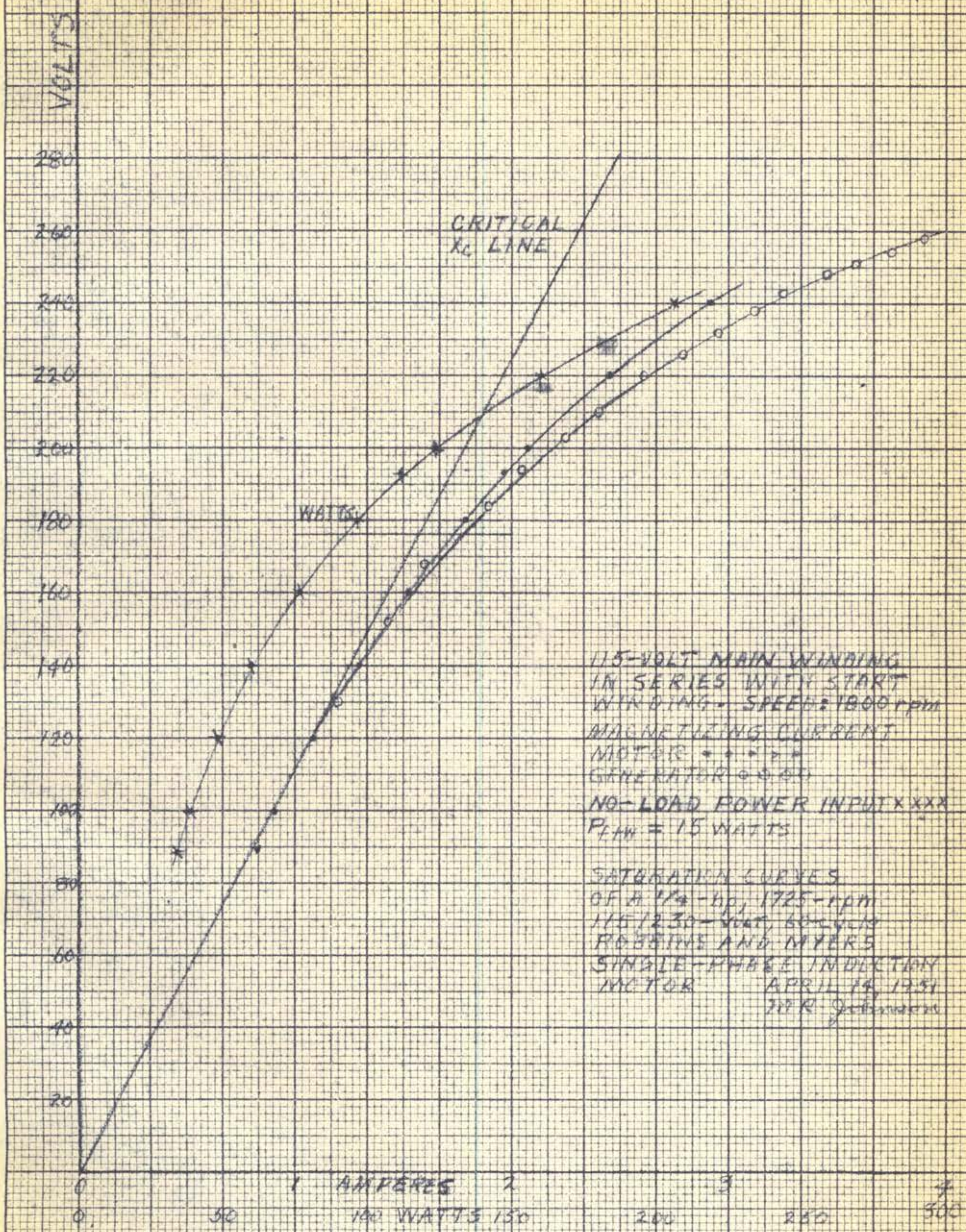

VOLTS
280
260
240
220
200
180
160
140
120
100
80
60
40
20
CRITICAL
X_C LINE
WATTS
115-VOLT MAIN WINDING
IN SERIES WITH START
WINDING - SPEED: 1800 rpm
MAGNETIZING CURRENT
MOTOR
GENERATOR
NO-LOAD POWER INPUT x x x
P_{f+w} = 15 WATTS
SATURATION CURVES
OF A 1/4-hp, 1725-rpm
115/230-volt, 60-cycle
ROBBINS AND MYERS
SINGLE-PHASE INDUCTION
MOTOR APRIL 14, 1951
W R Johnson
0
1
AMPERES
2
3
4
0
50
100 WATTS 150
200
250
300

FIG. 26

Therefore the total effective number of turns will be the same as for the 115-volt connection, but the effective resistance and reactance of the circuit will be somewhat greater.

Since the main and start windings are in space quadrature, the generated voltage of the two windings are in time quadrature. If the air-gap flux is constant, the generated voltage of each winding is proportional to the effective number of turns in the winding. The magnitude of the generated voltage of the two windings connected in series is therefore equal to the square root of the sum of the squares of the main winding generated voltage. The effective number of turns of the two windings in series bears this same relationship to the effective turns of individual windings. For the machine tested, the series connection has

$$\sqrt{(74.5)^2 + (86.8)^2} = 114.4 \text{ effective turns.}$$

The voltage of the two windings in series may be expressed in terms of the main winding voltage alone by means of the ratio of effective turns if the two windings:

$$a' = \frac{\text{start-winding effective turns}}{\text{main-winding effective turns}} = \frac{E_S}{E_M} \qquad (47)$$

$$E_{(M+S)} = \sqrt{E_M^2 + E_S^2} = E_M\sqrt{1+\left(\frac{E_S}{E_M}\right)^2} = E_M\sqrt{1+(a')^2} \qquad (48)$$

When the same magnetizing current flows through the two windings, the effective mmf is also greater than the mmf of the main winding alone by the factor $\sqrt{1+(a')^2}$. Therefore it is possible to calculate the saturation curve for the series connections of main and start windings from the no-load saturation curve of the main winding alone.

$$a' = \frac{86.8}{74.5} = 1.165 \; ; \quad \sqrt{1+(1.165)^2} = 1.548$$

From the no-load motor data of Table VIII (a) the current at 115 volts is 2.7 amperes. A corresponding point on the saturation curve for the series connection is then

$$1.548 \times 115 = 178 \text{ volts}$$
$$2.7/1.548 = 1.75 \text{ amperes}$$

Note that an actual test with the series connection gave 1.78 amperes at 180 volts (Table IX).

When the winding data are not available, the approximate value of a' may be obtained from the motor no-load saturation test if the generated voltage of the start winding is measured. The value of a' is approximately the generated voltage of the start winding divided by the applied voltage on the main winding.

To use the method for predicting generator characteristics which is shown in Chapter II, equations (31) through (46), the saturation curves of Tables VIII and IX are plotted as V_1/n versus I_1 in Fig. 26.

An example showing the use of these curves follows. A calculation of the voltage, current, and torque for a speed of 1800 rpm and a capacitance of 35 microfarads placed across the terminals of the 115-volt main winding in series with the start winding will be made.

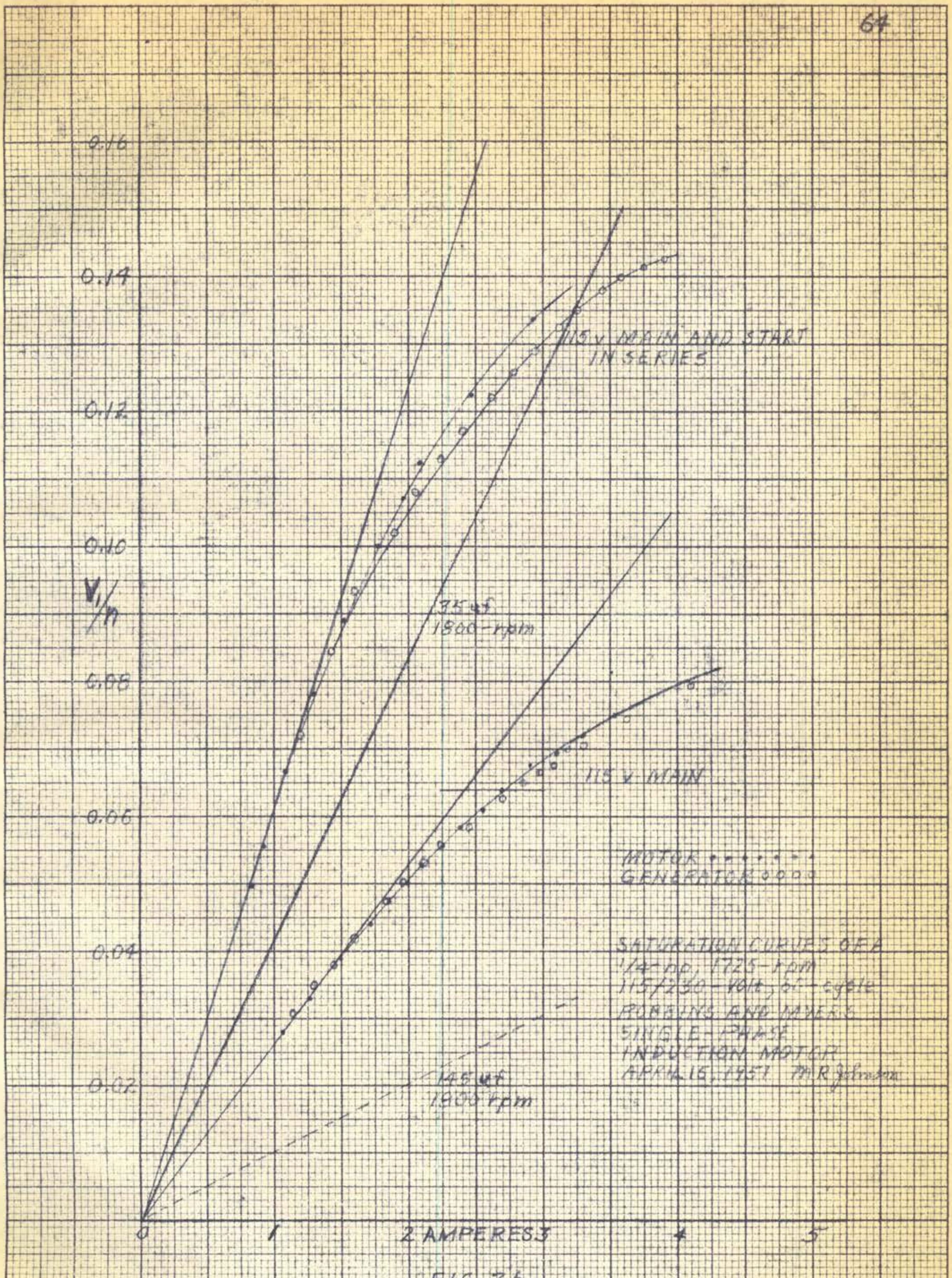

0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
V/n
0
1
2 AMPERES 3
4
5
115 v MAIN AND START
IN SERIES
75 uf
1800-rpm
115 V MAIN
MOTOR • • • • • • •
GENERATOR o o o o
SATURATION CURVES OF A
1/4-hp, 1725-rpm
115/230-volt, 60-cycle
ROBBINS AND MYERS
SINGLE-PHASE
INDUCTION MOTOR
APRIL 15, 1951 M R Johnson
145 uf
1800 rpm

FIG. 26

$$I_c = \frac{V_1}{n} \cdot \frac{2\pi \times 4 \times (1800)^2 \times 35 \times 10^{-6}}{120}$$

$$I_c = \frac{V_1}{n} \cdot (23.7)$$

This straight line is plotted on Fig. 26 and intersects the no-load saturation curve at the point $V_1/n = .134$, $I = 3.2$.

The voltage is then 241 volts at 1800 rpm. The input for this voltage and frequency is obtained from Fig. 25 and is 210 watts. The torque required to supply this power at 1800 rpm is

$$T = \frac{7.04 \times 210}{1800} = 0.83 \text{ lb.ft.}$$

Table IX (b) shows that during the actual test of generator action, 34 uf were required to generate 243 volts at 1800 rpm.

After the motor tests were completed, the machine was connected as shown in Fig. 27. A capacitor bank rated at 600 volts and variable in one microfarad steps from zero th 186 microfarads was used.

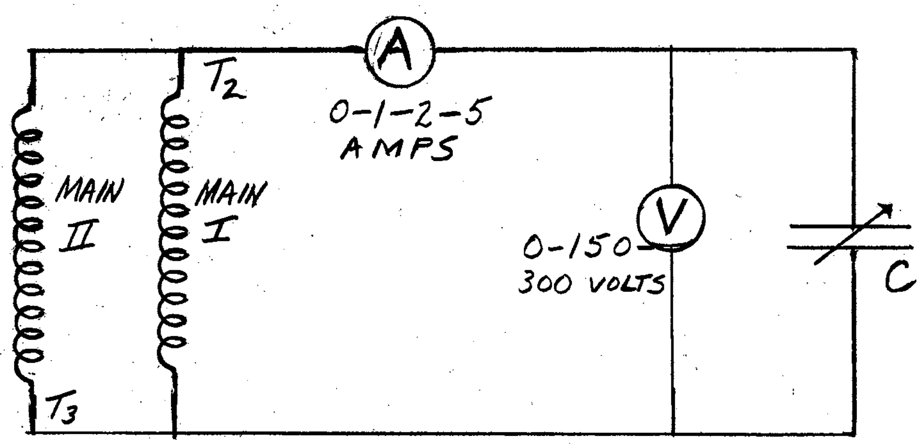


Fig. 27
Connection of a single-phase induction machine for steady-state, self-excited generator characteristics

The rotor was driven at 1800 rpm by a $\frac{1}{2}$-hp, 1800 rpm, 115-volt, d-c dynamometer. With the value of capacitance at zero (open-circuit), the voltage due to residual magnetism was measured and found to be 0.75 volts (rms). A cathode-ray oscilloscope was used to view the wave shape of the residual voltage. No departure from a true sine shape was noticeable. When the winding was shorted for a moment and subsequently opened, the residual voltage decreased to 0.5 volt. A repetition of the shorting procedure did not appreciably reduce the residual voltage.

To explore the conditions which allow an induction machine to build up its own voltage and become a self-excited generator, the following procedure was used. The rotor was driven at 1800 rpm and the capacitance of the variable capacitor shown in Fig. 27 was increased in small steps. Table X shows the effect of the added capacitance upon the terminal voltage of the machine before and after the critical value of capacitance was reached. The build-up is similar to that of a self-excited, direct-current generator. It was found that considerably more capacitance was required to cause the machine to build-up than was required to maintain stable operation. The exact amount required for build-up seemed to depend upon the previous conditions of machine operation.

The minimum value of capacitance at which generation was maintained was fairly definite for a particular speed. It was approximately the value which produced a straight line

Table X

Voltage build-up of a Robbins and Myers ¼-hp, 1725-rpm, 115/230-volt, 60-cycle, single-phase, capacitor-start, induction motor, operating as a capacitor-excited generator

Rotor Speed 1800 rpm

Before build-up		After build-up	
Capacitance (microfarads)	Voltage (rms)	Capacitance (microfarads)	Voltage (rms)
0	0.50	65	127
5	0.56	60	117
10	0.62	55	95
15	0.69	50	63
20	0.75	49	55
25	0.86	48	broke down
30	0.97		
35	1.16		
40	1.33		
45	1.63		
50	2.15		
55	3.00		
60	3.75		
65	built-up		
65	127.00		

[equation (38)] coincident with the air-gap line of the motor saturation curve. This could be referred to as the "critical capacitive reactance line" for the self-excited induction generator just as the air-gap line is referred to as the "critical field resistance line" of a self-excited, direct-current generator.

The lines indicating the critical X_c for the 115-volt main winding alone and for the connection of the 115-volt main winding in series with the start winding are shown on Figs. 24 and 25 respectively. On Fig. 26, the dotted line shows the reactance line for a 145 microfarad capacitor at 1800 rpm (60 cycles per second). This is the value of capacitance which is in series with the start winding to provide the desired starting torque as a motor [Fig. 22 (b) and (c)]. It is also effective as excitation for generator action with the main and start windings in series during deceleration of the machine after it has been disconnected from a source of electric power. Tests were made to determine the steady-state generator characteristic for this connection using the circuit shown in Fig. 28.

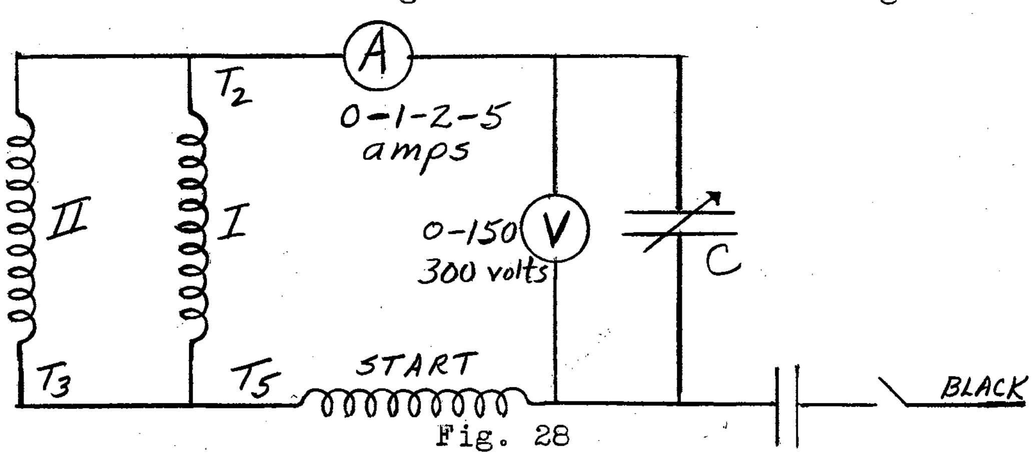


Fig. 28
Connection diagram for obtaining
steady-state generator characteristic

The steady-state operating point at 1800 rpm with 145 microfarads can be found by extending the straight line and the upper magnetization curve on **Fig.** 26 to the intersection. Equation (42) shows that the slope of the straight line varies inversely as the square of the speed. Therefore the minimum speed at which generation will be maintained with 145 microfarads is that value which gives a reactance line coincident with the air-gap line on Fig. 26. For the machine tested, this critical speed was calculated to be 730 rpm. A tabulation of the results of the steady-state generator characteristics at 1800 rpm is shown in Tables VIII (b) and IX (b). A plot of these data is shown on Figs. 24, 25, and 26.

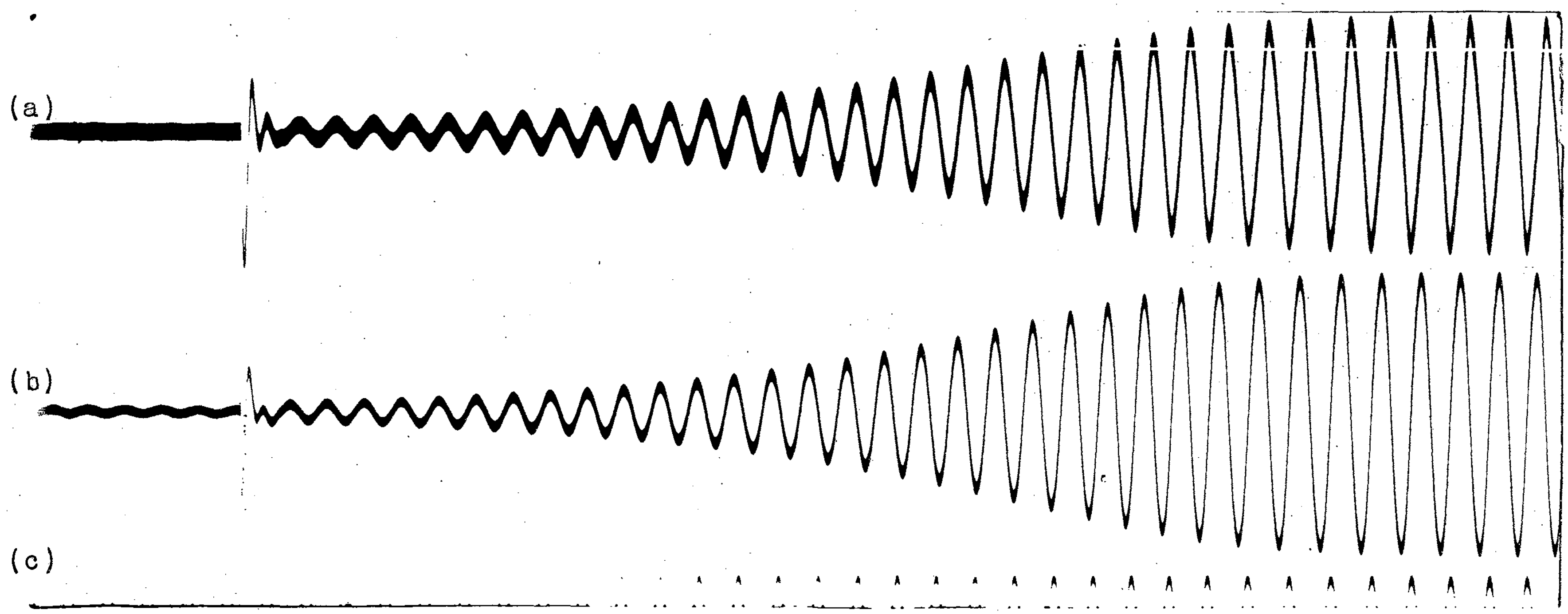
A magnetic oscillograph was used to record the instantaneous values of current and voltage for several conditions of generator operation.

Oscillogram #1 shows the build-up of voltage and current in the circuit of Fig. 28 when the speed was approximately 1800 rpm and the capacitance was 34 microfarads. A 60-cycle timing wave appears on the lower edge of each oscillogram. In approximately one-half second after the switch was closed the current reached a steady-state value of 3.32 amperes (rms).

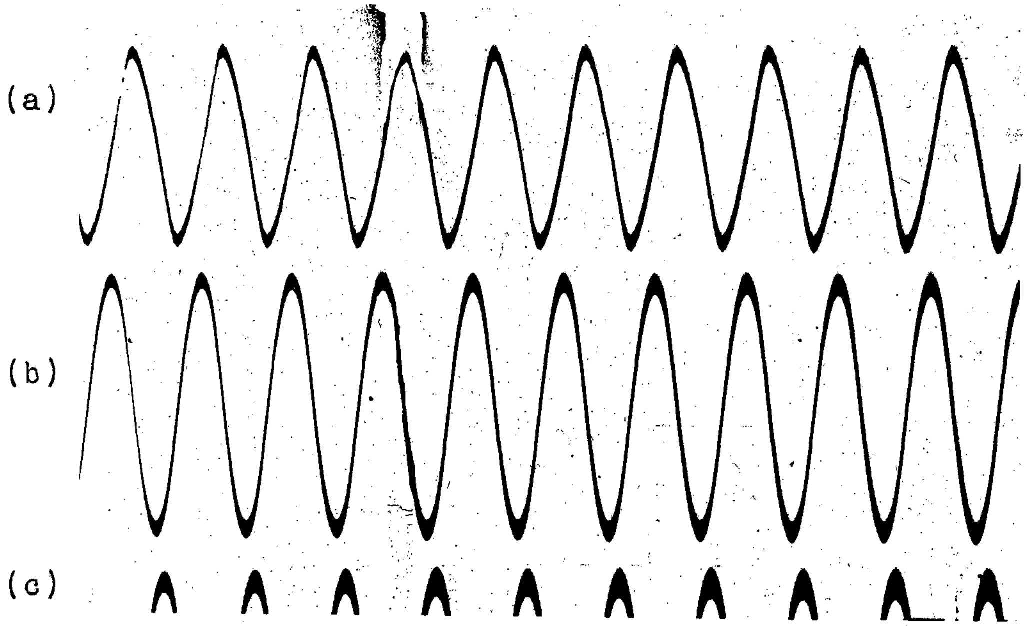
Oscillogram #2 shows the steady-state current and voltage at a speed of 1800 rpm and a capacitance of 27 microfarads. The effective current was 2.0 amperes and the voltage was 183 volts.

Oscillogram #3 shows the steady-state current and voltage at a speed of 1400 rpm and a capacitance of 49 microfarads. The current and voltage were 2.5 amperes and 165 volts respectively.

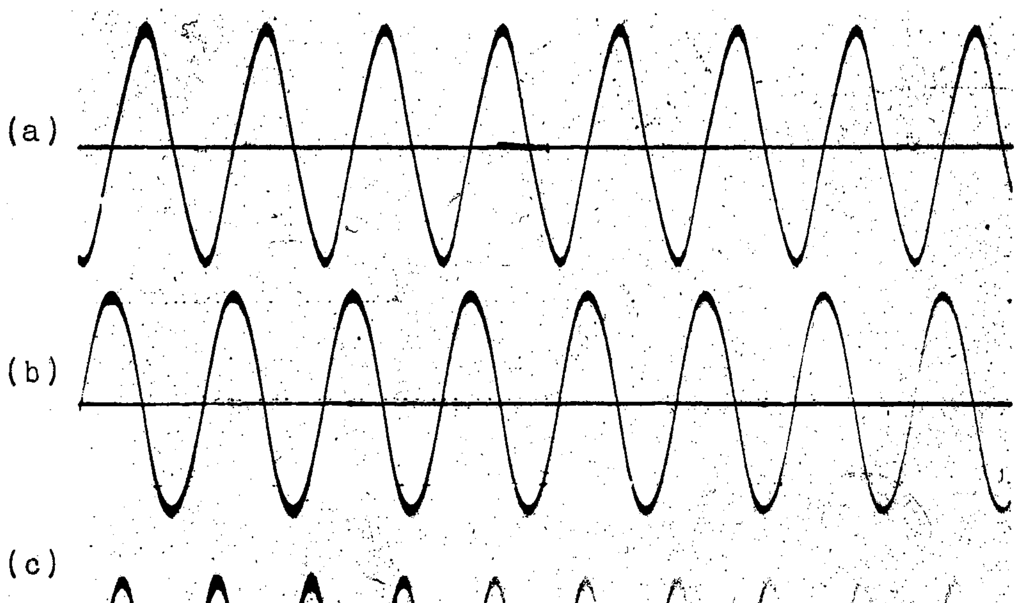
Rotor speed - 1800 rpm., Capacitance: 34 microfarads.
(a) Current - Steady-state value, 3.32 amperes (rms) or 4.7 amperes (peak).
(b) Voltage - Steady-state value, 238 volts (rms) or 336 volts (peak).
(c) Timing wave: 60 cycles per second; Film speed - 13.9 inches per second.



Oscillogram #1: Build-up of a self-excited induction generator. 1/4-hp., 1750-rpm, 115/230-volt, 60-cycle, single-phase, capacitor-start, Robbins and Myers, induction motor.



Oscillogram #2: Single-phase induction generator.
Rotor speed - 1800 rpm; Capacitance - 27 microfarads
Strady-state (a) current - 2.0 amperes (rms),
(b) voltage - 183 volts (rms), (c) film speed - 28.7
inches per second.



Oscillogram #3: Single-phase induction generator.
Rotor speed - 1400 rpm; Capacitance - 49 microfarads
Steady-state (a) current - 2.5 amperes (rms),
(b) voltage - 162 volts (rms), (c) film speed - 28.7
inches per second.

Oscillograms #4 and #5 were obtained to show the conditions of generation during deceleration of the machine. The circuit of Fig. 29 was used.

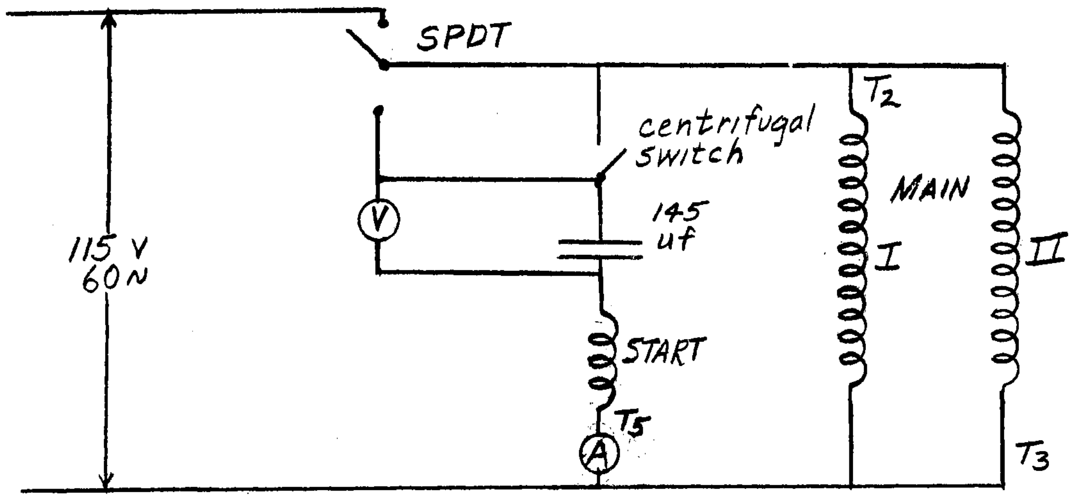
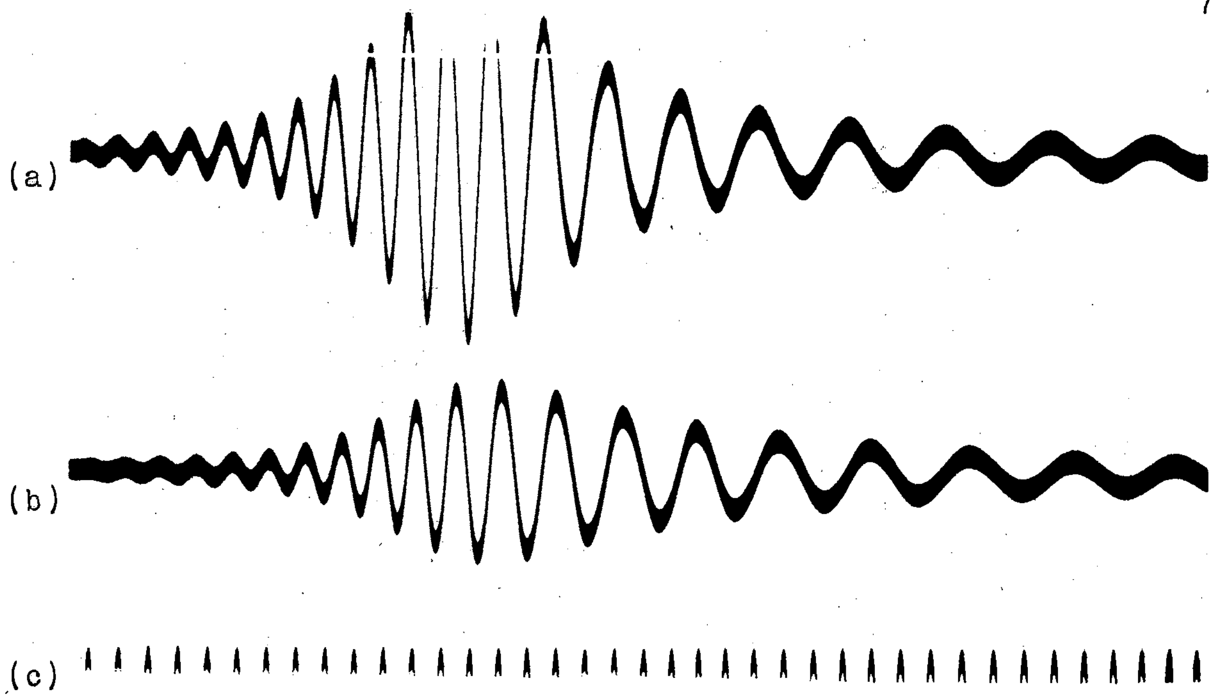
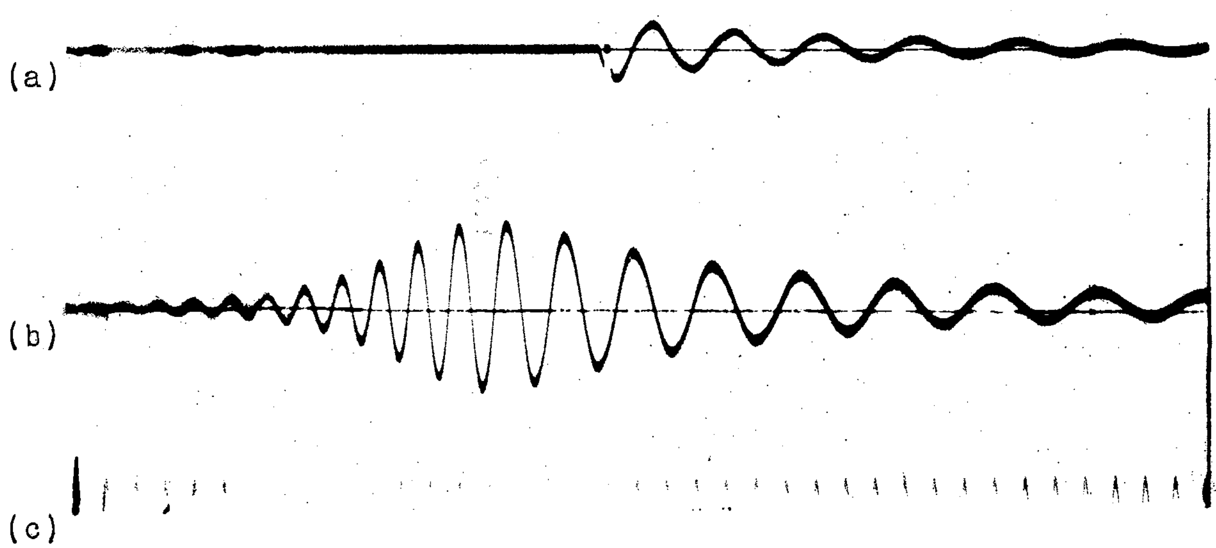


Fig. 29
Connection diagram for obtaining deceleration trantient of generator action

The procedure for the test was as follows: The single-pole, double-throw switch was closed to energize the motor, and the rotor was allowed to accelerate to normal no-load speed. The centrifugal switch opened at 1200 rpm and disconnected the start winding from the line. Next, the voltage was removed from the motor by opening the SPDT switch. In less than one second the switch was closed in the opposite position, completing a circuit from the capacitor to the main winding. This completed a circuit for generator action; the main winding in series with the start winding with the capacitor across the terminals for excitation. The oscillographic film was started immediately before the SPDT switch was closed to complete the generator circuit. The voltage and current built up rapidly



Oscillogram #4: Generator action during deceleration of a $\frac{1}{4}$-hp, 115/230-volt, single-phase, capacitor-start induction motor. No-load rotor speed - 1795 rpm, capacitance - 145 microfarads (electrolytic). (a) current - 7.1 amperes (peak), (b) voltage - 178 volts (peak), (c) film speed - 9.05 inches per second.



Oscillogram #5: Deceleration transient as in Oscillogram #4 except (a) current was applied to oscillograph element by means of centrifugal switch "cut-back" operation.

reaching a maximum (peak) of 178 volts in approximately 0.2 seconds. The rotor decelerated rapidly and, because of the lowering speed, the voltage began to decrease. One-half second after the switch was closed, the speed had dropped from approximately 1700 rpm to less than 600 rpm. Generator action continued at low amplitude even beyond this point.

In oscillogram #5, the current element was connected in series with the centrifugal switch so that the time of "cut-back" was recorded. The oscillogram shows that the speed decreased from 1700 rpm to 1000 rpm in one-third of a second.

The load characteristics of generator action of the 115-volt main winding operating at constant frequency was obtained. The circuit shown in Fig. 30 was used for the load test. A slide-wire resistor wound on a porcelain tubular form was used as a load. The inductive reactance of the resistor at 60 cps was approximately two per cent of the resistance.

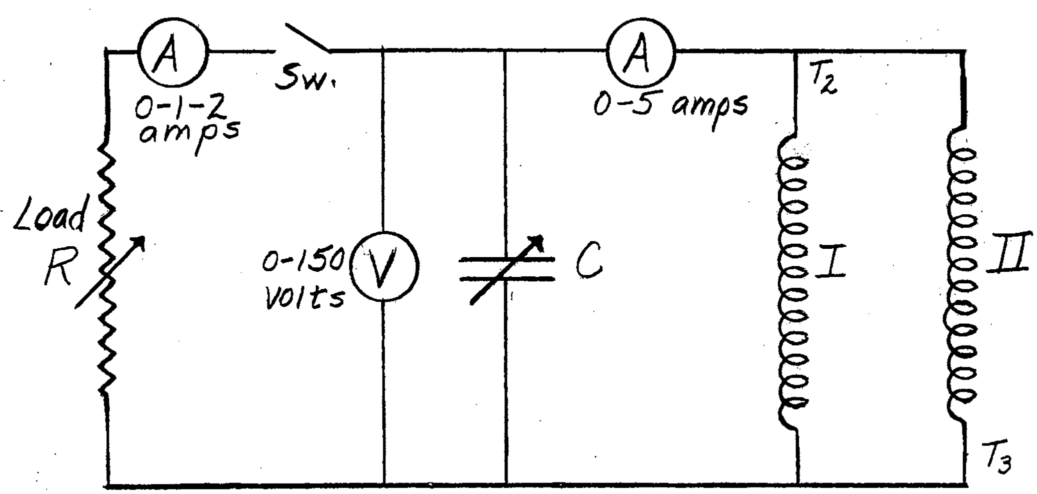


Fig. 30
Circuit diagram for generator load test

The rotor was driven at a speed required to maintain a frequency of 60 cps. The capacitance was adjusted to give 115 volts at no-load. Load was added by closing the switch to the resistor. The capacitance was readjusted to maintain 115 volts at the load. Readings of load current, main winding current, capacitance, and rotor speed were recorded for several points between no load and 1.9 amperes load current. Table XI (a) shows data from the test. Fig. 31 (a) is a plot of capacitance and main winding current versus load current.

A second load test was run to show voltage regulation with constant capacitance. The capacitor was adjusted to give 115 volts at no-load. Load was added in steps, and reading of load current, main winding current, terminal voltage and rotor speed were recorded. Table XI (b) shows data from this test. Fig. 31 (b) is a plot of terminal voltage and main winding current versus load current from no-load to break-down. The characteristic is very similar to that of a differentially compounded d-c generator.

Table XI

Load characteristic of a $\frac{1}{4}$-hp, 1725-rpm, 115/230-volt, 60-cycle, single-phase, Robbins and Myers capacitor-start motor, operated as a self-excited generator.

(a)

Rotor driven at speed necessary to maintain a frequency of 60 cycles per second. Voltage maintained at 115 volts by varying capacitance.

Current in load res. (amps)	Main wdg. current (amps)	Capacitance (uf)	Rotor speed (rpm)
0.00	2.75	59	1800+
0.46	2.80	61	1810
0.90	3.05	66	1830
1.64	3.56	74	1840
1.90	3.80	77	1850

(b)

Rotor driven at speed necessary to maintain a frequency of 60 cycles per second. Capacitance(59uf) adjusted to give rated voltage (115v) at no-load.

Current in load res. (amps)	Main wdg. current (amps)	Main wdg. voltage (volts)	Rotor speed (rpm)
0.00	2.75	115	1800+
0.20	2.62	111	1805
0.36	2.50	106	1810
0.50	2.31	99	1810
0.60	2.15	90	1810
0.70	1.91	80	1810
0.72	1.85	75	1810
0.74	1.60	60	1830
0.60	1.27	47	1830

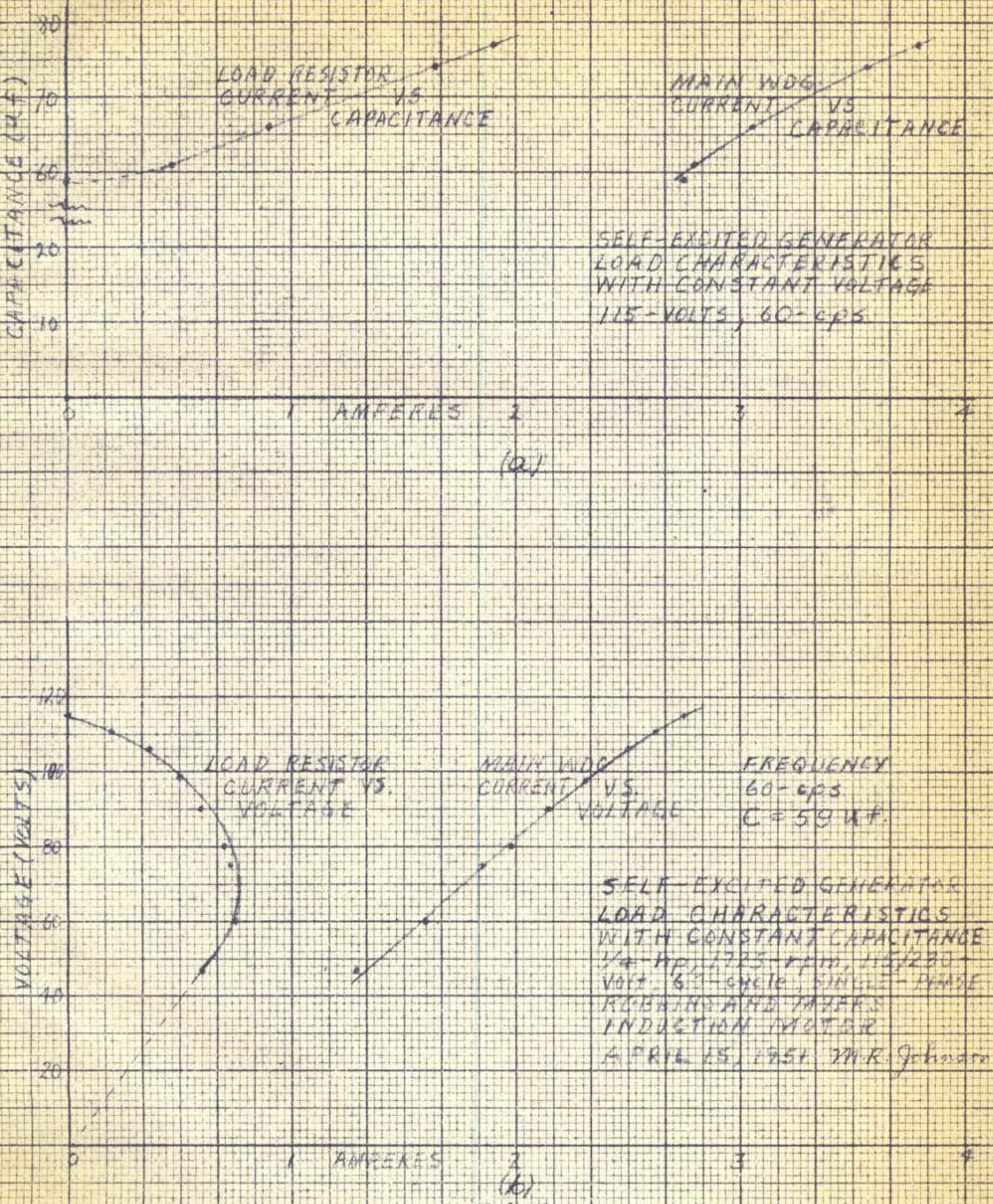

CAPACITANCE (µF)
80
70
60
20
10
LOAD RESISTOR CURRENT VS CAPACITANCE
MAIN WDG CURRENT VS CAPACITANCE
SELF-EXCITED GENERATOR LOAD CHARACTERISTICS WITH CONSTANT VOLTAGE 115-VOLTS, 60-cps
0
1
AMPERES
2
3
4
(a)
VOLTAGE (VOLTS)
120
100
80
60
40
20
LOAD RESISTOR CURRENT VS. VOLTAGE
MAIN WDG CURRENT VS. VOLTAGE
FREQUENCY 60-cps
C = 59 µF.
SELF-EXCITED GENERATOR LOAD CHARACTERISTICS WITH CONSTANT CAPACITANCE
1/4-hp, 1725-rpm, 115/230-Volt, 60-cycle, SINGLE-PHASE
ROBBINS AND MYERS INDUCTION MOTOR
APRIL 15, 1951 M.R. Johnson
0
1
AMPERES
2
3
4
(b)

FIG. 31

Conclusions

In general, both polyphase and single-phase induction generators have characteristics that may be determined by analysis of the equivalent circuit obtained from standard no-load, blocked-rotor and resistance tests. When operating in parallel with a system of controlled voltage and frequency, its performance may be predicted by using negative values of slip in the equivalent circuit. Since all excitation must be supplied from the line, it will not contunue to generate if the connection to the line is broken. The load and the power factor is uniquely determined by the internal characteristic of the machine and the slip.

Both polyphase and single-phase induction machines can be excited by means of static capacitors. Practical applications of capacitor-excited induction generators as steady-state power sources have not been made to any great extent. However, self-excited induction generator characteristics are important in some trantient conditions. These characteristics also can be determined from the no-load, blocked-rotor, and resistance tests using the method described in this thesis.

A great percentage of single-phase, capacitor-start motors have a capacitor of sufficient size to cause self-excitation during deceleration after the machine has been disconnected from the electric power source. Voltage across the capacitor due to generator action may rise to more than twice the voltage applied during the normal starting cycle. The value of the generated voltage is a function of the saturation curve of the machine, the capacitance, and the rotor speed.

This study of the single-phase induction generator has shown some rather interesting and little-known characteristics of an important and versatile electric machine. It has also shown that other studies are necessary to fully understand and apply some of the characteristics of the machine. Recent advancements in control devices for voltage, frequency and speed may open the way for application of the self-excited induction generator as a constant voltage, constant frequency, power source.

It is hoped that the information in this thesis will be helpful to those who are interested in the problem of induction generation.

List of Symbols

a	ratio of transformation between rotor and stator windings
a'	ratio of effective number of turns between the main and start windings of a single-phase motor
cps	cycles per second
C	capacitance of capacitor in farads
e	instantaneous generated voltage
E_1	rms volts/phase induced in stator winding
E_2	rms volts/phase induced in rotor winding at standstill
E'_2	rms volts/phase in rotor in terms of stator
f_1	stator frequency in cycles per second
f_2	rotor frequency in cycles per second
f	"a function of"
F	force on a conductor carrying current
i	instantaneous current
i_r	current in rotor conductor
I_1	rms amperes/phase stator current
I_2	rms amperes/phase rotor current
I'_2	rms amperes/phase rotor current in terms of stator
I_c	rms amperes through the capacitor
I_{bl}	rms amperes/phase blocked rotor current
I_{h+e}	rms amperes/phase iron loss component of stator current
I_m	vector sum of I_ϕ and I_{h+e}
I_ϕ	rms amperes/phase magnetizing current
$\mathcal{J}$	moment of inertia of the rotor about the axis of rotation
K_{d_1}	stator winding distribution factor

K_{d_2}	rotor winding distribution factor
K_{p_1}	stator winding pitch factor
K_{p_2}	rotor winding pitch factor
L_2	henrys per phase leakage rotor leakage inductance
n	speed of rotor in revolutions per minute
n_s	synchronous speed (rpm)
n_2	speed of revolving magnetic field with respect to rotor
N_1	series turns per phase on the stator winding
N_2	series turns per phase on the rotor winding
p_1	number of poles on stator
p_2	number of poles on rotor
P_{dev}	electro-magnetic power in watts converted into mechanical form
rpm	revolutions per minute
rms	root mean square
r_m	resistance in which iron loss is dissipated
R_1	ohms per phase effective resistance of stator winding
R_2	ohms per phase effective resistance of rotor winding
R_2'	ohms per phase effective resistance of rotor winding in terms of the stator
s	slip
t	time
T	torque in pound-feet
v	velocity
V_1	applied voltage (rms) per phase
x	distance (variable)
X_1	ohms per phase leakage reactance of stator winding at rated frequency f_1

X_2	ohms per phase leakage reactance of rotor winding at frequency f_1
X_2'	ohms per phase leakage reactance of rotor winding at frequency f_1 in terms of the stator
X_c	capacitive reactance of capacitor
X_m	ohms per phase magnetizing reactance
Z_2	ohms per phase rotor inpedance at standstill
α	angular acceleration
β	flux density
ϵ	base of natural logrithm, 2.7183
θ	rotor power factor angle
π	3.1416
ϕ	instantaneous flux in the air gap
ϕ_{max}	maximum air-gap flux linking a full pitch turn
τ	stator pole pitch
ω	angular velocity, radians/second
$\approx$	equals approximately
$\propto$	varies directly as
j	$\sqrt{-1}$, imaginary quantity (rotational operator)
∞	infinity

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