

FIELD CALIBRATION OF  
AERIAL MAPPING CAMERAS

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By

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## PREFACE

Most of the aerial mapping performed in this country is done with photographs made with cameras having wide angle lenses (A wide angle lens is generally defined as one whose focal length approximately equals one-half the diagonal of the format; the Bausch and Lomb 'Metrogon' lens and the Zeiss 'Topogon' lens are in this category). It is of utmost importance to the photogrammetrist to have complete calibration data of the lens in place in the camera. Once a camera-lens combination has been calibrated, great care must be taken to assure that its metrical characteristics are not disturbed. However, if they are disturbed, either by careless handling or by the replacing of camera parts, a re-calibration is necessary.

In the past, most camera calibrations have been performed by the Bureau of Standards, Department of Commerce. That organization is the only one in this country equipped with the precision equipment required for optical bench calibration. Since the facilities at the Bureau of Standards are limited and since it is desirable to have a method of calibration which can be accomplished by personnel in repair depots or in the field, the procedure shown in this thesis was developed. It is believed that the accuracy obtained with this method equals or surpasses that of other methods used in this or other countries. Its greatest advantage is its simplicity and the minimum of special instruments required. Also, it has the advantage of calibrating the cameras under nearly operating conditions without some of the disadvantages of operating conditions such as vibration, temperature changes, expense, etc..

This method is now being used by the Air Force for calibrating and re-calibrating all of its precision mapping cameras. Other government services and agencies, as well as camera manufacturers, also have accepted it as a suitable calibration technique.

Table of Contents

	<u>Page</u>
A. Introduction - - - - -	1
B. General - - - - -	2
C. Description of Camera Calibration Range - - - - -	5
D. Description of Field Work - - - - -	11
E. Measuring the Negative - - - - -	15
F. Determining the Equivalent Focal Length - - - - -	15
G. Determining the Point of Symmetry - - - - -	18
H. Determining the Calibrated Focal Length - - - - -	24
I. Plotting the Distortion Curve - - - - -	27
J. Measuring Angle Between Lines Connecting Opposite Fiducial Marks - -	29
K. Measuring the Flatness of the Locating Back - - - - -	32
L. Determining the Resolution of the Lens-Camera-Film Combination - - -	34
M. Calibrating Short Focal Length Cameras - - - - -	36
N. Accuracy of Measurements Required - - - - -	37
O. Miscellaneous - - - - -	39
Appendix I: General Rules - - - - -	41
Appendix II: Sample Computations - - - - -	42
Bibliography - - - - -	59

## A. INTRODUCTION

1. The calibration of aerial mapping cameras has always been considered essential when precise ground measurements were to be determined from measurements on films or plates exposed in these cameras. It has long been realized that certain constant relationships must exist between the lens, the focal plane, and the fiducial marks so that once the camera has been calibrated, the calibration data will remain unchanged for normal uses of the camera. With the advent of wide angle lenses and allied precision stereoscopic plotting instruments as well as rigorous analytical solutions of photogrammetric problems, it has become increasingly important to determine accurately camera calibration data.

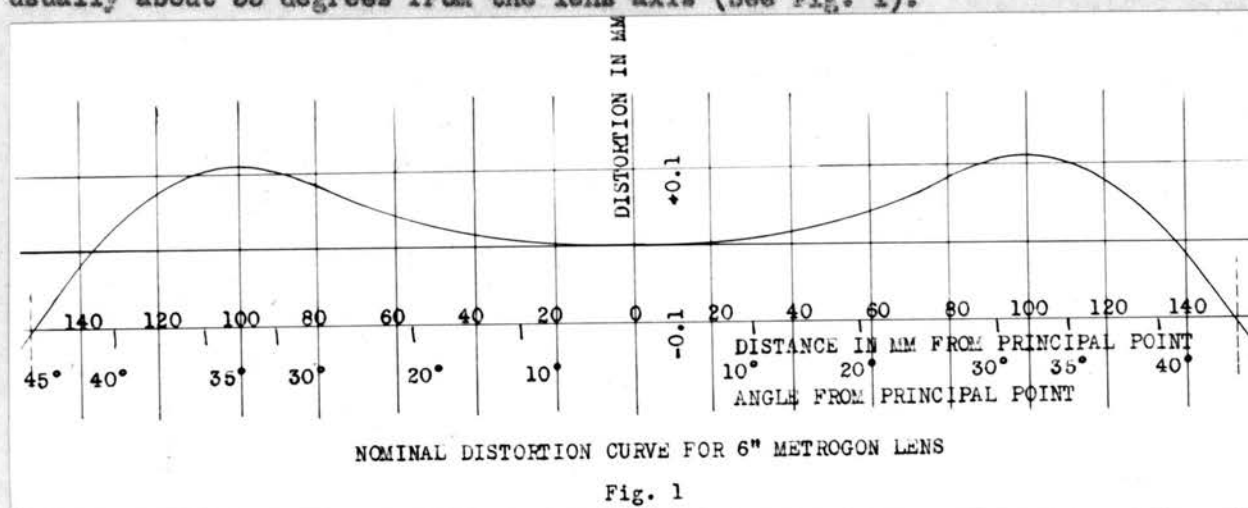
2. The purpose of this thesis is to give a step-by-step description of the procedure now used by the Aerial Photographic Branch, Engineer Research and Development Laboratories, Wright Field, in the field calibration of aerial cameras. No special laboratory calibration equipment was available at that office so it was necessary to establish an outdoor calibration range and to develop methods from which a complete calibration could be accomplished from measurements with a comparator, a dial indicator, and a theodolite, all standard precise measuring instruments.

3. Since the majority of mapping cameras in use today in the United States are equipped with Metrogon (or Topogon) lenses, most emphasis will be placed on the calibration of cameras having these types of wide angle lenses. It will be shown how to (a) construct a camera calibration range, (b) accomplish the field work, (c) measure the exposed film or plate, (d) compute the camera's equivalent focal length, (e) determine the Point of Symmetry (point about which the radial distortions are symmetrical), (f) determine the calibrated focal length, (g) compute the distortion along the two diagonals,

(h) construct the distortion curves, (i) measure the angles between lines connecting opposite fiducial marks, (j) measure the flatness of the locating back (vacuum or pressure back) of the camera, and (k) determine the resolution of the lens-camera-film combination.

#### B. GENERAL

1. Data derived from camera calibrations generally include the determination of the calibrated focal length of the lens in place in the camera, the error in the indicated position of the principal point, the radial distortion of the lens, the resolution of the lens, and the degree of flatness of the focal plane (locating back). The calibrated focal length of a camera is usually defined as an adjusted value of the equivalent focal length so computed as to distribute the effect of lens distortion over the entire field used in the aerial camera (The equivalent focal length referred to here is the perpendicular distance from the rear nodal point of the lens to the camera's focal plane). The calibrated focal length of a Metrogon lens is an adjusted value of the camera's equivalent focal length so computed as to cause the distortion curve to assume most nearly the shape of the nominal Metrogon curve, in which the negative distortion at  $45^\circ$  from the lens axis equals the positive maximum distortion of the lens. The point of maximum positive distortion is usually about 33 degrees from the lens axis (See Fig. 1).



Radial distortion is said to occur when the measured distance of an image from the principal point is not equal to the computed, theoretically distortion-free distance. If the measured distance is greater than the computed distortion-free distance, the distortion is positive and, if the measured distance is less than the computed distortion-free distance, the distortion is negative. The principal point of a camera is the foot of the perpendicular from the interior perspective center to the plane of the photograph. This is a point easy to define and extremely difficult to find. For an ideal lens, accurately mounted, a principal ray from a point on the optic axis (i.e., the optic axis) would intersect the focal plane of the camera at the principal point. However, the optic axis of the lens system is seldom an unbroken line and an axial ray may actually bend at each surface of each lens element and the ray which finally emerges may not be parallel to the incident ray.<sup>1</sup> This effect is presumably produced by slight decentering of the components of the lens system. Also the lens barrel may not be mounted with its axis perpendicular to the focal plane, which would cause another error in the point where the axial ray intersects the focal plane. In view of these facts, it is obvious that the principal point cannot be determined by merely locating the point where the axial ray intersects the focal plane. An indirect method for accurately determining the position of the principal point has been described by Dr. Francis E. Washer, National Bureau of Standards, U. S. Dept. of Commerce, in Research Paper RP 1428. Since special equipment is required for finding the position of the principal point by this method, it was not practicable to adopt this method for field calibrations. Therefore, it was necessary to devise a substitute method. The one recommended here determines the

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<sup>1</sup> NBC Research Paper RP1428 by Dr. F. E. Washer



location of the point in the focal plane about which all radial distortions are symmetrical. This point will be referred to as the Point of Symmetry, or Ps. It is not known just exactly what this point is since its position is influenced by the various factors which cause asymmetrical distortions. It has been found, however, that it is sufficiently close to the principal point to assume that a line from the interior perspective center of the lens to the Point of Symmetry equals the equivalent focal length of the camera and also that this line is perpendicular to the focal plane. It is the opinion of the writer that, regardless of whether or not the Point of Symmetry is the same as the principal point, it is the point of greater interest to the photogrammetrist since all radial distortions about it are approximately symmetrical.

2. In general, the procedure for calibrating an aerial camera is as follows: A row of easily identifiable targets is selected so that its distance from the camera station is greater than the hyperfocal distance of the camera to be calibrated. A theodolite is set over the camera station and the angles between the targets and a target centrally located (central target) are accurately measured. The aerial camera is placed horizontally over the camera station so that one diagonal of the focal plane frame is parallel to the ground and the camera is pointing at the central target. An exposure is made on a low shrink base film or a sensitized glass plate. By means of a comparator, the distance of each of the target images from the central target image is measured. The equivalent focal length is computed by using the central target and two other targets not more than twelve degrees off axis (the distortion of a Metrogon lens is negligible to  $12^\circ$ ). The Point of Symmetry is then computed by finding the point about which the distortions on the two sides of the curve are equal at approximately 33 degrees off axis. The calibrated focal length is determined and the lens distortion at each target is found by computing the theoretically distortion free distance to each target from the

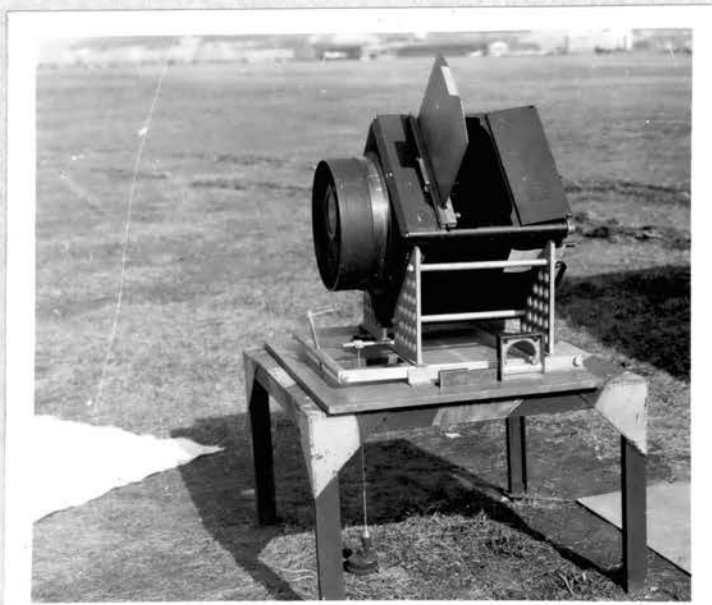
Point of Symmetry and comparing it with the measured distance. Distortion curves are then drawn. This procedure is repeated for the other diagonal and an average calibrated focal length and the resultant Point of Symmetry are found. The angle between lines connecting opposite fiducial marks is found, the flatness of the vacuum back is measured, and the resolution of the lens is determined.

### C. DESCRIPTION OF CAMERA CALIBRATION RANGE

1. Great care should be taken in selecting a site for a calibration range. By meeting the following simple requirements when selecting the site, a lot of time can be saved, errors will be reduced and computations will be simplified: (a) camera station and targets should be near the same elevation, (b) the targets should be about the same size and shape and easily identified, (c) the direction of the line of targets should be approximately east-west with the camera station to the south, and (d) the camera station should be easily accessible.

2. Any stationary object which can be easily and accurately identified and which is correctly located can be used as a target. A group of buildings, a row of telephone poles, a picket fence, or a group of trees usually make satisfactory targets. A camera calibration range recently constructed by the Aerial Photographic Branch, ERDL, at Wright Field is considered very satisfactory and is described here in detail.

3. This calibration range is located at the northwest edge of Wright Field inside of the steel wire fence surrounding the field. The camera station, consisting of a reference point and a camera platform, is located approximately 400 feet southeast of the line of targets. The reference point is a cross (+) marked on the cap of a two inch pipe driven firmly into the ground. The camera platform, located directly over the reference point, is 30 inches



T-5 camera on camera platform. A glass plate is in place in the camera. Note plate holder with dark slide partially removed.



T-5 camera on camera platform. Film magazine in place. Note camera sights, vacuum pump, and cloth on ground.

Fig. 2

square and 18 inches high (See Fig. 2). The four legs are made of 2" steel angles set in concrete. The top of the platform is made of 1/4 inch sheet steel and is welded to the four legs. A hole four inches in diameter is cut in the top directly over the reference point. The targets are selected posts of a high fence that borders Wright Field. The posts, made of steel and set in concrete, are two inches wide across the surface facing the reference point and are spaced approximately ten feet apart (See Fig. 3).



Typical Targets on Wright Field Calibration Range



Target No. 67 is Central Target

Fig. 3

A 20 inch section of each target post is painted black and a white target background made of two 8" x 20" x 1/2" sheets of Masonite is fastened to the fence, one sheet on each side of the painted area of the post. The target number is painted on one of the background sheets in black numerals 8" high and 1" thick (These numbers are not necessary as long as the central target can be identified but they are very convenient and are recommended for a permanent calibration range). A total of 125 posts have been prepared as targets and the targets are numbered consecutively from left to right (See Fig. 4). (Generally a smaller number of targets will suffice but this range was designed to be capable of calibrating the Pleon lens which has an angle of coverage of 138°.) A one (1) second theodolite was stationed over the reference point and readings of the horizontal angles were made on each target. A complete set of readings was made on each of three days and an average was taken of the three readings for each target. In the few cases where the angles failed to agree within six seconds, additional readings were made.

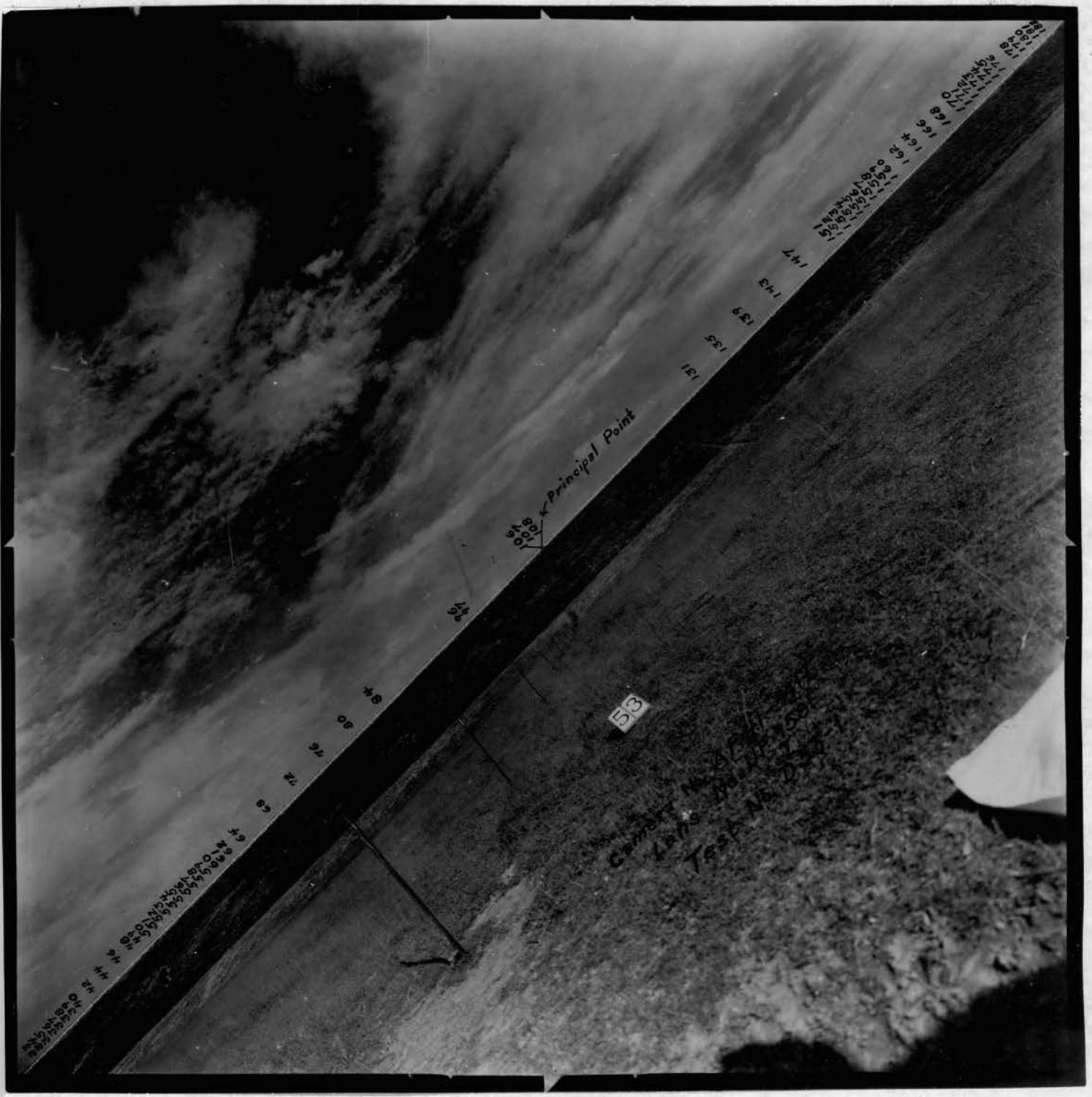
4. The calibration used prior to the one described above was set up at Huffman Dam about one half mile from Wright Field. This range consisted of a camera station and a line of guard rail posts on the road over the dam. The posts, which were about 570 feet from the camera station, were approximately 8 feet apart and were all in a straight line approximately at right angles to the axis of the camera lens. The intersection of the post with the top rail was the reference point for all measurements. Post number 107 was chosen as the central target (See Fig. 5).

5. If a calibration range is set up for calibrating a certain type lens (Metrogon, for example), the targets should be selected so as to be properly spaced along the distortion curve. A suggested spacing of the targets is given below. The angle shown is the angle at the camera station between the



A PORTION OF THE CAMERA CALIBRATION RANGE  
WRIGHT FIELD, DAYTON, OHIO

FIG. 4



Camera Calibration Range at Huffman Dam Near Wright Field  
 This Photograph was Made with a T-5 Camera and has been reduced slightly.

Fig. 5

central target and the targets to the right and left. Target No. 1 is on the extreme left and 51 is on the extreme right. Twenty-six is the central target (See Fig. 6).

#### D. DESCRIPTION OF FIELD WORK

1. Sensitized glass plates should be used for camera calibrations whenever possible. The plates should be thick enough to eliminate errors due to bending (The plates used at Wright Field are 1/4 inch thick and are coated with Super-Panchro Press emulsion). If film is used, rough and careless handling should be avoided so that no unusual stresses are caused.

2. The camera to be calibrated is set on the camera platform in such a manner that the axis of the lens is pointing toward the central target and the front node of the lens is directly over the reference point. The camera should be positioned so that the image of the row of targets will fall along one diagonal of the focal plane frame. The front node can be located by measuring from the focal plane a distance equal to the focal length (stamped on the lens holder) less the nodal separation of the lens (The average separation of the nodes of the 6" Metrogon lens is approximately 13.4 mm.). The front node should be positioned over the reference point within  $\pm 1/8$  inch. It should be remembered that the nodes of a Metrogon lens are reversed. (See Fig. 6). The lens axis can be directed accurately enough toward the central target by sighting down a straight edge of the camera or by means of very simple sights placed on the camera body (See Fig. 2). The lens axis can generally be directed toward the central target to within 20 minutes of the correct direction, although an error of several times that amount will cause no harm.

3. Two of the fiducial marks always appear along the edges of the film which photographs the ground directly in front of the camera and are generally very difficult to see on the negative, especially if the ground is dark. In order to assure that sharp fiducial marks appear on every negative, two pieces



Targets No.	Angle from Central Target	Targets No.	Angle from Central Target
1 and 51	46°	14 and 38	30
2	45	15	37
3	44	16	36
4	43	17	35
5	42	18	34
6	40	19	33
7	38	20	32
8	36	21	31
9	35	22	30
10	34	23	29
11	33	24	28
12	32	26	27
13	31	26	27

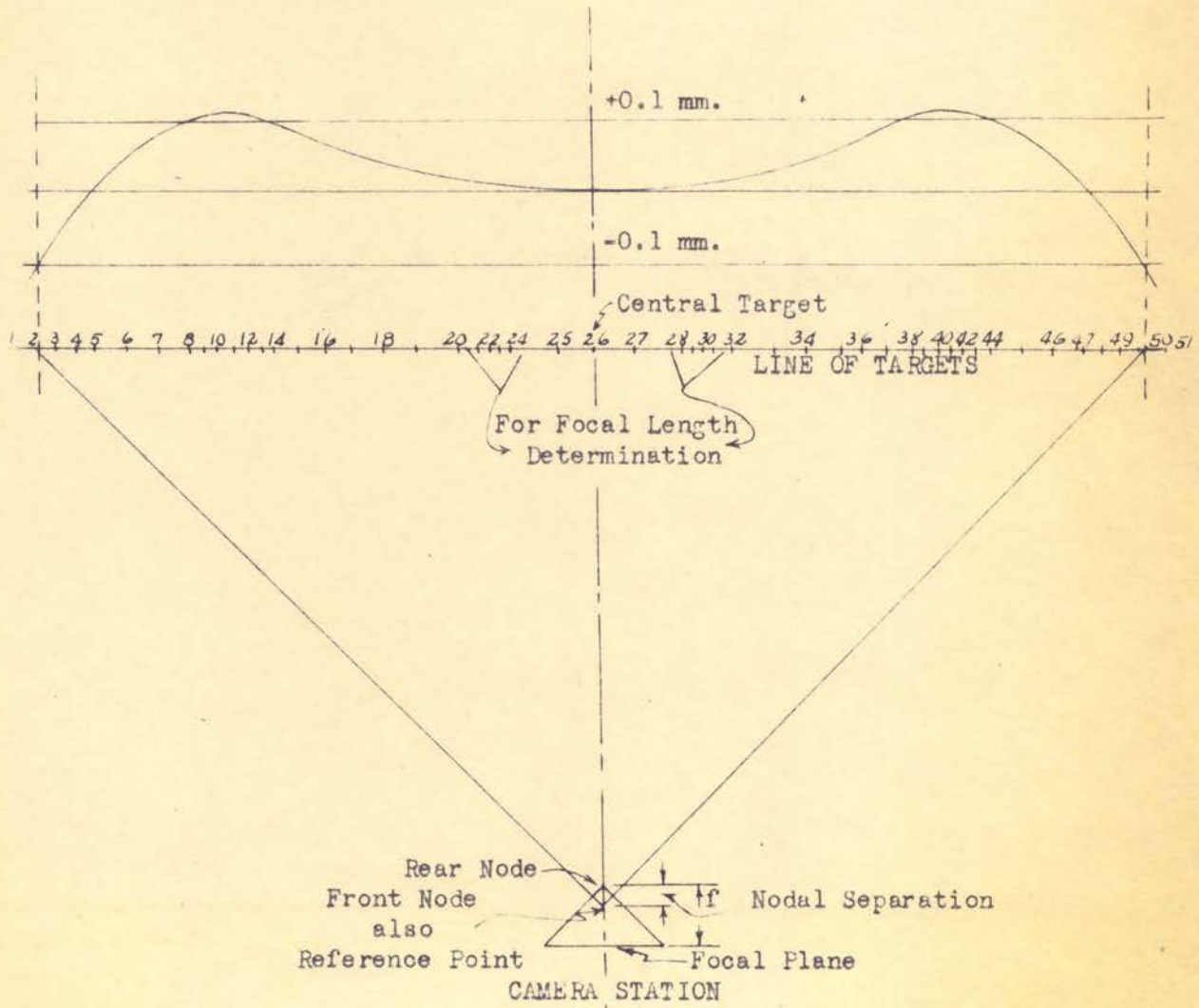


Fig. 6

of white cloth are placed on the ground in front of the camera so they will be photographed along the edges at the fiducial marks.

4. If the exposure is to be made on film, then a source of pressure differential must be provided for holding the film flat in the focal plane unless, of course, the camera is provided with a glass pressure plate. A satisfactory pressure differential can be supplied by an ordinary tire pump which has been modified slightly so a vacuum is created by an upward stroke. The exposure is made during the latter part of the stroke.

5. After the exposure is made along one diagonal, the camera is rotated 90°, realigned, and another exposure is made on a new plate. The plates are processed in the usual manner using a fairly high-contrast developer.

#### E. MEASURING THE NEGATIVE

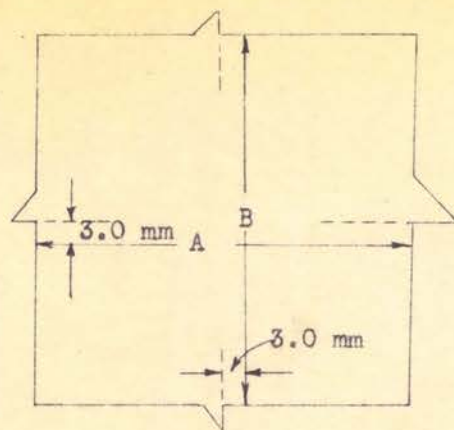
1. After the film or plate has been processed, the next step is to make comparator measurements along the diagonal between target images. A comparator capable of reading directly to 0.001 mm. is recommended although suitable results can be obtained by using a comparator which reads to 0.01 mm. The precision to which a point should be measured will be discussed in a latter section of this paper. The indicated position of the principal point is first located by one of several possible methods. The fastest method and the one most commonly used is to scratch very fine lines in the negative emulsion by laying a straight edge along opposite fiducial marks and scribing with a sharp needle. If great care is taken, this method might be sufficiently accurate but it is not recommended since there are other methods more suitable.

2. A simple and accurate method which can be used with any standard comparator is to position the negative so that the horizontal cross hair in the comparator telescope will be aligned with opposite fiducial marks as the negative, or telescope, is moved by the main screw of the comparator. The cross hairs are then moved in this manner until the vertical hair is approx-

imately in line with the two vertical fiducial marks. A small piece of acetate with a fine straight line on the lower surface is then positioned at this point so that the line corresponds exactly with the horizontal cross hair. It is fastened to the negative with cellulose tape. The negative is rotated  $90^\circ$  and the opposite fiducial marks are again aligned to the horizontal cross hair. The cross hairs are then moved as before until the vertical hair is positioned over the line on the acetate. The intersection of the cross hairs now indicates the intersection of lines between opposite fiducial marks and the acetate can be removed. The remaining problem is to prick the negative at the exact intersection of the cross hairs. This can be done quite accurately by using a piece of acetate with a very small dot or hole. The dot or hole is positioned at the intersection of the cross hairs and the acetate is fastened to the negative with cellulose tape. The plate can be permanently marked by pricking through the dot or hole with a small needle. This pricked point should be checked for accuracy before the plate is removed from the comparator. Of all the methods tried by the writer, the one just described is the most satisfactory.

3. After the indicated position of the principal point is marked, the plate is reoriented in the comparator so the diagonal to be measured is parallel to the principal motion of the comparator. Readings are taken on each target and on the indicated principal point. The angle between the line joining opposite fiducial marks and the line of targets is then measured.

4. Measurements also should be made of the focal plane frame between opposite fiducial marks. This information, which will be used for correcting mapping film for shrinkage, always should be included as part of the camera calibration data. In order to be consistent on each plate the measurements are usually made 3.0 mm. from the face of the fiducial marks as shown in Fig. 7.



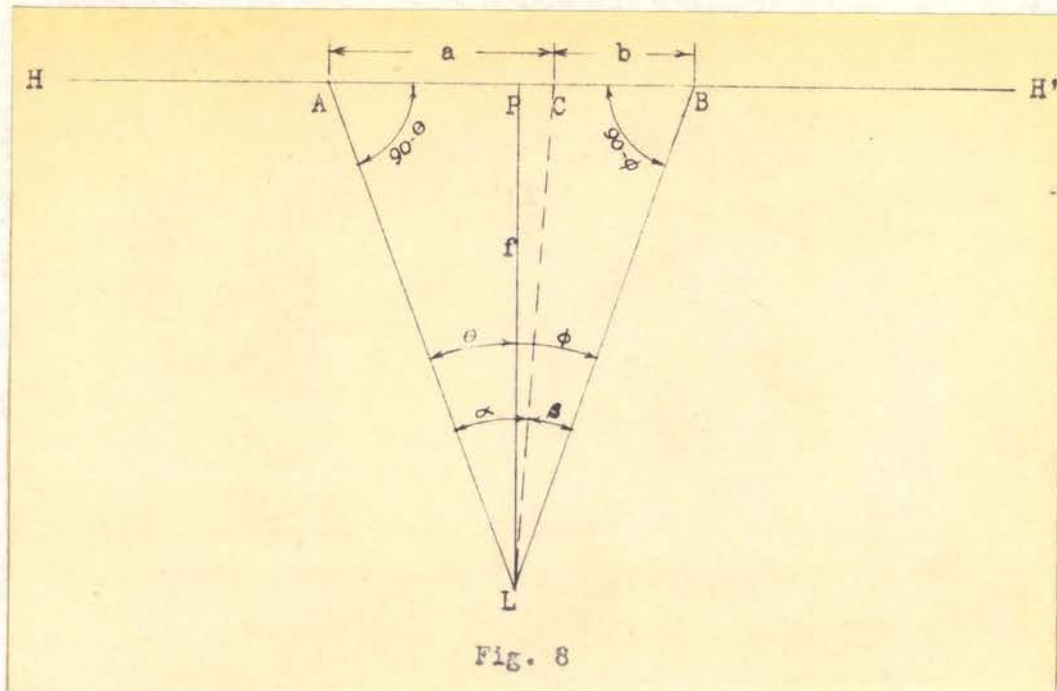
FOCAL PLANE OF T-5 CAMERA.

Fig. 7

5. If film is used for calibrating the camera, then some means must be employed for getting the image of the fiducial marks photographed on non-shrink base. One method is to lay a sensitized glass plate across the focal plane and expose it through the camera lens over a light box in the dark room. Sensitized aluminum foil has also been used successfully by shipping it in the magazine in place of the film and using vacuum to hold it flat during exposure. It is not necessary to correct each individual measurement for film shrinkage but instead a correction can be applied to the calibrated focal length as computed along each diagonal.

#### P. DETERMINING THE EQUIVALENT FOCAL LENGTH

1. The method shown below for computing the equivalent focal length is contained in Brigadier M. Hotine's book, "Surveying From Air Photographs". Figure 8 is a section containing the rear node of a wide angle lens  $L$  and the horizon trace  $HH'$ .  $A$  and  $B$  are two targets whose angles,  $\alpha$  and  $\beta$ , are less than  $12^\circ$  and greater than  $4^\circ$ . The principal point is assumed to be a small distance away from the horizon trace and  $P$  is the foot of the perpendicular from the principal point to the horizon trace. Therefore,  $PL$  is



perpendicular to  $HH'$ . Point  $C$  is the image of the central target and  $\alpha$  and  $\beta$  are the angles measured with the theodolite between the central target and targets  $A$  and  $B$ , respectively. Lengths  $a$  and  $b$  are the measurements obtained by the comparator. Angles  $\theta$  and  $\phi$  are unknown and are to be determined. By applying the Law of Sines to triangles  $ALC$  and  $CLB$ :

$$\frac{a}{\sin \alpha} = \frac{CL}{\sin(90 - \theta)}$$

and

$$\frac{b}{\sin \beta} = \frac{CL}{\sin(90 - \phi)}$$

$$a \cos \theta = CL \sin \alpha$$

$$b \cos \phi = CL \sin \beta$$

$$\frac{b \cos \phi}{a \cos \theta} = \frac{CL \sin \beta}{CL \sin \alpha}$$

$$\frac{\cos \phi}{\cos \theta} = \frac{a \csc \alpha}{b \csc \beta}$$

Now choose an auxiliary angle  $\lambda$  such that

$$\tan(45^\circ + \lambda) = \frac{a \csc \alpha}{b \csc \beta} = \frac{\cos \phi}{\cos \theta} \quad (1)$$

By expanding and rearranging the last equation

$$\frac{\tan 45^\circ + \tan \lambda}{1 - \tan 45^\circ \tan \lambda} = \frac{\cos \phi}{\cos \theta}$$

$$\frac{1 + \tan \lambda}{1 - \tan \lambda} = \frac{\cos \phi}{\cos \theta}$$

$$\cos \theta + \cos \theta \tan \lambda = \cos \phi - \cos \phi \tan \lambda$$

$$\tan \lambda (\cos \phi + \cos \theta) = \cos \phi - \cos \theta$$

$$\tan \lambda = \frac{\cos \phi - \cos \theta}{\cos \phi + \cos \theta} = \frac{-2 \sin 1/2(\phi + \theta) \sin 1/2(\phi - \theta)}{2 \cos 1/2(\phi + \theta) \cos 1/2(\phi - \theta)}$$

$$\tan \lambda = \tan 1/2(\theta - \phi) \tan 1/2(\theta + \phi)$$

or

$$\tan 1/2(\theta - \phi) = \tan \lambda \cot 1/2(\theta + \phi)$$

$$= \tan \lambda \cot 1/2(\alpha + \beta)$$

2. Referring now to the focal length computation form (See Fig. 20, Sample Computations),  $\lambda$  is computed from equation (1) and  $1/2(\theta - \phi)$  is computed from equation (2). Since  $1/2(\theta + \phi)$  is equal to  $1/2(\alpha + \beta)$ ,  $\theta$  and  $\phi$  are readily found by adding  $1/2(\theta - \phi)$  and  $1/2(\theta + \phi)$ .

Referring to Fig. 3,

$$FL = f = AL \cos \theta$$

$$\text{but } \frac{AL}{\cos \phi} = \frac{a + b}{\sin(\theta + \phi)}$$

$$\therefore f = \frac{(a + b) \cos \phi \cos \theta}{\sin(\theta + \phi)} \quad (3)$$

3. The focal length is usually computed for three combinations of targets A and B and an average is taken. However, if the first two computations agree within 0.02 mm., the third computation is unnecessary. A set of sample computations are contained in a latter section of this article.

4. When determining the equivalent focal length of a camera having a normal angle lens in which the distortion is negligible, it is recommended that targets A and B be selected so that angles  $\alpha$  and  $\beta$  are greater than  $22^\circ$ . This will give a stronger solution for the equivalent focal length.

5. When complete calibration data (i.e., the Ps, the C. F. L., and the distortion curves) are to be determined for a camera, it is not necessary to compute the camera's equivalent focal length if an approximate value for it is known. An approximate value for the focal length can be quickly determined by

assuming that PL and CL are equal and are perpendicular to HH'. The angle at L between these two lines is always small and the two distances are always near the same value. Then

$$f = \frac{a}{\tan \alpha} \quad \text{and} \quad f = \frac{b}{\tan \beta}$$

adding

$$2f = \frac{a}{\tan \alpha} + \frac{b}{\tan \beta}$$

$$f = \frac{a}{2 \tan \alpha} + \frac{b}{2 \tan \beta}$$

Also the G. F. L. stamped on the lens holder is sufficiently accurate to use for the value of  $f$  in the Point of Symmetry and Calibrated Focal Length computations shown in sections G and H, which follow. Since the calibrated focal length is normally found by adding to or subtracting from the camera's equivalent focal length a value which will cause the maximum positive distortion to equal the negative distortion at  $45^\circ$ , it is immaterial whether the computed equivalent focal length is used or a value near to it as long as the conditions just stated are met by the final values of the G. F. L.

#### G. DETERMINING THE POINT OF SYMMETRY

1. The Point of Symmetry determined for a single diagonal should probably be called an Axis of Symmetry since its location is defined by a single measurement--a distance along the row of targets from the central target. For convenience, however, the term "Point of Symmetry" will be used. Reference is made to Fig. 9, where

$f$  = Equivalent Focal Length (This is not necessarily the true equivalent focal length since that would be the perpendicular distance to the focal plane and not the distance to the Point of Symmetry. However, the difference in the two lengths will always be so small that it can be neglected.)

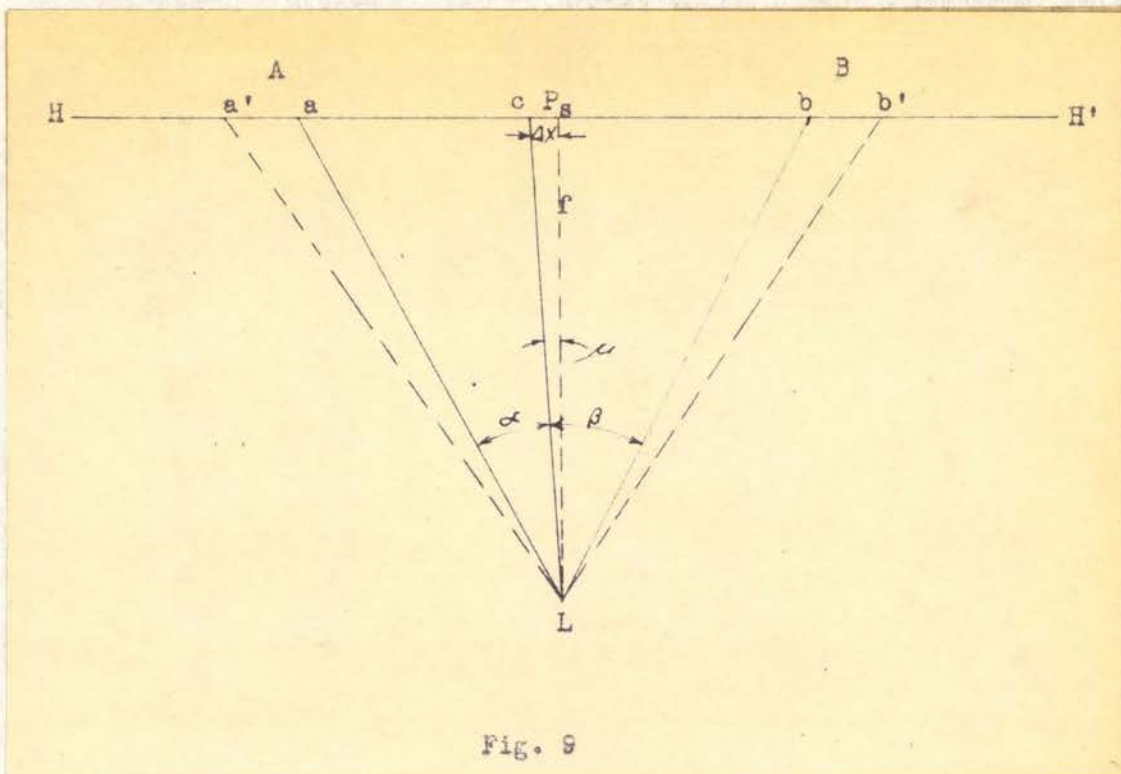


Fig. 9

$c$  = image of central target

$P_s$  = Point of Symmetry

$a$  and  $b$  = theoretically distortion-free positions of images of targets A and B

$a'$  and  $b'$  = positions of distorted photographic images of targets A and B

$\alpha$  and  $\beta$  = measured angles to targets A and B

$L$  = Rear nodal point of camera lens

$\Delta x$  = distance from central target to  $P_s$ .

$\mu$  = angle at  $L$  between central target and  $P_s$ .

$k = (a'c - cb')$

2. Two targets A and B whose angles  $\alpha$  and  $\beta$ , respectively, are between  $33^\circ$  and  $35^\circ$ , were selected because their images fall in the zone of maximum positive distortion of a Metrogon lens. An equation was written so that about a point a distance  $\Delta x$  from the central target, the distortions at A and B would be equal. Since the change in the distortion in the zone between  $32^\circ 30'$



and  $35^{\circ}30'$  is less than 0.005 mm., any targets selected in that zone can be safely used.

$$a'P_g - aP_g = P_g b' - P_g b$$

$$(a'o + \Delta x) - (ao + \Delta x) = (ob' - \Delta x) - (ob - \Delta x)$$

$$(a'o + \Delta x) - f \tan(\alpha + \mu) = (ob' - \Delta x) - f \tan(\beta - \mu).$$

Transposing and substituting  $k$  for  $(a'o - ob')$

$$k + 2\Delta x - f \left[ \tan(\alpha + \mu) - \tan(\beta - \mu) \right] = 0. \quad (4)$$

Equation (4), being transcendental, is not solvable explicitly for  $\mu$ . Both  $\alpha$  and  $\beta$  in the equation above are large and known;  $\mu$  is small, of the order of several minutes. It therefore seems desirable to expand the coefficient of  $f$  in equation (4) in a Taylor's series with the remainder, and thus reduce the equation to a solvable algebraic form. The expansion will be made around  $\alpha$  and  $\beta$  for the two terms in the bracket. Taylor's theorem is:

$$(x + y) = \phi(x) + y \phi'(x) + \frac{y^2 \phi''(x)}{2!} + \dots + \frac{y^{n-1} \phi^{(n-1)}(x)}{(n-1)!} + y^n \phi^{(n)}(x + \theta y)$$

where  $0 \leq \theta \leq 1$  and where  $\phi^{(n)}(x)$  represents the  $n$ th derivative of the function  $\phi$ , evaluated at  $x$ . This expansion is readily performed and, stopping with the fourth term in the expansion, the two terms of the coefficient of  $f$  in equation (4) become  $\tan(\alpha + \mu) = \tan \alpha + \mu \sec^2 \alpha + \frac{\mu^2}{2} \sec^2 \alpha \tan \alpha + \frac{\mu^3}{3} \sec^4(\alpha + \theta\mu) + 2 \tan^2(\alpha + \theta\mu) \sec^2(\alpha + \theta\mu)$ , and  $\tan(\beta - \mu) = \tan(\beta - \mu) \sec^2(\beta - \mu) + \mu^2 \sec^2 \beta \tan \beta - \frac{\mu^3}{3} \left[ \sec^4(\beta - \theta\mu) + 2 \tan^2(\beta - \theta\mu) \sec^2(\beta - \theta\mu) \right]$ , where  $0 \leq \theta \leq 1$ .

5. These two third order remainders can be evaluated approximately to determine whether they can be dropped. Because  $\alpha$  and  $\beta$  are approximately equal and because  $\mu$  is so small, it is seen that both remainder terms are approximately equal numerically. The total error incurred in dropping the two

remainders will therefore be equal to twice the remainder in  $\alpha$ .

Let  $R_\alpha$  be the remainder term in  $\alpha$

$$R_\alpha = \frac{\mu^3}{3} \left[ \sec^4(\alpha + \Theta\mu) + 2 \tan^2(\alpha + \Theta\mu) \sec^2(\alpha + \Theta\mu) \right]$$

Since  $\mu$  is of the order of several minutes, it is safe to assume  $\mu < 0.01$  radians.

Taking  $\Theta = 1$ ,  $\alpha = 35^\circ$  and  $\mu = 0.01$  radians

$$2R_\alpha = \frac{2 \times 10^{-6}}{3} (3.68) \cong 1.2 \times 10^{-6}$$

Thus,  $1.2 \times 10^{-6}$  is the largest possible value of the remainder term.

Substituting  $2R_\alpha$  for the remainder terms in the expansion of equation (4) and substituting 0 for the rest of the expansion (through the second order terms),

$$\begin{aligned} k + 2 \Delta x - f(C + 2 R_\alpha) &= 0 \\ \Delta x &= \frac{f(C + 2 R_\alpha) - k}{2} \\ &= \frac{fC - k}{2} + f R_\alpha \end{aligned}$$

The maximum error in  $\Delta x$  incurred by dropping the remainder is therefore  $f R_\alpha$

$$f \cong 152 \text{ mm.} \quad R_\alpha \cong 1.2 \times 10^{-6}$$

Maximum error in  $\Delta x \cong 10^{-4}$  mm. This error,  $10^{-4}$  mm., resulting from the 2nd order approximations is of higher order than the precision of plate measurement and therefore the third order remainders can be dropped.

4. A test will now be made to determine how much error in  $\Delta x$  will result from substituting  $\mu$  for  $\tan \mu$ . Let  $\mu = 0.01$  radians =  $0.573^\circ$ ;  $\tan 0.573^\circ = 0.0100020$

$$f \mu = 1.5400000$$

$$f \tan \mu = \underline{1.5403080}$$

$$\text{Difference} = 0.0003080 \text{ mm.}$$

This is also negligible so for values of  $\mu$  up to 0.01 radians,  $f \mu$  can be

substituted for  $f \tan \mu$ .

b. Now rewriting equation (4), in its expanded form but omitting the remainders:  $k + 2 \Delta x - f [\tan \alpha + \mu \sec^2 \alpha + \mu^2 \sec^2 \alpha \tan \alpha - \tan \beta + \mu \sec^2 \beta - \mu^2 \sec^2 \beta \tan \beta] = 0$

Collecting like terms and substituting  $f \mu$  for  $\Delta x$

$$\begin{aligned}
 k + 2 f \mu - f [(\tan \alpha - \tan \beta) + \mu (\sec^2 \alpha + \sec^2 \beta) + \\
 \mu^2 (\sec^2 \alpha \tan \alpha - \sec^2 \beta \tan \beta)] = 0 \\
 \mu^2 f [\sec^2 \beta \tan \beta - \sec^2 \alpha \tan \alpha] + \mu f [2 - \sec^2 \alpha - \sec^2 \beta] \\
 + [k - f(\tan \alpha - \tan \beta)] = 0 \quad (6)
 \end{aligned}$$

The equation is now in the standard quadratic form,  $A \mu^2 + B \mu + C = 0$  and can be solved in the usual manner, where

$$\begin{aligned}
 A &= f [\sec^2 \beta \tan \beta - \sec^2 \alpha \tan \alpha] \\
 B &= f [2 - \sec^2 \alpha - \sec^2 \beta] \\
 C &= [k - f(\tan \alpha - \tan \beta)] \\
 &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \text{ Radians} \\
 \text{and } \Delta x &= f \mu. \quad (7)
 \end{aligned}$$

c. It should be noted that, for a given calibration range, two targets could be selected and used for determining the Point of Symmetry of all cameras with similar lenses to be calibrated on that range. Thus, all of the elements of equation (6) would remain the same except  $\mu$ ,  $f$ , and  $k$ . Since  $f$  is determined for each camera and since  $k$  is easily found, the computations involved in determining the Point of Symmetry become very simple and rapid. The computations can be simplified further by setting up a table as shown in Fig. 10. This table is computed for targets 47 and 92 of the Bright Field Calibration Range for cameras having Nitrogen lenses whose equivalent focal lengths fall between 151.9 and 155.1 mm.  $\alpha = 34^\circ 16' 32''$  and  $\beta = 34^\circ 59' 44''$ .

Table of Constants for Point of Symmetry Computations

E.F.L.	A	B	Part of C*
151.9	3.6405873	-143.1802868	1.5082151
152.0	3.6429840	.2745464	1.5092080
2.1	3.6453807	.3686060	1.5102009
2.2	3.6477774	.4630655	1.5111938
2.3	3.6501741	.5573251	1.5121867
2.4	3.6525708	.6515847	1.5131796
2.5	3.6549675	.7458443	1.5141725
2.6	3.6573642	.8401038	1.5151654
2.7	3.6597609	.9343634	1.5161583
2.8	3.6621576	-144.0286230	1.5171512
2.9	3.6645543	.1228825	1.5181441
153.0	3.6669510	.2171421	1.5191370
3.1	3.6693477	.3114017	1.5201299
3.2	3.6717444	.4056612	1.5211228
3.3	3.6741411	.4999208	1.5221157
3.4	3.6765378	.5941804	1.5231086
3.5	3.6789345	.6884400	1.5241015
3.6	3.6813312	.7826995	1.5250944
3.7	3.6837279	.8769591	1.5260873
3.8	3.6861246	.9712187	1.5270802
3.9	3.6885213	-145.0654752	1.5280731
154.0	3.6909180	.1597378	1.5290660
4.1	3.6933147	.2539974	1.5300589
4.2	3.6957114	.3482569	1.5310518
4.3	3.6981081	.4425165	1.5320447
4.4	3.7005048	.5367761	1.5330376
4.5	3.7029015	.6310357	1.5340305
4.6	3.7052982	.7252952	1.5350234
4.7	3.7076949	.8195548	1.5360163
4.8	3.7100916	.9138144	1.5370092
4.9	3.7124883	-146.0080739	1.5380021
155.0	3.7148850	.1023355	1.5389950
5.1	3.7172817	.1965931	1.5399879

Fig. 10

\*  $f(\tan \alpha - \tan \beta)$

7. It is suggested that two sets of targets and two separate computations be used in determining the Point of Symmetry. An average value for  $\Delta x$  and  $\mu$  then would be used in the computations which follow. Since it requires only a few minutes to make a Point of Symmetry computation, and since the complete calibration depends on a correctly located  $F_p$ , it is believed to be good practice always to make two independent solutions for  $\Delta x$  and  $\mu$ .

#### H. DETERMINING THE CALIBRATED FOCAL LENGTH

1. After the average Point of Symmetry has been located, the value of the maximum distortion is found. Since the targets which were used to find the Point of Symmetry are located very close to the points of maximum distortion, it will be sufficiently accurate to determine the distortion at any one of these points and call it the maximum positive distortion of the lens. Reference is made to Fig. 9. It again is assumed that the line from L to  $F_p$  is perpendicular to the horizon trace  $HH'$ . The distortions at targets A and B can be found by the expressions  $aa' = (a'e + \Delta x) - f \tan(\alpha + \mu)$  and  $bb' = (cb' - \Delta x) f \tan(\beta - \mu)$ . The values of  $\mu$  and  $\Delta x$  have already been determined from equations (6) and (7), and the values  $a'e$  and  $cb'$  are measured directly from the film or plate.  $\alpha$  and  $\beta$  are known horizontal angles at the camera station to the two targets A and B respectively. If the values of  $\Delta x$  and  $\mu$  have been accurately determined, the distortion at target A ( $aa'$ ) will equal the distortion at target B ( $bb'$ ) in each Point of Symmetry computation. Although it is not necessary to determine both  $aa'$  and  $bb'$ , it is suggested that the two values be found as a check on the computations used for determining  $\mu$  and  $\Delta x$ .

2. As previously stated, the calibrated focal length of a Nitrogen lens is an adjusted value of the camera's equivalent focal length for which the negative distortion at  $45^\circ$  equals the maximum positive distortion. Therefore the next step is to determine the negative distortion at  $45^\circ$  out from the Point of



Select  $\Delta f$  so that  $d_p$  will equal  $(-d_n)$ ,

$$p - (f + \Delta f) \tan \alpha_p = -n + (f + \Delta f) \tan \alpha_n$$

$$p + n = (f + \Delta f) (\tan \alpha_n + \tan \alpha_p)$$

or

$$\Delta f = \frac{p + n}{\tan \alpha_n + \tan \alpha_p} - f = \frac{p + n - f \tan \alpha_n - f \tan \alpha_p}{\tan \alpha_n + \tan \alpha_p} \quad (8)$$

Since for a Retrogon lens,  $\alpha_n = 45^\circ$ ,

$$\tan \alpha_n = 1 \quad \& \quad f \tan \alpha_n = f$$

also

$$n = f \tan \alpha_n - d_n = f - d_n$$

and

$$p - f \tan \alpha_p = d_p$$

Substituting in equation (8)

$$\Delta f = \frac{d_p + f - d_n - f}{1 + \tan \alpha_p} = \frac{d_p - d_n}{1 + \tan \alpha_p} \quad (9)$$

$$CFL = f \pm \Delta f. \quad (10)$$

If  $d_p$  is smaller numerically than  $d_n$ , then  $\Delta f$  is negative. It should be noted that in equation (9),  $d_p$  and  $d_n$  are subtracted without regard to signs.

an  $\alpha_p$  is the average of  $\tan \alpha$  and  $\tan \beta$  in the point of symmetry computation.

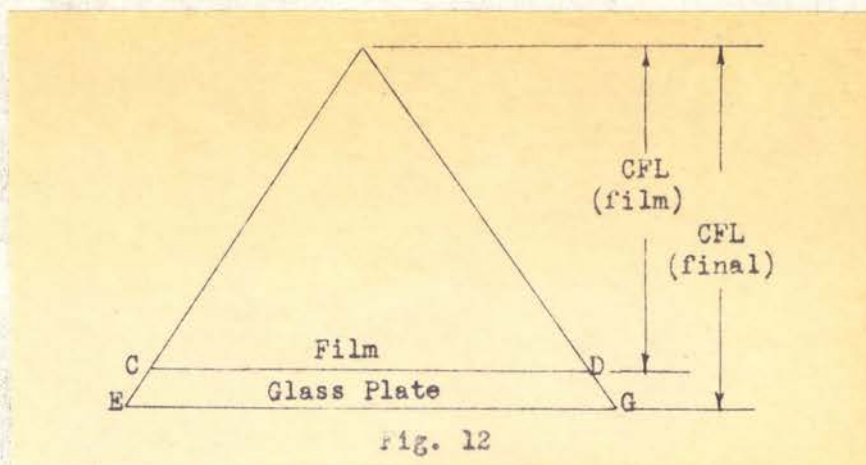
4. If film is used for making the exposure instead of glass plates, then a correction must be applied to the calibrated focal length to compensate for film shrinkage. Reference is made to Fig. 12, where

CD = Average of measurements on film across fiducial marks =  $(A + B)/2$   
(See Fig. 7)

EG = Average of measurements on non-shrink base across same fiducial marks.

CFL (film) = Calibrated Focal Length, uncorrected for film shrinkage.

CFL (final) = corrected Calibrated Focal Length.



$$\frac{\text{CFL (final)}}{\text{CFL (film)}} = \frac{\text{EG}}{\text{CD}}$$

$$\text{CFL (final)} = \frac{\text{EG} \times \text{CFL (film)}}{\text{CD}} \quad (11)$$

#### I. PLOTTING THE DISTORTION CURVE

1. The Point of Symmetry has been defined earlier as the point about which all radial distortions are symmetrical. The method just described for determining the calibrated focal length is based on average values of distortions on opposite sides of the  $P_s$ . It is therefore logical that the distortion curve plotted should be an average of the two sides of the complete distortion curve. The most direct method of obtaining this is to plot, as a common curve, values of distortion for both sides of the curve as ordinates against their respective distances from the  $P_s$  as abscissae. Since the maximum positive distortions are equal, and also the negative distortions at  $45^\circ$  are equal, the two halves of the curves should combine into a single average curve very readily.

2. Any type of standard graph paper with parallel lines closely spaced can be used for plotting the curve, but that most commonly used by the writer has 20 lines to the inch. Along the abscissa, each line (0.50 inch) rep-



resents one millimeter distance from the  $F_g$ , and along the ordinate, each line represents 0.005 mm. distortion. A typical distortion curve is shown in Fig. 25, Sample Computation.

3. The calibration methods described so far have been for one diagonal of the focal plane. It was mentioned earlier that plates were exposed on the calibration range for both diagonals of the focal plane. The location of the Point of Symmetry, the determination of the calibrated focal length, and the plotting of the average distortion curve must also be accomplished for the other diagonal. The calibrated focal lengths for the two diagonals should have the same value but, if there is a slight difference, an average is taken for the camera's calibrated focal length.

4. To go a step further in arriving at an average distortion curve for the camera lens, it has been found that time can be saved and results improved if the two halves of the two diagonals are plotted as a single curve. In plotting this curve, only every fourth target on each half diagonal is used. The targets are carefully selected so there is an even distribution of points along the curve. Occasionally, a few extra points are plotted if the location of the curve appears questionable.

5. In order to determine the resultant  $F_g$ , the angles between a set of fiducial marks and the row of targets across the two diagonals must be known. These angles can be measured to the required accuracy on the comparator or with any good protractor. The images of the targets will seldom be in a straight line so an "average" line through the targets is used. Reference is made to Fig. 15.

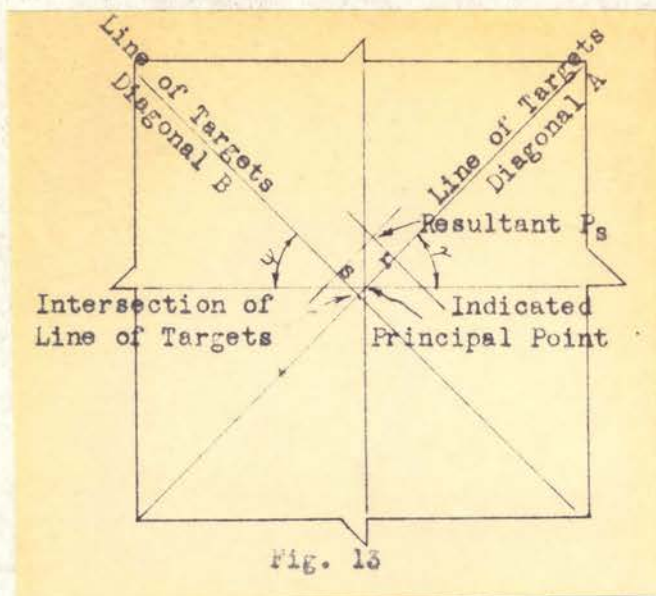


Fig. 13

$r$  = distance from indicated PP to  $P_s$  on diagonal A

$s$  = distance from indicated PP to  $P_s$  on diagonal B

$\gamma, \psi$  = Angles between line joining fiducial marks and lines of targets across diagonals A and B, respectively

$x_{P_s}$  = x-coordinate of resultant  $P_s$

$y_{P_s}$  = y-coordinate of resultant  $P_s$

From analytics, the Normal Form is

$$x_{P_s} \cos \gamma + y_{P_s} \sin \gamma = r$$

$$x_{P_s} \cos \psi + y_{P_s} \sin \psi = s.$$

By substituting the known values of  $r$ ,  $s$ ,  $\gamma$ , and  $\psi$  and solving the two equations simultaneously, the coordinates of the Point of Symmetry are found.

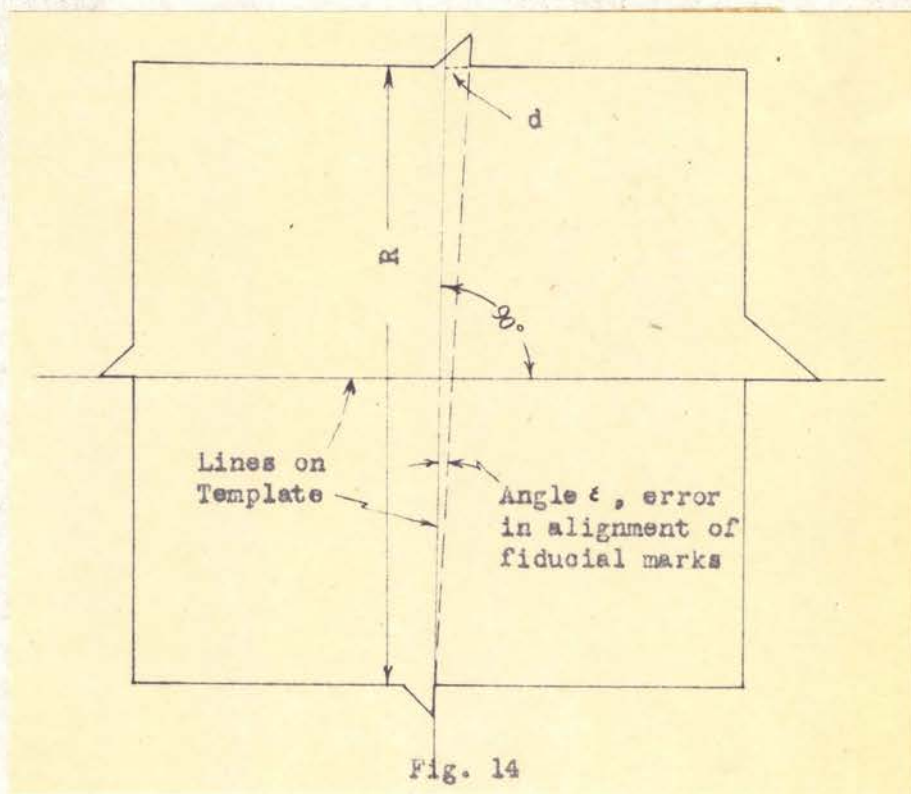
6. The values of  $x_{P_s}$  and  $y_{P_s}$  can also be found very readily by a graphical solution (See Fig. 27-IV, Sample Computations).

#### J. MEASURING ANGLE BETWEEN LINES CONNECTING OPPOSITE FIDUCIAL MARKS

1. Air Force specifications for precision mapping cameras (both the T-5 and T-9 cameras) state that "lines connecting opposite pairs of fiducial marks shall intersect at  $90^\circ$  plus or minus one minute of arc". The measuring of the angle between these lines can be accomplished in a number of ways but only two will be described here. By far the simpler method is to use a comparator having a rotating plate holder with a circle graduated to the required accuracy. The calibration plate is then fastened securely in the comparator, rotated, and translated until a pair of opposite fiducial marks are accurately aligned to the longitudinal cross hair. The graduated circle is read. The plate is

then rotated until the other pair of fiducial marks is aligned and the circle is read again. In order to successfully use the method just described, the angles should be read to an accuracy of 10 seconds. If a comparator having a circle graduated to the required accuracy is not available, the following secondary method is suggested.

2. This method required the use of a plane-parallel glass plate with two finely etched lines  $90^\circ$  apart. This template should be as large as the negative size of the camera to be calibrated, and the lines must be positioned to an accuracy of a few seconds of arc. The negative (film or plate) is placed in the comparator with the emulsion side up and the template is placed over the negative with the surface containing the lines in contact with the negative emulsion. The template is then moved over the negative until one line passes through opposite fiducial marks and the other line passes through one of the remaining fiducial marks and close to the other.



The perpendicular distance,  $d$  (see Fig. 14), from the face of the fiducial mark to the template line and the distance,  $R$ , between this fiducial mark and the one opposite, are measured. The error in alignment of the fiducial marks is the angle,  $\epsilon$ , whose tangent is  $d/R$ . This value can be checked by shifting the template until the lines pass through another set of three fiducial marks.

An alternate, and perhaps easier, method of using the glass template is to center the intersection of the template lines over the indicated position of the principal point which has previously been marked on the negative. The template then is rotated until lines are on or near the four fiducial marks. The distances  $aa'$ ,  $cc'$  and  $dd'$  (Fig. 15) are measured as before. Distances  $ac$ ,  $bc$ ,  $cc'$ , and  $dc$  are also measured.

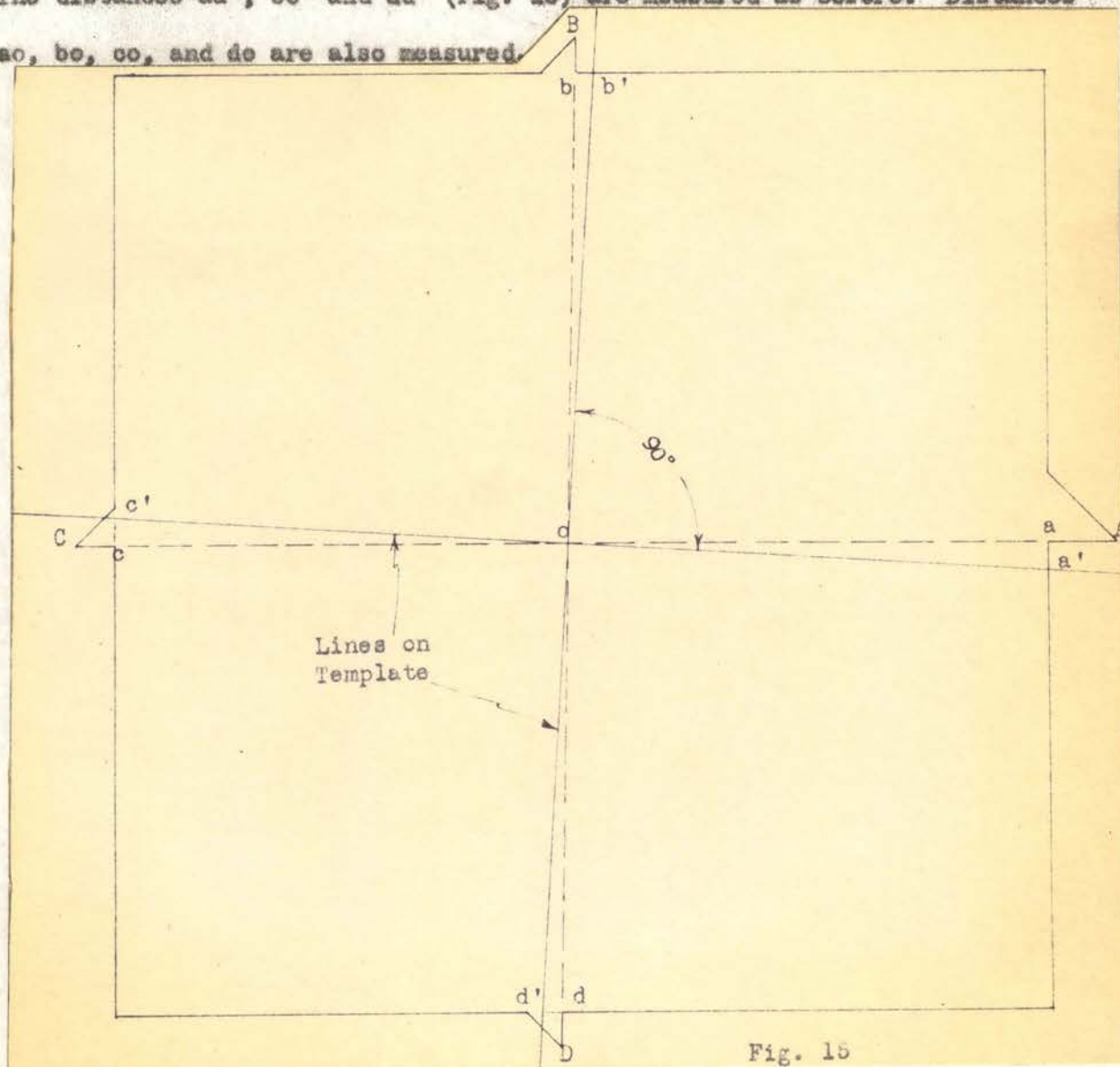


Fig. 15

Then

$$\tan \angle aoa' = \frac{aa'}{oa'}$$

$$\tan \angle ccc' = \frac{cc'}{oc'}$$

$$\tan \angle bob' = \frac{bb'}{ob'}$$

$$\tan \angle ddd' = \frac{dd'}{od'}$$

$$\angle aob = 90^\circ - \angle aoa' + \angle bob'$$

$$\angle ccd = 90^\circ - \angle ccc' + \angle ddd'$$

$$\angle boc = 90^\circ - \angle bob' + \angle ccc'$$

$$\angle doa = 90^\circ + \angle ddd' + \angle aoa'$$

3. The average of the differences between  $90^\circ$  and the four angles  $aob$ ,  $boc$ ,  $cod$ , and  $doa$ , is the error in the alignment of the fiducial marks. If this error is greater than one minute of arc, the fiducial marks should be re-set. This adjustment should be made and the results checked before the rest of the calibration is accomplished.

4. In measuring the distances described above, it is important that a systematic method of recording them be used so the small angles will be given proper signs. One method is to letter the fiducial marks, A, B, C, D, in a counterclockwise direction starting with the large fiducial mark as shown in Fig. 15. The inside corners of the fiducial marks are named a, b, c, d, and the points on the template lines adjacent to the corners, a', b', c', d'. The intersection of the lines is called o. The small distance between the fiducial mark and the line is then recorded by listing first the fiducial mark  $90^\circ$  counterclockwise to the fiducial mark being measured. For example, in Fig. 15, the small distance at A is recorded Baa'. The distance at B would be recorded Cbb', at C the distance would be Dcc', and at D it would be Add'. From this information, it is easy to reconstruct the location of the template with respect to the fiducial marks.

#### X. MEASURING THE FLATNESS OF THE LOCATING BACK

1. The most recent Air Force specifications for precision mapping cameras (type T-8) read as follows: "The surfaces of surfaces against which the film is held, shall, when in position for making an exposure, be such that

the overall range of flatness over the entire area is not greater than 0.001 of an inch, and will remain within that value for a period of not less than one year". Army Map Service (Corps of Engineers) specifications for aerial photography which accompany contracts state that, "The film shall be held flat in the focal plane to plus or minus 0.0005 inch either by an adequate vacuum behind the film or air pressure in front".

2. A simple and accurate way of measuring the flatness of the locating back is to mount it face up on top of a surface plate and to level it by means of a height gage and dial indicator (See Fig. 16).



Magazine from T-5 camera. The vacuum back is being measured for flatness with height gage and dial indicator. Magazine rests on small jacks which rest on surface plate.

Fig. 16

The locating back's position is adjusted until readings on three corners are exactly equal. Readings are then made to an accuracy of 0.0001 inch on

several other points across the frame as shown in Fig. 27-V, Sample Computations. By subtracting the value of the lowest reading from the other readings, the actual variations in the focal plane are found. The orientation of locating back should be referenced to the fiducial marks of the camera.

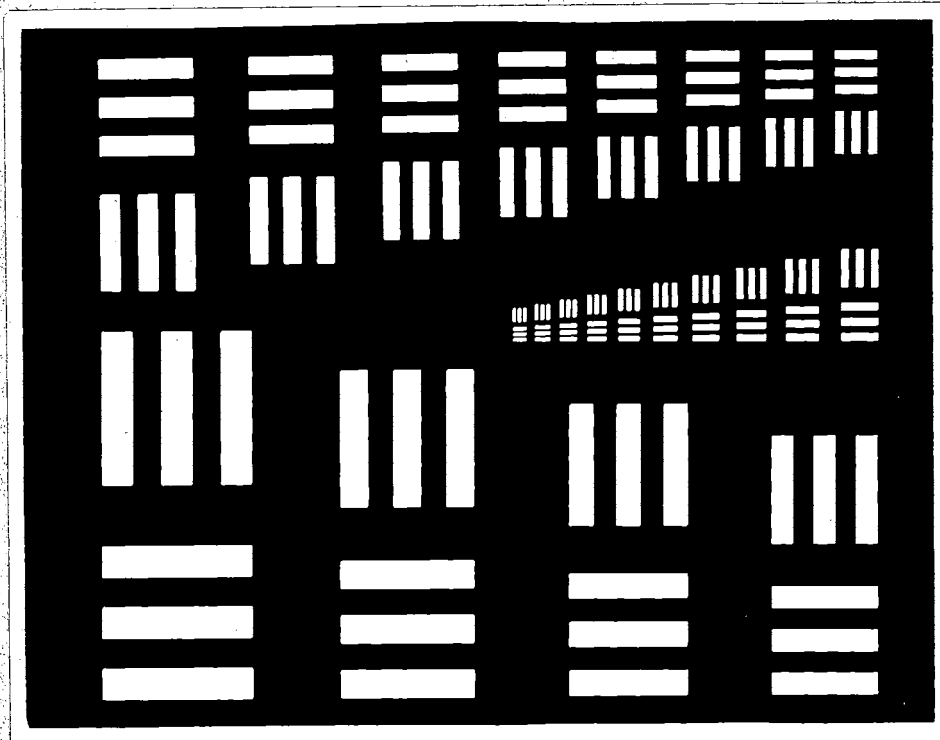
3. The flatness of focal plane frames should be checked periodically since there is always danger of the frame warping from age and from improper handling.

#### 1. DETERMINING THE RESOLUTION OF THE LENS-CAMERA-FILM COMBINATION

1. The determination of the resolution of the lens in an aerial mapping is considered an important phase of camera calibration. A method has been devised for accomplishing this on the same calibration range used for making the negatives required in the other phases of the calibration. This method seems to combine the advantages of the optical bench method and the aerial flight method without retaining the harmful disadvantages. It can be performed by any careful technician and with a minimum of special equipment.

2. Several types of useful information can be derived from the tests to be described, but only the two of major importance will be discussed. The first is the determination of the resolution across the entire lens image plane and the other is the checking of the focus of the lens in position in the camera. The data needed for the accomplishing of these tests are the same and merely require different analyses.

3. The field work required to obtain the necessary negatives is very simple after the resolution targets have been positioned. The targets used by this office are standard Air Force high-contrast line targets whose block sizes vary as the  $\sqrt{2}$  (See Fig. 17). Masonite sheets 36" x 48" in size were sprayed with black paint and the white blocks were painted on by the use of a silk screen template. A total of 13 targets is used by this office but a few more could be used to good advantage. One target is placed near the central target and others on each side of the central target at 5°, 10°, 15°, 20°, 30° and 40° outward. The targets must be in a straight line perpendicular to a



Line Resolution Target

Fig. 17

line from the camera to the central target and at a pre-computed distance from the camera. On the range at Wright Field, the line of targets is 400 feet from the camera station.

4. When making the exposures, the camera is pointed toward the targets in the same manner as described previously for the other calibration exposures. Exposures are made along each diagonal on aerial film having the same type of emulsion as that used for aerial photographs. In order to obtain an ideally exposed negative, it is best to make several exposures with different combinations of shutter speeds and aperture sizes. The negatives should be given normal development.

5. The selected negatives can be rapidly analyzed by placing on a light table and studying through a 15 to 20 power microscope. The resolution at each target image for each diagonal is read and recorded. These readings are



first checked for symmetry. If one side of the diagonal is obviously of higher resolution than the other the camera is returned to the depot or factory for realignment of the lens barrel. If the readings indicate symmetry then an average is made of the four readings at equal angles out. This gives the resolution at the center (on axis) and at 5°, 10°, 15°, 20°, 30° and 40° from the axis of the camera-lens-film combination being tested. This is compared with minimum acceptable values and the lens is either accepted or rejected, as the case may be.

6. The average resolution values are then analyzed to determine if the camera is set for the desired focus. Generally it is preferred to have the camera focussed for best definition over the entire negative although some users prefer to have the camera focussed at the center (on-axis). For a six-inch focal length Metrogon lens it has been found that best over-all definition occurs when the lens is 0.02 inch nearer the focal plane than for on-axis focussing.

7. Some of the advantages of a horizontal type resolution range as described above are: (a) No vibration at time of exposure, (b) Resolution target images in desired position on negative, (c) Selected lighting conditions, (d) Photographing through a long column of air (as compared to the optical bench method), (e) Simplicity and economy of method, and (f) Ease of standardization.

#### M. CALIBRATING SHORT FOCAL LENGTH CAMERAS

1. The methods described earlier for calibrating aerial mapping cameras can be used for calibrating cameras of any focal length. Changes may have to be made in the size and the spacing of the targets and in the distance of the camera station from the row of targets, for cameras having very short or very long focal lengths.

2. A number of cameras having 35 mm. focal length lenses have recently been calibrated by the writer on an indoor calibration range. This range consisted of a camera station and reference point located about 22 feet from a wall along which the targets were located (see Fig. 18A). The angles to each target from the central target were carefully measured and the front node of the camera was positioned over the reference point as before. Fiducial marks previously had been located in the camera so all exposures, measurements, computations, and adjustments were made exactly as described earlier.

#### N. ACCURACY OF MEASUREMENTS REQUIRED

1. In 1941, Mr. Gilbert C. Lorenz made a study of the tolerances of measurement required in the calibration of a camera equipped with a 6" Kestron lens. The study was based on data computed for a plate exposed on a calibration range whose camera station was approximately 600 feet from the central target. The angles from the camera station to the various posts were measured with a 10-second theodolite and the distances between images of the targets on the plates were read with a comparator reading to 0.001 mm. The following is copied directly from the report:

"B. 5. From the results of several calibrations the following errors in the factors mentioned earlier (location of node over reference point, measurements on the comparator, measurements of angles) may be expected:

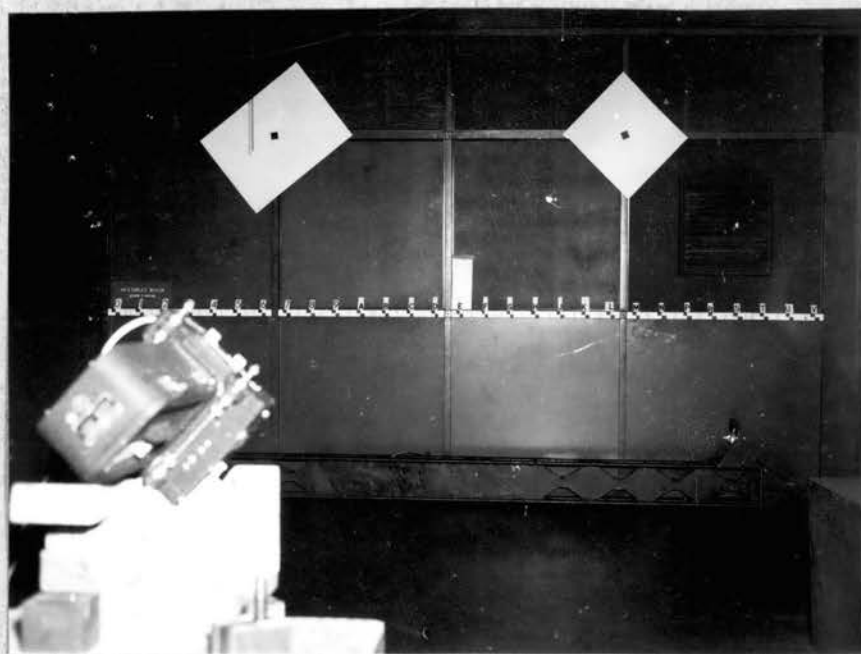
a. The positioning of the front node over the station depends upon the accuracy the node can be located. The maximum errors of locating the front node of the camera should be  $\pm 1/8"$  both along the axis and perpendicular to the axis. The probable errors of location should be approximately half the maximum values.

b. Readings of angles turned with the 10-second theodolite were estimated to 5 seconds of arc. The maximum departure of any angle from its mean in the measurement of these angles was 5 seconds. The probable error in any one angle is 1.5 seconds.

c. Although the comparator measures accurately to 0.001 mm., fuzziness of image makes it impossible to attain this accuracy when measuring distances on the plate. The maximum departure of any recording from its mean in one calibration was 0.005 mm. The probable error in any one reading for this calibration was 0.003 mm..



A. 35 mm Type O-15 Radar Recording Camera over reference point on indoor calibration range



B. Indoor calibration range. Large white sheets photograph along edges at fiducial marks.

Fig. 18

### C. CONCLUSIONS

1. In the computation of the focal length the following errors could result:

Factor	Maximum Tolerance	Probable Tolerance	Effect on Focal Length	
			Maximum	Probable
Positioning of node along axis	1/8 in.	1/16 in.	0.002 mm.	0.001 mm.
Position of node perpendicular to axis	1/8 in.	1/16 in.	none	none
Measurement of angles	5 sec.	1-1/2 sec.	0.014 mm.	0.003 mm.
Measurement on Comparator	0.005 mm.	0.003 mm.	0.025 mm.	0.016 mm.
Total			0.041 mm.	0.019 mm.

Since the final computed focal length is the average of four independent calculations, the probable error of the result should not be in error by more than 0.01 mm.

2. The positioning of the node of the lens over the station is the only factor that directly affects the distortion curve since errors in the measurement of angles and measurements on the comparator affect individual points only and are averaged out in the final distortion curve. However, care must be exercised in all measurements since it is difficult to arrive at the correct curve from a series of irregular points. Since the front node of the camera can be positioned with a maximum error of 1/8 inch both along the principal axis and perpendicular thereto, the effect of a displacement of this magnitude on the distortion curve will be negligible. The camera node must be located accurately from the available data on the lens design and the node must be positioned accurately over the station to maintain the accuracy of the distortion curve."

### C. MISCELLANEOUS

1. Camera Calibration data are generally used either (a) to correct image positions or (b) to determine whether or not a camera meets certain specifications. If the data are to be used for correcting image positions, then the corrections for distortion should be referenced to the Point of Symmetry. Positive distortion is subtracted from the measured distance between the  $P_3$  and the image in question, and negative distortion is added. In making diapositives for multiplex use, no corrections are presently made for error in the indicated position of the principal point if that error is less than a specified amount. It is the opinion of the writer that wide angle diapositive printers should be designed so that the Point of Symmetry on the aerial

negative can be centered exactly over the "Point of Symmetry" on the lower pressure plate of the printer. By placing the film in the printer as just described, the resulting diapositive would be corrected for distortion by the optimum provided by the printer lens.

2. If the distortion curves of diagonals A and B do not have the same maximum distortion, it may be necessary to determine distortions also between opposite fiducial marks if precise data are desired. These distortions can be determined in the same manner as for the distortion across the diagonals except that the values already found for  $\mu$ ,  $\Delta x$ , and the CFL can be used. After the new distortion curves are drawn, it might be advantageous to draw "Iso-Distortion" lines. This can be done by reading the distortion from the curves at predetermined intervals (say each 0.01 mm. of distortion) and then reading the distance from the  $P_s$  of that particular distortion. These distortions can then be plotted on polar coordinates and lines drawn through points of equal distortion.

3. Sufficient data are not available to indicate with any degree of accuracy the time required to make a complete calibration of, say, a T-5 camera. The break down given below is a fair estimate of the time required to calibrate a T-5 camera on the Wright Field Calibration Range. The time will vary with different ranges and different personnel.

	Hours	Minutes
Field Work (2 diagonals exposed)	0	30
Comparator Measurements (2 diagonals)	2	30
Computations & Curves (2 diagonals)	3	30
Totals	<u>5</u>	<u>30</u>

4. The writer wishes to express his appreciation to the following persons for their assistance in the preparation of this thesis; To:  
 Mr. A. H. Katz, Photographic Laboratory, Air Materiel Command, Wright Field for mathematical assistance in the Point of Symmetry computations; To:  
 Mr. R. G. Hickson, Aerial Photographic Branch, Engineer Research & Development

Laboratories, for Comparator measurements and sample computations; To: Mr. George E. Whiting, Aerial Photographic Branch, Engineer Research & Development Laboratories, for the field work and the preparation of enclosures; and To: Miss Phyllis J. Hill, Aerial Photographic branch, Engineer Research & Development Laboratories, for typing and general clerical assistance.

#### APPENDIX I

1. A few important general rules are repeated here for quick reference.

a. Angles on the calibration range should be accurate to plus or minus five (5) seconds.

b. Nodes of a Menzger lens are reversed.

c. The front node should be located within  $\pm 1/8$  inch of the camera station reference point.

d. When computing the equivalent focal length of a wide angle lens, targets should be selected which are less than  $12^\circ$  out from the central target.

e. When computing the equivalent focal length of a normal angle lens in which the distortion is negligible, targets should be selected which are at least  $30^\circ$  out from the central target.

f. When solving the equation

$$\mu = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

the negative value of  $\sqrt{B^2 - 4AC}$  is always used.

g. If  $\mu$  and  $\Delta x$  are positive, they are added to the values of angles and distances to the left of the central target and subtracted from the angles and distances to the right of the central target in the distortion computations.

h. Regarding the calibrated focal length computation, the value used for  $\tan \alpha_p$  is not too critical. Since the point of maximum distortion

usually falls between  $32^{\circ}30'$ , and  $35^{\circ}30'$ , and  $\alpha_p$  selected in that range will give usable results. However, an average of the four values of  $\tan \alpha_p$  used in the maximum positive distortion computation is recommended here.

i. Referring again to the calibrated focal length computation,  $\Delta f$  is subtracted from the equivalent focal length if  $d_p$  is smaller numerically than  $d_n$ , and added if  $d_p$  is larger than  $d_n$ . In the equation

$$\Delta f = \frac{d_p - d_n}{1 + \tan \alpha_p},$$

$d_p$  and  $d_n$  are subtracted, without regard to signs.

j. Regarding the resultant Point of Symmetry computation,  $r$  and  $s$  represent distance measurements, and always have positive signs.

k. It is again pointed out that generally only one distortion curve is plotted for each calibration performed. There may be special cases, however, where two curves are required as in Fig. 25, and there may even be instances where a curve is required for each half of each diagonal.

l. The calibrated focal length is usually shown to the nearest hundredth of a millimeter.

#### APPENDIX II--SAMPLE COMPUTATIONS

1. The calibration of aerial mapping cameras by the methods described in this paper is very simple and rapid if the work is organized properly and if computation forms are used. No suggestions will be made here concerning forms to be used since each office using such forms usually prefers to design its own.

2. A list of data and a set of sample computations for calibrating type T-5 Aerial Camera No. 41-4172 are shown on the pages which follow. Detailed computations are shown for one diagonal only, although the results for the complete calibration are given.

a. Fig. 19 is a list of horizontal angles measured on the right

Target No.	Angle from Target 67	Target No.	Angle from Target 67	Target No.	Angle from Target 67
	° ' "		° ' "		° ' "
30	49 31 57	55	25 32 57	81	21 51 01
31	48 49 58	56	24 23 36	82	23 02 01
32	48 07 34	57	23 11 38	83	24 11 38
33	47 22 54	58	19 31 48	84	25 19 30
34	46 39 02	59	18 17 22	85	26 26 38
35	45 07 20	60	17 02 40	86	27 32 32
36	44 19 26	61	11 56 25	87	28 37 13
37	43 30 18	62	10 37 03	88	30 42 57
38	42 38 39	63	9 19 02	89	31 39 14
39	41 47 34	64	7 59 43	90	32 44 33
40	40 55 25	65	2 40 48	91	33 42 55
41	40 02 20	66	1 20 18	92	34 39 44
42	39 07 27	67	0 00 00	93	35 36 14
43	38 12 09	68	3 59 55	94	36 31 05
44	37 15 40	69	5 19 40	95	37 24 52
45	36 17 02	70	6 40 31	96	38 17 48
46	35 17 25	71	7 59 35	97	39 08 50
47	34 16 32	72	9 18 28	98	39 59 02
48	33 14 40	73	10 37 09	99	40 48 26
49	32 12 52	74	11 55 55	100	41 36 03
50	31 09 31	75	13 12 52	101	42 22 57
51	30 03 39	76	14 28 52	102	43 08 46
52	28 58 38	77	16 59 29	103	43 53 51
53	27 50 20	78	18 14 26	104	46 43 19
54	26 42 18	79	19 27 20	105	47 23 58
		80	20 39 43		

Horizontal Angles on Wright Field Calibration Range

Fig. 19



### Field Calibration Range.

b. Fig. 20 shows the comparator readings on the negative.

c. Fig. 21 is the form used for computing the equivalent focal length by use of a computing machine.

d. Fig. 21a is the form used for computing the equivalent focal length by logarithms.

e. If the method suggested in Par. F-5 is used the results will be as shown below:

$$f = \frac{a}{2 \tan \alpha} + \frac{b}{2 \tan \beta}$$

<p style="text-align: center;"><u>Targets 63, 67, 72</u></p> $f = \frac{25.308}{2 \times 0.1640649} + \frac{25.272}{2 \times 0.1636957}$ $= 77.128 + 77.096 = 154.226$	<p style="text-align: center;"><u>Targets 62, 67, 73</u></p> $f = \frac{26.936}{2 \times 0.1874611} + \frac{26.910}{2 \times 0.1874912}$ $= 77.179 + 77.097 = 154.276$ <p style="text-align: center;"><u>Targets 61, 67, 74</u></p> $f = \frac{32.633}{2 \times 0.2114674} + \frac{32.588}{2 \times 0.2113154}$ $= 77.158 + 77.111 = 154.269$
--	---

$$\text{Aver. } f = 154.257$$

f. Locating the Point of Symmetry (See par. G-5) Targets 47 and 92 were chosen since they are sufficiently close to the points of maximum distortion on the curve.

$$\mu = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$A = f \left[ \sec^2 \beta \tan \beta - \sec^2 \alpha \tan \alpha \right]$$

$$B = f \left[ 2 - \sec^2 \alpha - \sec^2 \beta \right]$$

$$C = \left[ k - f (\tan \alpha - \tan \beta) \right]$$

and  $k = a^2c - cb^2$

$$f = 154.255$$

$$k = 105.445 - 106.591 = -1.106$$

$$\alpha = 34^\circ 18' 32''$$

$$\sec \alpha = 1.2101577$$

$$\sec^2 \alpha = 1.4644817$$

$$\tan \alpha = 0.6815258$$



Focal Length Computation Sheet  
(Machine Computation)

Camera No. AF 41-4172 Exposure No. A Date 5/3/48 Computer RGH

	Computation 1	Computation 2	Computation 3
a = distance from to Tar <u>62</u>	Tar <u>62</u> = <u>28936</u>	Tar <u>63</u> = <u>25308</u>	Tar <u>61</u> = <u>32633</u>
	to Tar <u>67</u>	to Tar <u>67</u>	to Tar <u>67</u>
b = distance from to Tar <u>67</u>	Tar <u>67</u> = <u>28910</u>	Tar <u>67</u> = <u>25272</u>	Tar <u>67</u> = <u>32588</u>
	to Tar <u>73</u>	to Tar <u>72</u>	to Tar <u>74</u>
a + b . . . . .	<u>57.846</u>	<u>50.580</u>	<u>65.221</u>
$\alpha$ . . . . .	<u>10° 37' 03"</u>	<u>9° 19' 02"</u>	<u>11° 56' 25"</u>
sin $\alpha$ . . . . .	<u>.1842516</u>	<u>.1619005</u>	<u>.2068920</u>
$\beta$ . . . . .	<u>10° 37' 09"</u>	<u>9° 18' 28"</u>	<u>11° 55' 55"</u>
sin $\beta$ . . . . .	<u>.1842801</u>	<u>.1617378</u>	<u>.2067497</u>
$(\alpha + \beta) = (\theta + \phi)$ . .	<u>21° 14' 12"</u>	<u>18° 37' 30"</u>	<u>23° 52' 20"</u>
sin $(\alpha + \beta)$ . . . .	<u>.3622211</u>	<u>.3193728</u>	<u>.4046983</u>
$1/2(\alpha + \beta) = 1/2(\theta + \phi) =$	<u>10° 37' 06"</u>	<u>9° 18' 45"</u>	<u>11° 56' 10"</u>
1. b x sin $\alpha$ . . . .	<u>5.3267138</u>	<u>4.0915494</u>	<u>6.7421965</u>
2. a x sin $\beta$ . . . .	<u>5.3323290</u>	<u>4.0932602</u>	<u>6.7468630</u>
3. $2 \div 1 = \tan(45^\circ + \lambda)$	<u>1.0010542</u>	<u>1.0004181</u>	<u>1.0006921</u>
4. $45^\circ + \lambda$ . . . .	<u>45° 01' 49"</u>	<u>45° 00' 43"</u>	<u>45° 01' 12"</u>
5. $\lambda$ . . . . .	<u>0° 01' 49"</u>	<u>0° 00' 43"</u>	<u>0° 01' 12"</u>
6. tan $\lambda$ . . . . .	<u>.0005285</u>	<u>.0002085</u>	<u>.0003491</u>
7. cot $1/2(\alpha + \beta)$ . .	<u>5.3340129</u>	<u>6.0982935</u>	<u>4.7305617</u>
8. $6x7 = \tan 1/2(\theta - \phi)$	<u>.0028190</u>	<u>.0012715</u>	<u>.0016514</u>
9. $1/2(\theta - \phi)$ . . . .	<u>0° 09' 41"</u>	<u>0° 04' 22"</u>	<u>0° 08' 41"</u>
10. $1/2(\theta + \phi)$ . . . .	<u>10° 37' 06"</u>	<u>9° 18' 45"</u>	<u>11° 56' 10"</u>
11. $9 + 10 = \theta$ . . . .	<u>10° 46' 47"</u>	<u>9° 23' 07"</u>	<u>12° 01' 51"</u>
12. $(\theta + \phi) - \theta = \phi$ . .	<u>10° 27' 25"</u>	<u>9° 14' 23"</u>	<u>11° 50' 29"</u>
13. cos $\phi$ . . . . .	<u>.9833916</u>	<u>.9870252</u>	<u>.9878194</u>
14. $(a + b) \times \cos \phi$	<u>56.8852705</u>	<u>49.9237346</u>	<u>63.8330580</u>
15. $14 \div \sin(\alpha + \beta)$	<u>157.0457118</u>	<u>156.3180541</u>	<u>157.7299880</u>
16. cos $\theta$ . . . . .	<u>.9823535</u>	<u>.9866141</u>	<u>.9780356</u>
17. $15 \times \cos \theta = f$	<u>154.274</u>	<u>154.226</u>	<u>154.266</u>

FIG. 21

Average  $f =$  154.255

Principal Distance Computation Sheet  
(Logarithmic)

Camera No. AF 41-4172

a = Target 62 to Target 67

b = Target 67 to Target 73

a = 28936 mm

$\alpha = 10^{\circ} 37' 03''$

b = 28910 mm

$\beta = 10^{\circ} 37' 09''$

a + b = 57846 mm

$(\alpha + \beta) = 21^{\circ} 14' 12''$

$$1/2(\theta + \phi) = 1/2(\alpha + \beta) = 10^{\circ} 37' 06''$$

(1)	log a	1.4614384	(13)	$1/2(\theta - \phi)$	$0^{\circ} 09' 41''$
(2)	log sin $\beta$	9.2654783	(14)	$1/2(\theta + \phi)$	$10^{\circ} 37' 06''$
(3)	(1)+(2)	10.7269168	(15)	(13)+(14)	$10^{\circ} 46' 47''$
			(16)	2(14)-(15)	$10^{\circ} 27' 25''$
(4)	log b	1.4610481			
(5)	log sin $\alpha$	9.2654111	(17)	log(a + b)	1.7622733
(6)	(4)+(5)	10.7264592	(18)	log cos $\phi$	9.9927264
			(19)	(17)+(18)	11.7549997
(7)	(3)-(6)	0.0004576	(20)	log sin $(\alpha + \beta)$	9.5589738
	log tan $(4b + \lambda)$		(21)	(19)-(20)	2.1960259
(8)	$45^{\circ} + \lambda$	$45^{\circ} 01' 49''$	(22)	log cos $\theta$	9.9922678
			(23)	(21)+(22)	2.1882937
(9)		$+ 0^{\circ} 01' 49''$			
(10)	log tan $\lambda$	6.7228283			
(11)	log cot $1/2(\alpha + \beta)$	0.7270560		$f = 154.274$	
(12)	(10)+(11)	7.4498843			
	log tan $1/2(\theta - \phi)$				

Fig. 21A

$$\begin{aligned} \beta &= 34^{\circ}39'44'' & \tan \beta &= 0.6914578 \\ \sec \beta &= 1.2157771 & \sec^2 \alpha \tan \alpha &= 0.9980865 \\ \sec^2 \beta &= 1.4781140 & \sec^2 \beta \tan \beta &= 1.0220535 \end{aligned}$$

$$A = 154.255(1.0220535 - 0.9980865) = + 3.6970296$$

$$B = 154.255(2 - 1.4644817 - 1.4781140) = - 145.4000997$$

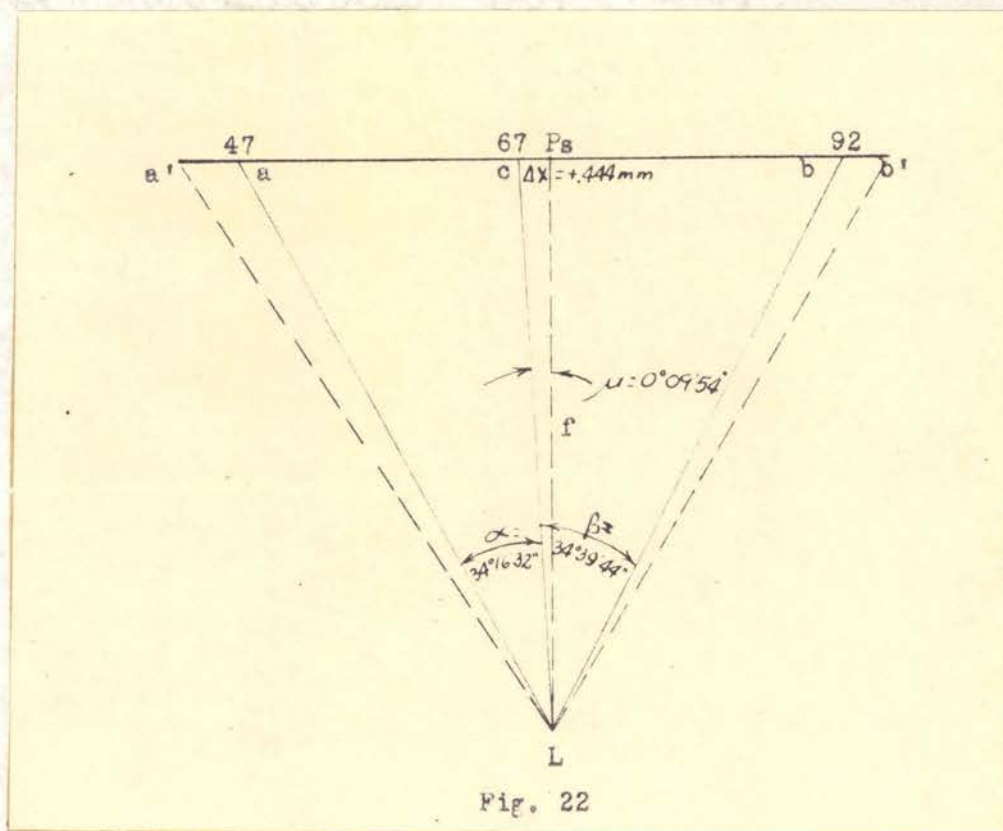
$$C = - 1.106 - 154.255 (0.6815288 - 0.6914578) = + 0.4255979$$

$$\mu = \frac{145.4000997 \pm \sqrt{(-145.4000997)^2 - 4(+3.6970296)(+0.4255979)}}{2(+3.6970296)}$$

$$\mu = 0.00292743 \text{ Radians} = 0^{\circ}10'04''$$

$$\Delta x = f \mu = (154.255) (0.00292743) = +0.452\text{mm.}$$

g. Determining Maximum Positive Distortions (See par. H-2). These computations offer a check on the computations in Par. f.



$$a'a = (a'e + \Delta x) - f \tan (\alpha + \mu).$$

$$bb' = (ob' - \Delta x) - f \tan (\theta - \mu).$$

Angle at L between target 47 and  $P_g = \alpha + \mu = 34^\circ 26' 36''$

Angle at L between target 92 and  $P_g = \theta - \mu = 34^\circ 29' 40''$

Distance from target 47 to  $P_g = a'e + \Delta x = 105.445 + 0.452 = 105.897$

Distance from target 92 to  $P_g = ob' - \Delta x = 106.551 - 0.452 = 106.099$

Tar	$\angle$ at L	Tan $\angle$	$f \tan \angle$	Measured Distance	Distortion
47	$34^\circ 26' 36''$	0.6858258	105.729	105.897	+ 0.105
92	$34^\circ 29' 40''$	0.6871582	105.995	106.099	+ 0.104

h. In making a second solution for the Point of Symmetry, targets 48 and 91 are used.

$$A = +0.0275148f = + 4.2133982$$

$$B = -0.8749597f = - 134.9689085$$

$$C = k + f (0.01181281) = -1.440 + 1.8221079 = + 0.3821079$$

$$\mu = \frac{134.9689085 - 134.9430490}{+8.4267964} = 0.00283137 \text{ Radians} = + 0^\circ 09' 44''$$

The values for  $a'a$  and  $bb'$  are again computed

Tar	$\angle$ at L	Tan $\angle$	$f \tan \angle$	Measured Distance	Distortion
48	$33^\circ 24' 24''$	0.6595455	101.738	101.839	+ 0.101
91	$33^\circ 33' 11''$	0.6632180	102.305	102.405	+ 0.100

Since the values of the distortions are slightly different from those computed above, when targets 47 and 92 were used, an average of the four values will be used in the calibrated focal length computations in par. j below.

$$\text{Average } d_p = \frac{+ 0.104 + 0.105 + 0.101 + 0.100}{4} = + 0.102$$

$$\text{Average } \tan \alpha = \frac{0.6858258 + 0.6871582 + 0.6595455 + 0.6632180}{4} = 0.6739369$$

i. Determining Negative Distortion at  $45^\circ$  from the Lens Axis (See par. H-2). From pars. g and h, above, the average value of

$$\mu = + 00^\circ 09' 54'' \text{ and } \Delta x = + 0.44 \text{ mm.}$$

Par.	$\angle$ at L	$\tan \angle$	$f \tan \angle$ mm.	Measured Distance	Distortion mm.
35	45° 17' 14"	1.0100769	158.809	158.636	-0.173
37	43 40 12	0.9546195	147.256	147.155	-0.099
39	41 57 28	0.8990706	138.686	138.666	-0.000
100	41 26 09	0.8327307	136.166	136.173	+0.007
102	42 58 52	0.9318969	143.750	143.691	-0.059
103	43 43 57	0.9567066	147.577	147.487	-0.090

The distortions just determined for the six points are plotted as ordinates against their respective distance from the  $P_3$  as abscissae, as shown in Fig. 27-I. A curve is drawn through these points and the average value of the distortion at 45° is found. For this example,  $d_n = -0.160$  mm.

j. Determining the Calibrated Focal Length.

$$\Delta f = \frac{d_p - d_n}{1 + \tan \alpha_p}$$

$$d_p = +0.102 \quad d_n = -0.160 \quad \tan \alpha = 0.6739369$$

$$\Delta f = \frac{0.102 - 0.160}{1 + 0.6739369} = 0.035; \text{CFL} = 154.256 - 0.035 = 154.220 \text{ mm.}$$

k. Plotting the Distortion Curve. As in Par. g above, the distortion at each target is found by comparing the measured distance from the  $P_3$ , with the computed, theoretically distortion-free distance. The measured distance is found by adding the average value of  $\Delta x (+0.444\text{mm.})$  to the distances between the  $P_3$  and the target images to the left of  $P_3$ , and by subtracting the average  $\Delta x$  from the distances between the  $P_3$ , and the target images to the right of  $P_3$ . The computed distances are found by the expression  $f \tan \alpha$  where the angles to the left of  $P_3$  are equal to the measured angles shown in Fig. 19, plus the average value of  $\mu (0^\circ 09' 54")$ . The angles to the right of  $P_3$  are equal to the measured angles minus the value of  $\mu$ .  $f$  is equal to the CFL just determined (154.220 mm.). The distortion computations for diagonal A are shown in Fig. 24. The computations for diagonal B are not shown. The distortion curves for diagonals A and B are shown in Fig. 25. However, it is not recommended that these curves be plotted separately.

In par. I-4 it is suggested that the two halves of the curves for the two diagonals be plotted as a single curve by using selected targets from each half diagonal to provide an even distribution of points along the final curve.

Fig. 26 shows the computations for the selected points along this curve. An average of the CFL's for diagonals A and B is used. CFL (diagonal B) = 154.200 mm.

$$\text{Average CFL} = \frac{154.22 + 154.20}{2} = 154.21 \text{ mm.}$$

$$\Delta x \text{ (diagonal B)} = -0.841 \text{ mm. } \mu \text{ (diagonal B)} = -0^{\circ}18'45''$$

1. Determining the Resultant Point of Symmetry. Shown below are the relative locations of the central target (67), the  $P_S$  and the IPP for diagonals A and B.

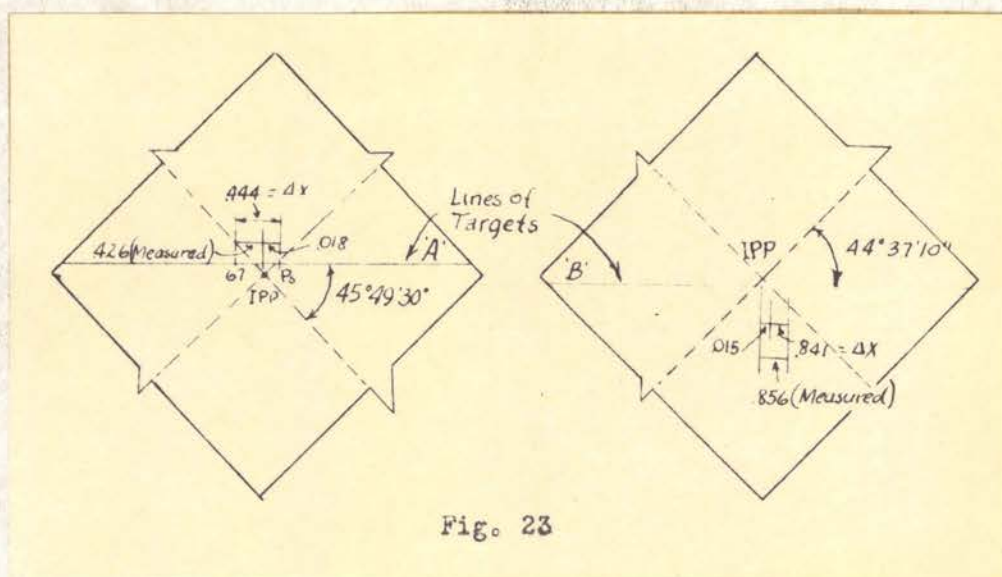


Fig. 23

$$\begin{aligned} r &= 0.018 \quad \text{Angle } \gamma = 45^{\circ}50' \\ s &= 0.015 \quad \text{Angle } \psi = 44^{\circ}37' \\ x_{P_S} \cos \gamma + y_{P_S} \sin \gamma &= r \\ x_{P_S} \cos \psi + y_{P_S} \sin \psi &= s \end{aligned}$$

$$\begin{aligned} \sin \gamma &= +0.71732 \quad \cos \gamma = +0.69675 \\ \sin \psi &= -0.70236 \quad \cos \psi = +0.71182 \\ +0.69675 x_{P_S} + 0.71732 y_{P_S} &= +0.018 \\ +0.71182 x_{P_S} - 0.70236 y_{P_S} &= +0.015 \\ \hline +0.72698 x_{P_S} - 0.71732 y_{P_S} &= +0.015 \\ +1.42373 x_{P_S} &= +0.033 \\ x_{P_S} &= +0.023 \quad (\text{Coords. of } P_S) \\ y_{P_S} &= +0.002 \end{aligned}$$



## DISTORTION COMPUTATION SHEET

Diagonal A

For

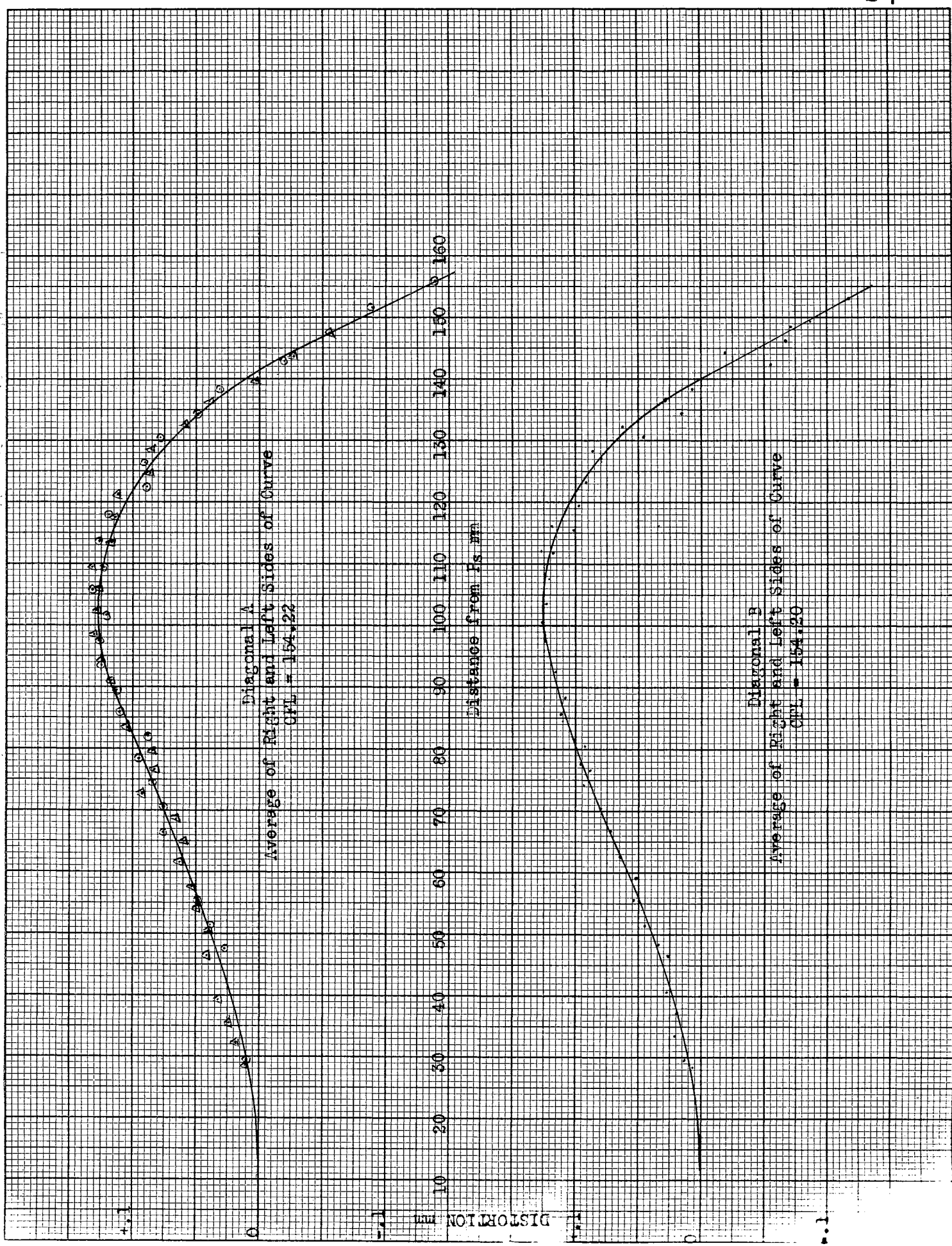
Camera No. AF 41-4172

Target	$\Delta$ at L	$\tan \Delta$	$f \tan \Delta$	Measured Distance	Distortion
35	45 17 14	1.0100755	155.774	155.536	-.138
36	44 29 20	.9823161	151.493	151.404	-.089
37	43 40 12	.9546195	147.221	147.165	-.056
38	42 48 33	.9263074	142.855	142.836	-.019
39	41 57 28	.8990706	138.654	138.636	+.031
40	41 05 19	.8720056	134.461	134.529	+.048
41	40 12 14	.8451619	130.344	130.422	+.073
42	39 17 21	.8181748	126.179	126.270	+.091
43	38 22 03	.7913671	122.091	122.179	+.088
44	37 25 34	.7652601	118.021	118.139	+.110
45	36 26 56	.7385615	113.904	114.030	+.126
46	35 27 19	.7121160	109.823	109.946	+.123
47	34 26 26	.6857545	105.757	105.889	+.132
48	33 24 34	.6596150	101.726	101.846	+.120
49	32 22 46	.6341162	97.793	97.920	+.127
50	31 19 26	.6085741	93.864	93.979	+.125
51	30 15 33	.5826177	89.851	89.964	+.113
52	29 08 32	.5573565	85.967	86.099	+.112
53	28 00 14	.5317965	82.014	82.133	+.089
54	26 52 12	.5066708	78.139	78.235	+.095
55	26 42 61	.4816721	74.268	74.353	+.085
56	24 33 30	.4569564	70.472	70.547	+.075
57	23 21 32	.4318370	66.666	66.681	+.075
58	19 41 42	.3879534	56.204	55.254	+.050
59	18 27 16	.3637115	51.465	51.505	+.035
60	17 12 34	.3097324	47.767	47.794	+.027
61	12 06 19	.2144776	33.077	33.077	0
62	10 46 57	.1904437	29.370	29.380	+.010
73	10 27 15	.1945118	28.456	28.466	+.011
74	11 46 01	.2083089	32.125	32.144	+.019
75	13 02 58	.2317775	35.745	35.769	+.023
76	14 18 58	.2551963	39.355	39.388	+.032
77	16 49 35	.3024204	46.639	46.680	+.041
78	18 04 32	.3263782	50.334	40.374	+.040
79	19 17 26	.350100	53.979	54.029	+.050
80	20 29 49	.3738239	57.651	57.704	+.053
81	21 41 07	.3973507	61.326	61.389	+.063
82	22 32 07	.4217711	65.046	65.104	+.068
83	24 01 44	.4468330	68.756	68.821	+.065
84	25 09 36	.4697118	72.439	72.533	+.094
85	26 16 44	.4937724	76.150	76.231	+.081
86	27 22 38	.5178465	79.882	79.952	+.090
87	28 27 19	.5419455	83.579	83.682	+.103
88	30 33 03	.5902407	91.027	91.143	+.116

Fig. 24

Target	$\angle$ at L	$\tan \angle$	$f \tan \angle$	Measured Distance	Distortion
89	31 29 20	.6128341	94.465	94.587	+.122
90	32 34 39	.6389735	98.542	98.673	+.131
91	33 33 01	.6661482	102.271	102.399	+.127
92	34 29 50	.6872096	105.981	106.107	+.126
93	35 26 20	.7116850	109.756	109.888	+.132
94	36 31 11	.7359997	113.506	113.622	+.116
95	37 14 58	.7604024	117.269	117.385	+.114
96	38 07 54	.7849050	121.052	121.173	+.111
97	38 58 56	.8092704	124.806	124.918	+.112
98	39 49 08	.8337273	128.577	128.663	+.086
99	40 38 32	.8583627	132.360	132.439	+.069
100	41 26 09	.8827307	136.155	136.173	+.038
101	42 13 03	.9073013	139.924	139.926	+.002
102	42 38 52	.9318989	143.717	143.691	-.026
103	43 23 57	.9567066	147.543	147.487	-.056

Fig. 24 (Concluded)



Diagonal A  
 Average of Right and Left Sides of Curve  
 CPL = 154.22

Diagonal B  
 Average of Right and Left Sides of Curve  
 CPL = 154.20

Diagonal A

Target No.	$\alpha$ at L	$\tan \alpha$	$f \tan \alpha$	Measured Distance	Distortion
35	45 17 14	1.0100765	155.764	155.636	-.128
39	41 57 28	.8990706	138.646	138.686	+.040
43	38 22 03	.7916671	122.083	122.179	+.096
47	34 26 26	.6857645	105.750	105.869	+.139
52	29 08 32	.5575585	85.981	86.099	+.118
54	26 52 12	.5066708	78.134	78.235	+.101
58	19 41 42	.3579534	55.200	55.264	+.064
61	12 06 19	.2144778	33.075	33.077	+.002
73	10 27 15	.1845118	28.454	28.468	+.014
78	18 04 32	.3263782	50.331	50.374	+.043
81	21 41 07	.3976507	61.322	61.389	+.067
84	25 09 36	.4697118	72.434	72.533	+.099
88	30 33 03	.6902407	91.021	91.143	+.122
91	33 33 01	.6631482	102.264	102.398	+.134
97	38 58 56	.8092704	124.798	124.918	+.120
101	42 13 03	.9073013	139.915	139.926	+.011
103	43 43 57	.9567066	147.534	147.487	-.047

Diagonal B

36	44 00 41	.9660730	148.978	148.897	-.081
41	39 43 35	.8309943	128.148	128.224	+.076
45	35 58 17	.7257799	111.923	112.041	+.118
48	32 55 55	.6477202	99.885	99.999	+.114
50	30 50 46	.5972109	92.096	92.200	+.104
56	24 04 51	.4469202	68.920	68.989	+.069
60	16 43 55	.3006222	46.359	46.382	+.023
75	14 47 37	.2640921	40.726	40.751	+.025
80	20 58 28	.3833524	59.117	59.165	+.048
82	23 20 46	.4316224	66.560	66.630	+.070
86	27 51 17	.5284614	81.494	81.590	+.096
90	33 03 18	.6507732	100.356	100.476	+.120
92	34 58 29	.6995602	107.878	107.994	+.116
95	37 43 37	.7736393	119.303	119.393	+.090
99	41 07 11	.8729620	134.619	134.626	+.007
102	43 27 31	.9475926	146.128	146.052	-.076

Fig. 26

For graphical solution, see Fig. 27-IV.

m. A set of sample computations is shown below in which the value of  $f$  in the Point of Symmetry and CPL computations was taken as 154.1 (as stamped on lens holder) instead of 154.255 as computed by the method given in Par. F. These computations are for the purpose of showing that the value of the calibrated focal length is practically unaffected by a relatively large change in the equivalent focal length.

Determination of Point of Symmetry (See par. G-5)

From table in par. G-6

$$A = + 3.6933147 \quad B = - 145.2539974$$

$$C = - 1.106 + 1.5300589 = + 0.4240589$$

$$\mu = \frac{+ 145.2539974 \pm \sqrt{(-145.2539974)^2 - 4(3.6933147)(0.4240589)}}{+ 7.3866294}$$

$$\begin{aligned} \mu &= \frac{+ 145.2539974 - 145.2324808}{+ 7.3866294} = + 0.002919667 \text{ Radians} \\ &= 0.16728460^\circ = 0^\circ 10' 02'' \end{aligned}$$

$$\Delta x = \mu f = + 0.480 \text{ mm.} \quad (\Delta x = 0.452 \text{ mm. when } f = 154.255).$$

Determination of maximum positive distortion (See par. H-2).

Par.	$\Delta$ at L	$\tan \Delta$	$f \tan \Delta$	Measured Distance	Distortion ( $d_p$ )
47	54° 26' 34"	0.6856115	105.684	105.895	+0.211
92	54° 29' 42"	0.6871525	105.890	106.101	+0.211

Determination of negative distortion at 45° (See par. H-3)

35	45 17 22	1.0101549	155.665	155.642	+0.023
37	45 40 20	0.9546936	147.118	147.171	+0.053
39	41 57 36	0.8991407	138.568	138.692	+0.124
100	41 28 01	0.6826617	136.018	136.163	+0.145
102	42 58 44	0.9318265	145.594	143.881	+0.087
103	45 43 49	0.9566323	147.417	147.477	+0.060

Camera No. AF 41-4172, I-B  
 Lens No. AF 5761  
 C.F.L., 154.21 mm  
 DATE 16 Mar, 1948

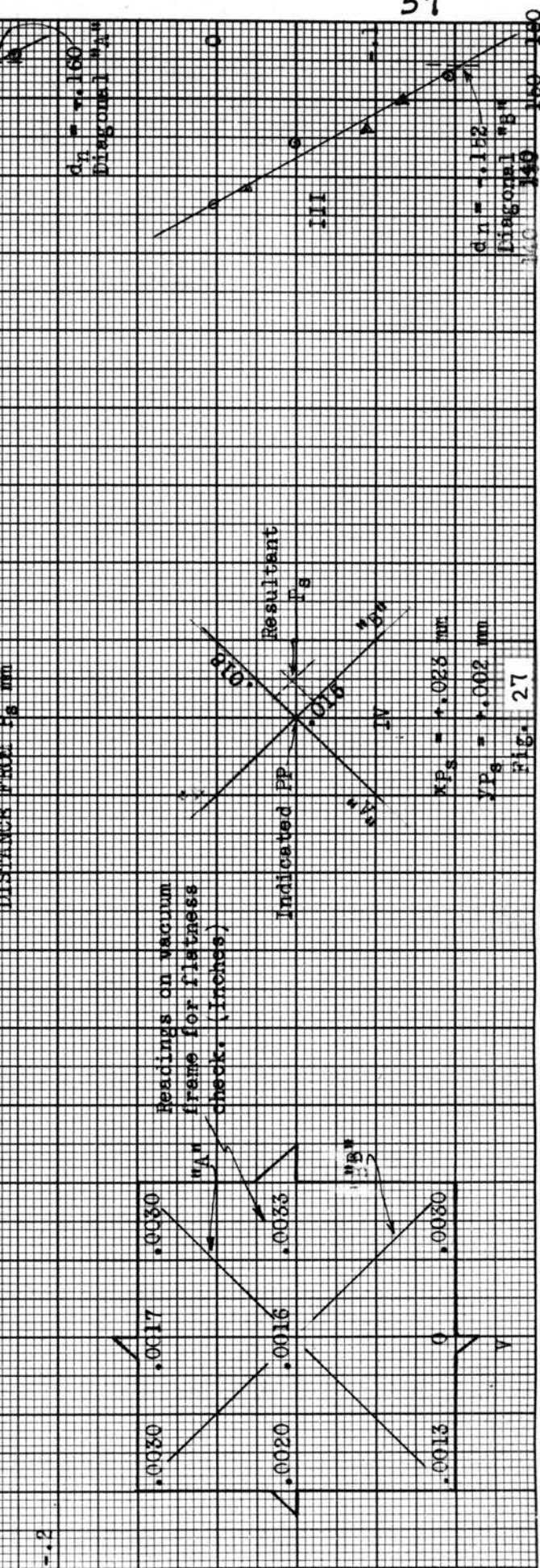
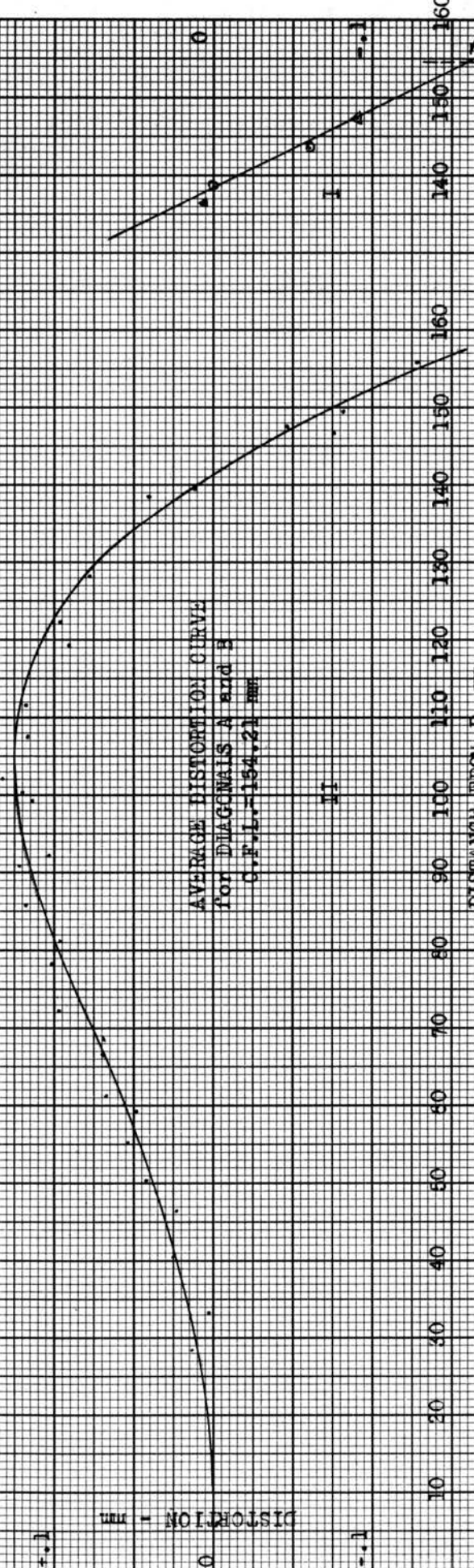


Fig. 27

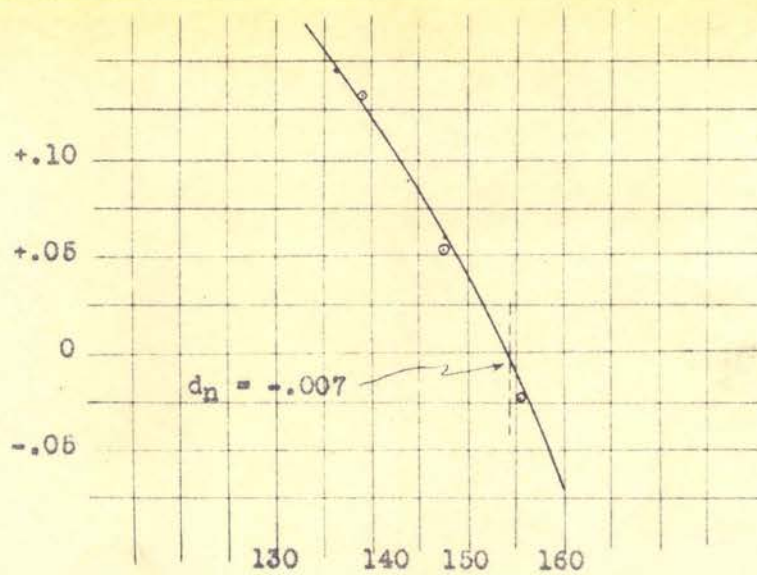


Fig. 28

Determination of Calibrated Focal Length (See par. H-3)

$$\Delta f = \frac{0.221 - 0.007}{1 + 0.6864820} = 0.121 \text{ mm.}$$

Since  $d_p$  is larger numerically than  $d_n$ ,  $\Delta f$  is positive.

$$\text{CFL} = 154.1 + 0.121 = 154.221 \text{ mm. (CFL} = 154.220 \text{ when } f = 154.225).$$

n. Measuring the vacuum back for flatness (See par. K-2). The measurements found for the magazine used with the camera being calibrated are shown in Fig. 27-V.

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Typist - Miss Phyllis J. Dill