DETERMINATION OF THE OPTIMUM GRID TANK L-C RATIO FOR THE CLASS C AMPLIFIER

By

DONALD G. HALLOCK

Bachelor of Science

Oklahoma Agricultural and Mechanical College

Stillwater, Oklahoma

1950

Submitted to the Faculty of the Graduate School of

the Oklahoma Agricultural and Mechanical College

in Partial Fulfillment of the Requirements

for the Degree of

MASTER OF SCIENCE

1951

DETERMINATION OF THE OPTIMUM GRID TANK L-C RATIO

FOR THE CLASS C AMPLIFIER

DONALD G. HALLOCK

MASTER OF SCIENCE

1951

THESIS AND ABSTRACT APPROVED:

Harold Fristoe
Thesis Adviser

A. Naeter
Faculty Representative

D. C. McIntosh
Dean of the Graduate School

ACKNOWLEDGEMENTS

The author wishes to express his indebtedness to Prof. H. T. Fristoe for suggesting the subject of this paper, and for his most helpful advice during the course of its writing. The author also wishes to thank Prof. D. L. Johnson for many helpful discussions on the work.

PREFACE

The Class C R-F amplifier has come to be very important in our present day system of communications. This importance has caused it to be the subject of much research. It is the purpose of this paper to delve further into the problem of design simplification. Specifically, an investigation of the grid tank circuit is carried out in order to determine an expression for the optimum grid tank L-C ratio. It is hoped that this information may be of value in expediting Class C amplifier design.

TABLE OF CONTENTS

INTRODUCTION

The development of the Class C radio frequency amplifier has made radio communications not only scientifically possible, but economically feasible. Plate efficiencies of 85% are theoretically obtainable, and this is of special value in R-F amplifiers where the output may run into hundreds of kilowatts.

Literature on the design of the Class C R-F amplifier is to be found in abundance in any technical library. The Wagener[1] and the Constant Current methods have proved valuable aids in determining circuit parameters, primarily those in the plate circuit, with accuracy and speed. Thomas[2] has set forth a method for determining the driving power required for a tube under certain specified conditions; but there still remains one gap in the fence of knowledge that has been built around the Class C amplifier. Very little exploration has been done on the input side of the question, although design of the plate circuit has been narrowed down to a few simple formulas.

Class C design is complicated by the fact that the d-c power input varies with the output, and it is possible to get maximum efficiency by adjusting the d-c potentials and the load resistance. Many variables are involved; grid bias, grid driving power, plate L/C ratio and plate supply voltage, to name but a few. The accepted method of solution is mainly graphical or cut-and-try, since the plate current is non-sinusodal and no simple set of equations can be evolved. The aforementioned design methods are only approximate, as they assume that the plate current pulse is a portion of a sine wave. The alternating plate voltage computed by these methods must be obtained by

---

1 Austin V. Eastman, Fundamentals of Vacuum Tubes, pp. 372-378

2 H. P. Thomas, Determination of Grid Driving Power in Radio Frequency Amplifiers, Proc, I.R.E., 21. p. 1134, August, 1933

choosing suitable values of L, C, and R in the plate tank circuit. Eastman has arrived at the following set of equations[3] which give optimum tank values:

(1) $$X_L = E_p^2 / P_o Q$$

where $Q = w_o L / R_L$, and,

(2) $$w_o C = 1 / w_o L$$

It was thought that perhaps there exists a similiar relation in the grid tank circuit. From personal experience it is known that simply changing the L/C ratio in the grid tank circuit would cause the amount of driving power delivered to the grid by the driving source to vary. It was assumed that some ratio of grid tank L-C would come closest to matching the input impedance of the driven stage to the preceding stage. Preliminary investigation indicated some validity to this assumption, but many other variables were discovered that further directly affected the matching.

It was originally the intent of the author to thoroughly investigate the coupling of the driver plate tank to the following grid tank circuit by means of a transmission line inductively fed at both ends (link coupling), and to propose a method by which optimum parameters could be chosen throughout the entire system. Preliminary research has shown the problem to be beyond the scope of any one paper, and it was decided that investigation should be concentrated on but one of the many phases of the problem.

It may be said that the grid circuit is only a continuation of the preceding stages plate circuit, or a form of load. If this is true, it but remains to set up the grid circuit so that the load it presents to the pre-

---

3 Eastman, op. cit., p.383.

ceding stage can be analyzed. In the Class A amplifier this is quite simple, as negligible grid current flows, and the equivalent load consists of possible a linear resistor-condensor combination. However, Class C operation dictates a bias of several times the cutoff value for the tube, and a driving voltage that will swing the grid positive for a small part of each cycle. When the grid is positive, grid current flows, and power is consumed in the bias source and in the grid-to-cathode section of the tube. Thus the load equivalent consists of a rectifier and two power consuming devices. As has been mentioned, this load is further complicated by virtue of the reflection due to the plate circuit. As the output of the stage is increased, the grid driving power must be increased, and more grid current flows; the output of the one stage has a direct effect upon the loading of the preceding stage.

In Equation (1), R_L is the equivalent series resistance in the tank circuit representing both power output and coil losses. For a perfect match, the coupling circuit and the grid tank circuit must reflect this value of R_L into the plate tank circuit. We can consider the resistance of the coil to be so small compared to the actual reflected load that we may reglect it completely and so simplify the analysis. The plate tank circuit cannot be designed until the value of R_L is known, and thus it would seem logical that the high power stage be designed before its driver. This practice also determines the power that the driver must furnish to the high power stage, and the driver can then be designed to furnish the required power.

Link coupling is commonly used between two R-F stages, as it is both efficient and practical. As it is a form of transmission line, losses due to radiation may be held to a minimum. Either coaxial cable or a balanced line may be used. If the stages are only a few inches apart, perhaps capacitive or inductive coupling can be used to advantages, but this method fails to be efficient as the distance between stages increases and as the frequency increases.

Some means of matching the link to the tank circuits is necessary if link coupling is to be used. The usual practice is to transformer couple the link by means of a few turns of wire around the "cold" end of the tank coil. This practice leads to a serious mismatch between the stages unless care is taken to make the link coils of the right impedance and to provide the right amount of coupling between the link coil and the tank. A mismatch means inefficiency of power transfer, and this often leads to building a driver stage of proportions much greater than are actually necessary to provide the required driving power.

The link can be tuned to resonance by means of a series or parallel condensor in the link circuit. This not only creates a better match, but when equivalent resistive loading in the grid tank circuit is obtained, only resistance will be reflected back into the driver plate tank. Several tests were made using this idea, and the results were very much better than was expected. A standing wave ratio of 1.1/1 was recorded, and the power from the driver required to satisfy the grid circuit was greatly reduced. If the load delivered by the high power stage were changed, thus requiring more or less grid driving power, the grid L-C ratio assumed a new value for optimum conditions, and consequently a new value of impedance was reflected back into the link circuit, and the link tuning capacitor had to be tuned again to bring the circuit back into resonance. The tuning procedure was time consuming and a standing wave meter in the link circuit was necessary to determine when optimum conditions were reached. There was no clue as to a possible way of determining the optimum L/C ratio in the grid tank circuit except that the power into the grid is an important factor.

The factor "Q" cannot be ignored in designating an optimum L/C ratio in the grid circuit, but when Q is used in a formula it must be thoroughly defined. The value of Q can be taken to be the Q of the inductance alone, the effective Q of the equivalent circuit, or it may be defined in its strictest sense as being proportional to the energy stored per cycle divided by the energy disapated per cycle[4]. The actual value of Q cannot be placed in the circuit as readily as can more tangible properties like resistance and reactance, but rather, the circuit must be designed with a definite value of Q in mind. Studies of optimum values of Q have been made. The Q for the plate circuit should be in the neighborhood of 12 to 15. Mention is made by K.L. Klippel[5] that some of the more experienced design engineers have concluded that an effective Q of 20 is the best for the grid circuit. He goes further to say that the L/C ratio in the grid circuit should be found by means of Equation 1, but no mathematical proof of this is offered.

A certain amount of consideration must be given to the fact that, although the power input to the grid circuit is in an a-c form, so to speak, the power disapated in the tube and in the bias source is due to a flow of rectified current. This fact leaves much to be desired in defining the effective Q of the grid circuit as a whole.

With the preceding discussion in mind, investigation of the coupling problem was begun. The entire circuit, from driver plate-tank to the grid-to-cathode impedance of the driven tube, was taken into consideration. The

---

4 M.I.T. Staff, Applied Electronics, p.579

5 K.L. Klippel, CQ Magazine, May, 1950, Vol. 6, No. 5, pp. 11-14.

mathematical results were not conclusively verified by the actual tests, but the test results do show that the right path of investigation was being followed.

THE THEORY OF LINK COUPLING

Link coupling serves, as its name implies, to magnetically link together two stages, in this case, two Class C R-F amplifiers. In so doing, the grid load of the driven stage is reflected back into the link, and from the link into the plate tank circuit of the driving stage. Should the tank circuit parameters then be changed, the stage is no longer operating into the load for which it was designed.

During the course of Class C amplifier design, a value of alternating plate voltage will be obtained. Proper design of the plate tank circuit will assure that this voltage is secured. Eastman[6] has arrived at the following equations for the tank parameters:

(1) $$w_0L = E_p^2/P_0Q$$

where $Q = w_0L / R_L$, and:

(2) $$w_0L = 1/w_0C$$

R_L is the equivalent total series resistance in the tank circuit representing both power output and coil losses. Let us see what effect the link coupling to the following circuit will have on the plate circuit.

CASE I.

An equivalent of the basic link coupling circuit is shown in Fig. 2. The grid circuit of the tube may be represented by a hypothetical resistance, R, equal approximately to the effective value of the a-c grid voltage divided by the d-c grid current.[7] Further simplification reduces the circuit to that

---

6 Eastman, op. cit., p. 383

7 Thomas, op. cit., p. 1134

of Fig. 3 by letting $X_{L_4} = X_{C_4}$, and assuming the resistance of the coils to be negligible.

The grid circuit of Fig. 2 was converted from the parallel form to an equivalent series circuit before being transformed into Fig. 3, as shown in Fig. 2b. The series circuit is easier to analyze, and in the rest of this paper we shall consider the grid circuit in the equivalent series form. R will then take on the equivalent value, R_e, equal to $w^2 L_4^2 / R$. [8]

Fig. 3 shows that there is an impedance, Z_b, reflected back into the plate tank circuit:

(3) $$Z_b = \frac{X_{M_1}^2 \left[\frac{X_{M_2}^2}{R_c} - j\omega(L_2+L_3) \right]}{X_{M_2}^4 / R_c^2 + \omega^2 (L_2+L_3)^2}$$

and Z_b may be broken down into real and imaginary components as follows:

(4) $$R_L = \frac{X_{M_1}^2 X_{M_2}^2}{R_c} \Big/ \left[\frac{X_{M_2}^4}{R_c^2} + \omega^2 (L_2+L_3)^2 \right]$$

(5) $$X_{C_b} = -X_{M_1}^2 \, \omega (L_2+L_3) \Big/ \left[\frac{X_{M_2}^4}{R_c^2} + \omega^2 (L_2+L_3)^2 \right]$$

The L-C ratio in the plate tank circuit is lowered by virtue of the capacitive reactance appearing in series with the plate inductance. The resistive component transferred to the plate circuit represents the output load, R_L.

It can be seen that any change in coupling at either the plate or the grid tank will change the plate L-C ratio, and also alter the value of the alternating plate voltage developed across the total plate tank circuit. The efficiency of the stage will drop off due to the plate load no longer matching the tube impedance. For instance, if X_{M_1} were increased, X_{C_b} and R_L would increase. The effective inductance in the plate circuit become smaller, and

---

8 Ryder, John D., Networks, Lines and Fields, pp. 106-107

E_p would diminish.

CASE II.

The reactances L_2 and L_3 in the link circuit can be made negligible by adding proper capacitance to the circuit, or in other words, by resonating the link circuit at the operating frequency. Equation (4) then becomes:

(6) $$X_{M_1}^2 R_c = X_{M_2}^2 R_L$$

and Equation (5) becomes zero.

The coupling can now be changed without altering the plate tank L-C ratio. Resistance alone is reflected back into the plate circuit. The value of E_p is still subject to change, but now it is due only to one variable, R_L, and the plate tank impedance is equal to $(wL)^2 / R_L$ when the tank is tuned to resonance.

CASE III.

Until now, no consideration has been made of the circuit that joins the two link coils. Let us specify this connection to be in the form of a transmission line with a characteristic impedance, Z_o. A maximum transfer of power occurs when this line is terminated in its characteristic impedance. The following equations express the link terminations when the grid and plate circuit components are reflected into the link circuit, assuming that $X_{C1} = X_{L1}$, and $X_{C_4} = X_{L4}$:

(7) $$Z_2 = \frac{X_{M_2}^2}{R_c} + j\omega L_3$$

at the grid tank end, and:

(8) $$Z_1 = \frac{X_{M_1}^2}{R_e} + j\omega L_2$$

at the plate tank end. R_e is the equivalent series resistance in the plate tank circuit. If the line is to be matched in impedance, Z_1 and Z_2 must be equal to the characteristic impedance of the line.

Equating the real and imaginary parts of Equations (7) and (8):

$$\frac{X_{M2}^2}{R_c} = \frac{X_{M1}^2}{R_e} \quad , \quad \omega L_2 = \omega L_3 \tag{9}$$

Thus the link coils should be of the same inductance. If we can consider $\frac{X_{M1}^2}{R_e}$ and $\frac{X_{M2}^2}{R_c}$ to be small compared to wL_2 and wL_3, and then $wL_2 = wL_3 = Z_o$. This is in accord with the network theorem which states that if the magnitude but not the angle of the load impedance may be varied, a maximum power transfer will occur when the absolute value of the load impedance is equal to the absolute value of the impedance of the supply network.

When the transmission line is terminated in such an impedance, reflection of capacitance into the tank circuits results. The L-C ratios and the Qs are lowered, and the tanks must be detuned from their own natural resonance points in order to bring about circuit resonance. The mutual impedances between the link and the tank coils must be carefully adjusted so that Z_1 and Z_2 in Equations (7) and (8) will be equal to Z_o. Then the tank circuits must be adjusted to compensate for the capacitance reflected back into them.

CASE IV

A tuned circuit may be used to terminate the transmission line at each end in place of the link coils. If the plate and grid circuits and the link tuned circuits are tuned to dynamic resonance, Equation (6) shows that the value of R_L appearing in the plate circuit is dependent only upon the mutual impedances between the grid and plate circuits and their respective link tuned circuits and R_c. It would thus appear that loading of the stage can be accomplished by varying the coupling of the link to the plate and grid circuits. It must be remembered, however, that the link circuit is still acting like a transmission line, and Z_o must be matched at each end. The imaginary parts of Equations (7) and (8) now become zero, and it can be seen that there is only one value of coupling at each end which will produce a specific Z_o at each end of the transmission line. Loading cannot be varied if maximum transfer of energy is to be maintained.

The characteristic impedance of a non-resonant transmission line is a complex quantity, having both a real and an imaginary part. Neither of the two previously discussed cases will completely satisfy Z_o. If the link is kept to a reasonably short length, (several wave lengths or more), the resistive and conductive components will have very little effect compared to the inductance and the capacitance, and Z_o will be very nearly resistive. Thus the condition of a tuned circuit at each end of the link will permit an almost perfect match, providing the mutual impedances of the link to the grid and plate tank circuits, and the value of R_c are adjusted to make the link terminations equal to Z_o.

CASE V

There remains the possibility of using the transmission line itself as a part of the tuned circuit, and thereby ridding ourselves of the trouble of matching Z_o. A piece of transmission line less than a quarter wave length long may be considered to be essentially capacitive, as the inductive component is very small compared to the capacitive component. A series capacitor may be added to the link circuit and tuned so that the link will resonate at the operating frequency. Fig. 4 shows the resultant link circuit, where C_3 is the capacitance of the line. The necessary series capacitance may be calculated from the expression:

$$X_{C_2} = X_{L_2} + \frac{X_{C_3} X_{L_3}}{X_{C_3} - X_{L_3}} \qquad (10)$$

Again we can see from Equation (6) that the value of R_L is dependent only upon the mutual impedances and R_c. Desired loading can now be accomplished by varying one or more of these parameters.

A relation can be found between the grid-cathode impedance R_c and the inductance L_4 of the grid tank circuit. The grid circuit is assumed to be tuned to resonance, and the rest of the circuit is assumed to be adjusted so that only resistance is transferred to the plate tank circuit. The following equations illustrate the method of relating L_4 and R_c.

C_3, L_3, and the resistance transferred from the grid circuit to the link, R_1, are combined into Z_1 (Fig. 5):

(11) $$Z_1 = \frac{\frac{R_1}{(\omega C_3)^2} - j\left[\frac{R_1^2}{\omega C_3} + \frac{L_3}{C_3}\left(\omega L_3 - \frac{1}{\omega C_3}\right)\right]}{R_1^2 + \left(\omega L_3 - \frac{1}{\omega C_3}\right)^2}$$

Then Z_t, the total impedance of the link circuit is equal to:

(12) $$Z_t = Z_1 - \frac{j}{\omega C_2} + j\omega L_2$$

The real part of Equation (12) is:

(13) $$\mathcal{R}e\, Z_t = \mathcal{R}e\, Z_1 = \frac{R_1}{\left[R_1^2 + \left(\omega L_3 - \frac{1}{\omega C_3}\right)^2\right](\omega C_3)^2}$$

If the link circuit is tuned to resonance, only the real part of Equation (12) will be transferred to the plate circuit, and this transferred resistance is R_L.

(14) $$R_L = \frac{X_{M_1}^2 (\omega C_3)^2 \left[R_1^2 + \left(\omega L_3 - \frac{1}{\omega C_3}\right)^2\right]}{R_1}$$

Substituting for X_{M_1} and R_1, and combining constants:

(15) $$BR_L = \frac{\omega^4 K_2^2 L_3^2 L_4^2 + R_c^2\left(\omega L_3 - \frac{1}{\omega C_3}\right)^2}{R_c L_4}$$

where $$B = \frac{K_2^2 L_3}{\omega^2 K_1^2 L_1 L_2 C_3^2}, \quad A = \left(\omega L_3 - \frac{1}{\omega C_3}\right)^2, \quad D = \omega^4 K_2^4 L_3^2$$

This equation may be arranged in the form:

(16) $$AR_c^2 - BR_L R_c L_4 + DL_4^2 = 0$$

from which expressions for R_c and L_4 may be obtained.

(17) $$R_c = \frac{BL_4R_L \pm \sqrt{B^2R_L^2L_4^2 - 4ADL_4^2}}{2A}$$

(18) $$L_4 = \frac{BR_cR_L \pm \sqrt{B^2R_L^2R_c^2 - 4ADR_c^2}}{2D}$$

Equations (17) and (18) will have a certain value when the transfer of power is at a maximum. This value is found by setting up an equation for the power output of the stage with R_L as one term. The derivative with respect to R can then be taken and set equal to zero, and the resulting equation is a solution of Equation (17) for maximum power transfer.

(19) $$P_o = \frac{E_p^2 R_L}{\omega^2 L_1^2} = \frac{E_p^2 (AR_c^2 + DL_4^2)}{\omega^2 L_1^2 B R_c L_4}$$

(20) $$\frac{dP_o}{dR_c} = \frac{E_p^2}{\omega^2 L_1^2 B L_4} \left(\frac{AR_c^2 - DL_4^2}{R_c^2} \right) = 0$$

$$\text{so} \quad AR_c^2 = DL_4^2$$

(21) $$R_c = \frac{BR_L L_4}{2A}, \qquad L_4 = \frac{BR_c R_L}{2D}$$

The derivative with respect to L_4 can be treated the same way, and again Equation (21) is found as the solution. This can readily be seen by examining Equations (17) and (18). All other circuit parameters remaining constant, R_c is directly proportional to L_4.

The equivalent series circuit of Fig. 2 may be expressed in terms of the original circuit:

$$R_c = \frac{1}{R \omega^2 C_4^2}$$

and if $w^2 C_4 L_4 = 1$:

(22) $$R_c = \frac{\omega^2 L_4^2}{R}$$

Substituting into Equation (21):

(23) $$L_4 = \frac{BR_L R}{2A\omega^2}$$

and by the same method:

(24) $$R = \frac{BR_L \omega^2 L_4}{2D}$$

Equations (23) and (24) may be added to give:

(25) $$\omega L_4 = R \cdot \frac{\sqrt{D}}{\omega\sqrt{A}}$$

The power used in the grid circuit can be found by:

(26) $$P_{in} = \frac{E_g^2}{R}$$

if all coil losses are neglected. Substituting Equation (26) in Equation (25):

(27) $$\omega L_4 = \frac{E_g^2}{P_{in}} \cdot \frac{\sqrt{D}}{\omega\sqrt{A}}$$

This is the form of equation which we have been seeking. It might prove rather difficult to show that $\frac{\sqrt{D}}{\omega\sqrt{A}}$ is equal to Q_2, and, indeed, we are not sure that this is the case. All we have shown in Equation (27) is that the grid inductance is a function of the effective grid voltage, the power input to the grid circuit and some constant.

The investigation so far has centered around the grid circuit load reflected back into the plate circuit. Equation (27) was obtained by going from grid circuit values to their equivalents in the plate circuit. After some manipulations, the result was transferred back to grid values. This is a lengthy and round-about method. Let us now look at the situation entirely from the grid circuit standpoint.

The plate and link circuits may be reflected into the grid circuit as a resistance, R_r, in Fig. 6. We shall assume, of course, that the plate and link circuits are so tuned that only resistance is reflected into the grid circuit. When the portion of the circuit to the left of a-b in Fig. 6 is tuned so as to be purely resistive, its impedance is very nearly equal to $w^2 L_4^2 / R_r$. Maximum power will be transferred to R if: $R = \frac{\omega^2 L_4^2}{R_R}$

The Q of the tank circuit is: $\frac{\omega L_4}{R_R}$

Therefore:

(28) $$R = \omega L_4 Q_2$$

The power input to the grid circuit is:

(29) $$P_{in} = \frac{E_g^2}{R}$$

Substituting Equation (29) in Equation (28):

(30) $$\omega L_1 = \frac{E_g^2}{Q_2 P_{in}}$$

The above demonstration gives us the equation for the value of the grid tank circuit inductance under specific conditions. It is thought, however, that the circuit used in this proof is too elementary. In the actual case of substituting a vacuum tube for the resistance R of Fig. 6 several differences occur. The tube is a non-linear element, power is consumed in the bias source, and, perhaps most important, the circuit is no longer strictly an a-c circuit. The d-c current resulting from rectification in the tube grid-to-cathode path will flow through the tank coil, and may have some effect on the Q of the circuit. Another important thought to keep in mind is the definition of Q. Are we justified in believing that it is the Q of the tank circuit alone that we wish to keep constant? Perhaps the Q of the entire circuit should be considered. In this case, Equation (30) would take on an entirely different form.

DESCRIPTION OF APPARTUS

The circuit in Fig. 1 was constructed for use in studying the functions of the grid tank circuit of a Class C R-F amplifier, and also to investigate the behavior of the link coupling circuit connecting it to the preceding stage.

The oscillator is a modified Pierce circuit with sufficient output to meet the maximum demands of the 2E26 grid circuit. A frequency of 3.24 megacycles was used in all the experiments. The 2E26 was operated at rated voltages on the plate and the control grid, but the screen grid was fed from the voltage divider R_L in order that the power output of the stage could be controlled. The plate tank circuit L and C were variable, and the Q of the coil was on the order of 250.

The link circuit consisted of a length of 50 ohm coaxial cable approximately three feet long. It was terminated at each end with a loop of a few turns of wire which were located at the low voltage ends of the tank coils. Inserted in the cable was a standing wave meter, calibrated also to read the power flowing through the cable.

The grid circuit of the 812 final stage was designed for flexibility. The tank circuit L and C were variable over a wide range. Provision for either self bias or a bias pack was made. D-c grid current and voltage were metered, as was the a-c grid voltage.

The 812 plate circuit was made to operate through a link coupling into a 73 ohm non-inductive load to disapate the power generated during the experiments. Good neutralization was accomplished by using the grid-plate capacitance of another 812 tube as C_n in Fig. 1.

The variable inductors and capacitors were calibrated closely on a Boonton Radio Corp. Q Meter, Type 160A. R-F voltage measurements were made on a Hewlett-Packard Meter, Model 410A, using the high impedance probe. All other meters and instruments used were of the best available grade.

Some of the experimentation was carried out using a non-inductive resistor in place of the grid circuit of the 812 tube. Preliminary investigation showed that variable coupling was necessary between the tank coils and the link coils. The link loops were constructed to fit inside the tank coils, and the loops were tapped so their inductance could be varied over a wide range.

The Q Meter was used as a constant current generator for a few of the tests. Very accurate measurements were possible, and by proper interpretation, certain circuit parameters could be set up that were heretofore not measureable. These included R_L, the resistance reflected from the grid circuit to the plate circuit.

EXPERIMENTS AND DATA

The first attempt to solve for the optimum L-C ratio in the grid circuit was experimental rather than mathematical. It was felt that a better understanding of the problem would be had if data from actual tests could be correlated and analyzed. Accordingly, the heretofore described driver stage and output stage were constructed. They were connected by a link of 50 ohm transmission line terminated in a link coil at each end. Provision was made to meter the standing wave ratio (SWR) and the power transfer in the link. The link coupling coils, and the coupling between them and the tank coils, were selected at random. The grid current in the driven stage was kept constant by varying the screen grid voltage in the driver stage. The grid L-C ratio was varied, and values of I_d, C_2, L_2, E_g, P_i, and P_r were taken for each value of I_2 / C_2. (For a description of the above notations see the appendix). The driver plate voltage was held constant for all the experiments.

Three conditions were set up and analyzed:

1. No plate voltage applied to the driven stage.

2. Plate voltage applied to the driven stage, but no power taken from it.

3. Full rated load on the driven stage. This load was a non-reactive dummy resistor, coupled to the output so that rated plate current was drawn by the driven stage.

Self-bias was used, the bias resistor being placed between the grid tank circuit and ground. The resistor was fully by-passed so as to place the bottom of the grid tank circuit at ground potential.

In all cases the power input to the driver stage remained essentially constant as the grid tank L-C ratio was varied. The grid voltage on the driven stage varied only a little and with no definite trend. Below is a compilation of the data taken for the three conditions:

Condition 1.

$L/C \times 10^4$	P_i	P_r	P_t
12.8	9.5	5.5	4.0
10.3	9.6	5.2	4.4
7.6	9.2	4.8	4.4
5.6	8.5	4.1	4.4
3.9	8.6	4.2	4.4
2.6	7.8	3.4	4.4
1.7	7.8	3.3	4.4
0.94	7.0	2.6	4.4
0.48	7.5	2.8	4.7

Condition 2.

$L/C \times 10^4$	P_i	P_r	P_t
12.8	-	-	-
10.5	12.0	7.1	4.9
7.8	9.9	5.0	4.9
5.5	9.0	4.4	4.6
3.8	8.2	3.5	4.7
2.6	8.3	3.5	4.8
1.6	7.2	2.4	4.8
0.97	6.8	2.5	4.3
0.48	7.6	2.5	5.1

Condition 3.

$L/C \times 10^4$	P_i	P_r	P_t
12.8	15.0	6.8	8.2
9.7	13.0	5.2	7.8
7.5	12.4	4.6	7.8
5.4	12.4	4.6	7.8
3.6	10.3	3.0	7.3
2.3	10.1	2.5	7.6

Table 1.

P_i, P_r and P_t are expressed in watts.

The reflected power decreased with a decreasing L-C ratio, showing that the changing L-C ratio was tending to create a better match between the plate tank and the grid tank. The total power delivered to the grid circuit remained fairly constant for each condition.

The results of this experiment indicate that, even though the power input to the grid circuit was not affected by the varying L-C ratio, there should theoretically be a greater efficiency as the L-C ratio is decreased. It is thought that at a standing wave ratio of 1/1 the power input to the grid would make a definite decrease. This was not proven in the above experiment, however.

The next step was to insert a series capacitor in the link circuit. The driver tank circuit, the driven stage grid circuit, and link circuit were carefully adjusted to produce a minimum SWR indication. The following data was taken:

SWR = 1.2/1 $P_i = P_t = 4$ watts $I_c = 0.035$ amp. $E_g = 89$ volts

$L_2 = 6.68 \times 10^{-6}$ henry

It is to be noted that the power required by the grid circuit is now only four watts under full load conditions for the stage, whereas Condition 3 of the data taken without the series capacitor in the link circuit shows the power consumed to be in the neighborhood of 7.8 watts. The efficiency of the coupling circuit is now much greater than before the addition of the series capacitor in the link circuit. The link was examined, and it was found that the series capacitor in the link circuit had tuned the circuit to resonance.

The minimum SWR was obtained by varying the plate and grid tank circuits as well as the link circuit. There was a definite setting of the L-C ratio in both the plate and grid circuits that produced a minimum SWR. This is to be expected of the plate circuit in view of Equations (1) and (2). It also shows that there must be a like relation in the grid circuit.

The power input to the grid circuit was calculated as a matter of interest. The equation used was set forth by Thomas[8] except for the factor

---

8 Thomas, op. cit., p. 1134

(0.9). This factor is said by Eastman[9] to give more exact results when used to correct the original equation.

(A) $$P = 0.9\sqrt{2}\, E_g I_c$$

The calculated and actual measured powers are very nearly in agreement. This calculation was made several times, and the results compared to the actual measured powers. In all cases the results were nearly equal. It may be said that Equation (A) is reliable when a quick check on the power input to the grid circuit is needed.

Equation (30) was used to calculate the value of the Q of the grid circuit. The value of Q calculated in this manner was 14.6. This seemed a reasonable value in view of the statement on page (vii) that the optimum grid circuit Q should be in the neighborhood of 20. Other values of Q calculated in the same manner from data taken when the power input and the grid bias of the driven stage were varied. Q ranged from 12 to 38. These results would indicate that an equation of the following form would predict the optimum L-C ratio for the grid circuit:

(B) $$\omega L_4 = \frac{E_g^2}{P_{in} Q_2}$$

A plot of relative L-C ratio vs. SWR was made using first the untuned link and then the resonant link. Two values of coupling were used for each condition, one very loose, and the other moderately tight. The tuned-link curves were smooth, and they indicated clearly an optimum value of L/C. The points on the untuned-link curves were scattered, and only a random curve could be drawn. No well defined optimum L-C ratio was shown. The SWR for the tuned link was much smaller on the whole than that for the untuned link. Thus it can be seen that tuning the link is advantageous in that the SWR can

---

9 Eastman, op. cit., p. 377

be brought to a minimum, and the resulting efficiency will decrease the amount of power the driver stage must supply to provide a specific excitation at the grid of the driven stage.

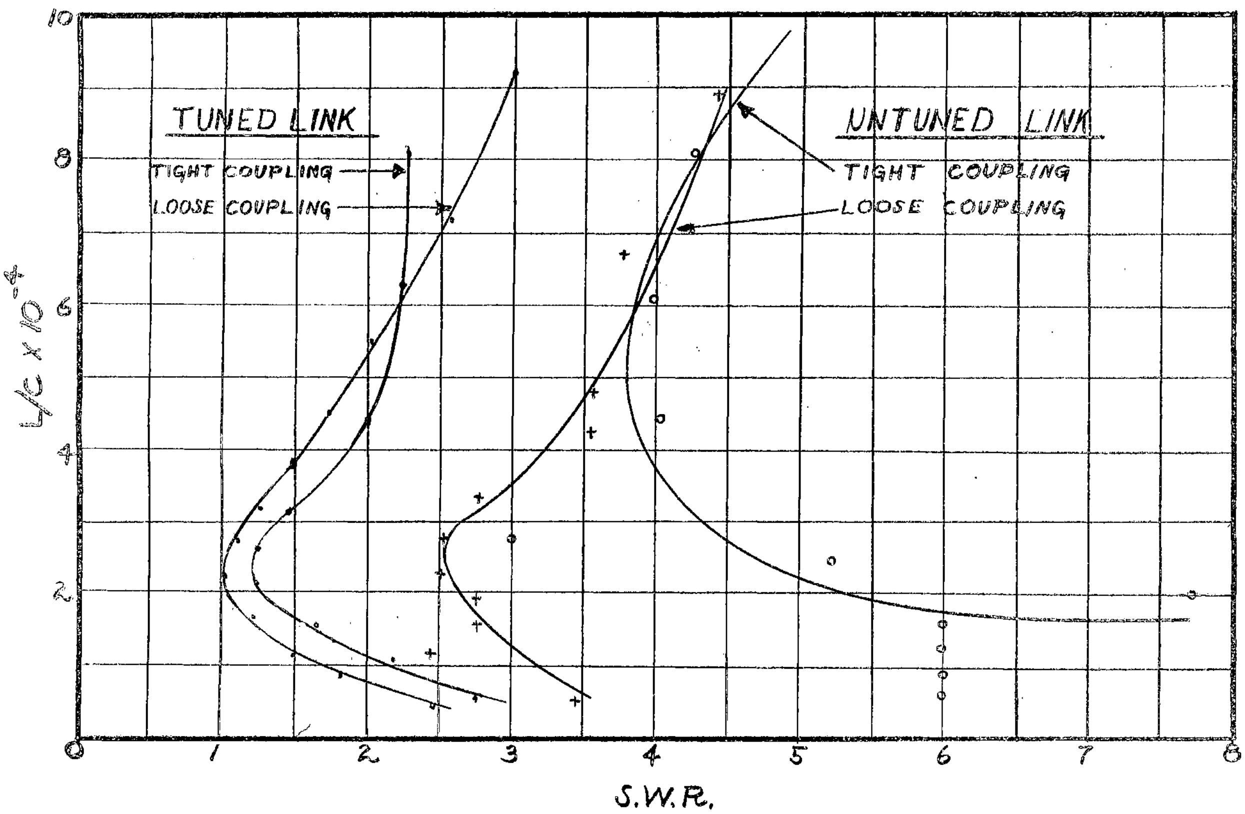


Fig. A. Relative grid tank L-C ratio vs. SWR.

The use of the previously constructed equipment, while good for rough checking upon the effects of varying the grid tank L-C ratio, proved too cumbersome to use in obtaining exact measurements. It was decided to use an impedance bridge to measure exact values of the plate tank circuit. The driving point impedance could then be measured. Several bridges were tried, but all proved too insensitive or too complicated to set up. Finally, the Model 160A Q Meter was chosen to act both as a signal source and as an indicator of the magnitude of the resistance that was reflected as the grid and link circuits were resonated. The circuit of the Q Meter is shown in Fig. 7. It is essentially a constant current device, the exciting voltage being developed across the 0.04 ohm resistor R_1. The resistor is so small compared to the other resistances introduced into the circuit that it may be neglected in most calculations. The coil, L_1, was external to the Q Meter circuit. The Q Meter condensor, C_1, was tuned until the circuit was resonated as evidenced by a maximum reading upon the Q scale of the meter. The Q of the coil was then noted, making sure that there were no outside agents influencing the reading. This value of Q gives the ratio of inductance to resistance for the coil alone. If by some means a resistance is reflected into the Q Meter circuit, the coil will act as if its ratio of inductance to resistance were decreased. Another way of stating this is to say that the Q of the circuit is decreased. The new value of Q can be read from the Q scale. The reflected resistance can then be calculated by:

$$R_R = \frac{(Q_1 - Q_2)R_1}{Q_2}$$

R_1 and Q_1 are the original values for the coil alone and Q_2 is the new value of Q. Any reactance reflected into the Q Meter circuit will be indicated by a change in the setting of C_1 necessary to resonate the circuit.

A circuit in the form of Fig. 8 was used to load the Q Meter, simulating the grid tank circuit and the grid-cathode load. The coupling was varied until maximum power transfer occured. This was indicated by a maximum voltage across R_2. The following data was taken:

E_g	E_c	R_2	Q	E_g/E_c
1.62	2.56	45.2	135	0.632
1.33	2.08	"	133	0.639
1.00	1.50	"	123	0.667
1.02	1.80	58.0	135	0.567
1.35	2.03	"	132	0.665
1.60	2.60	"	131	0.616
0.50	2.64	2.84	140	0.189
0.40	2.17	"	138	0.184
0.30	1.71	"	134	o.175
0.60	1.74	10.6	138	0.345
0.78	2.20	"	142	0.354
0.96	2.75	"	142	0.349

Table 2.

R is given in ohms X 1000, and E_c and E_g are in volts.

The fact that Q remained essentially constant for each value of R_2 employed is noteworthy. This can be explained in the following manner:

$$Q = \frac{\omega L_1}{R_3}$$

where R_3 is the resistance of the circuit of Fig. 8 reflected into the Q Meter circuit of Fig. 7. The Q Meter circuit may be converted into a constant voltage circuit and R_1 may then be considered to be the generator impedance. R_2 in Fig. 8 may be changed to its equivalent series form, R_4. The last two circuits are shown in Fig. 9, where R_3 is the result of the reflection of R_4 into the Q Meter circuit. Then: $X_M^2 = R_1 R_4 = R_3 R_4$

and thus: $R_1 = R_3$

under the conditions of maximum power transfer. R_1 is a constant, hence R_3 must be a constant also. The value of L_1 does not change, so Q must be a constant. The data shows that the above considerations may be taken for granted in making future measurements, as long as care is exercised in setting the value of coupling so that L_1 and L_2 are critically coupled. A maximum transfer of power then occurs.

The constant value of E_g / E_c for a specific value of R_2 can be explained:

$$P_o = G P_{in} \qquad \frac{E_c^2}{X_c^2} R_3 = G \frac{E_g^2}{R_2}$$

The proportionality constant G accounts for any losses in the circuits. As R_3, X_c, E_c and G are constant, the value of E_g / E_c should remain a constant for each value of R_2.

An effort was made to use the Q Meter to demonstrate the existence of an optimum L-C ratio in the grid tank circuit. The circuit of Fig. 8 was altered so as to make L_2 variable. The circuit was tuned to resonance at different L-C ratios, resonance being indicated as before by a minimum reading on the Q scale of the Q Meter. The coupling was then varied until critical coupling was indicated by a maximum reading of E_g. Values of E_g, E_c, Q and L/C were noted.

E_g	E_c	Q	Relative L/C
1.04	3.0	148	97
1.20	3.0	145	96
1.55	2.9	143	94
1.50	3.0	146	80
1.60	3.0	148	65
1.62	2.8	145	62
1.66	2.9	147	59

Table 3.

E_g and E_c are in volts.

The above table shows that Q and E_c remain fairly constant while the L-C ratio changes. This indicates an essentially constant input to the circuit. The power input, E_g^2 / R_2, increases as L/C decreases. Thus the ratio of L to C has a direct effect upon the efficiency of power transfer.

CONCLUSIONS

The investigation of the grid circuit of a Class C R-F amplifier and the coupling from it to the preceding plate tank circuit as covered by this paper is by no means complete. We may draw several conclusions from the data, but there is a need for much further work on the problem before a complete general analysis can be given

The mathematical analysis in this paper totally disregards the tube grid-cathode impedance as it might be encountered in actual practice. Instead, an equivalent resistance has been substituted for simplification of the problem. Most authors agree that the grid-cathode path of the tube may be represented by such a resistance, and Equation (A) of the Experiments and Data section of this paper would seem to indicate that this is a good assumption. Several questions may be asked at this point if we are conducting a rigorous investigation. Will the substitution of an equivalent resistance for the grid-cathode impedance suffice if the output of the tube is varying? Will this substitution hold for all values of bias, as the varying bias may change the shape of the grid current wave form? The grid current is not sinusoidal, so will an analysis on the basis of a sinusodial current be exact enough to be used in practice? Finally, we must not neglect the fact that the load on the grid tank circuit is composed of the tube grid-cathode impedance plus the impedance of the bias source. Rectification occurs in the tube, so we will have to consider a circuit that has both a-c and d-c currents flowing in it.

The mathematical theory presented herein points to a definite relation between the grid tank inductance and the load applied to the grid tank circuit. An equation involving both the grid tank inductance and the load impedance was derived for a circuit similiar to Fig. 2a, except that the link circuit was assumed to be tuned to resonance in the derivation. The two parameters in question were found to have a direct relation to each other. An analysis of the plate tank and grid tank circuits was made using for a coupling only the mutual impedance of the tank coils. Again it was found that the grid tank inductance was directly proportional to the load presented to the tank circuit. The mathematical theory was put to test in the laboratory by conducting several appropriate experiments.

The actual conduction of the experimental part of this problem was hindered both by lack of time and by inexperience on the part of the author. However, some light was shed on the techniques and equipment necessary to produce accurate and adequate data. Let us sum up the results of the experimentation before going into the limitations of the equipment.

The first important discovery was that the link circuit should be tuned to resonance for a maximum transfer of power. With the link untuned, the data taken at the grid tank was too unreliable to be of value. However, using the tuned link data was taken that showed a definite optimum L-C ratio for the grid tank circuit under specific conditions. Fig. A shows the difference between the tuned and untuned link, and also demonstrates the existence of the optimum L-C ratio. A way of determining the optimum L-C ratio from some easy-to-make measurements was the next problem.

It was stated in the introduction that an equation of the form of Equation (1) was possibly what we were looking for to define the optimum L-C ratio. The mathematical theory gives evidence of this in Equation (30). Accordingly, Equation (B) was set up and actual experimental values were the field as the optimum Q of the grid tank circuit, indicated that perhaps Equation (B) could be used to give an estimation of the optimum grid tank L-C ratio.

Table 3 shows that the efficiency of power transfer from the plate tank to the grid tank is a function of the L-C ratio in the grid tank. There is no indication of the relation of L/C to the power transferred, however.

The problem in constructing the equipment was mechanical rather than electrical. Time and facilities did not permit the design of more appropriate devices for conducting the experiments. An ideal version of the circuit of Fig. 1 would incorporate electrostatic shielding between the link coils and the tank circuits. This is necessary if we are to limit the coupling strictly to the magnetic properties of the coils. The tank and link coils should be made variable, and the coupling should be adjustable over a wide range. Provision should be made, if possible, to measure the circulating a-c currents in both the grid and plate tanks. These values would aid in calculating the power output of the driver stage and the power input to the driven stage.

All the variable components in the equipment should be directly calibrated. This will greatly facilitate the compilation of data and will lessen the chance of error in later computations.

The Model 160A Q Meter proved to be the most versatile of the instruments used in the experiments. Several bridges were tried in an attempt to measure the driving point impedance at the terminals of the plate tank circuit. Their main trouble was a lack of range. Long and involved calculations were necessary in order to determine a reading outside the range of the instrument. The Q Meter solved the problem nicely by providing in one instrument both a signal source of variable amplitude and an indicating device that gave values of reactance and resistance reflected back into plate tank circuit from the grid circuit. Simple calculations were involved in determining the actual value from a reading on the instrument. The sensitivity was very great in contrast to the sluggish action of the bridges previously tried.

The power output of the Q Meter is on the order of microwatts. It is this limitation that prevents its use as a driver stage. If it were possible to use it to drive a tube through suitable coupling networks, investigation of the coupling problem would be vastly simplified.

Measurements are always complicated by the introduction of R-F into the circuit. Experiments should be carried out on a large sheet of copper that is well grounded. All the instruments and equipment should be connected to the copper sheet as a reference point. The body of the operator has a great effect on the readings obtained. For this reason, care must be taken to read the instrument with the body in the same position at all times.

The load used on the grid circuit to compile Table 3 was 40k ohms. This is much above the equivalent resistance usually found in a tube grid-cathode circuit. Lower values of resistance were tried as a load, but the readings of the Q Meter were not then consistent. It is thought that perhaps the Q Meter was being overloaded. It is primarily an instrument for measuring rectance, and is not intended to furnish much power in so doing.

To sum up, an optimum L-C ratio exists in the grid tank circuit for a specific set of grid conditions. The expression for L in this case may be of the form of Equation (B), but it has not been proven that this is a general equation that will fit all circumstances. The experimentation also indicates that the grid tank inductance is directly proportional to the equivalent resistance of the tube grid-cathode path, but again there is not enough evidence to call Equation (B) a general equation.

It is hard to correlate the experimental data with Equation (B) as Q cannot be directly measured. An equation, or set of equations, that will express an optimum value of grid tank inductance in terms of quantities that can be readily measured is needed. If this can be found, and if the experimental data will fit the equation, for all conditions, then the equation may be of invaluable aid in the design of Class C R-F amplifiers.

APPENDIX I

EXPLANATION OF SYMBOLS

R_L - The resistance reflected into the plate tank circuit which represents the power output.

E_g- The effective value of the alternating grid voltage of the driven stage.

E_p - The effective value of the alternating plate voltage of the driver stage.

P_o - The power output of the driver stage.

P_{in} - The power input to the grid circuit of the driven stage.

w_o - The resonant frequency of a circuit.

Q_1 - The Q of the driver stage tank circuit.

Q_2 - The Q of the driven stage's grid tank circuit.

R_c - The equivalent series resistance in the grid circuit of the driven stage.

R_e - The equivalent series resistance in the plate tank circuit.

R_r - The reflected resistance appearing in the grid circuit due to the plate circuit impedance.

P_i - The incident power in the link circuit that is traveling towards the grid circuit of the driven stage.

P_r - The power that is reflected from the grid circuit back to the plate circuit of the driver stage.

P_t - The sum of P_i and P_r (P_r is considered negative) which gives the actual power consumed in the grid circuit.

APPENDIX II

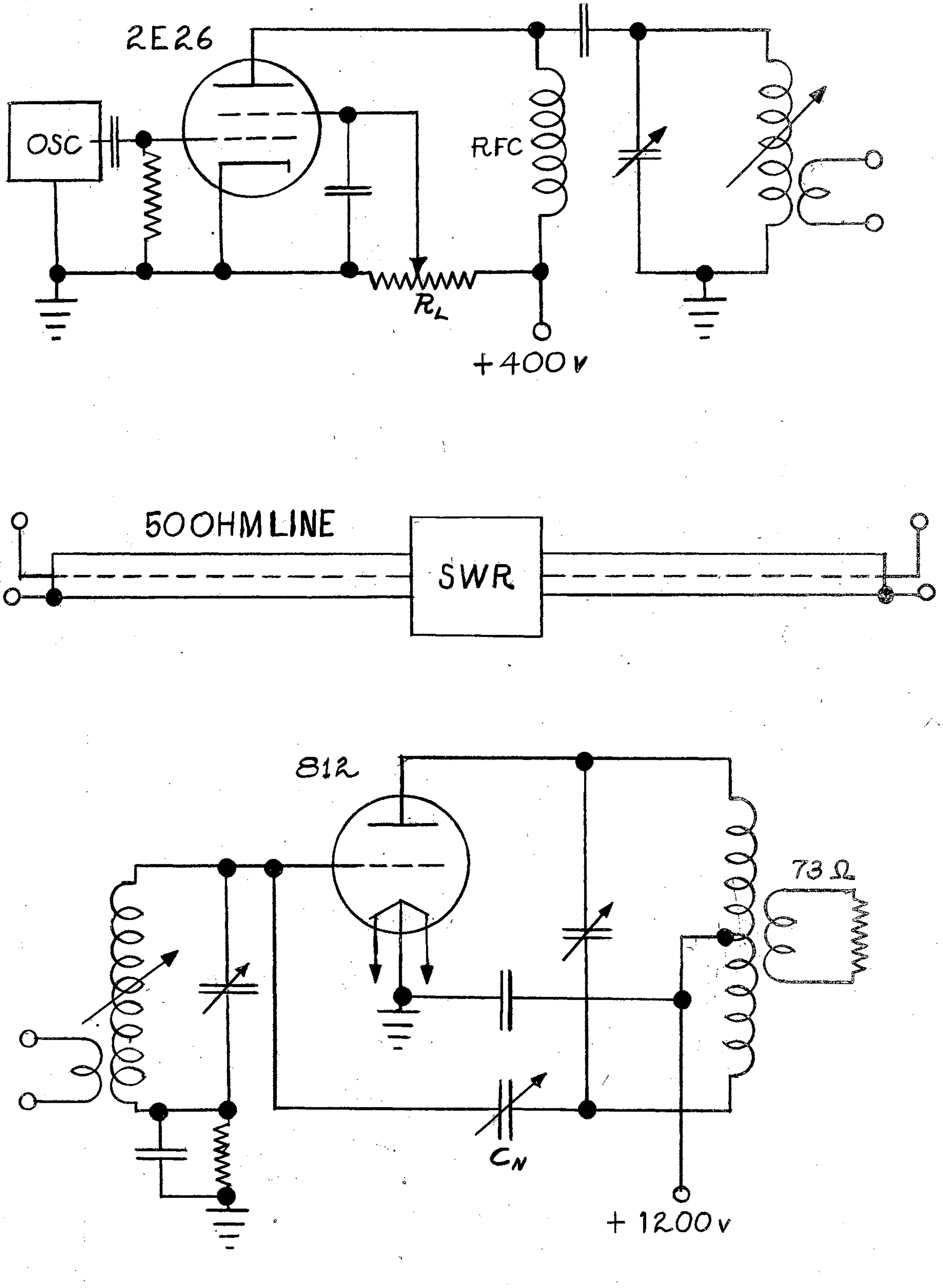


FIG. 1

FIG. 2

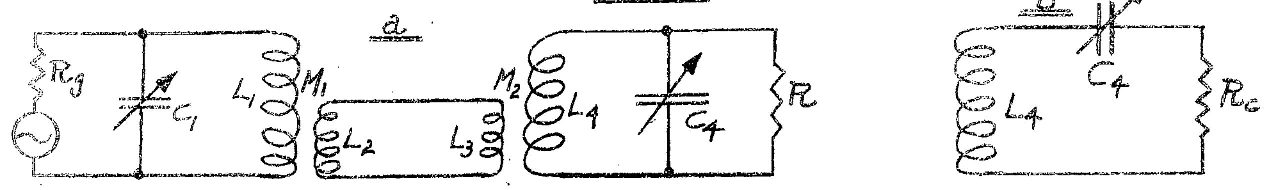


FIG. 3

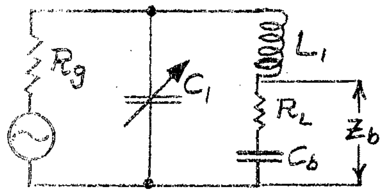


FIG. 4

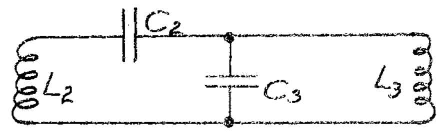


FIG. 5

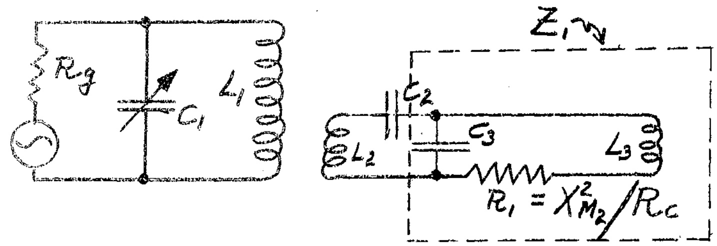


FIG. 6

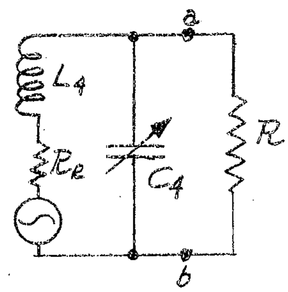


FIG. 7

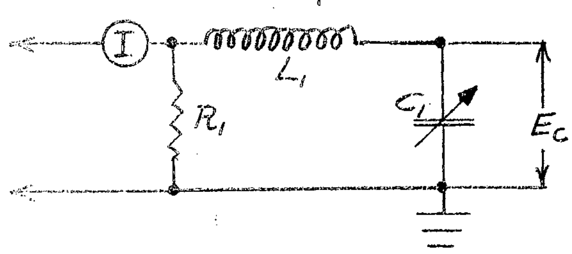


FIG. 8

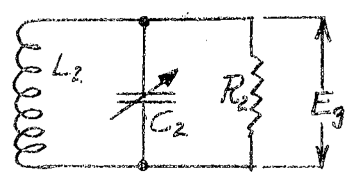
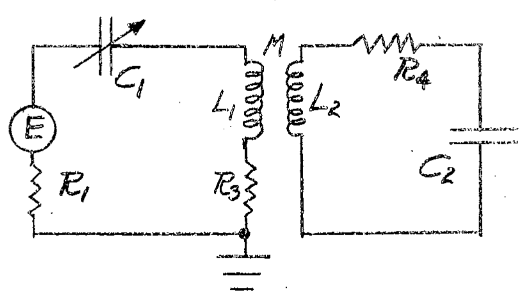


FIG. 9



BIBLOGRAPHY

Eastman, A. V. Fundamental of Vacuum Tubes. New York: McGraw-Hill Book Company, Inc., 1941.

Everitt, W. L. Communication Engineering. New York: McGraw-Hill Book Company, Inc., 1937.

Everitt, W. L. "Optimum Operating Conditions for Class C Amplifiers." Proceedings of the I. R. E., Vol. 22, No. 2 (February, 1934), 152.

M. I. T. Staff. Applied Electronics. New York: John Wiley and Sons, Inc., 1949.

Ryder, J. D. Networks, Lines and Fields. New York: Prentice-Hall, Inc., 1949.

Thomas, H. P. "Determination of Grid Driving Power in R-F Amplifiers." Proceedings of the I. R. E., Vol 21. No. 8. (August, 1933), 1134.

THESIS TITLE: Determination of the Optimum Grid Tank L-C Ratio for the Class C Amplifier

NAME OF AUTHOR: Donald G. Hallock

THESIS ADVISER: Harold Fristoe

The content and form have been checked and approved by the author and thesis adviser. "Instructions for Typing and Arranging the Thesis" are available in the Graduate School office. Changes or corrections in the thesis are not made by the Graduate School office or by any committee. The copies are sent to the bindery just as they are approved by the author and faculty adviser.

NAME OF TYPIST: Nancy Weed Herberger