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 GRADJATE COLLEGEA STUDY OF THE DEVELOPMENT OF THE ABILITY OF SELECTED STUDENTS TO VISUALIZE THE ROTATION AND DEVELOPMENT OF SURFACES

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## A STUDY OF THE DEVELOPMENT OF THE ABILITY OF SELECTED STUDENTS TO VISUALIZE THE ROTATION AND DEVELOPMENT OF SURFACES

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# A STUDY OF THE DEVELOPMENT OF THE ABILITY <br> OF SELECTED STUDENTS TO VISUALIZE THE <br> ROTATION AND DEVELOPMENT OF SURFACES 

## CHAPTER I

# IN TRODUCTION, STA TEMENT OF THE PROBLEM, AND NEED FOR THE STUDY 

## Introduction

Questions involving spatial orientation and
Visualization are common to many aptitude tests. A space factor was first isolated about 1938 by Louis L. Thurstone. Through further factor analysis he found that this space factor consisted of three factors which he described, as cited by Fruchter (1954), as:
$S_{1}$ - The ability to recognize the identity of an object when it is seen from different angles.
$S_{2}$ - The ability to imagine the movement or internal displacement among the parts of a configuration that one is thinking about.
$\mathrm{S}_{3}$ - The ability to think about spatial relations in which the body orientation of the observer is an essential part of the problem.

Thus, spatial ability is essential for success not
only in the obvious areas such as solid geometry and other
mathematical fields which use diagrams or models to represent space problems, but it is also one of the primary mental abilities determining a person's aptitude and plays an important role in areas such as the sciences, life sciences, engineering, and art. Siemankowski and MacKnight (1971) found a high correlation between spatial conceptualization and grades in college science courses. They also found that science (except chemistry), mathematics, and art majors have higher levels of spatial conceptualization than non-science majors. In examining sixty-four eminent scientists in physics, biology, and the social sciences, Roe (1952) found that the physicists and the psychologists scored very high on the spatial test and that the biologists and experimental physicists tended strongly to depend upon visual imagery in their thinking. One of the higher correlations Layton (1953) found between various tests and grades in dental school was that between scores on the Survey of Object Visualization Test and four year grades, indicating that spatial visualization may be a factor in success in dental school. In analyzing factors that could be used to predict success in first year engineering courses, Poole and Stanley (1972) found the ability to visualize and manipulate images to be the most relevant spatial ability.

Although spatial ability is one of the basic abilities necessary in so many areas, it is one that too
of ten has not been developed. For example, Siemankowski and MacKnight were surprised at the extremely low levels of ability of many of their subjects. Thus, it is important for educators to determine whether the ability to perform spatial operations is an innate characteristic or a result of learning and whether it is possible to increase such abilities through activities in the classroom.

## Statement of the Problem

This study is concerned with the aspect of spatial visualization that Piaget and Inhelder (1967) refer to as "the rotation and development of surfaces". By this is meant the ability to "rotate the sides of a solid into the frontal plane, and unfold or 'develop' the regular curved surfaces, such as the cylinder and the cone". (p. 273)

This study has two objectives:

1. To investigate the relationships among age, academic ability, sex, and the ability to visualize the results of rotating the sides of a solid into a plane and of unfolding curved surfaces.
2. To determine the effect that training has on such visualizations.

## Need for the Study

The results on aptitude tests show that many students have a very low level of ability in spatial perception and visualization. Courses such as solid
geometry, descriptive geometry, and mechanical drawing have been investigated to determine their effects on spatial abilities. Methods of instruction such as the use of solid models and programed instruction have also been tested. However, the results tend to be inconclusive. This may be due to the fact that spatial ability is not just one type of ability, but results from the coordination of many different types of tasks; thus no one general course or method is capable of affecting all of these factors. Developing a concept of space is a lengthy process which begins when a child first starts to look at objects around him. Thus, it is necessary to determine how the child progresses through the various stages required in reaching this goal, at what ages he is capable of attaining different types of behavior, and how he can be helped at each stage. Due to the influence of Plaget, many studies have considered such questions for some spatial tasks. They have investigated the ability of children to identify shapes, draw figures, conserve volume, length, and order, coordinate perspectives, and visualize cross sections. They have also studied the effects of various programs on these tasks. Such studies are needed for other activities that make up spatial abilities, including that of visualizing the results of rotating and developing surfaces. As more facets of spatial abilities are explored, it may be possible to produce a continuing
program that will enable students to develop their spatial abilities to the fullest. This study was designed to provide information on the ability of students to visualize the results of rotating and developing surfaces and on the effect that training has on the students' performances.

## CHAPTER II

THEORETICAL FRAMEWORK, REVIEW OF RELA TED<br>RESEARCH, AND HYPOTHESES

## Theoretical Framework

In The Child's Conception of Space Piaget and Inhelder (1967) have investigated the order in which children develop in their ability to deal with spatial concepts. This ability was found to evolve through the development of three spaces:

Topological- which involves the relations of proximity, separation, order, surrounding, and continuity.

Projective- which involves the shapes of figures, their relative positions, and apparent distances, in relation to a specific point of view.

Euclidean- which involves the construction of a frame of reference or a coordinate system and the conservation of size, distance, and angles.

Topological concepts emerge first, with most topological relations becoming integrated into a stable operational system about the age of seven. From these evolve the concepts of projective and euclidean space, which
develop concurrently and are mutually interdependent. Finally, at about the age of twelve, the concepts of all three spaces are coordinated into a fully developed operational system.

The rotation and development of surfaces is important since it involves the coordination of both projective and euclidean operations. Not only must the person be able to coordinate the different viewpoints of the object, but he must aiso be able to internalize the movements involved in rotating or developing the surfaces. This requires the construction of a coordinate system in order to preserve the relations or positions between the sides of the solid. Thus, success on such tasks is evidence of the integration of the three spaces into an operational system.

The ages at which these operations develop is reflected in Piaget and Inhelder's statements that "the age of 9 or thereabouts . . . marks a decisive turning point in the development of spatial concepts; that of the completion of the framework appropriate to comprehensive euclidean and projective systems." (p. 418) Furthermore, at Substage IIIB (approximately 9 to 11 years) "there appears a third type of image, one capable of anticipating the results of actions before they are carried out," (p. 296) so that "In Substage IIIB the correct solutions are arrived at, at least for the cylinder and cone. The cube and pyramid appear to offer rather more difficulty and
a completely correct development of the latter is sometimes not achieved until Stage IV." (p. 277)

While Piaget and Inhelder state that "the technique of rotating and unfolding surfaces is acquired in the course of a spontaneous and regular process of development, ${ }^{11}$ (p. 273) they also say that "imagining the rotation and development of surfaces depends largely on the actual process of unfolding solids, and the motor skills involved in such actions." (p. 276) This would seem to imply that such practice should increase the ability to visualize the results of such operations.

## Review of Related Research

There have been many criticisms made of Piaget's work, mostly directed to his lack of detailed information about the number and the mental abilities of his subjects. He has no statistical evidence to support his conclusions. His lack of controls causes some to feel that his informal questioning may have influenced his subjects. His emphasis on age is questioned due to the overlapping ages of subjects who are judged to be at different stages in their development. Even he seems to admit that there may be factors other than age when he says, in discussing the cube, that "one sometimes finds exceptional children able to give correct answers at the start of Substage IIIA as a result of possessing special aptitudes or having had experience in folding or making things at school." (p. 292)

The following review of literature presents some of the results that have been found regarding Piaget's theory in general and factors which affect the development of spatial abilities. It is divided into the following categories: research concerned with Piaget's evolution of spaces and developmental stages, research concerned with surface development and cross sections, and research concerned with the effects of age, ability, sex, and training.

## Piaget's Theory

Due to the problems found in Piaget's research discussed earlier, many studies have replicated his experiments in an attempt to determine whether there is statistical evidence for his conclusions. However, most of these studies involve tasks that are of concern in the earlier stages of development.

In surveying the research available in children's thinking, Wallach (1963) found evidence that linguistic factors and socio-economic factors may cause shifts in Piaget's age norms, but the same general developmental sequences outlined by Piaget have been obtained. He also noted that structural changes in thinking do occur between five and eight, when children develop the ability to understand concepts of conservation or constancy, and between twelve and fourteen, when they become capable of problem solving.

Several studies have found support for the devel-
opment of topological properties first, followed by projective and euclidean ones.

Peel (1959) reported on two such studies. E. I. Page, working with sixty children from three to eight years of age, replicated Piaget's experiment of having subjects draw or pick out an object from touching it, without being able to see it. Topological shapes were recognized more easily than euclidean ones by the younger children, but certain euclidean features were differentiated as early as some topological ones. E. Ferns replicated Piaget's experiment of having subjects copy geometric figures, with fifty-five children from three to eight years of age. She verified the sequence of stages. However, Piaget's age placements were not so clearly supported, since some of her subjects reached Piaget's stages at younger ages.

Rivoire (1961) developed a twenty-eight item test, measuring topological, projective, affine, and euclidean space which she administered to 144 middle-class subjects from four to fifteen years of age. Her results did not entirely support Piaget's stages, although age was a factor. She found that concepts of topological space were not developed until about six years of age, younger children (approximately four years old) did not use only topological relationships, projective concepts were developed earlier and euclidean concepts later than Piaget reported, and spatial development was not complete by fifteen.

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Significant differences were also found between items within each type of space. Some of the younger subjects correctly answered items about projective and euclidean space, while some older subjects missed topological items, indicating that topological space is not completely developed before some concepts of other spaces begin to develop.

Lovell (1959) supervised the replication of six of Piaget's experiments with 150 children, aged three to six years, and of varied socio-economic background and ability. In picking out shapes from their feel and copying drawings of geometric figures, topological properties tended to be used more than euclidean, except for those figures with curved edges. Figures with long straight sides and angles were the most difficult. Lovell concluded that specific features such as holes and corners may be of more influence than topological or euclidean features in general.

To test the influence of familiarity, Cousins and Abravanel (1971) had their fifty-six subjects, aged three to five years, pick the one of two comparison figures that was most like the given standard figure, with one choice having similar topological features (such as openness or hollowness) and the other euclidean (such as curvilinearity or rectilinearity). For the series in which the boundary was a familiar figure (such as a square or a circle) the majority of matches were based on the euclidean features for all ages. For those with an unfamiliar or free
boundary, there was a trend from topological to euclidean bases of matching with age. Thus, there was found to be some agreement with Piaget that topological properties are developed first, but there is an overlap between the use of topological and projective-euclidean concepts depending on the particular properties of the objects and on the subject's familiarity with the figure.

In the study cited above, Lovell also replicated Piaget's experiments on linear and circular order, knots, the projective straight Ine, and perspectives. The results tended to support Piaget's stages of development, but much more variance was found within the age groups and many of the subjects were able to perform the tasks at earlier ages than were Piaget's.

Dodwell (1963) also replicated several of Piaget's experiments, construction of a straight line, drawing shapes, points and continuity, horizontal and vertical, geometrical sections, similarity and proportion, and coordination of perspectives, using 194 children ranging in age from five to eleven years and with IQs from 80 to 136. Generally, Piaget's pattern of development was verified, but not in all respects. Categorization was harder due to a greater variety of responses. Subjects were found to be in different stages for different tasks and so could not be assigned to one particular developmental stage. While overall ability did increase with age, such growth was not
well-defined and could have been affected by other factors such as special interests or training.

Fishbein, Lewis, and Keiffer (1972) modified the experiment on perspectives to include possibilities other than development by age alone, considering the effect of three objects rather than just one, using eight photographs compared to four, and pointing to a photograph compared to turning the display. Using 120 middle-class subjects from three to nine years of age, they found that those as young as three and one half years old could succeed on certain of the tasks. Although performance generally increased with age, it was affected by social factors (egocentrism, nonegocentrism, empathy), cognitive factors (ability to see internal relationships between the objectsj, the complexity of the situation, and the method of responding. As a result, they felt that one should be extremely cautious when applying ages to stages of development.

Surface Development and Plane Sections
Guay and McDaniel (1977) compared the mathematics achievement of ninety students in grades two through seven With their responses on four spatial tests, including a multiple-choice surface development test. The high mathematics achievers scored significantly higher than the low achievers on each of the four tests. Grade level was found to be significant for all four tests, with test scores increasing with increases in grade levels. The males also

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scored significantly higher than the females on the surface development test and the coordination of viewpoints test.

The sectioning task is important since it also involves the coordination of both projective and euclidean operations.

One of the experiments in Dodwell's replications was that of sectioning solids. Of his 194 subjects, eighty two were incapable of identifying the cross sections, one was only partially correct, and twelve were able to perform correctly, while the other ninety-nine Were able to identify some of the cuts but not all. While there was a high correlation with age, there was an even greater correlation with mental age.

Boe (1966) individually tested seventy-two middleclass elghth, tenth, and twelfth grade students (thirteon to seventeen years of age) of varying ability on sixteen different cross sections tasks, using two tests. The solids used were a right rectangular prism, a right circular cylinder, a cube, and one nappe of a right circular cone. For each solid, the subject was to determine the boundary of the surface formed when the solid was sectioned by a longitudinal, a transverse, an oblique, and a parallel cut. On the first test the subjects were to draw their responses, while the second was a multiple-choice test. Only seven of her subjects had perfect scores on test one, three on test two, and no one on both tests, causing her to conclude that
students have not developed such abilities by the age of seventeen. Age (grade) was not found to be significant on either test, while ability level, type of solid, and type of cut were significant on both tests. Scores increased with increases in ability. The right circular cone and the oblique cut on the cube were the most difficult. Sex was also significant on the drawing test, with higher scores for the males.

Palow (1969), testing 1067 students in grades three through twelve (eight to nineteen years of age), of varying IQ and socio-economic backgrounds, found age, sex, and IQ to be significant. He supported Piaget's position that euclidean abilities are acquired about the age of twelve. He also found boys to perform better than girls and higher ability students better than average ability students.

Davis (1969) questionned Boe's choice of grades and her general conclusion that twelve year olds had not mastered geometric sections since they did not have perfect scores; so he replicated her experiment with modifications, using ninety students in grades six, eight, and ten, using only the multiple choice test, and adding a twenty-five minute work period before testing to be certain that the subjects understood the task. He found a significant difference between the sixth and eighth graders and between the sixth and tenth graders, but not between the eighth and tenth graders, with $50 \%$ of the eighth and tenth graders
correct on at least $87 \frac{1}{2} \%$ of the items. Thus, he agreed With Piaget that mastery of sectioning is achieved by the age of thirteen (eighth grade). He also found sex to be significant in favor of the boys. The scores of both the high and middle ability students were significantly higher than those of the low ability students.

Singletary (1972) investigated the effect of instruction on the sectioning task and its relationship to ability on sixty-three eighth grade students, ranging in age from thirteen to fifteen years, with IQs from 81 to 143 but in midale and low level ability classes. Two instructional groups were used. Both constructed and sectioned clay models and manipulated plastic and string models. One also had instruction in perspective drawing. The instruction lasted for nine days. He used the prism, cylinder, and cone, but replaced the cube with an ellipsoid and a square pyramid. The same four cuts were used that Boe had used. Ability was found to be significant for the drawing test, but the treatment was not significant for either test, except that those with instruction in perspective drawing performed better on the ellipsoid and pyramid than those in the other instructional group.

## Age

Age is important since it is the main variable which Piaget and Inhelder used and upon which they based their developmental stages. In most of the studies cited
above, age was a factor. Wallach found structural changes in thinking occurring between five and eight jears and again between twelve and fourteen years. In summarizing results of research, Fruchter (1954) concluded that spatial functions mature between the ages of eleven and fifteen. In the studies of Palow and Davis, which included students below twelve and above fourteen, age was found to be a significant variable. Dodwell also found age to be significant even though all his subjects were less than twelve years of age. Boe dia not find significance, but her subjects were all thirteen and older and even Davis did not find a difference between eighth and tenth graders. These studies seem to verify that a change does take place about twelve or thirteen which at least affects the responses on sectioning tasks.

## Ability

As was shown earlier, Piaget found that some children seem to have special aptitudes which enable them to perform spatial operations at earlier ages than others. It is possible that the problems Lovell and Dodwell found in trying to categorize students by ages were caused by differences in ability. In all the studies of plane sections, ability was found to be a significant variable. Davis, in analyzing this difference further, found it to be due to the difference between the lower ability level and the middle and upper levels. Guay and McDaniel found a
relationship between spatial ability and mathematical ability. Thus, ability seems to affect responses to spatial tasks.

## Sex

General studies of spatial abilities have often found sex to be a factor. According to Fruchter, boys have been found, with some consistencey, to excel girls on spatial tests. In a factor analysis of tests of mathematical ability given to 200 students from thirteen and one-half to fifteen years of age Blackwell (1940) found that, while the factor involving manipulation of spatial and verbal data was second in importance for both sexes, it plays a relatively larger part in the mathematical ability of boys than of girls. Hobson (1947) analyzed the results of the Chicago Tests of Primary Mental Abilities, given to 1097 ninth grade and 1436 eighth grade students over a period of two years. Although the IQs of the girls were found to be significantly higher, in Spatial Orientation the boys were significantly better. He also found a significant difference in class means between the eighth and ninth graders. Similar results with regard to $I Q$, the space factor, and sex were found by Herzberg and Lepkin (1954) in comparing 1049 sixteen to eighteen year olds on the Primary Mental Abilities Test, but differences were not found with respect to age. Thus, these studies not only verify the importance of sex, but they also support the
position that a change occurs at about the age of thirteen and that spatial ability is stable after that.

Many of the studies of Piagetian tasks have not considered sex, but in those by Guay and McDaniel, Boe, Palow, and Davis that did include sex as a variable, some relationships were found, favoring boys over girls.

Responses to the method of presentation may also differ between males and females. Moxness (1974) found that eighth grade females taught by a visually-rich method scored significantly higher on a test on probability than those taught by a verbally-rich method.

## Training

In early studies an attempt was made to determine if particular courses had an effect on spatial abilities.

Ranucci (1952) studied the effect of a course in solid geometry on high school seniors who had previously taken two years of algebra and one year of plane geometry. From comparing those who took the solid geometry course with those who did not, he concluded that the study of solid geometry did not improve space perception abilities.

Mendicino (1958) compared 150 tenth grade boys in a vocational curriculum who were taking a machine shop and mechanical drawing course with 150 who were in non-vocational curricula. From results on the Space Relations Test, he concluded that such experience had no more effect than the non-vocational curricula and that the increases in
test scores of both groups were due to general growth or development. However, the experiences in the course did seem to enable those with such a capacity at the beginning to use this capacity more effectively. Myers (1958) also found no significant differences on spatial relations tests between those entering college freshmen who had taken mechenical drawing and those who had not.

The effects of taking descriptive geometry have been somewhat more positive. According to Sedgwick (1962), Rugg had found that it increased students' ability in solving manipulation problems of a geometrical nature, but Sedgwick did not find it to affect spatial perception and concluded that improvement was a result of maturation and the general environment. A study that did find a difference was that by Blade and Watson (1955). They found significant improvement on the Spatial Relations Test for college freshmen who took a year course in engineering compared to non-engineering siudents. Myers (1953) also found significant gains on the Spatial Relations Test for 591 cadets at the U. S. Military Academy who had a year of training including descriptive geometry and engineering.

Since existing courses did not generally seem to be effective, attention was focused on methods and programs specifically designed to promote general spatial abilities.

Cohen (1959) had students in solid geometry construct models throughout the course, but found this did not
result in significant differences on the Space Relations Test.

Cleminson (1970) compared the responses of fifth grade students in three types of programs (problem solving method, process approach, and multitextbook approach) to several Fiagetian tasks, including one (coordination of perspectives) which was spatial. The method was not significant for this task, although he did find two results that were consistent with studies cited earlier. Boys were significantly better at this task than girls and the correlation between age and scores was low, indicating that Piaget's stages must not be associated only with age.

Brinkmann (1966) developed a three week programmed course in elementary geometry for use in the eighth grade, using a problem solving approach with drawings or diagrams and solids to manipulate. Gains for the experimental group on the Space Relations Test were significant, especially for the middle ability level. He concluded that "it appears reasonable to assume that the functional skill of individuals in spatial visualization can be improved when appropriate training is provided." (p. 184) That the difference in test scores between the sexes was not significant suggested to him that "girls can at least hold their own when provided with the opportunity to learn something about a particular area in which they are of ten assumed to possess less ability." (p. 184)

Wolfe (1970) also developed a program for training in spatial visualization, involving video tapes and student activities for seventh, eighth, and ninth grades. However, the only significant difference found was that between the elghth grade class that received training and the one that did not receive training.

Although some of these attempts have not proved to be effective, the last two studies indicate that it may be possible to have an effect on spatial abilities, at least at the eighth grade level, when appropriate training is provided. This view was supported by Peel in summarizing her experiment when she concluded that "it would seem that the more experience we can provide of materials . . at the appropriate level, the better." (p. 59) When subjects have some knowledge of a concept, they seem to be able to profit from such practice or training. Thus, the reason that Davis found a difference in the effect of practice between the sixth and eighth grades may have been that at the eighth grade the students were developing this ability and the practice period allowed them to solidify their understanding of the concepts involved.

## Hypotheses

This study is directed toward testing the following hypotheses:

1. There are no significant differences in the mean scores on a test over the rotation and development of

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surfaces among sixth, eighth, and tenth grades, for students with no training.
2. There are no significant differences in the mean scores among low, average, and high ability levels, for students with no training.
3. There is no significant difference in the mean scores between the sexes, for students with no training.
4. There is no significant difference in the mean scores between the sixth grade students with training and those without such training.
5. There is no significant difference in the mean scores between the eighth grade students with training and those without such training.

## CHAPTER III

## THE EXPERIMENT

## The Population

This study was conducted in the Putnam City Independent School District, which includes the western part of Oklahoma City, Oklahoma and two towns within the city limits of Oklahoma City. The district has fourteen elementary schools, four junior high schools, and two senior high schools. Two elementary schools, one junior high school, and one senfor high school were chosen for the study. These schools were selected because their students represent a wide range of socio-economic conditions. While predominantly middle-class, the families vary from unemployed to professional, with educational backgrounds from elementary school educations to advanced degrees.

The classes available in these schools were seven sixth grade classes, eight regular eighth grade classes, one eighth and ninth grade algebra class, and three tenth grade classes. The tenth grade classes were a first year algebra class, a plane geometry class, and an honors geometry class.

The academic abilities of these students were measured by the Short Form Test of Academic Achievement (SFTAA) which had been given by the school system. Those students for whom the schools did not have test results were eliminated from consideration. This left 122 sixth graders, 197 eighth graders, and 51 tenth graders. Their SFTAA scores ranged from 73 to 146 , with a mean of 105.1 .

The Criterion Test
In order to evaluate the subjects' abilities to develop and rotate surfaces of solids, a test was developed by the experimenter, based upon the procedures of Piaget and Inhelder. In their investigations, Piaget and Inhelder required their subjects to draw or to pick out the result of developing the cone and cylinder and of rotating the sides of a cube and a tetrahedron.

The criterion test consisted of three parts. For the first two parts, the four solids of Piaget and Inhelder (the cons, the cylinder, the cube, and the tetrahedron) were used so that a comparison could be made with their subjects. Eight other solids were also selected for the test and two solids to be used as examples. Models of these solids were constructed from yellow pasteboard. The solids used are given in Appendix A.

The first two parts of the test were identical except for the method of responding. In Part One, the subjects were required to draw the figure that would result
when the solid was developed or the sides rotated. Since the test was given to small groups of subjects, the models were held up by the experimenter and slowly turned around so that the subjects could see all the parts of the solid. Two examples were presented to be certain that the directions were understood. Each example solid was shown to the subjects, the subjects were asked to draw the result, and then the solid was cut open to show the correct result. The test solids were then presented, one at a time. Booklets of blank pages were given to the subjects, with one figure to be drawn on each page.

In Part Two, the same solids were presented in the same order as in Part One. For each solid, four possible figures and a choice of "none of the above" were given and the subjects were tc select the correct answer.

The third part of the test was designed to evaluate the ability to envision the result of the reverse operation. It contained fifteen figures, each of which was a figure consisting of six connected squares. For the first ten, the subject was to decide whether the figure could be folded to form a cube. The other five could be folded to form cubes and the subject was to decide which face would be opposite a particular face when the figure was folded. Parts Two and Three of the test are given in Appendices B and $C$.

For all groups, Part One was given first, followed
by Parts Two and Three, so that the given figures would not affect the drawings.

The drawing part of the test was given to a sixth grade class in Moore, Oklahoma to determine the types of errors that would be made: These were used in developing the choices for the multiple choice part of the test. The pilot study also gave the experimenter experience in administering the test, showed where changes were needed in the directions for the test, and provided estimates of the time needed for each item.

The Training Procedure
A two part training procedure was developed to proVide the active participation in folding solids which Piaget asserted is needed to be able to envision such actions.

For the first part, nine solids, different from those on the test, were chosen. Some of these solids were to be unfolded in different ways, resulting in fourteen solids being used. Models of the solids were made of pasteboard with tape applied on the edges to be cut. These edges were also marked with black lines so that the subjects could see clearly how the solid was to be opened. For each solid, a model was given to each subject. He was asked to inspect it carefully, noticing the number and shape of its faces, and then to draw the shape he thought would result when it was unfolded. He was then asked to
compare the solid to his drawing and to consider whether he had included all the faces and whether they were the correct shapes. The experimenter then cut the tape for the students and they were given time to check the results and manipulate the sides themselves. After each solid was finished, an incorrect drawing was shown and there was a discussion of why it was not possible to cut the solid to form the incorrect figure. The solids and drawings used are given in Appendices $D$ and $E$.

For the second part of the training procedure, ten figures, each consisting of six connected, numbered squares were chosen. These were also different from those on the test. For each figure, a copy was given to each subject. He was asked to imagine himself folding it and to determine if it would fold to form a cube. He was also asked to decide what numbers would be on the faces that would be opposite each other when the figure was folded. He was then given time to try to fold it and to investigate the results. The movement of the faces and the results were then discussed. The figures used are given in Appendix F.

## Selection of the Sample

The population was blocked by age (as measured by grade), sex, and ability (as measured by SFTAA scores). In order to differentiate between ability levels, an interval of sixteen (one standard deviation of the national distribution) was chosen for the average ability group. Since

Table 1
Elementary Statistics of the Subjects ' IQs by Treatment, Grade, Sex, and Ability

| Grade | Sex | Abil. | Treatment |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | $M$$F$ | $\begin{aligned} & \text { H } \\ & \text { A } \\ & \text { L } \\ & \text { H } \\ & \text { A } \\ & \text { L } \end{aligned}$ | Without training |  |  | With training |  |  |
|  |  |  | Mean | SD | No. of subj. | Mean | SD | No. of subj. |
|  |  |  | 116.7 103.5 86.3 | 4.2 3.8 7.2 | 6 6 6 | $\begin{array}{r} 120.2 \\ 103.3 \\ 87.7 \end{array}$ | 6.1 3.9 4.8 | $\begin{aligned} & 6 \\ & 6 \\ & 6 \end{aligned}$ |
|  |  |  | 125.8 100.8 86.5 | 10.7 5.0 6.7 | 6 6 | $\begin{array}{r} 119.8 \\ 101.8 \\ 88.3 \end{array}$ | 6.7 3.1 5.6 | 6 6 6 |
|  | M | H A L | 116.7 104.5 90.0 | 3.9 5.0 2.3 | 6 6 6 | $\begin{array}{r} 119.5 \\ 101.7 \\ 88.7 \end{array}$ | 6.6 4.2 3.7 | $\begin{aligned} & 6 \\ & 6 \\ & 6 \end{aligned}$ |
|  | F | $\begin{aligned} & \text { H } \\ & \text { A } \\ & \text { L } \end{aligned}$ | $\begin{array}{r} 121.3 \\ 102.7 \\ 89.7 \end{array}$ | 7.6 3.5 4.2 | 6 6 6 | $\begin{array}{r} 114.7 \\ 102.7 \\ 91.8 \end{array}$ | 3.3 2.5 2.0 | 3 6 6 |
| 10 | M F | H A L H A L | $\begin{array}{r} 115.7 \\ 105.5 \\ 87.5 \\ 122.3 \\ 10.8 \\ 91.7 \end{array}$ | 6.7 4.1 3.5 7.7 4.2 2.5 | $\begin{aligned} & 6 \\ & 6 \\ & 2 \\ & 6 \\ & 6 \\ & 6 \end{aligned}$ |  |  |  |

provided by the schools. All the subjects from a class receiving training met together for the training sessions and were then joined by the others from their class for testing. Thus, each group contained subjects of both sexes and of different ability levels. The groups receiving
training ranged in size from two to eight and the test was given to groups of six to thirteen subjects. The number of subjects involved from each class is given in table 2.

One week was allowed for each group of classes, With Monday through Wednesday available for training and the other two days for testing. The training procedure required approximately ninety minutes and the testing about forty-five minutes. Since the sixth grade classes were forty-five minutes in length, Monday, Tuesday, and part of Wednesday were used for the training program. Parts One and Two of the test were given on Thursday and Part Three on Friday. The eighth grade classes were longer, so that the training was completed on Tuesday and the test given on Thursday. One day was also sufficient for testing the tenth graders.

The first two weeks were spent at the elementary schools. The experimenter worked with the three classes in one school the first week and with the four classes in the other school the second week. The third and fourth weeks were spent at the junior high school, working with the subjects from the four classes of one teacher the first weok and the four classes of the second teacher and the subjects from the algebra class the second week. The tenth graders were tested on Monday of the fifth week. It was necessary to give the test to the entire tenth grade classes, with only the scores of the selected subjects

Table 2
Number of Subjects From Each Class

| Grade | Class | Number of students in the class with records | Number of subjects in group with treining | Number of subjects in group without training |
| :---: | :---: | :---: | :---: | :---: |
| 6 | 1 | 20 | 6 | 7 |
|  | 2 | 21 | 4 | 7 |
|  | 3 | 19 | 4 | 3 |
|  | 4 | 14 | 6 | 4 |
|  | 5 | 18 | 7 | 5 |
|  | 6 | 17 | 5 | 5 |
|  | 7 | 13 | 4 | 5 |
| 8 | 1 | 24 | 4 | 7 |
|  | 2 | 23 | 5 | 1 |
|  | 3 | 21 | 8 | 4 |
|  | 4 | 21 | 2 | 5 |
|  | 5 | 23 | 2 | 2 |
|  | 6 | 26 | 4 | 5 |
|  | 7 | 18 | 4 | 4 |
|  | 8 | 24 | 4 | 4 |
|  | Algebra | 17 | 0 | 4 |
| 10 | Algebra | 15 |  | 8 |
|  | Geometry | 21 |  | 16 |
|  | Honors Geometry | 15 |  | 8 |

being analyzed.
During the periods that the groups receiving trainIng were away from their regular classes no classes were studying any aspects of geometry. The sixth grade classes were working on fractions and decimals and the eighth grade classes were studying percents. The subjects were also asked not to discuss the experiment with their classmates.

While the three parts of the test were always given In the same order, the order of the items within each part of the test was randomly selected for each group being tested. The solids of Part One, the multiple choice figures for each solid of Part Two, and the figures in each section of Part Three were randomly ordered so that no two groups were given identical tests.

Parts Two and Three of the test were graded by the experimenter with the grade for each item being a one if correct and a zero if incorrect. The drawings of Part One were evaluated by the experimenter and a college mathematics instructor. Since the tests of the different groups were mixed together, the graders did not know to which group any subject belonged. The items were graded on a three point basis with a zero for little or no understanding of the correct figure, a one for some understanding, and a two for a correct drawing. When the grades differed, the drawing was discussed and a grade was agreed upon.

## Method of Analysis

The analysis of the ability of the subjects without training and of the effect of training was based on the scores on each part of the criterion test. The analysis of variance was chosen as the statistical test. In analyzing the ability of the group without training, the variables were age, ability, and sex. The effect of training at the sixth and eighth grades was investigated by comparing the group with training with the group without training, with ability and sex also being used as variables. In each part of the analysis, each part of the test was analyzed separately. Comparisons were made within levels of significant main effects using Tukey's test. Significant interactions were also investigated for differential results within levels of the variables involved, using tests of simple main effects.

The Pearson product-moment correlation coefficient was calculated to determine if there was a relationship between the responses on the drawing and multiple-choice parts of the test.

The Kuder-Richardson Formula No. 20 was applied to each part of the test to obtain a measure of its reliability.

The difficulty level of each item was also calculated.

## CHAPTER IV

PRESENTATION AND ANALYSIS OF THE DATA

The raw scores and the group means, standard deviations, and numbers of subjects for each part of the test are given in Appendices $G$ and $H$.

## Reliability

The Kuder-Richardson Formula No. 20 was applied to each part of the criterion test to obtain a measure of its reliability. The formula is $r=\frac{k}{k-1}\left[1-\frac{\sum p(l-p)}{s^{2}}\right]$, where $k=$ number of items on the test, $p=$ percent of correct responses on each item, and $s^{2}=$ variance of the test scores. (Downie, p. 246) The results for each of the parts are: Part One, $r=.73$; Part Two, $r=.55$; and Part Three, $r=$.70. The scores on Parts One and Three seem to be consistent from item to item and to be free of experimental error. The low reliability of Part Two could indicate that the use of multiple-choice items is not an adequate method for measuring the students ' understanding of the solids. This agrees with Piaget and Inhelder's statement that "this method cannot be used alone, for the children tend to pick out the right drawing much too easily, not through genuine
understanding but by guessing on the basis of certain details." (p. 274)

## Effect of Grade, Ability, and Sex

The analysis of variance was applied to the scores of the groups without training, using grade, ability, and sex as variables. Since the lack of tenth grade males was not related to the experimental procedure, an unweighted means analysis was used, using the computational procedure described by Kirk. Each part of the test was analyzed separately.

## Part One

The cell means and the summary of the analysis of variance for Part One are given in the following tables.

Table 3
Cell Means of Part One for Grade, Ability, and Sex

| Sex | Ability <br> level | 6 | 8 | 10 |
| :---: | :---: | :---: | :---: | :---: |
|  | Male | Average | 15.17 | 18.00 |
|  | Low | 12.83 | 16.67 | 17.00 |
|  | High | 15.00 | 17.50 | 20.00 |
|  | Average | 14.17 | 15.00 | 16.83 |
|  | Low | 9.33 | 14.33 | 10.67 |

Table 4
Unweighted Means Analysis of Variance of Part One for Grade, Ability, and Sex

| Source | Sum of squares | Degrees of freedom | Mean square | F |
| :---: | :---: | :---: | :---: | :---: |
| A (grade) | 250.12 | 2 | 125.06 | $6.51 \%$ |
| B (ability) | 308.87 | 2 | 154.43 | 8.05\%* |
| C (sex) | 125.01 | 1 | 125.01 | $6.52{ }^{*}$ |
| AB | 48.83 | 4 | 12.21 | . 64 |
| AC | 5.52 | 2 | 2.76 | . 14 |
| BC | 51.47 | 2 | 25.73 | 1.34 |
| ABC | 24.63 | 4 | 6.16 | . 32 |
| w. cell | 1649.83 | 86 | 19.18 |  |
| $\because$ significa | at . 01 le | * sign | cant at | level |

All three variables, grade, ability, and sex were found to be significant. The means of the levels of these variables are given in the following table.

Table 5
Means of the Levels of the Variables Grade, Ability, and Sex

| Grade |  | Ability |  | Sex |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 13.67 | High | 17.69 | Male | 17.00 |
| 8 | 16.61 | Average | 16.47 | Female | 14.76 |
| 10 | 17.41 | Low | 13.03 |  |  |

Tukey's test for making comparisons among means was applied to the variables of grade and ability to determine the sources of the effects. The differences among the means and the results of Tukey's test are given in the following tables.

Table 6
Differences in Means of the Grade Levels

|  | 8 | 10 |
| :---: | :---: | :---: |
| 6 | $2.94^{*}$ | $3.74^{* *}$ |
| 8 |  | .80 |

$\because$
significant at . 01 level
significant at .05 level

Table 7
Differences in Means of the Ability Levels

$\because$ significant at . 01 level

Thus, it was determined that the males scored significantly higher than the females, the eighth and tenth graders higher than the sixth graders, and the high and average ability subjects higher than the low ability subjects.

## Part Two

The cell means and the summary of the analysis of variance for Part two are given in the following tables.

Table 8
Cell Means of Part Two for Grade, Ability, and Sex

| Sex | Ability <br> level | Grade |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 6 | 8 | 10 |
| Female |  | 8.00 | 8.83 | 9.17 |
|  | Average | 7.17 | 8.33 | 9.67 |
|  | Low | 8.17 | 8.67 | 9.00 |
|  | High | 9.33 | 9.33 | 9.67 |
|  | Average | 6.50 | 7.67 | 8.83 |
|  | Low | 5.17 | 7.17 | 8.33 |

Table 9
Unweighted Means Analysis of Variance of Part Two for Grade, Ability, and Sex

| Source | Sum of <br> squares | Degrees of <br> freedom | Mean <br> square | $F$ |
| :--- | :---: | :---: | :---: | :---: |
| A (grade) | 48.20 | 2 | 24.10 | $6.44^{* *}$ |
| B (ability) | 30.65 | 2 | 15.32 | $4.09^{*}$ |
| C (sex) | 7.50 | 1 | 7.50 | 2.00 |
| AB | 8.45 | 4 | 2.11 | .56 |
| AC | .80 | 2 | .40 | .11 |
| BC | 25.65 | 2 | 12.82 | $3.42^{*}$ |
| ABC | 8.05 | 4 | 2.01 | .54 |
| W. cell | 322.00 | 86 | 3.74 |  |
| $\cdots$ significant at .01 level | $*$ significant at .05 level |  |  |  |

Grade, ability, and the interaction of ability and sex were found to be significant. The means of the levels of the three variables are given in the following table.

Table 10

## Means of the Levels of the Variables Grade, Ability, and Sex

| Grade |  | Ability |  | Sex |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 7.39 | High | 9.06 | Male | 8.52 |
| 8 | 8.33 | Average | 8.03 | Female | 8.00 |
| 10 | 9.12 | Low | 7.59 |  |  |

The differences among the means of the levels of the significant variables, grade and ability, and the results of applying Tukey's test to these variables are given in the following tables.

Table 11
Differences in Means
of the Grade Levels

|  | 8 | 10 |
| :---: | :---: | :---: |
| 6 | .94 | $1.74^{\text {米 }}$ |
| 8 |  | .79 |

示
significant at . 01 level

Table 12
Differences in Means of the Ability Levels


* significant at . 01 level

Thus, the tenth grade subjects scored significantly higher than the sixth grade subjects, and the high ability subjects higher than the low ability subjects. However,

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the effect of ability must be qualified, due to the interaction between ability and sex. To examine this relationship, a test of simple main-effects was applied, using the procedure described by Kirk. The means of the levels of ability and sex and the sumary of the analysis of variance are given in the following tables.

Table 13
Means of the Levels of the Variables Ability and Sex

|  | Male | Female |
| :---: | :---: | :---: |
| High | 8.67 | 9.44 |
| Average | 8.39 | 7.67 |
| Low | 8.50 | 6.89 |

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Table 14
Analysis of Variance for Ability and Sex

| Source | Sum of squares | Degrees of freedom | Mean square | F |
| :---: | :---: | :---: | :---: | :---: |
| B (ability) | 30.65 | 2 | 15.32 | 4.09* |
| B at $c_{1}$ (M) | . 70 | 2 | . 35 | . 09 |
| B at $c_{2}$ (F) | 55.60 | 2 | 27.80 | $7.422^{* *}$ |
| $c$ (sex) | 7.50 | 1 | 7.50 | 2.00 |
| $C$ at $b_{1}$ (H) | 4.90 | 1 | 4.90 | 1.31 |
| $C$ at $b_{2}(A)$ | 4.23 | 1 | 4.23 | 1.13 |
| $C$ at $\mathrm{b}_{3}$ (L) | 24.03 | 1 | 24.03 | 6.42* |
| BC | 25.65 | 2 | 12.82 | 3.42* |
| w. cell | 322.00 | 86 | 3.74 |  |
| *\% significant at . 01 level ${ }^{*}$ significant at . 05 level |  |  |  |  |
| Thus, ability was found to be significant for the |  |  |  |  |
| females and sex for the low ability subjects. |  |  |  |  |

Part Three
The cell means and the summary of the analysis of variance for Part Three are given in the following tables.

Table 15
Cell Means of Part Three for Grade, Ability, and Sex

| Sex | $\begin{aligned} & \text { Ability } \\ & \text { level } \end{aligned}$ | Grade |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 6 | 8 | 10 |
| Malo | High | 11.00 | 13.33 | 13.33 |
|  | Average | 8.83 | 11.83 | 13.50 |
|  | Low | 11.33 | 12.50 | 13.00 |
| Female | High | 11.50 | 13.83 | 13.50 |
|  | Average | 11.50 | 10.17 | 13.67 |
|  | Low | 7.83 | 12.83 | 11.67 |

Table 16
Unweighted Means Analysis of Variance of Part Three for Grade, Ability, and Sex

| Source | Sum of <br> squares | Degrees of <br> freedom | Mean <br> square | $F$ |
| :--- | ---: | :---: | ---: | :---: |
| A (grade) | 135.43 | 2 | 67.72 | 13.57 |
| B (ability) | 30.86 | 2 | 15.43 | 3.09 |
| C (sex) | 1.40 | 1 | 1.40 | .28 |
| AB | 31.59 | 4 | 7.90 | 1.58 |
| AC | .22 | 2 | .21 | .02 |
| BC | 19.28 | 2 | 9.64 | 1.93 |
| ABC | 48.17 | 4 | 12.04 | 2.41 |
| W. cell | 429.17 | 86 | 4.99 |  |
| wignificant at .01 level |  |  |  |  |

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The only significant variable found was grade. The means of the levels of the three variables are given in the following table.

Table 17

## Means of the Levels of the Variables Grade, Ability, and Sex

| Grade |  | Ability |  | Sex |  |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 10.33 | High | 12.75 | Male | 12.00 |
| 8 | 12.42 | Average | 11.58 | Female | 11.83 |
| 10 | 13.13 | Low | 11.34 |  |  |

The differences among the means of the levels of the significant variable, grade, and the results of applying Tukey's test to this variable are given in the following table.

Table 18
Differences in Means of the Grade Levels


Thus, the eighth and tenth graders scored significantly higher than the sixth graders.

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## Summary

The grade level was found to be significant.for all three parts of the test. For Parts One and Three, both the eighth and tenth graders scored significantly higher than the sixth graders, while on Part Two only the difference between the sixth graders and the tenth graders was significant. Ability level was significant on Parts One and Two. On Part One, both the high and average ability subjects scored significantly higher than the low ability subjects. On Part Two, the high ability subjects scored significantly higher than the low ability subjects, with this difference being due mostly to the significant differences within the females. Sex was also significant on Part One, with males scoring higher than females. On Part Two, the low ability males also scored significantly higher than the low ability females.

While there was improvement with increases in age on Part One, the largest increases were found for the high ability females both between the sixth and eighth grades and between the eighth and tenth grades and for the males of each ability level between the sixth and eighth grades. The scores of the low ability females also increased considerably between the sixth and eighth grades, but then decreased between the eighth and tenth grades. Scores also increased with increases in ability for each combination of sex and age. For the sixth grade females and for the tenth
grade males, the high and average ability subjects scored much higher than the corresponding low ability subjects, while for the tenth grade females there were large increases both between the low and average ability levels and between the average and high ability levels. The scores of the males were also the same or higher than those of the females for each cell, with the largest differences for the low ability sixth graders and the average and low ability eighth and tenth graders.

On Part Two, the scores also improved with increasing age, particularly for the average ability males and for the average and low ability females. All three of these groups improved both between the sixth and eighth grades and between the eighth and tenth grades. The largest increases with increases in ability within each combination of sex and age were found for the sixth grade females, with increases between both the low and average ability levels and between the averace and high ability levels. The high ability eighth grade females scored higher than the average and low ability eighth grade females. The average and low ability males at each grade had higher scores than the corresponding females, with the largest differences for the low ability sixth and eighth graders. However, for high ability subjects, the females were higher, especially in the sixth grade.

On Part Three, the largest increases by age were
found for the average ability males. The high ability males and females also had large increases in their scores between the sixth and eighth grades. The scores of the average ability females decreased between the sixth and eighth grades and then increased between the eighth and tenth grades, while those of the low ability females increased greatly between the sixth and eighth grades and then decreased some between the eighth and tenth grades. For the sixth and tenth grade females, both the high and average ability subjects scored higher than the corresponding low ability subjects. However, for the eighth grade females and the sixth grade males, the high and low ability subjects scored higher than the corresponding subjects of average ability. No general pattern was found between the sexes. The low ability sixth grade and the average ability eighth grade males scored much higher than the corresponding females. However, for the average ability sixth graders, the females scored much higher.

Thus, all groups generally had increases in scores with increases in age. However, the age intervals at which these increases occurred varied. The average ability males increased their scores both between the sixth and eighth grades and between the eighth and tenth grades. The scores of the high ability subjects and the low ability females increased more between the sixth and eighth grades, while those of the average ability females increased more between

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the eighth and tenth grades. While there was a general increase with increasing ability, this was more often found among the females. For the sixth grade males, the low ability subjects scored the highest and for the tenth grade males, the average ability subjects scored the highest on Parts Two and Three. For the sixth grade females, the high and average ability subjects had higher scores than the low ability subjects. The tenth grade females of high and average ability also tended to score higher than those of low ability, while for the eighth grade females, the high ability subjects generally scored higher than the average and low ability subjects. Males generally had higher scores than the corresponding females, especially for the average ability eighth graders and for the low ability subjects at each grade level. However, for high ability subjects, the females tended to have higher scores.

## Effect of Training at the Sixth Grade

To determine the effect of the training procedure for the sixth grade, the scores of the group with training and the group without training were analyzed using the analysis of variance, with ability and sex also used as variables. Each part of the test was analyzed separately.

Part One
The cell means and the summary of the analysis of variance for Part One are given in the following tables.

Table 19
Cell Means of Part One for Trestment, Ability, and Sex.

| Sex | Ability level | Treatment |  |
| :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { With } \\ & \text { training } \end{aligned}$ | Without training |
| Male | High | 19.00 | 15.50 |
|  | Average | 16.83 | 15.17 |
|  | Low | 15.67 | 12.83 |
| Female | High | 17.67 | 15.00 |
|  | Average | 15.83 | 14.17 |
|  | Low | 12.17 | 9.33 |
| Total |  | 16.19 | 13.67 |

Table 20
Analysis of Variance of Part One
for Treatment, Ability, and Sex

| Source | Sum of squares | Degrees of freedom | Mean square | F |
| :---: | :---: | :---: | :---: | :---: |
| A (treatment) | 115.01 | 1 | 115.01 | $6.34 *$ |
| B (ability) | 232.69 | 2 | 116.35 | $6.41{ }^{*}$ |
| $C$ (sex) | 58.68 | 1 | 58.68 | 3.23 |
| AB | 6.86 | 2 | 3.43 | . 19 |
| AC | . 35 | 1 | . 35 | . 02 |
| BC | 25.86 | 2 | 12.93 | . 71 |
| ABC | . 69 | 2 | . 35 | . 02 |
| w. cell | 1088.50 | 60 | 18.14 |  |

Thus, the training procedure was found to result in significantly higher scores.

Part Two
The cell means and the sumnary of the analysis of variance for Part Two are given in the following tables.

Table 21
Cell Means of Part Two for Treatment, Ability, and Sex

| Sex | Ability level | Treatment |  |
| :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { With } \\ & \text { training } \end{aligned}$ | Without training |
| Male | High | 10.00 | 8.00 |
|  | Average | 7.00 | 7.17 |
|  | Low | 8.83 | 8.17 |
| Female | High | 9.00 | 9.33 |
|  | Average | 7.83 | 6.50 |
|  | Low | 7.33 | 5.17 |
| Total |  | 8.33 | 7.39 |

Table 22
Analysis of Variance of Part Two for Treatment, Ability, and Sex

| Source | Sum of squares | Degrees of freedom | Mean square | F |
| :---: | :---: | :---: | :---: | :---: |
| A (treatment) | 16.06 | 1 | 16.06 | 3.64 |
| B (ability) | 54.53 | 2 | 27.26 | $6.19{ }^{\text {\% }}$ |
| $C$ (sex) | 8.00 | 1 | 8.00 | 1.82 |
| $A B$ | 2.19 | 2 | 1.10 | . 25 |
| AC | . 22 | 1 | . 22 | . 05 |
| BC | 22.58 | 2 | 11.29 | 2.56 |
| ABC | 14.69 | 2 | 7.35 | 1.67 |
| W. cell | 264.33 | 60 | 4.41 |  |

Although the group with training averaged a higher score than the group without training, the difference was not significant.

## Part Three

The cell means and the summary of the analysis of variance for Part Three are given in the following tables.

Table 23
Cell Means of Part Three for Treatment, Ability, and Sex

| Sex | Ability level | Treatment |  |
| :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { With } \\ & \text { training } \end{aligned}$ | Without training |
| Male | High | 13.00 | 11.00 |
|  | Average | 12.83 | 8.83 |
|  | Low | 10.67 | 11.33 |
| Female | High | 11.83 | 11.50 |
|  | Average | 13.17 | 11.50 |
|  | Low | 10.33 | 7.83 |
| Total |  | 11.97 | 10.33 |

Table 24
Analysis of Variance of Part Three for Treatment, Ability, and Sex

| Source | Sum of <br> squares | Degrees of <br> freedom | Mean <br> square | $F$ |
| :--- | :---: | :---: | ---: | :---: |
| A (treatment) | 48.35 | 1 | 48.35 | $7.94^{* *}$ |
| B (ability) | 45.19 | 2 | 22.60 | $3.71^{*}$ |
| C (sex) | 1.12 | 1 | 1.12 | .18 |
| AB | 13.03 | 2 | 6.51 | 1.07 |
| AC | .35 | 1 | .35 | .06 |
| BC | 35.08 | 2 | 17.54 | 2.88 |
| ABC | 27.03 | 2 | 13.51 | 2.22 |
| w. cell | 365.17 | 60 | 6.09 |  |
| $* *$ significent at .01 level | $*$ significant at .05 level |  |  |  |

The training procedure was found to result in significantly higher scores.

## Summary

For the sixth grade, the training procedure resulted in higher scores for all parts of the test, with the increases being significant on Parts One and Three. The scores for the groups receiving training were higher for almost all cells than for the corresponding groups without training, with the largest increases found for the high ability males and the low ability females on all three parts, the average ability females on Parts Two and Three, the low ability males and the high ability females on Part One, and the average ability males on Part Three.

It should also be noted that the means for the sixth graders with training increased enough that the mean on Part Two was equal to that of the eighth graders without training and the means on Parts One and Three were almost as high as those of the eighth graders.

## Effect of Training at the Eighth Grade

To determine the effect of the training procedure for the eighth grade, the scores of the group with training and that without training were analyzed with ability and sex also used as variables. Because of the smaller number of high ability females, an unweighted means analysis was

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used. Each part of the test was analyzed separately.

## Part One

The cell means and the sumnary of the analysis of variance for Part One are given in the following tables.

Table 25
Cell Means of Part One for Treatment, Ability, and Sex

| Sex | Ability <br> level | With <br> training | Without <br> training |
| :--- | :---: | :---: | :---: |
|  | Algh | 19.50 | 18.17 |
|  | Average | 17.00 | 18.00 |
|  | Low | 16.17 | 16.67 |

Table 26
Unweighted Means Analysis of Variance of Part One for Treatment, Ability, and Sex

| Source | Sum of <br> squares | Degrees of <br> freedom | Mean <br> square | $F$ |
| :--- | ---: | ---: | ---: | ---: |
| A (treatment) | 37.38 | 1 | 37.38 | $5.09^{*}$ |
| B (ability) | 62.49 | 2 | 31.24 | $4.26^{*}$ |
| C (sex) | 3.28 | 1 | 3.28 | .45 |
| AB | .54 | 2 | .27 | .04 |
| AC | 40.21 | 1 | 40.21 | $5.48^{*}$ |
| BC | .33 | 2 | .17 | .02 |
| ABC | 20.33 | 2 | 10.17 | 1.39 |
| W. cell | 418.33 | 57 | 7.34 |  |

*significant at . 05 level

The training procedure was found to result in significantly higher scores. However, there was also a significant interaction of treatment and sex. To examine this relationship a test of simple main-effects was applied. The means of the levels of treatment and sex and the summary of the analysis of variance are given in the following tables.

Table 27
Means of the Levels of the Variables Treatment and Sex


Table 28
Analysis of Variance for Treatment and Sex

| Source | Sum of squares | Degrees of freedom | Mean square | $F$ |
| :---: | :---: | :---: | :---: | :---: |
| A (treatment) | 37.38 | I | 37.38 | $5.09 \%$ |
| A at $c_{1}(M)$ | . 03 | 1 | . 03 | . 00 |
| A at $c_{2}(F)$ | 77.56 | 1 | 77.56 | $10.57^{* *}$ |
| $C$ (sex) | 3.28 | 1 | 3.28 | . 45 |
| $C$ at $a_{1}$ (with) | 10.26 | 1 | 10.26 | 1.40 |
| $C$ at $a_{2}$ (w. O) | 33.23 | 1 | 33.23 | $4.53 *$ |
| AC | 40.21 | 1 | 40.21 | $5.48 \%$ |
| w. cell | 418.33 | 57 | 7.34 |  |
| ** significant at . 01 level * significant at . 05 level |  |  |  |  |
| Thus, the training procedure was found to be |  |  |  |  |
| significantly effective for the females. |  |  |  |  |

Part Two
The cell means and the summary of the analysis of variance for Part Two are given in the following tables.

Table 29
Cell Means of Part Two for Treatment, Ability, and Sex

| Sex | Ability level | Treatment |  |
| :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { With } \\ & \text { training } \end{aligned}$ | Without training |
| Male | High | 8.83 | 8.83 |
|  | Average | 8.50 | 8.33 |
|  | Low | 7.83 | 8.67 |
| Female | High | 9.00 | 9.33 |
|  | Average | 7.83 | 7.67 |
|  | Low | 8.17 | 7.17 |
| Total |  | 8.30 | 8.33 |

Table 30
Unweighted Means Analysis of Variance of Part Two for Treatment, Ability, and Sex

| Source | Sum of <br> squares | Degrees of <br> freedom | Mean <br> square | F |
| :--- | ---: | ---: | ---: | ---: |
| A (treatment) | .01 | 1 | .01 | .00 |
| B (ability) | 14.33 | 2 | 7.17 | 2.38 |
| C (sex) | 1.55 | 1 | 1.55 | .52 |
| AB | .33 | 2 | .17 | .06 |
| AC | 1.04 | 1 | 1.04 | .34 |
| BC | 3.41 | 2 | 1.70 | .57 |
| ABC | 3.77 | 2 | 1.88 | .62 |
| W. cell | 171.83 | 57 | 3.01 |  |

No significent variables were found.

Part Three
The cell means and the summary of the analysis of variance for Part Three are given in the following tables.

Table 31
Cell Means of Part Three for Treatment, Ability, and Sex

| Sex | Ability level | Treatment |  |
| :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { With } \\ & \text { training } \end{aligned}$ | Wi thout training |
| Male | High | 14.33 | 13.33 |
|  | Average | 11.67 | 11.83 |
|  | Low | 10.83 | 12.50 |
| Female | High | 13.67 | 13.83 |
|  | Average | 13.67 | 10.17 |
|  | Low | 13.33 | 12.83 |
| Total |  | 12.85 | 12.42 |

Table 32
Unweighted Means Analysis of Variance of Part Three for Treatment, Ability, and Sex.

| Source | Sum of <br> squares | Degrees of <br> freedom | Mean <br> square | $F$ |
| :--- | ---: | ---: | ---: | :--- |
| A (treatment) | 4.15 | 1 | 4.15 | 1.38 |
| B (ability) | 45.31 | 2 | 22.65 | $7.55^{* *}$ |
| C (sex) | 4.15 | 1 | 4.15 | 1.38 |
| AB | 14.08 | 2 | 7.04 | 2.35 |
| AC | 10.05 | 1 | 10.05 | 3.35 |
| BC | 7.15 | 2 | 3.58 | 1.19 |
| ABC | 16.95 | 2 | 8.47 | 2.82 |
| w. cell | 171.00 | 57 | 3.00 |  |
| ي. significant at .01 level |  |  |  |  |

The treatment was not found to be significant

Summary
For the eighth grade, the training procedure was found to be significantly effective only for the females on Part One. The females of each ability level and the high ability males with training scored much higher than the corresponding subjects without training on this part. There were also increases in the scores of the low ability females on Part Two and of the high ability males and the average ability females on Part Three. As a result of the training, the mean for the females with training was higher than that for the males on Parts One and Three. For all three parts of the test, the females with training scored higher or almost as high as the corresponding males at each ability level.

For Part One, the mean for the eighth graders with training was higher than that for the tenth graders, due to the low ability eighth grade females scoring much higher than the low ability tenth grade females.

## Correlation of Parts One and Two

The Pearson product-moment correlation coefficient Was calculated from the scores on the 2076 items on each of Parts One and Two, resulting in an $r$ of. 34 . This indicates that the two methods are not equivalent methods of determining the subject's ability to rotate and develop
surfaces of solids.
In examining the corresponding items it was found that of the 1147 items drawn correctly, the same item was missed on the multiple choice part 185 times or $16 \%$ of the time. For the 530 items in which some understanding was shown in the drawing, the correct drawing was chosen in part two 300 times or $57 \%$ of the time. There were 399 items in which little or no understanding was shown in the drawings, but the correct drawing in part two was selected in 170 or $43 \%$ of these.

## Percentages of Correct Responses

The scores were investigated to determine if the subjects had mastered the ability to develop and rotate surfaces of solids. The percentages of subjects who scored $70 \%$ or higher for each part of the test were calculated. The percentages for each grade and treatment are given in table 33.

Table 33
Percentages Scoring $70 \%$ or Higher

| Grade | Treatment | Part 1 | Part 2 | Part 3 |
| :---: | :---: | :---: | :---: | :---: |
| 6 | With training | 53 | 56 | 75 |
|  | Without training | 36 | 39 | 58 |
| 8 | With training | 82 | 45 | 88 |
|  | Without training | 53 | 56 | 86 |
| 10 | Without training | 69 | 69 | 91 |

Less than $40 \%$ of the sixth graders without training scored $70 \%$ or better on each of Parts One and Two. For Part Three more than $70 \%$ of each group except the sixth graders without training scored $70 \%$ or better. The only other group which scored that well was the eighth graders with training on Part One.

## Responses to Individual Items

The individual items were investigated to determine which ones had been mastered and which were the most difficult. The percentages for each item by groups is given in Appendix I. For esch part of the test, comparisons were first made among those without training. The effect of training was then analyzed.

Part One
Only five solids were drawn correctly by more than $70 \%$ of the subjects without training, with $90 \%$ of them correct on the square pyramid (\#4), $87 \%$ on the cube (\#1), $81 \%$ on the triangular solid (\#9), $79 \%$ on the triangular prism (\#2), and $74 \%$ on the tetrahedron (\#12). The cube and the square pyramid were successfully drawn (by more than $70 \%$ of the subjects) by all grades, ability levels, and both sexes, the triangular solid by all but the sixth graders, and both the triangular prism and the tetranedron by all except the sixth graders, the females, and the low ability subjects. The frustum of a pyramid (\#6) was
successfully drawn by the tenth graders, the males, and the high ability subjects, while the right triangular solid (\#7) was drawn only by the tenth graders and the high ability subjects.

The three most difficult were the cone (\#8) with 10\% correct, the frustum of a cone (\#10) with $10 \%$, and the solid shaped like a slice of cake (\#11) with 13\%. The cylinder (\#5) was also difficult for the sixth graders, the females, and the low ability subjects, with each of these groups drawing it correctily only $28 \%$ of the time. The trapezoidal solid (\#3) was difficult for the sixth graders and the low ability subjects and the right triangular solid for the low ability subjects.

The relationships found within the variables of grade, ability, and sex were generally found to follow the same patterns for the individual solids as they did on part one overall, with a higher percentage of eighth and tenth graders being correct on most solids than sixth graders, of high and average ability subjects than low ability subjects, and of males than females.

The sixth graders were successful on only the cube and the square pyramid. The eighth graders were also able to draw the triangular prism, the triangular solid, and the tetrahedron, while the tenth graders added the frustum of a pyramid and the right triangular solid. The sixth graders had difficulty (less than $30 \%$ correct) with the
cone, the cake, the frustum of a cone, the trapezoidal solid, and the cylinder, while only the cone, the frustum of a cone, and the cake were that difficult for the eighth and tenth graders.

The low ability subjects were only able to draw the cube, the square pyramid, and the triangular solid. The average ability subjects added the triangular prism and the tetrahedron and the high ability subjects added the frustum of a pyramid and the right triangular solid. Six solids, the trapezoidal solid, the cylinder, the right triangular solid, the cone, the frustum of a cone, and the cake were difficult for the low ability subjects, while only the latter three were difficult for the avergge and high ability subjects. The right triangular solid varied the most by ability with $75 \%$ of the high ability subjects drawing it correctly to only $28 \%$ of the low ability subjects.

Only three solids, the cube, the square pyramid, and the triangular solid were drawn correctly by the females, while the males also were able to draw the triangular prism, the frustum of a pyramid, and the tetrahedron. Both sexes had difficulty with the cone, the frustum of a cone, and the cake, with the females also having problems with the cylinder.

At the sixth grade level, the training procedure resulted in overall gains of $20 \%$ or more for the right triangular solid, the triangular solid, and the tetra-
hedron. For these solids, gains were also found at most levels of both ability and sex. The percentages of correct drawings also increased more than $20 \%$ for the triangular prism for the females and the high and low ability subjects.

In addition to the two solids successfully drawn by the sixth graders without training, the sixth graders receiving training were able to draw the triangular prism, the frustum of a pyramid, the triangular solid, and the tetrahedron. However, they had difficulty with the same solids as those without training.

For the high ability subjects, increases of at least $20 \%$ were made on six solids, the triangular prism, the trapezoidal solid, the cylinder, the right triangular solid, the triangular solid, and the cake. The average gbilitw subjects gained on the frustum of a pyramid and the tetrahedron and the low ability subjects on the triangular prism, the right triangular solid, the triangular solid, and the tetrahedron.

Both sexes improved on the right triangular solid and the tetrahedron. The males also improved on the frustum of a pyramid, the triangular solid, and the cake while the femsies also improved on the triangular prism.

Although the subjects with training usually were better then those without it, the females and the high ability subjects with training performed much poorer on
the cube, as did the low ability subjects with training on the cylinder.

At the eighth grade level, the only solid in which the training procedure resulted in general improvement was the cylinder. This was due to the increase for the females and for the average and low ability groups. There were also increases on the cake for these same groups.

The eighth graders with training were able to draw one other solid, the frustum of a pyramid, in addition to those drawn by those without training. However, as in the sixth grade, they had difficulty with the same solids as those without training.

Both the average and low ability eighth graders with training increased their abilities to draw the trapezoidal solid, the cylinder, and the cake, with the average ability subjects also improving on the right triangular solid. Improvement was also found for the high ability subjects on the cone and the tetrahedron.

Little improvement was found for the males, while the females improved on the cylinder, the right triangular solid, and the cake. However, the females also performed worse on the square pyramid than those without training.

Part Two
Eight solids, the cube, the triangular prism, the trapezoidal solid, the square pyramid, the cylinder, the frustum of a pyramid, the right triangular solid, and the
tetrahedron, were chosen correctly by the subjects without training. For three of these, the cube, the frustum of a pyramid, and the right triangular solid, all grades, all ability levels, and both sexes were successful. The square pyramid and the cylinder were chosen correctly by all but the sixth graders, the triangular prism by all but the low ability subjects, and the trapezoidal solid by all but the sixth graders and the low ability subjects. The tetrahedron was recognized by the eighth and tenth graders, the males, and the high ability subjects. In addition, the triangular solid was successfully chosen by the tenth graders, the females, and the high ability subjects. The frustum of a cone was difficult for all groups.

As in Part One, increases were found with increases in grade and ability levels. The sixth graders were successful in choosing only the cube, the triangular prism, the frustum of a pyramid, and the right triangular solid. The eighth graders added the trapezoidal solid, the square pyramid, the cylinder, and the tetrahedron, while the tenth graders also added the triangular solid. All grades had difficulty with the frustum of a cone.

The low ability subjects recognized five solids, the cube, the square pyramid, the cylinder, the frustum of a pyramid, and the right triangular solid. The average ability subjects added the triangular prism and the trapezoidal solid, and the high ability subjects added the
triangular solid and the tetrahedron.
For the sixth graders, the training procedure did not result in large overall increases for any solid. However, there was improvement on the cube, the square pyramid, and the triangular solid for the females and the low ability subjects and on the frustum of a cone for the males and low ability subjects.

In addition to the four solids successfiully chosen by the sixth graders without training, those with training were able to recognize the correct figure for the square pyramid and the triangular solid. However, they still had difficulty with the frustum of a cone.

The main increases were found for the females and the low ability subjects. The females improved on the cube, the square pyramid, and the triangular solid. The low ability subjects improved on five solids, the cube, the square pyramid, the cone, the triangular solid, and the frustum of the cone. The males also improved on the frustum of a cone and the high ability subjects on the cake.

The eighth graders with training were successful on seven of the eight solids which were correctly chosen by those without training, but dropped below $70 \%$ on the tetrahedron. There was some improvement on the triangular prism, the triangular solid, and the cake, with the females and average ability subjects improving on the
triangular prism, the males and high ability subjects on the triangular solid, and the females, high ability, and low ability subjects on the cake.

Within the various groups, the results varied. The high ability subjects performed better on the triangular solid and the cake than the corresponding subjects without training but poorer on the trapezoidal solid, the average ability subjects better on the triangular prism and the cake but poorer on the square pyramid and the right triangular solic, and the low ability subjects better on the right triangular solid.

With training, the males improved on the triangular solid, while the females improved on the triangular prism and the cake but were worse on the square pyramid and the triangular solid. .

Part Three
Thirteen of the fifteen items on Part Three were answered correctly by the subjects without training. For the other two items, numbers 2 and 6 , the percentages were $55 \%$ and $51 \%$ respectively. Numbers 1, 3, 5, 7, 9, 10, and 15 were answered correctly by all grades, ability levels, and both sexes, numbers 4,13 , and 14 by all but the sixth graders, numbers 11 and 12 by all but the sixth graders and the low ability subjects, and number 7 by all but the sixth graders and the average ability subjects.

The largest difference was found between the sixth
grade and the other two grades. The sixth graders were successful on only seven of the items, numbers $1,3,5,7$, 9, 10 , and 15 , while both the eighth and tenth graders were successful on all thirteen items mentioned above. Both sexes and the high ability subjects were also successful on these same thirteen items. The average ability subjects did not master number 8 nor the low ability subjects numbers 11 and 12, but the percentages of correct answers for these were almost $70 \%$. No groups mastered numbers 2 and 6. However, the percentages of correct answers for these were over $50 \%$ for all groups except the sixth graders and the low ability subjects on both 2 and 6 and the average ability subjects on 6.

At the sixth grade level, the training procedure resulted in overall gains of $20 \%$ or more for four items, numbers $8,11,12$, and 13. For numbers 8 and 13 , increases also were found for the males and both high a: average ability subjects. Increases were found on number 11 for the males and both average and low ability subjects and on number 12 for both sexes and both average and low ability subjects.

In addition to the seven items answered correctly by the sixth graders without training, those with training were correct on five other items, numbers $4,8,11,12$, and 13.

The groups improving on the largest number of items
were the average ability subjects and the males. Increases of $20 \%$ or more were found for the high ability subjects on numbers $3,6,8,11,12$, and 13 and for the males on numbers $8,11,12$, and 13 . The high ability subjects improved on numbers 4, 8, and 13, the low ability subjects on numbers 11 and 12, and the females on numbers 6 and 12.

At the elghth grade level, there was no large overall improvement for those with training. They were successful on the same thirteen items as those without training. There was improvement for the females on number 10 , for the high ability subjects on number 2, for the average ability subjects on numbers 8 and 13, and for the low ability subjects on number 11. However, the low ability subjects also were much poorer on numbers 4 and 9.

## Common Errors

Part One
For those drawings in which some understanding was shown, the most common error made was that of omitting one surface of the solid, especially the top. Thus, the cube was of ten drawn $\sharp$, the frustum of a pyramid $\theta$, and the right triangular solid $\square$ A second type of error was that of making the bottom edges form a straight line, such as drawing the cone $\frac{0}{0}$ or the frustum of a pyramid $\square$. For those solids having circular surfaces, often only half of the circles were drawn, as in
drawing the cylinder The surfaces were also freequently found in the wrong order or wrong positions. For example, the right triangular solid was drawn
 the trapezoidal solid OD. These types of errors were found for ages and both treatments.

Of those incorrect on the cube, thirteen ( $57 \%$ of those incorrect) omitted one surface, drawing the figure
 - For the frustum of a pyramid, nine (18\%) omitted one surface, drawing it $\forall$ while another seven (ll\%) drew it $\xrightarrow{\square}$. Nine ( $12 \%$ ) drew the right triangular solid without one surface $\square$ and another nine ( $12 \%$ ) put the surfaces in the wrong positions, such as drawing For the triangular prism, ten (32\%) drew $\longrightarrow$. Seven ( $19 \%$ ) added an extra square to the tetrahedron, drawing it as a square pyramid (19\%) drew the triangular solid $\square$ The rapezoidal solid was drawn several ways. Twenty-five (23\%) omitted one surface, such as drawing or $\longrightarrow$, fourteen (13\%) had the wrong order, such as , and another seven ( $6 \%$ ) omitted one surface and had the other surfaces in the wrong order, such as The most common error made on the cylinder was drawing only half the circles. Thirty-one ( $62 \%$ ) drew it place 0 , while another seven ( $7 \%$ ) made both
mistakes, drawing $\longrightarrow$. For both the cone and the frustum of a cone, the edges were often dram as straight lines. Thirty-four (22\%) drew the cone $\frac{A}{o}$, while fifteen others ( $10 \%$ ) also had only a half circle 0 . The frustum of a cone was often drawn as a trapezoid or a rectangle with twenty-five ( $16 \%$ ) drawing it $\stackrel{0}{\circ}$, fourteen ( $9 \%$ ) $\xrightarrow[0]{0}$, ten ( $6 \%$ )
 Thirty-two (23\%) drew the cake while another four (3\%) indicated the curving by drawing
 Eighteen ( $13 \%$ ) made the edges of the sectors of the circles of diffferment lengths so that the ends of the arcs were along a straight line, drawing others ( $8 \%$ ) put the curving surface at the end of one of the sectors, often omitting other surfaces, such as drawing



Part Two
Many of the figures incorrectly chosen in Part Two reflect the same types of errors that were found on the drawings. For six of the solids, the figure most of ten chosen incorrectly was the same as the most common figure found in the incorrect drawings. The figure f as chosen for the cube by thirteen subjects ( $76 \%$ of those incorrect). Thirty-two ( $65 \%$ ) chose for the trapezoidal solid. Twelve others ( $24 \%$ ) thought that none of the figures were correct. Fifty-two ( $58 \%$ ) chose $A$ for the cone, while another twenty-six (29\%) chose 4 .

Twenty-nine ( $56 \%$ ) chose for the triangular solid and twelve (23\%) chose $\longrightarrow$. For the frustum of a cone ninety-seven (61\%) chose ${\underset{O}{0}}_{0}^{0}$ and thirty-four (22\%) chose $\xlongequal[0]{\circ}$. For the tetrahedron, thirty ( $54 \%$ ) chose $\otimes$ and twelve (21\%) chose $\theta$.

Only sixteen had drawn the square pyramid incorrectly. However, thirty-two missed it on part two, with seventeen of them ( $53 \%$ ) choosing $\mathbb{4}$ and seven others (22\%) choosing $\mathbb{\$}$. For the frustum of the pyramid and the cake, the most common incorrect drawing was one of the figures of ten chosen but not the most frequently. Fifteen ( $56 \%$ ) chose for the frustum of a pyramid and eight ( $30_{0}^{\circ}$ ) picked $母$, while thirty-two ( $46 \%$ ) chose $\$$ for the cake and twenty (29\%) picked $\quad$. Twenty-six (74\%) chose for the triangular prism. None of the given figures were judged to be correct by eighteen of the subjects ( $40 \%$ ) for the cylinder and eleven ( $55 \%$ ) for the right triangular solid. Nine others ( $2 \%$ ) chose $f$ for the cylinder and four (20\%) chose for the right triangular solid.

## Part Three

Items eleven through fifteen of Part Three were also analyzed to determine which squares were most frequently chosen incorrectly. For number eleven, eleven subjects ( $33 \%$ ) chose square one and another nine ( $27 \%$ ) chose square four. For number twelve, eleven ( $39 \%$ ) chose

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square one. Thirteen (41\%) chose square four on number thirteen, while squares two and five were each chosen by seven others (22\%). Squares four and three were chosen incorrectly on number fourteen by thirteen subjects (32\%) and twelve subjects ( $30 \%$ ) respectively, while square two was chosen by seven subjects ( $35 \%$ ) on number fifteen. These squares were mostly diagonal to the given square or one square further away from the diagonal square. However, for numbers eleven and fourteen, one of the squares frequently chosen incorrectly was the one adjacent to the given square.

## CHAPTER V

SU1MARY, FINDINGS, AND RECOMMENDA TIONS

Summary of the Study
This study was designed to investigate the capabilities of students to rotate and develop surfaces of solids and the effect of training on such abilities. It involved sixth, eighth, and tenth grade students of the Putnam City (Oklahoma) Independent School District and was conducted in the spring of 1976.

The students in each grade were blocked by ability and sex. Twelve subjects were randomly selected from each block in the sixth and eighth grades and six from each block in the tenth grade. Six of the subjects from each block in the sixth and eighth grades received both training and testing. The remaining subjects were tested only.

A criterion test was developed, consisting of three parts. The first two parts were identical, except for the method of responding. On Part One, the subjects were required to draw the result of rotating or developing the surfaces of twelve solids, while on Part Two, they had to select the correct figures. The twelve solids included the
four solids (the cone, the cylinder, the cube, and the tetrahedron) discussed by Piaget in The Child's Conception of Space. The third part consisted of fifteen figures, each of which consisted of six connected squares. For ten of the figures the subjects were to determine whether the figure could be folded to form a cube and for the other five they were to determine which face would be opposite a particular face when the figure was folded.

The training procedure was designed to provide the subjects with experience in folding and unfolding solids. Nine solids, different from those on the criterion test, were used. Models of each solid were given to the subjects to inspect and then to draw the result of developing or rotating the surfaces. Properties of the solids, such as the number and shape of the faces, were discussed. The models were then cut open and compared with the drawings. Incorrect drawings for each solid were shown and the reasons why they were incorrect were also discussed.

The criterion test was administered to all subjects. The scores of those subjects without training were compared to determine the abilities of students to rotate and develop surfaces, using age, sex, and ability as variables. Within each of the sixth and eighth grades, comparisons were made between those with training and those without training to determine the effect of the training procedure, with sex and ability also considered.

The responses on the drawing and the multiple choice parts of the criterion test were compared, percentages of correct responses were calculated, individual items were investigated to determine which had been mastered and which were the most difficult, and the types of errors frequently made in the drawings were determined.

## Findings and Conclusions

For those subjects who did not receive training, age was found to be a significant factor for all three parts of the test, with scores increasing as age increased. On Parts One and Three, both eighth and tenth graders scored significantly higher than sixth graders, while on Part Two, only the tenth graders scored significantly higher than the sixth graders. However, ability and sex were also found to be important factors. On Part One, both ability and sex were significant, with high and average ability subjects scoring significantly higher than low ability subjects and males scoring significantly higher than females. On Part Two, the high ability subjects scored significantly higher than the low ability subjects, with most of this difference resulting from differences Within the females. The low ability males also scored significantly higher than the low ability females on this part.

Increases in scores with increases in age were found for almost all combinations of ability and sex.

While these increases were of ten gradual, the average ability males had large increases both between the sixth and eighth grades and between the eighth and tenth grades. The scores of the high ability males and females and the 1ow ability females tended to increase more between the sixth and eighth grades and those of the average ability females more between the eighth and tenth grades. Increases in scores with increases in ability were found more often among the females. For sixth and tenth grade females, the high and average ability subjects tended to have higher scores than the corresponding low ability subjects, while for the eighth grade females, the high ability subjects tended to have higher scores than the average and low ability subjects. Males generally had higher scores than the corresponding females, especially for the average ability eighth graders and the low ability subjects of each grade. However, for the high ability subjects, the females tended to have higher scores.

Thus, this study tends to support Piaget's conclusion that age is a factor in the development of the ability to develop and rotate the surfaces of solids. However, ability and sex are also factors which affect this development.

Less than $40 \%$ of the sixth graders scored $70 \%$ or higher on Parts One and Two and only 5 $8 \%$ of them on Part Three. Although there was a significant increase between
the sixth and eighth grade means, only slightly more than half the eighth graders and $69 \%$ of the tenth graders scored this high on Parts One and Two. Therefore, these subjects are not considered to have mastered these tasks. However, on Part Three, $88 \%$ of the eighth and $91 \%$ of the tenth graders scored over $70 \%$. Thus, mastery of this part of the test does seem to be reached by the eighth grade (age fourteen). The only groups for which $70 \%$ or more of the group scored over $70 \%$ on Parts One or Two were the high ability eighth grade females and the high ability tenth grade males and females on both Parts One and Two, the average ability tenth grade males on Part One, and the high ability sixth grade females on Part Two.

For the sixth grade, the training procedure resulted in higher mean scores on all three parts of the test, with the increases being significant on Parts One and Three. Increases were found for almost all combinations of ability and sex, especially for the high ability males, the average ability females, and the low ability females. As a result of the training, the mean for the sixth graders with training was as high as that of the eighth graders without training on Part Two and almost as high on the other two parts. Slightly over half the sixth graders with training scored over $70 \%$ on Parts One and Two, While 75\% of them scored this high on Part Three. Therefore, the training procedure resulted in these sixth
graders mastering Part Three of the test. The high ability males and females also mastered Parts One and Two. For the eighth grade, the training procedure resulted in significantly higher mean scores on Part One. Over $80 \%$ of the eighth graders with training scored over 70\%. Thus, the training enabled them to master this part. As was found at the sixth grade level, the training was most effective for the high ability males, the average ability females, and the low ability females. The improvement in the scores of the females resulted in their means being higher than those of the males on Parts One and Three. For all ability levels, on each part of the test, the females with training scored almost as high or higher than the corresponding males.

The effectiveness of the training procedure supports Piaget's assertion that experience in folding and unfolding solids is necessary in the development of the ability to rotate and develop surfaces. That it was so effective for females, especially at the eighth grade Where it resulted in their scores being almost as high or higher than those of the corresponding males, seems to support the position that differences between the sexes are largely due to differences in experience.

The percentages of correct responses varied widely for the different solids. On Part One, only two solids (the cube and the square pyramid) were mastered by the
sixth graders without training. The eighth graders without training also mastered the triangular prism, the triangular solid, and the tetrahedron, and the tenth graders added the frustum of a pyramid and the right triangular solid. Less than $30 \%$ of the drawings of the cone, the cake, the frustum of a cone, the trapezoidal solid, and the cylinder were correct for these sixth graders, while only the cone, the frustum of a cone, and the cake were that difficult for the eighth and tenth graders. The same patterns with respect to age, ability, and sex were generally found for the individual solids as for Part One overall.

As a result of the training procedure, the sixth graders with training were able to master four solids (the triangular prism, the frustum of a pyramid, the triangular solid, and the tetrahedron) in addition to the two mastered by those without training. However, they also had difficulty with the same five solids as did those without training.

The eighth graders with training mastered one solid (the frustum of a pyramid) in addition to those mastered by the eighth graders without training. They also had difficulty with the same three solids as did those without training.

On Part Two, the sixth graders without training successfully selected four solids (the cube, the triangular
prism, the frustum of a pyramid, and the right triangular solid), the eighth graders added four others (the trapezoidal solid, the square pyramid, the cylinder, and the tetrahedron), and the tenth graders added one further solid (the triangular solid). All grades had difficulty with the frustum of a cone.

As a result of the training procedure, the sixth graders with training were able to successfully recognize two solids (the square pyramid and the triangular solid) in addition to the four successfully chosen by the sixth graders without training. However, they also had difficulty with the frustum of a cone.

While there was some improvement on a few of the solids for the eighth graders with training, they did not master any further solids and even dropped below $70 \%$ on the tetrahedron.

Of the four solids used by Piaget, the cube was the only one mastered by all three grades. The tetrahedron was mastered by the eighth and tenth graders and the cylinder was successfully chosen by the eighth and tenth graders on Part Two. This does not agree with the results reported by Piaget. His subjects were able to develop the cone and the cylinder more easily than to rotate the cube and the tetrahedron. Piaget attributed this to the fact that "the cylindrical and conic surfaces are not flat but curved, with the result that the curvature itself tends to suggest

## 84

the action of unrolling them. Apart from this, these solids have only two or three sides as compared with the four or six sides of the pyramid and cube." (Plaget and Inhelder, p. 286) While the present students did seem to be able to unroll the cone and cylinder, they had difficulty in preserving the equal height of the cone, of ten drawing the bottom edge as a straight line, and in preserving the entire circles, drawing only half a circle on the edge. Not only were these not mastered by age eleven, as Piaget seems to expect, but even at the tenth grade only $56 \%$ of the drawings of the cylinder and $16 \%$ of the cone were correct. Mastery of the cube was achieved by the sixth grade and of the tetrahedron by the eighth grade. The five most difficult solids for all groups were the four solids which had curved surfaces and the trapezoidal solid. Thus, it seems to be easier for students to rotate surfaces which are polygons.

On Part Three, the sixth graders without training mastered only seven of the fifteen items, while the eighth and tenth graders mastered all but two of them. The training procedure enabled the sixth graders with training to master twelve of the items. The two that were the most difficult were $\square \square$ and $母$, figures which can be folded into cubes, but which require some of the squares to be folded up and then around. The other item also not mastered by the sixth graders with training was 䀲,

Which uses the same figure as the latter one above.
For those drawings in which some understanding was shown, the most common error was that of omitting one of the surfaces, especially the top. Edges for the circular solids were also frequently drawn along a straight line, rather than being curved. For those solids having circular surfaces, often only half the circles were drawn. The surfaces were also frequently drawn in the wrong order or in the wrong positions. These types of errors were found for all ages and both treatments. For Part Two, many of the figures incorrectly chosen contained the same types of errors as were found in the drawings. For six of the twelve solids, the incorrect figure most of ten selected contained the same type of error as was most often made in the incorrect drawings of that solid. For the last five items of part three, the square most of ten incorrectly chosen was the one diagonal to the given square or one square further away from the diagonal square. On two of the figures, the square adjacent to the given square was also frequently chosen.

The calculation of the Pearson product-moment correlation coefficient between Parts One and Two resulted in an $r$ of .34 , indicating that the two methods are not equivalent methods of measuring the subjects' abilities to develop and rotate surfaces of solids.

It is possible that drawing ability could have
affected the results in Part one and that the training procedure merely improved the subjects' abilities to draw the mental images they had of the results. However, the significance of age on Part Two seems to indicate that it is more likely that the younger subjects have not developed the imagery necessary to perform the operation. Although the training procedure did not have a significant effect on the results on Part Two, there were Increases for some of the groups, so that the training could not have merely affected drawing ability. Also, the fact that the errors most frequently found in the drawings were of ten the same as those in the figures most frequently incorrectly chosen indicates that the subjects were drawing the images that they saw. In some cases it is possible that the subject knew what the result should be, but received a grade of one due to his inability to draw it. However, it was usually obvious when the subject was trying to draw the correct result and he was given full credit for his drawing.

The use of the multiple choice method of response is questionable due to its low reliability. It is also questionable due to the fact that for the 1147 items that were drawn correctly, the subjects missed the same item on the multiple choice part 185 times and for the 399 items in which the drawings showed no understanding of the result, the subjects were still able to choose the correct result 170 times. The latter seems to substantiate Piaget's
assertion that children can pick out the correct drawings on the basis of certain details. The existence of such details in more than one of the choices could also distract a subject who is capable of drawing the result himself. Thus, requiring the subjects to draw the results seems to be the more appropriate method of determining the subjects: abilities in developing and rotating surfaces of solids.

## Recommendations

Recommendations for the Classroom
Since experience in working with solids seems to be necessary for the development of the ability to develop and rotate surfaces of solids, opportunities for such experience should be provided in the classrooms. Models could be made available for the students to manipulate and projects could be devised in which the students would have to plan and build objects.

Females, in particular, should be encouraged to participate in activities that will give them experience In working with objects. Since, with training, they can further develop their abilities, they should be encouraged to take courses that will develop these abilities and to consider careers in areas that involve visual perception. The types of errors commonly made by the students should be considered in planning lessons on topics which involve solids so that difficulties due to such errors
might be forestalled.

Recommendations for Further Research
Since sixth graders were successful with the cube and the square pyramid, a basic understanding of this operation must start to develop at an earlier age. Plaget places the completion of the framework for such operations at about the age of nine. Thus, further research is needed to determine the abilities of jounger subjects and the effects of experience for them. The effects of such training at earlier ages upon the age at which mastery is later achieved should also be studied.

Older subjects should also be studied to determine When the more difficult solids are mastered and what effect training has at later ages.

The effects of training designed to develop other spatial abilities, especially those of females, could be investigated.

Longitudinal studies are also needed to determine if those with training maintain their superiority.

The training procedure also needs to be tested in the classroom. Since providing each student with a model might be impractical, small groups of students might work together on each model.

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## APPENDIX A

## SOLIDS USED IN TESTING

## Examples



Test Solids


Cube

2


Triangular prism

## 94




Cake

10


Frustum of a cone


Tetrahedron

## APPENDIX B

## PART TWO OF THE TEST

```
For each solid shown, check the figure
that will result when the solid is
unfolded. If none of the figures is
correct, check "none of the above."
```

Name

Example


None of the above


None of the above
$99$



None of the above

101


None of the above


None of the above $\qquad$


None of the above


None of the above


None of the above


None of the above


None of the above


None of the above


None of the above

## APPENDIX C

## PART THREE OF THE TEST

Determine whether each of the following figures can be folded to form a cube.

$\qquad$

2

$112$



Do not go on to the next page until asked to do so.

Each of the following figures 11 can be folded to form a cube. When it is folded, what side will be opposite the side marked with an $X$ ?



## APPENDIX D

## SOLIDS USED IN TRAINING

Heavy lines indicate where the solid was to be cut




## APPENDIX E

INCORRECT DRAWINGS USED IN TRAINING

$120$


## APPENDIX $F$

PLANE FIGURES USED IN TRAINING



## APPENDIX G

RAW SCORES

124

Part One

Sixth Graders with Training

| Sex | Ability level | Subj. no. |  | 2 | 3 |  | Sol |  | $\begin{gathered} \text { num } \\ 7 \end{gathered}$ | $\begin{gathered} \text { mber } \\ 8 \end{gathered}$ | 9 |  |  |  | Score |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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|  | Average | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \end{aligned}$ | 1 2 2 2 2 2 | $\begin{array}{ll}1 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2\end{array}$ | 1 1 1 2 0 1 | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 0 \\ & 2 \\ & 2 \\ & 1 \\ & 1 \\ & 2 \\ & 1 \end{aligned}$ | 2 2 2 2 2 2 | $\begin{aligned} & 0 \\ & 1 \\ & 0 \\ & 2 \\ & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \end{aligned}$ | 2 2 2 2 0 0 | $\begin{aligned} & 0 \\ & 1 \\ & 1 \\ & 0 \\ & 0 \\ & 1 \end{aligned}$ | 0 0 0 2 1 1 2 | 2 2 2 2 2 2 2 | 13 18 16 20 15 19 |
|  | Low | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \end{aligned}$ | 2 2 2 2 2 | $\begin{array}{ll}2 & 1 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2\end{array}$ | 1 1 1 0 1 1 | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 0 \\ & 0 \\ & 2 \end{aligned}$ | $\begin{aligned} & 1 \\ & 1 \\ & 1 \\ & 0 \\ & 1 \\ & 0 \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & 1 \\ & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & 1 \\ & 2 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 1 \\ & 0 \\ & 0 \\ & 1 \\ & 0 \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & \frac{1}{2} \\ & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 1 \\ & 1 \\ & 0 \\ & 0 \\ & 1 \\ & 0 \end{aligned}$ | 0 1 2 0 1 0 | 2 2 2 2 2 2 | $\begin{aligned} & 16 \\ & 19 \\ & 17 \\ & 12 \\ & 17 \\ & 13 \end{aligned}$ |
|  | High | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \end{aligned}$ |  | $\begin{array}{ll}2 & 2 \\ 1 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 1 & 2\end{array}$ | 1 1 2 1 1 1 | 2 2 2 2 2 2 | 2 2 1 2 1 1 0 | 1 2 2 2 2 2 | $\begin{aligned} & 2 \\ & 2 \\ & 1 \\ & 1 \\ & 2 \\ & 2 \\ & 1 \end{aligned}$ | $\begin{aligned} & 1 \\ & 0 \\ & 0 \\ & 1 \\ & 2 \\ & 0 \end{aligned}$ | 2 2 2 2 2 2 2 | 2 1 1 1 0 0 | 0 1 1 2 1 0 | 0 2 2 2 2 2 2 | 17 18 18 21 19 13 |
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Sixth Graders without Training


Eighth Graders with Training


Eighth Graders without Training


Tenth Graders

| Sex | \|Ability| <br> level | Subj. no. | 1 | 2 | 3 |  |  |  |  |  | 91 | 101 | 11 | 12 | Score |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| M | High | $\begin{aligned} & \frac{1}{2} \\ & 3 \\ & 4 \\ & 4 \\ & 5 \end{aligned}$ | $\begin{array}{ll} 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{array}$ |  | 2 1 <br> 2 1 <br> 2 2 <br> 2 2 <br> 2 1 <br> 2 1 | $\begin{array}{ll} 1 & 2 \\ 1 & 2 \\ 2 & 2 \\ 2 & 2 \\ 1 & 2 \\ 1 & 2 \end{array}$ | $\begin{aligned} & 1 \\ & \frac{1}{2} \\ & 2 \\ & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 1 \\ & 0 \\ & 0 \\ & 1 \\ & 1 \\ & 1 \end{aligned}$ | 2 2 2 2 2 2 | $\begin{aligned} & 1 \\ & 0 \\ & 1 \\ & 1 \\ & 2 \\ & 1 \end{aligned}$ | 1 2 2 2 1 1 1 |  | 19 18 21 22 21 19 |
|  | Average | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 5 \end{aligned}$ | 2 2 2 | 2 2 2 2 2 2 2 | 1 1 2 2 2 1 | 2 2 2 2 2 2 2 | $\begin{aligned} & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 0 \end{aligned}$ | 2 2 2 2 2 2 1 | 2 2 2 2 2 2 1 | 2 2 1 2 1 0 | 2 2 2 2 2 2 | 1 2 2 1 0 0 | 1 2 2 1 1 1 0 |  | 21 23 23 22 19 10 |
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|  | Average | $\begin{aligned} & 1 \\ & 2 \\ & 3 \\ & 4 \\ & 5 \\ & 6 \end{aligned}$ | $\begin{array}{ll} 0 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 1 \\ 2 & 2 \end{array}$ |  | $\begin{aligned} & 1 \\ & 2 \\ & 1 \\ & 2 \\ & 2 \\ & 2 \end{aligned}$ | 2222220 | $\begin{array}{ll} 0 & 0 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 1 & 2 \\ 0 & 0 \end{array}$ |  | $\begin{aligned} & 1 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \end{aligned}$ | $\begin{array}{ll} 1 & 2 \\ 0 & 2 \\ 1 & 2 \\ 2 & 2 \\ 1 & 1 \\ 1 & 2 \end{array}$ |  |  | $\begin{array}{ll} 0 & \\ 0 & 2 \\ 1 & 2 \\ 2 & 2 \\ 0 & \vdots \\ 0 & \end{array}$ |  | 91920241613 |
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|  | Low | 12345 |  |  |  | 212001 | $\begin{array}{ll} 2 & 2 \\ 2 & 1 \\ 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 2 & 1 \end{array}$ |  | $\begin{array}{ll} 2 & 2 \\ 1 & 2 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{array}$ | 2110110 | 100001 | 222002 | $\begin{array}{ll}0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0\end{array}$ |  |  | 1815130414 |
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Part Two

Sixth Graders with Training


Sixth Graders without Training


Eighth Graders with Training


Eighth Graders without Training


Tenth Graders


Part Three

Sixth Graders with Training



Sixth Graders without Training



Eighth Graders with Training



Eighth Graders without Training



Tenth Graders


$145$


## 246

| Grade | Sex | Abil. level | Treatment |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Without training |  |  |
|  |  |  | Mean | SD | No. of subj。 |
| All | M | H A L | 17.89 17.61 15.07 | 3.30 5.15 4.85 | 18 18 14 |
|  | F | $\begin{aligned} & \text { H } \\ & \text { A } \\ & \text { L } \end{aligned}$ | 17.50 15.33 11.44 | 3.25 3.76 5.56 | 18 18 18 |
|  | $\begin{aligned} & \mathrm{M} \\ & \mathrm{~F} \end{aligned}$ |  | 17.00 14.76 | 4.64 4.98 | 50 54 |
|  |  | H A L | 17.69 13.47 13.03 | 3.28 4.65 5.56 | 36 36 32 |
|  | A11 |  | 15.84 | 4.95 | 104 |

## 147

Part Two

$148$


## 149

Part Three

| Grade | Sex | Abil. <br> level | Treatment |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | SD | ining <br> No. of subj. |  | train SD | ning <br> No. of subj. |
| 6 | M F | $\begin{aligned} & \text { H } \\ & \text { A } \\ & \text { L } \\ & \text { H } \\ & \text { A } \\ & \text { L } \end{aligned}$ | 11.00 8.83 11.33 11.50 11.50 7.83 | $\begin{aligned} & 2.38 \\ & 3.80 \\ & 1.25 \\ & 1.12 \\ & 1.98 \\ & 3.93 \end{aligned}$ |  | $\begin{aligned} & 13.00 \\ & 12.83 \\ & 10.67 \\ & 11.83 \\ & 13.17 \\ & 10.33 \end{aligned}$ | $\begin{aligned} & 2.08 \\ & 1.07 \\ & 2.56 \\ & 1.34 \\ & 1.07 \\ & 1.89 \end{aligned}$ | 6 6 6 6 6 6 |
|  | M |  | 10.39 10.28 | 2.91 3.14 | $\begin{aligned} & 18 \\ & 18 \end{aligned}$ | 12.17 11.78 | $\begin{aligned} & 2.27 \\ & 1.87 \end{aligned}$ | $\begin{aligned} & 18 \\ & 18 \end{aligned}$ |
|  |  | H A L | 11.25 10.17 9.58 | 1.88 3.31 3.40 | 12 12 12 | $\begin{aligned} & 12.42 \\ & 13.00 \\ & 10.50 \end{aligned}$ | $\begin{aligned} & 1.85 \\ & 1.08 \\ & 2.25 \end{aligned}$ | 12 12 12 |
|  | All |  | 10.33 | 3.03 | 36 | 11.97 | 2.09 | 36 |
| 8 | M | $\begin{aligned} & \mathrm{H} \\ & \mathrm{~A} \\ & \mathrm{~L} \\ & \mathrm{H} \\ & \mathrm{~A} \\ & \mathrm{~L} \end{aligned}$ | $\begin{aligned} & 13.33 \\ & 11.83 \\ & 12.50 \\ & 13.83 \\ & 10.17 \\ & 12.83 \end{aligned}$ | $\begin{array}{r} 1.25 \\ 1.77 \\ 1.26 \\ .69 \\ 1.95 \\ 1.46 \end{array}$ | $\begin{aligned} & 6 \\ & 6 \\ & 6 \\ & 6 \\ & 6 \\ & 6 \end{aligned}$ | $\begin{aligned} & 14.33 \\ & 11.67 \\ & 10.83 \\ & 13.67 \\ & 13.67 \\ & 13.33 \end{aligned}$ | $\begin{aligned} & 1.11 \\ & 2.29 \\ & 1.95 \\ & 1.89 \\ & 1.60 \\ & 1.11 \end{aligned}$ | 6 6 6 3 6 6 |
|  | M |  | 12.56 12.28 | 1.57 2.13 | $\begin{aligned} & 18 \\ & 18 \end{aligned}$ | $\begin{aligned} & 12.28 \\ & 13.53 \end{aligned}$ | $\begin{aligned} & 2.38 \\ & 1.50 \end{aligned}$ | $\begin{aligned} & 18 \\ & 15 \end{aligned}$ |
|  |  | H A L | 13.58 11.00 12.67 | 1.04 2.04 1.37 | $\begin{aligned} & 12 \\ & 12 \\ & 12 \end{aligned}$ | $\begin{aligned} & 14.11 \\ & 12.67 \\ & 12.08 \end{aligned}$ | $\begin{aligned} & 1.45 \\ & 2.21 \\ & 2.02 \end{aligned}$ | $\begin{array}{r} 9 \\ 12 \\ 12 \end{array}$ |
|  | All |  | 12.42 | 1.88 | 36 | 12.85 | 2.12 | 33 |

150

| Grade | Sex | Abil. <br> level | Treatment |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Without training <br> Mean SD No. of subj. |  |  |
| 10 | M | $\begin{aligned} & \text { H } \\ & \text { A } \\ & \text { L } \end{aligned}$ | $\begin{aligned} & 13.33 \\ & 13.50 \\ & 13.00 \end{aligned}$ | $\begin{array}{r} 2.43 \\ 1.38 \\ .00 \end{array}$ | $\begin{aligned} & 6 \\ & 6 \\ & 2 \end{aligned}$ |
|  | F | $\begin{gathered} \text { H } \\ \text { A } \\ \text { L } \end{gathered}$ | 13.50 13.67 11.67 | 1.38 1.37 2.21 | 6 6 6 |
|  | $\underset{F}{M}$ |  | $\begin{aligned} & 13.36 \\ & 12.94 \end{aligned}$ | $\begin{aligned} & 1.84 \\ & 1.93 \end{aligned}$ | 14 18 |
|  |  | H | 13.42 13.58 12.00 | 1.98 1.38 2.00 | 12 12 8 |
|  | All |  | 13.13 | 1.90 | 32 |
| All | M | $\begin{aligned} & \text { H } \\ & \text { A } \\ & \text { L } \end{aligned}$ | $\begin{aligned} & 12.56 \\ & 11.39 \\ & 12.07 \end{aligned}$ | 2.36 3.20 1.33 | 18 18 14 |
|  | $F$ | $\begin{aligned} & \text { H } \\ & \text { A } \\ & \text { L } \end{aligned}$ | $\begin{aligned} & 12.94 \\ & 11.78 \\ & 10.78 \end{aligned}$ | $\begin{aligned} & 1.51 \\ & 2.30 \\ & 3.47 \end{aligned}$ | 18 18 18 |
|  | $\stackrel{M}{F}$ |  | $\begin{aligned} & 12.00 \\ & 11.83 \end{aligned}$ | $\begin{aligned} & 2.54 \\ & 2.71 \end{aligned}$ | $\begin{aligned} & 50 \\ & 54 \end{aligned}$ |
|  |  | H A L | $\begin{aligned} & 12.75 \\ & 11.58 \\ & 11.34 \end{aligned}$ | $\begin{aligned} & 1.99 \\ & 2.79 \\ & 2.82 \end{aligned}$ | 36 36 32 |
|  | All |  | 11.91 | 2.63 | 104 |

## APPENDIX I

## PERCENTAGES OF CORRECT RESPONSES

Part One




Part Two


| Grade | Treatment | Group | Number of subjects | 1 | 2 | 3 | 4 | $5^{\text {I }}$ | Item | $\begin{gathered} \text { numb } \\ 7 \end{gathered}$ | ${ }_{8} 8$ | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | W1th training | $\begin{gathered} \text { Male } \\ \text { Female } \end{gathered}$ | $\begin{aligned} & 18 \\ & 15 \end{aligned}$ | $\begin{array}{r} 89 \\ 100 \end{array}$ | $\begin{aligned} & 89 \\ & 93 \end{aligned}$ | $\begin{aligned} & 72 \\ & 73 \end{aligned}$ | $\begin{aligned} & 89 \\ & 60 \end{aligned}$ | $\begin{aligned} & 78 \\ & 87 \end{aligned}$ | $\begin{aligned} & 94 \\ & 93 \end{aligned}$ | $\begin{aligned} & 78 \\ & 93 \end{aligned}$ | $\begin{aligned} & 50 \\ & 27 \end{aligned}$ | $\begin{aligned} & 78 \\ & 53 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 61 \\ & 67 \end{aligned}$ | $\begin{aligned} & 61 \\ & 73 \end{aligned}$ |
|  |  | $\begin{gathered} \text { High } \\ \text { Average } \\ \text { Low } \end{gathered}$ | $\begin{array}{r} 9 \\ 12 \\ 12 \end{array}$ | $\begin{array}{r} 89 \\ 92 \\ 100 \end{array}$ | $\begin{array}{r} 100 \\ 92 \\ 83 \end{array}$ | $\begin{aligned} & 67 \\ & 83 \\ & 67 \end{aligned}$ | $\begin{aligned} & 89 \\ & 67 \\ & 75 \end{aligned}$ | $\begin{aligned} & 89 \\ & 83 \\ & 75 \end{aligned}$ | $\begin{array}{r} 89 \\ 100 \\ 92 \end{array}$ | $\begin{array}{r} 78 \\ 100 \\ 75 \end{array}$ | $\begin{aligned} & 44 \\ & 17 \\ & 58 \end{aligned}$ | $\begin{aligned} & 89 \\ & 42 \\ & 75 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 78 \\ & 58 \\ & 58 \end{aligned}$ | 78 83 42 |
|  |  | Total | 33 | 94 | 91 | 73 | 76 | 82 | 94 | 85 | 39 | 67 | 0 | 64 | 67 |
|  | Without training |  | $\begin{aligned} & 18 \\ & 18 \end{aligned}$ |  | $\begin{aligned} & 83 \\ & 61 \end{aligned}$ | $\begin{aligned} & 78 \\ & 78 \end{aligned}$ | $\begin{aligned} & 89 \\ & 89 \end{aligned}$ | $\begin{aligned} & 83 \\ & 89 \end{aligned}$ | $\begin{aligned} & 78 \\ & 94 \end{aligned}$ | $\begin{aligned} & 89 \\ & 89 \end{aligned}$ | $\begin{aligned} & 67 \\ & 33 \end{aligned}$ | $\begin{aligned} & 50 \\ & 78 \end{aligned}$ | $\begin{array}{r} 11 \\ 0 \end{array}$ | $\begin{aligned} & 67 \\ & 33 \end{aligned}$ | $\begin{aligned} & 78 \\ & 67 \end{aligned}$ |
|  |  | High <br> Average <br> Low | $\begin{aligned} & 12 \\ & 12 \\ & 12 \end{aligned}$ | 92 92 92 | $\begin{aligned} & 92 \\ & 58 \\ & 67 \end{aligned}$ | $\begin{aligned} & 92 \\ & 67 \\ & 75 \end{aligned}$ | $\begin{array}{r} 100 \\ 92 \\ 75 \end{array}$ | $\begin{aligned} & 83 \\ & 92 \\ & 83 \end{aligned}$ | $\begin{aligned} & 92 \\ & 83 \\ & 83 \end{aligned}$ | $\begin{aligned} & 92 \\ & 92 \\ & 83 \end{aligned}$ | $\begin{aligned} & 42 \\ & 75 \\ & 33 \end{aligned}$ | $\begin{aligned} & 67 \\ & 50 \\ & 75 \end{aligned}$ | $\begin{array}{r} 8 \\ 0 \\ 18 \end{array}$ | $\begin{aligned} & 58 \\ & 33 \\ & 58 \end{aligned}$ | 92 67 58 |
|  |  | Total | 36 | 92 | 72 | 78 | 89 | 86 | 86 | 89 | 50 | 64 | 6 | 50 | 72 |



Part Three

| Grade | Treatment | Group | Number subj. |  |
| :---: | :---: | :---: | :---: | :---: |
| 6 | With training | $\begin{aligned} & \mathrm{M} \\ & \mathrm{~F} \end{aligned}$ | $\begin{aligned} & 18 \\ & 18 \end{aligned}$ | 89 28 94 83 78 61 94 89 78 89 89 94 89 67 94 <br> 100 44 100 78 89 50 89 89 78 78 72 83 83 67 78 |
|  |  | $\begin{gathered} \mathrm{H} \\ \mathrm{~A} \\ \mathrm{~L} \end{gathered}$ | $\begin{aligned} & 12 \\ & 12 \\ & 12 \end{aligned}$ | 92 58 100 92 92 75 83 92 75 83 75 92 92 58 83 <br> 100 33 100 92 75 58 92 100 92 92 100 92 92 83 100 <br> 92 17 92 58 83 33 100 75 67 75 67 83 75 58 75 |
|  |  | Total | 36 | $\begin{array}{llllllllllllllllll}94 & 36 & 97 & 81 & 83 & 56 & 92 & 89 & 78 & 83 & 81 & 89 & 86 & 67 & 86\end{array}$ |
|  | Without training | $\begin{aligned} & \mathrm{M} \\ & \mathrm{~F} \end{aligned}$ | $\begin{aligned} & 18 \\ & 18 \end{aligned}$ | $\begin{array}{rlllllllllllll}100 & 39 & 83 & 72 & 72 & 50 & 89 & 56 & 78 & 78 & 61 & 67 & 50 & 67 \\ 89 & 56 & 89 & 67 & 72 & 28 & 94 & 83 & 78 & 67 & 56 & 56 & 72 & 56 \\ 67\end{array}$ |
|  |  | $\begin{aligned} & \mathrm{H} \\ & \mathrm{~A} \\ & \mathrm{~L} \end{aligned}$ | $\begin{aligned} & 12 \\ & 12 \\ & 12 \end{aligned}$ | 100 58 92 58 92 58 100 67 83 67 75 83 50 67 <br> 100 50 75 75 58 17 83 67 75 83 58 58 67 67 <br> 83              <br> 83 33 92 75 67 42 92 75 75 67 42 42 67 50 <br> 58              |
|  |  | Total | 36 | $\begin{array}{llllllllllllllllllllll}94 & 47 & 86 & 69 & 72 & 39 & 92 & 69 & 78 & 72 & 58 & 61 & 61 & 61 & 72\end{array}$ |




