OF A SFERIC DIRECTION FINDER

TRADACTIONE PARCHIGENT

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THE DEVELOPMENT

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#### SFERIC DIRECTION FINDER

Ву

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1948

Submitted to the School of Electrical Engineering Oklahoma Agricultural and Mechanical College In Partial Fulfillment of the Requirements

> for the Degree of MASTER OF SCIENCE

APPROVED BY:

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#### PREFACE

A tornado is a whirling wind accompanied by a funnel-shaped cloud and is the most violent of all storms. The wind in its vortex travels at a rate of 300 to 500 miles per hour. Tornadoes have been a menace to Oklahoma and its neighboring states for many years.

An extensive research program is being carried on by the School of Electrical Engineering of the Oklahoma Institute of Technology in an attempt to detect the formation of a tornado before it reaches the ground. When the results of this project are complete, they will be used to locate the formation of tornadoes, and the information will be passed on to a tornado tracking station. The purpose of this thesis is to develop a direction finder that will indicate the direction from which sferies are arriving. This direction finder is to be used in conjunction with equipment that records the waveform characteristics of a sferie discharge.

#### ACKNOWLEDGEMENT

The writer wishes to express his sincere appreciation to Professor H. L. Jones for his many helpful suggestions and careful scrutiny of this material and to Professor A. Naeter for his detailed reading of the subject matter.

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#### CHAPTER I

#### INTRODUCTION

Atmospheric disturbances are technically known as sferics. For the past twenty-five or thirty years scientists have carried on extensive research projects to determine the origin and characteristics of sferic disturbances. Many theories have been advanced as a result of these studies, and several systems have been devised to locate and track storms.

One of the theories that has been proposed is that the sferics accompanying a tornado differ from those in an ordinary thunderstorm, either in their waveform characteristics or the length of time for the discharge, or both, 1,2 This theory is being investigated at the present time by the School of Electrical Engineering of the Oklahoma Institute of Technology.

To those who are familiar with the tornado situation in Oklahoma and neighboring states, it is not surprising that this subject should be given extensive study by the Oklahoma Institute of Technology. A portion of an article published by the United States Department of Commerce is quoted.<sup>3</sup> "In all the world no place is more favorable for tornado formation than the relatively flat region lying east of the Rocky Mountains in the United States."

1 Miller, Carl William, <u>A Proposed Method of Identifying</u> and <u>Tracking Tornadoes</u>, Master of Science Thesis, Oklahoma Agricultural and Mechanical College, 1949.

<sup>2</sup> Hutchison, J.S., <u>A Study of Tornado Identification</u>, Master of Science Thesis, Oklahoma Agricultural and Mechanical College, 1949.

<sup>3</sup> U.S. Weather Bureau, <u>Tornadoes</u>, Daily Weather Map for June 13, 1949. A chart in this same publication shows by states, the total number of days in which one or more tornadoes were reported for a period of sixty-eight years, from 1880 to 1947 inclusive. The four states with the most tornado days were: Kansas 471, Texas 374, Iowa 326, and Oklahoma 225.

Tornadoes are the most violent of all storms, having wind velocities in their vortex ranging from 300 to 500 miles per hour. Property damages have been as high as forty-three million dollars in 1927, and fatalities in 1925 reached a high of 794.<sup>4</sup>

For eighteen months a superhetrodyne receiver, operating at 200,000 cycles per second, has been used by the Oklahoma Institute of Technology, to detect the electrical discharges that accompany atmospheric disturbances. The receiver output is applied to the vertical deflecting plates of a cathode-ray oscilloscope and the waveforms produced on its screen are photographed. These waveform pictures are studied in conjunction with the weather bureau reports, so that waveforms that are known to have been from a tornado can be compared with those known to have occurred during a thunderstorm.

To help in this work an amplifier that has a flat frequency response from 20 to 200,000 cycles per second is being built. The antenna for this apparatus is a short vertical whip antenna that has no directional characteristics. The amplifier will be used for the same purpose as the superhetrodyne receiver previously mentioned. This method would be sufficient if there is but

4 Ibid.

one atmospheric disturbance within the range of the amplifier at a given time. However, suppose there are two storms within the range of this equipment simultaneously, one a tornado and the other a thunderstorm. In this case the waveforms could be more intelligently interpreted if it were known which waveforms came from the tornado and which originated in the thunderstorm. The purpose of this thesis is to develop a direction finder that will indicate the direction from which each recorded waveform is arriving.

#### CHOICE OF ANTENNA SYSTEM

A sferic direction finder differs from the conventional direction finder in that it must show direction instantaneously. This property alone makes many types of antennas unsuited for sferic direction finding. The reason for requiring instantaneous direction finding is that sferics are of a very short duration, usually less than one-half of one second.

Since we need to know the direction from which each recorded waveform is arriving, antennas that have directional characteristics but have to be rotated mechanically cannot be used. Two antenna systems that are suited to our needs will be discussed briefly, pointing out which is to be used and the reason for choosing it.

#### 1. The Loop Antenna

Hertz, in one of his experiments, rotated a small loop of wire about its axis at a mean distance from an apparatus radiating electromagnetic energy. The loop contained a small spark gap. At certain positions a spark would pass across the gap, while at other positions nearby, the spark was not produced. This experiment indicated that the loop had directional characteristics.

In this brief discussion of the loop antenna it is sufficient to say that the polar diagram of a loop with dimensions that are small compared to the wavelength being received, is a figure eight, where the maximum energy received is in the plane of the loop and the minimum energy received is perpendicular to the plane of the loop, as shown in figure 5.<sup>5</sup> The polar diagram of the loop will be developed in detail in the theory section of this thesis. It should be pointed out that this antenna has 180 degrees ambiguity, that is, the e m.f induced into the antenna from a signal that makes an angle of 60 degrees with the plane of the loop, is identical to one from the same signal at 180 plus 60 degrees.

If the electromagnetic wave that approaches the loop is vertically polarized, the vertical members are active. However, if there is a horizontal component perpendicular to the plane of incidence of the wave, the horizontal members of the loop will respond and there will be a resultant error in bearing. If the initial wave is the predominate wave, the loop will give reasonably good readings of bearing. This type of error is known as polarization error or night effect.

#### 2. The Adcock System

An antenna that has two vertical members connected together at the bottom is commonly called an Adcock antenna. If the horizontal member connecting the vertical antennas is properly balanced or shielded, the error due to the horizontal component of the electric vector of an electromagnetic wave is less than that for a comparable loop antenna.

<sup>5</sup> Keen, R., <u>Wireless Direction Finding</u>, p. 72.

One of the main disadvantages of this antenna is that it has the same sensitivity as a one turn loop. An excellent description of the Adcock antenna system is given by Keen.<sup>6</sup>

### 3. Summary of the Choice of Antenna Systems

A balanced Adcock antenna has less error due to night effect than the loop; however, the loop can be made more sensitive. Making the assumption that the error due to night effect is negligible, the major consideration in choosing the antenna to be used is its sensitivity. Because of this assumption the loop antenna is chosen for the sferic direction finder.

Two loopsdesigned to be used with the U.S. Signal Corps BC-433-G were available and are used in this sferic direction finder.



#### CHAPTER II

#### THEORETICAL CONSIDERATIONS OF THE LOOP ANTENNA

#### BLOCK DIAGRAM OF EQUIPMENT

All sferic direction finders have three distinct parts, the antenna, the amplifier, and the bearing indicator. The antenna must have a sensitivity that is a function of the direction from which the electromagnetic wave is arriving.

Unless the electromagnetic field intensity is very strong or the loop has many turns, the induced voltage in the antenna is not great enough to actuate a bearing indicator. For this reason an amplifier must be placed between the antenna and the bearing indicator.

A bearing indicator must be used to show the direction from which the sferics are arriving.

#### THE LOOP ANTENNA

The directional characteristics of an antenna are best studied by the use of a plane polar diagram, which can be obtained with a transmitter used in conjuntion with a receiver. The current in a receiving antenna is measured while rotating the antenna in the field of a transmitter. The polar diagram of the receiving antenna is a polar plot of the measured currents versus the angle of rotation of the antenna. The polar diagram of the antenna as a transmitting antenna can be obtained by exchanging the roles of the transmitter and receiver.

The Rayleigh-Carson theorem relating the properties of transmitting and receiving antennas states that "whatever properties an antenna possesses as a transmitter or radiator are maintained by the same antenna when used as a receiver". In the mathematical development of the polar diagram of a loop antenna, it is considered as a transmitting antenna.

1. The Mathematical Development of the Field Pattern

One method for determining the energy radiated from an antenna is called the Poynting vector method. With this method the antenna is assumed to be at the center of a very large sphere and the field produced over the surface of the sphere by the assumed current is calculated. Under these circumstances the following assumptions are justified.<sup>1</sup>

1. Differences in radius vector to points on the antenna have a negligible effect on magnitudes.

2. Differences in direction of the radius to different points on the antenna are negligible.

3. Field components decreasing faster than  $\frac{1}{r}$  are negligible with respect to those decreasing as  $\frac{1}{r}$ .

4. For purposes of finding phase differences, in figure 2, r'cos m is taken as the difference in the radius vector to different points on the antenna.

In figure 2, q is the radiator, M is a point on the surface of the sphere where the field is being studied, r' is the radius to the radiating element.

The spherical notation illustrated in figure 3 will be used; where r is the radius vector to a point M, the angle  $\Theta$  is between OM and the Z axis, and the angle  $\phi$  is between the projection of OM on the XY plane and the X axis. These parameters are the spheri-

1 Ramo, Simon and Whinnery, John R., Fields and Waves in Modern Radio, p. 454.



cal coordinates of the point M. Their relations to rectangular coordinates are expressed as follows,

 $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta$ ,  $\sin \phi$ ,  $Z = r \cos \theta$ The mutually perpendicular vectors are  $l_r$ ,  $l_{\theta}$ ,  $l_{\phi}$ . Notations that are used in this derivation are:

A = Vector magnetic potential

- I = Current density
- Velocity
- V = Volume

w = Angular velocity

- j = -1
- $k = \frac{W}{V} = W u$
- N = Radiation vector

F = the Poynting vector

- $\epsilon$  = Dielectric constant
- t = Time
- n = Intrinsic impedance of a dielectric
- $\lambda = Wavelength$
- u = Permeability

In figure 2 the vector potential at the point M due to current in one element of the radiator q, is

$$\overline{A} = \int_{\nabla} \frac{i_{q} e^{j W (t - \frac{\mathbf{r}^{H}}{\nabla})}}{4\pi r^{H}} dV$$

From the preceding assumptions it follows

$$\overline{A} = \frac{e^{-jkr}}{4\pi r} \int_{V} \overline{I}_{q} e^{j'kr'} \cos \theta$$

When  $\cos m = \cos \Theta \cos \Theta' - \sin \Theta \sin \Theta' \cos (\phi - \phi')$ where  $\Theta$ ,  $\phi$  and  $\Theta' \phi'$  are the spherical coordinate angles of M and q respectively. The integral is independent of r and and is a function of the current, the antenna configuration and of the direction in which the field is being calculated. If the integral is defined as the radiation vector  $\overline{N}$ , then

 $\overline{N} = \int_{\nabla} \overline{i}_{q} e^{j'kr'} \cos m dv'$ 

and

$$\overline{A} = \frac{e^{-jkr}}{4\pi r} \overline{N}$$
$$= \frac{e^{-jkr}}{4\pi r} (\overline{l_r} N_r - \overline{l_{\theta}} N_{\theta} - \overline{l_{\theta}} N_{\theta})$$

The Poynting vector P is defined as

 $\overline{P} = \overline{E} \times \overline{H},$  $\overline{E} = \text{Electric field strength}$ 

Where

And  $\overline{H}$  = Magnetic intensity, and may be interpreted as the power-density flow per unit area. It can be shown upon examination of  $\overline{E}$  and  $\overline{H}$  that  $\overline{P}$  has a time average value.<sup>2</sup>

$$P_{\mathbf{r}} = \frac{1}{2} \mathbf{x} \frac{n}{2\lambda \mathbf{r}} \mathbf{x} \frac{1}{2\lambda \mathbf{r}} \left( \left| \mathbb{N}_{\Theta} \right|^{2} - \left| \mathbb{N}_{\phi} \right|^{2} \right)$$

The total time average power radiated is

$$W = \frac{n}{3\lambda^2} \int_0^{\pi} \int_0^{2\pi} \left( \left| N_{\Theta} \right|^2 - \left| N_{\phi} \right|^2 \right) \sin \Theta d \Theta d \phi.$$

This equation is independent of r.

If the currents in an antenna are circularly symmetrical about an axis, this axis may be used as a reference for a set of spherical coordinates. The vectors  $\overline{A}$  and  $\overline{N}$  can have only a component in the q direction and the total time average power radiated is

$$\mathbb{W} = \frac{2\pi n}{8\lambda^2} \int_0^{\pi} |\mathbb{N}\phi|^2 \sin \Theta d \Theta$$

These provisions hold for a circular loop antenna, that is,

<sup>2</sup> <u>Ibid.</u>, p. 455.

all the currents in the antenna are circularly symmetrical about an axis as shown in figure 4. In this case

 $e^{1} = \frac{\pi}{2}$ 

and H / has the same value for all values of  $\phi$ .

For  $\phi = 0$ 

 $\cos m = \sin \Theta \cos \phi$ 

and  

$$H_{\phi} = I \int_{0}^{2\pi} e^{jkq} \sin \Theta \cos \phi' \cos \phi' q d \phi'$$
  
 $j k \pi q^2 I \sin \Theta$ 

Therefore it is evident that the field pattern varies as a constant times the sin  $\Theta$ , where  $\Theta = 0$  is perpendicular at the plane of the loop.

 $N_{\phi} = C \sin \theta$ where  $C = j k \pi q^2 I$ .

Figure 5 shows the projection of the radiation vector,  $N_{\phi}$ , on a plane passed through the sphere upon whose surface the field is being studied, at the angle  $\phi = 0$ . The angle  $\Theta$  is measured with respect to a plane that is orthogonal to the plane of the loop.

It is standard practice to use the plane containing the loop as the reference to which the angle of radiation from the antenna is measured. An angle of 90 degrees must be added to the angle used in the preceding derivation to make the results comply with this convention. Making this change we have

 $N_{\phi} = C \sin(\Theta - 90)$ 

# = $C \cos \Theta_{1}$

where Q1, as shown in figure 5, is the angle between the new



reference plane and the radius along which Nø is to be calculated. Figure 5 is commonly called the figure eight or cosine diagram of a loop antenna.

Upon expanding C we get

but then since

and

 $C = j k \pi q^{2} I$   $k = w / u \epsilon$   $C = 2 \pi^{2} f q^{2} I / u \epsilon$   $f = \frac{1}{\lambda / u \epsilon}$ 

 $c = \frac{2\pi^2 q^2 I}{\lambda} .$ 

The area enclosed by the loop antenna is

$$A = q^2 \pi,$$

where q is the radius of the loop. Substituting A in place of  $q^2$  in the expansion of C gives

$$C = \frac{2\pi A I}{\lambda} .$$

The value of the radiation vector becomes

$$N_{\phi} = \frac{2\pi A I}{\lambda} \cos \Theta_1.$$

The emf induced into a one turn loop used as a receiving antenna can be shown to be

emf = 
$$\frac{2\pi A e}{\lambda} \cos \theta_1$$

where e is the field intensity in volts per meter, A is the area of the loop in square meters, and  $\lambda$  is the wavelength of the electromagnetic wave in meters.<sup>3</sup>

2. Graphical Development of the Field Pattern

The graphical development of the field pattern of a loop antenna will be made, the loop considered as the receiving antenna.

<sup>3</sup> Terman, Fredrick Emmons, <u>Radio Engineers Handbook</u>, p. 813.

The plane of the loop is a vertical plane, and only vertically polarized plane waves will be considered. For example, the electric vector of the electromagnetic wave has no horizontal component.

Figure 6 is a vector plot of a plane wave. Skilling<sup>4</sup> shows that the rate of flow of the electromagnetic energy is;

$$P = E \times H$$
,

where  $\overline{P}$  is known as Poynting's vector. From the theory of the cross product,  $\overline{P}$  is perpendicular to the plane containing both  $\overline{E}$  and  $\overline{H}$ .<sup>5</sup> E<sub>y</sub> and H<sub>z</sub> are the only components of an electromagnetic wave traveling in the x direction.<sup>6</sup>

The voltage induced in a loop antenna has the same value providing the shape of the loop is symmetrical about an axis, and the dimensions of the loop are small compared to those of the wavelength involved.

A square loop will be used in this graphical derivation of the field pattern of a loop antenna.

Faraday noted that the voltage induced into a coil of wire is proportional to the change in flux linking the coil,

 $e = -\frac{d \phi}{d t} \times 10^{-8}$  volts,

where e is the induced voltage,  $\phi$  is flux, and t is time. The minus sign is from Lenz's law, "The direction of an induced emf is such as to tend to oppose the change in flux-linkage which produces it".

<sup>4</sup> Skilling, Hugh Hildreth, Fundamentals of Electric Waves, pp. 131-133.

<sup>5</sup> Phillips, H.B., <u>Vector Analysis</u>, p. 10.

<sup>6</sup> Sarbacher, Robert I. and Edson, William A., Hyper and Ultrahigh Frequency Engineering, p. 64.



When a loop is placed in the field of the electromagnetic wave of figure 6 so that the plane of the loop is the XY plane, the magnetic vector is orthogonal to the plane of the loop. The magnitude of the flux in the immediate vicinity of the loop at a given time is uniform, as shown in figure 7, since the wavelength to be received is long compared to the dimensions of the loop.

A loop antenna is rotated in the magnetic field of figure 7 and the effects on the flux linking the loop is noted. The polar diagram of the loop is a polar plot of the flux linkage versus the angle of rotation of the antenna. It can be seen immediately that maximum flux linkage occurs in position A-A' where the plane of the loop is in the direction of travel of the wave, and that when the plane of the loop is perpendicular to the direction the wave is. traveling, there is no flux linkage and consequently, no induced voltage or signal received. This signifies that the loop has directional characteristics. Four other positions of the loop are shown at intervals of 30 degrees around the circumference of a circle whose radius is OA. In the position D-D! there are no flux lines through the loop, at E-E' there are 16; at F-F' there are 28; and 32 at A'-A; at B'-B there are 28; at C'-C there are 16, and none at D'-D. For one-half of a revolution of the loop the flux linkage, and hence the induced voltage, start at zero, increase to a maximum and then decrease to zero. During the next one-half of a revolution the growth of flux linkage recommences at D'-D, reaches a maximum at A-A' and falls to zero at D-D'. Although the same number of lines are linked in the C'-C and E'-E positions, the lines do not pass through the loop in the same direction as indicated by the arrows at C' and E'; therefore, the



induced voltages around the loop are in opposite directions. This change of direction starts as the loop passes through the position D'-D.

Using a negative sign to indicate the change of direction of current the results are tabulated as follows:

Position of frame	D-D1	R-€1	F-F'	A * - A	B'-B	C'-C	D'-D	E1-E	F'-F	A-A'	B-B'	C-C'
Linkage	0	-16	-28	-32	<b>-</b> \$8	<b>-1</b> 6	0	<b>41</b> 6	<b>+</b> 28	+32	+28	+16

Chart I.

It is purely arbitrary which half has a negative sign.

The polar diagram of a loop antenna is shown in figure 8, using a series of radial lines corresponding to the positions of the loop in figure 7, the lengths of which are proportional to the values given in Chart I.

This shows that the points all lie on the circumference of two circles, one for the positive linkage and one for the negative linkage. The polar plot is the figure eight or cosine diagram. 3. Crossed Loops

A sferic direction finder must give an indication of bearing in all directions. This requirement can be fulfilled by using two loop antennas positioned so that their planes are orthogonal. Two loops have to be used since a single loop will not give an indication of bearing in a plane perpendicular to the plane of the loop. The vector sum of the two field patterns of the two loops is shown in figure 9, providing the loops are sufficiently separated so that there is no coupling between them.

Referring to the block diagram of figure 1, it is seen that the output of each loop is connected through an amplifier to a





cathode-ray tube.

Before proceeding with the crossed-loops it will be necessary to establish a few facts concerning the application of the cathoderay tube as a bearing indicator.

The electron beam of a cathode-ray tube can be deflected with an electrostatic or electroagnetic field. A cathode-ray tube that employs electrostatic deflection will be used in this direction finder. A tube of this type has two pairs of orthogonal deflection plates. Figure 10 shows the conventional position of the tube; one set of plates deflects the electron beam in a horizontal plane, and the other set deflects the beam in a vertical plane. The beam or spot on the screen of the tube will be deflected in the plus Y direction when a direct voltage is applied between  $V_1$  and  $V_2$  so that V1 is positive with respect to V2. Reversing the polarity of the voltage on  $V_1$  and  $V_2$  will cause the spot to move in the minus Y direction. Similar statements can be made with reference to H1 and H2. The spot will move in accordance with both the vertical and horizontal plates if the two voltages are applied simultaneous-Suppose the two voltages are equal and of the proper value to ly. give one inch deflection in both the X and Y directions, and also assume that the spot is originally at the origin, O, of the cartesian coordinates. The resultant position of the spot will be:

> $R = \sqrt{X^{2} + Y^{2}} = \sqrt{1 + 1} = \sqrt{2} = 1.414 \text{ inches,}$  $\phi = \arctan \frac{Y}{X} = \arctan 1 = 45^{\circ}.$

The resultant equal 1.414 45°; that is, the spot will be 1.414 inches from the origin along a line making a 45° angle with the horizontal. Reversing the polarity of the applied voltages will



reverse the direction in which the spot moves.

The electron beam will trace a line along the Y axis of the screen of the cathode-ray tube if an alternating voltage is applied to the vertical deflection plates. Similarly, the electron beam will trace a line along the X axis if an alternating voltage is applied to the horizontal deflection plates. The deflection of the beam of a cathode-ray tube is proportional to instantaneous voltage applied to the deflection plates of the tube. Thus the length of the line traced on the screen of the cathode-ray tube by an alternating voltage is proportional to the peak to peak value of the voltage applied to its deflection plates.

The instantaneous values of an alternating voltage deflect the beam in a cathode-ray tube in the same manner as a direct voltage of the same magnitude. However, the alternating voltage retraces the path of the beam each cycle and will make a trace on the screen of the cathode-ray tube while the direct voltage will only show evidence of displacing the spot on the screen. If an alternating voltage is applied to both the vertical and horizontal deflection plates the resultant deflection of the beam of the cathode-ray tube can be calculated, using instantaneous values, by the same method employed with direct voltages. By representing the instantaneous magnitude of the voltage applied to the vertical deflection plates as  $E_y$  and that applied to the horizontal deflection plates as  $E_x$ , the magnitude of the resultant deflection at a given instant is,

 $R = \sqrt{E_y^2 + E_x^2}$ 

and the angle of R with respect to the X axis is,

 $\phi$  = arc tan  $\frac{E_y}{E_y}$ .

If the two voltages have equal magnitudes and are in time phase, the resultant deflection will be a line equal in length to 1.414 times the peak to peak magnitude of either voltage and the angle between the line and X axis will be 45 degrees.

Considering the plane of loop 1, figure 9, as the reference from which 0 is measured, the voltage induced into loop 1 from an electromagnetic wave is,

$$E_1 = B \cos \Theta$$
,

and the voltage induced into loop 2 from the same wave is,

where B is proportional to the strength of the electromagnetic field. Loop 1 is connected through an amplifier to the horizontal plates of a cathode-ray tube. The voltage applied to the plates of the cathode-ray tube is,

 $E_h = B A_h \cos \Theta_r$ 

where A<sub>h</sub> is the amplification of the amplifier between the horizontal plates of the cathode-ray tube and loop 1. Similar statements can be made concerning loop 2 and the vertical plates of the cathode-ray tube. The voltage applied to the vertical plates is,

#### $E_{\mathbf{v}} = B A_{\mathbf{v}} \sin \Theta$ .

Since the plates of the cathode-ray tube are orthogonal, the magnitude of the resultant line on the screen can be determined by a simple rectangular parallelogram of forces.

Vectorially, this is, as shown in figure 11,

$$E_{\rm R} = \sqrt{E_{\rm h}^2 + E_{\rm v}^2}$$
$$= \sqrt{B^2 A_{\rm h}^2 \cos^2 \Theta + B^2 A_{\rm v}^2 \sin^2 \Theta}$$



 $E_R = B A_h^2 - A_v^2.$ If  $A_h = A_v = A$ then  $E_R = B A$ . The direction of  $E_R$  with respect to  $E_h$  is tan a, but

 $\tan a = \frac{E_{v}}{E_{h}} = \frac{B A_{v} \sin \theta}{B A_{h} \cos \theta} = \frac{A_{v}}{A_{h}} \tan \theta$ 

if  $A_v = A_h$ 

$$\tan a = \tan \Theta = \frac{E_{\mathbf{v}}}{B_{\mathbf{h}}}$$

Comparing these results with the information previously developed on the cathode-ray tube gives

$$\frac{E_{\mathbf{v}}}{E_{\mathbf{h}}} = \tan a = \tan \Theta = \tan \phi$$

and

a = 0 = ø.

This illustrates that the resultant line on the face of the cathoderaye tube is a true indication of the direction from which the electromagnetic wave is arriving. It should be mentioned that a line making an angle of 45 degrees with the X-axis on the scope could indicate a wave arriving from either 45 or 225 degrees. This shows that there could be 180 degree ambiguity in the interpretation of the direction from which the wave is arriving. This is not considered a serious handicap due to the fact that only in very few instances are two storms exactly 180 degrees from each other with respect to the equipment.

#### 4. Errors Encountered in a Direction Finder

Night effect is one of the most bothersome error encountered in the use of the loop antenna. It is caused by an electromagnetic wave whose electric vector is not entirely in the vertical plane; that is, it has a horizontal component perpendicular to the vertical plane; this component energizes the horizontal members of the loop. Keen<sup>7</sup> shows that the error due to night effect can be as large as 90 degrees. The wave front of an electromagnetic wave is tilted forward as it travels over a surface that has low conductivity. By the time the wave has traveled a few miles along the earth's surface the electric vector may have a horizontal component. Reflection of a wave by the ionosphere may cause a shift in the direction of the electric vector resulting in an error in bearing indication. In fact, the name "night effect" was given this type of error because the reflections are more prominent at night than in the daytime.

Assuming the original electromagnetic wave is vertically polarized, the error in bearing will be small if the direct wave is the predominate wave being received.

Attempts have been made, with very little success, to cancel out the error due to night effect in a loop.<sup>3</sup> One method attempted was to place a dipole antenna near the loop to cancel out the effects of the horizontal component of the electric field, but no satisfactory results have been published, and this method is not used commercially. The use of a balanced Adcock antenna greatly reduces the error due to night effect, but its low sensitivity makes it unsuitable for a sferic direction finder.

#### Local Effects

Errors that are known as local effects are caused by trees, hills, large buildings, especially those with large conducting surfaces, power lines, telephone lines, railroad tracks, and

<sup>7</sup> Keen, R., Wireless Direction Finding, pp. 75-76.

<sup>&</sup>lt;sup>3</sup> Ibid., pp. 234-244.

similar objects near the site of the direction finder. The ideal site for a direction finder is at least one wavelength from such objects. The antenna will be tuned at 50,000 cycles per second, and this corresponds to a wavelength of 6000 meters. It is practically impossible to find such an ideal spot. It has been suggested that these effects be interpreted in terms of the angle they subtend at the antenna.<sup>9</sup> If the angle is four degrees, an error of three to four degrees should be used. Smith-Rose and Barfield made tests near telephone lines and found that errors up to 70 degrees were observed immediately under the telephone lines.<sup>10</sup> These errors decreased rapidly as the distance from the lines was increased, but was still a few degrees at 120 feet.

#### Direct Pickup

Direct pickup is obtained when voltages are induced into component parts of the direction finder as well as in the antenna. This error causes a shifting or broadening of the minimum and an overall distortion of the cosine diagram giving results that may resemble figure 12. In fact, all errors in a loop antenna can produce a distortion of the cosine diagram of an ideal pattern, and resulting in something similar to figure 12.

Direct pickup can be minimized by carefully shielding the parts of the equipment. Both electrostatic and magnetic shielding are usually necessary

#### Vertical or Antenna Effect

Figure 13 shows that the maximum induced voltage around the loop is 90 degrees out of phase with the flux wave that induces

10 Smith-Rose, R.L., and Barfield, R.H., "The Effect of Local Conditions on Radio Direction-Finding Installations", Journal of 1.E.E., Vol. 61 (1922), 179.

<sup>&</sup>lt;sup>9</sup> Ibid., p. 325.





it. In positions A and C the voltage induced in either vertical member of the loop is maximum, but as they oppose each other around the loop, the resultant induced voltage (e) is zero. Although the induced voltage in either leg of the loop is a minimum in position B, the total voltage around the loop is a maximum.

Figure 14 illustrates a conventional circuit for connecting the loop to a cathode follower. Without C2 and C3 in the circuit there is an unbalance of impedance to ground from the ends of the loop. The voltage at the terminals of the loop can never be equal because of this unbalance of impedance to ground. The effect called vertical or antenna effect causes the cosine diagram to shift its minimums or to give indefinite minimums and a distorted cosine diagram similar to figure 12.

This effect is remedied by making the impedance from each end of the loop to ground equal by adding  $C_2$  and  $C_3$ .







#### CHAPTER III

#### CIRCUIT CONSIDERATIONS

Circuit diagrams of the equipment are shown on two sheets for clarity. Figure 15 shows the power supplies and the cathoderay tube, and figure 16, the circuit of the loop and amplifier. Since the two loops and amplifiers are identical, only one diagram will be used to represent the two channels.

Waveforms due to sferic discharges are of a complex nature containing frequencies as high as several megacycles. These complex waveforms cannot be used to indicate the direction from which they originate due to interference from commercial radio signals and similar types of interference. The loop and a single stage of amplification in the amplifier are tuned to avoid interference from these signals.

When a sferic waveform is applied to a parallel L-C circuit, the voltage taken from the circuit will be a damped sinusoid with a frequency equal to the resonant frequency of the L-C circuit. The error due to interfering signals will be negligible providing the selectivity of the tuned circuit is large enough or the resonant frequency is remote from the interfering signals. A frequency of 50,000 cycles per second was chosen for the resonant frequency of this direction finder.

#### THE LOOP CIRCUIT

The loops that are to be used in this equipment are war surplus components designed to work in conjuntion with the Army BC-433-G radio compass.

The loop coils are enclosed in a waterproof aluminum shield

that is grounded, to eliminate electrostatic effects. The coils are wound in such a way that they are electrically symmetrical with respect to the shield. This is done so that the capacity to ground is symmetrical around the loop.

The loop is tuned to resonance at 50,000 cycles per second so that the maximum signal can be taken from the loop at that frequency, and the effects of interfering signals can be decreased.

#### THE AMPLIFIER

Reference to the amplifier will be interpreted to mean either the vertical or horizontal amplifier unless specific reference to one of them is made. The amplifier consists of a cathode follower, one tuned amplifier, one resistance coupled amplifier and a phase inverter.

In the following discussion the symbols to be used are:

$\mathbf{r}_{\mathbf{p}}$ =	plate resistance	$\frac{1}{2} \left( \frac{1}{2} \right) = \frac{1}{2} \left( \frac{1}{2} \right) \left( \frac{1}{2}$	
R <sub>L</sub> =	load resistance	an a	•
Sm =	transconductance		
u =	amplification factor		
A =	voltage gain		
eg =	alternating voltage between	the grid and catho	de
Z <sub>I</sub> , =	load impedance		
C (32) =	grid to plate capacity	•	
c <sub>pk</sub> =	plate to cathode capacity		
c <sub>gk</sub> =	grid to cathode capacity		
e <u>i</u> =	alternating voltage between and the ground	the control grid	
R <sub>gl</sub> =	grid leak resistance		

#### 1. The Cathode Follower

The loop is connected to the amplifier through a cathode follower, which is conventionally spoken of as an amplifier having again less than unity. The high input impedance and high stability of a cathode follower makes it an excellent circuit for coupling the loop to the amplifier. A high impedance across the loop is necessary to prevent loading of the tuned loop.

In the cathode follower circuit the load impedance is placed between cathode and ground; the plate of the tube is connected to alternating ground. This differs from the conventional amplifier in that this type of amplifier has its load impedance in the plate circuit. When the voltage between the grid and cathode of the cathode follower is made less negative, the plate current of the tube will increase. The increase in plate current causes the voltage between the cathode and ground to increase, and as a result the cathode voltage follows the grid voltage. This indicates that the gain of the stage can never be unity or greater. It can be shown that the gain of a cathode follower is

$$A = \frac{u R_{L}}{r_{p} + R_{L} (u + 1)}.$$

With good design the gain of the cathode follower can be made greater than 0.9.

If a cathode follower has a gain of 0.9 and a signal having a peak value of 10 volts is applied between the control grid and ground, the alternating voltage from cathode to ground is 9 volts; hence, the voltage across the grid to cathode capacitance is one

<sup>1</sup> Terman, Fredrick Emmons, <u>Radio Engineering</u>, p. 308.

volt. Applying the same signal between the control grid and ground of an amplifier with its cathode at alternating ground potential the voltage across the grid to cathode capacitance is 10 volts. The decrease in alternating voltage between the cathode and grid in the cathode follower, has the effect of reducing the capacitance between these two elements by a factor of (1 - A), where  $A = \frac{E_L}{e_1}$ 

and increasing the impedance of this capacitance by the factor  $(\frac{1}{1-A})$ . This makes this impedance high enough so that normally it can be neglected when calculating the input impedance of the stage. In this discussion only the input resistance will be considered. The input resistance of the cathode follower used in this equipment is equal to

$$R_1 = \frac{R_3}{A(1 - R_2)}$$

Another feature of the cathode follower is that the output impedance is very low compared to that of an amplifier. In this circuit this factor has very little significance, but will be mentioned briefly. The plate resistance of a cathode follower is equal to the plate resistance of the tube multiplied by a factor of  $(\frac{1}{u+1})$ .<sup>2</sup> The output resistance of a cathode follower is the plate resistance in parallel with the resistance from cathode to ground.

A 6J5 triode is used in the cathode follower circuit in this amplifier. Using the published values of plate resistance for the 6J5, plate resistance is 7700 ohms and a u of 20, the output resistance of this cathode follower is,

<sup>1</sup> Ibid., p. 309.

Output resistance =  $\frac{R_k}{\frac{r_p}{u+1}}$  $\frac{R_k}{R_k} + \frac{r_p}{u+1}$ 

$$\frac{r_p}{u+1} = \frac{7700}{21} = 367 \text{ ohms}$$
  
Output resistance =  $\frac{(367)(10620)}{(367) + (10620)} = 367 \text{ ohms}.$ 

This shows that the output resistance of a cathode follower is very much lower than that of an amplifier.

#### 2. The Tuned Amplifier

The output of the cathode follower is applied to the input of a tuned stage of amplification. The purpose of this stage is to increase the selectivity of the loop and to decrease the effects of interference from unwanted signals.

The gain of the stage is given by the relationship:

$$\Lambda = \operatorname{gm} Z_{\mathrm{L}}$$

where  $Z_{\rm L}$  is the load impedance of the stage. The load impedance is equal to the parallel impedance of the plate resistance of the tube, the impedance of the parallel resonant circuit, and the grid leak resistance of the following stage. The gain of the tuned amplifier is,

$$A = \frac{Em \ W L Q}{1 + \frac{WLQ}{r_p} + \frac{WLQ}{R_{gl}}} = 3726$$

where Q is the Q of the coil. This gain was not realized in the amplifier under discussion. This was due partially to the fact that the value of resistance and inductance used to calculate Q were those listed by the manufacturer and specifications were not given as to how and at what frequency they were measured. The

value of resistance of the coil that is listed by the manufacturer is the direct current resistance. Since there is no available equipment to measure the alternating current resistance at 50,000 cycles per second, the direct current resistance of the coil was used to calculate Q. Placing the coil close to the chassis caused its effective Q to be reduced. A gain of approximately 400 was realized for this stage with the half power points at 7.5 KC on each side of the mid-frequencies.

It is to be noted that because of its high transconductance, a 6AC7 tube was used in the circuit under discussion.

#### 3. The Resistance-Coupled Amplifier

The output of the tuned stage of amplification was connected to the input of a resistance coupled amplifier stage. The gain control was placed in the cathode circuit of the amplifier. A different phase shift for each setting of the gain control will result if the gain control is placed in a high impedance circuit. The gain control in the cathode circuit eliminates having a different phase shift through the stage for each setting of the gain control. A 200 ohm resistor was placed in series with the gain control to prevent the tube from ever operating without bias.

Phase shift is not a detriment in this amplifier. However, the vertical and horizontal amplifiers must have the same over-all phase shift in their circuits. The plate load resistor was chosen equal to the capacitative reactance of the shunting capacitance at 500 KC, and the coupling capacitor was chosen so that its reactance was small compared to the grid-leak resistor of the following stage at k KC. Theoretically, there should be no appreciable

phase shift through the amplifier at 50 KC. More gain through the stage could have been realized had the plate load resistor been made larger, but some phase shift would have been evident with a larger plate load resistor.

The tube used in this circuit is a 6SK7, which was chosen for this stage of amplification because of its remote cut-off characteristic.

#### 4. The Phase Inverter

The output of the resistance coupled amplifier was connected to the input of a phase inverter. A phase inverter of the floating para-phase type was chosen because it is self-balancing, and has some gain. The phase inverter was selected instead of a transformer because it is better adapted than a commercial transformer to the frequency used.

In figure 17 the phase inverter and its equivalent circuit is shown. The excitation of  $T_2$  is obtained from the voltage developed across  $R_0$  by the unbalance of currents of  $T_1$  and  $T_2$ through  $R_0$ . The plate current of  $T_1$  develops a voltage across  $R_0$ that is coupled to the input of  $T_2$ . Consequently, the current of  $T_2$  causes a current through  $R_0$  that cancels out part of the voltage developed across  $R_0$  due to  $T_1$ . It is this unbalance of currents that drives  $T_2$ . In the ratio of

$$\frac{E_1}{E_2} = 1 + \frac{2e_E!}{E_2}$$

it is to be noted that if  $\frac{2e_{g1}}{E_{Z}}$  is much less than unity, the tubes will, for all practical purposes, be balanced. This will be true if

$$R_c = R_o$$



and if the grid leak resistors of the following stages are equal.

It can be shown that the gain of T<sub>1</sub> is approximately

$$A_{1} = \frac{u_{1} R_{c}}{R_{c} + r_{p1}},$$

and that the gain of T2 is approximately

$$A_2 = \frac{u_2 A_c}{R_c + R_{p2}};$$

also that the ratio of

$$\frac{E_2}{E_1} = \frac{N}{N+3}$$

where N is the gain of  $T_{2.}^{3}$ 

#### THE BEARING INDICATOR

A cathode-ray tube is used for indicating the bearing, the output of the phase inverter being connected to the deflecting plates of this tube. The tube used, a 2AP1, has the four deflecting plates brought out to pins on the base. The tube is connected to the amplifier by a three foot cable. The cable is used so as to make it possible to place the cathode-ray tube adjacent to the oscilloscope, which is used to take pictures of the sferic waveforms. In this way it is possible to get both traces on the same film.

A Federal Telephone and Radio Corporation K-111 cable was used to transfer the signal to the deflection plates. The cable has two conductors in individual insulating tubing with a shield around the two insulators. This type of cable was employed because

<sup>3</sup> Wheeler, Myron S., "An Analysis of Three Self-Balancing Fhase Inverters", <u>Proceedings of IRE</u>, Vol.34, No.2, (Feb. 1946), 67. of its low distributed capacity and because the shield decreases the interference between the two signals which are fed to the deflection plates. The remainder of the connections was made through a six conductor cable. In the normal operating condition for the cathode-ray tube, the bias is increased to the point where the beam is just cut off. A pulse from the equipment that actuates the camera and records the waveforms of the sferics will develop a voltage across  $R_1$  through  $C_1$  that is sufficient to allow a trace to be drawn on the screen of the cathode-ray tube. This is done to prolong the life of the cathode-ray tube and to simplify the interpretation of the results.

#### THE POWER SUPPLIES

The equipment has both a low voltage power supply and a high voltage power supply. The low voltage power supply uses a 5Y3 tube connected as a full wave rectifier across the secondary of T<sub>1</sub> of figure 16. The filter is a pi type condenser input filter using two 40 microfarad condensers, and a smoothing choke. Across the output condenser are four one megohm potentiometers used as positioning controls for the deflection plates of the cathode-ray Due to the nature of the amplifier a regulated power supply tube. was not needed since no critical amplifier stages are present in the equipment. The power supply is used to supply the power to both amplifiers, and also the positioning voltage for the cathoderay tube. Positioning of this type was used so that varying the focus or intensity control for the cathode-ray tube would not change the positioning of the trace on the screen of the tube. It was necessary to use this type of positioning due to the fact

that in its normal state of operation, the electron beam of the cathode-ray tube will be cut off and will be triggered on by a pulse from the waveform amplifier.

The purpose of the high voltage power supply is to supply the voltages for the electrodes of the cathode-ray tube. It uses a 2X2, figure 16, as a half wave-rectifier across the secondary of T<sub>2</sub>. This power supply has both a positive and negative voltage output. These two outputs were necessary because of the type of positioning used on the cathode-ray tube. The accelerating anode and the deflecting plates should be approximately the same potential for best operating conditions in the cathode-ray tube. The filter across the positive voltage output of this power supply is a 0.5 microfarad, 600 volts working voltage, condenser, and the filter for the negative voltage output is a condenser input pi filter using two 0.1 microfarad, 2000 volts working voltage, condensers and a 300,000 ohm resistor. This type of filtering can be used because very little current is drawn from the supply.

#### CHAPTER IV

#### TESTS MADE ON THE DIRECTION FINDER

Preliminary tests were made on the direction finder by applying the signal from an oscillator to the circuit components of the equipment.

To determine the gain of the vertical and horizontal amplifiers, the output of an oscillator was applied between the contol grid and ground of the cathode follower, and the alternating voltage was measured at the deflection plates of the cathode-ray tube. The measured gain of each amplifier was approximately one million. The balance of the phase inverter was determined by applying a signal, having a given magnitude, to its input and measuring the output voltage of both tubes. The measured gain of the driver was 60, and that of the inverter, 58. This indicates a balance factor of approximately 0.965.

The ideal trace on the cathode-ray tube for indicating bearing was shown to be a straight line in the section on crossed loops.

The same alternating voltage was applied to the input of each amplifier and the trace observed on the screen of the cathode-ray tube was a small figure eight. This pattern indicated that the resonant circuit was not tuned to the same frequency.<sup>1</sup> The frequency of the tuned stage in the horizontal amplifier was adjusted and a straight line was obtained.

Further tests were made by discharging an 80 microfarad con-

<sup>&</sup>lt;sup>1</sup> Kessler, William J. and Knowles, Harold L., "Direction Finder for Locating Storms," Electronics XXI No. 5 (May 1948), 106-110.

denser at a distance of two feet from the crossed loops. The pattern traced on the screen indicated that the crossed loops are direction sensitive and the reading of bearing was approximately that of the angle of incidence.

Figure 18 shows four traces that indicate the readings obtained at 0, 45, 90, and 135 degrees.

A picture of the direction finder is shown in figure 19.



# TRACES INDICATING BEARING

Fig.18



# SFERIC DIRECTION FINDER

# PARTS LIST

Only parts for one of the amplifiers are listed since they are identical.

Component Part	Value of Component
<sup>C</sup> 1,12,13,15,17,20,23,24,25	0.001 mfd, 600 V.D.S.
°2,3	0.1 mfd, 2000 V.D.C.
C <sub>4</sub>	0.5 mfd, 600 V.D.C.
°5,6	40 mfd, 450 V.D.C.
<sup>C</sup> 7,8,9,10	0.005 mfd, 600 V.D.C.
C <sub>ll</sub>	0.1 mfd, 600 V.D.C.
<sup>C</sup> 14,18,21	8 mfd, 150 V.D.C.
C <sub>16</sub>	0 - 50 mnîd, variable
C <sub>19,22</sub>	30 mfd, 250 V
T	150 mb 100 ma
	Swoothing choke
12	Shooming onore
R <sub>1</sub>	620 ohm, 🛓 watt
R2	10 K, 1 watt
R3,4,7,12,13,19,31,32,33,34	l megohm, ½ watt
R5,8,14	200 ohm, È watt
R6,10	60 K, 🛓 watt
R9,21	50 % potentiometer (carbon)
<sup>R</sup> 11,16,17,18	15 K, 1 watt
R <sub>15</sub> ,	50 K, 2 Watt
R <sub>20</sub>	300 K, 1 watt
R <sub>22</sub>	90 K, ż watt
R <sub>23</sub>	100 K potentiometer (carbon)

R24	250 K, 1 watt
R <sub>25</sub>	200 K, 1 watt
R <sub>26</sub>	50 K, 1 watt
R <sub>27,28,29,30</sub>	l megohm potentiometer (carbon)
R <sub>35</sub>	10 K, 10 watt
Tl	Power transformer Secondary 700 V.R.M.S., 90 ma. Filaments 6.3V., 3.5A., 5V.,3A.
T <sub>2</sub>	Power transformer Secondary 700V.R.M.S., 70 ma. Filaments 6.3V, 2.5A.,5V.,3A.
vl	5¥3
V2	2X2
V <sub>3</sub>	2AP1
V <sub>4</sub>	6 <b>J</b> 5
V5,7,8	6AC7
V <sub>6</sub>	6SK7

IDO ZRAG U.S.A.

#### CONCLUSIONS

Several types of antennas could have been employed in this sferic direction finder. However, the major factor considered in choosing the antenna to be used was its sensitivity. A loop antenna employing many turns was the most sensitive antenna investigated and for this reason the loop was chosen as the direction sensitive component for this sferic direction finder. Both the mathematical and graphical development of the field pattern of a loop antenna demonstrates that this pattern takes the form of a figure eight. In the polar diagram of a loop antenna the maximum value is in the direction perpendicular to the plane of the antenna.

The resultant field pattern of two crossed loops is the vector sum of their individual polar diagrams. Figure 9 is a polar plot of the field pattern of a pair of crossed loops.

A cathode-ray tube, employing electrostatic deflection is used as the bearing indicator for this sferic direction finder. It was chosen because of its instantaneous response to voltages applied to the deflection plates.

This sferic direction finder was developed and built to indicate the direction from which sferic discharges were arriving. Sufficient equipment was not available to make extensive laboratory tests; however, the limited tests that were made indicated that the errors in the method of direction finding employed are small. The angle of incidence of the arriving signals could not

be measured accurately because the distance from which the tests were made was too short. Had equipment been available to make the tests from a distance of several yards, instead of three or four feet, more accurate results could have been obtained.

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