## SIEVE ANALYSIS

By

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Submitted to the Faculty of the Graduate School of the Oklahoma Agricultural and Mechanical College in Partial Fulfillment of the Requirements

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SIEVE ANALYSIS JOE BILL HOCOTT MASTER OF SCIENCE 1951

THESIS AND ABSTRACT APPROVED:

Thesis Adviser Faculty Representative

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The principal aim of this paper is to establish the relationship existing between particle dimensions and sieve dimensions for sieved particles. It is regarded as the first step in a more general program to provide a sufficient theoretical background for the optimum selection of screening equipment for solid-solid separations.

There is little doubt that the equally important problem involving elucidation of the relationship involving time required for particle passage through screens will have to be solved before the theory will be capable of dealing with even the simplest problems of solids-from-solids separation. It seems that this problem may best be attacked through study of probabilities for particle passage through screens. If so, much of the material of this paper will be relevant to the time-dependent study.

The properties of density, tendency to develop static electrical charges, moisture content, and possible others of a like nature generally exert less influence on screening characteristics. They have been ignored in this paper; the nature of the particles sieved in this study seemed such that careful control of these variables was not necessary.

The abrasive and corrosive properties of the materials to be screened are important considerations from the view-point of screen life rather than from that of screen operation. Yet it is in the proper balance of initial cost and operating cost that the general problem of optimum selection must find its solution. These properties have not been considered in this paper.

The presentation of the material in the order: Introduction, Derivation of Formulae, Experiments, Rate of Sieving, Nature of Particles, and Literature

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of Sieving, differs somewhat from that originally outlined and differs markedly from the sequence in which the topics were studied. Perhaps an elucidation of the development of the project will indicate why some of the now obvious weak points were not recognized and strengthened. The study of the screening operation was linked through the Nature of Particles portion to a preliminary study of particle shape effects in solids fluidization. The recognition or classification of solid particles as long, flat, or round, led at once to the equivalent separation problem and screening was selected as the most practical method to use. The "three-diemnsional" screen was developed to effect the desired classification and was finally successful. The success of this method of separation was accompanied by the presentation of a similar problem in the separation of wheat-vetch mixtures and soon by the more general grain separation problem. Meanwhile, it was deemed advisable to find or devise a suitable theory to explain the separations obtained and guide the attempts to obtain others. (At this time it was decided that the material should be presented as a master's thesis, rather than in the form of a monograph on the screening operation.) Simultaneously then the work was carried on in four directions: a literature survey of screens and screening was begun; the equipment was further developed; the mathematical analysis was continued; and the grain mixtures fractionated. The grain mixtures yielded what was considered to be a very good set of products in view of the equipment used. The literature survey and mathematical analysis were continued and other apparatus designed. This apparatus was too lately finished to contribute to this paper. The apparent completion of the literature survey marked a natural period for the presentation of the work done even though

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some of the details of the mathematical analysis were undiscovered at that time. While the manuscript was in preparation new library resources became available to us and revision of the literature survey was begun. The mathematical analysis was rounded out and some work on the rate of sieving done while the manuscript was being edited and such facts were included in the final form.

The attempt has been made throughout the work to keep the outlook as broad as possible and yet consistent with the scope of a thesis.

The author owes much to a great many people who have contributed to this paper and wishes it were possible to acknowledge all such debts explicitly here. He at least hopes that those people whose names are not mentioned will appreciate and understand his limitations. To his teachers, including especially his father whose influence was by far the greatest, he owes the background and training necessary for the production of the paper. Acknowledgement is due to Dr. William A. Klemm who guided the project throughout its life and to Dr. Charles L. Nickolls without whose advice and encouragement it would have probably never been finished; thanks are due Mr. Eugene McCroskey for advice concerning mechanical details and material aid in preparation of the pictures presented; other members of the staff at the Chemical Engineering Department of the Oklahoma Institute of Technology and fellow students have all given freely of their ideas. Thanks are due Mr. Boy Nollsch for information concerning threshing machines. To Mr. James D. Wolfe, Mr. Douglass C. Benton, Miss J. Jessie Hume, Mrs. J. Dean Hoffman, and Dr. Darrell Shreve we owe grateful thanks

for critically reading certain portions of the manuscript. Mr. Garland J. Shepherd has contributed many fine suggestions and much hard work to the making of machinery and to the final form of presentation of the paper. Mr. J. Dean Hoffman did much of the work involved in making the pictures. The librarians at the Oklahoma A. and M. College libraries, at Tulsa Fublic library, and at Little Rock Public library have all helped cheerfully and often beyond the call of duty. Mr. Gordon Smith of the Engineering Experiment Station of the Oklahoma Institute built the device pictured in figure (8) which represented a considerable advance in practical machinery at the time. A great deal of credit is due to the author's mother who typed the major portion of the final manuscript under very trying circumstances, and to Miss Clara Louise Pruess, Miss Marque Swartz, Mrs. Opal Jiles, and Miss Frances Ann Stewart, all of whom typed more or less of the material presented. Lithographic negatives were made by the Stillwater Photo-Engraving Company. The final printing of the material was done by Miss Ethel Kidd.

All errors discovered in the paper have been corrected. However, undiscovered errors probably remain in the paper and their discovery will be appreciated by the author, since the way to true facts lies in acknowledging and eliminating errors at the earliest possible date.

The author's own feeling for this paper at this time is that is is merely indicative of what can and should be done. If the paper promotes an interest in sieving problems, and, most important, if it leads to new data and better ideas concerning these problems, then it will have served the purpose for which it was primarily written.

July 3, 1951 Joe Bill Hocott

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## Historical Background

An attempt to ascertain the history of screens or sieves uncovers much confusion. There can be little doubt that sieving is a very ancient art.

Riegel<sup>1</sup> classifies them as follows:

"I. Stationary Screens

- A. Erect
- B. Shaking
- C. Blutergess

II. Screening Surface Moves

- A. Revolving cylindrical screens
- B. Flat screens
  - 1. Electrically vibrated
  - 2. Mechanically vibrated
    - a. Gyratory and Reciprocating combined with balls
    - b. High speed tapped
    - c. Positive circle throw
    - d. Reciprocating motion"

Each type represents a development of the art and each has its own history.

<sup>1</sup> E. Raymond Riegel, <u>Chemical Machinery</u>, p. 43.

While specific dates for the introduction of the various types of screens can not be given they can be roughly approximated. Those stationary screens classified above as "erect" and "shaking" have histories extending from as early as 4000 B. C. to the present. The "erect" screen is commonly seen today in construction work for screening concrete aggregate. The "shaking" screen of this classification is interpreted to mean what is more commonly called a "testing sieve" today and what was classically called a sieve or hand-riddle. The "Blutergess" is of relatively modern origin. It is also known as the "turbine sifter" because of its construction. Revolving "cylindrical screens" or trommels were the industrially important screens of the Nineteenth century. Vibrating "flat screens" are mentioned by Richards in Vol. III of his "Ore-Dressing"2 as new and untried in 1908. Fifteen years later they were discussed but briefly by Edward S. Wiard in "Handbook of Chemical Engineering"". They are the industrially important screens of today. Thus 33 pages of 70 in Taggart's "Handbook of Mineral Dressing"<sup>4</sup> (1945) are devoted to vibrating screens and only 15 pages to all other types combined. Many different means for causing the vibration have been employed in this forty-year period.

The "reciprocating motion, mechanically vibrated, flat screen" mentioned in this classification is usually called a shaking screen. This screen enjoyed its greatest popularity during the last quarter of the nineteenth century and the first quarter of the twentieth century.

- 2 Robert H. Richards, Ore-Dressing
- 5 Donald M. Liddel, Ed., Handbook of Chemical Engineering
- Arthur F. Taggart, Ed., Handbook of Mineral Dressing

The use of sieves is intimately associated with many industries and something of their history may be learned by study of the histories of these industries which include: (1) ore-dressing, (2) paper and pulp manufacture, (3) soil analysis, etc., (4) sugar industry, (5) pigment and paint industry, (6) ceramic industry, (7) cement industry, (8) fertilizer industry, (9) coal separation, (10) chemical manufacture, (11) coke (and gas) industry, (12) drugs, medicines, and cosmetics manufacture, (13) explosives industry, (14) rubber industry<sup>5</sup>, (15) seed cleaning and harvesting, (16) powder metallurgy, and (17) flour milling.

For example, Jacob<sup>6</sup> states, "A large square sieve was also used for separating---" and "Their best bread was of wheat, made from a specially sifted flour". The context indicates that this was prior to 3000 B. C.

Certain modifications of screening machinery are particular to single industries. The pulp and paper industry is probably the outstanding example of this point with its sliver screens, diaphram screens, and variety of rotary screens<sup>7</sup>.

Something of the history of sieves, etc. may be learned by study of the etymology of the words denoting the apparatus. Cassell's Latin-English and English-Latin Dictionary<sup>8</sup> which deals with "classical" Latin gives the following information:

<sup>5</sup> Theodore R. Olive and R. Norris Shreve, <u>Chem. & Met.'s Chemical Engineering Flow Sheets</u>, p. vii.

<sup>6</sup> Jacob, Six Thousand Years of Bread, pp. 22 & 35.

" G. S. Witham, Sr., Modern Pulp and Paper Making

<sup>8</sup> J. R. V. Marchant, M.A. and Joseph F. Charles, B.A., <u>Cassell's Latin</u> <u>English and English-Latin Dictionary</u>.

"Sieve - n. cribrum.

sift - v. tr. cribrare (Plin.) cribro,

cribrum,-i, n. (from root cre, cri)"

In KYKKO775 , English-Greek and Greek-English Dictionary<sup>9</sup> we find these definitions:

While not particularly good proof, these quotations are good indications that sieves were known to the Romans and Greeks in early times.

The Oxford English Dictionary<sup>10</sup> gives an excellent account of the terms sieve, riddle, sift, and screen showing the development of these terms from about 725 A. D. to 1200 A. D. up to date. The following uses are noted from quotations from this dictionary with the date for the quotation: (1) Cookery (1430 A. D.) (2) Manufacture of gunpowder (1508) (3) Grain-eleaning (1530) (1440) (4) Milling meal (1440) (5) Wax-purification (1577) (6) Ash-sifting (1683) (7) Lime and (8) Sand-sifting (1703).

 <sup>9</sup> XYKKOJIS, English-Greek and Greek-English Distionary
 <sup>10</sup> James A.H. Murray, Henry Bradley, W. A. Craigie and C.T. Onions, Eds. <u>The Oxford English Distionary</u> VIII p. 652, VIX p. 273.

Another mode of attacking the problem is found in study of the materials from which sieves are made and their history. These include: Cloth, wood, copper, brass, steel, stainless steels, monel metal, etc. Some of these materials are relatively new, others are so old that they have vague and uncertain pasts.

There are several good reasons for supposing that the first sieves were made from cloth. In fact, any piece of cloth is a sieve; how long the cloth was utilized by men until this fact was recognized is unknown. It was probably not too long.

The origin of cloth is also lost in antiquity having begun in the early stone age which ended about 4000 B.C. There is evidence from the Egyptian tombs that weaving was a highly developed art at that time.

An art whose time of origin was either the same as that of cloth-manufacture or earlier is basket weaving. The raw materials used for basketry occur naturally and several of them are usable in their natural state. Most cloths require spun threads. Conceivably, however, the first "cloth" was woven from grasses or similar material. At a much later date the use of bamboo sieves in paper making by the Chinese is recorded. Dard Hunter<sup>11</sup> gives the date for this accomplishment at about 150 A.D. He is of the opinion that the "wove" (cloth) mold was used some 50 years earlier. This "wove" mold is woven from China grass.

The use of metals in making sieves is obscured by time. It is known that the Egyptians made iron about 4000 B.C. and it is possible that an idea of their applications of it could be gained from study of their metal working

11 Dard Hunter, Paper-Making, Chapter IV.

tools. The Assyrians are said to have been skilled iron workers. Wire drawing was not discovered until about 1300 A.D.; until that time wire was made by hammering metal into thin sheets, cutting it into narrow strips, and hammering these round. It is probable that little time elapsed between the first making of wire and its weaving; the initiation of neither of these procedures has been established with any certainty.

## Manufacture of Sieves or Screens.

Sieves are manufactured in a very well defined range of sizes, roughly from about 4 inches (100 000  $\mu$ ) down to 37  $\mu$  (0.00146 inches). While these limits are rather arbitrary there are reasons for their existing.

Above the upper limit of about 4 inches sieving must compete with measurements by means of rulers, or calipers. The latter is more accurate and requires less power; it can not be made automatic without deviating from normal procedure. Secondly, above this limit particle dimensions are in many cases subject to fair control in manufacture and are hence known.

Two excellent reasons exist for the bottom limit. First, it is technologically almost impossible to manufacture a screen having more than 300 to 400 openings per linear inch with satisfactory precision; this reason might be soon outdated. The second reason is that the capacity of a screen varies inversely with the "diameter" of its aperture (and presumably cost of screening closely parallels capacity); thus in finer and finer sizes screening loses its principal advantages of high capacity and low cost. Certain developments may give screening some other advantages, but in fine screening these two will be much less important than other considerations. At this time it seems extremely unlikely that the limits of screening will be broadened beyond 6 or 8 inches to 5 or 6 microns. Perhaps reason can be advanced to justify this expansion.

Theoretically, of course, there are no limits beyond which screening can not be applied. Filtration, dialysis, and even diffraction phenomena can be regarded as "screening" processes --- but the nature of the screen opening and of the particle dimensions are rather vague for these cases, and other forces probably become exceedingly important.

Two methods of manufacturing account for the great majority of industrial screens now in use; these are the weaving of wire and the punching of metal plates. These two processes lead to products which are competitive in certain size ranges but definite trends have been established. Punched plates are usually preferred for the heavy-duty of sizes approximately 1/4 or 1/2 inch up; woven wire screens are manufactured in sizes from four inches to 37 by weaving "on immense power looms. They are manufactured from a wide variety of metals including steel, copper, brass, bronze, phosphor-bronze, monel, stainless steels, etc. They may be finished by galvanizing, tinning or lacquering where this is desirable<sup>w12</sup>. One of the principal problems in woven wire screens is the obtaining of a uniform aperture. Various devices, (usually some forms of crimping) have been employed to "lock" the wires in fixed relationship to each other.

Only the most general statements may be made concerning the manufacture of punched plate sieves. Taggart<sup>13</sup> lists the following types of punched plates available: "(1) round, staggered; (2) square, straight rows, staggered rows; (3) slotted, hit-and-miss sideways, hit-and-miss endways, diagonal, straight parallel rows, all full-open or burred".

<sup>12</sup> The W. S. Tyler Co., Woven Wire Screens, pp. 7, 98.

<sup>13</sup> Taggart, Op. cit. p. 7-14.

Among other methods of manufacture of screens which deserve mention are electrodeposition on a matrix made by any of several patented processes, and sintering together powdered metal particles in a stainless steel, electrically heated mold.

A number of patents have been issued for the manufacture of screens by electrodeposition.  $^{14}{}_{,15}{}_{,16}{}_{,17}{}_{,18}{}_{,19}$  At least one of these processes has reached the stage of commercial importance. The principal advantage of this method is the close tolerances which may be achieved in the smaller apertures. Screens of "400 mesh" made by this method are available. These possess those advantages and disadvantages, which punched plates have as compared with wovenwire screens.

While the powder metallurgy method is the subject of a patent<sup>20</sup>, no instance of the commercially available screen made therby is known to this author. The importance of the process as we see it may be appreciated by analogy. A fair screen may be made by packing steel balls of uniform size in a frame so that they are constrained to remain essentially fixed in position; the apertures

14 Edward O. Norris, "Electrolytic Production of Foraminous Sheets or Screens of Metal Such as Copper," U. S. Patent 2 166 366.

15 Willem Van de Pol, "Sieves," U. S. Patent 2 221 502.

16 Watson Beebe, "Electrolytic Production of Fine Mesh Metal Screens," U. S. Patents 2 225 733 and -4.

17 Edward O. Norris, "Electrolytic Production of Fine Metal Screens," U. S. Patents 2 226 381, -2, -3, and -4.

Edward O. Norris, "Reinforcing Thin Metallic Screens by Electrodeposition," U. S. Patent 2 246 380.

<sup>19</sup> Bertram G. Kathe, "Electrodeposition," U. S. Patent 2 381 911.

20 William G. Calkins, Roy E. Blue, and Ward W. Marvin, "Powdered-Metal. Products Such as Screen Material," U. S. Patent 2 267 372.

are the interstices between the spheres. If the spheres are very small they constitute a "powder" which while it can not be conveniently put in a frame can conceivably be fused into a screen plate by the sintering process of powder metallurgy. The utility of such a "screen" must be determined experimentally, of course. For a "screen" the particles should be but a single layer deep-which is an important consideration. For filters, however, multiple layers of particles is all right. The age-old sand filter is an excellent example.

## Relations Between Sieves and Other Equipment

Sieves exhibit two different types of relationships with other equipment. On the one hand is the relationship in purpose ranging from solid-solid separation in the dry state, through wet screening to filtration. There is no sharp dividing line between filter media and screening media. The jig contains a screen as an integral part of its mechanism and might be conceived of as a screening operation in a restricted sense. An interesting analogy can be drawn between screening and tabling since both have component elements which are restrictive spaces joining less restrictive spaces. These two devices again have a common purpose in many cases. On the other hand is the relationship in nature connecting the fixed openings of the sieve with the variable openings of crushers. A number of crushers employ screens as integral parts of the crusher. The stamp mill is an example.

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Van de Pol, Willem, "Sieves." U. S. Patent 2 221 502 (November 12, 1940).

Witham, G. S., Sr. Modern Pulp and Paper Making. New York: Reinhold Publishing Corporation.

A survey of the available and easily accessible literature revealed only one departure from the custom of designating particle size as the mean "dimension" of the retaining sieve and the last non-retaining sieve. The one exception involved an unelucidated graphical analysis based on the assumption of a rhombic section of the particles sieved.<sup>1</sup>

This procedure is inadequate in most cases for at least two reasons. First, only spherical particles can be defined precisely by a single dimension. Second, the restraint imposed by different types of sieves gives different kinds of information concerning particle dimensions.

The procedure used in the derivations of the formulae presented is quite general and could be applied to any type of sieve opening and to any particle. The general ideas have been retained as a model. The explicit specific cases have been discussed which seemed most interesting from the viewpoints of simplicity, approximation to existing equipment, and convenience. An infinite number of other specific formulae can be derived from the general formula presented; conditions will determine the profit with which they may be so derived.

These equations being independent of time as they are can be considered as a first step in the development of more general and more useful equations to provide a sufficient theoretical background for the optimum selection of screening equipment for solid-solid separations.

Gordon Rittenhouse, "Relationship of Shape to the Passage of Grains Through Sieves," Ind. Eng. Chem., Anal. Ed. 15, (1943) 153-5.

Limitations of time, equipment, and ability have governed the scope of the material studied. The heart of this paper is to be found in the formulae of this section and experimental evidence, both qualitative and quantitative, which is presented in their light.

Assuming that the dimensions of a particle, which will pass a series of sieves and will be rejected another series of sieves, are completely defined by the dimensions of the sieves will allow us to write certain mathematical equations which should be useful in defining the products obtained from such sets of sieves or in designing a set of sieves to effect certain separations or definitions of products. This first broad assumption might be expressed as a mathematical equation:

$$f(L_{9}W_{9}T_{9}d_{2}^{----etc_{0}}d_{1}^{\prime}_{9}d_{2}^{\prime}----etc_{0},d_{1}^{\prime\prime}_{9}d_{2}^{\prime\prime}----etc_{0}) = 0$$
(1)  
Where L = "length" of particle  
W = "width" of particle  
T = "thickness" of particle  
 $d_{1}d_{2}d_{1}^{\prime}_{9}d_{2}^{\prime}_{9}d_{1}^{\prime\prime}_{9}d_{2}^{\prime\prime}_{9}$  etc. = dimensions  
necessary to define the sieves.

That this equation is generally true has yet to be proved, but it has utility as a summary of ideas. The terms "length", "width", and "thickness" are to be taken cautiously, since it is difficult to define them for certain irregular particles. Equation (1) is usually too broad to be useful; the functional relationship between its variables has yet to be discovered. With this in mind we propose definition of the dimensions of the particle separately. Thus assuming that the thickness of a particle (which we shall define arbitraily as its least overall dimension) may be completely defined in terms of the

sieve through which it will pass and the dimensions of a sieve through which it will not pass we may write:

$$f(T_s_{1s}_{2}^{s}_{$$

Where T = "thickness" of the particle  $s_1 s_2 s_1 s_2 s_1 e, s_2 e, etc. = dimensions$ necessary to define the sieve system which in turn defines the particle "thickness"

This equation is useful only when "thickness" of the particle has definite physical meaning, and when the dimensions necessary to characterize the sieve system are known. Until we specifically state otherwise we shall assume that "thickness" of the particle refers to a definite physical dimension.

For the case where a single dimension serves to define the sieve through which the particle will pass and a single dimension serves to define the sieve through which the particle will not pass, equation (2) simplifies to:

$$\mathbf{f}(\mathbf{T}, \mathbf{s}_{1}, \mathbf{s}_{2}) = 0 \tag{3}$$

If f(T) represents some property of the particle dependent on T which determines that the particle will pass the sieve in one case and will not pass the sieve in the next case, it is logical to suppose that:

 $s_2 < f(T) < s_1$  (4)

(2)

Inequality (4) may be written in the form of an equation:

$$f(T) = \frac{s_1 + s_2}{2} + \frac{s_1 + s_2}{2}$$
(5)

If we limit ourselves to those cases where:

$$\mathbf{f}^{*}(\mathbf{T}) = \mathbf{T} \tag{6}$$

Substitution of the value of f (T) from equation (6) into equation (5) serves to give us a general expression for thickness of particles in terms of sieve dimensions alone:

$$T = \frac{s_1 + s_2}{2} + \frac{s_1 - s_2}{2}$$
(7)

Two procedures might be followed in determining the width of a particle in terms of the dimensions of sieves. Firstly, the particle might be constrained to move so that the width was the only dimension subject to the measuring restraint. Secondly, the thickness of the particle might be established, the sectional area (or other function of thickness and width) established by use of proper sieves, and the width calculated therefrom.

We will discuss only the second procedure outlined above in this paper. This treatment closely parallels that for definition of particle thickness.

Assume that the width of a particle (which we shall define as its second smallest dimension) may be completely defined in terms of the dimensions of a sieve through which the particle will pass and the dimensions of a sieve through which the particle will not pass, and the thickness of the particle, we may write:

$$f(T_*W_*d_1_*d_2, etc_* d_1^*, d_2^*, etc_*) = 0$$

Where: T = "thickness" of the particle W = "width" of the particle d<sub>1</sub>.d<sub>2</sub>.d<sub>1</sub>'.d<sub>2</sub>'. etc. = the dimensions necessary to define the sieve system.

As before, the number of dimensions required to define a sieve is dependent on the nature of the sieve. We shall limit our discussion to the relatively simple cases where each sieve is defined by a single dimension, round and square holes.

For the case where each sieve of the pair may be defined by a single dimension equation (8) simplifies to:

$$f(T_{\mathcal{W}}d_{1}d_{2}) = 0$$
(9)

Where:  $d_1$  and  $d_2$  = diameters of sieve openings, 1 and 2

### respectively.

If f(T,W) represents some property dependent on T and W, which determines that the particle will pass one sieve and fail to pass the other, we may write:

$$\mathbf{d}_{2} < \mathbf{f}(\mathbf{T}, \mathbf{W}) < \mathbf{d}_{1} \tag{10}$$

Inequality (10) may be written in the form of an equation:

$$f(T,W) = \frac{d_1 + d_2}{2} \pm \frac{d_1 - d_2}{2}$$
(11)

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(8)

For sieves consisting of round holes and particles of rectangular section, it may be demonstrated that:

$$W^2 + T^2 = d^2$$
 (12)

From the solution of this equation for  $W_{2}$  and setting up new inequalities and equations it follows that:

$$W = \sqrt{\frac{d_1^2 - r^2}{2}} + \sqrt{\frac{d_2^2 - r^2}{2}} + \sqrt{\frac{d_1^2 - r^2}{2}} - \sqrt{\frac{d_2^2 - r^2}{2}}$$
(13)

Alternately, solution of equation (12) for d and substitution in inequality (10) lead to identification of  $f(T_{,W})$  as  $\sqrt{W^2 + T^2}$  and make equation (11) read:

$$\sqrt{w^2 + r^2} = \frac{d_1 + d_2}{2} \pm \frac{d_1 - d_2}{2}$$
(14)

It may be demostrated that equations (13) and (14) are consistent. The choice between them should be governed by the problem being considered and the convenience of the application of the equations.

For sieves consisting of square holes and particles of rectangular crosssection, on the other hand, equation (12) must be replaced by a different equation. i.e. equation (9) takes the form:

$$W = \frac{d}{\cos B} = T \tan B$$
(15)

- Where B = the smallest angle which the side of the particle wakes with the side of the sieve.
  - d = length of one side of the square sieve opening.

For the sake of simplicity we may use in place of equation (15) a pair of equations:

$$W = d; (When T > d(\sqrt{2} - 1))$$
 (16)

or:

$$W = d\sqrt{2} - T;$$
 (When  $T \leq d(\sqrt{2} - 1)$ ) (17.)

When equation (17) is used to define equation (9) it follows that:

$$d_2 \sqrt{2} - T < W < d_1 \sqrt{2} - T$$

or alternately:

$$W = \frac{d_1 \sqrt{2} - T + (d_2 \sqrt{2} - T)}{2} + \frac{d_1 \sqrt{2} - T - (d_2 \sqrt{2} - T)}{2}$$
(19)

Equation (19) may be written also:

$$W = \sqrt{2} \left[ \frac{d_1 + d_2}{2} + \frac{d_1 - d_2}{2} \right] - T$$
 (19-A)

If we solve equation (17) for d we may equally well write for equation

(10)

$$d_2 < \frac{W + T}{\sqrt{2}} < d_1$$
 (20)

or alternately:

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$$\frac{W + T}{\sqrt{2}} = \frac{d_1 + d_2}{2} \pm \frac{d_1 - d_2}{2}$$
(21)

If, on the other hand equation (16) is used to define equation (9) it follows that:

$$d_2 < W < d_1 \tag{22}$$

or alternately:

$$W = \frac{d_1 + d_2}{2} \pm \frac{d_1 - d_2}{2}$$
(23)

This is entirely equivalent to equation (7) except that it defines W instead of T.

As before, two procedures might be followed in determining the length of a particle in terms of the dimensions of sieves. Firstly, the particle might be constrained so that the length is the only dimension subject to the measuring restraints. Secondly, the thickness and width of the particle might be established by any means, some function of length, width and thickness established by sieving with proper sieves and the length calculated therefrom.

Again, only the second procedure is to be discussed in this paper. This treatment closely parallels those preceding it for definition of particle thickness and particle width.

Assuming that the length of a particle (which we shall temporarily define as the third smallest dimension of a particle [largest dimension may be better]) may be completely defined in terms of the thickness and width of the particle and the dimensions of a sieve through which the particle will pass and the dimensions of a sieve through which the particle will not pass we may write:

$$f(L_sW,T_s, d_{1}, d_{2}, d_{3}, etc_{o} = 0 \qquad (24)$$

As previously stated, the number of dimensions required to define a sieve is dependent on the nature of the sieve.

Since the simplest sieve we have been able to devise to effect the separation of particles of different lengths (with the same cross-sections)

requires three dimensions for its definition, we shall limit our discussion to that case. Equation (24) simplifies to:

$$f(L_y W_y T_y d_{2y} d_{3y} d_{1y} d_{2y} d_{3y} d_{1y} d_{3y} d_{3y}$$

Following our usual procedure, if  $f(L_sW_sT)$  represents some property of a particle (dependent on L, W, and T), which determines that the particle will pass one sieve but will not pass another, we may write:

$$\mathbf{f}(\mathbf{d}_{1} \boldsymbol{\rho} \mathbf{d}_{2} \boldsymbol{\rho} \mathbf{d}_{3} \boldsymbol{\rho}) < \mathbf{f}(\mathbf{L}_{\mathfrak{s}} \mathbb{W}_{\mathfrak{s}} \mathbb{T}) \leq \mathbf{f}(\mathbf{d}_{1} \boldsymbol{\rho} \mathbf{d}_{2} \boldsymbol{\rho} \mathbf{d}_{3})$$
(26)

Where: f(d<sub>1</sub>,d<sub>2</sub>,d<sub>3</sub>) and f(d<sub>1</sub>,<sup>1</sup>d<sub>2</sub>,d<sub>3</sub>) = functions characterizing the sieve openings. f(L,W,T) = property defined above.

For equation (26) to be useful  $f(d_1, d_2, d_3)$ ,  $f(d_1, d_2, d_3)$ , and  $f(L_0, W_0, T)$ must be defined and evaluated. While attempts to achieve this definition have not been successful, an alternate procedure has yielded useful results. This procedure postulates:

$$\mathbf{L} = \mathbf{f}(\mathbf{d}_{1} \mathbf{d}_{2} \mathbf{d}_{3} \mathbf{d}_{1}^{*} \mathbf{d}_{2}^{*} \mathbf{d}_{3}^{*} \mathbf{d}_{3}^{*} \mathbf{T}_{p} \mathbf{W})$$
(27)

This is developed into the inequality:

$$f(d_{1}, d_{2}, d_{3}, T, W) < L < f(d_{1}, d_{2}, d_{3}, T, W)$$
 (28)

which is equivalent to:

$$L = \frac{f(d_1, d_2, d_3, T, W) + f(d_1, d_2, d_3, T, W)}{2}$$
 (Continued on next page)

$$\frac{f(d_{1},d_{2},d_{3},T,W) - f(d_{1},d_{2},d_{3},T,W)}{2}$$
(29)

The functions of equation (29) must be defined or evaluated to make the equation useful. From geometrical considerations it will be shown in the concluding paragraphs of this chapter that these functions can be defined for certain cases as indicated in Table I on page 32. Substitution of these values into equation (29) yields respectively equations (30) through (35) which follow. Two symbols appear in these equations which have not yet been defined these are:

c = the clearance between the baffle-plate and the sieve plate
 t = the thickness of the sieve-plate.

$$L = \frac{c_{1} + t_{1}}{T^{2} - t_{1}^{2}} \left[ T \sqrt{d_{1}^{2} - W^{2}} - t_{1} \sqrt{d_{1}^{2} - W^{2}} - (T^{2} - t_{1}^{2}) \right]$$

$$+ \frac{\partial_{2} + t_{2}}{T^{2} - t_{2}^{2}} \left[ T \sqrt{d_{2}^{2} - W^{2}} - t_{2} \sqrt{d_{2}^{2} - W^{2}} - (T^{2} - t_{2}^{2}) \right]$$

$$\frac{2}{2}$$

$$+ \frac{c_{1} + t_{1}}{T^{2} - t_{1}^{2}} \left[ T \sqrt{d_{1}^{2} - W^{2}} - t_{1} \sqrt{d_{1}^{2} - W^{2}} - (T^{2} - t_{1}^{2}) \right]$$

$$= \frac{c_{2} + t_{2}}{T^{2} - t_{2}^{2}} \left[ T \sqrt{d_{2}^{2} - W^{2}} - t_{2} \sqrt{d_{2}^{2} - W^{2}} - (T^{2} - t_{2}^{2}) \right]$$

$$(30)$$

Equation (30) is for right prismatic particles of rectangular crosssection and plane baffled, round-holed sieves, with thin sieve plates (where  $\leq 45^{\circ}$ ).

$$L = \frac{c_{1} + t_{1}}{T^{2} - t_{1}^{2}} \left[ Ts_{1} - t_{1} \sqrt{s_{1}^{2} - (T^{2} - t_{1}^{2})} \right]$$

$$+ \frac{c_{2} + t_{2}}{T^{2} - t_{2}^{2}} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$+ \frac{c_{1} + t_{1}}{T^{2} - t_{2}^{2}} \left[ Ts_{1} - t_{1} \sqrt{s_{1}^{2} - (T^{2} - t_{1}^{2})} \right]$$

$$= \frac{c_{1} + t_{2}}{T^{2} - t_{2}^{2}} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{2} + t_{2}}{T^{2} - t_{2}^{2}} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{2} + t_{2}}{T^{2} - t_{2}^{2}} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{1} + t_{1}}{2} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{1} + t_{2}}{T^{2} - t_{2}^{2}} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{1} + t_{2}}{2} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{1} + t_{2}}{2} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{1} + t_{2}}{2} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{1} + t_{2}}{2} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{1} + t_{2}}{2} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{1} + t_{2}}{2} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{2} + t_{2}}{2} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

$$= \frac{c_{2} + t_{2}}{2} \left[ Ts_{2} - t_{2} \sqrt{s_{2}^{2} - (T^{2} - t_{2}^{2})} \right]$$

Equation (31) is for right prismatic particles of rectangular cross-section and plane, baffled, square-holed sieves with thin sieve plates (where  $\alpha \leq 45^{\circ}$ ), when the particle's axis is parallel to a side of the square opening.

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$$\frac{c_{1} + t_{1}}{T^{2} - t_{1}^{2}} \left[ T(s_{1}\sqrt{2} - W) - t_{1}\sqrt{(s_{1}\sqrt{2} - W)^{2} - (T^{2} - t_{1}^{2})} \right]$$

$$\frac{c_{2} + t_{2}}{T^{2} - t_{2}^{2}} \left[ T(s_{2}\sqrt{2} - W) - t_{2}\sqrt{(s_{2}\sqrt{2} - W)^{2} - (T^{2} - t_{2}^{2})} \right]$$

(Continued on next page)

$$\pm \frac{\frac{c_{1} + t_{1}}{T^{2} - t_{1}^{2}} \left[ T(s_{1}\sqrt{2}-W) - t_{1}\sqrt{s_{1}}\sqrt{2}-(T^{2} - t_{1}^{2}) \right]}{2}$$

$$\pm \frac{\frac{c_{2} + t_{2}}{T^{2} - t_{2}^{2}} \left[ T(s_{2}\sqrt{2}-W) - t_{2}\sqrt{(s_{2}\sqrt{2}-W)^{2} - (T^{2} - t_{2}^{2})} \right]}{2}$$

$$(32)$$

Equation (32) is for right prismatic particles of rectangular cross-section and plane, baffled, square-holed sieves with thin sieve plates, (where  $\alpha \leq 45^\circ$ ) when the particle's axis is parallel to a diagonal of the square opening.  $4\sqrt{2}\left[c_{+}+\sqrt{d_{+}^{2}-W^{2}}-T\sqrt{2}\right]+\sqrt{2}\left[c_{+}+\sqrt{d_{-}^{2}-W^{2}}-T\sqrt{2}\right]$ 

L

$$= \frac{\sqrt{2 \left[c_{1} + \sqrt{d_{1}^{2} - W^{2}} - T\sqrt{2}\right] + \sqrt{2 \left[c_{2} + \sqrt{d_{2}^{2} - W^{2}} - T\sqrt{2}\right]}}{2}$$

$$= \frac{\sqrt{2 \left[c_{1} + \sqrt{d_{1}^{2} - W^{2}} - T\sqrt{2}\right] + \sqrt{2 \left[c_{2} + \sqrt{d_{2}^{2} - W^{2}} - T\sqrt{2}\right]}}{2}$$
(33)

Equation (33) is for right prismatic particles of rectangular cross-section and plane baffled circular-holed sieves with thick sieve plates, (where  $\alpha \ge 45^{\circ}$ ).

$$L = \frac{\sqrt{2} \left[ c_{1} + s_{1} - T \sqrt{2} \right] + \sqrt{2} \left[ c_{2} + s_{2} - T \sqrt{2} \right]}{2}$$

$$\frac{\sqrt{2} \left[ c_{1} + s_{1} - T \sqrt{2} \right] - \sqrt{2} \left[ c_{2} + s_{2} - T \sqrt{2} \right]}{2}$$
(34)

Equation (34) is for right prismatic particles of rectangular crosssection and plane, baffled, squared-holed sieves with thick sieve plates (where

**≩45°)** when the particle's axis is parallel to a side of the square opening.

$$\frac{\sqrt{2} \left[ \mathbf{o}_{1} + \mathbf{s}_{1} \sqrt{2} \mathbf{w} \mathbf{T} \sqrt{2} \right] + \sqrt{2} \left[ \mathbf{o}_{2} + \mathbf{s}_{2} \sqrt{2} \mathbf{w} \mathbf{T} \sqrt{2} \right]}{2}$$

$$\frac{\sqrt{2} \left[ \mathbf{o}_{1} + \mathbf{s}_{1} \sqrt{2} \mathbf{w} \mathbf{T} \sqrt{2} \right] - \sqrt{2} \left[ \mathbf{o}_{2} + \mathbf{s}_{2} \sqrt{2} \mathbf{w} \mathbf{T} \sqrt{2} \right]}{2}$$

$$(35)$$

Equation (35) is for right prismatic particles of rectangular crosssection and plane, baffled, squared-holed sieves with thick sieve plates (where  $\ll \geq 45^{\circ}$ ), when the particle's axis is parallel to a diagonal of the square opening.

The exhibition of the considerations leading to the values of the functions set forth in Table I is still necessary. Since similar considerations will probably be valuable in deriving other formulae they are presented in very general form.

Attention is focused upon the partiale and is concentrated upon that dimension or property of the particle which is to be determined by sloving. Then the following steps are taken:

(1) A critical sieve opening is disposed about the particle so that an infinitesimal change in the dimensions (or orientation) of the sieve will determine whether or not the particle may pass through the sieve opening. Thus the particle will pass through any sieve opening similar to but larger than the oritical sieve opening and will not pass through any sieve opening similar to but smaller than the critical sieve opening. (The orientation most favorable to the particle's passing through the

sieve opening is used in all of these derivations.) This step is illustrated in Figures 1, 2, and 3 (pp. 26, 27, and 28).

(2) From geometric considerations of the relationship between the critical sieve opening dimensions and particle dimensions, a functional relationship is discovered between them. This step may profitably be analyzed in greater detail.

(a) An equation is written relating the angle of inclination of the particle's long axis to the plane of the sieve opening, to the clearance between the baffle plate and the sieve plate and to the restrictive thickness of the sieve plate. The restrictive thickness may or may not be equal to the total thickness of the sieve plate. For those cases considered here the restrictive thickness is equal to the total sieve plate thickness when the angle of inclination between the particle's long axis and the sieve plate is less than forty five degrees and is equal to the restrictive thickness of the every case considered here the following equation obtains:

$$c + t_2 = L \sin \alpha$$
 (36)

When  $\alpha = 0^{\circ}$ , L is undefined by this equation, but it follows that  $t_{T} = -0$  as intuition dictates. For values of  $\alpha$  between 0° and 45°, the "thin-plate" formulae, equations (30), (31), and (32), are derived; for values of  $\alpha$  between 45° and 90°, the "thick-plate" formulae, equations (33), (34), and (35) are derived.

(b) An equation is written relating the total particle length to those portions lying on either side of a plane parallel to the







cross-section of the particle and containing a point of restriction in the plane of the top surface of the sieve plate. Thus in every case considered here the following equation obtains:

$$\mathbf{L} = \mathbf{X} + \mathbf{Y} \tag{37}$$

(\*) An equation is written relating the clearance between the baffle plate and the sieve plate to the angle of inclination between the particle's long axis and the plane of the sieve opening, the particle's thickness, and the length of that portion of the particle lying between the plane described above (b) and the plane of the bottom baffle surface. Again the same equation is obtained for all (six) cases considered here. This is:

$$\mathbf{a} = \mathbf{Y} \sin \mathbf{\alpha} + \mathbf{T} \cos \mathbf{\alpha} \tag{38}$$

(d) An equation is written relating a characteristic sieveplate opening and the sieve-plate restrictive thickness to the angle of inclination between the particle's long axis and the plane of the sieve-plate opening, and one or more of the particle's minor dimensions (thickness and width). Three different equations are obtained for the (six) cases considered; the same equations apply to both thick plates and thin plates. These equations are;

$$\sqrt{d^2 - W^2} = \frac{T}{d^2 - W} + \frac{t_m}{t_{an} \alpha}$$
(39)

for baffled round hole sieves,

$$s = \frac{T}{sln \alpha} + \frac{t_2}{tan \alpha}$$
(40)
## SIEVE ANALYSIS - DERIVATION OF FORMULAE

for baffled square hole sieves where the particle's optimum orientation is with its long axis parallel to the (plane) sides of the sieve opening, and

$$s\sqrt{2} - W = \frac{T}{\sin \alpha} + \frac{t_r}{\tan \alpha}$$
(41)

for baffled square hole sieves where the particle's optimum orientation is with its long axis at an angle of forty five degrees to the (plane) sides of the sieve opening.

(e) An equation is written relating that portion of the particle lying between the plane described above (b) and a plane parallel to it and containing the lower least (TW) face of the particle, to a characteristic sieve plate opening dimension, sieve plate thickness, and one or more of the particle's minor dimensions (thickness and width). Again, three different equations are obtained for the (six) cases considered; the same equations apply to both thick plates and thin plates. These equations are:

$$X = \sqrt{d^2 - W^2 - (T^2 - t^2)}$$
 (42)

for baffled round hole sieves,

$$I = \sqrt{s^2 - (I^2 - t^2)}$$
 (43)

for baffled square hole sieves where the particle's optimum orientation is with its long axis parallel to the (plane) sides of the sieve opening, and

$$K = \sqrt{(s\sqrt{2}-W)^2 - (T^2 - t^2)}$$
(44)

#### SIEVE ANALYSIS - DERIVATION FORMULAE

for baffled square hole sieves where the particle's optimum orientation is with its long axis at an angle of forty five degrees to the (plane) sides of the sieve opening.

The proper equations of (2)(a), (b), (c), (d) and (e) are then combined to eliminate those quantities (**Q**, X, and Y) which are difficult or impossible to evaluate and the resulting equation is solved for the dimension which is to be established by the sieving operation.

For convenience the definitions of the symbols appearing in Table I are repeated here:

- L = "length" of the particle
- W = "width" of the particle
- T = thickness of the particle
- c = the clearance between the baffleplate and the sieve plate.
- t = the thickness of the sieve plate
- d = the diameter of a circular sieve plate opening
- s = the side of a square sieve plate opening.
- tr = restrictive sieve-plate thickness.

#### SIEVE ANALYSIS - DERIVATION OF FORMULAE

#### TABLE I

#### Definition of Functions

- $f(d_{1},d_{2},d_{3},T,W), \text{ etc. of Equation (29)}$ Thin plates  $(\propto \leq 45^{\circ})$ Thick plates  $(\propto \leq 45^{\circ})$   $f = \frac{c+t}{T^{2}-t^{2}} \left[T\sqrt{d^{2}-W^{2}} + \sqrt{d^{2}-W^{2}} + \sqrt{d^{2}-W^{2}} T\sqrt{2}\right]$   $-t\sqrt{d^{2}-W^{2}-(T^{2}-t^{2})}$   $f = \sqrt{2}\left[0 + \sqrt{d^{2}-W^{2}} T\sqrt{2}\right]$   $f = \sqrt{2}\left[0 + s T\sqrt{2}\right]$   $f = \sqrt{\sqrt{s^{2}-(T^{2}-t^{2})}}$
- (2) Square openings

Circular

(1)

- (a) particle axis parallel to sides of square
- (b) particle axis parallel to a diagonal
- $f = \frac{a + t}{T^2 t^2} \left[ T(\sqrt{2} \text{ s-W}) \quad f = \sqrt{2} + \sqrt{2 W T} \sqrt{2} \right]$  $t \sqrt{(s\sqrt{2} W)^2 (T^2 t^2)}$

The apparatus shown in Figure 4 was made by cutting a flat ring 7-7/8 inches outside diameter, 5-5/8 inches inside diamenter and a flat disc 6 1/2 inches in diameter from 16 gauge sheet metal; four 3/16 inch bolts were brazed to the ring near its inner edge; slots were cut in the disc to fit over these bolts and a 6 1/2 inch disc of composition board was cut and bolted with small



# Figure (4)

First Three-dimensional Sieve

bolts to the metal disc; the composition board was then beveled to just fit inside the flat ring; holes were cut in the composition board to accommodate washers for spacing the disc from the flat ring: washers were used to effect this spacing; and washers and bolts were placed over the disc to hold the assembly rigidly. This assembly was then placed within a Tyler standard 8 inch brass sieve and the

outer edge was sealed to the brass sieve by means of "Scotch tape. The assembly was used in conjunction with several different Tyler Standard sieves.

This apparatus was the first working model of a three-dimensional sieve\* to be built, though two or three other previous designs were drawn up but not built due to complexity. (\*A three-dimensional sieve is defined as a sieve



Figure (5)

Sieve Fractionated Sawdust

restrictive in three directions. A sieve is defined as one or more restrictive spaces joining less restrictive spaces. together with the materials defining the space or spaces.)

The results obtained with the sieve in fractionating sawdust according to its shape are shown in Figures (5) and (6) on Pages 34 and 35. The four samples labeled A, B, C, and D in Figure (5) were all derived from the

same Tyler standard screen sized material (14/28), by sieving with a baffled Tyler standard 14 screen until practically no material passed the sieve in a 20 minute interval, raising the baffle-plate a few thousandths of an inch and resieving the oversize as before, then raising the baffle-plate a few thousandths of an inch farther and again resieving the oversize, etc.



Figure (6)

Sieve Fractionated Sawdust

The pictures labeled E and F (Figure 6) represent respectively the oversize and the undersize from a three-dimensional sieving with a baffled Tyler standard 48 screen of a Tyler standard screen size (48/65). The pictures labeled G and H (Figure 6) represent, similarly, respectively the undersize

and the oversize obtained by use of a baffled Tyler standard 28 screen from

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still a third Tyler standard screen size (28/48).

The results of these experiments indicate that the apparatus has considerable merit for fractionation of particles according to their shape. However, experience showed that clean cut samples of narrow length ranges could not be obtained by its use. This was attributed to curvature in the woven-wire sieve bottom and to lack of rigidity leading to vibration of the woven-wire. This led to rejection of the apparatus; at a later date it was found usable and



Figure (7) Three-dimensional Sieves

fairly satisfactory in conjunction with several of the larger Tyler standard sieves.

The next three-dimensional sieve used is pictured in Figure 7. Essentially, the apparatus consisted of a 6 inch disc bolted to an 7-7/8 disc or 8 inch pan with spacing washers between them. Again the original design was modified several times. The first model was cut full 7-7/8 inches from 16 gauge galvanized iron and drilled with 1/16

inch holes in the center portion. It was taped to a Tyler standard sieve with apertures much larger than 1/16 inch. The 6 inch plate was bolted to the 7-7/8 inch plate with 3/16 inch bolts, and the Tyler standard screen was protected by use of cardboard bearings. The second model, (II) was cut 7-3/4 inches from 16 gauge galvanized iron and encircled with a split rubber tube packing. A third modification (III) was 8 inch aluminum pans in place of the bottom plates and Tyler sieves.

Finally a fourth modification was made using a 7-3/4 inch black iron bot-





tom plate (A) bolted into a frame of aluminum made from one of the 8 inch pans (B) with a 6 inch black iron baffle-plate (C) bolted above it and with spacing washers between the plates. The first three forms are pictured in Figure (8) (page 37).

A seed mixture containing flax-seed, buckwheat, oats, wheat, weedseed, straw, etc. was fractionated by three methods: (a) the Tyler

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standard screens, (b) the set of three dimensional screens described, and (c)



Figure 9 - Tyler 10

combinations in series, of Tyler standard screens and three-dimensional screens. These methods can be compared by examination of Figures 9-28 which illustrate the fractions obtained.

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Alto Protection

Figures 9, 10, 11, and 12 represent respectively the seeds in Tyler



# Figure 10 - Tyler 10/14

standard screen scale fractions >10, 10/14, 14/28, and 28/48. All four fractions are definitely mixtures; the mixture shown in Figure 11 is practically binary, the others are more complex.

History .

The Figures 13, 14, 15, 16, 17, 18, 19, and 20 represent seeds found in the



# Figure 11 - Tyler 14/28

$$< [d = 1/8", c = 0.050", t = 0.069"], > [d = 1/16", c = 0.050", t = 0.069"],$$



# Figure 12 - Tyler 28/48

and < [d = 1/16", c = 0.016", t = 0.069"]. The only sample worthy of much notice in this group is that represented by Figure (18) which is relatively pure flaxseed and a very little buckwheat.



Figure 13 -> [d = 3/16", o = 0.099", t = 0.069"]



Figure 14 -> 
$$d = 3/16$$
",  $c = 0.050$ ",  $t = 0.069$ "



Figure 15 - 
$$\sqrt{d} = 3/16^*$$
,  $c = 0.050^*$ ,  $t = ?$ 



Figure 16 = 
$$< [d = 3/16", c = 0.050", t = 0.069"]$$



Figure 
$$17 - d = 1/8"$$
,  $c = 0.050"$ ,  $t = 0.069"$ 



Figure 18 - < 
$$d = 1/8^{"}$$
,  $c = 0.050^{"}$ ,  $t = 0.069^{"}$ 



Figure 19 -> 
$$d = 1/16^{\circ}$$
,  $c = 0.016^{\circ}$ ,  $t = 0.069^{\circ}$ 



# Figure 20 = $\langle d = 1/16^{n}, c = 0.016^{n}, t = 0.069^{n} \rangle$

The Figures 21, 22, 23, 24, 25, 26, 27, and 28 represent seeds found in sieved fractions obtained by sieving on two-dimensional (Tyler standard screen scale) and three-dimensional sieves in series.



Figure 21 -  $(d = 1/8^n, c = 0.050^n, t = 0.069^n) - Tyler 14$ 

Respectively the sieves used were:  $(21) \left[ > 14; < (d = 1/8", c = 0.050", t = 0.069") \right], (22) \left[ 10/14; > (d = 1/8", c = 0.050", t = 0.069") \right], (23) \left[ 10/14; < (d = 1/8", c = 0.050", t = 0.069") \right], (24) \left[ 10/14; < (d = 3/16", c = 0.050", t = 0.050", t = 0.050", c = 0.050", c$ 

t = ?], (25)  $10/14_{3}>(d = 3/16^{n}, c = 0.050^{n}, t = ?$ ], (26)  $14/28_{3}>(d = 1/8^{n}, c = 1/8^{n})$ 



Figure 22 =  $d = 1/8^{n}$ ,  $c = 0.050^{n}$ ,  $t = 0.069^{n}$  - Tyler 10/14

 $c = 0.050^{\circ}, t = 0.069^{\circ}$ ], (27)  $\left[\frac{14}{28} < (d = 1/8^{\circ}, c = 0.050^{\circ}, t = 0.069^{\circ})\right]$ (28)  $\left[\frac{28}{48} < (d = 1/16^{\circ}, c = 0.016^{\circ}, t = 0.069^{\circ})\right]$ . Among this group Figures (23), (24), and (26) represent, respectively, practically pure flax-seed,



Figure 23 -  $\sqrt{d} = 1/8\%$ , c = 0.050%, t = 0.069% - Tyler 10/14

practically pure buckwheat, and practically pure cats; while the remaining figures of this group represent seed mixtures they exhibit trends toward homogenity of shape. The advantages of series combination of two-dimensional





Figure 25 - >d = 3/16", c = 0.050", t = ? - Tyler 10/14



Figure 26 -> 
$$d = 1/8\%$$
,  $a = 0.050\%$ ,  $t = 0.069\%$  - Tyler 14/28



Figure 27 - 
$$\langle d = 1/8^{n}, c = 0.050^{n}, t = 0.069^{n} - Tyler 14/28$$



Figure 28 - < d = 1/16, a = 0.016, t = 0.065 - Tyler 28/48

and three-dimensional sieving is shown by these comparisons.

The apparatus represented by Figure (29) was built next. It consisted of: an inner box 2 inches by 4 inches by 6 inches in dimensions with axle, pulley, and crank attached, having ends made of alternating 1 inch by 4 inch by 1/16 inch metal strips and groups of spacer washers fastened on copper tubes and



Figure 29

bolted to the sides of the box, and a receiving box 6 inches by 8 inches by 10 inches in dimensions with slots to carry the axle of the inner box. This was the first one-dimensional sieve used by this writer.

A mixture of wheat and vetch was fractionated by means of this onedimensional sieve into three fractions. One of these fractions (s = 0.148"; Figure 30) was pure

vetch; the other two were still wheat-vetch mixtures.



# Figure 30 - {0.7 g., 0.0% wheat } [s = 0.148"]

The baffle-plates were then entirely removed from the three-dimensional sieves and they were used as two-dimensional drilled plate sieves (with round holes). The wheat-vetch mixtures were then fractionated further by use of the



two-dimensional slove series on onsy ainture, into three fractions in the one

59-A

Figure 31 - {0.7 g., 0.0% wheat} [S 0.148"]

B = 0.148"/0.125"; 4 = 0.185"/0.358"

case and two fractions in the other. Of these fractions and is " weight? . 0.123"; d = 0.165") was found to be pure weight and is shown in the protonytype, Figure (31). two-dimensional sieve series on each mixture, into three fractions in the one



case and two fractions in the other. Of these fractions one (s =  $0.148^{W}/0.123^{W}$ ; d  $\Rightarrow 0.156^{W}$ ) was found to be pure wetch and is shown in the photograph, Figure (31).

An interesting comparison may be made between the samples  $S = 0.148^{"}/$ 



Figure 33 - 11.1 g., 95.5% wheat  $S = 0.123^{*}/0.084^{*}$ ;  $d = 0.156^{*}/0.125^{*}$ 

0.125"; d = 0.156"/0.125"; (No baffle) and S = 0.123"/0.084"; d = 0.156"/ 0.125"; (No baffle) represented, respectively, by Figures (32) and (33), which were obtained by series sievings with one-dimensional and two-dimensional sieves alone and the following samples represented by Figures (34), (35), (36), (37),



(38), (39), and (40) which were resolved further by use of three-dimensional sieves.

It is noticeable that much of the wetch has been concentrated in the single

fraction represented by Figure (36) with relatively little wheat included; the



Figure 35 - {4.9 g., 57.1% wheat} [= -9.144"/0.123"; 4 = 0.125"; e = 0.111"/0.109"]

wheat has been concentrated in the single fraction represented by Figures (54); the existence of both fractions represented by Figures (35) and (37) in the form of mixtures resembling the original (unpictured) unresolved mixture and the unresolved mixture represented by Figure (32) indicated some limitations in the



apparatus used to effect the separation, for as will be indicated later where such a marked difference in shape of particles exists a practically perfect separation should be easily obtainable (with perfect apparatus).


Figure 37 - {4.2 g., 50.0% wheat}  $s = 0.148^{n}/0.125^{n}; d = 0.125^{n}; c = 0.100^{n}$ 

Similarly, the concentration of the wheat in the single fraction represent-



# Figure 36 - 37.4 g., 99.5% wheat = 0.123"; d = 0.125"; c = 0.108"

ed by Figure (38) is seen; it is not, perhaps, so striking as the preceding example, yet the progressive increase in vetch concentration in the samples represented by Figures (38), (39), and (40), respectively, is noticeable.



Figure 39 - {17.3 g., 97.7% wheat}  

$$s = 0.123^{n}; d = 0.125^{n}; c = 0.108^{n}/0.050^{n}$$



Figure 40 - 
$$\{35.0 \text{ g.}, 83.1\% \text{ wheat}\}$$
  
s = 0.123"; d = 0.125"; c = 0.050<sup>m</sup>

These last three fractions were obtained from the same portion of the original mixture by successive lowerings of the baffle-plate and resieving of the undersize fractions. Comparison with the sample represented by figure (33) indicates what advantages are to be gained by use of the three-dimensional sieve in series with the one-dimensional and two-dimensional sieves. These same results are presented less graphically but, perhaps, more exactly in table II following, in which the samples designated by Greek letters were further resolved and those samples designated by Roman letters represent pictured products. The operations designated by lower case Roman letters are defined as follows and represent single sieving operations, with the sieves designated;

a - s = 0.148 b - s = 0.125 c - s = 0.084 d - d = 0.156 e - d = 0.125 f - d = 0.094 g - d = 0.125; t = ?; c = 0.108 h - d = 0.125; t = ?; c = 0.050 i - d = 0.125; t = ?; c = 0.050 i - d = 0.125; t = ?; c = 0.100

						•								
									· · · ·		•			
Samp1e	Total Weight (g)	Weight % of Original Sample	% Wheat	Upon Operation	Sample	Figure	Total Weight (g)	Weight % of Original Sample	% Wheat	Sample	Figure	Total Weight (g)	Weight % of Original Sample	% wheat
α	177.9	100.0	73.9	a	М	30	0.7	0.4	0.0	B	-	177_2	00 G	71.9
B	177,2	99.6	74.2	Ъ	<u>ک</u>	÷	76.4	43.0	49-4	5		100.8	56 7	077
6	100.8	56.7	93.1	C	E	-	100.8	56.7	93.1	4			0.0	200T
σ.	76.4	43.0	49.4	đ	n	=	76.0	42.7	49.6	N.	31	0.4	0.2	0.0
E	100.8	56.7	93.1	đ	É.	ä	100-8	56.7	93.1	×	5	്രമ്മ	0.0	0.00
٤.	100.8	56.7	93,1	0	P	33	11,1	6.2	95.5	X		89.7	50.4	92.8
Ŋ	76.0	42.7	49.6	e	0	-	38.3	21.5	54.6	6	32	37.7	21.2	44.6
e	38.3	21.5	54.6	f	Π	6	0.0	0.0		•		38.3	21.5	54.6
A	89.7	50.4	92.8	£	V .	9	0.0	0.0		Ĭ.	ස	89.7	50.4	92.8
A	89.7	50.4	92.8	g	É	-	52.3	29,4	88.0	Q	38	37.4	21.0	99.5
0	38.3	21.5	54.6	i.	Ĩ	34	13.4	7.5	91.0	Ā		24.9	14.0	35.0
P	24.9	14.0	35.0	g	σ	÷	20.0	11.2	29.5	J	35	4.9	2.8	57.1
کر	52.3	29.4	88.0	h	R	39	17.3	9.7	97.7	5	40	35.0	19.7	83.1
0	۳ 20 <b>.</b> 0	11.2	29.5	j	ĸ	36	15.8	8 <b>.</b> 9	24.1	L	37	4.2	2.4	50.0

TABLE II

And Undersize

Yields Oversize

Experimental Testing of Formulae. The use of models in these studies is valuable. Models of a size convenient to work with should prove the value of the formulae as well as, or better than, their smaller counterparts, both for the conditions assumed and for those differing from them in various degrees.

Square holes were cut in two boards; one square was 0.59 0.01 inch on a side, the other square was 0.54 0.01 inch on a side. Several blocks of wood were cut which would just pass the larger opening as shown by careful hand guaging; that these particles would not pass the smaller opening was demonstrated. They were then measured carefully with a ruler graduated to thirty-seconds of an inch. The results are given in Table III on Page 72 together with the values calculated using formulae previously presented.

The agreement between the observed and calculated values is remarkable. It may be implied from these figures that the formulae 3-(19) and 3-(23) are reliable for these cases for which they were derived within the limits of experimental error.

A series of round holes were drilled in a piece of galvanized iron having diameters ranging from 24/64 to 32/64 inches in 1/64 inch intervals. Rectangular prismatic blocks were cut from suitable material and hand gauged with these holes; the widths and thicknesses of the particles were measured. The widths were calculated using formulae 30(13) and 30(14) and the results are presented in Table IV on Page 73.

# TABLE III

# RESULTS OBTAINED WITH TWO-DIMENSIONAL, SQUARE-APERTURED SIEVES (MODELS)

Observe	d Values			Calculated	Formula	
Sieve Di Sides of S	mensions quare Holes	Particle Dimensions Thickness Width		Width	%	
s <sub>1</sub>	S <sub>2</sub>	Т	W	W gal.	Error	
0.59 ± 0.01	0.54 ± 0.01	0.48 ± 0.01	0.56 ± 0.01	0.58 ± 0.04	0.0%	3-23
0.59 ± 0.01	0.54 ± 0.01	0.35 ± 0.01	0.56 ± 0.01	0.56 ± 0.04	k 0.0%	<b>3-</b> 23
0.59 ± 0.01	0.54 ± 0.01	0.29 ± 0.01	0.56 ± 0.01	0.55 ± 0.04	L 0.0%	3-23
0.59 ± 0.01	0.54 ± 0.01	0.21 ± 0.01	0.58 ± 0.01	0.58 ± 0.00	5 0 <b>.</b> 0%	3-19
0.59 ± 0.01	0.54 ± 0.01	0.04 ± 0.01	0.75 ± 0.01	0.75 ± 0.08	5 0.0%	3-19

:

# TABLE IV

# RESULTS OBTAINED WITH TWO-DIMENSIONAL ROUND-HOLED SIEVES (MODELS)

Observed V Sieve Dime	alues nsions	Particle Di	imensions			
Diameters of Si	eve-openings	Thickness	Width	Width	Error	Formula
đĵ	$d_2$	Т	W	W cal.		
0.50 ± 0.01	0.48 + 0.01	0.13 ± 0.01	0.49 - 0.01	0.48 ± 0.02	2.0%	3-(13)
0.50 ± 0.01	0.48 = 0.01	0.27 ± 0.01	0.44 🛓 0.01	0.41 ± 0.02	6.8%	3⇔(13)
0.50 ± 0.01	0.48 ± 0.01	0.33 ± 0.01	0.38 1 0.01	0.36 2 0.03	5.3%	3-(13)
0.48 ± 0.01	0.47 ± 0.01	0.31 ± 0.01	0.40 ± 0.01	0.37 2 0.03	7.5%	3-(13)
0.47 ± 0.01	0.45 ± 0.01	0 <b>.13 ± 0.01</b>	0.45 ± 0.01	0.44 ± 0.02	2.2%	3-(14)

The data of Table III (pg. 72) and Table IV (pg. 73) were obtained by the use of the apparatus pictured in Figure (41). An ordinary ruler was used in making all of the measurements noted.

The apparatus pictured in Figure (42) was devised and built. It consisted of a ring-stand and a support for holding either of two one and one-half inch diameter pipe segments perpendicular to the base of the ringstand. This support consisted of a 1 x 2 inch board eight



### Figure 41

inches long with a one and one-half inch hole out in it two inches from one end and a one-half inch hole out in it one inch from the other end; nuts were inserted in the wood adjacent to each hole and belts passed through them to hold the support a fixed (but adjustable) distance above the ring-stand and to hold a pipe segment firmly in the support.

The support was raised to the top of the ring-stand and secured there. A wooden right prism of uniform rectangular cross-section was then passed downward through the pipe segment until it was restricted by the ring-stand

base, points on the bottom edge of the pipe segment, and points on the top edge of the pipe segment. The prism was then cut off slightly below the upper point of restriction, so that it could be passed completely through the pipe segment. The diameter and length (thickness of "sieve-plate") of the pipe segment and the distance of its lower edge above the base of the ringstand were carefully measured with a ruler graduated to 1/16 inch and recorded. The length, width, and thickness



# Figure 42

of the wooden prism were likewise carefully measured with the same ruler and recorded. The support was then lowered until this "particle" could not be passed completely through the pipe segment, and the distance of the pipe segment's lower edge from the ring-stand base (the clearance "c") was remeasured and the "sieve-plate" thickness, the "clearance", the particle thickness and particle width were then substituted into the formulae (3-30) and (3-32) previously derived and a corresponding particle length was computed. The

percentage error of this computed length referred to the observed length was computed and recorded. The wooden prism was then cut shorter and the entire procedure repeated. This was done a number of times until  $L^2 + W^2 = d^2$ . A new wooden prism was then obtained and the procedure repeated. The results obtained are tabulated in Table V (pg. 77).

The apparatus pictured in Figure (43) was constructed by bending a four inch long, 0.98 inch wide, 0.016 inch thick aluminum plate three times at right angles to form approxi-

mately an open-ended, rightprismatic, square-sectioned surface. This "sieve" was tacked to a support with two countersunk tacks. The support was then fastened (adjustably) to a ringstand. The support was raised to a point near the top of the ring-stand and secured. A wooden right prism of uniform rectangular cross-section was then passed downward through the square siève surface until it was restricted by the ring-stand



Figure 43

TABLE V

RESULTS OBTAINED WITH THREE-DIMENSIONAL ROUND-HOLED SIEVES (MODELS)

đ	t	c	L	T	W	L cal.	E	$\mathbf{F}$
1.41"	2.28"	13 <b>.</b> 4"	15 <b>.</b> 9"	0 <b>. 50<sup>ii</sup></b>	0.75"	16.1 ± 0.1	1.3%	30
11	11	13.2"	11	90	ŧŧ.		1-	
11	n '	11.1"	14.2"	22	11	14.0 - 0.1	1.4%	30
11	<b>97</b> 1	11.3"	<b>11</b>	21	12			
11	Ħ	9.6 <sup>#</sup>	12 <b>.</b> 4"	88	11	12.2 ± 0.1	1.8%	30
¥1	- 17	9 <b>.</b> 3 <sup>8</sup>	90	83	17	A. 1	•	
87	<b>98</b> :-	7.2*	<b>10.</b> 0"	81	88	10.0 ± 0.1	0.0%	30
11° .	11	7 <b>.</b> 5"	11	88	88			
11	¥1	4.2"	6.3"	11	<u>tt</u>	6.5 🛨 0.1	3.2%	33
17	11	4.0"	88	Ħ	18		-	
11	<b>11</b>	2.4"	4•0"	88	17	4.3 🕇 0.3	7.5%	33
1T	11	2.0"	17	11	19	· · · ·		
11	11	1.1"	2.3"	<b>21</b>	18	2.3 ± 0.1	0.0%	33
11	<b>11</b> 1	1.2 <sup>%</sup>	13	17	97		·	
11	11	0•63 <sup>11</sup>	1.6"	88	91	1.53 ± 0.1	1.9%	33
11	11	0 <b>.</b> 531	97	11	17	•	·	
n	11	0.53"	1.2"	<b>\$</b> \$	89	1 <b>.</b> 34 ± 0.1	8•3%	33
TT -	tť	0.37"	#	88	'n			
1 <b>.41"</b>	2.28 <sup>#</sup>	12.2"	15.0 <sup>19</sup>	0 <b>.125"</b>	1 <b>.</b> 125"	15.1 ± 0.2	0.7%	30
11	Ħ	11.7"	11	11	Ħ		- •	
Ħ	11	10.2"	13.0"	11	Ħ	13.1 ± 0.2	0.8%	30
17	Ħ	9.9"	97	1T	11			,
1.41"	2 <b>.</b> 28"	12 <b>.</b> 2"	15.0"	0.13"	1 <b>.</b> 13"	15.1 📩 0.1	0.7%	30
Ħ	Ħ	- 11 <b>.</b> 7"	19	'n	Ħ		•	
11	11	10.2"	13.0"	18	11	$13.1 \pm 0.2$	0.8%	30
11		9 <b>.</b> 9"	Ĥ° -	n	11			
ŤŤ.	11	7.8"	10 <b>.5"</b>	13	11	10.6 ± 0.1	1.0%	30
11	1	7.6"	11	11	11		•	
11	tī	5 <b>,</b> 5"	8°04	T	Ħ	8.2 ± 0.1	2.5%	30
11	11	5.4"	13	Ħ	17			
11	11	4.1"	6.5"		17	6.8 ± 0.1	4.6%	33
11	11	<b>4.</b> 3"	17	11	Ħ			
11	11	2.2"	4.0"	n	19	3 <b>.</b> 9 <u>+</u> 0.1	2.5%	33
n	Ħ	2.1"	48	11	, TT		•	
Ħ	Ħ	0.75"	2.5"	11	n	2.1 1 0.1	16.0%	33
11	<b>FT</b>	1.0"	Ħ	FT	11			
11	17	0.63"	2.0"	· #1	11	1.9 ± 0.1	5.0%	33
<b>TT</b>	<u>H</u> . 1	0 <b>•75"</b>	13	11	88	· · ·		
11	Ħ.	0 <b>.75</b> "	'n	11	11	1.8 ± 0.1	10.0%	33
. 11	11	0,56"	Ħ	11	11			
Ħ	11	0,19"	1 <b>.</b> 13"	13	1.0 <sup>#</sup>	1.13 ± 0.1	0.0%	33
Ĥ	11	0.13"	22	11	- 11		/ -	

d = Diameter of Sieve-opening; t = Thickness of Sieve-plate; c = Clearance between Sieve-plate & Baffle; L = Length; T = Thickness; W = Width; L cal. = Calculated Length; E = % Error; F = Formula.

base, points on the bottom edge of the square sieve surface, and points on the top edge of the square sieve surface. It was then cut off slightly below the upper point of restriction so that it could be passed completely through the square sieve surface. The length of the sides and vertical depth (thickness of "sieve-plate") of the square sieve surface and the distance of its lower edge above the base of the ring-stand were carefully measured with a rule graduated to 1/16 inch and recorded. The length,width, and thickness of the wooden prism were likewise carefully measured and recorded. The support was then lowered until this "particle" could not be passed completely through the square sieve surface, and the distance of the square sieve surface's lower edge from the ring-stand base (the "clearance") was remeasured and recorded. The values of s, t, c, T, and W (i.e. the length of a side of the sieve opening, the "sieve-plate" thickness, the "clearance", the particle thickness and the particle width) were then substituted into the formulae (31), (32), (34), and (35) presented previously.

A corresponding particle length was computed and recorded. The % error of this computed length (relative to the observed length) was also computed and recorded. The wooden prism was then cut shorter and the entire procedure repeated. This was done a number of times. A new wooden prism was then obtained and the entire procedure repeated. The results obtained are given in Table VI (pg. 80).

When it is recalled that no provision was made in the formulae used for the precision with which the various quantities were measured, and that the apparatus used deviated more or less from the ideal, the difference between the observed and calculated lengths are quite reasonably explained as

experimental errors of observation. Within these limits as indicated by this experiment formulae 31 (pg. 22) 32, (pg. 22 and 23) 34, (pg. 23), and 35 (pg. 24) seem reliable.

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# TABLE VI

# RESULTS OBTAINED WITH THREE-DIMENSIONAL SQUARE-APERTURED SIEVES (MODELS)

Observed Values

Calculated Values

Sieve Dimensions

Side of Sieve Opening	Thickness of Sieve- Plate	Clearand Sieve- Bai	ce Betw <b>e</b> en -plate & Ifle	Thickness	Width	Length	Length	% Error	
S	t	cl	c <sub>2</sub>	T	W	L	Lcal.		F*
1.01±0.01	0.98±0.01	13.53±0.01	13.66±0.01	0.60±0.01	0.66±0.01	15.21±0.01	15.29±0.08	0.5%	31
1.01±0.01	0.98±0.01	5.06±0.01	4.97±0.01	0.63±0.01	0.66±0.01	5.94±0.01	6.20±0.05	4.4%	31
1.01±0.01	0.98±0.01	13.63±0.01	13.57±0.01	0.17±0.01	1.13±0.01	15.00±0.01	14.75±0.03	1.7%	32
1.01±0.01	0.98±0.01	6.75±0.01	6.73±0.01	0.17±0.01	1.13±0.01	8.00±0.01	7.81±0.02	2.4%	32
1.01±0.01	0.98±0.01	5.25±0.01	5.14±0.01	0.17±0.01	1.13±0.01	6.45±0.01	6.24±0.06	3.3%	32
1.01±0.01	0.98±0.01	1.69±0.01	1.58±0.01	0.17±0.01	1.13±0.01	2.44±0.01	2.40±0.08	1.6%	35
1.01±0.01	0.98±0.01	1.42 <u>+</u> 0.01	1.39±0.01	0.17±0.01	1.13±0.01	1.97±0.01	2.07±0.03	5.1%	35
1.01±0.01	0.98±0.01	7.94±0.01	7.75±0.01	0.50±0.01	0.77±0.01	9.95±0.01	9.55±0.11	4.0%	31
1.01±0.01	0.98±0.01	2.81±0.01	2.75±0.01	0.50±0.01	0.85±0.01	3.94±0.01	4.07±0.04	3.3%	31
1.01±0.01	0.98±0.01	1.34±0.01	1.25 <u>+</u> 0.01	0.50±0.01	0.94±0.01	2.19±0.01	2.25±0.07	2.7%	34

Particle Dimensions

\*F Formula

### Introduction

Taggart<sup>1</sup> presents an excellent review of some methods in use for calculating the capacity of vibrating screens. Five methods designated as follows are presented:

"Overflow Method" - A. D. Sinden

2. "Throughflow Method" - Smith Engineering Wks.

3. "Total-feed Method" - Link-Belt Co.

4. "Total-feed Method" - Allis-Chalmers Co.

5. "Total-feed Method" - Stephens-Adamson Co.

All five methods make use of "basic rates" determined from either empirical equations or tables. These basic rates are then modified by means of "factors" empirically related to various screen or material particle characteristics.

The "factors" employed in the various methods include those related to the following characteristics:

1. The sieve aperture

2. The percentage "clear" opening

3. The "intensity" of vibration

4. The screen length

million and the state of the state

5. The option of square or rectangular apertures

6. The "struck" weight density of the material particles

7. The moisture content of the material particles

8. The percentage "critical sized" particles in the feed

<sup>1</sup> Taggart, <u>Op. cit.</u>, pp. 7-63 to 7-70

9. The percentage "undersized" material in the screen's reject
10. The percentage "oversized" particles in the feed
11. The percentage "undersized" material in the feed
12. The percentage "half-sized" material in the feed

Careful review of the solution to a single problem by the five methods indicates better agreement about the factors determining screen capacity than is apparent from the answers obtained or the equations used. This is because each method involves certain implicit considerations in addition to those explicitly stated in the equations. None of the methods explicitly considers all of the "factors" listed previously; yet, except for overlapping of "critical-sized", "oversized", "undersized", and "half-sized" material, all methods seem to consider all the factors, provided they are applied by experienced designers.

Taggart concludes: "It is apparent from the screen sizes obtained by application of the various methods, ranging from 18 to 60 square feet of onedeck surface, that either the experiences of the different engineers have been markedly different or the manufacturers are displaying widely separated degrees of conservatism."<sup>2</sup>

Again, following a mathematical discussion of a spherical particle's probability for passing a square opening, Taggart writes, "Unfortunately, the number and nature of the qualifications that must be applied to this mathematical analysis in actual screening are such that no approach to a practical measure of capacity by a modification of it has yet been made."<sup>3</sup>

- 2 Taggart, Op. cit., p. 7~69.
- Taggart, Op. cit., p. 7-02.

Thus, the inexperienced designer for whom an explicit design procedure would have a maximum value is forced to choose between a not entirely satisfactory "practical" method and an entirely inadequate "theoretical" method.

The difference between the "Theoretical" and the "practical" solutions will vanish when both are developed to a satisfactory degree. Since the theoretical approach follows a logical pattern, and since it suffers from obvious defects, it will be developed somewhat in this paper.

### The Problem

Our ultimate goal is to find equations to predict within prescribed limits of accuracy the rate at which perfectly defined collections of particles may pass (or be rejected from) perfectly defined screens without reference to other data.

A most important factor in determination of screen capacity is that of relative particle and screen dimensions. A particle cannot be adequately defined by specification of a single dimension alone. The orientation of the particle relative to the screen surface is also most important.

### Probabilities In Screening

If "chance rules the approach of a particle to hole or imperforate surface"<sup>4</sup>, and if all possible particle orientations are equally probable, and if the passage of the particle through the hole or imperforate surface is possible for only some of the possible particle orientations, the probability of the particle's passage through the screen will be equal to the ratio of "favorable" particle orientations to all particle orientations,

4 Taggart, Op. cit., p. 7-01

From consideration of figure (1) on page 26 it is evident that relatively long particles must turn at an angle of  $\alpha_c$  or a larger angle to the horizontal in order to pass through the sieve opening.

If a single point in the particle is restricted to a fixed position in space, the locus of particle positions "at an angle of  $\alpha_c$  or a larger angle to the horizontal" (when the baffle is entirely removed) is a spherical segment (two spherical segments for assymetrical particles) and the locus of all possible particle positions is a hemisphere (or sphere for assymetrical particles). The probability of the fixed position then being a favorable position will be measured by the ratio of "clear opening" area to total surface area. The probability of the particle's passing the sieve will be the product of these two ratios.

$$P = \frac{2/3\pi R^2 h}{2/3\pi R^3} \quad (\% "clear" area)$$
(45)

which may be simplified to:

$$P = \frac{h}{R} \quad (\% \text{ "clear" area}) \tag{46}$$

From geometrical considerations

$$\frac{h}{R} = 1 - \cos \left(90 - \alpha_{\ell}\right) = 1 - \sin \alpha_{c} \tag{47}$$

whence the probability of the particle's passing the screen, P, may be written as:

$$P = 1 - \cos (90 - \alpha) (\% "clear" area) = (1 - \sin \alpha) (\% "clear" area) (48)$$

 $\alpha_c$  is determined by the relative values of the particle's thickness and its width, the diameter of the round aperture, and the thickness of the sieveplate. An equation expressing the relationship between these variables has been presented (Equation 39 p. 29). It is repeated here (slightly modified) for convenience:

$$\sqrt{d^2 - W^2} = \frac{T}{\sin \alpha_c} + \frac{t}{\tan \alpha_c}$$
(39)

Upon solving for  $\sin \alpha_c$  we obtain:

$$\sin \alpha_{c} = \frac{T\sqrt{d^{2} - W^{2}} \pm tr\sqrt{d^{2} - W^{2} - (T^{2} - t^{2}r)}}{d^{2} - W^{2} + t^{2}\bar{x}}$$
(39-A)

It may be demonstrated that the probabilities for spherical particles passing through round openings of diameter one inch are exactly the same as those tabulated in Table I, p. 7-02, of Taggart's <u>Handbook of Mineral Dress-</u> ing for spherical particles passing through square openings, one inch on a side. These values are reproduced in Table VII below with comparable values calculated from equations (48) and (39-A).

Consideration of Table VII shows that the shape of the particle is often as important as the relative size of the particle's "intermediate diameter" and the sieve aperture. There is little reliable evidence to favor the proposed theory or to prove it wrong beyond the statement of Taggart<sup>5</sup> that "feed factors affecting undersize recoevery are: --- (d) slabby or needlelike characteristics".

5 Taggart, Op. cit., P. 7-05.

# TABLE VII

Size of Grain, l/n, in.	Probable Chance per 1000 for Unrestricted Passage Through in. Circular Opening (Thickness, tr = 0.1 d)						
	Sphere	Right Prisms of Rectangular Section					
		T = W	T = W/2	T = W/4	T = 0		
0.001	998	898	898	899	89 <b>9</b>		
0.01	980	889	894	897	899		
0.1	810	802	850	875	900		
0.2	640	698	798	849 *	898		
0.3	490	589	741	817	896		
0.4	360	471	679	784	892		
0.5	250	337	605	744	885		
0.6	160	180	516	694	876		
0.7	90	6	398	625	861		
0.8	40	0	227	522	835		
0.9	10	0	0	315	775		
0.95	2.5	0	0	97	695		
0.99	0.1	0	0	0	422		
0.999	0.001	0	0	0	87		

Careful fractionation of material according to the procedure outlined in the section on separation of seed mixtures followed by rate studies should indicate the value of the theory.

It may well require modification of the assumption that "all possible particle orientations are equally probable" (p. SS) and of the assertion that "the probability of the fixed position then being a favorable position will be measured by the ratio of "elear opening" area to total surface area" (p. 84) to make the predictions of this theory approach experimental facts when these facts are available.

The presentation of this material is justified by its indication of the means by which the "relationship between sieve dimensions and particle dimensions" can be used "to find equations to predict within prescribed limits of accuracy the rate at which perfectly defined collections of particles may pass (or be rejected from) perfectly defined screens without reference to other date".

If the square openings of figures (2) and (3) pp. 27 and 28 are considered it is apparent that a degree of assymmetry is introduced by the nature of the sieve opening and particle orientation may vary from having the particle's long axis parallel to a side of the square to having this axis parallel to a diagonal of the square opening. If we designate the smallest angle between a side of the square opening and a vertical plane containing the particle's long axis as B, the "angle of horizontal approach", and if we designate the angle between the particle's second longest axis of symmetry and the horizontal (sieve) plane as \_,.the "angle of rotation", we may say that for the case of a right prismatic particle of rectangular cross-section passing a square sieve opening,

 $\alpha$ , the "critical angle of inclination" of the pariele's long axis to the (horizontal) sieve plane is dependent on B,  $\succ$ , T, W, s, and t, the angles of "horizontal approach" and of "rotation", the particle's thickness and width, and the length of a side and the thickness of the square aperture respectively, this should lead to an equation analogous to equation (39-A) for definition of for this case. This equation has not been derived.

### The Zone Concept

In its usual applications sieve analysis consists of a number of steps, First, it is not unusual for the particle to fall from a higher screen (or a distributor under the influence of gravity and with only the resistance of air friction; this process is well understood and may for many shapes of solids be predicted rather accurately. Second, the undersize particle must work its way through a layer of oversize particles and particles similar to itself to get to the screen, moving under the influence of gravity, such mechanical forces as are induced in the particles involved by shaking or vibrating actions, and frictional and static forces between particles. This second phase of the process may be considered as passage through a "sieve" comprised of the interstices of the particles forming the "bed". Third, for many particles, it is necessary for the particle to assume a favorable orientation with respect to the sieve opening while acting under the influence of the same forces enumerated above for the second pabse of its passage. Fourth, it must actually pass through the sieve aperture under the forces of gravity, frictional forces, and possible inertial and mechanical forces of sieve-particle interaction.

The fate of an oversize particle depends upon the nature of the screening operation. In a batch operation such as is involved in testing sleves

the particle certainly passes through the free-falling phase with the undersized particles; it may or may not pass through the second and third phases outlined for the undersized particle; it certainly does not pass through the fourth phase of behavior described above for the undersized particle. While these remarks apply equally well to continuous operation the transfer of the oversized particle parallel to the screen surface constitutes another step in the process while its ultimate rejection is the final step.

Any region in which the particles being sieved are subjected to a definite set of forces, will be called a sieve-zone. The zones corresponding to the phases of particle behavior outlined above will be designated as (1) the freefalling zone, (2) the stratification zone, (3) the orientation zone, (4) the undersize aperture-passage zone, and (5) the oversize rejection zone. Some overlapping is probable; the statification and orientation zones may constitute a single region---oversize (and undersize) transfer parallel to the soreen surface occur in this region in addition to the undersize transfer normal to the screen surfaces. Transition zones between the major zones are highly probable.

The relative magnitudes of transfer rates in the various zones are of primary importance in the analysis of sieving problems. It is likely that one or more zones may be neglected in most problems and only one zone considered in some problems without affecting the accuracy of the analysis. From consideration of a single particle which is subjected to force combinations we should expect to obtain rates of motion expressed as distance per unit time. A practical equation of this nature has yet to be found; certain factors which will enter into such an equation are listed in the following paragraphs.

The probability of a single particle's passing through a single sieve opening has already been discussed. Reason dictates further that increasing the number of particles, the number of sieve-openings, or the number of sieveparticle contacts should cause changes in the average rate of transfer per particle. The number of particles is a function of the particle characteristics: shape, size and density. The number of sieve-openings is a function of the sieve characteristics. The number of sieve-particle contacts depends both on particle characteristics and the nature of the screen's motion.

The vibration indused in the particles as a result of the screen vibration is usually a noticeably different vibration. The inclination of the screen to the horizontal may be expected to exert an effect on the relative parallel and normal particle transfer rates. These facts may be better visualized by constructing diagrams of particle trajectories. Dalle Valle<sup>6</sup> presents equations for computing such particle trajectories, both in its general form and for the particular problem of heavy spherical particles in air. Once the particle trajectory across the screen is visualized it seems apparent that the number of sieve-particle contacts will be governed by the relative magnitudes of the period of the particle vibration and the screen length. Probability of passage through the screen is probably a factor here as well.

Once the equations of particle motion have been obtained they may be combined with the "equation of continuity" to yield an equation involving a rate expressed as particles per unit time (or, of course, weight per unit time, or volume per unit time). This is the more usable rate usually sought in sieve design.

Dalle Valle, Micromeretics, pp. 23-27.

# The Equation of Continuity

Consider a differential section of a homogeneous sieve-zone represented by W, the width of the screen; dy, the differential side normal to the screen; and dx, the differential side parallel to the screen. Let  $U_x$  and  $U_y$  represent the components of particle velocity in the x and y directions and assume no motion in the direction of W. Let C represent the numerical concentration of particles of the specified size.

In the direction parallel to the screen the pariols velocity is  $U_x$  and the rate at which particles enter the section is  $CWU_x dy_o$ . At the opposite face, the rate at which particles leave the section is:

$$\begin{bmatrix} \mathbf{U}_{\mathbf{x}} \mathbf{C} & \Rightarrow & \frac{\partial \left( \mathbf{U}_{\mathbf{x}} \mathbf{G} \right)}{\partial \mathbf{x}} & d\mathbf{x} \end{bmatrix} \quad \mathbf{W} d\mathbf{y}$$

In the direction perpendicular to the screen the particle velocity is  $U_y$  and the rate at which particles enter the section is  $CWU_y dx_o$ . At the opposite face, the rate at which particle leave the section is:

$$\begin{bmatrix} \mathbf{u}^{\mathbf{y}}\mathbf{c} + \mathbf{y}^{\mathbf{y}} \\ \mathbf{y}^{\mathbf{y}}\mathbf{c} \end{bmatrix} = \mathbf{y}^{\mathbf{y}}$$

The total rate of input to the section is equal to  $CWU_X dy CWU_y dx_o$ The total rate of output from the section is equal to :

$$\begin{bmatrix} \mathbf{u}^{\mathbf{x}_{\mathbf{C}}} + \frac{\mathbf{y}(\mathbf{n}^{\mathbf{x}_{\mathbf{C}}})}{\mathbf{y}^{\mathbf{x}_{\mathbf{C}}}} & \mathbf{y}^{\mathbf{x}_{\mathbf{C}}} \end{bmatrix} \mathbf{M} \mathbf{q}^{\mathbf{x}_{\mathbf{C}}} + \begin{bmatrix} \mathbf{u}^{\mathbf{x}_{\mathbf{C}}} + \frac{\mathbf{y}(\mathbf{n}^{\mathbf{x}_{\mathbf{C}}})}{\mathbf{y}^{\mathbf{x}_{\mathbf{C}}}} & \mathbf{y}^{\mathbf{x}_{\mathbf{C}}} \end{bmatrix} \mathbf{M} \mathbf{q}^{\mathbf{x}_{\mathbf{C}}}$$

Also, the rate of accumulation is equal  $to(\frac{\partial c}{\partial \theta})$  dxdyW.

Substituting these values into the general material balance: Input -Output - Accumulation = 0; and simplifying gives us:

$$\frac{\partial \mathbf{C}}{\partial \theta} + \frac{\partial \mathbf{U}_{\mathbf{V}} \mathbf{G}}{\partial \mathbf{y}} + \frac{\partial \mathbf{U}_{\mathbf{X}} \mathbf{C}}{\partial \mathbf{x}} = 0 \quad (49)$$

This is a general equation to be used for analysis of screening problems. By setting the first term equal to zero we obtain the equation for continuous steady-state screening operations. On the other hand, by retaining this first term and setting the third term equal to zero we obtain the equation for batchtype or testing-sieve operation.

# Controlled Orientation

The importance of favorable sieve-particle orientation has been pointed out repeatedly. No commerically important machines employ devices for promoting or controlling the sieve-particle orientation. Certain experiments were performed to indicate the possible utilization of such devices.

Two cardboard boxes were made as shown in figure (44) (pg. 93). They were as nearly identical as was praticable in every detail except the portion containing the single slot; in one box this portion was made flat, corresponding roughly, with the conventional type of screen or "grizzly"; in the other box the vertex of the trough. A definite number of particles were introduced into one of the boxes. The box was then simply rotated about an axis parallel to the slot in it allowing the particles to fall first upon the slotted portion and then away from it. The revolutions were carefully counted and when a particle passed through the slot, the number of particles passed and the cumulative number of revolutions from the starting point were recorded. The particles were collected and the experiment repeated using the other box.

The experiment was performed five times with each box with ten needles; five times with each box with the first ten of thirty needles; five times in each box with ten flat cardboard plates, approximately threequarters of an inch square and eight one-thousands of an inch thick; and once with each box with the first ten of sixty-six cardboard plates (similar in dimensions to those mentioned above). The average period (number of revolutions) of particle passage relative to the most rapid particle passage (absolute value = two-tenths revolution per particle passed)

Figure 44

for each case is listed in Table VIII (p. 94). While it is impossible to draw any far-reaching conclusions from such meagre data, it is felt that the results are indicative of several facts. In some cases controlled orientation might lead to marked increase in the rate of sieving. Particle concentrations may exert a marked influence on the rate of sieving. Where a particle orienting

device serves as a particle concentrating device the advantages of controlled orientation might be out-weighed by the disadvantages of particle interference due to high particle concentration.

# Efficiency of Sieving

No really satisfactory definition of efficiency of sieving has been framed; muither has complete agreement been reached among various designers as to what factors comprise sieving efficiency. None of the definitions currently used consider the work input to the sieving machine.

# TABLE VIII

# PARTICLE PASSAGE PERIODS FOR PLATES AND RODS WITH CONTROLLED AND RANDOM PARTICLE ORIENTATIONS

Sieve	Ten Needles	lst Ten of Thirty Needles	Ten Flat Plates	lst Ten of Sixty-Six Flat Plates
Trough-bottomed	1	1	<b>11</b>	<b>1</b> 6
Flat-bottomed	41	9	645	69

To correct for the short-comings of existing definitions of sieving efficiency we propose the following definition. Sieving efficiency is the ratio of effective sieve-particle contacts to all sieve-particle contacts. An effective sieve-particle contact is one which determines correctly and finally the classification of a particle as above or below a certain size. The final classification of the particle as oversize or undersize can be accomplished only by removal of the particle from the screen. Such a definition focuses attention on the classification function of the screen in contrast to the separation function usually considered.

To the ratio of undersize material in the screen underflow to undersize material in the screen feed we suggest the term degree of separation be applied.

# Positive Rejection of Oversize

The conventional screening machine is very inefficient in terms of the efficiency definition proposed in the preceding section due to the fact that oversized particles may contact the screen thousands of times before being

finally rejected. To overcome this definiency the apparatus shown in figure (45) was devised and built. It consisted of two six inch rollers each mounted in a "half-box", a sloping platform, and a crank attached to one of the rollers.

One of the rollers was turned away from the other (at their tops) while slate rock was fed onto the rollers near their upper ends, the flat particles turned up on edge with the thin ones passing between the rollers into a



### Figure 45

tray and the thick ones sliding down and off the lower ends of the rollers into a box provided. The operation seemed satisfactory from the standpoint of both selectivity and speed of operation. Quantitative data has not been secured because changes in bearings and mechanism for turning both rollers are contemplated.

# Bibliography

Taggart, Arthur F. Handbook of Mineral Dressing. New York: John Wiley & Sons, Inc., 1945.

The nature of a thing may be defined<sup>1</sup> as: "The inherent or natural qualities of the thing; those peculiar characteristics and attributes which serve to distinguish one thing from another".

Particle may be defined<sup>2</sup> as: "A minute part or portion of matter, of an aggergation of which the whole mass consists; an atom, a molecule".

The term particle has been used by physicists at many levels to apply te discreet individual units. Quanta, corpuseles, and photons are described as "light particles". The ever growing list of "atomic particles" includes electrons, protons, neutrons, positrons, mesons, *Q*-particles, neutrinos, and neutrettos. The common "chemical particles include ions, atoms, molecules, and micelles. The "colloidal" particles are commonly understood to include all particles having sizes be tween about  $0.5 \, n\mu$  and  $0.2 \, \mu$ . The term "micromerities" has been defined as the "technology of fine particles"; it is "limited to particles ranging from  $10^{-1}$  to  $10^5$  microns"<sup>3</sup>.

Moreover, substances of selective restrictiveness are known for many of these particles. Many interesting analogies between well-known phenomena and sieving or filtering might be drawn; e.g.-polarization of light by a pane of glass, optical filters, refraction phenomena, reflection phenomena, and diffraction phenomena; use of screen-grids in "electronic tubes"; experiments on penetrating power of atomic particles; dyalysis of colleidal suspensions; and permeability of metals to gases.

1 Prof. Morris, Charles, The New Revised Encyclopaedic Distionary.

- <sup>2</sup> Prof. Morris, Charles, Op. cit.
- Dalle Valle, Micromeritics, p. 3.

However, most of these particles lie outside the scope of this paper which is primarily intended to deal with particles in the upper portion of the "micromeritic" range. If we restrict our discussion to such particles, some further classification is still possible.

Paricles may be classified as solid or fluid. Only solid particles are to be considered here. A reasonably extensive mathematical theory of solids exists. Solid particles may be rigid, elastic or plastic. We limit our attention to rigid solid particles.

Rigid solid particles may be homogeneous or heterogeneous, crystalline or non-crystalline, isotropic or anisotropic. These different classes will act alike when a particle of a fixed size, shape and density of each class is classified by a given sieve. They are convenient, however, in the prediction of particle properties.

The assumption was made that a rigid solid particle's behavior could be approximated by the behavior of a rectangular parellopiped in which the particle could just be inscribed. Practically, this was stated, "the behavior of a rigid solid particle can be approximated by the behavior of the least box in which the paritcle can be put. This assumption lends added interest to the following quotation from the Encyclopaedia Britannica.<sup>4</sup>

"- Furthermore, the fact that any solid can be approximated to as closely as desired by a polyhedral solid adds to the importance of the theory of such solids to a study of solids in general. A precise formulation of the fact just alluded to is the following; If S denotes any solid, then there exists a sequence of polyhedral solids,  $P_1$ ,  $P_2$ ,  $P_3$ ,  $\cdot$ ,  $\cdot$ ,  $P_n$ , such that every point of the solid  $P_n$  is a point of the interior of S and also of the interior of the

4 Encyclopaedia Britannica, Vol. 20, pp. 941-2.

solid  $P_{n+1}$ , and every point of the interior of S is a point of the interior of a polyhedral solid  $P_m$  of the sequence ".

The solences of crystallography and mineralogy are relatively old and highly developed sciences. The first deals for the most part with the study of homogeneous crystalline solids. The limitations on this elassfication of crystals is indicated by the following quotation from the <u>Encyclopaedia Britannica</u>.

"The actual form, or "habit" of crystals may vary widely in different crystals of the same substance, these differences depending largely on the conditions under which the growth has taken place. The material may have crystallized from a fused mass or from a solution; and in the latter case the solvent may be of different kinds and may contain other substances in solution, or the temperature may vary. Calcite affords a good example of a substance crystallizing in widely different habits, but all crystals are referable to the same type of symmetry and may be reduced to the same fundamental form.

When crystals are aggregated together, and so interfere with each other's growth, special structures and external shapes often result, which are sometimes characteristic of certain substances, especially amongst minerals",

Mineralogy deals for the most part with heterogeneous crystalline solids in the crystallographic sense just mentioned. Classification of crystals as isotropic or anistropic is for the most part an optical classification little affecting their external form.

The value of crystallographic considerations in predicting external form of solids in conjunction with solution of sieving problems is very limited. Nevertheless, it should be considered.

<sup>5</sup> Encyclopaedia Britannica, Vol. 6, p. 822.

The terms acicular and tabular have long been used to designate crystal shapes in a qualitative sense. Recently, an effort has been made to add an exact quantitative significance to the terms.  ${}^{6}{}_{o}{}^{7}{}_{o}{}^{8}{}_{o}{}^{9}$  The nature of the quantification is controversial, but the articles mentioned more or less agree upon the dependence of the terms upon three fundamental dimensions. It is these three dimensions which we have designated as "thickness", "width" and "length" in this paper.

### Bibliography

Dalle Valle, J. M. Micromeretics, 2nd ed. New York: Pitman Publishing Corporation, 1948.

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Smithson, Frank. "Particle Shape". <u>Nature</u>, Volume 155, (1945) 639. Tomkeleff, S. T. "Particle Shape". <u>Nature</u>, Volume 155, (1945) 639. Whittaker, E. J. W. "Particle Shape". <u>Nature</u>, Volume 155, (1945) 331-2.

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Yust, Walter (ed.). Encyclopaedia Britannica. 1946 Printing, Chicago: Encyclopaedia BritannicaInc., 1946.

<sup>6</sup> Whittaker, E. J. W., "Particle Shape", <u>Nature</u>, Vol. 155, (1945), pp. 331-2.

7 Smithson, Frank, "Particle Shape", Nature, Vol. 155, (1945) p. 639.
 8 Tomkeieff, S. T., "Particle Shape", Nature, Vol. 155, (1945), p. 639.
 9 Whittaker, E. J. W., "Particle Shape", Nature, Vol. 155, (1945), pp. 639-40.
The references directly quoted in this paper have been listed in the bibliographies at the ends of the chapters (pp. 10, 98, 102, and 106).

A complete literature survey of the subject seems impracticle if not impossible. A comprehensive survey of the literature within the limits of readily available library resources is desirable and practical.

All we care to claim is a thorough carrying out of the program outlined in the remainder of this chapter.

# Periodical Literature

A list of topics was made and modified as experience was gained. It included the following terms: bolting, coal screening (also, coal cleaning, coal preparation, coal sizing, coal mines and mining equipment), coke screening (also, coke preparation), flour sifters, minerals (conc. or sepn. of), ores (treatment of), paper or wood-pulp (screening, screening app., screens, screens (theory of), separation of solids, separators, sewage (screens for), shaking apparatus, sieves, sifting, threshing machines, and wire cloth.

The following index or abstract journals were searched for material listed under each of the topics listed above:

1. Chemical Abstracts (1907-1948)

2. Industrial Arts Index (1939-1948)

3. Nineteenth Century Readers' Guide

4. Readers' Guide to Periodical Literature

The years listed are those for which the index or abstract journals were consulted. Each article of pertinent or doubtful status was described on a note card and a card file was assembled; where abstracts were given they were included on the card.

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The libraries' periodicals' indexes were consulted and a card file of available articles was separated from the original file. These articles were then read and notes were made of their content.

#### Reference Works

The following list of handbooks and general references listed in the set of instructions furnished by the chemical engineering department<sup>1</sup> were consult-

ed: 😗

- 1. International Critical Tables
- 2. Perry (Chemical Engineers' Handbook)
- 3. Thorpe (and Watt) (Dictionary of Applied Chemistry)
- 4. Ullman (Enzyklopadie der Technischen Chemie)
- 5. Chemical Rubber Co. (Handbook of Chemistry and Physics)
- 6. Glazebrook (Dictionary of Applied Physics)
- 7. Kent (and Marks) Mechanical Engineering)

This list was supplemented by the following works listed in Mudge<sup>2</sup>:

- 1. Encyclopaedia American
- 2. Encyclopaedia Britannics
- 3. Compton's Pictured Encyclopedia
- 4. World Book
- 5. New Century Dictionary
- 6. Standard Dictionary
- 7. Oxford English Dictionary
- 8. Condensed Chemical Dictionary
- 9. Gardner, William, Chemical synonyms and trade names
- 10. Hackh, Chemical Dictionary
- 11. Kingzett, Chemical Encyclopaedia
- 12. Chemical Engineering Catalog
- 13. Liddell, Handbook of Chemical Engineering
- 14. Taggart, Handbook of Ore-dressing

<sup>2</sup> Mudge, T. G., <u>Guide to Reference Books</u>, Sixth Edition. Chicago: American Library Association, 1936 (with supplements through 1946)

Regulations Governing the Pursual of Study Leading to the M. S. Degree in Chemical Engineering at Oklahoma Agricultural & Mechanical College, mimeographed, undated and unsigned.

# Textbooks, Monographs and Theses

The textbooks examined were those of the writer's own library and includ-

ed:

- 1. Walker, Lewis, McAdams, and Gilliland
- 2. Badger and McCabe
- 3. Shreve
- 4. Tyler, Chemical Engineering Economics
- 5. Vilbrandt, Chemical Engineering Plant Design
- 6. Brown and Associates, Chemical Engineering

No monographs on the subject were discovered in the libraries searched. Theses of Oklahoma A & M College were examined.

All abstracted patents were checked in the official Gazette of the U. S.

### Patent Office.

The publications of the U.S. Bureau of Standards were examined in detail insofar as they were available.

#### Books

The card catalogs of the following libraries were checked against the list used for periodical literature:

- 1. Oklahoma A & M College Library, Stillwater, Oklahoma
- 2. Little Rock Public Library, Little Rock, Arkansas
- 3. University of Arkansas Chemistry Library, Fayetteville, Arkansas
- 4. Tulsa Public Library, Tulsa, Oklahoma

#### Accidentally Obtained Material

Several articles on sieves or sieving were called to our attention by friends. Others were obtained in non-technical reading, extraneous to our planned literature survey. There were catalogued with the other material in the card file.

#### Correspondence

Letters were exchanged with several manufacturer's representatives who provided some usable information. Letters were also exchanged with at least

one industrial research director concerning applications.

It was felt that in view of the material at hand a separate publication of the bibliography was advisable. This summary was included in this paper to spare any interested reader the effort of duplicating the search.

# Bibliography

Mudge, T. G. <u>Guide to Reference Books</u>, Sixth Editions. Chicago: American Library Association, 1936 (with supplements through 1946).

Anonymous. Regulations Governing the Pursual of Study Leading to the M. S. Degree in Chemical Engineering at Oklahoma Agricultural & Mechanical College. Mimeographed (not dated) THESIS TITLE: Sieve Analysis

NAME OF AUTHOR: Joe Bill Hocott

THESIS ADVISER: William A. Klemm

The content and form have been checked and approved by the author and thesis adviser. "Instructions for Typing and Arranging the Thesis" are available in the Graduate School office. Changes or corrections in the thesis are not made by the Graduate School office or by any committee. The copies are sent to the bindery just as they are approved by the author and faculty adviser.

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